AN ANALYTICAL APPROACH
TO n-PORT SYSTEM
DESIGN

By

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CHAPTER I

INTRODUCTION

1.1 Statement of the Problem. The design of a physical system which meets the requirements of a given or specified mathematical model is a problem of paramount importance to the engineer. Physical system design today is primarily an art, an art which draws upon the designer's intuition, his memory of similar situations encountered and his ability to predict the performance of various interconnection patterns of components or subsystems.

Analysis techniques, which are well developed for the linear lumped parameter case, are utilized only to the extent that they add to the designer's feel for the problem. Furthermore, with the exception of the area of filter design, the elegant synthesis theory which has been developed in the last few years has not been effectively utilized in system design.

The previous discussion of the state of the art of system design indicates: (i) that any technique which provides a straightforward procedure for design, even for a limited class of problems, would be worthwhile and (ii) that greater utilization of the existing analysis and synthesis methods is very desirable. This thesis presents a design procedure which is straightforward and which does directly utilize analysis and synthesis techniques.

1.2 Scope of Investigation. This study was motivated by the consideration of the idea of a double driver, a recent concept introduced by Blackwell and Grigsby (1). Another new concept which is basic to this study is the idea of a complex source (2). Both of these concepts will be used extensively in the work to follow and will be developed fully in Chapter II.

The systems considered in this thesis are linear with lumped parameters. Active as well as passive systems are considered.

A design procedure for a restricted class of design problems is developed. The class of design problem which is solvable by this procedure is that class for which a set of specifications is given at n ports. The class of systems is further restricted to those for which the geometry and component relations are partially specified. The specifications at the n ports are given in the form of a matrix relation between the through and the across variables at these ports. An analysis technique, unique in that it requires the selection and use of two different trees, is utilized to determine the matrix relation requirement for an n-port subsystem. This subsystem is initially completely unspecified as to geometry or type of components.

The design is then completed by synthesizing the unspecified subsystem by any available synthesis techniques. In particular, this realization appears to be a very good application for the results of the very extensive research of the last ten years in the area of n-port synthesis.

The results of the many contributors to the theory of passive n-port synthesis are given by Weinberg (3) and a later very good summary for active as well as passive networks is given by Weinberg and others (4). Both of these references provide an extensive bibliography to the theory of n-port synthesis and to the special case, two-port synthesis.

Before proceeding to a discussion of the design procedure, it is necessary to make a thorough investigation of systems containing double drivers. Chapter II is devoted to this investigation.

CHAPTER II

DOUBLE-DRIVER SYSTEMS

2.1 Introduction. An investigation of double-driver systems in the linear algebraic setting has been made (1). This chapter is devoted to an extension of the previous work on double-driver systems. The theory is extended to include energy storage elements, dependent sources, and multiterminal components. Additional topology requirements are established and the complex source concept is introduced.

2.2 Classification of Elements. Before proceeding with an investigation of the analysis of double-driver systems, a classification of the element types to be considered is in order. This classification begins with some formal definitions.

Definition 2.2.1. Double Driver. A double driver is defined as any two-terminal component for which both the through and the across variable are specified.

Definition 2.2.2. No-specification Element. An element for which no component equation exists in the primary mathematical model is defined as a no-specification element. This element is also called an unspecified element.

Definition 2.2.3. X Energy Storage Element. An element whose through and across variable are included in the matrix component equation

$$\underline{D}\,\underline{Y}_X = \frac{d}{dt}\underline{X}_X$$

is defined as an X energy storage element.

Definition 2.2.4. Y Energy Storage Element. An element whose through and across variable are included in the matrix component equation

$$\underline{K}\ \underline{X}_Y = \frac{d}{dt}\underline{Y}_Y$$

is defined as a Y energy storage element.

Definition 2.2.5. Algebraic Element. An element whose through and across variable are included in the matrix component equation

$$\underline{R}\ \underline{Y}_R = \underline{X}_R$$

is defined as an algebraic element.

In Definitions 2.2.3, 2.2.4, and 2.2.5 $\underline{D}$, $\underline{K}$, and $\underline{R}$ are square matrices of real constants.

The elements defined above plus the usual ideal drivers constitute the elements to be considered in this thesis. A summary of this classification of element types is shown in Table I.

The double driver is a component which may be either active or passive. It may be a component for which one variable is specified and a relation between the variables is known. Alternatively it may be an element for which both variables are specified explicitly.. The name double driver comes from the form of the component equations of this element. Regardless of the manner in which the variables are specified, the end result is two component equations in ideal driver form.

Since each of the double drivers in a system yield two component equations, it is desirable that a number of two-terminal elements equal to the number of double drivers have no component equations. The number of equations in the primary mathematical model and the number of unknowns are then in one to one correspondence. Throughout this thesis, the equality of the number of double drivers and the number of unspecified elements is assumed.

TABLE I

CLASSIFICATION OF ELEMENTS

Classification	Description	Number in Graph
N	unspecified	n_N
D	doubly specified	n_D
E	ideal across driver	n_E
J	ideal through driver	n_J
X	X energy storage	n_X
Y	Y energy storage	n_Y
R	algebraic	n_R

The coefficient matrices $\underline{D}$, $\underline{K}$, and $\underline{R}$ are not restricted to the diagonal case. Therefore energy storage element coupling is allowed within each classification. Furthermore resistive coupling is allowed. Hence, with some restrictions, dependent sources and the multiterminal components introduced by Koenig and Reed (5) are included. The restrictions are: (i) the coefficient matrix of any multiterminal representation must be partitionable into the direct sum of submatrices of $\underline{D}$, $\underline{K}$, and $\underline{R}$, (ii) through variable dependent through drivers are not included, and (iii) across variable dependent across drivers are not included. The last two restrictions are explicit only since it is often possible to find an algebraic multiterminal representation of a subsystem containing these drivers.

2.3 Topology Requirements. Consideration is now given to the requirements upon the topology of a double-driver system in order to ensure the existence of a unique solution of the equations in the mathematical model of this system. Necessary requirements for a unique solution in the case of a double-driver system containing ideal drivers and algebraic elements are known (1). After a very simple extension of these results, one obtains Theorem 2.3.1:

Theorem 2.3.1. Suppose a double-driver system containing e elements of the type shown in Table I has a v vertex, one-part linear graph G and has a primary mathematical model given by the following equations:

$$\begin{bmatrix} \underline{Q}_E & \underline{Q}_N & \underline{Q}_X & \underline{Q}_R & \underline{Q}_Y & \underline{Q}_D & \underline{Q}_J \end{bmatrix} \begin{bmatrix} \underline{Y}_E \\ \underline{Y}_N \\ \underline{Y}_X \\ \underline{Y}_R \\ \underline{Y}_Y \\ \underline{Y}_D \\ \underline{Y}_J \end{bmatrix} = 0 \tag{2.3.1}$$

$$\begin{bmatrix} \underline{B}_E & \underline{B}_N & \underline{B}_X & \underline{B}_R & \underline{B}_Y & \underline{B}_D & \underline{B}_J \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_N \\ \underline{X}_X \\ \underline{X}_R \\ \underline{X}_Y \\ \underline{X}_D \\ \underline{X}_J \end{bmatrix} = 0 \qquad (2.3.2)$$

$$\begin{bmatrix} -\underline{D} & 0 & 0 & \frac{d}{dt} & 0 & 0 \\ 0 & 0 & \frac{d}{dt} & 0 & 0 & -\underline{K} \\ 0 & -R & 0 & 0 & \underline{U} & 0 \end{bmatrix} \begin{bmatrix} \underline{Y}_X \\ \underline{Y}_R \\ \underline{Y}_Y \\ \underline{X}_X \\ \underline{X}_R \\ \underline{X}_Y \end{bmatrix} = 0 \qquad (2.3.3)$$

and

$$\begin{bmatrix} \underline{Y}_D & \underline{Y}_J & \underline{X}_E & \underline{X}_D \end{bmatrix} = \begin{bmatrix} \underline{J}_D(t) & \underline{J}_J(t) & \underline{E}_E(t) & \underline{E}_D(t) \end{bmatrix} \qquad (2.3.4)$$

The necessary conditions for a unique solution to Equations 2.3.1 through 2.3.4 are:

1. There exists some tree T_1 of G which contains the E-elements and the N-elements and for which the J-elements and the D-elements form a subgraph of the cotree.
2. There exists some tree T_2 of G which contains the E-elements and the D-elements and for which the J-elements

and the N-elements form a subgraph of the cotree.

Before proceeding to the proof of this theorem, two definitions are in order.

Definition 2.3.1. Impedance Element. The collection of X energy storage elements, Y energy storage elements, and algebraic elements are defined as impedance elements.

Definition 2.3.2. Set-aside Equations. Those equations in the primary mathematical model of a system which contain one variable that does not appear in any other equation of the primary mathematical model are defined as set-aside equations.

Proof of Theorem 2.3.1: Consider the complete set of equations as one matrix equation. The result is seen as

$$\begin{bmatrix} \underline{Q}_E & \underline{Q}_N & \underline{Q}_Z & \underline{Q}_D & \underline{Q}_J & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{B}_E & \underline{B}_N & \underline{B}_Z & \underline{B}_D & \underline{B}_J \\ 0 & 0 & \underline{C}_1 & 0 & 0 & 0 & 0 & \underline{C}_2 & 0 & 0 \\ 0 & 0 & 0 & \underline{U} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{U} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{U} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{U} & 0 \end{bmatrix} \begin{bmatrix} \underline{Y}_E \\ \underline{Y}_N \\ \underline{Y}_Z \\ \underline{Y}_D \\ \underline{Y}_J \\ \underline{X}_E \\ \underline{X}_N \\ \underline{X}_Z \\ \underline{X}_D \\ \underline{X}_J \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \underline{J}_D \\ \underline{J}_J \\ \underline{E}_E \\ \underline{E}_D \end{bmatrix} \qquad (2.3.5)$$

After applying several elementary transformations, one has

$$
\begin{bmatrix}
\underline{Q}_E & \underline{Q}_N & \underline{Q}_Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \underline{B}_J & \underline{B}_N & \underline{B}_Z & 0 & 0 & 0 & 0 \\
0 & 0 & \underline{C}_1 & 0 & 0 & \underline{C}_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \underline{U} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{U} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{U} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{U}
\end{bmatrix}
\begin{bmatrix}
\underline{Y}_E \\ \underline{Y}_N \\ \underline{Y}_Z \\ \underline{X}_J \\ \underline{X}_N \\ \underline{X}_Z \\ \underline{Y}_J \\ \underline{Y}_D \\ \underline{X}_E \\ \underline{X}_D
\end{bmatrix}
=
\begin{bmatrix}
-\underline{Q}_J \underline{J}_J - \underline{Q}_D \underline{J}_D \\
-\underline{B}_E \underline{E}_E - \underline{B}_D \underline{E}_D \\
0 \\
\underline{J}_J \\
\underline{J}_D \\
\underline{E}_E \\
\underline{E}_D
\end{bmatrix}
\qquad (2.3.6)
$$

By hypothesis this set of equations has a unique solution. Hence the coefficient matrix of Equation 2.3.6 is nonsingular. Therefore, the rows or any subset of the rows of this coefficient matrix are linearly independent. The linear independence of the first v-1 rows implies that the matrix $\left[\underline{Q}_E\ \underline{Q}_N\ \underline{Q}_Z\right]$ has rank v-1 which in turn implies that a subset of the columns of this matrix correspond to the branches of some tree T_1. Hence the J- and D- elements are in the cotree of T_1.

Similarly the linear independence of the next e-v+1 rows implies that $\left[\underline{B}_J\ \underline{B}_N\ \underline{B}_Z\right]$ has rank e-v+1 which in turn implies that a subset of the columns of this matrix correspond to the chords of the cotree of some tree T_2. Hence the E- and D- elements are in T_2.

Next consider the first n_E columns of the coefficient matrix.

Since the coefficient matrix is nonsingular, $\underline{Q}_E$ has maximum rank. Similarly $\underline{Q}_N$, $\underline{B}_J$, and $\underline{B}_N$ have maximum rank. This implies that the columns of $\underline{Q}_E$ and the columns of $\underline{Q}_N$ correspond to subgraphs of T_1 and that the columns of $\underline{B}_J$ and $\underline{B}_N$ correspond to subgraphs of the cotree of T_2. The proof is now complete.

The next question which naturally arises concerns the sufficiency of the above conditions. The question of sufficiency will be investigated through the development of a state model for the double-driver system. This will of course serve the dual purpose of deriving the state equations.

In the derivation of the state model, the usual assumption that the X-elements can be put in some tree and the Y-elements can be put in some cotree is made. T_1 and T_2 will in fact be chosen in accordance with this assumption.

Furthermore since n_N equals n_D, the number of algebraic elements in T_1 equals the number of algebraic elements in T_2 if the above assumption on energy storage elements is followed. For convenience of notation the same algebraic elements will be placed in T_1 as in T_2.

