A STOCHASTIC ANALYSIS OF THE COMPETITIVE BIDDING PROBLEM FOR CONSTRUCTION CONTRACTORS

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CONTRACTORS

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PREFACE

In FY 1969, $690,000,000 worth of U. S. government construction was performed under closed competitive bidding. Most construction performed by state and local governments is contracted in the same manner. Many corporations use the closed competitive bidding process to procure construction of facilities. The purpose of this research is to develop a dynamic mathematical model from which a contractor can optimize his expected utility when submitting a sealed bid for a construction contract.

There are several ways in which a contractor can increase his profit on a construction contract. First, he can perform the project using the cheapest acceptable materials and equipment. Secondly, he can schedule the project realistically, using a time-cost trade off, to minimize those costs associated with time. Thirdly, the contractor can optimize all feasible construction techniques. There has been much research accomplished on methods of optimizing the estimate and scheduling. Little has been accomplished on the third method of maximizing profit, leaving a fertile area for further research. The final method of maximizing profit is adding the maximum amount of profit to a contractor's estimate such that he will still be the low bidder. It is obvious to even those persons not associated with construction, that the monetary difference between the low bidder for a project and the next low bidder (commonly known as money "left on the table") is free profit lost to the contractor winning the bid. However, absolutely minimizing the difference between the low bidder and the next low bidder is impossible due to the stochastic nature of the construction industry. Therefore, this dissertation uses a utility maximization approach to reduce the difference between low bidder and the next lowest bidder, hence increasing the profit to the contractor.

This author has attempted to give credit to all sources from which material has been taken. He apologizes for any omissions of this character which may, unknowingly, have occurred.

The writer is greatly indebted to the following members of his Graduate Committee for their criticism and suggestions in the preparation of this work: Professor R. L. Janes, Civil Engineering faculty; Professor J. E. Shamblin, Industrial Engineering faculty; Professor J. L. Folks, Chairman of the Statistics Department; and Professor D. S. Ellifritt, Civil Engineering faculty.

In addition to members of his Graduate Committee, this author wishes to express his appreciation for the willing cooperation and assistance in obtaining references to Dr. L. R. Shaffer, Deputy Director of the U. S. Army Corps of Engineers Construction Engineering Research Laboratory and formerly Professor of Civil Engineering at the University of Illinois, Professor R. Stark at the University of Delaware, and Professor J. Douglas of Stanford University. The writer also wishes to express his appreciation to Mr. R. L. Peurifoy for the inspiration to become a construction engineer.

Finally, the writer wishes to acknowledge the tremendous moral support of his wife, Nancy, and his two sons, Hugh and Greg. It is hoped that the effort represented on the following pages is equal to theirs.

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CHAPTER I

INTRODUCTION

Bidding is defined as a competition for the right to perform services or acquire property. It assumes many forms in many industries. Auction bidding may be used to distribute or acquire object d'art, furniture, etc. Negotiated bidding may be used to acquire subcontractors for services, small construction projects by private industries, computer software systems, etc. Closed competitive bidding, the topic of this treatise, is the submission of sealed tenders to a public organization for the right to perform services or deliver products at a specified consideration. For brevity, "closed competitive bidding by sealed tenders" will be referred to as simply "bidding" for the remainder of this treatise.

Competitive bidding is fundamental to the economic system of the United States. A large portion of services and products provided to federal agencies must, by law, be acquired through competitive bidding. Legal statutes provide that, except under extremely limited circumstances, all government procurement shall be conducted under closed competitive bidding. This, along with the fact that most major corporations use competitive bidding for procurement, implies that a substantial portion of the Gross National Product expenditures is made through competitive bidding. To reduce the vastness of the subject, this thesis will deal only with bidding for construction contracts, although most models developed can be generalized to other procurement areas.

Of $833,469,000 worth of construction contracts performed for the U. S. Government in FY 1969, 87.2 per cent of these contracts were acquired through closed competitive bidding (44).

This thesis will analyze the competitive bidding problem from the contractor's point of view. The most direct benefit from the development of a rational approach to the bidding problem is an improvement in contractor profits. However, in a larger sense, better bidding policies make it more likely that the most efficient bidder will win the contract, which in turn is of extreme benefit to the sponsor.

An assumption used throughout this dissertation is that the bidding is conducted with asymmetrical information, e.g., only one contractor is utilizing a rational bid model. The bidding problem using an assumption of symmetrical information suggests a game theoretic approach, an approach seeking an equilibrium condition, rather than the decision theoretic approach used herein, and is a fertile area for further research. Rothkope (63) has conducted some research into this problem.

The Contractor Objective

In any decision process, a fundamental principle is the selection of an objective. The most common objective that one assumes is that of maximizing the contractor's expected profit for each contract. However, this objective is not always the objective used by each contractor.

The highly diverse nature of the construction industry has thus far defied any rational business model to define it. In addition to the multitude of types of construction accomplished, the engineering firms and contractors vary from the small home builder to the struggling, limited budget highway contractor to the multi-million dollar public construction firm. It offends the intuition to hypothesize that all contractors in such a diverse business would have the same objectives. For instance, it is not at all uncommon for a contractor to bid a job at five per cent above his estimated cost and find that he is 15 per cent higher than the low bidder; nor is it uncommon for a contractor to bid a job at 15 per cent markup and find he is the low bidder.

As stated before, the predominant and more satisfying objective is to maximize profit on a construction contract. Numerous other contractor objectives are prevalent from time to time. Although a more thorough discussion of these objectives will be conducted later some of the other contractor objectives will be outlined here by way of introduction so that the reader may understand other discussions to follow.

Because of business trend variations, tight money, stiff competition and other factors, a contractor might find that he has no construction to perform. This means that his equipment is idle and his constant supervisory force, although still on his payroll, has no job to perform. He may, in this instance, bid a job at a substantial loss, in order to meet his overhead requirements. It might be stated that this contractor has an objective to maintain a constant work volume.

Another contractor may bid a job at an extremely high profit margin. Since the lowest bidder will be awarded the contract, this contractor has a slim chance of being awarded the job. If by some chance he is awarded the contract, he should earn an extremely high profit. This contractor probably would be working to full capacity and submitted the bid merely "to keep his fingers in the pie" to maintain his good relations with the awarding agencies and associates.

Other, less likely objectives for a bidding firm might be (5:74):

a. To minimize the profit of competitors,

b. To maintain and improve quality of performance,

c. To reduce the variance of the profit random variable, and

d. To perform only a certain type of construction project.

Variables Involved

The profit obtained on any job is defined as the difference between the actual cost of performing the work and the bid price; in equation form:

$$y_1 = X_i - C_i^{'} \tag{1.1a}$$

where y_i is the profit for the i^{th} job, X_i is the bid price, and $C_i^{'}$ is the actual cost for the same job. However, in a competitive bidding situation, X_i must be lower than all of the other bids, X_{ij}, in order for the contractor to be awarded the job. This implies that Equation (1.1a) must be modified to:

$$y_i = \begin{cases} X_i - C_i^{'} & \text{if } X_i < X_{ij} \text{ for all } j \\ 0 & \text{if } X_i > X_{ik} \text{ for at least one } k. \end{cases} \tag{1.1b}$$

In the unlikely event that X_i is equal to X_{ik}, specific regulations for various agencies govern. This event is insignificant for the purposes of this paper. Figure 1 illustrates Equation (1.1b).

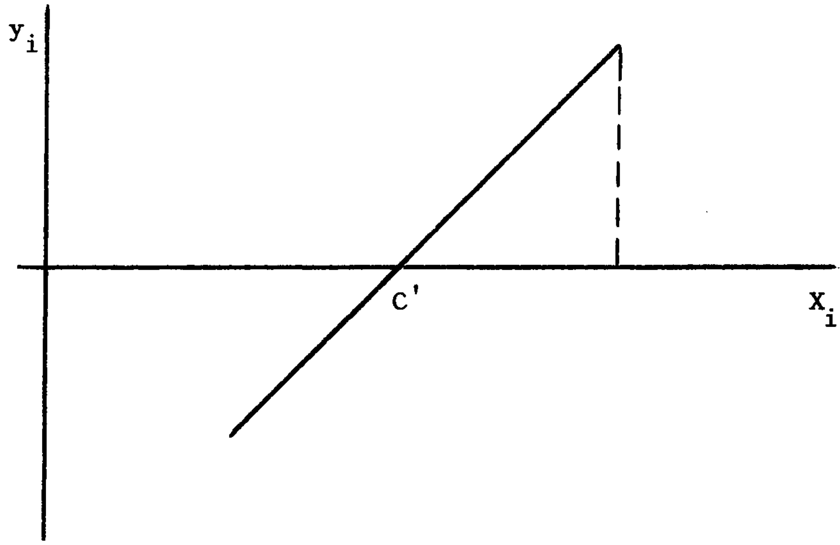


Figure 1. Profit Versus Bid Price for the i^{th} Job

It should be noted that there is essentially one variable (X_i) that can be controlled and there are three variables (X_{ij}, C_i', and y_i) that cannot be controlled in Equation (1.1b). The profit, y_i, in addition to being a function of X_i, is a function of variables whose values cannot be determined prior to the bid letting. In fact, Benjamin (5) has shown that the actual cost, C_i', and consequently the profit, y_i, are random variables with associated probability distributions.

One further, and more perplexing, problem is associated with the bidding problem. That is, "when will 'our' bid be lower than all other competitors?" Even if the actual cost, C_i', were not a random variable, the above question requires that the dependent variable, y_i, be a stochastic variable, which in turn implies that one can never <u>á priori</u> find the exact value of y_i and must resort to predicting the "expected value," $E(y_i)$. Thus, Equation (1.1b) may be restated as

$$E(y_i) = (X_i - C_i') \; P \; (X_i < X_{ij} \text{ for all } j) \tag{1.2}$$

where $P(X_i < X_{ij}$ for all j) is read as the probability that "our" bid is less than all other bids submitted for this specific job. Equation (1.2) is graphed as shown in Figure 2.

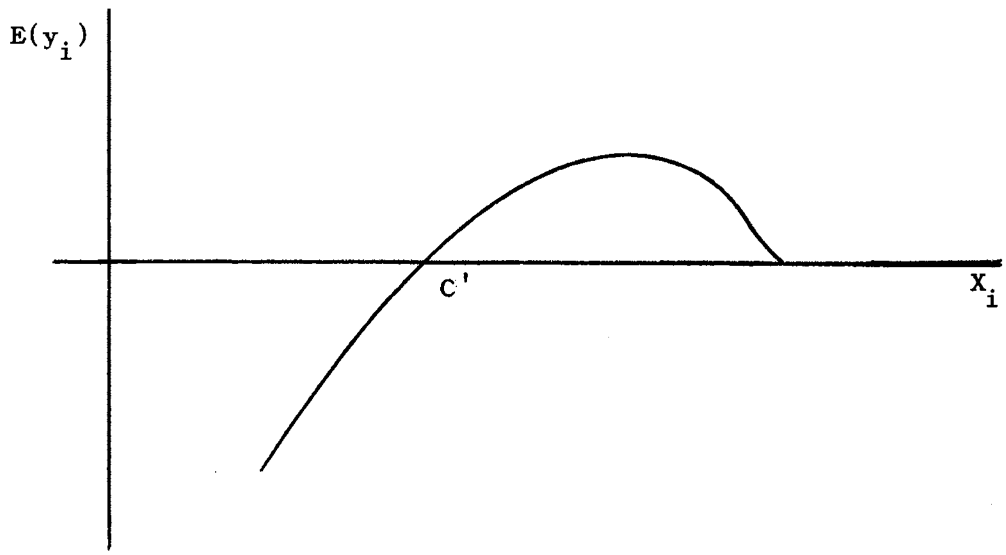


Figure 2. Expected Profit Versus Bid Amount

The Problem Restated

The problem can now be restated in more precise terms. Assuming the objective of maximizing profit as the sole objective, the problem is now simply to maximize profit over a series of jobs or for a specific time period with respect to X_i, or:

$$\max E(y_i) = \max(X_i - C_i') \; P \; (X_i < X_{ij} \text{ for all } j). \qquad (1.3)$$

Equation (1.3) may be subject to numerous constraints which will enter into the optimization procedure. A discussion of these constraints will be presented in Chapter V of this dissertation along with additional contractor objective considerations.

The fundamental purpose of this paper is to present a unique dynamic bidding model which provides a workable tool from which a contractor can determine his objective function to be used in bidding for a specific job, a method of determining his constraints, and a time dependent method of measuring his probability of winning the contract. Finally, a method of optimizing his bid is developed based upon his selected objective function.

The organization of this thesis is to first present a review of the most significant bidding models developed to date in Chapter II. Chapter III is a discussion of the shortcomings of these models. Chapter IV develops a method of treating the actual cost as a random variable. The development of a new dynamic model and the optimization of this model is accomplished in Chapter V. Chapter VI is a summary with suggestions for further research. Data utilized and lengthy computations are added as appendices.

CHAPTER II

REVIEW OF PREVIOUS MODELS

It should be noted at the onset that no one has adequately solved the competitive bidding problem to a point that a model is available by which a contractor can analytically calculate this optimum bid markup. From the shape of the curve shown in Figure 2, it is obvious that Equation (1.3) could be satisfied if functions were known for the probability that X_i will be less than the lowest bid of all the competitors and C_i' were not a stochastic variable. The procedure would simply utilize elementary calculus, that is, to equate the first derivative of $E(y_i)$ to zero and solve for the X_i which corresponds to the maximum point on the curve, e.g., rewriting Equation (1.2) here for convenience

$$E(y_i) = (X_i - C_i') \; P \; (X_i < X_{ij} \text{ for all } j) \qquad (1.2)$$

differentiate with respect to X_i and set $E'(y_i) = 0$, solve for the X_i^* that would correspond to the maximum point on the curve shown in Figure 2. (Note that fundamental conditions of differentiability would necessarily have to exist for this procedure to be valid. This is a minor problem at this stage of development.)

To simplify the procedure involved with the above analysis, assume that C_i' is a known constant, C_i, not a stochastic variable, as it is known to be. Equation (1.2) can now be normalized with respect to C_i, that is, the entire equation can be divided by C_i, giving the following equation:

$$E(y_i) = (r_i - 1)\ P\ (r_i < r_{ij} \text{ for all } j) \qquad (2.1)$$

where $r_i = X_i/C_i$ and $r_{ij} = X_{ij}/C_i$. This simply implies that $E(y_i)$, instead of being an absolute amount of money in dollars, is now a fraction (percentage if multiplied by 100) of the actual cost of the project. Figure 2 is then transformed into Figure 3.

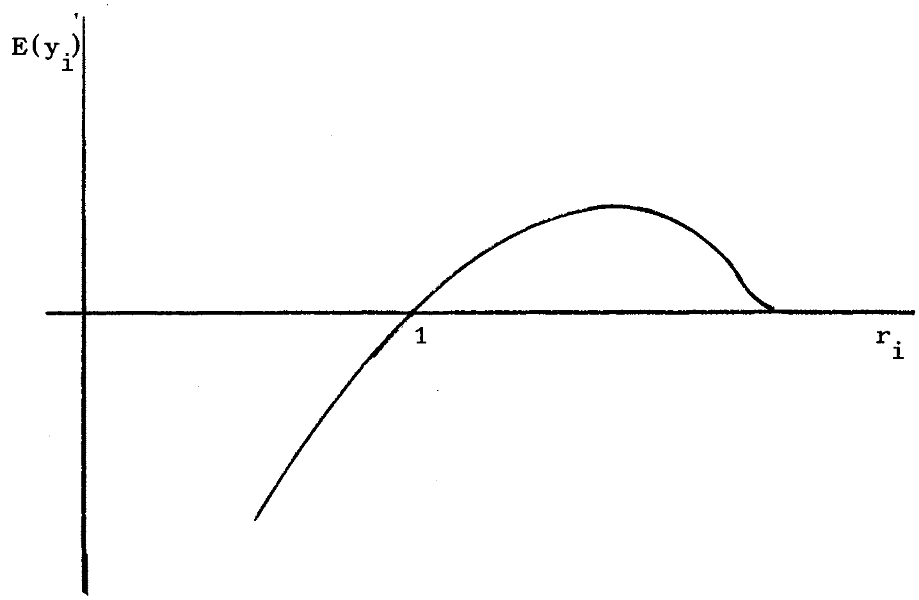


Figure 3. Normalized Profit Versus Normalized Bid

Since this is a one-to-one transformation, the maximization of $E(y_i)$ will be accomplished if the normalized $E(y_i)$ is maximized. The probability that r_i will be less than the lowest normalized bid will now be assumed to be given by the continuous and differentiable function, $G(r_i)$. In equation form

$$p(r_i < r_{ij} \text{ for all } j) = G(r_i) \text{ .} \tag{2.2}$$

Differentiating $E(y_i)$ with respect to r_i gives

$$E'(y_i) = (r_i - 1)\, G'(r_i) + G(r_i) \text{ .} \tag{2.3}$$

Equating this first derivative to zero gives the condition for optimality (9:97)

$$r_i^* = 1 - \frac{G(r_i)}{g(r_i)} \quad , \tag{2.4}$$

where $g(r_i)$ is the value of the first derivative of $G(r_i)$. (It should be noted that $G(r_i)$ is a complementary cumulative probability function, with a shape as shown in Figure 4, and its slope will always be less than or equal to zero; thus ratio of the two functions will always be negative.) The condition for optimality is shown graphically in Figure 5. Equation (2.4) shows that the optimum bid should always be:

$$X_i^* = \left(1 + \frac{G(r_i)}{|g(r_i)|}\right) C_i \text{ .} \tag{2.5}$$

Theoretically, the above procedure is flawless. As stated previously, C_i' is a stochastic variable which means that it is not known with certainty until the project in question has been completed. However, this is a minor problem when compared to finding an analytical expression for $G(r_i)$. No proven method is available to find this function. Thus, $G(r_i)$ is the key to the basic problem.

In summarizing the proposed competitive bidding models, the model presented by Lawrence Friedman in 1956 will be presented first. His model, although the first and yet the most outstanding work done towards a solution to date, contains several points as yet unresolved.

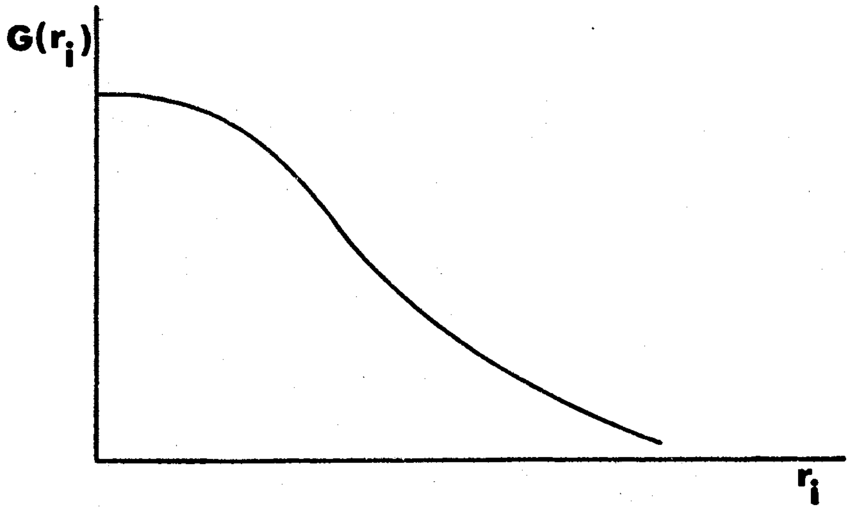


Figure 4. Probability Function (Representative) for $G(r_i)$ Versus r_i

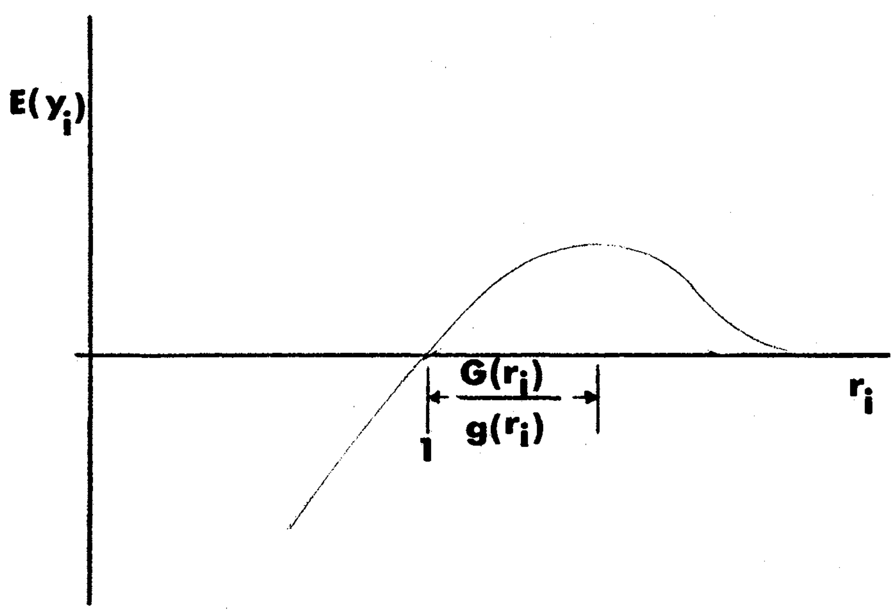


Figure 5. Optimality Condition Using Calculus

Additional models will be analyzed as they pertain to resolving the contended points in Friedman's original work.

Friedman's Model (21)

Lawrence Friedman earned the first Ph.D. in Operations Research in 1959 from Case Institute for his work in competitive bidding. Oddly enough, the work done on the subject since that time has mostly been modifications of his original model. There are several conceptual points in Friedman's work yet to be proven or disproven. However, regardless of their validity, his work is the logical place to start in any competitive bidding discourse.

Friedman was the first to outline the possible objectives that a firm might have when submitting a sealed bid. These have been lately extended as shown in Chapter I. However, he uses the objective stated in this treatise, that is to maximize profit. Benjamin (5:10), an advocate of utility theory, states that Friedman assumes a utility function linear with dollars. Friedman also recognizes that the actual cost of performing work is a random variable and not at all likely to be equal to the original cost estimate. Most researchers since 1959 have failed to utilize this in their models.

This author will present Friedman's model, with only the notation changed, to facilitate understanding by the reader and to maintain consistency in this thesis.

Let C_i' be defined as the actual cost of the i^{th} project, initially unknown at the time a bid is prepared and assumed to be a random variable. Let C_i represent the initial cost estimate for the i^{th} project. Now define S_i to be the ratio of the actual cost to the estimated cost for the i^{th} job, or

$$S_i = \frac{C_i'}{C_i} . \qquad (2.6)$$

Assume the continuous function h(S) to be the probability density function of S taken over all jobs. Then h(S)ΔS represents the probability that the ratio of the true cost to the estimated cost lies between S and S + ΔS. This is represented in Figure 6.

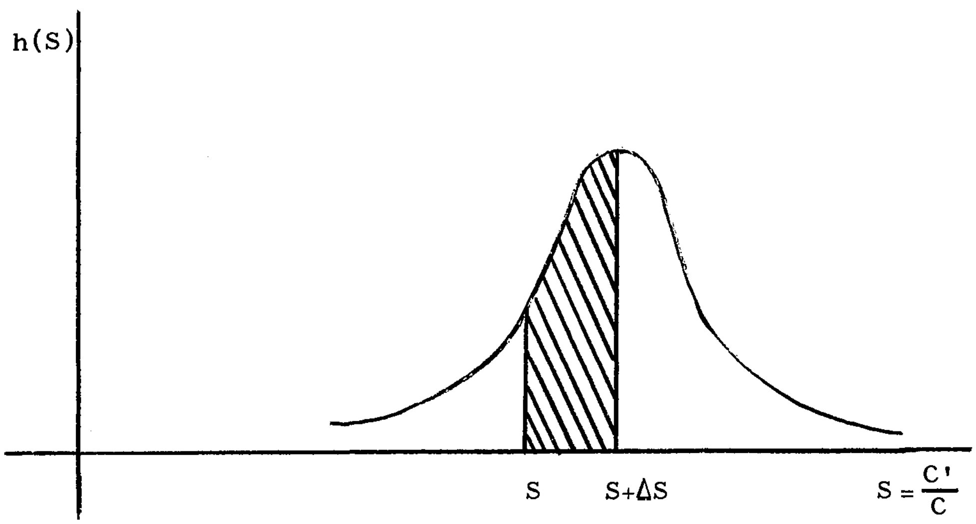


Figure 6. Reliability of the Cost Estimate (106:21)

The function h(S) can be determined <u>á priori</u> from histograms of past data. Therefore, a cost bias factor can be developed by taking the first moment of the developed distribution, e.g., an estimate of the mean actual cost can be given by

$$\hat{C}_i' = C_i \int_0^\infty S\, h(S)dS , \qquad (2.7)$$

where C_i is the estimated cost for the i^{th} job and $\hat{C}'_i$ is the estimated cost for the same job corrected for bias. The equation for the profit of the i^{th} job is given by

$$y_i = \int_0^{\infty} (X_i - SC_i)h(S)dS \qquad \text{if } X_i < X_{ij} \text{ for all } j, \qquad (2.8)$$

when X_{ij} is the bid of the j^{th} bidder for the i^{th} job.

In tackling the problem of finding the probability that $X_i < X_{ij}$ for every j, Friedman treats three cases. The first, winning over one bidder, is treated implicitly. The second, winning if all bidders are known, and the third, winning if all bidders are not known, are treated explicitly.

Winning if All Bidders are Known

If one assumes that all bidders will bid as they have done in the past (this assumption makes Friedman's model, and consequently all models to follow, static models), the probability of winning over a particular competitor is the area to the right of the ratio X_i/C_i of the j^{th} bidder's probability density function graph as shown in Figure 7. (Note that if this area is graphed as an inverse cumulative probability function, its shape will have the same characteristics as Figure 4.) Define r_j as the ratio of all of the j^{th} competitor's bids to "our" cost estimates for the same jobs. If all bidders are known, their bidding patterns can be as illustrated in Figure 8.

At this point in the model, Friedman makes his most contested assumption; the bids of all competitors are independent of each other. This assumption will be discussed at some length later in this paper

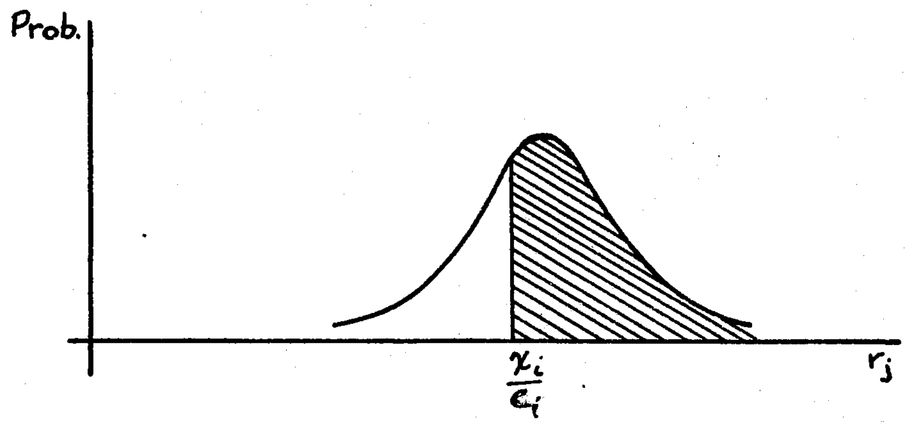


Figure 7. Bidding Pattern of Competitor j

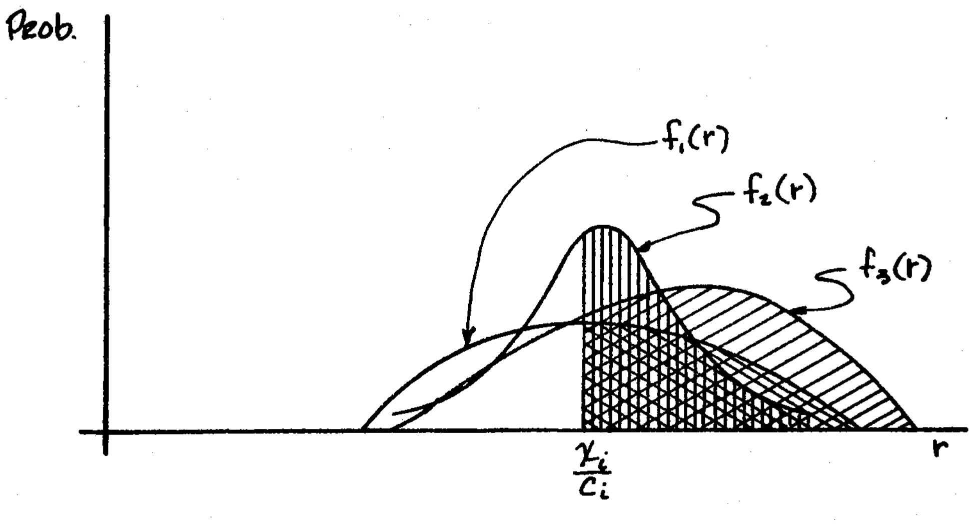


Figure 8. Bidding Patterns of Competitors

and should be kept in mind by the reader. However, following along with Friedman's model, this assumption, based on the laws of probability, implies that the joint probability that $X_i < X_{ij}$, $(j = 1, 2, \ldots, n)$, is equal to the product of the probabilities that $X_i < X_{ik}$ for each k. In equation form

$$p(X_i < X_{ij} \text{ for all } j) = p(X_i < X_{i1}) \cdot p(X_i < X_{i2}) \ldots p(X_i < X_{in})$$

or

$$p(X_i < X_{ij} \text{ for all } j) = \prod_{j=1}^{n} p(X_i < X_{ij}) \ . \qquad (2.9)$$

Simply stated, the probability that $X_i < X_{ij}$ for all j is the product of the areas to the right of X_i/C_i as shown in Figure 8. If $f_j(r)$ is the probability density function associated with each bidder's bidding pattern, assuming continuity for each function, then the expected profit, $E(y_i)$, can be stated as

$$E(y_i) = \int_0^{\infty} (X_i - C_i \cdot S)h(S)dS \cdot \int_{\frac{X_i}{C_i}}^{\infty} f_1(r)dr \ldots \int_{\frac{X_i}{C_i}}^{\infty} f_n(r)dr \ ,$$

or

$$E(y_i) = (X_i - \hat{C}_i') \cdot \prod_{j=1}^{n} \int_{\frac{X_i}{C_i}}^{\infty} f_j(r)dr \ , \qquad (2.10)$$

recalling that $\hat{C}_i'$ is the estimated cost corrected for bias.

<u>Winning if All Bidders are Not Known</u>

Friedman assumes that in a majority of situations, all bidders for a project will not be known prior to the opening of the sealed bids.

This presents a special problem in the above model; namely, Equation (2.10) cannot be calculated. As a partial solution (a solution in 1959), Friedman introduces the concept of the "average bidder." To find the probability density function for the average bidder, he simply develops a histogram with frequencies derived from data of all known competitors. In keeping with the consistent notation, let f(r) represent the probability density function of the "average bidder."

One more problem remains, that is: how many bidders will bid for this particular job? Friedman's solution to this problem is to perform a linear regression relating the expected number of bidders to "our" cost estimate. This is merely an assertion that the number of bidders expected to bid for a particular job is a function of the expected cost of the job. An example of this regression is shown in Figure 9. It should be noted that this is an assertion with no attempt at proof.

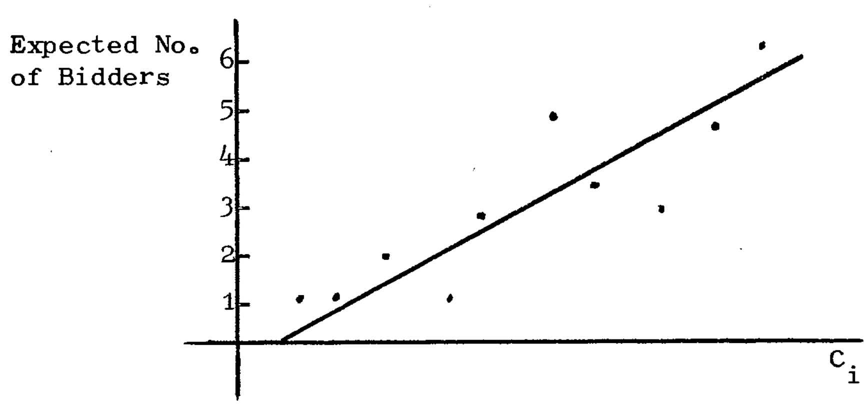


Figure 9. A Method of Obtaining an Estimate of the Number of Bidders Based on Previous Bidding History (110:21)

Friedman assumes that a probability density function that k bidders will bid for a contract can be found which he calls g(k). If this function is used, Equation (2.10) becomes

$$E(y_i) = (X_i - C_i') \cdot \sum_{k=0}^{\infty} g(k) \cdot \left(\int_{r_i}^{\infty} f(r)dr \right)^k . \qquad (2.11)$$

The summation sign is used since the probability density function for the expected number of bidders is a discrete distribution.

Optimization of X_i for Each Job

It is obvious that Equations (2.10 or 2.11) cannot be optimized as simply as the theoretical Equation (2.1). In fact, Friedman asserts that a solution does not exist in closed form. An optimum solution for this model can, however, be found through an iterative process. Casey and Shaffer (12) evaluated the applicability of this model for highway construction projects. Due to the unavailability of cost estimate data, which presents a real problem to the researcher, they assumed that the estimated cost of each project was 85 per cent of the bid amount, which was quite unrealistic. Actual markup for a particular job will normally vary from 1 to 15 per cent and is seldom constant (5, 9). Casey and Shaffer did, however, show how an iterative process could be used to obtain an "optimum" from this model if one makes a normality assumption throughout.

Shortcomings of Friedman's Model

Friedman's work was an outstanding contribution toward developing a rational approach to competitive bidding. Like Newton's development of "the method of fluxions" which paved the way for modern calculus, it was the first published rational approach to the problem. Hence, the long discourse on his model. However, there are numerous conceptual problems inherent in his model.

(1) The assumption that all bidders bid independently of one another has not been proven. In fact, this concept offends the intuition of a practical contractor. Marvin Gates (24), whose model will be presented later in this paper, asserts that this assumption does not follow his extensive experience in analyzing bid data. One can argue that in a competitive environment collusion among bidders would not be common. In fact, there would probably be very little, if any. Park (62) argues that the bids must be independent.

A counter argument would consider the fact that all bidders are bidding based on roughly the same materials and labor cost, perhaps the same subcontractor bids, etc. However, all arguments are superfluous; the mathematical definition of stochastic independence is all that is of consequence in order for the product of probabilities to be valid. That definition is:

> The probability of occurrence of event A is independent of the probability of occurrence of event B if, and only if, $P(A) \circ P(B) = P(A \text{ and } B)$. (95:115)

To prove that events satisfy this definition is difficult; indeed, many times impossible.

(2) Based on the above counterargument, it would appear that the concept of the "average bidder" is invalid since the probabilities cannot be multiplied together to obtain the

probability of winning. Gates (23) has proposed and Benjamin (5:18) has provided a derivation for a formula which more closely follows the "real world" results in competitive bidding. This formula and its assumptions will be discussed in the next section; the point being here that the absence of the independence assumption does not invalidate Friedman's "average bidder" concept. The basis upon which the idea that the "average bidder" concept is questionable lies in an assertion by Broemser (11) that, based on an idea presented by Howard (36, 37), it is necessary only to study the distribution of the low bidder for each job. This concept, to be presented in more detail (it should be noted that neither concept has been developed beyond an assertion) could render Friedman's concept of the "average bidder" invalid.

(3) Friedman's method assumes implicitly that each bidder will bid as he has done in the past. In other words, there is nothing in the model which would indicate any change in the bidding trends of competitors. This element of the problem has been omitted from every model developed to date. All bidding models based on Friedman's work are "static models."

(4) Friedman's model uses a straight line utility function based on dollars as shown in Figure 2. This concept has been challenged by Howard (37) and Benjamin (5). Benjamin examines in detail the risk associated with bidding and uses profit lotteries, linear, bi-linear and non-linear, to exhibit a contractor's willingness to accept the risk of a loss on a job.

(5) Friedman's model, as all following models, has no closed form of optimization.

Biasiolli's Modification of Friedman's Model (7)

Gerald Biasiolli's unpublished paper, written at St. Mary's University in 1966 has received little or no reference in more recent works on competitive bidding. However, this author, not necessarily granting significance to the model, feels that it provides a transition for models to follow.

Biasiolli modifies two aspects of Friedman's model and only treats the case in which the number and identity of the competitors are not known. The first disagreement involves the "average bidder" concept. Biasiolli asserts (again, merely an assertion; no proof) that the "average bidder" concept neglects to discriminate between the strata bid distributions, e.g., he feels that a distribution exists and is of pertinence, for the lowest bidder, the second lowest bidder, and on through the n^{th} bidder. If the probability of winning over each of these strata for various profit margins could be determined, then the expected profit would be given by

$$E(y_i) = (X_i - C_i') \prod_{k=1}^{n} p(r_i < r_{ik}) \qquad (2.12)$$

where all variables are as defined previously except that r_{ik} is the ratio of the "average bidder" bid in the k^{th} strata to "our" cost estimate when k = 1, 2, ..., n; k = 1 being the lowest bidder strata.

The second significant difference that Biasiolli made to Friedman's model was the method of determining probabilities. He felt that there was little evidence which would lead to the conclusion that a probability density function of a bidding model should follow a known distribution. Therefore, Biasiolli used a simulation technique based on non-parametric statistics to obtain the probability of winning over various competitors. This concept is mentioned here since it may have some significant merit. It has been also attempted by Gates in some of his earlier research. Table I shows the effect on a contractor's profit for eleven contracts of using Friedman's model and Biasiolli's modifications versus using no model at all.

TABLE I

COMPARISON OF PROFIT ATTAINED WHEN DIFFERENT BID TECHNIQUES WERE USED (7:32)

Bid Predictor	Old Technique	Friedman's Model	Stratified Bid Model
Profit for past eleven projects	$21,379.22	$59,721.28	$52,297.80
Profit increase resulting from use of the model	-0-	38,342.06	30,918.58
Percentage increase in profit from using the model	-0-	279%	244%
Percentage of contracts won	36%	45%	45%

It should be noted that Friedman's model yielded somewhat better results than did Biasiolli's modification. Biasiolli makes the observation:

This relatively small disagreement between Friedman's model and the stratified model can be attributed to the size of the sample. It is believed that when a much larger amount of past data is available, the stratified bid model becomes the more accurate predictor (7:29).

Howard's Model (37)

Howard's model differs from Friedman's model in basically two aspects. First, like Biasiolli, Howard feels that the "average bidder" concept is invalid when treating the case when all bidders are not known. Howard asserts that it is only necessary to bid lower than the lowest bidder among the competitors. In doing this, he immediately simplifies the data collection problem and makes Friedman's "independence among bidders" assumption unnecessary.

Secondly, Howard makes a vague attempt at developing a less static model by conditioning the probability of winning on prior experience to some degree. Just how this prior experience is used is not clearly developed in Howard's work and will be discussed to a great extent in this author's development of a dynamic model in Chapter V.

Howard's model can be written as:

$$E(y_i/e) = (X_i - C_i') \cdot p(r_i < r_{iL}/e) \qquad (2.13)$$

where r_{iL} represents the ratio X_{iL}/C_i as defined by the probability density function of all previous lowest bidders taken together in one distribution, e.g., the function described by Figure 10 is developed from the frequency histogram tabulated by taking the ratio of the bids of all previous low bidders to "our" cost estimate. The symbol "e" represents an experience factor. The variable C_i', as stated in Friedman's model, is still the cost estimate corrected for bias. Howard assumes the actual cost as a random variable as does Friedman. The advantages to Howard's assertion that it is necessary to bid less than the lowest bidder are:

(1) It simplifies the data collection problem. This is a major effort in applying Friedman's model.

(2) It reduces the problem to one of finding a single function to describe a single distribution.

(3) It eliminates the requirement of the independence assumption.

(4) On the surface, it is more pleasing to one's intuition.

The disadvantages to this approach are:

(1) It assumes that all bidders will bid as they have done in the past, thus allowing no method of corrections based on individual trends.

(2) Based on disadvantage (1), this assumption makes the model even more static than does Friedman's assumptions.

(3) Howard presents no method for finding $p(X_i < X_{iL}/e)$.

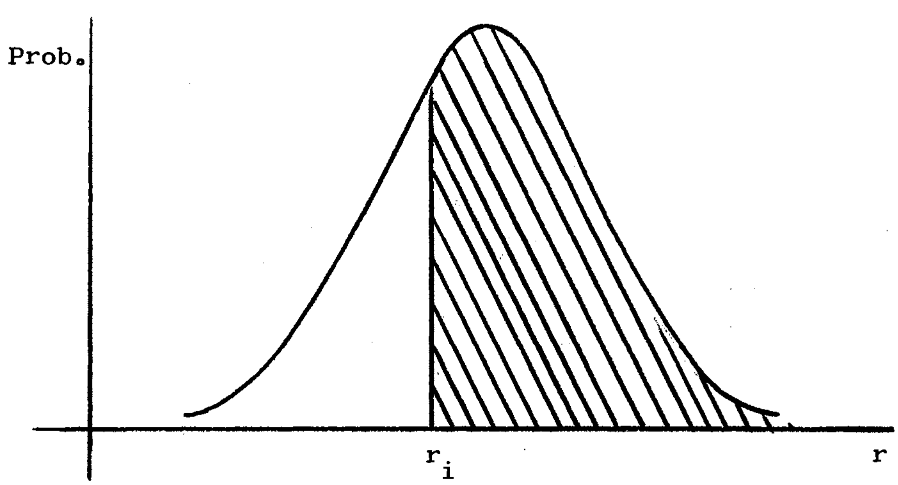


Figure 10. Probability Density Function of the Lowest Bidder

Broemser's Model (10)

Broemser, like Howard, assumes that it is only necessary to bid lower than the lowest bidder. His major contribution to the competitive bidding problem is the hypothesis of a single bid model which provides a method for calculating $p(X_i < X_{iL})$. Broemser's model is based on Christenson's (13) work in bidding for corporate securities. They are similar in both notation and conditions for optimality.

Broemser's model is as shown in Equation (2.1), repeated here for convenience

$$E(y_i) = (r_i - 1) \cdot p(r_i < r_{iL}) \tag{2.1}$$

where r_{iL} represents the ratio of previous lowest bids to "our" cost estimate for the same job. If $G_i(r)$ represents the function described on page 10, Equation (2.1) becomes

$$E(y_i) = (r_i - 1) \cdot G_i(r) \quad . \tag{2.1a}$$

The condition for optimality is expressed by Equation (2.4)

$$r_i^* = 1 + \frac{G_i(r)}{|g_i(r)|} \quad . \tag{2.4}$$

The uniqueness of the following model is that it gives a method for approximating $G_i(r)$. To accomplish this task, Broemser uses a multiple linear regression model. The dependent variable is the lowest competitor's bid expressed as a fraction of "our" cost. The independent variables are characteristics of the job which influence the profit that the contractor should expect from the job (5:23). These are:

$z_{jo} \equiv 1$

$z_{j1} \equiv (\text{estimated per cent of work not subcontracted})^{-1}$

$z_{j2} \equiv (\text{estimated per cent of work not subcontracted})$

$z_{j3} \equiv (\text{estimated per cent of work not subcontracted})^{2}$

$z_{j4} \equiv (\text{estimated job duration})^{-2}$

$z_{j5} \equiv (\text{estimated job duration})^{-1}$

$z_{j6} \equiv (\text{estimated job duration/estimated cost})^{-1}$

$z_{j7} \equiv (\text{estimated job duration/estimated cost})^{2}$

$z_{j8} \equiv (\text{estimated cost})^{-2}$ (97:9)

The regression coefficients, $\vec{\beta}$, are found by solving the normal matrix equation

$$\vec{\beta} = (Z^T Z)(Z^T L) \tag{2.14}$$

where the superscript "T" represents the transpose of the matrix, $\vec{\beta}$ is the vector of regression coefficients, Z is the matrix of n independent variables for each of m jobs, and L is the vector of the lowest competitor's bid, for each of the m jobs, expressed as a fraction of "our" cost estimate. The variance of the prediction is found by

$$\sigma^2 = \frac{(\vec{L} - Z\vec{\beta})^T (\vec{L} - Z\vec{\beta})}{m - n} \;. \tag{2.15}$$

A general contractor's bidding history over a one year period was examined by Broemser in developing this model. Benjamin performed sequential tests on the data subsequently for a different contractor and observed three specific shortcomings of the model (5:24). These were:

(1) The coefficient of multiple determination, R^2, varied within the range of about 0.25 to 0.50 as additional data were considered with time.

(2) The values of the regression coefficients varied depending upon the amount of bidding history that was considered in determining the coefficients.

(3) The success of this model, as measured by the cumulative profits obtained by applying the model to the data sequentially in time, varied with the amount of previous bidding history that was considered.

Broemser casts this single bid model into a constrained linear optimization problem. He used as constraints such items as limited bonding capacity, limited supervisory personnel, number of jobs acceptable and limited dollar volume.

The significance of this model is that it attempts to provide a rational approach to find the probability of winning a contract. To anyone familiar with construction, it is obvious that the model is not a complete solution since it contains as independent variables so few of the seemingly infinite variables that go into establishing the probability of winning and consequently profit.

Gates' Model (24)

Marvin Gates is probably the most authoritative writer on construction bidding today, having conducted research in the problem since the late 1950's. Essentially, his papers (23, 24, 25) present practical methods of applying previously developed models. However, he has developed some new concepts which shed light on the competitive bidding problem. Two aspects of Gates' works will be discussed. First, he contends that the independence assumption is not valid and presents a formula for handling the problem. This is an implicit rejection of the assumption by Howard and Broemser that one must only consider the probability density function of the lowest bidder; secondly, Gates uses non-parametric statistics for determining probabilities as does Biasiolli. However, Gates uses straight line assumptions to facilitate optimization procedures.

All Bidders Known Strategy; No Independence Assumption (24:84)

In developing a strategy to be used against bidders when all were known, Gates found that, in all of his bidding experience, that the probability of being lowest bidder in this situation differed greatly from the product of the probabilities of bidding lower than each bidder. Based on this, he rejected the independence assumption. (This is a valid reason for rejection noting that the stochastic definition of independence is an "if and only if" definition.) He found, based on experience, that if the probability of winning a bid at a specific price over the i^{th} bidder were p_i, then the probability of winning the bid over n of these bidders was closely approximated by the formula

$$p(\text{win}|r_j) = \frac{1}{\sum_{i=1}^{n}\left[\frac{1 - p_i}{p_i}\right] + 1} \qquad (2.16)$$

which can be simplified to

$$p(\text{win}|r_j) = \frac{1}{\sum_{i=1}^{n}\left(\frac{1}{p_i}\right) - (n - 1)} \quad . \qquad (2.17)$$

To emphasize the logic in this assumed formula, the following is a derivation supplied by Benjamin (5:16).

