COMPUTER CHARACTERIZATION OF n-PORT NETWORKS

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CHAPTER I

INTRODUCTION

An important result of linear-graph theory is the development of systematic methods for formulating the equations describing linear electrical networks. Koenig and Blackwell (1) have extended the use of linear-graph theory to obtaining the characteristics of multiport networks and to the analysis of complex systems containing electrical and mechanical components. As a result of this extension, systematic methods are available for obtaining the characteristics of multiport networks.

The existence of systematic methods for the analysis of electrical networks suggests the possibility of employing a digital computer to formulate and solve the necessary set of equations. Digital computers have been used for this purpose for certain types of electrical network problems. Branin (2) has described a program for use on the IBM 704 computer, which computes the d-c and transient response of transistor switching circuits of arbitrary configuration. This program has the important feature that the necessary equations are formulated from input data describing the circuit parameters and the circuit interconnections. The General Motors Research Laboratories DYANA program is intended for the analysis of mechanical and electrical network problems, and it includes the ability to formulate the necessary equations from input data (3). Reid (4) has presented a program somewhat similar to the DYANA program which accepts input data and from this formulates and solves the necessary equations.

The digital computer programs that have been developed for solving electrical network problems have been written so that multiport components may be included in the network. They have not, however, included the possibility of obtaining the multiport characteristics of a network from simple input data. This thesis is devoted to applying the digital computer to obtaining the multiport characteristics of a linear electrical network. The approach to determining the multiport characteristics is that used by Koenig and Blackwell (1). This method consists of applying conceptual voltage or current sources at the ports and determining the resulting currents or voltages at the ports as functions of the applied sources. The determinations of the relationship between voltages and currents at the ports of the network is made quite systematic by using linear-graph theory and it is this volt-ampere relationship that is used to characterize the multiport component. Koenig and Blackwell (1) have shown how to analyze an electrical network made up of an interconnection of multiport components using the multiport representation of the components. Therefore, to take advantage of these techniques, a mechanized method for determining multiport representations is quite desirable.

The type of network which is considered in this study is a linear network containing both two-terminal devices and multiport components. The two-terminal devices may be resistances, ideal voltage sources or ideal current sources. The multiport components may be subnetworks of two-terminal devices of the type just described, they may be devices described by h-parameters, or they may be ideal transformers.

The program employs the FORTRAN language and it is intended that the input data required be easily obtained. The input data consists of network interconnection information, the edges in the tree and the edges in the co-tree, network parameters, and a small number of constants. This type of program makes possible its application to a network of any configuration and relieves the user from the task of developing the necessary equations to solve.

CHAPTER II

AN ALGORITHM FOR FINDING THE $\underline{B}$ MATRIX

2.1 Introduction. The analysis of an electrical network requires that the mathematical expressions relating voltage and current for each component be known and also that it is possible to write mathematical expressions describing the interconnection of the components. The volt-ampere equations, Kirchhoff's voltage law equations and Kirchhoff's current law equations provide the needed mathematical expressions. When linear-graph theory is used, Kirchhoff's voltage and current law equations are included in the circuit and cut-set matrices respectively. Both of these matrices are obtained from the directed graph after the formulation tree has been selected. Only one of these matrices need be obtained from the directed graph as one is the negative transpose of the other if they are obtained from the same formulation tree (1).

In formulating the necessary set of equations to analyze an electrical network or to obtain its multiport characteristics, using linear-graph theory, the coefficient matrix of the fundamental circuit equations or the coefficient matrix of the cut-set equations is used (1). In this study the coefficient matrix of the fundamental circuit equations is selected because of a saving in computer program length. The coefficient matrix of the fundamental circuit equations shall be denoted by the symbol $\underline{B}$ in this thesis.

It is a simple matter to determine the $\underline{B}$ matrix for a given formulation tree by applying the definition of this matrix (5). To produce the B matrix by means of a digital computer requires that an algorithm be developed which reduces the process to a series of steps that can be programmed for execution by the digital computer. The objective of this chapter is to describe such an algorithm.

2.2 The B Matrix Algorithm. The information that is used in finding the B matrix is the directed linear graph, the formulation tree and the co-tree. From the definition of fundamental circuits it is known that each chord (elements of the co-tree) together with branches (elements of the tree) forms a circuit, and that each node in the circuit is incident to two and only two edges in the circuit (1). The problem is to determine which branches are in the circuit with a particular chord and also to determine the orientation of the branches in each circuit with respect to the chord in the circuit. It is obvious that a means must be found to furnish to the digital computer either as input data or as programmed instructions

(a) the interconnection information for the directed graph,

(b) the orientation information for the edges of the graph,

(c) the branches in the tree and the chords in the co-tree, and

(d) a series of steps which will yield the entries in the B matrix for each of the chords.

The first to be considered is the interconnection information, the orientation information and the tree and co-tree information. This set of information will depend upon the particular electrical network and its associated directed graph. It will, then, be supplied as input data and in this study it will be presented in matrix form. The definitions of the matrices used are now given.

Definition 2.2.1. The interconnection matrix, $\underline{K}$. Given a directed graph with n nodes numbered 1, 2, ..., n, b edges numbered 1, 2, ..., b and with the node of maximum degree having degree m. The matrix $\underline{K} = \left[k_{ij}\right]$ is defined by

k_{ij} = the number from the set $\{1, 2, \ldots, b\}$ identifying edge j incident at node i if node i is of degree m.

If node i is of degree $p < m$ then $\underline{K} = \left[k_{ij}\right]$ is defined by

$$k_{ij} = \begin{cases} \text{the number from the set } \{1, 2, \ldots, b\} \text{ identifying} \\ \text{edge } j \text{ incident at node } i \text{ for } 1 \leq j \leq p \\ 0 \text{ for } p < j \leq m. \end{cases}$$

Definition 2.2.2. The orientation matrix, $\underline{D}$. Given a directed graph with n nodes numbered 1, 2, ..., n and b edges numbered 1, 2, ..., b. The matrix $\underline{D} = \left[d_{ij}\right]$ is defined by

d_{i1} = the number from the set $\{1, 2, \ldots, n\}$ identifying the node <u>from</u> which edge i is directed,

d_{i2} = the number from the set $\{1, 2, \ldots, n\}$ identifying the node <u>to</u> which edge i is directed.

Definition 2.2.3. The branch matrix, $\underline{T}$. Given a directed graph with n nodes numbered 1, 2, ..., n and b edges numbered 1, 2, ..., b and a formulation tree contained in the graph. The matrix $\underline{T} = \left[t_j\right]$

is defined by

t_j = the number from the set $\{1, 2, \ldots, b\}$ identifying edge j in the formulation tree. $t_j < t_{(j+1)}$.

Definition 2.2.4. The chord matrix, $\underline{C}$. Given a directed graph with n nodes numbered 1, 2, ..., n and b edges numbered 1, 2, ..., b and a formulation tree contained in the graph. The matrix $\underline{C} = \left[c_j\right]$ is defined by

c_j = the number from the set $\{1, 2, \ldots, b\}$ identifying edge j in the co-tree. $c_j < c_{(j+1)}$.

To illustrate the matrices $\underline{K}$, $\underline{D}$, $\underline{T}$ and $\underline{C}$, the directed graph of Figure 2.2.1 will be considered where the formulation tree is shown by heavy lines. This graph has 5 nodes and 8 edges and the maximum degree of any node is 4. Thus $\underline{K}$ will be 5 x 4, $\underline{D}$ will be 8 x 2, $\underline{T}$ will be 1 x 4 and $\underline{C}$ will be 1 x 4.

The $\underline{K}$, $\underline{D}$, $\underline{T}$ and $\underline{C}$ matrices corresponding to the directed graph in Figure 2.2.1 are

$$\underline{K} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 8 & 7 & 0 & 0 \\ 1 & 2 & 6 & 8 \\ 3 & 5 & 0 & 0 \end{bmatrix} \qquad (2.2.1)$$

$$\underline{D} = \begin{bmatrix} 1 & 4 \\ 1 & 4 \\ 5 & 1 \\ 1 & 2 \\ 5 & 2 \\ 4 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix} \qquad (2.2.2)$$

$$\underline{T} = \begin{bmatrix} 1 & 3 & 5 & 7 \end{bmatrix} , \qquad (2.2.3)$$

$$\underline{C} = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix} . \qquad (2.2.4)$$

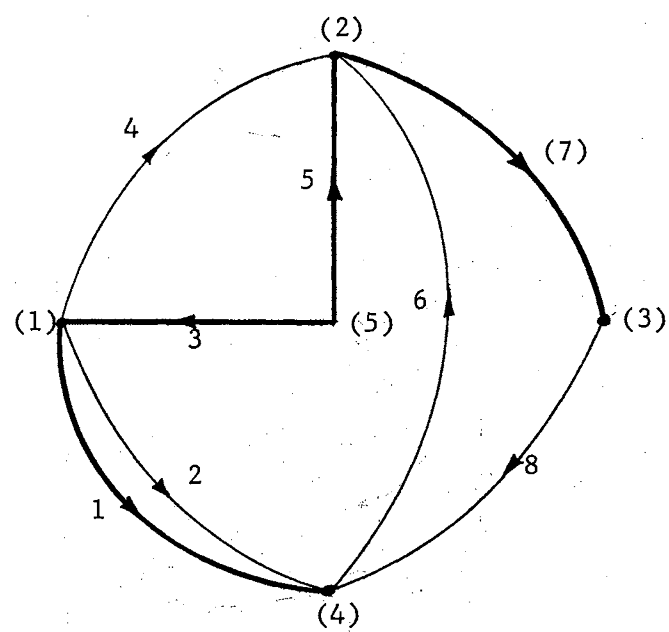


Figure 2.2.1. A Directed Graph.

Now with $\underline{K}$, $\underline{D}$, $\underline{T}$ and $\underline{C}$ available as input data for the computer, the problem is how to determine the entries in the $\underline{B}$ matrix by a series of steps that can be accomplished by the digital computer. The $\underline{K}$ matrix identifies the edges which are incident at each node. Now if one chord is selected and the entries which represent the other chords in the $\underline{K}$ matrix are made zero, a modified $\underline{K}$ matrix, $\underline{K}_1$, is obtained. The entries in $\underline{K}_1$ are either

(a) zero,

(b) numbers identifying branches of the formulation tree, or

(c) the number identifying the chord selected.

Examination of $\underline{K}_1$ will yield

(a) branches that are incident at a node with other branches,

(b) branches that are incident at a node with the chord selected, and

(c) branches that are incident at a node with no other edges.

This latter case will be evident for the row in the $\underline{K}_1$ matrix which represents such a condition will contain only one non-zero entry.

A branch which is incident at a node with no other edges may be removed from consideration for the circuit involving the chord selected because it cannot form a part of the circuit. To remove this branch from consideration, examine the rows of $\underline{K}_1$ and replace with zero all numbers which equal the number identifying this branch. This procedure may reveal other branches which cannot form a part of the circuit since other rows of $\underline{K}_1$ may now have all entries zero except one. These branches are removed from consideration in the same manner. By repeating the procedure at most n-2 times, where n is the number of nodes, a matrix $\underline{K}_2$, will be obtained containing rows that have

(a) all elements are zero, or

(b) two non-zero elements with the remaining elements zero.

The non-zero elements identify the edges in the circuit. One of these edges will be the chord selected and the rest of the edges will be branches.

The orientation of the branches with respect to the chord in the circuit is obtained by use of the $\underline{D}$ matrix. By means of this matrix it is possible to determine if the branches are oriented in the same direction or in the opposite direction to the chord. For those branches oriented in the same direction the $\underline{B}$ matrix entry is +1, for those oriented in the opposite direction the $\underline{B}$ matrix entry is -1, and for those branches that do not appear in the circuit the $\underline{B}$ matrix entry is zero.

The steps to follow in producing the $\underline{B}$ matrix are now given.

(a) Select the chord, c_j, from $\underline{C}$ that will form a circuit with some or all of the branches of $\underline{T}$.

(b) Make all entries in $\underline{K}$ zero that do not equal c_j or the entries of $\underline{T}$. Call the resulting matrix $\underline{K}_1$.

(c) Examine the rows of $\underline{K}_1$. If there is a row with only one non-zero entry, make this entry, and all others equal to it, zero.

(d) Repeat (c) until the rows contain either all zeros or only two non-zero entries. Call the resulting matrix $\underline{K}_2$. The non-zero entries in $\underline{K}_2$ will be either c_j or entries from $\underline{T}$ that form a circuit with c_j.

(e) Go to row c_j of $\underline{D}$ and note the entry in column 2. Call this entry n_1. Now from row n_1 of $\underline{K}_2$ select the non-zero entry $\neq c_j$ and call this entry t_1. Go to row t_1 of $\underline{D}$ and compare the column 1 entry, n_2 with n_1. If $n_2 = n_1$ the $\underline{B}$ matrix entry for t_1 is +1, if $n_2 \neq n_1$ the entry is -1. If $n_2 = n_1$ replace n_1 with the row t_1, column 2 entry of $\underline{D}$. If $n_2 \neq n_1$ replace n_1 with n_2. In either case, use the new n_1 and go to row n_1 of $\underline{K}_2$. Select the non-zero entry that has not been used and proceed as before. If both have been used the row of $\underline{B}$ corresponding to c_j is complete.

To illustrate this algorithm consider the $\underline{K}$, $\underline{D}$, $\underline{T}$ and $\underline{C}$ matrices of equations 2.2.1, 2.2.2, 2.2.3, and 2.2.4.

(a) Select $c_j = 4$.

(b)
$$\underline{K}_1 = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 4 & 5 & 0 & 7 \\ 0 & 7 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \end{bmatrix}$$

(c) Rows 3 and 4 each contain only one non-zero entry. Make all of the entries that are 1 and all that are 7 zero.

(d)

$$\underline{K}_2 \quad \begin{bmatrix} 0 & 0 & 3 & 4 \\ 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \end{bmatrix}$$

(e) By inspecting $\underline{K}_2$ it is apparent that branches 3 and 5 form a circuit with chord 4. Hence branches 1 and 7 are not in the circuit and the $\underline{B}$ matrix entries corresponding to these branches are both zero.

(f) Now $c_j = 4$ so go to row 4 of $\underline{D}$ where $n_1 = 2$. On row 2 of $\underline{K}_2$ find $t_1 = 5$. On row 5 of $\underline{D}$ $n_2 = 5$. Now $n_1 \neq n_2$ so the entry in $\underline{B}$ corresponding to branch 5 is -1. Since $n_2 \neq n_1$ replace 2 with 5 so that $n_1 = 5$. Go to row 5 of $\underline{K}_2$ and find branch $t_1 = 3$ that has not been used. On row 3 of $\underline{D}$ $n_2 = 5$, hence $n_2 = n_1$ and the entry in $\underline{B}$ corresponding to branch 3 is +1. Now make $n_1 = 1$ and go to row 1 of $\underline{K}_2$. Both of the non-zero entries have been used previously so the row in $\underline{B}$ corresponding to chord 4 is complete and is 0 1 - 1 0.

The steps that have been described in this algorithm consist of locating entries in matrices, examining for zero, and comparing one number with another. Each of these can be executed by a digital computer. A program that has been written for execution on the IBM 1620 computer is given in Appendix A. Part I of this three part program produces the $\underline{B}$ matrix. A program written for execution on the IBM 1410 computer is given in Appendix C. Part I of this nine part program produces the $\underline{B}$ matrix.

The task of producing this algorithm was undertaken after a search of the literature failed to reveal an algorithm suitable for the purpose intended. The objective was to produce the $\underline{B}$ matrix based on data easily obtained from the directed graph. The choice of $\underline{K}$, $\underline{D}$, $\underline{T}$ and $\underline{C}$ to designate the matrices defined in this chapter is arbitrary. The only precedent available to guide in designating matrices was in the case of the fundamental circuit matrix which is designated $\underline{B}$ to conform to the designation used by many writers in this field.

CHAPTER III

COMPUTER CHARACTERIZATION OF AN n-PORT NETWORK CONTAINING TWO-TERMINAL DEVICES

3.1 Introduction. By means of the algorithm described in Chapter II, it is possible to utilize a digital computer to produce the $\underline{B}$ matrix from input data which describes the circuit interconnections and edge orientation. The $\underline{B}$ matrix with any volt-ampere equations for the two-terminal devices and the voltages or currents for the ideal sources, makes it possible to determine the volt-ampere equations at the n ports. These equations characterize the network at these ports and, by proper attention to the sign of the parameters, these equations may be used to represent the network if it is a subnetwork of a larger network.

It is the purpose of this chapter to describe the method for obtaining the volt-ampere equations at the n ports and to describe the computer program which determines the parameters of the volt-ampere equations. At this point, the two-terminal devices are restricted to be linear resistances, ideal voltage sources or ideal current sources.

3.2 Partitioning of the $\underline{B}$ Matrix. In partitioning the $\underline{B}$ matrix it will be necessary to consider edges containing the following types of two-terminal devices:

(a) resistance elements (these may be in chords or branches).

(b) ideal current sources (these will be in chords).

(c) ideal voltage sources (these will be in branches).

The ideal sources mentioned in (b) and (c) are those that are a part of the network itself. The technique to be used to determine the volt-ampere equations at the n ports consists of placing conceptual sources (voltage or current) at the ports and finding the response (current or voltage) at the ports in terms of these sources. In this study the voltages are obtained in terms of the currents by applying conceptual current sources. Thus, in partitioning the $\underline{B}$ matrix, there are two types of ideal current sources to identify.

The $\underline{B}$ matrix is partitioned as follows:

$$\underline{B} = \begin{array}{c} \left[\begin{array}{c|c} \underline{B}_{11} & \underline{B}_{12} \\ \hline \underline{B}_{21} & \underline{B}_{22} \\ \hline \underline{B}_{31} & \underline{B}_{32} \end{array}\right] \begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array} \\ \begin{array}{cc} Y_1 & Y_2 \end{array} \end{array} \qquad (3.2.1)$$

X_1 = the chords containing resistance elements.

X_2 = the chords containing ideal current sources in the network itself.

X_3 = the chords containing ideal current sources at the ports.

Y_1 = the branches containing resistance elements.

Y_2 = the branches containing ideal voltage sources.

The partitioning of the $\underline{B}$ matrix in equation 3.2.1 is arbitrary in the order of placing the groups of chords and branches. The placing of the ideal current sources at the ports as the last group of chords has certain advantages because of the way in which the $\underline{B}$ matrix is produced by the computer. This will be considered in more detail when the computer program is discussed.

3.3 The Volt-Ampere Equations. In determining the desired volt-ampere equations, use will be made of the well known equations (1)

$$\underline{V}_C = -\underline{B}\,\underline{V}_T, \tag{3.3.1}$$

$$\underline{I}_T = -\underline{Q}\,\underline{I}_C, \tag{3.3.2}$$

$$\underline{Q} = -\underline{B}^T, \tag{3.3.3}$$

where $\underline{V}_C$ = the matrix of chord voltages,

$\underline{V}_T$ = the matrix of branch voltages,

$\underline{I}_C$ = the matrix of chord currents,

$\underline{I}_T$ = the matrix of branch currents, and

$\underline{Q}$ = the coefficient matrix of the cut-set equations.

Using the partitioned form of $\underline{B}$ and partitioning $\underline{V}_C$, $\underline{V}_T$, $\underline{I}_T$ and $\underline{I}_C$ in a corresponding manner, equations 3.3.1 and 3.3.2 may be written as

$$\begin{bmatrix} \underline{V}_{CR} \\ \underline{V}_{CI} \\ \underline{V}_{CIP} \end{bmatrix} = -\begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} \\ \underline{B}_{21} & \underline{B}_{22} \\ \underline{B}_{31} & \underline{B}_{32} \end{bmatrix} \begin{bmatrix} \underline{V}_{TR} \\ \underline{V}_{TE} \end{bmatrix}, \tag{3.3.4}$$

$$\begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TE} \end{bmatrix} = \begin{bmatrix} \underline{B}_{11}^T & \underline{B}_{21}^T & \underline{B}_{31}^T \\ \underline{B}_{12}^T & \underline{B}_{22}^T & \underline{B}_{32}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix}, \tag{3.3.5}$$

where $\underline{V}_{CR}$ = the matrix of chord voltages for resistance elements,

$\underline{V}_{CI}$ = the matrix of chord voltages for ideal current sources,

$\underline{V}_{CIP}$ = the matrix of chord voltages for ideal current sources at the ports,

$\underline{V}_{TR}$ = the matrix of branch voltages for resistance elements,

$\underline{V}_{TE}$ = the matrix of branch voltages for ideal voltage sources,

$\underline{I}_{CR}$ = the matrix of chord currents for resistive elements,

$\underline{I}_{CI}$ = the matrix of chord currents for ideal current sources,

$\underline{I}_{CIP}$ = the matrix of chord currents for ideal current sources at the ports,

$\underline{I}_{TR}$ = the matrix of branch currents for resistive elements, and

$\underline{I}_{TE}$ = the matrix of branch currents for ideal voltage sources.

Two additional sets of equations are available. These are the volt-ampere equations for the branches and chords containing resistance elements. They are

$$\underline{V}_{TR} = \underline{R}_T \, \underline{I}_{TR}, \tag{3.3.6}$$

$$\underline{V}_{CR} = \underline{R}_C \, \underline{I}_{CR}, \tag{3.3.7}$$

where $\underline{R}_T$ = the matrix of branch resistances, and

$\underline{R}_C$ = the matrix of chord resistances.

Using equations 3.3.4, 3.3.5, 3.3.6, and 3.3.7, it is possible to solve for $\underline{V}_{CIP}$ in terms of $\underline{I}_{CIP}$, $\underline{I}_{CI}$, and $\underline{V}_{TE}$. This is the desired set of equations. First write $\underline{V}_{CIP}$ in terms of the branch voltages.

