SPATIALLY VARIED STEADY FLOW IN A VEGETATED CHANNEL

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PREFACE

Spatially varied flow, in which water enters a channel all along its length, is the natural mode of flow for many natural and constructed channels. Spatially varied flow is quite unlike uniform or nonuniform flow. However, until recent years, most channels conveying spatially varied flow have been designed by uniform flow methods. This thesis deals with spatially varied steady flow in a vegetated channel. While it is only a small contribution in relation to the amount of work that must be done to fully understand spatially varied flow in open channels, perhaps some of the findings can help determine where additional research is needed and suggest possible avenues of approach to the problem of spatially varied unsteady flow.

The experiments reported herein were conducted in a 410-foot long bermudagrass-lined test channel located at the Stillwater Outdoor Hydraulic Laboratory.

An outdoor experiment of this type presents many problems. The Oklahoma wind affected the inflow during spatially varied flow experiments, so it was necessary to conduct experiments immediately following the dawn or at dusk. The necessity for extreme accuracy and precision necessitated great care and unusual procedures in referencing

the data gathering equipment. However, the results obtained were gratifying and more than justified the additional effort.

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CHAPTER I

INTRODUCTION

The Problem

Graded terraces and diversion channels are important soil and water conservation measures. Graded terraces are used principally to reduce erosion, retard runoff, and increase intake. The main uses of diversion channels are to protect bottom lands from runoff, divert excess water from active gullies, and to prevent the concentration of water on a long, gentle slope too flat for standard terracing. Much time and money are spent annually on the construction of terraces and diversion channels. From 1936 through 1961, 1,413,000 miles of graded terraces were constructed in the United States (1, p. 559).

To obtain the maximum return for the time and money invested in terracing systems, the most economical but practical and adequate combinations of size, shape, and grade should be used.

The method of design should be based upon the best concepts of hydraulics in order to prevent unnecessary and costly overdesign. At present, most terraces and diversion channels are designed using methods developed for uniform flow and modified by field observation and experience. Observation and experience play an overly important role, for when time-variable flow enters a channel all along its length as in a terrace or diversion channel, the use of uniform flow equations is unrealistic. The added water disturbs the energy or momentum content of the flow, and the uniform flow methods do not account for the water stored in the channel at the time of the peak inflow. At the time of the peak inflow a graded terrace system might contain an inch or more of runoff in storage.

The type of flow in which discharge enters the channel all along its length is called spatially varied flow. A theory has been developed to describe both the steady and the unsteady state, and the steady-state phenomenon has been investigated for the small, short channels used in water and sewage treatment plants and for large lateral spillway channels for dams. However, little work has been done toward applying the theory to terraces and diversion channels where the inflow per unit length is small and the energy loss due to the impact of the entering flow is probably small. Investigation of the spatially varied flow phenomenon in vegetated channels is prerequisite to placing the design of agricultural conservation channels on a sounder theoretical basis.

Objectives

1. To predict water surface profiles for spatially varied steady flow with increasing discharge in a vegetated channel using existing mathematical theories.
2. To determine experimentally the water surface profiles for spatially varied steady flow with increasing discharge in a vegetated channel for various inflow values and roughness conditions.
3. To compare the results obtained from objectives one and two.
4. To modify, if necessary, the existing equation to more accurately predict the actual water surface profiles.

Scope of Investigation

The investigation was limited to spatially varied steady flow with increasing discharge. Only one channel was available for testing. The range of discharge as well as the initial channel cross section, length, and slope were determined by available resources and facilities. During the course of the testing the slope of the channel could not be altered because of the time required to re-establish vegetation.

Definition of Terms

The terms used in this paper correspond to those presented in "Nomenclature for Hydraulics," published by the American Society of Civil Engineers (41, pp. 19-497). Any terms not appearing in "Nomenclature for Hydraulics" are defined where they occur.

Definition of Symbols

Unless otherwise defined in the text, the following symbols are used throughout this paper. Insofar as possible, these symbols correspond to those presented in "Nomenclature for Hydraulics" (41, pp. 12-18). The original workers' definitions are followed in some cases.

Symbol	Quantity	Dimensions
a	acceleration	$ft./sec.^2$
A	area	$ft.^2$
b_w	width, water surface	ft.
B	coefficient	nonhomogeneous
c	speed of sound	ft./sec.
C	coefficient, Chezy	$ft.^{1/2}/sec.$
C	coefficient, discharge	$ft.^{1/2}/sec.$
C	coefficient, exponent	nonhomogeneous
D	diameter	ft.
f	force	lb.
f	resistance coefficient, Darcy - Weisbach	dimensionless

Symbol	Quantity	Dimensions
F	total force on a body	lb.
F	force (basic quantity)	lb.
F_g	force, gravitational	lb.
F_p	force, pressure	lb.
F_s	force, shearing	lb.
g	gravitational acceleration	ft./sec.^{2}
h	head	ft.
h_L	head, resistance	ft.
h_p	head, pressure or piezometric	ft.
h_v	head, velocity	ft.
H	head, total (Bernoulli)	ft.
k	roughness height	ft.
L	length (weir, pipe, stream tube)	ft.
m	mass (basic quantity)	lb. sec.^{2}/ft.
m	coefficient in Bazin formula	nonhomogeneous
n	coefficient in Manning formula	*
n	coefficient in Kutter formula	nonhomogeneous
n	coefficient, exponent	dimensionless
N_C	Cauchy number	dimensionless
N_F	Froude number	dimensionless
N_M	Mach number	dimensionless
N_R	Reynolds number	dimensionless
N_W	Weber number	dimensionless
p	pressure	lb./ft.^{2}

*See Chapter II, Review of Literature

Symbol	Quantity	Dimensions
P	perimeter	ft.
q	discharge per foot of length	cfs./ft.
Q	discharge	cfs.
R	hydraulic radius (A/P)	ft.
s	distance along stream tube	ft.
S	slope (sine of inclination angle)	dimensionless
S_o	slope, bed	dimensionless
S_s	slope, shear	dimensionless
t	time	sec.
T	time (basic quantity)	sec.
v	local velocity	ft./sec.
V	mean velocity	ft./sec.
x	arbitrary direction	ft.
x	coordinate in direction of flow	ft.
y	depth of flow	ft.
y_c	depth to centroid, critical depth	ft.
y_m	depth, mean	ft.
z	vertical distance from a datum	ft.
α	angle	dimensionless
α	Coriolis coefficient	dimensionless
β	Boussinesq coefficient	dimensionless
γ	specific weight	lb./ft.^{3}
θ	inclination angle of channel bottom	dimensionless
μ	viscosity, dynamic	lb. sec./ft.^{2}
ν	viscosity, kinematic (μ/ρ)	ft.^{2}/sec.

Symbol	Quantity	Dimensions
ρ	density	lb. sec.^{2}/ft.^{4}
σ	surface tension	lb./ft.

CHAPTER II

REVIEW OF LITERATURE

Introduction

This chapter, Review of Literature, contains not only a review of previous thought and research, but also a development of some of the concepts presented. This development is thought necessary because preceding researchers were not in complete agreement on some of the concepts of fluid flow, and any conclusions reached in this section on these subjects must be supported by analysis or data.

The material in this chapter consists of a brief summary of some of the basic concepts of fluid mechanics necessary for considering gradually varied flow and spatially varied steady flow; a detailed review, analysis, and discussion of gradually varied flow, velocity distribution, and resistance; a review of spatially varied steady flow equations and methods for their solution; and a brief review of analysis and research on rectangular weirs.

One-Dimensional Flow

According to Sears and Zemansky (53, p. 237), when proper conditions are fulfilled the flow of a fluid is of a relatively simple type called laminar or streamline. If the flow is of a laminar or streamline type, every particle passing a point follows exactly the same path as the pre-ceeding particles which passed the same point. These paths are called lines of flow or streamlines. Flow will be of the streamline type provided the velocity is not too great and the obstructions, constrictions or bends in the conduit are not such as to cause the lines of flow to change their direction too abruptly. If these conditions are not fulfilled, the flow is of a much more complicated type called turbulent.

Rouse (48, pp. 35-36) defined a streamline as

> an imaginary curve connecting a series of particles in a moving fluid in such a manner that at a given instant the velocity vector of every particle on that line is tangent to it.

He defined a stream filament as

> a small filament or tube of fluid, bounded by streamlines and yet of inappreciable cross-sectional area. . . . This stream filament might be considered, in either steady or uniform flow, as the passage through space of a fluid particle, and as such is the basis of the one-dimensional treatment of certain flow problems. Indeed, elementary hydraulics is based largely upon this conception, a single filament being assumed to have the cross-sectional area of the entire flow.

Rouse (48, p. 37) stated that

> in certain types of fluid motion the stream filaments are arranged in a very orderly fashion, and may be made visible experimentally by the introduction of colored fluid at some point in the flow. More generally, however, there occurs a complex interlacing of the actual streamlines; the various particles not only follow completely different and intricate courses but suffer continuous distortion and subdivision, so that no particle exists as an individual for more than a short interval of time. In such cases it is often practicable to represent by streamlines or filaments the temporal average of conditions throughout the movement. Such representation does not ignore the actual complexity of the motion, but serves only as a convenient aid in visualizing the underlying pattern of the flow.

Bernoulli's Equation

Sears and Zemansky (53, p. 238) derived the Bernoulli equation for an incompressible, nonviscous fluid flowing with streamline flow. They considered a fluid-filled portion of a pipe consisting of two lengths of different diameters joined by a transition section. They then considered a cross section in each of the uniform diameter lengths and displaced the fluid some small distance. The net work done on the system was equated to the sum of the increases in the kinetic energy and gravitational potential energy of the system. The final equation is

$$\frac{V^2}{2g} + \frac{P}{\rho g} + y = \text{Constant} \qquad (1)$$

This is Bernoulli's equation applicable to streamline flow without resistance.

Rouse (48, pp. 42-49) used a more sophisticated approach in his derivation of Bernoulli's equation. Rouse's derivation follows:

> Consider a fluid with zero viscosity, surface tension, and compressibility. Weight and pressure will then be the only forces under consideration. Then the forces exerted in an axial direction upon an elementary cylinder of fluid as shown in Figure 1 will be the pressure at either end and the component of fluid weight acting parallel to the axis. The rate of pressure variation in any direction is the pressure gradient. The difference in pressure intensity on the two ends of the fluid cylinder is given by the pressure gradient in the axial direction times the distance between the two ends. The total force acting upon the fluid volume will be
>
> $$dF_x = p\ dA - (p + \frac{\partial p}{\partial x} dx)\ dA + \gamma dx \cos \alpha\ dA$$
>
> Introducing the rate of change of elevation, h, in the x direction ($\cos \alpha = - \partial h/\partial x$) this becomes
>
> $$dF_x = - \frac{\partial p}{\partial x} dxdA - \gamma \frac{\partial h}{\partial x} dxdA$$
>
> In words, the force per unit volume, f, acting in any direction is equal to the rate of decrease of the sum $(p + \gamma h)$ in that direction.
>
> $$\frac{dF_x}{dxdA} = f_x = - \frac{\partial}{\partial x} (p + \gamma h)$$
>
> This force per unit volume divided by the density of the fluid will equal the force per unit mass, or, in accordance with the Newtonian equation, the rate of acceleration of the fluid in the given direction
>
> $$a_x = \frac{f_x}{\rho}$$

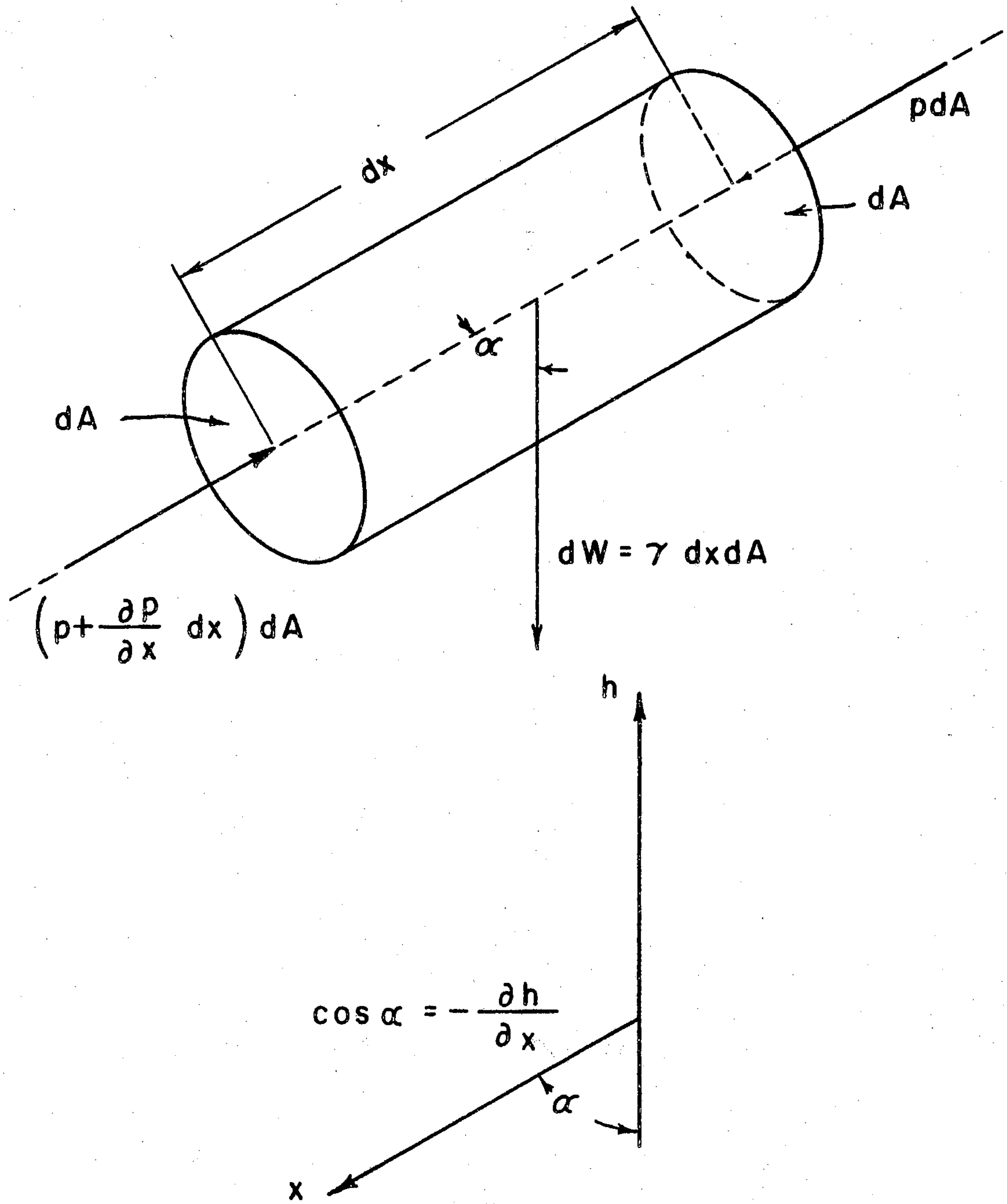

Figure 1. Elementary Forces Due to Pressure Gradient and Weight

$$a_x = \frac{dv_x}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial x}(p + \gamma h) \tag{2}$$

If any component of the substantial acceleration is zero, there can be no variation in the sum $(p + \gamma h)$ in that direction. In other words, the distribution of pressure intensity must be hydrostatic in any direction in which no acceleration takes place.

For the acceleration component along a streamline

$$a_s = \frac{dv_s}{dt} = \frac{\partial v_s}{\partial t} + \frac{\partial v_s}{\partial s}\frac{\partial s}{\partial t} = \frac{\partial v_s}{\partial t} + v\,\frac{\partial v_s}{\partial s}$$

$$= \frac{\partial v_s}{\partial t} + \frac{\partial (v^2/2)}{\partial s}$$

This equation may be combined with equation (2) to give

$$\frac{\partial v_s}{\partial t} + \frac{\partial (v^2/2)}{\partial s} = -\frac{1}{\rho}\frac{\partial}{\partial s}(p + \gamma h)$$

This may be rewritten

$$\rho\frac{\partial v_s}{\partial t} + \frac{\partial}{\partial s}\left(\rho\frac{v^2}{2} + p + \gamma h\right) = 0$$

The three terms within the parentheses may be set equal to the energy per unit volume, E_v. Then for steady flow along a streamline

$$\int_s dE_v = \rho\frac{v^2}{2} + p + \gamma h = f(t)$$

This equation states that while the velocity must not change with time the pressure intensity of the flow may vary with time, and this variation will exactly equal the change in E_v with time and will extend uniformly over the entire length of the streamline. It will have no effect whatever upon the velocity at any point. If E_v is not a function of time, along any streamline

$$E_v = \rho\frac{v^2}{2} + p + \gamma h = \text{Constant} \tag{3}$$

Each term of equation (3) has the dimension of energy per unit volume, the equation embodying a complete statement of the energy principle, or the essential balance between kinetic energy and potential energy over every part of a streamline in steady flow. Equation (3) is commonly known as the Bernoulli equation.

If each term in equation (3) is divided by the specific weight of the fluid, the result will be

$$E_w = \frac{v^2}{2g} + \frac{p}{\gamma} + h \tag{4}$$

Each term has the dimension of energy per unit weight of fluid. Since this is equivalent to length, the several terms are characterized as heads, and are called, respectively, the total head, the velocity head, the pressure head, and the geodetic or elevation head. Since the pressure head and elevation head represent potential energy as distinguished from the kinetic energy embodied in the velocity head, the sum $(p/\gamma + h)$ is known as the potential head. It follows that the sum of velocity and potential heads will not vary with distance along any streamline in steady flow. However, no restriction is placed upon variation from one streamline to another.

Dimensionless Groupings

Dimensional analysis is a powerful analytical tool that is very useful in model analysis and design. Through the years, some of the dimensionless groupings applicable to fluid flow have been evaluated and tabulated for a great number of experiments. Much can be learned about a particular flow condition by considering the numerical value of these dimensionless groupings in the light of previously recorded experiments. A short general discussion of the factors influencing fluid flow is presented by Murphy (38, pp. 164-170).

According to Murphy, a particular condition of flow will be influenced by the dimensions of the system, the properties of the fluid, and the applied forces aiding or retarding the flow. These factors may be indicated as

p	pressure	FL^{-2}
V	velocity	LT^{-1}
L	control distance	L
λ	outline dimensions	L
η	cross section dimensions	L
ρ	density	ML^{-3}
μ	viscosity	$ML^{-1}T^{-1}$
σ	surface tension	FL^{-1}
e	bulk modulus	FL^{-2}
g	acceleration of gravity	LT^{-2}

According to the Buckingham Pi Theorem, seven dimensionless terms are required to express a relationship among these variables. The following combination is usually chosen:

$$\frac{p}{\rho V^2} = f\ (\frac{\lambda}{L}, \frac{\eta}{L}, \frac{\rho VL}{\mu}, \frac{V^2}{gL}, \frac{\rho V^2 L}{\sigma}, \frac{\rho V^2}{e})$$

$$= f\ (\frac{\lambda}{L}, \frac{\eta}{L}, N_R, N_F, N_W, N_C)$$

The first two terms pertain to geometrical characteristics; the four following terms are the Reynolds number, the Froude number, the Weber number, and the Cauchy number, respectively.

The Reynolds number expresses the ratio of the inertial forces of an element of fluid to the viscous forces. It is of great value in pipe flow problems. It is useful in all flow problems in determining if a particular flow condition is in the laminar or turbulent mode. For pipe flow the Reynolds number is defined as

$$N_R = \frac{VD}{\nu}$$

It is usually defined for open channel flow as

$$N_R = \frac{VR}{\nu}$$

However, the open channel Reynolds number and that for pipe flow cannot be compared directly, because, for a pipe, D = 4R. For consistency the Reynolds number for pipe flow will be used in this paper for both pipe and open channel flow. Thus,

$$N_R = \frac{4VR}{\nu}$$

The value of the critical Reynolds number when flow changes from laminar to turbulent is approximately 2,000 by this definition (49, p. 129).

The Froude number is an expression of the ratio of the inertial forces to the gravitational force developed on an element of fluid. It is the most important criterion when designing models of prototypes in which gravitational forces cause fluid motion. The Froude number is usually defined as

$$N_F = \frac{V}{(gy_m)^{1/2}}$$

The Weber number expresses the ratio of surface-tension forces to inertial forces. It can be of major importance in small models in which free-surface flow occurs. The Weber number is usually defined as

$$N_W = \frac{V}{(\sigma/\rho L)}$$

The length term could be the depth of flow or the hydraulic depth, or some other length.

The Cauchy number is dimensionally equivalent to the ratio of the inertial force to the compressibility force. It is the criterion used when describing the motion of objects moving at a high speed in a fluid. The Cauchy number is defined as

$$N_C = \frac{\rho V^2}{e}$$

Another dimensionless group closely related to the Cauchy number is the Mach number. The Mach number is defined as the ratio of the velocity to the speed of sound.

$$N_M = \frac{V}{c}$$

It can be shown that the Mach number is the square root of the Cauchy number

$$N_M = V \left(\frac{\rho}{e}\right)^{1/2}$$

Gradually Varied Flow Equations for Open Channels

Introduction

The flow of water in open channels is usually nonuniform in both depth and velocity distribution. Equations derived to describe gradually varied flow express a relationship between the depth of water in a channel, the variation of this depth with distance along the bed of the channel, the mean velocity in a section, the variation of this velocity with distance, the slope of the bed, a coefficient of resistance, and a coefficient to account for the nonuniform velocity distribution. Both the energy and momentum concepts have been used to derive gradually varied flow equations. However, there is controversy between hydraulicians concerning the use of the momentum concept and the meaning of the resistance involved. There is some controversy concerning the form of the velocity distribution coefficient for use with the energy concept, and also concerning the equations used to describe the resistance in the two methods.

In classical mechanics, momentum is defined as the product of the mass of a body and the magnitude of its instantaneous velocity. Momentum is a vector quantity having both magnitude and direction. The principle of momentum is applied by summing the external forces acting upon a fluid body and equating them to the change of momentum of the fluid body. In deriving varied flow equations, the forces on a length of stream tube are frequently considered first and then an integration is carried out over all of the stream tubes between two cross sections. Some hydraulicians use the momentum principle to derive an equation of the Bernoulli form to describe gradually varied flow. Others state that the momentum principle can be used to derive a Bernoulli-type equation only for special conditions, if at all. Hydraulicians have different concepts of the resistance involved in the momentum approach. Some feel that it describes only the boundary shear, others that it describes all of the energy losses.

The energy approach is frequently applied by equating the rate of change of energy to the rate at which work is done upon an elementary free body of fluid in a stream tube as it passes between two cross sections. The resulting relationship is integrated along the stream tube and then over all of the stream tubes between the two cross sections. All of the equations derived by the energy principle are of the Bernoulli form. However, some variation is found in the velocity distribution coefficients used by different investigators.

The velocity distribution coefficients used in the gradually varied flow equations are of either mean-square or mean-cube form. The mean-square coefficient is defined as

$$\beta = \frac{1}{AV^2} \int_A v^2 dA \tag{5}$$

The mean-cube coefficient is defined as

$$\alpha = \frac{1}{AV^3} \int_A v^3 dA \tag{6}$$

Momentum Concept

Bahkmeteff (2, pp. 232-234) used the momentum concept in deriving an equation to describe phenomena such as the hydraulic jump where internal energy losses are quite high. The equation relates the change in momentum content of the entire fluid movement across two cross sections a short distance apart, each in reaches of uniform flow in a horizontal channel, to the difference in pressure force on the cross sections and the shearing force on the boundary of the channel between the two cross sections. The equation is not in Bernoulli form and no velocity distribution coefficient is included.

Keulegan (28, pp. 97-111) used the momentum concept in deriving a differential equation for gradually varied flow. The equation contains the mean-square velocity distribution coefficient and a friction coefficient that he stated is directly related to the wall friction.

Eisenlohr (15, pp. 633-644) derived a momentum equation by considering a channel to be divided into stream tubes and examining two cross sections in the channel a finite distance apart to see what happens when the fluid is allowed to displace and flow across these sections for unit time. The stream tubes are assumed to remain of constant area over the finite length. Forces are summed for a single stream tube between the two sections and then over all the stream tubes. The resulting equation has the form of a Bernoulli equation with a term for shearing stress and with the mean-square coefficient applied to the velocity-head term.

Eisenlohr was taken to task for his original paper by several hydraulicians. Kalinske (26, pp. 645-646) pointed out weaknesses in Eisenlohr's derivation. Kalinske thought that Eisenlohr should not have considered a finite length of stream tube, but should have integrated along the tube. Also, Kalinske disagreed with Eisenlohr's assumption that variation of the stream tube area was negligible, since he did not at the same time assume variation in the stream tube velocity to be negligible. Without this assumption concerning stream tube area, Eisenlohr's equation will not assume the form of a Bernoulli-type equation. Kalinske stated that the momentum concept deals only with the external forces on the fluid body.

Taylor (55, pp. 646-648) pointed out some of the flaws in Eisenlohr's basic assumptions. He presented no alternative derivation.

Van Driest (57, pp. 648-651) also touched upon errors in Eisenlohr's basic assumptions. He stated that Eisenlohr's momentum equation would be approximately correct provided that the difference between the mean-square coefficients approached zero faster than the difference in the mean velocities in the two sections under consideration. He also stated that the loss term for the momentum equation would describe only the losses due to external forces.

Rouse and McNown (50, pp. 651-657) stated that Eisenlohr's approach was oversimplified. They presented an alternate derivation that was more complete. Rouse and McNown derived their momentum equation as follows:

> Write the basic vector relationship between force per unit volume and the rate of change of momentum for an infinitesimal body of fluid and integrate over the entire volume. The basic vector relationship for an infinitesimal body of fluid is
>
> $$f = \frac{d(\rho v)}{dt} = \rho\frac{dv}{dt} = \rho a$$
>
> If this equation is written for the components in the three Cartesian coordinate directions, the integral expression for any direction, x, will be
>
> $$F_x = \sum(f_x) = \rho\int_{vol} a_x \, d(vol)$$
>
> The term on the left includes the x component of all forces acting upon every particle in the volume at a given instant. However, since every force

upon a particle within the volume requires the existence of an equal and opposite force upon the neighboring particles, all such internal forces will counterbalance each other so that only the external forces need be considered.

The term on the right of the equation may be made more explicit by considering the fluid volume to be composed of a great number of fluid filaments representing the temporal average of conditions throughout the flow. The surface of this volume will then consist of the walls of the outermost filaments and the sum of all the cross-sectional areas at either end of each one.

Consider the incremental volume of a stream tube or filament as shown in Figure 2. The external forces acting upon this elementary free body are the attraction of the earth and the pressure and shear exerted by the surrounding fluid, which may be resolved in any direction, x, and equated to the product of the mass of the element and the corresponding component of its acceleration.

$$\sum (dF)_x = dm \, a_x$$

The component of acceleration, a_x, of the incremental volume may be expressed in terms of a differential, ds

$$a_x = \frac{dv_x}{dt} = \frac{\partial v_x}{\partial s}\frac{ds}{dt} = \frac{v \partial v_x}{\partial s}$$

Furthermore, dm may be written as $\rho ds \, dA$, so that

$$\sum (dF)_x = \rho v \frac{\partial v_x}{\partial s} ds \, dA \tag{7}$$

Equation (7) expresses the equality between the impulse per unit time and the accompanying rate at which the momentum of the fluid element is changed.

Before it is possible to integrate over the volume, which means a double integration along the stream tube and then over the end areas, it is necessary to express dA in terms of a variable which is independent of s. This can be done by

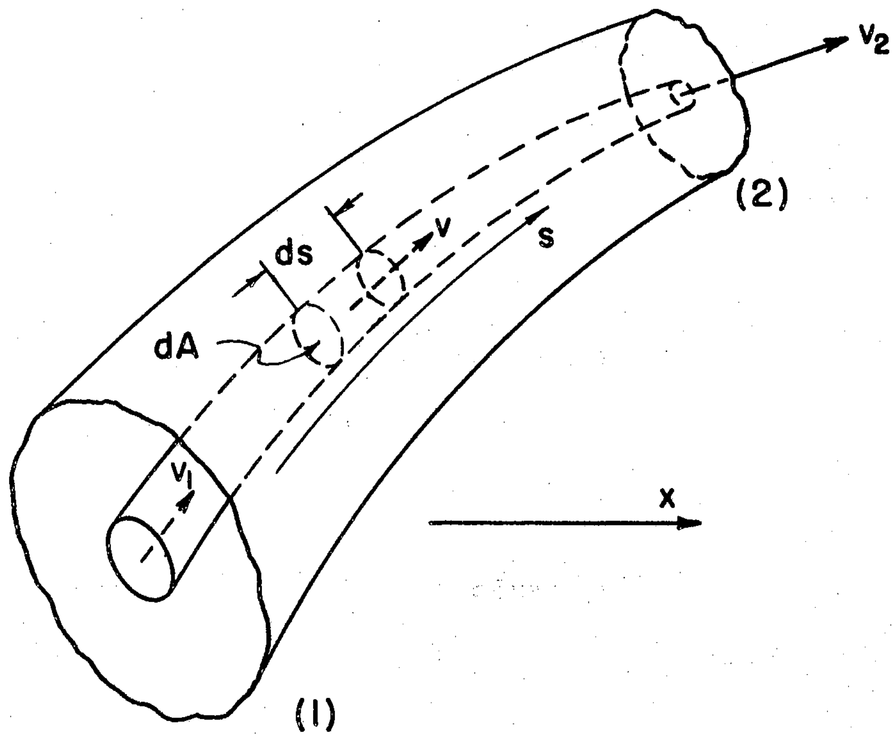

Figure 2. Incremental Volume of Stream Tube

using the relationship vdA = dQ, the rate of flow through the stream tube, which is necessarily the same at all cross sections. Thus,

$$\sum (dF)_x = \rho \frac{\partial v_x}{\partial s} \, ds \, dQ \tag{8}$$

and for a finite length of stream tube, $(s_2 - s_1)$

$$\int_{s_1}^{s_2} \sum (dF)_x = \rho (v_{x_2} - v_{x_1}) \, dQ$$

This may then be integrated across the volume to yield

$$\int_{vol} \sum (dF)_x = \int_Q \rho (v_{x_2} - v_{x_1}) \, dQ = \int_{A_2} \rho v_{x_2} \, v_2 \, dA - \int_{A_1} \rho v_{x_1} \, v_1 \, dA \tag{9}$$

Internal shears and pressures cancel in the process of summation. The left side represents the x component of the resultant of all external forces which can be evaluated only through measurement or arbitrary assumption as to type of variation. The right side, which represents the difference in flux of the x component of momentum past the two end sections, requires equally explicit knowledge as to the corresponding velocity distribution. Only if the flow at the two end sections is in essentially the same direction can the equation readily be applied to conditions in which the velocity varies across the flow, under which conditions it reduces to the alternative forms

$$\sum F_x = \rho Q \, (\beta_2 V_2 - \beta_1 V_1)$$

or

$$\sum F_x = \rho \beta_2 V_2^2 A_2 - \rho \beta_1 V_1^2 A_1 \tag{10}$$

A nearly identical derivation was presented at an earlier date by Rouse (48, pp. 52-54). According to Rouse (48, p. 54),

> In the general case of curvilinear flow, in which average values of neither velocity nor pressure intensity may be used, equation (9) must be followed strictly, actual curves of velocity and pressure distribution forming the basis for integration, and the actual volume of the fluid being used to determine the component of the fluid weight in the given direction. Yet such methods will require experimental measurement of velocity and pressure distribution at one section or the other, for the general principles of momentum, energy, and continuity have as yet provided no means of determining these characteristics by rational analysis.

In his closing discussion Eisenlohr (16, pp. 657-668) altered or clarified his original assumptions and methods of derivation. He also stated that the momentum equation in the form of a Bernoulli-type equation was only an approximation, due to the restriction of the use of the average area of the stream tube in the derivation.

Daugherty and Ingersoll (13, pp. 76-77) derived a momentum equation by considering an infinitesimal length of fluid in a horizontal pipe of uniform diameter and equating the shear and pressure forces to the momentum change of the free body. According to Daugherty and Ingersoll, if the fluid is incompressible, the resulting equation can be written in Bernoulli form even if the pipe is not of uniform diameter. The shearing force term seems to represent the entire head loss of the body of fluid. In a later section of their text (13, pp. 333-334), Daugherty and Ingersoll applied a mean-square velocity distribution coefficient to their original equation.

Chow (9, pp. 49-52) derived a momentum equation for gradually varied flow in open channels by considering two cross sections a finite distance apart and equating the external friction force, the resultant pressure force, and the weight component in the direction of flow to the change in momentum across the enclosed body of water per unit time. The initial equation is not in the Bernoulli form and includes the mean-square velocity distribution coefficient, a term for the weight component, a term for the difference in pressure on the two ends, and a term for external friction and resistance. Chow then assumed a rectangular channel and used the average depth in rearranging this equation into a Bernoulli-type equation with mean-square velocity distribution coefficients and a term for external losses only. The equation is quite similar to Eisenlohr's original momentum equation that he later stated was really only an approximation.

Energy Concept

Keulegan (28, pp. 97-111) used the energy concept in deriving a differential equation to describe gradually varied flow. The equation contains the mean-cube velocity distribution coefficient and a term for energy loss.

Eisenlohr (15, pp. 633-644) derived an energy equation by considering a channel to be divided into stream tubes and examining two cross sections in the channel a finite distance apart to see what happens when the fluid is allowed to displace and flow by these sections for unit time. The work done on the fluid in each stream tube is then summed for the finite length of tube and then over all the tubes. The resulting equation has the form of a Bernoulli equation with a term for energy loss and with a mean-cube velocity distribution coefficient.

Kalinske (26, pp. 645-646) made much the same comments about Eisenlohr's energy equation as he did about the momentum equation. Kalinske further stated that the energy principle as used by Keulegan and Eisenlohr probably should be called the power principle, since the terms calculated are really the flow of energy per unit time and the work done per unit time. Furthermore, Kalinske stated that the important thing to recognize in the use of this principle is that all external and internal energy losses and work done must be taken into account; in using the momentum principle, only external forces need be considered.

Van Driest (57, pp. 648-651) stated that not only could an energy equation containing mean-cube coefficients be obtained, but that an energy equation containing mean-square coefficients could also be obtained. According to Van Driest, this equation containing mean-square coefficients is easily obtained by considering the work done on an element of fluid in a stream tube as it moves between two sections. The work done is integrated across the cross sections. He stated that the work done and the energy changed is per pound of fluid which traverses a distance such that each element of it has a common displacement and occupies the cylindrical region that one pound would occupy at any section of the channel.

Rouse and McNown (50, pp. 651-657) presented an alternate to Eisenlohr's derivation. Their derivation of the energy equation follows:

> Equate the rate of change of energy to the rate at which work is done upon an elementary free body such as is shown in Figure 2 by the external forces of pressure, shear, and fluid weight. The component of these forces must be written in the direction of displacement, s. The energy of a particle will change as the result of both acceleration and dissipation. A term for energy change due to acceleration may be written in terms of an increase in kinetic energy, but a term for energy change due to dissipation can be written only in terms of the decrease in the total head of the element. Thus,
>
> $$\sum(dF)_s\ v = \rho ds\ dA \frac{d(v^2/2)}{dt} - \gamma ds\ dA \frac{dH}{dt}$$
>
> Rearranging the terms on the right side,
>
> $$\sum(dF)_s\ v = \rho ds\ dA \frac{\partial(v^2/2)}{\partial s} \frac{ds}{dt} - \gamma ds\ dA \frac{\partial H}{\partial s} \frac{ds}{dt}$$
>
> $$= \rho \frac{\partial(v^2/2)}{\partial s} ds\ dQ - \gamma \frac{\partial H}{\partial s} ds\ dQ$$
>
> Since dQ, unlike dA, is a constant along the stream tube, this equation may be integrated at once with respect to s:

$$\int_{s_1}^{s_2} \sum (dF)_s v = \frac{\rho}{2} [(v_2)^2 - (v_1)^2] \, dQ - \gamma(H_2 - H_1) \, dQ$$

This equation may be integrated over all of the stream tubes

$$\int_{A_1} (\frac{p_1}{\gamma} + z_1) \, v_1 \, dA - \int_{A_2} (\frac{p_2}{\gamma} + z_2) \, v_2 \, dA$$

$$= \int_{A_2} [(\frac{v_2}{2g})^2 - H_2] \, v_2 \, dA - \int_{A_1} [(\frac{v_1}{2g})^2 - H_1] \, v_1 \, dA$$

The foregoing general energy equation may be applied to a given state of flow only if the distribution of velocity and pressure is known at both end sections. Unlike the momentum equation, this energy equation involves only the magnitudes of the velocities. However, the velocity distribution here affects both sides of the equation, with the result that the energy principle may usually be applied only if both sections under consideration are located in essentially uniform zones. Then the pressure distribution is hydrostatic and the general energy equation reduces to the form of the Bernoulli equation

$$\alpha_1 (\frac{V_1}{2g})^2 + \frac{p_1}{\gamma} + z_1 = \alpha_2 (\frac{V_2}{2g})^2 + \frac{p_2}{\gamma} + z_2 + h_L \qquad (11)$$

in which

$$h_L = \frac{1}{A_1} \int_{A_1} H_1 \frac{v_1}{V_1} \, dA - \frac{1}{A_2} \int_{A_2} H_2 \frac{v_2}{V_2} \, dA$$

Eisenlohr (16, pp. 657-668) also attempted to derive an energy equation containing a mean-square coefficient. He considered two cross sections a finite distance apart and let the fluid in a stream tube displace a small distance.

He wrote an expression for the energy change of the fluid in the stream tube between the two cross sections and integrated along the tube. He then considered the fluid in every stream tube to have moved the same distance and integrated over all of the stream tubes. The resulting equation is of Bernoulli form with mean-square coefficients.

Eisenlohr then proceeded to develop a power equation which contains the mean-cube coefficient for velocity distribution. He did this by considering two cross sections a finite distance apart and letting the fluid in a stream tube flow by these sections for unit time. He wrote an equation for the power change between the end sections. The resulting equation was then integrated over all the stream tubes and divided by the weight of water flowing per unit time. The resulting equation of Bernoulli form contains the mean-cube coefficients and has the units of foot-pound per second per pound per second. The term for loss in a reach represents the average energy lost per second by each pound of water passing through the reach per second.

The energy concept was also discussed by Bakhmeteff (2, pp. 26-31), Rouse (48, pp. 47-52), Rouse (47, pp. 57-59), Rouse and Howe (49, pp. 69-72), Daugherty and Ingersoll (13, pp. 68-73, pp. 252-254), and Chow (9, pp. 39-40).

Discussion

There seems to be little agreement on the application of the momentum concept to gradually varied flow. Bakhmeteff (2, pp. 232-234), Rouse (48, pp. 52-54), Rouse and McNown (50, pp. 651-657), Rouse (47, pp. 55-57), and Ippen (47, pp. 506-507) did not derive the Bernoulli-type momentum equation. Eisenlohr (16, pp. 657-658) and Van Driest (57, pp. 648-651) wrote it with reservations. Keulegan (28, pp. 97-111), Daugherty and Ingersoll (13, pp. 76-77), and Chow (9, pp. 49-52) wrote the momentum equation in Bernoulli form although Chow's equation was derived for a rectangular channel.

A Bernoulli-type momentum equation may be obtained from equation (10) if appropriate assumptions are made. Consider equation (10) as applied to a finite length of the fluid in the condition of gradually varied flow in an open channel as shown in Figure 3.

$$\sum F_x = \rho \beta_2 V_2^2 A_2 - \rho \beta_1 V_1^2 A_1 \tag{10}$$

This can be written as

$$\sum F_x = \rho Q \, (\beta_2 V_2 - \beta_1 V_1)$$

The term on the left of the above equation can be considered as the sum of all of the external forces of pressure, gravitational acceleration, and bed shear acting in the x direction upon the body of water between sections (1) and (2).

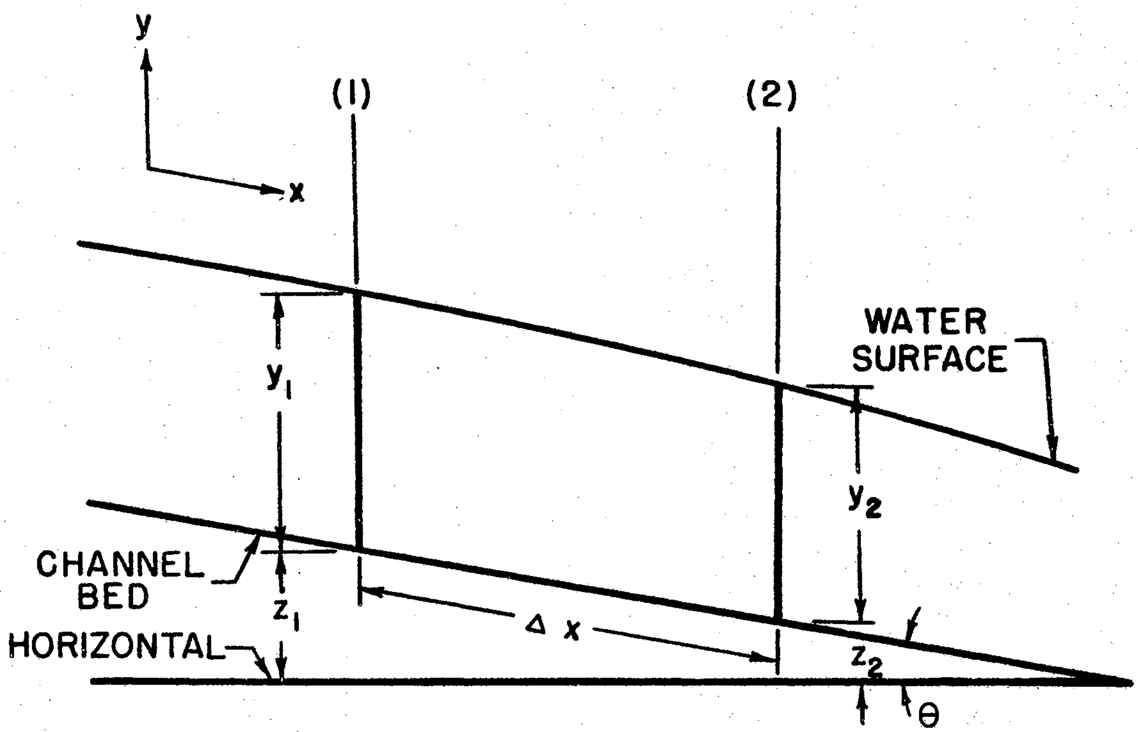

Figure 3. Gradually Varied Flow Diagram

$$\sum F_x = F_{p_x} + F_{g_x} - F_{s_x}$$

If hydrostatic pressure distribution can be assumed, then from Stoker (54, pp. 454-455), the resultant pressure force on the body can be written as

$$F_{p_x} = \gamma A_{avg} \; \Delta x \frac{dy}{dx}$$

The force caused by gravitational acceleration can be written as

$$F_{g_x} = \gamma A_{avg} \; \Delta x \tan \theta$$

For small inclination angles, $\tan \theta \approx \sin \theta = (z_1 - z_2)/\Delta x$

$$F_{g_x} = \gamma A_{avg} \; (z_1 - z_2)$$

If it is assumed that the variation from section (1) to section (2) is approximately linear in depth, area, and velocity, then

$$F_{p_x} = \gamma A_{avg} \; (y_1 - y_2)$$

$$\sum F_x = \rho A_{avg} \left(\frac{V_1 + V_2}{2}\right) (\beta_2 V_2 - \beta_1 V_1)$$

$$= \frac{\rho A_{avg}}{2} (\beta_2 V_2^2 - \beta_1 V_1^2 + \beta_2 V_1 V_2 - \beta_1 V_1 V_2)$$

If it is assumed that the difference between the velocity heads at sections (1) and (2) is proportionately larger than the difference between momentum coefficients at sections (1) and (2), (the assumption of Van Driest (57, p. 649)), then

$$\sum F_x = \frac{\rho A_{avg}}{2} (\beta_2 V_2^2 - \beta_1 V_1^2)$$

Then by grouping all terms,

$$\gamma A_{avg} (y_1 - y_2) + \gamma A_{avg} (z_1 - z_2) - F_{s_x}$$

$$= \frac{\rho A_{avg}}{2} (\beta_2 V_2^2 - \beta_1 V_1^2)$$

Rearranging and simplifying, the following Bernoulli-type momentum equation can be obtained:

$$\beta_1 \frac{V_1^2}{2g} + y_1 + z_1 = \beta_2 \frac{V_2^2}{2g} + y_2 + z_2 + \frac{F_{s_x}}{\gamma A_{avg}} \qquad (12)$$

The conditions assumed in the derivation should be kept in mind when using equation (12). As has been stated previously, there is not complete agreement concerning the validity of this equation.

Most of the hydraulicians cited agreed that the resistance in the momentum equation is only that of the boundary layer. However, Daugherty and Ingersoll (13, p. 77) implied that the loss terms in the momentum and energy equations could be interchanged.

There was general agreement on the form of velocity distribution coefficient to use with momentum equations. In all of the derivations in which a nonuniform velocity distribution was considered, the mean-square velocity distribution coefficient was used.

There seems to be more general agreement on the form and meaning of the energy equation than on the momentum equation. In all of the derivations cited, the final equation was developed into the form of the Bernoulli equation, or a derivative of this equation. The resistance term always represents all the losses in the channel.

Most of the investigators developed energy equations containing mean-cube velocity distribution coefficients. At least two of those cited, Van Driest (57, pp. 648-651) and Eisenlohr (16, pp. 657-668), developed energy equations containing mean-square velocity distribution coefficients. Each considered an element of a stream tube and wrote an equation for the work done on this element of stream tube as it moved between two sections. Then each considered the fluid in every stream tube to have moved the same distance and integrated across the entire cross section. The result is an energy equation with mean-square velocity distribution coefficients. However, because of the nonuniform velocity distribution, the fluid in some tubes is moving faster than in others, making it erroneous to obtain an average energy per pound of fluid in this manner. Therefore, the energy equations containing mean-square coefficients are invalid.

It is more proper to consider the energy change of the quantity of fluid flowing across a section per unit time. This method leads to the equations with mean-cube coefficients.

It seems that these Bernoulli-type energy equations containing the mean-cube velocity coefficients should really be called power equations because in reality the dimensions are energy per unit time per unit weight across a section per unit time, rather than energy per unit weight. This idea was mentioned by Kalinske (26, p. 645), Eisenlohr (16, pp. 664-668), and Ippen (47, p. 507).

The means for evaluating boundary shear and energy loss are also a matter of controversy. Keulegan (28, p. 110) stated that the Manning equation describes the magnitude of frictional force in channels, and hence Manning's *n* should be computed using a momentum equation. Eisenlohr (15, p. 640) stated that both the Chezy and Manning equations were momentum equations, and hence should be used only in evaluating boundary shear. He further stated that these equations would not yield practical results when used to evaluate energy loss except for uniform velocity distribution. He offered no equation to evaluate the energy loss under conditions of nonuniform velocity distribution.

According to Rouse and McNown (50, pp. 656-657), however, the Chezy and Manning equations were derived solely for the case of uniform flow, under which conditions boundary shear and energy loss are directly proportional. The customary loss coefficients of nonuniform flow, on the other hand, are simply means of evaluating the head loss in terms of boundary geometry and have no source in the momentum principle. Apparently Chow (9, p. 332) was of the same opinion concerning the loss coefficients.

According to Rouse and McNown (50, p. 656),

> The common failure to distinguish between the evaluation of boundary shear and energy dissipation in gradually varied flow is actually tantamount to assuming that $\alpha = \beta = 1$, for under such conditions (but only then) the energy and momentum equations become identical.

Conclusions

Considering the present apparent lack of understanding of the momentum concept as applied to gradually varied flow, it would seem best to use the energy or power concept for this type of flow and reserve the momentum concept for such phenomena as the hydraulic jump where there occur very high internal energy losses unpredictable by the energy equation. However, when considering spatially varied flow where turbulence caused by the entering fluid is quite high and energy losses are unpredictable, it may be necessary to use the momentum approach. Then the appropriate shearing loss term would probably have to be obtained from uniform or gradually varied flow by the use of equation (12).

The mean-square velocity distribution coefficient should be used with the momentum equations and the mean-cube coefficients with the energy or power equation of Bernoulli form.

Conclusions concerning resistance equations to use with the momentum and power equations will be given at the end of the section on resistance.

Velocity Distribution

Most ordinary calculations involving the total momentum content at a channel cross section or the total power crossing the section are based on the assumption of a uniform velocity distribution. The mean-square and mean-cube coefficients are assumed to be nearly one or of only theoretical interest. However, the velocity in a conduit is never uniform. Even away from the boundary layer the velocity is not uniformly distributed. This is particularly true of natural channels where there are irregularities in cross section and where there may be large or small obstructions to flow.

According to Chow (9, p. 27), Coriolis (12) was the first to propose the mean-cube coefficient to apply to the velocity head as computed from the mean velocity. The Coriolis, mean-cube, or velocity-head coefficient, α, is defined as

$$\alpha = \frac{1}{AV^3} \int_A v^3 \, dA \tag{6}$$

Boussinesq (7) proposed the mean-square coefficient as defined by equation (5)

$$\beta = \frac{1}{AV^2} \int_A v^2 \, dA \tag{5}$$

to be applied to the computation of the momentum content at a channel cross section.

Examination of equations (6) and (5) shows that the momentum coefficient will be less than the velocity head coefficient, since the local velocities are cubed in equation (6) and only squared in equation (5).

O'Brien and Johnson (42, p. 215) presented a graphical method for obtaining energy and momentum coefficients from velocity measurements in a conduit. The velocity data are plotted on a cross section of the channel. Lines of equal velocity are drawn and the areas between the lines are obtained with a planimeter. Values of velocity, velocity squared, and velocity cubed are plotted versus mass area. The areas between the latter two curves and the axis and bounding lines are then the integrals of equations (5) and (6), respectively. These areas may be found by the use of a planimeter. The total discharge may be found by the mass plotting of velocity versus area. With the total cross-sectional area known, the momentum and velocity-head coefficients are easily obtained.

