SYNTHESIS OF AN

ACTIVE TWO-PORT

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 Goal. Most of the work in synthesis has been in the area of two-ports containing linear, lumped, finite, passive and bilateral elements. The purpose of this work is to extend the scope of knowledge such that the terms passive and bilateral may be removed. That is, it is desirable to synthesize system functions that result from two-ports including active devices. To achieve this goal it is necessary to define and classify the active devices. Although the pole-zero structure of the passive two-port y and z parameters is well known from passive synthesis, the pole-zero structure of (i) the passive g and h parameters and (ii) the parameters of two-ports containing active devices must be determined. Further, the system functions, that is, the driving point and the transfer functions, that result from combining active and passive two-ports must be determined. The goal of this work is to achieve the above parameter investigation and then develop a synthesis procedure for the system functions. Further, it is desired to reduce the problem to a problem in passive synthesis where extensive work has been done.

1.2 Past and Present Synthesis Philosophy. Initially, network synthesis was concerned with the problem of synthesizing a driving point or transfer function of a passive two-port. For the past several years the work has been extended in two directions. An effort has been made to extend the theory to include n-ports which contain only resistors, inductors, capacitors and transformers, that is RLCM networks (1),(2). The other effort has been directed toward the synthesis of two-ports containing active devices as well as RLCM networks (3). The active devices allowed, however, were negative impedance converters and gyrators. The problem here is that these devices are complex and difficult to achieve in actual practice and therefore, have not gained wide acceptance. Their use and value in synthesis is that negative resistors, capacitors and inductors can be achieved. This simplifies many synthesis techniques. However, the difficult problem of synthesis with such active devices as transistors, tubes and field effect transistors, per se, still exists. Therefore, this work is in the synthesis area where the term active two-port refers to a two-port containing a transistor, tube or field effect transistor. The parameters of the active two-port are, primarily, the parameters of the active devices.

If the negative impedance two-ports are not to be allowed, then other characteristics must be employed. In this work the unsymmetrical and non-reciprocal characteristics of the allowed active devices are exploited in order to synthesize a system function. Further, the activeness of the device is used. Moreover, it seems that most active devices of the future will also exhibit the unsymmetrical and non-reciprocal nature of the devices employed today. Hence, the synthesis procedure developed in this work may also be applied to these future devices.

The difficulty of synthesis with active devices without modification is that the algebra becomes difficult. That is, the pole-zero structure of the parameters and the system functions becomes both difficult to determine and to follow as interconnections of the two-ports are made. The application of the root-locus techniques, which have solved numerous problems in the servo field, minimized this problem exceedingly well. In Chapters III and IV root-locus theory is used to prove theorems that would otherwise be algebraically difficult to prove. Further, this technique provides valuable insight into the variation of the pole-zero structure as magnitude factors and parameters are varied. The application of root-locus techniques quickly determines the feasibility of certain networks to perform as specified.

The synthesis of general RLC system functions relies upon a balanced bridge to obtain the desired characteristics. One such method as the Bott-Duffin algebraic algorithm gives exact component values that must be achieved exactly or the bridge balance does not exist and the specifications are not met. It seems desirable to make a trade off between exact mathematical algorithms and networks that will tolerate component variation. This has been done in this work. That is, the synthesis procedure does not specify the component value as exactly as an algorithm does, but the finished networks does not depend upon a balanced bridge. In fact, it seems desirable to develop procedures that exhibit tolerance of the same order of magnitude as that of the components.

In particular, an objective of this work is to achieve the synthesis of RLC functions using only RC networks and active devices. This was accomplished by using root-locus theory. Finally, if an RLC system function and an active device is the input to the procedure, the output is a specified passive RC two-port. The problem is considered to be solved at this point because it has been reduced to a passive synthesis problem.

Chapters II and III are concerned with the classification of active two-ports, the relationships of the various parameter sets and the pole-zero structure of the composite two-port parameters. Existing passive two-port theory was drawn upon extensively whenever possible. However, certain essential unknown relationships between the various passive parameters were derived and proven in order to make later theorems as general as possible. The actual synthesis procedures are developed in Chapters IV and V. These synthesis procedures do not rely upon the use of transformers.

CHAPTER II

TWO-PORTS

2.1 Introduction. This chapter, in general, discusses the broad definitions of passive, active and potentially unstable two-ports and the necessary interconnections leading to the required synthesis in latter chapters. In particular, the active two-port used in this work is defined in terms of the active device. The two-ports considered are linear and lumped; hence, the two-port parameters are rational functions.

2.2 Classification of Two-ports. Two-ports are broken down into two groups, active and passive. A representation of a two-port is shown in Figure 2.2.1 along with the voltage and current sign convention.

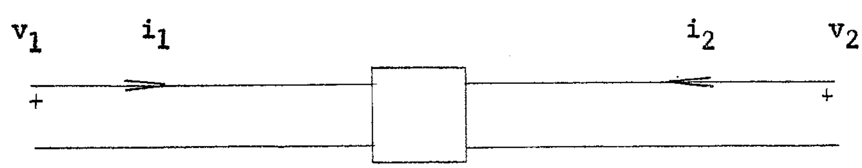


Figure 2.2.1. Two-port Voltage and Current Convention.

The two-ports, normally of interest in communication circuits, are degraded in that a common terminal exist between the two-ports. This constraint is important because these circuits in most environments will be subjected to spurious electromagnetic radiation which will be of the same order of magnitude as the signal and operated on by the two-port in the same manner as the signal, unless one terminal of all ports is maintained at ground potential. This is a real problem that must be considered in both active and passive circuits.

Using the voltage and current conventions indicated in Figure 2.2.1 a non-active or passive two-port is defined by

$$\mathrm{Re}\ (i_1 v_1^* + i_2 v_2^*) \geq 0. \qquad (2.2.1)$$

Or in matrix notation

$$\mathrm{Re}\left(\begin{bmatrix} v_1^* v_2^* \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \right) \geq 0. \qquad (2.2.2)$$

That is, the two-port is not capable of power gain at any frequency or for any termination. In this work the two-ports of interest which may be either active or passive have parameters that are frequency independent, unless explicitly stated otherwise, and hence, Equation 2.2.2 becomes

$$\begin{bmatrix} v_1 v_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \geq 0.$$

Using y-parameters the relationship between the voltages and currents for the two-port is

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{ij} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} , \qquad (2.2.4)$$

where

$$\begin{bmatrix} y_{ij} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} .$$

In general $\begin{bmatrix} y_{ij} \end{bmatrix}$ will not be from a reciprocal two-port, that is, $y_{12} \neq y_{21}$. However, $\begin{bmatrix} y_{ij} \end{bmatrix}$ may be written as the sum of a symmetric and skew-symmetric matrix (4), such as

$$\begin{bmatrix} y_{11} & \frac{y_{12} + y_{21}}{2} \\ \frac{y_{21} + y_{12}}{2} & y_{22} \end{bmatrix} + \begin{bmatrix} 0 & \frac{y_{12} - y_{21}}{2} \\ \frac{y_{21} - y_{12}}{2} & 0 \end{bmatrix} . \quad (2.2.5)$$

Substituting Equation 2.2.5 into Equation 2.2.4 and putting the results into Equation 2.2.3 gives,

$$\begin{bmatrix} v_1 v_2 \end{bmatrix} \begin{bmatrix} y_{11} & \frac{y_{12} + y_{21}}{2} \\ \frac{y_{21} + y_{12}}{2} & y_{22} \end{bmatrix} \begin{bmatrix} v_i \\ v_2 \end{bmatrix} + \quad (2.2.6)$$

$$\begin{bmatrix} v_1 v_2 \end{bmatrix} \begin{bmatrix} 0 & \frac{y_{12} - y_{21}}{2} \\ \frac{y_{21} - y_{12}}{2} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \geq 0.$$

Equation 2.2.6 will be true for all values of v_1 and v_2 if and only if the first matrix of Equation 2.2.6 is positive definite since the last matrix contributes nothing to the quadratic form in v_1 and v_2. The necessary and sufficient conditions for the matrix to be positive definite are given by Equation 2.2.7.a, Equation 2.2.7.b, and Equation 2.2.7.c.

$$y_{11} > 0. \quad (2.2.7.a)$$

$$y_{22} > 0. \quad (2.2.7.b)$$

$$y_{11}\, y_{22} - \frac{(y_{12} + y_{21})^2}{4} > 0. \quad (2.2.7.c)$$

Definition 2.2.1. A two-port is said to be passive if its y-parameters satisfy Equation 2.2.7.

Definition 2.2.2. A two-port is said to be active if its y-parameters violate any of the conditions of Equation 2.2.7.

The two-ports containing only RLCM elements always satisfy Equation 2.2.1, and hence, Equation 2.2.7. Further, they always are unconditionally passive. This type of two-port is used to introduce the frequency dependent nature of the overall two-port when interconnected with an active two-port that is frequency independent.

Definition 2.2.3. The degree of activeness, A, is defined to be

$$A = \frac{(y_{12} + y_{21})^2}{4y_{11}\, y_{22}} \quad . \tag{2.2.8}$$

Therefore a two-port is active if A is greater than one, whereas it is lossless if A is equal to unity and it is passive if A is less than one. That is, if A is less than one no power gain is possible. The size of A is an indication of how active the two-port is. Normally the parameter matrix of the active two-port will be the parameters of the active device without any external modification. However, an example will be given to show how a passive two-port may contain an active device. Example 2.2.1. Typical y-parameters for a transistor connected in the common emitter configuration are

$$y^a_{11} = 1 \times 10^{-3} = k_{11}, \tag{2.2.9.a}$$

$$y^a_{12} = -2 \times 10^{-6} = -k_{12}, \tag{2.2.9.b}$$

$$y^a_{21} = 1 \times 10^{-2} = k_{21}, \tag{2.2.9.c}$$

$$y^a_{22} = 1 \times 10^{-5} = k_{22} \; . \tag{2.2.9.d}$$

Its degree of activeness is given by Equation 2.2.8.

$$A \approx 2500.$$

If a feedback conductance, G_f is connected from the base to the collector the overall y-parameters become

$$y_{11} = y_{11}^a + G_f ,$$
$$y_{12} = y_{12}^a - G_f ,$$
$$y_{21} = y_{21}^a - G_f ,$$
$$y_{22} = y_{22}^a + G_f .$$

The expression for A is then

$$A = \frac{\left[(-2 \times 10^{-6} - G_f) \quad (1 \times 10^{-2} - G_f)\right]^2}{4\left[(1 \times 10^{-3} + G_f) \quad (1 \times 10^{-5} + G_f)\right]} . \tag{2.2.10}$$

If $G_f \gg y_{21}^a$, then Equation 2.2.10 becomes

$$A \approx \frac{G_f^4}{4G_f^2} = \frac{G_f^2}{4} .$$

For $G_f^2 < 4$ the two-port is passive although it contains an active element. This is now a conditionally passive two-port.

The classification of active two-ports is considered next. A two-port is said to be unconditionally stable or absolutely stable if

$$2 \operatorname{Re} (y_{11}(j\omega)) \operatorname{Re}(y_{22}(j\omega)) - \operatorname{Re}(y_{12}(j\omega)y_{21}(j\omega)) > \left|y_{12} (j\omega) y_{21}(j\omega)\right| . \tag{2.2.11}$$

and if Equation 2.2.11 fails, the two-port is potentially unstable (5). This condition guarantees that the $\operatorname{Re}(Y_{IN})$ of the terminated two-port will be greater than zero for all terminations at the specified frequency. If the two-port parameters are independent of frequency then the condition for unconditioned stability is

$$y_{11}\, y_{22} - y_{12}\, y_{21} > 0. \tag{2.2.12}$$

The active two-ports of interest here are those containing active devices such that Equation 2.2.7.c is violated but the activeness is such that Equation 2.2.11 is satisfied. However, after combining the unconditional passive two-port, $\left[y^{p}_{ij} \right]$, with the active but absolutely stable two-port, $\left[y^{a}_{ij} \right]$ the resulting two-port, $\left[y_{ij} \right]$, may be conditionally passive, as Example 2.2.1 illustrates, it may be active and absolutely stable or it may be potentially unstable. An example of the latter case is now given using the parameters of the active device in Example 2.2.1.

Using the device parameters of Equation 2.2.9 and analyzing the composite two-port obtained by connecting a capacitor from input to output the Equation 2.2.11 for stability becomes

$$\frac{2k_{11}\, k_{22} - \mathrm{Re}\left[-(k_{12} + j\omega c)\,(k_{21} - j\omega c)\right]}{\left|(k_{12} + j\omega c)\,(k_{21} - j\omega c)\right|} \geq 1. \tag{2.2.13}$$

Taking the real part of Equation 2.2.13 gives

$$\frac{2k_{11}\, k_{22} + k_{12}\, k_{21} + 2c^{2}}{\left[(k_{12}\, k_{21} + \omega^{2}c^{2})^{2} + (\omega c(k_{21} - k_{12}))^{2} \right]^{\frac{1}{2}}} \geq 1. \tag{2.2.14}$$

For instability the conditions on ωc are obtained by solving Equation 2.2.14 giving

$$\omega c \geq \left[\frac{(2k_{11}\, k_{22})^{2} + 4k_{11}\, k_{22}\, k_{21}\, k_{12}}{4k_{11}\, k_{22}} \right]^{\frac{1}{2}}.$$

Substituting numerical values for the k-parameters gives

$$\omega c \geq \left[2 \right]^{\frac{1}{2}} \times 10^{-4} \tag{2.2.15}$$

for a potentially unstable two-port. Or in otherwords, $Re(Y_{IN})$ of the resulting two-port will be negative for certain terminations. It is well to note that a potentially unstable two-port does not necessarily lead to an unstable system. If the resistance of the generator that is driving the resulting two-port is larger than $1/Re(Y_{IN})$ then the overall system is stable(6). Further, note that it was necessary to test the resulting two-port with Equation 2.2.11 since the resulting parameters were frequency dependent.

This leads to the definition of the initial active two-port of interest in this work and furthermore those that are encountered most often in practice.

Definition 2.2.4. A type I active two-port is a two-port that (i) satisfies Equation 2.2.7-a, Equation 2.2.7-b and Equation 2.2.12 but violates Equation 2.2.7-c. (ii) It is frequency independent. (iii) It is non-symmetrical and non-reciprocal.

This rules out such devices as tunnel diodes, unijunction transistors or any combination of devices such that y_{11} and y_{22} are negative. This definition includes such devices as the vacuum-tube, transistor and field-effect transistor. Except when explicitly stated otherwise an active two-port will be type I.

2.3 Topological Considerations. The interconnections that will allow the parameters of the passive and active two-ports to simply add will be discussed next. Also, the restrictions on the absolutely passive two-ports, imposed by the requirement of a common terminal between the two-ports, and the resulting topology will be considered.

Consider Figure 2.3.1. Using the y-parameters the relationships between the currents and voltages of the active and passive two-ports are respectively,

$$\begin{bmatrix} i_1^a \\ i_2^a \end{bmatrix} = \begin{bmatrix} y_{11}^a & y_{12}^a \\ y_{21}^a & y_{22}^a \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} , \qquad (2.3.1)$$

$$\begin{bmatrix} i_1^p \\ i_2^p \end{bmatrix} = \begin{bmatrix} y_{11}^p & y_{12}^p \\ y_{21}^p & y_{22}^p \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} . \qquad (2.3.2)$$

If in making the connections shown in Figure 2.3.1 the connections did not alter the original active and passive two-ports such that $\left[y_{ij}^a\right]$ and $\left[y_{ij}^p\right]$ change, then these matrices will continue to represent the individual two-ports after the inter-connection occurs.

From Figure 2.3.1

$$i_1 = i_1^a + i_1^p , \qquad (2.3.3.a)$$

$$i_2 = i_2^a + i_2^p . \qquad (2.3.3.b)$$

In terms of the y-parameters the relationship between i_1, i_2 and v_1, v_2 is

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} . \qquad (2.3.4)$$

Substituting Equation 2.3.1 and Equation 2.3.2 into Equation 2.3.3 and then inserting Equation 2.3.3 into Equation 2.3.4 gives

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1^a + i_1^p \\ i_2^a + i_2^p \end{bmatrix} = \begin{bmatrix} (y_{11}^a + y_{11}^p) & (y_{12}^a + y_{21}^p) \\ (y_{21}^a + y_{21}^p) & (y_{22}^a + y_{22}^p) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} . \qquad (2.3.5)$$

Or in other words, under the conditions described above the resulting y-parameters are the sum of the active and passive parameters.

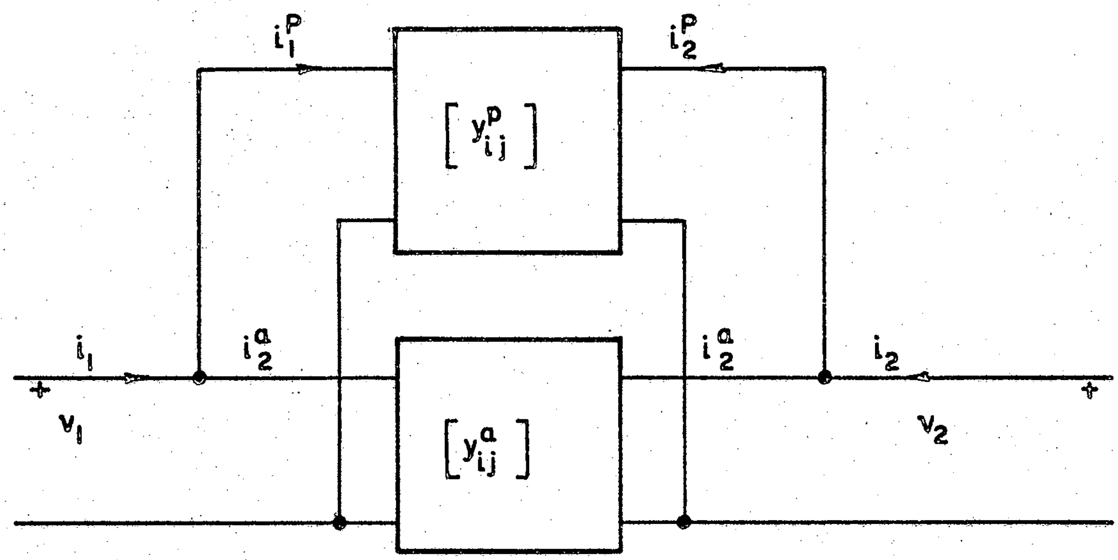


Figure 2.3.1. Two-ports Connected in Shunt-shunt.

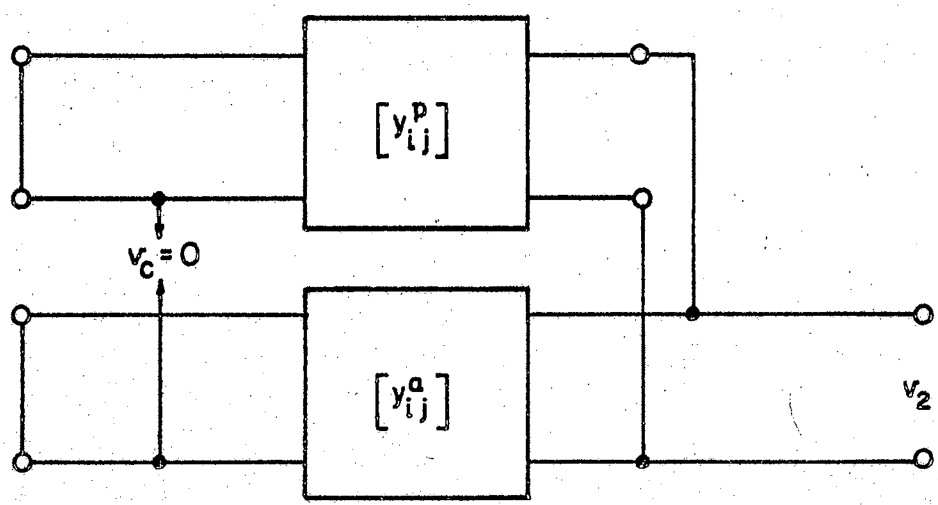


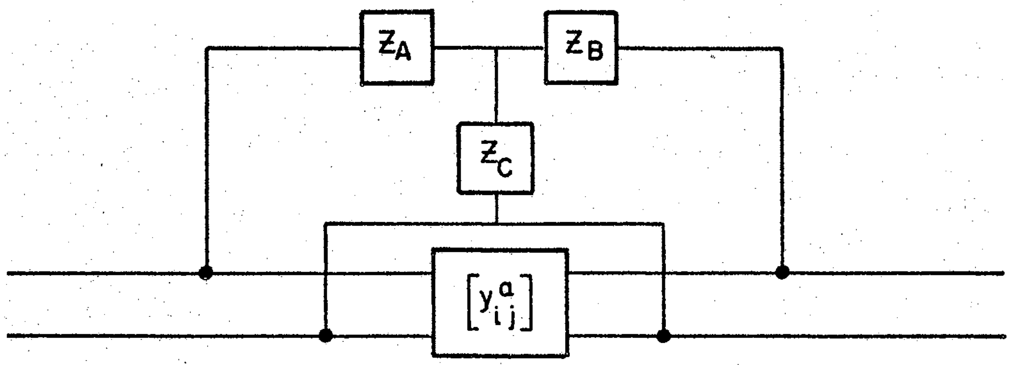
Figure 2.3.2. Validity Test for y_{12} and y_{22} for a Shunt-shunt Interconnection; v_c must be Zero.

The test to determine if the interconnections affect the original parameters is as follows. Connect the input ports together as desired and, depending upon which set of parameters are appropriate, either short circuit or open circuit the out-put ports. After applying a voltage v_2 determine if the voltage, v_c, between the points of the two-ports to be connected is zero. If it is, connecting the two-ports will not affect the original parameter. Figure 2.3.2. illustrates the test for y_{22} and y_{12}.

