OSU
Collection

Name: Arvind K. Shah Date of Degree: August, 1965

Institution: Oklahoma State University

Location: Stillwater, Oklahoma

Title of Study: A SURVEY OF AVAILABLE LITERATURE ON OPTIMUM DESIGN OF STRUCTURES

Pages in Study: 66 Candidate for Degree of Master of Science

Major Field: Civil Engineering

Scope of study: A survey of available literature on optimum design of structures is the scope of this report. The material presented in this report is an evaluation of the current literature pertaining to optimum design of structures. The principal methods of approach to achieve optimum design of structures are included in this report. The stress control method of getting optimum design is presented and its application to several problems is shown. Selected bibliography of the available literature with short descriptions of the contents is also included for further references and detailed study.

Findings and Conclusions: None.

ADVISER'S APPROVAL J. S. Kuo

A SURVEY OF AVAILABLE LITERATURE ON OPTIMUM DESIGN OF STRUCTURES

By

ARVIND K. SHAH

Bachelor of Engineering (Civil)
Maharaja Sayajirao University
Baroda, India

Submitted to the Faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
August, 1965

A SURVEY OF AVAILABLE LITERATURE ON
OPTIMUM DESIGN OF STRUCTURES

Report Approved:

J. V. Rao
____
Report Adviser

R L Janes
____

J H Boggs
____
Dean of the Graduate School

ACKNOWLEDGEMENTS

In completing the final phase of his graduate study, the writer wishes to express his sincere gratitude to the following individuals.

To his adviser, Dr. J. S. Kao, for his guidance, assistance, and cooperation for completion of this report.

To Dr. W. O. Carter, for suggesting this topic for study and his guidance and encouragement in the preparation of this report.

To Dr. D. M. MacAlpine, Dr. R. L. Janes, Prof. L. O. Bass, and Mr. R. K. Munshi for their valuable instruction and encouragement throughout his graduate work.

To Mr. M. N. Reddy and B. K. Agrawal for their help and encouragement in his studies.

To his parents for their complete understanding and sacrifices to provide for his education.

To Mr. and Mrs. R. V. Shah, his uncle and aunt, for their guidance, encouragement and understanding throughout his education.

To his friends who have provided financial assistance for his studies.

Finally, to his wife, Daksha, whose love, understanding, and continual encouragement gave him the incentive to continue graduate studies.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 The problems of optimum design are historically connected with the design of structures. The methods of determining the form of a structure have developed along with the designing of structures of various kinds. The criterion of choice of configuration depends on conditions necessary to be satisfied by the structure. Absolute criteria of optimum design, valid for all time, do not exist. The aims and trends of optimum design and of structural design are essentially the same -- for example, the tendency to determine the form of a structure, the acceptance of the relationship between the form and the internal forces, the tendency to reduce the weight and volume of the structure, and the necessity to determine in advance the conditions to be satisfied by the structure, as well as the quality of materials to be used.

Analytical optimum design is an improvement on and a complement of the first stage of design of structures; that is, choosing a form. The next stage, the control of strength and economic aspects of the structure, is carried out by the designer regardless of the way in which the form of the structure was determined. In optimum design only that part of the cost of a structure is considered directly which is connected with the volume of material. The influence of different conditions and of the lifetime of the structure on the overall cost, as also indirect expenditure, are disregarded.

The interdependence between internal forces and form is the fundamental factor in optimum design. The designer uses this dependence in a twofold way: by determining the form on the basis of the system of known internal forces, and by determining the internal forces appearing in an element of a given form. First, the form and dimensions of the structure are determined. The theory of optimum design aims at replacing intuitive drafting of a structure by analytical methods so that it may comply with strength conditions. Thus designing without the theory of optimum design consists in the formulation of assumptions and in varifying them by calculation; the optimum design makes it possible to determine the exact form directly on the basis of given strength conditions and designer indicates to the site engineer the form of the structure, the material to be used, and the methods of realization.

One of the objectives of optimum design is to obtain the desired structure from a minimum quantity of material. When the form of a structure is determined, which depends on constructional requirements and on the current state of knowledge in technical mechanics, the achievement of this aim is verified analytically on the designs prepared by calculations of the volume of materials used. By contrast, when the theory of optimum design is applied, the procedure is reversed. With the condition of constant volume for the minimum of deformability, maximum economy in materials is ensured in advance. These methods lead to the known mathematical problems of extremes of certain functions satisfying definite conditions.

1.2. Scope of report:

In this report the principal methods of approaches of designing structures are outlined in general. Stress control procedure is presented with its application to several problems.

The historical developments of the principal methods are described in Chapter II.

The general discussion, principles and assumptions, etc., of each method are outlined in brief in Chapter III.

In Chapter IV the stress control procedure is presented and the type of structural actions defined. The procedure is compared with Normal design procedure.

Chapter V consists of the application of the stress control procedure with numerical problems.

Selected references are given in the Bibliography with the purpose to collect maximum literature pertaining to optimum design of structures. Wherever possible the brief description of the contents in the respective references is also included for ready reference.

CHAPTER II

HISTORICAL SURVEY OF THE DEVELOPMENTS OF OPTIMUM DESIGN OF STRUCTURES

2.1. The development of optimum design of structures is connected with the development of technical civilization. The form of a building or other structure results from a design process. Various criteria for determining the form of a structure may be formulated; for instance, its intended use, its appearance or its strength.

It is difficult to fix historically the original attempts to determine the form of a structure as regards its strength. This must probably have coincided with the application of the notion of force and the laws of mechanics as a foundation for designing structures. It is generally accepted that this period began in 1638 with Galileo. Since that time theoretical mechanics has constituted a base for the development of structures.

Methods of design largely consist in trials and unintentional experiments, both in the sense of direct design and of working out calculation assumptions. Trials and experiments generally lead to the result sought. The best method of optimum design is to conduct planned trials on natural scale structures or on models. Such trials were carried out by Galileo and Euler. At present, the model method of testing the suitability of a structure for the purpose for which it was designed is still often considered the best for complicated systems.

Methods of verification by calculating the strength of a structure are of much later date than unintentional trials and are based on the relations observed between, on the one hand, the form, the dimensions and the properties of the material, and, on the other, the strength of the structure. For these relations, a number of strength theories have been established, among which three groups can be distinguished. They are based on the hypothesis concerning:

(1) the rupture of the material

(2) the acceptable value of the safety factor

(3) the quality of the structure.

The methods of verification of strength are now well developed and have found wide-scale uses. The extent of such methods is connected on the one hand with the existence of a deeper understanding of the properties of materials and, on the other hand, with the development of necessary mathematical methods for the analysis of more complicated structures. These include the method of admissible stresses or strains and the method of ultimate load. The common feature of these methods is that the form of the structure is assumed beforehand intuitively or on the basis of experience and then checked. If the internal forces satisfy the strength theory assumed, the structure is safe, but the reasons for assuming its form are not verified. Thus, this method has also an element of trial and error.

The modern development of design methods represents several independent directions, among which the following are of special interest.

1. The design of statically indeterminate structures of uniform strength.
2. Minimum strain energy for a given material volume.
3. Minimum weight design.
4. Other less known methods in which the form of the structure is the object of the research.

The design of structures of uniform strength is the object of studies by many Russian scientists. The historical facts of development of the method of uniform strength is started with Galileo. He tested a bent beam and obtained a result that the normal stresses in extreme fibers was uniform in every cross section. Afterward Bernoulli (1687), Newton (1687), Lagrange (1773), Young (1807), St. Venant (1864), and Russian scientists after 1900 have experimented and offered different recommendations. The method of uniform strength proved to be useful since parameters of the form can be deduced from the criterion of uniform strength.

The problem of determination of the form of a structure for minimum strain energy is being studied in Poland by Z. Wasiutynski and his associates. In 1939 he published some basic theorems with examples concerning the form of steel beams and historical survey of design methods. Again in 1950 and 1951 he published other papers on the subject. In the last decade a number of authors have contributed. The principle was applied to prestressed concrete beams by A. Brandt in 1957-58.

The minimum weight design is now based on the theory of plasticity. Research in this field is principally conducted by W. Prager at Brown University in the United States, and in recent years many authors have contributed on this subject. H. Hilton and M. Feign considered the problem of the best form on the basis of the probability. The general features of structure design in the plastic range have been determined by Drucker and Shield R. T.

CHAPTER III

GENERAL DISCUSSION OF STRUCTURAL DESIGN

3.1 General: A structure is designed to perform a certain function. To perform this function satisfactorily it must have sufficient strength and rigidity. Economy and good appearance are further objectives of major importance in structural design.

The complete design of a structure is likely to involve the following five stages:

1. Establishing the general layout to fit the functional requirements of the structure possible.
2. Consideration of the several possible solutions that may satisfy the functional requirements.
3. Preliminary structural design of the various possible solutions.
4. Selection of the most satisfactory solution, considering an economic, functional and aesthetic comparison of the various possible solutions.
5. Detailed structural design of the most satisfactory solution.

Both the preliminary designs of stage three and the final detailed design of stage five may be divided into three broad phases, although in practice these three phases are usually interrelated. First, the action of loads on the structure must be determined. Next the maximum stresses in the members and connections of the structure must be analyzed. Finally, the members and connections of the structure must be dimensioned, i.e., the make up of each part of the structure must be determined. Three steps are interrelated: (a) the weight of the structure itself is one of the loads that a structure must carry, and this weight is not definitely known until the structure is fully designed, in a statically indeterminate structure the stresses depend on the elastic properties of the members, which are not known until the main members are designed. Thus in a sense the design of any structure proceeds by successive approximations. For example, it is necessary to assume the weights of members in order that they may be properly designed. After the structure is designed, the true weights may be computed and unless the true weights correspond closely to those assumed, the process must be repeated.

The main three methods of approach to achieve the optimum design are as follows, which are on the basis of:

1. Uniform strength of indeterminate structures.
2. Minimum weight design.
3. Minimum strain energy for a given material volume.

3.2. Uniform Strength of Indeterminate Structures: In method of uniform strength, some of the parameters describing the form of a structure are assumed to be unknown and determined as a function of internal forces depending on strength theory adopted.

The material is assumed to be homogeneous, and elastic. Uniform strength is connected with uniform extremum stresses equal to the admissible stresses. The values of these stresses can be different for different members of the structure, and may depend on their sign. In most of the problems so far solved, the extreme stresses are assumed to be absolutely equal over the entire structure. The internal forces are determined by means of the equations of the theory of bending. The stability problem is disregarded, as also the influence of other secondary design problems on the form of the members and the influence of shear forces. The dead weight is assumed to be an external load. The live load, concentrated or uniformly distributed, may be arranged in any manner, provided the extremum value of the forces is reached in every cross section. The problem involves determination of a structure of minimum weight.

The design proceeds by establishing the conditions for which the first variation of the volume with respect to the inverse-rigidity becomes zero. Under such conditions, the variables are the coordinates of the points of change of sign of bending moment and normal forces. After determining the location of points where internal forces are equal to zero, such can be treated as fictitious hinges. The statically determinate structure thus obtained can easily be given such dimensions that definite stresses appear in every particular cross section. In the case of one load system, the location of the zero point is constant, and although they are considered to constitute the hinges, the relative rotations of neighboring parts are zero so that the continuity of the structure is preserved. If we consider a number of different load systems, the locations of zero points are different. In this case, the solution in the form of a minimum weight structure exists if consideration is given to the variability of the reactions which can act on the structure with arbitrary forces within certain limits. By means of zero points, statically indeterminate structures are replaced by statically determinate ones. The design, therefore, involves selecting from a set of structures of uniform strength the structure which satisfies the minimum material conditions.

3.3. Minimum Weight Design: The minimum weight design is now based on the theory of plasticity. Some parameters of form are determined from the condition that the ultimate state of the structure by the plastic flow of material is achieved under a given load for minimum material consumption.

The object, of the papers on research concerning the design of structures for minimum weight, is the determination of the form parameters which are: the cross sectional area of the elements of a frame or that of a continuous beam, constant over the length of each bar. The external load is assumed in the form of concentrated or continuously distributed forces. These loads are applied at fixed points and are constant in time in almost every work. The form parameters are determined from the condition of minimum material volume by considering the plastic collapse under a given load. For frames and beams, the collapse takes place as a result of such plastification of material that deformations increase indefinitely with constant values of external forces and a geometrical change is brought about in each part of the structure. In members with continuous variability of the thickness, for instance, this leads to the condition of simultaneous plasticity in the entire volume of structure.

In the works under consideration, it is assumed that the material is homogeneous, elastic, and perfectly plastic. The assumption of minimum volume corresponds to that of minimum weight. The influence of shear force on the form is disregarded; as is also the stability problem of bars and other secondary problems. As a consequence of the assumptions made, these works do not, in principle, concern concrete structures. The work concerning the determination of the form of structures is being continued notwithstanding the manufacturing difficulties and the cost. Many practical aspects of design have also to be disregarded. Some of the basic features of the form of a member connected with its practical application may, however, be taken into consideration by choosing certain form parameters beforehand. As an example, consider the use of an invariable geometrical configuration of a frame, and a constant cross section in each span of a continuous beam or frame. The structures to be designed are considered in the state of plastic collapse, the optimum form being determined for this state.

3.4. Minimum Strain Energy Design: The aim of the research in this field is to find such a structure that under a given load the state of strain or stresses is ordered in a prescribed way. The structure may be made of one or several materials working in a definite manner. The distribution of the strain and the stress is restricted by manufacturing and working conditions. The minimum strain energy design is based on the following assumptions:

a. The strain is elastic.

b. The load is prescribed.

c. The form is absolutely known.

d. The form undergoes certain restriction arising from the working conditions.

The load to be carried out by the structure is given in a definite form. The structural weight depends on the form, and may be taken into consideration with a considerable degree of accuracy. The design criterion is the condition of uniformity of the total unit strain energy or the unit strain energy due to shear for constant material volume. This criterion is equivalent to the condition of minimum total strain energy or the strain energy due to shear for a given material volume. This uniformity concerns every point of the structure at which it is possible, from the point of view of the conditions to be satisfied by the form.

If the inverse strain energy is assumed as a measure of the rigidity of the structure, this criterion is equivalent to the maximum rigidity design.