Choosing T_1 and T_2 as formulation trees for the cutset and circuit equations respectively and rewriting Equations 2.3.1, 2.3.2, and 2.3.3 accordingly, one has

$$\begin{bmatrix} \underline{U} & 0 & 0 & 0 & \underline{Q}_{15} & \underline{Q}_{16} & \underline{Q}_{17} & \underline{Q}_{18} \\ 0 & \underline{U} & 0 & 0 & \underline{Q}_{25} & \underline{Q}_{26} & \underline{Q}_{27} & \underline{Q}_{28} \\ 0 & 0 & \underline{U} & 0 & \underline{Q}_{35} & \underline{Q}_{36} & \underline{Q}_{37} & \underline{Q}_{38} \\ 0 & 0 & 0 & \underline{U} & \underline{Q}_{45} & \underline{Q}_{46} & \underline{Q}_{47} & \underline{Q}_{48} \end{bmatrix} \begin{bmatrix} \underline{Y}_E \\ \underline{Y}_N \\ \underline{Y}_X \\ \underline{Y}_{RT} \\ \underline{Y}_{RC} \\ \underline{Y}_Y \\ \underline{Y}_D \\ \underline{Y}_J \end{bmatrix} = 0 \qquad (2.3.7)$$

$$\begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & \underline{B}_{14} & \underline{U} & 0 & 0 & 0 \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & \underline{B}_{24} & 0 & \underline{U} & 0 & 0 \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} & \underline{B}_{34} & 0 & 0 & \underline{U} & 0 \\ \underline{B}_{41} & \underline{B}_{42} & \underline{B}_{43} & \underline{B}_{44} & 0 & 0 & 0 & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \\ \underline{X}_X \\ \underline{X}_{RT} \\ \underline{X}_{RC} \\ \underline{X}_Y \\ \underline{X}_N \\ \underline{X}_J \end{bmatrix} = 0 \qquad (2.3.8)$$

$$\frac{d}{dt} \begin{bmatrix} \underline{X}_X \\ \underline{Y}_Y \end{bmatrix} = \begin{bmatrix} \underline{D} & 0 \\ 0 & \underline{K} \end{bmatrix} \begin{bmatrix} \underline{Y}_X \\ \underline{X}_Y \end{bmatrix} \qquad (2.3.9)$$

and

$$\begin{bmatrix} \underline{X}_{RT} \\ \underline{Y}_{RC} \end{bmatrix} = \begin{bmatrix} \underline{R}_T & 0 \\ 0 & \underline{G}_C \end{bmatrix} \begin{bmatrix} \underline{Y}_{RT} \\ \underline{X}_{RC} \end{bmatrix} \qquad (2.3.10)$$

where T and C denote tree and cotree respectively. An inspection of Equations 2.3.7 through 2.3.10 shows that the equations involving the through variables of the E- and N-elements and the equations involving the across variables of the J- and N-elements need not be solved simultaneously with the remainder of the equations. These equations are set-aside equations.

Substituting Equations 2.3.7 and 2.3.8 less the set-aside equations into Equations 2.3.9 and 2.3.10, one has

$$\frac{d}{dt}\begin{bmatrix} \underline{X}_X \\ \underline{Y}_Y \end{bmatrix} = -\begin{bmatrix} \underline{D} & 0 \\ 0 & \underline{K} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & \underline{Q}_{35} & \underline{Q}_{36} & \underline{Q}_{37} & \underline{Q}_{38} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & \underline{B}_{24} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \\ \underline{X}_X \\ \underline{X}_{RT} \\ \underline{Y}_{RC} \\ \underline{Y}_Y \\ \underline{Y}_D \\ \underline{Y}_J \end{bmatrix} \qquad (2.3.11)$$

and

$$\begin{bmatrix} \underline{X}_{RT} \\ \underline{Y}_{RC} \end{bmatrix} = - \begin{bmatrix} \underline{R}_T & 0 \\ 0 & \underline{G}_C \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & \underline{Q}_{45} & \underline{Q}_{46} & \underline{Q}_{47} & \underline{Q}_{48} \\ \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & \underline{B}_{14} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \\ \underline{X}_X \\ \underline{X}_{RT} \\ \underline{Y}_{RC} \\ \underline{Y}_Y \\ \underline{Y}_D \\ \underline{Y}_J \end{bmatrix} \quad (2.3.12)$$

Solving Equation 2.3.12 for $\underline{X}_{RT}$ and $\underline{Y}_{RC}$, one has

$$\begin{bmatrix} \underline{X}_{RT} \\ \underline{Y}_{RC} \end{bmatrix} = - \begin{bmatrix} \underline{U} & \underline{R}_T\underline{Q}_{45} \\ \underline{G}_C\underline{B}_{14} & \underline{U} \end{bmatrix}^{-1} \begin{bmatrix} \underline{R}_T & 0 \\ 0 & \underline{G}_C \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \underline{Q}_{46} & \underline{Q}_{47} & \underline{Q}_{48} \\ \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \\ \underline{X}_X \\ \underline{Y}_Y \\ \underline{Y}_D \\ \underline{Y}_J \end{bmatrix} \quad (2.3.13)$$

Rewriting Equation 2.3.11

$$\frac{d}{dt} \begin{bmatrix} \underline{X}_X \\ \underline{Y}_Y \end{bmatrix} = - \begin{bmatrix} \underline{D} & 0 \\ 0 & K \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & 0 & \underline{Q}_{36} & \underline{Q}_{37} & \underline{Q}_{38} \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \\ \underline{X}_X \\ \underline{Y}_Y \\ \underline{Y}_D \\ \underline{Y}_J \end{bmatrix} + \begin{bmatrix} 0 & \underline{Q}_{35} \\ \underline{B}_{24} & 0 \end{bmatrix} \begin{bmatrix} \underline{X}_{RT} \\ \underline{Y}_{RC} \end{bmatrix} \right) \quad (2.3.14)$$

Substituting Equation 2.3.13 into Equation 2.3.14, one has

$$\frac{d}{dt}\begin{bmatrix}\underline{X}_X\\ \underline{Y}_Y\end{bmatrix} = -\begin{bmatrix}\underline{D} & 0\\ 0 & \underline{K}\end{bmatrix}\left(\begin{bmatrix}0 & 0 & 0 & \underline{Q}_{36} & \underline{Q}_{37} & \underline{Q}_{38}\\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & 0 & 0 & 0\end{bmatrix}\right. \tag{2.3.15}$$

$$\left. - \begin{bmatrix}0 & \underline{Q}_{35}\\ \underline{B}_{24} & 0\end{bmatrix}\begin{bmatrix}\underline{U} & \underline{R}_T\underline{Q}_{45}\\ \underline{G}_C\underline{B}_{14} & \underline{U}\end{bmatrix}^{-1}\begin{bmatrix}\underline{R}_T & 0\\ 0 & \underline{G}_C\end{bmatrix}\begin{bmatrix}0 & 0 & 0 & \underline{Q}_{46} & \underline{Q}_{47} & \underline{Q}_{48}\\ \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & 0 & 0 & 0\end{bmatrix}\right)\begin{bmatrix}\underline{X}_E\\ \underline{X}_D\\ \underline{X}_X\\ \underline{Y}_Y\\ \underline{Y}_D\\ \underline{Y}_J\end{bmatrix}$$

which for purposes of notation can be written

$$\frac{d}{dt}\begin{bmatrix}\underline{X}_X\\ \underline{Y}_Y\end{bmatrix} = \begin{bmatrix}\underline{C}_{11} & \underline{C}_{12} & \underline{A}_{11} & \underline{A}_{12} & \underline{C}_{13} & \underline{C}_{14}\\ \underline{C}_{21} & \underline{C}_{22} & \underline{A}_{21} & \underline{A}_{22} & \underline{C}_{23} & \underline{C}_{24}\end{bmatrix}\begin{bmatrix}\underline{X}_E\\ \underline{X}_D\\ \underline{X}_X\\ \underline{Y}_Y\\ \underline{Y}_D\\ \underline{Y}_J\end{bmatrix} \tag{2,3.16}$$

or

$$\frac{d}{dt}\begin{bmatrix}\underline{X}_X\\ \underline{Y}_Y\end{bmatrix} = \begin{bmatrix}\underline{A}_{11} & \underline{A}_{12}\\ \underline{A}_{21} & \underline{A}_{22}\end{bmatrix}\begin{bmatrix}\underline{X}_X\\ \underline{Y}_Y\end{bmatrix} + \begin{bmatrix}\underline{C}_{11} & \underline{C}_{12} & \underline{C}_{13} & \underline{C}_{14}\\ \underline{C}_{21} & \underline{C}_{22} & \underline{C}_{23} & \underline{C}_{24}\end{bmatrix}\begin{bmatrix}\underline{X}_E\\ \underline{X}_D\\ \underline{Y}_D\\ \underline{Y}_J\end{bmatrix} \tag{2.3.17}$$

Now Equation 2.3.17 is the state model in the form

$$\frac{d}{dt}\underline{Z} = \underline{A}\,\underline{Z} + \underline{C}\,\underline{F}(t) \tag{2.3.18}$$

where $\underline{A}$ and $\underline{C}$ are matrices of real constants and F(t) is given by Equation 2.3.4.

The conditions for the existence of a unique solution to Equation 2.3.18 are given by any good ordinary differential equations text. For example see Greenspan (6). Equation 2.3.18 does in fact have a unique solution for any closed interval of time on which $\underline{F}(t)$ is continuous.

Equation 2.3.13 shows that the state model given by Equation 2.3.17 can be developed only if the matrix

$$\begin{bmatrix} \underline{U} & \underline{R}_T\underline{Q}_{45} \\ \underline{G}_C\underline{B}_{14} & \underline{U} \end{bmatrix}$$

has an inverse, which means that the n_{RC} by n_{RC} determinant $|\underline{U} - \underline{G}_C\underline{B}_{14}\underline{R}_T\underline{Q}_{45}|$ must be nonzero.

The question of the sufficiency of the conditions of Theorem 2.3.1 is settled. These results are presented in the form of the following theorem:

Theorem 2.3.2. The primary mathematical model of a double-driver system which satisfies all the conditions of Theorem 2.3.1 possesses a unique solution if

1. $\begin{bmatrix} \underline{J}_D(t) & \underline{J}_J(t) & \underline{E}_E(t) & \underline{E}_D(t) \end{bmatrix}$ is continuous.
2. The determinant $|\underline{U} - \underline{G}_C\underline{B}_{14}\,\underline{R}_T\,\underline{Q}_{45}|$ is nonzero.

There are several points concerning the above proof of Theorem 2.3.2 which should be noted. First, as evidenced by Equation 2.3.10, coupling between the algebraic elements in the tree and the algebraic elements in the cotree is assumed to not exist. Second, for purposes of testing, the n_{RT} by n_{RT} determinant $|\underline{U} - \underline{R}_T\, \underline{Q}_{45}\, \underline{G}_C\, \underline{B}_{14}|$ being nonzero is equivalent to condition 2. Hence the smaller of these two determinants may be checked. Furthermore, if all the algebraic elements can be put in either the tree or the cotree, condition 2 is not needed. Finally, it is assumed that the required initial conditions are known.

The usual approach to the proof of a theorem such as Theorem 2.3.2 is the development of a set of mesh equations in the s domain. The coefficient matrix of the mesh variables is then shown to have an inverse for some value of s. An argument is then made concerning the existence of the inverse Laplace transform of the resulting mesh variable functions. This method relies heavily upon the positive definiteness of the branch impedance matrix and upon a theorem in matrix theory concerning the positive definiteness of a particular triple matrix product.

Considerable effort was spent attempting a proof based upon a mesh formulation. The author feels that the added conditions of Theorem 2.3.2 could be removed, at least for the case of a positive definite branch impedance matrix, if a successful mesh formulation proof could be accomplished. It must be pointed out however that the state model proof shown above is much less restrictive in one sense. The state model proof does not depend upon the positive definiteness and hence the implied symmetry of the branch impedance matrix.

The development of the mesh equations for a double-driver system are shown in Appendix A. Some discussion is included in this appendix of the difficulties encountered when a sufficiency proof based on the mesh formulation is attempted. The node formulation is also developed in Appendix A.

2.4 The Complex Source. The idea of a complex source is a very recent one (2). In the usual sense, the complex source is a conceptual source only and a brief definition of this concept is difficult. Rather it is an idea -- a way of thinking -- which must be established. A thorough understanding of this way of thinking is essential to the development of the design procedure.

The artifice of applying a conceptual source to a system to obtain defining characteristics of the system is quite useful. A very simple example is the finding of Thevenin's equivalent circuit for a one-port network. An arbitrary current source is applied at the port and the resulting port voltage is found as a function of the applied source. The result is the equation defining the Thevenin equivalent circuit.

There are many other examples of the use of arbitrary sources to obtain a representation of a system. The short-circuit admittance matrix or the open-circuit impedance matrix can be obtained by this method. Any mixed set of impedances, admittances, and transfer functions can be obtained by application of the proper arbitrary sources to an n-port network. In fact, in any system, almost any defining relation is possible. The only restriction is that it must be possible to find the set of variables which are chosen as dependent, as functions of the independent or arbitrary set of variables.

The complex source is an extension of the ideas discussed above. Consider the n-port system shown in Figure 2.4.1.