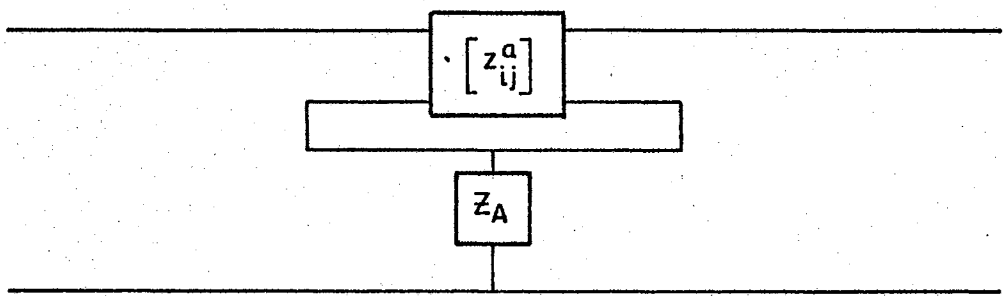
If the above tests are applied with positive results and if suitable topological arrangements are made then the z, h, and g parameters also add. That is, if the z-parameters are used, then a series-series connection is necessary, if the h-parameters are used then a series-shunt connection is necessary, while if the g-parameters are used a shunt-series connection is necessary.

Figure 2.3.3 shows the allowable topology for the y and z configurations. The requirement of a common terminal and the requirement that the parameters are invariant under the two-port interconnections are such that no useful g or h composite two-port results. Note that only a limited z-parameter passive two-port arrangement is possible, but that the most general common terminal is valid for the y-parameters. However, if transformers are allowed, as demonstrated for the h-parameters in Figure 2.3.4 , any configuration of a passive three terminal two-port is allowed for all parameter sets. Further, if the common terminal constraint is removed, then the addition of parameters is valid for the z-parameters and the most general three terminal network may be used. See Figure 2.3.5.

A general representation of interconnected passive and active two-



(a). A shunt-shunt connection for y-parameters.



(b). Series-series connection for z-parameters.

Figure 2.3.3. Valid Connections Between an Active and Passive Two-port for Parameters to Add.

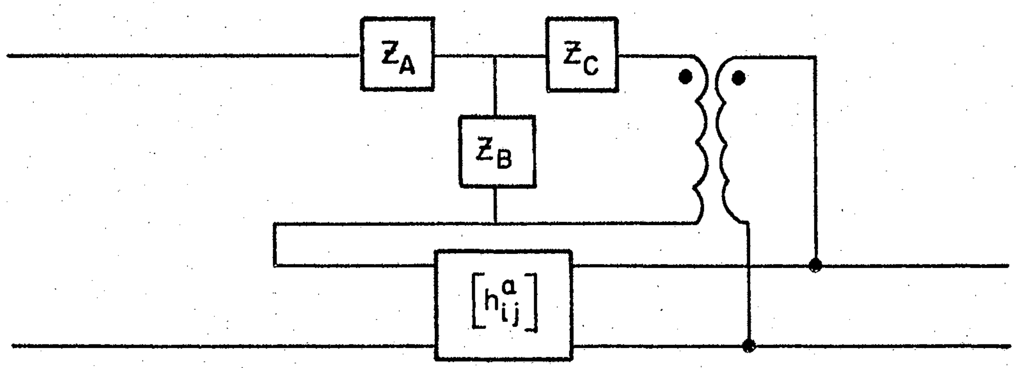


Figure 2.3.4. Valid Interconnection Between Two-ports Using a Transformer such that the h-parameters Add and a Common Ground Exist Between Input and Out-put.

ports is shown in Figure 2.3.6.a. The resulting two-port is shown in Figure 2.3.6.b. An expression for the generalized two-port parameter is

$$\lambda_{ij} = \lambda^{p}_{ij} + \lambda^{a}_{ij} \quad . \tag{2.3.6}$$

The interconnections in boxes one and two are shunt or series depending upon which set of parameters is used.

Because in most passive synthesis studies transformers are allowed and because one goal of this work is to extend the knowledge of two-port parameters such that active two-ports are included, the necessary realizibility conditions to be derived in Chapter III are determined in terms of the generalized two-port parameters. Hence, for the g and h parameters these conditions are valid and the interconnections of Figure 2.3.6 are valid only if transformers are used. However, one goal of this work is to achieve the actual synthesis of system functions without using transformers. Therefore the actual synthesis of driving point and transfer functions will be done in terms of y-parameters in order to circumvent the need for transformers. And yet, the most general three terminal passive network configuration may be used.

2.4 Relationships Between the Two-port Parameters. The relationship between the four sets of parameters y, z, h and g are discussed next and similarities in their pole-zero plots are shown. The pole-zero plots of the y and z parameters are well known from passive two-port synthesis; therefore, the pole-zero plots of the g and h parameters may be determined by relating the g and h parameters to the y and z parameters. Table I shows the interrelationships.

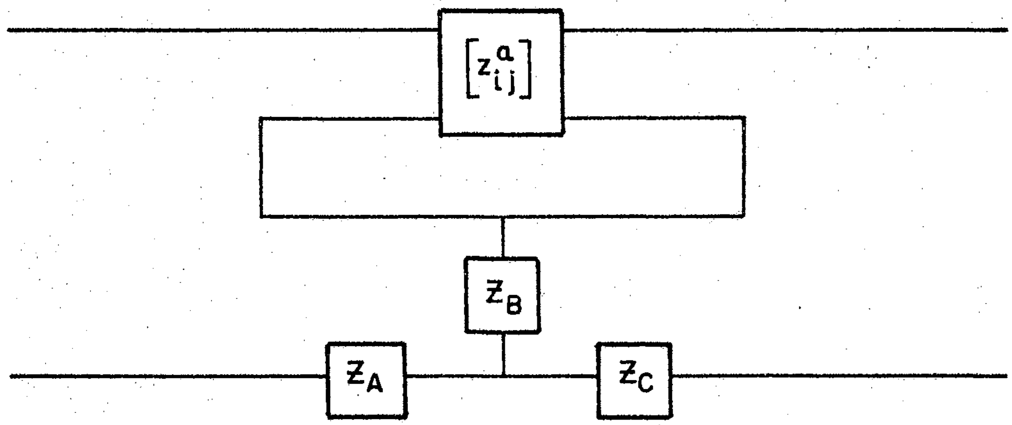


Figure 2.3.5. Valid Topology for the z-parameters to Add if the Common Terminal Requirement is Removed.

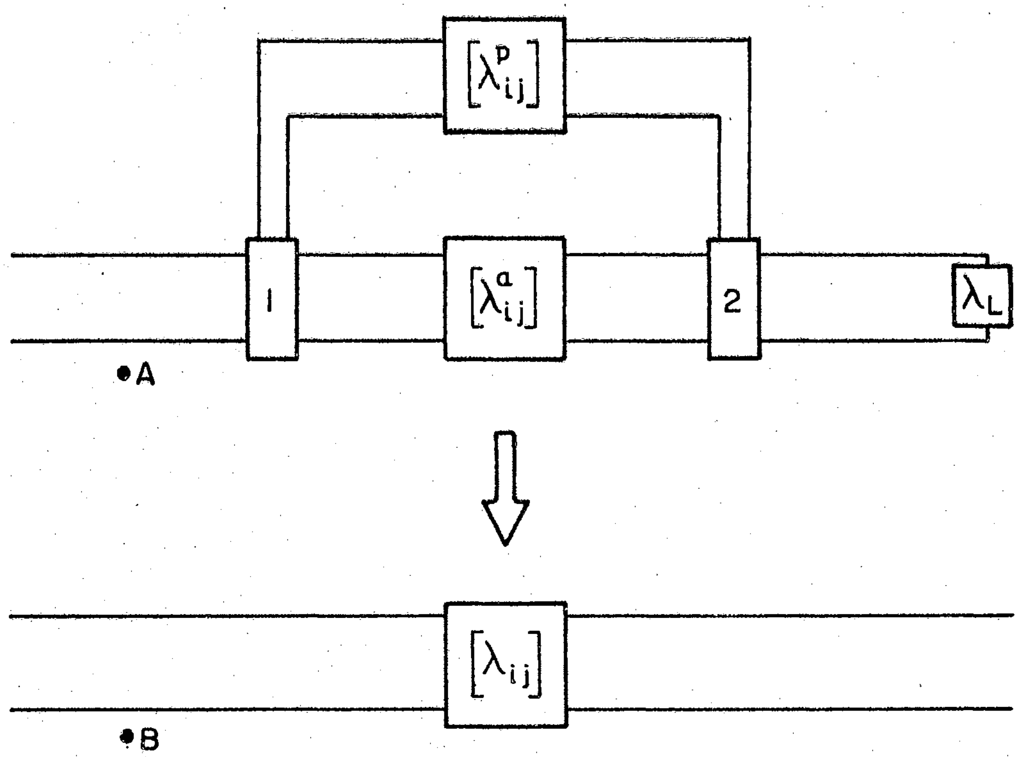


Figure 2.3.6. Generalized Interconnected Two-port.

TABLE I

PARAMETER RELATIONSHIPS

$h_{11} = 1/y_{11}$	$g_{11} = 1/z_{11}$
$h_{12} = -y_{12}/y_{11}$	$g_{12} = -z_{12}/z_{11}$
$h_{21} = y_{21}/y_{11}$	$g_{21} = z_{21}/z_{11}$
$h_{22} = 1/z_{22}$	$g_{22} = 1/y_{22}$

As an example of the information that may be gleamed from Table I, consider y_{11} and h_{11}. It is known that the poles and zeros of y_{11} are interlaced on the negative real axis and that the one nearest the origin, if not at the origin, is a zero(7). The number of poles, one of which may be at infinity, exactly equals the number of zeros. Because h_{11} is the reciprocal of y_{11} it follows that the number of zeros, one of which may be at infinity, exactly equals the number of poles, one of which may be at zero. The poles and zeros are interlaced on the negative real axis. Note, the description of h_{11} exactly fits the pole-zero pattern for a general z_{11}. In fact, for the passive two-port with a shorted out-put, the impedance seen looking into the input is exactly h_{11}. The specific two-port z_{11} is determined, however, with the out-put open-circuited and is not equal to the specific h_{11}, but the pole-zero pattern of a general z_{11} and h_{11} is identical.

Further conclusions can be drawn from Table I. For example h_{12} and g_{21} as defined in Table I are the open circuit voltage gain from the output port to the input port and the forward open circuit voltage gain of the two-port respectively. The characteristics of these two gain functions are well known from passive two-port theory. In particular, it is known, that the numerator cannot exceed the degree of the denominator(7); hence, the degree of the numerator of h_{12} and g_{21} cannot exceed the degree of the denominator. By relating h_{21} and g_{12} to the short circuit current gain functions it can be stated that the degree of the numerator of h_{21} and g_{21} cannot exceed the degree of the denominator. This is in contrast to the off-diagonal terms of the y and z parameters whose numerator degree may exceed the denominator degree by one.

CHAPTER III

GENERALIZED PARAMETERS

3.1 Introduction. In this chapter theorems are proven concerning the generalized two-port parameter, λ_{ij}, which resulted from the topology specified in Chapter II. The relationship between the general passive λ_{ij}^{p} pole-zero and the λ_{ij} resulting from combining an active and a passive two-port is developed. The relationship between the pole-zero plot of the parameter of the particular passive two-port used and the pole-zero plot of the resulting λ_{ij} is also determined. For the g, h and z parameters, the theorems hold if transformers are allowed and a common terminal is required whereas the theorems are true for the y-parameters without transformers. Further the theorems are also true for the z-parameters without transformers if a common terminal is omitted. The restriction on the passive two-port is that it contain only RC or RL elements.

3.2 Necessary Condition of λ_{11} and λ_{22}. Given that

$$\lambda_{ij} = \lambda_{ij}^{p} + \lambda_{ij}^{a} \quad , \tag{3.2.1}$$

then $$\lambda_{11} = \lambda_{11}^{p} + \lambda_{11}^{a} \quad , \tag{3.2.2}$$

where

$$\lambda_{11}^{p} = \frac{H_a \pi (s + a_i)}{\pi (s + p_i)} = \frac{H_a n(s)_{11}}{d(s)_{11}} \quad , \tag{3.2.3}$$

and $$\lambda_{11}^{a} = k_{11} > 0. \tag{3.2.4}$$

H_a is defined to be the magnitude factor and is a positive constant. When it is necessary to denote the magnitude factor for a particular set of parameters, say the y, the a subscript will be changed to y. a_i is a zero of λ_{11}^p and p_i is a pole of λ_{ij}^p. $n(s)_{ij}$ is the numerator polynomial of λ_{ij}^p and $d(s)_{ij}$ is the denominator polynomial of λ_{ij}^p. The π sign indicates multiplication. From passive two-port theory the following is known concerning RC or RL driving point function(7).

1. The a_i's and the p_i's are interlaced on the negative real axis of the s-plane.
2. $n_a - n_p = 0 \pm 1$,

   where

   n_a = degree of $n(s)_{11}$,

   n_p = degree of $d(s)_{11}$.
3. If $n_a - n_p = 0$, all the critical frequencies are finite and non-zero; whereas if $n_a - n_p = \pm 1$ there may be poles or zeros of λ_{11}^p at zero or infinity on the s-plane.
4. The number of zeros and the number of poles are exactly equal.
5. λ_{11}^p is positive real and a rational function.

Next a theorem on λ_{11} is stated and then proven.

Theorem 3.2.1. Given λ_{11} as defined by Equation 3.22, then:

1. The poles of λ_{11} will be the same as the poles of λ_{11}^p .
2. The zeros of λ_{11} will be shifted from the zeros of λ_{11}^p toward the poles of λ_{11}^p as k_{11} goes from zero to infinity.
3. The pole-zero plot of λ_{11} will satisfy the requirements on the pole-zero plot of a general λ_{11}^p with the exception of condition two above. Further, the degree of the denominator, $D(s)_{11}$, of λ_{11} can equal or be one less then the degree of

the numerator, $N(s)_{11}$ of λ_{11}, but the degree of $D(s)_{11}$ cannot exceed the degree of $N(s)_{11}$.

4. λ_{11} is positive real.

Proof:

Equation 3.2.2 may be written as

$$\lambda_{11} = \frac{H_a\ \pi(s + a_i) + k_{11}\ \pi(s + p_i)}{\pi(s + p_i)} = \frac{N_{11}(s)}{D_{11}(s)} . \tag{3.2.5}$$

Inspection of Equation 3.2.5 shows the poles of λ_{11} to be the same as the poles of λ_{11}^{p}. The numerator $N_{11}(s)$ is given by

$$N_{11}(s) = H_a\ \pi(s + a_i) + k_{11}\ \pi(s + p_i) , \tag{3.2.6}$$

$$= H_a\ \pi(s + a_i) \left[1 + \frac{(k_{11}/H_a)(\pi(s + p_i))}{\pi(s + a_i)} \right] . \tag{3.2.7}$$

The bracket term of Equation 3.2.6 may be treated by root-locus techniques to determine the roots of $N_{11}(s)$, which are the zeros of λ_{11}. Assume k_{11} may be varied from zero to infinity. From root-locus theory it is known that the roots of $N_{11}(s)$ will be located at the a_i's for $k_{11} = 0$; as k_{11} is increased the roots of $N_{11}(s)$ move toward the p_i's (8). Because the number of a_i's exactly equal the number of p_i's if the possible a_i or p_i at infinity is counted, and because all the critical frequencies are located on the negative real axis of the s-plane and are interlaced, the roots of $N_{11}(s)$ remain on the negative real axis for all positive values of k_{11}.

To prove part three, note that the degree of $N_{11}(s)$ is the degree of $n_{11}(s)$ or $d_{11}(s)$ which ever is larger, but that the degree of $D_{11}(s)$ is always the degree of $d_{11}(s)$. λ_{11} is positive real because adding a positive constant to a positive real function gives a positive real function.

Corollary 3.2.1. Given two polynomials in s, Q(s) and P(s), where the roots of Q(s) and P(s) are interlaced on the negative real axis, then the roots of M(s) are interlaced on the negative real axis of the complex s-plane, where $M(s) = aQ(s) + bP(s)$ and $a > 0$, $b > 0$.

Equation 3.2.6 may also be factored as

$$N_{11}(s) = k_{11}\pi(s + p_i) \left[1 + \frac{(H_a/k_{11})(\pi(s + a_i)}{\pi(s + p_i)} \right] . \tag{3.2.8}$$

Then the roots of the numerator of the bracket term move from the p_i's to the a_i's as k_{11} goes from infinity to zero respectively. Regardless of the factoring, the roots of the numerator of the bracket term will be the same for a given k_{11} and H_a and they will be located between the p_i's and the a_i's. Denote these roots as the α_i's. Then Equation 3.2.8 may be written as

$$N_{11}(s) = K_a \, \pi(s + \alpha_i) . \tag{3.2.9}$$

If $n_a = n_p$, then

$$K_a = H_a + k_{11}, \tag{3.2.10}$$

if $n_a = n_p + 1$, then

$$K_a = H_a \quad , \tag{3.2.11}$$

and if $n_a = n_p - 1$, then

$$K_a = k_{11}. \tag{3.2.12}$$

λ_{22} will be considered next. From Equation 3.2.1 an expression for λ_{22} is

$$\lambda_{22} = \lambda_{22}^{p} + \lambda_{21}^{a} \quad , \tag{3.2.13}$$

where

$$\lambda_{22}^{p} = \left(H_b\right) \left(\frac{\pi(s + b_i)}{\pi(s + p_i)} \right), \tag{3.2.14}$$

and

$$\lambda_{22}^{a} = k_{22} > 0. \tag{3.2.15}$$

b_i is the root of the numerator of λ_{22}^{p}. Because of the similarity between λ_{11} and λ_{22} and because the general pole-zero plots of λ_{11}^{p} and λ_{22}^{p} meet the same conditions, the following theorem may be stated concerning λ_{22}.

Theorem 3.2.2. Given λ_{22} with the specified topology, then:

1. The poles of λ_{22} will be the same as the poles of λ_{22}^{p}.
2. The zeros of λ_{22} will be shifted from the zeros of λ_{22} toward the poles of λ_{22}^{p}, with the amount of shift depending upon the size of λ_{22}^{a}.
3. The pole-zero plot of λ_{22} will satisfy the requirements on the pole-zero plot of a general λ_{22}^{p} with the exception that the degree of $D_{22}(s)$ can equal or be one less than the degree of $N_{22}(s)$ but not greater.
4. λ_{22} is a positive real function.

Denote the roots of $N_{22}(s)$ by β_i, then an expression for λ_{22} is

$$\lambda_{22} = \frac{K_b \, \pi(s + \beta_i)}{\pi(s + p_i)} , \tag{3.2.16}$$

where K_b is defined by equations similar to Equations 3.2.10, 3.2.11 and 3.2.12. β_i is the root of λ_{22} or the shifted root.

Although the above two theorems were developed for the case of no private poles in the passive two-port, the theorems, as stated, hold if private poles are allowed. Further, an interpretation of the above two theorems is that given a pole-zero plot, it could not be determined if it originated from a general passive RL or RC λ_{11}^{p} or from a composite

λ_{ii} unless a pole at infinity existed. That is, part three of the above two theorems rules out a pole at infinity for the rational function λ_{11} of the composite two-port.

3.3 Pole-zero Structure of the Off-diagonal Term λ_{12}. The non-diagonal term λ_{12} of the composite two-port is given by

$$\lambda_{12} = \lambda_{12}^{p} + \lambda_{12}^{a} = \frac{N_{12}(s)}{D_{12}(s)} , \tag{3.3.1}$$

where

$$\lambda_{12}^{p} = \frac{\pm\ a\pi(s + z_i)}{\pi(s + p_i)} = \frac{\pm\ an_{12}(s)}{d_{12}(s)} , \tag{3.3.2}$$

$$\lambda_{12}^{a} = \pm\ k_{12} , \tag{3.3.3}$$

and

$$\begin{aligned} a &> 0, \\ k_{12} &> 0. \end{aligned} \tag{3.3.4}$$

The positive sign holds for the z and h parameters and the negative sign holds for the y and h parameters respectively. This is true for both the active and passive parameters. See Appendix A. From passive two-port theory the following is known about RC and RL functions(1).

1. The p_i's are on the negative real axis.
2. The degree, n_z, of $n_{12}(s)$ may be 0, 1, 2 . . . . . . $n_p + 1$, for z and y parameters or 0, 1, 2 . . . . .. n_p for h and g, where n_p is the degree of $d_{12}(s)$.
3. The z_i's are on the negative real axis for series ladder networks, but for lattice structures the z_i's may be complex conjugates.

4. The p_i's are always on the negative real axis.

$N_{12}(s)$ may be expressed as

$$\pm N_{12}(s) = a\pi(s + z_i) + k_{12}\pi(s + p_i). \qquad (3.3.5)$$

In order to determine the roots of $N_{12}(s)$ by root-locus techniques Equation 3.3.5 may be written as

$$\pm N_{12}(s) = k_{12}\pi(s + p_i) \left[1 + \frac{(a/k_{12})(\pi(s + z_i))}{\pi(s + p_i)} \right]. \qquad (3.3.6)$$

It is important to note that for N_{12} a positive sign will always occur in the bracket term for the voltage and current convention shown in Figure 2.2.1. The zeros of $N_{12}(s)$ will start on the p_i's and move toward the z_i's as k_{12} is varied from infinity to zero, respectively.

To illustrate the wide range of values the roots of $N_{12}(s)$ may have and to demonstrate the root-locus method as applied to synthesis, several examples are presented.

