AN ECONOMIC ANALYSIS OF SELECTED PRICE PREDICTION MODELS

By

JAMES EZRA OSBORN

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Oklahoma State University
Stillwater, Oklahoma
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PREDICTION MODELS

Thesis Approved:

Odell L. Walker
Thesis Adviser

Nellis A. Briscoe

[illegible]

David L. Weeks

Arnold M. Sharp

Dean of the Graduate School

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CHAPTER I

INTRODUCTION

General Problem

The manager of an agricultural firm is faced with many decisions. In making these decisions, the manager must consider (1) achieving an objective, usually maximizing profits over time, (2) means for achieving the objective, (3) technical relationships, (4) market prices, (5) a rationale for organizing the use of resources, and (6) institutional arrangements. Generally, the manager combines the above elements into a plan and puts the plan into action. The burden of the outcome is assumed by the manager.

Plans with a goal of maximum profits will consider (1) the over-all quantities of resources to be combined, (2) the kinds and quantities of products, and (3) the production techniques and resource combinations to be used to produce the appropriate amount of each product. The optimum size of farm will be determined by the above interrelated considerations. A consistent plan that will maximize net revenue must be formulated for the entire farm. This can be accomplished when the manager enjoys perfect knowledge with which to formulate production plans.

Perfect knowledge exists when the manager is completely informed about (1) the production techniques, (2) input-output relationships, and (3) supply of factors and demand for products. The expected values would then correspond to the realized values. With perfect knowledge, management consists of adjusting resources for relative changes in prices, technology, weather and disease conditions. These conditions would be known in advance of their occurrence. Therefore, to maximize profit, the manager would need only to make appropriate adjustments in his production plans.

However, perfect knowledge does not exist in the real world. The usual managerial environment is characterized by imperfect knowledge; i.e., changes in prices, technology, weather and disease conditions do not occur in a regular or predictable manner. Therefore, the conditions for maximum profit are more difficult to determine. Some of the factors that lead to the difficulty are:

1. Managers obtain predictions of input-output relations and factor and product prices. Predictions are based on the manager's past experience, experiences of other managers and the results of research. Although managers base their predictions on the best available information, errors are made in predictions. With this information, managers will satisfy a priori the marginal conditions in their production plans. However, the a priori marginal conditions may extend production plans beyond or short of the level indicated by the a posteriori marginal conditions. Therefore, realized net revenue may not be maximized by the a priori marginal conditions.
2. Since the values are expected values, i.e., the values are not known with certainty, the manager may discount the values for uncertainty. The greater the uncertainty, the greater will be the discount. As a consequence, the manager will extend his

production plans to the point where maximum expected net revenue occurs even though this may be short of realized net revenue.

3. If the manager has goals other than profit maximization, such as security or growth of the firm, it is unlikely that he will fulfill the marginal conditions for maximum net revenue. With a different goal, he will derive another set of conditions which may or may not coincide with the expected or realized maximum net revenue.

Specific Problem

Normally at the time that production plans are formulated, and certainly before production plans are executed, factor prices are known with certainty. Therfore, the manager would need only adjust the production plans for given factor prices.

Physical laws that determine the input-output relationships can be determined with a reasonable degree of certainty. After discovery, the input-output relationships may undergo rather gradual changes where direction may be predicted.

Unlike factor prices, knowledge about product prices varies from near certainty to almost complete uncertainty. That is, product prices are known with various degrees of certainty before production plans are executed. Some product prices are still uncertain when the product is produced. The degree of product-price uncertainty has been decreased to some extent by institutional arrangements and market information. However, product prices are determined by a baffling combination of sociological, psychological, economic and cultural factors. Thus, present research techniques have been only slightly applicable to price uncertainty.

This study will investigate the effects of product-price uncertainty on production plans. Models will be developed to determine net revenue from predicted product prices. These models will be used to determine how critical price uncertainty is for selected enterprises. The physical laws and factor prices are assumed to be known with certainty.

Review of Literature

Research on uncertainty is made difficult by two empirical problems: the actual goals of decision makers under uncertainty and data to measure the nature of the uncertainty. In order to study uncertainty, several simplifying assumptions must be made.

In almost all empirical studies, it is assumed that managers maximize expected profits. A correspondence between utility and profits is assumed thereby. For example, Nerlove (1) tested the hypothesis that farmers revise the price they expect to prevail in the subsequent period in proportion to the error they made in predicting profit-maximizing price for the present period. He used an adaptive expectations model to predict price which was a weighted average of past prices, observed over 1909 to 1932 for corn, cotton, and wheat. However, Nerlove did not compare the results of the adaptive expectations model to the results of other models in terms of actual new revenue streams.

Darcovich and Heady (2) formulated 14 different expectation models in a manner similar to Nerlove. These models were tested with a series of data for efficiency of forecasting price and production outcomes. The results indicated that a weighted moving-average model was highly efficient with an imperfect degree of positive autocorrelation. However, there was no explicit analysis of the effect of the different models upon net revenue streams, since factor prices were ignored.

Another type of study attempts to maximize expected utility. Friedmann and Savage (3) have suggested this type of study. They considered Von Neumann and Morgenstern's (4) formulation that a consumer unit would choose the alternative with the probability that would maximize expected utility. Then, they observed that low-income consumer units buy, or are willing to buy, insurance and/or lottery tickets and that lotteries generally have multiple prizes. From these two considerations, they made a hypothesis about how a consumer unit chooses among alternatives. They hypothesized that the function describing the utility of money income has diminishing, increasing and diminishing, marginal utility, in that sequence. However, Friedmann and Savage did not test this hypothesis empirically.

In order to either maximize expected net revenue or expected utility, as the previous list of works has done, the probability distribution of prices must either be known or estimated. However, managers may recognize many possible outcomes (prices) which cannot be meaningfully described in probability terms. Major contributions have been made in this area by Von Neumann and Morgenstern (5). They have developed game theoretic models for this type of problem. The use of these models has been more frequent in the evaluation of business behavior under noncompetitive conditions than under uncertainty.

Studies have been made to evaluate business behavior under uncertain competitive conditions. Walker, et al. (6) have made a study for selected farming areas in Iowa. They have demonstrated how four game theoretic models can be applied to research data. However, they did not analyze the relation between the net revenue generated by the game theoretic models and the net revenue generated by other models.

Hildreth (7) presented an operational model for implementing a minimax decision rule within the context of a single product and a single input. It is assumed that the firm knows with certainty the extremities of a range of prices, as well as its production possibilities. The objective is to make the maximum possible loss as small as possible.

The review of literature indicates that the research on uncertainty has been one of the following types: (1) predictive with one or many models; or (2) nonpredictive. Generally the predictive studies analyze the relation between predicted prices and actual prices, whereas the nonpredictive studies analyze what managers could have done with respect to production.

Objectives

Farm managers use various methods to formulate production plans. These methods vary from rules of choice to optimizing principles. The rules of choice include habits, customs, etc., which are difficult to measure empirically. Therefore, this study will utilize the optimizing principles. Four models are developed in this study which differ in one or more of the following characteristics: (1) Information assumed available; (2) information required; and/or (3) the optimizing principle.

The outcome for each model is analyzed with respect to the average, variance and range of net revenue resulting from application to selected enterprises.

Given the models to be used, the following objectives for the study were chosen.

1. To study the importance of price uncertainty in making production decisions.
2. To demonstrate the use of alternative price prediction and decision models in planning production under price uncertainty.
3. To evaluate selected price prediction and decision models with respect to properties of income distribution over time for selected enterprises.

CHAPTER II

THEORETICAL FRAMEWORK

Models are developed in Chapter II that will enable the study to achieve the objectives stated in Chapter I. Each model will be discussed with respect to assumptions, factor-use levels, product levels, and net revenue. The discussion for each model is presented in the sequence that was given above.

The determinants of optimal factor-use level will be defined as:

(a) Product price,

(b) Factor prices,

(c) Production function,

(d) Manager's knowledge about (a), (b), and (c), and

(e) Manager's goal.

With the assumptions that the factor prices and production functions are known with certainty and the manager's goal can always be measured through net revenue, techniques will be presented in this chapter to predict product prices and determine the factor-use levels for maximum net revenue.

A production function of the following form will be assumed throughout this study:

$$(2.1) \qquad Y = \beta_o X^{\beta_1} V^{\beta_2} \qquad \beta_1 > 0;\ \beta_2 > 0;\ 0 < \beta_1 + \beta_2 < 1.$$

It is a Cobb-Douglas production function where Y represents output and X and V represent two inputs. Net revenue (NR) will be defined as the returns to factors of production other than X and V. More explicitly, net revenue is

(2.2) $NR = \text{Total Revenue} - \text{Specified Variable Costs}$

or

(2.3) $NR = PY - RX - KV = P\beta_o X^{\beta_1} V^{\beta_2} - RX - KV$

where P represents the price of the product (Y) and R and K represent the prices of inputs X and V, respectively.

If a manager's goals are to maximize net revenue, he will use more of the resources (X and V) as long as the resulting additions to net revenue for both resources are greater than zero.

Four models for determining net revenue, factor-use level and product level will be developed in the remaining portion of this chapter.

Model I

The well-known objective function of maximum net revenue with perfect knowledge will be called Model I. Perfect knowledge will imply that product price is known. Model I will be a norm model that will be used as the optimum. The other models will be compared to Model I.

Maximization of NR for time period t, as described here, is a mechanical process in which the production function is, as always, a restraint and prices are known.[1] Net revenue is maximum when X and V are used at levels for which

(2.4) $$\frac{\partial NR_{1t}}{\partial X} = P_{1t}\,\beta_o\,\beta_1\,X^{\beta_1 - 1}\,V^{\beta_2} - R_t = 0^{2/}$$

[1]Discounting for time will not be used in this study. Discounting is a procedure that reduces future dollars to a common time period for comparison. However, one objective of this study was to investigate the distribution of income over time. Therefore, the actual prices for each year were used in order to study the fluctuations in income from price changes.

[2]See Appendix A for the algebraic derivation of the equations used in this study.

and

$$\frac{\partial NR_{1t}}{\partial V} = P_{1t}\ \beta_o\ \beta_2\ X^{\beta_1}\ V^{\beta_2 - 1} - K_t = 0 \tag{2.5}$$[3]

The first subscript denotes the model being used. The second subscript denotes the time period. Simultaneous solution of equations (2.4) and (2.5) gives optimal factor-use levels ($\widetilde{X}_{1t}$ and $\widetilde{V}_{1t}$) for the time period t as

$$\widetilde{X}_{1t} = \left\{ \frac{\beta_1}{R_t} \left[P_{1t}\ \beta_o \left(\frac{\beta_2}{K_t} \right)^{\beta_2} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{1-\beta_2}{1-\beta}} \tag{2.6}$$

and

$$\widetilde{V}_{1t} = \left\{ \frac{\beta_2}{K_t} \left[P_{1t}\ \beta_o \left(\frac{\beta_1}{R_t} \right)^{\beta_1} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{1-\beta_1}{1-B}} \tag{2.7}$$

where $\beta = \beta_1 + \beta_2$. The optimal product level ($\widetilde{Y}_{1t}$) is derived by substituting $\widetilde{X}_{1t}$ and $\widetilde{V}_{1t}$ into equation (2.1) which is

$$\widetilde{Y}_{1t} = \beta_o\ \widetilde{X}_{1t}^{\ \beta_1}\ \widetilde{V}_{1t}^{\ \beta_2} \tag{2.8}$$

Similarly, optimal net revenue ($\widetilde{NR}_{1t}$) is derived by substituting $\widetilde{X}_{1t}$, $\widetilde{V}_{1t}$ and $\widetilde{Y}_{1t}$ into equation (2.3) to give

$$\widetilde{NR}_{1t} = \left[1-\beta\right] \left[P_{1t}\ \beta_o \left(\frac{\beta_1}{R_t} \right)^{\beta_1} \left(\frac{\beta_2}{K_t} \right)^{\beta_2} \right]^{\frac{1}{1-\beta}} \tag{2.9}$$

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[3]Equations (2.4) and (2.5) are necessary conditions for maximum net revenue. Since the production function is regularly concave from below, sufficiency is established.

Model II

The problem of product price uncertainty may be approached by several naive models. One set of naive models assumes that the past will continue into the future. Model II is a member of this set. It assumes that this year's price will exist next year.

The assumptions of Model II differ in only one consideration from Model I. Model II does not assume perfect knowledge about product prices. Product price for time period t is predicted. The expected price is obtained by lagging product price one year which is given by

$$P_{2t} = P_{t-1} \qquad t = 0, 1, \ldots, n. \tag{2.10}$$

The formulas analogous to equations (2.6) through (2.9) can be derived by substitution of P_{2t}, $\widetilde{X}_{2t}$, $\widetilde{V}_{2t}$, $\widetilde{Y}_{2t}$, and $\widetilde{NR}_{2t}$ for P_{1t}, $\widetilde{X}_{1t}$, $\widetilde{V}_{1t}$, $\widetilde{Y}_{1t}$, and $\widetilde{NR}_{1t}$. The equations are (2.11) and (2.12), optimum factor level; (2.13), optimum output; and (2.14), optimum net revenue.

Model III

Farmers may revise their expected price for the coming year in proportion to the error which they made in predicting price for the present period. In most cases, more recent prices would have more weight on expected price than past prices. Therefore, a product-price prediction model based on a declining weight system seems appropriate. Model III is such a model.

The assumptions of Model III are identical with those of Model II. Model III differs from Model II only in terms of predicted price. Model III uses Nerlove's adaptive expectation model as a predicted price for decision making. The expected price for any year is given by

$$P_{3t} = \sum_{i=0}^{m} \gamma(1-\gamma)^i \, P_{t-1-i} \qquad t = 0, 1, \ldots, n \quad i = 0, 1, \ldots, m \leq n-1 \tag{2.15}$$

Equation (2.15) shows P_{3t} to be a weighted average of the past values of P_t. The subscript t-1-i indicates that the price in period t is replaced by the price in year t-1-i weighted by $\gamma(1-\gamma)^i$. If t-1-i is negative, the subscript has no meaning; the price is beyond the period of observation. The weights decline as one goes from the present period to some previous period. The sum of the weights for any gamma value is bounded by unity since $0 \leq \gamma \leq 1$. However, in order to limit the necessary price observations, the upper limit m was restricted to be only large enough to ensure that this sum exceed 0.95. The upper limit on i was thus determined by the restriction,

$$0.95 \leq \sum_{i=0}^{m} \gamma(1-\gamma)^i \leq 1 \tag{2.16}$$

Gamma (γ) is determined by the restriction for a minimal value for Z where Z is given by

$$Z = \sum_{t=0}^{n} (\widetilde{NR}_{1t} - \widetilde{NR}_{3t})^2 \tag{2.17}$$

The net revenue for Model III ($\widetilde{NR}_{3t}$) is determined with P_{3t} which is given by (2.15). Thus, the coefficient of price expectation was determined by that adaptation to past price trends which would minimize the deviations of this model's net revenue stream from profit-maximizing revenues under certainty. Time period n is not the current production period.

Equations for optimum factor use (2.18) and (2.19), optimum output (2.20), and optimum net revenue (2.21) are analogus to equations (2.6) through (2.9). The substitution of P_{3t}, $\widetilde{X}_{3t}$, $\widetilde{V}_{3t}$, $\widetilde{Y}_{3t}$ and $\widetilde{NR}_{3t}$ for P_{1t}, $\widetilde{X}_{1t}$, $\widetilde{V}_{1t}$, $\widetilde{Y}_{1t}$ and $\widetilde{NR}_{1t}$ will give the needed equations.

