

**BOND IN PRESTRESSED CONCRETE**

by

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**Baroda, India**

**1962**

Submitted to the faculty of the Graduate School of  
the Oklahoma State University  
in partial fulfillment of the requirements  
for the degree of  
**MASTER OF SCIENCE**

May, 1965

BOND IN PRESTRESSED CONCRETE

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## ACKNOWLEDGEMENT

The writer, in completing the final phase of his graduate study, wishes to express his sincere appreciation and gratitude to the following individuals:

To Dr. R. L. Janes, his major advisor, for valuable guidance throughout the preparation of this report.

To Professor R. L. Flanders and Professor J. V. Parcher for their kind instructions during his studies.

To Mr. and Mrs. Odhermal T. Advani, his parents, for their continued encouragement and support throughout his education.

To Mr. and Mrs. Nevel O. Slack for their helpful interest.

To Miss Eileen Deal for her help in checking the proof.

Finally, to Mrs. Sue Ormiston for her excellent job of typing.

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## NOMENCLATURE

|                 |  |
|-----------------|--|
| $r, d$          | Radius and diameter of wire, respectively          |
| $f_s$           | Tensile stress in wire at any point                |
| $f_{se}$        | Pre-tension stress in wire                         |
| $\mu_s$         | Poisson's ratio of wire                            |
| $\mu_c$         | Poisson's ratio of concrete                        |
| $E_s$           | Elastic modulus of wire                            |
| $E_c$           | Elastic modulus of concrete                        |
| $\sigma_r$      | Radial stress at concrete and wire interface       |
| $\sigma_t$      | Tangential stress at concrete and wire interface   |
| $\phi$          | Coefficient of friction between steel and concrete |
| $u$             | Unit bond stress                                   |
| $l$             | Distance from end of pre-tensioned member          |
| $P$             | Prestressing force                                 |
| $A$             | Area of concrete section                           |
| $e$             | Eccentricity of prestressing force                 |
| $I$             | Moment of inertia of concrete section              |
| $L$             | Transfer Length                                    |
| $t_b$           | Thickness of specimens                             |
| $E_{pm}$        | Modulus of elasticity of plastic model             |
| $\epsilon_o$    | Maximum concrete strain                            |
| $\epsilon_{so}$ | Maximum retained strain                            |
| $\epsilon_{is}$ | Maximum value of loss of steel strain              |
| $\epsilon_s$    | True steel strain                                  |



|               |   |
|---------------|---|
| m             | Modular ratio   |
| p             | Percent steel   |
| $\epsilon_c$  | Concrete strain at a distance x from the end              |
| $\epsilon'_s$ | Loss of steel strain at a distance x from the end         |
| g             | Slip or pull-in   |
| $l_u$         | Embedment length  |
| $f_{sb}$      | Maximum steel stress at the time general bond slip occurs |

## INTRODUCTION

### 1.1 Bond in Reinforced Concrete

In reinforced concrete a comparatively small volume of steel is embedded in a large volume of concrete. Since steel is a much stronger and tougher material than concrete, it is possible for a small volume of properly embedded steel to carry a large proportion of the load imposed on the concrete encasing the steel. The transfer of load or stress from the concrete to the steel is made possible by the resistance established between the concrete and surface of embedded steel bar. The higher the resistance to relative motion or slippage under stress the more effective will be the desired interaction between the concrete and the steel. This resistance to slippage has been called bond or bond resistance. Without any bond resistance the embedded steel would be practically useless. With inadequate bond only a fraction of the desired interaction between concrete and steel would be obtained resulting in waste of material or an unsafe assembly. With adequate bond the primary requirement for proper interaction is met and both steel and concrete can be stressed as intended by the designer with economy and safety.

It is apparent, therefore, that definite information on the subject of bond resistance is necessary and of great practical importance. It is also apparent that

since it involves interaction between two such widely different materials as concrete and steel, each of which is subject to the influence of many variables, many combinations of variables are involved which make a proper study of bond resistance complicated and extensive.

Bond was intensively studied, by means of pull-out and beam specimens, at the University of Illinois by Abrams between 1909 and 1912, resulting in Bulletin 71, comprising 238 pages, published in 1913. Notwithstanding this extensive research, Abrams found it advisable in 1919 to conduct further research on bond with pull-out tests at the Structural Materials Research Laboratory at Lewis Institute, which was reported in Bulletin 17, published in 1925. In general, the results of these two researches provided the principal source of information on bond in relation to practical design problems. However, in recent years there has gradually developed among concrete technicians and designers a feeling that information on bond resistance is too incomplete to answer many questions of common interest in connection with modern concrete.

Some of these questions have been answered by an excellent series of papers by Messrs. Gilkey, Chamberlain and Beal, and by papers by Messrs. Davis and Kelly, Withey, Wernisch, Shank, Mylrea, and others. However, so many factors are involved and so many different combinations encountered in the assembly of steel and

concrete that many important questions will probably remain unanswered for years to come.

We shall, however, now study the nature of bond and a simple pull-out test and the conventional formula to calculate the bond stress.

## 1.2 Bond in Prestressed Concrete

Bond between concrete and steel must exist if concrete is to be prestressed by the pretensioning method for with this method the steel is tensioned before the concrete is placed and is released after the concrete has developed sufficient strength. The bond in prestressed concrete is of two types, Transfer and Flexural Bond.

A number of tests carried out by different people have been described to show the factors on which the bond is dependent.

## BOND IN REINFORCED CONCRETE

### 2.1 Nature of Bond in R.C.C.

For concrete and steel to work together in a beam it is necessary that stresses be transferred between the two materials. The term "bond" is used to describe the means by which slip between concrete and steel is prevented or minimized (as already described). Wherever the tensile or compressive stresses in a bar change, bond stresses must act along the surface of the bar to produce the change. Bond stresses are in effect longitudinal shearing stresses acting on the surface between the steel and concrete. They are normally evaluated in terms of pounds per square inch of bar surface.

In the case of smooth bars, the bond strength is largely dependent upon adhesion between the bar and concrete, but even after adhesion is broken, friction between the materials continues to provide a considerable bond resistance. Friction resistance is low for a smooth bar surface, such as that of cold-rolled steel, and is higher for rougher surfaces.

Deformed bars give a higher bond resistance by providing an interlock between the steel and concrete.

In this case the bond strength is primarily dependent upon the bearing (compressive) strength of the concrete against the lugs and the shear strength of the concrete between the lugs. Adhesion and friction become minor elements in this case. Longitudinal splitting of the concrete covering the bar can be the limiting strength factor when the cover is thin.

### 2.1-1 Conventional Bond Formula

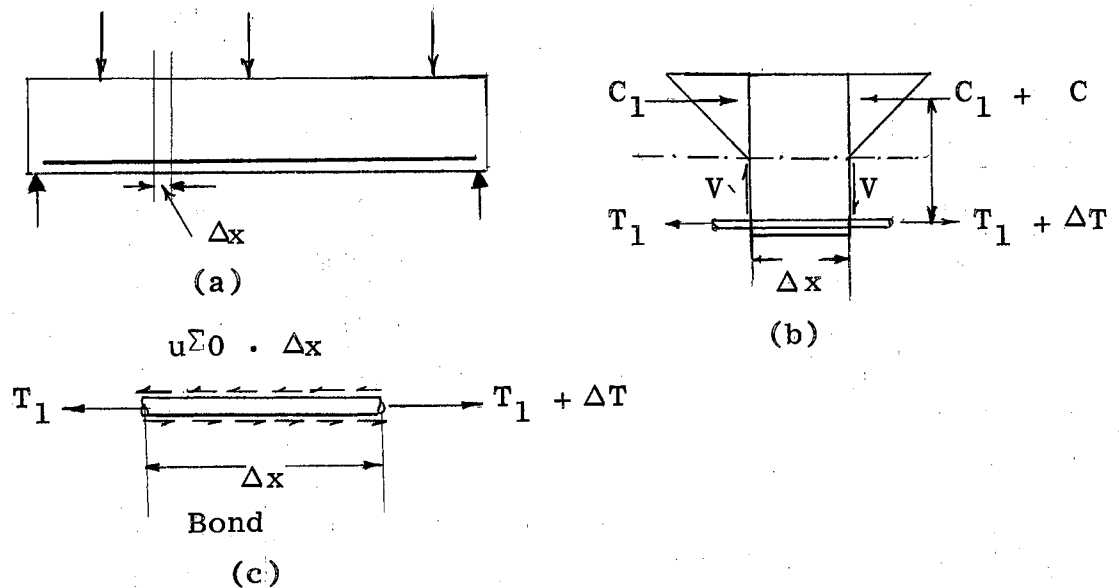


Fig. 2.1

Consider a short length  $\Delta x$  of the beam in Fig. (a) subject to a constant shear  $V$  and moments  $M_1$  and  $M_2 = M_1 + \Delta M$ . These moments will produce the bending stresses shown in the Fig. 2.1(b). Summation of moments about A gives

$$\sum M_A = V \Delta x - \Delta T \cdot jd = 0$$

$$\Delta T = \frac{V \cdot \Delta x}{jd} \quad (2.1)$$

In Fig. 2.1(c) the  $\Delta x$  length of bar is taken as the free body and the bond stresses on the surface of the bar are indicated. If the average unit bond stress is called  $u$  and the perimeter of the bar (or bars)  $\Sigma_0$ ,

$$\Delta T = u \Sigma_0 \cdot \Delta x \quad (2.2)$$

Equating these two values of  $\Delta T$  gives the conventional unit bond stress formula

$$\Delta T = u \Sigma_0 \Delta x = V \frac{\Delta x}{jd}$$

$$u = V / \Sigma_0 jd \quad (2.3)$$

The same relation results for a beam carrying a uniform load  $w$  if the shear  $V$  is taken as that at the center of the  $\Delta x$  element. Although this is an exact formula for the assumed conditions, the bond conditions in an actual beam, as indicated later, are quite different.

Strictly interpreted, this formula gives the average bond stress and this average is the critical bond stress only when all bars have the same diameter.

(b) Mixed bar sizes

When  $n_1$  bars of  $D_1$  diameter are used together with  $n_2$  bars of  $D_2$  diameter, the bond stresses  $u_1$  and  $u_2$  are different. Since  $\Delta f_s$  for a  $\Delta x$  length is the same for each bar, it follows that for individual bars:

$$\frac{\Delta T_1}{n_1} = \Delta f_s \cdot \pi \frac{D_1^2}{4} = u_1 \Sigma_{01} \Delta x = u_1 \pi D_1 \Delta x \quad (2.4)$$

$$\frac{\Delta T_2}{n_2} = \Delta f_s \cdot \pi \frac{D_2^2}{4} = u_2 \Sigma_{02} \Delta x = u_2 \pi D_2 \Delta x \quad (2.5)$$

The ratio between these two relations shows that  $\frac{D_1}{D_2} = \frac{u_1}{u_2}$ , or that the unit bond stress varies as the bar diameter. The total shear  $V$  can be subdivided into  $V_1$  and  $V_2$  parts in the ratio of the moments carried by the two groups of bars, that is, in the ratio of  $\frac{T_1}{T_2}$  or  $\frac{A_{s1}}{A_{s2}}$

$$V_1 = V \cdot \frac{A_{s1}}{A_s} \qquad V_2 = V \cdot \frac{A_{s2}}{A_s}$$

$$u_1 = \frac{V_1}{\Sigma O_1 jd} = \frac{V \cdot A_{s1}/A_s}{\Sigma O_1 jd} = \frac{V}{\frac{A_s}{A_{s1}} \cdot \Sigma O_1 \cdot jd} \quad (2.6)$$

But  $\frac{A_s}{A_{s1}} \cdot \Sigma O_1$  is simply the perimeter that would be obtained if the entire  $A_s$  were made from bars of diameter  $D_1$ . If this is designated as  $\Sigma O'_1$

$$u_1 = \frac{V}{\Sigma O'_1 jd} \quad (2.7)$$

$$u_2 = \frac{V}{\Sigma O'_2 jd} \quad (2.8)$$

If  $A'_s$  is made of the same diameter bars as  $A_s$ , the bond stress  $u'$  on  $A'_s$  multiplied by the ratio  $\frac{f'_s}{f_s}$ . Since this ratio is usually less than unity, the bond stress  $u'$  will usually be low.



## 2.1-2 Bond Pullout Tests

Permissible bond stresses have been established largely from pullout tests with some beam tests as confirmation. In the pullout test a bar is embedded in a cylinder or rectangular block of concrete and the force required to pull it out or make it slip excessively is measured. The Fig. 2.2(a) shows the type of test. Slip of the bar relative to the concrete is measured at the bottom (loaded end) and top (free end). The bond stress in such a specimen is very non-uniform. Even a very small load causes some slip and high bond stress near the loaded end, but leaves the upper part of the bar totally unstressed (as shown). As more load is applied, the slip at the loaded end increases and both the high bond stress and slip extend deeper into the specimen. With plain bars the bond stress will decrease to the friction or drag value wherever adhesion has been broken by slip; such slip is indicated by the heavy line in the right-hand sketches of the figure. The maximum bond is somewhat idealized in these sketches; its distribution depends on the type of bar and probably varies along the bar more than shown.

When the slip first reaches the unloaded end, the maximum resistance has nearly been reached. Failure will usually occur (1) by longitudinal splitting of the concrete in the case of deformed bars, or (2) by pulling the bar through the concrete in the case of a very smooth bar, or (3) by breaking the bar, if the embedment is long enough.

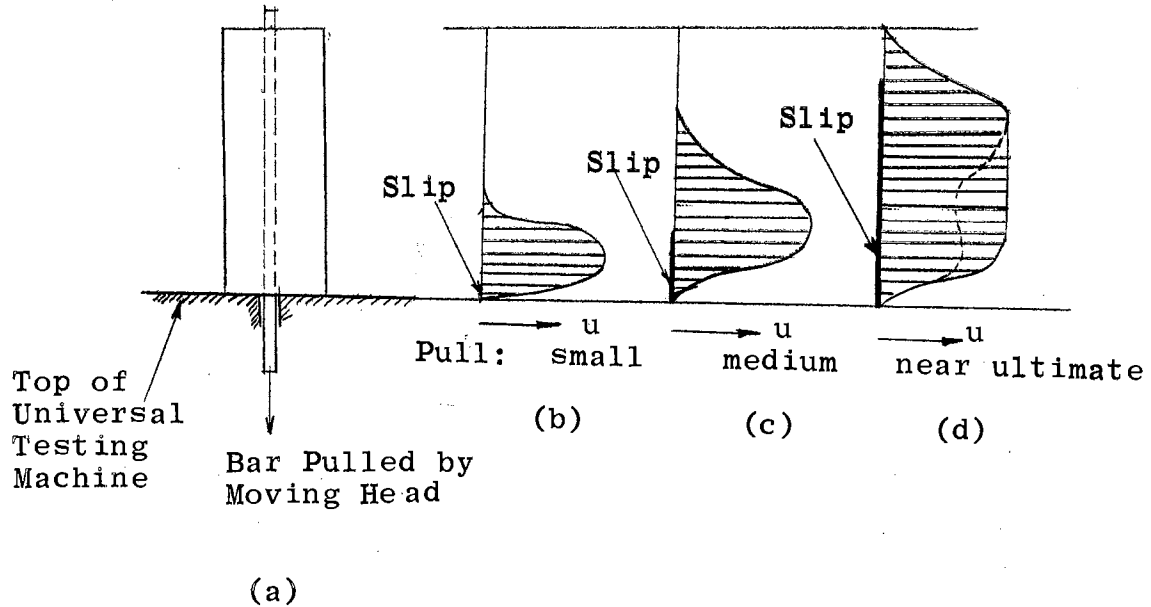


Fig. 2.2

### Bond Pullout Test With Bond Stress Distribution

The average bond resistance is always calculated just as though it were uniform over the bar embedment length. Actually, the bond stress varies greatly as slip develops and at any load the average is an average of values quite dissimilar, an average between ultimates and similar values. The very first slip of the loaded end of the bar represents essentially an ultimate bond stress over a short length of the bar, although the calculated average bond stress may be quite low. This test procedure can not measure maximum unit bond resistance unless an extremely short specimen is used (and such is not the customary test).

In the opinion of Professor Phil M. Ferguson of the University of Texas, the above mentioned test is defective in several ways. First, bond stress in a beam is usually critical in a tension zone, whereas the usual pullout specimen has the surrounding concrete in compression. Second, shearing stresses in a beam complicate the failure, whereas the pullout specimen carries no external shear. Third, in many pullout tests the specimens have been reinforced with spiral wire reinforcement to prevent a splitting failure. In spite of the fact that some beam tests seem to corroborate ordinary pullout tests, other types of beams do not. To Dr. Ferguson, it appears that the ultimate strength with deformed bars may be considerably lower than pullout tests have indicated, possibly 25% to 50% lower in some practical cases.

## BOND IN PRE-TENSIONED PRESTRESSED CONCRETE

### 3.1 Prestressed Concrete

Before actually describing the bond in pre-tensioned prestressed concrete, the author feels it necessary to describe prestressed concrete.

Prestressing, as the name indicates, consists of inducing compressive stress in the zone which will be tensile under the extreme loads. This compressive stress neutralizes the tensile stress that will be developed on loading, with the result that, in the tensile zone, the tensile stress in the concrete will be zero, or very small, and insufficient to cause any cracks.