Assume that A and B are the only bidders. The probability that A wins is given by

$$p(A|A + B) = \frac{p(A(A + B))}{p(A + B)} \quad . \qquad (2.17a)$$

Since the event that A wins is mutually exclusive of the event that B wins, the event (A(A + B)) which is read "A wins and A or B wins," is simply the event that A wins (note that this is the probability of an intersection of events); the probability of occurrence that A or B wins is the sum of the probability that A wins and the probability that B wins, e.g., Equation (2.17a) is transformed into

$$p(A|A + B) = \frac{p(A)}{p(A) + p(B)} \quad . \qquad (2.17b)$$

If one considers other competitors, say competitor C, D, E, ..., N, the sum of the probabilities that each will win exhausts all possibilities which implies that

$$p(A) + p(B) + p(C) + \ldots + p(N) = 1 \quad . \qquad (2.17c)$$

To solve for the unconditional probability that A wins, the probabilities of each of the other competitors winning must be expressed in terms of the conditional probability that A wins given that only two competitors are bidding and the unconditional probability that A wins. In the case of competitor B, solving Equation (2.17b),

$$p(B) = \frac{p(A) - p(A)p(A|A + B)}{p(A|A + B)} \tag{2.17d}$$

and in the case of competitor C,

$$p(C) = \frac{p(A) - p(A)p(A|A + C)}{p(A|A + C)} . \tag{2.17e}$$

Assuming that A, B, and C are the only competitors

$$p(A) + p(B) = p(C) = 1 \tag{2.17f}$$

and substituting into Equation (2.17f),

$$p(A) + \frac{p(A) - p(A)p(A|A + B)}{p(A|A + B)} + \frac{p(A) - p(A)p(A|A + C)}{p(A|A + C)} = 1 \tag{2.17g}$$

or solving for p(A)

$$p(A) = \frac{1}{\frac{1 - p(A|A + B)}{p(A|A + B)} + \frac{1 - p(A|A + C)}{p(A|A + C)} + 1} . \tag{2.17h}$$

Using an inductive proof, Equation (2.17h) can be generalized into Equation (2.17). Using Equation (2.17) in Friedman's model that treats the case in which all bidders are known, Equation (2.10) is converted into

$$E(y_i) = \frac{(X_i - C_i')}{\sum_{j=1}^{n} \frac{1}{\int_{r_i}^{\infty} f_j(r)dr} - (n - 1)} \tag{2.18}$$

which, complex as it may seem, is intuitively more satisfying than Friedman's product of probabilities.

This formula can be generalized into the strategy wherein all bidders are not known, by converting Equation (2.11) into

$$E(y_i) = \sum_{k=0}^{n} g(k) \frac{(X_i - C_i') \cdot \int_{r_i}^{\infty} f(r)dr}{k - (k - 1) \cdot \int_{r_i}^{\infty} f(r)\,dr} . \qquad (2.19)$$

Since f(r) is the probability density function of the "average bidder," it appears that Gates gives tacit approval to the concept of the "average bidder," in contention with Howard and Broemser. However, this does relieve the reader of the independence assumption required in Friedman's model.

A Method of Finding the Probability of Winning Over A Single Bidder - A Non-Parametric Method

Gates' method of determining the probability of winning over a single bidder is given here simply because it is a practical man's way of finding a solution to a complex problem. Its validity at this point will not be questioned. The method will be illustrated by an example taken directly from Gates' paper (24:80). However, several aspects of this example will be changed. First, all bids were not submitted by the same competitor; and secondly, the markup of "our" bid is assumed to be five per cent of the bid price since Gates did not know the actual cost estimates. The beauty of this example will be seen as the straight line approximations that can be generalized to other models.

Let C_k be "our" cost estimate and X_{k1} be the bid of one competitor for the k^{th} job. Table II is arranged from top to bottom in descending order based on the value of $r_{k1} = X_{k1}/C_k$.

TABLE II

BIDDING PATTERN OF ONE COMPETITOR*

Order No.	$r_k = X_{k1}/C_k$	$p = t/T$
1	1.102	0.033
2	1.064	0.067
3	1.060	0.100
4	1.053	0.133
5	1.050	0.167
6	1.039	0.200
7	1.037	0.233
8	1.036	0.267
9	1.034	0.300
10	1.031	0.333
11	1.029	0.367
12	1.029	0.400
13	1.012	0.433
14	1.012	0.467
15	1.008	0.500
16	1.006	0.533
17	0.994	0.567
18	0.989	0.600
19	0.977	0.633
20	0.975	0.667
21	0.974	0.700
22	0.953	0.733
23	0.953	0.767
24	0.927	0.800
25	0.906	0.833
26	0.903	0.867
27	0.894	0.900
28	0.886	0.933
29	0.846	0.967
30	0.821	1.000

*This table has been modified from the original table from Gates (80:22) to provide a better illustration of his technique.

If t is the order number of the ratio r_{k1} and T is the total number of variates considered, then the probability that the random variable r is less than the r corresponding to order number $t(r_t)$ is given by

$$p(r < r_t) = t/T \quad . \tag{2.20}$$

Hence, column three is the value of the probability in question. The resulting complementary cumulative probability function is plotted in Figure 10. Since a contractor is most often concerned with making a profit and assumes that his cost estimate is close to the actual cost, he will be concerned with that portion of the curve with $r > 1.00$. Gates approximates that portion of the curve with a straight line equation derived using elementary algebraic techniques. The equation which approximates this particular curve is given by Equation (2.21):

$$p(r < r_t) \sim 8.30 - 7.70r \quad . \tag{2.21}$$

While many theoretical flaws exist in this procedure, one can readily see that it gives results that can easily be manipulated. For instance, the elementary calculus procedure given on page eight can readily be applied to optimize a bid against a single bidder, or Equation (2.17) can be used in an iterative optimization procedure if all bidders are known.

The fundamental question here is whether or not the ogive shown in Figure 11 is the most valid method of handling the collected data. Non-parametric procedures are generally accepted as valid only when parametric methods cannot be used. In this case, it would appear more valid to describe the complementary cumulative frequency histogram, using a frequency analysis and apply the same approximating procedure. If this method were used, a practical method would probably have the edge over the multiple regression analysis presented by Broemser in both validity and acceptance by the industry. However, as seen in Figure 11, the straight line approximation is exceedingly inaccurate.

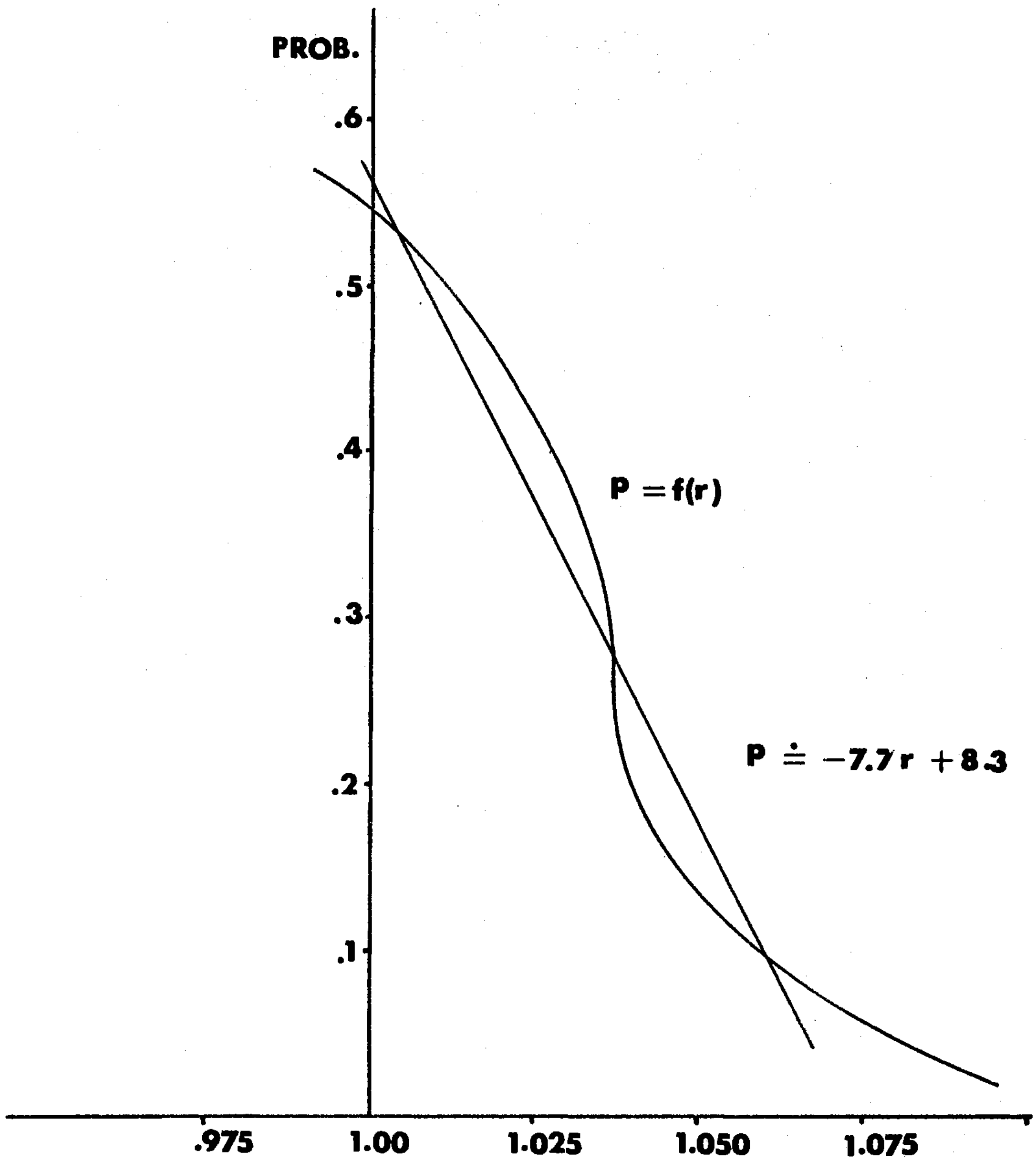


Figure 11. Probability Versus Markup of Bid

CHAPTER III

DISCUSSION OF PREVIOUS MODELS

Before making a comparison of the proposed models, it should be noted that one recent model is conspicuous by its absence. That is a model presented by Benjamin (4) in July, 1969. This model has been omitted summarily based on the following considerations:

(1) It presents an entirely new approach using utility theory which relates only to the contractor's objective and willingness to accept risk only as actual cost relates to estimated cost and not to the fundamental problems presented in this paper.

(2) This author considers Benjamin's model to be conservative to the extent that no contractor would bid a job if he were to use Benjamin's criteria.

(3) Benjamin's paper contains an excellent treatment of the actual cost as a random variable and will be discussed along with this author's views on the subject in Chapter IV.

There are three basic elements of the competitive bidding problem if the objective of profit maximization is used. They are represented by these questions:

(1) What will be the actual cost of the project?

(2) What is the probability of winning the contract at a given price?

(3) What is the optimum bid based on questions 1 and 2?

In comparing the proposed models, these three questions will be addressed.

What Will Be the Actual Cost of the Project?

Most researchers are in agreement that the cost of a project is a random variable. Friedman proposes the development of an _ā priori_ distribution of the ratio of cost to cost estimate in order to find a cost bias factor. Since the cost of performing the work is a random variable, the profit or loss is also a random variable at the time that a bid is prepared; the variation being caused by unforeseen costs that arise during the job. The variance of the probability distribution associated with Friedman's ratio is an indication of the riskiness of the job.

Common logic indicates that if this distribution is maintained current, the cost estimate should be modified concurrently until the estimate will approach the actual cost of the job or, using Friedman's terminology, the cost bias factor should approach one.

Therefore, the mode of handling the cost estimate as given by Friedman, Howard, et al., is a valid method which approximates the real world situation in an abstract sense. The researchers that fail to consider the cost as a random variable and treat the estimated cost as a constant for a particular job have treated the problem unrealistically.

What is the Probability of Winning the Contract at a Given Bid?

The models presented in this treatise have treated essentially two bidding situations in which a contractor might find himself. The first is a situation in which "local structure" of the bidding is known, i.e., a contractor can predict with a high degree of confidence who his competitors will be on a specific project. The second situation is represented by a competitive structure in which a contractor cannot predict with a high degree of accuracy who his competitors will be on a specific project. Within this second situation lies the possibility that the number of competitors may or may not be predictable with a high degree of accuracy.

For ease of reference in comparisons, the following types of models will be defined:

(1) The multi-distribution model (MD) is defined as the model in which all competitors are known with a high degree of certainty.

(2) The average bidder model (AD) is defined as a model in which the bids of all previous competitors are placed together into one distribution.

(3) The low-bidder model (LD) is defined as a model in which the bids of all previous competitors who were low bidders in competition with "our" bids are placed together into one distribution.

These definitions are after Shaffer and Micheau (72:8) with slight modifications.

The MD Model

On page 14, Friedman introduces the MD model. He developed distributions based on past data of the ratio of competitors bids to "our" cost estimate. He implies that these distributions can be related parametrically to known distributions. He then assumes that all bidders are independent of each other and asserts that the probability of winning is equal to the product of the probabilities of winning over each competitor. Park, Biasiolli and many others agree with this assertion.

Gates implicitly agrees in principle with the MD model in that the individual probabilities may be found _á priori_; on the other hand, he proposes that an ogive curve, based on non-parametric statistics, be used to compute the individual probabilities. Gates rejects the independence assumption and presents a formula that more adequately describes his experience in computing the probability of winning over several bidders.

The AD and the LD Models

These two types of models are discussed in the same sub-paragraph since both deal with the competitive situation in which the identity of the competitors on a specific project cannot be predicted with a high degree of confidence.

The AD model was first presented by Friedman's "average bidder" concept. As alluded to earlier, this concept may be used with Friedman's independence assumption or with Equation (2.17) provided by Gates. The unanswered question that remains is this: "Is the probability of winning equal to the probability of winning over the average bidder?" Friedman simply makes this assertion (21:107). Biasiolli agrees with the concept if extreme data are rejected from the distribution and the average bidder is stratified into the "lowest average bidder," "second lowest average bidder," etc. Park (61:147), in favoring the use of the

AD concept states:

> By using this concept, the general level of bids likely to result in maximum profits can be identified and used as a guide in setting an exact price, or in identifying the potentially profitable jobs.

(It should be noted that Park's model was not included in this treatise since it is merely a recapitulation of Friedman's work with experimental data included. He has presented a unique model (64) in a later article; however, the validity of it is severely questioned by this author and consequently it has been omitted.)

Howard has suggested that it is only necessary to bid lower than the lowest bidder among the competitors; hence, the LD model. The LD model consists of a distribution made up of all previous "lowest bidders" bids as a fraction of "our" cost estimate. This concept has been expounded by Casey and Shaffer (12) and Broemser (11). Benjamin (5:30) finds the LD model "more pleasing to one's intuition." Broemser does develop a method, not necessarily valid, for finding a probability distribution for the LD model through the use of multiple linear regression. For the AD model, Friedman assumes that a known distribution can be found whereas Casey and Shaffer assume normality as always.

In summary, one may assert that the MD model is valid if the local structure of the competitors is known with a high degree of certainty. However, there is no known distribution nor a method of finding a workable distribution for the MD model. The validity of using the AD or LD model is still in question. It appears from a cursory analysis that the AD model is the most conservative as it relates to winning the contract in question whereas the LD model should be used with a strategy of leaving as little money as possible on the table. Three methods have been noted in this study for finding a distribution function to approximate either of these distributions. They are:

(1) Broemser's multiple linear regression, single bid method.

(2) Gates' straight line approximation to a non-parametric ogive.

(3) Friedman's assumption that each distribution can be approximated by a known distribution.

Experimental Results by Shaffer and Micheau (72)

Shaffer and Micheau have used experimental data to study the reliability of the MD, AD, and LD model applications. Interestingly, in testing 50 project bids of one contractor, they found that this contractor met 118 different competitors but only 19 more than once. This experience matches the experimental data collected previously by this author. Based on this fact, unless a unique situation exists, the MD model is of little use.

In conducting these experiments, Shaffer and Micheau used a running average of distributions for the MD, AD, and LD models in an attempt to find an upper and lower bound for the bidding range in which a contractor should bid. In using the LD model, they have modified it to conform not only to the low bid distribution but to Biasiolli's stratified bidding model of the lowest, next lowest, third lowest, etc., without combining the distributions. Since this procedure has no basis other than experimental in all cases except those in which the MD, AD, and LD models have been used as described in this chapter, only those results will be presented.

To optimize the bid to be submitted, Shaffer and Micheau used an iterative technique with profit margins from zero to 30 per cent, picking the maximum expected profit in this range for each job. The number of bids won, the profit margin and the volume of work was noted. Table III shows the results using the models of this text. (Note: Data used in Table III considers only data available for the 50 jobs considered; neglecting the running averages for 10, 20, 30, and 40 jobs as presented in the stated reference. The interested reader may refer to this paper soon to be published in the Journal of the Construction Division, ASCE.

TABLE III

COMPARISON OF BIDDING MODELS

TYPE MODEL	NO. OF TIMES LOW BIDDER	WORK VOLUME	TOTAL PROFIT	PROFIT MARGIN
LD	4	$2,029,511	$ 67,748	3.44%
AD	8	3,956,050	209,068	5.28%
MD	2	1,236,659	70,000	5.66%
LD (excl. "our" bid)	6	3,246,376	146,139	4.19%

Before analyzing the results of these data, it should be noted that this experiment assumed that the estimated cost equaled the actual cost and was constant for each job. The optimization calculations were not included in the reference, and therefore, could not be verified.

It is evident from Table III that the AD model gave the best result in this experiment if one uses a linear objective function.

However, this should not be considered by the reader as proof that the AD model is most valid. This is only the result of one sample from an extremely large population. It should be considered only as an example in which the "average bidder" concept obtained the best results.

What is the Optimum Bid?

It has been noted that none of the presented models have a method for finding the optimum bid in closed form. If the probability of winning and the probability that the estimated cost equals the actual cost could be defined by a nice differentiable function, then the optimum bid might be found by calculus as stated in Equation (2.5). However, it should be obvious to the reader that this simple method cannot be used at the present "state of the art." Therefore, no method has yet been developed to find an optimum solution in closed form and only iterative or sequential search techniques can be used.

CHAPTER IV

THE PROFIT AS A RANDOM VARIABLE CONDITIONAL ON WINNING

The purpose of this chapter is to discuss and provide a solution to the acute problem of predicting the cost of a construction project prior to completing the project. The essential element of risk in a construction project is the inability to adequately estimate the true cost. Friedman (21), in the development of the first published bidding model recognized that the estimated cost in any industry is only the estimate of the mean of a random variable which describes the actual cost of a project. Since that time, most bidding models have treated the actual cost as a predetermined constant. Recently Benjamin (5:31) provided a treatment of the cost as a random variable but failed to provide a practical solution useable to the construction contractor. His method is solely dependent on the assumption of a cost distribution function.

The Cost Estimate

Since one of the major elements of uncertainty in construction contracting is the actual cost of construction, the problem should be discussed in terms of its basic elements. The cost of a construction project can be synthesized into seven elements. The cost estimate structure with these seven elements and profit is as shown in Figure 12. The majority of these seven costs are extremely sensitive to random influences of nature and human beings. Thus, these cost elements can be termed random variables. Over the course of time, the fact that variables behave randomly is not as imponderable as one might think. The fact that they behave randomly implies that each will follow a specific probability distribution. This fact can assist the contractor in more accurately predicting his cost as will be demonstrated in this chapter.

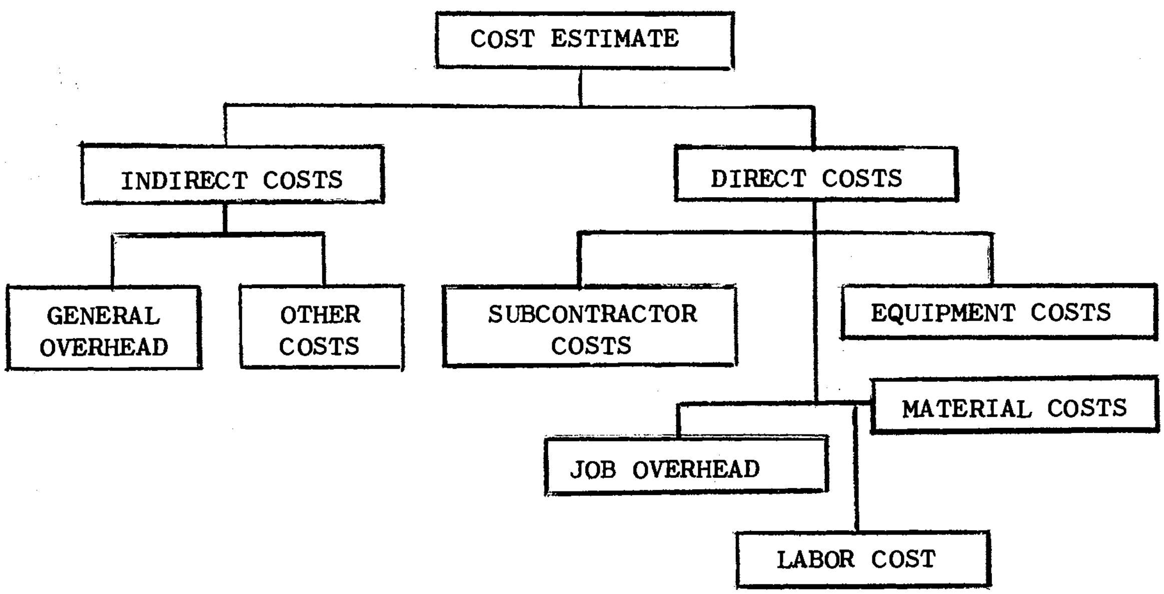


Figure 12. Cost Estimate Structure

The Direct Cost

First, a discussion of the specific cost elements and why they behave randomly is in order. The estimation of direct cost has been the subject of much study and literature over the past years. Although for specific contractors costs may be categorized differently, this author prefers to consider the direct cost of a project as consisting of the following five elements: subcontractor costs, material costs, equipment costs, labor cost, and job overhead costs.

The subcontractor costs are perhaps the most nearly stable costs estimated for a project. Once a contractor decides to estimate a job, subcontractors are requested to submit bids for certain phases of the job. Once a bid has been submitted to the contractor and a contract signed, the subcontractor is legally bound to perform the work at the quoted price. Only errors, accidents or acts of God can change the price of a subcontract and these must be either absorbed by the subcontractor, negotiated or handled by legal action. For the purpose of this discussion, subcontractor cost will be treated as a constant as it applies to the competitive bidding problem.

The cost of material in a properly prepared estimate will be quoted at the time that the estimate is made. Ideally, the quoted price should remain in effect for the duration of the project. However, there are many instances where external factors cause prices to change during the course of a project. These factors might be due to increased manufacturing costs, changes in the economy, shortages, strikes, etc. Therefore, unless special precautions are taken to insure a firm materials cost for the duration, which usually means paying premium prices, there exists the possibility of material price fluctuations during the construction of a project. In addition to price fluctuations, other factors affect the material cost of a project such as loss or damage to material on hand, miscalculations of quantities involved or expediting required due to inaccurate scheduling.

Hadley (97:40) gives an intuitive definition of a random variable as "any numerical quantity whose value will be determined by the outcome of a random experiment." The material costs of a project is a numerical quantity that varies; its true outcome depends on nature, the actions or nonactions of many individuals and human errors. The composite of these influences may be defined as a random experiment, thus qualifying the material cost as a random variable.

Labor cost, equipment cost, and job overhead costs are extremely sensitive to the duration of a project. The prescheduling and planning of the duration of a project has been researched extensively in recent years. It has been recognized that estimated durations are extremely stochastic in nature, the fact which led to the development of PERT (Program Evaluation and Review Techniques) as a scheduling vehicle. The time, cost, and knowledge associated with the use of PERT has hindered its use in the construction industry. A more common scheduling technique, CPM (Critical Path Method), which itself has not been widely adopted, is simply a deterministic form of PERT. Neither of these two scheduling techniques are adequate for scheduling and controlling a construction project. CPM will give one person's best estimate of a project. PERT, with its Beta distribution assumption for each activity will result in an expected duration with an associated variance for the duration of the project. Since the random influences of human error, weather, and a plethora of other factors affect the duration of construction projects; the time involved in construction satisfies the definition of a random variable.

There are two methods of estimating labor cost. The first, and probably most accurate, is a time-cost method. The labor force required for each activity is estimated, the time cost of this crew is calculated and the time of the activity is taken from the schedule. The labor cost for that activity is simply a product of the three estimates. If the labor force and costs are considered predetermined constants, the activity cost is the product of a constant by the time random variable which itself is a random variable. The labor cost for the project is a sum of these random variables, and consequently, a random variable itself.

The second method of estimating labor costs is by unit costs. Unit costs for various activities are calculated both rationally and from historical data. This method of labor cost estimating is generally less accurate than the previous method but is used extensively in building construction. The unit cost is simply an average labor cost for a certain amount of construction in place divided by that amount of construction measured in terms of some dimension, such as per cubic yard, per brick, etc. The unit cost, in actuality, is simply a mathematical manipulation of the time cost of labor, and therefore, can also be classed as a random variable.

The estimated equipment cost of a project involves numerous calculations, and its effect on the total job cost varies from slight as in the case of pure vertical construction to predominant in the case of pure horizontal construction. As mentioned previously, the estimate of equipment cost is very time-dependent. Basically, the equipment hourly cost is calculated from the fixed cost plus the operating cost of each item of construction equipment to be used on the project. The major element in the fixed cost of equipment is the depreciation of the equipment which is in itself time dependent upon the expected useful life of the item. The operating cost varies directly with the scheduled time of the job; therefore, time provides the random influence on the operating and fixed cost of the equipment.

The costs that are considered job overhead vary from contractor to contractor. Most generally, the job overhead costs are those that are incurred for the duration of the job which do not directly account for construction in place, such as supervision, inspection, bonding and insurance, etc. These costs are again a direct (not necessarily linear) function of time which is the randomizing influence.

The Indirect Costs

The indirect costs associated with a project are more elusive to the estimator than direct costs. They are difficult to identify and even more difficult to unitize. The indirect costs consist of the general overhead cost and "other costs."

The general overhead actual cost remains constant with time. However, it is roughly hyperbolic when related to the volume of work on hand. For instance, if a contractor has $300,000 of work in progress, his general overhead might be six per cent. This figure would consist of the proration of the cost of engineers, managers, office staff, physical plant, and other fixed business costs necessary for contractor operations. If this contractor's volume dropped to $100,000, then his overhead costs assuming no managerial corrections, would be 18 per cent of the cost of each project. Intuitively, since the actual general overhead costs per dollar volume are a function of the volume of work on hand, the estimate of the general overhead costs requires that a prediction of the average volume of work over a period of time be predicted in advance of estimating a job. The volume of work on hand depends on the state of the economy and the construction industry, the actions of competitors, the contractors estimating competency, and a multitude of other factors. Thus, these influences can only be considered by assuming that the general overhead cost is a random variable.

"Other costs" is a category of the cost estimate that depends upon the sophistication of the contractor and ideally should be zero. However, there will often be the case that certain costs associated with a project can more suitably be considered as "other costs" rather than categorized within other cost groups. For instance, a recent financial system developed under contract for Armco Steel Corporation (92) suggests that the contractor cost associated with labor costs (workman's compensation, health benefits, employer portion of F.I.C.A., etc.) be separated from direct labor cost to facilitate managerial cost control. This is an effort to make cost control compatible with necessary book-keeping procedures so that accounting procedures may be accomplished with one system rather than two or three systems. Since this procedure reduces contractor overhead, it is advisable to include these costs in the category of "other costs." Other items that might fall into this category might be mobilization costs, contingency costs and interest on money invested. Since this cost category and its use is nebulous, "other costs" will heuristically be assumed to be a random variable.

The Profit Random Variable

A random variable, to be completely defined, requires that a particular sample space upon which it is defined be designated and a function which relates a unique value to each point in the sample space be formulated (97:39). Based on this definition, the sample space upon which each of the seven elements of the cost estimate is defined consists of all real numbers greater than zero; the random experiment is the project itself and the function relating the outcome of the experiment to the sample space is unknown but can be assumed for the purpose of this discussion. Having met the defining conditions, let the set C be defined as:

$C_1 \equiv$ subcontractor costs

$C_2 \equiv$ job overhead costs

$C_3 \equiv$ labor costs

$C_4 \equiv$ equipment costs

$C_5 \equiv$ material costs

$C_6 \equiv$ general overhead costs

$C_7 \equiv$ other costs.

Let $h_i(c)\Delta c$ be defined as the probability that the random variable C_i will lie in the interval from c to c + Δc. If y is defined as profit and X is the total bid amount for the project, then

$$y = X - \sum_{i=1}^{7} C_i \; . \qquad (4.1)$$

Let X be considered a constant for any particular bid (this is valid since the profit is considered conditional on winning). Then y is only a function of the seven random variables and is therefore a random variable itself (97:54). The expected value of y is given by

$$E(y) = \mu_y = E\left(X - \sum_{i=1}^{7} C_i\right)$$

$$= E(X) - E\left(\sum_{i=1}^{7} C_i\right)$$

$$= X - \sum_{i=1}^{7} E(C_i)$$

$$\mu_y = X - \sum_{i=1}^{7} \mu_{C_i} = X - \mu_C \quad , \tag{4.2}$$

and the variance of y, assuming independence among the C_i, is given by

$$\sigma_y^2 = \sum_{i=1}^{7} \sigma_{C_i}^2 = \sigma_C^2 \quad . \tag{4.3}$$

Thus, y is a random variable with a mean, $\mu_y = X - \mu_C$ and a variance, $\sigma_y^2 = \sigma_C^2$.

The effect on the profit from a contract at a specific bid amount is shown in Figures 13 and 14. If the contract is bid at the mean cost, μ_C, the probability that the actual cost will be equal to or greater than the amount planned is 0.5 if one assumes a symmetric, unimodal distributed cost. If the anticipated profit is μ_y, bid at μ_C, y_2 represents the greater profit should the actual cost actually fall below the estimated cost, say at C_2. However, should the actual cost be above the true mean cost, say at C_1, y_1 illustrates the loss that will occur. Using the illustrated curve as the probability density function, it is quite apparent that the probability that the cost will fall as high as C_1 is quite small.

If no method of obtaining the distribution parameters is used during the bidding process, Figure 14 illustrates what might be the effect should a bid be submitted and won when the cost is estimated much below the true mean cost. If the estimator calculated his cost as C_2

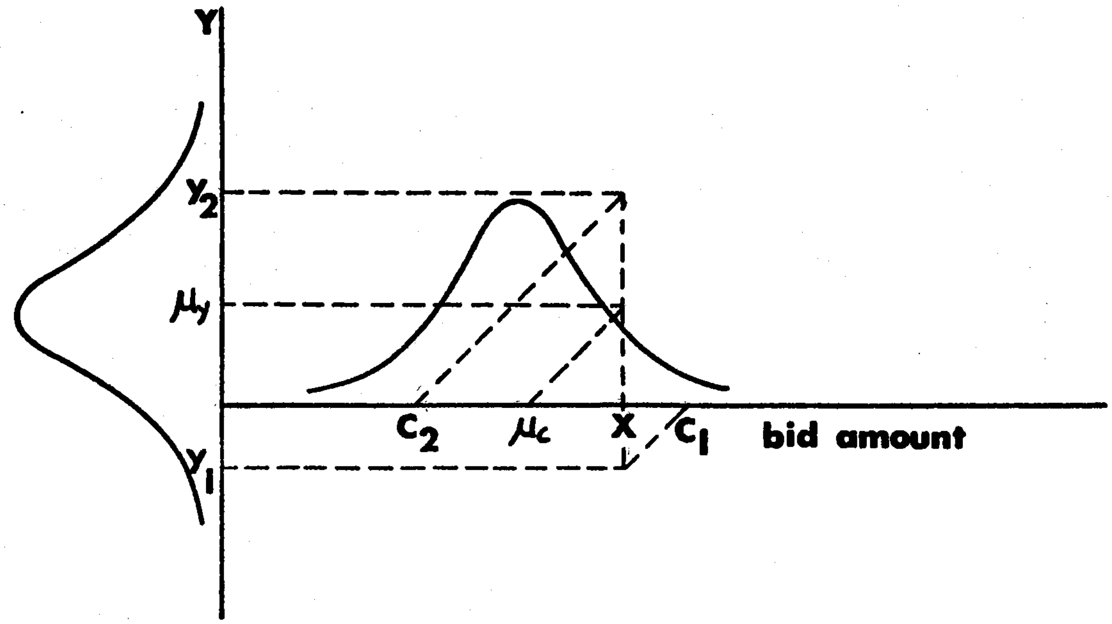


Figure 13. Effect on Profit if Bid at True Mean Cost

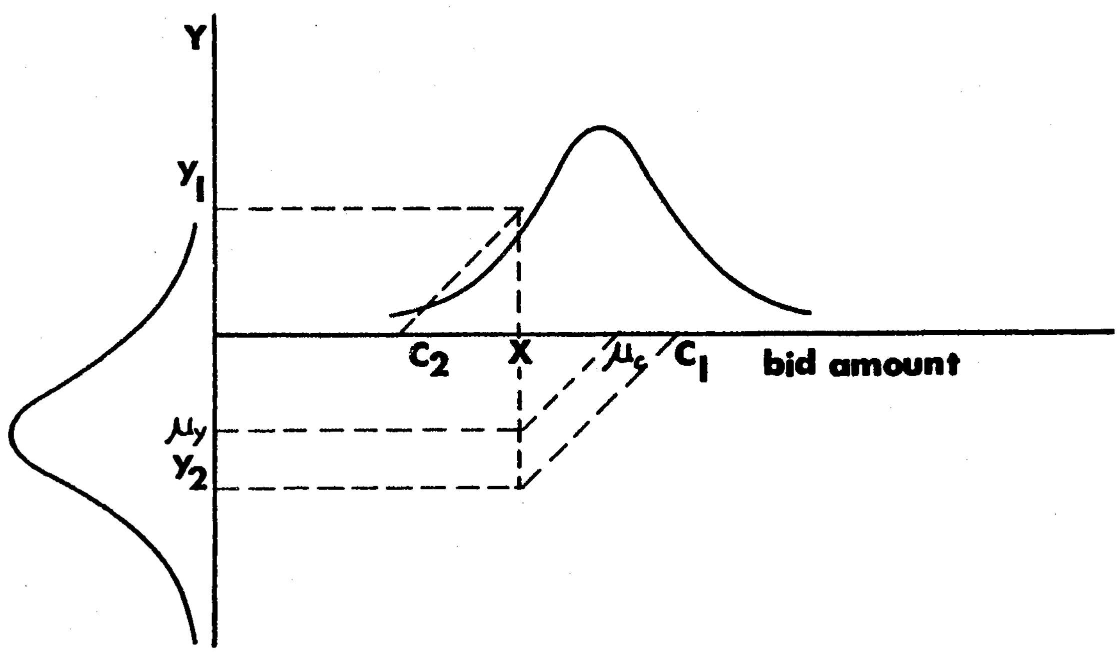


Figure 14. Effect on Profit if Bid at Below True Mean Cost

and bid the project at an amount X, the profit that he would expect to make is represented by y_1. However, under the assumed distribution, there is a high probability that the actual cost of the completed project will be as great as μ_c or even C_1, resulting in losses of μ_y and y_2, respectively.

The shortcomings of this type of analysis is obvious. The probability density function for the cost random variable, and hence, the profit is not known. There are no mathematical or physical characteristics associated with these individual random variables that would intuitively lead to the selection of any describing distribution such as the Gaussian, Beta or any other. A curve fitting approach for the selection of a distribution would, in all probability, be economically infeasible. Were the distribution of the costs known or assumed, the function could be included directly in the bid optimization as suggested by Friedman and Benjamin.

Based on the foregoing discussion and granting that the profit, should the bid be won, is a random variable, one must either ignore the random influences and treat the estimated cost as a known constant or find some means of handling it in a bidding process. Many authors have chosen to ignore the cost and hence the profit as a random variable when winning is assumed. The prudence of this assumption is questionable at this point. Should one choose not to ignore the random nature of the cost, then three alternatives exist. First, one may assume a type distribution for the cost and calculate the parameters associated with that distribution. This is an extremely hazardous method of treating the cost estimate as most often an estimator is overly optimistic and, rather than having his estimates distributed nicely according to some classical distribution, his estimate is skewed to the right as shown in Figure 15 (5:33). This implies that normally the estimator will estimate at a cost less than the mean true cost. A second method of analyzing the cost is to use multiple linear regression. The major drawback to this type is the extent to which one must research the variational characteristics. As discussed previously, the influence factors that affect the cost estimate are extremely numerous and the economic feasibility of this approach, with the inclusion of a significant number of variables, is questionable.

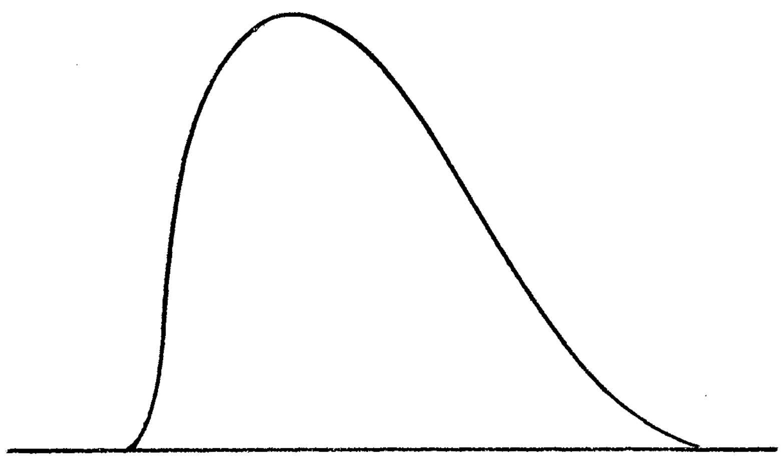


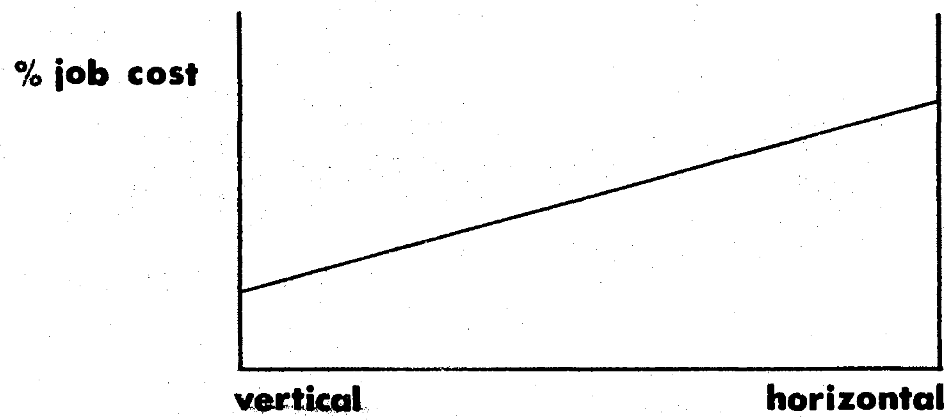
Figure 15. A Skewed Distribution

The third and final method, and the method proposed by this paper, of treating the estimated cost is essentially a compromise between the deterministic approach and the fully stochastic approach as proposed by Benjamin. The method proposed here is not a clear cut mathematical approach, but must be included in the total construction management system of a contractor. The method is called the variance minimization approach.

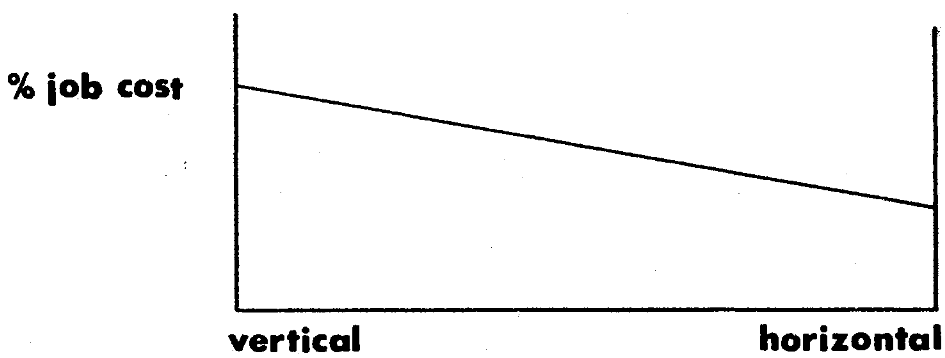
Cost Variance Minimization

Since the bidding process is essentially a four stage lottery -- which job to consider -- which objective to use in bidding -- how much markup should be added to win -- what will the job cost -- the deterministic approach to cost reduces it to a three stage problem. This approach greatly simplifies the optimization calculations also. But, as mentioned before, it is necessarily inaccurate to profit estimation. Therefore, to gain the benefits of the deterministic approach and to improve a contractor's business situation, the estimator's objective is to make the estimated cost approach the actual cost. Since the spread of any distribution about the mean actual cost is measured by its standard deviation, this objective becomes a minimization problem; that is, to minimize the variance of the cost estimate.

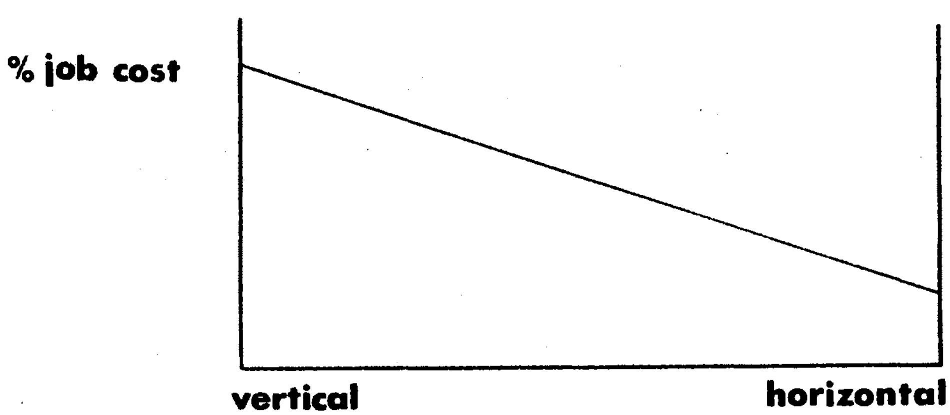
Equation (4.3) gives the variance of the cost estimate as a sum of the variances of the individual elements in the cost estimate. Those elements of the cost estimate with the greatest variance in cost are the elements which add the risk to construction contracting. Those elements depend primarily on the type of construction in which one is involved. For instance, Figures 16a, 16b, and 16c show qualitatively the amount of cost associated with various types of construction contractors. These graphs, developed intuitively while observing many cost estimates, indicate that the predominant direct costs of a building contractor are labor, materials, and subcontractor costs. For heavy construction, the direct cost is predominantly equipment cost and, perhaps, subcontractor cost. Therefore, since the variance in subcontractor cost can be taken as zero, a building contractor should expend most of his effort in reducing his labor and material cost variance



(a) Equipment Costs



(b) Labor Costs



(c) Materials Costs

Figure 16. Equipment, Labor, and Materials Costs

whereas a horizontal contractor should expend the same efforts on minimizing the variance of his equipment cost. Figure 17 shows a flow chart of the recommended estimating system whereby the cost variance can be minimized. This chart depicts, not a definite statistical method, but a portion of an integrated construction management system for a building contractor. The reports required for a heavy contractor are similar.

Based on the definition of the variance of a random variable, as each value of the variable approaches the mean, the variance tends to zero. The purpose of the system prepared in Figure 17 is to provide a feedback system to the estimator so that costs can be updated towards the mean cost. In addition, the complete system, with elements to be explained in Chapter V permit the contractor to establish a method of maximizing profit while at the same time maintaining a cost accounting and control system which has the same meaning to the bookkeeper, accountant, estimator and field staff (92). The system outputs only two reports.

The first, the weekly payroll report, is simply the standard payroll report used by most contractors and requires no major changes in operation procedures. The second, and most meaningful report is the monthly cost and unit cost report. Although this report is used in establishing job control, completed job analysis and tax analysis, its purpose in this chapter is to provide feedback to update the estimator's cost records.

Prior to instigating this cost updating system, a contractor must code each activity of a job. This is nothing new or unusual since most contractors code activities as a matter of record for cost control. There are numerous systems in use; numerical (92), alphameric, or a

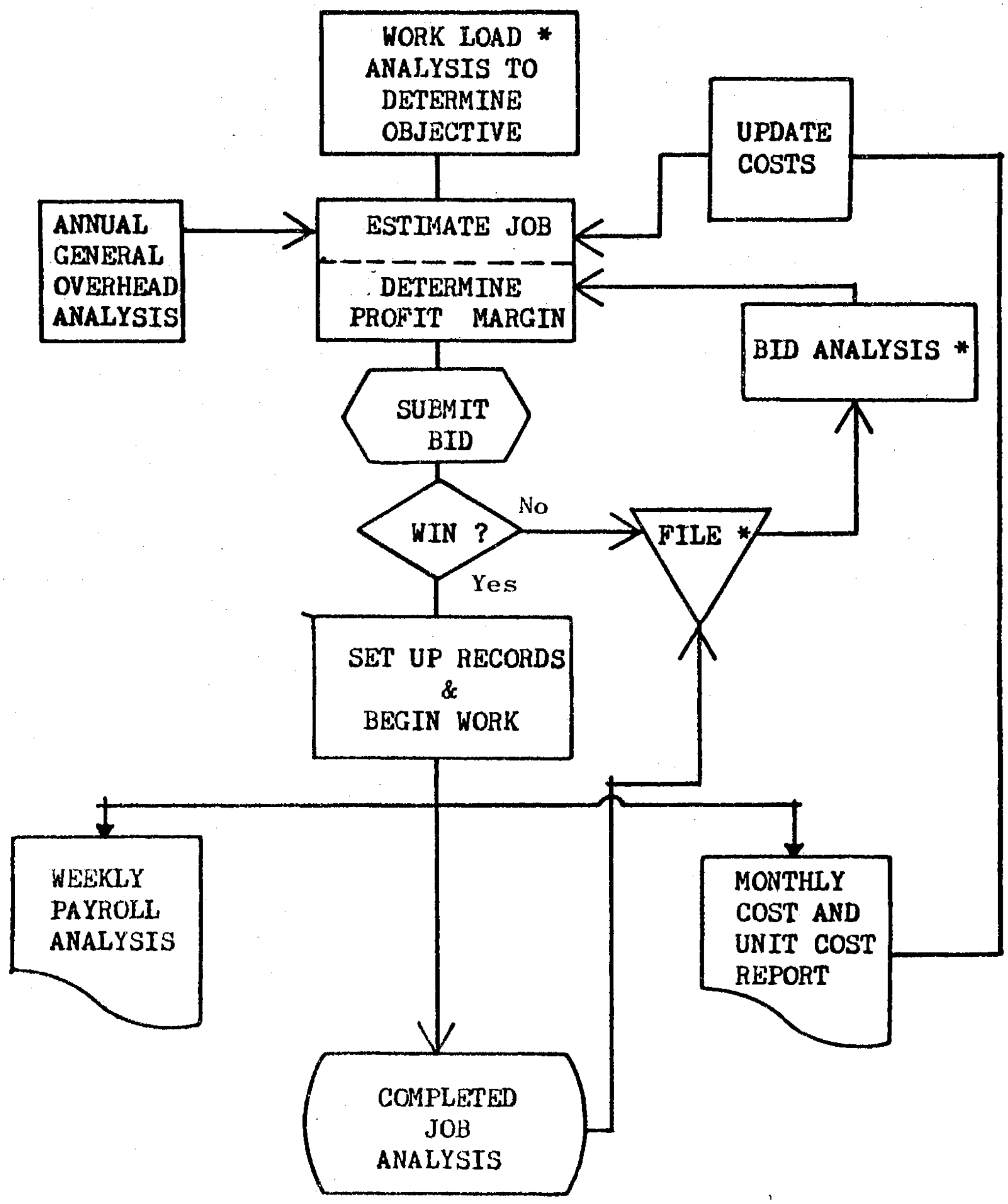


Figure 17. Integrated Cost Estimating Procedure for a Building Contractor

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*These items will be discussed in Chapter V.

combination of both. This author prefers the combination of major activities designated by alphameric characters with subactivities designated by numbers. Although this method is more difficult to program, it is easier for field personnel to learn; this being the most crucial element in the success of a report. As a matter of course, the estimated duration of each activity should be entered along with the estimated cost of the activity. Internally, the computer can calculate a straight line expected duration-cost curve; and when the actual cost exceeds this estimated cost by a certain percentage (allowing for the straight line approximation to a logrithmic growth curve), the job and activity can be "flagged" on the report. This is an aside however, since it provides for cost control and not directly to cost estimating feedback. The quantities of units in place may be entered by delivery voucher or separately, according to the characteristics of the activity.