The minimum strain energy design is a generalization of the uniform strength design, and makes it possible to tackle problems of more complicated state of stress. It enables us to determine the most convenient form in the case where the uniform strength design gives no unambiguous solution; for instance, if there is no possibility of selecting among all the lattices of uniform strength a truss with least material volume. The minimum strain energy design with constant material volume indicates in a unique manner the best truss. The advantages of minimum strain design are: (a) It is possible to reduce the design of statically indeterminate structures to that of statically determinate structures. (b) It is possible to reduce the design problem for a moving load to that for a fixed load. (c) It is possible to obtain a number of solutions of design problems of plane and three-dimensional trusses by analyzing the distribution of the nodes and the bars.

CHAPTER IV

STRESS CONTROL METHOD FOR DESIGN OF INDETERMINATE STRUCTURES

4.1. General:

The elastic indeterminate solution amounts to finding the stresses in a member consistent with the respective areas of the member chosen and with no support movements. The designer can assign both the area of a member and the stresses in the member. Satisfaction of these conditions will normally then involve some support movements. These support movements need only be considered at the time of construction. This procedure of design is defined as "stress control." Statically determinate structures can be designed to operate with any stress desired, since the structure is free to change its configuration to correspond. In general, however, the stress level and hence the strain level, cannot be set arbitrarily in the various portions of a statically indeterminate structure since the various portions must fit together in a consistent manner, so that simultaneously both the equilibrium and the deformation requirements for the structure are satisfied. It is, however, possible to control the erection of an indeterminate structure in such a manner that a desired stress (and strain) pattern is realized for one particular static load condition.

As quoted above, the structure can be designed to operate with any desired pattern of stress for one particular static load condition by adjusting its configuration as required through relative movements. When stress control is used to achieve maximum economy the values of the redundant should be such that it gives minimum volume of the material. The plastic design philosophy, however, offers the structure the opportunity to exercise its own stress control by introducing plastic deformations at its plastic hinges.

4.2. Normal and Stress Control Design Procedure:

In normal design analysis procedure, the following steps are commonly followed:

(1) Assume supports do not move.

(2) Assume trial proportions.

(3) Analyze structure.

(4) Design structure accounting for (a) maximum moment analysis and (b) assumed trial proportions.

(5) Calculate new trial proportions and repeat the procedure until minimum weight or volume is obtained.

In stress control procedure, the following steps are normally followed:

(1) Assume that any necessary support movement can be satisfied in construction.

(2) Assume any desired moment diagram.

(3) Assume any desired proportions.

(4) Design structure accounting for assumed moment and proportions.

(5) Compute weight or volume.

(6) If weight or volume is minimum, design is satisfactory and compute necessary support movement.

4.3. Structural Actions:

Stress control study indicates the trend and just say how good the assumptions are. In investigating, the study leads to recognized two distinct types of action of structures termed as "Normal" and "Hybrid" action, which are defined as follows.

4.3.a. Normal Action:

If the design moments, shears and thrust do not vary significantly with changes in sizes of design sections, the behavior is called normal. The convergence of a structure on a final design with desirable stress characteristic is rapid without change in configuration. If a set of moment is assumed in the range, which is termed as "normal range," and the section proportioned according to these assumptions an elastic analysis will yield the originally assumed value of moments.

4.3.b. Hybrid Action:

For any statically indeterminate structure for which direct design is not obtainable successive approximation leads to the most economical design which will be statically determinate if the original layout and loading involve hybrid action. Minimum volume is obtained when the structure changes its configuration. The convergence of a structure on a final design with desirable stress characteristic is slow, and if small change in design section changes the design moments, shear and thrust appreciably, the behavior is termed as "hybrid." Structures characterized by hybrid action are difficult to design and are often inefficient in any case. Study of them is made difficult by the inadequacy of traditional methods.

4.4. Conclusion Regarding Normal and Hybrid Action when Designed for Immovable Static Loads:

The effect of normal and hybrid action for the structures designed for immovable static loads are as follows:

(1) For every statically indeterminate structure, there is a statically determinate version which, if designed by itself, and for the volume used, minimum deflection.

(2) For some structures under some loadings, a normal range exists through which identical minimum volumes of materials and identical deflections are obtained.

(3) Through the normal range, direct design is possible without resorting to stress control.

(4) For any statically indeterminate structure for which a direct design is not obtainable, successive approximation leads to the most economical design which will be statically determinate if the original layout and loading involve hybrid action.

(5) Apparently, from the standpoint of strength and stiffness, there is no theoretical economy achieved by using statically indeterminate structures for static load systems.

Hybrid structures may be designed in many ways. In order that analysis may guide to design, it should precede design so that the designer may see in what ways the structure can act. Then, in a quite literal sense, he tells it how to act and makes it act in that way.

CHAPTER V

SEVERAL EXAMPLES SOLVED BY STRESS CONTROL PROCEDURE

5.1. The two span continuous beam as shown below, over supports A, B, and C, with left span and right span equal to 40'-0" and 20'-0" and uniformly distributed load of 1.2 k/ft., is to be designed by both elastic and stress control procedure and to compare the results.

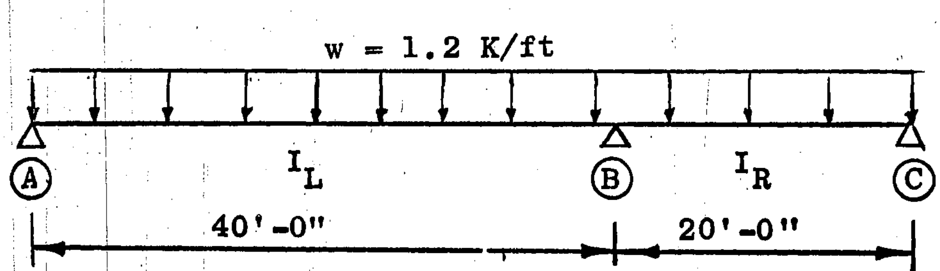


Fig. 5.1 Two Span Continuous Beam

Assume moment of inertia of left and right span equal to I_L and I_R.

(a) Elastic solution is carried out by moment distribution procedure.

(i) Fixed end moments:

Fixed end moment at B in span BA = $FEM_{BA} = + \frac{WL^2}{8}$

$$= \frac{1.2 \times 40^2}{8} = +240.0$$

Fixed end moment at B in span BC =

$$-FEM_{BC} = -\frac{WL^2}{8} = -\frac{1.2 \times 20^2}{8} = -60.0 \text{ k ft.}$$

(ii) Stiffness factors (K's):

K_{BA} = modified stiffness due to pinned end at A

$$= \frac{3EI_L}{40} .$$

K_{BC} = modified stiffness due to pinned end at C

$$= \frac{3EI_R}{20} .$$

(iii) Distribution factors (DF's):

ΣK_B = summation of stiffnesses at joint B

$$= K_{BA} + K_{BC}$$

$$= \frac{3EI_L}{40} + \frac{3EI_R}{20}$$

DF_{BA} = Distribution factor at B for span BA

$$= \frac{K_{BA}}{\Sigma K_B} = \frac{\frac{3EI_L}{40}}{\frac{3EI_L}{40} + \frac{3EI_R}{20}} = \frac{I_L}{I_L + 2I_R}$$

DF_{BC} = Distribution factor at B for span BC

$$= \frac{K_{BC}}{\Sigma KB} = \frac{2I_R}{I_L + 2I_R}$$

(iv) Carry over factors (COF's):

For both spans COF's are zero.

(v) Moment distribution procedure:

	← BA	Joint B	BC →
KS		$3\ EI_L/40$	$3\ EI_R/20$
DF_S		$\frac{I_L}{I_L + 2I_R}$	$\frac{2I_R}{I_L + 2I_R}$
COF_S		0.0	0.0
FEM_S		+ 240.0	- 60.0
		$-180 \left(\frac{I_L}{I_L + 2I_R} \right)$	$-180 \left(\frac{2I_R}{I_L + 2I_R} \right)$
FM (Final Moments)		$+240 - 180 \left(\frac{I_L}{I_L + 2I_R} \right)$	$- \left[60 + 180 \left(\frac{2I_R}{I_L + 2I_R} \right) \right]$

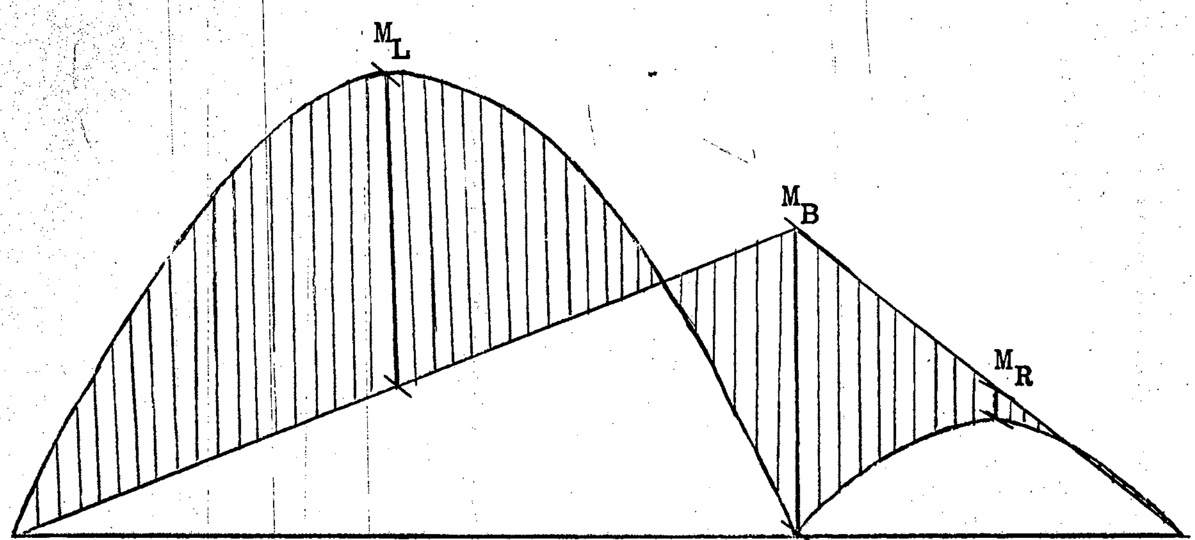


Fig. 5.2. Bending Moment Diagram

M_L = bending moment at the center of left span AB.

M_B = bending moment at support.

M_R = bending moment at the center of right span BC.

simple BM at center in span AB $= \frac{1.2 \times 40^2}{8} = 240.0$ Kft

simple BM at center in span BC $= \frac{1.2 \times 20^2}{8} = 60.0$ Kft

$$M_L = 240.0 - \frac{1}{2}\left[240 - 180\left(\frac{I_L}{I_L + 2I_R}\right)\right]$$

$$= 30\left[4 + 3\left(\frac{I_L}{I_L + 2I_R}\right)\right]$$

$$M_B = + 240 - 180\left(\frac{I_L}{I_L + 2I_R}\right)$$

$$= 60\left[4 - 3\left(\frac{I_L}{I_L + 2I_R}\right)\right]$$

$$M_R = 60 - \frac{1}{2}\left[60 + 180\left(\frac{2I_R}{I_L + 2I_R}\right)\right]$$

$$= 30\left[1 - 6\left(\frac{I_R}{I_L + 2I_R}\right)\right]$$

(vi) Extreme possibilities:

There are three possible extreme cases with the relation of I_L and I_R:

(1) $I_L \gg I_R$; $I_R/I_L \rightarrow 0$

(2) $I_L = I_R$

(3) $I_L << I_R$; $I_L/I_R \rightarrow 0$

The values of M_L, M_B, and M_R for all three cases are sketched below.

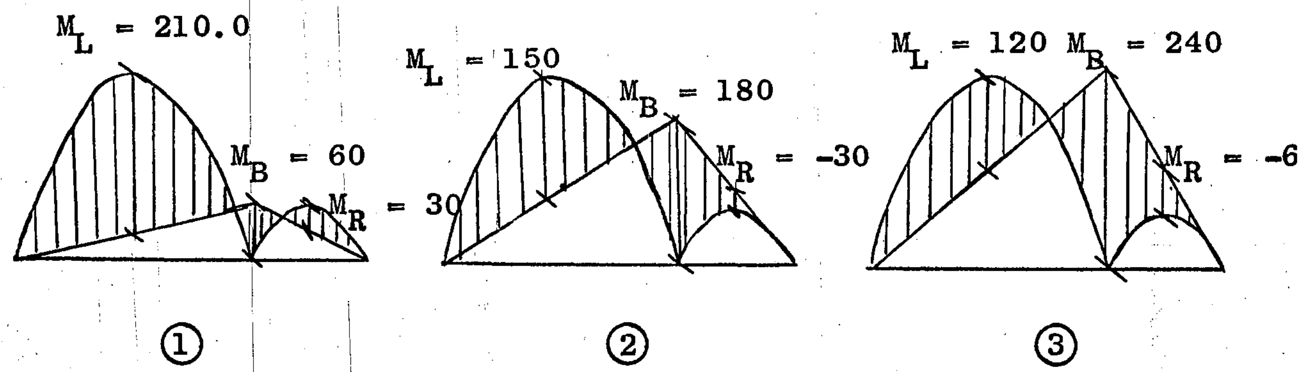


Fig. 5.3. Bending Moment Diagrams (Extreme Cases)

Equating M_L and M_B the ratio of $I_L/I_R = 1.6$ is obtained.

Therefore

M_L governs the left span when $I_L/I_R > 1.6$

M_B governs the left span when $I_L/I_R < 1.6$

M_B governs the right span for all the cases.

(vii) Minimum volume condition:

The value of I_L/I_R shall be obtained such that the minimum volume and hence minimum weight can be obtained.

Trial	$\frac{I_L}{I_R}$	M_L	M_B	M_R	Volume
1	10.0	195	-90	15	20x90+40x10x90 = 37800
2	4.0	180	-120	0	20x120+40x4x120 = 21600
3	2.0	165	-150	-15	20x150+40x2x150 = 15000
4	1.6	160	-160	-20	20x160+40x1.6x160 = 13450
5	1.0	150	-180	-30	20x180+40x180 = 10800
6	0.8	145.8	-188.4	-34	188.4x60 = 11310

In the above table minimum volume is obtained when I_L/I_R equals one. It is assumed that volume is directly proportional to moment and it is relative comparison. This type of behavior of a structure is called Normal action.