This set of equations is

$$\underline{V}_{CIP} = - \begin{bmatrix} \underline{B}_{31} & \underline{B}_{32} \end{bmatrix} \begin{bmatrix} \underline{V}_{TR} \\ \underline{V}_{TE} \end{bmatrix} ,$$

$$\underline{V}_{CIP} = - \underline{B}_{31} \underline{V}_{TR} - \underline{B}_{32} \underline{V}_{TE} . \qquad (3.3.8)$$

If equation 3.3.6 is substituted into equation 3.3.8, $\underline{V}_{CIP}$ may be expressed as

$$\underline{V}_{CIP} = -\underline{B}_{31} \underline{R}_{T} \underline{I}_{TR} - \underline{B}_{32} \underline{V}_{TE}. \qquad (3.3.9)$$

Using equation 3.3.5, $\underline{I}_{TR}$ may be expressed as

$$\underline{I}_{TR} = \begin{bmatrix} \underline{B}_{11}^{T} & \underline{B}_{21}^{T} & \underline{B}_{31}^{T} \end{bmatrix} \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} ,$$

$$\underline{I}_{TR} = \underline{B}_{11}^{T} \underline{I}_{CR} + \underline{B}_{21}^{T} \underline{I}_{CI} + \underline{B}_{31}^{T} \underline{I}_{CIP} , \qquad (3.3.10)$$

and when equation 3.3.10 is substituted into equation 3.3.9 the result is

$$\underline{V}_{CIP} = -\underline{B}_{31} \underline{R}_{T} \underline{B}_{11}^{T} \underline{I}_{CR} - \underline{B}_{31} \underline{R}_{T} \underline{B}_{21}^{T} \underline{I}_{CI} - \underline{B}_{31} \underline{R}_{T} \underline{B}_{31}^{T} \underline{I}_{CIP}$$

$$- \underline{B}_{32} \underline{V}_{TE} . \qquad (3.3.11)$$

It is possible to write $\underline{V}_{CR}$ in terms of branch voltages by making use of equation 3.3.4. The result is

$$\underline{V}_{CR} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} \end{bmatrix} \begin{bmatrix} \underline{V}_{TR} \\ \underline{V}_{TE} \end{bmatrix} ,$$

$$\underline{V}_{CR} = - \underline{B}_{11} \underline{V}_{TR} - \underline{B}_{12} \underline{V}_{TE} , \qquad (3.3.12)$$

and if this result is combined with equations 3.3.7 and 3.3.10 then

$$\underline{R}_C\,\underline{I}_{CR} = -\,\underline{B}_{11}\,\underline{R}_T\,(\underline{B}_{11}^T\,\underline{I}_{CR} + \underline{B}_{21}^T\,\underline{I}_{CI} + \underline{B}_{31}^T\,\underline{I}_{CIP})$$
$$-\,\underline{B}_{12}\,\underline{V}_{TE}\,. \qquad (3.3.13)$$

The solution for $\underline{I}_{CR}$ from equation 3.3.13 is

$$\underline{I}_{CR} = \left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T\right]^{-1} (-\underline{B}_{11}\,\underline{R}_T\,\underline{B}_{21}^T\,\underline{I}_{CI} - \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{31}^T\,\underline{I}_{CIP}$$
$$-\,\underline{B}_{12}\,\underline{V}_{TE}), \qquad (3.3.14)$$

and if this equation is substituted into equation 3.3.11 the equation for $\underline{V}_{CIP}$ becomes

$$\underline{V}_{CIP} = (\underline{B}_{31}\,\underline{R}_T\,B_{11}^T \left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T\right]^{-1} \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{31}^T - \underline{B}_{31}\,\underline{R}_T\,\underline{B}_{31}^T)\underline{I}_{CIP}$$
$$+ (\underline{B}_{31}\,\underline{R}_T\,\underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T\right]^{-1} \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{21}^T - \underline{B}_{31}\,\underline{R}_T\,\underline{B}_{21}^T)\underline{I}_{CI}$$
$$+ (\underline{B}_{31}\,\underline{R}_T\,\underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T\right]^{-1} \underline{B}_{12} - \underline{B}_{32})\,\underline{V}_{TE}. \qquad (3.3.15)$$

It is noted that this set of equations expresses the voltages, $\underline{V}_{CIP}$, at the ports in terms of the currents, $\underline{I}_{CIP}$, at the ports, the ideal current sources, $\underline{I}_{CI}$, in the network and the ideal voltage sources, $\underline{V}_{TE}$, in the network. This set of equations is of the form

$$\underline{V} = \underline{R}\,\underline{I} + \underline{E} \qquad (3.3.16)$$

where

$\underline{V} = \underline{V}_{CIP}$,

$\underline{R} = (\underline{B}_{31}\,R_T\,\underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T\right]^{-1} \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{31}^T - \underline{B}_{31}\,\underline{R}_T\,\underline{B}_{31}^T)$,

$\underline{I} = \underline{I}_{CIP}$, and

$$\underline{E} = (\underline{B}_{31}\, \underline{R}_T\, \underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\, \underline{R}_T\, \underline{B}_{11}^T\right]^{-1} \underline{B}_{11}\, \underline{R}_T\, \underline{B}_{21}^T - \underline{B}_{31}\, \underline{R}_T\, \underline{B}_{21}^T)\underline{I}_{CI}$$
$$+ (\underline{B}_{31}\, \underline{R}_T\, \underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\, \underline{R}_T\, B_{11}^T\right]^{-1} \underline{B}_{12} - \underline{B}_{32})\, \underline{V}_{TE} .$$

The n-port network may be characterized by this set of volt-ampere equations. If the n-port network is a subnetwork of a larger network, it may be represented by these equations if the signs associated with $\underline{R}$ and $\underline{E}$ correspond to the reference direction for the voltages and currents at the ports. In this study the signs for $\underline{R}$ and $\underline{E}$ correspond to port voltages which are positive at the terminal where current enters.

It is evident that the volt-ampere equations at the ports can be determined for any arbitrary network of two-terminal devices, within the restrictions imposed, by applying equation 3.3.15. The availability of the $\underline{B}$ matrix, partitioned as described in this chapter, is necessary. A computer program for determining the $\underline{B}$ matrix for any arbitrary directed graph has been presented in Chapter II. A method for incorporating this program into a larger program for determining the coefficient matrices of equation 3.3.15 will now be considered.

3.4 The Computer Program. The computer program for producing the $\underline{B}$ matrix places the rows and columns in ascending numerical order corresponding to the numbers assigned to the chords and branches. This feature must be kept in mind in numbering the edges of the graph so that the rows and columns of the $\underline{B}$ matrix will appear in the proper order for partitioning as previously described. A method for numbering the edges to achieve the desired result is presented here. If it is assumed that the network contains r resistances, i ideal current sources, e ideal voltage sources and p ports, number the edges as follows:

(a) edges containing resistances are numbered 1 through r.

(b) edges containing ideal current or voltage sources are numbered $r + 1$ through $r + i + e$.

(c) edges representing the conceptual sources at the ports are numbered $r + i + e + 1$ through $r + i + e + p$.

This numbering system will insure that the chords and branches containing resistance elements will appear first in the $\underline{B}$ matrix produced by the computer. The chords and branches containing sources in the network will appear next and the chords containing the conceptual current sources will appear last. The advantage of placing the chords containing the conceptual current sources last in the $\underline{B}$ matrix can now be seen, for it is possible to change the number of ports on a given network and not change the numbering of the edges representing elements in the network itself.

When the computer executes the $\underline{B}$ matrix program, the result will be stored as an array in the computer memory. With the chords and branches in the order just described, it is a simple matter to perform operations with the desired submatrices of the $\underline{B}$ matrix. This is accomplished by including, as a part of the input data, constants which identify

(a) the number of chords which contain resistance elements,

(b) the number of branches which contain resistance elements,

(c) the number of ideal current sources in the network,

(d) the number of ports, and

(e) the number of ideal voltage sources in the network.

In addition to the interconnection matrix, the orientation matrix, the branch matrix, the chord matrix, the degree of the node of maximum degree and the constants just defined, it is necessary to supply $\underline{R}_T$, the matrix of branch resistances, and $\underline{R}_C$, the matrix of chord resistances, as input data. $\underline{R}_T$ and $\underline{R}_C$ are diagonal matrices. In the case of $\underline{R}_T$, the diagonal entries are resistances appearing in branches taken in the same order as the branches corresponding to the columns of $\underline{B}_{11}$. The diagonal entries of the $\underline{R}_C$ matrix are resistances appearing in chords taken in the same order as the chords corresponding to the rows of $\underline{B}_{11}$. The $\underline{I}_{CI}$ and $\underline{V}_{TE}$ matrices may also be supplied as input data. In this particular program the coefficient matrices of $\underline{I}_{CI}$ and $\underline{V}_{TE}$ are determined with $\underline{I}_{CI}$ and $\underline{V}_{TE}$ considered as variables.

The limitation in memory size in the IBM 1620 computer available for execution of the program necessitates the division of the program into three parts. These are as follows:

I. Production of the $\underline{B}$ matrix.

II. Production of the coefficient matrix of $\underline{I}_{CIP}$.

III. Production of the coefficient matrices of $\underline{I}_{CI}$ and $\underline{V}_{TE}$.

The input data for Part I consists of the interconnection matrix (KONN), the orientation matrix (INTO), the branch matrix (NTRE), the chord matrix (KORD), the degree of the node of highest degree (MOD), and the constants indicating the number of the various types of elements in the branches and chords. The constants are not required in the production of the $\underline{B}$ matrix, but they are used in some tests that are included in the program. These tests involve checking the total number of branches and chords against the sum of the various types of devices appearing in the branches and chords. The output data for Part I is the $\underline{B}$ matrix in the form of punched cards.

The input data for Part II consists of the constants indicating the number of the various types of elements in the branches and chords, the $\underline{B}$ matrix from Part I, the $\underline{R}_T$ matrix and the $\underline{R}_C$ matrix. The output from Part II is a typed output, which is the coefficient matrix for $\underline{I}_{CIP}$, and a punched output for use in Part III.

The input data for Part III is the same as for Part II with the addition of the punched output from Part II. The output from Part III is a typed output which is the coefficient matrix for $\underline{I}_{CI}$ and the coefficient matrix for $\underline{V}_{TE}$.

The constants supplied as part of the input data are fixed point constants and are identified as follows:

KO = the number of chords.

MTRE = the number of branches.

NT = the number of branches containing resistances.

NC = the number of chords containing resistances.

NE = the number of ideal voltage sources.

NI = the number of ideal current sources.

NIE = the number of ports.

Flow charts for Parts I, II, and III of the program are shown in Appendix A along with a complete listing of the FORTRAN statements. The logical transfer statement, IF, is used throughout this program so that only the necessary operations are carried out.

A number of examples have been worked using this three part program. Some of these examples and the results are included here.

Example 3.4.1

The network and its associated directed graph shown in Figure 3.4.1 (the formulation tree is shown in heavy lines) will be considered.

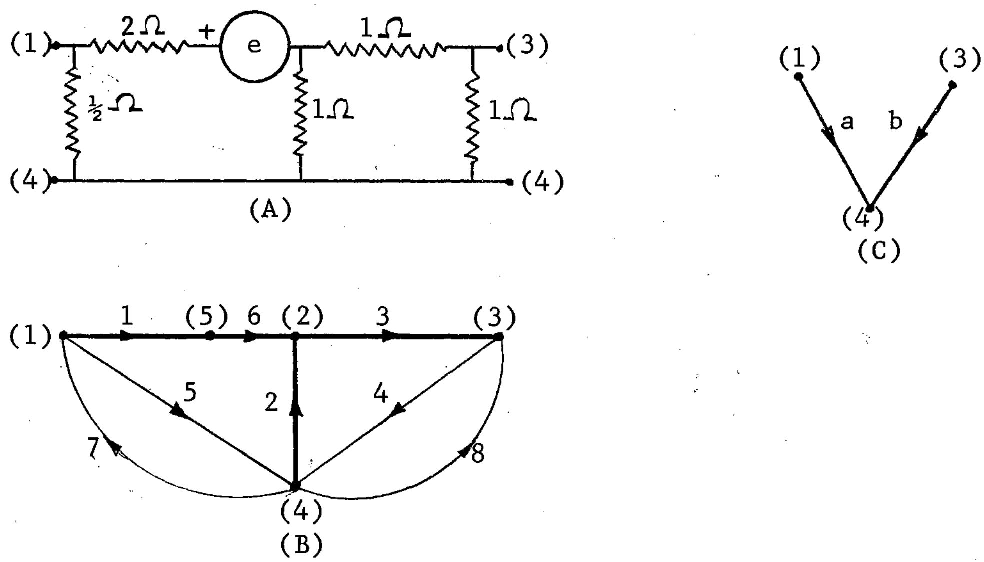


Figure 3.4.1. (A) The Electrical Network for Example 3.4.1.
(B) Its Associated Directed Graph.
(C) Its Terminal Graph.

Chords 7 and 8 in Figure 3.4.1 represent the conceptual current sources. The input data is as follows:

KO = 4, MTRE = 4, NT = 3, NE = 1, NC = 2, NI = 0, NIE = 2, MOD = 5

$$\text{Interconnection matrix (KONN)} = \begin{bmatrix} 1 & 5 & 7 & 0 & 0 \\ 6 & 3 & 2 & 0 & 0 \\ 3 & 4 & 8 & 0 & 0 \\ 7 & 5 & 2 & 4 & 8 \\ 1 & 6 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Orientation matrix (INTO)} = \begin{bmatrix} 1 & 5 \\ 4 & 2 \\ 2 & 3 \\ 3 & 4 \\ 1 & 4 \\ 5 & 2 \\ 4 & 1 \\ 4 & 3 \end{bmatrix}$$

$$\text{Branch matrix (NTRE)} = \begin{bmatrix} 1 & 2 & 3 & 6 \end{bmatrix}, \text{ Chord matrix (KORD)} = \begin{bmatrix} 4 & 5 & 7 & 8 \end{bmatrix}$$

$$\underline{R}_T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{R}_C = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$

The typed output from Part II is

.42105270

5.2631600E-02

909

5.2631600E-02

.63157890

909

The first two lines are the entries in the first row of the coefficient matrix of $\underline{I}_{CIP}$. The number 909 is simply a flag indicating a complete row has been typed. The two lines following the first 909 are the entries in the second row of the coefficient matrix of $\underline{I}_{CIP}$. There are only two rows since there are only two ports in this case. The coefficient matrix obtained by manual calculations is

$$\begin{bmatrix} 0.422 & 0.0527 \\ 0.0527 & 0.632 \end{bmatrix} .$$

The typed output from Part III is

111

.15789480

33

-.10526316

33

The number 111 is a flag to indicate that there are no current sources in the network. The next number is the first row of the coefficient matrix of $\underline{V}_{TE}$. There is only one column in this coefficient matrix since there is only one voltage source in the network. The number 33 is a flag to indicate that a complete row has been typed. The number following the first 33 is the second row of the coefficient matrix. The coefficient matrix obtained by manual calculations is

$$\begin{bmatrix} 0.157 \\ -0.1051 \end{bmatrix} .$$

The computer results for Example 3.4.1 may be summarized in the set of equations

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} .42105270 & .052631600 \\ .052631600 & .63157890 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} .15789480 \\ -.10526316 \end{bmatrix} e \quad (3.4.1)$$

The reference directions for the voltages and currents used in equation 3.4.1 are shown in Figure 3.4.1(C).

Example 3.4.2

The network and its associated directed graph shown in Figure 3.4.2 will be considered.

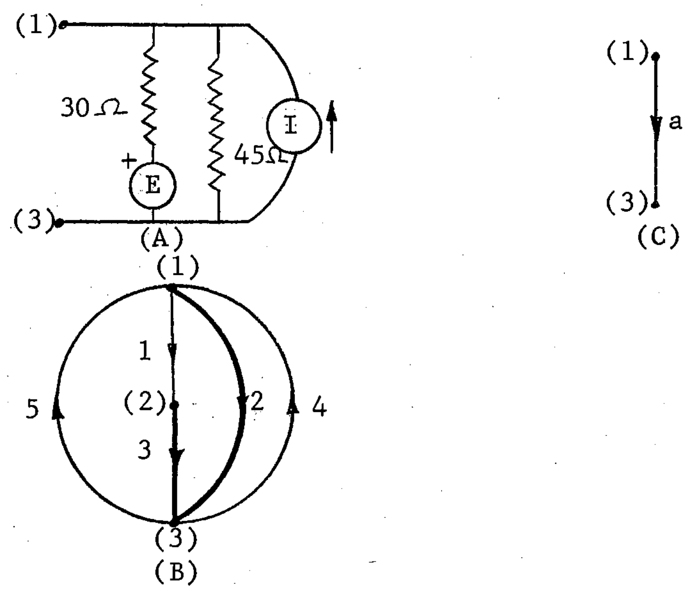


Figure 3.4.2. (A) The Electrical Network for Example 3.4.2.
(B) Its Associated Directed Graph.
(C) Its Terminal Graph.

Input data:

KO = 3, MTRE = 2, NT = 1, NE = 1, NC = 1, NI = 1, NIE = 1, MOD = 4

$$\text{KONN} = \begin{bmatrix} 5 & 1 & 2 & 4 \\ 1 & 3 & 0 & 0 \\ 5 & 3 & 2 & 4 \end{bmatrix} \qquad \text{INTO} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 3 \\ 3 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\text{NTRE} = \begin{bmatrix} 2 & 3 \end{bmatrix} \qquad \text{KORD} = \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$$

$$\underline{R}_T = 45 \qquad \underline{R}_C = 30$$

	Computer Results	Manual Calculations
Part II	18.000000	18
	909	
Part III	18.000000	18 (coefficient of $\underline{I}_{CI}$)
	11	
	.59999999	.6 (coefficient of $\underline{V}_{TE}$)
	33	

The computer results for Example 3.4.2 may be summarized in the equation

$$v_a = 18.000000 i_a + 18.000000 I + .59999999 E. \qquad (3.4.2)$$

The reference directions for the voltages and currents used in equation 3.4.2 are shown in Figure 3.4.2(C).

<u>Example 3.4.3</u>

The network and its associated directed graph shown in Figure 3.4.3

will be considered.

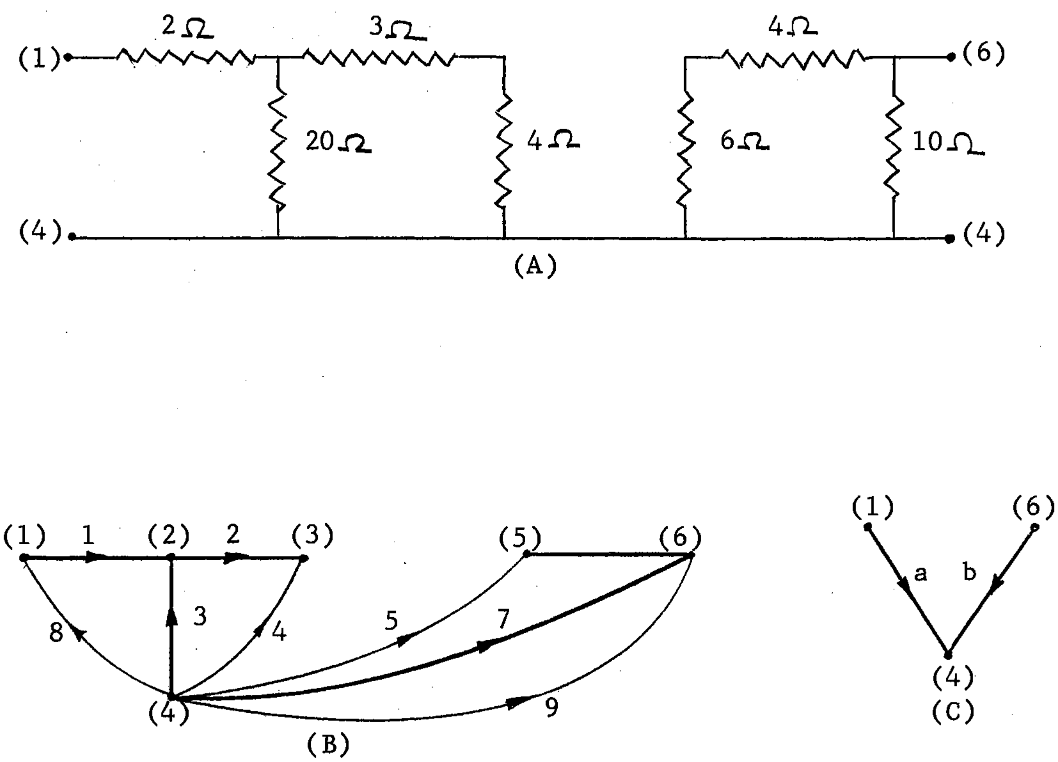


Figure 3.4.3. (A) The Electrical Network for Example 3.4.3.
(B) Its Associated Directed Graph.
(C) Its Terminal Graph.

Input Data:

KO = 4, MTRE = 5, NT = 5, NE = 0, NC = 2, NI = 0, NIE = 2, MOD = 6

$$\text{KONN} = \begin{bmatrix} 8 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 8 & 3 & 4 & 5 & 7 & 9 \\ 5 & 6 & 0 & 0 & 0 & 0 \\ 6 & 7 & 9 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{INTO} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 2 \\ 4 & 3 \\ 4 & 5 \\ 5 & 6 \\ 4 & 6 \\ 4 & 1 \\ 4 & 6 \end{bmatrix}$$

$$\text{NTRE} = \begin{bmatrix} 1 & 2 & 3 & 6 & 7 \end{bmatrix}$$

$$\text{KORD} = \begin{bmatrix} 4 & 5 & 8 & 9 \end{bmatrix}$$

$$\underline{R}_T = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\underline{R}_C = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$$

	Computer Results	Manual Calculations
Part II	6.0000000	$\begin{bmatrix} 6 & 0 \\ 0 & 5 \end{bmatrix}$
	-.00000000	
	909	
	-.00000000	
	5.0000000	

Since there are no sources in the network of Figure 3.4.3 Part III of the program was not executed.

The computer results for Example 3.4.3 may be summarized in the set of equations

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 6.0000000 & -.00000000 \\ -.00000000 & 5.0000000 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} . \qquad (3.4.3)$$

The reference directions for the voltages and currents used in equation 3.4.3 are shown in Figure 3.4.3(C).

The preceding examples are intended to demonstrate that the program written will produce accurate results. Within the limitations imposed, it is shown that the parameters of an n-port network can be determined from input data determined from the circuit interconnections and the circuit parameters. The next chapter considers the problem of determining the parameters of an n-port network consisting of two-terminal devices, multiport subnetworks and ideal transformers.

CHAPTER IV

COMPUTER CHARACTERIZATION OF AN n-PORT NETWORK CONTAINING MULTIPORT SUBNETWORKS

4.1 Introduction. An n-port network consisting of two-terminal devices may be contained in a larger network (1). It may be desirable in such cases to obtain the n-port representation of the larger network utilizing the multiport representation of the subnetwork. This chapter is devoted to the consideration of networks consisting of

(a) two-terminal devices limited to resistances, ideal current sources and ideal voltage sources,

(b) multiport subnetworks consisting of two-terminal devices listed under (a) and represented by volt-ampere equations at the ports, and

(c) ideal transformers.

The objective is to obtain the n-port representation of such a network utilizing a digital computer which is supplied with input data describing the circuit interconnections and parameters.

The multiport subnetworks embedded in the larger network may be represented by volt-ampere equations of the form

$$\underline{V} = \underline{R}\,\underline{I} + E, \tag{4.1.1}$$

and

$$\underline{I} = \underline{G}\,\underline{V} + \underline{H}. \tag{4.1.2}$$

For purposes of this study the form shown in equation 4.1.1 will be considered. This does not detract from the generality of the method developed, for equations in the form of 4.1.2 can be placed in the form of 4.1.1.

An ideal transformer is represented by

$$\begin{bmatrix} v_o \\ i_i \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_o \\ v_i \end{bmatrix}, \qquad (4.1.3)$$

where v_o = output voltage,

v_i = input voltage,

i_o = output current,

i_i = input current, and

n = turns ratio.

A somewhat more general representation will be used in this study. Specifically the (2, 1) element in the coefficient matrix will not be required to be the negative of the (1, 2) element as it is in equation 4.1.3. This increased generality is to allow the method developed to be extended readily to components represented by h-parameters.

<u>4.2 Partitioning the $\underline{B}$ Matrix.</u> The $\underline{B}$ matrix will be partitioned in a manner similar to that reported in Chapter III. The exception is that the partitioning will necessarily be finer since there are more types of circuit elements to be considered. The $\underline{B}$ matrix will be partitioned as follows:

$$\underline{B} = \begin{array}{c} \left[\begin{array}{c:c:c:c} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & \underline{B}_{14} \\ \hdashline \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & \underline{B}_{24} \\ \hdashline \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} & \underline{B}_{34} \\ \hdashline \underline{B}_{41} & \underline{B}_{42} & \underline{B}_{43} & \underline{B}_{44} \\ \hdashline \underline{B}_{51} & \underline{B}_{52} & \underline{B}_{53} & \underline{B}_{54} \end{array}\right] \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{array} \\ \begin{array}{cccc} Y_1 & Y_2 & Y_3 & Y_4 \end{array} \end{array} \qquad (4.2.1)$$

where X_1 = the chords containing ideal transformers.

X_2 = the chords containing two-terminal resistance elements.

X_3 = the chords containing multiport subnetworks.

X_4 = the chords containing ideal current sources in the network.

X_5 = the chords containing ideal current sources at the ports.

Y_1 = the branches containing ideal transformers.

Y_2 = the branches containing two-terminal resistance elements.

Y_3 = the branches containing multiport subnetworks.

Y_4 = the branches containing ideal voltage sources.

It will be necessary as in the case of networks consisting only of two-terminal devices, to number the edges in the directed graph properly. This will be discussed in more detail later.

<u>4.3 The Volt-Ampere Equations</u>. Using the $\underline{B}$ matrix partitioned

as shown in equation 4.2.1, the chord voltages may be written as

$$\begin{bmatrix} \underline{V}_{CH} \\ \underline{V}_{CR} \\ \underline{V}_{CM} \\ \underline{V}_{CI} \\ \underline{V}_{CIP} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & \underline{B}_{14} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & \underline{B}_{24} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} & \underline{B}_{34} \\ \underline{B}_{41} & \underline{B}_{42} & \underline{B}_{43} & \underline{B}_{44} \\ \underline{B}_{51} & \underline{B}_{52} & \underline{B}_{53} & \underline{B}_{54} \end{bmatrix} \begin{bmatrix} \underline{V}_{TH} \\ \underline{V}_{TR} \\ \underline{V}_{TM} \\ \underline{V}_{TE} \end{bmatrix}, \qquad (4.3.1)$$

and the branch currents may be written as

$$\begin{bmatrix} \underline{I}_{TH} \\ \underline{I}_{TR} \\ \underline{I}_{TM} \\ \underline{I}_{TE} \end{bmatrix} = \begin{bmatrix} \underline{B}_{11}^T & \underline{B}_{21}^T & \underline{B}_{31}^T & \underline{B}_{41}^T & \underline{B}_{51}^T \\ \underline{B}_{12}^T & \underline{B}_{22}^T & \underline{B}_{32}^T & \underline{B}_{42}^T & \underline{B}_{52}^T \\ \underline{B}_{13}^T & \underline{B}_{23}^T & \underline{B}_{33}^T & \underline{B}_{43}^T & \underline{B}_{53}^T \\ \underline{B}_{14}^T & \underline{B}_{24}^T & \underline{B}_{34}^T & \underline{B}_{44}^T & \underline{B}_{54}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \\ \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} \qquad (4.3.2)$$

where $\underline{V}_{CH}$ = the matrix of chord voltages for ideal transformers,

$\underline{V}_{CR}$ = the matrix of chord voltages for resistance elements,

$\underline{V}_{CM}$ = the matrix of chord voltages for multiport subnetworks,

$\underline{V}_{CI}$ = the matrix of chord voltages for ideal current sources,

$\underline{V}_{CIP}$ = the matrix of chord voltages for ideal current sources at the ports,

$\underline{V}_{TH}$ = the matrix of branch voltages for ideal transformers,

$\underline{V}_{TR}$ = the matrix of branch voltages for resistance elements,

$\underline{V}_{TM}$ = the matrix of branch voltages for multiport subnetworks,

$\underline{V}_{TE}$ = the matrix of branch voltages for ideal voltage sources,

$\underline{I}_{TH}$ = the matrix of branch currents for ideal transformers.

