DISTRIBUTION OF STRESS AROUND A TUNNEL CAVITY, USING THE FINITE ELEMENT METHOD

By

YOHANNES WOLDEMARIAM

Diploma, Civil Engineering
College of Technology
Loughborough, Leicestershire, England
July, 1955

Master of Science
Oklahoma State University
Stillwater, Oklahoma
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Thesis Approved:

William P. Dawkins

Thesis Adviser

Louis O. Bass

Donald E. Boyd

D. D. Durham

Dean of the Graduate College

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NOMENCLATURE

$[A]$	Rectangular matrix; function of position coordinates of a point within the element
$[B]$	Strain displacement transformation matrix
$[C]$	Rectangular matrix of elasticity coefficients
$\{e\}$	Column vector of total strains within an element
$[K]$	Structure stiffness matrix
$[k]$	Element stiffness matrix
$[M]$	Mass matrix
$[m]$	Rectangular matrix of displacement functions
$[P]$	Column vector of nodal point loads
$\{Q\}$	Column vector of displacements at nodal points
$\{q\}$	Column vector of displacements within an element
$[R]$	Rectangular matrix; function of position coordinates of nodal points
U_0	Strain energy density
V	Volume
$\{\alpha\}$	Column vector of generalized displacements
δU	Virtual strain energy
δW	Virtual work of external forces
$\{\varepsilon\}$	Column vector of elastic strains

$\{\epsilon_0\}$	Column vector of initial strains
$\{\sigma\}$	Column vector of element stresses

CHAPTER I

INTRODUCTION

1.1 General

Engineers concerned with the design and construction of buried structures have long been preoccupied with the problems of structure-medium interactions. With the advent of thermonuclear weapons, which may subject buried structures to severe ground motions and very high loads, a clear understanding of structure-medium interaction has become vitally important. Conventional methods, of empirical nature, used in the design of culverts and pipelines, cannot be used in the design of buried structures which have to withstand nuclear effects, as this would result in extremely costly structures. In an effort to reduce and control the magnitude of the forces applied to the buried structure by blast loadings, comprehensive studies were conducted both in the field and laboratory of structure-medium interactions.

A technique employed is to isolate the structure from the effects of nuclear blast by surrounding it with an energy absorbing medium called "packing." Such a packing material is normally elasto-plastic with a low yield strength and high compressibility. The packing is intended to absorb part of the shock energy and to redistribute the pressure around the structure, thereby reducing the loads applied to it.

1.2 Statement of Problem

This study is an effort to evaluate the feasibility of applying the finite element technique to the solution of structure-medium interaction problems and to investigate the magnitude and distribution of pressures around a lined or unlined horizontal tunnel subjected to a static uniform over-pressure on the ground surface. The structural liner considered in this study is made of reinforced concrete, although other materials could have served just as well, and is encased in a packing material placed between the liner and the face of the tunnel cavity.

An assumption made in the study is that the surrounding medium behaves as a continuous, elastic, isotropic and homogeneous medium. Using the well established theory of elasticity then, it is possible to determine the displacements and, hence, the stresses throughout the medium. A condition of plane-strain along the axis of the tunnel was considered, as most field observations from conventional installations such as culverts and pipelines approximately conform to such conditions.

1.3 Previous Work

The composition of various materials that could be satisfactorily used as back-packing for shock isolation of buried structures has interested several authorities and institutions for the past few years, and has been the subject of a number of experimental and theoretical studies.

During operation "plumbbob," in 1957, Vaile (1) studied the effects of violent ground motions on buried vertical concrete pipes, some of which had glass bottles for back-packing, while others had soil and served as control pipes. Vaile observed that the accelerations, velocities and displacements produced in the isolated pipes were markedly reduced compared to those of the control pipes. The reduction was due partly to the collapse and crushing of the bottles, which resulted in the dissipation of some of the shock energy.

Sevin (2) has performed studies on small scale structures and points out the effectiveness of the packing materials even when stressed only to the elastic region of the stress-strain curve. He also notes that "at sufficiently high stress levels this form could also be expected to behave as a dissipative isolation system." This study was primarily concerned with the redistribution of soil pressure around the structure-packing system.

Da Deppo and Werner (3) studied the influence on the response of a buried cylinder, with a crushable material placed adjacent to the cylinder. The packing material, they observed, greatly reduced the magnitude of the loads reaching the cylinder. The results of this study also indicated the effects of the softness of the packing material on the stress redistribution around the structure.

More recently, numerous studies have also dealt with the static and dynamic analyses of lined and unlined tunnels (4) and (5). The usual assumptions of linear elasticity, homogeneity and isotropy were made. It was observed, in these and other studies, that the displacements and stresses from a dynamic analysis obtained by an assumed plane stress wave of a certain amplitude were about 20% higher than the displacements and stresses obtained from a static analysis of the same amplitude (as that of plane wave stress). Because of the close agreements then, it is possible to predict the behavior of such cylindrical structures under dynamic conditions from studies of their behavior under static conditions.

1.4 The Finite Element Method

The finite element method is, basically, a process by which a complex structure is idealized as an assemblage of a finite number of structural elements of appropriate size and shape, interconnected by a finite number of nodes, and retaining all the material properties of the original structure. This model of the actual structure is used in all subsequent structural analyses and the accuracy of the results will depend on how well the behavior of the model simulates that of the actual structure; i.e., actions and deformations of the nodes are interpreted to be the actions and displacements of the corresponding points on the structures.

Idealization of a structure as an assemblage of structural elements is not new. The slope-deflection method of structural analysis, for example, is an idealization where the structural elements are one-dimensional in character. What is new in the finite element method is that the idealization is extended by the use of two- or three-dimensional elements to represent an elastic continuum.

Most of the approximate methods of structural analysis (such as finite differences) depend for their solution on approximate mathematical procedures. The only approximation involved in the finite element method, however, is that of replacing the actual structure by an articulated structural model, but there is no need to use an approximate mathematical analysis of the model.

Matrices representing element characteristics, such as force-displacement or stress-strain relationships, are obtained from an assumed displacement (or stress) function throughout the element. Once these matrices are derived, the behavior of the entire structure may then be determined by satisfying simultaneously the equilibrium, compatibility and force-deflection relationships at the nodes.

Of the force and displacement methods of analysis, the latter is more suitable for computer programming and has been used in this study.

1.5 Liner-Packing Materials and Configurations

Figures 1 and 2 show a liner-packing system and its finite element idealization, respectively. The liner material is shown to be concrete, but steel liners would work just as well.

1.5.1 Reinforced Concrete Liners

The behavior of a structure under load depends, to a large extent, on the stress-strain relationship of the materials from which it is made. For the liner under consideration, therefore, typical stress-strain curves for concrete under compressive loads are shown in Figure 3, for various cylinder strengths, f'_c. Each curve starts with an initial nearly straight elastic portion, then begins to curve to the horizontal and attains a maximum stress at a strain of approximately 0.002 in. per in., and follows a steep downward path beyond the peak stress, probably indicating internal disintegration of the material. It is noted that concretes of lower strength are less brittle than high strength concretes. Tests have shown that unit strains of 0.003 to

0.0045 occur before a beam fails. For concretes of cylinder strength f'_c over 6000 psi, the maximum observed strains range from 0.0025 to 0.0040. For high strength concretes, therefore, a maximum strain of 0.003 is considered the limit of usefulness.

The modulus of elasticity E_c, which is the slope of the initial straight portion, is seen to vary with the strength of the concrete; the higher the strength, the larger the modulus. The ACI Code (15) recommends the use of an empirical formula for the computation of the modulus of elasticity and is given by

$$E = 33\omega^{1.5} f'_c$$

where ω = weight of the concrete in pounds per cubic foot.

When stressed in one direction, concrete expands in the transverse direction and the Poisson's ratio, for concretes stressed up to 0.7 f'_c, varies from 0.15 to 0.20.

The tensile strength of concrete is very small compared to its compressive stress and is usually neglected.

1.5.2 Back-packing

The most important characteristic that a back-packing material should have is that it be highly compressible with a low yield stress. It should also be capable of dissipating a large portion of the shock energy. Several materials, with the desired characteristics, have been developed for use as back-packing; a few of which are foamed plastics, honeycombs, insulating concretes, and various granular materials.

The choice of back-packing materials will depend largely on the type and location of the structure, the assumed loading and, above all, the cost of material. Generally speaking, the granular materials are the least expensive. Typical stress-strain curves are shown in Figures 4 and 5 for elasto-plastic and plasto-elastic materials. To facilitate calculations, both of these curves have been approximated as shown on the same figures, although only the elastic range is of interest in this study.

The material used in this study is cellular concrete which is elasto-plastic with a yield stress of 40 psi..

1.5.3 Surrounding Medium

Soils and rocks, in general, possess non-linear stress-strain characteristics. A typical stress-strain curve for playa silt, in one-dimensional compression, is given in Figure 6. As a result of compaction and cementation that takes place, the initial part of the curve is concave downward up to the point A. As the stress is increased, the initial stiffness due to compaction and cementation is destroyed, and the curve begins to increase with the stress level and becomes concave upward. The one-dimensional modulus is determined as a tangent of such a stress-strain curve or from measurements of wave propagation through the medium.

1.5.4 Assumed Behavior of Materials

For the purpose of analysis, the above materials have been idealized as being continuous, isotropic and homogeneous. The study, however, will be restricted to the initial regions of the stress-strain curves where the behavior is almost linearly elastic in nature.

CHAPTER II

FORMULATION OF MODEL

2.1 Formation of Structure Matrices

A structural model assembled from a finite number of discrete elements is substituted for the continuous structure. These elements are separated by imaginary lines and are interconnected by a finite number of nodes, as shown in Figures 2, 7 and 8. The equations of elasticity for the continuous structure are formulated in matrix form using these elements.

2.2 Development of Element Stiffness Matrix

It is assumed that the displacements at any point within the element can be related to the displacements at the nodes by an equation of the form:

$$\{q\} = [A]\{Q\} \tag{2.1}$$

where $\{q\} = \{q_x, q_y\}$ is a column matrix of internal displacements at a point in the element, $\{Q\} = \{Q_1, Q_2, \ldots Q_n\}$ is a column matrix of displacements of the nodal points, and $[A] = [A(x, y, z)]$ is a rectangular matrix which is a function of the position coordinates of the point. With the displacements at any point within the element given, the corresponding total strains are obtained by differentiating Equation (2.1) and

$$\{e\} = [B]\{Q\} = \text{the total strain} \tag{2.2}$$

$$= \{\epsilon\} + \{\epsilon_0\} \tag{2.3}$$

where

$[B]$ = the strain-displacement transformation matrix;

$\{\epsilon\}$ = elastic strains

$\{\epsilon_0\}$ = initial strains due to temperature changes, pre-stress, etc.

Assuming linear elastic behavior for the material, the stresses are related to the elastic strains by the generalized Hooke's Law:

$$\{\sigma\} = [C](\{e\} - \{\epsilon_0\}) . \tag{2.4}$$

Substituting Equation (2.2) into Equation (2.4) gives

$$\{\sigma\} = [C][B]\{Q\} - [C]\{\epsilon_0\} . \tag{2.5}$$

where $\{\sigma\} = \{\sigma_x, \sigma_y, \sigma_z; \tau_{xy}, \tau_{xz}, \tau_{yz}\}$ is a column matrix of stresses, and $[C]$ is a square matrix of elasticity coefficients.

For virtual nodal displacements $\delta\{Q\}$, the corresponding virtual displacements at any point within the element are obtained from Equation (2.1).

$$\delta\{q\} = [A]\,\delta\{Q\} \tag{2.6}$$

and from Equation (2.2) the virtual strains become

$$\delta\{\epsilon\} = \delta\{e\} = [B]\,\delta\{Q\} . \tag{2.7}$$

The governing finite element equations for a unit length of the structure will be developed from a consideration of the principle of virtual work which takes the form:

$$\delta U = \delta W \tag{2.8}$$

where

δU = virtual strain-energy;

δW = virtual work of all external forces.

2.3 Virtual Strain Energy

The strain energy may be expressed as

$$U = \int_V U_0 \, dV \tag{2.9}$$

where U_0 = strain energy density, and V = volume.

If now, virtual displacements are imposed on the structure, there will be accompanying virtual strains, and the variation of the strain energy density will be given by:

$$\delta U_0 = \delta \{\epsilon\}^t \{\sigma\} . \tag{2.10}$$

Consequently, the variation of the strain energy will be

$$\delta U = \int_V \delta U_0 \, dV = \int_V \delta \{\epsilon\}^t \{\sigma\} \, dV . \tag{2.11}$$

Substitution of Equations (2.5) and (2.7) into Equation (2.11) yields

$$\delta U = \int_V \delta \{Q\}^t \left([B]^t [C] [B] \right) \{Q\} \, dV - \int_V \delta \{Q\} [B]^t [C] \{\epsilon_0\} \, dV . \tag{2.12}$$

2.4 Virtual Work of External Forces

The structure is assumed to be subjected to a system of surface forces $\{T\}$, body forces $\{X\}$, and a set of concentrated forces $\{P\}$

applied at the nodes interconnecting the boundaries of the elements. The virtual work of external forces, due to virtual displacements at the nodes, is then given by:

$$\delta W = \int_V \delta\{q\}^t \{X\}\, dV + \int_s \delta\{q\}^t \{T\}\, ds + \delta\{Q\}^t\{P\}, \qquad (2.13)$$

where s = surface area, and

$$\{X\} = \{X_x, X_y, X_z\} ;$$

$$\{T\} = \{T_x, T_y, T_z\} ;$$

$$\{P\} = \{P_x, P_y, P_z\} .$$

Substitution of Equation (2.6) into (2.13) yields

$$\delta W = \int_V \delta\{Q\}^t [A]^t \{X\}\, dV + \int_s \delta\{Q\}^t [A]^t \{T\}\, ds + \delta\{Q\}^t \{P\}. \qquad (2.14)$$

2.5 The Governing Equation

Equating the Equations (2.12) and (2.14) as required by Equation (2.8) results in

$$\int_V \delta\{Q\}^t [B]^t [C] [B] \{Q\}\, dV - \int_V \delta\{Q\}^t [B]^t [C] \{\epsilon_0\}$$
$$= \int_V \delta\{Q\}^t [A]^t \{X\} + \int_s \delta\{Q\}^t [A]^t \{T\}\, ds + \delta\{Q\}^t \{P\} \qquad (2.15)$$

This equation is valid for any value of virtual displacements. Whence

$$[k]\{Q\} = \{P\} + \int_V [B]^t [C] \{\epsilon_0\}\, dV + \int_V [A]^t \{X\}\, dV$$
$$+ \int_s [A]^t \{T\}\, ds \qquad (2.16)$$

where

$$[k] = \int_V [B]^t [C][B] = \text{element stiffness matrix.}$$

Addition of these elemental matrices yields the stiffness matrix for the entire structure. Equation (2.16) then becomes

$$[K]\{Q\} = \{P\} + \int_V [B]^t [C]\{\varepsilon_0\} dV + \int_V [A]^t \{X\} dV + \int_S [A]^t \{T\} ds \qquad (2.17)$$

where $[K]$ = structure stiffness matrix.

2.6 Displacement Function

The structural element considered is a quadrilateral element of unit thickness. A convenient way of calculating its stiffness properties is one proposed by Turner et al (7). The method consists of subdividing the quadrilateral into four triangular elements and combining the stiffnesses of the triangles to obtain that of the quadrilateral.

A typical triangular element subjected to inplane loads is shown in Figure 9. The displacements within the element can be expressed by an assumed displacement function as

$$\{q\} = [m(x, y, z)]\{\alpha\} \qquad (2.18)$$

where $[m(x, y, z)]$ is a rectangular matrix of displacement functions and $\{\alpha\}$ is a column matrix of generalized coordinates representing the amplitudes of the displacement functions.

The number of generalized coordinates chosen must be at least equal to the number of independent nodal point displacement components. Since a triangular element has six degrees of freedom of nodal displacements, the displacements at any point P(x, y) within the element will be taken as

$$\left.\begin{aligned} u_x &= \alpha_1 + \alpha_2 x + \alpha_3 y \\ u_y &= \alpha_4 + \alpha_5 x + \alpha_6 y \end{aligned}\right\} , \qquad (2.19)$$

expressed in standard matrix form, Equation (2.17) becomes

$$\{q\} = \left\{\begin{array}{c} u_x \\ \hline u_y \end{array}\right\} = \left[\begin{array}{c|c|c|c|c|c} 1 & x & y & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & x & y \end{array}\right] \left\{\begin{array}{c} \alpha_1 \\ \hline \alpha_2 \\ \hline \alpha_3 \\ \hline \alpha_4 \\ \hline \alpha_5 \\ \hline \alpha_6 \end{array}\right\} . \qquad (2.20a)$$

Symbolically,

$$\{q\} = [m] \{\alpha\} . \qquad (2.20b)$$

The six constants $\alpha_1 \ldots \alpha_6$ can be obtained from a consideration of the displacements at the nodes, given by:

$$\{Q\} = \left\{\begin{array}{c} u_1 \\ \hline u_2 \\ \hline u_3 \\ \hline u_4 \\ \hline u_5 \\ \hline u_6 \end{array}\right\} = \left[\begin{array}{c|c|c|c|c|c} 1 & x_1 & y_1 & & & \\ \hline & & & 1 & x_1 & y_1 \\ \hline 1 & x_2 & y_2 & & & \\ \hline & & & 1 & x_2 & y_2 \\ \hline 1 & x_3 & y_3 & & & \\ \hline & & & 1 & x_3 & y_3 \end{array}\right] \left\{\begin{array}{c} \alpha_1 \\ \hline \alpha_2 \\ \hline \alpha_3 \\ \hline \alpha_4 \\ \hline \alpha_5 \\ \hline \alpha_6 \end{array}\right\} \qquad (2.21a)$$

Symbolically,

$$\{Q\} = [R] \{\alpha\} \qquad (2.21b)$$

$\{\alpha\}$ is now calculated in terms of $\{Q\}$ and substituted into Equation (2.20) to yield:

$$
\begin{aligned}
\{q\} &= [m][R]^{-1}\{Q\} \\
&= [A]\{Q\} \\
&= \left[\begin{array}{c|c|c|c|c|c} d_1 & 0 & d_2 & 0 & d_3 & 0 \\ \hline 0 & d_1 & 0 & d_2 & 0 & d_3 \end{array}\right]\{Q\}
\end{aligned}
\qquad (2.22a)
$$

where

$$
\begin{aligned}
d_1 &= a_1 + b_1 x + c_1 y \\
d_2 &= a_2 + b_2 x + c_2 y \\
d_3 &= a_3 + b_3 x + c_3 y
\end{aligned}
\qquad (2.22b)
$$

and

$$
\begin{aligned}
a_1 &= \frac{1}{2\Delta}(x_2 y_3 - x_3 y_2) \\
b_1 &= \frac{1}{2\Delta}(y_2 - y_3) = y_{23}/2\Delta \\
c_1 &= \frac{1}{2\Delta}(x_3 - x_2) = y_{32}/2\Delta \quad .
\end{aligned}
\qquad (2.22c)
$$

a_2, b_2, c_2 and a_3, b_3, c_3 are obtained by a cyclic permutation of the subscripts in the order 1, 2, 3.

$$
2\Delta = \mathrm{DET}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = 2(\text{Area of triangle } ①, ②, ③).
$$

2.7 The Stiffness Matrix $[K]$

Equation (2.22a) is now differentiated to give the total strains at any point within the element

$$\{e\} = \begin{Bmatrix} e_x \\ \hline e_y \\ \hline \gamma_{xy} \end{Bmatrix} \begin{Bmatrix} \dfrac{\partial u_x}{\partial_x} \\ \hline \dfrac{\partial u_y}{\partial_y} \\ \hline \dfrac{\partial u_x}{\partial_y} + \dfrac{\partial u_y}{\partial_x} \end{Bmatrix} = \left[\begin{array}{c|c|c|c|c|c} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ \hline 0 & c_1 & 0 & c_2 & 0 & c_3 \\ \hline c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{array}\right] \begin{Bmatrix} u_1 \\ \hline u_2 \\ \hline u_3 \\ \hline u_4 \\ \hline u_5 \\ \hline u_6 \end{Bmatrix} \tag{2.23}$$

Symbolically,

$$\{e\} = [B] \{Q\}$$

The stiffness matrix for the triangular element can now be obtained from the relationship

$$[k] = \iiint [B]^t [C] [B] \, dx\,dy\,dz \ .$$

Since the element being considered is of unit width, the above reduces to

$$[k] = \iint [B]^t [C] [B] \, dx\,dy \ , \tag{2.24}$$

the integration being carried over the area of the triangle. The matrices $[B]$ and $[C]$ are independent of x and y and are, therefore, taken out of the integral sign and the resulting integral becomes the volume (vol) of the triangle.

Hence

$$[k] = [B]^t [C] [B] \cdot (\text{vol}) \ . \tag{2.25}$$

To obtain the stiffness matrix for the quadrilateral element, consider the quadrilateral element as built up from four triangular elements as shown in Figure 10. The stiffness matrices $[k]$ of the triangles are of order 6 x 6. By adding appropriate rows and columns of zeros, the order of these matrices is increased to 10 x 10. It is now possible to superimpose these stiffness matrices to obtain for the quadrilateral a relationship of the form:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_8 \\ \hline P_9 \\ P_{10} \end{Bmatrix} = \left[\begin{array}{ccccc|cc} a_{11} & a_{12} & \cdots & & a_{1,8} & a_{1,9} & a_{1,10} \\ a_{21} & a_{22} & \cdots & & a_{2,8} & a_{2,9} & a_{2,10} \\ \vdots & & & & & & \\ & & & & a_{8,8} & a_{8,9} & a_{8,10} \\ \hline a_{9,1} & & & & & a_{9,9} & a_{9,10} \\ a_{10,1} & & & & & a_{10,9} & a_{10,10} \end{array}\right] \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_8 \\ \hline u_9 \\ u_{10} \end{Bmatrix} \tag{2.26}$$

$$\left\{P''\right\} = \begin{Bmatrix} P \\ \hline P' \end{Bmatrix} = \left[\begin{array}{c|c} k_{11} & k_{12} \\ \hline k_{21} & k_{22} \end{array}\right] \begin{Bmatrix} Q \\ \hline Q' \end{Bmatrix}$$

where $\left\{P''\right\}$ is the column matrix of nodal forces. Since the forces applied to the quadrilateral element are acting at the nodes ①, ②, ③, and ④, it is necessary to impose the conditions $P_9 = 0$ and $P_{10} = 0$; i.e.,

$$\left\{P'\right\} = \begin{Bmatrix} 0 \\ \hline 0 \end{Bmatrix} .$$

Hence,

$$\{0\} = [k_{21}]\{Q\} + [k_{22}]\{Q'\}$$

which gives

$$\{Q'\} = -[k_{22}]^{-1}[k_{21}]\{Q\} , \tag{2.27}$$

$$\{P\} = \left([k_{11}] - [k_{12}][k_{22}]^{-1}[k_{21}]\right)\{Q\} ,$$

i. e.,

$$\{P\} = [k]\{Q\} . \tag{2.28}$$

The matrix $[k]$ in Equation (2.28) is the required stiffness matrix of the quadrilateral element.

A more convenient way of obtaining the $[k]$ matrix without having to invert any matrix is as follows. The rows (and columns) corresponding to $P_9 = 0$ and $P_{10} = 0$ are eliminated one at a time. Thus, for the 10th row:

$$0 = \sum_{j=1}^{9} a_{10,j} \cdot u_j + a_{10,10} \cdot u_{10} ,$$

from which

$$u_{10,10} = -\frac{1}{a_{10,10}} \sum_{j=1}^{9} a_{10,j} \cdot u_j ,$$

and

$$P_i = \sum_{j=1}^{9} \left(a_{ij} - \frac{a_{i,10} \cdot a_{10,j}}{a_{10,10}}\right) \cdot u_j \qquad (i = 1, 2 . . . 9),$$

$$= \sum_{j=1}^{9} a'_{ij} \cdot u_j .$$

The above procedure can be repeated to obtain

$$P_i = \sum_{j=1}^{8} \left(a'_{ij} - \frac{a'_{i,9}\, a'_{9,j}}{a'_{9,9}} \right) \cdot u_j \qquad (i = 1, 2 \ldots 8),$$

$$= \sum_{j=1}^{8} k_{ij} \cdot u_j \ . \tag{2.29}$$

The matrix $\left[k_{ij}\right]_{8\times 8}$ in Equation (2. 29) is the required stiffness matrix for the quadrilateral element in terms of the four external nodes. The proper matrix addition of all the elemental stiffnesses in accordance with the nodal numbering of each element produces the overall structural matrix $\left[K\right]$ shown in Equation (2.17).

CHAPTER III

STRUCTURAL IDEALIZATION

3.1 Description of Model

A uniform over-pressure of 100 psi was assumed to act on the surface of the ground and the investigation dealt with the following cases:

1. Stress distribution in the undisturbed medium.
2. Stress distribution around an unlined tunnel cavity.
3. Stress distribution around a tunnel with liner-packing system.
4. Stress distribution around a tunnel with a cracked liner (one or three cracks).

The tunnel cavity was considered at a depth of 5D below the ground surface, where D is the diameter of the tunnel cavity. It was assumed that (due to St. Venant's theory) the tunnel would not cause appreciable change on the stress distribution at distances of 2D or more from the tunnel cavity. The investigations for the lined and unlined tunnels were, therefore, based on the models shown in Figures 7 and 8, respectively. Due to symmetry of structure and loading, only one-half of the model is shown. In each case, the vertical boundary was considered fixed in the X-direction but free to displace in the Y-direction, and the horizontal boundary at the bottom was considered fixed in the Y-direction but free to displace in the X-direction.

3.2 Finite Element Idealization

The finite element idealization of the structure was carried out on the model. For each case, the model was divided by a number of radial lines and concentric circles. In both the lined and unlined tunnel studies, the radii of the circles, in the surrounding medium, were chosen with a weighting factor so as to obtain, wherever possible, a mesh of well proportioned quadrilateral elements. Due to the storage limitations of the computer, the maximum number of radial lines, for the lined tunnel, was kept at 49 which gave rise to a maximum of 945 nodes and 914 elements. Starting with the first line and going clockwise, every other line was extended beyond the tunnel cavity to end at the boundary of the model (see Figures 7 and 8). A systematic order was then followed in numbering the nodal points and elements, with the restriction that the maximum difference between the numbers of any two nodes in any one element be less than a predetermined number, 55 in this case. This, in essence, subdivided the structure into partitions containing a maximum of 55 nodes.

The unlined tunnel required only 25 radial lines starting at the cavity and ending on the boundary of the model (see Figure 8). Also shown in Figure 8 is the system followed in numbering the nodes and the elements.