The portion of the report of intimate concern to the estimator is the ACTIVITY CODE SUMMARY which is printed on the report after the final job summary. From this section of the report, the estimator can find both the mean and variance for each activity code for the month, the mean and variance for each activity code to date. From these data, he can compare his estimated and actual costs. If the monthly unit cost variance changes significantly, he must examine each job to determine the reason for the significant changes. Then, based on his judgment, he updates his cost book accordingly. The format for the MONTHLY COST AND UNIT COST REPORT is shown in Figure 18.

Finally, after a job is completed, the actual cost of each activity code is analyzed. The mean and variance of the unit costs are tabulated,

MONTHLY COST AND UNIT COST REPORT

JOBNR	CODE	ESTIMATED LABOR COST TO DATE	ACTUAL LABOR COST TO DATE	ESTIMATED MATERIAL COST TO DATE	ACTUAL MATL COST TODATE	TOTAL ESTIMATED COST TO DATE	TOTAL ACTUAL COST TO DATE	MONTHLY UNIT LABOR COST	MONTHLY UNIT MATL COST	TOTAL UNIT COST	TOTAL UNIT LABOR COST	TOTAL UNIT MATL COST	IN PLACE THIS MONTH
70-10	FC15	12550	13520	37500	33360	50050	46880	.370	.640	.728	.210	.518	420

****

JOB SUMMARY

JOBNR	ESTIMATED LABOR COST TO DATE	ACTUAL LABOR COST TO DATE	ESTIMATED MATERIAL COST TO DATE	ACTUAL MATL COST TO DATE
70-10	312020	321500	718730	617510

****

ACTIVITY CODE SUMMARY

ACTIVITY CODE	MONTHLY MEAN LABOR UNIT COST	MONTHLY LABOR UNIT COST VARIANCE	TOTAL MEAN LABOR UNIT COST	TOTAL LABOR UNIT COST VARIANCE	MONTHLY MEAN MATERIAL UNIT COST	MONTHLY MATERIAL UNIT COST VARIANCE	TOTAL MEAN MATL UNIT COST	TOTAL MATL UNIT COST VARIANCE
FC15	.237	.0015	.217	.0009	.721	.0072	.490	.0011

****

Figure 18. Monthly Cost and Unit Cost Report

the actual cost and estimated cost compared, and the estimated cost is corrected for bias.

Traditionally, the general overhead cost analysis has been a point of contention between accountants and estimators (92). Estimators desire to unitize it but lack an adequate tool with which to do so. Accountants tend to argue that it should not be unitized but should be added to the markup and separated from profit annually. The Associated General Contractors of America has even published percentage guidelines from time to time.

In order for a contractor to avoid bankruptcy, he must maintain a sufficient volume to cover his direct costs, "other costs," and his general overhead. Pending further research and relying for the time being on the present state of the art, the method of considering general overhead in this integrated system is to consider annually the general overhead per dollar volume as derived from the tax analysis and add this percentage to each bid cost as general overhead. This method tacitly assumes that volume remains constant from year to year. This percentage should be modified by the contractor if current information is known about future annual volume or expansions which will increase the general overhead cost.

This system differs from Friedman's original method of bias correction in that no distribution has been assumed and the individual elements of the cost estimate are constantly updated towards the mean -- thus reducing the variance. Benjamin's method of handling the profit random variable conditional on winning is also distribution oriented; and, when included in the final stage of the lottery, causes the bid model to be ultra conservative.

In summary, this chapter presents a compromise between those who treat the cost estimate as deterministic and those who treat the cost estimate stochastically. The latter deviate extremely from reality, whereas no practical method has been devised with which the former can be used. The procedure presented here gives the manager the tools necessary to make his bid conservative or non-conservative based on his judgment and his VT function as presented in Chapter V. It provides the contractor with a management tool whereby his entire organization can be controlled using one system. Finally, it is a practical procedure which is easy to be put into use.

CHAPTER V

A DYNAMIC BIDDING MODEL

Each model discussed in Chapter III was a static model in that neither the contractor's profit function nor his probability of winning changed with time. Because of its very nature, the construction industry cannot be described in a static sense. Each element of construction operations is ever changing with time. This is not to say that the contractor utilizing one of the models discussed in Chapter II does not have an advantage over the contractor who uses no rational approach to bidding. The models presented are unsuitable from the standpoint that they do not represent the dynamic situation in the construction industry.

The model presented herein adds three characteristics unavailable in any other bidding model. First, a method of analyzing a contractor's own objective is presented utilizing a volume-time (VT) function with associated constraints. Secondly, based on the contractor's objective, various utility functions are hypothesized. The third aspect of this model presents a dynamic approach to determining the most crucial aspects of the competitive bidding model, that of determining the probability of winning. Finally, a technique of optimization is presented.

Contractor Objectives

In all previous sealed bid models, the only contractor objective considered has been the intuitively pleasing objective of maximizing profit. Yet, it is not uncommon to go to a bid letting and see contracts won at five, ten, or even fifteen per cent below the cost estimated by a contractor. As paradoxical as this may seem, there are circumstances that make it feasible, or even necessary, that a contractor bid a job below cost. The reasons for this are varied. A contractor may bid a job below cost to pay for mobilization cost for a larger, more attractive job. He may bid below cost because he may feel it will put his firm in a better position to obtain negotiated work in the future. But, these reasons are rare. By far the most predominant reason for bidding a job below cost is overhead absorption, e.g., a contractor's volume reaches a point that a contractor must get the job or risk bankruptcy.

On the other hand, it is also not uncommon to see a bid with a markup of twenty per cent or more at a bid opening; the contractor submitting the bid knowing that he has very little chance of winning the contract.

Therefore, it is not unreasonable to suspect that the contractor utilizing this bid model might have an objective other than that of maximizing profit.

The mathematical model for determining the contractor's objective is taken from the VT function which has been developed from the work load diagram presented by Miller (99:114). The VT function is a functional representation of the volume of work on hand versus time.

The VT function, rather than using a logarithmic decay curve as in the workload diagram, approximates each project as a straight line for ease of computation and construction. Therefore, the current volume of work on hand in dollars can be represented in linear form by

$$V(t) = \sum_{k=1}^{m(t)} \alpha_k t + \beta_k \tag{5.1}$$

where m(t) is the number of jobs on hand at time t and $\alpha_k t + \beta_k$ is the equation of the k^{th} job. A sample work load diagram is shown in Figure 19 and a sample VT function is shown in Figure 20.

Each contractor will implicitly set a lower bound on his desired volume of work. This is necessary because a contractor must keep a certain volume of work just to pay his necessary overhead. If the contractor is normally involved in heavy horizontal construction, he must pay his straight time employees, make his equipment payments and pay his other fixed costs. A vertical contractor has the same considerations and in addition he must provide a certain amount of work for his normal subcontractors in order to remain competitive.

All contractors have an explicitly established upper bound. This may be a bound set by the manager or owner determined by the volume that he feels he is staffed to accomplish. Most often, however, this bound is set by the bonding capacity of the contractor which is based on fixed and quick assets. These two bounds, V_L, the lowest volume of work that a contractor desires to have on hand, and V_U, the highest volume that the contractor can accept, are two major factors that influence his bidding objectives.

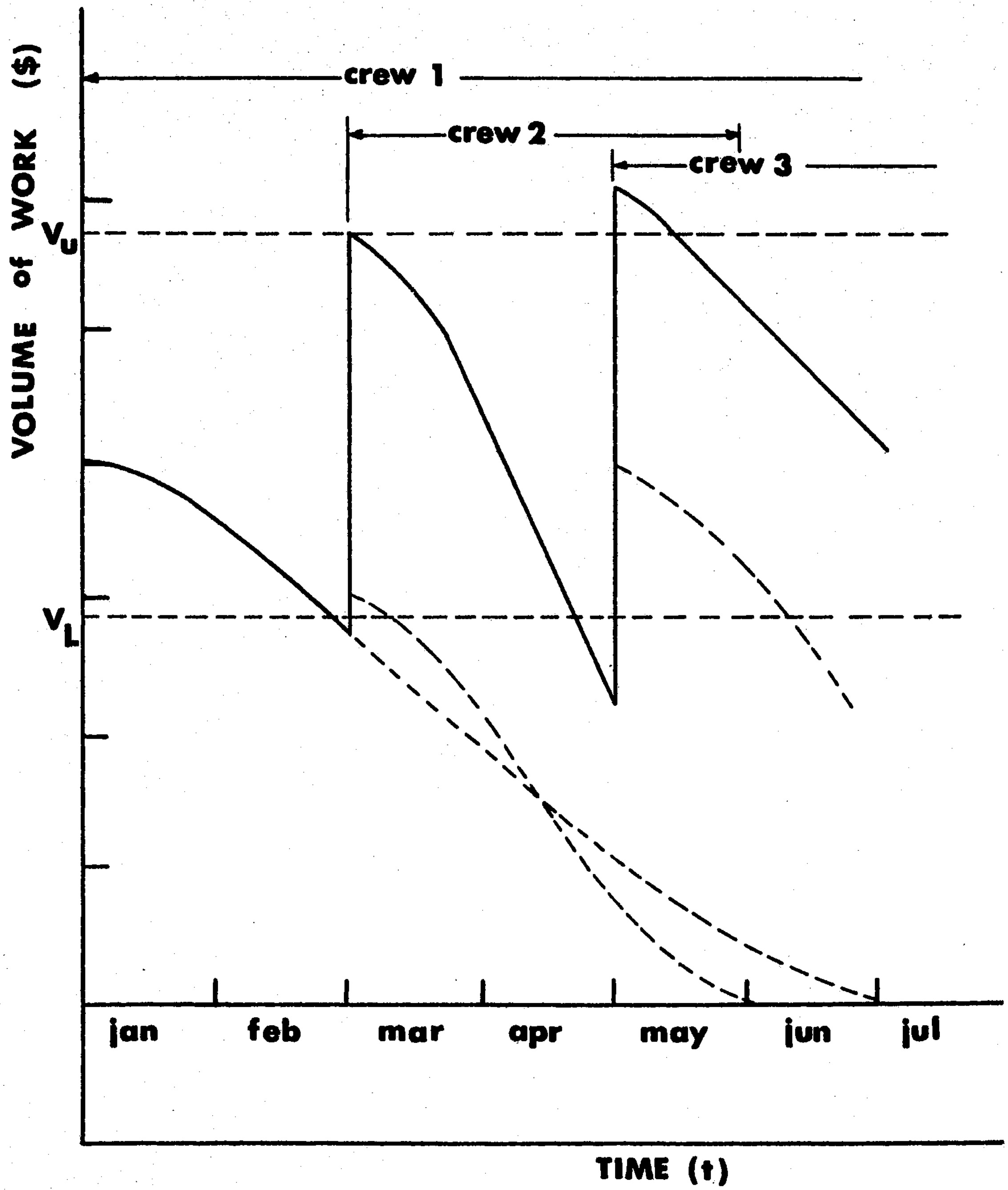


Figure 19. Work Load Diagram (From Miller (99:114))

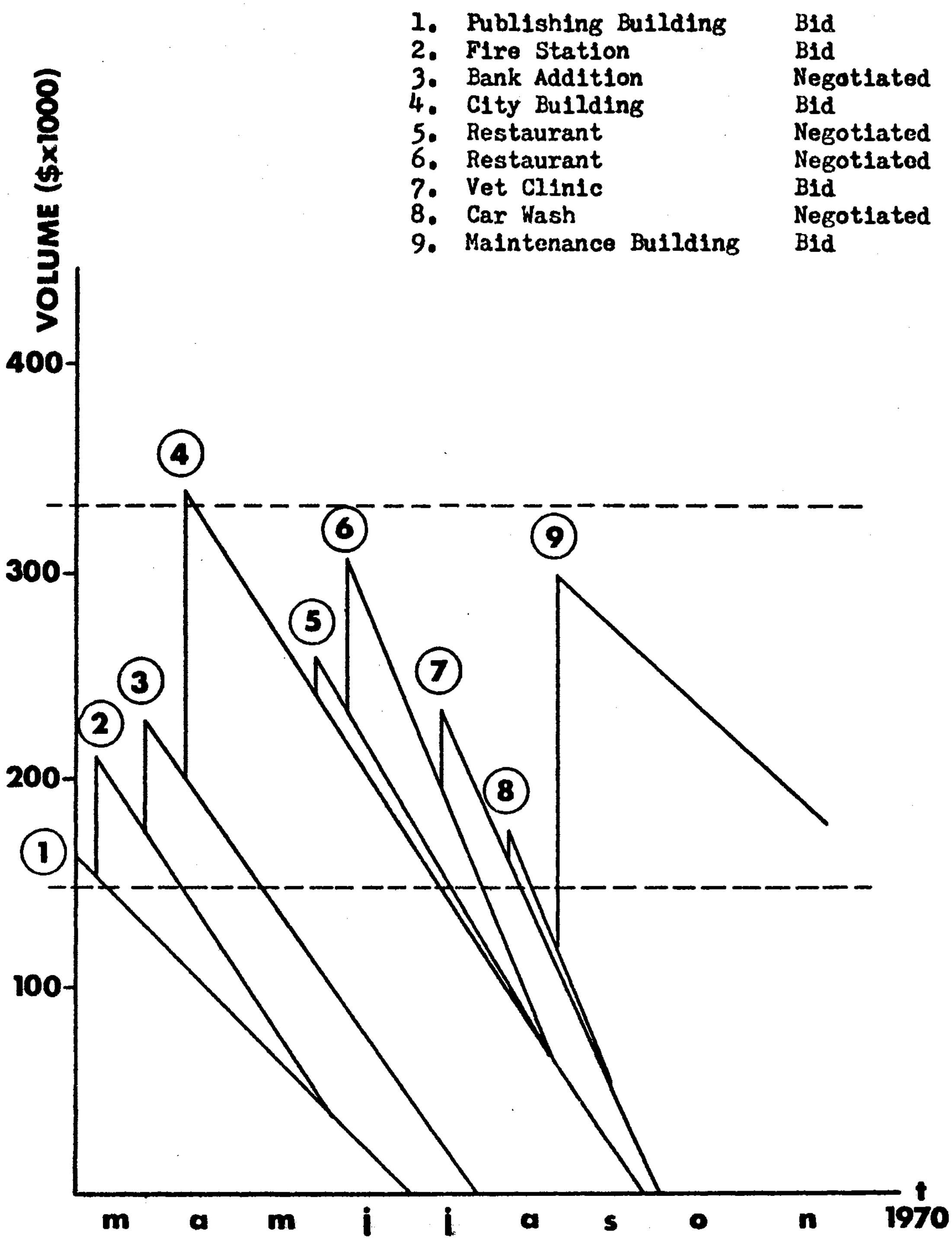


Figure 20. Example of Actual VT Function for a Small Building Contractor

In analyzing the contractor's objective prior to a particular bid letting, let the time in question be represented by t_o. Let $X_o^{'}$ represent a rough estimate of the value of job to be bid. Then if

$$V_L < \sum_{k=1}^{m(t_o)} \alpha_k t_o + \beta_k + X_o^{'} < V_U , \qquad (5.2)$$

the objective will be to maximize the expected profit for the contract. If, on the other hand,

$$\sum_{k=1}^{m(t_o)} \alpha_k t_o + \beta_k + X_o^{'} > V_U \qquad (5.3)$$

then two choices are open to the contractor. First, he can omit the job entirely. On the surface, this would appear the most feasible solution. However, other factors must be considered. The contractor must consider his public relations. A sound contracting business structure consists of a balanced amount of negotiated contracts with competitive contracts, the negotiated work forming a substantial base to the contractor's volume. Therefore, a contractor must bid work for sponsors in order to maintain his position for obtaining negotiated work. A contractor can, at times, exceed his normal bonding capacity. This is done normally with the acceptance of a certain risk. High risk performance bonds are inherently more costly and, if the normal bonding capacity is exceeded, the contractor is, in all probability, over-extending himself. Another factor to be considered is a contractor's strategic position. Even though he is working to capacity, he must still meet his competitors in competition. Not to do so would alert his competitors to the fact that he is working to capacity, thereby permitting an astute competitor to more accurately analyze his bidding trends. Therefore, if Equation (5.3) holds, the objective of the contractor will be to increase his profit margin to a point that he actually minimizes his probability of winning.

The third region of interest on the work load diagram is the region in which

$$V_L \geq \sum_{k=1}^{m(t_o)} \alpha_k t_o + \beta_k \quad . \tag{5.4}$$

If one assumes a constant general overhead cost, GOH, to a contractor, it is readily apparent that GOH is related to the volume of work on hand $V(t_o)$ by the equation

$$\gamma = GOH/V(t_o) \tag{5.5}$$

where γ is the general overhead cost per dollar volume. This function is represented in Figure 21. The function is an equilateral hyperbola and γ becomes extremely high as the volume of work decreases. Therefore, as a contractor's volume of work approaches the region of the VT function represented by Equation (5.4), he becomes most anxious to obtain more work. Thus, his objective changes to one of maximizing his probability of winning the contract.

By examining the general shape of the function which represents the probability of winning as shown in Figure 22, it is apparent that if one were to maximize his probability of winning the contract, he would bid at the least loss such that the probability of winning is equal to one. In many instances, this loss would be too large to absorb. Therefore, the third objective must necessarily be modified by

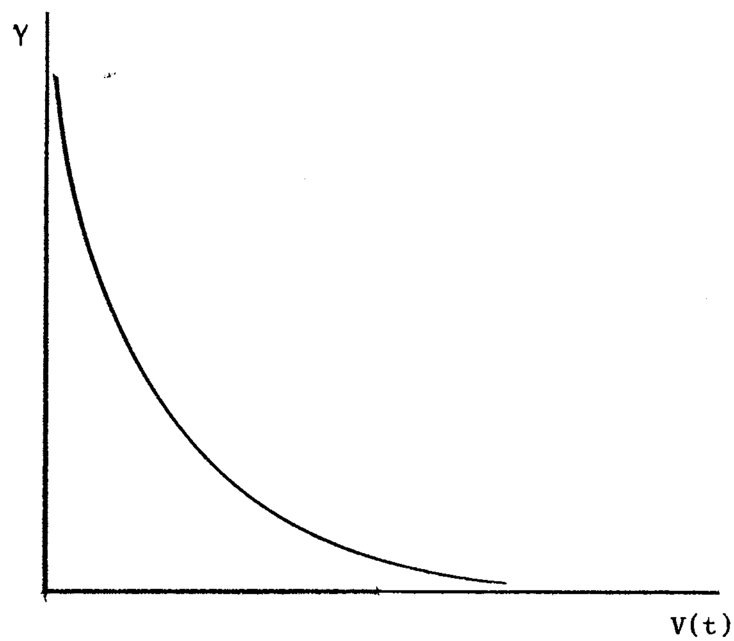


Figure 21. Overhead Cost Per Dollar Versus Volume

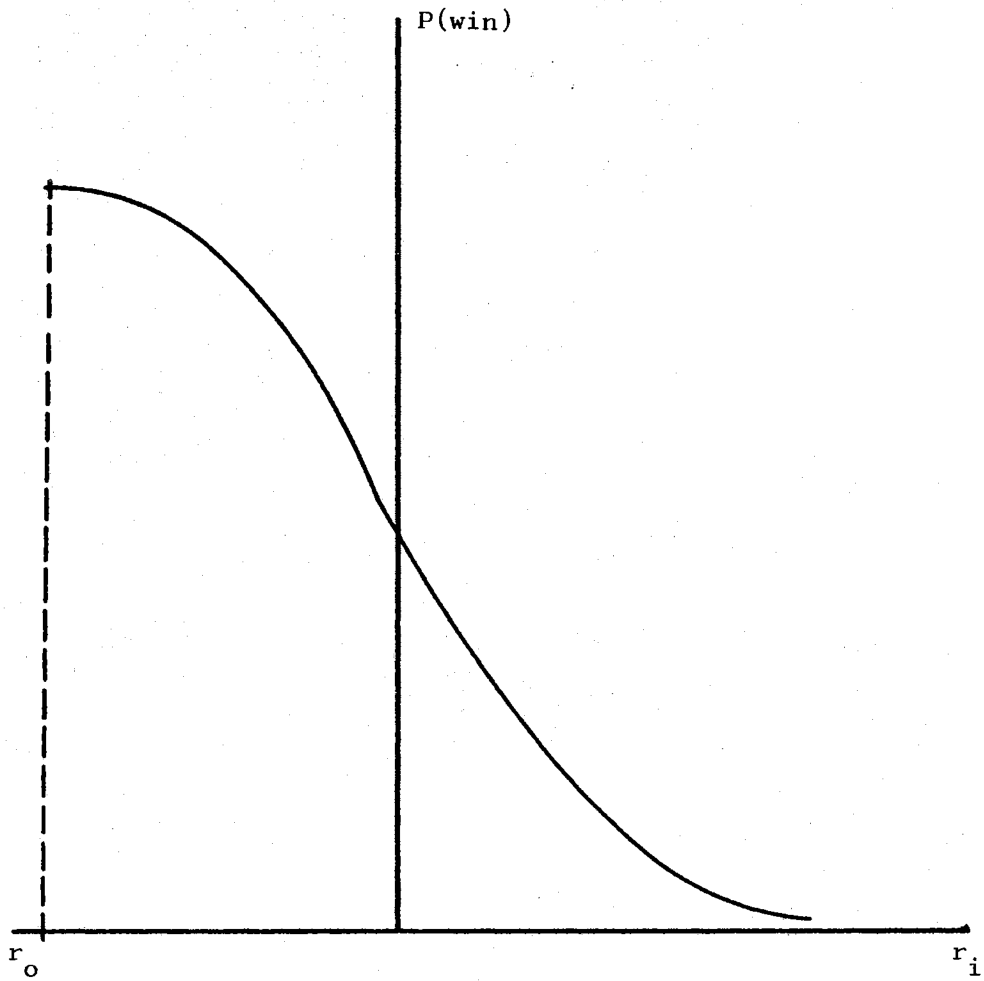


Figure 22. The Function Representing the Probability of Winning

some constraint. Three constraints are proposed in this paper. They are:

(1) Bid only so low as to absorb the direct cost plus overhead cost associated with the job. (Accept no loss.)

(2) Bid to accept a loss of a specific percentage.

(3) Bid only as low as necessary to absorb the direct cost associated with the contract.

There are additional factors that influence the choice of a bidding objective that can be analyzed from the VT function. The optimum volume of work on hand should be balanced to correspond to the organization of a contractor. For example, consider a building contractor with a bonding capacity of $350,000 and a field supervisory staff capable of handling four jobs. This contractor's optimum business situation is to have $350,000 of work on hand which consists of four jobs. In a dynamic situation, this will seldom be the case. If, at time t_o,

$$V(t_o) \in (V_L, V_U)$$

and he has four jobs in progress, none of which will be completed prior to the time that he must start on the job that has been announced, he must decide (1) not to bid the contract, (2) bid the job at a markup which will minimize his probability of winning, or (3) analyze the feasibility of establishing an additional job supervisory staff. If he decides on the latter, this changes his normal business situation and increases both his job and general overhead. This discussion shows that other constraints can be determined from the VT function. If A is defined as the number of projects capable of being managed by the field supervisory staffs available and $m(t_o)$ is the number of jobs in progress at time t_o, then the additional constraint that affects the contractor's objective is

$$m(t_o) \leq A \quad . \tag{5.6}$$

It should be noted that field supervisory staff capability is only an example constraint. There may be many others such as equipment available for similar jobs, specialty personnel for similar jobs, general supervision span of control, etc.

It should be noted, on the other hand, if $m(t_o)$ is much less than A, then idle field supervisory staff (or equipment, as the case may be) will increase general overhead per dollar volume and, like the situation denoted in Equation (5.4), cause the contractor's objective to sway to that of maximizing his probability of getting the job.

The Contractor Utility Functions

Having arrived at a rational method for selecting contractor objectives, they must be represented in such a manner so as to be compatible with the decision theoretic approach to be used in this bidding model. The common objective function used in most previous bid models has been the well known linear relation between bid, cost, and profit as represented in Figure 23. This objective function has two distinct disadvantages. The first disadvantage is that it represents only one contractor objective. The second is that it permits no contractor preference towards profit and loss. Therefore, in order to adequately demonstrate contractor objectives, it becomes necessary to use utility theory as presented by Hadley (97:81).

Utility theory has been developed in an attempt to more adequately describe "real world" decision processes. Basically the theory assumes

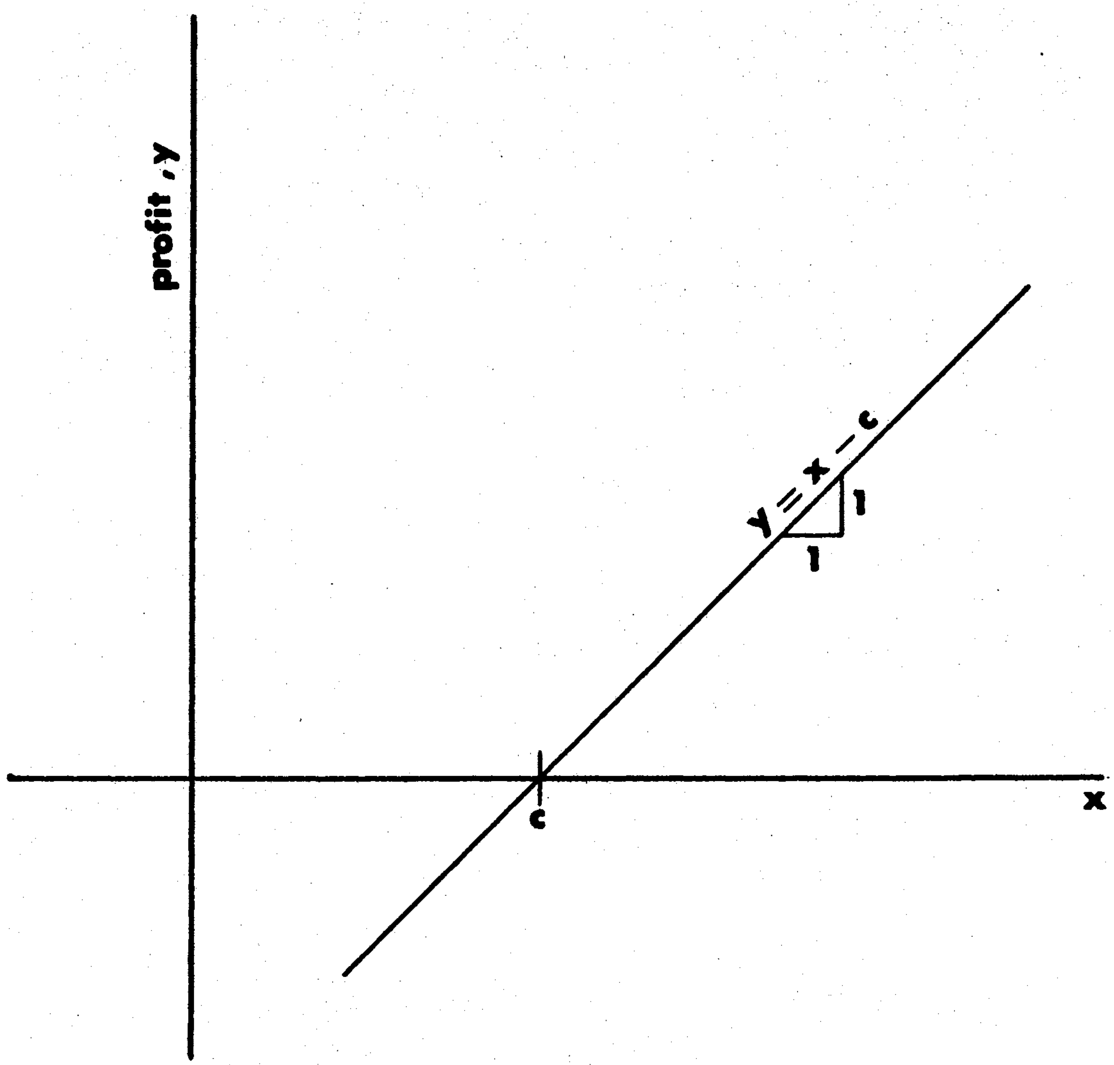


Figure 23. The Common Profit Objective Function

that the decision maker is rational in that he will choose the lottery that will maximize his expected utility (95:116). A "lottery" is completely defined by a set of prizes together with the probability distribution of the occurrences of the different prizes (5:42). Therefore, in order to define the optimal lottery, one must be able to describe a contractor's utility function. The formulation of this utility function depends upon the theory that has been developed based upon six axioms:

> (1) Orderability. It is possible to order preferences for different prizes in such a way that if A is preferred to B and B is preferred to C then A is preferred to C.
>
> (2) Reduction of Compound Lotteries. A rational decision maker is indifferent between a compound lottery and a single stage lottery having the same expected utility.
>
> (3) Continuity. If A is preferred to B and B is preferred to C, then there exists a probability, p, such that the decision maker is indifferent between receiving B with certainty and playing the lottery having the prize A occur with probability p and the prize C with probability (1-p). B is defined as the <u>certainty equivalent</u> of the lottery.
>
> (4) Substitutability. If the decision maker is indifferent between two prizes, then one prize may be substituted for the other without changing the lottery.
>
> (5) Transitivity. Preference and indifference among lotteries satisfy the transitivity property.
>
> (6) Monotonocity. If the prizes of two two-prize lotteries are the same one lottery is preferred to the other only if the probability of getting the more preferred prize is greater (5:43).

The question now becomes "How does one arrive at a contractor's utility function?" Assuming that a contractor is a rational decision maker obeying these six axioms, one can say that his utility function is a mathematical model for the contractor. Axiom number three provides, theoretically, a method of formulating a utility function. To determine the utility function $u_j = u_j(e_j)$ where e_j is the prize associated with the j^{th} lottery, one simply asks the decision maker what probability, u_j, must be associated with e_j in a lottery with prizes consisting of only e_1 and e_m such that he would be indifferent to selecting prize e_j with certainty and playing this lottery with the outcome equal to $u_j e_1 + (1 - u_j) e_m$. The number that he gives becomes the utility associated with e_j. One should note that, since the value assigned u_j comes from a probability, then the functional limits of the utility function will be zero to one inclusive. However, since any linear function of this utility function may also serve as the utility function, the limits are arbitrary.

The theoretical approach to deriving the utility function has little application in competitive bidding. The value of a bid for a construction contract is essentially continuous and the convex combination approach used by Hadley has little application. Contractors think in terms of dollars and the measure of utiles has little appeal. And finally, few contractors could actually state their preferences as a rational decision maker without a sound tool to assist them.

Hadley provides relief from the utilization of utiles as a measure of utility by his "criterion for using expected monetary values" as the utility measure.

> Given a set of lotteries in which the prizes are completely characterized by monetary values, then if the rational decision maker's utility is related to discounted monetary values by a linear equation, monetary values can be used as utilities, and if the decision maker selects the lottery which maximizes the expected discounted profit he will select the lottery which is highest on his preference list (95:123).

The method for constructing the utility function for a contractor is based on a subjective evaluation of the contractor objective as determined from the VT diagram presented previously in this chapter. It is inconceivable that identical utility functions would apply to the four objectives stated in part one of this chapter. Maximizing one's expected profit relates to quite a different utility function than does maximizing one's probability of winning based on self-prescribed constraints or minimizing the probability of winning.

Bidding for a construction contract can be considered as a four stage lottery. The four stages may be enumerated as:

(1) Selecting the project to consider.

(2) The selection of an objective.

(3) Deciding the amount to bid for the contract.

(4) Determination of the actual cost.

The lottery stage governing the selection of a project to consider will not be investigated in depth in this paper and is, therefore, a subject available for further research. This author's initial approach to this investigation would be to consider this lottery as a queueing problem, assuming Poisson arrivals for each job. With this state addition to the decision tree, the objective of the study would be to maximize the rate of return on investment over time. An interesting aspect to be discovered by this addition would be the derivation of a minimum overhead absorbing volume of work to keep on hand.

The second stage of the lottery might be considered as a deterministic stage since information is available from the VT function from which the contractor can select his objective. In a practical sense, it is meaningless to assign probabilities to each branch of the objective selection stage. Although the state space will initially be determined by nature, the deterministic approach is an adequate approximation. For each objective selected, at least one new objective function is necessary.

The third stage of the lottery is the selection of the optimum bid amount based on the objective selected. Rather than the selection of an absolute bid amount, this model is developed to select the optimum mark up,

$$r = x/c \tag{5.7}$$

where x is the bid amount selected and c is the estimated cost of the project. The action required is the selection of r, the state of nature is determined from the probability distributions of winning and the outcome is the expected utility based on the estimated cost.

The fourth stage of the lottery is the determination of the actual cost of the project. Considering the actual cost as a random variable as discussed in Chapter IV, the action is determined from stage three, the state space is determined by the distribution of the cost random variable and the outcome is the true expected utility considering the actual cost. The decision tree representing the three stage lottery is shown in Figure 24.

The Expected Utility Function for the Objective of Maximizing Profit

If one considers the expected profit function used in all previous models, the question arises as to whether or not it represents an adequate utility function for the objective of maximizing expected profit. If one lets $u = y/c$ then

$$u = (r - 1) \tag{5.8}$$

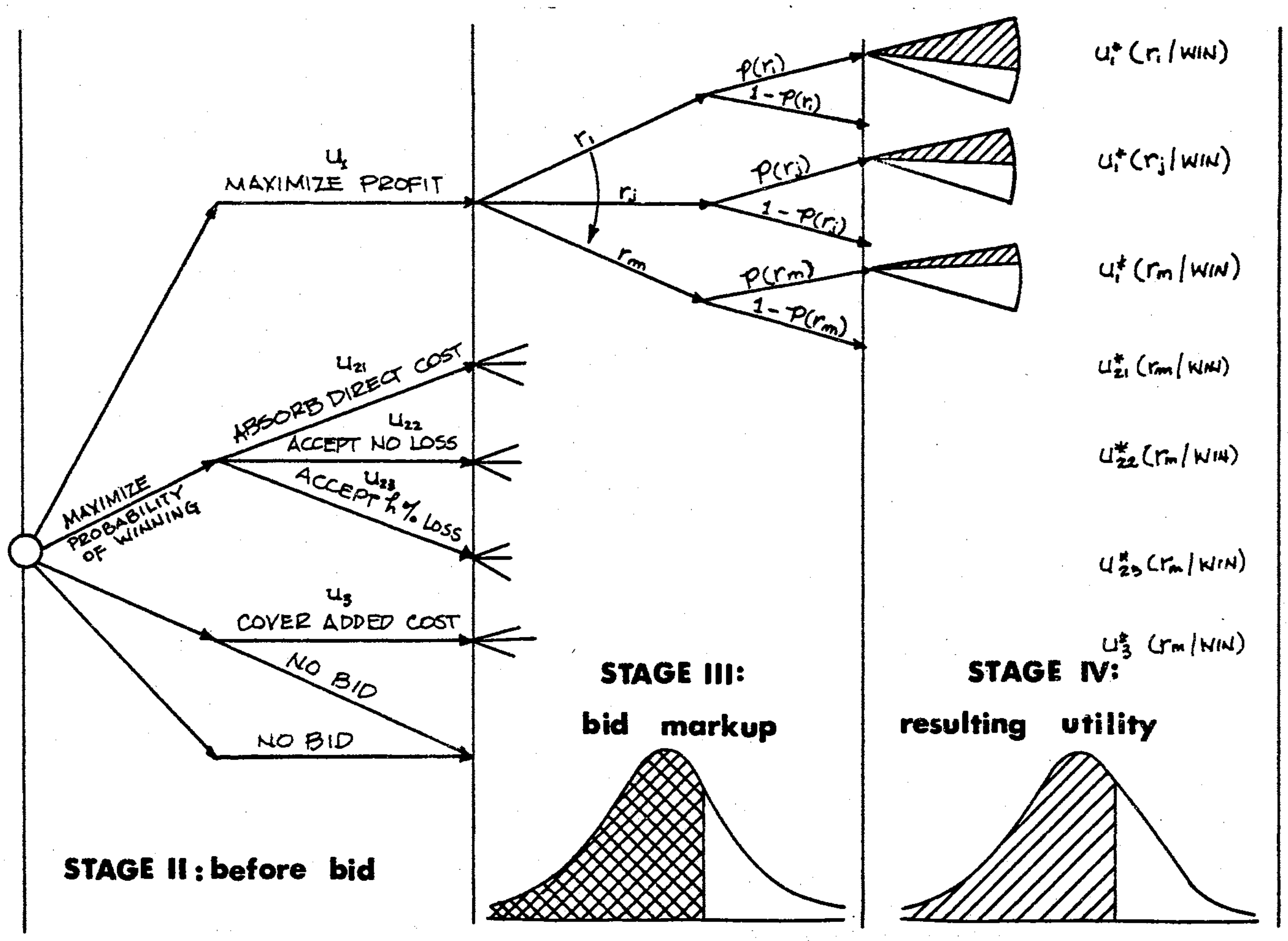


Figure 24. The Three Stage Bidding Lottery

is the equation of the expected profit as a fraction of the estimated cost. The feasible region for the bid markup, in order to maximize profit, is necessarily greater than one.

The state of the art in establishing a true utility function for any one contractor is still in a state of infancy. It is doubtful whether a contractor, much less a researcher questioning a contractor, could adequately define his utility function at any given instance. However, for the objective of maximizing profit, the utility function as given in Figure 25 is intuitively plausible since, in this instance, utility is linear with monetary value.

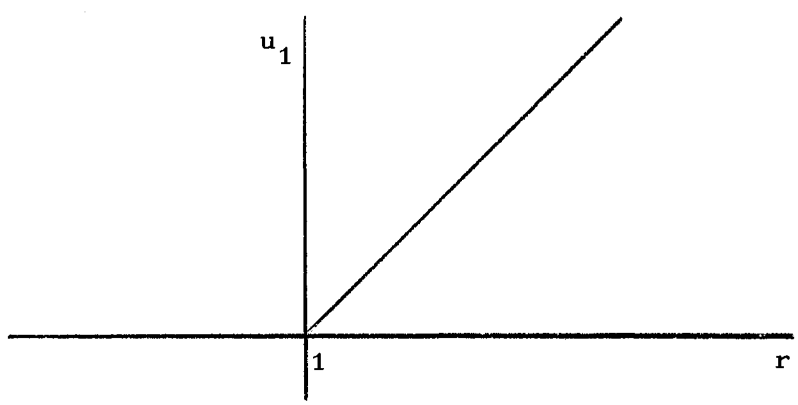


Figure 25. The Expected Profit Function for the Objective of Maximizing Profit

The Expected Utility Function for the Objective of Maximizing the Probability of Winning

If a contractor has decided, based on his VT function, that his objective is to maximize his probability of winning the contract, then as shown in Figure 24 he has three alternatives as defined in this paper. It is necessary to define these alternatives since one could maximize his probability of winning by bidding with a markup equal to that fraction of his estimated cost which from an á priori probability distribution would provide a probability of winning equal to one. However, experience has shown that to win a bid at this markup could result in as much as a 25 per cent loss; a loss which can be absorbed by few contractors for many jobs.

The three alternatives selected for discussion here are:

U_{21} : Bid at a markup so as to accept no loss.

U_{22} : Bid at a markup so as to accept a loss of the general overhead cost (GOH) associated with the project, e.g., the bid will be at the estimated direct cost (DC) of the project.

U_{23} : Bid at a markup so as to accept a certain percentage loss, h, of the estimated cost.

The root of each of the above utility functions is easy to define. However, the shape of the curve is not. The exact nature of the curve, at this stage of development of utility theory, is impossible to define; however, axiom three can possibly give some insight into its characteristics. According to axiom three, in order to establish a utility function for an individual, the following question should be posed to the individual.

Consider two events (in the bidding problem say two markups) r_1 and r_2. Let r_1 represent the root of the utility function, the markup where the utility of winning is zero. Consider r_2 as an arbitrary markup such that $r_2 > r_1$. Since the assumption has been made that the contractor is a rational decision maker, he has a preference of r_2 over r_1.

The question is then posed to the contractor, "At what probability, u_j, given an arbitrary markup, r_j, such that $r_1 < r_j < r_2$, would obtaining r_j with certainty be equivalent to obtaining by chance $u_j r_2 + (1 - u_j) r_1$?" The resulting probability is the utility attached to r_j by the contractor. This must be done for each markup. Consider now Figure 26.

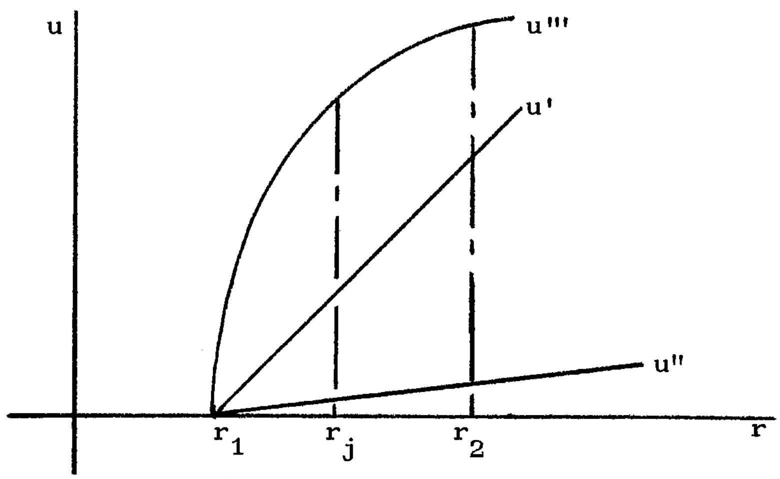


Figure 26. Utility Functions for Discussion

Let $r_j \rightarrow r_1$ from the right. Curve u' implies that the utility associated with r_j varies as the utility function associated with maximizing the expected profit. This in no way indicates that there is more importance attached to winning the bid than to making a large profit, which is the basic assumption here. Curve u" indicates that the utility associated with r_j is always less than that assigned by curve u'. Again, this curve does not show an increased preference to winning the contract. Curve u''', on the other hand, attaches a relatively high value to the utility associated with r_j in the vicinity of r_1, which implies a great importance assigned to winning the bid, the assurance that the optimum markup will remain greater than r_1 and a lesser importance is associated with making a large profit. Thus, a curve with the same general shape as curve u''' satisfies the basic requirements for a utility function describing the objective of maximizing the probability of winning.

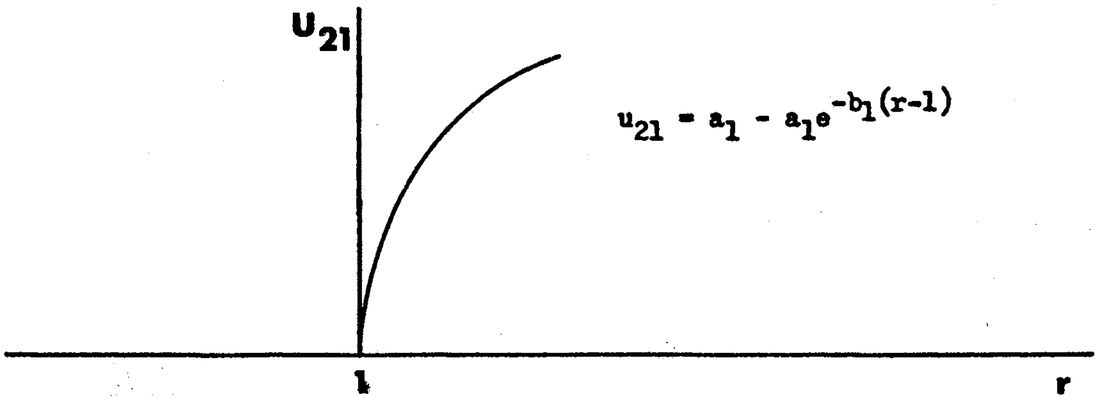
It should be noted that this discussion is purely qualitative and the shape of the utility function is purely conjecture, particularly as regards the degree of the curve. Should it be a straight line or a curve? For the purposes of this paper, the shape of this utility function will be assumed to be exponential in nature with its equation expressed as

$$u(r) = a - ae^{-b(r-k)} \qquad (5.9)$$

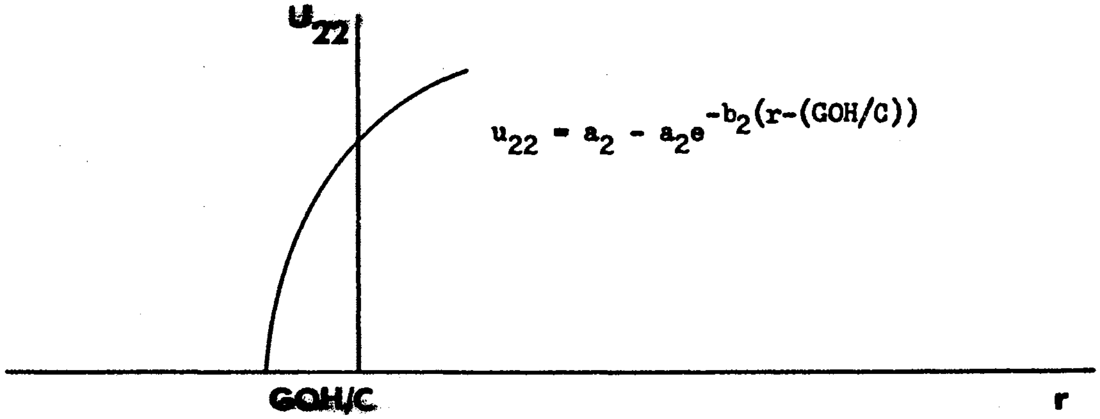
where a and b are parameters defined by the individual and k is the root of the utility function. Therefore, the assumed utility functions for the objective of maximizing the probability of winning are given in Figure 27.

