GENERATING DECISION CRITERIA:
THE FOUNDATION AND
APPLICATION OF
CARDINAL UTILITY
THEORY

By

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PREFACE

> It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard of their own interest. We address ourselves, not to their humanity, but to their self-love, and never talk to them of our own necessities but of their advantages. Nobody but a begger chooses to depend chiefly upon the benevolence of his fellow citizens (1).

And, so, in less harsh terms, the genesis of this thesis lies, firstly, in the goal accomplishment aims of the manager and, secondly, in the attainment of such goals through the allocation of scarce resources to the contributors of the organization.

In conducting the preliminary research on utility theory, there was no one reference source available that compressed the present state of the art, nor could the source be extended to a few outstanding works. Rather, the research covered over two hundred articles and books in establishing the current development level in this research area. As a consequence, the extensive citations presented are, hopefully, to accomplish two purposes. The first purpose is to document the development and application of utility theory in the allocation of scarce resources. The second purpose is to furnish a selected bibliography of the important contributions in utility theory so that this research may serve as a teaching aid as well as the intended purpose of research. The bibliographic references should conceivably reduce redundant literature search in any extension of this work.

Returning to Graduate School after eight and one-half years in industry was no lightly weighed decision. The firm push came from a company sponsored school at Princeton, New Jersey, where I felt humbled by the perspective of "what I didn't know," and soon thereafter came the decision to try for a more formal approach in the search for the techniques and tools of management. And, in this search a deep debt of gratitude is due most particularly to H. G. Thuesen, Professor and Head Emeritus, School of Industrial Engineering and Management, for pointing the way, in my undergraduate course in Engineering Economy, to a philosophy of learning both in and out of the classroom. Unfortunately, his retirement last year prevented his serving on my Graduate Committee; however, he did give, in immeasurable ways, guidance and counsel when paths looked rocky and forbidding. I look back to his counsel -- hopefully, look forward to more -- and regret only that his presence in the classroom will be lost to those who follow. Professor Thuesen stimulated me to seek answers surrounding the subjective areas of management. For my opportunity to study under his tutelage, I am forever indebted.

My deep debt to Professor Wilson J. Bentley, Head of

the School of Industrial Engineering and Management and Chairman of my Graduate Committee, who started all this by saying "Come ahead -- you can do it.", for his encouragement, his faith, his counsel, and for his particular ability to offer a measure of challenge, is sincerely acknowledged.

To a friend and master tutor, Professor Paul E. Torgersen, who provided a great amount of counsel and guidance, and provided the impetus to search for an objective measure of organization behavior, my debt is hardly less.

To the other members of my committee: Professors Ansel M. Sharp, J. Leroy Folks, and James E. Shamblin, my debts are deep and sincere. Their guidance and advice provided the touch-stones to connect logical reasoning with logical results.

No small debt of gratitude is due to the many others who provided the atmosphere and learning opportunity at Oklahoma State University; special recognition is due Professors Wolter J. Fabrycky and Luther G. Tweeten for their many helpful ideas and suggestions in narrowing down the gap of uncertainty over which all decision-makers must jump.

The financial assistance supplied through the Ford Foundation Forgivable Loan program aided significantly in allowing full-time devotion to graduate studies.

To Miss Velda Davis, I pay special tribute for more than the deciphering of atrocious handwriting into typescript. Her diligent and careful effort in the preparation of the thesis from rough draft into final form entailed details known only to and appreciated by those who travel the final steps towards a terminal degree.

To my parents, whose early sacrifices and continued faith and generosity have made this possible, I acknowledge the debt of a son.

To my father-in-law, Dr. A. E. Darlow, former Vice-President for Agricultural Sciences and Dean of Agriculture, Oklahoma State University, his advice, guidance and generosity in many ways will always be deeply appreciated.

My last, but certainly not least, acknowledgment of debt goes to my dear heart, my wife, without whose encouragement and love this task could never have been accomplished. To her and our three sons, who in their young years sometimes wonder what life is worth if even old men still have to go to school, I hope to make it seem worthwhile.

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CHAPTER I

ORGANIZATION GOALS

The essential executive functions in an organization are to (a) provide a system of communication, (b) to promote the securing of essential effort, and (c) to formulate and define purpose. While the first and third of these functions have been widely analyzed and revised, less has been recorded on the securing and maintaining of essential effort in the organization.

In a sense, the emphasis now appears to be the view that the efficiency of the organization is measured by its capacity to offer effective inducements in sufficient quantity to maintain the equilibrium of the cooperative system. Barnard (1, p. 93) indicates that it is efficiency in this sense and not the efficiency of material productiveness which maintains the vitality of the organization. In this context, the meaning of efficiency as applied to organization is the maintenance of an equilibrium of organizational activities through the satisfaction of the motives of individual contributors sufficient to produce such activities.

The appraisal of organization activity does not lend itself to isolated measurements. It is an appraisal of the whole system, the system of cooperative effort.

Cooperation is an expression of human will and purpose in a physical environment. It is never a creator of, and, to no degree, an operator on physical material. It is a creator and convertor of utility - a satisfying preference function. However, most possible cooperation is not undertaken or is not successful. To be successful, cooperation must create utility, and it must not be so allocated in the process of cooperative activity that no surplus remains to satisfy human motives. Barnard (2, p. 253) credits the system as:

> Each incoming contribution goes into a pool, as it were, and each outgoing distribution goes out of the pool, but the two cannot be identified. They cannot be identified because it is utilities, not the things to which utilities are ascribed, which are paid in and paid out; and utilities are created in the process. That is the reason for cooperation.

This means that efficiency of organization results from two controls: the control of output and income at the point of exchange, at the periphery of the organization; and coordination which is internal and the productive factor in the organization. Exchange is the distribution of utility; coordination is the creator of utility.

For the organization to survive, coordination must itself create a surplus of utilities. The necessity for conservatism in the distribution arises from the probability that the surplus, small perhaps in most successful organizations, will not be sufficient to permit dissipation by waste, and the organization will otherwise fail.

The equilibrium of the organization economy requires that it shall command and exchange sufficient utilities of various kinds so that it is able in turn to command and exchange services of which it is constituted. It does this by securing through the application of these services the appropriate supplies of utilities which, when distributed to the contributors, insure the continuance of appropriate contributions of utilities from them. Inasmuch as each of these contributors requires a surplus in his exchange; that is, a net inducement, the organization can survive only as it secures by exchange and creation a surplus of utilities in its own economy.

The nature of this economy must be strongly emphasized because fixed notions in current use so often conceal it. For example, it is said that a commercial organization cannot survive unless its income exceeds its outgo; a statement that begs the point. It is only true if no one will contribute the deficit in commercial goods for non-commercial reasons. But, this does not infrequently occur. Family pride, philanthropic motives, and national prestige often induce economic contributions for non-commercial motives that enable an organization that is economic in character to survive. And the fact is plain that organizations in large numbers that are unsuccessful economically nevertheless continue to exist, whatever may be the motives. They can exist, however, if the economic and other satisfactions which they produce or secure as a whole can pay for the economic and other services which they consume as a whole.

The appraisal of an organization is not a personal appraisal, nor, except incidentally, a market appraisal, nor the resultant of individual appraisals. The appraisal is and must be an appraisal based on its coordinative action - something unique to itself. The organization appraises physical possessions, social relations, personal contributions, on the basis of what it can do with them. The organization can create some utilities for itself by its action; it can gain some utilities by exchanges; it can transmute or transfer utilities. The ability of the organization to act and to survive depends upon the success of its action in maintaining the pool of utilities it can use.

CHAPTER II

ORGANIZATION THEORY AND THE THEORY OF THE FIRM

The assumption of economic rationality in the theory of the firm wherein scarce resources are allocated by the price mechanism is supported by two propositions: (a) firms operate with perfect knowledge, and (b) firms seek to maximize profits. A number of attempts have been made to adapt these propositions to higher levels of sophistication. First, it can be assumed that firms maximize the discounted value of future positive funds flow, and, secondly, that firms have perfect knowledge only up to a probability distribution of all future states of the environment.

There have been two basic challenges to the profit maximization criterion presumedly used by managers. First, is profit the only objective of the firm? Second, does maximization describe what the firms do about profits? Papandreau (3, pp. 183-219) argues that organizational objectives grow out of interaction among the various participants in the organization. This interaction produces a general "preference function," but Papandreau leaves little analytical thought beyond the profit maximization function he criticizes. An alternative to the preference function is suggested by Rothschild (4, pp. 297-320) who suggests that the primary motive of the manager is long run survival. In this view, decisions aim to maximize the security level of the organization. The security level implies that the probability of survival over the long run is maximized. Baumol (5, pp. 45-53) offers the suggestion, derived from empirical studies, that a security level of profit is maintained only as a constraint on sales maximization. Again, the analysis Baumol derives has the implication of long run survival as the firm seeks contributors to the organization who may be not only direct employees, but commercial funding firms to whom the firm looks for outside capital requirements.

The second attack on the assumption of profit maximization recognizes the importance of profits, but questions the assumption of maximization of profit. Gordon (6, p. 265), Simon (7, p. 99), and Margolis (8, p. 187) have all argued that profit maximizing should be replaced with a goal of making satisfactory profits. Satisfactory profits represent a level of aspiration - a security level - that is used as an evaluator of alternative policy and action. The aspiration or security level may change over time, but in the short run it defines a dichotomous utility function through which alternatives are good enough or not good enough. This critique of maximization of profit is linked with other proposals for revision of organization goals. Primary among these proposals is that of Simon who suggests that information is not given to the firm, but must be obtained; that alternatives are searched for and discovered sequentially, and that the order in which the environment is searched determines to a substantial extent the decisions that will be made. In this way, the theory of choice and the theory of search become closely entwined, thus take on prime importance in the general theory of decision-making.

Conflicts in Goal Measurement

A more conventional attempt to deal with the same problem is reflected in recent contributions to a normative theory of search proposed by Charnes and Cooper (9, pp. 450-458). In this theory, search activity is one of the competitors for internal resources, and expenditures for search are made up to the point where the marginal cost of search equals the marginal expected return from it. These elaborations of the theory are clearly more traditional than Simon (7) proposes, but they do not meet the requirement of satisfying the complaint. Unfortunately, the theory in all its standard forms ignores the situation that decisions are made in organizations. There is one relevant exception to the void. Marshall (10), as others before him, was impressed by the apparent increasing returns to scale. Marshall undertook to explain the historical reduction in production costs by introducing the concepts of internal and external economies, and it is this germ of an idea about the effect of organizational size on organizational performance, by the use of "internal economies," that became important to several decades of theory development (10). This development ignored other important organizational aspects of the firm.

In recent years there has been speculation about the effects of other organizational factors. First, the failure to view the firm as an organization has been criticized. Papandreau (3) has made perhaps the most detailed argument for expanding the framework of the firm. He views the firm as a cooperative system. The executive tasks are accomplished by a "peak coordinator." The firm has certain goals, and it is the peak coordinator's job to achieve these by allocating resources rationally. This involves three actions: (a) substantive planning - constructing the firm's budget, (b) procedural planning - constructing a system of communication and authority, and (c) executing both plans.

Within such a model of the firm, Papandreau sees certain areas of psychology playing a helpful role. The goals of the firm are strongly influenced by both internal and external forces. The internal influences come from such entities as stockholders, unions, government, and so forth. The preference function, previously mentioned, is a resultant of the influences which are exerted upon the firm. The peak coordinator's job is to maximize the preference function. The greatest difficulty with Papandreau's analysis is that it does not relate specifically to decision-making. It provides a general analysis of the firm from the standpoint of organization theory without specifying precisely how the model can be used for economically-oriented decisions.

The second indictment against current organization theory alleges that firms, in fact, do not equate marginal cost and marginal revenue in deciding on either output or price, rather they follow one or another of a series of rules of thumb in allocating resources. The cause for concern, although, appears to be more a lack of integrating tools of resource allocation due to substantial differences that have been found in decision-making processes. Many decision-making processes cannot offer proof acceptance of the theory of the firm since application capability is not wholly present in the decision environment. Many of the attacks on the theory of the firm are not so much proper critiques of existing theory as they are suggestions for the development of a theory appropriate with regard to the internal allocation of resources.

The objective to merge economic theory, i.e., the theory of the firm, with organization theory rests upon the integration of these theories through the particular attributes of utility theory.

To assess if such integration is possible, it is possible to examine three major divisions of interest in organization theory.

The first of these, the sociological division, was founded on the precepts of Weber, Durkeim, Pareto, and Michels and centers on the phenomena of bureaucracy. The second division is social psychology and has been built primarily on an experimental base with an emphasis on an "efficiency" criterion. The third is administrative in the sense that it focuses on the problems of the executive in dealing with an organization.

The early sociological theories of Weber and Durheim (11) emphasized the division of labor and specialization as broad social trends and the importance of organization growth in utilizing specialized competences. Weber placed considerable emphasis on the rationality or adaptable behavior of bureaucratic organization. To a certain extent, the early theorists and, to a much greater extent, modern sociological studies of organizations emphasized what Merton (12) has labeled the "unanticipated consequences of purposive social action." The major variables tend to be such things as subgoal differentiation and conflict, individual personality changes, and the organization life-cycle.

Social psychological approaches to organizational phenomena have tended to be less expansive in scope. In general, they have taken a relatively obvious criterion of efficiency, for example, productivity, in a simple task and examined experimentally the effect of some small set of independent variables in the efficiency of the organization.

In this tradition have been the studies of communication nets, simulated radar warning stations, and small problem solving groups. Somewhat less experimental have been the studies of morale and productivity, which also emphasize explicitly a criterion of efficiency (13)(14).

The final branch of organization theory dates, in a sense, from the earliest political and social philosophers. Speculation about centralization and decentralization and the problems of coordination can be found in pre-Christian writings. However, modern administrative theorists generally reject the earlier formulations with considerable vigor.

In particular, Barnard (2) and Simon (15) have argued against the excessively formal and nonoperational analyses of early administrative theory.

Much of the recent work in this branch of organization theory takes the decision-making process as its specific focus of concern. The theory tends to present a dichotomy; each dealing with a certain class of decisions of significance to organizations. The first stem views the organization as a system of contributors through which transfer payments are arranged among the contributors. The theory describes the decision to contribute and specifies the conditions of organizational survival in terms of maintaining an equilibrium of contributions from and payments to the contributors.

The second stem views the development of theory explaining how decisions are made in organizations with particular emphasis upon the region of executive influence and the impact of organizational position on individual goals and perceptions.

The Present Level of Combined Theory

Without attempting to describe the various forms that theoretical arguments have assumed within organization theory, three main points emphasize the present state of the disassociation of organization theory and economic theory.

1. Organization theory focuses on a set of problems that are different from those of the economic theory of the firm. Its problems are not specifically economic; virtually nothing is said about output levels, economies of scale, and budget determination.
2. Although it places considerable emphasis on the study of "process" - the study of what goes on in an organization - only the third branch of the theory focuses primarily upon organizational decision making processes.
3. Unlike the theory of the firm, there is little consideration of aggregation of resources. There is little reference of a commodity to aggregate.

As a consequence, existing organization theory provides little quantitative basis for merging economic theory into a modified theory of the firm. The sociological and social psychological approaches have emphasized questions that are only marginally relevant to either the objectives of conventional theories of the firm or the objective of predicting individual firm behavior. The decision-making approach has developed a substantial theory of decision-making processes in an organizational context, but has not applied the theory to the specific environmental conditions in which the business firm operates nor applied the theory in detail to the particular decision variables that characterize the firm's operations.

As a modification of Cyert and March's (16), "organizational slack" notion, the ability of the organization to survive, i.e., to obtain goals, represents its ability to implement an effective control system to essentially allocate resources in some predictive fashion to assure obtaining effort by necessary contributors.

As defined by Cyert and March (16, p. 34):

> Organizational slack represents the disparity between the resources available in the organization and the payments required to maintain coalition of members contributing to the firm. This difference between total resources and total necessary payments is what we call organizational slack.

While Cyert and March have reached a fairly refined level in their theory, their construct yields to sub-optimal characteristics of goal attainment, i.e., a production goal, inventory goal, profit goal, and market share goal. The optimal construct appears to be capable of synthesizing the subgoals into a utility function to represent the total resources of the firm and the demands placed on these resources by contributors.

CHAPTER III

UTILITY THEORY

To develop a theory of organization survival, some notions of utility must be presented.

The measurement of utility was considered possible by the Nineteenth Century economists Jevons, Walras, and Marshall (17). They argued that utility - the satisfaction derived from possession and/or consumption of commodities - was measurable in the same sense that the weight of objects is measurable. Marshall and Walras were the strongest proponents of measurable (cardinal) utility, and, although they were never satisfactorily able to quantify their theory, much of their work was accepted from the notion of maximizing total utility of commodities by way of the law of diminishing marginal utility. The conclusion was that the marginal utility of a commodity would be proportional to the exchange price. Marshall implied that the utility function was known. Unfortunately, it could never be developed satisfactorily. It remained for Walras to discover ordinal utility measurement. Walras determined that, given a indifference map of utility, no cardinal measurements would be required to establish the marginal rate of substitution of factors. The discovery that no scaling of utility preferences were required for establishing factor demand remained the chief source of utility doctrine until the now-classic Von Neumann-Morgenstern discovery of cardinal measurement in 1944. Pareto (17) refined Walras' ordinal theory and established ordinal utility theory in the manner of Figure 1. Pareto maintained that all consumer behavior can be described in terms of preferences, or rankings, in which a consumer need only state which of two collections of goods or activities he prefers without reporting on the magnitude of any numerical index of the strength of this preference.

Indifference Curves

The analytical device developed to represent ordinal preference is the indifference map developed from Figure 1 and illustrated as Figure 2. In this latter diagram, quantities of different commodities (goods or activities) are measured along the axes so that point 1 on difference curve AA' represents a collection of commodities consisting of one unit of wages and four units of other job emoluments. It represents no more than this, and this datum by itself contains no information about the consumer of utility. In particular, it does not mean that he is indifferent between four emolument units and one wage unit. Every possible combination of these two factors can be represented by a point on the indifference map. By definition, an indifference

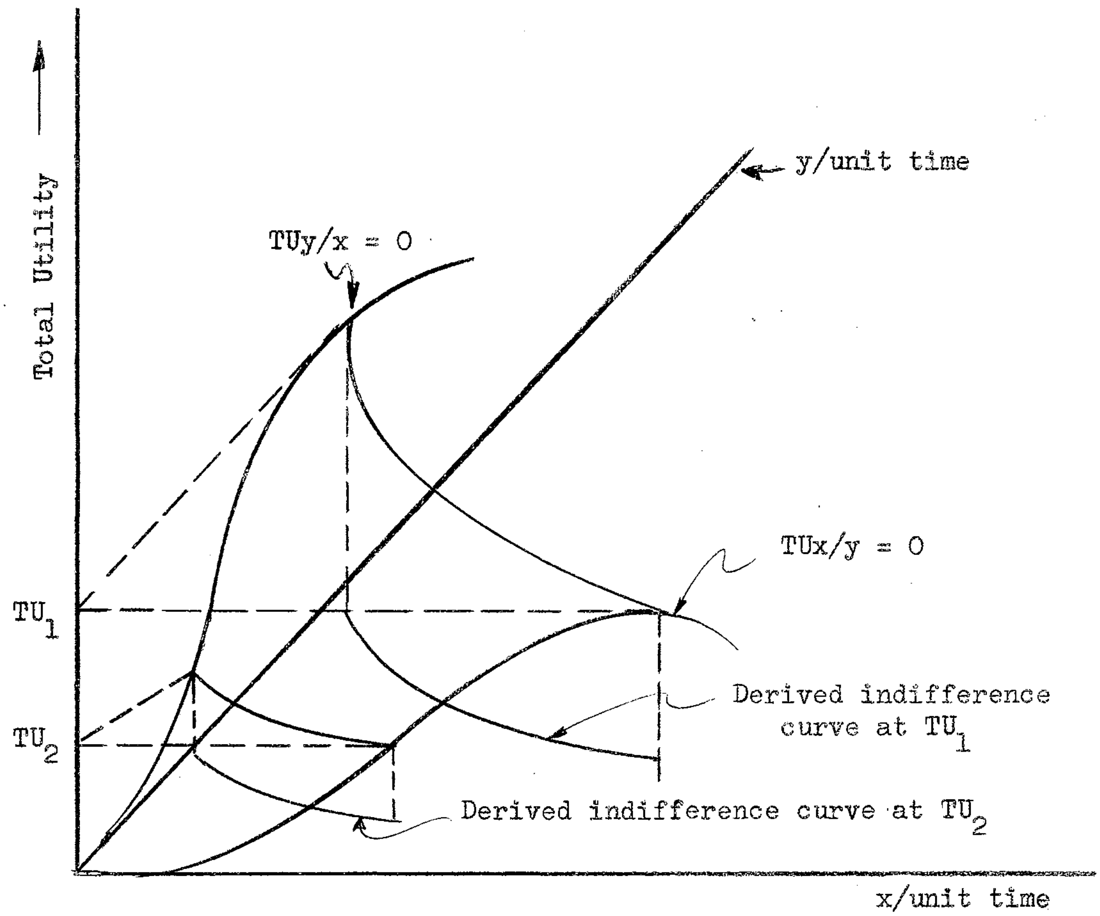


Figure 1. Development of Ordinal Indifference Curves

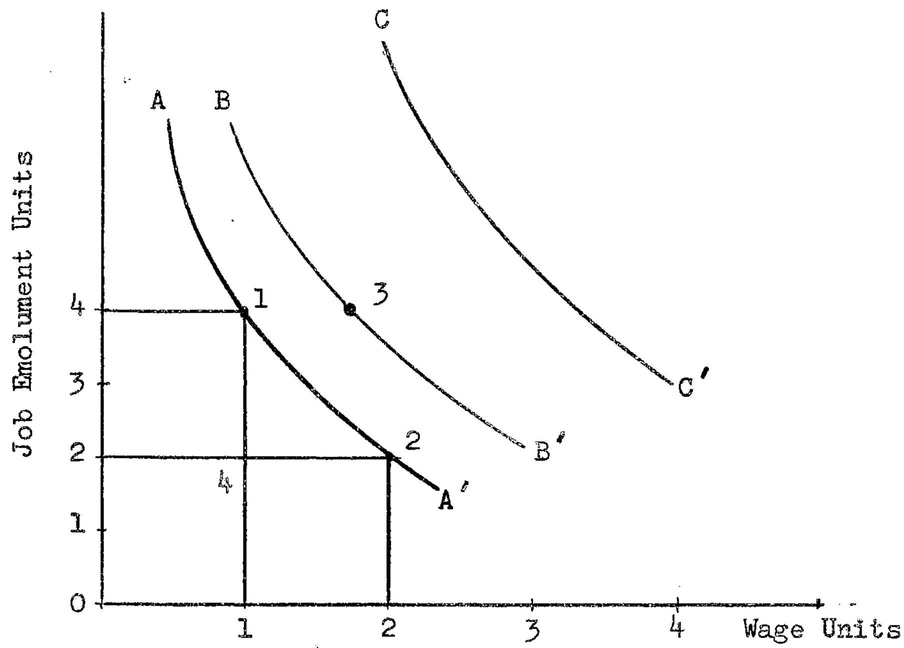


Figure 2. Indifference Map

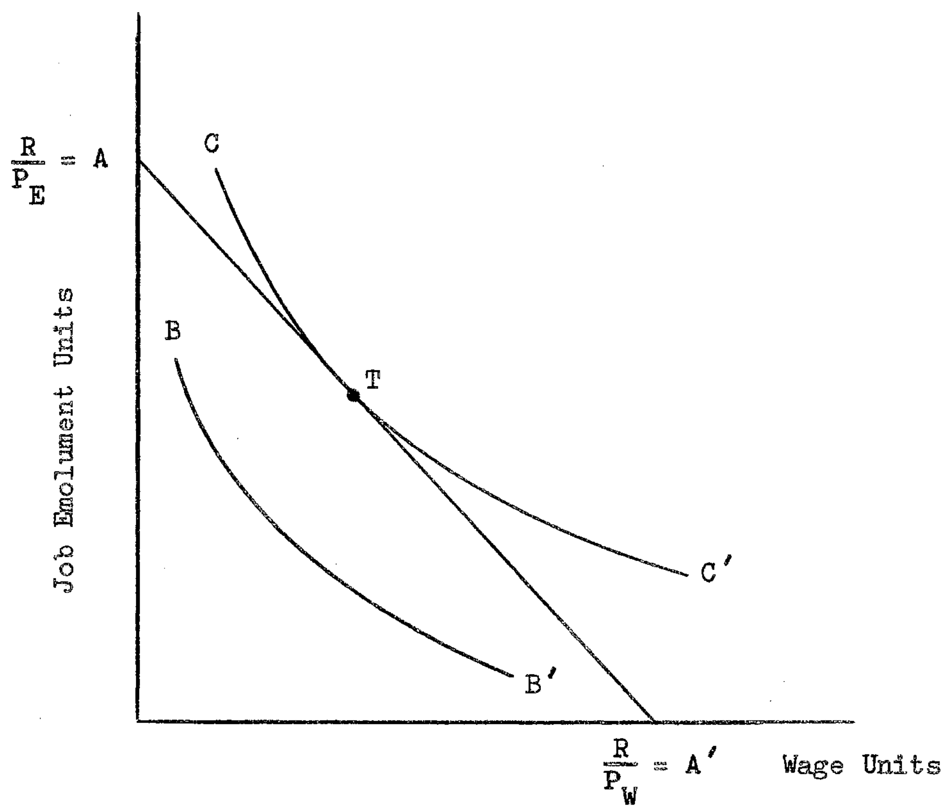


Figure 3. Equilibrium and Budget Restraint

curve is the locus of points each of which represents a collection of factors such that the consumer of utility is indifferent among any of these combinations. As an example, point 2 in Figure 2, means that the utility-consumer is indifferent between collections of factors represented by points 1 and 2. Each discrete combination of factors represented on curve AA' represents a fixed level of utility demand.