Example 3.3.1. $n_z = 2$ and the z_i's are complex. Figure 3.3.1 shows the root-locus plot for this case; that is, the movement of the roots of the numerator of the bracket term in Equation 3.3.6 is shown as k_{12} is assumed to vary from infinity to zero. The triangles indicate the direction of movement as well as a possible value, say γ, a root of $N_{12}(s)$ can take on. Note from Figure 3.3.1 that if p_1 happens to be at the origin the plot would remain the same. Further, p_2 could be at infinity and the plot would be the same, although γ_2 would have further to move from p_2 to z_1 as k_{12} goes from infinity to zero. If, however, the λ_{12}^p is a non-minimum phase function i.e., the zeros are in the right half plane, then the roots of $N_{12}(s)$ will move into the right half plane as k_{12} becomes small. Later, it will be shown that this can lead to

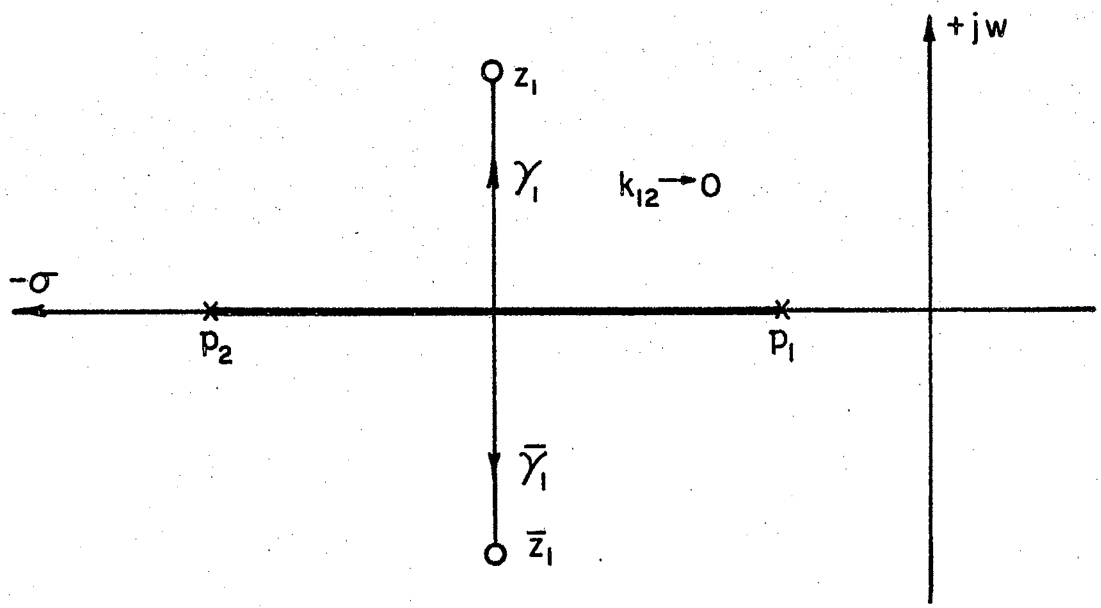


Figure 3.3.1. Root-locus Plot for the Roots of $N_{12}(s)$.

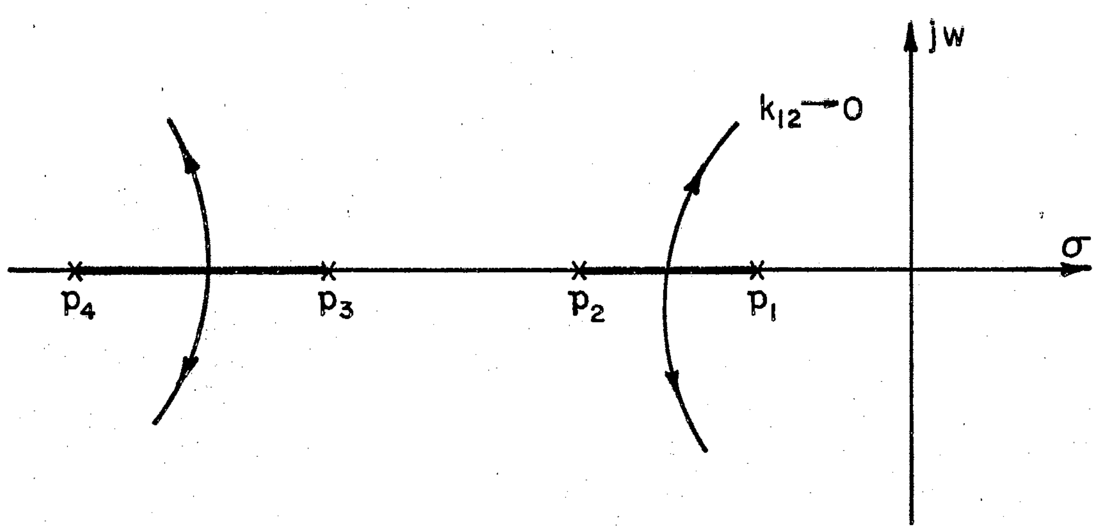


Figure 3.3.2. Root-locus Plot of $N_{12}(s)$ when λ_{12}^{p} is an all Pole Function.

potential instability of the composite two-port.

Example 3.3.2. λ_{12}^{p} , the reverse transfer function, is an all pole function. That is, the numerator polynomial of λ_{12}^{p} is a constant and the order of the denominator is n_p. λ_{12}^{p} then, has n_p zeros at infinity. Equation 3.3.6 may be written as

$$\pm N_{12}(s) = k_{12}\ \pi(s + p_i) \left[1 + \frac{a/k_{12}}{\pi(s + p_i)} \right] . \qquad (3.3.7)$$

For the case of $n_p = 4$ the root-locus plot appears as in Figure 3.3.2. Figure 3.3.2 demonstrates the important fact that the zeros of λ_{12} may be complex although the original passive two-port is a series ladder structure. Even though all the zeros of λ_{12}^{p} are at infinity, the roots of $N_{12}(s)$ move toward them. For $n_p \geq 3$, the roots can be in the right half plane and hence, the possibility of instability may exist in the over-all two-port.

For the all pole case, Equation 3.3.7 may be written as

$$\pm N_{12}(s) = k_{12} \left[\pi(s + p_i) + a/k_{12} \right] . \qquad (3.3.8)$$

From Equation 3.3.8 it can be seen that the degree of N_{12} is, theoretically, n_p. However, for a particular frequency range, the roots of $N_{12}(s)$ are very large for small values of k_{12}, and are considered to be at infinity. The degree of $N_{12}(s)$ is then zero. In fact, for the active devices under consideration k_{12} is extremely small and the problem appears to be the realization of passive components such that a will be sufficiently small so that the zeros may be placed in a usable region of the s-plane whenever it is desirable to generate zero of λ_{12} in this manner.

In general, $N_{12}(s)$ is expressed as

$$\pm N_{12}(s) = K_1 \pi(s + \gamma_i^1) \quad , \tag{3.3.9}$$

where γ_i^1 is a root of $N_{12}(s)$ after shifting. The movement of γ_i as a function of k_{12} could have been demonstrated by factoring Equation 3.3.5 as $N_{11}(s)$ was factored. This would show the movement of γ_i from z_i to p_i; however for later mathematical convenience the last method is preferred. The definition of K_1 of Equation 3.3.9 is dependent upon the degrees of $n_{12}(s)$ and $d_{12}(s)$ in a manner similar to K_a of Equation 3.2.9.

The above may be summerized in the form of two theorems.

Theorem 3.3.1. Given λ_{12} as defined above in Equation 3.3.1, then:

1. The pole-zero plot of λ_{12} will be the same as the pole-zero plot of a general two-port RL or RC transfer function.
2. In particular, the poles of λ_{12} will be the same as the poles of λ_{12}^p, but the zeros will be shifted from the zeros of λ_{12}^p.
3. Complex conjugates zeros of λ_{12} may result when λ_{12}^p is a series ladder structure.
4. A non-minimum phase λ_{12} may result from a minimum phase λ_{12}^p.

The next theorem views the problem from an active device standpoint.

Theorem 3.3.2. Given a two-port defined as in Theorem 3.3.1, then the zeros of λ_{12} will approach the zeros of y_{12}^p as the active feedback term λ_{12}^a goes to zero, but as λ_{12}^a increases, the zeros of λ_{12} will approach the poles of λ_{12}^p.

The importance of Theorem 3.3.2 is as follows. k_{12} is not a true variable as it is in the classical root-locus problem. Instead, for a given active device it is a constant; therefore, for a given passive two-port and a given active device, Theorem 3.3.2 furnishes insight into where the zeros of the composite λ_{12} are located.

3.4 Pole-zero Structure of the Off Diagonal Term λ_{21}. Equation 3.2.1 implies that

$$\lambda_{21} = \lambda_{21}^{p} + \lambda_{12}^{a} = \frac{N_{21}(s)}{D_{21}(s)} , \tag{3.4.1}$$

where

$$\lambda_{21}^{p} = \pm \frac{a\pi(s + z_i)}{\pi(s + p_i)} = \frac{\pm\, a n_{21}(s)}{d_{21}(s)} , \tag{3.4.2}$$

$$\lambda_{21}^{a} = \mp k_{21} , \tag{3.4.3}$$

and

$$a > 0, \tag{3.4.4.a}$$

$$k_{21} > 0. \tag{3.4.4.b}$$

The positive sign holds for the y and h active two-port parameters, and for the passive two-port z and g parameters. The negative sign occurs for the z and g, and for the y and h parameters for the active and passive two-port, respectively.

From passive RC and RL two-port theory it is known that

$$g_{12} = -g_{21} , \tag{3.4.5.a}$$

$$y_{12} = y_{21} , \tag{3.4.5.b}$$

$$z_{12} = z_{21} , \tag{3.4.5.c}$$

$$h_{12} = -h_{21} . \tag{3.4.5.d}$$

Therefore the conditions on λ_{12}^{p} as stated previously hold for λ_{21}^{p}, except for the negative sign indicated in Equation 3.4.5 for the g and h parameters.

$N_{21}(s)$ may be written as

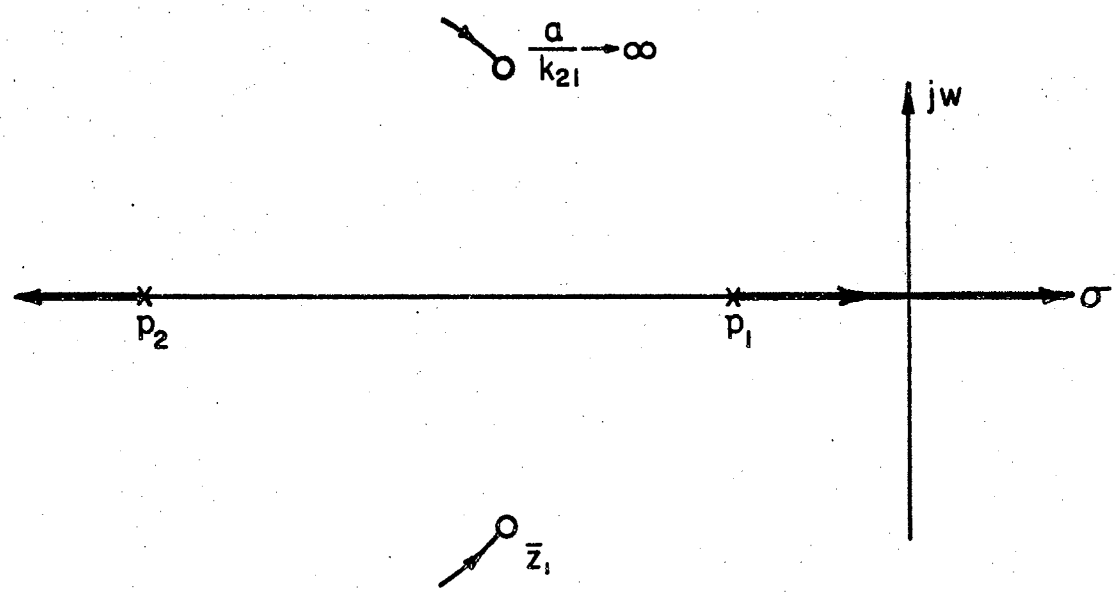
$$\pm N_{21}(s) = a\pi(s + z_i) - k_{21}\,\pi(s + p_i)\ , \tag{3.4.6}$$

$$\pm N_{21}(s) = k_{21}\pi(s + p_i)\left[1 - \frac{(a/k_{21})(\pi(s + z_i))}{\pi(s + p_i)}\right]. \tag{3.4.7}$$

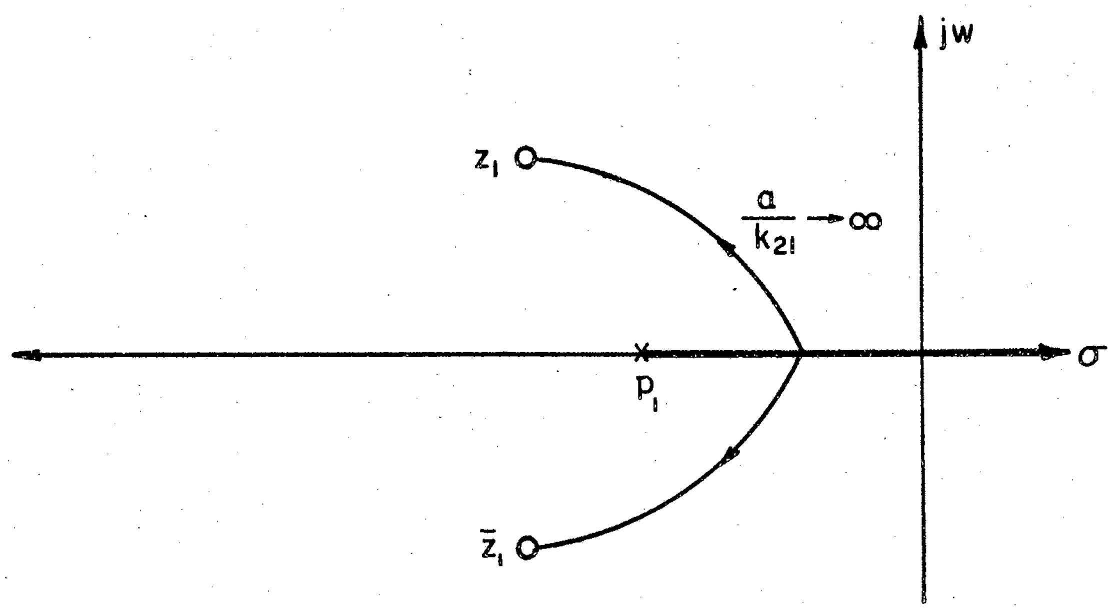
The positive sign on $N_{21}(s)$ of Equation 3.4.7 holds for the y and h parameters, whereas the negative sign is required for the z and g parameters. It is important to note that, regardless of the parameter set used, there will always be a sign difference between the one and the rational function. The roots of $N_{21}(s)$ are determined by the values of k_{21} such that the bracket term is zero. This occurs whenever the rational function has a modulus of one and an argument of $0 \pm 360^\circ$. This is in contrast to the classical root-locus plot where the roots occur on the s-plane whenever the argument is $180^\circ \pm 360^\circ$. Therefore care must be used in plotting the locus of the roots of $N_{21}(s)$ as k_{21} is varied because the plots differ in the two cases. The roots of $N_{21}(s)$ will continue to start on the p_i's and move toward the z_i's as k_{21} is varied from infinity to zero, respectively. The paths, however, will be different. For the following reasons, the root-loci on the real axis may be drawn in by inspection. The complex zeros of $\lambda^p{}_{21}$ are complex conjugates and make no net contribution to the argument. The poles are all real. Hence, the real line to the right of all critical frequencies will be a part of the root-loci. The real axis to the left of an even number of critical frequencies will also give the ration for an argument of 0°.

Several examples will be given to show the possible root location.

Example 3.4.1. $n_z = 2$ and the z_i's are complex. Figure 3.4.1 shows



(a). $n_p = n_z = 2$.



(b). $n_p = 1$, $n_z = 2$, and p_2 is at ∞.

Figure 3.4.1. Root-locus Plot of $N_{21}(s)$ for Complex Zeros in λ_{21}^{p}.

the root-locus plot for this case when $n_p = 2$. Figure 3.4.1.b is for the $n_p = 1$.

To demonstrate further the root-locus technique when a negative sign is involved and to illustrate the possible pole-zero structure of λ_{21}, another example is given.

Example 3.4.2. λ_{21} is an all pole function. Figure 3.4.2.a demonstrates this case for $n_z = 2$ and Figure 3.4.2.b demonstrates the case for the all pole structure with $n_z = 4$. The dotted line in Figure 3.4.2 shows the path of the roots for the case of the positive sign occurring in the bracket term. In contrast to the classical root-locus plot the roots for large values of s must satisfy the equation

$$\frac{a/k_{21}}{s^m} = 0^\circ \pm 360^\circ . \tag{3.4.8}$$

That is, for s large the term $1/s^m$ must produce an angle of $\pm 360^\circ$. Therefore if m is defined as in Equation 3.4.8, there will be m roots of $N_{21}(s)$ going to infinity guided by the asymptotes,

$$\frac{0^\circ}{m}, \quad \frac{\pm 360^\circ}{m}, \quad \frac{\pm 720^\circ}{m}, \quad \frac{\pm 1080^\circ}{m} \quad \ldots\ldots \tag{3.4.9}$$

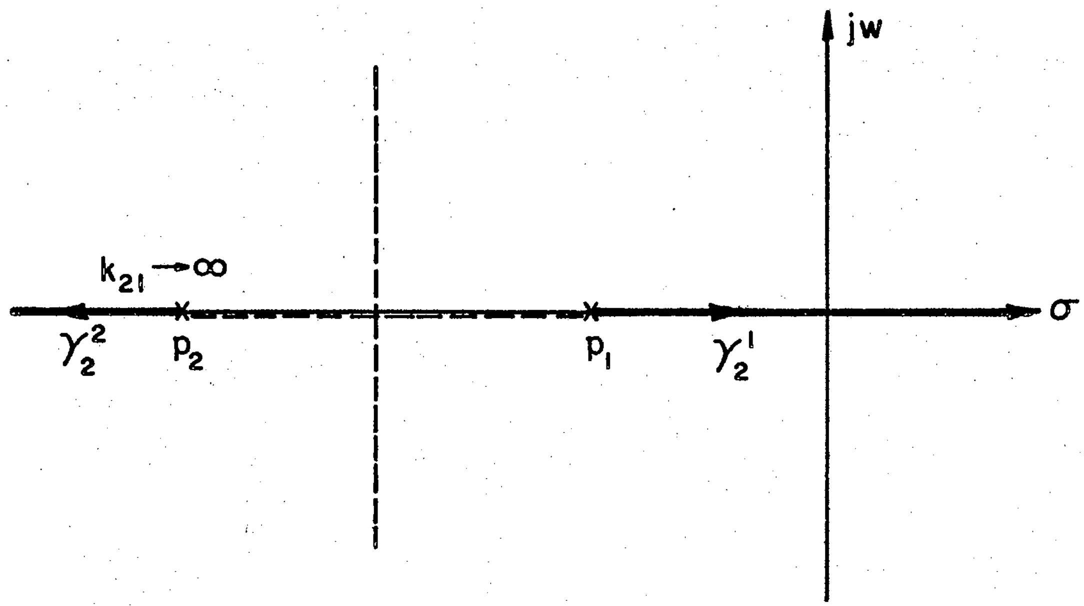
In general, if m is the amount that the degree of the denominator exceeds the numerator, the above asymptotic behavior holds.

In general then $N_{21}(s)$ may be expressed as

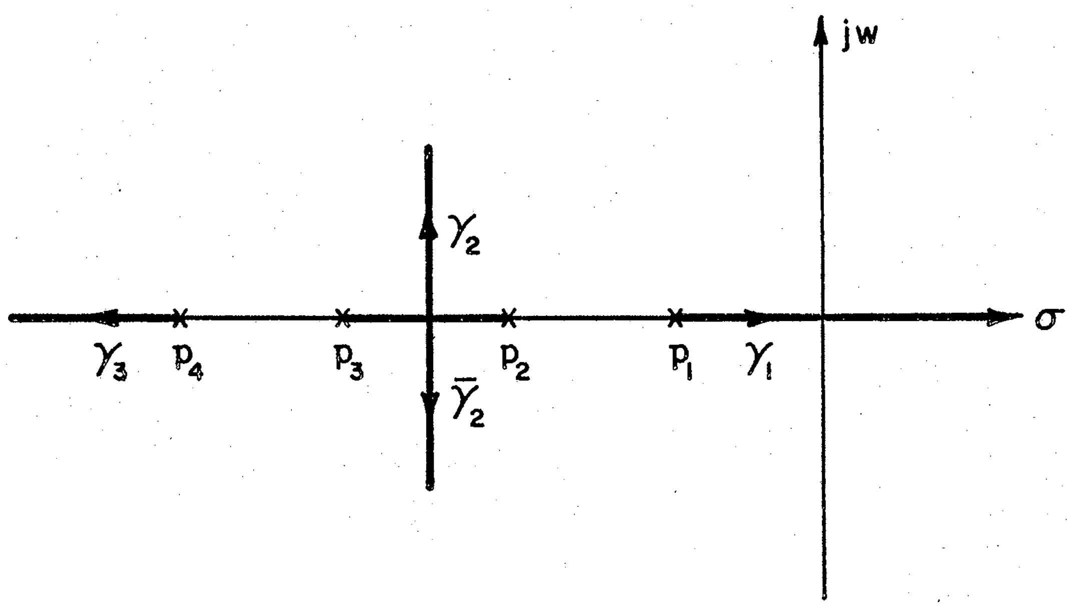
$$\pm N_{21}(s) = K_2 \, \pi(s + \gamma_i^2) \quad . \tag{3.4.10}$$

Once again K_2 is a positive constant defined in the manner of Equations 3.2.10, 3.2.11 and 3.2.12.

From the above examples the following conclusions can be made and stated in the form of two theorems.



(a). $n_p = 2$,



(b). $n_p = 4$.

Figure 3.4.2. Root-locus Plot of $N_{21}(s)$ when $n_{21}(s)$ is an all Pole Function.

Theorem 3.4.1. Given λ_{21} as defined above, then:

1. The pole-zero structure is, in general, the same as a pole-zero structure of a RC or RL transfer function, providing no zeros of λ_{21} occur on the positive real axis.
2. In particular, the poles of λ_{21} are the same as the poles of λ_{21}^{p}.
3. The zeros of λ_{21} are shifted from the poles of λ_{21}^{p} toward the zeros of λ_{21}^{p} depending inversely upon the size of k_{21}.
4. Complex zeros of λ_{21} can occur for passive series ladder structures.
5. λ_{21} may be non-minimum for both ladder and lattice structures of the passive two-port.

Theorem 3.4.2. For a composite circuit consisting of a passive two-port and an active two-port as defined above λ_{21} may have zeros on the positive real axis.

