AN EMPIRICAL STUDY OF THE MATHEMATICAL PROPERTIES OF ARRIVAL PATTERNS AT A SERVICE FACILITY; A CASE STUDY INVOLVING THE USE OF EMPIRICAL DATA IN THE SCHEDULING OF SERVICING CAPACITY

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PREFACE

This report presents the findings of a special project conducted by the investigator at a service facility in Stillwater, Oklahoma. The project included (1) an empirical study of arrival patterns, and (2) a case study of existing methods and procedures. In undertaking this project, the investigator was motivated by two main incentives: (1) gaining experience in empirical research, and (2) contributing usable information and ideas to the business community.

Several acknowledgements are in order. I am greatly indebted to Dr. W. W. Thompson, Jr., Associate Professor of Management, who encouraged me to undertake such a study in the first place, and who provided valuable direction and guidance as the advisor for the project; Dr. W. A. Meinhart, Assistant Professor of Management, and Mr. W. Lewis Zimmerman, Associate Professor of Finance, who took a keen interest in the project and served on the Report Committee; Mr. & Mrs. Glen H. Hartman, proprietors of Swim's Campus Shop, who allowed the investigator to conduct the study and provided cooperation throughout the project; Dr. E. L. Swearingen, Dean of the College of Business, who encouraged me to enter graduate school, and Dr. T. R. Brannen, Director of Master of Business Administration program, who was most helpful and provided valuable guidance throughout my graduate work at Oklahoma State University.

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CHAPTER I

INTRODUCTION

The contents of this report pertain to a special project undertaken by the investigator in Stillwater during first semester of the academic year 1964-5. The project was conducted for the following purposes:

1. To investigate the mathematical properties of arrival patterns at a service facility.

2. To study existing methods of operation with a view to using empirical data and general observations for purposes of scheduling servicing capacity and improving over-all operating efficiency.

Data was collected by direct observation and time-study over a period of nine weeks. Application of statistical tests indicated the utility of empirical data for policy recommendations.

Results of the study include a plan for scheduling servicing capacity and certain suggestions with respect to over-all operating efficiency.

The report is divided into two parts, each major purpose being treated separately. Chapter II deals with the description, analysis, and findings of the empirical study. Chapter III is concerned with utilization of empirical data in determining a servicing schedule and suggesting methods and procedures for general operations at the facility. Summary and conclusions of the project are presented in Chapter IV.

CHAPTER II

EMPIRICAL STUDY OF THE MATHEMATICAL PROPERTIES OF ARRIVAL PATTERNS AT A SERVICE FACILITY

A preliminary review of the selected facility indicated that the probability of the number of arrivals is small relative to the large potential population and that the arrival rate for various time intervals of the day is not uniform. It was decided to test two hypotheses (A and B) to suit the above observation. For purposes of statistical treatment and logical sequence, Hypothesis B will be stated and treated following a complete coverage of Hypothesis A.

Statement of Hypothesis A

That the observations with respect to the arrival pattern of customers within a specified time interval are distributed according to the Poisson probability function.

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!} \; ; \quad x = 0, 1, 2, \ldots, \infty.$$

Conditions

The empirical study was conducted under the following conditions: Swim's Campus Shop, 520 W. Elm, Stillwater, Oklahoma, was selected as the location for the project. This facility is conveniently located immediately to the east of the Oklahoma State University campus. Its clientele is composed largely of student, faculty, and university staff. Business schedule coincides with the university's semester schedule; i.e., business is closed during all official vacations. For a perspective of this facility see Appendix A.

Swim's food service operation was selected for observation. For purposes of simplicity and illustration, food service at the facility is described with respect to one customer in terms of the following steps:

1. Customer proceeds to the order counter where several pads of order blanks, several pencils, and a stack of 4" x 4" plastic squares with numbers are provided. The customer marks his choice of pre-printed food items on the order blank; selects a number; assigns it to the order blank; retains the plastic number square; places the order on the counter/hands it to a counter attendant; proceeds to find a place to sit down and wait for his order.

2. The order is placed on the kitchen counter by the counter attendant.

3. The order form is picked up by the kitchen attendant; food is prepared and placed on the kitchen counter along with the order form.

4. The counter attendant picks up the prepared order; inspects the order form for any drinks/counter items needed; provides any item from the fountain/counter; calculates the bill; and calls the number assigned by the customer into a microphone as a signal to the customer.

5. Customer proceeds to the order counter; pays the bill to the counter attendant; receives his food; and replaces the plastic square number on the counter.

The above arrival-service operation was described with respect to one customer, one counter attendant, and one kitchen attendant in order to illustrate the basic method of operation. Normal conditions of business are described in the following paragraph.

There are two basic structures of waiting line situations which describe the normal business conditions of Swim's food service. The first situation can be described as single-channel, single-phase. This occurs when a single line of waiting orders must be processed by one servicing operator manning a particular category of capital equipment. An example in this case may contain all grill orders. The other situation can be described as multiple-channel, single-phase. In this case, orders may involve grill and non-grill food items eligible for service simultaneously by more than one operator. It is clearly established, however, that under all circumstances the situation can be described as a single-phase; i.e., orders for processing are drawn on the one waiting line.

The above observations were recorded during peak periods of activity during which the number of kitchen and counter attendants is flexible. Also, the existing capital equipment in the kitchen, which may be broadly classified into two categories, grill and non-grill servicing equipment, is considered fixed.

Data Collection

Time duration and method of data collection are presented below. Data was collected over a period of nine weeks between November 16, 1964 and January 15, 1965 inclusive. It was decided to collect arrival data between 11:00 A.M. and 1:00 P.M. inclusive, four 30-minute continuous intervals, on all normal business days. Because of the non-representative nature of business on Saturdays and Sundays, it was decided to exclude these from the definition of normal business days. Arrival data for a total of 32 days was considered relevant for testing the hypothesis.

Direct observation was used to record the exact arrival times at the food service facility. An arrival was defined as a completed order blank for food items as filled in by a customer. The number of arrivals by time-interval by day of week is presented in Table 1.

Test of Hypothesis A

Hypothesis A: That the observations with respect to the arrival patterns of customers within a specified time interval are distributed according to the Poisson probability function.

$$f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} ; \; x = 0, 1, 2, \ldots, \infty.$$

Initially, graphical means were employed for testing the conformity between the actual distributions and the theoretical distributions for each of the four 30-minute time-intervals. The graphical tests are portrayed in Figures 1, 2, 3 and 4. These tests led the investigator to subject the data to the chi-square test of goodness of fit.

The distributions for each of the four 30-minute intervals were subjected to statistical tests in comparison to the Poisson, using the chi-square test of goodness of fit based on the following relationship:

$$\chi^2 = \sum_{0=1}^{K} \frac{(O_f - E_f)^2}{E_f}$$

TABLE I

EMPIRICAL DATA OF NUMBER OF ARRIVALS PER 30-MINUTE INTERVALS FOR 32 NORMAL BUSINESS DAYS

No.	Date		Day	11:00-11:30	11:31-12:00	12:01-12:30	12:31-1:00
1	Nov.	16	M	10	22	27	4
2	"	17	T	12	26	27	6
3	"	18	W	19	24	24	1
4	"	19	Th	10	22	32	11
5	"	20	F	16	29	20	8
6	"	23	M	19	25	18	7
7	"	24	T	10	13	23	6
8	"	30	M	15	17	33	3
9	Dec.	1	T	10	25	28	5
10	"	2	W	14	18	22	5
11	"	3	Th	10	32	32	10
12	"	4	F	26	18	24	8
13	"	7	M	14	22	27	5
14	"	8	T	13	16	20	12
15	"	9	W	10	25	17	6
16	"	10	Th	5	15	28	10
17	"	11	F	18	27	32	11
18	"	14	M	14	29	17	5
19	"	15	T	9	17	16	7
20	"	16	W	10	28	22	11
21	"	17	Th	20	28	29	6
22	"	18	F	28	27	31	4
23	Jan.	4	M	12	29	25	4
24	"	5	T	11	23	14	3
25	"	6	W	9	37	15	2
26	"	7	Th	10	18	36	3
27	"	8	F	6	34	31	9
28	"	11	M	14	27	39	0
29	"	12	T	15	22	28	7
30	"	13	W	9	37	28	4
31	"	14	Th	16	32	31	3
32	"	15	F	20	42	32	8
	Totals			434	806	828	194
	λ_t =			13.50	25.00*	25.00*	6.00

Mean arrival rate per time interval (30 Minutes)

* Rounded Off.

