SELECTION OF COMPATIBLE SPECIFICATIONS

FOR SYSTEM DESIGN

By

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PREFACE

This dissertation is the result of the author's interest in the development of a logical method of engineering design. Design engineers who do not have a comfortable background of experience to provide intuitive guidance are sometimes apt to approach a design problem rather haphazardly, and consequently there is need of a more basic understanding of the design process.

The author became interested in this area while working with Professors Charles F. Cameron and Daniel D. Lingelbach in the area of relay design. The requirements of the problem indicated a need for a logical, systematic method of determining exactly what parameters of a relay could be arbitrarily specified with assurance that the relay would be realizable. A "design map" developed by C. C. Freeny, who was also concerned with the project, has been used very successfully by the previously mentioned group, and several papers have been written on this subject.

The fundamental viewpoint taken in the thesis is that the set inclusion properties of a system of relations provide sufficient information to justify their use as the basis for a method of specification selection. In special cases where set inclusion alone is not sufficient, a very simple approach is provided to augment this information.

The author wishes to express his gratitude to Professors William A. Blackwell and Charles F. Cameron for their counsel throughout his graduate work.

The encouragement and guidance of Professor D. D. Lingelbach and the many hours of discussion with C. C. Freeny also are greatly appreciated. These men have the author's deep gratitude for helping to make this thesis a reality.

Finally, the author wishes to express his appreciation to his wife, Mary, for her many years of encouragement.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Once a design process has been well defined and stated in a logical theory, it is usually possible to train people or machines to perform this process, therefore allowing the more creative person to proceed to less clearly defined areas. It is the intent of the author to select a particular step of the design process and to attempt to develop a structure such that the process is reduced to a logical step-by-step procedure ultimately to be performed by a computer.

The development of a structure and of a theory of the design process has received attention in several different areas. The area of "systems" at present is so encompassing that at least one book has been written in an attempt to establish a definitive theory of systems. The book Systems Philosophy, which was published in 1962, used the terminology of modern mathematics to define a system. (1).

In the area of linear graph theory, work has been done on parameter selection in an electrical network, and a paper on the subject was presented at the Sixth Midwest Symposium on Circuit Theory in 1963.

In addition, this subject has been a topic of interest to the relay design group at Oklahoma State University for several years, and many papers and reports have been written on this subject.

As a direct result of this work, C. C. Freeny (2) defined a system symbolically in order to provide a foundation upon which to build a design theory.

Studies of a very similar nature (concerned with computer usage) are presently being directed by John C. Paul of North American Aviation, Inc.

As an indication that the problem of logical design is a widespread one, the following is quoted from a book on guided missile design:

> "The radar systems engineer is often asked to solve the following problem: 'Given a set of performance specifications based on the tactical problem requirements, derive a radar system that will meet the specifications.'"
>
> "For a variety of reasons, it is seldom possible to solve this problem in a straightforward fashion. Probably the most important reason is this: The performance specification - if properly derived - will seldom specify a task which simply cannot be performed by radar techniques; however, the performance specification will usually require the radar to perform a group of tasks which are not logically consistent with any one radar system mechanization."
>
> "The usual approach is to assume a generic type of radar system which experience and judgment deem reasonable. The assumed system

then is measured analytically against the over-all system requirements to determine whether it has the inherent potential for providing an acceptable problem solution. This process is repeated until the best match is found between the performance specification and the basic laws of nature governing what can be done by a given radar system." (3).

The last sentence of the above quotation tells the story; the process is trial and error, which places a premium on past experience rather than on formal knowledge of the subject. The authors of the book from which this passage was taken have recognized the problem but have attempted to solve it by simply sharing their experience with the reader.

The particular process to be discussed in this thesis is parameter selection. This is a very basic step of design, and a lack of knowledge of the structure involved has resulted in many designs which are products of art rather than science. A knowledge of the general system of relations and parameters, such as is presented in this thesis, removes the algebraic details, allowing the basic set structure to be considered. In many cases, this simplification allows the designer to view the entire system as one unit rather than as many subsystems which are difficult to fit together in one's mind.

Set theory is used extensively throughout the thesis because consideration of the set inclusion properties allows a great deal of simplification. It also removes information which is often necessary, but the amount of information which can be extracted from the set inclusion properties justifies its use.

The system, as defined in this thesis, concerns two basic concepts - the parameter and the relation. The parameter is one of a set of "things" for which there has been some accepted standard of measure. Length, weight, voltage, resistance, acceleration, and velocity are parameters for which exact measuring standards have been developed. Intelligence, kindness, love, and success are examples of parameters for which the measurement techniques have not, as yet, been agreed upon.

Associated with the measurement of the parameter is a set of "values" which may be expressed as real numbers, complex numbers, abstract symbols, etc. It is assumed that the parameter will be measured by selecting one of the members of the appropriate set and designating it as the measured "value" of the parameter. Therefore, it is clear that the parameter itself is a set of possible "values", and the definition given in the second chapter allows the consideration of the parameter as a set of "values" without regard to the exact nature or name of the value.

When more than one parameter is to be considered, the effect of the value of one parameter on the value of another is of considerable importance. If there is an effect, the parameters are said to be related.

To facilitate the study of sets of parameters, the cross-product of the sets is formed. This results in a set of n-tuples, which is all possible combinations of the values of the parameters involved and which is referred to as the scalar product set. The effect of each parameter on the others determines which of these many possible combinations will actually occur in such a way that they can be verified by measurement. The rule used to determine which combinations occur is called a relation.

In the previous discussion, only measurement of parameter values has been considered. To accomplish design, it is necessary to be able to obtain a desired value of a particular parameter or parameters. This means that the designer must know which values of the unspecified parameters will result in the desired values of the specified parameter. The deductive relation satisfies these requirements since values of exactly $n - 1$ of the parameters allow determination of the n^{th} parameter.

Just as parameters have values dependent on values of other parameters within a relation, there may exist other relations involving different parameters which also have an effect. The final result is a set of relations (ϕ_i) involving a set of parameters (P_n). To obtain a particular value of a parameter P_k under these conditions, the effect of each parameter of each relation which contains P_k must be taken into account. A convenient way of doing this is to form a grouping of elements with one position for each parameter involved. Then the set of allowed combinations of parameter values is determined by the requirement that every subcombination of values which appears in the grouping also appears in the set of allowed values of each relation having the corresponding set of parameters. The sets of elements obtained in this manner are called natural points, which, in common usage, are the solutions to sets of simultaneous equations.

This structure of the parameter and the relation is necessary to allow a more rigorous justification of the parameter selection process which is intended to provide a logical step-by-step procedure for one phase of design.

Whenever sets of relations are studied, the question of inconsistencies and redundancies arises. In the general sense, a knowledge of the functional form of the relations must be known in order to determine independence or dependence of the set. However, in all but a few special cases there exists a set structure which allows inconsistency and redundancy checks using only the set inclusion properties. The process of decomposability, defined in the second chapter, provides a method of determining independence if the exceptions suggested above are ruled out. The ability to determine decomposability of a set of relations is very important since it is this property upon which the remainder of the thesis is based.

A system is defined as a set of relations and corresponding parameters. An allowable specification set of a system is a set of parameters for which arbitrary values can be selected with assurance that values of the unselected parameters exist which will result in the desired system. It would be very desirable to have an easily applied, necessary and sufficient condition for the allowability of a set of parameters as specifications. However, if the information used to determine allowability is restricted to the set inclusion properties, only a sufficient condition can be obtained. A method is presented to be used for the determination of allowability since, if a certain condition is met, the set of parameters being tested is known to be an allowable set; and, even if this condition is not true, the process allows the designer to determine exactly which relations and parameters might be in conflict.

A system may be defined by a set of relations and a corresponding set of parameters. The word "defined" in this context means that the system in question can be distinguished from certain other systems. The number of relations and parameters necessary is a function of the degree of uniqueness desired. Obviously, as the number of restrictions (relations) is increased, the number of systems which will fit the requirements is decreased. Once a system has been selected and the parameters which must be considered are ascertained, there exists a unique maximal set of relations which is valid for the system under consideration. (A valid relation must have at least one solution.) Not all of these relations are needed to define the system, since a maximal independent set will uniquely specify the complete set of natural points, and any additional relations are simply combinations of the base set.

Selection of an allowed specification set is based on a single necessary and sufficient condition. This condition is that the set in question does not include a complete parameter set for any valid relation of the system. Any parameter set which satisfies this condition is an allowed specification set. The most obvious and straightforward method of parameter selection would therefore be to check the desired set to determine if a complete parameter set of any valid relation is restricted. This method is not practical since obtaining the complete set of valid relations would be a prohibitive task in all but the most trivial cases.

Several methods of checking the relations using only the information present in the defining relations are included in the chapter on applications.

The design map, developed by Freeny, has been used for several years with good results. The design map is, in essence, a matrix having columns corresponding to the relations of the system and rows corresponding to the parameters. If a set of parameters satisfies certain conditions on the design map, it is an allowable specification set. If a set does not, then it is necessary to determine allowability by an appeal to the functional form of the relations which show a possible conflict on the map.

In the search for a better method of parameter selection, use has been suggested more than once of a linear graph with edges representing parameters and circuits representing relations. The primary problem concerning this type of representation was this: When the linear graph of a system was drawn, there was not a one-to-one correspondence between the edges of the graph and the parameters of the system. This lack of a one-to-one correspondence is no longer a problem since in this thesis it is shown to be a function, not of the system itself, but of the particular set of relations used to define the system. By proper selection of the base set of relations, all systems can be represented by a linear graph.

The use of the linear graph as a design tool is limited since there is no exact correspondence between trees and allowable specification sets, nor between circuits and nonallowable sets. The graph does, however, allow the designer to identify by inspection those sets of parameters which will not satisfy the design map and, in the case of systems which satisfy certain restrictions, allows a complete listing of all allowable specification sets.

A third method of parameter selection which is suggested by the theorems in Chapter III which state that every decomposable system has at least one allowed specification set and that there is no set of parameters which will satisfy the sufficient condition for allowability for a nondecomposable system, is given in Chapter IV. The procedure is to select the specifications, in order of their importance, one at a time, checking each time to see that no conflict exists. This provides a step-by-step procedure for obtaining an allowable specification set, and each conflict of the selection is resolved in the order of the importance of the parameters. Use of this method requires that the designer be able to check decomposability quite rapidly, and an arrangement for doing this is given in an example.

The final chapter indicates some of the areas of application of the parameter selection process presented, and several possible areas of extension are noted.