Chow (9, p. 29) presented the following approximate formulas for velocity-head and momentum coefficients where it is possible to assume a logarithmic velocity distribution:

$$\alpha = 1 + 3\,\varepsilon^2 - 2\,\varepsilon^3$$

$$\beta = 1 + \varepsilon^2$$

$$\left(\varepsilon = \frac{v_m}{V} - 1\right) \qquad \text{where } v_m = \text{maximum velocity}$$

According to Chow, Rehbock (45) assumed a linear variation of velocity with depth in obtaining the following equations:

$$\alpha = 1 + \varepsilon^2$$

$$\beta = 1 + \frac{\varepsilon^2}{3}$$

For a logarithmic velocity distribution in a pipe Streeter (47, p. 401) arrived at the following equations for velocity-head and momentum coefficients:

$$\alpha = 1 + 2.93f - 1.55f^{3/2}$$

$$\beta = 1 + 0.98f$$

O'Brien and Johnson analyzed data from several previous investigations and obtained the values for α and β presented in Table I (42, p. 215). They pointed out that the highest value of α they had obtained was that for the Rhine River (Item 7 in Table I), which yielded an α of 1.35. The corresponding value of β was 1.121. The highest value of α they obtained was 2.08 (Item 18 in Table I), obtained upstream from a weir.

King (31, p. 7.11), in discussing the data in Table I, stated that in fairly straight uniform channels α appears to vary from about 1.04 to 1.10. Upstream from weirs or in the vicinity of obstructions or pronounced irregularities in alignment, α may have any value from 1.10 to 2.0 or even more.

TABLE I

VALUES OF VELOCITY-HEAD AND MOMENTUM COEFFICIENTS
FROM O'BRIEN AND JOHNSON (42, p. 215)

Item	Width	Maximum Depth	Hydraulic Radius	Area	Critical Depth	Mean Velocity	Coefficient α	Coefficient β	Source	Remarks
1	1.97	2.83	.73	5.59	0.65	1.05	1.20	1.07	34)	Rectangular channel 3 ft. above
2	3.28	2.88	1.07	9.64	0.71	1.16	1.22	1.08	34)	weir with obstructions upstream
3	3.28	2.87	1.07	9.62	0.72	1.20	1.41	1.12	34)	
4	3.3	1.41	.76	4.65	1.63	8.41	1.07	1.03	34	Simplon Tunnel, center of straight reach 164 feet long
5	34.6	10.6	6.11	250.5	4.67	3.32	1.10	1.05	34	Trapezoidal channel
6	6.52	4.92	2.07	31.2	2.52	4.88	1.07	1.03	34	Horseshoe conduit, straight reach
7	523.	12.51	8.0	4365.	6.27	3.36	1.35	1.121	34	Rhine River, 1,200 feet below bridge, long curve
8	8.5	4.54	2.28	36.92	2.25	2.91	1.06	1.01	19)	
9	8.75	4.01	2.13	32.4	2.16	2.87	1.04	1.014	19)	
10	9.	3.0	1.8	23.59	1.97	2.60	1.04	1.014	19)	Sudbury Aqueduct, bottom slope
11	8.9	2.03	1.35	15.24	1.74	2.16	1.04	1.01	19)	= 0.000189
12	8.75	1.51	1.07	10.92	1.64	1.87	1.04	1.012	19)	
13		0.87		.58	1.17	7.59	1.16		5	Bazin weir experiments
14		0.8		.40	0.47	0.68	1.14		40	Nikuradse experiments
15	4.22	2.5					1.07		36,52)	
16	4.22	5.0					1.08		36,52)	Computations by Lindquist of
17	4.22	5.0					1.6		36,52)	Schoder and Turner weir
18	4.22	5.0					2.08		36,52)	experiments.
19	4.22	10.1					1.8		36,52)	
20	4.22	9.0					2.0		36,52)	

According to Chow, in channels of complex cross section, the coefficients for velocity head and momentum can easily be as great as 1.6 and 1.2 respectively. He stated that the coefficients are usually higher in steep channels than in flat channels. Table II from Kolupaila (33, pp. 12-18) was presented as containing possible values for design (9, p. 28).

In a grass-lined V-shaped channel with 1 on 10 side slopes and a flow depth of less than 0.8 foot, Ree (43, p. 187) found velocity-head and momentum coefficients of 3.48 and 1.70, respectively.

Rouse (47, p. 59) quoted values of velocity-head and momentum coefficients as being as high as 2.0 and 1.33 respectively for parabolic velocity distributions in circular pipes.

Resistance

Introduction

The flow of liquid in a conduit or channel can follow one or another of three distinct modes of behavior: laminar flow, turbulent flow past a smooth surface, turbulent flow past a rough surface (3, p. 34). The three modes are revealed successively when the flow through a moderately rough pipe is changed from zero to some high velocity (20, p. 556).

Roughness does not appreciably affect the resistance in laminar flow because the velocity distribution is parabolic

TABLE II

DESIGN VALUES FOR VELOCITY-HEAD AND MOMENTUM COEFFICIENTS FROM KOLUPAILA (33, pp. 12-18)

Channels	Value of α			Value of β		
	Min.	Avg.	Max.	Min.	Avg.	Max.
Regular channels, flumes, spillways	1.10	1.15	1.20	1.03	1.05	1.07
Natural streams and torrents	1.15	1.30	1.50	1.05	1.10	1.17
Rivers under ice cover	1.20	1.50	2.00	1.07	1.17	1.33
River valleys, overflooded	1.50	1.75	2.00	1.17	1.25	1.33

and there is no velocity at the surface of contact. The resistance is dependent upon the viscosity of the fluid. An increase in velocity will eventually lead to turbulent flow past a smooth boundary or turbulent flow with a laminar boundary layer. The boundary roughness does not materially affect the resistance in this partially turbulent flow, because the roughness elements are shielded by the boundary layer. As the velocity is increased a point will eventually be reached where the laminar boundary layer is thinned sufficiently that the boundary roughness becomes exposed to the direct action of the moving fluid, and the flow goes into the fully turbulent mode. In this mode the resistance will depend upon the roughness of the boundary, and will be independent of the viscosity.

Pipe Flow Formulas

King and Brater (32, p. 6.6) presented a short discussion of the origin of pipe flow formulas. According to King, many empirical formulas have been developed from test data. One of the earliest was developed by Chezy in 1775. Most of the formulas were based on the assumption that the energy loss depends only on the velocity, the dimensions of the conduit, and the wall roughness. The work of Hagen, Poiseuille, and Reynolds showed that the density and viscosity of the fluid also affected the energy loss. Finally, principally as a result of the work of Nikuradse, it became generally recognized that the effect of roughness does not depend on the actual magnitude of the roughness, but on the ratio of the roughness size to the diameter of the pipe.

According to King and Brater (32, p. 6.16) of all the formulas that have been used to determine energy losses in pipes, only the Darcy-Weisbach formula permits the proper evaluation of all the factors that affect the loss

$$h_L = f \frac{L}{D} \frac{V^2}{2g} \qquad (13)$$

It is dimensionally correct and can be used for any liquid.

The Darcy-Weisbach equation can be derived analytically for the laminar flow of liquid in a pipe form the equation of Poiseuille (32, p. 6.8).

$$h_L = \frac{32\mu VL}{\gamma D^2} \qquad (14)$$

The equation (14) can be derived by considering a cylinder of fluid moving under conditions of laminar flow in a pipe of uniform diameter. If the Reynolds number is factored from equation (14) it can be seen that

$$h_L = \frac{64}{N_R} \frac{L}{D} \frac{V^2}{2g}$$

from which, for laminar flow:

$$f = \frac{64}{N_R}$$

However, for turbulent flow past a rough surface the resistance coefficient remains essentially constant over a wide range of velocities. This is known as the quadratic resistance law. For turbulent flow past smooth surfaces the resistance is proportional to the velocity raised to the 1.75 power.

According to Bakhmeteff (3, pp. 26-27) these equations for pipe resistance may all be derived from a general equation obtained by dimensional analysis

$$-\frac{dp}{dx} = \frac{V^n}{D^{3-n}} \rho \left(\frac{\mu}{\rho}\right)^{2-n}$$

The exponent, n, does not remain constant but changes with the Reynolds number. The exponential formulas with constant coefficients are thus useful and applicable only within given limited ranges. The usual procedure is to use the Darcy-Weisbach equation and to place in the resistance coefficient all the error caused by the constant exponent.

Open Channel Formulas

For laminar flow in a wide, open channel an equation similar to the equation of Poiseuille (14) can be written

$$S = \frac{3\mu V}{\gamma y^2} \tag{15}$$

Laminar flow is of only passing interest in this paper. Therefore, no further discussion will be presented.

Many empirical equations have been developed for computing the resistance of turbulent, uniform flow in open channels. In 1918 Houk (24, pp. 226-261) made a very comprehensive review of the existing open channel formulas. He divided the various formulas into four classes as follows: (1) German formulas developed on the assumption that a roughness factor is unnecessary, (2) Formulas of the exponential type in which roughness conditions are accounted for by a coefficient, (3) miscellaneous formulas, and (4) the formula of Bazin and that of Kutter.

The German formulas that did not include a roughness factor would have to be used with the assumption that the flow is of the laminar or partially turbulent modes. Among the authors of this type of formula, Houk included Siedek, Gröger, Hessle, Christen, Hagen and Gaukler, Hermanek, Matakiewicz, Lindboe, Teubert, and Harder.

The exponential formulas are those having the general form

$$V = C' R^x S^y$$

in which x and y are constants determined from experimental data and C′ is a coefficient. In some formulas the values of x and y are assumed to be the same for all classes of wetted perimeters and the coefficient C′ is to be varied according to the roughness, size, slope, and shape of the channel, and according to any other conditions that may affect the velocity. In others, the value of x and y are varied for the different classes of roughness, and the variation in the coefficient C′ is not supposed to be as great as it is in the former.

The most prominent of the equations with constant exponents is that of Chezy:

$$V = C \sqrt{RS} \tag{16}$$

The Chezy equation has been criticized by some investigators who state that C increases with S. Equations of the constant-exponent type were also proposed by Williams, and Williams and Hagen. Ellis and Barnes each proposed an equation with variable exponents x and y.

Among the authors of miscellaneous formulas Houk cited Manning, Biel, Schmeer, and Elliott. Houk presented the Manning formula in its abbreviated form as

$$V = \frac{1.49}{n} R^{0.67} S^{0.50} \tag{17}$$

A formula presented by Biel was distinct in that it contained a temperature correction. Houk dismissed the Schmeer and Elliott formulas without discussion.

Houk considered the Bazin and Kutter formulas to be the most valuable open channel formulas. Both of the formulas express a relationship for the Chezy coefficient in terms of other coefficients and parameters. The only essential difference between the two formulas is that the Kutter formula includes a slope correction.

The Bazin formula in English units is

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{R}}}$$

The Kutter formula is

$$C = \frac{\frac{1.811}{n} + 41.6 + \frac{0.00281}{S}}{1 + \left(41.6 + \frac{0.00281}{S}\right) \frac{n}{\sqrt{R}}}$$

The m and n are variable coefficients.

As the result of applying the various formulas to a range of channels Houk reached the following conclusions:

1. Of the German formulas which have been developed on the assumption that a roughness coefficient is not necessary, not one possesses sufficient merit to warrant its adoption as a general formula.

2. It is not possible to develop a satisfactory formula for velocities in open channels without introducing therein a variable term to allow for changes in roughness.

3. No exponential formula so far advanced could be recommended for general use.

4. The effect of temperature should not be introduced into a formula for the flow of water in open channels unless its magnitude is greater than that assumed by Biel.

5. Manning's formula in its original form is practically as good as Kutter's for channels of small or ordinary dimensions, but is inferior to Kutter's for large rivers. Although its algebraic form is somewhat more simple than Kutter's equation, it does not seem advisable to adopt it for use even in ordinary instances, since the latter equation is now in general use and, moreover, is applicable to extreme cases.

6. No definite effect of the slope on the Chezy coefficient is shown by the experimental data for small open channels.

7. Data available at present show a decrease in C with an increase in S in large rivers with flat slopes.

8. The Bazin formula is inferior to Kutter's for all types of open channels. The constancy of the factor m is less than that of the factor n in all instances.

9. Although the Kutter formula is not ideal, it is the best equation available at the present time.

Since the time of Houk's work no new resistance formulas for open channels have attained any degree of acceptance. The Manning formula in the form in English units of

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (18)$$

has come into wider acceptance because of its simplicity. The Kutter formula is less widely used, partly because of its more complex form and partly because the gagings of the Mississippi River by Humphreys and Abbot, on which the slope corrections were based, are known to be quite inaccurate (9, pp. 94-95). Brater deleted the Kutter formula when preparing the fifth edition of Handbook of Hydraulics (32, pp. 7.1-7.80).

According to Rouse (47, pp. 114-115), the Manning, Kutter, and Bazin formulas are applicable only when flow is in the fully turbulent mode, since the roughness parameters n and m of these formulas are directly comparable to the roughness parameter k and hence cannot logically be applied to conditions in which viscous action is appreciable. This same thought was expressed by Keulegan (29, pp. 707-741).

He found that the Manning formula described quite well the flow in rough channels when the relative roughness is large.

There has been much discussion concerning the dimensions of the Manning n. Directly from the Manning formula, the dimensions of n are seen to be $T/L^{1/3}$. The following discussion was presented by Chow (9, pp. 98-99):

> . . . Since it is unreasonable to suppose that the roughness coefficient would contain the dimension T, some authors assume that the numerator contains $g^{1/2}$, thus yielding the dimensions of $L^{1/6}$ for n. Also, for physical reasons, it will be seen that $n = [\phi\ (R/k)]\ k^{1/6}$, where. . . $\phi(R/k)$ is a function of R/k. If $\phi\ (R/k)$ is considered dimensionless, n will have the same dimensions as those of $k^{1/6}$, that is, $L^{1/6}$.
>
> On the other hand, it is equally possible to assume that the numerator of 1.486/n can absorb the dimensions of $L^{1/3}/T$, or that $\phi\ (R/k)$ involves a dimensional factor, thus leaving no dimensions for n. Some authors, therefore, preferring the simpler choice, consider n to be a dimensionless coefficient.
>
> It is interesting to note that the conversion of the units of the Manning formula is independent of the dimensions of n, as long as the same value of n is used in both systems of units. If n is assumed dimensionless, then the formula in English units gives the numerical constant $3.2808^{1/3} = 1.486$ since 1 meter = 3.2808 feet. Now, if n is assumed to have the dimensions of $L^{1/6}$, its numerical value in English units must be different from its value in metric units, unless a numerical correction factor is introduced for compensation. Let n be the value in metric units and n′ the value in English units. Then $n' = (3.2808^{1/6})\ n = 1.2190n$. When the formula is converted from metric to English units, the resulting form takes the numerical constant $3.2808^{(1/3\ +\ 1/6)} = 3.2808^{1/2} = 1.811$, since n has the dimensions of $L^{1/6}$. Thus, the resulting equation should be written $V = 1.811\ R^{2/3}S^{1/2}/n'$.

Since the same value of n is used in both systems, the practical form of the formula in the English system is

$$V = 1.811\ R^{2/3}S^{1/2}/1.2190n = 1.486\ R^{2/3}S^{1/2}/n$$

which is identical with the formula derived on the assumption that n has no dimensions.

In a search of early literature on hydraulics, the author has failed to find any significant discussion regarding the dimensions of n. It seems likely that this was not a problem of concern to the forefathers of hydraulics. It is most likely, however, that n was unconsciously taken as dimensionless in the conversion of the Manning formula, because such a conversion, as shown above, is more direct and simpler....

In computing resistance coefficients by the various formulas for data obtained under natural channel conditions, the loss has usually been computed by an equation of the form

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_L \qquad (19)$$

This equation is the same as the power equation (11) or the momentum equation (12) if the velocity is assumed to be uniformly distributed across the channel. However, in developing the empirical resistance formulas used in open channel work, equation (19) has been applied to natural channels with neither uniform flow nor uniform velocity distribution. The error from these assumptions has been left in the resistance coefficients.

Because the loss term in equation (19) describes neither shear loss nor power loss for nonuniform flow with nonuniform velocity distribution, and because the resistance equations developed therefrom to compute and systematize loss coefficients under these conditions are strictly empirical, it seems illogical to insist that these resistance equations are momentum equations rather than power equations, even though the shearing stress on the walls of a conduit or channel is sometimes used in demonstrating a derivation of these equations. Rather, it seems that these resistance equations could be used to systematize either shearing losses or power losses.

Resistance in Vegetated Channels

Probably the most extensive and complete information on resistance in small vegetation-lined channels available at present was presented by Ree and Palmer in 1949 (44). The work was conducted at the outdoor hydraulic laboratory of the Soil Conservation Service located at Spartanburg, South Carolina. Eleven different plant species adapted to the southeastern and south central parts of the United States were tested under various conditions of season, growth, and maintenance. The channel bed slopes ranged from 1 to 24 per cent, with most slopes between 3 and 6 per cent. Two general types of channel, trapezoidal and rectangular, were tested. The dimensions and vegetal conditions are presented in Table III.

Test data were analyzed using the Chezy, Manning, and Kutter resistance formulas. The slopes involved were large

TABLE III

DIMENSIONS AND VEGETAL-LINING CONDITIONS OF EXPERIMENTAL CHANNELS AT OUTDOOR HYDRAULIC LABORATORY AT SPARTANBURG, S.C. TABLE PHOTOGRAPHED FROM REE AND PALMER (44, p. 6)

Vegetation and channel No.	Nominal channel dimensions			Experiment number	Condition of vegetation
	Bed slope	Bottom width	Side slope		
	Percent	*Feet*			
Bermuda grass:					
B1–1	23.7	1.5	1:1 ...	1	Green, long.
				2	Dormant, long.
B1–2	20.0	1.5	1.5:1 ...	1	Green, long.
				2	Dormant, long.
B1–6	20.0	1.5	4:1	1	Green, short.[1]
B1–3	10.0	1.5	1.5:1 ...	1	Green, long.
				2	Dormant, long.
B1–5	10.0	1.5	4:1	1	Green, long.
				2	Green, short.[2]
B2–7	3.0	4.0	1.5:1 ...	1	Green, long.
				2	Dormant, long.
				3	Dormant. Long in test 1, short in tests 2 to 10.
B2–8	3.0	1.5	1.5:1 ...	1	Green, long.
				2	Green. Long in test 1, short in tests 2 to 10.
B2–18	3.0	1.5	4:1	1	Green, short.[1]
B2–19	3.0	1.0	Vertical	1	Dormant, short.[2]
		2.0	....do....	1	Do.
		3.5	....do....	1	Do.
		6.0	....do....	1	Do.
B2–17	1.0	1.5	4:1	1	Partially dormant, short.[2]
Supply canal	.2	4.0	1.5:1 ...	4	Green, short.[2]
Centipede grass:					
B1–4	10.0	1.5	1.5:1 ...	1	Green, long.
				2	Dormant, long.
Dallis grass and crabgrass:					
B2–6	6.0	2.0	3:1	1	Green, long.
Kudzu: B2–9 ...	3.0	4.0	1.5:1 ...	1	Green, long, first season.
	....	....	....	2	Green, long, second season.
	....	....	....	3	Dormant, mulch of vines and leaves.
	....	....	....	4	Green, cut.[1]
	....	(3)	(3)	5	Dead vines (test a). Green (tests b, c, and d).
Lespedeza:					
B2–2	6	2	3:1	1	Dead, uncut.
B2–5	6	2	3:1	1	Green, uncut.
B2–11B	3	2	Vertical	1	Do.
B2–15C	3	2	....do....	1	Dead, uncut.
B2–15B	3	2	....do....	1	Green, uncut.
B2–15A	3	2	....do....	1	Green, short.[1]
Sericea lespedeza:					
B2–1	6	2	3:1	1	Dormant, long.
B2–4	6	2	3:1	1	Green, medium long, woody.[4]
B2–10C	3	2	Vertical	1	Dormant, short.[5]
B2–10B	3	2	....do....	1	Green, long, not yet woody.
B2–14C	3	2	....do....	1	Dormant, long.
B2–14B	3	2	....do....	1	Green, long.
B2–14A	3	2	....do....	1	Green, short.[1]
Sudan grass:					
B2–3	6	2	3:1	1	Dead, long.
Grass mixture:					
B2–12C	3	2	Vertical	1	Green and dormant, short.
B2–12B	3	2	....do....	1	Green and dormant, long.
B2–16C	3	2	....do...	1	Green and dormant, short.
B2–16B	3	2	....do...	1	Green, long.
B2–16A	3	2	....do...	1	Green and dormant, short.
No vegetation:					
B2–3	6	2	3:1	1	
B2–13C	3	2	Vertical	1	
B2–13B	3	2	....do....	1	

1 Cut shortly before test.
2 Kept cut.
3 Changed by plowing.
4 Cut to 6-inch height 2 months before test.
5 Cut previous fall.

enough that the slope term in the Kutter formula was of such small magnitude that it was ignored. Where the flow was nonuniform, losses were computed using equation (19).

The usual procedure used in testing was to run a series of tests at different discharge values in each channel, usually beginning with a low discharge value and increasing the discharge for each succeeding test. Different types of vegetation reacted in different manners; the reaction being a function of the season and maintenance as well as of the type of vegetation.

Water flowing at slight depths through vegetation encounters resistance from stalks, stems, and foliage. A large proportion of the channel cross-sectional area may be blocked out by vegetation, and the resistance to flow will be high. As the discharge and hence the depth of flow is increased, the force exerted by the flowing water causes the vegetation to bend. The vegetation is bent over when the bending moment exceeds the resisting moment. The bending moment is a function of the depth and velocity of flow, the resisting moment a function of the length and type of vegetation. When sufficient bending moment is exerted to flatten vegetation to the channel bed and free a portion of the cross section of the channel, the resistance decreases sharply. If tests were run starting at high discharge values and going to low discharge values, the sequence of events probably would not be reversed, since the vegetation would not recover from the flattening produced by the high flows.

Test results for bermudagrass showed that while the discharge was low and flow was entirely within the area occupied by the grass, the Manning coefficient was practically constant. As the discharge was increased a point was reached where the flowing water exerted sufficient bending moment to start to bend and submerge the vegetation, and the resistance coefficient decreased rapidly with increased discharge. When the grass was completely submerged and lying flat the resistance reached a constant low value. Further changes in resistance were caused by roughening of the channel bed by erosion.

Sericea lespedeza in the tall green condition exhibited somewhat different resistance characteristics from highly flexible bermudagrass. The resistance increased slightly from an initial low-discharge value, reached a peak value, and then started to decline as discharge and depth increased. The physical explanation is as follows: The initial value of resistance coefficient was obtained with low discharge and the cross-sectional area of flow including only the lower part of the stalks below the first leaves. As the discharge was increased the water rose to include some of the lower leaves in the cross-sectional area of flow. Thus the resistance coefficient increased. The resistance coefficient increased until the fairly stiff plants started to bend and submerge and a portion of the flow area was cleared. Even after the plants bent, they did not lie flat as did the bermudagrass, but continued to offer considerable obstruction to the flow.

The product of mean velocity and hydraulic radius, VR, was used as a criterion for systematizing the resistance coefficients. The resistance of vegetation to flow was thought to be a function of the degree of flattening of the vegetation, which is influenced by the velocity and depth of flow. Results of a large number of tests with bermudagrass-lined channels of a range of shapes and slopes showed that the VR product could be used with a great deal of confidence. Log-log plottings of Manning's n versus the VR product yielded a straight-line relation with a negative slope from a VR value of 0.2 foot squaredper second to a VR value of 3 to 3.5 feet squared per second, the latter value depending upon the grass length. Then resistance ceased to be a function of VR. Tests with other vegetation yielded consistent relations, although few were of straight-line form over as large a portion of the range of data.

Spatially Varied Steady Flow with Increasing Discharge

Spatially varied flow is defined as flow having a non-uniform discharge resulting from the addition or diminution of fluid along the course of flow. Several hydraulicians have attempted to solve the problem of spatially varied steady flow with increasing discharge. The principle of the conservation of linear momentum has been used by nearly all of the investigators mentioned in this dissertation. However, these investigators have made various assumptions as to the effect of the entering water upon the main flow, and as to the amount and evaluation of the energy loss in the flow. Some have assumed that all of the momentum of the entering water will be lost; others have assumed that the component of the momentum of the entering water in the direction of flow will add to the momentum of the main flow. Some have assumed that the momentum losses will balance the shearing losses; others have assumed negligible shearing losses; and still others have assumed that a uniform flow resistance equation may be used to determine shearing losses.

According to Chow (9, p. 327), Hinds (22, pp. 881-927) was probably the first to develop a substantially correct theoretical analysis of spatially varied steady flow. Hinds assumed that: (1) all of the energy of impact of the entering fluid is lost, (2) the entering fluid has no component of momentum in the direction of the main flow. As Hinds stated, the first assumption is tantamount to assuming the collisions between water molecules to be completely inelastic and that the particles of fluid flow away together with approximately equal individual velocities. Hence, the velocity in the channel is uniform over the cross section. Because it was assumed that all energy of impact is lost, and that the average and individual velocities are equal, only the law of conservation of linear momentum is necessary in the treatment of the problem.

On the basis of the conservation of linear momentum, Hinds developed an equation for the case of uniform inflow by considering an incremental length of the channel, equating the momentum change across the length to the external forces acting on the length, and letting the length become infinitesimally small. The resulting equation can be integrated directly if an exponential velocity law is assumed. However, for more general conditions including nonuniform inflow it can be solved only by approximate methods. No term for shearing loss was included in the equation, although Hinds showed how to include a correction at the time of computing a water surface profile. For channels in which the control point is not located at the downstream end, Hinds developed a method for locating the control. The computations then proceed upstream and downstream from the control section.

The theory was verified by both model and prototype tests. The model spillway tested was 16 feet long and was of trapezoidal section with 10-inch bottom and 2 on 1 side slopes. Fifteen tests were run in the model; the maximum discharge was 31.1 cubic feet per second. The prototype testing consisted of measurements taken on the Arrowrock Reservoir spillway in the spring of 1923. The spillway consisted of six 62-foot sections separated by piers. The maximum discharge measured was approximately 10,000 cubic feet per second.

Camp (8, pp. 606-617) developed a theoretical analysis of spatially varied steady flow quite similar to that of Hinds. Camp assumed uniform inflow and a negligible momentum component of the inflow in the direction of the main flow. His derivation was based on the concept of the conservation of linear momentum. The only difference between the Hinds and Camp equations is that the latter contains a term for shearing loss.

Camp developed an analytical solution for rectangular channels. The resulting function was implicit in depth. The function was written in dimensionless form and a graphical solution was developed. He also developed a method to apply the solution to flow in channels with parallel sides extending below the water surface but with other bottom shapes.

The theory was applied to the channels used in water and sewage plants. Free outfall was assumed at the outlet end, and the control section was estimated to occur at a distance upstream from the end equal to three to four times the critical depth.

Tests were conducted on several small lateral spillway channels. Total discharge ranged from 1.237 to 73.6 cubic feet per second; total length of channel ranged from 20.0 feet to 49.0 feet.

Thomas (56, pp. 627-633) stated that the hydraulic theory underlying Camp's analysis does not apply to sharply curving streamlines and therefore fails in the vicinity of the outfall. Thomas wrote of experimental evidence that indicated a wider variation in the position of the effective control than suggested by Camp. According to Thomas, the uncertainty of the location of the control section frequently obscures the refinement of introducing the shearing loss term into the formulas. Consequently, the inclusion of the shearing loss term is often superfluous.

Thomas proposed an alternate method whereby a parabolic shape was used to approximate the water surface. The shearing loss was ignored. Tests were conducted in a small lateral spillway channel. The channel was 4 inches wide and 5 feet 11 inches long, with level top edges and adjustable bottom slope. The bottom slope was varied from 0 to 3.1 per cent. Calculated profiles were compared with experimental profiles and percentage errors were given. It was noted that with increased slope the velocity became larger and the percentage error increased.

Beij (6, pp. 193-213) conducted a study of spatially varied steady flow in roof gutters. He first approached the problem by using dimensional analysis, conducting tests, and obtaining empirical equations. However, he was able to analyze only level channels in this manner due to the complexities involved in analyzing sloping channels and so turned to a theoretical approach using the conservation of momentum. As a basis for his theoretical analysis he assumed uniform inflow and no momentum component in the direction of flow. For the general case he neglected the effects of surface tension and viscosity but included a shearing loss term and a slope term. A theoretical equation was derived by considering a short increment of channel and equating the momentum change across the free body to the external forces acting upon the body. The length of increment was then allowed to become infinitesimally small. The resulting differential equation could not be integrated in its general form.

Beij assumed negligible shearing loss and applied the differential equation to level channels. Under these conditions his equation can be integrated directly for a particular channel shape. He assumed critical depth at the outfall and developed particular solutions for the depth at the upstream end and for the capacity of rectangular, triangular, trapezoidal, and semicircular gutters. He then compared theoretical results for depth at the upstream end with experimental data.

Favre (37, pp. 520-522) developed a more complete equation that includes a shearing loss term and a component of inflow velocity in the direction of the axis of the channel. The Favre equation was used to predict water surface profiles in a model of the Boulder Dam that was tested at the Swiss Federal Hydraulic Research Laboratory at Zurich, Switzerland.

Jaeger (25, p. 181) cited De Marchi (14), who developed a graphical method of predicting water surface profiles for spatially varied flow based on the assumption that the momentum of the entering fluid has no component in the direction of flow, and that the component of the weight parallel to the sloping bed is balanced by the wall shear.

Li (35, pp. 255-274) also used the principle of the conservation of linear momentum in his analysis of spatially varied flow. He did not neglect the momentum component of the added fluid in the direction of flow as did Hinds and Camp. However, he did not deal with it directly. For conditions for which the shearing loss is of secondary importance, Li assumed that the momentum component of the entering fluid would balance the shearing force at the channel walls. A differential equation was derived. A closed form solution could not be obtained for the general case, but he developed rather ingenious methods of solution for channels of level or constant bottom slope with either parallel or sloping side walls. Various outlet conditions were treated.

Li assumed that the shearing loss would be important only for channels with level or gradual slope. For these conditions he treated only channels with level bottom, thereby removing the momentum component in the direction of flow. A differential equation was derived and solutions were developed for the percentage increase in the depth at the upper end of level channels with either parallel or sloping side walls.

Tests were conducted in both level and sloping channels. The test channel with level bottom was of rectangular cross section with 9-inch bottom and variable length of from 4.38 to 7.50 feet. Water was added to the channel over the level tops of both side walls. Total discharge ranged from 0.88 cubic foot per second to 3.06 cubic feet per second. The outlet end of the channel was continued 6 feet beyond the end of fluid introduction. The flow at the outlet of the test channel was subcritical. The test channel with sloping bottom was of rectangular cross section with 3-inch bottom, length of 4 feet 6 inches, and slope of 13 per cent. Water was added uniformly over a weir on one side and free discharge was allowed at the outlet.

Chow (9, pp. 329-332) used the momentum concept in deriving an equation for spatially varied flow. He assumed that the inflow occurs uniformly along the channel and that it possesses no momentum component in the direction of flow. A term for shearing losses is included. For nonuniform distribution of velocity the Coriolis coefficient rather than the Boussinesq coefficient is used because, according to Chow, the friction slope is evaluated by a formula for energy loss, such as the Manning formula. He then derived the same equation by an energy approach (9, pp. 332-333). However, this derivation seems to contain a fallacy in the term for the kinetic energy needed to speed up the added fluid. The Chow equation with Coriolis coefficient is as follows:

$$\frac{dy}{dx} = \frac{S_o - S_s - 2\,\frac{\alpha Q q}{gA^2}}{1 - \frac{\alpha Q^2}{gA^2 y_m}} \tag{20}$$

A finite difference method was devised whereby the equation can be used to predict the water surface profile in any channel provided a control point is known (9, pp. 341-346).

Woo and Brater (58, pp. 31-56) derived an equation nearly identical to the Chow momentum equation. No velocity distribution coefficients were included. The Darcy-Weisbach resistance equation was used to evaluate boundary resistance. A finite difference method quite similar to the Chow method was presented.

The Woo and Brater equation was tested by conducting experiments in which simulated rainfall fell on an impervious surface. The test flume was 29 feet 7 inches long and 6 1/4 inches wide. The bed slope was varied from 0 to 6 per cent. Rainfall intensities of 1.65, 2.95, and 5.04 inches per hour were simulated.

Numerical Integration of Initial Value Problems

The calculation of water surface profiles for either gradually varied flow or spatially varied flow involves a situation in which the water surface elevation is known only at some control point and the derivative of the depth with respect to the distance down the channel is a function of the depth and the distance. Mathematically this can be called an initial value problem. The calculation of the water surface elevation all along the channel must start from the known point and be projected to all points of the channel by use of the expression for the derivative.

Several methods have been developed to solve initial value problems. Open-type formulas express a relation for the ordinate at some value of the independent variable in terms of only previously calculated ordinates and slopes. Closed-type formulas involve previously calculated ordinates and slopes as well as the unknown slope at the projected point. Closed-type formulas usually must be solved by iteration, because they involve the value of the ordinate at the unknown point in both sides of the equation (21, p. 192).

A Taylor series expansion is sometimes used to obtain initial points from which to start either open-type or closed-type formulas that require more than one known value of the ordinate and slope (21, pp. 192-193). The Taylor series expansion has the disadvantage that it involves higher order derivatives of the ordinate. These may not be easily obtained for complicated equations.

The simplest method for solving initial value problems is that of Euler (39, pp. 224-227). Euler's method consists of projecting from a point where the ordinate is known to a point where the ordinate is unknown by simply evaluating the derivative at the known point, multiplying by the interval between values of the independent variable and adding this product to the value of the ordinate at the known point. The predicted ordinate is then taken as a known point and prediction is made to another unknown point. The solution is continued in this manner. Euler's method should be used with caution since it can be very inaccurate.

Euler's method can be modified to improve the accuracy by altering it to make it a closed-type formula (51, p. 119). In this modified Euler method the ordinate at the unknown point is calculated using the derivative at the point where the ordinate is known. Then the derivatives at the predicted point and the known point are averaged and a new prediction is made using this averaged slope. This modified Euler method can be iterated until the prediction does not change within some small limit. This method is well suited to the digital computer.

The Milne method is somewhat more complicated than the Euler method and requires knowledge of the ordinate and the derivative at four pivotal points to start the solution (51, p. 120). It is a closed-type formula involving both a predictor and a corrector.

The Adams method is an open-type formula developed from the Newton backward-difference formula (21, pp. 198-199). It can involve any number of pivotal points. If no differences were retained in the Newton backward-difference formula, the Adams method would be identical to the Euler method.

A closed-type formula developed from the Adams method is called the modified Adams method or Moulton's method (21, pp. 200-201).

The Runge-Kutta method is an open-type averaging method involving only one known point (39, pp. 232-236). It is self-starting and has no check on the computation. It has the advantage that the interval length can be changed readily in the middle of computing a series of points.

Frequently the solution of an initial value problem is started with one of the self-starting methods such as the Taylor series expansion, the Euler or modified Euler method, or the Runge-Kutta method. Then the solution is continued with a more sophisticated method such as that of Milne.

Rectangular Weirs

For many years experimenters worked to obtain an exact and general formula to describe the flow over weirs. Finally it became apparent that the number of variables involved is so great as to defy an exact analytical approach. The usual approach to the problem is to assume that gravitational forces are predominant and ignore the effects of viscosity, surface tension, weir height, shape and condition of crest, condition of approach channel, and the approach velocity in deriving an approximate equation. The resulting equations do not describe the flow over weirs with a great deal of accuracy, and corrections must be applied to account for the secondary effects.

The approximate equation for flow over weirs is derived from the theorem of Torricelli which states that

> the velocity of a fluid passing through an orifice in the side of a reservoir is the same as that which would be acquired by a heavy body falling freely through the vertical height measured from the surface of the fluid in the reservoir to the center of the orifice. (23, pp. 10-11)

Horton (23, pp. 10-13) presented a mathematical development of the general formula for weirs and orifices. The following development is patterned after that in Horton's paper.

Consider a rectangular opening in the side of a retaining vessel as shown in Figure 4. From Torricelli's theorem the velocity of flow through an elementary layer whose area is L dy will be

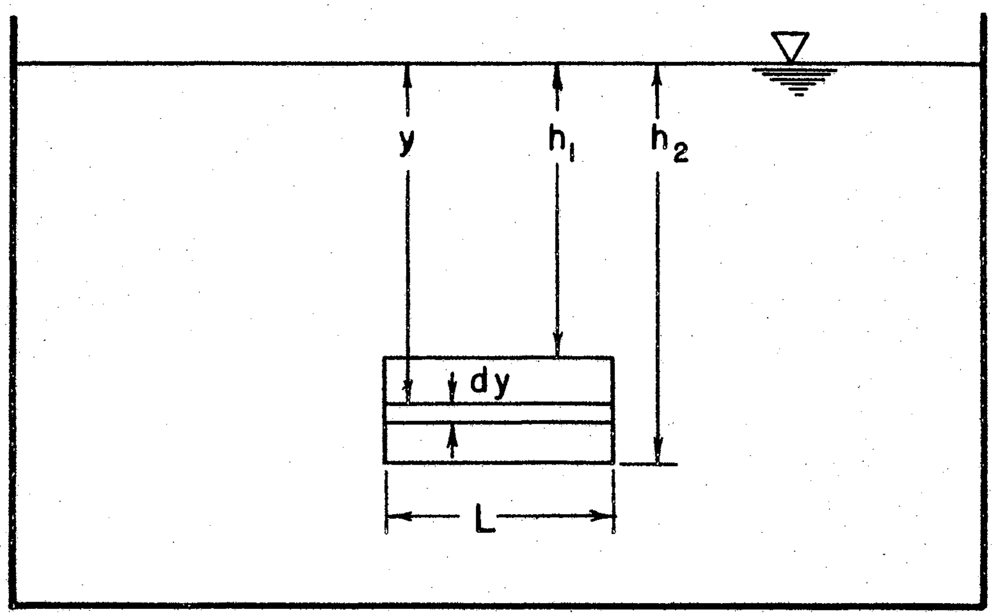

Figure 4. Rectangular Orifice

$$v = (2g\ y)^{1/2}$$

The discharge through the entire opening will be, per unit of time, neglecting contractions,

$$Q = \int_{h_1}^{h_2} (2g\ y)^{1/2}\ L\ dy$$

This equation can be considered a general approximate equation for the flow through any weir or orifice, if L is considered to be a variable. For a rectangular opening, L is a constant and the equation can be integrated into the following form:

$$Q = \frac{2}{3}\ L\ (2g)^{1/2}\ (h_2^{3/2} - h_1^{3/2}) \tag{21}$$

For a weir or notch, the upper edge will be at the surface, $h_1 = 0$, and if h_2 is replaced with h, then equation (21) can be written

$$Q = \frac{2}{3}\ L\ (2g)^{1/2}\ h^{3/2} \tag{22}$$

According to Horton (23, p. 130), practical weir formulas differ from equation (22) in that the velocity of approach must be considered and the discharge must be corrected by a contraction coefficient to allow for the diminished section of the nappe as it passes over the crest lip. Equation (22) is frequently written as

$$Q = C\ L\ h^{3/2} \tag{23}$$

This is the general equation for flow over horizontal-crested weirs.

The vertical contraction expresses the relation of the thickness of nappe in the plane of the weir crest to the depth on the crest. It comprises two factors, the surface curve or depression of the surface of the nappe, and the crest contraction or contraction of the under surface of the nappe at the crest edge. The latter factor varies with the form of the weir cross section. In general, variation of the vertical contraction is the principal source of variation in the discharge coefficients for various forms of weirs, according to Horton (23, pp. 13-14).

If the sides of the notch have sharp upstream edges so that the nappe is contracted in width, the weir is said to have end contractions. If the crest length is the same as the width of channel, the sides of the channel above the crest thus becoming the sides of the notch, the notch suffers no contraction in width, and the weir is said to have end contractions suppressed. The end contractions tend to reduce the effective length of a weir.

Weirs that operate with a negative pressure beneath the nappe do not have the same discharge characteristics as weirs that are fully aerated. The effect of these negative pressures is to increase the effective head operating on the weir and hence increase the discharge for a given measured head. Even with well-aerated nappes there is a tendency for the nappe to adhere at low heads. Little information is available on this condition. It is avoided if at all possible.

Corrections have been proposed for the velocity of approach, because the velocity head should be added to the potential head when computing the effective head on a weir.

Many experimenters have attempted to determine formulas to correct the approximate equation for the rectangular sharp-crested weir. Most of the experiments prior to 1907 were described by Horton (23). King (31, pp. 4.5-4.7) summarized the Horton information. Horton reported on early experiments in France that involved relatively small quantities of water and the results from which are of only limited use.

Francis (17) performed experiments at Lowell, Massachusetts, in 1852. The lengths of weirs were 8 and 10 feet; the weir heights were 2 and 5 feet; the range of heads was from 0.6 to 1.6 feet; the velocities of approach ranged from 0.2 to 1.0 foot per second.

Fteley and Stearns (19, pp. 1-118), in 1877 and 1879, experimented with two sharp-crested suppressed weirs, respectively 5 and 19 feet long, 3.17 and 6.55 feet high, and with maximum heads of 0.8 foot and 1.6 feet. Experiments were also conducted on a weir with end contractions.

Bazin (4) conducted 381 experiments on suppressed weirs in France in 1886. Heads varied from 0.3 foot to 1.7 feet, heights of weir ranged from 0.79 foot to 3.72 feet, and lengths of weir were 1.64, 3.28, and 6.56 feet.

The Frese (18) experiments were performed at Hanover, Germany, prior to 1890. Comparatively large volumes of water were used in testing weirs under a wide range of conditions.

The Rehbock (46, pp. 1143-1162) experiments were conducted at the Karlsruhe Hydraulic Laboratory in Germany. The quantities of water used were not large, but conditions were favorable for unusual accuracy and for conducting experiments under a wide range of conditions. Rehbock also presented the results of extensive experiments by the Swiss Society of Engineers and Architects.

The experiments of Schoder and Turner (52, pp. 999-1110) were performed at Cornell University between 1904 and 1920. With the published results of these experiments were included 1,162 experiments by others. In all, 2,438 separate volumetric measurements for 152 different heads were made. Heights of weir ranged form 0.5 to 7.5 feet, heads from 0.012 foot to 2.75 feet, and lengths of weir from 0.9 foot to 4.2 feet.

All of the preceding experimenters considered the velocity of approach when correcting equation (23). Francis developed an equation to determine the effective length of a weir when end contractions were not suppressed.

Cone (11) conducted weir experiments at the Fort Collins Hydraulic Laboratory. His testing program included 226 tests on rectangular notches of crest lengths 0.5, 1.0, 1.5, 2.0, 3.0, and 4.0 feet. End contractions were not suppressed. Cone presented a general equation similar to equation (23) in which the exponent on the head term was a linear function of the crest length, rather than the constant value of 3/2.

Cline (10, pp. 396-413) reanalyzed the data presented by Schoder and Turner and presented an empirical equation which considered the exponent on the head term in equation (23) a variable related to the head, rather than a constant. According to Cline it was possible to obtain good correlation between measured and computed discharge, even at low heads. He also stated that correction for velocity of approach depends entirely upon the physical dimensions of the weir, and can be applied directly.

Kandaswamy and Rouse (27, pp. 1-13) reported on various experiments conducted at the Massachusetts Institute of Technology and at the Iowa Institute of Hydraulic Research that showed the effect of the height of the weir.

Kindsvater and Carter (30, pp. 1-36) conducted tests on weirs with end contractions suppressed. The weir lengths ranged from 0.10 to 2.68 feet, heights from 0.30 to 1.44 feet, and heads from 0.10 to 0.72 foot. They chose equation (23) as their basic equation, but corrected the length and head terms to effective values. The discharge coefficient was considered a function of various dimensionless ratios.

Brater (32, pp. 5.11-5.12) plotted the discharge coefficient versus the head-over-height ratio for several of the previously mentioned experiments. His conclusions are as follows:

In general, it must be concluded that even among tests for which conditions appear to be quite similar, there are rather great differences in discharges for the same head, and that although the weir is a very useful measuring device, its limitations should be recognized and understood.

Two of the factors that seem to be of great importance in the flow over weirs are the crest condition and the condition of the upstream face of the weir (31, pp. 4.11-4.12). However, these conditions are difficult to evaluate quantitatively, and for a given weir the conditions will change with age, and a rating that is accurate when a weir is new may be completely inaccurate when the weir is older and perhaps slightly rounded or perhaps encrusted.

Head measurements on weirs should be made far enough upstream to be unaffected by the surface curve. However, the head should not be measured so far upstream as to be affected by head losses due to resistance. Cone (11, p. 1111) stated that head measurements should be made at least 4 h upstream, or sidewise from the end of the crest in the plane of the weir a distance of at least 2 h. King (31, p. 4.12) stated that head should be measured at least 2.5 h upstream from the weir. Brater (32, p. 5.30) changed this recommendation to 4 h for the horizontal-crested weir.

CHAPTER III

THEORETICAL ANALYSIS

Introduction

Much of the material which might otherwise appear in this chapter is presented in Chapter II, Review of Literature. Therefore, only two subjects, the derivation of an equation for spatially varied steady flow with increasing discharge, and the method of solution of this equation, are presented.

Spatially Varied Steady Flow With Increasing Discharge

Spatially varied steady flow with increasing discharge can be analyzed by the principle of the conservation of momentum. The change in momentum along an incremental length of channel can be equated to the sum of the forces acting on the body. If it is assumed that the inflow enters the channel with no velocity or momentum component in the direction of the main flow, and if the original concept of streamline flow is stretched somewhat further, then equation (10) is directly applicable to the problem.

$$\sum F_x = \rho \beta_2 V_2^2 A_2 - \rho \beta_1 V_1^2 A_1 \qquad (10)$$

The term on the left of equation (10) can be considered as the sum of all of the external forces of pressure, gravitational acceleration, and bed shear acting in the x direction upon the body of water between sections (1) and (2) of Figure 5.

$$\sum F_x = F_{p_x} + F_{g_x} - F_{s_x}$$

If hydrostatic pressure distribution can be assumed, then from Stoker (54, pp. 454-455), the resultant pressure force on the body can be written as

$$F_{p_x} = \gamma A_{avg} \, \Delta x \, \frac{dy}{dx}$$

The force caused by gravitational acceleration can be written as

$$F_{g_x} = \gamma A_{avg} \, \Delta x \tan \Theta$$

For small inclination angles, $\tan \Theta \approx \sin \Theta = S_o$

$$F_{g_x} = \gamma A_{avg} \, \Delta x \, S_o$$

The force caused by the bed shear can be written as

$$F_{s_x} = \gamma A_{avg} \, \Delta x \, S_s$$

If it can be assumed that the variation from section (1) to section (2) is approximately linear in depth, then

$$F_{p_x} = \gamma A_{avg} \, \Delta y$$

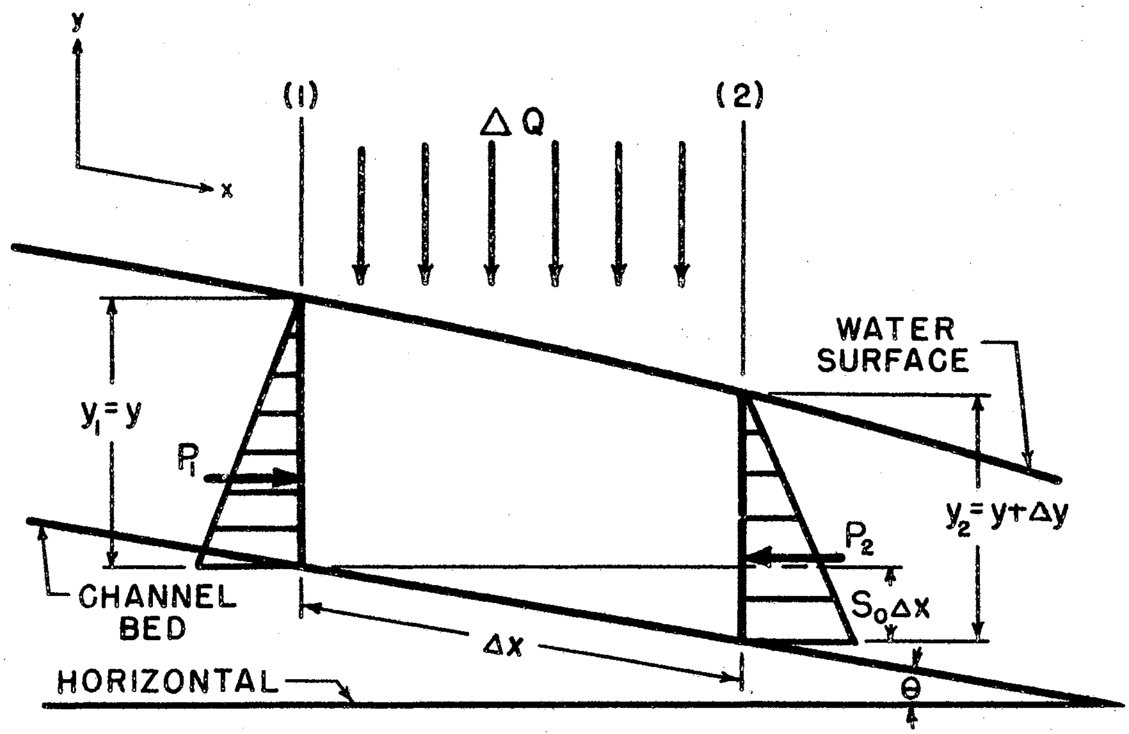

Figure 5. Spatially Varied Flow Diagram

Grouping all the terms,

$$\rho\beta_2 V_2^2 A_2 - \rho\beta_1 V_1^2 A_1 = \gamma A_{avg}\ \Delta y + \gamma A_{avg}\ \Delta x\ S_o$$

$$- \gamma A_{avg}\ \Delta x\ S_s$$

Rearranging and dividing by γA_{avg},

$$\Delta y = - \frac{\beta_2 V_2^2 A_2 - \beta_1 V_1^2 A_1}{g\ A_{avg}} + (S_o - S_s)\ \Delta x$$

If A_{avg} can be written as

$$A_{avg} = \frac{Q_1 + Q_2}{V_1 + V_2}$$

Then

$$\Delta y = - \frac{1}{g} \left(\frac{V_1 + V_2}{Q_1 + Q_2}\right) (\beta_2 V_2^2 A_2 - \beta_1 V_1^2 A_1) + (S_o - S_s)\ \Delta x$$

But since

$$V_1 A_1 = Q_1$$

$$V_2 A_2 = Q_2 = Q_1 + \Delta Q$$

Then

$$\beta_2 V_2^2 A_2 - \beta_1 V_1^2 A_1 = \beta_2\ (Q_1 + \Delta Q)\ V_2 - \beta_1 Q_1 V_1$$

$$= Q_1\ \left(\beta_2 V_2 - \beta_1 V_1 + \frac{\beta_2\ V_2}{Q_1}\ \Delta Q\right)$$

The expression for Δy can be written

$$\Delta y = - \frac{Q_1(V_1 + V_2)}{g\ (Q_1 + Q_2)} \ (\beta_2 V_2 - \beta_1 V_1 + \frac{\beta_2 V_2}{Q_1} \Delta Q)$$

$$+ (S_o - S_s)\ \Delta x \qquad (24)$$

This equation for spatially varied steady flow is nearly identical to an equation presented by Chow (9, p. 341) and King and Brater (32, p. 11.5). The only difference is in the inclusion of a variable momentum coefficient.

