PROCESS QUALITY CONTROL

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PROCESS QUALITY CONTROL

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PREFACE

This dissertation is concerned with a new approach to the general problem of quality control. Industry has accepted statistical quality control as a valuable tool for rapid, timely, and economical control of process output. Such widespread use of sampling techniques for obtaining quality data makes investigations of new quality control systems especially important.

The proposed control system, Process Quality Control (PQC), considers sequential inspection of process output as being representative of a three dimension random walk with absorbing barriers. All inspection is accomplished by go-not go gaging, i.e. attribute inspection. PQC is particularly unique in that both average size, $\overline{X}$, and range, R, is controlled by attribute inspection. No other existing quality control system has this characteristic.

I am greatly indebted to several persons for their assistance in this dissertation effort. Professor Wilson J. Bentley has been a constant source of encouragement and has offered valuable council for a number of years. Dr. James E. Shamblin contributed many important suggestions in my research work. His council and advice are greatly appreciated. I wish to express appreciation to

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SYMBOLISM

ASN = average sample size

m = specified dimension

N = number of paths

O.C. = operating characteristic

p = per cent parts in the lot that measure less than the lower tolerance limit

PA = probability of acceptance

PQC = process quality control

q = per cent parts in the lot that measure more than the upper tolerance limit

r = per cent parts in the lot that measure within tolerance limits

RX = undersize reject absorbing barrier

RY = oversize reject absorbing barrier

RZ = accept absorbing barrier

σ' = standard deviation

$\overline{X}$ = the average value of several trials or samples

CHAPTER I

INTRODUCTION

Quality control as a production tool has been growing at an increasing rate for many years. According to Duncan (1) as far back as the Middle Ages efforts to control quality were deemed important. The old medieval guilds controlled quality by requiring long periods of apprenticeship prior to acceptance as a journeyman. In later years the introduction of formal inspection of output, the development of standards, and legislative acts give evidence of continued concern for quality.

Beginning in the 1920's a new concept was developed which is now known as statistical quality control. The application of probability and statistical theory to quality control problems was made first by Dr. Walter A. Shewhart of the Bell Telephone Laboratories. The importance of his work is now recognized by all knowledgeable persons who are concerned with quality standards. His original work has grown into a well documented, widely used body of knowledge and therefore will not be re-documented in this paper.

In 1947 Wald (2) presented a new concept of quality control using sequential sampling techniques. His method requires continuous inspection of process output until such time as lot quality is either above or below computed limits. Use of Wald's method requires the selection of two probabilities and two per cent defect levels. These are shown below.

P_1 = Maximum allowable per cent defects for acceptance

P_2 = Minimum per cent defects for rejection

σ_1 = Probability of rejection at P_1

σ_2 = Probability of rejection at P_2

From these values accept and reject boundaries are computed. Figure 1 portrays Wald's method where an accept or reject decision is determined by continued inspection until a boundary is reached. The decision is dependent on one of the following conditions:

$D_n \geq R_n$ Reject

$D_n \leq A_n$ Accept

where

D_n = Number of defects in n trials.

Wald's method is significant in that a small sample size generally, but not always, will predict lot quality.

Other systems of sequential sampling have been developed by Burr (3, 4, 5), Shainin (6), and Shamblin (7). These latter works are modifications and extensions of Wald's method.

In this dissertation an extension of quality control theory is developed using sequential inspection procedures.

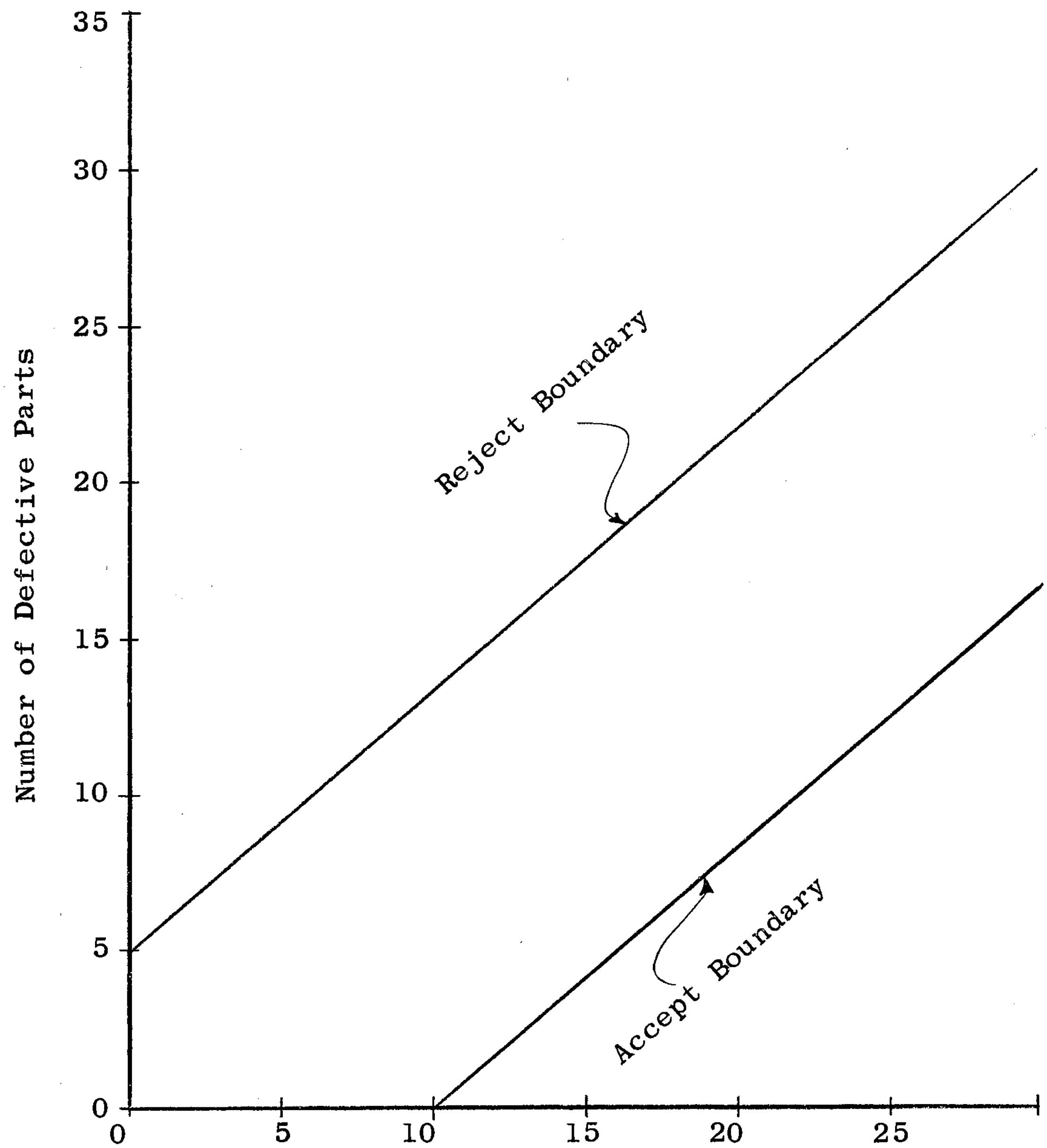
A three dimensional model having X, Y, and Z axis is employed to depict the cumulative effect of a "random walk" (8) condition representing the results of successive inspections of process output. The model size is variable depending on the location of absorbing barriers which represent reject and accept limits. Inspection of a part which is less than the lower tolerance limit represents a step in the X direction, parts having dimensions larger than upper tolerance limits indicates a step in the Y direction, and parts within tolerance limits indicates a step in the Z direction. Inspection is continued until an absorbing barrier is reached and a decision to accept or reject the process is made. The basic mathematical development of this system of statistical quality control is presented in the following chapter.

The term Process Quality Control, PQC, has been chosen to identify the proposed statistical quality control system. Inspection is accomplished by simple go-not go gaging of output which dictates movements within the PQC model. The system provides control of average part size, $\overline{X}$, and range, R, by attribute sampling only. There is no other system of attribute inspection which provides control for both $\overline{X}$ and R. The theory developed in this paper is unique in this respect.

In Chapter III the characteristics of PQC are presented. Particular emphasis is given to the effect of absorbing barrier locations since variable barrier positions provide numerous operating characteristic's curves.

Application of PQC to a typical process control problem is illustrated in Chapter IV. The ability of PQC to detect both controlled and out of control process output is simulated by use of random numbers.

Chapter V compares PQC with other systems of statistical quality control by concentrating on the advantages and disadvantages of each. The final chapter briefly summarizes the dissertation theory and suggests several areas of additional study.



Total Number Parts Inspected

Figure 1. Wald's Sequential Analysis

CHAPTER II

BASIC THEORY

Random Walk

The proposed Process Quality Control, PQC, system is that of developing a three dimensional model as the basis for a new approach toward achieving a simplified, more meaningful method of controlling output quality from a given production system. The model to be employed is best visualized as a three dimensional grid having X, Y, and Z planes and absorbing barriers in each plane at variable positions. Such a model lends itself readily to random walk concepts. The simplest random walk condition requires that any event be independent of past events which is the case in production processes; i.e., for any average size, $\overline{X}$, it is assumed that the probability of a part being above X is equal to the probability of its being below $\overline{X}$. For purposes here the model to be used will consider random walk movements as being in positive directions only and no negative or backtracking motions will be permitted. Thus, from an initial position at the model origin each successive step will increase the distance from the origin until a decision to accept or reject the process is made. Movements within the three dimensional model can occur along any of the three planes and will continue until one of three absorbing barriers (planes) is reached. The accept or reject decision is dependent upon which absorbing barrier is touched first. Once any barrier is reached the random walk ceases and the decision, accept or reject has been made. In effect, such a theoretical approach to process control is related directly to the per cent defective parts being produced by the process. The use of random walk theory is a convenient method for depicting the cumulative effect of a sequence of physical inspections of items from a production system. By proper positioning of absorbing barriers within a grid, the final accept or reject decision provides a realistic, convenient, and accurate measure of true operating characteristics of the process.

Inspection Procedure

Process Quality Control makes use of standard inspection procedures but requires only simple go-not go type gaging. A very important and unique characteristic of the three dimensional model is that not only is control of average size, $\overline{X}$, maintained but safeguards against range, R, variations greater than those permitted by tolerance limits are provided as well. The ability to control range is particularly unique when considering that attribute inspection only is employed to control the spread of part sizes. Shewhart Control Charts require that part sizes be measured exactly and the range calculated by subtraction before plotting.

Thus, PQC affords an advantage in that inspectors are not required to perform any mathematical calculations in order to control both $\overline{X}$ and R. A more detailed explanation of this concept will be outlined in Chapter IV.

As stated previously, the model design in three dimensions affords random walk movements in three distinct directions. Movements along planes are governed by one of three conditions. The three conditions are established by physical inspection and are specified as being above, below, or within specified tolerance limits. As related to the PQC model, parts below the lower tolerance limit indicates a step in the X direction, parts above the upper tolerance limit indicates a step in the Y direction, and parts within tolerance limits specifies a step in the Z direction. Therefore, repeated inspections of produced parts provide decisions for successive random steps in the three dimensional grid which ultimately results in reaching one of the absorbing barriers in the three planes and the accept or reject decision for the entire process.