CHAPTER II

PARAMETER RELATIONS

D2-1: A set $P = \{p_1, p_2 \ldots\}$ is a parameter if the following three conditions are satisfied.[1]

(1) There exists an ordered set A, such that $A \cap P = 0$.

(2) P can be placed in 1-1 correspondence with A.

(3) A contains at least two distinct elements.

Set A is called the indexing set and is necessary to provide an ordering of the elements of P. When dealing with abstract quantities such as voltage (E), current (I), and resistance (R), it is desired to establish an ordering which will allow these parameters to be related by conventional algebraic methods. Placing the elements of a parameter in a 1-1 correspondence with an indexing set (such as real numbers, complex numbers, etc.) allows the operation developed for the indexing set to be used on the parameters. This avoids any possible confusion of identity which might occur when each parameter is thought of as a "number".

[1]See Appendix A for a complete list of symbols.

Some confusion might arise concerning the equality of parameters. It is obvious that (E) and (I) are distinct parameters since they have a different name, but frequently several distinct parameters of the same type (name) are related (for example, when summing voltages around a loop in an electrical circuit). When this occurs, it is important to remember that each voltage is a distinct parameter regardless of the associated "name". Condition (3) of D2-1 requires at least two elements. This requirement limits the title "parameter" to those sets which allow a "choice"; i.e., this removes the "constants" in an equation from consideration. Parameters with different subscripts will be considered disjoint throughout this thesis. This does not imply that their indexing sets are disjoint. As will be shown later, the indexing sets of related parameters will usually be the same.

D2-2: Let $\{P_n\} = (P_1, P_2 \cdots\cdots P_n)$ be a set of disjoint parameters. The set $\mathcal{T}_n = \{(p_{1i}, p_{2i} \cdots\cdots p_{ni}): p_{1i} \in P_1, p_{2i} \in P_2 \cdots\cdots p_{ni} \in P_n\}$ is the product set of the parameters $\{P_n\}$.

D2-3: When the product set is written with elements of A replacing the corresponding elements of P, the resulting set Π_n of scalar n-tuples is called the scalar product set.

It should be noted that no ordering of parameters has been defined. However, the ordering used in the n-tuples will be retained as a convenient way to identify the individual parameters and their corresponding scalars.

D2-4: Two scalar n-tuples of the same product set are equal if the scalars of the respective parameters are equal regardless of order.

The scalar product set is the totality of distinct n-tuples that can be formed using the elements of the indexing sets. It is used to define a relation among parameters.

D2-5: If there exists a "rule" whereby the scalar product set can be divided into two nonempty equivalence classes, this "rule" is called a relation ϕ on the parameters involved in the product set.

In effect, a relation is the rule used to indicate the existence of a set. For example, let $\{X\}$ be a set for which a relation ϕ exists. Then for each $X \epsilon \{X\}$ either $\phi(X)$ is true or $\phi(X)$ is false. The subset of $\{X\}$ for which ϕ is true is the set "allowed" by the "rule". It is this set which will be called the graph of the relation.

D2-6: A set $G(P_n) = \{X: X \epsilon \Pi_n \text{ and } \phi(X) \text{ is true}\}$ is the graph of the relation ϕ on the set $\{P_n\}$.

D2-7: Two relations, ϕ_1 and ϕ_2 are equal if their graphs are equal.

The definition of equal relations allows the same relation to apply to more than one set of parameters.

Since the operations performed on the elements of the parameters are exactly those which can be performed on the indexing sets, it is required that the indexing sets for every parameter involved in a given relation should be equal. Therefore, the following definition is needed to limit the discussion to relations of this type.

D2-8: A relation ϕ on a set $\{P_n\}$ is said to be algebraic if each $P_i \in \{P_n\}$ is indexed by the same set.

All relations considered in the remainder of this thesis, unless otherwise noted, will be algebraic.

D2-8 is not as restrictive as it seems at first glance. In some relations one parameter will be allowed values only from a particular subset of the indexing set. This situation is taken into consideration by the relation itself. Only those n-tuples which contain the scalar from the proper subset of the indexing set are among those "allowed". Therefore, in general, the indexing sets of each parameter in the relation can be considered equal.

D2-9: The set of scalars which appears in the graph as an element of a parameter is called the range of the parameter.

The most useful property of a relation in the area of design is that of being deductive. When one knows the "value" of a parameter or a set of parameters, the relation is used to determine the corresponding "allowed value" of another parameter involved in the same relation. However, a set of parameters might contain an element X such that every combination of the scalars corresponding to the remaining parameters appears with every scalar of X. Then no proper subset of the parameters, not including X, could restrict the "allowed values" of X to any proper subset of the indexing set. For example, consider the relation E = IR. If a fourth parameter X were added to the set, the conditions for an algebraic relation could still be satisfied. The only restriction placed on the added parameter would concern the indexing set. In order to restrict consideration to those parameter sets not having trivial elements (such as X in the previous example), the following structure is necessary.

D2-10: A relation ϕ on a set $\{P_n\}$ is a deductive relation if the following conditions are satisfied.

(1) For every set of n - 1 elements of the indexing set, representing n - 1 parameters, there exists a unique set of n-tuples properly contained in $G(P_n)$.

(2) No proper subset of $\{P_n\}$ with ϕ satisfies condition (1).

D2-11: A deductive relation is a function if for every n - 1 element of the indexing set, representing n - 1 parameters, there exists a single n-tuple contained in $G(P_n)$.

Condition (1) of D2-10 allows a given set of n - 1 scalars to exist in more than one n-tuple. This is necessary to allow for relations which are not "single-valued". For example, the relation $y = X^2$, indexed with the real numbers, allows two different scalars of X to correspond to a single scalar of y. The requirement that the set of n-tuples be properly contained in the graph eliminates trivial parameters from consideration. Condition (2) of D2-10 requires that exactly n - 1 elements are needed to restrict the remaining element to a proper subset of its range.

The next classification is one of convenience.

D2-12: The set $\pi_n'(P_n')$, formed from the set $\pi_n(P_n)$ by deleting those scalars associated with parameters not contained in P_n', is called the projection of P_n' on ϕ. ϕ is the relation involving $\{P_n\}$.

D2-13: $\pi_n'(P_n')$ is a proper projection if $\{P_n'\} \subset \{P_n\}$.

T2-1: There are 2^n distinct projections of the elements of $\{P_n\}$ on ϕ.

Proof: The number of distinct projections is just the number of distinct subsets of $\{P_n\}$.

The deductive relation, as defined in D2-10, is the fundamental idea upon which this entire thesis is based. It will be assumed that all physical systems can be described by deductive relations; and to design the system, it is only necessary to determine the "specifications" (the scalars corresponding to n - 1 parameters which are allowed any scalar value contained in the respective ranges), and then to consult the graph to determine the n-tuple or n-tuples (the solutions) which are "allowed" by the relation. In the case of a single relation, the process is extremely simple. However, when several relations exist on parameter sets that are not disjoint, the problem of design becomes more complex.

The structure presently defined considers only a single relation. This structure will now be extended to consider many relations.

D2-14: Two relations ϕ_1 and ϕ_2 are connected if $P_{n1} \cap P_{n2} \neq 0$.

D2-14 is somewhat limited in usefulness because connectivity between relations is not transitive. For example, consider the three sets: $P_1 = (X_1\ X_2\ X_3)$, $P_2 = (X_3\ X_4\ X_5)$, and $P_3 = (X_5\ X_6\ X_7)$. P_1 is connected to P_2 and P_2 is connected to P_3, but P_1 is not connected to P_3. In the design process, however, sets of parameters which correspond in this manner are the rule rather than the exception. Therefore, the following definition is used to determine connectivity when more than two relations are involved.

D2-15: A set of relations $\{\phi_i\}$ on the parameter sets $\{P_{ni}\}$ is a connected set if for every proper subset $\{\phi_j\}$, $\{P_j\} \cap \{\overline{P}_j\} \neq 0$.
$(\{\overline{\phi}_j\} = \{\phi_i\} - \{\phi_j\})$.
$(\{\overline{P}_j\} = \{P_i\} - \{P_j\})$.

Each relation and its associated set of parameters have a graph which indicates "allowed" n-tuples. Since some parameters appear in more than one relation, it is possible that a particular scalar allowed for a parameter in one relation will not be allowed in the other relation or relations in which the same parameter appears. It is the problem of the designer to select a subset of each graph such that any scalar, or set of scalars, which is allowed (for a particular set of parameters) in one relation is allowed for each relation in which the parameters appear. The totality of n-tuples contained in these subgraphs can then be divided into equivalence classes containing one n-tuple from each graph. This idea is explained formally in the following definitions.

D2-16: Let $\left\{ \phi_i \right\}$ be a connected set of relations on the parameter sets $\left\{ P_{ni} \right\}$. Let X be a scalar n-tuple such that every parameter belonging to $\bigcup_{\forall i} \left\{ P_{ni} \right\}$ is represented once. X is a natural point if there exists $y_i \in G(P_{ni})$ such that $y_i \subseteq X$ for all i. The set of natural points shall be denoted $\pi_n(\phi_i)$.

Each natural point is an "allowed state" of the system represented by the relations. The graph of each relation has an allowed element included in each natural point. The question "Does a natural point exist?" is certainly an important one. It would be most useful in design to be able to determine quickly the question of existence.

However, the information required to answer this question in an "if" and "only if" manner is contained in the graph, and no simple method exists to extract it.

Relations for which no natural points exist are of little use in the design process. Such sets of relations shall be considered inconsistent.

D2-17: The relations $\{\phi_i\}$ are consistent if there exists a natural point.

D2-18: The relations ϕ_i and ϕ_j are naturally connected if $\pi_{ni}(P_{nij}) \subseteq \pi_{nj}(P_{nij})$ or $\pi_{nj}(P_{nij}) \subseteq \pi_{ni}(P_{nij})$ where $\{P_{nij}\} = \{P_{ni}\} \cap \{P_{nj}\} \neq 0$.

D2-19: The relations ϕ_i and ϕ_j are normally connected if $\pi_{ni}(P_{nij}) = \pi_{nj}(P_{nij})$.

Definitions D2-17, D2-18, and D2-19 provide three levels of restriction on connected relations. The first level requires the two graphs to contain at least one common element in the respective projections of $\{P_{nij}\}$. The second level requires that one projection contains the other. The most restrictive case requires that the projections be equal.

One of the most basic and important concepts involved with connected relations is that of dependence and independence. Conditions for these properties are usually expressed in terms of operations of the particular algebra being used. The same conditions can be expressed in relation theory by the following definition.

