SYNTHESIS OF CONTROLLED PROCESSES BY A NUMERICAL METHOD

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LIST OF SYMBOLS

Nomenclature is defined as used throughout the text. Important symbols are:

a_{ij} . . . . . . Constant coefficient of the control parameter u_{ij}.

f_i . . . . . . Function of the state of the process and time which defines the ith state equation of the fixed plant.

g_i . . . . . . Function of the control parameters u_{ij} which defines the control for the ith state equation, a control function.

m . . . . . . Number of process states used in least-squares procedure.

m_i . . . . . . Number of control parameters affecting the ith state equation.

n . . . . Number of state equations which describe the process.

t . . . . Independent variable, time.

u_{ij} . . . . . . Function of the state of the process and time which is for control affecting the ith state equation, a control parameter.

x_i . . . . . . The ith of a set of dependent variables which describe the state or the system, a state variable or phase coordinate.

INTRODUCTION

Control of physical processes which arise in engineering problems occupies an important role in present day technology. To control a process is to cause, in one way or another, the process to operate at its best according to some criteria. To this end, two more or less distinct questions are to be answered. One, what is this best, or "optimum," performance? Two, what realizable changes or additions to the process, i.e., what controls, are necessary to obtain this optimum mode of operation, which might be any mode since any operation is optimum to some criteria? The latter question in relation to dynamic processes of an engineering nature is the subject of this thesis.

Accounts of the historical background of control theory and practice can be found in most texts on the subject, the most complete probably being related by Newton, Gould and Kaiser [7].[1] Major topics concerning the present theory of controlled processes are treated in the books by Truxal [9], Gibson [4], Bellman [1] and Pontryagin, et. al. [8], only to name a few.

The scope of this thesis, but not necessarily the underlying concept, is confined to the continuous control of dynamic processes which are described by a system of ordinary differential equations; equations of the form

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[1]Numbers in brackets refer to references in Bibliography.

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n, t) + g_i(u_{i1}, u_{i2}, \ldots, u_{im_i}), \quad i = 1, 2, \ldots, n \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are the state variables, i.e., the variables which describe the state of the process at each value of the independent variable t, time. The relations f_i define the uncontrolled process. Control is brought to bear on the process through the relations g_i made up of the control parameters $u_{i1}, u_{i2}, \ldots, u_{im_i}$ where m_i denotes the number of control parameters which affect the ith state equation. Determination of suitable g_i as functions of the control parameters u_{ij} where

$$u_{ij} = u_{ij}(x_1, x_2, \ldots, x_n), \tag{2}$$

or alternatively,

$$u_{ij} = u_{ij}(x_1, x_2, \ldots, x_n, t) \tag{3}$$

is commonly called "synthesis." When more convenient, higher order equations will be used in lieu of the state model (1).

This thesis explores an idea which proposes to determine, by numerical methods, the relations g_i that control the system described by Equations (1) in such a manner as to give some desired $x_1, x_2, \ldots, x_n$. Taking, without loss of generality, the g_i to be of the form

$$g_i = a_{i1}u_{i1} + a_{i2}u_{i2} + \ldots + a_{im_i}u_{im_i} \tag{4}$$

where the u_{ij} are as in (2) or (3), synthesis is accomplished by selecting the constants a_{ij} such that Equations (1) are satisfied in a least-squares sense over a suitable range of time. In spirit, this constitutes the entire concept.

Examination of the method indicates no restriction, or even distinction, on the model (1) in the sense of its classification, to wit, whether it is linear, nonlinear, continuous, discontinuous, or characterized by time varying parameters. Indeed, elements of (1) need only be known graphically rather than as functional expressions. There are, of course, limitations. It appears there are no general ones, but rather the feasibility of each undertaking must be determined separately. Choosing an allowable set of control parameters u_{ij} which are capable of giving the desired mode of operation would be the paramount problem in many instances.

The work in this thesis might well be termed experimental in a mathematical sense as is indicated by a review of its contents, but the end results are intended to demonstrate the feasibility of an analytical method for parameter synthesis. Chapter I is devoted to formulation of the general problem and outlining the proposed method of solution. In Chapter II, a discussion of the synthesis method in relation to performance criteria is presented. Also, the mechanics of implementing the method are related as well as discussions of important considerations in its use. An illustrative problem is studied in Chapter III with the final objective of showing the method tractable and the intermediate objective of pointing out some considerations involved in its implementation. A second example problem is then attacked with the preceding objectives again in mind plus the idea of demonstrating the versatility of the method.

Results of the mathematical experimentation reported in this thesis encourage the conclusion that synthesis of a controlled process could, in many instances, be accomplished using the proposed method. The applicability of the method depends, to a great extent, on the availability of control parameters capable of giving the desired mode of operation.

The lack of certainty in applying the proposed synthesis method indicates its deficiency in conciseness, but this is more than made up for by its simplicity and generality. Although the mathematical concepts of the method are almost as old as numerical analysis itself,[2] this approach to the problem of synthesis is quite different as evidenced by the complete lack of published literature along these lines. The basic concept of the method was originally employed by Bernhart [2] to the problem of system identification with very good results.

The problem of synthesizing controlled processes is, in many respects, unsolved. Linear systems can be approached with some degree of confidence by transforming the describing equations to the complex domain and using one of several semi-graphical synthesis techniques, notably root locus plots [9] and Nyquist diagrams [4]. An analytical approach to synthesis based on minimizing error integrals also exists [7], but it too invariably ends up in the complex domain and is thus limited to linear plants and control. If the process demonstrates nonlinear characteristics, the synthesis problem is

[2]The books by Lanczos [6] and Hamming [5] on numerical methods are used as references for this thesis.

magnified many times over. No general approach exists, and the most common, the describing function technique [4], is an approximation to amplitude and frequency characteristics of the controlled process. Without reservation, it can be said that the proposed synthesis method, if successfully applied, is far superior in both generality and simplicity to any existing method of synthesis in use today.

CHAPTER I

THE SYNTHESIS PROCEDURE

1. The Problem.

The control of a dynamic process to perform in a desired manner is the object of this thesis. Specifically, this entails controlling a fixed plant described by

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n, t), \quad i = 1, 2, \ldots, n \tag{5}$$

with sets of allowable control parameters $u_{ij}(x_1, x_2, \ldots, x_n, t)$, $j = 1, 2, \ldots, m_i$. The dependent variables $x_i(t)$ describe the state of the process at any instant of time t, the independent variable. No restrictions are imposed on the f_i and u_{ij} other than prior knowledge of their numerical values in certain parts of phase space or, alternatively, over a period of time.

Control is effected such that the controlled plant is described by

$$\frac{dx_i}{dt} = f_i + a_{i1}u_{i1} + a_{i2}u_{i2} + \ldots + a_{im_i}u_{im_i} \ . \tag{6}$$

The synthesis method discussed in the following pages proposes to determine the coefficients a_{ij} which cause the state variables x_i to behave in a specified way.

2. Formulation of the Method.

If the process model (6) is rearranged as

$$a_{i1}u_{i1} + a_{i2}u_{i2} + \ldots + a_{im_i}u_{im_i} = \frac{dx_i}{dt} - f_i , \tag{7}$$

the right side is the fixed part of the model while the left side is changeable so far as the a_{ij} can be varied. At a given instant of time t_k, the process is at a particular point in phase space $(x_{1k}, x_{2k}, \ldots, x_{nk})$. Assuming the velocities of the phase coordinates at t_k known, the relationships appearing in Equation (7) which depend on the state of the system and time are numerically determined. The process can then be characterized, at $t = t_k$, by linear algebraic equations. The synthesis procedure is based on considering m such states, $m >> \sup(m_i)$, which are derived from specifications and solving the resulting set of overdetermined equations for the a_{ij} by what is most commonly called a least-squares approach [5, 6].