M. Freyssinet defines prestressing as follows: To prestress a structure is artificially to create in that structure, either prior to or simultaneously with the application of external loads, such permanent stresses that, in combination with the stresses due to external load, the total stresses remain everywhere, and for all state of load envisaged within the limits of stress that the material can support indefinitely.

The prestress in concrete members is created by pulling high tensile steel strands to the required degree (so as to give the required prestressing force) and anchoring them.

The concrete is then poured. When the concrete hardens, the tendons are cut out loose from the bulk-heads and the prestress is transferred to the concrete.

### 3.1-1 Failures in Pretensioned Concrete Beam

A pretensioned concrete beam may fail in one of three ways; flexure, shear or bond. Numerous acceptable methods for predicting ultimate strength in flexure are presently available and considerable current research is devoted to the problem of shear capacity. As yet, no practical method of analyzing the bond capacity of a member has been presented. The various analyses of shear and flexure all assume that bond failure will not occur first. Because test data for bond show great statistical variation it is generally agreed that design should be based on flexure and shear and that requirements for bond be so severe as to preclude the possibility of failure in bond.

### 3.2 Functions of Bond

Pretensioned steel in prestressed concrete members serves a dual function. Part of the available tensile strength of the steel is used first to establish a compressive prestress in the concrete. Secondly, if a member is loaded beyond cracking, all or part of the steel tensile strength may be utilized to assist the concrete in resisting the externally applied bending moment.

### 3.3 Nature of Bond

Bond in pretensioned concrete beams is of two types; transfer bond and flexural bond. Transfer bond utilizes a part of the available tensile strength of the steel to establish compression in the concrete. Flexural bond results from the action of external loads on beams. After cracking, the increase in steel stress above effective prestress develops flexural bond stress between the steel and concrete.

#### 3.3-1 Transfer Bond

Prestress transfer bond exists near beam ends after the load in the tensioned strand has been transferred to the concrete member. The length over which this transfer is made is termed the prestress transfer length, and depends mainly on the amount of prestress, surface condition of the strand, the strength of the concrete (described later in this report), and the method of steel stress release, which in these tests was gradual. Three factors which contribute to bond performance are adhesion between concrete and steel, friction between concrete and steel, and mechanical resistance between concrete and steel. In the transfer zone, reduction in the tensile strain of the steel does not equal the compressive strain in the concrete at the same point. There is relative movement of steel and concrete, and accordingly adhesion cannot contribute to prestress transfer. Friction is considered to be the

principal agent causing stress transfer from pretensioning steel to concrete. As the tension in the strand is released, the strand diameter tends to increase, thus producing high radial pressure against the concrete, which in turn produces high frictional resistance in the transfer zone. Mechanical resistance probably contributes little to prestress transfer in the case of individual smooth wires, but it may be a factor of some significance in the case of strand.

### 3.3-2 Flexural Bond

Flexural bond of significant magnitude exists only after the concrete beam has been loaded to cracking. When the concrete cracks, the bond stress in the immediate vicinity of the cracks rises to some limiting stress, slip occurs over a small portion of the strand length adjacent to the cracks and the bond stress near the cracks is then reduced to a low value. With continued increase in load, the high bond stress progresses as a wave from the original cracks toward the beam ends. The bond stress remaining behind the wave is always lower than the maximum value at the peak of the bond stress wave.

If the peak of the high bond stress wave reaches the prestress transfer zone, the increase in steel stress resulting from the bond slip decreases the strand diameter, reduces the frictional bond resistance, and precipitates general bond slip. Following loss of frictional resistance,

mechanical resistance is the only factor which can contribute to bond between concrete and steel. If the beam is prestressed with strand, the helical shape of the individual wires will provide sufficient mechanical resistance that the beam can support additional load even after slip of the strand at the beam ends. This mechanical resistance of strand to continued slip will be discussed later.

The investigation, which follows, was carried out during 1952-53 in the Research and Development Division of the Portland Cement Association.

#### 3.4 Scope of Tests

The study was divided into major series of tests to investigate the two categories of bond action described.

In the first test series, prestressed concrete prisms of small cross-section were instrumented to determine the distribution of prestress transfer bond between steel and concrete when the steel pre-tension was released. Variables incorporated into this first series were: four wire diameters and one strand, two concrete strengths, and three surface characteristics of wire and strand.

In the second test series, short beam specimens were loaded to failure with centerpoint loading to develop high flexural bond stresses. Steel strains were measured at several points throughout the length of each beam. Variables considered in this second series were: three wire diameters and one strand, clean and rusted wire, and several degrees of wire and strand pretension.



### 3.5 Prism Tests

#### 3.5-1 Variables and Test Procedure

Specimens for the first series were designed to evaluate only the transfer of stress from pretensioned wires and strands to the concrete through prestress transfer bond. Prisms 2 x 2 inches in cross-section and 72 or 96 inches long were used. Each prism contained a single pretensioned wire or strand at the longitudinal axis. The following wire diameters and surface conditions were used:

- (1) 0.162 inch diameter wire
  - (a) Clean-steel was thoroughly washed with carbon tetrachloride and then wiped with a cloth soaked in acetone
  - (b) Lubricated-steel was cleaned as described above and then liberally coated with a light rust-preventative oil
  - (c) Rusted-steel was rusted by seven days' exposure to 100 percent relative humidity. No attempt was made to remove the loose rust or scale from the steel
- (2) 0.100 inch wire
  - (a) Clean
  - (b) Lubricated
- (3) 0.197 inch wire
  - (a) Clean
  - (b) Lubricated

- (4) 0.276 inch wire
  - (a) Clean
  - (b) Lubricated
- (5) 5/16" strand
  - (a) Clean
  - (b) Lubricated

Before pretensioning, each wire was fitted with two SR-4 electric strain gages, and each strand was fitted with four gages. In each case, one waterproofed gage was placed at the midpoint of the prism. The waterproofing cylinders for these tests were 3/4 in. diameter and 3½ in. long. In subsequent tests these dimensions have been reduced substantially. The remaining gages were positioned outside the concrete forms. All steel reinforcement was pretensioned to 120,000 psi as determined from the several gages and checked with a calibrated pressure gage on the hydraulic system used to tension the wire. The coefficient of variation of the strain readings on any individual wire and strand from which the initial tension was determined was about 2 percent.

The concrete prisms were cast using a 7-bag mix of a blend of Type I cement and 3/8 in. maximum size aggregate. The slump was 3 in. and the cylinder strength of the concrete after five days' moist curing and 16 days' storage in the air of the laboratory was about 4500 psi. For the 0.162 in. clean wire an additional specimen was made with 6500 psi concrete having a slump of ½ inch.

Twenty-six SR-4 strain gages were placed along the sides of each prism after moist curing was completed. Just before the pretension was released, readings were taken on all gages on the concrete surface and on the steel. The tension was released from the steel and again complete strain readings were taken. These data established the pretension in the steel just prior to release, the tension retained in the steel at the center of the specimen after release, and the distribution of prestress.

It was the distribution of prestress as determined from strain measurements on the concrete surface from which the steel stress distribution, discussed later, was derived. This procedure is justified if, at all points along the length of a prism, the stress in the concrete is uniform over the entire cross-section. At any point the total concrete compression along the prism varies directly with the steel tension, even though compatibility of strains does not prevail in the end region where there is a stress gradient. Such an indirect procedure was necessary to avoid destroying the bond by waterproofing a number of gages attached to the steel. The procedure is believed to be reliable except within a very short region near the ends of the specimens.

### 3.6 Prestress Transfer Bond

Typical distribution of steel tension near an end of a prestressed prism is shown in Fig. 3.1. There are three factors which may contribute to bond between steel and concrete: adhesion between concrete and steel, friction between concrete and steel, and mechanical resistance such as is provided by the deformations on ordinary reinforcing steel.

The first of these, adhesion, can be present only if no slip has taken place between concrete and steel. If slip is absent, the reduction in tensile strain occurring in the steel at any point following the release of pretension must equal the increase in compressive strain in the concrete at the same point. Measured strains indicate that this condition is not only in the portion of the prism where the wire tension is constant, that is, in the central region. Within the region where there is a steel tension gradient (the outermost 15 in. in Fig. 3.1) the reduction in steel strain is greater than the corresponding concrete strain, with the difference increasing as the steel stress gradient increases when an end of the prism is approached. At the ends of the prism the reduction in steel strain is a maximum and the corresponding concrete strain is zero. Accordingly, slip must be present and adhesion may be discounted as a major factor contributing to prestress transfer bond between pretensioned steel and concrete. Furthermore, since the clean wire used in the tests is very smooth, mechanical resistance of any consequence is probably negligible.

It is reasonable to assume, therefore, that the friction between concrete and steel is largely responsible for the transfer of stress to the concrete.

As the tension is released and the wire starts to slip, the diameter of the wire increases in proportion to the reduction in tension, and a radial pressure is exerted on the surrounding concrete; thus, the frictional force restraining slippage of the wire is increased. This force is dependent upon the radial pressure developed and the coefficient of friction between the steel and the cement paste in intimate contact with the steel. The coefficient of friction will vary with the surface characteristics of the wire and possibly with the character of the paste.

An approximate elastic analysis of the frictional bonding phenomenon is of some interest, although values of the resulting maximum tensile and compressive stresses are so high that they are obviously beyond the elastic domain. Thus, the analysis provides only a qualitative guide regarding the type of stress transfer to be expected, and the type of stress transfer to be expected, and the resulting formula should not be considered as a design equation.

Reduction of tension at any point along a pretensioned wire from an initial value  $f_{se}$  to a value  $f_s$  will result in an increment of radius

$$\Delta r_1 = r (f_{se} - f_s) \frac{\mu_s}{E_s} \quad (3.1)$$

if the wire is free to expand. However, if a radial stress is imposed by the surrounding concrete, the actual increase

in radius must correspond to the radial displacement of the concrete. When the concrete stress is large in comparison with the wire radius, the elastic theory of a thick walled cylinder gives the following increase of the wire radius

$$\Delta r = r \cdot \sigma_r \cdot \frac{1 + \mu_c}{E_c} \quad (3.2)$$

where  $\sigma_r$ , the radial stress in the wire, is equal to the contact pressure between the steel and concrete. The magnitude of this radial stress is

$$\sigma_r = \frac{\Delta r_1 - \Delta r}{r} E_s = (f_{se} - f_s) \mu_s - \sigma_r (1 + \mu_c) \frac{E_s}{E_c} \quad (3.3)$$

$$\text{or } \mu_r = \frac{(f_{se} - f_s) \sigma_s}{1 + (1 + \mu_c) \frac{E_s}{E_c}} \quad (3.4)$$

For  $E_s = 29 \times 10^6$  psi,  $E_c = 3.5 \times 10^6$  psi,  $\mu_s = 0.3$  and  $\mu_c = 0.2$

$$\sigma_r = 0.0274 (f_{se} - f_s)$$

At the ends of a prism, the entire prestress of 120,000 psi is released and the corresponding radial compression  $\sigma_r = 0.0274 \times 120,000 = 3300$  psi. For the assumed elastic conditions, tangential tension in the concrete at the interface is also 3300 psi, so true elastic action would not be expected.

The bond stress at any point is equal to the slope of the stress transfer curve, as shown in Fig. 3.1, multiplied by  $\frac{r}{2}$ , or

$$u = \frac{df_s}{d\lambda} \cdot \frac{r}{2} \quad (3.5)$$

If the bond is due entirely to friction,

$$u = \phi \sigma_r \quad (3.6)$$

Solving Eq. (3.5) and (3.6) we get

$$d\lambda = \frac{r}{2\phi\sigma_r} \cdot df_s \quad (3.7)$$

Substituting  $\sigma_r$ , from Eq. (3.4), integrating, and using the boundary condition that  $f_s = 0$  for  $\lambda = 0$ , we get

$$\lambda = \frac{-r \left\{ 1 + (1 + \mu_c) \frac{E_s}{E_c} \right\} \log \frac{f_{se} - f_s}{f_{se}}}{2\phi u_s} \quad (3.8)$$

or

$$\log \frac{f_{se} - f_s}{f_{se}} = \frac{-2\phi \mu_s \cdot \lambda}{r \left\{ 1 + (1 + \mu_c) \frac{E_s}{E_c} \right\}} \quad (3.8a)$$

Eq. (3.8a) is plotted in Fig. 3.2 for a 0.197 inch diameter wire and various values of the coefficient of friction  $\phi$ . By comparison with observed stress transfer distributions, such as those given in Fig. 3.3, it is seen that the general shape of the theoretical curves is similar to that representing actual behavior. Thus, the fundamental

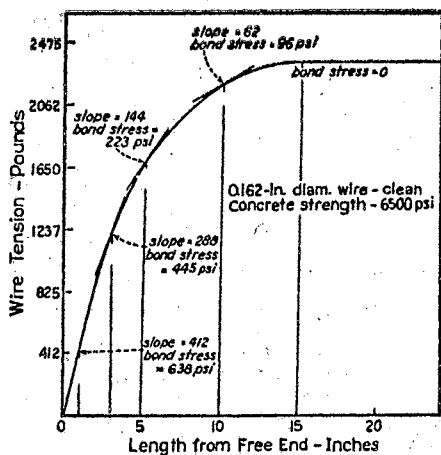


Fig. 3.1

Typical Distribution of Stress Transfer

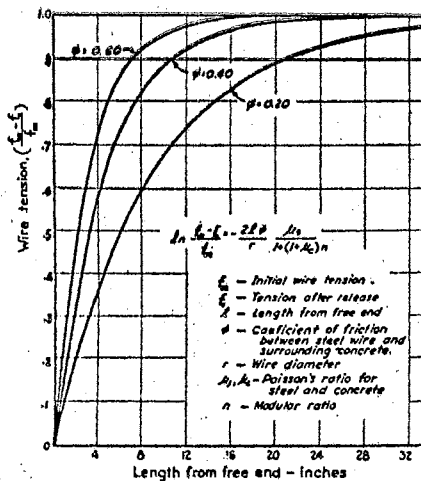


Fig. 3.2

Theoretical Stress Transfer Distribution for 0.197-inch Diameter Wire

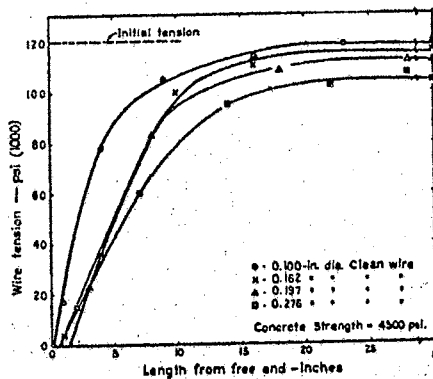


Fig. 3.3

Stress Transfer Distribution From Wires of Various Diameters



assumption that transfer of stress is effected largely through friction seems reasonable.

Microscopic examination of the exposed concrete and steel interface near the ends of the stress transfer prisms showed no indication of tensile failure of the paste in spite of the high computed values of tensile stress. It is believed that sufficient inelastic yielding of the concrete took place to relieve the high tensile stresses.

### 3.7 Presentation and Discussion of Test Results

#### 3.7-1 Influence of Wire Diameter

The distribution of steel stresses for various wire diameters is given in Fig. 3.3 as derived from measured strains on the surface of the 2" x 2" prisms. It is seen that the ability of pretensioned wire to transfer stress to the concrete through bond does not vary a great deal with wire size within the range of 0.100 to 0.276 in. diameter. The length of embedment necessary to transmit the stress fully to the concrete prisms is moderately greater as the wire diameter increases.

The final level of tension shown for the various wire diameters in Fig. 3.2 decreases as the wire diameter increases. The cross-section of all prisms was 4 sq. in. and all steel was pretensioned to 120,000 psi. Consequently, the prestress force imposed upon each specimen and the resulting concrete strain were greater as the wire diameter increased.

The compressive strain in the concrete resulting from full transfer of the prestress force is

$$\epsilon = \frac{f_{si} \cdot A_s}{A_s \cdot E_s + A_c \cdot E_c}$$

where  $f_{si}$  is the initial tensile stress in the steel.

For the small specimens used, this compression of the concrete resulted in a significant loss in steel tension. If there is no slip between the wire and the surrounding concrete, the reduction in steel strain from its initial pretensioned value should be the same as the increase in concrete strain resulting from the release of stress, and the corresponding reduction in steel stress will be equal to  $\epsilon \cdot E_s$ . The loss in steel stress computed in this way ranges from 1900 psi for the 0.100 in. diameter wire to 13,200 psi for the 0.276 in. wire. Measured and theoretical concrete and steel strains, together with loss in prestress, are compared in Table 1 for typical stress transfer prisms.

To avoid variation in retained tension, it would have been necessary to tension the wires progressively higher, or to increase the concrete cross section as the wire diameter increased. Thus, it was impossible to maintain consistency of the final tension level without adding other variables.

