CONVERGENCE OF THE MATRIX CARRY-OVER PROCEDURE IN PLANAR STRUCTURES LOADED NORMAL TO THE PLANE

by

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PREFACE

The problem and its solution presented in this thesis are the results of the author's studies at Oklahoma State University. Essentially, it represents the systematic solution for the statically indeterminate redundants in a rigid framework by an iterative procedure which is physically a systematic restoration of continuity at points where the redundants are located.

This research is the result of concepts expressed by Professor Jan J. Tuma in his lectures on Space Structures during the Summer of 1962 where he introduced the concept of the carry-over matrix. Again in the fall of 1963, Professor Tuma suggested the technique for formulation of the necessary equations which greatly simplified the process and enabled the author to write a much more general computer program.

The author has become indebted to many individuals and organizations for the opportunity to pursue his program of graduate study. Specifically, he must recognize and thank the following individuals and organizations:

To the National Science Foundation and the School of Civil Engineering at Oklahoma State University for the invitation to attend the Summer Institute in Structures and Soil Mechanics during the Summer of 1962. This Institute was primarily responsible for the author's return to his graduate studies.

To the National Science Foundation and South Dakota State University for providing a Science Faculty Fellowship and sabbatical leave which allowed the author to pursue his work on a full time basis from June 1963 until June 1964.

To the College of Engineering, South Dakota State University and Dean M. L. Manning, its School of Civil Engineering and its head Professor Emory E. Johnson for providing the facilities of the computer center for the author's use and for extending his educational leave so that he might complete his program.

To his major professor and adviser Professor Tuma, his committee members Dr. D. M. MacAlpine, Dr. O. H. Hamilton and Dr. R. W. Little, and his former committee members Dr. James W. Gillespie and Professor E. J. Waller for their council and instruction in the classroom and in private discussions.

To his wife, Delores; sons, Steven and Royal; and daughter, Pamela for the many sacrifices which they have made over the past several years to enable the author to continue his work to completion. This represents a personal debt which the author will never be able to repay.

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P. L. K.

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NOMENCLATURE

Superscripts	Absence of superscript indicates a quantity in the basic reference system or a quantity not ordinarily transformed. A superscript m indicates the quantity is in the member reference frame.
Subscript 1	When used as a subscript it will always represent the numeral one rather than the alphabetic character to avoid confusion.
Subscript A	Indicates the collection of all similiar quantities associated with loop A.
Superscripts in parentheses, as	
$S^{(n)}$	Indicates the value of the quantity after the n^{th} iteration.
A, B_1, B_2	General matrices associated with Eq. 12a.
b_{ijk}	Coefficient whose value is 1, 0 or -1 depending on the orientation of member ij relative to cut k.
BM_{ijx}	Internal bending moment at i of member ij in the x direction due to a statically determinate system of loads.
BS_{ij}	Entire collection of BM values on member ij.
C_{ij}	Carry-over matrix
c_{ijm}	Coefficient, 1 or -1, depending upon orientation of member ij and positive direction of loop m.
F	Flexibility matrix of entire structure.
F_{ij}	3x3 sub-matrix of F

f_{ij}	ij^{th} element of F, or flexibility matrix of member ij in the member frame of reference.
f_{ijxy} g_{ijyx}	Near and far end flexibility coefficient.
M_{ijx}	Internal moment at i of segment ij in x direction. M_{ij} indicates entire set of internal moments of segment ij.
$\overline{M}_{ijz}$	Internal deflection at i of segment ij in z direction.
N_{ijz}	Internal force at i of segment ij in z direction.
$\overline{N}_{ijx}$	Internal slope at i of segment ij in x direction.
P_{kz}	Applied force at k in z direction.
$\overline{P}_{ijy}$	Equivalent elastic joint force at i of segment ij in y direction.
$\overline{p}$, $\overline{p}_{jx}$	Rate of change of slope, distributed elastic weight. Subscript indicates point j, x direction.
Q_{kx}	Applied moment at k in x direction.
R	Residual matrix.
S_i	Redundant force and moment matrix at cut i.
s_{ij}	j^{th} component of S_i.
t_{ijk}	Linear transformation matrix from point i to member jk.
U, U*, V	General parameters used in error analysis.
X_{ij}^{m}	Individual member redundants, a subset of M .
x, y, z	Coordinate axes, basic reference system.
x^m, y^m, z^m	Coordinate axes, member reference system.
x_{oi}, y_{oi}	Coordinates from o to i.

Δ_i	Deflection of point i.
Δ_{iz}	Deflection of point i in z direction.
θ_i	Slope of elastic curve at i.
θ_{ix}	Slope of elastic curve at i in x direction.
τ_{ijx}	Angular load function.
σ_{ij}	Collection of all angular load functions associated with member ij.
ω_{oij}	Angle from positive x axis to positive x axis of member ij, or entire angular transformation matrix.
⓶	Indicates location of cut 2, or location of redundant S_2.
$[\ ]^T$	Indicates matrix transpose.

CHAPTER I

INTRODUCTION

1-1. Statement of the Problem

The analysis of a planar, elastic, rigidly connected framework loaded by forces normal to the plane or moments in the plane is investigated. A minimum set of internal forces and moments are chosen as the basic unknowns in the system. Compatibility of the system is realized using a set of equivalent elastic weights applied at the member ends. Utilizing these equivalent elastic weights and the continuity of the elastic curve around specifically defined paths results in the formulation of a sufficient set of equations involving the basic unknowns in the system. For the purposes of this study the problem is considered to be solved when all redundant reactions for each individual member have been found.

The structure is assumed to be a linear system whose supports are rigid but may have known initial deflection or rotations. Only the effects of bending and torsion are considered in the formulation.

Solution of the set of simultaneous equations is accomplished by systematic restoration of the continuity of the structure. This process leads to a multi-dimensional carry-over technique referred to as the matrix carry-over technique. Convergence of this process is investigated.

1-2. Analogy between the Matrix Carry-Over Technique and the Carry-Over Procedure in Continuous Beams

The matrix carry-over technique is an extension of the work of Tuma (1)* from a one-dimensional carry-over to, in this case, three dimensional carry-over.

By arranging the terms of the simultaneous equations in a particular way dictated by the structure, each step of the iterative procedure has a physical meaning which includes the following three steps:

1. Fictitious cuts are made at a sufficient number of locations such that the continuous structure is reduced to a series of determinate elements or trees.
2. From the end slopes and deflections of these simple structures starting values are computed. (The starting values are the internal force and moments at each cut to produce continuity at that cut when all of the other cuts are free of any force or moment.)
3. By means of a direct matrix carry-over procedure, the full continuity of the elastic curve is established.

Since continuity is restored to the structure in three directions simultaneously, the process involves a three

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*Numbers in parentheses refer to references in the Bibliography.

dimensional carry-over, hence the term matrix carry-over. Tuma (1) outlines an equivalent set of steps in the solution of the redundant elements in continuous beams.

1-3. Historical Background

In the formulation of the equations necessary for the solution of the redundant quantities in a structural system, the analyst may choose as unknowns either a set of forces and moments or a set of deflections and rotations. This choice of moments and forces as the basic unknowns generally leads to a method of analysis referred to as the flexibility approach. Using slopes and deflections as unknowns leads to the stiffness approach.

Basically, the flexibility approach requires that a sufficient number of internal redundants be selected as unknowns. Since the structure must exhibit known continuity of the elastic curve, this continuity produces the required relationships to determine the set of internal redundants.

Similiarly, the stiffness approach requires that a sufficient number of slopes and deflections be selected as the unknown quantities and then uses the conditions of equilibrium to produce the required equations necessary to compute the values of the selected set of slopes and deflections.

Usually moments and forces can be considered as primary objectives in the analysis of structural frameworks. Slopes and deflections are considered to be secondary products of the analysis. This does not imply that deflections and slopes are less important but merely that a structural framework without the necessary strength requires little further consideration.

The first formulation of an analysis procedure for general redundant structures began with Clapeyron (2) over a century ago with the formulation of the three moment equation. Maxwell (2) followed shortly with a more general solution utilizing flexibility influence coefficients. Mohr (2) contributed the concepts of the elastic weights which could be applied to a beam as loads and produced slopes and deflections instead of shears and moments. This technique is referred to as the conjugate beam method. Just prior to the turn of the century, Müller-Breslau (3) applied the distributed elastic weights of Mohr as a set of concentrated forces at a series of joints.

Baron and Michalos (4) and Kinney (2) utilized the distributed elastic weights recently in the solution of planar frames and also applied the technique to beams in space. Diwan (5) extends the method of Baron and Michalos using an equivalent elastic system concept.

Within the last decade, Tuma and many of his students have applied the concept of the elastic joint force, distributed elastic weights and the string polygon to various structures. Works by Tuma (1), Tuma and Oden (6) and Oden (7) represent a few of these contributions and contain a more extensive bibliography in this area than will be attempted here.

All of the investigations cited above give rise to sets of equations utilizing forces and moments as unknowns and are classified in the broad area of flexibility techniques.

Actually, at the present time, stiffness techniques are being used in a majority of the analytical procedures used in structural analysis. Using slopes and deflections as the basic unknowns was selected by Maney (8) whose formulation of the slope deflection equation produced a convienient manner of formulating a sufficient set of equations for the determination of all redundant elements in a structure. Mohr (2) is usually credited with the original use of this technique. Cross (9) produced an iterative technique, referred to as moment distribution which was in reality a rearrangement of the slope deflection equations. This method became an extremely popular method as it eliminated the necessity of actual solutions of large numbers of simultaneous equations. Southwell (10,11) developed a similiar technique independently of Cross and extended the technique to other engineering problems.

Modern high speed computing equipment has expanded the engineers capacity for the solutions of large numbers of simultaneous equations. Most computer analysis of large structural systems is accomplished using a generalized form of the slope deflection equations. As typical examples, Eisemann, Woo, and Namyet (12) describe a general formulation involving a space framework of well in excess of a thousand members, Carter (13) utilizes the technique in connection with the problem of critical buckling loads, and Fenves (14) has used similiar processes in the developement of a general computer program for structural analysis.

Theoretically, the comparison of which method is best, most efficient, or shortest can be answered at least in part by the results of Samuelson (15). He shows that mathematically the flexibility approach and stiffness approach are the duals of one another and thus require essentially the same technique.

Practically, the generalized slope deflection equation provides an almost automatic approach to the formulation if <u>all</u> of the slopes and <u>all</u> of the deflections at <u>each</u> joint are used as unknowns. This number is considerably greater than the minimum number of redundants necessary for the complete evaluation of the structural system. The method also provides rather simple procedures for including the effects of boundary conditions of a general nature. On the other hand, the flexibility technique usually leads quite naturally to the formulation of a smaller number of equations but requires a considerable amount of programming effort. Thus, if a trade must be made, usually the addition of a few unknowns is not as significant as an increase in programming effort.

Numerical techniques of solving the equations resulting from the particular choice of unknowns comprise a vast set of algorithms and procedures which is expanding at an extremely fast rate. Two general classifications of iterative techniques are available. The first is a technique which utilizes a certain definite process over and over again until the answers of the required accuracy are obtained. The common Gauss-Siedel process is of this form. The second technique is a process whose next step is dependent upon the previous step or the magnitudes of the quantities involved. The method referred to as the method of steepest descent is a process of this form.

Two rather recent books by Varga (16) and Faddeev and Faddeeva (17) contain extensive discussions of the above iterative procedures. The technique employed in this investigation is referred to by Varga as the Gauss-Siedel process and by Faddeev as the method of Nekrasov. In this study, it will be referred to as the Gauss-Siedel process. In any event, both of the references show that positive definiteness of a real, symmetric matrix is a necessary and sufficient condition for the convergence of the process. Faddeev and Faddeeva also state that as a rule convergence of a group Gauss-Siedel process is more rapid than the convergence of a corresponding point Gauss-Siedel technique. Southwell (10) and Temple (18) among others have shown that any flexibility or stiffness matrix is positive definite as a consequence of the positive nature of internal strain energy.

The physical nature of the problems solved by the matrix carry-over process in this investigation is such that it employs essentially the use of the Gauss-Siedel process taking blocks or groups of three equations at a time. Chapter V illustrates two different approaches to this iterative solution: specifically, the reduction of residual vectors to zero and straight forward convergence to the required solution.

CHAPTER II

MATHEMATICAL MODEL

2-1. Assumptions

The structure is assumed to be a system of members lying in a plane whose ends are rigidly connected with one another and whose supports are either fully fixed or have known initial deflection or rotations. Properties of each individual member are assumed to be known and are of such a nature that deflections in the plane or rotations normal to the plane do not occur. Deflections due to shearing forces are considered to be small compared with those due to moments and torsions and are neglected. Axial deformations are not considered. Members are identified by the numbers associated with the joints which coincide with the members end points. The member may have any shape or loading providing the above properties are realized.

Loads are stationary and may be either force vectors normal to the plane or moment vectors in the plane. The magnitude of the loading is constant.

All ordinary assumptions of linear elasticity are presumed.

2-2. Coordinate Systems

Two different coordinate systems are required. They are referred to as the basic or reference system for the entire structure and the member oriented system.

The basic system is a right handed orthogonal set of axes oriented in a convenient manner with the x and y axes in the plane of the structure and the positive z axis acting upward from the plane of the structure.

The member oriented system referenced to the undeformed structure has the z^m axis parallel to the z axis of the basic system but its x^m axis oriented such that the origin is at i and the positive x^m axis goes through j where $i < j$. The y^m axis is placed such that the system is right handed. The angle from the x axis of the basic system to the x^m axis of the member oriented system is designated ω_{oij} and is positive if it represents a right handed rotation about the z axis. See Figure 2-2.

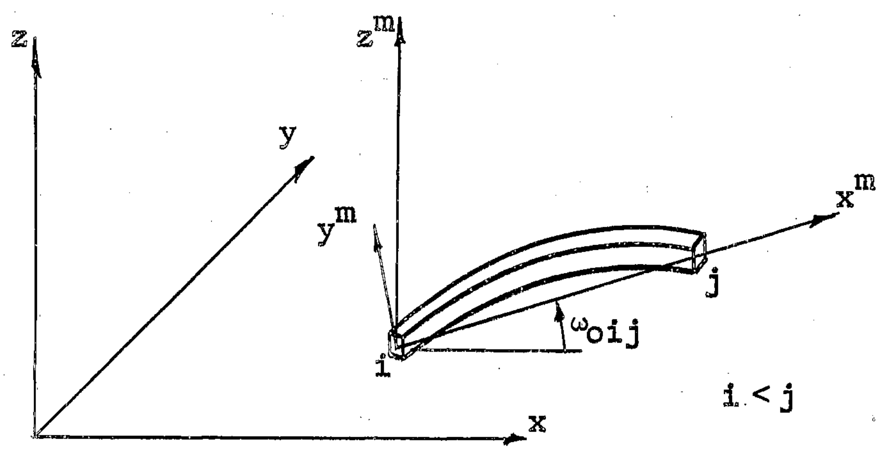


Figure 2-2: Coordinate Systems

Any values which are based on the member oriented reference system will be designated by a superscript m. Values referred to the basic system carry no superscript.

2-3. Definitions

Several terms are used throughout this paper which are of sufficient importance to be singled out here and defined rather carefully. They are terms borrowed from topological or linear graph treatments of various problems.

Tree - Every structure of the type considered in this paper can, with proper choice of redundants, be reduced to the consideration of a collection of statically determinate 'trees'. For the purposes of this paper, a tree will be considered one of the stable collections of members remaining after a sufficient number of cuts has been made to reduce the problem to a statically determinate one. The support of this tree is entirely contained at one joint referred to as its base. The number of trees is equal to the number of rigid supports.

Loop - A loop for the purposes of this paper will consist of an ordered sequence of joints which describes a complete path from either the base of one tree to the base of another tree or from one joint around a path completely contained in one tree and returning to the same joint. In the case of a loop contained in one tree the joint nearest the base is considered to be the beginning and end of the loop. In all cases a loop will contain one, and only one, cut.

Path. - A path is an ordered sequence of joints such that in traversing the path no member is traversed more than once.

With the definitions and with the assumptions regarding the end restraints of the structure, it is possible to make the following observation: For each 'cut' or location of redundants, one, and only one, loop can be found which satisfies the definition.

These definitions are illustrated in Figure 2-3. The configurations of the trees and loops are not unique for any given structure but depend upon the choice of the redundant cuts.

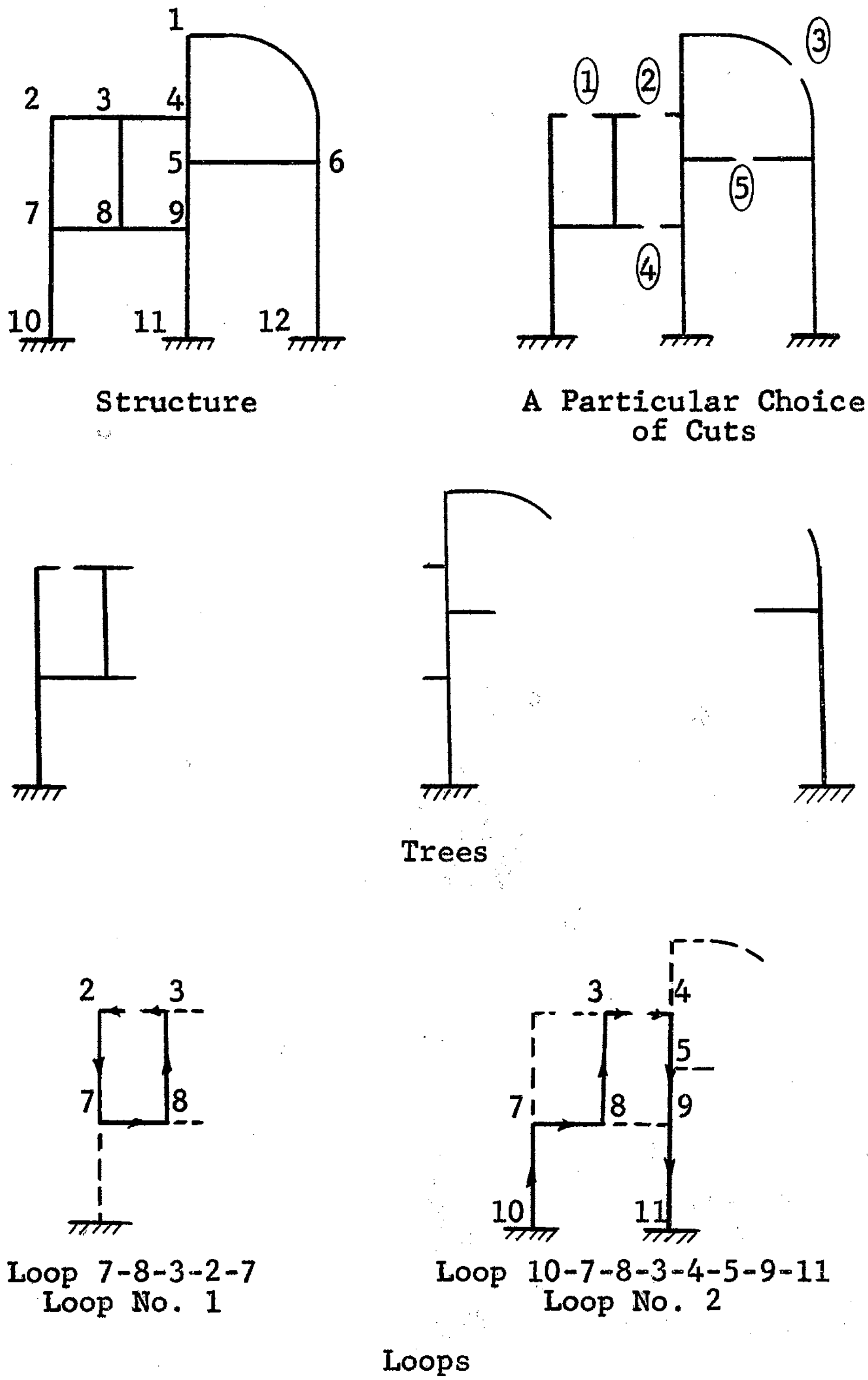


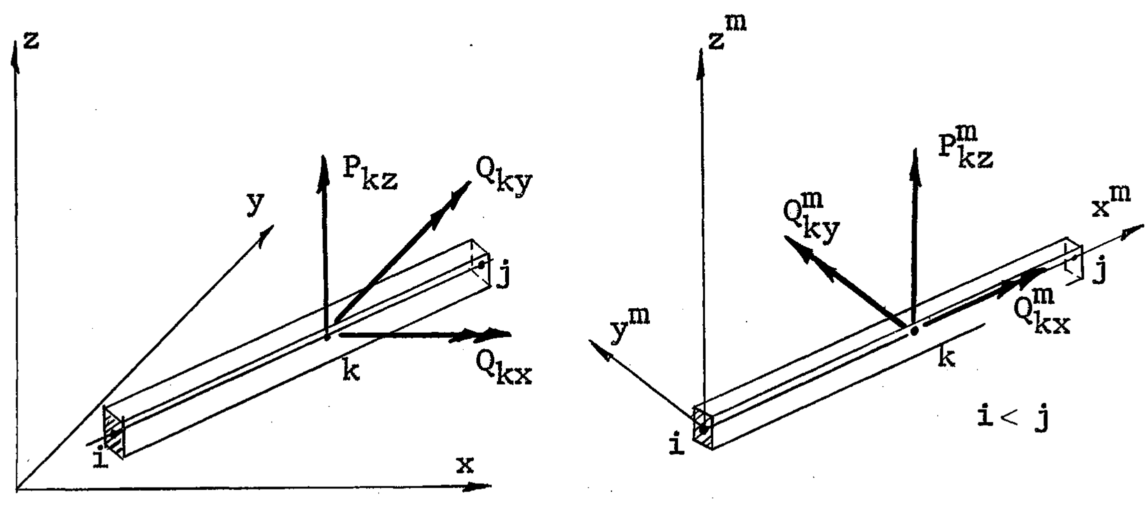
Figure 2-3: Trees and Loops

CHAPTER III

EQUILIBRIUM

3-1. Sign Convention and Notation, External and Internal Forces and Moments

External loads applied to the structure are force vectors normal to the plane and moment vectors in the plane. Forces are designated with single headed arrows and moments with double headed arrows. In either case they are positive when in the positive direction of the appropriate reference system, Figure 3-1.0.



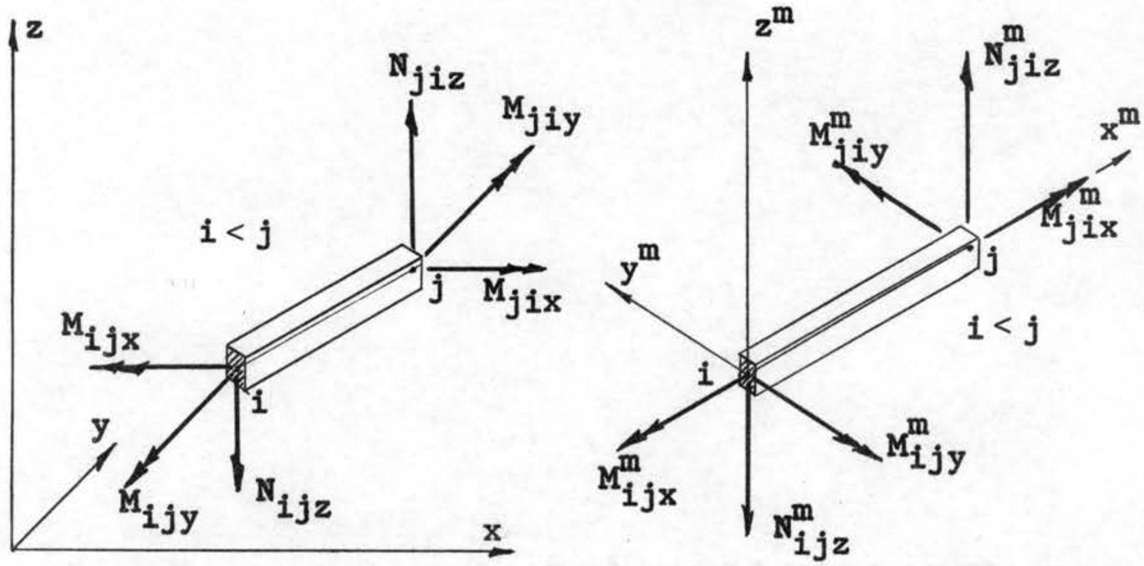
(a) Relative to Basic System

(b) Relative to Member System

Figure 3-1.0: Positive Applied Forces and Moments

Internal forces and moments, Figure 3-1.1, are also designated by single and double headed arrows as with applied forces and moments but consideration must be given to which face of the cut is termed the plus face. The positive face of any beam segment is always the face nearest end j of the beam ij where i < j. On this face internal forces and moments are plus when they coincide with the plus directions of the appropriate reference system.

Since reactions are treated as internal forces they require no special consideration.



(a) Relative to Basic System (b) Relative to Member System

Figure 3-1.1: Positive Internal Forces and Moments

3-2. Redundant Notation at the Cuts

The redundant forces and moments at each cut are designated by the matrix

$$\left[S_i\right] = \begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \end{bmatrix} = \begin{bmatrix} M_{ijy} \\ N_{ijz} \\ M_{ijx} \end{bmatrix} = \begin{bmatrix} M_{iky} \\ N_{ikz} \\ M_{ikx} \end{bmatrix} \qquad (1)$$

where: i identifies the cut

Furthermore, these elements follow the same sign convention given for internal forces but are applied at the origin of the basic coordinate system, Figure 3-2, using a hypothetical set of rigid arms as a portion of the member.

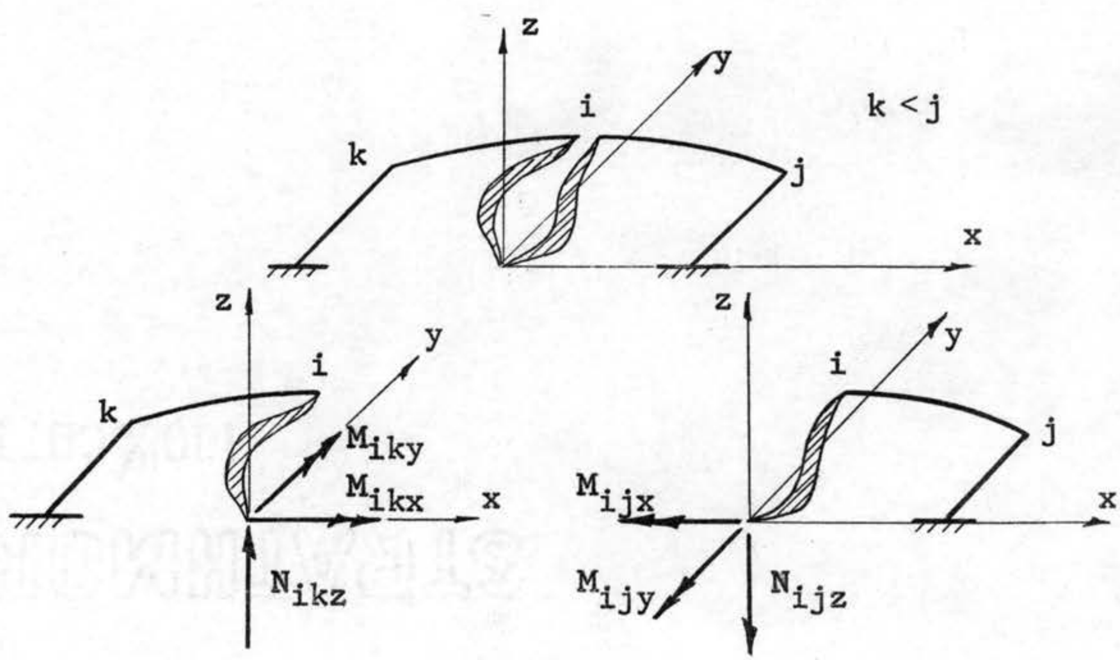


Figure 3-2: Positive Directions of Redundants

3-3. Redundant Notation, Single Member

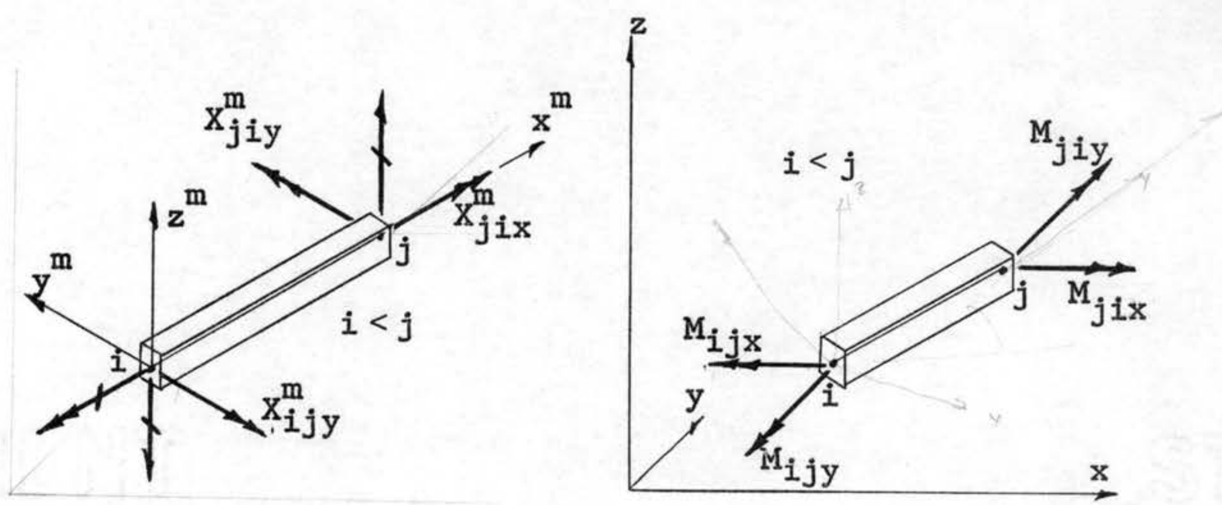
The redundant quantities associated with each member ij, $i < j$, are the torsional moment at end j and the bending moments at ends i and j. Figure 3-3 indicates these values in first the member frame of reference and then the basic reference system.

Obviously,

$$\begin{bmatrix} X^m_{ijy} \\ X^m_{jix} \\ X^m_{jiy} \end{bmatrix} = \begin{bmatrix} \cos\omega_{oij} & -\sin\omega_{oij} & 0 & 0 \\ 0 & 0 & \cos\omega_{oij} & \sin\omega_{oij} \\ 0 & 0 & -\sin\omega_{oij} & \cos\omega_{oij} \end{bmatrix} \begin{bmatrix} M_{ijy} \\ M_{ijx} \\ M_{jix} \\ M_{jiy} \end{bmatrix} \quad (2)$$

or

$$\left[X^m_{ij}\right] = \left[\omega_{oij}\right]^T \left[M_{ij}\right] \quad (3)$$



(a) Single Member Redundants Member Reference

(b) Single Member Moments Basic Reference

Figure 3-3: Single Member Redundants and Moments

It should be obvious that if the member loads plus member redundants given in equation 2 are available the undesignated forces in Figure 3-3a are easily evaluated from a consideration of static equilibrium of the individual member and their computation is not considered here.

3-4. Member Redundants in Terms of Loads and Redundants at the Cuts

Consider a portion of the structure shown in Figure 2-3 and repeated below in Figure 3-4.

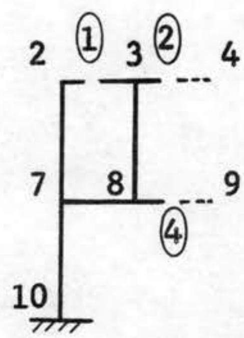


Figure 3-4: Typical Tree

Obviously, the redundants of each member are a function of the loads and the redundants at the cuts. Specifically, if all possible paths from any member are traversed in a direction away from the base beginning at the member end farthest from the base all cuts affecting this member will be encountered. In fact, if there are <u>two</u> paths to the same cut then those redundants <u>do not</u> affect the member redundants.

If the member contains a cut then the member redundants are functions of the loads and the redundants at that one cut only.

In Figure 3-4 the member redundants of member 7-10 are functions of the loads plus the redundants at cuts 2 and 4. The member redundants of member 7-8 are functions of the loads plus the redundants at cuts 1, 2 and 4.