Model IV

Uncertain product prices cause fluctuations in net revenue and fluctuations in net revenue may mean losses in potential net revenue to farmers, given imperfect knowledge for planning production. Therefore, farmers with strong reasons for risk aversion may prefer a minimax loss to an ordinary loss in net revenue. That is, some farmers may use a strategy designed to avoid high real-income losses or losses in income opportunities.

Assuming a minimax objective function, Hildreth (7, p. 1437) has derived conditions that will minimize the maximum loss in net revenue for a firm with a single input production process. However, a more general derivation can be obtained with the aid of a graph.[4] This derivation will be called Model IV.

Under the assumptions of Model I, a marginal cost curve can be derived for the product. Therefore, the manager can organize optimally because price is known. Now, assume that product price is unknown at decision time. However, assume that the manager knows the extremities of the range of past prices. Let the upper and lower extremities be denoted by P^* and P_*, respectively, in Figure 1.

Assume that the manager uses P, between P^* and P_*, to make production decisions. The output that would correspond to P is Y, somewhere between Y^* and Y_*. However, if P_* occurs, the manager would realize a loss in net revenue. The loss can be derived as follows: Production plans should have been developed with respect to P_* with a net revenue

[4]A derivation similar to the one presented was developed by Reutlinger (11).

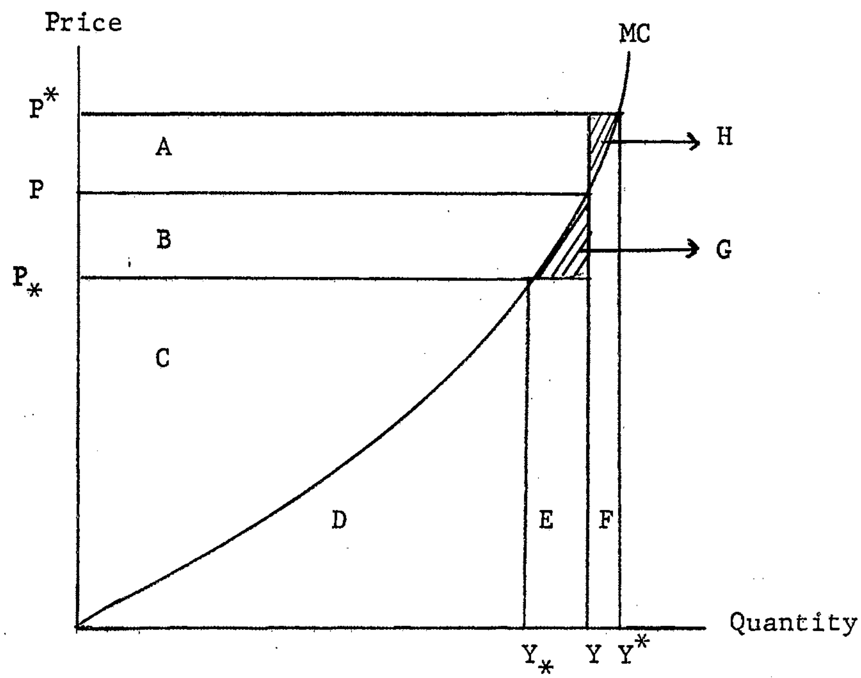

Figure 1. Losses Due to Forecasting.

of C. However, since the plans were developed on P, the net revenue is

(2.22) $$NR = C + D + E - (D+E+G) = C - G.$$

The loss (L) in net revenue is seen to be

(2.23) $$L = C - (C-G) = G.$$

Likewise, the loss in net revenue if P^* occurs is seen to be

(2.24) $$L = A + B + C + H - (A+B+C+D+E+G-D-E-G) = H.$$[5]

Given the extremities of the range of past prices and the losses corresponding to each extremity, the maximum loss can be made as small as possible by determining some level of product $(\hat{Y})$ such that the two losses are equal. If this condition were not true, another level of product would result in a greater possible loss in net revenue than $\hat{Y}$. For a marginal cost curve that is based on a Cobb-Douglas production

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[5] H is actually an "opportunity loss," whereas G represents an actual excess of expenses over receipts for the increment of production from Y_* to Y. In this study, each type of "loss" is given equal weight. Other formulations assigning different weights may be relevant.

function and increases at an increasing rate, $\hat{Y}$ will be less than the output ($\bar{Y}$) corresponding to the average of P^* and P_*. This situation is shown in Figure 2. For a marginal cost curve that is based on a Cobb-Douglas production function and increases at a decreasing rate, $\hat{Y}$ will be greater than $\bar{Y}$. In Figure 1, the loss in net revenue can be minimized by determining minimax product ($\hat{Y}$) such that

$$\text{Area } G = \text{Area } H. \tag{2.25}$$

In functional notation, $\hat{Y}$ can be determined by

$$L(P_*, \hat{Y}) = L(P^*, \hat{Y}), \tag{2.26}$$

if and only if the marginal cost curve is monotonic. Since $\hat{Y} = f(\hat{X}, \hat{V})$, (2.26) can be written as

$$L[P_*, f(\hat{X}, \hat{V})] = L[P^*, f(\hat{X}, \hat{V})]. \tag{2.27}$$

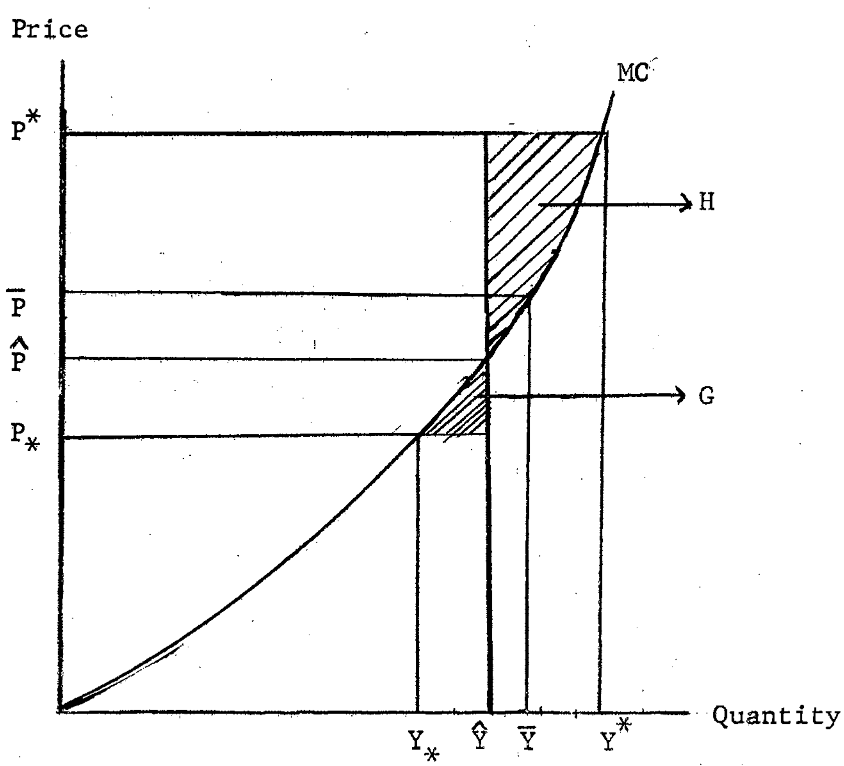

Figure 2. Relationship of $\bar{P}$ and $\hat{P}$ for Marginal Cost Curve Concave from Above.

By appropriate substitution, it can be shown that the loss is measured by

$$(2.28)\quad L[P,\ f(X,\ V)] = [1-\beta]\left[P\beta_c\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta}} - P\,\beta_o X^{\beta_1} V^{\beta_2} - RX - KV$$

Substitution of (2.28) into (2.27) gives the factor use level ($\hat{X}_{4t}$) of X in terms of V for time period t as

$$(2.29)\quad \hat{X}_{4t} = \left\{[1-\beta]\left[\beta_o^{\beta}\left(\frac{\beta_1}{R_t}\right)^{\beta_1}\left(\frac{\beta_2}{K_t}\right)^{\beta_2}\right]^{\frac{1}{1-\beta}} \frac{\left[P_t^{*\frac{1}{1-\beta}} - P_{*t}^{\frac{1}{1-\beta}}\right]}{\left[P_t^{*} - P_{*t}\right]}\right\}^{\frac{1}{\beta_1}} \hat{V}_t^{-\frac{\beta_2}{\beta 1}}$$

By analogy, the factor use ($\hat{V}_t$) of V in terms of X for time period t is

$$(2.30)\quad \hat{V}_{4t} = \left\{[1-\beta]\left[\beta_o^{\beta}\left(\frac{\beta_1}{R_t}\right)^{\beta_1}\left(\frac{\beta_2}{K_t}\right)^{\beta_2}\right]^{\frac{1}{1-\beta}} \frac{\left[P_t^{*\frac{1}{1-\beta}} - P_{*t}^{\frac{1}{1-\beta}}\right]}{\left[P_t^{*} - P_{*t}\right]}\right\}^{\frac{1}{\beta_2}} \hat{X}_t^{-\frac{\beta_1}{\beta_2}}$$

The levels of X and V can be determined by introducing the least-cost combination of X and V which is defined by the following necessary condition for the function used.

$$(2.31)\quad \frac{R}{\beta_o\beta_1 X^{\beta_1 - 1} V^{\beta_2}} = \frac{K}{\beta_o\beta_2 X^{\beta_1} V^{\beta_2 - 1}}$$

By solving (2.31) in terms of V and substituting the result into (2.29), the factor-use level ($\hat{X}_t$) of X for time period t can be found to be

$$(2.32)\quad \hat{X}_{4t} = \left\{[1-\beta]\,\frac{\left[P_t^{*\frac{1}{1-\beta}} - P_{*t}^{\frac{1}{1-\beta}}\right]}{\left[P_t^{*} - P_{*t}\right]}\right\}^{\frac{1}{\beta}} \left\{\beta_o\left(\frac{\beta_1}{R_t}\right)^{1-\beta_2}\left(\frac{\beta_2}{K_t}\right)^{\beta_2}\right\}^{\frac{1}{1-\beta}}$$

By analogy, the factor-use level $(\hat{V}_t)$ of V for time period t is seen to be

$$(2.33) \qquad \hat{V}_{4t} = \left\{ \frac{[1-\beta]\left[P_t^{*\frac{1}{1-\beta}} - P_{*t}^{\frac{1}{1-\beta}}\right]}{\left[P_t^{*} - P_{*t}\right]} \right\}^{\frac{1}{\beta}} \left\{ \beta_o \left(\frac{\beta_1}{R_t}\right)^{\beta_1} \left(\frac{\beta_2}{K_t}\right)^{1-\beta_1} \right\}^{\frac{1}{1-\beta}}$$

The minimax loss for time period t can be found by substituting (2.32) and (2.33) into (2.28).

The range of years that was used to discover P^* and P_* was determined in the following manner. The range was allowed to vary from two to fourteen years. A series of net revenue was computed for each range of years. The range that minimized the sum of squares given by

$$(2.34) \qquad \sum_{t=0}^{n} (\widetilde{NR}_{1t} - \widetilde{NR}_{4t})^2$$

was selected. $\widetilde{NR}_{4t}$ is the net revenue for output $\hat{Y}_{4t}$ with price P_{1t}. Thus, the price extremities were chosen from a subset of the observation period during which the deviations of this model's realized net revenue stream from profit maximizing revenues under certainty would have been minimized. Time period n is not the current production period.

The Cost of Uncertainty from Errors in Forecasting Price

The phrase "cost of uncertainty from errors in forecasting price" is called "cost of uncertainty" in the following discussion. The loss in net revenue if prices are not known is found by subtracting the realized net revenue obtained with a given model from the optimal net revenue of Model I. The loss is given by

$$(2.35) \qquad C_t = \widetilde{NR}_{1t} - NR_{it} \qquad i = 2, 3, 4$$

where C_t and NR_{it} represent the loss and the realized net revenue for

Model i in time period t, respectively. The realized net revenue is given by

$$NR_{it} = P_{1t}\, Y_{it} - R_t\, X_{it} - K_t\, V_{it} \qquad i = 2,\ 3,\ 4 \tag{2.36}$$

where Y_{it}, X_{it}, and V_{it} are the product and inputs predicted by the models. The total cost of uncertainty for Model i is seen to be

$$C = \sum_{t=0}^{n} (\widetilde{NR}_{1t} - NR_{it}) \tag{2.37}$$

Substitution of (2.36) into (2.37) gives

$$C = \sum_{t=0}^{n} (\widetilde{NR}_{1t} - P_{1t}\, Y_{it} + R_t\, X_{it} + K_t\, V_{it}) \tag{2.38}$$

Substitution of $P_{1t}\, \widetilde{Y}_{1t} - R_t\, \widetilde{X}_{1t} - K_t\, \widetilde{V}_{1t}$ for $\widetilde{NR}_{1t}$ in (2.38) gives the total cost of uncertainty as

$$C = \sum_{t=0}^{n} [P_{1t}(\widetilde{Y}_{1t} - Y_{it}) - R_t(\widetilde{X}_{1t} - X_{it}) - K_t(\widetilde{V}_{1t} - V_{it})]. \tag{2.39}$$

CHAPTER III

DATA AND PROCEDURES

The price data, production functions and procedures which were used to compute net revenue are discussed in this chapter. The criteria for evaluating the models are also presented.

Data

The enterprises used in this study are corn, alfalfa, steers and dairy cattle. The enterprises were chosen for two reasons: (1) It would seem desirable to strike some balance between livestock and crops in a study on uncertainty. Therefore, two crops and two livestock enterprises were selected, and (2) the four enterprises comprise a large portion of farmers' income.

Product prices were chosen for the period from 1934 to 1962. The input prices for alfalfa hay, dairy feed, milo, nitrogen, phosphorous, and potassium, were selected from 1949 to 1962. These periods were chosen in order to have an equal number of observations for each enterprise with a longer run of product prices which are assumed to be the sole source of uncertainty.

Input Prices

The source of nitrogen was assumed to be ammonium nitrate. Since nitrogen was used on corn, the average price paid by farmers in April was selected. Although the source of phosphorous chosen was 45 percent superphosphate, two different prices were observed. The phosphorous prices were the average prices paid by farmers in April for planting corn and in September for planting alfalfa. The price of potassium was an estimate derived from the price of 0-20-20 fertilizer using the customary NPK classification. Since the potassium was used for alfalfa, the price selected was the average price paid by farmers in September.

Since alfalfa hay was used in the steer and dairy enterprises, the annual average price received by farmers for No. 1 alfalfa hay in Kansas City was chosen. The milo prices were the annual average prices received by farmers. Also, milo was used in the steer enterprise. The dairy enterprise used 16 percent dairy feed for which annual average prices paid by farmers were selected.

Product Prices

The product prices used were average annual prices received by farmers. For corn and milk, U. S. price averages were used; for alfalfa, prices for No. 1 alfalfa hay in Kansas City were used. The annual average prices received by farmers in Chicago for choice steers was the source for the price of steers.

The basic price series used throughout this study and the original sources of the data are summarized in Appendix C. The data are not deflated. Since the tests are for farmer production responses from one production period to another period to actual prices paid and received, the decisions of managers are independent of real prices.

Production Functions

Production functions were selected for the four enterprises. A Cobb-Douglas type of production function was chosen for two reasons: (1) The majority of empirical estimates are of this form, and (2) it has good working characteristics. Each of the production functions is discussed with respect to origin, units of input and statistical characteristics.