TABLE 1

Comparison of Theoretical and Measured

Strains at the Center Section of Stress Transfer Prisms

$E_c = 3.5 \times 10^6$  psi,  $E_s = 29 \times 10^6$  psi, Initial Pretension = 120,000 psi

| Wire Diameter<br>in in. | Concrete Strain, millionths |          | Reduction in<br>Steel Strain,<br>millionths | Reduction in Steel Stress<br>psi |          |
|-------------------------|-----------------------------|----------|---|----------------------------------|----------|
|                         | Theoretical                 | Measured | Measured                                    | Theoretical                      | Measured |
| 0.100                   | 66                          | 65       | 96  | 1,900                            | 2,800    |
| 0.162                   | 170                         | 143      | 147   | 4,900                            | 4,300    |
| 0.197                   | 246                         | 258      | 287   | 7,100                            | 8,300    |
| 0.276                   | 455                         | 547      | 570   | 13,200                           | 16,500   |

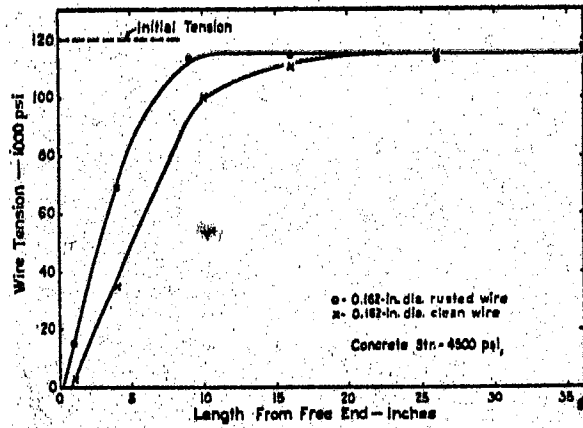


Fig. 3.4  
Stress Transfer Distribution  
For Clean and Rusted Wires

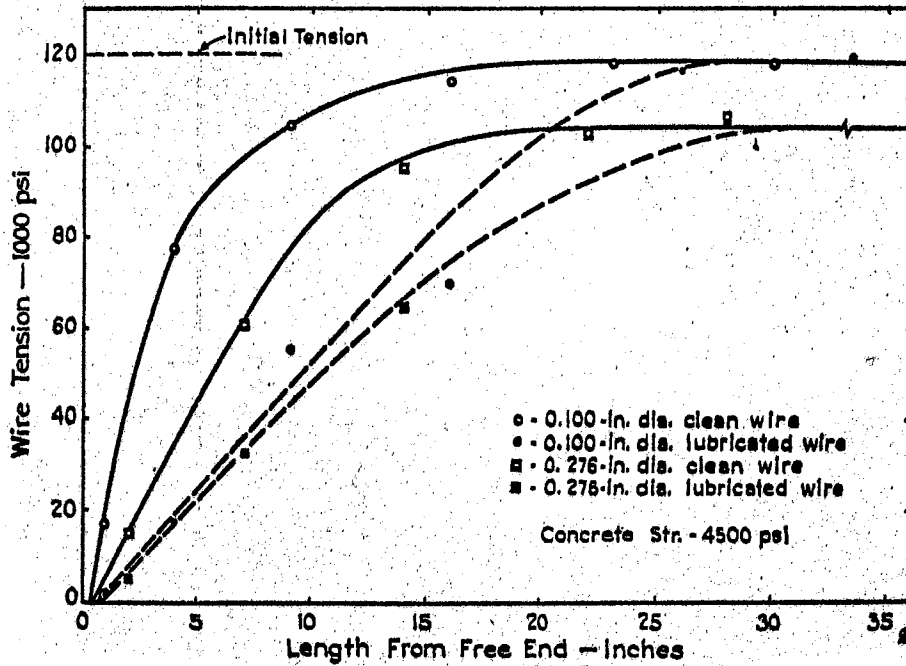


Fig. 3.5  
Stress Transfer Distribution For  
Clean and Lubricated Wires

### 3.7-2 Concrete Strength

The second variable briefly explored in this series of tests was concrete strength. Strengths of 4500 and 6500 psi were investigated, as representative of values likely to be encountered in practice. The tests carried out did not clarify the effect of cement paste quality on coefficient of friction between steel and cement paste. A paper on the "Influence of Concrete Strength on Strand Transfer Length" was presented at the Ninth Annual Convention of the Prestressed Concrete Institute. This is reproduced later.

### 3.7-3 Surface Condition

The two prisms, one prestressed with rusted wire and the other with clean wire of the same diameter, gave the stress transfer distributions as shown in Fig. 3.4. It is clearly seen from these curves that the rusted wire develops the full transfer of prestress at a more rapid rate and in a less distance from the free end. Cylinder strength at the time of release of prestressing force was about 4500 psi in both the cases.

Fig. 3.5 shows the stress transfer distributions for the smallest and the largest wire tested with the intermediate results obtained for the strand and the wires of other diameters. It can be assumed that the difference between stress transfer distributions for clean and lubricated wire is mainly due to difference in coefficient of friction.

## INVESTIGATION OF TRANSFER LENGTH

A description of "An Investigation of Transfer Length in Pretensioned Concrete Using Photoelasticity," as carried out by Don A. Linger and Suryaji R. Bhonsle is given in Prestressed Concrete Institute Journal of August, 1963.

The purpose of their study was to: first, determine the length of the tendons required to transfer the prestress force to the concrete beam; second, determine the stress distribution in the beam resulting from prestress transfer; third, determine the stress concentration and its value along the length of the beam required for the transferring of the prestress from the tendon to the beam.

### 4.1 Experimental Study

The problem was investigated by the use of photoelastic procedures with model beams of bi-refringent plastic which were prestressed with steel wires. A minimum thickness was used to eliminate the three dimensional effect. The stress concentration and stress distribution were indeterminate by strain gages and other methods.

The photoelastic method was used in order to show the pattern of stresses in the beam, since it is the concrete stress in the beam which determines the working bending moment and shear strength of a member.

## 4.2 Procedure

Various types of photoelastic materials for use in photoelastic analysis are available in solid plastic sheets. However, in their study, the problem of prestressing the model beam by means of predetermined wire (the wires in which the stresses are already measured by the use of strain gages) made it imperative to use a liquid plastic. The prestress wire could then be placed in the mold like prestressed cable in a concrete form and the plastic poured similar to the way in which the concrete is poured into a form to cast a prestressed concrete beam. Since this was a study of a structural system in which the ratio of the elastic moduli is important, it was therefore necessary to determine the shape of the stress-strain curve, the modulus of elasticity, and the creep characteristic of the photoelastic material as well as the normally required stress optical coefficient or photoelastic constant.

The specimens used to study the properties of the liquid plastic and the prestressed model beams were cast in a steel mold.

In order to construct the prestressed model beams, a wire was pulled between the two end blocks by means of clamps against the ends of the mold. At each casting two prestressed beams and one plain calibration beam (to study the photoelastic constant and modulus of elasticity of the plastic in each cast) were made.

Since creep is one of the properties of concrete which effects the distribution of stress, it was necessary to study the effective creep for the magnitude of stress which corresponds to the stress in the model of the prestressed concrete beams.

Each calibration beam was loaded in equal increments to determine the photoelastic constant and at the same time, the strains in the outer fibers were measured by the SR-4 strain gages. The curve of stress in the outer fiber versus strain in the outer fiber gave the value of modulus of elasticity.

#### 4.3 Photoelasticity

Isochromatic fringes are lines obtained when the stressed specimen is viewed through the polariscope and indicates points along which the principle stress difference ( $\sigma_1 - \sigma_2$ ) is zero or a multiple of the fringe constant of the specimen. To record the distribution of fringe order in the prestressed model beam the model was placed in the polariscope and photographs were taken of the stress pattern for the data reduction. Monochromatic light and quarter-wave plates were used to obtain sharp fringes and remove isoclinics.

#### 4.4 Determination of Prestress Force

From the fringe distribution for any prestressed model beam, the stress at any point along the depth is known and is equal to

$$\sigma_1 - \sigma_2 = \text{Fringe value} \times \text{Fringe value of model.}$$



The fringe value for each prestressed plastic model beam had been previously determined. If the fringe value is selected in the model beam where the stress distribution has no normal stress in the y direction and no shear in the x or y direction then,

$$\sigma_y = \sigma_z = 0 \qquad \tau_{xy} = 0$$

and

$$\sigma_1 = \sigma_x = \frac{P}{A} \pm \frac{Pey}{I} = (\text{Fringe value at distance } y) \times (\text{Fringe value of model}) \quad (4.1)$$

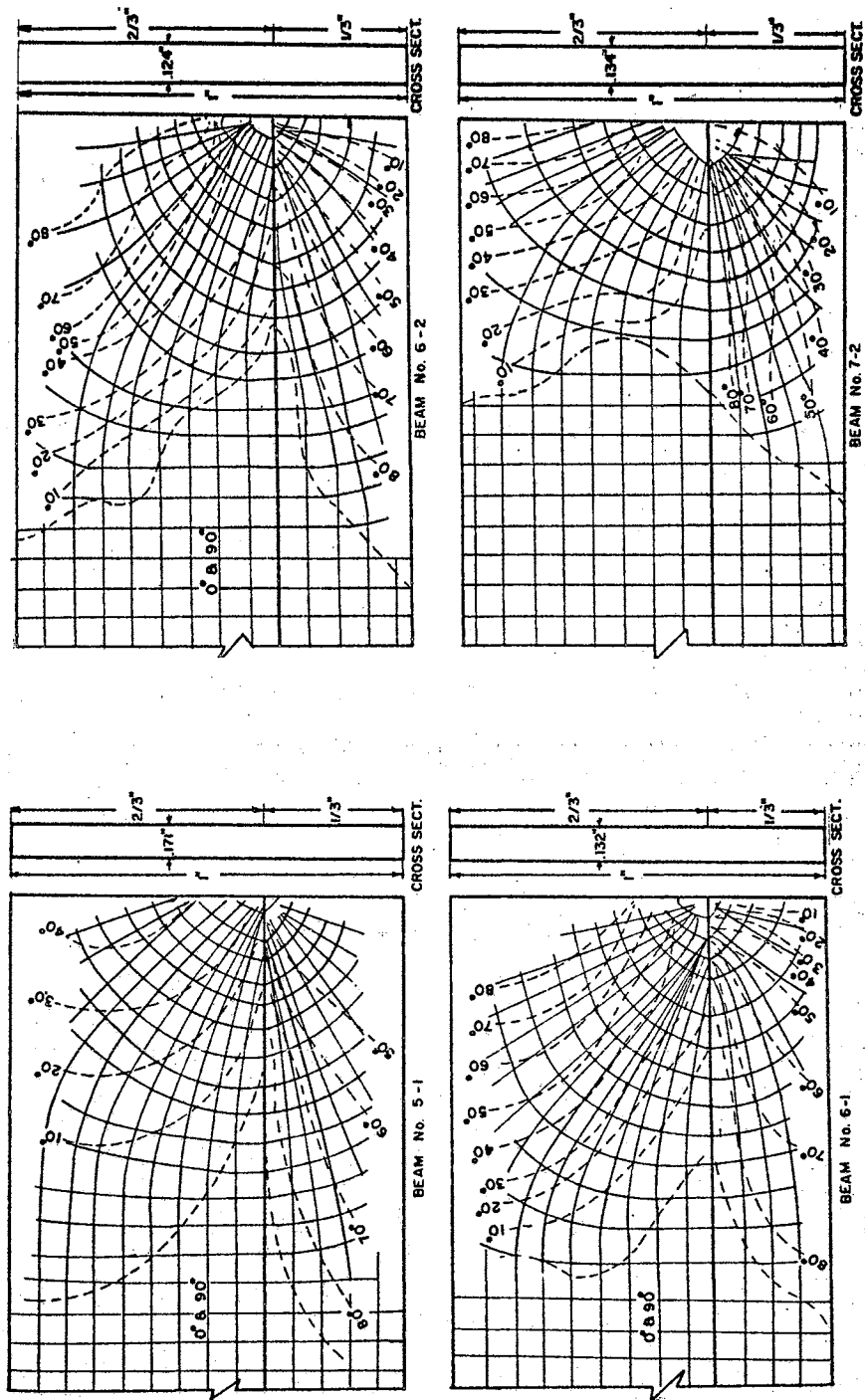
By knowing the cross section of the beam and distance of a point (where the fringe value is measured) from the neutral axis, the prestress force P is given as

$$P = \frac{\text{Fringe value at distance } y) \times (\text{Fringe value of model})}{\frac{1}{A} \pm \frac{e \cdot y}{I}} \quad (4.2)$$

The prestress force thus obtained is the net prestress force.

The test carried out to determine the creep indicated that the creep was considerably smaller than that required to produce an effect on stress distribution for the period of time required to run the tests.

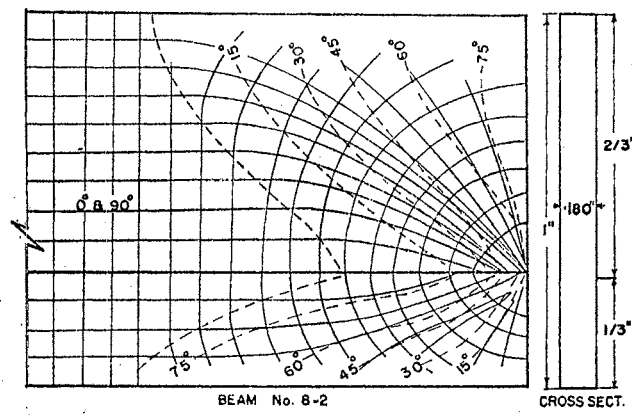
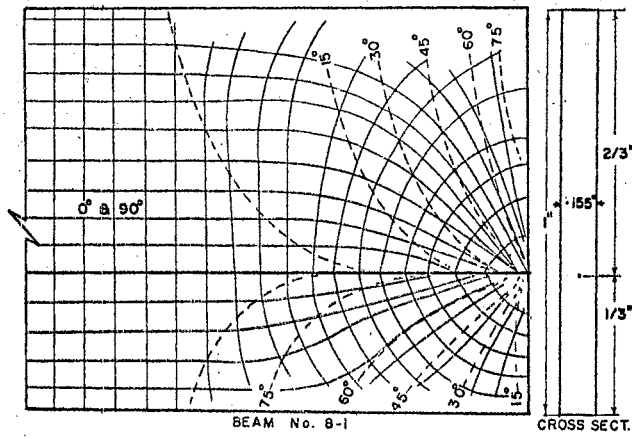
Tests conducted on two liquid plastic calibration beams showed that the average value of the photoelastic constant was about 62 lb. per square in. per in. per fringe which is lower than most of the photoelastic materials used in photoelastic study.



--Isoclinics and stress trajectories.

--Isoclinics and stress trajectories.

Fig. 4.1



Isoclinics and stress trajectories.

Fig. 4.2

Using white light source without quarter wave plates, the path of the isoclinics was traced. The stress trajectories were constructed from these isoclinics. These stress trajectories represent the direction of principal stresses. These stress trajectories indicate that the prestress force is dissipating in a manner similar to that of a concentrated normal load acting on the end of a beam. Figures 4.1 - 4.3 show the direction of principal stresses and the way in which the prestress force is transferred into the beam from the wire.

To show the variation in strain, and therefore in stress, the curve of maximum shear stress along the wire was plotted in Figs. 4.4 & 4.5. The curves show that the maximum value of stress concentration is between 0.1 and 0.3 inches from the end of the beam.