Finally, the relationship between these redundants may be stated as

$$\begin{bmatrix} M_{ijy} \\ M_{ijx} \\ M_{jix} \\ M_{jiy} \end{bmatrix} = \begin{bmatrix} 1 & x_{oi} & 0 \\ 0 & -y_{oi} & 1 \\ 0 & -y_{oj} & 1 \\ 1 & x_{oj} & 0 \end{bmatrix} \left[b_{ij1}I \,\vdots\, b_{ij2}I \,\vdots\, b_{ij3}I \,\vdots\, \cdots \,\vdots\, b_{ijm}I \right] \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \vdots \\ S_m \end{bmatrix} + \begin{bmatrix} BM_{ijy} \\ BM_{ijx} \\ BM_{jix} \\ BM_{jiy} \end{bmatrix} \quad (4a)$$

or

$$\left[M_{ij}\right] = \left[t_{oij}\right]^T \left[b_{ij}\right] \left[S\right] + \left[BS_{ij}\right] \quad (4b)$$

where: S_k = redundants at cut k, equation 1

x_{oi}, y_{oi} = coordinates of point i measured from the origin of the basic system to point i

b_{ijk} = 0 if there are 2 or 0 paths wholly contained in the tree containing member ij from the member to the cut k

b_{ijk} = 1 if the member containing cut k is numbered in the same order as the member ij when traversing the path in one direction, or member ij contains cut k

b_{ijk} = -1 if the member containing cut k is numbered in the opposite order as the member ij when traversing the path in one direction.

I = a 3x3 unit matrix.

BS_{ij} = basic system moments or moments due to the applied loads in the determinate system.

Thus, the b coefficients applicable to the tree shown in Figure 3-4 are

For Member 7-8	For Member 7-10
$b_{7,8,1} = -1$	$b_{7,10,2} = -1$
$b_{7,8,2} = 1$	$b_{7,10,4} = -1$
$b_{7,8,4} = 1$	all others 0
all others 0	

Some comments are appropriate at this point. Namely, for any given loads, member and set of redundant values at the cuts, equation 4a together with the three available equations of static equilibrium are sufficient to establish the equilibrium of the member in question. Furthermore, since all redundants at the cuts are referenced to the origin of the basic coordinate system the actual location of the cut between the member ends has no effect upon the formulation except for any necessary changes in the basic system moments. Also, the product of the b_{ij} matrix and the S_i matrix is actually nothing more than the sums (with the proper sign) of the S_i matrices actually affecting the moments in the member ij. Because all S_i matrices are referenced to the origin only one linear transformation matrix t is required.

CHAPTER IV

COMPATIBILITY

4-1. Sign Convention of Slopes and Angle Changes

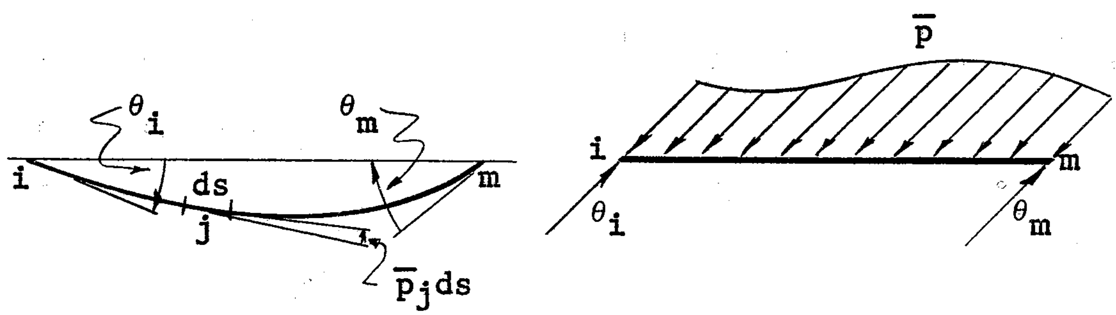
Distributed angle changes along the continuous elastic curve of a structure may be represented by vectors. In addition, if the slopes of the members are sufficiently small they may also be treated as a vector quantity. This assumption regarding the magnitudes of the slopes will be made.

As with internal forces, it is necessary to associate a direction along the curve with whatever angle changes are involved. Figure 4-1.1 indicates the manner in which these angle changes could be indicated for a segment of beam im. The distributed angle changes or the rates of change of slope are referred to as elastic weights.

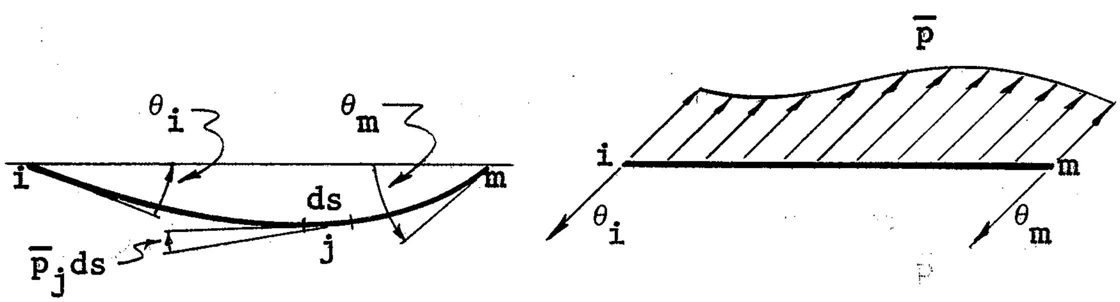
Obviously, traversing the curve in the opposite direction requires that the angle changes be reversed. For the purposes of this paper the positive direction of the path of each individual member will be taken from i to m if $i < m$.

Positive angle changes are defined as shown in Figure 4-1.2. Deflections are measured from the elastic curve to a horizontal reference and are plus if they are in the positive direction of the appropriate reference system.

Slopes are measured from the tangent of the elastic curve to a horizontal reference and are plus if they are in the positive direction of the appropriate coordinate system.



(a) Traversing the Path from i to m



(b) Traversing the Path from m to i

Figure 4-1.1: Vector Representation of Slopes and Angle Changes and Their Dependence Upon Direction of Path

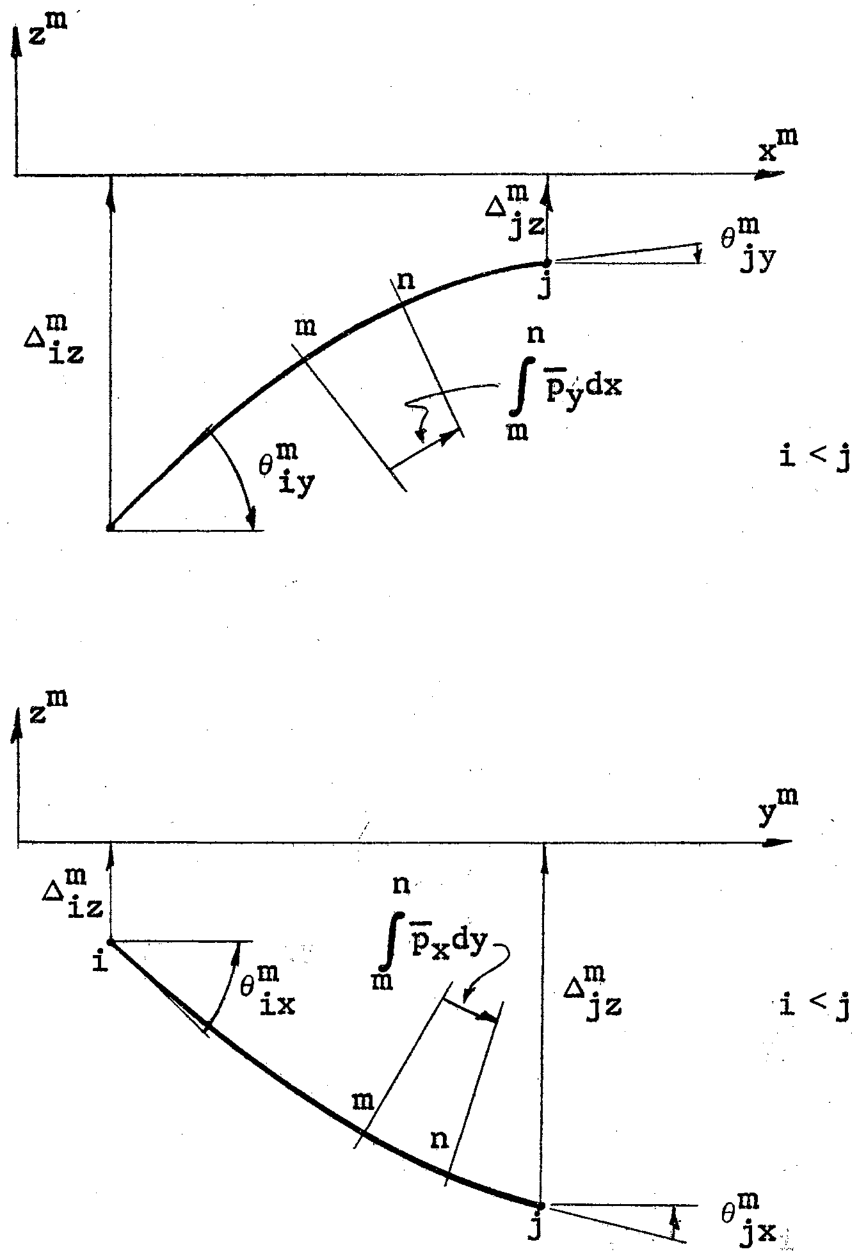


Figure 4-1.2: Positive Deflections, Slopes and Rates of Change of Slopes

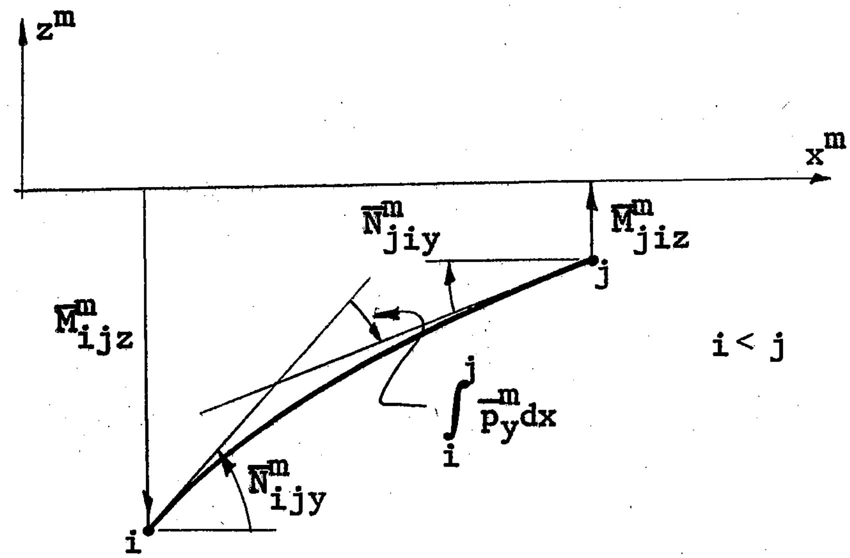
4-2. Analogy between Load, Shear and Moment and Distributed Angle Changes, Slope and Deflection

Figure 4-2.1 indicates the analogy between internal force and moment elements with slope and deflection. Figure 4-2.1a shows the elastic curve of a beam element ij subjected to bending about the y^m axis in a manner such that all end slopes, end deflections and distributed angle changes are plus.

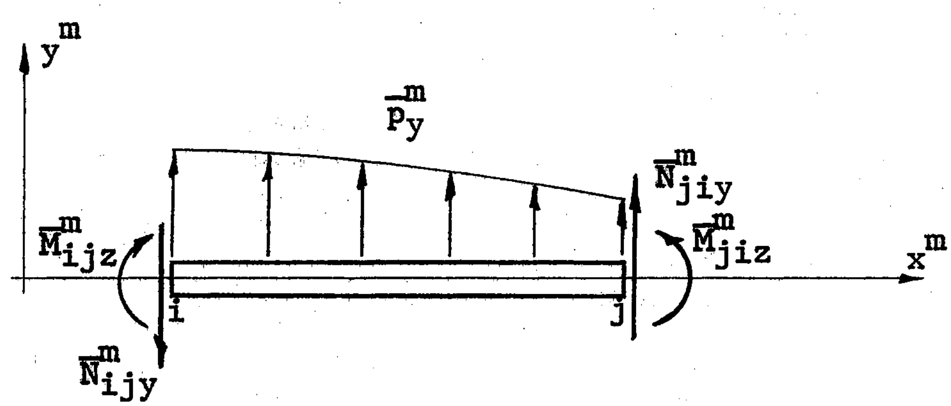
In order to complete the analogy it is necessary to introduce internal slopes and deflections similiar to shears and moments. Figure 4-2.1a illustrates such a set. From a comparison of Figure 4-1.2 and Figure 4-2.1 it is apparent that the positive internal slopes and deflections correspond to actual slopes and deflections at the far end and are in the positive directions of the coordinate axes. The set of positive near end internal slopes and deflections is opposite the direction of the coordinate axes.

Figure 4-2.1b indicates the geometric variables as loads and internal forces and moments on the member. From considerations of equilibrium of this analogous system

$$\begin{aligned}
\overline{M}^m_{ijz} &= \overline{M}^m_{jiz} + \int_1 \overline{p}^m_{ky} x^m_{ik} dx^m + \overline{N}^m_{jiy} x^m_{ij} \\
\overline{M}^m_{jiz} &= \overline{M}^m_{ijz} - \int_1 \overline{p}^m_{ky} x^m_{jk} dx^m + \overline{N}^m_{ijy} x^m_{ji} \\
\overline{N}^m_{jiy} &= \overline{N}^m_{ijy} - \int_1 \overline{p}^m_{ky} dx^m \\
\overline{N}^m_{ijy} &= \overline{N}^m_{jiy} + \int_1 \overline{p}^m_{ky} dx^m
\end{aligned} \qquad (5)$$



(a) actual geometry of deflected beam and positive sense of end slopes and deflections for purpose of the beam analogy



(b) equivalent elastic loads on beam segment

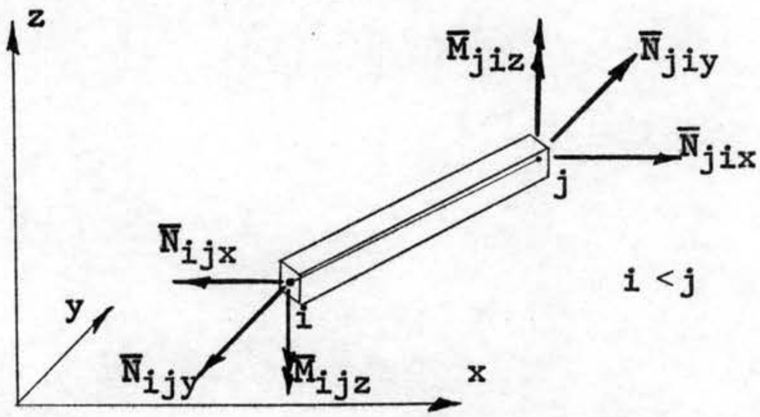
Figure 4-2.1: Angle Change-Force, Slope-Shear and Deflection-Moment Analogy for Bending about y axis

Entirely equivalent results are obtained from considerations of geometry. These equations are analogous to those obtained for shears and moments in a beam loaded with a positive set of distributed loads and represent the conjugate beam analogy for this particular coordinate direction. The equations will be referred to as the equations of elasto-static equilibrium.

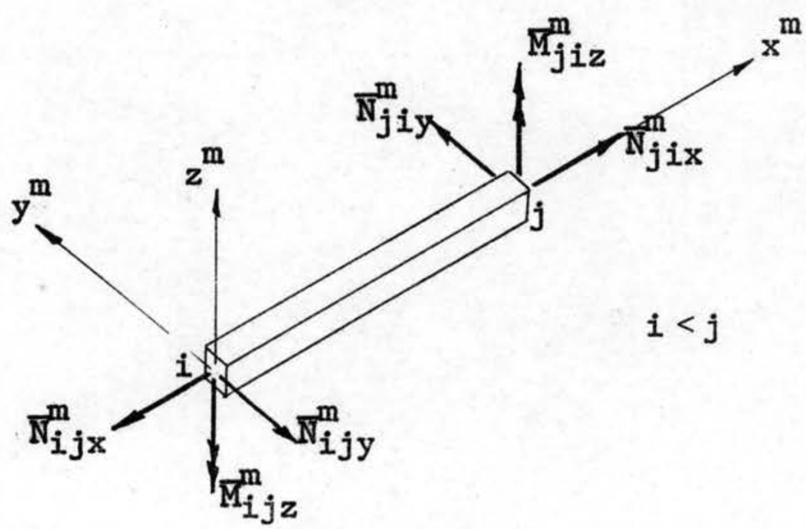
Physically, at i, the near end slopes and deflection represent the angle from the horizontal reference plane to the plane containing the tangent and bi-normal to the elastic curve at i and the displacement of the horizontal reference plane to point i. If these quantities are in a direction opposite the positive direction of the appropriate reference system, they are plus. Similiarly, at j, the far end slope and deflection represent an angle and displacement from the plane containing the tangent and bi-normal to the elastic curve at j to the horizontal reference plane. If these quantities are in the direction of the positive direction of the appropriate reference system, they are plus. Thus, the sense of these end effects is identical in form to internal shear and moment.

Identical analogies in the other two bending directions can be made. If the effects of shearing deformations, axial force deformations or uniform changes in temperature are to be included, they become analogous to a set of distributed moments rather than distributed forces. This class of problem is not investigated in this study.

Figure 4-2.2 indicates positive sets of internal slopes and deflections in the basic reference system and the member reference system.



(a) Positive Internal Deflections and Rotations
Basic System



(b) Positive Internal Deflections and Rotations
Member System

Figure 4-2.2: Positive Internal Deflections and Rotations
Member ij

4-3. Equivalent Elastic Weight Systems

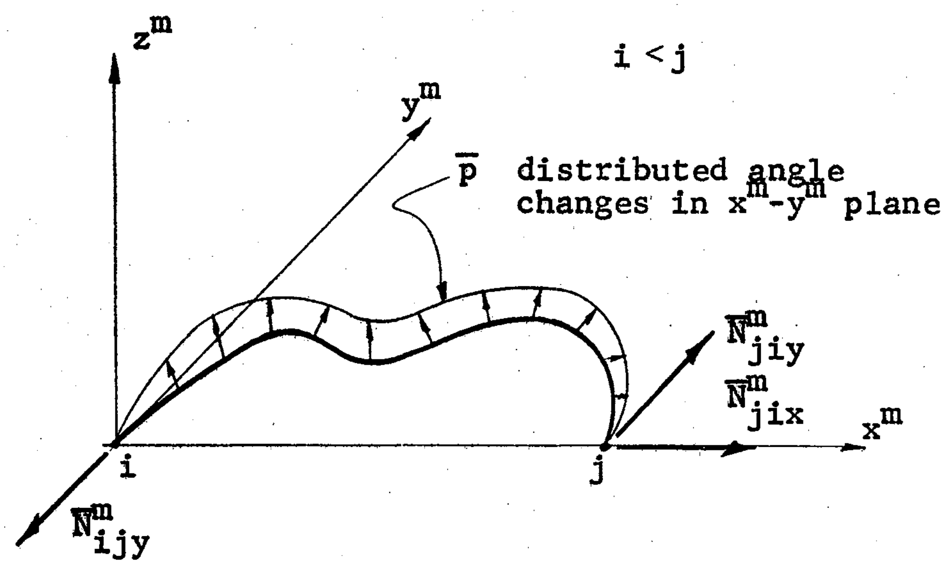
For any segment of a beam used in the class of structures considered in this investigation the distributed elastic weights are in the plane of the member. Since the distributed elastic weights are represented by an analogous set of distributed forces, the idea of replacing the distributed set with a statically equivalent concentrated set occurs quite naturally.

Many equivalent sets are possible. Since the distributed set of elastic weights is a function of the loads and redundants, the set of redundants for the member ij of Figure 3-3a will be used as a quide in the choice of this equivalent set. In short, the distributed set of angle changes will be replaced with the set $\bar{P}^m_{ijy}$, $\bar{P}^m_{jiy}$ and $\bar{P}^m_{jix}$ of Figure 4-3b.

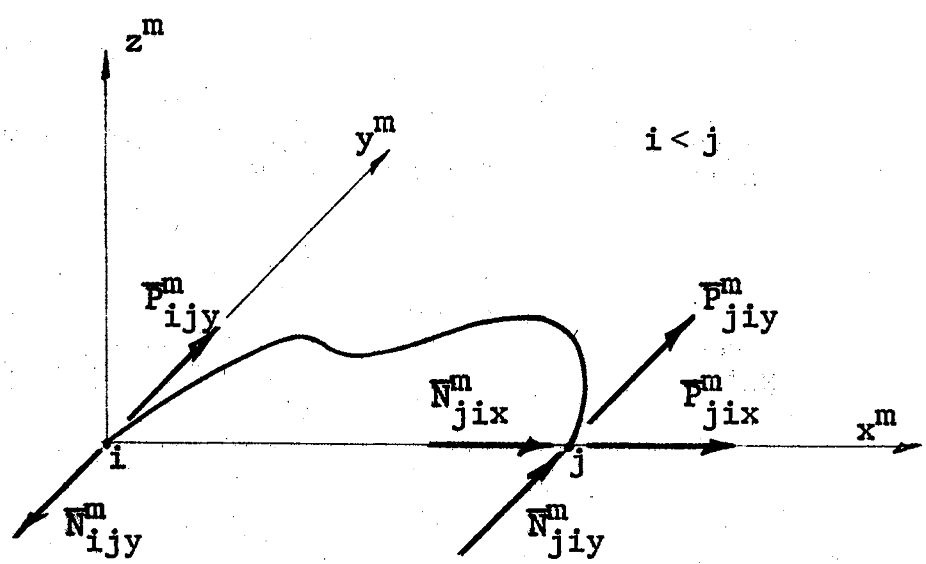
Additionally, if the beam ij is restrained as in Figure 3-3a such that $\bar{M}^m_{ijz} = \bar{M}^m_{jiz} = \bar{N}^m_{ijx} = 0$, then the internal slopes and deflections of the segment ij reduce to the three rotations shown in Figures 4-3a and 4-3b.

This restraint provides a convenient method of obtaining the equivalent elastic weights in terms of available data, that is

$$\begin{bmatrix} \bar{N}^m_{ijy} \\ -\bar{N}^m_{jix} \\ -\bar{N}^m_{jiy} \end{bmatrix} = \begin{bmatrix} \bar{P}^m_{ijy} \\ \bar{P}^m_{jix} \\ \bar{P}^m_{jiy} \end{bmatrix} = \begin{bmatrix} f_{ijyy} & g_{ijyx} & g_{ijyy} \\ g_{jixy} & f_{jixx} & f_{jixy} \\ g_{jiyy} & f_{jiyx} & f_{jiyy} \end{bmatrix} \begin{bmatrix} X^m_{ijy} \\ X^m_{jix} \\ X^m_{jiy} \end{bmatrix} + \begin{bmatrix} \tau^m_{ijy} \\ \tau^m_{jix} \\ \tau^m_{jiy} \end{bmatrix} \qquad (6)$$



(a) Beam Restrained such that $\overline{M}^m_{ijz} = \overline{M}^m_{jiz} = \overline{N}^m_{ijx} = 0$ and Distributed Elastic Weights



(b) Equivalent System of Elastic Weights Applied at Joints

Figure 4-3: Equivalent Sets of Elastic Weights

or

$$\left[P^{m}_{ij}\right] = \left[f_{ij}\right]\left[X^{m}_{ij}\right] + \left[\sigma^{m}_{ij}\right]$$

where the f and g flexibility coefficients and the τ functions are as defined in Table 4-3 and have been tabulated for a number of beams and loadings, see Tuma, et. al. (24). This also provides justification for the particular choice of redundant system associated with each member. If the flexibility coefficients are required, it is assumed that they are available and their actual calculation is not considered here.

f_{ijxy}	Near end angular moment flexibility. f indicates cause and effect at same end. First and third subscript indicate location and direction of cause. Fourth indicates direction of effect. Hence, f_{ijxy} represents rotation at i in x direction due to a unit moment at i in the y direction. Maxwell's reciprocal theorem implies that $f_{ijxy} = f_{ijyx}$
g_{jiyx}	Far end angular moment flexibility. g indicates cause is at end opposite effect. As for f above, g_{jiyx} represents the rotation at j in the y direction due to a unit moment at i in the x direction. Maxwell's reciprocal theorem implies $g_{jiyx} = g_{ijxy}$
τ_{ijy}	Angular load function. Represents the end rotations of basic determinate segment due to loads on segment. τ_{ijy} is the rotation due to loads on the span. The rotation is about the y axis at i of the segment ij.

Table 4-3: Flexibility Notation

4-4. Elasto-Static Equilibrium of a Closed Loop Containing a Single Member

All of the distributed elastic weights for the member jk may be replaced with the set given by equation 6. However, it is more convienient to transform these elastic weights to the basic reference system as follows:

$$\left[\overline{P}_{jk}\right] = \begin{bmatrix} \overline{P}_{jky} \\ \overline{P}_{jkx} \\ \overline{P}_{kjx} \\ \overline{P}_{kjy} \end{bmatrix} = \left[\omega_{ojk}\right]\left[\overline{P}^{m}_{jk}\right] \qquad (7)$$

Thus the advantages of the choice of equivalent elastic weights of Figure 4-2 becomes apparent. The angular transformation matrix is the transpose of the one previously used in equation 3.

Since the internal slopes and deflections at the far end of member ij are equal and opposite to those at the near end of member jk, their mutual contribution to the elasto-static equilibrium cancels and the only effective elasto-static forces are those shown in Figure 4-4.

For the system to be in equilibrium in the sense of equation 5

$$\begin{bmatrix} \overline{N}_{ijy} \\ \overline{M}_{ijz} \\ \overline{N}_{ijx} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ x_{oj} & -y_{oj} & -y_{ok} & x_{ok} \\ 0 & 1 & 1 & 0 \end{bmatrix} \left[\overline{P}_{jk}\right] + \begin{bmatrix} \overline{N}_{mky} \\ \overline{M}_{mkz} \\ \overline{N}_{mkx} \end{bmatrix} \qquad (8)$$

or

$$\left[\bar{N}_{ij}\right] = \left[t_{ojk}\right]\left[\bar{P}_{jk}\right] + \left[\bar{N}_{mk}\right]$$

Again the judicious choice of equivalent elastic weights and the extension of the member ends to the origin using the fictitious rigid arms becomes apparent in the appearance of the translational transformation matrix which is the same as the transpose of the one used in equation 4a.

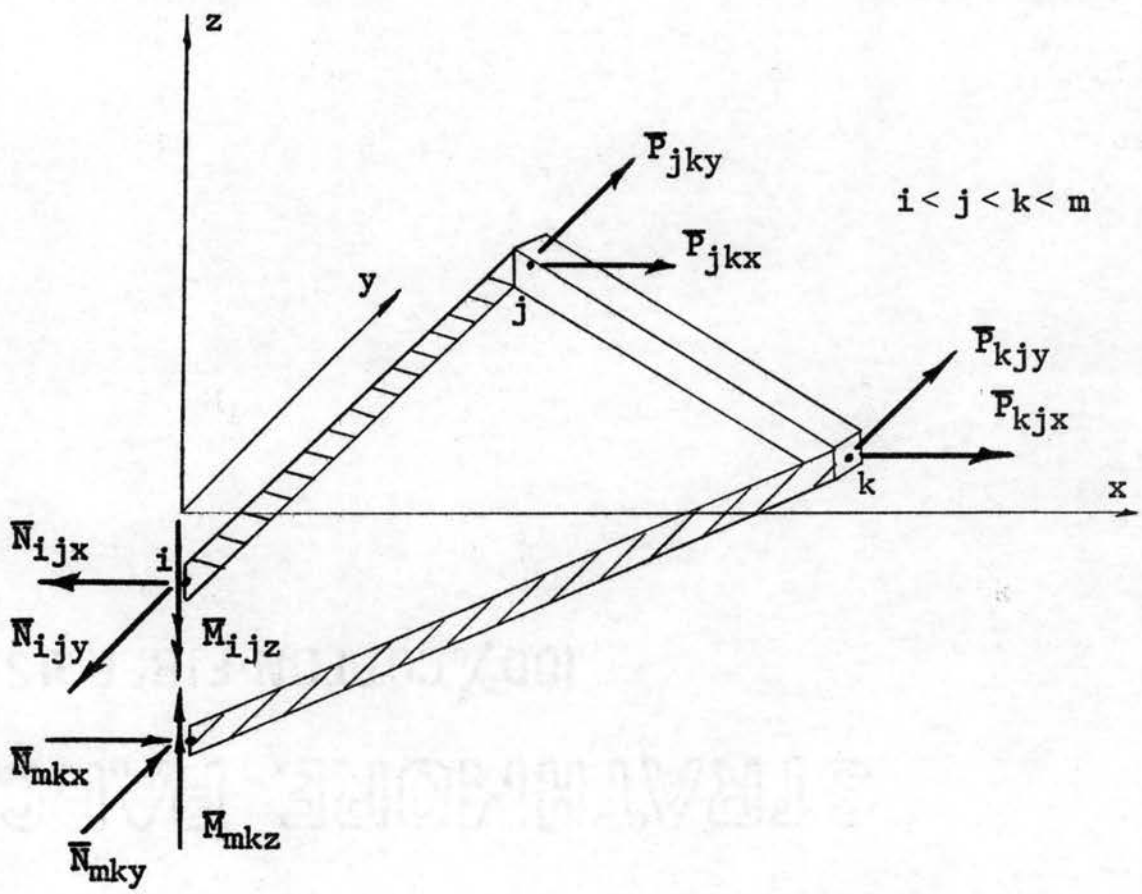


Figure 4-4: Elastic Weights, End Slopes and Displacements Single Member Loop

If the end slope and deflection at j are required rather than at i, then

$$\left[\bar{N}_{ji}\right] = \begin{bmatrix} 1 & 0 & 0 \\ x_{ji} & 1 & -y_{ji} \\ 0 & 0 & 1 \end{bmatrix} \left[\bar{N}_{ij}\right]$$

Similiarly, (9)

$$\left[\bar{N}_{ij}\right] = \begin{bmatrix} 1 & 0 & 0 \\ x_{ij} & 1 & -y_{ij} \\ 0 & 0 & 1 \end{bmatrix} \left[\bar{N}_{ji}\right]$$

where

$$\begin{bmatrix} 1 & 0 & 0 \\ x_{ij} & 1 & -y_{ij} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ x_{ji} & 1 & -y_{ji} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4-5. Elasto-Static Equilibrium of a Loop Containing Several Members

Let the loop A be a collection of members jkmn as shown in Figure 4-5.0. As before, assume the loop to be extended by rigid arms ij and nq to the origin of the basic coordinate system. Further assume that the positive x^m axis for each member including the rigid arms coincides with the direction around the path from i to q. Then equation 8 becomes

$$\left[\bar{N}_{ij}\right] - \left[\bar{N}_{qn}\right] = \left[t_{ojk} \mid t_{okm} \mid t_{omn}\right] \begin{bmatrix} \omega_{ojk} & & \\ & \omega_{okm} & \\ & & \omega_{omn} \end{bmatrix} \begin{bmatrix} \bar{P}^m_{jk} \\ \bar{P}^m_{km} \\ \bar{P}^m_{mn} \end{bmatrix} \quad (10)$$

or

$$\left[\overline{N}_{ij}\right] - \left[\overline{N}_{qn}\right] = \left[t_A\right]\left[\omega_A\right]\left[\overline{P}_A\right]$$