Although Theorem 3.4.2 is almost a restatement of part one on Theorem 3.4.1, its importance is that fact that it graphically demonstrates one difference between active and passive RLCM two-ports. For RLCM networks, a basic theorem states that no zeros may exist on the positive real axis for λ_{21}^{p} of grounded networks, while Theorem 3.4.2 states definitely that such zeros can occur in active two-port synthesis.

3.5 Zeros and Poles of λ_{ij}^{p} for First Cauer RC Form. In this section the relationships of the p-z structures between the various parameters are developed for the first Cauer form(7). Extensive use is made of the root-locus techniques to prove theorems that would otherwise be algebraically difficult to handle. One of the theorems which is proven is that for selected networks, all the parameters of a given set will have a common denominator. This is an important fact in later work. The first Cauer form is the selected network because it leads to an all pole function for the off diagonal terms and it has been worked with extensively in passive synthesis. The reason for an all pole function is simplicity.

It is known from passive synthesis that if the specified y-parameters do not have a common denominator, the private poles may be removed from y_{11} and y_{22} and represented as shunt networks with respect to a central network which will have y-parameters with common denominators (7). A similar statement applies to the z-parameters, except series networks are used to realize the private poles. No such general statements exist for the g and h parameters. Even if no private poles exist for the z and y parameters, the parameters of the g and h sets may not have a common denominator.

For example,

$$h_{11} = 1/y_{11} ,$$

$$h_{22} = 1/z_{22} ,$$

and

$$-h_{21} = h_{21} = -y_{12}/y_{11} = z_{12}/z_{22} .$$

If a cancellation occurs in y_{12} and y_{11}, then h_{11} and h_{12} will not have the same denominator. Further there is no guarantee that such a cancellation will not occur. Also, h_{11} and h_{21} are short circuit parameters, whereas h_{12} and h_{22} are open circuit parameters; therefore, it is not to be expected that the h-parameters will have a common denominator.

A similar argument applies for the g-parameters.

Next it is shown, by root-locus techniques, that for a particular network, Figure 3.5.1, of the first Cauer RC form all the parameters of a set have the same denominator. The following is concerned with a passive RC network and the superscript, p, will be dropped.

Necessary conditions on z_{11} for the first Cauer RC form, for the first and the last element are as follows.

1. If $z_{11}(s)$ has a zero at $s = \infty$ the first element is a capacitor.
2. If $z_{11}(s)$ is a constant at $s = \infty$ the first element is a resistor.
3. If $z_{11}(s)$ has a pole at $s = 0$ the last element is a capacitor.
4. If $z_{11}(s)$ is a constant at $s = 0$ the last element is a resistor.

Similar statements can be stated concerning z_{22}. Then from inspection of Figure 3.5.1 and the above necessary and sufficient condition it can be seen that the general form of z_{ii} is

$$z_{ii} = \frac{H_z s^n + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot + z^o}{s(s^{n-1} + \cdot \cdot \cdot \cdot \cdot \cdot \cdot + z^o)} \quad . \tag{3.5.1}$$

Also from passive two-port theory the conditions on y_{11} for the first and last element for the first Cauer form are:

1. If $y_{11}(s)$ has a zero at $s = 0$ the last element is a capacitor.
2. If $y_{11}(s)$ is a constant at $s = 0$ the last element is a resistor.
3. If $y_{11}(s)$ has a pole at $s = \infty$ the first element is a capacitor.

4. If $y_{11}(s)$ is a constant at $s = \infty$ the first element is a resistor.

Similar statements apply to y_{22}. From inspection of Figure 3.5.2 and the conditions for y_{ii}, the general expression for y_{ii} is

$$y_{ii} = \frac{H_y s^n + \cdots\cdots\cdots + y^o}{s^n + \cdots\cdots\cdots + y_o} \quad . \tag{3.5.2}$$

Analysis of the network of Figure 3.5.1 shows that

$$-y_{12} = -y_{21} = \frac{a}{\pi(s + p_i)} \quad .$$

The relationship between z_{11} and the y-parameters is

$$z_{11} = \frac{y_{22}}{D_y} = \frac{\dfrac{H_{y22}\,\pi(s + a_i^{22})}{\pi(s + p)}}{\dfrac{H_{y22}\,H_{y11}\pi(s + a_i^{22})\pi(s + a_i^{11}) - a^2}{\pi(s + p_i)^2}}$$

$$= \frac{H_{y22}\,\pi(s + a_i^{22})\,\pi\,(s + p_i)}{H_{y22}H_{y11}\pi(s + a_i^{22})\pi(s + a_i^{11}) - a^2} = \frac{K_s^{2n} + \cdots +}{s^{2n} + \cdots +} \tag{3.5.3}$$

D_y is the determinant of the y matrix. Now the numerator of Equation 3.5.3 has to be of degree of n and of the form given by Equation 3.5.1 hence cancellations must occur between the numerator and denominator. The denominator may be factored as

$$H_{y11}H_{y22}\pi(s + a_i^{22})\pi(s + a_i^{11}) \left[1 - \frac{\dfrac{a^2}{H_{y11}\,H_{y22}}}{\pi(s + a_i^{22})\pi(s + a_i^{11})} \right] . \tag{3.5.4}$$

A root-locus plot of the bracket term of Equation 3.5.4 is shown in Figure 3.5.3. The reasons for the plot of Figure 3.5.3 is that with no

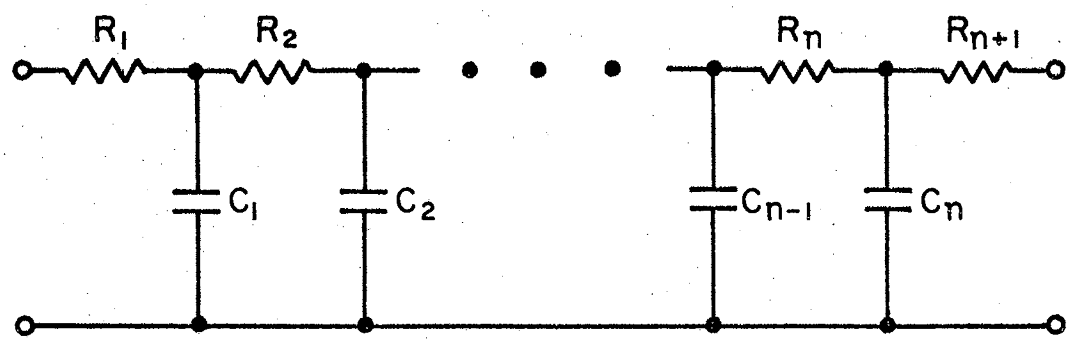


Figure 3.5.1. First Cauer RC Form From Which z_{11} May Be Determined.

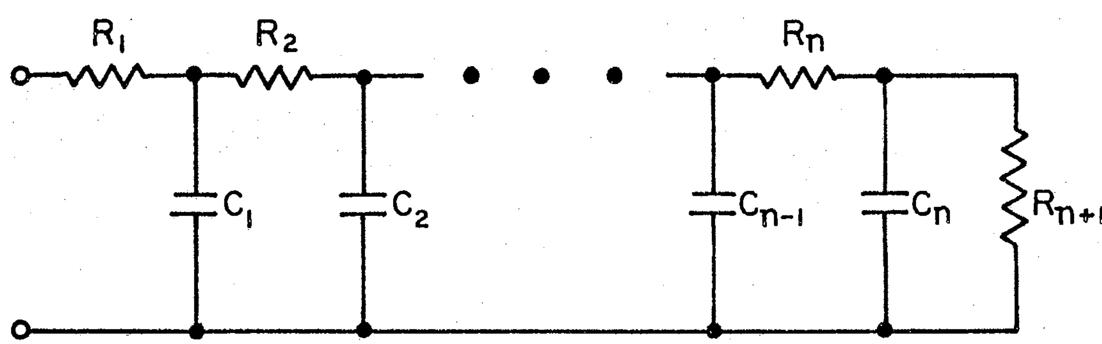


Figure 3.5.2. First Cauer Form With Output Shorted to Determine y_{11}.

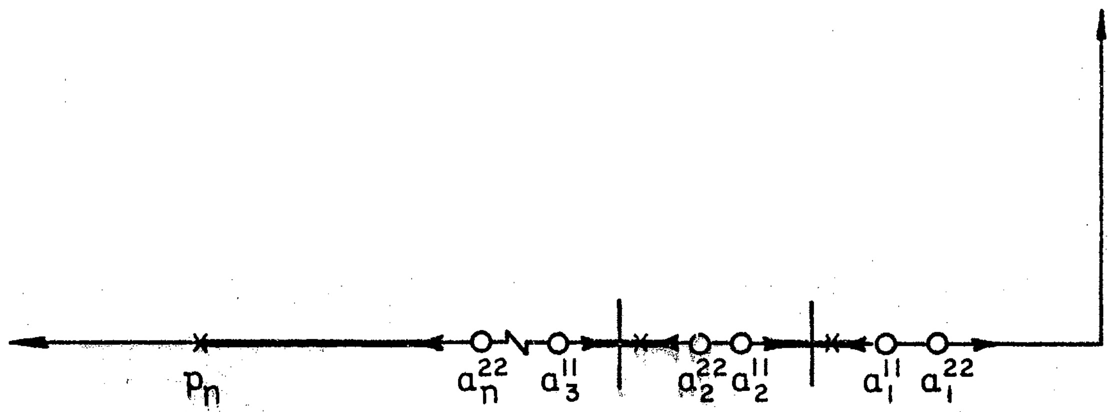


Figure 3.5.3. Root-locus Plot of $N_{12}(s)$ when λ_{12}^{p} is an All Pole Function.

private poles the p_i's are the same for y_{11} and y_{22}. Further they are interlaced on the negative real axis with the a_i^{ii}'s. Whether, $a_i^{11} > a_i^{22}$ or $a_j^{22} > a_j^{11}$, their values must lie between adjacent p_i's and further the root-locus plot is not affected which ever is the larger. Note the p_i's do not affect the root-locus plot, but are placed on the diagram to locate the a_i's and show possible cancellations. All the zeros in the numerator of the non-unity term in the bracket term of Equation 3.5.4 are at infinity; therefore, the poles of that term, the a_i's go to infinity according to Equation 3.4.9 as $a^2/H_{y11}H_{y22}$ is assumed to be varied. Actually $a^2/H_{y11}H_{y22}$ is a constant, but it must be such that n zeros of the denominator of Equation 3.5.3 ultimately cancel with the p_i's of the numerator and result in a rational function as required by Equation 3.5.1. From an inspection of Figure 3.5.3 it is seen that this is the only cancellation possible. Also one root of Equation 3.5.4 must move to the origin. Therefore Equation 3.5.3 becomes

$$z_{11} = \frac{\overset{n}{\pi}\,(s + a_i^{22})}{H_{y11}\, s \overset{n-1}{\underset{i-1}{\pi}} (s + g_i)} \quad . \tag{3.5.5}$$

Although the plot indicates the possibility of g_i being complex, this cannot occur since g_i is the root of a driving point RC impedance. Further they must be interlaced with the a_i^{ii}'s as this is a necessary p-z structure(7).

The above may be stated in the form of a theorem.

Theorem 3.5.1. Given a passive two-port containing a first Cauer form with resistors as end elements, then

$$z_{11} = \frac{\pi(s + a_i^{22})}{H_{y11}\; s\pi(s + g_i)} \quad , \tag{3.5.6}$$

where the a_i^{22}'s are the zeros of y_{22} and the g_i's are located between the a_i^{22}'s.

Theorem 3.5.2. Given a passive two-port as described above then

$$z_{22} = \frac{\pi(s + a_i^{11})}{H_{y22}\; s\pi(s + g_i)} \quad , \tag{3.5.7}$$

where the a_i^{11}'s are the zeros of y_{11} and the g_i's are interlaced with a_i^{11}'s along the negative real axis.

Theorem 3.5.3. Given the defined two-port, then

$$z_{12} = \frac{-y_{12}}{D_y} = \frac{\dfrac{a}{\pi(s + p)} \cdot \dfrac{\pi(s + p_i)^2}{1}}{H_{y11}H_{y22}\pi(s + a_i^{11})\pi(s + a_i^{22}) - a^2} \quad ,$$

$$= \frac{a\pi(s + p)}{H_{y11}H_{y22}\pi(s + a_i^{11})\;\pi(s + a_i^{22}) - a^2} \quad ,$$

$$z_{12} = \frac{a}{H_{y11}H_{y22}\ s\pi(s + g_i)} \quad .$$

The g_i's are the same as those defined in Theorem 3.5.1.

By using Theorem 3.5.1, Theorem 3.5.2 and Theorem 3.5.5 all the parameters may be expressed in terms of the y-parameters or in terms derived from the y-parameter. Let the y-parameter for the first Cauer form be as follows.

$$y_{11} = \frac{H_{y11}\ \pi(s + a_i^{11})}{\pi(s + p_i)} \quad , \tag{3.5.8}$$

$$y_{22} = \frac{H_{y22}\ \pi(s + a_i^{22})}{\pi(s + p_i)} \quad , \tag{3.5.9}$$

$$-y_{12} = -y_{12} = \frac{a}{\pi(s + p_i)} \quad , \tag{3.5.10}$$

where the y-parameters are for the two-port of Figure 3.5.1. Then from Theorem 3.5.1, Theorem 3.5.2, and Theorem 3.5.3,

$$z_{11} = \frac{\pi(s + a_i^{22})}{H_{y11}\, s\pi(s + g_i)} \quad , \tag{3.5.11}$$

$$z_{22} = \frac{\pi(s + a_i^{11})}{H_{y22}\ s\pi(s + g_i)} \quad , \tag{3.5.12}$$

$$z_{12} = z_{21} = \frac{a}{H_{y22}\, H_{y11}\, s\pi(s + g_i)} \quad . \tag{3.5.13}$$

From the parameter relationships indicated in Chapter II, Table I, the following results hold for the h-parameters.

$$h_{11} = 1/y_{11} = \frac{\pi(s + p_i)}{H_{y11}\ \pi(s + a_i^{11})} \tag{3.5.14}$$

$$h_{22} = 1/z_{22} = \frac{H_{y22}\ \pi(s + g_i)}{\pi(s + a_i^{11})} \tag{3.5.15}$$

$$h_{12} = -h_{21} = \frac{-y_{12}}{y + 1} = \frac{a}{H_{y11}\pi(s + a_i^{11})} \quad , \tag{3.5.16}$$

Then for the g-parameters

$$g_{11} = 1/z_{11} = \frac{H_{y11}\ \pi(s + g_i)}{\pi(s + a_i^{22})} \tag{3.5.17}$$

$$g_{22} = 1/y_{22} = \frac{\pi(s + p_i)}{H_{y22}\pi(s + a_i^{22})} \tag{3.5.18}$$

$$-g_{12} = g_{21} = \frac{z_{21}}{z_{11}} = \frac{\dfrac{\dfrac{a}{H_{y11}H_{y22}\ \pi(s + g_i)}}{\pi(s + a_i^{22})}}{H_{y11}\ \pi(s + g_i)} \tag{3.5.19}$$

$$= \frac{a/H_{y22}}{\pi(s + a_i^{22})} \quad .$$

The above may be summarized in the form of a Theorem.

Theorem 3.5.4. Given a two-port composed of the first Cauer network as described, then the following holds.

1. All the parameters of a particular set have the same poles.
2. All off-diagonal terms are all pole functions.

3. The poles of the h-parameters are the zeros of y_{11}.

4. The poles of the g-parameters are the zeros of y_{22}.

3.6 P-Z Structure of The Composite Two-port. In this section it is shown how the p-z structure of the parameters for the composite two-port can be controlled by the passive two-port parameters. This is an important feature since a large number of techniques exist for passive two-port synthesis, whereas active two-port synthesis techniques which do not allow gyrators and negative impedance converters are at a minimum.

In order to achieve a desired p-z structure for the composite two-port, the passive two-port must meet certain conditions. These conditions are developed next.

The diagonal terms are considered first. From Theorems 3.1.1 and 3.1.2 it is known that λ_{ii} will have the same poles as λ^{p}_{ii}, but the zeros will differ depending upon λ^{a}_{ii}. If $H \gg k_{ii}$ the zeros of λ_{ii} will remain close to the zeros of λ^{p}_{ii}, whereas if $k_{ii} \gg H$ the zeros of λ_{ii} will approach the p_i's. The objective of the following development is to enhance the above mentioned tendency. For example, if the active device is a transistor, where $y^{a}_{11} \gg y^{a}_{22}$, and if a symmetric passive two-port is used the zeros of y_{11} would tend to be at the poles of y^{p}_{11} while the zeros of y_{22} would tend to be at the zeros of y^{p}_{22}. A passive two-port with $H_b > H_a$ could also be used. Therefore, by synthesizing a passive pole-zero plot by any of the well known means, the p-z structure of the composite two-port, which contains an active two-port, may be constructed as desired.

If the numerator and denominator of λ^{p}_{11} are of the same degree, and if the particular parameter set has a common denominator, then the numerator of λ_{ii}, $N_{ii}(s)$, may be expressed as

$$N_{ii}(s) = H(s + c_i) \cdot \cdot \cdot \cdot (s + c_n) + k_{ii}(s + p_1)(s + p_2) \cdot \cdot \cdot \cdot (s + p_n) . \tag{3.6.1}$$

c_i is a root of $n_{ii}(s)$ and p_i is a root of $d_{ii}(s)$. If k_{ii} is larger than k_{jj}, say, then Equation 3.6.1 can be factored such as

$$N_{ii}(s) = k_{ii} \left[H/k_{ii}(s + c_1)(s + c_2) \cdot \cdot \cdot \cdot (s + c_n) + (s + p_1)(s + p_2) \cdot \cdot \cdot \cdot (s + p_3) \right] , \tag{3.6.2}$$

or

$$N_{ii}(s) = k_{ii} \left[(1 + \frac{H}{k_{ii}})s^n + (\frac{H}{k_{ii}} f_{n-1}(c_i) + f_{n-1}(p_i))s^{n-1} + \cdot \cdot \cdot + \frac{H}{k_{ii}} (c_1 c_2 \cdot \cdot c_n) + (p_1 p_2 \cdot \cdot p_n) \right] \tag{3.6.3}$$

If

$$1 \gg H/k_{ii} , \tag{3.6.4}$$

and

p_i and c_i are of the same order of magnitude, then

$$N_{ii} \approx k_{ii}\left[(s + p_1)(s + p_2) \cdot \cdot \cdot \cdot (s + p_n) \right] . \tag{3.6.5}$$

If k_{jj} is the smaller then Equation 3.6.1 may be written as

$$N_{jj}(s) = H \left[(1 + \frac{k_{jj}}{H})s^n + \frac{H}{k_{jj}} f_{n-1}(p_i) + f_{n-1}(c_i) \quad s^{n-1} + \cdot \cdot \cdot \cdot (c_1 c_2 \cdot \cdot c_n) + \frac{k_{jj}}{H} p_1 p_2 \cdot \cdot p_n \right] . \tag{3.6.6}$$

For

$$1 \gg k_{jj}/H \tag{3.6.7}$$

and if c_i and p_i are of the same order of magnitude then

$$N_{jj}(s) \quad H(s + c_1)(s + c_2) \cdot \cdot \cdot \cdot (s + c_n) \quad . \tag{3.6.8}$$

Since H is a magnitude factor of the passive network it may be arbitrarily determined without changing the poles and zeros of the passive two-port and hence the tendency of the zeros of N_{ii} to be at the poles of λ_{ii} and the zeros of N_{jj} to be at the zeros of λ_{jj}, depending upon which active parameter λ_{11}^a or λ_{22}^a is the larger, will be enhanced. For the y and h parameters N_{ii} will be N_{11}, whereas for the g and z parameters N_{ii} will be N_{22} for a transistor.

Next the off diagonal terms are considered. For these terms, in all cases $\lambda_{21} \gg \lambda_{12}$ for the active device. The requirement to place the zeros of λ_{12} close to the zeros of λ_{21}^p is

$$1 \gg k_{12}/a. \tag{3.6.9}$$

Also p_i and z_i should be of the same order of magnitude. Then the roots of $N_{12}(s)$ are removed some distance from the poles as indicated by the root-locus plots. Note for the case $k_{12} = 0$, the zeros of $N_{12}(s)$ are exactly the zeros of λ_{12}^p.

For the other off-diagonal term the condition is

$$1 \gg a/k_{21} \quad . \tag{3.6.10}$$

If p_i and z_i are of the same order of magnitude, then the zeros of $N_{21}(s)$ are close to the poles of p_1, $p_2 \cdot \cdot \cdot p_n$. Since a may be determined by the parameters of the passive network, the natural placement of the zeros of N_{ij} can be enhanced by the conditions indicated in Equation 3.6.9 and Equation 3.6.10. The above conditions indicated in Equation 3.6.4, Equation 3.6.7, Equation 3.6.9 and Equation 3.6.10 also hold whenever the numerator or denominator differ by one, but the constant factored out is not a function of the active two-port parameters whenever the denominator exceeds the numerator.

Hence by utilizing the natural anti-symmetrical and non-reciprocal character of the active device and by requiring the passive two-port to enhance the natural shifting tendencies of the numerator zeros of the composite parameters, it is possible to construct an actual pole-zero plot closely resembling a specified p-z plot for the composite parameters.

CHAPTER IV

SYNTHESIS OF DRIVING POINT IMMITTANCES

4.1 Introduction. In this chapter the actual techniques to synthesize a specified driving point admittance are developed using the y-parameters and the transistor as the active element. The technique is based on the non-reciprocal, non-symmetrical and active nature of the transistor, although the technique could also have been developed for the majority of other active devices since they also exhibit these characteristics. The y-parameters were chosen because the theorems of Chapters II and III hold for these parameters without the use of transformers and with a common ground. Further, the most general passive two-port consisting of a three terminal network may be used. The technique is developed specifically for the synthesis of RLC functions using only RC networks and active two-ports defined as in Chapter II. In particular, the problem of generating complex zeros in the driving point immittance is considered. The necessary conditions for realizibility of Λ_{in}, the generalized driving point immittance, are developed first and then interpreted in terms of the composite two-port y-parameters and the admittance, Y_{in} of the two-port. That is, the type of driving point admittance possible with a given two-port is first determined, then means of synthesizing it are developed. To synthesize a given driving point immittance, it is assumed that the two-ports are terminated in an element that is independent of frequency and whose units agree with those of λ_{22}. It is also assumed that all parameters of a set have the same denominator. While this is not true in general, it was proven in Chapter III that it is true for certain networks.