The Expected Utility Function for the Objective of Minimizing the Probability of Winning

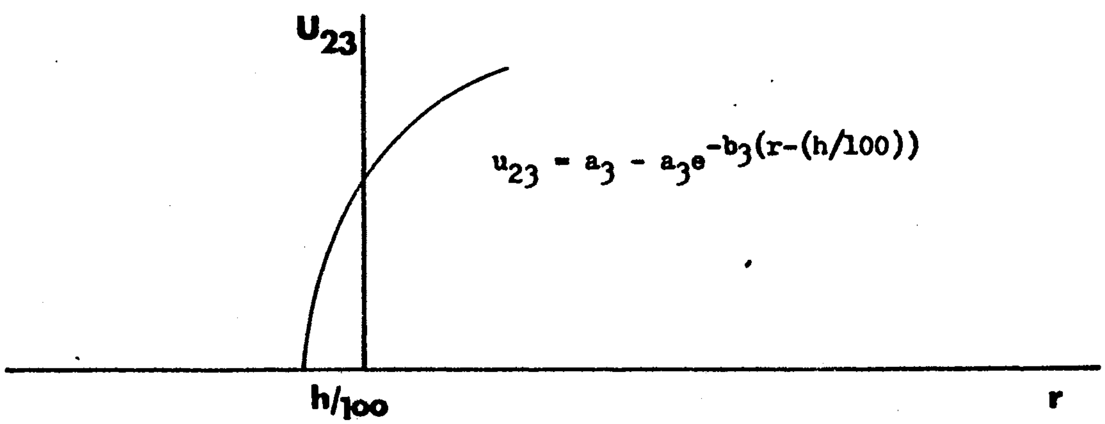
In the previous section, the reasons for selecting this objective were discussed. In the establishment of a utility function, the results of the selection of the objective of minimizing the probability of winning sum to one consequence, that is additional cost will be incurred should the contract be won. These costs may be cost associated with hiring additional crews or buying equipment, additional bond cost or intrinsic cost to cover the risk that a contractor would expose himself to should he be awarded the contract. Therefore, the root of this utility curve is simply the markup, r_i, that will cover the additional cost.



(a) The Utility Function Associated With Accepting No Loss



(b) The Utility Function Associated With Accepting a Loss of GOH



(c) The Utility Function Associated With Accepthing h% Loss

Figure 27. The Utility Functions Associated With the Objective of Maximizing the Probability of Winning

If one considers axiom three to establish the shape of the utility curve, there is little evidence which would indicate a unique characteristic of the curve. The shape of the utility function for the objective of maximizing profit was obviously a straight line with a one-to-one slope of profit versus markup. The utility function associated with the objective of maximizing the probability of winning was deduced to be capable of being represented by an exponential function. The characteristics of this utility function depend entirely on the personal preference of the contractor. With the root established (meaning essentially that the contractor will be paid handsomely should he be awarded the contract) his utility function might be a straight line with a one-to-one slope as the first utility function. On the other hand, even though his additional costs are covered, he still may have reservations about taking the job. Therefore, the utility function for this objective will be assumed linear with an undetermined slope which may have any value set by the contractor. The equation is given by

$$u_3(r) = m_1(r - c_1) \quad . \qquad (5.10)$$

where m_1 is the undetermined slope and c_1 is the additional cost decided upon by the contractor divided by his estimated cost. This curve is shown in Figure 28.

The Probability of Winning the Contract

In all previous static models, the probability of winning over any other contractor has been assumed to be a function of the markup, r, alone and the probability of winning over n competitors has been considered as a function of r and n. However, if one considers that each

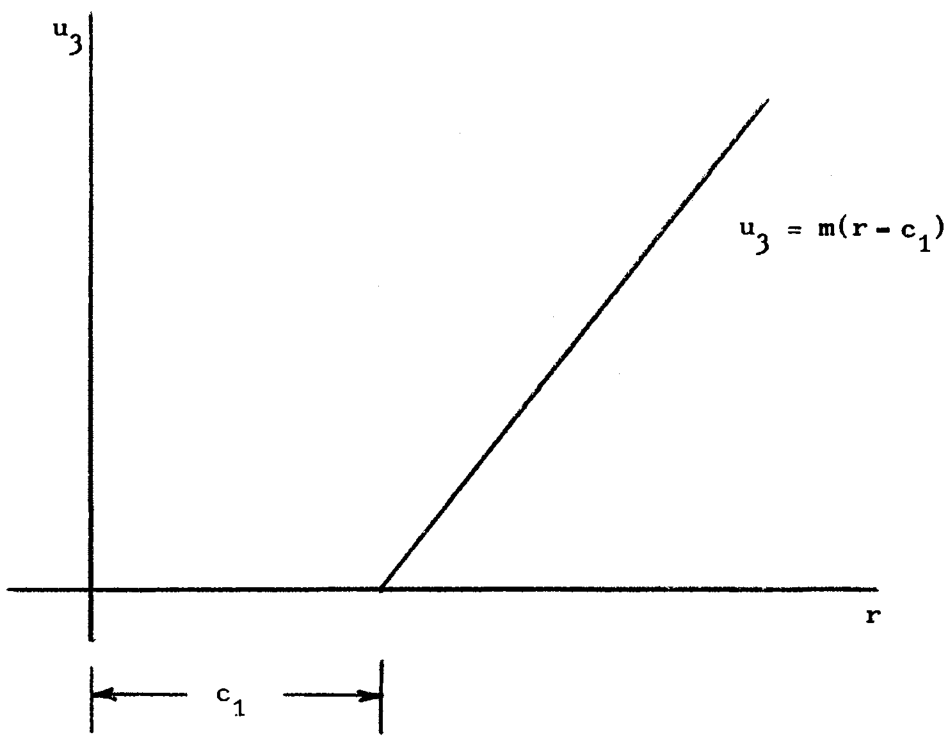


Figure 28. The Utility Function Associated With the Objective of Minimizing the Probability of Winning

individual contractor's business trends can be examined by a VT function as shown in Figure 20, it is readily apparent that the probability of winning can be considered as a function of n, r, and t.

This discussion will first treat the case of winning over a single competitor, extend it to the case of winning over n bidders, then extend it to the case where some, but not all, competitors are known.

The Probability of Winning Over One Known Competitor

The underlying assumption in this discussion is that the following data has been maintained on one competitor, say competitor k.

(1) The amount of money in dollars that competitor k has bid for each contract for which both competitor k and "our" contractor has bid.

(2) The cost estimate, in dollars, that "our" contractor has made for each job.

(3) "Our" contractor's estimated duration for each job.

(4) "Our" contractor subscribes to one of the many contractor information reports which give as a minimum the following information:

a - Contracts to be let in the near future.

b - The names of contractors expected to bid on a specific contract.

c - The results of the bid letting, e.g., the name of the contractor to which the contract has been awarded and the amount of the winning bid.

There are numerous national and local publications of this type, such as the "Dodge Reports" (105), "Southwest Construction News Report" (106), and others.

The purpose of elements (1) and (2) are to develop a static probability density function (P.D.F.) to be used in determining the probability that competitor k will bid at a markup as related to "our" estimated cost greater than some r_o. Since it is as easy to develop a complementary cumulative relative frequency histogram or assume a complementary cumulative probability density function (C.C.D.F.) using high speed data processing machines as to develop a P.D.F., the procedures in this paper will use the C.C.D.F. directly. The purpose of elements (1), (2), and (4) are to develop a relative VT function for competitor k.

Let the random variable R_k be defined as the random variable representing X_k/c, where X_k is a random variable representing the bid of competitor k, c is "our" cost estimate (considered here as a deterministic real number) and R_k is related to the set of real numbers $r \geq 0$ by the function

$$P[R_k \geq r] = F_{R_k}(r) \ . \tag{5.11}$$

The function, $F_{R_k}(r)$, to prevent confusion as this development continues, may be represented by a discrete complementary cumulative relative frequency histogram, or by an assumed or derived C.C.D.F.

$$F_{R_k}(r) = 1 - \int_o^r f_{R_k}(r)dr = \int_r^\infty f_{R_k}(r)dr \ . \tag{5.12}$$

As the number of data points for $F_{R_k}(r)$ become infinite, the function will have a characteristic shape as shown in Figure 29. It should be noted that as data are collected in time, the effect of individual data points tend to affect this function less and less as the number of points become large. Based on this fact, this author considers the

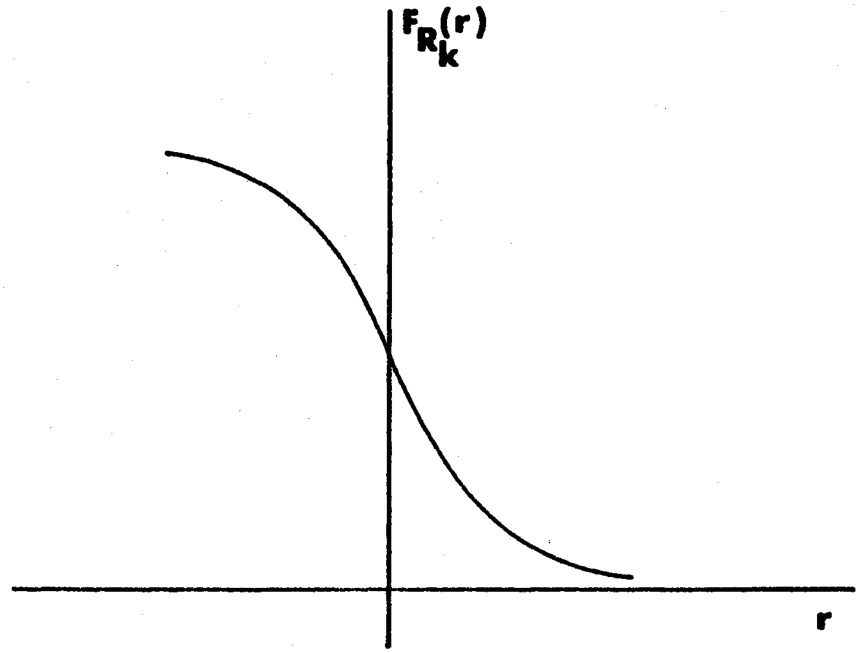


Figure 29. The Probability of Winning Over Competitor k

function to be a static function and is essentially the same function as developed in previous models. The purpose now is to transform $F_{R_k}(r)$ into a non-static model.

The basic assertion to transform $F_{R_k}(r)$ into a dynamic model is that, rather than being a function of r alone, $F_{R_k}(r)$ is indirectly a function of time. More specifically, $F_{R_k}(r)$, is a function of the business situation of competitor k as related to his VT function. Graphically, if one assumes a non-static situation, the function $F_{R_k}(r)$ is a surface varying with time such that for a specific time, t_1, and a markup, r_2, the probability that $R_k \geq r$ is the ordinate to the surface $F_{R_k}(r, V(t))$ where V(t) is a parameter, depending on time. Qualitatively, this function is described by Figure 30. The problem now becomes, how does $F_{R_k}(r)$ depend on V(t).

The model to be developed here consists of a number of assertions. It begins with a known marginal distribution function, adds an experiment which provides another variable about which no known or assumed marginal distribution exists and provides a practical solution for finding the conditional probability distribution function. The method for finding the expectation and variance of the conditional distribution is based on sound statistical assumptions. The assertion about the actual form of the conditional distribution, which is asserted to be the same as the marginal distribution function, is a conjecture which seems reasonable to this author.

Consider the three stage lottery as shown in Figure 24. After the objective and utility function have been selected, in Stage II, one refers to the marginal C.C.D.F., in this case $F_{R_k}(r)$, to determine the probability of winning at various markups. If some methods exists to

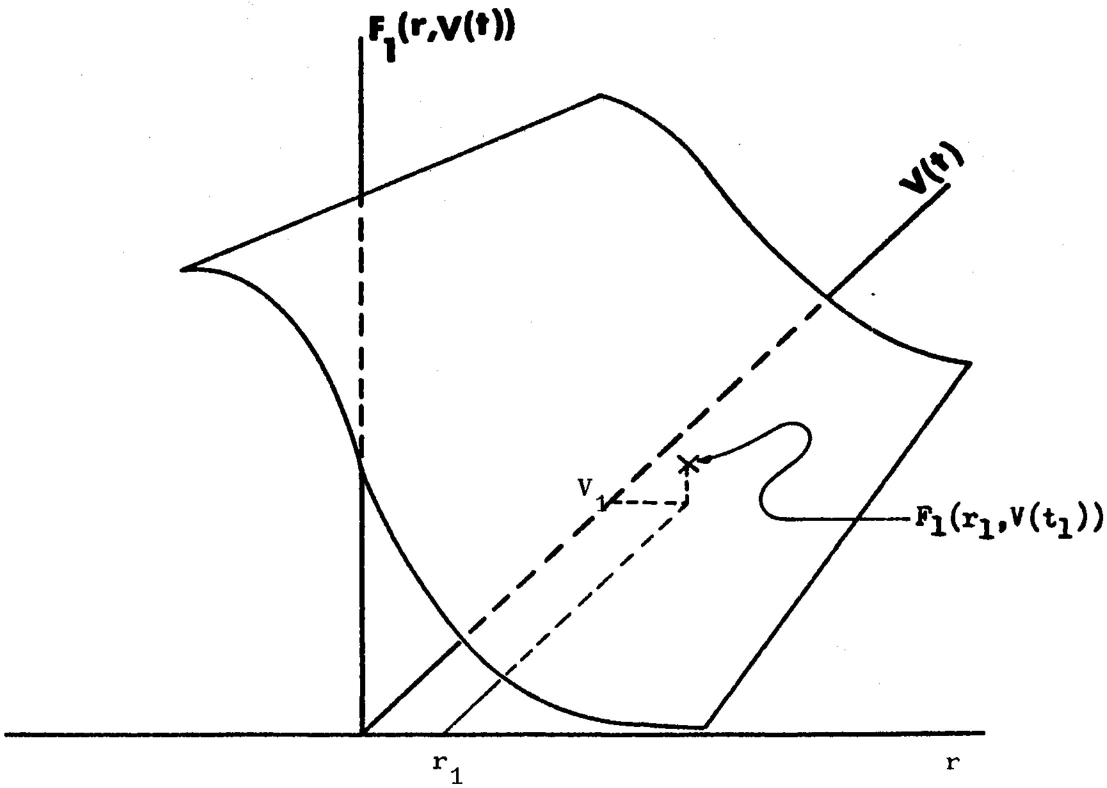


Figure 30. The Probability of Winning Over Competitor k as a Function of r and V(t)

obtain further information about $F_{R_k}(r)$, Stage III of the lottery will have an additional stage added to it. This stage consists of an experiment, in the purest sense of the definition, in which additional information is obtained about competitor k. Let the possible outcomes of this experiment be the set V_k such that

$$V_k = \{v_1, v_2, \ldots, v_m\}$$

which are functions of competitor k's business situation as determined from his relative VT function. The practical fact that contracts are bid in dollars and cents requires, from a theoretical viewpoint, that V_k be discontinuous, consisting of a countably infinite set of elements. The decision tree for Stage III will be modified as shown in Figure 31.

It was assumed at the beginning of this section that sufficient data have been maintained on competitor k to permit the evaluation of a relative VT function. Since the results of all public contracts must be available to the public by law, since there exist publications which announce the results of both public and private contracts when possible, and since most private project sponsors will release bid results to bidders as a courtesy, a VT function can be maintained on competitor k if "our" contractor is willing to expend the necessary effort and resources. This VT function will be relative in the sense that only competitive work will be included. As mentioned earlier, the sound contractor will maintain a volume of negotiated work in addition to competitive work. This negotiated work will not appear on his VT function. However, this will not affect the usefulness of the VT function since the resulting function will simply have its ordinate translated upward approaching the average volume of negotiated work.

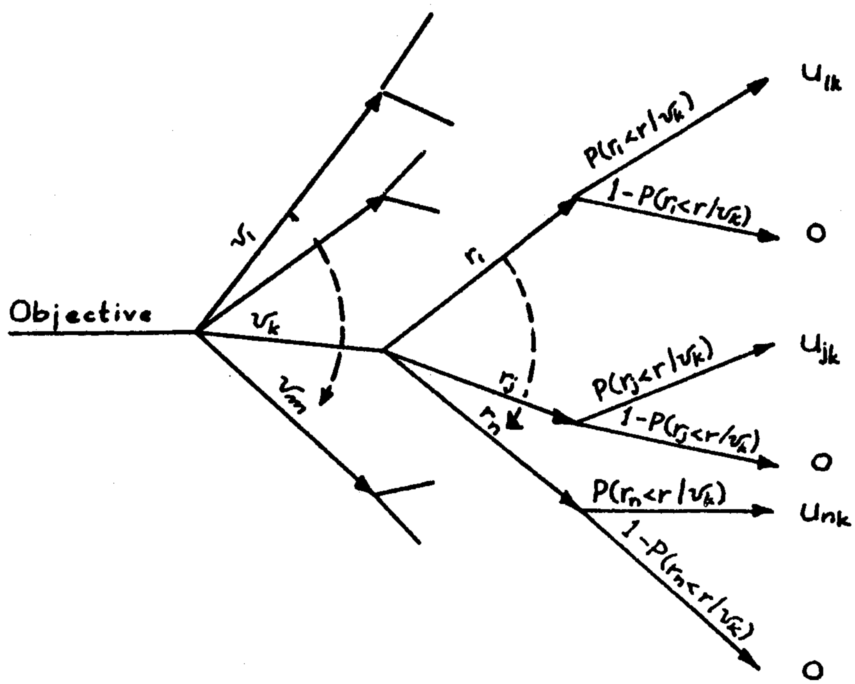


Figure 31. Stage III of the Bidding Process With an Experiment Added

That is, Figure 20, rather than beginning at an ordinate base of V = 0, will now have an ordinate base at V equal to some V_o.

To construct a model for updating F_{R_k} using the additional information obtained from the added experiment, assume that the value of the random variable R_k can be stated as a function of the random variable V_k, say

$$R_k = g(V_k) \tag{5.13a}$$

as shown in Figure 32. If this relation holds, then

$$F_{R_k}(r) = F_{R_k}(g(V_k)) \tag{5.13b}$$

and once a value, say v_1, is known for V_k, then $r_1 = g(v_1)$ and consequently $F_{R_k}(g(v_1))$, are uniquely determined. At this point, it is asserted that a curve can be calculated relating R_k to V_k and this curve will be a monotonic increasing curve. However, it is obvious to anyone familiar with the competitive bidding problem that factors other than competitor k's volume on hand affect the manner in which competitor k will bid. In this model, these factors will be considered as random influences since they cannot be enumerated. Therefore, the basic relationship between R_k and V_k must be stated as

$$R_k = g(V_k) + \epsilon \tag{5.14}$$

where ϵ is the error associated with the other random influences affecting competitor k's bidding pattern.

The next step in the development of this model is to establish a procedure by which the probability of winning, $F_{R_k}(r)$, can be updated using the additional information provided by the experiment V_k. The

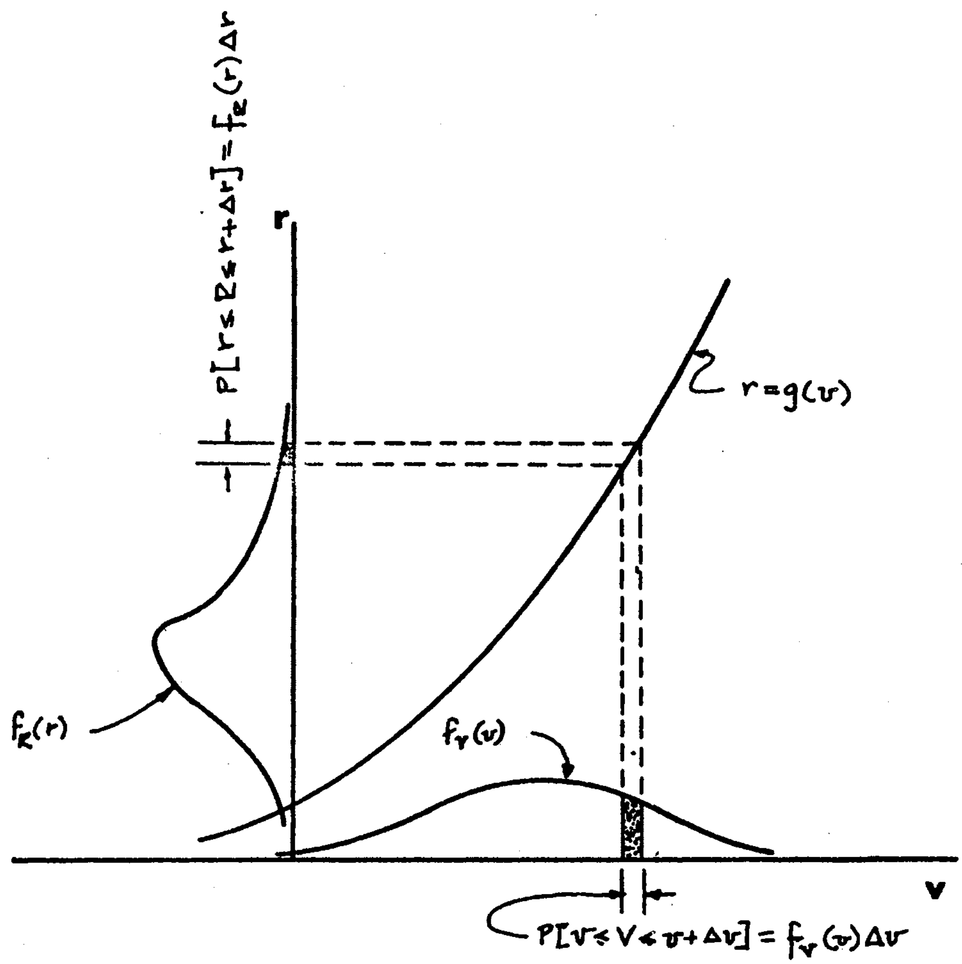


Figure 32. Complementary Cumulative Frequency Histogram for Competitor k

á priori distribution, $F_{R_k}(r)$, and its P.D.F., $f_{R_k}(r)$, are the marginal probability distributions of R_k over all V_k. The marginal probability distribution function for V_k is unknown as is the joint probability distribution of the two variables. Hence, a conditional C.C.D.F., $F_{R_k|V_k}(r)$, cannot be found by traditional methods.

To find $F_{R_k|V_k}(r)$, one must be able to calculate $g(V_k)$. It has been assumed that the marginal C.C.D.F., $F_{R_k}(r)$ is known. During the bidding process over time, matched data pairs, $\{(r_1, v_1), (r_2, v_2), \ldots, (r_n, v_n)\}$ can be observed by simply noting the value of V_k at the time of each bid and associating it with the value of competitor k's markup, R_k, at that time. The function $g(V_k)$ can then be calculated by the method of Least Squares. Once the function has been calculated, parameters concerning the conditional C.C.D.F. can then be calculated.

Least Squares theory, in its basic form, requires three assumptions:

(1) The values of the independent variable are fixed;

(2) The expectation of the error term in the least squares formulation be zero;

(3) The variance of $(R/V) = \sigma^2$ = constant.

Ideally, both variables, R_k and V_k, should be normally distributed in order to use the method of Least Squares for calculating $g(V_k)$. However, in this model, this is not the case and only the marginal distribution for R_k is known. Assumption (1) and (2) can reasonably be applied to this model and will, therefore, be considered assumptions in this model. However, assumption (3) requires further consideration. There is no evidence available to preclude the homoscedasticity assumption providing the variance of the conditional probability density function is less than the marginal variance. Huang (98:41) states that the best linear unbiased estimator of the variance of the error term in Equation (5.14) is

$$\sigma^2 = \frac{1}{n-2} \sum_{i=1}^{n} [r_i - g(v_i)]^2 \quad . \tag{5.15}$$

Therefore, the model for updating the marginal distribution proposed by this treatise is

$$F_{R_k|V_k}(r) = F_{R_k}(r) \tag{5.16}$$

where

$$E(r/v_k) = g(v_k) \tag{5.17}$$

and

$$\sigma^2(r/v_k) = \frac{1}{n-2} \sum_{i=1}^{n} [r_i - g(v_k)]^2 \; . \tag{5.18}$$

Thus, this model translates the mean of the marginal distribution of R_k over all V_k to the value predicted by the regression equation and contracts the distribution by the calculated variance. It implies that the conditional distribution retains the shape of the marginal distribution. It should be noted that this is an assertion, practical in nature, but has no theoretical basis. It does provide a workable method for updating the marginal C.C.D.F.

In the example that follows, the marginal C.C.D.F., $F_{R_k}(r)$, is a discrete complementary cumulative relative frequency histogram developed from historical data. The relation between the volume of work on hand for competitor k, v_k, and his markup, r_k, is found to be linear and given by an equation of the form

$$r = \alpha v + \beta \quad .$$

Therefore, the expected value of the conditional C.C.D.F. is

$$E(r|v) = \alpha v + \beta$$

and the conditional variance is given by the equation

$$\sigma^2_{r|v} = \frac{1}{n-2} \sum_{i=1}^{n} [r_i - (\alpha v_i + \beta)]^2$$

where n is the number of data pairs of r and v collected. To find the conditional C.C.D.F., the abscissa is first scaled by the ratio $\sigma_{r|v} \div \sigma_r$ then the mean of the marginal C.C.D.F. is translated to the mean of the conditional C.C.D.F., e.g., if r' is the calculated variate of the conditional C.C.D.F. and r is the original variate, then r' is given by

$$r' = \frac{\sigma_{r|v}}{\sigma_r} (r) - \left[\frac{\sigma_{r|v}}{\sigma_r} E(r) - E(r|v)\right] \quad . \tag{5.19}$$

For a discrete complementary cumulative relative frequency histogram with equally spaced intervals and letting $\Delta = r_{i+1} - r_i$, Equation (5.19) can be calculated by the recursion relation

$$r'_{i+1} = \left(\frac{\sigma_{r|v}}{\sigma_r}\right) \Delta + r'_i \quad . \tag{5.19a}$$

The data presented in Table IV and computed in Table V have been developed for competitor k over a three-year period. The values of r were taken from bid tabulations and the VT function has been derived

TABLE IV

DEVELOPMENT TABLE FOR V_k VERSUS t, r_k VERSUS t, AND r_k VERSUS V_k

DATE	BID	"our" EST	r_k	VOLUME ON HAND
02 23 68	258407	235817	1.0957	200000
04 09 68	12251	9867	1.2416	390000
04 24 68	198911	153700	1.2941	350000
07 02 68	225010	219500	1.0250	198000
09 27 68	4601	4129	1.1143	370000
10 31 68	245120	226225	1.0821	290000
02 05 69	194000	172127	1.1270	310000
05 08 69	41790	36491	1.1452	260000
05 15 69	266000	239944	1.1085	240000
05 22 69	166416	136239	1.2215	480000
08 26 69	340125	325418	1.0450	170000
10 27 69	89840	73003	1.2306	380000
09 20 70	116551	93620	1.2447	410000

TABLE V

DEVELOPMENT TABLE FOR COMPETITOR k COMPLEMENTARY CUMULATIVE RELATIVE FREQUENCY HISTOGRAM

r	$\Sigma r_k > r$	$\Sigma f/n > r$
1.00	13	1.000
1.02	13	1.000
1.04	12	.923
1.06	11	.846
1.08	11	.846
1.10	9	.692
1.12	7	.538
1.14	5	.385
1.16	4	.308
1.18	4	.308
1.20	4	.308
1.22	3	.231
1.24	3	.231
1.26	1	.077
1.28	1	.077
1.30	0	.000

from both bid tabulations and publications listed in assumption four. Figure 33 graphically illustrates competitor k's VT function superimposed on a time graph of his markup as related to "our" estimated cost. A least squares fit was used in finding r as a function of v. A linear relationship fit was found adequate.

By arbitrarily omitting one extreme point, a coefficient of determination of 0.74 and a correlation coefficient of 0.86 were found to exist. Figure 34 shows a graph of the points of r versus v which gives a visual indication of the dependence of r on v. Thus, the relationship for competitor k markup and volume is determined to be

$$R = (6.75 \times 10^{-4})V + 0.9320 \quad . \tag{5.20}$$

From Table V, the C.C.D.F. for competitor k has been developed and is shown in Figure 35. From Equations (5.16) and (5.19) the function $F_{R_k|V_k}(r)$ can be found. Given competitor k's volume, v_1, as taken from his VT function at any time, t, then the conditional expected value of r, $E(r|v_1)$, is calculated by Equation (5.17).

$$E(r|v_1) = (6.75 \times 10^{-4})v_1 + 0.932 \ .$$

The value of the conditional variance, $\sigma^2_{r|v}$, is given by Equation (5.18).

$$\sigma^2_{r|v} = \frac{1}{10} \sum_{i=1}^{10} \left\{ r_i - \left[(6.75 \times 10^{-4})v_i + 0.932 \right] \right\}^2 \quad .$$

The conditional C.C.D.F. for competitor k can now be given from Equation (5.16) as

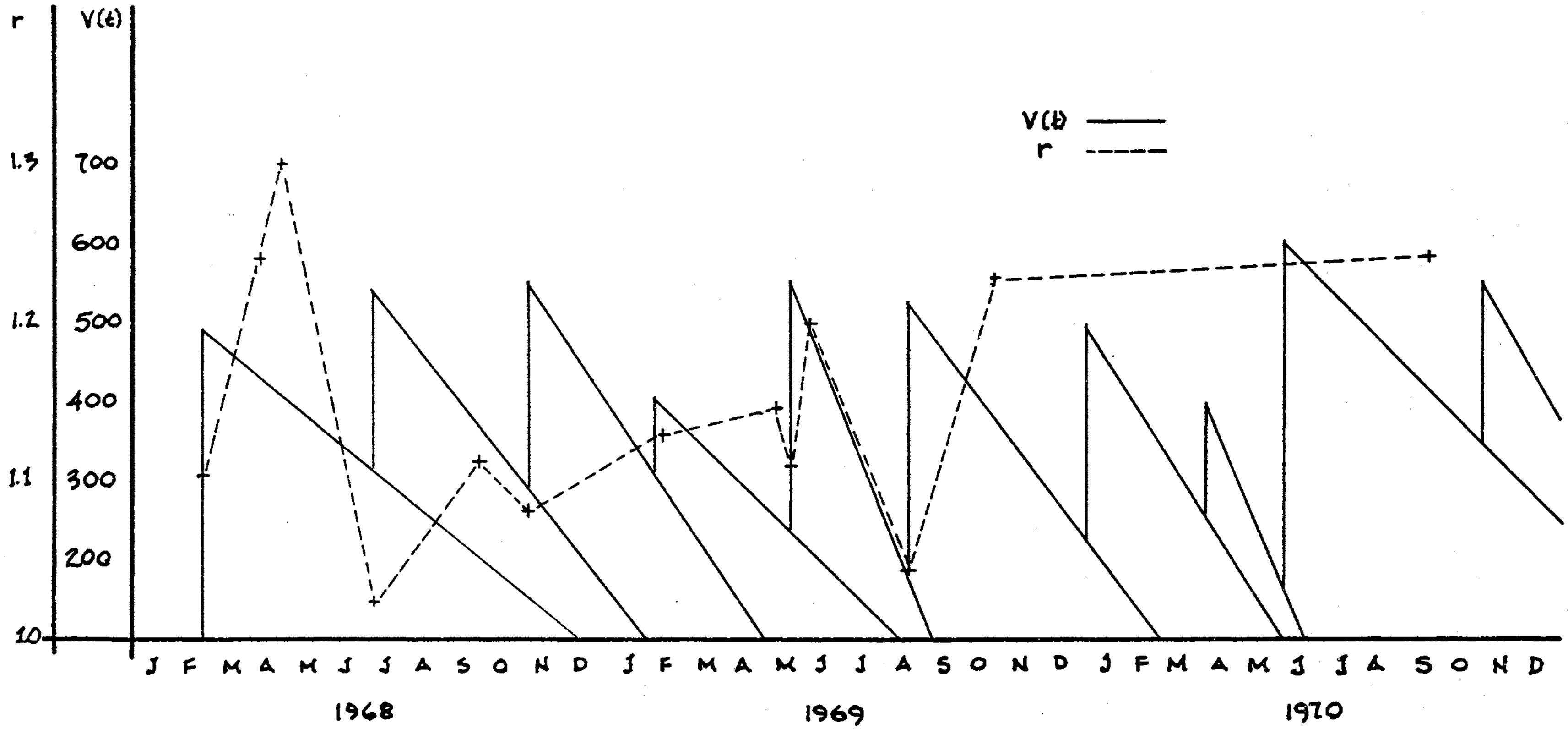


Figure 33. Volume Versus Time and r = X/C Versus Time for Competitor k

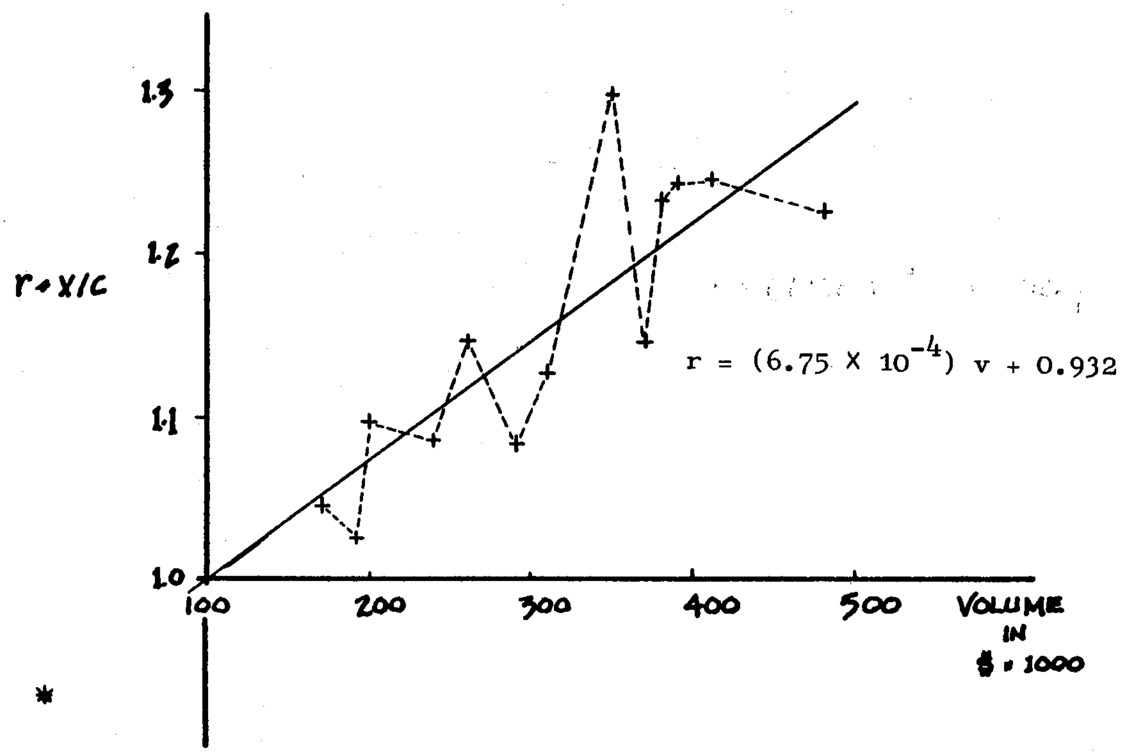


Figure 34. Bid Ratio of Competitor k Versus His Volume

$$F_{R_k|V_k}(r) = F_{R_k}(r')$$

where r' is given by Equation (5.19).

In this example, $E(r) = 1.1565$ and $\sigma_r^2 = 0.0056$. Suppose that at time, t_o, competitor k has a volume of work on hand equal to $400,000, $\sigma^2_{R|v} = 0.00168$ by Equation (5.18) and $E(r|400) = 1.202$. Therefore, r' from Equation (5.19)

$$r' = 0.5466\ r + 0.5699$$

and

$$F_{R_k|V_k}(r) = F_{R_k}(0.5466\ r + 0.5699) \quad .$$

The marginal C.C.D.F. for competitor k is shown in Figure 35a and the calculated conditional C.C.D.F.'s for V_k = $200,000, $300,000, and $400,000 are shown in Figures 35b, 35c, and 35d, respectively.

Thus, knowing competitor k's volume at the time of a bid letting provides a considerably "tighter" and more accurate probability distribution for winning over competitor k than does the original marginal C.C.D.F.

This example was taken from a competitor against whom "our" contractor has bid only 13 times in three years. This small amount of data is not at all uncommon in the construction industry. Later in this chapter a curve fitting procedure using an asymptotic distribution will be described in an effort to obtain more meaningful distributions.

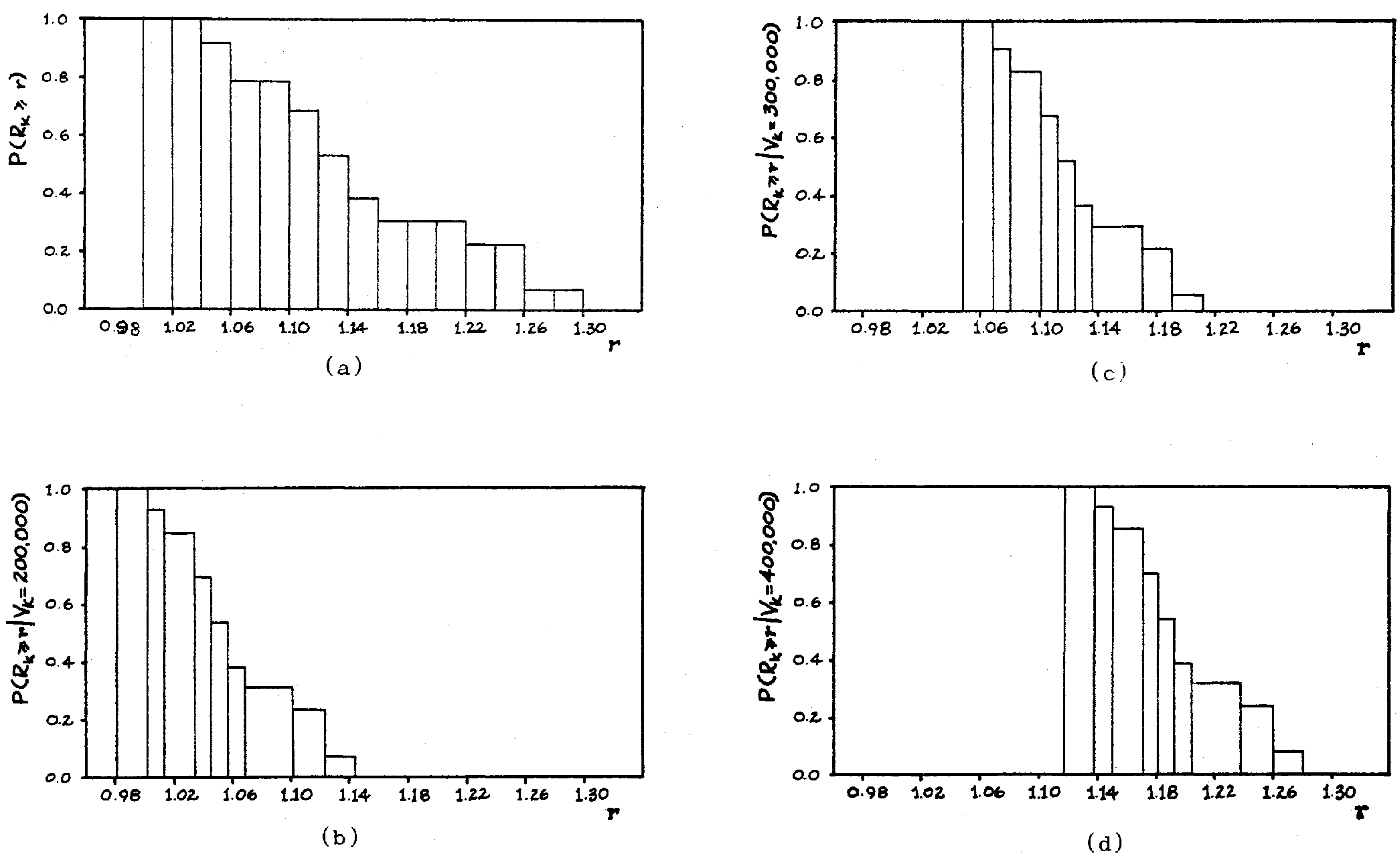


Figure 35. The Marginal and Conditional C.C.D.F. for Competitor k

It should be noted in the case of this particular contractor, "our" contractor in this study bid against 81 different contractors over a three-year period, meeting one particular competitor a maximum of 17 different times. He bid against only three specific competitors out of 81 at ten bid lettings or more.

The Probability of Winning Over n Competitors

Numerous methods have been proposed to develop the probability of winning over n competitors, both when n is known or n is unknown. Each of these methods have been discussed in Chapter II. If all competitors are known <u>and</u> sufficient data is available on each such that probability distribution functions can be developed for every competitor, then the results can be combined and the probability of winning can be expressed by:

$$p(R < r) = \frac{1}{\sum_{i=1}^{n} \frac{1}{\int_{r}^{\infty} f_i(r)dr} - (n - 1)} \qquad (5.21)^*$$

where f_i is the P.D.F. developed for the i^{th} bidder and n is the number of bidders. This method is seldom, if ever, possible. As mentioned

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*This is Gates' formula as derived in Chapter III by Benjamin.

previously, a small building contractor bid against less than four per cent of his competitors in excess of ten times over the course of three years. Johnson (39) found that out of 136 contractors bidding for 286 contracts let by the State Highway Department of Oklahoma, only five per cent of the contractors bid against the same competitor in excess of 11 times. Therefore, unfortunately, this straightforward method cannot receive universal utilization by a researcher. However, rarely in real world situations will idealized mathematical models apply. As a consolation to the reader, the science of operations research, rather than handing a decision maker a cut-and-dried solution to a problem, attempts to provide him with additional information upon which to make his decision. The situation in which a construction contractor usually finds himself is thus: he knows who and how many bidders will bid for a specific contract, and he has a sufficient data record on several key competitors. Therefore, the remainder of this chapter will develop a system from which an envelope of curves can be presented to the contractor which will provide a range in which the optimum bid will be. The decision maker then has a choice, based upon his own information, feelings, intuition or hunches, as to his bid markup.

Since the development of recent publications as mentioned in assumption four eliminates the number n as a random variable, the first envelope of curves can be developed simply by considering the distribution of the average bidder (AD) and the low bidder (LD) in terms of the ratios of bids to "our" cost estimate for all competitors and for all low bidders respectively. The equation for the probability of winning over the average bidder is given by:

$$p(\text{win}|\text{AD}) = \frac{1}{\sum_{i=1}^{n} \frac{1}{\int_r^\infty f_{AD}(r)dr} - (n - 1)}$$

$$= \frac{1}{\frac{n}{\int_r^\infty f_{AD}(r)dr} - (n - 1)} \tag{5.22}$$

$$\frac{\int_r^\infty f_{AD}(r)dr}{n - (n - 1) \int_r^\infty f_{AD}(r)dr}$$

and the equation for winning over the lowest bidder is given by:

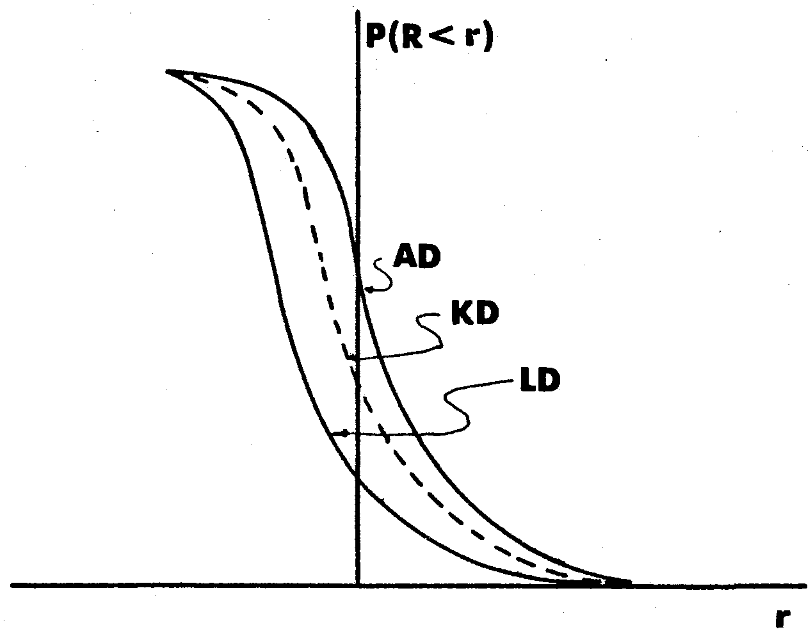
$$P(\text{win}|\text{LD}) = \int_r^\infty f_{LD}(r)dr \quad . \tag{5.23}$$

It is evident that the probability of winning over the AD is a function of n as well as r. Normally, the LD distribution will give the more conservative bid where as the AD will give the higher optimum bid given a small n. To complete the family of curves, the probability of winning, using the conditional distributions for the key bidders (KD) is combined with the AD to give the formula

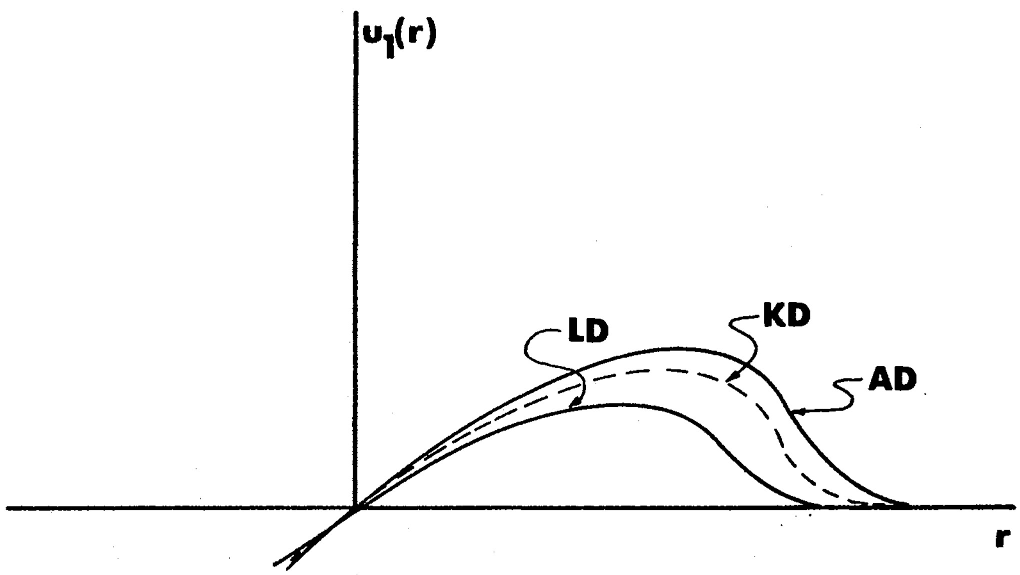
$$P(\text{win}|\text{AD and KD}) = \frac{1}{\frac{m}{\int_r^\infty f_{AD}(r)dr} + \sum_{i=1}^{n-m} \frac{1}{F_{R_i}|V_i} - (n - 1)} \tag{5.24}$$

where m is the number of bidders excluding the key bidders. The family of curves to be developed, idealistically, will be as shown in Figure 36.