D2-20: The relations $\{\phi_i\}$ are dependent if there exists $\phi \in \{\phi_i\}$ such that $\Pi_n(\phi_i) = \Pi_n(\phi_i - \phi)$.

D2-21: The relations $\{\phi_i\}$ are independent if they are not dependent.

The definition D2-20 requires that relations which are not consistent be independent. This is in agreement with standard algebraic definitions. The following theorem is an obvious result of D2-20.

T2-2: Let $\{\phi_i\}$ be a set of relations such that each relation involves a parameter not contained in any other parameter set of $\{\phi_i\}$. Then $\{\phi_i\}$ is an independent set.

Proof: Assume $\{\phi_i\}$ is dependent; then there exists $\phi \in \{\phi_i\}$ such that $\Pi_n(\phi_i) = \Pi_n(\phi_i - \phi)$. Let ϕ_1 be that relation. But $\Pi_n(\phi_1)$ involves a parameter not present in $\Pi_n(\phi_i - \phi_1)$. Therefore, $\Pi_n(\phi_i) \neq \Pi_n(\phi_i - \phi)$.

T2-3: Given a set of relations $\{\phi_i\}$. If there exists an order $\phi_1, \phi_2, \ldots \phi_n$, such that the i^{th} relation involves a parameter not contained in the parameter sets of the preceding relations, $\{\phi_i\}$ is an independent set.

Proof: $\{\phi_1\}$ is an independent set since two relations are required to satisfy D2-20; $\{\phi_1, \phi_2\}$ is an independent set since ϕ_2 contains a parameter not in ϕ_1 and, by definition D2-10, ϕ_1 contains a parameter not in ϕ_2. $\{\phi_1, \phi_2, \phi_3\}$ is an independent set since ϕ_3 contains a parameter not in ϕ_1 or ϕ_2; ϕ_1 and ϕ_2 both contain parameters not in ϕ_3 according to definition D2-10. This process is continued for all relations, thus, completing the proof.

Many sets of relations can be determined to be independent by satisfying T2-3. A straightforward procedure for obtaining the order needed to satisfy T2-3 is given by the following decomposition process:

Given the set $\{\phi_i\}$, form the set $\{\phi_i\}^1 = \{\phi_i\} - \{\phi_{i1}\}$, where each $\phi \in \{\phi_{i1}\}$ involves a parameter not contained in any parameter set of $\{\phi_i - \phi\}$. If $\{\phi_i\}^1$ is not empty, repeat the process replacing $\{\phi_i\}$ by $\{\phi_i\}^1$.

D2-22: A set $\{\phi_i\}$ is decomposable if the above process yields the empty set in a finite number of steps. Each $\{\phi_i\}^j$ is referred to as the j^{th} decomposition class and if $\{\phi_i\}^n$ is the last nonempty set, the set $\{\phi_i\}$ is said to be n-decomposable.

T2-4: An n-decomposable set $\{\phi_i\}$ is independent.

Proof: Order the relations starting with the n^{th} decomposition class. This order satisfies T2-3.

T2-4 gives a quick and simple method for verifying independence of a set of relations. However, a set of relations may be independent and not satisfy T2-3; therefore, a necessary and sufficient condition for independence would be very desirable. This condition cannot be obtained without further restrictions which will limit the types of relations to which the theorems can be applied, but they are sufficiently general to cover a very large group of engineering design problems.

T2-5: Let $\{\phi_i\}$ be an n-decomposable set. Then there do not exist disjoint subsets $\{\phi_j\}$ and $\{\phi_k\}$ such that $\bigcup\{P_{nj}\} = \bigcup\{P_{nk}\}$.

Proof: Let $\{\phi_j\}$ and $\{\phi_k\}$ be disjoint subsets of $\{\phi_i\}$ such that $\bigcup\{P_{nj}\} = \bigcup\{P_{nk}\}$. Let ϕ_1 be a relation belonging to $\{\phi_j\}$. If ϕ_1 is in the r^{th} decomposition class, $\{\phi_k\}$ contains a relation belonging to the decomposition classes (1, 2, 3 ... r - 1). If this relation is in the q^{th} class, $\{\phi_j\}$ contains a relation in the classes (1, 2, ... q - 1). This argument may be extended until one of the original subsets has a relation contained in the first decomposition class. Then there will exist a parameter in this relation not contained in any of the parameter sets of the other original subset.

D2-23: Let $\{\phi_i\}$ be a set of relations on $\{P_n\}$. Then $\{\phi_i\}$ is a restricted set if $\Pi_n(\phi_1) \subseteq \Pi_n(\phi_2)$ or $\Pi_n(\phi_1) \supseteq \Pi_n(\phi_2)$ for every subset $\{\phi_1\}$ and $\{\phi_2\}$ such that $\bigcup\{P_{n1}\} = \bigcup\{P_{n2}\}$.

T2-6: Every independent restricted set of relations is decomposable.

Proof: Let $\{\phi_i\}$ be a nondecomposable independent restricted set. Apply the decomposition process until a set $\{\phi_i'\}$ is obtained such that no relation belonging to $\{\phi_i'\}$ has a parameter that is not contained in a parameter set of the remaining parameters of $\{\phi_i'\}$. Select ϕ_1 belonging to $\{\phi_i'\}$. Then the $\bigcup\{P_{ni}'\} = \bigcup\{P_{ni}' - P_{n1}'\}$ and $\{\phi_i'\}$ is a dependent set by definition D2-23.

T2-7: A restricted set of relations is independent if and only if it is decomposable.

Proof: By theorems T2-6 and T2-4.

CHAPTER III

PARAMETER SYSTEMS

A formal definition of a system has, as yet, not been standardized. The most general and encompassing definition allows a system to be any subset of the universe. This definition, although not very strict, is consistent with present usage of the word. For the purpose of this thesis, a parameter system shall be defined and referred to as a system.

D3-1: A collection of n-parameters $\left\{ P_{ni} \right\}$ and i relations $\left\{ \phi_i \right\}$ is a parameter system S_{ni} if:

(1) For every $P \in \left\{ P_{ni} \right\}$ there exists $\phi_1 \in \left\{ \phi_i \right\}$ such that $P \in \left\{ P_{n1} \right\}$.

(2) For every $\phi_1 \in \left\{ \phi_i \right\}$ and for every $P \in \left\{ P_{n1} \right\}$ $P \in \left\{ P_{ni} \right\}$.

(3) $\left\{ \phi_i \right\}$ is algebraic.

D3-2: Two systems S_{ni} and S_{ni}' are equal if they have the same set of natural points.

D3-2 allows the same system to be represented by alternate relations. Although it is obvious that $\left\{ P_{ni} \right\}$ must always be the same, many possible sets of $\left\{ \phi_i \right\}$ will yield the same set of natural points.

D3-1 is not intended to be very restrictive; it serves only to restrict the discussion to sets of parameters and relations which could be used to define a subset of the universe.

The design engineer's concern for systems as defined above is related to the selection of a particular group of desired characteristics of the system and the eventual solution of the relations to obtain scalar values of the remaining parameters which are compatible with the original desires. The original desired characteristics are expressed as scalar values of the appropriate parameters and are referred to as specifications.

D3-3: A subset $\left\{ P_{ni}' \right\}$ of $\left\{ P_{ni} \right\}$ is an allowed specification set if every element of $\left\{ P_{ni}' \right\}$ can be assigned an arbitrary scalar (from its appropriate range) such that some element of $\Pi_n(\phi_i)$ contains this set.

D3-4: An allowed specification set is complete if it is maximal.

D3-5: Let $\left\{ P_{ni}' \right\}$ be an allowed specification set of $\left\{ P_{ni} \right\}$. Then $\left\{ P_{ni} - P_{ni}' \right\}$ is the solution set.

D3-3 says that there exists at least one system having the particular characteristics stated in an allowable specification set. This does not exclude the possibility of the existence of a solution for a nonallowable set since the particular scalars might allow this to occur. However, from a design standpoint only allowable sets are considered. The selection of allowed sets is a problem of the design itself. When only one relation is being considered, it is obvious that any n - 1 parameters form an allowed specification set. However, when many relations and two to three times as many parameters form a system, it is necessary to provide some structure so that a specification set may be considered from a standpoint of allowability. The design map developed by Freeny (2) provides a method which appears to be useful in this respect. This procedure will be formalized, together with alternate procedures, which will provide greater insight concerning parameter selection.

The previous chapter formalizes the structure of the parameter relations which make up a system. The primary result concerns the independence and dependence of relations. This important property will be used extensively in the material to follow.

D3-6: An element of $\left\{ P_{ni} \right\}$ which has been assigned a scalar value is a fixed parameter.

D3-7: A relation ϕ_k is restricted if any $P \in \left\{ P_{nk} \right\}$ is fixed.

D3-8: A relation ϕ_k is fixed if all $P \in \left\{ P_n \right\}$ are fixed.

T3-1: A relation ϕ_k is fixed if any $\sigma \left\{ P_{ni} - 1 \right\}$ parameters are restricted.

Proof: Consider the definition of a relation.

For convenience, let the set of fixed parameters related by ϕ_1 be designated by $\left\{ \Omega_1 \right\}$ and its complement by $\left\{ \overline{\Omega}_1 \right\}$. The following process will be used to provide a method of obtaining an allowable specification set.

Process P

Given a system S_{ni}:

(1) Fix an arbitrary $P_1 \in \{P_{ni}\}$.

(2) Form a new system with $\{P_{ni}'\} = \{P_{ni} - P_1\}$ and $\{\phi_i'\} = \{\phi_i - \phi_x\}$ where $\{\phi_x\} = \{\phi : \sigma\{\bar{\Omega}\} = 1\}$.

(3) If $\sigma\{\phi_x\} \neq 0$, repeat (2) using $\bigcup\{\bar{\Omega}\}$ as fixed parameters. If $\sigma\{\phi_x\} = 0$, repeat (1) and (2) using the new system.

(4) Repeat (3) until $\{\phi_i'\} = 0$.

This process, together with the next theorem, defines a sufficient condition for a specification set to be allowable.

T3-2: A set of parameters selected in (1) of Process P is an allowable specification set if $\sigma\{\phi_x\} \neq 0$ implies $\sigma\{\phi_x\} = \sigma \bigcup_{\forall i}\{\bar{\Omega}_i\}$.

Proof: If $\{\phi_x\} \leq 1$, any parameter that is fixed by the properties of a relation is fixed by only one relation and, therefore, has an allowable value. If $\sigma\{\phi_x\} > 1$, then $\sigma \bigcup_{\forall i}\{\bar{\Omega}_i\} = \sigma\{\phi_x\}$, which again implies that each parameter is fixed by only one relation and has an allowable value.