18 W 55 section is provided, thereby total weight is 3300 lbs.

(b) Stress control procedure:

(i) Procedure:

Support moment M_B is assumed as redundant and hence M_B is a variable parameter. Its value is to be found in such a way so as to result in minimum volume.

The equations shown below are as per para. 5 of elastic solution.

$M_L = 240 - M_B/2$ $\qquad$ $M_R = 60 - M_B/2.$

The different value of support moment M_B is Assumed and substituted for value of M_L and M_R and total volume is calculated.

(ii) Minimum volume condition:

Assume M_B	Compute M_L	Compute M_R	Volume Left Span	Volume Right Span	Total Volume
0	240	60	9600	1200	10800
10	235	55	9400	1100	10500
20	230	50	9200	1000	10200
30	225	45	9000	900	9900
40	220	40	8800	800	9600
50	215	35	8600	1000	9600
80	200	20	8000	1600	9600
100	190	10	7600	2000	9600
120	180	0	7200	2400	9600
140	170	-10	6800	2800	9600
160	160	-20	6400	3200	9600
170	155	-25	6800	3400	10200

It can be seen that constant volume for the values of M_B equals 40.0 to 160 K ft. However, M_B equals to 40 K ft. gives the minimum weight. Hence design moment for left span and right span is taken as 220.0 and 40.0 K ft. respectively. When M_B = 40.0, I_L/I_R equals 5.5.

A 21 W 55 and 14 B 17.2 are selected for left and right span assuming A 36 steel. Once again this beams were checked elastically by moment distribution and found that support moment is equal to 96.0 K ft. against 40.0 K ft. obtained above by stress control procedure. Therefore balance of 56 K ft. of moment will be adjusted through relative displacement of support.

The total weight of the structure works out to be 2544 lbs.

(c) Comparison of the results:

The comparison is shown below for both the methods graphically. A saving in weight of 756 lbs. was achieved by adopting stress control procedure, i.e., about 23% saving in weight of structure, thereby resulting in an economical solution.

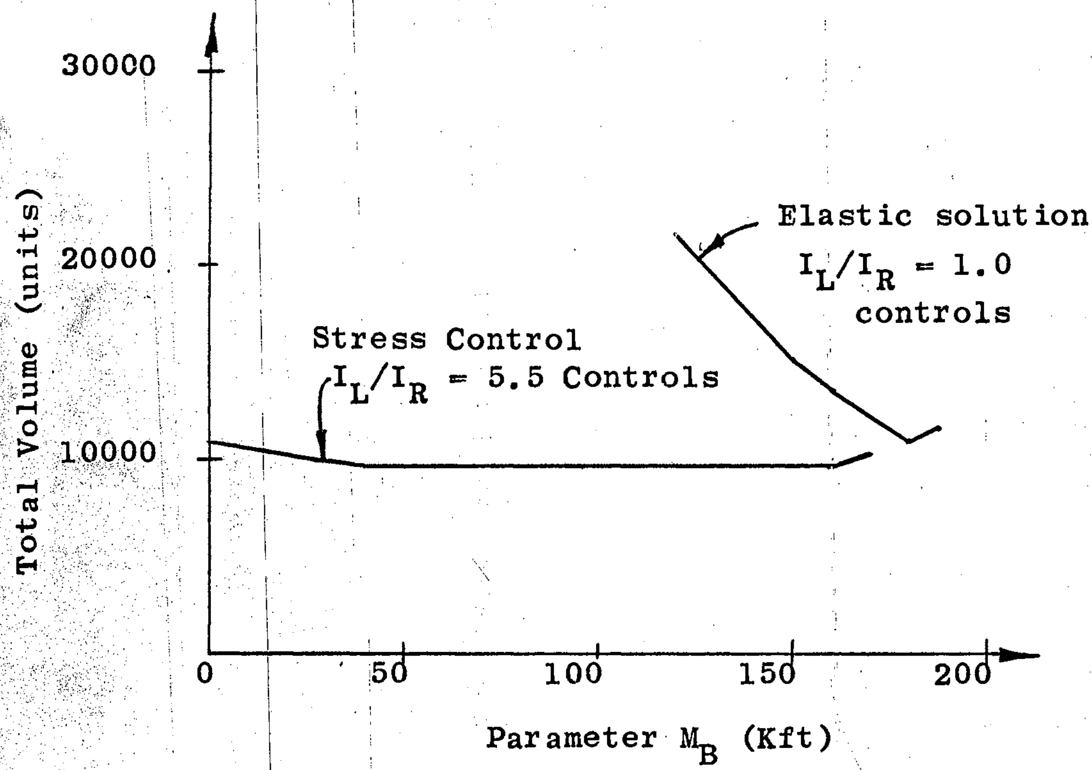


Fig. 5.4. Volume of Structure Against Value of Parameter M_B

5.2. A Knee Frame: A knee frame ABC, shown below, having beam AB equal to 20' and column BC equal to 20'-0". A load of 1.2 K/ft. is applied on beam AB. To find optimum solution.

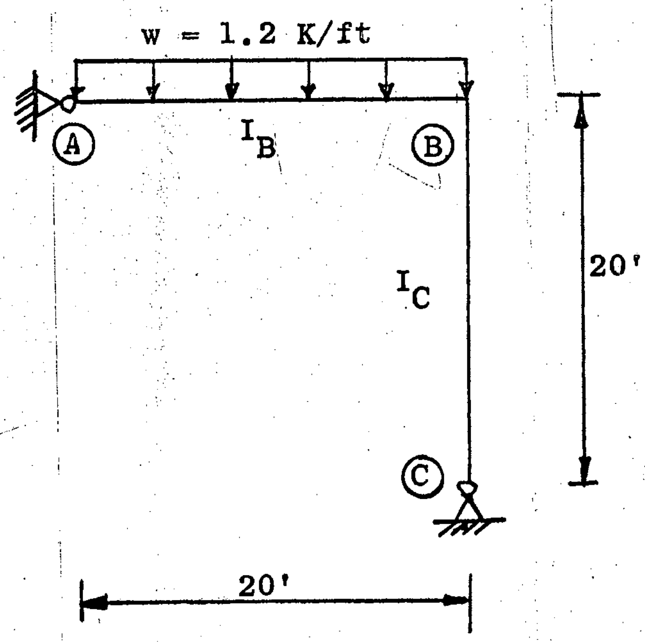


Fig. 5.5. A Knee Frame

Assume moment of inertia of beam AB and column BC equal to I_B and I_C respectively. Moment distribution procedure is followed to analyze the frame.

(i) Fixed end moments:

Fixed end moment at B in span BA $= FEM_{BA} = + \frac{WL^2}{8} = \frac{1.2 \times 20^2}{8} = 60.0$ K f

(ii) Stiffness factors (K_S):

K_{BA} = modified stiffness due to pinned end A

$$= \frac{3EI_B}{L} = \frac{3EI_B}{20}$$

K_{BC} = modified stiffness due to pinned end C

$$= \frac{3EI_C}{20}$$

(iii) Distribution factors (DF_S)

ΣK_B = summation of stiffnesses at joint B

$$= K_{BA} + K_{BC}$$

$$= \frac{3EI_B}{20} + \frac{3EI_C}{20}$$

DF_{BA} = Distribution factor at B for span BA.

$$= \frac{K_{BA}}{\Sigma KB} = \frac{3EI_B/20}{\frac{3EI_B}{20} + \frac{3EI_C}{20}} = \frac{I_B}{I_B + I_C}$$

DF_{BC} = Distribution factor at B for span BC

$$= \frac{K_{BC}}{\Sigma KB} = \frac{3EI_C/20}{\frac{3EI_B}{20} + \frac{3EI_C}{20}} = \frac{I_C}{I_B + I_C}$$

(iv) Carry over factors (COF_S):

For both beam and column COF_S are zero.

(v) Moment distribution procedure:

	BA	Joint B	BC
K_S		$\frac{3EI_B}{20}$	$\frac{3EI_C}{20}$
DF_S		$\frac{I_B}{I_B + I_C}$	$\frac{I_C}{I_B + I_C}$
COF_S		0.0	0.0
FEM_S		+60.0 $-60\ (\frac{I_B}{I_B + I_C})$	$-60\ (\frac{I_C}{I_B + I_C})$
Final Moments FM		$60 - 60\ (\frac{I_C}{I_B + I_C})$	$-60\ (\frac{I_C}{I_B + I_C})$

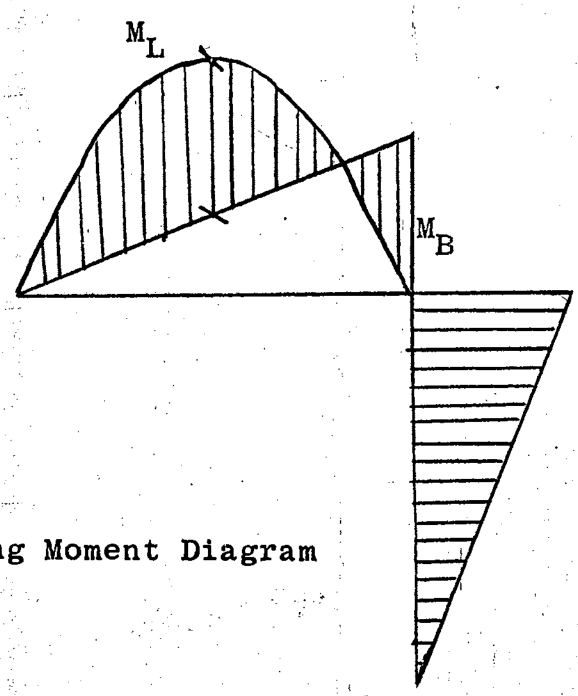


Fig. 5.6. Bending Moment Diagram

M_L = bending moment at center of beam AB.

M_B = bending moment at B for span BA and BC.

$$\text{simple BM at center of span AB} = \frac{1.2 \times 20^2}{8} = 60.0$$

$$M_L = 60 - \frac{1}{2} \times \frac{60\, I_C}{I_B + I_C}$$

$$= 30(2 - \frac{I_C}{I_B + I_C})$$

$$M_B = -60\,(\frac{I_C}{I_B + I_C}).$$

(vi) Extreme possibilities:

There are three possible extreme cases with the relation of I_B and I_C:

(1) $I_B >> I_C$ $\qquad I_C/I_B \rightarrow 0$

(2) $I_B = I_C$

(3) $I_B << I_C$ $\qquad I_B/I_C \rightarrow 0$

The values of M_L, M_B for all three cases are shown below.

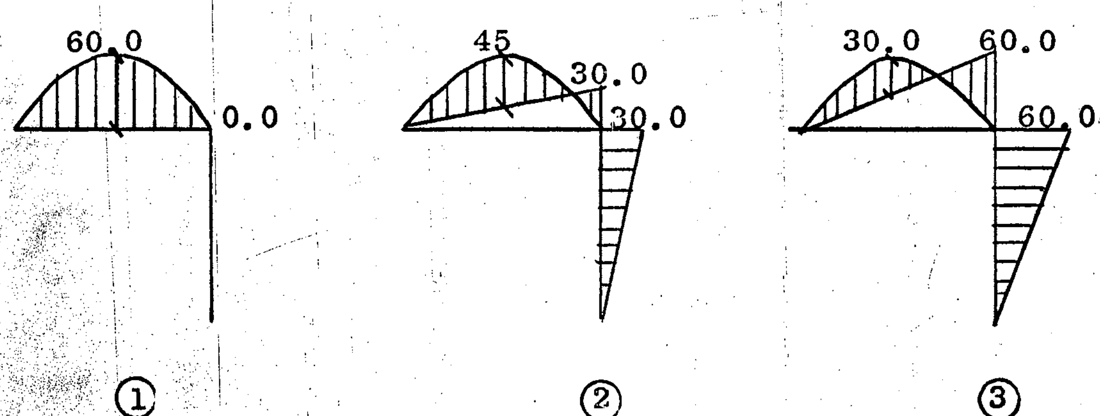


Fig. 5.7. Bending Moment Diagrams (Extreme Cases)

Equating M_L and M_B the ratio of $I_B/I_C = \frac{1}{2}$ is obtained.

Therefore

M_L governs the beam AB when $I_B/I_C > \frac{1}{2}$.

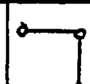
M_B governs the beam AB when $I_B/I_C < \frac{1}{2}$.

M_B governs the column BC for all cases.

(vii) Minimum volume condition:

The value of I_B/I_C shall be obtained such that the minimum volume is obtained.

Assume member BC is governed by bending moment only. Effect of direct force is neglected and volume is directly proportional to moment.

Trial No.	I_B/I_C	M_L	M_B	Total Volume	New I_B/I_C
1	1/2	40	40	40x20+2x40x20 = 2400	1.0
2	1.0	45	30	45x20+45x20 = 1800	1.5
3	1.5	48	24	48x20+2/3x48x20 = 1600	2.0
4	2.0	50	20	50x20+1/2x50x20 = 1500	2.5
5	2.5	51.6	17.2	51.6x20+$\frac{1}{2.5}$x20x51.6 = 1440	3.0
6	3.0	52.5	15.0	52.5x20+1/3x52.5x20 = 1400	3.5
Limit	∞	60.0	0	60 x 20 = 1200	

From above study it can be seen that minimum volume is obtained when structure changes its configuration. This behavior is termed as Hybrid behavior of structure.

5.3. Single Span Portal Frame With Hinged Base:

A portal frame with pinned base is shown having a span of 60'-0" and height of 20'-0". Assuming constant EI for the whole frame to find the optimum solution. The frame has 1.0 K/ft. load.

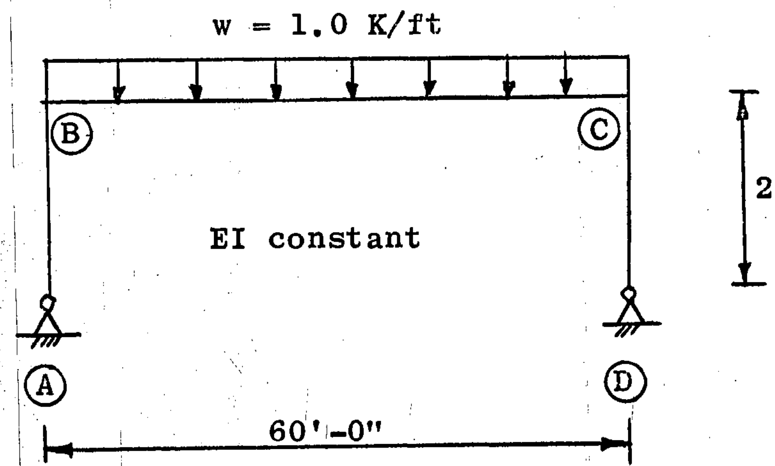


Fig. 5.8. A Portal Frame (Hinged Base

(i) Procedure:

Stress control procedure is adopted to find the solution. Horizontal force in the frame at A and D assumed as variable parameter and the equations of total volume of materials in terms of horizontal force H are formed and plotted for volume for different values of H, and minimum volume is achieved for a particular value of H.