Prior to further development of this discussion using actual data, there are integrals in Equations (5.22) through (5.24) which, although valid expressions, cause difficulty in application. Since it is almost as easy to develop complementary cumulative frequency histograms as it is to develop relative frequency histograms on high speed computers, and the expressions f(r) must either be handled discretely, assumed or integrated numerically, a more feasible approach is proposed in the next section of this chapter. That is to provide an analytic expression for the complementary cumulative frequency distributions directly. The purpose of this is threefold. First, with an analytic expression to replace the integrals, formulas (5.22) through (5.24) are easier to handle. Secondly, the variable r, which will be the independent variable in the optimization procedures, is a continuous variable and can assume any value greater than zero; e.g., the increments of markup may be as small as one hundredth of one per cent. Finally, although there generally exists a unique global optimum for the objective function, actual data when treated discretely, will result in small, local peaks and valleys when calculated. Thus, unimodality of the objective function cannot be assured for local conditions. Hence, no efficient search techniques can be used since all search techniques, with the exception of inefficient exhaustive or random search, require that the function be unimodal.



(a) The Probability of Winning Over the LD, KD, and AD Bidders



(b) The Expected Utility for the LD, KD, and AD Bidders

Figure 36. The Probability of Winning and the Expected Utility for the LD, KD, and AD Bidders

A Curve-Fitting Procedure; The Griffis-Weibull Method

According to Waloddi Weibull (102:293) any distribution function such that $P(X \leq x) = F(x)$, where X is a random variable, can be expressed by the function

$$F(x) = 1 - e^{-\Phi(x)} \tag{5.25}$$

where $\Phi(x)$ is a positive, nondecreasing function. Since for the competitive bidding problem, the random variable R is always expressed as $P(R \geq r)$, then, mathematically, the function $F(r)$ can be expressed by:

$$F(r) = 1 - (1 - e^{-\Phi(r)}) = e^{-\Phi(r)} \quad . \tag{5.26}$$

The problem, of course, is in finding the required function $\Phi(r)$. Rothkope (67) proposes in his discussion of bidding with symmetrical information that the function $F(r)$ can be represented by:

$$F(r) = e^{-a(r)^m} \quad . \tag{5.27}$$

This author has developed a method of fitting a curve to this type of Weibull distribution. This procedure is to assume that the probability of winning over any competitor, the average bidder or the low bidder can be approximated by the function

$$P(r) = e^{-a(r - r_o)^m} \tag{5.28}$$

where a and m are the Weibull parameters, as yet unknown, and r_o is the maximum value of r, such that the probability of winning is equal to one.

The mean and variance of this distribution function are of little value in this analysis but can be expressed as rather complex functions of Gamma functions.

Given that there exist data points (p_i, r_i) sufficient to develop a complementary cumulative relative frequency histogram, the basic approach to fitting this data with Equation (5.25) is the same as the least squares method.

The first step in this development is to take the natural logarithm of Equation (5.28) in an attempt to linearize the function. This results in

$$\log P(r) = \log P = - a(r - r_o)^m \quad . \tag{5.29}$$

Attempting to minimize the squared deviations of this log function will still result in a messy polynomial of degree m, therefore, it would be desirable to take the natural logarithm again, thus linearizing the partial derivatives. However, since the function P(r) is actually a probability and its value lies between zero and one, the log log P(r) does not exist. Realizing this approach is infeasible, an alternative approach must be used.

The function to be minimized is

$$h(a, m) = \sum_{i=1}^{n} [\log P_i + a(r_i - r_o)^m]^2 \quad . \tag{5.30}$$

This function is differentiable with respect to both a and m. However, differentiating with respect to m will be of little value since the expression will still contain an m degree polynomial in r as mentioned previously and for which a general solution procedure in closed form does not exist. Hence, the function is differentiated with respect to a, considering m as a varying parameter. A Fibonacci search is used to find an m such that Equation (5.30) is minimized. Thus, one necessary condition for a minimum with respect to the plane is

$$\frac{\partial h}{\partial a} = \sum_{i=1}^{n} (2)(r_i - r_o)^m [\log P_i + a(r_i - r_o)^m] = 0 \quad . \tag{5.31}$$

Solving for a gives the expression

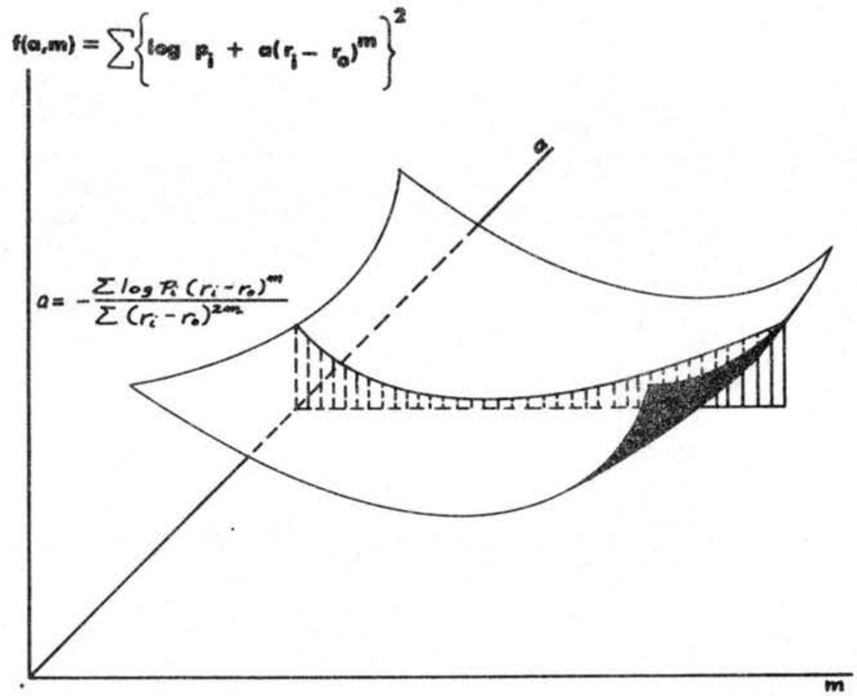
$$a = \frac{-\sum_{i=1}^{n} \log P_i (r_i - r_o)^m}{\sum_{i=1}^{n} (r_i - r_o)^{2m}} \quad . \tag{5.32}$$

Essentially Equation (5.32) defines a plane perpendicular to the a axis upon which the minimum value for Equation (5.30) must lie. This is illustrated in Figure 37a. To use a single variable Fibonacci search, the expression for the variable a as given by Equation (5.32) is substituted into Equation (5.30) resulting in

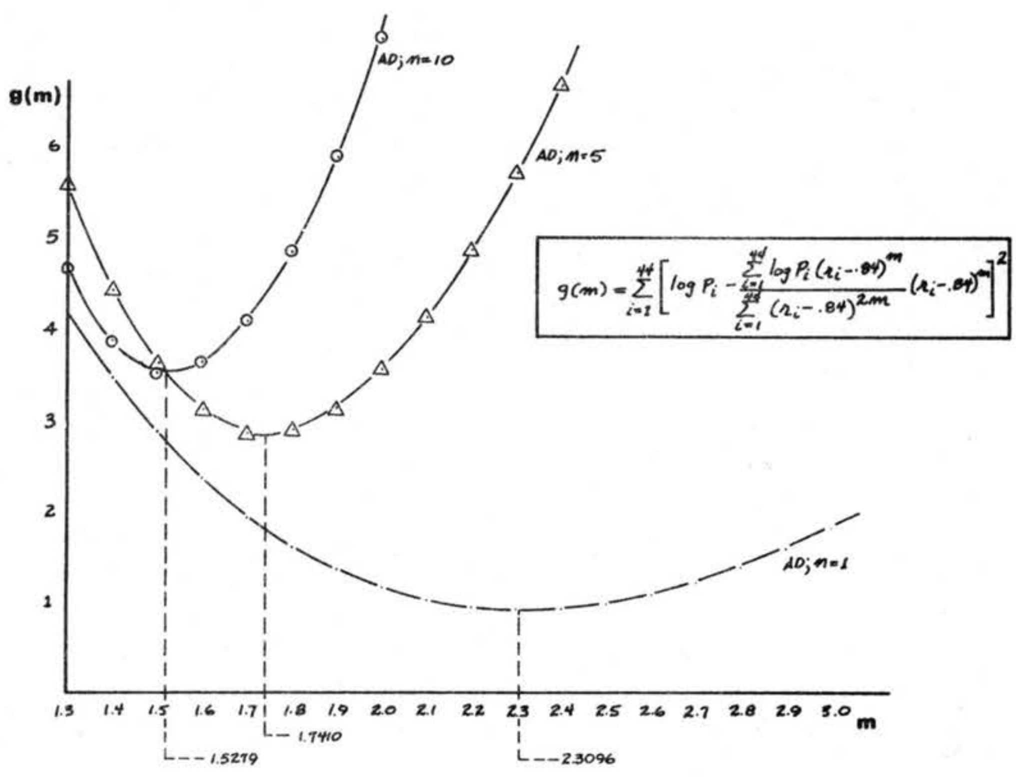
$$g(m) = \sum_{i=1}^{n} \left[\log P_i - \frac{\sum_{i=1}^{n} \log P_i (r_i - r_o)^m}{\sum_{i=1}^{n} (r_i - r_o)^{2m}} (r_i - r_o)^m \right]^2 \tag{5.33}$$

which is a function of m alone.

However, in order to use a Fibonacci search, it is essential that the function to be minimized be unimodal. Otherwise, the search



(a)



(b)

Figure 37. Graphical Display of Minimization Technique

procedure can possibly select a local "valley" of the function as the minimum value, missing the global minimum entirely. Since there is no guarantee that there exists a unique m which will minimize Equation (5.33) unimodality cannot be assured in this single variable search. However, in view of the fact that the right hand side of Equation (5.28) can be expanded into an infinite Taylor series with unique coefficients, it seems reasonable that there exists a unique m which will give the best fitting curve, thereby assuring unimodality of Equation (5.33). For the data used in this thesis, function values for Equation (5.33) have been calculated for m in the interval from 0.5 to 5.0 in increments of 0.01. The results have shown the function to be unimodal at least to the second decimal place. Figure 37b shows a representative plot of Equation (5.33) for the AD distribution with n = 1, 5, and 10.

When unimodality has been assured, the Fibonacci search algorithm as shown in Figure 38 can be used to find the optimal value for the Weibull parameter, m. The second Weibull parameter, a, can be found by substituting the optimal m into Equation (5.32). The efficiency of this search routine is evident in that less than thirty function evaluations were required to find the Weibull parameters for a single set of data with an interval of uncertainty of less than 0.001.

As a check to determine the validity of the assumptions, a Kolmogorov-Smirnov Goodness-of-fit test is made at the end of the algorithm. This test was selected since the distribution of its test statistic can be determined exactly and a confidence band for the distribution function established. For 35 or more data points, the critical value of the Kolmogorov-Smirnov test statistic, $d_c = 1.36$ at

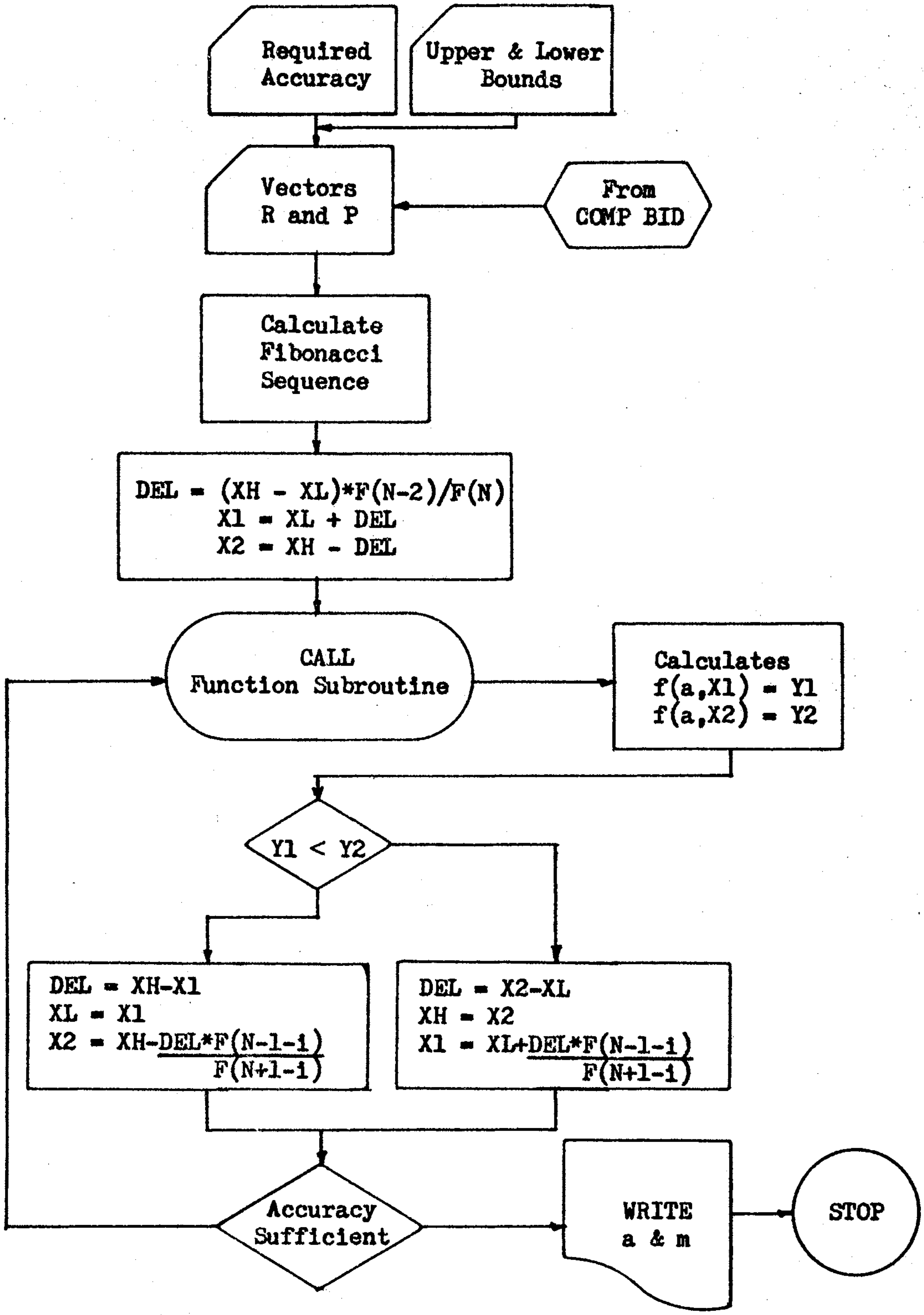


Figure 38. Algorithm for Fibonacci Search Routine

the 0.05 level of significance. The actual data plot for collected data of the low bidder over 38 different jobs is shown in Figure 39 and the plot of the fitted Weibull function is superimposed. The value of the Kolmogorov-Smirnov test statistic relating the actual data points of the LD C.C.D.F. to the Weibull function

$$P_{LD}(r) = e^{-90.2005(r-0.84)^{3.0534}}$$

is d = 0.3075 which is considerably less than the critical value of 1.36. Hence, the hypothesis that the LD distribution conforms to the Weibull distribution cannot be rejected.

Since the AD data vary with n, the data points have been calculated for n = 1 through 10 and plotted in Figure 40 for n = 1, 5, and 10. In fitting these data with a Weibull function, extreme data must be eliminated.

The various markups for the "average bidder" ranged from a low of 0.84 to a high value of 1.47. From the discussion of the VT function of competitor k, it is obvious that those bids with extremely high markups were either using an objective of minimizing the probability of winning since the contractors submitting these did not want the work, there was a mistake in the contractor's bid, or some other factor affected the contractor's objective.

The highest markup of the distribution of low bidders in this case was 1.21. Using the LD distribution as a basic distribution and integrating the basic P.D.F., it follows that the probability of a low bid occurring with a markup greater than 1.21 is

$$\int_{1.21}^{\infty} f_{LD}(r)dr = e^{-4.546} = 0.0173 \quad .$$

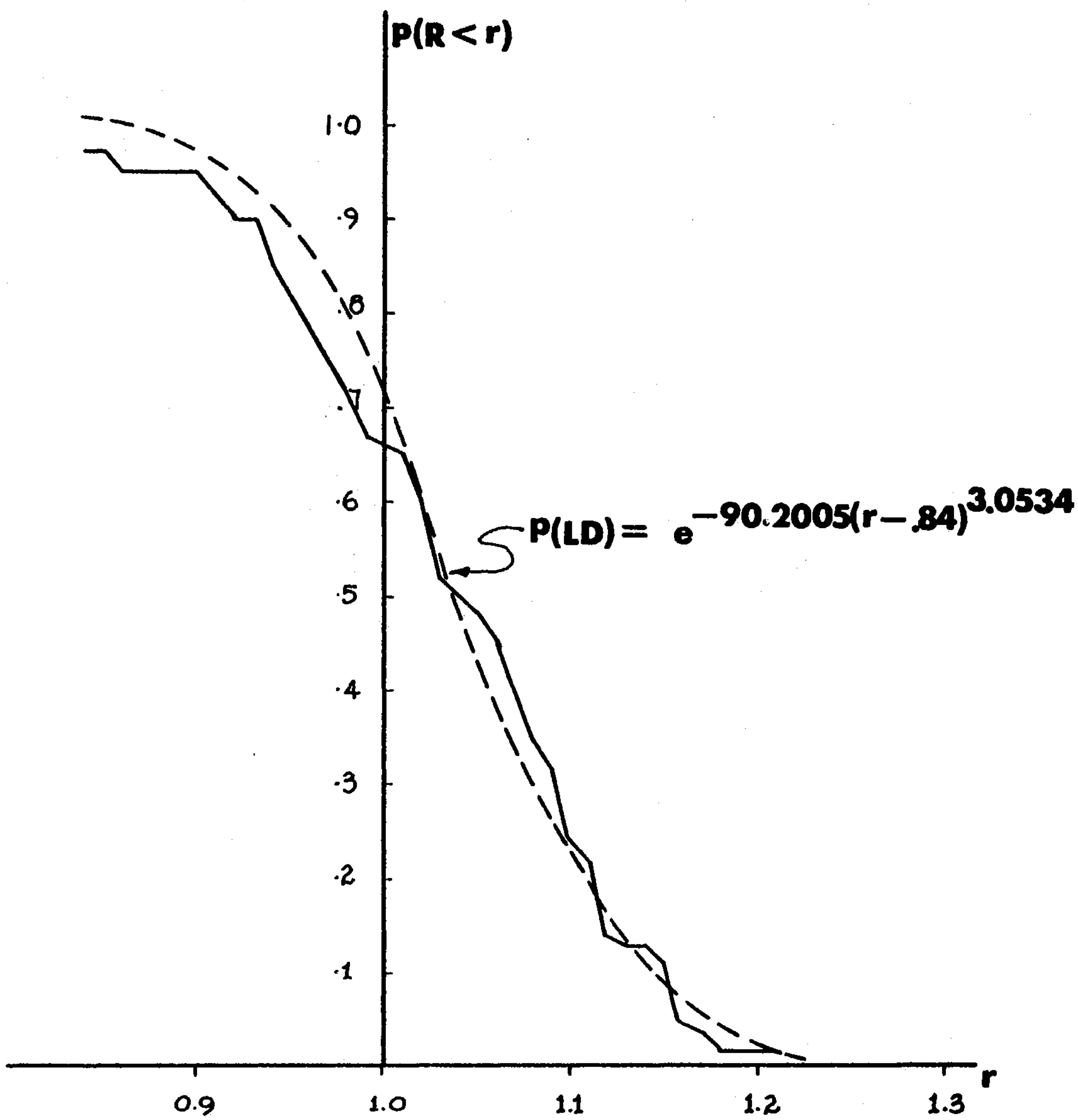


Figure 39. Actual Curve and Calculated Weibull Function for the LD Probability Distribution

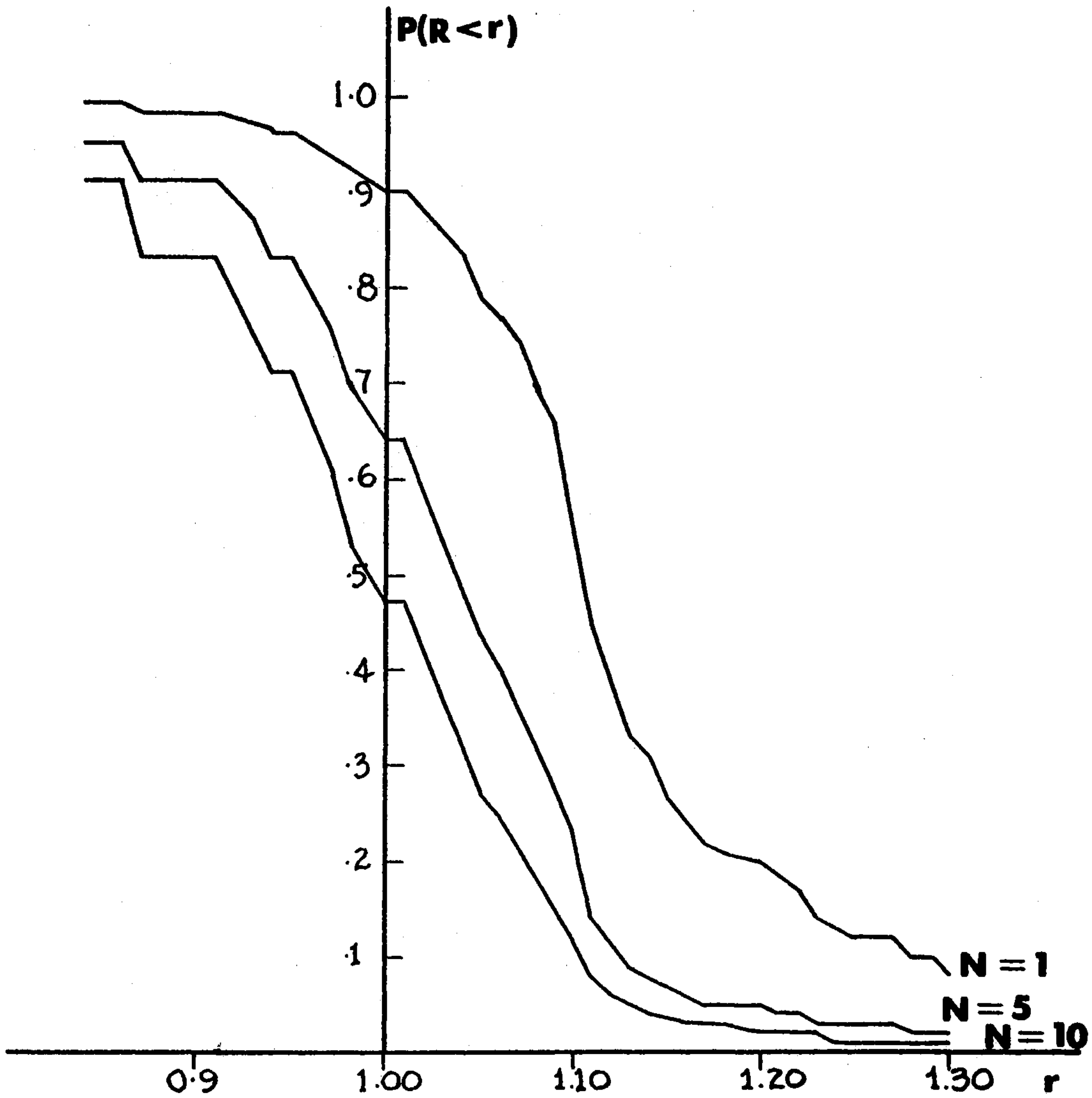


Figure 40. Actual Data Curves for the AD Function for N = 1, 5, 10

Using the actual low bidder data with the parameters

$$\mu = 1.04$$

$$\sigma = 0.09$$

it is seen that the maximum markup that could occur within three standard deviations of the mean is

$$1.04 + 3(0.09) = 1.31 \ .$$

Therefore, conservatively, all markups in the AD distribution in excess of 1.30 can be eliminated. The Weibull functions representing the AD distributions for n = 1, 5, and 10 are plotted in Figure 41.
The Kolmogorov-Smirnov test statistics calculated for each of these fits are shown in Table VI. These are all less than the critical test statistic of 1.36 at the .05 level of significance.

Thus the hypothesis that these functions can be fit by the calculated Weibull functions cannot be rejected.

The Probability of Winning Continued

It is apparent from the foregoing discussions that, with the number of bidders no longer an unknown and the identity of the individual bidders often known, the most accurate method of determining the probability of winning is to develop a C.C.D.F. and a VT function for each competitor. For a researcher, this is impractical. However, for a contractor working in one area for many years, this capability does exist. Even if a sufficient amount of data is not available for a C.C.D.F., subjective probabilities can be updated according to the developed VT function and the probability of winning can be found by

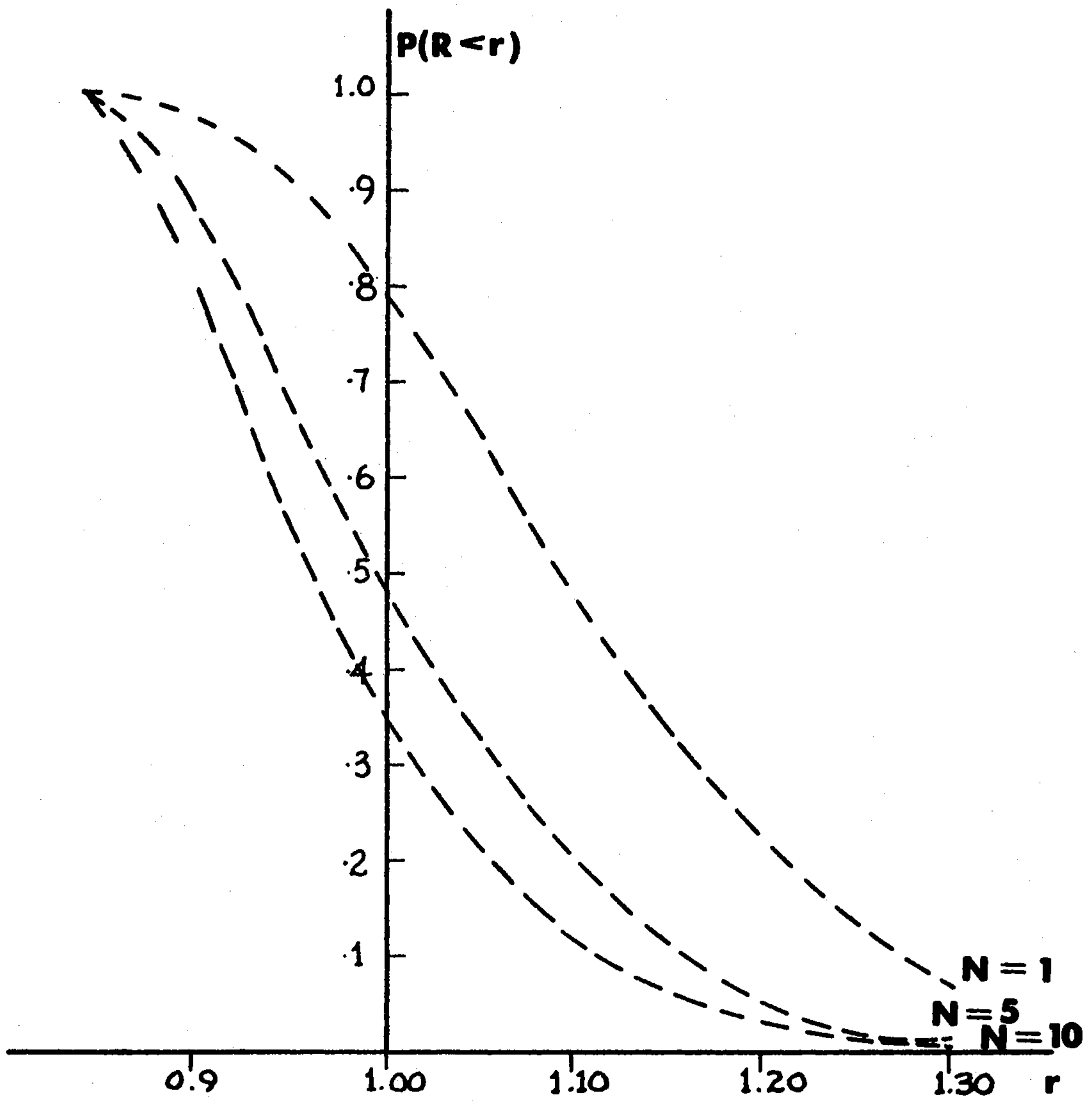


Figure 41. Calculated Weibull Distribution for the AD Function for N = 1, 5, 10

TABLE VI

RESULTS OF GOODNESS OF FIT TEST USING KOLOMOGOROV-SMIRNOV TEST STATISTIC

$$(P = e^{-a(r-0.84)^m})$$

	r ≤ 1.30				r ≤ 1.21			
DIST	NR PTS	a	m	d*	NR PTS	a	m	d
LD	--	--	--	--	38	-90.2005	3.0534	0.3075
AD n = 1	44	-16.0451	2.3096	1.0373	35	-42.1193	3.0916	0.5977
AD n = 2	44	-16.7360	2.0503	1.1783	35	-40.4911	2.7477	0.5770
AD n = 3	44	-16.9261	1.9087	1.2427	35	-38.3853	2.5448	0.5691
AD n = 4	44	-16.9913	1.8130	1.2450	35	-36.5174	2.4003	0.5758
AD n = 5	44	-17.0011	1.7410	1.2159	35	-35.0006	2.2900	0.5727
AD n = 6	44	-16.9779	1.6831	1.1740	35	-33.7252	2.2009	0.5606
AD n = 7	44	-16.9475	1.6353	1.1262	35	-32.6421	2.1265	0.5440
AD n = 8	44	-16.9161	1.5949	1.0768	35	-31.7158	2.0631	0.5253
AD n = 9	44	-16.8781	1.5596	1.0287	35	-30.8794	2.0071	0.5070
AD n = 10	44	-16.8301	1.5279	0.9837	35	-30.1709	1.9585	0.5428

*The value of d, the Kolomogorov-Smirnov Test Statistic at the .05 level of significance is 1.36.

Equation (2.17). However, for the purposes of this paper, the more universal approach will be continued.

Based on the curve fitting procedure presented, the probability of winning over the average bidder can be expressed by:

$$P[\text{win}|AD;n] = F_{AD}(r;n) = \frac{e^{-a_{AD;n}(r-r_o)^{m_{AD;n}}}}{n - (n - 1)e^{-a_{AD;n}(r-r_o)^{m_{AD;n}}}} \qquad (5.33)$$

and the probability of winning over the lowest bidder can be expressed by:

$$P[\text{win}|LD] = F_{LD}(r) = e^{-a_{LD}(r - r_o)^{m_{AD}}} \qquad (5.34)$$

with the parameters as determined by the Griffis-Weibull method.

If key bidders are known and sufficient data maintained on some, but not all key bidders, then there exists two methods of handling the probability of winning. One, $F_{AD}(r;\, n)$ and $F_{LD}(r)$ can be found omitting data for the key bidders and the optimum markup found as shown in the next section. Then the key bidders can be examined separately and "our" contractor can make a decision based on his utility function at that time. Secondly, a more analytical but less practical choice, the AD and key bidder distributions can be combined by the following equation:

$$P[\text{win}|AD \text{ and } KD;\, n] = F_{AD|KD}(r;n)$$

$$= \frac{1}{M\, e^{a_{AD;\,M}(r-r_o)^{m_{AD;\,M}}} + \sum_{i=1}^{n-M} \frac{1}{F_i(r)} - (n - 1)} \qquad (5.35)$$

where M is the number of expected competitors less the number of key bidders.

Optimization of the Utility Functions

The three utility functions to be optimized are:

(1) $u_1 = (r - 1)$.

(2) $u_2 = \xi - \xi e^{-\gamma(r-k)}$, where ξ and γ are constants determining the shape of the utility function and k is the root of the utility function depending on the self-imposed constraint of the contractor of accepting no loss, accepting the loss of his GOH or accepting a certain percentage loss in return for a higher assurance of winning the bid.

(3) $u_3 = \xi(r - c)$ where ξ is the slope of the utility function and c is the ratio of the additional cost incurred (either real or intrinsic) should the contract be won.

These utility functions may be subject to constraints mentioned earlier in this chapter, thus this may be classed as a non-linear constrained optimization problem. However, as a practical matter, the constraints, such as equipment, crews, supervisory staff, etc., are known or can be visually noted on his own VT function. Also, these constraints are difficult to define in terms of the independent variable and the marginal propensity of doing so is questionable. Therefore, the optimization of these three utility functions will be case as an unconstrained optimization problem although constraints will be considered by the contractor implicitly. The problem becomes, therefore, to find an r* which will maximize the expected value $u_i(r)$ for $i = 1, 2,$ or 3.

For all three of the utility functions listed, there exists two methods for finding the optimum value. Consider first the optimization of $u_1(r)$. Treating the maximization of $u_1(r)$ classically, one finds that the optimum using the LD distribution is given by differentiating the function

$$E(u_1(r)) = (r-1)e^{a_{LD}(r-r_o)^{m_{LD}}} \qquad (5.36)$$

with respect to r and setting the derivative equal to zero. The results give the implicit function of

$$r^* = 1 + \frac{1}{a_{LD}M_{LD}(r-r_o)^{m_{LD-1}}} \,. \qquad (5.37)$$

Using the AD function to find another r* so that an optimal range can be established, one differentiates

$$E(u_1(r) = (r-1)\,\frac{e^{-a_{AD;n}(r-r_o)^{m_{AD;n}}}}{n-(n-1)e^{-a_{AD;n}(r-r_o)^{m_{AD;n}}}} \qquad (5.38)$$

with respect to r and sets the derivative equal to zero, from which one obtains the unpleasing implicit function

$$r^* = 1 + \frac{n-(n-1)e^{-a_{AD;n}(r-r_o)^{m_{AD;n}}}}{amn(r-r_o)^{m-1} - [amn(r-r_o)^{m-1}+1](n-1)e^{-a_{AD;n}(r-r_o)^{m_{AD;n}}}} \,. \qquad (5.39)$$

Neither Equation (5.37) nor Equation (5.39) can be solved for r* explicitly. Although there exists numerous methods of approximating solutions for them, the most efficient method of solving for r* would probably be a Bolzano search (104). However, the resulting single value of r*, although unique, would give the contractor no information as to the sensitivity of his expected profit to a change in r.

Therefore, this author proposes to solve for the optimum markup for Equations (5.36) and (5.38) directly using a Fibonacci search (other sequential search techniques could be used) technique (102). Prior to doing so, however, the functions must be proven to be unimodal.

Consider the function

$$f(r) = (r-1)e^{-a(r-r_o)^m} . \qquad (5.40a)$$

This function is continuous and differentiable over all r which includes the interval $r \in [0.84, \infty)$, the interval of interest in this problem. Therefore, from well known theorems in basic calculus, there exists an absolute maximum and an absolute minimum on any closed interval. Considering the half closed interval $[0.84, \infty)$, $f(r) = 0$ when $r = 1$ and $f(r) \to 0$ as $r \to \infty$. Hence, on this half closed interval, there exists a point, r^*, such that $f'(r) = 0$. This is based on Rolle's Theorem as found in any standard basic calculus text. If a function is convex, the function is unimodal (100:101) although the converse is not necessarily true. Therefore, to prove unimodality, one can prove that the function is convex.

Since there exists a point r^* such that $f'(r) = 0$ on the interval of concern, let δ be a number, however small, greater than zero. Consider the two functions

$$f'(r^* - \delta) = [-a\,m\,(r^* - \delta - r_o)^{m-1}(r^* - \delta - 1) + 1]e^{-a(r^*-\delta-r_o)^m} \qquad (5.40b)$$

and

$$f'(r^* + \delta) = [-a\,m\,(r^* + \delta - r_o)^{m-1}(r^* + \delta - 1) + 1]e^{-a(r^*+\delta-r_o)^m} . \qquad (5.40c)$$

The first term in both Equations (5.40b and 5.40c),

$$[-a\,m\,(r - r_o)^{m-1}(r - 1) + 1] \qquad (5.40d)$$

strictly decreases as r increases due to the negative sign on a; the same holds true for the second term. Consequently, the following inequality holds

$$f'(r^* - \delta) > f'(r^*) > f'(r^* + \delta) \quad . \qquad (5.40e)$$

However, $f'(r^*) = 0$ which implies that $f'(r^* - \delta) > 0$ and $f'(r^* + \delta) < 0$. Thus, there does not exist an inflection point in the function for any r in the interval. Therefore, the function is convex and consequently unimodal. Hence, a Fibonacci search may be used to find the optimum r. The same proof can be given for the utility functions $u_2(r)$ and $u_3(r)$.

The results of the Fibonacci search for the optimum markup for utility function u_1 are shown in Table VII for the LD and AD distributions. A graph of the expected $u_1(r)$ curve using actual data is shown in Figure 42 and the expected $u_1(r)$ curves using the theoretical distributions are shown in Figure 43. Results are shown for $u_2(r)$ and $u_3(r)$ with assumed parameters in Tables VIII and IX, and Figures 44 through 47.

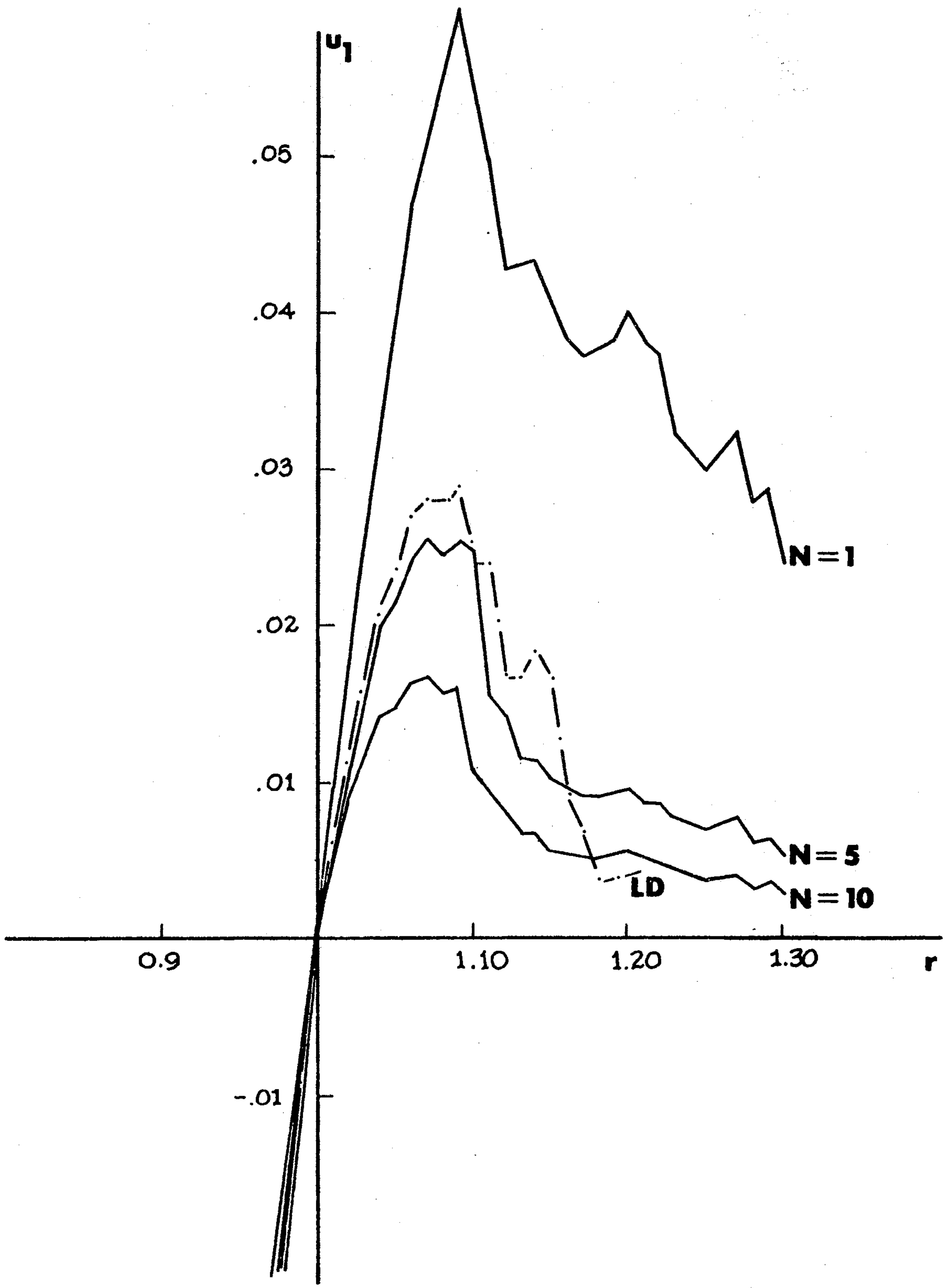


Figure 42. Expected Utility, $u_1 = (r-1)P(R<r)$, Using Actual Data for the LD Function and the AD Function for N = 1, 5, 10

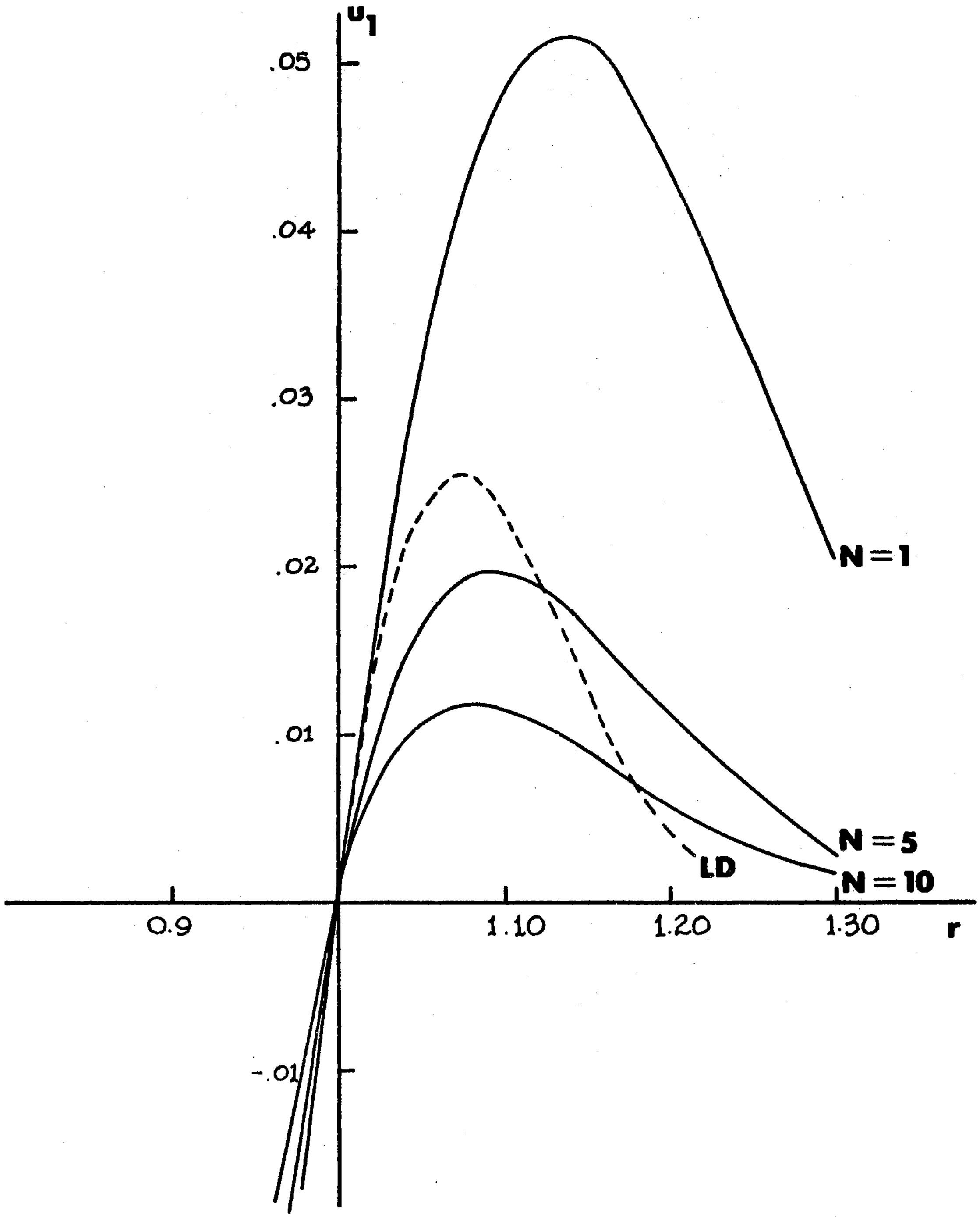


Figure 43. Expected Utility, $u_1 = (r-1)e^{-a(r-0.84)^m}$, for the LD Function and the AD Function for N = 1, 5, 10

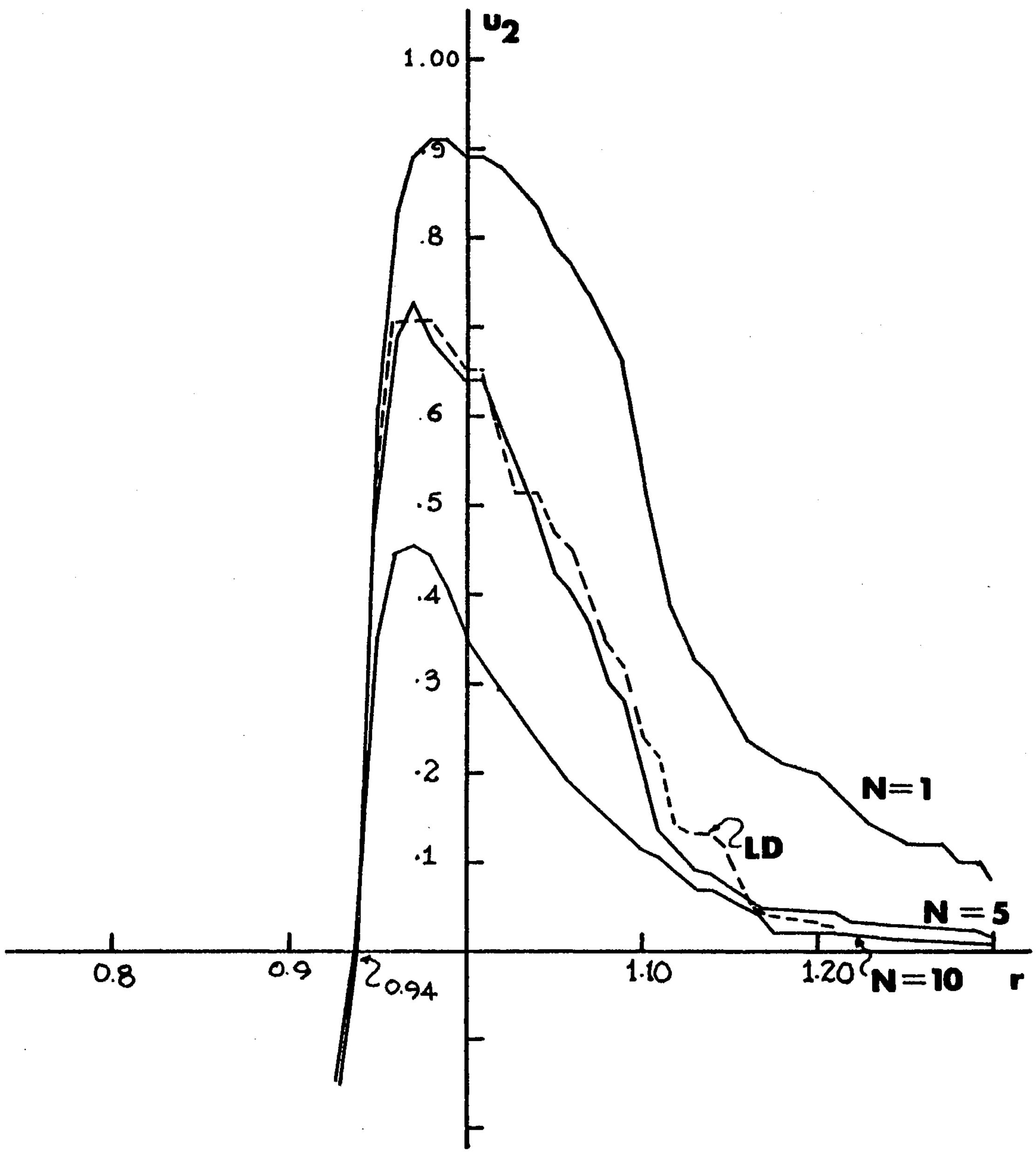


Figure 44. Expected Utility, $u_2 = (1-e^{-100(r-0.94)})(R<r)$, Using Actual Data for the LD Function and the AD Function for N = 1, 5, 10