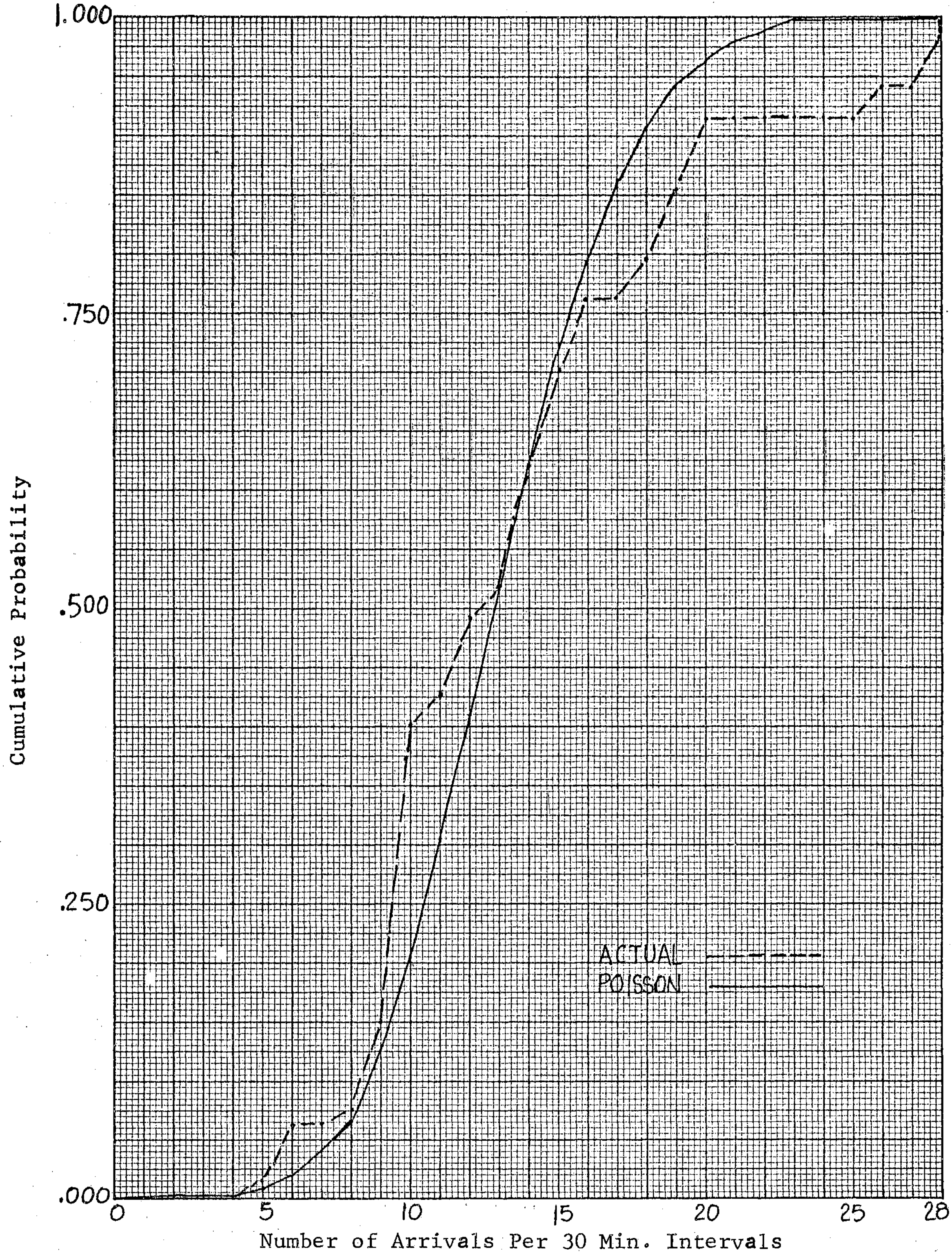


FIGURE 1

COMPARISON OF ACTUAL AND POISSON DISTRIBUTIONS: 11-11:30 A.M.