To study the effect on stress concentration of the amount of prestress force, modulus of elasticity, and cross sectional area, a curve was plotted using the two dimensionless terms

$$\frac{\tau_{\max}}{E_{pm}} \quad \text{and} \quad \frac{P}{t_b^2 E_{pm}}$$

The curve shows definite relationship between maximum shear stress concentration and the other variables as

$$\left[ \frac{\tau_{\max}}{E_{pm}} \right] = k \left[ \frac{P}{t_b^2 E_{pm}} \right]^{k_1} \quad (4.3)$$

$$\tau_{\max} = 0.95 \left[ \frac{P^{0.8} E_{pm}^{0.2}}{1.6 t_b} \right] \quad (4.4)$$

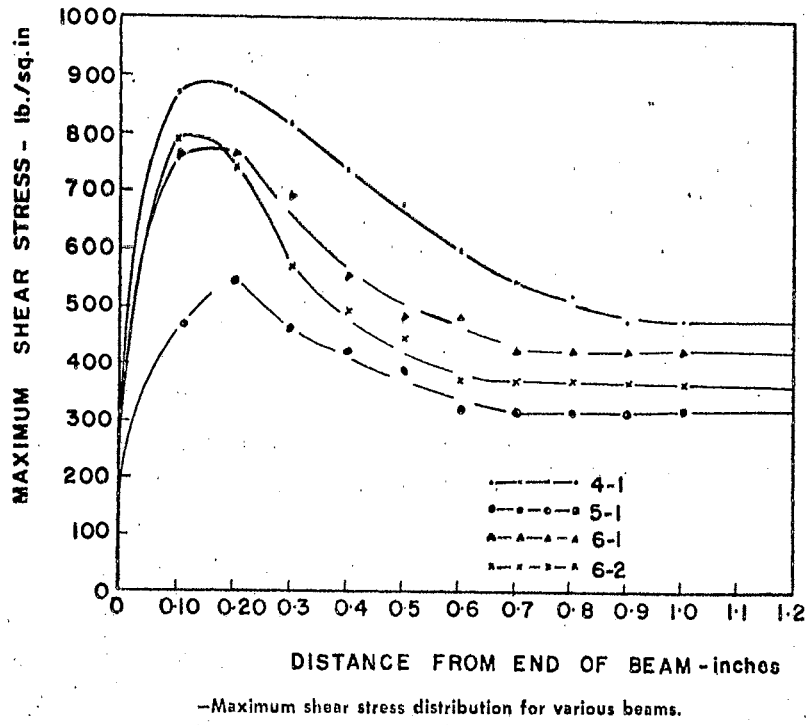


Fig. 4.3

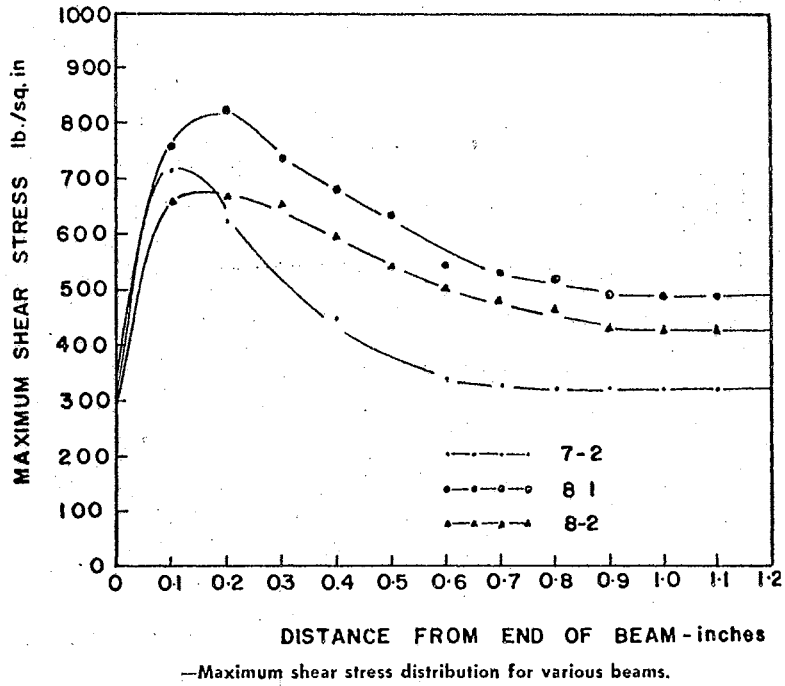
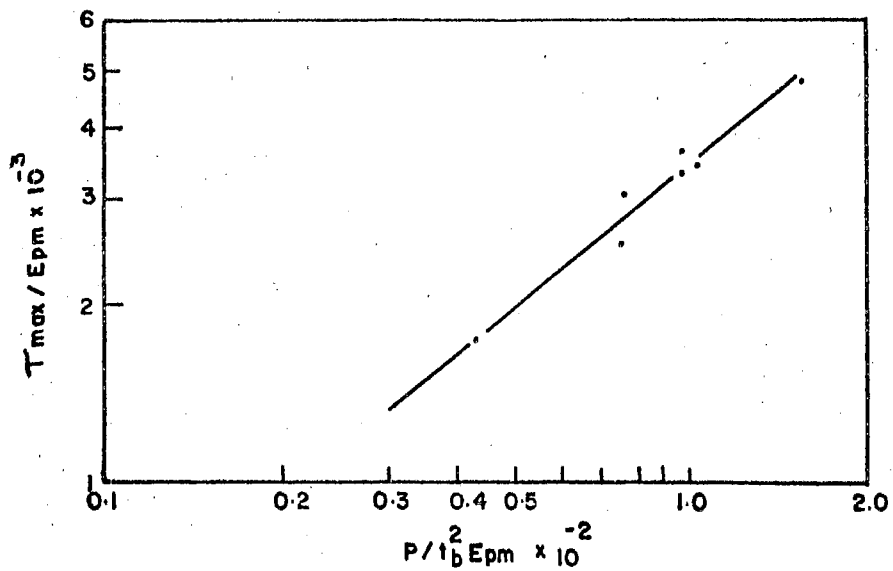
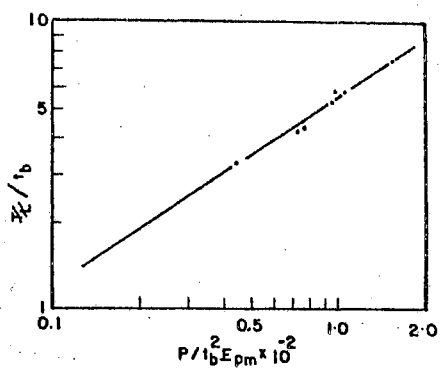


Fig. 4.4



-Relation of maximum shear stress with  $P$ ,  $t_b$ ,  $E_{pm}$ .

Fig. 4.5



Relation of transfer length to  $P$ ,  $t_b$ ,  $E_{pm}$ .

Fig. 4.6

#### 4.5 Transfer Length

The transfer length is different for each beam which clearly shows that there is an effect of prestress force, thickness of specimen, and modulus of elasticity of the beam. To determine the effect of these variables on transfer length a curve (Fig. 4.7) was plotted which gave a straight line on log-log coordinate graph paper. Therefore, the equation can be developed from the curve

$$\left[ \frac{L}{t_b} \right] = k \left[ \frac{P}{t_b^2 E_{pm}} \right]^{k_1} \quad (4.5)$$

$$L = 100 \left[ \frac{P^{0.62}}{E_{pm}^{0.62} t_b^{0.24}} \right] \quad (4.6)$$

The only assumption for this equation is that the Poisson's ratio for liquid plastic is similar to that of concrete.

THE USE OF X-RAYS IN MEASURING  
BOND STRESSES IN PRESTRESSED CONCRETE

At the World Conference on Prestressed Concrete held in San Francisco, California, July 29 through August 3, 1957, a paper on "The Use of X-Rays in Measuring Bond Stresses in Prestressed Concrete," was submitted by R. H. Evans and Alan Williams, professors from Leeds University, England. The tests carried out by them were focussed on "flexural bond stresses and the determination of the stress conditions in the transmission length zone of pretensioned prestressed concrete beams."

There are methods like pull-out, push-out tests; inserting electric strain gages in hollow reinforcement (limited to large diameter bars); steel strains determined by means of lugs; by electric resistance strain gages (waterproofing necessary interferes with the bond conditions). The distribution of concrete surface strains in the transfer region of pretensioned beams may be measured using the "Demac" strain gage. Due to shear deformations produced in the concrete from the wire outwards in the regions where there is a stress gradient, there is a "conning effect" or shear strain lag between the wire and the exterior surface. Thus, strains measured on the outside of the concrete are delayed a distance approximately equal to the



concrete cover. The large gauge length of eight inches is also a disadvantage as it makes the detection of small changes of strain at the end of the transmission length very difficult. The disadvantages of these earlier methods are avoided by the x-ray technique initiated by Evans and Robinson in 1951.

### 5.1 Test - Radiographic Strain Measurement Technique

The radiographic strain measurement technique involves embedding platinum markers, with a high x-ray absorption coefficient, in the prestressing tendons before casting the pretensioned specimen. This permits the investigation of strain conditions in the reinforcement without disturbing the surrounding concrete and interfering with the adhesion between the steel and concrete. In the case of pretensioned columns for investigating conditions in the transmission zone, radiographs taken before and after release of the prestressing force record the initial and final marker positions. Subsequent measurement of the x-ray films with a travelling microscope furnishes the change in the gauge length between adjacent markers and the slip between the tendon and the concrete. In the case of pretensioned beams, radiographs taken before and during loading enable the slip and the increase in strain to be determined as loading proceeds.

For plain wire tendons, rectangular platinum markers are used measuring 0.1 in. high x 0.06 in. deep x 0.006 in. wide. These are placed in slots cut in the wire 0.006 in. wide x 0.012 in. deep and fixed in position with "Araldite D"

adhesive. For the wire rope tendons, the markers are wedged between the two topmost wires in the strands and fixed with "Araldite D" adhesive. Platinum foil 0.010 in. thick is used for the markers which are cut 0.050 in. wide.

After casting a specimen part of the platinum marker protruding from the tendon is fixed in the concrete. This part of the marker moves with the concrete during subsequent loading and enables the slip between the concrete and the steel to be measured. Bond stresses are determined from the gradient of the steel stresses distribution curve which is obtained from the strain distribution curve.

## 5.2 Results

The distribution of slip at the end of a specimen pretensioned with a wire rope is given in Fig. 1. It will be observed from the curves that the pull-in immediately after transfer was 0.0125 in. and the transmission length 5.6 in. One week after transfer the pull-in had increased to 0.0135 in. and the transmission length to 5.8 inches.

Fig. 2 shows the distribution of slip in a specimen pretensioned with a 0.276 inch diameter rusted wire. The specimen was maintained constantly wet after two days so that the amount of shrinkage which occurred between casting and transfer could have been only very slight.

A similar specimen was prepared using a 0.276 inch diameter wire. After six weeks of drying the shrinkage which occurred between the setting of concrete and the transfer of the prestress affected the degree to which concrete grips the steel.

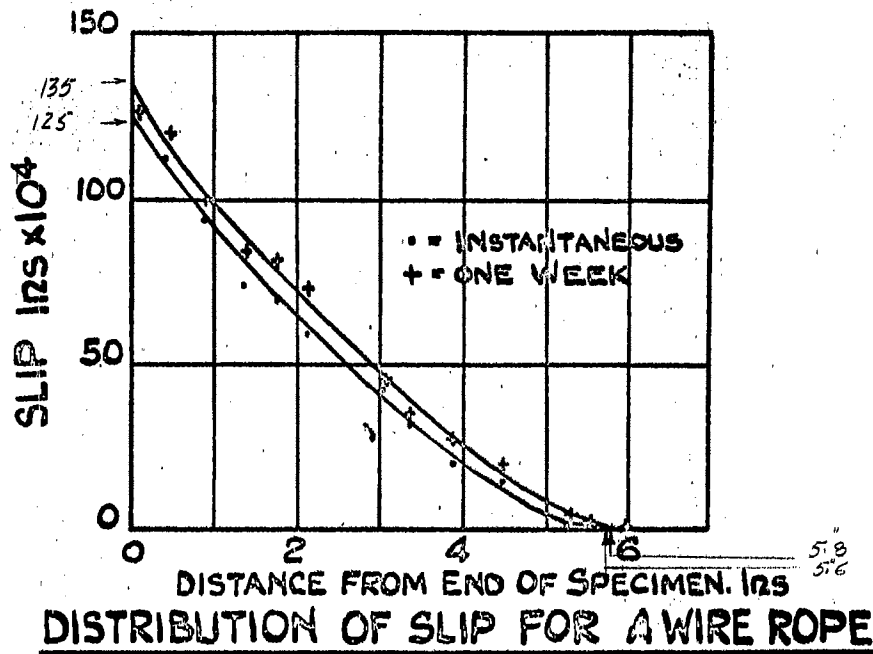


Fig. 5.1

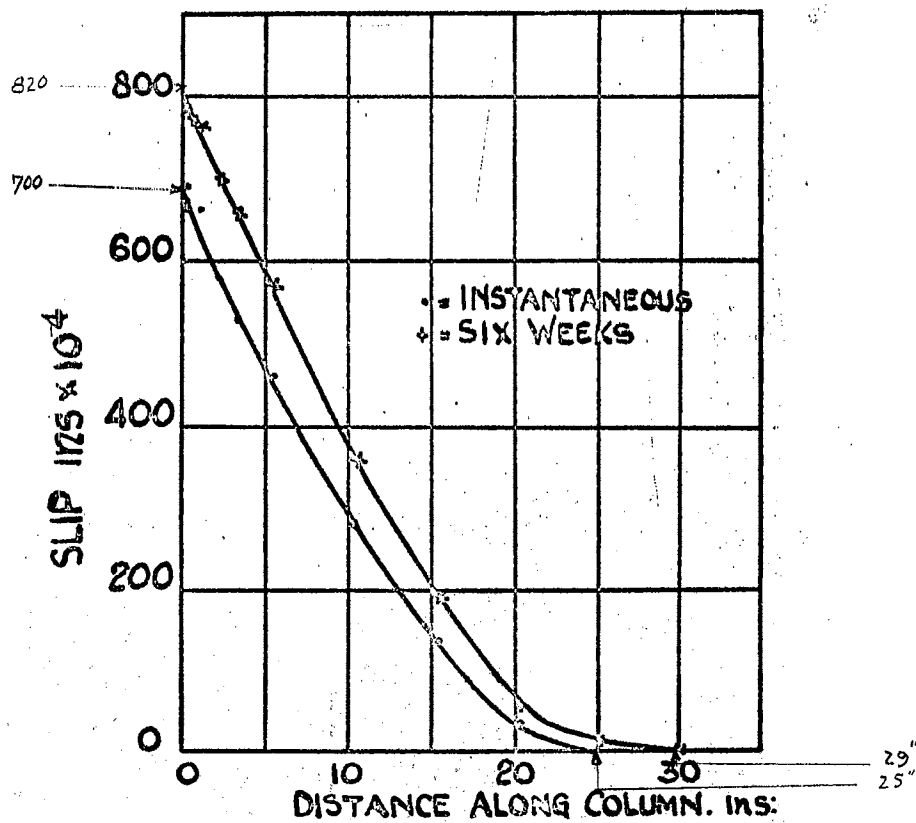


Fig. 5.2

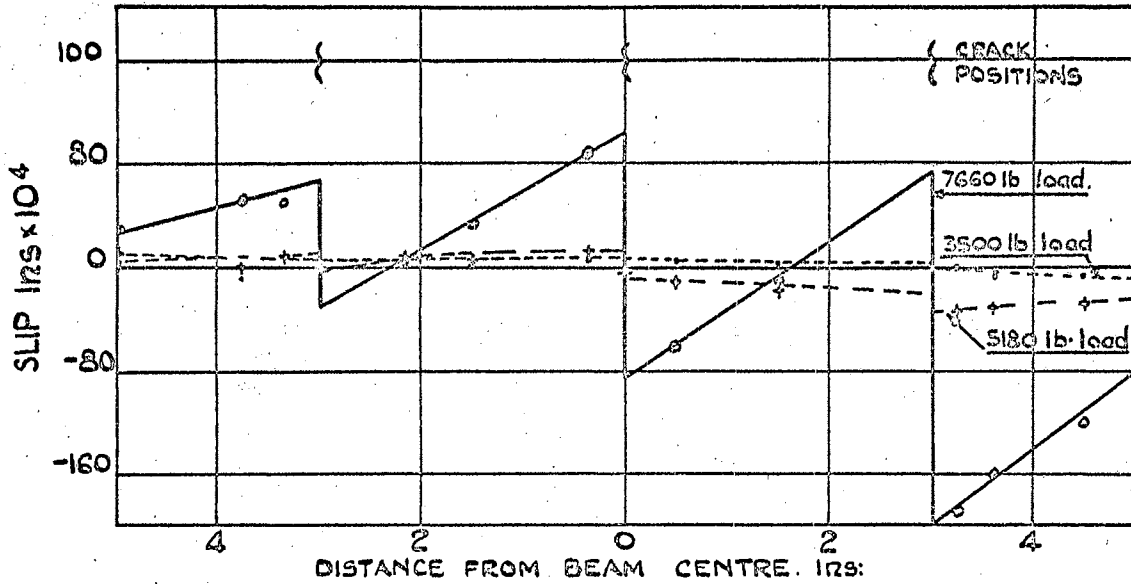


Slip and bond stress distribution curves for a beam pretensioned with a  $\frac{1}{2}$  inch diameter Macalloy deformed bar are given in Fig. 3 and 4. On either side of a crack the concrete slips over the reinforcement in opposite directions. Concrete slip to the left is plotted as positive. Hence, there is a break in the slip curve at a crack with positive slip to the left of the crack and negative slip to the right. The total ordinate of the slip curve at a crack is the width of the crack. Similarly, with the bond stress distribution curves, the bond stress opposing positive slip is plotted as positive. The beam was loaded centrally and the curves are plotted for a load just after cracking, twice the working load and at the ultimate load when the beam failed by concrete crushing. The curves of slip and bond stress illustrate how the bond stress builds up as the slip increases and the deformations in the bar wedge against the concrete. As the slip increases further, the concrete shears at the deformations and the bond stress reduces rapidly with the peak values in the bond stress curve moving away from the cracks where the slip is at a maximum.