If it is assumed that the consumer prefers combinations represented by points on higher indifference curves, e.g., he prefers factor combination $3 > 1$, the indifference map provides a complete and simple report on the utility consumer's ordering of all possible combinations of the two factors. For if two combinations are represented by points on the same indifference curve, the utility consumer is indifferent between them, and, in any other case, he prefers that collection which is represented by a point on a higher indifference curve.

Exchange Criteria

Since all combinations of consumption factors represented by points on an indifference curve, AA' have equal utility, the utility consumer's indifference curves are the contour lines or iso-utility lines of his utility preferences.

The important characteristics of indifference mapping now come to light.

The slope of an indifference curve has a significant interpretation. In Figure 2, arc 1-2 has the slope $\frac{1-4}{2-4}$. In moving from point 1 to point 2, the utility consumer loses 1 - 4 (or 2) emolument units and gains 1 - 2 (or 1) wage unit. Thus, the absolute value of the slope is $\frac{2}{1} = 2$, which indicates the utility loss of 2 emolument units can be replaced by a gain of 1 wage unit.

This absolute value of the slope, called the marginal rate of substitution (MRS), therefore, represents the number of units of the latter whose loss can be made up by a unit gain in the former. It is the consumer's psychological rate of exchange between two factors.

It is also possible to show that this slope is equal (in absolute units) to the fraction marginal utility of emoluments/marginal utility of wages; that is,

$$\text{slope AA}' = \text{MRS} = \frac{\Delta \text{ Emoluments}}{\Delta \text{ Wages}} = \frac{\text{MU of Wages}}{\text{MU of Emoluments}}.$$

This inverse relationship between ΔE and the marginal utility of E represents that $\Delta E = 2$ units of Wages which a utility consumer is willing to give up for $\Delta W = 1$ unit of Wages.

An important distinction to recognize in indifference mapping is that while MU appears in the analysis, only the ratio of two marginal utilities ever occurs in indifference analysis. In such a ratio, the marginal utility of one commodity is not measured in utiles, but in terms of the other commodity. The question in ordinal utility theory is how much of E an additional unit of W is worth or what is the MRS_{EW}.

Indifference curve analysis requires that certain assumptions be made concerning the choice behavior of utility consumers:

1. Nonsatiety: The consumer is not oversupplied with either factor; he prefers more emoluments and/or wages.
2. Transitivity: If A, B, and D are any three factor combinations, then if $A \supset B$ and $B \supset D$, then $A \supset D$. This condition simply requires that the utility consumer possesses a conceptually simple type of consistency of preferences.
3. Diminishing Marginal Rate of Substitution: Consider two points, 1 and 2, in Figure 2. If at point 2, the consumer has a relatively small amount of W compared with a large supply of E, then at 2 the marginal utility of the relatively scarcer W will be large in comparison to that of E, i.e., the consumer will be willing to give up only a relatively small amount of W in exchange for an additional unit of E.

With these three assumptions, certain properties of indifference curves develop:

1. An indifference curve which lies above and to the right of another indifference curve represents preferred combinations of factors.
2. Indifference curves have a negative slope.
3. Indifference curves can never meet or cross.

4. The absolute slope of the indifference curve diminishes toward the right.

Prediction of Utility Consumption

The interesting capabilities of ordinal utility theory now permit extensions of current theory to predict economic behavior of a manager in dispensing utility to contributors of an organization.

Since the axes of an indifference map represent only quantities of factors rather than real amounts of money, a resource budget or constraint line will permit an analysis of the quantity of utility limited resources may provide.

For example, Figure 3 (page 17) presents an indifference map together with a budget restraint line AA', which represents the various combinations of factors which may be purchased to equal the budget restraint.

If the price of factors is fixed, that is, they do not vary with the amount of goods purchased, the budget restraint line will possess the following properties:

1. It will be a straight line.
2. It will have a negative slope equal to the inverse ratio of the prices of the two factors, where $\frac{\Delta E}{\Delta W} = -\frac{P_W}{P_E}$.
3. The amount of budget restraint, R, is represented by the following equation where:

   E = quantity of emoluments

   W = quantity of wages

P_E = unit price of E

P_W = unit price of W

$$(P_W)W + (P_E)E = R$$

then

$$E = \frac{R}{E} - \frac{P_W W}{P_E}.$$

For the case where quantity discounts may be applied as a function of the learning progress function, the budget restraint line will be convex to the origin.

The point of equilibrium will occur at point T, Figure 4, at the point of tangency where the slope of the budget restraint line and that of the indifference curve are equal. Thus, equilibrium occurs when $\frac{P_W}{P_E} = \frac{MU_W}{MU_E} = MRS_{WE}$.

Growth and Decay of Factor Utility

Two important concepts are now developed to reveal the impact of changes in budget restraints and changes in the prices of factors as viewed by the executive in distributing utility to the individual contributors of the organization.

The budget restraint effect indicates that the growth or decay of budget resources definitely increases the utility mix of factors available for distribution to contributors. The primary change to be noted is a change in absolute budget resources which will result in a parallel shift of

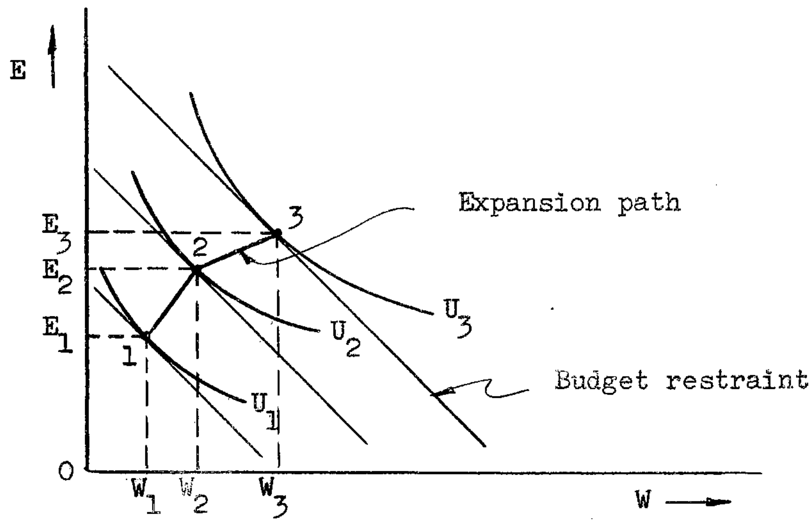


Figure 4. Budget Restraint Shift

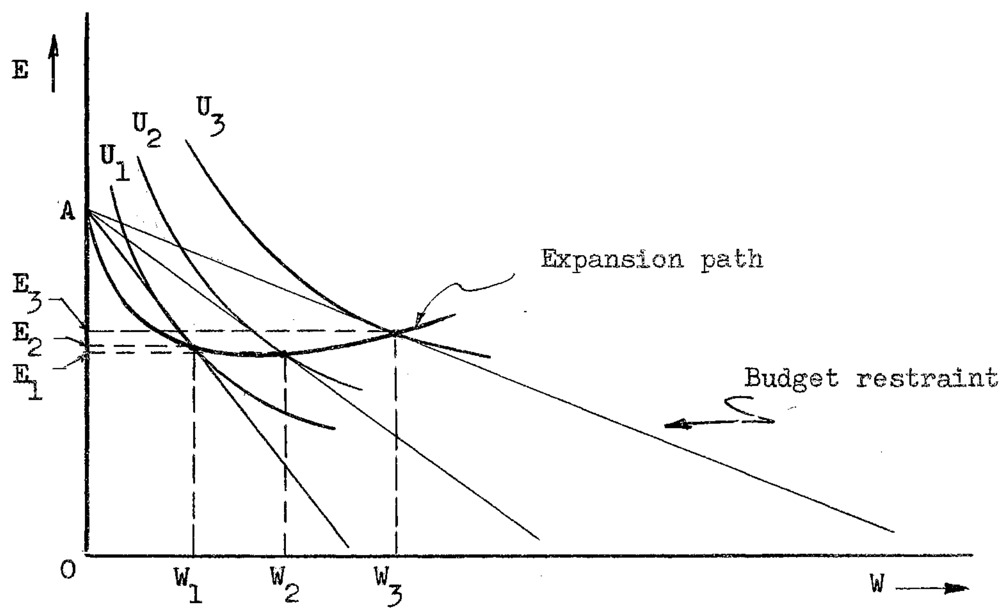


Figure 5. Price Change Shift

the budget restraint line as shown in Figure 4. The shift of factor combinations as budget restraint changes is depicted by the expansion path of factor combinations as budget resources change.

At this point, we have assumed factor prices have remained constant. If this assumption is relaxed, recognition of changes in the system of utility mix can be identified from both the change in budget restraint as well as from a change in factor price. Conventionally known as the Slutsky Theorem, the change in utility mix will first be examined for a price change with budget restraint fixed. If the price of job emoluments declines and the price of wages remains constant, then the manager can obtain more distributable utility in the form of job emoluments than before prices changed. Given a series of consecutive price changes, the change in the system of utility mix appears as in Figure 5.

If Figure 4 and 5 are each viewed independently, there are certain characteristics each analysis displays which is of interest in the distribution of utility.

In Figure 4, the expansion path is identified as the locus of points tangent to budget restraint lines and the iso-utility lines. The expansion path indicates how the relative proportion of the utility factors changes as greater amounts of utility are required by the organization if the optimal cost of incentives is to be maintained. The expansion path is shown to have a concave to origin slope to indicate as greater utility demands are made, the mix proportion shows that as utility demand increases, the rate of demand increase for job emoluments is greater than for wages. Wages become an "inferior" factor for as utility demand increases, more job emoluments are substituted for given wage increases. The reverse conclusion can easily be shown to indicate that organization activity can generate an inferior demand for job emoluments.

In Figure 5 (page 23), the expansion path is initiated at point A, rather than at origin, since as the price of job emoluments increases, the manager feels he can get less and less of this factor for a given budget restraint. Eventually, when the price of job emoluments goes high enough, the manager will be forced to discontinue its distribution entirely and the utility mix will consist of only one factor, wages.

If the effects of budget restraint changes and price changes are combined, then the effect of substitution of factors and the expansion effect of budget resources can both be identified to be equally applicable to (a) those organizations which have unlimited capital, and to (b) those organizations which have limited capital and must consume the utility of contributors only up to a given limit of available resources. Both types of situations are illustrated in Figure 6. With factor-price ratios as indicated by the slope of the iso-cost line and the utility demand indicated by the iso-utility curve, U_3, the optimal cost combination of E and W includes OE_3 units of E and OW_1 of W.

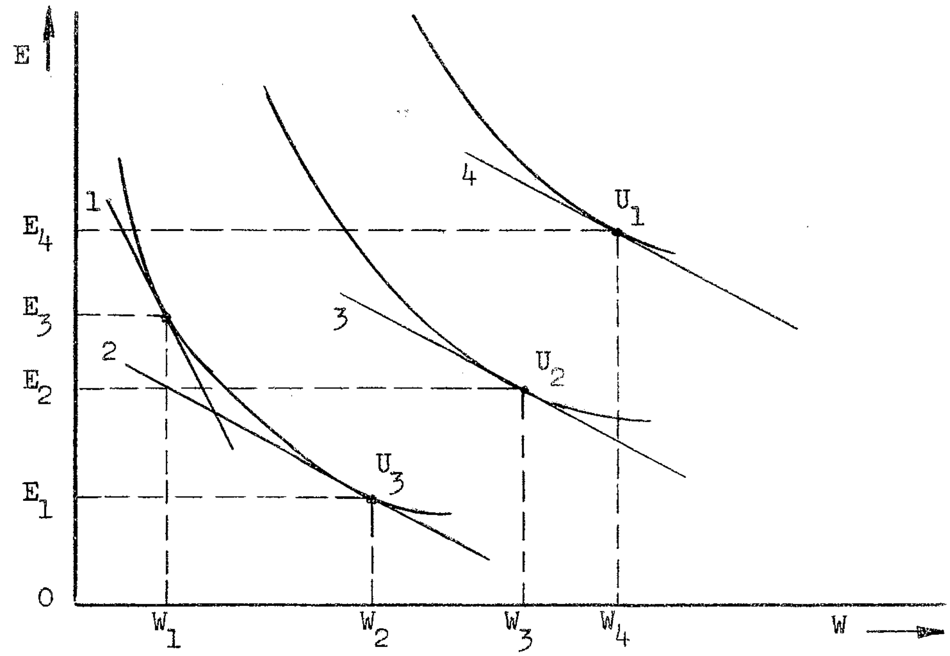


Figure 6. Substitution and Expansion Effects

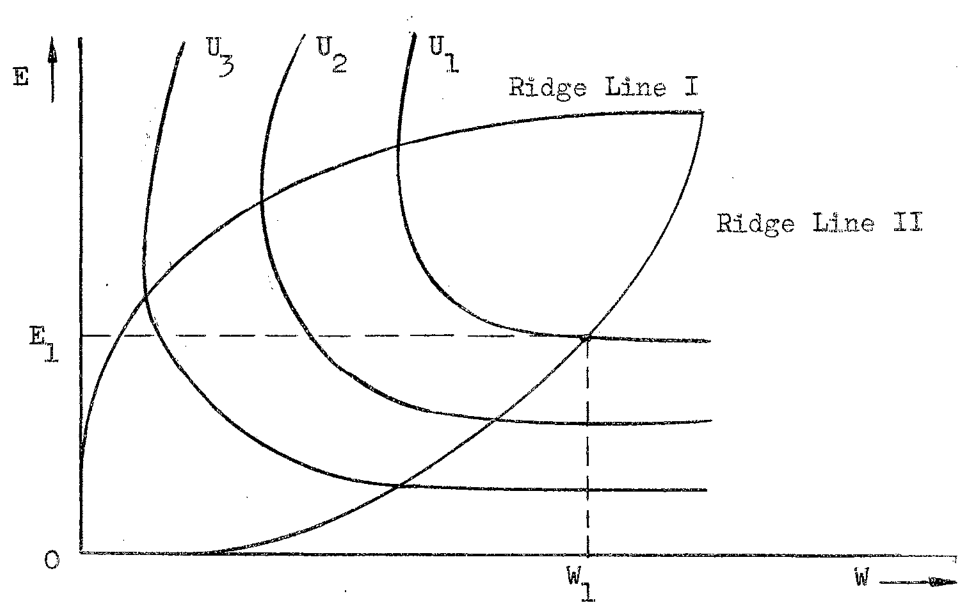


Figure 7. Input Relationships

If the price of factor W falls relative to the price of factor E as indicated by a new iso-cost line 2, an input of OE_1 units of E and OW_2 units of W will optimize costs for a U_3 level of utility. A substitution effect has taken place: input of W has increased by W_1 W_2 units while input of E has decreased by E_3 E_1 units. The substitution effect, use of more W and less E for the same output of product, results from changes in the relative prices of the factors. The expansion effect is demonstrated after a fall in the price of factor W, acquisition of additional utility can be accomplished because of either or both of the following: (a) the same cost outlay will allow the organization with limited resources to hire more factors, and (b) the lower cost ratio (due to a decrease in price of W and a substitution of this factor for E), allows utility acquisition to be extended to a higher iso-utility curve before the marginal cost of factor resources exceeds the value of their marginal utility for an organization with unlimited capital. The expansion effect is illustrated where utility acquisition is increased to U_2 after price changes and OE_2 units of E and OW_3 units of W are used. In respect to factor W, the substitution effect resulted in an increase in its use by W_1 W_2 units. In respect to factor E, the expansion effect resulted in use of another W_2 W_3 units. In respect to factor E, the expansion effect partially offset the substitution effect; while the substitution effect reduced the use of E by E_1 E_3 units, the expansion effect restores E_1 E_2 units to use.

Economic Complements and Substitutes

It is not impossible for the expansion effect to result in increased use of both factors. If in Figure 6 (page 26), the expansion effect is so great that it requires a utility level of U_1, use of E will increase to OE_4, while the input of W will increase to OW_4. This combination includes more of both factors than the original quantities OE_3 and OW_1. Factors E and W have now become economic complements. When two factors, E and W, can be used in acquiring utility and a decrease in the price of factor W leads to an increase in acquired utility and an increase in use of both factors E and W, the factors are economic complements. If more of factor W is used while less of factor E is used, with utility acquisition remaining the same or increasing, the two factors are economic substitutes.

Rational Choice in Factor Mix

Iso-utility curves for complementary, substitute, and limitational input factors are analyzed for a single iso-utility curve, U_1, Figure 7 (page 26). Input of E serves as a complementary and limitational factor relative to W. After reaching OW_1 units of W input, no amount of substitution of W for E will reduce the input level of E below OE_1 units. Down to this level of OE_1, E serves as a substitute for W. E serves as a complement for W beyond level OW_1, since no amount of substitution of W for E will permit utility to be increased to a higher U_x level. To apply combinations of factors in the region to the left of Ridge Line I requires that the quantity of W applied to generate a U_x level of utility is greater than required, or clearly a misappropriation of resources.

To display the usefulness of isoquant theory, it is deemed practical to present a theory of utility production in both theoretical and demonstration form in Chapter V.

Ordinal Utility Theory

The theoretical application of ordinal utility theory seems faultless. The difficulty, obviously, is that while ordinal theory lends itself to optimal decision patterns, the quantification of utility preferences is quite difficult under ordinal theory. In fact, the premise of ordinal theory lies in only an ordering of preferences. In a non-deterministic environment, the evaluation of ordinal alternate opportunities is outstandingly weak.

The essential development by Von Neumann and Morgenstern (18) in 1944 to allow for the quantification of utility preferences allowed for the refined techniques that are developed as tools of decision-making. As a consequence of the quantitative measurement of utility, the strength of both ordinal and cardinal utility theory will be presented in combined form.

In utilizing both cardinal and ordinal utility concepts, the objective is to allow for the probabilistic distribution of alternative outcomes in the decision environment of the manager as he seeks to maintain the stability of the organization.

The traditional mathematical device for dealing with risk and uncertainty has been to accept an available alternative so as to maximize expected value. The expected value of an alternative is found by multiplying the value of each possible outcome of alternative by its probability of occurrence and summing these products for all possible outcomes: In symbols, then

$$\text{Expected Value} = P_1 \$_1 + P_2 \$_2 + \ldots + P_n \$_n$$

where P_m represents the probability of occurrence and $\$_n$ represents the dollar value of the n^{th} possible outcome from a chosen alternative. Based upon this method of choosing among activities representing financial return, Daniel Bernoulli (19) discovered that the expected value criterion appears as a questionable test of acceptability when confronted with his St. Petersburg paradox.

The paradox is described as follows: A persons buys a chance to flip a coin until a head appears. Should heads appear on the first throw, he receives one dollar. Should heads appear on the 2^{nd}, 3rd, ..., n^{th} throw, he receives two dollars, four dollars ..., $\$2^{n-1}$, respectively. How much should he rationally pay for a chance to play the game?

The St. Petersburg game's expected value, assuming the coin to be fair, can be computed as follows:

$$EV = \sum_{t=1}^{\infty} (\tfrac{1}{2})^{t} (2)^{t-1} = \tfrac{1}{2} \sum_{1}^{\infty} 1^{t-1} = \infty.$$

One would expect a rational, expected return maximizing risk taker, therefore, to pay virtually anything for a chance at it. Why, asked Bernoulli, are takers so scarce at twenty dollars a throw?

A number of answers are possible. One is that an infinite series of tails would break the bank. The meaningful value of the game under this twenty dollar constraint then is:

$$EV = \tfrac{1}{2} \sum_{t=1}^{20} (1)^{t-1} = \tfrac{1}{2}(19) = \$9.50.$$

Another viewpoint is that the probability of a long series is close to zero. The probability of a long run of successive tails is:

$$P_{20} = (\tfrac{1}{2})^{20} = .9537 \times 10^{-33}$$

Yet, another factor lies in the assumed proportionality between money and satisfaction. Bernoulli favors this last reason. He contends that \$20.00 is not equal, but is less than 20 times as valuable as one dollar.