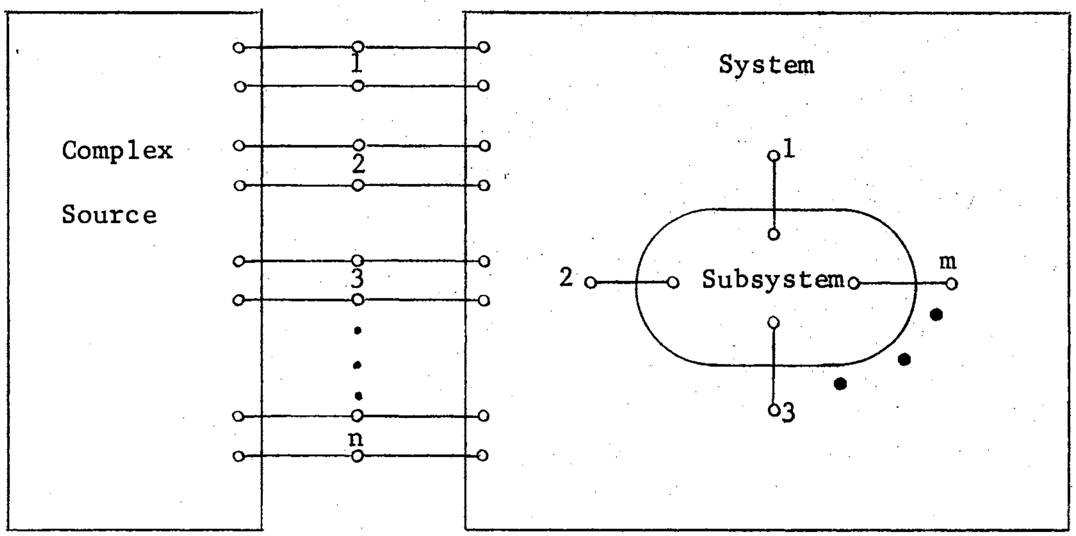

Figure 2.4.1. A System with a Complex Source

Suppose that as is usual, one through and one across variable is defined at each of the n ports of this system. Let one set S_1 of n of these variables be chosen as independent variables and let the remaining n variables be the dependent set S_2. Now suppose that it is desired that the system exhibit external terminal characteristics defined by

$$\underline{S}_2 = \underline{F}_1 \underline{S}_1 + \underline{S}_3 \tag{2.4.1}$$

Where $\underline{S}_1$, $\underline{S}_2$, and $\underline{S}_3$ are column matrices and $\underline{F}_1$ is an n by n matrix of functions. The entries in $\underline{S}_2$ and $\underline{S}_1$ could be time or s-domain variables. The entries in $\underline{F}_1$ would then be functions of the p operator or functions of the Laplace transform variable s respectively.

Further consider that with the exception of the subsystem shown, the geometry and component equations of the system are known. Nothing however is known about the subsystem. The problem is to determine the terminal characteristics required of the subsystem in order that the complete system satisfy Equation 2.4.1. The complex source is merely an arbitrary n-port driver which forces the system to have the desired external characteristics and at the same time allows one to determine the required terminal characteristics of the subsystem.

The required terminal characteristics of the complex source are then given by

$$\underline{S}_2^1 = \underline{F}_2 \, \underline{S}_1^1 + \underline{S}_4 \qquad (2.4.2)$$

Where $\underline{S}_2^1$ and $\underline{S}_1^1$ differ from $\underline{S}_2$ and $\underline{S}_1$ only in orientation of some of the entries and $\underline{F}_2$ and $\underline{S}_4$ differ from $\underline{F}_1$ and $\underline{S}_3$ only in the sign of some of the entries. The different signs are required in order to account for the differences in orientation.

Equation 2.4.2 and the associated discussion define the complex source. Equation 2.4.2 also shows that the complex source is related to the double driver. If the independent set S_1^1 is arbitrarily chosen, then the dependent set S_2^1 is given by Equation 2.4.2. Hence the complex source is in reality a set of n constrained double drivers. Therefore the theory of double-driver systems applies to systems of the type shown in Figure 2.4.1.

It is now possible to give a formal definition of a complex source.

Definition 2.4.1. Complex Source. An n-port system consisting of a set of n double drivers, with a known constraint relating the set of n variables chosen as dependent to the set of n variables chosen as independent, is defined as a complex source.

The details concerning the possible geometry of the unspecified subsystem or the determination of its terminal characteristics are not discussed in this chapter. The details will appear in Chapter III.

CHAPTER III

DESIGN OF AN n-PORT SYSTEM

3.1 Introduction. In this chapter a particular type of design problem is considered. The problem is limited to that class of design problems for which a portion of the system is already known and the designer is free to work with only an n-port subsystem. Utilizing the complex-source concept, a solution to this design problem is given and the technique is demonstrated by examples. The role of n-port network synthesis in the developed design procedure is discussed at some length.

3.2 The Design Procedure. Quite often in the design of practical systems, the designer is faced with the problem of modifying or replacing a portion of an existing system in order to cause the system to exhibit particular characteristics at several ports or terminal-pairs. This problem is now considered in detail.

Figure 2.4.1 is illustrative of the type of system which is considered in this design procedure development. If, as discussed in Section 2.2, the number of unspecified elements is made equal to the number of double drivers, the unspecified subsystem must be represented by n elements. This restriction does not mean that the subsystem must be realized by n elements. It only means that for purposes of solution of the terminal requirements of the subsystem that an n element representation is assumed. Assuming an n element representation of the subsystem requires one to assume that the subsystem is connected into the system at n + 1 points.

Now suppose that Figure 2.4.1 is redrawn and that the through and across variables at the system terminals are defined as shown in Figure 3.2.1.

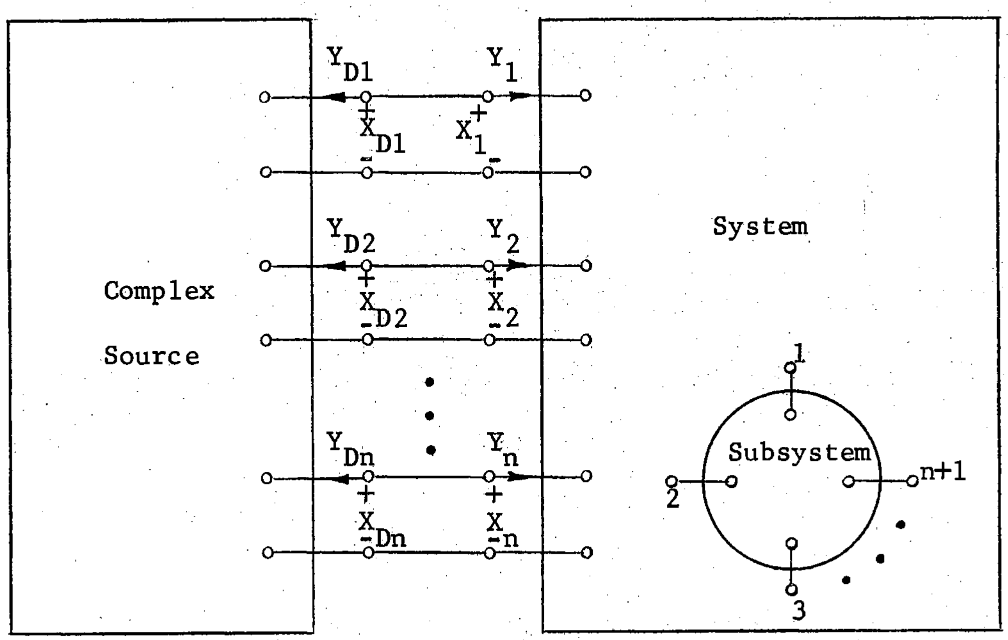

Figure 3.2.1. System to be Designed

Figure 3.2.1 shows an n-port system consisting of X-, Y-, R-, E-, and J-elements and a completely unspecified n + 1 terminal subsystem. Any n + 1 terminal, n element representation can be assumed for the unspecified subsystem and double-driver theory can be applied to this system when it is connected to a complex source as shown in Figure 3.2.1.

Next suppose that among the 2n variables X_1, X_2, ..., X_n, Y_1, Y_2,

..., Y_n the set of all the across variables is chosen as the independent set. Further suppose that the desired terminal characteristics are given by

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1n} \\ W_{21} & W_{22} & \cdots & W_{2n} \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ W_{n1} & W_{n2} & \cdots & W_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{bmatrix} + \begin{bmatrix} J_1 \\ J_2 \\ \cdot \\ \cdot \\ \cdot \\ J_n \end{bmatrix} \tag{3.2.1}$$

or more compactly

$$\underline{Y} = \underline{W}\ \underline{X} + \underline{J} \tag{3.2.2}$$

where in these equations as throughout this chapter the s-domain functional dependence is implied.

Now the complex source or n constrained double drivers are utilized to find the terminal characteristics which the subsystem must possess in order to force the overall system to exhibit the desired characteristics given by Equation 3.2.1. The complex source is defined by

$$\underline{Y}_D = \underline{W}_D\ \underline{X}_D + \underline{J}_D \tag{3.2.3}$$

where $\underline{Y}_D$, $\underline{X}_D$ and $\underline{J}_D$ are n dimensional column matrices. Now since $X_{Di} = X_i$ and $Y_{Di} = -Y_i$, it is readily seen that

$$\underline{Y}_D = -\underline{W}\ \underline{X}_D - \underline{J} \tag{3.2.4}$$

is the required complex source specification.

The analysis techniques which are discussed in Chapter II and Appendix A can be used to solve for the through and across variables of the N-elements (the unspecified subsystem) as functions of the double drivers $\underline{X}_D$ and $\underline{Y}_D$. The result of this straightforward analysis is in the form

$$\begin{bmatrix} X_{N1} \\ X_{N2} \\ \cdot \\ \cdot \\ \cdot \\ X_{Nn} \\ Y_{N1} \\ Y_{N2} \\ \cdot \\ \cdot \\ \cdot \\ Y_{Nn} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1p} \\ h_{21} & h_{22} & \cdots & h_{2p} \\ & & & \\ & & & \\ \cdot & & & \\ & & & \\ \vdots & \cdot & & \cdot \\ & & & \\ \cdot & & & \\ & & & \\ & & & \\ h_{p1} & h_{p2} & \cdots & h_{pp} \end{bmatrix} \begin{bmatrix} X_{D1} \\ X_{D2} \\ \cdot \\ \cdot \\ \cdot \\ X_{Dn} \\ Y_{D1} \\ Y_{D2} \\ \cdot \\ \cdot \\ \cdot \\ Y_{Dn} \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_n \\ I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_n \end{bmatrix} \qquad (3.2.5)$$

where $p = 2n$ and V_i and I_i result from the existence of any ideal drivers in the system.

Equation 3.2.5 can be partitioned according to across and through variables to obtain

$$\begin{bmatrix} \underline{X}_N \\ \underline{Y}_N \end{bmatrix} = \begin{bmatrix} \underline{H}_{11} & \underline{H}_{12} \\ \underline{H}_{21} & \underline{H}_{22} \end{bmatrix} \begin{bmatrix} \underline{X}_D \\ \underline{Y}_D \end{bmatrix} + \begin{bmatrix} \underline{V} \\ \underline{I} \end{bmatrix} \qquad (3.2.6)$$

which may be rewritten as

$$\underline{X}_N = \underline{H}_{11}\,\underline{X}_D + \underline{H}_{12}\underline{Y}_D + \underline{V} \qquad (3.2.7)$$

and

$$\underline{Y}_N = \underline{H}_{21}\,\underline{X}_D + \underline{H}_{22}\underline{Y}_D + \underline{I} \qquad (3.2.8)$$

Substituting Equation 3.2.4 into Equations 3.2.7 and 3.2.8, one obtains

$$\underline{X}_N = \underline{H}_{11}\,\underline{X}_D + \underline{H}_{12}(-\underline{W}\,\underline{X}_D - \underline{J}) + \underline{V} \qquad (3.2.9)$$

and

$$\underline{Y}_N = \underline{H}_{21}\underline{X}_D + \underline{H}_{22}(-\underline{W}\,\underline{X}_D - \underline{J}) + \underline{I} \qquad (3.2.10)$$

Now solving Equation 3.2.9 for $\underline{X}_D$ and substituting this result into Equation 3.2.10, one obtains the desired result, namely

$$\underline{Y}_N = (\underline{H}_{21}-\underline{H}_{22}\underline{W})(\underline{H}_{11}-\underline{H}_{12}W)^{-1}\underline{X}_N + (\underline{H}_{21}-\underline{H}_{22}W)(\underline{H}_{11}-\underline{H}_{12}\underline{W})^{-1}(\underline{H}_{12}\underline{J}-\underline{V})$$

$$+ \underline{I} - \underline{H}_{22}\,\underline{J} \qquad (3.2.11)$$

or

$$\underline{Y}_N = \underline{W}_N\,\underline{X}_N + \underline{J}_N \qquad (3.2.12)$$

which is hereafter referred to as the admittance design equation.

As discussed in Section 2.4, any set of n of the terminal variables can be chosen as independent. Another desirable choice for example is the set of all the through variables. In this case the terminal specifications of the system are

$$\underline{X} = \underline{Z}\ \underline{Y} + \underline{E} \tag{3.2.13}$$

and the complex source specifications are

$$\underline{X}_D = -\underline{Z}\ \underline{Y}_D + \underline{E} \tag{3.2.14}$$

Substituting Equation 3.2.14 into Equations 3.2.7 and 3.2.8, one obtains

$$\underline{X}_N = \underline{H}_{11}(-\underline{Z}\ \underline{Y}_D + \underline{E}) + \underline{H}_{12}\ \underline{Y}_D + \underline{V} \tag{3.2.15}$$

and

$$\underline{Y}_N = \underline{H}_{21}(-\underline{Z}\ \underline{Y}_D + \underline{E}) + \underline{H}_{22}\ \underline{Y}_D + \underline{I} \tag{3.2.16}$$

and proceeding in a manner analogous to the development of the admittance design equation, one obtains the impedance design equation

$$\underline{X}_N = (-\underline{H}_{11}\ \underline{Z} + \underline{H}_{12})(-\underline{H}_{21}\ \underline{Z} + \underline{H}_{22})^{-1}\ \underline{Y}_N + (-\underline{H}_{11}\ \underline{Z} + \underline{H}_{12})(-\underline{H}_{21}\underline{Z} + \underline{H}_{22})^{-1} (-\underline{H}_{21}\ \underline{E} - \underline{I}) + \underline{H}_{11}\underline{E} + \underline{V} \tag{3.2.17}$$

or

$$\underline{X}_N = \underline{Z}_N\ \underline{Y}_N + \underline{E}_N \tag{3.2.18}$$

The design is now completed by utilizing any available synthesis techniques to realize the subsystem specified by Equation 3.2.12, or Equation 3.2.18 or any analogous derived terminal specification. In the majority of the cases of interest $\underline{E}$, $\underline{J}$, $\underline{V}$ and $\underline{I}$ are zero and hence $\underline{E}_N$ and $\underline{J}_N$ are zero. It should be pointed out that $\underline{E}_N$ and $\underline{J}_N$ being zero does not imply that the system is passive but only that it does not contain any independent drivers.