Theorem T3-3 specifies the number of arbitrary parameter selections which are required to fix a system.

T3-3: Every complete specification set contains n - i elements.

Proof: Process P must be completed to fix the entire system. This process terminates with $\left\{ \phi_i' \right\} = 0$; therefore exactly i relations have been fixed. Each time a relation was fixed, the fixed parameters used in the next step were $\bigcup \left\{ \bar{\Omega} \right\}$, not arbitrary parameters. This implies that exactly n - i parameters were selected.

Note that, if at any time during the application of Process P, $\sigma \left\{ \phi_x \right\}$ was not equal to $\sigma \bigcup \left\{ \bar{\Omega} \right\}$ or zero, some parameter has been fixed by two separate relations. Since there is nothing to require that the scalars determined by the two relations are equal, the set of selected parameters cannot, in general, satisfy the system. The following theorems show some of the properties of sets which will satisfy Theorem T3-2.

T3-4: Let $\left\{ \phi_i \right\}$ be an independent set of relations on the parameters $\left\{ P_{ni} \right\}$. Then there exists $P \in \left\{ P_{ni} \right\}$ such that $\left\{ \phi_i \right\}$ on $\left\{ P_{ni} - P \right\}$ is independent.

Proof: Consider any element in the last decomposition set which appears in more than one relation. Removal of this parameter does not affect the decomposability of $\left\{ P_{ni} \right\}$, and therefore $\left\{ P_{ni} - P \right\}$ is independent.

T3-5: Let $\{\phi_i\}$ be an independent set of relations on $\{P_{ni}\}$. Then any subset of $\{\phi_i\}$ on the corresponding parameters is independent.

Proof: Any subset of a decomposable set is decomposable.

It is important to know that there is at least one allowable specification set for a given system. The following theorems give sufficient conditions for the existence of specification sets.

T3-6: Let $\{\phi_i\}$ be an independent set of relations on the parameters $\{P_{ni}\}$. Then there exists a set $\{P_k\} \subset \{P_{ni}\}$ such that $\{P_k\}$ is an allowable specification set.

Proof: Perform Process P on the n^{th} decomposition class, fixing first the parameters which appear in more than one relation. The condition $\sigma\{\phi_x\} \neq 0$, $\sigma\{\phi_x\} \neq \sigma \bigcup \{\bar{\Omega}\}$ cannot occur, and the process may be continued until each relation in the n^{th} decomposition class has been fixed. Repeat, using the n - 1 decomposition class and each n - m decomposition class in turn until the empty set is obtained.

T3-7: Let $\{\phi_i\}$ be an independent set of relations on the parameters $\{P_{ni}\}$. Then any subset $\{P_k\}$ of

$\left\{ P_{ni} \right\}$ such that $\left\{ \phi_i \right\}$ on $\left\{ P_{ni} - P_k \right\}$ is independent is an allowable specification set.

Proof: If $\left\{ \phi_i \right\}$ on $\left\{ P_{ni} - P_k \right\}$ is independent, then there exists a set which satisfies Theorem T3-2. The union of P_k and this set is an allowable specification set.

The next theorem does not imply a necessary condition for an allowable specification set; however, it does give a necessary condition for a set which will satisfy Theorem T3-2.

T3-8: Let $\left\{ \phi_i \right\}$ be a dependent set of relations on the parameters $\left\{ P_{ni} \right\}$. Then no set $\left\{ P_k \right\}$ exists that will satisfy Theorem T3-2.

Proof: Let $\left\{ \phi_i' \right\}$ be a subset of $\left\{ \phi_i \right\}$ such that every parameter involved appears in at least two relations. $\left\{ \phi_i' \right\}$ is not empty since $\left\{ \phi_i \right\}$ is not decomposable. Perform Process P on $\left\{ \phi_i \right\}$. To fix every relation in $\left\{ \phi_i' \right\}$ will require that $\sigma \left\{ \phi_x \right\} \neq 0$, $\sigma \cup \left\{ \overline{\Omega} \right\} \neq \sigma \left\{ \phi_x \right\}$ since fixing n - 1 of the relations in $\left\{ \phi_i' \right\}$ will fix all of the parameters involved in $\left\{ \phi_i' \right\}$.

When applying some of the rules given for determining specification sets, it is desirable to reduce the number of relations and parameters which must be considered. A method for reduction of a system is given in the following theorem.

T3-9: Given an independent system S_{ni}, let $\left\{ \phi_k \right\}$ be a subset of $\left\{ \phi_i \right\}$ involving the parameters $\left\{ P_k \right\}$. Let $\left\{ P_k' \right\}$ be the set of parameters which appears only in the relations $\left\{ \phi_k \right\}$. Further, let there exist an ordering of $\left\{ \phi_k \right\}$ and $\left\{ P_k' \right\}$ such that $P_1 \in \left\{ \phi_1 \right\}$, $P_2 \in \left\{ \phi_1, \phi_2 \right\}$ ..... $P_n \in \left\{ \phi_1, \phi_2 \cdots\cdots \phi_n \right\}$. If the parameters appear only in those relationships indicated and if $\left\{ P_k' \right\}$ contains no elements of the desired specification set, the parameters $\left\{ P_k' \right\}$ and the relations $\left\{ \phi_k \right\}$ may be omitted from the system for the purpose of checking the specification set.

Proof: Removal of $\left\{ \phi_k \right\}$ from $\left\{ \phi_i \right\}$ removes only the parameters $\left\{ P_k' \right\}$ which, since they are not members of the specification set, may assume any value within their respective ranges. Selection of a specification set for the system $\left\{ \phi_i - \phi_k \right\}$ and $\left\{ P_{ni} - P_k' \right\}$ will fix all relations and parameters in the system. When $\left\{ \phi_k \right\}$ and $\left\{ P_k' \right\}$ are considered, ϕ_n is fixed since n - 1 parameters are fixed. This in turn fixes ϕ_{n-1}, and the process is continued until ϕ_1 is fixed.

The process of reduction given in Theorem T3-9 is most useful when a specification set has not yet been decided upon, but several parameters are known to be excluded from consideration.

The application of the procedures which have been explained is much easier than the definitions and theorems indicate. A convenient way to apply Process P and to determine if Theorem T3-2 is satisfied has been developed by Freeny. Called a "design map", this procedure has been used for several years by persons in the relay design group at Oklahoma State University. In essence, the design map is a matrix having columns corresponding to the relations of the system and rows corresponding to the parameters. An entry appears in the ijth position only if the relation of the jth column involves the parameter of the ith row. This matrix serves as a chart to tabulate the selections as shown in the example of Figure 3-1.

System Relations

(1) $(X_1\ X_2\ X_3\ X_4\ X_5)$

(2) $(X_4\ X_5\ X_6)$

(3) $(X_2\ X_3\ X_4\ X_7)$

Design Map

P_{ni}	(1)	(2)	(3)
X_1	X_1		
X_2	X_2		X_2
X_3	X_3		X_3
X_4	X_4	X_4	X_4
X_5	X_5	X_5	
X_6		X_6	
X_7			X_7

Figure 3-1. Design Map

A discussion of the use of this map is presented in the following chapter.

An alternate method of graphic representation of a system is the relation graph. The relation graph is obtained by selecting a linear graph such that there is a one-to-one correspondence between the edges of the graph and the parameters of the system, and such that the parameter set for every relation appears as a circuit in the linear graph.[2]

The idea of the relation graph was originally suggested about three years ago by John C. Paul. However, the usefulness of the graph could not be realized since there seemed to be no way to assure the existence of a relation graph for every system. In particular, the problem was the lack of a linear graph with the required one-to-one correspondence between edges and parameters. This lack of a one-to-one correspondence will be shown to be a function, not of the system itself, but of the particular set of relations which are used to define the system. The following theorems are used as justification for using the relation graph as a design tool.

D3-9: Given a system S_{ni}; let G_R be a linear graph such that there exists a one-to-one correspondence between the edges of G_R and the elements of $\{P_{ni}\}$. If for every relation $\phi_k \varepsilon \{\phi_i\}$ there exists a circuit of G_R such that there is a one-to-one correspondence between the elements of $\{P_k\}$ and the edges of the corresponding circuit, G_R is called a relation graph of S_{ni}.

[2]See Appendix.

From D3-2 it is known that the relation graph is not unique. Also, the set $\left\{ \phi_i \right\}$ cannot be obtained from G_R since a one-to-one correspondence does not always exist between the elements of $\left\{ \phi_i \right\}$ and the circuits of G_R. An example of a relation graph is shown in Figure 3-2.

System Relations

(1) $(x_1\ x_2\ x_3\ x_4\ x_5)$

(2) $(x_4\ x_5\ x_6)$

(3) $(x_2\ x_3\ x_4\ x_7)$

Relation Graph

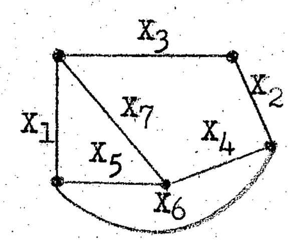


Figure 3-2. Relation Graph

When a linear graph is found such that every relation of the system appears as a circuit, it is generally the case that circuits appear in the linear graph which are not valid relations for the system. The set of circuits of a linear graph is, in fact, the set generated by a set of fundamental circuits (the system relations) and the operation of union minus intersection. Those circuits of this set which are not valid relations of the system will be known as implied relations.

D3-10: Any circuit of a relation graph which is not a valid relation of the corresponding system is an implied relation.

It is obvious that an implied relation which is properly contained in, or properly contains, an element of $\left\{ \phi_i \right\}$ is not a valid relation.

Theorem T3-10 provides for the existence of a relation graph for every system.

T3-10: Let S_{ni} be a system. Then there exists a relation graph of S_{ni}.

Proof: From linear graph theory, a sufficient condition for the existence of a linear graph corresponding to a set of elements E is the satisfaction of the following postulates.

(1) Every subset of E either is or is not a circuit.

(2) No proper subset of a circuit is a circuit.

(3) The union minus intersection of two circuits is either a circuit or a disjoint union of circuits.

Postulates (1) and (3) are satisfied by all systems if the implied relations are considered as relations. This leaves postulate (2) to be satisfied before the existence of the relation graph is assured. Therefore, it is sufficient to show that every system can be expressed by a set of relations which will satisfy postulate (2).