The paradox lies in the expected return criterion's symmetrical treatment of extreme possible outcomes.

Another extreme, however, is of more specific concern to the manager -- the possibility of a serious financial reverse. Is a loss of $20,000 exactly twice as unpleasant as a loss of one-half that amount. What is the value of the last dollar that stands between success and failure of the organization.

Symmetrical treatment of the value of one's first and last unit of money wealth by the expected return criterion can defend an insurance company's sale of a policy (on which it has a positive expected return) to a policy holder as being a rational act; but, it cannot provide the same justification for the policy's buyer. Indeed, it can always justify one who offers (but never one who takes) an unfair bet. Thus, neither the person who refuses to stake his fortune on a try at the St. Petersburg game, nor the one who accepts an unfair bet from an insurance company can be classified, by this criterion, as a rational investor.

The Von Neumann-Morgenstern Utility Index

Since Bernoulli first proposed a cardinal utility value for money, later to be suppressed by Pareto (17), individual utility values are taken into consideration implicitly in every decision that an individual makes. Von Neumann and Morgenstern merely proposed a method for extracting and recording these values so that they can be explicitly used as a guide to action -- and what is particularly important, as a guide to consistent action. With utility values explicitly stated, alternatives can be viewed in the light of generating proper levels of utility.

Von Neumann and Morgenstern (18) pointed out that the usual assumption that economic man can always say whether he prefers one state to another or is indifferent between them needs only to be slightly modified in order to imply cardinal utility. The modification consists of adding that economic man can also completely order probability combinations of states. Thus, suppose that an economic man is indifferent between the certain possession of \$15.00 and a 50-50 chance of gaining \$20.00 or nothing. It can be assumed that his indifference between these two choices means that they have the same utility for him. By defining the utility of \$0.00 as zero utiles and the utility of \$20.00 as 20 utiles, two arbitrary definitions, defining the two undefined constants which are permissible since cardinal utility is measured only up to a linear transformation, are available. The utility of \$15.00 may be calculated by using the concept of expected utility as follows:

$$U(\$15.00) = 0.5U(\$20.00) + 0.5U(\$0.00)$$
$$U(\$15.00) = 0.5(20) + 0.5(0)$$
$$U(\$15.00) = 10 \text{ utiles}$$

where: U(X) represents the utility of X dollars.

Stated symbolically:

$$U(B) = P_a[U(A)] + (1 - P_a)[U(C)]$$

where

$U(A)$ = utility of A

$U(B)$ = utility of B

$U(C)$ = utility of C

P_a = Probability of A being received

$(1 - P_a)$ = Probability of B being received.

The Formal Proof of Cardinal Utility Measurement

The formal proof of the Von Neumann-Morgenstern cardinal utility index rests on five assumptions:

<u>Assumption 1. Transitivity</u>: If an individual is indifferent between two alternatives, A and B, and he is also indifferent between B and C, then he will be indifferent between A and C.

<u>Assumption 2. Continuity of Preferences</u>: This is the assumption that if an alternative, A, is preferred to another alternative, B, when P(A) = 1, and if B is preferred to A when P(A) = 0, then there exists some value of P(A) whereby the decision-maker is indifferent between his choice for A and B.

<u>Assumption 3. Independence</u>: If four alternatives exist, A, B, C, and D, and if the decision maker is indifferent between alternative A and B, and he is also indifferent between alternatives

C and D, and if A I B represents an indifferent choice between A and B, then (A,C)I(B,D). For any probability P, then P(A,C)I P(B,D).

Assumption 4. Desire for High Probability of Success: For any alternatives A I B, if $P_a > P_b$ to represent the probability of obtaining A is greater than the probability of attaining B, then A is preferred to B when $P_a(A) > P_b(B)$.

Assumption 5. Compound Probabilities: For any alternatives A and B, where A is preferred to B, and any probability numbers P_1, P_2, P_3, P_4, then in the classical notation of a lottery ticket array devised by Von Neumann-Morgenstern (18):

$$P_1[P_2(A,B),\ P_3(A,B)]\ I\ [P_4(A,B)]$$

where if a lottery ticket holder wins not a prize, but another lottery ticket $[P_2(A,B)]$ which contains a high probability of A being won. If the player loses, then he receives a consolation lottery ticket offering the same (A,B), but with a low probability of winning A (and, consequently, a high probability of winning B). Then, what is the probability of eventually winning the superior prize A? There is a probability P_1 of winning the better lottery ticket which offers A with probability P_2, so the probability of getting E in this manner is $(P_1)(P_2)$. However, if he loses, which has a probability of $(1 - P_1)$, the ticket holder still has the probability, P_3, of getting A, so that there is a probability $(1 - P_1)(P_3)$ of obtaining A. The total probability of obtaining A is then $P_1 P_2 + (1 - P_1)(P_3)$, which is P_4.

The interpretation of this last assumption is to say that an individual's psychology is such that he will evaluate a compound lottery ticket (or a compound decision event) in terms of the probabilities of winning the utlimate prize (or objective).

Some Theorems of Applications

In a traditional demonstration, a lottery ticket is now chosen to be used as a standard against which other alternatives can be evaluated. The ticket is assumed to offer to the winner the prize E, eternal bliss; to the loser, D, damnation, so that any alternative, A, which is brought to be evaluated against this standard ticket will be presumedly no better than E and no worse than D.

By Assumption 2, for any such A, there will be a probability number $P_1 (0 < P_1 < 1)$ such that $AI[P_a(E,D)]$. The following is proof:

<u>Theorem 1</u>. Possibility of Predicting: Given any two lottery tickets $P(A,B)$ and $P'(A',B')$ and a person whose preferences never violate Assumption 1-5, if there is obtained (say by his introspection) the four probabilities, numbers P_a, P_a', P_b, P_b' so chosen that:

$$AI[P_a(E,D)] \quad \text{and} \quad BI[P_b(E,D)], \text{ etc.}, \tag{1}$$

then from these probabilities it is possible to predict which of the two lottery tickets will be preferred.

Proof: The first lottery ticket is evaluated in terms of E and D. This is done by replacing A and B by their equivalents in terms of the standard lottery ticket to obtain

$$[P(A,B)]\, I\, [P_a(E,D)], [P_b(E,D)] \text{ by Assumption 3} \tag{2}$$

$$\therefore [P(A,B)]\, I\, [P_4(E,D)] \text{ by Assumption 1 and 5} \tag{3}$$

where P_4 is the probability $PP_a + (1-P)P_b$. Similarly, the second lottery ticket can be evaluated in terms of E and D as:

$$[P'(A,B)]\, I\, [P_4'(E,D)] \text{ where } P_4' = P'P_{a'} + (1-P')P_{b'}. \tag{4}$$

Therefore, by Assumption 4, the individual must prefer $[P(A,B)]$ to $P'(A',B')$ if and only if

$$P_4 = [PP_a + (1-P)P_b] > P'P_{a'} + (1-P')P_{b'} = P_4' \tag{5}$$

and indifference between these tickets exists if and only if $P_4 = P_4'$. But, by hypothesis, P, P′ are numbers given by the terms of the two lottery tickets, and P_a, P_b, P_a', and P_b' were found out by observing or questioning the lottery ticket holder. Then, P_4 and P_4' can be evaluated directly and the higher of these two numbers must, by Assumption 4, correspond to the preferred lottery ticket.

Now, the construction of the Von Neumann-Morgenstern index is completed and used to predict correctly the choice of lottery ticket. As previously indicated, the following linguistic convention is used for evaluating the utility of a lottery ticket in terms of the utilities of its prizes:

$$U[P(A,B)] = PU(A) + (1-P)U(B).$$

That is, if P = three-fourths, so that the odds of winning are 3 to 1, the utility of the lottery ticket is evaluated at three-fourths the utility of a win plus one-fourth the utility of a defeat. But this only represents a convention. To show that it is usable, one must first re-state, in terms of the present notation, how these utility numbers can be found, and then one must prove that they must always assign a higher utility number to the preferred lottery ticket.

To find the utility of any alternative, A, one first assigns arbitrary "utility" numbers:

$$U(E) > U(D) \tag{6}$$

to eternal bliss (E) and damnation (D) in our standard lottery ticket. Now, U(A) by reference to Equation (1) is found and defined:

$$U(A) = U[P_a(E,D), U(B) = U[P_b(E,D], \text{ etc.,} \tag{7}$$

so that by Equation (6):

$$U(A) = P_a U(E) + (1 - P_a)\,U\,(D), \text{ etc.} \tag{8}$$

Hence, by finding P_a in Equation (1), the utility number U(A) can be computed.

Finally, it can be proven:

<u>Theorem II. Validity of the Prediction</u>: These utility numbers rank lottery tickets correctly so that $U[P(A,B)] > U[P'(A', B']$ if and only if the former is the preferred lottery ticket; i.e., if and only if Equation (5) holds.

<u>Proof</u>: The utility of the first lottery ticket is:

$$U[P(A,B)] = PU(A) + (1 - P)\,U\,(B) \text{ by Equation (6)} \tag{9}$$

$$= PU[P_a(E,D) + (1 - P)U\,[P_b(E,D)] \text{ by Equations (1) and (7)} \tag{10}$$

$$= P[P_a U(E) + (1 - P_a)\,U\,(D)] + (1 - P)[P_b U(E) + (1 - P_b)\,U(D)] \text{ by Equation (6)} \tag{11}$$

which permits upon multiplication and rearranging terms:

$$\text{Equation (11)} = [P\,P_a + (1 - P)(P_b\,)]\,U\,(E) + [P(1 - P_a) + (1 - P_a) + (1 - P)(1 - P_b)]\,U\,(D). \tag{12}$$

$$\text{Equation (12)} = [P\,P_a + (1 - P)P_b]\,U\,(E) + [1 - P\,P_a - - (1 - P)P_b]\,U\,(D) \tag{13}$$

$$= P_4 \; U(E) + (1 - P_4)\, U(D) \tag{14}$$

where P_4 is defined as in Equation (5).

Similarly, the utility of the second lottery ticket is:

$$U[P'(A,B) = P_4' \, U(E) + (1 - P_4')\, U(D)\,. \tag{15}$$

At this point by comparing Equations (14) and (15), since $U(E) > U(D)$ by Equation (6a), the first lottery ticket will have the higher utility value if and only if $P_4 > P_4'$. It has been shown in Equation (5) that this condition also assures that that lottery ticket will be preferred.

Thus, it has been shown that the convention of Equation (6) will always assign a higher utility number to the preferred lottery ticket as is required.

Construction of a Cardinal Utility Index

Consider a lottery ticket which offers two prizes: first prize is an all-expense paid three-week vacation in Europe, booby prize is one month's subscription to the local newspaper. Suppose the odds in winning are one in one thousand; that is, the probability of winning is 0.001 and the probability of losing (booby prize won) is 0.999. Suppose also that the prospective ticket purchaser is interviewed, and from psychological information as to how he views the utility of these two prizes (the method of establishing this utility comparison will follow), the vacation is assigned 4000 utiles (the unit of utility measurement) and the paper subscription at one utile. Then, the N - M utility convention requires that the lottery ticket be evaluated at

$$(0.001)(4000) + (0.999)(1) = 4.999 \text{ utiles.}$$

More generally, if a lottery ticket offers two prizes, A with probability P and B with probability (1 - P), and if their respective utilities at U(A) and U(B), then the utility of the lottery ticket, L, is defined to be

$$U(L) = P\,U(A) + (1 - P)\,U(B). \tag{16}$$

This simple calculation is all that is required in the N - M evaluation of the utility of a lottery ticket, once the individual's utility evaluation of the prizes is known. The crucial question is how to determine the utility of these prizes.

In principle, this is accomplished by an extension of the preceding convention in Equation (16). For this purpose a special (artificial) lottery ticket is designed that will serve as a standard of comparison. Consider two extreme prizes, E and D, representing eternal bliss and damnation. The standard lottery ticket, which is designated as S(P), offers an individual E with probability P and D with probability (1 - P), where the probability number P is unspecified and is left to vary over a range of values. If two arbitrary utility numbers are assigned to E and D, pay U(E) = 100 and U(D) = 1.

Now, the next step requires that any ordinary prize, A, be assigned a utility value. For some values of P in the standard lottery ticket, S(P), the individual will prefer S(P) to A, and for other values of P, the reverse will be true. For example, if P = 1 (certainty of eternal bliss), he will surely prefer S(P) to A, and if P = 0 (certainty of damnation), he will prefer A to S(P). It is, therefore, plausible that there will be some in-between value of P_a, at which an individual is indifferent between A and $S(P_a)$. Once this indifference probability has been established, there is no difficulty in finding the utility of A. For A must have the same utility value as $S(P_a)$, since an indifference exists on the part of the individual. But, the utility of this standard lottery ticket, $U[S(P_a)]$, is easily calculated with the aid of the N - M convention of Equation (16):

$$U[S(P_a)] = P_a U(E) + 1(1 - P_a) U(D) = 0.4(100) + 0.6(1) = 40.6$$

utiles if the indifference probability is found to be $P_a = 0.4$.

Thus, in order to find a utility number which represents some individual's attitude toward any prize, X, he is interviewed or observed to find out the probability, P_x, at which he is indifferent between the standard lottery ticket, $S(P_x)$ and X. The utility of X is then evaluated using the standard of Equation (16) to determine the utility of $S(P_x)$.

Expected Utility Versus Expected Payoff

One feature of the N - M utility convention Equation (16) should be pointed out. According to this rule, a lottery ticket is evaluated at the expected value of its utilities and not at the expected value of the prizes themselves.

Consider a lottery ticket whose prizes, A and B, are amounts of money. Let these amounts and their respective utilities be shown as:

	A	B
Prize Value, dollars	$1,000	$100
Prize Value, utility	80	20
Probability	P	1 - P

The standard expected dollar value of the lottery ticket is

$$P(\$1000) + (1 - P)(\$100)$$

$$\text{if} \qquad P = 0.3$$

then the expected dollar value of the lottery ticket is

$$0.3(\$1000) + 0.7(\$100) = \$370.$$

In terms of the N - M utility concept, the expected utility value would be

$$P(80) + (1 - P)(20) = 24 + 14 = 38 \text{ utiles.}$$

Utility as a Preference Indicator

The use of utility as a preference indicator for alternative choices has the virtue of allowing for the consideration of diminishing or increasing marginal utility. For example, a gain of $1000 may not be worth 10 times the gain of $100, particularly if there is a desperate need for $100. And, likewise, the probable gain of $100,000 profit to a manager may not be offset by an equiprobable loss of $100,000 if the organization has a minimum security requirement of no loss greater than $50,000 since this would completely deplete the working capital of the organization.

The second use of utility measures as a guide in decision-making is that it may be a device to promote consistent action. The N - M utility concept proposes a system for determining an individual's utility function so that uncertain events may be evaluated in some consistent manner. With utility values explicitly stated, alternatives can be evaluated and selected that follow an individual's true preferences to achieve consistency in decision-making.

Why be concerned with consistency? One reason in support of consistency is that it permits a person to work in the most effective manner toward some goal. Inconsistency causes a person to meander, act in opposite ways to previous actions, possibly nullifying earlier gains. As pointed out by Davidson (18, p. 2) if a person makes decisions inconsistent with the view of maximizing expected utility, he does not have a rational pattern of preferences and expectations.

Inconsistency can also lead to frustration; i.e., acting one way one minute and another the next creates confusion and tension within the individual. As Dean M. R. Lohmann, College of Engineering, Oklahoma State University, has said before his classes many times, "One of the characteristics of a good leader is that he makes it easy for people to follow him." Dean Lohmann has elaborated extensively on the virtue of communications in accomplishing objectives. The elaboration emphasizes that managers are required to communicate in a manner so that they are understood by those to whom they communicate. The essence of it all is that using inconsistent language in communicating about objectives, incentives, and methods of accomplishment increases the difficulty of determining: "What's he trying say?" "This week we go north." "Last week we went south." "Does anyone know where we are trying to go?"

The fact that consistent action by maximizing expected utility is advanced as a recommended, or normative, guide does not intimate that all people are consistent. It is a commonplace observance that they are not. However, this does not destroy the need for pointing out such illogic or inconsistency and saying, "This is a more effective way to work toward some goal, it is only a tool, use it if you will."

Jacob Marschak (20, p. 186) notes:

> It is not asserted that norms are obeyed by all or even a sizable portion of people, just as logicians and mathematicians do not assert that all or a majority of the people are immune to errors of logic or arithematic. It is merely recommended that these errors be avoided. Recommended norms and habits are not the same thing.

The concept of using expected utility value as a decision guide also has another advantage over expected monetary value. It has been a criticism of expected monetary value that it overlooks the consequence of variance on individual preference; that is, expected value is really a weighted value. As such a measure, it focuses attention on the mean value. Yet, there may be varying ranges of possible outcomes from a large loss to a large gain, which strongly influence an individual's decision, regardless of the mean value. Expected utility value overcomes this objection by incorporating these variances directly into the computations. A large loss may be assigned a large negative utility by the individual, or he may assign a very large positive utility to a large increment of wealth, thus bringing the influence of variance into the decision.

A specific manner to allow for variance is implied by Marschak (20) and can take the form:

$$\bar{\bar{U}}_x = \delta(\bar{U}_x - k\sigma^2_{u_x})$$

where $\bar{\bar{U}}_x$ = expected corrected value of utility

δ = an unspecified constant

$\bar{U}_x$ = expected uncorrected value of utility

k = weight given the variance of utility values for U_x where $X = n$ when $n = 1, 2, \ldots, n$ for each n^{th} uncorrected utility outcome of a course of action.

Then, it is possible to solve for k for indifference of choice between two alternatives. The determined value of k is compared with the subjective weight, k_s, which has been given to the variance by the decision-maker. Then, if

$k_s = k$ each alternative is equally acceptable,

$k_s > k$ the alternative with the smaller variance is preferred,

$k_s < k$ the alternative with the larger variance is preferred.

Contradictions

A number of critical comments have been made by Allais (22), Edwards (23), Friedman and Savage (24), and Mosteller and Nagee (25) implying that there is no reliability attached to measures of cardinal utility because of the inconsistencies which a number of experiments have shown.

The most interesting and relevant arguments against the expected utility maxim involve specific cases in which individuals, after careful deliberation, choose alternatives inconsistent with the maxim. The situations are reasonably simple, the human choice fairly definite, the contradiction between choice and maxim apparently escapable. Either the conclusion must be that the expected utility maxim is not the criterion of rational behavior, or else the human being has a natural propensity toward irrationality, even in thoughtful situations. Markowitz (24, p. 219) notes three vivid examples that have been devised to show either (a) that rational and reasonable men choosing among simple alternatives contradict the expected utility maximum, or (b) that the "wrong" choices which were made prove individuals do act irrationally. Markowitz shows with simple clarity that human beings do act in an irrational manner. The examples will not be cited except to note the difficulty that individuals face in assessing alternative outcomes having probability distributions accounts for a major portion of the irrationality. It is no further afield to mention the analogous situation that prevailed in many industrial situations before discounted cash flow concepts were accepted as aids in decision-making. Before their advent, did managers make irrational decisions? Not necessarily; they made rational decisions given that the same decision would have been reached by any other individual given the same knowledge, experience, and objectives.

Since 1959, beginning with Grayson's (26) major contribution in the application of utility theory, the refinement of utility measurement as an aid in decision-making has had increasing acceptance. The most notable contributions in support of the theoretical as well as empirical applications of utility theory have been made by Farrar (27) and

Kaufman (28). Chapter IV will demonstrate the application of the model for determining choice behavior under the expected utility maxim.

CHAPTER IV

THE HEURISTIC APPLICATION OF UTILITY MODELS IN THE ENVIRONMENT OF CHOICE ACTIVITY

As noted earlier, in every type of organization, for whatever purpose the organization serves, several incentives are necessary and accompanied by a degree of persuasion so that, to the recipient, available incentives appear adequate in order to secure and maintain the contribution of required effort to the organization. The difficulties of securing a means of offering incentives, of avoiding the conflict of incentives, and of making effective persuasive efforts are readily conceded. The difficulty lies in the determination of the precise combination of incentives and of persuasion that will be both effective and feasible consonant with the resources of the organization. The delicacy of the administration of a scheme of inducements causes it to be the most unstable of the elements of the cooperative system. External variables, as well as internal variables, affect the stability of the scheme of inducements. In short, the tendency of an organization to fail is ever-present. The tendency towards instability, the loss of equilibrium, requires a deliberate attendance and growth of the scheme of inducements under an antagonistic environment. The efficiency of the organization in this environment is measured by its survival. Survival requires the maintenance of an equilibrium of organizational activities through the satisfaction of the motives of individuals sufficient to induce these activities. An organization, then, is a system of cooperative human activities, the functions of which are the creation and distribution of utilities to those whose contribution is required by the organization.

The equilibrium of the organization economy requires that it shall generate and exchange sufficient utility so that it is able to turn to command and exchange the personal services of which it is constituted. The accomplishment of equilibrium requires that through the application of these services the appropriate supplies of utilities which, when distributed to the contributors, insure the continuance of appropriate contributions of utilities from them. And, as each individual contributor requires a surplus of utility for his act of exchange of energy with the organization, the organization can survive only as it secures by exchange (creation) a surplus of utilities in its own internal economy. If the operations of the organization result in deficit, that is, if it is unable to meet the demand for utility by the contributors, it is less and less able to acquire the contributions through which its activity functions. The demand on the organization is for utility, it cannot supply more than it has. If it has enough utility, it has the resources for survival. The securing of the appropriate combination of the elements of the organization to produce utility is the basis for the endurance of cooperative systems. The function of management is to provide for the securing and distribution of utility appropriate to the demands of the contributors to the organization.

Unfortunately, the executive in the role of decision maker attempting to maintain an organizational equilibrium faces an environment fraught with uncertainty. When time considerations are associated with perfect knowledge of the future, problems of error in decisions and planning do not arise since the environment is deterministic. Perfect knowledge of the future does not exist; and, therefore, decision making must take place in an environment of uncertainty.

Uncertainty, Risk, and Expectations

When faced with the lack of perfect knowledge of the future, decision-making activities must take place in an environment of uncertainty. A decision-maker is faced with two types of eventualities or outcomes which affect plans for the future. One of these is risk; the other is uncertainty.

Risk refers to variability or outcomes which are measurable in an empirical or quantitative manner. Empirical probabilities can be established a priori when the characteristic parameters of the outcome distribution are known beforehand or the statistical probability of variability can be established when (a) the sample size is large enough, (b) the observations are repeated in the relevant population, and (c) the observations are independently distributed as random variables. The concept of risk can be defined to mean that the parameters of the probability distribution can be established for outcomes that involve risk.

Uncertainty is present when knowledge of the future is less than perfect in the sense that the parameters of the probability distribution cannot be determined. Uncertainty is, therefore, entirely of a subjective nature. It simply refers to anticipations of the future and is particularly associated to the mind of the individual decision-maker. Uncertainty arises because the decision-maker must formulate an "image of the future" in his mind, but has no quantitative method by which these predictions can be verified except ex poste.

