EFFECTS OF A WINGTIP-MOUNTED PROPELLER
ON WING LIFT, INDUCED DRAG, AND
SHED VORTEX PATTERN

## By

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## PREFACE

The subject of induced drag is one that is both intriguing and frustrating to an aerodynamicist. It is the penalty that must be paid for producing lift using a wing having a finite span. Induced drag is drag that would be present even in a perfect (inviscid) fluid. Also present is the trailing vortex which produces the induced drag.

It was desired to determine whether it was possible to combine the swirling of a propeller slipstream with the trailing wing vortex in ways such that the wing loading would be affected and the induced drag either increased or decreased. This paper reports the results of a wind tunnel testing program designed to examine this idea.

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## IIST OF SYMBOLS

| A | Aspect Ratio |  |
| :---: | :---: | :---: |
| A | Area | sq．ft． |
| $\hat{a}_{\theta}$ | Unit angular vector |  |
| b | Span of wing（or other lifting surface） | $f t$. |
| $\mathrm{b}_{\mathrm{V}}$ | Vortex span | ft。 |
| $\mathrm{C}_{\mathrm{D}}$ | Coefficient of drag，$\frac{D}{\text { qS }}$ |  |
| $\mathrm{C}_{\mathrm{D}_{\mathrm{i}}}$ | Induced drag coefficient |  |
| $\mathrm{C}_{\mathrm{D}}{ }_{\mathrm{p}}$ | Effective parasite drag coefficient |  |
| $\begin{aligned} & C_{D_{r}} \\ & C_{I} \end{aligned}$ | $\begin{aligned} & \text { Residual drag coefficient }=C_{D_{p}}-c_{d_{O_{\text {mir }}}} \\ & \text { Coefficient of lift, } \frac{I}{q S} \end{aligned}$ |  |
| $\mathrm{C}_{\mathrm{I}_{\max }}$ | Maximum lift coefficient |  |
| $\begin{aligned} & \mathrm{C}_{L_{\alpha}} \\ & \mathrm{C}_{\mathrm{M}} \end{aligned}$ | Slope of lift curve， $\mathrm{dC}_{\mathrm{L}} / \mathrm{d} \alpha$ Coefficient of pitching moment，$\frac{M}{q S c}$ | degree |
| $\mathrm{C}_{\mathrm{M}}^{\mathrm{c} / 4} \textrm{}$ | Coefficient of the moment about the wing quarter－chord |  |
| $\mathrm{C}_{\text {T }}$ | Thrust Coefficient，$\frac{\text { Thrust }}{\rho \mathrm{N}^{2} \mathrm{~d}^{4}}$ |  |
| c | Chord of wing（or airfoil section） | ft． |
| $c_{d_{0}}$ | Profile（two－dimensional airfoil section）drag coefficient |  |
| D | Drag；component of force parallel to the freestream velocity direction | 1b。 |
| $D^{\prime}$ | Drag per foot of span，D／b | 1b．／ft． |
| $\mathrm{D}_{\mathrm{i}}$ | Induced drag | 13. |
| d | Propeller（or impeller）diameter | ft。 |


| $d_{V}$ | Distance from wing to position of＂fully rolled－up＂vortex | ft。 |
| :---: | :---: | :---: |
| e | Rate of expansion in x －direction |  |
| e | Span efficiency factor $\frac{1}{\pi A} \frac{d C_{D}}{\mathrm{dC}_{L^{2}}}$ |  |
| $e_{W}$ | Wing efficiency factor，$\frac{1}{\pi \mathrm{Am}} \approx \frac{1}{1+\delta}$ |  |
| F | Thrust | 1b。 |
| F | Force | 1b． |
| f | Rate of expansion（divergence）in $y$－direction |  |
| $g$ | Rate of expansion（divergence）in z－direction |  |
| h | Semi－height of end－plate，measured from wing chord to tip of end－plate （total height of a symmetrical end－ plate is 2h） | ft． |
| î | Unit vector in X －direction |  |
| J | Advance ratio $\frac{a}{N d}$ |  |
| 今 | Unit vector in y－direction |  |
| K | Constant |  |
| $\hat{k}$ | Unit vector in z－direction |  |
| L | Lift；component of force normal to the freestream velocity direction | 1 b |
| $L^{\prime}$ | Lift per foot of span，L／b | 1b．／ft． |
| 1 | Length | ft。 |
| M | Pitching moment，about the y－axis | 1b．／ft． |
| $\mathrm{M}_{\mathrm{C} / 4}$ | Moment about the wing quarter－chord line | 1b．／ft． |
| m | Slope $\left(\frac{\Delta C_{D}}{\Delta C_{L}{ }^{2}}\right)$ of straight line fit to wing data when plotted $C_{D}$ vs．$C_{L}{ }^{2}$ |  |
| N | Rotational speed of propelier | V．／sec． |