$\underline{I}_{TR}$ = the matrix of branch currents for resistance elements,

$\underline{I}_{TM}$ = the matrix of branch currents for multiport subnetworks,

$\underline{I}_{TE}$ = the matrix of branch currents for ideal voltage sources,

$\underline{I}_{CH}$ = the matrix of chord currents for ideal transformers,

$\underline{I}_{CR}$ = the matrix of chord currents for resistance elements,

$\underline{I}_{CM}$ = the matrix of chord currents for multiport subnetworks,

$\underline{I}_{CI}$ = the matrix of chord currents for ideal current sources, and

$\underline{I}_{CIP}$ = the matrix of chord currents for ideal current sources at the ports.

The volt-ampere equations for the resistances, the multiport subnetworks and the ideal transformers will be used with equations 4.3.1 and 4.3.2 to obtain the desired characterization of the network. The volt-ampere equations for the resistances are given in equations 3.3.6 and 3.3.7 in Chapter III. The set of volt-ampere equations for the multiport subnetworks is

$$\begin{bmatrix} \underline{V}_{TM} \\ \underline{V}_{CM} \end{bmatrix} = \begin{bmatrix} \underline{R}_{TM1} & \underline{R}_{CM1} \\ \underline{R}_{TM2} & \underline{R}_{CM2} \end{bmatrix} \begin{bmatrix} \underline{I}_{TM} \\ \underline{I}_{CM} \end{bmatrix} + \begin{bmatrix} \underline{E}_{TM} \\ \underline{E}_{CM} \end{bmatrix}, \qquad (4.3.3)$$

where $\underline{R}_{TM1}$ = the matrix of r-parameters relating branch voltages and currents,

$\underline{R}_{CM1}$ = the matrix of r-parameters relating branch voltages and chord currents,

$\underline{R}_{TM2}$ = the matrix of r-parameters relating chord voltages and branch currents,

$\underline{R}_{CM2}$ = the matrix of r-parameters relating chord voltages and currents,

$\underline{E}_{TM}$ = the matrix of voltages which are constants appearing in the volt-ampere equations for the voltages of the branches assigned to multiport subnetworks, and

$\underline{E}_{CM}$ = the matrix of voltages which are constants appearing in the volt-ampere equations for the voltages of the chords assigned to multiport subnetworks.

The set of volt-ampere equations for the ideal transformers is

$$\begin{bmatrix} \underline{V}_{TH} \\ \underline{I}_{CH} \end{bmatrix} = \begin{bmatrix} 0 & \underline{N}_{12} \\ \underline{N}_{21} & 0 \end{bmatrix} \begin{bmatrix} \underline{I}_{TH} \\ \underline{V}_{CH} \end{bmatrix} , \qquad (4.3.4)$$

where $\underline{N}_{12}$ = the matrix of constants relating branch and chord voltages assigned to ideal transformers, and

$\underline{N}_{21}$ = the matrix of constants relating branch and chord currents assigned to ideal transformers.

Equations 3.3.6, 3.3.7, 4.3.1, 4.3.2, 4.3.3 and 4.3.4 can be combined to yield $\underline{V}_{CIP}$ in terms of $\underline{I}_{CIP}$, $\underline{I}_{CI}$, $\underline{V}_{TE}$, $\underline{E}_{TM}$ and $\underline{E}_{CM}$. The details of combining these equations are shown in Appendix B. The desired equation is

$$\underline{V}_{CIP} = \underline{B}_p\,(\underline{G}\ \underline{H}\ \underline{J} - \underline{F})\ \underline{I} + \underline{B}_p\,(\underline{G}\ \underline{H}\ \underline{L})\underline{E} - \underline{B}_Q\underline{E}, \qquad (4.3.5)$$

where $\underline{B}_p = \begin{bmatrix} \underline{B}_{51} & \underline{B}_{52} & \underline{B}_{53} \end{bmatrix}$,

$$\underline{G} = \begin{bmatrix} \underline{N}_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \underline{R}_{CM1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_T\underline{B}_{12}^T\underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1}\underline{B}_{13}^T\underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} 0 & \underline{B}_{21}^T & \underline{B}_{31}^T \\ 0 & \underline{B}_{22}^T & \underline{B}_{32} \\ 0 & \underline{B}_{23}^T & \underline{B}_{33} \end{bmatrix},$$

$$\underline{A} = \begin{bmatrix} \underline{U} - \underline{N}_{21}\underline{B}_{11}^T \end{bmatrix}^{-1} \underline{N}_{21},$$

$$\underline{H} = \left\{ \begin{bmatrix} \underline{U} & 0 & 0 \\ 0 & \underline{R}_C & 0 \\ 0 & 0 & \underline{R}_{CM2} \end{bmatrix} + \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{N}_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \underline{R}_{CM1} \end{bmatrix} \right.$$

$$+ \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_T\underline{B}_{12}^T\underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1}\underline{B}_{13}^T\underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} 0 & \underline{B}_{21}^T & \underline{B}_{31}^T \\ 0 & \underline{B}_{22}^T & \underline{B}_{32}^T \\ 0 & \underline{B}_{23}^T & \underline{B}_{33}^T \end{bmatrix}$$

$$\left. + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \underline{R}_{TM2}\underline{B}_{13}^T\underline{A} & 0 & \underline{R}_{TM2} \end{bmatrix} \begin{bmatrix} 0 & \underline{B}_{21}^T & \underline{B}_{31}^T \\ 0 & \underline{B}_{22}^T & \underline{B}_{32}^T \\ 0 & \underline{B}_{23}^T & \underline{B}_{33}^T \end{bmatrix} \right\}^{-1},$$

$$\underline{J} = \left\{ \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_T\underline{B}_{12}^T\underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1}\underline{B}_{13}^T\underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \underline{R}_{TM2}\underline{B}_{13}^T\underline{A} & 0 & \underline{R}_{TM2} \end{bmatrix} \right\} \begin{bmatrix} \underline{B}_{41}^T & \underline{B}_{51}^T \\ \underline{B}_{42}^T & \underline{B}_{52}^T \\ \underline{B}_{43}^T & \underline{B}_{53}^T \end{bmatrix},$$

$$\underline{F} = \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1}\ \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{B}_{41}^T & \underline{B}_{51}^T \\ \underline{B}_{42}^T & \underline{B}_{52}^T \\ \underline{B}_{43}^T & \underline{B}_{53}^T \end{bmatrix},$$

$$\underline{I} = \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix},$$

$$\underline{L} = \begin{bmatrix} \underline{B}_{13} & \underline{B}_{14} & 0 \\ \underline{B}_{23} & \underline{B}_{24} & 0 \\ \underline{B}_{33} & \underline{B}_{34} & \underline{U} \end{bmatrix},$$

$$\underline{B}_Q = \begin{bmatrix} \underline{B}_{53} & \underline{B}_{54} & 0 \end{bmatrix}, \text{ and}$$

$$\underline{E} = \begin{bmatrix} \underline{E}_{TM} \\ \underline{V}_{TE} \\ \underline{E}_{CM} \end{bmatrix}.$$

The set of equations shown in equation 4.3.5, with the proper sign on the coefficients of $\underline{I}$ and $\underline{E}$ is the set desired to represent the n-port network.

4.4 The Computer Program. In order to program the set of equations of 4.3.5 it is necessary to number the edges of the directed graph properly for the reasons set forth in Chapter III. The method of numbering the edges is similar to that reported in Chapter III. If the directed graph contains h edges assigned to ideal transformers, r edges assigned to resistances, m edges assigned to multiport subnetworks, i edges assigned to ideal current sources, e edges assigned to ideal voltage sources and p edges assigned to the conceptual sources at the ports, a method for numbering the edges to insure the proper order for the rows and columns of $\underline{B}$ is

(a) number the edges assigned to ideal transformers 1 through h,

(b) number the edges assigned to resistances h + 1 through h + r,

(c) number the edges assigned to multiport subnetworks h + r + 1 through h + r + m,

(d) number the edges assigned to ideal sources h + r + m + 1 through h + r + m + i + e, and

(e) number the edges assigned to the conceptual sources at the ports h + r + m + i + e + 1 through h + r + m + i + e + p.

It is evident, as in Chapter III, that placing the chords assigned to the conceptual sources last in the $\underline{B}$ matrix allows the number of ports to be changed without changing the numbering of the edges in the network.

The procedure for producing the program is the same as was used in Chapter III. The $\underline{B}$ matrix is produced from interconnection data and the desired submatrices of $\underline{B}$ and then used with the appropriate circuit parameters in calculating the solution. In order that the proper submatrices of $\underline{B}$ may be selected a number of constants are supplied as input data. The constants include those mentioned in Chapter III and, in addition, contain

(a) the number of chords which contain ideal transformers,

(b) the number of chords which contain multiport subnetworks,

(c) the number of branches which contain ideal transformers, and

(d) the number of branches which contain multiport subnetworks.

The parameters which must be supplied as input data are

(a) $\underline{R}$, the matrix of two-terminal resistances (this matrix includes both branch and chord resistances with the branch resistances appearing first in ascending numerical order of the edge numbers),

(b) $\underline{R}_M$, the matrix of r-parameters shown in equation 4.3.3, and

(c) $\underline{N}_{12}$ and $\underline{N}_{21}$, the matrices of constants shown in equation 4.3.4.

As in the program described in Chapter III, the voltage and current sources are treated as variables. The coefficient matrices for the sources are determined and are printed as output data.

The program that has been prepared to compute the coefficients of $\underline{I}$ and $\underline{E}$ in equation 4.3.5 is written in FORTRAN IV language for execution on the IBM 1410 computer. The length of the program is such that is impractical to execute on the IBM 1620 computer. The program is divided into 9 parts and each part is executed in sequence and supplies data for the succeeding parts.

The card input data for Part I consists of the matrices $\underline{K}$, $\underline{D}$, $\underline{T}$ and $\underline{C}$, the degree of the node of highest degree, the fixed point constants named in Chapter III, plus the fixed point constants for the additional types of circuit elements to be considered. These constants are

NCH = the number of chords containing ideal transformers,

NCM = the number of chords containing multiport subnetworks,

NTH = the number of branches containing ideal transformers,

and

NTM = the number of branches containing multiport subnetworks.

All of these constants are stored on magnetic tape for use in succeeding parts of the program. The $\underline{B}$ matrix is the output of Part I. It is typed and is also stored on magnetic tape for use by other parts of the program.

The card input data for Part II consist of the matrices $\underline{R}$, $\underline{R}_M$, $\underline{N}_{12}$ and $\underline{N}_{21}$. This data is stored on magnetic tape for future use. The output of Part II is used in Part III etc. through Part IX where the output consists of the desired coefficient matrices and is typed with appropriate headings. The signs for the coefficient matrices conform to voltages at the ports which are positive at the terminal at which current enters. A flow chart and a complete listing of the FORTRAN statements for this program is included in Appendix C. It will be noted that the IF statement is used so that only operations necessary for a particular problem are carried out.

A number of examples have been worked using this program and some of them are included here to demonstrate the use of the program.

Example 4.4.1

The network in Figure 4.4.1 will be considered .

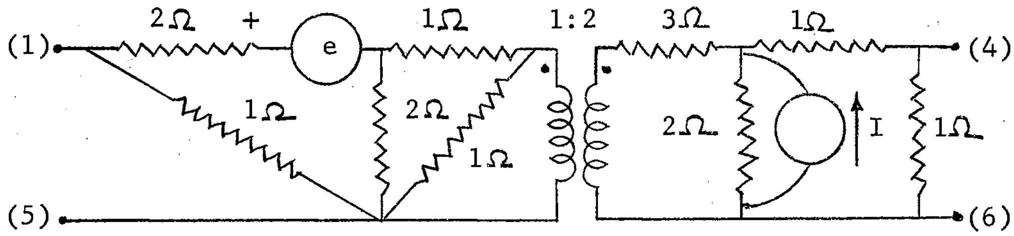
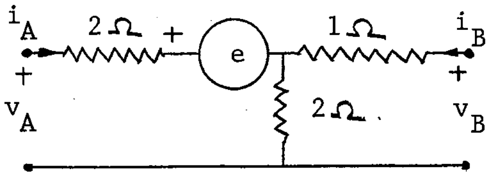
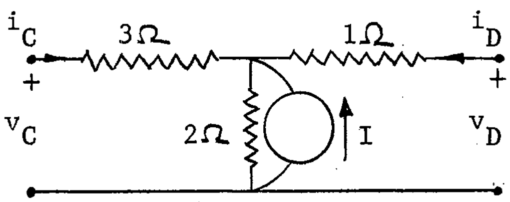


Figure 4.4.1. The Electrical Network for Example 4.4.1.

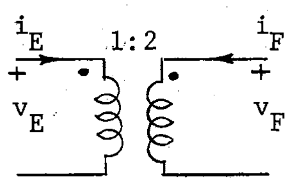
The following will be represented as multiport subnetworks:



$$\begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} + \begin{bmatrix} e \\ 0 \end{bmatrix} .$$



$$\begin{bmatrix} v_C \\ v_D \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} i_C \\ i_D \end{bmatrix} + \begin{bmatrix} 2I \\ 2I \end{bmatrix} .$$



$$\begin{bmatrix} v_E \\ i_F \end{bmatrix} = \begin{bmatrix} 0 & 1/2 \\ -1/2 & 0 \end{bmatrix} \begin{bmatrix} i_E \\ v_F \end{bmatrix}$$

Using these multiport subnetworks the directed graph corresponding to Figure 4.4.1 is shown in Figure 4.4.2.

In Figure 4.4.2, the formulation tree is shown in heavy lines and the relationship between edge voltages and the voltages shown in the multiport representations is

$$v_1 = v_e,$$

$v_2 = v_F$,

$v_6 = v_A$,

$v_7 = v_B$,

$v_8 = v_C$, and

$v_9 = v_D$.

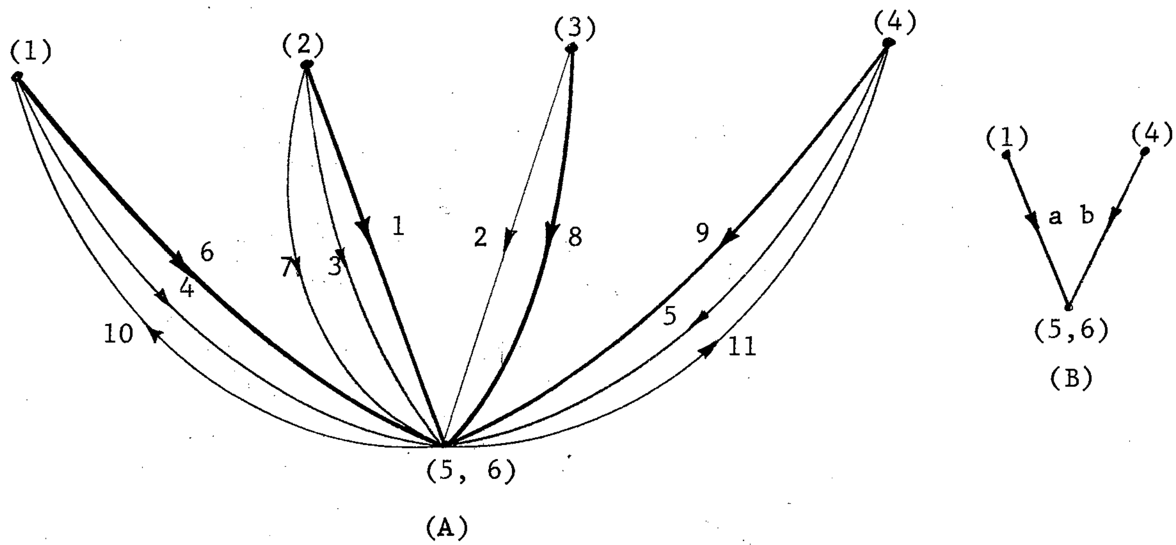


Figure 4.4.2. (A) The Directed Graph Corresponding to Figure 4.4.1.
(B) The Terminal Graph.

The input data is

KO = 7, MTRE = 4, NT = 0, NE = 0, NC = 3, NI = 0, NIE = 2,

MOD = 11, NCH = 1, NCM = 1, NTH = 1, NTM = 3

$$\mathrm{KONN} = \begin{bmatrix} 10 & 4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 4 & 6 & 7 & 3 & 1 & 2 & 8 & 9 & 5 & 11 \end{bmatrix},$$

$$\mathrm{INTO} = \begin{bmatrix} 2 & 5 \\ 3 & 5 \\ 2 & 5 \\ 1 & 5 \\ 4 & 5 \\ 1 & 5 \\ 2 & 5 \\ 3 & 5 \\ 4 & 5 \\ 5 & 1 \\ 5 & 4 \end{bmatrix}, \qquad \mathrm{NTRE} = \begin{bmatrix} 1 & 6 & 8 & 9 \end{bmatrix}, \quad \mathrm{KORD} = \begin{bmatrix} 2 & 3 & 4 & 5 & 7 & 10 & 11 \end{bmatrix},$$

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \underline{N}_{12} = \begin{bmatrix} 1/2 \end{bmatrix}, \ \underline{N}_{21} = \begin{bmatrix} -1/2 \end{bmatrix},$$

$$\underset{\sim}{R}_M = \begin{bmatrix} 4 & 0 & 0 & 2 \\ 0 & 5 & 2 & 0 \\ 0 & 2 & 3 & 0 \\ 2 & 0 & 0 & 3 \end{bmatrix} .$$

The computer results are shown in Table 4.4.1. These results may be summarized in the set of equations

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} .74074 & .018518 \\ .018518 & .71296 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

$$+ \begin{bmatrix} .25925 & .037037 & -.018518 & -.14814 \\ -.018518 & -.074074 & .28703 & .046296 \end{bmatrix} \begin{bmatrix} e \\ 2I \\ 2I \\ 0 \end{bmatrix} . \qquad (4.4.1)$$

The results obtained manually by another method are

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 0.74 & 0.0185 \\ 0.0185 & 0.713 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

$$+ \begin{bmatrix} 0.259 & 0.037 & -0.0185 & -0.148 \\ -0.0185 & -0.0742 & 0.287 & 0.0463 \end{bmatrix} \begin{bmatrix} e \\ 2I \\ 2I \\ 0 \end{bmatrix} . \qquad (4.4.2)$$

The reference directions for the voltages and currents used in equation 4.4.1 and 4.4.2 are shown in Figure 4.4.2(B).

<u>Example 4.4.2</u>

The network shown in Figure 3.4.1 will be considered. The input data is the same as shown in Chapter III with the additional data

NCH = 0, NCM = 0, NTH = 0, NCH = 0.

The computer results are shown in Table 4.4.2. These results may be summarized in the set of equations

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} .42105 & .052631 \\ .052631 & .63157 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} .15789 \\ -.10526 \end{bmatrix} e \quad . \qquad (4.4.3)$$

The reference directions for the voltages and currents used in equation 4.4.3 are shown in Figure 3.4.1(C).

Example 4.4.3

The network shown in Figure 3.4.2 will be considered. The input data is the same as shown in Chapter III with the additional data

NCH = 0, NCM = 0, NTH = 0, NCH = 0.

The computer results are shown in Table 4.4.3. These results may be summarized in the set of equations

$$v_a = 18i_a + 18\ I + .59999E. \qquad (4.4.4)$$

The reference directions for the voltages and currents used in equation 4.4.4 are shown in Figure 3.4.2(C).

It is noted that the computer results for Example 4.4.1 compare well with the results obtained by manual means. It is also noted that the results for Example 4.4.2 and 4.4.3 are the same as those obtained by the program described in Chapter III. Hence it is demonstrated that the program of Chapter III is contained in the program for use on the IBM 1410 computer.

4.5 Application of Program to Transistor Problem. The computer program that has been devised may be applied to problems involving

TABLE 4.4.1

COMPUTER OUTPUT FOR EXAMPLE 4.4.1

```
     MON$$      EXEQ PART1,MJB

    B MATRIX

  0  0 -1  0
 -1  0  0  0
  0 -1  0  0
  0  0  0 -1
 -1  0  0  0
  0  1  0  0
  0  0  0  1

    INTERCONNECTION MATRIX

 10  4  6  0  0  0  0  0  0  0  0
  7  3  1  0  0  0  0  0  0  0  0
  2  8  0  0  0  0  0  0  0  0  0
  9  5 11  0  0  0  0  0  0  0  0
 10  4  6  7  3  1  2  8  9  5 11

    ORIENTATION MATRIX

  2  5
  3  5
  2  5
  1  5
  4  5
  1  5
  2  5
  3  5
  4  5
  5  1
  5  4

    BRANCH MATRIX

  1  6  8  9
     MON$$      EXEQ PART2,MJB

    R MATRIX

   .1000E 01   .0000E-99   .0000E-99
   .0000E-99   .1000E 01   .0000E-99
   .0000E-99   .0000E-99   .1000E 01

    H12 MATRIX

   .5000E 00

    H21 MATRIX

  -.5000E 00

    RM MATRIX

   .4000E 01   .0000E-99   .0000E-99   .2000E 01
   .0000E-99   .5000E 01   .2000E 01   .0000E-99
   .0000E-99   .2000E 01   .3000E 01   .0000E-99
   .2000E 01   .0000E-99   .0000E-99   .3000E 01
     MON$$      EXEQ PART3,MJB
     MON$$      EXEQ PART4,MJB
     MON$$      EXEQ PART5,MJB
     MON$$      EXEQ PART6,MJB
     MON$$      EXEQ PART7,MJB
     MON$$      EXEQ PART8,MJB
     MON$$      EXEQ PART9,MJB

         COEF MATRIX OF PORT CURRENTS

     .74074E 00   .18518E-01
     .18518E-01   .71296E 00

         COEF MATRIX OF VOLTAGE DRIVERS

     .25925E 00   .37037E-01  -.18518E-01  -.14814E 00
    -.18518E-01  -.74074E-01   .28703E 00   .46296E-01
```

TABLE 4.4.2

COMPUTER OUTPUT FOR EXAMPLE 4.4.2

```
        MON$$       EXEQ PART1,MJB

      B MATRIX

   0   1   1   0
  -1   1   0  -1
   1  -1   0   1
   0  -1  -1   0

      INTERCONNECTION MATRIX

   1   5   7   0   0
   6   3   2   0   0
   3   4   8   0   0
   7   5   2   4   8
   1   6   0   0   0

      ORIENTATION MATRIX

   1   5
   4   2
   2   3
   3   4
   1   4
   5   2
   4   1
   4   3

      BRANCH MATRIX

   1   2   3   6
        MON$$       EXEQ PART2,MJB

      R MATRIX

    .2000E 01    .0000E-99    .0000E-99    .0000E-99    .0000E-99
    .0000E-99    .1000E 01    .0000E-99    .0000E-99    .0000E-99
    .0000E-99    .0000E-99    .1000E 01    .0000E-99    .0000E-99
    .0000E-99    .0000E-99    .0000E-99    .1000E 01    .0000E-99
    .0000E-99    .0000E-99    .0000E-99    .0000E-99    .5000E 00
        MON$$       EXEQ PART3,MJB
        MON$$       EXEQ PART4,MJB
        MON$$       EXEQ PART5,MJB
        MON$$       EXEQ PART6,MJB
        MON$$       EXEQ PART7,MJB
        MON$$       EXEQ PART8,MJB
        MON$$       EXEQ PART9,MJB

              COEF MATRIX OF PORT CURRENTS

        .42105E 00   .52631E-01
        .52631E-01   .63157E 00

              COEF MATRIX OF VOLTAGE DRIVERS

        .15789E 00
       -.10526E 00
```

TABLE 4.4.3

COMPUTER OUTPUT FOR EXAMPLE 4.4.3

```
        MON$$       EXEQ PART1,MJB

      B MATRIX

  -1  1
   1  0
   1  0

      INTERCONNECTION MATRIX

   5  1  2  4
   1  3  0  0
   5  3  2  4

      ORIENTATION MATRIX

   1  2
   1  3
   2  3
   3  1
   3  1

      BRANCH MATRIX

   2  3
        MON$$       EXEQ PART2,MJB

      R MATRIX

    .4500E 02    .0000E-99
    .0000E-99    .3000E 02
        MON$$       EXEQ PART3,MJB
        MON$$       EXEQ PART4,MJB
        MON$$       EXEQ PART5,MJB
        MON$$       EXEQ PART6,MJB
        MON$$       EXEQ PART7,MJB
        MON$$       EXEQ PART8,MJB
        MON$$       EXEQ PART9,MJB

              COEF MATRIX OF PORT CURRENTS

        .18000E 02

              COEF MATRIX OF CURRENT DRIVERS

        .18000E 02

              COEF MATRIX OF VOLTAGE DRIVERS

        .59999E 00
```

transistors by properly representing the transistor. Using the common hybrid model to represent a transistor, the set of equations relating voltages and currents is (6)

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} . \qquad (4.5.1)$$

Now equation 4.5.1 may be written as

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & 0 \\ 0 & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 & h_{12} \\ h_{21} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} , \qquad (4.5.2)$$

and it may be seen that the second term on the right side of equation 4.5.2 has the same form as the ideal transformer coefficients. The first term on the right hand side is a matrix containing a resistance, h_{11}, and a conductance h_{22}. This suggests a representation as shown in Figure 4.5.1(b).