4.2 P-Z Structure for $\wedge_{in}$. The first theorem to be proven concerns the location of the poles of $\wedge_{in}$.

Theorem 4.2.1. The driving point admittance, $\wedge_{in}$, of a two-port, obtained by connecting a two-port containing only RC or RL elements to an active two-port such that the respective parameters add, cannot have complex poles.

For the composite two-port defined,

$$\wedge_{in} = \lambda_{11}\left[1 - \frac{\lambda_{12}\lambda_{21}}{\lambda_{11}\lambda_{22}}\right] \quad , \tag{4.2.1}$$

$$= \left[\frac{H_a\pi(s + a_i)}{\pi(s + p_i)} + k_{11}\right]\left[1 - \frac{\left(\frac{\pm a\pi(s + z_i)}{\pi s + p_i} \pm k_{12}\right)\left(\frac{\pm a\pi(s + z_i)}{\pi(s + p_i)} \mp k_{21}\right)}{\left(\frac{H_a\pi(s + a_i)}{\pi(s + p_i)} + k_{11}\right)\left(\frac{H_b\pi(s + b_i)}{\pi(s + p_i)} + k_{22} + \lambda_L\right)}\right]. \tag{4.2.2}$$

Note that λ_L is contained in λ_{22}, but the source immittance is not included in λ_{11}. The denominator, Q(s), of Equation 4.2.2. becomes

$$Q(s) = \pi(s + p_i)\Big[(H_a\pi(s + a_i) + k_{11}\pi(s + p_i)) \\ (H_b\pi(s + b_i) + \pi(s + p_i)(k_{22} + \lambda_L))\Big] \tag{4.2.3}$$

By Theorem 3.1.1 and Theorem 3.2.2 the a_i's and p_i's are interlaced on the negative real axis, as are the b_i's and p_i's. Then by Corollary 3.2.1 it is known that the roots of the polynomials in the two bracket terms of Equation 4.2.3 have roots on the negative real axis of the s-plane. The p_i's are real and positive by hypothesis; therefore the roots of Q(s) must be on the negative real axis.

The next theorem concerns the possibility of complex zeros in the numerator of Λ_{in}.

Theorem 4.2.2. The driving point admittance of a composite two-port, defined as in Theorem 4.2.1, can have at least one set of complex zeros if

1. $\overset{2}{\gamma_1}$ and $\overset{1}{\gamma_1} > \alpha_1$ and β_1, where the subscript denotes roots counted from the origin.
2. The passive network is selected such that K_a, K_b, K_1 and K_2 are defined by equations similar to Equation 3.2.10 and Equation 3.2.12.

The driving point admittance is given by

$$\Lambda_{in} = \lambda_{11}\left[1 - \frac{\lambda_{12}\lambda_{21}}{\lambda_{11}\lambda_{22}}\right] , \tag{4.2.4}$$

$$= \frac{K_a \pi(s+\alpha_i)}{\pi(s+p_i)}\left[1 + \frac{K_1K_2\pi(s+\overset{1}{\gamma_i})\,\pi(s+\overset{2}{\gamma_i})}{K_aK_b\pi(s+\alpha_i)\,\pi(s+\beta_i)}\right] =$$

$$= \frac{P(s)}{Q(s)} . \tag{4.2.5}$$

α_i is the shifted root due to k_{11} and β_i is the shifted root due to k_{22} and λ_L. Further, note from Appendix A that the signs of λ_{12}^p and λ_{12}^a for the various parameters are such that a positive sign will always result between the positive one and the rational function in the bracket term. This is important because the standard root-locus technique may be applied to determine the roots of the numerator of the bracket term. By the hypothesis the root-locus plot around the origin is as shown in Figure 4.2.1. From Figure 4.2.1.a it can be seen that complex roots will result for the proper selection of K, where

$$K = \frac{(k_{12} + a)\ (k_{21} + a)}{(k_{11} + H_a)\ (k_{22} + H_b + \lambda_L)} = \frac{K_1 K_2}{K_a K_b} \tag{4.2.6}$$

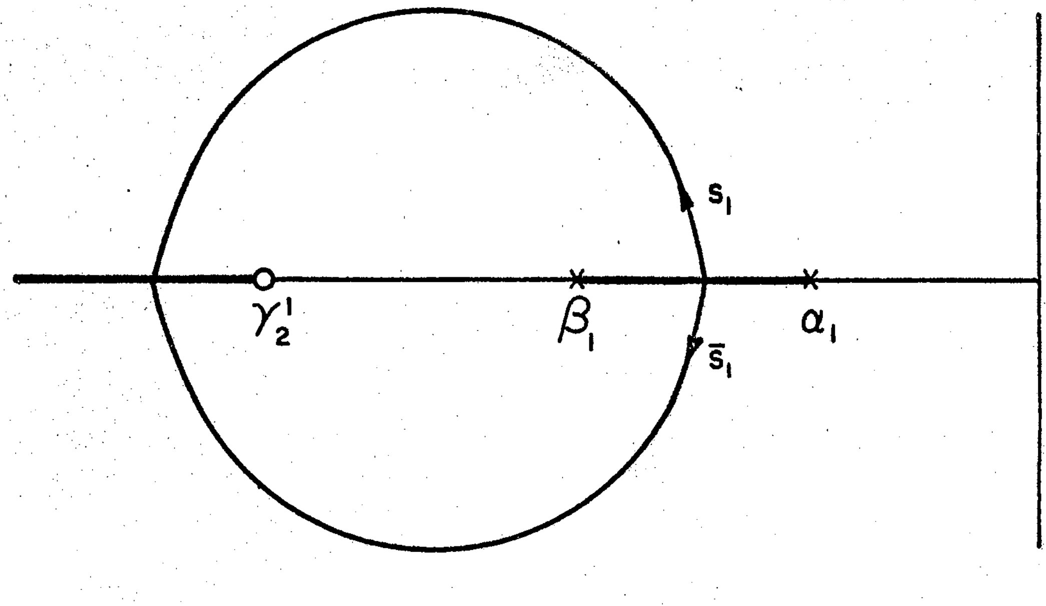
or

$$K = \frac{(k_{12} + a)\ (k_{21} + a)}{(k_{11})\ (k_{22} + \lambda_L)} = \frac{K_1 K_2}{K_a K_b} . \tag{4.2.7}$$

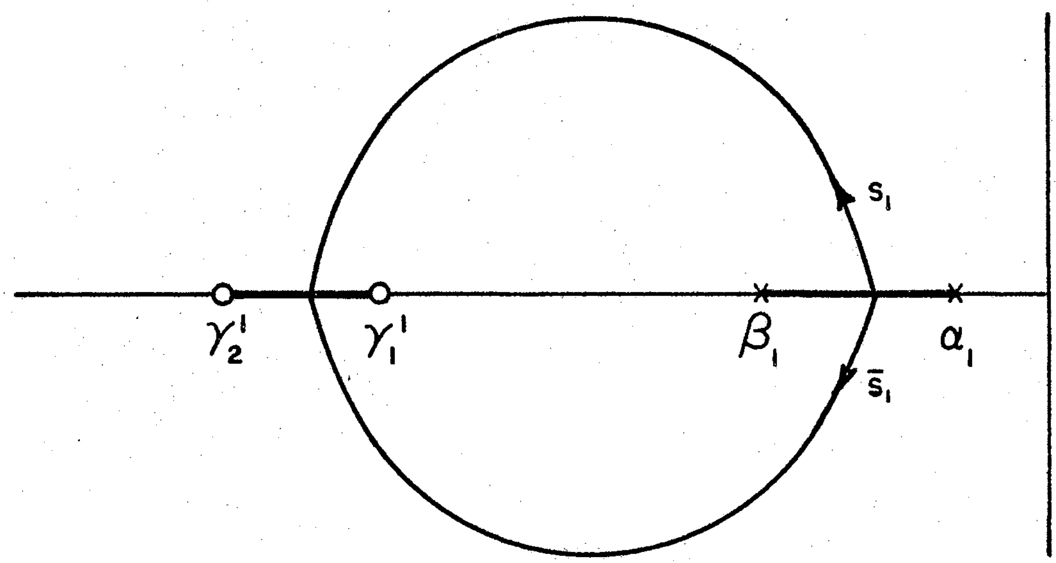
Part two of the hypothesis was necessary in order that K be a function of $k_{ij}\ k_{ji} / k_{ii}\ k_{jj}$. Therefore, if an active two-port with the proper degree of activeness, A, is selected, then K may be made any value desired and complex conjugate zeros will result.

The activeness is used to arrange the ultimate zero location of Λ_{in} after the non-reciprocal and non-symmetrical character of the active two-port was utilized to construct the necessary p-z structure for, λ_{ij}, the composite parameters. Or in other words, the p-z structure of λ_{ij} from which the p-z plot of the rational function in the bracket term is obtained, is analogous to the open loop pole-zero structure of a servo-system. The ultimate zeros of Λ_{IN} correspond to the closed-loop poles.

Theorem 4.2.3. If λ_{12}^{p} does not have complex zeros, then the necessary and sufficient conditions for the driving point immitance, Λ_{IN},



(a) γ_1^1 is at ∞ .



(b) γ_1^1 is finite .

Figure 4.2.1. Root-locus Plot of the Bracket Term of Λ_{IN}.

of a two-port defined as in Theorem 4.2.2 to have complex poles is as follows:

1. The p-z structure of the rational function in the bracket term must have two zeros together on the negative real axis, the first zero being an odd critical frequency with respect to the origin.

2. K may be selected to be any desired value.

From root-locus theory it is known that the locus of the roots on the real axis lies only to the right of a critical frequency; hence the root-locus lies between the two-zeros(8). Since it terminates on the zeros and because it started on poles, which do not lie between the zeros, the loci must have moved onto the real axis from the complex plane. If K is properly selected the ultimate zeros will be on the root-locus in the complex plane off the negative real axis.

4.3 Degree of P(s) and Q(s). The possible degree, n_p, of P(s) and the possible degree of n_Q, of Q(s) are discussed next. Assuming no cancellation occurs in the rational function of the bracket term, Equation 4.2.5 may be expanded as

$$\frac{P(s)}{Q(s)} = \frac{(K_a\pi(s + \alpha_i))(K_b\pi(s + \beta_i))}{K_b\pi(s + p_i)\ \pi(s + \beta_i)} + \frac{(K_1\pi_1(s + \overset{1}{\gamma_1}))(K_2\pi(s + \overset{2}{\gamma_2}))}{K_b\pi(s + p_i)\ \pi(s + \beta_i)} . \tag{4.3.1}$$

By Theorems 3.2.1, 3.2.2, 3.3.1, and 3.4.1 the following is known,

1. The degree, n_α, of $\pi(s + \alpha_i)$ is n_p or $n_p + 1$, where n_p is the degree of $\pi(s + p_i)$, the denominator of the original two-port

parameter.

2. The degree, n_β, of $\pi(s + \beta_i)$ is n_p or $n_p + 1$.

3. The degree, n_γ, of $\pi(s + \gamma_i)$ or $\pi(s + \gamma_i^2)$ is 0, 1, 2, $\cdots$ $n_p + 1$.

Because n_γ cannot exceed n_α or n_β, the degree of P(s) is given by

$$n_P = n_\alpha + n_\beta \quad , \tag{4.3.2}$$

if no cancellation occurs between numerator and denominator of Equation 4.3.1. From one and two above the maximum value of n_β and n_α is $n_p + 1$. The maximum value n_p can obtain is then,

$$n_p = n_\beta + n_\alpha = 2\ n_p + 2 \quad . \tag{4.3.3}$$

Similarly from one and two above the minimum value is

$$n_p = 2\ n_p. \tag{4.3.4}$$

n_Q is given by

$$n_\beta + n_p \quad . \tag{4.3.5}$$

From one and two above the maximum value of n_Q is

$$n_Q = 2\ n_p + 1 \quad . \tag{4.3.6}$$

Whereas its minimum value is

$$n_Q = n_p + n_p = 2\ n_p. \tag{4.3.7}$$

The minimum value of n_p and the maximum value of n_Q cannot occur at the same time because the former is dependent upon $n_\alpha = n_p$, while the latter requires $n_\alpha = n_p + 1$. For similar reasons the maximum of n_p and the minimum of n_Q cannot occur concurrently. Inspection of Equation 4.2.9, Equation 4.3.7 and consideration of the above conditions on n_α, n_β and

n_γ reveals the following theorem. This theorem assumes no cancellations occur in the bracket term; however, it will be shown that the theorem holds regardless of cancellations.

Theorem 4.3.1. Given $\wedge_{IN}$ with the specified topology, then

$$n_P - n_Q = 0 \text{ or } 1. \qquad (4.3.8)$$

This is an important necessary realizibility condition for the synthesis of the RLC driving point function using only RC networks and active two-ports. Because a general RLC network may have a $\wedge_{IN}$ such that the degree of the numerator may be one less than the degree of the denominator it is immediately evident that all RLC driving point immittances cannot be synthesized. Further, Theorem 4.3.1 is not changed if cancellation occurs after expansion between terms in P(s) and Q(s) because the degree of each will be reduced by the same amount.

The above theorem was developed assuming that no cancellations occurred between the numerator and the denominator of the rational function of the bracket term. Cancellations of this type will be broken down into (i) Alpha cancellations and (ii) Beta cancellations. In order to show the effect of Alpha cancellation, assume $\pi(s + G_i)$ is common to $\pi(s + \alpha_i)$ and $\pi(s + \gamma_i)$. Then $\wedge_{IN}$ may be written as

$$\wedge_{IN} = \frac{\pi(s + G_i)(K_a\pi'(s + \alpha_i)K_b\pi(s + \beta_i) + K_1\pi'(s + \gamma_i^1)K_2\pi(s + \gamma_i^2))}{\pi(s + G_i)\; K_a\pi'(s + \alpha_i)\; K_b\pi(s + \beta_i)}$$

$$\cdot \frac{K_a\pi(s + \alpha_i)}{\pi(s + p_i)} \qquad , \qquad (4.3.9)$$

where the prime indicates the common factors have been extracted. Then,

$$\wedge_{IN} = \frac{\pi(s + G_i)(K_a\pi'(s + \alpha_i)K_b\pi(s + \beta_i) + K_1\pi'(s + \gamma_i^1)K_2\pi'(s + \gamma_i^2)}{K_b\pi(s + p)\ \pi(s + \beta_i)}$$

$$= \frac{K_aK_b\pi(s + \alpha_i)\pi(s + \beta_i) + K_1K_2\pi(s + \gamma_i^1)\pi(s + \gamma_i^2)}{K_b\pi(s + p_i)\ \pi\ (s + \beta_i)} \quad . \qquad (4.3.10)$$

Comparison of Equation 4.3.1 and Equation 4.3.10 shows that the degrees of P(s) and Q(s) are unaffected by Alpha cancellations and hence, Theorem 4.3.1 is not affected.

Beta cancellations occur whenever $\pi(s + \beta_i)$ and $\pi(s + \gamma_i)$ have common terms, say $\pi(s + G_i)$. Because n_β and n_α determine the degree of P(s), n_p is decreased as n_β is decreased by cancellation. But the denominator degree is given by n_β and n_p and is also decreased by the same amount. Hence, Theorem 4.3.1 holds for Beta cancellations, although the degree of P(s) and Q(s) is decreased by cancellation of the β_i terms.

Cancellation of p_i terms and the α terms, both of which are outside the bracket, are considered next. If some terms, say $\pi(s + G_i)$, are common between $\pi(s + \alpha)$ and $\pi(s + p)$, then Equation 4.2.5 may be written as

$$\frac{H_a\pi'(s + \alpha)\pi(s + G_i)\pi(s + \beta_i) + K\pi(s + \gamma_i^1)\pi(s + \gamma_j^2)}{H_a\pi(s + \alpha)\pi(s + G_i)\pi(s + \beta_i)}$$

$$\cdot \quad \frac{H_a\ \pi'(s + \alpha)\ \pi(s + G_i)}{\pi'(s + p_i)\ \pi(s + G_i)} \quad , \qquad (4.3.11)$$

$$= \frac{H_a\pi(s + \alpha)\pi(s + \beta) + K\pi(s + \gamma_i^1)\pi(s + \gamma_j^2)}{\pi(s + p_i)\ \pi(s + \beta_i)} \quad . \qquad (4.3.12)$$

Inspection of Equation 4.3.11 and Equation 4.3.12 indicates the degree of P(s) and Q(s) will not be altered. Further common terms for the p_i's and the numerator of the bracket term of Equation 4.3.11, and γ_i terms must be common to the α's and β's. If a β_i term is common, then a β cancellation would result and the degree of P(s) and Q(s) would be reduced equally as before. Therefore,

$$n_p = (n_\alpha + n_\beta) - (\text{number of } \beta \text{ cancellations}) \quad , \tag{4.3.13}$$

$$n_Q = (n_p + n_\beta) - (\text{number of } \beta \text{ cancellations}) \quad . \tag{4.3.14}$$

In achieving a desired $\wedge_{IN}$, procedures that involve cancellations should be avoided since this assumes that device and component values are known specifically. They are not. In discreet solid state devices typical parameter variation may be of the order 100%, or even greater while better than 1% tolerance is not uncommon for passive components. However for solid circuits the tolerance of passive devices is 20% to 50% and hence, dependence upon cancellation should be avoided.

4.5 Synthesis of Y_{IN}. This section develops the actual synthesis technique to obtain a given driving point admittance. The conditions imposed on the passive parameters in Section 3.6 to obtain the desired composite parameter p-z structure for a particular active device are restated explicitly in terms of the y-parameters and under the assumption that the active device is a transistor. The steps to achieve the specified p-z plot of Y_{IN} are outlined and several examples are worked in order to (i) show how synthesis may be achieved by the root-locus method and (ii) to demonstrate how the root-locus technique may be used to quickly determine if a given passive structure can be used to synthesize a given Y_{IN}.

The driving point admittance is given by

$$Y_{IN} = y_{11}\left[1 - \frac{y_{12}\ \ y_{21}}{y_{11}(y_{22} + G_L)}\right] , \tag{4.4.1}$$

$$= \frac{H_a\pi(s + \alpha_i)}{\pi(s + p_i)}\left[1 + \frac{K_1\pi(s + \gamma_i^1)\ K_2(s + \gamma_i^2)}{K_aK_b\pi(s + \alpha_i)\pi(s + \beta_i)}\right], \tag{4.4.2}$$

$$= \frac{H_a\pi(s + \alpha_i)}{\pi(s + p_i)} \cdot \frac{K_3^1\ \pi(s + s_i)}{\pi(s + \alpha_i)(s + \beta_i)} ,$$

$$= \frac{P(s)}{Q(s)} = \frac{K_3\pi(s + s_i)}{\pi(s + q_i)} . \tag{4.4.3}$$

The roots of Q(s), the q_i's, are achieved by selecting the passive network such that

$$\pi(s + p_i)\pi(s + \beta_i) = \pi(s + q_i) . \tag{4.4.4}$$

From Equation 4.3.14 the degree of Q(s) is given by

$$n_Q = (n_p + n_\beta) - (\text{number of beta cancellations}) . \tag{4.4.5}$$

However, if beta cancellations are not allowed,

$$n_Q = n_p + n_\beta . \tag{4.4.6}$$

The roots of P(s), the s_i's, which are the zeros of the numerator of the bracket term, are achieved by selecting the γ_i's such that as the roots of P(s) move from the α_i's and β_i's to the γ_i's, as K_1K_2/K_aK_b is varied, their paths cross the desired s_i points on the s-plane. The constant to achieve this is then determined.

In order for β_i to be the desired value, Equation 3.6.7 implies that for the transistor

$$y_{22}^{a}/H_b = k_{22}/H_b \qquad \ll \qquad 1. \tag{4.4.7}$$

This condition places the zeros, the β_i's, of the composite y_{22} close to the zeros of y_{22}^{p}. This still leaves the α_i's and γ_i's to be determined. The α's may be placed at the desired point by invoking Equation 3.6.4.

$$H_a/y_{11}^{a} \;=\; H_a/k_{11} \qquad << \qquad 1, \tag{4.4.8}$$

for the y-parameters and for a transistor as the active device. This places the α_i's close to the poles of y_{11}^{p}.

Further the degree of n_p is given by

$$n_p = n_\alpha + n_\beta \;. \tag{4.4.9}$$

The γ_i's are placed in the desired location by making the γ_i's close to the zeros of y_{12}^{p} and placing the zeros of y_{12} in the desired location. The γ_i's will be as desired if

$$k_{12}/a \qquad << \qquad 1 \;, \tag{4.4.10}$$

according to Equation 3.6.9.

The γ_i^2's will be close to the poles of y_{12}^{p} if,

$$a/k_{21} \qquad << \quad 1 \;. \tag{4.4.11}$$

Since the p_i's must be selected to (i) place the γ_i^2's and the α_i's and (ii) to satisfy Equation 4.4.4, inconsistencies may arise. The addition of private poles in the more complex case will give a greater degree of freedom.

Instructions for root-locus synthesis techniques are as follows;

1. Plot the p-z structure of Y_{IN} on the s-plane.
2. Determine n_p, n_α, n_β, n_a, n_b and n_γ from the above equations for number of beta cancellations.
3. Write down the general form of the passive parameters.
4. Place the p_i's and the β_i's such that Equation 4.4.4 is satisfied and such that the β_i's meet the necessary conditions derived in Chapter III.
5. Indicate the γ_i^2's and the α_i's close to the p_i's.
6. Place the zeros of y_{12}^p at the points the γ_i's are needed to achieve the desired root-locus track.
7. Sketch the root-locus.
8. Determine the needed value of K_1K_2/K_aK_b using a spirule.