| n | Normal distance from a point to a vortex filament | ft． |
| :---: | :---: | :---: |
| p | Pressure | 1b．$/ \mathrm{ft} \mathrm{o}^{2}$ |
| $\mathrm{P}_{\mathrm{a}}$ | Ambient（freestream）pressure | 1b．／ft．${ }^{2}$ |
| ${ }^{p}$ L | Pressure acting on lower surface | Ib．／ft。 ${ }^{2}$ |
| $\mathrm{p}_{\mathrm{t}}$ | Total（stagnation）pressure， $p+q F_{c} \approx p_{a}+q$ | lb．／ft。 ${ }^{\text {a }}$ |
| $\mathrm{p}_{u}$ | Pressure acting on upper surface | 1b．／ft．${ }^{\text {2 }}$ |
| q | Dynamic pressure，$\frac{1}{2} \rho V^{2}=\frac{1}{5} Y_{p} M^{2}$ | lb．／ft。 ${ }^{2}$ |
| $\mathrm{q}_{\mathrm{a}}$ | Free－stream dynamic pressure | 1b．／ft。 ${ }^{\text {2 }}$ |
| $q_{S S}$ | Dynamic pressure in propeller slipstream | lb．／ft。 ${ }^{2}$ |
| R | Propeller radius，d／2 | $f t$. |
| $\mathrm{R}^{\prime}$ | Resultant force per foot of span of lifting surface（having components $L^{\prime}$ and $\left.D^{\prime}\right), \rho V_{\text {res }} T$ | lb．$/ \mathrm{ft}$ 。 |
| $\overline{\mathrm{R}}$ | Vector position of point in flow field |  |
| Re | $\begin{aligned} & \text { Reynolds' number, } \frac{\rho V \ell}{\mu}, \text { Eff. Re }= \\ & (\operatorname{Re}) x\left(T . F_{0}\right) \end{aligned}$ |  |
| $r$ | Radius | ft。 |
| $r_{\text {c }}$ | Radius of rotational core of rolled－up trailing vortex | $f t$. |
| S | Wing area | sq．ft． |
| $S_{p}$ | Portion of wing area in propeller slipstream | sq．ft． |
| $\mathrm{T}_{\mathrm{C}}$ | Thrust coefficient，$\frac{\text { Thrust }}{e_{2} \mathrm{~V}^{2} \mathrm{~d}^{2}}$ |  |
| t | Thickness of airfoil，esp．max． thickness | ft。 |
| u | X－component of velocity | ft．／sec． |
| V | Velocity $=\|\bar{V}\|$ | fto／sec． |


| $\mathrm{V}_{\mathrm{a}}$ | Freestream velocity | ft．／sec． |
| :---: | :---: | :---: |
| $V_{\text {min }}$ | Minimum level－flight velocity， corresponds to $C_{I}$ | ft．／sec． |
| $V_{\text {max }}$ | Maximum level－flight speed | ft．／sec． |
| $\mathrm{V}_{\text {res }}$ | Resultant（of $\mathrm{V}_{\mathrm{a}}$ and w ）velocity | ft．／sec． |
| $\mathrm{V}_{S}$ | Velocity along a surface or boundary | ft．／sec． |
| $V_{\text {SS }}$ | Velocity in propeller slipstream | ft．／sec． |
| $V_{\theta}$ | Velocity on a cylindrical surface at angle $\theta$ from stagnation point | ft．／sec． |
| $\overline{\mathrm{V}}$ | Total velocity vector | ft．／sec． |
| $\bar{V}_{0}$ | Translational component of velocity vector | ft．／sec． |
| $\bar{V}^{\prime}$ | Rotational component of velocity vector | ft．／sec． |
| $\bar{V}^{\prime \prime}$ | Vectorial representation of divergence of velocity | ft．／sec． |
| V | Y－component of velocity | $f t . / s e c$. |
| W | Airplane weight | 1b。 |
| w | Z－component of velocity | ft．／sec． |
| W | Downwash velocity | ft．／sec． |
| X | Dimension parallel to wind tunnel centerline | fot。 |
| Y | Dimension measured spanwise from the plane of symmetry normal to tunnel centerline | fto |
| z | Dimension normal to x and y | 化。 |
| $\alpha$ | Angle of attack | degrees |
| $\alpha_{i}$ | Induced angle of attack $18.24 \frac{\mathrm{C}^{\mathrm{L}}}{\mathrm{A}}(1+\tau)$ | Degrees |
| $\alpha_{0}$ | ```Airfoil section (two-dimensional) angle of attack``` | degrees |
| $\beta$ | Propeller blade pitch angle | degrees |


| $\beta$ | Angle between radial line and the normal to the vortex filament | degrees |
| :---: | :---: | :---: |
| $\Gamma$ | Circulation, $\oint_{S} V_{S} \mathrm{ds}$ | $\mathrm{ft} .{ }^{2} / \mathrm{sec}$. |
| $\Gamma_{0}$ | Circulation at mid-span | $\mathrm{ft}.{ }^{2} / \mathrm{sec}$. |
| $\gamma$ | Airplane approach angle, tan ${ }^{-1} \frac{D-F}{L}$ | degrees |
| $\gamma$ | Strength of vortex sheet | ft./sec. |
| $\triangle$ | Increment |  |
| $\delta$ | Glauert (induced drag) factor |  |
| $\delta_{e}$ | End-plate deflection angle | degrees |
| $\delta_{f}$ | Flap deflection angle | degrees |
| $\varepsilon$ | (negative) downwash angle | radius |
| $\mu$ | Viscosity slu | /ft. sec. |
| $p$ | Density (fluid, air) | slugs/ft. ${ }^{3}$ |
| $\sigma$ | Angle between path of integration and velocity vector | degrees |
| $\Sigma$ | Summation |  |
| $\tau$ | ```Glauert (induced angle of attack.) factor``` |  |
| $\theta$ | Angle of rotation (or of position) | degrees |
| $\bar{\Omega}$ | Angular velocity of rotation | rad./sec. |
| $\omega$ | $=\|\bar{\Omega}\|$ |  |
| $\zeta$ | Vorticity, $\frac{\Gamma}{A}$ | 1/sec. |

## SUBSCRIPTS

| a | Freestream or ambient |
| :--- | :--- |
| $c / 4$ | Quarter chord point or line |
| D | Drag |
| e | End-plates |
| i | Induced |
| L | Lift |
| L | Lower surface |
| $\mathrm{M}_{\mathrm{c} / 4}$ | Pitching moment about the quarter-chord line |
| 0 | Oval end-plate |
| p | Equivalent parasite |
| res | Resultant |
| r | Round end-plate |
| V | Upper surface |
| p | Propeller |
| O | Center of span |

## CHAFTER I

## INTRODUCTION

The use of a wing, having a finite span, to produce lift results in three penalties which would not exist if the lifiting surface had an infinite span. These three penalties are:

1. Decrease in lift near the wing tips (and, therefore, a lower $C_{L}$ of the wing at any angle of attack and a lower $C_{I_{\text {max }}}$ )。
2. Increase in wing drag by the amount of the induced drag.
3. Creation of downwash behind the wing (e.g., at the tail surfaces) which is not constant, but is a function of the wing lift coefficient.