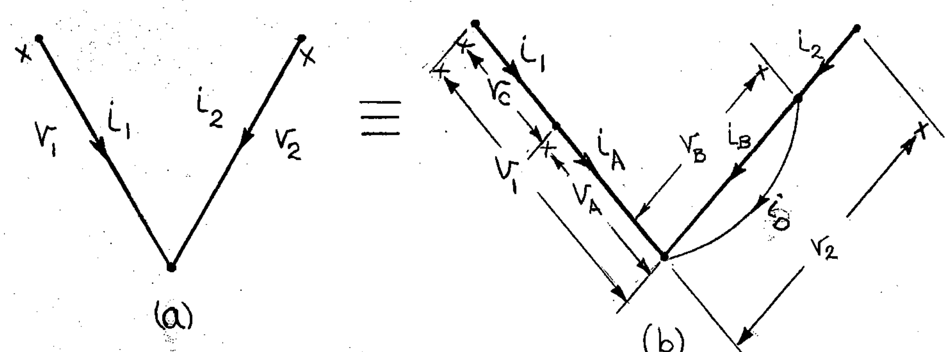


Figure 4.5.1. Equivalent Directed Graph for the Representation of a Transistor.

The volt-ampere equations are

$$v_C = h_{11} i_1,$$

$$v_A = h_{12} v_B,$$

$$i_D = h_{22} v_2,$$

$$i_B = h_{21} i_A.$$

Now

$$v_1 = v_C + v_A,$$

$$i_2 = i_B + i_D,$$

$$v_B = v_2, \text{ and}$$

$$i_1 = i_A.$$

so

$$v_1 = h_{11} i_1 + h_{12} v_2,$$

$$i_2 = h_{21} i_1 + h_{22} v_2,$$

or

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} . \tag{4.5.3}$$

Thus the representation of Figure 4.5.1(b) is equivalent to that of Figure 4.5.1(a). With this representation of a transistor, the multiport representation of a transistor amplifier circuit in the mid-frequency range may be obtained using the program described in Section 4.4. The technique for achieving this is shown in Example 4.5.1.

Example 4.5.1

The transistor amplifier shown in Figure 4.5.2 will be considered.

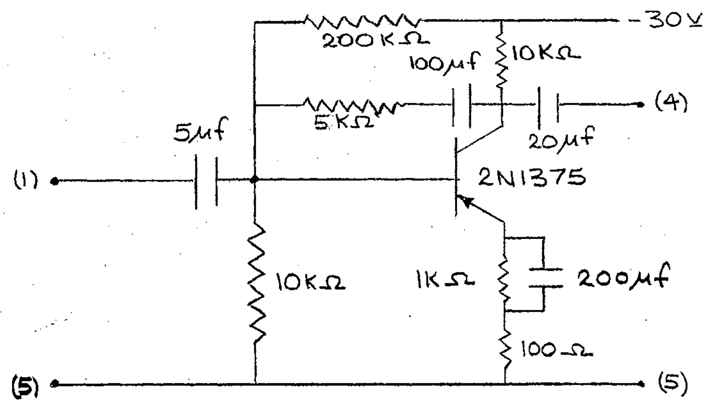


Figure 4.5.2. The Transistor Amplifier for Example 4.5.1.

The transistor characteristics are

$$\begin{bmatrix} v_{in} \\ i_{out} \end{bmatrix} = \begin{bmatrix} 2000 & 6 \times 10^{-4} \\ 70 & 4 \times 10^{-5} \end{bmatrix} \begin{bmatrix} i_{in} \\ v_{out} \end{bmatrix} .$$

The directed graph for the mid-frequency range case is shown in Figure 4.5.3.

The volt-ampere equations for the edges of Figure 4.5.3(A) are

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \times 10^{-4} \\ 70 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} ,$$

$$v_3 = 2000\ i_3,$$

$$v_4 = 5000\ i_4,$$

$$v_5 = 25{,}000\ i_5,$$

$v_6 = 10{,}000\ i_6,$

$v_7 = 100\ i_7,$

$v_8 = 200{,}000\ i_8$, and

$v_9 = 10{,}000\ i_9.$

Edges 1, 2, 3 and 5 represent the transistor and edges 10 and 11 represent the conceptual current sources.

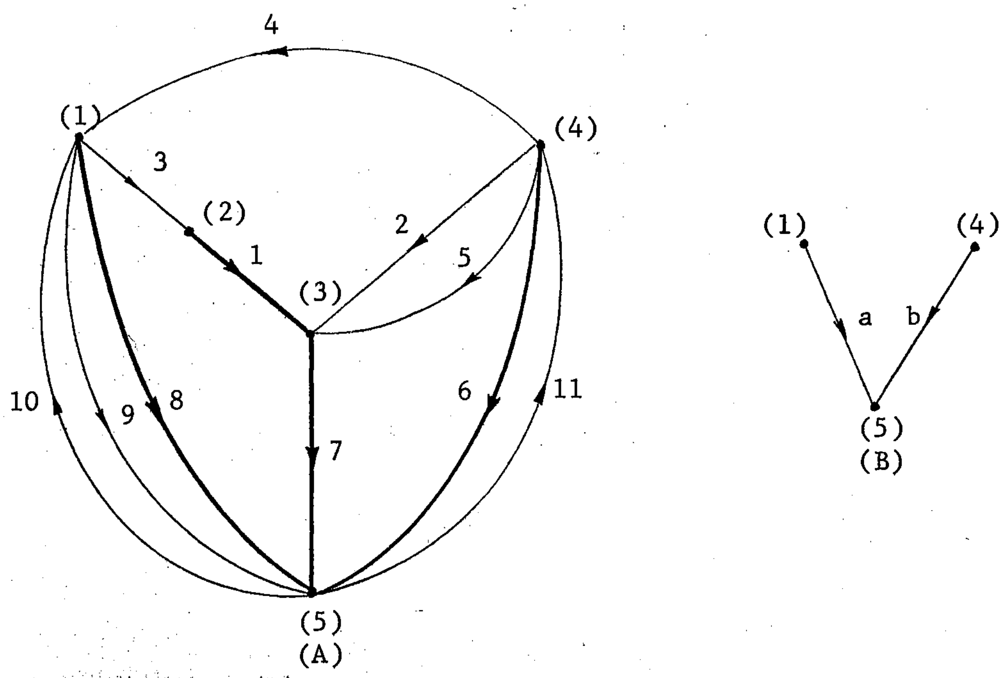


Figure 4.5.3. (A) Directed Graph for Mid-frequency Range of Amplifier of Figure 4.5.2.
(B) The Terminal Graph.

The input data is as follows

KO = 7, MTRE = 4, NT = 3, NE = 0, NC = 4, NI = 0, NIE = 2, MOD = 6,

NCH = 1, NCM = 0, NTH = 1, NTM = 0.

$$\mathrm{KONN} = \begin{bmatrix} 10 & 9 & 8 & 3 & 4 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 5 & 7 & 0 & 0 \\ 4 & 2 & 5 & 6 & 11 & 0 \\ 10 & 9 & 8 & 7 & 6 & 11 \end{bmatrix} .$$

$$\mathrm{INTO} = \begin{bmatrix} 2 & 3 \\ 4 & 3 \\ 1 & 2 \\ 4 & 1 \\ 4 & 3 \\ 4 & 5 \\ 3 & 5 \\ 1 & 5 \\ 1 & 5 \\ 5 & 1 \\ 5 & 4 \end{bmatrix} .$$

$$\mathrm{NTRE} = \begin{bmatrix} 1 & 6 & 7 & 8 \end{bmatrix} .$$

$$\mathrm{KORD} = \begin{bmatrix} 2 & 3 & 4 & 5 & 9 & 10 & 11 \end{bmatrix} .$$

$$\underline{N}_{12} = \begin{bmatrix} 6 \times 10^{-4} \end{bmatrix} .$$

$$\underline{N}_{21} = \begin{bmatrix} 70 \end{bmatrix} .$$

$$\underline{R} = \begin{bmatrix} 10 \times 10^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 200 \times 10^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \times 10^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \times 10^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 25 \times 10^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 \times 10^3 \end{bmatrix}.$$

The computer results are shown in Table 4.5.1. These results may be summarized in the set of equations

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 187.20 & 123.15 \\ -4599.8 & 255.08 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}. \tag{4.5.4}$$

The results obtained manually by another method are

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 189 & 124 \\ -4560 & 257 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}. \tag{4.5.5}$$

The reference directions for the voltages and currents used in equations 4.5.4 and 4.5.5 are shown in Figure 4.5.3(B).

These results demonstrate the applicability of the program to a transistor amplifier in the mid-frequency range. This ability to accommodate active devices greatly increases the versatility of the program.

TABLE 4.5.1

COMPUTER OUTPUT FOR EXAMPLE 4.5.1

```
     MON$$       EXEQ PART1,MJB

  B MATRIX

 0 -1  1  0
 1  0  1 -1
 0 -1  0  1
 0 -1  1  0
 0  0  0 -1
 0  0  0  1
 0  1  0  0

  INTERCONNECTION MATRIX

10  9  8  3  4  0
 3  1  0  0  0  0
 1  2  5  7  0  0
 4  2  5  6 11  0
10  9  8  7  6 11

  ORIENTATION MATRIX

 2  3
 4  3
 1  2
 4  1
 4  3
 4  5
 3  5
 1  5
 1  5
 5  1
 5  4

  BRANCH MATRIX

 1  6  7  8
     MON$$       EXEQ PART2,MJB

  R MATRIX

  .1000E 05   .0000E-99   .0000E-99   .0000E-99   .0000E-99   .0000E-99
  .0000E-99
  .0000E-99   .1000E 03   .0000E-99   .0000E-99   .0000E-99   .0000E-99
  .0000E-99
  .0000E-99   .0000E-99   .2000E 06   .0000E-99   .0000E-99   .0000E-99
  .0000E-99
  .0000E-99   .0000E-99   .0000E-99   .2000E 04   .0000E-99   .0000E-99
  .0000E-99
  .0000E-99   .0000E-99   .0000E-99   .0000E-99   .5000E 04   .0000E-99
  .0000E-99
  .0000E-99   .0000E-99   .0000E-99   .0000E-99   .0000E-99   .2500E 05
  .0000E-99
  .0000E-99   .0000E-99   .0000E-99   .0000E-99   .0000E-99   .0000E-99
  .1000E 05

  H12 MATRIX

  .6000E-03

  H21 MATRIX

  .7000E 02
     MON$$       EXEQ PART3,MJB
     MON$$       EXEQ PART4,MJB
     MON$$       EXEQ PART5,MJB
     MON$$       EXEQ PART6,MJB
     MON$$       EXEQ PART7,MJB
     MON$$       EXEQ PART8,MJB
     MON$$       EXEQ PART9,MJB

        COEF MATRIX OF PORT CURRENTS

     .18720E 03   .12315E 03
    -.45998E 04   .25508E 03
```

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Summary. A motivating force in this investigation has been the need for a mechanized method for determining the multiport representation of an electrical network. This is needed in order to better take advantage of existing formulation techniques involving the interconnection of multiterminal components. This investigation has led to the development of two digital computer programs for calculating the n-port representation of electrical networks of arbitrary configuration. One of these programs is for use on the IBM 1620 computer and is limited to two-terminal devices which may be

(a) resistances,

(b) ideal current sources, or

(c) ideal voltage sources.

The maximum number of each variable which the program, as it is now written, will accommodate is included in Appendix A. A part of this list is restated here. The program will accommodate a maximum of four ports, three ideal voltage sources and three ideal current sources, five resistances in the tree and five in the co-tree, and a total of fifteen edges.

The second program is written for the IBM 1410 computer and will accommodate ideal transformers, multiport subnetworks and two-terminal devices. The limitations on the two-terminal devices are the same as for the first program and the multiport subnetworks must consist of these same types of two-terminal devices. The program for the IBM 1410 computer will accommodate transistors which are represented by h-parameters. Appendix C includes the maximum value that each of the variables may have for the program as it is now written. A part of this list is restated here. The program will accommodate four ports, three ideal voltage sources and three ideal current sources, six resistances in the tree and six in the co-tree, two ideal transformers or two transistors, and three edges for multiport components in the tree and three in the co-tree and a total of twenty edges.

In order that the networks considered may be of arbitrary configuration, an algorithm for finding the $\underline{B}$ matrix which can be programmed for execution by a digital computer was developed. This algorithm is described in Chapter II.

The technique for obtaining the volt-ampere equations at the ports of the network is not new. It involves using the $\underline{B}$ matrix and the parameters of the network along with conceptual current sources at the ports. The topological limitations are that it must be possible to place all of the ideal voltage sources in the formulation tree, all of the ideal current sources in the co-tree, and one edge representing the ideal transformer in the formulation tree while the other edge is in the co-tree. The application of the digital computer to the solution of these equations requires a particular ordering for the $\underline{B}$ matrix and for the network parameters. The method for obtaining this ordering is included in Chapters III and IV.

A number of examples that have been worked using the two programs are included in this study to demonstrate how the input data is obtained from the network. These examples also serve to illustrate that the programs do achieve the correct results.

5.2 Suggestions for Further Investigation and Program Improvements. It is apparent that the size of the networks which may be accommodated by the programs described in this study is limited. This may be improved to a certain extent by dividing the programs into more parts. For example, it may be feasible to make the program of Chapter III into four or more parts instead of three and as a result increase the size of the network which may be accommodated. It may also be feasible to divide the program of Chapter IV into ten or more parts instead of nine. Such a procedure could increase the size of the networks which could be handled but there would be a limit to this size. However, it does appear that an investigation to determine the optimum number of parts for each of the computer programs would be useful.

The possibility of applying the programs to handle a network of any size by dividing the network into parts is one which could be the subject of further investigation. It is feasible to divide the network into parts, obtain the multiport representation of each part and then combine the multiport representations to obtain the multiport representation of the entire network. A useful investigation would be to consider the possibility of determining the optimum method for dividing the network. A desirable feature would be to write the program so that the determination of the multiport representations of the various parts of the network and the combination of these into the multiport representation of the entire network would be accomplished by the computer.

An area of investigation which could lead to added versatility of the programs is to increase their scope so that inductors and capacitors may be included in the networks. The techniques developed for the programs described in this study should be suitable for sinusoidal steady-state analysis, while considerable additional research would be necessary to adapt the computer to state-variable formulation or the determination of the s-domain representation.

A number of changes that may make the programs more convenient to use have been suggested through application of the programs. These changes are

(a) allow the $\underline{T}$ matrix elements to be placed in random order and include instructions in the programs which would cause the elements of the matrix to be placed in proper order,

(b) include instructions so that the $\underline{C}$ matrix would be generated from $\underline{T}$ and the total number of edges in the graph,

(c) read the diagonal matrices $\underline{R}_T$ and $\underline{R}_C$ into the computer as column matrices and arrange the multiplication and addition instructions so that they are considered as square matrices,

(d) arrange the program so that the parameters of the two-terminal passive elements may be supplied as either resistances or conductances, and

(e) include instructions so that the g- and h-parameters may be obtained if desired.

5.3 Conclusions. This investigation has demonstrated that the digital computer can be applied to the problem of obtaining the n-port representation of electrical networks whose elements are

(a) resistances,

(b) ideal current sources,

(c) ideal voltage sources,

(d) multiport subnetworks consisting of two-terminal devices listed under (a), (b), and (c),

(e) ideal transformers, or

(f) transistors.

Although the programs produced as a result of this investigation are limited in the class of devices which may be considered, they may be applied to networks of considerable complexity. It has been shown how a transistor, which is described by means of h-parameters, may be represented by a coupling similar in form to an ideal transformer with a series resistor and a parallel resistor. This increases the class of networks which may be accommodated.

Based on experience gained in using these programs, it is concluded that considerable use can be made in the determination of equivalent resistance of networks that are quite complex, the analysis of multiport network problems and the investigation of the variation of the parameters in the multiport representation as the elements in the network are varied.

It is also concluded that the availability of a mechanized method of obtaining the multiport representation of a network can reduce the effort required in determining the multiport representation of a network of great complexity, even though the repetitive combination of multiport subnetworks would be manual at this time.

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APPENDIX A

APPENDIX A

THE COMPUTER PROGRAM FOR CHARACTERIZATION OF n-PORT NETWORK OF TWO-TERMINAL DEVICES

PART I

This program is written in FORTRAN without format for execution by an IBM 1620 computer. The entire program is written using fixed point variables and constants. It is the first part of a three part program.

The input data is read into the machine as shown in Table A-1.

TABLE A-1

ORDER OF INPUT DATA FOR PART I OF THE PROGRAM

Card Group	Variable Name and Order on Card
1	KO, MTRE, NT, NE, NC, NI, NIE, MOD
2	KONN (one row per card)
3	INTO (one row per card)
4	NTRE
5	KORD

The input data consists of punched cards which contain the B matrix. Each card will contain one element from B, the first being the (1, 1) element, the second being the (1, 2) element, etc. Through the (KO, MTRE) element.

The variable names are shown in Table A-3.

The program contains two diagnostics. These are as follows:

1. If MTRE $\neq$ NT + NE the typewriter will type 99 and control will be transferred to statement 1 which is the first READ statement. If MTRE = NT + NE the program execution is performed normally.
2. If KO $\neq$ NC + NI + NIE the typewriter will type 999 and control will be transferred to statement 1. If MTRE = NC + NI + NIE the program execution is performed normally.

When the output data (LOOP) has been punched into cards control is transferred to statement 1.

PARTS II AND III

These parts of the program are written in FORTRAN without format for execution by an IBM 1620 computer. Both fixed point and floating point variables are used in this program.

The input data is read into the machine as shown in Table A-2.

The output data for Part II consists of both typed and punched data. The typed data is the coefficient matrix of the port currents. It appears in a single column. The first element is the (1, 1) element, the next is the (1, 2) element, etc. through the (1, NIE) element then the number 909 is typed. The next element is (2, 1) etc. until all of the elements have been typed. The number 909 is typed after a complete row of elements have been typed. The punched data is used as input data for Part III.

TABLE A-2

ORDER OF INPUT DATA FOR PARTS II AND III OF THE PROGRAM

Card Group	Variable Name and Order on Card
Part II	
1	KO, MTRE, NT, NE, NC, NI, NIE, MOD
2	LOOP (output from Part I)
3	RT (one row per card)
4	RC (one row per card)
Part III	
1	KO, MTRE, NT, NE, NC, NI, NIE, MOD
2	LOOP (output from Part I)
3	RT (one row per card)
4	R1 (output from Part II)
5	U (output from Part II)
6	RC (one row per card)

Note: If there are no resistances in the branches, card group 3 is omitted in Parts II and III. If there are no resistances in the chords, card group 4 in Part II and group 6 are omitted. If R1 or U or both are not punched out of Part II, they are omitted as input data for Part III.

The output data for Part III is typed. It consists of the coefficient matrices of the ideal current sources in the network and of the ideal voltage sources in the network. The coefficient matrix for the ideal current sources appears in a single column. The number 11 is typed to indicate that a complete row has been typed. If there are no ideal current sources in the network, the number 111 is typed. The coefficient matrix for the ideal voltage sources appears in a single column. The number 33 is typed to indicate that a complete row has been typed. If there are no ideal voltage sources in the network, the number 333 is typed.

The variable names are shown in Table A-3.

In Part II and Part III of the program, when typing and punching are completed, control is transferred to statement 1, the first READ statement.

The maximum number of each variable which the program, as it is now written, will accommodate is as follows:

KO = 8.
MTRE = 7.
MOD = 8.
NC = 5.
NT = 5.
NI = 3.
NE = 3.
NIE = 4.

TABLE A-3

VARIABLES USED IN PROGRAM

KO = the number of chords.

MTRE = the number of branches.

NT = the number of branches containing resistances.

NE = the number of ideal voltage sources.

NC = the number of chords containing resistances.

NI = the number of ideal current sources.

NIE = the number of ports.

MOD = the degree of the node of maximum degree.

KONN (I,J) = the interconnection matrix, $\underline{K}$.

INTO (I, 2) = the orientation matrix $\underline{D}$.

NTRE (I) = the branch matrix, $\underline{T}$.

KORD (I) = the chord matrix, $\underline{C}$.

KONNM(I,J) = a modified $\underline{K}$ matrix, $\underline{K}_2$.

NBR(1) = the sum of the elements in the ith row of $\underline{K}_2$.

NOT(I) = the branches which are not in the circuit with the ith chord.

MESH(I) = the branches forming a circuit with the ith chord.

LOOP(I,J) = the $\underline{B}$ matrix.

RT(I,J) = the $\underline{R}_T$ matrix.

RC(I,J) = the $\underline{R}_C$ matrix.

R1 (I,J) = $\underline{B}_{31}\underline{R}_T\,\underline{B}_{11}^T$.

R2 (I,J) = $\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T$.

U (I,J) = $\left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{R}_{11}^T\right]^{-1}$.

R3 (I,J) = $\underline{B}_{31}\,\underline{R}_T\,\underline{B}_{31}^T$.

R4 (I,J) = $\underline{B}_{31}\,\underline{R}_T\,\underline{B}_{11}^T\left[\underline{R}_C + \underline{B}_{11}\,\underline{R}_T\,\underline{B}_{11}^T\right]^{-1}\underline{B}_{11}\,\underline{R}_T\,\underline{B}^T_{31}$.

R5 (I,J) = $\underline{B}_{11}\ \underline{R}_T\ \underline{B}_{21}^T$.

R6 (I,J) = $\underline{B}_{31}\ \underline{R}_T\ \underline{B}_{21}$.

R7 (I,J) = $\underline{B}_{31}\ \underline{R}_T\ \underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\ \underline{R}_T\ \underline{B}_{11}^T\right]^{-1} \underline{B}_{11}\ \underline{R}_T\ \underline{B}_{21}^T - \underline{B}_{31}\ \underline{R}_T\ \underline{B}_{21}^T$.

R8 (I,J) = $\underline{B}_{31}\ \underline{R}_T\ \underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\ \underline{R}_T\ \underline{B}_{11}^T\right]^{-1} \underline{B}_{12}$.

R9 (I,J) = $\underline{B}_{31}\ \underline{R}_T\ \underline{B}_{11}^T \left[\underline{R}_C + \underline{B}_{11}\ \underline{R}_T\ \underline{B}_{11}^T\right]^{-1} \underline{B}_{12} - \underline{B}_{32}$.

COFF = coefficients of $\underline{I}_{CIP}$.