Since knowledge of the future is so imperfect, managers normally expect that a range of outcomes, rather than a single outcome, is possible. Anticipations of the future can be formed, but there is no way that the manager can assemble enough homogeneous observations to predict the relevant probability distribution. While subjective probabilities may be assigned to these anticipations, no method exists by which actual values may be numerically derived and assigned these anticipations. The ultimate assignment of a probability distribution is known in decision theory as the assignment of subjective or personal probabilities. The idea has been discussed at some length by Savage (29, pp. 27 and 57) who has proposed a technique to establish subjective probabilities which is modified for presentation. It is most important to recognize that the derivation of these probabilities does not imply objectivity, or authority, although some individuals often try to give them this interpretation. The developed probabilities are merely a form of language, permitting subjective judgment to be put into a more precise form to allow for further evaluation of the generation of a store of utility values.

Suppose the following choices were offered to a manager faced with a decision: Select either the real world alternative or a hypothetical alternative which has the following respective outcomes:

1. Real World Alternative (Increase Plant Size):

   Profit Increases = \$1000 with subjective probability P_1

   No Profit Increase = \$0 with subjective probability P_2.

2. Hypothetical Alternative

   Profit Increase = \$1000 with known probability of 0.25

   No Profit Increase = \$0 with known probability of 0.75.

The manager is asked which of these alternatives he would prefer. If the manager feels there is a "good" chance of making the plant addition increase his profit, he will choose this alternative. If he feels the proposal to increase plant size has "little" probability of making a profit, he will select the hypothetical alternative. By revising the associated probabilities in the hypothetical alternative, it is possible to find a point, after several revisions in the probabilities in hypothetical alternative have been made, where the manager is indifferent in his preferences for the two alternatives. Say this indifference point occurs at a probability of 0.80 and 0.20 for the profit results of $1000 and $0, respectively, for the hypothetical alternative. Thus, it can be inferred that the "subjective probabilities" he associates with the results of a plant expansion are also 0.80 and 0.20, respectively, for a profit gain of $1000 and $0.

This technique is demonstrated only for the value it has in the numerical development and ranking of subjective probabilities in order to utilize the following action on utility measures. Competency in the use of this technique is assumed in the following development. A cautionary word of advice is in order. The technique is deceptively simple and requires an appropriate level of familiarity and competency analogous to the system of ranking and rating in establishing benchwork jobs in wage and salary administration.

The Development of a Utility Index

In accordance with the theorems of Chapter III and empirical studies made by Farrar (27), Grayson (26), and Kaufman (28), it is possible to measure utility. If a person can express preferences over a series of alternatives, then it is possible to associate utility values to the alternatives provided that there is an element of consistency in the individual's preference for utility.

Given that a man is offered two alternatives:

1. Obtain $100 for certain

or

2. Have a 50-50 chance of winning $500 or $0.

And, if he replies that these two alternatives are about equal, i.e., he is indifferent between them, then these alternatives have the same utility. Another individual given this same choice of alternatives may feel that he has to have a 50-50 chance of winning $700 or $0 before he feels the alternatives are equal. This second individual has a different utility function.

A series of such alternatives can be given to an individual, using different amounts of money and different probabilities. His responses can be plotted on a graph converting dollars into a utility function, measured in terms of utiles.

In deriving a utility measure of money, certain arbitrary values of utility are assigned. This does not matter since this scale is unique to a particular individual, thus the consistency axioms determine a linear utility function only up to its zero point and its value in utiles. Hence, the assigned values will be \$0 = 0 utiles and \$500 = 25 utiles.

From Chapter III, Equation (8):

$$U(A) = PU(B) + (1-P)U(C),$$

then

$$U(\$100) = .50(25) + .50(0)$$

solving for $U(\$100) = 12.5$ utiles.

If this information were plotted on a graph, the representation would be similar to that shown in Figure 8. But this is merely a plot of one gamble, and, since only a limited number of utility-for-money values can be obtained, a number of gambles are presented to the individual. These results are also plotted. If there is an element of consistency in the individual's preference for money, the results can be extended to form a general utility function for him. Inconsistency of choice displays itself in the utility values obtained. As noted earlier, inconsistency has been demonstrated by a number of writers. Much of the inconsistency stems not from human preference and the expected utility-maxim, but, rather, from misapplication or misinterpretation of the maxim. A concise demonstration of this appears at the end of this chapter. For the current

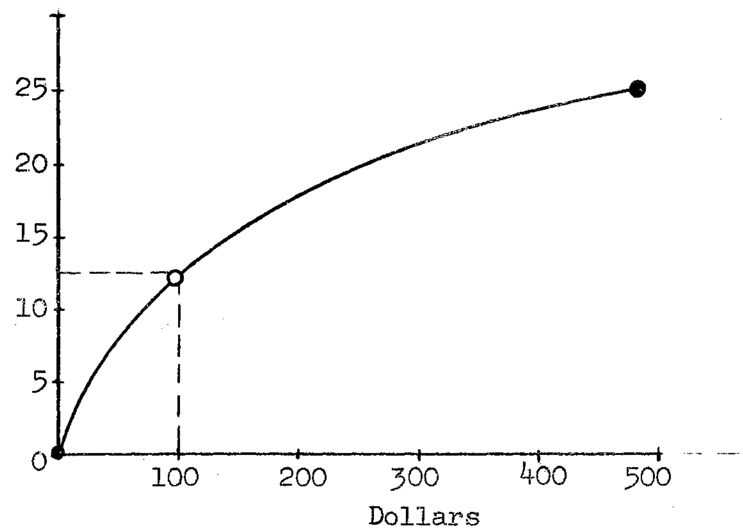


Figure 8. A Partially Developed Utility Function

discussion, normative application of the utility maxim requires that if people prefer A to B to C, then they should not prefer C to A, if they are consistent. Inconsistency when recognized is brought to the attention of the individual somewhat as follows: "Look, this demonstrates the difficulty of attempting consistent behavior. You said you would choose this alternative, but a few moments later, with an identical position, you said you would not take this alternative. This is inconsistent. Which of these two alternatives really represents your position?"

After having this called to his attention, the individual may modify his preferences to bring them into consistent order. These inconsistencies are disturbing, but not unexpected. Modifying them is a process whereby the individual removes inconsistent behavior influences, so that he may select alternatives on a consistent basis. This pattern is not unlike that which often is observed in industry where project design is evaluated on a "consistency" basis. For example, if a 10,000 foot oil well requires a 20 horsepower pumping system, the chief engineer is skeptical of the design criteria for the next oil well, ceteris paribus, if the project engineer recommends a 40 horsepower pumping system. The project engineer may volunteer, "That doesn't look right (consistent), let me go over my calculations again." Inconsistency, unfortunately, does not wear a badge, "Look, I am inconsistency," to warn the unsuspecting decision-maker of his foibles.

The Experimental Development of a Utility Function

In order to develop the utility function of an individual, a series of decisions must confront him so that a measure of utility for various sums of money may be determined. Since the type of decisions he must make are in the guise of a hypothetical game or gamble for which alternative decisions must be made, it is quite essential that the frame of reference for decisions among alternatives accomodate the physical, social and economic environment in which the decision-maker normally operates. The utility function should be determined under choice-making situations that closely approximate the actual decision environment. In constructing the gamble or hypothetical situation, three basic ingredients are necessary: the capital investment, the payoff or return, and the probability of success.

For the monetary considerations of capital investment and return, present day values after taxes should be considered, since this is the realistic situation of the decision-maker. The rate at which cash flows are discounted will depend on the firm's weighted cost of capital as a minimum, the maximum discount rate will be the investment opportunity rate available to the firm. A complete discussion of these techniques may be found in Thuesen and Fabrycky (30) and Bierman and Smidt (31).

Developing the Indifference Probability

As an example, a decision-maker is offered an opportunity to accept or reject an investment opportunity where the required capital investment is $40,000, the total return (or positive cash flow) is $130,000, and the probability of success of the investment is 0.70. That is, there is a subjective probability that the venture will return $130,000; there is also a (1 - 0.70) or 0.30 probability of failure of the venture. In symbols:

Expected Utility = 0.70[Utility ($90,000)]

+ 0.30[Utility -$40,000]

Expected Utility = Utility Gain + Utility Loss.

Now, if this investment opportunity is acceptable to the decision-maker, the probability of success and failure are adjusted until he rejects this investment opportunity.

If probability of success is 0.10 and that of failure is 0.90, then the expected gain is negative if the decision-maker rejects this opportunity to risk a $40,000 investment. Finally, it is possible to adjust the probability of success to the point where the decision-maker is not sure whether to accept or reject. In effect, he is indifferent about the loss or gain and essentially feels the net effect of the investment borders on an expected utility value of zero. If this indifference level were found to be:

$$P(\text{Success}) = 0.60$$

$$P(\text{Failure}) = 0.40.$$

Then

$$(0.60)[\text{Utility } (\$90{,}000)] + (.40)[\text{Utility } (-\$40{,}000)]$$

$$= 0 \text{ utiles.}$$

The zero utiles represent status quo, which is merely a convenient convention without the loss of general application.

It is necessary in building the utility function of an individual to be exceedingly clear about the representation of capital investment, cash flows and the concept of probability. The last item is the most difficult to display to the decision-maker. A number of various demonstrations may be necessary in order to convey the meaning of probability to the decision-maker. The use of marble boards and dice may effectively introduce him to probability concepts. The necessary requirement is that he is able to attach the notion of probability to his decision environment. For an oil marketer, the frame of reference is in terms of profitably operating service stations. A 0.10 probability means that 1 out of 10 service stations will not be profitable for any number of reasons, such as traffic density, station density, and so forth. To a manufacturer of automobiles, certain sytles and models sell better than others; some models lose money, some do not. How does he view probability?

Could he conceive of one model in ten being rejected by the buying public? Could he conceive of eight models in ten being successful? Does this mean the same to him as an 0.80 chance of success for any model? The nomenclature seems trivial, but the area of greatest difficulty in establishing a utility function is to have a decision-maker grasp the "odds" - the probability - of success or failure of a project as it influences his acceptance, rejection or indifference evaluation of an alternative.

Description of Evaluation Form

A form which can be used to obtain the indifference probabilities appears as Figure 9 together with psuedo values that represent the empirical results of a test. The partial display is presented only to indicate the form and overlap of investment values to enable an evaluation of consistent behavior. The levels of investment and return must be realistic in terms of opportunities that the decision-maker faces. Since a measure of his utility preference during the development of a utility function is to fairly represent his "real world" preference, the development of plausible opportunities should be given considerable thought.

As an example, the manager of market development for a major glass company might be confronted with, "You have been considering the construction of a major warehouse on the West Coast. The cost will be $150,000; the return might be

Indifference Reactions for Investment Opportunities

Name ____________________ Date ____________________

Organization ____________________ Interviewer ____________________

Position ____________________

Brief Description of Authority and Responsibility ____________________

Investment*	Total Return	Net Return	Derived Indifference Probability	Investment*	Total Return	Net Return	Derived Indifference Probability
$10	$ 20	$ 10	.90	$50	$100	$ 50	
	40	30	.70		120	70	
	60	50	.40		140	90	
	80	70	.35		160	110	
	100	90	.30		180	130	
20	40	20	.90	60	120	60	
	60	40	.70		140	80	
	80	60	.60		160	100	
	100	80	.55		180	120	
	120	100	.50		200	140	
30	60	30	None**	80	160	80	
	80	50	.90		180	100	
	100	70	.80		200	120	
	120	90	.70		220	140	
	140	110	.50		240	160	
40	80	40		100	200	100	
	100	60			220	120	
	120	80			240	140	
	140	100			260	160	
	150	110			280	180	

*In thousand of dollars.

**None interprets that no probability of success is acceptable.

Figure 9. Form for Data Collection

$500,000 because of better customer service, inventory cost control and so on. Would you recommend this warehouse be built if there was a 0.90 probability of the $500,000 return occurring? Yes. All right, then, would you feel the warehouse was a good investment if there was only a 0.10 chance of the return being $500,000? No. All right, now what if there was a 0.50 probability of the investment returning $500,000? Not quite sure? Let us say you might be indifferent." Perhaps in a more detailed verbal exchange, the objective would be to build a set of indifference values that represent plausible situations faced by the decision-maker. Figure 10 portrays what would possibly result as indifferent probabilities are evaluated as a utility function.

Plotting Utility Functions

If the results from the indifference probabilities were developed as follows:

Investment	Total Payoff	Net Revenue	Derived Indifference Probability
$10,000	$ 20,000	10,000	0.80
	30,000	20,000	0.60
	40,000	30,000	0.60
	60,000	50,000	0.30
	100,000	90,000	0.20
$20,000	40,000	20,000	0.90
	50,000	30,000	0.85
	120,000	100,000	0.40

the next step would be to assign 0.0 utility to $00. Next, the loss of $10,000 is arbitrarily set at -1 utile. The

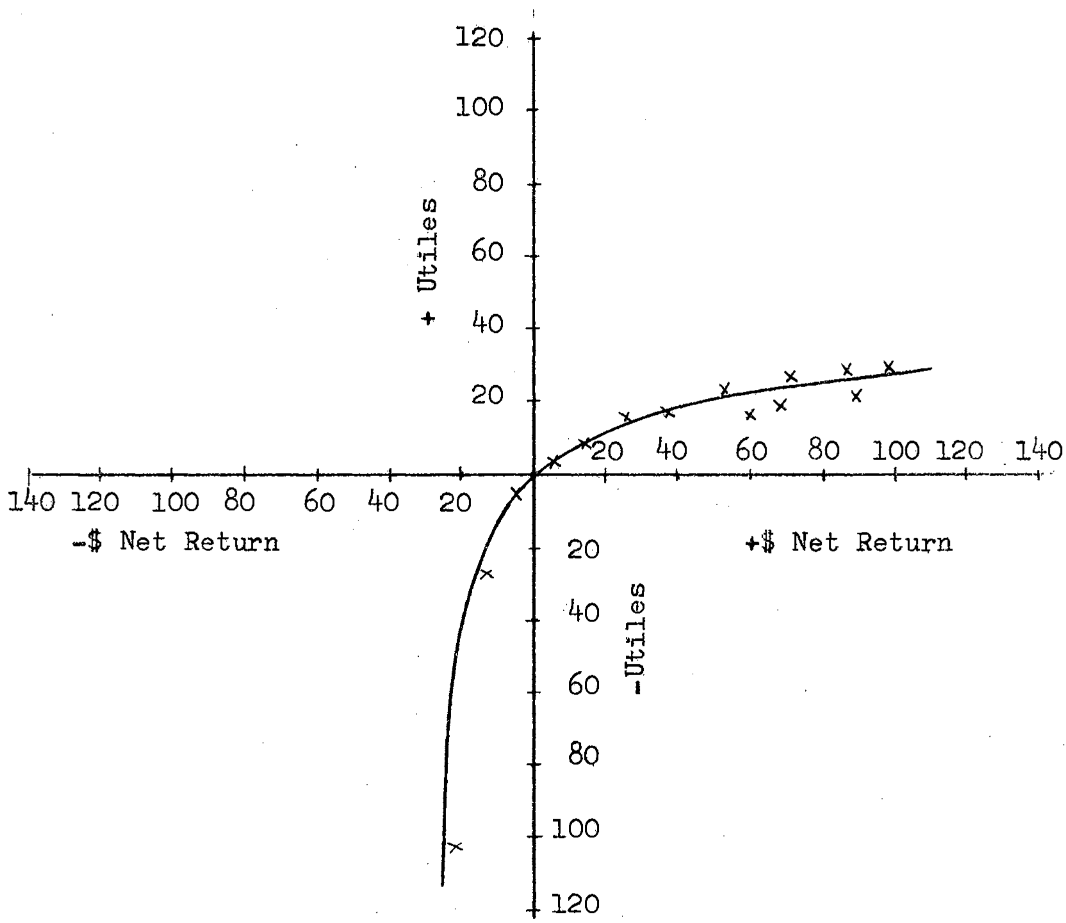


Figure 10. Rough Sketch of Data of Figure 9

equation for the first point plot is:

$$.80[\text{Utility } (\$10{,}000)] + .20[\text{Utility } (-\$10{,}000)] = 0.$$

As the utility of $10,000 was set at -1 utile, then, by solution for the utility of $10,000, 0.25 utiles represents the utility of $10,000.

The three utile points (-1, 0, 0.25) are plotted against the three net revenue points (-$10,000, $0, $10,000). In a similar manner, the utility values for other net revenue points at the $10,000 investment level are obtained and plotted over the range of -$10,000 and $90,000.

The next series of indifferent probabilities at the $20,000 investment level are plotted. The overlapping net revenues from the previous investment level serve as an internal check on the consistency of the decision-maker. This series of points require that an arbitrary utility level for -$20,000 be set. This is done by reference to the first set of points to extrapolate a utility value for the loss of the investment sum. This arbitrary value may require adjustment during the plotting of this series to determine if the overlapping net revenues can be made to possess consistent utility values.

A plotted utility curve developed by Grayson (26, p. 306) through indifference probability evaluation is reproduced as Figure 11. The general interpretation of this utility curve indicates the decision-maker requires high

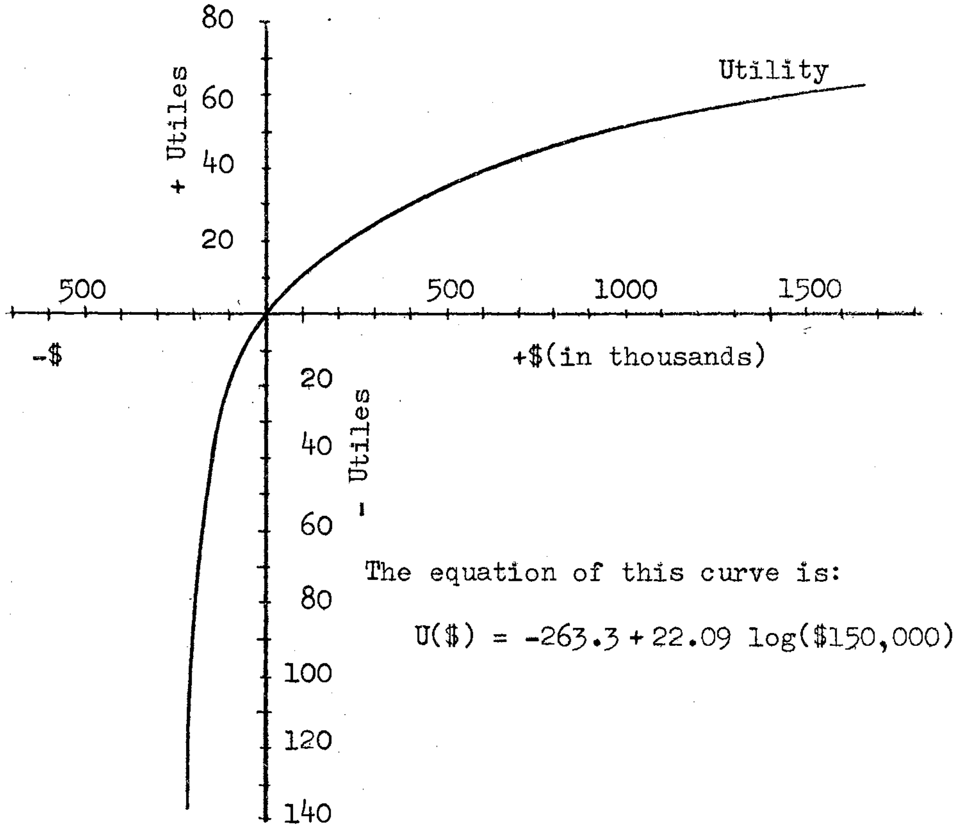


Figure 11. Plot of Derived Utility Values

success probabilities before he is willing to risk investment funds. Also, the interpretation indicates that the possible loss of $200,000 could never be counterbalanced by any increment of gain. The $200,000 loss may possibly represent the loss of ownership of the firm.

The Use of Utility Functions

The potential for evaluating the probability distributions of various alternatives becomes apparent when the weight of a gain or loss of money represents something other than the expected monetary value of an alternative. Not only does decision-making by the utility maxim provide for consistency for choosing alternative methods so as to provide for consistent behavior in reaching organization objectives, it also provides for a method of delegating decision-making authority in the allocation of organization assets. It seems plausible, as refinements are made in the development of utility functions, that utility functions will represent objective criteria of the firm so that the president of an organization can say to a subordinate, "Here is our company's utility function for money. It reflects the company's preferences for large losses, large gains, and for those expected values between these extremes."

The chances for consistent action throughout the firm would be greatly improved. Personal observations in completing this research in utility concepts uncovered two interesting observations made by industrialists who perhaps may not have been consistent with goals of the organization.

Observation 1: "We are here to produce oil. That's my goal. If production is increasing, I'm happy." This executive, however, was producing oil at a cost of $4.40 per barrel against a market value of $3.12 a barrel.

Observation 2: "Costs are our concern -- keep costs down and profit will take care of itself." The executive with this objective would reach the break-even point where costs would exceed a declining revenue in 18 months.

A Comparison of Expected Money Value and Expected Utility Value

Using the utility function of Figure 11, the following comparison is made, in tabular form, of three mutually exclusive alternatives (Table I on following page).

From the data of Table I, the following comparisons may be made:

Results	Alternative No. 1	Alternative No. 2	Alternative No. 3
Expected Money Value*	$75,000	$62,000	$55,500
Expected Utility Value	4.1 Utiles	5.1 Utiles	5.3 Utiles

TABLE I

TABULATION OF EXPECTED MONEY VALUE AND EXPECTED UTILITY VALUE

Alternative No. 1 Investment = $50,000			Alternative No. 2 Investment = $40,000			Alternative No. 3 Investment = $20,000		
(A) Net Revenue	(B) Utility of (A)	(C) Probability of Occurrence (A)	(A) Net Revenue	(B) Utility of (A)	(C) Probability of Occurrence (A)	(A) Net Revenue	(B) Utility of (A)	(C) Probability of Occurrence (A)
$- 50,000	- 7	0.40	$- 40,000	- 3	0.10	$- 20,000	- 1	0.05
100,000	+ 8	0.20	60,000	+ 5	0.60	50,000	+ 5	0.65
150,000	+12	0.30	100,000	+ 8	0.30	80,000	+ 7	0.30
200,000	+17	0.10						
75,000	4.1		62,000	5.1		55,500	5.3	

where Expected Money Value = EMV = $\sum_{i=1}^{n} (p_i)(\$E_i)$

where: p_i = subjective probability of i^{th} event occurring

$\$E_i$ = money value of i^{th} event

where Expected Utility Value = EUV = $\sum_{i=1}^{n} (p_i)(U\,E_i)$

where: p_i = subjective probability of i^{th} event

UE_i = utility value of i^{th} event.