Corn

Heady, et al. (8, p. 304) have estimated, using data from an experiment in Western Iowa, the production function for corn to be

$$Y = 34.405\ X^{0.135}\ V^{0.077} \tag{3.1}$$

where Y, X, and V are bushels of corn, pounds of phosphorous (P_2O_5) and nitrogen per acre, respectively. Concentrated (45 percent) superphosphate and ammonium nitrate were the sources of phosphorous and nitrogen, respectively. The coefficient of determination (R^2) of 15.07 percent seems low, but its calculated t value is significant at the one percent level of probability. The calculated t value for β_1 is significant at the one percent level of probability. The calculated t value for β_2 is significant between the 10 and 20 percent level of probability.

Alfalfa

Based on data from an experiment in North-Central Iowa, Heady, et al. (8, p. 317) have estimated the production function for alfalfa to be

$$Y = 0.879\ X^{0.054}\ V^{0.131} \tag{3.2}$$

where Y, X, and V are tons of alfalfa, pounds of potassium (K_2O) and phosphorous per acre, respectively. Potassium chloride and concentrated superphosphate were the sources of potassium and phosphorous, respectively. The coefficient of determination is 53.71 percent. The computed t values for R^2, β_1, and β_2 are significant at the one percent level of probability.

Steers

Tefertiller (9, p. 23) has estimated the production function for steers to be

$$Y = 0.945\ X^{0.557}\ V^{0.218} \tag{3.3}$$

where Y, X, and V are pounds of grain, milo, and alfalfa hay, respectively. The coefficient of determination is 99.5 percent. The calculated t values for β_1 and β_2 are significant at the one percent level of probability.

Milk

Heady, et al. (10, p. 904) have estimated the production function of milk to be

$$Y = 15.749\ X^{0.276}\ V^{0.121}\ A^{0.366} \tag{3.4}$$

where Y, X, V, and A are the pounds of milk, concentrate and hay per cow for one lactation period and the ability of the cow to produce, respectively. The value of A was set at 10,000 pounds for the purposes of this study. The coefficient of determination is 73.02 percent. The computed t values for R^2, β_1, and β_2 are significant at the one percent level of probability.

The basic statistics for the four production functions are summarized in Table I.

TABLE I

VALUES OF R^2 AND REGRESSION COEFFICIENTS WITH THE COMPUTED "t" VALUES FOR β_1, β_2, AND β_3

Equation	R^2	β_o	β_1 Value	β_1 "t"	β_2 Value	β_2 "t"	β_3 Value	β_3 "t"
1	15.07*	34.405	0.135	2.85*	0.077	1.62**	-	-
2	53.71*	0.879	0.054	9.01*	0.131	4.29*	-	-
3	99.50***	0.945	0.557	15.40*	0.218	5.57*	-	-
4	73.02*	15.749	0.276	5.65*	0.121	2.84*	0.366	5.13*

*$P < 0.01$.
**$0.10 < P < 0.20$.
***t value was not available.

Computing Techniques

In order to reduce errors and to increase the rate of computation, the optimal formulas given in Chapter II were written in FORTRAN language. When all computations for all four models and enterprises were put into one program, the capacity of the IBM 1620 was exceeded. Therefore, the computations were divided into three programs.

The first program (Appendix B-I) was written to compute the results needed for the four enterprises of Model I. This program computes optimal levels of inputs (X1 and X2) and product (Y) with the resulting optimal net revenue (BNR).

In writing in FORTRAN, a statement cannot exceed 72 spaces. Therefore, many of the original equations were divided to comply with the requirements of FORTRAN. For example, equation (2.6) would require 81 spaces. In order to reduce the length of this statement, the last two exponents were computed in a previous statement. The results of these computations were called A1 and A2. The statement was reduced to 51 spaces by these operations.

The second program (Appendix B-II) computed all the results needed for Models II and III. The program computed predicted prices (EXP), optimal levels of inputs (X1 and X2) and product (Y) based on predicted prices, realized net revenue (BNRI) the net revenue received from Y based on actual prices, and predicted net revenue (ENR).

The second program was an iterative one. That is, since the net revenue curves for various gamma values are not known, then the net revenues for the period must be computed for selected gamma values. This can be done by (1) including the incremental changes in gamma in the program and/or (2) giving the incremental changes in gamma to the machine on cards or through the typewriter. Utilization of Sense Switches will enable the operator to use any combination of the above. However, the latter method with cards was selected for convenience and flexibility. The selected incremental changes in gamma were 0.1. Further, it may not be desired to type the results for all gamma values. If this is the situation, the program may be reduced by deleting some of the computations. The program was compiled two ways. First, a short program was compiled that would compare the optimal net revenue (from program one) with predicted optimal net revenue (ENR) for each gamma value. After each computation was completed, the gamma value and BENR (sum of squares between ENR and the net revenue from program one) were printed. Then, the gamma value which minimized the sum of squares was selected to be used in the long program. This program could be modified so that prices for many enterprises could be predicted with various gamma values.

The third program (Appendix B-III) was written to compute the necessary results for Model IV. The program selects the upper price (PH) and lower price (PL) for a specified range of years from two to twelve. Then, the program computes the inputs (X4 and X5), product (Y) and the price that is predicted (EP) by the minimax model. Finally, the net revenue is computed for the lower price (BPLR), the upper price (BPHR), and the known price (BNR).

Program three is also iterative with respect to the range of years. The method is similar to that used in program two. That is, there is a short and a long program. The short program compares the net revenue from Model I with the net revenue from Model IV for each range of years. The range with the smallest sum of squares between the net revenues is selected to be run through the long program to obtain all the corresponding inputs and product. As before, the program may be modified to give the predicted price of the minimax model which may be used in extension and outlook work. However, due to the complexity of the minimax price, its usefulness may be restricted in this type of work. Decision makers using the minimum price must understand and accept the objective for which the price is designed.

Method of Analysis

The models developed in Chapter II have been applied to the price data discussed in this chapter. The predicted prices will be compared to the actual prices with respect to variation and the degree of relation. Realized net revenue from the three prediction models will be compared with the net revenue from Model I which is assumed optimal. The comparison of net revenues will be made with respect to variation and average net revenue over time.

CHAPTER IV

PRESENTATION AND INTERPRETATION OF THE EMPIRICAL RESULTS

The estimating procedures outlined in Chapter III have been used to predict prices and to compute net revenues for the four enterprises. The results are presented and explained in this chapter and Chapter V.

Predicted Prices

The purposes of this section are to examine the nature of prices obtained from different models and to tentatively test the usefulness of the price prediction techniques.[1] Three price-prediction techniques are used. Their usefulness will be measured with respect to variance, coefficient of variation and correlation. Actual results of using prices, examined in the following section, is probably the most valuable test of usefulness.

A suggested price can be derived from Model IV after the upper and lower prices are determined. The suggested price is called a minimax price. The minimax price is not a predicted price. Production plans are not based on the minimax price. Rather, production plans are based on the price extremities. The minimax price, if used in Model I, will yield the same output as Model IV would yield with the price extremities. A

[1]See Appendix Tables D-I, D-II, D-III, and D-IV for the predicted prices.

good approximation for the minimax price is given by the mean of the upper and lower prices. However, from Chapter II, the mean will always be greater than the minimax price if the marginal cost curve is increasing at an increasing rate. The mean will always be less than the minimax price if the marginal cost curve is increasing at a decreasing rate.

Models III and IV reduced the variance and coefficient of variation of price for all enterprises (Table II). Since Model III uses a weighted average of past prices, it will not allow all of the extreme fluctuations in actual prices to influence the predicted prices. Likewise, since Model IV uses the extremities of a price range, it will not allow all of the extreme fluctuations in actual prices to influence the minimax price.

In particular Model IV reduced the coefficient of variation for alfalfa and milk by a substantial amount. The variance and coefficient of variation were reduced by Model III for all enterprises. The decreases in the variance and coefficient of variation for the other enterprises were not substantial.

The correlation coefficients indicated a strong relation between predicted and actual prices for corn (Table III). Also, the relation between the predicted prices was strong for corn. The variance and coefficient of variation for the actual and predicted prices of corn yield the same conclusion since their values for all the models are similar. The relation between actual and predicted prices for alfalfa is not as strong. Also, the relation between the predicted prices was weak for alfalfa. Except for Model IV, the relation between predicted and actual steer prices appeared to be strong. Model IV had the highest

TABLE II

STANDARD DEVIATIONS AND COEFFICIENTS OF VARIATION FOR ACTUAL AND PREDICTED PRICES BY MODELS

Model	Corn		Alfalfa		Steers		Milk	
	Std.Dev.	C.V.	Std.Dev.	C.V.	Std.Dev.	C.V.	Std.Dev.	C.V.
Model I	0.23	18.05	3.36	11.02	3.98	15.00	0.26	6.09
Model II	0.22	16.95	3.44	11.37	4.06	15.00	0.30	7.14
Model III	0.21	16.56	2.70	9.23	3.35	12.67	0.15	3.70
Model IV	0.22	15.25	1.10	3.64	3.05	11.19	0.15	3.58

TABLE III

CORRELATION COEFFICIENTS FOR ACTUAL AND PREDICTED PRICES BY ENTERPRISES

Models	Corn	Alfalfa	Steers	Milk
Models I and II	0.87	0.19	0.55	0.14
Models I and III	0.85	0.46	0.48	-0.08
Models I and IV	0.87	0.19	0.26	0.45
Models II and III	0.99	0.67	0.96	0.85
Models II and IV	0.82	0.11	0.63	-0.07
Models III and IV	0.83	0.40	0.80	0.06

correlated predicted prices with the actual price for milk. The other predicted prices for milk have low correlation coefficients with actual prices.

The price prediction techniques do reasonably well in predicting price for corn. For predicting over-all prices, Model III appears to be the best technique except for milk. However, the relation between the predicted prices and actual prices for steers, alfalfa, and milk is weak.

Net Revenue

The purpose of this section is to test and discuss the price-prediction techniques and decision models. The evaluation consists of a comparison of the characteristics, average, variance, and range of the net revenue distributions by enterprises and models.

Realized net revenue (realized net revenue will be called net revenue during the remainder of this discussion) refers to the net revenue from an enterprise when production plans are optimal with respect to predicted prices and actual prices are used to value output. That is, it is the net revenue from production plans based on predicted prices, when actual prices received for the product are assumed. The costs of uncertainty for the three price-prediction models are obtained by subtracting the respective net revenues from the net revenue of Model I.

Corn

The total cost of uncertainty varied from $1.94 for Model III to $4.91 for Model IV, which is an average cost of $0.13 and $0.35, respectively, per acre per year (Table IV). Average net revenue per year was least for Model IV and greatest for Model III. The flow of net revenue was more variable for Model IV than it was for the other models. Model III had the least-fluctuating flow of net revenue.

The major portion of the cost of uncertainty was realized from 1950 through 1952 for Models II and III. Since both models only considered the preceding period, the extreme fluctuations in actual price were

TABLE IV

REALIZED NET REVENUE AND COST OF UNCERTAINTY PER ACRE FROM CORN FOR THE FOUR SELECTED MODELS FROM 1949 TO 1962

	Model				Cost of Uncertainty		
Year	Model I	Model II	Model III[a]	Model IV[b]	Model II	Model III	Model IV
				- Dollars -			
1949	98.81	98.79	98.69	97.75	0.02	0.12	1.06
1950	126.88	126.06	126.01	126.65	0.82	0.87	0.23
1951	139.32	139.14	139.02	139.31	0.18	0.30	0.01
1952	123.75	123.58	123.64	123.52	0.17	0.11	0.23
1953	118.66	118.65	118.65	118.65	0.01	0.01	0.01
1954	112.64	112.62	112.63	112.64	0.02	0.01	0.00
1955	105.04	104.98	104.99	104.69	0.06	0.05	0.35
1956	92.12	91.92	91.93	91.29	0.83	0.19	0.83
1957	83.92	83.83	83.83	83.31	0.09	0.09	0.61
1958	83.09	83.09	83.09	82.76	0.00	0.00	0.33
1959	75.84	75.77	75.78	75.25	0.07	0.06	0.59
1960	68.52	68.44	68.44	67.90	0.08	0.08	0.62
1961	73.70	73.66	73.65	73.66	0.04	0.05	0.04
1962	73.87	73.87	73.87	73.87	0.00	0.00	0.00
				Total	2.39	1.94	4.91
				Sum of Squares	1.448	0.933	3.256

[a]The SSE was minimized when a gamma value of 0.9 was used.

[b]The SSE was minimized when a range of five years was used.

included in the predicted prices. Model IV considered a range of five years. Therefore, the cost of uncertainty was relatively uniform throughout the period for Model IV.

Models II and III reduced the variance and coefficient of variation relative to Model I (Table V). The reduction is small. However, it demonstrates that optimal organization does not imply minimum variation in net revenue.[2]

Therefore, complete knowledge about the prices of corn does not substantially effect the level of net returns. Operationally, this means that the manager may base production plans on predicted corn prices without substantially increasing the cost of uncertainty. "Predicted prices" refers to the prices predicted by the three price-prediction techniques, and will throughout this discussion.

Alfalfa

Model III and Model II had the greatest and least, respectively, cost of uncertainty (Table VI). The total cost of uncertainty was $0.93 for Model III and $0.90 for Model II which is an average of $0.07 and $0.06, respectively, per acre per year. Although Model II had the least total cost of uncertainty, the flow of net revenue from it fluctuated the most. The flow of net revenue fluctuated least for Model IV. Average net revenue per year was a minimum for Model III and a maximum for Model II.

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[2]The standard deviation and coefficient of variation are presented in Appendix Table E-IX. See Appendix Tables E-I and E-II for the levels of inputs (phosphorous and nitrogen) and bushels of corn, respectively, per acre. Note the changes in inputs and outputs from Model I.

TABLE V

COMPARISON OF THE NET REVENUE FOR THE FOUR MODELS FROM CORN USING FOUR SELECTED MEASURES[a]

Model	Standard Deviation	Coefficient of Variation	Minimum Cost of Uncertainty	Minimum Sum of Squares For the Cost of Uncertainty
Model I	3	3	1	1
Model II	2	2	3	3
Model III	1	1	2	2
Model IV	4	4	4	4

[a]The ranks are from low to high in each case.

TABLE VI

REALIZED NET REVENUE AND COST OF UNCERTAINTY PER ACRE FROM ALFALFA FOR THE FOUR SELECTED MODELS FROM 1949 TO 1962

	Model				Cost of Uncertainty		
Year	Model I	Model II	Model III[a]	Model IV[b]	Model II	Model III	Model IV
				- Dollars -			
1949	42.13	42.13	42.13	42.08	0.00	0.00	0.05
1950	46.83	46.79	46.78	46.83	0.04	0.05	0.00
1951	55.94	55.76	55.72	55.78	0.18	0.22	0.16
1952	58.48	58.46	58.44	58.29	0.02	0.04	0.19
1953	47.25	47.00	47.04	47.23	0.25	0.21	0.02
1954	43.18	43.14	43.14	43.08	0.04	0.04	0.10
1955	41.53	41.52	41.52	41.35	0.01	0.01	0.18
1956	50.83	50.66	50.64	50.83	0.17	0.19	0.00
1957	42.28	42.14	42.18	42.15	0.14	0.10	0.13
1958	41.29	41.29	41.29	41.25	0.00	0.00	0.04
1959	44.36	44.34	44.34	44.36	0.02	0.02	0.00
1960	45.21	45.21	45.20	45.21	0.00	0.01	0.00
1961	49.15	49.12	49.11	49.13	0.03	0.04	0.02
1962	48.66	48.66	48.66	48.64	0.00	0.00	0.02
				Total	0.90	0.93	0.91
				Sum of Squares	0.148	0.146	0.126

[a]The SSE was minimized when a gamma value of 0.9 was used.