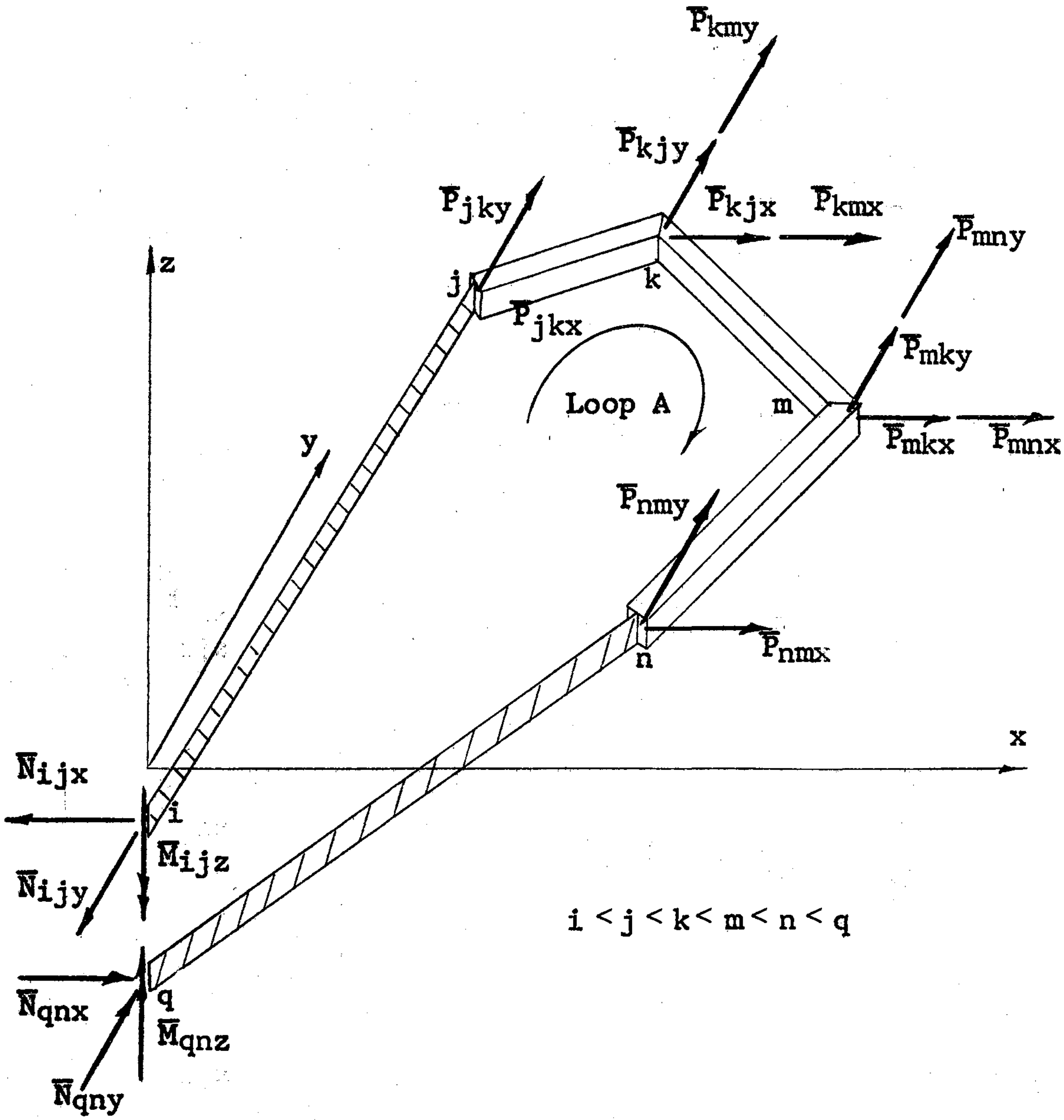


Figure 4-5.0: Elastic Weights, End Slopes and Displacements, Multiple Member Loop

At this point the assumption is made that there are no relative displacements between i and m, then

$$\left[t_A\right]\left[\omega_A\right]\left[\bar{F}_A\right] = \left[0\right] \qquad (10b)$$

From equations 7, 6 and 4a, equation 10b may be written in terms of the redundants at the cuts, the τ functions and the basic system moments, as follows:

$$0 = t_A \omega_A f_A \omega_A^T t_A^T b_A S + t_A \omega_A f_A \omega_A^T BS_A + t_A \omega_A \sigma_A \qquad (11)$$

where

$$t_A = \left[\begin{array}{c|c|c|c} t_{oij} & t_{ojk} & \cdots\cdots & t_{onq} \end{array}\right]$$

$$\omega_A = \begin{bmatrix} \omega_{oij} & & & \\ & \omega_{ojk} & & \\ & & \ddots & \\ & & & \omega_{onq} \end{bmatrix}$$

$$f_A = \begin{bmatrix} f_{ij} & & & \\ & f_{jk} & & \\ & & \ddots & \\ & & & f_{nq} \end{bmatrix}$$

$$b_A = \begin{bmatrix} b_{ij} \\ b_{jk} \\ \vdots \\ b_{nq} \end{bmatrix} \qquad S = \begin{bmatrix} S_1 \\ \vdots \\ S_m \end{bmatrix}$$

$$BS_A = \begin{bmatrix} BS_{ij} \\ BS_{jk} \\ \vdots \\ BS_{nq} \end{bmatrix} \qquad \sigma_A = \begin{bmatrix} \sigma_{ij} \\ \sigma_{jk} \\ \vdots \\ \sigma_{nq} \end{bmatrix}$$

While equation 11 looks rather formidable it should be noted that it can be broken down into a member by member summation process around the loop A, namely,

$$\sum b_{ijk} t_{oij} \omega_{oij} f_{ij} \omega_{oij}^{T} t_{oij}^{T} S_k = - \sum t_{oij} \omega_{oij} f_{ij} \omega_{oij}^{T} BS_{ij} - \sum t_{oij} \omega_{oij} \sigma_{ij} \tag{12}$$

where the summation is over the m members in the loop.

Some important computational aspects should be pointed out at this time. The contribution of member ij to this set of equations takes on a particularly efficient form, as follows:

$$b_{ij1}\left[A\right]\left[S_1\right] + b_{ij2}\left[A\right]\left[S_2\right] + \cdots + b_{ijk}\left[A\right]\left[S_k\right] = \left[B\right] \tag{13}$$

In other words, the contribution of the member ij to the coefficients of the redundant matrices S_1 to S_k is identical except for signs as determined by the coefficient b_{ijk}.

As a result of the conclusions arrived at in Para. 4-1 all elastic weights must be reversed in sign if the path is traversed in a manner opposite to its assumed plus direction. Thus, if a path ijkm is traversed from i to m and member km is such that $m < k$ then the sign of the terms in equations 12 or 13 must be reversed to properly account for the effect of member km on the formulation.

Equation 12 is therefore modified to reflect this possibility, as

$$\sum c_{ijm} b_{ijk} \left[A\right]\left[S_k\right] = -\sum c_{ijm}\left[B_1\right] - \sum c_{ijm}\left[B_2\right] \quad (12a)$$

where: c_{ijm} = 1 if positive axis of member coincides with positive path around loop m, otherwise, -1

b_{ijk} = 0, 1 or -1 as indicated after eq. 4b

$A = t_{oij}\omega_{oij}f_{ij}\omega^T_{oij}t^T_{oij}$

$B_1 = t_{oij}\omega_{oij}f_{ij}\omega^T_{oij}BS_{ij}$

$B_2 = t_{oij}\omega_{oij}\sigma_{ij}$

S_k = redundants at cut k, see eq. 1

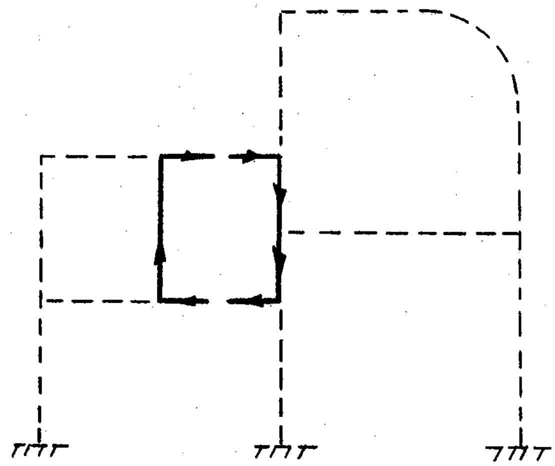
BS_{ij} = moments in member ij, statically determinate system

σ_{ij} = angular load functions, member ij, eq. 6

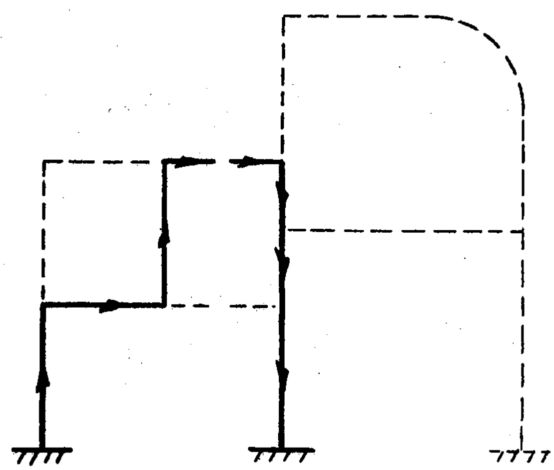
Because of the restriction on the formation of the loops, equation 12a properly summed for all loops results in the formation of the flexibility matrix since the loop containing the redundant S_k includes all of the members whose internal moments are functions of S_k. Furthermore, since equation 12a is merely an expansion of equation 10, the coefficient matrix of the matrix S_k represents the rotations and deflections at the cut containing S_k for unit values of the redundants S_k. The loop also contains all of the members whose internal moments are functions of both S_i and S_k and the resulting coefficient matrix of the matrix S_i represents the rotations and deflections at the cut containing S_k for unit values of the redundants S_i. These are by definition the flexibility coefficients.

Equation 12a could be used around any arbitrary closed loop, as shown in Figure 4-5.1a. This represents the sum of the deflections and angle changes around the loop. However, this is nothing more than the sum of the angle changes and deflections around the loop of Figure 4-5.1b minus the sum of the angle changes and deflections around the loop of Figure 4-5.1c. Hence, the deflection properties around any loop containing more than one redundant cut can be made from a linear combination of the basic loops defined in this paper. These combinations do not result in the flexibility matrix and for that reason are not considered further. If chosen properly they do represent a perfectly satisfactory set of simultaneous equations involving the redundant matrix as the unknowns, but since they are not the flexibility matrix convergence of the iterative technique cannot be assured.

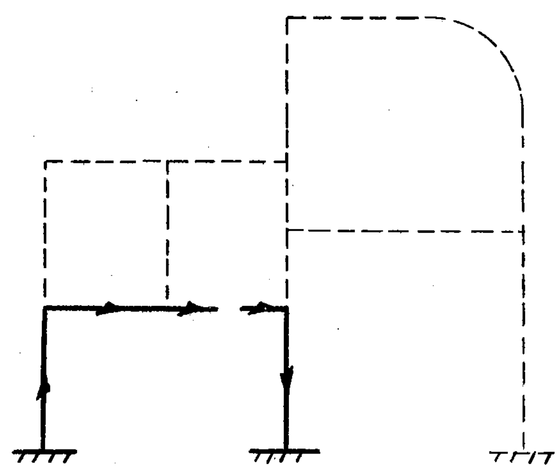
Finally, since the matrices A, B_1 and B_2 contain nothing but the parameters associated with member ij, the terms A, B_1 and B_2 for member ij represent its entire contribution to the flexibility matrix. If for example $c_{ijm} = 1$, $c_{ijn} = -1$ and $c_{ijq} = 0$, this means that the member is a part of the m^{th} loop and it is traversed in the plus direction, the member is also a member of the n^{th} loop but is traversed in



(a) Arbitrary Closed Loop Containing
More than One Cut



(b) Closed Loop Containing
One Cut



(c) Closed Loop Containing
One Cut

Figure 4-5.1: Linearly Dependent Closed Loops the negative direction and the member is not contained in the q^{th} loop.

The significance of the preceding paragraph is simply that the matrix multiplication need be performed only once for each member and then properly added to the flexibility matrix and the constant vector associated with the problem. The above ideas are shown schematically in Figure 4-5.2 for member 5-9 of Figure 2-3. If the loops are traversed such that the member containing the cut is traversed in a positive direction then $b_{ijm} = c_{ijm}$ for all loops.

Flexibility Matrix / Constant Matrix

	$b_{591} = 0$	$b_{592} = 1$	$b_{593} = -1$	$b_{594} = 0$	$b_{595} = -1$	Constant Matrix
$c_{591} = 0$	0	0	0	0	0	0
$c_{592} = 1$	0	A	-A	0	-A	$-B_1-B_2$
$c_{593} = -1$	0	-A	A	0	A	$+B_1+B_2$
$c_{594} = 0$	0	0	0	0	0	0
$c_{595} = -1$	0	-A	A	0	A	$+B_1+B_2$

each partition of the flexibility matrix represents a 3x3 matrix and each partition of constant matrix represents a 3x1 matrix.

member and redundants refer to those shown in Figure 2-3

Figure 4-5.2: Contribution of Member 5-9 to the Flexibility Matrix and the Constant Matrix

CHAPTER V

SOLUTION OF PROBLEM REDUNDANTS BY CARRY-OVER TECHNIQUES

5-1. General

The set of equations formulated by equation 12a can be written

$$\begin{bmatrix} F_{11} & F_{12} \cdots\cdots & F_{1m} \\ F_{21} & F_{22} \cdots\cdots & F_{2m} \\ \vdots & \vdots & \vdots \\ F_{m1} & F_{m2} \cdots\cdots & F_{mm} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_m \end{bmatrix} \qquad (13)$$

or

$$\begin{bmatrix} F \end{bmatrix} \begin{bmatrix} S \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix}$$

where: $\begin{bmatrix} F_{ij} \end{bmatrix}$ = a 3x3 flexibility matrix which is physically the deflections at cut i due to unit causes at cut j

$\begin{bmatrix} S_i \end{bmatrix}$ = redundant matrix at cut i

$\begin{bmatrix} Z_i \end{bmatrix}$ = constant vector, sum of terms 2 and 3 of equation 12a. Physically, the initial displacements at cut i due to the basic system moments

5-2. Solution by the Matrix Carry-Over Method

If equation 13 is rewritten as follows

$$\begin{bmatrix} S_1 \\ S_2 \\ \cdot \\ \cdot \\ \cdot \\ S_m \end{bmatrix} = \begin{bmatrix} F_{11}^{-1} Z_1 \\ F_{22}^{-1} Z_2 \\ \cdot \\ \cdot \\ \cdot \\ F_{mm}^{-1} Z_m \end{bmatrix} + \begin{bmatrix} 0 & -F_{11}^{-1} F_{12} & \cdot \; \cdot & -F_{11}^{-1} F_{1m} \\ -F_{22}^{-1} F_{21} & 0 & \cdot \; \cdot & -F_{22}^{-1} F_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ -F_{mm}^{-1} F_{m1} & -F_{mm}^{-1} F_{m2} & \cdot \; \cdot & 0 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \cdot \\ \cdot \\ \cdot \\ S_m \end{bmatrix} \qquad (14)$$

or in a somewhat shorter form, called here the carry-over form, after Tuma (1)

$$\begin{bmatrix} S \end{bmatrix} = \begin{bmatrix} SV \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} S \end{bmatrix}$$

where: $\begin{bmatrix} SV_i \end{bmatrix}$ = starting values, physically the solution of the problem for S_i if all $S_j = 0$, $j \neq i$

$\begin{bmatrix} C_{ij} \end{bmatrix}$ = the carry-over coefficient, physically the induced forces at cut i due to unit causes at cut j while maintaining compatibility at i

Now, if the first set of values for the redundant S is assumed to be SV, the second approximation is given by equation 14 using the first values of S on the right hand side, or

$$\begin{bmatrix} S \end{bmatrix}^{(2)} = \begin{bmatrix} SV \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} S \end{bmatrix}^{(1)}$$

and, after n iterations

$$\begin{bmatrix} S \end{bmatrix}^{(n)} = \begin{bmatrix} SV \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} S \end{bmatrix}^{(n-1)} \qquad (15)$$

Let the last term be defined as the residual matrix, or

$$\left[S\right]^{(2)} = \left[SV\right] + \left[R\right]^{(1)}$$

$$\left[S\right]^{(3)} = \left[S\right]^{(2)} + \left[C\right]\left[R\right]^{(1)} = \left[S\right]^{(2)} + \left[R\right]^{(2)}$$

and after n trials

$$\left[S\right]^{(n)} = \left[SV\right] + \left[R\right]^{(1)} + \left[R\right]^{(2)} + \cdots\cdots + \left[R\right]^{(n-1)} \quad (16)$$

$$\text{where: } \left[R\right]^{(n)} = \left[C\right]\left[R\right]^{(n-1)}$$

For convergence the n^{th} residual matrix must approach zero with n sufficiently large.

Equation 16 represents a form of iteration used by Cross (9) in his moment distribution technique or Tuma (1) is his carry-over technique with one rather important difference. With these techniques it is not necessary to process each set of values in its entirety each step of the way. As in the carry-over technique or moment distribution technique only the large residuals need be iterated initially. If their carry-over effects are small, these carry-over effects may be accumulated and their resulting feedback accounted for all in one operation.

5-3. Block Gauss-Siedel Iterative Procedure

If equation 14 is used directly as the iterative procedure, then

$$\left[S\right]^{(n)} = \left[SV\right] + \left[C\right]\left[S\right]^{(n-1)} \quad (17)$$

Additionally, if each new set of values for the S matrix is computed from the most recent set of values available, then in component form, equation 17 becomes

$$\left[S_j\right]^{(n+1)} = \left[SV_j\right] + \sum_{k=1}^{j-1} \left[C_{jk}\right]\left[S_k\right]^{(n+1)} + \sum_{k=j+1}^{m} \left[C_{jk}\right]\left[S_k\right]^{(n)} \quad (18)$$

This form is preferable to equation 17 in that only one matrix S need be retained at any time. This is the block Gauss-Siedel process referred to by Varga (16).

5-4. Point Gauss-Siedel Iterative Procedure

For reference purposes, a simpler iterative technique might be used on equation 13. It is referred to by Varga (16) as the point Gauss-Siedel process and is essentially identical to the technique of equation 18 except it deals with one equation at a time and can be written

$$f_{ii}\, s_i^{(n+1)} = z_i + \sum_{k=1}^{i-1} f_{ik}\, s_k^{(n+1)} + \sum_{k=i+1}^{3m} f_{ik}\, s_k^{(n)} \quad (19)$$

Use of this process in the solution of the structural problem presented here is physically the restoration of continuity at a cut in only one direction rather than simultaneously in all directions as given by the processes of Para. 5-2 and 5-3.

5-5. Convergence of the Iterative Methods

Because of the restriction placed on the formulation of the coefficient matrix associated with the redundants the following statements can be made:

a. The coefficient matrix is the flexibility matrix as discussed in Para. 4-5 and is therefore real and symmetric.

b. The positive nature of the internal strain energy is sufficient assurance that the flexibility matrix is positive definite, see Southwell (10) or Temple (18).

c. Positive definiteness of the flexibility matrix is a necessary and sufficient condition for the convergence of all three of the techniques discussed in Para. 5-2, 5-3 and 5-4. Proofs of these are found in the references given in the paragraphs indicated.

No general conclusions can be reached regarding which of the three iterative processes converges the most rapidly. However, while no general statement can be made regarding the convergence, physical interpretations favor the block process. That is, it seems reasonable to assume that simultaneous restoration of continuity in three directions should usually converge to the answer in a more rapid fashion than working with the one-dimensional counterpart. A similiar conclusion is reached by Faddeev and Faddeeva (17).

For the purposes of desk calculation, the carry-over technique is ideally suited since the analyst can tell immediately how the iteration is progression by the convergence of the residual matrix to zero. However, round off errors may accumulate or simple mistakes go uncorrected unless frequent use is made of equation 15 to check the progress. Automatic computation on the other hand favors equation 18 as this avoids the round off problem with little increase in actual computation time.

CHAPTER VI

APPLICATION

6-1. Two-Bay Framework

The two-bay frame shown in Figure 6-1.0 has members whose properties are given in Table 6-1. All members have equal EI values and all members have EI = GJ.

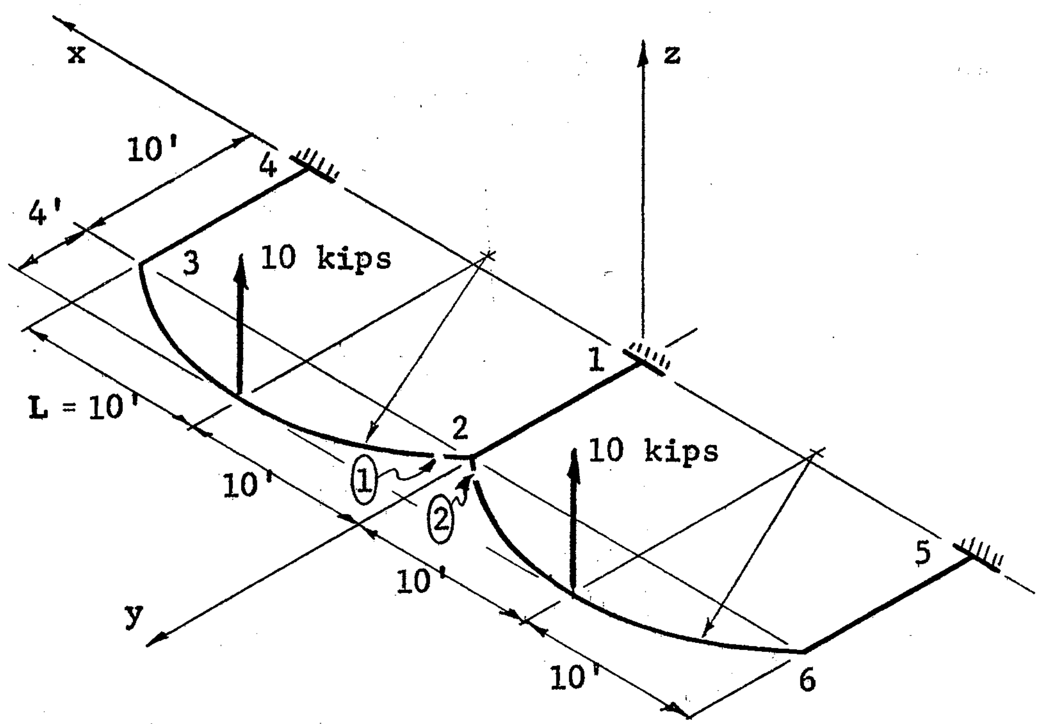


Figure 6-1.0: Dimensions, Loads and Redundant Locations Two-Bay Framework

Figures 6-1.1 thru 6-1.5 illustrate the step by step formulation of the terms involved in the flexibility matrix and the constant matrix. Figure 6-1.6 represents the step by step formulation of the flexibility matrix. All of the calculations are shown to emphasize the repetitive nature of the calculations. Finally, the application of the carry-over technique is shown in Figure 6-1.7.

Table 6-1: Two-Bay Framework, Basic System Moments, Flexibilities and Angular Load Functions

Member	BS Values, Kip-feet			
	BM_{ijy}	BM_{ijx}	BM_{jix}	BM_{jiy}
1-2	0	0	0	0
2-3	0	0	-40	-100
3-4	-100	-40	-140	-100
5-6	-100	140	40	-100
2-6	0	0	-40	100

Member	Near and Far End Flexibilities from Appendix B					
	f_{ijyy}	f_{jiyx}	g_{jiyy}	f_{jixx}	g_{jixy}	f_{jiyy}
1-2	.3333	0	.1667	1.000	0	.3333
2-3	.7975	-.2914	.3059	2.2069	.2914	.7975
3-4	.3333	0	.1667	1.000	0	.3333
5-6	.3333	0	.1667	1.000	0	.3333
2-6	.7975	.2914	.3059	2.2069	-.2914	.7975

Member	Angular Load Functions, from Appendix B		
	τ_{ijy}	τ_{jix}	τ_{jiy}
1-2	0	0	0
2-3	32.000	44.150	20.348
3-4	0	0	0
5-6	0	0	0
2-6	32.000	-44.150	20.348

Note: L/EI deleted from Near and Far End Flexibilities and from the Angular Load Functions.

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -10 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} .3333 & 0 & .1667 \\ 0 & 1.000 & 0 \\ .1667 & 0 & .3333 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -10 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 33.333 & -5.000 \\ 0 & -5.000 & 1.000 \end{bmatrix}$$

$[B_1] = [B_2] = 0$ since angular load functions and basic system moments equal zero

$c_{122} = c_{121} = 1$ since path around loops coincides with positive axis of member

$b_{122} = b_{121} = 1$ since both redundants 1 and 2 are oriented in a positive manner relative to member 1-2

Figure 6-1.1: Equation 12a, Two-Bay Framework, Loop No. 1, Member 1-2 (also Loop No. 2, Member 1-2)

$$
\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -10 & -10 & 20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .7975 & .2914 & .3059 \\ .2914 & 2.207 & -.2914 \\ .3059 & -.2914 & .7975 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -10 & 1 \\ 0 & -10 & 1 \\ 0 & 20 & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} 2.2068 & 22.068 & 0 \\ 22.068 & 656.26 & -27.898 \\ 0 & -27.898 & 2.207 \end{bmatrix}
$$

$$
\begin{bmatrix} B_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -10 & -10 & 20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .7975 & .2914 & .3059 \\ .2914 & 2.207 & -.2914 \\ .3059 & -.2914 & .7975 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -40 \\ -100 \end{bmatrix} = \begin{bmatrix} -100.34 \\ -770.5 \\ -59.14 \end{bmatrix}
$$

$$
\begin{bmatrix} B_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -10 & -10 & 20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 32.000 \\ 44.150 \\ 20.348 \end{bmatrix} = \begin{bmatrix} 52.348 \\ -34.540 \\ 44.150 \end{bmatrix}
$$

$c_{231} = 1$ since path around loop coincides with positive member axis

$b_{231} = 1$ since member contains redundant 1

$b_{232} = 0$ since member moments are not functions of redundant 2

Figure 6-1.2: Equation 12a, Two-Bay Framework, Loop No. 1, Member 2-3

$$\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 20 & -10 & 0 & 20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} .3333 & 0 & .1667 \\ 0 & 1.000 & 0 \\ .1667 & 0 & .3333 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 20 & 0 \\ 0 & -10 & 1 \\ 0 & 0 & 1 \\ 1 & 20 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.000 & 20.000 & 0 \\ 20.00 & 433.33 & -5.000 \\ 0 & -5.000 & 1.000 \end{bmatrix}$$

$$\begin{bmatrix} B_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 20 & -10 & 0 & 20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} .3333 & 0 & .1667 \\ 0 & 1.000 & 0 \\ .1667 & 0 & .3333 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -100 \\ -40 \\ -140 \\ -100 \end{bmatrix} = \begin{bmatrix} -100.00 \\ -1633.33 \\ -90.00 \end{bmatrix}$$

$\begin{bmatrix} B_2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$ since all angular load functions are equal to zero

$c_{341} = 1$ since path around loop coincides with positive member axis

$b_{341} = 1$ since redundant 1 is oriented positively relative to member 3-4

$b_{342} = 0$ since member moments are not functions of redundant 2

Figure 6-1.3: Equation 12a, Two-Bay Framework, Loop No. 1, Member 3-4

$$
[A] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -10 & -10 & -20 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\begin{bmatrix} .7975 & -.2914 & .3059 \\ -.2914 & 2.207 & .2914 \\ .3059 & .2914 & .7975 \end{bmatrix}
\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & -10 & 1 \\ 0 & -10 & 1 \\ 1 & -20 & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} 2.2068 & -22.068 & 0 \\ -22.068 & 656.26 & -27.898 \\ 0 & -27.898 & 2.207 \end{bmatrix}
$$

$$
[B_1] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -10 & -10 & -20 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\begin{bmatrix} .7975 & -.2914 & .3059 \\ -.2914 & 2.207 & .2914 \\ .3059 & .2914 & .7975 \end{bmatrix}
\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ -40 \\ 100 \end{bmatrix}
= \begin{bmatrix} 110.34 \\ -770.50 \\ -59.14 \end{bmatrix}
$$

$$
[B_2] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -10 & -10 & -20 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\begin{bmatrix} 32.000 \\ -44.150 \\ 20.348 \end{bmatrix}
= \begin{bmatrix} -52.348 \\ -34.540 \\ 44.150 \end{bmatrix}
$$

$c_{262} = 1$ since path around loop coincides with positive member axis

$b_{261} = 0$ since member moments are not functions of redundant 1

$b_{262} = 1$ since redundant is contained in member 2-6

Figure 6-1.4: Equation 12a, Two-Bay Framework, Loop No. 2, Member 2-6

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -20 & 0 & -10 & -20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} .3333 & 0 & .1667 \\ 0 & 1.000 & 0 \\ .1667 & 0 & .3333 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -20 & 0 \\ 0 & 0 & 1 \\ 0 & -10 & 1 \\ 1 & -20 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0000 & -20.000 & 0 \\ -20.000 & 433.33 & -5.000 \\ 0 & -5.000 & 1.000 \end{bmatrix}$$

$$[B_1] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -20 & 0 & -10 & -20 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} .3333 & 0 & .1667 \\ 0 & 1.000 & 0 \\ .1667 & 0 & .3333 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -100 \\ 140 \\ 40 \\ -100 \end{bmatrix} = \begin{bmatrix} -100.00 \\ 1633.33 \\ 90.00 \end{bmatrix}$$

$[B_2] = [0]$ since all angular load functions are equal to zero

c_{562} = -1 since path around loop is opposite positive sense of member axis

b_{561} = 0 since member moments are not functions of redundant 1

b_{562} = -1 since redundants 2 are oriented in negative sense relative to member 5-6

Figure 6-1.5: Equation 12a, Two-Bay Framework, Loop No. 2, Member 5-6

(a) After member 1-2, Loop No. 1, is added to a 6x6 and 6x1 null matrix

$$\begin{bmatrix} 1.000 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 33.333 & -5.000 & 0 & 33.333 & -5.000 \\ 0 & -5.000 & 1.000 & 0 & -5.000 & 1.000 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) After member 2-3, Loop No. 1, is added to (a)

$$\begin{bmatrix} 3.2068 & 22.068 & 0 & 1.000 & 0 & 0 \\ 22.068 & 689.59 & -32.898 & 0 & 33.333 & -5.000 \\ 0 & -32.898 & 3.207 & 0 & -5.000 & 1.000 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 57.99 \\ 805.04 \\ 14.99 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(c) After member 3-4, Loop No. 1, is added to (b)

$$\begin{bmatrix} 4.2068 & 42.068 & 0 & 1.000 & 0 & 0 \\ 42.068 & 1122.92 & -37.898 & 0 & 33.333 & -5.000 \\ 0 & -37.898 & 3.207 & 0 & -5.000 & 1.000 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 157.99 \\ 2438.37 \\ 104.99 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Figure 6-1.6: Evolution of Flexibility Matrix, Member by Member, Two-Bay Framework

(d) After member 1-2, Loop No. 2, is added to (c)

$$\begin{bmatrix} 4.2068 & 42.068 & 0 & 1.000 & 0 & 0 \\ 42.068 & 1122.92 & -37.898 & 0 & 33.333 & -5.000 \\ 0 & -37.898 & 3.207 & 0 & -5.000 & 1.000 \\ 1.000 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 33.333 & -5.000 & 0 & 33.333 & -5.000 \\ 0 & -5.000 & 1.000 & 0 & -5.000 & 1.000 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 157.99 \\ 2438.37 \\ 104.99 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(e) After member 2-6, Loop No. 2, is added to (d)

$$\begin{bmatrix} 4.2068 & 42.068 & 0 & 1.000 & 0 & 0 \\ 42.068 & 1122.92 & -37.898 & 0 & 33.333 & -5.000 \\ 0 & -37.898 & 3.207 & 0 & -5.000 & 1.000 \\ 1.000 & 0 & 0 & 3.2068 & -22.068 & 0 \\ 0 & 33.333 & -5.000 & -22.068 & 689.59 & -32.898 \\ 0 & -5.000 & 1.000 & 0 & -32.898 & 3.207 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 157.99 \\ 2438.37 \\ 104.99 \\ -57.99 \\ 805.04 \\ 14.99 \end{bmatrix}$$

(f) After member 5-6, Loop No. 2, is added to (e)

$$\begin{bmatrix} 4.2068 & 42.068 & 0 & 1.000 & 0 & 0 \\ 42.068 & 1122.92 & -37.898 & 0 & 33.333 & -5.000 \\ 0 & -37.898 & 3.207 & 0 & -5.000 & 1.000 \\ 1.000 & 0 & 0 & 4.2068 & -42.068 & 0 \\ 0 & 33.333 & -5.000 & -42.068 & 1122.92 & -37.898 \\ 0 & -5.000 & 1.000 & 0 & -37.898 & 3.207 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 157.99 \\ 2438.37 \\ 104.99 \\ -157.99 \\ 2438.37 \\ 104.99 \end{bmatrix}$$

Figure 6-1.6: Continued

Premultiplying each set of 3 equations by the inverse of the 3x3 matrix on the diagonal yields, from Figure 6-1.6,

$$\begin{bmatrix} 1 & 0 & 0 & .51483 & .32443 & -.11107 \\ 0 & 1 & 0 & -.02771 & -.03244 & .01111 \\ 0 & 0 & 1 & -.24963 & -1.4808 & .33776 \\ .51483 & -.32443 & .11107 & 1 & 0 & 0 \\ .02771 & -.03244 & .01111 & 0 & 1 & 0 \\ .24963 & -1.4808 & .33776 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} -12.44 \\ 5.00 \\ 70.00 \\ 12.44 \\ 5.00 \\ 70.00 \end{bmatrix}$$

In carry-over form,

$$\begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \end{bmatrix} = \begin{bmatrix} -12.44 \\ 5.00 \\ 70.00 \end{bmatrix} + \begin{bmatrix} -.51483 & -.32443 & .11107 \\ .02771 & .03244 & -.01111 \\ .24963 & 1.4808 & -.33776 \end{bmatrix} \begin{bmatrix} s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} \qquad [S_1] = [SV_1] + [C_{12}][S_2]$$

$$\begin{bmatrix} s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 12.44 \\ 5.00 \\ 70.00 \end{bmatrix} + \begin{bmatrix} -.51483 & .32443 & -.11107 \\ -.02771 & .03244 & -.01111 \\ -.24963 & 1.4808 & -.33776 \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \end{bmatrix} \qquad [S_2] = [SV_2] + [C_{21}][S_1]$$

Which, when solved by the carry-over technique yield,

(after 2 cycles) (after 6) (solution)

$$\begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \end{bmatrix} = \begin{bmatrix} -12.44 \\ 5.00 \\ 70.00 \end{bmatrix} \underset{C_{12}}{+} \begin{bmatrix} -.25 \\ -.27 \\ -13.14 \end{bmatrix} \underset{C_{12}}{+} \begin{bmatrix} -1.50 \\ .14 \\ 4.10 \end{bmatrix} = \begin{bmatrix} -14.14 \\ 4.87 \\ 60.86 \end{bmatrix} = \cdots\cdots = \begin{bmatrix} -15.01 \\ 4.89 \\ 60.52 \end{bmatrix} \quad \begin{bmatrix} -15.06 \\ 4.90 \\ 60.56 \end{bmatrix}$$

$$\begin{bmatrix} s_{21} \\ s_{22} \\ s_{23} \end{bmatrix} = \begin{bmatrix} 12.44 \\ 5.00 \\ 70.00 \end{bmatrix} \underset{C_{21}}{+} \begin{bmatrix} .25 \\ -.27 \\ -13.14 \end{bmatrix} \underset{C_{21}}{+} \begin{bmatrix} 1.50 \\ .14 \\ 4.10 \end{bmatrix} = \begin{bmatrix} 14.19 \\ 4.87 \\ 60.86 \end{bmatrix} = \cdots\cdots = \begin{bmatrix} 15.01 \\ 4.89 \\ 60.52 \end{bmatrix} \quad \begin{bmatrix} 15.06 \\ 4.90 \\ 60.56 \end{bmatrix}$$

$\nearrow C_{12}$ indicates carry-over operation

Figure 6-1.7: Solution of Two Bay Framework by the Matrix Carry-Over Technique

Once the redundants at the cuts are evaluated, equation 4b can be used in the following form to obtain the internal redundants for each member

$$\left[M_{ij}\right] = \begin{bmatrix} 1 & x_{oi} & 0 \\ 0 & -y_{oi} & 1 \\ 0 & -y_{oj} & 1 \\ 1 & x_{oj} & 0 \end{bmatrix} \left[b_{ij1}S_1 + b_{ij2}S_2\right] + \begin{bmatrix} BM_{ijy} \\ BM_{ijx} \\ BM_{jix} \\ BM_{jiy} \end{bmatrix} \qquad (4b)$$

for member 1-2 this becomes, from Table 6-1, Figure 6-1.1 and Figure 6-1.7

$$\left[M_{12}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -10 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -15.06 & + & 15.06 \\ 4.90 & + & 4.90 \\ 60.56 & & 60.56 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 121.12 \\ 23.12 \\ 0 \end{bmatrix}$$

since this is in the basic reference system it should probably be rotated to the member reference frame, or

$$\left[X^m_{ij}\right] = \left[\omega_{oij}\right]^T \left[M_{ij}\right]$$

$$\begin{bmatrix} X^m_{12y} \\ X^m_{21x} \\ X^m_{21y} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 121.12 \\ 23.12 \\ 0 \end{bmatrix} = \begin{bmatrix} -121.12 \\ 0 \\ -23.12 \end{bmatrix}$$

In this case, the summation of the S_i matrices is obtained prior to multiplication by the ω and t matrices. This operation is possible because all redundants are referenced to the origin of the basic coordinate system.