Let S_{ni} be a system defined by $\left\{\phi_i\right\}$. Also assume that $\phi_1 * \phi_2 = \phi_3$ (* indicates the operation union minus intersection), ϕ_1, $\phi_2 \in \left\{\phi_i\right\}$, such that ϕ_3 contains, or is contained in, some ϕ_k of $\{\phi_i\}$. Add the relations ϕ_1 and ϕ_2, obtaining ϕ_4. ϕ_4 is not equal to ϕ_3 since a valid relation cannot contain or be a subset of another valid relation. Replace ϕ_1 or ϕ_2 in $\{\phi_i\}$ by ϕ_4, and the implied relation ϕ_3 will no longer exist. Then either there exists a relation graph or there exist ϕ_5 and ϕ_4 such that $\phi_5 * \phi_4 = \phi_7$ is contained in ϕ_4. If ϕ_7 is contained in ϕ_4, it must also be contained in ϕ_1 and ϕ_2 since ϕ_4 contains at least one element belonging to both ϕ_1 and ϕ_2 and all elements of ϕ_3. Therefore, the relation graph exists.

If more than one conflict of the type just discussed appears, each may be removed in an identical manner. If a finite number of conflicts exist, the system has a relation graph.

It should be noted that, although Theorem T3-10 provides a method for obtaining a set of relations defining the system such that a relation graph can be obtained, it is necessary to know the algebraic form of the relations in order to accomplish the generation of another valid relation. Examples of the process are given in the following chapter.

The key to the use of the relation graph as an aid to design is given by Theorem T3-11.

T3-11: Given a system S_{ni} with relation graph G_R, then no parameter set of any relation or implied relation will satisfy Theorem T3-2.

Proof: Case 1: Let $\phi_k \in \{ \phi_i \}$. Theorem T3-2 cannot be satisfied.

Case 2: Let ϕ_k be an implied relation. Then ϕ_k can be expressed as $\phi_j * \phi_q$ where $\phi_j \in \{ \phi_i \}$ and ϕ_q is an implied relation. Further, ϕ_q can be expressed as $\phi_L * \phi_R$ where $\phi_L \in \{ \phi_i \}$ and ϕ_R is an implied relation. If ϕ_k is fixed, all parameters of ϕ_j are fixed except those belonging to $\phi_j * \phi_q$. All parameters of ϕ_L are fixed except those belonging to $\phi_j * \phi_q$ and $\phi_L * \phi_R$. Therefore the remaining system will have ϕ_j and ϕ_L such that $P_j \subseteq P_L$ and nondecomposability exists.

This theorem says that any set of parameters which appears as a circuit in the relation graph will not satisfy Theorem T3-2. In terms of the design map, these are sets which will not "map through." One of the most desirable characteristics of the relation graph is that of being able to tell at a glance whether or not a particular set of parameters will satisfy Theorem T3-2.

A listing of all trees of the relation graph is readily obtained by the Cauchy expansion.[3] This is a list of all possible maximal subsets of $\left\{ P_{ni} \right\}$ that can be formed without including a relation or an implied relation.

[3]See Appendix.

CHAPTER IV

EXAMPLES AND APPLICATIONS

The following discussion is meant to aid in the understanding of the parameter selection process, and no attempt is made to justify each statement algebraically.

A system may be defined by a set of relations and a corresponding set of parameters. The word "defined" in this context means that the system in question can be distinguished from certain other systems. The number of relations and parameters necessary is a function of the degree of uniqueness desired. Obviously, as the number of restrictions (relations) is increased, the number of systems which will fit the requirements is decreased. Once a system has been selected and the parameters which must be considered are ascertained, there is a unique maximal set of relations which are valid for the system under consideration. (A valid relation must have at least one natural point.) Not all of these relations are needed to define the system, since a maximal independent set will uniquely specify the complete set of natural points, and any additional relations are simply combinations of the base set.

Selection of an allowed specification set is based on a <u>single</u> necessary and sufficient condition. This condition is that the set in question does not include a complete parameter set for any <u>valid</u> relation of the system. Any parameter set which satisfies this condition is an allowed specification set. The most obvious and straightforward method of parameter selection would therefore be to check the desired set to determine whether a complete parameter set of any valid relation is restricted. This method is not practical since obtaining the complete set of valid relations would be a prohibitive task in all but the most trivial cases.

The design map is a technique for checking the relations, using only the information from the base set. It is not unique. In fact, there are as many design maps for a system as there are maximal independent sets of relations. Any one of these design maps may be used, but the logical choice would be the one corresponding to the defining set of relations, since any other map would necessitate determination of additional valid relationships.

The following is an example of the use of the design map and shows some of its limitations.

Consider a relay defined by the following relations:

$$(1) \quad \eta = (157.5) \frac{R_c(X_o + \alpha)\sqrt{2P_o}}{ENS}$$

$$(2) \quad t_s = (10^{-8}) \frac{N^2S^2}{(X_o + \alpha)(R_c)} \ln \frac{1}{1 - \eta} +$$

$$(8.66 \times 10^{-3}) \left[\frac{18\, MX_o^2\, R_c}{E^2 \eta\, (1 - \eta)\, \left[1 - \nabla^2 (1 + KX_o/P_o)\right]} \right]^{1/3}$$

$$(3) \quad P = E^2/R_c$$

$$(4) \quad R_c = \frac{(.865 \times 10^{-6})\, S^2(1 - \beta - \sigma)(1 + \beta + \sigma)\ell\, g_r}{\delta^4}$$

$$(5) \quad N = \frac{(.637)\,\ell\,S(1 - \beta - \sigma)g_n}{\delta^2}$$

where

P = coil power

η = stability factor

P_o = restoring force on the armature

E = supply voltage

N = coil turns

M = effective mass of armature

K = effective spring constant of spring system

ℓ = coil length

δ = diameter of coil wire

t_s = seating time of armature

R_c = coil resistance

X_o = length of armature working air gap

S = outside coil width

α = air equivalent of the nonforce producing part of the magnetic circuit when using a series representation

∇ = ratio of the nonforce producing air equivalent of the magnetic circuit to the total air equivalent of the magnetic circuit

σ = ratio of total thickness of core and inside coil insulation to the outside coil width

β = ratio of core width to outside coil width

g_r = winding space factor for resistance

g_n = winding space factor for turns

The parameters β, α, σ, g_n, g_r and $1 - \nabla^2 (1 + KX_o/P_o)$ will be considered constants in the example.

A design map of this system is shown in Figure 4-1. A specification set, R_c, N, S, E, X_o, η, M, has been selected for test. The result, shown in Figure 4-1, leaves the two parameters δ and ℓ undetermined; and therefore, an appeal to the relations themselves is necessary to determine whether the set is an allowable specification set.[4] To do this, consider a system defined by the relations (4) and (5). Then it is only necessary to determine whether R_c, S, and N form an allowable specification set for the two-relation system.

Let $A = R_c$

$B = S$

$C = N$

K_1 = constant

K_2 = constant

then relation (4) becomes $A = \dfrac{K_1 B^2}{\delta^4}$

and relation (5) becomes $C = \dfrac{K_2 \quad B}{\delta^2}$

solving for ℓ, $\ell = \dfrac{K_3 C^2}{A}$

solving for δ, $\delta = K_4 \quad \dfrac{BC}{A}$

[4]The order of selection is indicated by the number within the squares. The O indicates a selected parameter, the □ indicates a parameter fixed by a relation and the △ indicates parameters which are fixed by two or more relations.

	o	f_1	f_2	f_3	f_4	f_5
11	δ				δ	δ
7	η	η	η			
4	E	E	E	E		
12	ℓ				ℓ	ℓ
9	m		m			
$\boxed{5}$	P			$\boxed{P}$		
$\boxed{8}$	P_o	$\boxed{P_o}$				
1	R_c	R_c	R_c	R_c	R_c	
3	S	S			S	S
$\boxed{10}$	t_s		$\boxed{t_s}$			
6	X_o	X_o	X_o			
2	N	N	N			N

Figure 4-1. Design Map I

$\boxed{5}$	δ				$\boxed{\delta}$	
9	η	η	η			
6	E	E	E	E		
$\boxed{3}$	ℓ				$\boxed{\ell}$	$\boxed{\ell}$
11	m		m			
$\boxed{7}$	P			$\boxed{P}$		
$\boxed{10}$	P_o	$\boxed{P_o}$				
1	R_c	R_c	R_c	R_c	R_c	R_c
4	S	S			S	
$\boxed{12}$	t_s		$\boxed{t_s}$			
8	X_o	X_o	X_o			
2	N	N	N			N
	o	f_1	f_2	f_3	f_4	f_6

Figure 4-2. Design Map II

Therefore, both ℓ and δ have been determined, and the original set of parameters is an allowable specification set. This method of checking a set of parameters which does not map through is accomplished quickly, and positive results are obtained. However, note that the functional form of the relations involving the parameters in question must be known.

Now if equations (4) and (5) are combined, the result is equation (6).

$$(6) \quad R_c = (2.14 \times 10^{-6}) \frac{N^2 g_r}{2 g_n{}^2} \frac{(1 + \beta + \sigma)}{(1 - \beta - \sigma)}$$

If relation (5) in the previous system is replaced by relation (6), a new set of equations is obtained which defines the same system. The map for this system is shown in Figure 4-2. The same set of parameters that was tested on the map of Figure 4-1 is shown to map through in Figure 4-2. Therefore, the set is known to be an allowable specification set. The point of the example is as follows: A given set of parameters may map through on one map and not on another apparently equivalent map. Although they are both maps of a set of relations defining a given system, they do not contain the same information. For example, the first map shows that it might be possible to solve equation (4) and (5) and obtain a relation involving only R_c and N. Whether or not this can be done cannot be determined from the map itself; and, therefore, no set of parameters containing both R_c and N would map through the design map in Figure 4-1.

The map in Figure 4-2 does not contain this possibility, and the set containing R_c and N maps through. The difference in information contained in the two maps is a result of the particular functional form of the relations themselves.

It is important to remember that the design map contains only a limited amount of information about the system; and, therefore, positive results in all cases cannot be obtained without resorting to additional information. However, it is advantageous to use the map to determine the particular relations which must be investigated, rather than to generate the complete set of valid relations.

It is readily apparent that the design map will reject all sets of parameters which include the complete parameter set of any relations or any possible relation. To explain the phrase "any possible relation", consider a system defined by the two relations $(X_1\ X_2\ X_3\ X_4)$ and $(X_1\ X_2\ X_5\ X_6)$. If the functional form of the relations is not known, it cannot be determined which of the following parameter sets is a valid relation.