The sizes of the elements in the surrounding medium grow progressively larger due to the weighting factor used. In the case of the unlined cavity, the elements in the surrounding medium adjacent to the cavity are quadrilateral, while in the lined tunnel these elements are triangular and serve as a transition zone from the smaller elements in the packing to the larger elements in the medium.

CHAPTER IV

THE COMPUTER PROGRAM

4.1 Description

The nodal displacements were determined by solving Equation (2.17) on an IBM computer system 360/50. The main program is made sufficiently flexible to handle two dimensional plane strain or plane stress problems with a maximum of 950 nodes and 930 elements.

Referring to the program flow chart given in Figure 11, the elastic constants of materials used were first read in as data, followed by the automatic generation and storage of the nodal coordinates in the global system. For the lined tunnel, the generation of the nodal coordinates was carried out in two stages: the nodes in the liner-packing system first, followed by the nodes in the surrounding medium. In each stage, only the first and last nodal numbers on each radial line were read in as input, after which, the numbering of the intermediate nodes and the corresponding coordinates, together with information as to whether load or displacement is prescribed and its magnitude, were generated automatically. For nodes not on radial lines, the coordinates and other relevant information were read in from punched cards.

The generation of the elements, material properties, and the associated nodal numbers was automatic and accomplished in two stages, as mentioned above. Each element, whether quadrilateral or triangular, had four nodal numbers associated with it; in the case of a triangular element, one of the nodal numbers was recorded twice. In a string of quadrilateral elements, only the first and last elements and their four nodal numbers were read in, after which, the intermediate elements with their four nodes and material properties were generated. For triangular elements, all the pertinent data had to be provided as input. All the input or generated data were then printed out for checking. A listing of typical input data is shown in Appendix C.

Next in order of execution was the calculation of the element stiffnesses. Each quadrilateral element is assumed to be made up of four triangles by introducing an extra node in the middle of the quadrilateral. The coordinates of the auxiliary node are the average of the other four nodal coordinates. The stiffness matrix obtained by adding the stiffnesses of the four triangles is of order 10 x 10. This is reduced to an 8 x 8 matrix by eliminating the extra node introduced to yield the required element stiffness matrix for the quadrilateral. These element stiffnesses are then added in accordance with the nodal numbering to obtain the structure stiffness matrix. The structure load column matrix is formed next by adding all the element nodal loads according to the nodal numbering. Before going on with the calculation of displacements, the structure stiffness matrix and load vector were revised to account for the prescribed displacements. The resulting simultaneous equations were solved for the other displacements using the Gaussian elimination method for banded matrices. Finally, the displacements of each node were printed.

The average global stresses in each quadrilateral element were calculated from the average of the strains of the four triangles making up the quadrilateral element. In order to do this, the element stiffness matrix had to be regenerated for each element and used to calculate the displacement of the middle node. A 10 x 1 displacement vector was thus produced which made it possible to calculate the average of the strains on the four triangular elements. The global stresses were then determined from Equation (2.5), with $\{\varepsilon_0\}$ set to zero. The global stresses were used to calculate the principal stresses and directions.

The output consisted of the coordinates of the middle node, the global stresses, the principal stresses and directions, for each element in the mesh. A typical computer output is shown in Appendix D.

4.2 Input of Parameters and Elastic Constants

The material properties and other pertinent parameters entered as data are given below.

Concrete Liner		
Modulus of Elasticity	3×10^6	psi
Cylinder Strength (f'_c)	4000	psi
Poisson's Ratio	0.15	
Steel Reinforcement		
Modulus of Elasticity	30×10^6	psi
Poisson's Ratio	0.3	
Packing Material (Cellular Concrete)		
Modulus of Elasticity	1.5×10^4	psi
Poisson's Ratio	0.1	
Surrounding Medium		
Modulus of Elasticity	4.0×10^4	psi
Poisson's Ratio	0.33	

Inner Radius of Liner	36 in.
Outer Radius of Liner	44 in.
Radius of Tunnel Cavity	68 in.

A flow chart showing the logic of computations is shown on page 38 and a program for one of the cases considered is given in Appendix B.

CHAPTER V

DISCUSSION OF RESULTS

5.1 General

As mentioned in Chapter III, the output for each of the cases studied consisted of the stresses in the continuum together with the principal stresses and principal directions for each element of the mesh. The principal stresses were plotted, to a convenient scale, directly on the mesh and from these, contours of principal stress directions and contours of major and minor principal stresses were drawn. In the following few sections, the results obtained for each case will be discussed in turn.

5.2 Stress Distribution in Undisturbed Medium

This case was considered primarily to serve as a check on the computational procedures and the program developed. The finite element mesh was made up of square elements and the output for this case showed that the directions of the principal stresses coincided with the axes of the coordinate system used and that the principal stress in the Y-direction was 100 psi compression, as would be expected. No figures are given for this case as it is self-explanatory.

5.3 Stress Distribution Around an Unlined Tunnel Cavity

Figure 12 shows the relative magnitudes and the directions of principal stresses for this case. Figure 13 shows the direction contours of the principal stresses. In Figures 14 and 15 are given the contours of equal stresses of the major and minor principal stresses, respectively. These figures indicate that at distances far from the tunnel cavity, the minor principal stress is not appreciably different from the applied loading, both in magnitude and direction. The stresses at the crown, side and bottom of the tunnel are about 55 psi, 233 psi and 56 psi, all compression, respectively.

5.4 Stress Distribution Around a Tunnel with a Liner-Packing System

The magnitudes and directions of the principal stresses for this case are as shown in Figure 16. Figure 17 shows the direction contours of the principal stresses, and Figures 18 and 19 show contours of equal stress, of major and minor principal stresses. It is observed, again, that the stresses are all compressive and that the directions and magnitudes of the stresses at sections far from the tunnel boundary do not differ by a great deal from the applied loading of 100 psi. The maximum compressive stresses at the crown, the side, and bottom of the cavity are 77 psi, 130 psi, and 77 psi, respectively. The presence of a liner-packing system has increased the stresses at the crown and bottom of the cavity by about 40%, while the stresses on the side have been reduced by about 45%, compared to those of an unlined tunnel. The compressive stresses in the packing at the crown, the side and the bottom of the cavity are about 80 psi, 42 psi and 77 psi, respectively.

5.5 Stress Distribution Around a Tunnel with a Cracked Liner

There were two studies made of this case; a liner with just one crack at the crown and a liner with three cracks, one at the crown and one on each side of the crown along a radial line making an angle of 30° with the vertical through the crown. The cracks were introduced into the liner by specifying the ultimate moment that would develop at the section when it fails. The moment is produced by a couple whose forces are applied at the nodes of the steel elements. The two cases are discussed separately.

5.5.1 Liner with One Crack

Figure 20 shows the principal stresses and their directions plotted on the mesh. Contours of principal stress directions and contours of equal stresses for the major and minor stresses are shown in Figures 21, 22, and 23. The stresses far from the cavity are about the same as the applied compressive stress of 100 psi. The compressive stresses at the crown, side, and bottom of the cavity are about 124 psi, 128 psi, and 90 psi, respectively. The crack at the crown of the liner has increased the stresses there by about 60%, at the bottom of the cavity by about 15%, but has not affected the side stresses as compared to those of an uncracked liner. The stresses far from the cavity are still nearly equal to the applied loading. Compressive stresses in the packing range from 130 psi to 320 psi for elements at the crown, about 42 psi at the side, and about 100 psi at the bottom.

5.5.2 Liner with Three Cracks

In Figure 24, the magnitude and directions of the principal stresses are given. Figure 25 shows the direction contours, and Figures 26 and 27 show the equal stress contours for the principal major and minor stresses. Far from the cavity, the stresses are nearly the same as the applied loads. Some tensile stresses have developed at the cavity boundary about 25° - 30° from the crown and the maximum tensile stress produced is about 122 psi. The maximum compressive stress in the crown, the sides, and the bottom are 530 psi, 120 psi, and 86 psi. While there are no appreciable increases in the stresses on the side and bottom of the cavity, there seems to be a sharp rise in the stress at the crown. The stresses in the packing at the crown range from 600 psi to 1400 psi, about 60 psi at the side, and about 88 psi at the bottom.