As can be seen, if the expected money value of an alternative is the objective, then Alternative No. 1 is preferred. However, if the utility maxim is applied, then the weight given a possible loss of -\$50,000 is not offset by a gain of \$50,000 since the loss of \$50,000 has greater disutility than the gain of \$50,000 possesses utility. Consequently, the expected utility value of Alternative No. 3, 5.3 utiles, possesses the largest expected utility value and would be the choice of the utility maximizer. A number of other investment decision criteria could also be modified with a utility index to consider payout time, average annual earning power, and per cent profit at the organization's current rate of return.

Empirical Studies and Comments

Grayson (26) has done the most complete empirical studies to date in measuring utility functions of decision-makers who drill oil and gas wells. His study utilized the oil industry environment because of the inherent acknowledged risk and uncertainty attached to the drilling of oil and gas wells. Yet, even though the individuals whose utility was measured were usually confronted with risk and uncertainty, Grayson (26, p. 313) observed these difficulties:

1. Some operators do not use numerical probabilities in their decisions, and they found it strange to try to reach a decision on the basis of probabilities.

2. Some operators could not view probabilities as objective -- they further discounted the given probabilities.

3. When probabilities were similar to those they actually experienced in their environment, they gave consistent evaluations to the alternatives. However, when the probabilities ranged away from a familiar level, they had difficulty in pursuing consistent mental processes.

4. When dollar values exceeded an individual's customary maximum range of investment funds he experienced great difficulty in attaching a value of indifference to say, $300,000 when he customarily invested funds to a maximum of $50,000.

On the plus side, several decision-makers felt the utility function would serve a useful purpose in defining consistent behavior for the firm. One firm, Eason Oil Company, has incorporated the utility function into its general evaluation criteria of investment opportunities (28, p. 57).

Misapplication and Misinterpretation of the Utility Maxim

A great deal of theoretical conflict as well as empirical misinterpretation exists in the application of the utility maxim. In order to present a summation and possible resolution of the conflict, two situations are analyzed:

Situation 1

Alternative A: Certainty of \$1,000

Alternative B: A 0.10 probability of \$5,000
A 0.89 probability of \$1,000, and
A 0.01 probability of \$00.

Situation 2

Alternative C: A probability of 0.11 of getting \$1,000, and a probability of 0.89 of getting \$00.

Alternative D: A probability of 0.10 of getting \$5,000, and
a probability of 0.90 of getting \$00.

Given the above choices, in Situation 1 subjects often prefer A to B, and in Situation 2, these same subjects prefer D to C. This choice of A > B and D > C clearly contradicts the expected utility maxim.

This may be seen by letting

$$U(\$5000) = U(5)$$
$$U(\$1000) = U(1)$$
$$U(\$\ 00\ ) = U(0)$$

Then the choices indicate that the preference of A > B is:

$$1U(1) > [0.1\,U(5) + .89\,U(1) + .01\,U(0)] ,$$

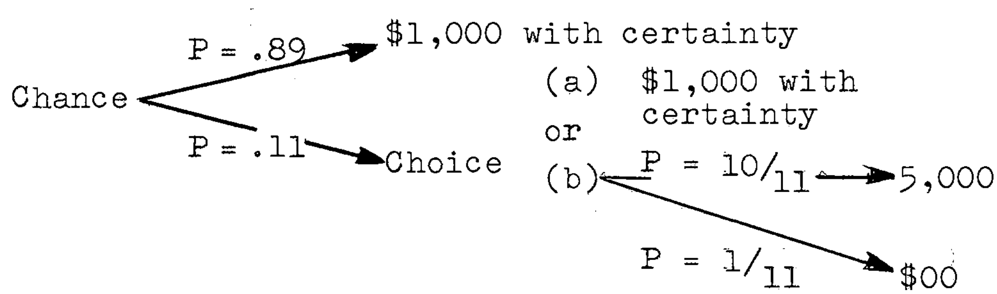
but, if $[0.89\,U(0) - .89\,U(1)]$ is added to both sides of the inequality, then

$$0.11\,U(1) + 0.89\,U(0) > 0.1\,U(5) + 0.9\,U(0)$$

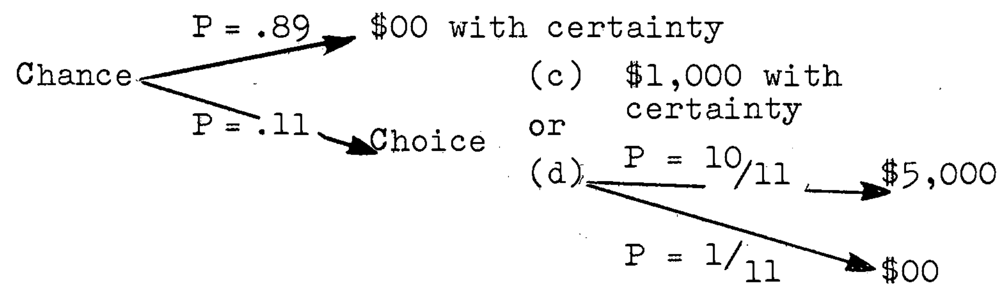
which indicates that an individual who prefers A > B, then must prefer C > D, a contradiction of his earlier choice of D > C. The expected utility maxim must not be valid, or is it? At stake is the utility maxim unless individuals reverse themselves given the same alternative choices in another format. If this is the case, that individuals are indeed mislead by concepts of expected utility values, then the utility maxim is still acceptable.

Using the previous Alternatives of A, B, C, and D, the same probabilities of outcomes are arrayed in a slightly different manner.

Situation 1:



Situation 2:



It can be shown now that these new "forms" of decision choices exactly represent those in the original format, but now these same individuals overwhelmingly select, for Situation 1, $a > b$, and for Situation 2, $c > d$, a reversal of the earlier preference of $A > B$ and $D > C$.

To show the equality of expected values for the previous choice situations, the following is given:

$$A = \text{certainty of } \$1000$$

$$a = \$1000(0.89 + 0.11) = \text{certainty of } \$1000$$

$$B = 0.10(\$5000) + 0.89(\$1000) + 0.01(\$00)$$

$$b = 0.11(\tfrac{10}{11})(5000) + 0.89(\$1000) + 0.11(\tfrac{1}{11})(\$00)$$

$$= 0.10(5000) + 0.89(\$1000) + 0.01(\$00)$$

$$C = 0.11(\$1,000) + 0.89(\$00)$$

$$c = 0.11(\$1,000) + 0.89(\$00)$$

$$D = 0.11(\$5,000) + 0.90(\$00)$$

$$d = 0.11(\tfrac{10}{11})(\$5000) + 0.11(\tfrac{1}{11})(\$00) + .89(\$00)$$

The explanation for the inconsistency primarily results from the greater focus brought to bear on the weight effect of a probability distribution that can be visualized when "almost the same," i.e., 0.10 and 0.11, probabilities are identified more dramatically as in the arrow diagram of choice situations.

The General Source of Conflict

The error more specifically lies in the misinterpretation of the notion $[aP + (1-a)Q]$ wherein probabilities are mixed. If event P has a probability distribution (a) and event Q has a probability distribution (1 - a), and P is considered to be exactly as good as Q, then the expected utility maxim asserts that having either of these with certainty is exactly equal to the equiprobable chance of receiving either one. The probability distribution in the latter case is $(\frac{1}{2})P + (\frac{1}{2})Q$, which is considered exactly as good as P or Q, which implies diversification does no good.

This is a non-sequitur. The probability distribution associated with the flip of a coin to choose between event P and event Q is not the same probability distribution of utility values which results from investing resources equally in each event; assuming this is a realistic possibility. As an extreme example, if there were 10 events all with the same mean and same probability distribution, and if the utility derived from an investment in each of these events were uncorrelated, then diversification among the 10 events would considerably reduce the utility variance to be realized since

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}}$$

where:

$s_{\bar{x}}$ = variance of the mean utility of the events

s_x = variance of utility of an event

n = number of events.

Choosing a single event at random would result in exactly the same probability distribution of utility as investing resources in an event outright. Clearly, the mixing of probabilities and the mixing of utility should not be confused.

It can be argued that the assumption that "if P is exactly as good as Q, then either is exactly as good as (a P) + (1 - a)Q" should be doubted or rejected in the case of probabilities because there are other things to which the assumption clearly does not apply. But, the fact that the assumption does not apply to buying a suit of clothes, planning a trip or blending wheat flour does not affect its reasonableness or unreasonableness when applied to probabilities. The fact that individuals may prefer scotch to beer does not mean that people will prefer a mixture of 0.50 scotch and 0.50 beer. So the assumption does not necessarily hold. But, the assumption is still consistent with the statement that "If a drink of scotch is preferred to a drink of beer, then a 50-50 chance of a drink of scotch is preferred to the certainty of a drink of beer."

The Resolution of Conflict

The area of conflict over contradictions versus the expected utility maxim will be cleared if conflicts can be viewed from a set of evaluatory principles set forth as axioms. The following axioms would seem to serve this purpose:

<u>Axiom I</u>: If P and Q are two probability distributions of utility outcomes, then either P is preferred to Q, or Q is preferred to P, or both are considered equal in preference.

<u>Axiom II</u>: If P is considered at least as good as Q, and Q is considered at least as good as R, then P is considered to be at least as good as R.

<u>Axiom III</u>: If a probability distribution of utility P is preferred to a probability distribution of utility Q, and if R is any probability distribution at all, then

A probability (a) of obtaining P and a probability of (1 - a) of obtaining R is preferred to a probability (a) of obtaining Q and (1 - a) of obtaining R, given that (a) > 0.

In other words, if P is preferred to Q, if (a) is greater than zero, and if R is any distribution whatever, then

$$aP + (1 - a)R > aQ + (1 - a)R$$

which says that $aP + (1 - a)R$ is the over-all probability if an individual would choose P if the opportunity arises; $aQ + (1 - a)R$ is the over-all probability of Q if an individual would choose Q if the opportunity arises. If it is assumed that different ways of generating the same probability distributions are equally good, then the statement that P should be chosen instead of Q, whatever (a) or R, is the same statement that $aP + (1 - a)R$ should be preferred to $aQ + (1 - a)R$.

In the following game matrix, the previous arguments become apparent if P is preferred to Q:

	(a)	(1 - a)
Choice A	P	R
Choice B	Q	R

since Choice A will always be made regardless of R, a, and (1 - a).

CHAPTER V

THE ACQUISITION AND DISTRIBUTION OF UTILITY

As mentioned in Chapters I and II, the essential goal of a manager is organization survival. In effect, a vital, and, indeed, necessary element for survival is the willingness of contributors to contribute their individual efforts to the organization seeking to survive. As Barnard (2, p. 140) has so aptly put it, "The net satisfactions which induce a man to contribute his efforts to an organization result from the positive advantages as against the disadvantages which are entailed." The organization, and more specifically, the manager must seek to generate those types of incentives particularly and singularly attractive to the particular contributors whose contributions are vital to the organization's survival.

As noted by Barnard (2, pp. 139-160), the economy of incentives requires an extraordinary degree of delicacy in the application of incentives to assure that an individual recognizes them as being sufficiently adequate to maintain his contribution of effort to the organization. However, since all incentives have a direct or an indirect cost of acquisition, the manager seeks under the constraint of limited resources to balance the distribution of utility consistent with its supply. Since an economy of distribution is required, the distribution of utility must be proportioned to the value and effectiveness of the contributory efforts required by the organization.

At the outset, the manager is faced with the requirement to produce a "store" of utility from the efforts of the contributors, so that, in turn, he may dispense the "produced" utility to contributors in such a fashion so as to insure continuity of the distribution process necessary for organization survival.

The organization can be represented as a production unit whose product is utility. It is the manager who decides how much and what combination of input factors will generate the sufficient amount of utility to maintain equilibrium. A manager, by his decisions, transforms inputs into outputs according to the capability of the contributors. The difference between total output utility and total input utility, which are each determined by the summation of individual utilities, represents whether or not the organization can acquire, convert and distribute a sufficient amount of utility so that it may survive.

The analysis of such a system for the production of utility is first developed for the relatively simple case in which two utility inputs are combined for a single utility output. The system is then extended for the general case of n inputs.

Inputs and the Production of Utility

The input to any organization is viewed as the efforts of individual contributors. The effort to contribute financial resources, the effort to operate a milling machine, and the effort to contribute any necessary factor of production are specifically what permit the organization to achieve its objectives. For a given time period of production, inputs are classified as fixed or variable. Given a time period, a fixed input is necessary for production, but its quantity is invariant with respect to the quantity of output produced. The costs of fixed utility inputs are incurred by the manager regardless of "short-run" optimizing decisions. The necessary quantity of a variable utility input depends upon the quantity of utility output. The distinction between fixed and variable inputs is temporal. Inputs which are fixed for one period of time are variable when considered for a longer time period. All utility inputs are variable, given a sufficient period of time for adjustment.

The assumption underlying the development of rational behavior for the decision-maker is that he attempts to maximize the residual utility of the organization's activities so that he has, firstly, enough utility to satisfy current demands of the contributors and, secondly, a surplus of utility to accommodate the uncertain stability of organization's environment.

The Production Function

Consider a simple production function in which a decision-maker utilizes two variable inputs, X_1 and X_2, and one or more fixed inputs in order to produce a Q quantity of utility. The production function states the output quantity, q, as a function of variable inputs x_1 of X_1 and x_2 of X_2, or

$$q = f(x_1, x_2). \tag{1}$$

Unlike the utility functions of Chapter III, the utility production function for Equation (1) is assumed to be a single-valued continuous function with continuous first and second order partial derivatives defined only for non-negative values of utility input and utility output.

The decision-maker may use any number of combinations of X_1 and X_2 for the production of a given level of utility output. Since Equation (1) is continuous, the possible combinations of inputs are infinite. The best utilization of any particular input combination is an economic problem, since the acquisition of utility is assumed to have an associated cost. The selection of the best input combination for the production of a particular output level depends upon the costs of input factors and the revenue received from the output activity. This combination is subject to economic analysis.

Input and output utility levels are taken as rates of flow of utility per unit of time. The period of time for which such flows, i.e., the short-run production function, are defined is subject to three general restrictions:

1. It must be sufficiently short so that levels of fixed inputs cannot be altered.
2. It must be sufficiently short so that the production is not altered through technological improvements.
3. It must be sufficiently long to allow the completion of the necessary processes to generate output utility.

The analysis to accommodate the "long run" relaxes restriction number one and allows the time period to be of sufficient length to allow for the variation of all inputs.

Productivity Curves

In the short run, the total productivity of utility of input factor X_1, when producing a Q quantity of utility when input factor x_2 of X_2 is held constant and assigned the fixed value x_2^0 is

$$q = f(x_1, x_2^0) . \tag{2}$$

The input level of x_2^0 is treated as a parameter, and q becomes a function of x_1 alone. The relation between q and x_1 can be altered as levels of x_2 are changed. Normally, an increase of x_2 will reduce the amount of x_1 required to produce utility at each feasible output level. Leftwich (32, pp. 107-135) gives a very complete graphical presentation of this development, and Allen (33, pp. 190 and 341) gives a complete mathematical explanation.

In a similar manner, the average productivity (AP) of X_1, its total productivity divided by its quantity, is

$$AP = \frac{q}{x_1} = \frac{f(x_1\ x_2^o)}{x_1}. \tag{3}$$

The marginal productivity (MP) of X_1 is the rate of change of its total productivity with respect to its variations in its quantity, i.e., the partial derivative of Equation (1) with respect to x_1, and is

$$MP = \frac{\partial q}{\partial x_1} = f_1(x_1 x_2^o). \tag{4}$$

For the production of utility in an organization, production functions satisfy the almost universal law of diminishing marginal utility: the MP of X_1 will eventually decline as x_1 is increased with x_2 remaining fixed. It is intuitively seen that if the capital investment in production equipment is held fixed and the variable input of labor is increased; then at some level of labor input, continued increases in labor serve to reduce the total output as individuals become so numerous that they impede effective movement. As this situation occurs, it is conceivable that the output of utility is likewise hindered.

Another characteristic of production functions is that the ability of one input factor to substitute for another input has a relationship known as the Marginal Rate of Substitution (MRS). This refers to the amount by which one resource factor (X_2) is decreased as the input of another resource factor (X_1) is increased by one unit

$$MRS_{x_1 x_2} = \frac{dx_2}{dx_1} .$$

The Cobb-Douglas Function

While a number of production functions for describing the output of utility of an organization have application, the Cobb-Douglas function appears to have more general applicability for utility functions since it assumes constant elasticity of utility production with diminishing marginal returns. The Cobb-Douglas function follows the general form

$$Y = a X_1^b \; X_2^c \; X_3^d \; \ldots . \; X_i^n \tag{4}$$

where

X_i^n = an input factor, i, raised to the n^{th} power.

Characteristics of the Cobb-Douglas Function

The virtuosity of the Cobb-Douglas function has been demonstrated by Heady (34, pp. 59, 68, 143) to have particular importance in generating response relationships between input factors and products when utility functions are emphasized as a basis for decision-making.

A production function denoting constant returns to scale is said to be homogeneous of the first degree denoting that, if input of each factor is multiplied by a constant amount, the product will be increased in a like ratio. The marginal and average productivity of all factors depends only on the ratio between the amounts of the factors and <u>not on the amounts</u> of the factors. If a production function is defined as $Y = f(X_1, X_2)$ and the input of both X_1 and X_2 is multiplied by a constant, k, the right side of the equation becomes $f(kX_1, kX_2)$ and the production of utility will be increased by the same proportion (to kY) only if the function is homogeneous of the first degree. In other words, the condition must exist for

$$f(kX_1, kX_2) = kf(X_1, X_2) = kY \tag{5}$$

denoting that the utility is increased by the same constant ratios as all input factors.

It is possible to illustrate the difference between functions which denote constant returns to scale and those which illustrate economies of scale and diseconomies of scale. For example, a production function for an organization might be represented by

$$Y = 10C^{0.25}L^{.75} \tag{6}$$

where

Y = utility

C = capital input

L = labor input.

If each factor is multiplied by a constant, k, the right side of the equation becomes

$$10(kC)^{0.25}(kL)^{0.75}$$

which becomes

$$10k^{0.25}C^{0.25}k^{0.75}L^{0.75}$$

which simplifies to

$$k(10C^{0.25}L^{0.75}).$$

Accordingly, the product will also be increased by the same ratio, k, since the power of k is one. Thus,

$$kY = k(10C^{0.25}L^{0.75})$$

to represent constant returns to scale.

As has been demonstrated, the Cobb-Douglas function for

$$Y = X_1^{\alpha} X_2^{1-\alpha}$$

represents constant returns to scale if

$$\alpha + (1 - \alpha) = 1 .$$

For economies of scale, then

$$\alpha + 1 - \alpha > 1$$

which is demonstrated by

$$Y = X_1^{0.7} X_2^{0.8}$$

then

$$(kX_1)^{0.7}(kX_2)^{0.8} = k^{1.5}X_1^{0.7} X_2^{0.8}$$

and $$\therefore k^{1.5}Y = k^{1.5}X_1^{0.7}X_2^{0.8} \qquad (7)$$

for diseconomies of scale, then

$$\alpha + (1 - \alpha) < 1$$

which is demonstrated by

$$Y = X_1^{0.5} X_2^{0.3} ,$$

then

$$(kX_1)^{0.5}(kX_2)^{0.3} = k^{0.8}X_1^{0.5} X_2^{0.3}$$

and $$\therefore k^{0.8} Y = k^{0.8} X_1^{0.7}X_2^{0.8} .$$

As an example, given a utility production function

$$Y = 1.50L^{0.06}C^{0.40}R^{0.30}$$

where

L = labor

C = capital

R = raw material

since scale relationships refer to an increase in all resource inputs in fixed proportions, the increase in utility is the increase resulting, say from a 10 per cent increase in all input factors, which results in a 7.6 per cent increase in produced utility. The calculations are

$$Y_1 = 1.50(2)^{0.06}(4)^{0.40}(5)^{0.30} = 4.407$$

where

L = 2

C = 4

R = 5.

When each input is increased by 10 per cent, then

$$Y_2 = 1.5(2.2)^{0.06}(4.4)^{0.40}(5.5)^{0.30} = 4.743.$$

Or an increase in produced utility of 7.6 per cent when inputs have been increased by 10 per cent. The same result could have been obtained by adding the exponents of the input variables which equal 0.76 which is the elasticity of production and is interpreted as a 1.0 per cent change in inputs represents a 0.76 per cent change in output. The optimum levels of input will be discussed later.

Utility and Limited Resources

Consider the simplest case of a manager who uses a single input (X) for the production of two outputs (Q_1 and Q_2). Implicitly, the utility production function is,

$$U(q_1, q_2, x) \tag{8}$$

where

q_1 = quantity of Q_1 output

q_2 = quantity of Q_2 output

x = quantity of X input

and it is assumed that Equation (8) can be solved for

$$x = u(q_1, q_2).$$

The cost of production in terms of input, X, is a function of the two outputs,

$$C = p_1 q_1 + p_2 q_2 \tag{9}$$

where

C = cost constraint

p_1 = cost of q_1

p_2 = cost of q_2.

Then, the Marginal Rate of Substitution ($MRS_{q_2 q_1}$) is developed

$$\frac{du}{dq_1} = \frac{\partial u}{\partial q_1} + \frac{\partial u}{\partial q_2}\frac{dq_2}{dq_1} \tag{10}$$

since

$$\frac{dq_2}{dq_1} = - \frac{\frac{\partial u}{\partial q_1}}{\frac{\partial u}{\partial q_2}} \tag{11}$$

and for maximum utility

$$(MUq_1)(p_1) = (MUq_2)(p_2) \tag{12}$$

where

MU_{q_1} = marginal utility of q_1

MU_{q_2} = marginal utility of q_2

p_1 = price of q_1

p_2 = price of q_2

or

$$\frac{MU_{q_1}}{MU_{q_2}} = \frac{p_2}{p_1} . \tag{13}$$

Note: The proof of Equation (12) is elementary and can be found in Leftwich (32, p. 62).

Then, from Equation (9)

since

$$q_2 = \frac{C - p_1 q_1}{p_2}, \tag{14}$$

Equation (10) reduces by substitution to

$$\frac{du}{dq_1} = \frac{\partial u}{\partial q_1} + \frac{\partial u}{\partial q_2}\left(-\frac{p_1}{p_2}\right) = 0 \ . \tag{15}$$

The second order function, y'', to satisfy the maximum condition is obtained if

$$f_1 = \frac{\partial u}{\partial q_1} \ ; \ f_2 = \frac{\partial u}{\partial q_2} \ ; \ f_{11} = \frac{\partial^2 u}{\partial q_1^2} \ ; \ f_{22} = \frac{\partial^2 u}{\partial q_2^2}$$

and $f_{12} = \dfrac{\partial^2 u}{\partial q_1 \partial q_2}$,

then $$\frac{\partial^2 u}{\partial q_1^2} = f_{11} + 2 f_{12}\left(-\frac{p_1}{p_2}\right) + f_{22}\left(-\frac{p_1}{p_2}\right)^2 < 0 \tag{16}$$

if Equation (16) is multiplied by p^2_2 , then

$$\frac{\partial^2 u}{\partial q^2} = \left(f_{11}p^2_2 - 2 f_{12} \ p_1 \ p_2 + f_{22} \ p^2_1\right) < 0, \tag{17}$$

then

$$\frac{d^2 q_2}{dq^2_1} = \frac{1}{f^3_2}\left(f_{11}f^2_2 - 2 f_{12} \ f_1 \ f_2 + f_{22} \ f^2_1 \right) \tag{18}$$

since

$$f_1 = p_1 \frac{f_2}{p_2} \tag{19}$$

it is possible to substitute to obtain the slope of the indifference curve

$$\frac{d^2 q_2}{dq^2_1} = -\frac{1}{f_2 \, p^2_2}\left(f_{11}p^2_2 - 2 f_{12} \ p_1 \ p_2 + f_{22} \ p^2_1\right) \tag{20}$$

if Equation (11) is obtained by substitution of Equation (17), then a positive value of the slope of the utility curve is obtained to indicate it is convex to the origin.