Probability of Reaching An Absorbing Barrier

Since parts which fall within tolerance limits specify steps in the Z direction, if a process were capable of producing only parts within tolerance limits nothing but Z steps would ever be made and the accept barrier would be reached with a minimum number of inspections. However, no process is ever in perfect control and some parts above and below tolerance limits will be produced due to chance variation. Thus, due to chance alone, steps other than those in the Z direction will result. The PQC system then must consider some means of positioning the absorbing barriers in the grid in such a way that specific quality levels will be established as successive inspections are made. One should keep in mind that at any time during the random walk that a barrier is touched a decision to accept or reject the process has been made. For a realistic situation it is obvious that the two reject barriers must be positioned closer to the grid origin than the accept barrier. For a given per cent defective parts there must be a relationship between this per cent defective and the barrier positions in the grid. In effect, the position of an absorbing barrier establishes various operating characteristics. This concept will be developed in detail in Chapter III. For the present the discussion will be limited to a method of determining the probability of acceptance for any model size. Figure 2 is a 3 x 3 x 4 PQC model where maximum values for the variables X, Y, and Z are:

$$X_{max} = 3$$

$$Y_{max} = 3$$

$$Z_{max} = 4$$

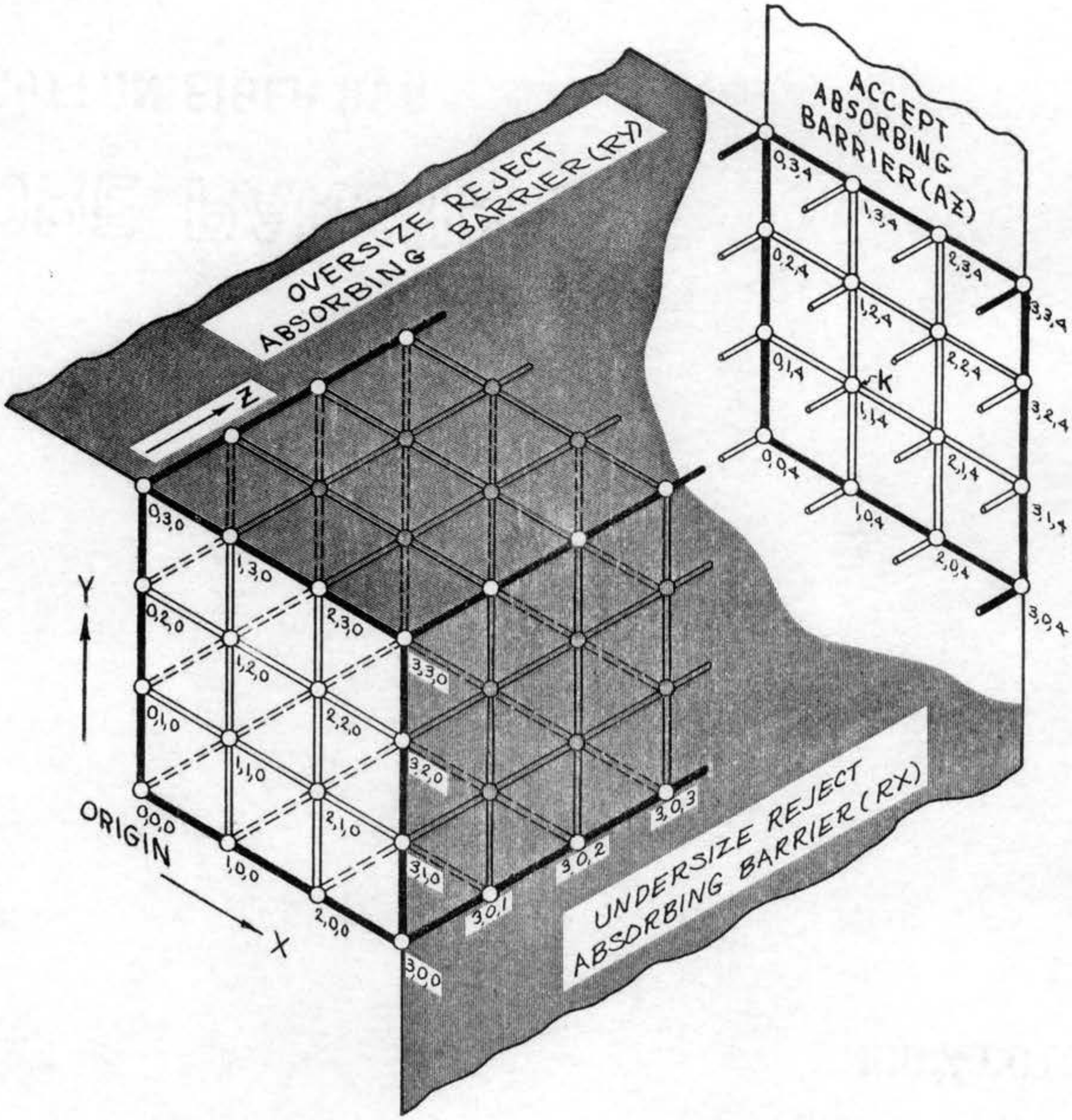


Figure 2. Process Quality Control Model

Reference to Figure 2 will be helpful in formulating two expressions relating to the probability of acceptance, PA. The first probability of acceptance expression is limited to the probability of reaching a specific point on the accept barrier by following a specified path. The second part is the development of an expression for calculating PA for the general case.

Probability of Acceptance, Specific Case

The probability of absorption by the accept barrier at point K in Figure 2 by an exact path is

$$PA(K) = p^i \; q^j \; r^h$$

where

PA(K) = probability of acceptance at the exact point K.

K = the point on the accept barrier where $X = 1, Y = 1, Z = 4$.

p = probability of a single step in the X direction, or per cent parts in the lot that measure less than the lower tolerance limit.

q = probability of a single step in the Y direction, or per cent parts in the lot that measure more than the upper tolerance limit.

r = probability of a single step in the Z direction, or per cent parts in the lot that measure within tolerance limits.

i = number of steps in the X direction.

j = number of steps in the Y direction.

h = number of steps in the Z direction.

If the restriction of reaching point K following an exact path is eliminated it is obvious that many other paths might be followed to reach this same point. Then, the probability of absorption at point K is

$$PA(K) = N\ p^i\ q^j\ r^h$$

where

N = number of paths from the origin to this point without previously touching a barrier at any other point.

Following the above reasoning it is apparent that the probability of acceptance in exactly K steps following any effective path is

$$PA(K) = N\ p^i\ q^j\ r^h. \qquad K = i + j + h$$

The above indicates that any meaningful expression for calculating PA must include a method for determining the number of paths to any point on the accept barrier. Therefore, consideration of this problem will be necessary before a general expression for calculating PA can be formulated.

Computing Number of Paths, N

The number of different ways to reach any point in the three dimensional model can be determined by combinatorial procedures. The number of ways to divide $i + j + h$ things into three groups with i things alike, j things alike, and h things alike is

$$\frac{(i + j + h)!}{i!\ j!\ h!} \qquad (8)$$

or

$$\binom{i + j + h}{i,\ j,\ h}$$

In finding the number of paths to a point on the accept barrier, or any other point as well, it is necessary to determine the number of steps in the i direction, the number of steps in the j direction, and the number of steps in the h direction. Calculation of the number of ways, N, to any point can then be made by direct substitution in the given formula. For example to find the number of ways to point K where

$$i = 1$$

$$j = 1$$

$$h = 4$$

$$N = \binom{1 + 1 + 4}{1,\ 1,\ 4} = \frac{6!}{1!\ 1!\ 4!} = 30$$

This method will be employed for all cases where N must be determined. And, as will be shown in the following section this method for computing N is valid in every case.

Probability of Acceptance, General Case

By referring to Figure 2 it is apparent that there are nine points on the accept barrier, which if reached, would result in process acceptance. These nine points specifically have the following X, Y, Z values: 0, 0, 4; 1, 0, 4; 2, 0, 4; 0, 1, 4; 1, 1, 4; 2, 1, 4; 0, 2, 4; 1, 2, 4; 2, 2, 4. Other points on the accept barrier are: 3, 0, 4; 3, 1, 4; 3, 2, 4; 3, 3, 4; 0, 3, 4; 1, 3, 4; 2, 3, 4. However, by observation it is seen that these latter points cannot be reached without first touching a reject barrier on a previous step. Thus, the only points which permit process acceptance are the nine former points.

Note that it is possible to reach any one of the accept points on the accept barrier from only one other point. For example, accept point K can be reached only from the previous point 1, 1, 3 (X = 1, Y = 1, Z = 3) and in no other way. Generalized this means that to reach any accept point X, Y, Z one would first have to reach point X, Y, Z-1 on the prior step. This condition is <u>not</u> to say that there is only one possible path to point K but only to indicate that the number of paths to it are equal to the number of paths to the previous point (in this case, point 1, 1, 3). The same general reasoning is applicable to all points on the accept barrier. Applying the above to the given example it is true that

$$N(0, 0, 4) = N(0, 0, 3)$$
$$N(1, 0, 4) = N(1, 0, 3)$$
$$N(2, 0, 4) = N(2, 0, 3)$$
$$N(0, 1, 4) = N(0, 1, 3)$$

etc.

Notice that there are zero effective paths to the seven accept points on the accept barrier that are common to one or more reject barriers. This general condition will always exist for points common to more than one absorbing barrier. This means that PA for these common points is always zero.

The probability of acceptance is the summation of probabilities for all points on the accept barrier. The general expression for probability of acceptance must then be a means of summing the various probabilities associated with accept points on the accept barrier.

For reasons of simplifying the probability of acceptance expression it is necessary to define precisely all barriers and their locations in the grid. In Figure 2 each absorbing barrier is identified as

RX = undersize reject absorbing barrier

RY = oversize reject absorbing barrier

AZ = accept absorbing barrier

Furthermore, barrier locations will be related to the minimum distance or steps from the grid origin to each individual barrier. In addition each barrier is perpendicular to the minimum distance line from the origin, considering all possible plane angles, i.e., the absorbing barriers are either parallel or perpendicular to the three axis X, Y, and Z. With reference to the given example it is possible to define and locate each absorbing barrier as follows:

RX = 3 (A plane intercepting the X axis at a right angle three steps from the origin.)

RY = 3 (A plane intercepting the Y axis at a right angle three steps from the origin.)

RZ = 4 (A plane intercepting the Z axis at a right angle four steps from the origin.)

The indicated barrier definitions will simplify the probability of acceptance expression.

One additional economy of reference needs to be made. It is apparent that all points in the accept barrier have a common Z value in all cases. (In Figure 2, Z is always equal to four.) Thus, for any point in the accept barrier in Figure 2, X and Y values only will be required and the Z value of four will be implied. For example, point 1, 1, 4 = 1, 1. Generalized this means that for any given model size, points on the accept barrier can be defined by reference to X and Y values only.

Now, recalling that probability of acceptance is the summation of the probabilities of acceptance at each point on the accept barrier, from Figure 2

$$PA = PA_{00} + PA_{10} + PA_{20} + PA_{01} + PA_{11} + PA_{21} + PA_{02} + PA_{12} + PA_{22}$$

$$= \sum_{i=0}^{RX-1} PA_{i,\,Y=0} + \sum_{i=0}^{RX-1} PA_{i,\,Y=1} + \sum_{i=0}^{RX-1} PA_{i,\,Y=2}$$

where

RX = number of steps in the X direction from the origin to the undersize absorbing barrier.

$PA_{i,\,Y}$ = probability of acceptance at a specific point on the accept barrier.

i = any X value for accept points.

Note: The probability of reaching points on the accept barrier which are common with reject barriers are not included since their respective PA values are always zero.

Further development of a general expression for PA must then consider not only p, q, and r but a term for calculating the number of paths to various accept points on the accept barrier. With the preceding in mind it follows that for the given example

$$PA = \sum_{i=0}^{RX-1} p^i q^0 r^{AZ} \begin{pmatrix} i + 0 + (AZ-1) \\ i,\ 0,\ (AZ-1) \end{pmatrix} + \sum_{i=0}^{RX-1} p^i q^1 r^{AZ} \begin{pmatrix} i + 1 + (AZ-1) \\ i,\ 1,\ (AZ-1) \end{pmatrix} + \sum_{i=0}^{RX-1} p^i q^2 r^{AZ} \begin{pmatrix} i + 2 + (AZ-1) \\ i,\ 2,\ (AZ-1) \end{pmatrix} .$$

Furthermore, the general expression is

$$PA = \sum_{j=0}^{RY-1} \sum_{i=0}^{RX-1} p^i q^j r^{AZ} \begin{pmatrix} i + j + (AZ-1) \\ i,\ j,\ (AZ-1) \end{pmatrix}$$

where

RY = number of steps in the Y direction from the origin to the oversize absorbing barrier.

j = any Y value for accept points.