The following assumptions were made in deriving equation (24):

1. The flow is such that it can be represented by at least temporal streamlines.
2. The flow at the two ends of a reach is in essentially the same direction.
3. The pressure in the flow is approximately hydrostatically distributed.
4. The angle of inclination of the channel bottom is relatively small.
5. The variation in depth and area between the two ends of a reach is approximately linear.

These assumptions are approximately the same as those used in deriving equation (12) for gradually varied flow, with the exception of the restrictive assumption concerning the variation in momentum coefficient in gradually varied flow.

Equation (24) in its present form can be solved by numerical integration. If a constant Boussinesq coefficient is assumed, a somewhat simpler differential equation can be obtained. While this differential equation is no more amenable to solution than equation (24), it can be classified and examined for possible discontinuities.

Consider equation (24) and assume

$$\beta_1 = \beta_2 = \beta$$

and

$$\frac{V_1 + V_2}{Q_1 + Q_2} = A_{avg} = A + \frac{1}{2} \Delta A$$

Let

$$Q_1 = Q \qquad\qquad Q_2 = Q + \Delta Q$$

$$V_1 = V \qquad\qquad V_2 = V + \Delta V$$

Then equation (24) can be written as

$$\Delta y = - \frac{\beta Q}{g} \frac{(V + \Delta V - V + \frac{(V + \Delta V)}{Q} \Delta Q)}{A + \frac{1}{2} \Delta A}$$

$$+ (S_o - S_s) \Delta x$$

Simplifying and ignoring the product of the increments,

$$\Delta y = - \frac{\beta Q}{g} \left(\frac{\Delta V + \frac{V}{Q} \Delta Q}{A + \frac{1}{2} \Delta A}\right) + (S_o - S_s) \Delta x$$

$$= - \frac{\beta Q}{g} \left(\frac{Q \Delta V + V \Delta Q}{Q (A + \frac{1}{2} \Delta A)}\right) + (S_o - S_s) \Delta x$$

But since

$$V + \Delta V = \frac{Q + \Delta Q}{A + \Delta A}$$

or

$$\Delta V = \frac{Q + \Delta Q}{A + \Delta A} - V$$

The preceding equation can be written as

$$\Delta y = - \frac{\beta}{g} \left(\frac{Q(\frac{Q + \Delta Q}{A + \Delta A} - V) + V \Delta Q}{A + 1/2 \Delta A}\right) + (S_o - S_s) \Delta x$$

This equation can be simplified to the following:

$$\Delta y = - \frac{\beta}{g} \left(\frac{2Q \quad \Delta Q - QV \quad \Delta A + V \Delta Q \quad \Delta A}{(A + 1/2 \Delta A)(A + \Delta A)}\right) + (S_o - S_s) \Delta x$$

If the products of the increments and the product of the area and the increment of area can be assumed to be relatively small, then the preceding equation can be written as

$$\Delta y = - \frac{\beta}{g} \left(\frac{2Q \Delta Q - QV \Delta A}{A^2}\right) + (S_o - S_s) \Delta x$$

Divide by Δx

$$\frac{\Delta y}{\Delta x} = - \frac{\beta}{g} \left(\frac{2Q}{A^2} \frac{\Delta Q}{\Delta x} - \frac{Q^2}{A^3} \frac{\Delta A}{\Delta x}\right) + S_o - S_s$$

Assume the necessary conditions to form the derivative and let x become infinitesimally small. Then

$$\frac{dy}{dx} = - \frac{\beta}{g} \left(\frac{2Q}{A^2} \quad \frac{dQ}{dx} - \frac{Q^2}{A^3} \frac{dA}{dx}\right) + S_o - S_s$$

But

$$\frac{1}{A}\frac{dA}{dx} = \frac{1}{A}\frac{dA}{dy}\frac{dy}{dx} = \frac{b_w}{A}\frac{dy}{dx} = \frac{1}{y_m}\frac{dy}{dx}$$

And

$$\frac{dQ}{dx} = q$$

Thus

$$\frac{dy}{dx} = - \frac{2\beta Qq}{gA} + \frac{\beta Q^2}{gA^2 y_m}\frac{dy}{dx} + S_o - S_s$$

This equation can be rearranged into the following form:

$$\frac{dy}{dx} = \frac{S_o - S_s - 2\beta Qq/gA^2}{1 - \beta Q^2/gA^2 y_m} \tag{25}$$

Equation (25) is identical to the Chow equation (20) except for the presence of the Coriolis coefficient in equation (20) and the Boussinesq coefficient in equation (25).

The resistance losses due to bed shear in spatially varied flow can be estimated by any empirical resistance formula. The Manning formula (18) can be rearranged into the following form:

$$S_s = \frac{Q^2_{avg}\ n^2}{2.21\ A^2_{avg}\ R^{2/3}_{avg}} \tag{26}$$

The Manning coefficient to use in equation (26) should be obtained under uniform flow conditions, or if it is obtained under nonuniform flow conditions where velocity distribution is not uniform, equations (10) or (12) should be used in calculating bed shear losses.

Solution of Spatially Varied Steady Flow Equation

The differential equation (25) describing spatially varied steady flow is a first order, first degree, nonlinear ordinary differential equation. It can be solved analytically only for the simplest cases in which the shearing resistance can be assumed to be negligible. Furthermore, the denominator in equation (25) corresponds to $(1 - \beta N_F^2)$. At the critical depth the Froude number, N_F, will be one, and if the Boussinesq coefficient, β, should be approximately one, then the denominator will be zero and the derivative undefined. For a channel in which the only point of known water surface elevation is near a free outfall where the depth is near critical, the denominator can be of such small magnitude as to adversely affect the accuracy of the computation. This influence might extend for some distance upstream from the control point.

This same effect should be kept in mind when working with equation (24). However, equation (24) shows no possibility of having a zero denominator, and the only possible difficulty would be with the relative magnitudes of the various terms. The only way to be certain about this possibility would be to apply the equation to a given situation and calculate the individual terms in equation (24).

Equation (24) can be solved as follows: Starting from a control point or location, x_n, where the ordinate, y_n, is known, estimate the depth, y_{n+1}, some incremental distance upstream or downstream, depending upon whether the bottom slope is subcritical or supercritical. Then solve for Δy in equation (24). Use this Δy to reestimate y_{n+1}, and recalculate Δy. When the calculated Δy values cease to change within some small limit, assume that the iterative process has been carried far enough and take y_{n+1} as a known ordinate and proceed to calculate y_{n+2}. Continue this procedure until the water surface elevation has been calculated all along the channel.

The preceding method is approximately equivalent to a modified Euler method with iteration. It is readily adapted to a digital computer.

CHAPTER IV

EXPERIMENTAL SETUP

General Description

The experimental setup was a full-size outdoor model as shown in Figures 6 and 7. It consisted of an asymmetrical V-shaped test channel approximately 410 feet long with design side slopes of 3 on 1 and 6.6 on 1 and with design bottom slope of 0.001. The maximum depth was approximately 2.7 feet. Free outfall occurred normally at the lower end of the channel, although a set of end sills could be used to block the end of the channel and raise the water surface elevation in the lower portion of the channel. The usual outlet condition is shown in Figure 8. The channel was lined with bermudagrass which was clipped with a rotary power lawnmower. Figure 9 shows a typical stand and vegetal condition.

Flow was measured and introduced into Forebay 1 with the two-foot modified Parshall flume shown in Figure 10. The flow could then enter the channel either at the upper end or all along the upper 399.2 feet over the adjustable weir shown in Figure 11, depending upon the position of a set of movable entrance gates. This flow introduction scheme and the possible

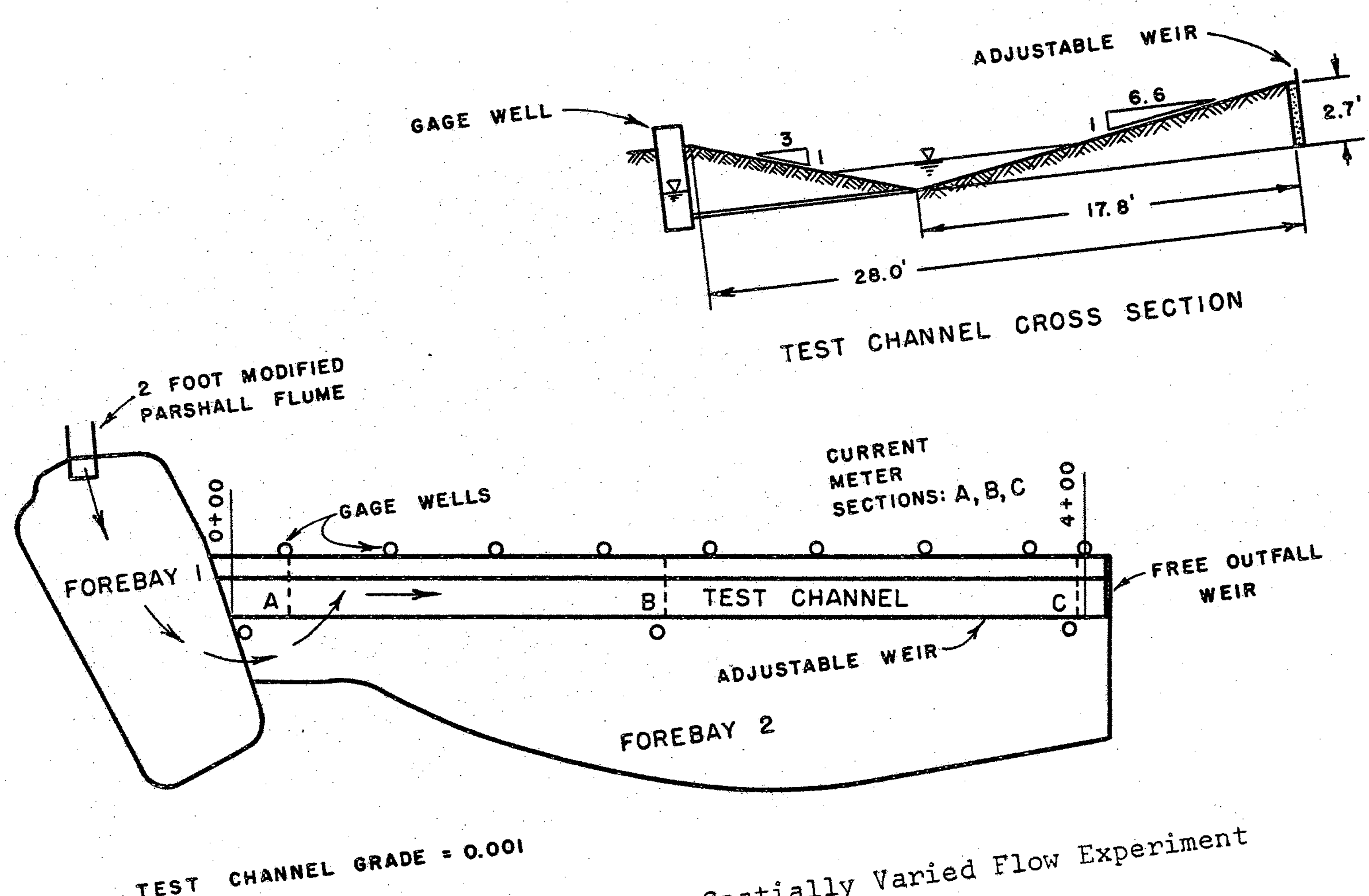

Figure 6. Layout for Spatially Varied Flow Experiment

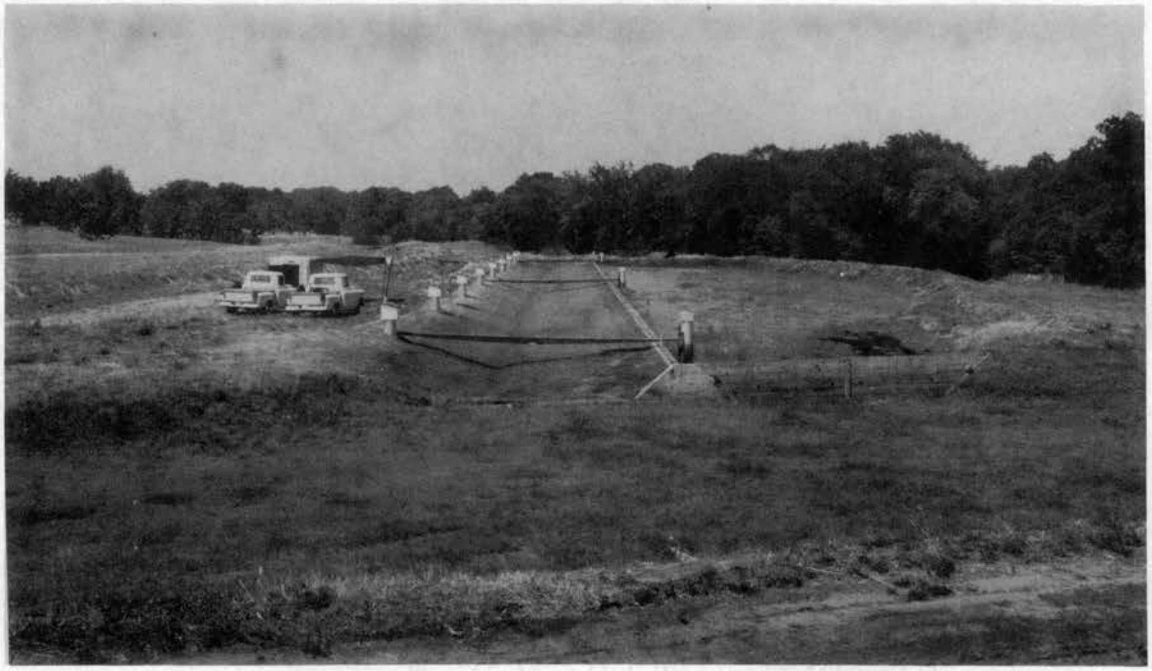

Figure 7. Over-All View, Spatially Varied Flow
Experimental Setup

Figure 8. Outlet Weir, FC 31

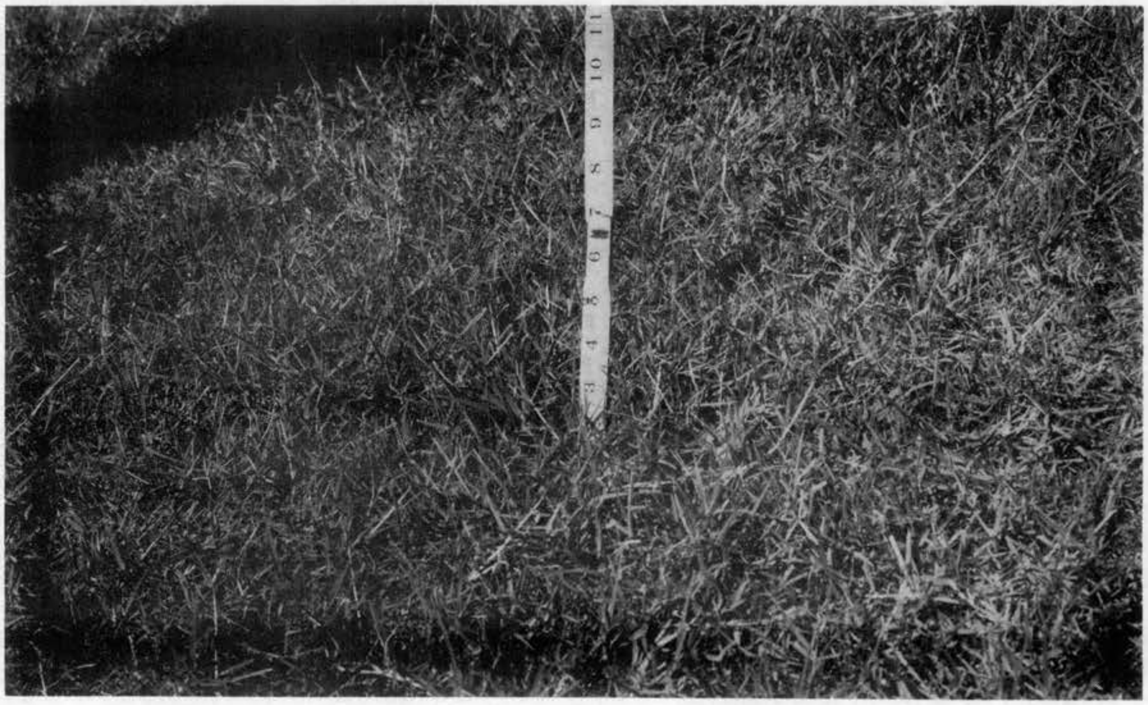

Figure 9. Grass Condition in 1964, FC31

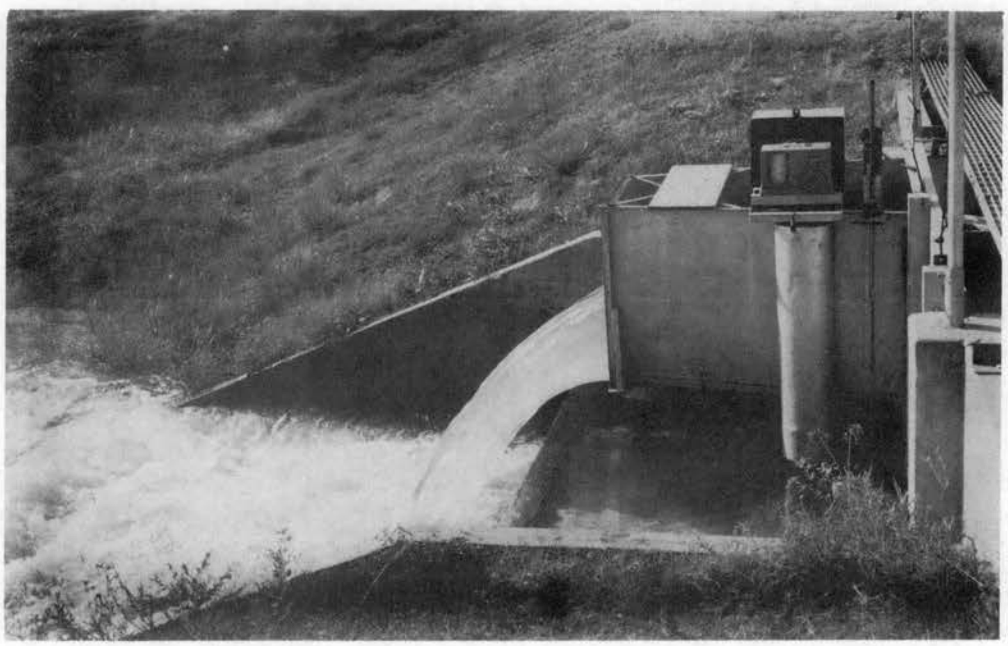

Figure 10. Two-Foot Modified Parshall Flume

Figure 11. Spatially Varied Flow into FC 31

exit conditions allowed uniform, nonuniform, or spatially varied flow experiments to be conducted. The total system capacity was 40 cubic feet per second.

The water surface profile was measured with the gage wells equipped with manual point gages and FW-1 recorders as shown in Figure 12. The farthest downstream well was approximately 10 feet from the outlet; the next was 25 feet upstream from this well; all the rest were on a 50-foot spacing. For spatially varied flow experiments the head on the adjustable weir was measured with the three gage wells in Forebay 2 and with a special point gage that was moved down the weir. This gage is shown in Figure 13.

During the 1964 testing season, three current metering stations were installed in the channel. They were located 27, 200, and 396 feet, respectively, from the upper end of the adjustable weir. Figure 14 shows a current meter in place at one of the stations.

Detailed Description

Test Channel and Outlet Structure:

The design shape and slope of channel were not exactly realized in the field. The final grading left a few irregularities, some settling occurred, and some erosion took place during establishment of the vegetal cover.

A concrete retaining wall with apron was installed at the downstream end of the channel. The retaining wall was

Figure 12. Gage Well and Equipment

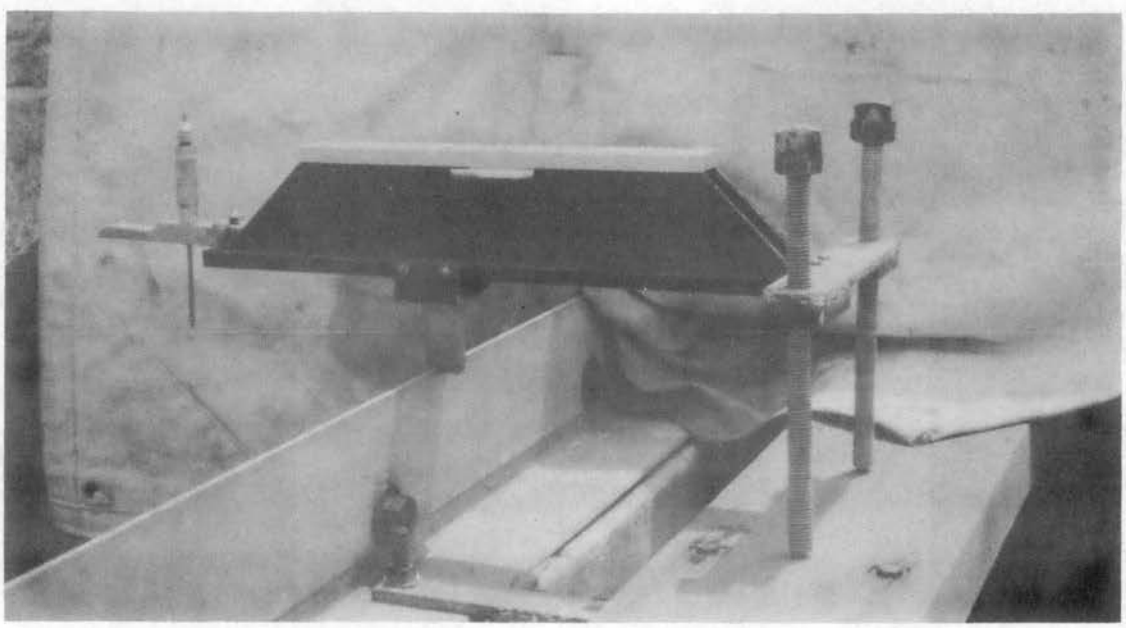

Figure 13. Direct-Measuring Weir-Head Point Gage

Figure 14. Current Meter and Velocity Direction Vane

built to the design cross section of the channel. The apron and downstream channel were low enough that free outfall could occur over the wall. A galvanized angle, 2 x 2 1/2 x 3/16, was mounted on the retaining wall to serve as a weir lip. The angle was attached to the top of the wall with the 2 1/2-inch side flat and the 2-inch side about one inch upstream from the downstream face of the retaining wall. Vertical slots were constructed on the downstream face of the retaining wall so that 2-inch lumber of various widths could be dropped into the slots and used to raise the downstream water level in the channel. This structure is shown in Figure 8.

Gage Wells:

The gage wells were of 16-inch steel pipe. Each was connected to the channel with one 1 1/2-inch galvanized pipe. This pipe was installed with the invert at the design channel bottom at each station. Each gage well was equipped with an FW-1 recorder with 5:12 pen-float ratio and 6-hour time scale. A 4-inch float was used. A 3-foot Lory point gage accurate to 0.001 foot was mounted in each gage well. Figure 12 shows a gage well and equipment.

Referencing Systems:

A bench mark was mounted in a concrete monument sunk into the ground near the channel and approximately 200 feet from the upper end. Two permanent mounts for engineers' levels were located approximately 100 and 300 feet, respectively, from the upper end of the channel. The levels were used for referencing the elevation of the gage wells and for leveling the adjustable weir. The mounts were constructed of 3-inch pipe set in concrete. Shades were provided so that direct rays of the sun would not heat the level bubbles unevenly.

Prior to the 1964 testing season a manifold system was installed such that the point gages in each well could be referenced to a common water surface. A plastic pipe was laid beside the wells and a tee, valve, and inlet were provided at each well. By plugging the pipe to the channels the wells could be filled by a pump feeding one end of the manifold. Then the wells were allowed to drain to the elevation of an outlet slightly above the elevation of the highest point of the manifold system. This provided a common water surface relatively unaffected by wind.

Inflow Introduction and Measurement:

The adjustable weir was 399.23 feet long and was mounted on a concrete retaining wall. The weir plate was aluminum and was 1/8 inch thick and 3 1/4 inches tall. It was notched for adjustment and was bolted on 3 x 3 galvanized angles mounted on the wall. The weir plate and angles were installed in sections approximately 12 feet long. Waterproof tape and calking compound were used to seal joints between the sections and between the weir plate and angle. The adjustable weir and wall are shown in Figures 11 and 13.

Three gage wells similar in form and equipment to those connected to the channel were installed in Forebay 2 near the upper end, middle, and lower end of the weir to provide information on the water elevation in Forebay 2.

The direct-measuring weir-head gage shown in Figure 13 was used to obtain the head on the adjustable weir. It was constructed of a precision level and a micrometer depth gage capable of measuring to 0.001 inch. The depth gage was clamped to the level. The end of the depth gage rod was sharpened and used as the point gage. When in use, the point was approximately 6 5/8 inches upstream from the upper edge of the adjustable weir.

A pair of leveling wells were constructed to aid in leveling the adjustable weir. These consisted of two small round plexiglass wells connected with clear plastic tubing and with micrometer depth gages mounted on top. The wells had angle irons on the bottom such that they sat astride the weir plate. The difference in elevation between two points on the weir could be determined to be approximately 1 or 2 thousandths of an inch.

Velocity Distribution Measurement:

Three current metering stations were installed in the channel in the summer of 1964. At each station the channel was spanned with an open-web steel joist on which was mounted a 3 x 3 angle running the length of the joist and marked at half-foot intervals. This angle supported a small rider and clamp. The rider was equipped with a mount for a Lory point gage. A Gurley pygmy current meter was mounted on a rod on the point gage. Use of the point gage allowed exact knowledge of the depth of the current meter while taking a series of readings. For testing spatially varied flow, a vane and protractor were used to determine the direction of current. This latter arrangement is shown in Figure 14. The vane was 0.10 foot above the meter at Stations A and B and the same elevation as the meter at Station C. A headphone and a stopwatch were used at the upstream and middle current metering stations. A signal counter and stopwatch set was used at the downstream station.

The gage well and current meter station locations are presented in Table IV. The reference point is the end of the adjustable weir nearest the upstream end of the channel. The distance from this reference point was used in all computations involving distance down the channel.

Rating of Adjustable Weir:

A length of the adjustable weir was mounted in a model basin to obtain a rating curve. The basin was 5.66 feet wide and the weir extended all the way across the basin. The configuration was as shown in Figure 15. This was intended to match as closely as possible the approach conditions in the field installations. The head was obtained with a Lory point gage in a small gage well whose inlet was greater than 4 h upstream from the weir, and also with the direct-measuring weir-head gage. The weir was divided into four parts of equal length, and the direct-measuring gage was set at the center of each length when obtaining readings.

TABLE IV

LOCATIONS OF GAGE WELLS, CURRENT METER STATIONS, AND OUTLET WEIR

Gage Well Number	Nominal Station	Distance From Upper End Of Weir (ft.)
1	0 + 25	23.6
2	0 + 75	73.6
3	1 + 25	123.6
4	1 + 75	173.6
5	2 + 25	223.6
6	2 + 75	273.6
7	3 + 25	323.6
8	3 + 75	373.6
9	4 + 00	399.2
Lip Of Outlet Weir	4 + 10	409.6
Current Meter Station A	- -	26.8
Current Meter Station B	- -	199.8
Current Meter Station C	- -	396.2

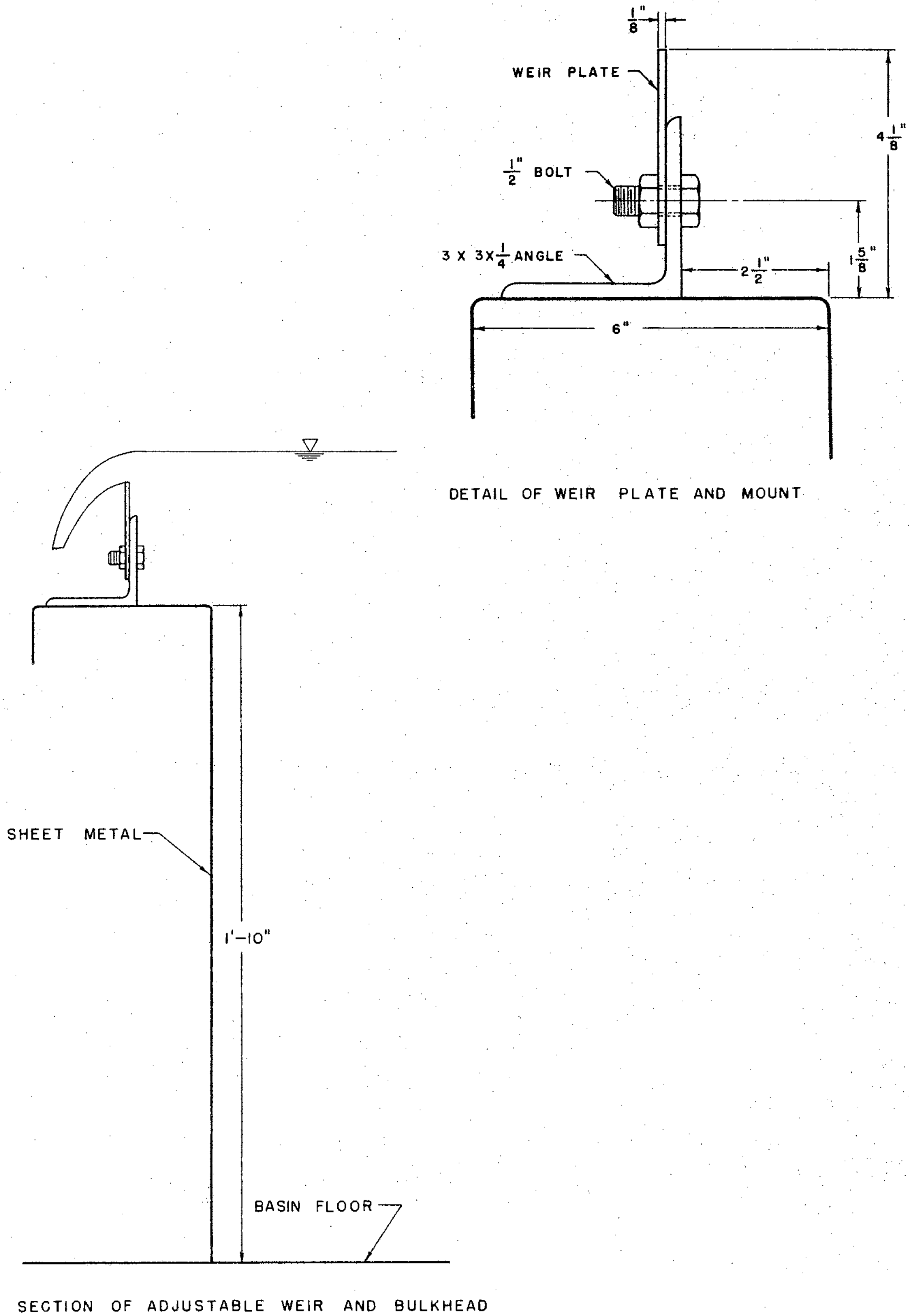

Figure 15. Setup to Rate Adjustable Weir

CHAPTER V

EXPERIMENTAL PROCEDURE

General Procedure

The general experimental procedure was to conduct uniform and nonuniform flow experiments to determine a resistance-vegetal condition relation for the test channel and then to conduct spatially varied steady flow experiments to test water surface profile predictions made using theoretical equations solved by digital computer with the resistance-vegetal condition relation as input information. The first experiments were conducted in 1963. Analysis of the spatially varied flow data indicated the need for obtaining a rating curve for the adjustable weir. This was done in the winter of 1963-64. Experiments were continued in the summer of 1964 and included repetitions of the 1963 experiments as well as measurements of velocity distribution for nonuniform flow and spatially varied steady flow.

Details of the 1963 Experimental Procedure

The 1963 testing schedule is presented in Table V. The first three experiments in 1963 were uniform flow experiments. End sills were used to raise the outlet until a condition

TABLE V

SUMMARY OF TESTS CONDUCTED, FC 31, 1963

Expt. No.	Test No.	Date	Length Of Culms (in.)	Type Of Test	Water Temp. °F.	Discharge Through Parshall Flume (cfs)	Leakage (cfs)	Corrected Discharge (cfs)
1	1	8-20-63	3.08	Uniform	83	2.035	0.000	2.035
	2	8-20-63	3.08	Uniform	82	5.043	0.000	5.043
	3	8-20-63	3.08	Uniform	82	9.832	0.000	9.832
	4	8-20-63	3.08	Uniform	80	20.154	0.000	20.154
	5	8-20-63	3.08	Uniform	80	29.92	0.000	29.92
2	1	8-23-63	4.00	Uniform	81	2.050	0.000	2.050
	2	8-23-63	4.00	Uniform	82	5.087	0.000	5.087
	3	8-23-63	4.00	Uniform	82	9.002	0.000	9.002
	4	8-23-63	4.00	Uniform	81	20.32	0.000	20.32
	5	8-23-63	4.00	Uniform	81	29.98	0.000	29.98
3	1	8-26-63	4.19	Uniform	83	2.144	0.000	2.144
	2	8-26-63	4.19	Uniform	83	5.076	0.000	5.076
	3	8-26-63	4.19	Uniform	82	8.907	0.000	8.907
	4	8-26-63	4.19	Uniform	81	20.04	0.000	20.04
	5	8-26-63	4.19	Uniform	81	29.73	0.000	29.73
4	1	8-27-63	4.30	Nonuniform	83	2.200	0.000	2.200
	2	8-27-63	4.30	Nonuniform	83	5.131	0.000	5.131
	3	8-27-63	4.30	Nonuniform	84	9.016	0.000	9.016
	4	8-27-63	4.30	Nonuniform	84	20.26	0.000	20.26
	5	8-27-63	4.30	Nonuniform	84	29.90	0.000	29.90
5	1	8-29-63	3.04	Nonuniform	82	2.248	0.000	2.248
	2	8-29-63	3.04	Nonuniform	82	5.197	0.000	5.197
	3	8-29-63	3.04	Nonuniform	82	9.511	0.000	9.511
	4	8-29-63	3.04	Nonuniform	81	20.50	0.000	20.50
	5	8-29-63	3.04	Nonuniform	81	30.35	0.000	30.35
6	1	9-4-63	3.83	Spat. Var.	79	5.01	.65	4.36
	2	9-6-63	3.95	Spat. Var.	77	4.95	.65	4.30
	3	9-6-63	3.95	Spat. Var.	78	9.57	.65	8.92
	4	9-6-63	3.95	Spat. Var.	81	19.53	.65	18.88
	5	9-11-63	4.12	Spat. Var.	80	28.16	.37	27.79
	6	9-11-63	4.12	Spat. Var.	81	38.46	.37	38.09
	7	10-10-63	3.86	Spat. Var.	74	5.15	.24	4.91
	8	10-10-63	3.86	Spat. Var.	72	9.34	.24	9.10
	9	10-11-63	3.87	Spat. Var.	72	19.81	.24	19.57

approximating uniform flow was obtained. Five test flows, approximately 2, 5, 10, 20, and 30 cubic feet per second were run at each of three roughness conditions: just after mowing, a few days later, and at an arbitrary maximum length. The vegetal condition of the channel lining was described by determining the average length of the vegetative and flowering culms of the bermudagrass. A single measurement was made of each vegetative or flowering culm arising at a node of a stolon. The measurement was made from the node to the apex of the longest blade or racene of the inflorescence. Some annual bristlegrass was present. Measurements of the length to the apex of the longest leaf or inflorescence of each individual plant were taken in a similar manner and were averaged with the bermudagrass measurements. In the early part of the season, these measurements were taken at several locations throughout the length of the channel. By the latter part of the testing season a system had been devised in which the grass was measured in 24 two-inch squares. The square was placed at random three times between each profile station, twice on the long flat slope and once on the short steep slope.

The water surface profile was determined by using the average of ten point gage readings taken successively in each gage well down the channel. Twenty readings were taken on the Parshall flume, ten before and ten after taking the channel readings. This same procedure was used on the nonuniform and spatially varied flow experiments.

Two nonuniform flow experiments of five tests each were conducted at minimum and maximum grass lengths. The only difference in testing procedure from that of the uniform flow experiments was that no end sills were used.

A spatially varied flow experiment of nine tests with total discharges of 5, 10, 20, 30, and 40 cubic feet per second was conducted in 1963. The repetition of some tests was necessary because the early tests indicated that the inflow was not uniformly distributed as had been desired and that a measurement of the head on the adjustable weir was necessary to determine inflow distribution. During the later tests, head measurements were obtained at 25-foot intervals down the weir using the direct-measuring point gage. The shape of the weir crest proved to be such that both an adhering and a springing-free condition were obtained. This dual behavior did not cause unusual difficulties because the transition occurred at about 12 to 13 cubic feet per second. Only at the 10 cubic feet per second discharge was there any mixed flow. The situation was handled by including the position of the adhering and springing-free flow in the test notes and watching to see that these positions remained stable during the test.

Some seepage through the dikes around Forebay 2 was observed during preliminary tests of the adjustable weir. This made it necessary to estimate the amount of leakage to be subtracted from the discharge measured with Parshall flume for the spatially varied flow tests. The surface area of Forebays 1 and 2 at the elevation of the adjustable weir was determined. Then immediately after conducting a spatially varied flow test the rate of fall of the water in Forebays 1 and 2 was measured. This rate of fall multiplied by the surface area gave the rate of leakage. The leakage rate was determined in this manner several times during the 1963 testing season. The data are included in Table V.

Engineers' levels on the permanent mounts were used in 1963 to reference the elevations of the point gages in the channel gage wells and of those in the wells in Forebay 2. The levels were also used to level the adjustable weir. The length of sight ranged from approximately 30 to 105 feet.

Bottom elevation readings were taken across the channel at half-foot intervals at each profile station. This was done five times during 1963. Examination of the data showed negligible erosion, and the data were averaged.

Weir Rating Procedure

A 5.66-foot length of the adjustable weir was tested in the model basin under both adhering and springing-free conditions. During the earlier tests the head on the weir was obtained at the gage well located upstream from the weir. However, this did not prove entirely satisfactory, and during later tests the head was obtained with both the gage well and the direct-measuring weir-head gage. Readings with the direct-measuring gage were taken along the weir at four places spaced so that each reading was for an equal length of weir. These head readings were averaged. Discharges were measured using orifice plates located in the pipeline leading to the testing basin.

Details of the 1964 Experimental Procedure

Experience in 1963 indicated the engineers' levels used with a length of sight of up to 105 feet to be unsatisfactory for precision referencing and for leveling of the adjustable weir. In 1964 the manifold system connecting the gage wells was used to reference the gage wells. The system proved to be quite satisfactory. The engineers' levels were used in referencing only for short lengths of sight such as obtaining the reference of the gage well at Station 1 + 75 from the elevation of the bench mark so that the other gages could be referenced, and for the referencing of the wells of Forebay 2 to the nearest channel wells. The adjustable weir was leveled by using the portable gage wells to establish reference points at 50-foot intervals and using an engineers' level to set the elevations between these reference points. With this method it was possible to obtain accuracy of approximately ± 0.001 foot in leveling the weir.

A summary of the experiments conducted in 1964 is presented in Table VI. The experimental procedure was changed somewhat

TABLE VI

SUMMARY OF TESTS CONDUCTED, FC 31, 1964

Expt. No.	Test No.	Date	Length Of Culms And Branches (in.)	Type Of Test	Water Temp. °F.	Discharge Through Parshall Flume (cfs)	Leakage (cfs)	Corrected Discharge (cfs)
7	1	7-21-64	2.89	Uniform	85	2.166	0.000	2.166
	2	7-21-64	2.89	Uniform	85	5.105	0.000	5.105
	3	7-21-64	2.89	Uniform	84	9.675	0.000	9.675
	4	7-21-64	2.89	Uniform	84	20.40	0.000	20.40
	5	7-21-64	2.89	Uniform	85	33.38	0.000	33.38
8	1	7-22-64	2.92	Nonuniform	86	2.119	0.000	2.119
	2	7-22-64	2.92	Nonuniform	86	5.109	0.000	5.109
	3	7-22-64	2.92	Nonuniform	86	9.422	0.000	9.422
	4	7-22-64	2.92	Nonuniform	85	20.50	0.000	20.50
	5	7-22-64	2.92	Nonuniform	85	33.35	0.000	33.35
9	1	7-28-64	2.45	Uniform	83	1.964	0.000	1.964
	2	7-28-64	2.45	Uniform	83	4.629	0.000	4.629
	3	7-28-64	2.45	Uniform	83	8.181	0.000	8.181
	4	7-28-64	2.45	Uniform	82	20.30	0.000	20.30
	5	7-28-64	2.45	Uniform	82	32.89	0.000	32.89
10	1	7-29-64	2.48	Nonuniform	82	2.268	0.000	2.268
	2	7-29-64	2.48	Nonuniform	82	5.046	0.000	5.046
	3	7-29-64	2.48	Nonuniform	84	9.174	0.000	9.174
	4	7-29-64	2.48	Nonuniform	83	20.58	0.000	20.58
	5	7-29-64	2.48	Nonuniform	83	33.48	0.000	33.48
11	1	8-4-64	2.80	Uniform	86	2.264	0.000	2.264
	2	8-4-64	2.80	Uniform	86	4.801	0.000	4.801
	3	8-4-64	2.80	Uniform	86	8.701	0.000	8.701
	4	8-4-64	2.80	Uniform	83	20.48	0.000	20.48
	5	8-4-64	2.80	Uniform	84	33.42	0.000	33.42
12	1	8-5-64	2.94	Nonuniform	86	2.379	0.000	2.379
	2	8-5-64	2.94	Nonuniform	86	4.976	0.000	4.976
	3	8-5-64	2.94	Nonuniform	85	9.021	0.000	9.021
	4	8-5-64	2.94	Nonuniform	84	20.40	0.000	20.40
	5	8-5-64	2.94	Nonuniform	83	33.06	0.000	33.06
13	1	8-27-64	*	Nonuniform	80	1.961	0.000	1.961
	2	8-28-64	*	Nonuniform	77	5.048	0.000	5.048
	3	8-28-64	*	Nonuniform	82	8.983	0.000	8.983
	4	8-31-64	2.28	Nonuniform	81	18.98	0.000	18.98
	5	8-31-64	2.28	Nonuniform	80	29.04	0.000	29.04
14	1	9-22-64	**	Spat. Var.	75	4.08	0.29	3.79
	2	9-23-64	**	Spat. Var.	74	9.59	0.29	9.30
	3	9-24-64	**	Spat. Var.	—	19.50	0.29	19.21
	4	9-24-64	**	Spat. Var.	75	29.99	0.29	29.70
	5	9-25-64	**	Spat. Var.	73	39.62	0.29	39.33

*Estimated 2.28
**Estimated 2.35

from that used in 1963. An attempt was made to remove the effect of vegetal condition when comparing uniform and nonuniform flow experiments by conducting a uniform flow experiment on one day and a nonuniform flow experiment on the following day. The vegetal condition was assumed to change very little between the two experiments. The intention was to conduct three sets of uniform and nonuniform flow experiments at three grass lengths. However, the vegetation length data in Table VI show that Experiments 7 and 8 and Experiments 11 and 12 were conducted at approximately the same grass length. The water surface and discharge measurements were obtained in the same manner as in 1963. The method used to describe the vegetal condition differed from that used in 1963 in that the length of all vegetative and flowering culms and branches in each two-inch square were measured and averaged. Very few weeds or undesirable grasses were present in 1964. For nearly all determinations the two-inch square was placed randomly three times in each reach. An alternate method was tried whereby the material in a six-inch square was clipped, dried, and weighed. The same pattern of 24 samples, three per reach with two taken from the long slope and one taken from the short slope, was tried. The samples were air-dried for several days and then were oven-dried at approximately 180 to 190 degrees Fahrenheit for 24 hours and weighed. A check on a sample left for 12 more hours showed no further weight change.

Following these experiments the three current metering stations were installed and a nonuniform flow experiment of five tests was conducted and velocity distribution information was obtained. Velocity observations were taken at different depths at the marked vertical stations across the channel. For the tests with smaller discharges at Stations A and B, and for all tests at Station C, the half-foot stations were used. For the tests with larger discharges at stations A and B, one-foot stations were used. The observations were taken at intervals of one-tenth of the depth along each vertical except near the edges of the channel, where the interval between vertical settings would have been quite small. Observations could not be obtained closer to the bottom than approximately 0.2 foot because of grass tangling in the current meter cups.

A current direction vane and protractor were mounted on the current metering rod and a spatially varied steady flow experiment of five tests was conducted. Velocity direction and magnitude observations were taken at approximately the same vertical stations as for the nonuniform flow experiment. Observations were taken closer to the surface and closer to the bottom than in the previous experiment in an attempt to better define the isovels in those regions. The head on the adjustable weir was obtained every 25 feet down the weir using the direct-measuring point gage. There was no problem with mixed flow because with the new improved leveling system there were only very small differences in head down the weir.

Leakage determinations were made during the spatially varied flow tests in the same manner as in 1963. Before the start of the testing season the dike around Forebay 2 was raised and strengthened, and the leakage rate was decreased from that observed in 1963. The observed leakage rates from 1964 are included in Table VI.

Bottom elevation readings were taken across the channel at half-foot intervals at each profile station as in 1963. This was done four times during the 1964 testing season and the elevations were averaged.

CHAPTER VI

PRESENTATION, ANALYSIS, AND DISCUSSION OF DATA

Introduction

The object of all testing prior to the spatially varied steady flow experiments was to provide knowledge of resistance and velocity distribution in the channel and to enable measurement of inflow during the spatially varied flow experiments. This information was necessary for computing theoretical water surface profiles to compare with the observed profiles obtained from the spatially varied flow experiments. Therefore, the data from these prior experiments are not presented in order of collection, but rather as it seems logical to mention them in leading up to the profile prediction methods and the comparison of the results from these methods with the observed profiles.

Velocity Distribution

The data from the current meter measurements of the velocities in the channel during nonuniform flow and spatially varied steady flow Experiments 13 and 14 in 1964 were used to determine Boussinesq coefficients. The analysis was performed by a method similar to that of O'Brien and Johnson described in Chapter II.

The depth and velocity data obtained at each vertical station during Experiment 13 were plotted log depth versus velocity. A smooth curve was drawn through each set of points, and the depth values corresponding to desired isovels were taken from the plots and plotted on a drawing of the cross section. The isovels were then drawn similarly to those shown in Figure 16. The areas within the isovels, or between the isovels and the water surface, were planimetered and tabulated. These data are presented in Table A-1 in Appendix A.

The corresponding portion of the analysis of the spatially varied flow data from Experiment 14 was slightly more complicated than that for Experiment 13. The velocity observations were corrected for the component down the channel. The vane was 0.10 foot higher than the meter at Stations A and B, so the correction could not be made directly for these stations. The angle-from-axis-of-channel and depth data were plotted for each vertical station, the angle at the depth of a given velocity reading was determined from this plot, and the velocity was multiplied by the cosine of the angle. The depth and this velocity component were then plotted log depth versus velocity and the analysis proceeded as for Experiment 13. The velocity-area data from Experiment 14 are presented in Table A-2 in Appendix A.

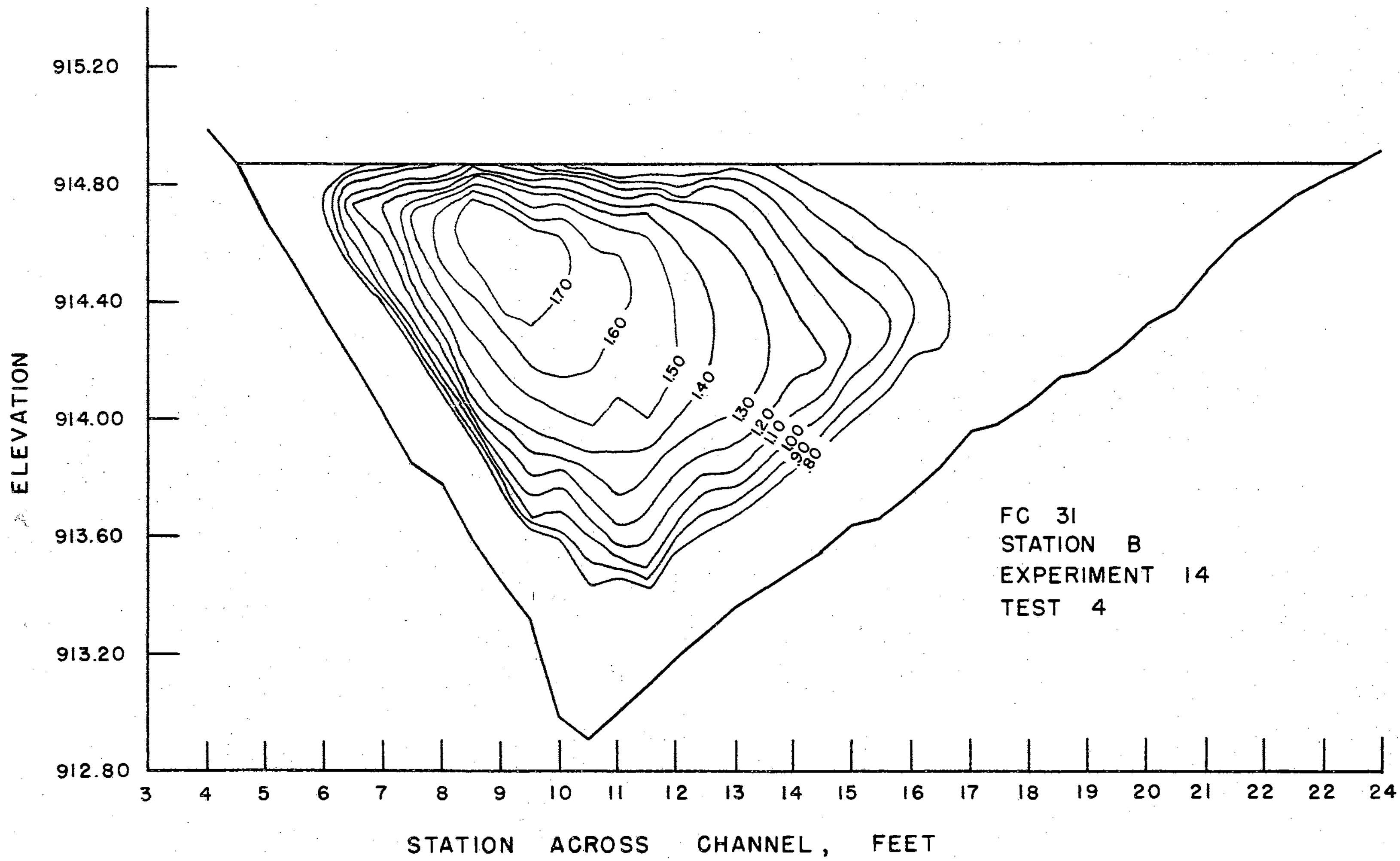

Figure 16. Velocity Distribution at Station B, Experiment 14, Test 4

When the data were taken the current meters could not go within approximately 0.2 foot of the bottom because of grass tangling in the cups, so velocity data were unavailable for this region. Using the method of O'Brien and Johnson and assuming the velocity to be linearly distributed from the last known isovel to the bottom yielded discharges greater than the total measured flow. This led to the belief that the grassed portion of the channel was carrying very little flow and that perhaps some effective bottom elevation or effective cross-sectional area could be determined. Various schemes such as a trial and error method of finding a constant value to raise the channel bottom were tried. The method finally chosen was to plot velocity versus area similarly to Figure 17, and to integrate between zero area and the area of the lowest defined isovel to obtain the discharge within that portion of the cross section. This was always less than the total measured discharge. The difference was computed and was assumed to be conveyed at one-half the value of the lowest defined isovel. The additional area needed to convey this residual discharge was computed. This value was added to the value of the total enclosed area at the lowest known isovel to give an effective area which was plotted at zero velocity as shown in Figure 17. With this point determined it was possible to calculate velocity distribution coefficients by the method of O'Brien and Johnson.