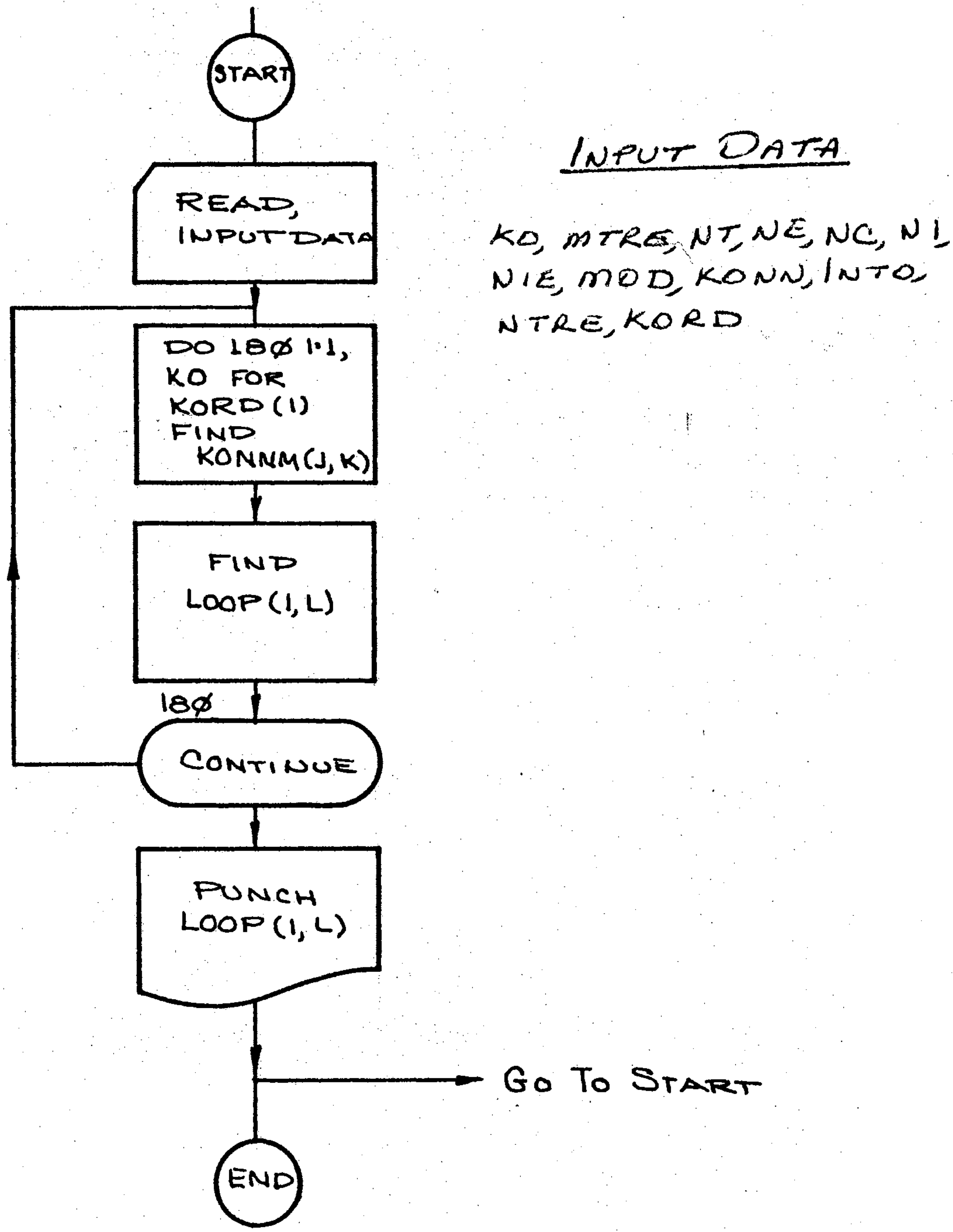

Figure A-1. Flow Chart for Part I of the Program of Chapter III.

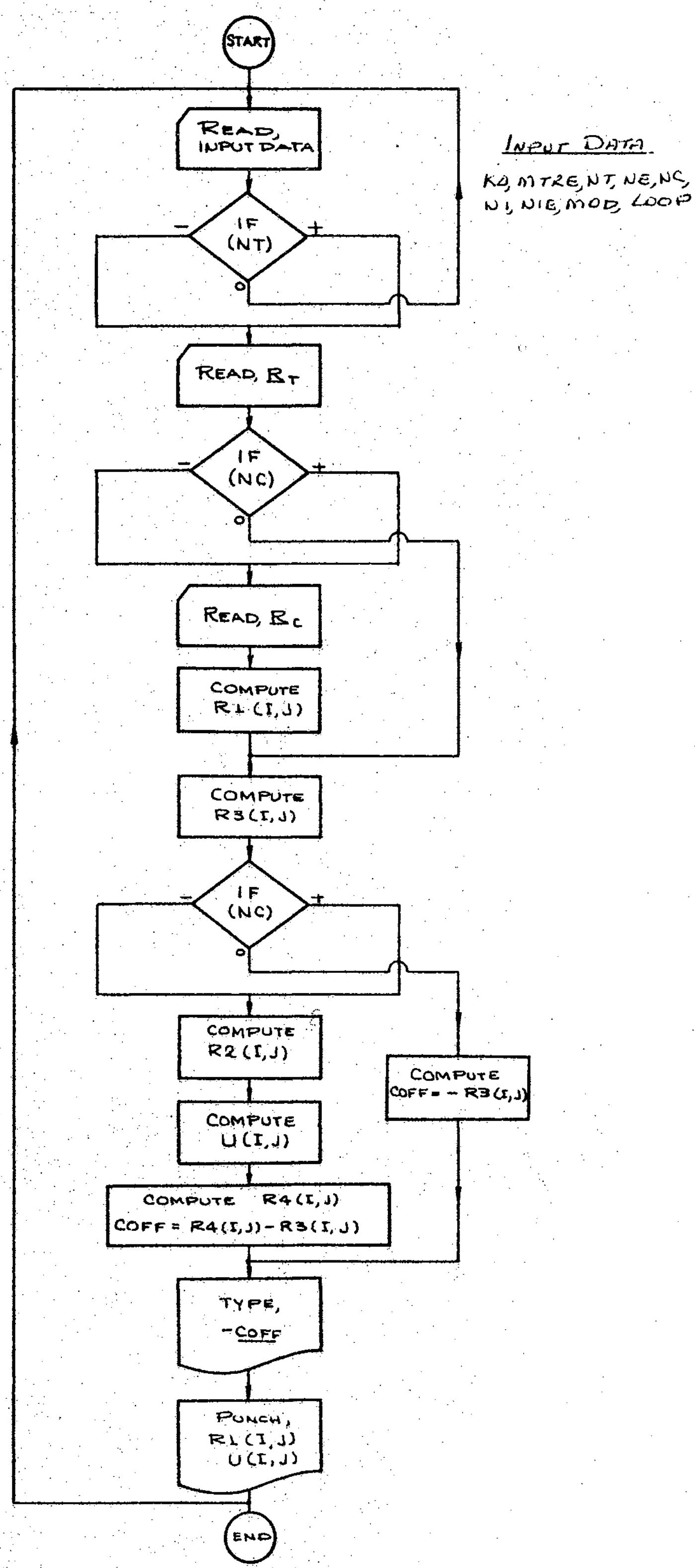

Figure A-2. Flow Chart for Part II of the Program of Chapter III.

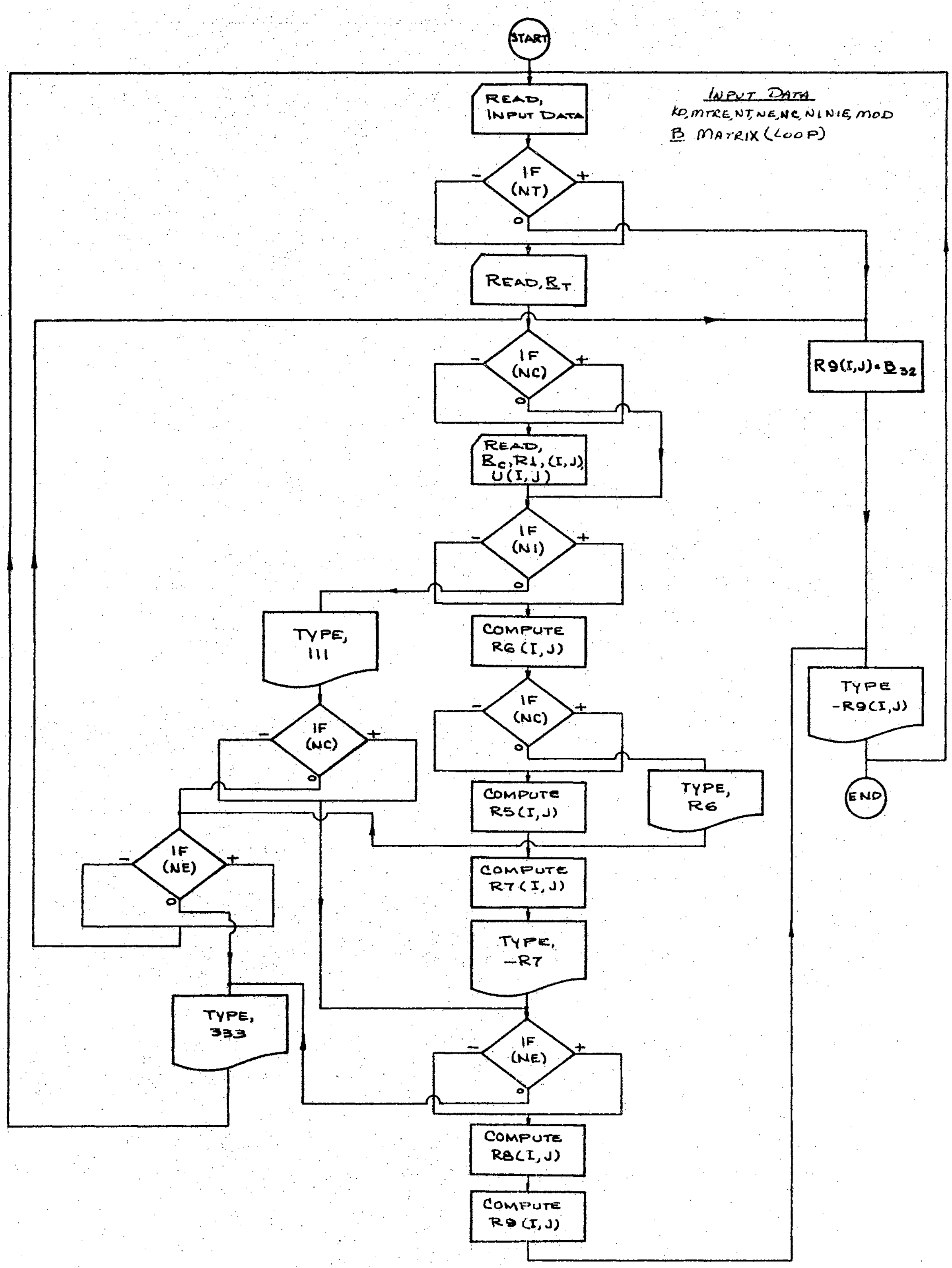

Figure A-3. Flow Chart for Part III of the Program of Chapter III.

TABLE A-4

FORTRAN STATEMENTS FOR IBM 1620 PROGRAM

```
C     COMP CHAR OF N-PORT NETWORK PART ONE
      DIMENSION KONN(8,8),INTO(15,2),NTRE(7),KORD(8),NBR(8),NOT(8)
      DIMENSION KONNM(8,8),MESH(8),LOOP(8,8)
1     READ,KO,MTRE,NT,NE,NC,NI,NIE,MOD
      NV=MTRE+1
      DO2I=1,NV
      DO2J=1,MOD
2     READ,KONN(I,J)
      NX=KO+MTRE
      DO3I=1,NX
      DO3J=1,2
3     READ,INTO(I,J)
      DO4I=1,MTRE
4     READ,NTRE(I)
      DO5I=1,KO
5     READ,KORD(I)
      IF(MTRE-(NT+NE))6,8,6
6     NZ=99
      TYPE,NZ
      GOTO1
8     IF(KO-(NC+NI+NIE))9,10,9
9     NZ=999
      TYPE,NZ
      GOTO1
10    DO35I=1,KO
      DO35J=1,MTRE
35    LOOP(I,J)=0
      DO180I=1,KO
      DO36J=1,NV
      DO36K=1,MOD
36    KONNM(J,K)=0
      DO37J=1,NV
37    MESH(J)=0
      DO45J=1,NV
      DO45K=1,MOD
      IF(KORD(I)-KONN(J,K))45,40,45
40    KONNM(J,K)=KONN(J,K)
45    CONTINUE
      DO55J=1,NV
      DO55K=1,MOD
      DO55L=1,NV
      IF(NTRE(L)-KONN(J,K))55,50,55
50    KONNM(J,K)=KONN(J,K)
55    CONTINUE
      DO80LL=1,MTRE
      DO60J=1,NV
      NOT(J)=0
      NBR(J)=0
      DO60K=1,MOD
60    NBR(J)=NBR(J)+KONNM(J,K)
      LA=1
      DO70J=1,NV
      DO70K=1,MOD
      IF(KONNM(J,K))61,70,61
61    IF(NBR(J)-KONNM(J,K))70,65,70
65    NOT(LA)=KONNM(J,K)
      NA=LA
       LA=LA+1
70    CONTINUE
      DO80J=1,NV
      DO80K=1,MOD
      DO80LB=1,NA
      IF(KONN(J,K))71,80,71
71    IF(KONN(J,K)-NOT(LB))80,75,80
75    KONNM(J,K)=0
80    CONTINUE
82    MESH(1)=KORD(I)
      LM=KORD(I)
      LN=INTO(LM,2)
      LP=2
104   DO115J=1,MOD
      IF(KONNM(LN,J)-LM)105,115,105
105   IF(KONNM(LN,J))110,115,110
```

TABLE A-4 (Continued)

```
110   LQ=KONNM(LN,J)
115   CONTINUE
      IF(KORD(I)-LQ)120,135,120
120   MESH(LP)=LQ
      DO130K=1,2
      IF(INTO(LQ,K)-LN)125,130,125
125   LR=INTO(LQ,K)
130   CONTINUE
      LM=LQ
      LN=LR
      LP=LP+1
      GOTO104
135   DO180J=1,NV
      IF(MESH(J))140,180,140
140   KD=MESH(J)
      IF(J-1)145,145,150
145   KE=INTO(KD,2)
      GOTO180
150   DO165M=1,MTRE
      IF(NTRE(M)-KD)165,160,165
160   KG=M
165   CONTINUE
      IF(INTO(KD,1)-KE)170,175,170
170   LOOP(I,KG)=(-1)
      KE=INTO(KD,1)
      GOTO180
175   LOOP(I,KG)=1
      KE=INTO(KD,2)
180   CONTINUE
       DO190I=1,KO
       DO190J=1,MTRE
190   PUNCH,LOOP(I,J)
      GOTO1
      END

C     COMP CHAR OF N-PORT NETWORK PART TWO

      DIMENSION LOOP(8,8),RT(5,5),RC(5,5),R1(4,5),R2(5,5),U(5,5),R3(4,4)
      DIMENSION R4(4,4)
1     READ,KO,MTRE,NT,NE,NC,NI,NIE,MOD
      K1=NC+NI+1
      K2=NC+1
      K3=NC+NI
      NV=MTRE+1
       DO15I=1,KO
       DO15J=1,MTRE
15    READ,LOOP(I,J)
      IF(NT)18,475,18
18    DO20I=1,NT
      DO20J=1,NT
20    READ,RT(I,J)
      IF(NC)22,206,22
22    DO25I=1,NC
      DO25J=1,NC
25    READ,RC(I,J)
      M1=0
      DO205I=K1,KO
      M1=M1+1
      DO205J=1,NC
      R1(M1,J)=0.0
      DO205K=1,NT
      DO205L=1,NT
      X=LOOP(I,L)
      Y=LOOP(J,K)
205   R1(M1,J)=R1(M1,J)+(X*RT(L,K)*Y)
206   M1=0
      DO210I=K1,KO
      M1=M1+1
      M2=0
      DO210J=K1,KO
      M2=M2+1
      R3(M1,M2)=0.0
```

TABLE A-4 (Continued)

```
      DO210K=1,NT
      DO210L=1,NT
      X=LOOP(I,L)
      Y=LOOP(J,K)
210   R3(M1,M2)=R3(M1,M2)+(X*RT(L,K)*Y)
      IF(NC)236,445,236
236   DO240I=1,NC
      DO240J=1,NC
      Z=0.0
      DO240K=1,NT
      DO240L=1,NT
      X=LOOP(I,L)
      Y=LOOP(J,K)
      Z=Z+(X*RT(L,K)*Y)
      IF(K-NT)240,237,240
237   R2(I,J)=RC(I,J)+Z
240   CONTINUE
      DO260I=1,NC
      DO260J=1,NC
      IF(I-J)245,250,245
245   U(I,J)=0.0
      GOTO260
250   U(I,J)=1 0
260   CONTINUE
      M=1
      DO285L=1,NC
      M=M+1
      IF(M-NC)265,265,275
265   DO270I=M,NC
270   R2(L,I)=R2(L,I)/R2(L,L)
275   DO280J=1,NC
280   U(L,J)=U(L,J)/R2(L,L)
      DO285J=1,NC
      IF(L-J)281,285,281
281   IF(M-NC)282,282,284
282   DO283I=M,NC
283   R2(J,I)=R2(J,I)-(R2(L,I)*R2(J,L))
284   DO285IQ=1,NC
      U(J,IQ)=U(J,IQ)-(U(L,IQ)*R2(J,L))
285   CONTINUE
      DO290I=1,NIE
      DO290J=1,NIE
      R4(I,J)=0.0
      DO290K=1,NC
      DO290L=1,NC
290   R4(I,J)=R4(I,J)+(R1(I,L)*U(L,K)*R1(J,K))
      DO310I=1,NIE
      DO310J=1,NIE
      COFF=(-1.)*(R4(I,J)-R3(I,J))
      TYPE,COFF
      IF(J-NIE)310,306,310
306   KZ=909
      TYPE,KZ
310   CONTINUE
      GOTO480
445   DO450L=1,NIE
      DO450I=1,NIE
      TYPE,R3(L,I)
      IF(I-NIE)450,446,450
446   KZ=909
      TYPE,KZ
450   CONTINUE
475   GOTO1
480   DO485I=1,NIE
      DO485J=1,NC
485   PUNCH,R1(I,J)
      DO486I=1,NC
      DO486J=1,NC
486   PUNCH,U(I,J)
      GOTO1
      END
```

TABLE A-4 (Continued)

```
C       COMP CHAR OF N-PORT NETWORK  PART THREE
        DIMENSION LOOP(8,8),RT(5,5),RC(5,5),R1(4,5),U(5,5),R5(5,3),R6(4,3)
        DIMENSION R7(4,5),R8(4,3),R9(4,3)
1       READ,KO,MTRE,NT,NE,NC,NI,NIE,MOD
        K1=NC+NI+1
        K2=NC+1
        K3=NC+NI
        K4=NT+1
        DO15I=1,KO
        DO15J=1,MTRE
15      READ,LOOP(I,J)
        IF(NT)18,616,18
18      DO20I=1,NT
        DO20J=1,NT
20      READ,RT(I,J)
        IF(NC)21,499,21
21      DO25I=1,NIE
        DO25J=1,NC
25      READ,R1(I,J)
        DO30I=1,NC
        DO30J=1,NC
30      READ,U(I,J)
        DO35I=1,NC
        DO35J=1,NC
35      READ,RC(I,J)
499     IF(NI)500,635,500
500     M1=0
        DO505I=K1,KO
        M1=M1+1
        M2=0
        DO505J=K2,K3
        M2=M2+1
        R6(M1,M2)=0.0
        DO505K=1,NT
        DO505L=1,NT
        X=LOOP(I,L)
        Y=LOOP(J,K)
505     R6(M1,M2)=R6(M1,M2)+(X*RT(L,K)*Y)
        IF(NC)510,625,510
510     DO515I=1,NC
        M1=0
        DO515J=K2,K3
        M1=M1+1
        R5(I,M1)=0.0
        DO515K=1,NT
        DO515L=1,NT
        X=LOOP(I,L)
        Y=LOOP(J,K)
515     R5(I,M1)=R5(I,M1)+(X*RT(L,K)*Y)
        DO520I=1,NIE
        DO520J=1,NI
        Z=0.0
        DO520K=1,NC
        DO520L=1,NC
        Z=Z+(R1(I,L)*U(L,K)*R5(K,J))
        IF((L+K)-(NC+NC))520,518,520
518     R7(I,J)=Z-R6(I,J)
520     CONTINUE
        DO525I=1,NIE
        DO525J=1,NI
        R7(I,J)=(-1.)*R7(I,J)
        TYPE,R7(I,J)
        IF(J-NI)525,522,525
522     KZ=11
        TYPE,KZ
525     CONTINUE
526     IF(NE)600,640,600
600     DO605I=1,NIE
        M1=0
        DO605J=K4,MTRE
        M1=M1+1
        R8(I,M1)=0.0
```

TABLE A-4 (Continued)

```
      DO605K=1,NC
      DO605L=1,NC
      X=LOOP(K,J)
605   R8(I,M1)=R8(I,M1)+(R1(I,L)*U(L,K)*X)
      M1=0
      DO610I=K1,KO
      M1=M1+1
      M2=0
      DO610J=K4,MTRE
      M2=M2+1
      X=LOOP(I,J)
610   R9(M1,M2)=(R8(M1,M2)-X)*(-1.)
      DO615I=1,NIE
      DO615J=1,NE
      TYPE,R9(I,J)
      IF(J-NE)610,608,610
608   KZ=33
      TYPE,KZ
615   CONTINUE
      GOTO1
616   DO620I=K1,KO
      DO620J=K4,MTRE
      X=LOOP(I,J)
      TYPE,X
      IF(J-MTRE)620,618,620
618   KZ=33
      TYPE,KZ
620   CONTINUE
      GOTO1
625   DO630I=1,NIE
      DO630J=1,NI
      TYPE,R6(I,J)
      IF(J-NI)630,628,630
628   KZ=11
      TYPE,KZ
630   CONTINUE
631   IF(NE)616,640,616
635   KZ=111
      TYPE,KZ
      IF(NC)526,631,526
640   KZ=333
      TYPE,KZ
      GOTO1
      END
```

APPENDIX B

APPENDIX B

DEVELOPMENT OF THE VOLT-AMPERE EQUATIONS
FOR AN n-PORT NETWORK CONTAINING
MULTIPORT SUBNETWORKS

The voltage at the ports, $\underline{V}_{CIP}$, may be written in terms of the branch voltages by using equation 4.3.1. The result is

$$\underline{V}_{CIP} = - \begin{bmatrix} \underline{B}_{51} & \underline{B}_{52} & \underline{B}_{53} \end{bmatrix} \begin{bmatrix} \underline{V}_{TH} \\ \underline{V}_{TR} \\ \underline{V}_{TM} \end{bmatrix} - \underline{B}_{54}\, \underline{V}_{TE}. \qquad \text{B-1}$$

Using equations 3.3.6, 4.3.3 and 4.3.4 it is possible to write

$$\begin{bmatrix} \underline{V}_{TH} \\ \underline{V}_{TR} \\ \underline{V}_{TM} \end{bmatrix} = \begin{bmatrix} \underline{N}_{12} & 0 & 0 & 0 \\ 0 & \underline{R}_{T} & 0 & 0 \\ 0 & 0 & \underline{R}_{TM1} & \underline{R}_{CM1} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{TR} \\ \underline{I}_{TM} \\ \underline{I}_{CM} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \underline{E}_{TM} \end{bmatrix}, \qquad \text{B-2}$$

and this may be rearranged to be

$$\begin{bmatrix} \underline{V}_{TH} \\ \underline{V}_{TR} \\ \underline{V}_{TM} \end{bmatrix} = \begin{bmatrix} \underline{N}_{12} & 0 \\ 0 & 0 \\ 0 & \underline{R}_{CM1} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CM} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \underline{R}_{T} & 0 \\ 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TM} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \underline{E}_{TM} \end{bmatrix} . \qquad \text{B-3}$$

The currents $\underline{I}_{TR}$ and $\underline{I}_{TM}$ can be expressed in terms of the chord currents by making use of 4.3.2. The desired expression is

$$\begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TM} \end{bmatrix} = \begin{bmatrix} \underline{B}_{12}^T & \underline{B}_{22}^T & \underline{B}_{32}^T & \underline{B}_{42}^T & \underline{B}_{52}^T \\ \underline{B}_{13}^T & \underline{B}_{23}^T & \underline{B}_{33}^T & \underline{B}_{43}^T & \underline{B}_{53}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \\ \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} . \qquad \text{B-4}$$