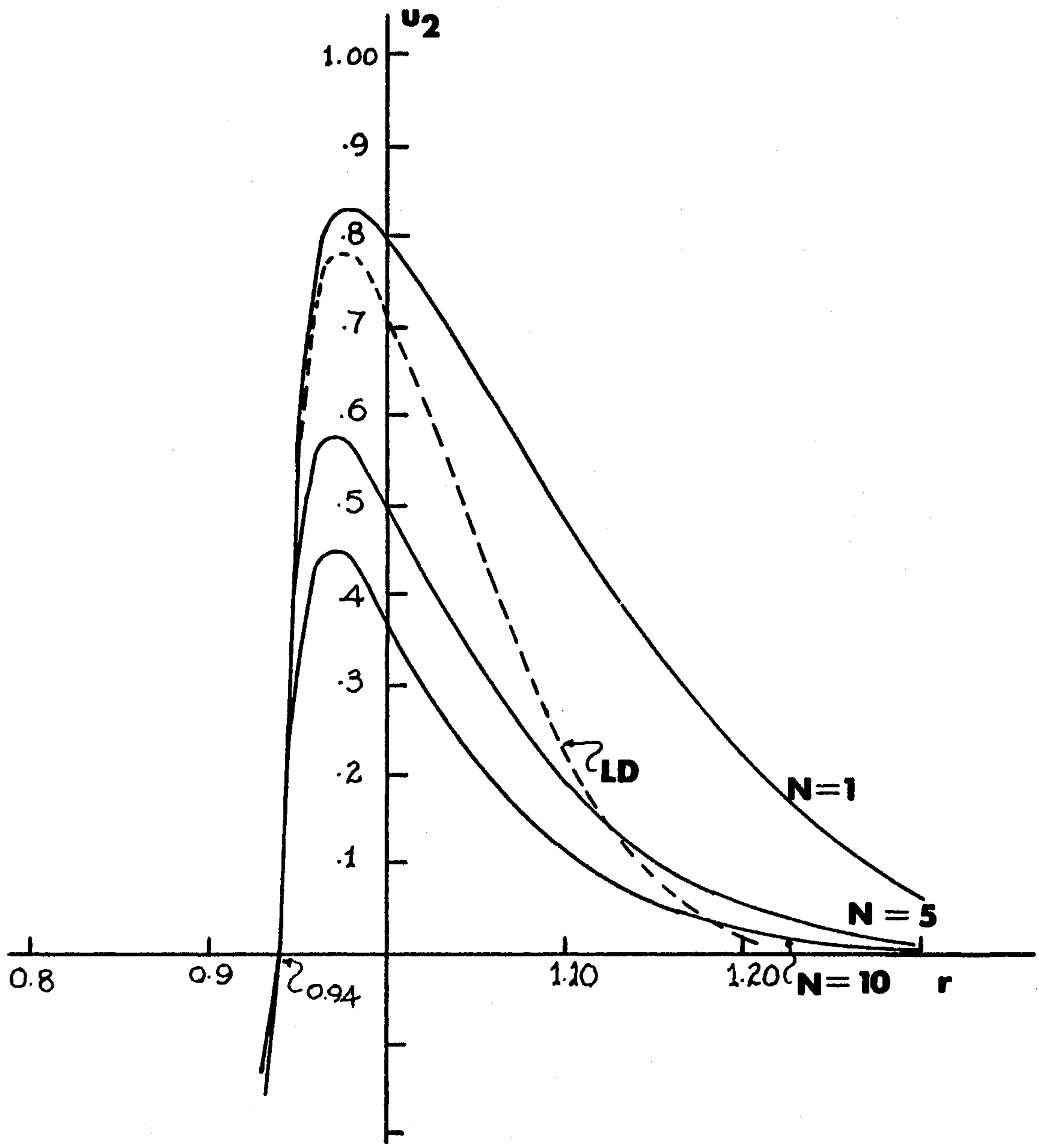


Figure 45. Expected Utility, $u_2 = (1-e^{-100(r-0.94)})e^{-a(r-0.84)^m}$, for the LD Function and the AD Function for N = 1, 5, 10

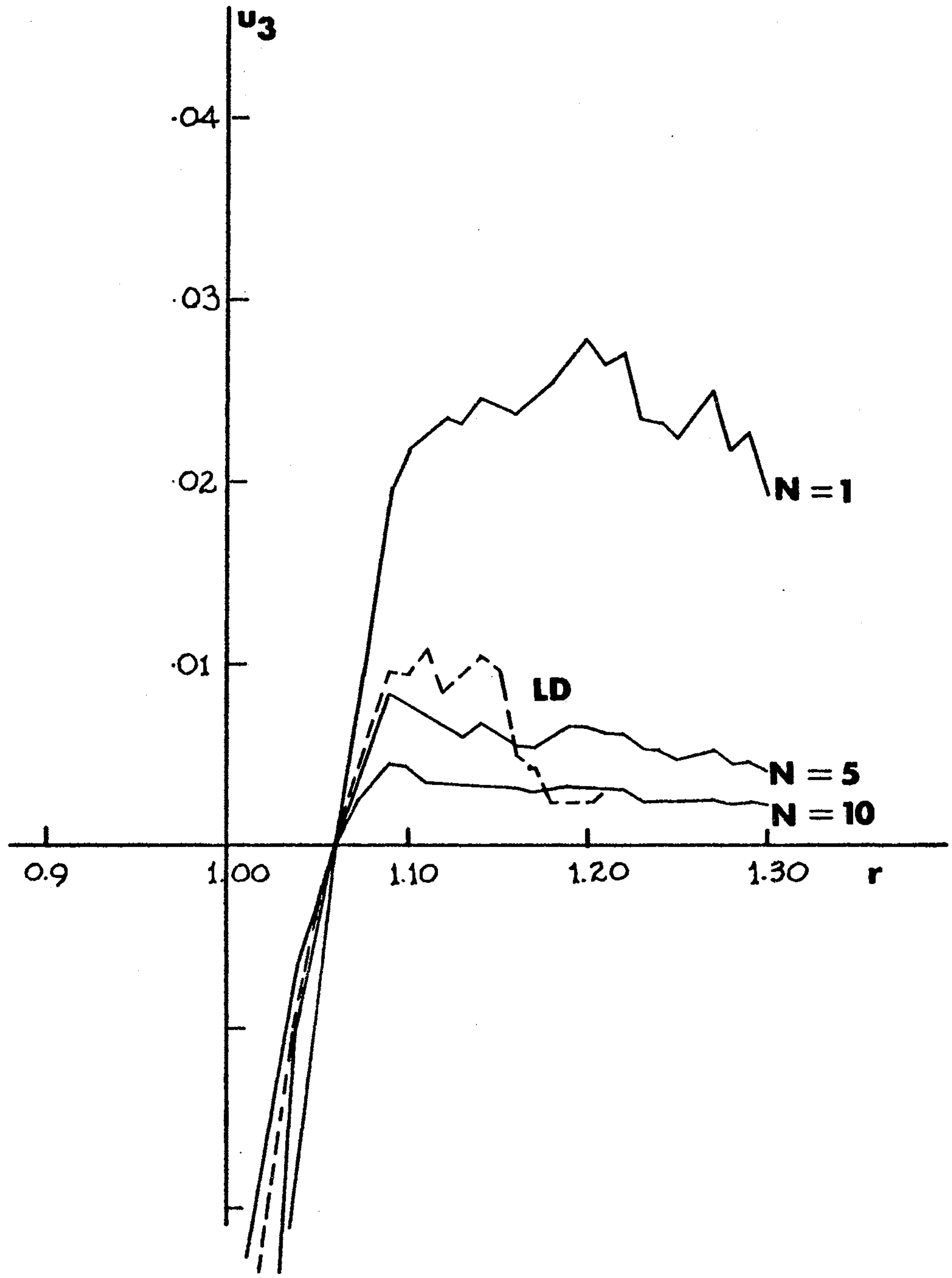


Figure 46. Expected Utility, $u_3 = (r-1.06)P(R<r)$, Using Actual Data for the LD Function and the AD Function for N = 1, 5, 10

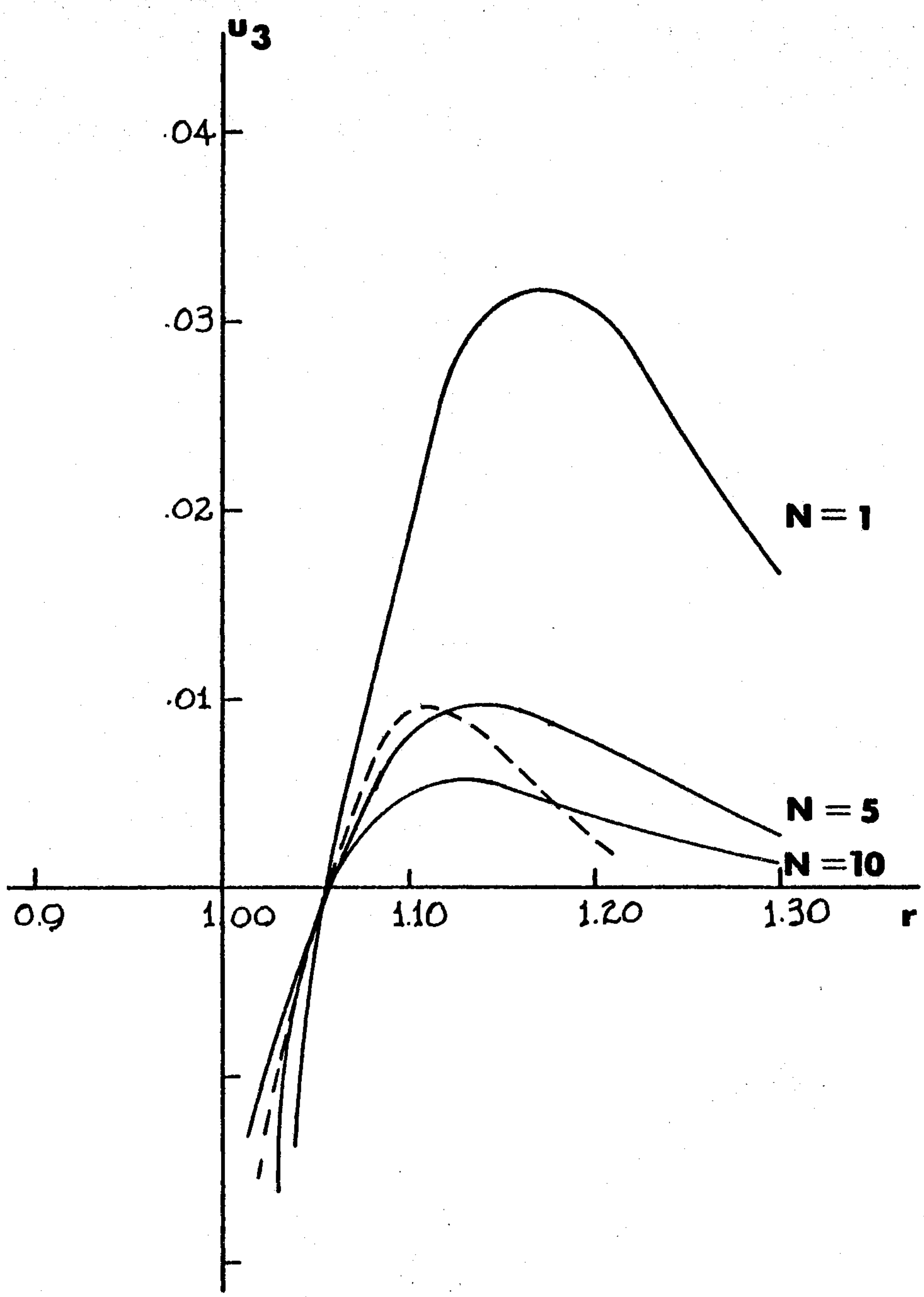


Figure 47. Expected Utility, $u_3 = (r-1.06)e^{-a(r-0.84)^m}$, for the LD Function and the AD Function for N = 1, 5, 10

TABLE VII

OPTIMUM BID FOR $U_1 = (r - 1)P(WIN)$ FOR LD AND AD FOR n = 1, 10

	ACTUAL DATA		CALCULATED DATA	
DISTRIBUTION	OPT. BID	EXP. UTILITY	OPT. BID	EXP. UTILITY
LD	1.09	0.0288	1.073	0.0254
AD n = 1	1.09	0.0594	1.134	0.0519
AD n = 2	1.09	0.0443	1.114	0.0351
AD n = 3	1.09	0.0354	1.104	0.0274
AD n = 4	1.09	0.0294	1.098	0.0228
AD n = 5	1.07	0.0254	1.094	0.0197
AD n = 6	1.07	0.0225	1.090	0.0173
AD n = 7	1.07	0.0202	1.088	0.0156
AD n = 8	1.07	0.0184	1.085	0.0141
AD n = 9	1.07	0.0168	1.084	0.0129
AD n = 10	1.07	0.0155	1.082	0.0120

TABLE VIII

OPTIMUM BID FOR $U_2 = (1 - e^{-100(r - 0.94)})P(WIN)$ FOR LD AND AD FOR n = 1, 10

	ACTUAL DATA		CALCULATED DATA	
DISTRIBUTION	OPT. BID	EXP. UTILITY	OPT. BID	EXP. UTILITY
LD	0.97	0.7127	0.97	0.7955
AD n = 1	0.99	0.9039	0.98	0.8273
AD n = 2	0.97	0.8426	0.97	0.7361
AD n = 3	0.97	0.7975	0.97	0.6732
AD n = 4	0.97	0.7569	0.97	0.6240
AD n = 5	0.97	0.7203	0.97	0.5837
AD n = 6	0.97	0.6871	0.97	0.5495
AD n = 7	0.97	0.6568	0.97	0.5201
AD n = 8	0.97	0.6290	0.97	0.4944
AD n = 9	0.97	0.6035	0.97	0.4716
AD n = 10	0.97	0.5800	0.97	0.4510

TABLE IX

OPTIMUM BID FOR $U_3 = (r - 1.06)P(WIN)$ FOR LD AND AD FOR n = 1, 10

	ACTUAL DATA		CALCULATED DATA	
DISTRIBUTION	OPT. BID	EXP. UTILITY	OPT. BID	EXP. UTILITY
LD	1.11	0.0110	1.11	0.0095
AD n = 1	1.20	0.0280	1.19	0.0318
AD n = 2	1.20	0.0136	1.16	0.0198
AD n = 3	1.09	0.0118	1.15	0.0147
AD n = 4	1.09	0.0098	1.15	0.0118
AD n = 5	1.09	0.0084	1.15	0.0098
AD n = 6	1.09	0.0073	1.15	0.0085
AD n = 7	1.09	0.0069	1.14	0.0075
AD n = 8	1.09	0.0059	1.14	0.0067
AD n = 9	1.09	0.0053	1.14	0.0061
AD n = 10	1.09	0.0049	1.14	0.0055

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A method by which a construction contractor can analyze his objectives prior to bidding for a contract, develop his utility functions representing his objectives, determine the probabilities of winning the contract with a dynamic probability function and finally optimize the amount of his bid has been presented. The bidding process has been developed as a three stage lottery, using a conditional update of probabilities associated with winning the contract. A compromise has been made between considering the estimated cost as a deterministic variable and considering it as a stochastic variable. This was done using a cost variance minimization approach outlined in Chapter IV.

Basically, a mathematical model representing a contractor, his competitors and the state of nature has been developed both analytically and with experimental data. This is the concept of Operations Research or scientific management which is so rarely applied to construction operations. Considering this as a mathematical model, it is apparent that absolute answers are not the result. It is merely one more package of information available to the decision maker to add to his own information system, experience, and guesses, and should be considered as such.

Six additions were made to the state of the art of competitive bidding by this work.

(1) A Volume-Time function was introduced whereby a contractor can graphically determine his objectives when bidding for a contract according to his volume of work on hand at any time.

(2) Utility functions were proposed to correspond to selected contractor objectives.

(3) A method of determining the probability of winning a contract was presented. This method is a fully time dependent process which, using a decision theoretic strategy, updates the *á priori* distribution of a particular competitor to his present probability density function.

(4) If all competitors are not known or if sufficient bidding data is not available to obtain *á priori* probability density functions on all competitors, a method of combining the probability distribution of the "Average Bidder" and the "Low Bidder" with particular "Key Bidders" has been developed.

(5) The Griffis-Weibull technique of fitting a complementary cumulative probability distribution function to a Weibull asymptotic distribution has been developed. This was done to obtain a unimodal function to which sequential search techniques would apply. This technique is not only applicable to the competitive bidding problem but to any other problem of science which requires fitting data to a Weibull distribution.

(6) Finally, a nonconstrained optimization procedure using a Fibonacci Sequential search technique has been illustrated.

The results of experimental data utilized indicate that the expected utility using actual data and data fitted to a Weibull curve decreases significantly as the number of bidders increase.

Using actual data, the optimum markup for a contractor's bid as a percentage of his estimated cost varied from nine per cent for one through four expected bidders to seven per cent for five to ten expected bidders. Using data fitted to a Weibull distribution, the optimum bid varied from eleven per cent with two expected bidders to eight per cent with ten expected bidders. These optimum bids were based on a contractor's normal linear utility function, e.g., profit maximization.

This author feels that the data fitted to a Weibull distribution gives the more accurate optimum markups since it is an asymptotic distribution. Although there is no method other than extensive experimentation to prove this assertion, the Weibull fitted data does provide the more conservative bidding policy.

The reader, at this point, has probably noted from Table VII that the expected utility (profit) for the normal contractor objective function appears extremely small. It should be remembered that this is percentage of estimated contract cost. It does not represent the percentage of return on the contractor's investment. In actuality, a contractor will rarely have more than 20 per cent of the estimated cost invested in a construction project. Using this figure, a two per cent return on the estimated cost results in a ten per cent return on invested capital.

This author has treated the competitive bidding problem with asymetric information as a three stage lottery. As mentioned in Chapter V, it is actually a four stage lottery; the stage omitted in this work is the first stage. The first stage should answer the question: "Which projects should a contractor consider?" This could be treated as a queueing problem and the utility function would incorporate the volume of work and the specific projects necessary to absorb the contractor's overhead and required return on investment. This is a fertile area for further research.

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APPENDIX A

BID COLLECTION DATA FORM

Bid Collection Data Sheet

Contractor File No.__________ Date of Award __________

Brief Description of Project

Contractor Bid	Contractor Est.	Mat'l Est.	Labor Est.	Eq. Est.
		*	*	*

Est. Dur.	Actual Cost	JOH Est.	GOH Est.	Other
	**	*	*	*

Rank	Name of Bidder	Bid	Remarks

* Not necessary in the bid analysis; useful as a management tool.

** Helpful in correcting cost bias.

APPENDIX B

COMPUTED RESULTS FOR THE UTILITY FUNCTION

$$u = (r - 1)p(win)$$

THIS IS AN LO FUNCTION

THE EQUATION OF THE WEIBULLFUNCTION IS EXP(- 90.2576*(R - 0.84)** 3.0539)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.97	1.00	-0.0300	-0.1552	-0.1600	0.0048
0.85	0.97	1.00	-0.0299	-0.1455	-0.1500	0.0045
0.86	0.95	1.00	-0.0494	-0.1330	-0.1399	0.0069
0.87	0.95	1.00	-0.0480	-0.1235	-0.1297	0.0062
0.88	0.95	1.00	-0.0452	-0.1140	-0.1194	0.0054
0.89	0.95	0.99	-0.0404	-0.1045	-0.1089	0.0044
0.90	0.95	0.98	-0.0334	-0.0950	-0.0983	0.0033
0.91	0.92	0.97	-0.0535	-0.0828	-0.0876	0.0048
0.92	0.90	0.96	-0.0605	-0.0720	-0.0768	0.0048
0.93	0.90	0.94	-0.0439	-0.0630	-0.0661	0.0031
0.94	0.85	0.92	-0.0734	-0.0510	-0.0554	0.0044
0.95	0.82	0.90	-0.0788	-0.0410	-0.0449	0.0039
0.96	0.82	0.87	-0.0501	-0.0328	-0.0348	0.0020
0.97	0.75	0.84	-0.0872	-0.0225	-0.0251	0.0026
0.98	0.72	0.80	-0.0803	-0.0144	-0.0160	0.0016
0.99	0.67	0.76	-0.0896	-0.0067	-0.0076	0.0009
1.00	0.65	0.72	-0.0654	0.0	0.0	0.0
1.01	0.65	0.67	-0.0183	0.0065	0.0067	-0.0002
1.02	0.60	0.62	-0.0188	0.0120	0.0124	-0.0004
1.03	0.52	0.57	-0.0478	0.0156	0.0170	-0.0014
1.04	0.52	0.52	0.0042	0.0208	0.0206	0.0002
1.05	0.47	0.46	0.0063	0.0235	0.0232	0.0003
1.06	0.45	0.41	0.0376	0.0270	0.0247	0.0023
1.07	0.40	0.36	0.0374	0.0280	0.0254	0.0026
1.08	0.35	0.31	0.0350	0.0280	0.0252	0.0028
1.09	0.32	0.27	0.0498	0.0288	0.0243	0.0045
1.10	0.24	0.23	0.0113	0.0240	0.0229	0.0011
1.11	0.22	0.19	0.0290	0.0242	0.0210	0.0032
1.12	0.14	0.16	-0.0173	0.0168	0.0189	-0.0021
1.13	0.13	0.13	0.0024	0.0169	0.0166	0.0003
1.14	0.13	0.10	0.0281	0.0182	0.0143	0.0039
1.15	0.11	0.08	0.0299	0.0165	0.0120	0.0045
1.16	0.05	0.06	-0.0120	0.0080	0.0099	-0.0019
1.17	0.04	0.05	-0.0071	0.0068	0.0080	-0.0012
1.18	0.02	0.04	-0.0152	0.0036	0.0063	-0.0027
1.19	0.02	0.03	-0.0058	0.0038	0.0049	-0.0011
1.20	0.02	0.02	0.0014	0.0040	0.0037	0.0003
1.21	0.02	0.01	0.0069	0.0042	0.0028	0.0014

THIS IS AN AD FUNCTION, K = 1

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.0451*(R - 0.84)** 2.3096)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.99	1.00	-0.0100	-0.1584	-0.1600	0.0016
0.85	0.99	1.00	-0.0096	-0.1485	-0.1499	0.0014
0.86	0.99	1.00	-0.0081	-0.1386	-0.1397	0.0011
0.87	0.98	1.00	-0.0151	-0.1274	-0.1294	0.0020
0.88	0.98	0.99	-0.0106	-0.1176	-0.1189	0.0013
0.89	0.98	0.98	-0.0043	-0.1078	-0.1083	0.0005
0.90	0.98	0.98	0.0039	-0.0980	-0.0976	-0.0004
0.91	0.98	0.97	0.0139	-0.0882	-0.0869	-0.0013
0.93	0.97	0.94	0.0298	-0.0679	-0.0658	-0.0021
0.94	0.96	0.92	0.0356	-0.0576	-0.0555	-0.0021
0.95	0.96	0.91	0.0534	-0.0480	-0.0453	-0.0027
0.96	0.95	0.89	0.0629	-0.0380	-0.0355	-0.0025
0.97	0.94	0.87	0.0743	-0.0282	-0.0260	-0.0022
0.98	0.92	0.84	0.0773	-0.0184	-0.0169	-0.0015
0.99	0.91	0.82	0.0918	-0.0091	-0.0082	-0.0009
1.00	0.90	0.79	0.1078	0.0	0.0	0.0
1.01	0.90	0.76	0.1350	0.0090	0.0076	0.0014
1.02	0.88	0.74	0.1434	0.0176	0.0147	0.0029
1.04	0.83	0.68	0.1529	0.0332	0.0271	0.0061
1.05	0.79	0.65	0.1437	0.0395	0.0323	0.0072
1.06	0.77	0.62	0.1549	0.0462	0.0369	0.0093
1.07	0.74	0.58	0.1564	0.0518	0.0409	0.0109
1.08	0.69	0.55	0.1380	0.0552	0.0442	0.0110
1.09	0.66	0.52	0.1395	0.0594	0.0468	0.0126
1.10	0.55	0.49	0.0607	0.0550	0.0489	0.0061
1.11	0.45	0.46	-0.0085	0.0495	0.0504	-0.0009
1.12	0.39	0.43	-0.0382	0.0468	0.0514	-0.0046
1.13	0.33	0.40	-0.0686	0.0429	0.0518	-0.0089
1.14	0.31	0.37	-0.0598	0.0434	0.0518	-0.0084
1.15	0.27	0.34	-0.0720	0.0405	0.0513	-0.0108
1.16	0.24	0.32	-0.0752	0.0384	0.0504	-0.0120
1.17	0.22	0.29	-0.0695	0.0374	0.0492	-0.0118
1.18	0.21	0.26	-0.0550	0.0378	0.0477	-0.0099
1.20	0.20	0.22	-0.0197	0.0400	0.0439	-0.0039
1.21	0.18	0.20	-0.0190	0.0378	0.0418	-0.0040
1.22	0.17	0.18	-0.0096	0.0374	0.0395	-0.0021
1.23	0.14	0.16	-0.0215	0.0322	0.0371	-0.0049
1.24	0.13	0.14	-0.0147	0.0312	0.0347	-0.0035
1.25	0.12	0.13	-0.0092	0.0300	0.0323	-0.0023
1.26	0.12	0.11	0.0051	0.0312	0.0299	0.0013
1.27	0.12	0.10	0.0182	0.0324	0.0275	0.0049
1.28	0.10	0.09	0.0101	0.0280	0.0252	0.0028
1.29	0.10	0.08	0.0209	0.0290	0.0229	0.0061
1.30	0.08	0.07	0.0107	0.0240	0.0208	0.0032

THIS IS AN AD FUNCTION, N = 2

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.7360*(R - 0.84)** 2.0503)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.98	1.00	-0.0198	-0.1568	-0.1600	0.0032
0.85	0.98	1.00	-0.0165	-0.1470	-0.1498	0.0028
0.86	0.98	0.99	-0.0143	-0.1372	-0.1392	0.0020
0.87	0.96	0.99	-0.0267	-0.1249	-0.1284	0.0035
0.88	0.96	0.98	-0.0167	-0.1153	-0.1173	0.0020
0.89	0.96	0.96	-0.0039	-0.1057	-0.1061	0.0004
0.90	0.96	0.95	0.0117	-0.0961	-0.0949	-0.0012
0.91	0.96	0.93	0.0300	-0.0865	-0.0838	-0.0027
0.93	0.94	0.89	0.0549	-0.0659	-0.0621	-0.0038
0.94	0.92	0.86	0.0616	-0.0554	-0.0517	-0.0037
0.95	0.92	0.83	0.0888	-0.0462	-0.0417	-0.0044
0.96	0.90	0.81	0.0995	-0.0362	-0.0322	-0.0040
0.97	0.89	0.77	0.1121	-0.0266	-0.0232	-0.0034
0.98	0.85	0.74	0.1089	-0.0170	-0.0149	-0.0022
0.99	0.83	0.71	0.1247	-0.0083	-0.0071	-0.0012
1.00	0.82	0.68	0.1416	0.0	0.0	0.0
1.01	0.82	0.64	0.1757	0.0082	0.0064	0.0018
1.02	0.79	0.61	0.1776	0.0157	0.0122	0.0036
1.04	0.71	0.54	0.1701	0.0284	0.0216	0.0068
1.05	0.65	0.51	0.1475	0.0326	0.0253	0.0074
1.06	0.63	0.47	0.1540	0.0376	0.0283	0.0092
1.07	0.59	0.44	0.1479	0.0411	0.0308	0.0104
1.08	0.53	0.41	0.1190	0.0421	0.0326	0.0095
1.09	0.49	0.38	0.1156	0.0443	0.0339	0.0104
1.10	0.38	0.35	0.0319	0.0379	0.0347	0.0032
1.11	0.29	0.32	-0.0288	0.0319	0.0351	-0.0032
1.12	0.24	0.29	-0.0498	0.0291	0.0350	-0.0060
1.13	0.20	0.27	-0.0688	0.0257	0.0346	-0.0090
1.14	0.18	0.24	-0.0588	0.0257	0.0339	-0.0082
1.15	0.16	0.22	-0.0634	0.0234	0.0329	-0.0095
1.16	0.14	0.20	-0.0619	0.0218	0.0317	-0.0099
1.17	0.12	0.18	-0.0548	0.0210	0.0303	-0.0093
1.18	0.12	0.16	-0.0427	0.0211	0.0288	-0.0077
1.20	0.11	0.13	-0.0163	0.0222	0.0255	-0.0033
1.21	0.10	0.11	-0.0142	0.0208	0.0238	-0.0030
1.22	0.09	0.10	-0.0072	0.0204	0.0220	-0.0016
1.23	0.08	0.09	-0.0130	0.0173	0.0203	-0.0030
1.24	0.07	0.06	-0.0080	0.0167	0.0186	-0.0019
1.25	0.06	0.07	-0.0041	0.0160	0.0170	-0.0010
1.26	0.06	0.06	0.0046	0.0166	0.0154	0.0012
1.27	0.06	0.05	0.0123	0.0172	0.0139	0.0033
1.28	0.05	0.04	0.0080	0.0147	0.0125	0.0022
1.29	0.05	0.04	0.0141	0.0153	0.0112	0.0041
1.30	0.04	0.03	0.0085	0.0125	0.0100	0.0025

THIS IS AN AC FUNCTICN, N = 3

THE EQUATICN OF THEWEIBULL FUNCTICN IS EXP(- 16.9261*(R - 0.84)** 1.9087)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	CIFFERENCE	EXPECTED UTILITY ACTUAL CATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.97	1.00	-0.C294	-C.1553	-0.1600	0.0047
0.85	0.97	1.00	-0.0268	-C.1456	-0.1496	0.0040
0.86	0.97	C.S9	-0.0198	-0.1359	-0.1387	0.0028
0.87	0.94	0.98	-0.C369	-C.1225	-0.1273	0.0048
0.88	C.94	0.96	-0.0220	-C.1131	-0.1157	0.0026
0.8S	C.94	C.95	-0.0036	-C.1037	-0.1040	0.0004
0.90	0.94	0.92	C.0181	-C.C942	-0.0924	-0.0018
0.91	0.94	C.90	0.0426	-0.0848	-0.0810	-0.0038
0.93	C.92	0.84	0.0721	-C.0641	-0.0590	-0.0050
0.94	0.89	0.81	0.C774	-C.C533	-0.0487	-0.0046
0.95	0.8S	0.78	0.1105	-0.0444	-0.0389	-0.0055
0.96	0.86	0.74	0.1197	-C.C345	-0.0298	-0.0048
0.97	0.84	0.71	0.13C8	-C.C252	-0.0213	-0.0039
0.S8	0.79	C.67	0.1208	-C.C159	-0.0134	-0.0024
0.99	0.77	0.64	C.1354	-C.C077	-0.0064	-0.0014
1.00	0.75	0.60	0.1509	0.C	0.0	0.0
1.01	0.75	C.56	0.1874	C.CC75	0.0056	0.0019
1.02	0.71	0.53	0.1831	C.C142	0.0105	0.0037
1.04	0.62	0.46	0.1629	0.0248	0.0183	0.0065
1.05	0.56	0.42	0.1335	C.C278	0.0211	0.0067
1.C6	0.53	C.39	0.1371	C.C316	0.0234	0.0082
1.07	C.45	0.36	0.1277	C.0341	0.0251	0.0089
1.08	0.43	0.33	0.C966	C.C341	0.0263	0.0077
1.C9	0.39	0.30	0.C919	C.C354	0.0271	0.0083
1.10	0.29	0.27	0.C153	C.C289	0.0274	0.0015
1.11	0.21	0.25	-0.C346	C.C236	0.0274	-0.0038
1.12	0.18	0.23	-0.C496	0.0211	0.0270	-0.C059
1.13	0.14	0.20	-0.C621	C.C183	0.0264	-0.0081
1.14	0.13	0.18	-0.0524	C.C182	0.0256	-0.0073
1.15	0.11	0.16	-0.C539	C.0165	0.0245	-0.0081
1.16	0.10	0.15	-0.C509	C.C152	0.0234	-0.0081
1.17	0.09	0.13	-0.0441	C.C146	0.0221	-0.0075
1.18	C.C8	0.12	-0.C340	C.0147	0.0208	-0.0061
1.20	0.08	0.C9	-0.0131	C.C154	0.0180	-0.0026
1.21	0.07	0.08	-0.0109	0.0143	0.0166	-0.0023
1.22	C.C6	0.07	-0.C053	C.C141	0.0152	-0.0012
1.23	0.05	0.06	-0.C090	C.C118	0.0139	-0.0021
1.24	C.05	C.05	-0.0052	C.0114	0.0126	-0.0012
1.25	C.04	0.C5	-0.C022	C.01C9	0.0114	-0.0005
1.26	0.04	0.04	0.C040	C.C113	0.0103	0.C010
1.27	C.04	0.03	0.CC94	C.C117	0.C092	0.0026
1.28	C.04	0.03	0.0065	C.01C0	C.CC82	0.C018
1.2S	0.C4	0.03	0.01C7	C.C104	0.0073	0.0031
1.3C	C.03	0.C2	0.0068	C.C085	0.0064	0.0020

THIS IS AN AD FUNCTICN, N =   4

THE EQUATION OF THEWEIBULL FUNCTICN IS EXP(- 16.9913*(R - 0.84)** 1.8130)

THIS IS A COMPARISCN CF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	CIFFERENCE	EXPECTEC UTILITY ACTUAL CATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.96	1.CC	-0.C388	-C.1538	-0.1600	0.0062
0.85	C.96	1.00	-0.0348	-C.1442	-0.1494	0.0052
0.86	0.96	0.99	-0.0248	-C.1346	-0.1380	0.0035
0.87	0.92	0.97	-0.C464	-C.1202	-0.1262	0.0060
0.88	0.92	0.95	-0.0270	-C.11C9	-C.1142	0.0032
0.89	C.92	0.93	-0.0038	-0.1017	-0.1021	0.0004
0.9C	C.92	C.90	0.C229	-C.C925	-0.0902	-0.0023
0.91	0.92	0.87	0.0525	-C.C832	-0.0785	-0.0047
0.93	0.89	0.81	0.C841	-C.0623	-0.0564	-0.0059
0.94	0.86	0.77	C.0871	-C.C514	-0.0462	-0.0052
0.95	0.86	0.73	0.1242	-C.C429	-0.0366	-0.0062
0.96	0.83	0.70	0.1310	-C.C330	-0.0278	-0.0052
0.97	0.8C	0.66	0.1399	-C.C239	-0.C197	-0.0042
0.98	0.74	0.62	0.1238	-0.0148	-0.0124	-0.0025
0.99	0.72	0.58	0.1368	-C.0C72	-0.0058	-0.0014
1.00	0.69	0.54	0.1505	C.C	0.0	0.C
1.01	0.69	0.50	0.1877	C.0C69	0.0050	0.0019
1.02	0.65	C.47	0.1788	0.C129	0.0094	0.0036
1.04	0.55	0.40	0.1505	C.C220	0.0160	0.0060
1.05	C.48	0.37	0.1180	0.0242	0.0183	0.0059
1.06	0.46	0.34	0.1199	C.C273	0.0201	0.0072
1.07	0.42	0.31	0.1094	C.C291	0.0214	0.0077
1.C8	0.36	0.28	C.C789	C.C286	0.C223	0.0063
1.09	0.33	C.25	0.C742	C.C294	0.0227	0.0067
1.1C	0.23	0.23	0.0059	0.0234	0.0228	0.00C6
1.11	0.17	C.21	-0.C357	C.C187	0.0226	-0.0039
1.12	0.14	0.18	-C.0467	C.C165	0.0221	-0.0056
1.13	0.11	0.17	-0.0555	0.0143	0.0215	-0.0072
1.14	0.10	0.15	-C.0463	C.C141	0.C206	-0.0065
1.15	0.08	0.13	-0.C463	C.C127	0.0196	-0.0070
1.16	0.07	0.12	-0.0429	C.C117	0.0186	-0.0069
1.17	0.07	0.10	-C.C368	C.C112	0.0174	-0.0062
1.18	C.0E	0.C9	-0.0281	C.C112	C.0163	-C.0051
1.20	0.06	0.07	-0.0107	C.C118	0.0139	-0.0021
1.21	0.05	0.06	-0.0C87	C.C1C9	0.0128	-0.0018
1.22	0.05	C.05	-C.C041	C.01C7	0.0116	-0.0009
1.23	0.04	0.C5	-0.0C68	C.CC90	0.C105	-0.0016
1.24	0.04	0.04	-0.0037	C.CC86	0.0095	-0.C009
1.25	C.C3	0.03	-C.CC13	C.CC82	0.CC86	-0.0003
1.26	0.03	0.03	0.CC35	C.CC86	0.0C77	C.0009
1.27	C.03	0.03	0.CC77	C.0089	0.0068	0.0021
1.28	0.C3	0.02	0.CC54	C.CC76	0.0060	0.0015
1.29	0.03	0.02	C.CC86	C.C078	0.0C53	0.0025
1.30	C.C2	0.02	0.0C56	C.C064	0.0047	0.0017

THIS IS AN AD FUNCTION, N = 5

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 17.0011*(R - 0.84)** 1.7410)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.95	1.00	-0.0481	-0.1523	-0.1600	0.0077
0.85	0.95	0.99	-0.0425	-0.1428	-0.1492	0.0064
0.86	0.95	0.98	-0.0295	-0.1333	-0.1374	0.0041
0.87	0.91	0.96	-0.0554	-0.1180	-0.1252	0.0072
0.88	0.91	0.94	-0.0319	-0.1089	-0.1127	0.0038
0.89	0.91	0.91	-0.0044	-0.0998	-0.1003	0.0005
0.90	0.91	0.88	0.0265	-0.0907	-0.0881	-0.0027
0.91	0.91	0.85	0.0603	-0.0817	-0.0762	-0.0054
0.93	0.87	0.77	0.0927	-0.0606	-0.0541	-0.0065
0.94	0.83	0.73	0.0532	-0.0497	-0.0441	-0.0056
0.95	0.83	0.69	0.1330	-0.0414	-0.0347	-0.0066
0.96	0.79	0.65	0.1372	-0.0317	-0.0262	-0.0055
0.97	0.76	0.61	0.1438	-0.0227	-0.0184	-0.0043
0.98	0.70	0.57	0.1226	-0.0139	-0.0115	-0.0025
0.99	0.67	0.54	0.1340	-0.0067	-0.0054	-0.0013
1.00	0.64	0.50	0.1461	0.0	0.0	0.0
1.01	0.64	0.46	0.1833	0.0064	0.0046	0.0018
1.02	0.59	0.42	0.1709	0.0119	0.0085	0.0034
1.04	0.49	0.36	0.1377	0.0198	0.0143	0.0055
1.05	0.43	0.33	0.1041	0.0215	0.0163	0.0052
1.06	0.40	0.30	0.1052	0.0241	0.0177	0.0063
1.07	0.36	0.27	0.0945	0.0254	0.0188	0.0066
1.08	0.31	0.24	0.0656	0.0246	0.0194	0.0053
1.09	0.28	0.22	0.0613	0.0252	0.0197	0.0055
1.10	0.20	0.20	0.0003	0.0196	0.0196	0.0000
1.11	0.14	0.18	-0.0349	0.0155	0.0193	-0.0038
1.12	0.11	0.16	-0.0433	0.0136	0.0188	-0.0052
1.13	0.09	0.14	-0.0497	0.0117	0.0181	-0.0065
1.14	0.08	0.12	-0.0412	0.0115	0.0173	-0.0056
1.15	0.07	0.11	-0.0405	0.0103	0.0164	-0.0061
1.16	0.06	0.10	-0.0371	0.0095	0.0154	-0.0059
1.17	0.05	0.08	-0.0314	0.0091	0.0144	-0.0053
1.18	0.05	0.07	-0.0239	0.0091	0.0134	-0.0043
1.20	0.05	0.06	-0.0090	0.0095	0.0113	-0.0018
1.21	0.04	0.05	-0.0072	0.0088	0.0103	-0.0015
1.22	0.04	0.04	-0.0033	0.0087	0.0094	-0.0007
1.23	0.03	0.04	-0.0053	0.0073	0.0085	-0.0012
1.24	0.03	0.03	-0.0028	0.0070	0.0076	-0.0007
1.25	0.03	0.03	-0.0008	0.0066	0.0068	-0.0002
1.26	0.03	0.02	0.0031	0.0069	0.0061	0.0008
1.27	0.03	0.02	0.0065	0.0072	0.0054	0.0018
1.28	0.02	0.02	0.0047	0.0061	0.0048	0.0013
1.29	0.02	0.01	0.0072	0.0063	0.0042	0.0021
1.30	0.02	0.01	0.0048	0.0051	0.0037	0.0014

THIS IS AN AD FUNCTION, N = 6

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9779*(R - 0.84)** 1.6831)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.94	1.00	-0.0571	-0.1509	-0.1600	0.0091
0.85	0.94	0.99	-0.0499	-0.1414	-0.1489	0.0075
0.86	0.94	0.98	-0.0340	-0.1320	-0.1368	0.0048
0.87	0.89	0.95	-0.0637	-0.1158	-0.1241	0.0083
0.88	0.89	0.93	-0.0365	-0.1069	-0.1113	0.0044
0.89	0.89	0.90	-0.0052	-0.0980	-0.0986	0.0006
0.90	0.89	0.86	0.0294	-0.0891	-0.0861	-0.0029
0.91	0.89	0.82	0.0666	-0.0802	-0.0742	-0.0060
0.93	0.84	0.74	0.0989	-0.0590	-0.0521	-0.0069
0.94	0.80	0.70	0.0969	-0.0480	-0.0422	-0.0058
0.95	0.80	0.66	0.1387	-0.0400	-0.0331	-0.0069
0.96	0.76	0.62	0.1404	-0.0304	-0.0248	-0.0056
0.97	0.72	0.58	0.1448	-0.0217	-0.0173	-0.0043
0.98	0.66	0.54	0.1195	-0.0131	-0.0108	-0.0024
0.99	0.63	0.50	0.1295	-0.0063	-0.0050	-0.0013
1.00	0.60	0.46	0.1402	0.0	0.0	0.0
1.01	0.60	0.42	0.1770	0.0060	0.0042	0.0018
1.02	0.55	0.39	0.1622	0.0110	0.0078	0.0032
1.04	0.45	0.32	0.1259	0.0179	0.0129	0.0050
1.05	0.39	0.29	0.0924	0.0193	0.0146	0.0046
1.06	0.36	0.27	0.0931	0.0215	0.0159	0.0056
1.07	0.32	0.24	0.0827	0.0225	0.0167	0.0058
1.08	0.27	0.21	0.0556	0.0216	0.0172	0.0044
1.09	0.24	0.19	0.0517	0.0220	0.0173	0.0047
1.10	0.17	0.17	-0.0030	0.0169	0.0172	-0.0003
1.11	0.12	0.15	-0.0335	0.0132	0.0169	-0.0037
1.12	0.10	0.14	-0.0400	0.0116	0.0164	-0.0048
1.13	0.08	0.12	-0.0449	0.0099	0.0157	-0.0058
1.14	0.07	0.11	-0.0370	0.0098	0.0149	-0.0052
1.15	0.06	0.09	-0.0359	0.0087	0.0141	-0.0054
1.16	0.05	0.08	-0.0325	0.0080	0.0132	-0.0052
1.17	0.04	0.07	-0.0274	0.0076	0.0123	-0.0047
1.18	0.04	0.06	-0.0207	0.0076	0.0114	-0.0037
1.20	0.04	0.05	-0.0078	0.0080	0.0096	-0.0016
1.21	0.04	0.04	-0.0061	0.0074	0.0087	-0.0013
1.22	0.03	0.04	-0.0027	0.0073	0.0079	-0.0006
1.23	0.03	0.03	-0.0044	0.0061	0.0071	-0.0010
1.24	0.02	0.03	-0.0022	0.0058	0.0064	-0.0005
1.25	0.02	0.02	-0.0005	0.0056	0.0057	-0.0001
1.26	0.02	0.02	0.0028	0.0058	0.0050	0.0007
1.27	0.02	0.02	0.0057	0.0060	0.0045	0.0015
1.28	0.02	0.01	0.0041	0.0051	0.0039	0.0012
1.29	0.02	0.01	0.0062	0.0053	0.0035	0.0018
1.30	0.01	0.01	0.0042	0.0043	0.0030	0.0013