An Example

Let $u = q_1 q_2$

where

q_1 = output utility of product q_1

q_2 = output utility of product q_2.

$$C = \$100 = p_1 q_1 - p_2 q_2$$

where

p_1 = \$2, price of X in q_1.

p_2 = \$5, price of X in q_2.

$$100 - 2q_1 - 5q_2 = 0$$

$$q_2 = \frac{-100 + 2q_1}{-5} = 20 - \frac{2}{5} q_1$$

$$u = q_1 \left(20 - \frac{2}{5} q_1 \right) = 20q_1 - \frac{2}{5} q^2{}_1$$

$$\frac{du}{dq_1} = 20 - \frac{4}{5} q_1 = 0$$

$$20 = \frac{4}{5} q_1$$

$\therefore q_1 = 25$

and $q_2 = 20 - \frac{2}{5}(25) = 10.$

The marginal utility of this Cobb-Douglas function is

$$\frac{\partial u}{\partial q_1} = q_2 = 10$$

$$\frac{\partial u}{\partial q_2} = q_1 = 25$$

$$\frac{MU_{q_1}}{p_1} = \frac{10}{2} = 5 \text{ utiles}$$

and

$$\frac{MU_{q_2}}{p_2} = \frac{25}{5} = 5 \text{ utiles}$$

and

$$\frac{dq_1}{dq_2} = -\frac{\frac{\partial u}{\partial q_2}}{\frac{\partial u}{\partial q_1}} = -\frac{25}{10} = -2.5 = MRS_{q_1 q_2}$$

Total utility = $u = q_1 q_2 = (25)(10) = 250$ utiles

and

$$q_2 = \frac{250}{q_1}$$

$$q_1 = \frac{250}{q_2}$$

which represents a rectangular hyperbola.

And if

$$MRS_{q_1 q_2} = -\frac{p_2}{p_1}$$

Then the decision-maker should produce 25 units of q_1 or 10 units of q_2 to maximize utility under a budget constraint.

Optimization With the Lagrange Multiplier

However, a more convenient method is to use an undetermined Lagrange multiplier to optimize a function under constrained conditions.

The general equation of utility, using the same notation as before, is

$$U = u(q_1, q_2, \ldots, q_n) \tag{21}$$

$$C = p_1 q_1 + p_2 q_2 + \ldots, p_n q_n \tag{22}$$

or

$$C - \sum_{i=1}^{n} p_i q_i = 0. \tag{23}$$

The optimal condition requires that utility be maximized subject to the following:

$$M = u(q_1, q_2, \ldots, q_n) + \lambda [C - \sum_{i=1}^{n} p_i q_i] \tag{24}$$

where

λ = the unspecified Lagrange multiplier.

To solve the constrained-maximization problem of the decision-maker who desires to maximize utility for a specified input of X, the partial derivatives of Equation (24) are set equal to zero:

$$\frac{\partial M}{\partial q_1} = \frac{\partial u}{\partial q_1} - \lambda p_1 = 0 \tag{25}$$

$$\frac{\partial M}{\partial q_2} = \frac{\partial u}{\partial q_2} - \lambda p_2 = 0$$

$$\vdots$$

$$\frac{\partial M}{\partial q_n} = \frac{\partial u}{\partial q_n} - \lambda p_n = 0$$

$$\frac{\partial M}{\partial \lambda} = C - \sum_{i=1}^{n} p_i q_i = 0.$$

Solving for the Lagrange multiplier, λ,

$$\lambda = \frac{\frac{\partial u}{\partial q_1}}{p_1} = \frac{\frac{\partial u}{\partial q_2}}{p_2} = \frac{\frac{\partial u}{\partial q_2}}{p_2} = \ldots = \frac{\frac{\partial u}{\partial q_n}}{p_n} \tag{26}$$

And in the frame of reference of Equation (26), λ represent the marginal utility of money since

$$\frac{\frac{\partial u}{\partial q_1}}{\frac{\partial u}{\partial q_2}} = \frac{p_1}{p_2} \tag{27}$$

or

$$\frac{\partial q_2}{\partial q_1} = \frac{p_1}{p_2} . \tag{28}$$

An Example

From previous notation the example is:

$$U = q_1 q_2 \tag{29}$$

$$C = \$100 = \$2q_1 - \$5q_2 \tag{30}$$

$$M = q_1 q_2 + \lambda(\$100 - \$2q_1 - \$5q_2) \tag{31}$$

$$\frac{\partial M}{q_1} = q_2 - \$2\lambda = 0 \ , \tag{32}$$

then

$$\lambda = \frac{q_2}{\$2} \quad \text{and} \quad q_2 = \$2\lambda \tag{33}$$

$$\frac{\partial M}{\partial q_2} = q_1 - \$5\lambda = 0 \ , \tag{34}$$

then

$$\lambda = \frac{q_1}{\$5} \quad \text{and} \quad q_1 = \$5\lambda \tag{35}$$

$$\frac{\partial M}{\partial \lambda} = \$100 - \$2q_1 - \$5q_2 = 0 \ . \tag{36}$$

Then, by substituting the values for q_1 and q_2 into Equation (36)

$$100 - \$2(5\lambda) - 5(2\lambda) = 0 \tag{37}$$

$$100 = 20\lambda$$

$$\lambda = 5 \ .$$

Then, by substitution in Equations (33) and (34) and solving for q_1 and q_2 ,

$$q_1 = 25 \ , \quad \text{and } q_2 = 10 \ . \tag{38}$$

However, a secondary procedure allows the development of the output demand requirements for q_1 and q_2 where

$$\frac{\partial M}{\partial \lambda} = 100 - p_1 q_1 - p_2 q_2 = 0 \tag{39}$$

$$\frac{\partial M}{\partial \lambda} = 100 - p_1 q_1 - p_2 \left(\frac{p_1}{p_2}\right) q_1 , \tag{40}$$

since $\frac{q_2}{q_1} = \frac{p_1}{p_2}$ because the utility curve and the price ratio line have equal slopes at the optimum values of q_1 and q_2, then

$$q_2 = \frac{p_1}{p_2} q_1 ,$$

therefore:

$$100 - p_1 q_1 - p_1 q_1 = 0 \tag{41}$$

$$100 = 2p_1 q_1 , \tag{42}$$

which indicates the demand for q_1 in generating utility is represented by

$$q_1 = \frac{100}{2p_1} \tag{43}$$

by a similar computation, the demand for $q_2 = \frac{\$100}{2p_2}$

if $p_1 = \$2$ and $p_2 = \$5$, then $q_1 = 25$ units and $q_2 = 10$ units.

The General Equilibrium Model

For the general equilibrium model, the utility function to maximize is represented by

$$M = u(q_1 , q_2 , \ldots , q_n) + \lambda[y - (p_1 q_1 + p_2 q_2 + \ldots + p_n q_n].$$

The partial derivative of each variable is equated to zero:

$$\frac{\partial M}{\partial q_1} = \frac{\partial u}{\partial q_1} - \lambda p_1$$

$$\frac{\partial M}{\partial q_2} = \frac{\partial u}{\partial q_2} - \lambda p_2$$

$$\vdots$$

$$\frac{\partial M}{\partial q_n} = \frac{\partial u}{q_n} - \lambda p_n$$

$$\frac{\partial M}{\partial \lambda} = C - \sum_{i=1}^{n} p_i q_i$$

where, $\frac{\partial u}{\partial q_i}$ = Marginal Utility of the i^{th} output (MU_i)

and

$$\frac{MU_i}{p_i} = \lambda = \text{marginal utility for money.} \tag{44}$$

and any pair of output factors in Equation (44) may be divided to obtain:

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2} \tag{45}$$

where

$$\frac{MU_1}{MU_2} = -MRS_{2\,1} = \frac{p_1}{p_2}\ , \tag{46}$$

since

$$-\frac{dq_2}{dq_1} = \frac{p_1}{p_2}\ . \tag{47}$$

The Expansion Path

For the Cobb-Douglas function where the output requires two variable input factors, x_1 and x_2, then

$$u = q = f(x_1^{a} x_2^{b}) \tag{48}$$

and the

$$MRS_{x_1 x_2} = \frac{dx_1}{dx_2} = -\frac{\frac{\partial q}{\partial x_2}}{\frac{\partial q}{\partial x_1}} = \frac{-b\, x_1^{a}\ x_2^{b-1}}{a\, x_1^{a-1}\ x_2^{b}} \tag{49}$$

$$MRS_{x_1 x_2} = -\frac{b}{a}\ \frac{x_1}{x_2}\ . \tag{50}$$

Equation (50) then represents the equation for the expansion path of input factor combinations to produce utility at the least cost. The equation of the expansion path is a linear function and goes through the origin.

An Example of a Minimum Cost Solution

Given that:

$$\text{Cost of output} = C = p_1 x_1 + p_2 x_2 \tag{51}$$

where

p_1 = input price, x_1

p_2 = input price, x_2.

Quantity to be produced = 10 units.

Then, if

$$\frac{dC}{dx_1} = p_1 + p_2 x_2 \frac{dx_2}{dx_1} = 0$$

$$\frac{dx_2}{dx_1} = - \frac{p_1}{p_2} . \tag{52}$$

For the budget line constraint

$$x_1 = \frac{C}{p_1} - \frac{p_2}{p_1} x_2 \tag{53}$$

which is derived from Equation (51).

The slope of the constraint line $= - \frac{p_1}{p_2}$.

Now, for the utility production function

$$q = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} , \tag{54}$$

then solving for x_1

$$x_1^{\frac{1}{3}} = \frac{x_2^{-\frac{2}{3}}}{q^{-1}} . \tag{55}$$

Therefore,

$$x^{\frac{1}{3}} = x_2^{-\frac{2}{3}} q \tag{56}$$

or

$$x = q^3 x_2^{-2} . \tag{57}$$

Then

$$\frac{dx_1}{dx_2} = -2(q^3)x_2^{-3} \tag{58}$$

For input factor prices, $p_1 = 2$ and $p_2 = 4$

$$\frac{dx_1}{dx_2} = -\frac{p_2}{p_1} = -2\ , \tag{59}$$

then

$$(-2)q^3 x_2^{-3} = 2 \tag{60}$$

is the expansion path of inputs or the input cost minimizing function.

Another similar approach follows:

$$q = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \tag{61}$$

$$\frac{dq}{dx_2} = \frac{2}{3} x^{\frac{1}{3}} x_2^{-\frac{1}{3}} \tag{62}$$

$$\frac{dq}{dx_1} = \frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} \tag{63}$$

$$\frac{\frac{dq}{dx_2}}{\frac{dq}{dx_1}} = \frac{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}}{\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}} = -\frac{2x_1}{x_2} = -2 \tag{64}$$

$$\therefore x_1 = x_2 \tag{65}$$

for the least cost equation.

From Equation (61)

$$q^3 = x_1 x_2^2$$

$$\frac{dx_1}{dx_2} = \frac{-2(x_1 x_2^2)}{x_2^3} = -\frac{2x_1}{x_2} = -2$$

which is identical to Equation (64).

If

$$q = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \tag{66}$$

given: q = 10 units required

p_1 = \$2.00

p_2 = \$4.00 ,

then

$$x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} = 10$$

since

$$x_1 = x_2$$

$$x_1 = 10 \quad ; \; x_2 = 10.$$

$$\text{Let } Z = p_1 x_1 + p_2 x_2 + \ldots + p_n x_n + FC + \lambda [q_o - q(x_1, x_2, \ldots, x_m)] \tag{67}$$

where

FC = fixed costs

q_o = output quantity,

then

$$\frac{\partial Z}{\partial x_1} = p_1 + \lambda q'(x_1) = 0 \tag{68}$$

$$\frac{\partial Z}{\partial x_2} = p_2 + \lambda q'(x_2) = 0$$

$$\vdots$$

$$\frac{\partial Z}{\partial x_n} = p_n + \lambda q'(x_n)$$

$$\frac{\partial Z}{\partial \lambda} = q_o - q(x_1, x_2, \ldots x_n)\,.$$

Then

$$\frac{p_1}{p_2} = \frac{q'(x_1)}{q'(x_2)} = -\frac{dx_2}{dx_1}\,. \tag{69}$$

Then, the minimum cost to produce 10 utility units is

$$C = p_1 x_1 + p_2 x_2$$

or

$$C = \$2(10) + (\$4)(10) = \$60.$$

The relationship is displayed as Figure 12.

An Example - Producing a Given Utility Output for a Minimum Price

If $q = q(x_1, x_2, \ldots, x_n)$

and

$$Z = p_1 x_1 + p_2 x_2, \ldots, p_n + FC + \lambda\,[q_o - q(x_1, x_2, \ldots, x_n) \tag{70}$$

where

FC = fixed cost

q_c = required output.

Then

$$\frac{\partial Z}{\partial x_1} = p_1 + \lambda\, q'(x_1) = 0$$

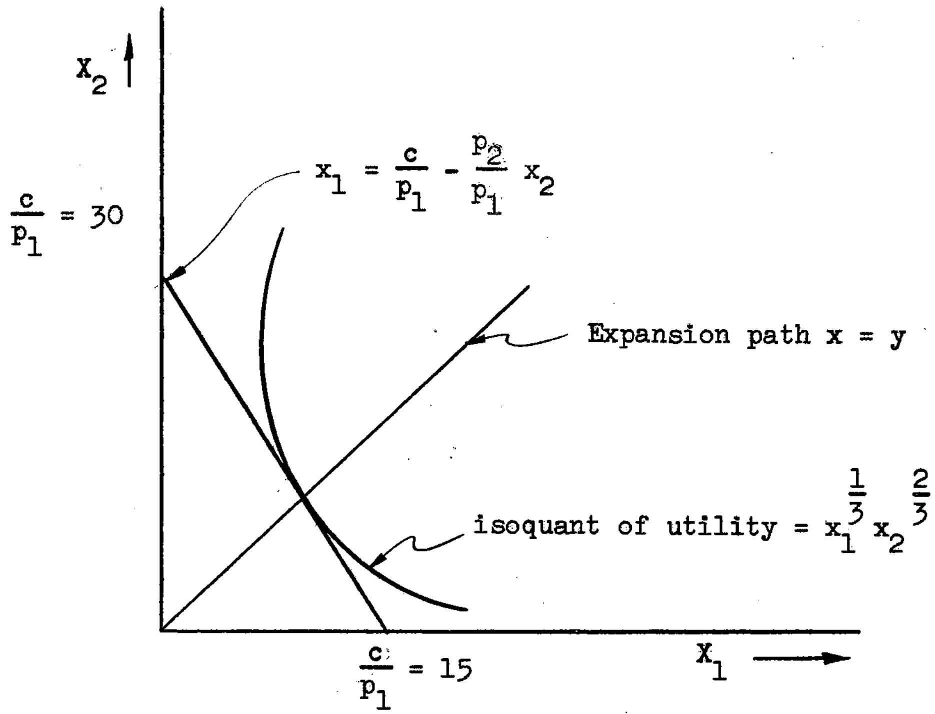


Figure 12. The Expansion Path for Minimum Cost

$$\frac{\partial Z}{\partial x_2} = p_2 + \lambda q'(x_2) = 0$$

$$\frac{\partial Z}{\partial x_n} = p_n + \lambda q'(x_n) = 0$$

$$\frac{\partial Z}{\partial \lambda} = q_o - q(x_1, x_2, \ldots, x_n).$$

If the utility function is given as

$$q = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

where

$q_o = 16$

$p_1 = \$2$

$p_2 = \$8$.

Then from Equation (70)

$$Z = \$2x_1 + \$8x_2 + \lambda(16 - x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}) \tag{71}$$

$$\frac{\partial Z}{\partial x_1} = 2 - \lambda \tfrac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} = 0 \tag{72}$$

$$\frac{\partial Z}{\partial x_2} = 8 - \lambda \tfrac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} = 0 \tag{73}$$

$$\frac{\partial Z}{\partial \lambda} = 16 - x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 0 . \tag{74}$$

From Equations (72) and (73)

$$\frac{dx_1}{dx_2} = \frac{8 - \lambda \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}}{2 - \lambda \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}} = \frac{x_1}{x_2} = \frac{p_2}{p_1} = 4 \tag{75}$$

or

$$x_1 = 4x_2 .$$

Then,

$$16 - (4x_2)^{\frac{1}{2}}(x_2)^{\frac{1}{2}} = 0 \qquad (76)$$

or

$$2x_2 = 16.$$

Then,

$$x_2 = 8 \text{ and } x_1 = 32$$

$$q = 32^{\frac{1}{2}}8^{\frac{1}{2}} = 16$$

since

$$(4x_2)^{\frac{1}{2}}(x_2)^{\frac{1}{2}} = 2x_2 = 16.$$

$\therefore$ Minimum cost = (32)($2) + (8)($8) = 128.

An Example - The Maximization of Residue of Output-Input Utility

If total utility output is represented by:

$$TUO = p_1 q_1 + p_2 q_2 \qquad (77)$$

and if the residue utility is represented by

$$RU = p_1 q_1 + p_2 q_2 - p_3 X \qquad (78)$$

or

RU = output less input.

$$\frac{d(TUO)}{dq_1} = p_1 + p_2 \frac{dq_2}{dq_1} = 0 \tag{79}$$

or

$$\frac{dq_2}{dq_1} = -\frac{p_1}{p_2} = MRS_{q_2 q_1}, \tag{80}$$

or since

$$p_1 q_1 = -p_2 q_2 + TUO \tag{81}$$

$$q_1 = \frac{TUO}{p_1} - \frac{p_2}{p_1} q_2 \tag{82}$$

or

$$\frac{dq_1}{dq_2} = -\frac{p_2}{p_1} = 0. \tag{83}$$

And, it is obvious that Equation (82) represents the linear equation of the output mix utility given some level of TUO.

The next step is to locate the point of tangency for optimum production of utility.

If the utility production function is represented by

$$q_1 = x_1^{\frac{1}{2}} \tag{84}$$

$$q_2 = 2x_2^{\frac{1}{2}} \tag{85}$$

where

X = a common resource input

and

x_1 = input for q_1 and x_2 = input for q_2.

And, since the x_3 exponents for q_1 and q_2 are < 1, diseconomies of scale result; i.e., diminishing returns are present.

From Equations (84) and (85)

$$x_1 = q^2_1$$

$$x_2 = \frac{q^2_2}{4} .$$

And, if the total resource available is X, then

$$X = x_1 + x_2 = q^2_1 + \frac{q^2_2}{4} \tag{86}$$

which reflects a utility production function.

To maximize TUO, let

$$TUO = p_1 q_1 + p_2 q_2 + \dots + p_n q_n + \mu \, [X - h(q_1, q_2, \dots, q_n) \tag{87}$$

$$\therefore TUO = p_1 q_1 + p_2 q_2 + \mu \left(X_o - q^2_1 - \frac{q^2_2}{4} \right) . \tag{88}$$

To maximize TUO, then

$$\frac{\partial (TUO)}{\partial q_1} = p_1 - 2\mu q_1 = 0 \tag{89}$$

$$\frac{\partial (TUO)}{\partial q_1} = p_2 - \tfrac{1}{2} \mu q_2 \tag{90}$$

$$\frac{\partial (TUO)}{\partial \mu} = X - q^2_1 - \frac{q^2_2}{4} , \tag{91}$$

and Equation (91) represents the equation for an input restraint.

From Equations (89) and (90)

$$\frac{\frac{2\mu q_1}{\frac{\mu q_2}{2}}}{} = \frac{p_1}{p_2} \tag{92}$$

or

$$\frac{p_1}{p_2} = \frac{4q_1}{q_2}. \tag{93}$$

If

$$p_1 = \$8$$

$$p_2 = \$2,$$

then

$$\frac{4q_1}{q_2} = \$8/\$2 \tag{94}$$

and the expansion path of utilizing inputs is

$$q_1 = q_2. \tag{95}$$

If X is assumed to be limited to 160 utility units of input, then $160 - q^2_1 - \frac{q^2_1}{4} = 0$

solving for $q^2_1 = 128$

$$q_1 = 11.3$$

$$\therefore q_2 = 11.3.$$

Therefore, the maximum utility output for an input $X_o = 80$.

$$TUO = p_1 q_1 + p_2 q_2$$

$$TUO = (8)(11.3) + (2)(11.3) = 123.1 \text{ utiles.}$$

If residual utility (RU) is to be maximized, for simplicity a single output factor is assumed, then

$$RU = pq - p_1 x_1 - p_2 x_2 - \ldots - p_n x_n \tag{96}$$

where

$$q = q(x_1, x_2, x_3, \ldots, x_n).$$

$$RU = p[q(x_1, x_2, \ldots, x_n)] - p_1 x_1 - p_2 x_2 - \ldots - p_n x_n$$

$$\frac{\partial(RU)}{\partial x_1} = p[q'(x_1)] - p_1 = 0 \tag{97}$$

$$\frac{\partial(RU)}{\partial x_2} = p[q'(x_2)] - p_2 = 0 \tag{98}$$

$$\vdots$$

$$\frac{\partial(RU)}{\partial x_n} = p[q'(x_n)] - p_n = 0 \tag{99}$$

Since $p[q'(x_i)]$ = Marginal Utility Value, then under the assumption of perfect competition

$$MUV_i = p_i \tag{100}$$

where

MUV_i = marginal utility value of the i^{th} input factor in terms of output value

p_i = cost of input in utiles

and then

$$\frac{MUV_1}{p_1} = \frac{MUV_2}{p_2} = \frac{MUV_n}{p_n} = 1 \tag{101}$$

For the General Case

Output is represented by multiple output factors, input has a multiple of input factors, all represented by:

$$q_1 = q_1(x_1, x_2, \ldots, x_n) \tag{102}$$

$$q_2 = q_2(x_1, x_2, \ldots, x_n) \tag{103}$$

$$\vdots$$

$$q_m = q_n(x_1, x_2, \ldots, x_n). \tag{104}$$

The total function is stated as

$$TUO(q_1, q_2, \ldots, q_s, x_1, x_2, \ldots, x_n) \tag{105}$$

$$RU = \sum_{i=1}^{m} p_1 q_1 + \lambda TUO(x_1, x_2, \ldots, x_n). \tag{106}$$

The notation can be simplified to let $q_{s+j} = -x_j (j = 1, \ldots, n)$ and rewrite Equation (105)

$$TUO(q_1, \ldots, q_m) = 0 \tag{107}$$

where

$m = n + s$, and output levels of factors $(q_1, \ldots, q_s)$ are positive and input levels $(q_{s+j}, \ldots, q_n)$ are negative.