The above general expression for calculating the probability of acceptance is valid in all cases where absorbing barriers are at right angles to the axis.

Using the general expression an example problem is computed, i.e. calculation of PA for reaching the accept barrier where

$$p = 0.05$$
$$q = 0.05$$
$$r = 0.90$$
$$RX = 3$$
$$RY = 3$$
$$AZ = 4$$

$$PA = (.05)^0(.05)^0(.90)^4 \binom{0 + 0 + (4-1)}{0,\ 0,\ (4-1)}$$

$$+ (.05)^1(.05)^0(.90)^4 \binom{1 + 0 + (4-1)}{1,\ 0,\ (4-1)}$$

$$+ (.05)^2(.05)^0(.90)^4 \binom{2 + 0 + (4-1)}{2,\ 0,\ (4-1)}$$

$$+ (.05)^0(.05)^1(.90)^4 \binom{0 + 1 + (4-1)}{0,\ 1,\ (4-1)}$$

$$+ (.05)^1(.05)^1(.90)^4 \binom{1 + 1 + (4-1)}{1,\ 1,\ (4-1)}$$

$$+ (.05)^2(.05)^1(.90)^4 \binom{2 + 1 + (4-1)}{2,\ 1,\ (4-1)}$$

$$+ (.05)^0(.05)^2(.90)^4 \binom{0 + 2 + (4-1)}{0,\ 2,\ (4-1)}$$

$$+ (.05)^1(.05)^2(.90)^4 \binom{1 + 2 + (4-1)}{1,\ 2,\ (4-1)}$$

$$+ (.05)^2(.05)^2(.90)^4 \binom{2 + 2 + (4-1)}{2,\ 2,\ (4-1)}$$

$$= (.6561)\ \frac{3!}{3!0!} + (.03281)\ \frac{4!}{1!3!} + (.0164)\ \frac{5!}{2!3!}$$

$$+ (.03281)\frac{4!}{1!3!} + (.00164)\frac{5!}{1!1!3!} + (.000049)\frac{6!}{2!1!3!}$$

$$+ (.00164)\frac{5!}{2!3!} + (.000049)\frac{6!}{1!2!3!} + (.000004)\ \frac{7!}{2!2!3!}$$

$$= .99090$$

A high probability should be expected for the example above. By changing p, q, r or model size, different probabilities of acceptance will result and thus the desired operating characteristics for any process can be controlled by proper design of the PQC system.

<u>Computing Average Sample Size, ASN</u>

As previously stated, the random walk ceases when any one of the three absorbing barriers are touched. Since there are numerous points on the absorbing barriers which if reached results in an accept or reject decision, it is obvious that the number of inspections, or steps, required to touch a barrier is a variable. Consider as an example, the number of steps to reach any point on the undersize reject barrier, RX, in Figure 2. The number of steps to the closest point (3, 0, 0) on the RX plane is three. But, to reach point 3, 2, 3 on this same barrier eight steps are required. The same general reasoning is applicable for all points on all absorbing barriers. Logic then leads to the conclusion that average sample size is a variable which is dependent on per cent defective and positions of absorbing barriers (number of steps).

For reasons of economy in process control activities consideration must be given to average sample size. In any control system it is desirable to maintain a given quality level using a minimum amount of inspection time, i.e. sample as little as possible. Therefore, it is of value to determine the amount of sampling required in the proposed system. Due to the variability in the number of inspections which may be required to reach any of the absorbing barriers, the only meaningful statement must consider average sample size, ASN.

Effect of Quality on Probability of Acceptance, PA, and Average Sample Size, ASN

As previously determined

$$PA = \sum_{j=0}^{RY-1} \sum_{i=0}^{RX-1} p^i q^j r^{AZ} \binom{i + j + (AZ-1)}{i,\ j,\ (AZ-1)}$$

Since that part of the equation which determines the number of paths, $\binom{i + j + (AZ-1)}{i,\ j,\ (AZ-1)}$, does not change with changes in quality no direct consideration need be given to it. However, the remaining part of the equation, $p^i q^j r^{AZ}$, is obviously affected by quality changes.

Now from probability theory it is known that

$$p + q + r = 1.$$

Furthermore,

$$AZ \geq 1$$

$$i \geq 0$$

$$j \geq 0$$

and recalling that

$$PA = \sum_{j=0}^{RY-1} \sum_{i=0}^{RX-1} p^i q^j r^{AZ} \binom{i + j + (AZ-1)}{i,\ j,\ (AZ-1)}$$

then

$$0 \leq PA \leq 1.$$

From the equation then

$$PA = 1, \text{ when } r = 1 \quad p = q = 0,\ i = j = 0$$

$$PA = 0, \text{ when } r = 0 \quad p + q = 1.$$

The above analysis results in the following conclusions.

1. As r (per cent good parts) becomes small, PA becomes small.
2. If r = 0, a minimum number of inspections are required for process rejection.
3. As p and/or q gets large, PA becomes small, i.e., as the quality level decreases, p and/or q gets large, r becomes small and PA becomes small.

Average Sample Size, ASN

By careful consideration of the PQC system it becomes apparent that average sample size is equal to the summation of the probabilities of reaching all barrier points multiplied times the number of steps (or inspections) to the points.

Or,

$$ASN = \sum_{\substack{\text{For all}\\ \text{points on}\\ \text{absorbing barriers}}} P_{ijZ} S$$

and

P_{ijZ} = probability of reaching a point on any absorbing barrier.

S = number of steps to reach a point on any absorbing barrier.

Breaking the above equation into separate segments representing the three individual absorbing barriers, then

$$ASN = \sum_{\substack{\text{For all} \\ \text{points on} \\ \text{undersize} \\ \text{reject barrier}}} P_{RX} S_X + \sum_{\substack{\text{For all} \\ \text{points on} \\ \text{oversize} \\ \text{reject barrier}}} P_{RY} S_Y + \sum_{\substack{\text{For all} \\ \text{points on} \\ \text{accept} \\ \text{barrier}}} P_{AZ} S_Z$$

where

P_{RX} = probability of reaching a point on absorbing barrier RX.

P_{RY} = probability of reaching a point on absorbing barrier RY.

P_{AZ} = probability of acceptance or reaching a point on absorbing barrier AZ.

Or

$$\begin{aligned} ASN = & \sum_{j=0}^{RY-1} \sum_{i=0}^{RX-1} p^i q^j r^{AZ} \binom{i + j + (AZ-1)}{i,\ j,\ (AZ-1)} (i + j + AZ) \\ & + \sum_{j=0}^{RY-1} \sum_{h=0}^{AZ-1} p^{RX} q^j r^h \binom{j + h + (RX-1)}{j,\ h,\ (RX-1)} (j + h + RX) \\ & + \sum_{i=0}^{RX-1} \sum_{h=0}^{AZ-1} p^i q^{RY} r^h \binom{i + h + (RY-1)}{i,\ h,\ (RY-1)} (i + RY + h). \end{aligned}$$

h = a step in the Z direction.

The preceding formula cannot meaningfully be further generalized and will be used in the algorithm in the form given.

CHAPTER III

CHARACTERISTICS OF PROCESS QUALITY CONTROL

The general mathematical expression for calculating the probability of acceptance, PA, applies equally well to various inspection sample sizes and variations in per cent defectives. This ability to determine PA for various process control demands is necessary if the overall system is to have wide application. Thus, the three dimensional model must be variable in size in order to provide the necessary flexibility of application. The preceding statement correctly implies that the three dimensional model size can be altered dependent upon the position of absorbing barriers within it.

Effect of Location of Absorbing Barriers

For ease of understanding the effect of changes in the location of absorbing barriers, consideration will be given first to the accept barrier position only. Following this discussion the effect of moving reject barrier locations will be described.

Accept Barrier Location

If the accept barrier, AZ, is moved away from the origin while reject barriers remain in a fixed location, the effect is to decrease PA for any p and/or q value greater than zero. Obviously if p = q = 0, then PA = 1.00 regardless of accept barrier location. PA will always decrease under the stated conditions since the number of steps to AZ has been increased as has been ASN. And, as ASN increases the opportunity for touching a reject barrier increases and PA must decrease. Note here that p and q values are not altered for the above example and PA values are changed only by relocating AZ. This concept is important due to its relationship to the operating characteristics curve for PQC. Also, the reasoning above leads to the reverse conclusion that by bringing AZ barrier closer to the origin, PA is increased.

Reject Barrier Location

Now consider the effect of RX and RY barrier positions on PA values. Since, the model is symmetrical with respect to reject barrier positions, the effect of shifting RX and RY will be considered jointly. If AZ remains fixed when reject barriers are moved away from the origin, PA and ASN increase, where p, q > 0. This statement is seen to be true by noting that for given p and q values, and where AZ is fixed, there is reduced opportunity to touch a reject barrier during a random walk. Consequently, PA increases as RX and RY barriers are positioned further from the origin.

Calculated Effect of Absorbing Barrier Locations

In order to verify the above discussion a number of calculations for various barrier positions and two different sets of p, q, and r values are included in Table I. Note first that in rows 1 and 2 RX = RY = 2 for both cases but AZ changes from AZ = 25 to AZ = 40. This increase in the AZ value is equivalent to increasing the location of the accept barrier at a distance farther from the origin. As stated above such a change should decrease the probability of acceptance, PA, for any given quality level. A comparison of PA values in rows 1 and 2 verifies this statement for either of the two process quality levels shown in the table. Added verification of the effect of increasing AZ values is found by considering row 4 where RX = RY = 2 but an additional number of steps have been included to reach the accept barrier, i.e. AZ has increased to 55. In this case PA continues to decrease. Thus, it is always true that as the AZ barrier recedes from the origin while reject barriers remain fixed, the probability of acceptance decreases for any given process quality. (See Figure 3.) Now consider the effect of relocating the symmetrical reject barriers on probability of acceptance. For rows 4, 5, and 6 in Table I note that AZ is fixed while RX and RY values vary from 2 through 4. These data will be used to verify the stated results of reject barrier locations, i.e., the probability of acceptance increases when reject barrier distances are moved away from the origin and the accept barrier remains fixed, and vice versa.