(ii) Minimum volume condition:

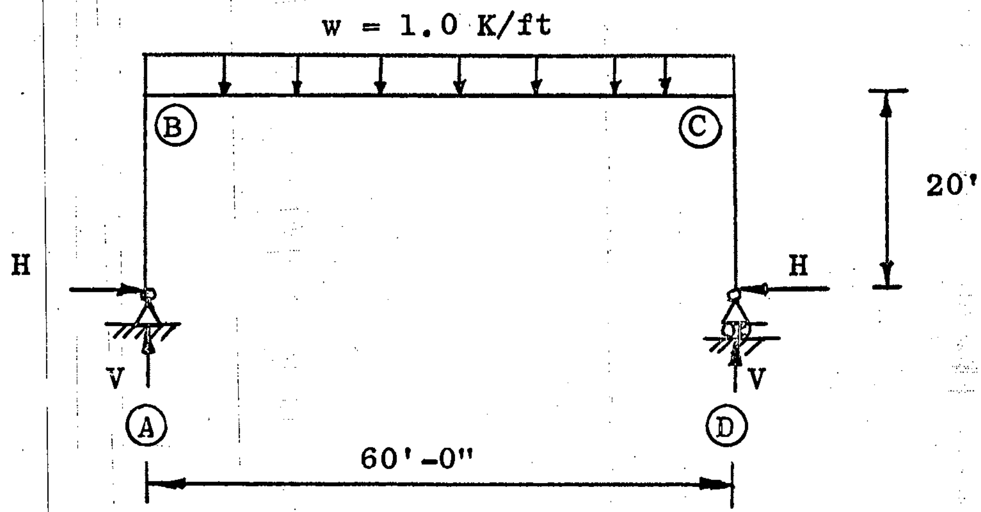


Fig. 5.9. Basic Structure and Redundant H

V - Vertical reaction at A and B due to applied load.

H - Horizontal reaction at A and D due to applied load.

V_T = Total volume of material for the structure.

V_{T_1} and V_{T_2} - Volume of material for the structure for alternative (1) and (2).

Assume volume is directly proportional to moment.

$$\text{simple BM in span BC} = \frac{1.0 \times 60 \times 60}{8} = 450 \text{ K ft.}$$

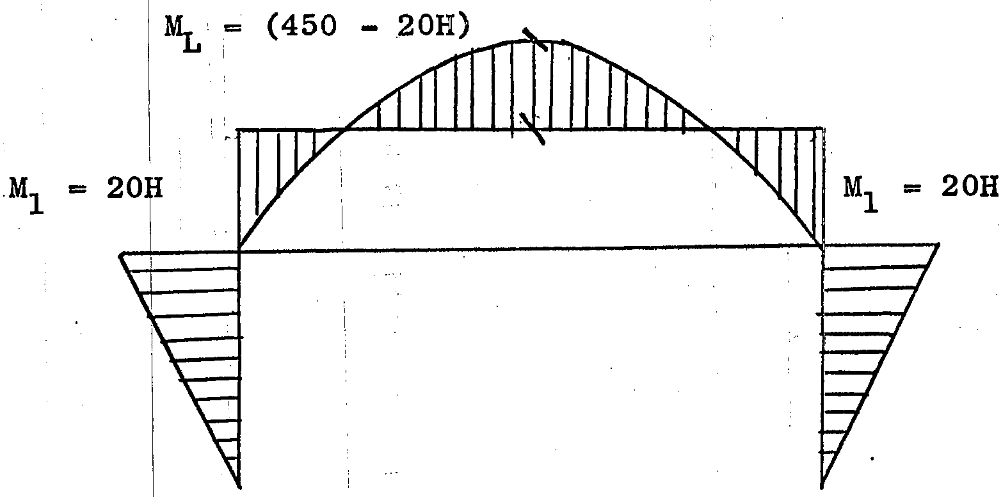


Fig. 5.10 Bending Moment Diagram

$V_T = 2 \times 20 \times M_1 + 60(M_1 \text{ or } M_L)$

M_1 - moment at haunch B or C = 20H

M_L - moment at center of span BC = (450 - 20H)

$V_T = 2 \times 20 \times 20H + 60 \left[20H \text{ or } (450 - 20H) \right]$

$V_{T_1} = 800H + 1200H = 2000H$ (1)

$V_{T_2} = 800H + 27000 - 1200H = 27000 - 400H$ (2)

H	V_{T_1}	V_{T_2}
0	0	27000
5	10000	25000
10	20000	23000
15	30000	21000

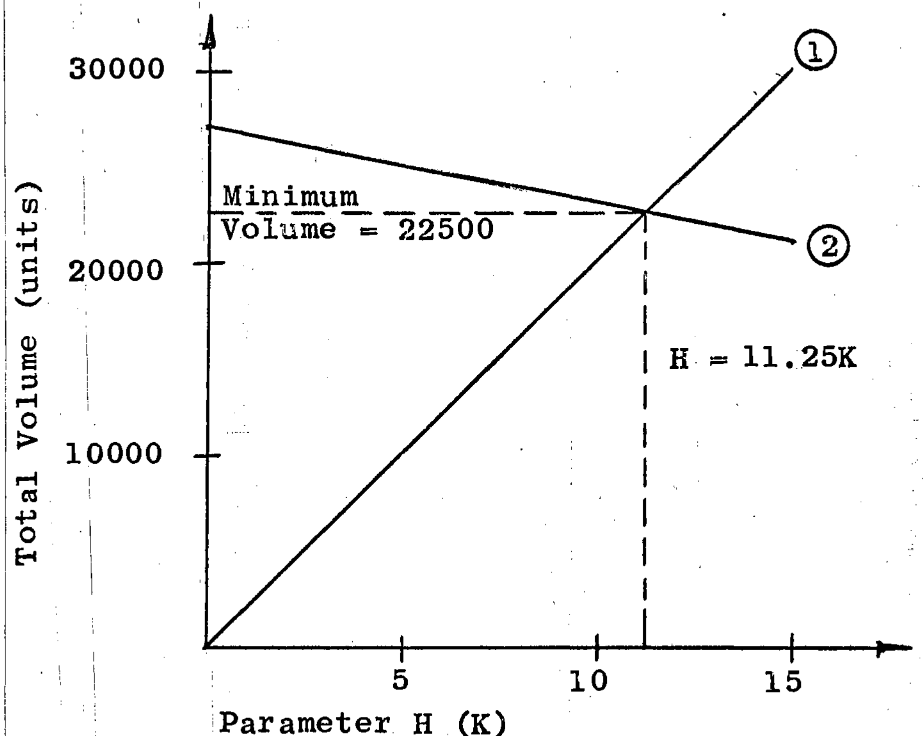


Fig. 5.11. Volume of Structure against Value of Parameter H

(iii) Design:

The value of H = 11.25 K results in minimum volume.

∴ Moment M_1 at haunch = 20 x 11.25 = 225.0 K ft.

Assuming A 36 steel, select 21 W 55 section.

∴ Total weight = $\frac{55 \times 100}{2000}$ = 2.77 tons.

Checking the provision by elastic analysis by using strain energy principals, the value of H works out to be equal to 12.25 K and M_1 = 245.0 K ft. and 21 W 62 section is required giving thereby 3.1 tons of weight.

To balance the moment of 20.0 K ft. support adjustment is required by 1.49" pulling outwards. Thus about 10% saving in weight of the structure is obtained.

5.4. Single Span Gable Frame With Hinged Base:

A single span, pinned base gable frame as shown of

68'-6" span and 11'-9" column height and 4'-6" gable height with 450 lbs/ft. of load to be designed by stress control to achieve optimum solution.

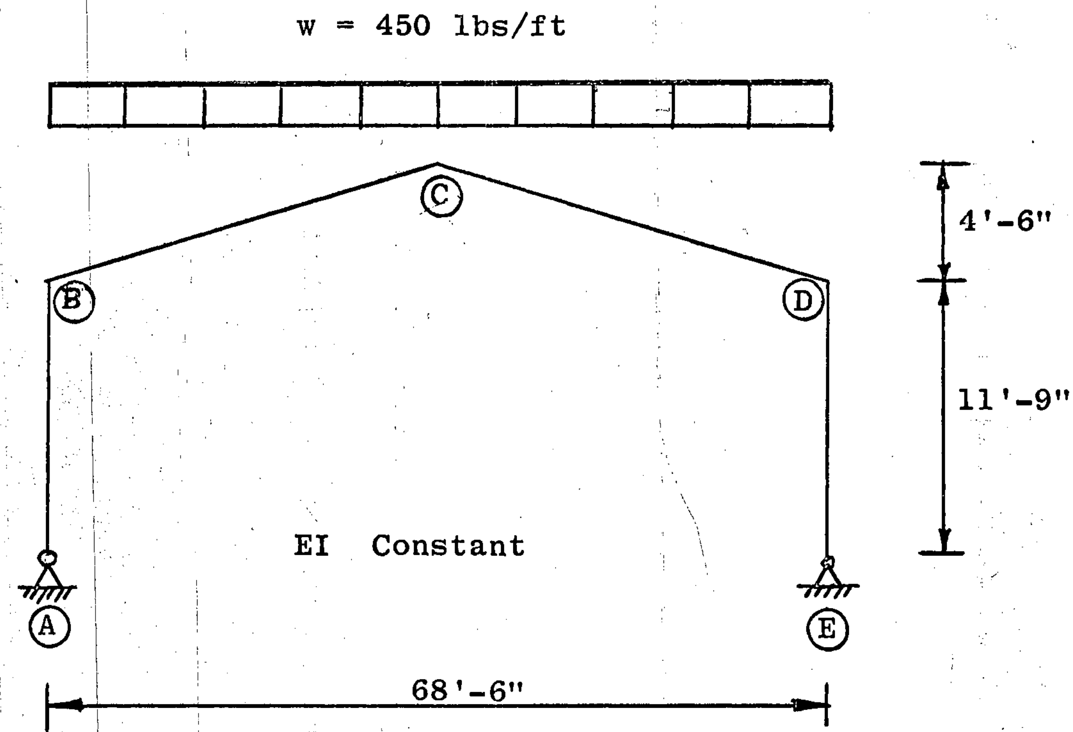


Fig. 5.12 A Gable Frame (Hinged Base)

(i) Procedure:

The value of variable parameter H, (horizontal force at A and E) is found such that to result with minimum volume.

(ii) Minimum volume condition:

Assume constant section throughout the frame.

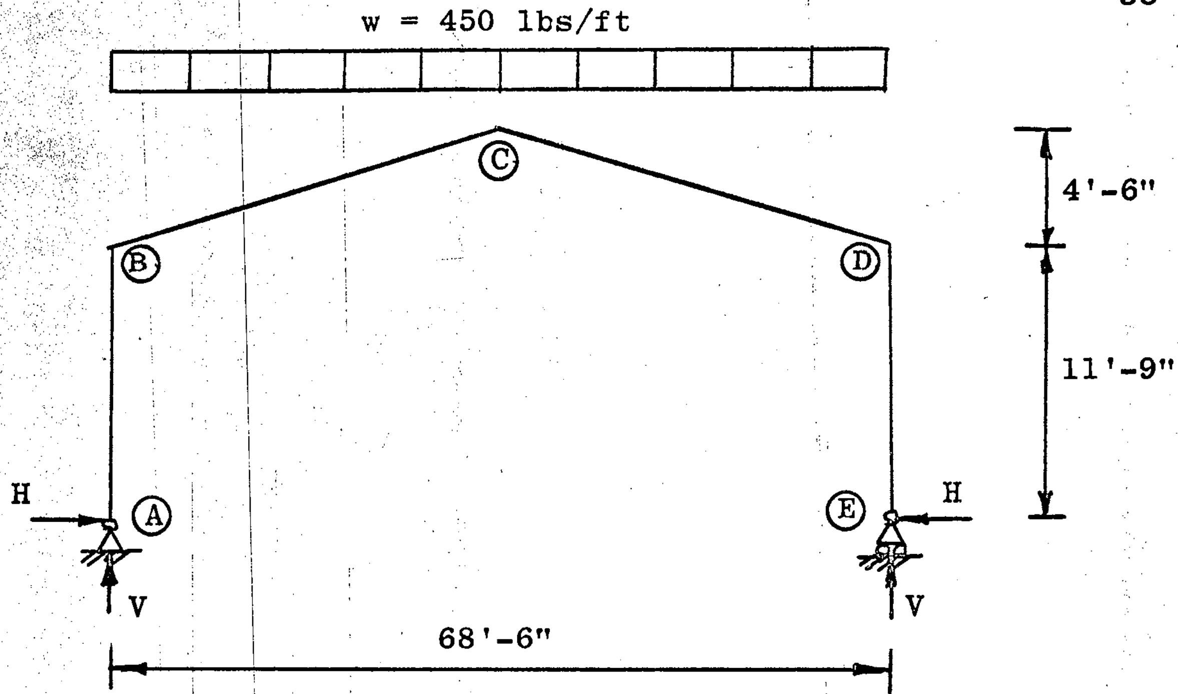


Fig. 5.12. Basic Structure and Redundant H

V - Vertical reaction at A and E due to applied load.

H - Horizontal reaction at A and D due to applied load.

V_T = Total volume of material for the structure.