### 5.3 Discussion of Results

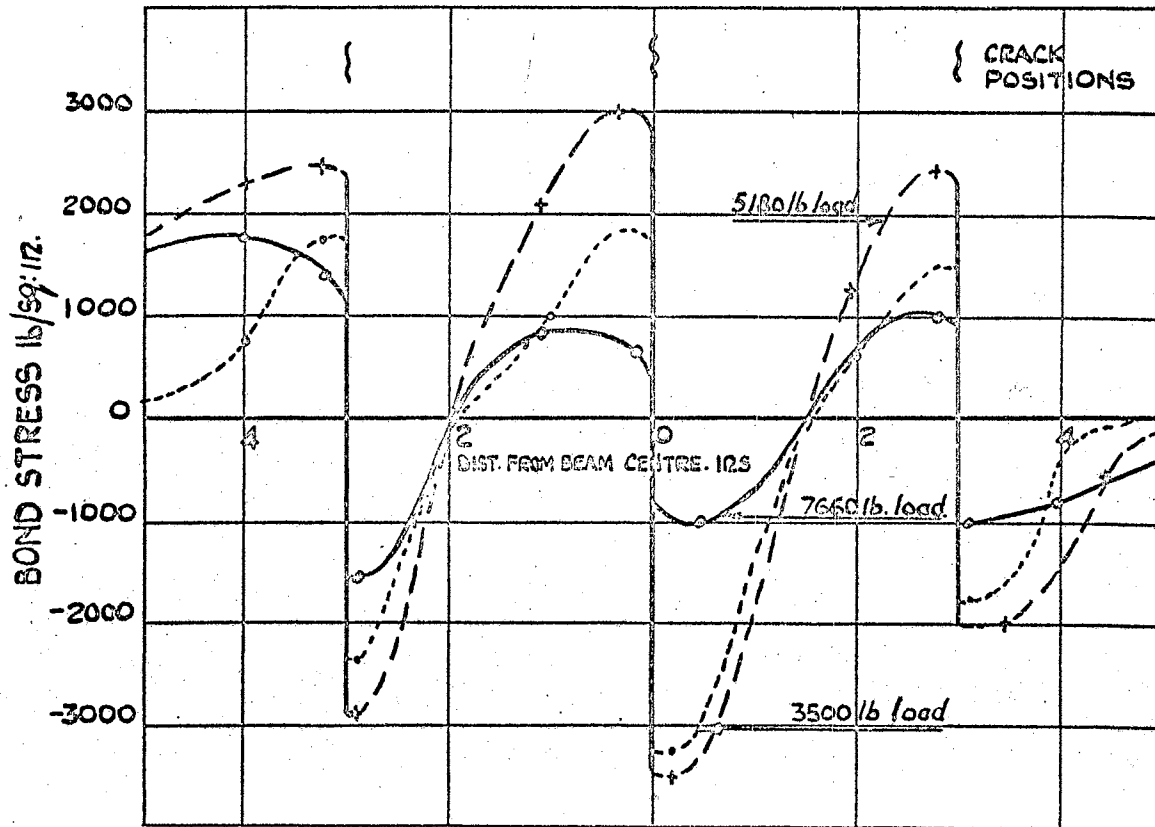
#### 5.3-1 Transmission Length and Pull-In

On releasing a prestressed wire for a pretensioned beam or column, the wire at the end of the unit pulls into the concrete surrounding it and slip continues into the



SLIP DISTRIBUTION FOR BEAM PRETENSIONED WITH A 1/2-12-DIA MACALLOY DEFORMED BAR

Fig. 5.3



BOND STRESS DISTRIBUTION FOR A BEAM PRETENSIONED WITH A 1/2-12-DIA MACALLOY DEFORMED BAR

Fig. 5.4

concrete for some distance. At the end of the prestressed column, the prestressing tendon loses the whole of its initial pretension. At points within the column, the tendon loses less of its pretension till, at the end of the transmission length, the retained tension in the wire reaches its maximum. This maximum retained tension depends on the concrete section and the value of the Young's Modulus for the concrete. The strain associated with the maximum retained tension is given by

$$\epsilon_{so} = \frac{1}{1 + mp} \epsilon_{is} \quad (5.1)$$

The maximum concrete strain  $\epsilon_o = \frac{mp}{1 + mp} \epsilon_{is}$

Fig. 5(a) illustrates the buildup of steel and concrete strains. The curve of loss of steel strain (i.e. the steel strain referred to the strain in the tendon before release) is seen from Fig. 5(b) to have a maximum value of  $\epsilon_{is}$  at the end of the column and a minimum value of  $\epsilon_o$  at the end of the transmission length. Thus at the end of transmission length the concrete strain and the loss of steel strain are equal and constant at the value of  $\epsilon_o$ . The loss of steel strain is given by

$$\epsilon'_s = \epsilon_{is} - \epsilon_s$$

where  $\epsilon_s$  = true steel strain

The slip (g) at any point is caused by the differential strain between the steel and concrete from the end of the transmission length to the particular point.

$L$  = transmission length

$\epsilon_c$  = concrete strain at a distance  $x$  from the end

$\epsilon'_s$  = loss of steel strain at a distance  $x$  from the end

$$g = \int_x^L (\epsilon'_s - \epsilon_c) dx \quad (5.2)$$

= the area shown hatched in Fig. 5(c)

Slip is only zero when  $\epsilon'_s = \epsilon_c$  i.e. beyond the transmission length. Hence, the point at which the slip curve meets the  $x$ -axis indicates the end of the transmission length.

In a very long column "L" may be replaced by infinity,

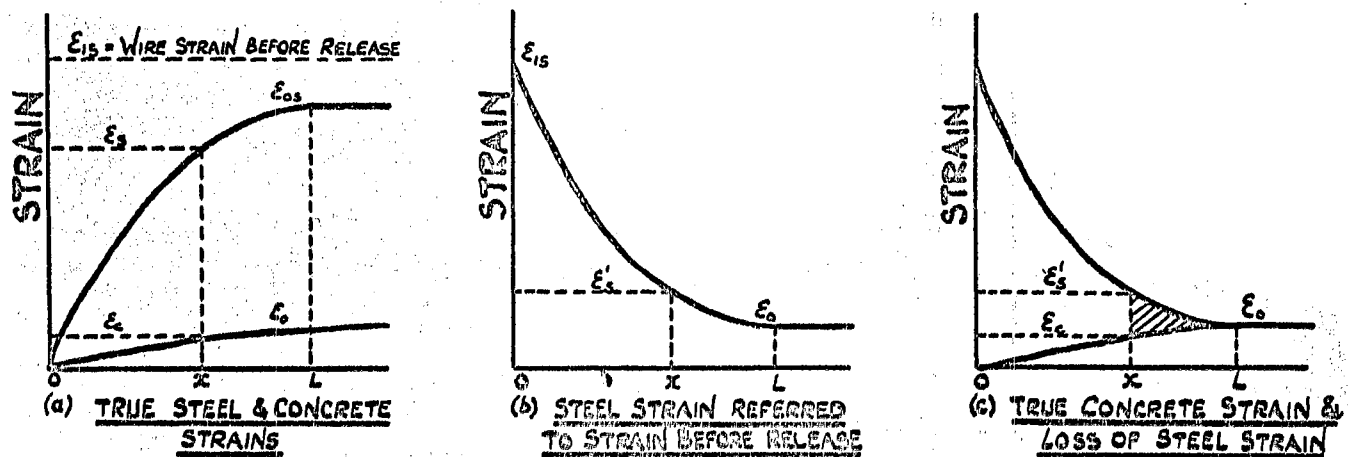
i.e.

$$\begin{aligned} g &= \int_x^{\infty} (\epsilon'_s - \epsilon_c) dx \\ &= \int_x^{\infty} \left\{ \epsilon'_s - mp (\epsilon_{is} - \epsilon'_s) \right\} dx \\ &= \int_x^{\infty} \left\{ (\epsilon_{is} - \epsilon_s) (1 + mp) - mp \epsilon_{is} \right\} dx \\ &= \int_x^{\infty} \left\{ \epsilon_{is} - \epsilon_s (1 + mp) \right\} dx \end{aligned} \quad (5.3)$$

G. Marshall has shown that the retained steel strain can be expressed by the exponential relationship

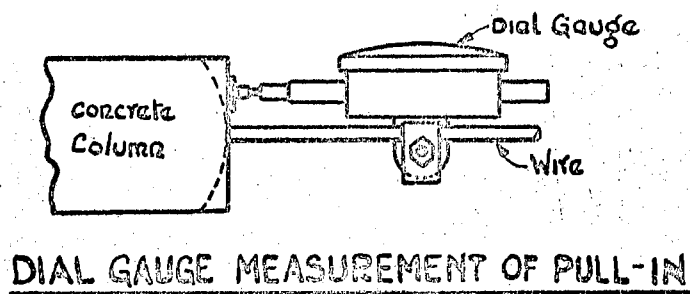
$$\epsilon_s = \epsilon_{s0} \left( 1 - e^{-\frac{4a}{d} x} \right) \quad (5.4)$$





STEEL AND CONCRETE STRAINS IN TRANSMISSION LENGTH.

Fig. 5.5



DIAL GAUGE MEASUREMENT OF PULL-IN

Fig. 5.6

where

$\epsilon_{s0}$  = maximum retained steel strain

$d$  = diameter of wire

$a$  = constant for a particular wire

Substituting the value of  $\epsilon_s$  in  $g$  and integrating we obtain

$$g = \frac{d}{4a} \epsilon_{is} \cdot e^{\frac{-4a}{d} x} \quad (5.5)$$

Hence, the end slip or pull-in is given by

$$g_0 = \frac{d}{4a} \epsilon_{is}$$

$$\frac{g}{g_0} = e^{\frac{-4a}{d} x} = e^{\frac{-\epsilon_{is}}{g_0} x}$$

and

$$g = g_0 e^{\frac{-\epsilon_{is}}{g_0} x} \quad (5.6)$$

It has been assumed that the stress in the concrete is uniform over the entire cross section. This is not strictly correct as the stress is not uniform in the transmission zone where there is a stress gradient. Nor are the actual slip curves obtained perfectly exponential as is suggested by equation (5.6)

However, from equation (5.6)

$$\log_e \frac{g_0}{g} = \frac{\epsilon_{is}}{g_0} \cdot L$$

$$L = \frac{g_0}{\epsilon_{is}} \cdot \log_e \frac{g_0}{g} \quad (5.7)$$

It is found for wire rope

$$L = 2.6 \frac{g_o}{\epsilon_{is}} \quad (5.8)$$

For rusted wire

$$L = 1.8 \frac{g_o}{\epsilon_{is}} \quad (5.9)$$

For bright wire

$$L = 3.6 \frac{g_o}{\epsilon_{is}} \quad (5.10)$$

#### 5.4 The Effect of Time on Transmission Length

Several investigators have recorded that the transmission length in pretensioned concrete increases with elapse of time. Time effects may be summarized as the general increase in slip with time, leading to a bodily movement of the build-up of strain curve into the center of the concrete. The effect of radial pressure exerted on the concrete is already described in the chapter on "Nature of Bond."

## INFLUENCE OF CONCRETE STRENGTH ON STRAND TRANSFER LENGTH

A paper on the above mentioned subject, was presented by Kaar, LaFraugh and Moss at the Ninth Annual Convention of the Prestressed Concrete Institute and was published in the Prestressed Concrete Institute Journal in October, 1963. The paper reports an investigation of the influence of concrete strength on the stress transfer length of seven-wire strand at the time of transfer of prestress, carried out at Portland Cement Association Research and Development Laboratories. Strands of 1/4, 3/8, 1/2 and 6/10 in. diameter were used to prestress rectangular section members having concrete strengths of 1660, 2500, 3330, 4170 and 5000 psi at the time of prestress transfer. Concrete strength was found to have little influence on transfer length for strands up to 1/2 inch diameter.

Periodic measurements were made of the concrete compressive strains in all specimens, beginning immediately after release of the strands and continuing for one year, to determine the initial transfer length as well as time effects. The increase in transfer length during one year was generally less than 10 percent.

In all tests except those of the specimens containing 6/10 in. diameter strand, unpitted rust-free strand was used.

Prestress transfer length is defined as the distance over which the stress in a pretensioned tendon is transferred by bond to the concrete.

### 6.1 Test

The beams were designed using the maximum stresses permitted by 1963 ACI Code, so that the transfer lengths determined would be the maximum likely to occur in members designed in conformance with the code.

Measurements made along the length of each specimen determined compressive strains set up in the concrete by the prestress force transferred from the strand to the concrete by bond. From these measurements it was possible to trace the build-up of prestress from the ends of the members and so measure the transfer length for each combination of strand size and concrete strength. These measurements were repeated at intervals during the year following transfer of prestress to determine whether transfer length increased with time.

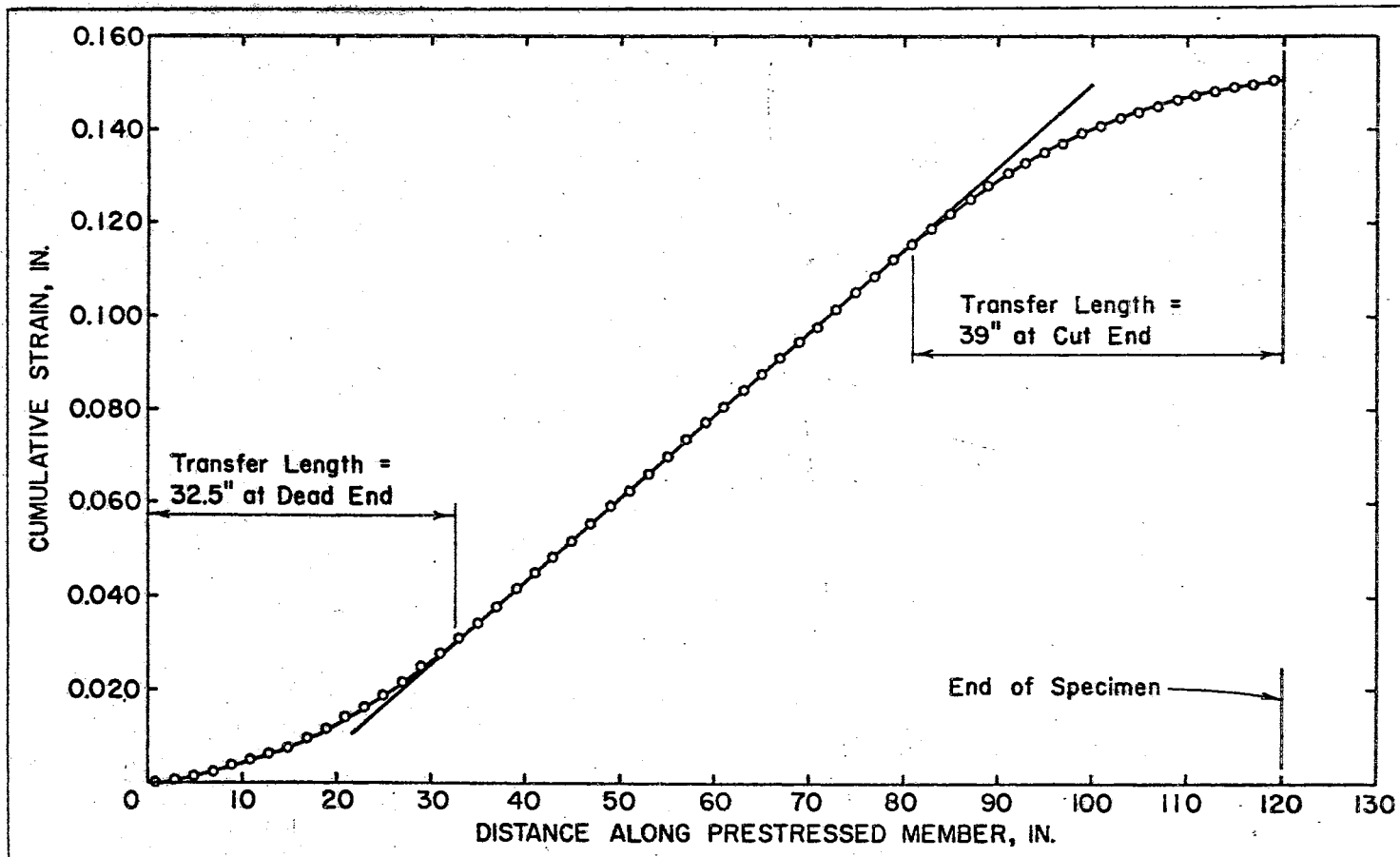
The average strain over a 10 in. gage length was measured by Whittemore strain gage. In view of the rapid change in strain which occurs near the end of a pretensioned prestressed member, it was necessary to evaluate the strain at each point along the length of the member. The actual

strain at particular points can be obtained from the average strains measured by the 10 in. gage by plotting the cumulative strain from the end of the member and then measuring the slope of the curve at the points in question. The actual strain at any particular point will then be equal to the slope of the cumulative strain curve at that point. A curve of cumulative strains is shown in Fig. 6.1. The tangents to this cumulative curve clearly define strains starting near zero at each end and increasing throughout the prestress transfer region to uniform prestress strain determined from tangents drawn to the cumulative curve of Fig. 6.1 is shown in Fig. 6.2. The distance from the end of the member to the point at which the tangents drawn to the cumulative curve deviate from the straight-line portion of the curve is the transfer length.