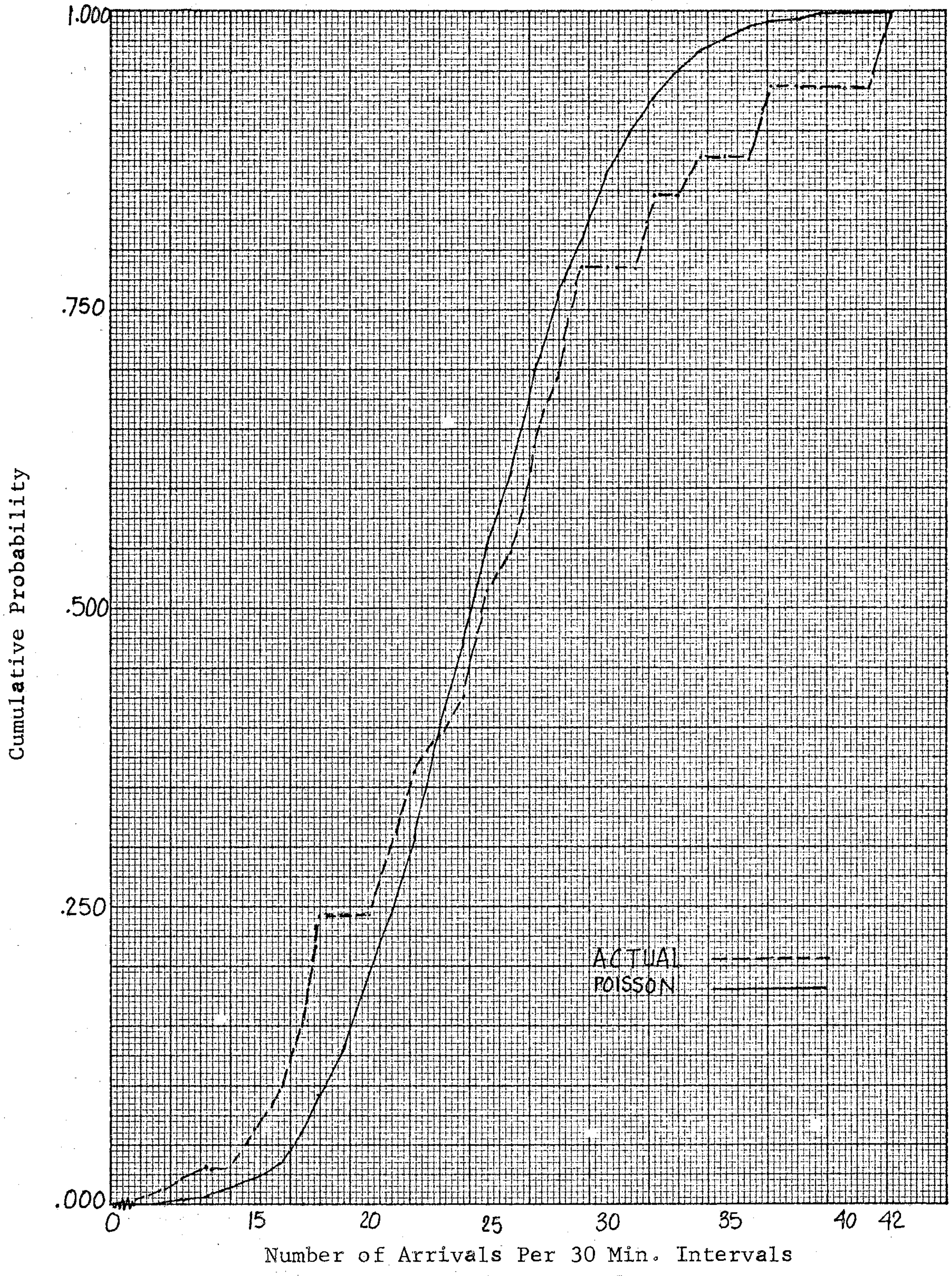


FIGURE 2

COMPARISON OF ACTUAL AND POISSON DISTRIBUTIONS: 11:31-12:00 Noon

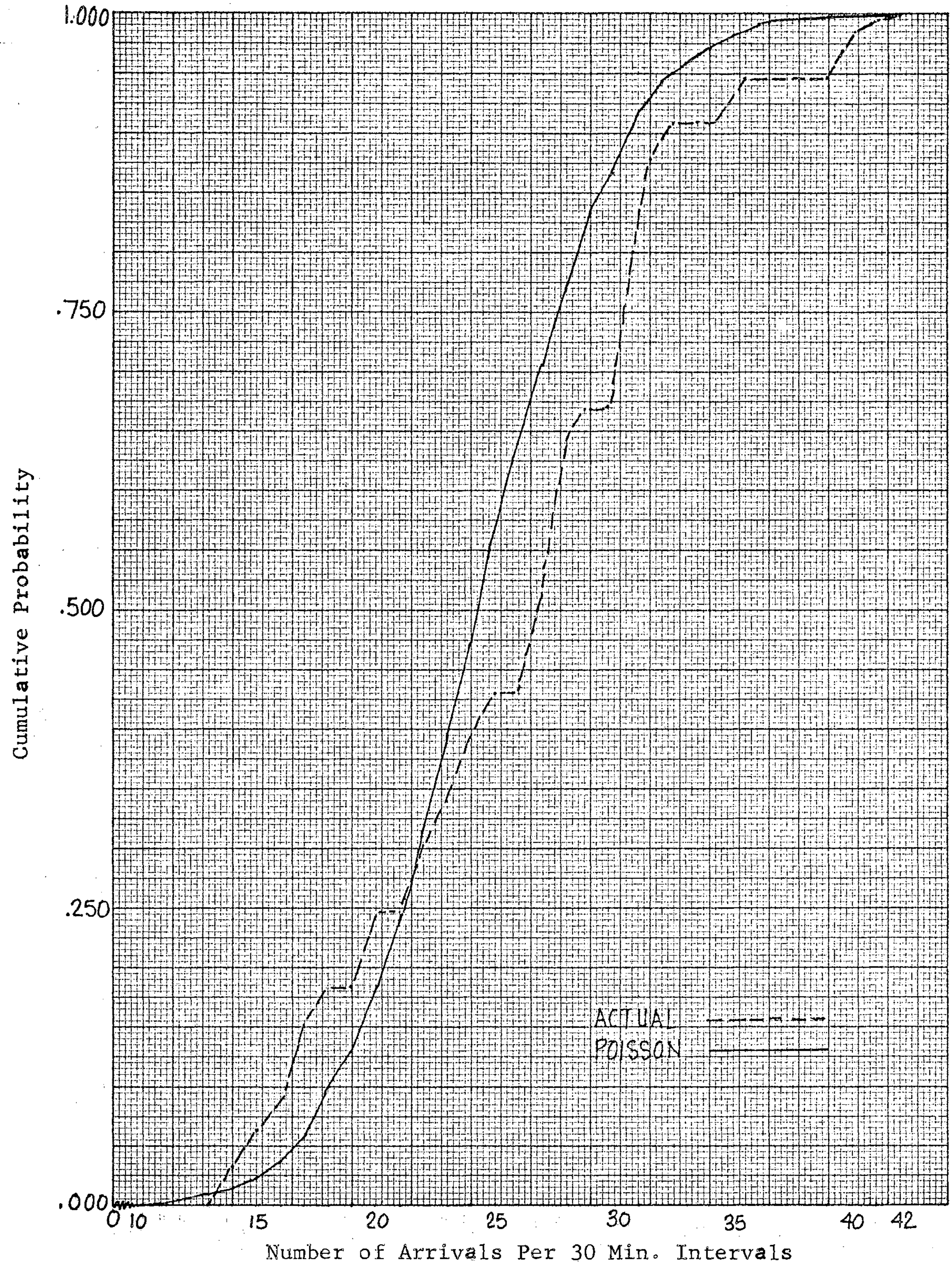
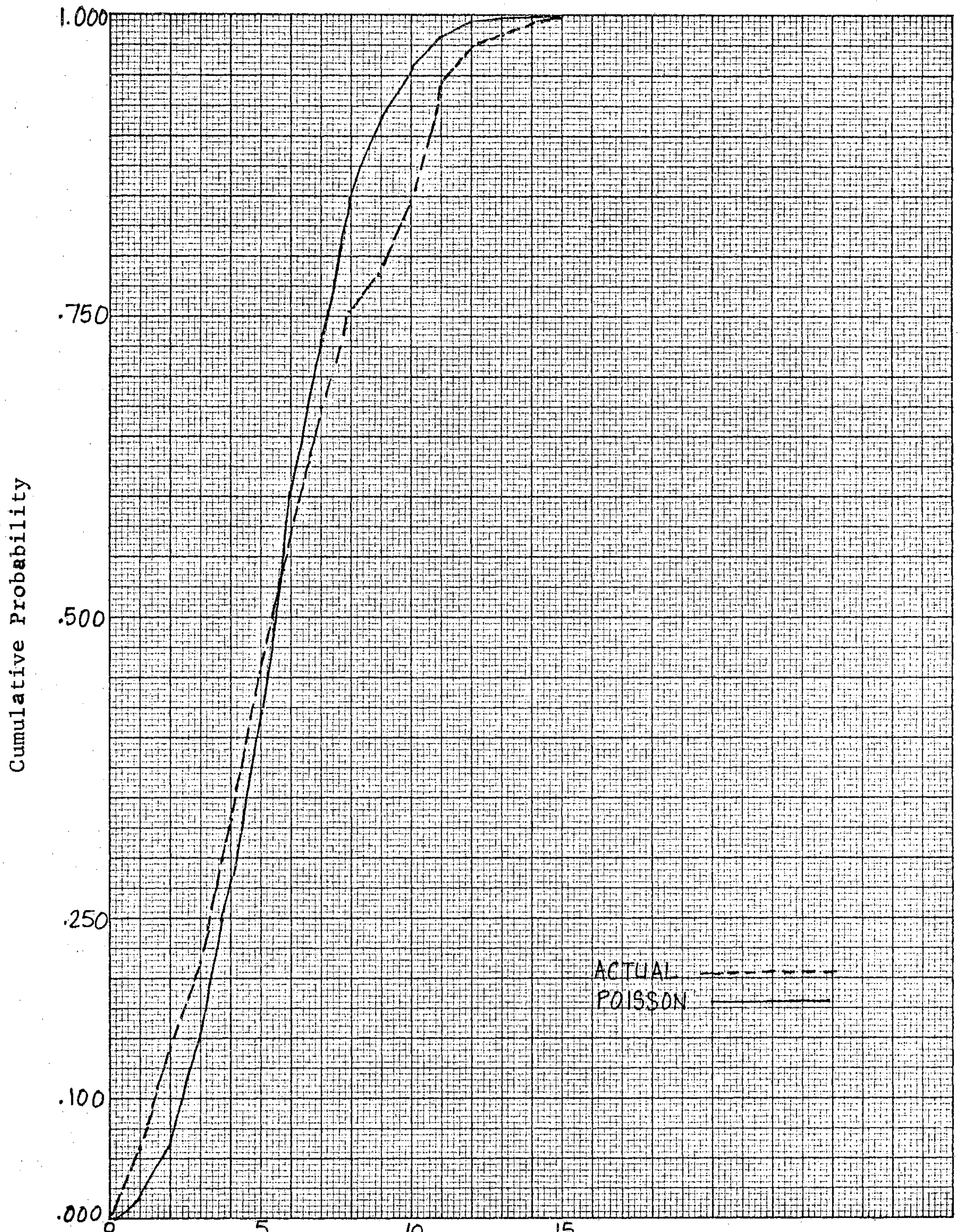


FIGURE 3

COMPARISON OF ACTUAL AND POISSON DISTRIBUTIONS: 12:01-12:30 P.M.



Number of Arrivals Per 30 Min. Intervals

FIGURE 4

COMPARISON OF ACTUAL AND POISSON DISTRIBUTIONS: 12:31-1:00 P.M.

where

O_f = observed value

E_f = expected value

This procedure consisted of comparing the actual frequencies of arrivals with the expected frequencies of arrivals calculated by using the assumed Poisson distribution with parameter set equal to sample values. The number of class intervals, k, was obtained by grouping expected frequencies such that the sum in each class was greater than or equal to 3 ($E_f \geq 3$). Because of the inherent properties of the Poisson probability function, the degrees of freedom were calculated by using k-2.

It was found that the distributions of arrivals for all four time-intervals were described by the Poisson probability function at the 30, 10, 20, and 20 per cent confidence levels respectively. Although confidence levels are not conclusive, graphical comparisons indicate a degree of conformity. It is suggested that the lack of a sufficient number of observations has led to an inadequate number of degrees of freedom which in turn has affected the confidence levels at which the hypothesis can be accepted.

The details of the chi-square test may be consulted in Appendixes B, C, D and E. A summary of the results is shown in Table II below.

TABLE II

SUMMARY OF RESULTS OF THE CHI-SQUARE TEST OF GOODNESS OF FIT

30-Minute Interval	$x^2 = \frac{(O_f - E_f)^2}{E f}$	k-2	P	x^2	RESULT
11:00-11:30	8.33	7	.30	8.38	Accepted
11:31-12:00	9.56	6	.10	10.60	Accepted
12:01-12:30	7.47	6	.20	8.56	Accepted
12:31- 1:00	7.26	6	.20	8.56	Accepted

Statement of Hypothesis B

Acceptance of Hypothesis A and scrutiny of arrival data led the investigator to test Hypothesis B for purposes of possible use of empirical data in policy formulation.

Hypothesis B: That the parameters (λ_i) of the various arrival distributions are significantly different.

Hypothesis B was accepted since the various parameter values differ significantly among the four 30-minute intervals as shown in Table III.

TABLE III

COMPARISON OF PARAMETER VALUES OF DIFFERENT TIME-INTERVALS

Time Interval	λ
11:00-11:30	13.50
11:31-12:00	25.00
12:01-12:30	25.00
12:31- 1:00	6.00

Test of Steady State

In order to confirm the utility of the empirical data for policy recommendation, it was decided to determine whether the system under review was a stable data generating process. It was decided to use control chart technique for conducting the steady state test based on the following relationships:

$u = \sigma^2 = np$;   ∵ Poisson is assumed

$\bar{c} = \sigma^2 c = \frac{\Sigma (np)}{m}$; $\bar{c}$ = Mean of arrival population

np = Total per sample

m = Number of samples each containing n opportunities of probability p of the event.

$\sigma \bar{c} = \sqrt{\bar{c}}$

$\bar{c} + 3\,\sigma \bar{c}$ = upper control limit

$\bar{c} - 3\,\sigma \bar{c}$ = lower control limit

Test control data and relevant computations are shown in Appendix F and control charts for each time-interval are presented in Figure 5, 6, 7 and 8 below.

Control charts in Figures 5 and 6 indicate the system to be out of control. An examination of the particular days when upper control limits were exceeded led the investigator to decide that these days were not representative of the time period. The three particular points in question were removed from the data, and revised control charts were constructed as shown in Figures 9 and 10; and the process was found to be in control within the revised limits. It was concluded that the system under review is a stable data generating process.

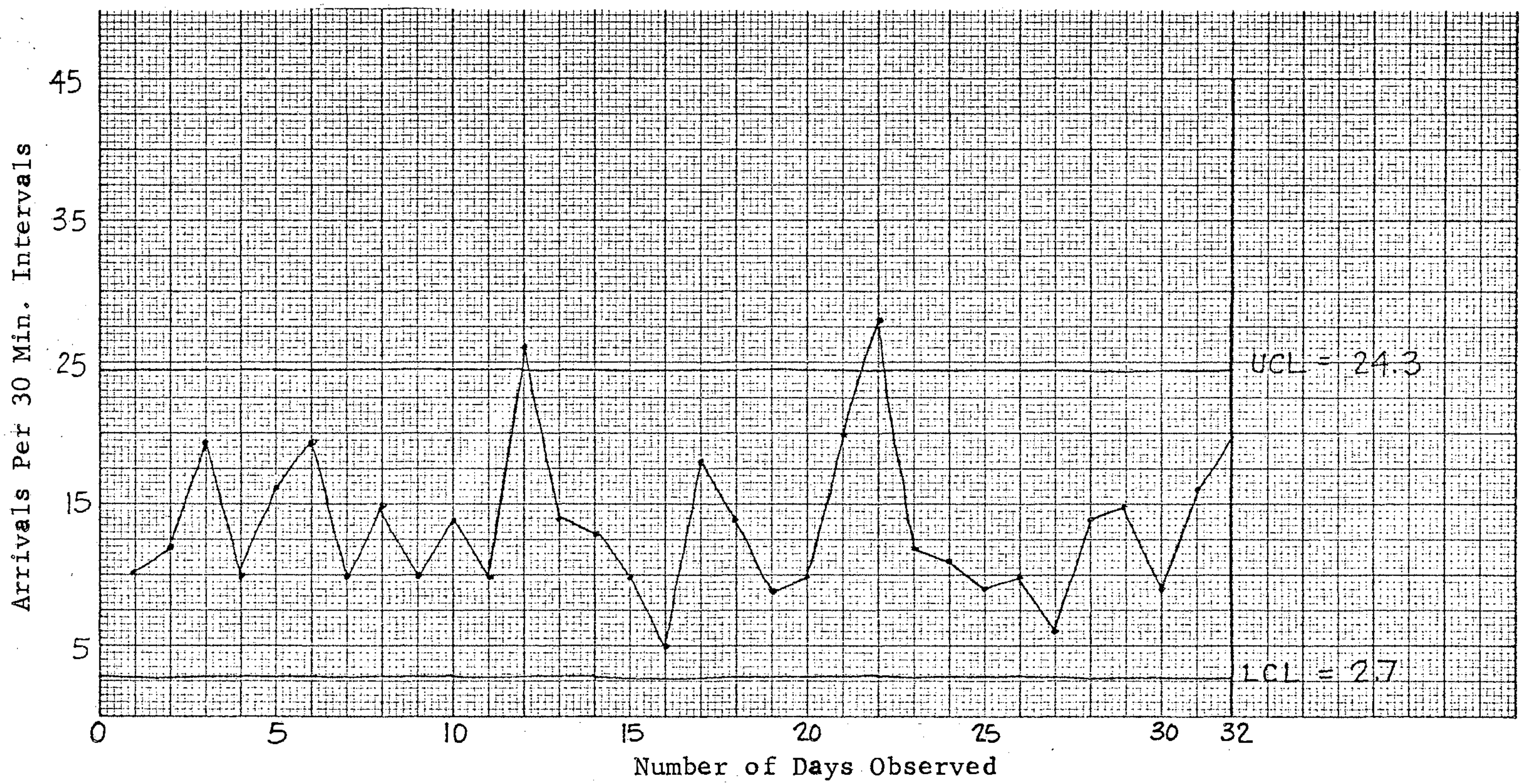


FIGURE 5

CONTROL CHART FOR ARRIVALS BETWEEN 11-11:30 A.M.