The design procedure is now demonstrated by a simple 3-port example. The system is an electric network and the usual electric network notation is used throughout.

Example 3.2.1. Consider the 3-port network shown in Figure 3.2.2.

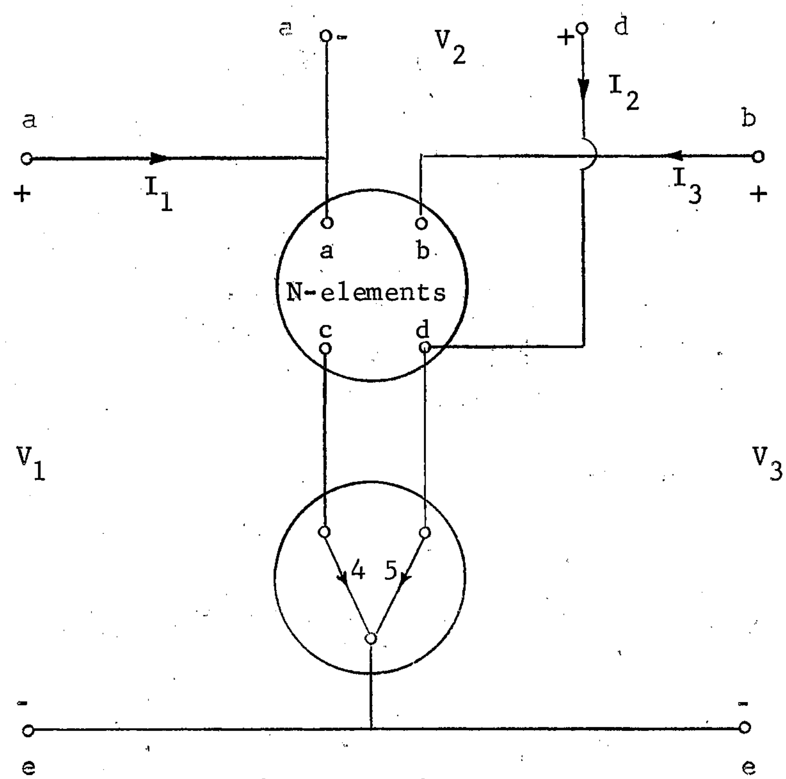

Figure 3.2.2. A 3-port Network

Figure 3.2.2 shows a 3-port network which consists of a 3-port unspecified subnetwork and a subnetwork whose multiterminal representation is given by elements 4 and 5 as shown and the relation

$$\begin{bmatrix} I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} V_4 \\ V_5 \end{bmatrix} \tag{3.2.19}$$

Suppose that the desired terminal characteristics of the network are

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{2s^2 + 7s + 6}{2s + 1} & \frac{2s^2 + 3s + 5}{2s + 1} & -(s + 1) \\ \frac{2s^2 + 2s + 5}{2s + 1} & \frac{2s^2 + s + 6}{2s + 1} & -s \\ -(s + 1) & -s & s + 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{3.2.20}$$

Choosing some arbitrary 3-element representation for the unspecified subsystem and picking the formulation trees T_1 and T_2 as discussed in Section 2.3, one obtains the linear graphs of Figure 3.2.3.

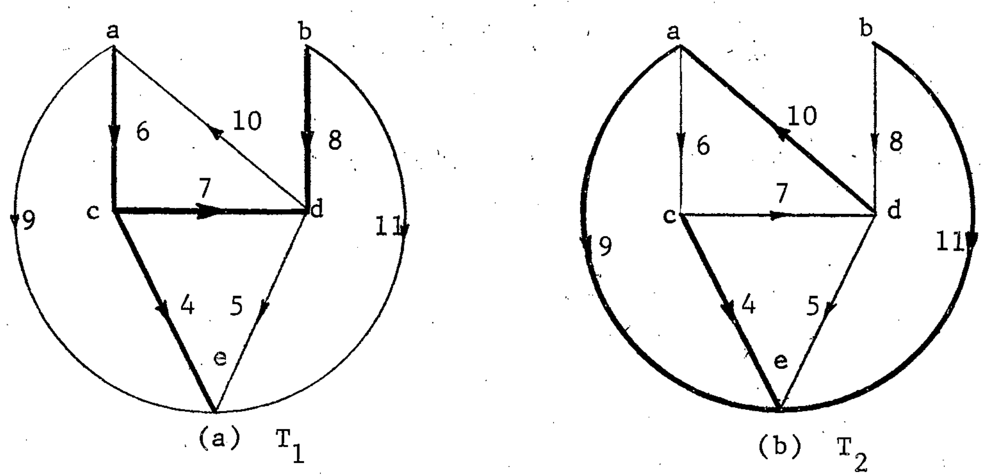

Figure 3.2.3. Linear Graphs Showing Formulation Trees

Where

1. Elements 6, 7, and 8 are N-elements.

2. Elements 9, 10 and 11 are D-elements.

3. Elements 4 and 5 are R-elements.

Writing the circuit equations for T_2 and the cutset equations for T_1, one obtains

$$\text{Set aside}\left\{\begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\right. \begin{bmatrix} V_9 \\ V_{10} \\ V_{11} \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = 0 \qquad (3.2.21)$$

and

$$\text{Set aside}\left\{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}\right. \begin{bmatrix} I_6 \\ I_7 \\ I_8 \\ I_4 \\ I_5 \\ I_9 \\ I_{10} \\ I_{11} \end{bmatrix} = 0 \qquad (3.2.22)$$

Equations 3.2.19, 3.2.21 and 3.2.22 plus the specifications of the double drivers constitute the primary mathematical model of the network. Next substitute the first equation of Equation 3.2.21 into Equation 3.2.19 and then substitute this result into the last equation of Equation 3.2.22 to obtain the nodal equation

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} V_4 + \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} V_9 \\ V_{10} \end{bmatrix} \right) = - \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_9 \\ I_{10} \\ I_{11} \end{bmatrix} \tag{3.2.23}$$

which can be solved for V_4 to obtain

$$V_4 = -1/2\, I_9 - 1/2\, I_{11} - 3/2 V_9 - 3/2 V_{10} \tag{3.2.24}$$

Substituting Equation 3.2.24 back into the set aside equations and utilizing Equations 3.2.19, one obtains

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} 5/2 & 3/2 & 0 & 1/2 & 0 & 1/2 \\ -5/2 & -5/2 & 0 & -1/2 & 0 & -1/2 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1/2 & -1/2 & 0 & -1/2 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_9 \\ V_{10} \\ V_{11} \\ I_9 \\ I_{10} \\ I_{11} \end{bmatrix} \tag{3.2.25}$$

which is in the desired form of Equation 3.2.6.

If Equation 3.2.25 is partitioned according to Equation 3.2.6 and this result and the specifications of Equation 3.2.20 are substituted into Equation 3.2.11, one obtains after some algebra

$$\underline{Y}_N = \begin{bmatrix} 2 & 1 & -1 \\ 1 & \frac{s+1}{s} & -1 \\ -1 & -1 & s+1 \end{bmatrix} \tag{3.2.26}$$

which with the representation assumed for elements 6, 7, and 8 define the required subnetwork.

In order to complete the design, it is necessary to synthesize the subnetwork. The specifications of Equation 3.2.26 are first transformed to a Lagrangian tree representation as shown in Figure 3.2.4.

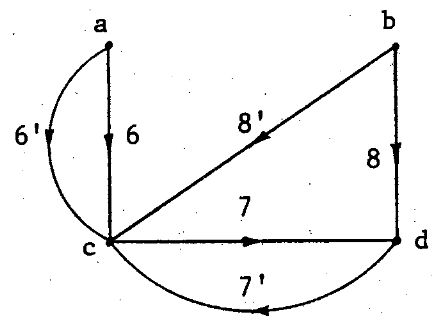

Figure 3.2.4. Transformation Graph

From Figure 3.2.4 the transformation

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_6' \\ V_7' \\ V_8' \end{bmatrix} \tag{3.2.27}$$

is written. Hence the terminal specifications for the N-element subnetwork can be written for the Lagrangian tree as

$$\underline{Y}'_N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & \frac{s+1}{s} & -1 \\ -1 & -1 & s+1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & s+\frac{1}{s} & -s \\ -1 & -s & s+1 \end{bmatrix} \qquad (3.2.28)$$

which is readily synthesized as seen in Figure 3.2.5.

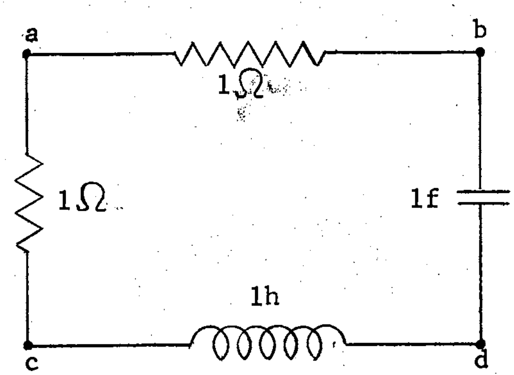

Figure 3.2.5. N-element Subnetwork Realization

The design is now complete and if the subnetwork of Figure 3.2.5 is connected into the network of Figure 3.2.2 the terminal specifications of Equation 3.2.20 result.

Two very pertinent points concerning the synthesis of the subsystem are apparent. Since the designer is free to pick the tree to represent the N-element subsystem, it is advantageous to pick a Lagrangian tree and thereby avoid the transformation step in the realization. Furthermore only that portion of n-port synthesis which is concerned with n + 1 terminal realizations is pertinent since the unspecified subsystem is always n + 1 terminal. These are advantages rather than disadvantages since n-port passive network synthesis of short-circuit admittance matrices using n + 1 terminals is a completely solved problem and much is known about n-port active network synthesis with n + 1 terminals. In contrast the theory of n-port network synthesis with more than n + 1 terminals is far from complete. A summary of n-port network synthesis which is pertinent to this study is now appropriate.

3.3 Synthesis of n-port Networks with n + 1 Terminals. This discussion of synthesis begins with several definitions and proceeds with several appropriate theorems. No attempt is made to give credit to the originator in every case. Rather a reference which is felt to be appropriate is given. The references cited provide proofs for the theorems.

Definition 3.3.1. Positive Real Matrix. A positive real matrix $[Z(s)]$ is defined as an nxn symmetric matrix each entry of which is a rational function of s with real coefficients and for which the function

$$Z(s) = \sum_1^n \sum_1^n Z_{ik} n_i n_k$$

is a positive real function for any choice of the real numbers n_1, n_2, ..., n_n (7).

Weinberg (3) gives a more usable but quite lengthy method to test for a positive real matrix.

Definition 3.3.2. Positive Semidefinite Matrix. A matrix of real numbers $\left[a\right]$ is defined as positive semidefinite if and only if

$$\sum_1^n \sum_1^n a_{ik} x_i x_k \geqslant 0$$

for all finite values of the real variables x_1, x_2, ..., x_n (3).

Definition 3.3.3. Dominant Matrix. A real symmetric matrix is dominant if each of its main diagonal elements is not less than the sum of the absolute values of all the other elements in the same row (8).

Theorem 3.3.1. An nxn real matrix is realizable as the short-circuit conductance matrix of an n-port, n + 1 terminal, resistive network with a Lagrangian tree representation if and only if the matrix is dominant and every off-diagonal element is nonpositive (3).

Theorem 3.3.2. An nxn symmetric matrix of short-circuit admittances $\left[Y(s)\right]$ is realizable by an n-port RLC network containing only n + 1 terminals, one of which is a ground for all the ports, if and only if every element of $\left[Y(s)\right]$ is of the form

$$Y_{ij} = \pm\left(C_{ij}s + g_{ij} + \frac{1}{L_{ij}s}\right)$$

where C_{ij}, g_{ij} and L_{ij} are real nonnegative constants and the sign is positive for i = j but negative otherwise. Furthermore the matrix formed by letting s take on any positive real value k is dominant (9).

Theorem 3.3.3. An nxn symmetric matrix of open-circuit impedances [Z(s)] or short-circuit admittances [Y(s)] is realizable by an n-port network (containing R's, L's, C's, M's and ideal transformers) if and only if it is a positive real matrix (3).

Theorem 3.3.4. A real symmetric nxn matrix is realizable by an n-port network of resistances and ideal transformers if and only if it is positive semidefinite (3).

Theorem 3.3.5. A real nonsingular nxn matrix is realizable as the open-circuit impedance matrix of an n + 1 terminal network (with a common ground) if and only if its inverse is a dominant matrix with nonpositive off-diagonal elements (3).