This fact has long been recognized。 At the turn of the century, Lanchester postulated the type of flow that a real finite span wing would experience. His sketches predicted the formation and shedding of vortices which wrap up into large-scale vortices trailing downstream from each wing tip. Figure 1 is a reproduction of his sketohes published in Aerodynamics (27).


Figure I. Lanchester's Sketches of Vortex Motion in the Periptery

Prandtl first discussed the problem of the threedimensional flow over a wing of finite span in 1911 and published his treatment of the probiem in 1918. The Prandtl wing theory is the basis for most of the work which has been done to date on the finitemspan lifting wing.

Not only have the problems been long recognized, but the history of the attempts to improve the effectiveness of a finite lifting wing predates the Wright brother's first powered flight. Pope (37) reports that Lanchester secured a patent in 1897 covering the use of end-plates on wings!

The principal objectives of various schemes to alter the flow around finite-span wings are:
a) The increase of wing lift (i.e., increase $\mathrm{C}_{\mathrm{I}_{\text {max }}}$ and $\mathrm{C}_{\mathrm{I}_{\alpha}}$ ).
b) The decrease of wing drag (by reduction of induced drag).

The maximum lift coefficient of the wing is always less than the maximum coefficient of lift of the wing sections in two-dimensional flow. The reason for this difference is that the loss of lift near the wing tips causes the maximum lift coefficient of sections near the tips to be less than if the flow were two-dimensional. Alsog the stall of the wing effectively begins when some portion of the wing stalls; that is, there is separation
locally at some point on the airfoil. Although most of the wing may be unstalled, the wing $C_{L}$ will not continue to increase as $\alpha$ increases after a portion of the wing stalls.

It is, however, the induced drag which, in the past, has been the target of most of the improvement devices. The induced drag varies inversely with the square of the wingspan for a given wing loading. It is at low speeds that the induced drag is particularly important. At the velocity for maximum range of an airplane the induced drag is equal in magnitude to the skin friction drag of the wing and airplane. At speeds less than this speed the induced drag is greater than the drag of the rest of the airplane。

Included among the desirable traits for an airplane wing are:
a) High $C_{L_{\max }}$.
b) Low values of induced drag (for takeoff and climb).
c) High value of induced drag (for approach and landing).

The physical span of an aircraft wing is limited by prace tical considerations. As mentioned above, the earliest (and the most often repeated) approach to the goal of making a wing perform as though the span were greater, has been the modification of wingtip geometry (e.g. g through the use of end-plates, tip-bodies, etc。). In this paper.
these methods are examined in Chapter II and shown to be inferior. However, it is possible to attain many of the desired wing traits by the use of rotating propellers at the wingtips. This application of energy to the flow field through the use of mechanical rotors is the subject of the experimental program reported in this paper. These experiments are examined in detail in Chapters III and IV.

## CHAPIER II

## EVALUATION OF END-PLATES

It could be said that an infinite-span wing would not be necessary if it were possible to force the air to behave as though the span were infinite; i.e., if it were possible to maintain two-dimensional flow over the threedimensional wing. The (apparently) obvious approach to this objective is through the use of end-plates.

## Conventional End-Plates

In 1927, Hemke (21) reported a systematic investigation of the effect of end-plates on the drag of wings. Hemke reached the following conclusions:

Calculations show that the induced drag of monoplanes and multiplanes may be decreased by attaching end-plates to the ends of the wing. The frictional drag of the end-plates may be calculated approximately. The reduction of the induced drag exceeds the additional frictional drag due to the end-plates at all but the small, values of lift. For given dimensions of wings and end-plates the reduction of drag less the friction of drag of the endplates varies directly as the square of the absolute lift coefficient. The average reduction of drag decreases as the aspect ratio decreases. Calculations and experiments agree quite satisfactorily for single wings equipped with end-plates.

Wind tunnel tests show that the coefficients used in calculating a frictional drag of the
end-plates may be reduced by fairing the endplates. The shape of the end-plate determines to some extent the reduction of induced drag ...

Recent experiments have shown that much higher lift coefficients can be obtained than have been the case up to now with the conventional airfoils ...

Although these conclusions seem favorable to the idea of attaching end-plates to the wing, the period following the publication of this report was not marked by the application of end-plates to aircraft wings. The key sentence in the conclusions is the one which states that the reduction of the induced drag exceeds the additional frictional drag due to the end-plates in all but the small values of the Iift. It is at these "small values of the lift" (actually, at small values of lift coefficient) that the airplane flies at high speed. This fact means that the additions of end-plates to the wing will limit the high speed performance of the aircraft.

The models reported on by Hemke were rather small (four inches chord) and the Reynolds Number of the tests, although not stated in the report, were probably quite low. In order to determine whether Hemke's conclusions were valid, a brief wind tunnel investigation was conducted.

This test and all of the powered model testing, reported in the following chapters, were performed in the Walter H. Beech Memorial Wind Tunnel on the campus of Wichita State University. This wind tunnel is a horizontal, single-return, closed-throat tunnel. A plan of the tunnel is shown in Figure 2.


Figure 2. Plan of Walter H . Beech Wind Tunnel

The test section is 7 feet by 10 feet with corner fillets which reduce the cross-sectional area to 68 square feet. The test section is 12 feet long. The tunnel is powered by a 1500 horsepower motor, and the speed is varied by controlling the propeller pitch. The six component pyramidal balance is mounted below the test section floor. The balance readings, together with angles of attack and yaw, test section dynamic pressure, ambient pressure and temperature and run number are recorded by an on-line card punch. Because the mounting of the reflection plane model of the wing required that the wing be rotated 90 degrees from the conventional position, the wing lift was measured by the side-force balance, the pitching moment of wing was measured by the yawing moment balance, and drag was measured by the drag balance. Only these three components were measured.