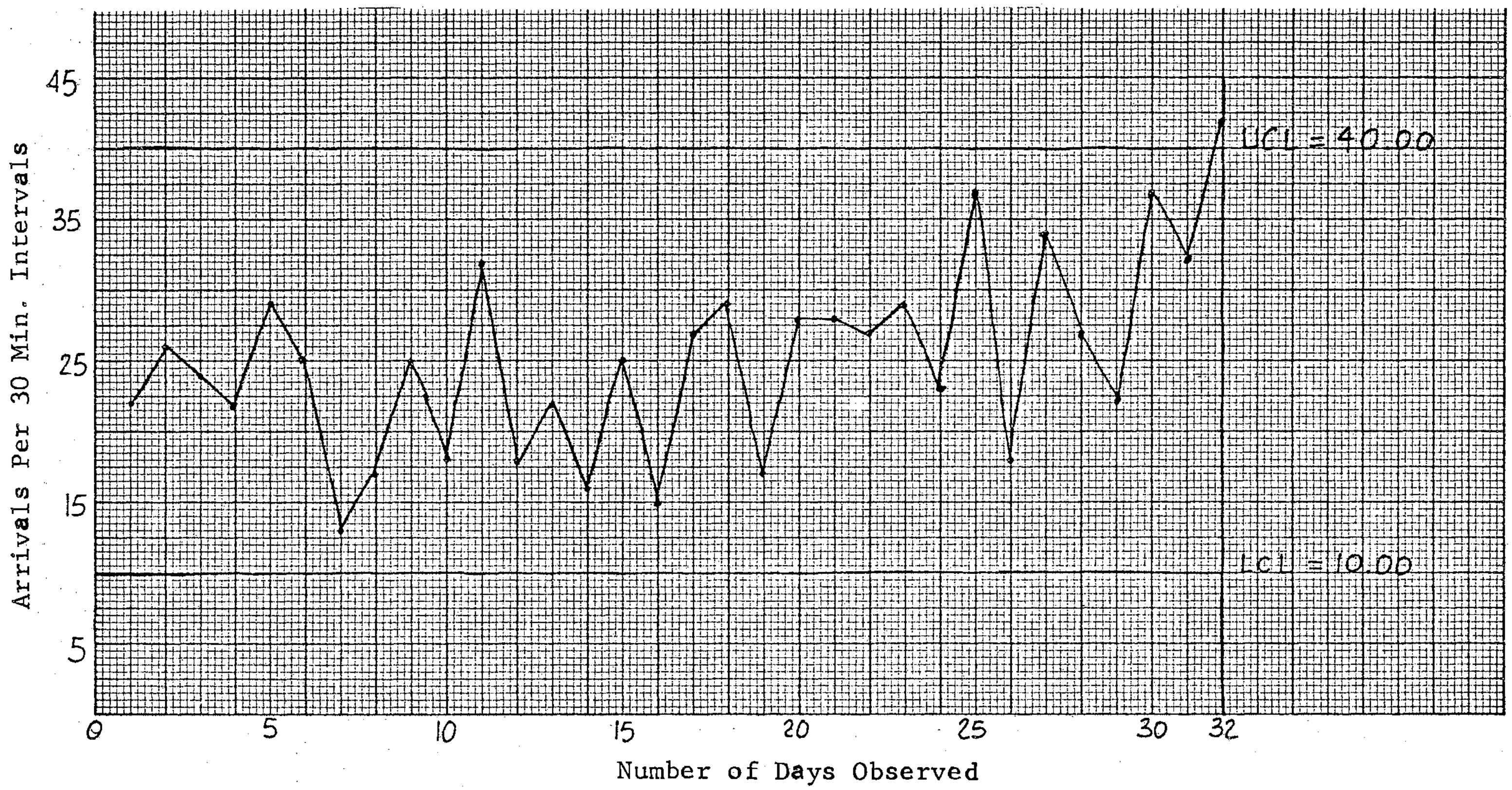


FIGURE 6

CONTROL CHART FOR ARRIVALS BETWEEN 11:31-12:00 Noon

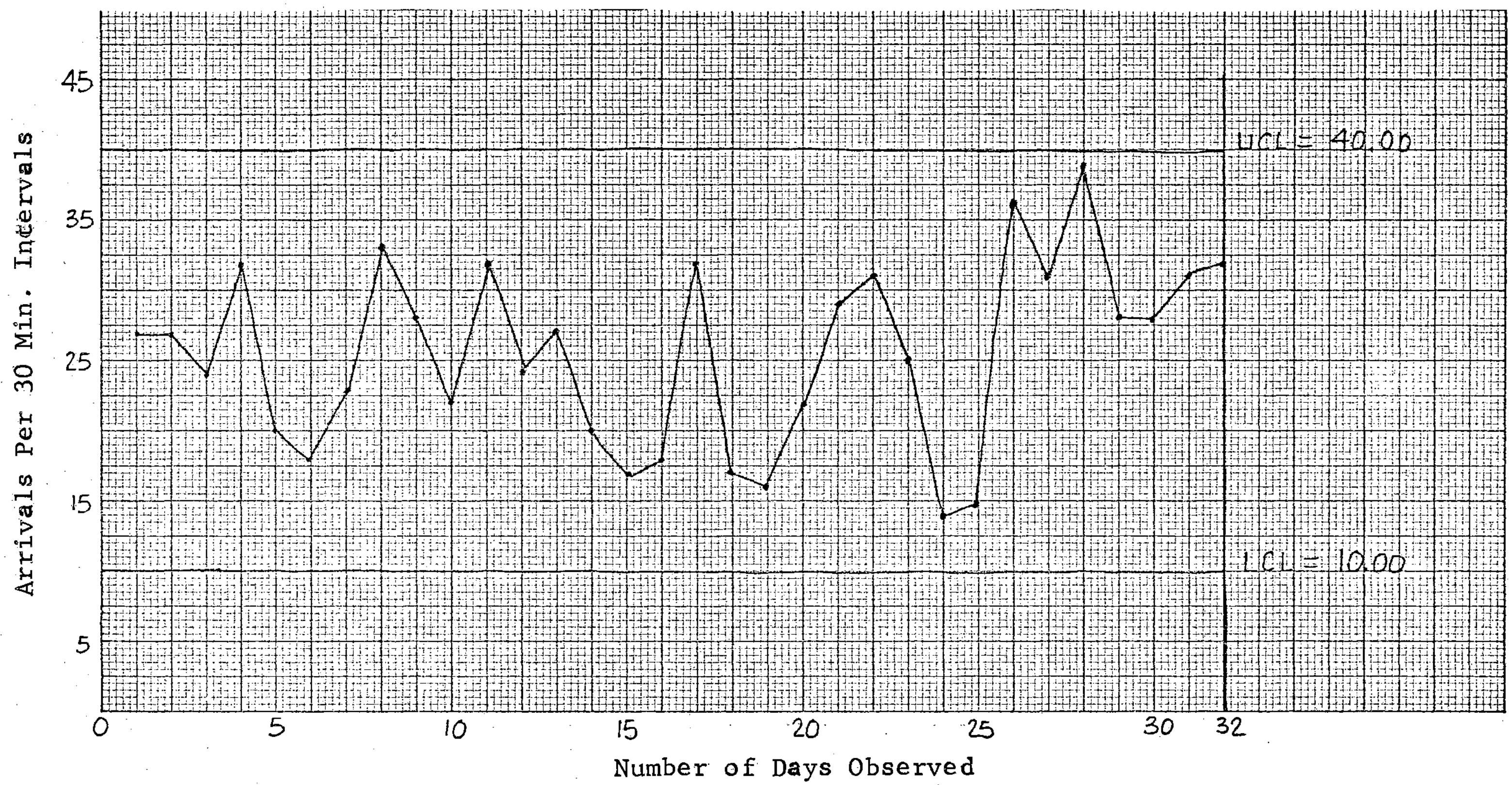


FIGURE 7

CONTROL CHART FOR ARRIVALS BETWEEN 12:01-12:30 P.M.