If equations 4.3.2 and 4.3.4 are used together it is possible to obtain

$$\underline{I}_{CH} = \underline{N}_{21}\, \underline{I}_{TH} = \underline{N}_{21} \begin{bmatrix} \underline{B}_{11}^T & \underline{B}_{21}^T & \underline{B}_{31}^T & \underline{B}_{41}^T & \underline{B}_{51}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \\ \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} ,$$

$$\underline{I}_{CH} = \underline{N}_{21}\, \underline{B}_{11}^T\, \underline{I}_{CH} + \underline{N}_{21} \begin{bmatrix} \underline{B}_{21}^T & \underline{B}_{31}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix}$$

$$+ \underline{N}_{21} \begin{bmatrix} \underline{B}_{41}^T & \underline{B}_{51}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} . \qquad \text{B-5}$$

The solution for $\underline{I}_{CH}$ is

$$\underline{I}_{CH} = \left[\underline{U} - \underline{N}_{21}\,\underline{B}_{11}^{T}\right]^{-1} \underline{N}_{21} \left[\underline{B}_{21}^{T}\;\underline{B}_{31}^{T}\right] \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} + \left[\underline{U} - \underline{N}_{21}\,\underline{B}_{11}^{T}\right]^{-1} \underline{N}_{21} \left[\underline{B}_{41}^{T}\;\underline{B}_{51}^{T}\right] \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix}, \qquad \text{B-6}$$

and if $\underline{A} = \left[\underline{U} - \underline{N}_{21}\,\underline{B}_{11}^{T}\right]^{-1} \underline{N}_{21}$ B-7

then equation B-6 may be rewritten as

$$\underline{I}_{CH} = \underline{A} \left[\underline{B}_{21}^{T}\;\underline{B}_{31}^{T}\right] \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} + \underline{A} \left[\underline{B}_{41}^{T}\;\underline{B}_{51}^{T}\right] \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix}. \qquad \text{B-8}$$

Now, if equation B-8 is combined with B-4 the result is

$$\begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TM} \end{bmatrix} = \begin{bmatrix} \underline{B}_{12}^{T} \\ \underline{B}_{13}^{T} \end{bmatrix} \left\{ \underline{A} \left[\underline{B}_{21}^{T}\;\underline{B}_{31}^{T}\right] \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} + \underline{A} \left[\underline{B}_{41}^{T}\;\underline{B}_{51}^{T}\right] \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} \right\},$$

$$+ \begin{bmatrix} \underline{B}_{22}^{T} & \underline{B}_{32}^{T} \\ \underline{B}_{23}^{T} & \underline{B}_{33}^{T} \end{bmatrix} \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} + \begin{bmatrix} \underline{B}_{42}^{T} & \underline{B}_{52}^{T} \\ \underline{B}_{43}^{T} & \underline{B}_{53}^{T} \end{bmatrix} \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} \qquad \text{B-9}$$

and this may be rearranged to be

$$\begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TM} \end{bmatrix} = \begin{bmatrix} \underline{B}_{12}^{T}\underline{A} & \underline{U} & 0 \\ \underline{B}_{13}^{T}\underline{A} & 0 & \underline{U} \end{bmatrix} \left\{ \begin{bmatrix} \underline{B}_{21}^{T} & \underline{B}_{31}^{T} \\ \underline{B}_{22}^{T} & \underline{B}_{32}^{T} \\ \underline{B}_{23}^{T} & \underline{B}_{33}^{T} \end{bmatrix} \begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} \right.$$

$$+ \begin{bmatrix} \underline{B}_{41}^{T} & \underline{B}_{51}^{T} \\ \underline{B}_{42}^{T} & \underline{B}_{52}^{T} \\ \underline{B}_{43}^{T} & \underline{B}_{53}^{T} \end{bmatrix} \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} \Bigg\} . \qquad \text{B-10}$$

Using equation 4.3.1 it is possible to obtain

$$\begin{bmatrix} \underline{V}_{CH} \\ \underline{V}_{CR} \\ \underline{V}_{CM} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{V}_{TH} \\ \underline{V}_{TR} \\ \underline{V}_{TM} \end{bmatrix} - \begin{bmatrix} \underline{B}_{13} \\ \underline{B}_{24} \\ \underline{B}_{34} \end{bmatrix} \underline{V}_{TE} , \qquad \text{B-11}$$

and if this is combined with B-3, the result is

$$\begin{bmatrix} \underline{V}_{CH} \\ \underline{V}_{CR} \\ \underline{V}_{CM} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{N}_{12} & 0 \\ 0 & 0 \\ 0 & \underline{R}_{CM1} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CM} \end{bmatrix}$$

$$- \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \underline{R}_{T} & 0 \\ 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TM} \end{bmatrix}$$

$$- \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \underline{E}_{TM} \end{bmatrix} - \begin{bmatrix} \underline{B}_{14} \\ \underline{B}_{24} \\ \underline{B}_{34} \end{bmatrix} \underline{V}_{TE} . \qquad \text{B-12}$$

The volt-ampere equations 3.3.7 and 4.3.3 may be utilized to yield

$$\begin{bmatrix} \underline{V}_{CH} \\ \underline{V}_{CR} \\ \underline{V}_{CM} \end{bmatrix} = \begin{bmatrix} \underline{U} & 0 & 0 \\ 0 & \underline{R}_C & 0 \\ 0 & 0 & \underline{R}_{CM2} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \underline{R}_{TM2} \end{bmatrix} \underline{I}_{TM} + \begin{bmatrix} 0 \\ 0 \\ \underline{E}_{CM} \end{bmatrix}, \qquad \text{B-13}$$

and combining this result with equation B-12, the result is

$$\begin{bmatrix} \underline{U} & 0 & 0 \\ 0 & \underline{R}_C & 0 \\ 0 & 0 & \underline{R}_{CM2} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{N}_{12} & 0 \\ 0 & 0 \\ 0 & \underline{R}_{CM1} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CM} \end{bmatrix}$$

$$- \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \underline{R}_T & 0 \\ 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{I}_{TR} \\ \underline{I}_{TM} \end{bmatrix} \qquad \text{B-14}$$

$$- \begin{bmatrix} 0 \\ 0 \\ \underline{R}_{TM2} \end{bmatrix} \underline{I}_{TM} - \begin{bmatrix} \underline{B}_{13} & \underline{B}_{14} & 0 \\ \underline{B}_{23} & \underline{B}_{24} & 0 \\ \underline{B}_{33} & \underline{B}_{34} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{E}_{TM} \\ \underline{V}_{TE} \\ \underline{E}_{CM} \end{bmatrix} .$$

If equation B-10 is combined with B-14, then

$$\begin{bmatrix} \underline{U} & 0 & 0 \\ 0 & \underline{R}_C & 0 \\ 0 & 0 & \underline{R}_{CM2} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{N}_{12} & 0 \\ 0 & 0 \\ 0 & \underline{R}_{CM1} \end{bmatrix} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CM} \end{bmatrix}$$

$$
- \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix}
\begin{bmatrix} 0 & 0 \\ \underline{R}_{T} & 0 \\ 0 & \underline{R}_{TM1} \end{bmatrix}
\begin{bmatrix} \underline{B}^{T}_{12}\underline{A} & \underline{U} & 0 \\ \underline{B}^{T}_{13}\underline{A} & 0 & \underline{U} \end{bmatrix}
$$

$$
\left\{ \begin{bmatrix} \underline{B}^{T}_{21} & \underline{B}^{T}_{31} \\ \underline{B}^{T}_{22} & \underline{B}^{T}_{32} \\ \underline{B}^{T}_{23} & \underline{B}^{T}_{33} \end{bmatrix}
\begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix}
+ \begin{bmatrix} \underline{B}^{T}_{41} & \underline{B}^{T}_{51} \\ \underline{B}^{T}_{42} & \underline{B}^{T}_{52} \\ \underline{B}^{T}_{43} & \underline{B}^{T}_{53} \end{bmatrix}
\begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} \right\}
$$

$$
- \begin{bmatrix} 0 \\ 0 \\ \underline{R}_{TM2} \end{bmatrix}
\begin{bmatrix} \underline{B}^{T}_{13}\underline{A} & 0 & \underline{U} \end{bmatrix}
\left\{ \begin{bmatrix} \underline{B}^{T}_{21} & \underline{B}^{T}_{31} \\ \underline{B}^{T}_{22} & \underline{B}^{T}_{32} \\ \underline{B}^{T}_{23} & \underline{B}^{T}_{33} \end{bmatrix}
\begin{bmatrix} \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} \right.
$$

$$
\left. + \begin{bmatrix} \underline{B}^{T}_{41} & \underline{B}^{T}_{51} \\ \underline{B}^{T}_{42} & \underline{B}^{T}_{52} \\ \underline{B}^{T}_{43} & \underline{B}^{T}_{53} \end{bmatrix}
\begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} \right\}
- \begin{bmatrix} \underline{B}_{13} & \underline{B}_{14} & 0 \\ \underline{B}_{23} & \underline{B}_{24} & 0 \\ \underline{B}_{33} & \underline{B}_{34} & \underline{U} \end{bmatrix}
\begin{bmatrix} \underline{E}_{TM} \\ \underline{V}_{TE} \\ \underline{E}_{CM} \end{bmatrix} \qquad \text{B-15}
$$

and this may rearranged be

$$
\begin{bmatrix} \underline{U} & 0 & 0 \\ 0 & \underline{R}_{C} & 0 \\ 0 & 0 & \underline{R}_{CM2} \end{bmatrix}
\begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix}
= - \left\{ \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix}
\begin{bmatrix} \underline{N}_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \underline{R}_{CM1} \end{bmatrix} \right.
$$

$$
+ \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_{T}\underline{B}^{T}_{12}\underline{A} & \underline{R}_{T} & 0 \\ \underline{R}_{TM1}\underline{B}^{T}_{13}\underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix}
\begin{bmatrix} 0 & \underline{B}^{T}_{21} & \underline{B}^{T}_{31} \\ 0 & \underline{B}^{T}_{22} & \underline{B}^{T}_{32} \\ 0 & \underline{B}^{T}_{23} & \underline{B}^{T}_{33} \end{bmatrix}
$$

$$
\left. + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \underline{R}_{TM2}\underline{B}_{13}^{T}\,\underline{A} & 0 & \underline{R}_{TM2} \end{bmatrix} \begin{bmatrix} 0 & \underline{B}_{21}^{T} & \underline{B}_{31}^{T} \\ 0 & \underline{B}_{22}^{T} & \underline{B}_{32}^{T} \\ 0 & \underline{B}_{23}^{T} & \underline{B}_{33}^{T} \end{bmatrix} \right\} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix}
$$

$$
- \left\{ \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_{T}\underline{B}_{12}^{T}\,\underline{A} & \underline{R}_{T} & 0 \\ \underline{R}_{TM1}\underline{B}_{12}^{T}\underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \right.
$$

$$
\left. + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \underline{R}_{TM2}\underline{B}_{13}^{T}\underline{A} & 0 & \underline{R}_{TM2} \end{bmatrix} \right\} \begin{bmatrix} \underline{B}_{41}^{T} & \underline{B}_{51}^{T} \\ \underline{B}_{42}^{T} & \underline{B}_{52}^{T} \\ \underline{B}_{43}^{T} & \underline{B}_{53}^{T} \end{bmatrix} \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix}
$$

$$
- \begin{bmatrix} \underline{B}_{13} & \underline{B}_{14} & 0 \\ \underline{B}_{23} & \underline{B}_{24} & 0 \\ \underline{B}_{33} & \underline{B}_{34} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{E}_{TM} \\ \underline{V}_{TE} \\ \underline{E}_{CM} \end{bmatrix} . \qquad \text{B-16}
$$

The solution for $\begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix}$ may be obtained from equation B-16 and it is

$$
\begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix} = \underline{V} = -\underline{D}\ \underline{J}\ \underline{I} - \underline{D}\ \underline{G}\ \underline{E} \qquad \text{B-17}
$$

where $\underline{D}$, $\underline{J}$, $\underline{I}$, $\underline{G}$ and $\underline{E}$ have the meanings given in Chapter IV.

It is possible to combine equation B-10 with equation B-3 to obtain

$$\begin{bmatrix} \underline{V}_{TH} \\ \underline{V}_{TR} \\ \underline{V}_{TM} \end{bmatrix} = \left\{ \begin{bmatrix} \underline{N}_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \underline{R}_{CM1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} 0 & \underline{B}_{21}^T & \underline{B}_{31}^T \\ 0 & \underline{B}_{22}^T & \underline{B}_{32}^T \\ 0 & \underline{B}_{23}^T & \underline{B}_{33}^T \end{bmatrix} \right\} \begin{bmatrix} \underline{V}_{CH} \\ \underline{I}_{CR} \\ \underline{I}_{CM} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{B}_{41}^T & \underline{B}_{51}^T \\ \underline{B}_{42}^T & \underline{B}_{52}^T \\ \underline{B}_{43}^T & \underline{B}_{53}^T \end{bmatrix} \begin{bmatrix} \underline{I}_{CI} \\ \underline{I}_{CIP} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \underline{E}_{TM} \end{bmatrix} \qquad \text{B-18}$$

This result may be combined with B-1 and B-17 to yield the desired solution for $\underline{V}_{CIP}$. This is

$$\underline{V}_{CIP} = \underline{B}_P \, (\underline{G}\ \underline{H}\ \underline{J} - \underline{F})\ \underline{I} + \underline{B}_P \, (\underline{G}\ \underline{H}\ \underline{L})\ \underline{E} - \underline{B}_Q\ \underline{E}, \qquad \text{B-19}$$

where $\underline{B}_P$, $\underline{G}$, $\underline{H}$, $\underline{J}$, $\underline{F}$, $\underline{I}$, $\underline{L}$, $\underline{E}$ and $\underline{B}_Q$ have the same meanings given in Chapter IV.

APPENDIX C

APPENDIX C

THE COMPUTER PROGRAM FOR CHARACTERIZATION OF AN n-PORT NETWORK CONTAINING MULTIPORT SUBNETWORKS

This program is written in FORTRAN IV with format for execution by an IBM 1410 computer. It is divided into 9 parts. Card input data is required for Parts I and II. The output data from Parts I through VIII is placed on magnetic tape for use in the succeeding parts of the program. The output of Part I is also printed as is the output of Part IX. The card input data for Part II is printed for information purposes.

The card input data is arranged as shown in Table C-1.

TABLE C-1

ORDER OF CARD INPUT DATA FOR THE PROGRAM

Card Group		Variable Name and Order on Card
	Part I	
1		KO, MTRE, NT, NC, NI, NIE, MOD, NCH, NCM, NTH, NTM (I3 form)
2		KONN (one row per card, I3 form)
3		INTO (one row per card, I3 form)
4		NTRE (I3 form)
5		KORD (I3 form)

TABLE C-1 (Continued)

Card Group	Variable Name and Order on Card
Part II	
1	R (E12.4 form)
2	H12 (E12.4 form)
3	H21 (E12.4 form)
4	RM (E12.4 form)

Note: The program is written so that if any of the variables R, H12, H21, or RM are blank, the particular variable or variables are omitted as input data.

The order and labeling of the printed output data is

(a) B MATRIX,

(b) INTERCONNECTION MATRIX,

(c) ORIENTATION MATRIX,

(d) BRANCH MATRIX,

(e) R MATRIX,

(f) H12 MATRIX,

(g) H21 MATRIX,

(h) RM MATRIX,

(i) COEF MATRIX OF PORT CURRENTS,

(j) COEF MATRIX OF CURRENT DRIVERS, and

(k) COEF MATRIX OF VOLTAGE DRIVERS.

If any of the above are not applicable for a particular problem, this is denoted by their absence from the output data.

The variable names are shown in Table C-2. The maximum value that the variables may have is

NT = 6,

NE = 3,

NC = 6,

NI = 3,

NIE = 4,

NCH = 2,

NCM = 3,

NTH = 2,

NTM = 3,

MOD = 11,

KO = 10, and

MTRE = 10.

TABLE C-2

VARIABLES USED IN PROGRAM

KO = the number of chords.

MTRE = the number of branches.

NT = the number of branches containing resistances.

NE = the number of ideal voltage sources.

NC = the number of chords containing resistances.

NI = the number of ideal current sources.

NIE = the number of ports.

MOD = the degree of the node of maximum degree.

NCH = the number of chords containing ideal transformers.

NCM = the number of chords containing multiport subnetworks.

NTH = the number of branches containing ideal transformers.

NTM = the number of branches containing multiport subnetworks.

KONN (I, J) = the interconnection matrix, $\underline{K}$.

INTO (I, 2) = the orientation matrix, $\underline{D}$.

NTRE (I) = the branch matrix, $\underline{T}$.

KORD (I) = the chord matrix, $\underline{C}$.

KONNM (I, J) = a modified $\underline{K}$ matrix, $\underline{K}_2$.

NBR(I) = the sum of the elements in the ith row of $\underline{K}_2$.

NOT(I) = the branches which are not in the circuit with the ith chord.

MESH(I) = the branches forming a circuit with the ith chord.

LOOP(I, J) = the $\underline{B}$ matrix.

R (I, J) = the matrix of branch and chord resistances, $\underline{R}$.

H12(I, J) = the matrix of ideal transformer constants relating voltages, $\underline{N}_{12}$.

TABLE C-2 (Continued)

H21(I, J) = the matrix of ideal transformer constants relating currents, $\underline{N}_{21}$.

RM(I, J) = the matrix of r-parameters for the multiport subnetworks, $\underline{R}_M$.

$$A(I, J) = \underline{U} - \underline{N}_{21} \underline{B}_{11}^T .$$

$$A1(I, J) = \left[\underline{U} - \underline{N}_{21} \underline{B}_{11}^T\right]^{-1} .$$

$$A2(I, J) = \left[\underline{U} - \underline{N}_{21} \underline{B}_{11}^T\right]^{-1} \underline{N}_{21}.$$

$$A4(I, J) = \underline{R}_{TM1} \underline{B}_{13}^T \left[\underline{U} - \underline{N}_{21} \underline{B}_{11}^T\right]^{-1} \underline{N}_{21}.$$

$$A5(I, J) = \underline{R}_{TM2} \underline{B}_{13}^T \left[\underline{U} - \underline{N}_{21} \underline{B}_{11}^T\right]^{-1} \underline{N}_{21}$$

$$R1(I, J) = \begin{bmatrix} \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} .$$

$$R2(I, J) = \begin{bmatrix} \underline{R}_{TM2} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM2} \end{bmatrix} .$$

$$R3(I, J) = \begin{bmatrix} \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{B}_{21}^T & \underline{B}_{31}^T \\ \underline{B}_{22}^T & \underline{B}_{32}^T \\ \underline{B}_{23}^T & \underline{B}_{33}^T \end{bmatrix} .$$

$$R4(I, J) = \begin{bmatrix} \underline{R}_{TM2} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM2} \end{bmatrix} \begin{bmatrix} \underline{B}_{21}^T & \underline{B}_{31}^T \\ \underline{B}_{22}^T & \underline{B}_{32}^T \\ \underline{B}_{23}^T & \underline{B}_{33}^T \end{bmatrix} .$$

TABLE C-2 (Continued)

$$R5(I, J) = \begin{bmatrix} \underline{B}_{11} \\ \underline{B}_{21} \\ \underline{B}_{31} \end{bmatrix} \underline{N}_{12}.$$

$$R6(I, J) = \begin{bmatrix} \underline{B}_{13} \\ \underline{B}_{23} \\ \underline{B}_{33} \end{bmatrix} \underline{R}_{CM1} .$$

$$R7(I, J) = \underline{H}^{-1}.$$

$$R8(I, J) = \underline{H} .$$

$$R9(I, J) = \begin{bmatrix} \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{B}_{21}^T & \underline{B}_{31}^T \\ \underline{B}_{22}^T & \underline{B}_{32}^T \\ \underline{B}_{23}^T & \underline{B}_{33}^T \end{bmatrix} .$$

$$R10(I, J) = \begin{bmatrix} \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} \begin{bmatrix} \underline{B}_{41}^T & \underline{B}_{51}^T \\ \underline{B}_{42}^T & \underline{B}_{52}^T \\ \underline{B}_{43}^T & \underline{B}_{53}^T \end{bmatrix} .$$

$$R11(I, J) = \begin{bmatrix} \underline{B}_{12} & \underline{B}_{13} \\ \underline{B}_{22} & \underline{B}_{23} \\ \underline{B}_{32} & \underline{B}_{33} \end{bmatrix} \begin{bmatrix} \underline{R}_T \underline{B}_{12}^T \underline{A} & \underline{R}_T & 0 \\ \underline{R}_{TM1} \underline{B}_{13}^T \underline{A} & 0 & \underline{R}_{TM1} \end{bmatrix} .$$

$$R12(I, J) = \underline{G}.$$

$$R13(I, J) = R11(I, J) + R2(I, J).$$

$$R14(I, J) = \underline{J}.$$

$$R15(I, J) = \underline{G}\ \underline{H}\ \underline{J}.$$

TABLE C-2 (Continued)

R16(I, J) = R15(I, J) - R10 (I, J).

R17(I, J) = $\left[\underline{B}_P\right]$ R16(I, J) .

R18(I, J) = $\underline{B}_P \, \underline{G} \, \underline{H}$.

R19(I, J) = $\underline{L}$.

R20(I, J) = $\underline{B}_P \, \underline{G} \, \underline{H} \, \underline{L}$.

R21(I, J) = $\underline{B}_P \, \underline{G} \, \underline{H} \, \underline{L} - \underline{B}_Q$.

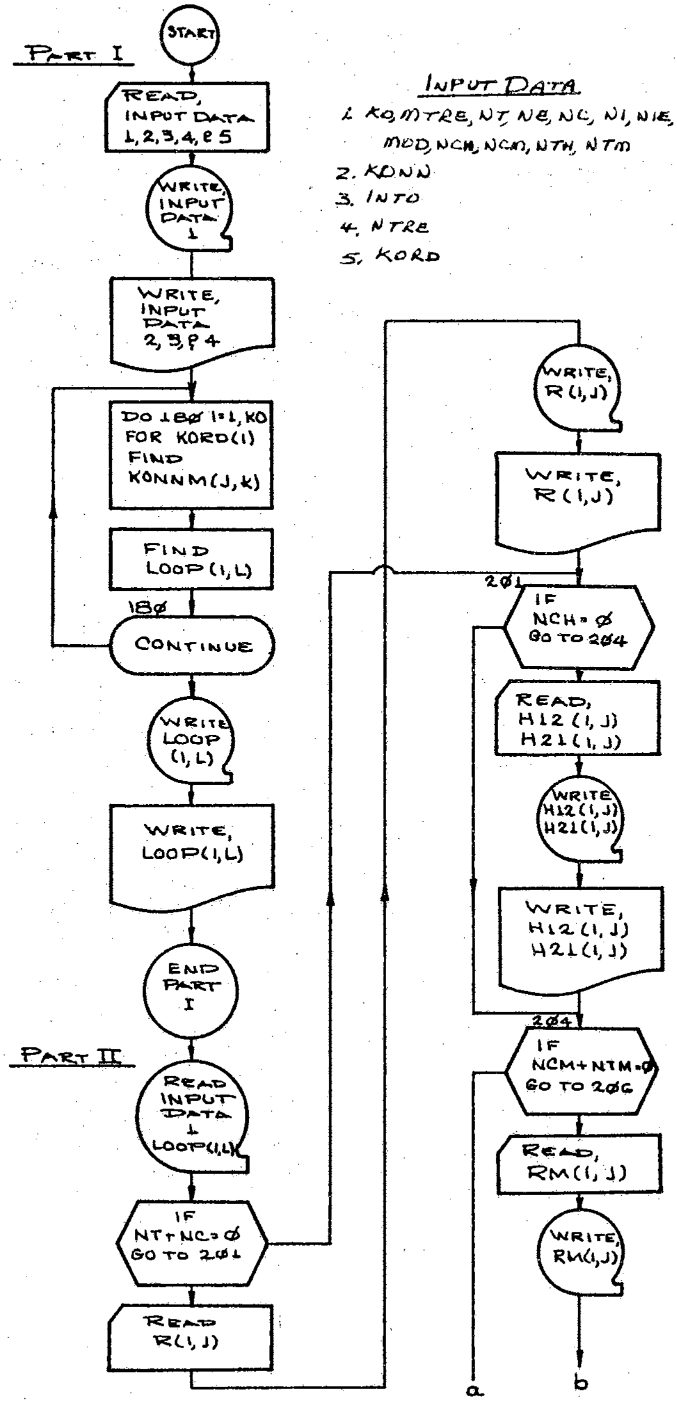

Figure C-1. Flow Chart for the Program of Chapter IV.