One important advantage of this method is that very quickly the required passive two-port may be determined and then the synthesis can proceed in the passive domain where synthesis techniques are extremely well formulated. As in all root-locus techniques for high degree polynomials the actual determination of numerical values is loborious, although today, many computer techniques using root-locus plots are available whenever numerical values are needed(9). As in all root-locus techniques the amount of cut and try effort or the amount of engineering judgement necessary is decreased as experience in the technique increases. This is certainly in contrast to existing passive synthesis techniques that are strictly algorithms. However, these later techniques often require networks that are balanced bridges or rely on pole zero cancellations. These last conditions in turn require component values to be extremely accurate. Another advantage is that for a given Y_{IN}

p-z structures, it may be quickly determined that a particular passive network will not work. Further, it may be said that a specified driving point admittance with complex zeros can be achieved by passive RC synthesis techniques which, in general, do not rely on a balanced bridge network or p-z cancellation.

Next several examples of the root-locus technique will be given to demonstrate how this method places the actual synthesis in the passive domain. The method is also used to determine if the driving point admittance can be achieved by a particular passive network topology. In particular, the synthesis of biquadratic driving point admittances is considered. From Theorem 4.2.1 it is known that only a Y_{IN} with real poles may be synthesized with the specified topology.

Example 4.4.1. The problem is to synthesize a driving point admittance given by

$$Y_{IN} = \frac{(s + \bar{s}_1)(s + s_1)}{(s + q_1)(s + q_2)} = \frac{P(s)}{Q(s)} \quad . \qquad (4.4.12)$$

Following the above instruction, the p-z structure of Y_{IN} is drawn in Figure 4.4.1. Further, for a biquadratic with no beta cancellations n_p is given by

$$n_p = n_\alpha + n_\beta = 2, \qquad (4.4.13)$$

and

$$n_Q = n_p + n_\beta = 2. \qquad (4.4.14)$$

Therefore

$$n_\alpha = n_p = n_\beta = n_p = 1 \ , \qquad (4.4.15)$$

$$n_a = n_b = n_\gamma = 1 \ . \qquad (4.4.16)$$

n_a is one because n_α must be one and n_a cannot be lower than n_α. Then the form of the passive two-port parameters must be

$$y_{11}^{p} = \frac{H_a(s + a_1)}{s + p} , \quad -y_{12}^{p} = -y_{21}^{p} = \frac{a(s + z_1)}{(s + p_1)} ,$$

$$\text{and } y_{22}^{p} = \frac{H_b(s + b)}{(s + p)} . \qquad (4.4.17)$$

In accordance with statement four of Section 4.4,

$$p_1 = q_2. \qquad \beta_1 = q_1. \qquad (4.4.18)$$

Note that p_1 and β_1 must be in the order indicated by Equation 4.4.18 because β_1 must be placed as desired by locating a zero of y_{22}^{p} at that point and from passive synthesis it is known that the first critical frequency of y_{22}^{p} is a zero(7).

The bracket term of Equation 4.4.2 will furnish the root-locus plot and in terms of this example it becomes

$$1 + \frac{K_1K_2(s + \overset{1}{\gamma_1})(s + \overset{2}{\gamma_1})}{K_aK_b(s + \alpha_1)(s + \beta_1)} . \qquad (4.4.19)$$

The p-z plot of the rational function of Equation 4.4.19 is shown in Figure 4.4.2. The placement of $\overset{1}{\gamma_1}$ is discussed next. $\overset{1}{\gamma_1}$ will be located by the placement of z_1 which, in contrast to a_1 or b_1, may be to the right or left of p_1. It should be placed such that the needed root-locus plot is obtained. Then, its placement is as Figure 4.4.2 indicates. Note in Figure 4.4.2 that only β_1, p_1, s_1, and $\bar{s}_1$ show up in the p-z plot of Y_{IN}. Further, p_1 was only shown in Figure 4.4.2 in order to place $\overset{2}{\gamma_1}$, and α_1, as p_1 is not actually involved in the root-locus plot. The necessary constant K_1K_2/K_aK_b should then be determined.

Next a sequence, Figure 4.4.3, of root-locus plots is shown in order to illustrate how the p-z plot of Figure 4.4.2 was obtained. p_1 is indicated in all the parameter plots since it is common to all the passive parameters. In part a, α_1 starts on p_1 and moves toward a_1; similarly β_1 starts on p_1 and moves toward b_1. However because the active device, a transistor in this case, is extremely non-symmetric with $k_{11} \gg k_{22}$, for a given H_a and H_b, α_1 will be close to p_1 and β_1 will be close to b_1. Even if a symmetric passive network is used, the orders of magnitude between k_{11} and k_{22} will prevail. A similar argument applies to Figure 4.4.3 part c and d with respect to the ultimate location of γ_1^1 and γ_1^2. However, in plotting the root-locus of y_{21} it must be remembered a negative sign is involved and a plot such as indicated in part d applies, rather than part c. Further, both α_1 and γ_1^2 are close to the p_1; however γ_1^2 will be closer if

$$k_{21}/a \gg k_{11}/H_a \quad .$$

From passive synthesis it is known that(7)

$$H_a \geq a,$$

and for the transistor

$$k_{21} > k_{11} \quad .$$

Therefore, if the various parts of Figure 4.4.3 are added Figure 4.4.2 will result.

Although further refinements are necessary, such as determining the exact location of α_1, β, γ_1^1 and γ_1^2 and what A is needed for the active device, the crux of the problem has been solved and now, the problem is to synthesize a passive two-port with parameters given by Equation 4.4.17 or

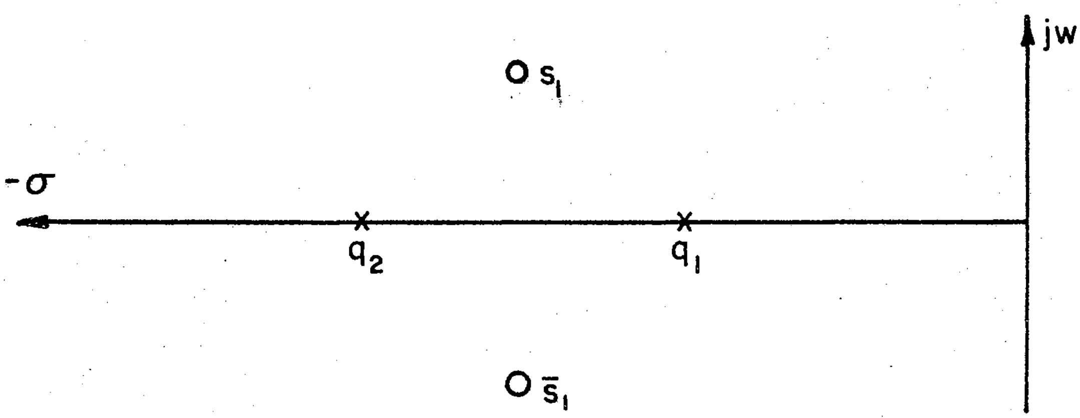


Figure 4.4.1. Pole-zero Structure of Equation 4.4.12.

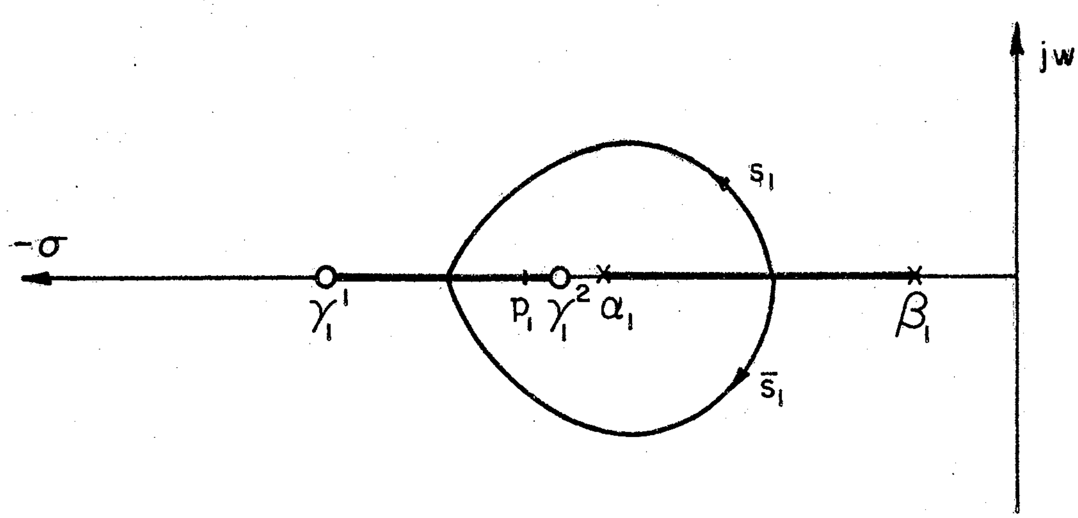
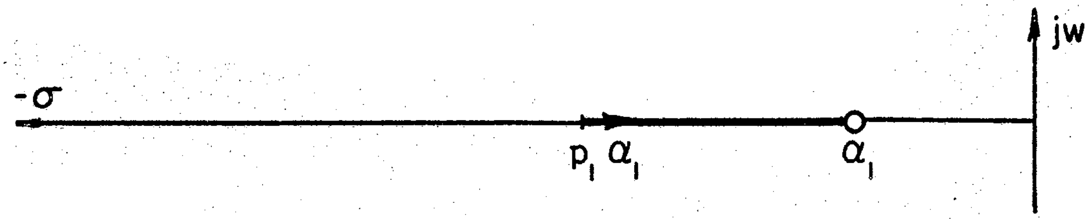
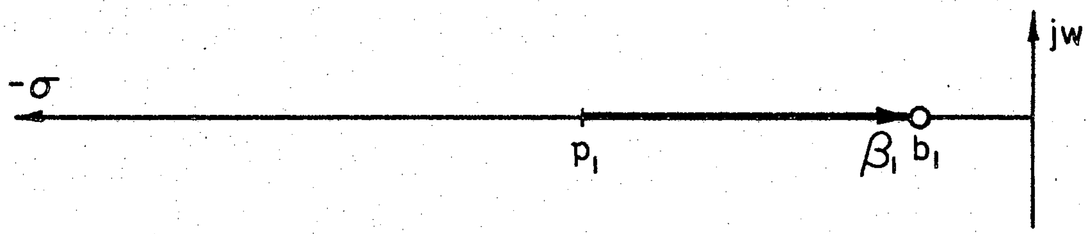


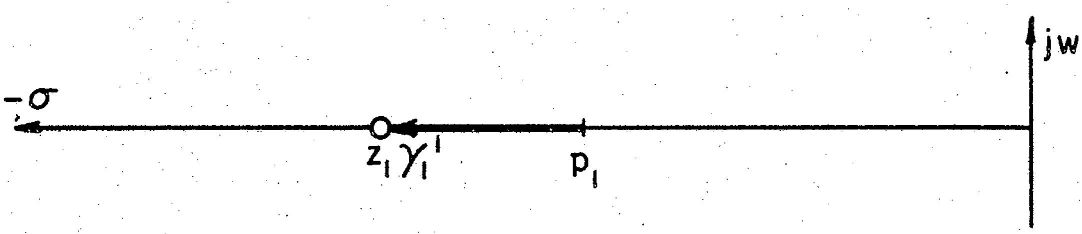
Figure 4.4.2. Pole-Zero Plot of the Rational Function of Equation 4.4.19 and the Resulting Root-locus Plot.



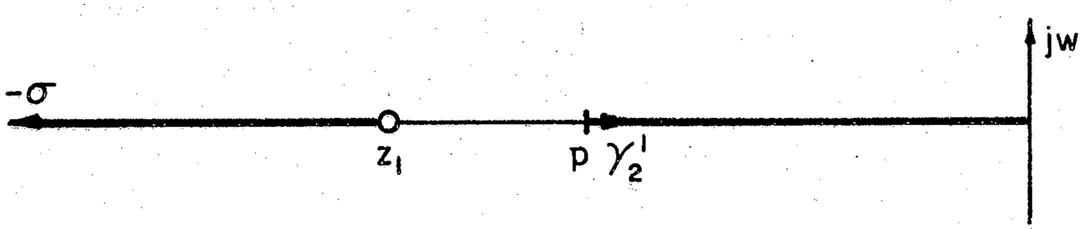
(a). The zeros of y_{11} is obtained from y_{11}^{p} for a given k_{11}.



(b). The zero of y_{22} is obtained from y_{22}^{p} for a given k_{22}.



(c). The zero of y_{12} is obtained for a given k_{12}.



(d). The zero of y_{21} is obtained for a given k_{21}.

Figure 4.4.3. The Root-locus Plot of the Zeros of y_{ij}. The dark line indicates the root-locus, while the triangles indicates the directions of the roots and the anticipated stopping point.

$$\left[y_{ij}^{p} \right] = \begin{bmatrix} \dfrac{H_a\ (s + a_1)}{s + p_1} & \dfrac{-a(s + z_1)}{s + p_1} \\ \dfrac{-a(s + z_1)}{s + p_1} & \dfrac{H_b(\ s + b_1)}{s + p_1} \end{bmatrix} .$$

The next example demonstrates how the root-locus method may be used to reject certain passive network topology. The same Y_{IN} is specified. Initially, it appears desirable to allow $\gamma_1^{\ 1}$ and $\gamma_2^{\ 1}$ to be complex conjugates equal to the desired s_1 and $\bar{s}_1$. This could be achieved by using a bridged-tee passive network and making y_{22}^{p} have complex zeros. Here the bridged-tee is used to achieve complex zeros, and not to obtain a rejection band pass network. Inspection of the bracket term of Equation 4.2.5 shows that if beta cancellations are allowed to cancel the γ_i^2 terms, then the roots of P(s) will move from the α_i's and the β_i's toward the predetermined complex points γ_1^1 and $\bar{\gamma}_1^{\ 1}$. It will be shown in the next example, however that these cancellations cannot occur.

Example 4.4.2. Test the bridged-tee of Figure 4.4.4 to determine if a biquadratic driving point admittance can be achieved by using it as the passive two-port. For Figure 4.4.4 the y-parameters are, in general,

$$y_{11}^{p} = \frac{H_a(s + a_1)(s + a_2)}{(s + p_1)(s + p_2)} , \tag{4.4.20}$$

and because of symmetry

$$y_{22}^{p} = \frac{H_a(s + a_1)(s + a_2)}{s + p_1} , \tag{4.4.21}$$

$$y_{12}^{p} = \frac{-a(s + z_1)(s + \bar{z}_1)}{(s + p_1)(s + p_2)} . \tag{4.4.22}$$

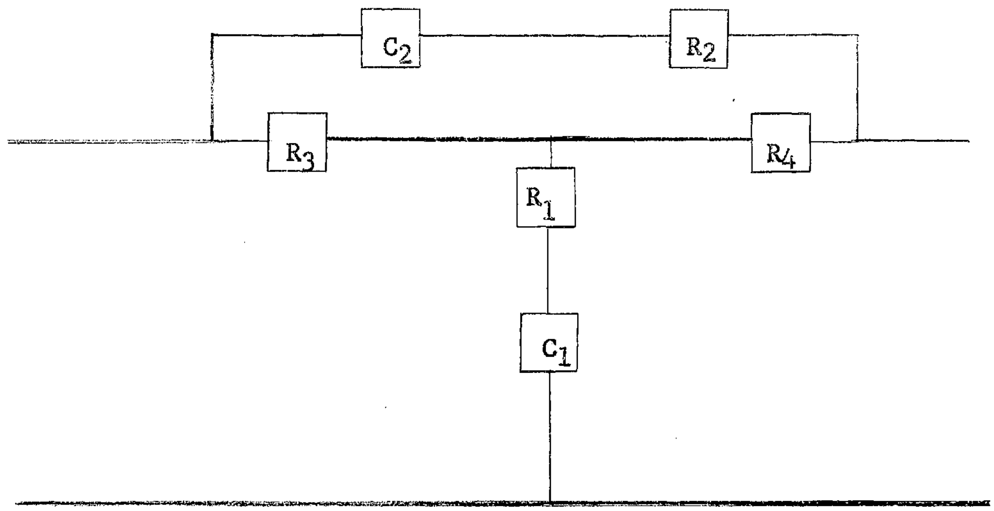


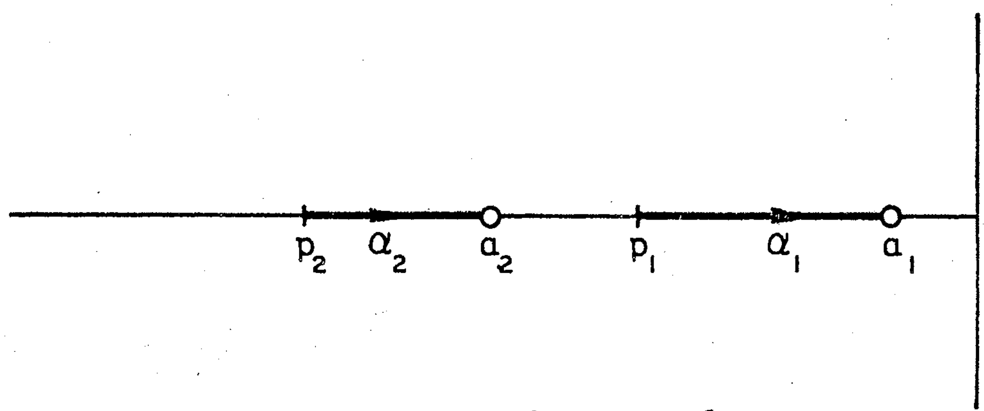
Figure 4.4.4. A Bridged-tee that can have Complex Zeros in y_{12}^{P}.

From Equation 4.3.13 and Equation 4.3.14 and inspection of the equations of the passive parameters gives

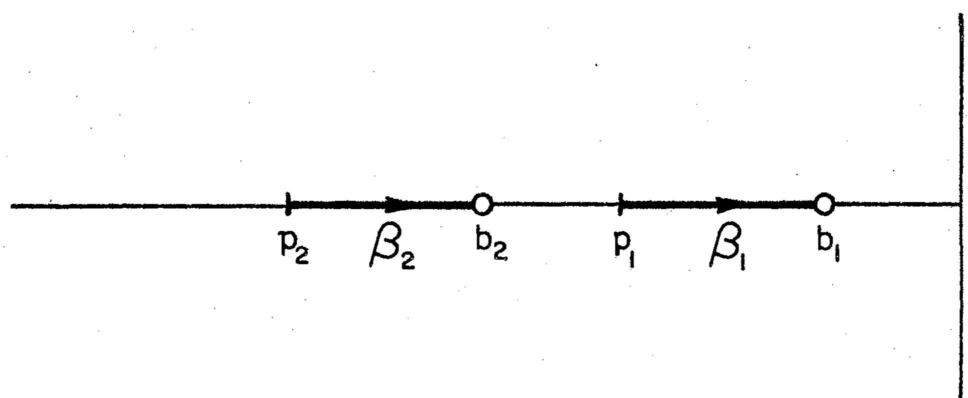
$$n_P = n_\alpha + n_\beta - \text{number of } \beta \text{ cancellations} = 2 \;, \tag{4.4.23}$$

$$n_Q = n_\beta + n_p - \text{number of } \beta \text{ cancellations} = 2 \;. \tag{4.4.24}$$

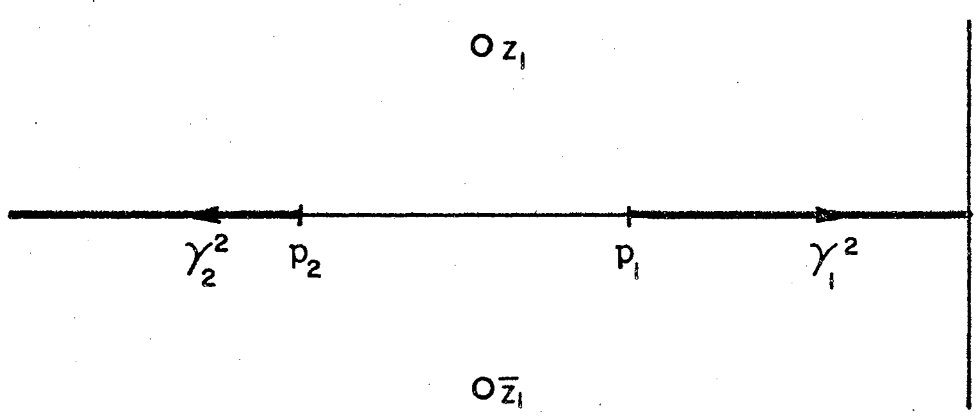
Since $n_\alpha + n_\beta = n_\beta + n_p = 4$, then two beta cancellations must occur. Inspection of the root-locus plots, Figure 4.4.5, of the zero movement of y_{ij} shows that only one γ_1^2 can cancel with a beta; hence Y_{IN} will be a cubic. That is, part c shows that only one γ_1^2 will overlap with a β_i on the real axis to obtain a beta cancellation. In fact, a necessary condition for a beta cancellation is that the γ_i be on the real axis since the β_i's must be on the real axis. If beta cancellations are not allowed Figure 4.4.6 shows the movement of the zeros of Y_{IN} for the bridged-tee.



(a). Root-locus plot for zeros of y_{11}.



(b). Root-locus plot for zeros of y_{22}.



(c). Root-locus plot for zeros of y_{21}.

Figure 4.4.5. Root-locus Plots of the Zeros of y_{ij}.

