OPTIMUM DESIGN OF PRESTRESSED PLATE GIRDERS AND PRESTRESSED COMPOSITE GIRDERS

By

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GIRDERS AND PRESTRESSED
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Champa Lal Mehta

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A	Cross-section area of the steel section
A_c	Cross-section area of the concrete slab
A_v	Cross-section area of the tendon
A_w	Cross-section area of the web of the girder
A_1, A_2	Cross-section areas of the compressed chord and stretched chord of the girder, in loaded state, respectively
A_3	Transformed area (steel) of the concrete slab
a	Asymmetry of the steel section = e_2/e_1
D	$V^*h/M_{0\,max,\,s}$
E_c, E_s, E_v	Moduli of elasticity for concrete slab, steel section and prestressing tendon, respectively
$e_1 = e_{1,s}$; $e_2 = e_{2,s}$	Distances from the centroidal axis to the chord 1 and chord 2, respectively, for the steel section
$e_{1,c}$; $e_{2,c}$; $e_{3,c}$	Distances from the centroidal axis to the chord 1, chord 2 and extreme fiber of the concrete slab, respectively, for the composite section
$F(x)$	Lagrangian function
f	Distance from the centroidal axis of the steel section to the centroid of the tendon
$f(x)$	Function of x
$g_i(x)$	i-th constraint
$H_1, \ldots, H_{10}$	Expressions, Table III
h	Depth of the steel section
h_c	Thickness of the concrete slab

$I = I_s$	Moment of inertia of the steel section
I_c	Moment of inertia of the composite section
ℓ	Span of a simply supported girder
ℓ_v	Length of the tendon
M_0	Moment due to applied loads at a general section
M_{0br}	Moment in the steel girder at anchoring brackets, due to applied loads
$M_{0br,s}$, $M_{0br,c}$	Moments in the composite girder at anchoring brackets, due to loads carried by steel section and composite section, respectively
M_{0max}	Maximum moment in the steel girder due to applied loads
$M_{0max,s}$; $M_{0max,c}$	Maximum moments in the composite girder, due to loads supported by steel section and composite section, respectively
N	Axial force
N_{ji}, P_{ji}	Negative and positive signs of the coefficients in the Lagrange multipliers matrix, respectively
n_c	E_s/E_c
n_v	E_s/E_v
$n_{v\ell}$, n_{vu}	Lower and upper prestress accuracy factors, respectively
O_w, O_p, O_b	Objective functions for weight, price of the materials and bearing capacity of the girder, respectively
P	Concentrated load
p	Distributed load
p_{vs}	Ratio of prices per unit volume of the materials used in the tendon and in the girder, respectively
R_1, R_2, R_3, R_v	Design stresses for chord 1, chord 2, concrete slab (transformed) and prestressing tendon, respectively

$S_1 = S_{1,s}$; $S_2 = S_{2,s}$	Section moduli; $I/e_1 = I_s/e_{1,s}$; $I/e_2 = I_s/e_{2,s}$, respectively
$S_{1,c}$, $S_{2,c}$, $S_{3,c}$	Section moduli; $I_c/e_{1,c}$, $I_c/e_{2,c}$, $I_c/e_{3,c}$, respectively
s, t	Lengths locating ends of the prestressing tendon
U	Strain energy
V^*	Standard value of the prestress force
$W_1, \ldots, W_6$	Expressions, Table III
X	Redundant force in the tendon for steel girder
X_s, X_c	Redundant forces in the tendon for steel section and composite section, respectively
$Y, Y_1, \ldots, Y_8$	Expressions, Table III
$Z_1, \ldots, Z_9$	Expressions, Table III
α, α_1, α_2, α_3, α_v, α_w	Non-dimensional coefficients for cross-section areas, defined by Eqs (3.2) and (3.3)
β	$h_c/2h$
β_v, β_x, $\beta_{x,s}$, $\beta_{x,c}$	Non-dimensional coefficients for forces in the tendon at various stages of load, defined by Eqs (3.2) and (3.3)
δ	Web thickness
ϵ, ρ_1, ρ_2, ρ_c, ρ_v	Ratios of design stresses, defined by Eq (2.4)
κ	Parameter defining location of the tendon, $f/e_2 = f/e_{2,s}$
λ	Web slenderness parameter
μ_{br}, $\mu_{br,s}$, $\mu_{br,c}$	Coefficients at moments defined by Eqs (2.3) and (3.4)
μ_{avg}, $\mu_{avg,s}$, $\mu_{avg,c}$	Coefficients at moments defined by Eqs (2.3) and (3.30)
η	Ratio of the maximum bending moments = $M_{0max,c}/M_{0max,s}$
η_{avg}	$\eta\, \mu_{avg,c}/\mu_{avg,s}$

CHAPTER I

INTRODUCTION

1.1 Discussion

Structural optimization can be identified as the rational process of structural design, as opposed to the empiricism of conventional design methods. Structural optimization is one of the few fields of engineering in which the impact of modern computer-based techniques has not yet reached the level of practical problems. Intensive research over the past ten years has produced a multitude of approaches, some of which have materialized into powerful computer programs. The basic problems of structural design, however, have received no clear answer.

No current method of structural optimization can be guaranteed to find the global optimum unless the problem is proved to be convex. This can be done only in special cases. It is known that a fully stressed design, one in which each element of the structure sustains a limiting stress under at least one of the specified load conditions, may not be unique and may not be optimal. On the other hand, there is no way to determine a priori that the optimum design will not be fully stressed. With these uncertainties, in both optimum design and fully stressed design and the considerably greater computational effort for optimization, it will be very difficult for optimization methods to displace the fast and familiar fully stressed design methods on problems which are controlled by stress conditions alone. However, for problems in which deformations or other constraints are active, the fully stressed approach is obviously unreasonable. Nevertheless, in practice many designs seem to be fully stressed.

Of the two most important materials used in construction--concrete and steel--the principle of prestressing has been used much more extensively in concrete structures. However, the principle of prestressing is not limited to concrete structures and may be applied equally well to steel structures. The aim of prestressing principles in steel structures is not to overcome tensile deficiencies of the material, as is done for concrete, but to build opposing stresses into the member to counteract the stresses caused by external forces. Prestressed steel structures have been used to some extent in Europe with considerable economy in material, but comparatively little use has been made of them in the United States.

This study presents a method of determining the optimum design for a prestressed plate girder and a prestressed composite plate girder of constant depth. A search of fully stressed designs is made to determine the optimum design for a selected criterion. The Kuhn-Tucker theorem (1), a necessary condition for optimality, is used to establish the relationship between the optimum design and the fully stressed design.

1.2 Survey of Structural Optimization Studies

The present survey confines itself to the development of structural optimization in the field of elastic behavior. Even in this reduced area, the amount of material published is so extensive as to discourage a systematic classification. Here, only the most significant trends are followed, in their historical sequence, and only the principal contributors are mentioned. For more extensive surveys of the literature, reference is made to (2) and (3).

The first contribution, in terms of a mathematical model solvable by some optimization technique, came from the studies of Michell (4). The subject of Michell's paper was to find the minimum weight or minimum material design of statically determinate structures subjected to single load situations. It was concluded that the stresses in the members of a determinate truss must be at their limiting values for the structure to have the minimum weight. Michell provided the basis for the development of the modern concepts of optimum design of structures. This concept of fully stressed design has been used throughout the aerospace industries for the past three decades; see, for example, Shanley (5) and Cox (6).

In the '50's, Vinogradov (7) and Radtsig (8) consolidated the formulation of the minimum weight problem by associating the energy theorems with the classical Muller-Breslau equations. The examples reported are limited to one or two degrees of redundancy. Heyman (9) and others obtained slightly different results, starting with the same formulation of the problem.

In the '60's, following the expansion of research in optimization, several techniques became available for the solution, at least in theory, of the structural optimization problem. Prager, Schield (10), and others derived the optimality criteria from classical extremum principles.

Schmit (11), introducing the concept of synthesis, clearly defined the structural optimization problem as a non-linear programming problem. At the same time, Best (12), Gellatly (13) and others developed programs for optimization of complex structures, using stress rate convergence criteria, which are slight modifications of conventional design methods. With the stress rate method, the direction of movement in the solution space is given at each step by a vector which has components proportional to the amount by which each member area must be modified in order for the member to become fully stressed. The separation of the fully stressed solution from the minimum weight solution was detected by several authors. The quantitative definition of the problem, however, was given by Razani (14), followed later by Kicher (15).

Linear programming was used for structural problems by Moses (16) and Cornell, Reinschmidt, and Brotchie (17), through Kelley's cutting plane method. Brown and Ang (18) employed Rossen's gradient projection algorithm for weight optimization of simple plane frames. This approach requires a great deal of computational work. Schmit and Fox (19) used the Fiacco-McCormic technique to convert a constraint problem to that of an unconstrained problem. They introduced penalty functions to add the constraints to the objective function. The method of feasible directions was used by Karnes and Tocher (20) for plane stress problems, using the finite element approach. Toakley (21) considered a finite set of members extracted from the steel tables and adapted Gomory's first algorithm for integer programming. The number of variables, according to his formulation, is the product of the number of members in the structure and the number of profiles in the table.

The research effort in structural optimization has sharply increased over the past few years. Many techniques and algorithms are now available for the general problem. An efficient solution procedure in future years will depend in part on the creation of a well defined and well justified link between structural optimization and structural design.

The optimum design of prestressed steel structures has been partially solved, using simplifying assumptions, by Tochacek (22), Vasilev (23), Vedenikov (24) and others. A detailed study of the development and use of prestressed steel flexural members has been reported in References (22) and (25). Design of prestressed composite steel girders and the effect of creep and shrinkage in concrete slabs has been studied by Szilard (26), Hoadley (27) and others.

1.3 Approach of This Study

A prestressed plate girder and a prestressed composite girder of constant cross-section and subjected to one critical inplane load are considered. Prestressing is induced by a tendon of high strength material located parallel and close to the chord of the girder which would be in tension under the applied loads (the stretched chord). For convenience, weight of the girder (steel section) is selected as the criterion for optimization.

A set of non-linear constraints is derived from strength and continuity conditions. All constraints are considered to be tight (equality constraints) in order to obtain a fully stressed design. It is shown that the maximum number of independent variables is 7 for a prestressed plate girder and 9 for a prestressed composite girder. The number of constraints is one less than the number of variables involved in the problem. Using geometric relations, the constraints are reduced to one equation with two variables in the case of a steel girder and two equations with three variables in the case of a composite girder. The governing equations, coupled with an optimization condition or a side constraint, are treated numerically by a sequential search technique, the Golden Section search (28). The complexity of these equations excludes any explicit solution.

CHAPTER II

CROSS-SECTION PROPERTIES AND LOADS

2.1 General

A plate girder is a deep flexural member employed to carry loads which cannot be supported economically by rolled beams. The use of a plate girder gives the designer the advantage of selecting component parts of convenient and economical size. Further, prestressing induces favorable distribution of internal stresses and thus increases the load bearing capacity (at yield) by as high as 30% to 35% (25). For many practical reasons, plate girders are fabricated in different shapes (Fig 1a - d). For optimization purposes, different profiles may be replaced by an idealized section, as shown in Fig 1e.

Prestressed tendons may be placed below, coaxial with, or above the stretched chord (chord 2), in a simply supported girder, as indicated in Fig 2. The location of the tendon is indicated by the value of the parameter κ as shown in Figs 1 and 2a.

2.2 Cross-Section Properties

2.2.1 Discussion

Section properties of a plate girder can be evaluated by simple derivations employing geometrical relations, conditions for locating centroidal axes, and rules for determining moment of inertia and

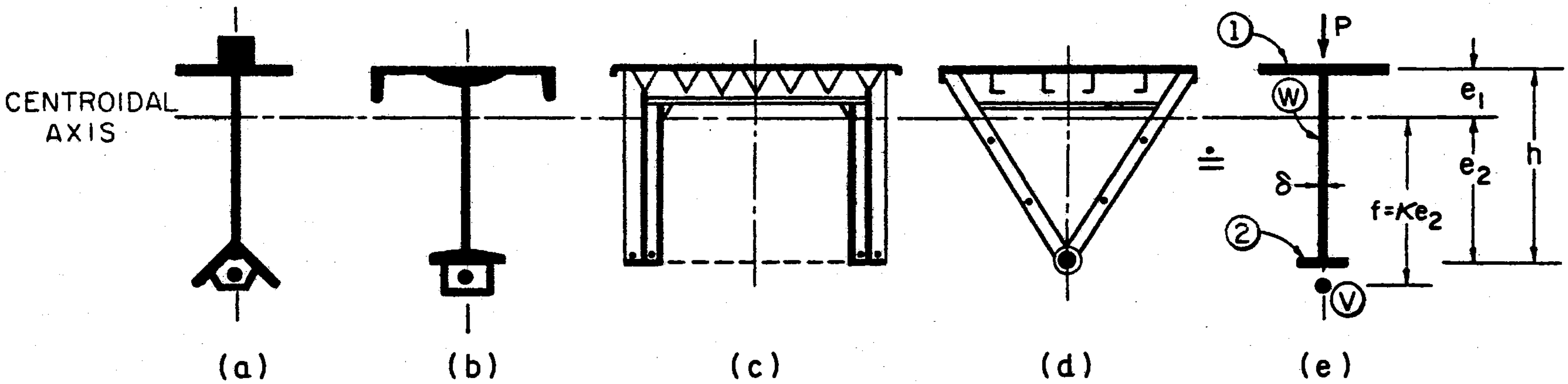


Figure 1. Typical Prestressed Steel Sections (a) Through (d) and Idealized Section (e)

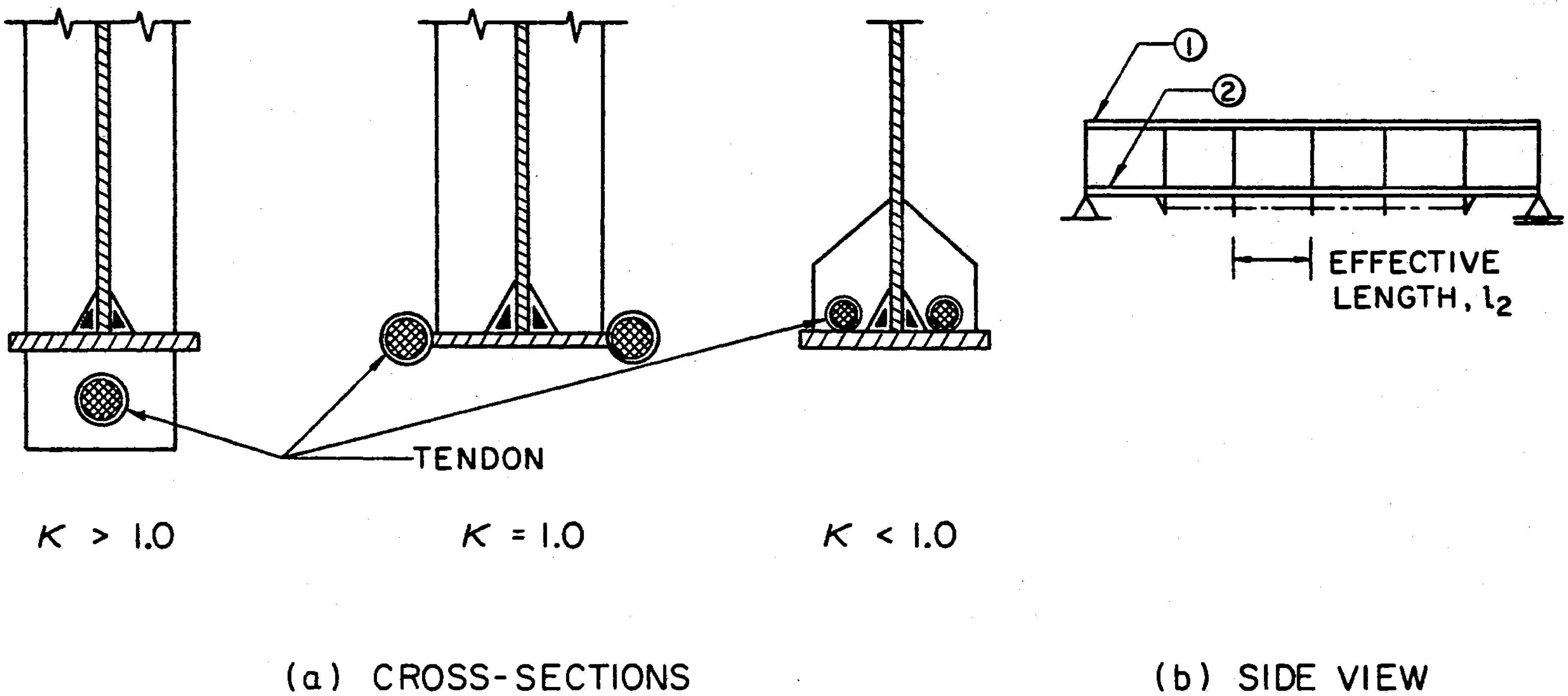


Figure 2. Location of Prestressing Tendons in a Simply Supported Girder

section moduli with respect to the horizontal centroidal axis. To simplify the analysis chord (flange) areas are assumed to be concentrated at their respective centroids. This assumption is acceptable since the thickness of the plates used is usually very small compared to the depth of the girder. However, the concrete slab in the composite girder is considered to have a finite thickness.

Where essential, the following subscripts are used:

avg	average value;
br	anchoring bracket;
c	composite section;
s	steel section;
v	prestressing;
w	girder-web;
0	primary system (a statically determinate beam without tendon);
1, 2	compressed and stretched chord (steel section) in loaded state, respectively;
3	concrete slab.

2.2.2 Steel Section

All of the section properties for the idealized I-section, Fig 1e, can be expressed in terms of four parameters. The following quantities will be employed:

A = total cross-section area of the girder;

a = e_2/e_1 (Fig 1) = section asymmetry;

h = depth of the girder;

α_2 = ratio of stretched chord area to total cross-section area of the girder.

The section properties for the steel girder are given in Table I, using the symbols shown in Figs 1e and 3.

2.2.3 Composite Section

For evaluating the section properties for the composite section, the transformed composite section is used, as shown in Fig 3. These properties are based on the modular ratio,

$$n_c = \frac{E_s}{E_c},$$

where E_s and E_c are the moduli of elasticity of steel and concrete, respectively.

In this study, the superimposed load is considered as live load only following the procedure of Hoadley (27). If all of the superimposed load is acting permanently, the effects of creep in the concrete may be accounted for by increasing the modular ratio to $3n_c$ as suggested by Szilard (26).

In addition to the quantities used for steel sections, the following definitions are required to express section properties for the transformed composite section:

$$\alpha_3 = \frac{A_3}{A},$$

where A_3 = transformed concrete area = A_c/n_c;

$$\beta = \frac{h_c}{2h},$$

where h_c = thickness of the concrete slab.

Section properties in terms of A, a, h, α_2, α_3, and β for the composite section are given in Table I.

TABLE I

EXPRESSIONS FOR SECTION PROPERTIES

Quantities (1)	Section Properties: Steel Section (2)	Section Properties: Composite Section[2] (3)
Section Areas	$A_1 = \left(\alpha_2 - \frac{1-a}{1+a}\right)A$ $A_2 = \alpha_2 A$ $A_w = 2\left(\frac{1}{1+a} - \alpha_2\right)A$	$A_3 = \alpha_3 A$
Distances to the Critically Stressed Fibers from Centroidal Axes	$e_1 = e_{1,s} = \frac{h}{1+a}$ $e_2 = e_{2,s} = \frac{ah}{1+a}$	$e_{1,c} = \frac{W_4 h}{(1+\alpha_3)(1+a)}$ $e_{2,c} = \frac{W_2 h}{(1+\alpha_3)(1+a)}$ $e_{3,c} = \frac{W_3 h}{(1+\alpha_3)(1+a)}$
Moment of Inertia	$I = I_s = \frac{A h^2 W_1}{6(1+a)^2}$ [1]	$I_c = \frac{A h^2}{6(1+a)^2}\left(W_1 + \frac{W_5}{1+\alpha_3}\right)$
Section Moduli	$S_1 = S_{1,s} = \frac{Ah W_1}{6(1+a)}$ $S_2 = S_{2,s} = \frac{Ah W_1}{6(1+a)}$	$S_{1,c} = \frac{Ah}{6(1+a)}\left\{\frac{W_1(1+\alpha_3)+W_5}{W_4}\right\}$ $S_{2,c} = \frac{Ah}{6(1+a)}\left\{\frac{W_1(1+\alpha_3)+W_5}{W_2}\right\}$ $S_{3,c} = \frac{Ah}{6(1+a)}\left\{\frac{W_1(1+\alpha_3)+W_5}{W_3}\right\}$

[1]The expressions for W_1 through W_5 are given in Table 3.

[2]The expressions for the quantities related to the steel section of the composite girder (when concrete is not effective) are the same as those for the steel girder, column (2).

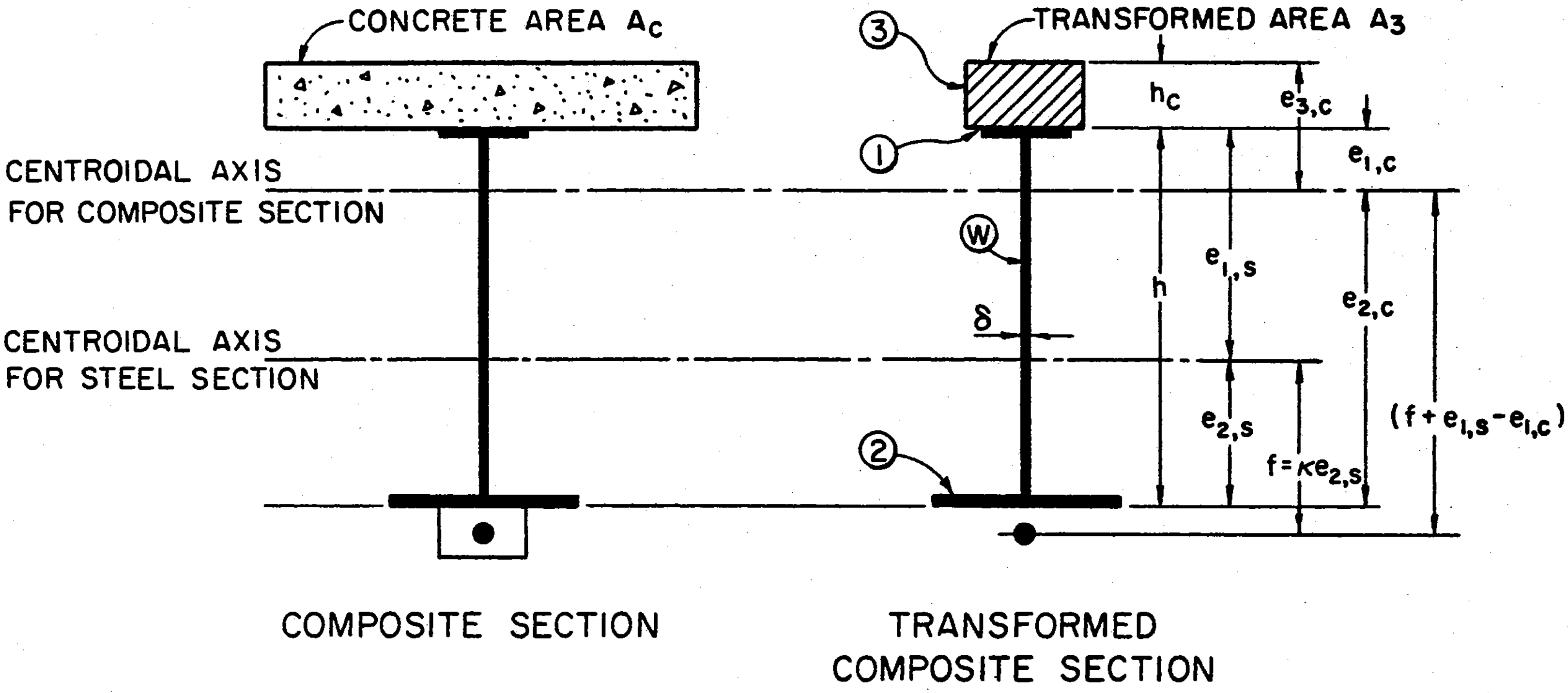


Figure 3. Prestressed Composite Section and its Idealization

2.3 Loads

One of the important features of the present approach is that only the shape of the bending moment diagram is required for the analysis and not the absolute values of the bending moment at any section. Six basic bending moment diagrams are illustrated in Fig 4a.