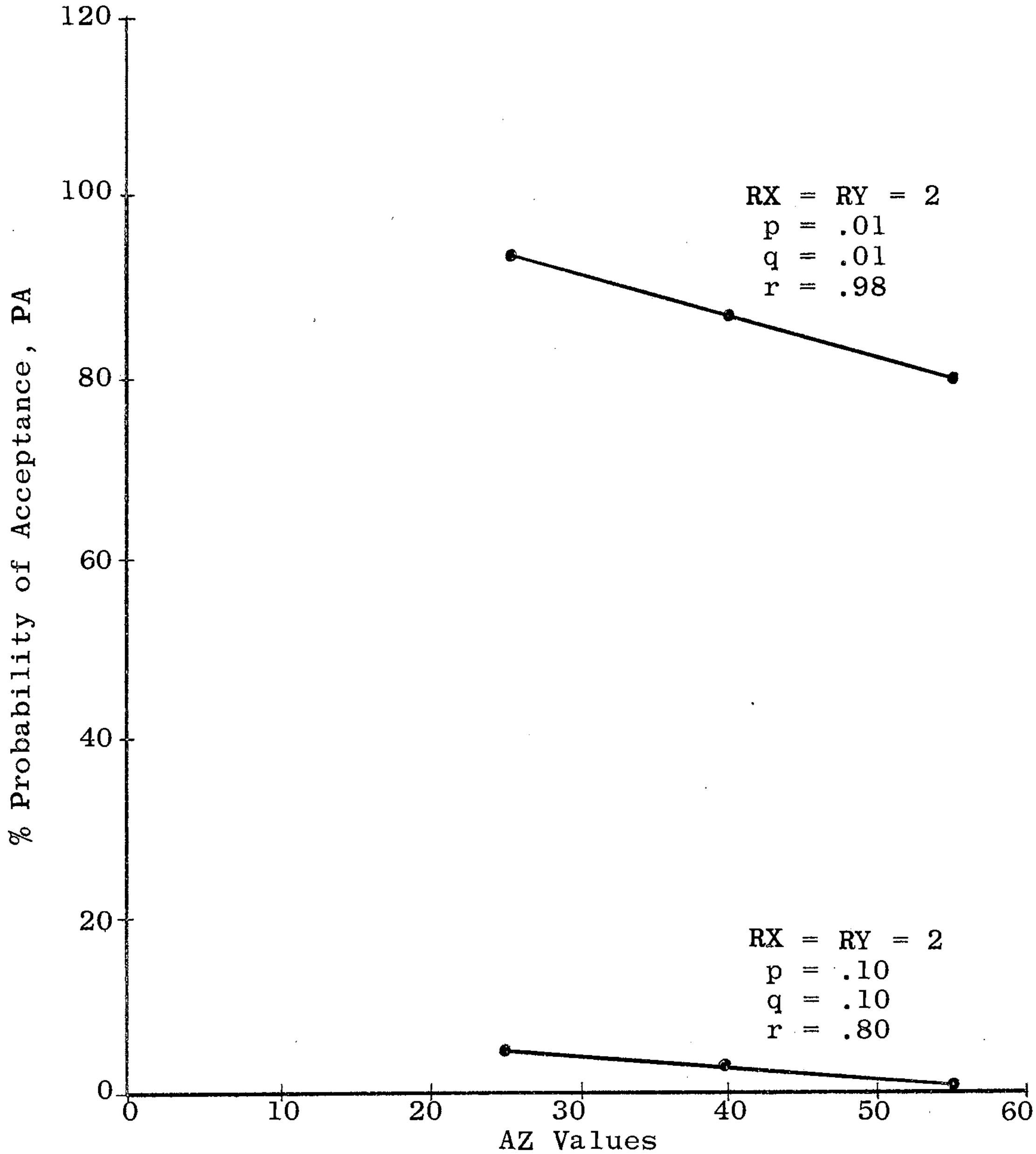


Figure 3. Effect of Accept Barrier Location on PA

TABLE I

EFFECT OF ABSORBING BARRIER LOCATIONS

	Barrier Positions			p = .01 q = .01 r = .98		p = .10 q = .10 r = .80	
Row No.	AZ	RX	RY	PA	ASN	PA	ASN
1	25	2	2	.944	25	.047	12
2	40	2	2	.875	39	.003	12
3	40	3	3	.982	41	.024	20
4	55	2	2	.792	52	.000	12
5	55	3	3	.960	55	.002	21
6	55	4	4	.994	56	.013	29

In Table I note in rows 4, 5, and 6 that AZ = 55 and RX = RY = 2, 3, and 4 for successive rows. As RX and RY increases the probability of acceptance increases for a given quality level. The general results from increasing RX and RY values while holding AZ constant is depicted in Figure 4. For any process quality level it will always be found that PA will increase as reject barriers are located farther from the origin.

From the above analysis it is possible to state several general conclusions relating to model size, or barrier positions, for any quality level.

1. As the accept barrier position becomes large, when the reject barrier positions remain fixed, probability of acceptance, PA, decreases and vice versa.

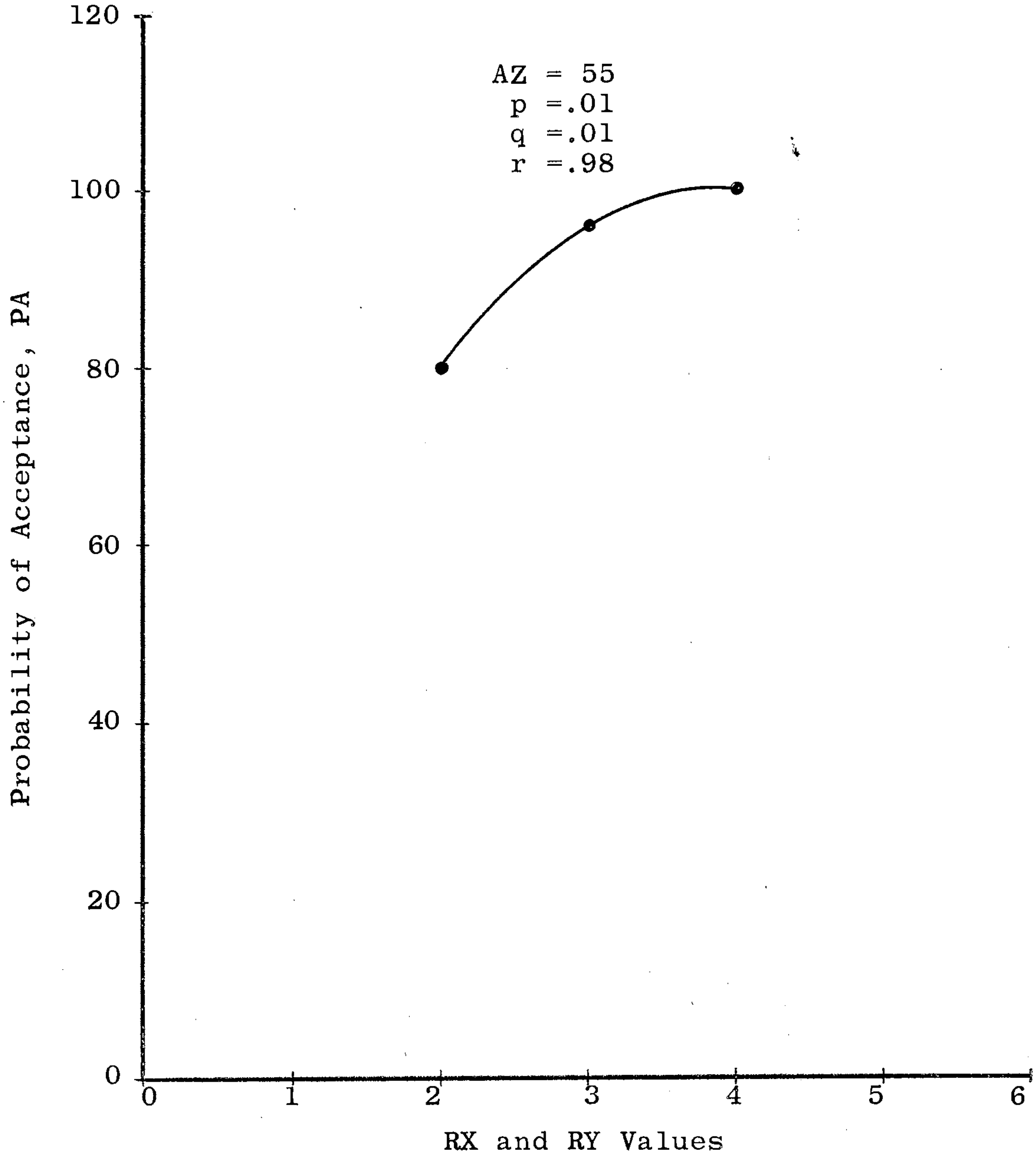


Figure 4. Effect of Reject Barrier Locations on PA.

2. As the reject barrier positions become large, when the accept barrier position remains fixed, probability of acceptance, PA, increases and vice versa.

3. Or, the PQC model size directly affects the probability of acceptance for any quality level, and therefore, affords a means of controlling operating characteristics of the PQC system.

Barrier Positions and ASN

One additional aspect of model size should be considered; namely, the effect of barrier position on average sample size, ASN. By inspection of the PQC model it is seen that ASN will increase as the model size increases. Verification of the above statement is available by analysis of Table I. Consider first the effect of increasing AZ with fixed RX and RY values. And, secondly the effect of increasing RX and RY values with AZ remaining constant. In rows 1, 2, and 4 AZ values are successively 25, 40, and 55 and RX = RY = 2 in all three cases. For the quality level of p = 0.01, q = 0.01, r = 0.98, the corresponding values for ASN are 25, 39, and 52 (see Figure 5) and increase as AZ increases. This condition will always exist as it does for the quality level p = 0.10, q = 0.10, r = 0.80 which is included in the table. However, in the latter case the increases are small and do not show in Figure 5 due to round-off of ASN values.

Next consider the effect of increasing RX and RY values

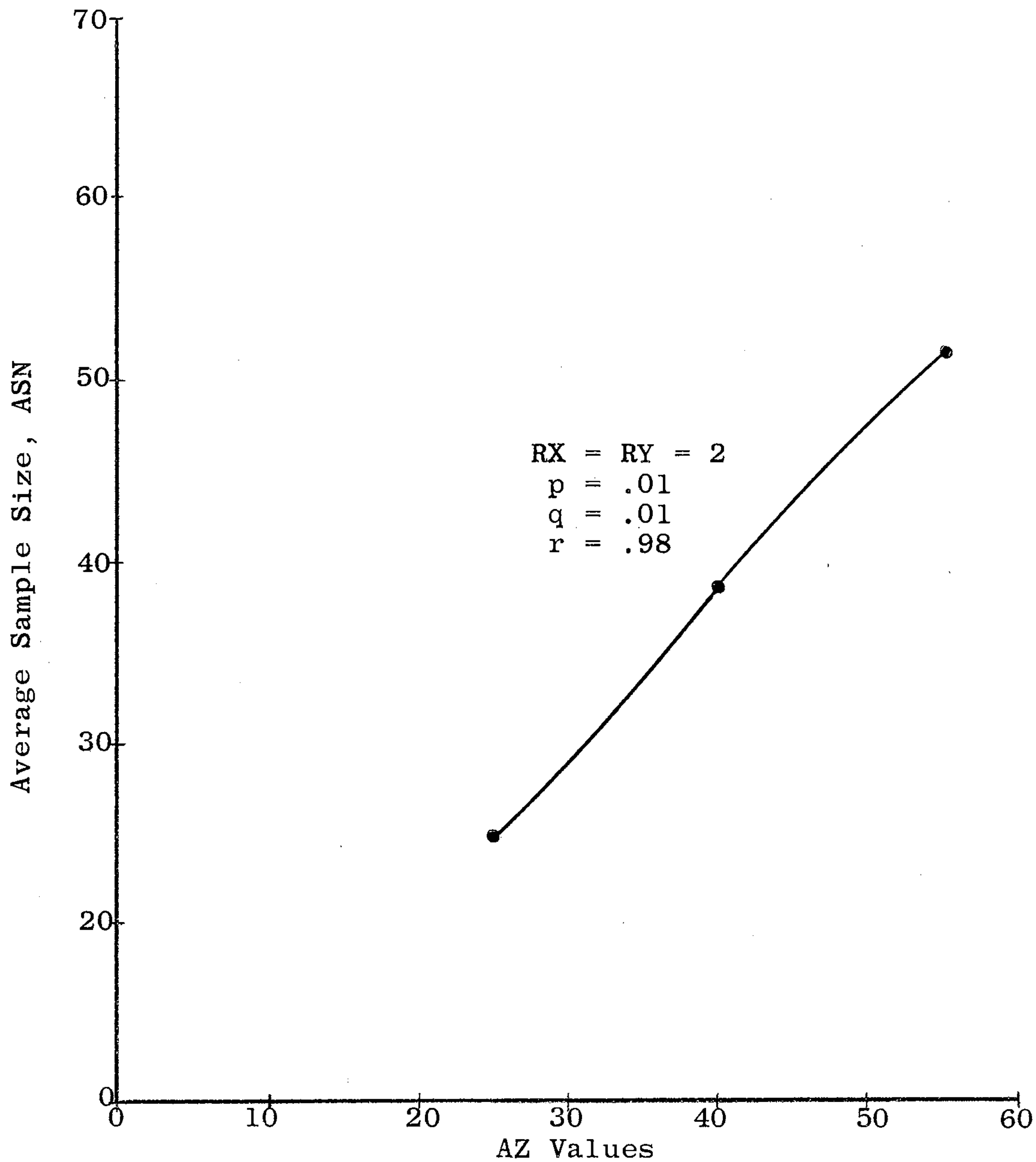


Figure 5. Effect of Accept Barrier Location on ASN

when AZ is constant. Such an effect can be seen from rows 4, 5, and 6 in Table I. Stated briefly, as RX and RY becomes large, ASN becomes large. (See Figure 6.)