Then maximization of the Residual Utility function

$$RU = \sum_{i=1}^{m} p_i q_i + \lambda F(q_1, \ldots, q_m) \tag{108}$$

and setting each of the (m + 1) derivatives equal to zero results in:

$$\frac{\partial J}{\partial q_i} = p_i + \lambda F_i = 0 \qquad (i = 1, \ldots, m) \tag{109}$$

$$\frac{\partial J}{\partial \lambda} = F(q_1, \ldots, q_m) = 0, \tag{110}$$

then setting any two of the first m equations equal to each other and solving for the MRS, the results equal

$$\frac{p_j}{p_k} = \frac{F_j}{F_k} = -\frac{\partial q_k}{\partial q_j} \qquad (j, k = 1, \ldots, m) \tag{111}$$

which says that the $MRS_{q_k q_j}$ for any pair of output factors must equal the negative inverse price ratio, or output utility ratio if all other inputs and outputs are held constant. That is: $MRS_{12} = -\frac{p_2}{p_1}$. Now, if the j^{th} variable is an input and the k^{th} variable is an output, then if

$$p_j = r_{j-s} \quad \text{and} \quad \frac{dq_j}{dx_{j-s}} = -1,$$

then

$$\frac{r_{j-s}}{p_k} = \frac{q_k}{\partial x_{j-s}} \tag{112}$$

or

$$r_{j-s} = p_k \frac{\partial q_k}{\partial x_{j-s}} \qquad \begin{array}{l}(k = 1, \ldots, s)\\ (j = s + 1, \ldots, m)\end{array}$$

which says that the marginal productivity of an input with respect to every output must be equated to its price, or

$$\frac{\left(\frac{dq_1}{x_1}\right) p_{q_1}}{p_{x_1}} = \frac{\left(\frac{dq_2}{dx_1}\right) p_{q_2}}{p_{x_1}} = \ldots = \frac{\left(\frac{dq_j}{dx_i}\right) p_{q_j}}{p_{x_1}} = 1 , \tag{113}$$

and this condition must hold for any one input used for the production of several output variables. <u>This is the output - input rule</u>.

Likewise if i and j are inputs, then

$$\frac{dx_j}{dx_i} = \frac{p_i}{p_j} = MRS_{x_j x_i} . \tag{114}$$

The requirement is that the input utility price of i must equal j for optimal input-input allocation. <u>This is the input - input rule</u>.

And, if i and j are outputs, then

$$\frac{dq_j}{dq_i} = \frac{p_i}{p_j} \tag{115}$$

or

$$MRS_{ji} = \frac{p_i}{p_j} \tag{116}$$

or

$$\left(\frac{dq_j}{dq_i}\right) p_j = p_i . \tag{117}$$

Then, if (MRS_{ji}) pj > pi, allocate more utility to produce more q_j; if $(MRS_{ji})\ p_j < p_i$, allocate more utility to produce less q_j. This is the output-output rule.

Rule Summary

In the output-input rule, the RU can be maximized, but there is no assurance costs are being minimized.

In the input-input rule, cost is being minimized, but no assurance is given that RU is being maximized.

In the output-output rule, RU is being maximized, but no assurance is given that costs are minimized.

By using a combination of the input-input rule and the output-output rule, assurance can be obtained that utility output is maximized with a minimum of input so that

Rule 1: $$MRS_{x_1 y_1} = \frac{p_{y_1}}{p_{x_1}} \qquad \text{output-input rule} \tag{116}$$

Rule 2: $$MRS_{x_1 x_2} = \frac{p_{x_2}}{p_{x_1}} \qquad \text{input-input rule} \tag{117}$$

Rule 3: $$MRS_{y_1 y_2} = \frac{p_{y_2}}{p_{y_1}} \qquad \text{output-output rule} \tag{118}$$

where

x = input factor

y = output factor.

By using Rules 2 and 3 above, the following is obtained:

$$\frac{\left(MRS_{y_1 y_2}\right) p_{y_1}}{p_{y_2}} = \frac{\left(MRS_{x_1 x_2}\right) p_{x_1}}{p_{x_2}} = 1 \tag{119}$$

for any combination of j inputs and k outputs.

Of particular interest is that Equation (119) can be discounted for future receipt of utility analogously to the method of discounting cash flows. The procedure is to allow

$$\frac{\left(MRS_{y_1 y_2}\right) p_{y_1}}{p_{y_2}} = \frac{\left(MRS_{x_1 x_2}\right) p_{x_1}}{p_{x_2}} = \frac{1}{(1 + UDR)^n}$$

where

UDR = utility discount rate in decimal form.

n = unit time before receipt of utility.

CHAPTER VI

THE STABILITY AND INPUT-OUTPUT ANALYSIS OF A UTILITY SYSTEM

In Chapter V, methods of allocating utility resources were developed. This is only an identification of a portion of the problem. Not only must the decision-maker allocate utility to the individual contributors to the organization, but, in sum, he must have enough total utility to dispense. Essentially, for the organization to be stable it must have at least a sufficient supply of utility to meet the demand for utility made by the contributors.

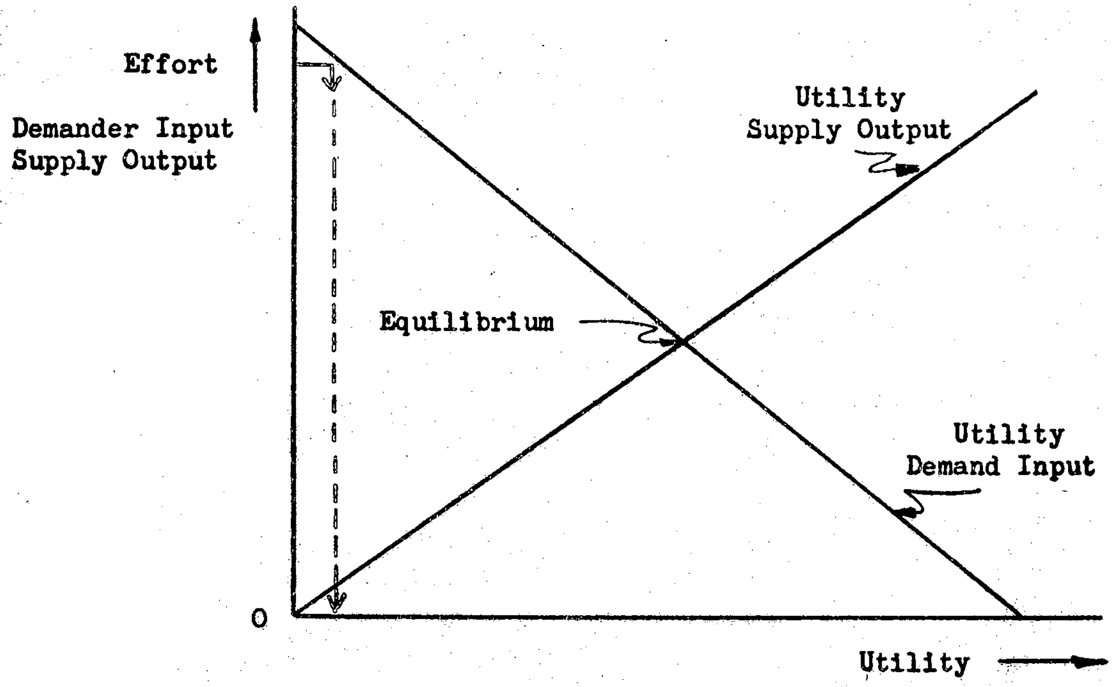
Equilibrium in the Supply and Demand of Utility

Equilibrium is characterized by the acquiescence of suppliers and demanders of utility in the status quo: no participant in the exchange of utility has an incentive to modify the supply and demand requirements of utility. The organization is stable - in equilibrium - if supply and demand are equal. Unfortunately, the existence of an equilibrium point does not guarantee that it will be attained. There is no reason to assume that an initial exchange of utility will happen to be the equilibrium point where the demand for utility exactly equals the supply which the organization is willing and able to give up, indicated in Figure 13. In no more realistic fashion is this recognized than in Wage and Salary Administration where a continued knowledgeable effort is required to assure that the labor force is in equilibrium with the organization's requirement for effort. Moreover, changes in the evaluation of utility preferences will generally shift the demand curve, and changes in the effectiveness with which the organization utilizes effort will shift the supply curve. Both of these types of changes tend to disturb the established equilibrium. The changes define a new equilibrium, but, again, there exists no guarantee it will be attained.

Assumptions for Stability Under Dynamic Change

If the condition exists such that the demand for utility exceeds its supply, the assumption is made that an increase in effort will be made to acquire more utility.

In the organization, the contributors may exert this effort within the organization or affiliate with another organization. If the condition exists whereby more utility is supplied than is being demanded, then it will be assumed that the organization will reduce the effort in generating the surplus utility.



(Note: This figures requires a specific interpretation as follows. The ability of an organization to generate an increasing supply of utility as additional effort is contributed is seen intuitively. The demand for the utility which an organization dispenses to its contributors is viewed, by the contributor, from the position that a large required effort input makes the contributor demand a small amount of utility from this organization. The organization faces such a demand for the utility it may distribute. In practical terms, if an organization were to offer double the present starting salary of any of its competitors, the utility of such a salary would create a heavy demand by job applicants.) From another viewpoint, if the required effort is large, the individual assigns less utility to the utility being offered by the organization.

Figure 13. Equilibrium of Utility Supply and Demand

A Digression on Difference Equations

The use of difference equations is applicable to the study of dynamic stability, since decisions to modify input-output relationships are assumed to be discrete as decisions extend over time periods.

Discrete analysis over time periods is generally identified as being suitable for manipulation by difference equations.

Initially, a difference equation indicates changes taking place in a function as influenced by changes in the relevant time periods.

For example, a growth rate equation is represented by:

$$C = 100\left(\frac{U_t - U_{t-1}}{U_{t-1}}\right) \tag{1}$$

where

C = change in total utility in per cent

U_t = total utility at time t

U_{t-1} = total utility in the preceding time period.

Thus, stipulating at utility growth rate of five per cent per time period equals

$$5 = 100\left(\frac{U_t - U_{t-1}}{U_t}\right) \tag{2}$$

or Equation (2) equals

$$100\ U_t - 100\ U_{t-1} = 5\ U_{t-1} \tag{3}$$

$$100\ U_t = 105\ U_{t-1}$$

Then Equation (3) can be represented by

$$U_t = AU_{t-1} \tag{4}$$

where

$A = 1.05.$

By repeated multiplication of Equation (4), it is possible to obtain:

$$U_2 = AU_1$$

$$U_3 = AU_2 = A(AU_1) = A^2 U_1$$

and, in general

$$U_t = A^{t-1}U_1 \quad \text{for } t = 1, 2, 3, \ldots \tag{5}$$

Equation (5) is the solution for Equation (4). Since Equation (4) relates the dependent variable, U, in terms of different time periods, it is called a difference equation. Since the lag in time periods is one period, it is called a first order difference equation.

In general, a difference equation involves a function F of a dependent variable U and an integer variable t, which is represented by

$$F(t, U_t, \ldots, U_{t-k}) = 0 \tag{6}$$

where the difference equation is said to be of the k^{th} order when the maximum difference of the subscripts of U are equal to k. It is linear when the dependent variables appear to no higher power than one. It has constant coefficients when all the coefficients of U are constants.

The general n^{th} order linear difference equation with constant coefficients is written as

$$U(t) = \alpha_1 U(t-1) + \alpha_2 U(t-2) + \ldots + \alpha_n U(t-n) + f(t)$$

where

α_i = a constant for each lag period $(i = 1, 2, \ldots, n)$

$f(t)$ = a function of t

or

$$U_t = \alpha U_{t-1} + f(t) \tag{8}$$

where

$t = 1, 2, 3, \ldots,$

where Equation (8) represents the general first order difference equation with constant coefficients.

Now, it can be said $\overline{U}_t$ is a particular solution of Equation (8) if it satisfies the equation for all permissible values of t.

For example, $\overline{U}_t = 20\alpha^t$ is a particular solution of $U_t = \alpha U_{t-1}$ since, by substitution,

$$20\alpha^t = \alpha(20\alpha^{t-1}) = 20\alpha^t. \tag{9}$$

And, in fact, there is a whole family of particular solutions identified by

$$U_t = k\alpha^t \tag{10}$$

where

k = an arbitrary constant,

which is the general solution of

$$U_t = \alpha U_{t-1}. \tag{11}$$

A particular solution is obtained by the specification of k. The specific value of k is determined from the initial condition; i.e., the value of U_t for the first period considered.

For example, from Equation (4), the general solution is

$$U_t = k\alpha^t$$

for

$$t = 1, 2, 3 \ldots \quad . \tag{12}$$

The initial condition in this case is $U_t = U_t$ for $t = 1$ so that

$$U_1 = k\alpha \tag{13}$$

or

$$k = \frac{U_1}{\alpha}, \tag{14}$$

so the general solution may be represented by

$$U_t = \left(\frac{U_1}{\alpha}\right)\alpha^t = U_1\alpha^{t-1}. \qquad (15)$$

While the above solution has been demonstrated for clarity, an easier method is presented.

First, let $f(t) \equiv 0$ in Equation (8). The result is identified as the homogeneous form of Equation (8) and is represented by

$$u_t = \alpha u_{t-1} \qquad (t = 1, 2, 3, \ldots) \qquad (16)$$

where

u = the dependent variable in the homogeneous equation. (35).

The general solution is found as

$$u_t = k\alpha^t \qquad t = (1, 2, 3, \ldots). \qquad (17)$$

Let

$$U_t = k\alpha^t + \overline{U}_t \qquad (18)$$

where

$\overline{U}_t$ is a particular solution of Equation (8).

That is, $\overline{U}_t$ is a function of t that satisfies the equation

$$\overline{U}_t = \alpha\overline{U}_{t-1} + f(t) \qquad (19)$$

to result in

$$k\alpha^t + \overline{U}_t = \alpha(k\alpha^{t-1} + \overline{U}_{t-1}) + f(t) \qquad (20)$$

or

$$k\alpha^t + \overline{U}_t = k\alpha^t + \alpha\overline{U}_{t-1} + f(t). \tag{21}$$

If $k\alpha^t$ is subtracted from both sides of Equation (21)

$$\overline{U}_t = \alpha\overline{U}_{t-1} + f(t), \tag{22}$$

which is true for all t by definition of $\overline{U}_t$ as a particular solution.

Thus,

$$U_t = k\alpha^t + \overline{U}_t \tag{23}$$

is a solution to Equation (8) and, since it contains an arbitrary constant k, it is a general solution.

If, in determining future utility levels U_t, the condition is $U_t = U_o$ for $t = 0$, then k can be determined. To evaluate k, set t equal 0 in Equation (23) to obtain

$$U_o = k\alpha^o + \overline{U}_o \tag{24}$$

where

$\overline{U}_o = \overline{U}_t$

then

$$U_t = (U_o - \overline{U}_o)\alpha^t + \overline{U}_t. \tag{25}$$

The Use of Difference Equations in Stability Analysis

The analysis of stability is done for a one period lag in adjusting to the demand for a supply of utility made by the contributors to the organization. Essentially, the decision-maker would like to obtain and maintain equilibrium in the supply and demand for utility. He may be concerned about fluctuations taking place; in one period an under supply of utility may cause a response that will over compensate for such shortage.

Taking into account a one-time period adjustment cycle, the demand and supply of utility can be represented as

$$D = \text{demand for utility, } U_D$$

$$S = \text{supply of utility, } U_S$$

where each, respectively, are effected by the effort required to obtain and produce utility, then

$$D(E_t) = S(E_{t-1}) \tag{26}$$

where

E_t = effort in current period

E_{t-1} = effort in prior period.

Essentially, the decision-maker questions, "Did I supply enough utility last period to satisfy the demand for this period; if I didn't, perhaps my supply of utility needs to be adjusted."

It is assumed that the demand and supply functions are linear where effort (E) is the variable. Then

$$D = A + aE$$
$$S = B = bE ,$$

or Equation (26) can be rewritten

$$A + aE_t = B + bE_{t-1}. \tag{27}$$

Then

$$E_t = \frac{b}{a} E_{t-1} + \frac{B - A}{a}, \tag{28}$$

the homogeneous form is $e_t = \frac{b}{a} p_{e-1}$ (29)

where

$$\alpha = \frac{b}{a} \quad , \quad \text{and } f(t) = \frac{B - A}{a} \; .$$

The solution follows where

$p_t = k\left(\frac{b}{a}\right)^t$ is the general solution, and (30)

the particular solution $\overline{E}_t$ of Equation (28) is assumed to be constant, say $\overline{E} = C$; then, substituting in Equation (8)

$$C = \left(\frac{b}{a}\right) C + \frac{B - A}{a} \tag{31}$$

$$C = \frac{\frac{B - A}{a}}{\frac{(a - b)}{a}} = \frac{B - A}{a - b} , \tag{32}$$

then

$$\bar{E} = \frac{B - A}{a - b}, \tag{33}$$

and the general solution is

$$E_t = k\left(\frac{b}{a}\right)^t + \frac{B - A}{a - b}. \tag{34}$$

If the effort, E, at $t = 0$ is given as E_o, then from Equation (28), the solution is

$$E_t = \left(E_o - \bar{E}\right)\left(\frac{b}{a}\right)^t + \bar{E} . \tag{35}$$

It is assumed that the demand for utility is inversely related to the effort required to obtain utility; that is, a contributor is a utility maximizer with limited effort resources. Then the utility demand function has a negative slope as in Figure 14; i.e., $a < 0$. The slope of the utility supply function is positive, $b > 0$, since the decision-maker reasons that the acquisition of utility requires an expenditure of effort by the contributors to the organization. Then, Equation (35) exhibits three types of alternation:

<u>Alternate 1</u>: $\left|\frac{a}{b}\right| > 1$. Then $b < |a|$, then the magnitude of $\left(\frac{a}{b}\right)^t$ increases indefinitely as $t \to \infty$. This explosive situation is shown in Figure 14. The organization will never achieve

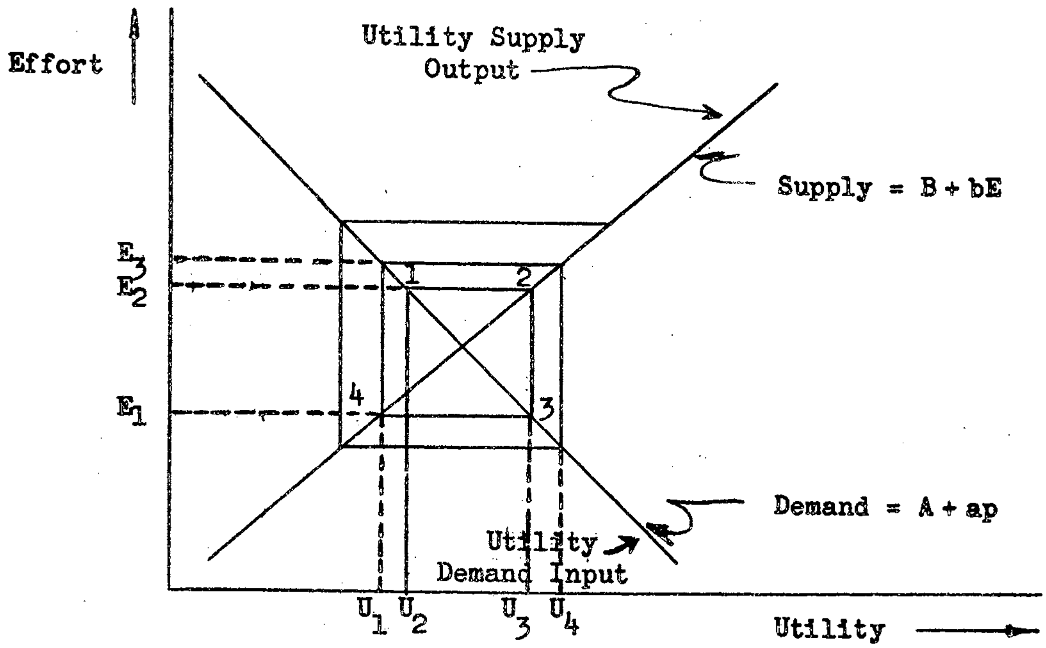


Figure 14. An Explosive System

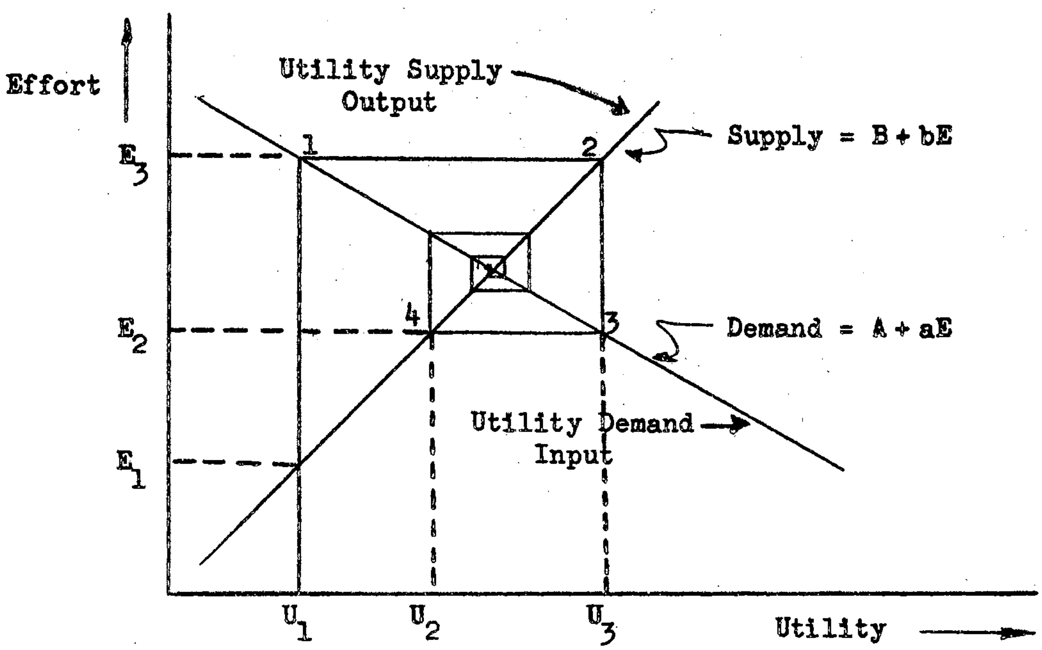


Figure 15. A Dynamically Stable System

stability in the allocation of utility.

Alternate 2: $|\frac{a}{b}| = 1$. Then $b = |a|$ and both the supply and demand utility functions have the same absolute slope and regular alternation will result from too much utility in one period to too little utility in the next period. The fluctuation can never be accommodated in this situation.