V_{T_1} and V_{T_2} - volume of material for the structure for alternative (1) and (2).

$$\text{simple BM in BD} = \frac{0.450 \times 68.5^2}{8} = 264.0 \text{ K ft.}$$

$$V_T = 2 \times 11.75\ M_1 + 68.5\ (M_1 \text{ or } M_L)$$

M_1 = moment at haunch B or D = 11.75 H

M_L = moment at center of span BD = (264 - 16.25H)

$$V_{T_1} = 2 \times 11.75 \times 11.75H + 68.5 \times 11.75H$$
$$= 276H + 806H = 1082H \qquad \text{(i)}$$

$$V_{T_2} = 276H + 68.5(264 - 16.25H) = 18100 - 836H \qquad \text{(ii)}$$

H	V_{T_1}	V_{T_2}
0	0	18100
5	54100	13920
10	18020	9740

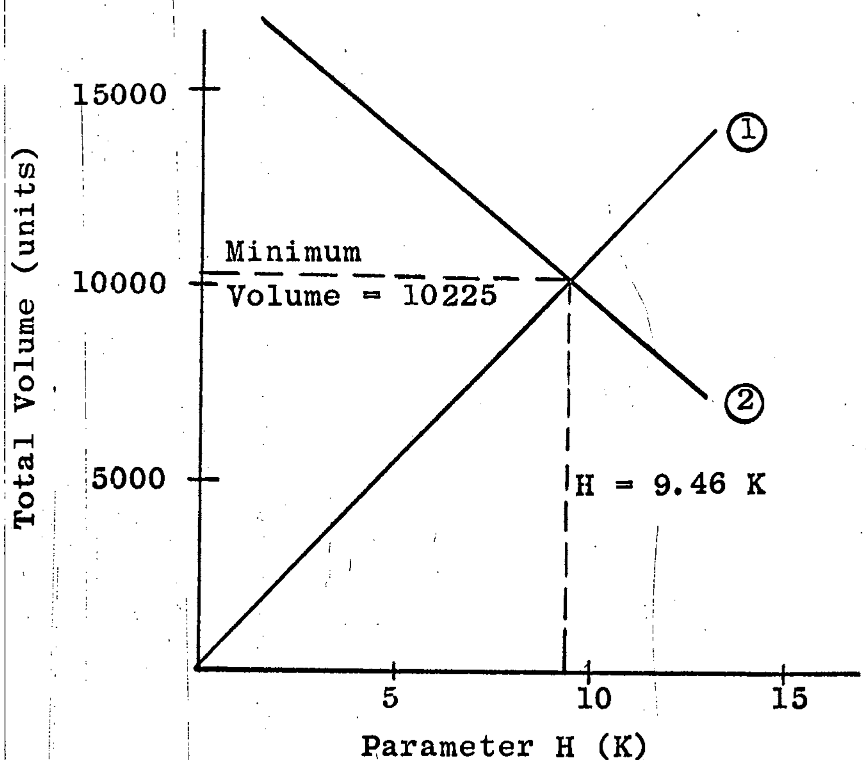


Fig. 5.14. Volume of Structure Against Values of Parameter H

(iii) Design

Value of H - 9.46 K results in minimum volume.

.·. Moment M_1 at haunch = 9.46 x 11.75 = 110.0 K ft.

Using A 36 steel provide 16 W 36 giving total weight of 1.66 tons.

Now the frame is analyzed on elastic procedure by principal of strain energy gives the value of H = 11.75K and moment = 138.0 K ft.

The difference in moment = 27 K ft. shall be adjusted through relative displacement of support by 4.94" pulling outwards, thus saving about 20% in weight.

5.5. Cantilever truss:

To find the optimum design for the cantilevered truss shown.

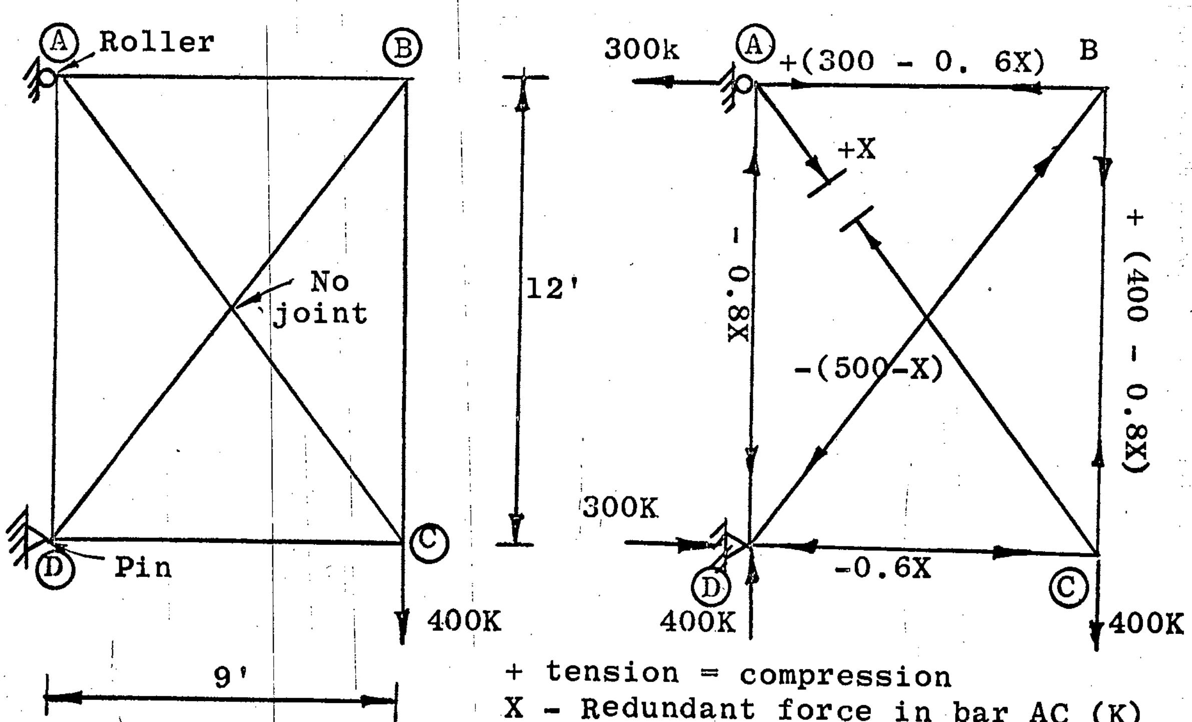
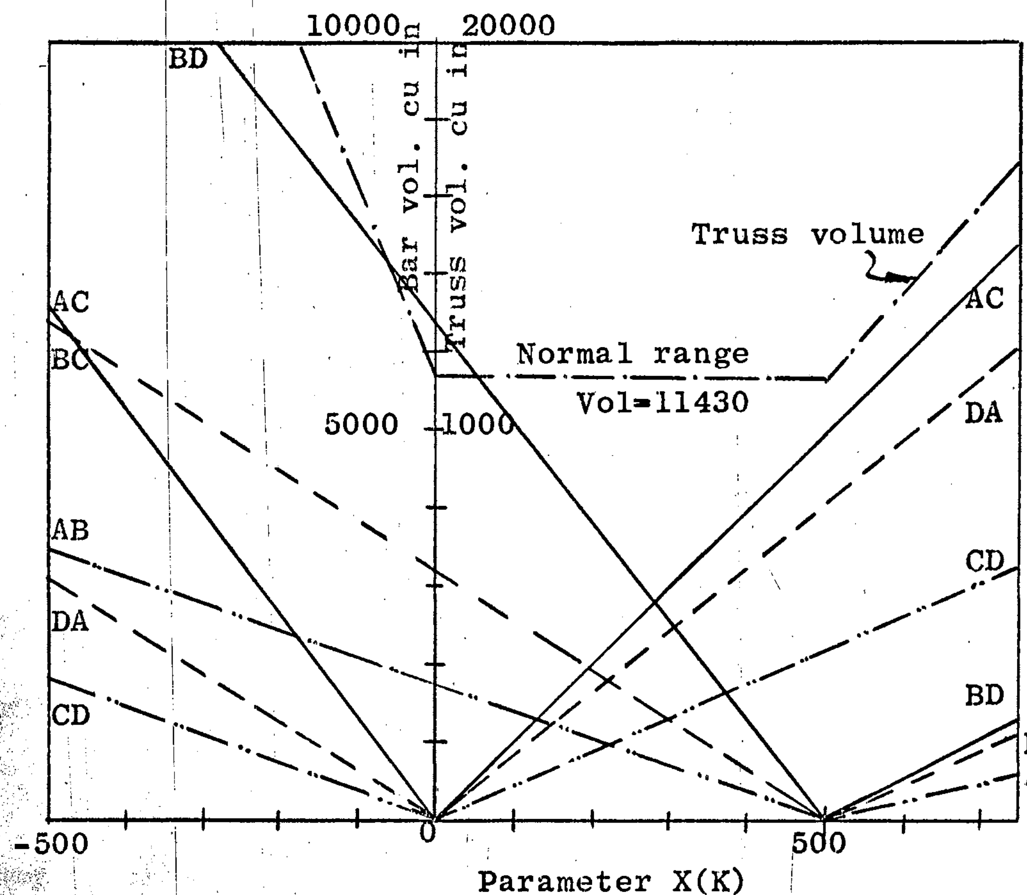


Fig. 5.15. Cantilever Truss

The frame is internally statically indeterminate to one degree and hence force in AC is assumed as redundant and its value to be found in such a way so as to result in minimum volume. Assume that all the members are fully stressed.

Assume E = 30,000 Ksi, and allowable stress in compression and tension is 14 and 18 Ksi respectively. Stability effect is neglected and all joints are assumed pin connected.



Scale - Bar volume, 1" = 2500 cu in   Truss volume 1" = 5000 cu in   Force X 1" = 250K

Fig. 5.16. Volume of Truss and Bars Against Value of Variable Force X

It is noted in the graph that minimum volume curve for the truss is between x = 0 and x = 500 kips. The volume of material is the same for any value selected for the redundant between these limits. This range of limits is called Normal Range, and the action of the structure is known as Normal Action.

For values of redundant selected within Normal Range, direct design is possible without using stress control.

TABLE SHOWING DIFFERENT VALUES OF x - AND VOLUME OF BARS AND TRUSS

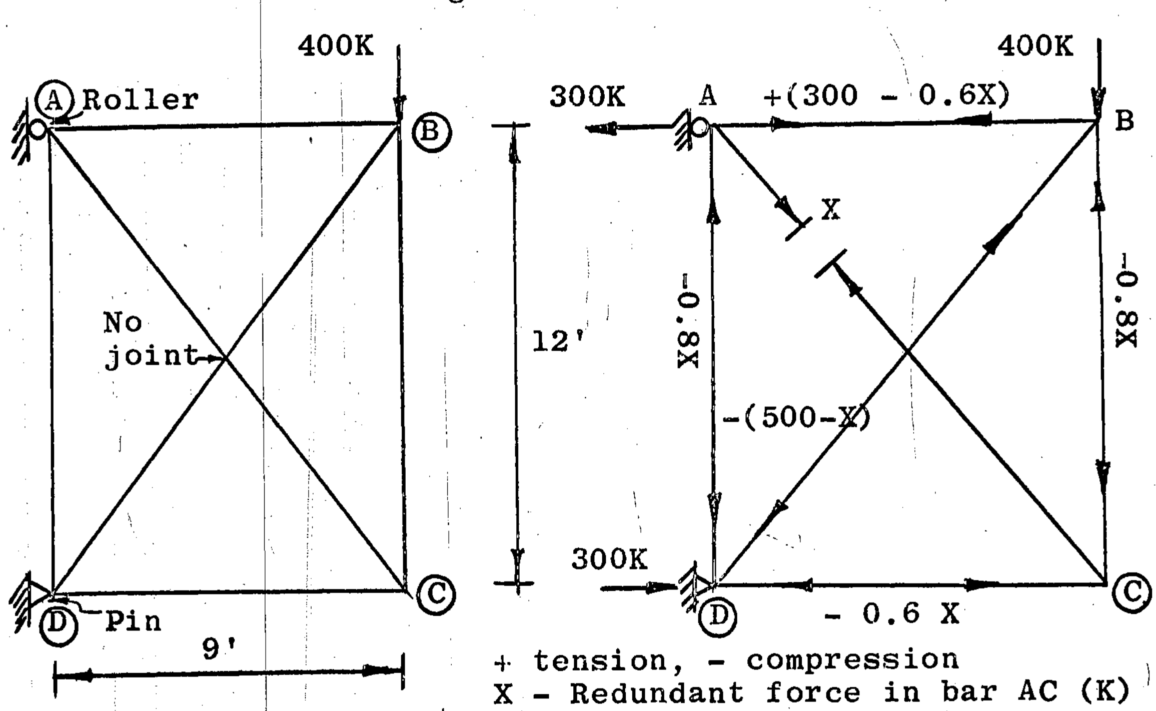
x in k		-500	-100	0.0	+100	+200	+300	+400	+500	550
AB	Force K	600	360	300	240	180	120	60	0.0	-30
108"	Area sq in	33.4	20.0	16.7	13.3	10.0	6.7	3.3	0.0	2.2
long	Volume cu in	3600	2160	1800	1440	1808	720	360	0.0	230
BC	Force K	800	480	400	320	240	160	80	0.0	-40
144"	Area sq in	44.5	26.7	22.2	17.8	13.4	8.9	4.5	0.0	2.9
long	Volume cu in	6400	3840	3200	2560	1920	1280	640	0.0	410
CD	Force K	300	60	0.0	-60	-120	-180	-240	-300	-330
108"	Area sq in	16.6	3.3	0.0	4.3	8.6	12.8	17.2	21.5	23.6
long	Volume cu in	1800	360	0.0	460	925	1380	1850	2310	2540
DA	Force K	400	80	0.0	-80	-160	-240	-320	-400	-440
144"	Area sq in	22.2	4.4	0.0	5.7	11.4	17.2	22.9	28.6	31.5
long	Volume cu in	3200	640	0.0	820	1645	2470	3290	4120	4520
AC	Force K	-500	-100	0.0	100	200	300	400	500	550
180"	Area sq in	358	7.1	0.0	5.6	11.1	16.7	22.1	27.8	30.6
long	Volume cu in	6430	1280	0.0	1000	2000	3000	4000	5000	5500
BD	Force K	-1000	-600	-500	-400	-300	-200	-100	0.0	+50
180"	Area sq in	71.5	42.8	35.7	28.6	21.4	14.3	7.5	0.0	2.8
long	Volume cu in	12850	7720	6430	5150	3860	2580	1290	0.0	500
Total Volume	cuin.	34280	16000	11430	11430	11430	11430	11430	11430	13700

If X is selected within this range and the areas are determined to satisfy the desired stress than the two sides of cut in the redundant match perfectly and no adjustment is required in length of bar, and hence provide area at X = 250 where equal force in the diagonal is obtained.