## 6.2 Conclusions

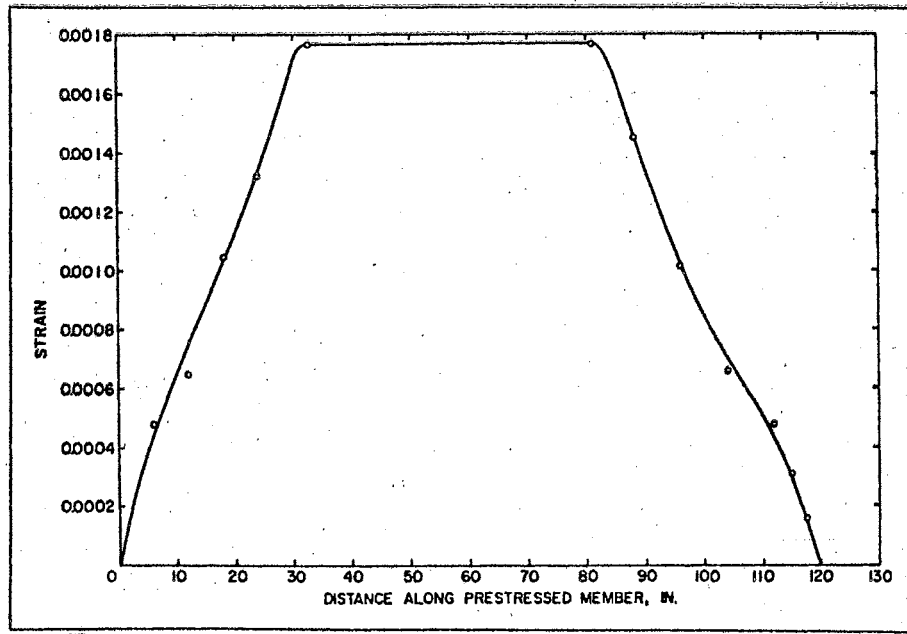
Study of the test data in this investigation lead to the following conclusions.

1. For concrete cylinder strengths of 1500 to 5500 psi, concrete strength at transfer of prestress has little influence on the transfer length of clean seven-wire strands of up to and including 1/2 in. diameter.
2. For strands of 6/10 in. diameter slip occurred over a distance of from 5 to 12 inches at the end of members adjacent to the flame-cutting process when concrete strength at transfer was less than 3000 psi,



-Cumulative Prestress Concrete Strain-Specimen 6/10-4170 at 56 Days.

Fig. 6.1



-Prestress Strain Obtained from Cumulative Strain Curve for Specimen 6/10-4170 at 56 Days.

Fig. 6.2



and the total transfer length was increased correspondingly.

3. For strands of up to 1/2 in. diameter, the transfer lengths measured at the ends of the specimens adjacent to the flame-cutting operation were approximately 20 percent greater than the transfer lengths measured at the opposite ends of the specimens. For 6/10 in. diameter strand, the increase was 30 percent.
4. The average increase in transfer length over a period of one year following prestress transfer was 6 percent for all sizes of strand. The maximum increase in transfer length in this same period was 19 percent. The increase in transfer length with time was apparently independent of the concrete strength at the time of transfer.

## BOND, ANCHORAGE AND RELATED FACTORS

The studies were made by Carl A. Menzel at the Research Laboratory of the Portland Cement Association to show the effect of some of the many factors influencing the results of pull-out bond tests.

A number of tests on bond and the factors relating to bond were carried out, such as

1. Performance of beams with various types of web reinforcement in anchorage zone.
2. Effect on beam performance of hooks at ends of longitudinal reinforcing bars.
3. Effect on beam performance of plates at ends of plain longitudinal reinforcing bars.
4. Effect of prestressing plain bars anchored by heavy end plates.
5. Effect of concentrating or distributing bond action in the anchorage zone.
6. Relative performance of different types of reinforcing bars in beam and pull-out tests.
7. Effect on beam performance of two methods for reducing settlement under top bars.
8. Bond tests of concrete containing various amounts of entrained air.

- (a) Series I. Pull-out and beam tests with portland and air entraining cement.
- (b) Series II. Beam tests in which air content of concrete was varied by adding vinsol resin in solution at mixer.

To describe these tests is not within the scope of this report. Hence, a summary of beam tests is given.

Comparison of the first few beam tests with their corresponding pull-out tests indicated that the bond value of a reinforcing bar would not be fully developed unless some sort of auxiliary reinforcement was used in the anchorage zone which would minimize cracking and distortion, maintain beam action and hold the concrete together around the bar. Diagonal tension cracks in particular reduced the effective length of embedment and induced cracking and disruption of the concrete around the bar so that the bar was no longer held securely enough to prevent appreciable slipping in the region where firm anchorage is needed to resist the pull resulting from beam action.

The tests were made with different types of auxiliary reinforcement to determine which type was most effective in holding the concrete together and in maintaining beam action. Closed loop stirrups inclined at 60 degrees with the main reinforcing bar were found to be most effective for this purpose and were used frequently in subsequent beam

tests. However, since it is a common practice to place stirrups vertically at 90 degrees with the main reinforcing bar, many beams were reinforced with vertical loops for direct comparison with beams reinforced with the more effective inclined loop stirrups.

Tests were also made in which the ends of the main reinforcing bar were anchored by the conventional hooks. These tests revealed some of the helpful and harmful effects of compound stress development introduced by hooked anchorage.

Other tests of beams were made in which plain and deformed bars were anchored by several types of end plates instead of with hooks. The tests with end plates revealed a more favorable distribution of tensile stress in the concrete of the anchorage zone than would normally be obtained without special end anchorage. This was particularly noticeable in beams where the bond between the steel and the concrete in the normal anchorage zone was deliberately reduced or eliminated so that the anchorage was virtually concentrated at the extreme ends of the beam. Although such beams developed greater deflections at a given load and relatively few but wide cracks in the central portion between load points, they carried much higher maximum loads before diagonal tension distress developed in the anchorage zone than was possible when the bars were anchored by surface bond alone. These higher loads were attained even though the auxiliary reinforcement normally

required in the anchorage zone of conventional beams was omitted. However, the maximum load of beams without auxiliary reinforcement was very sensitive to the location of the end plate. For example, the maximum beam load decreased about 10 percent when the end plate was moved 2 inches from the extreme end of the beam to the center of the support and it decreased about 40 percent when it was moved 2 inches from the center of the support to a point directly above the inner edge of the bearing plate at the support.

In some of the tests with end plate anchorage the bars were placed in tension before applying load to the beam. This placed the concrete in compression on the tension side of the beam. As a result higher loads could be applied before cracks developed on the tension side of the beam between the load points. These beams failed by compression of the concrete between load points at the top of the beam rather than by diagonal tension or other disruption of the concrete in the anchorage zone.

Further information on the effect of variations in force distribution and in the concentration of bond stress in the anchorage zone was obtained from the performance of beams in which a special bar with turned lugs was used. With this bar the amount and distribution of the mechanical grip established by the lugs was varied by covering a portion of the lugs or by filling in the spaces between the lugs. The effect of auxiliary reinforcement was determined

by making some beams without such reinforcement and others with vertical and inclined loop stirrups. The effect of bond stress distribution was also studied by varying the number of plain round bars in a beam and by covering a portion of the surface of these bars to prevent bonding in a critical region of the anchorage zone.

The effect of settlement of concrete under the main reinforcing bar was one of the important factors studied in the different groups of beam tests. This was because earlier studies with pull-out specimens had shown that with good workable concrete of 5-6 in. slump the bond resistance of horizontal bars decreased significantly as the depth of concrete under the bar increased. Furthermore, they showed that the effects of settlement and the effectiveness of the two remedial measures for reducing settlement was determined for beam specimens by comparing the performance of two similar beams, one cast inverted with the main reinforcing bar near the top of the beam (usually 9 inches of concrete under the bar) and the other cast upright with the bar near the bottom of the beam (usually 2 inches of concrete under the bar). Both beams were tested in an identical manner with the bar in the lower portion of the specimens so that any difference in the performance of the beams could be attributed directly to the difference in settlement of the concrete under the bars.

The results of the beam tests verified the indications of the pull-out specimens regarding the adverse effects of

settlement on bond resistance and beam performance. They demonstrated conclusively that either of the methods for reducing settlement under horizontal reinforcing bars (aluminum powder in 5-6 inch slump concrete or vibration of 2-3 inch slump concrete) was effective in improving the bond resistance and the load carrying capacity of beams where settlement could be expected to be an important factor.

An important phase of the bond studies was the determination of the performance of beams with different types of bars - plain straight bars, twin-twisted stretched plain "Isteg" bars, commercial deformed bars, special bars prepared with knurled surface or with turned lugs or permanently stretched to increase yield strength. The effect of the variations in the properties of the main reinforcing bar was determined under different test conditions which included variations in the type of auxiliary reinforcement, amount of settlement, age, slump and richness of concrete, method of placing, etc. Tests with a series of bars with turned lugs showed how beam performance was improved by increasing the gripping power of the lugs and how the adverse effect of settlement of concrete under the bar in deep beams could be minimized by improving the gripping action of the lug deformations.

The beam tests repeatedly demonstrated that careful attention to a variety of details is necessary to realize the most effective use of steel in reinforced concrete beams.

Obviously, this requires not only definite knowledge of the effect of variations in the properties of the materials used and in the design and construction of the beam assembly but also the prompt application of this knowledge so that the trend toward refinement in design and more efficient use of materials can be accelerated.

The results of this investigation on relatively small beams indicate that marked improvements in the performance of large reinforced concrete beams can be realized through improved bond resistance, anchorage, and interaction between steel and concrete by

1. Using superior deformed bars of high yield strength steel.

Such bars have closely spaced and properly shaped projections which can develop a positive mechanical grip and a high degree of interaction with the enveloping concrete throughout the beam over a wide range of steel stress.

2. Providing effective auxiliary reinforcement in the anchorage zone.

Close loop stirrups included at 60 degrees with the main reinforcement are much more than similar vertical stirrups in holding the concrete together in this vital region of the beam.



3. Using concrete of 2 to 3 in. slump instead of 5 to 6 in. slump and placing by internal vibration instead of by hand-rodding.

This produces concrete of higher strength (about 10 percent) and low settling characteristics and improves the degree of contact between the concrete and the reinforcing steel.

4. Using concrete mixtures to be placed by hand-rodding with a very small quantity of aluminum powder.

The swelling produced by 2 grams of powdered aluminum per 100 lb. of cement minimizes the tendency of 5-6 in. slump concrete to settle under reinforcing bars. Although the swelling action reduced the compressive strength of the concrete about 15 percent it resulted in a substantial improvement in beam performance in all cases where settlement was an important factor.

Optimum beam performance, as judged by maximum load-carrying capacity, minimum load deflection and minimum width of cracks at a given load, can be obtained by the simultaneous use of practices 1, 2 and 3 above with low-slump vibrated concrete or of practices 1, 2 and 4 with workable concrete of higher slump placed by hand-rodding.

## CONSIDERATION OF FLEXURAL BOND STRESS

There has been much confusion as to the available means of evaluating flexural bond stresses and in particular, great abuse of the familiar expression  $u = \frac{V}{\Sigma O} \cdot a$  in which  $V$  is total shear,  $\Sigma O$  is total perimeter and "a" is the distance from the centroid of the steel to the resultant of the resisting forces developed in the concrete.

All expressions for bond stress are basically founded on the equilibrium of a segment of reinforcing. See Fig.

8.1.

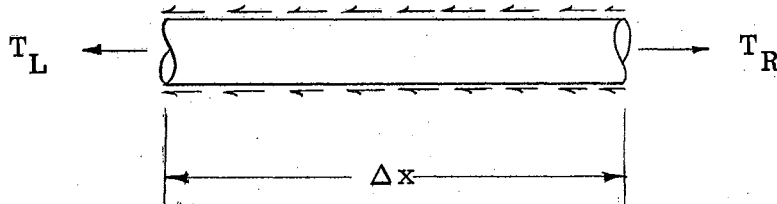


Fig. 8.1 Free-body Diagram of an Element of Reinforcement

From which

$$T_R - T_L = \Delta T \quad T = u \Sigma O \cdot \Delta x$$

$$u_{avg} = \frac{\Delta T}{\Delta x \cdot \Sigma O} \quad (8.1)$$

or

$$u = \frac{1}{\Sigma O} \cdot \frac{dT}{dx} \quad (8.2)$$

Various forms are derived by appropriate substitutions for  $T$ . Derivations for both prestressed and conventionally reinforced beams will be considered for the following conditions: precrack, post crack and ultimate.

### 8.1 Before Cracking

#### 8.1-1 Non-Prestressed

The forces acting on a segment of a conventionally reinforced uncracked beam in flexure are shown in Fig. 8.2. Elastic strains in the concrete produce forces  $T_c$  and  $C_c$  in the concrete. The resultant of these,  $C$ , is at an elevation "a" above the steel centroid.

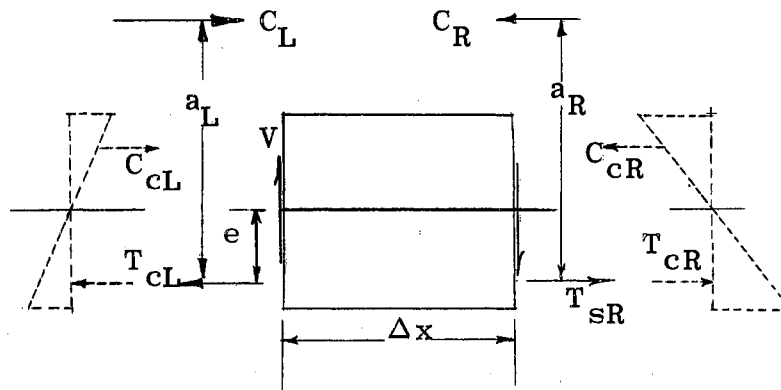


Fig. 8.2 Forces Acting on an Element of A Non-Prestressed Beam

The steel forces,  $T_s$ , are given by

$$T_{sL} = M_L \cdot \frac{e_n \cdot A_s}{I_c} \quad T_{sR} = M_R \cdot \frac{e_n \cdot A_s}{I_c}$$

Substituting into equation (8.1)

$$u = \frac{M_R - M_L}{\Delta x} \cdot \frac{e_n \cdot A_s}{\Sigma \sigma \cdot I_c} = \frac{V}{\Sigma \sigma} \cdot \frac{e_n \cdot A_s}{I_c} \quad (8.3)$$

This expression may be written in the form  $u = \frac{V}{\Sigma \sigma \cdot a}$

as follows:

$$a_L = \frac{M_L}{T_{sL}} = \frac{I_c}{e_n \cdot A_s} \quad a_R = \frac{M_R}{T_{sR}} = \frac{I_c}{e_n \cdot A_s}$$

Therefore "a" is a constant and eq. (8.3) becomes

$u = \frac{V}{\Sigma \sigma \cdot a}$ . Values of "a" are extremely large in usual cases and bond stresses trivial.

### 8.1-2 Prestressed

A segment of the member remote from the transfer zone is shown in Fig. 8.3. Upon release of prestress the concrete strains shown in Fig. 8.3(a) produce a total resisting force,  $C_e$  in the concrete equal and opposite to the effective prestress,  $T_e$ , acting in the steel. Flexure due to dead and live load sets up in the additional internal forces shown in Fig. 8.3(b). The resultant of the tensile and compressive forces in the concrete,  $T_c$  and  $C_c$ , is  $C$ , located a distance  $b$  above the centroid of the steel. This figure is similar to the non-prestressed case, and by the same argument

$$b_R = b_L = \frac{I_c}{e_n \cdot A_s}$$

The combined internal forces are given in Fig. 8.3(c).  
The change in the steel tension across the segment is

$$\Delta T = T_{sR} - T_{sL} \quad (8.4)$$

where

$$T_{sL} = M_L \cdot \frac{e_n A_s}{I_c} \quad T_{sR} = M_R \cdot \frac{e_n A_s}{I_c} \quad (8.5)$$

Substituting into (8.1)

$$u = \frac{M_R - M_L}{\Delta x \cdot \Sigma O} \cdot \frac{e_n \cdot A_s}{I_c} = \frac{V}{\Sigma O} \cdot \frac{e_n \cdot A_s}{I_c} \quad (8.6)$$

an expression identical to equation (8.3)

This expression can be written as

$$u = \frac{V}{\Sigma O \cdot b} \quad (8.7)$$

which is obviously quite different from the widely used

$$\text{form } u = \frac{V}{\Sigma O \cdot a}$$

The total moment arm,  $a$  is given by

$$a_L = \frac{M_L}{T_e + T_{sL}} = \frac{M_L}{T_e + \frac{M_L \cdot e_n A_s}{I_c}} \quad (8.8)$$

$$a_R = \frac{M_R}{T_e + T_{sR}} = \frac{M_R}{T_e + \frac{M_R \cdot e_n A_s}{I_c}} \quad (8.9)$$

It can be seen that "a" is a variable, a function of both prestressed and applied moment. Near the end of a member,

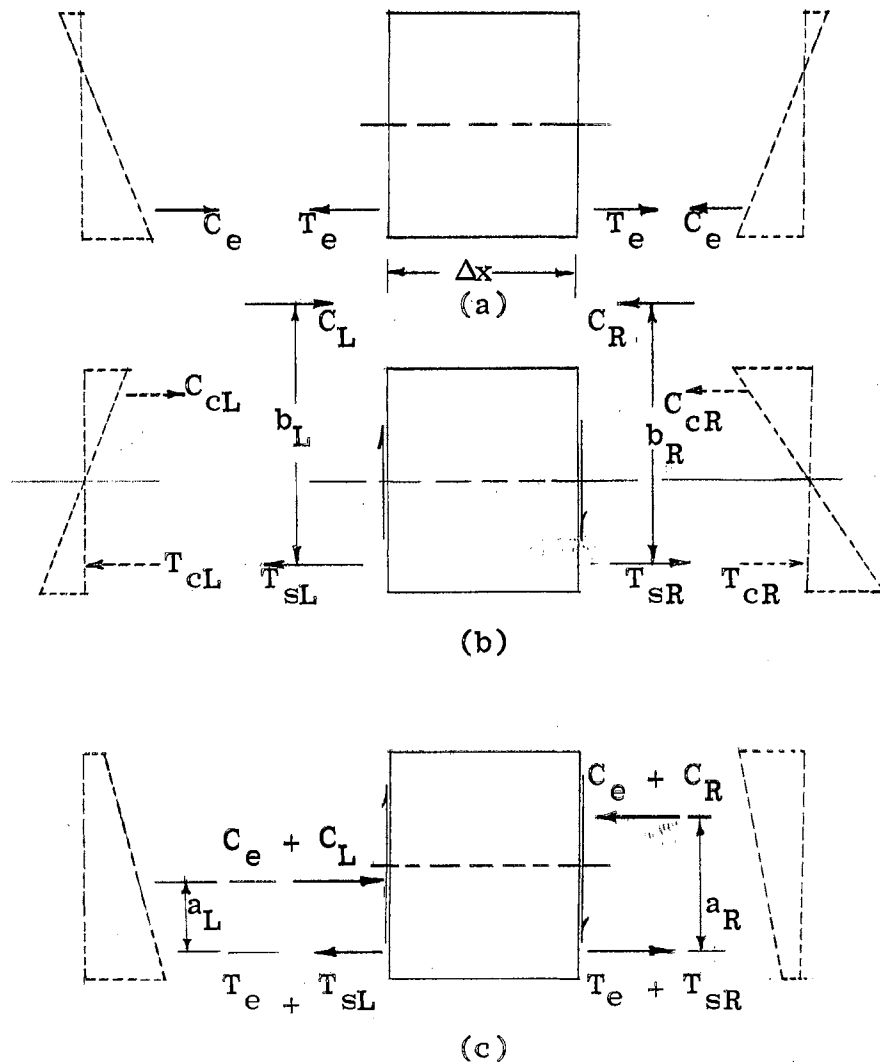


Fig. 8.3 Forces Acting on an Element of a Prestressed Beam

as  $M$  approaches zero, " $a$ " also approaches zero. If the expression  $u = \frac{V}{\Sigma 0 \cdot a}$  were correct, flexural bond stress would be a minimum under the load and infinite at the support.