The underlying concept is to choose the a_{ij} so that the sum of the squares of the differences in the fixed and variable parts of Equation (7), the residues, at each of the m states is a minimum with respect to the a_{ij}, i.e., to minimize[1]

$$\left[(a_{i1}u_{i1} + \ldots + a_{im_i}u_{im_i} + f_i - \frac{dx_i}{dt})^2\right] = \left[(\text{residue})^2\right] \tag{8}$$

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[1]The notation $[u]$, introduced by Gauss [6], indicates the sum of the m values of u taken at the m states.

with respect to the a_{ij}. This is accomplished by taking the partial derivative of Equation (8) with respect to a_{ij}, $j = 1, 2, \ldots, m_i$, and setting the result equal to zero. For each i this gives m_i equations to be solved for the minimizing a_{ij}. Formally,

$$\frac{\partial}{\partial a_{ij}} \left[(\text{residue})^2 \right] = 2 \left[u_{ij}(a_{i1}u_{i1} + \ldots + a_{im_i}u_{im_i} + f_i - \frac{dx_i}{dt}) \right] = 0$$

which results in the m_i equations

$$a_{i1}\left[u_{ij}u_{i1}\right] + a_{i2}\left[u_{ij}u_{i2}\right] + \ldots + a_{im_i}\left[u_{ij}u_{im_i}\right] = \left[u_{ij}\left(\frac{dx_i}{dt} - f_i\right)\right], \qquad j = 1, 2, \ldots, m_i \tag{9}$$

to be solved for the a_{ij}, $j = 1, 2, \ldots, m_i$.

In essence, that constitutes the synthesis procedure. The remainder of the chapter calls attention to three important considerations in implementing the method. Namely, the specified state variables, the allowable control parameters and the performance of the synthesized system. Additional comments of a more direct nature which are based on both fact and experiment are reserved for Chapter II.

3. Specifications.

In the approach to synthesis presented above, the states of the process at times t_k, $k = 1, 2, \ldots, m$, were assumed specified and given by the phase coordinates $(x_{1k}, x_{2k}, \ldots, x_{nk})$. Further, the velocities of the coordinates at $t = t_k$ were assumed known. Prior knowledge of all these from specifications is, of course, not usually the case.

Generally speaking, the procedure demands the time-domain specification of at least one state variable. Obviously, if more are specified, the specifications must be compatible. The remaining state variables and their velocities are "specified" either through any of the Equations (6) or by operations on the specified x_i, particularly differentiation or integration.

It is clear then that the time history of at least one state variable must be known, but not necessarily as a functional relationship of time. Further, specifications which call for the process to be at a few desired points at given instants are acceptable, because intelligent interpolation between those states can supply the m states needed.

If the f_i and u_{ij} of Equation (6) do not depend explicitly on time, specifying a phase space trajectory would be suitable in most cases. Otherwise, the coordinates must be converted to the time-domain.

It is apparent some caution must be exercised in deciding on a desired system performance, since the specified response must at least be closely realizable by the fixed plant plus control. This consideration depends, to a great extent, on the allowable control parameters.

4. Control Parameters.

In this thesis, it is postulated that a person implementing the synthesis procedure is restricted to some allowable set of control parameters either by his own choice or by physical considerations.

The primary problem, as related above, is the sufficiency of the control parameters to give the desired performance.

Secondary to this, the task of selecting the proper parameters out of an allowable set, or rather omitting the proper ones, must be faced in many instances. That is to say, it is possible to include one or more control parameters whose very presence forbids realization of the desired control. To successfully meet this problem, an understanding of the analysis and behavior of differential equations is a prerequisite.

5. Performance of the Synthesized System.

After the synthesis of control is complete, it is, of course, necessary to determine the response of the synthesized system in relation to the desired performance. As the method might be, and rightly so, likened to the fitting of a surface through a group of points, it is natural to expect the residuals as defined by Equation (8) to be an indication of any differences in the synthesized and specified responses.

This is not the case. It appears that the only such measure is obtained by solving for the actual response of the synthesized system and comparing with the desired response. This is not a difficult matter to do numerically and would most likely be done even if there did exist a performance criterion such as residuals in surface fitting.

CHAPTER II

DISCUSSION OF THE METHOD

1. Performance Criteria.

The presentation of the synthesis procedure in Chapter I made no attempt to show or support by evidence any justification for the method. This chapter, and particularly this section, is devoted to discussion of this aspect. But it is in the third and final chapter, the chapter where results of actual use of the synthesis method are related, that justification becomes apparent. Many remarks in this chapter are derived from experiences of the author during compilation of these results.

It was mentioned earlier that the synthesis method can be likened to the problem of fitting a surface through a set of data points. If the sum indicated in Equation (8) were replaced by a time integral, the method could be noted as a minimization of an integral-error-squared. But this is, in the sense of a finite sum, what a least-squares surface fitting procedure accomplishes.

As far as the mechanics are concerned, the relation to least-squares surface fitting holds, but interpretation of the results is not so simple nor concise. The difference has its origin from finding coefficients of a differential equation by solving for those coefficients from algebraic equations. That is to say, the coefficients do not necessarily perform the same function in a differential equation as they do in its algebraic counterparts.

For instance, it is possible to perform the procedure, obtain a very good fit (small residues), and the resulting differential equation be unstable! Or possibly a stable singular point is introduced not at all where wanted, but such that the system response falls under its influence. Also, the resulting differential equation solution could be stable and approach the desired final value, but the response of the uncontrolled system would be more satisfactory than the controlled response.

These possibilities are not meant to discredit the proposed method of synthesis, but rather to emphasize the unimportance of the role residues assume in the course of the procedure. The method is not formulated on the basis of any particular performance criteria other than the commission to cause the process to behave in a specified way. And the best criteria for the goodness of its performance is to obtain the response by solving the describing equations and compare it with the specifications. The possible difficulties related above are not necessary evils and can be held to a minimum or even eliminated by careful study of the effect of the control parameters on the system and the compatibility of the specifications and control.

2. Control Parameters.

Using the proposed synthesis procedure, the determination of a process control starts with the selection of an allowable set of control parameters. These could be dictated by the physical situation, or they could be most anything the designer can invent. The sensitive area in this phase is the question of compatibility of control and specifications. That is, will one or more of the control parameters in the allowable set be capable of performing the desired control, once provided with the proper coefficients?

If the specifications are not particularly strict nor the control parameters particularly limited, this consideration is of little consequence. To a great extent, the detection and solution of the dilemma, if it occurs, is a matter of judgment. Thus, the need for a general understanding of the behavior of differential equations.

One particularly perplexing problem occurs when the final value of the controlled system is not the same as the specifications and thus usually not the same as even the uncontrolled response. This is attributable to the incompatibility of the specifications and control function and can sometimes be detected by a singular point analysis of the controlled system. In this respect, it is obvious that the controlled process equations must have a singular point at the desired final value of the system. Generally, this consideration poses few problems. The possibility of the control function introducing other singular points, stable or otherwise, under whose influence the controlled response falls is often just that, a possibility, because the location of a singular point so introduced usually depends on the thus far undetermined coefficients. Once recognized, the problem can be attacked by reevaluating and changing the control function.

In a situation where the designer must use a complete set of specified control parameters, the method proceeds without much further concern over the type of control. But if the final set of control parameters is not fixed, additional problems arise. Generally speaking, it is obvious that the desired control is the one using the least control parameters while giving a desirable response. For the arbitrary control problem then, it is necessary to eliminate any control parameters which contribute little or nothing to the control's performance. Many of these often can be eliminated by a study of the describing equations and specifications, but possibly not all of them.