When X = 0 the truss is reduced to statically determinate truss composed of AB, BC and BD. When X = 500, the structure is reduced to statically determinate truss composed of CD, DA and AC

5.6. Cantilever Truss:

To find the design for the cantilevered truss shown

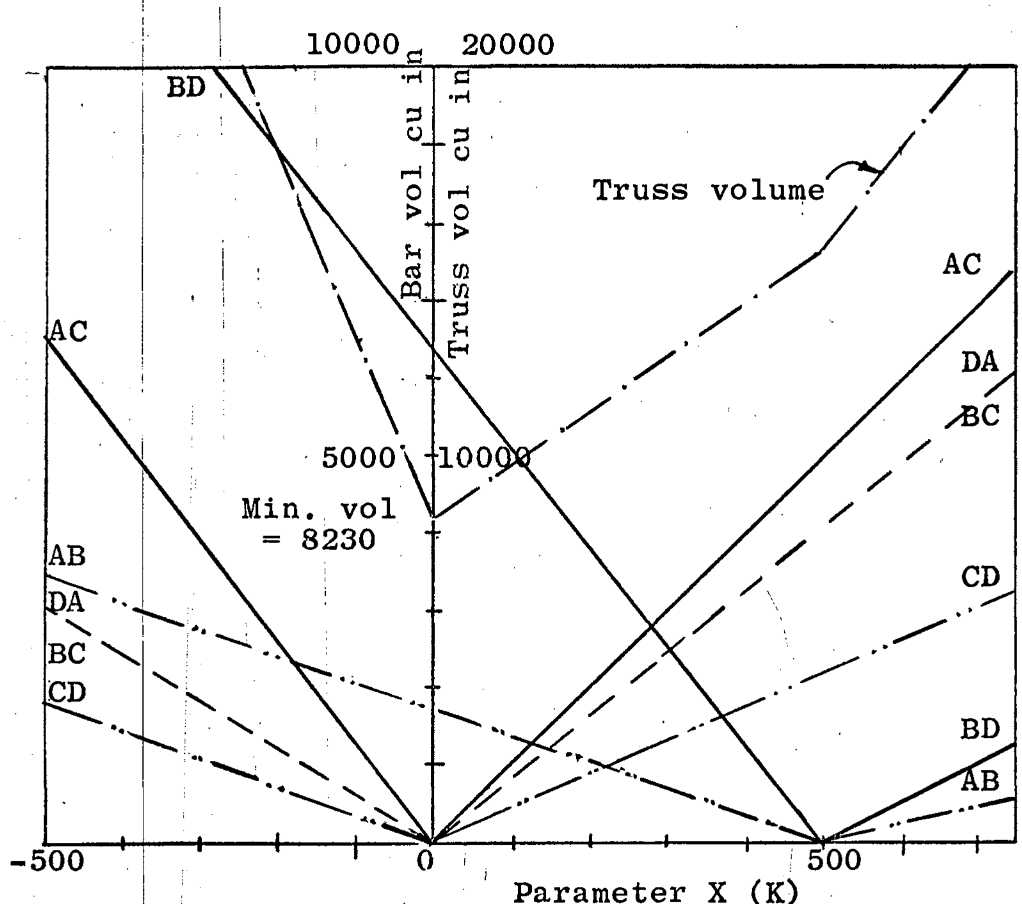


+ tension, - compression
X - Redundant force in bar AC (K)

Fig. 5.17. Cantilever Truss

assume E = 30,000 ksi and allowable stress in compression and tension as 14 and 18 ksi respectively. Stability effect is neglected and all joints are assumed as pin connected.

The frame is internally statically indeterminate to one degree. Assumed bar AC as redundant.



Scale - Bar volume, 1" = 2500 cu in; Truss volume 1" = 5000 cu in Force X 1" = 250 K

Fig. 18. Volume of Truss and Bars Against Value of Variable Force X

From the graph below, it is seen that minimum volume for the truss is obtained when X = 0. In other words, the minimum volume is obtained when the truss is reduced to the two bar statically determinate truss formed by AB and BD. This structural behavior is termed as Hybrid action. However, design this with equal force in diagonal from the applied load, i.e., when X = 250K.

TABLE SHOWING DIFFERENT VALUES OF X - AND VOLUME OF BARS AND TRUSS

X_K		-500	-100	0	+100	+200	+300	+400	+500	+550
AB	Force K	600	360	300	240	180	120	60	0.0	-30
108"	Area sq in	33.4	20.0	16.7	13.3	10.0	6.7	3.3	0.0	2.2
long	Volume cu in	3600	2160	1800	1440	1080	720	360	0.0	230
BC	Force K	400	80	0.0	-80	-160	-240	-320	-400	-440
144"	Area sq in	22.2	4.4	0.0	5.7	11.4	17.2	22.9	28.6	31.5
long	Volume cu in	3200	640	0.0	820	1645	2470	3290	4120	4520
CD	Force K	300	60	0.0	-60	-120	-180	-240	-300	-330
108"	Area sq in	16.6	3.3	0.0	4.3	8.6	12.8	17.2	21.5	23.6
long	Volume cu in	1800	360	0.0	460	925	1380	1850	2310	2540
DA	Force K	400	80	0.0	-80	-160	-240	-320	-400	-440
144"	Area sq in	22.2	4.4	0.0	5.7	11.4	17.2	22.9	28.6	31.5
long	Volume cu in	3200	640	0.0	820	1645	2470	3290	4120	4520
AC	Force K	-500	-100	0.0	100	200	300	400	500	550
180"	Area sq in	35.8	7.1	0.0	5.6	11.1	16.7	22.1	27.8	30.6
long	Volume cu in	6430	1280	0.0	1000	2000	3000	4000	5000	5500
BD	Force K	-1000	-600	-500	-400	-300	-200	-100	0.0	+50
180"	Area sq in	71.5	42.8	35.7	28.6	21.4	14.3	7.5	0.0	2.8
long	Volume cu in	12850	7720	6430	5150	3860	2580	1290	0.0	500
Total Volume	cu. in.	31080	12800	8230	9690	11155	12620	14080	15550	17810

$$\Delta_1 = \sum F_1 \cdot \frac{F}{A} \cdot \frac{L}{E}$$

F_1 - Force in members due to X = 1.

F - Force in members when X = 250.

L - Length of bars

A - Area of bars.

E - Modulus of Elasticity

Δ_1 - Elongation of bar AC

Bar	L(in)	$\frac{F}{A}$(ksi)	F_1	$F_1 \cdot \frac{F}{A} L$
AB	108	+18	-0.6	-1167
BC	144	-14	-0.8	+1612
CD	108	-14	-0.6	+907
DA	144	-14	-0.8	+1612
AC	180	+18	+1.0	+3240
BD	180	-14	-1.0	-2520

$$\sum F_1 \cdot \frac{F}{A} \cdot L = +3684.$$

$$\therefore \Delta_1 = \frac{3684}{30 \times 10^3} = 0.1228 \text{ in (together)}.$$

The above calculation shows that the two sides of the cut of bar AC have come together by 0.1228 inches. If, however, member AC has been made 0.1228 inches too short, the two sides of the cut would fit together perfectly. Therefore, if in the fabrication of this truss member AC were made 0.1228 inches too short and all other members were made perfectly, then during construction AC could be forced into place. This would create a prestressed condition in the truss such that, when the 400 Kip load was applied, the desired stress condition would be obtained exactly.

5.7. Two Span Pinned Base Continuous Gable Frame:

To design two bay pinned end continuous gable frame as shown by stress control procedure:

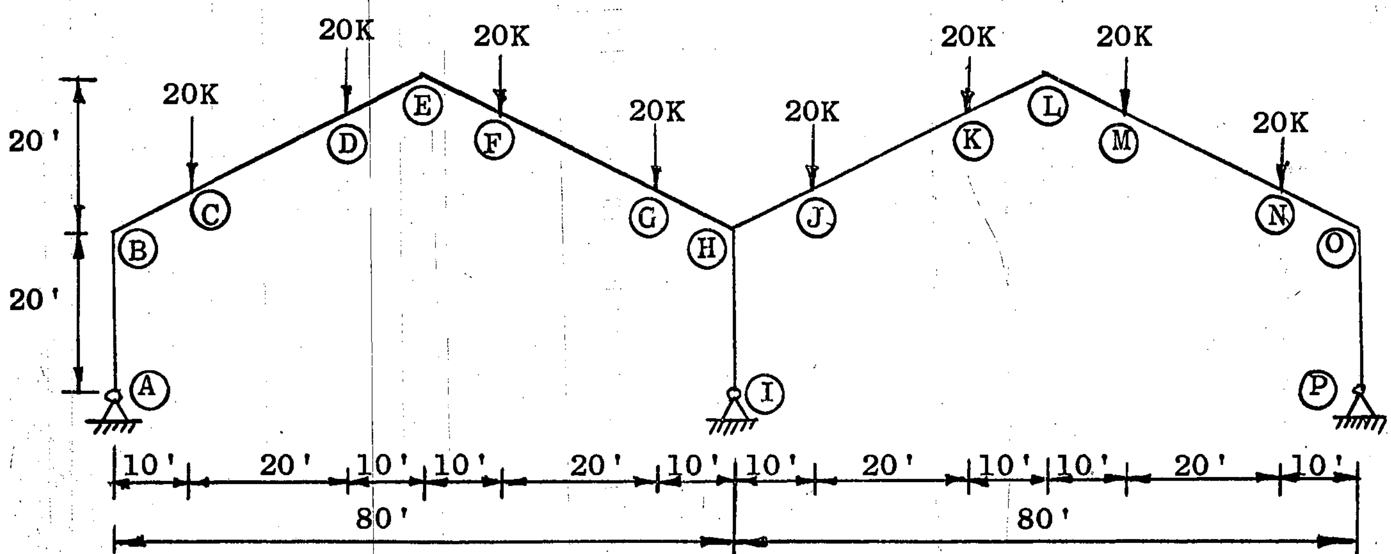


Fig. 5.19. Two Span Continuous Gable Frame (Pinned End)

The frame is symmetrical with symmetrical loading and therefore there will not be any moment on central column and that joint H will not be rotated.

As the symmetry is available for both structure and loading its advantage is taken in selecting the variable parameters. Moment at B and H is assumed same.

Summation of moment at H

$$\Sigma M_H = -80V + 20H + 3200 = M_B = 20H$$

$\therefore$ V = 40K, where V is vertical reaction at A and P

H is horizontal reaction at A and P

M_B is moment at B.

Assume constant cross section throughout and volume is directly proportional to moment. Only half of the structure is considered in developing equations for volume with respect to variable parameter horizontal force H and compared

V_T = total volume

$= 20M_B + 80\ (M_B \text{ or } M_D)$.

M_D = moment at D

$= -35H + 30V - 400 = -35H + 1200 - 400 = -35H + 800$

$V_T = 20 \times 20H + 80\ (20H \text{ or } 800 - 35H)$.

V_{T_1} and V_{T_2} are total volume for alternative possibilities.

$V_{T_1} = 20 \times 20H + 80 \times 20H$

$= 2000H$ (1)

$V_{T_2} = 400H + 80\ (800 - 35H)$

$= 400H + 64000 - 2800H$

$= 64000 - 2400H$ (2)

H	V_{T_1}	V_{T_2}
0	0.0	64000
5	10,000	52000
10	20,000	40000
20	40,000	16000
26.7	53,400	0.0

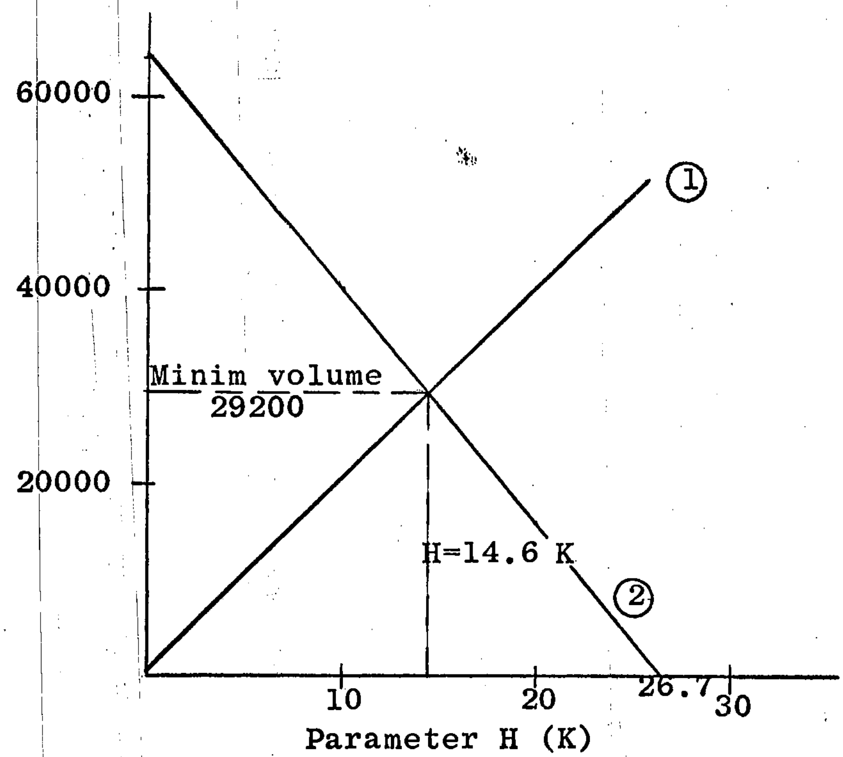


Fig. 5.20. Volume of Structure Against Parameter H

H = 14.6 K.

Moment at B = 14.6 x 20 = 292.0 Kft.

Using A 36 steel, select 24 W 68

The same problem is solved on elastic analysis by strain energy. The value of H is found to be 18.82 K, the moment 376.45 K ft. and section 27 W 94, hence the difference in moment of 84.45 K ft. which is to be adjusted during construction by relative displacement of support.

The same problem is solved on plastic analysis by mechanism method and the value of H works out to be 26.9 K considering load factor of 1.85 and moment is 538 K ft. Section required for this moment is 24 W 76.

Thus it is seen that stress control solution is very near to plastic analysis and time and labor can be saved in simple problems.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Although design rather than analysis is the real problem in machines and structures, far more research effort is spent on analysis. The reason is primarily the specific nature of the problem posed and the greater possibility of obtaining an unambiguous solution.