THIS IS AN AD FUNCTION, N = 7

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9475*(R - 0.84)** 1.6353)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.93	1.00	-0.0660	-0.1494	-0.1600	0.0106
0.85	0.93	0.99	-0.0570	-0.1401	-0.1486	0.0085
0.86	0.93	0.97	-0.0382	-0.1308	-0.1361	0.0053
0.87	0.87	0.95	-0.0717	-0.1137	-0.1231	0.0093
0.88	0.87	0.92	-0.0410	-0.1050	-0.1099	0.0049
0.89	0.87	0.88	-0.0063	-0.0962	-0.0969	0.0007
0.90	0.87	0.84	0.0315	-0.0875	-0.0843	-0.0032
0.91	0.87	0.80	0.0717	-0.0787	-0.0723	-0.0065
0.93	0.82	0.72	0.1034	-0.0575	-0.0503	-0.0072
0.94	0.77	0.68	0.0988	-0.0465	-0.0405	-0.0059
0.95	0.77	0.63	0.1421	-0.0387	-0.0316	-0.0071
0.96	0.73	0.59	0.1415	-0.0292	-0.0236	-0.0057
0.97	0.69	0.55	0.1439	-0.0207	-0.0164	-0.0043
0.98	0.62	0.51	0.1152	-0.0124	-0.0101	-0.0023
0.99	0.59	0.47	0.1240	-0.0059	-0.0047	-0.0012
1.00	0.56	0.43	0.1336	0.0	0.0	0.0
1.01	0.56	0.39	0.1698	0.0056	0.0039	0.0017
1.02	0.51	0.36	0.1533	0.0102	0.0072	0.0031
1.04	0.41	0.30	0.1154	0.0164	0.0118	0.0046
1.05	0.35	0.27	0.0825	0.0175	0.0134	0.0041
1.06	0.32	0.24	0.0830	0.0194	0.0144	0.0050
1.07	0.29	0.22	0.0730	0.0202	0.0151	0.0051
1.08	0.24	0.19	0.0478	0.0193	0.0155	0.0038
1.09	0.22	0.17	0.0444	0.0195	0.0155	0.0040
1.10	0.15	0.15	-0.0051	0.0149	0.0154	-0.0005
1.11	0.10	0.14	-0.0318	0.0115	0.0150	-0.0035
1.12	0.08	0.12	-0.0371	0.0100	0.0145	-0.0045
1.13	0.07	0.11	-0.0409	0.0085	0.0139	-0.0053
1.14	0.06	0.09	-0.0335	0.0084	0.0131	-0.0047
1.15	0.05	0.08	-0.0322	0.0075	0.0124	-0.0048
1.16	0.04	0.07	-0.0289	0.0069	0.0115	-0.0046
1.17	0.04	0.06	-0.0242	0.0066	0.0107	-0.0041
1.18	0.04	0.05	-0.0182	0.0066	0.0099	-0.0033
1.20	0.03	0.04	-0.0068	0.0069	0.0083	-0.0014
1.21	0.03	0.04	-0.0052	0.0064	0.0075	-0.0011
1.22	0.03	0.03	-0.0023	0.0063	0.0068	-0.0005
1.23	0.02	0.03	-0.0037	0.0052	0.0061	-0.0008
1.24	0.02	0.02	-0.0018	0.0050	0.0054	-0.0004
1.25	0.02	0.02	-0.0003	0.0048	0.0048	-0.0001
1.26	0.02	0.02	0.0026	0.0050	0.0043	0.0007
1.27	0.02	0.01	0.0050	0.0052	0.0038	0.0014
1.28	0.02	0.01	0.0037	0.0044	0.0033	0.0010
1.29	0.02	0.01	0.0055	0.0045	0.0029	0.0016
1.30	0.01	0.01	0.0037	0.0037	0.0026	0.0011

THIS IS AN AD FUNCTION, N = 8

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9161*(R - 0.84)** 1.5949)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.93	1.00	-0.0748	-0.1480	-0.1600	0.0120
0.85	0.93	0.99	-0.0639	-0.1388	-0.1484	0.0096
0.86	0.93	0.97	-0.0423	-0.1295	-0.1355	0.0059
0.87	0.86	0.94	-0.0793	-0.1118	-0.1221	0.0103
0.88	0.86	0.91	-0.0455	-0.1032	-0.1086	0.0055
0.89	0.86	0.87	-0.0077	-0.0946	-0.0954	0.0008
0.90	0.86	0.83	0.0330	-0.0860	-0.0827	-0.0033
0.91	0.86	0.78	0.0757	-0.0774	-0.0706	-0.0068
0.93	0.80	0.70	0.1064	-0.0561	-0.0487	-0.0074
0.94	0.75	0.65	0.0994	-0.0450	-0.0390	-0.0060
0.95	0.75	0.61	0.1438	-0.0375	-0.0303	-0.0072
0.96	0.70	0.56	0.1410	-0.0281	-0.0225	-0.0056
0.97	0.66	0.52	0.1417	-0.0199	-0.0156	-0.0042
0.98	0.59	0.48	0.1104	-0.0118	-0.0096	-0.0022
0.99	0.56	0.44	0.1182	-0.0056	-0.0044	-0.0012
1.00	0.53	0.40	0.1268	0.0	0.0	0.0
1.01	0.53	0.37	0.1623	0.0053	0.0037	0.0016
1.02	0.48	0.33	0.1447	0.0096	0.0067	0.0029
1.04	0.38	0.27	0.1061	0.0152	0.0109	0.0042
1.05	0.32	0.25	0.0742	0.0160	0.0123	0.0037
1.06	0.30	0.22	0.0745	0.0177	0.0132	0.0045
1.07	0.26	0.20	0.0651	0.0184	0.0138	0.0046
1.08	0.22	0.18	0.0416	0.0174	0.0141	0.0033
1.09	0.20	0.16	0.0386	0.0176	0.0141	0.0035
1.10	0.13	0.14	-0.0064	0.0133	0.0139	-0.0006
1.11	0.09	0.12	-0.0302	0.0102	0.0135	-0.0033
1.12	0.07	0.11	-0.0345	0.0089	0.0130	-0.0041
1.13	0.06	0.10	-0.0375	0.0075	0.0124	-0.0049
1.14	0.05	0.08	-0.0306	0.0074	0.0117	-0.0043
1.15	0.04	0.07	-0.0292	0.0066	0.0110	-0.0044
1.16	0.04	0.06	-0.0261	0.0061	0.0102	-0.0042
1.17	0.03	0.06	-0.0217	0.0058	0.0095	-0.0037
1.18	0.03	0.05	-0.0163	0.0058	0.0087	-0.0029
1.20	0.03	0.04	-0.0060	0.0061	0.0073	-0.0012
1.21	0.03	0.03	-0.0046	0.0056	0.0066	-0.0010
1.22	0.02	0.03	-0.0020	0.0055	0.0059	-0.0004
1.23	0.02	0.02	-0.0032	0.0046	0.0053	-0.0007
1.24	0.02	0.02	-0.0014	0.0044	0.0047	-0.0003
1.25	0.02	0.02	-0.0001	0.0042	0.0042	-0.0000
1.26	0.02	0.01	0.0024	0.0044	0.0037	0.0006
1.27	0.02	0.01	0.0045	0.0045	0.0033	0.0012
1.28	0.01	0.01	0.0033	0.0038	0.0029	0.0009
1.29	0.01	0.01	0.0049	0.0040	0.0025	0.0014
1.30	0.01	0.01	0.0033	0.0032	0.0022	0.0010

THIS IS AN AD FUNCTION, N = 9

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.6781*(R - 0.84)** 1.5596)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.92	1.00	-0.0833	-0.1467	-0.1600	0.0133
0.85	0.92	0.99	-0.0706	-0.1375	-0.1481	0.0106
0.86	0.92	0.96	-0.0462	-0.1283	-0.1348	0.0065
0.87	0.84	0.93	-0.0865	-0.1098	-0.1211	0.0112
0.88	0.84	0.89	-0.0497	-0.1014	-0.1073	0.0060
0.89	0.84	0.85	-0.0092	-0.0929	-0.0939	0.0010
0.90	0.84	0.81	0.0340	-0.0845	-0.0811	-0.0034
0.91	0.84	0.77	0.0790	-0.0760	-0.0689	-0.0071
0.93	0.78	0.67	0.1084	-0.0548	-0.0472	-0.0076
0.94	0.73	0.63	0.0993	-0.0436	-0.0377	-0.0060
0.95	0.73	0.58	0.1444	-0.0364	-0.0291	-0.0072
0.96	0.68	0.54	0.1397	-0.0271	-0.0216	-0.0056
0.97	0.64	0.50	0.1388	-0.0191	-0.0149	-0.0042
0.98	0.56	0.46	0.1055	-0.0112	-0.0091	-0.0021
0.99	0.53	0.42	0.1125	-0.0053	-0.0042	-0.0011
1.00	0.50	0.38	0.1203	0.0	0.0	0.0
1.01	0.50	0.34	0.1551	0.0050	0.0034	0.0016
1.02	0.45	0.31	0.1367	0.0090	0.0062	0.0027
1.04	0.35	0.25	0.0980	0.0141	0.0101	0.0039
1.05	0.29	0.23	0.0671	0.0147	0.0114	0.0034
1.06	0.27	0.20	0.0675	0.0163	0.0122	0.0040
1.07	0.24	0.18	0.0586	0.0168	0.0127	0.0041
1.08	0.20	0.16	0.0367	0.0159	0.0129	0.0029
1.09	0.18	0.14	0.0341	0.0160	0.0129	0.0031
1.10	0.12	0.13	-0.0073	0.0120	0.0127	-0.0007
1.11	0.08	0.11	-0.0286	0.0092	0.0123	-0.0031
1.12	0.07	0.10	-0.0321	0.0080	0.0118	-0.0039
1.13	0.05	0.09	-0.0345	0.0067	0.0112	-0.0045
1.14	0.05	0.08	-0.0281	0.0067	0.0106	-0.0039
1.15	0.04	0.07	-0.0266	0.0059	0.0099	-0.0040
1.16	0.03	0.06	-0.0237	0.0054	0.0092	-0.0038
1.17	0.03	0.05	-0.0197	0.0052	0.0085	-0.0033
1.18	0.03	0.04	-0.0147	0.0052	0.0078	-0.0026
1.20	0.03	0.03	-0.0054	0.0054	0.0065	-0.0011
1.21	0.02	0.03	-0.0041	0.0050	0.0059	-0.0009
1.22	0.02	0.02	-0.0017	0.0049	0.0053	-0.0004
1.23	0.02	0.02	-0.0028	0.0041	0.0047	-0.0006
1.24	0.02	0.02	-0.0012	0.0039	0.0042	-0.0003
1.25	0.01	0.01	-0.0000	0.0037	0.0037	-0.0000
1.26	0.01	0.01	0.0022	0.0039	0.0033	0.0006
1.27	0.01	0.01	0.0041	0.0040	0.0029	0.0011
1.28	0.01	0.01	0.0030	0.0034	0.0026	0.0008
1.29	0.01	0.01	0.0044	0.0035	0.0023	0.0013
1.30	0.01	0.01	0.0030	0.0029	0.0020	0.0009

THIS IS AN AD FUNCTION, N = 10

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.8301*(R - 0.84)** 1.5279)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.91	1.00	-0.0917	-0.1453	-0.1600	0.0147
0.85	0.91	0.99	-0.0771	-0.1362	-0.1478	0.0116
0.86	0.91	0.96	-0.0500	-0.1272	-0.1342	0.0070
0.87	0.83	0.92	-0.0933	-0.1080	-0.1201	0.0121
0.88	0.83	0.88	-0.0537	-0.0997	-0.1061	0.0064
0.89	0.83	0.84	-0.0106	-0.0914	-0.0925	0.0012
0.90	0.83	0.80	0.0349	-0.0831	-0.0796	-0.0035
0.91	0.83	0.75	0.0818	-0.0747	-0.0674	-0.0074
0.93	0.76	0.65	0.1099	-0.0535	-0.0458	-0.0077
0.94	0.71	0.61	0.0988	-0.0424	-0.0364	-0.0059
0.95	0.71	0.56	0.1445	-0.0353	-0.0281	-0.0072
0.96	0.66	0.52	0.1380	-0.0262	-0.0207	-0.0055
0.97	0.61	0.47	0.1357	-0.0183	-0.0142	-0.0041
0.98	0.53	0.43	0.1008	-0.0107	-0.0087	-0.0020
0.99	0.50	0.40	0.1071	-0.0050	-0.0040	-0.0011
1.00	0.47	0.36	0.1143	0.0	0.0	0.0
1.01	0.47	0.33	0.1483	0.0047	0.0033	0.0015
1.02	0.42	0.29	0.1294	0.0085	0.0059	0.0026
1.04	0.33	0.24	0.0909	0.0131	0.0095	0.0036
1.05	0.27	0.21	0.0612	0.0137	0.0106	0.0031
1.06	0.25	0.19	0.0616	0.0150	0.0114	0.0037
1.07	0.22	0.17	0.0532	0.0155	0.0118	0.0037
1.08	0.18	0.15	0.0327	0.0146	0.0119	0.0026
1.09	0.16	0.13	0.0304	0.0146	0.0119	0.0027
1.10	0.11	0.12	-0.0077	0.0109	0.0117	-0.0008
1.11	0.08	0.10	-0.0270	0.0083	0.0113	-0.0030
1.12	0.06	0.09	-0.0300	0.0072	0.0108	-0.0036
1.13	0.05	0.08	-0.0320	0.0061	0.0103	-0.0042
1.14	0.04	0.07	-0.0260	0.0060	0.0097	-0.0036
1.15	0.04	0.06	-0.0245	0.0054	0.0090	-0.0037
1.16	0.03	0.05	-0.0217	0.0049	0.0084	-0.0035
1.17	0.03	0.05	-0.0179	0.0047	0.0077	-0.0030
1.18	0.03	0.04	-0.0134	0.0047	0.0071	-0.0024
1.20	0.02	0.03	-0.0048	0.0049	0.0058	-0.0010
1.21	0.02	0.03	-0.0036	0.0045	0.0053	-0.0008
1.22	0.02	0.02	-0.0015	0.0044	0.0047	-0.0003
1.23	0.02	0.02	-0.0024	0.0037	0.0042	-0.0006
1.24	0.01	0.02	-0.0010	0.0035	0.0038	-0.0002
1.25	0.01	0.01	0.0000	0.0034	0.0034	0.0000
1.26	0.01	0.01	0.0020	0.0035	0.0030	0.0005
1.27	0.01	0.01	0.0037	0.0036	0.0026	0.0010
1.28	0.01	0.01	0.0028	0.0031	0.0023	0.0008
1.29	0.01	0.01	0.0040	0.0032	0.0020	0.0012
1.30	0.01	0.01	0.0028	0.0026	0.0018	0.0008

APPENDIX C

COMPUTED RESULTS FOR THE UTILITY FUNCTION

$$u = (1 - e^{-100(r - 0.94)})p(win)$$

THIS IS AN LD FUNCTION

THE EQUATION OF THE WEIBULLFUNCTION IS EXP(- 90.2576*(R - 0.84)** 3.0539)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.97	1.00	-0.0300	*******	*******	*******
0.85	0.97	1.00	-0.0299	*******	*******	*******
0.86	0.95	1.00	-0.0494	*******	*******	*******
0.87	0.95	1.00	-0.0480	*******	*******	52.5740
0.88	0.95	1.00	-0.0452	*******	*******	18.1719
0.89	0.95	0.99	-0.0404	*******	*******	5.9624
0.90	0.95	0.98	-0.0334	*******	*******	1.7895
0.91	0.92	0.97	-0.0535	*******	*******	1.0217
0.92	0.90	0.96	-0.0605	-5.7502	-6.1365	0.3864
0.93	0.90	0.94	-0.0439	-1.5465	-1.6218	0.0753
0.94	0.85	0.92	-0.0734	0.0	0.0	0.0
0.95	0.82	0.90	-0.0788	0.5183	0.5682	-0.0498
0.96	0.82	0.87	-0.0501	0.7090	0.7524	-0.0433
0.97	0.75	0.84	-0.0872	0.7127	0.7956	-0.0829
0.98	0.72	0.80	-0.0803	0.7068	0.7857	-0.0788
0.99	0.67	0.76	-0.0896	0.6655	0.7545	-0.0890
1.00	0.65	0.72	-0.0654	0.6484	0.7136	-0.0652
1.01	0.65	0.67	-0.0183	0.6494	0.6677	-0.0183
1.02	0.60	0.62	-0.0188	0.5998	0.6186	-0.0188
1.03	0.52	0.57	-0.0478	0.5199	0.5677	-0.0478
1.04	0.52	0.52	0.0042	0.5200	0.5158	0.0042
1.05	0.47	0.46	0.0063	0.4700	0.4637	0.0063
1.06	0.45	0.41	0.0376	0.4500	0.4124	0.0376
1.07	0.40	0.36	0.0374	0.4000	0.3626	0.0374
1.08	0.35	0.31	0.0350	0.3500	0.3150	0.0350
1.09	0.32	0.27	0.0498	0.3200	0.2702	0.0498
1.10	0.24	0.23	0.0113	0.2400	0.2287	0.0113
1.11	0.22	0.19	0.0290	0.2200	0.1910	0.0290
1.12	0.14	0.16	-0.0173	0.1400	0.1573	-0.0173
1.13	0.13	0.13	0.0024	0.1300	0.1276	0.0024
1.14	0.13	0.10	0.0281	0.1300	0.1019	0.0281
1.15	0.11	0.08	0.0299	0.1100	0.0801	0.0299
1.16	0.05	0.06	-0.0120	0.0500	0.0620	-0.0120
1.17	0.04	0.05	-0.0071	0.0400	0.0471	-0.0071
1.18	0.02	0.04	-0.0152	0.0200	0.0352	-0.0152
1.19	0.02	0.03	-0.0058	0.0200	0.0258	-0.0058
1.20	0.02	0.02	0.0014	0.0200	0.0186	0.0014
1.21	0.02	0.01	0.0069	0.0200	0.0131	0.0069

THIS IS AN AD FUNCTICN, N = 1

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.C451*(R - 0.84)** 2.3096)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISCN OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.99	1.00	-C.0100	*******	*******	*******
0.85	0.99	1.00	-0.0096	*******	*******	77.8984
0.86	0.99	1.00	-0.CC81	*******	*******	24.1086
0.87	0.98	1.00	-0.0151	*******	*******	16.5828
0.88	C.98	C.99	-0.0106	*******	*******	4.2529
0.89	0.98	C.98	-0.0043	*******	*******	0.6277
0.90	0.98	0.98	0.0039	*******	*******	-0.2083
0.91	C.98	0.97	0.0139	*******	*******	-0.2658
0.93	0.97	0.94	0.0298	-1.6667	-1.6155	-0.0512
C.94	0.96	0.92	0.0356	C.0	0.0	0.0
0.95	0.96	C.91	0.0534	C.6068	0.5731	0.0337
0.96	0.95	0.89	0.0629	0.8214	0.7670	0.0544
0.97	C.94	0.87	0.0743	0.8932	0.8226	0.0706
0.98	0.92	0.84	0.0773	C.9031	0.8273	0.0758
0.99	0.91	0.82	0.0918	C.9039	0.8127	0.0912
1.00	0.90	0.79	0.1078	C.8978	0.7903	0.1075
1.01	0.90	0.76	0.1350	C.8992	C.7643	0.1349
1.02	0.88	0.74	0.1434	0.8797	0.7363	0.1434
1.04	0.83	0.68	0.1529	C.8300	0.6771	0.1529
1.05	0.79	0.65	0.1437	C.7900	0.6463	0.1437
1.06	0.77	0.62	0.1549	0.7700	0.6151	0.1549
1.07	0.74	0.58	0.1564	C.7400	C.5836	0.1564
1.08	0.69	0.55	0.1380	C.6900	0.5520	0.1380
1.09	0.66	0.52	0.1395	C.6600	0.5205	0.1395
1.10	0.55	0.49	0.0607	C.5500	0.4893	0.0607
1.11	0.45	0.46	-0.0085	C.4500	0.4585	-0.0085
1.12	0.39	0.43	-0.0382	C.3900	0.4282	-0.0382
1.13	0.33	0.40	-0.0686	C.3300	0.3986	-0.0686
1.14	0.31	0.37	-0.0598	C.3100	0.3698	-0.0598
1.15	0.27	0.34	-0.C72C	C.2700	0.3420	-0.0720
1.16	0.24	0.32	-0.C752	C.2400	0.3152	-0.0752
1.17	0.22	0.29	-0.0695	0.2200	0.2895	-0.0695
1.18	0.21	0.26	-0.C550	C.2100	C.2650	-0.0550
1.20	0.20	0.22	-0.0197	C.2000	0.2197	-0.0197
1.21	0.18	0.20	-0.C190	C.1800	0.1990	-0.0190
1.22	0.17	0.18	-0.CC96	C.1700	0.1796	-0.0096
1.23	0.14	0.16	-0.0215	C.1400	0.1615	-0.0215
1.24	0.13	0.14	-0.0147	C.1300	0.1447	-0.0147
1.25	0.12	0.13	-0.0092	C.1200	0.1292	-0.0092
1.26	0.12	0.11	0.0051	C.1200	0.1149	0.0051
1.27	0.12	0.10	C.0182	C.12C0	0.1018	0.0182
1.28	0.10	0.09	0.0101	C.1000	0.0899	0.01C1
1.29	0.1C	0.08	C.C2C9	C.1000	0.0791	0.0209

THIS IS AN AD FUNCTICN, N = 2

THE EQUATION OF THEWEIBULL FUNCTICN IS EXP(- 16.7360*(R - 0.84)** 2.0503)

U2 = 1.0 - EXP(-100.C*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISCN CF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	CIFFERENCE	EXPECTED UTILITY ACTUAL CATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.98	1.00	-0.0198	*******	*******	*******
0.85	0.98	1.00	-0.0185	*******	*******	*******
0.86	0.98	0.99	-0.0143	*******	*******	42.6658
0.87	0.96	0.99	-0.0267	*******	*******	29.2175
0.88	0.96	0.98	-0.0167	*******	*******	6.7192
0.89	0.96	0.96	-0.0039	*******	*******	0.5699
C.90	0.96	0.95	0.0117	*******	*******	-0.6294
0.91	0.96	0.93	0.0300	*******	*******	-0.5728
0.93	0.94	0.89	0.0549	-1.6182	-1.5238	-0.0944
0.94	0.92	0.86	0.0616	C.0	0.0	0.0
0.95	0.92	0.83	0.0888	0.5835	0.5273	0.0562
0.96	0.90	0.81	0.0995	0.7823	0.6963	0.0861
0.97	0.89	0.77	0.1121	C.8426	0.7361	0.1065
0.98	0.85	0.74	0.1089	C.8362	0.7293	0.1069
0.99	0.83	0.71	0.1247	0.8292	0.7054	0.1239
1.00	0.82	0.68	0.1416	C.8162	0.6749	0.1413
1.01	0.82	0.64	0.1757	C.8174	0.6419	0.1756
1.02	0.79	0.61	0.1776	C.7855	0.6079	0.1776
1.04	0.71	0.54	0.1701	0.7094	0.5393	0.1700
1.05	0.65	0.51	0.1475	C.6529	0.5054	0.1475
1.06	0.63	0.47	0.1540	C.6260	0.4721	0.1540
1.07	0.59	0.44	0.1479	C.5873	0.4394	0.1479
1.08	0.53	0.41	0.1190	C.5267	0.4077	0.1190
1.C9	C.49	0.38	0.1156	C.4925	0.3770	0.1156
1.10	0.38	0.35	0.0319	C.3793	0.3474	0.0319
1.11	0.29	0.32	-0.0288	C.2903	0.3191	-0.0288
1.12	0.24	0.29	-0.0498	C.2422	0.2921	-0.0498
1.13	0.20	0.27	-0.0688	C.1976	0.2665	-0.0688
1.14	0.18	0.24	-0.0588	C.1834	0.2423	-0.0588
1.15	0.16	0.22	-0.0634	C.1561	0.2195	-0.0634
1.16	0.14	0.20	-0.0619	C.1364	0.1982	-0.0619
1.17	0.12	0.18	-0.0548	C.1236	0.1784	-0.0548
1.18	0.12	0.16	-0.0427	C.1173	0.1600	-0.0427
1.20	0.11	0.13	-0.0163	C.1111	0.1274	-0.0163
1.21	0.1C	0.11	-0.0142	C.C989	0.1131	-0.0142
1.22	0.09	0.10	-0.0072	C.C929	0.1001	-0.0072
1.23	0.C8	0.C9	-0.0130	C.C753	0.0882	-0.0130
1.24	0.07	0.C8	-0.0080	C.C695	0.0775	-0.0080
1.25	0.06	0.C7	-0.0041	C.0638	0.0679	-0.0041
1.26	0.06	0.C6	0.0046	C.C638	0.0592	0.0046
1.27	0.06	0.05	0.0123	C.C638	0.0515	0.0123
1.28	0.05	0.C4	0.0080	C.0526	0.0446	0.0080
1.29	0.05	0.04	C.0141	C.C526	0.0386	0.0141

THIS IS AN AD FUNCTION, N = 3

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9261*(R - 0.84)** 1.9087)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6.

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.97	1.00	-0.0294	*******	*******	*******
0.85	0.97	1.00	-0.0268	*******	*******	*******
0.86	0.97	0.99	-0.0198	*******	*******	58.9456
0.87	0.94	0.98	-0.0369	*******	*******	40.4590
0.88	0.94	0.96	-0.0220	*******	*******	8.8564
0.89	0.94	0.95	-0.0036	*******	*******	0.5276
0.90	0.94	0.92	0.0181	*******	*******	-0.9686
0.91	0.94	0.90	0.0426	*******	*******	-0.8140
0.93	0.92	0.84	0.0721	-1.5724	-1.4485	-0.1239
0.94	0.89	0.81	0.0774	0.0	0.0	0.0
0.95	0.89	0.78	0.1105	0.5619	0.4920	0.0699
0.96	0.86	0.74	0.1197	0.7468	0.6432	0.1035
0.97	0.84	0.71	0.1308	0.7975	0.6732	0.1243
0.98	0.79	0.67	0.1208	0.7786	0.6600	0.1186
0.99	0.77	0.64	0.1354	0.7660	0.6315	0.1345
1.00	0.75	0.60	0.1509	0.7481	0.5977	0.1505
1.01	0.75	0.56	0.1874	0.7493	0.5621	0.1872
1.02	0.71	0.53	0.1831	0.7094	0.5264	0.1831
1.04	0.62	0.46	0.1629	0.6194	0.4564	0.1629
1.05	0.56	0.42	0.1335	0.5563	0.4228	0.1335
1.06	0.53	0.39	0.1371	0.5274	0.3903	0.1371
1.07	0.49	0.36	0.1277	0.4868	0.3591	0.1277
1.08	0.43	0.33	0.0966	0.4259	0.3293	0.0966
1.09	0.39	0.30	0.0919	0.3929	0.3010	0.0919
1.10	0.29	0.27	0.0153	0.2895	0.2742	0.0153
1.11	0.21	0.25	-0.0346	0.2143	0.2489	-0.0346
1.12	0.18	0.23	-0.0496	0.1757	0.2252	-0.0496
1.13	0.14	0.20	-0.0621	0.1410	0.2031	-0.0621
1.14	0.13	0.18	-0.0524	0.1303	0.1826	-0.0524
1.15	0.11	0.16	-0.0539	0.1098	0.1636	-0.0539
1.16	0.10	0.15	-0.0509	0.0952	0.1461	-0.0509
1.17	0.09	0.13	-0.0441	0.0859	0.1301	-0.0441
1.18	0.08	0.12	-0.0340	0.0814	0.1154	-0.0340
1.20	0.08	0.09	-0.0131	0.0769	0.0900	-0.0131
1.21	0.07	0.08	-0.0109	0.0682	0.0791	-0.0109
1.22	0.06	0.07	-0.0053	0.0639	0.0693	-0.0053
1.23	0.05	0.06	-0.0090	0.0515	0.0605	-0.0090
1.24	0.05	0.05	-0.0052	0.0474	0.0526	-0.0052
1.25	0.04	0.05	-0.0022	0.0435	0.0457	-0.0022
1.26	0.04	0.04	0.0040	0.0435	0.0395	0.0040
1.27	0.04	0.03	0.0094	0.0435	0.0340	0.0094
1.28	0.04	0.03	0.0065	0.0357	0.0292	0.0065
1.29	0.04	0.03	0.0107	0.0357	0.0251	0.0107

THIS IS AN AD FUNCTION, N = 4

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9913*(R - 0.84)** 1.8130)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.96	1.00	-0.0388	*******	*******	*******
0.85	0.96	1.00	-0.0348	*******	*******	*******
0.86	0.96	0.99	-0.0248	*******	*******	73.9304
0.87	0.92	0.97	-0.0464	*******	*******	50.8811
0.88	0.92	0.95	-0.0270	*******	*******	10.8857
0.89	0.92	0.93	-0.0038	*******	*******	0.5584
0.90	0.92	0.90	0.0229	*******	*******	-1.2258
0.91	0.92	0.87	0.0525	*******	*******	-1.0014
0.93	0.89	0.81	0.0841	-1.5291	-1.3846	-0.1445
0.94	0.86	0.77	0.0871	0.0	0.0	0.0
0.95	0.86	0.73	0.1242	0.5418	0.4633	0.0785
0.96	0.83	0.70	0.1310	0.7143	0.6010	0.1133
0.97	0.80	0.66	0.1399	0.7569	0.6240	0.1330
0.98	0.74	0.62	0.1238	0.7283	0.6068	0.1215
0.99	0.72	0.58	0.1368	0.7117	0.5759	0.1358
1.00	0.69	0.54	0.1505	0.6906	0.5405	0.1501
1.01	0.69	0.50	0.1877	0.6917	0.5041	0.1875
1.02	0.65	0.47	0.1788	0.6468	0.4681	0.1787
1.04	0.55	0.40	0.1505	0.5496	0.3992	0.1505
1.05	0.48	0.37	0.1180	0.4847	0.3667	0.1180
1.06	0.46	0.34	0.1199	0.4556	0.3357	0.1199
1.07	0.42	0.31	0.1094	0.4157	0.3063	0.1094
1.08	0.36	0.28	0.0789	0.3575	0.2786	0.0789
1.09	0.33	0.25	0.0742	0.3267	0.2525	0.0742
1.10	0.23	0.23	0.0059	0.2340	0.2282	0.0059
1.11	0.17	0.21	-0.0357	0.1698	0.2055	-0.0357
1.12	0.14	0.18	-0.0467	0.1378	0.1845	-0.0467
1.13	0.11	0.17	-0.0555	0.1096	0.1651	-0.0555
1.14	0.10	0.15	-0.0463	0.1010	0.1473	-0.0463
1.15	0.08	0.13	-0.0463	0.0846	0.1310	-0.0463
1.16	0.07	0.12	-0.0429	0.0732	0.1161	-0.0429
1.17	0.07	0.10	-0.0368	0.0659	0.1026	-0.0368
1.18	0.06	0.09	-0.0281	0.0623	0.0904	-0.0281
1.20	0.06	0.07	-0.0107	0.0588	0.0696	-0.0107
1.21	0.05	0.06	-0.0087	0.0520	0.0607	-0.0087
1.22	0.05	0.05	-0.0041	0.0487	0.0529	-0.0041
1.23	0.04	0.05	-0.0068	0.0391	0.0459	-0.0068
1.24	0.04	0.04	-0.0037	0.0360	0.0397	-0.0037
1.25	0.03	0.03	-0.0013	0.0330	0.0342	-0.0013
1.26	0.03	0.03	0.0035	0.0330	0.0294	0.0035
1.27	0.03	0.03	0.0077	0.0330	0.0253	0.0077
1.28	0.03	0.02	0.0054	0.0270	0.0216	0.0054
1.29	0.03	0.02	0.0086	0.0270	0.0184	0.0086

THIS IS AN AD FUNCTION, N = 5

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 17.0011*(R - 0.84)** 1.7410)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.95	1.00	-0.0481	*******	*******	*******
0.85	0.95	0.99	-0.0425	*******	*******	*******
0.86	0.95	0.98	-0.0295	*******	*******	87.9670
0.87	0.91	0.96	-0.0554	*******	*******	60.6521
0.88	0.91	0.94	-0.0319	*******	*******	12.8364
0.89	0.91	0.91	-0.0044	*******	*******	0.6464
0.90	0.91	0.88	0.0265	*******	*******	-1.4221
0.91	0.91	0.85	0.0603	*******	*******	-1.1502
0.93	0.87	0.77	0.0927	-1.4882	-1.3290	-0.1592
0.94	0.83	0.73	0.0932	0.0	0.0	0.0
0.95	0.83	0.69	0.1330	0.5231	0.4391	0.0840
0.96	0.79	0.65	0.1372	0.6845	0.5659	0.1187
0.97	0.76	0.61	0.1438	0.7203	0.5837	0.1367
0.98	0.70	0.57	0.1226	0.6842	0.5638	0.1204
0.99	0.67	0.54	0.1340	0.6646	0.5315	0.1331
1.00	0.64	0.50	0.1461	0.6413	0.4955	0.1457
1.01	0.64	0.46	0.1833	0.6423	0.4591	0.1831
1.02	0.59	0.42	0.1709	0.5944	0.4235	0.1709
1.04	0.49	0.36	0.1377	0.4940	0.3564	0.1377
1.05	0.43	0.33	0.1041	0.4293	0.3252	0.1041
1.06	0.40	0.30	0.1052	0.4010	0.2958	0.1052
1.07	0.36	0.27	0.0945	0.3627	0.2682	0.0945
1.08	0.31	0.24	0.0656	0.3080	0.2424	0.0656
1.09	0.28	0.22	0.0613	0.2797	0.2184	0.0613
1.10	0.20	0.20	0.0003	0.1964	0.1961	0.0003
1.11	0.14	0.18	-0.0349	0.1406	0.1756	-0.0349
1.12	0.11	0.16	-0.0433	0.1134	0.1567	-0.0433
1.13	0.09	0.14	-0.0497	0.0897	0.1394	-0.0497
1.14	0.08	0.12	-0.0412	0.0824	0.1237	-0.0412
1.15	0.07	0.11	-0.0405	0.0689	0.1094	-0.0405
1.16	0.06	0.10	-0.0371	0.0594	0.0965	-0.0371
1.17	0.05	0.08	-0.0314	0.0534	0.0848	-0.0314
1.18	0.05	0.07	-0.0239	0.0505	0.0744	-0.0239
1.20	0.05	0.06	-0.0090	0.0476	0.0567	-0.0090
1.21	0.04	0.05	-0.0072	0.0421	0.0492	-0.0072
1.22	0.04	0.04	-0.0033	0.0394	0.0427	-0.0033
1.23	0.03	0.04	-0.0053	0.0315	0.0369	-0.0053
1.24	0.03	0.03	-0.0028	0.0290	0.0318	-0.0028
1.25	0.03	0.03	-0.0008	0.0265	0.0273	-0.0008
1.26	0.03	0.02	0.0031	0.0265	0.0234	0.0031
1.27	0.03	0.02	0.0065	0.0265	0.0200	0.0065
1.28	0.02	0.02	0.0047	0.0217	0.0171	0.0047
1.29	0.02	0.01	0.0072	0.0217	0.0145	0.0072

THIS IS AN AD FUNCTION, N =   6

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9779*(R - 0.84)** 1.6831)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6.

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.94	1.00	-0.0571	*******	*******	*******
0.85	0.94	0.99	-0.0499	*******	*******	*******
0.86	0.94	0.98	-0.0340	*******	*******	*******
0.87	0.89	0.95	-0.0637	*******	*******	69.8196
0.88	0.89	0.93	-0.0365	*******	*******	14.6943
0.89	0.89	0.90	-0.0052	*******	*******	0.7672
0.90	0.89	0.86	0.0294	*******	*******	-1.5764
0.91	0.89	0.82	0.0666	*******	*******	-1.2716
0.93	0.84	0.74	0.0989	-1.4493	-1.2793	-0.1700
0.94	0.80	0.70	0.0969	0.0	0.0	0.0
0.95	0.80	0.66	0.1387	0.5057	0.4180	0.0877
0.96	0.76	0.62	0.1404	0.6571	0.5357	0.1214
0.97	0.72	0.58	0.1448	0.6871	0.5495	0.1376
0.98	0.66	0.54	0.1195	0.6451	0.5278	0.1173
0.99	0.63	0.50	0.1295	0.6234	0.4948	0.1286
1.00	0.60	0.46	0.1402	0.5985	0.4587	0.1398
1.01	0.60	0.42	0.1770	0.5995	0.4226	0.1768
1.02	0.55	0.39	0.1622	0.5498	0.3877	0.1621
1.04	0.45	0.32	0.1259	0.4486	0.3227	0.1259
1.05	0.39	0.29	0.0924	0.3854	0.2929	0.0924
1.06	0.36	0.27	0.0931	0.3581	0.2651	0.0931
1.07	0.32	0.24	0.0827	0.3217	0.2391	0.0827
1.08	0.27	0.21	0.0556	0.2706	0.2150	0.0556
1.09	0.24	0.19	0.0517	0.2444	0.1927	0.0517
1.10	0.17	0.17	-0.0030	0.1692	0.1722	-0.0030
1.11	0.12	0.15	-0.0335	0.1200	0.1535	-0.0335
1.12	0.10	0.14	-0.0400	0.0963	0.1363	-0.0400
1.13	0.08	0.12	-0.0449	0.0759	0.1208	-0.0449
1.14	0.07	0.11	-0.0370	0.0697	0.1067	-0.0370
1.15	0.06	0.09	-0.0359	0.0581	0.0940	-0.0359
1.16	0.05	0.08	-0.0325	0.0500	0.0825	-0.0325
1.17	0.04	0.07	-0.0274	0.0449	0.0723	-0.0274
1.18	0.04	0.06	-0.0207	0.0424	0.0631	-0.0207
1.20	0.04	0.05	-0.0078	0.0400	0.0478	-0.0078
1.21	0.04	0.04	-0.0061	0.0353	0.0414	-0.0061
1.22	0.03	0.04	-0.0027	0.0330	0.0357	-0.0027
1.23	0.03	0.03	-0.0044	0.0264	0.0308	-0.0044
1.24	0.02	0.03	-0.0022	0.0243	0.0265	-0.0022
1.25	0.02	0.02	-0.0005	0.0222	0.0227	-0.0005
1.26	0.02	0.02	0.0028	0.0222	0.0194	0.0028
1.27	0.02	0.02	0.0057	0.0222	0.0165	0.0057
1.28	0.02	0.01	0.0041	0.0182	0.0141	0.0041
1.29	0.02	0.01	0.0062	0.0182	0.0119	0.0062

THIS IS AN AD FUNCTICN, N = 7

THE EQUATICN OF THEWEIBULL FUNCTICN IS EXP(- 16.9475*(R - 0.84)** 1.6353)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISCN OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL CATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.93	1.00	-0.C660	*******	*******	*******
0.85	0.93	0.99	-0.0570	*******	*******	*******
0.86	0.93	C.97	-0.0382	*******	*******	*******
0.87	0.87	0.95	-0.0717	*******	*******	78.5349
0.88	0.87	C.92	-0.0410	*******	*******	16.5098
0.89	0.87	0.88	-0.0063	*******	*******	0.9313
0.90	0.87	0.84	0.0315	*******	*******	-1.6896
0.91	0.87	C.80	0.0717	*******	*******	-1.3684
0.93	0.82	0.72	0.1034	-1.4125	-1.2349	-0.1776
0.94	C.77	0.68	0.C988	0.0	0.0	0.0
0.95	0.77	0.63	0.1421	C.4894	0.3996	0.0898
0.96	0.73	0.59	0.1415	C.6319	0.5096	0.1223
0.97	0.69	0.55	0.1439	C.6568	0.5201	0.1367
0.98	0.62	0.51	0.1152	C.6102	0.4971	0.1131
C.99	0.59	0.47	0.1240	C.5869	0.4637	0.1232
1.0C	C.56	0.43	0.1336	C.5611	0.4279	0.1332
1.01	0.56	0.39	0.1698	C.5620	0.3924	0.1696
1.02	0.51	0.36	0.1533	C.5115	0.3582	0.1532
1.04	0.41	0.30	0.1154	C.4109	0.2954	0.1154
1.05	0.35	0.27	0.0825	C.3496	0.2670	0.0825
1.06	0.32	0.24	0.0830	0.3235	0.2405	0.0830
1.07	0.29	0.22	C.C730	C.2891	0.2160	0.0730
1.08	0.24	0.19	0.0478	C.2413	0.1935	0.0478
1.C9	0.22	0.17	0.0444	0.2171	0.1727	0.0444
1.10	0.15	0.15	-0.0051	0.1486	0.1537	-0.0051
1.11	0.10	0.14	-0.0318	0.1047	0.1365	-0.C318
1.12	0.C8	0.12	-0.0371	C.C837	0.1208	-0.0371
1.13	0.07	0.11	-0.0409	0.C657	0.1066	-0.04C9
1.14	0.06	0.09	-0.0335	C.C603	0.0938	-0.0335
1.15	C.05	C.C8	-0.0322	C.C5C2	0.0824	-0.0322
1.16	0.04	0.07	-0.0289	C.0432	0.0721	-0.0289
1.17	0.04	0.C6	-0.0242	C.C387	0.0630	-0.0242
1.18	0.04	0.05	-0.0182	C.C366	0.0548	-0.0182
1.20	0.03	0.04	-0.0068	C.0345	0.0413	-0.0068
1.21	0.03	0.04	-0.0052	C.0304	0.0356	-0.C052
1.22	0.03	0.03	-0.0023	0.0284	0.0307	-0.0023
1.23	0.02	0.03	-0.0037	0.0227	0.0264	-0.0037
1.24	0.02	0.02	-0.0018	C.0209	0.0227	-0.0018
1.25	0.02	0.02	-C.0003	C.C191	0.C194	-0.0003
1.26	0.02	0.02	0.0026	0.0191	0.0165	0.0026
1.27	C.02	0.C1	0.0050	C.C191	0.0141	0.0050
1.28	C.02	0.01	0.0037	0.0156	0.0120	0.0037
1.29	0.02	0.01	0.CC55	C.0156	0.0101	0.0055

THIS IS AN AD FUNCTICN, N = 8

THE EQUATION OF THEWEIBULL FUNCTICN IS EXP(- 16.9161*(R - 0.84)** 1.5949)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISCN OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.93	1.00	-0.C748	*******	*******	*******
0.85	0.93	0.99	-0.0639	*******	*******	*******
0.86	0.93	C.97	-0.0423	*******	*******	*******
0.87	0.86	0.94	-0.0793	*******	*******	86.8601
0.88	0.86	0.91	-0.0455	*******	*******	18.2917
0.89	0.86	0.87	-0.0077	*******	*******	1.1334
C.9C	0.86	0.83	0.C330	*******	*******	-1.7680
0.91	0.86	0.78	0.0757	*******	*******	-1.4448
0.93	0.80	0.70	0.1064	-1.3775	-1.1947	-0.1828
0.94	0.75	C.65	0.C994	C.C	0.0	0.0
0.95	0.75	0.61	0.1438	C.4741	0.3832	0.0909
0.96	0.70	C.56	0.1410	0.6C85	0.4865	0.1219
0.97	0.66	0.52	0.1417	C.6290	0.4944	0.1346
0.98	0.59	0.48	0.1104	C.5789	0.4706	0.1084
0.99	0.56	0.44	0.1182	0.5545	0.4371	0.1174
1.0C	0.53	0.40	0.1268	C.5281	0.4016	0.1265
1.01	0.53	0.37	0.1623	C.5289	0.3667	0.1622
1.02	0.48	0.33	0.1447	C.4781	0.3335	0.1446
1.04	0.38	C.27	0.1061	C.379C	0.2729	0.1061
1.05	C.32	0.25	0.C742	0.3198	0.2457	0.0742
1.06	0.30	0.22	0.C745	C.2950	0.2205	0.0745
1.07	0.26	0.20	0.0651	C.2624	0.1973	0.0651
1.C8	0.22	0.18	0.0416	C.2177	0.1760	0.0416
1.09	0.20	0.16	0.C386	C.1953	0.1566	0.0386
1.10	0.13	0.14	-0.0064	G.1325	0.1390	-0.0064
1.11	0.09	0.12	-C.C302	C.C928	0.1230	-0.0302
1.12	0.07	0.11	-0.0345	C.C740	0.1085	-0.0345
1.13	0.06	0.10	-0.0375	C.0580	0.0955	-0.0375
1.14	0.05	C.C8	-0.C306	C.0532	0.0838	-0.0306
1.15	0.04	0.C7	-0.0292	C.0442	0.0733	-0.0292
1.16	0.04	0.06	-0.0261	0.0380	0.0640	-0.0261
1.17	C.03	0.06	-0.0217	C.C341	0.0558	-0.0217
1.18	0.03	0.05	-0.0163	C.C322	0.0484	-0.0163
1.20	0.03	0.04	-0.0060	C.C303	0.0363	-0.0060
1.21	0.03	0.03	-0.0046	C.C267	0.0313	-0.0046
1.22	0.02	0.03	-0.0020	C.0250	0.0269	-C.0020
1.23	0.C2	0.C2	-0.0032	C.C199	0.0231	-0.0032
1.24	0.02	0.02	-0.0014	C.0183	0.0198	-0.0014
1.25	0.02	0.C2	-0.0001	C.0168	0.0169	-0.0001
1.26	0.02	0.01	0.0024	C.C168	0.0144	0.0024
1.27	0.02	0.01	0.0045	C.0168	0.0122	0.0045
1.28	0.01	0.01	0.0C33	C.0137	0.0104	0.0033
1.29	0.01	0.01	0.0C49	C.C137	0.0088	0.0049

THIS IS AN AD FUNCTION, N = 9

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.8781*(R - 0.84)** 1.5596)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.92	1.00	-0.0833	*******	*******	*******
0.85	0.92	0.99	-0.0706	*******	*******	*******
0.86	0.92	0.96	-0.0462	*******	*******	*******
0.87	0.84	0.93	-0.0865	*******	*******	94.7568
0.88	0.84	0.89	-0.0497	*******	*******	20.0032
0.89	0.84	0.85	-0.0092	*******	*******	1.3495
0.90	0.84	0.81	0.0340	*******	*******	-1.8250
0.91	0.84	0.77	0.0790	*******	*******	-1.5073
0.93	0.78	0.67	0.1084	-1.3441	-1.1578	-0.1863
0.94	0.73	0.63	0.0993	0.0	0.0	0.0
0.95	0.73	0.58	0.1444	0.4597	0.3684	0.0913
0.96	0.68	0.54	0.1397	0.5867	0.4659	0.1208
0.97	0.64	0.50	0.1388	0.6035	0.4716	0.1319
0.98	0.56	0.46	0.1055	0.5507	0.4472	0.1035
0.99	0.53	0.42	0.1125	0.5255	0.4138	0.1117
1.00	0.50	0.38	0.1203	0.4988	0.3787	0.1200
1.01	0.50	0.34	0.1551	0.4995	0.3446	0.1549
1.02	0.45	0.31	0.1367	0.4488	0.3122	0.1366
1.04	0.35	0.25	0.0980	0.3517	0.2537	0.0980
1.05	0.29	0.23	0.0671	0.2948	0.2276	0.0671
1.06	0.27	0.20	0.0675	0.2711	0.2037	0.0675
1.07	0.24	0.18	0.0586	0.2403	0.1817	0.0586
1.08	0.20	0.16	0.0367	0.1983	0.1616	0.0367
1.09	0.18	0.14	0.0341	0.1774	0.1434	0.0341
1.10	0.12	0.13	-0.0073	0.1196	0.1268	-0.0073
1.11	0.08	0.11	-0.0286	0.0833	0.1119	-0.0286
1.12	0.07	0.10	-0.0321	0.0663	0.0985	-0.0321
1.13	0.05	0.09	-0.0345	0.0519	0.0864	-0.0345
1.14	0.05	0.08	-0.0281	0.0475	0.0757	-0.0281
1.15	0.04	0.07	-0.0266	0.0395	0.0661	-0.0266
1.16	0.03	0.06	-0.0237	0.0339	0.0576	-0.0237
1.17	0.03	0.05	-0.0197	0.0304	0.0500	-0.0197
1.18	0.03	0.04	-0.0147	0.0287	0.0434	-0.0147
1.20	0.03	0.03	-0.0054	0.0270	0.0324	-0.0054
1.21	0.02	0.03	-0.0041	0.0238	0.0279	-0.0041
1.22	0.02	0.02	-0.0017	0.0223	0.0239	-0.0017
1.23	0.02	0.02	-0.0028	0.0178	0.0205	-0.0028
1.24	0.02	0.02	-0.0012	0.0163	0.0175	-0.0012
1.25	0.01	0.01	-0.0000	0.0149	0.0150	-0.0000
1.26	0.01	0.01	0.0022	0.0149	0.0127	0.0022
1.27	0.01	0.01	0.0041	0.0149	0.0108	0.0041
1.28	0.01	0.01	0.0030	0.0122	0.0092	0.0030
1.29	0.01	0.01	0.0044	0.0122	0.0078	0.0044