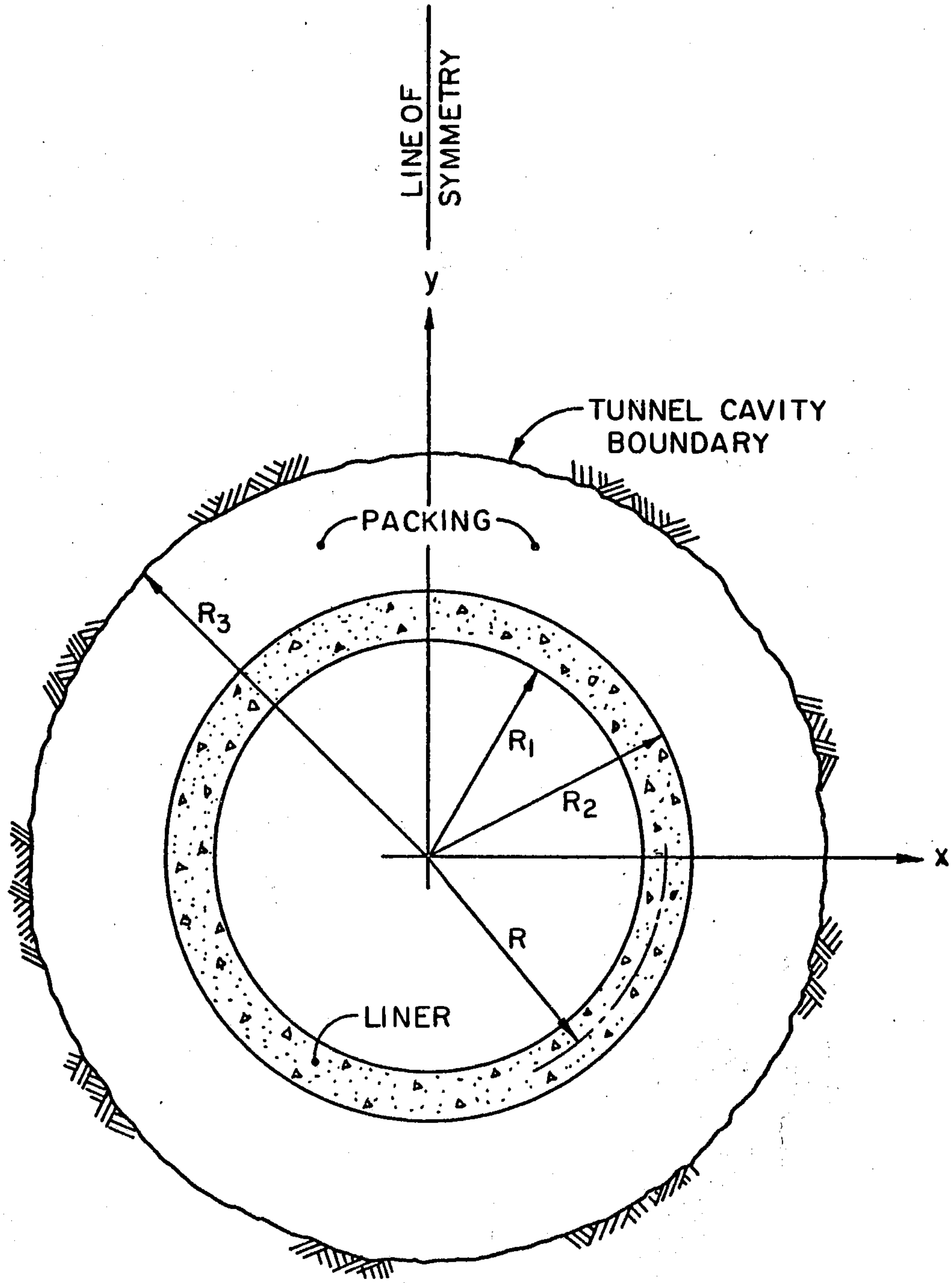

Figure 1. Liner-Packing System

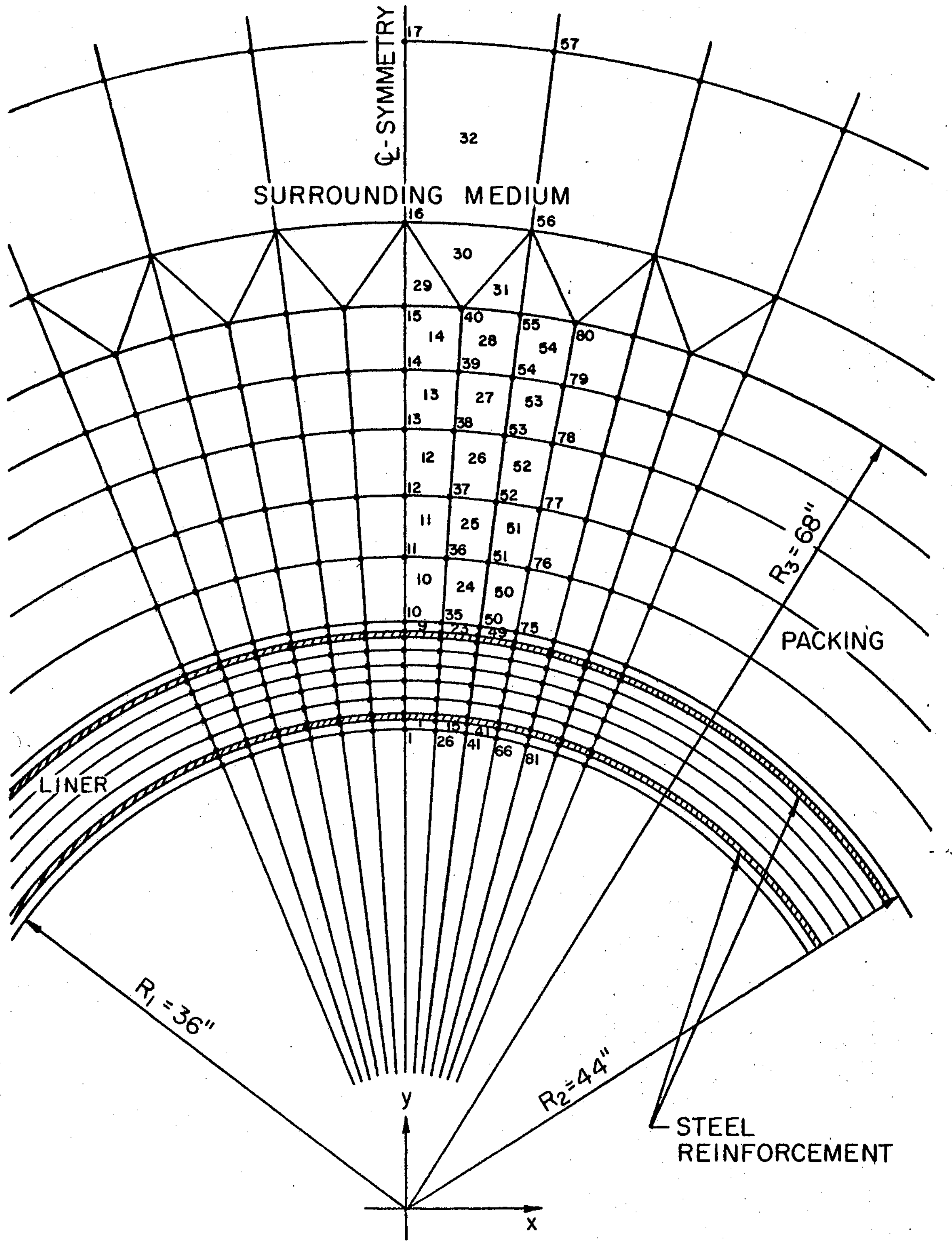

Figure 2. Finite Element Idealization of Liner-Packing System

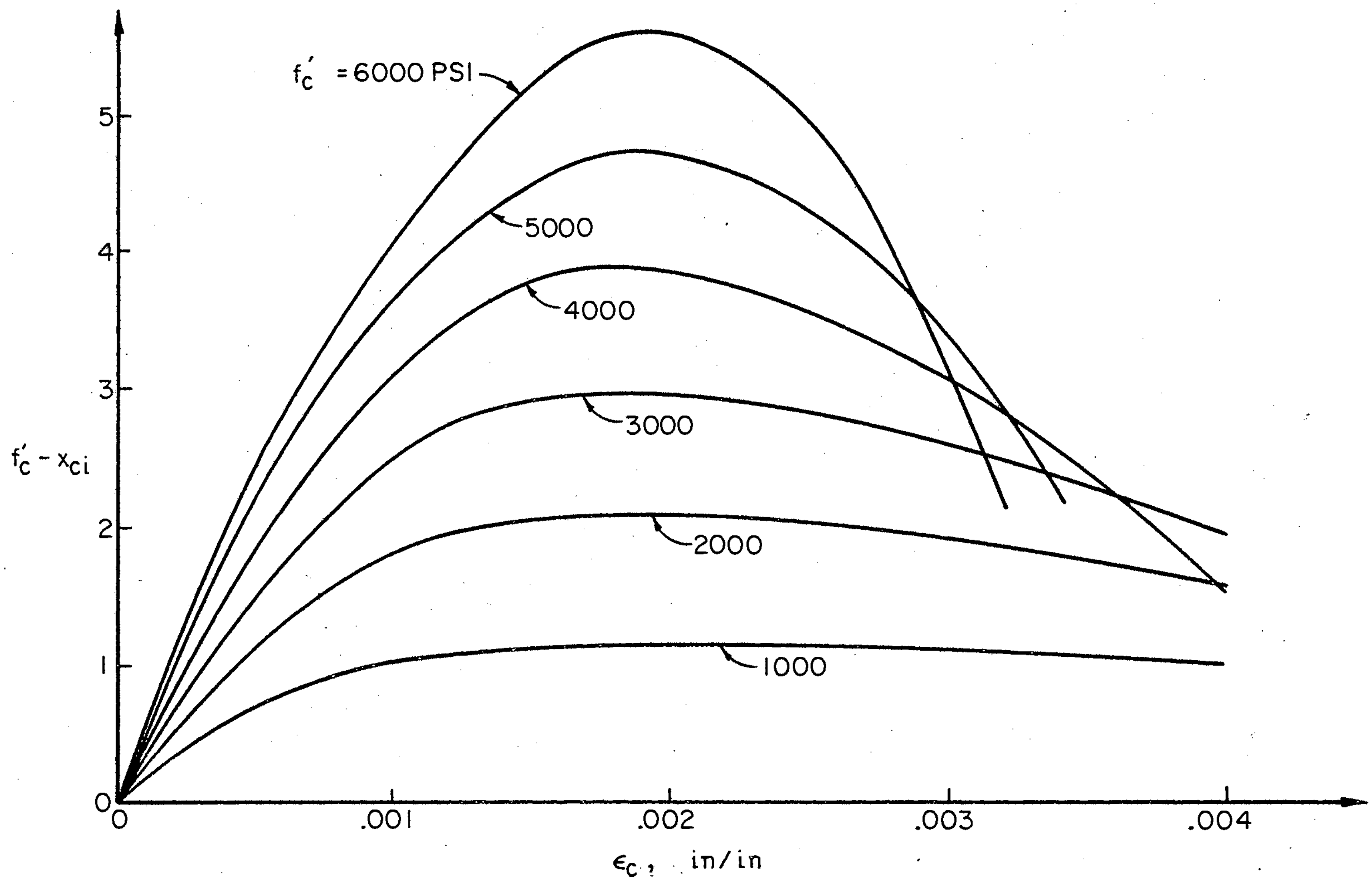

Figure 3. Typical Concrete Stre[illegible]-Strain Curve

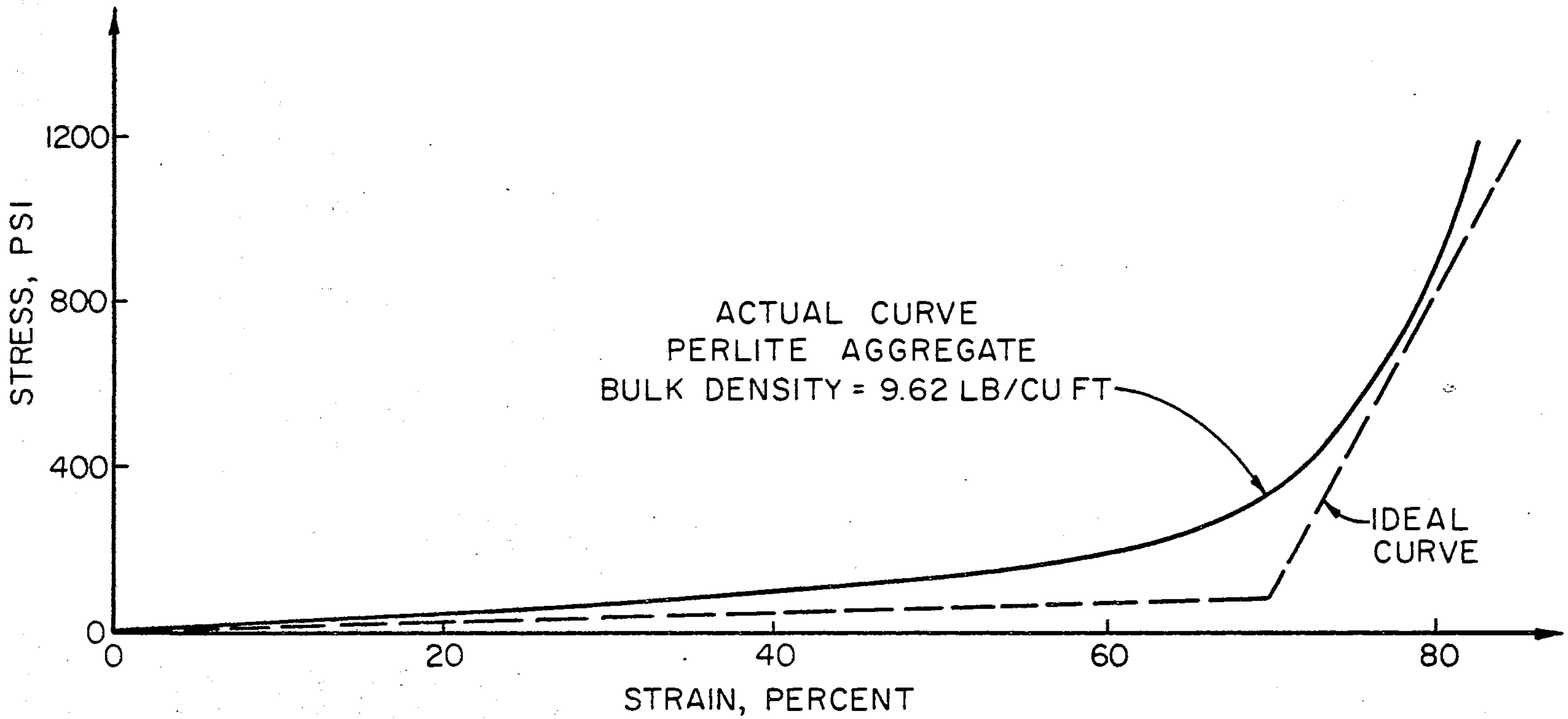

Figure 4. Typical Stress-Strain Curve for a Plasto-Elastic Material

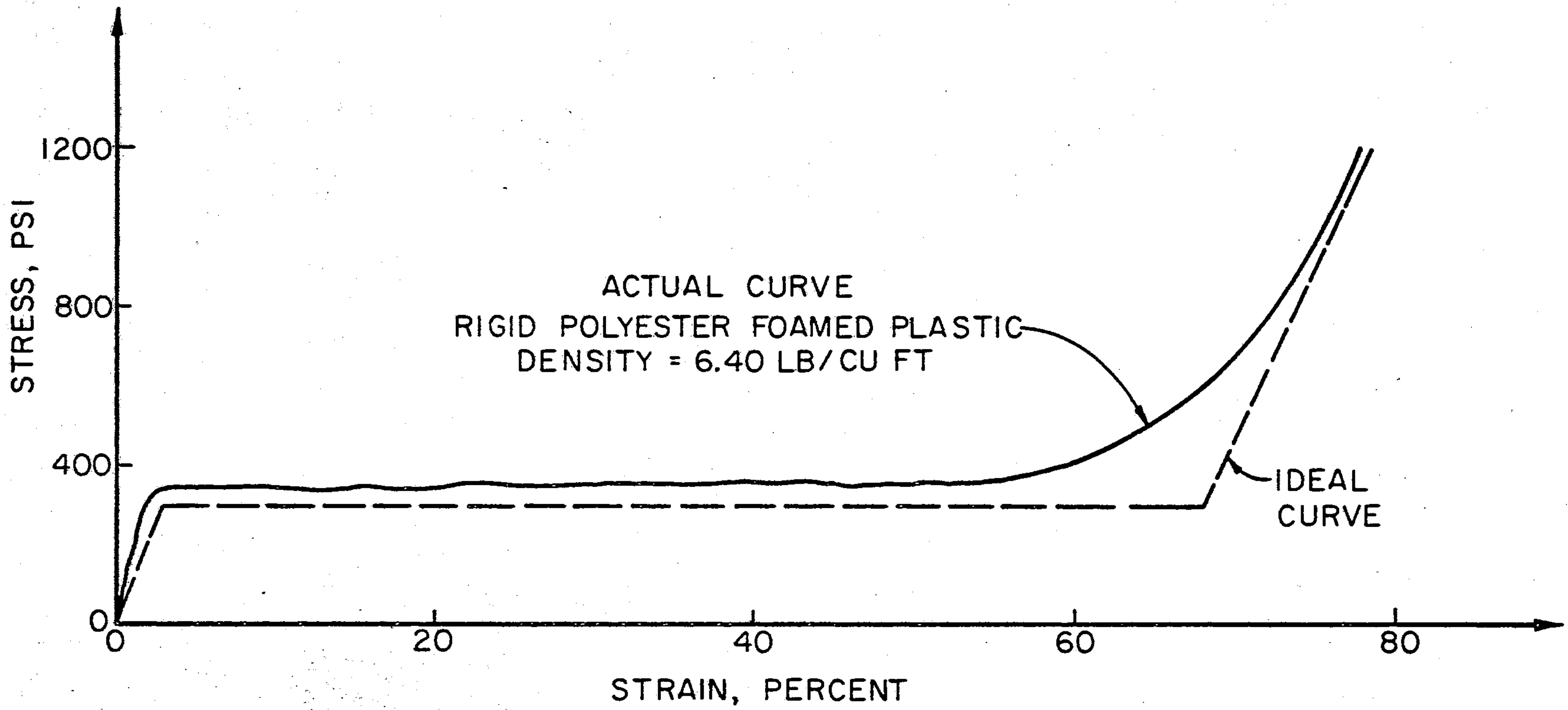

Figure 5. Typical Stress-Strain Curve for an Elasto-Plastic Material

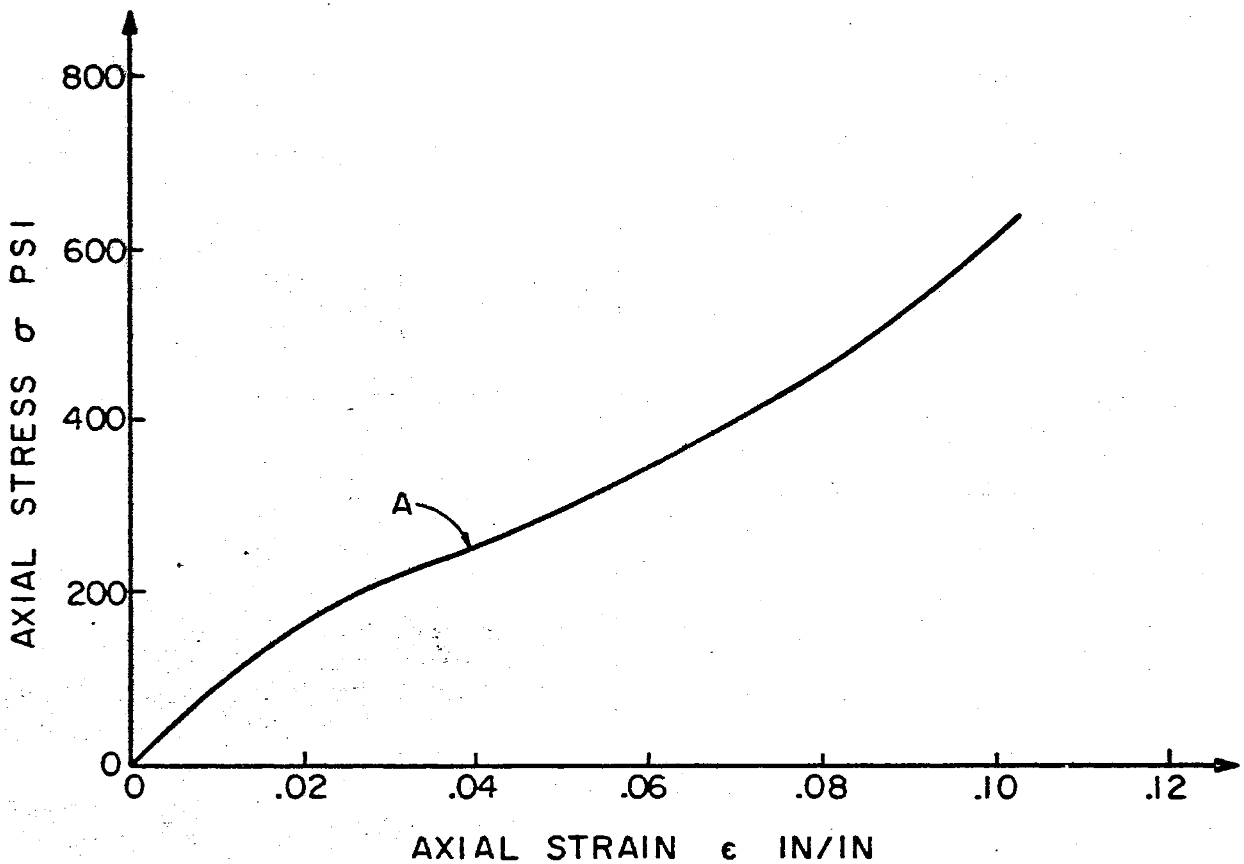

Figure 6. Stress-Strain Curve for Playa Silt in One-Dimensional Compression

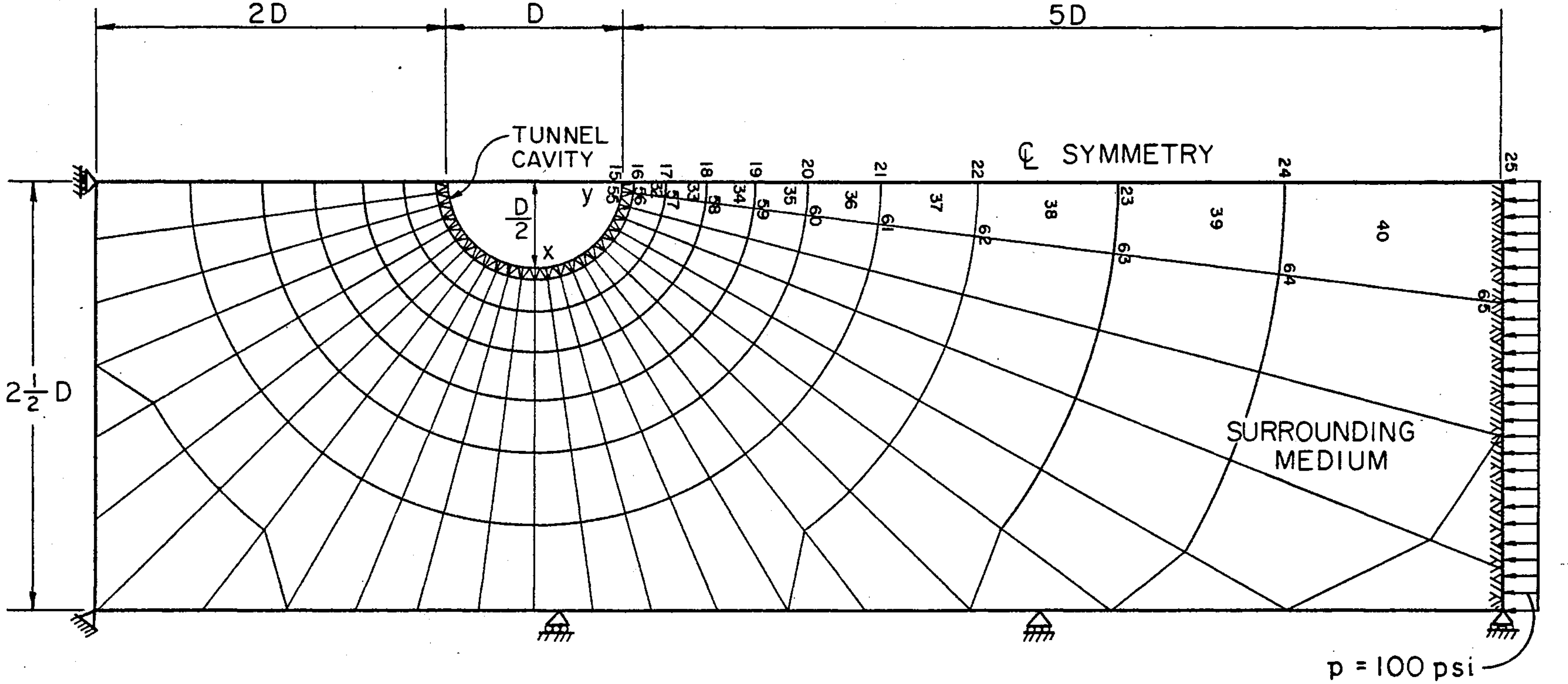

Figure 7. Finite Element Idealization of Surrounding Medium (Tunnel with Liner-Packing System)

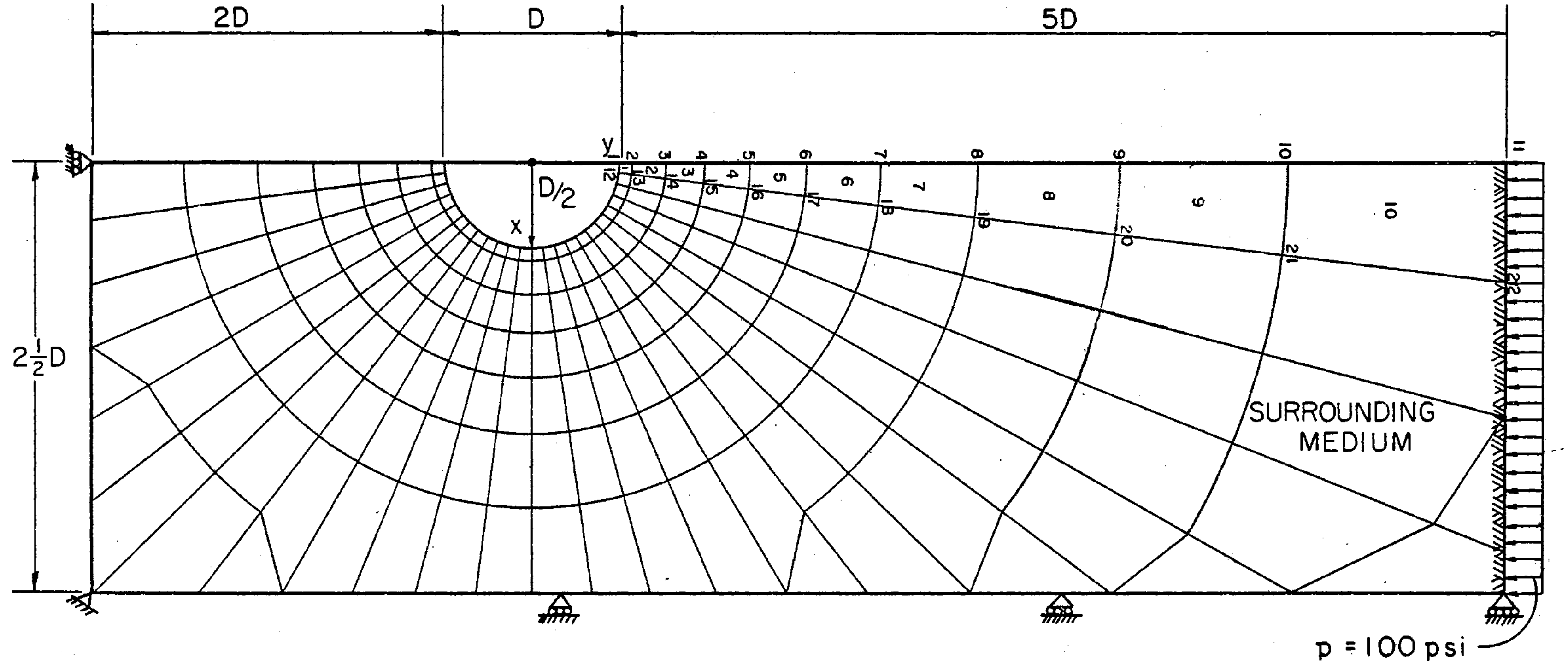

Figure 8. Finite Element Idealization of Surrounding Medium (Tunnel without Liner)

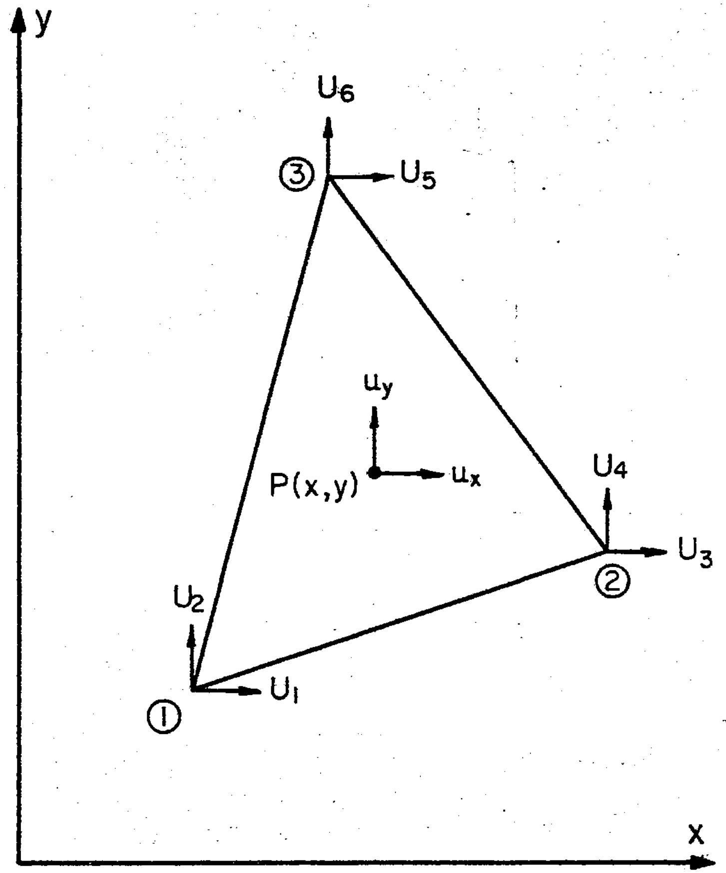

Figure 9. Triangular Element

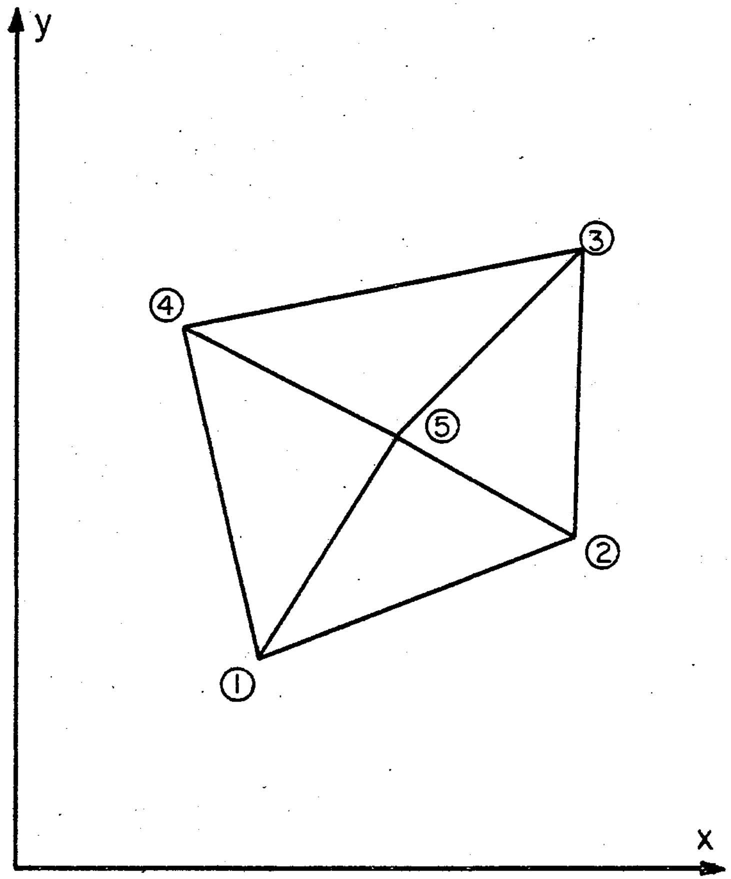

Figure 10. Quadrilateral Element

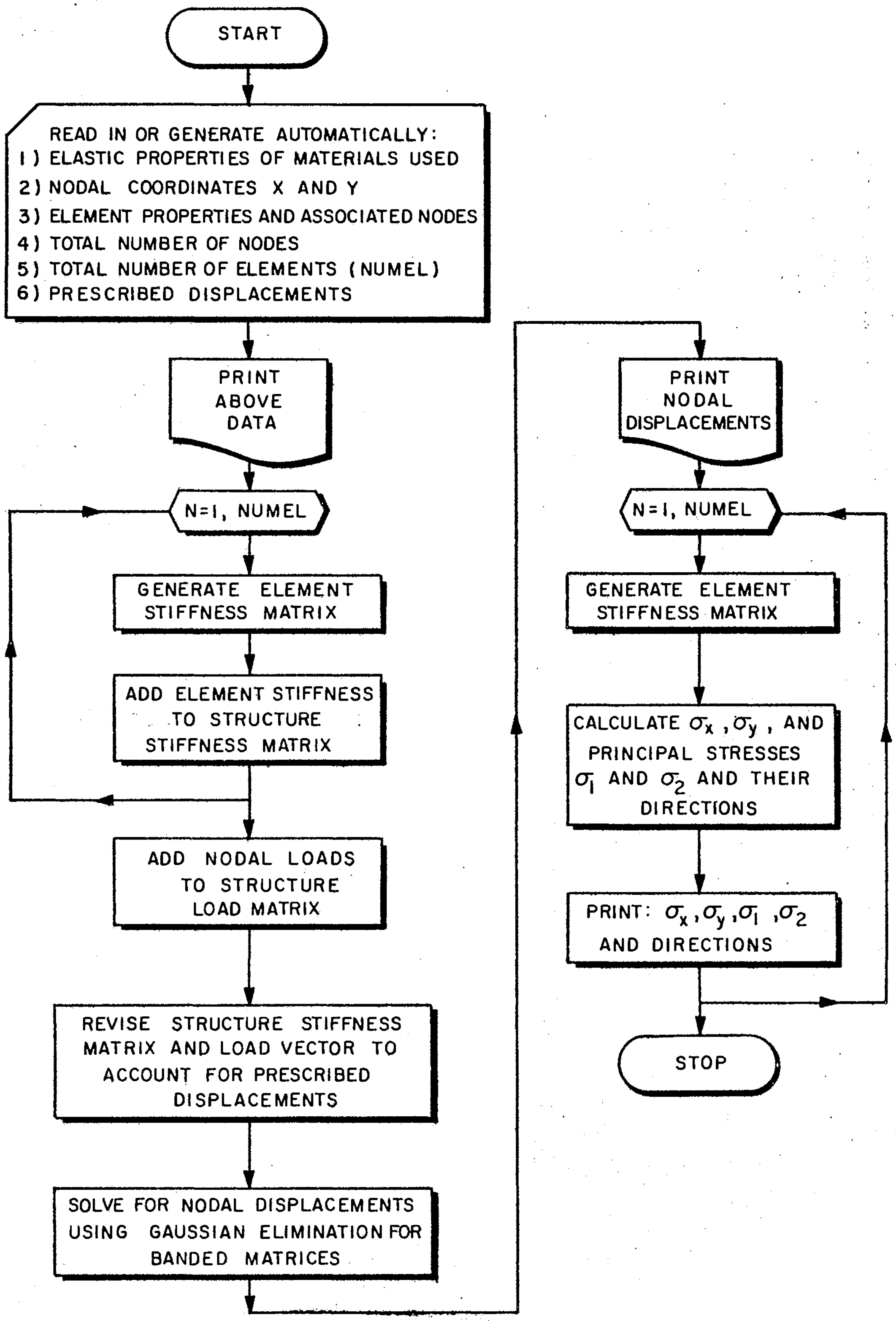

Figure 11. Flow Chart for Computer Program

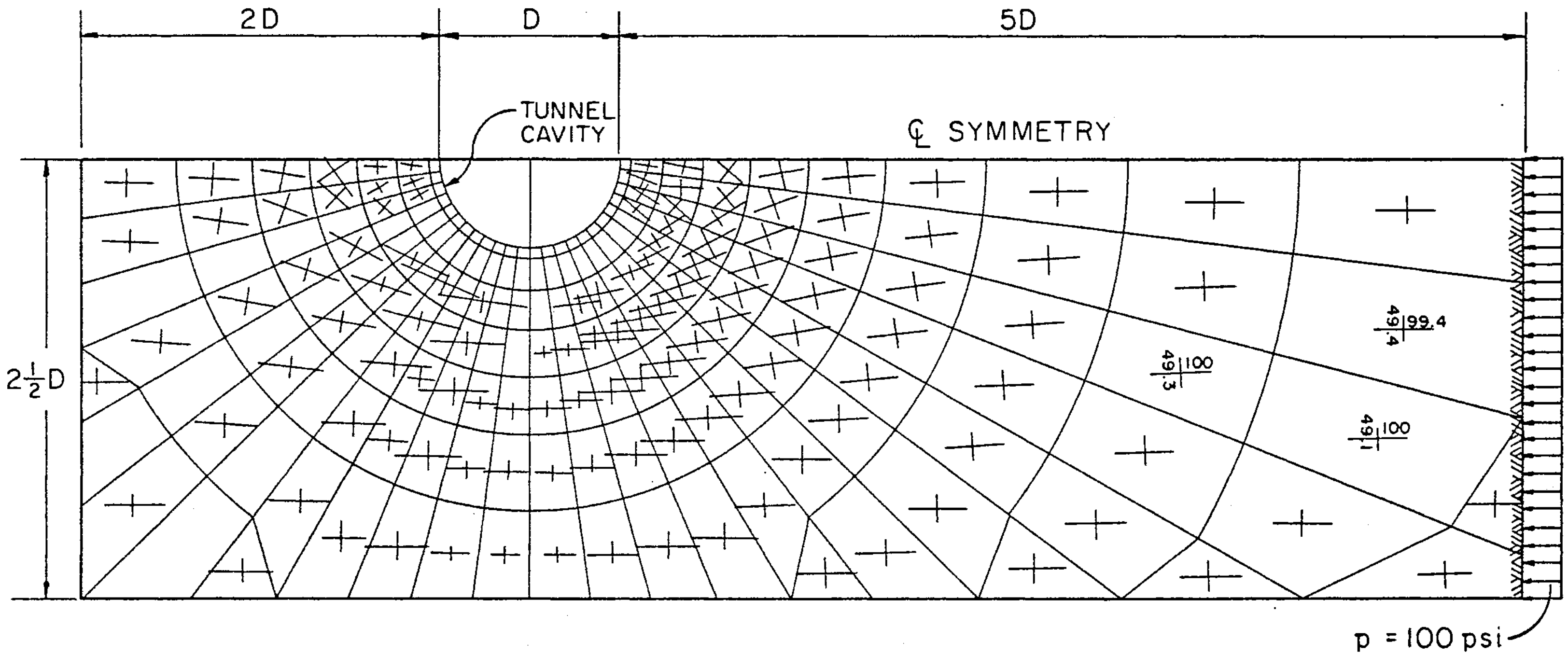

Figure 12. Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel without Liner)

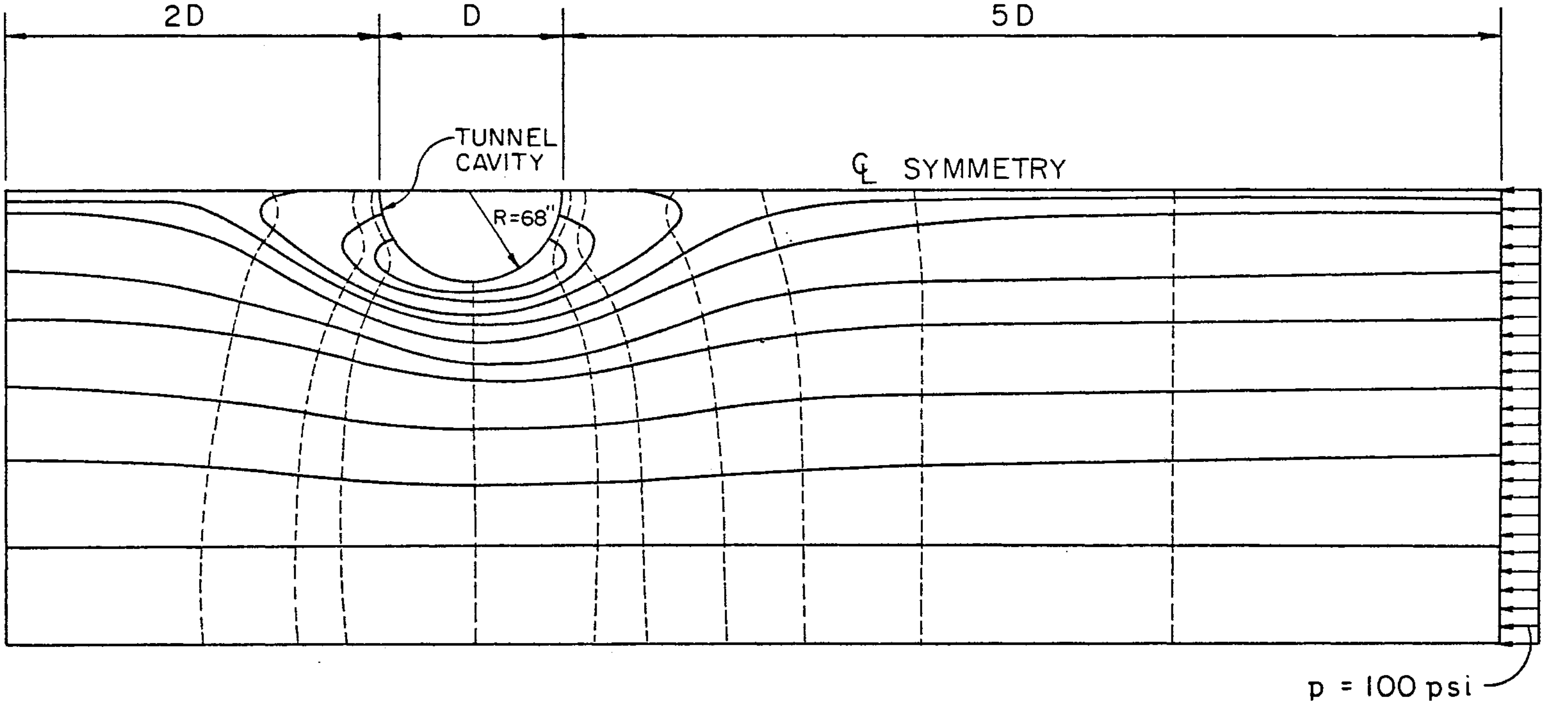

Figure 13. Principal Stress Directions in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel without Liner)