The end-plate test program consisted of a reflectionplane wing model (referred to as the auxiliary model) tested in the wind tunnel in the following configurations:

1. The basic constant-chord wing with a plain square wingtip.
2. The wing with a round end-plate.
3. The wing with an oval end-plate.

The model details are sketched in Figure 3, and Figure 4 shows the model under test in the wind tunnel. The model is a plastic-foam model with a wooden span and covered with fiberglas.


Wing Without End-Plate


Figure 3. Auxiliary Model and End-Plates


The effective Reynolds Number of the test was
$1.6 \times 10^{6}$.
The addition of end-plates to a wing should have two principal effects:

1. Decrease in induced drag.
2. Increase in lift at any angle of attack
(greater than angle of zero lift) result-
ing in increased slope of the lift curve
and increased $C_{L_{\text {max }}}$.
These effects can be seen in Figures 5 and 6 .
The basic wing has the dimensions:
Span $=80$ inches
Chord $=18$ inches
Area $=1440$ square inches
Aspect Ratio $=4.44$
The characteristics of the wing in the three configurations tested are summarized in Table I.

It will also be noted that the basic wing has a lower total drag than the wing with round end-plates in the range of $C_{L}$ from 0 to 0.2 and a lower drag than with oval end-plate in the range $0<C_{L}<0.3$.

As expected, the end-plates improved the induced drag characteristics of the wing, increasing the effective aspect ratio. The increased $C_{L_{\text {max }}}$ will result in decreasing the minimum flight speed. It is necessary to examine whether this method of accomplishing these ends is an efficient one. If, instead of putting end-plates on the


Figure 5. Supplementary Model Characteristics


Figure 6. Supplementary Model Drag

TABIE I
SUMMARY OF CHARACTERISTICS OF AUXILIARY MODEL

| Wing Characteristic | Basic Wing | With Round End-Plate | With Oval End=Plate |
| :--- | :--- | :--- | :--- |
| Lift-Curve Slope | 0.072 per deg | 0.0815 | 0.0815 |
| $C_{L_{\text {max }}}$ | 1.185 | 1.249 | 1.229 |
| $V_{\min } /\left(V_{\text {min }}\right)$ | 0.97 | 0.98 |  |
| $C_{D}$ equation | 1.0 | $C_{D}=0.0159+0.0764 C_{L}{ }^{2}$ | $C_{D}=0.0188+0.0784 C_{L}{ }^{2}$ |
| $e_{w}$ | $C_{D}=0.011+0.0987 C_{L}{ }^{2}$ | 0.938 | 0.914 |
| Eff.Aspect Ratio $=A_{W}$ | 3.23 | 4.17 | 4.06 |
| $C_{D}$ at $C_{L}=1$ | 0.109 | 0.725 |  |

wing, the same area were added to the wing by extending the wing span, the following results would obtain (Prime indicates extended span wing):

Round end-plate area $=214$ sq. in. (each)
Semispan increment $=11.89 \mathrm{in}$.

$$
A^{\prime}=\frac{\left(b^{\prime}\right)^{2}}{S^{\prime}}=\frac{(103.78)^{2}}{1868}=5.77
$$

If it is assumed that this extended span wing would have the same span efficiency factor as the basic wing, i.e., $e_{w}=0.725$, then the effective aspect ratio would be $\left(A^{\prime} e_{W}\right)=5.77(.725)=4.19$. Then the drag equation would be:

$$
\begin{aligned}
& C_{D}^{\prime}=0.11+\frac{1}{4.19 \pi} C_{L}{ }^{2} \\
& C_{D}^{\prime}=.011+.076 C_{L}^{2}
\end{aligned}
$$

These calculations show that if, instead of installing the round end-plate, the same area were added to the wingtip in the form of an extension of the span, the result would be an effective aspect ratio almost exactly the same as that resulting from the end-plate. The coef.ficient of drag of the extended wing will be less than that of the basic wing with round end plates at any given ${ }^{C}$.

The drag of the two wings have been compared on the basis of same flight velocities or on the basis of the same wing-loading (gross weight of extended wing airplane is increased). In each case, the drag of the extended
wing is less than that of the wing plus end-plates.
Calculations may be repeated for the oval end-plate:
Oval end-plate area $=190.7$ square inches
Extended wing area $=1821.4$ square inches

$$
\begin{aligned}
A^{\prime}=5.625, \mathrm{~A}^{\prime} \mathrm{e}_{\mathrm{W}}= & 4.08 \text { (compared to } 4.06 \\
& \text { with oval end-plate) }
\end{aligned}
$$

$$
C_{D}^{\prime}=.011+.078 C_{L}^{2}
$$

Again, it can be seen that extension of the basic wing results in lower drag coefficient. The extended wing has less drag.

Full airplane model tests with rectangular and endplates were previously conducted in the same wind tunnel at an effective Reynold's Number of $1,840,000$. Three of the configurations tested are shown in Figure 7. These tests were reported by Morris (33) and the results are reproduced in Figure 8. It can be seen that only at the high lift coefficients is the drag of the wing with endplates lower than that of the basic wing (in this case, $\left.C_{L}>0.7\right)$ 。

These results confirm the conclusions reached by Riley (42):

Substantial increases may be obtained in the maximum lift-drag ratio of wing-body combinations or complete airplanes, for which the total drag of the components other than the wing is large relative to the wing drag, by the use of appropriately designed endplates. Except possibly for the smaller endplates area, however, the increases obtained are not likely to be as large as those which would be obtained by utilizing the end-plate area as a simple addition to the wing span,


Figure 7. Goodyear Racer Being Tested
With End-Plates


Figure 8. Effect of End-Plates on Drag of Goodyear Racer
thus increasing the wing geometric aspect ratio.

The maximum lift coefficient of the wing experienced an increase when the end-plates were added. The rate of increase, however, decreased with increasing end-plate area.

Similarly, Hoerner (22) stated:
Basically, the end-plates have at least the drag of a pair of added wingtip extensions (having the same area). Any such added area naturally produces more lift for the same price of viscous drag. Practical application of end-plates in airplanes and/or guided missiles, therefore, seems to be restricted to such designs where the plates can also be utilized for stabilizing or control purposes.