It would indeed be nice if the synthesis procedure would yield zero or very nearly zero coefficients for those control parameters which are not necessary. But this is not the case, for the procedure will generally assign non-zero coefficients to all the parameters, a failing which can be traced to the previously discussed differences incurred using algebraic and differential equations in the same context. The heartening aspect of the situation is the fact that addition of such a parameter to a set of control parameters which exercise fairly good control on the system will not improve on that response. Since computation time to execute the synthesis procedure for all but very complex systems is just a matter of minutes, a process of elimination based on successive addition of control parameters to the control function is feasible and should indicate proper combinations of parameters.

3. Specifications.

The need for compatibility between specifications and the allowable control function has been emphasized in the previous section. This section is intended to indicate the mechanics of determining the specified state variables and their velocities from incomplete specifications as outlined in Chapter I. Although difficult, the discussion will be kept as general as possible. Several considerations of a more specific nature appear in the following chapter.

It is assumed one state variable is completely specified in the time domain (either given as such or determined by interpolation between certain specified points). Some of the phase coordinates (and their velocities) will be either first or higher order time derivatives or single or multiple time integrals of the specified variable. These variables are determined by differentiating or integrating the specified variable the required number of times. Several numerical routines are available for such undertakings [6]. Integration techniques are looked upon as quite successful while differentiation is approached with misgivings and caution [6]. For the purpose at hand no such adverse feelings are necessary, for if the derivatives are anywhere near being compatible with the specified variable, the errors are "averaged out" in the least-squares procedure. Here by errors is meant errors in compatibility, because the derivatives are not specified, thus no true errors.

The method used to obtain derivatives for the work in this thesis was purposely crude so as to illustrate the point. The time history of the specified variable was graphically represented on a sufficient scale. Realizing the derivatives vary continuously, the slope of the specified variable at several selected instants was estimated using a manual angle measuring device, the values were plotted against time, and finally a smooth curve was drawn through the plotted points. The data points needed for the least-squares procedure were then read from the resulting graph. Second derivatives of the specified variable were obtained by the same operation on the first derivative. Of course this type of differentiation would give increasingly erroneous results as the order of differentiation was increased, but using one of the more sophisticated numerical techniques should allow acceptable differentiation several times over and generally would be more efficient than a graphical method since it could be executed by a computer. By the same tokens expressed above, integration techniques could be similarly loose.

Other state variables could possibly be given in terms of the specified variable and others derivable from it as above by some of the state equations (5). An example of this appears in the next chapter. If more than one variable is specified, the foregoing remarks hold true, but the overall problem is obviously simplified. Of course, all specifications must be compatible. Generally speaking, the case of a specified phase trajectory requires, in addition to the trajectory, the velocities of the phase coordinates and the time associated with the state points used in the least-squares procedure. If the relations f_i and u_{ij} in Equations (6) do not contain time explicitly, the latter information is not needed. Other than what has been stated above, the utility and discussion of state space specifications depend on the particular problem.

The popular design term "constraints" has not been directly mentioned thus far, but a word might be in order to clarify the situation of the synthesis procedure in relation to contraints imposed on a controlled process by specifications. Constraints on the response of the system are included in specifying the state variables. Indeed, the specifications which will generally give the state variables are constraints (overshoot, maximum velocity, etc.). Often constraints in a design arise from changes in the operation of system components at certain states of the system (saturation, etc.). Although these constraints could be worked into the specified variables, the synthesis method easily allows for these changes and possibly makes the constraints unnecessary. There seems to be no general type of constraint that could not be incorporated in the method, but this consideration and all the others previously discussed lead to the same conclusion — the applicability of the proposed synthesis method to a problem depends only on that particular problem, not on whether it belongs or does not belong to a certain group.

4. Computing Aspects.

The computations and programming involved in executing the synthesis procedure are not complicated and generally of the common-knowledge type, but it might be well to point out a few items connected with the computational aspects of the procedure. Since the object of the work reported in this thesis was to show the proposed synthesis method tractable and not to see how fast it would give an answer, no computing times will be quoted, but they are very nominal. Fourth order Runge-Kutta formulas [5] were used to solve differential equations during the course of the research.

Computational difficulties associated with solving for least-square coefficients [5] sometimes occur when the number of Equations (9) becomes larger than six or seven although this depends on the control parameters to a great extent. Other than the usual remedies for such difficulties, time scaling the state variables could possibly help. The coefficient matrix of Equations (9) is noted to be symmetric which decreases execution time. To improve results, the possibility of weighting data at certain points is always present [5].

For a process incorporating a graphical relation in its state model, it might be worthwhile to approximate that relation by a closed form expression, possibly a polynomial. Otherwise, actual data points must be provided for the computer. The so-called Forsythe polynomials [3] deserve attention in any data fitting problem of this type.

5. Sensitivity.

There are two distinct questions to be answered that come under the general heading of sensitivity. First, what can be said, in relation to the synthesis method, about the sensitivity of the controlled system response to disturbances from its desired state space trajectory? Second, what effect does the number of state points used in the least-squares procedure have on the ultimate control function? The answer to the first question is, "Nothing." The synthesis method is only concerned with providing proper coefficients for a set of differential equations, while the sensitivity of the process response is governed mainly by the form of the differential equations regardless of their coefficients.

The second question can also be answered rather easily by relying on a good understanding of what has gone before under the topic of specifications. The answer is that, in the general case, the number of state points used in the synthesis procedure does have a marked effect on the ensuing coefficients. This is due to the nature of the specifications and is best explained by the following hypothetical case. Suppose a control is synthesized using a certain number of specified state points, and the controlled response is identical with that specified. If more state points were taken from this response and added to those already used, it is quite obvious that the synthesis procedure would result in the same control. But if the additional points are not a part of the original controlled response, synthesis is being effected for a totally different problem, i.e., one with different specifications, and thus a different control function. Then this question can also be considered as one of compatibility but in a somewhat different light than before, and the degree of compatibility of one set of data points to another is a definite measure of the sameness of their respective control functions.

As a result of the above answer, it might seem that the synthesis method loses still more of any rigor it might possess. This is not completely true, for although the coefficients are very sensitive to the specifications, the controlled response is not. For instance, control was synthesized for a particular problem using two completely different sets of data points but from the same general specifications. The resulting coefficients for one case differed from their counterparts in the other case by as much as twice one or the other, but the associated system responses were of the same degree of goodness. The number of state points needed in a particular problem is surely not fixed, but it is clear that too few can be used. Also too many data points could give poor results, because if incompatibilities are present in the specifications, the more added, the more affect they have on the synthesis.