6-2. Summary of Problems

In the appendix the final results of sixteen different problems are included along with an analysis of the convergence of the individual redundants at the cuts as well as an analysis of the convergence of the individual member redundants.

To analyse the variation of the iterated answers, the percentage deviation from the basis value is computed as follows:

$$\% \text{ Deviation of } U = \frac{U - U^*}{V} \times 100$$

where: V = the absolute value of the maximum basis value of all quantities in the problem

U = value being investigated

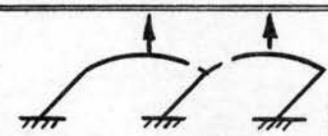
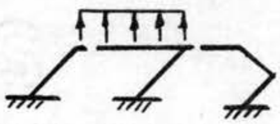
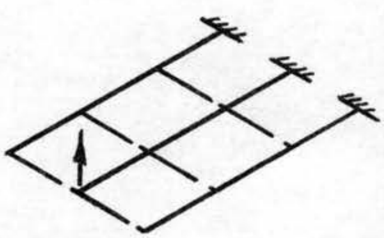
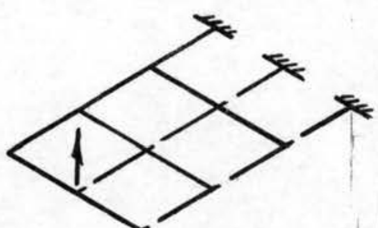
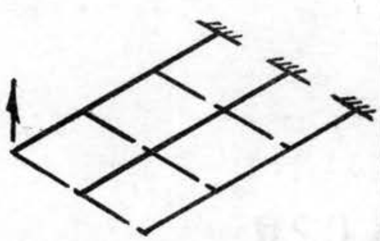
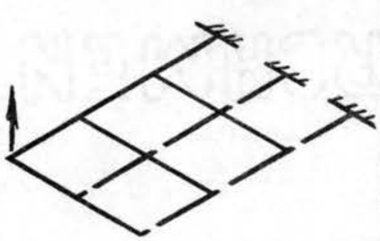
U^* = basis value of U

In the case of redundants at the cuts, the values are mixed values regarding the nature of their units. That is, some values are shears and some moments. For this reason, whenever a shear value is encountered it is multiplied by the absolute value of the largest coordinate used in the problem prior to its comparison by the above formula.

Problem 1 is the problem used as a sample problem in Para. 6-1. Problem 2 is a problem solved by Diwan (5). Problems 3 thru 14 all involve a 2 bay, 3 story framework with symmetrical, unsymmetrical and anti-symmetrical loadings as well as a variation of relative torsional to bending stiffnesses and two different choices of redundants. Problem 15 is an extension of problem 3 to three bays and three stories. Problem 16 is a hexagonal framework symmetrically loaded.

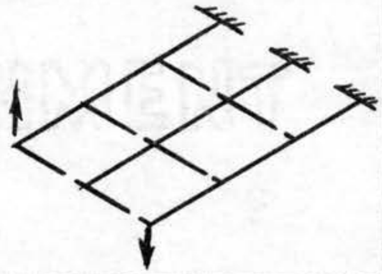
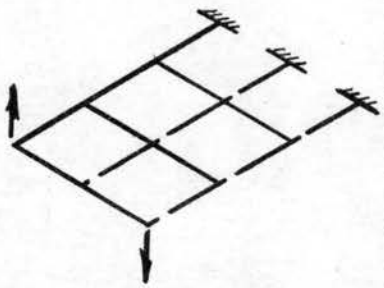
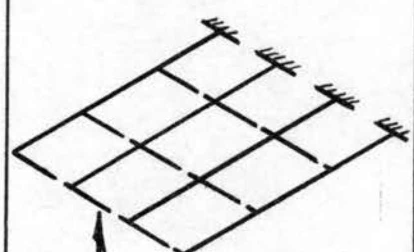
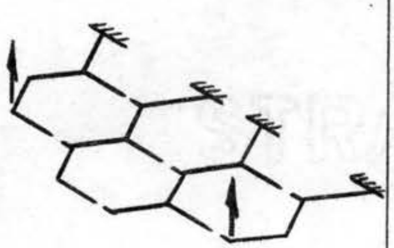
Table 6-2.1 contains a summary of the significant data relative to the convergence of the carry-over process that is contained in the Appendix A. Figure 6-2 illustrates graphically the nature of the convergence for problem 3. A considerable amount of additional data relative to the problems but not specifically associated with the convergence of the carry-over process is also included in Appendix A. Some general details involving the nature of the input and output data as well as the overall nature of the computer program used to obtain the solutions shown is included in Appendix C.

Table 6-2.2 shows a comparison in the speed of convergence of the point vs. the block Gauss-Siedel techniques for problems no. 1, 2 and 3.

Prob.	Geometry, Loads, and Redundants	EI/GJ	Maximum % Error*			
			S Values		X Values	
			Error	Cycles	Error	Cycles
1		1.0	0.0	6	0.0	6
2		see App. A	0.01	7	0.01	7
3		1.0	1.54	20	0.35	20
4		2.0	0.83	20	0.21	20
5		0.5	3.91	20	0.70	20
6		1.0	2.29	50	1.02	50
7		1.0	0.66	20	0.13	20
8		2.0	0.40	20	0.09	20
9		0.5	1.51	20	0.33	20
10		1.0	1.36	50	0.27	50

*for calculation of error see Para. 6-2

Table 6-2.1: Summary of Problem Results

Prob.	Geometry, Loads, and Redundants	EI/GJ	Maximum % Errors* S Values Error	Maximum % Errors* S Values Cycles	Maximum % Errors* X Values Error	Maximum % Errors* X Values Cycles
11		1.0	0.18	20	0.07	20
12		2.0	0.12	20	0.04	20
13		0.5	0.96	20	0.49	20
14		1.0	0.27	50	0.24	50
15		1.0	0.72	20	0.50	20
16		1.0	0.01	10	0.01	10

* for calculation of error see Para. 6-2

Table 6-2.1: (continued)

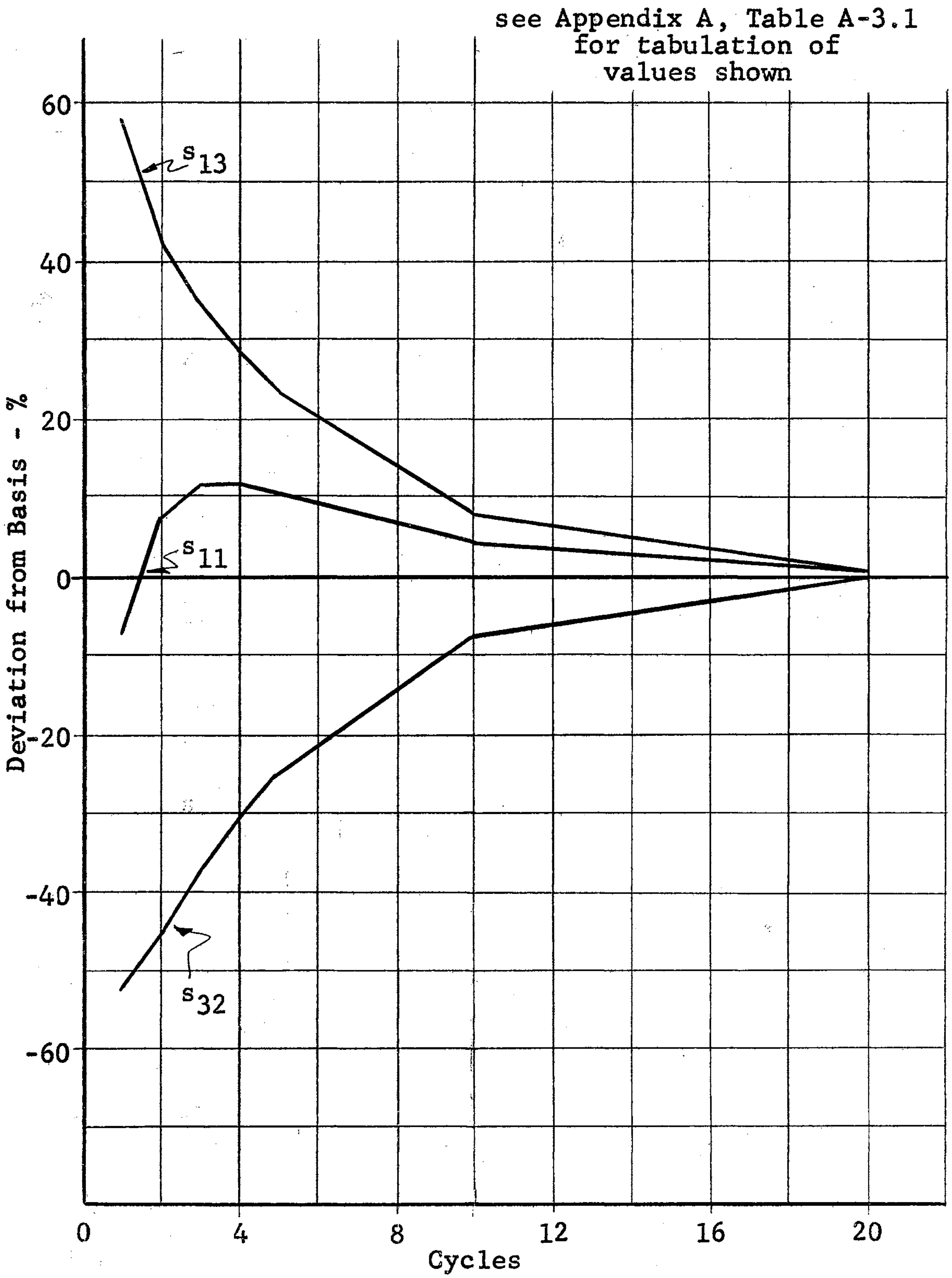


Figure 6-2: Convergence of Some Typical Redundant Matrix Elements of Problem No. 3

Table 6-2.2: Comparison of Convergence Rates of Block vs. Point Gauss-Siedel Techniques

Prob. No.	Iterative Procedure	Iteration No. where All Values Changed Less Than the Indicated Percentage During the Previous Cycle*		
		.01%	.001%	.0001%
1	point block	- 5	- 6	41 8
2	point block	- 7	- 9	47 12
3	point block	111 31	174 41	243 49

*Percentages are computed individually for each unknown using its own most recent value as a basis of comparison.

CHAPTER VII

SUMMARY AND CONCLUSIONS

7-1. Summary

The following principal formulas were developed for the solution of the problem stated in Para. 1-1. Briefly, the internal redundant moments in each member are given by equation 4b, as follows:

$$\left[M_{ij}\right] = \left[t_{oij}\right]^T \left[b_{ij}\right]\left[S\right] + \left[BS_{ij}\right] \qquad (4b)$$

Then the flexibility matrix, F, was formulated from equation 12a

$$\sum c_{ijm} b_{ijk} \left[A\right]\left[S_k\right] = -\sum c_{ijm}\left[B_1\right] - \sum c_{ijm}\left[B_2\right] \qquad (12a)$$

Finally, the solution was obtained by iterating the flexibility matrix in a manner described as a block Gauss-Siedel process. Physically, this process represented a systematic restoration of the continuity at each cut in a cyclic manner until continuity of the elastic curve was achieved. It is therefore an extension of the one-dimensional carry-over of Tuma (1) and for that reason was termed the matrix carry-over technique. This iterative procedure was given by

$$\left[S_i\right]^{(n+1)} = \left[SV_i\right] + \sum_{k=1}^{i-1}\left[C_{ik}\right]\left[S_k\right]^{(n+1)} + \sum_{k=i+1}^{m}\left[C_{ik}\right]\left[S_k\right]^{(n)} \qquad (18)$$

Equation 4b was then used to obtain the redundant moments in each member.

7-2. Conclusions

1. The matrix carry-over technique was found to converge in a rapid manner. For the class of problems investigated, the member moments converged to within 0.70% after 20 cycles of iteration if the redundants were chosen such as to make the individual trees relatively compact. Problems 1, 2 and 3 indicated the convergence to be approximately 4 times faster than the point Gauss-Siedel process.

2. Formulation of the problem by equation 12a proved to be relatively simple. By defining the redundants in each member in the same manner as the equivalent elastic weights, the terms of equation 12a become highly repetitious. Use of the origin of the basic coordinate system as a reference for redundants at the cuts and deflection matrix at the cuts produced similiar simplifications. As can be seen from Figures 6-1.1 thru 6-1.5, the only quantities involved in equation 12a were the coordinates, flexibility coefficients and statically determinate loads for the individual member.

A general computer program was developed to solve any type of structure satisfying the statement of the problem. This includes problems having a large variety of different linear graphs. Samuelson (19) was able to program problems subject to the requirement that they have the same linear graph.

3. Choices of compact trees indicated better or faster rates of convergence than trees formed in a less compact manner. This is indicated by a comparison of problems 3, 7 and 11 with problems 6, 10 and 14. The latter group of problems was identical with the first except for the choice of redundant cuts.

4. Since the formulation resulted in a minimum set of simultaneous equations, the method can be applied to a structure having stiffness variations within each member by replacing the members with a number of straight segments of constant section. This would be accomplished with no increase in the number of simultaneous equations.

Both the technique of formulation and its accompanying simplifications as well as the use of the multi-dimensional carry-over technique are believed to be original with this investigation.

7-3. Extensions of the Technique

Probably the most important result of the problem studied in this work is actually a by-product. This is the formulation procedure which produces a minimum set of simultaneous equations. Therefore, four immediate extensions should be investigated:

1. Produce the analogous technique for the solution of planar structures with loads in the plane. This extension is in a one-to-one correspondence with the techniques used for the problem considered here. The only differences are the terms within the t and ω matrices and the flexibility factors for the individual members.

2. Investigate the errors introduced by replacing curved members or members having a varying stiffness with short segments of constant section properties. This procedure would eliminate the need of a large number of tables or formulas for the proper evaluation of member flexibilities.

3. Extend the technique to a three dimensional structure with arbitrary loading. This would utilize the same logical process involved in determining the b_{ij} and c_{ij} factors but would deal with all 6 internal force and moment elements at each redundant cut.

4. Investigate the technique of introducing hinges and other discontinuities into the structure. A 'simple' solution to this problem would greatly increase the value of all of the methods described.

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APPENDIX A

PROBLEM SOLUTIONS AND ERROR ANALYSIS OF CONVERGENCE

The following pages contain the results of sixteen different problems formulated and solved by the iterative technique described in this thesis. All results are shown in a similiar fashion, that is, Figure A-1, Table A-1.1 and Table A-1.2 summarize problem 1; Figure A-2, Table A-2.1 and Table A-2.2 summarize problem 2; etc. A summary of all of these problems in contained in Table 6-2.1.

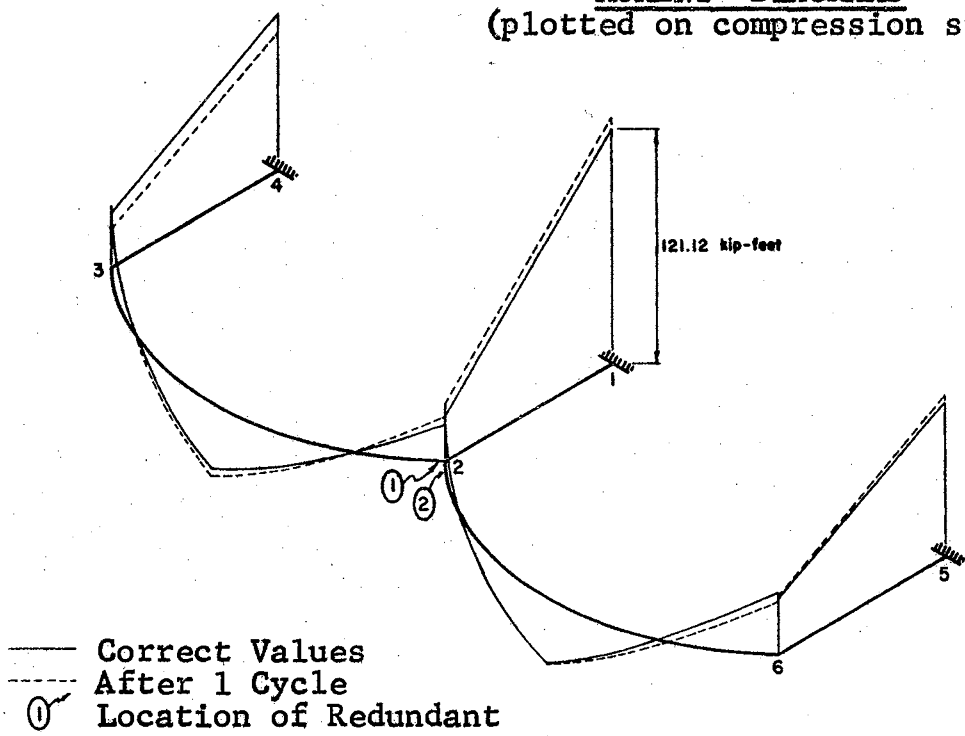
In all instances, the figure associated with the problem contains the problem dimensions, loads, member properties and a sketch showing the variation of bending moments throughout the structure. Wherever possible, bending moments after 1, 5, 10 or 20 cycles of iteration are also shown to indicate the regularity of the convergence. For brevity, a similiar sketch of torsional moments is not shown. The convergence of the torsional moments are similiar to those shown for bending moments.

Tables A-1.1, A-2.1, etc. contain an analysis of the convergence of the redundant matrix for each problem.

Tables A-1.2, A-2.2, etc. contain an analysis of the convergence of the actual member moments and torsions computed from the redundant matrices at each indicated cycle of iteration.

Percentage deviation is always computed as discussed in Para. 6-2.

MOMENT DIAGRAMS
(plotted on compression side)



MEMBER PROPERTIES

All members have equal EI
All members have EI=GJ

LOADS AND DIMENSIONS

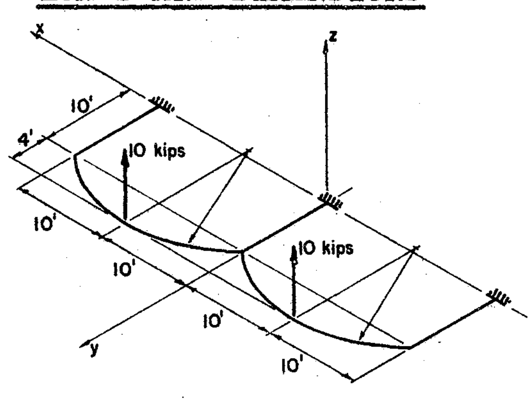


Figure A-1: Problem 1, Member Properties, Dimensions, Loads, and Comparison of Member Moments after 1 Cycle of Iteration with Correct Values

TABLE A-1.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 1
AFTER 1, 2, 3, 4, 5, 6, 8 AND 10 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	6	8	10	K,K-FEET
1	1	2.67	.88	.23	.06	.01	.00	.00	.00	-15.06
1	2	1.96	-.61	-.21	-.06	-.01	.00	.00	.00	4.90
1	3	9.62	.41	-.04	-.02	.00	.00	.00	.00	60.56
2	1	-2.41	-.51	-.12	-.03	-.01	.00	.00	.00	15.06
2	2	-3.55	-.60	-.12	-.03	-.01	.00	.00	.00	4.90
2	3	-3.77	-.40	-.06	-.01	.00	.00	.00	.00	60.56

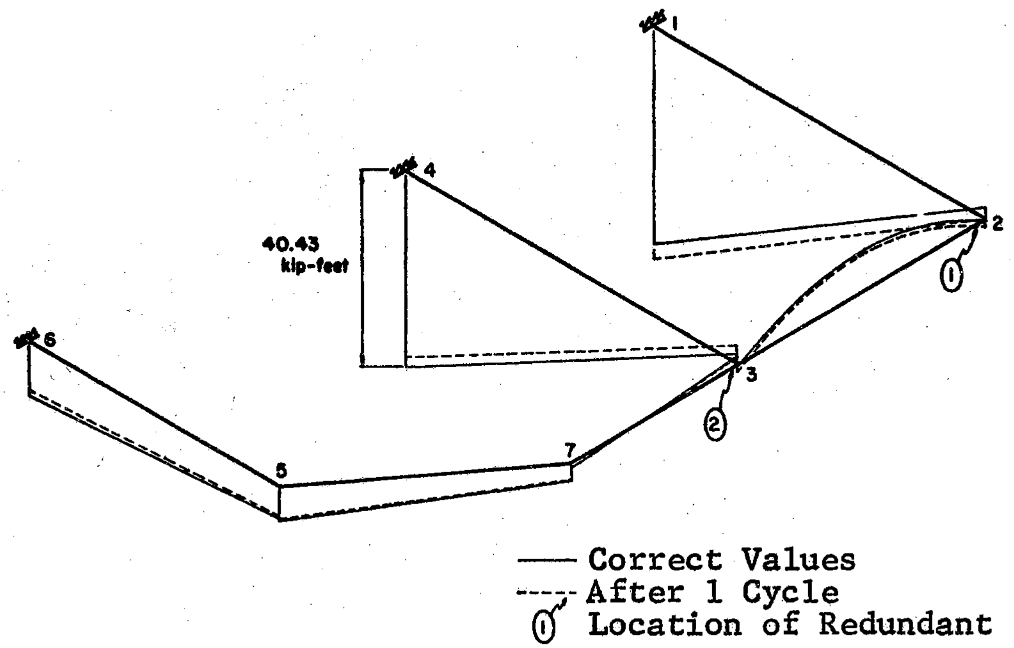
** BASIS IN THIS PROBLEM TAKEN AFTER 13 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .00001 PERCENT DURING LAST ITERATION.

TABLE A-1.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 1, AFTER 5 AND 6 CYCLES OF ITERATION

MEMBER*			ITERATION 5	6	BASIS** KIP-FEET
1	2	N	.01	.00	-121.12
		F	.00	.00	-23.05
		T	.01	.00	.00
2	3	N	.01	.00	-15.06
		F	.00	.00	-16.99
		T	.00	.00	-28.47
2	6	N	.01	.00	-15.06
		F	.00	.00	-16.99
		T	.00	.00	28.47
3	4	N	.00	.00	-28.47
		F	.00	.00	-79.44
		T	.00	.00	16.99
5	6	N	.00	.00	-79.44
		F	.00	.00	-28.47
		T	.00	.00	-16.99

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 13 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



MEMBER PROPERTIES

Member	EI	EI/GJ
1-2	1.3333	1.0
2-3	1.2000	1.2
3-4	1.3333	1.5
3-7	1.3333	1.0
5-7	.8333	2.0
5-6	1.0000	1.0

LOADS AND DIMENSIONS

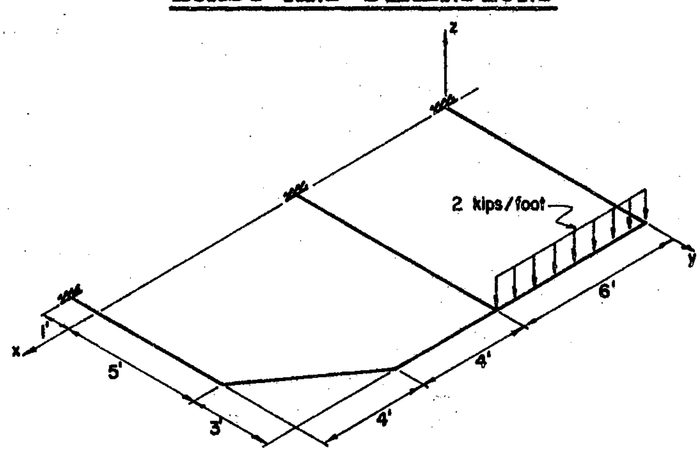


Figure A-2: Problem 2, Diwan's Problem, Member Properties, Dimensions, Loads and Comparison of Member Moments after 1 Cycle of Iteration with Correct Values

TABLE A-2.1
PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 2
AFTER 1, 2, 3, 4, 5, 6, 9 AND 12 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	O	1	2	3	4	5	7	9	12	K,K-FEET
1	1	1.68	.24	.06	.02	.01	.00	.00	.00	.28
1	2	-2.19	.20	.18	.08	.03	.01	.00	.00	-5.92
1	3	-3.83	-.22	.04	.03	.01	.00	.00	.00	-45.09
2	1	1.35	.40	.14	.06	.02	.00	.00	.00	-6.65
2	2	-1.64	-.40	-.13	-.05	-.02	.00	.00	.00	.76
2	3	-1.36	-.20	-.05	-.02	-.01	.00	.00	.00	10.48

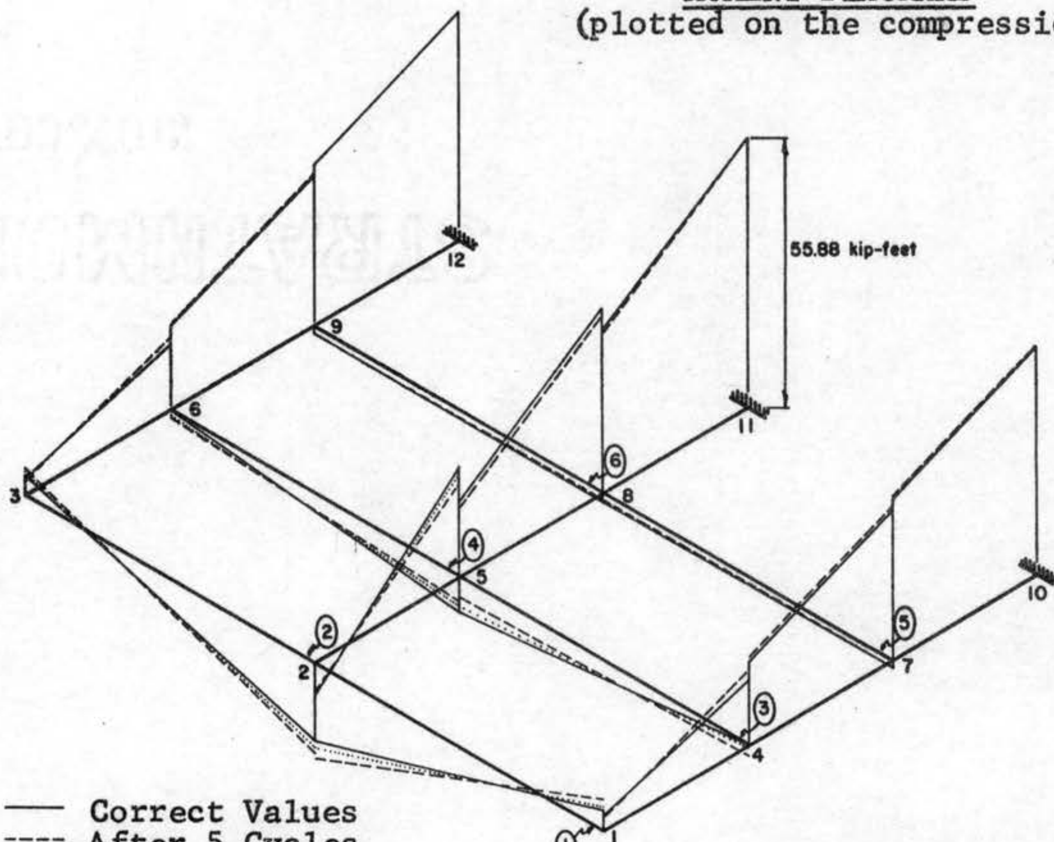
** BASIS IN THIS PROBLEM TAKEN AFTER 17 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .00001 PERCENT DURING LAST ITERATION.