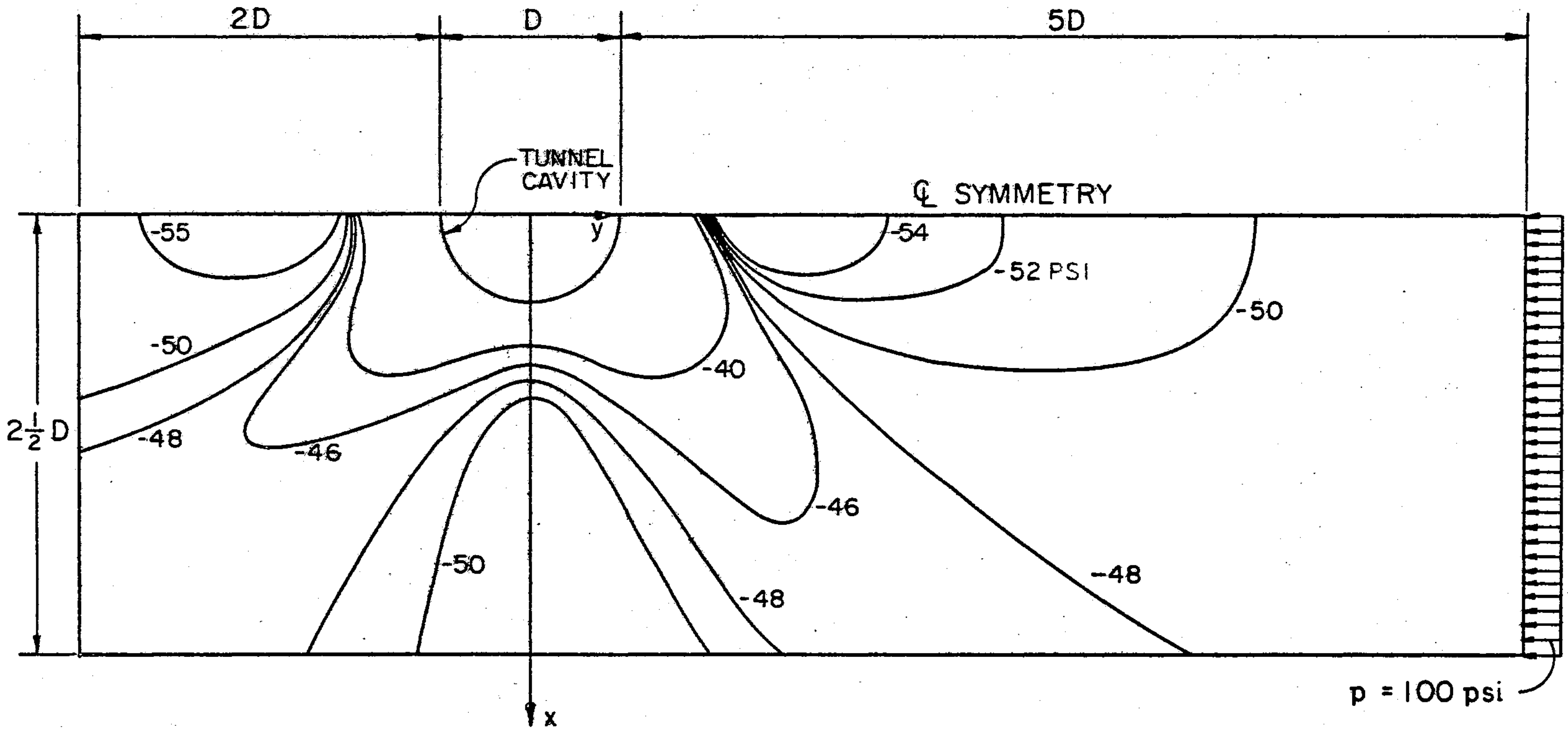

Figure 14. Major Principal Stress in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel without Liner)

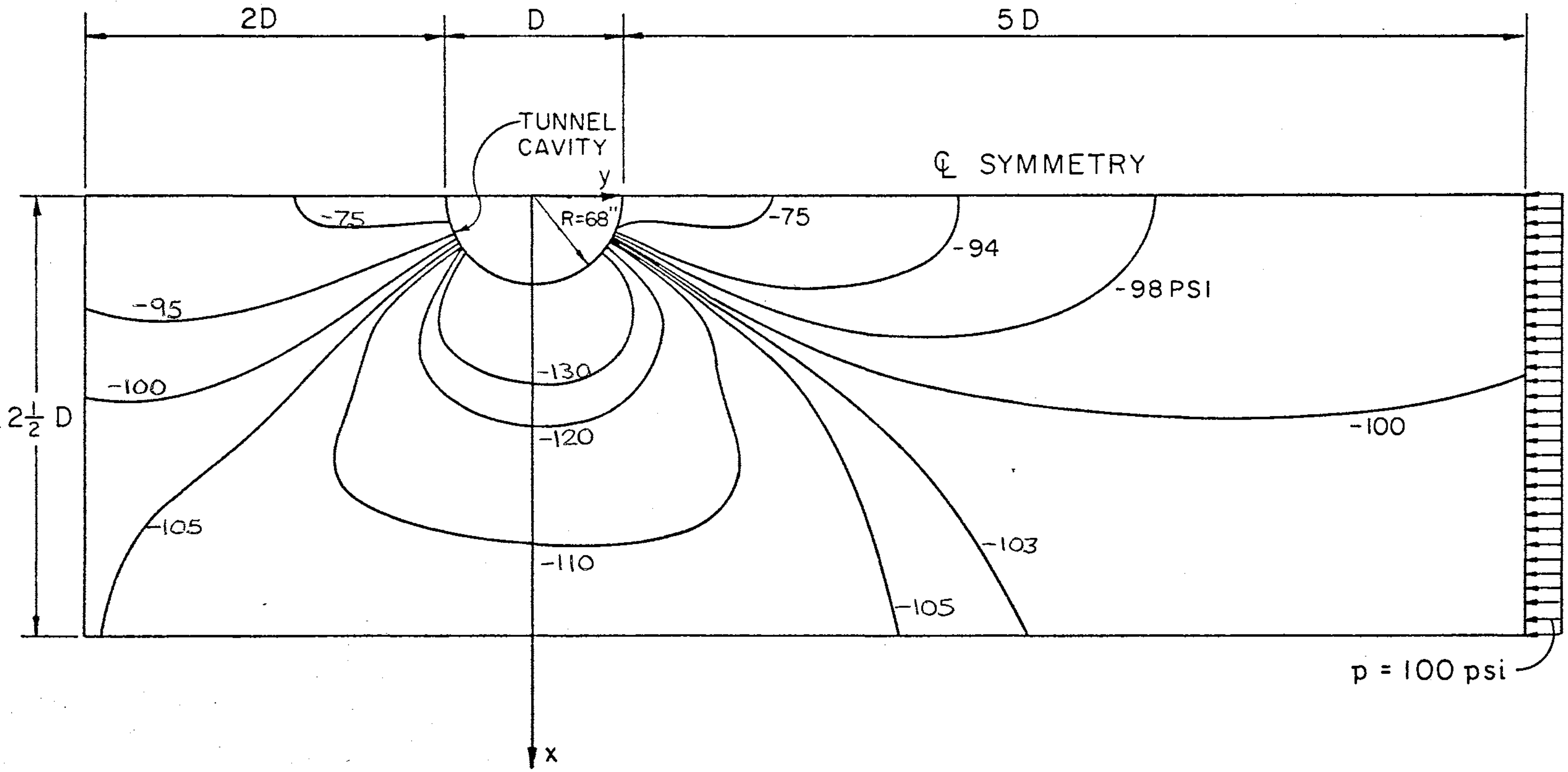

Figure 15. Minor Principal Stress in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel without Liner)

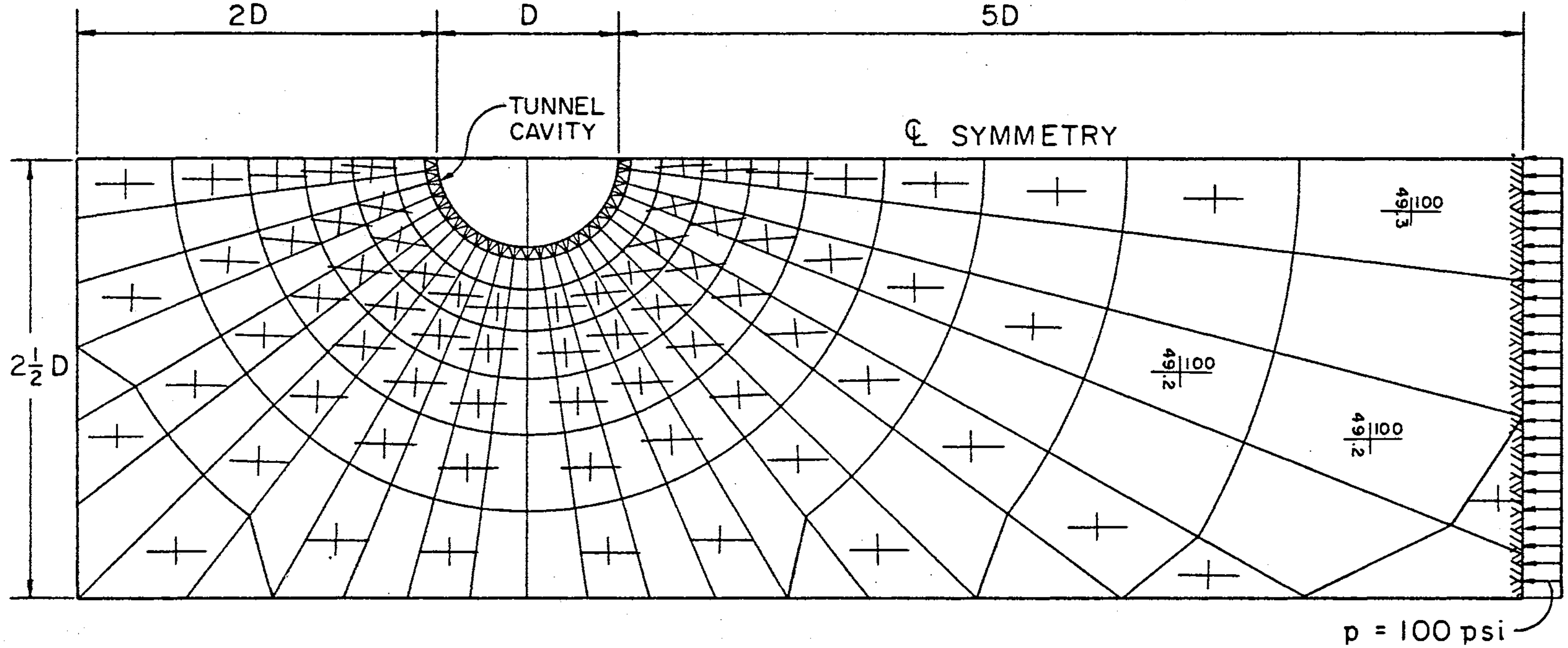

Figure 16. Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Liner-Packing System)

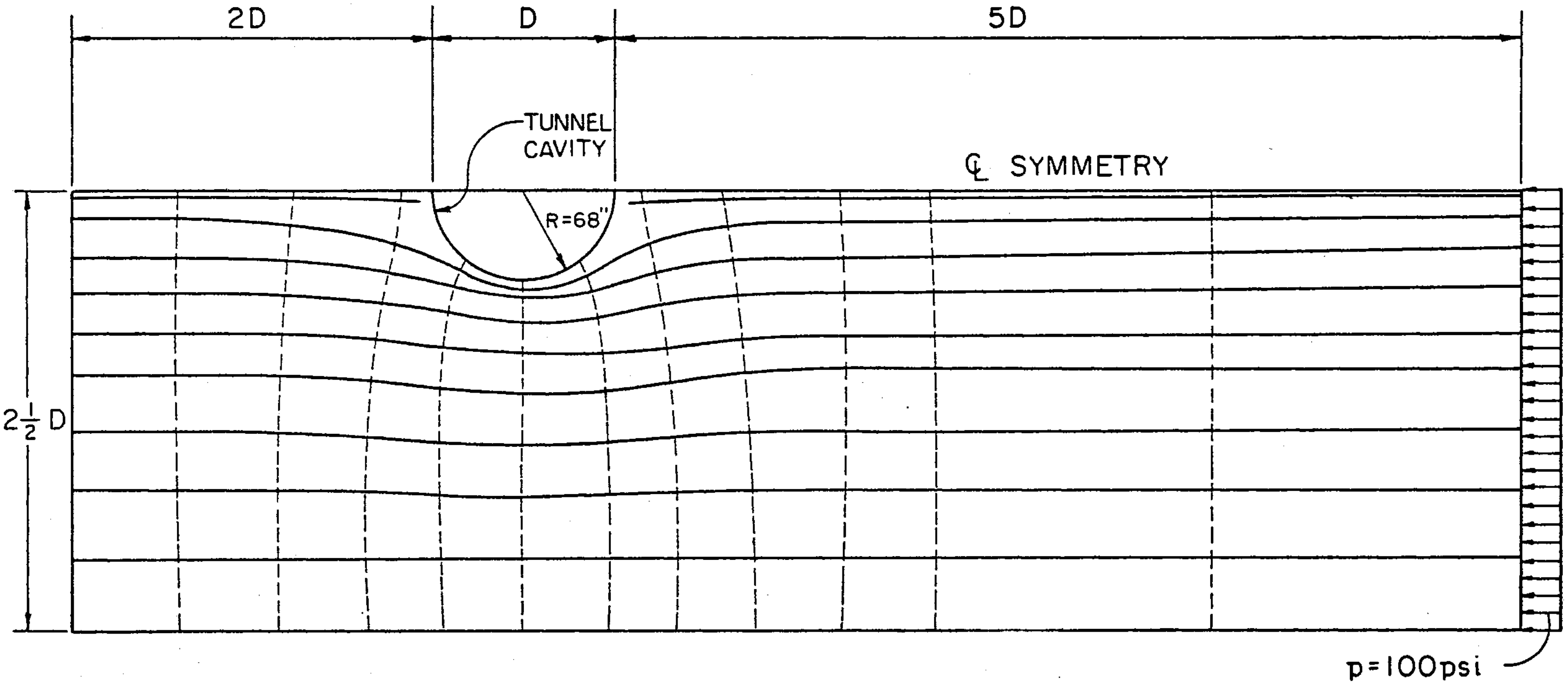

Figure 17. Principal Stress Directions in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Liner-Packing System)

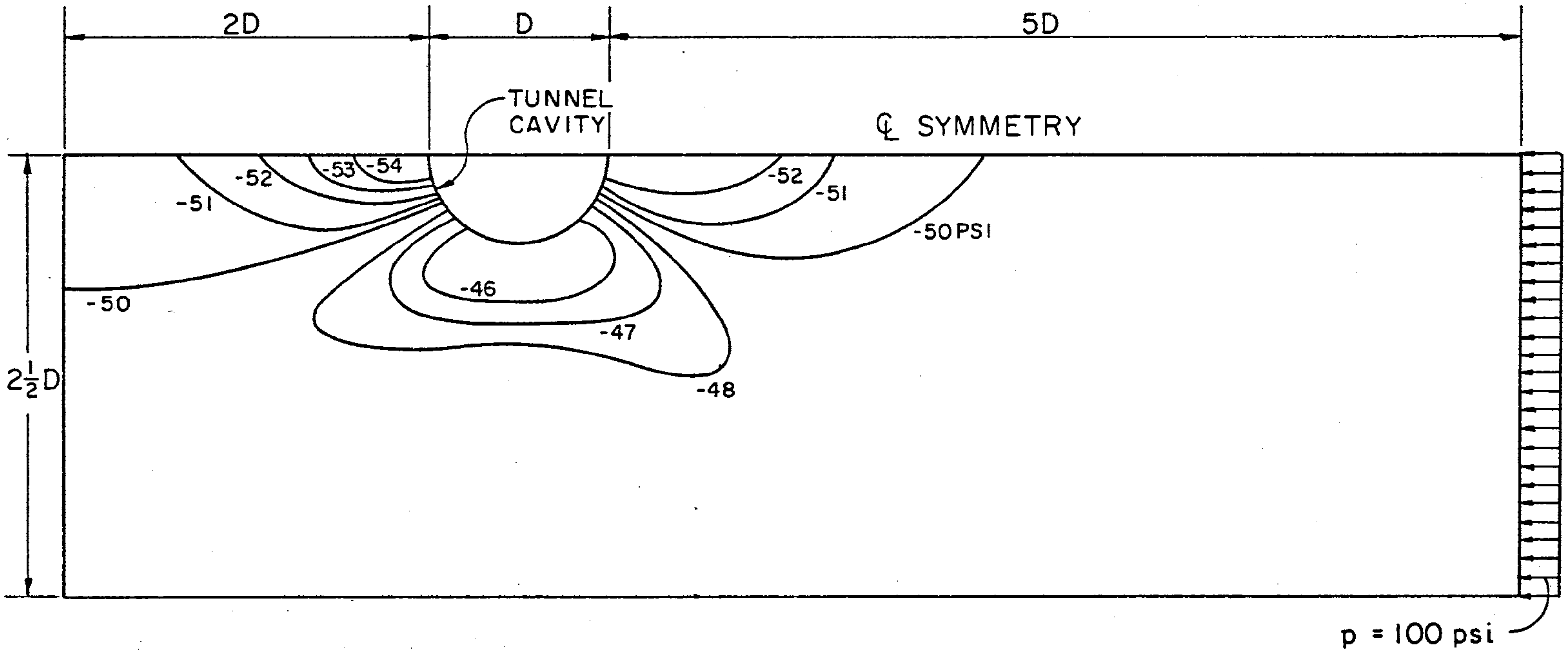

Figure 18. Major Principal Stress in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Liner-Packing System)

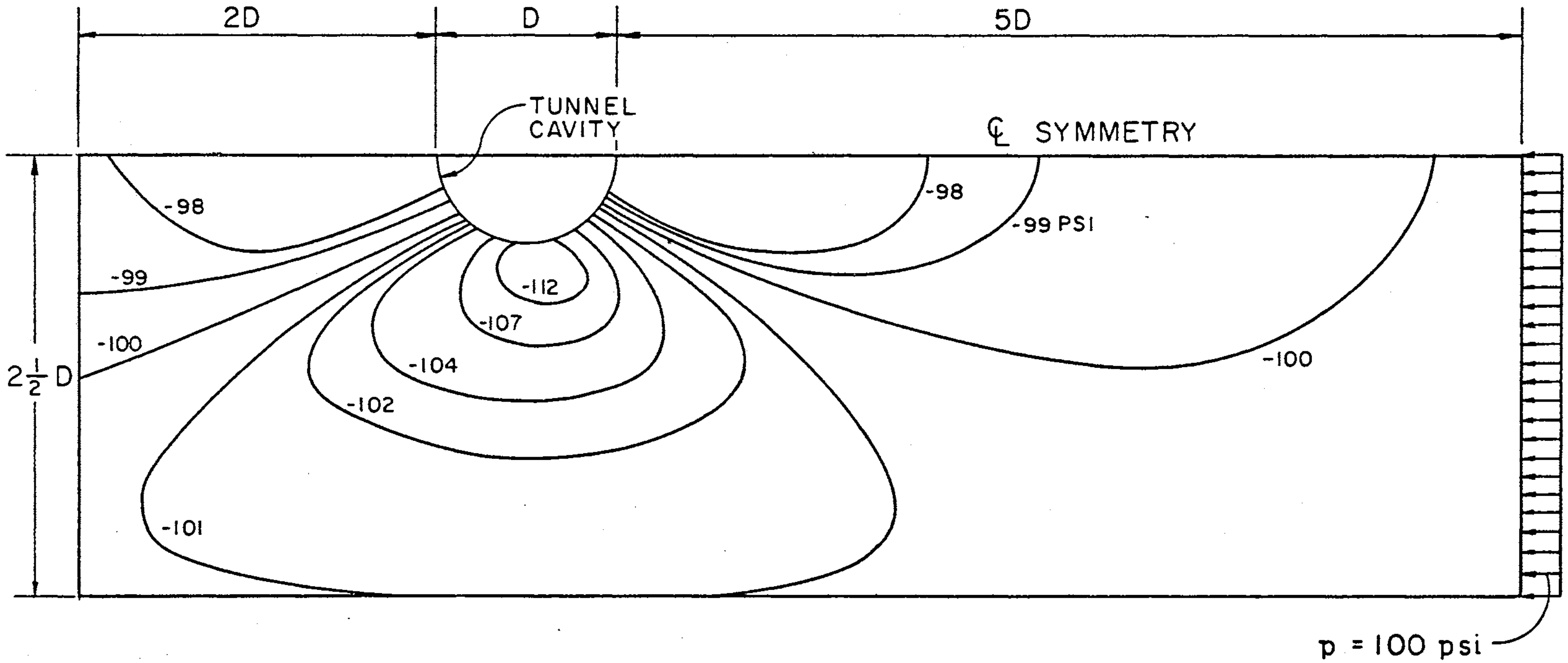

Figure 19. Minor Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Liner-Packing System)

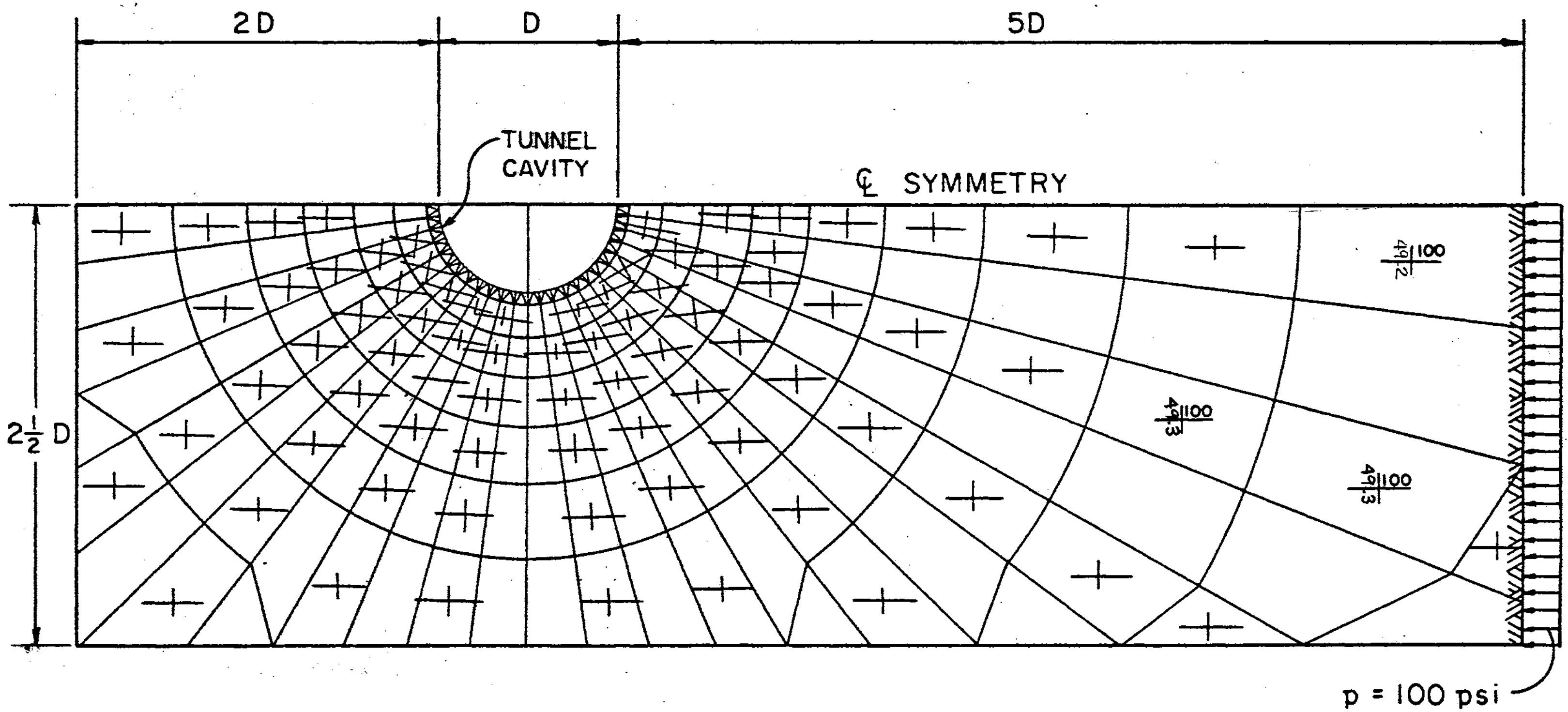

Figure 20. Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--One Crack)

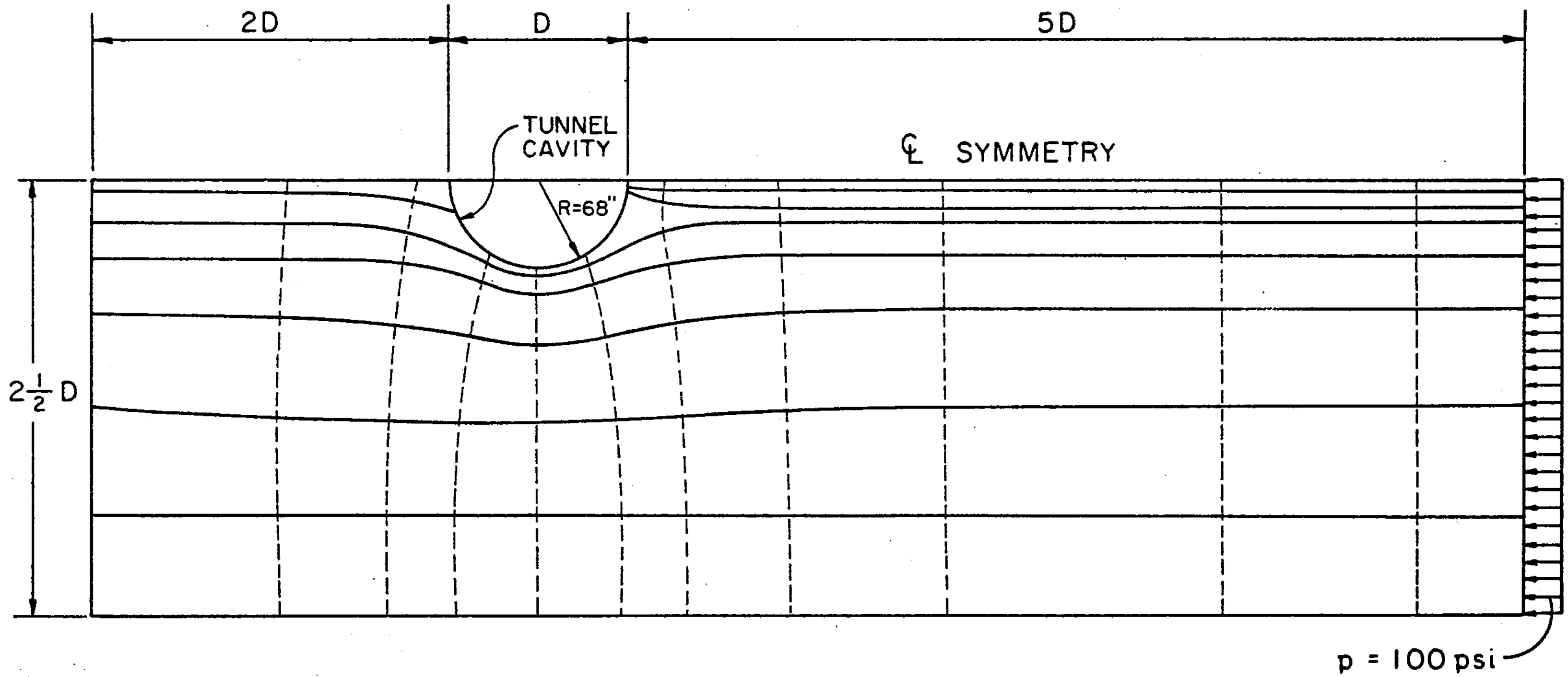

Figure 21. Principal Stress Directions in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--One Crack)