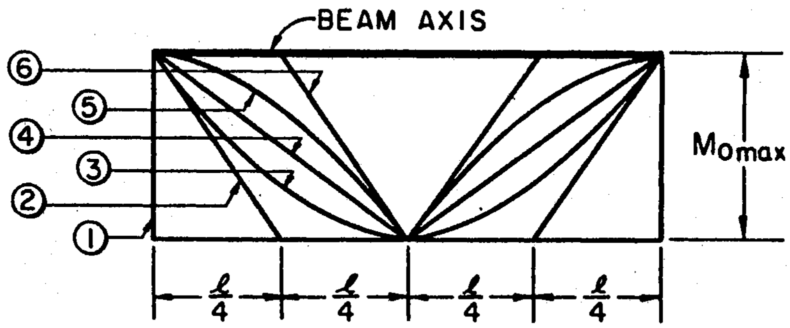
Types of supports are of secondary importance, because the same bending moment diagram could correspond to a simply supported beam, a cantilever, a simple beam with overhangs, etc., as shown in Fig 4b. Each system is naturally subjected to different loads (29), (30). For convenience, an explanation to follow will refer to a simply supported beam.

2.4 Prestressing Tendon

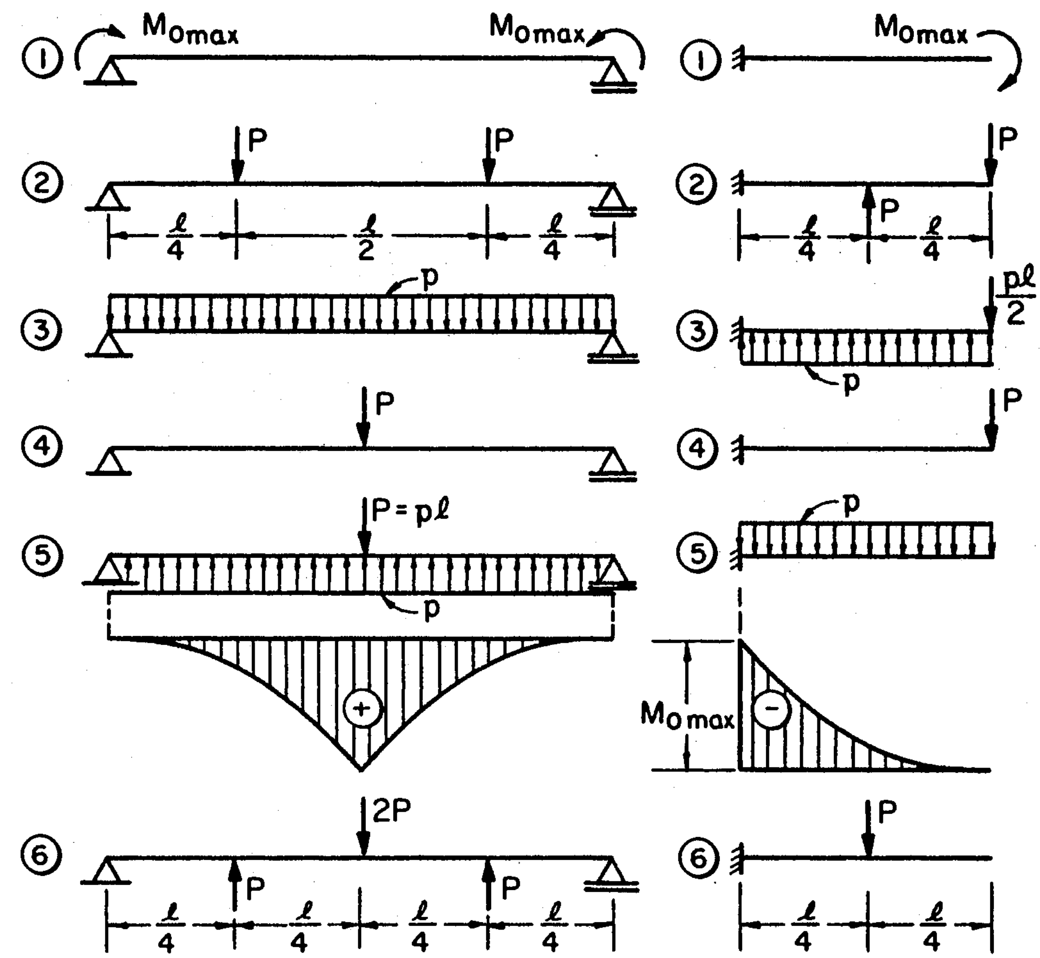
Since a girder is capable of carrying part of the load without any assistance of the prestressing tendon, a girder prestressed by a short length tendon, as illustrated in Fig 2b, placed appropriately will require less material than a girder prestressed by a full length tendon. Often a full length tendon is preferred because of the difficulty in providing anchorages between the supports and for other practical considerations. In this study, both the full length tendon and the short length tendon are investigated.

2.5 Increase of the Tendon Force Due to Applied Loads

It is assumed in the analysis that the plate girder is prestressed in the fabricating shop in the unloaded state. The brackets for anchoring the tendon are assumed to be rigid. The force (V^*) in the tendon



(a) SHAPES OF BENDING MOMENT DIAGRAMS



(b) CORRESPONDING LOADS

Figure 4. Bending Moment Diagrams and Loads

produced by prestressing is increased due to the deformation of the prestressed girder under the applied loads. The increase of the tendon force is denoted by X.

A simply supported steel girder with a short tendon is shown in Fig 5a. With no applied load, the bending moment diagram (M_v) and the axial force diagram (N_v) will be as shown in Fig .5b. When loads are applied, the bending moment diagram due to the applied load only will be as shown in Fig 5c (M_p), while the bending moment diagram (M_v) and the axial force diagram (N_v) due to the tendon force will increase as shown in Fig 5c.

An expression for the redundant force X is derived by making the strain energy of the prestressed girder stationary with respect to X. The following expression is obtained for a steel girder prestressed by a short length tendon. Similar expressions for the composite girder may be obtained.

The strain energy U due to both bending and axial effect is

$$U = \frac{1}{2E_s I}\left\{\int_0^s M_0^2\,dx + \int_s^{s+\ell_v} (M_0 - Xf)^2\,dx + \int_{s+\ell_v}^{\ell} M_0^2\,dx\right\} + \frac{X^2\ell_v}{2E_s A} + \frac{X^2\ell_v}{2E_v A_v}\,, \tag{2.1}$$

where

A_v = cross-section area of the tendon;

E_v = modulus of elasticity of the tendon;

f = eccentricity of the tendon for a prestressed steel girder, Figs 1e and 3;

I = moment of inertia of the steel girder;

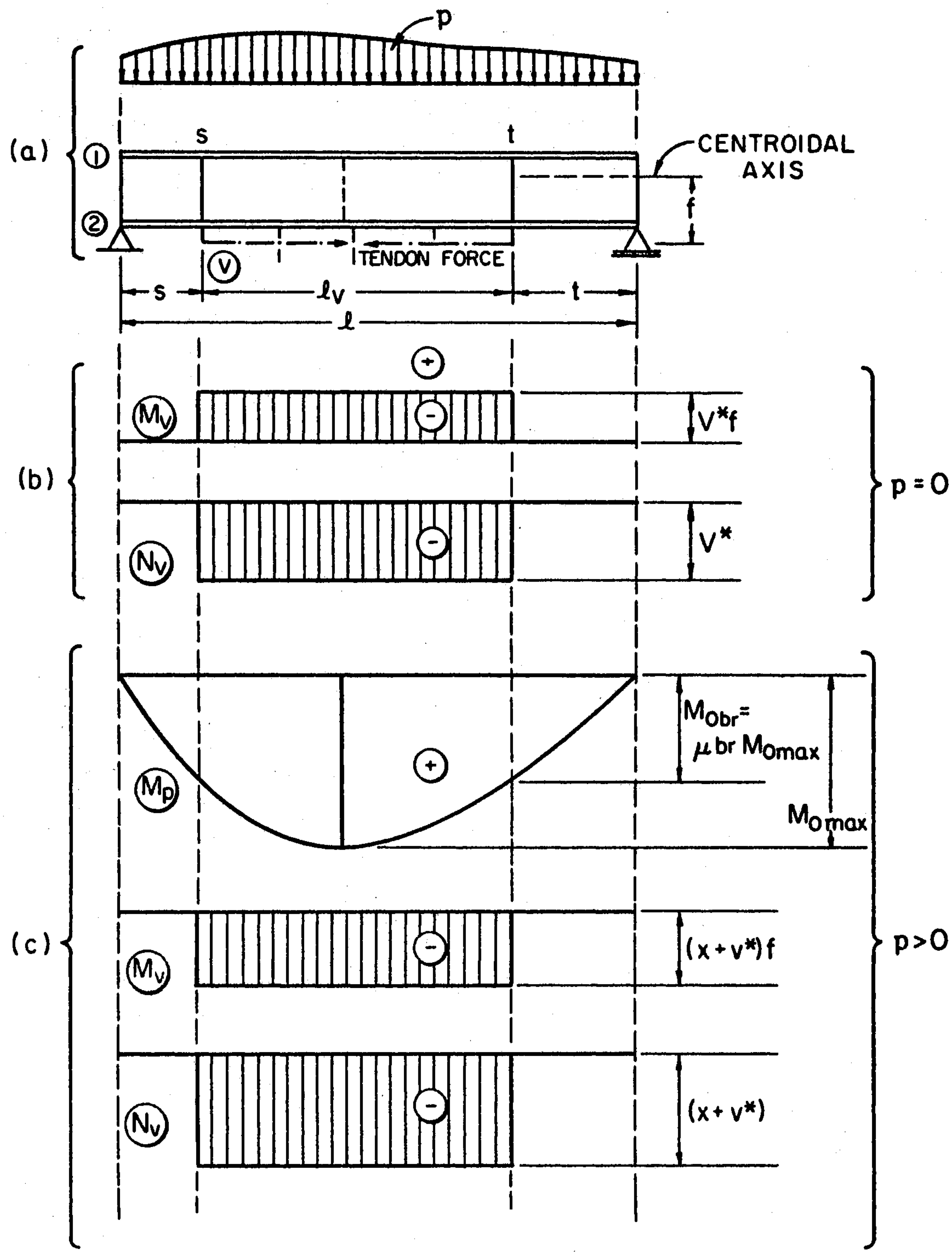


Figure 5. Bending Moment and Axial Force Diagrams for a Steel Girder Prestressed by a Short Tendon

ℓ = total length of the girder;

ℓ_v = length of the tendon;

M_0 = moment due to applied load at the general section of the girder;

s,t = lengths locating ends of the prestressing tendon, and other symbols have been defined previously.

Differentiating Eq (2.1) with respect to the redundant force X, setting the result equal to zero, and reducing the terms yields

$$X = \frac{\frac{1}{\ell_v}\int_s^{s+\ell_v} M_0\,dx}{f + \frac{I}{Af}\left(1 + \frac{E_s A}{E_v A_v}\right)} \tag{2.2}$$

The numerator in Eq (2.2) represents the average value of the bending moment in the prestressed length of the girder due to the applied loads, which is denoted as M_{0avg}.

Define for convenience

$$\mu_{avg} = \frac{M_{0avg}}{M_{0max}}\,; \quad \mu_{br} = \frac{M_{0br}}{M_{0max}}\,; \tag{2.3}$$

where

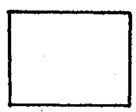
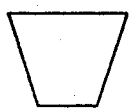
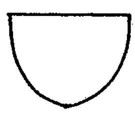
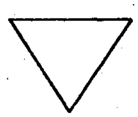
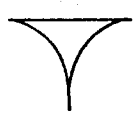
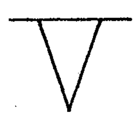
M_{0max} = maximum moment in the girder due to applied loads;

M_{0br} = moment in the girder at anchoring brackets due to applied loads.

It is always possible to express μ_{avg} as a function of μ_{br} for a given bending moment diagram. Similarly, s/ℓ, t/ℓ, and ℓ_v/ℓ can be obtained as functions of μ_{br}. These relationships for the six basic bending moment diagrams of Fig 4 are presented in Table II.

TABLE II

FORMULAS FOR μ_{avg} AND $\frac{\ell_v}{\ell}$

No.	Shape of Bending Moment Diagram	Full Length Tendon	Short Length Tendon		
		μ_{avg}	μ_{avg}	$\frac{\ell_v}{\ell}$	$\frac{s}{\ell} = \frac{t}{\ell}$
1		1.0	1.0	1.0	
2		0.75	$\frac{2 + \mu_{br}}{2} - \frac{1}{2(2 - \mu_{br})}$	$1 - \frac{\mu_{br}}{2}$	
3		0.667	$\frac{2 + \mu_{br}}{3}$	$\sqrt{1 - \mu_{br}}$	$\frac{1}{2}\left(1 - \frac{\ell_v}{\ell}\right)$
4		0.5	$\frac{1 + \mu_{br}}{3}$	$1 - \mu_{br}$	
5		0.333	$\frac{1 + \mu_{br} + \sqrt{\mu_{br}}}{3}$	$1 - \sqrt{\mu_{br}}$	
6		0.25	$\frac{1 + \mu_{br}}{2}$	$\frac{1 - \mu_{br}}{2}$	

2.6 Design Concepts

Prestressed structures may be proportioned according to the Concept of Limit States or according to the Concept of Allowable Stresses. For either of these procedures the magnitudes of the applied loads (p in Fig 5) and allowable stresses are prescribed by specifications or codes. From these quantities the required tendon force V^* is determined. These standard effects are used directly in the Allowable Stress Concept for proportioning the prestressed structures. In the case of Limit States Concept the design effects are obtained by modifying the standard effects by appropriate factors (load factors, prestress accuracy factors, homogeneity factors, working condition factors, etc.). In this study, the solution of the problem is developed in such a way that either of the concepts could be used. However, the derivations presented herein are based on the Limit States Concept.

2.6.1 Design Effects for Limit States Concept

The modified standard load mentioned above will produce a moment diagram in the girder such as shown in Figs 4 and 5. From these diagrams the maximum bending moment M_{0max} may be determined. The tendon force V^* is modified by prestress accuracy factors (31) in order to produce the maximum possible stress condition when prestress effects and applied load effects are combined. For example, when no load is applied, in order to produce maximum compressive stress in chord 2, a prestress force of magnitude $n_{vu}V^*$ is used (where n_{vu} is a prestress accuracy factor greater than 1.0). At the same location, when the design load is applied, the maximum tensile stress in chord 2 is determined from the combination of M_{0max} and a prestress force equal to $n_{v\ell} V^*$ (where $n_{v\ell}$ is a prestress accuracy factor less than 1.0) and the redundant force X obtained from Eq 2.2.

2.6.2 Design Stress

The maximum permissible stress in the girder is specified by code values. In order to simplify the derivations, the stresses at each point in the girder are expressed in terms of a reference stress (R_2) of chord 2. The stresses at other points in the girder are expressed in terms of R_2 by

$$\begin{aligned} R_2^+ &= R_2 \;;\; R_2^- = \rho_2 R_2 \;;\; R_1^- = \rho_1 R_2 \;; \\ R_c^- &= \frac{\rho_c}{n_c} R_2 \;; \quad R_v^+ = \rho_v R_2 \;; \end{aligned} \tag{2.4}$$

where

R_2^+ = tensile design stress in chord 2,

R_2^- = compressive design stress in chord 2,

R_1^- = compressive design stress in chord 1,

R_c^- = compressive design stress for concrete slab,

R_v^+ = tensile design stress for the tendon,

ρ_1, ρ_2, ρ_c, ρ_v are proportionality factors, and

n_c = modular ratio for steel and concrete.

2.7 Stability and Other Design Considerations

The present study does not consider in depth such problems as constructional details of the prestressed girder; suitable materials and their characteristics; introducing, measuring and losses of prestressing; checks for buckling and deformations; etc. For details of methods of treating these effects see Refs (22), (29), and (31). However, a brief discussion of flange and web buckling as well as control of deflection is presented below.

Chord 1 under full load and chord 2 in the unloaded state, in a simple beam are compressed. Chord 1 is frequently supported against the loss of stability by cross-beams, horizontal bracing trusses, slabs, etc. The stress in chord 1 can be controlled by selecting appropriate value of $\rho_1 \leq 1.0$. Buckling of chord 2 could be prevented by measures similar to those mentioned for chord 1 and by arranging the diaphragms connecting the tendon to chord 2 (Fig 2). If the holes in the diaphragms are only slightly larger than the diameter of the tendon, the effective length of the compressed chord is approximately equal to the distances between the diaphragms (Fig 2).

Detailed investigations of web stability, which depends, among other things, on the arrangement of the stiffeners could considerably complicate obtaining an optimum design. Web buckling can be controlled by selection of an appropriate value of the ratio of the depth of the girder to the web thickness. This ratio is called the web slenderness parameter and is given by

$$\lambda = \frac{h}{\delta} ,$$

where

δ = thickness of the web plate.

This parameter commonly varies between 100 and 200 for steel (29). Higher values correspond to a web stiffened by both vertical and horizontal stiffeners in most highly stressed regions. An appropriate choice of λ also helps to control shear stresses and deflection.

CHAPTER III

FORMULATION OF THE SOLUTION

3.1 General

In this study linear elastic behavior of materials and small deformation theory are considered. It is assumed that a plate girder is prestressed in the unloaded state. There are various methods by which prestressing may be accomplished. However, the basic concept is to tension one or more prestressing tendons, parallel and close to the stretched girder chord, to the desired stress level by jacking against the beam and then to fasten them, at each end of the beam for a full length tendon or at the intermediate projecting brackets for a short length tendon, by means of appropriate anchoring devices (32).

For the composite girder, the shop-fabricated prestressed steel plate girders are first erected and then the concrete slab is placed. It is assumed that the forms for placing the concrete are supported by the steel girder, so that the steel section alone carries the weight of the concrete and forms. Any additional dead load as well as live load that may appear after the slab cures is carried by the composite action of the steel girder and the slab. Shear connectors are assumed to transmit horizontal shear between the slab and steel section.

In the idealized cross-section, Fig 1e, one straight tendon near chord 2, and parallel to it, is assumed. Its position can be described by the parameter κ as indicated in Figs 1, 2, and 3, where

$$\kappa = f/e_2 = f/e_{s2} \tag{3.1}$$

3.2 Selected Parameters

Certain parameters encountered in the analysis and design of the prestressed girders must be selected a priori by the designer. The values selected are governed by the intended use of the design, by the character of the constructional arrangements, loading and materials, by the required accuracy of prestressing, etc.

The following sections list and define the parameters which are assumed to have been selected by the designer.

3.2.1 Steel Girder

The selected parameters are:

R_2 = reference design stress;

ρ_1, ρ_2, ρ_v = ratios of the design stresses defined by Eq (2.4);

n_v = modular ratio for steel and prestressing tendon where $n_v = E_s/E_v$ and E_v is the modulus of elasticity for the tendon;

$n_{vu}, n_{v\ell}$ = prestress accuracy factors;

κ = tendon location parameter;

h = depth of the girder.[1]

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[1]Alternatively either λ or δ could be selected.

3.2.2 Composite Girders

In addition to the parameters selected for the steel girder, the following parameters are required:

ρ_c = ratio of the design stresses for concrete;

n_c = modular ratio for steel and concrete;

β = one-half of the ratio of depth of the slab and the steel section;

η = ratio of the maximum bending moments;

where

$$\eta = M_{0max, c}/M_{0max, s}$$

and

$M_{0max,c}$ = maximum moment carried by the composite section;

$M_{0max,s}$ = maximum moment carried by the steel section.

3.3 Unknown Quantities

All other quantities needed for the design and, possibly, for checking the deflections or stability of the girder may be expressed in terms of selected parameters, the maximum bending moments and the quantities listed in Sec 2.2, as follows:

3.3.1 Steel Section

$$\begin{aligned} A &= \alpha\, M_{0max}/hR_2, & A_v &= \alpha_v A, \\ A_1 &= \alpha_1 A, & V^* &= \beta_v\, M_{0max}/h, \\ A_2 &= \alpha_2 A, & X &= \beta_x\, M_{0max}/h, \\ A_w &= \alpha_w A, & & \end{aligned} \tag{3.2}$$

where α, α_1, α_2, α_w, α_v, β_v, β_x are proportionality factors. Relative tendon length ℓ_v/ℓ and relative distances s/ℓ, t/ℓ, locating the ends of the tendon, are expressed as functions of μ_{br} as shown in Table II.

3.3.2 Composite Section

$$
\begin{aligned}
A &= \alpha\, M_{0max,s}/hR_2, & A_v &= \alpha_v A, \\
A_1 &= \alpha_1 A, & V^* &= \beta_v\, M_{0max,s}/h, \\
A_2 &= \alpha_2 A, & X_s &= \beta_{x,s}\, M_{0max,s}/h, \\
A_3 &= \alpha_3 A, & X_c &= \beta_{x,c}\, M_{0max,s}/h, \\
A_w &= \alpha_w A, & & \qquad (3.3)
\end{aligned}
$$

where

X_s = increase in the tendon force due to loads carried by steel section,

X_c = increase in the tendon force due to loads carried by composite section,

and α_3, $\beta_{x,s}$, $\beta_{x,c}$ are proportionality factors. Relative tendon length ℓ_v/ℓ for the composite section may be expressed as a function of the ratio $\mu_{br,s}$ where

$$\mu_{br,s} = M_{0br,s}/M_{0max,s} \tag{3.4}$$

where

$M_{0br,s}$ = moment at the location of the anchoring brackets due to loads carried by steel section. This relationship will be derived later.

3.4 Approach

Fully stressed designs of statically indeterminate structures are governed only by strength and continuity conditions, which are expressed as equality constraints. The optimum result (e.g., the minimum weight) is obtained by a search of all fully stressed designs. Other conditions such as deflections, buckling, shear, etc., need not be considered in the fully stressed design if they are properly controlled by the selected parameters. These conditions need not be satisfied as equality constraints in the fully stressed design and can be checked independently later on.

The variables expressed by Eqs (3.2) and (3.3) can be defined in any manner, so long as they are truly independent. For convenience, the following variables will be used:

	Steel Girder	Composite Girder
area of the steel section	A	A
area of the tendon	A_v	A_v
asymmetry of the steel section	a	a
increment in tendon force	X	X_s, X_c
force due to prestressing in the tendon	V*	V*
chord 2 parameter	α_2	α_2
concrete slab parameter	–	α_3
ratios of the moments	μ_{br}	$\mu_{br,s}$

It is shown subsequently that the maximum number of independent variables is 7 for a prestressed plate girder and 9 for a prestressed composite girder. The number of constraints (which are considered as equality constraints for the optimum design) are found to be one less than the number of variables involved in the problem. To obtain a unique optimum design, the optimization (e.g., minimization of the weight) is executed with respect to the "extra" variable. For convenience, α_2 is considered to be the extra variable.

In certain situations, a side constraint defining the bound on a variable, or bounds on certain mathematical combinations of design variables, become active. For instance, parameters defining the chord areas, α_2 in the case of prestressed steel girder, and α_1 (function of a and α_2) in the case of prestressed composite girder, are generally rather small quantities. In the free optimum design (without any side constraints), these quantities may attain values smaller than the construction or buckling aspects permit. In this case, the extra variable is obtained from the side constraint instead of the optimization condition.

Several optimization techniques have been investigated to solve the problem. However, only that one described subsequently was found to be feasible. Its principal idea is to reduce the number of the equations involved by eliminating some variables and to replace numerous equations, in that way, by one or two governing equations.

For the prestressed steel girder (with a short tendon), six equations are reduced to a single governing equation in terms of variables a and α_2. Similarly, for the prestressed composite girder (with a short tendon), eight equations are replaced by two governing equations in terms of variables a, α_2, and α_3. These one or two equations (with two or three variables), coupled with an optimization condition (or a side constraint), are then treated numerically, because their complexity excludes an explicit solution.

3.5 Derivation of the Governing Equations for a Prestressed Steel Girder[2]

Stress distributions for various loading stages at a maximum moment section are shown in Fig 6a. The most highly stressed fibers, f_1, f_2, . . . , are indicated in Fig 7, for a symmetrical load. The following derivations are valid for symmetrical or unsymmetrical load.

3.5.1 Tendon of Full Length $\ell_v = \ell$

The stresses in the most highly stressed fibers may be expressed as follows:

Compression in chord 1 at maximum moment section, fibers f_1, under the full load (Fig 7a) is

$$\frac{n_{v\ell} V^* + X}{A} - \frac{(n_{v\ell} V^* + X) f}{S_1} + \frac{M_{0max}}{S_1} = \rho_1 R_2. \qquad (3.5)$$

Tension in chord 2 at maximum moment section, fibers f_2, under the full load (Fig 7a) is

$$-\frac{n_{v\ell} V^* + X}{A} - \frac{(n_{v\ell} V^* + X) f}{S_2} + \frac{M_{0max}}{S_2} = \rho_1 R_2. \qquad (3.6)$$

Compression in chord 2 at support section, fibers f_3, under the full load (Fig 7a) is

$$\frac{n_{vu} V^* + X}{A} + \frac{(n_{vu} V^* + X) f}{S_2} = \rho_2 R_2. \qquad (3.7)$$

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[2]The governing equations and expressions for the design parameters for accurate prestressing are furnished in Appendix A.