Garbell (15) has discussed these cases of using wingtip fins instead of centrally-located vertical fins.

For most of the history of flight, the major emphasis has been on increasing the cruise and top-speed flight velocities. These flight regimes are at low value of lift coefficient. It is logical, then, that end-plates have not been built onto aircraft for the reasons stated above. However, in recent years, increasing interest in STOL aircraft has made it reasonable to consider schemes which might sacrifice high-speed performance to attain low-speed capability. It was this attitude which led Lowry and Vogler (28) to investigate the use of end-plates on wings equipped with jet flaps and for Morris and Ten Eyck (34) to consider the combination of end-plates and circulation control to improve the landing and take-off distances of a reconnaissance-type airplane.

The Morris and Ten Eyck analysis determined the
effects of wing aspect ratio, end-plates, and circulation control on the distances to land and take-off over a 50 foot obstacle. Assumed conditions included:

| Gross Weight | 2100 pounds |
| :---: | :---: |
| Standard sea-level atmsophere Runway friction coefficient: | here $\quad 0.06$ for take-off |
|  | t: 0.40 for landing |
| Maximum $C_{L}$ for take-off: | No C.C., $29^{\circ}$ flap: 2.24 |
|  | With circulation control: |
|  | 4.0 |
| Maximum $\mathrm{C}_{\mathrm{L}}$ for landing: | No C.C., $60^{\circ}$ flap: 2.0 |
|  | With circulation control: |
|  | 3.0 |
| Wing area | 174 square feet |
| Taper ratio | 1.5 |

Two types of end-plates:
(1) End-plates of $9 \% \mathrm{~S}$ were added to wingtip.
(2) Wingtip turned up at each end so that turned-up area was $9 \%$ of remaining area.

Equivalent parasite drag coefficient 0.028
Addition of end-plates increased e by $40 \%$.
The results of this analysis are shown graphically in Figures 9 and 10. Examination of these figures leads to the following conclusions:
l. Addition of end-plates would not shorten landing distance.


Figure 9. Effect of End-Plates on Take-Off Distance


Figure 10. Effect of End-Plates on Landing Distance (over 50 ft. obstacle)
2. End-plates used in conjunction with circulation control would decrease take-off distance.
3. The area used in forming end-plates could be used to better advantage in the form of increased wing area.

The reason for the degradation of the landing performance with end-plates is that the decrease in $D / L$ produces a flatter approach (i.e., smaller angle of glide, $\gamma$ ), as shown in Figure 11.

The approach angle is $\gamma=\operatorname{arc} \tan \frac{(D-F)}{L}$. In the final approach for minimum landing distance, $F$ is negligible so that $\gamma=\operatorname{arc} \tan \frac{D}{\mathrm{~L}}$. Since landing distance is the distance from a 50-foot obstacle to the aircraft stopped position, the angle of approach has a marked effect on landing distance. It is desirable for $D / L$ to be large for landing. Figure 12 illustrates the effect of $D / L$ on the landing distance.

## Variable-Geometry End-Plates

These opposing D/L requirements for take-off and landing led Clements ( 6 and 7) to devise variablegeometry end-plates for the purpose of controlling $C_{D_{i}}$ (and, thus, $D / L$ ) to increase the angle of glide for landing and to decrease the take-off roll. It was his idea that, by making the end-plates of symmetrical airfoils with hinged sections, the induced drag could be increased


Figure 11. Forces on Aircraft in Approach Flight


Figure 12. Effect of Approach Angle on Landing Distance
for approach flight or could be reduced for take-off and climb. Clements used the model shown in Figure 13 . Both the angle of incidence of the symmetrical-airfoil endplates and the angle of deflection of the end-plate flaps could be varied (inboard or outboard). These deflection angles are shown in Figure 14.

Clements' reasoning was a bit vague.
The portion of the end-plate above the wing is deflected to give a reduced pressure on the inboard side of the plate, and the portion below the wing is deflected to give an increased pressure on the inboard side. In this way the force distribution of the wing is extended. The vortex flow of the wing tip is replaced by the vortex flow of the plates, or, in effect, the wing aspect ratio is increased.

If, on the other hand, an increase in drag is desired, the end-plates are deflected in a manner opposite that described above. The endplates will not supplement the wing lift distribution. The induced drag will not be reduced and will perhaps be increased because of a more rapidly diminishing lift distribution in the region of the end plates (7).

A more rigorous analysis of end-plate action may be considered. The following visualization of the vortex pattern was suggested by Mangler (29). Figure 15 shows the lifting wing represented by a horseshoe vortex. The downwash velocity at any station (on the span) at a distance $y_{1}$ inboard of the wingtip is $w=\frac{K}{y_{1}}$.

The presence of end-plates on the wing will change the pattern of the horseshoe vortex to that of Figure 16. If the end-plates are symmetric (2h high), the strength of each of the trailing vortices will be one-half the strength of the bound vortex. Then, the downwash velocity


Figure 13. Clements' Variable Geometry End-Plates;

$\delta_{f}$ End-Plate Flap Deflection
$\delta_{e}$ End-Plate Deflection

+ Increase Drag
- Decrease Drag

Note that for a given deflection sign the deflections above and below the wing are opposite.