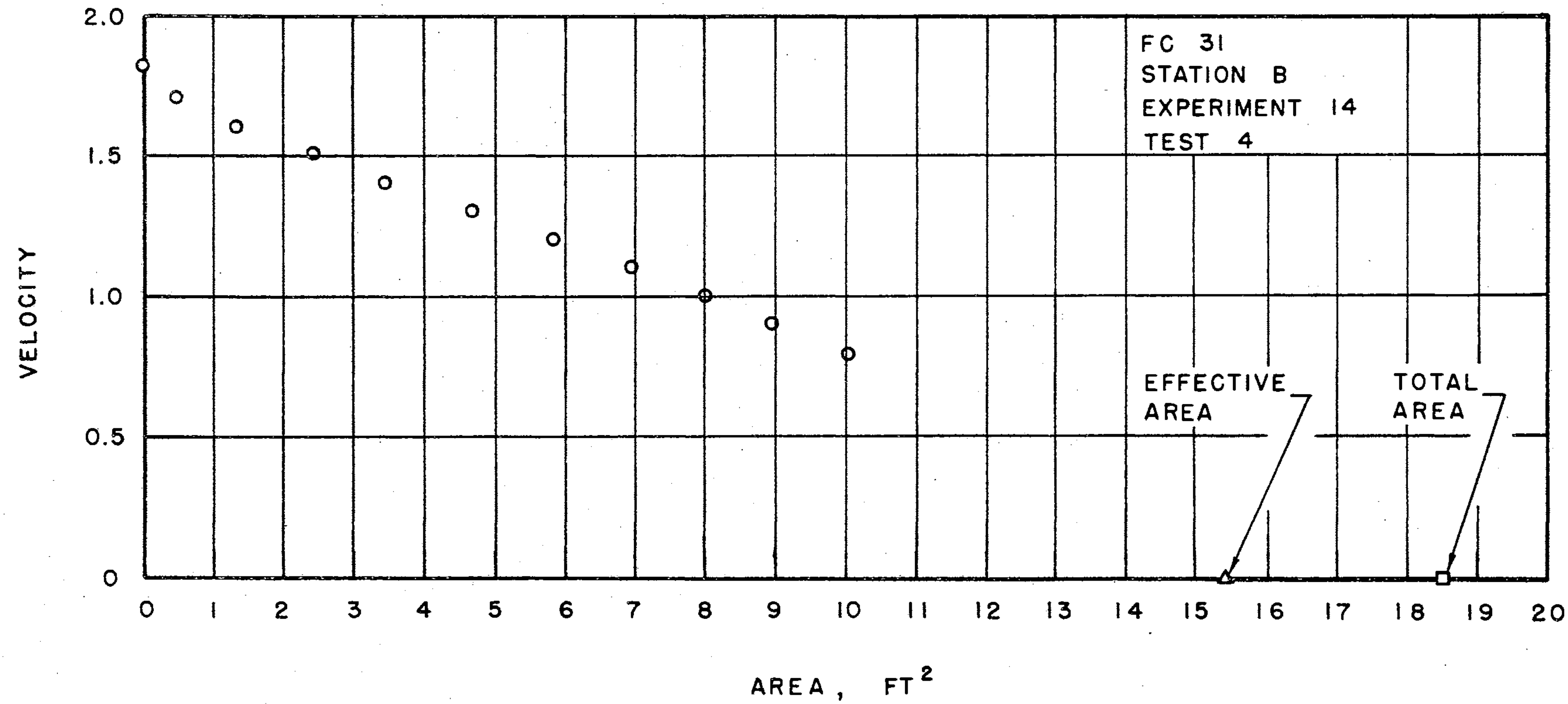

Figure 17. Velocity and Area Data from Station B, Experiment 14, Test 4

A part of the O'Brien and Johnson method was replaced by two computer programs. The velocity and area data were obtained as in the O'Brien and Johnson method. The velocity was then related to area by a least-squares polynomial using the Polyfit Fortran IV program for the IBM 1410 listed in Table B-1 in Appendix B. This program fitted the data with a least-squares polynomial of up to degree four. The coefficients of the polynomial obtained from the Polyfit program were used in the Alphabet 3 program listed in Table B-2 in Appendix B which integrated beneath the fitted polynomial down to the value of the last known isovel by Simpson's rule, then determined the residual discharge and the effective area. The program determined the Boussinesq and Coriolis coefficients by squaring and cubing the velocity-area relationships, integrating the resulting relationships over the effective area, and dividing the integration sums by the product of the total area and the mean velocity squared and by the product of the total area and the mean velocity cubed, respectively.

Polynomials of second, third, and fourth degree were tried in the program. The best fit of most of the data was obtained by using the maximum observed velocity at each station as the velocity for zero area and using a polynomial of degree four. The maximum observed velocity was not used for the data from Station C of Test 4 of Experiment 14, because it was so high in relation to the other maximums at Station C from Experiment 14 as to be of questionable value.

Part of the input to the Alphabet 3 program included the number of increments into which the integration interval was broken. A range of values was tried with some trial data. Breaking the interval into about twenty-five increments gave good results.

The Boussinesq coefficient results as well as the discharge, total area, and effective area values from Stations B and C for Experiment 13 are presented in Table VII. The data from Station A were so erratic as to be useless and are not presented. Apparently the observer at that station was not picking up all of the signals on his headset at the higher velocities. The Boussinesq coefficient results and the discharge, total area, and effective area values from Experiment 14 are presented in Table VIII.

The Boussinesq coefficients presented in Tables VII and VIII are considerably higher than most of the values cited in Chapter II, Review of Literature. Only Ree (43, p. 187) presented a value, 1.70, within the range of those presented in Tables VII and VIII. Ree's value was also for a small grassed channel. These high values of the Boussinesq coefficient for small grassed channels are caused by the vegetation blocking a sizable portion of the cross section. Comparatively little water flows in the grassed portion of the channel. This effect can be seen by examining Figure 16 and noting the large area outside the last known isovel.

TABLE VII

BOUSSINESQ COEFFICIENTS FROM NONUNIFORM FLOW
EXPERIMENT 13, FC 31

		Station A x = 27			Station B x = 200			Station C x = 396		
Test No.	Discharge cfs	Total Area ft.^{2}	Effective Area ft.^{2}	Beta	Total Area ft.^{2}	Effective Area ft.^{2}	Beta	Total Area ft.^{2}	Effective Area ft.^{2}	Beta
1	1.961	Data Erratic, Not Presented			4.064	3.175	1.754	2.548	2.053	1.801
2	5.048				6.837	5.298	1.638	3.972	3.236	1.705
3	8.983				9.160	7.926	1.482	5.370	4.481	1.673
4	18.978				13.776	12.289	1.383	7.820	7.104	1.433
5	29.041				17.630	15.653	1.356	10.524	9.350	1.395

TABLE VIII

BOUSSINESQ COEFFICIENTS FROM SPATIALLY VARIED STEADY FLOW EXPERIMENT 14, FC 31

Test No.	Station A x = 27				Station B x = 200				Station C x = 396			
	Disch. cfs	Total Area ft.^{2}	Effec. Area ft.^{2}	Beta	Disch. cfs	Total Area ft.^{2}	Effec. Area ft.^{2}	Beta	Disch. cfs	Total Area ft.^{2}	Effec. Area ft.^{3}	Beta
1	0.289	3.714	1.958	2.494	1.912	5.109	3.128	1.950	3.766	3.939	2.980	1.794
2	.687	6.695	5.064	1.789	4.720	8.626	6.978	1.592	9.237	5.831	4.536	1.692
3	1.381	11.493	8.545	1.669	9.712	13.788	11.980	1.483	19.084	8.546	7.267	1.535
4	2.079	16.000	13.628	1.482	14.884	18.564	15.390	1.513	29.485	11.138	9.109	1.502
5	2.746	19.326	12.418	1.677	19.722	22.668	19.470	1.467	39.037	13.420	10.463	1.478

The Boussinesq values from Experiment 14 varied considerably with distance down the channel. This was a result of two factors. First, there was a very low mean velocity at Station A for the size of the cross section as compared to that at Station C. The range of the variables was not the same from one end of the channel to the other. Second, the inflow currents from the side had a stronger influence at the upper end of the channel since they were larger there in relation to the mean velocity in the channel. Figure 16 shows the effect of the inflow at Station B. There was a rather large area to the right of the cross section that had almost no effective velocity component down the channel. The area of maximum velocity was located at the left of the point of maximum depth in the channel, whereas for nonuniform flow this area occurred almost directly over the point of maximum depth.

Because the Boussinesq coefficient data showed considerable variation it was thought desirable to try to relate the Boussinesq coefficient to various parameters rather than to simply use an average value when computing resistance or water surface profiles. Possible relations were sought between the Boussinesq coefficient and area, mean velocity, discharge, hydraulic radius, Reynolds number, and Froude number, singly and in various combinations. The only attempt that showed any real promise was a log-log plotting of the Boussinesq coefficient and discharge, as shown in Figures 18 and 19 for Experiments 13 and 14, respectively. The data from each station for each experiment were transformed to logarithms and fitted using the LS02 program presented in Table B-3 in Appendix B. The resulting coefficients and exponents of the relation

$$\text{Beta} = C_3 Q^{C_4} \tag{27}$$

are presented in Table IX and the fitted lines are plotted in Figures 18 and 19. The coefficient and exponent from Experiment 13 varied only slightly from Station B to Station C, so the data were lumped and an average line fitted. The resulting coefficient and exponent are also presented in Table IX and the average line drawn on Figure 18. The loss of the data from Station A for Experiment 13 was a definite handicap in this analysis.

Only a fair fit of the data would have been obtained by lumping all of the data from Experiment 14. An attempt was made to relate the coefficient and exponent to distance down the channel. The three data points each for the coefficient and exponent were found to plot linearly on log-log paper as shown in Figures 20 and 21. A line was fitted through each set of data using the LS02 program. The resulting relationships were

$$C_3 = 1.598\ x^{0.04106} \tag{28}$$

$$C_4 = -0.5026\ x^{-0.2862} \tag{29}$$

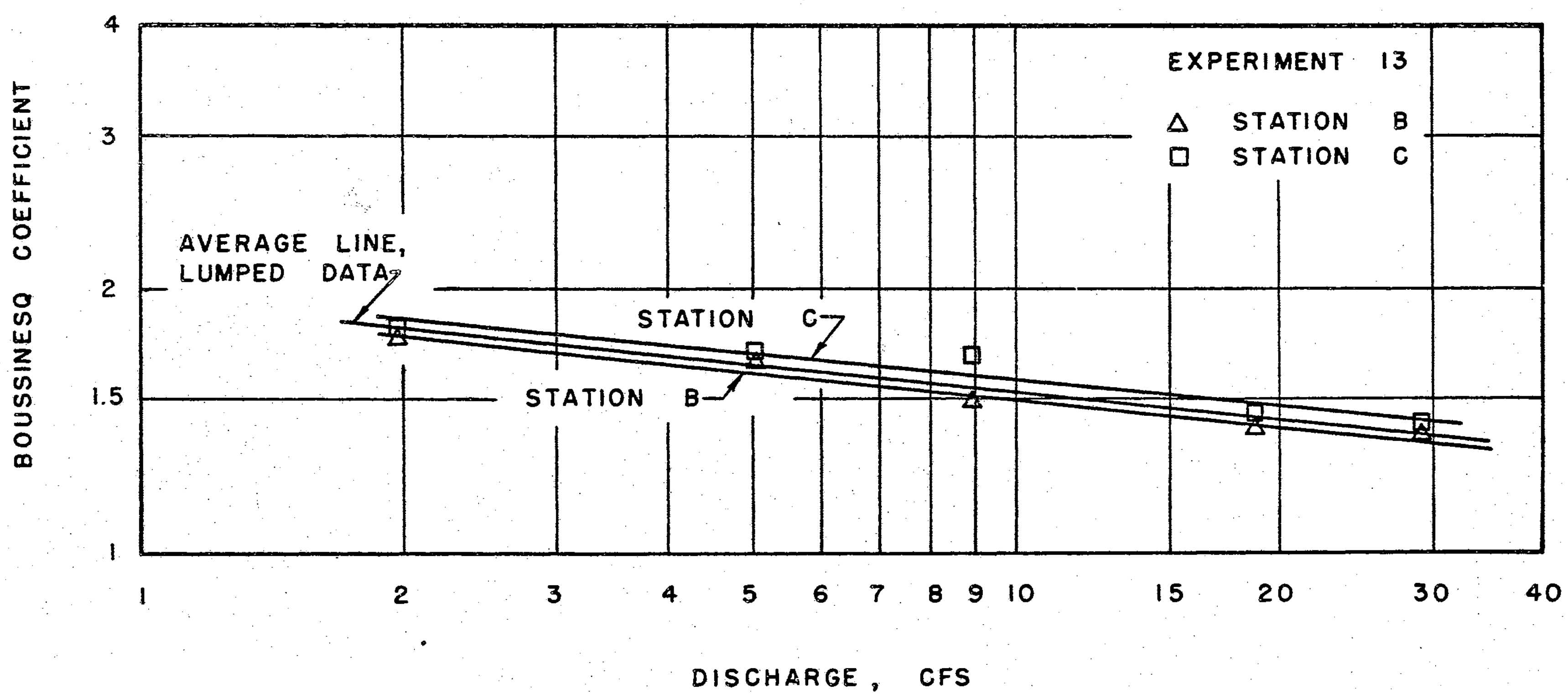

Figure 18. Boussinesq Coefficient and Discharge From Nonuniform Flow Experiment 13

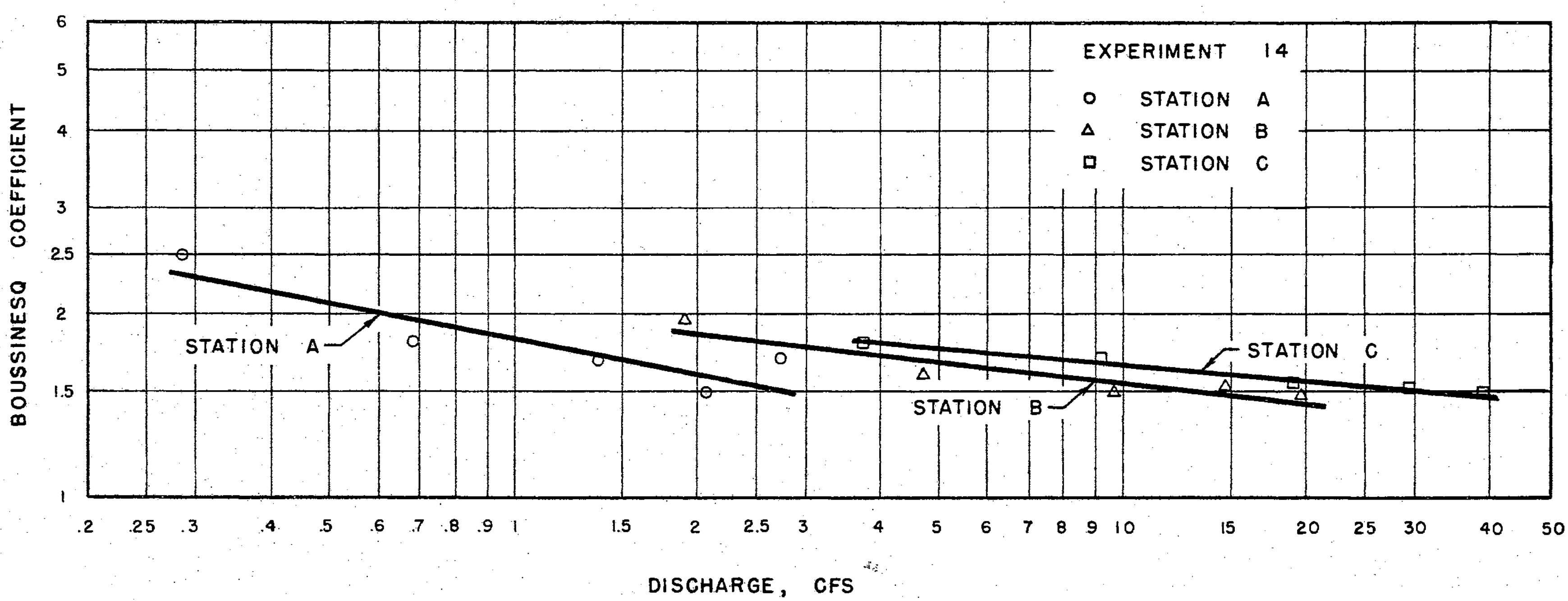

Figure 19. Boussinesq Coefficient and Discharge From Spatially Varied Steady Flow Experiment 14

TABLE IX

COEFFICIENT AND EXPONENT IN RELATIONSHIP

$$BETA = C_3 Q^{C_4}$$

Experiment	Station	Coefficient C_3	Exponent C_4
13	A	--	--
	B	1.888	-0.1018
	C	1.980	-.1006
13	B & C Lumped	1.934	-.1012
14	A	1.824	-.1935
	B	2.012	-.1153
	C	2.024	-.08779

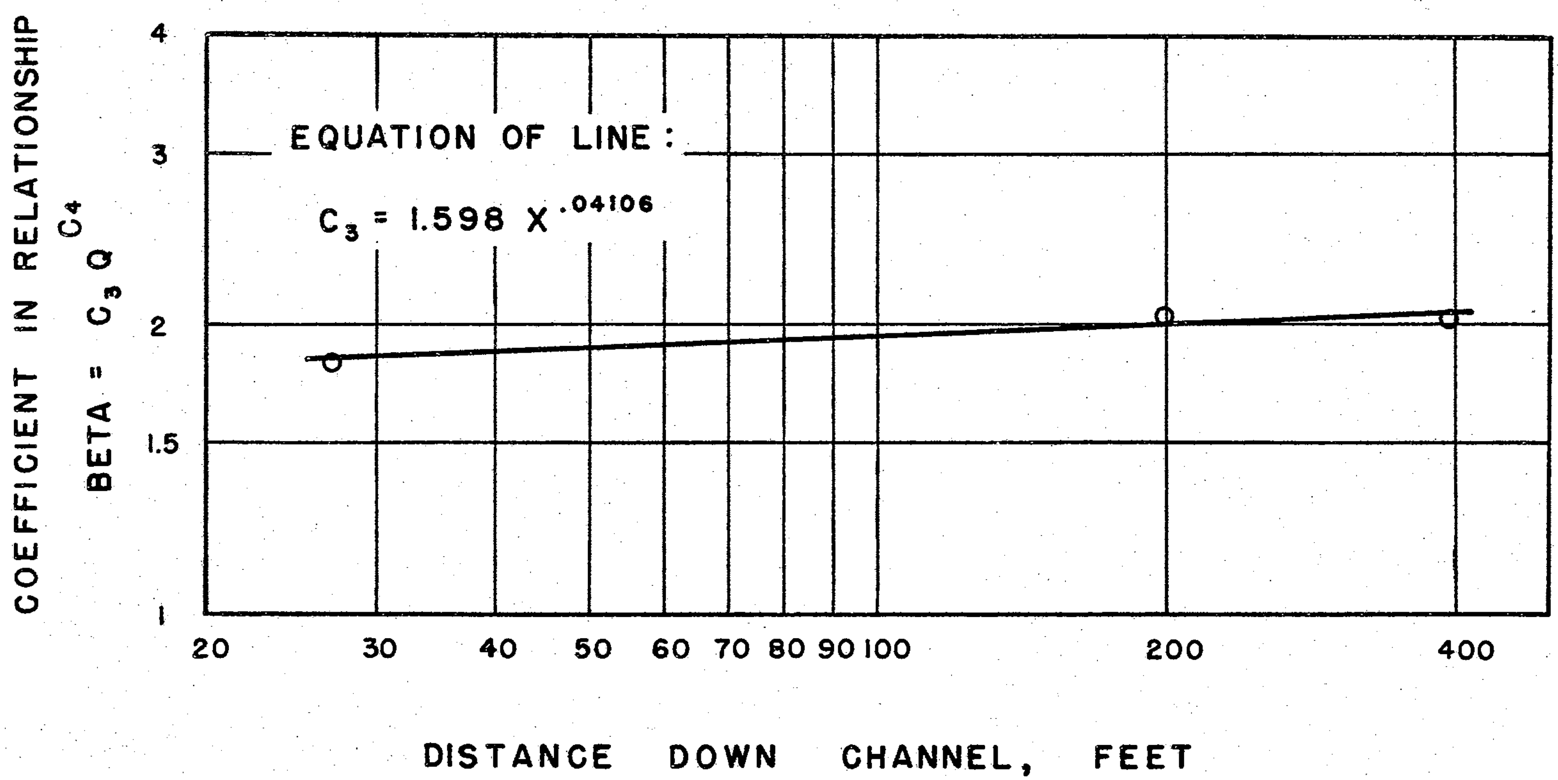

Figure 20. Coefficient in Relationship Beta = $C_3Q^{C_4}$ and Distance Down Channel, Experiment 14

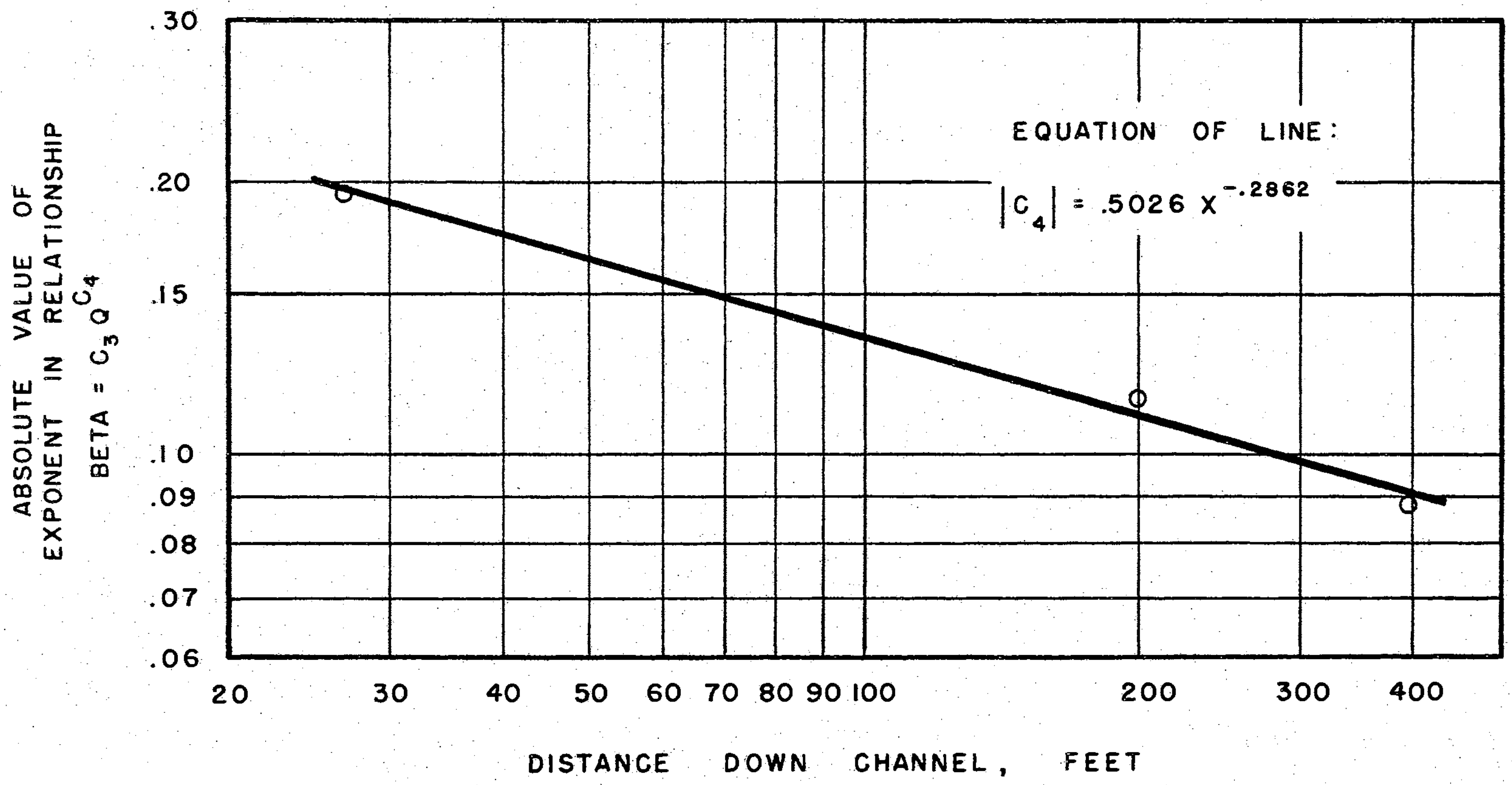

Figure 21. Exponent in Relationship Beta = $C_3Q^{C_4}$ and Distance Down Channel, Experiment 14

The fitted lines are drawn in Figures 20 and 21. The fit was so close that final fitted lines of Boussinesq coefficient versus discharge on Figure 19 would have been difficult to distinguish from the lines individually fitted to each station. Therefore, the final lines are not shown.

Resistance

The resistance of the test channel was determined using the water surface elevation and cross section data from the uniform flow and nonuniform flow experiments in 1963 and 1964, presented in Tables A-3 through A-8 in Appendix A, and the corresponding discharge and temperature data presented in Tables V and VI. Resistance was computed by two different methods. One method consisted of ignoring the variation in velocity across the channel and using Equation (19)

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_L \qquad (19)$$

to determine a head loss value. The other method consisted of assuming that the conditions of flow were such that Equation (12)

$$\beta_1 \frac{V_1^2}{2g} + y_1 + z_1 = \beta_2 \frac{V_2^2}{2g} + y_2 + z_2 + \frac{F_{s_x}}{\gamma A_{avg}} \qquad (12)$$

was applicable and using the Boussinesq coefficient relationship from Experiment 13

$$\text{Beta} = C_3 Q^{C_4} \qquad (27)$$

One of the most important restrictions on Equation (12) is that the variation between β_1 and β_2 be less than the variation between V_1 and V_2. Since Equation (27) gives the Boussinesq coefficient as a function of discharge alone, this requirement is fulfilled and Equation (12) is indeed applicable.

By the conclusions reached in Chapter II, Review of Literature, it is possible to use the Manning formula, Equation (18)

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (18)$$

with either Equation (19) or (12), so long as the resulting resistance coefficients are not used indiscriminately. The slope term simply has a different meaning and value in each case.

The amount of data to be analyzed was quite vast and a Fortran IV program for the IBM 1410 was used to carry out the analysis. The Retardance 3 program presented in Table B-4 of Appendix B was used to compute resistance using Equation (12). A slightly modified version computed resistance coefficients using Equation (19). The programs computed resistance using the data from three profile stations at a time. A resistance value was computed for the reach between the upstream and middle stations, the reach between the upstream and downstream stations, the reach between the middle and downstream stations, and finally, an average for the entire three-station reach. The output included Manning's n, the Chezy coefficient, the Darcy-Weisbach resistance coefficient, Reynolds number, velocity, hydraulic radius, and other variables.

The output was examined for possible relationships to systemize the data. One of the methods tried was a log-log plotting of Manning's n versus velocity times hydraulic radius. This method was presented by Ree and Palmer (44, pp. 21-23) and is cited in Chapter II, Review of Literature. The range of the 1963 and 1964 data computed using both Equation (19) and Equation (12) plotted in a straight band for each experiment. The averaged resistance values for the three-station reach gave the narrowest band. The results from analysis of the 1963 and 1964 nonuniform flow experiments using Equation (19) are presented in Figures 22 through 27. The results obtained using Equation (12) are presented in Figures 28 through 33. Comparison of the two figures for the same experiment shows the effect of considering the Boussinesq coefficient when computing the resistance for nonuniform flow in the test channel, FC 31. Consideration of the Boussinesq coefficient made very little difference in the resistance values at the upstream end of the channel. However, use of Equation (12) and the Boussinesq coefficient rather than Equation (19) tended to decrease the resistance values near the downstream end of the channel where the flow was accelerating. The combination of these two effects decreased the width of the band of data in Figures 28 through 33.

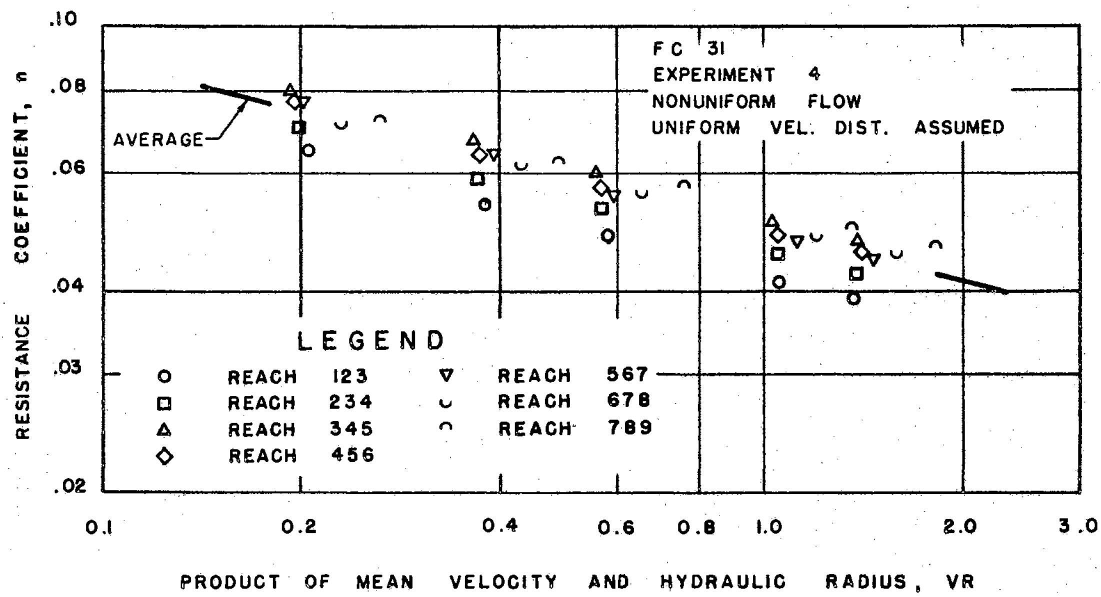

Figure 22. Resistance Computed Assuming Uniform Velocity Distribution, Experiment 4

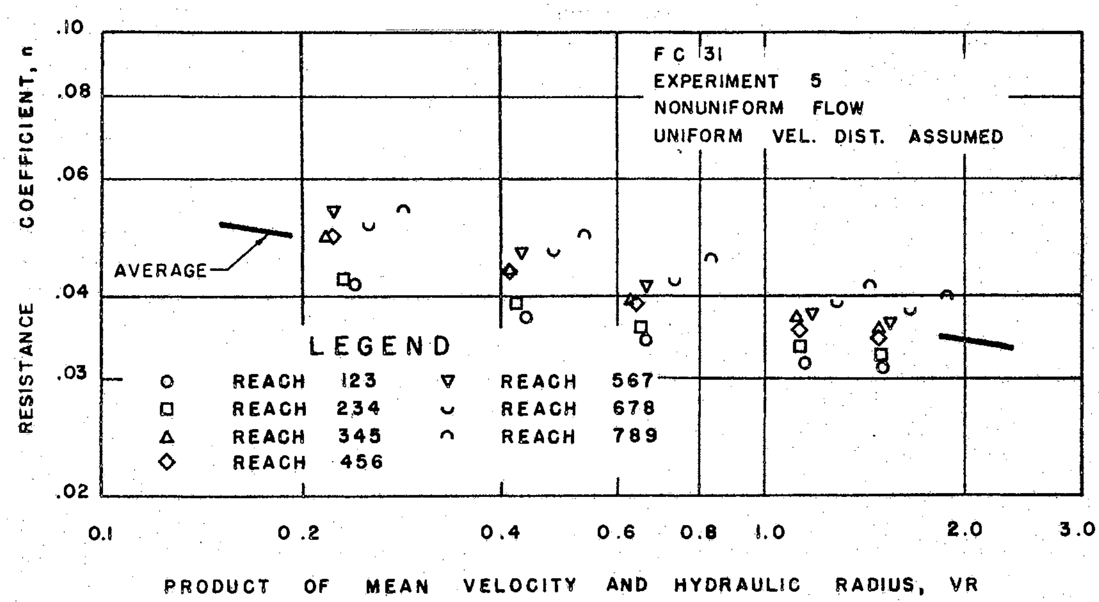

Figure 23. Resistance Computed Assuming Uniform Velocity Distribution, Experiment 5

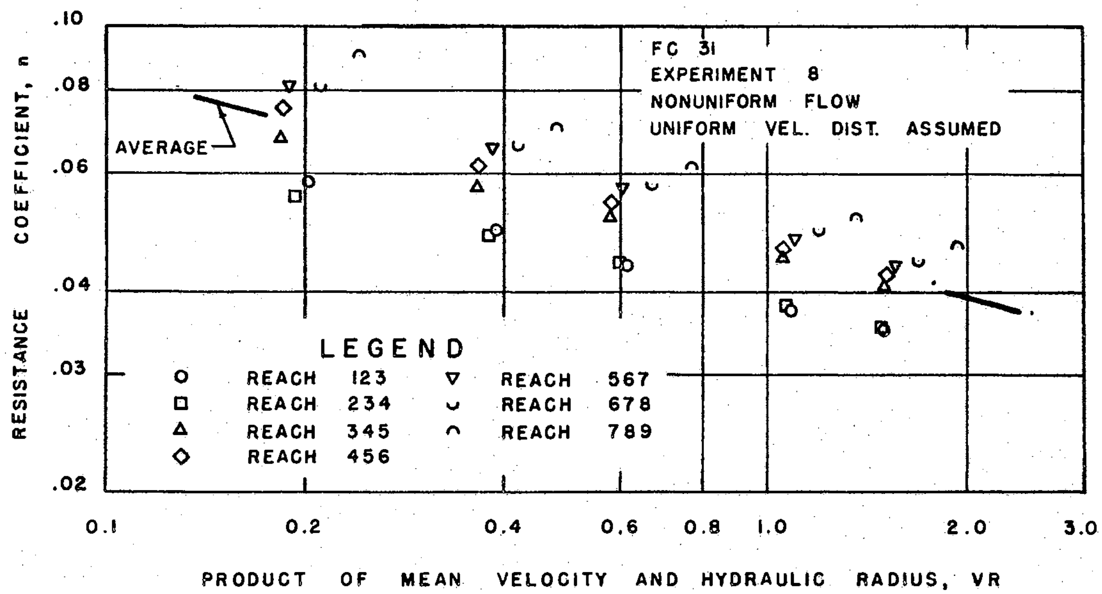

Figure 24. Resistance Computed Assuming Uniform Velocity Distribution, Experiment 8

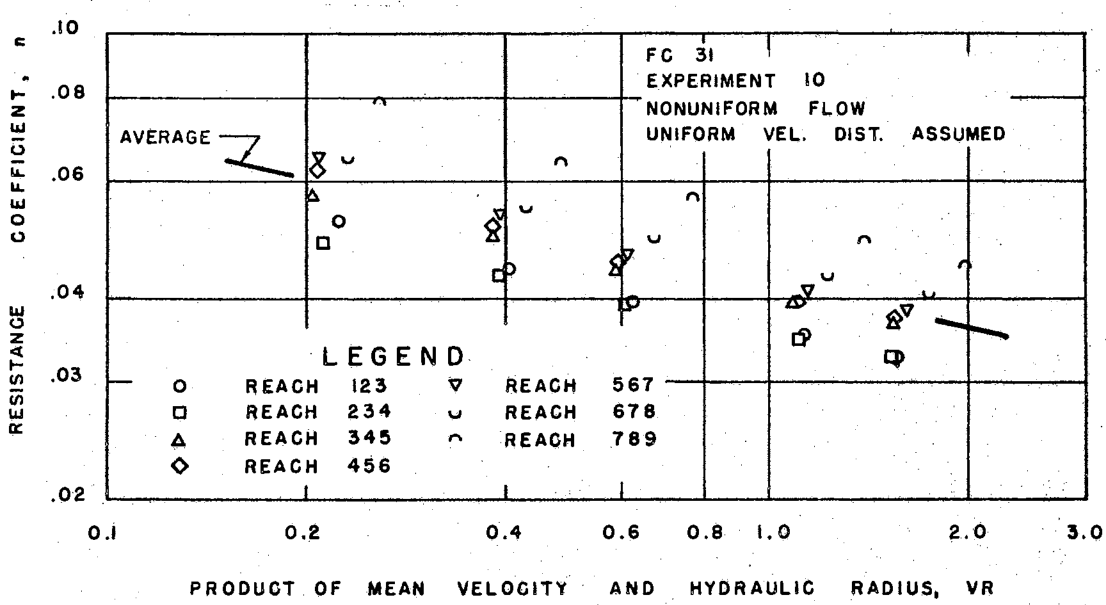

Figure 25. Resistance Computed Assuming Uniform Velocity Distribution, Experiment 10

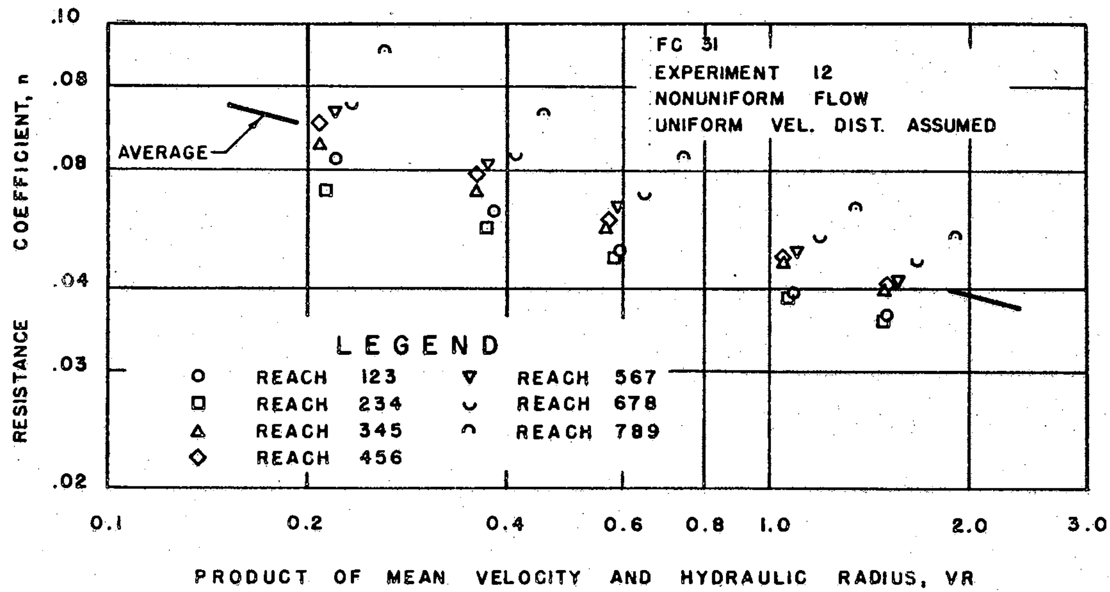

Figure 26. Resistance Computed Assuming Uniform Velocity Distribution, Experiment 12

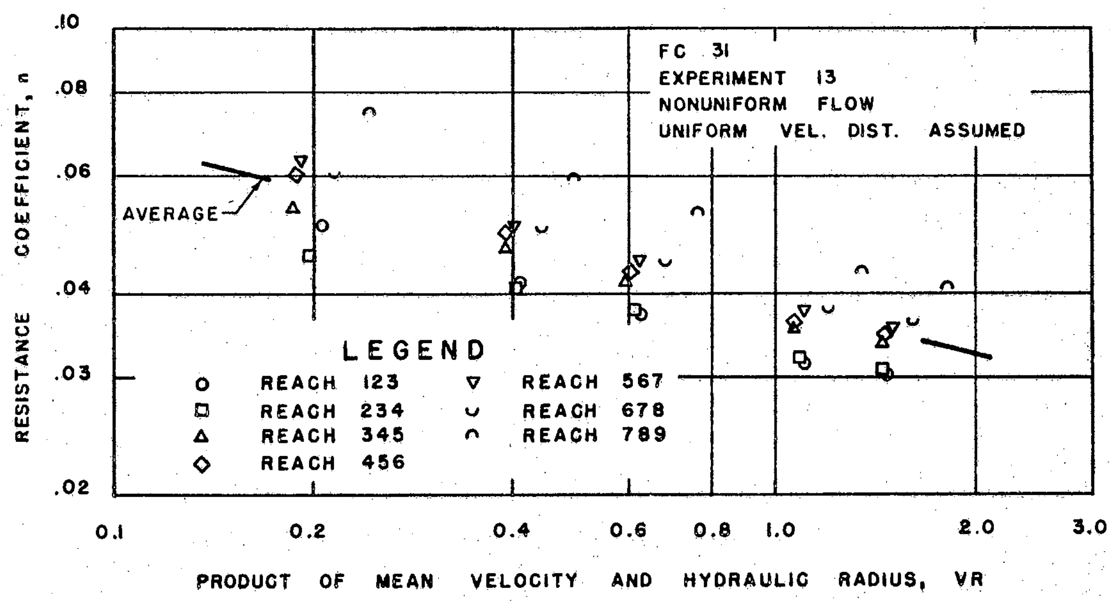

Figure 27. Resistance Computed Assuming Uniform Velocity Distribution, Experiment 13

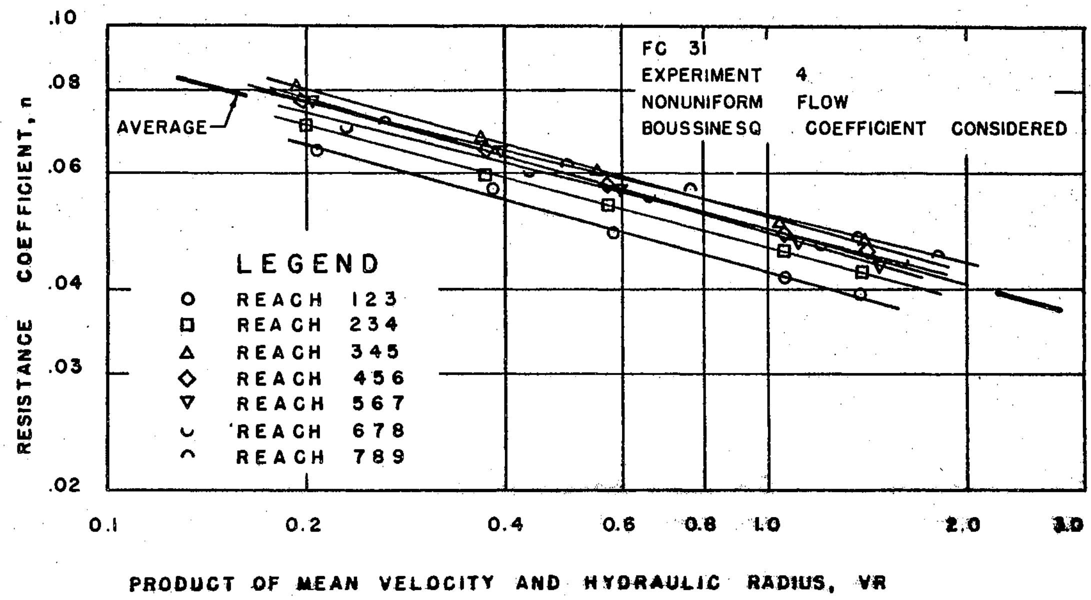

Figure 28. Resistance Computed Considering Boussinesq Coefficient, Experiment 4

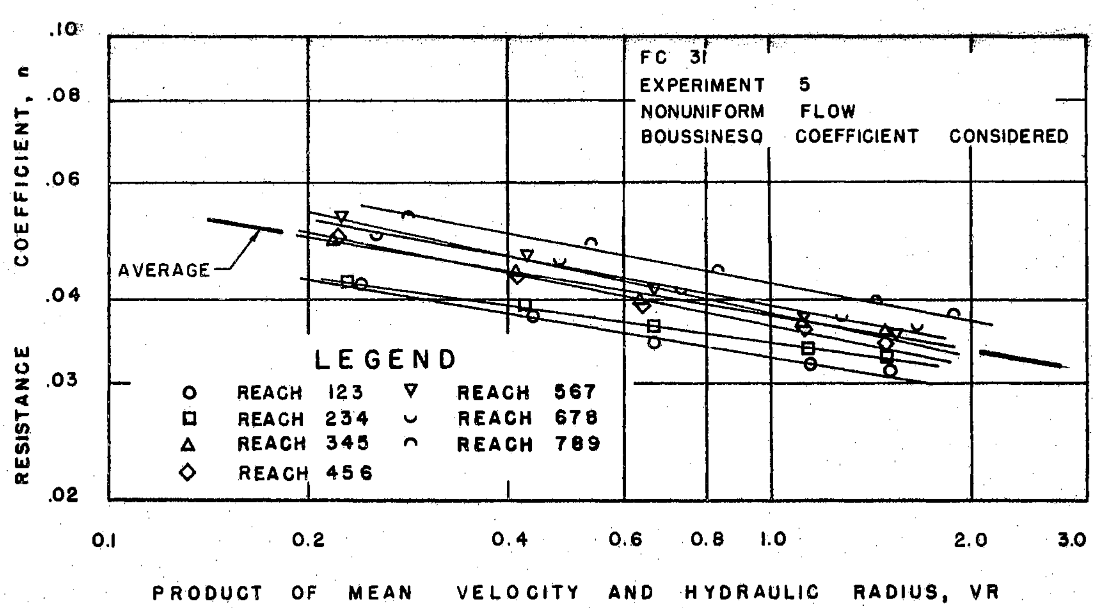

Figure 29. Resistance Computed Considering Boussinesq Coefficient, Experiment 5

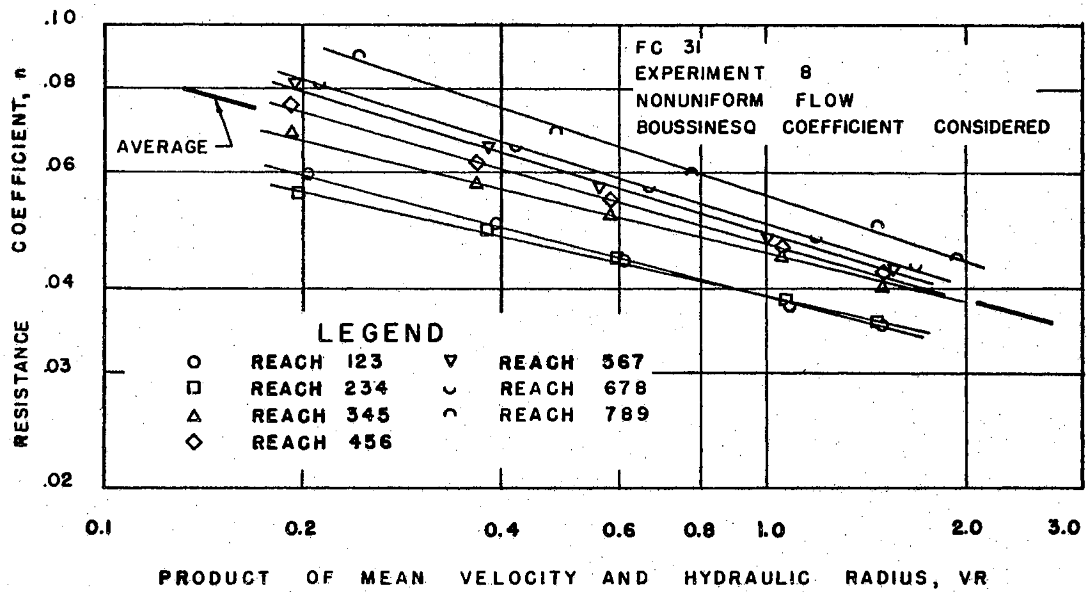

Figure 30. Resistance Computed Considering Boussinesq Coefficient, Experiment 8

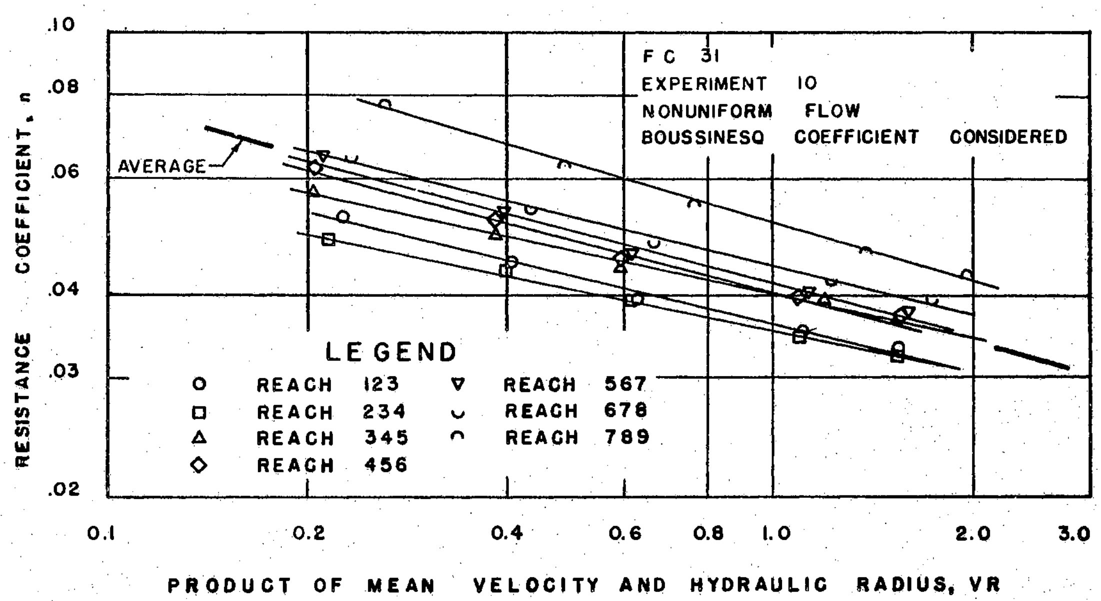

Figure 31. Resistance Computed Considering Boussinesq Coefficient, Experiment 10

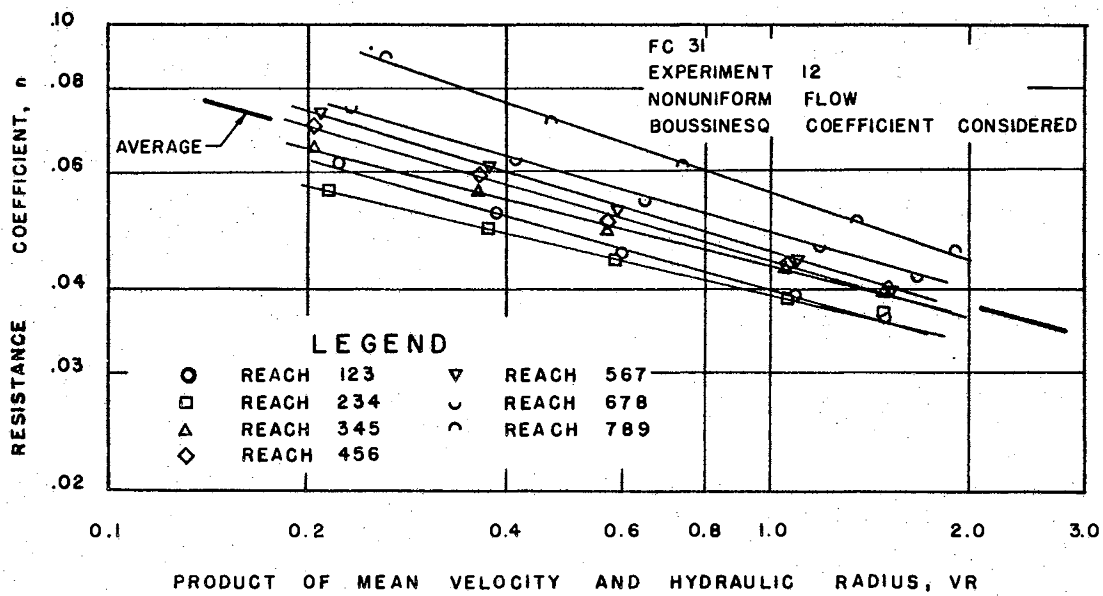

Figure 32. Resistance Computed Considering Boussinesq Coefficient, Experiment 12

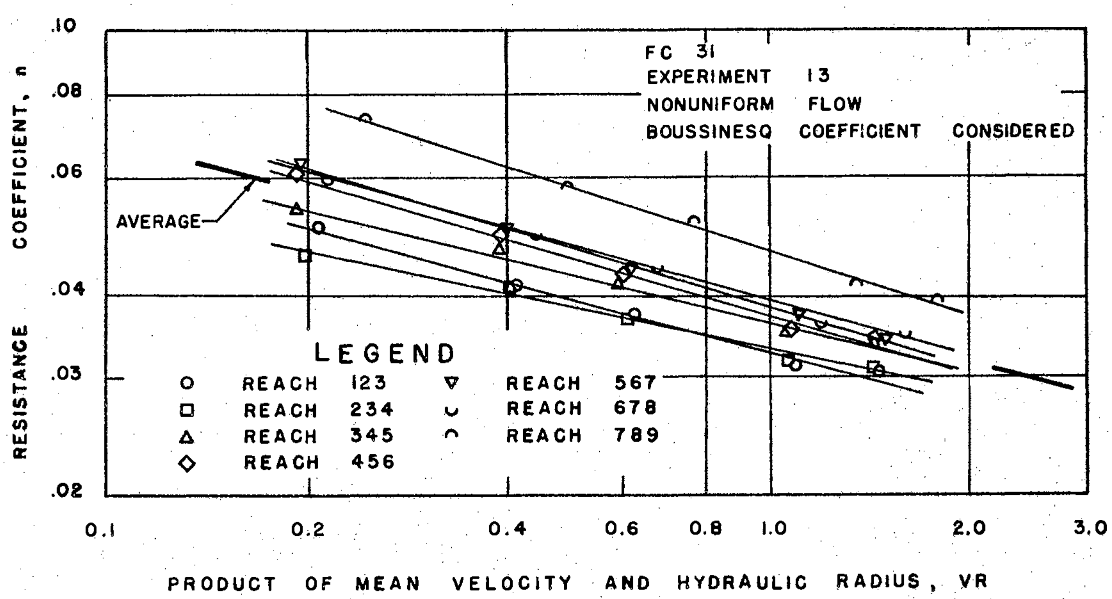

Figure 33. Resistance Computed Considering Boussinesq Coefficient, Experiment 13

An average line was fitted to each n - VR log-log plot using the LS02 program, giving the equation

$$n = C_1 (VR)^{C_2} \tag{30}$$

The coefficients and exponents of these fitted lines are presented in Tables X and XI and the average lines are shown in Figures 22 through 33. The coefficients and exponents from the uniform flow experiments are also included in Tables X and XI. Examination of the data in Tables X and XI showed that considering the Boussinesq coefficient in computing resistance for the nonuniform flow experiments tended to decrease the value of the coefficient and increase the absolute value of the exponent. For the nearly uniform flow tests, exactly the opposite effects were noted.