TABLE A-2.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 2, AFTER 5 AND 7 CYCLES OF ITERATION

MEMBER*			ITERATION 5	7	BASIS** KIP-FEET
1	2	N	-.02	.00	45.09
		F	.01	.00	-2.27
		T	.01	.00	.28
2	3	N	.01	.00	.28
		F	.04	.01	.76
		T	-.01	.00	2.27
3	4	N	-.02	.00	-2.16
		F	.03	.01	40.43
		T	-.01	.00	-2.86
3	7	N	.02	.00	-2.11
		F	.01	.00	.92
		T	.01	.00	4.43
5	6	N	.00	.00	6.70
		F	-.01	.00	11.24
		T	.00	.00	-3.95
5	7	N	.00	.00	7.18
		F	.02	.00	3.39
		T	.00	.00	2.99

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 17 CYCLES

MOMENT DIAGRAMS
(plotted on the compression side)



—— Correct Values
---- After 5 Cycles
.... After 10 Cycles
(1) Location of Redundant

MEMBER PROPERTIES

All members have equal EI.
All members have EI=GJ

LOADS AND DIMENSIONS

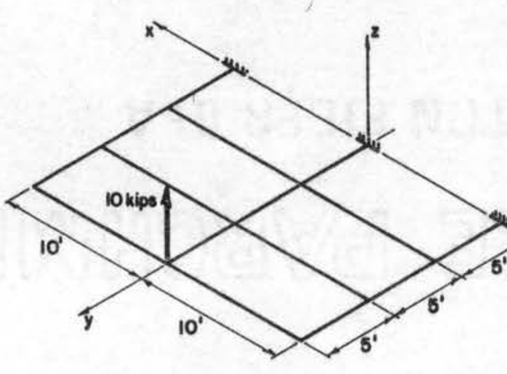


Figure A-3: Problem 3, Member Properties, Dimensions, Loads, and Comparison of Member Moments after 5 and 10 Cycles of Iteration with Correct Values

TABLE A-3.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 3
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	-7.09	7.40	11.53	11.74	10.50	4.04	.46	.04	15.94
1	2	30.12	40.77	36.38	30.84	25.63	8.87	.78	.03	2.03
1	3	58.12	42.32	34.92	28.90	23.73	7.89	.61	.01	33.84
2	1	2.38	9.41	9.66	8.66	7.53	3.59	.58	.07	15.94
2	2	-13.54	-21.11	-20.73	-19.27	-17.49	-9.11	-1.54	-.18	-2.03
2	3	-10.96	-17.99	-18.43	-17.53	-16.13	-8.62	-1.49	-.18	-33.84
3	1	-11.73	-13.47	-13.31	-11.76	-9.82	-3.38	-.33	-.02	7.23
3	2	-52.53	-45.43	-37.64	-30.83	-24.96	-7.89	-.57	-.01	.72
3	3	-46.97	-38.09	-31.10	-25.32	-20.45	-6.39	-.43	.00	10.86
4	1	-12.99	-10.70	-9.03	-7.63	-6.50	-3.07	-.49	-.06	7.23
4	2	26.44	23.06	20.96	18.95	16.96	8.61	1.43	.16	-.72
4	3	18.83	18.61	17.68	16.28	14.69	7.50	1.24	.14	-10.86
5	1	.93	-.24	-.94	-1.16	-1.17	-.66	-.11	-.01	1.55
5	2	-2.84	-3.68	-3.14	-2.73	-2.48	-1.43	-.25	-.03	.02
5	3	-5.01	-3.26	-2.78	-2.50	-2.25	-1.08	-.15	-.01	2.36
6	1	-3.79	-2.92	-2.09	-1.51	-1.15	-.49	-.08	-.01	1.55
6	2	7.39	4.97	3.18	2.26	1.79	.96	.19	.02	-.02
6	3	1.58	1.68	1.63	1.55	1.46	.88	.17	.02	-2.36

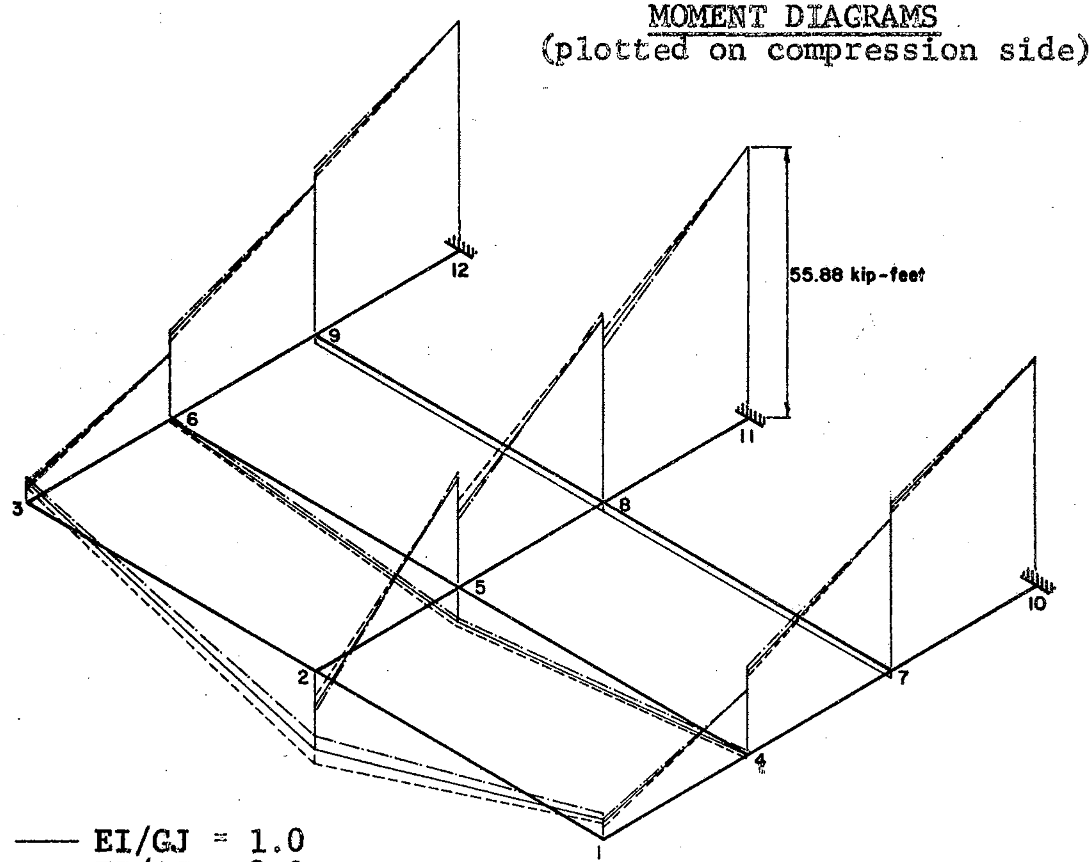
** BASIS IN THIS PROBLEM TAKEN AFTER 116 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .00001 PERCENT DURING LAST ITERATION.

TABLE A-3.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 3, AFTER 5, 10, 20 AND 30 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	30	BASIS** KIP-FEET
1	2	N	-3.99	-1.13	-.04	.01	-4.34
		F	6.36	2.45	.28	.02	15.94
		T	-1.15	-.59	-.10	-.01	3.42
1	4	N	1.15	.59	.10	.01	-3.42
		F	-4.02	-1.20	-.06	.01	-13.56
		T	-3.99	-1.13	-.04	.00	-4.34
2	3	N	4.56	2.17	.35	.04	15.94
		F	-2.50	-1.51	-.27	-.03	-4.34
		T	.82	.30	.03	.00	-3.42
2	5	N	-1.98	-.89	-.13	-.01	6.84
		F	6.73	2.74	.34	.03	-22.88
		T	-1.80	-.27	.07	.01	.00
3	6	N	.82	.30	.03	.00	-3.42
		F	-2.71	-1.54	-.28	-.03	-13.56
		T	2.50	1.51	.27	.03	4.34
4	5	N	4.13	1.14	.03	-.01	.03
		F	-5.95	-2.04	-.20	-.01	7.23
		T	-2.31	-.69	-.03	.00	3.65
4	7	N	-1.71	-.51	-.03	.00	-17.21
		F	-1.85	-.71	-.07	.00	-30.96
		T	.14	.01	-.01	.00	-4.32
5	6	N	-3.94	-1.86	-.30	-.03	7.23
		F	2.91	1.61	.28	.03	.03
		T	2.05	1.07	.18	.02	-3.65
5	8	N	2.37	.99	.13	.01	-15.57
		F	2.61	1.29	.19	.02	-38.09
		T	.21	-.09	-.02	.00	.00
6	9	N	-.66	-.48	-.10	-.01	-17.21
		F	-.76	-.58	-.13	-.02	-30.96
		T	-.41	-.11	-.01	.00	4.32
7	8	N	.29	.18	.03	.00	1.38
		F	-.71	-.40	-.07	-.01	1.55
		T	-.86	-.37	-.04	.00	2.28
7	10	N	-.99	-.34	-.03	.00	-33.23
		F	-.62	-.25	-.02	.00	-47.06
		T	.43	.18	.02	.00	-2.93
8	9	N	-.69	-.30	-.05	-.01	1.55
		F	.03	.09	.02	.00	1.38
		T	.52	.34	.06	.00	-2.28
8	11	N	1.23	.58	.09	.01	-33.53
		F	.61	.40	.07	.01	-55.88
		T	.23	.01	.00	.00	.00
9	12	N	-.24	-.24	-.06	-.01	-33.23
		F	.01	-.14	-.05	-.01	-47.06
		T	-.44	-.20	-.03	.00	2.93

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 100 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



—— EI/GJ = 1.0
---- EI/GJ = 2.0
—-— EI/GJ = 0.5

MEMBER PROPERTIES

All members have equal EI
All members have EI = 2GJ

LOADS AND DIMENSIONS

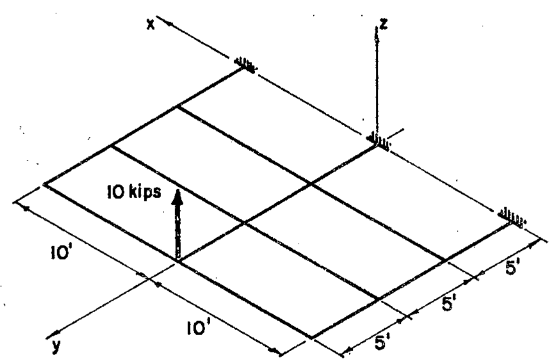


Figure A-4: Problem 4, Member Properties, Dimensions, Loads and Comparison of Member Moments for EI/GJ Variation of .5, 1.0 and 2.0

TABLE A-4.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 4
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

$		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	-21.74	-7.93	-1.34	1.83	3.16	2.33	.38	.06	18.74
1	2	-.68	11.18	11.53	10.63	9.52	4.44	.70	.10	2.21
1	3	27.35	14.11	11.75	10.25	8.92	4.01	.63	.09	35.75
2	1	-9.98	-.33	2.71	3.65	3.70	1.80	.30	.04	18.74
2	2	4.85	-7.01	-9.13	-9.26	-8.65	-4.17	-.70	-.10	-2.21
2	3	-.03	-7.64	-8.91	-8.84	-8.20	-3.95	-.66	-.10	-35.75
3	1	-8.09	-6.53	-6.54	-6.38	-5.92	-2.77	-.43	-.06	8.15
3	2	-38.80	-27.82	-20.72	-16.41	-13.45	-5.41	-.83	-.12	.76
3	3	-27.70	-19.04	-14.51	-11.66	-9.63	-3.93	-.60	-.09	10.08
4	1	-13.39	-9.89	-7.95	-6.51	-5.36	-2.07	-.33	-.05	8.15
4	2	26.67	20.71	16.88	14.08	11.82	4.89	.80	.12	-.76
4	3	15.84	13.71	11.90	10.28	8.82	3.78	.62	.09	-10.08
5	1	1.81	2.11	1.56	1.04	.64	-.03	-.02	.00	1.68
5	2	.18	.70	1.10	1.13	.96	.15	-.01	.00	.04
5	3	.41	1.21	.78	.46	.26	-.06	-.02	.00	1.54
6	1	-1.99	-.81	-.40	-.28	-.25	-.17	-.03	.00	1.68
6	2	3.67	1.36	.18	-.16	-.17	.09	.03	.00	-.04
6	3	-1.40	-1.06	-.62	-.29	-.07	.15	.03	.00	-1.54

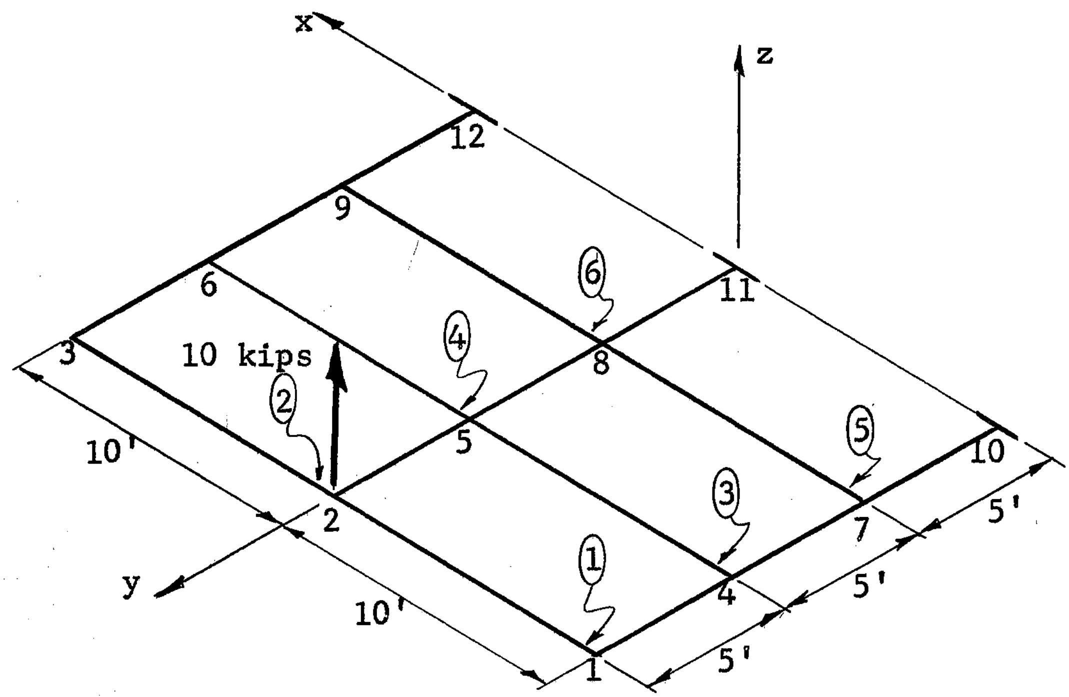
** BASIS IN THIS PROBLEM TAKEN AFTER 42 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-4.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 4, AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	-2.05	-.40	-.05	-3.36
		F	2.01	1.48	.21	18.76
		T	-.39	-.27	-.04	2.60
1	4	N	.39	.27	.04	-2.60
		F	-1.65	-.66	-.09	-13.66
		T	-2.05	-.40	-.05	-3.36
2	3	N	2.37	1.14	.16	18.76
		F	-1.32	-.62	-.09	-3.36
		T	.29	.14	.02	-2.61
2	5	N	-.67	-.41	-.06	5.21
		F	3.21	1.40	.20	-22.67
		T	.36	-.34	-.05	.00
3	6	N	.29	.14	.02	-2.61
		F	-1.56	-.74	-.11	-13.67
		T	1.32	.62	.09	3.36
4	5	N	1.96	.53	.07	.53
		F	-3.79	-1.75	-.24	8.13
		T	-.42	-.21	-.03	2.45
4	7	N	-1.22	-.46	-.06	-16.12
		F	-.38	-.25	-.04	-30.97
		T	-.09	.13	.02	-2.83
5	6	N	-3.44	-1.31	-.18	8.13
		F	1.61	.75	.11	.54
		T	.60	.33	.05	-2.45
5	8	N	2.18	.87	.12	-17.77
		F	.66	.51	.08	-38.05
		T	.71	.11	.01	.00
6	9	N	-.96	-.41	-.06	-16.12
		F	-.28	-.26	-.04	-30.98
		T	-.29	-.14	-.02	2.83
7	8	N	.00	-.08	-.01	1.33
		F	.42	-.01	-.01	1.68
		T	-.04	-.07	-.01	1.36
7	10	N	-.34	-.18	-.03	-32.33
		F	.30	-.01	.00	-47.37
		T	-.09	.05	.01	-1.50
8	9	N	-.16	-.11	-.02	1.68
		F	-.23	-.07	-.01	1.33
		T	-.01	.07	.01	-1.36
8	11	N	.63	.37	.06	-35.33
		F	-.65	.03	.01	-55.26
		T	.14	.02	.00	.00
9	12	N	-.29	-.19	-.03	-32.34
		F	.35	-.02	-.01	-47.37
		T	-.06	-.07	-.01	1.50

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

LOADS, DIMENSIONS AND LOCATION
OF REDUNDANTS



MEMBER PROPERTIES

All members have equal EI
All members have EI = 0.5GJ

Figure A-5: Problem 5, Member Properties, Dimensions and Loads

TABLE A-5.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 5
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	8.73	22.38	23.06	20.21	16.97	6.76	.66	-.07	13.17
1	2	64.28	71.06	62.47	52.91	44.16	16.51	1.07	-.32	1.83
1	3	88.47	71.34	59.98	50.03	41.42	14.84	.67	-.38	31.54
2	1	12.95	15.62	13.63	11.98	10.79	6.34	1.41	.17	13.17
2	2	-31.50	-32.81	-30.37	-28.30	-26.39	-16.53	-3.91	-.51	-1.83
2	3	-21.42	-26.22	-26.01	-25.04	-23.75	-15.48	-3.82	-.53	-31.53
3	1	-16.51	-19.24	-17.45	-14.52	-11.86	-4.11	-.10	.13	6.20
3	2	-64.73	-59.54	-50.94	-42.30	-34.64	-11.40	.04	.46	.70
3	3	-66.66	-57.69	-48.45	-40.04	-32.75	-10.74	.03	.43	11.90
4	1	-10.83	-8.40	-7.29	-6.79	-6.46	-4.18	-.96	-.12	6.20
4	2	24.17	21.14	20.42	19.99	19.31	12.75	3.01	.39	-.70
4	3	20.63	20.76	20.69	20.20	19.35	12.63	3.01	.39	-11.90
5	1	-1.19	-3.28	-3.56	-3.22	-2.80	-1.43	-.30	-.04	1.46
5	2	-9.93	-11.30	-9.94	-8.62	-7.52	-3.88	-.80	-.09	.05
5	3	-13.01	-10.19	-8.50	-7.30	-6.33	-3.07	-.54	-.04	3.56
6	1	-5.71	-4.31	-2.95	-2.19	-1.80	-.98	-.24	-.03	1.46
6	2	12.98	9.53	6.86	5.40	4.58	2.65	.70	.11	-.05
6	3	6.10	5.60	4.89	4.29	3.83	2.34	.63	.10	-3.56

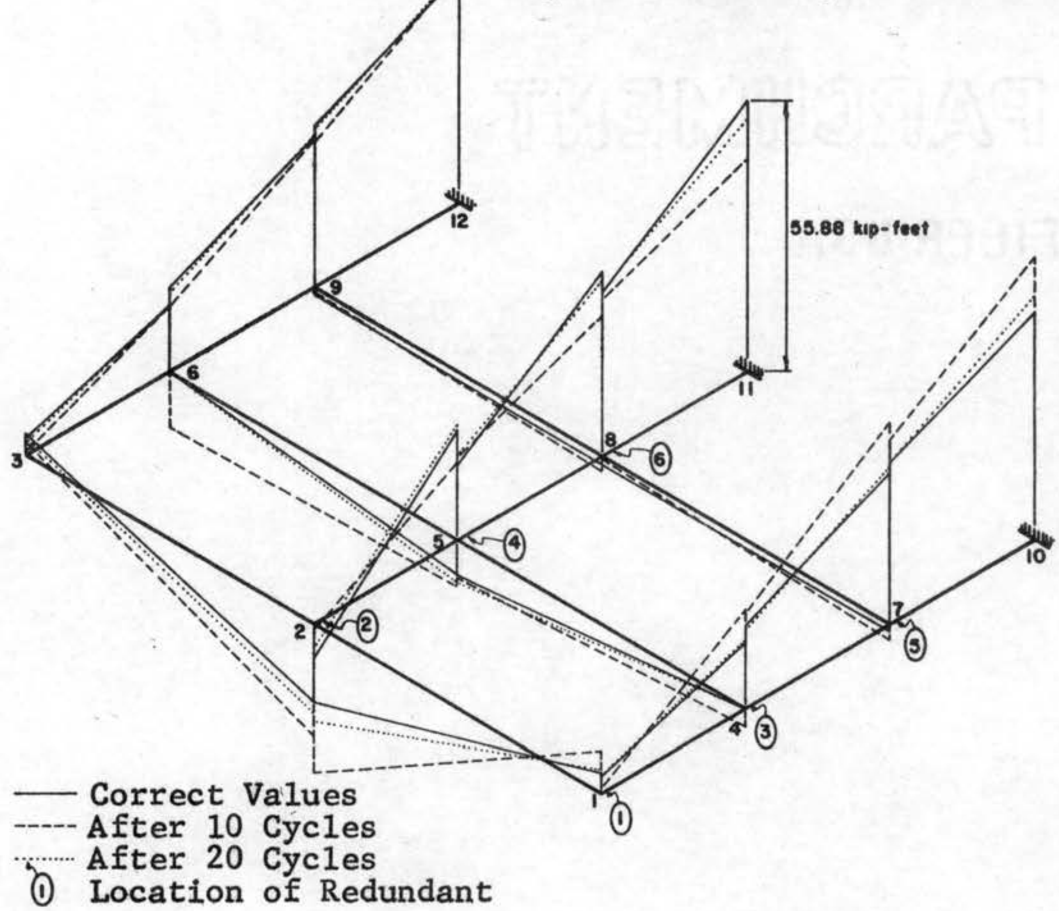
** BASIS IN THIS PROBLEM TAKEN AFTER 61 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-5.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 5, AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	-7.10	-2.47	-.11	-5.11
		F	9.60	3.84	.41	13.15
		T	-1.51	-.91	-.20	4.03
1	4	N	1.51	.91	.20	-4.03
		F	-6.84	-2.25	-.07	-13.16
		T	-7.10	-2.47	-.11	-5.11
2	3	N	5.98	3.48	.70	13.22
		F	-3.73	-2.53	-.58	-5.21
		T	1.50	.60	.06	-4.05
2	5	N	-3.01	-1.51	-.25	8.08
		F	10.19	4.66	.65	-23.57
		T	-3.61	-.37	.29	.07
3	6	N	1.50	.60	.06	-4.05
		F	-3.36	-2.41	-.58	-13.27
		T	3.73	2.53	.58	5.21
4	5	N	6.42	2.06	.03	-.82
		F	-6.75	-2.39	-.13	6.24
		T	-5.51	-1.84	-.07	4.97
4	7	N	-1.33	-.42	.00	-18.13
		F	-3.09	-1.35	-.18	-30.79
		T	-.68	-.41	-.09	-5.94
5	6	N	-3.57	-2.29	-.47	6.16
		F	3.53	2.35	.51	-.73
		T	3.57	2.25	.49	-4.89
5	8	N	1.12	.57	.09	-13.71
		F	4.18	2.19	.42	-38.39
		T	-.43	-.27	-.06	.00
6	9	N	.21	-.16	-.09	-18.16
		F	-1.10	-.84	-.24	-30.82
		T	.20	.18	.07	5.94
7	8	N	1.23	.64	.12	1.01
		F	-1.55	-.79	-.15	1.45
		T	-2.15	-.99	-.15	3.33
7	10	N	-.94	-.36	-.03	-34.12
		F	-1.31	-.58	-.09	-47.00
		T	.55	.22	.03	-4.92
8	9	N	-.99	-.53	-.12	1.45
		F	.69	.42	.11	1.02
		T	1.26	.79	.19	-3.31
8	11	N	.78	.41	.08	-31.75
		F	1.61	.84	.17	-56.00
		T	.13	-.01	-.02	.00
9	12	N	.17	-.06	-.05	-34.13
		F	-.30	-.26	-.08	-47.00
		T	-.49	-.24	-.04	4.92

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



MEMBER PROPERTIES

All members have equal EI
All members have EI=GJ

Note: Problem 6 is identical with problem 3 except for choice of redundants

LOADS AND DIMENSIONS

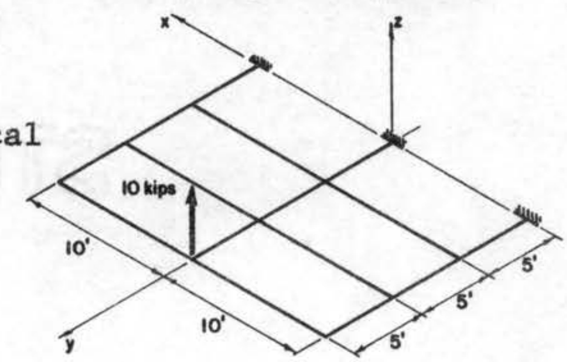


Figure A-6: Problem 6, Member Properties, Dimensions, Loads and Comparison of Member Moments after 10 and 20 Cycles of Iteration with Correct Values

TABLE A-6.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 6
AFTER 1, 2, 3, 5, 10, 20, 30 AND 50 CYCLES OF ITERATION

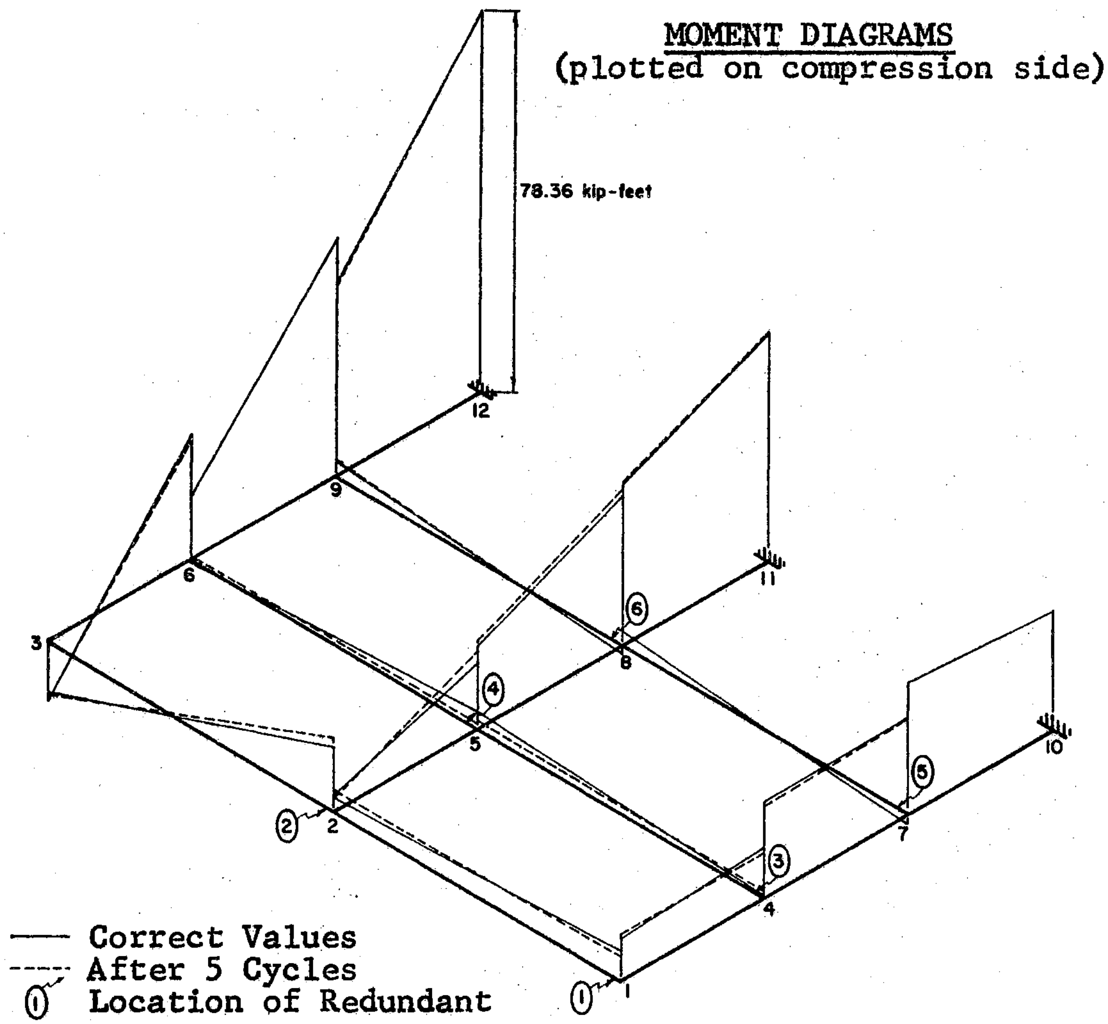
S		ITERATION								BASIS**
I	J	1	2	3	5	10	20	30	50	K,K-FEET
1	1	-4.56	-11.46	-15.33	-16.33	-10.98	-4.44	-1.87	-.37	-15.94
1	2	-20.39	-27.63	-35.04	-32.14	-19.84	-7.47	-2.95	-.52	-2.03
1	3	-10.27	-19.10	-29.85	-29.35	-16.49	-5.84	-2.22	-.36	-33.84
2	1	10.93	9.21	8.98	8.52	5.13	1.85	.70	.12	.00
2	2	65.64	59.51	52.46	41.62	26.35	10.29	4.14	.77	-5.94
2	3	61.36	54.48	47.15	35.57	20.99	7.88	3.07	.53	-82.32
3	1	5.69	-9.02	-15.05	-16.45	-12.15	-6.06	-3.01	-.75	-23.17
3	2	2.65	-26.56	-29.10	-26.77	-19.09	-9.57	-4.78	-1.19	-2.75
3	3	13.56	-17.40	-23.33	-20.66	-11.62	-5.22	-2.50	-.59	-44.70
4	1	6.19	10.43	8.56	6.08	3.95	1.94	.96	.24	.00
4	2	54.33	40.43	40.20	38.22	28.15	14.20	7.10	1.77	-4.50
4	3	48.91	35.34	30.31	23.82	15.20	7.27	3.53	.85	-60.60
5	1	-4.82	-13.33	-15.59	-15.12	-11.31	-6.16	-3.27	-.88	-24.72
5	2	-27.48	-27.24	-25.11	-21.83	-16.54	-9.42	-5.14	-1.42	-2.77
5	3	-9.81	-17.37	-16.97	-12.88	-6.88	-3.33	-1.70	-.44	-47.06
6	1	21.10	11.14	6.23	3.17	2.15	1.31	.74	.21	.00
6	2	21.33	32.88	37.30	36.50	27.78	15.49	8.36	2.29	-4.47
6	3	28.70	22.23	18.46	13.91	8.94	4.66	2.43	.64	-55.88

** BASIS IN THIS PROBLEM TAKEN AFTER 160 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN 1 DIGIT IN FIFTH SIGNIFICANT FIGURE DURING THE PREVIOUS 10 CYCLES OF ITERATION.