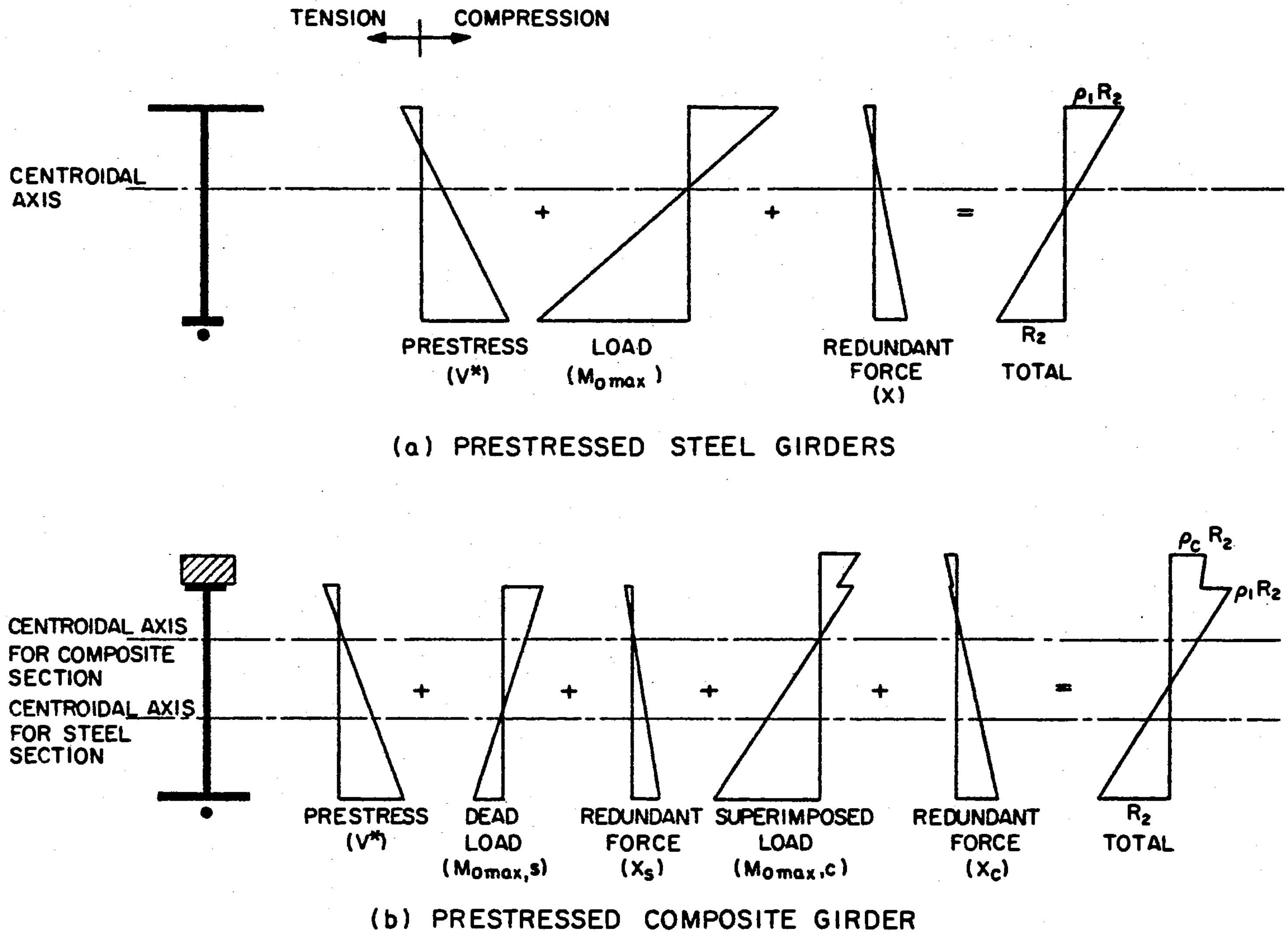


Figure 6. Stress Distributions for Various Stages of Loads at a Maximum Moment Section

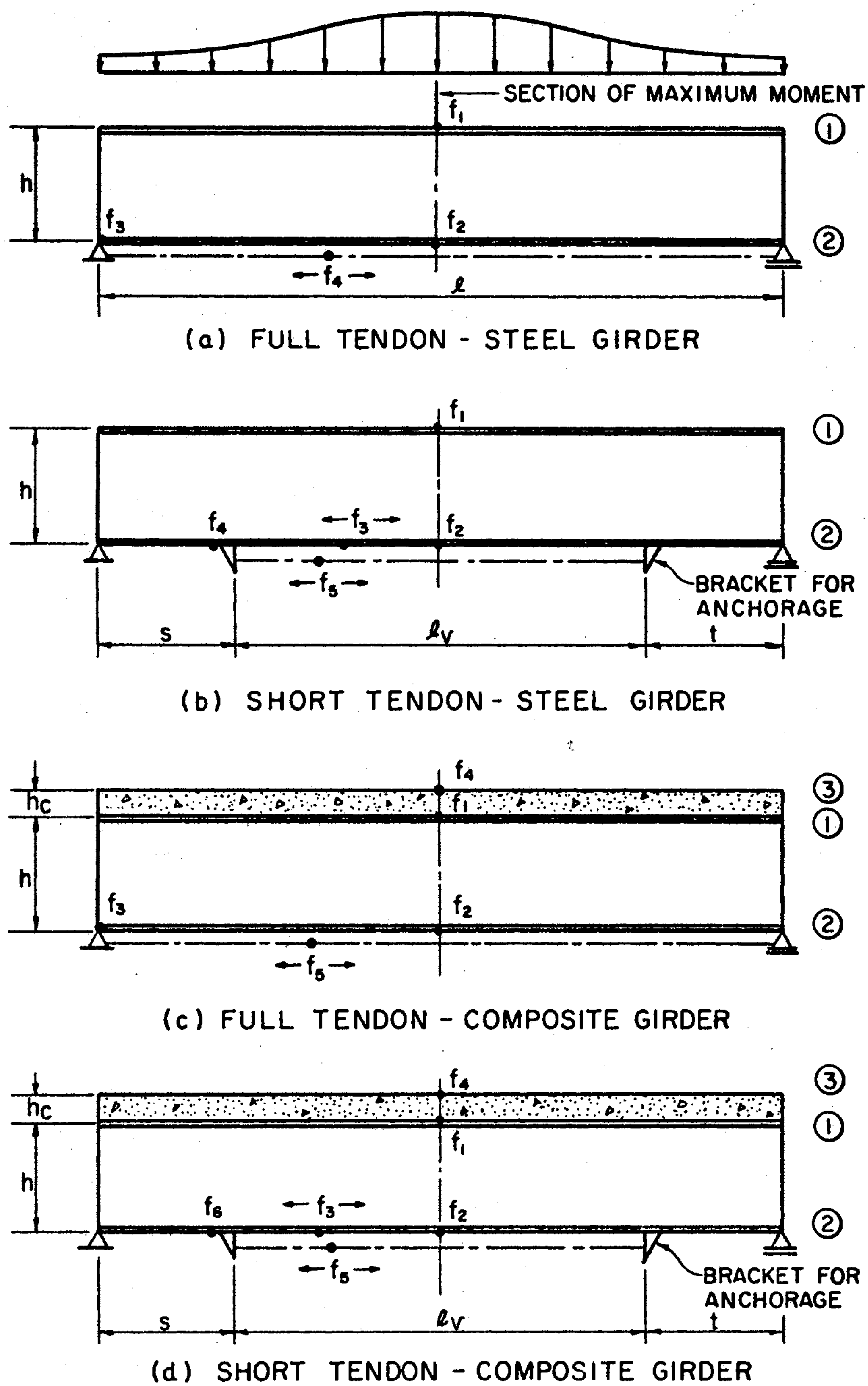


Figure 7. Critically Stressed Sections Under Various Stages of Loads

Stress in the tendon, fibers f_4, under the full load (Fig 7a) is

$$\frac{n_{vu} V^* + X}{A_v} = \rho_v R_2 . \tag{3.8}$$

By substituting Eq (2. 3) in Eq (2. 2), the increment of the tendon force X is obtained

$$X = \frac{\mu_{avg} M_{0max}}{f + \frac{s_2 e_2}{fA}\left(1 + \frac{E_s A}{E_v A_v}\right)} . \tag{3.9}$$

Substituting section properties, Table I, in Eqs (3. 5) and (3. 6), yields

$$\frac{n_{v\ell} V^* + X}{A} = - R_2\left(\frac{1-\rho_1 a}{1+a}\right). \tag{3.10}$$

Eq (3. 10) substituted in Eq (3. 6), yields

$$A = \frac{M_{0max}}{hR_2} \frac{6a(1+a)^2}{W_1\{1 + a - Y_2(1-\rho_1 a)\}} . \tag{3.11}$$

Expressions for W_1 and Y_2 are given in Table III. Substituting Eq (3. 11) in Eq (3. 7), one obtains

$$n_{vu} V^* + X = \frac{M_{0max}}{h} \frac{6a\rho_2(1+a)^2}{Y_2 W_1\{1+a - Y_2(1-\rho_1 a)\}} . \tag{3.12}$$

Substitution of Eq (3. 11) in Eq (3. 10), gives

$$n_{v\ell} V^* + X = -\frac{M_{0max}}{h} \frac{6a(1-\rho_1 a)(1+a)^2}{W_1(1+a)\{1 + a - Y_2(1-\rho_1 a)\}} . \tag{3.13}$$

Eq (3. 12) and Eq (3. 13) yield

$$V^* = \frac{6a(1+a)^2}{(n_{vu} - n_{v\ell}) W_1\{1+a - Y_2(1-\rho_1 a)\}}\left(\frac{\rho_2}{Y_2} + \frac{1-\rho_1 a}{1+a}\right) \tag{3.14}$$

From Eqs (3. 12) and (3. 13), it follows that

TABLE III

LIST OF PRINCIPAL EXPRESSIONS

$$W_1 = 2\left\{\alpha_2(1+a)^2 - 1 + 2a\right\}$$

$$W_2 = a + \alpha_3(1+a)(1+\beta)$$

$$W_3 = 1 + \beta(1+a)(2+\alpha_3)$$

$$W_4 = 1 - \alpha_3\beta(1+a)$$

$$W_5 = 6\alpha_3\left\{1+\beta(1+a)\right\}^2$$

$$W_6 = 6\left\{\alpha_3(1+\beta+\alpha\beta) + a\kappa\,(1+\alpha_3)\right\}$$

$$Y = W_1(1+\alpha_3) + W_5$$

$$Y_1 = \frac{6(1+a)}{(1+\rho_2)}\left(\frac{a}{W_1} + \frac{\eta W_2}{Y}\right)$$

$$Y_2 = \left(1 + \frac{6a^2\kappa}{W_1}\right)$$

$$Y_3 = -\alpha_3 - \frac{6a^2\kappa(1+\alpha_3)}{W_1} + \frac{W_2W_6}{Y}$$

$$Y_4 = \frac{6(1+a)}{W_1} + \frac{6\eta W_4(1+a)}{Y}$$

$$Y_5 = 1 - \frac{W_3W_6}{Y}$$

$$Y_6 = \frac{6\eta(1+a)W_3}{Y}$$

$$Y_7 = -\alpha_3 + \frac{6a\kappa(1+\alpha_3)}{W_1} - \frac{W_4W_6}{Y}$$

$$Y_8 = \frac{\eta W_1 W_2}{aY}$$

$$Z_1 = \frac{(n_{vu} - n_{v\ell})}{(1+\rho_2)}$$

$$Z_2 = \frac{\rho_2 Y_1}{Y_2} - \frac{Y_3}{Y_2Y_5}(\rho_c Y_1 - Y_6)$$

$$Z_3 = \rho_2\left(1 - \frac{\rho_c}{\rho_2}\,\frac{Y_3}{Y_5}\right)$$

$$Z_4 = \frac{Y_4 + (1+\rho_2)\dfrac{Y_1Y_7}{Y_3}}{1 - \dfrac{6a\kappa}{W_1} - \dfrac{Y_2Y_7}{Y_3}}$$

$$Z_5 = \frac{\left(\rho_1 + \dfrac{Y_7}{Y_3}\right)}{1 - \dfrac{6a\kappa}{W_1} - \dfrac{Y_2Y_7}{Y_3}}$$

$$Z_6 = \frac{Y_3Y_6}{Y_2Y_5} + \frac{Y_1}{Y_2}(1+\rho_2)$$

$$Z_7 = \left(\frac{1}{Y_2} + \frac{\rho_c Y_3}{Y_2Y_5} - \frac{\rho_2(n_{vu} - n_{v\ell})}{Y_2\, n_{vu}}\right)$$

$$Z_8 = \left(\frac{W_1}{W_1 - 6a\kappa}\right)\left(\frac{Y_6Y_7}{Y_5} - Y_4\right)$$

$$Z_9 = \left(\frac{W_1}{W_1 - 6a\kappa}\right)\left(\rho_1 - \frac{\rho_c Y_7}{Y_5}\right) + \frac{(n_{vu} - n_{v\ell})}{n_{vu}}\,\frac{\rho_2}{Y_2}$$

$$D = \frac{V^*h}{M_{0\,max}} \quad \text{(also refer to Eq (3.37))}$$

$$H_1 = 6a^2\kappa^2 Z_1Z_3 + W_1Z_1Z_3 + W_1\kappa Z_1Y_2$$

$$H_2 = n_{vu} - Z_1Z_3 + \frac{Z_1\rho_c Y_2(1+\alpha_3)}{Y_5}$$

$$H_3 = 6a^2\kappa^2 Z_2 + W_1Z_2 + W_1Y_1\kappa$$

$$H_4 = \frac{(\rho_c Y_1 - Y_6)(1+\alpha_3)}{Y_5} - Z_2$$

$$H_5 = 6\,\mu_{avg,s}(1+a)a\kappa$$

$$H_6 = \frac{W_1\left\{Z_4Y_2Y_5 + (1+\rho_2)Y_1Y_5 + Y_3Y_6\right\}}{6a(1+a)\left\{Y_5(1+Y_2Z_5) + \rho_c Y_3\right\}}$$

$$H_7 = \frac{W_1}{6a(1+a)}\left(\frac{Z_6 - Z_8}{Z_7 + Z_9}\right)$$

$$H_8 = Y_5(1+Y_2Z_5) + \rho_c Y_3$$

$$H_9 = Z_4Y_2Y_5 + (1+\rho_2)Y_1Y_5 + Y_3Y_6$$

$$H_{10} = Z_5\left\{(1+\rho_2)Y_1Y_5 + Y_3Y_6\right\} - Z_4(Y_5 + \rho_c Y_3)$$

$$X = -\frac{M_{0max}}{h}\,\frac{6a(1+a)^2}{(n_{vu}-n_{v\ell})W_1\{1+a-Y_2(1-\rho_1 a)\}}$$

$$\cdot\left\{n_{vu}\left(\frac{1-\rho_1 a}{1+a}\right)+n_{v\ell}\frac{\rho_2}{Y_2}\right\}. \qquad (3.15)$$

Substituting Eqs (3.11) and (3.12) in Eq (3.8), one obtains

$$\frac{A}{A_v} = \left(\frac{\rho_v}{\rho_2}\right)\cdot Y_2\,. \qquad (3.16)$$

Eq (3.16) substituted in Eq (3.9) leads to

$$X = \frac{M_{0max}}{h}\,\frac{\mu_{avg}(1+a)}{a\kappa+\frac{W_1}{6a\kappa}\left(1+\epsilon\frac{Y_2}{\rho_2}\right)}\,, \qquad (3.17)$$

where $\epsilon = \rho_v n_v$.

Equating Eqs (3.15) and (3.17), one obtains the governing equation in terms of a and α_2,

$$\frac{6a(1+a)\left\{n_{vu}\left(\frac{1-\rho_1 a}{1+a}\right)+n_{v\ell}\frac{\rho_2}{Y_2}\right\}}{(n_{vu}-n_{v\ell})W_1\{1+a-Y_2(1-\rho_1 a)\}}+\frac{\mu_{avg}}{a\kappa+\frac{W_1}{6a\kappa}\left(1+\epsilon\frac{Y_2}{\rho_2}\right)} = 0\,.$$

(3.18)

Expressions for the design quantities α, α_1, . . . , β_v, . . . , are accumulated in Table IV.

3.5.2 Tendon of Short Length $\ell_v < \ell$

The constraints for the critically stressed fibers, f_1, f_2, and f_5 (Fig 7b), are expressed by Eqs (3.5), (3.6), and (3.8), respectively. The expression for X is given by Eq (3.8).

Compression in chord 2 at any section in the prestressed length, fibers f_3, in the unloaded state (Fig 7b) is

TABLE IV

EXPRESSIONS FOR SIZING PARAMETERS FOR A PRESTRESSED STEEL GIRDER FORMULATED AS FUNCTIONS OF a AND α_2

Tendon of Full Length	Tendon of Short Length
$\alpha = \dfrac{6a(1+a)^2}{W_1\{1+a-Y_2(1-\rho_1 a)\}}$	$\alpha = \dfrac{6a(1+a)^2}{W_1\{1+a-Y_2(1-\rho_1 a)\}}$
$\alpha_1 = \alpha_2 - \dfrac{1-a}{1+a}$	$\alpha_1 = \alpha_2 - \dfrac{1-a}{1+a}$
$\alpha_w = 2(\dfrac{1}{1+a} - \alpha_2)$	$\alpha_w = 2(\dfrac{1}{1+a} - \alpha_2)$
$\alpha_v = \dfrac{n_v\rho_2}{\epsilon Y_2}$	$\alpha_v = \dfrac{n_v(\rho_1 a - 1)}{\epsilon(1+a)}$
$\beta_v = \dfrac{6a(1+a)^2}{(n_{vu}-n_{v\ell})W_1\{1+a-Y_2(1-\rho_1 a)\}}(\dfrac{\rho_2}{Y_2} + \dfrac{1-\rho_1 a}{1+a})$	$\beta_v = \dfrac{6a\rho_2(1+a)^2}{W_1 Y_2\{1+a-Y_2(1-\rho_1 a)\}}$
$\beta_x = \dfrac{\mu_{avg}(1+a)}{a\epsilon + \dfrac{W_1}{6a\epsilon}(1+\epsilon\dfrac{Y_2}{\rho_2})}$	$\beta_x = \dfrac{\mu_{avg}(1+a)}{a\epsilon + \dfrac{W_1}{6a\epsilon}\left\{1-\epsilon\dfrac{(1+a)}{(1-\rho_1 a)}\right\}}$
Governing Equation (3.18)	$\mu_{br} = \dfrac{1+a}{1+a-Y_2(1-\rho_1 a)}$ Governing Equation (3.22)

$$\frac{n_{vu}V^*}{A} + \frac{n_{vu}V^* f}{S_2} = \rho_2 R_2 . \qquad (3.19)$$

Tension in chord 2 at the anchorage-location-section, fibers f_4, in the non-prestressed length under the full load (Fig 7b) is

$$\frac{\mu_{br} M_{0max}}{S_2} = R_2 . \qquad (3.20)$$

Derivations in this case are similar to those in Sec (3.5.1), except that the value of μ_{avg} is now a function of μ_{br} (Table II) instead of a constant value as for a full length tendon. The quantity μ_{br} is determined from Eq (3.20). Substituting the expression for A, the same as Eq (3.11), for S_2 (Table I) in Eq (3.20), yields

$$\mu_{br} = \frac{1+a}{1+a - Y_2(1-\rho_1 a)} . \qquad (3.21)$$

The governing equation in terms of a and α_2 would read

$$\frac{6a(1+a)^2}{W_1\left\{1+a-Y_2(1-\rho_1 a)\right\}}\left\{\frac{(1-\rho_1 a)}{(1+a)} + \frac{n_{v\ell}}{n_{vu}}\cdot\frac{\rho_2}{Y_2}\right\}$$

$$+ \frac{\mu_{avg}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}\left\{1 + \frac{\varepsilon}{\left(1 - \frac{n_{v\ell}}{n_{vu}}\right)\frac{\rho_2}{Y_2} - \frac{1-\rho_1 a}{1+a}}\right\}} = 0 \qquad (3.22)$$

Expressions for design parameters are furnished in Table IV.

3.6 Derivation of the Governing Equations for a Prestressed Composite Girder[4]

Stress distributions for various loading stages at a maximum moment section are shown in Fig 6b. Critically stressed fibers f_1,

[4]The governing equations and expressions for the design parameters for accurate prestressing are presented in Appendix A.

$f_2, \ldots,$ are indicated in Fig 7, for a symmetrical load. For the composite girder this study is restricted to symmetrical load only, because of the complexity of a general case. Expressions W_1, W_2, $\ldots$ Y, $Y_1, \ldots$ Z_1, $Z_2, \ldots$ H_1, $H_2, \ldots$ are explained in Table III.

3.6.1 Tendon of Full Length $\ell_v = \ell$

The stresses in the most highly stressed fibers may be expressed as follows:

Compression in chord 1 at maximum moment section, fibers f_1, under the total load (Fig 7c) is

$$\frac{n_{v\ell} V^* + X_s}{A} + \frac{X_c}{A(1+\alpha_3)} - \frac{(n_{v\ell} V^* + X_s) f}{S_{1,s}} - \frac{X_c (e_{c2} - e_{s2} + f)}{S_{1,c}} + \frac{M_{0max,s}}{S_{1,s}} + \frac{M_{0max,c}}{S_{1,c}} = \rho_1 R_2 . \tag{3.23}$$

Tension in chord 2 at maximum moment section, fibers f_2, under the total load (Fig 7c) is

$$-\frac{n_{v\ell} V^* + X_s}{A} - \frac{X_c}{A(1+\alpha_3)} - \frac{(n_{v\ell} V^* + X_s) f}{S_{2,s}} - \frac{X_c (e_{c2} - e_{s2} + f)}{S_{2,c}} + \frac{M_{0max,s}}{S_{2,s}} + \frac{M_{0max,c}}{S_{2,c}} = R_2 . \tag{3.24}$$

In Eq (3.25) it is assumed that the concrete at the support section will not crack because the tension due to the force X_c is small. Therefore, the assumption of the transformed section is still valid.

Compression in chord 2 at the support section, fibers f_3, under the total load (Fig 7c) is

$$\frac{n_{vu}V^* + X_s}{A} + \frac{X_c}{A(1+\alpha_3)} + \frac{(n_{vu}V^* + X_s)f}{S_{2,s}}$$

$$+ \frac{X_c(e_{c2} - e_{s2} + f)}{S_{2,c}} = \rho_2 R_2 . \qquad (3.25)$$

Compression in the outer concrete, fibers f_4, at maximum moment section under the total load (Fig 7c) is

$$\frac{X_c}{A(1+\alpha_3)} - \frac{X_c(e_{c2} - e_{s2} + f)}{S_{3,c}} + \frac{M_{0max,c}}{S_{3,c}} = \rho_c R_2 . \qquad (3.26)$$

Tension in the tendon, fibers f_5, under the total load (Fig 7c) is

$$\frac{n_{vu}V^* + X_s + X_c}{A_v} = \rho_v R_2 . \qquad (3.27)$$

Increment of the tendon force due to loads carried by the steel section is

$$X_s = \frac{\mu_{avg,s}\, M_{0max,s}}{f + \frac{S_{2,s}\, e_{2,s}}{fA}\left(1 + \frac{E_s A}{E_v A_v}\right)} . \qquad (3.28)$$

Increment of the tendon force due to loads supported by the composite section is

$$X_c = \frac{\mu_{avg,c}\, M_{0max,c}}{(e_{2,c} - e_{2,s} + f) + \frac{S_{2,c}\, e_{2,c}}{(e_{2c} - e_{2s} + f)\, A(1+\alpha_3)}\left(1 + \frac{E_s A(1+\alpha_3)}{E_v A_v}\right)} . \qquad (3.29)$$

where

$$\mu_{avg,s} = \frac{M_{0avg,s}}{M_{0max,s}}, \qquad \mu_{avg,c} = \frac{M_{0avg,c}}{M_{0max,c}}, \qquad (3.30)$$

and

$$M_{0avg,s} = \frac{1}{\ell_v}\int_s^{s+\ell_v} M_{0,s}\,dx = $$ average moment in the pre-stressed length of the steel section,

$$M_{0avg,c} = \frac{1}{\ell_v}\int_s^{s+\ell_v} M_{0,c}\,dx = $$ average moment in the pre-stressed length of the composite section.