(1) $(X_3\ X_4\ X_5\ X_6)$

(2) $(X_2\ X_3\ X_4\ X_5\ X_6)$

(3) $(X_1\ X_3\ X_4\ X_5\ X_6)$

None of these relations will map through on the design map, although two of them may be allowable specification sets. The sets of parameters which give positive results on the design map may be only a small portion of the total number of allowable specification sets.

The relation graph is simply a convenient method for obtaining the set of relations generated by the operation union minus intersection performed on the parameter sets. It can be used in the same manner as the design map if desired, but requires only a quick visual inspection to determine whether or not a set maps through. The following theorem concerns this problem.

T4-1: No circuit of a relation graph will satisfy Theorem T3-2 of Chapter III.

Proof: Case 1: Let $\phi_k \in \{\phi_i\}$. Then P_k does not satisfy Theorem T2-1.

Case 2: Let $\phi_k \notin \{\phi_i\}$. ϕ_k can be expressed as $\phi_j * \phi_q$ where $\phi_j \in \{\phi_i\}$ and $\phi_q \in \{\phi_i\}$. Further, ϕ_q can be expressed as $\{\phi_L * \phi_R\}$ where $\phi_L \in \{\phi_i\}$ and $\phi_R \notin \{\phi_i\}$. If ϕ_k is fixed, all parameters of ϕ_j are fixed except those belonging to $\{\phi_j \cap \phi_q\}$. All parameters of ϕ_L are restricted except those belonging to $\{\phi_j \cap \phi_q\}$ and $\{\phi_L \cap \phi_R\}$. Therefore, the remaining system will have ϕ_q and ϕ_L such that $P_q \subseteq P_L$, and nondecomposability exists.

Since no set of parameters which are contained in a circuit of the relation graph will map through the design map, a very large list of possible test sets is removed from consideration by inspection of the relation graph. Since a tree contains no circuits, intuition might indicate that the set of trees of the relation graph would be a complete set of allowable specification sets. However, this is not the case. The following example shows how a tree can exist which is not an allowable set and also how some circuits may be allowable sets.

Consider the system defined by the following relations:

(1) $(X_1\ X_2\ X_3\ X_4)$

(2) $(X_1\ X_2\ X_5\ X_6)$

(3) $(X_1\ X_2\ X_4\ X_5\ X_7)$

The relation graph is shown in Figure 4-3.

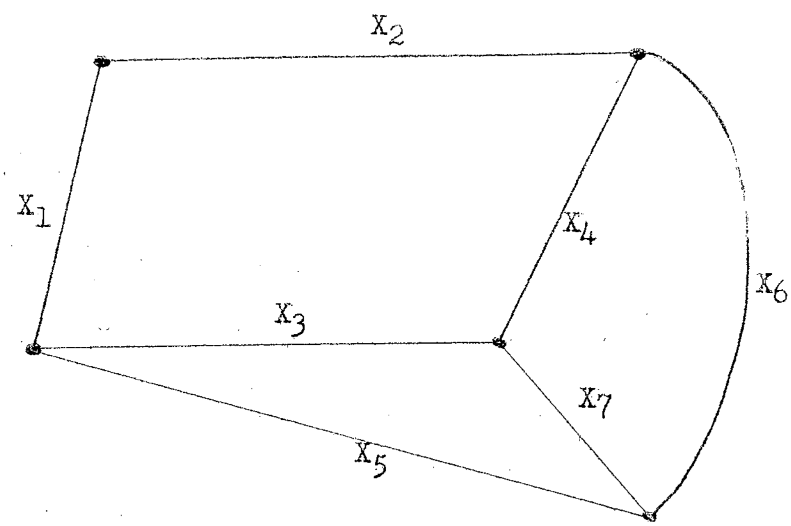


Figure 4-3. Relation Graph

The set $(X_3\ X_4\ X_5\ X_6)$ is a circuit of the relation graph. However, unless both relations (1) and (2) can be placed in the form $f(X_1\ X_2) + f(\text{remaining variables}) = 0$, this is not a valid relation. If the actual solution of relations (1) and (2) yielded a relation involving $(X_2\ X_3\ X_4\ X_5\ X_6)$, the set $(X_3\ X_4\ X_5\ X_6)$ would be an allowable specification set. Therefore some circuits of the relation graph are allowable sets. The set $(X_3\ X_4\ X_5\ X_6)$ and the set $(X_1\ X_2\ X_4\ X_5\ X_7)$ yield the circuit of the relation graph $(X_1\ X_2\ X_3\ X_6\ X_7)$. The set $(X_1\ X_3\ X_6\ X_7)$ is a tree of the graph, but not necessarily an allowable set, since the actual relation generated by (1) and (2) might be $(X_2\ X_3\ X_4\ X_5\ X_6)$, which, with (3), might yield the set $(X_1\ X_3\ X_6\ X_7)$, thereby making the tree an unallowable set. It is unfortunate that trees of the relation graph exist which are not allowable sets since a complete listing of the trees can be obtained by the Cauchy expansion process. This method may still be used to obtain sets of allowable specification sets since in practice very few of the trees are not allowable.

If a particular set of parameters is being investigated, a definite answer concerning this set is usually desired. Since a definite conclusion in many cases requires that the functional form of the equation be known, a general method for allowability testing cannot be obtained. Therefore, with any design method used, the most efficient procedure would be to use one of the methods previously discussed to locate the contradiction, if one exists, and then to resort to the equations in order to determine allowability. The contradiction can be located by either performing Process P on the design map or by inspection of the relation graph.

When using the design map or the relation graph to select specifications, it is desirable to know the order of importance of the parameters. When a conflict is found, one of the parameters must be deleted from the specifications in order to assure a solution.

The key to checking specification sets lies in the fact that every independent set of relations will have at least one allowable specification set. Also, it is known that there does not exist any set of parameters which will map through a dependent set of relations.

The obvious result of this is that any subset of parameters which when deleted from the system leaves the relations independent is an allowable specification set. Therefore, any subset of the test set may be checked for allowability without the necessity of trying to map it through with the rest of the set. By the same reasoning, when any set of parameters is deleted, leaving a dependent system, it will not map through. If this is the case, there is no point in trying to include it in any test set.

The above discussion suggests a method for generating an allowed specification set; that is, selecting the parameters one by one, testing each time to determine whether an independent system remains. If the desired parameters are selected in order of their importance in the design, then the best possible specification set is obtained.

An example of this process is given in the sample selection problem which follows.

Consider a relay system defined by the following relations which were given in the previous example concerning the relay:

(1) $(\eta, E, P_o, R_c, S, X_o, N)$

(2) $(\eta, E, M, R_c, t_s, X_o\ N)$

(3) (E, P, R_c)

(4) (δ, ℓ, R_c, S)

(5) (δ, ℓ, S, N)

If the set $(N, R_c, \delta, X_o, S, P_o, \eta)$ is selected for test, the results are negative since both f_4 and f_5 specify values for ℓ as shown in Figure 4-4. The original system is independent as shown by the decomposition sets:

a. (1, 2, 3) (first decomposition set)

b. (4, 5) (second decomposition set)

The system with the test set removed is

(1) (E)

(2) (E, M, t_s)

(3) (E, P)

(4) (ℓ)

(5) (ℓ)

which is dependent since the decomposition process yields

a. (2, 3) (first decomposition set)

b. (1) (second decomposition set)

c. (4, 5) (this set is nondecomposable)

The result indicates that the set will not map through, and this fact is also exemplified in the results of the mapping in Figure 4-4. If it is assumed that the desirability of the parameters as members of the specification set is indicated by their order as given above, each subset may be checked for allowability. This process is indicated below.

Step 1: The system with N removed is

(1) $(\eta, E, P_o, R_c, S, X_o)$

(2) $(\eta, E, M, R_c, t_s, X_o)$

(3) (E, P, R_c)

(4) (δ, ℓ, R_c, S)

(5) (δ, ℓ, S)

which has the decomposition classes

a. (1, 2, 3)

b. (4)

c. (5)

3	δ				δ	δ
	η	η	η			
	E	E	E	E		
6	ℓ				ℓ	ℓ
	M		M			
	P			P		
	P_o	P_o				
2	R_c	R_c	R_c	R_c	R_c	
5	S	S			S	S
	t_s		t_s			
4	X_o	X_o	X_o			
1	N	N	N		N	
	o	f_1	f_2	f_3	f_4	f_5

Figure 4-4. Design Map III

Step 2: The system with (N, R_c) removed is

(1) (η, E, P_o, S, X_o)

(2) (η, E, M, t_s, X_o)

(3) (E, P)

(4) (δ, ℓ, S)

(5) (δ, ℓ, S)

which, when an attempt at decomposition is made, yields

a. (1, 2, 3) (first decomposition class)

b. (4, 5) (nondecomposable)

The implication of Step 2 is that no set containing both N and R_c will map through. Since N is considered more desirable than R_c, R_c will be removed from the set of specifications.

Step 3: The system with (N, δ) removed is

(1) $(\eta, E, P_o, R_c, S, X_o)$

(2) $(\eta, E, M, R_c, t_s, X_o)$

(3) (E, P, R_c)

(4) (ℓ, R_c, S)

(5) (ℓ, S)

which has decomposition classes

a. (1, 2, 3)

b. (4)

c. (5)

Step 4: The system with (N, δ, X_o) removed is

(1) (η, E, P_o, R_c, S)

(2) (η, E, M, R_c, t_s)

(3) (E, P, R_c)

(4) (ℓ, R_c, S)

(5) (ℓ, S)

which has decomposition classes

a. (1, 2, 3)

b. (4)

c. (5)

Step 5: The system with (N, δ, X_o, S) removed is

(1) (η, E, P_o, R_c)

(2) (η, E, M, R_c, t_s)

(3) (E, P, R_c)

(4) (ℓ, R_c)

(5) (ℓ)

which has decomposition classes

a. (1, 2, 3)

b. (4)

c. (5)

Step 6: The system with (N, δ, X_o, S, P_o) removed is

(1) (η, E, R_c)

(2) (η, E, M, R_c, t_s)

(3) (E, P, R_c)

(4) (ℓ, R_c)

(5) (ℓ)

which has decomposition classes

a. (2, 3)

b. (1)

c. (4)

d. (5)

Step 7: The system with (N, δ, X_o, S, P_o, η) removed is

(1) (E, R_c)

(2) (E, M, R_c, t_s)

(3) (E, P, R_c)

(4) (ℓ, R_c)

(5) (ℓ)

which has decomposition classes

a. (2, 3)

b. (1)

c. (4)

d. (5)

Step 7 indicates that an allowable specification set has been obtained, retaining as many parameters as possible of the original test set. An additional parameter may be selected to replace R_c.