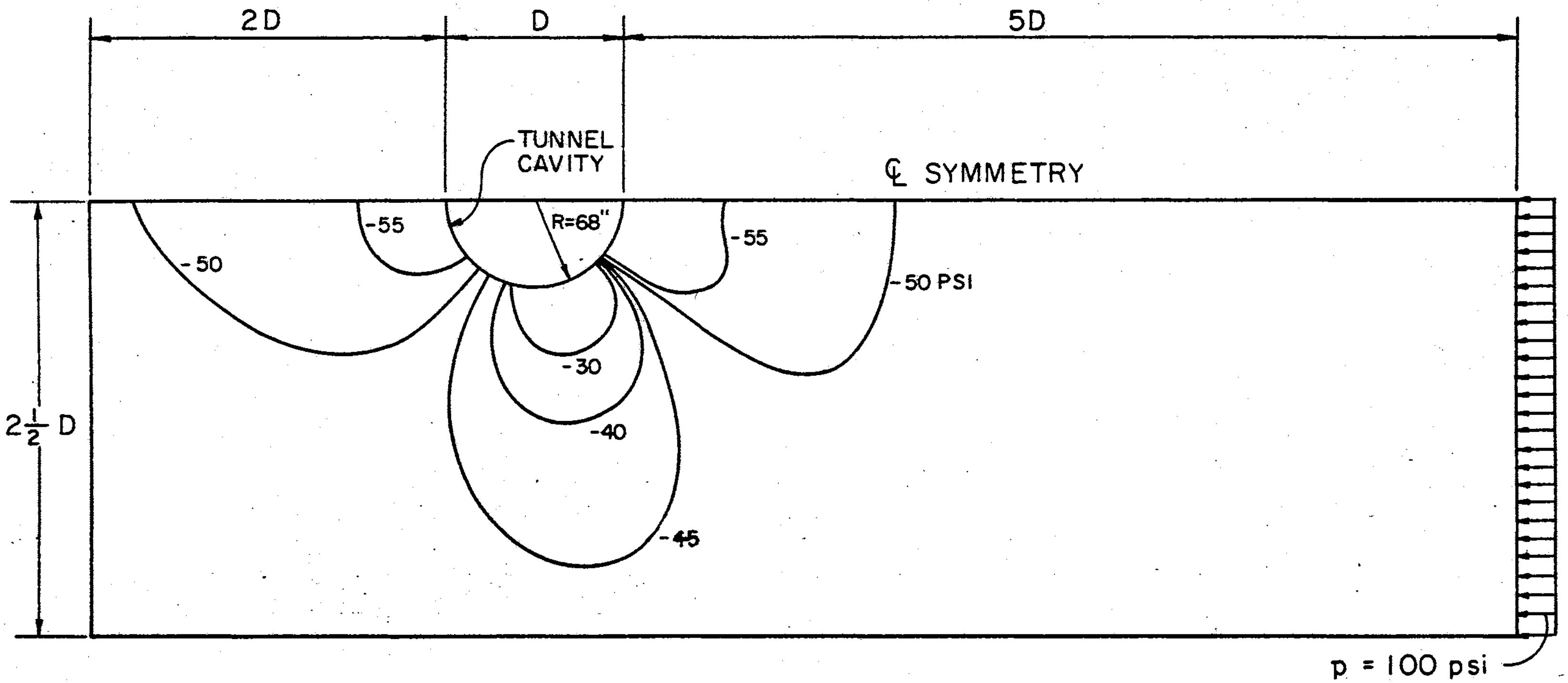

Figure 22. Major Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--One Crack)

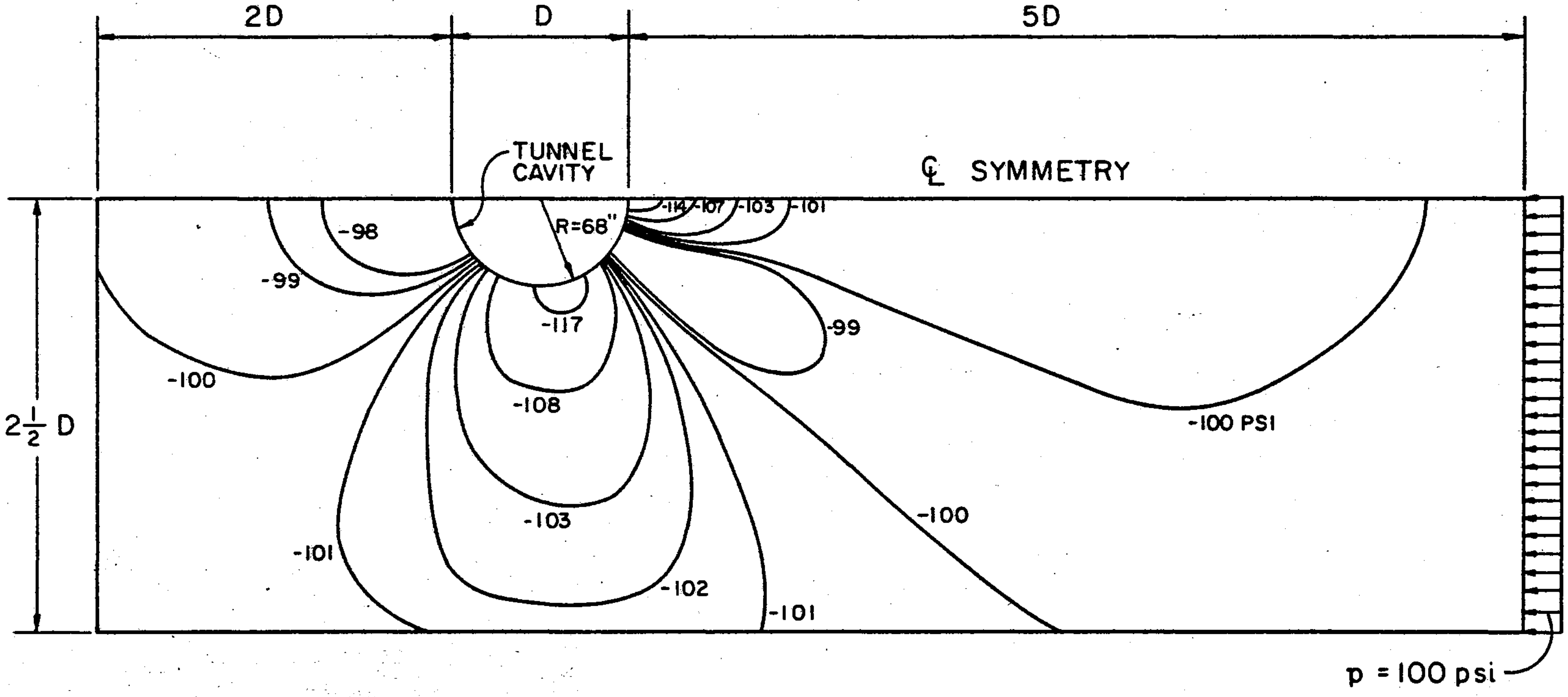

Figure 23. Minor Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--One Crack)

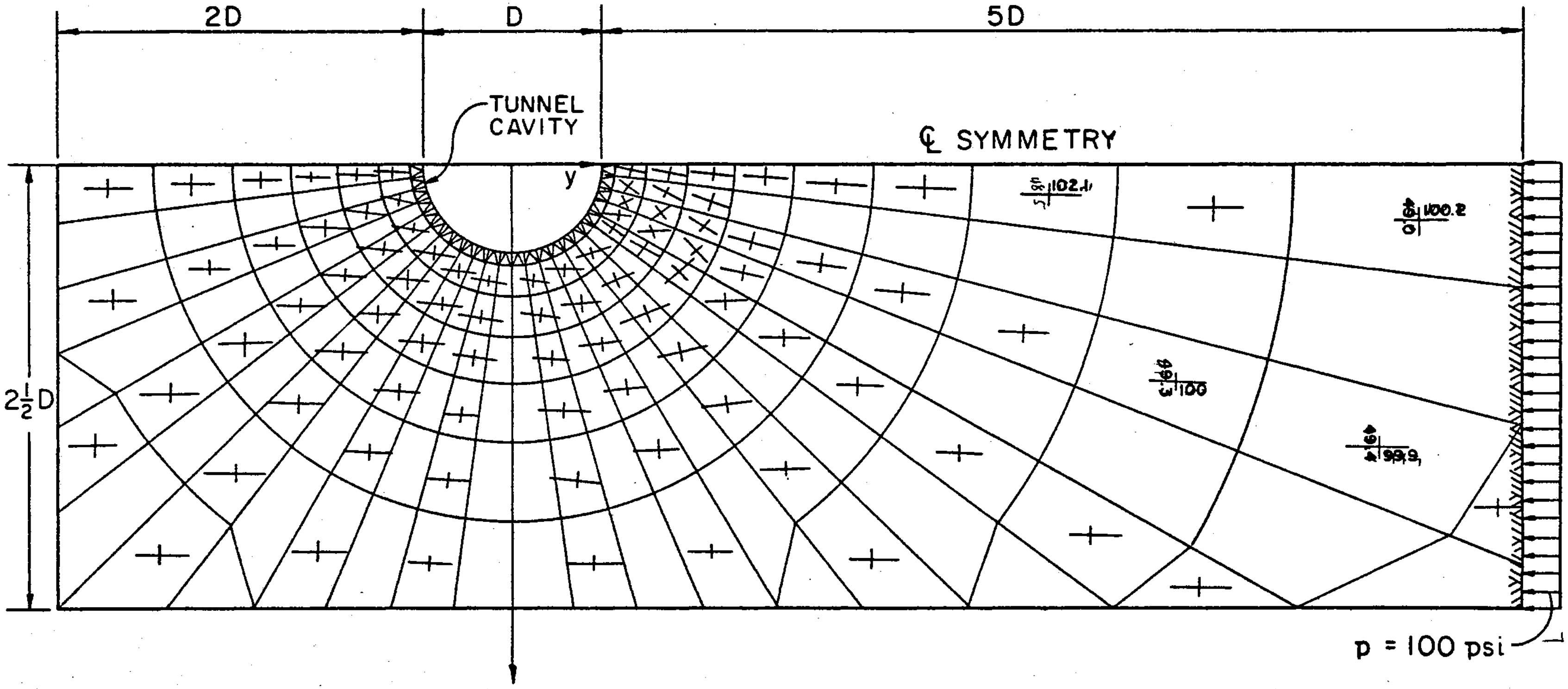

Figure 24. Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--Three Cracks)

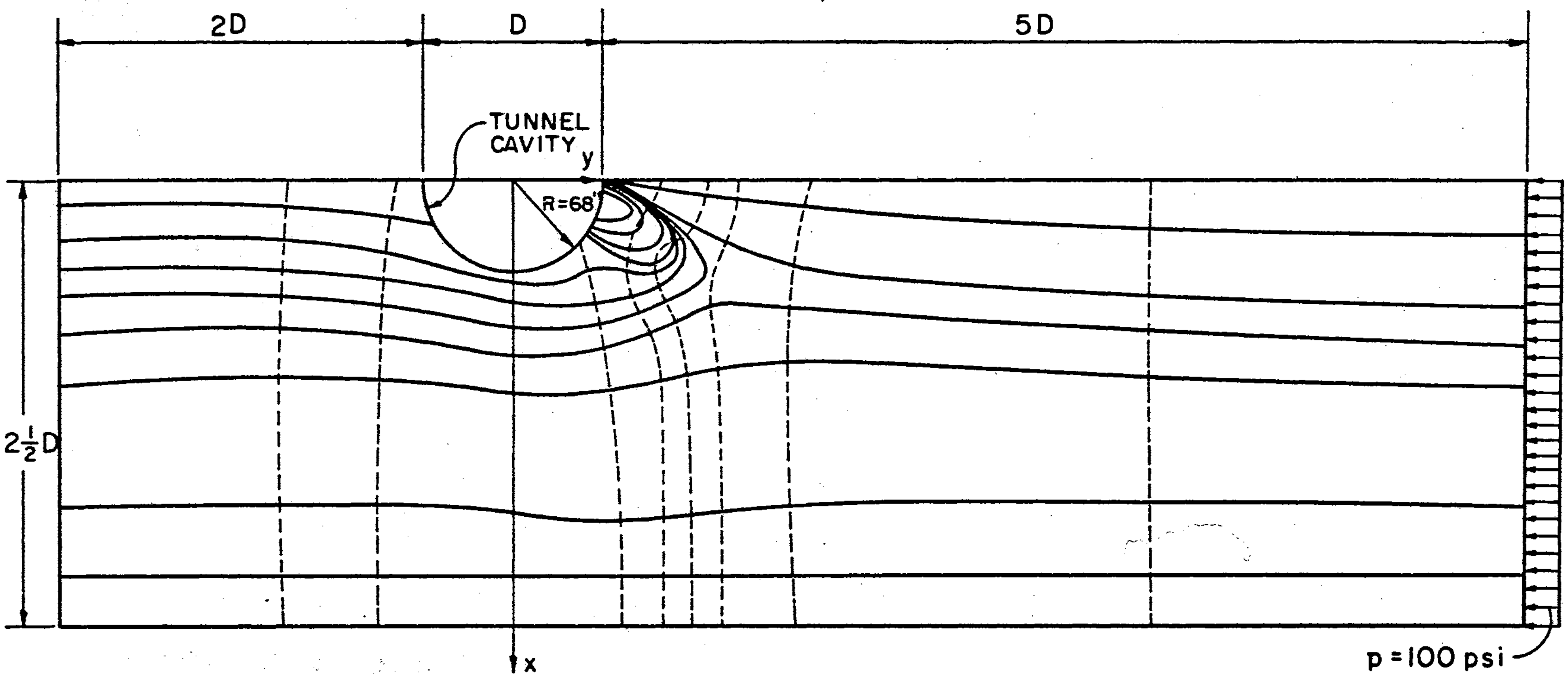

Figure 25. Principal Stress Directions in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--Three Cracks)

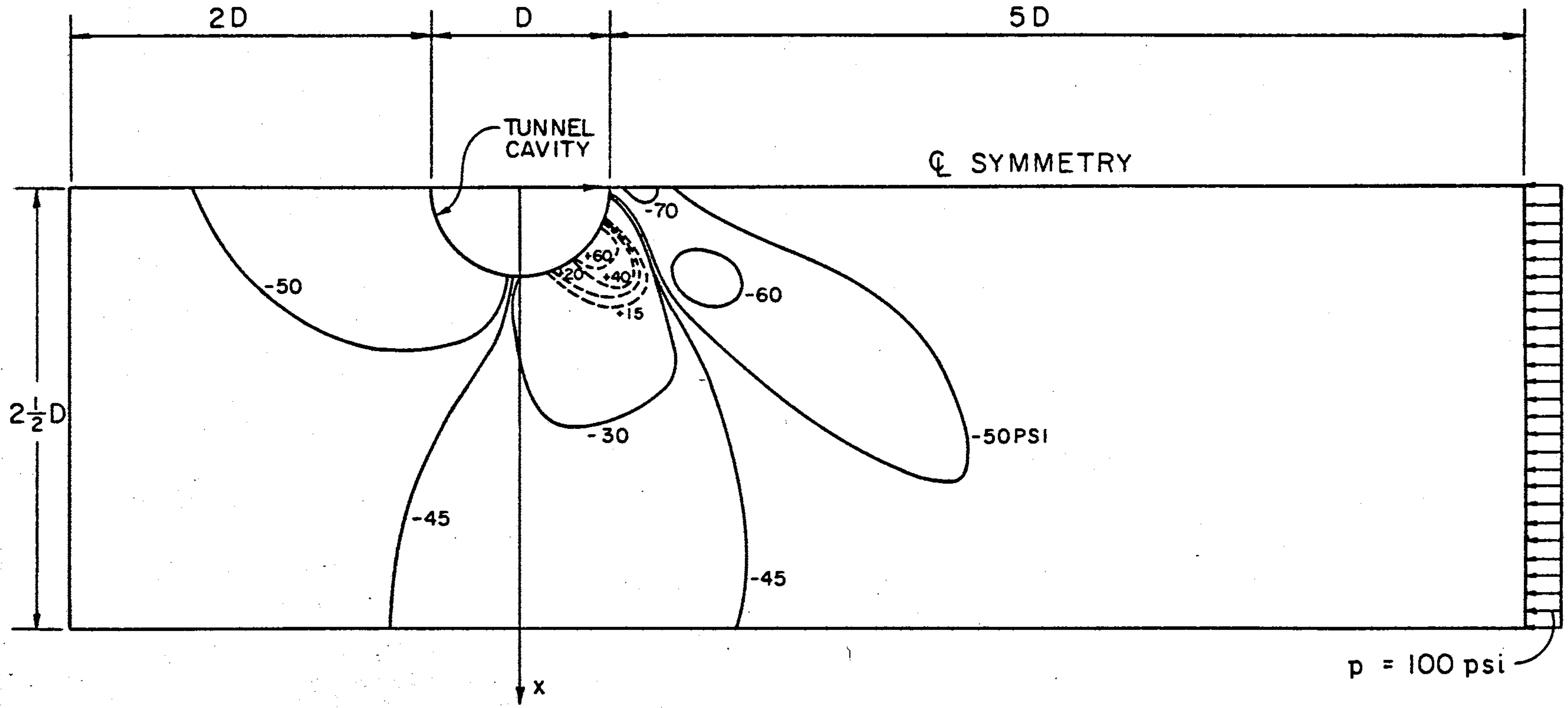

Figure 26. Major Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--Three Cracks)

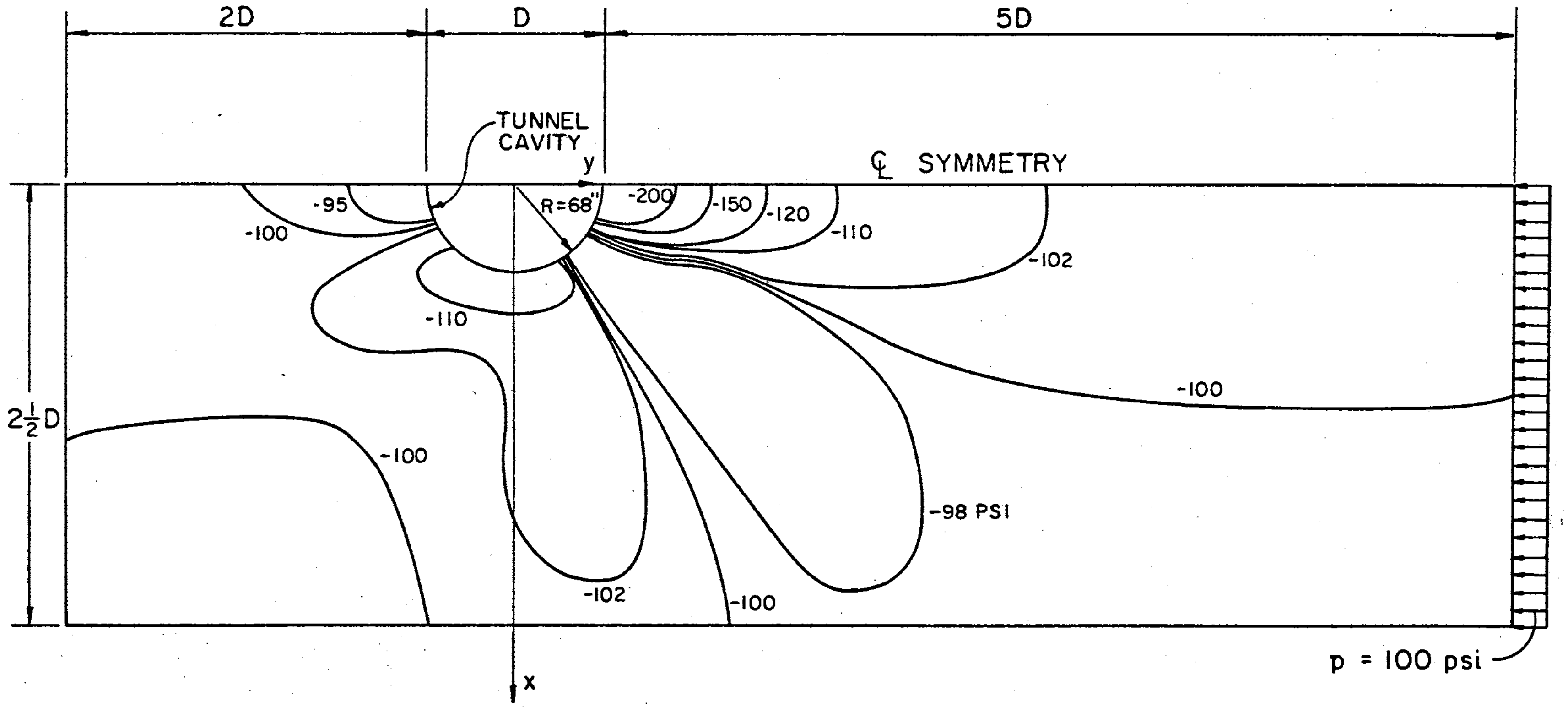

Figure 27. Minor Principal Stresses in Surrounding Medium due to a Uniform Over-Pressure of 100 psi (Tunnel with Cracked Liner--Three Cracks)

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

The objective of this study was to evaluate the feasibility of using the finite element method in the investigation of stress distribution around a tunnel cavity with or without lining due to a uniform over-pressure applied at the surface of the surrounding medium, under conditions of plane strain. It was assumed that the tunnel was at a depth of 5 tunnel diameters, which was just over 28 feet. For the cases studied the surrounding medium was assumed to be continuous and linearly elastic. For the purposes of analysis, the actual situation was approximated by a structural model made up of a finite number of elements interconnected at a finite number of nodes. This model was to simulate the actual situation, in its actions and deformations. The boundary of the surrounding medium was assumed to be about 2 diameters from cavity boundaries.

6.2 Conclusions

It was stated in Chapter III that the study was restricted to the nearly linear initial regions of the stress-strain curves of the materials used. Stress conditions obtained for some of the studied cases, however, show that this assumption is violated. The yield stress of the packing medium is about 40 psi. For the tunnel with an uncracked liner, the stresses at the top and bottom of the packing medium range from 2 to $2\frac{1}{2}$ times the yield stress of the material. This ratio is much higher for the cracked liner. In the case of a liner with three cracks, the stresses in the surrounding medium, next to the liner at about 30° from the vertical, are tensile. This state of stress is undesirable since rocks and soils are either weak in tension or incapable of resisting tension. The analysis is, therefore, unrealistic.

These situations can be remedied by improvement on the analysis. For cases where tensile stresses occur and cracks take place, any crack can be represented by separating the elements on each side of the crack and assigning them different nodal numbers (12). Another approach would be iterative in nature, consisting of the following steps (14):

a. An analysis is carried out and elements are noted in which tension occurs.

b. Principal tensile stresses are eliminated and the effect of their omission balanced by external "equivalent" nodal forces.

c. The elastic analysis is repeated to remove the balancing nodal forces and a check for tensile stresses is made again.

d. Steps b. and c. are repeated until no tension remains.

For non-linear elastic cases, the stresses can be calculated by applying the load in a stepwise fashion until the desired loading is reached. References (8) and (9) give details of the various approaches that can be used to account for yielding of the material.

The major drawback to using the finite element method in this study was that for the large number of nodal displacements to be calculated, the time required for computer run was excessive. An attempt was made by the writer to apply the finite element method to the determination of the dynamic response of a tunnel liner-packing system. The approach taken is summarized in Appendix A. This had to be abandoned at an early stage, however, because the computer time required made the cost prohibitive. It is hoped that this would set the stage for further investigation in this area.

It must be concluded, therefore, that with the present computer facilities at Oklahoma State University, the finite element method does not lend itself feasible cost-wise to the type of studies attempted in this investigation.

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APPENDIX A

DYNAMIC RESPONSE OF TUNNEL LINER-PACKING SYSTEM

The possibility of investigating the dynamic response of a tunnel liner-packing system using the finite element technique was examined. A computer program was written for this study, but had to be abandoned early, as the required computer run-time was prohibitive. The following is a brief outline of the procedure used.

A.1 Derivation of Element Stiffness Matrix

The liner-packing system was idealized as shown in Figure 2. It was assumed that the displacements $\{q\}$ at any point in the continuous structure could be related to a finite number of displacements at some arbitrarily selected points on the structure and expressed in the form

$$\{q\} = [A]\{Q\} \qquad (A.1)$$

where

$\{q\} = \{q_x, q_y, q_z\}$ = matrix of internal displacements;

$\{Q\} = \{Q_1, Q_2, Q_3 \ldots Q_n\}$ = matrix of nodal displacements;

$[A] = [A](x, y, z)$ = function of position coordinates.

Strictly speaking, Equation (A.1) is not valid, but would give a very good approximation if a large number of displacements calculated from dynamic equations are considered (9).

Differentiation of Equation (A. 1) yields the total strains at the point:

$$\{e\} = [B]\{Q\} = \{\varepsilon\} + \{\varepsilon_0\} \tag{A.2}$$

where $[B]$ is a strain-displacement transformation matrix, $\{\varepsilon\}$ is the elastic strain component, and $\{\varepsilon_0\}$ is a prestrain component which may be caused by temperature changes or some mechanical means.

Using Hooke's Law, the stresses are expressed as

$$\{\sigma\} = [C]\{\varepsilon\} = [C]\left([B]\{Q\} - \varepsilon_0\right). \tag{A.3}$$

The virtual work principle, modified to include inertia forces, is given by

$$\delta U = \delta W - \int_V \rho\delta\{q\}^t\{\ddot{q}\}\,dv \tag{A.4}$$

where the second term on the right hand side of the equality represents the virtual work due to inertia forces.

The virtual strain energy is given by:

$$\delta U = \int_V \delta\{\varepsilon\}^t\{\sigma\}\,dv \tag{A.5}$$

and the virtual work of all external, surface and body forces is given by:

$$\delta W = \delta\{Q\}^t\{P_e\} + \int_S \{q\}^t\{T\}\,ds + \int_V \delta\{q\}^t\{X\}\,dv. \tag{A.6}$$

From Equations (A. 1) and (A. 2), the virtual displacements and the virtual strains become

$$\delta\{q\} = [A]\,\delta\{Q\} \tag{A.7}$$

$$\delta\{\varepsilon\} = \delta\{e\} = [B]\,\delta\{Q\}. \tag{A.8}$$

Substituting Equations (A.7) and (A.8) into (A.4), noting that

$$\{q\} = [A]\{\ddot{Q}\} ,$$

and rearranging, the following is obtained:

$$\int_V \rho\{Q\}^t [A]^t [A] \{\ddot{Q}\} dv + \int_V \delta\{Q\}^t [B]^t [C] [B] \{Q\} dv$$

$$= \delta\{Q\}^t \{P_e\} + \int_S \delta\{Q\}^t [A]^t \{T\} ds$$

$$+ \int_V \delta\{Q\}^t [A]^t \{X\} dv + \int_V \delta\{Q\}^t [B]^t [C] \{\varepsilon_0\} dv. \qquad (A.9)$$

Since the virtual displacements are arbitrarily chosen, Equation (A.9) may be expressed as

$$[M]\{\ddot{Q}\} + [K]\{Q\} = \{P\} \qquad (A.10)$$

where

$$[M] = \int_V \rho [A]^t [A] dv \qquad \text{(mass matrix)}, \qquad (A.11)$$

$$[k] = \int_V [B]^t [C] [B] dv \qquad \text{(stiffness matrix)}, \qquad (A.12)$$

and

$$\{P\} = \{P_e\} + \int_V [A]^t \{T\} ds + \int_V [A]^t \{X\} dv + \int_V [B]^t [C] \{\varepsilon_0\} dv. \qquad (A.13)$$

Equation (A.10) represents the equation of motion.