An attempt was made to relate resistance to the vegetal condition of the channel. The average culm length data for 1963 presented in tabular form in Table A-9 in Appendix A were plotted versus date as in Figure 34. Lines were drawn to connect the data points on the culm length versus date plot. The culm length at the time of a given experiment could then be read from this plot. Similarly, the average length of culms and branches for 1964 and the sample weight data presented in Tables A-10 and A-11 in Appendix A were plotted versus date as in Figure 35. Whereas the length of culms and branches showed reasonable variation and trend in relation to the time of mowing, the vegetation sample weights

TABLE X

COEFFICIENT AND EXPONENT IN RESISTANCE

RELATIONSHIP $n = C_1(VR)^{C_2}$, FC 31, 1963

Experiment	Average Culm Length (in.)	Resistance Computed Assuming Uniform Velocity Distribution C_1	C_2	Resistance Computed Considering Boussinesq Coefficient C_1	C_2
Uniform Flow					
1	3.08	0.03580	-0.2110	0.03593	-0.2099
2	4.00	.03954	- .2468	.03964	- .2456
3	4.19	.04474	- .2709	.04474	- .2713
Nonuniform Flow					
4	4.30	.04919	- .2540	.04866	- .2606
5	3.04	.03811	- .1595	.03769	- .1677

TABLE XI

COEFFICIENT AND EXPONENT IN RESISTANCE RELATIONSHIP

$n = C_1(VR)^{C_2}$, FC 31, 1964

Experiment	Average Culm And Branch Length (in.)	Resistance Computed Assuming Uniform Velocity Distribution C_1	C_2	Resistance Computed Considering Boussinesq Coefficient C_1	C_2
Uniform Flow					
7	2.89	0.03995	-0.2798	0.04003	-0.2792
9	2.45	.03663	- .2411	.03667	- .2417
11	2.80	.04057	- .2872	.04058	- .2872
Nonuniform Flow					
8	2.92	.04673	- .2581	.04630	- .2645
10	2.48	.04195	- .2240	.04153	- .2309
12	2.94	.04615	- .2559	.04572	- .2624
13	2.28	.03814	- .2497	.03775	- .2566

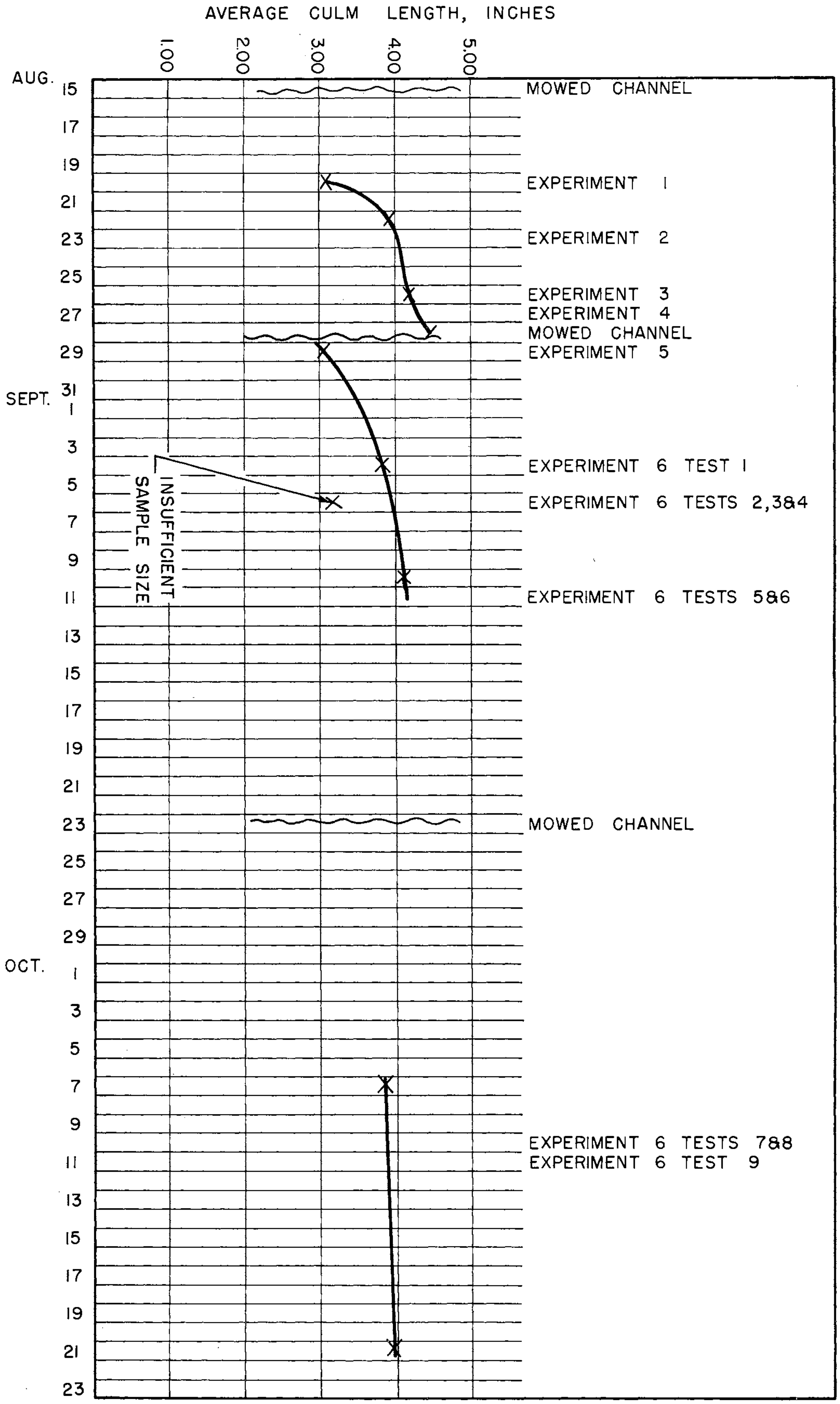

Figure 34. Average Culm Length, FC 31, 1963

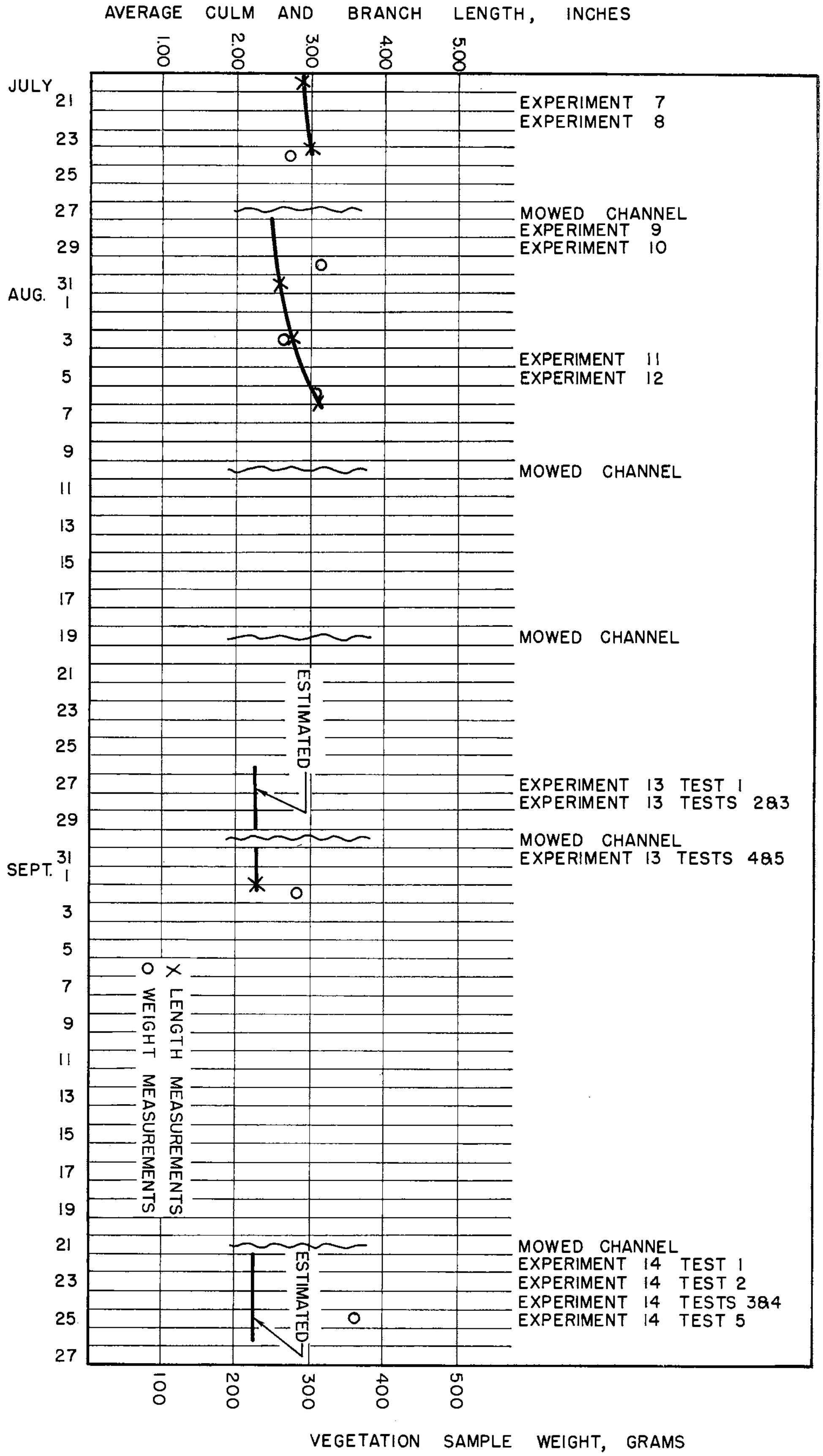

Figure 35. Average Culm and Branch Length and Vegetation Sample Weights, FC31, 1964

were extremely variable and of very little value. Apparently the sample was not sufficiently large or some unknown factor was acting. Therefore, the sample weight data were disregarded and lines were drawn to connect the data points of culm and branch length versus date.

The coefficients and exponents in Table X from both uniform and nonuniform flow experiments in 1963 were plotted versus average culm length as in Figures 36 and 37. The coefficients and exponents for the 1964 data were plotted versus average culm and branch length as in Figures 38 and 39. The culm length data collected in 1963 seemed to have much more promise of being related to resistance than the culm and branch length data collected in 1964. The culm length showed a wider range and the data were much less clustered.

The coefficients and exponents for the uniform flow and nonuniform flow average lines in 1964 seemed to be unlike. The spatially varied flow experiments with free outfall were reasoned to be more like the nonuniform flow experiments than the uniform flow experiments, so the uniform flow data from 1964 were deleted. The 1963 uniform flow data were also deleted for the sake of consistency, although the 1963 data showed less tendency to fall into two groups than did the 1964 data. Possibly, deleting the last three-station group of data down the channel when computing the average line for each experiment would have made the uniform flow and nonuniform flow data more alike. Examination of the data in Figures 24

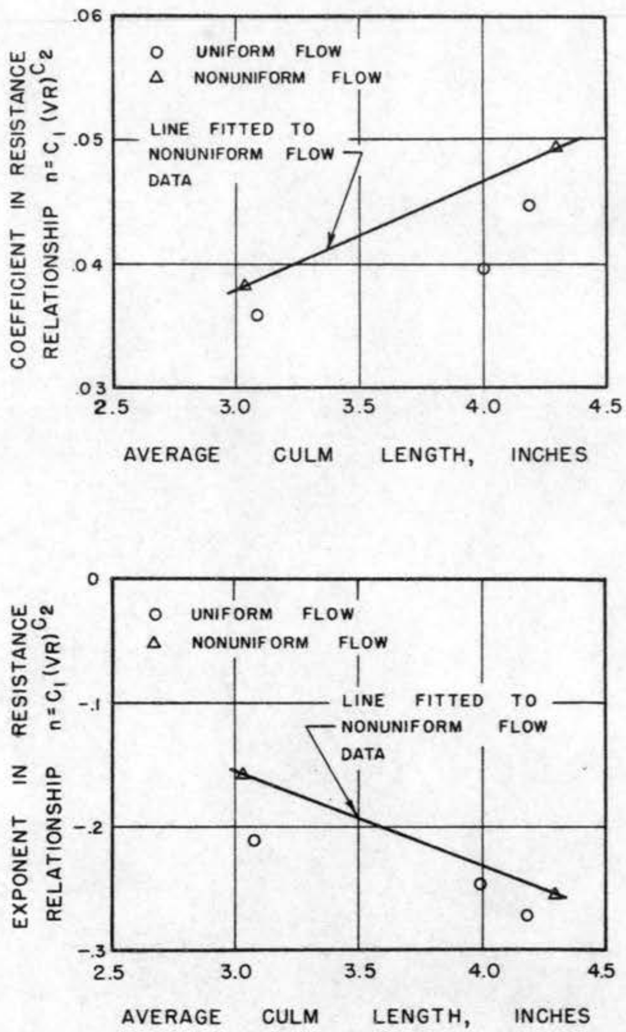

Figure 36. C_1 and C_2 in $n = C_1(VR)^{C_2}$, Resistance Assuming Unif. Vel. Dist., FC 31, 1963

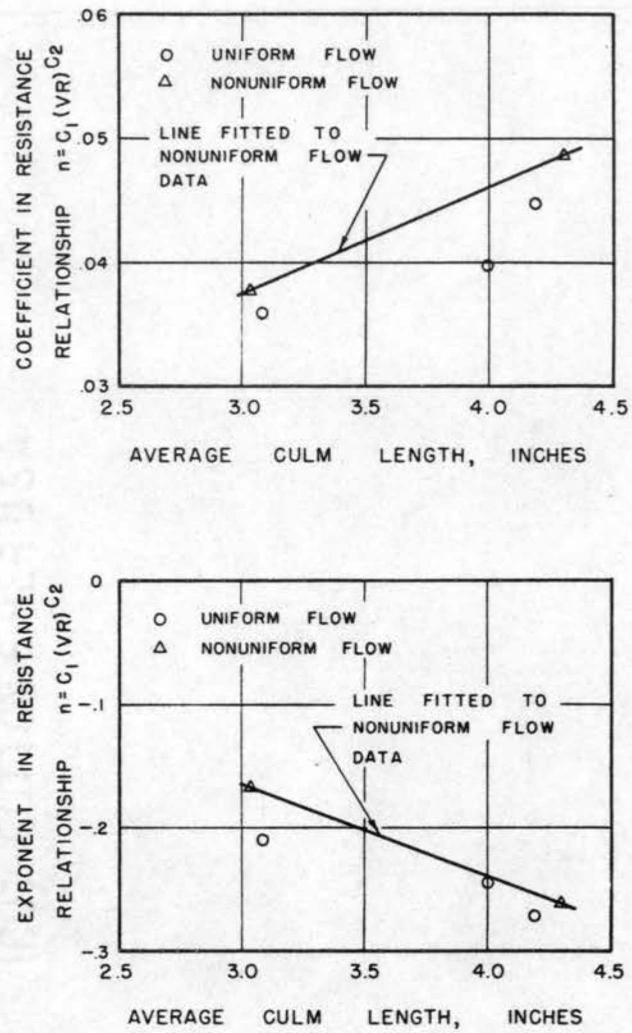

Figure 37. C_1 and C_2 in $n = C_1(VR)^{C_2}$, Resistance Considering Boussinesq Coefficient, FC 31, 1963

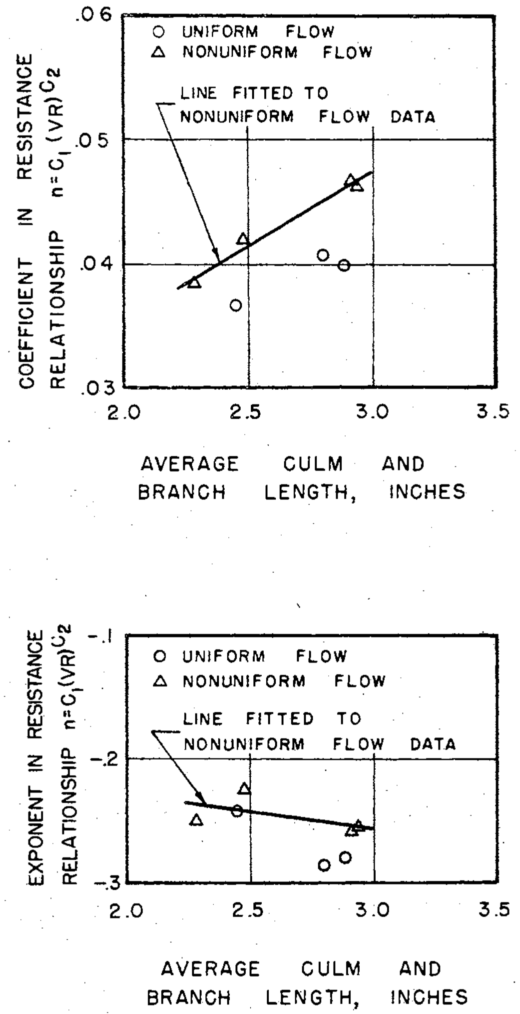

Figure 38. C_1 and C_2 in $n = C_1(VR)^{C_2}$, Resistance Assuming Unif. Vel. Dist., FC 31, 1964

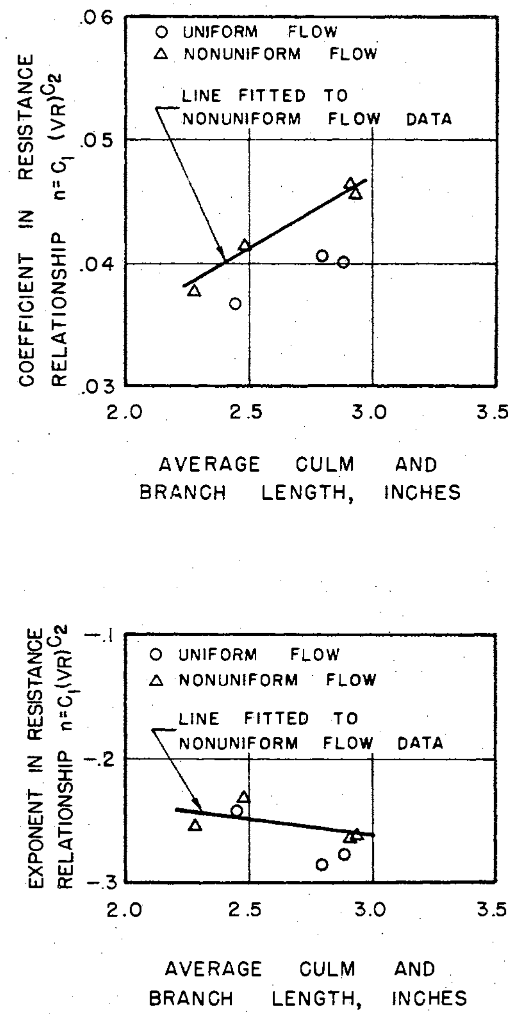

Figure 39. C_1 and C_2 in $n = C_1(VR)^{C_2}$, Resistance Considering Boussinesq Coefficient, FC 31, 1964

through 27 and 30 through 33 showed that the resistance data from reach combination 789 were considerably higher than that from the other reach combinations for 1964. The data in Figures 22, 23, 28, and 29 showed the effect to be less pronounced for the 1963 data.

Examination of the data from the nonuniform flow experiments in Figures 36 through 39 led to the decision to relate the coefficient and exponent in the retardance relationship linearly to vegetation length. This was done using the Fortran IV Multivariate program listed in Table B-4 in Appendix B. The resulting equations were of the form

$$C_1 = B_1 + B_2 \text{ (Vegetation length)} \tag{31}$$

$$C_2 = B_3 + B_4 \text{ (Vegetation length)} \tag{32}$$

The equations from the data analyzed assuming uniform velocity distribution are presented in Table XII, and the fitted lines are plotted in Figures 36 and 38. The equations from the data analyzed considering the Boussinesq coefficient are presented in Table XIII, and the fitted lines are plotted in Figures 37 and 39.

The Manning's n data plotted in Figures 22 through 33 varied considerably within each experiment. The general trend was for the resistance to increase from the upstream to the downstream end of the channel. This trend was investigated. The data from each reach combination computed considering the Boussinesq coefficient were fitted separately using the LS02

TABLE XII

EQUATIONS RELATING COEFFICIENT AND EXPONENT IN RESISTANCE RELATIONSHIP $n = C_1(VR)^{C_2}$ TO LENGTH OF VEGETATION, RESISTANCE COMPUTED ASSUMING UNIFORM VELOCITY DISTRIBUTION, NONUNIFORM FLOW, FC 31, 1963 AND 1964

1963:

$$C_1 = 0.01138 + 0.008794 \text{ (Culm Length)} \quad (33)$$

$$C_2 = 0.06850 - 0.07500 \text{ (Culm Length)} \quad (34)$$

1964:

$$C_1 = 0.01122 + 0.01206 \text{ (Culm and Branch Length)} \quad (35)$$

$$C_2 = -0.1771 - 0.02631 \text{ (Culm and Branch Length)} \quad (36)$$

TABLE XIII

EQUATIONS RELATING COEFFICIENT AND EXPONENT IN RESISTANCE RELATIONSHIP $n = C_1(VR)^{C_2}$ TO LENGTH OF VEGETATION, RESISTANCE COMPUTED CONSIDERING BOUSSINESQ COEFFICIENT, NONUNIFORM FLOW, FC 31, 1963 AND 1964

1963:

$$C_1 = 0.01122 + 0.008706 \text{ (Culm Length)} \quad (37)$$

$$C_2 = 0.05644 - 0.07373 \text{ (Culm Length)} \quad (38)$$

1964:

$$C_1 = 0.01094 + 0.01201 \text{ (Culm and Branch Length)} \quad (39)$$

$$C_2 = -0.1858 - 0.02555 \text{ (Culm and Branch Length)} \quad (40)$$

program. The resulting lines are shown in Figures 28 through 33 and the coefficients and exponents are presented in Tables XIV and XV. The distance down the channel to the middle of each reach combination is also given in Tables XIV and XV. The coefficient and exponent were plotted versus distance down the channel as in Figures 40 through 43. The trend in coefficients was for a slight increase with distance down the channel. The trend in exponents was for a slight decrease. The general trend in the coefficients and exponents and also in the vegetation length data led to the consideration and use of a multivariable relationship to relate the coefficient and exponent in the resistance relationship to vegetation length and distance down the channel. The Multivariate program was used to fit the data. The 1963 and 1964 data were fitted separately because of the difference in the methods of measuring the vegetation lengths. The equations relating the coefficient and exponent to distance down the channel and vegetation length were of the form

$$C_1 = B_1 + B_2 x + B_3 \text{ (Vegetation length)} \qquad (41)$$

$$C_2 = B_4 + B_5 x + B_6 \text{ (Vegetation length)} \qquad (42)$$

These equations for the 1963 and 1964 nonuniform flow data analyzed considering the Boussinesq coefficient are presented in Table XVI. The resulting final fitted resistance lines for Experiment 13 are presented in Figure 44.

TABLE XIV

COEFFICIENTS AND EXPONENT IN RESISTANCE RELATIONSHIP $n = C_1(VR)^{C_2}$ FITTED TO EACH 3- SECTION REACH, RESISTANCE COMPUTED CONSIDERING BOUSSINESQ COEFFICIENT, FC 31, 1963

Experiment	Reach Combination	Distance Down Channel (ft.)	Culm Length (in.)	Coefficient	Exponent
4	123	73.6	4.30	0.04243	-0.2723
	234	123.6	4.30	.04636	- .2614
	345	173.6	4.30	.05171	- .2708
	456	223.6	4.30	.04958	- .2716
	567	273.6	4.30	.04873	- .2875
	678	323.6	4.30	.04946	- .2506
	789	365.5	4.30	.05200	- .2414
5	123	73.6	3.04	.03301	- .1652
	234	123.6	3.04	.03452	- .1410
	345	173.6	3.04	.03807	- .1661
	456	223.6	3.04	.03678	- .1996
	567	273.6	3.04	.03867	- .2128
	678	323.6	3.04	.03986	- .1776
	789	365.5	3.04	.04279	- .1825

TABLE XV

COEFFICIENT AND EXPONENT IN RESISTANCE RELATIONSHIP $n = C_1(VR)^{C_2}$ FITTED TO EACH 3-SECTION REACH, RESISTANCE COMPUTED CONSIDERING BOUSSINESQ COEFFICIENT, FC 31, 1964

Experiment	Reach Combination	Distance Down Channel (ft.)	Culm And Branch Length (in.)	Coefficient	Exponent
8	123	73.6	2.92	0.03899	-0.2604
	234	123.6	2.92	.03905	- .2257
	345	173.6	2.92	.04537	- .2445
	456	223.6	2.92	.04695	- .2813
	567	273.6	2.92	.04888	- .3047
	678	323.6	2.92	.05061	- .3031
	798	365.5	2.92	.05510	- .3322
10	123	73.6	2.48	.03634	- .2408
	234	123.6	2.48	.03580	- .2014
	345	173.6	2.48	.04006	- .2206
	456	223.6	2.48	.04085	- .2575
	567	273.6	2.48	.04202	- .2681
	678	323.6	2.48	.04448	- .2435
	789	365.5	2.48	.05160	- .2928
12	123	73.6	2.94	.04039	- .2775
	234	123.6	2.94	.03949	- .2340
	345	173.6	2.94	.04378	- .2485
	456	223.6	2.94	.04470	- .2815
	567	273.6	2.94	.04612	- .2948
	678	323.6	2.94	.04919	- .2842
	789	365.5	2.94	.05646	- .3340
13	123	73.6	2.28	.03306	- .2651
	234	123.6	2.28	.03327	- .2099
	345	173.6	2.28	.03659	- .2378
	456	223.6	2.28	.03728	- .2909
	567	273.6	2.28	.03842	- .2960
	678	323.6	2.28	.03925	- .2763
	789	365.6	2.28	.04664	- .3212

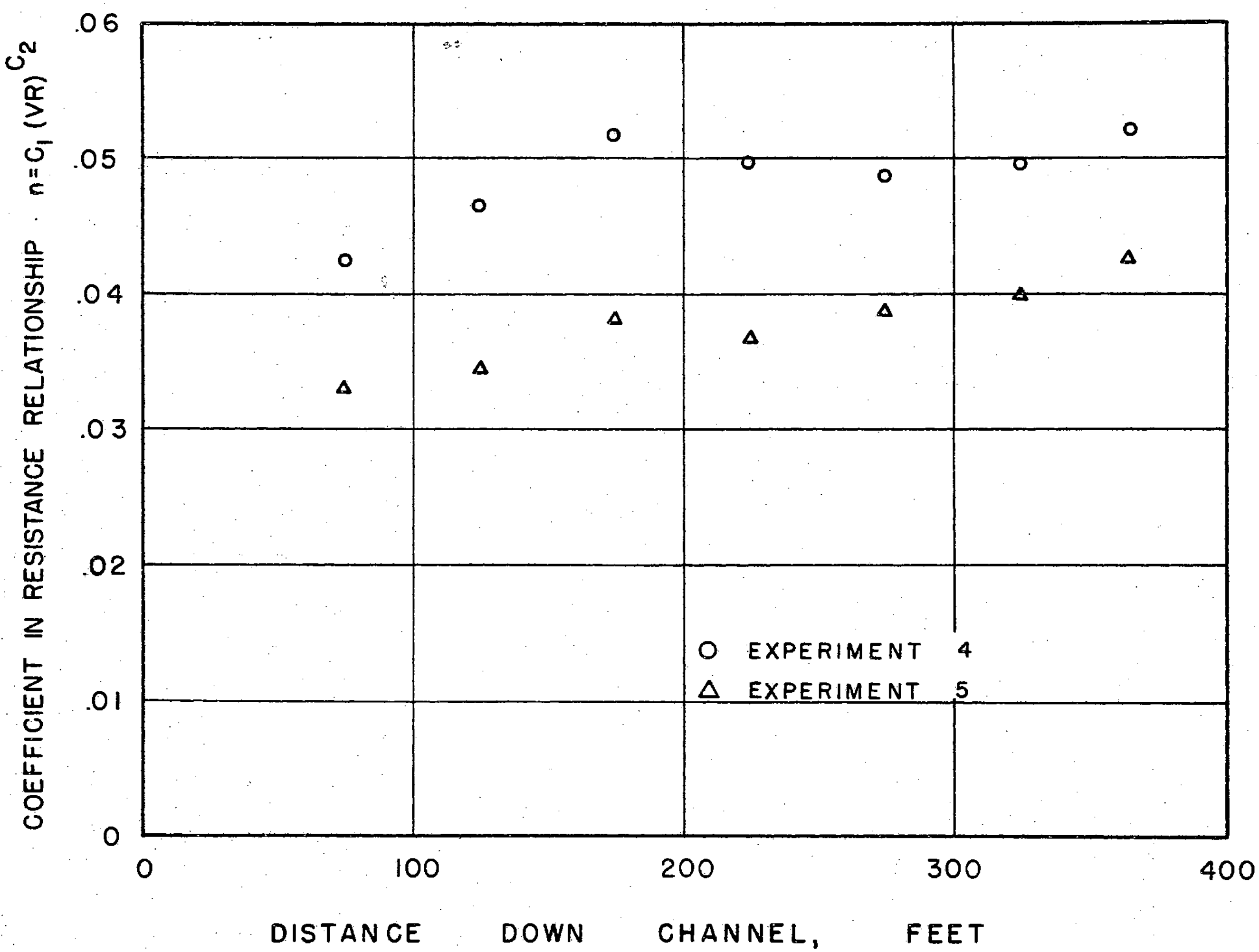

Figure 40. Coefficient in Resistance Relationship $n = C_1 (VR)^{C_2}$ and Distance Down Channel, Resistance Computed Considering Boussinesq Coefficient, FC 31, 1963

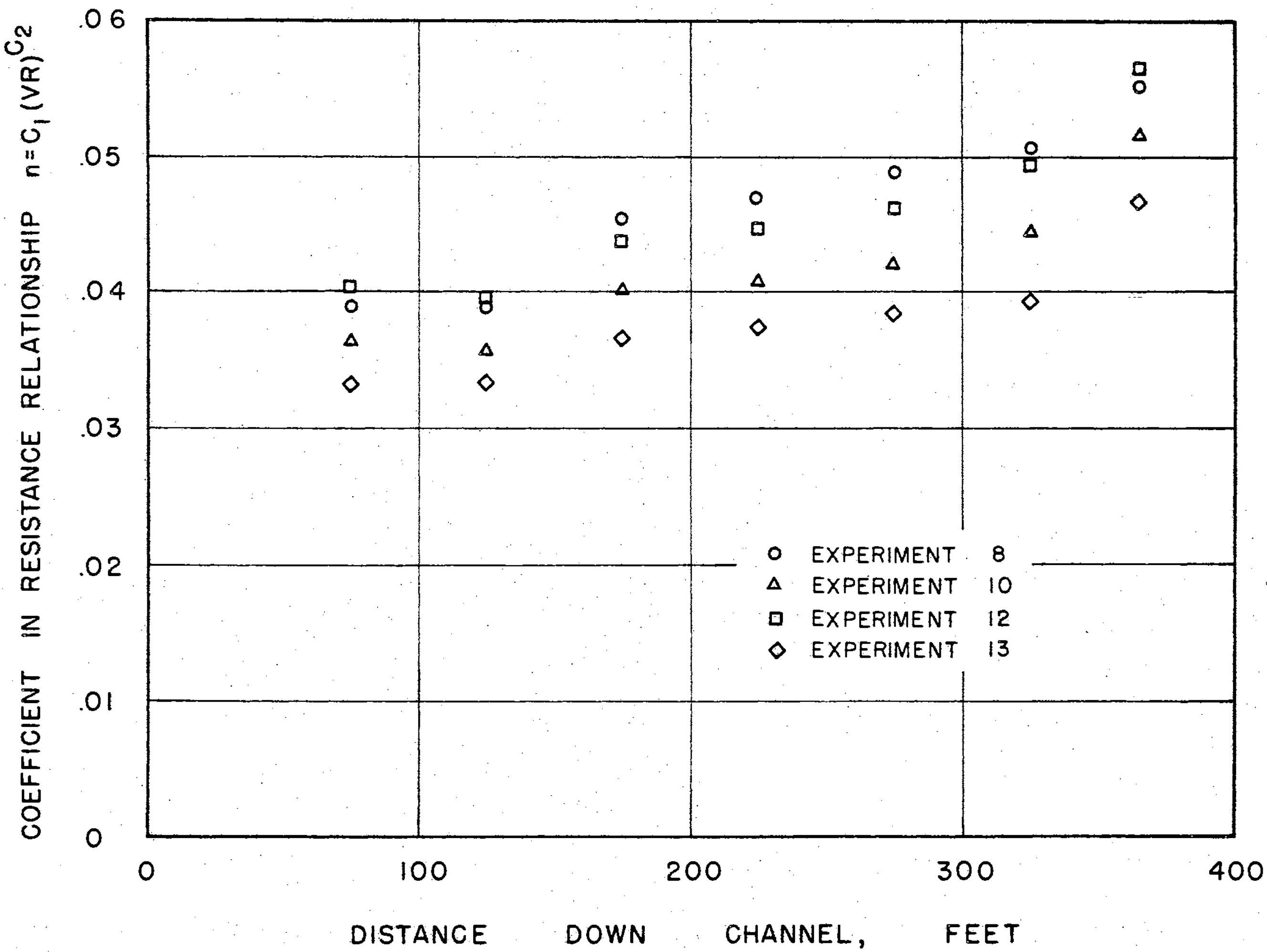

Figure 41. Coefficient in Resistance Relationship $n = C_1(VR)^{C_2}$ and Distance Down Channel, Resistance Computed Considering Boussinesq Coefficient, FC 31, 1964

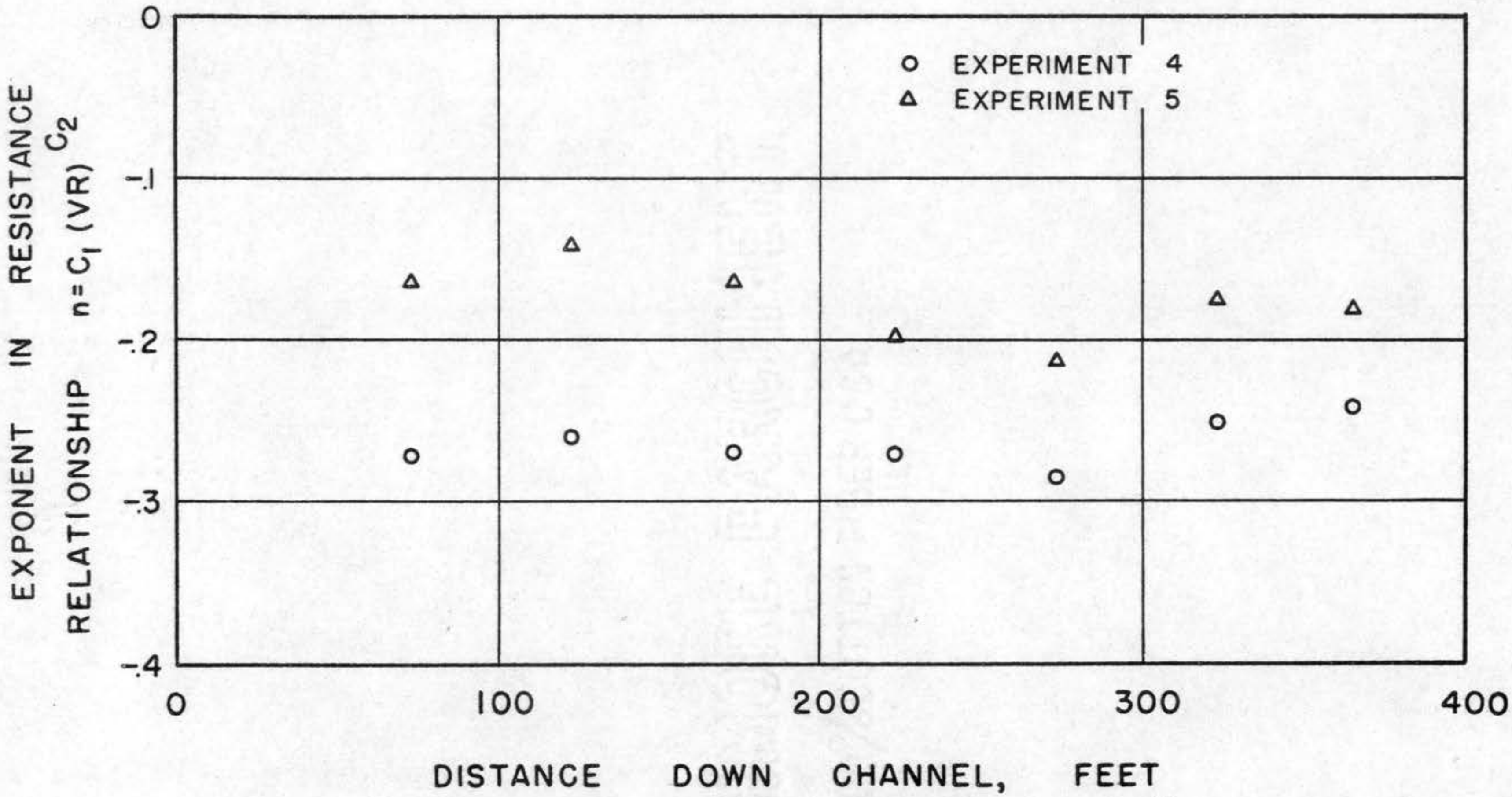

Figure 42. Exponent in Resistance Relationship $n = C_1 (VR)^{C_2}$ and Distance Down Channel, Resistance Computed Considering Boussinesq Coefficient, FC 31, 1963

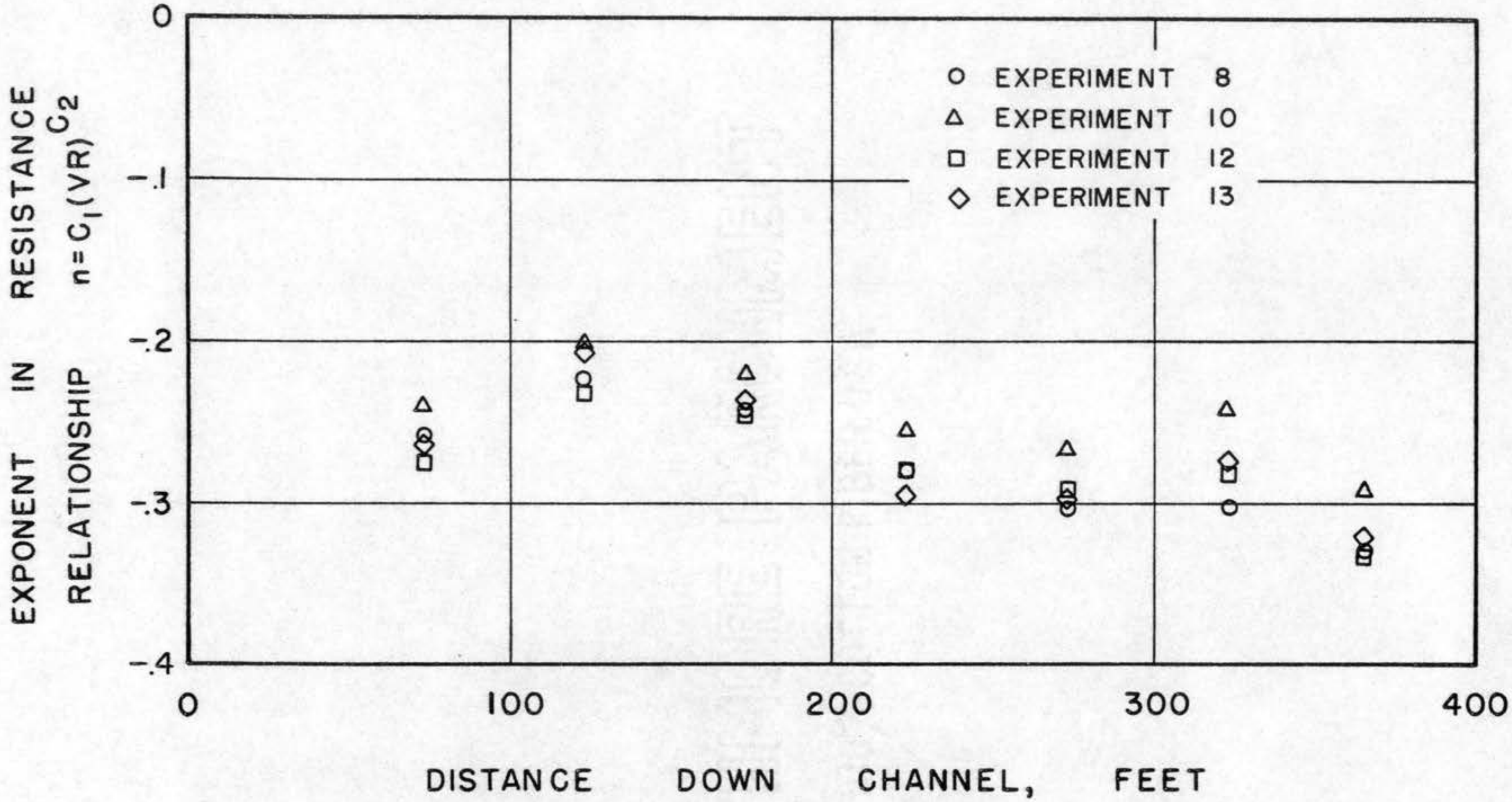

Figure 43. Exponent in Resistance Relationship $n = C_1(VR)^{C_2}$ and Distance Down Channel, Resistance Computed Considering Boussinesq Coefficient, FC 31, 1964

TABLE XVI

EQUATIONS RELATING COEFFICIENT AND EXPONENT IN RESISTANCE RELATIONSHIP $n = C_1(VR)^{C_2}$ TO DISTANCE DOWN CHANNEL AND LENGTH OF VEGETATION, RESISTANCE COMPUTED CONSIDERING BOUSSINESQ COEFFICIENT, NONUNIFORM FLOW, FC 31, 1963 AND 1964

1963:

$$C_1 = 0.005427 + 0.00002631\,x + 0.008681\,(\text{Culm Length}) \qquad (43)$$

$$C_2 = 0.03905 - 0.00002857\,x - 0.06925\,(\text{Culm Length}) \qquad (44)$$

1964:

$$C_1 = 0.0002846 + 0.00004775\,x + 0.01204\,(\text{Culm and Branch Length}) \qquad (45)$$

$$C_2 = -0.1412 - 0.0002515\,x - 0.02701\,(\text{Culm and Branch Length}) \qquad (46)$$

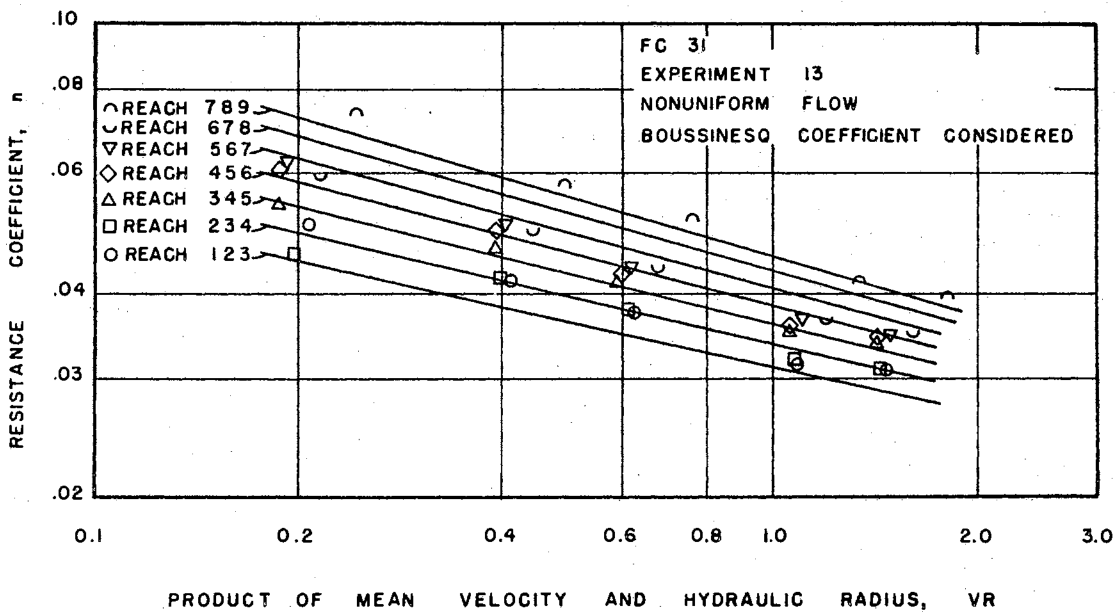

Figure 44. Final Fitted Lines of Resistance, Experiment 13, FC 31

Inflow Distribution

The data from the rating tests of the adjustable weir were analyzed using Equation (23)

$$Q = C\ L\ h^{3/2} \tag{23}$$

The equation was rearranged into the following form:

$$C = Q/(L\ h^{3/2}) \tag{47}$$

and the discharge coefficient was determined for each test. These coefficient values were plotted versus head as in Figure 45, and separate rating curves were drawn for the springing-free and adhering conditions. Some of the data points were determined using the gage well and Lory point gage and some using the direct-measuring weir-head gage. The measurements obtained with the Lory gage were used only for purposes of extending the springing-free rating curve.