From equations (8.3) and (8.5) it should be noted that bond stresses before cracking are very small for prestressed members and are independent of the degree of prestress except insofar as the prestressing determines the cracking load.

## 8.2 After Cracking

### 8.2-1 Non-Prestressed

In the design of conventionally reinforced members bond stresses are computed for the cracked section at design load by  $u = \frac{V}{\Sigma O \cdot j \cdot d}$ , where  $jd$  has the same meaning as "a" and is compared with an allowable value. The derivation follows from a figure similar to Fig. 8.2 except that the tensile concrete is inoperative, its load being transferred to the steel. Thus both  $T_s$  and  $C$  increase sharply at cracking and "a" assumes a value which is constant so long as both steel and concrete are elastic and is less than the effective depth. Hence, either equation (8.1) or (8.3) may be used in the evaluation of the bond stress so long as  $I_c$  of the cracked section is used.

Of course, bond stresses computed by these formulas are fictitious quantities. The very opening of a crack requires the destruction of adhesive bond in the immediate vicinity. The abrupt change in the direction of the reinforcing as the crack widens, gives rise to high normal forces and consequent highly effective frictional bonding. Throughout the cracked zone, the neutral axis must undulate between cracks. This alone would cause wide variation in bond stress. The design formulas and allowable stresses merely serve to correlate beam design with test results to assure satisfactory overall behavior.

### 8.2-2 Prestressed

In the case of prestressed members after cracking, the theoretical moment of inertia and position of the neutral axis vary along the length of the beam as a rather complicated function of the applied moment, ruling out the use of equation (8.6). Average bond stresses can be found only by the computation of steel force at successive sections and the application of these values to equation (8.1). The evaluation of the steel forces can be performed with reasonable ease by the method presented by Warner in Fritz Laboratory Report 223.20, Lehigh University, November, 1958.

Bond stresses between cracking and ultimate loads are of little practical interest. The most severe bond stresses must be present at ultimate load and it is only these values which are significant in design since the bond capacity of the member is required to exceed the ultimate capacity of the member.

### 8.3 At Ultimate Load

Conditions in both conventionally reinforced and prestressed beams are essentially the same as ultimate load.

In under-reinforced beams the maximum bond stresses at ultimate do not occur at the section of maximum moment and shear because of the plasticity of the steel at high stress levels. The free body diagram of Fig. 8.4 illustrates the point. The applied loading is such that the externally applied moment is a maximum at the right face. As ultimate



load is approached the steel yields plastically. The crack widens and the neutral axis progresses upward. At the left face, where the moment is somewhat less, the same effect occurs but lags behind the action at the right. After the steel has yielded at both faces, then, there is no differential of steel force and bond stress is zero. The difference in moment is accommodated by the difference in the moment arms  $a_R$  and  $a_L$ .

Since the maximum bond stresses do not occur close to the section under maximum moment and certainly not in the uncracked portion where they have been shown to be negligible, the greatest flexural bond stresses must develop at a section somewhere between the two.

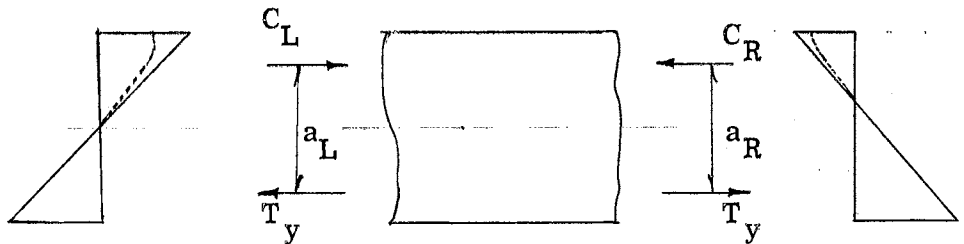


Fig. 8.4 Equilibrium of Element of Beam at Ultimate

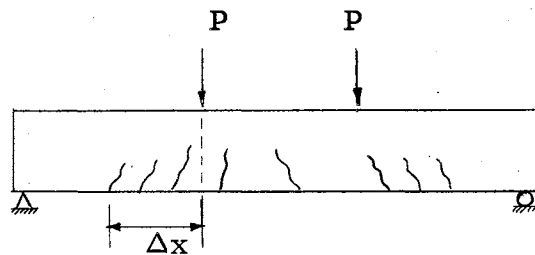


Fig. 8.5 Sketch of Cracked Beam

It is most unlikely that a practical procedure for evaluating the maximum stress can be found because of the uncertainties of crack spacing and local effects. It would be quite reasonable, however, to apply equation (8.1) to the entire region between the section of maximum moment and the outermost crack to obtain a value of average bond stress which could be used in the comparison of experimental results.

Consider the under-reinforced pretensioned beam of Fig. 8.5 for example. As ultimate load is approached the strand force at B approaches  $T$  while just to the left of A it may be considered essentially  $T_e$ , the effective prestress.

Then

$$u_{avg} = \frac{T_{ult} - T_e}{\Delta x \Sigma_0} \quad (8.10)$$

Values given by this equation for measured lengths  $\Delta x$  would be somewhat higher than actual because the strand force actually approaches  $T_e$  some distance beyond the outermost crack, but nevertheless would offer a sound basis for comparison.

It is again emphasized that  $u = \frac{V}{a \Sigma_0}$  is not valid at ultimate load for either conventional or prestressed members.

#### 8.4 The Problem of Anchorage Bond

The anchorage developed in a pretensioned member is in large measure dependent upon both the transfer of prestress

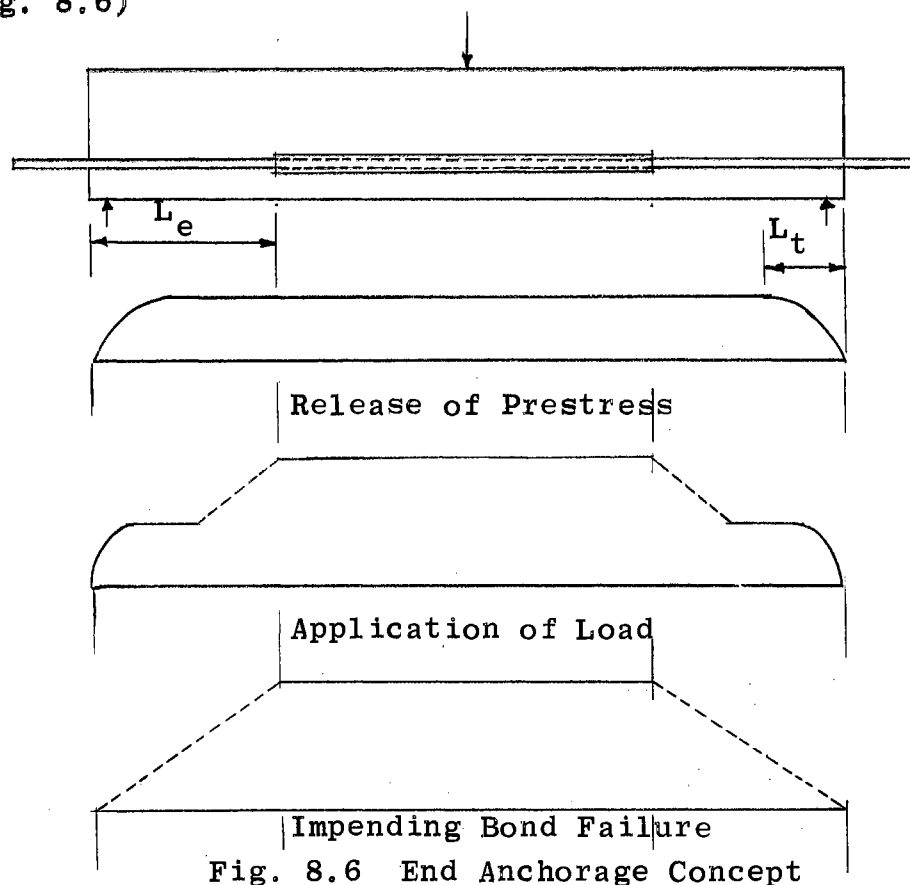
and flexural action. The transfer length provides full anchorage for the strand at effective prestress. When flexure causes an increase in strand tension, additional embedment length is required to transfer the additional force.

The increase in strand tension due to flexure is quite small in the uncracked portions. If, however, the outermost crack were to occur close to the transfer zone, the relatively large increase in strand tension could not be transferred in the short distance between the crack and the end of the transfer zone. The pull-out action would penetrate into the elastic portion of the transmission length, with the result that bond would become entirely frictional and mechanical over the entire length from the end of the member to the crack. The slightest additional increase in strand tension would then result in slip of the strand.

Consider a pretensioned beam as ultimate load is approached. The steel at the section of maximum moment is first to yield plasticly. At adjacent cracks steel strain is considerably less, but stress is essentially the same. While it is conservative to do so, it is not unrealistic to assume, then, that at ultimate load the strand force in an over-reinforced beam is the ultimate strength of the strand over the entire cracked portion. In this context the beam may be thought of as a post-tensioned member whose end anchorage is developed by

bond over the embedment length  $L_e$  where  $L_e$  is the distance from the end of the member to the outermost flexural crack.

(Fig. 8.6)

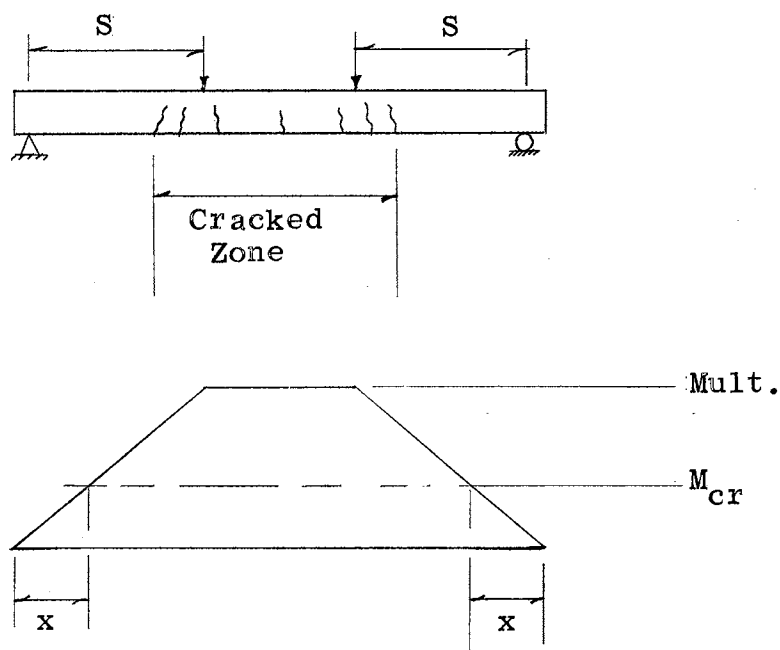


Bond in the central portion is destroyed by encasing the strand in a tube or by similar device. When the prestress is released to this member, the prestressing force in the strand builds up to its effective level over the transfer length  $L_t$ . Should the required transfer length be greater than the available embedment length,  $L_e$ , bond failure must occur at release of prestress. But, if the embedment length is great enough so that the beam withstands the transfer of prestress and the beam is subsequently loaded, increasing the strand force at the unbounded interior of the beam, the tendency for the strand

to slip and screw inward is likewise increased. If at a given load the embedment length is not sufficient to develop a total bond force equal to the strand tension, the strand must slip. The length of embedment is the "ultimate anchorage length" which will provide just sufficient anchorage bond to develop its full tensile capacity.

### 8.5 Available Anchorage Length in a Beam at Ultimate Load

To assure adequate anchorage, the embedment length available to provide anchorage must exceed the ultimate anchorage length. The available embedment is the distance from the outermost flexural crack crossing the reinforcement to the point of initial contact between the concrete and steel.



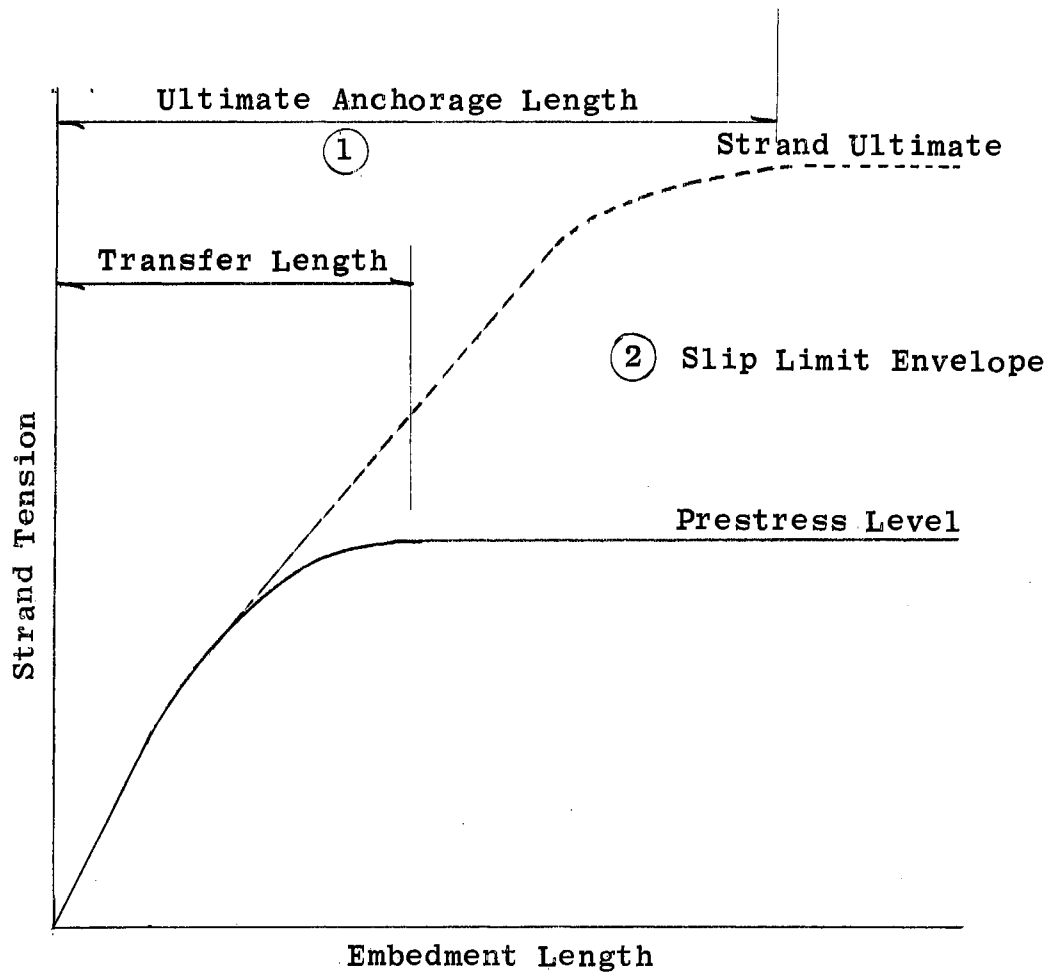


Fig. 8.7 The Slip-Limit Envelope

For the simple loading given (neglecting dead loads)

$$x = \frac{S}{M_{ult}/M_{cr}} \quad (8.11)$$

$x$  = Distance from the support to the outermost crack originating at the bottom fiber.