CHAPTER III

ILLUSTRATIONS OF THE METHOD

1. Control of a Linear Oscillator.

As an example problem to thoroughly study the synthesis method, it was proposed to control the response x of the fixed plant (a linear oscillator)

$$\frac{d^2x}{dt^2} + 2D\frac{dx}{dt} + K^2 x\ , \quad x(0) = 1.0\ , \quad \frac{dx}{dt}(0) = 0.0$$

with control parameters consisting of combinations of x and $\frac{dx}{dt}$. In the state variable description ($x = x_1$),

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = -2Dx_2 - K^2x_1 + a_{21}x_1^a x_2^b + a_{22}x_1^c x_2^d + \ldots \quad .$$

Two cases were considered: overdamped, $D = 5$, $K^2 = 9$; and underdamped, $D = 1$, $K^2 = 17$. No linear terms were included in the allowable set of control functions, because including them would reduce the problem to one with no fixed plant. The uncontrolled responses of the two systems are shown in Figure 1. In the same figure, the response used as the specified x_1 is shown. The specified response

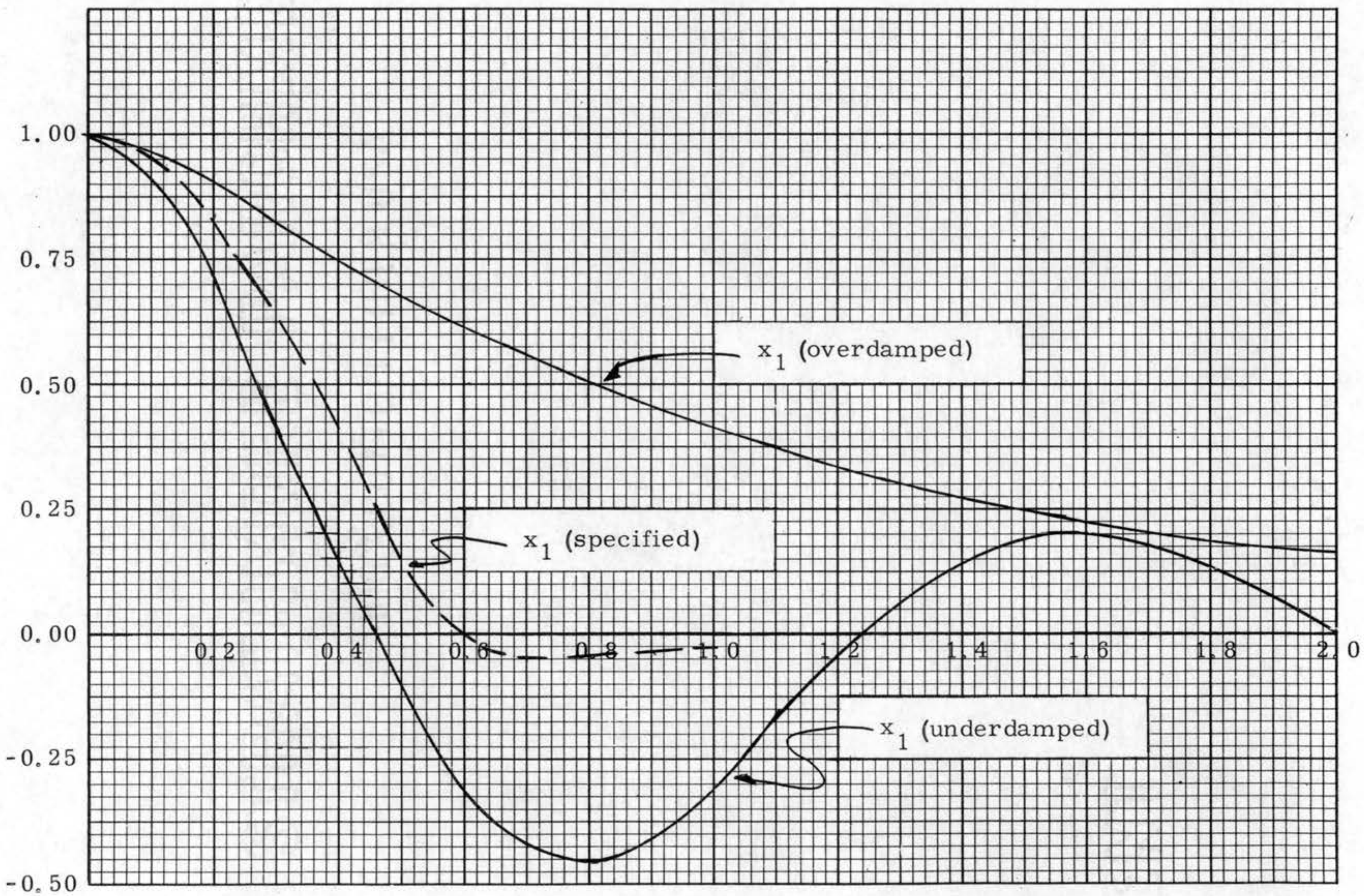

Figure 1. Uncontrolled and Specified Responses for Linear Oscillator.

was chosen for no particular reason other than it seemed compatible and also provided a stiff test for the method. It originated as nothing more than a rough sketch. It is noted that the allowed control gives a singular point at $x_1 = 0$, $x_2 = 0$, or (0, 0), as required by the specification. The remaining state variables x_2 and $\frac{dx_2}{dt}$ were specified from x_1 using the graphical differentiation procedure outlined in the preceding chapter.

Forty-one data points between $t = 0.0$ and $t = 1.0$ were used. The specified response was truncated at $t = 1.0$ for several reasons. Computationally, the values of the specified variables past $t = 1.0$ are small and do not significantly contribute to the least-squares sums. Mathematically, if the response is stable and follows the specified response up to $t = 1.0$, it will continue to approach the desired final value (0, 0). And finally, since the linear terms are not changed, the overdamped case will have a nodal singular point at (0, 0), and the underdamped case will have a focal singular point at (0, 0). Thus, a complete set of specifications for one would not be strictly suitable for the other.

This problem illustrates choosing a suitable set of control parameters from a large number of allowable parameters. The procedure used in this instance was to consider all combinations of x_1 and x_2 of second degree and above and up to and including those of fifth degree (eighteen in all). The least-squares procedure was repeated eighteen times for each case with a new parameter added to the control function each time. Responses for each case were found and studied. Using these results and some analysis, many parameters could be discarded. Further trial and error led finally to some desirable results.[1] Of course many of the trials gave favorable results, even in the first elimination. In view of the desire for the simplest control possible, several good and/or interesting results are shown in Figures 2 through 5.

Other than the specified response being quite demanding, the only major problem occurred in synthesizing the underdamped case. Many control functions would introduce an unstable singular point near the origin in addition to the stable one at (0, 0). Quite often the controlled response would be very good until it started approaching the desired final value, then it would become unstable. It is noticed the controlled systems of Figure 5 do not possess singular points except at (0, 0). If the results shown are not considered satisfactory, the controlled linear oscillator described by

$$\frac{dx_1}{dt} = x_2$$

$$\begin{aligned}\frac{dx_2}{dt} = {} & -10.0\, x_2 - 9.0\, x_1 + 178.02\, x_1^2 + 58.95\, x_1 x_2 + \\ & 0.03\, x_2^2 + 323.78\, x_1^3 + 60.68\, x_1^2 x_2 + 3.99\, x_1 x_2^2 + \\ & 0.46\, x_2^3 + 141.02\, x_1^4\end{aligned}$$

responds almost exactly as specified although the control function is quite complicated.

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[1] Although no computing times were to be mentioned a note that the actual computing time needed for the work outlined is less than three hours seems to be in order.