Active n-port network synthesis is not as neat a package as that presented above for the passive n + 1 terminal case. A very good summary and bibliography of active n-port network synthesis is given by Cruz (4). The majority of the work on active synthesis that has been done does not consider the special case of realization with only n + 1 terminals. There are two notable exceptions. Brown (10) considers the realization of a short-circuit admittance matrix of an n + 1 terminal network containing tubes, transistors, transformers, etc., as well as RLC elements. Huelsman (11) gives brief consideration to the use of the gyrator in the realization of a short-circuit admittance matrix with an n + 1 terminal network.

Finally it should be pointed out that the synthesis of hybrid matrices has received recent attention (12). This recent work gives the designer some motivation for the development of a hybrid design equation.

3.4 Other Design Considerations. Heretofore the n-element subsystem has been assumed to be connected. Attention is now directed to the possibility that the subsystem consists of two or more disjoint parts. For example suppose that the n unspecified elements are in two disjoint sets of k and m elements respectively. The derivation of the $\underline{Y}_N$ matrix proceeds simply. First assume k + 1 and m + 1 terminal representations for the two sets of elements. Proceed as normal to obtain Equation 3.2.6. Partition Equation 3.2.6 and substitute into Equation 3.2.11 to obtain

$$\underline{Y}_N = \begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} \\ \underline{Y}_{21} & \underline{Y}_{22} \end{bmatrix} \qquad (3.4.1)$$

where $\underline{Y}_{11}$ is k x k and $\underline{Y}_{22}$ is mxm.

Now if $\underline{Y}_{12} = \underline{Y}_{21} = 0$, $\underline{Y}_{11}$ and $\underline{Y}_{22}$ can be synthesized individually in a normal fashion. Constants which appear in $\underline{Y}_{12}$ or $\underline{Y}_{21}$ can be realized with dependent sources. Symmetric occurrences of constants times s or constants times 1/s require electrostatic or electromagnetic coupling.

Next consideration is given to a somewhat different approach to the design problem. The possibility of compensating or changing the terminal characteristics of an n-port system with a known multiterminal representation by connecting a second n-port into the first in some fashion is investigated. These ideas are best illustrated by an example.

Example 3.4.1. Suppose one has a 3-port network as shown in Figure 3.4.1 and a known multiterminal representation as given by Equation 3.4.2.

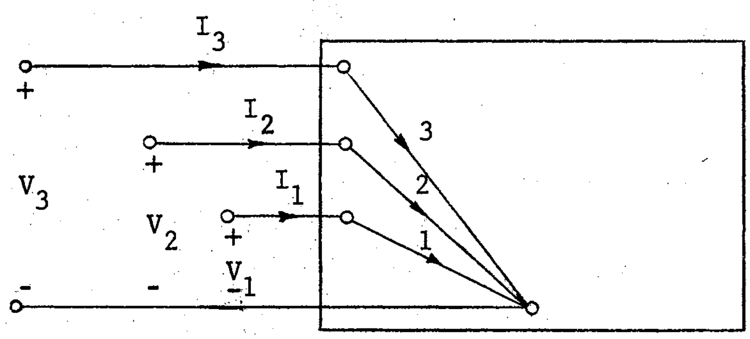

Figure 3.4.1. Network with Known Multiterminal Representation

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{3.4.2}$$

Now suppose that it is desired that the terminal characteristics be changed to

$$\begin{bmatrix} I'_1 \\ I'_2 \\ I'_3 \end{bmatrix} = \begin{bmatrix} y'_{11} & y'_{12} & y'_{13} \\ y'_{21} & y'_{22} & y'_{23} \\ y'_{31} & y'_{32} & y'_{33} \end{bmatrix} \begin{bmatrix} V'_1 \\ V'_2 \\ V'_3 \end{bmatrix} \tag{3.4.3}$$

by connecting a second 3-port into the existing 3-port as shown by Figure 3.4.2.

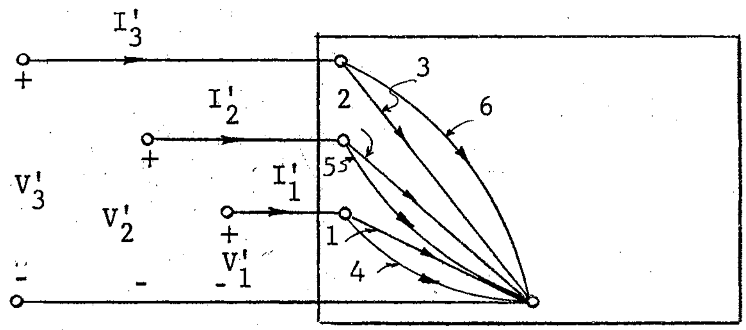

Figure 3.4.2. Modified Network

The problem is to find the short-circuit admittance matrix of the connected 3-port which will cause the network to have the terminal characteristics of Equation 3.4.3. When the three double drivers, constrained by Equation 3.4.3, are connected to the network the resulting linear graph is shown in Figure 3.4.3.

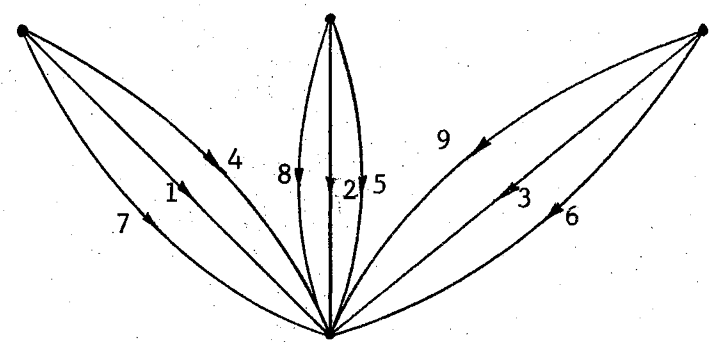

Figure 3.4.3. Graph of Network

where elements 7, 8, and 9 are double drivers.

From Figure 3.4.3 it is readily seen that

$$\begin{bmatrix} V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_7 \\ V_8 \\ V_9 \end{bmatrix} \tag{3.4.4}$$

and

$$\begin{bmatrix} I_4 \\ I_5 \\ I_6 \end{bmatrix} = - \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} - \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} \tag{3.4.5}$$

or

$$\begin{bmatrix} I_4 \\ I_5 \\ I_6 \end{bmatrix} = - \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_7 \\ V_8 \\ V_9 \end{bmatrix} - \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} \tag{3.4.6}$$

From Equations 3.4.4 and 3.4.6 one obtains

$$\begin{bmatrix} V_4 \\ V_5 \\ V_6 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & \bigcirc & \\ 0 & 0 & 1 & & & \\ \hline -y_{11} & -y_{12} & -y_{13} & -1 & 0 & 0 \\ -y_{21} & -y_{22} & -y_{23} & 0 & -1 & 0 \\ -y_{31} & -y_{32} & -y_{33} & 0 & 0 & -1 \end{array}\right] \begin{bmatrix} V_7 \\ V_8 \\ V_9 \\ I_7 \\ I_8 \\ I_9 \end{bmatrix} \tag{3.4.7}$$

which is in the form of Equation 3.2.6.

Now substituting Equation 3.4.7 and the terminal specifications of Equation 3.4.3 into the admittance design equation (Equation 3.2.11), one obtains

$$\underline{Y}_N = \begin{bmatrix} y'_{11} & -y_{11} & y'_{12} & -y_{12} & y'_{13} & -y_{13} \\ y'_{21} & -y_{21} & y'_{22} & -y_{22} & y'_{23} & -y_{23} \\ y'_{31} & -y_{31} & y'_{32} & y'_{33} & y'_{33} & -y_{33} \end{bmatrix} \tag{3.4.8}$$

which is an expected result since it is well known that the y parameters of n-ports in parallel add. This example then serves as a double-driver derivation of this well known result.

Double-driver techniques can be used to derive relations similar to Equation 3.4.8 for various multiterminal representations of the known network and various interconnection patterns of the N-element representation. However no additional information is gained, at least in the passive case, by developing symbolic relations for all the many possible configurations because any result which is obtained can be transformed to Equation 3.4.8 by suitable tree transformations. Of course parameters other than the short-circuit admittance parameters can also be considered. A rather detailed discussion of h-parameters for the two-port case is given in Chapter IV.

3.5 Summary of Design Procedure. There are several points concerning the design scheme developed in this chapter which are worth enumerating. These are:

1. A straightforward but somewhat unconventional analysis technique is utilized to derive the required terminal characteristics of the unspecified subsystem.
2. The first step in n-port synthesis requiring the determination of the port structure is circumvented by the design procedure developed in this thesis. This is true because the designer establishes the port structure when he chooses a tree for the multiterminal representation of the N-elements. Avoiding this step in the synthesis is a very valuable asset since time is saved and the designer is allowed to work in a much better founded theory of network synthesis.
3. The Lagrangian tree representation of the N-elements will in general lead to a more readily synthesizable result.

4. The synthesis theory associated with the admittance design equation is more complete.

CHAPTER IV

DESIGN OF TWO-PORT SYSTEMS

4.1 Introduction. In this chapter the n-port design procedure is specialized to the two-port system. Short-circuit admittance and open-circuit impedance design is considered briefly. Because of the practical significance of h-parameters in transistor work, h-parameter design is considered in detail.

4.2 S-C Admittance and O-C Impedance Design. Because of the frequency with which two-port systems occur in practice, the results of Chapter III are specialized for the two-port case. Very little additional discussion is needed for two-port design using the admittance or impedance design equations. The results of Equation 3.2.6 are readily applied, with n equal two, to find the terminal characteristics required of the unspecified two port as given by either Equation 3.2.11 or Equation 3.2.17.

The discussion of synthesis in Chapter III is readily extended to the two-port case also. All of the theorems stated for passive network synthesis are applicable. Furthermore the literature which is available for passive two-port network synthesis is exhaustive. An extensive list of the properties of the elements of the short-circuit admittance matrix and the open-circuit impedance matrix of an RLC two-port network can be found in most synthesis texts. For example see Chapter 10 of Van Valkenburg (13). In his discussion of the synthesis of active n-port networks, Cruz (4) gives special attention to a summary of two-port theory.

Theorem 4.2.1 is a special case of Theorem 3.3.1 and will be used later in a proof.

Theorem 4.2.1. Necessary and sufficient conditions for the realization of a symmetric second-order real matrix as the short-circuit conductance (open circuit resistance) matrix of a common-ground two-port network containing only resistances are:

1. All main diagonal elements are nonnegative and all off-diagonal elements are nonpositive (all elements are nonnegative).
2. Each of the main diagonal elements is not less than the absolute value of the off-diagonal elements.

The above discussion indicates that for admittance and impedance design all of the structure of Chapter III carries over to two-port design. In addition there is the added convenience of a firmer network synthesis foundation.

4.3 h-parameter Design. Because of the extensive use of h-parameters in transistor work and the associated two-port network theory, an h-parameter design equation is now developed. Blackwell and Grigsby (2) considered h-parameter design briefly and developed a design equation for systems containing no independent drivers. This work is extended to the general case in the text to follow.

Consider a system as shown in Figure 3.2.1 with n set equal to two. This figure is redrawn for convenience as Figure 4.3.1.

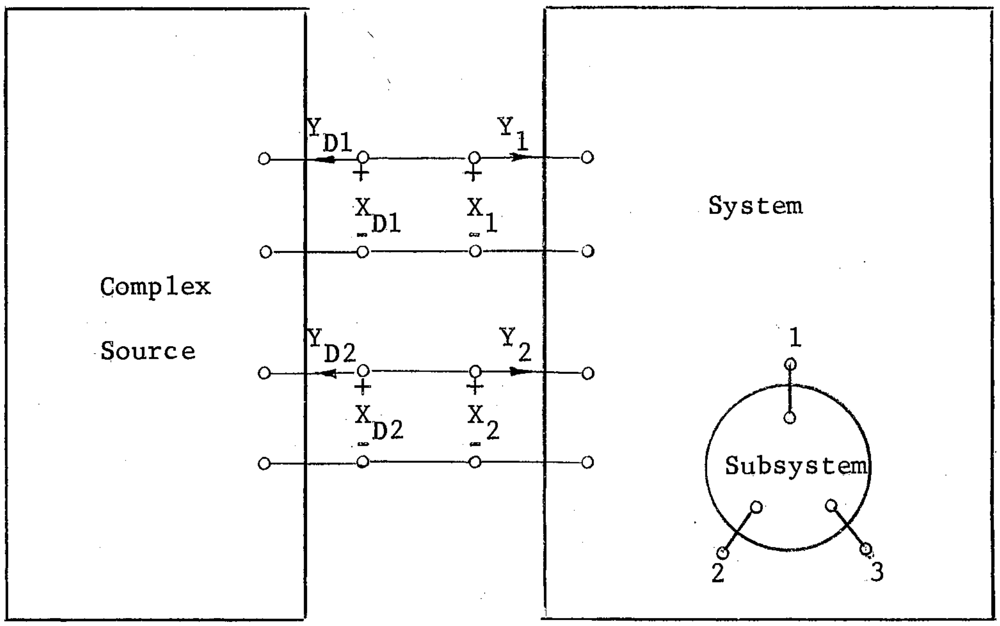

Figure 4.3.1. Two-port System

If Y_1 and X_2 are chosen as the independent variables and a desired terminal relation is assumed, one has

$$\begin{bmatrix} X_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} Y_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} E_1 \\ J_2 \end{bmatrix} \qquad (4.3.1)$$

and since the through variables of the complex source are oriented oppositely to those at the terminals of the system, the terminal characteristics of the complex source are

$$\begin{bmatrix} X_{D1} \\ Y_{D2} \end{bmatrix} = \begin{bmatrix} -h_i & h_r \\ h_f & -h_o \end{bmatrix} \begin{bmatrix} Y_{D1} \\ X_{D2} \end{bmatrix} + \begin{bmatrix} E_1 \\ -J_2 \end{bmatrix} \tag{4.3.2}$$

or more compactly

$$\underline{S}_2 = \underline{H}_1 \underline{S}_1 + \underline{S}_3 \tag{4.3.3}$$