TABLE A-6.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 6, AFTER 10, 20, 30 AND 50 CYCLES OF ITERATION

MEMBER*			ITERATION 10	20	30	50	BASIS** KIP-FEET
1	2	N	-3.57	-.86	-.15	.04	-4.34
		F	17.53	7.09	2.99	.60	15.94
		T	-5.35	-2.60	-1.16	-.25	3.42
1	4	N	5.35	2.60	1.16	.25	-3.42
		F	-5.20	-1.37	-.40	-.02	-13.56
		T	-3.57	-.86	-.15	.03	-4.34
2	3	N	9.33	4.14	1.86	.41	15.94
		F	2.40	1.15	.59	.15	-4.34
		T	3.21	1.24	.55	.11	-3.42
2	5	N	-8.55	-3.84	-1.71	-.37	6.84
		F	5.46	1.63	.49	.04	-22.88
		T	-8.19	-2.95	-1.12	-.18	.00
3	6	N	3.21	1.24	.55	.12	-3.42
		F	-.26	-.26	-.09	-.01	-13.56
		T	-2.40	-1.15	-.59	-.15	4.34
4	5	N	2.65	.34	-.12	-.11	.03
		F	1.85	2.57	1.82	.60	7.23
		T	-6.97	-3.22	-1.50	-.34	3.65
4	7	N	1.77	1.84	1.09	.32	-17.21
		F	-8.38	-3.24	-1.45	-.31	-30.96
		T	-.92	-.51	-.27	-.06	-4.32
5	6	N	3.74	2.42	1.41	.41	7.23
		F	1.03	.48	.21	.05	.03
		T	4.18	1.91	.90	.21	-3.65
5	8	N	-5.69	-3.50	-1.91	-.52	-15.57
		F	9.29	4.05	1.87	.42	-38.09
		T	-6.30	-3.10	-1.54	-.37	.00
6	9	N	3.92	1.66	.81	.20	-17.21
		F	-.90	-.81	-.42	-.11	-30.96
		T	-3.43	-1.63	-.80	-.19	4.32
7	8	N	1.38	.32	.02	-.04	1.38
		F	-1.33	.16	.41	.21	1.55
		T	-6.21	-2.95	-1.47	-.36	2.28
7	10	N	-2.18	-.29	.02	.05	-33.23
		F	-10.97	-5.31	-2.72	-.70	-47.06
		T	.46	-.20	-.25	-.10	-2.93
8	9	N	1.54	1.17	.76	.25	1.55
		F	-.77	-.35	-.19	-.05	1.38
		T	3.59	1.91	.97	.25	-2.28
8	11	N	-.51	-.81	-.57	-.20	-33.53
		F	14.27	7.43	3.87	1.02	-55.88
		T	-3.43	-2.10	-1.19	-.34	.00
9	12	N	2.69	1.10	.55	.14	-33.23
		F	-3.29	-2.12	-1.15	-.32	-47.06
		T	-2.66	-1.28	-.62	-.14	2.93

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 100 CYCLES, PROBLEM 3



MEMBER PROPERTIES

All members have equal EI
All members have EI = GJ

LOADS AND DIMENSIONS

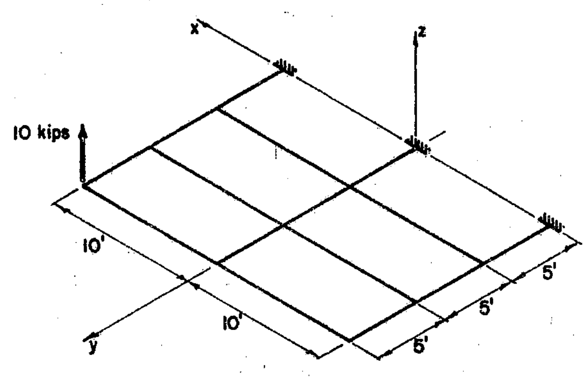


Figure A-7: Problem 7, Member Properties, Dimensions, Loads and Comparison of Member Moments after 5 Cycles of Iteration with Correct Values

TABLE A-7.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 7
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	5.71	-.64	-3.53	-4.46	-4.36	-1.69	-.16	-.01	-2.72
1	2	-10.63	-14.85	-13.23	-11.39	-9.61	-3.25	-.22	.00	.34
1	3	-28.99	-15.67	-12.36	-10.21	-8.44	-2.73	-.15	.01	13.82
2	1	-.67	-5.90	-6.20	-5.48	-4.61	-1.84	-.24	-.02	-13.22
2	2	10.76	16.56	15.14	13.19	11.31	4.83	.66	.06	2.37
2	3	12.27	15.35	14.02	12.35	10.68	4.63	.64	.06	47.66
3	1	.90	3.40	4.74	4.75	4.16	1.34	.11	.00	-3.77
3	2	17.22	16.18	13.79	11.47	9.31	2.70	.13	-.01	-.31
3	3	18.27	14.00	11.17	9.06	7.30	2.11	.08	-.01	6.18
4	1	7.46	7.09	6.15	5.06	4.11	1.58	.21	.02	-3.46
4	2	-20.46	-17.94	-15.43	-13.11	-11.05	-4.54	-.61	-.06	.41
4	3	-15.52	-14.41	-12.83	-11.10	-9.47	-3.95	-.53	-.05	17.03
5	1	-1.23	-.79	-.07	.33	.49	.33	.05	.00	-3.07
5	2	.22	.53	.86	1.02	1.09	.72	.11	.01	-.52
5	3	1.26	.91	.92	.93	.91	.49	.06	.00	4.59
6	1	-.43	.70	.90	.80	.65	.25	.04	.00	1.52
6	2	-.02	-1.72	-1.60	-1.33	-1.10	-.53	-.08	-.01	-.50
6	3	-1.84	-1.28	-1.15	-1.05	-.96	-.49	-.08	-.01	6.95

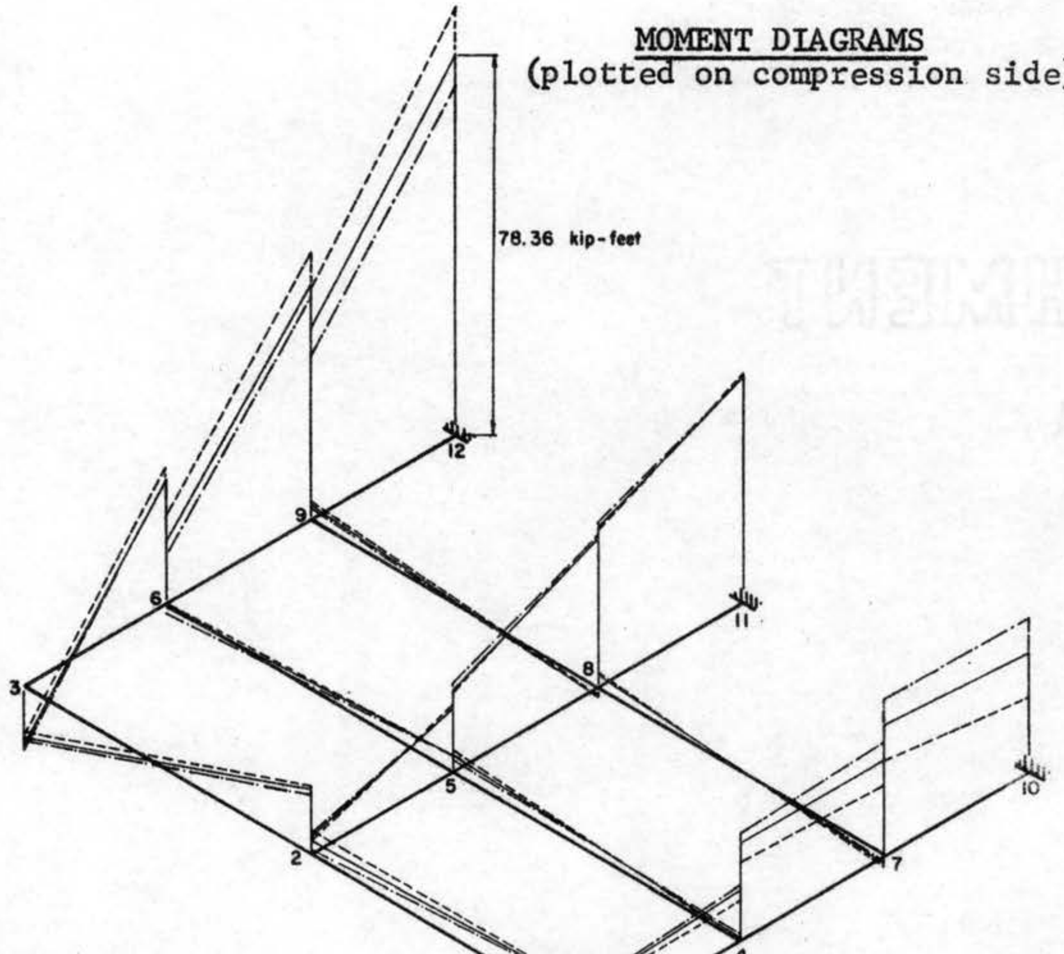
** BASIS IN THIS PROBLEM TAKEN AFTER 41 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-7.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 7,
AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	1.25	.30	.00	-6.10
		F	-2.64	-1.02	-.09	-2.72
		T	.71	.31	.04	8.76
1	4	N	-.71	-.31	-.04	-8.76
		F	1.24	.35	.01	-10.44
		T	1.25	.30	.00	-6.10
2	3	N	-2.79	-1.11	-.13	-13.23
		F	1.77	.83	.11	10.45
		T	-.38	-.12	-.01	12.17
2	5	N	1.09	.43	.05	-3.42
		F	-3.14	-1.19	-.12	-13.57
		T	-.15	-.08	-.04	-10.51
3	6	N	-.38	-.12	-.01	12.17
		F	1.90	.85	.11	-25.99
		T	-1.77	-.83	-.11	-10.45
4	5	N	-1.25	-.29	.01	-.64
		F	2.53	.81	.06	-3.77
		T	.67	.19	.00	9.30
4	7	N	.57	.16	.00	-19.75
		F	.63	.27	.02	-19.87
		T	.00	.01	.00	-6.74
5	6	N	2.49	.95	.11	-3.45
		F	-1.97	-.87	-.11	.61
		T	-1.27	-.55	-.07	12.95
5	8	N	-1.21	-.45	-.05	-17.22
		F	-1.32	-.62	-.07	-30.96
		T	-.18	.06	.01	-10.19
6	9	N	.63	.29	.04	-13.04
		F	.68	.35	.05	-49.17
		T	.20	.04	.00	-11.06
7	8	N	-.15	-.09	-.01	2.10
		F	.29	.20	.03	-3.06
		T	.33	.15	.01	7.17
7	10	N	.30	.12	.01	-27.04
		F	.14	.08	.01	-24.58
		T	-.14	-.08	-.01	-4.64
8	9	N	.39	.15	.02	1.52
		F	-.05	-.06	-.01	-3.49
		T	-.35	-.19	-.03	9.44
8	11	N	-.63	-.28	-.03	-33.24
		F	-.30	-.20	-.03	-47.06
		T	-.09	.01	.00	-5.61
9	12	N	.33	.16	.03	-39.72
		F	.16	.11	.02	-78.36
		T	.25	.10	.01	-7.57

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



— EI/GJ = 1.0
----- EI/GJ = 2.0
-·-·- EI/GJ = 0.5

MEMBER PROPERTIES

All members have equal EI
All members have EI = 2GJ

LOADS AND DIMENSIONS

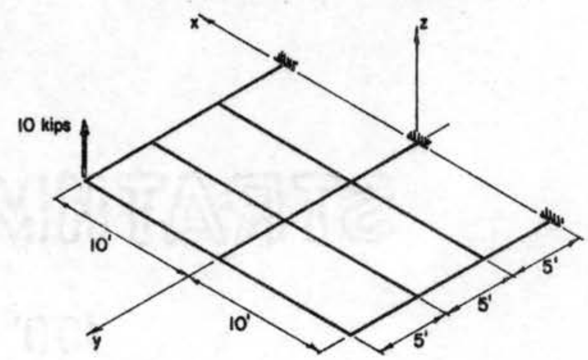


Figure A-8: Problem 8, Member Properties, Dimensions, Loads and Comparison of Member Moments for EI/GJ Variation of .5, 1.0 and 2.0

TABLE A-8.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 8
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	10.80	5.78	2.54	.53	-.60	-1.09	-.19	-.03	-4.87
1	2	-3.50	-4.53	-3.42	-2.94	-2.74	-1.81	-.34	-.05	.11
1	3	-20.66	-6.13	-3.69	-2.83	-2.46	-1.58	-.30	-.05	9.31
2	1	6.45	.51	-1.42	-2.07	-2.14	-.96	-.15	-.02	-13.87
2	2	-4.03	5.15	6.34	6.13	5.49	2.27	.35	.05	2.31
2	3	1.05	5.76	6.24	5.92	5.27	2.16	.33	.05	45.06
3	1	.12	.09	1.14	1.86	2.19	1.36	.21	.03	-4.61
3	2	14.26	9.25	6.61	5.37	4.65	2.37	.40	.06	-.46
3	3	11.78	6.61	4.57	3.64	3.13	1.67	.29	.05	3.08
4	1	5.95	5.11	4.49	3.83	3.18	1.11	.16	.03	-3.54
4	2	-16.40	-13.66	-11.16	-9.15	-7.47	-2.66	-.39	-.06	.30
4	3	-10.53	-9.27	-7.94	-6.66	-5.53	-2.04	-.31	-.05	13.17
5	1	-.69	-1.63	-1.42	-1.04	-.71	-.04	.01	.00	-2.93
5	2	2.96	.17	-.75	-.95	-.88	-.19	.00	.00	-.53
5	3	1.26	-.04	-.31	-.35	-.31	-.03	.01	.00	2.92
6	1	-2.07	-1.36	-.78	-.40	-.17	.06	.02	.00	1.24
6	2	5.26	2.75	1.69	1.03	.60	.00	-.01	.00	-.50
6	3	1.83	1.40	.79	.40	.17	-.06	-.02	.00	4.46

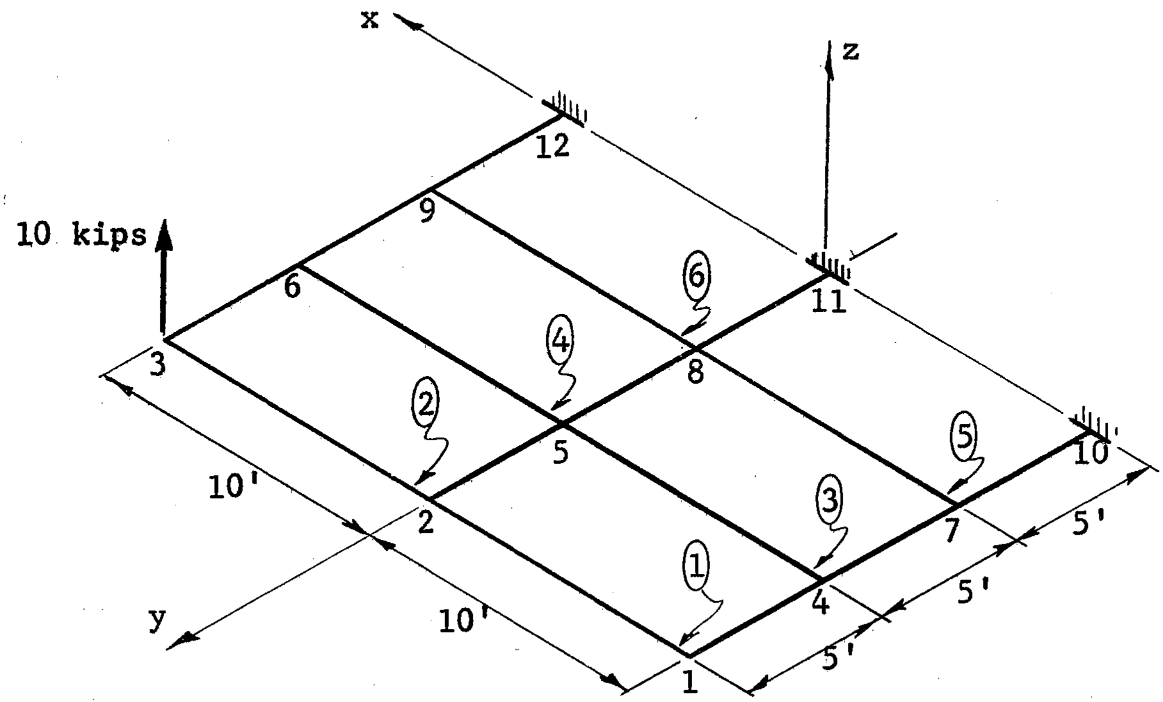
** BASIS IN THIS PROBLEM TAKEN AFTER 49 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-8.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 8, AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	.63	.06	.02	-5.91
		F	-.29	-.55	-.08	-4.88
		T	.14	.12	.02	7.74
1	4	N	-.14	-.12	-.02	-7.74
		F	.32	.19	.03	-8.25
		T	.63	.06	.02	-5.91
2	3	N	-1.09	-.48	-.06	-13.88
		F	.78	.28	.04	9.28
		T	-.12	-.05	-.01	10.34
2	5	N	.26	.17	.02	-2.60
		F	-1.14	-.51	-.08	-13.67
		T	-.80	.06	.02	-9.01
3	6	N	-.12	-.05	-.01	10.34
		F	.82	.33	.04	-28.08
		T	-.78	-.28	-.04	-9.28
4	5	N	-.47	-.11	-.02	.00
		F	1.11	.68	.09	-4.59
		T	.02	.04	.01	7.70
4	7	N	.31	.14	.02	-15.95
		F	-.02	.05	.01	-14.17
		T	.17	-.05	-.01	-5.91
5	6	N	1.63	.56	.07	-3.53
		F	-.92	-.33	-.04	-.54
		T	-.28	-.14	-.02	10.15
5	8	N	-.85	-.33	-.05	-16.12
		F	-.18	-.17	-.03	-30.98
		T	-.28	-.06	.00	-7.94
6	9	N	.54	.19	.02	-17.93
		F	.20	.12	.02	-54.85
		T	.14	.06	.01	-8.74
7	8	N	-.07	.04	.00	2.40
		F	-.37	-.02	.00	-2.93
		T	-.01	.02	.00	5.58
7	10	N	-.01	.03	.01	-19.75
		F	-.18	-.03	.00	-15.31
		T	.10	-.01	.00	-3.51
8	9	N	-.09	.03	.01	1.25
		F	.12	.03	.00	-3.73
		T	-.01	-.03	.00	6.94
8	11	N	-.18	-.13	-.02	-32.34
		F	.23	.00	.00	-47.37
		T	.00	.00	.00	-3.77
9	12	N	.19	.09	.01	-47.91
		F	-.04	.03	.00	-87.32
		T	.02	.03	.00	-5.01

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

LOADS, DIMENSIONS AND LOCATION OF REDUNDANTS



MEMBER PROPERTIES

All members have equal EI
All members have EI = 0.5GJ

Figure A-9: Problem 9, Member Properties, Dimensions and Loads

TABLE A-9.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 9
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	2.17	-5.96	-8.17	-7.71	-6.50	-2.23	-.04	.08	-1.03
1	2	-14.53	-22.55	-20.96	-18.09	-15.03	-4.68	.24	.26	.46
1	3	-33.49	-22.51	-19.02	-15.98	-13.13	-3.79	.40	.27	15.88
2	1	-7.98	-11.21	-9.52	-7.83	-6.62	-3.22	-.52	-.03	-12.14
2	2	28.23	28.21	23.75	20.24	17.54	8.77	1.50	.10	2.29
2	3	25.35	25.43	22.00	19.06	16.63	8.44	1.51	.12	47.41
3	1	2.38	6.29	6.61	5.45	4.19	.97	-.17	-.09	-2.59
3	2	19.97	21.22	18.20	14.49	11.14	2.23	-.65	-.28	-.13
3	3	23.62	20.25	16.42	12.99	10.07	2.08	-.61	-.26	9.23
4	1	8.71	7.38	6.01	5.07	4.42	2.21	.36	.02	-3.61
4	2	-23.62	-19.78	-17.16	-15.19	-13.51	-6.85	-1.15	-.07	.57
4	3	-20.76	-18.78	-16.89	-15.07	-13.38	-6.79	-1.16	-.08	21.13
5	1	-2.39	-.32	.84	1.18	1.20	.67	.11	.01	-2.73
5	2	-3.45	.48	2.15	2.80	2.95	1.79	.28	.01	-.42
5	3	-.08	1.06	1.81	2.20	2.28	1.30	.17	.00	6.58
6	1	2.51	2.96	2.15	1.49	1.11	.51	.10	.01	1.27
6	2	-8.72	-7.86	-5.50	-3.91	-3.01	-1.44	-.29	-.03	-.37
6	3	-7.92	-5.60	-4.12	-3.19	-2.60	-1.31	-.27	-.03	10.14

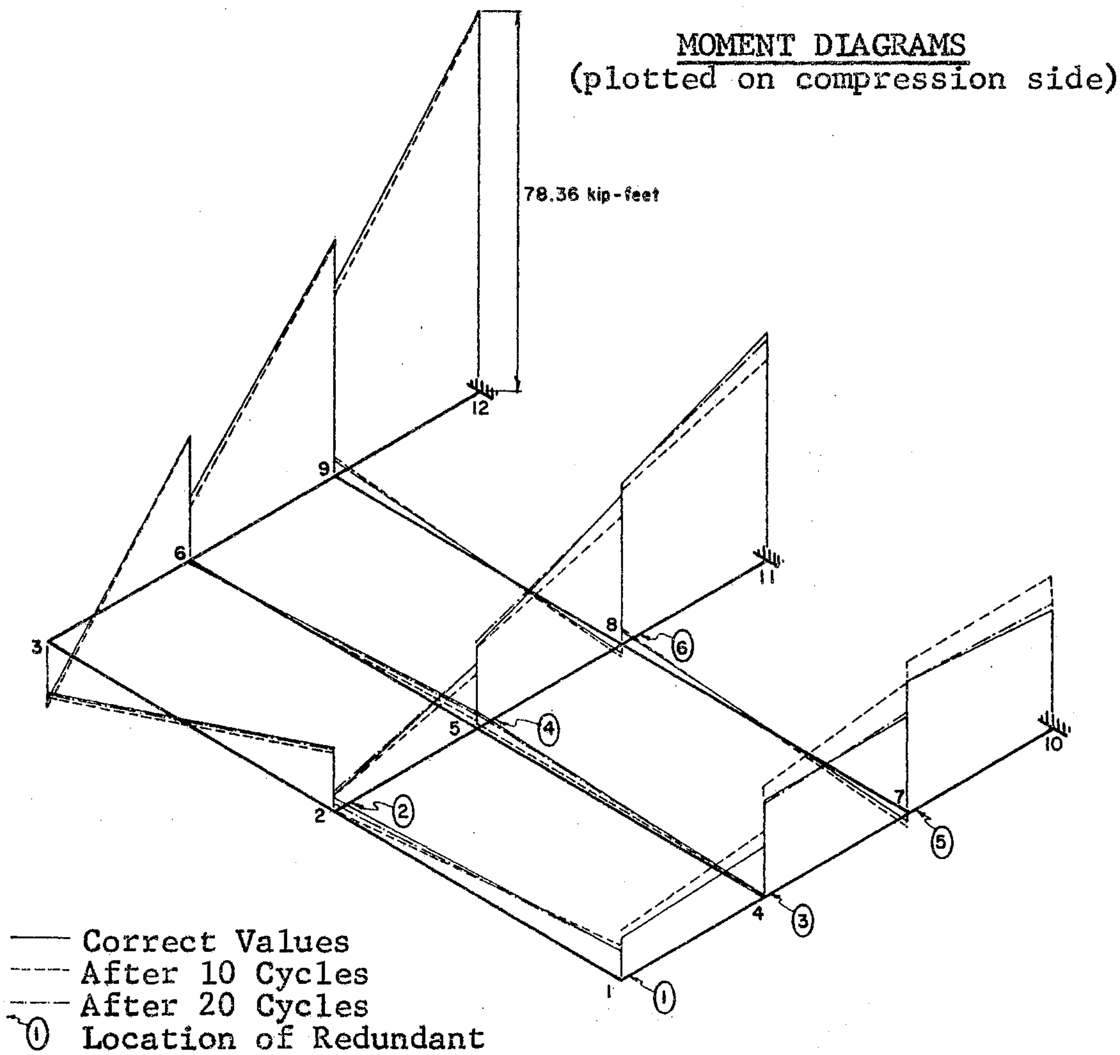
** BASIS IN THIS PROBLEM TAKEN AFTER 58 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-9.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 9, AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION			BASIS**
			5	10	20	KIP-FEET
1	2	N	2.41	.66	-.07	-5.67
		F	-4.37	-1.53	-.08	-.99
		T	1.25	.59	.10	9.00
1	4	N	-1.25	-.59	-.10	-9.00
		F	2.14	.51	-.09	-11.33
		T	2.41	.66	-.07	-5.67
2	3	N	-4.39	-2.12	-.33	-12.15
		F	3.34	1.72	.29	10.80
		T	-.61	-.23	-.01	13.05
2	5	N	1.86	.81	.10	-4.05
		F	-5.39	-2.20	-.21	-13.19
		T	-.02	-.59	-.25	-11.16
3	6	N	-.61	-.23	-.01	13.05
		F	3.25	1.69	.30	-25.48
		T	-3.34	-1.72	-.29	-10.80
4	5	N	-2.22	-.41	.11	-1.27
		F	2.84	.70	-.05	-2.63
		T	1.81	.44	-.06	10.47
4	7	N	.33	.07	-.03	-21.80
		F	1.19	.60	.06	-23.46
		T	.19	.24	.04	-6.94
5	6	N	2.92	1.45	.22	-3.60
		F	-3.03	-1.55	-.25	2.07
		T	-2.89	-1.46	-.24	15.42
5	8	N	-.69	-.30	-.03	-18.14
		F	-2.44	-1.26	-.19	-30.80
		T	.06	.16	.03	-12.13
6	9	N	.36	.24	.06	-10.06
		F	1.25	.65	.13	-45.75
		T	-.31	-.17	-.04	-12.87
7	8	N	-.51	-.34	-.05	1.42
		F	.80	.45	.07	-2.73
		T	.87	.47	.05	8.65
7	10	N	.32	.13	.00	-32.11
		F	.52	.27	.03	-31.69
		T	-.32	-.10	-.01	-5.51
8	9	N	.73	.33	.06	1.28
		F	-.59	-.29	-.06	-2.43
		T	-1.05	-.54	-.10	11.98
8	11	N	-.52	-.24	-.04	-34.12
		F	-.95	-.49	-.08	-47.00
		T	.00	.05	.02	-8.13
9	12	N	.20	.12	.03	-33.77
		F	.43	.22	.05	-71.31
		T	.28	.13	.02	-10.44

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



MEMBER PROPERTIES

All members have equal EI
All members have EI = GJ

Note: Problem 10 is identical with Problem 7 except for choice of redundants.

LOADS AND DIMENSIONS

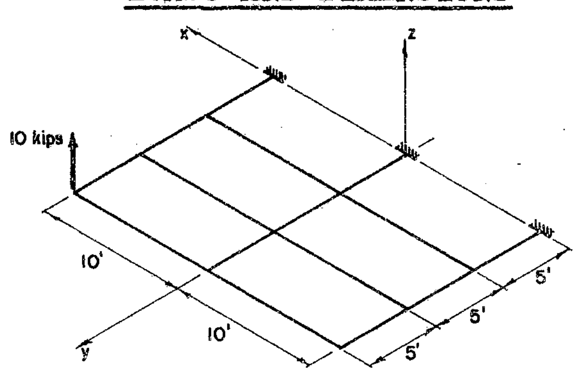


Figure A-10: Problem 10, Member Properties, Dimensions, Loads, and Comparison of Member Moments after 10 and 20 Cycles of Iteration with Correct Values

TABLE A-10.1
PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 10
AFTER 1, 2, 3, 5, 10, 20, 30 AND 50 CYCLES OF ITERATION

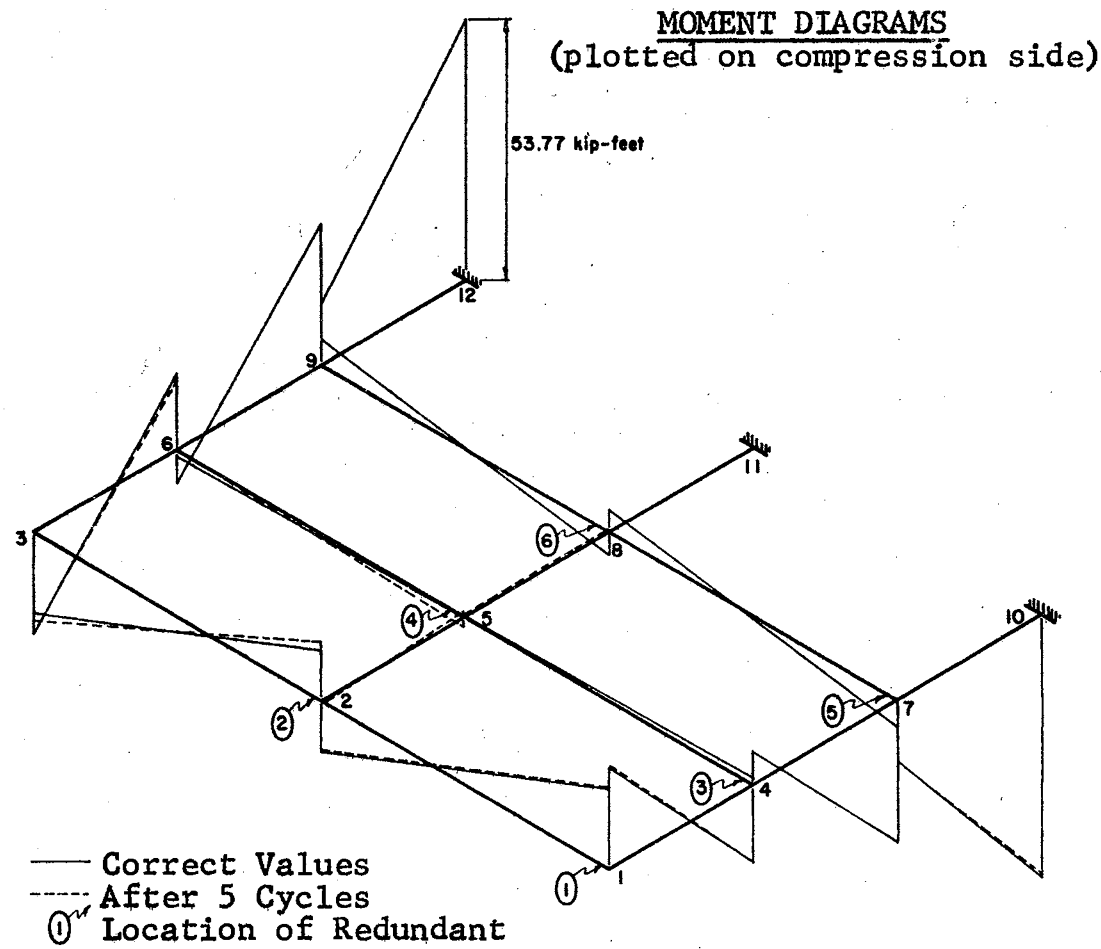
S		ITERATION								BASIS**
I	J	1	2	3	5	10	20	30	50	K,K-FEET
1	1	-5.75	-5.84	-4.49	-5.73	-4.82	-2.18	-.98	-.19	2.70
1	2	9.53	2.64	-9.70	-11.89	-7.83	-3.23	-1.38	-.25	-.34
1	3	23.03	8.95	-10.20	-14.82	-6.73	-2.33	-.96	-.17	-13.83
2	1	-20.36	-12.85	-3.52	2.55	2.02	.74	.31	.05	10.50
2	2	59.98	44.57	25.84	13.65	10.16	4.63	2.02	.38	-2.03
2	3	63.61	46.58	28.35	14.01	7.80	3.31	1.41	.26	-33.82
3	1	-13.63	-11.89	-11.98	-11.41	-8.35	-4.07	-1.98	-.43	6.45
3	2	1.64	-22.55	-23.90	-20.58	-13.70	-6.54	-3.15	-.69	-.03
3	3	23.74	-17.17	-25.21	-20.20	-8.29	-3.35	-1.58	-.34	-20.03
4	1	-21.87	-1.04	3.53	4.50	2.91	1.33	.63	.14	10.20
4	2	87.09	44.43	33.64	27.29	19.74	9.67	4.68	1.02	-2.74
4	3	80.61	46.89	32.36	19.38	10.27	4.75	2.27	.49	-44.66
5	1	-20.27	-16.70	-14.49	-11.92	-8.42	-4.42	-2.26	-.52	9.51
5	2	-32.74	-28.38	-23.81	-18.52	-12.99	-7.00	-3.63	-.85	.49
5	3	-14.23	-23.79	-22.18	-14.36	-5.31	-2.28	-1.14	-.26	-24.61
6	1	12.40	8.15	5.18	2.84	1.80	1.02	.54	.13	5.62
6	2	32.25	33.04	33.04	29.82	21.39	11.35	5.84	1.36	-2.76
6	3	49.30	31.41	21.88	12.84	6.61	3.27	1.65	.38	-47.03

** BASIS IN THIS PROBLEM TAKEN AFTER 77 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN 1.0 PERCENT DURING LAST ITERATION.