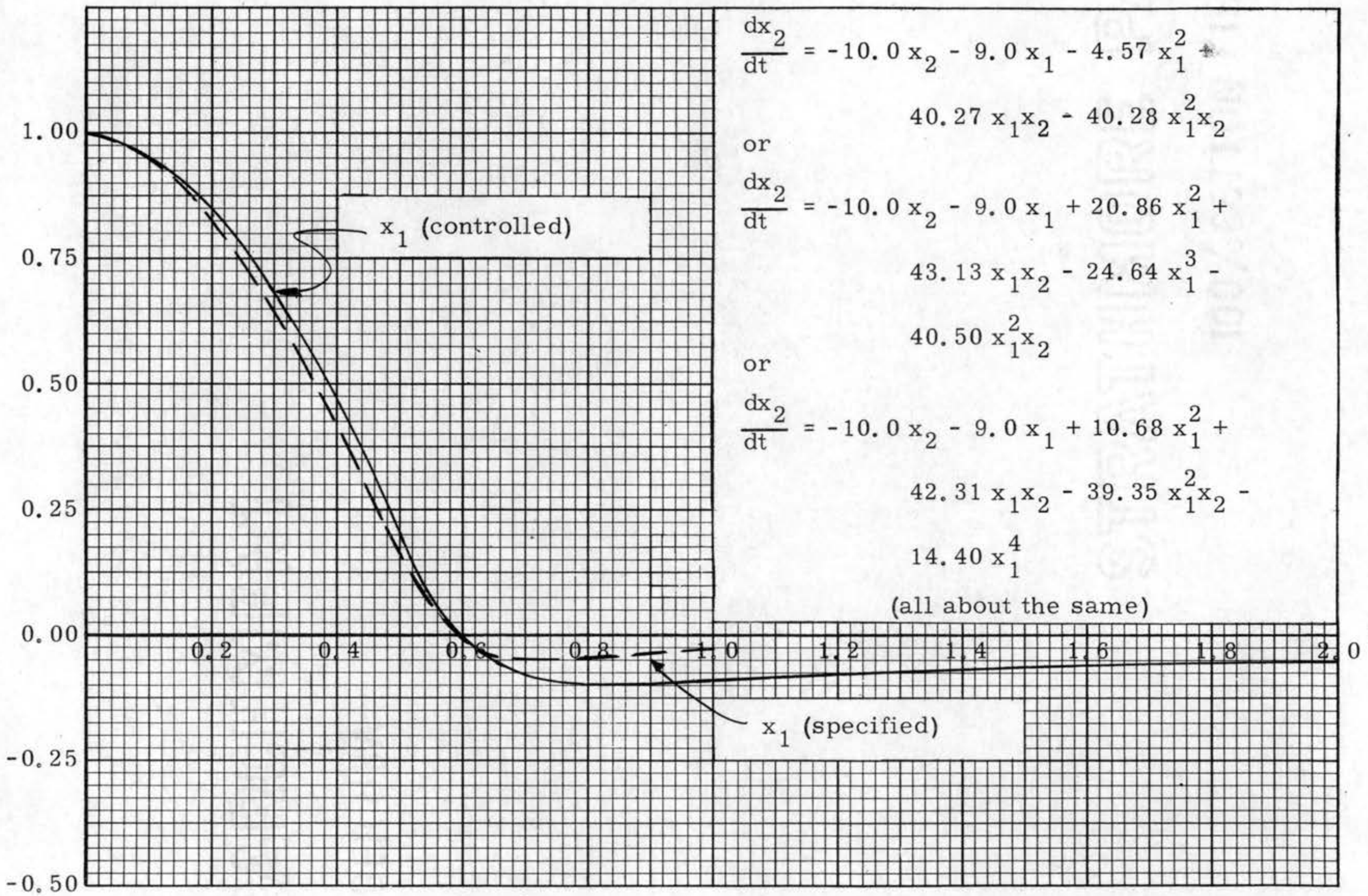

Figure 2. Controlled Response for Overdamped Linear Oscillator.

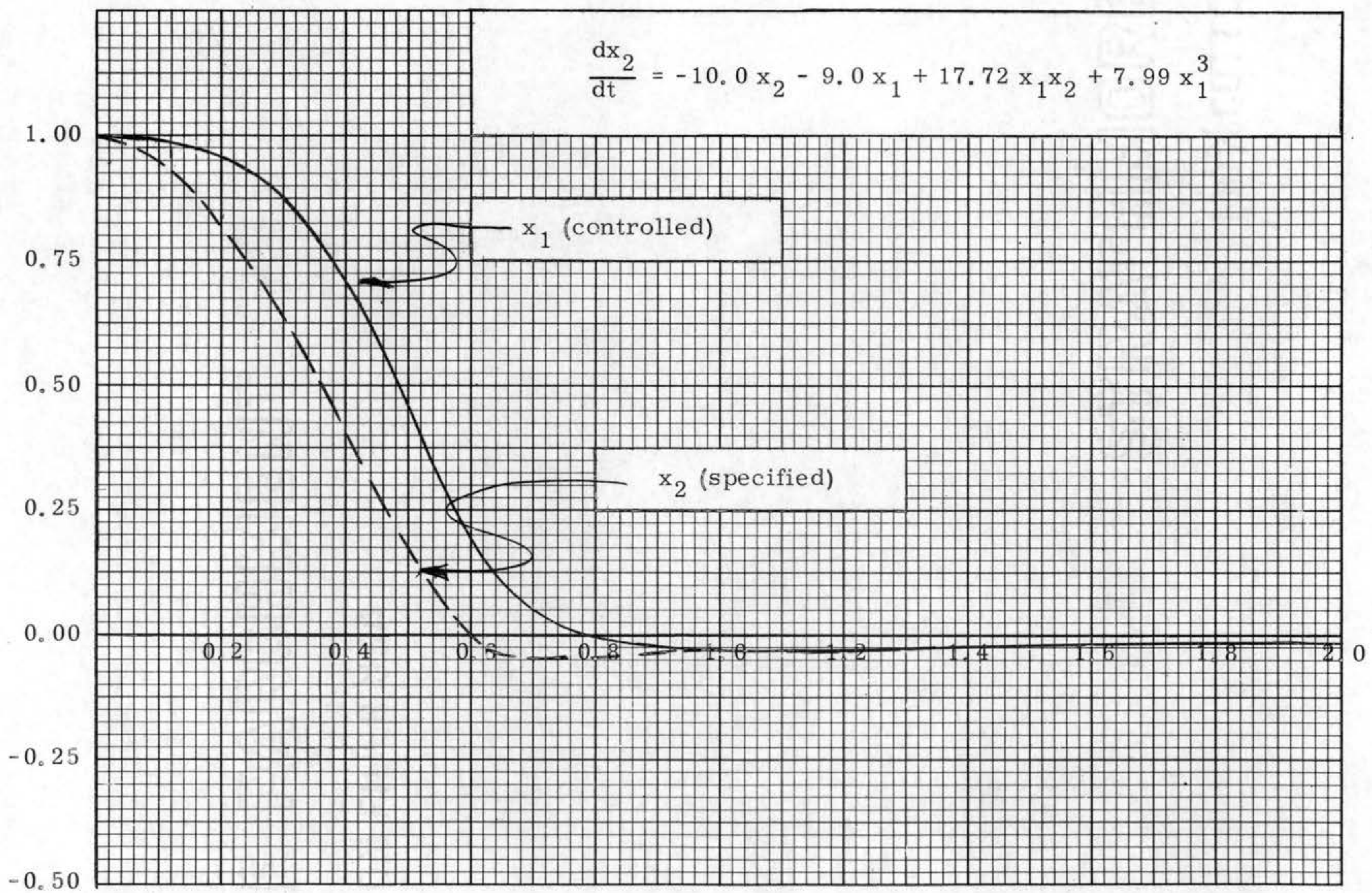

Figure 3. Controlled Response for Overdamped Linear Oscillator.