$S$  = Shear span

Comparing with the laboratory tests it is observed that the reduced ratio of  $\frac{M_{ult}}{M_{cr}}$  should be used. Designating the reduced ratio as  $c$ :

$$c = \frac{S}{x} \quad (8.12)$$

The tests carried out at Fritz Laboratory showed  $c = 1.6$ , but no empirical expression could be found for  $c$ .

### 8.6 Conclusions and Recommendations from the Tests Carried Out at Fritz Laboratory

1. The test results indicate that no practical slip limit envelope exists.
2. The recommended criteria to assure safety against anchorage failure is as follows:
  - a) The location of the outermost crack is first established. The moment diagram, or the curve of maximum moments is plotted with a maximum ordinate of 1.6. The intersections of this curve with a level line of ordinate 1.0 locate the outermost crack.

- b) The available embedment length is the distance from the point of initial contact between the concrete and strand to the outermost crack.
  - c) If the available embedment length determined is four feet or greater (for 7/16" strand or smaller) the member is satisfactory in bond.
  - d) If the available embedment length determined is less than four feet, the beam is unsafe with respect to bond. This condition may be remedied as follows:
    - i) Use a post-tensioned, grouted beam
    - ii) Provide positive anchorage by means of a mechanical device.
    - iii) Increase the overhang at the beam supports to provide the minimum embedment length.
    - iv) Use a smaller strand, the ultimate anchorage length of which is equal to or less than the available anchorage length.
3. Friction appears to be the major component determining ultimate bond strength.
4. The ages and strengths of concrete investigated in the research at Fritz Laboratory, four to forty days and 4,250 to 6,940 psi respectively, do not appear to be the principal variables affecting bond strength.
5. The degree of vibration and resultant compaction of the wet concrete appear to be the most significant variables affecting the bond strength of high strength, low slump concrete. Thorough vibration gave consistently



better results whereas insufficient vibration was the most probable cause of the exceptional, poor results.

6. The bond resistance of strand was not seen to be affected by either gradual or sudden release.
7. Transfer length was not observed to increase in the time interval investigated, maximum of which was 136 hours.
8. The bond strength of strand does not appear to be significantly affected by the degree of initial steel pretension.

## FLEXURAL BOND TESTS

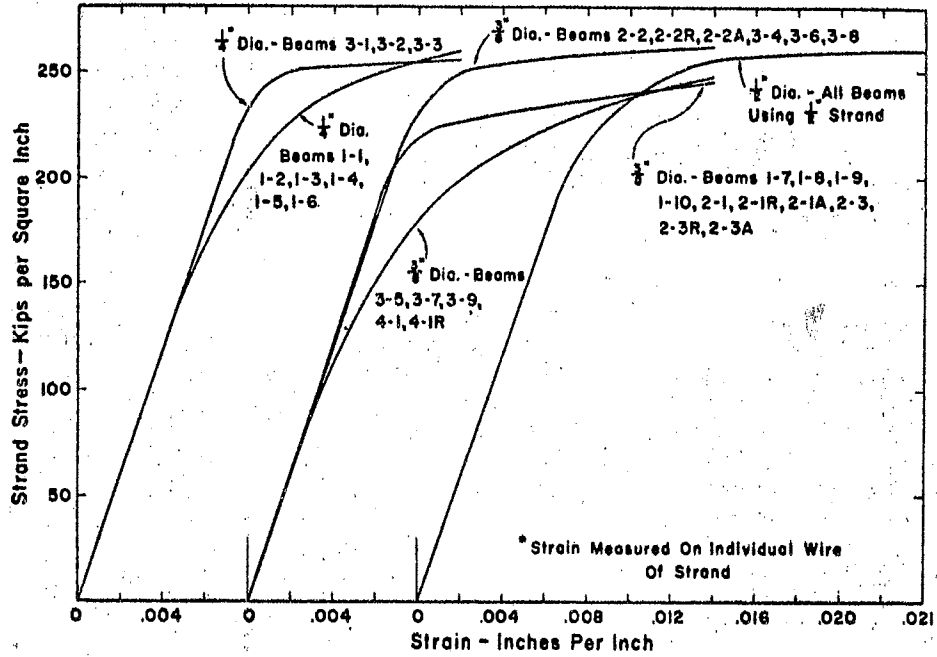
The flexural bond tests, carried out by Hanson and Kaar at the Research and Development Laboratories of the Portland Cement Association and reported in the Journal of the American Concrete Institute in January, 1959, are summarized here.

### 9.1 Test

The test program involved 47 beam tests divided into four series such as (1) various shear-span lengths covering a wide range (containing separately 1/4, 3/8 or 1/2 in. diameter strand), (2) various concrete strengths e.g. 3700, 5420 and 7230 psi and using rusted or clean or clean with strand wise anchor, (3) geometrically similar beams (b and d variable, but b/d constant), (4) beams prestressed with clean, smooth strand or rusted strand.

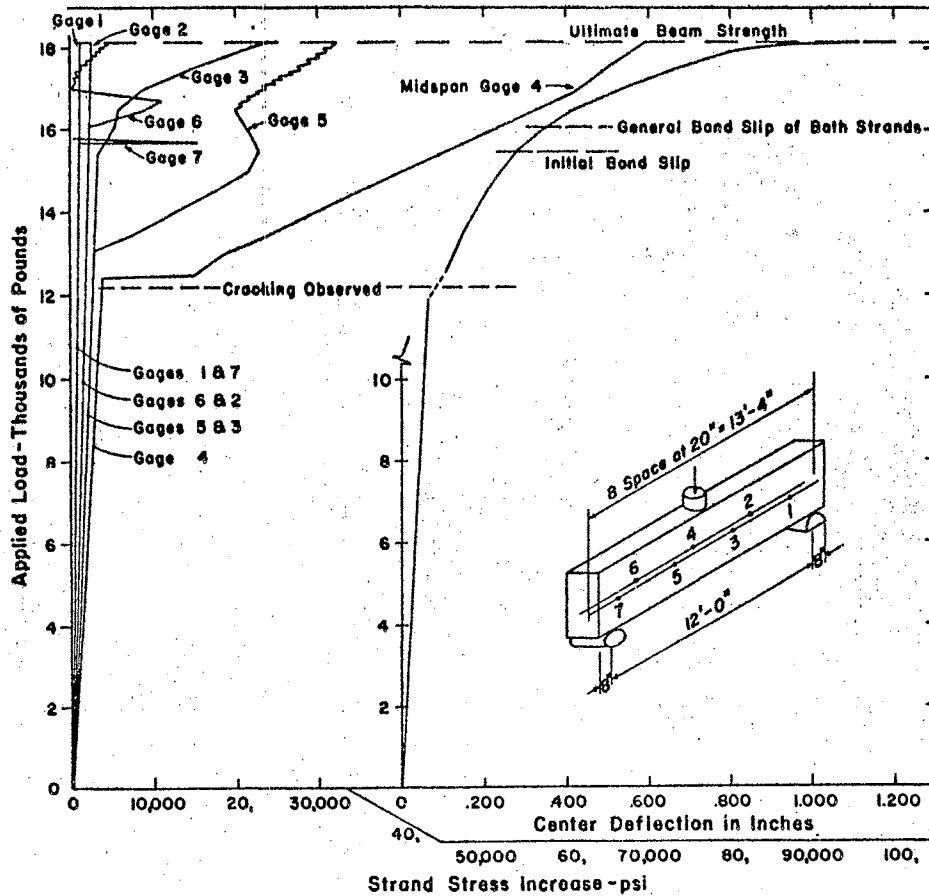
As bond failure is progressive rather than instantaneous, the test beams were instrumented to record the continuous development of flexural strains and the subsequent evidence of bond slip.

All beams were tested to failure in a hydraulic testing machine. The individual beam tests were generally conducted in about 30 minutes, with a continuously increasing load.



—Stress-strain curves of strand

Fig. 9.1



—Load versus strand stress increase and deflection, Beam 3-10

Fig. 9.2

The steel strains as indicated by the SR-4 gages were recorded continuously, and dial gage readings of deflection and strand slip were made without interrupting the loading.

Test results are shown in Fig. 9.2. Strains have been converted to stresses, using the applicable stress-strain curve of Fig. 9.1.

## 9.2 Discussion of Test Results

### 9.2-1 Interrelation of Variables

If a strand is to be stressed to fracture at ultimate load of a beam, then there is a critical embedment length for each size of strand, which must be provided if bond slip is to be avoided. However, if a beam contains a high percentage of steel, or if the concrete strength is low, then flexural failure may occur by crushing of the concrete while the steel is stressed below its ultimate strength. In this case, bond slip will not necessarily occur even if the embedment length provided is less than the critical length needed for the size of strand used to develop the ultimate strand length. Thus concrete strength, percentage of steel, and the embedment length are interrelated in design.

Mode of failure: Thirteen of the beams failed in flexure without prior slippage of the strand along its entire embedment length. The remainder failed in flexure after a general bond slip of the strands. The moment sustained at general bond slip and the ultimate moment sustained were both of interest in that investigation.

Strand embedment length: Beams having an embedment length from load to beam end of 80 in. or more failed in flexure by crushing of the concrete after yielding of the steel, before general bond slip could occur. As the embedment length decreased, failure occurred at progressively lower moments due to slippage of the strands.

Percentage of steel: High steel percentages reduce the possibility of bond failure for a given embedment length because the steel stress at flexural failure is less in a beam with a high steel percentage than in a beam with a low  $p$ .

Reduction of concrete strength: The effect of reduction of concrete strength in a particular beam is to decrease the steel stress at flexure failure, with a correspondingly lower average bond stress over the embedment length. If the reduction in average bond stress due to drop in steel stress is greater than the reduction in the bond strength due to drop in concrete strength, then a failure due to general bond slip will be less likely in the beam with reduced concrete strength.

Strand surface condition: The beams prestressed by rusted strand performed as well as, or better than, the beams prestressed by clean, smooth strand.

Provision of end anchors: A comparison of anchored and unanchored beams for three concrete strengths used shows little difference in performance. The end anchors did not become effective until slip has occurred along the

entire embedment length, and they were therefore unable to delay the onset of general bond slip. After the anchors became effective the beams acted as post-tensioned beams without bond, and their ultimate moments of resistance are no greater than those of the beams depending only on the mechanical interlocking effect between the strand and concrete after general bond slip.

### 9.3 Calculation of Bond Stresses

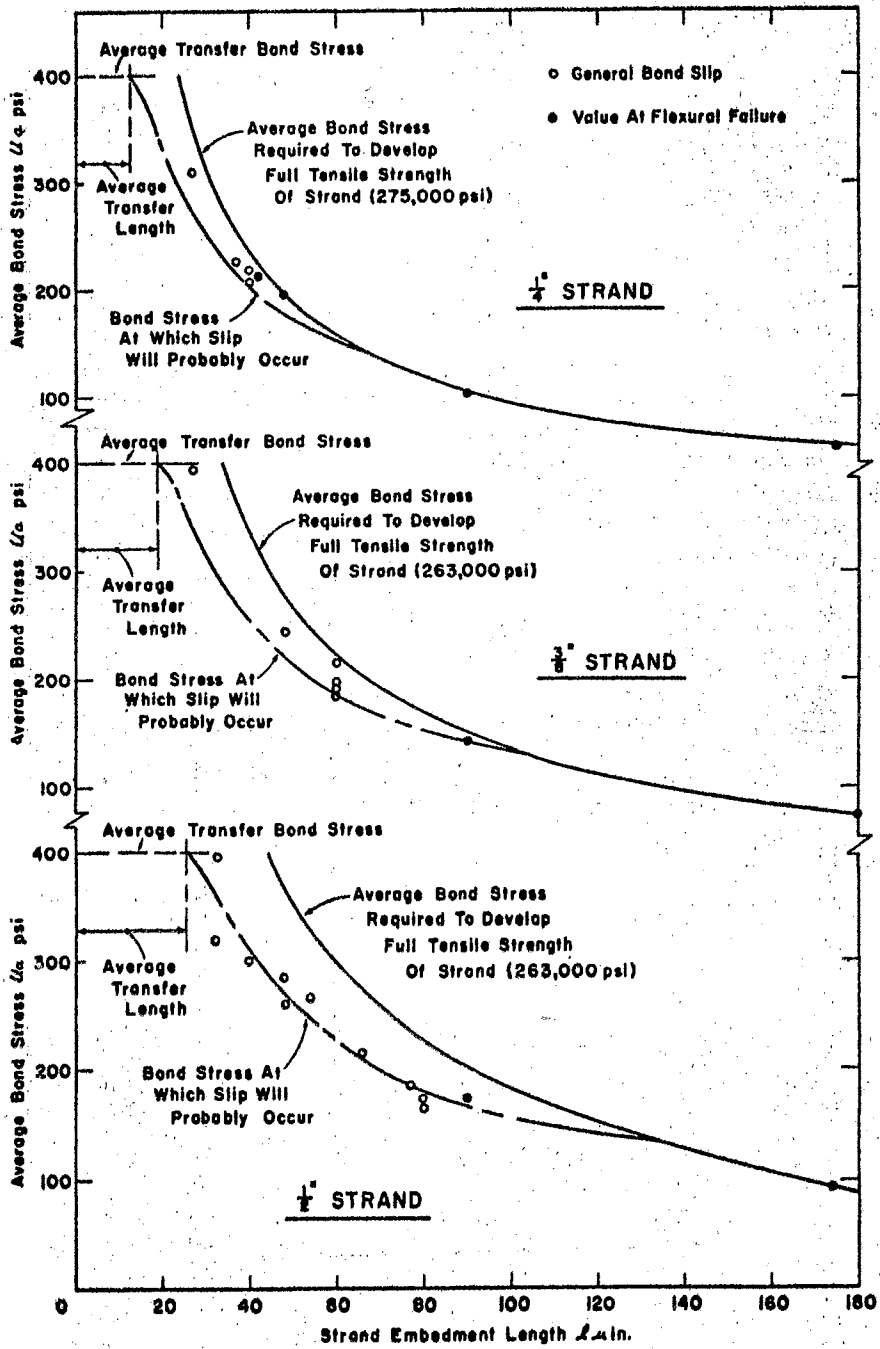
It is not feasible to measure the magnitude and complex distribution of bond stresses along the entire length of a prestressing strand. However, average bond stress can be calculated.

Consider the forces acting along the length of a prestressed strand between its free end and a section of maximum stress. The force due to tensile stress in the steel will then equal  $f_s A_s$ . The force due to bond stresses on the surface of the embedded strand will be given by  $u_a \cdot l_u \Sigma O$ . To satisfy the conditions of static equilibrium these forces must be equal.

$$u_a \cdot l_u \cdot \Sigma O = f_s \cdot A_s$$

$$u_a = \frac{f_s \cdot A_s}{l_u \Sigma O} \quad (9.1)$$

Given the maximum steel stress at the time general bond slip occurs,  $f_{sb}$ , and the embedment length,  $l_u$ , we can calculate the average bond stress along the embedded



—Relation of average bond stress  $u_a$  to strand embedment length

Fig. 9.3

length of strand.

$$u_a = \frac{f_{sb} \cdot A_s}{l_u \cdot 10} \quad (9.2)$$

The value of  $u_a$  was calculated for the various beams tested, and the results are shown in Fig. 9.3.

The values of average bond stress have been plotted as broken lines. The average bond stress was calculated by using flexural bond stress wave immediately prior to general bond slip.

Summary: A general bond slip occurs in a pretensioned beam when the peak of the flexural bond stress wave reaches the transfer zone.



## CONCLUSION

Some of the conclusions made from the tests carried out (as described in this report) are as follows:

The ability of pretensioned wire to transfer stress to the concrete through bond does not vary a great deal with wire size within the range of 0.100 to 0.276 in. diameter. The length of embedment necessary to transmit the stress fully to the concrete prisms is moderately greater as the wire diameter increases.

A rusted wire develops the full transfer of prestress at a more rapid rate and in somewhat less distance from the free end.

According to the theoretical analysis by Dinsmore, Deutsch and Montemayor from Fritz Laboratories, the bond stresses, before cracking, are independent of the degree of prestressing except in so far as the prestressing determines the cracking load. According to the experiments carried out by Linger and Bhonsle as published in 1963, using photoelasticity; the prestress force, thickness of specimen and modulus of elasticity of the beam effect the transfer length.

According to Evans and Williams, using x-rays, the transmission length depends upon the maximum retained tension in the wire. The maximum retained tension depends on the

concrete section and the value of Young's modulus for the concrete.

According to Kaar, LaFraugh and Mass, concrete strength between 1500 to 5500 psi, at transfer of prestress has little influence on the transfer length of clean seven-wire strand up to and including 1/2 in. diameter. The increase in transfer length with time is apparently independent of the concrete strength at the time of transfer.

According to tests carried out by Menzel, some sort of auxiliary reinforcement (closed loops at  $60^{\circ}$  were the most effective) used in the anchorage zone reduces cracking and distortion, to hold the concrete together around the bar; because the diagonal tension cracks in particular reduced the effective length of embedment.

Also that bond resistance can be improved by using superior deformed bars of high yield strength steel.

According to tests by Hanson and Kaar, a general bond slip occurs in a pretensioned beam when the peak of the flexural bond stress wave reaches the transfer zone.

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