The rating curves shown in Figure 45 were used with the heads on the adjustable weir measured in the field using the direct-measuring weir-head point gage to determine the inflow distribution into the channel for the spatially varied flow experiments. The heads measured at two consecutive places along the weir were averaged. The weir coefficient corresponding to this average head was determined from the appropriate rating curve. The coefficient was multiplied by the weir length between measurements, and the resulting product by the average head raised to the three-halves power to give the increment of inflow along that length of weir. The total

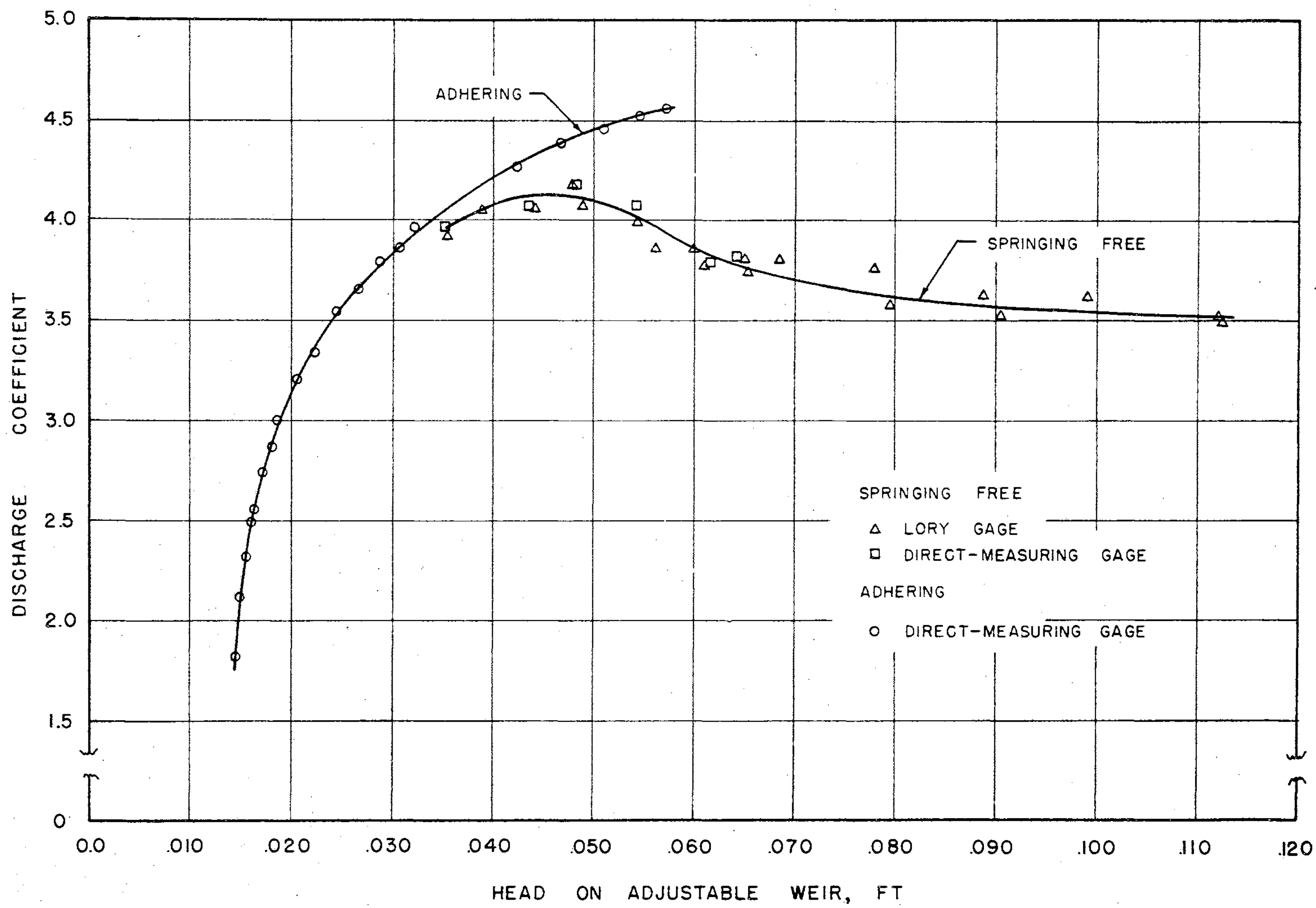

Figure 45. Rating Curves for Adjustable Weir

calculated inflow was determined, and each increment of inflow was corrected by multiplying by the total corrected discharge from Tables V or VI and dividing by the total calculated discharge over the weir. The measurement spacing, measured heads, calculated increment of discharge, corrected increment of discharge, and total discharge at a given distance down the channel are presented in Tables A-12 and A-13 in Appendix A. Test 1 of Experiment 6 was not included because an insufficient number of head measurements was obtained to define the inflow distribution accurately. Better agreement was obtained in 1964 than in 1963 between the calculated discharges over the weir and the measured discharges corrected for leakage. This was attributed to the rebuilding of the Forebay 2 dike prior to the 1964 testing season, the improved weir leveling techniques developed in 1964, and the greater number of head measurements obtained in 1964.

Water Surface Profile for Spatially Varied Flow

Methods of Computing Theoretical Profiles

The observed water surface profiles for spatially varied steady flow Experiments 6 and 14 are presented in Tables A-14 and A-15 in Appendix A. Also presented in Tables A-14 and A-15 are four computed water surface profiles for the conditions corresponding to each spatially varied flow test. Each of the four computed profiles for each test was computed with a different technique, each representing a different degree of refinement of the theory of spatially varied steady flow and of the information on resistance and velocity distribution. Briefly, the four methods were as follows:

Method 1

This method consisted of assuming uniform velocity distribution and using Equation (19)

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_L \tag{19}$$

for nonuniform flow with uniform velocity distribution to calculate water surface profiles for spatially varied steady flow. This method was approximately equivalent to assuming $\beta_1 = \beta_2 = 1.00$ and deleting the term containing ΔQ from Equation (24), which gives

$$\Delta y = \frac{Q_1(V_1 + V_2)}{g(Q_1 + Q_2)} (V_2 - V_1) + (S_o - S_s) \Delta x \tag{48}$$

and ignoring the difference between Q_1 and $(Q_1 + Q_2)/2$. The head loss term in Equation (19) was computed by multiplying Δx by a slope value obtained using Equation (26)

$$S_s = \frac{Q_{avg}^2 \, n^2}{2.21 \, A_{avg}^2 \, R_{avg}^{2/3}} \tag{26}$$

Values of Manning's coefficient were obtained by using the resistance relationship, Equation (30)

$$n = C_1 \ (VR)^{C_2} \qquad (30)$$

with Equations (33) through (36), presented in Table XII, which related C_1 and C_2 to vegetation length. Equations (33) through (36) were determined assuming uniform velocity distribution. For the 1963 tests the vegetation lengths were determined from Figure 34. Because no vegetation length measurements were taken at the time of conducting the 1964 spatially varied flow tests, and because the 1964 spatially varied flow tests were conducted shortly after mowing the channel, the average of the culm and branch lengths after two previous mowings, 2.35 inches, was used.

A starting point for the water surface computations was needed. Because the test channel, FC 31, was a mild channel with free outfall, the water surface elevation at the farthest downstream gage well was chosen. The measured inflow distributions presented in Tables A-12 and A-13 were used. The measured channel cross section data presented in Tables A-7 and A-8 were used. The computer program Hydel 2 listed in Table B-6 computed tables of area and hydraulic radius from the cross section data.

Method 2

Method 2 consisted of assuming uniform velocity distribution and using Equation (24), which gave

$$\Delta y = - \frac{Q_1(V_1 + V_2)}{g(Q_1 + Q_2)} (V_2 - V_1 + \frac{V_2}{Q_1} \Delta Q) + (S_o - S_s) \Delta x \tag{49}$$

The change from Equation (19) to Equation (49) was the only difference between Method 1 and Method 2; all of the other variables remained the same.

Method 3

Nonuniform velocity distribution was considered in Method 3. Equation (24)

$$\Delta y = \frac{Q_1(V_1 + V_2)}{g(Q_1 + Q_2)} (\beta_2 V_2 - \beta_1 V_1 + \frac{\beta_2 V_2}{Q_1} \Delta Q) + (S_o - S_s) \Delta x \tag{24}$$

and the Boussinesq coefficient relationship, Equation (27),

$$\text{Beta} = C_3 Q^{C_4} \tag{27}$$

were used. The values for C_3 and C_4 in Table IX obtained by lumping the data from Stations B and C for nonuniform flow Experiment 13 were used in Equation (27). The shear slope, S_s, was computed using Equation (26)

$$S_s = \frac{Q_{avg}^2 n^2}{2.21 A_{avg}^2 R_{avg}^{2/3}} \tag{26}$$

Values of Manning's n were obtained by using the resistance relationship, Equation (30)

$$n = C_1 \, (VR)^{C_2} \tag{30}$$

with Equations (37) through (40), presented in Table XIII, which related C_1 and C_2 to vegetation length. Equations (37) through (40) were determined considering the Boussinesq coefficient. The starting point, inflow distribution data, and channel cross section data remained the same as for Methods 1 and 2.

Method 4

Method 4 was the most refined technique available. It consisted of using Equations (24) and (26) with the relationships for C_3 and C_4 as a function of distance down the channel developed from data from spatially varied flow Experiment 14 and given in Equations (28) and (29)

$$C_3 = 1.598 \, x^{0.04106} \tag{28}$$

$$C_4 = -0.5026 \, x^{-0.2862} \tag{29}$$

The shear slope in Equation (24) was obtained using Equations (26) and (30) and Equations (43) through (46) presented in Table XVI which relate C_1 and C_2 to vegetation length and distance down the channel and which were developed from the resistance data with Boussinesq coefficient considered. Other variables remained the same as for the previous methods.

Four digital computer programs were written to accomplish the four methods. All used the Euler method with iteration as explained in Chapter III, Theoretical Analysis, to project up the channel from the starting point at the farthest downstream gage well. The programs were set to project at one-foot intervals. The iterations were continued until a calculated Δy at a station agreed with the preceeding calculated Δy at that station within 0.00001 foot. The inflow distribution was read into the computer in tabular form and the programs interpolated linearly between two locations where the total discharge was known. The tables of area and hydraulic radius obtained with the Hydel 2 program were used as input for the profile computation programs. The predictor programs interpolated between the cross section stations and also within each table of areas and hydraulic radii, since these were set up for elevation intervals of 0.01 foot. The program used with Method 4, SVF 5F, is presented in Table B-7 in Appendix B. The programs used with Methods 1 through 3 were simplifications of this program.

Discussion of Observed and Computed Profiles and Methods of Computation

The observed and calculated water surface profiles from the test for which the differences between the observed and calculated water surface profiles were greatest, Test 5 of

Experiment 14, are presented in Figure 46. Even for this test, plotting elevation versus distance down the channel revealed very little. Therefore, another means of comparing the results of the different prediction methods was sought. The scheme chosen was to compute the differences between the calculated and observed profiles and to plot these differences versus distance down the channel. The differences are presented in Tables A-14 and A-15 and are plotted in Figures 47 through 50.

The differences plotted as in Figures 47 through 50 showed that the predicted profiles agreed with the observed profiles in order of the refinement of the prediction method. Method 2 was better than Method 1; Method 3 was better than Method 2, and Method 4 was better than Method 3. The largest differences for all methods occurred near the downstream end at either nominal station 3 + 75 or 3 + 25. From those stations upstream, the difference plots of Methods 1, 2, and 3 were nearly parallel. There was a general tendency for the parallel lines to become farther apart with increased discharge, although the differences obtained with Method 3 showed little or no increase with increased discharge. This meant that out of the zone of curvature of the streamlines the results obtained with any of Methods 1, 2, or 3 should be nearly the same, but that in a zone of curvature of streamlines the approximations involved in Methods 1 and 2 would become increasingly inaccurate as discharge increases.

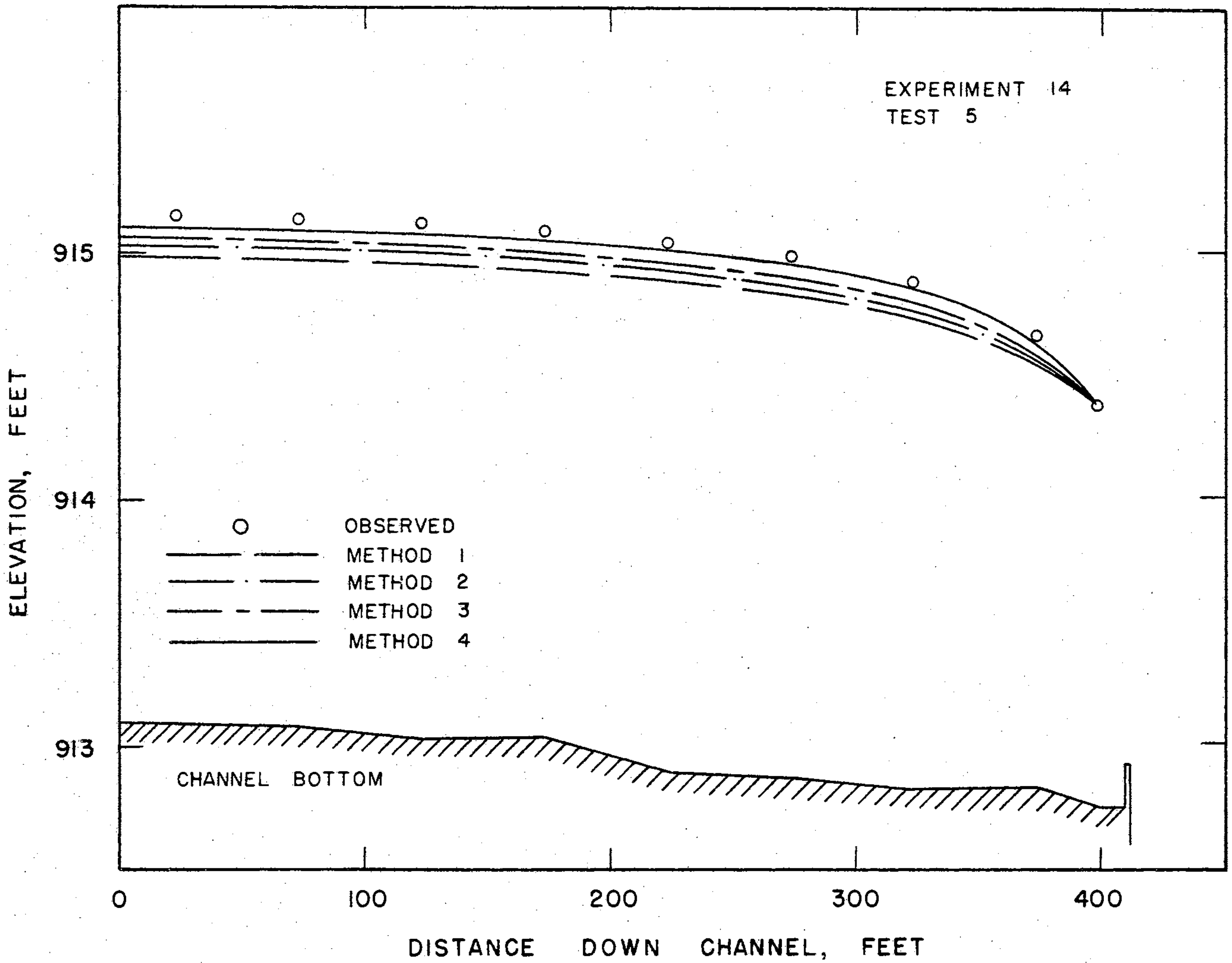

Figure 46. Observed and Calculated Water Surface Profiles, Experiment 14, Test 5

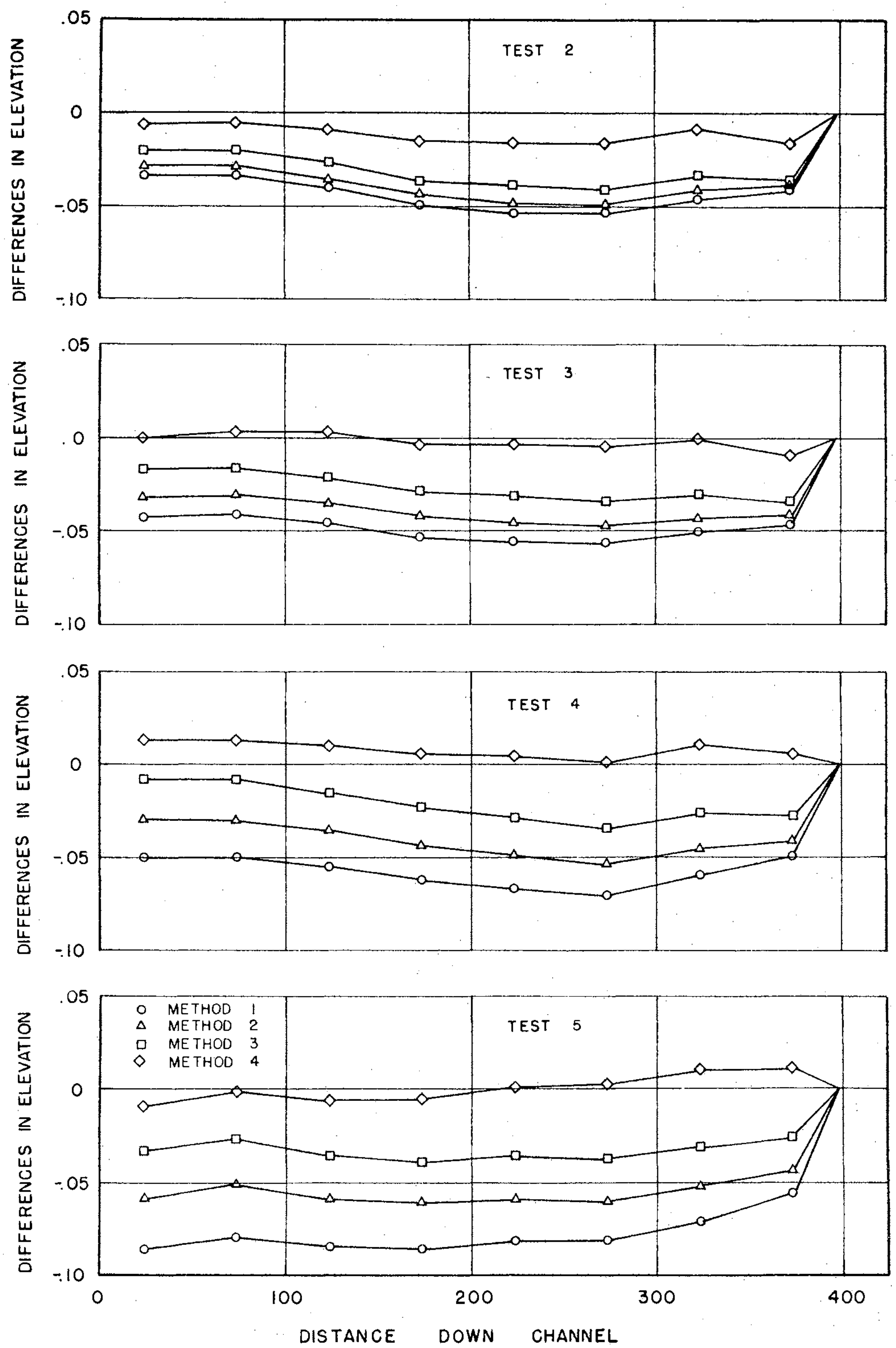

Figure 47. Differences Between Observed and Calculated Water Surface Profiles, Experiment 6, Tests 2-5

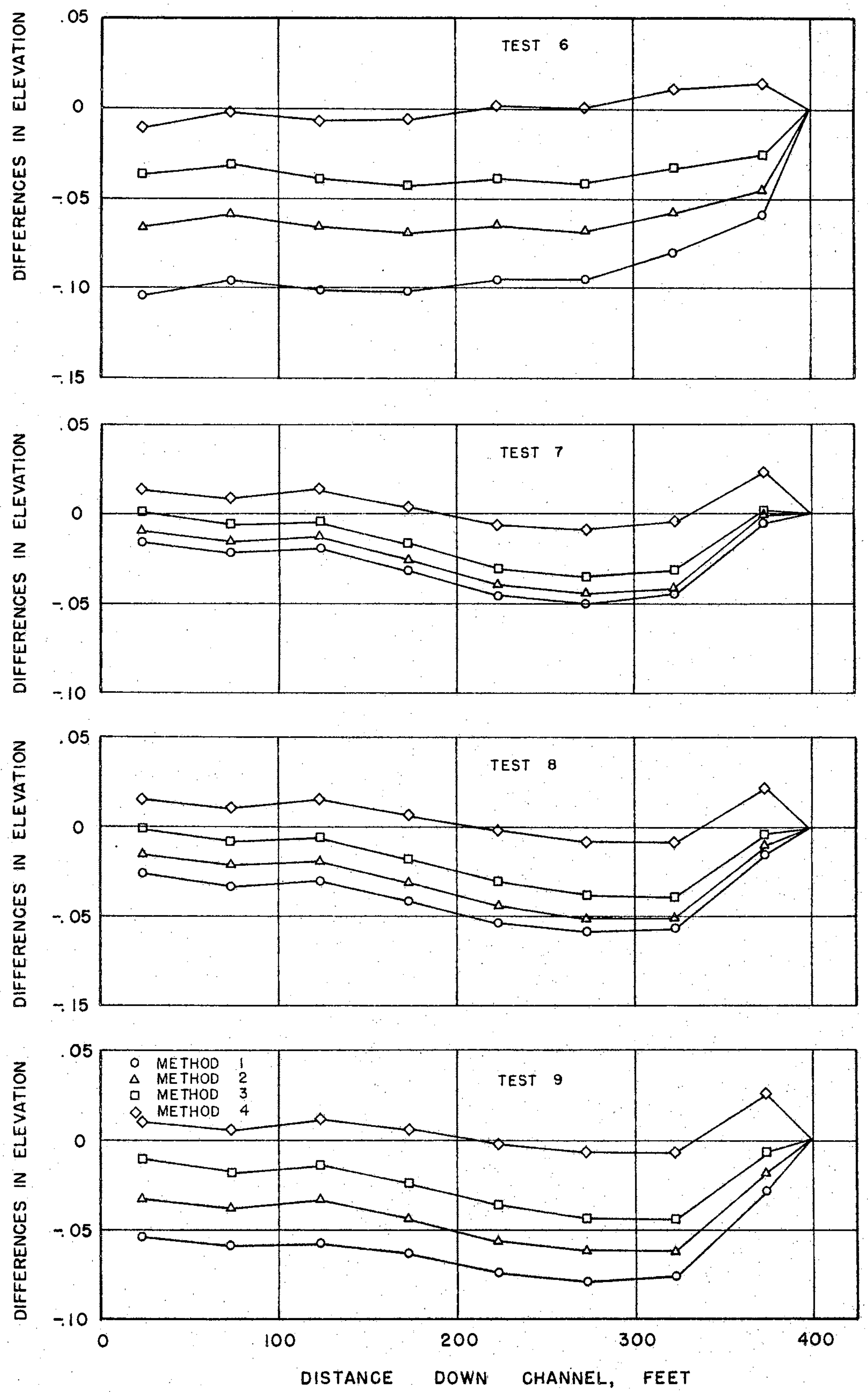

Figure 48. Differences Between Observed and Calculated Water Surface Profiles, Experiment 6, Tests 6-9

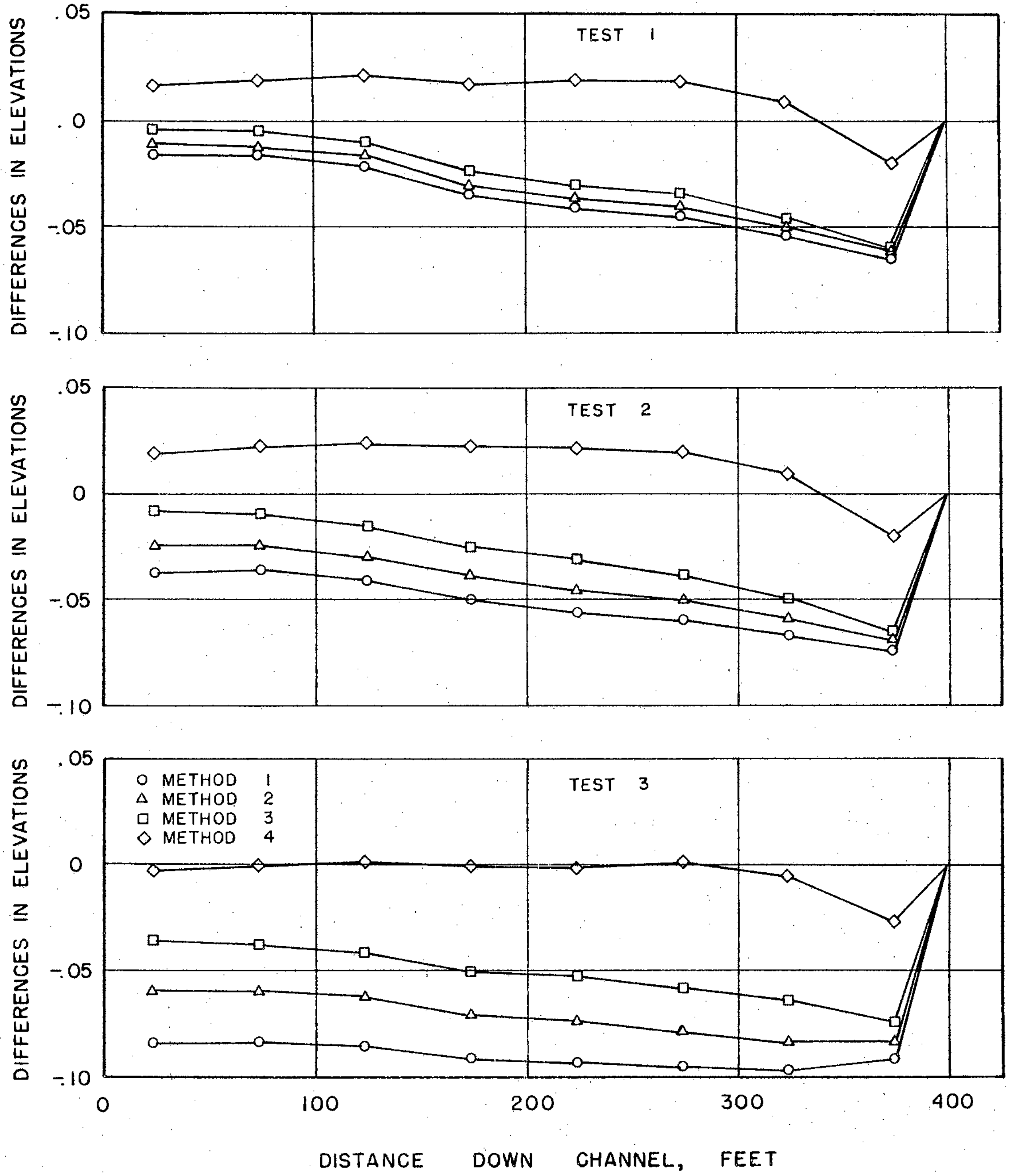

Figure 49. Differences Between Observed and Calculated Water Surface Profiles, Experiment 14, Tests 1-3

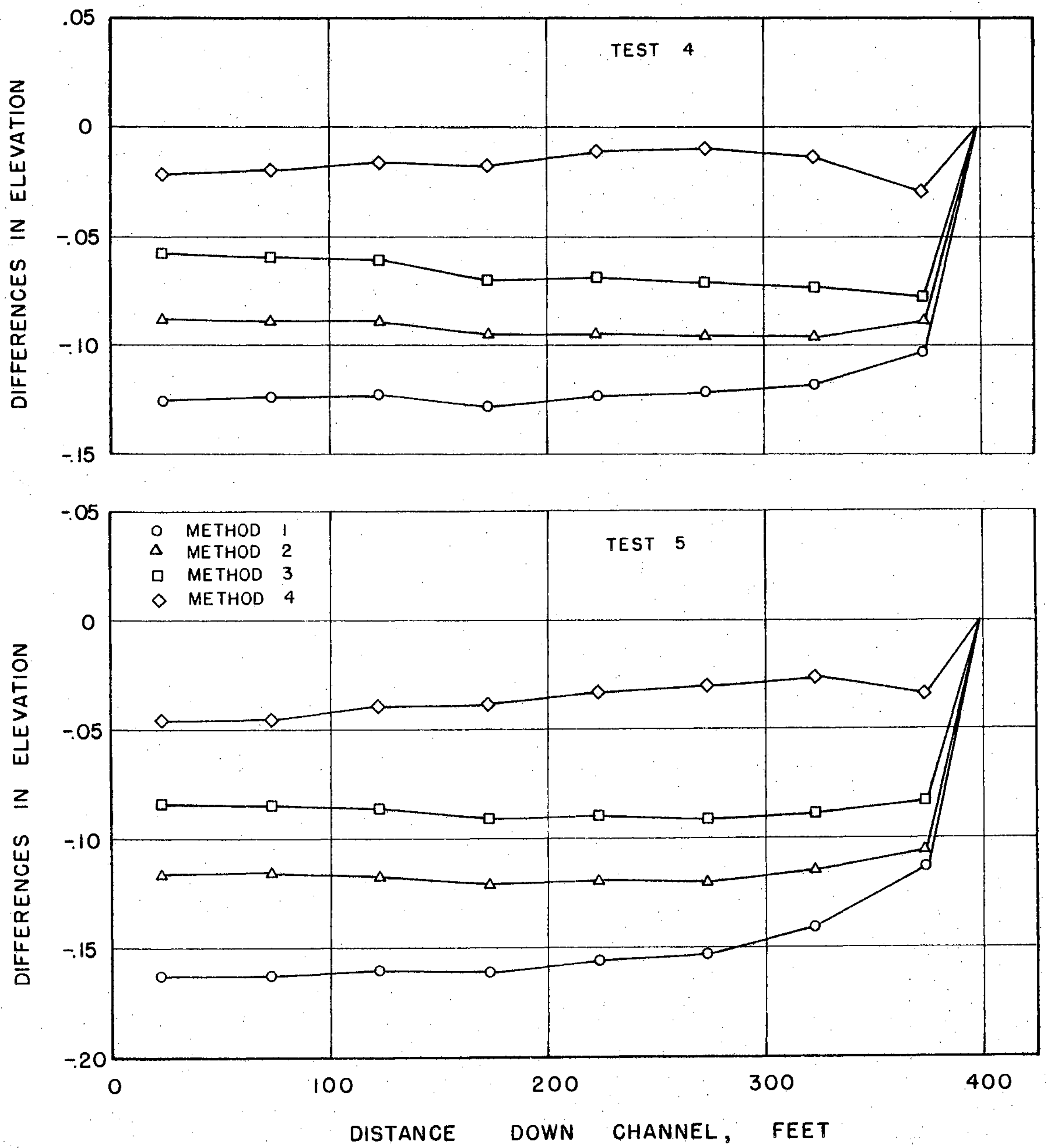

Figure 50. Differences Between Observed and Calculated Water Surface Profiles, Experiment 14, Tests 4 and 5

The difference plots from Method 4 did not vary exactly as did those from Methods 1, 2, or 3, mainly because of the consideration of the variation of resistance with distance down the channel. Upstream from nominal stations 3 + 75 or 3 + 25, the profiles obtained with Method 4 tended to become more parallel to the observed profiles than did those obtained with Methods 1, 2, or 3.

The differences in water surface elevations obtained near the downstream end of the channel from Methods 3 and 4 were attributed in large part to the consideration of the variation of resistance with distance down the channel, because the Boussinesq coefficients from nonuniform flow Experiment 13 used in Method 3 and those from spatially varied flow Experiment 14 used with Method 4 were of the same order of magnitude at both Stations B and C. This initial better prediction with Method 4 made the Method 4 profiles fit better all the way up the channel.

The plotted differences between the observed and computed profiles for Experiment 14 revealed the effect caused by estimating a constant culm and branch length. The differences for Method 4 were mainly positive for Test 1 and decreased to a negative value for Test 5. The bermudagrass grows considerably in four days, as shown in Figures 34 and 35. Probably the estimated culm and branch length of 2.35 inches was too long for Tests 1 and 2, about right for Test 3, and was too short for Tests 4 and 5. Apparently, the decrease of the differences was not caused by the increase of the discharges from Test 1 to Test 5, because Tests 7, 8, and 9 of Experiment 6, conducted within 36 hours, did not show the same effect with increasing discharge.

The large differences near the downstream end between the observed and computed profiles in 1964 were noted and contemplated. An initial hypothesis was that these differences might be caused by the steep slope between the observed low points in the channel. This steep slope would have the effect of decreasing the Δy values and lowering the profiles. However, this would be in direct contradiction to the results obtained for Tests 7, 8, and 9 of Experiment 6 in 1963, where the differences show a larger increase from nominal station 3 + 75 to 3 + 25 than from 4 + 00 to 3 + 75, and where the observed low points in the channel show no such break as in 1964. No explanation was thought satisfactory for explaining the differences. However, rather than using the minimum bottom elevations at each station it probably would have been better to have used an averaging method, such as averaging the lowest point at a cross section and the two elevations adjacent to it.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Summary

Spatially varied flow, where water enters a channel all along its length, is the usual mode of flow for many natural and constructed channels. Theoretical equations describing spatially varied steady flow have been obtained by use of the principle of the conservation of linear momentum. However, the theory had not been tested previously for applicability to small agricultural conservation channels.

The problem was investigated at the Stillwater Outdoor Hydraulic Laboratory in an asymmetrical V-shaped bermudagrass-lined test channel approximately 410 feet long with design side slopes of 3 on 1 and 6.6 on 1, maximum depth of 2.7 feet, and design bottom slope of 0.001. Free outfall occurred at the outlet, although there was a provision for outlet sills. Flow could enter the channel either at its upper end or all along the upper 399.2 feet over an adjustable weir. Thus it was possible to conduct uniform flow, nonuniform flow, and spatially varied steady flow experiments in the channel. Three current meter stations were located 27, 200, and 396 feet, respectively, from the upstream end of the channel.

Uniform and nonuniform flow experiments were conducted in the channel to determine the resistance characteristics of the channel. Discharges of 2, 5, 10, 20, and 30 cubic feet per second were used during each experiment. Velocity distribution measurements were taken during one nonuniform flow experiment. Spatially varied steady flow experiments with total discharges of 5, 10, 20, 30, and 40 cubic feet per second were conducted to provide a check for water surface profile predictions made using theoretical equations solved by digital computer with the resistance and velocity distribution characteristics as input information.

The first experiments were conducted in 1963. Three uniform, two nonuniform, and one spatially varied flow experiment were conducted. Analysis of the spatially varied flow data indicated the need for obtaining a rating curve for the adjustable weir. This was done in the winter of 1963-64. Experiments were continued in the summer of 1964. Three uniform, four nonuniform, and one spatially varied steady flow experiment were conducted. Measurements of velocity distribution were taken during one nonuniform flow experiment and during the spatially varied steady flow experiment.

Water surface profiles were computed by four different methods, each representing a different degree of refinement of the theory of spatially varied steady flow and of the information on resistance and velocity distribution. The results were compared.

Conclusions

1. The spatially varied steady flow equation as developed from the momentum concept yields a good prediction of water surface profiles if suitable Boussinesq coefficient and resistance relationships are used.
2. Boussinesq coefficients and resistance coefficients determined from steady nonuniform flow can be used with reasonable accuracy in predicting spatially varied steady flow water surface profiles.
3. The use of Boussinesq coefficients and resistance coefficients computed considering Boussinesq coefficients is essential in computing spatially varied steady flow water surface profiles where there is appreciable curvature of flow.
4. The limiting factor in predicting spatially varied steady flow profiles in small vegetated channels is not the theory nor the computational method, but rather the estimation of Boussinesq and resistance coefficients and possibly of the hydraulic elements of the channel.
5. Boussinesq coefficients in small vegetation-lined channels are much larger than commonly quoted text book values of 1.1.
6. For fairly dense clipped bermudagrass sod, average culm length seems to be a satisfactory criterion for relating resistance to vegetal condition.

Suggestions for Future Research

The original statement of the problem and the findings of these experiments suggest several topics that need investigation.

The effect on the predicted profiles of using different integration increments of distance up the channel, and the differences in predicted profiles between the various methods for other bottom slopes could be easily investigated.

The difference between the resistance values obtained under the nearly uniform and the nonuniform flow conditions in the experimental channel should be studied.

The extent of the variation of the Boussinesq coefficient with culm length in the experimental channel is of interest. If significant variation is found, this should be considered when computing resistance using the Boussinesq coefficient.

Because of the importance of the Boussinesq coefficient in computing water surface profiles in zones of appreciable streamline curvature in small vegetation-lined channels, more research needs to be conducted on velocity distribution and methods of estimating the Boussinesq coefficient for different types of flow in vegetation-lined channels.

The most important area for future research is in obtaining experimental and predicted spatially varied unsteady flow profiles for small vegetation-lined channels. Some work is being done on the general problem of spatially varied unsteady flow, but very little is being done on small vegetation-lined channels. The research facility at Stillwater offers unique opportunities for working with this problem.

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APPENDIX A

EXPERIMENTAL DATA AND THEORETICALLY PREDICTED WATER SURFACE PROFILES

TABLE A-1

VELOCITY DISTRIBUTION DATA FROM EXPERIMENT 13

Test	Section A Velocity	Section A Area	Section B Velocity	Section B Area	Section C Velocity	Section C Area
1	Data Erratic,		1.40	0.000	2.36	0.000
	Not Presented		1.10	.370	1.75	.329
			1.00	.599	1.50	.569
			.90	.881	1.25	.694
			.75	1.267	1.00	.839
			.50	1.793	.90	.898
2			1.79	.000	3.40	.000
			1.50	.823	3.00	.249
			1.25	1.737	2.50	.730
			1.10	2.298	2.00	1.166
			1.00	2.693	1.50	1.652
			.90	3.056	1.25	1.846
			.75	3.473	1.00	1.921
3			2.11	.000	4.18	.000
			1.75	1.514	4.00	.118
			1.50	2.664	3.50	.703
			1.25	3.758	3.00	1.268
			1.00	4.732	2.50	1.809
			.75	5.512	2.00	2.319
					1.50	2.683
					1.00	2.955
4			2.68	.000	4.95	.000
			2.50	.971	4.50	.720
			2.25	2.640	4.00	1.786
			2.00	4.116	3.50	2.572
			1.75	5.698	3.00	3.360
			1.50	7.006	2.50	4.068
			1.25	8.198	2.00	4.629
			1.00	9.122	1.50	5.045
5			3.09	.000	5.17	.000
			3.00	.170	5.00	.723
			2.75	2.304	4.75	1.443
			2.50	4.439	4.50	2.170
			2.25	6.237	4.00	3.376
			2.00	7.830	3.50	4.516
			1.75	9.443	3.00	5.522
			1.50	10.893	2.00	6.746
			1.00	12.613		

TABLE A-2

VELOCITY DISTRIBUTION DATA FROM EXPERIMENT 14

	Section A		Section B		Section C	
Test	Velocity	Area	Velocity	Area	Velocity	Area
1	0.306	0.000	1.05	0.000	2.55	0.000
	.275	.118	1.00	.119	2.50	.085
	.250	.258	.90	.421	2.25	.368
	.225	.406	.80	.773	2.00	.634
	.200	.586	.75	1.279	1.75	.965
	.175	.803	.70	1.474	1.50	1.268
	.150	1.002	.60	1.802	1.25	1.516
			.50	2.142	1.00	1.711
			.40	2.457		
2	.298	.000	1.30	.000	3.94	.000
	.240	.595	1.20	.424	3.50	.540
	.225	.766	1.10	.960	3.25	.855
	.200	1.345	1.00	1.548	3.00	1.189
	.175	1.842	.90	2.284	2.75	1.530
	.150	2.291	.80	3.048	2.50	1.803
	.130	2.531	.70	3.650	2.25	2.074
			.60	4.124	2.00	2.386
			.50	4.564	1.75	2.655
					1.50	2.855
3	.297	.000	1.59	.000	4.95	.000
	.275	.265	1.50	.272	4.50	.622
	.250	.986	1.40	1.016	4.00	1.655
	.225	2.168	1.30	1.850	3.50	2.522
	.200	3.490	1.20	2.740	3.00	3.414
	.175	4.542	1.10	3.736	2.50	4.141
	.150	5.225	1.00	4.668	2.00	4.651
			.90	5.612	1.50	5.078
			.80	6.532		
			.70	7.170		
			.60	7.866		

TABLE A-2 (CONTINUED)

Test	Section A Velocity	Section A Area	Section B Velocity	Section B Area	Section C Velocity	Section C Area
4	0.307	0.000	1.82	0.000	5.25	0.540
	.300	.030	1.70	.499	5.00	1.227
	.275	.266	1.60	1.359	4.50	2.469
	.250	.923	1.50	2.438	4.00	3.530
	.225	2.839	1.40	3.468	3.50	4.603
	.200	5.071	1.30	4.666	3.00	5.450
	.175	6.677	1.20	5.819	2.50	6.290
			1.10	6.953	2.00	6.867
			1.00	8.000	1.50	7.442
			.90	8.961		
			.80	10.019		
5	.347	.000	1.92	.000	5.74	.000
	.275	1.207	1.80	.710	5.00	2.373
	.250	4.046	1.70	1.752	4.50	3.815
	.240	5.768	1.60	3.084	4.00	5.261
	.225	7.632	1.50	3.998	3.50	6.564
	.200	9.547	1.40	5.270	3.00	7.513
	.175	10.601	1.30	6.635	2.50	8.373
	.150	11.463	1.20	7.949	2.00	9.031
			1.10	9.145		
			1.00	10.465		
			.90	11.665		
			.80	13.033		

TABLE A-3

WATER SURFACE ELEVATIONS FROM UNIFORM FLOW EXPERIMENTS 1, 2, AND 3, 1963

Experiment 1

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.000	914.302	914.602	915.040	915.253
73.6	913.958	914.266	914.570	915.012	915.218
123.6	913.910	914.223	914.535	914.987	915.184
173.6	913.852	914.176	914.494	914.954	915.142
223.6	813.803	914.139	914.463	914.927	915.107
273.6	913.750	914.104	914.432	914.903	915.073
323.6	913.706	914.074	914.405	914.878	915.035
373.6	913.643	914.028	914.363	914.843	914.982
399.2	913.622	914.017	914.354	914.838	914.976

Experiment 2

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.051	914.346	914.602	915.027	915.294
73.6	914.007	914.305	914.572	914.990	915.256
123.6	913.958	914.264	914.534	914.956	915.222
173.6	913.895	914.212	914.491	914.907	915.175
223.6	913.842	914.166	914.454	914.867	915.134
273.6	913.784	914.127	914.422	914.832	915.102
323.6	913.731	914.090	914.390	914.790	915.058
373.6	913.652	914.030	914.344	914.734	914.999
399.2	913.618	914.011	914.330	914.720	914.989

Experiment 3

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.133	914.435	914.657	915.072	915.329
73.6	914.087	914.394	914.613	915.028	915.286
123.6	914.038	914.350	914.568	914.983	915.240
173.6	913.976	914.300	914.514	914.929	915.185
223.6	913.924	914.255	914.463	914.879	915.135
273.6	913.873	914.218	914.419	914.833	915.090
323.6	913.829	914.183	914.378	914.785	915.040
373.6	913.772	914.137	914.317	914.716	914.968
399.2	913.746	914.121	914.294	914.695	914.948

TABLE A-4

WATER SURFACE ELEVATIONS FROM NONUNIFORM FLOW EXPERIMENTS 4 AND 5, 1963

Experiment 4

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.143	914.407	914.623	915.004	915.238
73.6	914.093	914.355	914.568	914.946	915.176
123.6	914.043	914.301	914.513	914.888	915.115
173.6	913.967	914.223	914.432	914.801	915.026
223.6	913.897	914.147	914.353	914.716	914.937
273.6	913.824	914.070	914.273	914.631	914.847
323.6	913.742	913.977	914.175	914.520	914.725
373.6	913.594	913.792	913.963	914.272	914.458
399.2	913.479	913.637	913.775	914.041	914.209

Experiment 5

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.010	914.258	914.484	914.853	915.091
73.6	913.960	914.206	914.430	914.797	915.033
123.6	913.915	914.158	914.380	914.745	914.978
173.6	913.855	914.097	914.315	914.674	914.906
223.6	913.801	914.040	914.256	914.610	914.838
273.6	913.743	913.981	914.194	914.544	914.770
323.6	913.667	913.898	914.101	914.436	914.648
373.6	913.549	913.749	913.932	914.230	914.424
399.2	913.442	913.604	913.747	913.999	914.161

TABLE A-5

WATER SURFACE ELEVATIONS FROM UNIFORM FLOW EXPERIMENTS 7, 9, AND 11, 1964

Experiment 7

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.095	914.380	914.633	915.035	915.430
73.6	914.048	914.338	914.590	914.995	915.397
123.6	914.016	914.307	914.558	914.965	915.371
173.6	913.977	914.271	914.518	914.928	915.341
223.6	913.938	914.233	914.478	914.889	915.311
273.6	913.896	914.197	914.436	914.852	915.282
323.6	913.852	914.158	914.388	914.804	915.242
373.6	913.795	914.109	914.326	914.748	915.200
399.2	913.770	914.093	914.303	914.730	915.187

Experiment 9

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.039	914.292	914.493	914.933	915.264
73.6	913.991	914.249	914.448	914.887	915.219
123.6	913.954	914.215	914.410	914.849	915.182
173.6	913.917	914.181	914.372	914.806	915.141
223.6	913.879	914.148	914.333	914.764	915.101
273.6	913.845	914.117	914.296	914.722	915.064
323.6	913.813	914.087	914.255	914.668	915.010
373.6	913.770	914.048	914.199	914.596	914.946
399.2	913.753	914.034	914.176	914.574	914.927

Experiment 11

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.118	914.355	914.581	915.038	915.340
73.6	914.062	914.302	914.527	914.994	915.292
123.6	914.021	914.263	914.487	914.959	915.251
173.6	913.978	914.224	914.446	914.921	915.212
223.6	913.935	914.183	914.403	914.882	915.167
273.6	913.892	914.147	914.360	914.848	915.131
323.6	913.849	914.104	914.313	914.806	915.074
373.6	913.779	914.042	914.241	914.748	915.004
399.2	913.738	914.014	914.210	914.728	914.976

TABLE A-6

WATER SURFACE ELEVATIONS FROM NONUNIFORM FLOW EXPERIMENTS 8, 10, 12 AND 13, 1964

Experiment 8

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.104	914.372	914.605	914.972	915.261
73.6	914.053	914.319	914.552	914.914	915.198
123.6	914.015	914.276	914.506	914.868	915.149
173.6	913.974	914.228	914.453	914.807	915.085
223.6	913.920	914.167	914.387	914.734	915.007
273.6	913.864	914.103	914.317	914.658	914.925
323.6	913.792	914.014	914.216	914.540	914.792
373.6	913.647	913.837	914.014	914.310	914.542
399.2	913.455	913.616	913.760	914.011	914.226

Experiment 10

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.077	914.310	914.528	914.910	915.202
73.6	914.022	914.253	914.471	914.849	915.138
123.6	913.978	914.206	914.423	914.799	915.086
173.6	913.934	914.157	914.370	914.741	915.024
223.6	913.883	914.102	914.310	914.675	914.955
273.6	913.828	914.043	914.250	914.609	914.884
323.6	913.770	913.974	914.170	914.510	914.770
373.6	913.647	913.823	913.996	914.302	914.538
399.2	913.464	913.612	913.757	914.024	914.241

Experiment 12

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.143	914.364	914.780	914.966	915.256
73.6	914.084	914.304	914.520	914.903	915.189
123.6	914.037	914.255	914.469	914.850	915.132
173.6	913.994	914.207	914.415	914.789	915.069
223.6	913.941	914.150	914.356	914.721	914.995
273.6	913.887	914.089	914.291	914.652	914.921
323.6	913.827	914.015	914.208	914.548	914.801
373.6	913.691	913.853	914.019	914.324	914.557
399.2	913.489	913.626	913.763	914.033	914.228

TABLE A-6 (CONTINUED)

Experiment 13

Station	Test 1	Test 2	Test 3	Test 4	Test 5
23.6	914.016	914.282	914.486	914.792	915.046
73.6	913.961	914.227	914.434	914.737	914.986
123.6	913.916	914.180	914.383	914.688	914.934
173.6	913.874	914.130	914.329	914.628	914.873
223.6	913.824	914.075	914.272	914.572	914.810
273.6	913.765	914.013	914.209	914.502	914.738
323.6	913.711	913.946	914.133	914.412	914.637
373.6	913.602	913.806	913.977	914.231	914.438
399.2	913.438	913.612	913.756	914.006	914.202