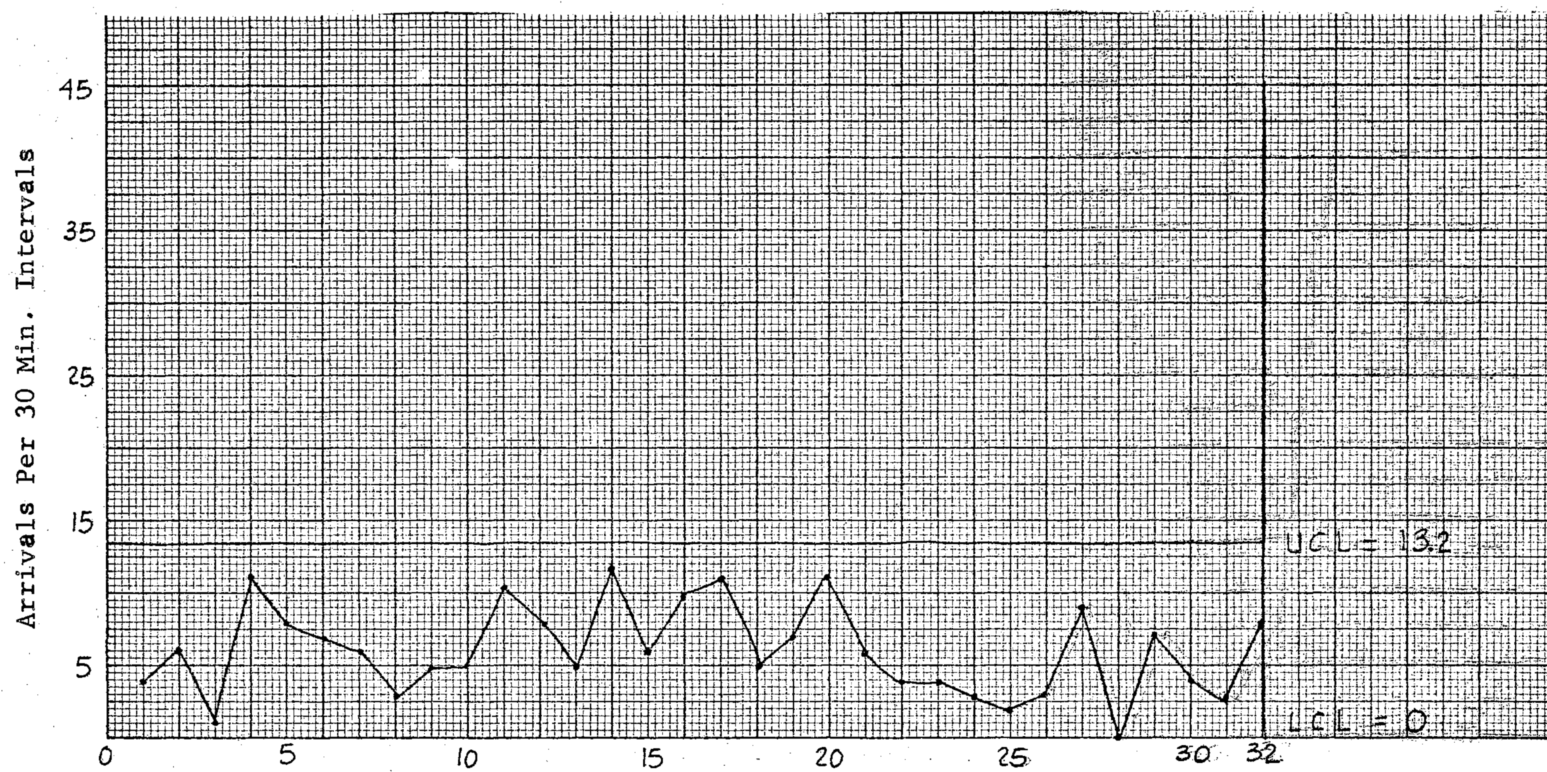


FIGURE 8

CONTROL CHART FOR ARRIVALS BETWEEN 12:31-1:00 P.M.

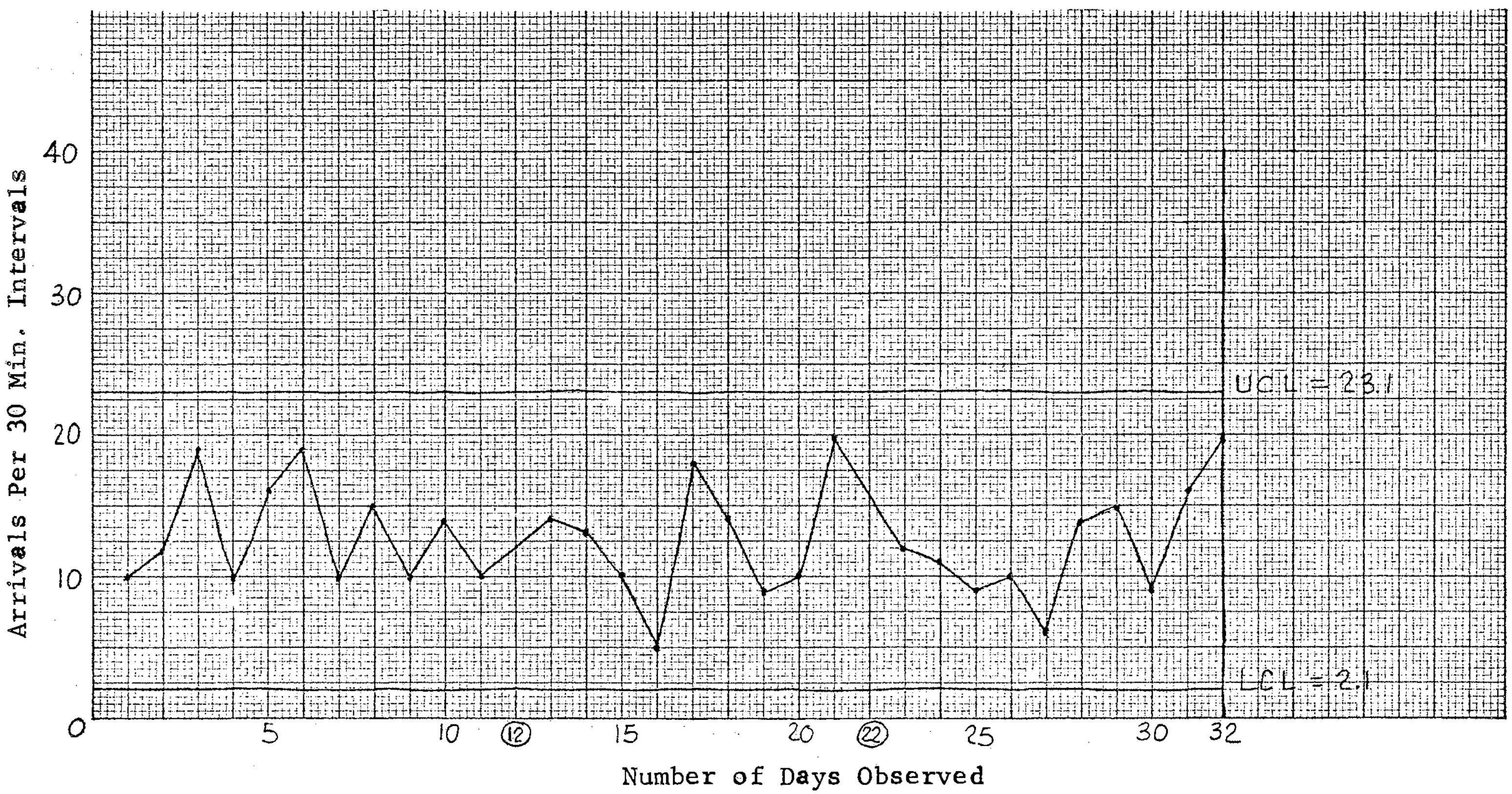


FIGURE 9

REVISED CONTROL CHART FOR ARRIVALS BETWEEN 11-11:30 P.M.

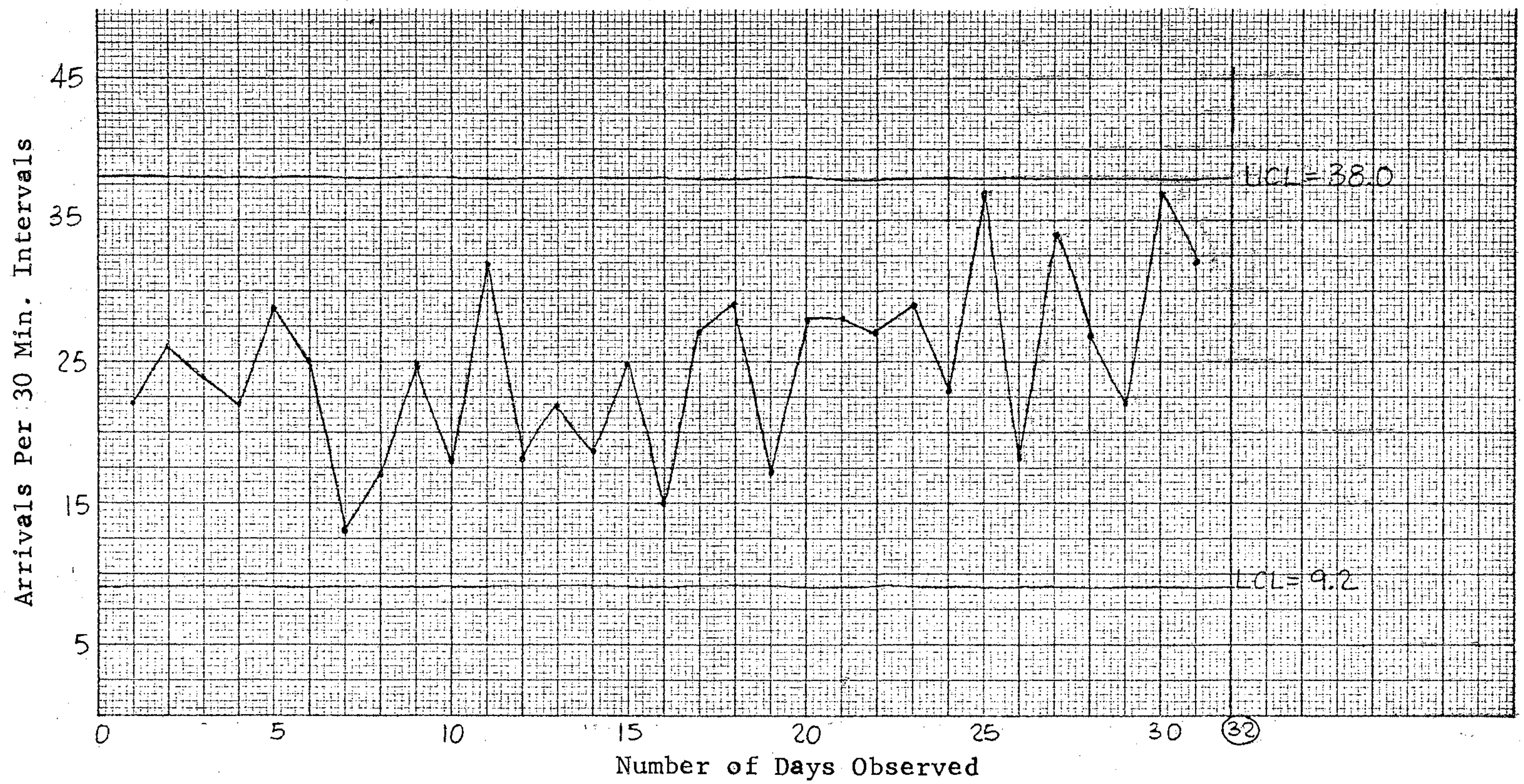


FIGURE 10

REVISED CONTROL CHART FOR ARRIVALS BETWEEN 11:31-12:00 Noon

Effect of a Particular Day

In order to investigate the effect of a particular day of the week with respect to arrival rates, it was decided to organize data by day of week per time-interval (Table IV).