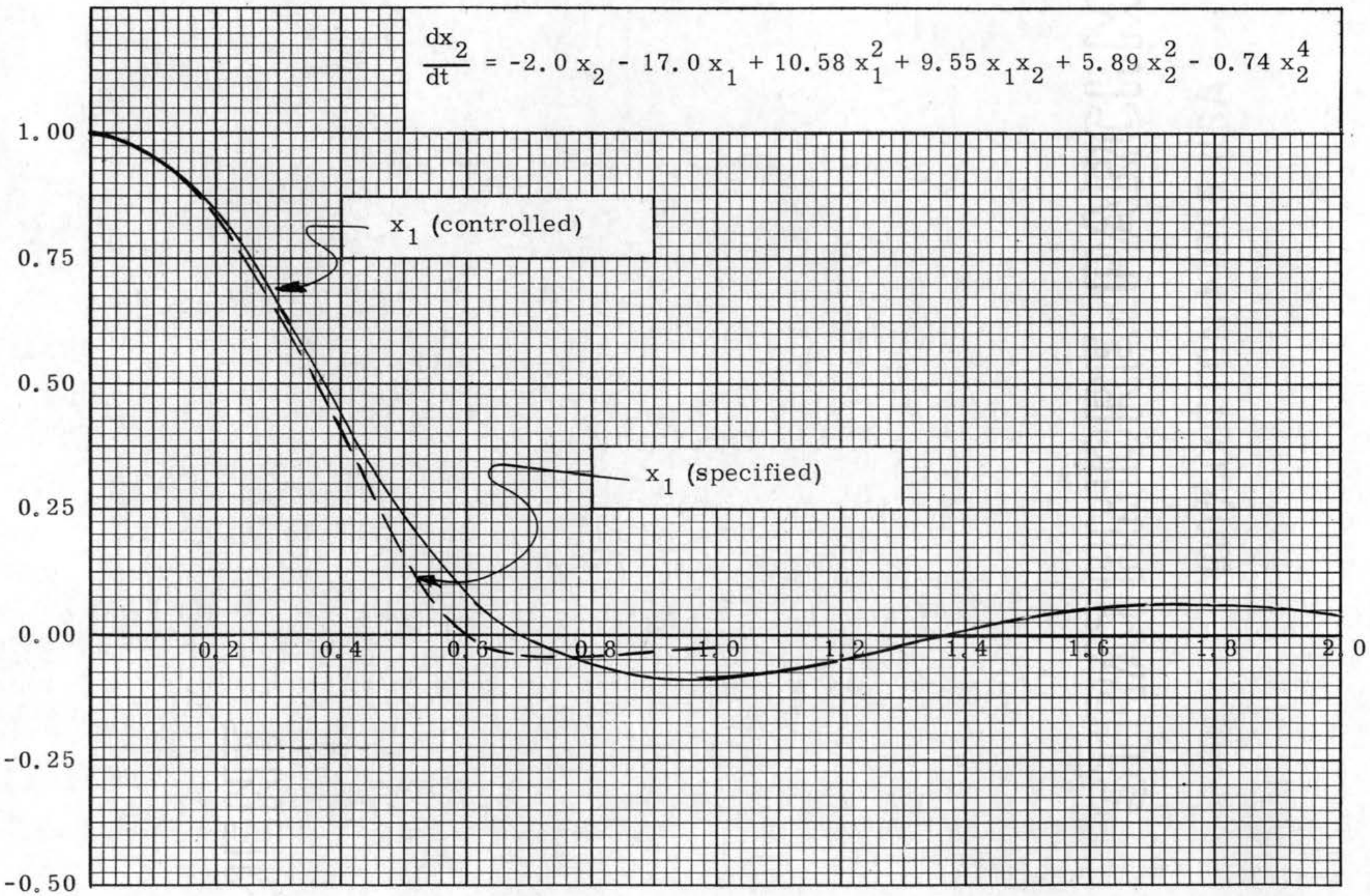

Figure 4. Controlled Response for Underdamped Linear Oscillator.

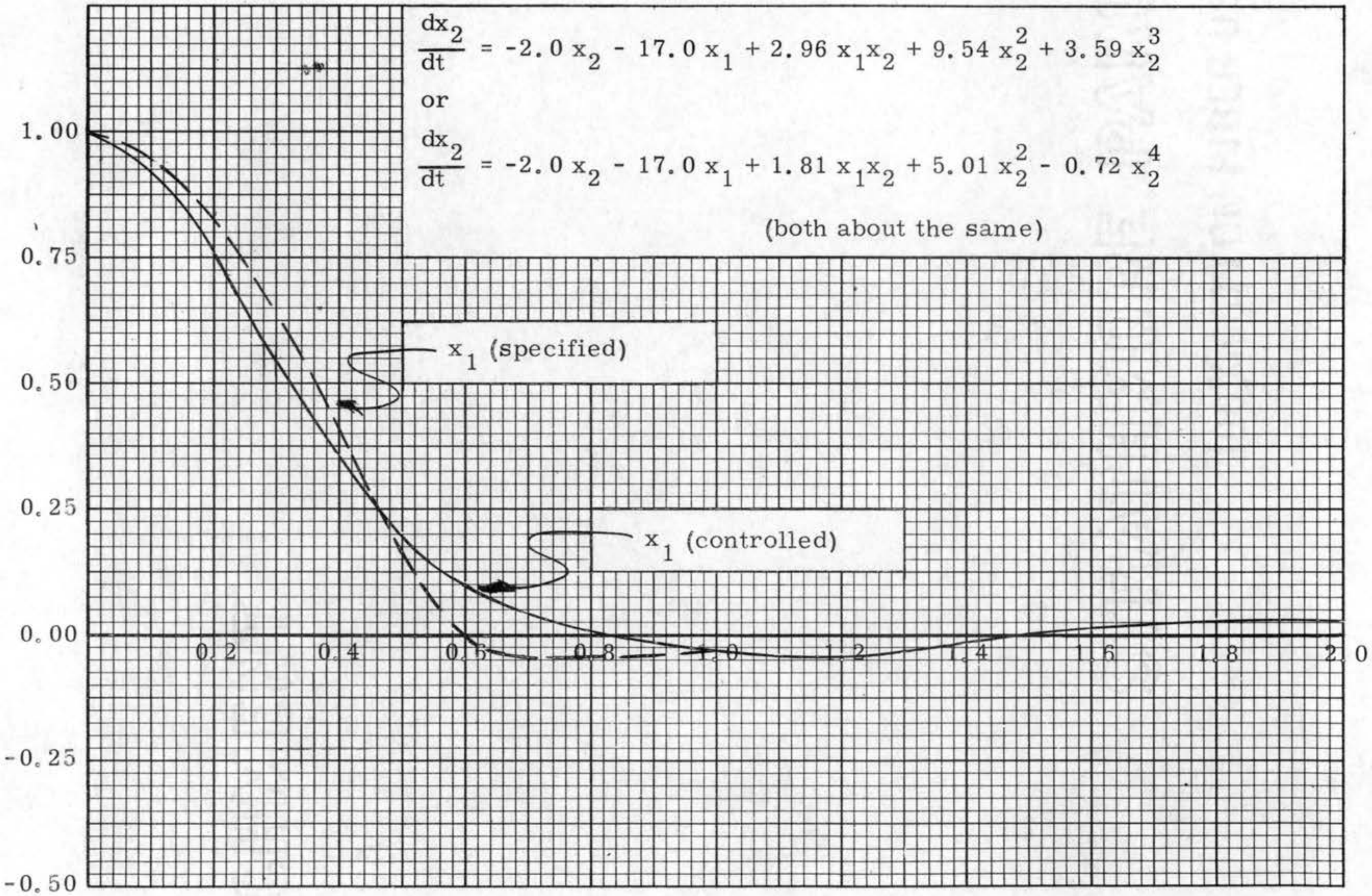

Figure 5. Controlled Response for Underdamped Linear Oscillator.