Although it may seem complex, in practice, this procedure is accomplished very easily and quickly and will allow the designer to pinpoint contradictions in the specifications without having to map the entire set through.

It is possible for a single parameter, when removed from the system, to leave a dependent system. It follows, then, that any set containing this parameter will not map through. The procedure in the example allows a quick check for this possibility.

In general, this procedure provides the quickest, most efficient method for obtaining an allowable specification set. Since each conflict is pinpointed in its order of importance, no confusion occurs concerning which alternate set to try. If this method is to be used, a convenient way to check decomposability is needed. One approach to this problem is to form a linear graph having one vertex for each relation in the system and one edge connecting two vertices for each parameter common to the two corresponding relations. An example of this type of graph is shown in Figure 4-5. For convenience, this shall be called a decomposition graph.

System:

a. $(X_1\ X_2\ X_3\ X_4\ X_5)$

b. $(X_4\ X_5\ X_6)$

c. $(X_2\ X_3\ X_4\ X_7)$

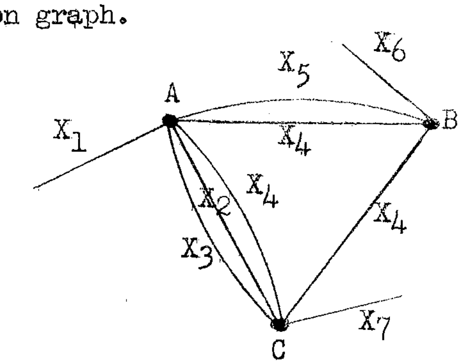


Figure 4-5. Decomposition Graph

Since the process used in decomposition is selection of parameters which appear in only one relation, the decomposability of the system in Figure 4-5 is obvious. Note that a decomposition graph which has no single vertex edge always depicts a dependent system, but the converse is not true. An example is shown in Figure 4-6. Clearly, if relation A (vertex A) is removed from the system, the remaining relations are nondecomposable. This graph is very useful on simple systems but becomes very complex on large systems.

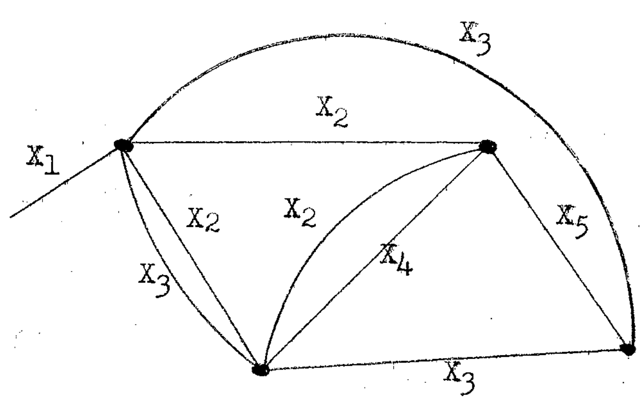
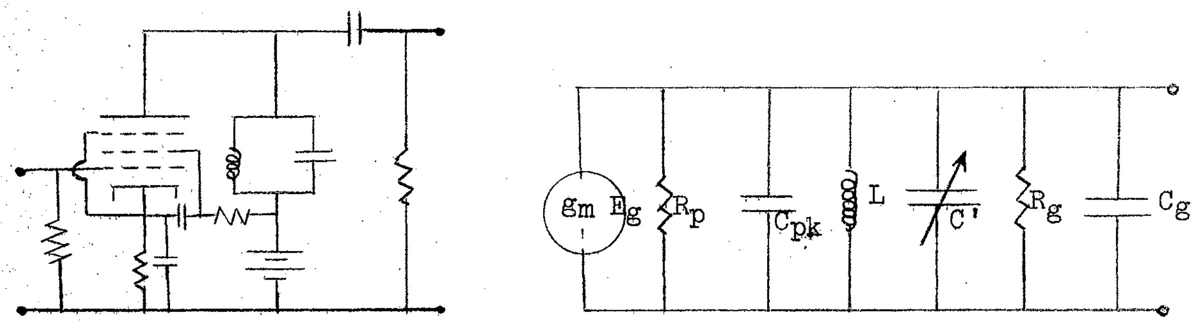


Figure 4-6. Alternate Decomposition Graph

A method for testing the decomposability of a system quickly and without the necessity of redrawing a graph or a design map is given in the following example.

Consider the single-stage, single-tuned amplifier shown in Figure 4-7.



Single-tuned Amplifier   Equivalent Circuit

Figure 4-7. Tuned Amplifier

Assume the amplifier can be represented by the following relations:

(1) $K = \dfrac{E_o}{E_i}$ (K, E_o, E_i)

(2) $K = -g_m w_o L Q_e$ ((K, g_m, w_o, L, Q_e)

(3) $Q_e = \dfrac{Q}{1 + w_o L Q(\frac{1}{r_p} + \frac{1}{R_g})}$ $(Q_e, Q, w_o, L, r_p, R_g)$

(4) $Q = \dfrac{w_o L}{R_L}$ (Q, w_o, L, R_L)

(5) $C = C_{pk} + C' + C_g$ (C, C_{pk}, C', C_g)

(6) $w_o = \frac{1}{\sqrt{LC}}$ (w_o, L, C)

(7) $\delta = \frac{w}{w_o} - 1$ (δ, w, w_o)

(8) $B = \frac{f_o}{Q_e}$ (B, f_o, Q_e)

(9) $w = 2\pi f$ (w, f)

(10) $w_o = 2\pi f_o$ (w_o, f_o)

(11) $\theta = \pi - \tan^{-1} 2\delta Q_e$ (θ, δ, Q_e)

where

E_o = output voltage

E_i = input signal

g_m = transconductance

r_p = plate resistance

R_g = grid resistor

R_L = load resistor

C_{pk} = plate to cathode capacitance

C = an equivalent capacitance

C_g = an equivalent capacitance

B = bandwidth

f = frequency

f_o = resonant frequency

θ = phase shift

k = amplification

L = inductance

Q = Q of the circuit

Q_e = equivalent Q of the amplifier

If the parameters are grouped according to the number of relations in which they appear, the table shown in Figure 4-8 is obtained.

Since the relations tested in the first group in Figure 4-8 all have a unique parameter, they will all appear in the first decomposition class. To form the second decomposition class, it is necessary to consider only those relations listed in the second and following groups.

The parameters of the second group all appear in two relations, and three possibilities exist concerning these two relations. First, both might be in the first decomposition class. If this is the case, no further consideration is necessary. Second, only one of the relations might be a member of the first decomposition class. This occurrence obviously qualifies the other relation for the second decomposition class. The only other possibility is that neither relation is included in the first decomposition class. This possibility excludes both relations from the second decomposition class on the basis of the particular parameter. However, one or both relations may be included in the second decomposition class by consideration of a different parameter.

The parameters of the third group are included in three relations. If all three relations are in the first decomposition class, no further consideration is necessary. When only two relations belong, the third is a member of the second decomposition class. Membership in the

E	D	C	B	A	P	Relations
		X	X	X	E_o	(1)
		X		X	E_i	(1)
			X		g_m	(2)
		X		X	r_p	(3)
			X		R_g	(3)
		X	X		R_L	(4)
				X	C_{pk}	(5)
			X		C'	(5)
			X	X	C_g	(5)
		X		X	B	(8)
			X		f	(9)
					θ	(11)
					K	(1)(2)
					Q	(3)(4)
		X		X	C	(5)(6)
					δ	(7)(11)
		X	X	X	w	(7)(9)
					f_o	(8)(10)
					L	(2)(3)(4)(6)
		X	X		Q_e	(2)(3)(8)(11)
		X		X	w_o	(2)(3)(4)(6)(7)(10)

Figure 4-8. Modified Design Map

first decomposition class of one or more of the relations implies the exclusion of those remaining from the second decomposition class on the basis of the parameter in question.

The pattern is the same for the remaining groups. Obviously each time a relation is added to a particular decomposition class, all other relations involving the parameter which is being checked must be included in a previous decomposition class.

As an example, assume the system is to be tested for independence by the method just described. The relations listed after the first set of parameters form the first decomposition class. In considering the second set of parameters, it is seen that relations (6), (7) and (10) are in the second decomposition class. Inspection of the remaining groups of parameters adds no information since all relations have been placed in a decomposition class.

The decomposition classes are

a. (1, 2, 3, 4, 5, 8, 9, 11)

b. (6, 7, 10)

which verify the independence of the systems.

Since there are twelve relations and twenty-one parameters, nine parameters can be specified. Suppose the set of specifications (E_o, E_i, r_p, C_{pk}, C_g, B, C, w, w_o) is desired. To check the allowability, column A of Figure 4-8 is used. First, each parameter of the specification set is removed from consideration (indicated by X in column A). The system remaining without these parameters is then checked for decomposability. The relations listed in the first group (2, 3, 4, 5, 9, 11) compose the first decomposition class.

In the second set of parameters, relation (2) listed after K is in the first decomposition class, implying that (1) is in the second. The same can be said of (11) and (7) listed after δ. Neither (8) nor (10) after f_o is in the first decomposition set; and, therefore, they must be considered for the third.

From the third group of parameters it is seen that relations (6) and (8) belong to the second decomposition class, thereby, making relation (10) a member of the third. The decomposition classes can be listed as

a. (2, 3, 4, 5, 9, 11)

b. (1, 6, 7, 8)

c. (10)

and the set (E_o, E_i , r_p, C_{pk}, C_g, B, C, w, w_o) is an allowable specification set. Note that if any relation is missing from the classes A, B, and C, this implies that a complete parameter set for that relation was specified and that the decomposition is void.

As a second example, consider the set (E_o, g_m, R_g, R_L, C', C_g, f, Q_e, w) marked in column B of Figure 4-8. Inspection of the second set of parameters shows the relations (2), (4), (7) and (10) to be in the second decomposition class. The remaining parameter L implies that relation (6) is in the third decomposition class.

The decomposition classes are

a. (1, 3, 5, 8, 11)

b. (2, 4, 7, 10)

c. (6)

Note that relation (9) is not included. Inspection shows that the entire parameter set of (9) was included in the specification set and that, therefore, it is not an allowable set.