Thus, the effect of barrier positions is that it controls all operating characteristics of the PQC system. Therefore, for any desired process quality level PQC can be employed successfully by proper model design.

PQC Operating Characteristics Curve

A process control system must have the capability of measuring performance over a range of possible quality levels. This ability to measure performance is depicted by the operating characteristic curve. Or, stated somewhat differently, the operating characteristic curve shows the long-run percentage of lots that would be accepted if a great many lots of any stated quality level were submitted for inspection (9).

As shown previously in this chapter the proposed PQC system can measure performance for any quality level by proper positioning of absorbing barriers in the model. One O.C. curve is shown in Figure 7 where the absorbing barrier distances are:

$$RX = 4$$

$$RY = 4$$

$$AZ = 55$$

In the given case for a process producing 4% defective parts, the process would be accepted 94% of the time and at $1\sigma'$

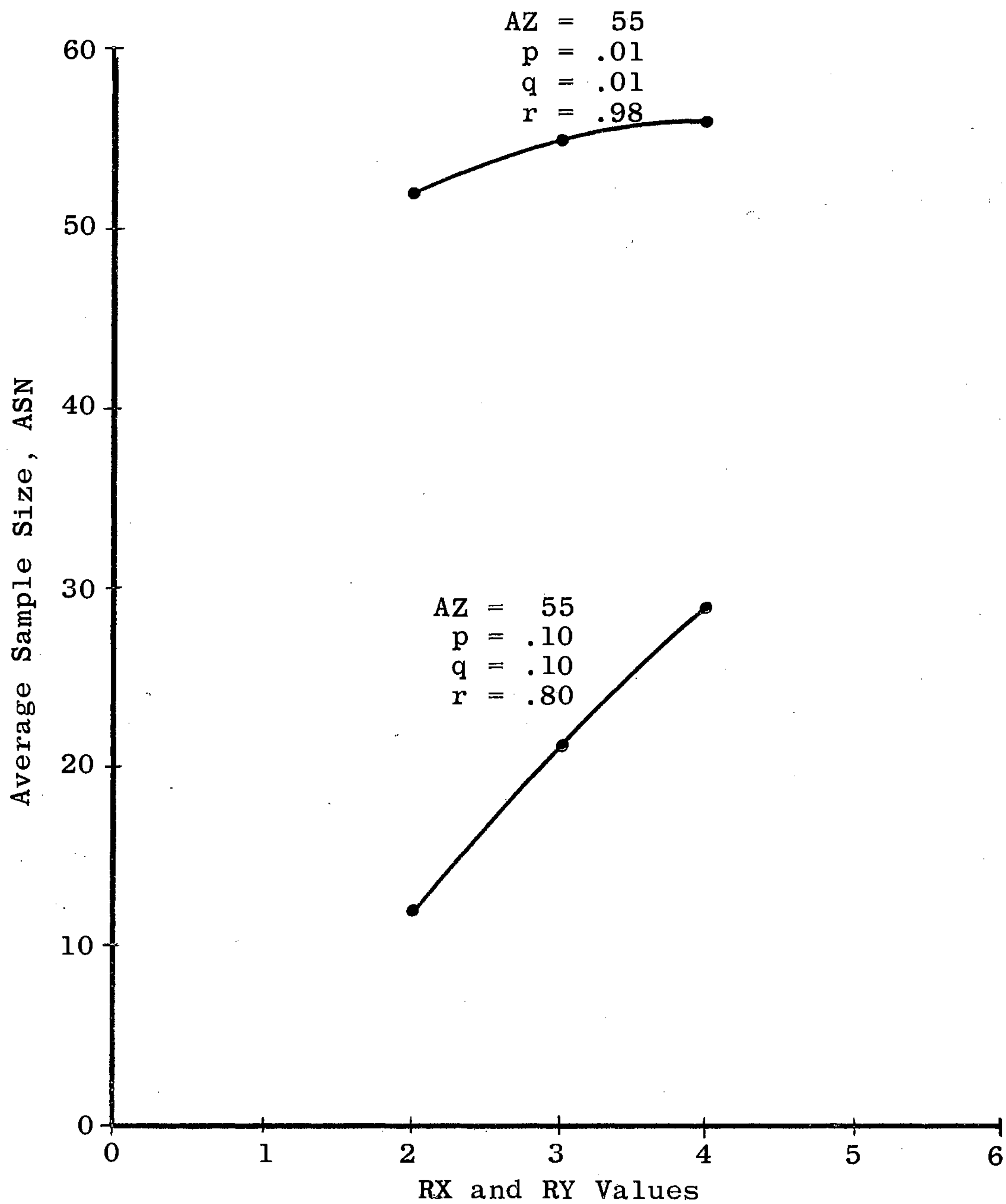


Figure 6. Effect of Reject Barrier Locations on ASN

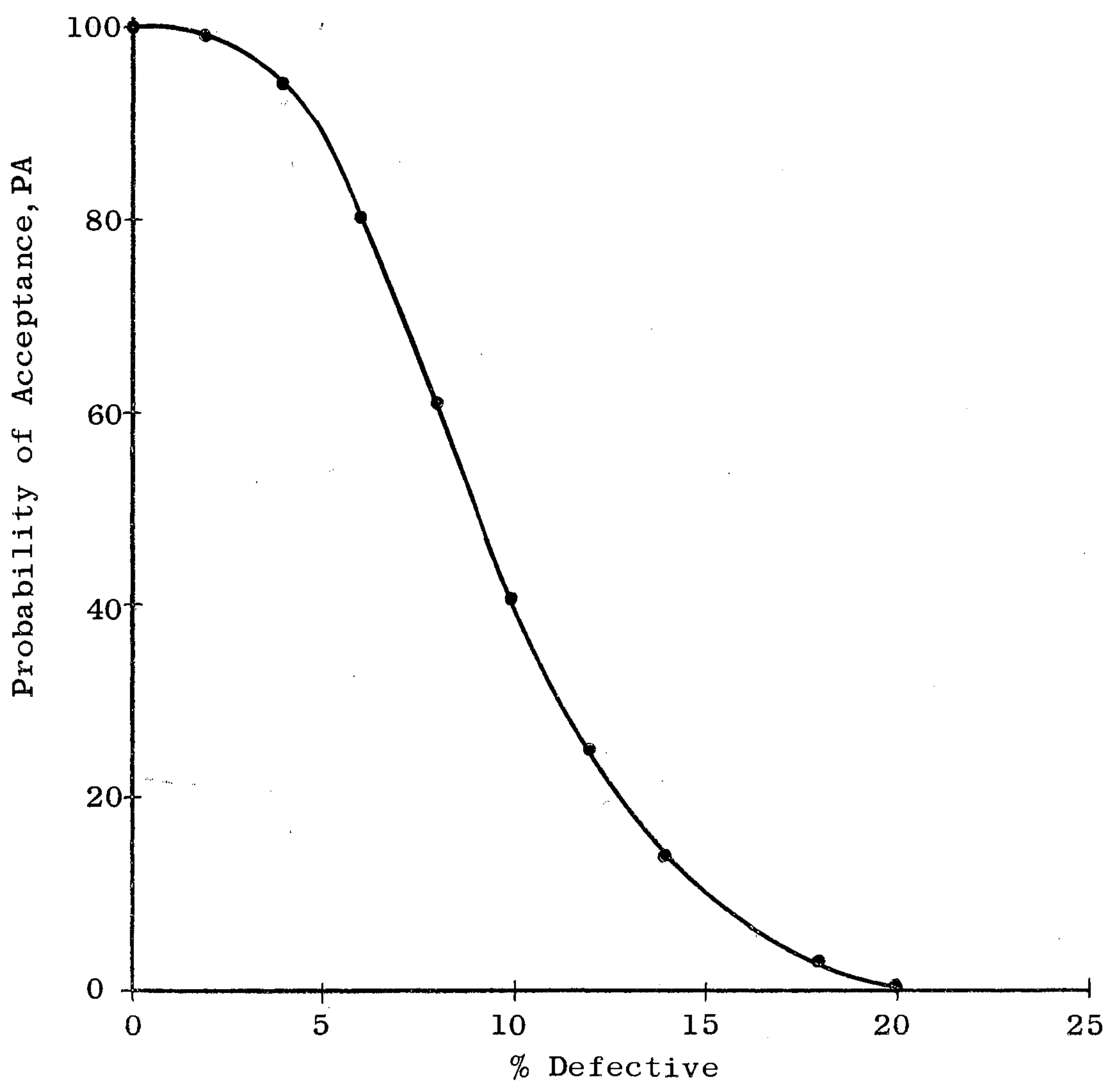


Figure 7. PQC Operating Characteristic Curve

limits the process would be accepted 7% of the time. Since the PQC system affords unlimited types of operating characteristic curves, the proposed system is readily adaptable to meet any desired quality demand.

PQC Algorithm

Computations of PQC data for various quality levels and model sizes can be accomplished by the following computer program. The program as given was prepared for an I.B.M. 1410 computer and is written in Fortran II language. Adaptation of the program to other computers can be made readily. The algorithm as given generates the data used specifically in this paper.