Eqs (3.24) and (3.25) yield

$$A = \frac{M_{0max,s}}{hR_2}\,Y_1 - Z_1Y_2\left(\frac{V^*}{R_2}\right). \tag{3.31}$$

Eqs (3.27) and (3.31) substituted in Eq (3.25) yield

$$\frac{X_c\,Y_3}{(1+\alpha_3)} + \rho_v R_2 A_v Y_2 = \rho_2 Y_1\,\frac{M_{0max,s}}{h} + \rho_2 Y_2 Z_1 V^*. \tag{3.32}$$

Substituting Eq (3.31) in Eq (3.26) leads to

$$X_c = \frac{M_{0max,s}}{h}(\rho_c Y_1 - Y_6)\left(\frac{1+\alpha_3}{Y_5}\right) + \frac{Z_1Y_2(1+\alpha_3)\rho_c V^*}{Y_5}. \tag{3.33}$$

Substitution of Eq (3.33) in Eq (3.32) yields

$$A_v = \frac{1}{\rho_v R_2}\left(\frac{M_{0max,s}}{h}\,Z_2 + V^* Z_1 Z_3\right). \tag{3.34}$$

From Eqs (3.31) and (3.34), it follows that

$$\frac{A}{A_v} = \rho_v\,\frac{Y_1 + D Z_1 Y_2}{Z_2 + D Z_1 Z_3}. \tag{3.35}$$

where

$$D = \frac{V^* h}{M_{0max,s}}$$

Substituting Eqs (3.28), (3.31), (3.33), (3.34), and (3.35) in Eq (3.27), results in a quadratic equation for D.

$$D^2 H_1 H_2 + D(H_2 H_3 + H_1 H_4 + H_5 Z_1 Z_3) + H_3 H_4 + H_5 Z_2 = 0. \quad (3.36)$$

The valid root of this equation (with the positive sign of the radical) is obtained as

$$D = \frac{1}{2H_1 H_2} \Big\{ -(H_2 H_3 + H_1 H_4 + H_5 Z_1 Z_3)$$

$$+ \sqrt{(H_2 H_3 + H_1 H_4 + H_5 Z_1 Z_3)^2 - 4H_1 H_2 (H_3 H_4 + H_5 Z_2)} \Big\}. \quad (3.37)$$

Substituting Eqs (3.27), (3.31), (3.33), and (3.34) in Eq (3.23) yields the first governing equation,

$$\left(1 - \frac{6a\kappa}{W_1}\right) \left\{ Z_2 + D Z_1 Z_3 - D(n_{vu} - n_{v\ell}) \right\} + \frac{Y_7}{Y_5} (\rho_c Y_1 - Y_6 + D Z_1 Y_2 \rho_c)$$

$$+ Y_4 - \rho_1 Y_1 - D Z_1 Y_2 \rho_1 = 0. \quad (3.38)$$

Simultaneous solution of Eqs (3.29) and (3.33) results in the second governing equation

$$\frac{\eta_{avg} \mu_{avg,s} (1+a)}{\frac{W_6}{6} + \frac{Y}{W_6} \left\{ 1 + \epsilon (1+\alpha_3) \left(\frac{Y_1 + D Y_2 Z_1}{Z_2 + D Z_1 Z_3} \right) \right\}}$$

$$- \frac{\rho_c Y_1 - Y_6}{Y_5} - \frac{D Z_1 Y_2 \rho_c}{Y_5} = 0. \quad (3.39)$$

Where

$$\eta_{avg} = \eta \frac{\mu_{avg,c}}{\mu_{avg,s}}. \quad (3.40)$$

Numerical values of $\mu_{avg,s}$ and $\mu_{avg,c}$ for six basic bending moment diagrams can be obtained from Table II (read $\mu_{avg,s}$ or $\mu_{avg,c}$ as needed for μ_{avg} in this table).

In the above two governing equations, (3.38) and (3.39), the expression D is given by Eq (3.37). Expressions for the design parameters are furnished in Table V.

3.6.2 Tendon of Short Length $\ell_v < \ell$

Constraints for the critically stressed fibers, f_1, f_2, f_4, f_5 (Fig 7d), are described by Eqs (3.23), (3.24), (3.26), and (3.27), respectively. Expressions for redundant tendon forces X_s and X_c are given by Eqs (3.28) and (3.29), respectively.

Compression in chord 2 at any section in the prestressed length, fibers f_3, under unloaded state (Fig 7d) is

$$\frac{n_{vu}V^*}{A} + \frac{n_{vu}V^*f}{S_{2,s}} = \rho_2 R_2 . \tag{3.41}$$

Tension in chord 2 at the anchorage-location in the non-prestressed length, fibers f_6, under the full load (Fig 7d) is

$$\frac{M_{0br,s}}{S_{2,s}} + \frac{M_{0br,c}}{S_{2,c}} = R_2 . \tag{3.42}$$

Because of the linearity of Eq (3.41) in V^*, it is possible to obtain V^* as a function of $M_{0max,s}/h$ without developing a quadratic equation in D, as in the previous case. The following two governing equations are developed by similar manipulations, but without using Eq (3.42).

$$\frac{Z_7Z_8 + Z_6Z_9 - (Z_6 - Z_8)\left\{\frac{\rho_2}{Y_2} + \frac{\rho_c(1+\alpha_3)}{Y_5}\right\}}{(Z_7 + Z_9)} + \frac{Y_6}{Y_5}(1+\alpha_3)$$

$$- \frac{\mu_{avg,s}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}\left\{1 + e\,\frac{(Z_6 - Z_8)}{Z_7Z_8 + Z_6Z_9}\right\}} = 0 , \tag{3.43}$$

TABLE V

EXPRESSIONS FOR SIZING PARAMETERS FOR A PRESTRESSED COMPOSITE GIRDER FORMULATED AS FUNCTIONS OF a, α_2 AND α_3

Tendon of Full Length	Tendon of Short Length
$\alpha = Y_1 + D Z_1 Y_2$	$\alpha = \frac{Z_6 - Z_8}{Z_7 + Z_9}$
$\alpha_1 = \alpha_2 - \frac{1-a}{1+a}$	$\alpha_1 = \alpha_2 - \frac{1-a}{1+a}$
$\alpha_w = 2\left(\frac{1}{1+a} - \alpha_2\right)$	$\alpha_w = 2\left(\frac{1}{1+a} - \alpha_2\right)$
$\alpha_v = \frac{n_v(Z_2 + D Z_1 Z_3)}{\epsilon(Y_1 + D Z_1 Y_2)}$	$\alpha_v = \frac{n_v(Z_7 Z_8 + Z_6 Z_9)}{Z_6 - Z_8}$
$\beta_v = D$ (refer to Eq (3.37))	$\beta_v = \frac{\rho_2}{n_{vu} Y_2}\left(\frac{Z_6 - Z_8}{Z_7 + Z_9}\right)$
$\beta_{x,s} = \dfrac{\mu_{avg,s}(1+a)}{\kappa a + \frac{W_1}{6a\kappa}\left\{1 + \epsilon \frac{Y_1 + D Z_1 Y_2}{Z_2 + D Z_1 Z_3}\right\}}$	$\beta_{x,s} = \dfrac{\mu_{avg,s}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}\left\{1 + \epsilon \frac{(Z_6 - Z_8)}{(Z_7 Z_8 + Z_6 Z_9)}\right\}}$
$\beta_{x,c} = \frac{(\rho_c Y_1 - Y_6)(1+\alpha_3)}{Y_5} + \frac{(1+\alpha_3) D Z_1 Y_2 \rho_c}{Y_5}$	$\beta_{x,c} = \frac{(1+\alpha_3)}{Y_5}\left\{\rho_c\left(\frac{Z_6 - Z_8}{Z_7 + Z_9}\right) - Y_6\right\}$
Governing Eqs (3.38) and (3.39)	Refer to Table VI for $\mu_{br,s}$. Governing Eqs (3.43) and (3.44).

and

$$\frac{\rho_c}{Y_5}\left(\frac{Z_6 - Z_8}{Z_7 + Z_9}\right) - \frac{Y_6}{Y_5} - \frac{\eta_{avg}\,\mu_{avg,\,s}(1+a)}{\frac{W_6}{6} + \frac{Y}{W_6}\left\{1 + 6\,\frac{(1+\alpha_3)(Z_6 - Z_8)}{Z_7 Z_8 + Z_6 Z_9}\right\}} = 0. \quad (3.44)$$

Expressions for the design parameters are furnished in Table V. The parameter η_{avg} needed in Eq (3.44) is obtained by the use of Eq (3.42). Manipulations similar to those involved in obtaining Eq (3.31) leads to an expression for A. Substituting the expression for A into the expressions for $S_{2,\,s}$ and $S_{2,\,c}$ (Table I) in Eq (3.42) yields

$$\mu_{br,\,s} + \mu_{br,\,c}\,Y_8 - H_7 = 0. \quad (3.45)$$

Since the quantities $\mu_{br,\,s}$ and $\mu_{br,\,c}$ depend upon the shape of the bending moment diagrams, Eq (3.45) must be solved for individual cases. The loads supported by the steel section usually are the self-weight of the girder and the concrete slab. They can be considered as uniformly distributed over the entire span. Thus, from Table II,

$$\frac{\ell_v}{\ell} = \sqrt{1 - \mu_{br,\,s}}\,, \quad (3.46)$$

and

$$\mu_{avg,\,s} = \frac{1}{3}(2 + \mu_{br,\,s}). \quad (3.47)$$

For the loads carried by the composite section,

$$\frac{\ell_v}{\ell} = \text{a function of } (\mu_{br,\,c}). \quad (3.48)$$

By equating the right sides of Eqs (3.46) and (3.48) for the relative tendon length ℓ_v/ℓ, the quantity $\mu_{br,\,c}$ is obtained as a function of $\mu_{br,\,s}$. Similarly, $\mu_{avg,\,c}$ can be expressed as a function of $\mu_{br,\,s}$.

Substituting $\mu_{br,s}$ in Eq (3.45) for $\mu_{br,c}$, a quadratic or linear equation for $\mu_{br,s}$ results (see Table VI). The solution of the latter equation is used to express other needed quantities, such as η_{avg}, ℓ_v/ℓ, etc. The expressions for $\mu_{br,s}$ (solution of Eq (3.45)) and the expressions for $\mu_{avg,c}$ and $\mu_{br,c}$ as functions of $\mu_{br,s}$ are presented in Table VI.

3.7 Objective Functions

It has been mentioned in Sec (3.2) that any one of three quantities, depth h, slenderness $\lambda = h/\delta$, or thickness of the web δ, can be freely selected. To simplify most of the aforementioned expressions, they have been formulated in terms of the depth h. However, if the depth h is selected, the optimization condition leads to zero web area, $A_w = 0$, since there is no constraint of any kind on the quantity δ. Thus, a side constraint on α_w must be introduced. To remove the necessity of any side constraint on α_w from the optimum design, the objective functions can be expressed in terms of either λ or δ instead of h, since h does not appear in the governing equations. Here, web-slenderness ratio λ is employed. The depth h can be expressed in terms of λ using Eq (3.2) for A and the following equations:

$$\lambda = h/\delta; \qquad\qquad \alpha_w A = h\delta; \tag{3.49}$$

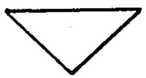
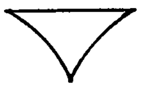
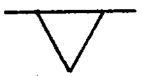
$$h = \sqrt[3]{\alpha\,\alpha_w}\;\sqrt[3]{\frac{M_{0max}\lambda}{R_2}} \tag{3.50}$$

For the composite girder, M_{0max} is replaced by $M_{0max,s}$ in Eq (3.50).

In composite construction, a concrete slab of certain specified dimensions (based on spacing of girders, loads, constructional regards,

TABLE VI

EXPRESSIONS FOR $\mu_{br,s}$, $\mu_{br,c}$ AND $\mu_{avg,c}$

No.	Bending Moment Diagram[3]	$\mu_{br,s} = \mu_{br,s}(a, \alpha_2, \alpha_3)$	$\mu_{br,c}$	$\mu_{avg,c}$
1		$(H_7 - Y_8)$	1.0	1.0
2		$2Y_8\sqrt{Y_8^2 + 2Y_8 - H_7 + 1} - (2Y_8 + 2Y_8^2 - H_7)$	$2(1 - \sqrt{1 - \mu_{br,s}})$	$2 - \frac{5-4\mu_{br,s}}{4\sqrt{1-\mu_{br,s}}}$
3		$\left(\frac{H_7}{1+Y_8}\right)$	$\mu_{br,s}$	$\frac{1}{3}(2 + \mu_{br,s})$
4		$\frac{1}{2}\left\{Y_8\sqrt{Y_8^2 + 4Y_8 - 4H_7 + 4} - Y_8^2 - 2Y_8 + 2H_7\right\}$	$1 - \sqrt{1 - \mu_{br,s}}$	$1 - \frac{1}{2}\sqrt{1 - \mu_{br,s}}$
5		$\frac{1}{(1-Y_8)^2}\left\{2Y_8\sqrt{1-H_7 + H_7Y_8} + H_7(1-Y_8) - 2Y_8\right\}$	$(1 - \sqrt{1 - \mu_{br,s}})^2$	$\frac{1}{3}(4 - \mu_{br,s} - 3\sqrt{1 - \mu_{br,s}})$
6		$2Y_8\sqrt{Y_8^2 + Y_8 - H_7 + 1} - (2Y_8^2 + Y_8 - H_7)$	$1 - 2\sqrt{1 - \mu_{br,s}}$	$1 - \sqrt{1 - \mu_{br,s}}$

[3] Solve Eq (3.45) for any other case of loading.

etc.) is provided in most cases, regardless of the section properties of the steel girder. Therefore, contribution of the concrete slab is ignored in the objective functions. Substituting h, Eq (3.50), in the expression for area A, Eq (3.2), gives

$$A = \sqrt[3]{\frac{\alpha^2}{\alpha_w}} \sqrt[3]{\frac{M_{0max}^2}{R_2^2 \lambda}} . \tag{3.51}$$

Three types of objective functions are considered.

3.7.1 Minimum Volume or Weight

The volume of the steel girder and tendon is given by

$$\text{Volume} = A\ell + A_v \ell_v . \tag{3.52}$$

Substituting Eq (3.51) in Eq (3.52) yields

$$\text{Volume} = \sqrt[3]{\frac{\alpha^2}{\alpha_w}} \left(1+\alpha_v \frac{\ell_v}{\ell}\right) \sqrt[3]{\frac{M_{0max}^2}{R_2^2 \lambda}}\ \ell . \tag{3.53}$$

Since ℓ, M_{0max}, R_2, and λ are given or selected quantities, the objective function for this case, to be minimized, is

$$O_w = \sqrt[3]{\frac{\alpha^2}{\alpha_w}} \left(1 + \alpha_v \frac{\ell_v}{\ell}\right). \tag{3.54}$$

Where O_w is identified with that of minimum weight if the unit weights of both materials are equal, which is at least approximately true.

3.7.2 Minimum Price of the Consumed Material

The objective function for this case, to be minimized, is derived from Eq (3.54) and reads

$$O_p = \sqrt[3]{\frac{\alpha^2}{\alpha_w}} \left(1 + p_{vs}\alpha_v \frac{\ell_v}{\ell}\right). \qquad (3.55)$$

Here, p_{vs} is the ratio of prices per unit volume of materials used in the tendon to that used in the girder.

3.7.3 Maximum Load Bearing Capacity of the Girder

The objective function represents the load bearing capacity (expressed, e.g., by M_{0max}) of the girder for a given volume of material (for the steel section), assuming that the material necessary for the tendon is provided.

$$O_\ell = \sqrt[3]{\frac{\alpha^2}{\alpha_w}} \text{ , (to be minimized).} \qquad (3.56)$$

If the volume of the tendon is neglected in the objective functions, all three objective functions considered, Eqs (3.54), (3.55), (3.56), become identical.

CHAPTER IV

OPTIMUM DESIGN - MINIMUM WEIGHT VERSUS FULLY STRESSED

4.1 General

The concept of fully stressed design can be applied to many basic structural problems. However, attempts to extend the concept to problems involving multiple load conditions and external or side constraints have led to complications and even erroneous results. Therefore, the equivalence between fully stressed design and minimum weight design should be examined rigorously. There are many cases where the equivalence of the two designs can be argued from a physical stand point.

Here, the equivalence between both approaches is proved for a steel girder prestressed by a full length tendon. The problems of a steel girder prestressed by a short length tendon and composite girders prestressed by full length and short length tendons are considerably more complex. However, these problems are basically similar to that of a steel girder with a full length tendon. The proof is based on the Kuhn-Tucker theorem, an extension of the classical Lagrangian approach to account for inequality constraints, Reference (1).

4.2 The Kuhn-Tucker Theorem

In the optimization problem,

$$\text{minimize } f(x),$$

$$\text{subject to } g_i(x) \le 0;\ i = 1, \ldots m \qquad (4.1)$$

$$\text{and } x > 0,$$

with the Lagrangian function

$$F(x,\mu) = f(x) + \sum_{i=1}^{m} \mu_i\, g_i(x), \qquad (4.2)$$

the conditions

$$\frac{\partial f(x)}{\partial x_j} + \sum_{i=1}^{m} \mu_i \frac{\partial g_i(x)}{\partial x_j} = 0;\quad j = 1, \ldots, n; \qquad (4.3)$$

$$\mu_i g_i(x) = 0,\quad i = 1, \ldots, m; \qquad (4.4)$$

$$\mu_i \mp 0 \qquad (4.5)$$

are necessary for x to be a local minimum.[1]

In the above expressions

$f(x)$ = objective function,

$g_i(x)$ = i-th constraint,

m = number of constraints,

n = number of independent variables,

x = a vector of independent variables,

μ_i = i-th Lagrange multiplier used to incorporate the effects of a constraint on the minimization of the objective function.

---

[1]In addition, the Kuhn-Tucker Constraint Qualification must be satisfied. In practice, we generally assume that this rather complex condition is satisfied without checking, (1).

The solution of Eq (4.3) will yield expressions for the Lagrange multipliers μ_i in terms of the independent variables x_j.

Two important inferences, which provide the key to the establishment of relations between the minimum weight design and the fully stressed design, can be drawn from the Kuhn-Tucker necessary conditions for minimization:

1. If the i-th Lagrange multiplier is nonzero, it can be seen from Eq (4.4) that the i-th constraint is satisfied as the equality constraint, $g_i(x) = 0$.

2. The relation, Eq (4.5), shows that the Lagrange multipliers are non-negative.

4.3 Example - Prestressed Steel Girder

4.3.1 Objective Function and Constraints

A plate girder prestressed by a tendon of full length is considered. To simplify the derivation, the prestress accuracy factors are assumed to be equal to unity. As explained in Sec (3.7), the depth of the girder h can be expressed by the web-slenderness λ; $h = \sqrt{\lambda \alpha_w A}$. All of the required section properties (Table II) can be expressed in terms of A, a, α_w, and λ. Here, instead of chord 2 parameter α_2, the web parameter α_w is used to simplify the required differentiation of the section properties. The necessary section properties are:

$$e_2 = \frac{a}{1+a}\sqrt{\lambda \alpha_w A},$$

$$S_1 = \frac{W_1}{6(1+a)}\sqrt{\lambda \alpha_w A^3}, \qquad (4.6)$$

$$S_2 = \frac{W_1}{6a(1+a)}\sqrt{\lambda \alpha_w A^3},$$

where

$$W_1 = 6a - \alpha_w(1+a)^2.$$

In this case the independent variables (x) are A, a, α_w, T and A_v, where

$T = V^* + X$ = the total force in the tendon under the full load condition.

The weight of the girder is proportional to the sum of the cross-section areas of the girder and the tendon. Therefore, the objective function

$$f(x) = A + A_v. \tag{4.7}$$

The constraints are obtained from Sec (3.5.1) and expressed as inequalities:

$$g_1(x) = \frac{T}{A} - \frac{T\,f}{S_1} + \frac{M_{0max}}{S_1} - \rho_1 R_2 \le 0, \tag{4.8}$$

$$g_2(x) = -\frac{T}{A} - \frac{T\,f}{S_2} + \frac{M_{0max}}{S_2} - R_2 \le 0, \tag{4.9}$$

$$g_3(x) = \frac{T}{A} + \frac{T\,f}{S_2} - \rho_2 R_2 \le 0, \tag{4.10}$$

$$g_4(x) = \frac{T}{A_v} - \rho_v R_2 \le 0. \tag{4.11}$$

4.3.2 The Lagrange Multiplier Matrix

The necessary condition expressed by Eq (4.3) can be written in the matrix form (known as the Lagrange multiplier matrix):

$$
\begin{bmatrix}
\frac{\partial g_1}{\partial A} & \frac{\partial g_2}{\partial A} & \frac{\partial g_3}{\partial A} & \frac{\partial g_4}{\partial A} \\
\frac{\partial g_1}{\partial a} & \frac{\partial g_2}{\partial a} & \frac{\partial g_3}{\partial a} & \frac{\partial g_4}{\partial a} \\
\frac{\partial g_1}{\partial \alpha_w} & \frac{\partial g_2}{\partial \alpha_w} & \frac{\partial g_3}{\partial \alpha_w} & \frac{\partial g_4}{\partial \alpha_w} \\
\frac{\partial g_1}{\partial T} & \frac{\partial g_2}{\partial T} & \frac{\partial g_3}{\partial T} & \frac{\partial g_4}{\partial T} \\
\frac{\partial g_1}{\partial A_v} & \frac{\partial g_2}{\partial A_v} & \frac{\partial g_3}{\partial A_v} & \frac{\partial g_4}{\partial A_v}
\end{bmatrix}
\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix}
=
\begin{bmatrix}
-\frac{\partial f}{\partial A} \\
-\frac{\partial f}{\partial a} \\
-\frac{\partial f}{\partial \alpha_w} \\
-\frac{\partial f}{\partial T} \\
-\frac{\partial f}{\partial A_v}
\end{bmatrix}
\qquad (4.12)
$$

Expressions for the coefficients of the Lagrange multiplier matrix are presented in Table VII. These coefficients are very complex, but only their signs are required to determine whether the Lagrange multipliers are, or are not, non-zero. These signs are obtainable by either detailed observation of the expressions or by intuition. For example, the first coefficient, $\partial g_1/\partial A$, represents the rate of change of stress in chord 1 under the full load with respect to the cross-section area A. Since the stress decreases with the increase in area A, keeping other variables constant, the sign of the coefficient must be negative. Similar arguments may be applied to other coefficients and can be verified by detailed study.