2. Control of a Multivariable, Nonlinear, Time Varying Parameter Feedback Process.

The example problem of this section is presented to give an idea of the type of process the synthesis method can handle. As evidenced by the title, it was designed to include several difficulties, all of which would not be admissible in any other method. Figure 6 shows the process in block diagram form. In state variables,

$$\frac{dx_1}{dt} = x_3$$

$$\frac{dx_2}{dt} = x_4$$

$$\frac{dx_3}{dt} = -0.1\, x_4^2\, (\text{sign}\, x_4) - 0.2\, x_2^3 + 0.1\, x_3 - a_{31}u_{31}\, \overbrace{(x_3 - x_4)}^{\dot{z}} - a_{32}u_{32}\, \overbrace{(x_1 - x_2)}^{z} - a_{33}\, \overbrace{(x_1 - h)}^{e(t)}$$

$$\frac{dx_4}{dt} = -0.1\, x_4^2\, (\text{sign}\, x_4) - 0.2\, x_2^3 + 0.1\, x_3\ .$$

The coefficients a_{3j} were found using the proposed synthesis method such that the state variable x_1 would behave as shown in Figure 7 when the system input h(t) is as shown in the same figure.

The plant studied bears no intentional resemblance to any actual process and was derived solely from the author's imagination. An attempt, and a seemingly successful one at that, was made to include most every general classification of process describing equations. The specification of x_1 was thought to be quite compatible

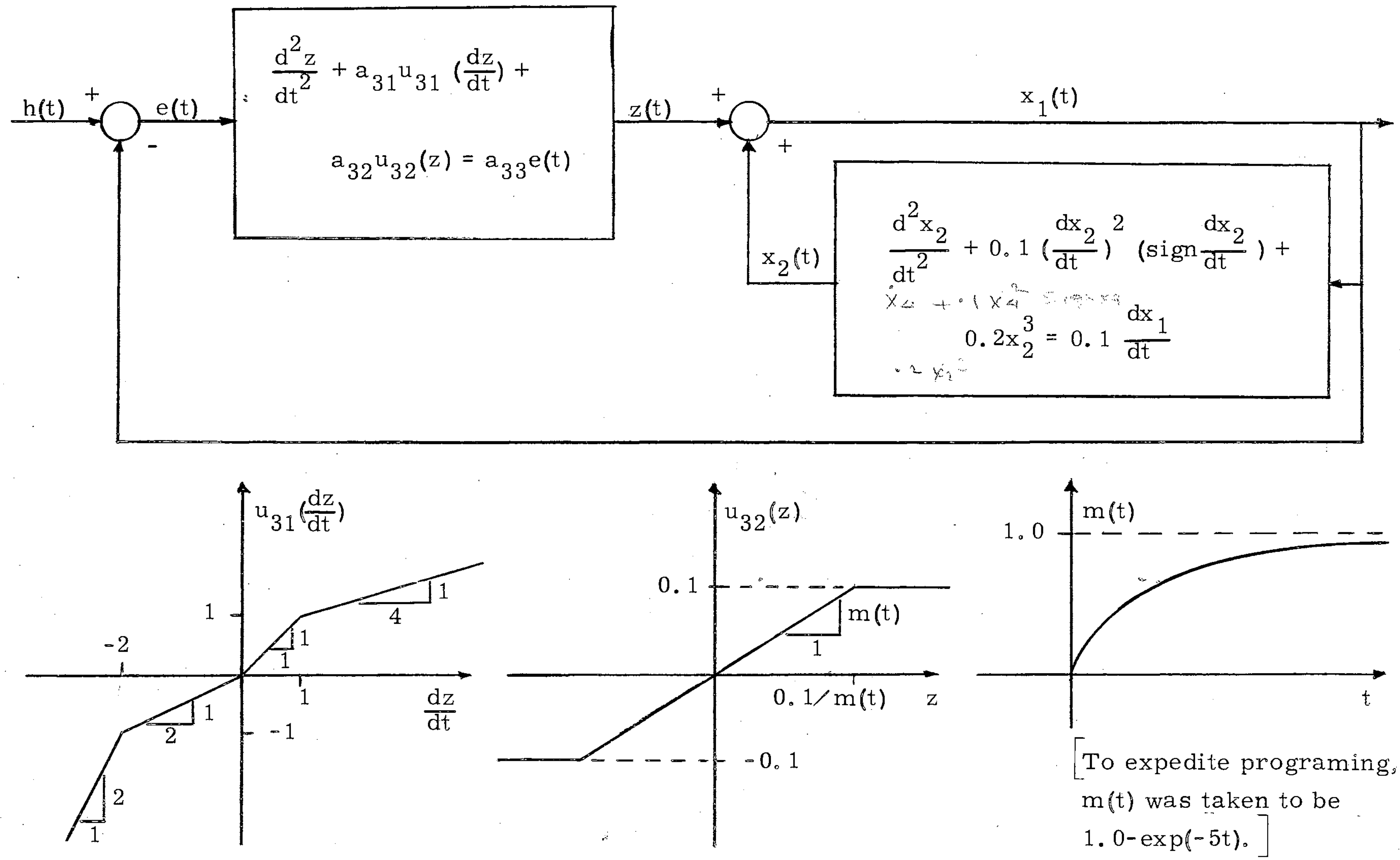

Figure 6. System for Second Illustrative Example.

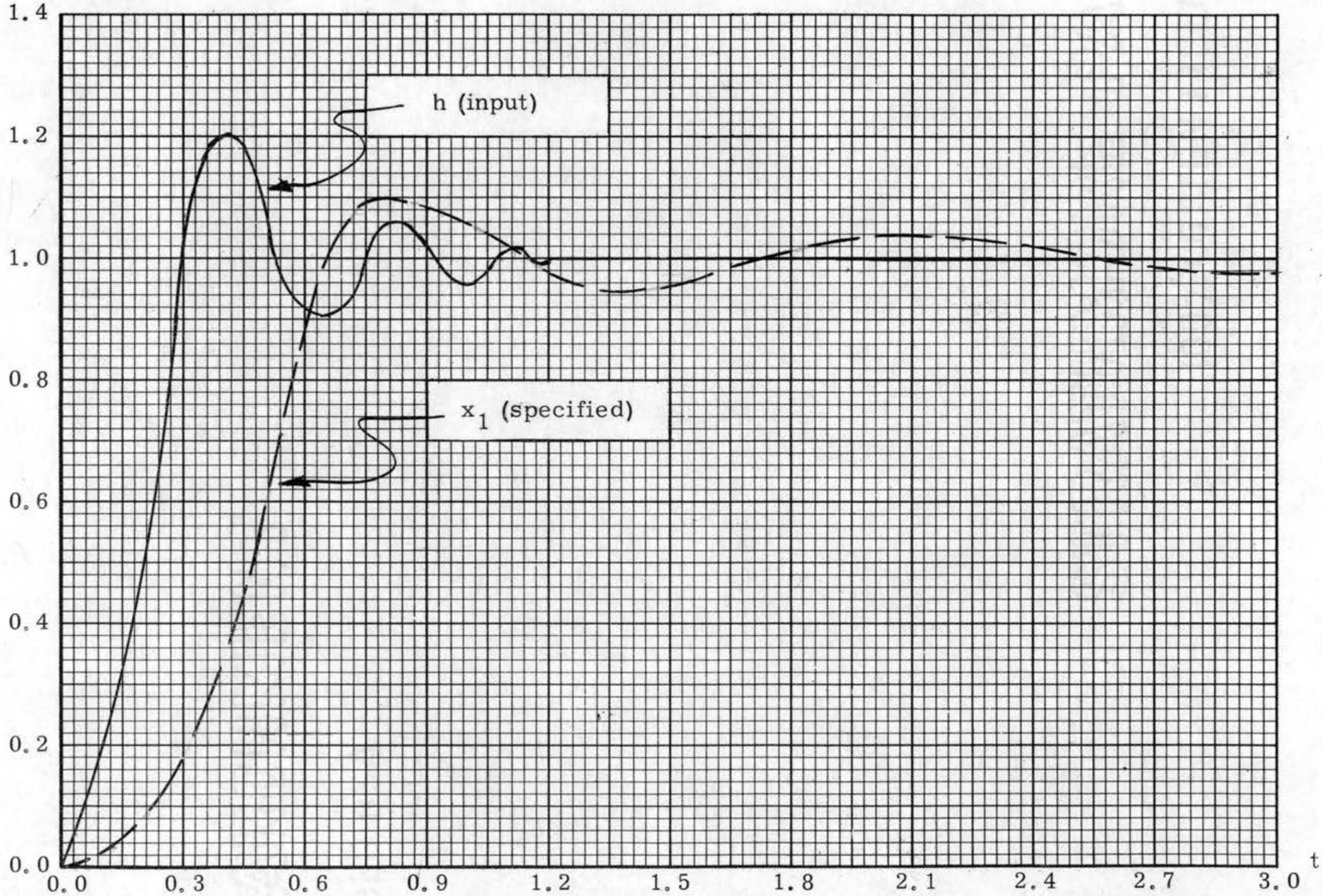

Figure 7. Input and Specified Response for Second Example.