Now in the usual fashion, one can solve for the through and across variables of the N-element subsystem as functions of the double drivers. This result is simply a special case of Equation 3.2.5.

$$\begin{bmatrix} X_{N1} \\ X_{N2} \\ Y_{N1} \\ Y_{N2} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} X_{D1} \\ X_{D2} \\ Y_{D1} \\ Y_{D2} \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} \tag{4.3.4}$$

Rewriting Equation 4.3.4 in a more convenient fashion, one has

$$\begin{bmatrix} X_{N1} \\ Y_{N2} \\ Y_{N1} \\ X_{N2} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{14} & h_{13} & h_{12} \\ h_{41} & h_{44} & h_{43} & h_{42} \\ h_{31} & h_{34} & h_{33} & h_{32} \\ h_{21} & h_{24} & h_{23} & h_{22} \end{bmatrix} \begin{bmatrix} X_{D1} \\ Y_{D2} \\ Y_{D1} \\ X_{D2} \end{bmatrix} + \begin{bmatrix} V_1 \\ I_2 \\ I_1 \\ V_2 \end{bmatrix} \tag{4.3.5}$$

Equation 4.3.5 can be partitioned according to the sets S_1 and S_2 to obtain

$$\begin{bmatrix} \underline{S}_{N2} \\ \underline{S}_{N1} \end{bmatrix} = \begin{bmatrix} \underline{H}_{11} & \underline{H}_{12} \\ \underline{H}_{21} & \underline{H}_{22} \end{bmatrix} \begin{bmatrix} \underline{S}_2 \\ \underline{S}_1 \end{bmatrix} + \begin{bmatrix} \underline{S}_4 \\ \underline{S}_5 \end{bmatrix} \tag{4.3.6}$$

where $\underline{H}_{ij}$ refers to a different submatrix from that referred to in Equation 3.2.6.

Substituting Equation 4.3.3 into Equation 4.3.6 and rewriting, one obtains

$$\underline{S}_{N2} = \underline{H}_{11}(\underline{H}_1\underline{S}_1 + \underline{S}_3) + \underline{H}_{12}\,\underline{S}_1 + \underline{S}_4 \tag{4.3.7}$$

and

$$\underline{S}_{N1} = \underline{H}_{21}(\underline{H}_1\,\underline{S}_1 + \underline{S}_3) + \underline{H}_{22}\,\underline{S}_1 + \underline{S}_5 \tag{4.3.8}$$

Solving Equation 4.3.8 for $\underline{S}_1$ and substituting this result into Equation 4.3.7, one obtains

$$\underline{S}_{N2} = (\underline{H}_{11}\,\underline{H}_1 + \underline{H}_{12})(\underline{H}_{21}\,\underline{H}_1 + \underline{H}_{22})^{-1}(\underline{S}_{N_1} - \underline{H}_{21}\underline{S}_3 - \underline{S}_5) + \underline{H}_{11}\underline{S}_3 + \underline{S}_4 \tag{4.3.9}$$

or

$$\underline{S}_{N2} = (\underline{H}_{11}\,\underline{H}_1 + \underline{H}_{12})(\underline{H}_{21}\,\underline{H}_1 + \underline{H}_{22})^{-1}\underline{S}_{N1} - (\underline{H}_{11}\underline{H}_1 + \underline{H}_{12})(\underline{H}_{21}\underline{H}_1 + \underline{H}_{22})^{-1}$$
$$(\underline{H}_{21}\,\underline{S}_3 + \underline{S}_5) + \underline{H}_{11}\,\underline{S}_3 + \underline{S}_4 \tag{4.3.10}$$

which is the desired design equation in the form

$$\underline{S}_{N2} = \underline{h}_N\,\underline{S}_{N1} + \underline{S}_{N3} \tag{4.3.11}$$

where $\underline{S}_{N3}$ is the contribution due to ideal drivers and $\underline{h}_N$ is the h-parameter matrix of the unspecified two-port. This h-parameter matrix is in the usual form and can be written as

$$\underline{h}_N = \left(\begin{bmatrix} h_{11} & h_{14} \\ h_{41} & h_{44} \end{bmatrix} \begin{bmatrix} -h_i & h_r \\ h_f & -h_o \end{bmatrix} + \begin{bmatrix} h_{13} & h_{12} \\ h_{43} & h_{42} \end{bmatrix} \right) \left(\begin{bmatrix} h_{31} & h_{34} \\ h_{21} & h_{24} \end{bmatrix} \begin{bmatrix} -h_i & h_r \\ h_f & -h_o \end{bmatrix} + \begin{bmatrix} h_{33} & h_{32} \\ h_{23} & h_{22} \end{bmatrix} \right)^{-1} \quad (4.3.12)$$

As pointed out in Chapter III, the ideal driver contribution is zero in a majority of the cases of interest. When $\underline{S}_{N3}$ is zero, the unspecified subsystem is completely determined by Equation 4.3.12.

As previously stated, a recent work considers synthesis of nth-order hybrid matrices (12). Theorems 4.3.1 and 4.3.2 are taken from this work and specialized for the case n equal two. Consider the following properties on the second-order hybrid matrix $[h(s)]$.

Property 4.3.1. $[h]$ is positive real.

Property 4.3.2. $h_{21} = -h_{12}$.

Property 4.3.3. For any choice of complex numbers x_1 and x_2.

$$\text{Re} \sum_{1}^{2} \sum_{1}^{2} h_{ij}\, x_j\, x_i^* = 0, \text{ for } \text{Re}(s) = 0.$$

Theorem 4.3.1. $[h(s)]$ is the hybrid matrix of an RLC, ideal transformer two-port network if and only if it satisfies Properties 4.3.1 and 4.3.2.

Theorem 4.3.2. $[h(s)]$ is the hybrid matrix of an LC, ideal transformer two-port network if and only if it satisfies Properties 4.3.1, 4.3.2 and 4.3.3.

As a direct consequence of Theorem 4.3.1, one can write a theorem for resistive networks.

Theorem 4.3.3. A real second order hybrid matrix is realizable by a two-port network of resistances and ideal transformers if and only if it satisfies Property 4.3.2 and is positive semidefinite.

Since a fairly large class of practical design problems are resistive in nature (mid-band amplifier design for example) it is appropriate to investigate the properties of the hybrid parameters of a grounded two-port resistive network. In particular a test for realizability which can be applied directly to the hybrid matrix is desirable. This test is given as Theorem 4.3.4.

Theorem 4.3.4. A real second-order hybrid matrix $\left[h\right]$ is realizable as a resistive grounded two-port network if and only if

1. h_{11}, h_{12}, and h_{22} are nonnegative.
2. $h_{12} \leqslant 1$.
3. $h_{21} = -h_{12}$.
4. $\det \underline{h} \geqslant h_{12}$.

Proof:

For the necessary conditions consider the relation between the r-parameters and the h-parameters.

$$\underline{h} = \begin{bmatrix} \dfrac{\det \underline{r}}{r_{22}} & \dfrac{r_{12}}{r_{22}} \\ \dfrac{-r_{21}}{r_{22}} & \dfrac{1}{r_{22}} \end{bmatrix} \tag{4.3.13}$$

Conditions 1, 2, and 3 follow directly from Theorem 4.2.1.

To obtain condition 4, expand the determinant of Equation 4.3.13.

$$\det \underline{h} = \frac{1}{r_{22}^2} (\det \underline{r} + r_{12} r_{21})$$

$$= \frac{r_{11}}{r_{22}}$$

By Theorem 4.2.1 $r_{11} \geqslant r_{12}$, hence $\det \underline{h} \geqslant h_{12}$.

For the sufficient conditions consider the inverse of the relation of Equation 4.3.13.

$$\underline{r} = \begin{bmatrix} \frac{\det \underline{h}}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix} \tag{4.3.14}$$

Now show that subject to the conditions of Theorem 4.3.4 this matrix satisfies Theorem 4.2.1 and hence is realizable as a resistive grounded two-port network.

Consider the determinant of $\underline{h}$

$$\begin{aligned} \det \underline{h} &= h_{11}h_{22} - h_{12}h_{21} \\ &= h_{11}h_{22} + h_{12}^2 \end{aligned} \tag{4.3.15}$$

Conditions 1 and 3 in conjunction with Equation 4.3.15 imply that all the elements in the matrix of Equation 4.3.14 are nonnegative and Condition 3 implies symmetry. Conditions 2 and 3 imply $1/h_{22}$ is not less than the off-diagonal elements. Conditions 3 and 4 imply $\frac{\det \underline{h}}{h_{22}}$ is not less than the off-diagonal elements. Hence the matrix of Equation 4.3.14 does satisfy the conditions for realizability and the proof is complete.

Leaving h-parameter synthesis now, the development of a significant result in h-parameter design is given. Suppose that a three-terminal representation of a two-port network is given by Figure 4.3.2 and Equation 4.3.16.

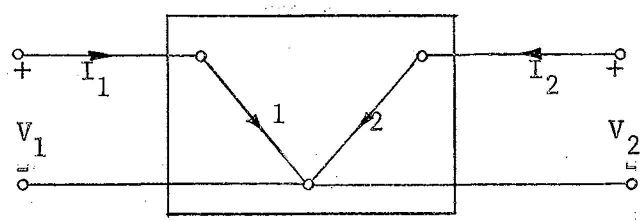

Figure 4.3.2. Three Terminal Network

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{4.3.16}$$

From Equation 4.3.16, one can write

$$\underline{h} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \tag{4.3.17}$$

Now suppose it is desired to change the parameters of the network to those of Equation 4.3.18 by adding an N-element two-port representation as shown in Figure 4.3.3.

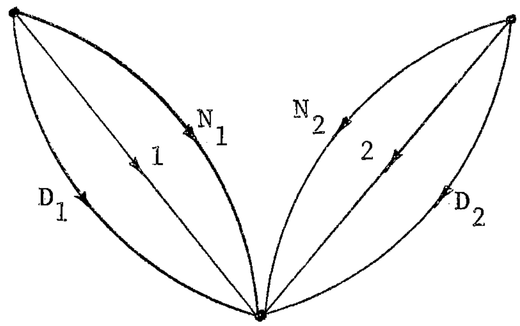

Figure 4.3.3. Graph of Modified Two-port

$$\underline{h}' = \begin{bmatrix} h_i' & h_r' \\ h_f' & h_o' \end{bmatrix} \tag{4.3.18}$$

From Figure 4.3.3, one can write

$$\begin{bmatrix} V_{N1} \\ V_{N2} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{D1} \\ V_{D2} \end{bmatrix} \tag{4.3.19}$$

and

$$\begin{bmatrix} I_{N1} \\ I_{N2} \end{bmatrix} = - \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \begin{bmatrix} I_{D1} \\ I_{D2} \end{bmatrix} \tag{4.3.20}$$

From Equation 4.3.16, one can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{h_i} & \frac{-h_r}{h_i} \\ \frac{h_f}{h_i} & \frac{\det \underline{h}}{h_i} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{4.3.21}$$

Substituting Equations 4.3.19 and 4.3.21 into Equation 4.3.20, one obtains

$$\begin{bmatrix} I_{N1} \\ I_{N2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{h_i} & \frac{h_r}{h_i} \\ \frac{h_f}{h_i} & -\frac{\det \underline{h}}{h_i} \end{bmatrix} \begin{bmatrix} V_{D1} \\ V_{D2} \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} I_{D1} \\ I_{D2} \end{bmatrix} \qquad (4.3.22)$$

From Equations 4.3.19 and 4.3.22, one obtains

$$\begin{bmatrix} V_{N1} \\ I_{N2} \\ I_{N1} \\ V_{N2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-h_f}{h_i} & -1 & 0 & \frac{\det \underline{h}}{h_i} \\ \frac{1}{-h_i} & 0 & -1 & \frac{h_r}{h_i} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{D1} \\ I_{D2} \\ I_{D1} \\ V_{D2} \end{bmatrix} \qquad (4.3.23)$$

When Equation 4.3.23 is partitioned according to Equation 4.3.6 and this result and Equation 4.3.18 are substituted into Equation 4.3.12, the result is

$$\underline{h}_N = \begin{bmatrix} -h_i' & h_r' \\ \frac{h_f h_i'}{h_i} - h_f' & h'_o - \frac{h_f h_r' + \det \underline{h}}{h_i} \end{bmatrix} \begin{bmatrix} \frac{h_i'}{h_i} - 1 & \frac{h_r - h_r'}{h_i} \\ 0 & 1 \end{bmatrix}^{-1} \qquad (4.3.24)$$

or

$$\underline{h}_N = \frac{1}{h_i'-h_i} \begin{bmatrix} -h_i' h_i & h_i'(h_r-h_r') + h_r'(h_i'-h_i) \\ h_f h_i' - h_f' h_i & \frac{(h_r'-h_r)(h_f h_i' - h_f' h_i) + (h_i'-h_i)(h_o' h_i - h_f h_r' - \det \underline{h})}{h_i} \end{bmatrix} \qquad (4.3.25)$$

or more compactly

$$\underline{h}_N = \begin{bmatrix} h_{Ni} & h_{Nr} \\ h_{Nf} & h_{No} \end{bmatrix} \tag{4.3.26}$$

Several observations concerning the result of Equation 4.3.25 are in order. First the inverse in Equation 4.3.24 and consequently $\underline{h}_N$ does not exist if h_i' equals h_i which means that the short-circuit driving-point impedance at port one cannot remain invariant with a connection as shown in Figure 4.3.3. Secondly, although all work has been done in the s-domain, Equation 4.3.25 applies directly to resistive compensation of that class of ω-domain systems for which all of the elements of $\underline{h}$ and $\underline{h}'$ are real numbers. Furthermore, if the elements of $\underline{h}$ and $\underline{h}'$ are nonnegative, the short-circuit driving-point impedance at port one and the short-circuit current gain of the system can only be reduced by resistive N-elements when connected as shown in Figure 4.3.3. These latter two parameters and their variations are of considerable interest in some design problems.