TABLE IV

MEAN ARRIVAL RATE PER 30-MINUTE INTERVAL BY PARTICULAR DAYS

Day / Interval	M	T	W	T	F
11:00-11:30	14	11.4	11.8	11.8	19
11:31-12:00	24.4	20.2	28.1	24.5	29.5
12:01-12:30	26.5	22.2	21.3	31.3	28.3
12:31- 1:00	4	6.5	4.8	7.1	8
Mean Arrival Rate Per Hour By Day	34.5	30.2	33.0	37.4	42.4

It was concluded that the effect of any particular day on total demand within the two-hour period under consideration was not significant.

RESULTS

1. The following hypotheses with respect to arrival patterns were accepted:

   A. That the observations with respect to the arrival pattern of customers within a specified time interval are distributed according to the Poisson probability function; however, the level of confidence at which the hypothesis is accepted is probably not adequate at present. It is probable that additional data would increase the level of confidence.

B. That the parameters (λ_i) of the various Poisson distributions are significantly different; it appears to be so from an examination of Table III above.

2. The system under review appears to be a stable data generating process.

3. The effect of particular days of the week on arrival rates appears not to be significant.

The three results of the empirical study of arrival patterns may be used as bases for policy recommendation in the scheduling of servicing capacity. In addition, the hypotheses relating to the Poisson approximations could be employed in the generation of simulated data for future studies of the system. This, however, would be contingent upon additional data substantiating, at a higher confidence level, the appropriateness of the Poisson assumption.

CHAPTER III

CASE STUDY INVOLVING THE USE OF EMPIRICAL DATA IN THE SCHEDULING OF SERVICING CAPACITY

Purpose

The case study is concerned with the following purposes:

1. To use empirical data in the scheduling of servicing capacity.

2. To use general observations with a view to improving overall operating efficiency.

Data Collection

It was decided to obtain actual labor utilization data by using time study. Actual labor utilization time is considered to include actual physical involvement of an operator with a task (capital time in between actual labor utilization times is not considered).

All the elements necessary in servicing one order for food and drink items constitute actual labor utilization time per order as shown in Table V below.

Because of the variety of food items catered at Swim's, actual labor utilization times were obtained by stop-watch readings for every major food item. A series of times obtained per major item was used to arrive at expected times.

TABLE V

ELEMENTS OF ACTUAL LABOR TIME PER ORDER

Element No.	Activity
1	Take order form from order counter to kitchen counter
2	Processing in kitchen
3	Carry food and order form from kitchen counter to order counter area
4	Inspect order form for any drinks/item needed from counter area; place food in tray; provide silver
5	Fetch drinks/item
6	Calculate bill and call number on microphone
7	Make change and hand over food

By inspecting records of food items served during lunch intervals, a list of typical orders was drawn up to determine the expected actual labor utilization time per order as shown in Table VI. (Elements referred to in this table are same as described in Table V above.)

Scheduling of Servicing Capacity

Using the empirical data on arrival rates, the results obtained through statistical tests and analysis in Chapter I and data on actual labor utilization times, the following schedule for servicing capacity was determined as presented in Table VII.

A minimum of one attendant at the counter and one attendant in the kitchen is essential at all times. This requirement is imposed by nature of the basic method of operation at this servicing facility. As λ changes from the first time interval to the second time interval, it is necessary to increase manpower allocation from one to two, both

TABLE VI

TYPICAL ORDER EXPECTED ACTUAL LABOR UTILIZATION TIMES

No.	Typical Order	Elements 2 and 5	Elements 1, 3, 4, 6 and 7	Total
1	Hamburger Steak, Fries, Coke	.90	.50	1.40
2	Ham Steak, Milk	.77	.50	1.27
3	Cheeseburger, Coke	.39	.50	.89
4	Beefburger, Fries, Coke	.36	.50	.86
5	Steak Sandwich, Dr. Pepper	.36	.50	.86
6	Bacon & Tomato Sandwich, Fries, Coke	.56	.50	1.06
7	Grilled Cheese, Fries, Milk	.33	.50	.83
8	Bacon & Cheese Sandwich, Malt	.44	.50	.94
9	Chicken Salad, Pie, Milk	.76	.50	1.26
10	Chili, Fries, Coke	.47	.50	.97
11	Giantburger, Pie, Milk	.40	.50	.90
12	Grilled Steak, Dr. Pepper	.77	.50	1.27
	Total	6.51	6.00	12.51

$$E(x) = \frac{12.51}{12} = 1.042 \text{ mts.} = \text{Actual Labor Utilization Time Per Order}$$

TABLE VII

SCHEDULING OF SERVICING CAPACITY

Time Interval	λ	Actual Labor Utilization Time	Manpower in Kitchen	Manpower at Counter
11:00-11:30	13.5	14.04	1	1
11:31-12:00	25.0	26.00	2	2
12:01-12:30	25.0	26.00	2	2
12:31- 1:00	6.0	6.24	1	1

at the counter and in the kitchen. This is necessary in order to maintain effective servicing of incoming orders.

The determination of manpower allocation was based on the consideration that a 30-minute time-interval consists of two components: actual labor utilization time; and an allowance factor. The allowance factor includes capital utilization time for food and drink items, set-up times, and reserves for other subsidiary times such as customer approach to order counter.

The peak period at Swim's occurs during 11:31 A.M. and 12:30 P.M. This is supported by empirical data; and it was observed that customers tend to arrive in small groups (very frequently). In order to meet the demand during the peak period and provide speedy and efficient service, it is necessary to schedule manpower as suggested above.

Another distinct feature of the schedule presented in Table VII is that a maximum of two attendants at the counter and two attendants in the kitchen is sufficient to meet the current demand at Swim's. It can be shown that an increase in this number of scheduling servicing capacity will lead to diseconomies of scale due to available capital equipment and available space at the two points of service (the counter and the kitchen).

Figure 11 illustrates the policy with respect to scheduling of servicing capacity as λ changes through the various time intervals. It has been confirmed that labor is flexible at this service facility and allocations can be made conviently to suit demand requirements through various time-intervals. An examination of relevant data leads to the recommendation to double manpower allocation during the two hours under consideration as λ changes by 15. (This value is used to illustrate the scheduling of the servicing capacity in Figure 11.)

Study of Existing Methods and General Conditions

This section is concerned with noting all features considered as potential improvement areas as a result of the investigator's general observations during the period of empirical study (November 16, 1964 through January 15, 1965).

Seating Capacity and Arrangement. At present 90 customers can be accommodated at one time. The effective seating capacity is, however, much less than 90 (approximately 60) since one- and two-party customers tend to block at least two effective seats when four-seater booths are occupied. The northwest section of the facility is considered suitable for providing a different seating arrangement. This will feature an around-the-wall seating arrangement and will allow an open number occupation of seats. The suggested change is shown in perspective (Appendix G). It is estimated that