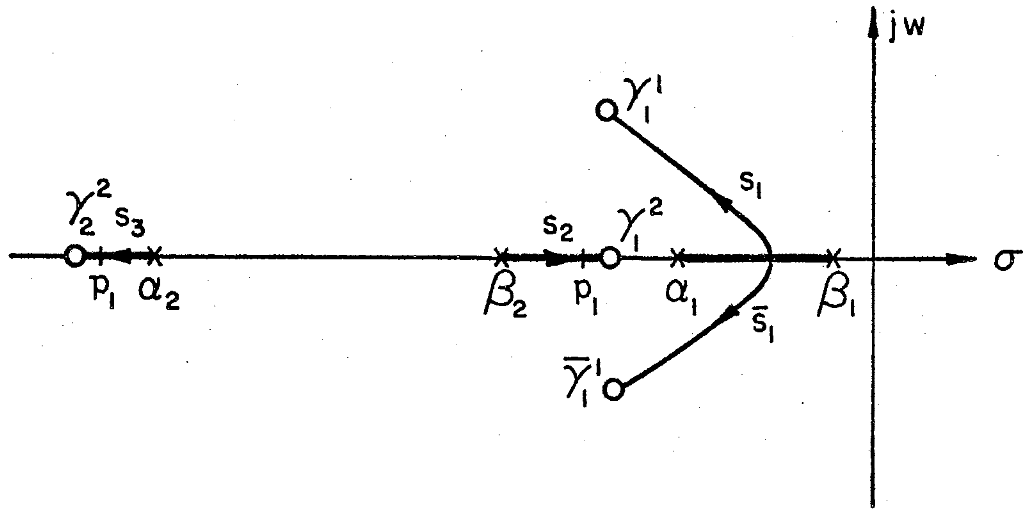


Figure 4.4.6. Root-locus Plot of the Zeros of Y_{IN} for a Bridged-tee with no beta Cancellations.

Figure 4.4.6 is for analysis or if a fourth degree numerator with one set of complex zeros is desired the figure indicates how they may be placed. From inspection of Figure 4.4.6, Y_{IN} is given by

$$Y_{IN} = \frac{H(s + \bar{s})(s + s_1)(s + s_2)(s + s_3)}{(s + p_1)(s + p_2)(s + \beta_1)(s + \beta_2} \quad . \tag{4.4.25}$$

The placement of the α_i's and the β_i's is justified by the fact that they must be interlaced with the p_i's on the negative real axis. Due to the non-reciprocal nature of the active two-port, it is known that the γ_i^1's and the γ_i^2's will be adjacent to the z_i's and p_i's respectively.

From Example 4.4.1 and Example 4.4.2 the conclusion can be drawn that to achieve a biquadratic driving point admittance it is necessary to start with real poles and zeros in the passive parameters, if no beta cancellations are allowed. Further, $n_a = n_b = n_p = 1$.

By proceeding according to the instructions for synthesis, various topological networks may be found to be acceptable for the synthesis of a specified Y_{IN}. As in all synthesis techniques, there is no one answer, instead it will be found that a number of passive networks will suffice. In particular, it seems desirable to place the zeros of y_{12} in the desired s_i locations, as in the case of bridged-tee, and then drive the zeros of the bracket term into them. In this way the pole-zero sensitivity will be minimized to variations in A.

Although the synthesis instructions were given in terms of the y-parameters, a procedure using the z-parameters could also have been developed if a terminal common to the two two-ports was not required. However, if z-parameters had been used with a transistor for the active element, then the root-locus plot for λ_{11} and λ_{22} would be changed to this extent. Since $\lambda_{11}^{a} < \lambda_{22}^{a}$ for the z-parameters, it is desirable to place the zeros of λ_{11} on the zeros of λ_{11}^{p} and the zeros of λ_{22} on the poles of z_{22}. The procedure was stated in order to synthesize RLC functions using only a transistor and RC elements; however, the procedure holds using only RL elements, but the allowable passive pole-zero structure would be different.

Although the active element considered was a transistor, any active device exhibiting non-reciprocal and non-symmetrical characteristics may be employed and the synthesis instructions will hold. The placement of the poles and zeros will be dependent upon the relative magnitudes of the active parameters and which set is used.

4.5 Stability Considerations of the Composite Two-port. An expression for the driving point immittance is

$$\wedge_{IN} = \lambda_{11}\left[1 - \frac{\lambda_{12}\lambda_{21}}{\lambda_{11}(\lambda_{22} + \lambda_L)}\right] = \frac{H\pi(s + s_i)}{\pi(s + q_i)} = \frac{P(s)}{Q(s)} . \qquad (4.5.1)$$

From Theorem 4.2.1 it is known that the roots of Q(s) are on the negative real axis; but, the roots of P(s) may be complex and in the right half plane. Since $\wedge_{IN}$ is either an admittance or an impedance, then its reciprocal is either an impedance or an admittance respectively. Further, since a two-port is considered to be potentially unstable if either its driving point admittance or impedance has poles in the right half plane. The two-port will be potentially unstable if $\wedge_{IN}$ has zeros in the right half plane. In particular, if λ_{21} or λ_{12} has zeros in the right half plane, the zeros of $\wedge_{IN}$ will tend toward them for large values of **A** which will result, ultimately, in potential instability. λ_{21} is particularly susceptible to having zeros in the right half plane because of the minus sign occuring in the root-locus plot. That is, a zero of λ_{21} will always move from the first pole on the negative real axis toward the right half plane.

A zero of λ_{21} or λ_{12} in the right half plane is neither necessary nor sufficient for the composite two-port to be potentially unstable. For, even if λ_{12} or λ_{21} has a zero in the right half plane, if **A** is small enough the zero of $\wedge_{IN}$ will be closer to the α_i's or the β_i's which must be in the left half plane. If λ_{21} and λ_{12} have no zeros in the right half plane, and if the numerator of the rational function of the bracket term is less than the denominator by m, say, then the zeros of $\wedge_{IN}$ will leave m of the α_i's and β_i's and asymptotically approach infinity at angles defined by $180 \pm 2\pi n$. Zeros of $\wedge_{IN}$ can occur, then, without zeros of λ_{12} and λ_{21} being in the right half plane. However, if the numerator and denominator of the bracket term are of the same order and all the zeros of λ_{12} and λ_{21} are in the left half plane, then this is a sufficient condition for absolute stability.

CHAPTER V

SYNTHESIS OF TRANSFER FUNCTIONS

5.1 Introduction. In this chapter the actual synthesis technique is developed for transfer functions in terms of the y-parameters. The active device considered is the transistor. Initially, the possible transfer function for the composite two-port is determined in terms of the generalized parameters. These are then interpreted in terms of the y-parameters and a synthesis procedure is determined. In particular, the generation of complex poles using only RC or RL networks and active devices is considered. In completing the synthesis procedure developed, which includes active devices, a passive RC two-port is specified that will, when synthesized, give the composite transfer function. Hence, the actual synthesis algorithm is performed in the passive domain.

5.2 Two-port Transfer Functions. For a composite two-port defined as in Chapter III, the transfer function, in terms of the generalized parameters, is given by

$$\frac{v_2}{v_1} = \frac{G_\lambda}{\lambda_o} = \frac{-\lambda_{21}}{1 - \dfrac{\lambda_{12}\lambda_{12}}{\lambda_{11}\lambda_{22}}} \cdot \frac{1}{\lambda_{11}\lambda_{22}} \qquad (5.2.1)$$

$$\frac{G_\lambda}{\lambda_o} = \frac{-(\lambda_{21}^p + \lambda_{21}^a}{\left[\dfrac{-(\lambda_{12}^a + \lambda_{12}^p)(\lambda_{21}^a + \lambda_{21}^p}{(\lambda_{11}^p + \lambda_L + \lambda_{11}^a)(\lambda_{22}^p + \lambda_s + \lambda_{22}^a)} + 1\right] \cdot \lambda_{11}\lambda_{22}} \cdot \qquad (5.2.2)$$

$$= \frac{\dfrac{-K_2\pi(s+\gamma_i^2)}{\pi(s+p_i)}}{\left[1+\dfrac{K_1K_2\,\pi(s+\gamma_i^1)\pi(s+\gamma_i^2)}{K_aK_b(s+\alpha_i)\pi(s+\beta_i)}\right]} \cdot \frac{1}{\left[\dfrac{K_aK_b\pi(s+\alpha_i)\pi(s+\beta_i)}{\pi(s+p_i)\pi(s+p_i)}\right]} . \tag{5.2.3}$$

The γ_i's are the shifted roots of λ_{ij} due to k_{ij} and the α_i's and β_i's are the shifted roots due to k_{11} and λ_s, and k_{22} and λ_L respectively. Note that only λ_{11} differs from the previous definitions. λ_{11} now has the source immittance included such that the units of λ_s agree with those of λ_{11}. λ_0, a constant, is $-G_s, R_L$, G_s, $-R_L$, or unity if the parameter set used is the g,y,z or h respectively. v_2/v_1 is then defined to be the voltage gain from source to load. The K's are defined as in Chapter III, Equation 3.2.10, Equation 3.2.11 and Equation 3.2.12 and hence, depend upon the degrees of the numerator and denominator of the passive parameters.

Equation 5.2.3 may be written as

$$\frac{G_\lambda}{\lambda_0} = \frac{-K_2\,\pi(s+\gamma_i^2)\,\pi(s+p_i)}{\left[K_aK_b\pi(s+\alpha_i)\pi(s+\beta_i)+K_1K_2\pi(s+\gamma_i^1)\pi(s+\gamma_i^2)\right]} \tag{5.2.4}$$

$$= \frac{H\pi(s+\gamma_i^2)\,\pi(s+p_i)}{\pi(s+r_i)} = \frac{U(s)}{V(s)} .$$

Theorem 5.2.1. G_λ, the voltage transfer function for a composite two-port consisting of an active two-port and a two-port containing only RL or RC elements can have complex poles and zeros.

Proof:

By Theorem 4.2.2 the denominator of Equation 5.2.4 can have complex roots and by Theorem 3.4.1 the numerator may have complex roots.

Because $n_\gamma \leq n_\beta$ or n_α, the degree of the denominator is given by

$$n_V = n_\beta + n_\alpha \quad , \tag{5.2.5}$$

if no cancellations occur. In which case, the degree of the numerator is

$$n_U = n_{\gamma 2} + n_p \quad . \tag{5.2.6}$$

Theoretically, $n_{\gamma 2}$ may range from zero up to n_p, so that $n_V - n_U =$ 0, 1, 2, $\cdots\cdots$ $n_p + 2$.

However, for the active two-port defined in Chapter II, the tendency is for

$$n_{\gamma 2} = n_p \quad , \tag{5.2.7}$$

unless k_{21} is reduced until the active two-port is no longer active. Since this is undesirable it will be assumed that Equation 5.2.7 holds. That is, the synthesis procedure should be such that the root shifting should enhance the natural tendencies.

The effect of specifying λ_L and λ_S on the zero shifting of λ_{ii} is as follows. The zeros, of the composite two-port parameters, λ_{jj}, which exhibited proximity with the zeros of λ^p_{jj}, will move toward the poles of λ^p_{ii} since λ_L and λ_S are positive constants. In fact, for cascaded stages and where $\lambda^a_{ii} < \lambda^a_{jj}$, the zeros of λ_{jj} will be in the neighborhood of the zeros of λ_{ii}, which are close to the poles of λ^p_{ii}. For examples, if the transistor is the active device, the zeros of h_{22} and y_{22} will move into the neighborhood of the p_i's whereas, it is the zeros of z_{11} and g_{11} that are moved from the a_i's to the p_i's. Since λ_S and λ_L affect the pole-zero structure of the voltage transfer function only by adding to λ^a_{11} and λ^a_{22} respectively, the active two-port may be considered to be a two-port with a λ_{11}^{a} and a λ_{22}^{a} which are the result of the sum of the original two-port λ_{ii} and λ_S and λ_L. If $\lambda_S = \lambda_{22}$ and $\lambda_L = \lambda_{11}$, which occurs for cascaded stages, then the net effect is to create an active two-port that is symmetrical. Therefore, one characteristic of an active two-port is no longer available to shape the pole-zero structure as before. Instead, the α_i's and the β_i's will both be close to the p_i's as will be the γ_i^2's. This only leaves the zeros of the γ_i's to be placed by selecting the location of the zeros of the passive two-port parameters.

5.3 General Synthesis Techniques. The instructions for synthesizing voltage transfer functions are as follows:

1. Plot the pole-zero structure of G_λ on the s-plane.
2. Determine n_α, n_β, n_γ, n_a, n_b, n_z and n_p from Equation 5.2.5, Equation 5.2.6 and Equation 5.2.7 and from passive considerations.
3. Write down the general form of the passive parameters.
4. Place the p_i's such that the α_i's, β_i's and γ_i's are in the desired location and such that the p_i's meet the real pole specifications.
5. Place the zeros of y_{12}^{p} at the points the γ_i's are needed to achieve the desired root-locus track.
6. Sketch the root-locus of the individual parameters, λ_{ij}, as indicated in Chapter III, to verify the resulting pole-zero plot of instructions four and five.
7. Sketch the root-locus of G_λ.
8. Determine the necessary gain constant.

With respect to instruction number two, a passive requirement is that n_z must not exceed n_p for the h^p and g^p parameters; however, for the z^p and y^p parameters, n_z may be one larger than n_p. Further, for the y^p_{11}, g^p_{11} and h^p_{11} parameters the zeros of y_{ii}, g_{11} and h_{22} will be to the right of the given p_i, while the zeros of z_{ii}, g_{22} and h_{11} will be to the left of the given p_i. This occurs because the locus of the zero movement lies between the poles and zeros on the real axis and the first critical frequency for the first group above is a zero, while it is a pole for the latter group.

It is immediately apparent from inspection of instruction number four that certain specified voltage transfer functions may lead to incompatible constraints. While certainly the inability to synthesize all voltage transfer functions is undesirable, the ability to determine those that cannot be realized is also important. Also, since the p_i's always appear in G_λ, regardless of whether they are desired, their effect on the circuit may be minimized by placing them some distance further from the origin than the dominant specified critical frequencies. Furthermore, by permutting the parameters defining λ_0, G may become a current transfer function, a transimpedance or a transadmittance function. Because of their similarity, G_λ will be considered to be a voltage transfer function i.e., a transvoltage function.

5.4 Synthesis of Filters. The preceding synthesis procedure will be used to synthesize and determine what filters may be synthesized using RC elements and active elements. The band pass filter is of particular importance to the synthesis problem. Band pass filters such as the Tchebysheff, the Butterworth and the Bessel are of particular interest because of their wide spread use and because their frequency response approaches the ideal. Depending upon the given specifications, one type is preferred over the other. For example, the Bessel filter has good transient response, no overshoot, but a very poor steady state cutoff response. On the other hand the first two types have sharp cutoff characteristics, but a transient overshoot. However, all three types have two things in common. (i). The low-pass filters are all pole transfer functions. (ii). The poles are, in general, complex. These voltage transfer functions are normally obtained by using inductors and capacitors to achieve the complex poles. At low servo frequencies the inductors become prohibitively large and in solid state networks the inductors are not available so other means must be used to obtain the complex poles. With active two-ports defined as in Chapter II, the only possible way to achieve complex roots using only resistors and capacitors is to use feed-back loops or in essence, use the topological two-port configurations specified in Chapter II.

In order to achieve complex roots without the use of transformers and to maintain a common terminal between the input port and the output port it is necessary to develop the procedure for the transvoltage function in terms of the y-parameters. In terms of the y-parameters Equation 5.2.3 becomes

$$\frac{G_y}{G_s} = \frac{-K_2\,\pi(s+\gamma_i^2)}{\left[1+\dfrac{K_1K_2\pi(s+\gamma_i^1)\pi(s+\gamma_i^2)}{K_aK_b\pi(s+\alpha_i)\pi(s+\beta_i)}\right]\left[\dfrac{K_aK_b\pi(s+\alpha_i)\pi(s+\beta_i)}{\pi(s+p_i)}\right]} \tag{5.4.1}$$

$$= \frac{-K_2\pi(s+\gamma_i^2)\,\pi(s+p_i)}{K_3'\,\pi(s+r_i)} = \frac{U(s)}{V(s)}\,, \tag{5.4.2}$$

where the r_i's are the roots of $K_aK_b \, \pi(s + \alpha_i)\pi(s + \beta_i) + K_1K_2 \, \pi(s + \gamma_i^1)\pi(s + \gamma_i^2)$. For Equation 5.4.2 to be an all pole function $n_p = 0 = n_\gamma$. However if $n_p = 0$, then from Chapter III and passive synthesis, it is known that n_α and n_β are equal to one at most. Therefore the highest order filter possible is a second order filter. The order of an all pole filter is the degree of the denominator polynomial. Once again it is assumed that no cancellations are to be allowed between U(s) and V(s). If $n_\alpha = 1 = n_\beta$, then $n_a = 1 = n_b$. This means that the passive two-port parameters must have a pole at infinity since $n_p = 0$. Because no private poles are allowed, the passive two-port must have the form

$$\begin{bmatrix} y_{ij}^p \end{bmatrix} = \begin{bmatrix} H_a(s + a_i) & -a(s + z_i) \\ -a(s + z_i) & H_b(s + b_i) \end{bmatrix} . \qquad (5.4.3)$$

However, by Theorem 4.2.2 if the numerator degree of the passive two-port exceeds the denominator degree the ultimate placement of the roots of the bracket term of Equation 5.4.1 will not be a function of A of the active two-port. Hence, there is no guarantee that complex roots will exist.

Therefore, in order to achieve a filter with the all poles characteristics, say, of the low-pass Butterworth filter, n_p must be allowed to be larger than one. The effect of these poles must be minimized by making the poles five to ten times larger than the cutoff frequency. An example of the synthesis procedure for a Butterworth filter will now be worked out.

Example 5.4.1. Synthesize a low-pass second order Butterworth filter with a normalized cutoff frequency of one radian per second.

The degrees of the various polynomials are determined as instruction number two indicates. In order to achieve minimum pole-zero sensitivity with respect to the active and passive network the γ_i's will be forced to terminate on the z_i's of the passive network. This requires that $n_\gamma = 2$ and in order to have a minimum number of unwanted poles $n_p = 2$. From passive synthesis it is known that n_α and n_β cannot be less than two. The general form of the passive two-port will then be

$$\left[y^p_{ij}\right] = \begin{bmatrix} \dfrac{H_a(s + a_1)(s + a_2)}{(s + p_1)(s + p_2)} & \dfrac{a(s + s_1)(s + \bar{s}_1)}{(s + p_1)(s + p_2)} \\ \dfrac{a(s + s_1)(s + \bar{s}_1)}{(s + p_1)(s + p_2)} & \dfrac{H_a(s + a_1)(s + a_2)}{(s + p_1)(s + p_2)} \end{bmatrix} . \qquad (5.4.4)$$

Note that a symmetric two-port is specified since the only requirement on the a_i's, b_i's and p_i's is that they will be removed from the dominant critical frequency. The p_i's will arbitrarily be set four times larger than the filter cutoff frequency, say, at 4.1 and 4.2. From a root-locus plot it is known that the α_i's and β_i's will lie somewhere between the a_i's and p_i's. The a_i's, interlaced with the p_i's, will be placed at 4.1 and 4.3. Now a should be selected such that $k_{21} > a$ so that the γ_i^2's, which appear in the numerator of G_λ, will also be four times larger than the cut-off frequency. Further, a should also be selected such that $a > k_{12}$. This will insure that the γ_i's will rest on the desired zeros of $-y^p_{12}$ in accordance with Equation 3.6.9.

Rewritting Equation 5.4.1 for this example gives

$$\frac{G_y}{G_s} = \frac{-K_2(s + \gamma_1{}^2)(s + \gamma_2{}^2)}{\left[1 + \frac{K_1K_2(s + \gamma_1{}^1)(s + \gamma_2{}^1)(s + \gamma_2{}^1)(s + \gamma_2{}^2)}{K_aK_b\ (s + \alpha_1)^2\ (s + \alpha_2)^2}\right]} \cdot$$

$$\frac{1}{\dfrac{K_aK_b(s + \gamma_1)^2(s + \gamma_2)^2}{(s + p_1)\ (s + p_2)}} \quad . \qquad (5.4.5)$$

Because it is necessary that r_1 and $\bar{r}_1$ terminate on their ultimate destination it is desirable to have K as large as possible. This is in contrast to other procedures where the roots passed through the desired location and a particular value of K was determined.

$$K = \frac{(a + k_{12})(k_{21} - a)}{(H_a + k_{11} + G_S)(H_a + k_{11} + G_L)} \quad . \qquad (5.4.6)$$

Inspection of Equation 5.4.6 shows that k_{22} has been replaced with k_{11} because the non-symmetrical nature of the active two-port was destroyed as is often the case for cascaded transistor stages. Because the α_i's and β_i's do not affect the root-locus plot, it is not necessary to select H_a such that α_i and β_i terminate on the pole. Therefore, H_a may be selected such that the largest K will result. From passive synthesis it is known that a cannot exceed H_a, so H_a should be chosen such that a may have the proper magnitude(9). The two-port y-parameters for a good planar transistor whose output parameter includes the effect of the next stages is

$$\begin{bmatrix} y^a_{ij} \end{bmatrix} = \begin{bmatrix} 1 \times 15^{-4} & -1 \times 10^{-6} \\ 5 \times 15^{-2} & 1 \times 10^{-4} \end{bmatrix} \quad . \qquad (5.4.7)$$

By selecting H_a to equal k_{11} and by magnitude scaling the expression for the passive two-port that will achieve a Butterworth filter low pass response becomes

$$\left[y_{ij}^{p}\right] = \begin{bmatrix} \dfrac{1 \times 10^{-4}(s + 4.1)(s + 4.3)}{(s + 4.2)(s + 4.4)} & \dfrac{-8 \times 10^{-5}(s^2 + 2^{\frac{1}{2}}s + 1)}{(s + 4.2)(s + 4.4)} \\ \dfrac{-8 \times 15^{5}(s^2 + 2^{\frac{1}{2}}s + 1)}{(s + 4.2)(s + 4.4)} & \dfrac{1 \times 10^{-4}(s + 4.1)(s + 4.3)}{(s + 4.2)(s + 4.4)} \end{bmatrix} . \tag{5.4.8}$$

The zeros of y_{12}^{p} are selected to be the zeros of the Butterworth polynomial and the poles of y_{12}^{p} are the same as the poles of y_{11}^{p}. From the initial discussion in this section and from Example 5.4.1 it is apparent that the synthesis of all pole functions or filters, without allowing pole-zero cancellations to occur, is difficult. Computer runs of the frequency response for the transvoltage function of the above two-ports verified this. That is, the response was not ideal. This is particularly true of high order filters. One reason for this is that the p_i's show up as zeros in the transvoltage function. It is then necessary to make them larger than the cutoff frequency by a number of octaves. This creates difficulties which may be observed from inspection of Equation 5.4.9.

$$1 + \frac{K_1K_2\ \pi(s + \gamma_1^{\ 1})\ \pi(s + \gamma_1^{\ 2})}{K_aK_b\ \pi(s + \alpha_i)\ \pi(s + \beta_i)} \quad . \tag{5.4.9}$$

The root-locus plot of Equation 5.4.9 determines the poles of V(s). It is necessary in a n^{th} order filter for n roots to have close proximity to the n $\gamma_i^{\ 1}$'s. For this to occur K_1K_2/K_aK_b must be larger than the product of the α_i's; however, the α_i's are located between the poles and zeros of y^p_{11} and the larger the p_i's are, the larger the α_i's will be. For the extreme case the roots of V(s) will be closer to the α_i's than the γ_i^1's. Therefore in order to achieve an all pole transvoltage function, pole-zero cancellation must be allowed if the poles are driven into their ultimate destination.