TABLE A-10.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 10,
AFTER 10, 20, 30 AND 50 CYCLES OF ITERATION

MEMBER*			ITERATION 10	20	30	50	BASIS** KIP-FEET
1	2	N	-.23	.03	.05	.02	-6.10
		F	2.92	1.34	.61	.14	-2.72
		T	-.68	-.55	-.26	-.06	8.76
1	4	N	.68	.55	.26	.07	-8.76
		F	-.90	-.10	-.02	.01	-10.44
		T	-.23	.03	.05	.02	-6.10
2	3	N	1.71	.89	.43	.11	-13.23
		F	.74	.30	.14	.03	10.45
		T	.76	.26	.12	.02	12.17
2	5	N	-1.44	-.81	-.38	-.09	-3.42
		F	.62	.14	.04	.01	-13.57
		T	-1.21	-.44	-.18	-.02	-10.51
3	6	N	.76	.26	.12	.03	12.17
		F	.28	-.04	-.02	-.01	-25.99
		T	-.74	-.30	-.14	-.02	-10.45
4	5	N	-.25	-.21	-.13	-.04	-.64
		F	2.14	1.16	.62	.17	-3.77
		T	-1.42	-.73	-.35	-.08	9.30
4	7	N	.52	.62	.33	.09	-19.75
		F	-2.25	-.71	-.33	-.08	-19.87
		T	-.48	-.18	-.08	-.01	-6.74
5	6	N	1.59	.79	.41	.10	-3.45
		F	.11	.11	.06	.02	.61
		T	.95	.45	.22	.06	12.95
5	8	N	-1.75	-1.04	-.53	-.14	-17.22
		F	2.24	.94	.45	.11	-30.96
		T	-1.77	-.82	-.40	-.09	-10.19
6	9	N	1.23	.41	.20	.05	-13.04
		F	.01	-.23	-.12	-.04	-49.17
		T	-.86	-.40	-.20	-.05	-11.06
7	8	N	.33	.02	-.02	-.01	2.10
		F	.05	.22	.18	.06	-3.06
		T	-1.66	-.75	-.37	-.09	7.17
7	10	N	-.58	.04	.05	.02	-27.04
		F	-3.22	-1.40	-.71	-.18	-24.58
		T	-.15	-.15	-.10	-.03	-4.64
8	9	N	.72	.41	.24	.07	1.52
		F	-.24	-.09	-.05	-.01	-3.49
		T	.89	.51	.26	.06	9.44
8	11	N	-.32	-.31	-.19	-.05	-33.24
		F	4.01	2.01	1.03	.27	-47.06
		T	-1.10	-.63	-.34	-.09	-5.61
9	12	N	.90	.28	.14	.03	-39.72
		F	-.79	-.61	-.32	-.09	-78.36
		T	-.62	-.31	-.15	-.03	-7.57

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES, PROBLEM 7



MEMBER PROPERTIES

All members have equal EI
All members have EI = GJ

LOADS AND DIMENSIONS

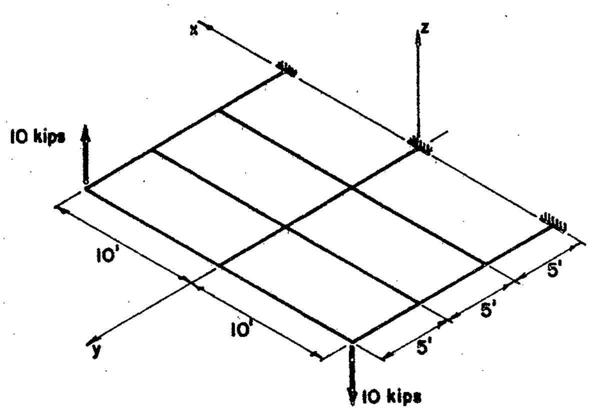


Figure A-11: Problem 11, Member Properties, Dimensions, Loads and Comparison of Member Moments after 5 Cycles of Iteration with Correct Values

TABLE A-11.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 11
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

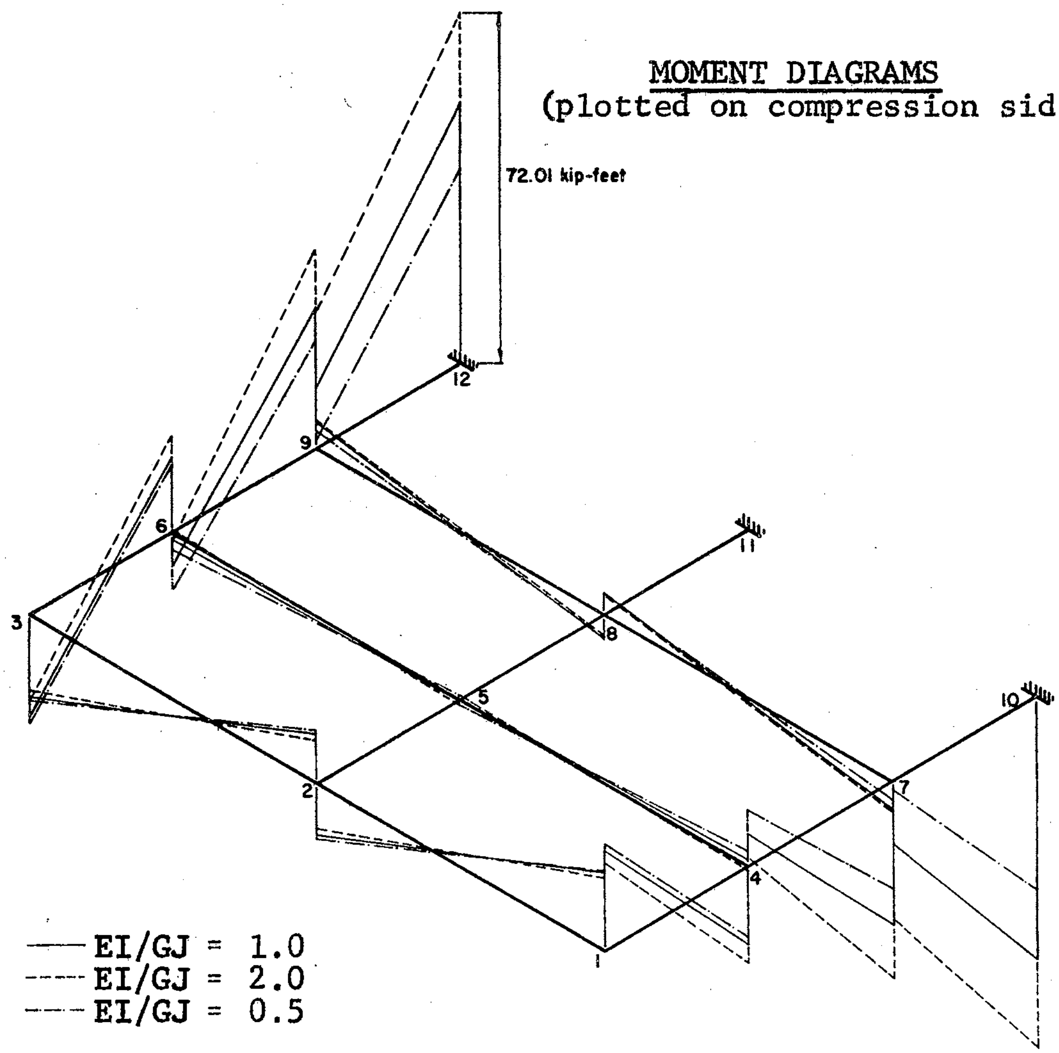
S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	4.94	3.08	.88	-.45	-.97	-.40	.00	.01	10.50
1	2	.10	-.57	-.49	-.68	-.79	-.16	.09	.02	2.70
1	3	-12.96	-.99	.06	.08	-.02	.11	.11	.02	61.48
2	1	.27	-3.97	-4.30	-3.74	-3.00	-.88	-.06	.00	-10.50
2	2	9.23	14.06	12.06	9.84	7.92	2.48	.18	.00	2.70
2	3	13.00	13.90	11.61	9.51	7.69	2.44	.18	.00	61.48
3	1	-5.06	-2.14	.03	.90	1.04	.22	-.02	-.01	-.31
3	2	-2.23	.07	.66	.80	.69	-.15	-.12	-.02	.10
3	3	2.47	.73	.20	.10	.05	-.25	-.11	-.02	23.22
4	1	4.43	5.11	4.56	3.65	2.80	.76	.05	.00	.31
4	2	-17.18	-15.12	-12.39	-9.90	-7.81	-2.31	-.16	.00	.10
4	3	-13.70	-12.11	-10.17	-8.26	-6.59	-2.00	-.14	.00	23.22
5	1	-1.39	-1.36	-.62	-.13	.11	.15	.01	.00	-4.58
5	2	-1.22	-1.21	-.40	.09	.33	.32	.03	.00	-1.02
5	3	-.80	-.39	-.11	.06	.17	.16	.01	.00	11.53
6	1	-2.75	-.52	.24	.40	.37	.12	.01	.00	4.58
6	2	4.05	.06	-.73	-.82	-.73	-.29	-.03	.00	-1.02
6	3	-1.99	-1.06	-.88	-.78	-.68	-.27	-.02	.00	11.53

** BASIS IN THIS PROBLEM TAKEN AFTER 48 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-11.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 11, AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	-.51	-.33	-.06	-16.54
		F	-1.12	-.46	.00	10.50
		T	.87	.31	.02	20.92
1	4	N	-.87	-.31	-.02	-20.92
		F	-.57	-.24	-.05	15.55
		T	-.51	-.33	-.06	-16.54
2	3	N	-3.43	-1.00	-.07	-10.50
		F	2.60	.89	.07	16.54
		T	-.26	-.05	.00	20.92
2	5	N	1.13	.35	.02	.00
		F	-2.19	-.66	-.02	.00
		T	-2.31	-.54	-.06	-21.00
3	6	N	-.26	-.05	.00	20.92
		F	2.76	.90	.07	-15.56
		T	-2.60	-.89	-.07	-16.54
4	5	N	.66	.36	.06	-1.26
		F	1.20	.26	-.01	-.32
		T	-.46	-.16	-.03	22.26
4	7	N	-.11	-.08	-.02	-6.71
		F	-.07	.04	-.01	29.30
		T	.15	.03	.01	-17.80
5	6	N	3.20	.87	.06	.31
		F	-2.75	-.89	-.06	1.26
		T	-1.59	-.53	-.04	22.26
5	8	N	-1.06	-.30	-.01	.00
		F	-1.14	-.48	-.03	.00
		T	-.31	.08	.01	-20.38
6	9	N	1.17	.37	.03	6.71
		F	1.21	.44	.04	-29.30
		T	.15	.00	.00	-17.80
7	8	N	-.12	-.07	-.01	5.58
		F	.12	.18	.01	-4.58
		T	.07	.06	.00	16.61
7	10	N	-.14	-.02	-.01	12.69
		F	-.23	-.02	-.01	53.77
		T	.03	-.04	.00	-12.21
8	9	N	.42	.13	.01	4.58
		F	-.13	-.09	-.01	-5.58
		T	-.50	-.20	-.02	16.62
8	11	N	-.57	-.22	-.01	.00
		F	-.25	-.17	-.01	.00
		T	-.01	.04	.00	-11.21
9	12	N	.71	.23	.02	-12.69
		F	.47	.19	.02	-53.77
		T	.28	.09	.01	-12.22

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES



MEMBER PROPERTIES

All members have equal EI
All members have EI = 2GJ

LOADS AND DIMENSIONS

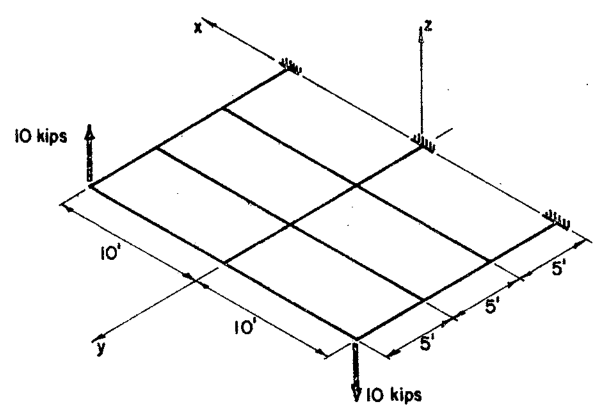


Figure A-12: Problem 12, Member Properties, Dimensions, Loads and Comparison of Member Moments for EI/GJ Variations of .5, 1.0 and 2.0

TABLE A-12.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 12
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	3.60	4.38	3.33	2.08	1.09	-.27	-.05	-.01	9.01
1	2	-6.24	-.15	1.91	2.14	1.72	-.07	-.09	-.01	2.42
1	3	-16.25	-.88	1.62	2.05	1.78	.02	-.08	-.01	54.37
2	1	4.13	.64	-.57	-1.03	-1.10	-.41	-.04	-.01	-9.01
2	2	-3.50	3.92	4.50	4.06	3.41	1.01	.11	.01	2.42
2	3	1.71	4.51	4.48	3.99	3.33	.97	.10	.01	54.37
3	1	-5.13	-4.15	-2.42	-1.11	-.27	.42	.06	.01	-1.06
3	2	-1.89	-2.97	-2.67	-1.91	-1.15	.36	.11	.02	-.16
3	3	1.30	-1.57	-1.98	-1.65	-1.15	.18	.08	.01	16.25
4	1	1.05	1.96	2.21	2.06	1.75	.48	.05	.01	1.06
4	2	-9.64	-9.01	-7.40	-5.89	-4.60	-1.19	-.12	-.02	-.16
4	3	-7.03	-6.34	-5.32	-4.28	-3.35	-.90	-.09	-.01	16.25
5	1	.05	-1.31	-1.32	-1.05	-.76	-.09	.00	.00	-4.17
5	2	5.02	.74	-.51	-.83	-.82	-.21	.00	.00	-1.03
5	3	2.35	.73	-.01	-.28	-.34	-.09	.00	.00	7.37
6	1	-4.73	-2.78	-1.56	-.84	-.44	-.01	.00	.00	4.17
6	2	11.13	5.45	2.91	1.59	.88	.05	.00	.00	-1.03
6	3	2.12	1.62	.90	.47	.24	-.01	.00	.00	7.37

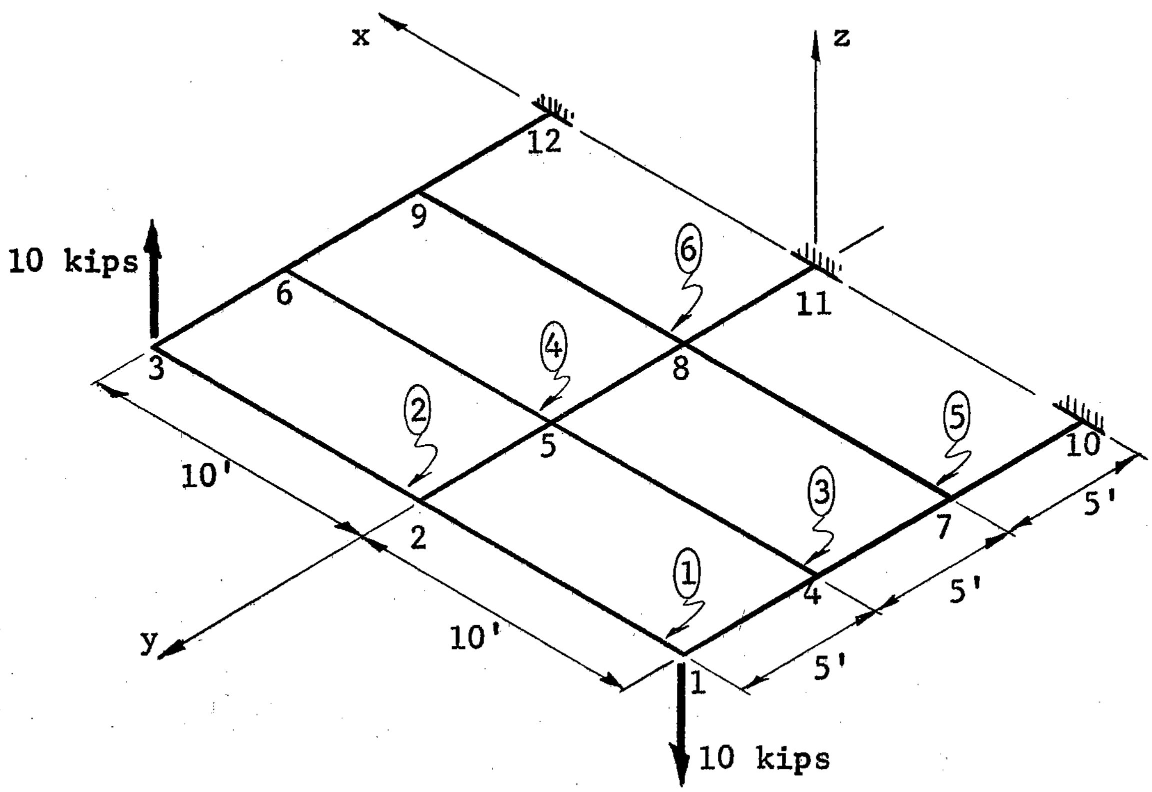
** BASIS IN THIS PROBLEM TAKEN AFTER 43 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-12.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 12,
AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	-.04	-.17	.00	-15.19
		F	.83	-.20	-.03	9.00
		T	.05	.07	.01	18.07
1	4	N	-.05	-.07	-.01	-18.07
		F	-.48	-.06	.01	19.83
		T	-.04	-.17	.00	-15.19
2	3	N	-.83	-.30	-.03	-9.01
		F	.88	.20	.02	15.19
		T	-.06	-.03	.00	18.07
2	5	N	.11	.10	.01	.00
		F	-.31	-.17	-.03	.00
		T	-1.66	-.10	.01	-18.02
3	6	N	-.06	-.03	.00	18.07
		F	.79	.22	.02	-19.83
		T	-.88	-.20	-.02	-15.19
4	5	N	.37	.14	-.01	.54
		F	-.21	.31	.04	-1.06
		T	-.29	-.05	.00	17.85
4	7	N	-.19	-.01	.01	1.98
		F	-.34	-.08	.00	40.68
		T	.33	-.03	.00	-14.65
5	6	N	1.31	.36	.03	1.07
		F	-.99	-.23	-.02	-.54
		T	-.22	-.08	-.01	17.85
5	8	N	-.38	-.14	-.02	.00
		F	.06	-.03	-.01	.00
		T	-.13	-.06	.00	-15.89
6	9	N	.58	.15	.01	-1.97
		F	.28	.10	.01	-40.68
		T	.11	.03	.00	-14.65
7	8	N	-.16	.04	.00	6.13
		F	-.57	-.07	.00	-4.17
		T	-.05	-.02	.00	12.52
7	10	N	-.28	-.06	.00	28.16
		F	-.22	-.08	.00	72.01
		T	.17	.01	.00	-8.52
8	9	N	-.33	.00	.00	4.17
		F	.11	.02	.00	-6.13
		T	-.04	-.02	.00	12.52
8	11	N	.04	-.03	-.01	.00
		F	.06	.03	.00	.00
		T	.11	.01	.00	-7.54
9	12	N	.24	.09	.01	-28.16
		F	.16	.05	.00	-72.01
		T	.00	.01	.00	-8.52

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

LOADS, DIMENSIONS AND LOCATION OF REDUNDANTS



MEMBER PROPERTIES

All members have equal EI
All members have EI = 0.5GJ

Figure A-13: Problem 13, Member Properties, Dimensions and Loads

TABLE A-13.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 13
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	7.60	2.23	-.75	-1.48	-1.28	.03	.27	.08	11.11
1	2	10.26	1.63	-.28	-.74	-.51	1.22	.90	.23	2.75
1	3	-6.10	1.83	1.38	.99	.96	1.72	.93	.22	63.29
2	1	-5.50	-9.01	-7.47	-5.77	-4.55	-1.66	-.08	.04	-11.11
2	2	26.60	25.92	20.45	16.22	13.13	4.90	.30	-.10	2.75
2	3	27.31	25.04	20.00	16.08	13.09	4.94	.35	-.09	63.29
3	1	-4.66	-.16	1.21	.94	.37	-.60	-.30	-.07	1.02
3	2	-2.33	2.13	1.89	.63	-.57	-2.34	-.96	-.19	.44
3	3	2.17	1.59	.46	-.49	-1.24	-2.23	-.90	-.18	30.36
4	1	7.65	6.87	5.37	4.21	3.40	1.22	.06	-.03	-1.02
4	2	-23.34	-19.09	-15.54	-12.80	-10.62	-3.91	-.22	.08	.44
4	3	-20.82	-17.79	-15.00	-12.51	-10.41	-3.88	-.23	.08	30.36
5	1	-4.17	-2.11	-.52	.17	.41	.30	.01	-.01	-4.00
5	2	-10.11	-4.90	-1.74	-.10	.68	.76	.03	-.03	-.79
5	3	-6.60	-3.49	-1.52	-.35	.27	.42	-.02	-.03	16.72
6	1	.91	2.29	1.75	1.13	.76	.27	.02	-.01	4.00
6	2	-6.59	-7.02	-4.82	-3.17	-2.22	-.84	-.09	.01	-.79
6	3	-8.82	-5.60	-3.74	-2.64	-1.98	-.79	-.08	.01	16.72

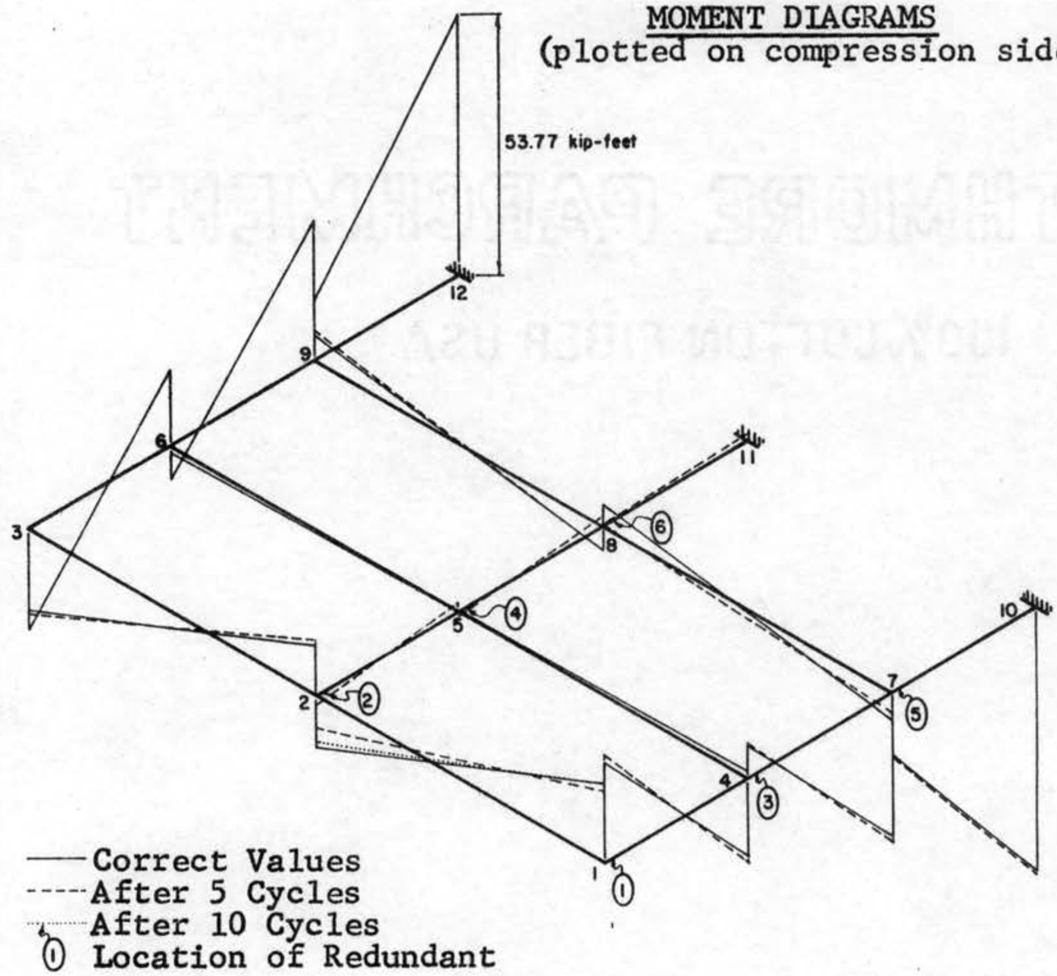
** BASIS IN THIS PROBLEM TAKEN AFTER 56 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN .001 PERCENT DURING LAST ITERATION.

TABLE A-13.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 13, AFTER 5, 10 AND 20 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	BASIS** KIP-FEET
1	2	N	-1.38	-1.14	-.41	-16.44
		F	-2.17	-.08	.30	11.16
		T	2.37	.82	.07	22.02
1	4	N	-2.37	-.82	-.07	-22.02
		F	-1.97	-1.35	-.43	14.17
		T	-1.38	-1.14	-.41	-16.44
2	3	N	-7.33	-2.72	-.19	-11.08
		F	6.76	2.60	.24	16.38
		T	-.08	.04	.06	22.04
2	5	N	2.45	.78	.01	-.02
		F	-4.99	-1.35	.15	.06
		T	-5.16	-2.64	-.49	-22.24
3	6	N	-.08	.04	.06	22.04
		F	6.97	2.70	.27	-14.23
		T	-6.76	-2.60	-.24	-16.38
4	5	N	1.09	1.43	.44	-3.36
		F	.69	-.85	-.38	.98
		T	-1.28	-1.00	-.33	25.91
4	7	N	-.69	-.35	-.10	-11.73
		F	-.10	.26	-.05	22.29
		T	-.29	.29	.03	-19.81
5	6	N	5.48	2.00	.14	-1.04
		F	-5.92	-2.26	-.18	3.42
		T	-5.36	-2.08	-.18	25.95
5	8	N	-.92	-.27	.01	.01
		F	-2.86	-1.42	-.10	.02
		T	-.38	.21	.02	-24.27
6	9	N	1.61	.62	.09	11.73
		F	2.96	1.16	.15	-22.31
		T	-.84	-.35	-.06	-19.80
7	8	N	-.09	-.34	-.02	3.86
		F	.67	.49	.04	-4.01
		T	.09	.30	-.02	20.63
7	10	N	-.19	-.04	-.03	1.66
		F	.03	.15	-.01	39.62
		T	-.38	-.06	.01	-15.95
8	9	N	1.22	.44	.05	4.00
		F	-1.16	-.46	-.06	-3.85
		T	-2.00	-.82	-.09	20.65
8	11	N	-.77	-.30	-.02	.00
		F	-1.15	-.58	-.05	.01
		T	.18	.16	.03	-16.25
9	12	N	.96	.34	.06	-1.66
		F	1.12	.42	.06	-39.62
		T	.32	.12	.00	-15.95

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



MEMBER PROPERTIES

All members have equal EI
All members have EI = GJ

Note: Problem 14 is identical with problem 11 except for choice of redundants

LOADS AND DIMENSIONS

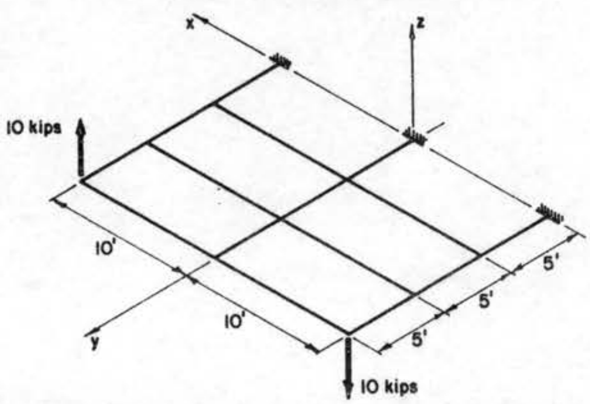


Figure A-14: Problem 14, Member Properties, Dimensions, Loads and Comparison of Member Moments after 5 and 10 Cycles of Iteration with Correct Values

TABLE A-14.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 14
AFTER 1, 2, 3, 5, 10, 20, 30 AND 50 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	5	10	20	30	50	K,K-FEET
1	1	-23.93	-10.45	-1.11	3.22	2.42	.94	.37	.06	89.52
1	2	-10.92	-.06	5.50	6.84	4.54	1.68	.63	.09	7.30
1	3	-6.93	-.55	3.82	5.58	3.72	1.36	.50	.06	88.54
2	1	-21.70	-13.09	-6.18	-2.44	-1.18	-.43	-.16	-.02	20.99
2	2	17.33	5.13	-5.57	-9.21	-6.08	-2.28	-.87	-.12	.00
2	3	14.43	3.64	-5.16	-7.67	-4.88	-1.79	-.67	-.09	-.03
3	1	-36.96	-11.60	-1.44	2.93	2.40	1.11	.51	.10	89.85
3	2	-24.29	.42	4.17	5.43	4.01	1.82	.84	.16	7.21
3	3	-15.54	-1.68	1.45	3.03	2.46	1.06	.46	.08	65.34
4	1	-23.79	-12.90	-6.32	-2.02	-.91	-.40	-.18	-.03	20.36
4	2	25.61	8.99	-1.67	-7.46	-5.75	-2.64	-1.22	-.23	-.01
4	3	13.28	4.50	-1.16	-4.26	-3.24	-1.42	-.64	-.12	-.05
5	1	-25.21	-8.18	-2.00	.99	1.26	.82	.46	.11	94.44
5	2	-6.19	-2.15	-.62	.69	1.32	1.10	.67	.17	8.22
5	3	-5.10	-1.47	-.41	.64	.92	.51	.26	.06	53.80
6	1	-19.57	-7.82	-2.85	-.30	-.05	-.12	-.09	-.02	11.20
6	2	31.16	11.97	3.28	-1.69	-2.61	-1.92	-1.13	-.27	-.01
6	3	8.98	3.49	.73	-1.03	-1.17	-.68	-.36	-.08	-.03

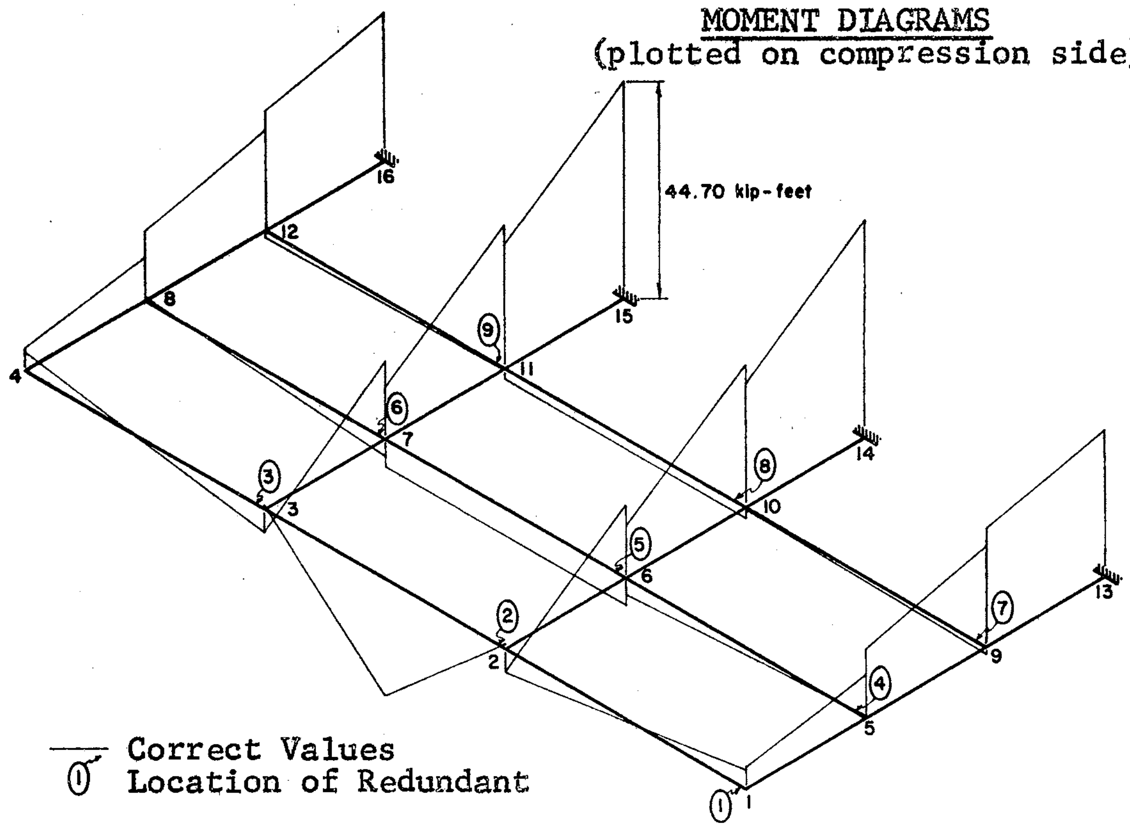
** BASIS IN THIS PROBLEM TAKEN AFTER 70 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN 5 DIGITS IN THE FOURTH SIGNIFICANT FIGURE DURING THE PREVIOUS 10 CYCLES OF ITERATION.