Let P and N represent positive and negative signs of the coefficients, respectively; let subscripts j and i at P and N indicate the location of the distinct coefficients in the matrix (j-th row and i-th column). The Lagrange multiplier matrix, Eq (4.12), in terms of the signs of coefficients reads:

TABLE VII

COEFFICIENTS IN THE LAGRANGE MULTIPLIER MATRIX

Operator \ g_i	g_1	g_2	g_3	g_4
$\frac{\partial}{\partial A}$	$-\frac{T}{A^2}\left(1-\frac{6\kappa a}{W_1}\right)-\frac{3}{2}\frac{M_{0\,max}}{S_1 A}$	$\frac{T}{A^2}\left(1+\frac{6a^2\kappa}{W_1}\right)-\frac{3}{2}\frac{M_{0\,max}}{S_2 A}$	$-\frac{T}{A^2}\left(1+\frac{6a^2\kappa}{W_1}\right)$	0
$\frac{\partial}{\partial a}$	$\frac{T}{A}\frac{6\kappa\alpha_w(1-a^2)}{W_1^2}-\frac{6M_{0max}}{\lambda\alpha_w A^3}\frac{\left\{6-\alpha_w(1+a)^2\right\}}{W_1^2}$	$-\frac{T}{A}\frac{12\kappa a\left\{3a-\alpha_w(1+a)\right\}}{W_1^2}+$ $\frac{6M_{0max}}{\lambda k A^3}\frac{\left\{6a^2-\alpha_w(1+a)^2\right\}}{W_1}$	$\frac{T}{A}\frac{12\kappa a\left\{3a-\alpha_w(1+a)\right\}}{W_1^2}$	0
$\frac{\partial}{\partial\alpha_w}$	$\frac{T}{A}\frac{6\kappa a(1+a)^2}{W_1}-$ $\frac{M_{0\,max}\ \lambda\alpha_w A}{4S_1^2(1+a)}\left\{\frac{2a}{\alpha_w}-(1+a)^2\right\}$	$\frac{T}{A}\frac{6a^2\kappa(1+a)^2}{W_1^2}-$ $\frac{M_{0\,max}\ \lambda\alpha_w A^3}{4S_1^2 a(1+a)}\left\{\frac{2a}{\alpha_w}-(1+a)^2\right\}$	$-\frac{T}{A}\frac{6a^2\kappa(1+a)^2}{W_1^2}$	0
$\frac{\partial}{\partial T}$	$\frac{1}{A}\left\{1-\frac{6\kappa a}{W_1}\right\}$	$-\frac{1}{A}\left(1+\frac{6a^2\kappa}{W_1}\right)$	$\frac{1}{A}\left(1+\frac{6a^2\kappa}{W_1}\right)$	$\frac{1}{A_v}$
$\frac{\partial}{\partial A_v}$	0	0	0	$-\frac{T}{A_v^2}$

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} & 0 \\ N_{21} & P_{22} & P_{23} & 0 \\ P_{31} & P_{32} & N_{33} & 0 \\ N_{41} & N_{42} & P_{43} & P_{44} \\ 0 & 0 & 0 & N_{55} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{4.13}$$

From the 5th row of the matrix,

$$\mu_4 > 0 . \tag{4.14}$$

From the 1st row,

$$N_{11}\mu_1 + N_{12}\mu_2 + N_{13}\mu_3 = -1 . \tag{4.15}$$

To satisfy the above condition, Eq (4.15), at least one of the Lagrange multipliers must be non-zero; because of relation, Eq (4.5), they cannot be negative. Therefore, at least

$$\mu_1 \text{ or } \mu_2 \text{ or } \mu_3 > 0. \tag{4.16}$$

From the 2nd row,

$$N_{21}\mu_1 + (P_{22}\mu_2 + P_{23}\mu_3) = 0 .$$

Therefore,

$$\mu_1 > 0 \text{ and } \mu_2 \text{ or } \mu_3 > 0 . \tag{4.17}$$

Using a similar argument, the 3rd row reveals

$$\mu_3 > 0 . \tag{4.18}$$

Similar simple arguments are not available to prove that μ_2 is non-zero. However, it can be shown that μ_2 is non-zero as follows. Assume $\mu_2 = 0$; then, from Eq (4.12)

$$\frac{\partial g_1/\partial a}{\partial g_1/\partial \alpha_w} = \frac{\partial g_3/\partial a}{\partial g_3/\partial \alpha_w} \tag{4.19}$$

Substitution of the required expressions in Eq (4. 19) from Table VII, yields

$$\frac{6\kappa\alpha_w(1-a^2) - \dfrac{6\,M_{0max}}{T\sqrt{\lambda\alpha_w A}}\,W_1}{6W_1\kappa a(1+a) - \dfrac{9\,M_{0max}}{T\sqrt{\lambda\alpha_w A}}\left\{\dfrac{2a}{\alpha_w} - (1+a)^2\right\}} = -\,\frac{6a-2\alpha_w(1+a)}{a(1+a)}\,. \quad (4.20)$$

The left side of Eq (4. 20) is a function of the web-slenderness λ (a selected parameter) and other quantities, while the right side is a function of the section asymmetry a and web parameter α_w only. It can be shown that the relative proportion of the cross-section (in other words a and α_w) based on flexural strength considerations does not depend on the web slenderness λ. This fact is substantiated by the governing Eq (3. 18), which is a function of section asymmetry a and web parameter α_w and not a function of λ. Hence, Eq (4. 20) is not true in general, which proves that μ_2 is non-zero. Thus, all of the Lagrange multipliers have been proved to be non-zero, and hence, in this case, the minimum weight design is a fully stressed design.

CHAPTER V

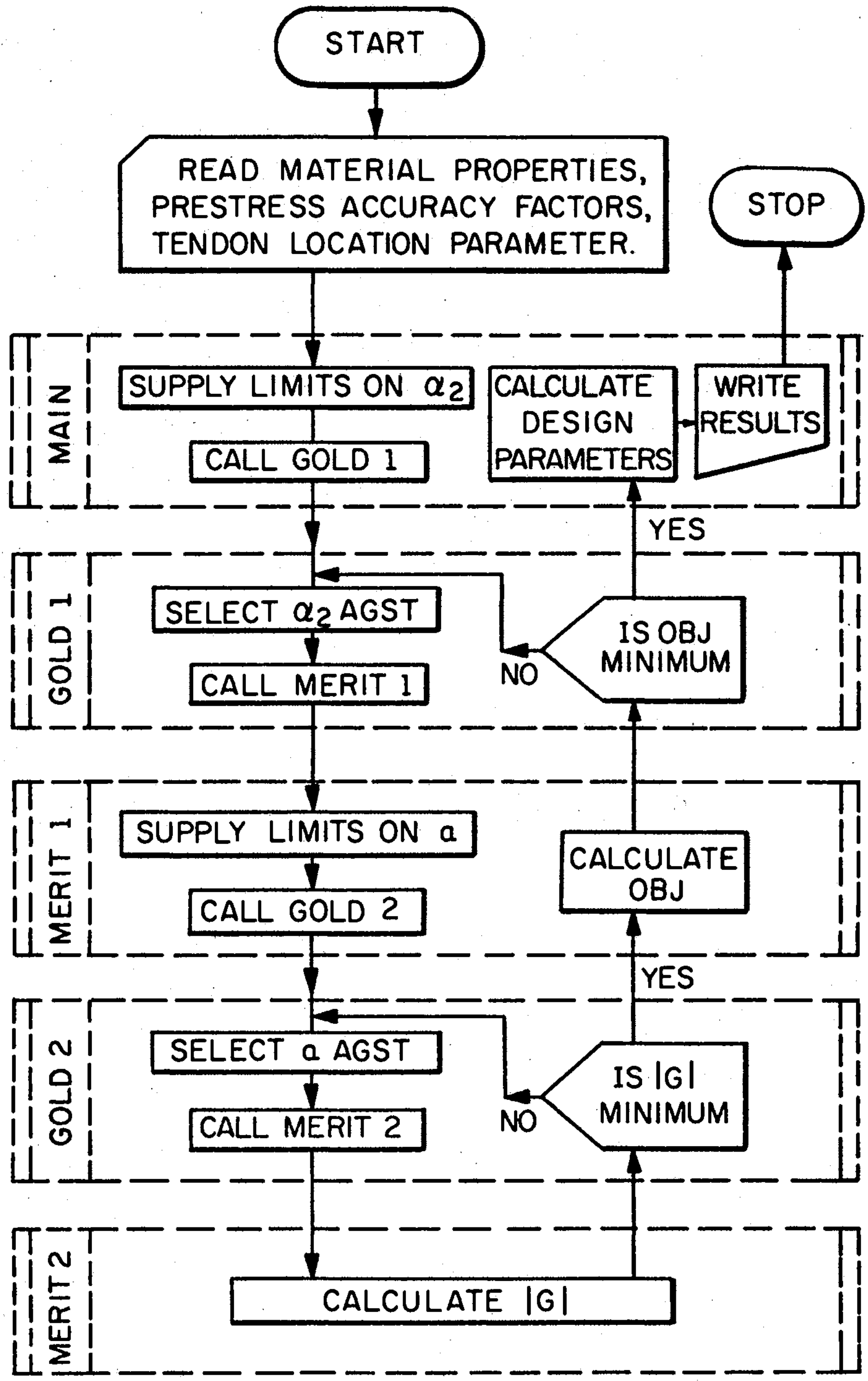
COMPUTER SOLUTION AND EXAMPLES

5.1 General

The governing equations developed in Chapter III are solved using a sequential search technique, the Golden Section search (28). The same search procedure is employed to optimize the objective function with respect to the "extra" variable. Calculations were made on the Oklahoma State University IBM Model 360/65 Computer. The flow charts of the computer programs for a prestressed plate girder and a prestressed composite girder are presented in Figs 8 and 9, respectively. The listing of the programs is given in Appendix B. To cover all aspects of the computation in the present study, the explanations which follow deal with the more involved problem of a prestressed composite girder.

5.2 Solution of the Equations by the Search Technique

An equation of the form $G = 0$ can be solved by minimizing the absolute value $|G|$ with respect to the involved variable (since minimum $|G|$ is equal to zero). The minimization can be performed by a single variable search technique. The search procedure used is based on the elimination technique which by "bold moves" shrinks the region in which the minimum must lie. For a set of two simultaneous equations, successive use of the single variable search technique was

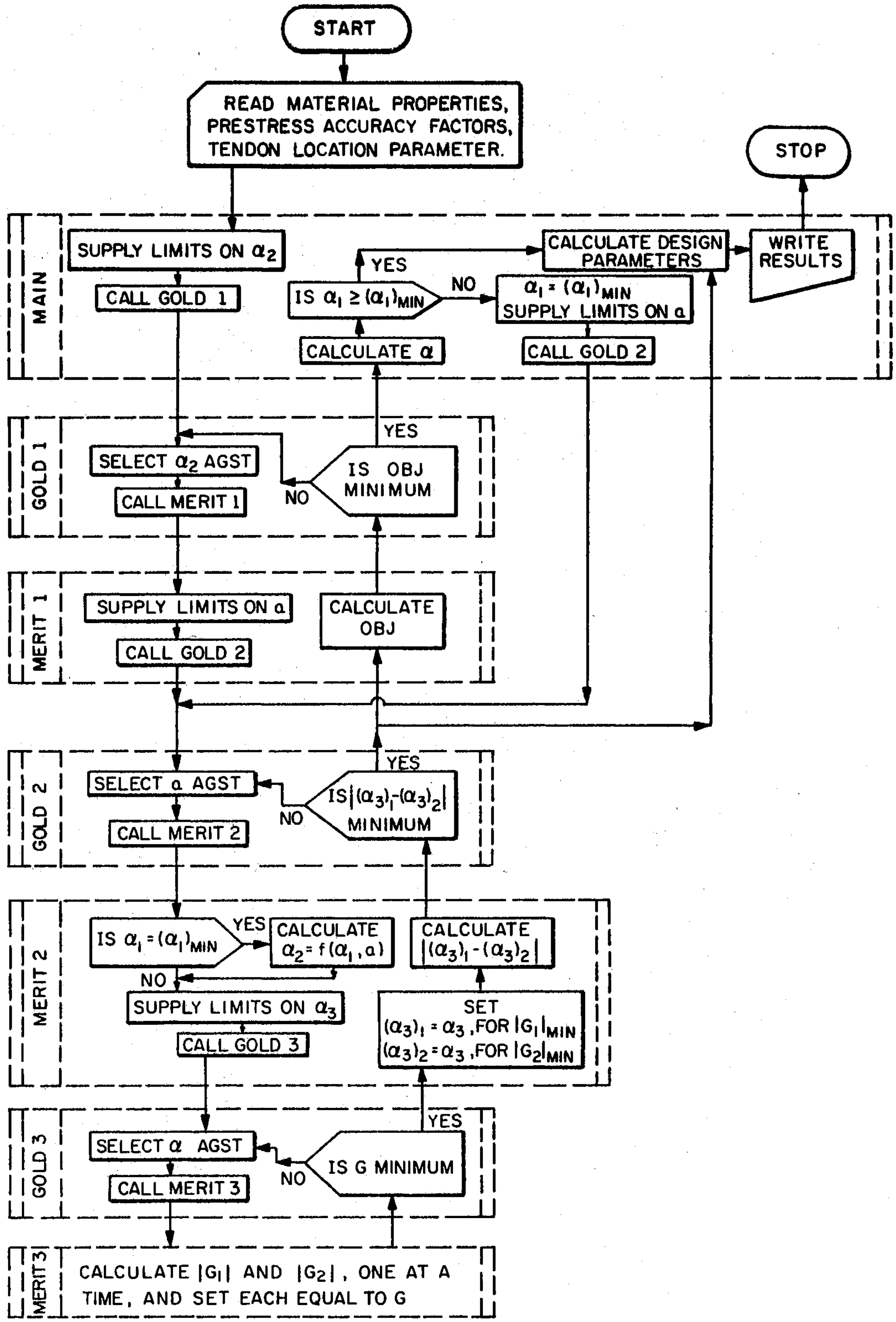


AGST = ACCORDING TO GOLDEN SECTION TECHNIQUE

OBJ = OBJECTIVE FUNCTION

G = GOVERNING EQUATION

Figure 8. Computer Flow Chart for Prestressed Steel Girder



AGST = ACCORDING TO GOLDEN SECTION TECHNIQUE
OBJ = OBJECTIVE FUNCTION
G = GOVERNING EQUATION

Figure 9. Computer Flow Chart for Prestressed Composite Girder

found very effective and is used in lieu of the multivariable search procedures (e.g., gradient method, grid search, etc.) (28).

There are two governing equations and one optimization condition involving three independent variables, section asymmetry a, chord 2 parameter α_2 and concrete slab parameter α_3. These are of the form

$$G_1(a, \alpha_2, \alpha_3) = 0 \tag{5.1}$$

$$G_2(a, \alpha_2, \alpha_3) = 0 \tag{5.2}$$

$$O(a, \alpha_2, \alpha_3), \text{ to be minimized.} \tag{5.3}$$

Eqs (5.1) and (5.2) can be solved simultaneously to obtain a pair of roots (a, α_3) for a selected value of α_2. A set of values of chord 2 parameter α_2 are selected by the elimination technique, and for every value of α_2 and the corresponding values of a and α_3 satisfying Eqs (5.1) and (5.2), the objective function is evaluated and compared to obtain its minimum value.

A set of values of section asymmetry a within given limits is determined by the elimination technique. For selected values of a and α_2, the roots $(\alpha_3)_1$ and $(\alpha_3)_2$ of Eqs (5.1) and (5.2), respectively, are obtained. The absolute difference of the roots $\left|(\alpha_3)_1 - (\alpha_3)_2\right|$ is minimized (the minimum being zero) with respect to section asymmetry a keeping chord 2 parameter α_2 constant. Thus a pair of values of section asymmetry a and concrete slab parameter $\alpha_3 \left[\doteq (\alpha_3)_1 \doteq (\alpha_3)_2\right]$ is determined for a selected value of chord 2 parameter α_2.

After the minimum value of the objective function is determined, chord 1 parameter α_1, is calculated. If α_1 violates the side constraint (the minimum bound), the minimization of the objective function with respect to α_2 is ignored and α_1 is assumed equal to its minimum bound. The procedure is repeated to obtain the corresponding values of section asymmetry a and concrete slab parameter α_3.

Invariably all of the search techniques (except an exhaustive search) require the function to be unimodal (having only one valley in the interval to be explored). The functions encountered in the problems of the present study were investigated in the entire range of the parameters and found to be unimodal. A typical solution of the governing equations is depicted in Fig 10.

5.3 Illustrative Examples

5.3.1 Prestressed Steel Girder

Consider the design of a simply supported plate girder, prestressed by a short tendon, subjected to an unsymmetrical load as shown in Fig 11a.

Quantities μ_{avg}, $\frac{\ell_v}{\ell}$, $\frac{s}{\ell}$, and $\frac{t}{\ell}$ are obtained as function of the parameter μ_{br}, as explained in Sec (2.5):

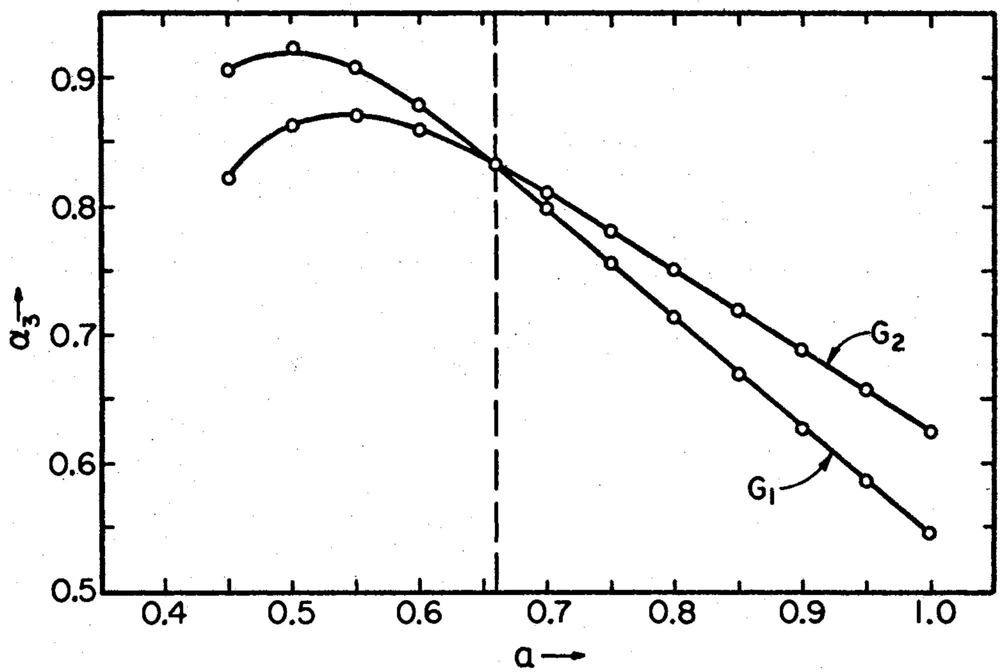
$$\mu_{avg} = \frac{5}{(5 - 4\,\mu_{br})}\left(\frac{7}{2} - \frac{2}{5}\,\mu_{br}{}^2\right),$$

$$\frac{\ell_v}{\ell} = 1 - \frac{4}{5}\,\mu_{br}\,,$$

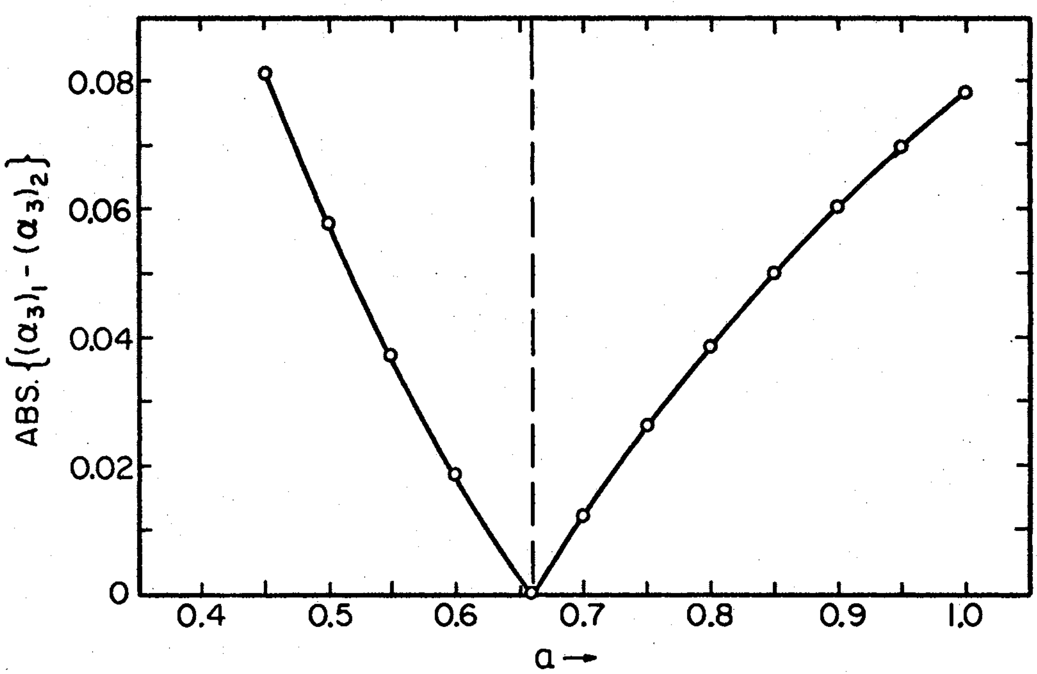
$$\frac{s}{\ell} = \frac{3}{10}\,\mu_{br}\,,$$

$$\frac{t}{\ell} = \frac{1}{2}\,\mu_{br}\,.$$

The following materials are assumed (31): carbon steel ASTM A36 for the girder; $R_2 = 29$ ksi; $E_s = 30,000$ ksi; a wire-rope



(a) SECTION ASYMMETRY a VERSUS CONCRETE SLAB PARAMETER α_3.



(b) SECTION ASYMMETRY a VERSUS ABSOLUTE DIFFERENCE OF THE SLAB PARAMETERS α_3 OBTAINED FROM THE TWO GOVERNING EQUATIONS.

Figure 10. A Typical Solution of the Governing Equations

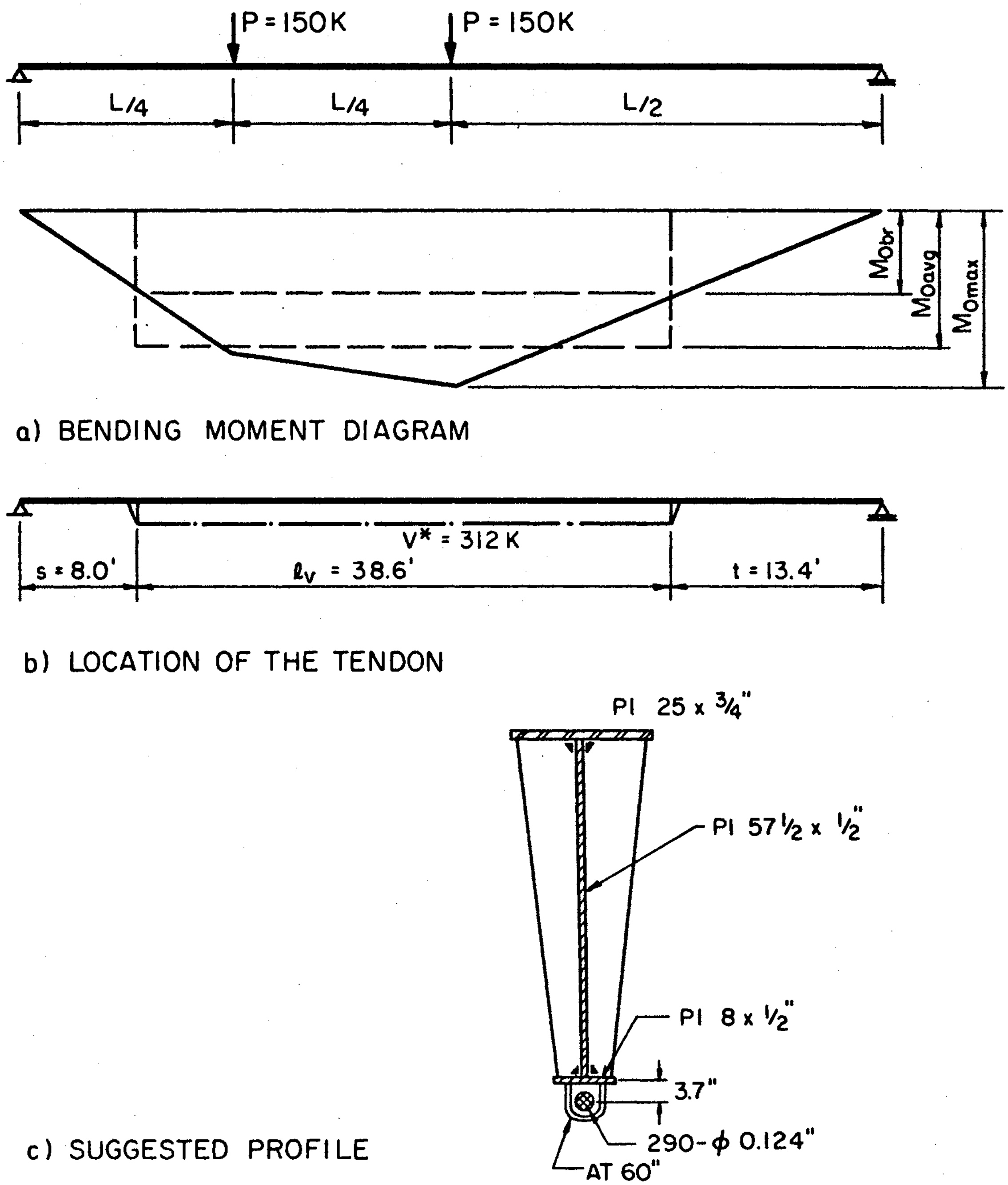


Figure 11. Design of a Prestressed Steel Girder

with parallel straight wires (zinc-coated, with standard ultimate tensile stress 220 ksi): $R_v = 141.5$ ksi, $E_v = 29{,}000$ ksi; hence, $\epsilon = 5.05 \doteq 5$. Furthermore, $\kappa = 1.1$; $\rho_1 = \rho_2 = 1.0$; $n_{v\ell} = 0.9$, $n_{vu} = 1.1$, are assumed.