In column C of Figure 4-8, the set (E_o, E_i , r_p, R_L, B, C, w, Q_e, w_o) is indicated as a test set. Inspection of the second group of parameters shows the second decomposition class to contain the relations (1), (4) and (7). The relations (8) and (10), listed after f_o, must be considered for the third decomposition set. Consideration of the relations involving L indicates that relation (6) belongs to the third decomposition class. Relations (8) and (10) will not decompose, and additional information is necessary to determine allowability.

CHAPTER V

SUMMARY AND CONCLUSIONS

The basic problem with which this thesis is concerned is the determination of which parameters of a system can be used as specifications without generating inconsistencies. When only a few relations are involved, the designer can easily spot any inconsistencies by inspection. However, when the number of relations increase, the picture quickly becomes so complex that the determination of the existence of inconsistencies is quite difficult. The general approach used requires the development of a formal structure which would allow discussion of parameters, relations and systems in a concise mathematical manner, rather than in a wordy philosophical manner.

First, the parameter is defined as a set of "values". This definition allows consideration of the parameter as an element of a set without regard to the exact nature or name of the values. Next, sets of related parameters are defined as relations. The relations provide the "rules" by which it is possible to ascertain the value of a particular parameter when the values of the related parameters are known. Finally, a system is defined to complete the structure. Throughout this discussion, many properties of the relation and system are defined and derived. The proofs of these properties are for the most part original, although the general structure follows a pattern similar to the system theory developed by Freeny (2). The two structures differ primarily in the definition of the relation and although an attempt was made to correlate the two structures wherever possible by using similar terms, the reader is cautioned against drawing conclusions of correspondence without thorough consideration since, although many of the results are the same, the method for obtaining them is quite different.

Next, the selection of an allowed specification is pursued under the assumption of a single necessary and sufficient condition. This condition is that the set in question does not include a complete parameter set for any valid relation of the system. Since it is obvious that obtaining a complete set of valid relations for a complex system would be an almost insurmountable task, the properties of set inclusion are shown to contain sufficient information to determine allowability or allow selection of the least difficult method for obtaining the necessary information. "Process P" is a formalization of the method of selection used by Freeny in his "design map." This process of selection is formalized and justified by theorems which, to this author's knowledge, are unique to this thesis. These theorems, relating the existence of an allowable specification set to the decomposability of the system, allowed the development of a systematic approach for the selection of system specifications. This method allows the designer to develop a system step by step, resolving all inconsistencies as they occur. The primary advantage of this method over the previous design map is the savings in time and effort since checking of nonmaximal specification sets is possible with the new method.

Next, the relation graph is defined. The use of a linear graph to represent a system has been discussed by several authors, but the lack of a one-to-one correspondence between the elements of the graph and the parameters of the system prevented any useful contributions in this area. An original theorem, proving the existence of a relation graph for every system is given. However, the results of the investigation are disappointing since an exact correlation between the relations of the system and the circuits of the graph cannot be obtained in the general case. During the early investigation performed for this thesis, the author studied the various forms of the relation graph which could be obtained from an electrical circuit. These studies indicated that a typical pattern of construction might be formed which would allow generation of a relation graph with a one-to-one correspondence between circuits and relations in all cases. The author feels that there is sufficient justification for continued work in this area. In the special cases in which the relation graph defines a unique system, it provides the simplest and easiest method of parameter selection yet developed. Also, a complete listing of all allowable specification sets is readily obtainable.

Several examples of the different methods of parameter selection are given in the fourth chapter. These examples point out many of the less obvious limitations of the various methods and provides the reader with a better intuitive "feel" for the problem of parameter selection. For example, the discussion of the design map shows that several maps are required for each system to gain complete information. Also, it is shown that the relation graph lacks the required correspondence between circuits and relations in many cases.

The application of the material in this thesis requires only that a set of defining relations for the system be known. The form of the relations, whether linear or nonlinear, is immaterial.

The author feels that many design engineers lack a fundamental knowledge of systems of relations and as a result, tend to write and solve equations in a haphazard manner without full knowledge of the correct procedure which will allow a solution in a minimum number of steps. Also, many trivial problems are thought to be complex or unsolvable at first investigation since the exact information needed for obtaining a solution may be present but unknown to the designer.

Obviously, parameter selection concerns only a very small portion of the over-all design problem. Several areas worthy of continued study are indicated in the following paragraphs.

First, it would be desirable to develop the theory using matrices. Since matrix theory is almost universally used in present day circuit theory, the problem of parameter selection might well be simplified using this medium. As an example of this application, consider T2-3. If a matrix were formed, having a row for each parameter and a column for each relation, a condition relating the rank of the matrix to the decomposability of the system might be proven. This would then allow a check for decomposability to be performed by a knowledge of the rank of the matrix.

Next, to continue the integration of design theory and circuit theory, an investigation of possible applications of the relation graph and hopefully, a way to circumvent the present problem could be made. This problem has such great possibilities if solved that it is deserving of future study.

One further problem which is perhaps the most important is the extension of parameter selection to the case where the ranges of individual parameters have been restricted. When specifications are given in this manner (and they frequently are), more than the usual number of parameters can be specified. For example, if the correct values happened to be selected, <u>all</u> of the parameters in the relation $E = IR$ could be specified, whereas only two can be specified in the general case. The restriction of the range of the parameters, in addition to allowing more parameters to be selected, would allow optimization techniques to be developed. This development, although a long problem, appears to be solvable and would be extremely valuable.

APPENDIX A

SET THEORY

The fundamentals of set theory concern three undefined concepts. These are:

(1) element

(2) set

(3) "belongs to"

In general, sets are indicated by the use of capital letters or are enclosed in brackets. Lower case letters are commonly used to denote elements. However, since there are sometimes sets of sets, care must be taken to ascertain the indicated concept.

The concept "belongs to" relates sets and elements. It is generally written in the manner $a \in A$, which is read, the element a belongs to the set A. The negation of this statement is indicated by a slash mark through the belongs to notation, $a \notin A$.

The slash through a symbol is a general notation of negation. Some fundamental definitions follow:

DA-1: A set A is a subset of a set B if all the elements of A are also elements of B. This is written $A \subseteq B$.

DA-2: If $A \subseteq B$ and $B \subseteq A$, then set A and set B are said to be equal and are written as $A = B$.

DA-3: If $A \subseteq B$ and $A \neq B$, A is a proper subset of B and is written $A \subset B$.

DA-4: The set which consists of no elements is the empty set. It is considered to be a subset of every set.

DA-5: Given two sets A and B, the set C consisting of the elements X such that

(1) $X \in A$

or

(2) $X \in B$

is the union of A and B and is written $C = A \cup B$.

DA-6: Given two sets A and B, the set C consisting of the elements X such that

(1) $X \in A$

and

(2) $X \in B$

is called the intersection of A and B and is written $A \cap B$.

DA-7: The number of distinct elements in a set is referred to as the cardinality of the set. This, in notation form, is written $\sigma(A)$.

Additional symbols used in this thesis are defined in the following list:

A = indexing set

P = parameter

π_n = product set of parameter

$\overline{\pi}_n$ = scalar product set

ϕ = relation

$\{\phi_i\}$ = set of i relations

$\{P_n\}$ = set of n parameters involved in relation ϕ

$\{P_{ni}\}$ = i sets of n parameters involved in the relations ϕ_i

$\Pi_n(\phi_i)$ = natural points of ϕ_i on P_{ni}

S_{ni} = system of n parameter and i relations

APPENDIX B

GRAPH THEORY

Graph theory, as used by the members of the electrical engineering profession, concerns the use of a geometric figure to represent a physical circuit. The following definitions will provide a satisfactory basis to interpret the material in the thesis.

DB-1: A line segment together with its distinct end points is an edge. Edge and element are synonymous.

DB-2: A vertex is an end point at an edge.

DB-3: A linear graph is a collection of edges, no two of which have a point in common that is not a vertex.

DB-4: A subgraph is a subset of the edges of the graph.

DB-5: A vertex and an edge are incident with each other if the vertex is an endpoint of the edge.

DB-6: The degree of a vertex is the number of edges incident at the vertex.

DB-7: If the edges of a graph can be ordered such that each vertex in common with the preceding edge and the other vertex in common with the succeeding edge (each edge appearing only once), the sequence is an edge train.

DB-8: If the degree of each nonterminal vertex of an edge train is 2 and each terminal vertex is 1, the edge train is a path.

DB-9: If the terminal vertices of an edge train coincide and all vertices are of degree 2, the edge train is a circuit.

DB-10: A graph is connected if there exists a path between any two vertices of the graph.

DB-11: A tree of a graph is a maximal connected subgraph containing all the vertices of the graph.

APPENDIX C

TREE LISTING

It might sometimes be desirable to obtain a listing of all sets of parameters which will give positive results on the design map. If the system is represented by a linear graph, the trees of the graph, with few exceptions, are those sets of parameters which map through the design map. A method for obtaining a listing of the trees of a graph was suggested by Paul.(4). This method is explained in the following discussion.

Given a linear graph with V vertices and E edges, form a listing of any V - 1 vertex cut sets. (A vertex cut set corresponding to a particular vertex is just a listing of the edges incident to the vertex.) Obtain the Cauchy product of the cut sets and the result is a list of all trees of the graph. (The Cauchy product is explained in the example.)

Consider the system shown in Figure C-1.

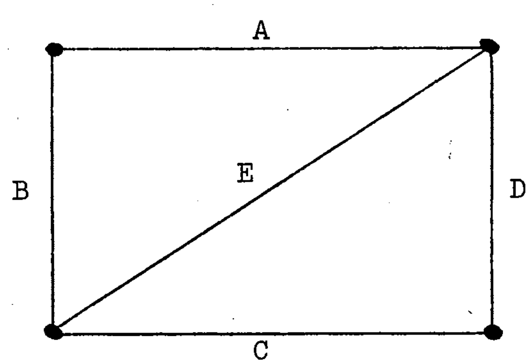


Figure C-1. Linear Graph

The vertex cut sets are (A, E, D), (A, B), (B, C, E), and (C, D). Select V - 1 of these sets (A, B), (B, C, E), (C, D) and arrange them in the following manner:

$$(A + B)(B + C + E)(C + D).$$

Perform ordinary multiplication on the line shown, obtaining

$$(AB + AC + AE + BB + BC + BE)(C + D)$$

and

$$ABC + ACC + AEC + BBC + BCC + BEC + ABD + ACD + AED$$
$$+ BBD + BCD + BED.$$

Remove all products which contain the same edge twice and all products which appear twice. The result is

$$ABC + AEC + BEC + ABD + ACD + AED + BCD + BED.$$

Each of the products in this listing is a tree of the graph and each is an allowable specification set.

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