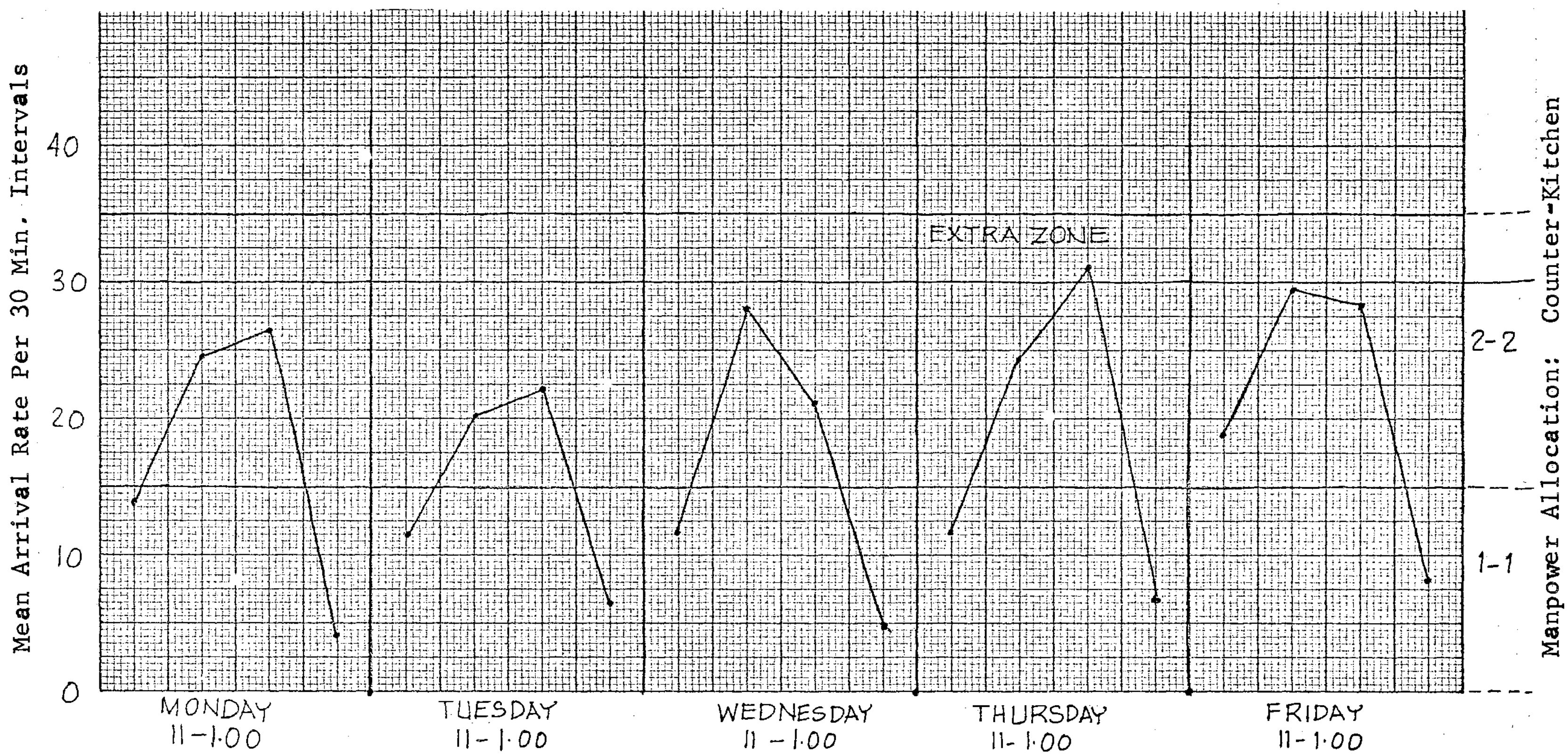


FIGURE 11

SCHEDULING OF SERVICING CAPACITY BETWEEN 11-1:00 P.M.

this change will lead to an increase in the effective seating capacity from 60 to 100.

Division of Work

It was observed that there is no division of tasks among attendants in this area (considering two counter attendants). The attendants tend to perform all varieties of tasks necessary in the servicing operation. Tasks are selected at random by the attendants. It was noticed, particularly during rush hours, that the attendants cross paths (work flow lines) to the point of hinderance. Time delays were noticed particularly with respect to "change-making" and "drink-fetching" operations. The situation is considerably worsened because of limited space between the order counter and the kitchen counter and the position of capital equipment during peak periods of activity when the number of attendants at the counter was observed to be three.

In the previous section, it was established that maximum of two attendants is sufficient to meet existing demand. Considering a maximum allocation of two attendants at the order counter, it is suggested that a program of division of labor be instituted for improving operating efficiency. A task list is presented below (Table VIII) as a solution.

In order for the proposed task list to be effective, a change in the current counter layout is suggested. This involves that the cash register must be moved as close to the point where orders are placed by the customer as possible.

TABLE VIII

DIVISION OF TASKS AT COUNTER AREA

No.	Task	Attendant (A or B)
1	Take order form from counter to kitchen counter	A
2	Carry food and order form from kitchen counter to order counter area	B
3	Place in tray; provide silver; fetch any drinks/item needed per order	B
4	Calculate bill; call number on microphone; make change; deliver food	A

Customer Convenience

The facility does not display to customers coming in for the first time any information concerning its method of service. Although the designated place for placing orders is identified, this is not considered sufficient because for new customers the counter attendant must invariably explain the method. Particular reference is made to the customer-number identification system in this respect.

With respect to the order-filling activity, it was observed that during peak periods clusters of customers tend to form since the order-blanks are stacked/scattered on the counter. For customer convenience, it is suggested that a special order rack be provided for the customer. The rack should display Swim's method of food service (i.e., how to place order, etc.) and provide a parallel placing of at least four stacks of order blanks and numbers so that four customers may be able to conveniently read and fill out order blanks simultaneously during rush hours.

Housekeeping

During peak periods it was observed that the general appearance of the facility deteriorates since used dishes, cups, baskets, and trays are removed by the customer from the table and placed on the wooden space between adjoining booths. Also, any used trays, dishes, etc. cleared away from customer area by attendants are placed on the main counter's east wing.

It is suggested that provision for maintaining a clean appearance be provided. Any available attendant, other than counter attendants during the peak period, should be allocated the task of clearing dishes, trays, etc. every 10 minutes.

Summary of Recommendations

1. Scheduling of servicing capacity based on analysis of demand and actual labor utilization as presented in Table VII, p. 26 should be adopted.

2. A program of division of work involving a maximum of two attendants at the counter area as presented in Table VIII, p. 30 should be instituted.

3. Suggestion for the proposed seating arrangement to increase effective seating capacity should be considered (See Apendix G).

4. Suggestions for providing greater convenience to the customer and improving housekeeping should be considered.

CHAPTER IV

SUMMARY AND CONCLUSIONS

A special project was undertaken at Swim's Campus Shop in Stillwater, Oklahoma, for purposes of investigating the mathematical properties of arrival patterns and studying existing methods of operations with a view to using empirical data and general observations for making policy recommendations for improvement.

The empirical data was subjected to statistical tests and it was found that the observations with respect to arrival patterns within specified time-intervals were distributed according to the Poisson probability function. It appeared that: the arrival rates for different time-intervals were significantly different; the demand situation was stable; and the effect of particular days on demand was not significant.

A case study which involved time study of actual labor utilization at the facility and analysis of existing operating methods was conducted in order to use the results of the analysis of empirical data for policy recommendations.

The conclusions of the project consist of the following policy recommendations: (1) Scheduling of servicing capacity considering the arrival rate per 30-minute interval and actual labor utilization time. An allocation of two attendants is necessary to meet an interval demand of 15 customers. Allocation of manpower should be doubled as demand per interval is greater than 15. Given the existing capital equipment, the physical structure of the facility, and current demand, the maximum number of attendants is four (two at the counter and two in the kitchen). (2) A program of division of work where the tasks for each of the two counter attendants is clearly designated to allow for smooth work flow and operating efficiency. (3) A proposed seating arrangement which will lead to an increase in the effective seating capacity by approximately 60 per cent. (4) Suggestions for installing an information rack equipped with all material necessary to place an order will enhance customer convenience, particularly during rush hours, and improve housekeeping through programmed activity for maintaining appearance.

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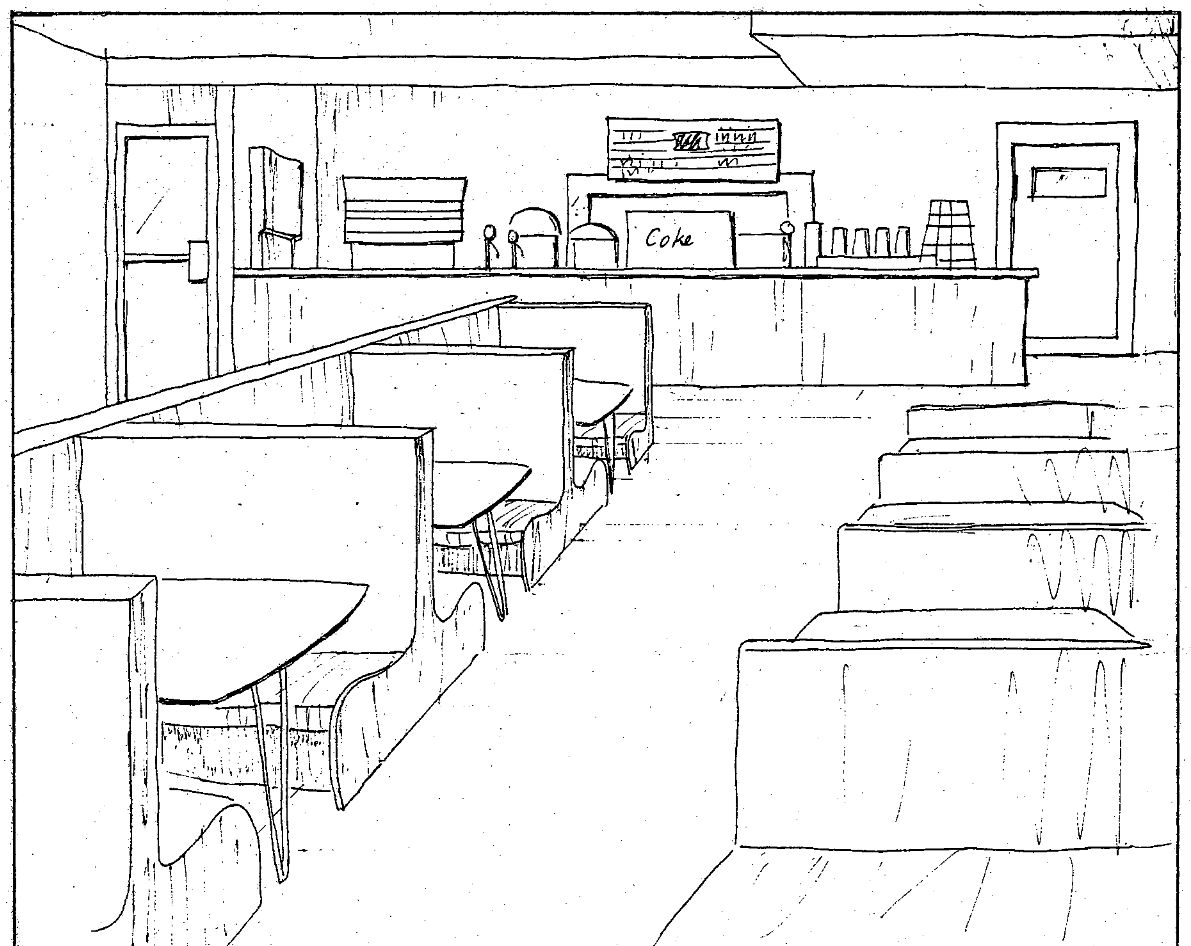
APPENDIX A



EXISTING VIEW: SWIM'S CAMPUS SHOP

APPENDIX B

CALCULATION OF x^2 FOR ACTUAL ARRIVALS DISTRIBUTION AND THEORETICAL POISSON DISTRIBUTION

11:00 - 11:30 A.M.