Example 5.4.2. Synthesize a high-pass second order Butterworth filter with a normalized cutoff frequency of one radian per second. The transvoltage function of a second order Butterworth filter with a normalized cutoff frequency of one radian per second is given by,

$$\frac{s^2}{s^2 + \sqrt{2}\, s + 1} .$$

Although this function cannot be realized exactly because the poles of the passive parameters appear in the numerator, the poles, p_1 and p_2, may be placed very near the origin, say at .06 and .1. Providing the a_i's are interspaced with the p_i's, they may be arbitrarily placed along the negative real axis, say at .05 and .09. As in Example 5.4.1, the passive two-port is symmetric. The numerator of y^p_{12} is the second order Butterworth polynomial. By an argument similar to the one given for Example 5.4.1, the passive two-port must have the general form given by Equation 5.4.4. Therefore,

$$\left[y^p_{ij}\right] = \begin{bmatrix} \dfrac{(s+.05)(s+.09)}{(s+.06)(s+.1)} & \dfrac{-a(s+\sqrt{2}s+1)}{(s+.06)(s+.1)} \\ \dfrac{-a(s+\sqrt{2}s+1)}{(s+.06)(s+.1)} & \dfrac{(s+.05)(s+.09)}{(s+.06)(s+.1)} \end{bmatrix} . \quad (5.4.10)$$

The figures are drawn according to the synthesis instructions as follows. The idealized pole-zero plot of a second order high pass filter is shown

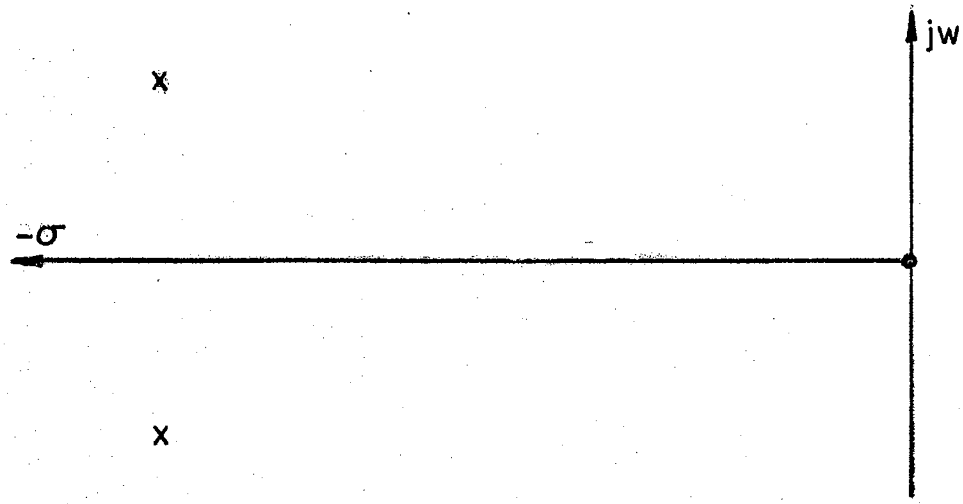


Figure 5.4.1. The Idealized Pole-Zero Plot of a Second Order Butterworth Filter.

$$G_s = \frac{s^2}{s^2 + \sqrt{2}\, s + 1}$$

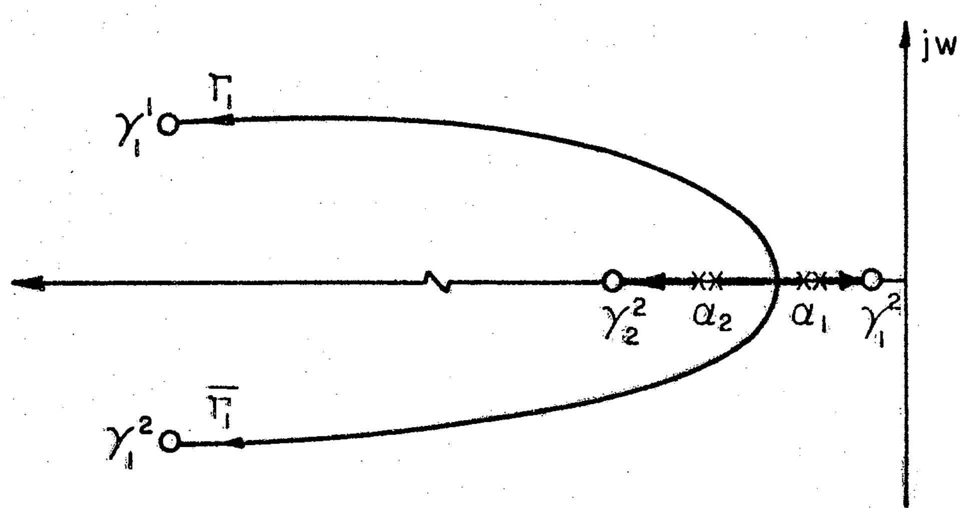
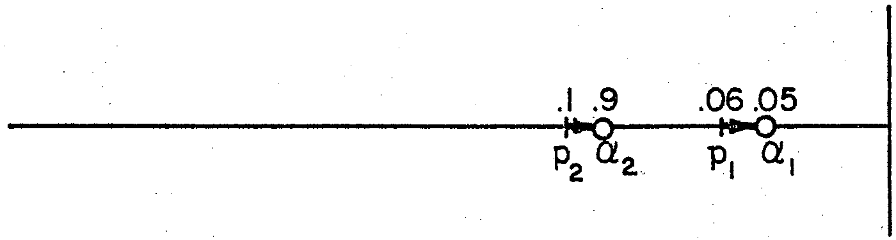
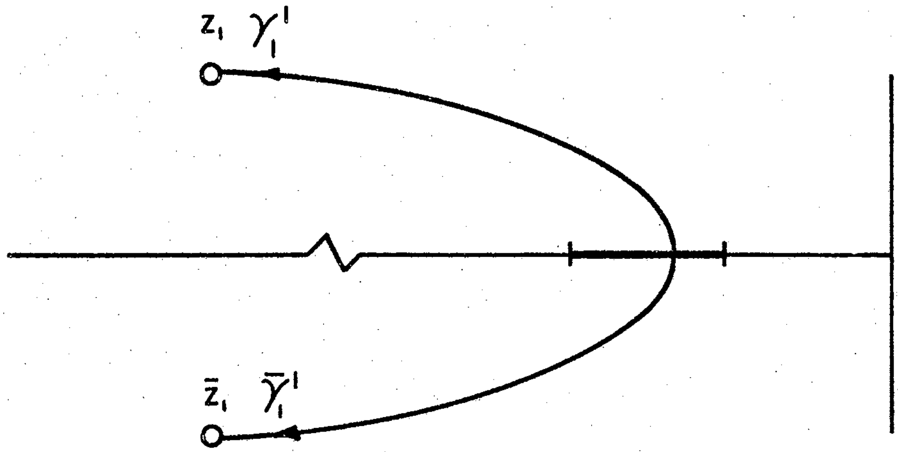


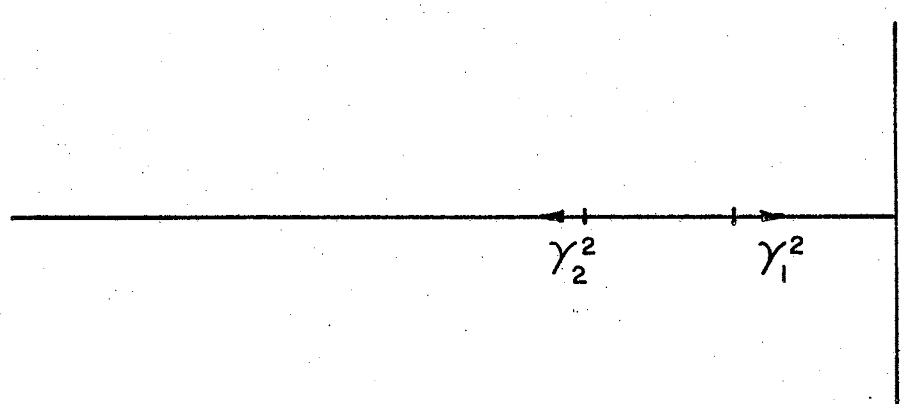
Figure 5.4.2. The Root-locus Plot of the Bracket Term of Equation 5.4.11 for a Second Order Low-pass Butterworth Filter.



(a). y_{11} and y_{22}.



(b). y_{12}.



(c). y_{21}.

Figure 5.4.3. The Root-locus Plot and the Pole-zero Structure of the Individual Parameters.

in Figure 5.4.1. Assuming an active two-port such as given by Equation 5.4.7, the transvoltage function becomes,

$$\frac{G_y}{G_s} = \frac{K_2(s + \gamma_1^{\,2})(s + \gamma_2^{\,2})}{\left[1 + \dfrac{K_1K_2(s + \gamma_1^{\,1})(s + \gamma_2^{\,1})(s + \gamma_2^{\,2})(s + \gamma_1^{\,2})}{K_aK_b\ (s + \alpha_1)^2\ (s + \alpha_2)^2}\right]} \cdot \frac{1}{\dfrac{K_aK_b\ (s + \alpha_1)^2\ (s + \alpha_2)^2}{(s + p_1)(s + p_2)}} \qquad (5.4.11)$$

Figure 5.4.2 shows the root-locus plot of Equation 5.4.11. The individual parameter plots are shown in Figure 5.4.3.

Once again it is desirable to make K_1K_2/K_aK_b as large as possible. Further, $\gamma_1^{\,2}$, $\gamma_2^{\,2}$, p_1 and p_2 may be made as close to the origin as desired. This allows the poles of G_y to ultimately end on the $\gamma_i^{\,1}$'s as desired. Figure 5.4.4 shows the magnitude frequency response of the composite two-port obtained by programing Equation 5.4.11 on a computer. See Appendix C. Inspection of this figure shows the roll off is about 10db/ octive instead of the idealized 12db/ octive for an ideal filter. This is due to the zeros not being exactly at zero. The response is down 3db at one radian. Therefore it appears feasible to achieve a high-pass filter response using only RC elements and active devices. Further, the problem has been reduced to that of synthesizing a passive twoport given by Equation 5.4.10.

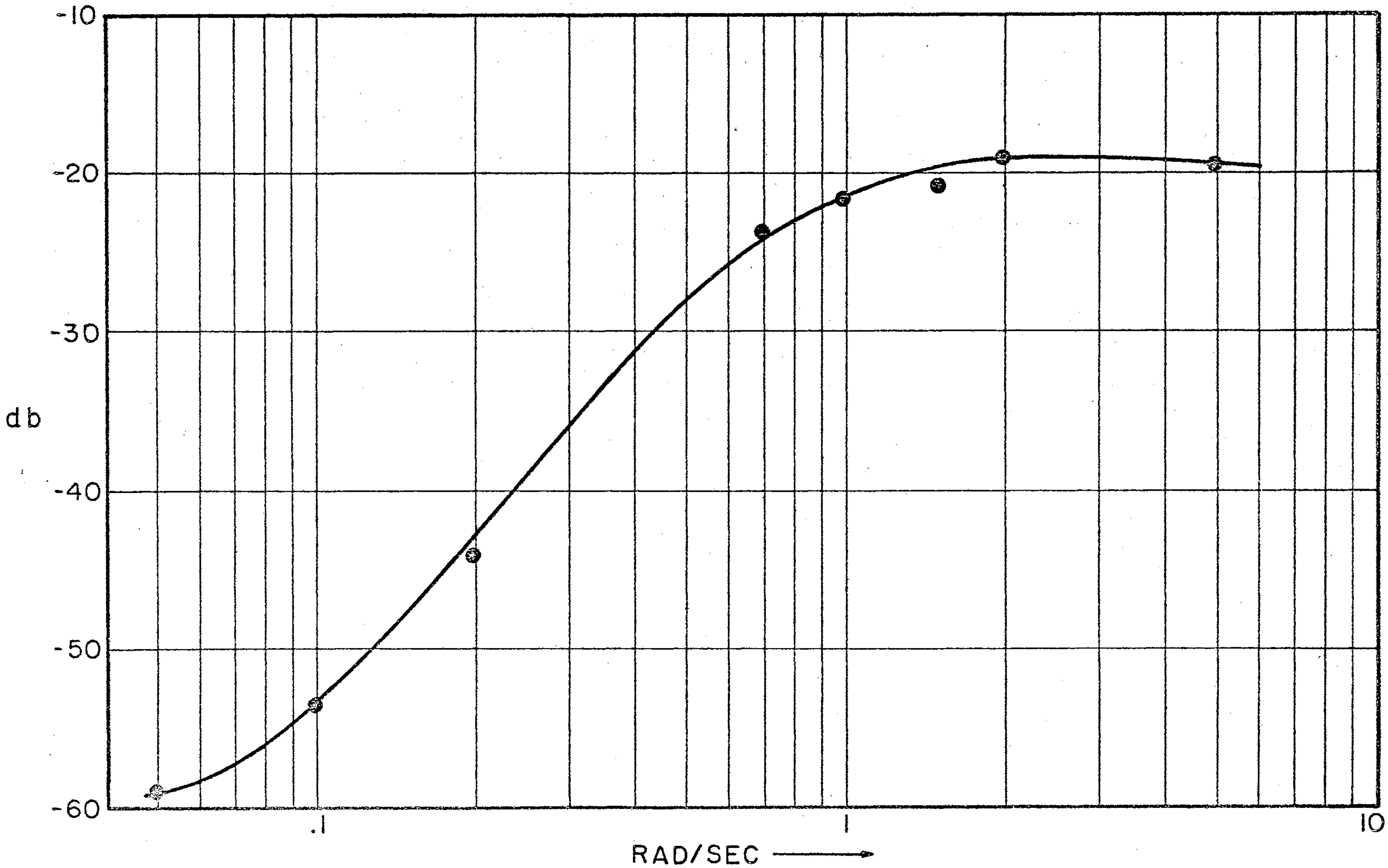


Figure 5.4.4. Magnitude Response of Example 5.4.2.

CHAPTER VI

SUMMARY

6.1 Results and Conclusions . The general results of this thesis are as follows. Active two-ports were classified and the pole-zero structure of the generalized composite two-port parameters were determined. The necessary conditions which the overall composit two-port system functions must meet were derived. A synthesis technique based on the theory of the root-locus was developed to synthesize these system functions. The end result of the synthesis method is that a passive two-port is specified. Or in other words the problem of active synthesis is reduced to passive synthesis where extensive methods and techniques exist.

In particular, a restricted class of RLC system functions are synthesized using only RC or RL elements and active two-ports. The active two-ports contain only unmodified active devices that do not exhibit negative impedance. The non-reciprocal and unsymmetrical nature of the active devices are exploited. The procedure ultimately specifies an RC or RL passive two-port. Further, the final network is not dependent upon a balanced bridge. The method of synthesis developed also indicates when certain passive network topology will not suffice.

The actual synthesis procedure is accomplished using the y-parameters. This circumvents the need for transformers and further, a grounded network may be used. This is an important fact in communication networks.

Specifically, the problem of biquadratic driving point admittances was considered and an example was worked out. Also, an approximate second order Butterworth high pass transfer function was synthesized. The actual frequency response of the composite two-port transfer function was determined on an IBM 1620 computer. The frequency reponse was very close to that of an ideal second order filter.

6.2 Recommendations for Further Study. In the strictly passive area there is a need to develop synthesis procedures using h and g parameters. In particular, the networks that have common denominators for the h and g parameter sets, respectively, need to be determined. Also, as more work is being done with two-ports, instead of just two parameters of the four, the sufficient condition for the synthesis of a passive RC or RL two-port needs to be determined.

One area for future study would be the actual computer programing of the root-locus synthesis technique. Because the root-locus technique has been programmed numerous times to solve servo problems with good results, the same would be true here.

The study could be broadened to include the ABCD parameters for cascaded stages, since these parameters would also add. However, RLC functions could not be synthesized with RC two-ports and active two-ports. The method would furnish a systematic approach to filter design of cascaded stages. Particularly, whenever the stages are not isolated as in transistor circuits.

Further, it appears that the extension of the root-locus method of synthesis would be appropriate for the case where the active two-port parameters are frequency dependent. For the high frequency case the composite two-port λ_{ij}, will have a denominator polynomial whose roots are known. The numerator will be the sum of two polynomials and may be factored such that the movement of the roots may be determined by root-locus techniques. Also, their ultimate destination will be a function of the magnitude factors and hence, the ultimate pole-zero structure can be arranged as desired. This will allow the input impedance to be compensated for high frequency drop off and unilaterization to be achieved. Further, for the high frequency case it seems desirable to allow transformers.

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APPENDIX A

APPENDIX A

SIGN CONVENTION

A.1 Sign Convention on the Passive Parameters.

$$-y^p_{12} = \frac{a\pi(s + z_i)}{\pi(s + p_i)} = -y^p_{12}$$

$$z^p_{12} = \frac{+a\pi(s + z_i)}{\pi(s + p_i)} = z^p_{21}$$

$$h^p_{12} = \frac{a\pi(s + z_i)}{\pi(s + p_i)} = -h^p_{21}$$

$$-g^p_{12} = \frac{a\pi(s + z_i)}{\pi(s + p_i)} = g^p_{21}$$

$$y^a_{12} = -k^y_{12}$$

$$z^a_{12} = k^z_{12}$$

$$h^a_{12} = k^h_{12}$$

$$g^a_{12} = -k^g_{12}$$

$$y^a_{21} = k^y_{21}$$

$$z_{21}^{a} = -k_{21}^{z}$$

$$h_{21}^{a} = k_{21}^{h}$$

$$g_{21}^{a} = -k_{21}^{z}$$

The requirements on the constants are as follows:

$a > 0$, $H_a > 0$, $H_b > 0$ and $k_{ij} > 0$.

APPENDIX B

APPENDIX B

SYMBOLS

B.1 Statements Concerning Symbols.

a_i denotes zeros of λ_{11}^p.

α_i denotes zeros of λ_{11}, that is, they have been shifted.

b_i denotes zeros of λ_{22}^p.

β_i denotes zeros of λ_{22}.

z_i denotes zeros of y_{12}^p and y_{21}^p.

p_i denotes poles of λ_{ij}^p.

γ_i^1 denotes zeros of λ_{12}.

γ_i^2 denotes zeros of λ_{21}.

s_i denotes zeros of Y_{IN}.

r_i denotes poles of G_λ.

q_i denotes poles of Λ_{IN}.

$n(s)_{ij}$ = numerator of λ_{ij}^p .

$d(s)_{ij}$ = denominator of λ_{ij}^p.

$N(s)_{ij}$ = numerator of λ_{ij}.

$D(s)_{ij}$ = denominator of λ_{ij}.

$P(s)$ = numerator of Λ_{IN}.

$Q(s)$ = denominator of Λ_{IN}.

$U(s)$ = numerator of G_λ.

$V(s)$ = denominator of G_λ.

n_a = degree of numerator of λ_{11}^{p}.

n_b = degree of numerator of λ_{22}^{p}.

n_p = degree of denominator of λ_{ij}^{p}.

n_α = degree of numerator of λ_{11}.

n_β = degree of numerator of λ_{22}.

n_z = degree of numerator of λ_{ij}, $i \neq j$.

$n_{\gamma_1}{}^{1}$ = degree of numerator of λ_{12}.

$n_{\gamma_1}{}^{2}$ = degree of numerator of λ_{21}.

n_P = degree of numerator of Λ_{IN}.

n_Q = degree of denominator of Λ_{IN}.

n_U = degree of numerator of G_λ.

n_V = degree of denominator of G_λ.

APPENDIX C.

APPENDIX C.

COMPUTER PROGRAM

C.1 Fortran Program for Computation of the Magnitude of Equation 5.4.11 as a Function of ω.

```
        ω  =  .05.
  10    A  =  .00036*(1. + (ω/.06)**2)*(1. + (ω/.1)**2) .
        B  =  .00036*(1. + (ω/.06)**2)*(1. + (ω/.1)**2).
        C  =  40.*SQRT((1. +  (1.4*ω + 1.)**2)*(1. + (1.4*ω-1.)**2)).
        D  =  .006*SQRT((1. + (ω/.06)**2)*(1. + (ω/.1)**2)).
        GFCN = 8.686*LOG(A/(B + C*D)).
        Punch 12, GFCN.
  12    FORMAT  (F8.4).
        ω  =  ω + .05.
        IF(ω -1.1)10,10,14.
  14    END.
```

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Professional Experience: Entered the Navy, 1951; attended fire-control, submarine, sonar and submarine fire-control schools for a year and half. Served as a fire-control technician aboard submarines until 1955. Field engineer for Sun Oil Company, summer 1959, Beaumont, Texas. Employed school year 1959-1960 by Oklahoma State University as lab instructor. During summer 1960, taught Differential Equations to a National Science Foundation group. Taught senior and graduate level courses in Communication Theory as Asst. Professor at the University of Kentucky from 1960-1962. Employed by Texas Instruments, Dallas, Texas, in the summers of 1961, 1962 and 1964, as a Senior Engineer. Worked in the Modular Electronics group the first two summers and in the Anti-radiation Missles Group the last summer. Employed as a circuit consultant for Metals and Control, Versailles, Kentucky, 1960-1962. Taught junior level courses at Oklahoma State University from 1962-1965 as an instructor.

Professional Organizations: Member of the Institute of Electrical and Electronics Engineers.