Figure 14. Deflection Nomenclature for Clements' End-Plates


Figure 15. (a) Horseshoe Vortex Representing Lifting Wing
(b) Downwash at Distance $\mathrm{y}_{\mathrm{I}}$ Inboard of Wingtip

(b)

Figure 16. (a) Vortex Pattern Simulating Wing with End-Plates
(b) Downwash Produced by Wing with End-Plates
at the station of distance $y_{1}$ inboard of the wing tip is:

$$
\begin{aligned}
w^{\prime}=w^{\prime \prime} & =\frac{K / 2}{r}=\frac{K}{2 \sqrt{y_{1}^{2}+h^{2}}} \\
w_{e}=\sqrt{\left(w^{\prime}\right)^{2}+\left(w^{\prime \prime}\right)^{2}} & =\sqrt{\frac{K^{2}}{4\left(y_{1}^{2}+h^{2}\right)}+\frac{K^{2}}{4\left(y_{1}^{2}+h^{2}\right)}} \\
& =\frac{K}{\sqrt{2\left(y_{1}^{2}+h^{2}\right)}}
\end{aligned}
$$

(The subscript e refers to the case with end-plates.)

$$
\frac{D_{i}^{\prime}}{L^{\prime}}=\frac{w}{V_{a}} \quad ; \quad \frac{D^{\prime}{ }_{i}{ }_{e}}{L^{\prime}}=\frac{w_{e}}{V_{a}}
$$

For a given $\mathrm{L}^{\prime}$ :

$$
\begin{aligned}
& \frac{D_{i}^{\prime}}{L^{\prime}}{ }^{\prime} \frac{L^{\prime}}{D_{i}^{\prime}}=\frac{w_{e}}{V_{a}} \frac{V_{a}}{W}=\frac{K}{\sqrt{2} \sqrt{y_{1}^{2}+h^{2}}} \frac{y_{1}}{K} \\
& \frac{D_{i}^{\prime}}{D_{i}^{\prime}}=\frac{Y_{1}}{\sqrt{2} \sqrt{y_{1}^{2}+h^{2}}}=\frac{1}{\sqrt{2} \sqrt{1+\left(h / y_{1}\right)^{2}}}
\end{aligned}
$$

This ratio, $D_{i}^{\prime}{ }_{e / D}{ }_{i}$ is always smaller than 1.O. Note that as the height, $h$, of the end-plates increases, the value of $D_{i}^{\prime}$ decreases. The limiting case is, of course, that of infinite end-plates ( $h \rightarrow \infty$ ) in which case the induced drag approaches zero. This flow is, then, twodimensional.

In the consideration of a vortex sheet instead of a single vortex filament, the reasoning is identical since the result of each trailing vortex filament within the
sheet is the same as that of the single vortex. Thus, the cumulative effect of all the vortices in the vortex sheet will have the same trend. The presence of a neutral endplate decreases the induced drag.

The symmetrical airfoil (or a thin flat plate) will produce side-forces directed inboard on the end-plate above the wing and outboard below the wing. Examination of the pressure field tends to confirm this conclusion (see Figure 17).


Figure 17. Side Forces on Neutral End-Plates

It will be noted that these sideforces are in the direction specified by Clements (7) for the reduction of wing induced drag.

Figure 18 contrasts the downwash pattern for an

elliptically-loaded wing (a) with wings in which the lift had been shifted outboard (b) or inboard (c). For ease in visualization, these wings have been approximated, in each case, by a finite number of trailing vortices. At any spanwise station, the downwash due to a single trailing vortex may be obtained by integrating the Biot-Savart equation from 0 to $\infty$, giving $w=\frac{\Gamma}{4 \pi n}$. Then the downwash at the station due to all the trailing vortices is $\sum_{i=1}^{\infty} \frac{\Gamma}{4 \pi n_{i}} ; n$ is the spanwise distance to the trailing vortex.

Shifting the trailing vortex strength outboard decreases the inboard downwash and decreases the average downwash velocity. The spanwise position of the rolled-up vortex will also move outboard. The decrease in average downwash velocity causes a corresponding decrease in the induced drag. Shifting the load inboard increases the average downwash, increases the induced drag, and moves the rolled-up vortex inboard. These statements may be summarized:

| Direction of | Average <br> Downwash | Vortex | Induced |
| :---: | :--- | :---: | :---: |
| Wingloading Shift | Velocity | Span | Drag |
| Inboard | Increases | Decreases | Increases |
| Outboard | Decreases | Increases Decreases |  |

This shifting of lift distribution may be done in a variety of ways:

Wing twist - (wash-in and wash-out)
Changing of airfoil section

Flap deflection.
It is well-known that wash-out (decreasing the angle of attack toward the tip) increases the drag of a wing.

As stated above, Clements proposed shifting the lift distribution (and, thus affecting the induced drag) by using adjustable end-plates. His model is shown in Figure 13. It consisted of an aspect ratio 5 wing of Clark-Y section equipped with full-span slots and fullspan $30 \%$ chord slotted flaps. In all tests, the flaps were deflected 50 degrees because Clements was particularly interested in the landing and take-off configurations (high $C_{L}$ ). The end-plates had an NACA 0012 section and were equipped with $30 \%$ chord plain flaps. The end-plates had a square planform; the end-plate chord equaled the wing chord and the end-plates extended one-half chord above the wing and one-half chord below the wing.

The incidence of the end-plates was varied by rotating the end-plate above a vertical axis through the end-plate quarter-chord. Also the end-plate flaps were deflected. The sign convention is shown in Figure 14. The tests were performed at a Reynolds Number of 350,000 .

Several values of end-plate and flap deflections were investigated. The results of the best combinations are presented in Figure 19. Values abstracted from these results are given in Table II.


Figure 19. Wing Characteristics With Adjustable End-Plates

## TABLE II

CANTED ADJUSTABIE END-PLATES

|  | $C_{I}$ | $C_{D}$ | $\alpha$ | $C_{M}$ | ${d C_{M / d C}}^{M}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Wing alone | 1.4 | 0.308 | 8.2 | -0.246 | -0.068 |
| End-Plate and end-plate <br> flap neutral | 1.4 | 0.284 | 7.4 | -0.149 | -0.101 |
| End-Plate deflected $-5^{\circ}$ <br> end-plate flap neutral | 1.4 | 0.269 | 6.0 | -0.205 | -0.060 |
| End-Plate neutral, <br> end-plate flap at $+30^{\circ}$ | 1.4 | 0.345 | 8.3 | -0.128 | -0.148 |

Examination of these results gives credence to Clements' statement that "Canted Adjustable End-Plates can be used to control the drag of wings. The drag can be increased or decreased depending upon the end-plate or end-plate flap deflection" (7). This method of constructing endplates appears quite promising.