Many forward steps have been taken toward the highly desired goal of direct design of structures as contrasted with usual procedure of an informed guess followed by analysis. The complexity of elastic analysis is so great that it is not surprising to find design based on arbitrary rules despite more efforts to develop scientific procedures.

The assumptions of perfect plasticity opens the possibility of direct design in the strict sense in comparison with the preliminary guess and repeated analysis without confirming the best solution. The form of a structural member is the result of a compromise between the material cost and the working cost. The form of minimum volume, even if not directly applicable, may constitute a criterion for the evaluation of other solutions, and gives important indications concerning the form of the member designed.

With the help of computers, more and more variables are considered by the different authors and procedures are given to find an optimum solution. These are listed in the Bibliography.

Stress control procedure is followed in this report to solve several examples. The procedure of selecting unknown redundants and developing equations of the volume with unknown parameters and after plotting for various values of parameter minimum volume can be found. The method is quick and saves time in calculation in comparison to conventional elastic procedure. The method is applied for simple structures but it can be extended to more complicated structures with the use of computers to solve equations.

Plastic methods are now of more use to solve complicated structures resulting in minimum weight. The development of practice for plastic design may permit multi-story structures. The plastic design philosophy, however, offers the structure the opportunity to exercise its own stress control by introducing plastic deformations at its plastic hinges.

A SELECTED BIBLIOGRAPHY

Baker, J. F. "The design of steel frames." The Structural Engineer, Vol. 27 (1949), pp. 397-431.

Bass, Louis. Lecture Notes on the Course of Limit Design of Structures. Oklahoma State University, Spring, 1965.

Beedle, L. S. Plastic Design of Steel Frames. New York: John Wiley & Sons, Inc., 1961.

Benjamin, J. R. Statically Indeterminate Structures. New York: McGraw Hill, 1959.

Biernawski, A. A theorem on the potential of a spatial gating with four loaded nodes. Bull Acad. Polonaise Sci Ser Sci Tech 10, 1, 1-10, 1962.

Brandt, A. Some theorems on statically determinate prestressed beams designed for minimum potential. Bull Acad. Polonaise Ser Sci Tech 10, 2, 57-62, 1962.

Brotchie, J. F. "Direct Design of Plate and Shell Structures." Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 88, Part 2 (December, 1962), pp. 127-148.

A concept for the direct design of structures is introduced, and is applied to the design of plates and shells. A desired behavior is selected as a design criterion, and the proportions of the structure are determined directly to satisfy it. The concept may be applied to homogeneous reinforced concrete and prestressed concrete structures and it allows optimum behavior and maximum material economy to be obtained.

Brotchie, J. F. "A concept for the direct design of structures." Transactions, Institute of Engineers, Sydney, Australia, September, 1963, P. 210.

This paper introduces a new general concept for design in which a desired behavior is used to determine directly the required proportions of a structure. Its application to plates, shells and framed structures, of reinforced concrete, prestressed concrete or metal, is outlined. The concept allows the required structure for optimum behavior, in any given sense, or for maximum material economy to be directly and simply obtained.

Brotchie, J. F. "Direct design of framed structures." Journal of Structural Division, Proceedings of American Society of Civil Engineers, Vol. 90, Part 2 (December, 1964), pp. 243-247.

A general, rational procedure for the design of structure is considered, and its application to framed structures is outlined. The problem is defined in such a way that the required proportions become the dependent variables, for which the solution may be uniquely determined from design criteria such as: required behavior at the various loadings involved, minimum weight or minimum cost, and conventional constraints imposed by the site, the codes, or the function of structure.

Carter, W. O. Lecture Notes on the Course of Welded Steel Structures. Oklahoma State University, Spring, 1965.

Charlton, T. N. "Some Notes on the Analysis of Redundant Systems by Means of the Conception of Conservation of Energy." Franklin Institute Journal, Vol. 250 (December, 1950), pp. 543-551.

The principles of the analysis of conservative redundant systems capable of deformation are considered. It is well known that reference to both the conditions for equilibrium and geometrical compatibility is necessary to enable a unique solution to be obtained in a particular case. The setting up of one or other of these conditions in a general case using the concept of energy is discussed with reference to the law of conservation of energy. The use of complementary energy for the fulfillment of the conditions for geometrical compatibility is demonstrated. The direct application of the conditions of equilibrium and geometrical compatibility is considered in an appendix.

Chentsov, N. G. Minimum Weight Struts. (in Russian), Proc. CLGI, 1936.

Cilley, F. H. "The exact design of statically indeterminate frameworks--An exposition of its possibility, but futility." Transactions of the American Society of Civil Engineers, Vol. XLIII (June, 1900), pp. 353-443.

The object of this paper is not to consider the more exact calculation of actual structures but rather to examine certain questions in connection with ideal frameworks, of the greatest practical importance because at the foundation of actual designs. In his detailed comparison of statically determinate and indeterminate frameworks, the author claims that determinate structures are more economical, theoretically.

Cissel, J. H., Stress Analysis and Design of Elementary Structures. New York: John Wiley & Sons, Inc., 1948.

This textbook is divided into two parts which deal with stress analysis and design. Part two of the book gives principles of designs and its stages for the selection of material, general arrangement of parts and its configuration.

Cox, H. L. and H. S. Smit. Structure of Minimum Weight. Aeronautical Research Committee Reports and Memoranda, No. 1923-1943.

Cross, Hardy. "The relation of analysis to structural design." Transactions of the American Society of Civil Engineers, Vol. 101 (1936), pp. 1363-1408.

This paper presents a classification for the analysis, with the idea of suggesting a convenient arrangement of certain familiar characteristics. The paper describes various types of behavior of structure and stresses. The paper has indicated four types of structural action, the characteristics of which make separate discussion. The author has described in detail each type of structural action.

Drucker, D. C., and W. Prager. "Extended limit design theorems for continuous media." Quarterly Journal of Applied Mathematics, Vol. 9 (1952), pp. 381-389.

In the present paper the general limit design problem is concerned. The actual history of loading is assumed to be completely specified rather than only the extreme values of all loads. The writers have given a number of theorems which suggest behavior of structures.

Drucker, D. C., and R. T. Shield. "Design for minimum weight." Proceedings 9th International Congr. Appl. Mech., Brussels.

Drucker, D. C., and R. T. Shield. "Bounds of minimum weight design." Quarterly of Applied Mathematics, Vol. 15 (1957), pp. 269-281.

A somewhat limited design procedure for elastic perfectly plastic structure was developed by the authors in the previous article quoted above. It is extended here to provide upper and lower bounds on the minimum weight of three dimensional structures and is specialized to safe one and two dimensional structures in which either direct stresses or bending stresses are negligible. The present paper dealing with beams, plates, shells and space structures where the minimum weight is sought without regard to problems and cost of manufacture and construction.

Faulks, J. "Minimum weight design and theory of plastic collapse." Quarterly of Applied Mathematics, Vol. 10 (1953), pp. 347-358.

This paper examines the problem of assigning economical sections to the members of a structure whose geometrical form is given. The criterion of failure is taken to be that of the plastic theory of collapse, and the criterion of minimum weight is employed to determine the best design. A geometrical analogue of the equations involved is used to clarify their significance, and such proofs as they are in text, are cast into geometrical terms. A method of solution is suggested at the end of the paper, but the primary concerns of the paper are the general features of the problem.

Faulks, J. "The minimum weight design of structural frames." Proc. Roy. Soc. (A) 223, London, 1954, pp. 482-494.

Farrar, D. J. "The design of compression structures for minimum weight." Journal Royal Aeronautical Society, Vol. 53 (November, 1949), pp. 1041-1052.

Francis, A. J. "Direct design of elastic statically indeterminate triangulated frameworks for single systems of loads." Australian Journal of Applied Science, Vol. 4 (1953), pp. 175-185.

The paper describes a method of design of elastic statically indeterminate triangulated framework for a single system of loads. The elastical indirect approach based on indeterminate structural theory is replaced by a direct approach which yields an economical solution in a very short time. The general principle of prestressing follows naturally from the method and is shown to lead to appreciable economy in material. In the method described in this paper, the structure is designed by first assuming that the members are fully stressed. The stresses in some members are then adjusted as necessary to satisfy the condition of continuity of deformation. A suitable force system that is one satisfying condition of equilibrium is chosen. The method is more rapid since one begins with an ideally stressed structure.

Francis, A. J. "Direct design of non-linear redundant triangulated frameworks." Australian Journal of Applied Science, Vol. 6 (1955), pp. 13-31.

The paper extends the method to deal with any non-linear structure the behavior of whose members is reversible. It is shown that pre-stressing can be used in such structures with advantage. Elastoplastic structures are treated as a special case of non-linear behavior. The paper shows that the method may be extended to deal with structures with certain restrictions having non-linear characteristics. The method at present is applicable only to problems involving only one system of loading.

Gerard, G. Minimum Weight Analysis of Compression Structures. New York University Press, 1960.

Greenberg, H. J., and W. Prager. "Limit design of beams and frames." Transactions, American Society of Civil Engineers, Vol. 117 (1952), pp. 447-484.

The paper is concerned with the limit design of statically indeterminate beams or frames under the action of given loads. A method for solving the problem of determining the load required to cause collapse in a structure of known dimension is indicated.

Grinter, L. E. Theory of Modern Steel Structures. New York: MacMillan Company, 1949.

Gurevich, Y. I. Problems of rational variability of the cross sections of statically indeterminate bar systems. (in Russian) Proc. Khabarovsk Inst Zhelezn Transp., 1954.

Heyman, J. "Plastic design of beams and plane frame for minimum material consumption." Quarterly of Applied Mathematics, Vol. 8 (1951), pp. 373-381.

This paper is concerned with the design of plane frames in such a way that the material consumption is minimum. The method of solution is to set up linear inequalities for the variables involved, and to solve these inequalities. Three slightly different classes of problems are treated: collapse design under fixed loads, collapse design under varying loads, and shake-down design under varying loads. Illustrative examples of each are given.

Heyman, J. "Plastic design of plane frames for minimum weight." Structural Engineer, Vol. 31 (1935), pp. 125-129.

Heyman, J. "Minimum weight of frames under shakedown loading." Journal of the Engineering Mechanics Division , Proceedings of the American Society of Civil Engineers, Vol. 84 (1958), pp. 1790-1-25.

The first part of the paper discusses the minimum weight design of framed structures under both fixed and independently varying loads. A simple numerical example is given, for which the complete solution is obtained, and the results corresponding to the two types of loading are compared. An iterative method is then presented for the minimum weight design of a frame of any degree of complexity; this iterative method may lead to results which are slightly inexact.

Heyman, J. "On the absolute minimum weight design of framed structures." Quarterly Journal of Mechanics and Applied Mathematics, Vol. 12 (1959), pp. 314-324.

The strength of frames follows the bending moment diagram if the cross section of the members of a frame varied continuously, then provided design is carried out in a certain way, the frame will have absolute minimum material consumption. The theorem is applied to determine the minimum weight of some simple structures and a trial and error "relaxation" technique is developed. The writer discusses in detail about the method and states theorems also.

Heilbon, C., R. Contini and W. Horsfall. Optimum structural design data for compression surface structure. Lockheed Aircraft Corporation Structures Research Memorandum No. 115, May, 1945.

Hilton, H. H., and M. Feign. "Minimum weight analysis based on structural reliability." Journal of the Aerospace Sciences, Vol. 27 (September, 1960), pp. 641-652.

An analytical investigation is presented for the proportioning of probabilities of failure among structural components in terms of a preassigned probability of failure of the entire structure, such that the total structural weight is a minimum. It is shown in the discussion that for a given total structural probability of failure of a multicomponent structure, as any components becomes heavier it should be assigned a higher probability of failure in order to achieve minimum weight for the entire structure.

Hu, T. C., and R. T. Shield. Uniqueness in the Optimum Design of Structures. Brown University Tech. Report DA-4795/2, 1959.

Hu, T. C. Optimum Design for Structures of Perfectly Plastic Materials. Ph.D. Thesis, Brown University, 1960.

Kalaba, R. "Design of minimum weight structures for given reliability and cost." Journal of the Aerospace Sciences, Vol. 29 (March, 1962), p. 355.

The purpose of this paper is to provide an improved computational procedure and to show how to generalize the considerations of the article given by Hilton and Feigan in their article, "Minimum weight analysis based on structural reliability," so as to include the cost of material.

Kaufman, S., and T. Hop. A study of rational design of the cross section of a prestressed beam (in Russian) Ar-h Inzyn Lad S. No. 1, 1959.

Kenedi, R. M., W. S. Smith and F. O. Fahmy. "Light structures research and its application to economic design." Transactions Inst Engrs Shipbuilders Scot, Vol. 9 (1956), pp. 207-264.

Kitcher, T. P., and L. A. Schmit. "Synthesis of material and configuration selection." Journal of Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 88, Part I (June, 1962), pp. 79-102.

Several combinations of material and configuration are proposed for a structure, and structural synthesis is used to select that combination giving a minimum weight design. Structural analysis provides a means of obtaining a minimum weight, optimum design for each candidate combination. Several synthesis provides a means of obtaining an optimum balanced design for several combinations of material and configuration. The method shows a selection based on minimum weight. Structural synthesis has been defined as the rational directed evolution of a structural configuration.

Kitcher, T. P., and L. A. Schmit. Some Further Results of Structural Design by Systematic Synthesis. Cleveland, Ohio: Case Inst. of Tech., August, 1960 (unpublished notes).

Klein, B. "Direct use of external principle in solving certain optimizing problems involving inequalities." Journal Operations Research Soc. of Amer., Vol. 3, No. 2

(May, 1955), pp. 168-175 and 548.

Kosko, E. "Effect of local modification in redundant structures." Journal Inst. of the Aeronautical Sciences, Vol. 21, No. 3 (March 1954), pp. 206-207.

Krishnan, S., and K. V. Shetty. "Methods in optimum structural design for compression elements." Journal Aero Soc India, Vol. 11 (May, 1959), pp. 23-29.

Laushey, L. M. "Direct design of optimum indeterminate truss." Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 84, Part 2 (December, 1958), pp. 1867-1-35.