Many other appropriate restrictions can be seen when Equation 4.3.25 is utilized for the particular problem at hand. This is best illustrated by an example.

Example 4.3.1. Consider the grounded-emitter amplifier of Figure 4.3.4. When this amplifier is operated at mid-band the terminal characteristics are

$$\underline{h} = \begin{bmatrix} 2400 & 0.5 \\ 18 & 4 \times 10^{-3} \end{bmatrix} \tag{4.3.27}$$

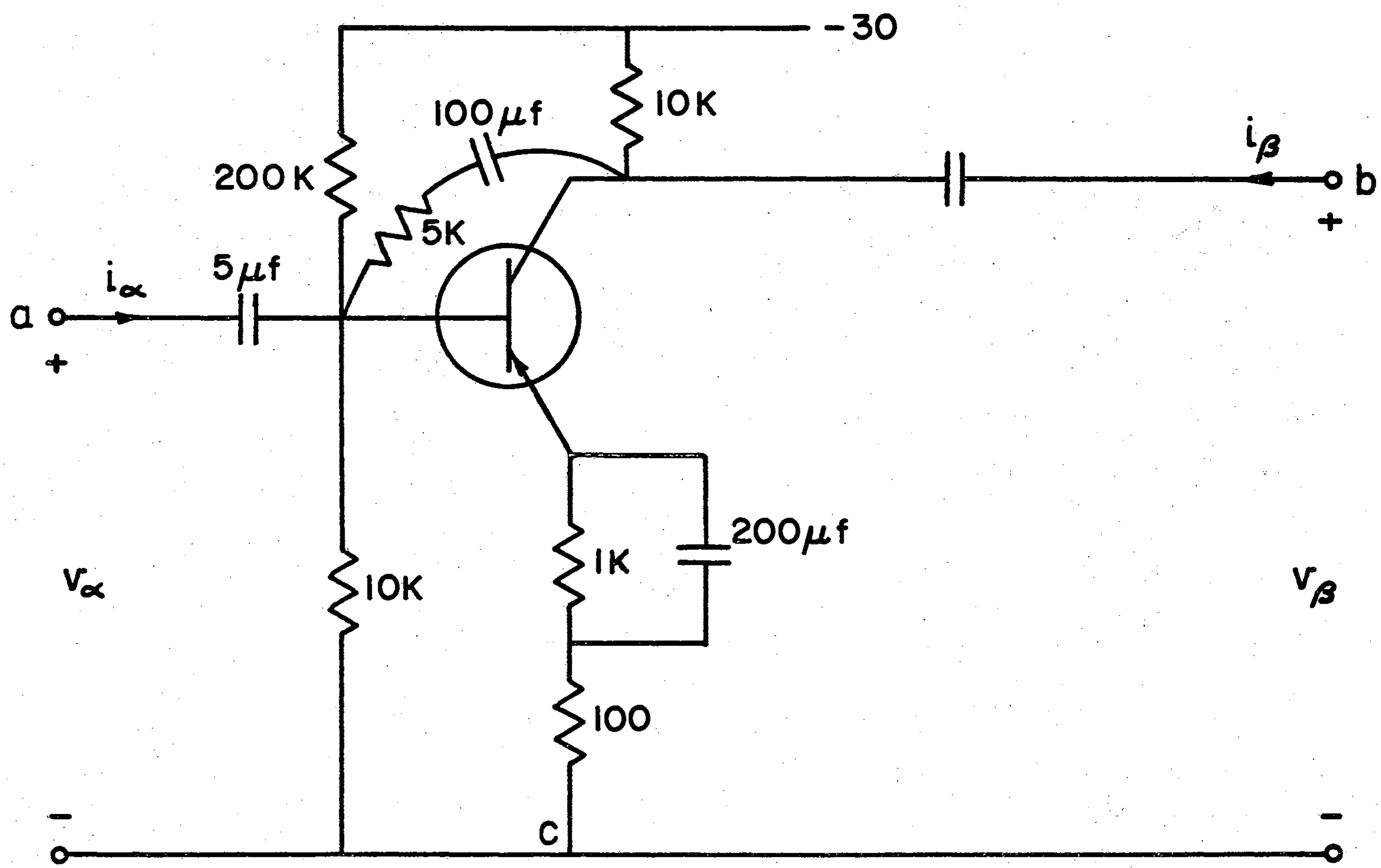

Figure 4.3.4. Transistor Amplifier

The matrix $\underline{h}$ is to be adjusted to some new matrix $\underline{h}'$ by the use of resistive N-elements and Equation 4.3.25.

Since the elements of $\underline{h}'$ are not independent, one can ensure realizability by fixing these elements one at a time. Suppose that one first chooses h_i' to be 2000. Then from Equation 4.3.25, one obtains

$$h_{Ni} = \frac{-(2000)(2400)}{2000-2400} = 12000 \qquad (4.3.28)$$

From Theorem 4.3.4, $-1 \leqslant h_{Nf} \leqslant 0$, for resistive networks. Utilizing this restriction on h_{Nf} and applying Equation 4.3.25, one obtains

$$\frac{h_f\, h_i' - (h_i - h_i')}{h_i} \leqslant h_f' \leqslant \frac{h_i'}{h_i}\, h_f \qquad (4.3.29)$$

or

$$14.833 \leqslant h_f' \leqslant 15 \qquad (4.3.30)$$

as the only realizable values of h_f' for the h_i' which was chosen above.

Suppose h_f' is chosen to be 15. This conveniently makes

$$h_{Nf} = h_{Nr} = 0 \qquad (4.3.31)$$

However h_{Nr} is zero only if h_r' is chosen properly. No freedom now exists in the choice of h_r'. The only allowable value of h_r' is found by setting h_{Nr} of Equation 4.3.25 equal to zero and solving. This yields h_r' equal 0.41667.

Next substituting all known values into the expression for h_{No} of Equation 4.3.25, one obtains

$$h_{No} = h_o' - 3.37 \times 10^{-3} \qquad (4.3.32)$$

which establishes the limit on h_o' since h_{No} must be nonnegative.

Choose h_o' equal 4.37×10^{-3}. Then from Equation 4.3.32,

$$h_{No} = 10^{-3} \tag{4.3.33}$$

The results of this example are

$$\underline{h}' = \begin{bmatrix} 2000 & 0.41667 \\ 15 & 4.37 \times 10^{-3} \end{bmatrix} \tag{4.3.34}$$

when

$$\underline{h}_N = \begin{bmatrix} 12000 & 0 \\ 0 & 10^{-3} \end{bmatrix} \tag{4.3.35}$$

and the matrix of Equation 4.3.35 is readily synthesized as shown in Figure 4.3.5.

One might choose to begin a problem of the above type with a choice for h_f' rather than h_i'. This choice would complicate the problem somewhat but not excessively. The pertinent point is that much time can be saved by avoiding random choices of the elements in $\underline{h}'$.

The design of two-port systems through h-parameters has been considered in detail. It is felt that Equation 4.3.25 will prove to be a significant result in two-port design. Theorem 4.3.4, although simple, presents a result which to the author's knowledge is original.

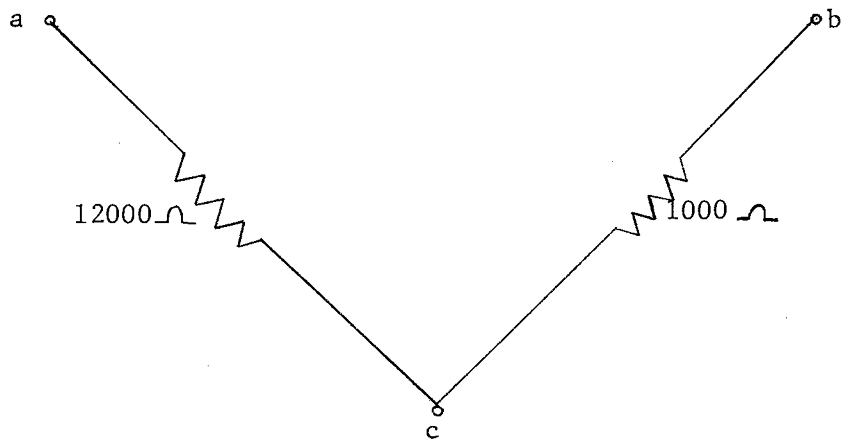

Figure 4.3.5. N-elements for Transistor Amplifier

CHAPTER V

SUMMARY

5.1 Results and Conclusions. The results of this thesis are primarily twofold. First, a theory is developed for the analysis of double-driver systems which contain algebraic elements, energy storage elements and multiterminal components as well as double drivers and the usual ideal drivers. Secondly, an analytical design procedure is demonstrated for an often encountered class of n-port system design problems and the role of n-port network synthesis in this design procedure is established.

The concept of an arbitrary n-port double driver or complex source is introduced and utilized to produce, by analysis techniques, a design equation. The design equation produced in this fashion represents a much more straightforward and time saving approach to the design problem considered than any more conventional approach which might be attempted. The design procedure of this thesis circumvents the difficult step of n-port synthesis required by the determination of the port structure and utilizes that portion of the theory of n-port network synthesis which is most firmly founded. Furthermore the design procedure presented herein allows the designer to attack a problem which is much too complicated to attempt by synthesis techniques alone.

A technique for modifying or compensating an existing n-port system with a known n + 1 multiterminal representation is given. This technique presents equations for the system parameters of the n-port system which is to be connected into the existing system at the n + 1 terminals in order to effect the desired modification of the overall system characteristics. The parameters of the system to be connected are given as functions of the parameters of the existing system and the parameters which are desired for the final resulting system. Since the connection of this technique is basically that of parallel connection of N-elements, this technique can aptly be called parallel modification.

This parallel modification technique allows connection to external terminals only. Modification or compensation by connecting into internal terminals or by varying existing parameters must be approached by the use of the design equations and no general results can be developed. Rather each case presents the designer with a new problem.

The design procedure developed in this thesis suffers from the disadvantage, which is common to most design techniques, that the geometry of the system is assumed. When compensating an existing system however, the designer normally will have the choice of several possible connections for the N-element subsystem. The selection of the realizable connection or the optimum of several realizable connections is then the problem of interest.

5.2 Recommendations for Further Study. It is now appropriate to mention several topics which are areas for further research. First, as introduced above, the development of a procedure for the systematic selection of the topology, according to some optimum criteria, of the N-element subsystem within a system of specified topology is a problem whose solution would be extremely valuable. This problem appears difficult but solvable. Perhaps the use of a digital computer would be appropriate.

A digital computer program for use in the parallel modification technique would also be useful. This program should calculate the multiterminal representation of the existing system and then calculate the terminal parameters of the modification system. Further this program might make some simple realizability tests. A program which provides a partial solution to this problem in the resistive case has been written (14).

The extension of the state model techniques of this thesis to develop a state model design procedure is an interesting problem which appears solvable. This latter problem also points out a very wide area of research which has recently opened up due to the extensive interest in state models. This area of research is the synthesis of state models.

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APPENDIX A

APPENDIX A

s-DOMAIN MESH AND NODAL EQUATIONS FOR DOUBLE-DRIVER SYSTEMS

A.1 Mesh Equations. Assume that a double-driver system containing elements of the type shown in Table I has an impedance matrix. That is assume that Equation 2.3.3 can be written

$$\begin{bmatrix} \underline{X}_X(t) \\ \underline{X}_R(t) \\ \underline{X}_Y(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{p}\underline{D} & 0 & 0 \\ 0 & \underline{R} & 0 \\ 0 & 0 & p\underline{L} \end{bmatrix} \begin{bmatrix} \underline{Y}_X(t) \\ \underline{Y}_R(t) \\ \underline{Y}_Y(t) \end{bmatrix} + \begin{bmatrix} \underline{X}_X(0+) \\ 0 \\ 0 \end{bmatrix} \qquad (A.1.1)$$

where the operational notation

$$p\ y(t) = \frac{d\ y(t)}{dt} \text{ and } \frac{1}{p}\ y(t) = \int_0^t y(u)\ d\ u$$

has been used.