THIS IS AN AD FUNCTION, N = 10

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.8301*(R - 0.84)** 1.5279)

U2 = 1.0 - EXP(-100.0*(R(I) - 0.94))

THE OBJECTIVE IS TO MAXIMIZE THE PROBABILITY OF WINNING S.T. LOSS LESS THAN OR EQUAL TO 6,

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.91	1.00	-0.0917	*******	*******	*******
0.85	0.91	0.99	-0.0771	*******	*******	*******
0.86	0.91	0.96	-0.0500	*******	*******	*******
0.87	0.83	0.92	-0.0933	*******	*******	*******
0.88	0.83	0.88	-0.0537	*******	*******	21.6086
0.89	0.83	0.84	-0.0106	*******	*******	1.5585
0.90	0.83	0.80	0.0349	*******	*******	-1.8713
0.91	0.83	0.75	0.0818	*******	*******	-1.5610
0.93	0.76	0.65	0.1099	-1.3124	-1.1235	-0.1889
0.94	0.71	0.61	0.0988	0.0	0.0	0.0
0.95	0.71	0.56	0.1445	0.4462	0.3549	0.0913
0.96	0.66	0.52	0.1380	0.5665	0.4472	0.1193
0.97	0.61	0.47	0.1357	0.5800	0.4510	0.1290
0.98	0.53	0.43	0.1008	0.5251	0.4261	0.0990
0.99	0.50	0.40	0.1071	0.4994	0.3930	0.1064
1.00	0.47	0.36	0.1143	0.4725	0.3585	0.1140
1.01	0.47	0.33	0.1483	0.4733	0.3251	0.1482
1.02	0.42	0.29	0.1294	0.4229	0.2936	0.1293
1.04	0.33	0.24	0.0909	0.3280	0.2371	0.0909
1.05	0.27	0.21	0.0612	0.2734	0.2121	0.0612
1.06	0.25	0.19	0.0616	0.2508	0.1892	0.0616
1.07	0.22	0.17	0.0532	0.2216	0.1683	0.0532
1.08	0.18	0.15	0.0327	0.1821	0.1493	0.0327
1.09	0.16	0.13	0.0304	0.1626	0.1321	0.0304
1.10	0.11	0.12	-0.0077	0.1089	0.1166	-0.0077
1.11	0.08	0.10	-0.0270	0.0756	0.1026	-0.0270
1.12	0.06	0.09	-0.0300	0.0601	0.0901	-0.0300
1.13	0.05	0.08	-0.0320	0.0469	0.0789	-0.0320
1.14	0.04	0.07	-0.0260	0.0430	0.0690	-0.0260
1.15	0.04	0.06	-0.0245	0.0357	0.0601	-0.0245
1.16	0.03	0.05	-0.0217	0.0306	0.0523	-0.0217
1.17	0.03	0.05	-0.0179	0.0274	0.0454	-0.0179
1.18	0.03	0.04	-0.0134	0.0259	0.0393	-0.0134
1.20	0.02	0.03	-0.0048	0.0244	0.0292	-0.0048
1.21	0.02	0.03	-0.0036	0.0215	0.0251	-0.0036
1.22	0.02	0.02	-0.0015	0.0201	0.0216	-0.0015
1.23	0.02	0.02	-0.0024	0.0160	0.0184	-0.0024
1.24	0.01	0.02	-0.0010	0.0147	0.0158	-0.0010
1.25	0.01	0.01	0.0000	0.0135	0.0134	0.0000
1.26	0.01	0.01	0.0020	0.0135	0.0114	0.0020
1.27	0.01	0.01	0.0037	0.0135	0.0097	0.0037
1.28	0.01	0.01	0.0028	0.0110	0.0082	0.0028
1.29	0.01	0.01	0.0040	0.0110	0.0070	0.0040

APPENDIX D

COMPUTED RESULTS FOR THE UTILITY FUNCTION

$$u = (r - 1.06)p(win)$$

THIS IS AN LD FUNCTION

THE EQUATION OF THE WEIBULLFUNCTION IS EXP(- 90.2576*(R - 0.84)** 3.0539)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.97	1.00	-0.0300	-0.2134	-0.2200	0.0066
0.85	0.97	1.00	-0.0299	-0.2037	-0.2100	0.0063
0.86	0.95	1.00	-0.0494	-0.1900	-0.1999	0.0099
0.87	0.95	1.00	-0.0480	-0.1805	-0.1896	0.0091
0.88	0.95	1.00	-0.0452	-0.1710	-0.1791	0.0081
0.89	0.95	0.99	-0.0404	-0.1615	-0.1684	0.0069
0.90	0.95	0.98	-0.0334	-0.1520	-0.1573	0.0053
0.91	0.92	0.97	-0.0535	-0.1380	-0.1460	0.0080
0.92	0.90	0.96	-0.0605	-0.1260	-0.1345	0.0085
0.93	0.90	0.94	-0.0439	-0.1170	-0.1227	0.0057
0.94	0.85	0.92	-0.0734	-0.1020	-0.1108	0.0088
0.95	0.82	0.90	-0.0788	-0.0902	-0.0989	0.0087
0.96	0.82	0.87	-0.0501	-0.0820	-0.0870	0.0050
0.97	0.75	0.84	-0.0872	-0.0675	-0.0754	0.0079
0.98	0.72	0.80	-0.0803	-0.0576	-0.0640	0.0064
0.99	0.67	0.76	-0.0896	-0.0469	-0.0532	0.0063
1.00	0.65	0.72	-0.0654	-0.0390	-0.0429	0.0039
1.01	0.65	0.67	-0.0183	-0.0325	-0.0334	0.0009
1.02	0.60	0.62	-0.0188	-0.0240	-0.0248	0.0008
1.03	0.52	0.57	-0.0478	-0.0156	-0.0170	0.0014
1.04	0.52	0.52	0.0042	-0.0104	-0.0103	-0.0001
1.05	0.47	0.46	0.0063	-0.0047	-0.0046	-0.0001
1.06	0.45	0.41	0.0376	0.0	0.0	0.0
1.07	0.40	0.36	0.0374	0.0040	0.0036	0.0004
1.08	0.35	0.31	0.0350	0.0070	0.0063	0.0007
1.09	0.32	0.27	0.0498	0.0096	0.0081	0.0015
1.10	0.24	0.23	0.0113	0.0096	0.0091	0.0005
1.11	0.22	0.19	0.0290	0.0110	0.0096	0.0014
1.12	0.14	0.16	-0.0173	0.0084	0.0094	-0.0010
1.13	0.13	0.13	0.0024	0.0091	0.0089	0.0002
1.14	0.13	0.10	0.0281	0.0104	0.0082	0.0022
1.15	0.11	0.08	0.0299	0.0099	0.0072	0.0027
1.16	0.05	0.06	-0.0120	0.0050	0.0062	-0.0012
1.17	0.04	0.05	-0.0071	0.0044	0.0052	-0.0008
1.18	0.02	0.04	-0.0152	0.0024	0.0042	-0.0018
1.19	0.02	0.03	-0.0058	0.0026	0.0034	-0.0008
1.20	0.02	0.02	0.0014	0.0028	0.0026	0.0002
1.21	0.02	0.01	0.0069	0.0030	0.0020	0.0010

THIS IS AN AC FUNCTION, N = 1

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.0451*(R - 0.84)** 2.3096)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.99	1.00	-0.0100	-0.2178	-0.2200	0.0022
0.85	0.99	1.00	-0.0096	-0.2079	-0.2099	0.0020
0.86	0.99	1.00	-0.0081	-0.1980	-0.1996	0.0016
0.87	0.98	1.00	-0.0151	-0.1862	-0.1891	0.0029
0.88	0.98	0.99	-0.0106	-0.1764	-0.1783	0.0019
0.89	0.96	0.98	-0.0043	-0.1666	-0.1673	0.0007
0.90	0.98	0.98	0.0039	-0.1568	-0.1562	-0.0006
0.91	0.98	0.97	0.0139	-0.1470	-0.1449	-0.0021
0.93	0.97	0.94	0.0298	-0.1261	-0.1222	-0.0039
0.94	0.96	0.92	0.0356	-0.1152	-0.1109	-0.0043
0.95	0.96	0.91	0.0534	-0.1056	-0.0997	-0.0059
0.96	0.95	0.89	0.0629	-0.0950	-0.0887	-0.0063
0.97	0.94	0.87	0.0743	-0.0846	-0.0779	-0.0067
0.98	0.92	0.84	0.0773	-0.0736	-0.0674	-0.0062
0.99	0.91	0.82	0.0918	-0.0637	-0.0573	-0.0064
1.00	0.90	0.79	0.1078	-0.0540	-0.0475	-0.0065
1.01	0.90	0.76	0.1350	-0.0450	-0.0382	-0.0068
1.02	0.88	0.74	0.1434	-0.0352	-0.0295	-0.0057
1.04	0.83	0.68	0.1529	-0.0166	-0.0135	-0.0031
1.05	0.79	0.65	0.1437	-0.0079	-0.0065	-0.0014
1.06	0.77	0.62	0.1549	0.0	0.0	0.0
1.07	0.74	0.58	0.1564	0.0074	0.0058	0.0016
1.08	0.69	0.55	0.1380	0.0138	0.0110	0.0028
1.09	0.66	0.52	0.1395	0.0198	0.0156	0.0042
1.10	0.55	0.49	0.0607	0.0220	0.0196	0.0024
1.11	0.45	0.46	-0.0085	0.0225	0.0229	-0.0004
1.12	0.39	0.43	-0.0382	0.0234	0.0257	-0.0023
1.13	0.33	0.40	-0.0686	0.0231	0.0279	-0.0048
1.14	0.31	0.37	-0.0598	0.0248	0.0296	-0.0048
1.15	0.27	0.34	-0.0720	0.0243	0.0308	-0.0065
1.16	0.24	0.32	-0.0752	0.0240	0.0315	-0.0075
1.17	0.22	0.29	-0.0695	0.0242	0.0318	-0.0076
1.18	0.21	0.26	-0.0550	0.0252	0.0318	-0.0066
1.20	0.20	0.22	-0.0197	0.0280	0.0308	-0.0028
1.21	0.18	0.20	-0.0190	0.0270	0.0298	-0.0028
1.22	0.17	0.18	-0.0096	0.0272	0.0287	-0.0015
1.23	0.14	0.16	-0.0215	0.0238	0.0275	-0.0037
1.24	0.13	0.14	-0.0147	0.0234	0.0260	-0.0026
1.25	0.12	0.13	-0.0092	0.0228	0.0245	-0.0017
1.26	0.12	0.11	0.0051	0.0240	0.0230	0.0010
1.27	0.12	0.10	0.0182	0.0252	0.0214	0.0038
1.28	0.10	0.09	0.0101	0.0220	0.0198	0.0022
1.29	0.10	0.08	0.0209	0.0230	0.0182	0.0048
1.30	0.08	0.07	0.0107	0.0152	0.0166	0.0026

THIS IS AN AD FUNCTION, N = 2

THE EQUATION OF THEWEIBULL FUNCTICN IS EXP(- 16.7360*(R - 0.84)** 2.0503)

U3 = (R - 1.C6)

THIS IS A COMPARISCN CF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.98	1.00	-0.0198	-C.2156	-0.2200	0.0044
0.85	C.98	1.00	-0.0185	-C.2058	-0.2097	0.0039
0.86	C.98	0.99	-0.0143	-C.1960	-0.1989	0.0029
C.87	0.96	0.99	-0.0267	-0.1825	-0.1876	0.0051
0.88	C.96	0.98	-0.0167	-C.1729	-0.1759	0.0030
0.89	0.96	C.96	-0.0039	-C.1633	-0.1640	0.0007
C.90	0.96	C.95	0.0117	-C.1537	-0.1518	-0.0019
0.91	0.96	0.93	0.C300	-C.1441	-0.1396	-0.0045
0.93	0.94	0.89	0.0549	-C.1224	-0.1153	-0.0071
0.94	0.92	0.86	0.0616	-C.1108	-0.1034	-0.0074
0.95	0.92	0.83	0.0888	-C.1015	-0.0918	-0.0098
0.96	0.90	0.81	0.0995	-C.0905	-0.0805	-C.0100
0.97	0.89	0.77	0.1121	-C.0798	-0.0697	-0.0101
0.98	0.85	0.74	C.1C89	-C.C681	-0.C594	-0.0087
0.99	0.83	0.71	0.1247	-0.0584	-0.0497	-0.0087
1.00	0.82	0.68	0.1416	-C.0491	-0.0406	-0.0085
1.01	0.82	0.64	0.1757	-C.04C9	-0.0321	-0.0088
1.02	0.79	0.61	0.1776	-C.0314	-0.0243	-0.0071
1.04	C.71	0.54	0.1701	-C.0142	-0.0108	-0.0034
1.05	0.65	0.51	0.1475	-C.C065	-0.0051	-0.0015
1.C6	0.63	0.47	0.1540	C.C	0.0	0.C
1.07	0.59	0.44	0.1479	C.CC59	0.0044	0.0015
1.C8	0.53	0.41	0.1190	0.0105	0.0082	0.0024
1.09	0.49	0.38	0.1156	C.0148	0.0113	0.0035
1.10	0.38	0.35	0.0319	C.C152	0.0139	0.0013
1.11	0.29	0.32	-0.0288	C.0145	0.0160	-0.0014
1.12	0.24	0.29	-C.0498	C.C145	0.0175	-0.0030
1.13	0.20	0.27	-0.C688	C.C138	0.0187	-0.0048
1.14	0.18	C.24	-0.C588	C.C147	0.0194	-0.0047
1.15	0.16	0.22	-0.0634	C.0140	0.0198	-0.0057
1.16	0.14	0.20	-0.0619	C.C136	0.0198	-0.0062
1.17	C.12	0.18	-0.C548	C.0136	0.0196	-0.0060
1.18	0.12	0.16	-0.0427	C.C141	0.0192	-0.0051
1.20	0.11	0.13	-0.C163	0.0156	0.0178	-0.0023
1.21	0.1C	0.11	-0.C142	C.C148	C.0170	-0.0021
1.22	0.09	0.10	-0.0072	0.C149	0.0160	-0.0011
1.23	0.08	0.C9	-0.0130	C.C128	0.0150	-0.0022
1.24	0.07	0.C8	-0.0080	C.C125	0.0140	-0.0014
1.25	0.06	0.07	-0.0041	0.0121	0.0129	-0.0008
1.26	0.06	0.06	0.0046	C.C128	0.0118	0.0009
1.27	C.06	0.05	0.0123	C.C134	0.0108	0.0026
1.28	0.05	C.04	0.0080	0.0116	0.C098	0.0018
1.29	0.C5	C.04	0.0141	C.C121	0.CC89	0.0032
1.30	0.04	0.03	0.0085	C.C1C0	0.0080	0.0020

THIS IS AN AD FUNCTION, N = 3

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9261*(R - 0.84)** 1.9087)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.97	1.00	-0.0294	-0.2135	-0.2200	0.0065
0.85	0.97	1.00	-0.0268	-0.2038	-0.2095	0.0056
0.86	0.97	0.99	-0.0198	-0.1941	-0.1981	0.0040
0.87	0.94	0.98	-0.0369	-0.1790	-0.1861	0.0070
0.88	0.94	0.96	-0.0220	-0.1696	-0.1736	0.0040
0.89	0.94	0.95	-0.0036	-0.1602	-0.1608	0.0006
0.90	0.94	0.92	0.0181	-0.1508	-0.1479	-0.0029
0.91	0.94	0.90	0.0426	-0.1413	-0.1349	-0.0064
0.93	0.92	0.84	0.0721	-0.1190	-0.1096	-0.0094
0.94	0.89	0.81	0.0774	-0.1067	-0.0974	-0.0093
0.95	0.89	0.78	0.1105	-0.0978	-0.0856	-0.0122
0.96	0.86	0.74	0.1197	-0.0864	-0.0744	-0.0120
0.97	0.84	0.71	0.1308	-0.0755	-0.0638	-0.0118
0.98	0.79	0.67	0.1208	-0.0634	-0.0538	-0.0097
0.99	0.77	0.64	0.1354	-0.0540	-0.0445	-0.0095
1.00	0.75	0.60	0.1509	-0.0450	-0.0359	-0.0091
1.01	0.75	0.56	0.1874	-0.0375	-0.0281	-0.0094
1.02	0.71	0.53	0.1831	-0.0284	-0.0211	-0.0073
1.04	0.62	0.46	0.1629	-0.0124	-0.0091	-0.0033
1.05	0.56	0.42	0.1335	-0.0056	-0.0042	-0.0013
1.06	0.53	0.39	0.1371	0.0	0.0	0.0
1.07	0.49	0.36	0.1277	0.0049	0.0036	0.0013
1.08	0.43	0.33	0.0966	0.0085	0.0066	0.0019
1.09	0.39	0.30	0.0919	0.0118	0.0090	0.0028
1.10	0.29	0.27	0.0153	0.0116	0.0110	0.0006
1.11	0.21	0.25	-0.0346	0.0107	0.0124	-0.0017
1.12	0.18	0.23	-0.0496	0.0105	0.0135	-0.0030
1.13	0.14	0.20	-0.0621	0.0099	0.0142	-0.0043
1.14	0.13	0.18	-0.0524	0.0104	0.0146	-0.0042
1.15	0.11	0.16	-0.0539	0.0099	0.0147	-0.0048
1.16	0.10	0.15	-0.0509	0.0095	0.0146	-0.0051
1.17	0.09	0.13	-0.0441	0.0095	0.0143	-0.0049
1.18	0.08	0.12	-0.0340	0.0098	0.0138	-0.0041
1.20	0.08	0.09	-0.0131	0.0108	0.0126	-0.0018
1.21	0.07	0.08	-0.0109	0.0102	0.0119	-0.0016
1.22	0.06	0.07	-0.0053	0.0102	0.0111	-0.0009
1.23	0.05	0.06	-0.0090	0.0088	0.0103	-0.0015
1.24	0.05	0.05	-0.0052	0.0085	0.0095	-0.0009
1.25	0.04	0.05	-0.0022	0.0083	0.0087	-0.0004
1.26	0.04	0.04	0.0040	0.0087	0.0079	0.0008
1.27	0.04	0.03	0.0094	0.0091	0.0071	0.0020
1.28	0.04	0.03	0.0065	0.0079	0.0064	0.0014
1.29	0.04	0.03	0.0107	0.0082	0.0058	0.0025
1.30	0.03	0.02	0.0068	0.0068	0.0051	0.0016

THIS IS AN AC FUNCTION, N = 4

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9913*(R - 0.84)** 1.8130)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.96	1.00	-0.0388	-0.2115	-0.2200	0.0085
0.85	0.96	1.00	-0.0348	-0.2018	-0.2092	0.0073
0.86	0.96	0.99	-0.0248	-0.1922	-0.1972	0.0050
0.87	0.92	0.97	-0.0464	-0.1757	-0.1845	0.0088
0.88	0.92	0.95	-0.0270	-0.1664	-0.1713	0.0049
0.89	0.92	0.93	-0.0038	-0.1572	-0.1578	0.0006
0.90	0.92	0.90	0.0229	-0.1479	-0.1443	-0.0037
0.91	0.92	0.87	0.0525	-0.1387	-0.1308	-0.0079
0.93	0.89	0.81	0.0841	-0.1157	-0.1048	-0.0109
0.94	0.86	0.77	0.0871	-0.1029	-0.0924	-0.0105
0.95	0.86	0.73	0.1242	-0.0943	-0.0806	-0.0137
0.96	0.83	0.70	0.1310	-0.0826	-0.0695	-0.0131
0.97	0.80	0.66	0.1399	-0.0717	-0.0591	-0.0126
0.98	0.74	0.62	0.1238	-0.0594	-0.0495	-0.0099
0.99	0.72	0.58	0.1368	-0.0502	-0.0406	-0.0096
1.00	0.69	0.54	0.1505	-0.0415	-0.0325	-0.0090
1.01	0.69	0.50	0.1877	-0.0346	-0.0252	-0.0094
1.02	0.65	0.47	0.1788	-0.0259	-0.0187	-0.0072
1.04	0.55	0.40	0.1505	-0.0110	-0.0080	-0.0030
1.05	0.48	0.37	0.1180	-0.0048	-0.0037	-0.0012
1.06	0.46	0.34	0.1199	0.0	0.0	0.0
1.07	0.42	0.31	0.1094	0.0042	0.0031	0.0011
1.08	0.36	0.28	0.0789	0.0072	0.0056	0.0016
1.09	0.33	0.25	0.0742	0.0098	0.0076	0.0022
1.10	0.23	0.23	0.0059	0.0094	0.0091	0.0002
1.11	0.17	0.21	-0.0357	0.0085	0.0103	-0.0018
1.12	0.14	0.18	-0.0467	0.0083	0.0111	-0.0028
1.13	0.11	0.17	-0.0555	0.0077	0.0116	-0.0039
1.14	0.10	0.15	-0.0463	0.0081	0.0118	-0.0037
1.15	0.08	0.13	-0.0463	0.0076	0.0118	-0.0042
1.16	0.07	0.12	-0.0429	0.0073	0.0116	-0.0043
1.17	0.07	0.10	-0.0368	0.0072	0.0113	-0.0040
1.18	0.0[illegible]	0.09	-0.0281	0.0075	0.0109	-0.0034
1.20	0.06	0.07	-0.0107	0.0082	0.0097	-0.0015
1.21	0.05	0.06	-0.0087	0.0078	0.0091	-0.0013
1.22	0.05	0.05	-0.0041	0.0078	0.0085	-0.0007
1.23	0.04	0.05	-0.0068	0.0066	0.0078	-0.0011
1.24	0.04	0.04	-0.0037	0.0065	0.0071	-0.0007
1.25	0.03	0.03	-0.0013	0.0063	0.0065	-0.0002
1.26	0.03	0.03	0.0035	0.0066	0.0059	0.0007
1.27	0.03	0.03	0.0077	0.0069	0.0053	0.0016
1.28	0.03	0.02	0.0054	0.0059	0.0048	0.0012
1.29	0.03	0.02	0.0086	0.0062	0.0042	0.0020
1.30	0.02	0.02	0.0056	0.0051	0.0038	0.0014

THIS IS AN AD FUNCTICN, N = 5

THE EQUATION OF THEWEIBULL FUNCTICN IS EXP(- 17.0011*(R - 0.84)** 1.7410)

U3 = (R - 1.06)

THIS IS A COMPARISON CF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	CIFFERENCE	EXPECTED UTILITY ACTUAL CATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.95	1.00	-0.0481	-C.2C94	-0.2200	0.0106
0.85	0.95	0.99	-0.0425	-C.1999	-0.2088	0.0089
0.86	C.95	C.98	-0.0295	-C.1904	-0.1963	0.0059
0.87	C.91	0.96	-0.0554	-C.1724	-0.1829	0.0105
0.88	0.91	0.94	-0.0319	-C.1633	-0.1691	0.0057
0.89	0.91	0.91	-0.0044	-C.1543	-0.1550	0.0007
0.90	0.91	0.88	0.0265	-C.1452	-0.1409	-0.0042
0.91	0.91	0.85	0.0603	-C.1361	-0.1271	-0.0090
0.93	0.87	C.77	0.C927	-C.1126	-0.1005	-0.0120
0.94	0.83	0.73	0.0932	-C.C993	-0.0881	-0.0112
0.95	0.83	0.69	0.1330	-0.C910	-0.0764	-0.0146
0.96	0.79	0.65	0.1372	-C.0792	-0.0654	-0.0137
C.97	0.76	0.61	0.1438	-C.C682	-0.0553	-0.0129
0.98	0.70	0.57	0.1226	-C.C558	-0.0459	-0.0098
0.99	0.67	0.54	0.1340	-C.C468	-0.0375	-0.0094
1.00	0.64	0.50	0.1461	-C.C386	-0.0298	-0.0088
1.01	0.64	0.46	0.1833	-C.0321	-0.0230	-0.0092
1.02	0.59	0.42	C.1709	-C.C238	-0.0169	-0.0068
1.04	0.49	C.36	0.1377	-0.0099	-0.0071	-0.0028
1.05	0.43	0.33	0.1041	-0.0043	-0.0033	-0.0010
1.06	0.40	0.30	0.1052	C.C	0.0	0.0
1.07	0.36	0.27	0.C945	0.0036	0.0027	0.0009
1.C8	0.31	0.24	0.C656	C.0062	0.0048	0.0013
1.09	0.28	0.22	0.0613	C.CC84	0.0066	0.0018
1.10	0.20	0.20	0.0003	C.CC79	0.0078	0.0000
1.11	0.14	0.18	-0.0349	C.0070	0.0C88	-0.0017
1.12	0.11	0.16	-0.0433	0.0068	0.C094	-0.0026
1.13	0.09	0.14	-0.0497	C.0063	0.0098	-0.0035
1.14	0.C8	0.12	-0.0412	C.C066	0.0099	-0.0033
1.15	0.07	0.11	-0.0405	C.0062	0.0098	-0.0036
1.16	0.06	0.10	-0.0371	C.C059	0.CC96	-0.0037
1.17	0.05	0.08	-0.0314	C.C059	0.0093	-0.0035
1.18	C.05	0.C7	-0.0239	C.C061	0.0C89	-0.0029
1.20	0.05	0.06	-0.0090	C.0067	0.0079	-0.0013
1.21	0.04	0.05	-0.0072	0.0063	0.0074	-0.0011
1.22	0.04	0.04	-0.0033	C.C063	0.0068	-0.0005
1.23	0.03	0.04	-0.0053	C.C054	0.0063	-0.0009
1.24	0.C3	0.C3	-0.0028	0.0052	0.0057	-0.0005
1.25	0.C3	0.03	-0.0008	C.CC50	0.0052	-0.0001
1.26	0.03	0.02	0.0031	C.C053	0.0047	0.0006
1.27	C.C3	0.02	0.0065	C.0056	0.0042	0.0014
1.28	0.02	0.02	0.0047	C.0048	0.0038	0.0010
1.29	0.02	0.01	0.0072	0.0050	0.0C33	0.0017
1.30	0.02	0.01	0.0048	C.C041	0.0029	0.0012

THIS IS AN AD FUNCTION, N = 6

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9779*(R - 0.84)** 1.6831)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.94	1.00	-0.0571	-0.2074	-0.2200	0.0126
0.85	0.94	0.99	-0.0499	-0.1980	-0.2085	0.0105
0.86	0.94	0.96	-0.0340	-0.1886	-0.1954	0.0068
0.87	0.89	0.95	-0.0637	-0.1693	-0.1814	0.0121
0.88	0.89	0.93	-0.0365	-0.1604	-0.1669	0.0066
0.89	0.89	0.90	-0.0052	-0.1515	-0.1523	0.0009
0.90	0.89	0.86	0.0294	-0.1425	-0.1378	-0.0047
0.91	0.89	0.82	0.0666	-0.1336	-0.1236	-0.0100
0.93	0.84	0.74	0.0989	-0.1097	-0.0968	-0.0129
0.94	0.80	0.70	0.0969	-0.0960	-0.0844	-0.0116
0.95	0.80	0.66	0.1387	-0.0880	-0.0727	-0.0153
0.96	0.76	0.62	0.1404	-0.0760	-0.0620	-0.0140
0.97	0.72	0.58	0.1448	-0.0651	-0.0520	-0.0130
0.98	0.66	0.54	0.1195	-0.0526	-0.0430	-0.0096
0.99	0.63	0.50	0.1295	-0.0439	-0.0349	-0.0091
1.00	0.60	0.46	0.1402	-0.0360	-0.0276	-0.0084
1.01	0.60	0.42	0.1770	-0.0300	-0.0212	-0.0088
1.02	0.55	0.39	0.1622	-0.0220	-0.0155	-0.0065
1.04	0.45	0.32	0.1259	-0.0090	-0.0065	-0.0025
1.05	0.39	0.29	0.0924	-0.0039	-0.0029	-0.0009
1.06	0.36	0.27	0.0931	0.0	0.0	0.0
1.07	0.32	0.24	0.0827	0.0032	0.0024	0.0008
1.08	0.27	0.21	0.0556	0.0054	0.0043	0.0011
1.09	0.24	0.19	0.0517	0.0073	0.0058	0.0016
1.10	0.17	0.17	-0.0030	0.0068	0.0069	-0.0001
1.11	0.12	0.15	-0.0335	0.0060	0.0077	-0.0017
1.12	0.10	0.14	-0.0400	0.0058	0.0082	-0.0024
1.13	0.08	0.12	-0.0449	0.0053	0.0085	-0.0031
1.14	0.07	0.11	-0.0370	0.0056	0.0085	-0.0030
1.15	0.06	0.09	-0.0359	0.0052	0.0085	-0.0032
1.16	0.05	0.06	-0.0325	0.0050	0.0083	-0.0033
1.17	0.04	0.07	-0.0274	0.0049	0.0080	-0.0030
1.18	0.04	0.06	-0.0207	0.0051	0.0076	-0.0025
1.20	0.04	0.05	-0.0078	0.0056	0.0067	-0.0011
1.21	0.04	0.04	-0.0061	0.0053	0.0062	-0.0009
1.22	0.03	0.04	-0.0027	0.0053	0.0057	-0.0004
1.23	0.03	0.03	-0.0044	0.0045	0.0052	-0.0007
1.24	0.02	0.03	-0.0022	0.0044	0.0048	-0.0004
1.25	0.02	0.02	-0.0005	0.0042	0.0043	-0.0001
1.26	0.02	0.02	0.0028	0.0044	0.0039	0.0006
1.27	0.02	0.02	0.0057	0.0047	0.0035	0.0012
1.28	0.02	0.01	0.0041	0.0040	0.0031	0.0009
1.29	0.02	0.01	0.0062	0.0042	0.0027	0.0014
1.30	0.01	0.01	0.0042	0.0034	0.0024	0.0010

THIS IS AN AD FUNCTION, N = 7

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9475*(R - 0.84)** 1.6353)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.93	1.00	-0.0660	-0.2055	-0.2200	0.0145
0.85	0.93	0.99	-0.0570	-0.1961	-0.2081	0.0120
0.86	0.93	0.97	-0.0382	-0.1868	-0.1944	0.0076
0.87	0.87	0.95	-0.0717	-0.1662	-0.1799	0.0136
0.88	0.87	0.92	-0.0410	-0.1575	-0.1649	0.0074
0.89	0.87	0.88	-0.0063	-0.1487	-0.1498	0.0011
0.90	0.87	0.84	0.0315	-0.1400	-0.1350	-0.0050
0.91	0.87	0.80	0.0717	-0.1312	-0.1205	-0.0108
0.93	0.82	0.72	0.1034	-0.1069	-0.0934	-0.0134
0.94	0.77	0.68	0.0988	-0.0929	-0.0810	-0.0119
0.95	0.77	0.63	0.1421	-0.0852	-0.0695	-0.0156
0.96	0.73	0.59	0.1415	-0.0731	-0.0589	-0.0141
0.97	0.69	0.55	0.1439	-0.0622	-0.0493	-0.0129
0.98	0.62	0.51	0.1152	-0.0497	-0.0405	-0.0092
0.99	0.59	0.47	0.1240	-0.0414	-0.0327	-0.0087
1.00	0.56	0.43	0.1336	-0.0337	-0.0257	-0.0080
1.01	0.56	0.39	0.1698	-0.0281	-0.0196	-0.0085
1.02	0.51	0.36	0.1533	-0.0205	-0.0143	-0.0061
1.04	0.41	0.30	0.1154	-0.0082	-0.0059	-0.0023
1.05	0.35	0.27	0.0825	-0.0035	-0.0027	-0.0008
1.06	0.32	0.24	0.0830	0.0	0.0	0.0
1.07	0.29	0.22	0.0730	0.0029	0.0022	0.0007
1.08	0.24	0.19	0.0478	0.0048	0.0039	0.0010
1.09	0.22	0.17	0.0444	0.0065	0.0052	0.0013
1.10	0.15	0.15	-0.0051	0.0059	0.0061	-0.0002
1.11	0.10	0.14	-0.0318	0.0052	0.0068	-0.0016
1.12	0.08	0.12	-0.0371	0.0050	0.0072	-0.0022
1.13	0.07	0.11	-0.0409	0.0046	0.0075	-0.0029
1.14	0.06	0.09	-0.0335	0.0048	0.0075	-0.0027
1.15	0.05	0.08	-0.0322	0.0045	0.0074	-0.0029
1.16	0.04	0.07	-0.0289	0.0043	0.0072	-0.0029
1.17	0.04	0.06	-0.0242	0.0043	0.0069	-0.0027
1.18	0.04	0.05	-0.0182	0.0044	0.0066	-0.0022
1.20	0.03	0.04	-0.0068	0.0048	0.0058	-0.0009
1.21	0.03	0.04	-0.0052	0.0046	0.0053	-0.0008
1.22	0.03	0.03	-0.0023	0.0045	0.0049	-0.0004
1.23	0.02	0.03	-0.0037	0.0039	0.0045	-0.0006
1.24	0.02	0.02	-0.0018	0.0038	0.0041	-0.0003
1.25	0.02	0.02	-0.0003	0.0036	0.0037	-0.0001
1.26	0.02	0.02	0.0026	0.0038	0.0033	0.0005
1.27	0.02	0.01	0.0050	0.0040	0.0030	0.0011
1.28	0.02	0.01	0.0037	0.0034	0.0026	0.0008
1.29	0.02	0.01	0.0055	0.0036	0.0023	0.0013
1.30	0.01	0.01	0.0037	0.0029	0.0021	0.0009

THIS IS AN AD FUNCTION, N = 8

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.9161*(R - 0.84)** 1.5949)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.93	1.00	-0.0748	-0.2036	-0.2200	0.0164
0.85	0.93	0.99	-0.0639	-0.1943	-0.2077	0.0134
0.86	0.93	0.97	-0.0423	-0.1850	-0.1935	0.0085
0.87	0.86	0.94	-0.0793	-0.1633	-0.1784	0.0151
0.88	0.86	0.91	-0.0455	-0.1547	-0.1629	0.0082
0.89	0.86	0.87	-0.0077	-0.1461	-0.1474	0.0013
0.90	0.86	0.83	0.0330	-0.1375	-0.1323	-0.0053
0.91	0.86	0.78	0.0757	-0.1289	-0.1176	-0.0114
0.93	0.80	0.70	0.1064	-0.1042	-0.0904	-0.0138
0.94	0.75	0.65	0.0994	-0.0900	-0.0781	-0.0119
0.95	0.75	0.61	0.1438	-0.0825	-0.0667	-0.0158
0.96	0.70	0.56	0.1410	-0.0704	-0.0563	-0.0141
0.97	0.66	0.52	0.1417	-0.0596	-0.0468	-0.0127
0.98	0.59	0.48	0.1104	-0.0472	-0.0383	-0.0088
0.99	0.56	0.44	0.1182	-0.0391	-0.0308	-0.0083
1.00	0.53	0.40	0.1268	-0.0318	-0.0242	-0.0076
1.01	0.53	0.37	0.1623	-0.0265	-0.0184	-0.0081
1.02	0.48	0.33	0.1447	-0.0191	-0.0133	-0.0058
1.04	0.38	0.27	0.1061	-0.0076	-0.0055	-0.0021
1.05	0.32	0.25	0.0742	-0.0032	-0.0025	-0.0007
1.06	0.30	0.22	0.0745	0.0	0.0	0.0
1.07	0.26	0.20	0.0651	0.0026	0.0020	0.0007
1.08	0.22	0.18	0.0416	0.0044	0.0035	0.0008
1.09	0.20	0.16	0.0386	0.0059	0.0047	0.0012
1.10	0.13	0.14	-0.0064	0.0053	0.0056	-0.0003
1.11	0.09	0.12	-0.0302	0.0046	0.0061	-0.0015
1.12	0.07	0.11	-0.0345	0.0044	0.0065	-0.0021
1.13	0.06	0.10	-0.0375	0.0041	0.0067	-0.0026
1.14	0.05	0.08	-0.0306	0.0043	0.0067	-0.0024
1.15	0.04	0.07	-0.0292	0.0040	0.0066	-0.0026
1.16	0.04	0.06	-0.0261	0.0038	0.0064	-0.0026
1.17	0.03	0.06	-0.0217	0.0037	0.0061	-0.0024
1.18	0.03	0.05	-0.0163	0.0039	0.0058	-0.0020
1.20	0.03	0.04	-0.0060	0.0042	0.0051	-0.0008
1.21	0.03	0.03	-0.0046	0.0040	0.0047	-0.0007
1.22	0.02	0.03	-0.0020	0.0040	0.0043	-0.0003
1.23	0.02	0.02	-0.0032	0.0034	0.0039	-0.0005
1.24	0.02	0.02	-0.0014	0.0033	0.0036	-0.0003
1.25	0.02	0.02	-0.0001	0.0032	0.0032	-0.0000
1.26	0.02	0.01	0.0024	0.0034	0.0029	0.0005
1.27	0.02	0.01	0.0045	0.0035	0.0026	0.0009
1.28	0.01	0.01	0.0033	0.0030	0.0023	0.0007
1.29	0.01	0.01	0.0049	0.0032	0.0020	0.0011
1.30	0.01	0.01	0.0033	0.0026	0.0018	0.0008

THIS IS AN AD FUNCTION, N = 9

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.8781*(R - 0.84)** 1.5596)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.92	1.00	-0.0833	-0.2017	-0.2200	0.0183
0.85	0.92	0.99	-0.0706	-0.1925	-0.2073	0.0148
0.86	0.92	0.96	-0.0462	-0.1833	-0.1926	0.0092
0.87	0.84	0.93	-0.0865	-0.1605	-0.1769	0.0164
0.88	0.84	0.89	-0.0497	-0.1521	-0.1610	0.0089
0.89	0.84	0.85	-0.0092	-0.1436	-0.1452	0.0016
0.90	0.84	0.81	0.0340	-0.1352	-0.1297	-0.0054
0.91	0.84	0.77	0.0790	-0.1267	-0.1149	-0.0118
0.93	0.78	0.67	0.1084	-0.1017	-0.0876	-0.0141
0.94	0.73	0.63	0.0993	-0.0873	-0.0754	-0.0119
0.95	0.73	0.58	0.1444	-0.0800	-0.0641	-0.0159
0.96	0.68	0.54	0.1397	-0.0679	-0.0539	-0.0140
0.97	0.64	0.50	0.1388	-0.0572	-0.0447	-0.0125
0.98	0.56	0.46	0.1055	-0.0449	-0.0364	-0.0084
0.99	0.53	0.42	0.1125	-0.0370	-0.0292	-0.0079
1.00	0.50	0.38	0.1203	-0.0300	-0.0228	-0.0072
1.01	0.50	0.34	0.1551	-0.0250	-0.0172	-0.0078
1.02	0.45	0.31	0.1367	-0.0180	-0.0125	-0.0055
1.04	0.35	0.25	0.0980	-0.0070	-0.0051	-0.0020
1.05	0.29	0.23	0.0671	-0.0029	-0.0023	-0.0007
1.06	0.27	0.20	0.0675	0.0	0.0	0.0
1.07	0.24	0.18	0.0586	0.0024	0.0018	0.0006
1.08	0.20	0.16	0.0367	0.0040	0.0032	0.0007
1.09	0.18	0.14	0.0341	0.0053	0.0043	0.0010
1.10	0.12	0.13	-0.0073	0.0048	0.0051	-0.0003
1.11	0.08	0.11	-0.0286	0.0042	0.0056	-0.0014
1.12	0.07	0.10	-0.0321	0.0040	0.0059	-0.0019
1.13	0.05	0.09	-0.0345	0.0036	0.0061	-0.0024
1.14	0.05	0.08	-0.0281	0.0038	0.0061	-0.0023
1.15	0.04	0.07	-0.0266	0.0036	0.0059	-0.0024
1.16	0.03	0.06	-0.0237	0.0034	0.0058	-0.0024
1.17	0.03	0.05	-0.0197	0.0033	0.0055	-0.0022
1.18	0.03	0.04	-0.0147	0.0034	0.0052	-0.0018
1.20	0.03	0.03	-0.0054	0.0038	0.0045	-0.0007
1.21	0.02	0.03	-0.0041	0.0036	0.0042	-0.0006
1.22	0.02	0.02	-0.0017	0.0036	0.0038	-0.0003
1.23	0.02	0.02	-0.0028	0.0030	0.0035	-0.0005
1.24	0.02	0.02	-0.0012	0.0029	0.0032	-0.0002
1.25	0.01	0.01	-0.0000	0.0028	0.0028	-0.0000
1.26	0.01	0.01	0.0022	0.0030	0.0025	0.0004
1.27	0.01	0.01	0.0041	0.0031	0.0023	0.0009
1.28	0.01	0.01	0.0030	0.0027	0.0020	0.0007
1.29	0.01	0.01	0.0044	0.0028	0.0018	0.0010
1.30	0.01	0.01	0.0030	0.0023	0.0016	0.0007

THIS IS AN AD FUNCTION, N = 10

THE EQUATION OF THEWEIBULL FUNCTION IS EXP(- 16.8301*(R - 0.84)** 1.5279)

U3 = (R - 1.06)

THIS IS A COMPARISON OF THE PROBABILITIES AND THE EXPECTED PROFIT USING ACTUAL DATA POINTS AND THE WEIBULL FUNCTION

R	P	WEIBULL PROB	DIFFERENCE	EXPECTED UTILITY ACTUAL DATA	EXPECTED UTILITY WEIBULL CALCULATED DATA	DIFFERENCE
0.84	0.91	1.00	-0.0917	-0.1998	-0.2200	0.0202
0.85	0.91	0.99	-0.0771	-0.1907	-0.2069	0.0162
0.86	0.91	0.96	-0.0500	-0.1817	-0.1916	0.0100
0.87	0.83	0.92	-0.0933	-0.1578	-0.1755	0.0177
0.88	0.83	0.88	-0.0537	-0.1495	-0.1592	0.0097
0.89	0.83	0.84	-0.0106	-0.1412	-0.1430	0.0018
0.90	0.83	0.80	0.0349	-0.1329	-0.1273	-0.0056
0.91	0.83	0.75	0.0818	-0.1246	-0.1123	-0.0123
0.93	0.76	0.65	0.1099	-0.0993	-0.0850	-0.0143
0.94	0.71	0.61	0.0988	-0.0847	-0.0729	-0.0119
0.95	0.71	0.56	0.1445	-0.0776	-0.0618	-0.0159
0.96	0.66	0.52	0.1380	-0.0655	-0.0517	-0.0138
0.97	0.61	0.47	0.1357	-0.0549	-0.0427	-0.0122
0.98	0.53	0.43	0.1008	-0.0428	-0.0347	-0.0081
0.99	0.50	0.40	0.1071	-0.0352	-0.0277	-0.0075
1.00	0.47	0.36	0.1143	-0.0284	-0.0216	-0.0069
1.01	0.47	0.33	0.1483	-0.0237	-0.0163	-0.0074
1.02	0.42	0.29	0.1294	-0.0169	-0.0117	-0.0052
1.04	0.33	0.24	0.0909	-0.0066	-0.0047	-0.0018
1.05	0.27	0.21	0.0612	-0.0027	-0.0021	-0.0006
1.06	0.25	0.19	0.0616	0.0	0.0	0.0
1.07	0.22	0.17	0.0532	0.0022	0.0017	0.0005
1.08	0.18	0.15	0.0327	0.0036	0.0030	0.0007
1.09	0.16	0.13	0.0304	0.0049	0.0040	0.0009
1.10	0.11	0.12	-0.0077	0.0044	0.0047	-0.0003
1.11	0.08	0.10	-0.0270	0.0038	0.0051	-0.0014
1.12	0.06	0.09	-0.0300	0.0036	0.0054	-0.0018
1.13	0.05	0.08	-0.0320	0.0033	0.0055	-0.0022
1.14	0.04	0.07	-0.0260	0.0034	0.0055	-0.0021
1.15	0.04	0.06	-0.0245	0.0032	0.0054	-0.0022
1.16	0.03	0.05	-0.0217	0.0031	0.0052	-0.0022
1.17	0.03	0.05	-0.0179	0.0030	0.0050	-0.0020
1.18	0.03	0.04	-0.0134	0.0031	0.0047	-0.0016
1.20	0.02	0.03	-0.0048	0.0034	0.0041	-0.0007
1.21	0.02	0.03	-0.0036	0.0032	0.0038	-0.0005
1.22	0.02	0.02	-0.0015	0.0032	0.0034	-0.0002
1.23	0.02	0.02	-0.0024	0.0027	0.0031	-0.0004
1.24	0.01	0.02	-0.0010	0.0027	0.0028	-0.0002
1.25	0.01	0.01	0.0000	0.0026	0.0026	0.0000
1.26	0.01	0.01	0.0020	0.0027	0.0023	0.0004
1.27	0.01	0.01	0.0037	0.0028	0.0020	0.0008
1.28	0.01	0.01	0.0028	0.0024	0.0018	0.0006
1.29	0.01	0.01	0.0040	0.0025	0.0016	0.0009
1.30	0.01	0.01	0.0028	0.0021	0.0014	0.0007

VITA

Fletcher Hughes Griffis, Jr.

Candidate for the Degree of

Doctor of Philosophy

Thesis: A STOCHASTIC ANALYSIS OF THE COMPETITIVE BIDDING PROBLEM FOR CONSTRUCTION CONTRACTORS

Major Field: Engineering

Biographical:

Personal Data: Born in Wauchula, Florida, April 22, 1938, the son of Fletcher H. and Eva Griffis.

Education: Attended grade school in Wauchula and Avon Park, Florida; graduated from high school in DeLand, Florida, in 1956; received the Bachelor of Science Degree from the United States Military Academy at West Point, New York, in 1960; received the Master of Science in Construction Engineering Degree from Oklahoma State University in 1965; graduated from the United States Army Command and General Staff College at Fort Leavenworth, Kansas, in 1970; completed requirements for the Doctor of Philosophy degree in July, 1971.

Professional Experience: A professional officer in the United States Army Corps of Engineers for eleven years; assignments include student, Airborne School, Ranger School, U. S. Army Engineer School, 1960; Platoon Leader and Company Commander, 307th Engineer Battalion (Airborne), 1961-62; Company Commander, 11th Engineer Battalion (Combat), 1963, Engineer Officer Advanced Course, The Army Engineer School, 1964; Instructor and Assistant Professor, Department of Mathematics, United States Military Academy at West Point, 1966-68; Chief, Construction Management and Highway Construction Section, U.S. Army Engineer Construction Agency Vietnam, 1968; Operations Officer, 36th Engineer Battalion (Construction), 1969; presently Major, U.S. Army Corps of Engineers.

Professional Activities: National Society of Professional Engineers, Member-at-large; American Society of Civil Engineers, Member; Chi Epsilon, Member; Sigma Tau, Member; Registered Professional Engineer (Oklahoma), 1966.

Publications:

"Cable-Tramway System on Hill 651" The Military Engineer, May, 1964.

"Optimizing Haul Fleet Size Using Queueing Theory" Journal of the Construction Division, American Society of Civil Engineers, January, 1969.