A.2 Mass Matrix, $[M]$

It was pointed out earlier that Equation (A.1) was not valid, because, for general dynamic conditions, the matrix $[A]$ does not exist, except for very few special cases. The mass matrix $[M]$ as given by Equation (A.11) is, therefore, approximate. In order to facilitate computations in generating the mass matrix, it was further assumed that the mass of the elements was equally distributed among the nodes of the element, resulting in a diagonal mass matrix.

A.3 Stiffness Matrix, $\left[K\right]$

The discussion given in page 14 of the text also applies to this case.

A.4 Description of Forcing Function

A high energy explosion creates a shock wave which, for mathematical analysis, can be expressed as a displacement wave. The wave may produce elastic or plastic deformations, depending on the type of soil through which it propagates. Since the nature of the wave is not known precisely, it was idealized by the sketch shown in Figure 28. By varying the parameters shown in the sketch, a wide range of pulse shapes can be simulated.

The effect of the shock wave on the system is given by specifying the displacements of the nodes on the tunnel cavity. The time it takes for the wave front from the moment it arrives at one end of the diameter of symmetry to reach the j^{th} node is given by

$$t_{ex} = \frac{R_3 + X_j}{V} \tag{A.14}$$

where

R_3 = radius of tunnel cavity;

V = velocity of wave propagation;

X_j = the X-coordinate of the j^{th} node.

It is assumed that for $t \leq t_{ex}$ each node on the cavity boundary is unaffected and for $t > t_{ex}$ each node translates in the direction of the wave freely.

The translation of the j^{th} node is obtained from the displacement curve as one of the following expressions:

letting $t_a = t - t_{ex}$, then for

$$t_a \leq t_1 \quad \begin{cases} S_{xj} = (t_a/t_1)A_1 \\ S_{yj} = 0 \end{cases} ;$$

$$t_1 \leq t_a \leq t_2 \quad \begin{cases} S_{xj} = A_1 \\ S_{yj} = 0 \end{cases} ;$$

$$t_2 \leq t_a \leq t_3 \quad \begin{cases} S_{xj} = A_1 + \left(\frac{t_a - t_2}{t_3 - t_2}\right)(A_2 - A_1) \\ S_{yj} = 0.0 \end{cases} ;$$

$$t_3 \leq t_a \quad \begin{cases} S_{xj} = A_2 \\ S_{yj} = 0 \end{cases} .$$

It is assumed that successive wave fronts follow within a few milliseconds of each other or that the system has come to rest after the passage of the last front, and the effects are superimposed.

A.5 Methods of Solution

In evaluating the response of a structural system to arbitrary dynamic loads, the actual distributed mass system requires partial differential equations to describe its equilibrium state. However, the assumption (11) that the mass properties of the system are separate from its elastic properties, permits the idealization of the structure as a lumped mass system whose equilibrium state can be formulated by a finite number of simultaneous differential equations. These equations may then be solved by either of the following methods:

1. The modal superposition method which requires the determination of the eigenmodes of the system and superimposing these to calculate the response of the system.

2. The step-by-step method (10) which essentially uses recurrence formulas that involve the direct numerical integration of the equilibrium equations. The latter is a very useful method which can be used to deal with linear, damped or non-linear systems, and is the one followed in this study.

A.6 The Step-by-Step Recurrence Formulas

Referring to Equation (A.10), the equilibrium of the lumped mass system, without damping, at time t can be expressed as:

$$\left[M\right]\left\{\ddot{Q}\right\}_t + \left[K\right]\left\{Q\right\}_t = \left\{P\right\}_t \tag{A.15}$$

where

$\left\{\ddot{Q}\right\}_t$ = acceleration of the system at "t";

$\left\{Q\right\}_t$ = displacement of the system at "t";

$\left\{P\right\}_t$ = force on the system at "t";

$\left[M\right]$ = mass matrix;

$\left[K\right]$ = stiffness matrix.

It will be assumed that the variation of the acceleration for each mass point of the system is linear over the time increment, Δt, as shown in Figure 29.

A simple integration over the time interval for all mass points yields the following matrix equations for the velocity and displacement at the end of the time interval, Δt.

$$\{\dot{Q}\}_t = \{\dot{Q}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{Q}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{Q}\}_t$$

$$= \{a\} + \frac{\Delta t}{2}\{\ddot{Q}\}_t \qquad (A.16)$$

and

$$\{Q\}_t = \{Q\}_{t-\Delta t} + \Delta t\{\dot{Q}\}_{t-\Delta t} + \frac{(\Delta t)^2}{3}\{\ddot{Q}\}_{t-\Delta t} + \frac{(\Delta t)^2}{6}\{\ddot{Q}\}_t$$

$$= \{b\} + \frac{(\Delta t)^2}{6}\{\ddot{Q}\}_t \qquad (A.17)$$

where

$$\{a\} = \{\dot{Q}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{Q}\}_{t-\Delta t} \qquad (A.18)$$

and

$$\{b\} = \{Q\}_{t-\Delta t} + \Delta t\{\dot{Q}\}_{t-\Delta t} + \frac{(\Delta t)^2}{3}\{\ddot{Q}\}_{t-\Delta t}\,. \qquad (A.19)$$

Substitution of Equations (A.16) and (A.17) into Equation (A.15) yields

$$[M]\{\ddot{Q}\}_t + [K]\left(\{b\} + \frac{(\Delta t)^2}{6}\{\ddot{Q}\}_t\right) = \{P\}_t$$

or rearranging

$$\left[[M] + \frac{(\Delta t)^2}{6}[K]\right]\{\ddot{Q}\}_t = \left\{\{P\}_t - [K]\{b\}\right\}. \qquad (A.20)$$

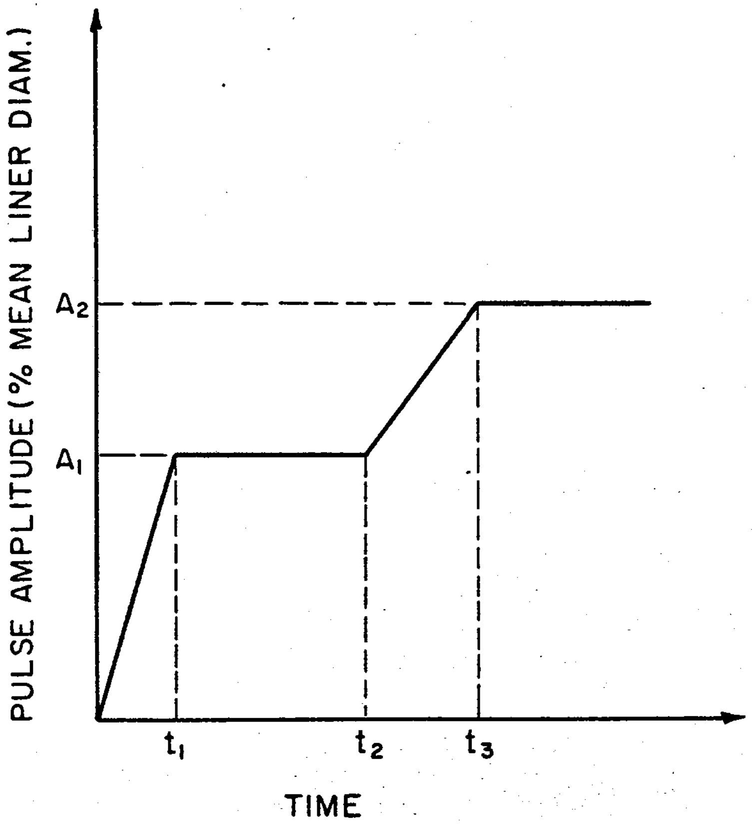

Figure 28. Forcing Function (5)

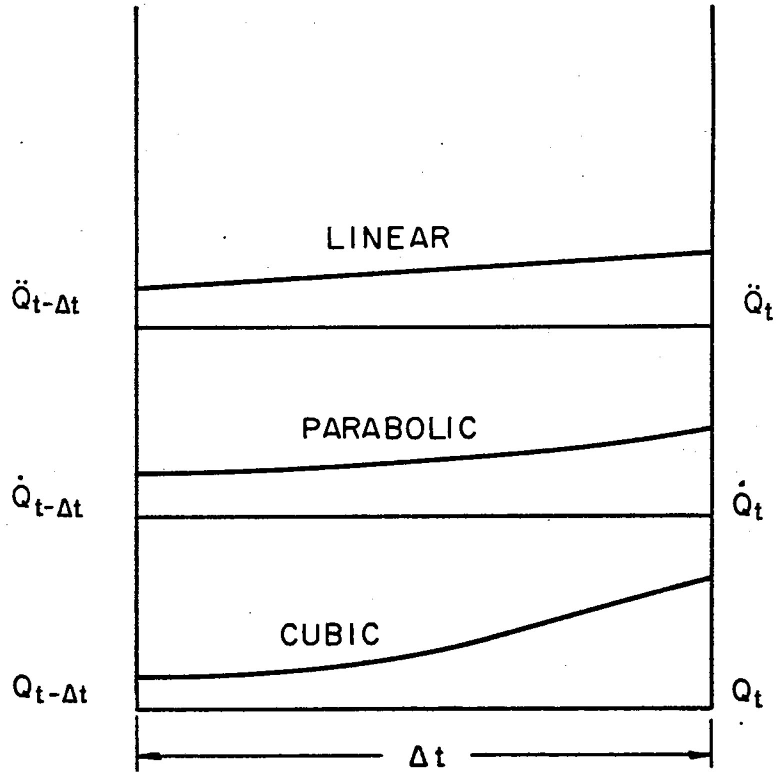

Figure 29. Displacement Functions

Equation (A. 20) is the required recurrence formula which expresses the accelerations at the end of an interval in terms of displacements, velocities, and accelerations at the beginning of the interval. Using Equations (A. 16) and (A. 17), it is now possible to calculate velocities and displacements to be used in determining the accelerations at the end of the next time interval, and so on.

It was at this stage of development that the study had to be discontinued because of the excessive computer time needed.

APPENDIX B

COMPUTER PROGRAM

```
FORTRAN IV G LEVEL 1, MOD 4              MAIN              DATE = 70188          20/30/07

          C***********************************************************************
          C*                                                                    *
          C*    STRESS DISTRIBUTION AROUND A TUNNEL CAVITY                      *
          C*    USING THE FINITE ELEMENT METHOD                                 *
          C*                                                                    *
          C*                                                                    *
          C*    PROGRAMMER                                                      *
          C*                                                                    *
          C*    YOHANNES WOLDEMARIAM                                            *
          C*    GRADUATE STUDENT                                                *
          C*    SCHOOL OF CIVIL ENGINEERING                                     *
          C*    OKLAHOMA STATE UNIVERSITY                                       *
          C*    STILLWATER , OKLAHOMA                                           *
          C*                                                                    *
          C*    PARAMETERS USED IN THIS PROGRAM                                 *
          C*                                                                    *
          C*    NUMNP      = TOTAL NUMBER OF NODAL POINTS                       *
          C*    NUMEL      = TOTAL NUMBER OF ELEMENTS                           *
          C*    NP         = INDICATOR  NP=1 UNDISTURBED MEDIUM                 *
          C*                            NP=2 UNLINED TUNNEL                     *
          C*                            NP=3 LINED TUNNEL                       *
          C*    X,Y        = NODAL POINT COORDINATES                            *
          C*    K1,K2,K3 = NUMBER OF DIVISIONS IN LINER-PACKING SYSTEM          *
          C*    NR         = TOTAL NUMBER OF RADIAL LINES                       *
          C*    DP         = CONCRETE COVER                                     *
          C*    DS         = DIAMETER OF STEEL                                  *
          C*    PS         = PERCENTAGE OF STEEL                                *
          C*    FC , FY  = STRENGTHS OF CONCRETE AND STEEL RESPECTIVELY         *
          C*    E          = MODULUS OF ELASTICITY                              *
          C*    PR         = POISSON'S RATIO                                    *
          C*    S          = ELEMENT STIFFNESS MATRIX                           *
          C*    C          = ELASTICITY MATRIX                                  *
          C*    P          = ELEMENT LOAD VECTOR                                *
          C*    A          = STRUCTURE STIFFNESS MATRIX                         *
          C*    B          = STRUCTURE LOAD VECTOR                              *
          C*    UXTYPE,UYTYPE  = PRESCRIBED LOAD OR DISP. IN X OR Y DIRECTION   *
          C*    UX ,UY    = VALUE OF PRESCRIBED QUANTITY                        *
          C*    IX(N,J)   = NUMBER OF NODE J OF ELEMENT NUMBER N                *
          C*    D,DD,H,HH,F,TP = VARIOUS MATRICES EMPLOYED IN PROGRAM           *
          C*    XX(J),XXX(J),YY(J),YYY(J) = NOMENCLATURES SYNONIMOUS WITH THE   *
          C*                                COORDINATES OF NODE J IN VARIOUS    *
          C*                                PARTS OF PROGRAM                    *
          C*                                                                    *
          C***********************************************************************
          C*                                                                    *
          C*                                                                    *
          C***********************************************************************
          C*                              MAIN  PROGRAM                         *
          C***********************************************************************
0001            IMPLICIT REAL*8(A-H,O-Z)
0002            REAL*8 DFLOAT,DCOS,DSIN
0003            COMMON TYPE(8),HED(18),E(8),X(950) ,Y(950) ,UX(950) ,UY(950) ,VOL,
               1UXTYPE(950) ,UYTYPE(950) ,PR(8),
               2MTYPE(930),NUMNP,NUMEL,NUMAT,N
0004            COMMON /ARG/ XXX(5),YYY(5),S(10,10),DD(3,3),HH(6,10),P(10),XX(4),
               1YY(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),
               2DK(2,2),TYPE1,TYPE2,TEST1,TEST2,SIG(10),IX(930,4),LM(4),NR
0005            COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
```

```
                C*********************************************************************
                C*         READ  AND WRITE  OF CONTROL INFORMATION & MAT  PROPERTIES *
                C*********************************************************************
0006                  READ(5,1000) HED,TYPE,NUMAT
0007             1000 FORMAT(18A4,/,8A3,/,I5)
0008                  WRITE(6,2000) HED,TYPE,NUMAT
0009             20000FORMAT(1H1,4X,18A4,//5X,8A3/,3X,
                     130H  NUMBER OF DIFF. MATERIALS...I3,//)
0010                  READ(5,1001) ( E(M),PR(M),M=1,NUMAT)
0011             1001 FORMAT(2E14.4)
0012                  WRITE(6,2001)
0013             20010FORMAT(6X,50HMAT MODULUS OF   POISSON'S
                     16X,49HNO. ELASTICITY      RATIO
0014                  WRITE(6,2002) ( M,E(M),PR(M),M=1,NUMAT)
0015             2002 FORMAT(5X,I3,1P2D12.3)
                C*********************************************************************
                C*                    INPUT OF NODAL POINTS DATA                     *
                C*********************************************************************
0016                  READ(5,50) NP
0017               50 FORMAT(6X,I5)
0018                  IF(NP-2) 510,322,60
0019               60 PI=3.141592653589793
0020                  READ (5,402) SP,DS
0021              402 FORMAT(2F10.4)
                C
0022                  E(2)=((PI*DS)/(4.*SP))*E(2)
                C
0023                  READ (5,430) R1,R2,R3,DP,NR,K1,K2,K3
0024              430 FORMAT(4E14.4,4I6)
0025                  READ(5,433) FC,FY
0026              433 FORMAT( 10X, 2F13.3)
0027                  PB=(0.72*FC*90000)/(FY*(90000+FY))
0028                  AS= PI*(DS**2)/4.0
0029                  DEP= R2-R1-DP-(DS/2.)
0030                  PS=AS/(DEP*SP)
0031                  IF(PS.GT.PB)GO TO 434
0032                  MU=PS*FY*SP*(DEP**2)*(1.-0.59*PS*FY/FC)
0033                  GO TO 436
0034              434 MU=AS*FY*(DEP-DP)
0035              436 FO= MU/(DEP-DP-(DS/2.))
0036                  FS=FO/2
0037                  PRINT 438
0038              438 FORMAT(//,10X,59H       AS              PS           PB            FC
                     1          FY)
0039                  PRINT 439, AS, PS, PB, FC, FY
0040              439 FORMAT(/,10X, 1P5D13.4)
0041                  WRITE(6,435)
0042              435 FORMAT(//,13X,2HR1,12X,2HR2,12X,2HR3,12X,2HDP,12X,2HDS,8X,2HNR,4X,
                     12HK1,4X,2HK2,4X,2HK3,/)
0043                  WRITE(6,432) R1,R2,R3,DP,DS,NR,K1,K2,K3
0044              432 FORMAT(5X,1P5D14.4,4I6)
0045                  DR1=DP/DFLOAT(K1)
0046                  DR2=(R2-R1-2.0*(DP+DS))/DFLOAT(K2)
0047                  DR3=(R3-R2)/DFLOAT(K3)
0048                  ALPHA=PI/DFLOAT(NR-1)
0049                  NRM=((NR-1)/2)+1
0050                  NUMNP=0
0051                  NRS=NRM+1
```

```
            NRN=NR+1
            NRC=((NR-1)/6)+1
            NCR=NRC+1
      C
            DO 320 I=1,NRN
            READ(5,450) N1,N2
        450 FORMAT( 10X,2I6)
            IF(I-NCR) 451,453,452
        451 THETA=ALPHA*DFLOAT(I-1)
            GO TO 453
        452 THETA=ALPHA*DFLOAT(I-2)
        453 DO 310 J=N1,N2
            NUMNP=NUMNP+1
            JR=N1+K1
            IF(J.GT.JR) GO TO 455
            R=R1+DR1*DFLOAT(J-N1)
            GO TO 445
        455 JR=N1+K1+1
            IF(J.GT.JR) GO TO 465
            R=R1+DP+DS
            GOTO445
        465 JR=N1+K1+K2+1
            IF(J.GT.JR) GO TO 470
            R=R1+DP+DS+DR2*DFLOAT(J-N1-K1-1)
            GOTO445
        470 JR=N1+K1+K2+2
            IF(J.GT.JR) GO TO 485
            R=R2-DP
            GOTO445
        485 IF(I.NE.NCR) GO TO 486
            R=R2
            GO TO 445
        486 JR=N2-K3
            IF(J.GT.JR) GO TO 490
            R=R2-DP+DR1*DFLOAT(J-K1-K2-N1-2)
            GOTO445
        490 R=R3-DR3*DFLOAT(N2-J)
        445 IF(I.EQ.NRS.OR.I.EQ.NRN) GO TO 446
            X(J)=R*DSIN(THETA)
            Y(J)=R*DCOS(THETA)
            GO TO 466
        446 IF(I.EQ.NRN) GO TO 447
            X(J)=R
            Y(J)=0.0
            GO TO 466
        447 X(J)=0.0
            Y(J)=-R
        466 IF(I.EQ.1.OR.I.EQ.NRC.OR.I.EQ.NCR) GO TO 469
            IF(X(J).EQ.0.0) GO TO 467
            UXTYPE(J)=TYPE1
            UX(J)=0.0
            GO TO 468
        467 UXTYPE(J)=TYPE2
            UX(J)=0.0
            GO TO 468
        469 IF(I.GT.1) GO TO 471
            JRL= N1+2*K1+K2+2
            IF(J.GT.JRL) GO TO 467
```

```
0109      471  UXTYPE(J)=TYPE1
0110           JK1= N1+K1
0111           JK2= N1+K1+1
0112           JK3= N1+K1+K2+1
0113           JK4= N1+K1+K2+2
0114           IF(J.EQ.JK1.OR.J.EQ.JK2.OR.J.EQ.JK3.OR.J.EQ.JK4) GO TO 473
0115           UX(J)=0.0
0116           GO TO 468
0117      473  IF(I.EQ.NCR) GO TO 475
0118           IF(J.EQ.JK1.OR.J.EQ.JK2) GO TO 472
0119           GO TO 474
0120      472  UX(J)=-FS*DCOS(THETA)
0121           UYTYPE(J)=TYPE1
0122           UY(J)= FS*DSIN(THETA)
0123           GO TO 310
0124      474  UX(J)= FS*DCOS(THETA)
0125           UYTYPE(J)=TYPE1
0126           UY(J)=-FS*DSIN(THETA)
0127           GO TO 310
0128      475  IF(J.EQ.JK3.OR.J.EQ.JK4) GO TO 472
0129           GO TO 474
0130      468  UYTYPE(J)=TYPE1
0131           UY(J)=0.0
0132      310  CONTINUE
0133      320  CONTINUE
0134           IF(NP.EQ.3) GO TO 330
0135      322  READ(5,325) R3,NR
0136      325  FORMAT( 10X, F10.3, I6 )
0137           PI=3.141592653589793
0138           ALPHA=PI/DFLOAT(NR-1)
0139           NUMNP=0
0140           NRM=((NR-1)/2)+1
0141      330  DIA=2.*R3
0142           NR1=(3*(NR-1)/4)+1
0143           READ(5,326) D1,D2,D3,RT,F1,F2,F3
0144      326  FORMAT( 10X, 7F10.3)
0145           WRITE(6,328)
0146      328  FORMAT(10X,66H     D1         D2         D3            RT       F1
              1  F2         F3,/)
0147           WRITE(6,326) D1,D2,D3,RT,F1,F2,F3
0148           XT=(F2+0.5)*DIA
0149           YT=(F1+0.5)*DIA
0150           YB=(F3+0.5)*DIA
0151           READ(5,327) PRS
0152      327  FORMAT( 10X,F10.3 )
0153           DO 375 I=1,NR
0154           IF(NP.EQ.2) GO TO 336
0155           K=MOD(I,2)
0156           IF(K) 336,375,336
0157      336  READ(5,450) N1,N2
0158           THETA=ALPHA*DFLOAT(I-1)
0159           DO 374 J=N1,N2
0160           IF(NP.EQ.2) GO TO 338
0161           IF(J.EQ.N1) GO TO 374
0162      338  NUMNP=NUMNP+1
0163           IF(J.EQ.N2) GO TO 361
0164           IF(J-N1) 342,342,344
0165      342  R=R3
```

```
      GO TO 354
  344 IF(J-N1-1) 346,346,348
  346 R=R3+D1
      GO TO 354
  348 IF(J-N1-2) 332,332,352
  332 R=R+D2
      GO TO 354
  352 KE=(J-N1-3)
      RI=D3*(RT**KE)
      R=R+RI
  354 IF(I.EQ.NRM.OR.I.EQ.NR) GO TO 356
      X(J)=R*DSIN(THETA)
      Y(J)=R*DCOS(THETA)
      GO TO 358
  356 IF(I.EQ.NR) GO TO 357
      X(J)=R
      Y(J)=0.0
      GO TO 358
  357 X(J)=0.0
      Y(J)=-R
  358 IF(X(J).EQ.0.OR.X(J).EQ.XT) GO TO 359
      UXTYPE(J)=TYPE1
      UX(J)=0.0
      GO TO 362
  359 UXTYPE(J)= TYPE2
      UX(J)=0.0
  362 UYTYPE(J)=TYPE1
      UY(J)=0.0
      GO TO 374
  361 IF(I-NRM) 341,347,349
  341 IF(THETA-(PI/3.)) 373,345,345
  373 XN=YT*DTAN(THETA)
      IF(XN-XT) 343,343,345
  343 X(N2)=XN
      Y(N2)=YT
      GO TO 355
  345 BETA=(PI/2.)-THETA
      X(N2)=XT
      Y(N2)=XT*DTAN(BETA)
      GO TO 355
  347 X(N2)= XT
      Y(N2)=0.0
      GO TO 355
  349 THETA=PI-THETA
      IF(THETA-(PI/3.)) 305,305,353
  305 IF(I.EQ.NR) GO TO 306
      IF(I.EQ.NR1) GO TO 307
      XN=YB*DTAN(THETA)
      IF(XN-XT) 351,351,353
  351 X(N2)=XN
      GO TO 308
  307 X(N2)=XT
  308 Y(N2)=-YB
      GO TO 355
  306 X(N2)=0.0
      Y(N2)=-YB
      GO TO 355
  353 BETA=(PI/2.)-THETA
```

```
      X(N2)=XT
      Y(N2)=-XT*DTAN(BETA)
  355 IF(X(N2).EQ.0.OR.X(N2).EQ.XT) GO TO 382
      UXTYPE(N2)=TYPE1
      UX(N2)=0.0
      GO TO 363
  382 UXTYPE(N2)=TYPE2
      UX(N2)=0.0
  363 IF(Y(N2).EQ.YT.OR.Y(N2).EQ.(-YB)) GO TO 365
  364 UYTYPE(N2)=TYPE1
      UY(N2)=0.0
      GO TO 374
  365 IF(Y(N2).EQ.YT) GO TO 366
      UYTYPE(N2)=TYPE2
      UY(N2)=0.0
      GO TO 374
  366 IF(NP.EQ.3) GO TO 386
      IF(I-3) 367,368,368
  367 GAMMA=0.0
      GO TO 369
  368 GAMMA=ALPHA*DFLOAT(I-2)
  369 DELTA=ALPHA*DFLOAT(I)
      GO TO 377
  386 IF(I-5) 387,388,388
  387 GAMMA=0.0
      GO TO 376
  388 GAMMA=ALPHA*DFLOAT(I-3)
  376 DELTA=ALPHA*DFLOAT(I+1)
  377 X1=YT*DTAN(GAMMA)
      X2=YT*DTAN(DELTA)
      IF(X2-XT) 370,370,371
  370 PR1=-PRS*0.5*(X2-X1)
      GO TO 372
  371 PR1=-PRS*0.5*(XT-X1)
  372 UYTYPE(N2)=TYPE1
      UY(N2)=PR1
  374 CONTINUE
  375 CONTINUE
      READ(5,380) NS
  380 FORMAT( 10X, I6 )
      DO 390 I=1,NS
      NUMNP=NUMNP+1
      READ(5,385) J,X(J),Y(J),UXTYPE(J),UX(J),UYTYPE(J),UY(J)
  385 FORMAT(I5,5X,2F10.3,6X,A4,F10.3,6X,A4,F10.3)
  390 CONTINUE
      GO TO 560
  510 READ(5,515) N1,N2,XT,YT,PRS
  515 FORMAT( 10X,2I6,3F10.3)
      WRITE(6,525)
  525 FORMAT(10X,44H    N1       N2         XT          YT         PRS,/)
      WRITE(6,530) N1,N2,XT,YT,PRS
  530 FORMAT(10X,I5,3X,I5,3X,3F10.3,//)
      DR1=YT/N1
      DR2=XT/N2
      NR=N1+1
      KN=N2+1
      NUMNP=KN*NR
      DO 550 I=1,NR
```

```
      L1=KN*(I-1)+1
      L2=KN*I
      DO 545 J=L1,L2
      X(J)=DR2*DFLOAT(J-L1)
      Y(J)=DR1*DFLOAT(I-1)
      IF(J.EQ.L1.OR.J.EQ.L2) GO TO 517
      UXTYPE(J)=TYPE1
      UX(J)=0.0
      IF(I.EQ.1.OR.I.EQ.NR) GO TO 516
      GO TO 518
  516 IF(I.NE.NR) GO TO 520
      UYTYPE(J)=TYPE1
      UY(J)=-PRS*DR2
      GO TO 545
  517 UXTYPE(J)=TYPE2
      UX(J)=0.0
      IF(I.EQ.1.OR.I.EQ.NR) GO TO 519
  518 UYTYPE(J)=TYPE1
      UY(J)=0.0
      GO TO 545
  519 IF(I.NE.NR) GO TO 520
      UYTYPE(J)=TYPE1
      UY(J)=-0.5*PRS*DR2
      GO TO 545
  520 UYTYPE(J)=TYPE2
      UY(J)=0.0
  545 CONTINUE
  550 CONTINUE
      NUMEL=0
      DO 220 I=1,N1
      N3=N2*(I-1)+1
      N4=N2*I
      DO 210 J=N3,N4
      IX(J,1)=J+I-1
      IX(J,2)=IX(J,1)+1
      IX(J,3)=IX(J,2)+KN
      IX(J,4)=IX(J,1)+KN
      MTYPE(J)=1
      NUMEL=NUMEL+1
  210 CONTINUE
  220 CONTINUE
      GO TO 595
  560 LK=NR-1
      NUMEL=0
      DO 590 I=1,LK
      KK=MOD(I,2)
  563 READ(5,564) KF,KL
  564 FORMAT( 10X,2I6)
      M=KF-1
  565 READ(5,566) N,(IX(N,J),J=1,4)
  566 FORMAT( 10X, 5I8 )
  567 M=M+1
      NUMEL=NUMEL+1
      IF(N-M) 580,571,568
  568 DO 570 K=1,4
  570 IX(M,K)=IX(M-1,K)+1
  571 IF(NP.EQ.2) GO TO 573
      IF(KK-1) 572,572,576
```

```
0340        572  JL=KF+K1
0341             JH=JL+K2+1
0342             JP=JH+K1+1
0343             IF(M.EQ.JL.OR.M.EQ.JH) GO TO 574
0344             IF(M.GE.JP) GO TO 575
0345        573  MTYPE(M)=1
0346             GO TO 577
0347        574  MTYPE(M)=2
0348             GO TO 577
0349        575  MTYPE(M)=3
0350             GO TO 577
0351        576  MTYPE(M)=4
0352        577  IF(N-M) 580,578,567
0353        578  IF(KL-N) 580,585,565
0354        580  WRITE(6,2005) N
0355        2005 FORMAT(21HO EL.NO CARD ERROR N=I5)
0356             CALL EXIT
0357        585  IF(NP.EQ.2) GO TO 590
0358             IF(KK-1) 587,590,590
0359        587  KK=KK+2
0360             GO TO 563
0361        590  CONTINUE
0362        595  NM=0
0363             DO 160 N=1,NUMEL
0364             DO 633 M =1,4
0365             DO 633 MM=1,4
0366             KK=IABS(IX(N,M)-IX(N,MM))
0367             IF(KK-NM) 633,633,631
0368        631  NM=KK
0369             IF(NM-55) 633,634,634
0370        634  WRITE(6,628) N
0371        628  FORMAT(33HO BAND WIDTH EXCEEDS ALLOWABLE N=I5)
0372             CALL EXIT
0373        633  CONTINUE
0374        160  CONTINUE
0375             MBAND=2*NM+2
0376             WRITE(6,440)
0377        440 OFORMAT (    50H1            NODE      X-ORDINATE  Y-ORDINATE  X-LOAD
                1           29H OR DISPL.  Y-LOAD OR DISPL.   )
0378             WRITE(6,2004) (J,X(J),Y(J),UXTYPE(J),UX(J),UYTYPE(J),UY(J),
                1J=1,NUMNP )
0379        2004 FORMAT( 10X,I5,3X,1P2D12.3,4X,A4,1PD12.4,2X,A4,1PD12.4)
0380             WRITE(6,605)
0381        605  FORMAT(1H1,10X , 56HEL.NO       J1        J2        J3        J4
                1    MAT.NO)
0382             WRITE(6,650) (N,(IX(N,J),J=1,4),MTYPE(N),N=1,NUMEL)
0383        650  FORMAT( 10X,I5,5I10)
           C
           C*********************************************************************
           C*                       FORM STIFFNESS MATRIX                       *
           C*********************************************************************
           C
0384             CALL STIFF
           C
           C*********************************************************************
           C                      SOLVE FOR DISPLACEMENTS                       *
           C*********************************************************************
           C
```

```
0385           CALL BANSOL
         C
         C*************************************************************************
         C*                            PRINT DISPLACEMENTS                        *
         C*************************************************************************
0386           PRINT 2009
0387      2009 FORMAT (    45H1          NODE NO.        X-DISPLACEMENT
              1            20H       Y-DISPLACEMENT,/)
0388           K=0
0389           DO 350 NB=1,NUMBLK
0390           DO 350 NM=111,220,2
0391           K=K+1
0392           PRINT 2010,K,A(NM,NB),A(NM+1,NB)
0393      2010 FORMAT (10X,I5,10X,1PD15.6,10X,1PD15.6)
0394           IF(K-NUMNP) 350,360,360
0395       350 CONTINUE
         C
         C*************************************************************************
         C                                COMPUTE STRESS                          *
         C*************************************************************************
         C
0396       360 CALL STRESS
         C
         C
0397           STOP
0398           END
```