A computer program (Appendix B) can be used to find the coefficients for the design quantities, given by Eq (3.2). The resulting coefficients are:

$$\alpha = 2.116, \qquad \alpha_1 = 0.361, \qquad \alpha_2 = 0.076,$$

$$\alpha_w = 0.564, \qquad \alpha_v = 0.067, \qquad a = 1.796,$$

$$\beta_v = 0.443, \qquad \frac{\ell_v}{\ell} = 0.642, \qquad \mu_{br} = 0.447,$$

$$\frac{s}{\ell} = 0.134, \qquad \frac{t}{\ell} = 0.223.$$

For the design load $P = 150$ kip and span $\ell = 60'$, the design bending moment $M_{0max} = 40{,}500$ kip-in. The self weight of the girder is neglected in the preliminary design.

For $\lambda = 115$, Eq (3.50) yields $h = 57''$. Other necessary design quantities are obtained from Eq (3.2) as

$$A = 51.4 \text{ in.}^2, \qquad A_1 = 18.7 \text{ in.}^2,$$

$$A_2 = 3.95 \text{ in.}^2, \qquad A_w = 28.75 \text{ in.}^2,$$

$$A_v = 3.48 \text{ in.}^2, \qquad V^* = 312 \text{ kips},$$

$$\ell_v = 38.6 \text{ ft}, \qquad s = 8.0 \text{ ft},$$

$$t = 13.4 \text{ ft}.$$

A possible cross-section is pictured in Fig 11c. The computer time required for this problem was approximately 12 seconds on the IBM 360/65.

5.3.2 Prestressed Composite Girder

In this example the design of a composite plate girder prestressed by a short length tendon and subjected to a concentrated superimposed load at mid-span is considered, Fig 12a.

Required expressions for various parameters are available in Tables V and VI. In addition to the material properties and some parameters used in the previous example, Sec (5.3.1), other quantities assumed are: modular ratio $n_c = 10.0$, concrete design stress = 2.32 ksi; hence $\rho_c = 0.8$; bending moment ratio $\eta = 4.0$, and concrete thickness parameter $\beta = 0.05$.

A computer program (Appendix B) is used to find the coefficients for the design quantities given by Eq (3.3). The resulting coefficients are:

$$\alpha = 5.508, \qquad \alpha_1 = 0.036, \qquad \alpha_2 = 0.215,$$

$$\alpha_w = 0.749, \qquad \alpha_3 = 0.950, \qquad \alpha_v = 0.120,$$

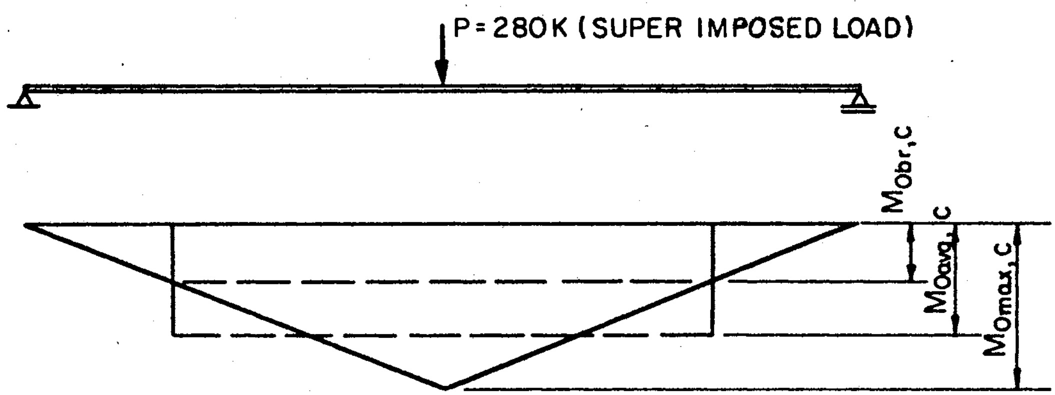
$$\frac{\ell_v}{\ell} = 0.628, \qquad a = 0.697, \qquad \beta_v = 1.938.$$

For the design load (superimposed) $P = 280$ kips, and span ℓ = 60 ft, the design moments $M_{0max,\ c} = 50,400$ kip-in and $M_{0max,\ s}$ 12,600 kip-in, for $\lambda = 120$, Eq (3.50) yields $h = 60$ in, hence $h_c = 6.0$ in. Other needed design quantities are obtained from Eq (3.3) as

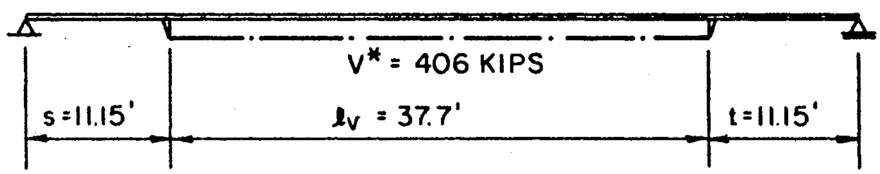
$$A = 40.0 \text{ in.}^2, \qquad A_1 = 1.44 \text{ in.}^2,$$

$$A_2 = 8.56 \text{ in.}^2, \qquad A_w = 30.0 \text{ in.}^2,$$

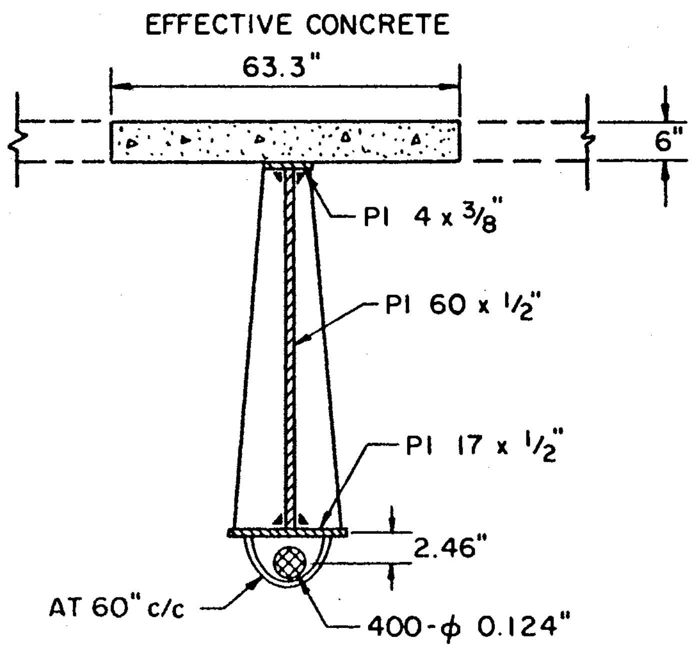
$$A_v = 4.8 \text{ in.}^2, \qquad A_3 = 38.0 \text{ in.}^2,$$



a) BENDING MOMENT DIAGRAM FOR SUPERIMPOSED LOAD



b) LOCATION OF THE TENDON



c) SUGGESTED PROFILE

Figure 12. Design of a Prestressed Composite Girder

$$A_c = 380 \text{ in.}^2, \qquad V^* = 406 \text{ kips,}$$

$$\frac{\ell_v}{\ell} = 37.7 \text{ ft.}$$

A possible profile is depicted in Fig 12c.

The proposed design should be checked for deflections, shear and stability and to verify the assumed moment ratio η. The computer time required for this problem was approximately 45 seconds on the IBM 360/65.

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

A method has been developed for the optimum design of a prestressed plate girder and a prestressed composite girder of a constant cross-section, subjected to one critical inplane load. Prestressing is induced by a tendon of a high-strength material located parallel to the stretched chord of the girder. The tendon may be of full length or of short length (the former being equal to and the latter being shorter than the girder span). Strength conditions are considered as equality constraints, which lead to fully stressed designs. Governing equations, which replace continuity and strength conditions, have been derived for a general distribution of load along the span in the case of a prestressed steel girder and for any symmetrical load in the case of a prestressed composite girder. The optimum design is obtained by a search of the fully stressed designs.

A brief historical sketch of structural optimization with related discussions is presented in Chapter I. Chapter II deals with cross-section properties, design stresses, load conditions and increment of the tendon force. Governing equations for a prestressed steel girder and a prestressed composite girder are developed and expressions for the objective functions are derived in Chapter III. The relationship between a fully stressed design and the minimum weight design is established by the use of Kuhn-Tucker necessary conditions in Chapter IV. Computational aspects of the problem and two illustrative examples are treated in Chapter V.

Governing equations and various needed expressions for the case of accurate prestressing are accumulated in Appendix A. Computer programs are furnished in Appendix B. Some representative cases regarding various material properties, tendon locations, and bending moment ratios for a uniformly distributed load and a concentrated load at mid-span have been considered and the results are presented in Appendix C.

6.2 Conclusions

A fully stressed design obtained by an iterative procedure based on stress rate convergence criteria (12) and (13) has severe difficulties with convergence and there does not exist an explicit optimization condition. These shortcomings have been eliminated in the present approach. It has been shown for the problems of this study that an optimum design is a fully stressed design, within a certain practical range of the parameters involved. The suggested procedure is inexpensive in application. The trial-and-error methods of proportioning are entirely eliminated. If the technological means can be developed for reducing the fabrication cost, the use of optimization techniques in the design of prestressed girders may be promising.

The following observations are based on the numerical results presented in Tables X through XVII, Appendix C.

For a prestressed steel girder:

1. Girders with short length tendons are about 2% to 3% lighter than those with full length tendons.

2. The smallest part of the section is chord 2 (less than 12% of the total area A).

3. About 50% of the material used is needed for the web of the girder.

For a prestressed composite girder:

1. The reduction in weight of the steel section, for the use of the short length tendon instead of the full length tendon, varies from 8% to 12%.

2. The smallest part of the section is chord 1 (less than 8% of the total area A).

3. More than 60% of all steel utilized is needed in the web of the girder. This distribution of the material is reasonable because the steel chords 1 and 2 are partially replaced by the concrete slab and prestressing tendon, respectively.

4. The steel section is lighter for a low strength concrete slab compared to that for a high strength concrete slab, but a larger concrete area is needed in the former case.

6.3 Suggestions for Further Work

During this study, many interesting topics were noted which could merit further investigations. Some suggestions are:

1. Study multiple loading conditions by selecting appropriate constraints from each case of loading and derive the governing equations.

2. Solve the same problem by a non-linear programming approach and compare the computation time.

3. Study a varying cross-section expressed as a function of loading.

4. Extend the approach to the elasto-plastic and plastic range of the materials.

5. Develop the optimization methods for girders with different tendon layouts, for large span trusses, prestressed concrete beams, etc.

6. Investigate the optimum design of statically indeterminate structures prestressed either by prestressing tendons, by the enforced deformation of redundant restrains or by combinations of the two.

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APPENDIX A

EXPRESSIONS FOR SIZING PARAMETERS AND GOVERNING EQUATIONS FOR ACCURATE PRESTRESSING

$(n_{vu} = n_{v\ell} = 1.0)$

TABLE VIII

EXPRESSIONS FOR SIZING PARAMETERS FOR A PRESTRESSED STEEL GIRDER FORMULATED AS FUNCTIONS OF a AND α_2

Tendon of Full Length	Tendon of Short Length
$\alpha = \frac{6a(1+a)}{W_1(1+\rho_2)}$	$\alpha = \frac{6a(1+a)^2}{W_1\{1+a-Y_2(1-\rho_1 a)\}}$
$\alpha_1 = \alpha_2 - \frac{1-a}{1+a}$	$\alpha_1 = \alpha_2 - \frac{1-a}{1+a}$
$\alpha_w = 2(\frac{1}{1+a} - \alpha_2)$	$\alpha_w = 2(\frac{1}{1+a} - \alpha_2)$
$\alpha_v = \frac{n_v \rho_2}{\varepsilon Y_2}$	$\alpha_v = \frac{n_v(\rho_1 a - 1)}{\varepsilon(1+a)}$
$\beta_v = \frac{6a\rho_2(1+a)}{W_1 Y_2(1+\rho_2)} - \frac{\mu_{avg}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}(1+\varepsilon\frac{Y_2}{\rho_2})}$	$\beta_v = \frac{6a\rho_2(1+a)^2}{W_1 Y_2\{1+a-Y_2(1-\rho_1 a)\}}$
$\beta_x = \frac{\mu_{avg}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}(1+\varepsilon\frac{Y_2}{\rho_2})}$	$\beta_x = \frac{\mu_{avg}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}\left[1 - \frac{\varepsilon(1+a)}{1-\rho_1 a}\right]}$

TABLE IX

EXPRESSIONS FOR SIZING PARAMETERS FOR PRESTRESSED COMPOSITE GIRDER FORMULATED AS FUNCTIONS OF a, α_2 AND α_3

Tendon of Full Length	Tendon of Short Length
$\alpha = Y_1$	$\alpha = \dfrac{H_9}{H_8}$
$\alpha_1 = \alpha_2 - \dfrac{1-a}{1+a}$	$\alpha_1 = \alpha_2 - \dfrac{1-a}{1+a}$
$\alpha_w = 2\left(\dfrac{1}{1+a} - \alpha_2\right)$	$\alpha_w = 2\left(\dfrac{1}{1+a} - \alpha_2\right)$
$\alpha_v = \dfrac{n_v Z_2}{\epsilon Y_1}$	$\alpha_v = \dfrac{n_v}{\epsilon} \dfrac{H_{10}}{H_9}$
$\beta_v = Z_2 - \dfrac{(1+\alpha_3)(Y_1\rho_c - Y_6)}{Y_5} - \dfrac{\mu_{avg,\,s}(1+a)}{a\kappa + \dfrac{W_1}{6a\kappa}\left(1+\epsilon \dfrac{Y_1}{Z_2}\right)}$	$\beta_v = \dfrac{\rho_2}{Y_2} \dfrac{H_9}{H_8}$
$\beta_{x,\,s} = \dfrac{\mu_{avg,\,s}(1+a)}{a\kappa + \dfrac{W_1}{6a\kappa}\left(1+\epsilon \dfrac{Y_1}{Z_2}\right)}$	$\beta_{x,s} = \dfrac{\mu_{avg,\,s}(1+a)}{a\kappa + \dfrac{W_1}{6a\kappa}\left(1+\epsilon \dfrac{H_9}{H_{10}}\right)}$
$\beta_{x,\,c} = \dfrac{(1+\alpha_3)(Y_1\rho_c - Y_6)}{Y_5}$	$\beta_{x,c} = \dfrac{(1+\alpha_3)}{Y_3}\left\{Z_4Y_2 + (1+\rho_2)Y_1 - (1+Y_2Z_5)\dfrac{H_9}{H_8}\right\}$

GOVERNING EQUATIONS

A.1 Prestressed Steel Girder

A.1.1 Tendon of the Full Length $\ell_v = \ell$

$$\frac{\rho_2}{Y_2}\left(1 - \frac{6a\kappa}{W_1}\right) - \rho_1 + \frac{(1+\rho_2)}{a} = 0 \qquad (A.1)$$

A.1.2 Tendon of Short Length $\ell_v < \ell$

$$\frac{6a\left\{Y_2(1-\rho_1 a) + \rho_2(1+a)\right\}}{W_1 Y_2\left\{1+a-Y_2(1-\rho_1 a)\right\}} + \frac{\mu_{avg}}{a\kappa + \frac{W_1}{6a\kappa}\left\{1 - \epsilon\frac{(1+a)}{1-\rho_1 a}\right\}} = 0 \qquad (A.2)$$

A.2 Prestressed Composite Girder

A.2.1 Tendon of the Full Length

$$Z_2\left(1 - \frac{6a\kappa}{W_1}\right) + \frac{Y_7}{Y_5}(Y_1\rho_c - Y_6) + Y_4 - \rho_1 Y_1 = 0 \qquad (A.3)$$

$$\frac{\eta_{avg}\,\mu_{avg,s}(1+a)}{\frac{W_6}{6} + \frac{Y}{W_6}\left\{1+\epsilon\frac{Y_1(1+\alpha_3)}{Z_2}\right\}} - \frac{Y_1\rho_c - Y_6}{Y_5} = 0 \qquad (A.4)$$

A.2.2 Tendon of Short Length

$$\frac{\rho_2}{Y_2}\;\frac{Z_9}{Z_8} + \frac{\mu_{avg,s}(1+a)}{a\kappa + \frac{W_1}{6a\kappa}\left\{1+\epsilon\frac{Z_9}{Z_{10}}\right\}} + \frac{(1+\alpha_3)}{Y_3}\left\{Z_4Y_2 + \right.$$

$$\left.(1+\rho_2)Y_1 - (1+Y_2Z_5)\frac{Z_9}{Z_8}\right\} - \frac{Z_{10}}{Z_8} = 0 \qquad (A.5)$$

$$\frac{1}{Y_3}\left\{Z_4Y_2 + (1+\rho_2)Y_1 - (1+Y_2Z_5)\frac{Z_9}{Z_8}\right\} - \frac{\eta_{avg}\,\mu_{avg,s}(1+a)}{\frac{W_6}{6} + \frac{Y}{W_6}\left\{1+\epsilon\frac{(1+\alpha_3)Z_9}{Z_{10}}\right\}} = 0 \qquad (A.6)$$

In expressions for $\mu_{br,s}$, Table VI, replace H_7 by H_6 for accurate prestressing.

APPENDIX B

COMPUTER PROGRAMS

```
C
C
C       CPTIMUM DESIGN OF A PRESTRESSED PLATE GIRDER
C
C       OKLAHOMA STATE UNIVERSITY       CHAMPA LAL MEHTA
C
C
C       GENERAL  COMMENTS
C
C       THE PROGRAM HAS BEEN WRITTEN TO GIVE THE OPTIMUM VALUES OF THE
C       DESIGN PARAMETERS FOR A PRESTRESSED STEEL GIRDER
C
C       ONE DATA CARD IS REQLIRED
C
C
C       DESCRIPTION OF PARAMETERS
C
C
C        R2        = DESIGN STRESS FOR CHORD 2 (TENSION)
C        RAW1      = DESIGN STRESS FOR CHORD 1 (COMP.) / R2
C        RAW2      = DESIGN STRESS FOR CHORD 2 (COMP.) / R2
C        RAWV      = DESIGN STRESS FOR TENDON (TENSION) /R2
C        ENV       = RATIO OF ELASTICITY MODULI  =  E(S)/E(V)
C        EPS       = RAWV * ENV  =  (EPSILCN)
C        RKA       = TENDCN LCCATICN PARAMETER
C        ASM       = SECTICN ASYMMETRY
C        AVG,BR    = COEFFICIENTS AT BENDING MOMENTS
C        RNVU      = UPPER VALUE CF PRESTRESS ACCURACY FACTOR
C        RNVL      = LOWER VALUE CF PRESTRESS ACCURACY FACTOR
C        ALP       = COEFFICIENT FOR CROSS-SECTION AREA
C        ALP1      = CHCRD 1 PARAMETER
C        ALP2      = CHORD 2 PARAMETER
C        ALPW      = WEB PARAMETER
C        ALPV      = TENDCN AREA PARAMETER
C        BV        = COEFFICIENT FOR PRESTRESSING FORCE
C        BX        = COEFFICIENT FOR REDUNDANT FORCE IN THE TENDON X
C        RLEN      = COEFFICIENT FOR LENGTH OF THE TENDCN
C        OBJW      = OBJECTIVE FUNCTION FOR WEIGHT OF THE GIRDER
C        D1        = LOWER BCUND ON ALP2
C        D2        = UPPER BOUND ON ALP2
C        D3 = D6   = FRACTICNAL REDUCTION OF INTERVAL OF UNCERTAINTY
C        D4        = LOWER BCUND CN ASM
C        D5        = UPPER BCUND ON ASM
C
C
C       SUBROUTINES  REQUIREC
C
C        1. GOLD1, 2. MERIT1, 3. GOLD2, 4. MERIT2
C
C
      CCMMON /BLOK1/ RAW1,RAW2,ENV,EPS,RKA,ASM
      CCMMCN /BLOK2/ BR,AVG,ALP2,ALPW,W1,Y2,RNVU,RNVL
      CCMMON /BLOK3/ G
C
    1 FCRMAT (7F10.5)
    2 FORMAT (1H1,16X,'DATA USED',//)
    3 FORMAT (16X,'RAW1',6X,'RAW2',7X,'ENV',7X,'EPS',/)
    4 FORMAT (10X,4F10.5)
```

```
  5 FORMAT (17X,'RKA',6X,'RNVU',6X,'RNVL',/)
  6 FORMAT (10X,3F10.5)
  7 FORMAT (//,16X,'OPTIMUM DESIGN PARAMETERS',/)
  8 FORMAT (//,16X,'RESIDUE LEFT AFTER ITTERATION',/)
 11 FORMAT (/,17X,'ALP',6X,'ALP1',6X,'ALP2',6X,'ALPW',6X,'ALPV',
   17X,'ASM',/)
 12 FORMAT (10X,6F10.5)
 13 FORMAT (/,15X,'BETAV',5X,'BETAX',6X,'RLEN',7X,'S/L',7X,'T/L',
   16X,'OBJW',/)
 15 FORMAT (13X,'G = ',F10.5,////)
C
    READ (5,1) RAW1,RAW2,ENV,EPS,RKA,RNVU,RNVL
    WRITE (6,2)
    WRITE (6,3)
    WRITE (6,4) RAW1,RAW2,ENV,EPS
    WRITE (6,5)
    WRITE (6,6) RKA,RNVU,RNVL
    WRITE (6,7)
    D1  = 0.04
    D2  = 0.15
    D3  = 0.001
    CALL GOLD1 (D1,D2,D3,B1,B2)
    ALP  = 6.*ASM*(1.+ASM)**2./(W1*(1.+ASM+Y2*(RAW1*ASM-1.)))
    ALP1 = ASM/(1.+ASM)-ALPW/2.
    BV   = 6.*ASM*RAW2*(1.+ASM)**2./(RNVU*W1*Y2*(1.+ASM+Y2*(RAW1
   1       *ASM-1.)))
    ALPV = ENV*(BV*(RNVU-RNVL)+ALP*(RAW1*ASM-1.)/(ASM+1.))/(ALP*EPS)
    BX   = AVG*(1.+ASM)/(ASM*RKA+W1/(6.*ASM*RKA)*(1.+ALP*EPS/
   1       (BV*(RNVU-RNVL)+ALP*(RAW1*ASM-1.)/(ASM+1.))))
C
C     FOLLOWING  3  CARDS SHOLD BE CHANGED FOR DIFFERENT LOADS
C
    RLEN = 1.-0.8*BR
    RSL  = 0.3*BR
    RTL  = 0.5*BR
C
    OBJW = (ALP**2./ALPW)**0.33333*(1.+ALPV*RLEN)
    WRITE (6,11)
    WRITE (6,12) ALP,ALP1,ALP2,ALPW,ALPV,ASM
    WRITE (6,13)
    WRITE (6,12) BV,BX,RLEN,RSL,RTL,OBJW
    WRITE (6,8)
    WRITE (6,15)  G
    STOP
    END
```

```
      SUBROUTINE GOLD1 (XL,XR,F,YSMALL,XSMALL)
C
C      SUBROUTINE  GOLD1  SELECTS THE VALUE OF  X  (ALP2) AND CALLS
C      SUBROUTINE MERIT1 TO CALCULATE  Y  (OBJECTIVE FUNCTION)
C
      XLEFT = XL
      XRIGHT = XR
   13 SPAN = XR - XL
      DELTA = ABS(SPAN)
   14 X1 = XL + 0.381966*DELTA
      X2 = XL + 0.618034*DELTA
      CALL MERIT1 (X1,Y1)
      CALL MERIT1 (X2,Y2)
    9 IF(ABS(XL - XR) - ABS(F*SPAN))4,4,8
    8 DELTA = 0.618034*DELTA
      IF (Y2-Y1) 1,10,2
    1 XL = X1
      X1 = X2
      Y1 = Y2
      X2 = XL + 0.618034*DELTA
      CALL MERIT1 (X2,Y2)
      GO TO 9
    2 XR = X2
      Y2 = Y1
      X2 = X1
      X1 = XL + 0.381966*DELTA
      CALL MERIT1 (X1,Y1)
      GO TO 9
    4 IF(Y2 - Y1)5,5,6
    5 YSMALL=Y2
      XSMALL=X2
      GO TO 7
    6 YSMALL=Y1
      XSMALL=X1
      GO TO 7
   10 XL = X1
      XR = X2
      DELTA = XR - XL
      GO TO 14
    7 RETURN
      END


      SUBROUTINE MERIT1 (ALP22,OBJ)
C
C      SUBROUTINE  MERIT1  CALLS SUBROUTINE GOLD2 TO DETERMINE THE
C      VALUE OF ASM AND CALCULATES OBJECTIVE FUNCTION
C
      COMMON /BLOK1/ RAW1,RAW2,ENV,EPS,RKA,ASM
      COMMON /BLOK2/ BR,AVG,ALP2,ALPW,W1,Y2,RNVU,RNVL
      COMMON /BLOK3/ G
      ALP2 = ALP22
      D4   = 1.0
      D5   = 4.0
      D6   = 0.001
      CALL GOLD2 (D4,D5,D6,B3,B4)
      G = B3
      ASM  = B4
      ALP  = 6.*ASM*(1.+ASM)**2./(W1*(1.+ASM+Y2*(RAW1*ASM-1.)))
      BV   = 6.*ASM*RAW2*(1.+ASM)**2./(RNVU*W1*Y2*(1.+ASM+Y2*(RAW1
     1      *ASM-1.)))
      ALPV = ENV*(BV*(RNVU-RNVL)+ALP*(RAW1*ASM-1.)/(ASM+1.))/(ALP*EPS)
C
C      FOLLOWING CARD SHOULD BE CHANGED FOR DIFFERENT BENDING MOMENT
C      DIAGRAMS
C
      RLEN = 1.-0.8*BR
C
      OBJ  = (ALP**2./ALPW)**0.33333*(1.+ALPV*RLEN)
C
      RETURN
      END
```

```
      SUBROUTINE GOLD2 (XL,XR,F,YSMALL,XSMALL)
C
C      SUBROUTINE  GOLD2  SELECTS THE VALUE OF  X  (ASM) AND CALLS
C      SUBROUTINE MERIT2 TO CALCULATE  Y  (RESIDUE OF THE GOVERNING
C      EQUATION)
C
      XLEFT = XL
      XRIGHT = XR
   13 SPAN = XR - XL
      DELTA = ABS(SPAN)
   14 X1 = XL + 0.381966*DELTA
      X2 = XL + 0.618034*DELTA
      CALL MERIT2 (X1,Y1)
      CALL MERIT2 (X2,Y2)
    9 IF(ABS(XL - XR) - ABS(F*SPAN))4,4,8
    8 DELTA = 0.618034*DELTA
      IF (Y2-Y1) 1,10,2
    1 XL = X1
      X1 = X2
      Y1 = Y2
      X2 = XL + 0.618034*DELTA
      CALL MERIT2 (X2,Y2)
      GO TO 9
    2 XR = X2
      Y2 = Y1
      X2 = X1
      X1 = XL + 0.381966*DELTA
      CALL MERIT2 (X1,Y1)
      GO TO 9
    4 IF(Y2 - Y1)5,5,6
    5 YSMALL=Y2
      XSMALL=X2
      GO TO 7
    6 YSMALL=Y1
      XSMALL=X1
      GO TO 7
   10 XL = X1
      XR = X2
      DELTA = XR - XL
      GO TO 14
    7 RETURN
      END


      SUBROUTINE MERIT2 (ASMM,G)
C
C      SUBROUTINE  MERIT2  CALCULATES RESIDUE OF THE GOVERNING EQUATION
C
      COMMON /BLOK1/ RAW1,RAW2,ENV,EPS,RKA,ASM
      COMMON /BLOK2/ BR,AVG,ALP2,ALPW,W1,Y2,RNVU,RNVL
      ASM = ASMM
      ALPW = 2.*(1./(1.+ASM)-ALP2)
      W1  = 2.*(2.*ASM-1.+ALP2*(1.+ASM)**2.)
      Y2  = 1.+6.*RKA*ASM**2./W1
      BR  = (ASM+1.)/(ASM+1.+Y2*(RAW1*ASM-1.))
C
C      FOLLOWING CARD SHOULD BE CHANGED FOR DIFFERENT BENDING MOMENT
C      DIAGRAMS
C
      AVG = 5./(5.-4.*BR)*(7./12.-0.4*BR**2.)
C
      ALP  = 6.*ASM*(1.+ASM)**2./(W1*(1.+ASM+Y2*(RAW1*ASM-1.)))
      BV   = 6.*ASM*RAW2*(1.+ASM)**2./(RNVU*W1*Y2*(1.+ASM+Y2*(RAW1
     1      *ASM-1.)))
C
C      EVALUATE RESIDUE OF GOVERNING EQUATION,  G
C
      G    = ALP*(RAW1*ASM-1.)/(ASM+1.)-RNVL*BV-AVG*(1.+ASM)/(ASM*RKA
     1      +W1/(6.*ASM*RKA)*(1.+ALP*EPS/(BV*(RNVU-RNVL)+ALP*
     2      (RAW1*ASM-1.)/(ASM+1.))))
      G    = ABS(G)
      RETURN
      END
```