PROCESS QUALITY CONTROL ALGORITHM

```
FORTRAN40KRUN
      DIMENSION TFAC(100)
      MY=1964
      MM=3
      MD=16
100   FORMAT(2X,9H DATE IS 315)
      PRINT100,MM,MD,MY
101   FORMAT(2X,21H PA FOR RIGHT PLANES)
      PRINT101
      MAXS=75
      DO1MF=1,MAXS
      TFAC(MF)=1.0
      DO1MM=1,MF
      C=MM
  1   TFAC(MF)=TFAC(MF)*C
      I5=1
      IXA=55
      IYR=4
      IZR=4
  3   FORMAT(2X,5H)
      PRINT3
  2   FORMAT(2X,7H IXA = I3,7H IYR = I3,7H IZR = I3/)
      PRINT 2, IXA,IYR,IZR
      DO50IPY=1,10
      PY=IPY
```

```
      PY=PY/100.0
      PZ=PY
      PX=1.0-PY-PZ
      AANS=0.0
      SUMPA=0.0
      DO20JZ=1,IZR
      IZ=JZ-1
      DO20JY=1,IYR
      IY=JY-1
      N1=IXA-1+IZ+IY
      N2=IXA-1
      IF(IZ)10,10,11
10    IF(IY)12,12,13
12    PATH=1.0
      GOTO16
13    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IY))
      GO TO 16
11    IF(IY)14,14,15
14    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IZ))
      GO TO 16
15    PATH = TFAC(N1)/(TFAC(N2)*TFAC(IY)*TFAC(IZ))
16    CONTINUE
      PA = PATH*(PX**IXA)*(PY**IY)*(PZ**IZ)
      SUMPA=SUMPA+PA
      IAA=IZ+IY+IXA
      AA=IAA
      AA=AA*PA
```

```
      AANS=AANS+AA
20    CONTINUE
      SUMPZ=0.0
      ZANS=0.0
      DO30JX=1,IXA
      IX=JX-1
      DO30JY=1,IYR
      IY=JY-1
      N1=IZR-1+IX+IY
      N2=IZR-1
      IF(IX)31,31,32
31    IF(IY)33,33,34
33    PATH=1.0
      GO TO 37
34    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IY))
      GO TO 37
32    IF(IY)35,35,36
35    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IX))
      GO TO 37
36    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IY)*TFAC(IX))
37    CONTINUE
      PR=PATH*(PZ**IZR)*(PY**IY)*(PX**IX)
      SUMPZ=SUMPZ+PR
      IAA=IZR+IY+IX
      AA=IAA
      AA=AA*PR
      ZANS=ZANS+AA
```

```
30    CONTINUE
      SUMPY=0.0
      YANS=0.0
      DO40JZ=1,IZR
      IZ=JZ-1
      DO40JX=1,IXA
      IX=JX-1
      N1=IYR-1+IZ+IX
      N2=IYR-1
      IF(IX)41,41,42
41    IF(IZ)43,43,44
43    PATH=1.0
      GO TO 47
44    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IZ))
      GO TO 47
42    IF(IZ)45,45,46
45    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IX))
      GO TO 47
46    PATH=TFAC(N1)/(TFAC(N2)*TFAC(IZ)*TFAC(IX))
47    CONTINUE
      PR=PATH*(PY**IYR)*(PZ**IZ)*(PX**IX)
      SUMPY=SUMPY+PR
      IAA=IZ+IYR+IX
      AA=IAA
      AA=AA*PR
      YANS=YANS+AA
```

```
40      CONTINUE
        ANS=AANS+ZANS+YANS
99      FORMAT(2X,6H PA = F5.3,7H PRZ = F5.3,7H PRY = F5.3,
        112H PX PY PZ = 3F6.3,7H ANS = F6.3)
        PRINT99,SUMPA,SUMPZ,SUMPY,PX,PY,PZ,ANS
50      CONTINUE
        STOP
        END
```

CHAPTER IV

APPLICATION OF PROCESS QUALITY CONTROL

The use of any method of control of quality from a production process has two major objectives; namely:

1. A high probability of accepting process output which is in control.
2. A low probability of accepting process output which is not in control.

In addition there are other important considerations for selecting an adequate quality control system. Some of these considerations are:

3. Average sample size should be small.
4. Maximum sample size should be small.
5. Method of inspection should be fast, accurate, but simple.
6. Require minimum calculations by inspector.
7. Afford protection against variation both in process average, $\overline{X}$, and range, R.
8. Indicate process trend toward oversize or undersize.
9. Minimize record keeping.

The PQC system proposed in this dissertation meets the above criteria satisfactorily in all cases. The general PQC model can be employed effectively to control a manufacturing process for specific tolerance limits and quality levels.

<u>General Application of PQC</u>

The PQC model can be designed so that barrier positions will yield any operating characteristics curve desired. The variability of operating characteristics was discussed previously in Chapter III.

For the usual case some desired quality level is stated in terms of a specific PA for a particular fraction defective. For example consider the case where the quality control engineer has specified a quality standard such that:

PA_1 = 99.5% and a 1% defect level, $p_1 = q_1 = 0.005$

PA_2 = 1.5% at the reject level, $p_2 = q_2 = 0.10$

PA_1 = probability of acceptance when process is in perfect control; $p = q = 0$.

PA_2 = probability of acceptance when process is beyond control limits.

Since a wide range of PA values for various PQC model designs are obtainable from the use of the FORTRAN program included in Chapter III, the specified control can be assured by selection of that system which meets the stated conditions. In the example problem, the desired quality protection is afforded by a PQC model where

RX = 4

RY = 4

AZ = 55

The operating characteristics curve for this particular PQC model is shown in Figure 7, page 34.

By use of the above procedure great flexibility in model design, and consequently operating characteristics, is obtainable. Once the quality standard is established for a production process a PQC system can be designed to meet the stated standard.

Inspection Procedure

The PQC system lends itself readily to standard inspection techniques. Specifically, it is designed for use in simple go-not go type gaging. Thus, special training for inspectors is minimized and inspection costs can be kept low. The inspector sequentially inspects production directly from the production process and establishes if parts are above, below, or within tolerance limits and records findings until an accept or reject decision is made. Since it is desirable to simplify the required record keeping procedure, an easily applied system should be employed. Such a system is shown in the Inspector's Report, Figure 8. The Inspector's Report will include the necessary instructions to the inspector as to the number of good parts, etc. Then by go-not go gaging of process output a tally of parts above, under, and within stated

Inspector's Report

Part No.____________ Operation No.__________

Work Order No.______ Department_____________

Inspector No._______ Date___________________

Instructions:

Reject when number of undersize parts = 4

Reject when number of oversize parts = 4

Accept when number of good parts = 55

No. Undersize Parts	No. Oversize Parts	No. Good Parts

Accept________ Reject________

Figure 8. Inspector's Report

tolerance limits can be made and the accept or reject decision determined.

It is obvious that the proposed method for recording inspection data can be readily applied to the attribute inspection scheme of PQC. Furthermore, a minimum amount of training would be necessary for satisfactory results. Also, as is true for most quality control systems, those persons actually applying the system need not understand the basic theory of PQC.

Problem Simulation

For illustrating PQC, a system of simulated inspection is used. A random number table provides data which approximates inspection results where the process is producing parts according to a normal distribution. For example, by use of a random number table from 00-99 and where per cent defective is stated to be 8%, then random numbers 00, 01, 02, 03 represent undersize parts; random numbers 96, 97, 98, 99 represent oversize parts; and random numbers 04-95 represent parts within tolerance limits.

Consider a situation where

$PA_1 = 99.5\%$ $\quad p_1 = q_1 = 0.005$

$PA_2 = 1.5\%$ $\quad p_2 = q_2 = 0.10$

$RX = 4$

$RY = 4$

$AZ = 55$

Then for this PQC model a summary of several simulated inspection trials is given in Table II.

TABLE II

SUMMARY OF SIMULATED INSPECTION DATA

Trial No.	No. Inspections	Occurrence of p	Occurrence of q	Occurrence of r	Decision
1	57	1	1	55	Accept
2	56	1	0	55	Accept
3	56	0	1	55	Accept
etc.					

Note that for the given PQC model it is always necessary to inspect a minimum of 55 parts if the process is accepted. Also, due to the high PA value and the low per cent defective it should be expected that the process will be accepted.

Now that the general method of simulation has been shown, three examples will be given to illustrate the versatility and reliability of PQC.

Variations in Average Size, $\bar{X}$, and Range, R.

It has been stated previously that PQC provides protection against shifts in both average size and range. To illustrate the ability of the above PQC model to provide such protection, three representative problems will be developed. The problems specifically are (1) where $\bar{X} = m$ (m = nominal dimension), (2) where $\bar{X}$ shifts upward (or downward), and (3) where R increases while $\bar{X}$ remains at

$\bar{X}$ = m. If PQC controls these three conditions then it is obvious that it will provide adequate protection for any production process.

Problem: An automated centerless grinder is producing wrist bins with a nominal diameter m = 1.000" $\pm$ 0.010." The standard deviation of the process is known to be 0.0043."

Case 1: $\bar{X}$ = m = 1.000

Tolerance = $\pm$ 0.010"

σ' = 0.0043"

With the information given the normal distribution curve is shown in Figure 9. Since the distance to control limits is $\pm$ 2.33σ', reference to a table of areas under the normal curve indicates that 1% of parts are oversize and 1% of parts are undersize. This condition is indicated in Figure 9.

From a random number table representative inspection data is obtained and presented in Table III for several samples. For all three samples the process was accepted as would be anticipated for the given conditions.

Case 2: $\bar{X}$ Shifts Upward, R Remains Constant

For a determinable reason, such as tool wear, a shift in $\bar{X}$ upward (or downward) may unknowingly occur and PQC must be capable of detecting such a shift. Such a case is indicated where

$\bar{X}$ = 1.006" (R remains constant)

σ' = 0.0043."

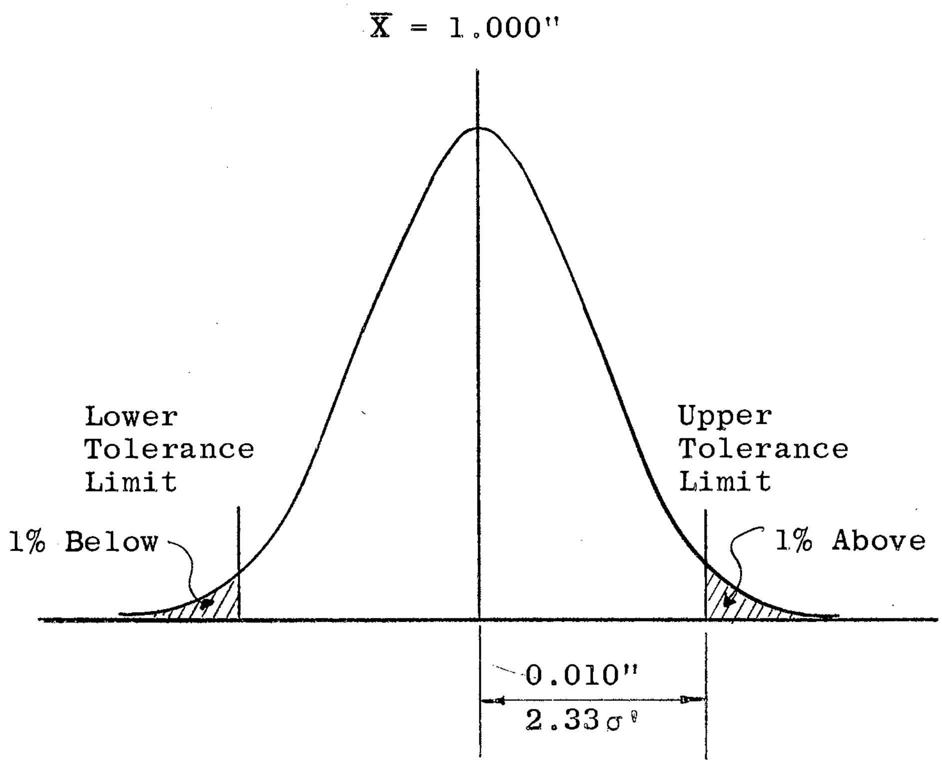


Figure 9. Normal Distribution Curve, $\overline{X} = m = 1.000$

TABLE III

CASE 1: $\overline{X} = m$

Sample 1 No.	Sample 1 No.	Sample 1 No.	Sample 2 No.	Sample 2 No.	Sample 2 No.	Sample 3 No.	Sample 3 No.	Sample 3 No.
96	36	75	22	61	35	50	88	16
47	05	31	17	03	26	86	56	38
97	56	62	68	28	00 U	54	53	64
81	80	66	65	28	15	48	27	43
56	30	54	84	28	39	61	59	59
51	03	53	19	26	25	96	90	98
72	30	56	36	08	70	48	72	53
76	98	68	27	93	99 O[2]	95	95	44
45	53	53	59	22	58	03	84	09
16	28	40	46	53	71	36	29	42
94	70	78	16	64	96	93	12	72
54	58	91	77	39	30	89	96	40
31	96	69	23	78	24	41	88	76
04	90	16	02	76	57	26	17	66
82	74	00 U[1]	77	58	35	18	31	26
98	80	93	78	54	27	87	85	84
83	55	62	43	74	33	00 U	94	02
36	09	43	76	23	72	42	57	17
89	40	Accept	71	68	48	31	24	Accept
					Accept			

[1]Number 00 = undersize part

[2]Number 99 = oversize part

For this condition the normal distribution curve changes as is indicated in Figure 10. The shift in $\overline{X}$ to $\overline{X}'$ as shown indicates that from $\overline{X}$ to the upper tolerance limit, which is unchanged, is 0.004" or $0.930\sigma'$ $(0.0040 \div 0.0043)$. Or, by referring to a table of areas under the normal curve, it is found that 18% of all parts are above the upper tolerance limit and that 0% of parts are below the lower tolerance limit.