TOTAL MEMORY REQUIREMENTS 002BDC BYTES

```
                C
                C
                C
0001                  SUBROUTINE STIFF
0002                  IMPLICIT REAL*8(A-H,O-Z)
0003                  COMMON TYPE(8),HED(18),E(8),X(950) ,Y(950) ,UX(950) ,UY(950) ,VOL,
                     1UXTYPE(950) ,UYTYPE(950) ,PR(8),
                     2MTYPE(930),NUMNP,NUMEL,NUMAT,N
0004                  COMMON /ARG/ XXX(5),YYY(5),S(10,10),DD(3,3),HH(6,10),P(10),XX(4),
                     1YY(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),
                     2DK(2,2),TYPE1,TYPE2,TEST1,TEST2,SIG(10),IX(930,4),LM(4),NR
0005                  COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
                C***********************************************************************
                C*                            INITIALIZATION                           *
                C***********************************************************************
0006                  REWIND 2
0007                  NB=55
0008                  ND=2*NB
0009                  ND2=2*ND
0010                  NUMBLK=0
                C
0011                  DO 50 N=1,ND2
0012                  B(N)=0.0
0013                  DO 50 M=1,ND
0014               50 A(N,M)=0.0
                C***********************************************************************
                C*                 FORM STIFFNESS MATRICES IN BLOCKS                   *
                C***********************************************************************
0015               60 NUMBLK=NUMBLK+1
0016                  NH=NB*(NUMBLK+1)
0017                  NM=NH-NB
0018                  NL=NM-NB+1
0019                  KSHIFT=2*NL-2
                C
0020                  DO 210 N=1,NUMEL
0021                  IF(MTYPE(N))210,210,65
0022               65 DO 80 I=1,4
0023                  IF(IX(N,I)-NL) 80,70,70
0024               70 IF(IX(N,I)-NM) 90,90,80
0025               80 CONTINUE
0026                  GO TO 210
0027               90 CALL QUAD
0028                  MTYPE(N)=-MTYPE(N)
0029                  IF(VOL)142,142,144
0030              142 WRITE(6,2000) N
0031             2000 FORMAT(26HONEGATIVE AREA ELEMENT NO. I4)
0032                  CALL EXIT
0033              144 IF(IX(N,3)-IX(N,4)) 145,165,145
                C
0034              145 DO 150 II=1,9
0035                  CC=S(II,10)/S(10,10)
0036                  P(II)=P(II)-CC*P(10)
0037                  DO 150 JJ=1,9
0038              150 S(II,JJ)=S(II,JJ)-CC*S(10,JJ)
                C
0039                  DO 160 II=1,8
0040                  CC=S(II,9)/S(9,9)
0041                  P(II)=P(II)-CC*P(9)
```

```
0042              DO 160 JJ=1,8
0043         160  S(II,JJ)=S(II,JJ)-CC*S(9,JJ)
           C
           C***********************************************************************
           C*              ADD ELEMENT STIFFNESSES TO TOTAL STIFFNESS              *
           C***********************************************************************
           C
0044         165  DO 166 I=1,4
0045         166  LM(I)=2*IX(N,I)-2
           C
0046              DO 200 I=1,4
0047              DO 200 K=1,2
0048              II=LM(I)+K-KSHIFT
0049              KK=2*I-2+K
0050              B(II)=B(II)+P(KK)
0051              DO 200 J=1,4
0052              DO 200 L=1,2
0053              JJ=LM(J)+L-II+1-KSHIFT
0054              LL=2*J-2+L
0055              IF(JJ) 200,200,175
0056         175  IF(ND-JJ) 180,195,195
0057         180  WRITE(6,2001) N
0058        2001  FORMAT(29HOBAND WIDTH EXCEEDS ALLOWABLE I4)
0059              CALL EXIT
0060         195  A(II,JJ)=A(II,JJ)+S(KK,LL)
0061         200  CONTINUE
0062         210  CONTINUE
           C
           C***********************************************************************
           C*                     ADD CONCENTRATED FORCES WITHIN BLOCK             *
           C***********************************************************************
           C
0063              DO 250 N=NL,NM
0064              K=2*N-KSHIFT
0065              IF(UYTYPE(N).NE.TYPE1) GO TO 240
0066              B(K) = B(K)+ UY(N)
0067         240  IF(UXTYPE(N).NE.TYPE1) GO TO 250
0068              B(K-1)= B(K-1)+ UX(N)
0069         250  CONTINUE
           C
           C***********************************************************************
           C      BOUNDARY CONDITIONS
           C***********************************************************************
           C
0070          310 DO 400 M=NL,NH
0071              IF (M-NUMNP) 315,315,400
0072          315 U=UX(M)
0073              N=2*M-1-KSHIFT
0074              IF ( UXTYPE(M) .NE. TYPE2 ) GO TO 370
0075              CALL MODIFY ( A, B, ND2, MBAND, N, U  )
0076          370 N = N + 1
0077              U = UY(M)
0078              IF ( UYTYPE(M) .NE. TYPE2 ) GO TO 400
0079              CALL MODIFY ( A, B, ND2, MBAND, N, U  )
0080          400 CONTINUE
           C
           C***********************************************************************
           C*    WRITE BLOCK OF EQUATIONS ON TAPE AND SHIFT UP LOWER BLOCK         *
```

```
            C*********************************************************************
0081               WRITE (2) ( B(N),(A(N,M),M=1,MBAND),N=1,ND )
            C
0082               DO 420 N=1,ND
0083               K=N+ND
0084               B(N)=B(K)
0085               B(K)=0.0
0086               DO 420 M=1,ND
0087               A(N,M)=A(K,M)
0088           420 A(K,M)=0.0
            C*********************************************************************
            C*                      CHECK FOR LAST BLOCK                           *
            C*********************************************************************
0089               IF(NM-NUMNP) 60,480,480
0090           480 CONTINUE
0091           500 RETURN
0092               END
```


TOTAL MEMORY REQUIREMENTS 000834 BYTES

```
              C
              C
              C
0001                SUBROUTINE QUAD
0002                IMPLICIT REAL*8(A-H,O-Z)
0003                REAL*8 DCOS,DSIN
0004                COMMON TYPE(8),HED(18),E(8),X(950) ,Y(950) ,UX(950) ,UY(950) ,VOL,
                   1UXTYPE(950) ,UYTYPE(950) ,PR(8),
                   2MTYPE(930),NUMNP,NUMEL,NUMAT,N
0005                COMMON /ARG/ XXX(5),YYY(5),S(10,10),DD(3,3),HH(6,10),P(10),XX(4),
                   1YY(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),
                   2DK(2,2),TYPE1,TYPE2,TEST1,TEST2,SIG(10),IX(930,4),LM(4),NR
0006                COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
0007                I=IX(N,1)
0008                J=IX(N,2)
0009                K=IX(N,3)
0010                L=IX(N,4)
0011                MAT=MTYPE(N)
              C**********************************************************************
              C*                     STRESS -STRAIN  RELATIONS  -LINEAR ELASTIC        *
              C**********************************************************************
0012                IF(TYPE(4)-TEST1)10,30,10
0013             10 IF(TYPE(4)-TEST2)20,40,20
0014             20 WRITE(6,2000)
0015           2000 FORMAT(38HO PLANE STRESS OR STRAIN TYPE ERROR    )
0016                CALL EXIT
              C
0017             30 COMM=E(MAT)/(1.0-PR(MAT)*PR(MAT))
0018                C(1,1)=COMM
0019                C(1,2)=COMM*PR(MAT)
0020                C(1,3)=0.0
0021                C(1,4)=0.0
0022                C(2,1)=C(1,2)
0023                C(2,2)=C(1,1)
0024                C(2,3)=0.0
0025                C(2,4)=0.0
0026                C(3,1)=0.0
0027                C(3,2)=0.0
0028                C(3,3)=0.0
0029                C(3,4)=0.0
0030                C(4,1)=0.0
0031                C(4,2)=0.0
0032                C(4,3)=0.0
0033                C(4,4)=COMM*0.5*(1.0-PR(MAT))
0034                GO TO 50
0035             40 COMM=E(MAT)/((1.0+PR(MAT))*(1.0-2.0*PR(MAT)))
0036                C(1,1)=COMM*(1.0-PR(MAT))
0037                C(1,2)=COMM*PR(MAT)
0038                C(1,3)=0.0
0039                C(1,4)=0.0
0040                C(2,1)=C(1,2)
0041                C(2,2)=C(1,1)
0042                C(2,3)=0.0
0043                C(2,4)=0.0
0044                C(3,1)=C(1,2)
0045                C(3,2)=C(1,2)
0046                C(3,3)=0.0
0047                C(3,4)=0.0
```

```
0048            C(4,1)=0.0
0049            C(4,2)=0.0
0050            C(4,3)=0.0
0051            C(4,4)=COMM*0.5*(1.0-2.0*PR(MAT))
          C*********************************************************************
          C*                    FORM QUADRILATERAL STIFFNESS MATRIX            *
          C*********************************************************************
0052         50 XXX(5)=(X(I)+X(J)+X(K)+X(L))/4.0
0053            YYY(5)=(Y(I)+Y(J)+Y(K)+Y(L))/4.0
0054            DO 94 M=1,4
0055            MM=IX(N,M)
0056            XXX(M)=X(MM)
0057         94 YYY(M)=Y(MM)
          C
0058            DO 100 II=1,10
0059            P(II)=0.0
0060            DO 95 JJ=1,6
0061         95 HH(JJ,II)=0.0
0062            DO 100 JJ=1,10
0063        100 S(II,JJ)=0.0
0064            IF(K-L)125,120,125
0065        120 CALL TRISTF(1,2,3)
0066            XXX(5)=(XXX(1)+XXX(2)+XXX(3))/3.0
0067            YYY(5)=(YYY(1)+YYY(2)+YYY(3))/3.0
0068            GO TO 130
0069        125 VOL=0.0
0070            CALL TRISTF(4,1,5)
0071            CALL TRISTF(1,2,5)
0072            CALL TRISTF(2,3,5)
0073            CALL TRISTF(3,4,5)
          C
0074            DO 138 II=1,6
0075            DO 138 JJ=1,10
0076        138 HH(II,JJ)=HH(II,JJ)/4.
          C
0077        130 RETURN
0078            END
```


TOTAL MEMORY REQUIREMENTS 00073E BYTES

```
                C
                C
0001                  SUBROUTINE TRISTF(II,JJ,KK)
0002                  IMPLICIT REAL*8(A-H,O-Z)
0003                  COMMON TYPE(8),HED(18),E(8),X(950) ,Y(950) ,UX(950) ,UY(950) ,VOL,
                     1UXTYPE(950) ,UYTYPE(950) ,PR(8),
                     2MTYPE(930),NUMNP,NUMEL,NUMAT,N
0004                  COMMON /ARG/ XXX(5),YYY(5),S(10,10),DD(3,3),HH(6,10),P(10),XX(4),
                     1YY(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),
                     2DK(2,2),TYPE1,TYPE2,TEST1,TEST2,SIG(10),IX(930,4),LM(4),NR
0005                  COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
                C***********************************************************************
                C*                           INITIALIZATION                            *
                C***********************************************************************
0006                  LM(1)=II
0007                  LM(2)=JJ
0008                  LM(3)=KK
0009                  XX(1)=XXX(II)
0010                  XX(2)=XXX(JJ)
0011                  XX(3)=XXX(KK)
0012                  XX(4)=XXX(II)
0013                  YY(1)=YYY(II)
0014                  YY(2)=YYY(JJ)
0015                  YY(3)=YYY(KK)
0016                  YY(4)=YYY(II)
                C
0017               85 DO 100 I=1,6
0018                  DO  90 J=1,10
0019                  F(I,J)=0.0
0020               90 H(I,J)=0.0
0021                  DO 100 J=1,6
0022              100 D(I,J)=0.0
                C***********************************************************************
                C*                   FORM INTEGRAL(G)T*(C)*(G)                         *
                C***********************************************************************
0023                  COMM=XX(2)*(YY(3)-YY(1))+XX(1)*(YY(2)-YY(3))+XX(3)*(YY(1)-YY(2))
0024                  VOL=VOL+COMM*0.5
0025                  IF(COMM)107,500,107
0026              107 D(2,2)=C(1,1)*COMM*0.5
0027                  D(2,3)=C(1,4)*COMM*0.5
0028                  D(2,5)=D(2,3)
0029                  D(2,6)=C(1,2)*COMM*0.5
0030                  D(3,3)=C(4,4)*COMM*0.5
0031                  D(3,5)=D(3,3)
0032                  D(5,5)=D(3,3)
0033                  D(6,6)=C(2,2)*COMM*0.5
                C
0034              108 DO 110 I=1,6
0035                  DO 110 J=1,6
0036              110 D(J,I)=D(I,J)
                C***********************************************************************
                C*              FORM COEFFICIET-DISPLACEMENT TRANSFORMATION MATRIX     *
                C***********************************************************************
0037                  DD(1,1)=(XX(2)*YY(3)-XX(3)*YY(2))/COMM
0038                  DD(1,2)=(XX(3)*YY(1)-XX(1)*YY(3))/COMM
0039                  DD(1,3)=(XX(1)*YY(2)-XX(2)*YY(1))/COMM
0040                  DD(2,1)=(YY(2)-YY(3))/COMM
0041                  DD(2,2)=(YY(3)-YY(1))/COMM
```

```
0042           DD(2,3)=(YY(1)-YY(2))/COMM
0043           DD(3,1)=(XX(3)-XX(2))/COMM
0044           DD(3,2)=(XX(1)-XX(3))/COMM
0045           DD(3,3)=(XX(2)-XX(1))/COMM
         C
0046           DO 120 I=1,3
0047           J=2*LM(I)-1
0048           H(1,J)=DD(1,I)
0049           H(2,J)=DD(2,I)
0050           H(3,J)=DD(3,I)
0051           H(5,J+1)=DD(2,I)
0052       120 H(6,J+1)=DD(3,I)
         C*********************************************************************
         C*                FORM ELEMENT STIFNESS MATRIX (H)T*(D)*(H)          *
         C*********************************************************************
0053           DO 130 J=1,10
0054           DO 130 K=1,6
0055           IF(H(K,J))128,130,128
0056       128 DO 129 I=1,6
0057       129 F(I,J)=F(I,J)+D(I,K)*H(K,J)
0058       130 CONTINUE
         C
0059           DO 140 I=1,10
0060           DO 140 K=1,6
0061           IF(H(K,I))138,140,138
0062       138 DO 139 J=1,10
0063       139 S(I,J)=S(I,J)+H(K,I)*F(K,J)
0064       140 CONTINUE
         C*********************************************************************
         C*                       FORM STRAIN TRANSFORMATIOA MATRIX           *
         C*********************************************************************
         C
0065       400 DO 410 I=1,6
0066           DO 410 J=1,10
0067       410 HH(I,J)=HH(I,J)+H(I,J)
         C
0068       500 RETURN
0069           END
```


TOTAL MEMORY REQUIREMENTS 000796 BYTES

```
      SUBROUTINE BANSOL
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
      NN=110
      NL=NN+1
      NH=NN+NN
      REWIND 1
      REWIND 2
      NB=0
      MM=MBAND
      GO TO 150
C
C***********************************************************************
C*         REDUCE EQUATIONS BY BLOCKS - SHIFT BLOCK OF EQUATIONS       *
C***********************************************************************
C
  100 NB=NB+1
      DO 125 N=1,NN
      NM=NN+N
      B(N)=B(NM)
      B(NM)=0.0
      DO 125 M=1,MM
      A(N,M)=A(NM,M)
  125 A(NM,M)=0.0
C***********************************************************************
C*                 READ NEXT  BLOCK OF EQUATIONS INTO CORE             *
C***********************************************************************
C
      IF(NUMBLK-NB) 150,200,150
C
  150 READ (2) ( B(N),(A(N,M),M=1,MM),N=NL,NH)
C
      IF(NB) 200,100,200
C***********************************************************************
C*                          REDUCE BLOCK OF EQUATIONS                  *
C***********************************************************************
  200 DO 300 N=1,NN
      IF(A(N,1)) 225,300,225
  225 B(N)=B(N)/A(N,1)
      DO 275 L=2,MM
      IF(A(N,L)) 230,275,230
  230 Q=A(N,L)/A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-Q*A(N,K)
      B(I)=B(I)-A(N,L)*B(N)
      A(N,L)=Q
  275 CONTINUE
  300 CONTINUE
C***********************************************************************
C*               WRITE BLOCK OF REDUCED EQUATIONS ON TAPE 1            *
C***********************************************************************
      IF(NUMBLK-NB) 375,400,375
  375 WRITE (1) (B(N),(A(N,M),M=2,MM),N=1,NN)
      GO TO 100
C
```

```
      C*********************************************************************
      C*                        BACK SUBSTITUTION                          *
      C*********************************************************************
      C
  400 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,MM
      L=N+K-1
  425 B(N)=B(N)-A(N,K)*B(L)
      NM=N+NN
      B(NM)=B(N)
  450 A(NM,NB)=B(N)
      NB=NB-1
      IF(NB) 475,500,475
  475 BACKSPACE 1
      READ (1) (B(N),(A(N,M),M=2,MM),N=1,NN)
      BACKSPACE 1
      GO TO 400
      C
  500 RETURN
      END
```


TOTAL MEMORY REQUIREMENTS 000746 BYTES

```
      SUBROUTINE MODIFY(A,B,NEQ,MBAND,N,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(220,110),B(220)
C
      DO 250 M=2,MBAND
      K=N-M+1
      IF(K)235,235,230
  230 B(K)=B(K)-A(K,M)*U
      A(K,M)=0.0
  235 K=N+M-1
      IF(NEQ-K) 250,240,240
  240 B(K)=B(K)-A(N,M)*U
      A(N,M)=0.0
  250 CONTINUE
      A(N,1)=1.0
      B(N)=U
      RETURN
      END
```


TOTAL MEMORY REQUIREMENTS 0002CE BYTES

```
      SUBROUTINE STRESS
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TYPE(8),HED(18),E(8),X(950) ,Y(950) ,UX(950) ,UY(950) ,VOL,
     1UXTYPE(950) ,UYTYPE(950) ,PR(8),
     2MTYPE(930),NUMNP,NUMEL,NUMAT,N
      COMMON /ARG/ XXX(5),YYY(5),S(10,10),DD(3,3),HH(6,10),P(10),XX(4),
     1YY(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),
     2DK(2,2),TYPE1,TYPE2,TEST1,TEST2,SIG(10),IX(930,4),LM(4),NR
      COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
C
C*********************************************************************
C     COMPUTE ELEMENT STRESS
C*********************************************************************
C
      MPRINT = 0
      PI=3.141592653589793
      NN=110
C
      DO 90 M = 1,NUMEL
C
      N = M
      MTYPE(N) = IABS(MTYPE(N))
      CALL QUAD
C
      DO 10 I = 1,4
      II = 2 * I
      KK=2*IX(N,I)
      NB=KK/NN
      NJ=MOD(KK,NN)
      IF(NJ) 5,6,5
    5 NB=NB+1
      JJ=NN+NJ
      GO TO 8
    6 JJ=2*NN
    8 P(II-1)=A(JJ-1,NB)
      P(II)=A(JJ,NB)
   10 CONTINUE
C
      DO 20 I = 1,2
      XX(I) = P(I+8)
      DO 20 K = 1,8
   20 XX(I) = XX(I) - S(I+8,K) * P(K)
C
      COMM = S(9,9) * S(10,10) - S(9,10) * S(10,9)
      IF (COMM) 30,40,30
   30 P(9) = (S(10,10) * XX(1) - S(9,10) * XX(2)) / COMM
      P(10) = (-S(10,9) * XX(1) + S(9,9) * XX(2)) / COMM
C
   40 DO 50 I = 1,6
      TP(I) = 0.0
      DO 50 K = 1,10
   50 TP(I) = TP(I) + HH(I,K) * P(K)
C
   52 XX(1) = TP(2)
      XX(2) = TP(6)
      XX(3) = 0.0
      XX(4) = TP(3) + TP(5)
```

```
FORTRAN IV G LEVEL 1, MOD 4            STRESS           DATE = 70188        20/30/07          PAGE 0002

             56 DO 60 I = 1,4
                SIG(I) = 0.
                DO 60 K = 1,4
             60 SIG(I) = SIG(I) + C(I,K) * XX(K)
                RAD= ((SIG(1)-SIG(2))/2.)**2 + (SIG(4))**2
                TMAX=DSQRT(RAD)
                SAVR= ((SIG(1) + SIG(2))/2.)
                SIG1= SAVR + TMAX
                SIG2= SAVR - TMAX
                TAN2A=2.*SIG(4)/(SIG(1)-SIG(2))
                ANG=DATAN(TAN2A)*90.0/PI
          C
          C*********************************************************************
          C*    OUTPUT STRESSES                                                *
          C*********************************************************************
          C
             62 IF (MPRINT) 80,70,80
             70 PRINT 2000
                MPRINT = 62
             80 MPRINT = MPRINT - 1
          C
                PRINT 2001, N,XXX(5),YYY(5),(SIG(I),I=1,4),SIG1,SIG2,ANG
          C
             90 CONTINUE
          C
                RETURN
          C
           2000FORMAT (    50H1      EL.       X          Y        X-STRESS       Y-STRESS
               1           48H      Z-STRESS      XY-SHEAR     SIG1          SIG2
               2           13H            ANG)
           2001 FORMAT ( 5X, I3, 2F8.2, 7D13.4 )
                END
```

```
FORTRAN IV G LEVEL 1, MOD 4            BLK DATA         DATE = 70188        20/30/07          PAGE 0001

                BLOCK DATA
                IMPLICIT REAL*8(A-H,O-Z)
                COMMON TYPE(8),HED(18),E(8),X(950) ,Y(950) ,UX(950) ,UY(950) ,VOL,
               1UXTYPE(950) ,UYTYPE(950) ,PR(8),
               2MTYPE(930),NUMNP,NUMEL,NUMAT,N
                COMMON /ARG/ XXX(5),YYY(5),S(10,10),DD(3,3),HH(6,10),P(10),XX(4),
               1YY(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),
               2OK(2,2),TYPE1,TYPE2,TEST1,TEST2,SIG(10),IX(930,4),LM(4),NR
                COMMON /BANARG/ A(220,110),B(220),MBAND,NUMBLK
                DATA TYPE1/4HLOAD/,TYPE2/4HDISP/,TEST1/3HESS/,TEST2/3HAIN/
                END
```