DATA USED

RAW1	RAW2	ENV	EPS
1.C000C	1.00000	1.03400	5.00000
RKA	RNVU	RNVL	
1.1000C	1.1000C	0.90C00	

OPTIMUM DESIGN PARAMETERS

ALP	ALP1	ALP2	ALPW	ALPV	ASM
2.11579	0.36058	0.07584	C.56365	0.06756	1.79645
BETAV	BETAX	RLEN	S/L	T/L	OBJW
C.44297	0.20415	0.64223	0.13416	0.22360	2.08171

RESIDUE LEFT AFTER ITTERATION

G = 0.00019

```
C
C
C      OPTIMUM DESIGN OF PRESTRESSED COMPOSITE GIRDER
C
C      OKLAHOMA STATE UNIVERSITY       CHAMPA LAL MEHTA
C
C
C      GENERAL  COMMENTS
C
C      THIS PROGRAM HAS BEEN WRITTEN TO GIVE THE OPTIMUM VALUES OF THE
C      DESIGN PARAMETERS FOR A PRESTRESSED COMPOSITE GIRDER
C
C      TWO DATA CARDS ARE REQUIRED
C
C
C      DESCRIPTION OF PARAMETERS
C
C       R2         = DESIGN STRESS FOR CHORD 2 (TENSION)
C       RAW1       = DESIGN STRESS FOR CHORD 1 (COMP.) / R2
C       RAW2       = DESIGN STRESS FOR CHORD 2 (COMP.) / R2
C       RAWV       = DESIGN STRESS FOR TENDON (TENSION) /R2
C       RAWC       = DESIGN STRESS FOR CONCRETE SLAB (TRANSFORMED INTO
C                    STEEL AREA) /R2
C       ENV        = RATIO OF ELASTICITY MODULI  =  E(S)/E(V)
C       EPS        = RAWV * ENV  =  (EPSILON)
C       RKA        = TENDON LOCATION PARAMETER
C       ASM        = SECTION ASYMMETRY
C       B          = THICKNESS OF CONCRETE SLAB/(2*DEPTH OF STEEL SECTION)
C       RAT        V RATIO OF MAXIMUM MOMENTS
C       BRS,AVGS,AVGC = COEFFICIENTS AT BENDING MOMENTS
C       RNVU       = UPPER VALUE OF PRESTRESS ACCURACY FACTOR
C       RNVL       = LOWER VALUE OF PRESTRESS ACCURACY FACTOR
C       ALP        = COEFFICIENT FOR CROSS-SECTION AREA
C       ALP1       = CHORD 1 PARAMETER
C       ALP2       = CHORD 2 PARAMETER
C       ALPW       = WEB PARAMETER
C       ALP3       = CONCRETE SLAB PARAMETER
C       ALPV       = TENDON AREA PARAMETER
C       ALPP1      = LOWER BOUND ON ALP1
C       BV         = COEFFICIENT FOR PRESTRESSING FORCE
C       BXS,BXC    = COEFFICIENTS FOR REDUNDANT FORCES IN THE TENDON,
C                    X(S) AND X(C), RESPECTIVELY
C       RLEN       = COEFFICIENT FOR LENGTH OF THE TENDON
C       OBJW       = OBJECTIVE FUNCTION FOR WEIGHT OF THE GIRDER
C       D1         = LOWER BOUND ON ALP2
C       D2         = UPPER BOUND ON ALP2
C       D3=D6=D9   = FRACTIONAL REDUCTION OF INTERVAL OF UNCERTAINTY
C       D4         = LOWER BOUND ON ASM
C       D5         = UPPER BOUND ON ASM
C       D7         = LOWER BOUND ON ALP3
C       D8         = UPPER BOUND ON ALP3
C
C
C      SUBROUTINES  REQUIRED
C
C       1. GOLD1, 2. MERIT1, 3. GOLD2, 4. MERIT2, 5. GOLD3, 6. MERIT3
C
C
```

```
      CCMMON /BLOK1/ RAW1,RAW2,RAWC,ENV,EPS,RKA,B,RAT,RNVU,RNVL
      COMMCN /BLOK2/ BRS,AVGS,ASM,ALP1,ALP2,ALPW,ALP31,ALP32,IFS,KK
      CCMMON /BLOK4/ W1,Y,Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8
      COMMON /BLOK5/ D,Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,H8,H9,H10
      COMMON /BLOK6/ G1,G2
      CCMMON /BLOK7/ ALP33
C
    1 FORMAT (5F10.5)
    3 FORMAT (1H1,16X,'DATA USED',//)
    4 FORMAT (16X,'RAW1',6X,'RAW2',6X,'RAWC',7X,'ENV',7X,'EPS',/)
    5 FORMAT (10X,5F10.5)
    6 FORMAT (17X,'RKA',9X,'B',7X,'RAT',6X,'RNVU',6X,'RNVL',/)
    8 FORMAT (//,16X,'OPTIMUM DESIGN PARAMETERS',/)
   11 FORMAT (/,17X,'ALP',6X,'ALP1',6X,'ALP2',6X,'ALPW',6X,'ALP3',
     16X,'ALPV',/)
   12 FORMAT (10X,6F10.5)
   13 FORMAT (/,17X,'ASM',5X,'BETAV',4X,'BETAXS',4X,'BETAXC',6X,
     1'RLEN',6X,'OBJW',/)
   15 FORMAT (//,16X,'RESICUE LEFT AFTER ITTERATION',/)
   16 FORMAT (13X,'G1 = ',F10.5,3X,'G2 = ',F10.5,3X,'ALP33 = ',F10.5,//)
C
      ALPP1 = C.02
      KK  = 1
      READ (5,1) RAW1,RAW2,RAWC,ENV,EPS
      READ (5,1) RKA,B,RAT,RNVU,RNVL
      WRITE (6,3)
      WRITE (6,4)
      WRITE (6,5)  RAW1,RAW2,RAWC,ENV,EPS
      WRITE (6,6)
      WRITE (6,5) RKA,B,RAT,RNVU,RNVL
      WRITE (6,8)
      D1    = 0.08
      D2    = 0.30
      D3    = 0.001
      CALL GOLD1 (D1,D2,C3,B1,B2)
C
C      DETERMINE  ALP1  ANC CCMPARE WITH ITS LOWER BOUND
C
      ALP1 = ASM/(1.+ASM)-ALPW/2.
      IF (ALP1-ALPP1) 21,22,22
   21 ALP1 = ALPP1
      KK  = 2
      D4  = 0.5
      D5  = 1.0
      D6  = 0.001
      CALL GOLD2 (D4,D5,D6,B3,B4)
   22 CCNTINUE
      ALP = (Z6-Z8)/(Z7+Z9)
      ALP2 = 1./(1.+ASM)-ALPW/2.
      ALP3 = (ALP31+ALP32)/2.
      ALPV = ENV/EPS*(Z7*Z8+Z6*Z9)/(Z6-Z8)
      BV   = RAW2/(RNVU*Y2)*(Z6-Z8)/(Z7+Z9)
      BXS = AVGS*(1.+ASM)/(ASM*RKA+W1/(6.*ASM*RKA)*(1.+EPS*(Z6-Z8)/
     1       (Z7*Z8+Z6*Z9)))
      BXC = (1.+ALP3)/Y5*(RAWC*(Z6-Z8)/(Z7+Z9)-Y6)
C
C      FOLLOWING CARD SHOULD BE CHANGED FOR DIFFERENT BENDING MOMENT
C
```

```
      RLEN = (1.-BRS)**0.5
C
      OBJW = (ALP**2./ALPW)**0.33333*(1.+ALPV*RLEN)
      WRITE (6,11)
      WRITE (6,12) ALP,ALP1,ALP2,ALPW,ALP3,ALPV
      WRITE (6,13)
      WRITE (6,12) ASM,BV,BXS,BXC,RLEN,CBJW
      WRITE (6,15)
      WRITE (6,16) G1,G2,ALP33
      STOP
      END


      SUBROUTINE GOLD1 (XL,XR,F,YSMALL,XSMALL)
C
C       SUBROUTINE  GOLD1  SELECTS THE VALUE OF  X  (ALP2) AND CALLS
C       SUBROUTINE MERIT1 TC CALCULATE  Y  (OBJECTIVE FUNCTION)
C
      XLEFT = XL
      XRIGHT = XR
   13 SPAN = XR - XL
      DELTA = ABS(SPAN)
   14 X1 = XL + 0.381966*DELTA
      X2 = XL + 0.618034*DELTA
      CALL MERIT1(X1,Y1)
      CALL MERIT1(X2,Y2)
    9 IF(ABS(XL - XR) - ABS(F*SPAN))4,4,8
    8 DELTA = C.618034*DELTA
      IF (Y2-Y1) 1,10,2
    1 XL = X1
      X1 = X2
      Y1 = Y2
      X2 = XL + 0.618034*DELTA
      CALL MERIT1(X2,Y2)
      GC TO 9
    2 XR = X2
      Y2 = Y1
      X2 = X1
      X1 = XL + 0.381966*DELTA
      CALL MERIT1(X1,Y1)
      GO TO 9
    4 IF(Y2 - Y1)5,5,6
    5 YSMALL=Y2
      XSMALL=X2
      GC TO 7
    6 YSMALL=Y1
      XSMALL=X1
      GC TO 7
   10 XL = X1
      XR = X2
      DELTA = XR - XL
      GO TO 14
    7 RETURN
      END
```

```
      SUBROUTINE MERIT1 (ALP22,OBJ)
C
C      SUBROUTINE  MERIT1  CALLS SUBROUTINE GOLD2 TO DETERMINE THE
C      VALUE OF ASM AND CALCULATES OBJECTIVE FUNCTION
C
      COMMON /BLOK1/ RAW1,RAW2,RAWC,ENV,EPS,RKA,B,RAT,RNVU,RNVL
      COMMON /BLOK2/ BRS,AVGS,ASM,ALP1,ALP2,ALPW,ALP31,ALP32,IFS,KK
      COPMCN /BLOK4/ W1,Y,Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8
      COMMON /BLOK5/ D,Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,H8,H9,H10
      COMMON /BLOK7/ ALP33
      ALP2 = ALP22
      D4  = 0.5
      D5  = 1.0
      D6  = 0.001
      CALL GOLD2 (D4,D5,D6,B3,B4)
      ALP33 = B3
      ALP3 = (ALP31+ALP32)/2.
      ALP  = (Z6-Z8)/(Z7+Z9)
      ALPV = ENV/EPS*(Z7*Z8+Z6*Z9)/(Z6-Z8)
      RLEN = (1.-BRS)**0.5
      OBJ  = (ALP**2./ALPW)**0.33333*(1.+ALPV*RLEN)
      RETURN
      END


      SUBROUTINE GOLD2 (XL,XR,F,YSMALL,XSMALL)
C
C      SUBROUTINE  GOLD2  SELECTS VALUE OF  X  (ASM) AND CALLS
C      SUBROUTINE MERIT2 TO CALCULATE  Y  (DIFFERENCE OF ROOTS OF
C      GOVERNING EQUATICNS)
C
      XLEFT = XL
      XRIGHT = XR
   13 SPAN = XR - XL
      DELTA = ABS(SPAN)
   14 X1 = XL + 0.381966*DELTA
      X2 = XL + 0.618034*DELTA
      CALL MERIT2 (X1,Y1)
      CALL MERIT2 (X2,Y2)
    9 IF(ABS(XL - XR) - ABS(F*SPAN))4,4,8
    8 DELTA = 0.618034*DELTA
      IF (Y2-Y1) 1,10,2
    1 XL = X1
      X1 = X2
      Y1 = Y2
      X2 = XL + 0.618034*DELTA
      CALL MERIT2 (X2,Y2)
      GC TO 9
    2 XR = X2
      Y2 = Y1
      X2 = X1
      X1 = XL + 0.381966*DELTA
      CALL MERIT2 (X1,Y1)
      GO TO 9
    4 IF(Y2 - Y1)5,5,6
    5 YSMALL=Y2
      XSMALL=X2
      GO TO 7
    6 YSMALL=Y1
      XSMALL=X1
      GC TC 7
   10 XL = X1
      XR = X2
      DELTA = XR - XL
      GO TO 14
    7 RETURN
      END
```

```
      SUBROUTINE MERIT2 (ASMM,ALP33)
C
C      SUBROUTINE  MERIT2  CALLS SUBROUTINE GOLD3 TO DETERMINE VALUE
C      OF ALP3 AND CALCULATES DIFFERENCE OF ROOTS OF THE TWO GOVERNING
C      EQUATIONS. IF SIDE CONSTRAINT ON ALP1 IS ACTIVE (KK = 2),
C      CALCULATES ALP2.
C
      COMMON /BLOK2/ BRS,AVGS,ASM,ALP1,ALP2,ALPW,ALP31,ALP32,IFS,KK
      COMMON /BLOK6/ G1,G2
      ASM=ASMM
      GO TO (70,80), KK
   80 ALP2 = ALP1+(1.-ASM)/(1.+ASM)
   70 CONTINUE
      DO 40  IFS = 1,2
      D7  = 0.01
      D8   = 1.5
      D9  = 0.001
      CALL GOLD3 (D7,D8,D9,B5,B6)
      GO TO (50,60), IFS
   50 ALP31 = B6
      G1  = B5
      GO TO 40
   60 ALP32 = B6
      G2  = B5
   40 CONTINUE
      ALP33=ABS (ALP31-ALP32)
      RETURN
      END


      SUBROUTINE GOLD3 (XL,XR,F,YSMALL,XSMALL)
C
C      SUBROUTINE  GOLD3  ESTABLISHES FEASIBLE LOWER LIMIT (XLEFT)
C      AND UPPER LIMIT (XRIGHT) ON ALP3 (WHICH GIVES REAL POSITIVE
C      VALUE OF BRS IN SUBROUTINE MERIT3).  SELECTS VALUE OF  X
C      (ALP3) AND CALLS SUBROUTINE MERIT3 TO CALCULATE  Y  (RESIDUE
C      OF THE GOVERNING EQUATIONS)
C
      COMMON /BLOK2/ BRS,AVGS,ASM,ALP1,ALP2,ALPW,ALP31,ALP32,IFS,KK
      COMMON /BLOK3/ XLEFT,XRIGHT,JIM,KKK
      IF (IFS .EQ. 2) GO TO 3
      JIM = 1
      X1 = XL
      X2 = XR
      KKK= 1
      CALL MERIT3 (X1,Y1)
      KKK = 2
      CALL MERIT3 (X2,Y2)
      JIM = 2
    3 XL = XLEFT
      XR = XRIGHT
   11 SPAN = XR-XL
      DELTA = ABS(SPAN)
   14 X1 = XL + 0.381966*DELTA
      X2 = XL + 0.618034*DELTA
      CALL MERIT3 (X1,Y1)
      CALL MERIT3 (X2,Y2)
    9 IF(ABS(XL - XR) - ABS(F*SPAN))4,4,8
    8 DELTA = 0.618034*DELTA
      IF (Y2-Y1) 1,10,2
    1 XL = X1
      X1 = X2
      Y1 = Y2
      X2 = XL + 0.618034*DELTA
      CALL MERIT3 (X2,Y2)
      GO TO 9
    2 XR = X2
      Y2 = Y1
      X2 = X1
      X1 = XL + 0.381966*DELTA
      CALL MERIT3 (X1,Y1)
      GO TO 9
    4 IF(Y2 - Y1)5,5,6
    5 YSMALL=Y2
      XSMALL=X2
      GO TO 7
    6 YSMALL=Y1
      XSMALL=X1
      GO TO 7
   10 XL = X1
      XR = X2
      DELTA = XR - XL
      GO TO 14
    7 RETURN
      END
```

```
      SUBROUTINE MERIT3 (ALPP,G)
C
C      SUBROUTINE  MERIT3  CALCULATES RESIDUES OF GOVERNING EQUATIONS
C
      COMMON /BLOK1/ RAW1,RAW2,RAWC,ENV,EPS,RKA,B,RAT,RNVU,RNVL
      COMMON /BLOK2/ BRS,AVGS,ASM,ALP1,ALP2,ALPW,ALP31,ALP32,IFS,KK
      COMMON /BLOK3/ XLEFT,XRIGHT,JIM,KKK
      CCMMON /BLOK4/ W1,Y,Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8
      COMMON /BLOK5/ D,Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,H8,H9,H10
      ALP3 = ALPP
      DDEL = 0.1
      NFL  = 1
      NFC  = 1
      ALPW = 2.*(1./(1.+ASM)-ALP2)
  100 CONTINUE
      W1   = 2.*(2.*ASM-1.+ALP2*(1.+ASM)**2.)
      W2   = ASM+ALP3*(1.+B)*(1.+ASM)
      W3   = 1.+B*(2.+ALP3)*(1.+ASM)
      W4   = 1.-ALP3*B*(1.+ASM)
      W5   = 6.*ALP3*(1.+B*(1.+ASM))**2.
      W6   = 6.*(ALP3*(1.+B+ASM*B)+ASM*RKA*(1.+ALP3))
      Y    = W1*(1.+ALP3)+W5
      Y1   = 6.*(1.+ASM)/(1.+RAW2)*(ASM/W1+RAT*W2/Y)
      Y2   = 1.+6.*RKA*ASM**2./W1
      Y3   = -ALP3-(1.+ALP3)*6.*RKA*ASM**2./W1+W2*W6/Y
      Y4   = 6.*(1.+ASM)/W1+6.*W4*RAT*(1.+ASM)/Y
      Y5   = 1.-W3*W6/Y
      Y6   = 6.*W3*RAT*(1.+ASM)/Y
      Y7   = -ALP3+6.*ASM*RKA*(1.+ALP3)/W1-W4*W6/Y
      Y8   = RAT*W1*W2/(ASM*Y)
      Z1   = (RNVU-RNVL)/(1.+RAW2)
      Z2   = RAW2*Y1/Y2-Y3*(RAWC*Y1-Y6)/(Y2*Y5)
      Z3   = RAW2*(1.-RAWC*Y3/(RAW2*Y5))
      Z4   = (Y4+(1.+RAW2)*Y1*Y7/Y3)/(1.-6.*ASM*RKA/W1-Y2*Y7/Y3)
      Z5   = (RAW1+Y7/Y3)/(1.-6.*ASM*RKA/W1-Y2*Y7/Y3)
      Z6   = Y3*Y6/(Y2*Y5)+Y1*(1.+RAW2)/Y2
      Z7   = 1./Y2+RAWC*Y3/(Y2*Y5)-RAW2*(RNVU-RNVL)/(RNVU*Y2)
      Z8   = (W1/(W1-6.*ASM*RKA))*(Y6*Y7/Y5-Y4)
      Z9   = (W1/(W1-6.*ASM*RKA))*(RAW1-RAWC*Y7/Y5)
     1       +(RNVU-RNVL)*RAW2/(RNVU*Y2)
      H7   = W1/(6.*ASM*(1.+ASM))*(Z6-Z8)/(Z7+Z9)
      H8   = Y5*(1.+Z5*Y2)+RAWC*Y3
      H9   = Z4*Y2*Y5+(1.+RAW2)*Y1*Y5+Y3*Y6
      H10  = Z5*((1.+RAW2)*Y1*Y5+Y3*Y6)-Z4*(Y5+RAWC*Y3)
      H    = H7
C
C      FOLLOWING CARD SHOULD BE CHANGED FOR DIFFERENT BENDING MOMENT
C      DIAGRAMS
C
      HH2 = Y8*Y8+4.*Y8-4.*H+4.
C
      IF (HH2 .LT. 0.0) GO TO 25
C
C      FOLLOWING CARD SHOULD BE CHANGED FOR DIFFERENT BENDING MOMENT
C      DIAGRAMS
C
      BRS = (Y8*HH2**0.5-Y8*Y8-2.*Y8+2.*H)/2.
C
```

```
      IF (BRS .LE. 1.0  .AND.  BRS .GE. 0.0) GO TO 26
   25 FJ = 1.0
      NFC = 2
      IF (NFL .EQ. 2) DDEL = DDEL/2.
      IF (DDEL .LT. 0.001) GC TO 65
      GO TC 27
   26 GO TO (28,52),JIM
   28 IF (NFC .EQ. 1) GO TC 56
      NFL =2
      FJ =-1.
      DDEL = DDEL/2.
      IF (DDEL .LT. 0.001) GO TO 56
   27 GO TO (45,55),KKK
   45 ALP3 = ALP3 + DDEL*FJ
      GO TC 100
   55 ALP3 = ALP3 - DDEL*FJ
      GO TC 100
   65 ALP3 = ALP3 + 0.002
   56 GO TO (53,54),KKK
   53 XLEFT = ALP3
      RETURN
   54 XRIGHT = ALP3
      RETURN
   52 CCNTINUE
      AVGS = (2.+BRS)/3.
C
C      FOLLOWING CARD SHOULD BE CHANGED FOR DIFFERENT BENDING MOMENT
C      DIAGRAMS
C
      AVGC = 1.-(1.-BRS)**C.5/2.
C
      RATAV = RAT*AVGC/AVGS
      GO TO (10,20), IFS
   10 CCNTINUE
C
C      EVALUATICN OF RESICUE OF THE  FIRST GOVERNING EQUATION  G1
C
      G1  = (Z7*Z8+Z6*Z9-(Z6-Z8)*(RAW2/Y2+RAWC*(1.+ALP3)/Y5))/(Z7+Z9)
     1      +(1.+ALP3)*Y6/Y5-AVGS*(1.+ASM)/(ASM*RKA+W1/(6.*ASM*RKA)*
     2      (1.+EPS*(Z6-Z8)/(Z7*Z8+Z6*Z9)))
      G   = ABS(G1)
      RETURN
   20 CONTINUE
C
C      EVALUATION OF RESIDLE OF THE SECCND GOVERNING EQUATION  G2
C
      G2  = RAWC/Y5*(Z6-Z8)/(Z7+Z9)-Y6/Y5-RATAV*AVGS*(1.+ASM)/(W6/6.
     1      +Y/W6*(1.+EPS*(1.+ALP3)*(Z6-Z8)/(Z7*Z8+Z6*Z9)))
      G   = ABS(G2)
      RETLRN
      END
```