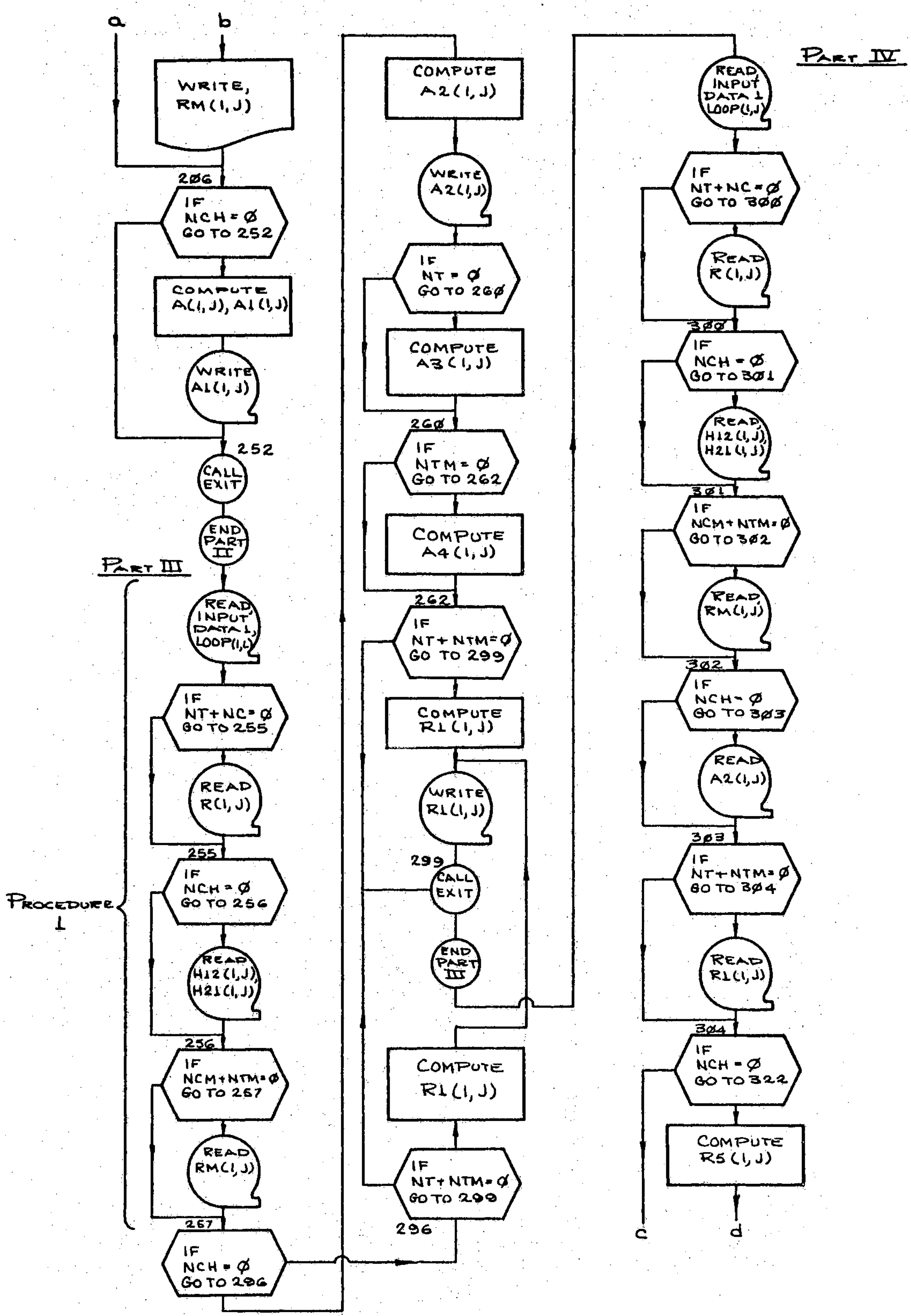

Figure C-1 (Continued)

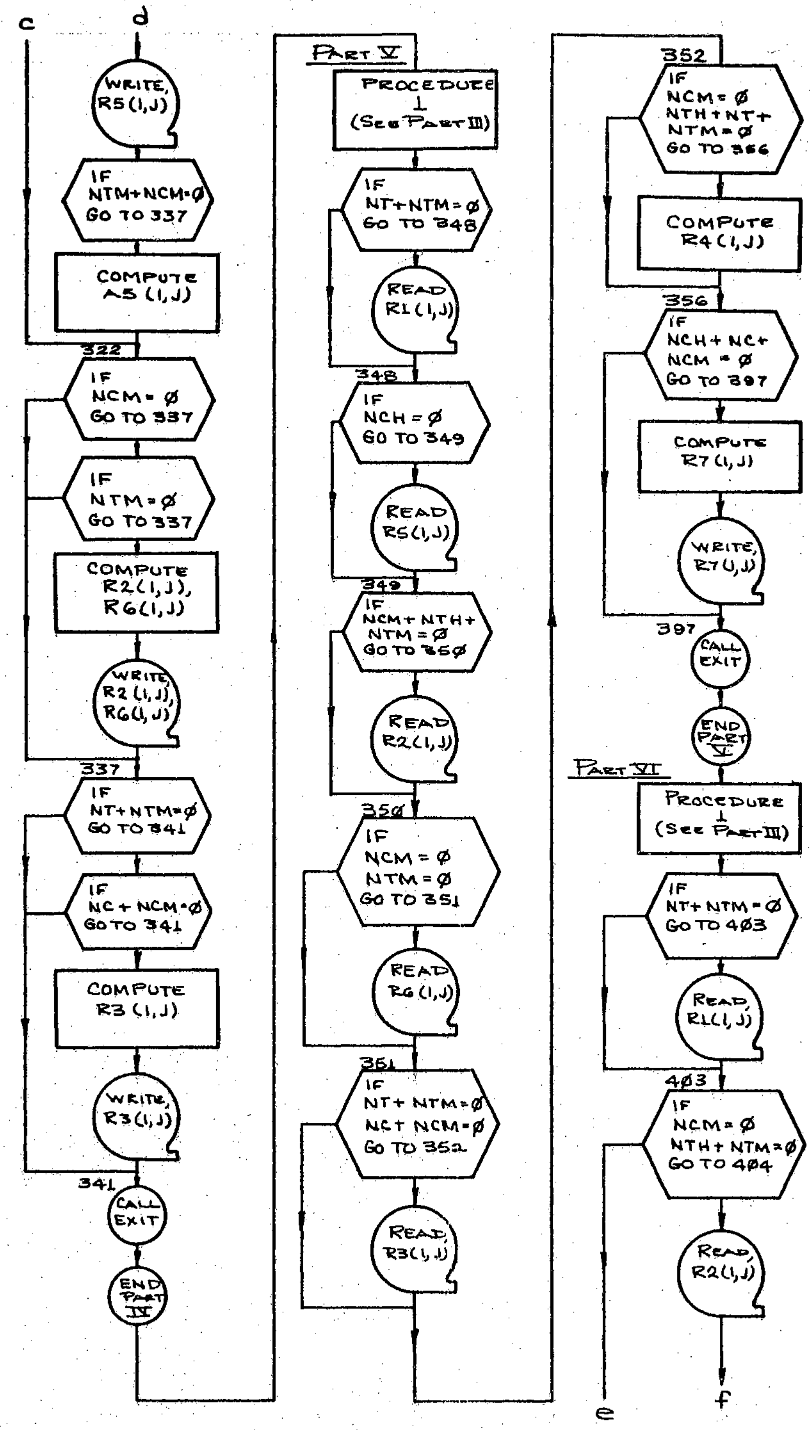

Figure C-1 (Continued)

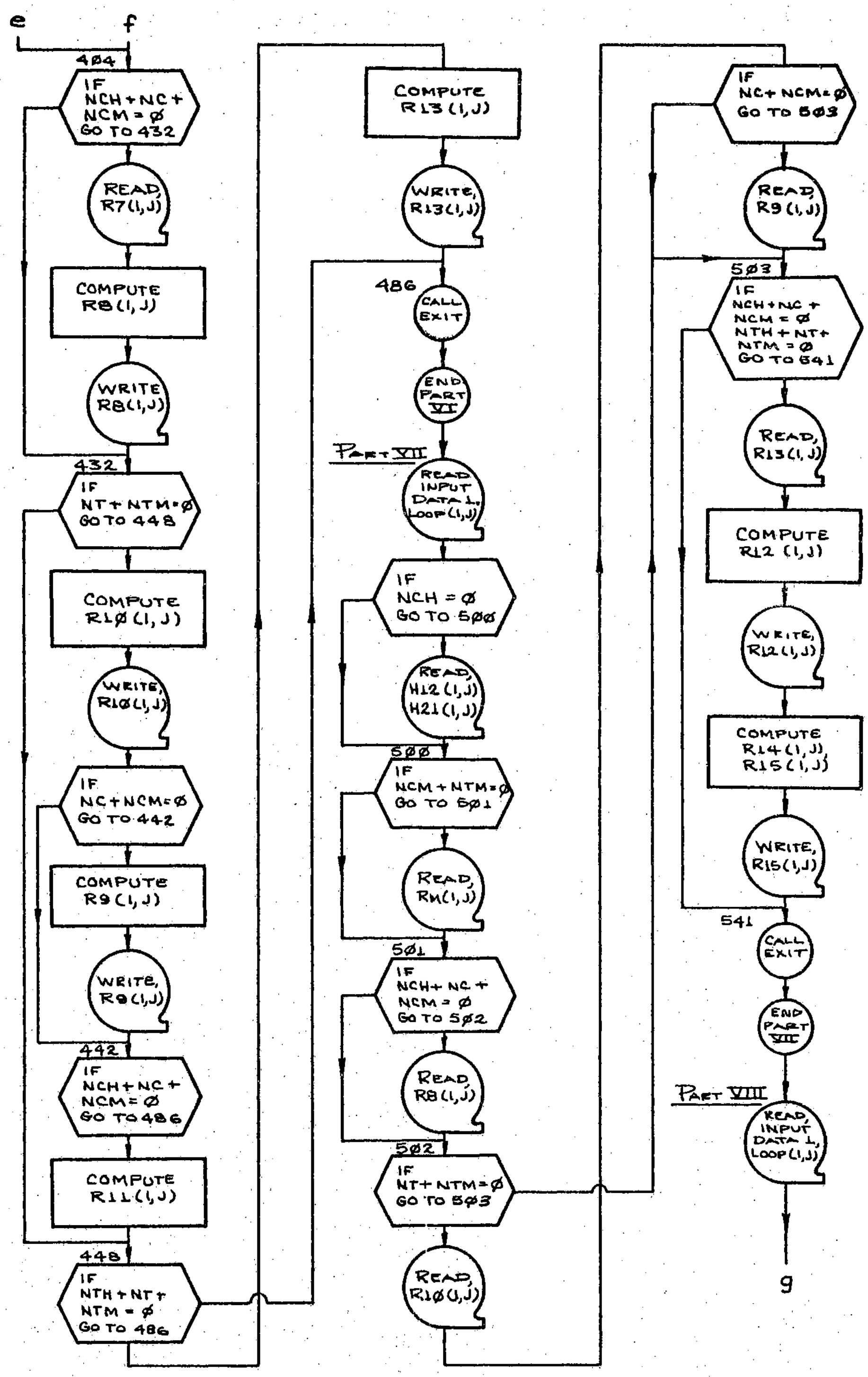

Figure C-1 (Continued)

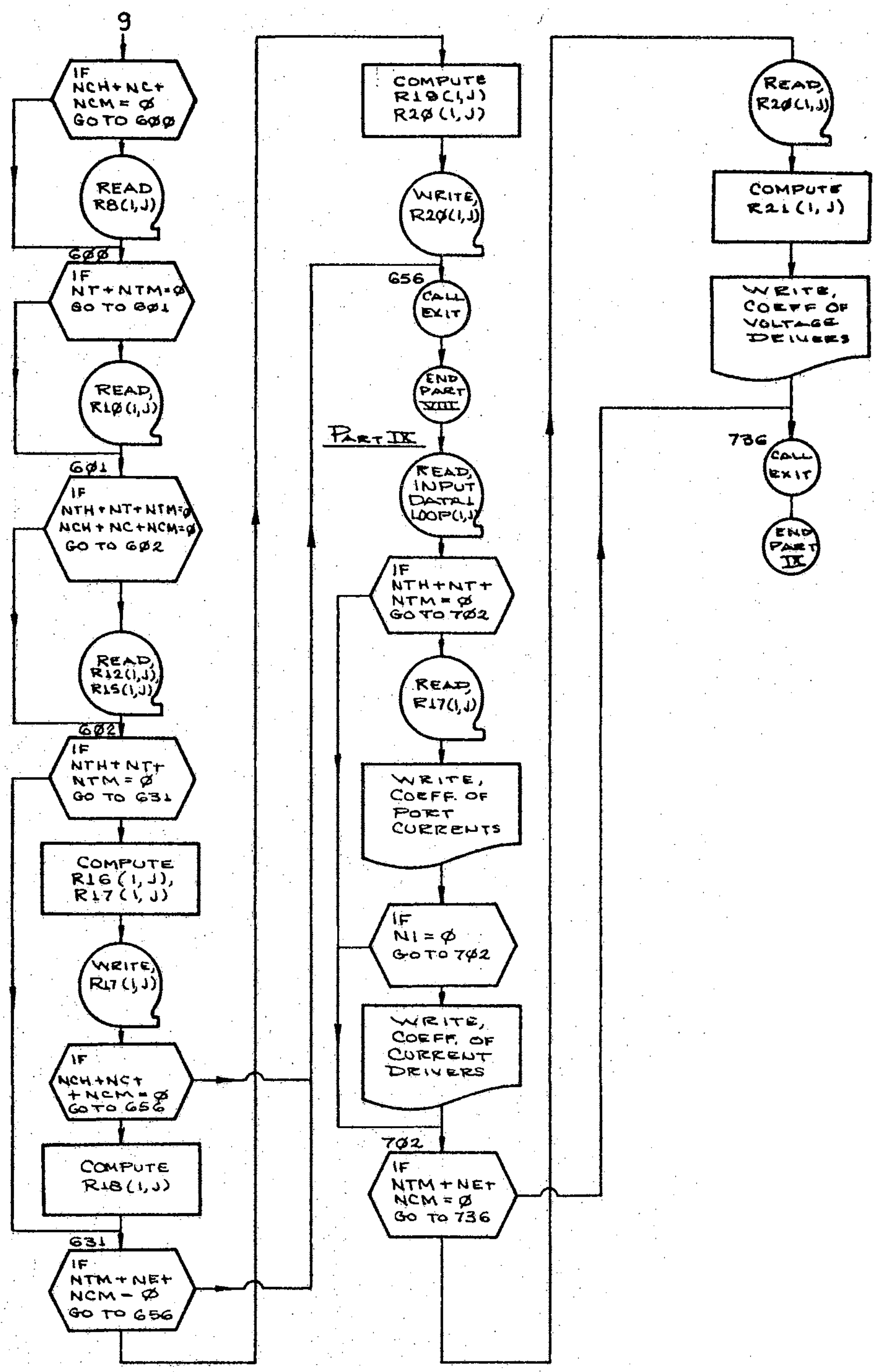

Figure C-1 (Continued)

TABLE C-3

FORTRAN STATEMENTS FOR IBM 1410 PROGRAM

```
MON**     EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM1
      DIMENSION KONN(11,11),INTO(20,2),NTRE(10),KORD(10),NBR(10),NOT(10)
      DIMENSION KONNM(11,11),MESH(10),LOOP(10,10)
1     FORMAT(12I3)
2     FORMAT(11I3)
21    FORMAT(/4X,8HB MATRIX//)
3     FORMAT(10I3)
22    FORMAT(/4X,22HINTERCONNECTION MATRIX//)
4     FORMAT(11I3)
5     FORMAT(/4X,18HORIENTATION MATRIX//(2I3))
6     FORMAT(/4X,13HBRANCH MATRIX//(11I3))
      READ(1,1)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      REWIND6
      WRITE(6)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      NV=MTRE+1
      DO20I=1,NV
20    READ(1,2)(KONN(I,J),J=1,MOD)
      NX=KO+MTRE
      DO19I=1,NX
19    READ(1,2)(INTO(I,J),J=1,2)
      READ(1,2)(NTRE(I),I=1,MTRE)
      READ(1,2)(KORD(I),I=1,KO)
      DO35I=1,KO
      DO35J=1,MTRE
35    LOOP(I,J)=0
      DO180I=1,KO
      DO36J=1,NV
      DO36K=1,MOD
36    KONNM(J,K)=0
      DO37J=1,NV
37    MESH(J)=0
      DO45J=1,NV
      DO45K=1,MOD
      IF(KORD(I)-KONN(J,K).EQ.0)KONNM(J,K)=KONN(J,K)
45    CONTINUE
      DO55J=1,NV
      DO55K=1,MOD
      DO55L=1,NV
      IF(NTRE(L)-KONN(J,K).EQ.0)KONNM(J,K)=KONN(J,K)
55    CONTINUE
      DO80LL=1,MTRE
      DO60J=1,NV
      NOT(J)=0
      NBR(J)=0
      DO60K=1,MOD
60    NBR(J)=NBR(J)+KONNM(J,K)
      LA=1
      DO70J=1,NV
      DO70K=1,MOD
      IF(KONNM(J,K).EQ.0)GOTO70
      IF(NBR(J)-KONNM(J,K).NE.0)GOTO70
      NOT(LA)=KONNM(J,K)
      NA=LA
      LA=LA+1
70    CONTINUE
      DO80J=1,NV
      DO80K=1,MOD
      DO80LB=1,NA
      IF(KONN(J,K).EQ.0)GOTD80
       IF(KONN(J,K)-NOT(LB).NE.0)GOTO80
      KONNM(J,K)=0
80    CONTINUE
      MESH(1)=KORD(I)
      LM=KORD(I)
      LN=INTO(LM,2)
      LP=2
104   DO115J=1,MOD
      IF(KONNM(LN,J)-LM.EQ.0)GOTO115
      IF(KONNM(LN,J).EQ.0)GOTO115
      LQ=KONNM(LN,J)
115   CONTINUE
```

TABLE C-3 (Continued)

```
      IF(KORD(I)-LQ.EQ.0)GOTO135
      MESH(LP)=LQ
      DO130K=1,2
      IF(INTO(LQ,K)-LN.NE.0)LR=INTO(LQ,K)
130   CONTINUE
      LM=LQ
      LN=LR
      LP=LP+1
      GOTO104
135   DO180J=1,NV
      IF(MESH(J).EQ.0)GOTO180
      KD=MESH(J)
      IF(J-1.GT.0)GOTO150
      KE=INTO(KD,2)
      GOTO180
150   DO165M=1,MTRE
      IF(NTRE(M)-KD.EQ.0)KG=M
165   CONTINUE
      IF(INTO(KD,1)-KE.EQ.0)GOTO175
      LOOP(I,KG)=(-1)
      KE=INTO(KD,1)
      GOTO180
175   LOOP(I,KG)=1
      KE=INTO(KD,2)
180   CONTINUE
      WRITE(3,21)
      DO181I=1,KO
181   WRITE(3,3)(LOOP(I,J),J=1,MTRE)
      WRITE(3,22)
      DO182I=1,NV
182   WRITE(3,4)(KONN(I,J),J=1,MOD)
      WRITE(3,5)((INTO(I,J),J=1,2),I=1,NX)
      WRITE(3,6)(NTRE(J),J=1,MTRE)
      WRITE(6)((LOOP(I,J),J=1,MTRE),I=1,KO)
      CALLEXIT
      END

     MON$$      EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM2
      DIMENSION LOOP(10,10),R(12,12),H12(2,2),H21(2,2),RM(6,6),A(2,2)
      DIMENSION A1(2,2)
7     FORMAT(6E12.4)
8     FORMAT(2E12.4)
9     FORMAT(2E12.4)
10    FORMAT(6E12.4)
30    FORMAT(6E12.4)
23    FORMAT(/4X,8HR MATRIX//)
31    FORMAT(6E12.4)
32    FORMAT(2E12.4)
24    FORMAT(/4X,10HH12 MATRIX//)
25    FORMAT(/4X,10HH21 MATRIX//)
26    FORMAT(/4X,9HRM MATRIX//)
33    FORMAT(6E12.4)

      REWIND4
      REWIND5
      REWIND6
      READ(6)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      WRITE(5)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      K2=NTH+NT
      K4=NTH+NT+NTM
      K6=NT+NTM
      K19=NT+NC
      READ(6)((LOOP(I,J),J=1,MTRE),I=1,KO)
      WRITE(5)((LOOP(I,J),J=1,MTRE),I=1,KO)
      IF(NT+NC.EQ.0)GOTO201
      WRITE(3,23)
      DO200I=1,K19
      READ(1,7)(R(I,J),J=1,K19)
      WRITE(3,30)(R(I,J),J=1,K19)
200   WRITE(5)(R(I,J),J=1,K19)
201   IF(NCH.EQ.0)GOTO204
      WRITE(3,24)
```

TABLE C-3 (Continued)

```
      DO202I=1,NTH
      READ(1,8)(H12(I,J),J=1,NCH)
      WRITE(3,31)(H12(I,J),J=1,NCH)
202   WRITE(5)(H12(I,J),J=1,NCH)
      WRITE(3,25)
      DO203I=1,NCH
      READ(1,9)(H21(I,J),J=1,NTH)
      WRITE(3,32)(H21(I,J),J=1,NTH)
203   WRITE(5)(H21(I,J),J=1,NTH)
204   IF(NCM+NTM.EQ.0)GOTO206
      WRITE(3,26)
      NM=NCM+NTM
      DO205I=1,NM
      READ(1,10)(RM(I,J),J=1,NM)
      WRITE(3,33)(RM(I,J),J=1,NM)
205   WRITE(5)(RM(I,J),J=1,NM)
206   IF(NCH.EQ.0)GOTO252
      DO210I=1,NCH
      DO210J=1,NTH
      A(I,J)=0.0
      X=LOOP(I,J)
      A(I,J)=A(I,J)+(H12(I,J)*X)
      IF(I.EQ.J)GOTO207
      A(I,J)=(-1.)*A(I,J)
      GOTO210
207   A(I,J)=1.0-A(I,J)
210   CONTINUE
      DO220I=1,NCH
      DO220J=1,NCH
      IF(I.EQ.J)GOTO215
      A1(I,J)=0.0
      GOTO220
215   A1(I,J)=1.0
220   CONTINUE
      M=1
      DO250L=1,NCH
      M=M+1
      IF(M.GT.NCH)GOTO235
      DO230I=M,NCH
230   A(L,I)=A(L,I)/A(L,L)
235   DO240J=1,NCH
240   A1(L,J)=A1(L,J)/A(L,L)
      DO250J=1,NCH
      IF(L.EQ.J)GOTO250
      IF(M.GT.NCH)GOTO244
      DO243I=M,NCH
243   A(J,I)=A(J,I)-(A(L,I)*A(J,L))
244   DO250IQ=1,NCH
      A1(I,IQ)=A1(I,IQ)-(A1(L,IQ)*A(J,L))
250   CONTINUE
      DO251I=1,NCH
251   WRITE(4)(A1(I,J),J=1,NCH)
252   CALLEXIT
      END

     MON$$     EXEQ FORTRAN,SOF,SIU,D8,04,,,MAINPGM3
      DIMENSION LOOP(10,10),R(12,12),H12(2,2),H21(2,2),RM(6,6),A1(2,2)
      DIMENSION A2(2,2),A4(6,2),R1(9,11),A3(6,3)
      REWIND4
      REWIND5
      REWIND6
      READ(5)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      READ(5)((LOOP(I,J),J=1,MTRE),I=1,KO)
      K1=NTH+1
      K7=NT+1
      K2=NTH+NT
      K3=K2+1
      K4=NTH+NT+NTM
      K6=NT+NTM
      K19=NT+NC
      IF(NT+NC.EQ.0)GOTO255
      READ(5)((R(I,J),J=1,K19),I=1,K19)
```