However, analysis of the results, in the same manner as employed for the standard end-plates previously, shows that the canted adjustable plates are not as good as they appear to be. Data from Figure 19 were used in plotting $C_{D}$ Vs $C_{I}^{2}$ in Figure 20. The principal results are tabulated in Table III. Also included in Table III are results for an "Extended Wing" which resulted from extending the span of the wing, instead of using end-plates, by


Data abstracted from reference 6 .


Figure 20. Drag of Adjustable End-Plates

TABLE III
SUMMARY OF WING PERFORMANCE WITH CLEMENTS' CANTED ADJUSTABLE END-PLATES

| Configuration | ${ }^{C^{L}}{ }_{\text {max }}$ | $C_{D}$ Equation | $e_{w}$ | Effective Aspect Ratio | $D / L$ at $C_{L}=1$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Basic Wing | 1.97 | $C_{D}=0.155+0.077 C_{L}{ }^{2}$ | 0.828 | 4.14 | 0.234 |
| Wing with Neutral End-Plates | 2.01 | $C_{D}=0.167+0.059 C_{L}{ }^{2}$ | 1.08 | 5.40 | 0.225 |
| "Best Take-off" <br> Configuration | 2.055 | $C_{D}=0.158+0.058 C_{L}{ }^{2}$ | 1.09 | 5.46 | 0.212 |
| "Best Landing" Configuration | 2.0 | $C_{D}=0.239+0.055 C_{L}{ }^{2}$ | 1.162 | 5.81 | 0.28 |
| Extended Wing <br> (no end-plates) | 1.97 | $C_{D}=0.155+0.055 C_{L}{ }^{2}$ | 0.82 | 5.64 | 0.200 |
| Extended Wing <br> + Drag Brake | 1.97 | $C_{D}=.0 .239+.055 C_{L}{ }^{2}$ | 0.82 | 5.64 | 0.294 |

adding the same area to the wingtips. The actual aspect ratio of this extended wing would be 7.0. The last configuration in Table III is the extended wing combined with a retractable flat-plate drag brake, having an area of about $7 \%$ of the wing area.

Table III shows that this higher-aspect-ratio-extended-wing with the drag brake retracted would produce lower drag than the "Best take-off Configuration" for take-off and climb. With the drag brake extended, the drag of the "Best Landing Configuration" is exceeded.

Thus, it appears that even with this most promising of the various arrangements of end-plates, the additional area could have been used more effectively in the form of extensions of the wing lifting surface. Improvements in effective aspect ratio must come from other schemes, such as the use of wingtip rotors described in the next chapters.

## CHAPTER III

## WINGTIP-MOUNTED PROPELLERS

As reviewed in the previous chapter, a myriad of fixed and variable-goemetry end-plates have been proposed but with little effectiveness. The exception to this statement is the combination of end-plates with some method of circulation-control producing high values of $C_{L}$, particularly when forced to use wings of low geometric aspect ratio.

Rather than continue to manipulate wing geometry to approach two-dimensional flow it would seem logical to use some energy source for the task of directing the flow. The most obvious source of energy is the main aircraft powerplant. This paper will be restricted to propellerdriven aircraft -- the propellers being driven by piston engines or by gas turbines.

The purpose is to determine whether the engine-driven propellers which propel the aircraft can, at the same time, be used to control the wing loading and the downwash and drag of the wing.

Objectives and Proposed Solution

The objectives, previously set forth, of controlling
the wing loading and the downwash are:
a) High $C_{I_{\text {max }}}$
b) Low values of induced drag (for take-off and climb).
c) High values of induced drag (for approach and landing).

To these highly desirable characteristics should be added the requirements that the system for attaining these ends should:

1. Add no weight or structural penalty.
2. Add no additional profile drag increment.
3. Add no complexity to the control of the aircraft.

These requirements can be met by placing the propellers which propel the aircraft at the wingtips. The engines would be mounted in pods, or nacelles, fitted to the ends of the wing.

Possible advantages of such an arrangement include:

1. The rotational component of the propeller slip-stream is available for amplifying or attenuating the wing vortex system. This component has, heretofore, been considered the result of lost energy.
2. The propellers are necessary for propulsion of the aircreft, i.e., this is not a case of adding a piece of equipment or structure;
rather, it is a case of locating it to best advantage.
3. Placing the engines and propellers at the wing-tips will relieve the wing shear and bending-moments and could result in a lighter structure.
4. Placing the engines at the wingtips and the fuel in the outboard parts of the wings would greatly improve the safety and chances for crew survival in cases of crash landing. Possible disadvantages of this arrangement include:
l. Difficulty (or impossibility) of trimming the aircraft for one-engine-out operation.
5. Production of aero-elastic problems created by the changing of the torsional moment of inertia of the wing and the interaction of bending and torsional modes of flutter or vibration.

The main objective of this paper is to report the results of an experimental program designed to test and evaluate the effectiveness of this scheme.

## Experimental Investigation

The test program consisted of wind tunnel testing of a wing, with and without propellers mounted at the tip of the wing. These propellers were powered by a motor contained within a streamlined pod at the wingtip.

## Model Description

The wing tested is a tapered wing with an aspect ratio of about 8 . The dimensions of the model are given in Table IV and Figure 21 is a sketch of the model.

In order to provide a periptery not influenced by the tunnel walls, the test was a reflection-plane test. The reflection-plane was the floor of the tunnel test section as shown in Figures 22 and 23. The wing model that was tested is a hollow, cast magnesium wing having an unbroken taper in thickness ratio and in planform. The wingtip, ailerons, flaps and inserts are mahogany. The model was mounted with no dihedral and with the vertical quarterchord line collinear with the balance centerline. Figures 22(a) and 23(a) show the wing with standard plain tip installed in the tunnel.