TABLE A-14.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 14, AFTER 10, 20, 30 AND 50 CYCLES OF ITERATION

MEMBER*			ITERATION 10	20	30	50	BASIS** KIP-FEET
1	2	N	1.40	.42	.12	.01	-16.54
		F	-5.59	-2.20	-.90	-.17	10.50
		T	1.89	.77	.33	.06	20.92
1	4	N	-1.89	-.77	-.33	-.07	-20.92
		F	1.60	.54	.18	.02	15.55
		T	1.40	.42	.12	.00	-16.54
2	3	N	-2.87	-1.20	-.52	-.11	-10.50
		F	-.50	-.28	-.14	-.04	16.54
		T	-.88	-.37	-.16	-.03	20.92
2	5	N	2.77	1.14	.49	.10	.00
		F	-1.91	-.63	-.21	-.03	.00
		T	2.72	1.00	.38	.06	-21.00
3	6	N	-.88	-.37	-.16	-.03	20.92
		F	.31	.09	.03	.00	-15.56
		T	.50	.28	.14	.03	-16.54
4	5	N	-.76	-.18	-.02	.01	-1.26
		F	.01	-.42	-.36	-.13	-.32
		T	2.10	.91	.41	.09	22.26
4	7	N	-.50	-.36	-.23	-.07	-6.71
		F	2.60	1.07	.45	.09	29.30
		T	.63	.25	.10	.01	-17.80
5	6	N	-.60	-.49	-.31	-.10	.31
		F	-.27	-.12	-.05	-.01	1.26
		T	-1.20	-.53	-.24	-.05	22.26
5	8	N	1.39	.81	.45	.12	.00
		F	-3.06	-1.27	-.55	-.11	.00
		T	2.10	.93	.43	.09	-20.38
6	9	N	-.89	-.44	-.21	-.05	6.71
		F	.46	.20	.10	.03	-29.30
		T	.77	.40	.20	.05	-17.80
7	8	N	-1.50	-.44	-.12	.00	5.58
		F	2.60	.65	.11	-.03	-4.58
		T	1.50	.74	.37	.09	16.61
7	10	N	1.10	.32	.08	.00	12.69
		F	2.15	1.21	.64	.17	53.77
		T	-.86	-.19	-.01	.01	-12.21
8	9	N	.65	.02	-.09	-.05	4.58
		F	-.01	.04	.03	.01	-5.58
		T	-.89	-.46	-.24	-.06	16.62
8	11	N	-.67	-.07	.06	.04	.00
		F	-2.74	-1.61	-.89	-.24	.00
		T	.15	.30	.22	.07	-11.21
9	12	N	-.43	-.25	-.13	-.04	-12.69
		F	.59	.40	.24	.07	-53.77
		T	.78	.35	.16	.03	-12.22

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES, PROBLEM 11

MOMENT DIAGRAMS
(plotted on compression side)



— Correct Values
(1) Location of Redundant

MEMBER PROPERTIES

All members have equal EI
All members have EI = GJ

LOADS AND DIMENSIONS

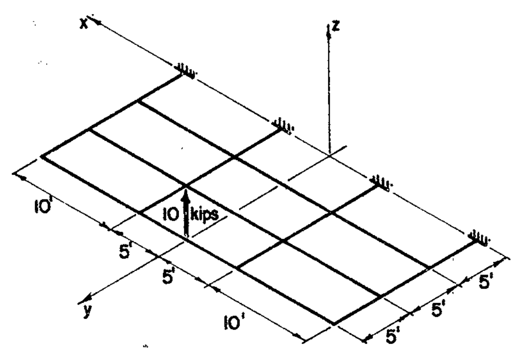


Figure A-15: Problem 15, Member Properties, Dimensions, Loads and Final Moment Diagram

TABLE A-15.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 15
AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	-11.99	-12.25	-10.94	-9.44	-7.95	-2.89	-.32	-.02	8.99
1	2	-17.25	-18.79	-16.33	-13.65	-11.28	-3.97	-.37	-.01	.86
1	3	-23.47	-19.63	-16.21	-13.34	-10.94	-3.73	-.32	.00	17.60
2	1	-2.66	-2.27	-1.37	-.72	-.34	.00	.03	.01	23.87
2	2	.00	2.75	3.27	3.60	3.74	2.89	.72	.11	5.00
2	3	.00	.68	1.69	2.40	2.80	2.56	.66	.09	75.00
3	1	2.78	4.13	3.67	3.12	2.63	1.20	.36	.08	8.99
3	2	-7.25	-7.33	-5.71	-4.49	-3.62	-1.56	-.48	-.11	-.86
3	3	-10.60	-7.71	-5.72	-4.40	-3.48	-1.43	-.44	-.11	-17.60
4	1	7.25	10.41	10.46	9.26	7.74	2.64	.26	.01	5.46
4	2	15.55	18.07	16.22	13.69	11.26	3.73	.31	.00	.42
4	3	20.32	16.44	13.87	11.55	9.48	3.12	.24	.00	9.38
5	1	-.24	.48	.29	.07	-.06	-.05	-.03	-.01	5.55
5	2	-.31	-2.47	-3.20	-3.43	-3.48	-2.66	-.65	-.09	.00
5	3	-.48	-1.75	-2.38	-2.71	-2.85	-2.27	-.54	-.07	.00
6	1	-6.71	-5.30	-4.17	-3.31	-2.65	-1.11	-.32	-.07	5.46
6	2	10.48	7.81	5.89	4.57	3.63	1.51	.45	.11	-.42
6	3	8.39	6.16	4.70	3.69	2.96	1.27	.39	.09	-9.38
7	1	-1.35	-.75	-.17	.22	.44	.41	.09	.01	-.34
7	2	-2.73	-1.02	-.10	.33	.54	.51	.11	.02	-.11
7	3	.64	1.11	.99	.87	.79	.41	.06	.01	3.32
8	1	-.49	.02	.15	.15	.11	.02	.00	.00	1.92
8	2	1.85	.42	-.45	-.75	-.79	-.47	-.13	-.03	.00
8	3	.75	.19	-.17	-.34	-.40	-.32	-.09	-.02	.00
9	1	-1.09	-1.11	-.87	-.66	-.51	-.20	-.05	-.01	-.34
9	2	1.90	1.72	1.20	.82	.60	.22	.06	.01	.11
9	3	1.85	1.00	.62	.43	.32	.14	.05	.01	-3.32

** BASIS IN THIS PROBLEM TAKEN AFTER 60 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN 2 DIGITS IN THE FOURTH SIGNIFICANT FIGURE DURING THE PREVIOUS 10 CYCLES OF ITERATION.

TABLE A-15.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 15, AFTER 5, 10, 20 AND 30 CYCLES OF ITERATION

MEMBER*			ITERATION 5	10	20	30	BASIS** KIP-FEET
1	2	N	5.58	1.81	.10	-.01	-3.95
		F	-7.03	-2.62	-.32	-.02	4.68
		T	.57	.40	.10	.01	4.66
1	5	N	-.57	-.40	-.10	-.01	-4.66
		F	5.74	1.82	.11	-.01	-8.97
		T	5.58	1.81	.10	.00	-3.95
2	3	N	-2.66	-1.62	-.35	-.04	-1.13
		F	1.52	1.61	.45	.08	-1.13
		T	-1.57	-.56	-.10	-.01	.00
2	6	N	2.14	.96	.20	.03	4.66
		F	-6.26	-2.88	-.41	-.03	-16.03
		T	4.37	1.01	-.03	-.01	-5.81
3	4	N	2.39	1.14	.33	.08	4.68
		F	-1.66	-.61	-.20	-.05	-3.95
		T	.22	.21	.06	.01	-4.66
3	7	N	-1.79	-.77	-.16	-.03	4.66
		F	2.32	1.72	.50	.09	-16.03
		T	.86	-.48	-.12	.00	5.81
4	8	N	.22	.21	.06	.01	-4.66
		F	-1.80	-.66	-.21	-.05	-8.97
		T	1.66	.61	.20	.05	3.95
5	6	N	-5.92	-1.83	-.08	.01	-.78
		F	6.68	2.34	.26	.01	3.38
		T	3.31	1.06	.06	.00	5.23
5	9	N	2.43	.76	.05	.00	-14.20
		F	2.44	.89	.09	.01	-20.59
		T	-.34	-.02	.01	.00	-4.73
6	7	N	1.85	1.40	.31	.03	5.55
		F	-2.05	-1.58	-.41	-.07	5.55
		T	-.89	-.82	-.19	-.02	.00
6	10	N	-2.06	-.99	-.16	-.02	-10.80
		F	-2.21	-1.25	-.24	-.03	-29.41
		T	-.46	.07	.02	.00	-3.63
7	8	N	-2.42	-1.01	-.29	-.06	3.38
		F	1.65	.67	.22	.05	-.78
		T	.90	.44	.15	.03	-5.23
7	11	N	.54	.45	.16	.03	-10.80
		F	.67	.61	.22	.05	-29.41
		T	.49	.09	.00	.00	3.63
8	12	N	-.90	-.21	-.05	-.02	-14.20
		F	-.90	-.24	-.07	-.02	-20.59
		T	.01	-.07	-.02	.00	4.73
9	10	N	-.17	-.18	-.04	-.01	1.30
		F	.44	.40	.08	.01	.20
		T	1.02	.40	.05	.00	3.87
9	13	N	1.42	.48	.04	.00	-24.46
		F	1.12	.33	.02	.00	-30.30
		T	-.51	-.20	-.02	.00	-3.43

TABLE A-15.2 (CONTINUED)

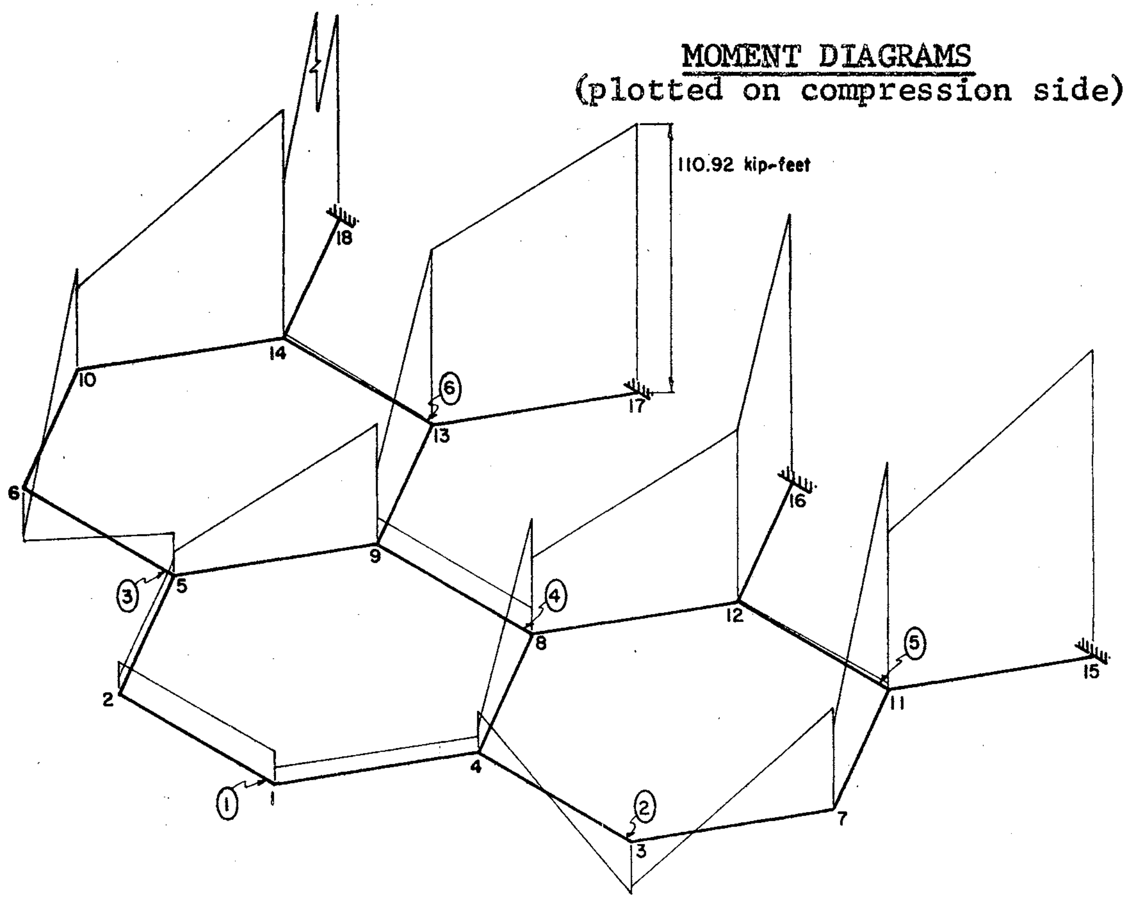
MEMBER*		ITERATION			BASIS**
	5	10	20	30	KIP-FEET
10 11 N	.63	.31	.07	.01	1.92
F	-.25	-.22	-.07	-.02	1.92
T	-.23	-.27	-.08	-.01	.00
10 14 N	-.96	-.58	-.11	-.02	-25.54
F	-.37	-.29	-.06	-.01	-44.70
T	-.28	-.02	.01	.00	-1.92
11 12 N	-.52	-.22	-.06	-.01	.20
F	.15	.03	.01	.00	1.30
T	.21	.11	.04	.01	-3.87
11 15 N	.23	.24	.10	.02	-25.54
F	-.41	.00	.05	.01	-44.70
T	.23	.10	.02	.00	1.92
12 16 N	-.68	-.14	-.02	-.01	-24.46
F	-.34	-.05	.00	.00	-30.30
T	-.14	-.09	-.03	.00	3.43

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE

* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS

** BASIS MOMENTS ARE RESULTS AFTER 60 CYCLES

MOMENT DIAGRAMS
(plotted on compression side)



— Correct Values
(1) Location of Redundant

MEMBER PROPERTIES

All members have equal EI
All members have EI = GJ

LOADS AND DIMENSIONS

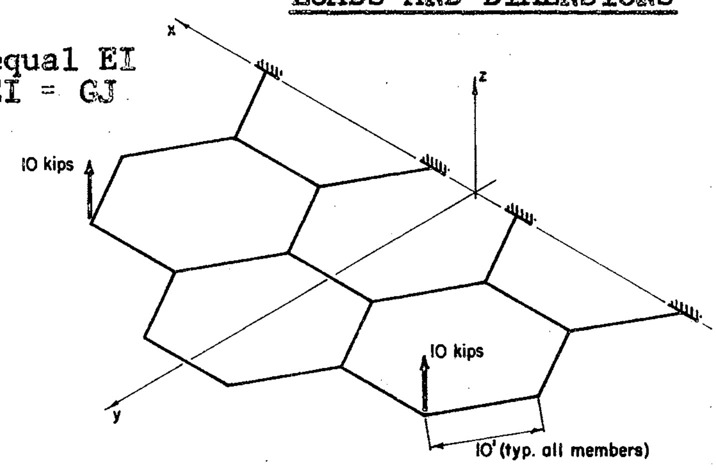


Figure A-16: Problem 16, Member Properties, Dimensions, Loads and Final Moment Diagram

TABLE A-16.1

PERCENTAGE DEVIATION OF REDUNDANT VECTOR FROM BASIS, PROBLEM 16

AFTER 1, 2, 3, 4, 5, 10, 20 AND 30 CYCLES OF ITERATION

S		ITERATION								BASIS**
I	J	1	2	3	4	5	10	20	30	K,K-FEET
1	1	12.03	7.42	4.03	2.00	.92	.00	.00	.00	-13.53
1	2	.00	.00	.00	.00	.00	.00	.00	.00	.00
1	3	.00	.00	.00	.00	.00	.00	.00	.00	.00
2	1	12.77	5.62	2.41	.94	.31	-.01	.00	.00	-57.85
2	2	8.64	4.04	2.08	.97	.40	-.01	.00	.00	-3.99
2	3	2.76	2.46	1.49	.76	.34	-.01	.00	.00	-112.49
3	1	12.77	5.62	2.41	.94	.31	-.01	.00	.00	-57.85
3	2	-8.64	-4.04	-2.08	-.97	-.40	.01	.00	.00	3.99
3	3	-2.76	-2.46	-1.49	-.76	-.34	.01	.00	.00	112.49
4	1	-1.19	-2.31	-1.72	-1.01	-.53	.00	.00	.00	-11.76
4	2	.00	.00	.00	.00	.00	.00	.00	.00	.00
4	3	.00	.00	.00	.00	.00	.00	.00	.00	.00
5	1	1.87	.81	.34	.16	.09	.01	.00	.00	.25
5	2	5.98	2.26	.65	.15	.02	.01	.00	.00	.11
5	3	.45	-.69	-.62	-.37	-.18	.01	.00	.00	-8.11
6	1	1.87	.81	.34	.16	.09	.01	.00	.00	.25
6	2	-5.98	-2.26	-.65	-.15	-.02	-.01	.00	.00	-.11
6	3	-.45	.69	.62	.37	.18	-.01	.00	.00	8.11

** BASIS IN THIS PROBLEM TAKEN AFTER 50 CYCLES OF ITERATION. REPRESENTS VALUES WHICH CHANGED LESS THAN 1 DIGIT IN THE SIXTH SIGNIFICANT FIGURE DURING THE PREVIOUS 10 CYCLES OF ITERATION.

TABLE A-16.2
PERCENTAGE DEVIATION OF MOMENTS FROM BASIS, PROBLEM 16, AFTER 5 AND 10 CYCLES OF ITERATION

MEMBER*			ITERATION 5	ITERATION 10	BASIS** KIP-FEET
1	2	N	.82	.00	-13.53
		F	.82	.00	-13.53
		T	.00	.00	.00
1	4	N	.41	.00	-6.76
		F	.41	.00	-6.76
		T	.71	.00	-11.72
2	5	N	.41	.00	-6.76
		F	.41	.00	-6.76
		T	-.71	.00	11.72
3	4	N	.00	.00	21.89
		F	.14	-.01	-17.98
		T	-.05	.00	-8.84
3	7	N	.05	-.01	18.59
		F	-.09	.00	-41.57
		T	-.02	.00	14.54
4	8	N	-.38	.00	-9.88
		F	-.25	.00	-49.77
		T	.56	.01	-.56
5	6	N	.14	-.01	-17.98
		F	.00	.00	21.89
		T	.05	.00	8.84
5	9	N	-.38	.00	-9.88
		F	-.25	.00	-49.77
		T	-.56	-.01	.56
6	10	N	.05	-.01	18.59
		F	-.09	.00	-41.57
		T	.02	.00	-14.54
7	11	N	-.02	.00	-33.40
		F	-.16	.01	-93.56
		T	-.09	.00	-28.71
8	9	N	-.47	.00	-11.76
		F	-.47	.00	-11.76
		T	.00	.00	.00
8	12	N	.13	.00	-31.28
		F	.26	.00	-71.17
		T	.09	.00	32.61
9	13	N	.13	.00	-31.28
		F	.26	.00	-71.17
		T	-.09	.00	-32.61
10	14	N	-.02	.00	-33.40
		F	-.16	.01	-93.56
		T	.09	.00	28.71
11	12	N	.06	.00	-1.95
		F	.07	.00	-.85
		T	-.17	.00	-9.06

TABLE A-16.2 (CONTINUED)

MEMBER*			ITERATION 5	10	BASIS** KIP-FEET
11	15	N	.02	.00	-64.81
		F	-.13	.00	-126.08
		T	.07	.00	60.41
12	16	N	-.05	.00	-72.13
		F	.10	.00	-110.92
		T	.13	.00	-49.08
13	14	N	.07	.00	-.85
		F	.06	.00	-1.95
		T	.17	.00	9.06
13	17	N	-.05	.00	-72.13
		F	.10	.00	-110.92
		T	-.13	.00	49.08
14	18	N	.02	.00	-64.81
		F	-.13	.00	-126.08
		T	-.07	.00	-60.41

PERCENTAGE = 100X(VALUE-BASIS VALUE)/MAX.BASIS VALUE
* N, F AND T REFER TO NEAR, FAR AND TORSIONAL MOMENTS
** BASIS MOMENTS ARE RESULTS AFTER 30 CYCLES

APPENDIX B

FLEXIBILITY DATA - CIRCULAR BEAM

The following two pages contain flexibility coefficients and angular load functions for a circular beam of constant cross section. This beam configuration is used in example problem 1. The coefficients were evaluated from equations presented by Reddy (20) suitably modified to satisfy the sign conventions indicated in Figure B-1.

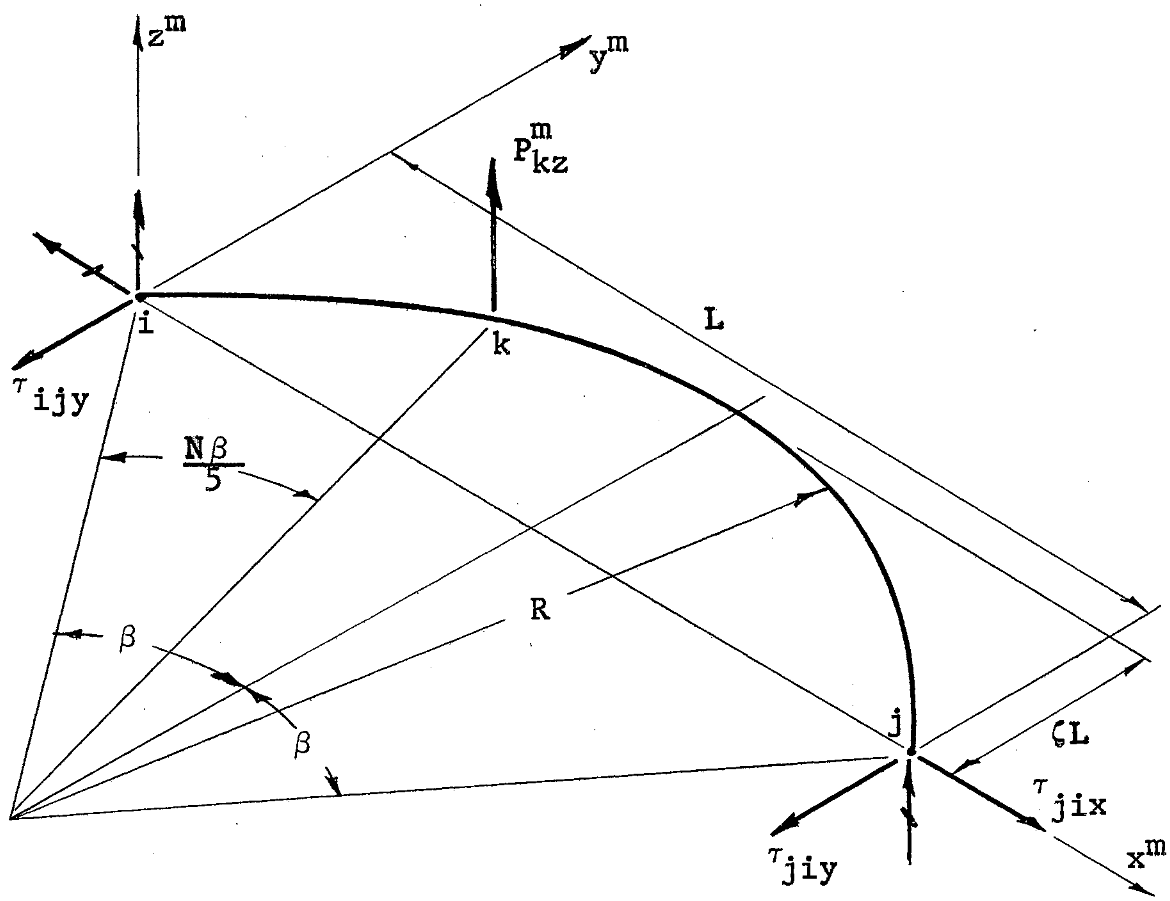


Figure B-1: Geometry and Definition Sketch of Loads and Angular Load Functions, Circular Beam

Angular Load Functions

Positive in direction shown, right hand rule.

$$\tau_{kmn} = P^m_{kz}(\text{TKMN})L^2/EI$$

where: (TKMN) is the appropriate coefficient tabulated in Table B-1

TABLE B-1 FLEXIBILITY COEFFICIENTS AND ANGULAR LOAD FUNCTION COEFFICIENTS, CIRCULAR BEAM, CONSTANT SECTION, EI=GJ

GEOMETRIC DATA
RADIUS = .725 L
ζ = .20
β = 43.6028 DEGREES

FLEXIBILITY FACTORS
F(JIXX) = 1.10347 L/EI
F(JIYY) = F(IJYY) = .39877 L/EI
G(JIYY) = G(IJYY) = .15296 L/EI
F(JIXY) = F(IJYX) =-G(JIXY) =-G(IJYX) =-.14568 L/EI

SEE PREVIOUS PAGE FOR DEFINITION SKETCH AND EXPLANATION OF COEFFICIENTS.

LOCATION	LOAD FUNCTION COEFFICIENT	VERTICAL FORCE
N = 1	TJIX	.00000
	TJIY	.00000
	TIJY	.00000
N = 1	TJIX	.01617
	TJIY	.01284
	TIJY	.02946
N = 2	TJIX	.03913
	TJIY	.02582
	TIJY	.05308
N = 3	TJIX	.06520
	TJIY	.03738
	TIJY	.06969
N = 4	TJIX	.09033
	TJIY	.04612
	TIJY	.07869
N = 5	TJIX	.11035
	TJIY	.05087
	TIJY	.08000
N = 6	TJIX	.12112
	TJIY	.05078
	TIJY	.07404
N = 7	TJIX	.11872
	TJIY	.04541
	TIJY	.06166
N = 8	TJIX	.09960
	TJIY	.03476
	TIJY	.04413
N = 9	TJIX	.06079
	TJIY	.01930
	TIJY	.02300
N = 10	TJIX	.00000
	TJIY	.00000
	TIJY	.00000

APPENDIX C

COMPUTER ANALYSIS

The pages which follow indicate the procedure used in assembling the computer program used to determine the information shown in Appendix A.

A macro flow diagram, Figure C-1, illustrates the basic logic of the process. Required input data are indicated below. Actual output consisted of much more data than would normally be produced. Since it was necessary to iterate by several methods, it was necessary to print the actual formulated flexibility matrix both before and after it had been conditioned for the carry-over process. Also, to study the convergence, the redundant matrix was printed out after 1, 2, 3, 4, 5 and 10 cycles of iteration and every 10th. cycle thereafter. Final member moments were obtained from these iterated redundant vectors upon analysis of the nature of the convergence. In an actual problem, the only necessary output would be the final member moments.

Since it was not the purpose of this thesis to develop an efficient program, the programming effort terminated when results were obtained. In retrospect, many of the internal details of the program logic could be made somewhat more efficient and this is left to another time. For this reason the details of the program are omitted. The program was written for a computer having a total storage capacity of 40,000 decimal digits, therefore the structure was restricted in size to 24 joints, 12 loops (36 redundant elements) and 30 members.

INPUT DATA

Joints and Coordinates

I J_1 J_2 J_3 J_4 J_5 J_6 X Y

First number I indicates joint number.
Next 6 numbers J indicate joints connected with I.
X and Y are coordinates of I in the basic system.
Up to 24 such joints are possible and may be read in any order.

Flexibility and Stiffness Properties

I-J f_{ijyy}, g_{jixy}, etc. EI EI/GJ

I-J indicates member.
Next six numbers are all six flexibility factors, if member is straight program will compute proper values.
EI is member moment of inertia, or reference moment of inertia if member has non-uniform section.
EI/GJ is the ratio of bending stiffness to torsional stiffness, or reference values if member has non-uniform section.
Up to 30 such cards are possible in any order.

Redundants

I_1 J_1 I_2 J_2 I_3 J_3 I_4 J_4

I_i J_i represent member containing redundants.
Up to 12 such pairs are possible in any order.

Angular Load Functions and Basic System Moments

τ values, 3; BS values, 4

Up to 30 cards, one per member, in any order.

Loops

I_1 I_2 I_3 $I_4 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot I_n$

Each I represents a joint in the loop and must be ordered to coincide with either a clockwise or counterclockwise traversal of the loop. Up to 24 joints are possible in any of 12 loops. May be read in any order.

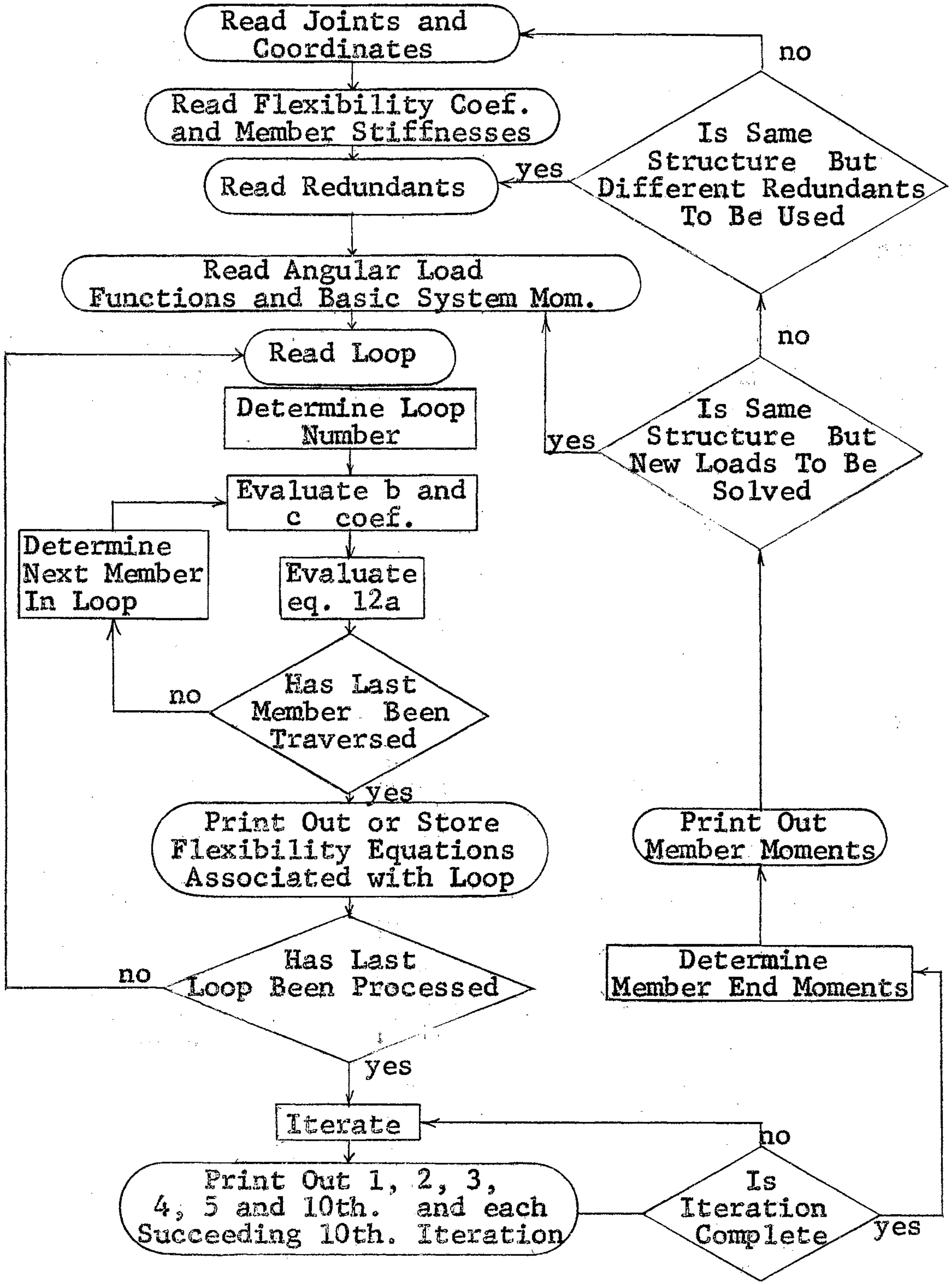


Figure C-1: Macro Flow Diagram of Computer Program

VITA

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