TABLE A-7

BOTTOM ELEVATIONS AT HALF-FOOT INTERVALS ACROSS FC 31, 1963 (READ ROW-WISE)

NOMINAL STATION 0+25							
915.54	915.35	915.20	915.00	914.78	914.66	914.46	914.31
914.12	913.94	913.77	913.69	913.58	913.41	913.19	913.10
913.21	913.29	913.40	913.53	913.60	913.65	913.76	913.85
913.92	914.01	914.10	914.15	914.19	914.23	914.30	914.40
914.44	914.49	914.58	914.68	914.74	914.79	914.90	914.97
915.08	915.13	915.16	915.23	915.35	915.39	915.41	915.49
915.49							
NOMINAL STATION 0+75							
915.48	915.26	915.17	914.97	914.81	914.63	914.51	914.31
914.14	913.94	913.85	913.69	913.61	913.45	913.11	913.07
913.20	913.31	913.43	913.47	913.52	913.58	913.68	913.70
913.77	913.81	913.88	913.98	914.04	914.10	914.17	914.23
914.35	914.35	914.45	914.52	914.62	914.73	914.69	914.88
914.97	914.99	915.07	915.15	915.17	915.19	915.25	915.29
915.36							
NOMINAL STATION 1+25							
915.53	915.36	915.20	914.96	914.79	914.68	914.51	914.27
914.09	913.94	913.79	913.60	913.48	913.34	913.07	913.04
913.18	913.29	913.36	913.39	913.44	913.53	913.52	913.61
913.65	913.71	913.75	913.77	913.82	913.89	914.00	914.12
914.21	914.27	914.31	914.37	914.43	914.52	914.59	914.71
914.76	914.84	914.91	914.97	915.00	915.06	915.14	915.24
915.29							
NOMINAL STATION 1+75							
915.36	915.23	915.02	914.82	914.64	914.50	914.29	914.17
913.96	913.84	913.70	913.51	913.42	913.26	913.07	913.03
913.16	913.31	913.36	913.38	913.42	913.52	913.53	913.60
913.65	913.69	913.73	913.80	913.87	913.90	914.02	914.07
914.13	914.18	914.27	914.35	914.43	914.51	914.55	914.64
914.76	914.82	914.93	915.02	915.08	915.16	915.20	915.21
915.28							
NOMINAL STATION 2+25							
915.28	915.12	914.92	914.76	914.56	914.33	914.18	914.06
913.87	913.71	913.58	913.52	913.40	913.16	912.99	912.90
912.98	913.06	913.11	913.22	913.24	913.32	913.37	913.48
913.48	913.55	913.59	913.63	913.73	913.80	913.86	913.95
913.99	914.08	914.17	914.22	914.26	914.37	914.46	914.54
914.65	914.75	914.83	914.90	914.97	915.04	915.12	915.18
915.26							
NOMINAL STATION 2+75							
915.10	914.92	914.71	914.50	914.32	914.19	914.02	913.80
913.66	913.55	913.45	913.35	913.19	913.06	912.93	912.87
912.97	913.10	913.18	913.20	913.29	913.32	913.37	913.47
913.57	913.64	913.69	913.76	913.82	913.93	914.00	914.05
914.15	914.20	914.25	914.34	914.44	914.48	914.55	914.63
914.68	914.74	914.79	914.88	914.94	915.01	915.07	915.18
915.23							
NOMINAL STATION 3+25							
915.15	914.97	914.75	914.60	914.38	914.17	913.99	913.77
913.63	913.50	913.36	913.28	913.16	913.07	912.94	912.83
912.96	913.07	913.17	913.26	913.34	913.32	913.37	913.42
913.53	913.60	913.67	913.76	913.89	913.96	914.04	914.14
914.23	914.30	914.38	914.50	914.53	914.60	914.74	914.79
914.81	914.91	914.98	915.03	915.13	915.18	915.23	915.28
915.36							
NOMINAL STATION 3+75							
915.04	914.89	914.71	914.49	914.30	914.21	914.01	913.80
913.76	913.54	913.41	913.23	913.07	912.97	912.80	912.81
912.96	913.09	913.21	913.26	913.36	913.40	913.50	913.59
913.67	913.74	913.81	913.90	913.96	914.06	914.13	914.22
914.29	914.38	914.45	914.51	914.58	914.63	914.73	914.84
914.91	914.95	915.00	915.04	915.12	915.19	915.28	915.33
915.40							
NOMINAL STATION 4+00							
914.93	914.75	914.59	914.41	914.30	914.08	913.92	913.77
913.59	913.47	913.32	913.17	913.06	912.89	912.75	912.74
912.88	912.96	913.00	913.10	913.18	913.24	913.39	913.43
913.53	913.62	913.73	913.83	913.94	913.96	914.06	914.10
914.24	914.27	914.31	914.36	914.49	914.57	914.66	914.74
914.72	914.80	914.81	914.88	914.95	914.98	915.03	915.06
915.13							

TABLE A-8

BOTTOM ELEVATIONS AT HALF-FOOT INTERVALS ACROSS FC 31, 1964 (READ ROW-WISE)

			NOMINAL STATION 0+25				
915.53	915.34	915.18	914.99	914.77	914.62	914.43	914.30
914.12	913.93	913.76	913.67	913.58	913.38	913.22	913.09
913.20	913.28	913.36	913.53	913.59	913.65	913.74	913.85
913.92	913.99	914.06	914.12	914.18	914.21	914.27	914.36
914.43	914.49	914.58	914.66	914.72	914.81	914.88	914.96
915.06	915.12	915.16	915.21	915.33	915.37	915.39	915.47
915.46							
			NOMINAL STATION 0+75				
915.47	915.26	915.16	914.96	914.82	914.63	914.51	914.32
914.14	913.96	913.86	913.70	913.60	913.45	913.14	913.08
913.18	913.32	913.42	913.47	913.52	913.58	913.68	913.69
913.77	913.81	913.89	913.98	914.04	914.10	914.17	914.22
914.32	914.35	914.45	914.52	914.61	914.72	914.68	914.86
914.96	914.98	915.06	915.14	915.15	915.18	915.23	915.29
915.36							
			NOMINAL STATION 1+25				
915.50	915.34	915.17	914.95	914.80	914.68	914.50	914.28
914.09	913.92	913.83	913.62	913.46	913.35	913.06	913.03
913.15	913.26	913.35	913.39	913.44	913.50	913.52	913.60
913.64	913.71	913.74	913.78	913.82	913.88	913.98	914.10
914.19	914.26	914.30	914.37	914.40	914.50	914.58	914.68
914.74	914.81	914.88	914.95	914.97	915.04	915.11	915.21
915.27							
			NOMINAL STATION 1+75				
915.38	915.24	915.02	914.85	914.65	914.54	914.28	914.18
913.97	913.83	913.70	913.51	913.43	913.29	913.08	913.04
913.14	913.29	913.36	913.38	913.42	913.52	913.54	913.61
913.64	913.70	913.72	913.80	913.87	913.88	913.98	914.07
914.13	914.18	914.25	914.35	914.42	914.53	914.56	914.63
914.76	914.80	914.91	915.00	915.06	915.14	915.17	915.22
915.28							
			NOMINAL STATION 2+25				
915.28	915.13	914.90	914.75	914.57	914.33	914.17	914.06
913.88	913.71	913.59	913.52	913.38	913.16	913.01	912.90
912.98	913.07	913.11	913.22	913.25	913.30	913.37	913.46
913.47	913.55	913.59	913.64	913.72	913.80	913.88	913.94
914.00	914.08	914.18	914.23	914.28	914.38	914.47	914.55
914.67	914.75	914.82	914.89	914.94	915.02	915.12	915.16
915.24							
			NOMINAL STATION 2+75				
915.08	914.90	914.68	914.47	914.32	914.18	913.98	913.80
913.64	913.54	913.45	913.34	913.18	913.08	912.95	912.88
913.00	913.10	913.18	913.21	913.28	913.32	913.38	913.48
913.58	913.65	913.70	913.77	913.84	913.92	914.01	914.07
914.16	914.21	914.26	914.34	914.45	914.50	914.56	914.64
914.68	914.73	914.78	914.87	914.93	915.00	915.06	915.16
915.21							
			NOMINAL STATION 3+25				
915.16	914.98	914.77	914.63	914.39	914.17	914.02	913.80
913.66	913.52	913.41	913.30	913.18	913.10	912.96	912.83
912.92	913.06	913.13	913.22	913.33	913.33	913.36	913.42
913.50	913.61	913.68	913.76	913.89	913.96	914.03	914.10
914.20	914.30	914.36	914.48	914.53	914.58	914.72	914.80
914.79	914.91	914.98	915.01	915.11	915.17	915.22	915.25
915.32							
			NOMINAL STATION 3+75				
915.05	914.91	914.76	914.51	914.31	914.22	914.04	913.84
913.77	913.60	913.44	913.28	913.12	913.01	912.86	912.84
912.93	913.08	913.20	913.26	913.35	913.43	913.46	913.60
913.65	913.74	913.82	913.88	913.96	914.04	914.10	914.22
914.28	914.36	914.44	914.53	914.57	914.64	914.73	914.80
914.90	914.94	914.98	915.04	915.11	915.18	915.26	915.31
915.38							
			NOMINAL STATION 4+00				
915.02	914.77	914.61	914.45	914.34	914.21	913.96	913.81
913.64	913.50	913.38	913.21	913.11	912.97	912.78	912.76
912.86	912.96	913.00	913.07	913.16	913.23	913.35	913.42
913.48	913.61	913.70	913.80	913.91	913.95	914.06	914.08
914.17	914.28	914.30	914.36	914.44	914.54	914.62	914.70
914.72	914.77	914.80	914.81	914.93	914.93	915.01	915.04
915.08							

TABLE A-9

CULM LENGTH AND DENSITY DATA, FC 31, 1963

Date	Area Measured (in.^{2})	Grass	Total Culm Length (in.)	Total Number Of Culms	Avg. Culm Length (in.)	Culm Density culms/in.^{2}
8/20/63	288	Bermuda	1,908.25	627		
		Crabgrass	16.50	6		
		Bristlegr.	135.00	36		
			2,059.75	669	3.08	2.32
8/22/63	64	Bermuda	551.00	154		
		Bristlegr.	143.50	23		
			694.50	177	3.92	2.77
8/26/63	96	Bermuda	833.50	203		
		Crabgrass	37.25	7		
		Bristlegr.	21.25	3		
			892.00	213	4.19	2.22
8/28/63	96	Bermuda	1,267.50	299		
		Crabgrass	116.75	17		
		Bristlegr.	100.00	16		
			1,484.25	332	4.47	3.46
8/29/63	32	Bermuda	143.25	49		
		Crabgrass	2.75	1		
		Bristlegr.	39.50	11		
			185.50	61	3.04	1.91
9/4/63	64	Bermuda	460.00	121		
		Bristlegr.	4.75	1		
			464.75	122	3.81	1.91
9/6/63	24	Bermuda	263.75	83	3.18	3.46
9/10/63	96	Bermuda	876.25	214		
		Crabgrass	30.25	7		
			906.50	221	4.10	2.30
10/7/63	96	Bermuda	859.50	224	3.84	2.33
10/21/63	96	Bermuda	1,001.00	255		
		Bristlegr.	20.00	4		
			1,021.00	259	3.94	2.70

TABLE A-10

BERMUDAGRASS CULM AND BRANCH LENGTH AND DENSITY DATA, FC 31, 1964

Date	Area Measured (in.^{2})	Total Culm And Branch Length (in.)	Total Number Of Culms And Branches	Avg. Culm And Branch Length (in.)	Culm And Branch Density C and B/in.^{2}
7/20/64	52	534.88	187	2.86	3.60
7/23-24/64	96	1,579.25	531	2.97	5.53
7/31/64	96	1,210.75	477	2.54	4.97
8/3/64	96	1,432.25	528	2.71	5.50
8/6-7/64	96	1,937.00	626	3.09	6.52
9/1-2/64	96	1,444.00	632	2.28	6.58

TABLE A-11

VEGETATION SAMPLE WEIGHTS, FC 31, 1964

Date	Area Sampled (in.^{2})	Dry Weight (grams)
7/24/64	864	270.9
7/30/64	864	312.6
8/3/64	864	262.0
8/6/64	864	304.6
8/30/64	864	282.5
9/2/64	864	282.2
9/25/64	864	360.9

TABLE A-12

INFLOW DISTRIBUTION, EXPERIMENT 6

Test 2
Flow Adhering

Distance From Upper End Of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	——			0.000
		0.237	0.240	
25	0.021			.240
		.387	.392	
75	.017			.632
		.306	.310	
125	.017			.942
		.384	.389	
175	.020			1.331
		.480	.486	
225	.021			1.817
		.646	.654	
275	.027			2.471
		.804	.814	
325	.027			3.285
		.732	.741	
375	.024			4.026
		.229	.232	
395	.021			4.258
		.040	.041	
399.23	——			4.299
Total		4.245	4.299	

TABLE A-12 (CONTINUED)

Test 3
Flow Adhering Except From 290 to 350

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	——			0.000
		0.467	0.484	
25	0.029			.484
		.837	.867	
75	.026			1.351
		.760	.787	
125	.026			2.138
		.887	.919	
175	.030			3.057
		1.048	1.085	
225	.032			4.142
		1.284	1.330	
275	.037			5.472
		1.400	1.450	
325	.036			6.922
		1.367	1.416	
375	.035			8.338
		.517	.535	
397	.030			8.873
		.045	.047	
399.23	——			8.920
Total		8.612	8.920	

TABLE A-12 (CONTINUED)

Test 4
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	——			0.000
		1.170	1.125	
25	0.051			1.125
		2.234	2.148	
75	.047			3.273
		2.194	2.110	
125	.050			5.383
		2.340	2.250	
175	.052			7.633
		2.430	2.336	
225	.053			9.969
		2.600	2.500	
275	.058			12.469
		2.742	2.636	
325	.058			15.105
		2.705	2.601	
375	.057			17.706
		1.016	.977	
395	.052			18.683
		.205	.197	
399.23	——			18.880
Total		19.636	18.880	

TABLE A-12 (CONTINUED)

Test 5
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.071			0.000
		1.794	1.738	
25	.075			1.738
		1.835	1.777	
50	.072			3.515
		1.757	1.702	
75	.071			5.217
		1.739	1.684	
100	.070			6.901
		1.762	1.707	
125	.072			8.608
		1.798	1.741	
150	.074			10.349
		1.876	1.817	
175	.076			12.166
		1.889	1.830	
200	.075			13.996
		1.798	1.741	
225	.071			15.737
		1.739	1.684	
250	.070			17.421
		1.720	1.666	
275	.070			19.087
		1.720	1.666	
300	.070			20.753
		1.753	1.698	
325	.072			22.451
		1.794	1.738	
350	.073			24.189
		1.876	1.817	
375	.077			26.006
		1.526	1.478	
395	.075			27.484
		.317	.307	
399.23	———			27.791
Total		28.693	27.791	

TABLE A-12 (CONTINUED)

Test 6
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)

Windy And High Head.

Uniform Inflow

Assumed

TABLE A-12 (CONTINUED)

Test 7
Flow Adhering

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.030			0.000
		0.504	0.369	
30	.025			.369
		.269	.197	
50	.024			.566
		.316	.232	
75	.023			.798
		.286	.210	
100	.022			1.008
		.265	.194	
125	.022			1.202
		.307	.225	
150	.025			1.427
		.373	.273	
175	.027			1.700
		.427	.313	
200	.028			2.013
		.437	.320	
225	.027			2.333
		.455	.333	
250	.030			2.666
		.543	.398	
275	.033			3.064
		.584	.428	
300	.032			3.492
		.551	.404	
325	.031			3.896
		.509	.373	
350	.030			4.269
		.462	.339	
375	.028			4.608
		.342	.251	
395	.027			4.859
		.069	.051	
399.23	——			4.910
Total		6.699	4.910	

TABLE A-12 (CONTINUED)

Test 8
Flow Adhering Except From 300 To 350

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.036			0.000
		0.737	0.728	
30	.030			.728
		.400	.395	
50	.030			1.123
		.473	.467	
75	.029			1.590
		.441	.435	
100	.027			2.025
		.416	.411	
125	.027			2.436
		.444	.438	
150	.029			2.874
		.520	.513	
175	.032			3.387
		.584	.577	
200	.033			3.964
		.584	.577	
225	.032			4.541
		.615	.607	
250	.035			5.148
		.715	.706	
275	.038			5.854
		.747	.738	
300	.037			6.592
		.702	.693	
325	.036			7.285
		.652	.644	
350	.034			7.929
		.622	.614	
375	.033			8.543
		.468	.462	
395	.032			9.005
		.096	.095	
399.23	———			9.100
Total		9.216	9.100	

TABLE A-12 (CONTINUED)

Test 9
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.058			0.000
		1.556	1.480	
30	.054			1.480
		.972	.925	
50	.051			2.405
		1.166	1.109	
75	.051			3.514
		1.160	1.104	
100	.050			4.618
		1.153	1.097	
125	.050			5.715
		1.190	1.132	
150	.053			6.847
		1.246	1.185	
175	.055			8.032
		1.297	1.234	
200	.057			9.266
		1.307	1.244	
225	.055			10.510
		1.323	1.259	
250	.058			11.769
		1.403	1.335	
275	.061			13.104
		1.430	1.361	
300	.060			14.465
		1.409	1.341	
325	.059			15.806
		1.375	1.308	
350	.057			17.114
		1.326	1.262	
375	.056			18.376
		1.038	.988	
395	.055			19.364
		.218	.207	
399.23	——			19.571
Total		20.569	19.571	

TABLE A-13

INFLOW DISTRIBUTION, EXPERIMENT 14

Test 1
Flow Adhering

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.025			0.000
		0.119	0.122	
10	.021			.122
		.194	.199	
30	.021			.321
		.189	.193	
50	.021			.514
		.209	.214	
75	.018			.728
		.222	.227	
100	.022			.955
		.246	.252	
125	.020			1.207
		.230	.235	
150	.021			1.442
		.230	.235	
175	.020			1.677
		.135	.138	
190	.020			1.815
		.192	.196	
210	.021			2.011
		.144	.147	
225	.021			2.158
		.226	.231	
250	.020			2.389
		.226	.231	
275	.021			2.620
		.236	.242	
300	.021			2.862
		.236	.242	
325	.020			3.104
		.251	.257	
350	.022			3.361
		.228	.233	
375	.019			3.594
		.039	.040	
380	.019			3.634
		.151	.155	
399.23	——			3.789
Total		3.703	3.789	

TABLE A-13 (CONTINUED)

Test 2
Flow Adhering

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.037			0.000
		0.276	0.273	
10	.035			.273
		.498	.493	
30	.033			.766
		.477	.472	
50	.033			1.238
		.563	.557	
75	.031			1.795
		.584	.578	
100	.034			2.373
		.610	.604	
125	.032			2.977
		.590	.584	
150	.033			3.561
		.587	.581	
175	.032			4.142
		.350	.346	
190	.033			4.488
		.479	.474	
210	.033			4.962
		.357	.353	
225	.033			5.315
		.577	.571	
250	.032			5.886
		.580	.574	
275	.033			6.460
		.593	.587	
300	.033			7.047
		.587	.581	
325	.033			7.628
		.603	.597	
350	.034			8.225
		.571	.565	
375	.031			8.790
		.106	.105	
380	.031			8.895
		.409	.405	
399.23	———			9.300
Total		9.397	9.300	

TABLE A-13 (CONTINUED)

Test 3
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.061			0.000
		0.562	0.529	
10	.058			.529
		1.064	1.001	
30	.056			1.530
		1.039	.978	
50	.055			2.508
		1.261	1.187	
75	.053			3.695
		1.272	1.197	
100	.056			4.892
		1.296	1.219	
125	.055			6.111
		1.283	1.207	
150	.055			7.318
		1.272	1.197	
175	.054			8.515
		.757	.712	
190	.054			9.227
		1.018	.958	
210	.055			10.185
		.770	.725	
225	.055			10.910
		1.265	1.190	
250	.054			12.100
		1.265	1.190	
275	.055			13.290
		1.273	1.198	
300	.055			14.488
		1.273	1.198	
325	.054			15.686
		1.290	1.214	
350	.056			16.900
		1.265	1.190	
375	.052			18.090
		.244	.230	
380	.053			18.320
		.946	.890	
399.23	——			19.210
Total		20.415	19.210	

TABLE A-13 (CONTINUED)

Test 4
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.079			0.000
		0.787	0.795	
10	.076			.795
		1.515	1.530	
30	.075			2.325
		1.480	1.495	
50	.074			3.820
		1.796	1.814	
75	.072			5.634
		1.825	1.843	
100	.075			7.477
		1.861	1.879	
125	.074			9.356
		1.839	1.857	
150	.074			11.213
		1.825	1.843	
175	.073			13.056
		1.089	1.100	
190	.073			14.156
		1.483	1.498	
210	.075			15.654
		1.117	1.128	
225	.074			16.782
		1.832	1.850	
250	.073			18.632
		1.832	1.850	
275	.074			20.482
		1.854	1.872	
300	.075			22.354
		1.861	1.879	
325	.074			24.233
		1.872	1.890	
350	.075			26.123
		1.836	1.854	
375	.072			27.977
		.355	.358	
380	.072			28.335
		1.350	1.363	
399.23	——			29.698
Total		29.409	29.698	

TABLE A-13 (CONTINUED)

Test 5
Flow Springing Free

Distance From Upper End of Weir (ft.)	Head (ft.)	Calculated ΔQ (cfs)	Adjusted ΔQ (cfs)	Total Adjusted Discharge At Station (cfs)
0	0.095			0.000
		1.032	1.036	
10	.094			1.036
		2.023	2.032	
30	.092			3.068
		1.986	1.995	
50	.091			5.063
		2.409	2.419	
75	.088			7.482
		2.434	2.445	
100	.093			9.927
		2.478	2.489	
125	.090			12.416
		2.434	2.445	
150	.091			14.861
		2.421	2.432	
175	.090			17.293
		1.450	1.456	
190	.090			18.749
		1.954	1.962	
210	.091			20.711
		1.465	1.471	
225	.090			22.182
		2.416	2.426	
250	.090			24.608
		2.421	2.432	
275	.091			27.040
		2.454	2.465	
300	.091			29.505
		2.450	2.461	
325	.091			31.966
		2.486	2.497	
350	.093			34.463
		2.466	2.477	
375	.090			36.940
		.487	.489	
380	.091			37.429
		1.894	1.902	
399.23	——			39.331
Total		39.160	39.331	

TABLE A-14

OBSERVED AND CALCULATED WATER SURFACE PROFILES, EXPERIMENT 6, SPATIALLY VARIED FLOW

Test 2

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.050	914.017	914.022	914.030	914.043	-0.033	-0.028	-0.020	-0.007
73.6	914.042	914.009	914.014	914.022	914.037	-.033	-.028	-.020	-.005
123.6	914.034	913.994	913.999	914.008	914.025	-.040	-.035	-.026	-.009
173.6	914.021	913.972	913.977	913.985	914.006	-.049	-.044	-.036	-.015
223.6	913.999	913.946	913.951	913.960	913.983	-.053	-.048	-.039	-.016
273.6	913.967	913.913	913.918	913.926	913.951	-.053	-.049	-.041	-.016
323.6	913.902	913.856	913.860	913.868	913.894	-.046	-.042	-.034	-.008
373.6	913.758	913.717	913.719	913.721	913.742	-.041	-.039	-.037	-.016
399.2	913.586	913.586	913.586	913.586	913.586	-.000	-.000	-.000	-.000

Test 3

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.314	914.272	914.283	914.298	914.315	-.042	-.031	-.016	+.001
73.6	914.306	914.265	914.276	914.290	914.309	-.041	-.030	-.016	+.003
123.6	914.296	914.251	914.261	914.275	914.297	-.045	-.035	-.021	+.003
173.6	914.280	914.227	914.238	914.252	914.277	-.053	-.042	-.028	-.003
223.6	914.252	914.197	914.207	914.221	914.249	-.055	-.045	-.031	-.003
273.6	914.213	914.157	914.166	914.179	914.209	-.056	-.047	-.034	-.004
323.6	914.137	914.087	914.094	914.107	914.137	-.050	-.043	-.030	-.000
373.6	913.965	913.919	913.924	913.931	913.956	-.047	-.041	-.034	-.009
399.2	913.746	913.746	913.746	913.746	913.746	-.000	-.000	-.000	-.000

TABLE A-14 (CONTINUED)

Test 4

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.673	914.623	914.643	914.665	914.686	-0.050	-0.030	-0.008	+0.013
73.6	914.665	914.615	914.635	914.657	914.678	-.050	-.030	-.008	+.013
123.6	914.653	914.598	914.618	914.638	914.663	-.055	-.035	-.015	+.010
173.6	914.631	914.569	914.588	914.608	914.637	-.062	-.043	-.023	+.006
223.6	914.598	914.532	914.550	914.570	914.603	-.066	-.048	-.028	+.005
273.6	914.530	914.480	914.497	914.516	914.552	-.070	-.053	-.034	+.002
323.6	914.452	914.393	914.407	914.426	914.463	-.059	-.045	-.026	+.011
373.6	914.242	914.193	914.202	914.215	914.248	-.049	-.040	-.027	+.006
399.2	913.973	913.973	913.973	913.973	913.973	-.000	-.000	-.000	+.000

Test 5

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.946	914.860	914.888	914.913	914.937	-.086	-.058	-.033	-.009
73.6	914.930	914.851	914.879	914.903	914.928	-.079	-.051	-.027	-.002
123.6	914.916	914.832	914.858	914.881	914.910	-.084	-.058	-.035	-.006
173.6	914.884	914.799	914.824	914.846	914.879	-.085	-.060	-.038	-.005
223.6	914.838	914.757	914.780	914.803	914.839	-.081	-.058	-.035	+.001
273.6	914.777	914.697	914.718	914.740	914.780	-.080	-.059	-.037	+.003
323.6	914.668	914.598	914.617	914.638	914.679	-.070	-.051	-.030	+.011
373.6	914.433	914.378	914.391	914.408	914.445	-.055	-.043	-.025	+.012
399.2	914.133	914.133	914.133	914.133	914.133	-.000	-.000	-.000	+.000

TABLE A-14 (CONTINUED)

Test 6

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	915.158	915.054	915.092	915.121	915.148	-0.104	-0.066	-0.037	-0.010
73.6	915.141	915.045	915.082	915.110	915.139	-.096	-.059	-.031	-.002
123.6	915.126	915.025	915.060	915.087	915.119	-.101	-.066	-.039	-.007
173.6	915.093	914.991	915.024	915.050	915.087	-.102	-.069	-.043	-.006
223.6	915.044	914.949	914.979	915.005	915.046	-.095	-.065	-.039	+.002
273.6	914.981	914.886	914.913	914.939	914.982	-.095	-.068	-.042	+.001
323.6	914.860	914.780	914.802	914.827	914.872	-.080	-.058	-.033	+.012
373.6	914.606	914.547	914.561	914.581	914.621	-.059	-.045	-.025	+.015
399.2	914.284	914.284	914.284	914.284	914.284	-.000	-.000	-.000	+.000

Test 7

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.079	914.063	914.070	914.080	914.093	-.016	-.009	+.001	+.014
73.6	914.077	914.055	914.061	914.071	914.086	-.022	-.016	-.006	+.009
123.6	914.059	914.040	914.046	914.055	914.073	-.019	-.013	-.004	+.014
173.6	914.049	914.017	914.023	914.032	914.053	-.032	-.026	-.017	+.004
223.6	914.033	913.988	913.994	914.003	914.027	-.045	-.039	-.030	-.006
273.6	914.001	913.951	913.957	913.966	913.992	-.050	-.044	-.035	-.009
323.6	913.934	913.890	913.894	913.902	913.930	-.044	-.040	-.032	-.004
373.6	913.748	913.743	913.747	913.749	913.771	-.005	-.001	+.001	+.023
399.2	913.610	913.610	913.610	913.610	913.610	-.000	-.000	+.000	+.000

TABLE A-14 (CONTINUED)

Test 8

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.313	914.287	914.298	914.312	914.329	-0.026	-0.015	-0.001	+0.016
73.6	914.311	914.278	914.289	914.303	914.322	-.033	-.022	-.008	+.011
123.6	914.292	914.262	914.273	914.286	914.308	-.030	-.019	-.006	+.016
173.6	914.279	914.237	914.248	914.261	914.286	-.042	-.031	-.018	+.007
223.6	914.259	914.205	914.215	914.229	914.257	-.054	-.044	-.030	-.002
273.6	914.222	914.163	914.171	914.184	914.214	-.058	-.051	-.038	-.008
323.6	914.148	914.091	914.097	914.109	914.140	-.057	-.051	-.039	-.008
373.6	913.937	913.922	913.926	913.933	913.959	-.015	-.011	-.004	+.022
399.2	913.753	913.753	913.753	913.753	913.753	-.000	-.000	-.000	+.000

Test 9

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.692	914.638	914.659	914.681	914.702	-.054	-.033	-.011	+.010
73.6	914.689	914.630	914.651	914.671	914.695	-.059	-.038	-.018	+.006
123.6	914.667	914.613	914.633	914.653	914.679	-.053	-.034	-.014	+.012
173.6	914.647	914.584	914.603	914.623	914.653	-.063	-.044	-.024	+.006
223.6	914.621	914.547	914.565	914.585	914.619	-.074	-.056	-.036	-.002
273.6	914.574	914.495	914.512	914.531	914.568	-.079	-.062	-.043	-.006
323.6	914.483	914.408	914.421	914.440	914.477	-.075	-.062	-.043	-.006
373.6	914.237	914.209	914.219	914.230	914.264	-.028	-.018	-.007	+.027
399.2	914.000	914.000	914.000	914.000	914.000	-.000	-.000	-.000	+.000

TABLE A-15

OBSERVED AND CALCULATED WATER SURFACE PROFILES,
EXPERIMENT 14, SPATIALLY VARIED FLOW

Test 1

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.011	913.994	913.999	914.007	914.027	-0.017	-0.012	-0.004	+0.017
73.6	914.003	913.986	913.990	913.998	914.022	-.017	-.013	-.005	+.019
123.6	913.989	913.967	913.972	913.979	914.010	-.022	-.017	-.010	+.021
173.6	913.972	913.937	913.941	913.948	913.989	-.035	-.031	-.024	+.017
223.6	913.944	913.903	913.907	913.914	913.963	-.041	-.037	-.030	+.019
273.6	913.910	913.865	913.869	913.876	913.928	-.045	-.041	-.034	+.018
323.6	913.863	913.809	913.812	913.818	913.872	-.054	-.051	-.045	+.009
373.6	913.752	913.688	913.690	913.691	913.731	-.064	-.062	-.061	-.021
399.2	913.599	913.599	913.599	913.599	913.599	-.000	-.000	-.000	-.000

Test 2

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.319	914.281	914.294	914.310	914.338	-.038	-.025	-.009	+.019
73.6	914.310	914.273	914.285	914.300	914.332	-.037	-.025	-.010	+.022
123.6	914.297	914.255	914.267	914.281	914.320	-.042	-.030	-.016	+.023
173.6	914.276	914.226	914.237	914.250	914.298	-.050	-.039	-.026	+.022
223.6	914.248	914.191	914.201	914.215	914.269	-.057	-.047	-.032	+.021
273.6	914.208	914.148	914.157	914.169	914.227	-.060	-.051	-.039	+.019
323.6	914.148	914.081	914.088	914.099	914.157	-.067	-.060	-.049	+.009
373.6	914.004	913.931	913.935	913.938	913.983	-.073	-.069	-.066	-.021
399.2	913.811	913.811	913.811	913.811	913.811	-.000	-.000	-.000	-.000

TABLE A-15 (CONTINUED)

Test 3

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.672	914.588	914.612	914.636	914.669	-0.084	-0.060	-0.036	-0.003
73.6	914.663	914.579	914.603	914.625	914.662	-.084	-.060	-.038	-.001
123.6	914.647	914.561	914.584	914.605	914.648	-.086	-.063	-.042	+.001
173.6	914.624	914.532	914.553	914.574	914.623	-.092	-.071	-.050	-.001
223.6	914.589	914.496	914.515	914.536	914.591	-.093	-.074	-.053	-.002
273.6	914.542	914.447	914.464	914.484	914.543	-.095	-.078	-.058	+.001
323.6	914.465	914.368	914.382	914.401	914.460	-.097	-.083	-.064	-.005
373.6	914.287	914.196	914.204	914.213	914.260	-.091	-.083	-.074	-.027
399.2	914.054	914.054	914.054	914.054	914.054	-.000	-.000	-.000	-.000

Test 4

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	914.940	914.815	914.852	914.882	914.918	-.125	-.088	-.058	-.022
73.6	914.931	914.807	914.842	914.871	914.911	-.124	-.089	-.060	-.020
123.6	914.912	914.789	914.823	914 850	914.895	-.123	-.089	-.062	-.017
173.6	914.887	914.759	914.791	914.817	914.869	-.128	-.096	-.070	-.018
223.6	914.847	914.723	914.752	914.778	914.835	-.124	-.095	-.069	-.012
273.6	914.793	914.671	914.696	914.721	914.782	-.122	-.097	-.072	-.010
323.6	914.702	914.584	914.605	914.628	914.688	-.118	-.097	-.074	-.014
373.6	914.500	914.397	914.411	914.422	914.470	-.103	-.089	-.078	-.030
399.2	914.244	914.244	914.244	914.244	914.244	-.000	-.000	-.000	-.000

TABLE A-15 (CONTINUED)

Test 5

Station	(1) Observed	(2) Method 1	(3) Method 2	(4) Method 3	(5) Method 4	(6) Diff. (2)-(1)	(7) Diff. (3)-(1)	(8) Diff. (4)-(1)	(9) Diff. (5)-(1)
23.6	915.150	914.987	915.033	915.066	915.104	-0.163	-0.117	-0.084	-0.046
73.6	915.140	914.978	914.023	915.055	915.096	-.163	-.117	-.085	-.046
123.6	915.120	914.959	915.002	915.033	915.080	-.161	-.118	-.087	-.040
173.6	915.091	914.929	914.969	914.999	915.052	-.162	-.122	-.092	-.039
223.6	915.048	914.891	914.928	914.958	915.015	-.157	-.120	-.090	-.033
273.6	914.990	914.836	914.869	914.898	914.959	-.154	-.121	-.092	-.031
323.6	914.887	914.745	914.772	914.798	914.860	-.142	-.115	-.089	-.027
373.6	914.666	914.553	914.560	914.583	914.632	-.113	-.106	-.083	-.034
399.2	914.396	914.396	914.396	914.396	914.396	-.000	-.000	-.000	-.000

APPENDIX B

LISTINGS OF COMPUTER PROGRAMS

TABLE B-1

LISTING OF POLYFIT FORTRAN IV PROGRAM

```
      MON$$      JOB  211140007      MCCOOL  JANUARY,1965
      MON$$      ASGN MGO,A2
      MON$$      ASGN MJB,A3
      MON$$      MODE GO,TEST
      MON$$      EXEQ FORTRAN,SOF,SIU,08,05,,,POLYFIT
C       POLYFIT
C       THIS PROGRAM IS FOR THE 1410
C       CALCULATES A POLYNOMIAL OF UP TO DEGREE 4 TO FIT OBSERVED DATA.
C       GIVES OBSERVED AND CALCULATED VALUES OF Y, THEN RUNS REGRESSION
C       ON YOBS AND YCAL.
C       N=DEGREE OF EQUATION
C       K=NUMBER OF OBSERVATIONS
        DIMENSION X(100),Y(100),A(7,7),B(6),YCAL(100)
    150 FORMAT(21X,F10.3,5X,F10.3,6X,F10.3,5X,F10.3)
    175 FORMAT(12X,19HEQ. OF XOP1 VS XCP2,//,12X,2HY=,F9.4,1H+,F9.4,1HX,3X
       1,12HCORR. COEF.=,F9.3,3X,10HSTD. DEV =,F9.3,//)
    300 FORMAT(2I3)
    310 FORMAT(10F8.3)
    323 FORMAT(5X,9HPOLY EQ  ,2HY=,F10.3,3H  +,F10.3,3HX +,F10.3,9H(X**2)
       1 +,F10.3,9H(X**3)  +,F10.3,9H(X**4)  +,F10.3,6H(X**5),//)
    324 FORMAT(I3,6E12.6)
    400 FORMAT(27X,4HXPR2,10X,6HXOBSP1,9X,6HXCALP1,9X,3HDEV,//)
    500 FORMAT(1H1)
    600 FORMAT(//)
      2 WRITE(3,500)
        READ(1,300)N,K
        M=K
        JJ=N
        WRITE(3,600)
        READ(1,310)(Y(I),X(I),I=1,M)
        DO 40 KK=1,JJ
        N=KK
      5 NP1=N+1
        DO 10 J=1,NP1
        DO 10 K=1,NP1
        K1=J+K-2
        A(J,K)=0.
        DO 10 I=1,M
     10 A(J,K)=A(J,K)+X(I)**K1
        A(1,1)=M
        NP2=N+2
        DO11J=1,NP1
     11 A(J,NP2)=0.
        DO14I=1,M
     14 A(1,NP2)=A(1,NP2)+Y(I)
        DO15J=2,NP1
        DO15I=1,M
        K2=J-1
     15 A(J,NP2)=A(J,NP2)+((X(I))**K2)*(Y(I))
        DO 16 I=1,6
     16 B(I)=0.
        DO 420 K=1,N
        KP1=K+1
        L=K
        DO 402 II=KP1,NP1
        IF(ABS(A(II,K)).LE.ABS(A(L,K)))GO TO 402
    401 L=II
    402 CONTINUE
        IF(L.LE.K)GO TO 420
    405 DO 410 J=1,NP2
        TEMP=A(K,J)
        A(K,J)=A(L,J)
    410 A(L,J)=TEMP
    420 CONTINUE
        DO 102 I=1,N
    200 REC=1./A(I,I)
        IP1=I+1
        DO 111 J=IP1,NP2
    111 A(I,J)=A(I,J)*REC
```

TABLE B-1 (CONTINUED)

```
      DO 102 K=IP1,NP1
      IF(A(K,I).EQ.0.)GO TO 102
   12 REC=1./A(K,I)
      DO 101 J=IP1,NP2
  101 A(K,J)=A(K,J)*REC-A(I,J)
  102 CONTINUE
      B(NP1)=A(NP1,NP2)/A(NP1,NP1)
      NNN=0
      DO 103 MM=1,N
      I=NP1-MM
      B(I)=A(I,NP2)
      NNN=NNN+1
      DO 103  J=1,NNN
      M3=NP2-J
  103 B(I)=B(I)-A(I,M3)*B(M3)
      GO TO (51,52,53,54,55),N
   51 WRITE(3,323)B(1),B(2)
      WRITE(2,324)KK,B(1),B(2)
      GO TO 22
   52 WRITE(3,323)B(1),B(2),B(3)
      WRITE(2,324)KK,B(1),B(2),B(3)
      GO TO 22
   53 WRITE(3,323)B(1),B(2),B(3),B(4)
      WRITE(2,324)KK,B(1),B(2),B(3),B(4)
      GO TO 22
   54 WRITE(3,323)B(1),B(2),B(3),B(4),B(5)
      WRITE(2,324)KK,B(1),B(2),B(3),B(4),B(5)
      GO TO 22
   55 WRITE(3,323)B(1),B(2),B(3),B(4),B(5),B(6)
      WRITE(2,324)KK,B(1),B(2),B(3),B(4),B(5),B(6)
C     CALCULATING YCAL(I)
   22 WRITE(3,600)
      WRITE(3,400)
      DO 21 I=1,M
      YCAL(I)=B(1)+B(2)*X(I)+B(3)*X(I)*X(I)+B(4)*X(I)*X(I)*X(I)+B(5)*X(I
     1)*X(I)*X(I)*X(I)
      DEV=Y(I)-YCAL(I)
   21 WRITE(3,150)X(I),Y(I),YCAL(I),DEV
      WRITE(3,600)
C     REGRESSION OF YOBS VS YCAL
C     LEAST SQUARES EQUATION   Y = A + BX
      SUMX=0.
      SUMY=0.
      SXSQ=0.
      SUMXY=0.
      SYSQ=0.
      C=0.
      DO 20 I=1,M
      C=C+1.
      SUMY=SUMY+Y(I)
      SUMX=SUMX+YCAL(I)
      SXSQ=SXSQ+YCAL(I)**2
      SUMXY=SUMXY+YCAL(I)*Y(I)
      SYSQ=SYSQ+Y(I)**2
   20 CONTINUE
      SLYSQ=SYSQ-((SUMY**2)/C)
      SLXSQ=SXSQ-((SUMX**2)/C)
      SLXY=SUMXY-((SUMX*SUMY)/C)
      Z=((SUMY/C)-(SUMXY/SUMX))/((SUMX/C)-(SXSQ/SUMX))
      W=(SUMY-Z*SUMX)/C
      R=SLXY/(SQRT(SLXSQ*SLYSQ))
      SDSQ=SLYSQ-((SLXY**2)/SLXSQ)
      SSQ=SDSQ/(C-2.0)
      S =SQRT(SSQ)
      WRITE(3,175)W,Z,R,S
   40 CONTINUE
      GO TO 2
      END
     MON$$      EXEQ LINKLOAD
                PHASEENTIREPROG
                CALL POLYFIT
     MON$$      EXEQ ENTIREPROG,MJB
```

TABLE B-2

LISTING OF ALPHABET 3 FORTRAN IV PROGRAM

```
       MONSS     JOB  211140007     MCCOOL, JANUARY, 1965
       MONSS     ASGN MGO,A2
       MONSS     ASGN MJB,A3
       MONSS     MODE GO,TEST
       MONSS     EXEQ FORTRAN,SOF,SIU,8,5,,,ALPHABET3
C       ALPHABET3
C       THIS PROGRAM IS FOR THE 1410
C       PROGRAM IS FOR FINDING BOUSSINESQ AND CORIOLIS COEFFICIENTS
C       PROGRAM USES SIMPSON RULE TO FIND AREA UNDER CURVES OF V VS.A,
C       V**2 VS.A, AND V**3 VS.A
C       THIS PROGRAM TAKES OUTPUT FROM POLYFIT PROGRAM
C       B(I) ARE COEFFICIENTS OF A POLYNOMIAL FIT OF VELOCITY TO AREA
C       B(I) ARE READ IN ORDER OF INCREASING POWERS OF A
C       NN=DEGREE OF POLYNOMIAL
C       N=TOTAL NUMBER OF TESTS TIMES NUMBER OF SECTIONS IN EACH
C       EXPERIMENT
C       NE=EXPERIMENT NUMBER
C       NT=TEST NUMBER
C       TION=SECTION NUMBER
C       QT=TOTAL DISCHARGE AT SECTION
C       AT=TOTAL AREA
C       XFIRST =ZERO
C       XLAST=AREA WITHIN LAST ISOVEL
C       DELX=INTERVAL OF AREA FOR INTEGRATION BY SIMPSON RULE
        DIMENSION B(6)
    300 FORMAT(3F8.3)
    301 FORMAT(A4,I3,A4,A5,I3,A4,A8,A3,A6,F7.3,A7,F7.3)
    317 FORMAT(2I3)
    318 FORMAT(3HA1=,E12.6,6HALPHA=,E12.6,5HBETA=,E12.6)
    319 FORMAT(1H1//57X,17HSVF    EXPERIMENT ,I3/)
    320 FORMAT(31X,4HTEST,5X,7HSECTION,7X,2HQT,10X,2HAT,10X,2HA1,8X,
       15HALPHA,6X,4HBETA//)
    321 FORMAT(31X,I3,8X,A3,6X,3(F7.3,5X),F6.3,5X,F6.3/)
    322 FORMAT(1H1)
    324 FORMAT(I3,6E12.6)
      1 READ(1,317) N,NE
        WRITE(3,319) NE
        WRITE(3,320)
        I=0
      2 READ(1,301)EXP,NE,SKIP1,TEST,NT,SKIP2,SEC,TION,DISCH,QT,AREA,AT
        READ(1,300) DELX,XFIRST,XLAST
        READ(1,324) NN,(B(J),J=1,6)
        NNN=NN+1
        NNNN=NNN+1
        DO 6 J=NNNN,6
      6 B(J)=0.
        K=((XLAST-XFIRST)/DELX)+1.
        L=(K-1)/2
        ALPHA=0.
        BETA=0.
        Q=0.
        X1=XFIRST
        X2=X1+DELX
        X3=X2+DELX
        CALL VALUE(B(1),B(2),B(3),B(4),B(5),B(6),X1,Y1,YS1,YC1)
        CALL VALUE(B(1),B(2),B(3),B(4),B(5),B(6),X2,Y2,YS2,YC2)
        CALL VALUE(B(1),B(2),B(3),B(4),B(5),B(6),X3,Y3,YS3,YC3)
        DD 40 II=1,L
        ALPHA=ALPHA+DELX*(YC1+4.*YC2+YC3)/3.
        BETA=BETA+DELX*(YS1+4.*YS2+YS3)/3.
        Q=Q+DELX*(Y1+4.*Y2+Y3)/3.
        X1=X3
        Y1=Y3
        YS1=YS3
        YC1=YC3
        X2=X3+DELX
        X3=X2+DELX
        CALL VALUE(B(1),B(2),B(3),B(4),B(5),B(6),X2,Y2,YS2,YC2)
        CALL VALUE(B(1),B(2),B(3),B(4),B(5),B(6),X3,Y3,YS3,YC3)
```

TABLE B-2 (CONTINUED)

```
40 CONTINUE
   IF((K-(2*L)-1).EQ.0) GO TO 50
   ALPHA=ALPHA+DELX*(YC1+YC2)/2.
   BETA=BETA+DELX*(YS1+YS2)/2.
   Q=Q+DELX*(Y1+Y2)/2.
   X1=X2
   Y1=Y2
50 QR=QT-Q
   A1=2.*QR/Y1+X1
   S=Y1/(X1-A1)
   ALPHA=ALPHA+S*S*S*((((A1/4.-A1)*A1+1.5*A1*A1)*A1-A1*A1*A1)*A1-
  1(((X1/4.-A1)*X1+1.5*A1*A1)*X1-A1*A1*A1)*X1)
   ALPHA=(AT*AT/(QT*QT*QT))*ALPHA
   BETA=BETA+S*S*(((A1/3.-A1)*A1+A1*A1)*A1-((X1/3.-A1)*X1+A1*A1)*X1)
   BETA=(AT/(QT*QT))*BETA
   WRITE(2,301)EXP,NE,SKIP1,TEST,NT,SKIP2,SEC,TION,DISCH,QT,AREA,AT
   WRITE(2,324)NN,(B(J),J=1,NNN)
   WRITE(2,318)A1,ALPHA,BETA
   WRITE(3,321)NT,TION,QT,AT,A1,ALPHA,BETA
   I=I+1
   IF(I.LT.N) GO TO 2
   WRITE(3,322)
   GO TO 1
   END
  MON$$     EXEQ FORTRAN,SOF,S1U,8,5
   SUBROUTINE VALUE(B1,B2,B3,B4,B5,B6,X,Y,YS,YC)
   Y=((((B6*X+B5)*X+B4)*X+B3)*X+B2)*X+B1
   YS=Y*Y
   YC=Y*Y*Y
   RETURN
   END
  MON$$     EXEQ LINKLOAD
            PHASEENTIREPROG
            CALL ALPHABET3
  MON$$     EXEQ ENTIREPROG,MJB
```

TABLE B-3

LISTING OF LS02 FORTRAN IV PROGRAM

```
     MON$$      JOB  211140007 D K MCCOOL       ,AUGUST, 1964,LINEARLS02
     MON$$      ASGN MGO,A2
     MON$$      ASGN MJB,A3
     MON$$      MODE GO,TEST
     MON$$      EXEQ FORTRAN,SOF,SIU,16,20,,,LINEARLS02
C     PROGRAM TO TRANSFORM DATA AND FIT LINEAR EQUATION BY LEAST SQUARE
C     THIS PROGRAM IS FOR THE 1410
C     THIS PROGRAM WRITTEN BY W. R. GWINN
C     IDENT IS TWO CARDS CONTAINING HEADING
C     NOE=NUMBER OF EQUATIONS
C     CONTROL CARD, ZERO=NO TRANSFORMATION
C     LOGX=0 OR 1               LOGX
C     ONX=0 OR 1                1/X
C     XN=0 OR EXPONENT          X**XN
C     LOGY=0 OR 1               LOGY
C     ONY=0 OR 1                1/Y
C     XONY=0 OR 1               X/Y
C     N=NUMBER OF POINTS
C     SIX DIGIT ACCURACY, FIVE DIGITS IF LOG TRANSFORM
C     CAUTION.  WATCH NUMBER OF DIGITS TO LEFT OF DECIMAL.
C     BOTH PUNCH AND PRINT OUTPUT
      INTEGER ONX, XONY,ONY,SUMX,SUMY,SUMXY,SUMXX,SUMYY,SXY,SXX,SYY
      REAL NP
      DIMENSION X(100),Y(100),IDENT(16)
  100 FORMAT( 8A10/ 8A10/ 22X,I3, 7X, I2, 5X, I2, 4X, E12.6, 7X, I2, 5X,
     1 I2, 5X, I2)
  101 FORMAT (18X, I4/(32X, F8.3, F10.4))
  102 FORMAT ( 8A10/ 8A10//)
  103 FORMAT (1H1, 27X, 8A10/27X, 8A10//)
  104 FORMAT (3E12.6, 43X, 1H1)
  105 FORMAT (44X,12HINTERCEPT = ,E12.6, 9H SLOPE = , E12.6/ 25X,31H STA
     1NDARD ERROR OF INTERCEPT = , E12.6,27H STANDARD ERROR OF SLOPE = ,
     2 E12.6//15X, 1HX,13X, 1HY,8X,11HESTIMATED Y, 1X, 14HDEVIATION OF Y
     3, 2X, 16HSQ. OF DEVIATION, 5X,7HINPUT X,7X,7HINPUT Y,5X,11HINPUT E
     4ST.Y/)
  106 FORMAT (29HSTANDARD ERROR OF INTERCEPT =, E12.6,25HSTANDARD ERROR
     1OF SLOPE =,E12.6// 8X, 1HX, 13X, 1HY, 8X, 11HESTIMATED Y, 1X, 14HD
     2EVIATION OF Y, 2X,16HSQ. OF DEVIATION/)
  107 FORMAT ( 4E14.6,E17.6)
  108 FORMAT ( 7X, 4E14.6, E17.6, E16.6,2E14.6)
  109 FORMAT (/     11HINTERCEPT =,E12.6,1X,7HSLOPE =,E12.6,1X,23HSTANDA
     1RD ERROR OF EST.=,E12.6/21X,27HCORRELATION COEFFICIENT R =,E12.6
     2///)
  110 FORMAT (/27X, 11HINTERCEPT =,E12.6,1X,7HSLOPE =,E12.6,1X,23HSTANDA
     1RD ERROR OF EST.=,E12.6/48X,27HCORRELATION COEFFICIENT R =,E12.6)
  111 FORMAT (1H1)
   17 CONTINUE
      READ(1,100)IDENT,NOE,LOGX,ONX,XN,LOGY,ONY,XONY
    1 WRITE (3,103) IDENT
      WRITE (2,102)IDENT
      READ (1,101) N, (X(I),Y(I),I=1,N)
      NP = FLOAT (N)
      L = 4
      K = 4
      SB1 = 0.0
      SB0 = 0.0
      IF (LOGX.EQ.0) GO TO 3
      L = 1
      J = 0
      DO 2 I=1,N
      J = J + 1
    2 X(J) =.434294*ALOG(X(J))
    3 IF (ONX.EQ.0) GO TO 5
      L = 2
      J = 0
      DO 4 I=1,N
      J = J + 1
    4 X(J) = 1./X(J)
    5 IF (XN.EQ.0.0) GO TO 7
```