Again inspection results may be simulated by use of random numbers. Since 18% of parts are oversize, the random number assignment must be such that numbers 82-99 represent oversize parts. All other numbers, i.e. 00-81, represent parts within tolerance limits. No undersize parts would be expected. Several samples are shown in Table IV. Thus, all three samples are rejected for the given quality level and provides evidence of PQC to detect shifts in $\overline{X}$.

Case 3: $\overline{X} = m$, R Increases

Due to various causes the range of part size being produced by a process may increase. PQC is unique in its ability to control for range by attribute sampling. To illustrate the situation where range increases assume that the standard deviation changes from 0.0043 to

$$\sigma' = 0.0075.$$

Note that $\overline{X} = m$ remains unchanged. The normal distribution curve is shown in Figure 11. Because of the change in

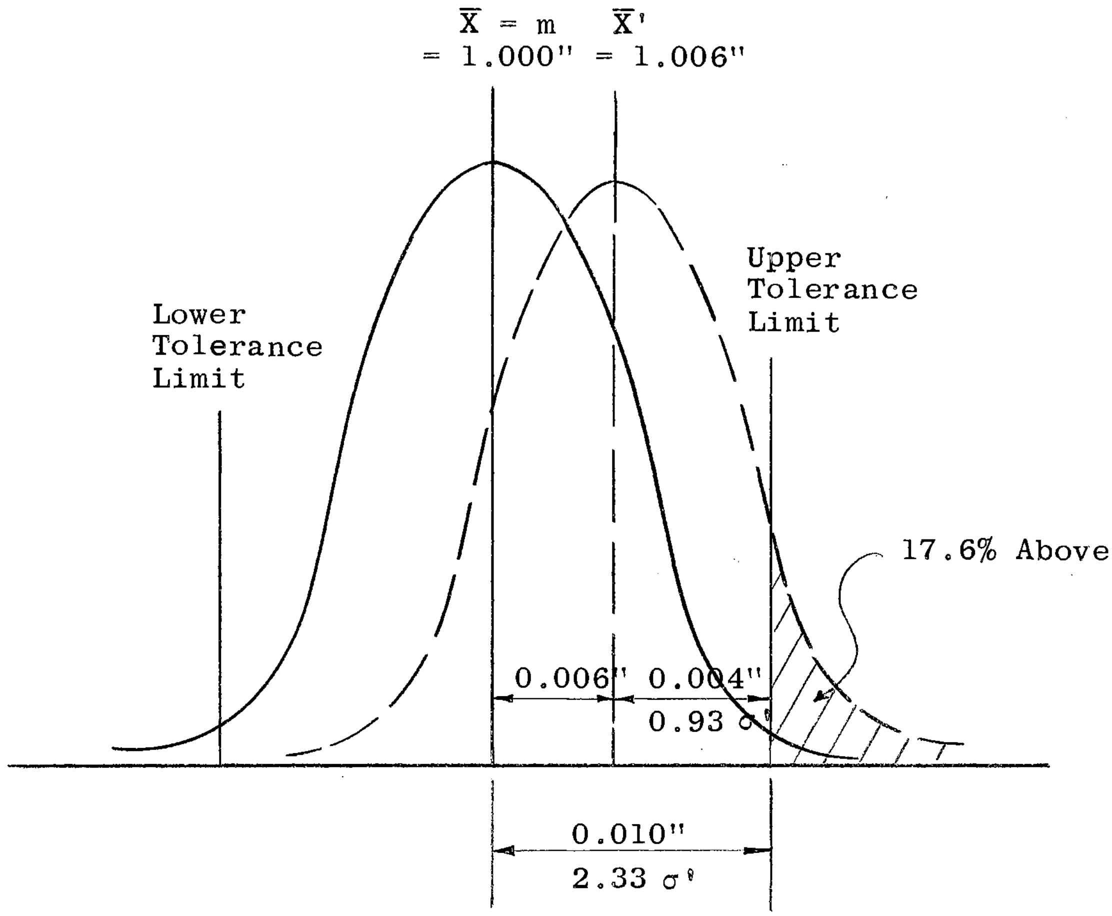


Figure 10. Normal Distribution Curve, $\overline{X}'$ = 1.006

standard deviation, tolerance limits are now only 1.33σ' from $\overline{X}$. Again from a table of areas under the normal curve it can be established that 9% of parts are above the upper tolerance limit and 9% of parts are below the lower tolerance limit.

TABLE IV

CASE 2: $\overline{X}$ = 1.006", R REMAINS CONSTANT

Sample 1			Sample 2			Sample 3		
No.	No.	No.	No.	No.	No.	No.	No.	No.
79	56	09	60	95 0	97 0	68	35	37
18	41	61	12	20	97 0	95 0	13	55
05	77	65	99 0	47	Reject	23	79	09
95 0[1]	80	61				92 0	93 0	61
17	20	68						87 0
82 0	75	66						Reject
06	82 0	93 0						
53	46	Reject						
35	40							
76	66							
22	44							
42	52							
92 0	37							
26	56							
29	08							

[1]Numbers 82-99 = oversize parts

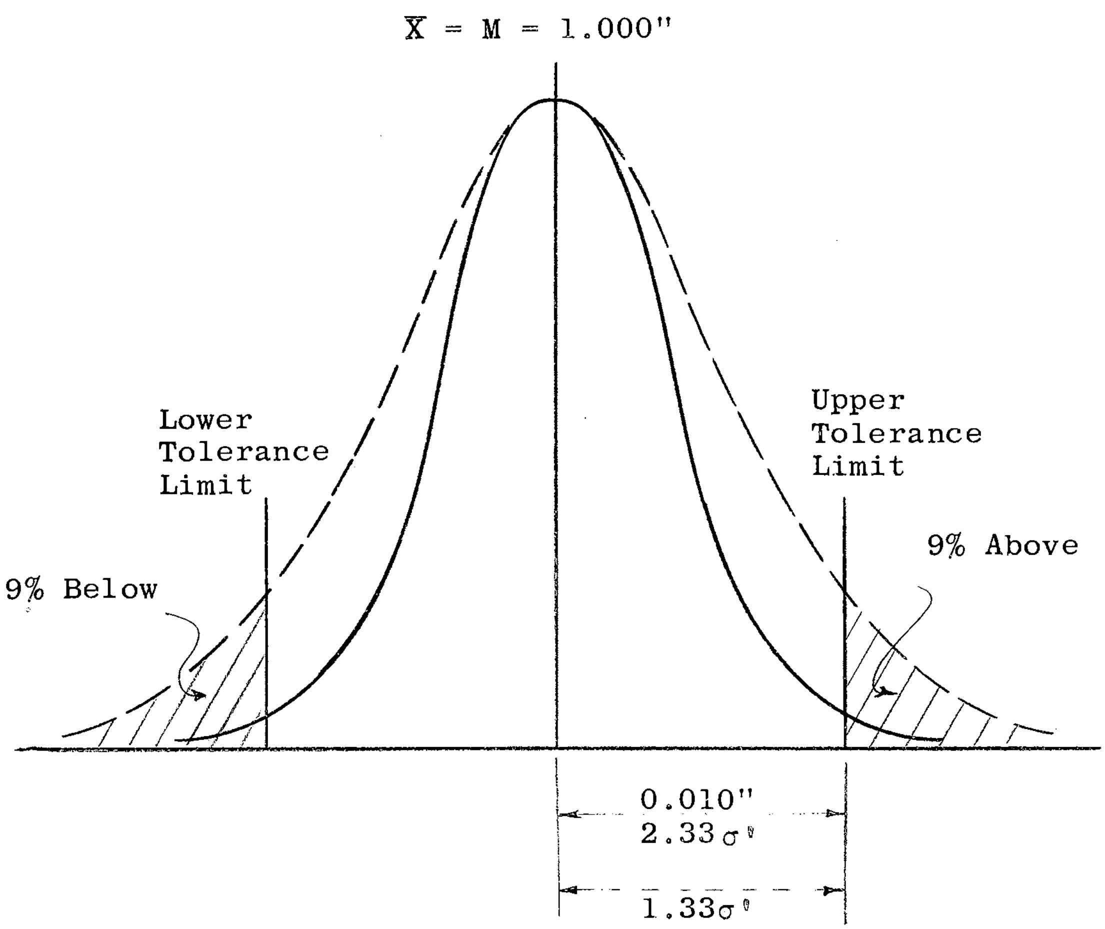


Figure 11. Normal Distribution Curve, $\overline{X} = m$, R Increases

The random number table will again be used to simulate inspection of production for the above conditions and the results tabulated in Table V. Random numbers 00-08 indicate undersize parts, numbers 91-99 indicate oversize parts, and numbers 08-90 indicate parts within tolerance limits. Thus, due to change in **R** when $\overline{X}$ remains fixed, **PQC** rejects all three of the samples and demonstrates the value of **PQC** as a method of controlling production quality for a change in **R**.

TABLE V

CASE 3: $\overline{X}$ = m, RANGE INCREASES

Sample 1			Sample 2			Sample 3		
No.	No.	No.	No.	No.	No.	No.	No.	No.
00 U[1]	40	26	36	79	38	70	61	76
83	98 0[2]	61	67	88	20	45	97 0	38
63	40	70	68	54	46	80	22	03 U
22	23	04 U	44	19	51	63	61	29
55	72	48	10	90	67	52	98 0	63
87	51	67	13	70	11	52	99 0	53
64	39	26	85	99 0	52	01 U	46	05 U
81	73	43	57	00 U	49	41	50	70
07 U	96 0	18	95 0	65	17	60	47	53
83	53	55	06 U	97 0	95 0	23	86	66
20	97 0	03 U			Reject			99 0
69	86	Reject						Reject
22	38							

[1]Numbers 00-08 = undersize parts
[2]Numbers 91-99 = oversize parts

General Observation

The previous cases illustrate all possible individual situations with regard to production variations. Of course the situation could arise where both $\overline{X}$ has shifted and R has increased. However, a combination of such events would lead to a reject decision sooner, i.e. require fewer inspections, than if only one of the out of control conditions existed.

It should be noted that PQC detects trends towards undersize and oversize parts and increase in range. Specifically, when relatively few inspections results in a decision to reject, the inspector can immediately note one of several conditions. These conditions are:

1. If all unacceptable parts inspected tend to be oversize, then it is likely that excessive tool wear has occurred or there is some other assignable cause.
2. If all unacceptable parts inspected tend to be undersize, then it is likely that the process is out of adjustment or there is some other assignable cause.
3. If unacceptable parts inspected tend to be both undersize and oversize then an increase in R has occurred and indicates bearing wear or some other assignable cause.

Thus, the inspector is able to detect very rapidly a tendency to produce bad product and corrective steps may be taken.

It has been shown that PQC meets all of the criteria stated at the beginning of this chapter. These goals were obtained by go-not go inspection and validates the statement that PQC can be used to control production processes both accurately and economically while fulfilling all requirements of a good quality control system.

CHAPTER V

COMPARISON OF PQC WITH OTHER QUALITY CONTROL SYSTEMS

The proposed system of quality control is difficult to compare with existing techniques. Each system has special applications which have been found to be satisfactory for specific situations. A meaningful, general comparison of the various quality control systems will be related to a statement of advantages and disadvantages of each of three prior systems and to PQC.

There are in existence three quality control techniques that will be discussed. Two of these systems are well documented and have been in use for many years. They are:

1. Shewhart's Control Charts.
2. Wald's Sequential Analysis.

A third and quite recently developed quality control system is:

3. Sequential Process Control.

The advantages and disadvantages of each of the above will be discussed and related to PQC.