[b]The SSE was minimized when a range of five years was used.

Again, for Models II and III the major portion of the cost of uncertainty was realized in the early years. The cost of uncertainty was relatively uniform for Model IV since it used a range of five years.

The variance and coefficient of variation was reduced by all three price-prediction models relative to Model I (Table VII). Again, optimal organization with known prices does not imply minimum variation in net revenue.[3]

In conclusion, it does not appear that complete knowledge of the actual price of alfalfa significantly affects the level of net revenue. Thus, a manager may make production plans based on predicted alfalfa prices without a substantial increase in the cost of uncertainty.

Steers

Average net revenue per steer was least for Model IV and greatest for Model III. The net revenue flows were extremely variable for all models (Table VIII). However, the flow of net revenue for Model IV was the most variable. Model II had the least variability in the flow of net revenue. The total cost of uncertainty was greatest for Model IV and least for Model III. Model IV had a total cost of uncertainty of $69.57, which represents a yearly average of $4.97 per steer. The total cost of uncertainty for Model III was $50.91, which gives an average of $3.64 per steer.

Due to the movements of steer prices, all of the models incurred the major portion of the cost of uncertainty from 1951 through 1953 and from

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[3]See Appendix Table E-IX for the standard deviation and coefficient of variation. The levels of inputs (potassium and phosphorous) and alfalfa per acre are given in Appendix Tables E-III and E-IV, respectively.

TABLE VII

COMPARISON OF THE NET REVENUE FOR THE FOUR MODELS FROM ALFALFA USING FOUR SELECTED MEASURES[a]

Model	Standard Deviation	Coefficient of Variation	Minimum Cost of Uncertainty	Minimum Sum of Squares For the Cost of Uncertainty
Model I	4	4	1	1
Model II	2	2	2	4
Model III	1	1	4	3
Model IV	3	3	3	2

[a]The ranks are from low to high in each case.

TABLE VIII

REALIZED NET REVENUE AND COST OF UNCERTAINTY PER STEER FOR THE FOUR SELECTED MODELS FROM 1949 TO 1962

	Model				Cost of Uncertainty		
Year	Model I	Model II	Model III[a]	Model IV[b]	Model II	Model III	Model IV
				- Dollars -			
1949	25.66	16.42	23.86	25.62	9.24	1.80	0.04
1950	47.87	43.45	44.16	47.41	0.42	3.71	0.46
1951	65.32	54.01	49.29	50.60	11.31	16.03	14.72
1952	25.94	24.35	25.94	25.37	1.59	0.00	0.57
1953	9.86	-8.55	-5.97	-7.36	18.41	15.83	17.22
1954	13.82	13.77	13.40	5.24	0.05	0.42	8.58
1955	14.82	14.29	14.45	4.45	0.53	0.37	10.37
1956	10.68	10.55	10.61	10.19	0.13	0.07	0.49
1957	21.52	20.91	20.69	21.49	0.61	0.83	0.03
1958	47.62	42.61	39.93	40.63	5.01	7.69	6.99
1959	48.80	48.72	46.71	45.47	0.08	2.09	3.33
1960	49.07	47.55	48.93	49.01	1.52	0.14	0.06
1961	25.80	23.35	23.96	21.25	2.45	1.84	4.55
1962	21.19	21.14	21.10	19.03	0.05	0.09	2.16
				Total	51.40	50.91	69.57
				Sum of Squares	589.021	592.471	780.460

[a]The SSE was minimized when a gamma value of 0.7 was used.

[b]The SSE was minimized when a range of three years was used.

1958 through 1959. The cost was distributed over a longer period of time for Model IV since it used the extremities in the previous three years.

Model III reduced the variance of net revenue relative to Model I (Table IX). However, all of the price prediction models increased the coefficient of variation relative to Model I. The increase was relatively large for Model IV.[4]

Therefore, incomplete knowledge about steer prices does substantially affect the level of net returns. That is, production plans based on predicted steer prices will substantially depress net revenue which will, in turn, increase the cost of uncertainty.

Milk

The total cost of uncertainty varied from $6.86 for Model IV to $18.21 for Model II, which is an average of $0.49 and $1.30, respectively, per cow per lactation period (Table X). The flow of net revenue from Model II was the most variable. Also, Model II had the lowest average net revenue per cow. Model IV had the least variability in the net revenue flow and the highest average net revenue per cow.

The major portion of the cost of uncertainty was incurred in the first half of the fourteen-year period for all three models. The lower cost of later years was probably due to the programs in milk marketing which tended to stabilize milk prices in the last one-half of the fourteen-year period.

Models II and III reduced the variance of net revenue relative to Model I. Also, Model II reduced the coefficient of variation (Table XI).

[4] Appendix Table E-IX gives the standard deviation and coefficient of variation. See Appendix Tables E-V and E-VI for levels of inputs (milo and alfalfa hay) and gains, respectively, per steer.

TABLE IX

COMPARISON OF THE NET REVENUE FOR THE FOUR MODELS FROM STEERS USING FOUR SELECTED MEASURES[a]

Model	Standard Deviation	Coefficient of Variation	Minimum Cost of Uncertainty	Minimum Sum of Squares For the Cost of Uncertainty
Model I	2	1	1	1
Model II	3	3	3	2
Model III	1	2	2	3
Model IV	4	4	4	4

[a]The ranks are from low to high in each case.

TABLE X

REALIZED NET REVENUE AND COST OF UNCERTAINTY PER DAIRY COW PER LACTATION PERIOD FOR THE FOUR SELECTED MODELS FROM 1949 to 1962

Year	Model: Model I	Model: Model II	Model: Model III[a]	Model: Model IV[b]	Cost of Uncertainty: Model II	Cost of Uncertainty: Model III	Cost of Uncertainty: Model IV
				- Dollars -			
1949	302.80	294.07	301.48	302.74	8.73	1.32	0.06
1950	285.70	285.76	285.43	285.61	0.03	0.36	0.18
1951	340.84	336.46	337.01	340.49	4.38	3.83	0.35
1952	370.55	369.92	366.74	368.62	0.63	3.81	1.93
1953	335.13	332.44	334.95	335.09	2.69	0.18	0.04
1954	295.00	293.78	293.84	293.43	1.22	1.16	1.57
1955	311.96	311.94	311.91	310.63	0.02	0.05	1.33
1956	312.45	312.28	312.21	311.75	0.17	0.24	0.70
1957	337.00	336.95	336.49	336.60	0.05	0.51	0.40
1958	327.00	326.93	326.88	327.00	0.07	0.12	0.00
1959	325.22	325.20	324.98	325.17	0.02	0.24	0.05
1960	336.42	336.36	335.94	336.25	0.06	0.48	0.17
1961	328.95	328.95	328.68	328.93	0.00	0.27	0.02
1962	308.98	308.84	308.98	308.92	0.14	0.00	0.06
				Total	18.21	12.57	6.86
				Sum of Squares	104.580	33.130	8.804

[a]The SSE was minimized when a gamma value of 0.5 was used.

[b]The SSE was minimized when a range of five years was used.

TABLE XI

COMPARISON OF THE NET REVENUE FOR THE FOUR MODELS FROM MILK USING FOUR SELECTED MEASURES[a]

Model	Standard Deviation	Coefficient of Variation	Minimum Cost of Uncertainty	Minimum Sum of Squares For the Cost of Uncertainty
Model I	2	3	1	1
Model II	4	4	4	4
Model III	1	1	3	3
Model IV	3	2	2	2

[a]The ranks are from low to high in each case.

Although Model IV had the least total cost of uncertainty, it increased the variance and decreased the coefficient of variation.[5]

The conclusions for milk are not as clear as for the steer and crop enterprises. However, it appears that incomplete knowledge about the price of milk does not significantly affect the level of net revenue. Therefore, the manager may make production plans from predicted prices of milk without substantially increasing the cost of uncertainty.

The rationale for the results is discussed in the next chapter.

Comparison of the Models Using a Common Base

The criteria for choosing the range of years from which expected prices are to be predicted were given in Chapter II. The effect of differing ranges was evaluated by equalizing the range of years to a common basis of two years for Models III and IV. The results are given in Tables XII, XIII, XIV and XV.

By using a common range of years, the same information was used by all of the models to make production decisions. If the restriction is relaxed, Model IV will use a different set of data. That is, Model IV does not allow large changes in price to occur. However, Models II and III do allow large changes to occur in predicted prices.

The change in the range of years did not effect the order of the costs of uncertainty for the models for corn, i.e., the cost of uncertainty remained greatest for Model IV and least for Model III.

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[5]The standard deviation and coefficient of variation are given in Appendix Table E-IX. The levels of inputs (concentrate and alfalfa hay) and milk production per cow are given in Appendix Tables E-VII and E-VIII, respectively.

TABLE XII

THE COST OF UNCERTAINTY PER ACRE FROM CORN FOR THE SELECTED PRICE PREDICTION MODELS USING A COMMON RANGE OF YEARS FROM 1949 TO 1962

	Model[a]		
Year	Model II	Model III	Model IV
		- Dollars -	
1949	0.02	0.12	2.04
1950	0.82	0.87	0.90
1951	0.18	0.30	0.75
1952	0.17	0.11	0.04
1953	0.01	0.01	0.11
1954	0.02	0.01	0.04
1955	0.06	0.05	0.10
1956	0.83	0.19	0.33
1957	0.09	0.09	0.27
1958	0.00	0.00	0.03
1959	0.07	0.06	0.08
1960	0.08	0.08	0.18
1961	0.04	0.05	0.00
1962	0.00	0.00	0.01
Total	2.39	1.94	4.88
Sum of Squares	1.448	0.933	5.781

[a]The range of years from III and IV is two years.

TABLE XIII

THE COST OF UNCERTAINTY PER ACRE FROM ALFALFA FOR THE SELECTED PRICE PREDICTION MODELS USING A COMMON RANGE OF YEARS FROM 1949 TO 1962

	Model[a]		
Year	Model II	Model III	Model IV
		- Dollars -	
1949	0.00	0.00	0.06
1950	0.04	0.05	0.03
1951	0.18	0.22	0.32
1952	0.02	0.04	0.11
1953	0.25	0.21	0.19
1954	0.04	0.04	0.19
1955	0.01	0.01	0.03
1956	0.17	0.19	0.14
1957	0.14	0.10	0.03
1958	0.00	0.00	0.05
1959	0.02	0.02	0.01
1960	0.00	0.01	0.01
1961	0.03	0.04	0.04
1962	0.00	0.00	0.00
Total	0.90	0.93	1.21
Sum of Squares	0.148	0.146	21.69

[a]The range of years for Models III and IV is two years.

TABLE XIV

THE COST OF UNCERTAINTY PER STEER FOR THE SELECTED PRICE PREDICTION MODELS USING A COMMON RANGE OF YEARS FROM 1949 TO 1962

	Model[a]		
Year	Model II	Model III	Model IV
		- Dollars -	
1949	9.24	6.39	2.29
1950	0.42	4.03	0.46
1951	11.31	13.24	16.96
1952	1.59	0.68	0.01
1953	18.41	18.00	26.60
1954	0.05	0.00	4.19
1955	0.53	0.34	0.36
1956	0.13	0.10	0.49
1957	0.61	0.72	0.32
1958	5.01	5.96	6.99
1959	0.08	0.46	1.96
1960	1.52	0.96	1.14
1961	2.45	2.20	4.55
1962	0.05	0.05	0.90
Total	51.40	53.13	67.22
Sum of Squares	589.021	598.974	1,094.199

[a]The range of years for Models III and IV is two years.

TABLE XV

THE COST OF UNCERTAINTY PER YEAR FROM ONE DAIRY COW FOR THE SELECTED PRICE PREDICTION MODELS USING A COMMON RANGE OF YEARS FROM 1949 TO 1962

Year	Model[a]		
	Model II	Model III	Model IV
		- Dollars -	
1949	8.73	6.93	3.99
1950	0.03	0.02	2.64
1951	4.38	4.83	4.00
1952	0.63	1.25	3.37
1953	2.69	2.01	1.50
1954	1.22	1.25	3.72
1955	0.02	0.03	0.19
1956	0.17	0.30	0.22
1957	0.05	0.16	0.19
1958	0.07	0.01	0.02
1959	0.02	0.04	0.00
1960	0.06	0.15	0.09
1961	0.00	0.02	0.01
1962	0.14	0.06	0.15
Total	18.21	17.06	20.09
Sum of Squares	104.580	78.664	66.487

[a]The range of years for Models III and IV is two years.

For alfalfa, the cost of uncertainty became greatest for Model IV. The flow of net revenue fluctuated greatest for Model IV and least for Model III rather than Model II and Model III, respectively.

The cost of uncertainty and the fluctuation of net revenue continued to be greatest for Model IV for steers. However, the least cost of uncertainty changed from Model III to Model II.

The greatest change occurred in the dairy enterpise. The cost of uncertainty became greatest for Model IV. However, the fluctuation of net revenue continued to be least for Model IV. Model II had the least cost of uncertainty.

In summary, the change in the range of years to a common base tended to increase the cost of uncertainty for Model IV in a greater proportion than for Model III. This result was due primarily to the prediction procedure. Last year's price was weighted by 0.9. However, Model IV essentially weighted both years equally since a good approximation for the minimax price is given by the mean of the upper price and lower price.

CHAPTER V

FACTORS DETERMINING COST OF PRICE CERTAINTY

The purpose of this chapter is to explain: (1) The relative stability of net revenue from the crops and milk; and (2) the instability of net revenue from steers. This purpose is accomplished with a concept which is developed and explained. The concept is the elasticity of the marginal cost curve.