with the control parameters and input in the sense discussed in Section 2 of Chapter II. This was intended since the primary obstacles were designed to be the system describing equations. Unexpectedly, the state variable x_2 approaches a final value other than zero which, if anything, makes the problem harder.

This problem illustrates the case where state variables must be specified through the state equations. After determining x_3 and $\frac{dx_3}{dt}$ from x_1, the last of the above describing equations was solved, giving x_2, x_4 and $\frac{dx_4}{dt}$. Thus, the specifications were completed. One hundred and twenty-one state points between t = 0.0 and t = 3.0 were used. The remainder of the procedure is straightforward. The resulting control is

$$\frac{dx_3}{dt} = -0.1\, x_4^2\,(\text{sign } x_4) - 0.2\, x_2^3 + 0.1\, x_3 - 9.35\, u_{31}(x_3 - x_4) - 8.92\, u_{32}(x_1 - x_2) - 24.72\,(x_1 - h) .$$

The response x_1 of the controlled system is shown in Figure 8 along with the specified x_1 (x_2 is also shown).

In arriving at the specified x_1, the type of singularity at (1, 0) could have been chosen as either a focus or a node. It is noted that both would have about the same effect on the least-squares procedure with the focal type having the greatest effect compared to a straight line specification (unlike the first example, the state points as the process approaches its final value must be included in the specification, because the values of x_1 are not zero). For this reason, the specified x_1 was chosen as shown with no guarantee that the synthesized control would provide this type of response. In fact, the controlled

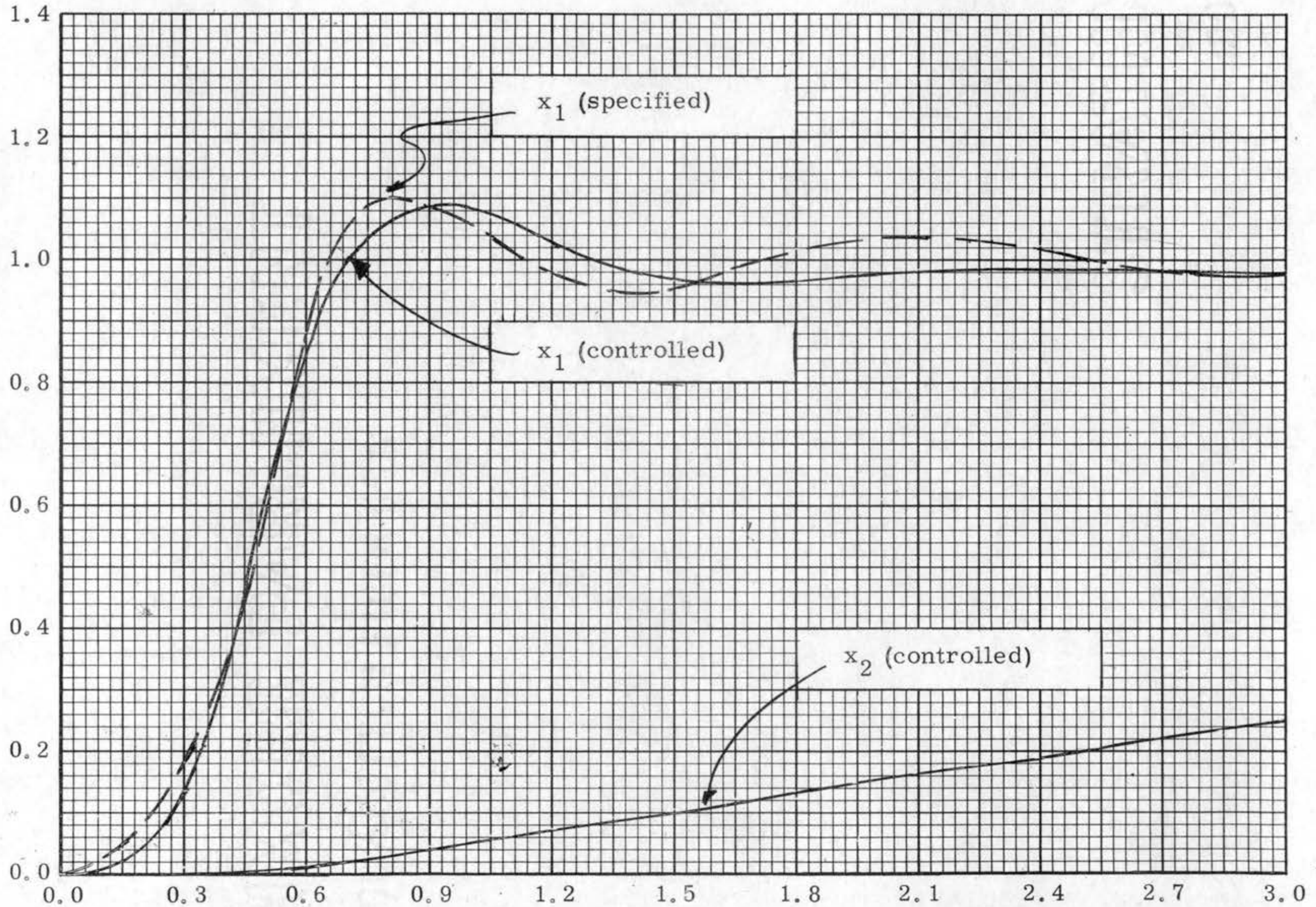

Figure 8. Controlled and Specified Response for Second Example.

response approaches a stable focal point at (.983, 0) rather than at (1, 0) as specified. The performance of the synthesized control might or might not be satisfactory for any particular problem, but it seems safe to state that this is a very successful attempt to synthesize a control for a process sufficiently complicated to preclude the use of any existing synthesis technique.

SUMMARY AND CONCLUSIONS

It was the purpose of this thesis to show that the synthesis method set forth in Chapter I is useful for synthesizing control of dynamic processes such as can be described by Equations (5) and controlled as per Equations (6). Due to the nature of the method, this objective could only be accomplished by studying the results of the procedure when applied to specific problems. Limitations and considerations in implementing the method which became apparent during the course of this mathematical experimentation were discussed in Chapter II.

Results of two such attempts at synthesis of control were presented in Chapter III. These results were considered very good in light of the problems studied, because analytic synthesis of control for either system could not be accomplished by any other synthesis method now in existence. It appears that the proposed synthesis method is feasible and could become a useful tool in the synthesis of controlled processes.

A few words about visualized extensions of the work reported here follow. To start, the synthesis of processes described statistically [4] seems feasible. The concept of the method surely deserves study in relation to problems in adaptive control [1]. The method's standing relative to the synthesis of optimum switched processes [8] should be clarified. Finally, more investigation of the synthesis procedure itself would be justified in order to further establish pertinent considerations.

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