TABLE B-3 (CONTINUED)

```
      L = 3
      J = 0
      DO 6 I=1,N
      J = J+ 1
      IF(X(J).GT.0.0) X(J) = X(J)**XN
    6 IF(X(J).LT.0.0) X(J) = X(J)**(IFIX(XN))
    7 IF (LOGY.EQ.0) GO TO 9
      K = 1
      J = 0
      DO 8 I=1,N
      J = J +1
    8 Y(J) = .434294*ALOG(Y(J))
    9 IF (ONY.EQ.0) GO TO 11
      K = 2
      J = 0
      DO 10 I=1,N
      J = J + 1
   10 Y(J) = 1./Y(J)
   11 IF (XONY.EQ.0) GO TO 13
      K = 3
      J = 0
      DO 12 I=1,N
      J = J +1
   12 Y(J) = X(J)/Y(J)
   13 SUMY = 0
      SUMXY = 0
      SUMX = 0
      SUMXX = 0
      SUMYY = 0
      J = 0
      DO 14 I=1,N
      J = J + 1
      NX = IFIX((X(J)*1000000.)+0.5)
      NY = IFIX((Y(J)*1000000.)+0.5)
      SUMX = SUMX+NX
      SUMY = SUMY + NY
      SUMXY = SUMXY + (NX*NY)
      SUMXX = SUMXX + (NX*NX)
   14 SUMYY = SUMYY + (NY*NY)
      SXY = SUMXY - ((SUMX*SUMY)/N)
      SXX = SUMXX - ((SUMX*SUMX)/N)
      SYY = SUMYY - ((SUMY*SUMY)/N)
      B1=(FLOAT((SXY*1000000)/SXX))*.000001
      B0=((FLOAT(SUMY/N))-B1*(FLOAT(SUMX/N)))*.000001
      IF (NP.LE.2.) GO TO 15
      SSY=(FLOAT((SYY-((SXY*SXY)/SXX))/(N-2)))*.000000000001
      SY = SQRT(SSY)
      SSB1 = SSY/((FLOAT(SXX))*.000000000001)
      SB1 = SQRT(SSB1)
      SSB0 = SSY*((1./NP) +((FLOAT(((SUMX/N)*(SUMX/N)*1000000)/SXX))*
     1.000001))
      SB0 = SQRT(SSB0)
   15 R=SQRT((FLOAT(((SXY*1000000)/SXX)*((SXY*1000000)/SYY)))
     1*.000000000001)
      WRITE (3,105) B0,B1,SB0,SB1
      WRITE (2,106) SB0,SB1
      J = 0
      SUMDES = 0.0
      DO 16 I=1,N
      J = J + 1
      YHAT = B0 + (B1*X(J))
      DEVY = Y(J) - YHAT
      SQDEV = DEVY * DEVY
      SUMDES = SUMDES + SQDEV
      GO TO (20,21,22,23),L
   19 WRITE (2,107) X(J),Y(J),YHAT,DEVY,SQDEV
   16 WRITE (3,108) X(J),Y(J),YHAT,DEVY,SQDEV,XIN,YIN,YHATIN
      SEREST = 0.0
      IF (NP.LE.2.) GO TO 18
      SEREST = SQRT(SUMDES/(NP-2.))
   18 WRITE (2,104) B0, B1,SEREST
      WRITE (2,109) B0,B1,SEREST,R
      WRITE (3,110) B0,B1,SEREST,R
      NOE = NOE - 1
      IF (NOE.GE.1) GO TO 1
      WRITE (3,111)
      GO TO 17
   20 XIN = 10.**(X(J))
      GO TO 24
   21 XIN = 1./X(J)
      GO TO 24
   22 XIN = X(J)**(1./XN)
      GO TO 24
   23 XIN = X(J)
   24 GO TO (25,26,27,28),K
   25 YIN = 10.**(Y(J))
      YHATIN = 10.**(YHAT)
```

TABLE B-3 (CONTINUED)

```
      GO TO 19
   26 YIN = 1./Y(J)
      YHATIN = 1./YHAT
      GO TO 19
   27 YIN = X(J)/Y(J)
      YHATIN = X(J)/YHAT
      GO TO 19
   28 YIN = Y(J)
      YHATIN = YHAT
      GO TO 19
      END
     MONS$      EXEQ LINKLOAD
                PHASEENTIREPROG
                CALL LINEARLS02
     MONS$      EXEQ ENTIREPROG,MJ8
```

TABLE B-4

LISTING OF RETARDANCE 3 FORTRAN IV PROGRAM USED TO COMPUTE RESISTANCE CONSIDERING BOUSSINESQ COEFFICIENT

```
      MON$$     JOB  211140007      MCCOOL    FEBRUARY, 1965  RETARDANCE3
      MON$$     ASGN MGO,A2
      MON$$     ASGN MJB,A3
      MON$$     MODE GO,TEST
      MON$$     EXEQ FORTRAN,SOF,SIU,6,3,,,MANNINGSN
C     RETARDANCE3 PROGRAM FOR 1410
C     RETARDANCE PROGRAM ALTERED TO UTILIZE BOUSSINESQ COEFFICIENTS
C     THIS IS AN ADAPTATION OF A PROGRAM WRITTEN BY W. R. GWINN
C     DELTX =DISTANCE BETWEEN READINGS ACROSS CHANNEL
C     SCALE = MODEL LENGTH SCALE
C     NGT = TOTAL NUMBER OF GAGES
C     NT = TOTAL NUMBER OF TESTS
C     VISNUE(I) = KINEMATIC VISCOSITY (FT2/SEC) (I=1 FOR 33 DEGREES F )
C     STAU = UPSTREAM STATION DISTANCE IN FEET
C     NYU = NUMBER OF UY READINGS,UPSTREAM (EQUAL OR LESS THAN 100)
C     NOGAU  = UPSTREAM GAGE NUMBER (0 TO 9 ONLY)
C     ZEROU = UPSTREAM GAGE ZERO (PRDTO FEET)(SHOULD BE THE SAME FOR
C              BOTTOM READINGS AND WATER SURFACE ELEVATION)
C     UY = ELEVATION OF GROUND SURFACE ACROSS  SECTION,UPSTREAM SECTION
C       NOTE, UY AND DY , MAINCHANNEL READ IN FIRST WITH PEAKS FOLLOWING
C                        MAIN CHANNEL
C     STAD = DOWNSTREAM STATION DISTANCE IN FEET ( SHOULD BE GREATER THAN
C             STAU)
C     NYD = NUMBER OF DY READINGS,DOWNSTREAM (EQUAL OR LESS THAN 100)
C     NOGAD = DOWNSTREAM GAGE NUMBER ( 0 TO 9 ONLY)
C     ZEROD = DOWNSTREAM GAGE ZERO  ETC.
C     DY = ELEVATION OF GROUND SURFACE ACROSS  SECTION, DOWNSTREAM SECTION
C     STA3 = THIRD STATION DOWNSTREAM (PROTOTYPE FEET)
C     NY3 = NUMBER OF Y3 READINGS
C     NOGA3 = THIRD GAGE DOWNSTREAM NUMBER (0 TO 9 ONLY)
C     ZERO3 = THIRD GAGE DOWNSTREAM ZERO  ETC.
C     Y3(I) = ELEVATION OF GROUND SURFACE ACROSS SECTION, THIRD GAGE DOWNS
C     NOTEST = TEST NUMBER
C     NEX = EXPERIMENT NUMBER
C     MO = MONTH
C     DAY =  DAY OF MONTH
C     YEAR = LAST TWO DIGITS OF YEAR
C     QM = MODEL DISCHARGE (C.F.S.)
C     TEMP = WATER TEMPERATURE (DEGREES F )
C     UELEVM = UPSTREAM WATER SURFACE ELEVATION (MODEL FEET)
C     DELEVM = DOWNSTREAM WATER SURFACE ELEVATION (MODEL FEET)
C     ELEVM3 = WATER SURFACE ELEVATION THIRD STATION DOWNSTREAM (MODEL FT)
C     TIMEFA = TIME FACTOR (BETWEEN 0 AND 1)( 0 = BEFORE READINGS,
C              1 = AFTER TEST BOTTOM READINGS USED)
C     DURFLO = DURATION OF FLOW (MIN.)
C     IDENT = IDENTIFICATION (13SPACES)
C     NREACH = REACH NUMBER (COMPOSED OF GAGE NUMBERS)
C     QP = PROTOTYPE DISCHARGE (C.F.S.)
C     AVHYRP = AVERAGE HYDRAULIC RADIUS (PROTOTYPE)
C     AVVEL = AVERAGE VELOCITY
C     AVVR = AVERAGE VELOCITY TIMES THE HYDRAULIC RADIUS
C     AVAREA = AVERAGE AREA
C     CHEZY = CHEZY C
C     ROUGHN = MANNINGS N (PROTOTYPE)
C     WETPER = WETTED PERIMETER
C     CENDEP = CENTER DEPTH ( FEET )
C     SCOUR = RATE OF SCOUR (IN/HR )
C     KN = KUTTERS N
C     AVHYRM = AVERAGE HYDRAULIC RADIUS (MODEL)
C     RENOLD = REYNOLDS NUMBER
C     F = DARCY-WEISBACH RESISTANCE COEFFICIENT
C     ROUGNM = MANNINGS N (MODEL)
        REAL  KN,MAXD3,MAXDU,MAXDD,MAXD3E,MAXDUE,MAXDDE,IDENT1,IDENT2,
      1IDENT3
```

TABLE B-4 (CONTINUED)

```
      INTEGER DAY,YEAR,TEMP,BRYU,ERYU,BRYD,ERYD,BRY3,ERY3,DURFLO
      DIMENSION UY(100),VISNUE(68),Y3(100),DY(100)
   90 FORMAT (14X,F8.2,24X,I5,12X,I3,5X,F9.3/(9F8.3))
   91 FORMAT (7X,F6.2,13X,F7.2,17X,I2,20X,I4/(8E10.4))
   92 FORMAT (I3,4I2,E10.5,I3,   3(I2,E10.5),F4.2,I3,A6,A6,A1)
   93 FORMAT (////19X,8HCHANNEL ,A6,A6,A1,17H   EXPERIMENT NO.,I3)
   94 FORMAT(I3,I8,1X,3I3,F9.2,F9.2, F6.3,F7.2,F8.2,F7.3,I5,7X,1H1,/I3,
     1I8,F6.2,F8.5,F7.2,F7.4,F8.4,F7.3,F13.0,F9.4,3X,1H2)
   95 FORMAT (80X)
   96 FORMAT (1H1  /46X,8HCHANNEL ,A6,A6,A1,16H  EXPERIMENT NO.,I3)
   97 FORMAT ( 6X,I3,I6,1X,3I3,F9.2,F8.2,F7.2,F8.2,F8.3,F7.4,F7.2,2I5,
     1F8.2,F7.2,F8.4,F7.4,F9.3)
   99 FORMAT (////1X)
  100 FORMAT (//1X)
  101 FORMAT(1H1)
  103 FORMAT(80HTEST  REACH                               MEAN   MEAN         WE
     1TTED  HYD.  WATER        1/80H NO.    NO.      DATE   DISCHARGE    AREA
     2   VEL.   BETA  PERIM. RADIUS TEMP.        1//80H
     3                                 VALUE                FRICTION   2/
     580HTEST  REACH CENTER             CHEZY MANNING KUTTER     OF      REYNOL
     6DS     FACTOR     2/80H NO.    NO.  DEPTH    SLOPE     C        N       N
     7       VR        NO.           F         2/)
  104 FORMAT(2E12.6)
  106 FORMAT (/19X,4HDATE,14X,27HMEAN     MEAN    WETTED  HYD.,17X,4HDUR.,
     17X,4HRATE,27X,5HVALUE/ 5X,122HTEST  REACH     OF     DISCHARGE  AREA
     2    VEL.    PERIM. RADIUS  SLOPE  CENTER  OF  WATER  OF      CHEZY  M
     3ANNING KUTTER       OF / 6X,                                     19HNO.
     4  NO.    TESTING,13X,1HA,7X,1HV,7X,1HP,6X,1HR,12X,28HDEPTH  FLOW TE
     5MP. SCOUR      C,7X,1HN,7X,1HN,7X,  2HVR,// 7X,1H1,5X,1H2,7X,1H3,8X,
     61H4,7X,1H5,7X,1H6,7X,1H7,6X,1H8,7X,1H9,6X,2H10,4X,2H11,3X,2H12,4X,
     72H13,5X,2H14,6X,2H15,6X,2H16,7X,2H17///28X,35HC.F.S.  SQ.FT.  F.P.
     8S.   FT.     FT.,13X,21HFT.  MIN. DEG.F IN/HR/)
  200 WRITE (3,101)
      READ (1,91)DELTX,SCALE,NGT,NT, VISNUE
      READ(1,104)COEFF,EXPON
   23 NOT=NT
      NOL = 40
      READ (1,90)STAU,NYU,NOGAU,ZEROU,(UY(I),I=1,NYU)
      DO 301 I = 1,NYU
  301 UY(I) = UY(I) + ZEROU
      READ (1,90) STAD,NYD,NOGAD,ZEROD,(DY(I),I=1,NYD)
      DO 302 I = 1,NYD
  302 DY(I) = DY(I) + ZEROD
      IF (NGT.LT.3)GO TO 21
      READ (1,90) STA3,NY3,NOGA3,ZERO3,(Y3(I),I=1,NY3)
      DO 303 I = 1,NY3
  303 Y3(I) = Y3(I) + ZERO3
   21 K=1
      INDEX=0
   11 READ (1,92) NOTEST,NEX,MO, DAY,YEAR   ,QM, TEMP,NOGAU,UELEVM,NOGAD,
     1DELEVM,NOGA3,ELEVM3,TIMEFA,DURFLO,IDENT1,IDENT2,IDENT3
      IF (ELEVM3.EQ.0.0)GO TO 13
      ELEV3P=(ELEVM3*SCALE)+ZERO3
      CALL AREAHR(ELEV3P,AREA3,HYRAD3,Y3,DELTX       ,NY3,MAXD3)
   13 NREACH=(NOGAU*10)+NOGAD
      QP= QM*(SCALE**2.5)
      UELEVP=(UELEVM*SCALE)+ZEROU
      DELEVP=(DELEVM*SCALE)+ZEROD
      CALL AREAHR(UELEVP,UAREA,UHYRAD,UY,DELTX       ,NYU,MAXDU)
      CALL AREAHR(DELEVP,DAREA,DHYRAD,DY,DELTX       ,NYD,MAXDD)
      IF (TIMEFA.NE.0.0)GO TO 24
      SCOURD = 0.0
      SCOURU = 0.0
      SCOUR3 = 0.0
      SCOURA = 0.0
   14 IF (DAREA.EQ.0.0) GO TO 5
      IF (UAREA.EQ.0.0) GO TO 5
      BETAU=COEFF*QP**EXPON
 1000 BETAD=BETAU
      BETA=BETAU
      SLOPE=(UELEVP+((( BETAU*QP*QP)/(UAREA*UAREA))/64.3)-DELEVP-
     1((( BETAD*QP*QP)/(DAREA*DAREA))/64.3))/(STAD-STAU)
      IF(SLOPE.LE.0.0)GO TO 5
      AVAR23=(UAREA*(UHYRAD**.666667   )+(DAREA*(DHYRAD**.666667   )))/2.
      ROUGHN=(1.486/QP)*AVAR23*(SLOPE**.5)
      AVR16=((UHYRAD**.166667   )+(DHYRAD**.166667   ))/2.
      CHEZY = (1.486/ROUGHN)*AVR16
      AVAREA=AVAR23/(AVR16**4.0)
      AVVEL = QP/AVAREA
      AVHYRP=AVR16**6.0
      C=41.65 +(.00281/SLOPE)
      KN=(((((C-CHEZY)*(C-CHEZY))+(7.244*CHEZY*C/(AVHYRP**.5)))**.5)-
     1CHEZY+C)/((2.*CHEZY*C)/(AVHYRP**.5))
    6 AVVR = AVHYRP*AVVEL
      AVHYRM=AVHYRP/SCALE
```

TABLE B-4 (CONTINUED)

```
      ROUGNM=ROUGHN/(SCALE**.166667)
      SCOUR=(SCOURU+SCOURD)/2.
      RENOLD=4.0*AVVR/((SCALE**1.5)*(VISNUE(TEMP-32)))
      WETPER = 0.0
      IF (AVHYRP.NE.0.0) WETPER = AVAREA/AVHYRP
      CENDEP = (MAXDU + MAXDD)/2.
   18 IF (NOL.NE.40) GO TO 8
      WRITE (2,93)IDENT1,IDENT2,IDENT3,NEX
      WRITE(2,103)
      WRITE (3,96)IDENT1,IDENT2,IDENT3,NEX
      WRITE (3,106)
      NOL = 0
    8 WRITE (3,97)NOTEST     ,NREACH,MO,DAY,YEAR,QP,AVAREA,AVVEL,WETPER,
     1AVHYRP,SLOPE,CENDEP,DURFLO,TEMP,SCOUR,CHEZY,ROUGHN,KN,AVVR
      F=257.2/(CHEZY*CHEZY)
      WRITE(2,94)NOTEST,NREACH,MO,DAY,YEAR,QP,AVAREA,AVVEL, BETA,WETPER,
     1AVHYRP,TEMP,NOTEST,NREACH,CENDEP,SLOPE,CHEZY,ROUGHN,KN,AVVR,RENOLD
     2,F
      NOL=NOL+1
      A=(FLOAT(NOL))/4.0
      B = FLOAT(NOL/4)
      IF (A.NE.B) GO TO 9
      WRITE (3,95)
      WRITE(2,95)
      IF (NOL.NE.40) GO TO 9
    9 IF (ELEVM3.EQ.0.0) GO TO 19
      GO TO (15,16,17,19),K
   15 IF (ELEVM3.EQ.0.0) GO TO 19
      ABETA=BETA
      AMAXD = MAXDD
      ASTAD = STAD
      AELEVP=DELEVP
      AAREA =DAREA
      AHYRAD=DHYRAD
      ASCOUR=SCOURD
      SCOURD= SCOUR3
      STAD = STA3
      MAXDD = MAXD3
      DELEVP = ELEV3P
      DAREA = AREA3
      DHYRAD = HYRAD3
      NREACH = (NOGAU*10)+NOGA3
      AHRP = AVHYRP
      AV = AVVEL
      AVR = AVVR
      AA  = AVAREA
      AS = SLOPE
      AC = CHEZY
      ANP = ROUGHN
      ANK=KN
      AHRM = AVHYRM
      AR = RENOLD
      ANM = ROUGNM
      K = 2
      AVMAXD = CENDEP
      GO TO 14
    5 ROUGHN = 0.0
      SLOPE = 0.0
      F  = 0.0
      CHEZY  = 0.0
      AVAREA = 0.0
      BETA=0.0
      AVVEL  = 0.0
      AVHYRP = 0.0
      KN     = 0.0
      INDEX=1
      GO TO (6,6,6,89),K
   89 AVVR=0.0
      AVHYRM=0.0
      WETPER=0.0
      ROUGHM=0.0
      GO TO 18
   24 READ (1,90) STAU,BRYU,ERYU,ZEROU,(UY(I),I=BRYU,ERYU)
      READ (1,90) STAD,BRYD,ERYD,ZEROD,(DY(I),I=BRYD,ERYD)
      IF (NGT.LT.3) GO TO 25
      READ (1,90) STA3,BRY3,ERY3,ZERO3,(Y3(I),I=BRY3,ERY3)
      CALL AREAHR(ELEV3P,AREA3E,HYRA3E,Y3,DELTX       ,NY3,MAXD3E)
      IF (HYRAD3.NE.0.0) SCOUR3=(((AREA3E-AREA3)*720.)/(((AREA3/HYRAD3)+
     1(AREA3E/HYRA3E))/2.))/FLOAT (DURFLO)
      MAXD3=  (MAXD3*(1.0-TIMEFA))+(MAXD3E*TIMEFA)
      AREA3 = (AREA3*(1.0-TIMEFA))+(AREA3E *TIMEFA)
      HYRAD3= (HYRAD3*(1.0-TIMEFA))+(HYRA3E*TIMEFA)
      IF(HYRAD3.EQ.0.0) SCOUR3=  HYRA3E* 720./ FLOAT(DURFLO)
   25 CALL AREAHR(UELEVP,UEAREA,UEHYRA,UY,DELTX       ,NYU,MAXDUE)
      IF(UHYRAD.NE.0.0) SCOURU=(((UEAREA-UAREA)*720.)/(((UAREA/UHYRAD)+
     1(UEAREA/UEHYRA))/2.))/FLOAT(DURFLO)
      MAXDU = (MAXDU*(1.0-TIMEFA))+(MAXDUE *TIMEFA)
```

TABLE B-4 (CONTINUED)

```
      UAREA = (UAREA*(1.0-TIMEFA))+( UEAREA*TIMEFA)
      UHYRAD= (UHYRAD*(1.0-TIMEFA))+(UEHYRA*TIMEFA)
      IF (UHYRAD.EQ.0.0) SCOURU = (UEHYRA*720.)/FLOAT(DURFLO)
      CALL AREAHR(DELEVP,DEAREA,DEHYRA,DY,DELTX       ,NYD,MAXDDE)
      IF (DHYRAD.NE.0.0) SCOURD = (((DEAREA-DAREA)*720.)/(((DAREA/DHYRAD
     1)+(DEAREA/DEHYRA))/2.))/FLOAT(DURFLO)
      IF (DHYRAD.EQ.0.0) SCOURD = (DEHYRA*720.)/FLOAT(DURFLO)
      DAREA= (DAREA *(1.0-TIMEFA)) + (DEAREA * TIMEFA)
      DHYRAD=(DHYRAD*(1.0-TIMEFA)) + (DEHYRA * TIMEFA)
      SCOURA = (SCOURU+SCOURD+SCOUR3)/3.0
      MAXDD = (MAXDD *(1.0-TIMEFA)) + (MAXDDE *TIMEFA)
      GO TO 14
   16 UELEVP= AELEVP
      SCOURU = ASCOUR
      UAREA =  AAREA
      MAXDU = AMAXD
      UHYRAD = AHYRAD
      BSTAD = STAU
      NREACH = (NOGAD*10) + NOGA3
      STAU = ASTAD
      K = 3
      ABETA=ABETA+BETA
      AHRP = AHRP + AVHYRP
      AV = AV + AVVEL
      AVR = AVR + AVVR
      AA = AA + AVAREA
      AS = AS + SLOPE
      AC = AC + CHEZY
      ANP = ANP + ROUGHN
      ANK=ANK+KN
      AHRM = AHRM + AVHYRM
      AR = AR + RENOLD
      ANM = ANM + ROUGNM
      AVMAXD = AVMAXD + CENDEP
      GO TO 14
   17 NREACH  = (NOGAU*100) + (NOGAD*10) + NOGA3
      STA3 = STAD
      STAD = STAU
      STAU = BSTAD
      SCOUR = SCOURA
      BETA=(ABETA+BETA)/3.
      AVHYRP = (AHRP + AVHYRP)/3.
      AVVEL = (AV + AVVEL)/3.
      AVVR = (AVR + AVVR) /3.
      AVAREA = (AA + AVAREA)/3.
      SLOPE = (AS + SLOPE)/3.
      CHEZY = (AC + CHEZY)/3.
      ROUGHN = (ANP + ROUGHN)/3.
      KN=(ANK+KN)/3.
      AVHYRM = (AHRM + AVHYRM)/3.
      RENOLD = (AR + RENOLD)/3.
      ROUGNM = (ANM + ROUGNM)/3.
      CENDEP= (AVMAXD + CENDEP)/3.
      WETPER=AVAREA/AVHYRP
      K = 4
      IF(INDEX.EQ.1) GO TO 5
      GO TO 18
   19 NQT=NQT-1
      IF(NQT.NE.0) GO TO 21
      NGT = NGT - 1
      IF(NGT.LT.3) GO TO 200
      GO TO 23
      END
    MON$$      EXEQ FORTRAN,SOF,S1U,6,3
      SUBROUTINE AREAHR(ELEV,AREA,HYRAD,Y    ,DELTX      ,NY,CENDEP)
      DIMENSION Y(100)
      CENDEP = 0.0
      I = 0
      AREA = 0.0
      HYRAD = 0.0
      WTPER = 0.0
      DELTXS  = DELTX *DELTX
    1 I = I + 1
      IF (I.EQ.NY) GO TO 100
      Y1 = ELEV - Y(I)
      Y2 = ELEV - Y(I+1)
      IF (Y2.GE.Y1) CENDEP = AMAX1(CENDEP,Y2)
      IF (Y2.GE.0.0) GO TO 2
      IF (AREA.EQ.0.0) GO TO 1
      GO TO 4
    2 IF (Y1.LT.0.0) GO TO 3
      AREA = AREA + (((Y1 + Y2)*DELTX)/2.0)
      WTPER=WTPER+(((Y2-Y1)*(Y2-Y1))+DELTXS)**.5
      GO TO 1
    3 X = (Y2/(Y2 - Y1))*DELTX
      AREA = AREA + ((Y2 * X)/2.0)
      WTPER = WTPER + (((X * X)+(Y2 *Y2))**.5)
```

TABLE B-4 (CONTINUED)

```
      GO TO 1
    4 X = (Y1/(Y1 - Y2))*DELTX
      AREA  = AREA + ((Y1*X)/2.0)
      WTPER = WTPER + (((X*X) + (Y1*Y1))**.5)
      HYRAD = AREA/WTPER
  100 CONTINUE
      RETURN
      END
     MON$$      EXEQ LINKLOAD
                PHASECHANNEL
                CALL MANNINGSN
     MON$$      EXEQ CHANNEL,MJB
```

TABLE B-5

LISTING OF MULTIVARIATE FORTRAN IV PROGRAM

```
    MON$$     JOB  211140007          MCCOOL       1964
    MON$$     ASGN MGO,A2
    MON$$     ASGN MJB,A3
    MON$$     MODE GO
    MON$$     EXEQ FORTRAN,,,08,05,,,RESPONSURF
C      MULTIVARIATE PROGRAM
C      THIS PROGRAM IS FOR THE 1410
C      MULTIVARIABLE FUNCTIONAL RELATIONSHIPS
C      M= NO. OF OBSERVATIONS
C      N= NO. OF COLUMNS OF INDEPENDENT VARIABLES
C                            COLUMNS
C      X1          X2          X3             X4-----------ETC---  Y
C      TO PASS THROUGH THE ORIGIN READ X0 IN AS 0.
C      TO ELIMINATE FORCING THE PLANE THROUGH THE ORIGIN READ X0 IN AS 1.
C                        NORMAL EQUATIONS
C      B1*X0*X0      +     B2*X0*X1     +  --BK*X0*X(K-1) =  X0*Y
C      B1*X1*X0      +     B2*X1*X1     +  --BK*X1*X(K-1) =  X1*Y
C      B1*X(K-1)*X0 +     B2*X(K-1)*X1  + --------        =  X(K-1)*Y
       DIMENSIONX(9,50),A(9,9),B(8)
   300 FORMAT(2I3,F5.2)
   309 FORMAT(10X,F12.0,12X,F12.0)
   310 FORMAT(1X,7E12.6)
   311 FORMAT(1X,9F12.4)
   323 FORMAT(41X,1HB,I3,9X,E12.6)
   500 FORMAT(1H1)
   600 FORMAT(//)
     2 WRITE(3,500)
       READ(1,300)N,M,X0
       NM1=N-1
       NP1=N+1
       NP2=N+2
       DO 1 J=1,M
     1 X(1,J)=X0
       READ(1,309)((X(I,J),I=2,NP2),J=1,M)
       WRITE(3,311)((X(I,J),I=1,NP2),J=1,M)
  1000 CONTINUE
C       THESE CARDS PRINT OUT XX AND XY MATRIX
C      INSERT BEFORE 1003
       WRITE(3,600)
       WRITE(3,310)((X(I,J),I=1,NP2),J=1,M)
  1003 DO 20 I=1,NP1
       DO 20 J=1,NP2
    11 IF(J.GE.I) GO TO 6
       J=J+1
       GO TO 11
     6 A(I,J)=0.
       DO 15 K=1,M
    15 A(I,J)=A(I,J)+X(I,K)*X(J,K)
    20 A(J,I)=A(I,J)
C      THESE CARDS PRINT OUT COEFFICIENT MATRIX
C      INSERT BEFORE 1002
       WRITE(3,600)
       WRITE(3,310)((A(I,J),J=1,NP2),I=1,NP1)
  1002 DO16 I=1,6
    16 B(I)=0.
       DO 420 K=1,N
       KP1=K+1
       L=K
       DO 402 II=KP1,NP1
       IF(ABS(A(II,K)).LE.ABS(A(L,K)))GO TO 402
   401 L=II
   402 CONTINUE
       IF(L.LE.K)GO TO 420
```

TABLE B-5 (CONTINUED)

```
  405 DO 410 J=1,NP2
      TEMP=A(K,J)
      A(K,J)=A(L,J)
  410 A(L,J)=TEMP
  420 CONTINUE
      DO 102 I=1,N
      IF(A(I,I).EQ.0.)I=I+1
  200 REC=1./A(I,I)
      IP1=I+1
      DO 111 J=IP1,NP2
  111 A(I,J)=A(I,J)*REC
      DO 102 K=IP1,NP1
      IF(A(K,I).EQ.0.)GO TO 102
   12 REC=1./A(K,I)
      DO 101 J=IP1,NP2
  101 A(K,J)=A(K,J)*REC-A(I,J)
  102 CONTINUE
      B(NP1)=A(NP1,NP2)/A(NP1,NP1)
      NNN=0
      DO 103 MM=1,N
      I=NP1-MM
      B(I)=A(I,NP2)
      NNN=NNN+1
      DO 103  J=1,NNN
      M3=NP2-J
  103 B(I)=B(I)-A(I,M3)*B(M3)
      WRITE (3,600)
      WRITE(3,323)(I,B(I),I=1,NP1)
      WRITE(2,323)(I,B(I),I=1,NP1)
C     THESE CARDS COMPUTE CALCULATED VALUES AND DEVIATIONS AND
C     STANDARD DEVIATIONS
C     INSERT BEFORE STATEMENT NO. 1001
      WRITE(3,600)
      SRSQ=0.
      DO 76 K=1,M
      YCAL=0.
      DO 77 J=1,NP1
      YSUM=B(J)*X(J,K)
   77 YCAL=YCAL+YSUM
      RES=X(NP2,K)-YCAL
      WRITE(3,311) X(NP2,K),YCAL,RES
      RESSQ=RES**2
   76 SRSQ=SRSQ+RESSQ
      D=N
      C=M
      VAR=(1./(C-(D+XO)))*SRSQ
      SDEV=SQRT(VAR)
      WRITE(3,600)
      WRITE(3,311)SDEV
 1001 GO TO 2
      END
    MON$$        EXEQ LINKLOAD
                 PHASECALCRF
                 CALL RESPONSURF
    MON$$        EXEQ CALCRF,MJB
```

TABLE B-6

LISTING OF HYDEL 2 FORTRAN IV PROGRAM

```
      MON$$      JOB  211140007  MCCOOL FALL,1964
      MON$$      ASGN MGO,A2
      MON$$      ASGN MJB,A3
      MON$$      MODE GO,TEST
      MON$$      EXEQ FORTRAN,SOF,SIU,8,6,,,HYDEL2
C      HYDEL2
C      THIS PROGRAM IS FOR THE 1410
C      THIS PROGRAM COMPUTES HYDRAULIC ELEMENTS FOR ANY CHANNEL WITH
C      BOTTOM READINGS AT EVEN INTERVALS.  THE ONE EXCEPTION IS A
C      CHANNEL WITH A VERTICAL SIDE.
C      HYDEL2 GIVES TABULAR OUTPUT OF AREA AND HYDRAULIC RADIUS WHICH CAN
C      BE USED IN  THE SVF SERIES PROGRAMS
C      DELTX = INTERVAL AT WHICH READINGS ARE TAKEN
C      NY = NUMBER OF BOTTOM READINGS
C      CHANGY = TABLE INTERVAL
C      HIGH = HIGHEST ELEVATION IN TABLE
C      HIGH CAN BE GREATER THAN ANY BOTTOM READING
C      Y(I)=BOTTOM READINGS IN SEQUENCE.  SHOULD BE ROUNDED TO SAME
C      ACCURACY AS CHANGY.
C      BOTU=LOWEST ELEVATION IN CROSS SECTION
       DIMENSION AREAU(300),HYRADU(300),Y(60)
    95 FORMAT(I3)
   110 FORMAT(8F10.3)
   115 FORMAT(4F10.3,4X,I3)
   120 FORMAT(5E14.8)
       READ(1,95) NS
       WRITE(2,95)NS
       DO 300 K=1,NS
       READ(1,115) DISTU,HIGH,CHANGY,DELTX,NY
       READ(1,110)(Y(I),I=1,NY)
     1 DO 2 N=1,300
       AREAU(N)=0.
     2 HYRADU(N)=0.
       BOT=10000.
       DO 12 I=1,NY
    12 BOT=AMIN1(BOT,Y(I))
       ELEV=BOT
       NYU=((HIGH-BOT)/CHANGY)+1.0
       I=0
     3 I=I+1
       IF(Y(I+1).GT.Y(I))GO TO 4
       IF(Y(I+1).LT.Y(I)) GO TO 5
       IF(Y(I+1).EQ.Y(I)) GO TO 6
     4 IF(Y(I).GE.HIGH)GO TO 13
       SLOPE=(Y(I+1)-Y(I))/DELTX
       MIN=((Y(I)-BOT)/CHANGY)+2.0
       MAX=((Y(I+1)-BOT)/CHANGY)+1.0
       GO TO 7
     5 IF(Y(I+1).GE.HIGH)GO TO 13
       SLOPE=(Y(I)-Y(I+1))/DELTX
       MIN=((Y(I+1)-BOT)/CHANGY)+2.0
       MAX=((Y(I)-BOT)/CHANGY)+1.0
       GO TO 7
     6 IF(Y(I).GE.HIGH)GO TO 13
       MAX=((Y(I+1)-BOT)/CHANGY)+1.0
       N=MAX
       IF(N.EQ.1) N=MAX+1
       HYRADU(N)=HYRADU(N)+DELTX
       GO TO 9
     7 DIFF=0.
       DO 8 N=MIN,MAX
       AREAU(N)=AREAU(N)+CHANGY*((CHANGY/SLOPE)+2.*DIFF)/2.
       HYRADU(N)=HYRADU(N)+SQRT(CHANGY*CHANGY*(1.+(1./(SLOPE*SLOPE))))
       DIFF=CHANGY/SLOPE+DIFF
       IF(N.GE.NYU) GO TO 13
     9 IF(MAX.GE.NYU) GO TO 13
     8 CONTINUE
       IF(N.GE.NYU) GO TO 13
       MAX=MAX+1
```

TABLE B-6 (CONTINUED)

```
      DO 10 N=MAX,NYU
   10 AREAU(N)=AREAU(N)+CHANGY*DELTX
   13 IF((I+1).LT.NY)GO TO 3
      DO 11 N=2,NYU
      ELEV=ELEV+CHANGY
      AREAU(N)=AREAU(N-1)+AREAU(N)
   11 HYRADU(N)=AREAU(N)/(HYRADU(N)+(AREAU(N-1)/HYRADU(N-1)))
      BOTU=BOT
      DYU=CHANGY
      WRITE(2,115)DISTU,HIGH,DYU,BOTU,NYU
      WRITE(2,120)(AREAU(I),I=1,NYU)
  300 WRITE(2,120)(HYRADU(I),I=1,NYU)
      END
     MON$$      EXEQ LINKLOAD
                PHASEENTIREPROG
                CALL HYDEL2
     MON$$      EXEQ ENTIREPROG,MJB
```

TABLE B-7

LISTING OF SVF 5F FORTRAN IV PROGRAM USED TO CALCULATE SPATIALLY VARIED FLOW PROFILES USING METHOD 4

```
       MON$$     JOB  211140007      MCCOOL    SVF 5F      MAY, 1965
       MON$$     ASGN MGO,A2
       MON$$     ASGN MJB,A3
       MON$$     MODE GO,TEST
       MON$$     EXEQ FORTRAN,SOF,SIU,8,5,,,SVF5
C       SVF 5F
C       THIS PROGRAM IS FOR THE 1410
C       PROGRAM TO COMPUTE SPATIALLY VARIED STEADY FLOW PROFILE IN FC 31
C       USING TABLES OF AREA AND HYDRAULIC RADIUS FOR EACH CROSS SECTION
C       STATION
C       AREA AND HYDRAULIC RADIUS TABLES ARE COMPUTED WITH HYDEL 2 PROGRAM
C       READ IN TABLES FROM DOWNSTREAM END
C       STORE TABLES ON TAPE
C       DISTU=DISTANCE OF CROSS SECTION FROM UPPER END OF CHANNEL
C       HIGH =ELEVATION AT WHICH TABLES STOP
C       BOTU=ELEVATION OF CHANNEL BOTTOM
C       DYU=VERTICAL INTERVAL BETWEEN TABLE VALUES
C       NYU=NUMBER OF VALUES IN TABLE
C       PROGRAM USES ACTUAL INFLOW DISTRIBUTION.
C       PROGRAM COMPUTES CHANGE IN DEPTH BETWEEN TWO POINTS USING AN
C       EULER METHOD WITH ITERATION
C       SET CLOSENESS OF ITERATION AT STATEMENT 25.
C       SOLUTION STARTS FROM SOME DOWNSTREAM ELEVATION AND WORKS UPSTREAM.
C       PRINT OUT INTERVAL CAN BE CONTROLLED.
C       DITCH=CHANNEL NO.
C       DATE=DAY,MONTH,YEAR,ETC.
C       NS=NUMBER OF CROSS SECTION STATIONS
C       EXP=EXPERIMENT NO.
C       TEST=TEST NO.
C       QD=TOTAL DISCHARGE
C       DELX=DISTANCE BETWEEN COMPUTATION POINTS (NEGATIVE IF WORKING
C       UPSTREAM)
C       GL=GRASS LENGTH
C       NO=NUMBER OF DISCHARGE READINGS
C       MANNINGS N=C1*((V*R)**C2)
C       C1=B1+B2*X+B3*GL
C       C2=B4+B5*X+B6*GL
C       BOUSSINESQ COEFFICIENT, BETA=C3*((Q)**C4)
C       C3=C5*((X)**C6)
C       C4=C7*((X)**C8)
C       START=STARTING POINT FOR PROFILE(MUST BE CROSS SECTION STATION)
C       ELEV=ELEVATION AT START
C       Q(I)=TOTAL DISCHARGE AT X(I)
        DIMENSION AREAU(295),HYRADU(295)
C       THIS PHASE READS IN TABLES OF AREA AND HYDRAULIC RADIUS
     95 FORMAT(I3)
    115 FORMAT(4F10.3,4X,I3)
    125 FORMAT(5E14.8)
        REWIND 6
        READ(1,95) NS
        DO1K=1,NS
        READ(1,115)DISTU,HIGH,DYU,BOTU,NYU
        READ(1,125)(AREAU(I),I=1,NYU)
        READ(1,125)(HYRADU(I),I=1,NYU)
        WRITE(6) DISTU,BOTU,DYU,NYU,(AREAU(I),I=1,NYU)
      1 WRITE(6)(HYRADU(I),I=1,NYU)
        REWIND 6
        CALL NEXTPH
        END
       MON$$     EXEQ FORTRAN,SOF,SIU,8,5,,,SVF5P2
        INTEGER EXP,TEST
C       THIS PHASE READS IN PRELIMINARY INFORMATION AND PRINTS HEADER
        DIMENSIONX(22),Q(22)
```

TABLE B-7 (CONTINUED)

```
100 FORMAT(8X,A8,A3,9X,A8,A8,A2,21X,I3)
105 FORMAT(3X,I3,2X,I3,3X,F7.3,6X,F6.2,4X,F5.2,4X,I3)
110 FORMAT(8F10.3)
115 FORMAT(6E12.6)
120 FORMAT(1H1//30X,14HPROGRAM SVF 5F,  8X,8HCHANNEL ,A8,A3,9X,5HDATE
   1,A8,A8,A2//30X,8HEXP. NO.,I3,10X, 8HTEST NO.,I3,10X,2HQ=,F7.3,10X,
   26HDELX= , F6.2//60X,3HGL=,F5.2//35X,16HN=C1*((V*R)**C2),30X,
   317HBETA=C3*((Q)**C4)//35X,16HC1=B1+B2*X+B3*GL,32X,15HC3=C5*((X)**C
   46)/35X, 16HC2=B4+B5*X+B6*GL,   32X, 15HC4=C7*((X)**C8)//35X,3HB1=,
   5E12.6,33X, 3HC5=,E12.6/35X,3HB2=,E12.6, 33X, 3HC6=,E12.6/35X,3HB3=
   6,E12.6, 33X,3HC7=,E12.6/35X, 3HB4=,E12.6,33X,3HC8=,E12.6/35X,3HB5=
   7,E12.6/35X,3HB6=,E12.6///33X, 1HX,6X,9HDISCHARGE, 4X,9HELEVATION,
   84X, 5HDEPTH,4X, 8HVELOCITY,4X, 16HHYDRAULIC RADIUS/)
125 FORMAT(1H1//3X, 14HPROGRAM SVF 5F,  8X,8HCHANNEL ,A8,A3,9X,5HDATE
   1,A8,A8,A2//3X, 8HEXP. NO.,I3,10X, 8HTEST NO.,I3,10X,2HQ=,F7.3,10X,
   26HDELX= , F6.2//33X,3HGL=,F5.2//8X, 16HN=C1*((V*R)**C2),30X,
   317HBETA=C3*((Q)**C4)//8X, 16HC1=B1+B2*X+B3*GL,32X,15HC3=C5*((X)**C
   46)/8X,  16HC2=B4+B5*X+B6*GL,   32X, 15HC4=C7*((X)**C8)//8X, 3HB1=,
   5E12.6,33X, 3HC5=,E12.6/8X, 3HB2=,E12.6, 33X, 3HC6=,E12.6/8X, 3HB3=
   6,E12.6, 33X,3HC7=,E12.6/8X,  3HB4=,E12.6,33X,3HC8=,E12.6/8X, 3HB5=
   7,E12.6/8X, 3HB6=,E12.6////6X,  1HX,6X,9HDISCHARGE, 4X,9HELEVATION,
   84X, 5HDEPTH,4X, 8HVELOCITY,4X, 16HHYDRAULIC RADIUS/)
  1 READ(1,100)DI,TCH,DA,T,E,NS
    READ(1,105)EXP,TEST,QD,DELX,GL,NO
    READ(1,115)B1,B2,B3,B4,B5,B6
    READ(1,115)C5,C6,C7,C8
    READ(1,110)START,ELEV
    READ(1,110)(X(J),Q(J),J=1,NO)
    WRITE(3,120)DI,TCH,DA,T,E,EXP,TEST,QD,DELX,GL,B1,C5,B2,C6,B3,C7,
   1B4,C8,B5,B6
    WRITE(2,125)DI,TCH,DA,T,E,EXP,TEST,QD,DELX,GL,B1,C5,B2,C6,B3,C7,
   1B4,C8,B5,B6
    WRITE(4)START,ELEV,QD,DELX,GL,B1,B2,B3,B4,B5,B6,C5,C6,C7,C8,NO,
   1(X(J),Q(J),J=1,NO)
    REWIND 4
    CALL NEXTPH
    END
   MON$$      EXEQ FORTRAN,SOF,SIU,8,5,,,SVF5P3
    DIMENSIONX(22),Q(22),AREAD(295),HYRADD(295),AREAU(295),HYRADU(295)
119 FORMAT(3X,20HERROR, XD .GT. X(NO))
120 FORMAT(30X,20HERROR, XD .GT. X(NO))
125 FORMAT(29X,F7.2,4X,F7.3,6X,F8.3,4X,F6.3,4X,F6.3,F16.3)
130 FORMAT(1H1)
131 FORMAT(80X)
135 FORMAT(2X,F7.2,4X,F7.3,6X,F8.3,4X,F6.3,4X,F6.3,F16.3)
    READ (4)START,ELEV,QD,DELX,GL,B1,B2,B3,B4,B5,B6,C5,C6,C7,C8,NO,
   1(X(J),Q(J),J=1,NO)
    J=NO
    XD=START
    CHANGY=+.001
    GO TO 303
  4 IF(XD.GT.X(J)) GO TO 7
    IF(XD.EQ.X(J)) GO TO 44
    IF(XD.GT.X(J-1)) GO TO 44
    J=J-1
    GO TO 4
 44 YD=ELEV-BOTU
    MD=YD/DYU+1.
    DM=MD
    PROPU=(YD-DYU*(DM-1.))/DYU
    AD=AREAU(MD)+PROPU*(AREAU(MD+1)-AREAU(MD))
    RD=HYRADU(MD)+PROPU*(HYRADU(MD+1)-HYRADU(MD))
    QD=Q(J-1)+((XD-X(J-1))/(X(J)-X(J-1)))*(Q(J)-Q(J-1))
    VD=QD/AD
    BD=BOTU
    ACCX=5.
  5 ACCX=ACCX-DELX
    IF(ACCX.GE.5.) GO TO 300
    GO TO 301
300 WRITE(3,125) XD,QD,ELEV,YD,VD,RD
    WRITE(2,135)XD,QD,ELEV,YD,VD,RD
    ACCX=0.0
301 XU=XD+DELX
    IF(XU.LT.25.) GO TO 2
    IF(XU.LE.X(J-1)) J=J-1
    QU=Q(J-1)+((XU-X(J-1))/(X(J)-X(J-1)))*(Q(J)-Q(J-1))
    IF(XU.GE.DISTU) GO TO 304
    DISTD=DISTU
    BOTD=BOTU
    DYD=DYU
    NYD=NYU
    DO 302 I=1,NYD
    AREAD(I)=AREAU(I)
302 HYRADD(I)=HYRADU(I)
303 READ(6)DISTU,BOTU,DYU,NYU,(AREAU(K),K=1,NYU)
    READ(6)(HYRADU(K),K=1,NYU)
    IF(XD.GT.DISTU) GO TO 304
```

TABLE B-7 (CONTINUED)

```
      IF(XD.EQ.DISTU) GO TO 4
      GO TO 303
  304 PROP=(XU-DISTU)/(DISTD-DISTU)
      BU=PROP*BOTD +(1.-PROP)*BOTU
      XA=(XD+XU)/2.
      C1=B1+B2*XA+B3*GL
      C2=B4+B5*XA+B6*GL
      C3U=C5*((XU)**C6)
      C4U=C7*((XU)**C8)
      C3D=C5*((XD)**C6)
      C4D=C7*((XD)**C8)
      BETAU=C3U*((QU)**C4U)
      BETAD=C3D*((QD)**C4D)
   20 YU=YD+CHANGY
      MU=YU/DYU+1.
      MD=YU/DYD+1.
      UM=MU
      PROPU=(YU-DYU*(UM-1.))/DYU
      DM=MD
      PROPD=(YU-DYD*(DM-1.))/DYD
      AU=PROP*(AREAD(MD)+PROPD*(AREAD(MD+1)-AREAD(MD)))+(1.-PROP)*
     1(AREAU(MU)+PROPU*(AREAU(MU+1)-AREAU(MU)))
      RU=PROP*(HYRADD(MD)+PROPD*(HYRADD(MD+1)-HYRADD(MD)))+(1.-PROP)*
     1(HYRADU(MU)+PROPU*(HYRADU(MU+1)-HYRADU(MU)))
      VU=QU/AU
      RA=(RD+RU)/2.
      VA=(VD+VU)/2.
      RUFFA=C1*((VA*RA)**C2)
      SE=(RUFFA*RUFFA*VA*VA)/(2.21*(RA**(4./3.)))
      SO=(BD-BU)/DELX
      DELY=QU*(VU+VD)*(BETAD*VD-BETAU*VU+BETAD*VD*(QD-QU)/QU)/(32.15*
     1(QU+QD))+(SO*DELX)-(SE*DELX)
      DIFF=ABS(DELY-CHANGY)
   25 IF(DIFF.LT..00001) GO TO 50
      CHANGY=DELY
      GO TO 20
   50 XD=XU
      QD=QU
      ELEV=ELEV+CHANGY-(SO*DELX)
      YD=YU
      RD=RU
      AD=AU
      BD=BU
      VD=VU
      GO TO 5
    2 IF(ACCX.EQ.0.) GO TO 3
      WRITE(3,125)XD,QD,ELEV,YD,VD,RD
      WRITE(2,135)XD,QD,ELEV,YD,VD,RD
      GO TO 3
    7 WRITE(3,120)
      WRITE(2,119)
    3 WRITE(3,130)
      WRITE(2,131)
      REWIND 6
      REWIND 4
      CALL PHASE(002)
      END
     MON$$      EXEQ LINKLOAD
                PHASEENTIREPROG
                CALL SVF5
                PHASE
                BASE1SVF5
                CALL SVF5P2
                PHASE
                BASE1SVF5P2
                CALL SVF5P3
     MON$$      EXEQ ENTIREPROG,MJB
```

VITA

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Doctor of Philosophy

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