Marginal Cost Elasticity of Output

Using the original production function

$$Y = \beta_o X^{\beta_1} V^{\beta_2} \qquad \beta_1 > 0;\ \beta_2 > 0;\ 0 < \beta_1 + \beta_2 < 1, \tag{5.1}$$

the level of inputs which maximize net revenue are

$$\tilde{X} = \left\{ \frac{\beta_1}{R} \left[P\beta_o \left(\frac{\beta_2}{K}\right)^{\beta_2} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{1-\beta_2}{1-\beta}} \tag{5.2}$$

and

$$\tilde{V} = \left\{ \frac{\beta_2}{K} \left[P\beta_o \left(\frac{\beta_1}{R}\right)^{\beta_1} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{1-\beta_1}{1-\beta}} \tag{5.3}$$

where $\beta = \beta_1 + \beta_2$. Factoring of product price from equations (5.2) and (5.3) gives

$$\tilde{X} = \left\{ \frac{\beta_1}{R} \left[\beta_o \left(\frac{\beta_2}{K}\right)^{\beta_2} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{1-\beta_2}{1-\beta}} \quad P^{\frac{1}{1-\beta}} \tag{5.4}$$

$$(5.5) \qquad \tilde{V} = \left\{ \frac{\beta_2}{K} \left[\beta_0 \left(\frac{\beta_1}{R}\right)^{\beta_1} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{1-\beta_1}{1-\beta}} P^{\frac{1}{1-\beta}}$$

Substitution of equations (5.4) and (5.5) into equation (5.1) yields the supply curve of a firm for enterprise Y as

$$(5.6) \qquad \tilde{Y} = \left\{ \beta_0 \; \frac{\beta_1}{R} \left[\beta_0 \left(\frac{\beta_2}{K}\right)^{\beta_2} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{\beta_1(1-\beta_2)}{1-\beta}}$$

$$P^{\frac{1}{1-\beta}} \left\{ \frac{\beta_2}{K} \; \beta_0 \left[\left(\frac{\beta_1}{R}\right)^{\beta_1} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{\beta_2(1-\beta_1)}{1-\beta}} P^{\frac{\beta_2}{1-\beta}}$$

Factoring of all common terms in (5.6) gives

$$(5.7) \qquad \tilde{Y} = \beta_0^{\frac{1}{1-\beta}} \left(\frac{\beta_1}{R}\right)^{\frac{\beta_1}{1-\beta}} \left(\frac{\beta_2}{K}\right)^{\frac{\beta_2}{1-\beta}} P^{\frac{\beta_1+\beta_2}{1-\beta}}$$

Simplification of (5.7) yields

$$(5.8) \qquad \tilde{Y} = \left[\beta_0 \left(\frac{\beta_1}{R}\right)^{\beta_1} \left(\frac{\beta_2}{K}\right)^{\beta_2} \right]^{\frac{1}{1-\beta}} P^{\frac{\beta_1+\beta_2}{1-\beta}}$$

Since the firm in this study is a price taker, i.e., the firm sells at market price and the assumption was made that the manager's goal is measured by maximum net revenue, marginal cost (MC) may be substituted for product price in (5.8). The result is the marginal cost curve for the firm for producing product Y. The curve is represented by

$$(5.9) \qquad Y = \left[\beta_0 \left(\frac{\beta_1}{R}\right)^{\beta_1} \left(\frac{\beta_2}{K}\right)^{\beta_2} \right]^{\frac{1}{1-\beta}} \left[MC\right]^{\frac{\beta}{1-\beta}}$$

The marginal cost elasticity of output (E_{MC}) is defined as

$$E_{MC} = \frac{\partial Y}{\partial MC} \frac{MC}{Y} \tag{5.10}$$

Substitution for $\frac{\partial Y}{\partial MC}$ and $\frac{MC}{Y}$ in (5.10) gives

$$E_{MC} = \frac{\beta}{1-\beta} \tag{5.11}$$

Cost of Uncertainty and Marginal Cost Elasticity

The cost of uncertainty was defined in Chapter II as the difference between optimal net revenue from Model I and realized net revenue. Consider a marginal cost curve for $0 < \beta < 1/2$ as shown in Figure 3. The cost of uncertainty if expected price is higher than actual is

$$P_\alpha Y_\alpha - \int_0^{Y_\alpha} MCdY - P_\alpha Y_u + \int_0^{Y_u} MCdY = F. \tag{5.12}$$

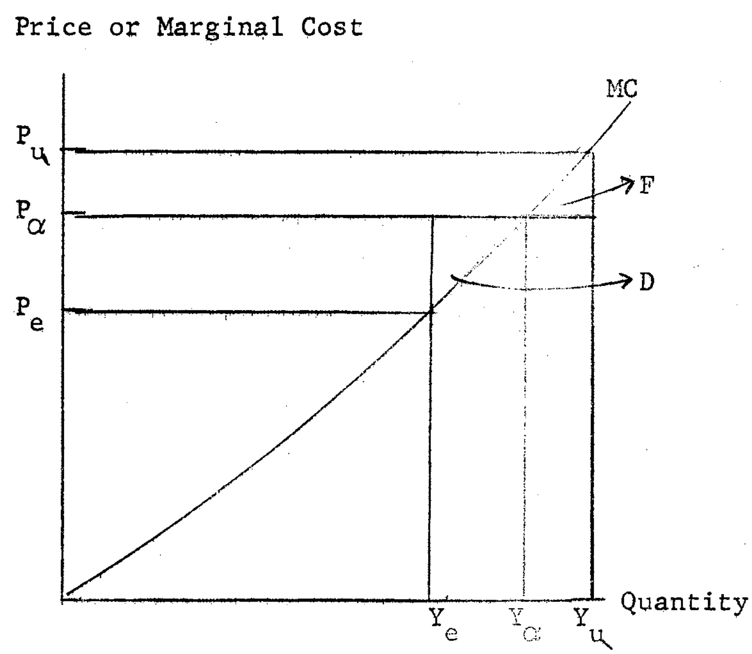

Figure 3. Hypothetical Marginal Cost Curve.

Likewise, the cost of uncertainty if expected price is lower than actual is

$$P_\alpha Y_\alpha - \int_0^{Y_\alpha} MCdY - P_\alpha Y_u + \int_0^{Y_e} MCdY = D. \tag{5.13}$$

Notice that $F < D$ for $0 < \beta < 1/2$.

Now consider one enterprise with a given price range, P_e to P_u, around P_α. Also, consider two situations of marginal cost elasticity of output for β and β' where $0 < \beta < \beta' < 1/2$.

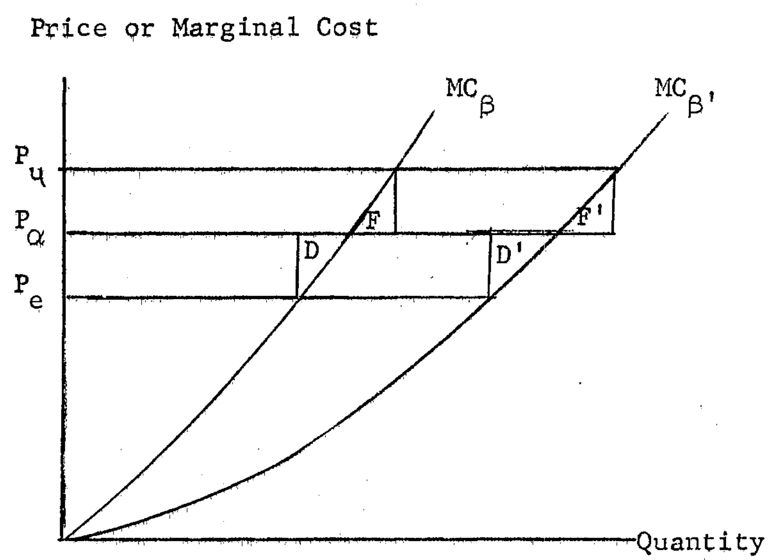

Figure 4. Marginal Cost Curves with Different β Values.

It can be seen that as β becomes larger and E_{mc} becomes more elastic, the cost of uncertainty increases. The shift in the marginal cost curves is due to changes in β_1 and β_2. The curve is a straight line when $\beta = 1/2$ and concave downward for $1/2 < \beta < 1$.[1]

Now, consider one enterprise with a price range of $P_\alpha - 2\sigma$ to $P_\alpha + 2\sigma$. Also, consider marginal cost curves with β and β' where $0 < \beta < \beta' < 1/2$.

It can be seen in Figure 5, where $0 < \sigma < \sigma'$, that the cost of uncertainty increases with increases in the variation of price. Therefore, variation in net revenue increases as price variation increases and E_{MC} becomes more elastic, for fixed capital and labor.

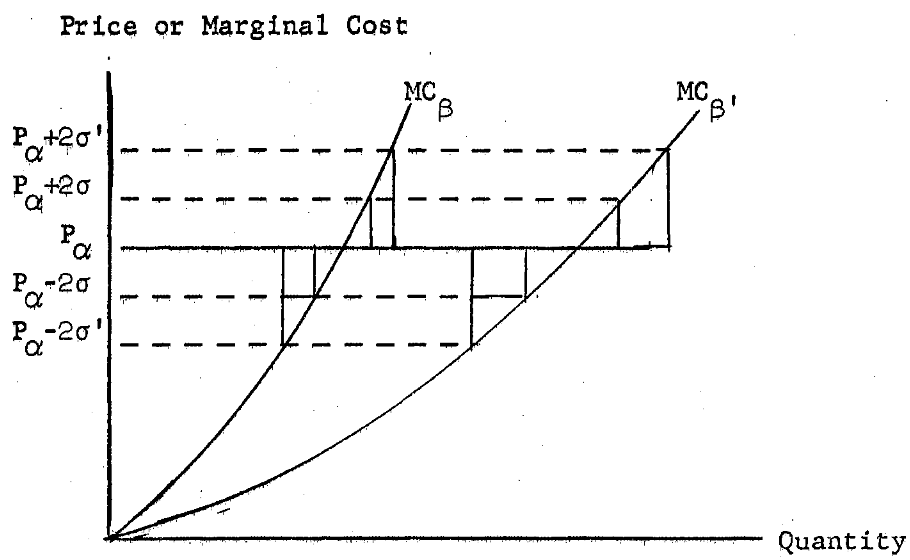

Figure 5. Marginal Cost Curve with Different β Values and Variation in Price.

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[1] If the substitution $\left[\beta_0\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta}} = S$ is made in equation (5.8), then

$$MC = \frac{1}{S}\, Y^{\frac{1-\beta}{\beta}}$$

and

$$\frac{\partial^2(MC)}{\partial Y^2} = \frac{1}{S}\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-2\beta}{\beta}\right)Y^{\frac{1-3\beta}{\beta}}$$

Therefore, MC is concave upward for $0 < \beta < 1/2$, a straight line for $\beta = 1/2$, and concave downward for $1/2 < \beta < 1$.

Analysis of Enterprises

A ranking of the various enterprises according to price variations, marginal elasticities and net revenue variation is given in Table XVI. The ranks are from low to high in each case. Although the variation in corn prices was greater than the variation in steer prices, the variations in net revenue for corn was less than the variation in net revenue for steers. However, the elasticity coefficient for steers was larger than the coefficient for corn. Therefore, high variations in prices is not sufficient to yield high variations in net revenue as in corn. However, high price variations and a high elasticity coefficient will yield high variations in net revenue as in steers. Also, low variations in prices and low elasticity coefficients will yield low variations in net revenue as in alfalfa.

TABLE XVI

RANKING OF THE ENTERPRISES WITH RESPECT TO PRICE VARIATIONS AND MARGINAL COST ELASTICITIES

Enterprise	Price Variation	Marginal Cost Elasticity	Net Revenue Variation	Average Rank
Corn	4	2	3	3
Alfalfa	2	1	2	1
Steers	3	4	4	4
Milk	1	3	1	2

In conclusion, the elasticity coefficient is the critical factor to observe for high variations in net revenue. The less diminishing are returns to scale, the higher the cost of uncertainty. Less critical, but essential for high variations in net revenue, is the variation in prices of the enterprise. Finally, when expected price is biased downwards so that it turns out to be lower than actual price more often than not, the cost of uncertainty will be higher than the reverse type of bias for $\beta < 1/2$ as found in the enterprises studied. This last effect does not seem to be important for the data observed.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND LIMITATIONS

The purpose of this chapter is threefold. First, a summary of the actions taken to meet the objectives set forth in Chapter I is given. Second, the conclusions based on the results of the actions are presented. Third, some of the major limitations of the study are given.

Summary

The actions which were used to accomplish the objectives stated in Chapter I were:

1. Production functions (Cobb-Douglas) were selected for four enterprises (corn, alfalfa, steers, and milk).
2. Models were developed with two different objective functions. The two objective functions were to maximize net revenue and to minimize maximum loss in net revenue from price uncertainty. Maximum net revenue was the objective of Models I, II, and III. The objective of Model IV was minimizing maximum loss in net revenue.
3. Product prices and resource prices were obtained for 28 and 14 years, respectively. The prices were average prices received and paid by farmers in the United States.
4. Programs were written for a high speed computer. These programs were used to compute net revenue and predicted price for each model for each year.

Conclusions

The conclusions which can be derived from the theory in this study and from the results are:

1. The fluctuations and loss in net revenue from price uncertainty is relatively small for alfalfa, corn and milk. That is, the loss in net revenue may be rather large in some years for alfalfa, corn, and milk, but the contribution of price uncertainty is small. However, price uncertainty does have a depressing affect upon net revenue for steers. Consideration of other uncertainties would increase this depressing action on net revenues for steers.
2. Model III was the best price prediction model used in this study. The joint reasons for this conclusion are:
   a. Minimum variance and coefficient of variation for net revenue.
   b. Highest average net revenue for corn and steers.
   c. Consistent in having less fluctuations in net revenue.

Likewise, Model IV was the worst price prediction model for the following reasons:

a. Maximum variance and coefficient of variation for net revenue for corn, alfalfa, and steers.
b. Least average net revenue for all enterprises except milk.

These remarks indicate that future research on such models be oriented toward the following two areas:

1. Production functions for crops and milk.
2. Price prediction models for livestock.

Limitations of the Study

The greatest short-coming for the study is the type of production function used. Some of the limitations of a Cobb-Douglas function are: (1) Unlimited substitutability between factors of production; (2) a combination of increasing and decreasing returns to size cannot be displayed; and (3) constant elasticity of production with respect to any input. However, these limitations of the Cobb-Douglas production function were partially compensated by the simplicity of the results for each model.

The price data are another limiting factor. The price data were composed of prices from several areas. Therefore, some of the fluctuations in net revenue from price variations were eliminated by the aggregation process.

Since soil and weather conditions vary from one area to another, implications of the results from corn and alfalfa are limited. The reason being that the production functions for corn and alfalfa were for specified soil types in Iowa. However, the steer and milk results are more general. Areas which approximate the weather situations in Oklahoma and Iowa, respectively, may have similar losses in net revenue from price uncertainty for steers and milk.