Taking the Laplace transform of Equation A.1.1, one obtains

$$\begin{bmatrix} \underline{X}_X(s) \\ \underline{X}_R(s) \\ \underline{X}_Y(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s}\underline{D} & 0 & 0 \\ 0 & \underline{R} & 0 \\ 0 & 0 & s\underline{L} \end{bmatrix} \begin{bmatrix} \underline{Y}_X(s) \\ \underline{Y}_R(s) \\ \underline{Y}_Y(s) \end{bmatrix} + \begin{bmatrix} \frac{1}{s}\underline{X}_X(0+) \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \underline{L}\,\underline{Y}_Y(0+) \end{bmatrix} \qquad (A.1.2)$$

which can be written more concisely as

$$\underline{X}_Z(s) = Z(s)\underline{Y}_Z(s) + \frac{1}{s}\underline{X}_Z(0+) - \underline{L}_Z\underline{Y}_Z(0+) \qquad (A.1.3)$$

where $\underline{L}_Z$ is the matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \underline{L} \end{bmatrix}$$

From this point on, assume that the functional notation for s is implied.

Choose T_1 and T_2 as formulation trees for the cutset and circuit equations respectively. Furthermore since n_N equals n_D, the number of Z-elements in T_1 equals the number of Z-elements in T_2. Hence for convenience, choose T_1 and T_2 to include the same Z-elements. The cutset and circuit equations are then

$$\begin{bmatrix} \underline{U} & 0 & 0 & \underline{Q}_{14} & \underline{Q}_{15} & \underline{Q}_{16} \\ 0 & \underline{U} & 0 & \underline{Q}_{24} & \underline{Q}_{25} & \underline{Q}_{26} \\ 0 & 0 & \underline{U} & \underline{Q}_{34} & \underline{Q}_{35} & \underline{Q}_{36} \end{bmatrix} \begin{bmatrix} \underline{Y}_E \\ \underline{Y}_N \\ \underline{Y}_{ZT} \\ \underline{Y}_{ZC} \\ \underline{Y}_J \\ \underline{Y}_D \end{bmatrix} = 0 \qquad (A.1.4)$$

and

$$\begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13} & \underline{U} & 0 & 0 \\ \underline{B}_{21} & \underline{B}_{22} & \underline{B}_{23} & 0 & \underline{U} & 0 \\ \underline{B}_{31} & \underline{B}_{32} & \underline{B}_{33} & 0 & 0 & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \\ \underline{X}_{ZT} \\ \underline{X}_{ZC} \\ \underline{X}_J \\ \underline{X}_N \end{bmatrix} = 0 \qquad (A.1.5)$$

From Equations A.1.4 and A.1.5 the non-set aside equations are

$$\begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{X}_{ZT} \\ \underline{X}_{ZC} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} \end{bmatrix} \begin{bmatrix} \underline{X}_E \\ \underline{X}_D \end{bmatrix} \qquad (A.1.6)$$

and

$$\underline{Y}_{ZT} = - \begin{bmatrix} \underline{Q}_{34} & \underline{Q}_{35} & \underline{Q}_{36} \end{bmatrix} \begin{bmatrix} \underline{Y}_{ZC} \\ \underline{Y}_J \\ \underline{Y}_D \end{bmatrix} \qquad (A.1.7)$$

Partitioning Equation A.1.3 according to the tree and cotree elements, one has

$$\begin{bmatrix} \underline{X}_{ZT} \\ \underline{X}_{ZC} \end{bmatrix} = \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} \underline{Y}_{ZT} \\ \underline{Y}_{ZC} \end{bmatrix} + \frac{1}{s} \begin{bmatrix} \underline{X}_{ZT}(0+) \\ \underline{X}_{ZC}(0+) \end{bmatrix} - \begin{bmatrix} \underline{L}_{11} & \underline{L}_{12} \\ \underline{L}_{21} & \underline{L}_{22} \end{bmatrix} \begin{bmatrix} \underline{Y}_{ZT}(0+) \\ \underline{Y}_{ZC}(0+) \end{bmatrix} \qquad (A.1.8)$$

In the interest of more concise notation assume that all initial conditions are zero. Now substitute Equation A.1.7 into Equation A.1.8 to obtain

$$\begin{bmatrix} \underline{X}_{ZT} \\ \underline{X}_{ZC} \end{bmatrix} = \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} -\underline{Q}_{34} & -\underline{Q}_{35} & -\underline{Q}_{36} \\ \underline{U} & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{Y}_{ZC} \\ \underline{Y}_{J} \\ \underline{Y}_{D} \end{bmatrix} \qquad \text{(A.1.9)}$$

Substitute Equation A.1.9 into Equation A.1.6 to obtain

$$\begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} -\underline{Q}_{34} & -\underline{Q}_{35} & -\underline{Q}_{36} \\ \underline{U} & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{Y}_{ZC} \\ \underline{Y}_{J} \\ \underline{Y}_{D} \end{bmatrix} = - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} \end{bmatrix} \begin{bmatrix} \underline{X}_{E} \\ \underline{X}_{D} \end{bmatrix} \qquad \text{(A.1.10)}$$

or

$$\begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} -\underline{Q}_{34} \\ \underline{U} \end{bmatrix} \underline{Y}_{ZC}$$

$$= - \begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} \end{bmatrix} \begin{bmatrix} \underline{X}_{E} \\ \underline{X}_{D} \end{bmatrix} + \begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} \underline{Q}_{35} & \underline{Q}_{36} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{Y}_{J} \\ \underline{Y}_{D} \end{bmatrix} \qquad \text{(A.1.11)}$$

which is the desired mesh formulation.

If the necessary topology of Theorem 2.3.1 is also sufficient, Equation A.1.11 must have a unique solution. Therefore one must first prove the nonsingularity of the mesh impedance matrix

$$\underline{Z}_{m} = \begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} -\underline{Q}_{34} \\ \underline{U} \end{bmatrix} \qquad \text{(A.1.12)}$$

The author has been unable to prove that $\underline{Z}_{m}$ is nonsingular even in a limited sense. However a summary showing some of the efforts and pointing out some of the difficulties encountered is in order.

The attempt which appears to have the most promise is centered about the assumption that the branch impedance matrix of Equation A.1.12 is positive definite and that $\underline{Z}_{12}$ and $\underline{Z}_{21}$ are zero. This latter assumption is not severely restrictive in that it only requires that no coupling exist between the Z-elements in the tree and the Z-elements in the cotree.

Equation A.1.12 can now be rewritten as

$$\underline{Z}_m = \begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & 0 \\ 0 & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} \underline{U} & -\underline{B}_{13}^T - Q_{34} \\ 0 & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{B}_{13}^T \\ \underline{U} \end{bmatrix} \tag{A.1.13}$$

or

$$\underline{Z}_m = \begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & -\underline{Z}_{11}(\underline{B}_{13}^T + \underline{Q}_{34}) \\ 0 & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} \underline{B}_{13} \\ \underline{U} \end{bmatrix}^T \tag{A.1.14}$$

or more concisely

$$\underline{Z}_m = \underline{B}_1 \underline{Z}_1 \underline{B}_1^T \tag{A.1.15}$$

The approach now is to attempt to prove that $\underline{Z}_1$ and hence $\underline{Z}_m$ is positive definite, if it is given that the branch impedance matrix and hence $\underline{Z}_{11}$ and $\underline{Z}_{22}$ is positive definite for some value of s. Unfortunately however, to the author's knowledge, very little is known about the positive definite property as applied to nonsymmetric matrices. Without a very extensive investigation of the positive definiteness of nonsymmetric matrices and the ramifications thereof, it does not seem possible to carry this proof through to a conclusion by this approach. A little thought will convince one that many of the well known properties of positive definite symmetric matrices are not necessarily true when nonsymmetric matrices are considered.

An approach whereby $\underline{Z}_1$ is written as the sum of a symmetric and a skew-symmetric matrix also meets with difficulty. It does not appear possible to prove that the resulting symmetric matrix is positive definite in general.

There is another approach which might yield the desired results. An attempt is made to utilize the orthogonality of the cutset and the circuit matrices and the fact that any two basis circuit matrices are related by a nonsingular transformation in order to write Equation A.1.12 as

$$\underline{Z}_m = \begin{bmatrix} \underline{B}_{13} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} \begin{bmatrix} \underline{B}_{13} \\ \underline{U} \end{bmatrix}^T \begin{bmatrix} \underline{T}_{11} & \underline{T}_{12} \\ \underline{T}_{21} & \underline{T}_{22} \end{bmatrix} \tag{A.1.16}$$

or

$$\underline{Z}_m = \underline{B}_1 \, \underline{Z} \, \underline{B}_1^T \, \underline{T} \tag{A.1.17}$$

which is readily seen to be nonsingular if $\underline{T}$ is a nonsingular transformation matrix and $\underline{Z}$ is a positive definite symmetric branch impedance matrix.

The difficulty with this latter approach seems to be that the resulting form of $\underline{Z}_m$ is similar to Equation A.1.13 with the transformation matrix on the wrong side of $\underline{B}_1^T$.

Brown and Veerkamp (15) have used this latter approach in a proof involving a mesh formulation using two different trees. However the systems which they considered did not contain unspecified elements.

Furthermore an assumption that the system contains no ideal through drivers allows the mesh impedance matrix to be put in the form of Equation A.1.17 very readily.

A.2 Nodal Equations. Assume that a double-driver system containing elements of the type shown in Table I has an admittance matrix. That is assume that it is possible to rewrite Equation 2.3.3 as

$$\begin{bmatrix} \underline{Y}(t)_X \\ \underline{Y}(t)_R \\ \underline{Y}(t)_Y \end{bmatrix} = \begin{bmatrix} p\,\underline{C} & 0 & 0 \\ 0 & \underline{G} & 0 \\ 0 & 0 & \frac{1}{p}\underline{K} \end{bmatrix} \begin{bmatrix} \underline{X}(t)_X \\ \underline{X}(t)_R \\ \underline{X}(t)_Y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \underline{X}(0+)_Y \end{bmatrix} \qquad \text{(A.2.1)}$$

where the p operator notation has again been used.

Taking the Laplace transform of Equation A.2.1, one obtains

$$\begin{bmatrix} \underline{Y}(s)_X \\ \underline{Y}(s)_R \\ \underline{Y}(s)_Y \end{bmatrix} = \begin{bmatrix} s\,\underline{C} & 0 & 0 \\ 0 & \underline{G} & 0 \\ 0 & 0 & \frac{1}{s}\underline{K} \end{bmatrix} \begin{bmatrix} \underline{X}(s)_X \\ \underline{X}(s)_R \\ \underline{X}(s)_Y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{s}\underline{X}(0+)_Y \end{bmatrix} - \begin{bmatrix} \underline{C}\,\underline{X}(0+)_X \\ 0 \\ 0 \end{bmatrix} \qquad \text{(A.2.2)}$$

In a manner similar to the mesh development assume zero initial conditions, implied functional notation, and tree-cotree partitioning of Equation A.2.2 to obtain

$$\begin{bmatrix} \underline{Y}_{ZT} \\ \underline{Y}_{ZC} \end{bmatrix} = \begin{bmatrix} \underline{W}_{11} & \underline{W}_{12} \\ \underline{W}_{21} & \underline{W}_{22} \end{bmatrix} \begin{bmatrix} \underline{X}_{ZT} \\ \underline{X}_{ZC} \end{bmatrix} \qquad \text{(A.2.3)}$$

Rewriting Equations A.1.6 and A.1.7 in a manner suitable for the usual node formulation substitutions, one has

$$\underline{X}_{ZC} = -\begin{bmatrix}\underline{B}_{11} & \underline{B}_{12} & \underline{B}_{13}\end{bmatrix}\begin{bmatrix}\underline{X}_E \\ \underline{X}_D \\ \underline{X}_{ZT}\end{bmatrix} \tag{A.2.4}$$

and

$$\begin{bmatrix}\underline{U} & \underline{Q}_{34}\end{bmatrix}\begin{bmatrix}\underline{Y}_{ZT} \\ \underline{Y}_{ZC}\end{bmatrix} = -\begin{bmatrix}\underline{Q}_{35} & \underline{Q}_{36}\end{bmatrix}\begin{bmatrix}\underline{Y}_J \\ \underline{Y}_D\end{bmatrix} \tag{A.2.5}$$

Substituting Equation A.2.4 into Equation A.2.3 and this result into Equation A.2.5, one obtains

$$\begin{aligned}&\begin{bmatrix}\underline{U} & \underline{Q}_{34}\end{bmatrix}\begin{bmatrix}\underline{W}_{11} & \underline{W}_{12} \\ \underline{W}_{21} & \underline{W}_{22}\end{bmatrix}\begin{bmatrix}\underline{U} \\ -\underline{B}_{13}\end{bmatrix}\underline{X}_{ZT} \\ &= -\begin{bmatrix}\underline{Q}_{35} & \underline{Q}_{36}\end{bmatrix}\begin{bmatrix}\underline{Y}_J \\ \underline{Y}_D\end{bmatrix} + \begin{bmatrix}\underline{U} & \underline{Q}_{34}\end{bmatrix}\begin{bmatrix}\underline{W}_{11} & \underline{W}_{12} \\ \underline{W}_{21} & \underline{W}_{22}\end{bmatrix}\begin{bmatrix}0 & 0 \\ \underline{B}_{11} & \underline{B}_{12}\end{bmatrix}\begin{bmatrix}\underline{X}_E \\ \underline{X}_D\end{bmatrix}\end{aligned} \tag{A.2.6}$$

which is the desired nodal formulation.

Of course any attempt to prove the sufficiency of the conditions of Theorem 2.3.1 by use of the nodal formulation experiences the exact same difficulties as the mesh approach.

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