Number Arrivals	Observed Frequency (O_f)	Expected Frequency (E_f)	Deviation ($O_f - E_f$)	$\frac{(O_f - E_f)^2}{E_f}$
0	0	0		
1	0	0		
2	0	0		
3	0	.032		
4	0	.064		
5	1	.160		
6	1	.352		
7	0	.704		
8	0	1.216		
9	3	1.792	+0.680	.11
10	8	2.432		
11	1	2.976	+3.592	2.38
12	2	3.360	-1.360	.55
13	1	3.488	-2.488	1.77
14	4	3.360	+0.640	.12
15	2	3.040	-1.040	.35
16	2	2.560		
17	0	2.016	-2.565	1.45
18	1	1.504		
19	2	1.088		
20	2	.736	+1.672	.84
21	0	.480		
22	0	.288		
23	0	.160		
24	0	.096		
25	0	.032		
26	1	.032		
27	0	0		
28	1	0	+0.912	.76
TOTALS	32	31.968	+0.032	$x^2 = 8.33$

$$\lambda = 13.5$$

$$E > 3$$

$$x^2 = 8.33 < x^2_{.30}(7) = 8.38$$

APPENDIX C

CALCULATION OF x^2 FOR ACTUAL ARRIVALS DISTRIBUTION AND THEORETICAL POISSON DISTRIBUTION

11:31 - 12:00 Noon

Number Arrivals	Observed Frequency (O_f)	Expected Frequency (E_f)	Deviation ($O_f - E_f$)	$\frac{(O_f - E_f)^2}{E_f}$
0	0	0		
1	0	0		
2	0	0		
3	0	0		
4	0	0		
5	0	0		
6	0	0		
7	0	0		
8	0	0		
9	0	0		
10	0	.032		
11	0	.032		
12	0	.064		
13	1	.096		
14	0	.192		
15	1	.320		
16	1	.512		
17	2	.704		
18	3	1.024		
19	0	1.344	+3.680	3.14
20	0	1.632		
21	0	1.984	-3.616	3.61
22	4	2.272		
23	1	2.432	+0.296	.02
24	1	2.528		
25	3	2.560	-1.088	.23
26	1	2.432		
27	3	2.272	-0.704	.10
28	2	2.016		
29	3	1.760	+1.224	.40
30	0	1.440		
31	0	1.184		
32	2	.928	-1.552	.68

Number Arrivals	Observed Frequency (O_f)	Expected Frequency (E_f)	Deviation (O_f-E_f)	$\frac{(O_f-E_f)^2}{E_f}$
33	0	.672		
34	1	.512		
35	0	.384		
36	0	.224		
37	2	.192		
38	0	.096		
39	0	.096		
40	0	.032		
41	0	.032		
42	1	.000	+1.760	1.38
TOTALS	32	31.938	- 0 -	9.56

$\lambda = 25$

$E \geqslant 3$

$x^2 = 9.56 < x^2_{.10(6)} = 10.60$

APPENDIX D

CALCULATION OF x^2 FOR ACTUAL ARRIVALS DISTRIBUTION

AND THEORETICAL POISSON DISTRIBUTION

12:01 - 12:30 P.M.

Number Arrivals	Observed Frequency (0_f)	Expected Frequency (E_f)	Deviation (0_f-E_f)	$\frac{(0_f - E_f)^2}{E_f}$
0	0	0		
1	0	0		
2	0	0		
3	0	0		
4	0	0		
5	0	0		
6	0	0		
7	0	0		
8	0	0		
9	0	0		
10	0	.032		
11	0	.032		
12	0	.064		
13	0	.096		
14	1	.192		
15	1	.320		
16	1	.512		
17	2	.704		
18	1	1.024		
19	0	1.344	+1.680	.65
20	2	1.632		
21	0	1.984	-1.616	.72
22	2	2.272		
23	1	2.432	-1.704	.62
24	2	2.528		
25	1	2.560	-2.088	.85
26	0	2.432		
27	3	2.272	-1.704	.62
28	4	2.016		
29	1	1.760	+1.224	.40
30	0	1.440		
31	3	1.184		
32	4	.928	+3.448	3.36

Number Arrivals	Observed Frequency (0_f)	Expected Frequency (E_f)	Deviation (0_f-E_f)	$\frac{(0_f - E_f)^2}{E_f}$
33	1	.672		
34	0	.512		
35	0	.384		
36	1	.224		
37	0	.192		
38	0	.096		
39	1	.096		
40	0	.032		
41	0	.032		
42	0	.000	+0.760	.25
TOTALS	32	31.938	- 0 -	7.47

$\lambda = 25$

$E \geqslant 3$

$x^2 = 7.47 < x^2_{.20(6)} = 8.56$

APPENDIX E

CALCULATION OF x^2 FOR ACTUAL ARRIVALS DISTRIBUTION AND THEORETICAL POISSON DISTRIBUTION

12:31 - 1:00 P.M.

Number Arrivals	Observed Frequency (O_f)	Expected Frequency (E_f)	Deviation $(O_f - E_f)$	$\frac{(O_f - E_f)^2}{E_f}$
0	1	.064		
1	1	.480		
2	1	1.440		
3	4	2.848	+2.168	.97
4	4	4.288	-0.288	.01
5	4	5.152	-1.152	.25
6	4	5.120	-1.120	.24
7	3	4.416	-1.416	.45
8	3	3.296	-0.296	.02
9	1	2.208		
10	2	1.312	-0.520	.07
11	3	.736		
12	1	.352		
13	0	.160		
14	0	.096		
15	0	.000	+2.656	5.25
TOTALS	32	31.966	+0.032	7.26

$\lambda = 6$

$E \geq 3$

$x^2 = 7.26 < x^2_{.20(6)} = 8.56$

APPENDIX F

ORGANIZATION OF TEST CONTROL DATA AND

COMPUTATION OF CONTROL LIMITS

11:00 - 11:30		11:31 - 12:00		12:01 - 12:30		12:31 - 1:00	
Sample Number	np	Sample Number	np	Sample Number	np	Sample Number	np
1	10	1	22	1	27	1	4
2	12	2	26	2	27	2	6
3	19	3	24	3	24	3	1
4	10	4	22	4	32	4	11
5	16	5	29	5	20	5	8
6	19	6	25	6	18	6	7
7	10	7	13	7	23	7	6
8	15	8	17	8	33	8	3
9	10	9	25	9	28	9	5
10	14	10	18	10	22	10	5
11	10	11	32	11	32	11	10
12	26	12	18	12	24	12	8
13	14	13	22	13	27	13	5
14	13	14	16	14	20	14	12
15	10	15	25	15	17	15	6
16	5	16	15	16	18	16	10
17	18	17	27	17	32	17	11
18	14	18	29	18	17	18	5
19	9	19	17	19	16	19	7
20	10	20	28	20	22	20	11
21	20	21	28	21	29	21	6
22	28	22	27	22	31	22	4
23	12	23	29	23	25	23	4
24	11	24	23	24	14	24	3
25	9	25	37	25	15	25	2
26	10	26	18	26	36	26	3
27	6	27	34	27	31	27	9
28	14	28	27	28	39	28	0
29	15	29	22	29	28	29	7
30	9	30	37	30	28	30	4
31	16	31	32	31	31	31	3
32	20	32	42	32	32	32	8

$\bar{\bar{c}}$ = 13.5 $\bar{\bar{c}}$ = 25.0 $\bar{\bar{c}}$ = 25.0 $\bar{\bar{c}}$ = 6.0

c = $\bar{c}$=3.6   c = 5.0   c = 5.0   c = 2.4

11:00 - 11:30		11:31 - 12:00		12:01 - 12:30		12:31 - 1:00	
Sample Number	np	Sample Number	np	Sample Number	np	Sample Number	np
UCL = 24.3		UCL = 40.0		UCL = 40.0		UCL = 13.2	
LCL = 2.7		LCL = 10.0		LCL = 10.0		LCL = --	

Revised

$\bar{c}$ = 12.6	$\bar{\bar{c}}$ = 23.6
c = 3.5	c = 4.8
UCL = 23.1	UCL = 28.0
LCL = 2.1	LCL = 9.2

APPENDIX G

PROPOSED SEATING ARRANGEMENT

VITA

Mohammed Akmal Saifie

Candidate for the Degree of

Master of Business Administration

Report: AN EMPIRICAL STUDY OF THE MATHEMATICAL PROPERTIES OF ARRIVAL PATTERNS AT A SERVICE FACILITY; A CASE STUDY INVOLVING THE USE OF EMPIRICAL DATA IN THE SCHEDULING OF SERVICING CAPACITY

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Biographical:

Personal Data: Born in Gonda, India, February 7, 1937, the son of Mohammed Muslim and Tayyab Saifie.

Education: Attended grade school in Kanpur and Meerut, India; graduated from St. Patrick's High School in 1954; received the Bachelor of Science degree from the Oklahoma State University, with a major in Production Management, in May, 1964, the requirements having been completed in January, 1964; completed requirements for the Master of Business Administration degree in May, 1965.

Professional experience: Joined John Lewis & Co., Oxford Street, London, England, in September, 1965, for two and a half years, and worked up to the position of Assistant, Incentive Plan Operations; Joined Burma-Shell Oil Co., Karachi, Pakistan, August, 1959, and worked as Office Supervisor, Efficiency Research Department, for one year; Employed for the summer, 1962, by Kerr-McGee Oil Industries, Oklahoma City, Oklahoma, as a Systems Analyst; Employed for the summer, 1964, by Esso Standard Eastern, New York City, New York, as a Systems Analyst; During undergraduate years at College of Business, Oklahoma State University, worked as a part-time statistics lab instructor.