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APPENDIXES

APPENDIX A

Consider a one period production function of the form

(A.1) $$Y = \beta_o X^{\beta_1} V^{\beta_2} \qquad \beta_1 > 0;\ \beta_2 > 0;\ 0 < \beta_1 + \beta_2 < 1$$

where Y is the product procuced and X and V are the factors of production. Let P, R, and K be the prices of Y, X, and V, respectively. Net revenue (NR) to other factors of production is given by

(A.2) $$NR = \text{Total Revenue} - \text{Specified Total Cost}$$
$$= PY-RX-VK = P\beta_o X^{\beta_1} V^{\beta_2} - RX-KV.$$

The factor use level which will maximize NR is found by setting the following partial derivatives to zero and solving the equations simultaneously for X and V;

(A.3) $$\frac{\partial NR}{\partial X} = P\,\beta_o\,\beta_1\,X^{\beta_1 - 1}\,V^{\beta_2} - R = 0$$

(A.4) $$\frac{\partial NR}{\partial V} = P\,\beta_o\,\beta_2\,X^{\beta_1}\,V^{\beta_2 - 1} - K = 0$$

Solving (A.3) for X gives

(A.5) $$X = \left[\frac{R}{P\,\beta_o\,\beta_1\,V^{\beta_2}}\right]^{\frac{1}{\beta_1 - 1}} = \left[\frac{P\,\beta_o\,\beta_1\,V^{\beta_2}}{R}\right]^{\frac{1}{1-\beta_1}}$$

Substitution of (A.5) into (A.4) gives

(A.6) $$K = P\,\beta_o\,\beta_2\left[\frac{P\,\beta_o\,\beta_1\,V^{\beta_2}}{R}\right]^{\frac{\beta_1}{1-\beta_1}} V^{\beta_2 - 1}$$

(A.6a) $$K = \beta_2 \left[P\ \beta_o \left(\frac{\beta_1}{R}\right)^{\beta_1} \right]^{\frac{1}{1-\beta_1}} V^{\frac{-1+\beta_1+\beta_2}{1-\beta_1}}$$

The optimal factor use level ($\widetilde{V}$) of V is found by solving (A.6) for V which is given by

(A.7) $$\widetilde{V} = \left\{ \frac{\beta_2}{K} \left[P\ \beta_o \left(\frac{\beta_1}{R}\right)^{\beta_1} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{1-\beta_1}{1-\beta_1-\beta_2}}$$

By analogy, the optimal factor use level ($\widetilde{X}$) of X is seen to be

(A.8) $$\widetilde{X} = \left\{ \frac{\beta_1}{R} \left[P\ \beta_o \left(\frac{\beta_2}{K}\right)^{\beta_2} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{1-\beta_2}{1-\beta_1-\beta_2}}$$

The optimal level of product produced ($\widetilde{Y}$) is found by substituting (A.7) and (A.8) into (A.1) which gives

(A.9) $$\widetilde{Y} = \beta_o\ \widetilde{X}^{\beta_1}\ \widetilde{V}^{\beta_2}$$

Optimal net revenue ($\widetilde{NR}$) is found by substituting (A.7), (A.8), and (A.9) into (A.2) which gives

(A.10) $$\widetilde{NR} = P\widetilde{Y} - R\widetilde{X} - K\widetilde{V}$$

(A.10a) $$\widetilde{NR} = P\ \beta_o\ \widetilde{X}^{\beta_1}\ \widetilde{V}^{\beta_2} - R\widetilde{X} - K\widetilde{V}$$

(A.10b) $$\widetilde{NR} = P\ \beta_o \left\{ \frac{\beta_1}{R} \left[\frac{P\beta_o\beta_2^{\beta_2}}{K^{\beta_2}} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{\beta_1(1-\beta_2)}{1-\beta_1-\beta_2}} \left\{ \frac{\beta_2}{K} \left[\frac{P\beta_o\beta_1^{\beta_1}}{R^{\beta_1}} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{\beta_2(1-\beta_1)}{1-\beta_1-\beta_2}}$$

$$-R \left\{ \frac{\beta_1}{R} \left[\frac{P\beta_o\beta_2^{\beta_2}}{K^{\beta_2}} \right]^{\frac{1}{1-\beta_2}} \right\}^{\frac{1-\beta_2}{1-\beta_1-\beta_2}} - K \left\{ \frac{\beta_2}{K} \left[\frac{P\beta_o\beta_1^{\beta_1}}{R^{\beta_1}} \right]^{\frac{1}{1-\beta_1}} \right\}^{\frac{1-\beta_1}{1-\beta_1-\beta_2}}$$

Expansion and simplification of (A.10b) gives

$$\text{(A.11)} \qquad \widetilde{NR} = P\,\beta_o \left(\frac{\beta_1}{R}\right)^{\frac{\beta_1}{1-\beta_1-\beta_2}} \left(\frac{\beta_2}{K}\right)^{\frac{\beta_2}{1-\beta_1-\beta_2}} \left(P\,\beta_o\right)^{\frac{\beta_1+\beta_2}{1-\beta_1-\beta_2}} - \left[\frac{\beta_1^{\frac{1-\beta_2}{1-\beta_1-\beta_2}}}{R^{\frac{\beta_1}{1-\beta_1-\beta_2}}}\right]$$

$$\left(\frac{\beta_2}{K}\right)^{\frac{\beta_2}{1-\beta_1-\beta_2}} \left(P\,\beta_o\right)^{\frac{1}{1-\beta_1-\beta_2}} - \left[\frac{\beta_2^{\frac{1-\beta_1}{1-\beta_1-\beta_2}}}{K^{\frac{\beta_2}{1-\beta_1-\beta_2}}}\right] \left(\frac{\beta_1}{R}\right)^{\frac{\beta_1}{1-\beta_1-\beta_2}} \left(P\,\beta_o\right)^{\frac{1}{1-\beta_1-\beta_2}}$$

By factoring $\left(P\,\beta_o\right)^{\frac{1}{1-\beta_1-\beta_2}}$, $\left(\frac{\beta_1}{R}\right)^{\frac{\beta_1}{1-\beta_1-\beta_2}}$ and $\left(\frac{\beta_2}{K}\right)^{\frac{\beta_2}{1-\beta_1-\beta_2}}$ in (A.11), $\widetilde{NR}$ is given by

$$\text{(A.12)} \qquad \widetilde{NR} = \left(P\,\beta_o\right)^{\frac{1}{1-\beta_1-\beta_2}} \left(\frac{\beta_1}{R}\right)^{\frac{\beta_1}{1-\beta_1-\beta_2}} \left(\frac{\beta_2}{K}\right)^{\frac{\beta_2}{1-\beta_1-\beta_2}} \left[1-\beta_1-\beta_2\right]$$

Further simplification of (A.12) given $\widetilde{NR}$ as

$$\text{(A.13)} \qquad \widetilde{NR} = \left[1-\beta_1-\beta_2\right] \left[P\,\beta_o \left(\frac{\beta_1}{R}\right)^{\beta_1} \left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}}.$$

The discussion presented with graphs in Chapter II will not be repeated. Rather, the brief algebraic expressions will be derived. By using graphs, it was shown that the loss in net revenue from uncertain product prices could be minimized with

$$\text{(A.14)} \qquad L(P_*, \hat{Y}) = L(P^*, \hat{Y})$$

With the functional relationship of Y, X, and V, it was shown that (A.14) could be written as

(A.14a) $$L\left[P_*,\ f(\hat{X},\ \hat{V})\right] = L\left[P^*,\ f(\hat{X},\ \hat{V})\right]$$

Further, it was shown that

(A.15)
$$L\left[P, f(X,V)\right] = \left[1-\beta_1-\beta_2\right]\left[P\beta_o\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}} - P\beta_o X^{\beta_1} V^{\beta_2} + RX + KV$$

Substitution of (A.15) into (A.14a) gives

(A.16) $$\left[1-\beta_1-\beta_2\right]\left[P_*\beta_o\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}} - P_*\beta_o\hat{X}^{\beta_1}\hat{V}^{\beta_2} + R\hat{X} + K\hat{V}$$

$$= \left[1-\beta_1-\beta_2\right]\left[P^*\beta_o\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}} - P^*\beta_o\hat{X}^{\beta_1}\hat{V}^{\beta_2} + R\hat{X} + K\hat{V}$$

Simplification and factoring of (A.16) gives

(A.17)
$$\beta_o\hat{X}^{\beta_1}\hat{V}^{\beta_2}\left[P^*-P_*\right] = \left[1-\beta_1-\beta_2\right]\left[\beta_o\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}}\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]$$

Further simplification of (A.17) yields

(A.18)
$$\hat{X}^{\beta_1}\hat{V}^{\beta_2} = \left[1-\beta_1-\beta_2\right]\left[\beta_o^{\beta_1+\beta_2}\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}}\frac{\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]}{\left[P^* - P_*\right]}$$

The factor use level of X in terms of V that will minimize the maximum (minimax) loss is given by

(A.19)
$$\hat{X} = \left\{\left[1-\beta_1-\beta_2\right]\left[\beta_o^{\beta_1+\beta_2}\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}}\frac{\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]}{\left[P^* - P_*\right]}\right\}^{\frac{1}{\beta_1}}\hat{V}^{\frac{\beta_2}{-\beta_1}}$$

By analogy, the factor use level of V in terms of X is given by

(A.20)
$$\hat{V} = \left\{ \left[1-\beta_1-\beta_2\right] \left[\beta_o^{\beta_1+\beta_2} \left(\frac{\beta_1}{R}\right)^{\beta_1} \left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{1-\beta_1-\beta_2}} \frac{\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]}{\left[P^* - P_*\right]} \right\}^{\frac{1}{\beta_2}} \hat{X}^{-\frac{\beta_1}{\beta_2}}$$

Equations (A.19) and (A.20) give the factor use level combinations of X and V that will yield the minimax loss. Therefore, the system cannot be solved for X and V unless another equation is introduced. Since cost has not been considered, the introduction of the least cost combination will be used.

The least cost combination of X and V is the combination that allows each input to add the same amount to total cost for an equal increment of output. The condition is stated as

(A.21)
$$\frac{R}{MPP_X} = \frac{K}{MPP_V}$$

The marginal physical products of $X(MPP_X)$ and $V(MPP_V)$ can be found by taking the first partial derivative of the production function (Y) with respect to each resource which results in

(A.22)
$$MPP_X = \frac{\partial Y}{\partial X} = \beta_o \beta_1 X^{\beta_1 - 1} V^{\beta_2}$$

and

(A.23)
$$MPP_V = \frac{\partial X}{\partial V} = \beta_o \beta_2 X^{\beta_1} V^{\beta_2 - 1}$$

Substitution of (A.22) and (A.23) into (A.21) gives

(A.24)
$$\frac{R}{\beta_o \beta_1 X^{\beta_1 - 1} V^{\beta_2}} = \frac{K}{\beta_o \beta_2 X^{\beta_1} V^{\beta_2 - 1}}$$

Solving (A.24) for X gives

(A.25)
$$X = \left(\frac{\beta_1}{R}\right)\left(\frac{K}{\beta_2}\right) V$$

By analogy, V is

$$V = \left(\frac{\beta_2}{K}\right)\left(\frac{R}{\beta_1}\right)X \tag{A.26}$$

Substitution of (A.26) into (A.19) gives

$$\hat{X} = \left[1-\beta_1-\beta_2\right]^{\frac{1}{\beta_1}}\left[\beta_o^{\ \beta_1+\beta_2}\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right]^{\frac{1}{(1-\beta_1-\beta_2)\beta_1}} \left\{\frac{\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]}{\left[P^* - P_*\right]}\right\}^{\frac{1}{\beta_1}}\left[\hat{X}\left(\frac{\beta_2}{K}\right)\left(\frac{R}{\beta_1}\right)\right]^{-\frac{\beta_2}{\beta_1}} \tag{A.27}$$

The factor use level of X that gives the minimax loss is found by simplifying (A.27) to

$$\hat{X} = \left\{\left[1-\beta_1-\beta_2\right]\frac{\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]}{\left[P^* - P_*\right]}\right\}^{\frac{1}{\beta_1+\beta_2}}\left\{\beta_o\left(\frac{\beta_1}{R}\right)^{1-\beta_2}\left(\frac{\beta_2}{K}\right)^{\beta_2}\right\}^{\frac{1}{1-\beta_1-\beta_2}} \tag{A.28}$$

By analogy, the factor use level of $\hat{V}$ that gives the minimax loss is

$$\hat{V} = \left\{\left[1-\beta_1-\beta_2\right]\frac{\left[P^{*\frac{1}{1-\beta_1-\beta_2}} - P_*^{\frac{1}{1-\beta_1-\beta_2}}\right]}{\left[P^* - P_*\right]}\right\}^{\frac{1}{\beta_1+\beta_2}}\left\{\beta_o\left(\frac{\beta_1}{R}\right)^{\beta_1}\left(\frac{\beta_2}{K}\right)^{1-\beta_1}\right\}^{\frac{1}{1-\beta_1-\beta_2}} \tag{A.29}$$

The minimax loss can be found by substituting (A.28) and (A.29) into (A.14).

APPENDIX B-I

```
 1  Format (23HOPT ORG FOR KNOWN PRICE)
    PRINT 1
    DIMENSION Y(15), XI(15), X2(15), P(3Ø), R1(15), R2(15), BNR(15)[1]
    READ 3, B1, B2, B3, I1, I2[2]
    DO 1Ø K=1, I1
1Ø  READ 3, P(K)
    DO 11 K=1, I2
11  READ 3, R1(K), R2(K)
    A1=(1.Ø/(1.Ø-B3))
    A2=(1.Ø-B3)/(1.Ø-B2-B3)
    A3=(1.Ø/(1.Ø-B2))
    A4=(1.Ø-B2)/(1.Ø-B2-B3)
    DO 12 K=1, I2
    X1(K)=((B2/R1(K))*(P(K)*B1*(B3/R2(K))**B3)**A1)**A2
12  X2(K)=((B3/R2(K))*(P(K)*B1*(B2/R1(K))**B2)**A3)**A4
    DO 13 K=1, I2
13  Y(K)=B1*(X1(K)**B2*(X2(K)**B3)
    DO 14 K=1, I2
14  BNR(K)=(P(K)*Y(K))-(R1(K)*X1(K))-(R2(K)*X2(K))
 2  FORMAT(49H    PRODUCT      INPUT 1      INPUT 2      NET REVENUE)
    PRINT 2
    DO 15 K=1, I2
15  PRINT 4, Y(K), X1(K),X2(K),BNR(K)
 3  FORMAT (F1Ø.5, F1Ø.5, F1Ø.5, I5, I5)
 4  FORMAT (F1Ø.5, 2X F1Ø.5, 2X F1Ø.5, 2X F1Ø.5)
    END
```

[1]Y, X, P, R, and BNR are the product, inputs, price of the product, price of the inputs and optimal net revenue, respectively.

[2]B1, B2, B3, I1 and I2 represent β_o, β_1, β_2, the number of years of product prices and the number of years of input, respectively.

APPENDIX B-II

```
 1  FORMAT (38HNET REVENUE FOR NERLOVES WT AVE PRICES)
    PRINT 1
    DIMENSION P(3Ø), R1(15), R2(15), EXP(15), BNR(15)[1]
    DIMENSION X1(15), X2(15), Y(15), BNR1(15), ENR(15)[2]
    READ 4, B1, B2, B3, I1, I2
    DO 1Ø K=1, I1
1Ø  READ 4, P(K)
    DO 11 K=1, I2
11  READ 4, R1(K), R2(K)
    DO 12 K=1, I2
12  READ 5, BNR(K)
2Ø  READ 5, NER,B
    DO 13 K=1, I2
    EXP(K)=Ø.Ø
    DO 13 I=1, NER
    N=K+I
13  EXP(K)=EXP(K)+(B*((1.Ø-B)**(I-1))*P(N))
    A1=(1.Ø/(1.Ø-B3))
    A2=(1.Ø-B3)/(1.O-B2-B3)
    A3=(1.Ø/(1.Ø-B2))
    A4=(1.Ø-B2)/(1.Ø-B2-B3)
    DO 14 K=1,I2
    X1(K)=((B2/R1(K))*(EXP(K)*B1*(B3/R2(K))**B3)**A1)**A2
14  X2(K)=((B3/R2(K))*(EXP(K)*B1*(B2/R1(K))**B2)**A3)**A4
    DO 15 K=1, I2
15  Y(K)=B1*(X1(K)**B2)*(X2(K)**B3)
    DO 16 K=1,I2
    BNR 1(K)=(P(K)*Y(K))-(R1(K)*X1(K))-(R2(K)*X2(K))
16  ENR(K)=(EXP(K)*Y(K))-(R1(K)*X1(K))-(R2(K)*X2(K))
    BENR =Ø.Ø
    DO 17 K=I, I2
17  BENR=BENR+((BNR(K)-ENR(K))**2)
    PRINT 6, NER,B,BENR
 2  FORMAT(42H    INPUT 1        INPUT 2        PRODUCT)
    PRINT 2
    DO 18 K=1, I2
18  PRINT 7, X1(K),X2(K),Y(K)
 3  FORMAT(4ØH   EPRICE      NET REVENUE     ENET REVENUE)
    PRINT 3
    DO 19 K=1, I2
19  PRINT 8, EXP(K),BNR1(K),ENR(K)
    IF (SENSE SWITCH 1) 2Ø,21
 4  FORMAT (F1Ø.5, F1Ø.5, F1Ø.5, I5, I5)
 5  FORMAT (I5,F1Ø.5)
 6  FORMAT (4H NER=I5,2X 2HB=,F1Ø.5,2X 5H BENR= E14.8)
 7  FORMAT (E14.8,2X E14.8,2X E14.8)
 8  FORMAT (F10.5,2X E14.8,2X E14.8)
21  END
```

[1]EXP is the weighted average price which is the expected price.