APPENDIX C

TYPICAL INPUT DATA

```
                                  80/80  LIST

     00000000011111111112222222222333333333344444444445555555555666666666677777777778
     12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
0001               FINITE ELEMENT ANALYSIS    LINER WITH THREE CRACKS
0002 PLANE STRAIN ANALYSIS
0003      4
0004      0.3500E 07     0.1500E 00
0005      0.3000E 08     0.3000E 00
0006      0.1500E 05     0.1000E 00
0007      0.4000E 05     0.3300E 00
0008              3
0009     12.0        0.625
0010      0.3600E 02     0.4400E 02     0.6800E 02     0.8000E 00     49      1       5       5
0011                4000.          60000.
0012                   1    15
0013                  26    40
0014                  41    55
0015                  66    80
0016                  81    95
0017                 106   120
0018                 121   135
0019                 147   161
0020                 162   176
0021                 188   197
0022                 198   212
0023                 213   227
0024                 237   251
0025                 252   266
0026                 275   289
0027                 290   304
0028                 313   327
0029                 328   342
0030                 350   364
0031                 365   379
0032                 387   401
0033                 402   416
0034                 424   438
0035                 439   453
0036                 461   475
0037                 476   490
0038                 498   512
0039                 513   527
0040                 535   549
0041                 550   564
0042                 572   586
0043                 567   601
0044                 609   623
0045                 624   638
0046                 646   660
0047                 661   675
0048                 684   698
0049                 699   713
0050                 722   736
0051                 737   751
0052                 760   774
0053                 775   789
0054                 798   812
0055                 813   827
```

```
      00000000011111111112222222222333333333344444444445555555555666666666677777777778
      12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
0056               835   849
0057               850   864
0058               872   886
0059               887   901
0060               909   923
0061               924   938
0062                 6.0      14.0      26.9       1.308     5.0       2.0       2.0
0063               100.
0064                15    25
0065                55    65
0066                95   105
0067               135   146
0068               176   186
0069               227   236
0070               266   274
0071               304   312
0072               342   349
0073               379   386
0074               416   423
0075               453   460
0076               490   497
0077               527   534
0078               564   571
0079               601   608
0080               638   645
0081               675   683
0082               713   721
0083               751   759
0084               789   797
0085               827   834
0086               864   871
0087               901   908
0088               938   945
0089                 1
0090    187        340.      748.         DISP     0.0        LOAD -1510.0
0091                 1    14
0092                     1       1      26      27       2
0093                    14      14      39      40      15
0094                15    28
0095                    15      26      41      42      27
0096                    28      39      54      55      40
0097                29    40
0098                    29      15      40      16      16
0099                    30      16      40      56      56
0100                    31      40      55      56      56
0101                    32      16      56      57      17
0102                    40      24      64      65      25
0103                41    54
0104                    41      41      66      67      42
0105                    54      54      79      80      55
0106                55    68
0107                    55      66      81      82      67
0108                    68      79      94      95      80
0109                69    80
0110                    69      55      80      56      56
```

APPENDIX D

TYPICAL OUTPUT

NODE	X-ORDINATE	Y-ORDINATE	X-LOAD OR DISPL.		Y-LOAD OR DISPL.	
1	0.0	3.600D 01	LOAD	0.0	LOAD	0.0
2	0.0	3.680D 01	LOAD	-1.0616D 04	LOAD	0.0
3	0.0	3.743D 01	LOAD	-1.0616D 04	LOAD	0.0
4	0.0	3.846D 01	LOAD	0.0	LOAD	0.0
5	0.0	3.949D 01	LOAD	0.0	LOAD	0.0
6	0.0	4.052D 01	LOAD	0.0	LOAD	0.0
7	0.0	4.154D 01	LOAD	0.0	LOAD	0.0
8	0.0	4.258D 01	LOAD	1.0616D 04	LOAD	0.0
9	0.0	4.320D 01	LOAD	1.0616D 04	LOAD	0.0
10	0.0	4.400D 01	LOAD	0.0	LOAD	0.0
11	0.0	4.880D 01	DISP	0.0	LOAD	0.0
12	0.0	5.360D 01	DISP	0.0	LOAD	0.0
13	0.0	5.840D 01	DISP	0.0	LOAD	0.0
14	0.0	6.320D 01	DISP	0.0	LOAD	0.0
15	0.0	6.800D 01	DISP	0.0	LOAD	0.0
16	0.0	7.400D 01	DISP	0.0	LOAD	0.0
17	0.0	8.800D 01	DISP	0.0	LOAD	0.0
18	0.0	1.149D 02	DISP	0.0	LOAD	0.0
19	0.0	1.501D 02	DISP	0.0	LOAD	0.0
20	0.0	1.961D 02	DISP	0.0	LOAD	0.0
21	0.0	2.563D 02	DISP	0.0	LOAD	0.0
22	0.0	3.350D 02	DISP	0.0	LOAD	0.0
23	0.0	4.380D 02	DISP	0.0	LOAD	0.0
24	0.0	5.727D 02	DISP	0.0	LOAD	0.0
25	0.0	7.480D 02	DISP	0.0	LOAD	-4.9238D 03
26	2.355D 00	3.592D 01	LOAD	0.0	LOAD	0.0
27	2.407D 00	3.672D 01	LOAD	0.0	LOAD	0.0
28	2.448D 00	3.734D 01	LOAD	0.0	LOAD	0.0
29	2.515D 00	3.837D 01	LOAD	0.0	LOAD	0.0
30	2.582D 00	3.940D 01	LOAD	0.0	LOAD	0.0
31	2.650D 00	4.043D 01	LOAD	0.0	LOAD	0.0
32	2.717D 00	4.146D 01	LOAD	0.0	LOAD	0.0
33	2.785D 00	4.248D 01	LOAD	0.0	LOAD	0.0
34	2.825D 00	4.311D 01	LOAD	0.0	LOAD	0.0
35	2.878D 00	4.391D 01	LOAD	0.0	LOAD	0.0
36	3.192D 00	4.870D 01	LOAD	0.0	LOAD	0.0
37	3.506D 00	5.349D 01	LOAD	0.0	LOAD	0.0
38	3.820D 00	5.827D 01	LOAD	0.0	LOAD	0.0
39	4.133D 00	6.306D 01	LOAD	0.0	LOAD	0.0
40	4.447D 00	6.785D 01	LOAD	0.0	LOAD	0.0
41	4.699D 00	3.569D 01	LOAD	0.0	LOAD	0.0
42	4.803D 00	3.649D 01	LOAD	0.0	LOAD	0.0
43	4.885D 00	3.710D 01	LOAD	0.0	LOAD	0.0
44	5.019D 00	3.813D 01	LOAD	0.0	LOAD	0.0
45	5.154D 00	3.915D 01	LOAD	0.0	LOAD	0.0
46	5.288D 00	4.017D 01	LOAD	0.0	LOAD	0.0
47	5.423D 00	4.119D 01	LOAD	0.0	LOAD	0.0
48	5.557D 00	4.221D 01	LOAD	0.0	LOAD	0.0
49	5.639D 00	4.283D 01	LOAD	0.0	LOAD	0.0
50	5.743D 00	4.362D 01	LOAD	0.0	LOAD	0.0
51	6.370D 00	4.838D 01	LOAD	0.0	LOAD	0.0
52	6.996D 00	5.314D 01	LOAD	0.0	LOAD	0.0
53	7.623D 00	5.790D 01	LOAD	0.0	LOAD	0.0
54	8.249D 00	6.266D 01	LOAD	0.0	LOAD	0.0
55	8.876D 00	6.742D 01	LOAD	0.0	LOAD	0.0
56	9.659D 00	7.337D 01	LOAD	0.0	LOAD	0.0
57	1.149D 01	8.725D 01	LOAD	0.0	LOAD	0.0
58	1.500D 01	1.139D 02	LOAD	0.0	LOAD	0.0
59	1.959D 01	1.488D 02	LOAD	0.0	LOAD	0.0
60	2.560D 01	1.944D 02	LOAD	0.0	LOAD	0.0
61	3.345D 01	2.541D 02	LOAD	0.0	LOAD	0.0
62	4.373D 01	3.322D 02	LOAD	0.0	LOAD	0.0
63	5.717D 01	4.343D 02	LOAD	0.0	LOAD	0.0

EL.NO	J1	J2	J3	J4	MAT.NO
1	1	26	27	2	1
2	2	27	28	3	2
3	3	28	29	4	1
4	4	29	30	5	1
5	5	30	31	6	1
6	6	31	32	7	1
7	7	32	33	8	1
8	8	33	34	9	2
9	9	34	35	10	1
10	10	35	36	11	3
11	11	36	37	12	3
12	12	37	38	13	3
13	13	38	39	14	3
14	14	39	40	15	3
15	26	41	42	27	1
16	27	42	43	28	2
17	28	43	44	29	1
18	29	44	45	30	1
19	30	45	46	31	1
20	31	46	47	32	1
21	32	47	48	33	1
22	33	48	49	34	2
23	34	49	50	35	1
24	35	50	51	36	3
25	36	51	52	37	3
26	37	52	53	38	3
27	38	53	54	39	3
28	39	54	55	40	3
29	15	40	16	16	4
30	16	40	56	56	4
31	40	55	56	56	4
32	16	56	57	17	4
33	17	57	58	18	4
34	18	58	59	19	4
35	19	59	60	20	4
36	20	60	61	21	4
37	21	61	62	22	4
38	22	62	63	23	4
39	23	63	64	24	4
40	24	64	65	25	4
41	41	66	67	42	1
42	42	67	68	43	2
43	43	68	69	44	1
44	44	69	70	45	1
45	45	70	71	46	1
46	46	71	72	47	1
47	47	72	73	48	1
48	48	73	74	49	2
49	49	74	75	50	1
50	50	75	76	51	3
51	51	76	77	52	3
52	52	77	78	53	3
53	53	78	79	54	3
54	54	79	80	55	3
55	66	81	82	67	1
56	67	82	83	68	2
57	68	83	84	69	1
58	69	84	85	70	1
59	70	85	86	71	1
60	71	86	87	72	1
61	72	87	88	73	1
62	73	88	89	74	2
63	74	89	90	75	1

NODE NO.	X-DISPLACEMENT	Y-DISPLACEMENT
1	-1.940117D 00	1.844326D 00
2	-1.800966D 00	1.844262D 00
3	-1.689101D 00	1.844378D 00
4	-1.501745D 00	1.844162D 00
5	-1.317465D 00	1.843740D 00
6	-1.133847D 00	1.843357D 00
7	-9.495885D-01	1.842966D 00
8	-7.626980D-01	1.842238D 00
9	-6.512243D-01	1.841037D 00
10	-5.121245D-01	1.840511D 00
11	0.0	1.043168D 00
12	0.0	4.757753D-01
13	0.0	1.013347D-01
14	0.0	-1.759456D-01
15	0.0	-3.916387D-01
16	0.0	-4.486675D-01
17	0.0	-5.576502D-01
18	0.0	-6.804069D-01
19	0.0	-7.821159D-01
20	0.0	-8.838751D-01
21	0.0	-1.000214D 00
22	0.0	-1.142552D 00
23	0.0	-1.322269D 00
24	0.0	-1.552703D 00
25	0.0	-1.849714D 00
26	-1.948005D 00	1.427844D 00
27	-1.806823D 00	1.418039D 00
28	-1.696030D 00	1.409507D 00
29	-1.514190D 00	1.397085D 00
30	-1.331908D 00	1.384999D 00
31	-1.149748D 00	1.372858D 00
32	-9.676352D-01	1.360662D 00
33	-7.860797D-01	1.348777D 00
34	-6.756086D-01	1.342055D 00
35	-5.346938D-01	1.332939D 00
36	-6.443926D-02	8.031676D-01
37	3.400119D-02	3.852796D-01
38	4.096852D-02	5.057170D-02
39	3.097327D-02	-2.063279D-01
40	8.576402D-03	-4.111328D-01
41	-1.981586D 00	1.015359D 00
42	-1.842963D 00	9.967989D-01
43	-1.733566D 00	9.820257D-01
44	-1.554666D 00	9.582206D-01
45	-1.375474D 00	9.344278D-01
46	-1.196200D 00	9.107200D-01
47	-1.017153D 00	8.871158D-01
48	-8.384817D-01	8.635674D-01
49	-7.296390D-01	8.492106D-01
50	-5.912593D-01	8.309999D-01
51	-1.398805D-01	4.580585D-01
52	2.215424D-02	1.609371D-01
53	5.746018D-02	-8.695361D-02
54	4.745914D-02	-2.885672D-01
55	7.884142D-03	-4.495024D-01
56	2.020152D-02	-4.950976D-01
57	1.796850D-02	-5.834169D-01
58	1.218328D-02	-6.910525D-01
59	7.148320D-03	-7.859814D-01
60	4.551697D-03	-8.843116D-01
61	3.215118D-03	-9.984223D-01
62	2.370338D-03	-1.138900D 00

EL.	X	Y	X-STRESS	Y-STRESS	Z-STRESS	XY-SHEAR	SIG1	SIG2	ANG
1	1.19	36.36	0.1028D 05	0.4564D 03	0.1611D 04	-0.2585D 04	0.1092D 05	-0.1822D 03	-0.1388D 02
2	1.21	37.07	0.4627D 04	0.7247D 03	0.1606D 04	0.3787D 03	0.4664D 04	0.6883D 03	0.5492D 01
3	1.24	37.90	0.7085D 04	0.7974D 02	0.1075D 04	0.2473D 04	0.7870D 04	-0.7055D 03	0.1761D 02
4	1.27	38.93	0.1893D 04	-0.6042D 03	0.1933D 03	0.6222D 03	0.2039D 04	-0.7506D 03	0.1325D 02
5	1.31	39.96	-0.1010D 03	-0.9954D 03	-0.1645D 03	0.2191D 03	-0.5017D 02	-0.1046D 04	0.1305D 02
6	1.34	40.99	-0.2026D 04	-0.1452D 04	-0.5218D 03	0.7855D 03	-0.9030D 03	-0.2576D 04	-0.3497D 02
7	1.38	42.01	-0.6353D 04	-0.2287D 04	-0.1296D 04	0.2642D 04	-0.9866D 03	-0.7653D 04	-0.2621D 02
8	1.40	42.84	-0.4840D 04	-0.2822D 04	-0.2299D 04	0.4295D 03	-0.2734D 04	-0.4928D 04	-0.1153D 02
9	1.43	43.55	-0.9392D 04	-0.2565D 04	-0.1794D 04	-0.1857D 04	-0.2093D 04	-0.9865D 04	0.1427D 02
10	1.52	46.35	-0.3945D 03	-0.2078D 04	-0.2472D 03	-0.1762D 03	-0.3763D 03	-0.2096D 04	-0.5911D 01
11	1.67	51.15	-0.2418D 03	-0.1556D 04	-0.1798D 03	-0.2910D 03	-0.1802D 03	-0.1617D 04	-0.1195D 02
12	1.83	55.94	0.3222D 02	-0.1106D 04	-0.1074D 03	-0.1458D 03	0.5061D 02	-0.1124D 04	-0.7186D 01
13	1.99	60.73	0.4430D 02	-0.8333D 03	-0.7890D 02	-0.9132D 02	0.5370D 02	-0.8427D 03	-0.5879D 01
14	2.15	65.53	-0.4125D 01	-0.6615D 03	-0.6656D 02	-0.6645D 02	0.2524D 01	-0.6682D 03	-0.5714D 01
15	3.57	36.21	0.9236D 04	-0.1595D 03	0.1362D 04	-0.5211D 03	0.9265D 04	-0.1883D 03	-0.3165D 01
16	3.64	36.91	0.2584D 04	-0.8318D 03	0.5256D 03	0.5485D 03	0.2670D 04	-0.9178D 03	0.8902D 01
17	3.72	37.74	0.5426D 04	-0.5117D 03	0.7371D 03	0.8311D 03	0.5540D 04	-0.6259D 03	0.7820D 01
18	3.82	38.76	0.2839D 04	-0.3955D 03	0.3665D 03	0.1639D 04	0.3524D 04	-0.1081D 04	0.2269D 02
19	3.92	39.79	0.1991D 03	-0.8253D 03	-0.9393D 02	0.1786D 04	0.1545D 04	-0.2171D 04	0.3700D 02
20	4.02	40.81	-0.2329D 04	-0.1221D 04	-0.5325D 03	0.1838D 04	0.1446D 03	-0.3695D 04	-0.3662D 02
21	4.12	41.84	-0.4749D 04	-0.1038D 04	-0.8680D 03	0.1389D 04	-0.5754D 03	-0.5211D 04	-0.1841D 02
22	4.20	42.66	-0.2545D 04	-0.7024D 03	-0.9741D 03	0.7050D 03	-0.4635D 03	-0.2783D 04	-0.1871D 02
23	4.27	43.37	-0.7960D 04	-0.1115D 04	-0.1361D 04	0.8247D 03	-0.1017D 04	-0.8058D 04	-0.6774D 01
24	4.55	46.15	-0.3227D 03	-0.1247D 04	-0.1569D 03	-0.3479D 03	-0.2064D 03	-0.1363D 04	-0.1849D 02
25	5.02	50.93	-0.2735D 03	-0.1026D 04	-0.1300D 03	-0.4360D 03	-0.7396D 02	-0.1226D 04	-0.2460D 02
26	5.49	55.70	-0.7953D 02	-0.8505D 03	-0.9300D 02	-0.3476D 03	0.5402D 02	-0.9840D 03	-0.2102D 02
27	5.96	60.47	-0.1619D 02	-0.6813D 03	-0.6975D 02	-0.2370D 03	0.5964D 02	-0.7571D 03	-0.1774D 02
28	6.43	65.25	-0.4333D 02	-0.5589D 03	-0.6022D 02	-0.1664D 03	0.5698D 01	-0.6079D 03	-0.1642D 02
29	1.48	69.95	-0.1632D 03	-0.5070D 03	-0.2212D 03	-0.7059D 02	-0.1492D 03	-0.5209D 03	-0.1116D 02
30	4.70	71.74	-0.1693D 03	-0.5351D 03	-0.2325D 03	-0.8034D 02	-0.1525D 03	-0.5520D 03	-0.1186D 02
31	7.66	69.55	-0.1852D 03	-0.3803D 03	-0.1866D 03	-0.1088D 03	-0.1367D 03	-0.4289D 03	-0.2406D 02
32	5.29	80.65	-0.9238D 02	-0.3520D 03	-0.1467D 03	-0.6159D 02	-0.7851D 02	-0.3659D 03	-0.1269D 02
33	6.62	101.02	-0.5433D 02	-0.2153D 03	-0.8897D 02	-0.2822D 02	-0.4952D 02	-0.2201D 03	-0.9663D 01
34	8.65	131.93	-0.4732D 02	-0.1477D 03	-0.6436D 02	-0.1090D 02	-0.4615D 02	-0.1489D 03	-0.6126D 01
35	11.30	172.36	-0.4779D 02	-0.1209D 03	-0.5568D 02	-0.4333D 01	-0.4754D 02	-0.1212D 03	-0.3379D 01
36	14.76	225.24	-0.4806D 02	-0.1097D 03	-0.5206D 02	-0.1873D 01	-0.4801D 02	-0.1098D 03	-0.1739D 01
37	19.30	294.41	-0.4824D 02	-0.1046D 03	-0.5044D 02	-0.8810D 00	-0.4823D 02	-0.1046D 03	-0.8953D 00
38	25.23	384.88	-0.4851D 02	-0.1021D 03	-0.4970D 02	-0.4494D 00	-0.4851D 02	-0.1021D 03	-0.4805D 00
39	32.98	503.22	-0.4884D 02	-0.1008D 03	-0.4937D 02	-0.2378D 00	-0.4884D 02	-0.1008D 03	-0.2623D 00
40	43.31	659.15	-0.4895D 02	-0.1002D 03	-0.4921D 02	-0.1032D 00	-0.4895D 02	-0.1002D 03	-0.1155D 00
41	5.93	35.89	0.6314D 04	-0.1379D 02	0.9451D 03	-0.1383D 03	0.6317D 04	-0.1681D 02	-0.1251D 01
42	6.04	36.60	0.2218D 04	-0.2083D 03	0.6030D 03	0.9567D 03	0.2550D 04	-0.5401D 03	0.1913D 02
43	6.18	37.41	0.3863D 04	-0.4875D 03	0.5063D 03	0.1217D 04	0.4180D 04	-0.8046D 03	0.1461D 02
44	6.34	38.43	0.2290D 04	-0.8371D 03	0.2179D 03	0.1922D 04	0.3204D 04	-0.1751D 04	0.2544D 02
45	6.51	39.44	0.6892D 03	-0.9930D 03	-0.4557D 02	0.2237D 04	0.2238D 04	-0.2542D 04	0.3470D 02
46	6.68	40.46	-0.9700D 03	-0.1042D 04	-0.3019D 03	0.2120D 04	0.1114D 04	-0.3127D 04	0.4451D 02
47	6.85	41.48	-0.2661D 04	-0.1094D 04	-0.5631D 03	0.1724D 04	0.1641D 02	-0.3771D 04	-0.3278D 02
48	6.98	42.29	-0.1633D 04	-0.1078D 04	-0.8133D 03	0.9206D 03	-0.3939D 03	-0.2317D 04	-0.3661D 02
49	7.10	42.99	-0.5105D 04	-0.1059D 04	-0.9246D 03	0.1105D 04	-0.7767D 03	-0.5387D 04	-0.1433D 02
50	7.55	45.76	-0.2728D 03	-0.6626D 03	-0.9354D 02	-0.3806D 03	-0.4011D 02	-0.8953D 03	-0.3144D 02
51	8.34	50.49	-0.2797D 03	-0.5550D 03	-0.8347D 02	-0.4435D 03	0.4700D 02	-0.8817D 03	-0.3638D 02
52	9.12	55.22	-0.1724D 03	-0.4927D 03	-0.6651D 02	-0.3967D 03	0.9526D 02	-0.7604D 03	-0.3401D 02
53	9.90	59.96	-0.1052D 03	-0.4334D 03	-0.5386D 02	-0.3003D 03	0.7295D 02	-0.6115D 03	-0.3068D 02
54	10.68	64.69	-0.8690D 02	-0.3757D 03	-0.4626D 02	-0.2249D 03	0.3598D 02	-0.4986D 03	-0.2865D 02
55	8.26	35.43	0.2695D 04	-0.2257D 03	0.3704D 03	0.2547D 03	0.2717D 04	-0.2478D 03	0.4946D 01
56	8.42	36.12	0.1365D 04	-0.5814D 03	0.2350D 03	0.1188D 04	0.1927D 04	-0.1144D 04	0.2534D 02
57	8.61	36.93	0.2257D 04	-0.8660D 03	0.2087D 03	0.1662D 04	0.2976D 04	-0.1585D 04	0.2339D 02
58	8.84	37.93	0.1803D 04	-0.1172D 04	0.9466D 02	0.2241D 04	0.3005D 04	-0.2374D 04	0.2821D 02
59	9.08	38.93	0.1212D 04	-0.1293D 04	-0.1206D 02	0.2380D 04	0.2649D 04	-0.2729D 04	0.3112D 02
60	9.31	39.94	0.4768D 03	-0.1246D 04	-0.1153D 03	0.2170D 04	0.1950D 04	-0.2719D 04	0.3417D 02
61	9.55	40.94	-0.4218D 03	-0.1037D 04	-0.2189D 03	0.1656D 04	0.9547D 03	-0.2414D 04	0.3974D 02
62	9.73	41.74	-0.3142D 03	-0.7327D 03	-0.3141D 03	0.8977D 03	0.3984D 03	-0.1445D 04	0.3844D 02

VITA

Yohannes Woldemariam

Candidate for the Degree of

Doctor of Philosophy

Thesis: DISTRIBUTION OF STRESS AROUND A TUNNEL CAVITY, USING THE FINITE ELEMENT METHOD

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Mogadisho Somalia, January 17, 1931 the son of Woldemariam Kifle and Letehaimanot Adhonom.

Education: Graduated from the Haile Selassie I Secondary School, Addis Abeba, Ethiopia, in 1949; having passed the University of London Matriculation in 1949, joined the College of Technology, Loughborough, Leicestershire, England, and was awarded the Pass Diploma in Civil Engineering in 1955; received the degree of Master of Science from Oklahoma State University, Stillwater, Oklahoma, in May, 1966; completed the requirements for the degree of Doctor of Philosophy at Oklahoma State University, Stillwater, Oklahoma, in May, 1971.

Professional Experience: Engineering design and drafting with the British Reinforced Concrete Engineering Co., Ltd., Stafford, England, 1956; design and drafting with F. C. Construction Co., Derby, England, 1957; the Blue Nile Geodetic Survey, Ethiopia, November, 1958 to August, 1962; instructor at Haile Selassie I University, February, 1962 to January, 1965; instructor at Tristate College, Angola, Indiana, September, 1966 to June, 1967; graduate teaching assistant, Oklahoma State University, January, 1968 to August, 1968; instructor at Langston University, Langston, Oklahoma, September, 1968 to May, 1970.