A method is proposed for the direct design of indeterminate truss. Analysis of trial structure is avoided. The direct design method concentrates on design, not on the mathematical analysis of a trial structure. The principle of potential work is introduced to obtain the maximum compatible stresses for the bars. Redundants are selected to yield the minimum weight of truss. The final bar areas follow directly by dividing the forces in static equilibrium by the stresses satisfying continuity. The relative weights of alternative structures and the optimum structure are revealed by the direct design method. The stresses in a statically determinate structure depend only on the loading and geometrical configuration and can be computed before the proportioning of the sections. A statically indeterminate structure on the contrary cannot be analyzed except on the basis of a fully proportioned structure, since the stresses are controlled by the elastic distortions and these, in turn, by the distribution of material. The design of such structure is a cut and try process. The method has been illustrated with examples.

Livesley, R. K. "The automatic design of structural frames." Quarterly Journal of Mechanics and Applied Mathematics, Vol. 9 (1956), pp. 257-278.

This paper considers the problem of finding the lightest structural frame of given geometrical form which will support a given set of loads. Following the development of a geometrical analogue and an iterative method of solution, an analytical technique is presented which gives an exact solution in any particular numerical case. In determining the lightest possible frame capable of carrying given loads theory of plastic collapse is used. The method is very suitable for use on an electronic computer. The author has also discussed the method and

the help of computers to solve the problem in detail.

Michalos, J. Theory of Structural Analysis and Design. New York: Ronald Press Company, 1958.

Micks, W. R. Bibliography of Literature on Optimum Design of Structures and Related Topics. California: The Rand Corporation, 1958.

The well-arranged list of the literature on the subject is useful for the reference.

"Minimum weight design of structural frames." Proc. Royal Aeronaut. Soc. Vol. 223 (1954), pp. 482-494.

Moses, Fred. "Optimum structural design using linear programming." Journal of the Structural Division, The American Society of Civil Engineers, Vol. 90 (December, 1964), pp. 89-104.

A systematic approach is presented for finding optimum (lightest or cheapest) design for a wide class of elastic structures. The cutting plane method for non-linear operations research problems is used, and the optimization of a structural design can thus be transformed into a series of linear programming problems. Solutions are obtained by simplex method thereby making available an efficient and rapid alogrithm for finding optimum designs. An efficient and rapid procedure for optimizing structural designs, with particular reference to conventional elastic structure is presented. It is assumed that basic configuration of the structure has been chosen and the size of the individual members must be found.

Mroz, Z. On a problem of minimum weight design. Brown University, 1960, and Quart. Appl. Math., Vol. 19, 1961, pp. 127-135.

Norris, C. H., and J. B. Wilbur. Elementary Structural Analysis. New York: McGraw-Hill, 1960, p. 539.

The author suggests the method of stress control for indeterminate structures and its application with several examples. He lists the effect of Normal and Hybrid action for the structures and states further that every statically indeterminate structure there is a statically determinate version which, if designed by itself, will have the minimum volume of material and for the volume used, minimum deflection.

Notes on the Problem of the Optimum Structures. The College of Aeronautics, Great Britain, Notes No. 73, January, 1958.

O'Connell, R. F. "A digital method for redundant structural anslysis." Journal of Aerospace Sciences, Vol. 29 (December, 1962), pp. 1414-1420.

A digital method of structural analysis is developed using equivalent electrical circuit analogies in the systemization of the original computer program. This method is shown to possess particular advantages in the analysis of redundant structural systems, in that the redundancies need not be determined in order to form a statically determinate system. The method is completely independent of such redundancies. In addition, all of the procedures except the development of the equivalent electrical analogy may be carried out in a simple, routine manner with very little prior experience in the method.

Parcel and Mooreman. Analysis of Statically Indeterminate Structures. New York: John Wiley & Sons, Inc., 1955, p. 333.

Pearson, C. "Structural design by high speed computing machines." Proceedings 1st National Conference on Electronic Computation, Structural Division ASCE, Kansas City, Mo., 1958, pp. 417-436.

Prager, W. "Minimum weight design of a portal frame." ASCE Engineering Mechanics Division Journal, Vol. 82 (1956), p. 1073-1-10.

In this paper, the weight per unit length of a structural member has been assumed to be proportional to its full plastic moment. The present paper is concerned with the minimum weight of a structural member is proportional to the α th power of its fully plastic moment. The positive exponent α being smaller than unity. For $\alpha = 2/3$, a chart is developed that gives at a glance, the minimum weight design for various geometries and loading conditions of portal frame. The method is restricted to frames that consist of straight prismatic members subjected to concentrated load only.

Prager, W., and J. Heyman. "Automatic minimum weight design of steel frames." Franklin Institute Journal, Vol. 266 (November, 1958), pp. 339-364.

The automatic plastic design of structural frames can be treated by the method of linear programming. The number of variables, however, increases so fast with

with the complexity of the frame that only simple frames can be handled by this method even on a large electronic computer. In the present paper a method is proposed which considers alternately two different requirements that a frame must satisfy, and thereby greatly reduces the size of the problem. Part I of the paper presents the method with reference to a simple numerical example. Part II establishes the general applicability of the proposed method. Part III presents some lemmas of practical importance, and some discussion, with examples, of special considerations that may arise in the design of actual frames.

Pugsley, Alfred. "The economy of structures." Journal Royal Aero Soc, Vol. 31 (March, 1959), pp. 153-162.

Rabinovich, J. M. On the theory of statically indeterminate lattices. Centr Inst Transp Stroit, 1933.

A work concerning the problem of choosing the best from a set of structures of uniform strength.

Readey, W. B. "Optimum design of indeterminate frames." Journal of the Aeronautical Sciences, Vol. 21 (September, 1954), pp. 615-620.

The paper presents methods for obtaining the optimum section property distributions, for indeterminate frames and beams, directly from the design conditions. An optimum section property distribution is assumed to be one which results in constant extreme fiber strains, where the tension strain is not necessarily assumed to be equal to the compression strain. As a consequence of the constant bending strains, the change in geometry, due to given strain, is determined when the inflexion points are located. A method is presented, for determining optimum inflexion point locations which result in optimum design. In general the solution is achieved by an iteration process, but a direct solution is presented for certain beam problems.

Rogers, Paul. "Economy in structural design." Consulting Engineer, Vol. 3 (January, 1954), pp. 32-35.

The methods described in this article can be used to bring about considerable savings without diminishing the factor of safety of the structure. The methods described are intended specifically for use in structural design of power plants even though structures serving any purpose should be economically designed.

Rozvany, George I. N. "Optimum synthesis of prestressed structures." Journal of Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 90, Part 2 (December, 1964), pp. 189-211.

The concept of reversed deformation is described. By using this technique it is shown that any prestressed, indeterminate structure can be designed for one given load system without solving compatibility equations. Stresses in the members and the fully stressed shape of the structures are chosen prior to design and an optimum design is selected. This method is particularly suitable for the design of non-linear and highly redundant structures on the basis of permissible stress criterion. The method is illustrated with examples.

Schmidt, L. C. "Fully stressed design of elastic redundant truss under alternative load systems." Australian Journal of Applied Science, Vol. 9 (1958), pp. 337-348.

The present paper derives the conditions for minimum weight design. These conditions require the minimum of all fully stressed designs to be found. The method of fully stressed design is presented; a fully stressed design being one in which each member reaches its maximum allowable stress under at least one of the alternative load systems. The effects of instability have been ignored in this paper, the basis of design being with allowable stress. It has been shown that the minimum weight design of a statically indeterminate truss, subject to alternative load system, resolves itself to the determination of a series of fully stressed designs. The minimum of these will be the least weight design. Such a procedure does not seem practicable by ordinary design office method, but any one fully stressed design will give a good efficient structure. The method of successive approximation is used.

Schmidt, Lucian C. "Minimum weight layout of elastic, statically determinate, triangulated frames under alternative load systems." Journal of the Mechanics and Physics of Solids, Vol. 10 (1962), pp. 139-149.

The Michell theory for the absolute minimum weight design of triangulated frames under a single load system is extended to allow for alternative load systems. Only minimum weight statically determinate frames, designed to maximum allowable stresses, are discussed.

Schmit, L. A. "Structural design by systematic synthesis."

Proceedings. 2nd National Conference of Electronic Computation. Structural Division ASCE, Pittsburgh, Pennsylvania, 1960, pp. 105-132.

Schmit, Lucian and William Morrow. "Structural synthesis with buckling constraints." Journal of Structural Division. Proceedings of the American Society of Civil Engineers, Vol. 89, Part I (April 1963), pp. 107-126.

The writers define structural synthesis as a systematic process of optimum design. The techniques are made possible by the use of digital computers. The technique uses the area of the members as design parameters. The configuration and material are treated as continuous variables. Minimum weight design are presented for nine combinations of material and configuration all subject to the same loading conditions. Buckling effect is also considered and by matrix analysis structures have been solved.

Schmit, L. A., and Robert H. Mallet. "Structural synthesis and design parameter hierarchy." Journal of the Structural Division. Proceedings, American Society of Civil Engineers, Vol. 89, Part 2 (August, 1963), p. 269.

The structural synthesis concept may be used to size members automatically to select the geometrical configuration or layout, and to determine the combination of attainable material properties in order to achieve a minimum weight design. An improved synthesis technique is presented and the digital computer program is outlined. Multiple synthesis paths show convergence to essentially identical optimum structural design in most cases. The author has discussed in detail the method and its application.

Shanley, F. R. "Principles of structural design for minimum weight." Journal of the Aeronautical Sciences, Vol. 16 (March, 1949), pp. 133-149.

The usual procedure in structural analysis involves the determination of the strength of a certain structure under specified, loading conditions. From the design point of view the real problem is to determine the lightest practical arrangement of material which will transmit the required loads through specified distances. This may be accomplished through the use of the structural Index. Methods of comparing various materials or weight basis are developed. The structural Index provides a sound and convenient basis for presentation of test data, comparison of various designs for given material, comparison of different materials on an optimum basis and on loading conditions.

The author recommends that more emphasis be placed on the "design" approach, through the use of the structural Index, instead of on the "stress analysis" approach. The latter is best adapted to checking the accuracy of a structure already designed, while the structural index method permits the selection of the best structure for a given force transmission requirement.

Shanley, F. R. Weight Strength Analysis of Aircraft Structures. New York: Dover Publications, Inc., 1960.

This textbook in its first part discusses the principles of optimum structural design and part two and three for structural weight equations and material properties and behavior. The author emphasizes the design approach. He states briefly the procedure as follows: (a) Presentation of test data for various types of construction, resulting in different materials which will generally assist in planning research programs; (b) Comparisons of various designs for a given material, resulting in determination of optimum proportions for minimum weight; (c) Comparison of various materials on an optimum basis thereby allowing for the fact that optimum type of design varies with the material and the loading conditions; (d) Determination of a rational, allowable stress for structures subject to buckling.

Shield, R. T. Optimum Design Methods for Structures Plasticity. Pergamon Press, 1960.

Smirnov, A. F. Bars and Arches of Minimum Weight Subjected to Bending. (in Russian) Proc MIIT, 1950.

Sved, G. "The minimum weight of certain redundant structures." Australian Journal of Applied Science, Vol. 5 (1954), pp. 1-9.

A method has been presented whereby the minimum weight pin jointed frame structure corresponding to any configuration can be determined in a straight forward manner. The minimum weight structure is always a statically determinate structure. The method presented is limited to structures which are designed for a single load system. The paper shows that prescribing the stresses still permits the selection of the forces in the redundant members. For prescribed external forces, geometry of the structure and stresses, the cross sectional areas of the members, and the weight of the structure become the function of the forces in the redundant members, by proper selection of these forces the weight of structure can be made a minimum. The method has been illustrated with two examples.

Vargo, L. G. "Nonlinear minimum weight design of planer structures." Journal of Aeronautical Sciences, Vol. 23 (1956), pp. 956-960.

A direct method has been developed for determining the minimum weight fully plastic moments of a planer structure subject to set of concentrated loads. A general nonlinear relationship between the fully plastic moments and areas of the series of sections available to the designer is included in the method. Solution for the two span beam and rectangular bent under concentrated loads are obtained, and a comparison with the results of a linearity assumption is made. When geometrically similar sections are used, the linearity assumption results in up to a 15 per cent weight penalty. Consideration is given to a possible extension of the nonlinear theory to planer structures under distributed loads and space frames.

Vinogradov, A. I. Statical indetermination of bar systems of minimum weight (in Russian). Issl. Teor Sooruzh 6, 1954.

Vinogradov, A. I. Problems of the analysis of minimum weight structures (in Russian). Proc. Khark Kirov's Inst. Inzh Zhelezn Transp 25, 1-173-1955.

Wang, C. K. Statically Indeterminate Structures. New York: McGraw Hill, Inc., 1953.

Wasiutynski, Z. The Strength Design (in Polish). Acad Tech Sci, Warszawa, 1939.

Wasiutynski, Z. Application of Science to the Determination of the Forms of Structure (in Polish). Nauka Polska 8, 1-79-107, 1960.

Wasiutynski, Z. and Brandt Andrzej. "The present state of knowledge in the field of optimum design of structures." Applied Mechanics Review, Vol. 16 (1963), pp. 341-350.

The author has surveyed the developments on optimum design and information with several references. The author has suggested four main approaches to achieve optimum design. The author remarks in conclusion that in spite of differences in the design criteria and methods, the arrangement is always the same.

VITA

Arvind K. Shah

Candidate for the Degree of

Master of Science

Report: A SURVEY OF AVAILABLE LITERATURE ON OPTIMUM DESIGN OF STRUCTURES

Major Field: Civil Engineering

Biographical:

Personal Data: Born February 23, 1939, in Akrund Gujarat, India, the son of Keshavlal Vithaldas and Champaben K. Shah.

Education: Graduated from the Liberty High School, Bombay, India, in June, 1955; received the Bachelor of Engineering (Civil) from the Maharaja Sayajirao University, Baroda, India, in June, 1960. Completed requirements for the Master of Science Degree in August, 1965.

Professional Experience:

(1) Junior Engineer in Designs Circle (Roads and Building). Government of Gujarat, Ahmedabad, India, from August 31, 1960, to December 18, 1961.

(2) Surveyor Assistant Grade I in Military Engineer Services, Ministry of Defense, Government of India from December 20, 1961, to January 11, 1964.

(3) Structural Assistant in R. K. Chokski and Argon Engineers, Architects and Engineers, Ahmedabad, India, from February 5, 1964 to September 1, 1964.