[2]BNR1 and ENR are the realized net revenue and expected net revenue from production plans based on expected product prices (EXP).

APPENDIX B-III

```
  1  FORMAT (24HNET REVENUE FOR HILDRETH)
     PRINT 1
     DIMENSION P(3Ø),R1(15),R2(15),PL(15),PH(15),X4(15),X5(15)[1]
     DIMENSION X1(15),X2(15),Y(15),BNR(15),PB(15),PB1(15),BNR1(15)[2]
     DIMENSION BPLR(15),BPHR(15)[3]
     READ 4, B1,B2,B3, I1,I2
     DO 1Ø K=1, I1
 1Ø  READ 4, P(K)
     DO 11 K=1,I2
 11  READ 4, R1(K),R2(K)
     DO 12 K=1, I2
 12  READ 4, BNR1(K)
     B4=(1.Ø-B2-B3)**(1.Ø/(B2+B3))
     B5=(1.Ø/(1.Ø-B2-B3))
     B6=(1.Ø-B2-B3)/(B2+B3)
     B7=(1.Ø/(B2+B3))
100  READ 5, NER
     NER1=NER-1
     DO 14 K=1, I2
     PL(K)=P(K+1)
     DO 14 L=1,NER1
     N=K+L+1
     IF(PL(K)-P(N))14,14,13
 13  PL(K)=P(N)
 14  CONTINUE
     DO 16 K=1,12
     PH(K)=P(K+1)
     DO 16 L=1,NER1
     N=K+L+1
     IF(PH(K)-P(N))15,16,16
 15  PH(K)=P(N)
 16  CONTINUE
     DO 17 K=1,I2
 17  PB(K)=(PH(K)**(1.Ø/(1.Ø-B2-B3))-(PL(K)**(1.Ø/(1.Ø-B2-B3))))
     DO 18 K=1, I2
 18  PB1(K)=(PB(K)/(PH(K)-PL(K)))**(1.Ø/(B2+B3))
     DO 19 K=1,I2
     X4(K)=(B1*((B3/R2(K))**B3)*((B2/R1(K))**(1.Ø-B3)))**B5
 19  X5(K)=(B1*((B2/R1(K))**B2)*((B3/R2(K))**(1.Ø-B2)))**B5
     DO 2Ø K=1,I2
     X1(K)=B4*PB1(K)*X4(K)
 20  X2(K)=B4*PB1(K)*X5(K)
     DO 21 K=1,I2
 21  Y(K=B1*(X1(K)**B2)*(X2(K)**B3)
     DO 3Ø K=1, I2
     Y3(K)=B1*((B2/R1(K))**B2)*((B3/R2(K))**B3)
 30  Y2(K)=Y(K)**B6
     DO 31 K=1,I2
 31  EP(K)=((1.Ø/Y3(K))**B7)*Y2(K)
     DO 22 K=1, I2
     BPLR(K)=(PL(K)*Y(K))-(R1(K)*X1(K))-(R2(K)*X2(K))
     BPHR(K)=(PH(K)*Y(K))-(R1(K)*X1(K))-(R2(K)*X2(K))
```

APPENDIX B-III (Continued)

```
22   BNR(K)=(P(K)*Y(K))-(R1(K)*X1(K))-(R2(K)*X2(K))
     BENR=Ø.Ø
     DO 23 K=1,I2
23   BENR=BENR+((BNR1(K)-BNR(K))**2)
     PRINT 6, NER,BENR
 2   FORMAT(42H    INPUT 1      INPUT 2      PRODUCT)
     PRINT 2
     DO 24 K=1,I2
24   PRINT 7,X1(K),X2(K),Y(K)
 3   FORMAT(49H   PL         PH      BNR        BPLR      BPHR)
     PRINT 3
     DO 25 K=1,I2
25   PRINT 8, PL(K),PH(K),BNR(K),BPLR(K),BPHR(K)
32   FORMAT(13HMINIMAX PRICE)
     PRINT 32
     DO 33 K=1,I2
33   PRINT 1,EP(K)
     IF (SENSE SWITCH 1)1ØØ,1Ø1
 4   FORMAT(F1Ø.5,F1Ø.5,F1Ø.5,I5,I5 )
 5   FORMAT(I5)
 6   FORMAT(4HNER= I5,2X 5HBEAR= E14.8)
 7   FORMAT(E14.8,2X E14.8,2X E14.8)
 8   FORMAT (F1Ø.5,2X F1Ø.5,2X F1Ø.5,2X F1Ø.5,2X F1Ø.5)
1Ø1  END
```

---

[1]PL and PH are the low and high prices of a range of prices.

[2]PB, PB1, X4 and X5 are intermediate computations.

[3]BPLR and BPHR are expected net revenues if the low price and high price would have been realized, respectively.

APPENDIX TABLE C-I

FACTOR PRICES USED IN THIS STUDY

Year	Nitrogen[1]	Phosphorous[2]	Phosphorous[3]	Potassium[4]	Milo[5]	Dairy Feed[6]	Alfalfa[7]
	(lb.)	(lb.)	(lb.)	(lb.)	(lb.)	(cwt.)	(ton)
				- Dollars -			
1949	0.1185	0.0717	0.0730	0.1059	1.97	3.57	27.20
1950	0.1205	0.0746	0.0749	0.0997	1.87	3.69	29.65
1951	0.1277	0.0803	0.0802	0.1010	2.19	4.24	34.60
1952	0.1320	0.0819	0.0827	0.1026	2.71	4.27	36.05
1953	0.1352	0.0847	0.0864	0.0768	2.43	3.79	30.00
1954	0.1415	0.0867	0.0867	0.0748	2.27	3.81	27.85
1955	0.1372	0.0866	0.0860	0.0722	2.00	3.55	26.90
1956	0.1302	0.0853	0.0850	0.0695	2.00	3.70	31.60
1957	0.1243	0.0851	0.0862	0.0672	1.80	3.56	27.20
1958	0.1298	0.0880	0.0885	0.0685	1.69	3.57	26.80
1959	0.1266	0.0878	0.0870	0.0685	1.68	3.62	28.35
1960	0.1255	0.0882	0.0889	0.0698	1.50	3.57	28.90
1961	0.1272	0.0896	0.0889	0.0698	1.59	3.61	30.90[5]
1962	0.1260	0.0889	0.0887	0.0708	1.67	3.74	30.70[5]

[1]Source: Prices paid by farmers for actual nitrogen in amonium nitrate in April in the U. S., <u>Agricultural Prices</u>, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

[2]Source: Prices paid by farmers for actual phosphorous in 45 percent superphosphate in April in the U. S., <u>Agricultural Prices</u>, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

[3]Source: Prices paid by farmers for actual phosphorous in 45 percent superphosphate in September in the U. S., <u>Agricultural Prices</u>, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

[4]Source: Prices paid by farmers for actual potassium in 0-20-20 (except for 1949-1952 when 0-14-7 was used) in September in the U. S., <u>Agricultural Prices</u>, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture. (The price was obtained by subtracting the total cost per ton of 0-20-0 from the total cost of 0-20-20 and dividing the result by 400, except for 1949-1952 when 0-14-0 was used with 0-14-7.)

[5]Source: Annual average prices received by farmers, <u>Agricultural Prices</u>, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

[6]Source: Annual average prices paid by farmers for 16 percent dairy feed, <u>Agricultural Prices</u>, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

[7]Source: Annual average prices received by farmers for No. 1 alfalfa hay in Kansas City, <u>Agricultural Statistics</u>, U. S. Department of Agriculture.

APPENDIX TABLE C-II

PRODUCT PRICES USED IN THIS STUDY

Year	Corn[1] (bu.)	Alfalfa[2] (ton)	Steers[3] (cwt.)	Milk[4] (cwt.)
		- Dollars -		
1934	0.82	21.28	6.94	1.55
1935	0.65	13.43	10.79	1.72
1936	1.04	19.37	8.82	1.88
1937	0.51	18.38	11.79	1.99
1938	0.47	13.58	9.14	1.73
1939	0.57	15.41	9.81	1.69
1940	0.62	13.40	10.48	1.82
1941	0.75	15.90	11.36	2.19
1942	0.92	20.45	13.90	2.58
1943	1.12	29.25	15.34	3.12
1944	1.03	27.20	15.73	3.21
1945	1.23	27.85	16.00	3.19
1946	1.53	32.15	19.32	3.99
1947	2.16	32.50	26.22	4.27
1948	1.28	27.65	30.96	4.88
1949	1.24	27.20	26.07	3.95
1950	1.52	29.65	29.68	3.89
1951	1.66	34.60	35.96	4.58
1952	1.52	36.05	33.18	4.85
1953	1.48	30.00	24.14	4.32
1954	1.43	27.85	24.66	3.97
1955	1.35	26.90	23.16	4.01
1956	1.21	31.60	22.30	4.14
1957	1.12	27.20	23.83	4.21
1958	1.12	26.80	27.42	4.13
1959	1.04	28.35	27.83	4.16
1960	0.96	28.90	26.24	4.24[1]
1961	1.02	30.90[1]	23.80[5]	4.23[1]
1962	1.02	30.70[1]	23.40[5]	4.11[1]

[1]Source: Annual average prices received by farmers, Agricultural Prices, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

[2]Source: Annual average prices received by farmers for No. 1 alfalfa hay in Kansas City, Agricultural Statistics, U. S. Department of Agriculture.

[3]Source: Annual average prices received by farmers for choice steers in Chicago, Agricultural Statistics, U. S. Department of Agriculture.

[4]Source: Annual average prices received by farmers, Agricultural Statistics, U. S. Department of Agriculture.

[5]Source: Annual average prices received by farmers for steers and heifers, Agricultural Prices, Crop Reporting Board, Statistical Reporting Service, U. S. Department of Agriculture.

APPENDIX TABLE D-I

PREDICTED PRICES PER BUSHEL FOR CORN FOR THE THREE SELECTED MODELS FOR THE YEARS 1949 TO 1962

			Model IV		
Year	Model II	Model III	Minimax Price	Lower Price	Upper Price
			- Dollars -		
1949	1.28	1.35	1.57	1.03	2.16
1950	1.24	1.23	1.68	1.23	2.16
1951	1.52	1.48	1.68	1.24	2.16
1952	1.66	1.63	1.68	1.24	2.16
1953	1.52	1.52	1.45	1.24	1.66
1954	1.48	1.47	1.45	1.24	1.66
1955	1.43	1.42	1.54	1.43	1.66
1956	1.35	1.34	1.50	1.35	1.66
1957	1.21	1.21	1.36	1.21	1.52
1958	1.12	1.12	1.30	1.12	1.48
1959	1.12	1.11	1.27	1.12	1.43
1960	1.04	1.04	1.19	1.04	1.35
1961	0.96	0.96	1.08	0.96	1.21
1962	1.02	1.00	1.04	0.96	1.12

APPENDIX TABLE D-II

PREDICTED PRICES PER TON FOR ALFALFA FOR THE THREE SELECTED MODELS FOR THE YEARS 1949 TO 1962

			Model IV		
Year	Model II	Model III	Minimax Price	Lower Price	Upper Price
			- Dollars -		
1949	27.65	27.81	29.82	27.20	32.50
1950	27.20	26.97	29.82	27.20	32.50
1951	29.65	29.13	29.82	27.20	32.90
1952	34.60	33.81	30.84	27.20	34.60
1953	36.05	35.56	31.54	27.20	36.05
1954	30.00	30.24	31.54	27.20	36.05
1955	27.85	27.65	31.88	27.85	36.05
1956	26.90	26.72	31.39	26.90	36.05
1957	31.60	30.86	31.39	26.90	36.05
1958	37.20	27.32	29.23	26.90	31.60
1959	26.80	26.57	29.17	26.80	31.60
1960	28.35	27.93	29.17	26.80	31.60
1961	28.90	28.56	29.17	26.80	31.60
1962	30.90	30.41	28.83	26.80	30.90

APPENDIX TABLE D-III

PREDICTED PRICES PER HUNDRED WEIGHT FOR STEERS FOR THE THREE SELECTED MODELS FOR THE YEARS 1949 TO 1962

Year	Model II	Model III	Model IV Minimax Price	Model IV Lower Price	Model IV Upper Price
			- Dollars -		
1949	30.96	28.40	25.68	19.32	30.96
1950	26.07	26.40	28.60	26.07	30.96
1951	29.68	28.20	28.60	26.07	30.96
1952	35.96	33.05	31.33	26.07	35.96
1953	33.18	32.65	32.94	29.68	35.96
1954	24.14	26.13	30.52	24.14	35.96
1955	24.66	24.42	28.95	24.40	33.18
1956	23.16	22.91	23.92	23.16	24.66
1957	22.30	22.03	23.50	22.30	24.66
1958	23.83	22.82	23.08	22.30	23.83
1959	27.42	25.60	24.97	22.30	27.42
1960	27.83	26.74	25.89	23.83	27.83
1961	26.24	25.94	27.04	26.24	27.83
1962	23.80	23.93	25.88	23.80	27.83

APPENDIX TABLE D-IV

PREDICTED PRICES PER HUNDRED WEIGHT FOR MILK FOR THE THREE SELECTED MODELS FOR THE YEARS FROM 1949 TO 1962

Year	Model II	Model III	Model IV Minimax Price	Model IV Lower Price	Model IV Upper Price
			- Dollars -		
1949	4.88	4.30	4.02	3.19	4.88
1950	3.95	4.08	4.02	3.19	4.88
1951	3.89	3.93	4.38	3.89	4.88
1952	4.58	4.19	4.38	3.89	4.88
1953	4.85	4.46	4.38	3.89	4.88
1954	4.32	4.31	4.37	3.89	4.85
1955	3.97	4.08	4.37	3.89	4.85
1956	4.01	3.98	4.41	3.97	4.85
1957	4.14	3.99	4.41	3.97	4.85
1958	4.21	4.02	4.14	3.97	4.32
1959	4.13	4.01	4.09	3.97	4.21
1960	4.16	4.02	4.11	4.01	4.21
1961	4.24	4.07	4.18	4.13	4.24
1962	4.23	4.08	4.18	4.13	4.24

APPENDIX TABLE E-I

OPTIMAL LEVELS OF PHOSPHOROUS AND NITROGEN IN POUNDS PER ACRE FOR CORN FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

	Model I		Model II		Model III		Model IV	
Year	Phosphorous	Nitrogen	Phosphorous	Nitrogen	Phosphorous	Nitrogen	Phosphorous	Nitrogen
				- Pounds -				
1949	236.71	81.51	246.44	84.86	262.79	90.49	319.39	109.98
1950	292.13	102.93	225.59	79.48	223.56	78.77	331.52	116.80
1951	298.01	106.64	266.47	95.36	257.51	92.15	303.63	108.66
1952	259.54	91.64	290.26	102.49	283.79	100.21	295.73	104.42
1953	240.64	85.80	248.93	88.75	248.39	88.56	233.70	83.32
1954	223.16	77.82	233.11	81.29	230.87	80.51	226.39	78.94
1955	208.34	74.84	224.14	80.51	222.19	79.81	247.05	88.74
1956	185.49	69.16	213.15	79.47	211.89	79.00	244.29	91.08
1957	169.39	66.00	186.85	72.80	186.95	72.84	217.31	84.67
1958	162.18	62.57	162.18	62.57	161.61	62.35	195.37	75.38
1959	148.37	58.56	163.07	64.34	160.94	63.52	191.72	75.67
1960	133.44	53.37	147.71	59.08	147.13	58.85	175.74	70.29
1961	141.29	56.64	130.82	52.44	130.41	52.28	152.50	61.14
1962	142.73	57.31	142.73	57.31	139.96	56.20	146.16	58.69