TABLE C-3 (Continued)

```
255    IF(NCH.EQ.0)GOTO256
       READ(5)((H12(I,J),J=1,NCH),I=1,NTH)
       READ(5)((H21(I,J),J=1,NTH),I=1,NCH)
256    IF(NCM+NTM.EQ.0)GOTO257
       NM=NCM+NTM
       READ(5)((RM(I,J),J=1,NM),I=1,NM)
257    IF(NCH.EQ.0)GOTO296
       READ(4)((A1(I,J),J=1,NCH),I=1,NCH)
       DO258I=1,NCH
       DO258J=1,NCH
       A2(I,J)=0.0
       DO258K=1,NCH
258    A2(I,J)=A2(I,J)+(A1(I,J)*H21(I,J))
       WRITE(6)((A2(I,J),J=1,NCH),I=1,NCH)
       IF(NT.EQ.0)GOTO260
       DO259I=1,NT
       DO259J=1,NCH
       A3(I,J)=0.0
       DO259K=1,NCH
       M=0
       DO259L=K1,K2
       M=M+1
       X=LOOP(K,L)
259    A3(I,J)=A3(I,J)+(R(I,M)*X*A2(K,J))
260    IF(NTM.EQ.0)GOTO262
       DO261I=1,NTM
       DO261J=1,NCH
       A4(I,J)=0.0
       DO261K=1,NCH
       M=0
       DO261L=K3,K4
       M=M+1
       X=LOOP(K,L)
261    A4(I,J)=A4(I,J)+(RM(I,M)*X*A2(K,J))
262    IF(K6.EQ.0)GOTO299
       DO265I=1,K6
       DO265J=1,K4
265    R1(I,J)=0.0
       IF(NT.EQ.0)GOTO271
       DO270I=1,NT
       DO270J=1,NCH
270    R1(I,J)=A3(I,J)
271    IF(NTM.EQ.0)GOTO281
       M=0
       DO275I=K7,K6
       M=M+1
       DO275J=1,NCH
275    R1(I,J)=A4(M,J)
276    IF(NTM.EQ.0)GOTO281
       M1=0
       DO280I=K7,K6
       M1=M1+1
       M2=0
       DO280J=K3,K4
       M2=M2+1
280    R1(I,J)=RM(M1,M2)
281    IF(NT.EQ.0)GOTO294
       DO285I=1,NT
       M=0
       DO285J=K1,K2
       M=M+1
285    R1(I,J)=R(I,M)
286    DO287I=1,K6
287    WRITE(6)(R1(I,J),J=1,K4)
       GOTO299
294    IF(NTH+NTM.NE.0)GOTO286
       GOTO299
296    IF(NT+NTM.EQ.0)GOTO299
       DO298I=1,K6
       DO298J=1,K4
298    R1(I,J)=0.0
       GOTO276
```

TABLE C-3 (Continued)

```
299   CALLEXIT
      END
     MON$$      EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM4
      DIMENSION R1(9,11),A5(6,2),R2(3,11),R3(11,9),R5(11,2),R6(11,3)
      DIMENSION LOOP(10,10),R(12,12),H12(2,2),H21(2,2),RM(6,6),A2(2,2)
      REWIND4
      REWIND5
      REWIND6
      READ(5)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      READ(5)((LOOP(I,J),J=1,MTRE),I=1,KO)
      K4=NTH+NT+NTM
      K2=NTH+NT
      K3=K2+1
      K14=NCH+NC+NCM
      K15=NC+NCM
      K6=NT+NTM
      K18=NCH+NC
      K19=NT+NC
      K28=NTH+1
      IF(NT+NC.EQ.0)GOTO300
      READ(5)((R(I,J),J=1,K19),I=1,K19)
300   IF(NCH.EQ.0)GOTO301
      READ(5)((H12(I,J),J=1,NCH),I=1,NTH)
      READ(5)((H21(I,J),J=1,NTH),I=1,NCH)
301   IF(NCM+NTM.EQ.0)GOTO302
      NM=NCM+NTM
      READ(5)((RM(I,J),J=1,NM),I=1,NM)
302   IF(NCH.EQ.0)GOTO303
      READ(6)((A2(I,J),J=1,NCH),I=1,NCH)
303   IF(NT+NTM.EQ.0)GOTO304
      READ(6)((R1(I,J),J=1,K4),I=1,K6)
      WRITE(4)((R1(I,J),J=1,K4),I=1,K6)
304   IF(NCH.EQ.0)GOTO322
      DO305I=1,K14
      DO305J=1,NCH
      R5(I,J)=0.0
      DO305K=1,NTH
      X=LOOP(I,K)
305   R5(I,J)=R5(I,J)+(X*H12(J,K))
      WRITE(4)((R5(I,J),J=1,NCH),I=1,K14)
       IF(NCM.EQ.0)GOTO337
      DO310I=1,NCM
      DO310J=1,K4
310   R2(I,J)=0.0
      IF(NTM.EQ.0)GOTO337
      M1=NTM
      DO315I=1,NCM
      M1=M1+1
      DO315J=1,NTH
      A5(I,J)=0.0
      DO315K=1,NCH
      M=0
      DO315L=K3,K4
      M=M+1
      X=LOOP(K,L)
315   A5(I,J)=A5(I,J)+(RM(M1,M)*X*A2(K,J))
      DO320I=1,NCM
      DO320J=1,NTH
320   R2(I,J)=A5(I,J)
322   IF(NCM.EQ.0)GOTO337
      IF(NTM.EQ.0)GOTO337
      DO325I=1,NCM
      DO325J=K28,K4
325   R2(I,J)=0.0
      M1=NTM
      DO330I=1,NCM
      M1=M1+1
      M2=0
      DO330J=K3,K4
      M2=M2+1
```

TABLE C-3 (Continued)

```
330   R2(I,J)=RM(M1,M2)
      WRITE(4)((R2(I,J),J=1,K4),I=1,NCM)
      DO335I=1,K14
      M=NTM
      DO335J=1,NCM
      M=M+1
      R6(I,J)=0.0
      M1=NTH+NT
      DO335K=1,NTM
      M1=M1+1
      X=LOOP(I,M1)
335   R6(I,J)=R6(I,J)+(X*RM(K,M))
      WRITE(4)((R6(I,J),J=1,NCM),I=1,K14)
337   IF(NT+NTM.EQ.0)GOTO341
      IF(NC+NCM.EQ.0)GOTO341
      DO340I=1,K14
      M2=NCH
      DO340J=1,K15
      M2=M2+1
      R3(I,J)=0.0
      DO340K=1,K4
      M1=NTH
      DO340L=1,K6
      M1=M1+1
      X=LOOP(I,M1)
      Y=LOOP(M2,K)
340   R3(I,J)=R3(I,J)+(X*R1(L,K)*Y)
      WRITE(4)((R3(I,J),J=1,K15),I=1,K14)
341   CALLEXIT
      END

     MON**      EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM5
      DIMENSION LOOP(10,10),R(12,12),H12(2,2),H21(2,2),RM(6,6),R1(9,11)
      DIMENSION R2(3,11),R3(11,9),R4(3,9),R5(11,2),R6(11,3),R7(11,11)
      REWIND4
      REWIND5
      REWIND6
      READ(5)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      READ(5)((LOOP(I,J),J=1,MTRE),I=1,KO)
      K4=NTH+NT+NTM
      K6=NT+NTM
      K14=NCH+NC+NCM
      K15=NC+NCM
      K18=NCH+NC
      K21=NI+NIE
      K19=NT+NC
      K8=NCH+1
      K16=K18+1
      IF(NT+NC.EQ.0)GOTO345
      READ(5)((R(I,J),J=1,K19),I=1,K19)
345   IF(NCH.EQ.0)GOTO346
      READ(5)((H12(I,J),J=1,NCH),I=1,NTH)
      READ(5)((H21(I,J),J=1,NTH),I=1,NCH)
346   IF(NCM+NTM.EQ.0)GOTO347
      NM=NCM+NTM
      READ(5)((RM(I,J),J=1,NM),I=1,NM)
347   IF(NT+NTM.EQ.0)GOTO348
      READ(4)((R1(I,J),J=1,K4),I=1,K6)
      WRITE(6)((R1(I,J),J=1,K4),I=1,K6)
348   IF(NCH.EQ.0)GOTO349
      READ(4)((R5(I,J),J=1,NCH),I=1,K14)
349   IF(NCM.EQ.0)GOTO350
      IF(NTH+NTM.EQ.0)GOTO350
      READ(4)((R2(I,J),J=1,K4),I=1,NCM)
      WRITE(6)((R2(I,J),J=1,K4),I=1,NCM)
350   IF(NCM.EQ.0)GOTO351
      IF(NTM.EQ.0)GOTO351
      READ(4)((R6(I,J),J=1,NCM),I=1,K14)
351   IF(NT+NTM.EQ.0)GOTO352
      IF(NC+NCM.EQ.0)GOTO352
      READ(4)((R3(I,J),J=1,K15),I=1,K14)
352   IF(NCM.EQ.0)GOTO356
```

TABLE C-3 (Continued)

```
      IF(K4.EQ.0)GOTO356
      DO355I=1,NCM
      M=NCH
      DO355J=1,K15
      M=M+1
      R4(I,J)=0.0
      DO355K=1,K4
      X=LOOP(M,K)
355   R4(I,J)=R4(I,J)+(R2(I,K)*X)
356   IF(K14.EQ.0)GOTO397
      DO360I=1,K14
      DO360J=1,K14
360   R7(I,J)=0.0
      IF(NCH.EQ.0)GOTO376
      DO365I=1,NCH
      DO365J=1,NCH
      IF(I.EQ.J)GOTO363
      R7(I,J)=R5(I,J)
      GOTO365
363   R7(I,J)=1.+R5(I,J)
365   CONTINUE
      IF(K15.EQ.0)GOTO396
      DO370I=K8,K14
      DO370J=1,NCH
370   R7(I,J)=R5(I,J)
      IF(NC.EQ.0)GOTO381
      DO375I=1,NCH
      M=0
      DO375J=K8,K18
      M=M+1
375   R7(I,J)=R3(I,M)
376   M=NT
      DO380I=K8,K18
      M=M+1
      M1=0
      M2=NT
      DO380J=K8,K18
      M1=M1+1
      M2=M2+1
380   R7(I,J)=R(M,M2)+R3(I,M1)
381   IF(NCM.EQ.0)GOTO396
      DO385I=1,K18
      M=0
      M1=NC
      DO385J=K16,K14
      M=M+1
      M1=M1+1
385   R7(I,J)=R6(I,M)+R3(I,M1)
      M=0
      DO390I=K16,K14
      M=M+1
      M1=0
      DO390J=K8,K14
      M1=M1+1
390   R7(I,J)=R3(I,M1)+R4(M,M1)
      M=NTM
      DO395I=K16,K14
      M=M+1
      M1=NTM
      M2=0
      DO395J=K16,K14
      M1=M1+1
      M2=M2+1
395   R7(I,J)=R7(I,J)+RM(M,M1)+R6(I,M2)
396   WRITE(6)((R7(I,J),J=1,K14),I=1,K14)
397   CALLEXIT
      END

     MON$$     EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM6
      DIMENSION LOOP(10,10),R(12,12),H12(2,2),H21(2,2),RM(6,6)
      DIMENSION R1(9,11),R2(3,11),R7(11,11),R8(11,11),R9(9,9),R10(9,7)
      DIMENSION R11(11,11),R13(11,11)
```

TABLE C-3 (Continued)

```
      REWIND4
      REWIND5
      REWIND6
      READ(5)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      WRITE(4)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      READ(5)((LOOP(I,J),J=1,MTRE),I=1,KO)
      WRITE(4)((LOOP(I,J),J=1,MTRE),I=1,KO)
      K4=NTH+NT+NTM
      K6=NT+NTM
      K14=NCH+NC+NCM
      K15=NC+NCM
      K18=NC+NCH
      K19=NT+NC
      K21=NI+NIE
      K16=K18+1
      IF(NT+NC.EQ.0)GOTO400
      READ(5)((R(I,J),J=1,K19),I=1,K19)
400   IF(NCH.EQ.0)GOTO401
      READ(5)((H12(I,J),J=1,NCH),I=1,NTH)
      WRITE(4)((H12(I,J),J=1,NCH),I=1,NTH)
      READ(5)((H21(I,J),J=1,NTH),I=1,NCH)
      WRITE(4)((H21(I,J),J=1,NTH),I=1,NCH)
401   IF(NCM+NTM.EQ.0)GOTO402
      NM=NCM+NTM
      READ(5)((RM(I,J),J=1,NM),I=1,NM)
      WRITE(4)((RM(I,J),J=1,NM),I=1,NM)
402   IF(NT+NTM.EQ.0)GOTO403
      READ(6)((R1(I,J),J=1,K4),I=1,K6)
403   IF(NCM.EQ.0)GOTO404
      IF(NTH+NTM.EQ.0)GOTO404
      READ(6)((R2(I,J),J=1,K4),I=1,NCM)
404   IF(K14.EQ.0)GOTO432
      READ(6)((R7(I,J),J=1,K14),I=1,K14)
      DO406I=1,K14
      DO406J=1,K14
      IF(I.EQ.J)GOTO405
      R8(I,J)=0.0
      GOTO406
405   R8(I,J)=1.0
406   CONTINUE
      M=1
      DO430L=1,K14
      M=M+1
      IF(M.GT.K14)GOTO416
      DO415I=M,K14
415   R7(L,I)=R7(L,I)/R7(L,L)
416   DO420J=1,K14
420   R8(L,J)=R8(L,J)/R7(L,L)
      DO430J=1,K14
      IF(L.EQ.J)GOTO430
      IF(M.GT.K14)GOTO424
      DO423I=M,K14
423   R7(J,I)=R7(J,I)-(R7(L,I)*R7(J,L))
424   DO430IQ=1,K14
      R8(J,IQ)=R8(J,IQ)-(R8(L,IQ)*R7(J,L))
430   CONTINUE
      WRITE(4)((R8(I,J),J=1,K14),I=1,K14)
432   IF(NT+NTM.EQ.0)GOTO448
      DO435I=1,K6
      M=K14
      DO435J=1,K21
      M=M+1
      R10(I,J)=0.0
      DO435K=1,K4
      X=LOOP(M,K)
435   R10(I,J)=R10(I,J)+(R1(I,K)*X)
      WRITE(4)((R10(I,J),J=1,K21),I=1,K6)
      IF(NC+NCM.EQ.0)GOTO442
      DO440I=1,K6
      M=NCH
      DO440J=1,K15
      M=M+1
```

TABLE C-3 (Continued)

```
      R9(I,J)=0.0
      DO440K=1,K4
      X=LOOP(M,K)
440   R9(I,J)=R9(I,J)+(R1(I,K)*X)
      WRITE(4)((R9(I,J),J=1,K15),I=1,K6)
442   IF(K14.EQ.0)GOTO486
      DO445I=1,K14
      DO445J=1,K4
      M=NTH
      R11(I,J)=0.0
      DO445K=1,K6
      M=M+1
      X=LOOP(I,M)
445   R11(I,J)=R11(I,J)+(X*R1(K,J))
448   IF(K4.EQ.0)GOTO486
      IF(K18.EQ.0)GOTO457
      DO455I=1,K18
      DO455J=1,K4
455   R13(I,J)=R11(I,J)
457   IF(NCM.EQ.0)GOTO458
      M=0
      DO485I=K16,K14
      M=M+1
      DO485J=1,K4
485   R13(I,J)=R11(I,J)+R2(M,J)
458   IF(K18.EQ.0)GOTO486
      WRITE(4)((R13(I,J),J=1,K4),I=1,K14)
486   CALLEXIT
      END

     MON$$      EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM7
      DIMENSION LOOP(10,10),H12(2,2),H21(2,2),RM(6,6),R8(11,11)
      DIMENSION R9(9,9),R12(11,11),R10(9,7),R13(11,11),R14(11,7)
      DIMENSION R15(11,7)
      REWIND4
      REWIND5
      REWIND6
      READ(4)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      READ(4)((LOOP(I,J),J=1,MTRE),I=1,KO)
      K4=NTH+NT+NTM
      K9=NTH+NT
      K18=NCH+NC
      K21=NI+NIE
      K14=NCH+NC+NCM
      K6=NT+NTM
      K15=NC+NCM
      K1=NTH+1
      K8=NCH+1
      K16=K18+1
      K11=K9+1
      IF(NCH.EQ.0)GOTO500
      READ(4)((H12(I,J),J=1,NCH),I=1,NTH)
      READ(4)((H21(I,J),J=1,NTH),I=1,NCH)
500   IF(NCM+NTM.EQ.0)GOTO501
      NM=NCM+NTM
      READ(4)((RM(I,J),J=1,NM),I=1,NM)
501   IF(K14.EQ.0)GOTO502
      READ(4)((R8(I,J),J=1,K14),I=1,K14)
      WRITE(6)((R8(I,J),J=1,K14),I=1,K14)
502   IF(K6.EQ.0)GOTO503
      READ(4)((R10(I,J),J=1,K21),I=1,K6)
      WRITE(6)((R10(I,J),J=1,K21),I=1,K6)
      IF(K15.EQ.0)GOTO503
      READ(4)((R9(I,J),J=1,K15),I=1,K6)
503   IF(K14.EQ.0)GOTO541
      IF(K4.EQ.0)GOTO541
      READ(4)((R13(I,J),J=1,K4),I=1,K14)
      DO504I=1,K4
      DO504J=1,K14
504   R12(I,J)=0.0
      IF(NTH.EQ.0)GOTO506
      DO505I=1,NTH
```

TABLE C-3 (Continued)

```
      DO505J=1,NCH
505   R12(I,J)=H12(I,J)
506   IF(NT.EQ.0)GOTO511
      IF(K15.EQ.0)GOTO521
      M=0
      DO510I=K1,K9
      M=M+1
      M1=0
      DO510J=K8,K14
      M1=M1+1
510   R12(I,J)=R9(M,M1)
511   IF(NTM.EQ.0)GOTO521
      IF(NC.EQ.0)GOTO516
      M=NT
      DO515I=K11,K4
      M=M+1
      M1=0
      DO515J=K8,K18
      M1=M1+1
515   R12(I,J)=R9(M,M1)
516   IF(NCM.EQ.0)GOTO521
      M=0
      M2=NT
      DO520I=K11,K4
      M=M+1
      M2=M2+1
      M1=NTM
      M3=NC
      DO520J=K16,K14
      M1=M1+1
      M3=M3+1
520   R12(I,J)=RM(M,M1)+R9(M2,M3)
521   WRITE(6)((R12(I,J),J=1,K14),I=1,K4)
      DO530I=1,K14
      M=K14
      DO530J=1,K21
      M=M+1
      R14(I,J)=0.0
      DO530K=1,K4
      X=LOOP(M,K)
530   R14(I,J)=R14(I,J)+(R13(I,K)*X)
      DO540I=1,K4
      DO540J=1,K21
      R15(I,J)=0.0
      DO540K=1,K14
      DO540L=1,K14
540   R15(I,J)=R15(I,J)+(R12(I,L)*R8(L,K)*R14(K,J))
      WRITE(6)((R15(I,J),J=1,K21),I=1,K4)
541   CALLEXIT
      END
    MON$$     EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM8
      DIMENSION LOOP(10,10),R8(11,11),R10(9,7),R15(11,7),R16(11,7)
      DIMENSION R17(4,7),R18(4,11),R19(11,9),R20(4,9),R12(11,11)
      REWIND4
      REWIND5
      REWIND6
      READ(4)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
      READ(4)((LOOP(I,J),J=1,MTRE),I=1,KO)
      K4=NTH+NT+NTM
      K9=NTH+NT
      K18=NCH+NC
      K21=NI+NIE
      K14=NCH+NC+NCM
      K6=NT+NTM
      K15=NC+NCM
      K22=K14+NI
      K25=NTM+NE
      K23=K25+NCM
      K1=NTH+1
      K16=K18+1
      K26=K25+1
```

TABLE C-3 (Continued)

```
      IF(K14.EQ.0)GOTO600
      READ(6)((R8(I,J),J=1,K14),I=1,K14)
600   IF(NT+NTM.EQ.0)GOTO601
      READ(6)((R10(I,J),J=1,K21),I=1,K6)
601   IF(K4.EQ.0)GOTO602
      IF(K14.EQ.0)GOTO602
      READ(6)((R12(I,J),J=1,K14),I=1,K4)
      READ(6)((R15(I,J),J=1,K21),I=1,K4)
602   IF(K4.EQ.0)GOTO631
      DO605I=1,K4
      DO605J=1,K21
605   R16(I,J)=0.0
      IF(K6.EQ.0)GOTO611
      M=0
      DO610I=K1,K4
      M=M+1
      DO610J=1,K21
610   R16(I,J)=R15(I,J)-R10(M,J)
611   IF(NTH.EQ.0)GOTO616
      DO615I=1,NTH
      DO615J=1,K21
615   R16(I,J)=R15(I,J)
616   M=K22
      DO620I=1,NIE
      M=M+1
      DO620J=1,K21
      R17(I,J)=0.0
      DO620K=1,K4
      X=LOOP(M,K)
620   R17(I,J)=R17(I,J)+(X*R16(K,J))
      DO621I=1,NIE
      DO621J=1,K21
621   R17(I,J)=(-1.)*R17(I,J)
      WRITE(5)((R17(I,J),J=1,K21),I=1,NIE)
      IF(K14.EQ.0)GOTO656
      M=K22
      DO630I=1,NIE
      M=M+1
      DO630J=1,K14
      R18(I,J)=0.0
      DO630K=1,K14
      DO630L=1,K4
      X=LOOP(M,L)
630   R18(I,J)=R18(I,J)+(X*R12(L,K)*R8(K,J))
631   IF(K23.EQ.0)GOTO656
      DO635I=1,K14
      DO635J=1,K23
635   R19(I,J)=0.0
      IF(K25.EQ.0)GOTO641
      DO640I=1,K14
      M=K9
      DO640J=1,K25
      M=M+1
640   R19(I,J)=LOOP(I,M)
641   IF(NCM.EQ.0)GOTO651
      M=0
      DO650I=K16,K14
      M=M+1
      M1=0
      DO650J=K26,K23
      M1=M1+1
      IF(M.EQ.M1)R19(I,J)=1.0
650   CONTINUE
651   DO655I=1,NIE
      DO655J=1,K23
      R20(I,J)=0.0
      DO655K=1,K14
655   R20(I,J)=R20(I,J)+(R18(I,K)*R19(K,J))
      WRITE(5)((R20(I,J),J=1,K23),I=1,NIE)
656   CALLEXIT
      END
```

TABLE C-3 (Continued)

```
        MON**     EXEQ FORTRAN,SOF,SIU,08,04,,,MAINPGM9
         DIMENSION LOOP(10,10),R17(4,7),R20(4,9),R21(4,9)
20       FORMAT(4X,6E12.5)
19       FORMAT(4X,3E12.5)
18       FORMAT(4X,4E12.5)
21       FORMAT(/10X,28HCOEF MATRIX OF PORT CURRENTS//)
22       FORMAT(/10X,30HCOEF MATRIX OF CURRENT DRIVERS//)
23       FORMAT(/10X,30HCOEF MATRIX OF VOLTAGE DRIVERS//)
         REWIND4
         REWIND5
         READ(4)KO,MTRE,NT,NE,NC,NI,NIE,MOD,NCH,NCM,NTH,NTM
         READ(4)((LOOP(I,J),J=1,MTRE),I=1,KO)
         K21=NI+NIE
         K14=NCH+NC+NCM
         K22=NCH+NC+NCM+NI
         K23=NTM+NE+NCM
         K9=NTH+NT
         K25=NTM+NE
         K4=NTH+NT+NTM
         K26=K25+1
         K27=NI+1
         IF(K4.EQ.0)GOTO702
         READ(5)((R17(I,J),J=1,K21),I=1,NIE)
         WRITE(3,21)
         DO700I=1,NIE
700      WRITE(3,18)(R17(I,J),J=K27,NIE)
         IF(NI.EQ.0)GOTO702
         WRITE(3,22)
         DO701I=1,NIE
701      WRITE(3,19)(R17(I,J),J=1,NI)
702      IF(K23.EQ.0)GOTO736
         IF(K14.EQ.0)GOTO716
         READ(5)((R20(I,J),J=1,K23),I=1,NIE)
         DO705I=1,NIE
         DO705J=1,K23
705      R21(I,J)=0.0
         IF(K25.EQ.0)GOTO711
         M=K22
         DO710I=1,NIE
         M=M+1
         M1=K9
         DO710J=1,K25
         M1=M1+1
         X=LOOP(M,M1)
710      R21(I,J)=R20(I,J)-X
711      IF(NCM.EQ.0)GOTO721
         DO715I=1,NIE
         DO715J=K26,K23
715      R21(I,J)=R20(I,J)
         GOTO721
716      M=K22
         DO720I=1,NIE
         M=M+1
         M1=K9
         DO720J=1,K25
         M1=M1+1
         X=LOOP(M,M1)
720      R21(I,J)=(-1.)*X
721      DO730I=1,NIE
         DO730J=1,K23
730      R21(I,J)=(-1.)*R21(I,J)
         WRITE(3,23)
         DO735I=1,NIE
735      WRITE(3,20)(R21(I,J),J=1,K23)
736      CALLEXIT
         END
```

VITA

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