Alternate 3: $|\frac{a}{b}| < 1$. Then, $b > |a|$ and $\left(\frac{a}{b}\right)^t$ will decrease in magnitude as t increases; $|\frac{a}{b}|^t \rightarrow 0$ as $t \rightarrow \infty$. This indicates a damped alternation in the supply and demand of utility which will tend to bring the situation to a equilibrium state (Figure 15).

Using Figure 14, a detailed explanation of the explosive condition is developed. First, assume that the supply of utility does not equal the equilibrium quantity. The reason may be reflected in attrition rates higher than can be attributed to chance causes alone. At U_{21} the level of effort E_2 faced by the contributor represents his demand for organization utility. If the effort is at a high level, the utility demand is low since effort and utility both account for the attractiveness in contributing to the organization. The manager facing this E_2 level of effort recognizes that if he possessed this level of effort in the next period, the utility level would increase to U_1. And, if he produces this level of utility, there is an increased demand for it by the contributors. Their demand increases from U_2 to U_3. At utility level U_2, the manager recognizes a lowered effort, E_1, which requires that he reduce his utility supply to U_1 in this second period. Such a reduction in utility output is recognized by the contributors who match the reduction by reasoning that the scarce utility increases their effort in an attempt to obtain it. Hence, the effort level rises to E_3 at which point the manager anticipates in the next period he has the effort capability to produce a U_4 level of utility. The cycle continues and explodes since equilibrium will never be reached.

Using Figure 15, again assuming supply does not equal the equilibrium quantity, rather it equals U_1, the corresponding effort that contributors are willing to contribute is E_3 at point 1 on the demand curve. The amount of demanded utility equals the available supply U_1. The corresponding effort E_3 and E_1 are not equal. The manager reasons that more utility from this firm will be demanded as the effort requirement is reduced. (Note: This statement is not to be construed as saying that the contributor will not demand more utility as his effort increases, for he will. The contributor is being viewed here not from his individual position of a supplier of effort, but rather a demander of utility.) The effort, E_3, that contributors are willing to contribute is viewed as "If you require a great deal of my effort, I don't want very much of the utility you have to offer." The effort, E_3, permits the manager to supply U_3 in the succeeding period of activity. He reasons that if E_3 were available, U_3 could be supplied to the contributors. However, the contributors view this new level of utility, U_3, as very attractive and all of it is demanded by contributors who drop the effort level to E_1. The manager recognizes a decrease in effort which reduces the organization's utility supply to U_2 in the next period. At the utility supply level, U_2, effort E_2 is required by the supply curve and the demand for this level of utility is instantly recognized on the utility demand curve wherein the contributor reasons that "If the organization increases, the effort demanded of me, then I want less of its utility in this second period." This adjustment continues until equilibrium is reached.

The lagged analysis in the foregoing development for the stable and explosive systems has been represented to show the organization lagging the demands of the contributors. Where an explosive system exists, stability can be generated if the organization will lead the demands of the contributors.

Given the equiprobable chance of each type of alternation occurring, it is readily apparent that if stability is an objective of a manager, and the organization exists in a competitive environment, stability is an elusive goal. Given the chance occurrence of Alternate 1, 2, or 3, it is quite likely, if the survival of an organization is contingent upon stability; then 0.666 of all organizations will fail due to chance. This is not an uncommon observation in organization survival rates.

Input-Output Analysis

In the analysis of a sector of an organization to develop insights into its behavior, Leontief's (36) technique of input-output analysis seems desirable and appropriate. The technique seeks to take account of general equilibrium phenomena in the empirical analysis of production. The type of analysis proposed deals with the production and consumption of utility. Demand theory plays no role in the hard core of input-output analysis. The problem is essentially technological. This portion of the investigation seeks to determine what amount of utility can be produced, and the quantity of utility which can be consumed in the production processes, given the quantities of available resources and the state of technological development.

Descriptive Input-Output Activity

Input-output analysis seeks to take account of the interdependence of the resources which consitute an identified organization. The interdependence arises out of the fact that each contributor employs the outputs of other contributors as its input resource. Its output, in turn, is often used by other contributors as a productive input factor; sometimes by the very contributors from which inputs were obtained initially.

As an example, the contributor on the assembly line builds a car; the car is the input for sales, sales output is sales revenue which is the input to the accounting department; accounting output is a paycheck for the contributor on the assembly line; accounting output is also a paycheck for the salesman and a paycheck for the accountant; accounting output may also go to surplus funds.

The basic problem is to see what can be left over for final surplus utility and how much of each contributor's output will be used up in the course of the productive activities which must be undertaken to obtain these net utility outputs. Most significantly, it can be used for organization planning in the sense, for example, of determining what contributor or group of contributors consume less utility than they produce as viewed by the decision-maker.

Essentially, then, input-output analysis is an empirical technique developed by Leontief (36) as a method of determining interdependence among various sectors of an organization. In contrast, to the normative characteristic of most economically oriented analysis, input-output is mainly positive in nature. Rather than to predict what ought to be, given specified objectives and means, it mainly describes conditions as they existed at a particular point in time. The conditions explained are largely the interdependence coefficients among the various producing sectors of an organization. The interdependence coefficients computed can be used then to predict utility output and input in various sectors of the organization under different conditions of demand.

Most work in input-output analysis has been in terms of open models or systems. A system is said to be open if vectors are not included in the flow, input-output, and interdependence matrices which relate all sectors to each other. An open system is represented by an autonomous sector which does not have flows back to the producing sectors. An open system represents the possibility of a surplus of produced utility. In a closed system, these final utility demands would be included in the flow tables; i.e., surplus utility would be non-existent. In this analysis, an open model approach has primary considerations.

A major interpretative problem of input-output analysis relates to the fixed-mix characteristics of inputs and outputs of the various sectors. For diagnostic and highly aggregated types of analyses, these limitations may be no more severe than for alternative empirical procedures as discussed in Chapter V.

The strength of input-output analysis permits the establishment of interdependence of sector utility output and consumption as it relates to total utility demand within the organization.

The Input-Output Model

The data for Leontief-type input-output analyses requires that the organization under consideration be divided into relevant groups or sectors. In parallel terminology, each of these sectors can be viewed as an activity. These sectors or activities are interdependent because some use the outputs from other sectors. Output from each of the sectors is designated

$$X_1, X_2, \ldots, X_n$$

where

n = the total number of sectors

and

X_1 = total utility output from sector 1.

Final utility demands are represented by demands, U_i, made by autonomous sectors, and represents available surplus utility in the sense that the quantity for any one demand sector does not depend on the magnitude of any other U_i quantity.

Quantities of intersector utility flows are designated as X_{ij}, denoting the quantity of utility moving from the i^{th} producing sector to the j^{th} consuming sector. Final demand, U_i, indicates the amount of utility which does not move between sectors, but moves into final surplus utility. The total utility output includes total utility production from the respective producing sectors within the organization.

The Model

In matrix form, the model appears as:

		Consuming Sectors 1	2	...	j	...	n	Final Surplus Utility U_i	Total Utility Output X_i
Producing Sectors	1	x_{11}	x_{12}	...	x_{1j}	...	x_{1n}	U_1	X_1
	2	x_{21}	x_{22}	...	x_{2j}	...	x_{2n}	U_2	X_2
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$	$\vdots$
	i	x_{i1}	x_{i2}	...	x_{ij}	...	x_{in}	U_i	X_i
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	n	x_{n1}	x_{n2}	...	x_{nj}	...	x_{nn}	U_n	X_n

where $\sum_{i}^{n} x_{1n}$ = intermediate utility demand + $U_1 = X_1$.

Then:

$$x_{11} + x_{12} + \ldots + x_{1j} + \ldots + x_{1n} + U_1 = X_1$$

or

$$X_1 - x_{11} - x_{12} - x_{1i} - \ldots - x_{1n} = U_1 .$$

In matrix notation:

$$Z = \begin{bmatrix} x_{11} & \ldots & x_{1n} \\ \vdots & & \vdots \\ x_{n1} & \ldots & x_{nn} \end{bmatrix} \qquad X = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix} \qquad U = \begin{bmatrix} U_1 \\ \vdots \\ U_n \end{bmatrix}$$

If

$$I = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Then

$$ZI + U = X$$

$$X - ZI = U.$$

Input-output coefficients equal:

$$a_{ij} = \frac{x_{ij}}{x_j}$$

where

$x_{12} = a_{12} X_2$ means that the total quantity of output from sector 1 used by sector j = the amount of sector 1 output used per unit of output from sector 2 multiplied by total output from sector 2.

Where:

$$
\begin{bmatrix}
a_{11} & -a_{12} & \cdots & -a_{1i} & -a_{1n} \\
 & \vdots & & \vdots & \\
-a_{21} & \cdots & \cdots & \cdots & -a_{2n} \\
\vdots & \vdots & & \vdots & \vdots \\
-a_{i1} & \cdots & \cdots & \cdots & -a_{in} \\
\vdots & \vdots & & \vdots & \vdots \\
-a_{in} & \cdots & \cdots & -a_{ni} & -a_{nn}
\end{bmatrix}
\begin{bmatrix}
X_1 \\ \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_n
\end{bmatrix}
=
\begin{bmatrix}
U_1 \\ \\ U_2 \\ \vdots \\ U_i \\ \vdots \\ U_n
\end{bmatrix}
$$

when all $a_{ii} = 1$; that is when $i = j$, one utile of output from a particular sector is required for each utile of input to this same sector.

All other a_{ij} represent values of output from the i^{th} sector required to produce one utile of product in the j^{th} consuming sector.

To relate output of one sector to quantities of demand for all other sectors, the following is required:

Let

$$
A = \begin{bmatrix} a_{1i} \\ \vdots \\ a_{nn} \end{bmatrix}
$$

the matrix of input-output coefficients.

Then

$$AX = U$$

$$X = UA^{-1}$$

where A^{-1} is the inverse matrix defined as

$$A^{-1} = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \cdots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix}$$

Then

$$\begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \cdots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ \vdots \\ U_n \end{bmatrix}$$

The element c_{11} indicates the amount by which output of sector 1 will change as final utility demand for the output of this sector is increased by one utile. Or, c_{ij} expresses the amount of utility from sector i used per utile of final demand for the product from sector j.

An Example - Input-Output Analysis

To demonstrate the foregoing with an elementary analysis, assume a simple economy of three sectors: finance, manufacturing, and sales. Utility output of the sectors is denoted by X_1, X_2, X_3. For simplicity there is no intra-sector consumption.

Producing Sector	Consuming Sector 1	2	3	Final Surplus Utility U	Total Utility Output X
1	-	100	40	360	500
2	100	-	0	100	200
3	50	80	-	270	400

Sector 2 consumes 100 utiles of output from sector 1

Sector 3 consumes 40 utiles of output from sector 1.

Final demand for utility output from sector 1 is 360 utiles; i.e., exogeneous demand is 360 utiles. Total output from sector 1 = 500 utiles.

The matrix of input-output coefficients is

Producing Sector	Consuming Sector 1	2	3
1	1	-0.5	-0.1
2	-0.2	1	0
3	-0.1	-0.4	1

$$a_{12} = \frac{\text{utility flow from sector 1 to sector 2}}{\text{total utility output of sector 2}}$$

$= \frac{100}{200} = .5$, the negative sign is affixed to conform to previous equation manipulation.

The interdependence coefficient is:

$$A^{-1} A = I$$

$$C = A^{-1}$$

$$X = CU$$

since

$$A^{-1}AX = A^{-1}U$$

$$c_{ij} = \frac{\text{cofactor } (-1)^{i+j}}{|A|}$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} 1 & -0.5 & -0.1 \\ -0.2 & 1 & 0 \\ -0.1 & -0.4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A| = 1 \begin{bmatrix} 10 \\ -0.41 \end{bmatrix} + .5 \begin{bmatrix} -0.2 & 0 \\ -0.1 & 1 \end{bmatrix} - 0.1 \begin{bmatrix} -0.2 & 1 \\ -0.1 & -0.4 \end{bmatrix} = .842$$

where

$$c_{11} = \frac{\begin{bmatrix} 10 \\ -0.41 \end{bmatrix}}{0.842} = 1.1337 \quad ; \quad c_{21} = \frac{\begin{bmatrix} -0.20 \\ -0.11 \end{bmatrix} (-1)}{0.842} = 0.2267.$$

$A^{-1} = |A| \text{adj } A.$

$$c = \begin{bmatrix} 1.1333 & 0.6122 & 0.1133 \\ 0.2267 & 1.2244 & 0.0226 \\ 0.1020 & 0.5102 & 1.0204 \end{bmatrix}$$

The interdependence coefficients are:

Producing Sector	Consuming Sector 1	2	3
1	1.1333	0.6122	0.1133
2	0.2267	1.2244	0.0226
3	0.1020	0.5102	1.0204

$$X_1 = 1.134U_1 + 0.61U_2 + 0.11U_3$$

$$X_2 = 0.23U_1 + 1.22U_2 + 0.023U_3$$

$$X_3 = 0.10U_1 + 0.51U_2 + 1.02U_3$$

which can be interpreted as:

$$X_1 = (1.134)(360) + 0.61(100) + 0.11(270) = 500$$

or interpretatively, if final utility demand U_1 increases by one utile, X_1 output must be increased by 1.134 utiles since the utility demands have measured the demands on X_1 by other industries in the input-output coefficient matrix.

If final utility demands for surplus, U_3, increases by one utile, X_1 utility output increases by 0.11 utile.

If a utile of final demand for sector 2 utility exists, then an output of 0.61 utile is required from sector 1 to be consumed by sector 2 to produce one utile of sector 2 output.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The study covered in the prior chapters was successful in several aspects. Possibly the most challenging development was the exploration of an integrating tool to allow the current concepts of organization theory and microeconomic theory to be applied in a particular manner to allow the decision-maker to acquire and distribute utility to insure the survival of the organization.

A Classification of the Decision Environment

The decision-maker is faced with three distinct decision environments given that he has an ability to weigh uncertain events. These are:

1. The decision environment to allocate utility resources to individual contributors so as to optimize the return from such expenditures.
2. The decision environment to allocate utility resources in accordance with constraints imposed by the environment of limited resources.
3. The decision environment to stabilize the supply and demand for utility.

Each of these three environments are interdependent on the assumption that objective measures of utility are possible. If they are, the structure of the organization becomes better identified. It becomes better identified as the decision-maker can be brought closer to the point of recognizing what various sectors of the organization contribute towards its survival.

The technique of input-output analysis demonstrates that a operational scheme is available to evaluate the contributory effect of each sector of an organization. The sector interdependency in generating a surplus of utility allows the organization to examine sectors; e.g., Plant A, the Accounting Department, Production Department, or the Research Laboratory, from the point of view in determining where scarce resource allocation may be most beneficial to the entire organization. While admittedly a theoretical tool, input-output analyis would aid in developing an objective measure of the utility contribution made to the organization by the various sectors.

The general weakness in any of the approaches derived is the lack of empirical application of these quantitative techniques. The advance of knowledge hopefully precedes its application. In a number of areas, isolated use has been made implicitly of many of these techniques. There appears to be no currently applied ordered body of knowledge to integrate these techniques into fruitful application. As mentioned earlier, the developments indicate certain methods by which managers may be able to analyze not only their objectives, but the route by which these objectives may be achieved. The tools of analysis seem at the stage of development similar to the early stages of development of non-stochastic inventory control. Much theoretical work remains to be done to present these techniques as workable tools of management. Some of the current obstacles have been cited. It remains now for refinements to take place in developing utility functions and the functional relationships within an organization to relate the supply and demand of utility.

As a further observation developed through the research effort devoted to this thesis, an inordinate amount of time was necessary to assure as complete knowledge of the application of utility theory to the decision-making environment as might be possible. As cited in the final chapter, the extensions of this work seem possible in several distinct areas. It has been the intention in this work to provide a consistent and documented basis from which further research may proceed. Accordingly, the literature search conducted in completing this thesis should be helpful in providing a concise reference source for those who wish to proceed in advancing research activity in the application of utility theory in the managerial decision-making process.

CHAPTER VIII

RECOMMENDATIONS FOR FURTHER INVESTIGATION

The recommendations for further investigation can be identified in a number of distinct areas, all of which require ultimate regard for the adaptability of these techniques by the individual who is responsible for allocating resources to achieve certain objectives.

Extension of the investigation is to determine what survival mechanism is utilized by the manager; i.e., what satisfies the short-run and long-run goals for survival. A number of elements can be considered, such as attrition rates, comparative financial and physical productivity ratio analysis, and comparative growth rates that have never been given adequate attention in assessing stability.

The most relevant weakness found in this study is the abstract relation that is given probability theory. Subjective probability distributions many times are viewed as unrelated to decision patterns of the manager. Research in this area is bringing theoretical probability theory to the level of interpretative application by a decision-maker. The results of informal empirical tests with students exposed to instruction in probability theory indicate even they have difficulty in deciding among probabilistic alternatives.

The development of utility production functions has been done on a limited basis. Investigations would prove fruitful in developing physical production functions for organizations. A bare minimum of work has been done in this area. Such studies are a forerunner of the development of a utility production function.

Since personal utility preference functions have been developed, it would be an outstanding contribution if the technique of factorial analysis could be applied to contributors of an organization. Ordinal surveys of preferences are widely circulated, yet no attack has been made on attempting the identification of interaction coefficients associated with contributory effort.

A feasible extension of input-output analysis would seem to be in the area of lagged interdependence coefficients to show change of utility during progressive time periods. The transitional states might well be examined through the Markov process. The value of such an investigation would be to identify and manipulate a dynamic input-output model in terms of resource (labor and capital) allocation to achieve resource investment criteria in the ultimate production of utility.

BIBLIOGRAPHY

1. Smith, Adam. _Wealth of Nations_. Edited by Edwin Cannon. New York: Random House, 1937, p. 14.

2. Barnard, Chester I. _The Functions of the Executive_. Cambridge, Massachusetts: Harvard University Press, 1938.

3. Papandreau, A. "Some Basic Problems in the Theory of the Firm," in _A Survey of Contemporary Economics_. Edited by B. F. Haley. Vol. 2. Homewood, Illinois: Richard D. Irwin, 1952, pp. 183-219.

4. Rothschild, K. W. "Price Theory and Oligopoly," _Economic Journal_, 42 (1947), pp. 297-320.

5. Baumol, W. J. _Business Behavior, Value and Growth_. New York: Macmillan, 1959, pp. 45-53.

6. Gordon, R. A. "Short-Period Price Determination," _American Economic Review_, 38 (1948), pp. 265-288.

7. Simon, H. A. "A Behavioral Model of Rational Choice," _Quarterly Journal of Economics_, 69 (1952), pp. 99-118.

8. Margolis, J. "The Analysis of the Firm: Rationalism, Conventionalism, and Behaviorism," _Journal of Business_, 31 (1958), pp. 187-199.

9. Charnes, A., and W. W. Cooper. "The Theory of Search: Optimum Distribution of Search Effort," _Management Science_, 5 (1958), pp. 450-458.

10. Marshall, A. _Principles of Economics_. 8th ed. London: Macmillan, 1936, pp. 278-290.

11. Sraffa, P. "The Laws of Return Under Competitive Conditions," _Economic Journal_, 36 (1926), pp. 535-550.

12. Merton, Robert K. et al. _Reader in Bureaucracy_. Glencoe, Illinois: Free Press, 1952.

13. Argyris, C. Understanding Organizational Behavior. Homewood, Illinois: Dorsey, 1960.

14. Blair, P. Dynamics of Bureaucracy. Chicago: University of Chicago Press, 1955.

15. Simon, Herbert A. "Comments on the Theory of Organizations," American Political Science Review, 46, No. 4, December, 1952, pp. 1130-39.

16. Cyert, Richard M, and James G. March. A Behavioral Theory of the Firm. Englewood Cliffs, New Jersey: Prentice-Hall, 1963.

17. Pareto, V. Manuale di economia politica, con una introduzione ulla scienza sociale. Milan, Italy: Societa Editrice Libraria, 1906.

18. Von Neumann, John, and Oskar Morgenstern. Theory of Games and Economic Behavior. 2nd ed. Princeton, New Jersey: Princeton University Press, 1947.

19. Bernoulli, D. "Specimen Theoriae Novae de Mensura Sortis," Comentarii Academiae Scientiarum Imperiales Petropolitane (1738), pp. 175-192. (Translated by L. Sommer in Econometrica, Vol. 22 (1954), pp. 23-36.

20. Davidson, Donald, Patrick Suppes, and Sidney Siegel. Decision Making: An Experimental Approach. Stanford, California: Stanford University Press, 1957.

21. Marschak, Jacob. "Probability in the Social Sciences," in Mathematical Thinking in the Social Sciences. Edited by Paul F. Lazarsfeld. Glencoe, Illinois: The Free Press, 1954.

22. Allais, Maurice. Fondements d'une théorie positive des choix comportant un risque et critique des postulats et axiomes de l'ecole américaine. Paris: Imprimeria Nationale, 1955.

✓23. Edwards, Ward. "The Theory of Decision Making," Psychological Bulletin, Vol. 51, No. 4 (July, 1954). (Note: This article contains an outstanding Bibliography on Utility Theory.)

✓24. Friedman, Milton, and L. J. Savage. "The Utility Analysis of Choice Involving Risk," Journal of Political Economy, Vol. 56, No. 4 (August, 1948), p. 279; "The Expected Utility Hypothesis and the

Measurability of Utility," Journal of Political Economy, Vol. 60 (1952), p. 463.

25. Mosteller, F., and P. Nagee. "An Experimental Measurement of Utility," Journal of Political Economy, Vol. 59 (1951), p. 371.

26. Grayson, Jackson C. Jr. Decisions Under Uncertainty. Boston: Harvard Business School, 1960.

27. Farrar, Donald Eugene. The Investment Decision Under Uncertainty. Englewood Cliffs: Prentice-Hall, 1962.

28. Kaufman, Gordon M. Statistical Decision and Related Techniques in Oil and Gas Exploration. Englewood Cliffs: Prentice-Hall, 1963.

29. Savage, Leonard J. The Foundations of Statistics. New York: John Wiley and Sons, Inc., 1954.

30. Thuesen, H. G., and W. J. Fabrycky. Engineering Economy. 3rd ed. Englewood Cliffs, New Jersey: 1964.

31. Bierman, Harold Jr., and Seymour Smidt. The Capital Budgeting Decision. New York: The Macmillan Company, 1960.

32. Leftwich, Richard H. The Price System and Resource Allocation. New York: Holt, Rinehart and Winston, 1961.

33. Allen, R. G. D. Mathematics for Economists. London: Macmillan, 1942.

34. Heady, Earl O. Economics of Agricultural Production and Resource Use. Englewood Cliffs: Prentice-Hall, 1961.

35. Goldberg, S. Introduction to Difference Equations. New York: John Wiley and Sons, Inc., 1958.

36. Leontief, Wassily. The Structure of American Economy. New York: Oxford Press, 1949.

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