APPENDIX TABLE E-II

OPTIMAL LEVELS OF CORN IN BUSHELS PER ACRE FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

Year	Model I	Model II	Model III	Model IV
		- Bushels -		
1949	101.16	102.03	103.43	107.80
1950	105.97	100.31	100.12	108.85
1951	106.54	104.04	103.29	106.97
1952	103.36	105.84	105.34	106.26
1953	101.79	102.52	102.48	101.16
1954	100.00	100.93	100.72	100.31
1955	98.78	100.32	100.14	102.42
1956	96.65	99.54	99.42	102.47
1957	95.12	97.13	97.14	100.29
1958	94.18	94.18	94.11	97.98
1959	92.58	94.45	94.19	97.76
1960	90.61	92.58	92.51	96.06
1961	91.73	90.24	90.18	93.23
1962	91.94	91.94	91.56	92.40

APPENDIX TABLE E-III

OPTIMAL LEVELS OF POTASSIUM AND PHOSPHOROUS IN POUNDS PER ACRE FOR ALFALFA FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

	Model I		Model II		Model III		Model IV	
Year	Potassium	Phosphorous	Potassium	Phosphorous	Potassium	Phosphorous	Potassium	Phosphorous
				- Pounds -				
1949	26.47	92.80	27.00	94.68	27.20	95.36	29.63	103.88
1950	31.25	100.53	28.11	90.43	27.82	89.49	31.47	101.24
1951	36.84	112.14	30.48	92.78	29.83	90.80	30.70	93.43
1952	37.91	113.68	36.05	108.10	35.04	105.07	31.31	93.87
1953	40.92	87.92	51.27	110.15	50.41	108.32	43.52	93.51
1954	38.40	80.07	42.07	87.72	42.49	88.60	44.74	93.30
1955	38.26	77.64	39.93	81.02	39.78	80.71	47.13	95.64
1956	48.65	96.14	39.92	78.90	39.59	78.23	48.25	95.35
1957	41.86	78.86	50.31	94.80	48.87	92.08	99.90	94.02
1958	40.10	75.02	40.84	76.39	41.06	76.82	44.60	83.43
1959	43.82	81.99	40.21	76.52	39.78	75.71	44.62	84.92
1960	43.09	81.76	42.08	79.86	41.31	78.40	43.59	82.72
1961	47.52	89.20	43.78	42.16	43.15	80.98	44.29	83.12
1962	45.72	88.20	46.08	88.91	45.19	87.18	42.33	81.66

APPENDIX TABLE E-IV

OPTIMAL LEVELS OF ALFALFA IN TONS PER ACRE FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

Year	Model I	Model II	Model III	Model IV
		- Tons -		
1949	1.90	1.91	1.91	1.94
1950	1.94	1.90	1.90	1.94
1951	1.98	1.92	1.90	1.91
1952	1.99	1.97	1.96	1.92
1953	1.93	2.01	2.01	1.96
1954	1.91	1.94	1.94	1.96
1955	1.89	1.91	1.91	1.97
1956	1.97	1.90	1.90	1.97
1957	1.91	1.97	1.96	1.97
1958	1.89	1.90	1.90	1.93
1959	1.92	1.90	1.89	1.93
1960	1.92	1.91	1.90	1.92
1961	1.95	1.92	1.92	1.93
1962	1.94	1.94	1.94	1.92

APPENDIX TABLE E-V

OPTIMAL LEVELS OF MILO AND ALFALFA IN POUNDS PER STEER FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

Year	Model I Milo	Model I Hay	Model II Milo	Model II Hay	Model III Milo	Model III Hay	Model IV Milo	Model IV Hay
				- Pounds -				
1949	3,224.86	1,828.26	6,923.60	3,925.19	4,714.41	2,672.73	3,013.69	1,708.55
1950	6,337.04	3,133.77	3,560.93	1,760.94	3,767.23	1,862.96	5,374.63	2,657.86
1951	7,383.28	3,658.04	3,146.15	1,558.76	2,506.85	1,242.01	2,668.34	1,322.03
1952	2,369.62	1,396.29	3,388.31	1,996.55	2,327.76	1,371.63	1,836.94	1,082.41
1953	1,004.75	637.05	4,130.57	2,618.94	3,843.95	2,437.21	4,000.43	2,536.43
1954	1,506.78	963.08	1,370.61	876.04	1,949.43	1,246.00	3,883.91	2,482.45
1955	1,834.32	1,071.52	2,424.42	1,416.23	2,322.03	1,356.42	4,943.05	2,887.50
1956	1,321.63	654.76	1,565.69	774.68	1,490.46	738.40	1,804.84	894.16
1957	2,960.20	1,533.40	2,204.12	1,141.75	2,086.78	1,080.96	2,784.48	1,442.38
1958	6,975.83	3,443.33	3,738.83	1,845.52	3,086.06	1,523.30	3,240.56	1,599.57
1959	7,191.20	3,329.84	6,732.15	3,117.28	4,963.96	2,298.53	4,438.64	2,055.28
1960	8,098.85	3,301.82	10,519.04	4,288.50	8,808.31	3,591.06	7,633.46	3,112.08
1961	4,016.38	1,622.97	6,197.56	2,504.36	5,888.54	2,379.50	7,087.80	2,864.10
1962	3,140.67	1,332.96	3,386.40	1,437.26	3,465.32	1,470.75	4,913.34	2,085.32

APPENDIX TABLE E-VI

OPTIMAL LEVELS OF GAIN IN POUNDS PER STEER FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

Year	Model I	Model II	Model III	Model IV
		- Pounds -		
1949	437.50	790.94	587.21	415.13
1950	716.82	458.57	479.03	630.91
1951	807.27	416.78	349.50	366.83
1952	347.47	458.43	342.70	285.24
1953	181.58	543.10	513.66	529.80
1954	249.02	231.39	304.03	518.71
1955	284.39	353.01	341.40	613.15
1956	212.80	242.43	233.58	270.93
1957	401.43	319.41	306.15	382.84
1958	771.90	476.04	410.26	426.09
1959	779.37	740.52	584.77	536.22
1960	831.18	1,017.88	887.07	793.92
1961	481.72	674.21	648.01	748.12
1962	402.41	426.60	434.29	569.24

APPENDIX TABLE E-VII

OPTIMAL LEVELS OF GRAIN AND HAY IN POUNDS PER DAIRY COW PER LACTATION PERIOD FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

Year	Model I		Model II		Model III		Model IV	
	Grain	Hay	Grain	Hay	Grain	Hay	Grain	Hay
				- Pounds -				
1949	3,800.00	4,479.48	4,098.25	4,731.45	4,476.95	5,168.66	4,002.74	4,621.18
1950	3,543.01	3,879.87	4,139.36	4,532.92	3,831.23	4,195.49	3,749.08	4,105.53
1951	3,677.35	3,963.89	3,245.22	3,498.09	2,857.75	3,080.43	3,417.21	3,683.47
1952	3,969.82	4,137.24	3,282.80	3,421.24	3,120.37	3,251.96	3,354.64	3,496.11
1953	4,045.00	4,495.03	4,041.11	4,490.71	4,257.80	4,731.51	4,141.46	4,602.22
1954	3,542.05	4,263.90	4,216.66	5,075.99	4,061.70	4,880.45	4,148.68	4,994.16
1955	4,019.91	4,666.47	4,742.07	5,504.79	4,135.39	4,800.53	4,630.74	5,375.55
1956	3,863.07	3,978.72	4,094.02	4,216.59	3,624.26	3,732.76	4,285.83	4,414.14
1957	4,330.39	4,985.45	4,161.12	4,790.58	3,962.06	4,561.41	4,672.52	5,379.33
1958	4,190.11	4,909.71	4,110.48	4,816.40	4,013.87	4,703.20	4,214.68	4,938.50
1959	4,109.75	4,620.91	3,048.49	4,552.02	3,866.39	4,347.28	3,995.35	4,492.28
1960	4,310.91	4,684.20	4,176.84	4,538.52	3,950.86	4,292.98	4,093.67	4,448.15
1961	4,168.39	4,283.65	4,095.10	4,208.33	3,908.15	4,016.21	4,095.00	4,208.23
1962	3,779.31	4,049.88	3,902.10	4,181.47	3,740.78	4,008.59	3,894.29	4,173.10

APPENDIX TABLE E-VIII

OPTIMAL LEVELS OF MILK PRODUCTION PER LACTATION PERIOD PER COW FOR THE FOUR SELECTED MODELS FOR THE YEARS 1949 TO 1962

	Model I	Model II	Model III	Model IV
		- Pounds -		
1949	12,714.77	14,614.62	13,458.25	12,872.99
1950	12,185.84	12,309.31	12,570.22	12,462.49
1951	12,343.60	11,084.93	11,167.48	11,989.15
1952	12,672.52	12,203.32	11,517.02	11,852.90
1953	12,876.06	13,886.18	13,131.72	12,988.04
1954	12,325.23	13,030.55	13,013.78	13,123.75
1955	12,903.46	12,818.54	13,049.40	13,539.02
1956	12,518.14	12,257.83	12,204.92	13,045.17
1957	13,277.02	13,131.20	12,816.52	13,684.04
1958	13,132.57	13,299.58	12,910.38	13,163.10
1959	12,966.92	12,905.25	12,656.38	12,822.36
1960	13,160.64	12,996.56	12,712.65	12,893.17
1961	12,898.56	12,918.64	12,572.58	12,807.90
1962	12,469.47	12,708.08	12,418.83	12,618.76

APPENDIX TABLE E-IX

THE STANDARD DEVIATION AND COEFFICIENT OF VARIATION OF NET REVENUE FOR THE FOUR MODELS BY ENTERPRISES

Model	Corn Std. Dev.	Corn C.V.	Alfalfa Std. Dev.	Alfalfa C.V.	Steers Std. Dev.	Steers C.V.	Milk Std. Dev.	Milk C.V.
Model I	23.01	23.41	5.31	11.32	17.68	57.84	21.27	6.74
Model II	22.94	23.37	5.29	11.28	18.00	67.62	22.15	6.89
Model III	22.90	23.33	5.28	11.26	16.69	61.96	21.11	6.56
Model IV	23.13	23.62	5.30	11.30	18.68	72.95	21.66	6.72

APPENDIX F

Size of Enterprise

The cost of uncertainty for corn, alfalfa, steers, and milk is rather small on a unit basis in proportion to the cost of production. However, the results are more significant if a size of enterprise is assumed.

It was assumed that a net revenue of $10,000 must be realized for an enterprise. Using this as a basis, any of the following four sizes would produce a net revenue of $10,000 in 1962: (1) 135 acres of corn; (2) 205 acres of alfalfa; (3) 470 steers; or (4) 32 dairy cows.

The results are presented in Appendix Tables F-I, F-II, F-III, and F-IV. The cost of uncertainty ranges from $184.50 for Model II for alfalfa, to $32,697.90 for Model IV for steers. The cost of uncertainty for the specific size of steer enterprise is several times greater than the cost of uncertainty for the other enterprises.

The results exemplify the importance of the elasticity of the marginal cost curve. The inelastic marginal cost curve for steers caused the loss in net revenue from a small error in price prediction to multiply several times. The relatively elastic marginal cost curve for the other enterprises did not allow an extreme cost of uncertainty from errors in price prediction.

APPENDIX TABLE F-I

COST OF UNCERTAINTY FOR THE SELECTED MODELS FOR A FARM WITH 135 ACRES OF CORN

Year	Model II	Model III	Model IV
		- Dollars -	
1949	2.37	16.20	143.10
1950	110.70	117.45	31.05
1951	24.30	40.50	1.35
1952	22.95	14.85	31.05
1953	1.35	1.35	1.35
1954	2.37	1.35	0.00
1955	8.10	6.75	47.25
1956	112.05	25.65	112.05
1957	12.15	12.15	82.35
1958	0,00	0.00	44.55
1959	9.45	8.10	79.65
1960	10.80	10.80	83.70
1961	5.40	6.75	5.40
1962	0.00	0.00	0.00
Total	321.99	261.90	662.85

APPENDIX TABLE F-II

COST OF UNCERTAINTY FOR THE SELECTED MODELS FOR A FARM WITH 205 ACRES OF ALFALFA

Year	Model II	Model III	Model IV
		- Dollars -	
1949	0.00	0.00	10.25
1950	8.20	10.25	0.00
1951	36.90	45.10	32.80
1952	4.10	8.20	38.95
1953	51.25	43.05	4.10
1954	8.20	8.20	20.50
1955	2.05	2.05	36.90
1956	34.85	38.95	0.00
1957	28.70	20.50	26.65
1958	0.00	0.00	8.20
1959	4.10	4.10	0.00
1960	0.00	2.05	0.00
1961	6.15	8.20	4.10
1962	0.00	0.00	4.10
Total	184.50	190.65	186.55

APPENDIX TABLE F-III

COST OF UNCERTAINTY FOR THE SELECTED MODELS FOR A FARM WITH 470 STEERS

Year	Model		
	Model II	Model III	Model IV
		- Dollars -	
1949	4,342.80	846.00	18.80
1950	197.40	1,743.70	216.20
1951	5,315.70	7,534.10	6,918.40
1952	747.30	0.00	267.90
1953	8,652.70	7,440.10	8,093.40
1954	23.50	197.40	4,032.60
1955	249.10	173.90	4,873.90
1956	61.10	32.90	230.30
1957	286.70	390.10	14.10
1958	2,354.70	3,741.20	3,285.30
1959	37.60	982.30	1,565.10
1960	714.40	65.80	28.20
1961	1,151.50	864.80	2,138.50
1962	23.50	42.30	1,015.20
Total	24,158.00	24,054.66	32,697.90

APPENDIX TABLE F-IV

COST OF UNCERTAINTY FOR THE SELECTED MODELS FOR A FARM WITH 32 DAIRY COWS

Year	Model		
	Model II	Model III	Model IV
		- Dollars -	
1949	279.36	42.24	1.92
1950	0.96	11.52	5.76
1951	140.16	122.56	11.20
1952	20.16	121.92	61.76
1953	86.08	5.76	1.28
1954	39.04	37.12	50.24
1955	0.64	1.60	42.56
1956	5.44	7.68	22.40
1957	1.60	16.32	12.80
1958	2.24	3.84	0.00
1959	0.64	7.68	1.60
1960	1.96	15.36	5.44
1961	0.00	8.64	0.64
1962	4.48	0.00	1.92
Total	582.76	402.24	219.52

VITA

James Ezra Osborn

Candidate for the Degree of

Doctor of Philosophy

Thesis: AN ECONOMIC ANALYSIS OF SELECTED PRICE PREDICTION MODELS

Major Field: Agricultural Economics

Biographical:

Personal Data: Born near Tuttle, Oklahoma, November 9, 1936, the son of Roy and Ida Osborn.

Education: Attended grade school and high school at Tuttle, Oklahoma; graduated from Tuttle High School in 1955; received the Bachelor of Science Degree from Oklahoma State University, Stillwater, Oklahoma, with a major in Agricultural Economics, in June, 1959; completed requirements for the Doctor of Philosophy Degree in August, 1964.

Professional Experience: Graduate student at Oklahoma State University from September, 1959 to August, 1962. Employed as a Research Assistant at the University of North Carolina at Raleigh from September, 1962 to March 1, 1964. Assistant Professor at Auburn University since March 1, 1964.