<u>Shewhart's Control Charts</u>

The most widely used quality control system in industry is Shewhart's Control Charts. This control system is based on control limits which ordinarily are computed from the standard deviation of the process output. In some cases control limits are established from the range or the average of the process. Note that for Shewhart's method the control limits are not determined by tolerance specifications. Thus the system has limited application where tolerances are less than the control limits as established by the process standard deviation. This condition poses no problem for PQC since no control limits are required other than design tolerance limits. It is also interesting to note that for PQC the standard deviation of the process output need not be known and the system can be used beginning with the first output from the process.

There are other advantages and disadvantages of Shewhart's Control Charts. The more important ones of general concern are listed below.

Advantages:

1. Sample size is small.
2. Theory of the system is simple.
3. The direction of output deviation is indicated.

Disadvantages:

1. Accurate measurement of output is required.
2. Inspectors are required to perform mathematical calculations.

3. Two charts, $\overline{X}$ and R, are required for controlling average size and range.
4. Difficult to apply when tolerance limits equal (or are greater than) standard deviation of the process.
5. Process standard deviation must be established before control limits can be computed.

Shewhart's Control Charts have sufficient advantages to make them extremely useful and valuable for many industrial situations for economic control of quality. The chief drawbacks to their use is indicated above in that precise measurements, some mathematical computations, and limited applicability of the system are all present.

<u>Wald's Sequential Analysis</u>

Sequential sampling of process output was presented by Wald in 1947. His method provides for sequential inspection of items by go-not go gaging and is based on an allowable per cent defective. Sequential Analysis has the small average sample size inherent in item-by-item sampling but it does not provide for a maximum number of inspections (usually up to $2\frac{1}{2}$ or 3 times the average sample size). There are no mathematical calculations required of the inspector and a minimum amount of record keeping is required. However, it does not specifically protect against variations in range or process average nor does it differentiate between process tendencies toward oversize or undersize values. This characteristic makes timely corrective action difficult. A general statement of the characteristics of Sequential Analysis can best be summarized by a listing of its advantages and disadvantages.

Advantages:

1. Average samples size, ASN, is small.
2. Simple go-not go inspection procedures are employed.
3. Inspector is not required to make mathematical calculations.

Disadvantages:

1. The system does not indicate reason for reject.
2. There is no definite maximum sample size.
3. Data is not generated for calculating $\overline{X}$ and/or σ'.

The single most significant value of Sequential Analysis is that it ordinarily requires fewer inspections than does single and double sampling plans. However, the basic theory of the system is difficult to understand and widespread adoption of Sequential Analysis has not occurred.

<u>Sequential Process Control</u>

Sequential Process Control (SPC) is more similar to PQC than either of the other previously discussed quality control systems. SPC employs a two dimensional model having X and Y axis with absorbing barriers. The absorbing barriers are lines representing reject or accept decisions. In SPC inspection of process output is accomplished by go-not go gaging and depicts a random walk condition. Inspection of a part smaller than the specified nominal dimension represents a step in the X direction and a part size larger than the nominal dimension indicates a step in the Y direction. The preceding statement is illustrated in Figure 12.

Various operating characteristics curves can be developed in the SPC system and any desired quality levels can be controlled for $\overline{X}$ (but not R).

SPC has been developed only recently and its application is quite limited. However its characteristics are such that it compares favorably with other existing process control systems. The main disadvantage of SPC is that it offers very limited protection for variations in range but does control for average size very well.

The advantages and disadvantages of SPC are listed below.

Advantages:

1. Average sample size is small.
2. Maximum sample size is small.
3. Simple go-not go inspection methods are used.
4. The inspector is not required to perform mathematical calculations.
5. The direction of manufacturing process error is indicated.

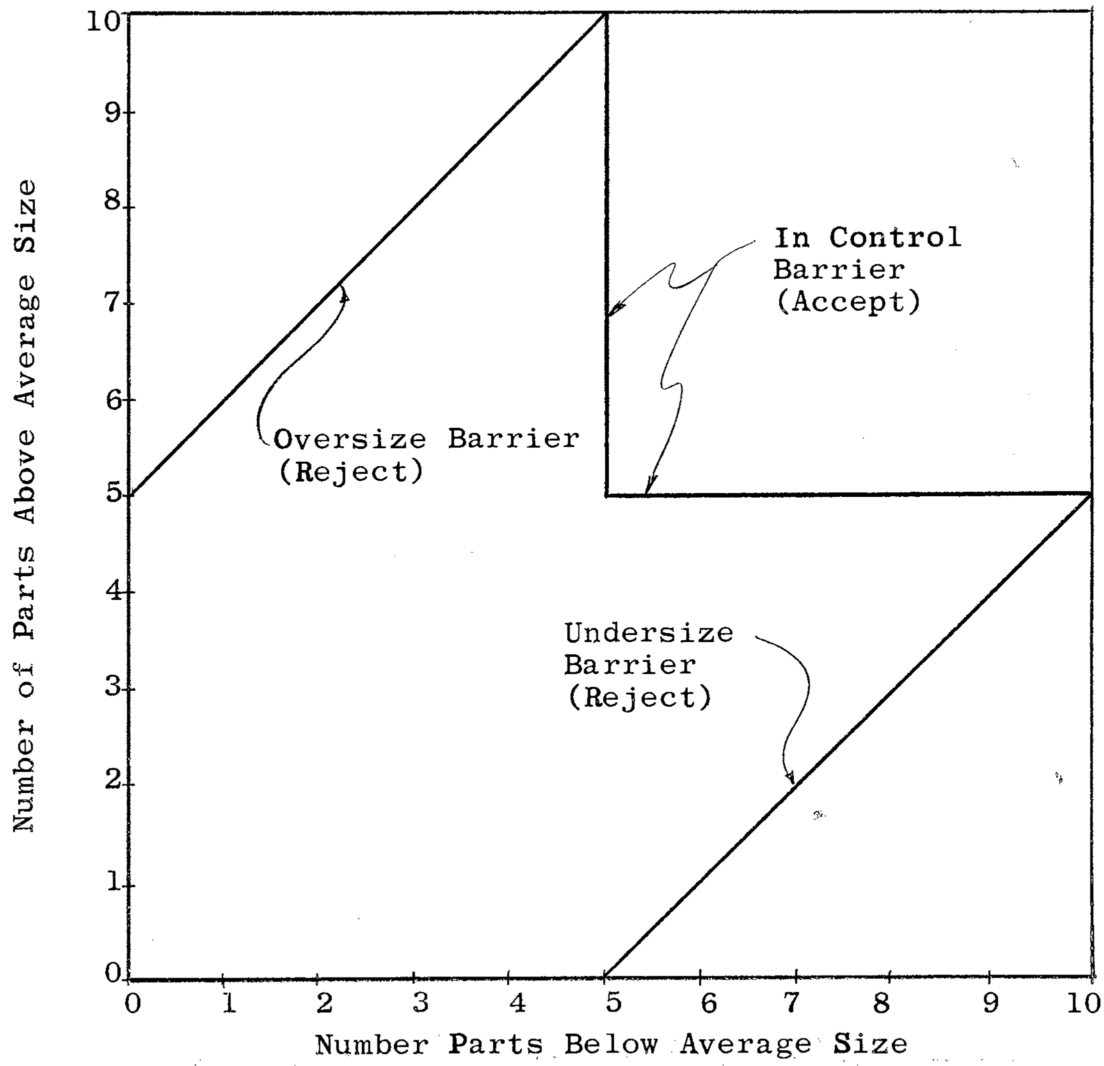


Figure 12. Sequential Process Control Model

Disadvantages:

1. Data for calculating $\overline{X}$ and σ' are not generated directly.
2. Variation in part size, i.e. range is not under control.

SPC is a unique system of quality control and provides an economical means of controlling process output. Where range control is not a problem SPC gives evidence of a significant improvement over existing quality control systems.

Process Quality Control

Process Quality Control as developed in this paper is a new and useful application of probability and statistical theory to the problem of process control. It is not intended to supplant all of the existing methods of process control. Rather it is one more step that complements and adds to those systems already in existence. The chief advantage of PQC over SPC is that it specifically affords control over range, R, and for average size, $\overline{X}$, as well. The specific advantages and disadvantages for the proposed system are given below:

Advantages:

1. Average sample size is reasonably small.
2. Maximum sample size is specifically defined.
3. Simple go-not go inspection methods are used.

4. Requires the use of only one form for control of both $\overline{X}$ and R.
5. The inspector is not required to perform mathematical calculations.
6. The direction of manufacturing process error is indicated.
7. Variation of part sizes, R, is specifically controlled.

Disadvantages:

1. Data for calculating $\overline{X}$ and σ' is not generated directly.
2. Average sample size is larger than that required for SPC.

Thus, PQC affords significant advantages with reasonably insignificant disadvantages as a tool for controlling process output. It is simple and easily applied by non-technical personnel with minimal instruction. Continued development and actual experience from its use will ultimately lead to improved process control in a practical sense.

CHAPTER VI

SUMMARY AND CONCLUSIONS

This final chapter will be composed of two sections. The first section will summarize briefly the PQC concepts. Proposals for additional study will be presented in the second section.

Summary

Process Quality Control makes use of a three dimensional grid or matrix as a model. The model provides a theoretical means of accumulating the results of repeated inspections of process output. Movements within the model are determined by go-not go gaging of parts for one of three possible conditions. These conditions are stated as being (1) below lower tolerance limit, (2) above upper tolerance limit, or (3) within tolerance limits. Respective to these three conditions movements within the model are in X, Y, or Z directions respectively but in positive directions only. Thus by sequential inspection of process output a series of positive steps or movements within the model leads to one of three pre-established absorbing barriers. Once an absorbing barrier is reached an accept or reject decision is made. The absorbing barriers are located symmetrically in the model at right angles to each other. The barriers in the X and Y planes are reject barriers while the barrier in the Z plane is the accept barrier. The positions of absorbing barriers are variable, i.e. with relation to the model origin they can be located so that any desired operating characteristics curve can be obtained. Therefore, by variation of absorbing barrier locations (model configuration) a PQC system can be designed to provide protection for any desired quality level.

The general mathematical expression for probability of acceptance, PA, was developed as being:

$$PA = \sum_{j=0}^{RY-1} \sum_{i=0}^{RX-1} p^i q^j r^{AZ} \begin{pmatrix} i + j + (AZ-1) \\ i,\ j,\ (AZ-1) \end{pmatrix}$$

This expression provides a means for calculating PA for all values of p, q, and r and all model configurations. Such an expression is obviously necessary for establishing various operating characteristic curves that are required for a quality control system that will be applicable to a wide range of problems. The algorithm for calculating PA, ASN, etc. is included in Chapter II.

Thus PQC affords a new and unique method of production control of average size, $\overline{X}$, and range, R, that is based upon attribute inspection only. This ability to specifically control R by attribute inspection has not been possible by previous quality control systems.

Proposals for Additional Study

There are two specific but related areas of study that should prove to be both interesting and worthwhile. Each of these suggestions will be discussed briefly.

1. Investigate the effect of non-symmetrical placement of absorbing barriers upon PQC. The theory outlined in this dissertation considers absorbing barriers which are at right angles to each other. By non-symmetrical alignment of absorbing barriers it can readily be seen that PA will be affected directly and result in an alteration of the O.C. curve.
2. Consider the effect of non-continuous absorbing barriers; i.e. barriers that are "stair-stepped" in shape, on PQC. As visualized at this time such a model would be small at the origin and become progressively larger toward the accept absorbing barrier.

Experimentation with model configuration should lead to more economical inspection in that ASN could possibly be reduced.

One other minor proposal must be made in regard to the algorithm. The algorithm as given is in Fortran II language. Conversion to Fortran IV will provide some savings in computer time when calculating O.C. curves.

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