DATA USED

RAW1	RAW2	RAWC	EWV	EPS
1.C0000	1.CCCOC	0.80000	1.C3400	5.00000
RKA	**B**	**RAT**	**RNVU**	**RNVL**
1.1000C	C.C5C00	4.CCOOC	1.100C0	0.90000

OPTIMUM DESIGN PARAMETERS

ALP	ALP1	ALP2	ALPW	ALP3	ALPV
5.50758	0.C3615	0.21482	0.74902	C.95019	0.12054
ASM	**BETAV**	**BETAXS**	**BETAXC**	**RLEN**	**CBJW**
0.69683	1.93847	C.29581	C.59146	0.62787	3.69410

RESIDUE LEFT AFTER ITTERATICN

G1 = C.00024 G2 = C.CCC17 ALP33 = 0.07503

APPENDIX C

CHARACTERISTIC QUANTITIES FOR PRE-STRESSED STEEL GIRDERS (TABLES X THROUGH XIII) AND COMPOSITE GIRDERS (TABLES XIV THROUGH XVII)

TABLE X

CHARACTERISTIC QUANTITIES FOR A PLATE GIRDER, PRESTRESSED BY A FULL LENGTH TENDON, SUBJECTED TO A UNIFORMLY DISTRIBUTED LOAD THROUGHOUT THE SPAN

Assumed Parameters: $\rho_2 = 1.0$, $n_v = 1.0$, $\epsilon = 5.0$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_1	α	α_1	α_2	α_w	α_v	a	β_v	β_x	σ_w
	0.7	3.0099	0.4449	0.0460	0.5090	0.0443	2.3268	0.4912	0.1252	2.7266
	0.8	2.7832	0.3986	0.0573	0.5542	0.0487	2.0368	0.5020	0.1270	2.5415
0.9	0.9	2.5501	0.3701	0.0780	0.5518	0.0534	1.8252	0.5006	0.1276	2.3971
	1.0	2.3933	0.3387	0.0940	0.5674	0.0575	1.6479	0.5100	0.1288	2.2854
	0.7	2.9725	0.4305	0.0452	0.5242	0.0415	2.2536	0.4431	0.1271	2.6702
	0.8	2.7029	0.3947	0.0643	0.5410	0.0461	1.9867	0.4479	0.1285	2.4911
1.0	0.9	2.4688	0.3695	0.0880	0.5424	0.0508	1.7836	0.4506	0.1293	2.3535
	1.0	2.3586	0.3283	0.0969	0.5749	0.0542	1.6027	0.4631	0.1316	2.2464
	0.7	2.8881	0.4287	0.0527	0.5186	0.0395	2.2054	0.4041	0.1279	2.6240
	0.8	2.6342	0.3917	0.0710	0.5373	0.0439	1.9436	0.4056	0.1296	2.4491
1.1	0.9	2.4753	0.3508	0.0834	0.5658	0.0476	1.7304	0.4170	0.1320	2.3176
	1.0	2.2943	0.3276	0.1054	0.5669	0.0517	1.5712	0.4160	0.1331	2.2105

TABLE XI

CHARACTERISTIC QUANTITIES FOR A PLATE GIRDER, PRESTRESSED BY A FULL LENGTH TENDON, SUBJECTED TO A CONCENTRATED LOAD AT MID-SPAN

Assumed Parameters: $\rho_2 = 1.0$, $n_v = 1.0$, $\kappa = 5.0$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_1	α	α_1	α_2	α_w	α_v	a	β_v	β_x	O_w
	0.7	3.0002	0.4471	0.0491	0.5038	0.0447	2.3224	0.5254	0.0938	2.7310
	0.8	2.8042	0.3942	0.0554	0.5504	0.0486	2.0253	0.5348	0.0955	2.5445
0.9	0.9	2.5539	0.3698	0.0794	0.5508	0.0536	1.8186	0.5370	0.0958	2.4014
	1.0	2.3668	0.3452	0.1002	0.5546	0.0580	1.6487	0.5349	0.0961	2.2870
	0.7	2.9794	0.4297	0.0461	0.5242	0.0416	2.2448	0.4788	0.0954	2.6748
	0.8	2.7011	0.3953	0.0662	0.5384	0.0463	1.9809	0.4796	0.0963	2.4945
1.0	0.9	2.4596	0.3721	0.0915	0.5363	0.0512	1.7800	0.4821	0.0968	2.3574
	1.0	2.3476	0.3310	0.1002	0.5687	0.0545	1.6000	0.4900	0.0985	2.2481
	0.7	2.8896	0.4264	0.0522	0.5213	0.0396	2.1955	0.4328	0.0960	2.6265
	0.8	2.6321	0.3925	0.0732	0.5342	0.0441	1.9379	0.4378	0.0972	2.4531
1.1	0.9	2.4362	0.3603	0.0920	0.5476	0.0483	1.7334	0.4431	0.0984	2.3198
	1.0	2.2932	0.3285	0.1077	0.5637	0.0520	1.5662	0.4490	0.0998	2.2145

TABLE XII

CHARACTERISTIC QUANTITIES FOR A PLATE GIRDER, PRESTRESSED BY A SHORT LENGTH TENDON, SUBJECTED TO A UNIFORMLY DISTRIBUTED LOAD THROUGHOUT THE SPAN

Assumed Parameters: $\rho_2 = 1.0$, $n_v = 1.0$, $\kappa = 5.0$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_1	α	α_1	α_2	α_w	α_v	$\frac{\ell_v}{\ell}$	a	β_v	β_x	O_w
	0.7	2.7769	0.4946	0.0406	0.4648	0.0545	0.7333	2.6625	0.5119	0.1940	2.6524
	0.8	2.6077	0.4365	0.0410	0.5226	0.0592	0.7330	2.3087	0.5223	0.1976	2.4542
0.9	0.9	2.4348	0.3946	0.0497	0.5556	0.0642	0.7333	2.0527	0.5285	0.2000	2.3052
	1.0	2.2454	0.3718	0.0695	0.5587	0.0699	0.7333	1.8661	0.5306	0.2007	2.1887
	0.7	2.6830	0.4874	0.0414	0.4711	0.0527	0.7404	2.6102	0.4607	0.1999	2.5782
	0.8	2.5264	0.4283	0.0409	0.5307	0.0572	0.7404	2.2643	0.4706	0.2043	2.3880
1.0	0.9	2.3635	0.3864	0.0498	0.5637	0.0620	0.7404	2.0146	0.4778	0.2074	2.2466
	1.0	2.2003	0.3584	0.0659	0.5757	0.0673	0.7405	1.8266	0.4821	0.2093	2.1348
	0.7	2.6043	0.4798	0.0408	0.4794	0.0510	0.7474	2.5650	0.4175	0.2053	2.5109
	0.8	2.4361	0.4256	0.0438	0.5305	0.0557	0.7475	2.2350	0.4261	0.2097	2.3296
1.1	0.9	2.2649	0.3884	0.0559	0.5556	0.0608	0.7475	1.9960	0.4323	0.2128	2.1932
	1.0	2.1217	0.3571	0.0697	0.5732	0.0657	0.7476	1.8066	0.4377	0.2157	2.0854

TABLE XIII

CHARACTERISTIC QUANTITIES FOR A PLATE GIRDER, PRESTRESSED BY A SHORT LENGTH TENDON, SUBJECTED TO A CONCENTRATED LOAD AT MID-SPAN

Assumed Parameters: $\rho_2 = 1.0$, $n_v = 1.0$, [illegible] $= 5.0$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_1	α	α_1	α_2	α_w	α_v	$\frac{\ell_v}{\ell}$	a	β_v	β_x	O_w
	0.7	2.9160	0.4639	0.0403	0.4957	0.0497	0.4980	2.4696	0.5614	0.1070	2.6428
	0.8	2.6928	0.4171	0.0498	0.5330	0.0546	0.4980	2.1610	0.5691	0.1084	2.4521
0.9	0.9	2.4572	0.3910	0.0712	0.5378	0.0601	0.4980	1.9401	0.5716	0.1088	2.3061
	1.0	2.3054	0.3587	0.0857	0.5557	0.0646	0.4975	1.7512	0.5778	0.1098	2.1910
	0.7	2.8305	0.4570	0.0436	0.4993	0.0475	0.5031	2.4093	0.5119	0.1090	2.5825
	0.8	2.6037	0.4136	0.0557	0.5306	0.0524	0.5031	2.1144	0.5192	0.1105	2.3994
1.0	0.9	2.4237	0.3765	0.0696	0.5540	0.0570	0.5026	1.8861	0.5267	0.1120	2.2599
	1.0	2.2478	0.3519	0.0895	0.5586	0.0619	0.5026	1.7118	0.5306	0.1128	2.1484
	0.7	2.7594	0.4502	0.0459	0.5039	0.0455	0.5081	2.3569	0.4703	0.1107	2.5294
	0.8	2.5585	0.4032	0.0558	0.5411	0.0500	0.5076	2.0648	0.4796	0.1128	2.3539
1.1	0.9	2.3480	0.3747	0.0756	0.5496	0.0551	0.5082	1.8532	0.4838	0.1139	2.2171
	1.0	2.1805	0.3501	0.0956	0.5542	0.0598	0.5083	1.6825	0.4882	0.1150	2.1094

TABLE XIV

CHARACTERISTIC QUANTITIES FOR A COMPOSITE PLATE GIRDER, PRESTRESSED BY A FULL LENGTH TENDON, SUBJECTED TO A UNIFORMLY DISTRIBUTED LOAD THROUGHOUT THE SPAN

Assumed Parameters: $\rho_1 = 1.0$, $\rho_2 = 1.0$, $n_v = 1.0$, $\epsilon = 5.0$, $\beta = 0.05$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_c	η	α	α_1	α_2	α_w	α_3	α_v	a	β_v	$\beta_{x,s}$	$\beta_{x,c}$	O_w
0.9	0.6	3	5.7604	0.0637	0.1984	0.7379	0.9544	0.0853	0.7625	1.7932	0.1518	0.3317	3.8592
		4	6.2887	0.0224	0.2544	0.7232	1.2264	0.0946	0.6234	2.1693	0.1577	0.4312	4.1544
		5	6.6026	0.0203	0.3271	0.6526	1.4701	0.1055	0.5305	2.5472	0.1563	0.5262	4.4857
	0.8	3	7.2138	0.0770	0.1120	0.8110	0.4775	0.0716	0.9324	1.9036	0.1521	0.3356	4.2900
		4	7.4730	0.0489	0.1789	0.7722	0.6809	0.0827	0.7700	2.2495	0.1548	0.4599	4.5107
		5	7.8023	0.0288	0.2390	0.7322	0.8548	0.0922	0.6526	2.6270	0.1568	0.5513	4.7670
1.0	0.6	3	5.5393	0.0604	0.2190	0.7205	0.9877	0.0835	0.7262	1.6617	0.1574	0.3262	3.7834
		4	6.0320	0.0218	0.2799	0.6983	1.2681	0.0934	0.5897	2.0223	0.1641	0.4285	4.0837
		5	6.3550	0.0203	0.3535	0.6262	1.5147	0.1044	0.5001	2.3867	0.1637	0.5294	4.4290
	0.8	3	6.8649	0.0746	0.1325	0.7929	0.5024	0.0701	0.8906	1.7240	0.1564	0.3542	4.1761
		4	7.1718	0.0376	0.1985	0.7640	0.7149	0.0809	0.7228	2.0876	0.1618	0.4430	4.3972
		5	7.4737	0.0230	0.2632	0.7138	0.8913	0.0909	0.6127	2.4365	0.1641	0.5535	4.6661
1.1	0.6	3	5.3944	0.0511	0.2328	0.7161	1.0145	0.0812	0.6924	1.5507	0.1638	0.3216	3.7172
		4	5.8223	0.0213	0.3023	0.6764	1.3044	0.0922	0.5612	1.8986	0.1702	0.4252	4.0268
		5	6.3535	0.0201	0.3531	0.6268	1.5004	0.0998	0.5003	2.2488	0.1707	0.5251	4.4085
	0.8	3	6.5402	0.0746	0.1540	0.7714	0.5276	0.0691	0.8530	1.5920	0.1599	0.3484	4.0768
		4	6.9218	0.0278	0.2162	0.7560	0.7447	0.0793	0.6829	1.9450	0.1685	0.4359	4.3029
		5	7.1644	0.0208	0.2884	0.6908	0.9266	0.0902	0.5778	2.2790	0.1704	0.5533	4.5831

TABLE XV

CHARACTERISTIC QUANTITIES FOR A COMPOSITE PLATE GIRDER, PRESTRESSED BY A FULL LENGTH TENDON, SUBJECTED TO A CONCENTRATED LOAD AT MID-SPAN

Assumed Parameters: $\rho_1 = 1.0$, $\rho_2 = 1.0$, $n_v = 1.0$, $\epsilon = 5.0$, $\beta = 0.05$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_c	η	α	α_1	α_2	α_w	α_3	α_v	a	β_v	$\beta_{x,s}$	$\beta_{x,c}$	O_w
0.9	0.6	3	5.6744	0.0659	0.2083	0.7258	0.9758	0.0864	0.7507	1.8707	0.1507	0.2436	3.8459
		4	6.2193	0.0227	0.2630	0.7143	1.2475	0.0956	0.6125	2.2640	0.1569	0.3252	4.1449
		5	6.5342	0.0210	0.3366	0.6424	1.4929	0.1066	0.5202	2.6582	0.1556	0.4045	4.4825
	0.8	3	7.0801	0.0788	0.1209	0.8003	0.4923	0.0727	0.9192	1.9588	0.1513	0.2686	4.2601
		4	7.4971	0.0309	0.1769	0.7922	0.6985	0.0823	0.7453	2.3714	0.1573	0.3195	4.4806
		5	7.7244	0.0254	0.2465	0.7281	0.8728	0.0930	0.6379	2.7660	0.1567	0.3932	4.7476
1.0	0.6	3	5.5603	0.0481	0.2175	0.7345	1.0027	0.0832	0.7103	1.7341	0.1591	0.2462	3.7679
		4	5.9676	0.0206	0.2889	0.6904	1.2928	0.0944	0.5769	2.1269	0.1637	0.3120	4.0736
		5	6.3729	0.0211	0.3540	0.6249	1.5184	0.1041	0.5005	2.5070	0.1630	0.3967	4.4392
	0.8	3	6.7050	0.0767	0.1444	0.7789	0.5226	0.0716	0.8733	1.8069	0.1554	0.2557	4.1411
		4	7.1309	0.0275	0.2020	0.7704	0.7339	0.0812	0.7029	2.1852	0.1631	0.3298	4.3695
		5	7.3524	0.0216	0.2749	0.7035	0.9153	0.0923	0.5957	2.5596	0.1637	0.4142	4.6438
1.1	0.6	3	5.3901	0.0422	0.2345	0.7233	1.0330	0.0813	0.6774	1.6285	0.1649	0.2344	3.7030
		4	5.7459	0.0211	0.3133	0.6656	1.3328	0.0934	0.5477	1.9952	0.1697	0.3188	4.0174
		5	6.3774	0.0201	0.3531	0.6268	1.5045	0.0994	0.5003	2.3745	0.1701	0.3846	4.4177
	0.8	3	6.4091	0.0713	0.1641	0.7647	0.5511	0.0704	0.8302	1.6648	0.1599	0.2636	4.0385
		4	6.8113	0.0254	0.2263	0.7483	0.7672	0.0804	0.6655	2.0314	0.1684	0.3367	4.2761
		5	7.0260	0.0212	0.3029	0.6759	0.9541	0.0919	0.5605	2.4036	0.1696	0.4146	4.5641

TABLE XVI

CHARACTERISTIC QUANTITIES FOR A COMPOSITE PLATE GIRDER, PRESTRESSED BY A SHORT LENGTH TENDON, SUBJECTED TO A UNIFORMLY DISTRIBUTED LOAD THROUGHOUT THE SPAN

Assumed Parameters: $\rho_1 = 1.0$, $\rho_2 = 1.0$, $n_v = 1.0$, $\epsilon = 5.0$, $\beta = 0.05$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_c	η	α	α_1	α_2	α_w	α_3	α_v	$\frac{\ell_v}{\ell}$	a	β_v	$\beta_{x,s}$	$\beta_{x,c}$	O_w
0.9	0.6	3	5.2040	0.0665	0.1668	0.7667	1.0954	0.1078	0.7316	0.8177	1.8762	0.2386	0.5028	3.5396
		4	5.6262	0.0220	0.2233	0.7546	1.4116	0.1210	0.7321	0.6649	2.2651	0.2488	0.6589	3.7822
		5	5.8716	0.0200	0.2960	0.6840	1.6971	0.1361	0.7327	0.5674	2.6576	0.2471	0.8244	4.9621
	0.8	3	6.4568	0.0776	0.0869	0.8355	0.5679	0.0907	0.7312	0.9817	1.9601	0.2394	0.5296	3.9256
		4	6.7514	0.0315	0.1439	0.8246	0.7998	0.1041	0.7314	0.7979	2.3424	0.2487	0.6825	4.0992
		5	6.9437	0.0241	0.2120	0.7639	0.9985	0.1179	0.7308	0.6836	2.7408	0.2476	0.8427	4.3245
1.0	0.6	3	4.8918	0.0657	0.1899	0.7444	1.1550	0.1089	0.7369	0.7791	1.7298	0.2500	0.5080	3.4347
		4	5.3125	0.0212	0.2495	0.7293	1.4803	0.1223	0.7364	0.6282	2.1092	0.2617	0.6638	3.6870
		5	5.5595	0.0200	0.3251	0.6549	1.7747	0.1379	0.7365	0.5324	2.4938	0.2609	0.8298	3.4808
	0.8	3	5.9174	0.0894	0.1176	0.7930	0.6120	0.0930	0.7367	0.9452	1.7888	0.2459	0.5373	3.7766
		4	6.1795	0.0483	0.1827	0.7691	0.8572	0.1076	0.7360	0.7631	2.1633	0.2543	0.6930	3.9666
		5	6.3821	0.0342	0.2498	0.7160	1.0680	0.1226	0.7371	0.6452	2.5315	0.2581	0.8561	4.1934
1.1	0.6	3	4.6790	0.0580	0.2060	0.7359	1.2045	0.1088	0.7409	0.7422	1.6124	0.2619	0.5111	3.3481
		4	5.0427	0.0200	0.2731	0.7069	1.5452	0.1238	0.7407	0.5960	1.9756	0.2744	0.6687	3.6038
		5	5.2877	0.0200	0.3515	0.6285	1.8486	0.1400	0.7405	0.5020	2.3525	0.2745	0.8358	3.9106
	0.8	3	5.3370	0.1245	0.1613	0.7142	0.6550	0.0974	0.7423	0.9289	1.6431	0.2468	0.5471	3.6636
		4	5.7481	0.0576	0.2149	0.7265	0.9100	0.1103	0.7402	0.7281	2.0150	0.2616	0.7006	3.8590
		5	6.1117	0.0200	0.2769	0.7031	1.1207	0.1215	0.7362	0.5912	2.4099	0.2699	0.7826	4.0955

TABLE XVII

CHARACTERISTIC QUANTITIES FOR A COMPOSITE PLATE GIRDER, PRESTRESSED BY A SHORT LENGTH TENDON, SUBJECTED TO A CONCENTRATED LOAD AT MID-SPAN

Assumed Parameters: $\rho_1 = 1.0$, $\rho_2 = 1.0$, $n_v = 1.0$, $\epsilon = 5.0$, $\beta = 0.05$, $n_{vu} = 1.1$, $n_{v\ell} = 0.9$

κ	ρ_c	η	α	α_1	α_2	α_w	α_3	α_v	$\frac{\ell_v}{\ell}$	a	β_v	$\beta_{x,s}$	$\beta_{x,c}$	O_w
0.9		3	5.1888	0.0674	0.1774	0.7552	1.1013	0.1066	0.6072	0.8017	1.9087	0.2478	0.4159	3.5043
	0.6	4	5.5930	0.0285	0.2389	0.7325	1.4173	0.1199	0.5961	0.6523	2.3106	0.2573	0.5500	3.7452
		5	5.9062	0.0200	0.3067	0.6732	1.6900	0.1333	0.5862	0.5543	2.7167	0.2584	0.6902	4.0192
		3	6.4526	0.0733	0.0941	0.8326	0.5740	0.0895	0.6039	0.9592	1.9944	0.2502	0.4376	3.8832
	0.8	4	6.7947	0.0232	0.1493	0.8275	0.8027	0.1020	0.5944	0.7760	2.3854	0.2622	0.5671	4.0525
		5	6.9094	0.0254	0.2235	0.7511	1.0024	0.1171	0.5853	0.6692	2.7819	0.2604	0.7088	4.2641
1.0		3	4.8840	0.0664	0.2019	0.7317	1.1619	0.1075	0.6132	0.7613	1.7666	0.2592	0.4188	3.4050
	0.6	4	5.3358	0.0200	0.2604	0.7195	1.4802	0.1199	0.6015	0.6123	2.1585	0.2728	0.5505	3.6534
		5	5.5883	0.0200	0.3368	0.6432	1.7693	0.1352	0.5914	0.5188	2.5513	0.2734	0.6938	3.9398
		3	6.1009	0.0571	0.1091	0.8339	0.6181	0.0891	0.6115	0.9012	1.8267	0.2637	0.4391	3.7406
	0.8	4	6.3411	0.0286	0.1824	0.7890	0.8543	0.1030	0.5961	0.7335	2.2256	0.2693	0.5697	3.9351
		5	6.5359	0.0200	0.2595	0.7205	1.0593	0.1164	0.5830	0.6136	2.6394	0.2691	0.6327	4.1639
1.1		3	4.8296	0.0320	0.1985	0.7965	1.1944	0.1040	0.6210	0.7146	1.6480	0.2796	0.4152	3.3193
	0.6	4	5.0635	0.0200	0.2863	0.6937	1.5468	0.1213	0.6065	0.5795	2.0286	0.2854	0.5531	3.5761
		5	5.3231	0.0200	0.3643	0.6157	1.8425	0.1369	0.5960	0.4877	2.4147	0.2870	0.6962	3.8760
		3	5.5136	0.0878	0.1518	0.7604	0.6636	0.0931	0.6157	0.8797	1.6872	0.2646	0.4463	3.6221
	0.8	4	5.5080	0.0361	0.2148	0.7490	0.9502	0.1165	0.6279	0.6968	1.9385	0.2958	0.5915	3.6941
		5	6.1266	0.0200	0.2835	0.6965	1.1225	0.1202	0.5941	0.5829	2.4430	0.2859	0.7139	4.0469

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