In addition to the plain tip, the model was tested with a wingtip pod. The purpose of the pod is to house an electric motor which serves as drive for the propeller and impeller. The podis a streamlined body of revolution 192 inches long with a fineness ratio of 4.6. Figure 22(c) shows the pod with a dummy spinner nose. The spinners of the propeller and the impeller have the same contour.

It was planned to use two propellers in the tests -one of standard conformation and of relatively high efficiency (high ratio of axial velocity to angular velocity) and the other having fairly high rotational velocity

## TABLE IV

MODEL PHYSICAL DIMENSIONS

Wing

| Semi-area | 3.992 sq. ft. |
| :--- | :--- |
| Semi-span | 4.011 ft. |
| Aspect Ratio | 8.06 |
| Mean aerodynamic chord | 0.995 ft. |
| Airfoil sections |  |
| $\quad$ Root | NACA 23018 |
| $\quad$ Tip | NACA 23012 |
| Twist | None |
| Flap chord | $25 \%$ chord |
| Flap span | $62.5 \%$ span |
| Aileron chord | $30 \%$ chord |
| Aileron span | $25 \%$ span |
| ngtip Pod | $19 \frac{1}{2}$ in. |
| Length | $4 \frac{1}{4}$ in. |
| Maximum diameter | 14.2 sq. in。 |
| Frontal area | 40.5 sq. in. |
| Planform area | 4.302 ft. |
| Wing semi-span, including pod |  |
| Wing aspect ratio, including pod 8.78 |  |




compared to the axial velocity. The purpose in using this second propeller was to obtain data bearing on the observations of Lippisch ${ }^{l}$. He stated that a limitation to the use of a propeller in affecting the wing trailing vortex was the efficiency of the propeller -- about $80 \%$ of the energy provides axial acceleration of the air and only about $20 \%$ is available to produce rotation with which to supplement or to counteract the trailing vortex and the attendant downwash pattern.

Some years ago, the writer tested, for the Beech Aircraft Corporation, a propeller invented by Tom Quick. This Quick propeller was an impeller similar to that shown in Figures 22(d) and 23(d). Quick maintained that the air would be thrown radially outward producing at the nose of the impeller a region of pressure that would be lower than that which existed on the rear portion of the streamlined housing, producing thrust. The tests indicated that the device does produce thrust, but the propulsive efficiency peaks at less than fifty per cent. This type of propeller was chosen to serve as the inefficient propeller for these tests. The Quick propeller will be referred to as the impeller and the four-bladed propeller will be called the propeller.

[^0]The impeller is shown on the wing in Figures 22(d) and 23(d); the propeller is shown in Figures 22(b) and 23(b). They are compared in Figure 24.

The impeller has the same basic contour as the propeller spinner. Added to that contour are twelve radial blades having a maximum depth of one inch giving an overall impeller diameter of 5.25 inches. Figure 26 is a drawing of the impeller.

The blade planform and blade sections of the fourblade propeller are shown in Figure 27. This propeller is a right-hand rotation propeller. The pitch angle of the propeller blades was arbitrarily set at $15.5^{\circ}$ (at $\mathrm{r} / \mathrm{R}=60 \%$ ) using the jig shown in Figure 25. This angle corresponds to a very low pitch. Propeller activity factor is about 90 per blade and the total propeller diameter is 15 inches.

The motor installation is shown in Figure 28. The motor is a synchronous motor rated at 10 horsepower at 12,000 r.p.m. and is three inches in diameter and six inches long. The power source is a variable frequency ( 0 to 400 cycles/sec.) generator; the frequency of the generated power is varied to change the motor speed. Speed is monitored by beating an induced signal from a loop on the motor shaft against a known input. Figure 29 shows the motor controls including the signal generator, oscilloscope, and TV monitor. As shown in Figure 28, the power leads and tachometer wires, as well as the coolingwater tubing and thermo-couple leads were carried to the


Figure 24. Impeller and Propeller.


Figure 25. Propeller on Pitch-Angle Jig


Figure 27. Propeller Blade


Figure 28. Motor Installation


Figure 29. Motor Instrumentation
motor through the hollow wing.
Downstream of the wing, a tuft grid was mounted for the observation of the flow field direction. This grid is shown in Figures 22 ( $\mathrm{a}, \mathrm{b}$, and c ), and in Figure 23(a) the remotely-operated camera can be seen upstream of the tunnel propeller (the camera hood has been removed and is on the floor).

In order to correct the values of lift, drag, and pitching moment observed during the power-on tests, the lift, drag, and moment produced by the propellers was determined using the propeller mount shown in Figure 30.

The dynamic tare was found using the dummy spinner as shown in Figure 30(a). The propellers were operated as shown in Figures $30(b$ and $c)$. In each case, the mount was rotated through the same angles of attack as those at which the wing had been tested. The propeller speeds were the same as during the wing tests.

## Test Program

The test program included the following types of tests and model configurations.

Test Method:

1. Photographs of tuft-grid downstream of the model.
2. Balance measurement of three-components (lift, drag, and pitching moment) through

```
range of angles of attack from -10' to
```

$+26^{\circ}$ 。

The tuft grid pictures were taken at nominal angles of attack of $12^{\circ}$ and $0^{\circ}$. The flap deflections were either O or $40^{\circ}$. These angles were chosen from preliminary balance tests which indicated that the angle of zero lift of the basic wing was approximately zero and at $\alpha=12^{\circ}$ the $C_{L}$ was about 1.0.
Model Configurations:

1. Wing with plain tip (Figure 22(a) and 23(a)).
2. Wing with pod and dummy spinner (Figure 22(c)).
3. Wing with pod and impeller (Figures 22(d) and 23(d)).
a) Impeller stationary
b) Impeller turning in the same direction as the wing trailing vortex (two speeds)
c) Impeller turning in the countervortex direction (two speeds)
4. Wing with pod and propeller (Figures 22(b) and $23(b$ and $c)$ ).
a) Propeller stationary (two positions)
b) Propeller windmilling (countervortex direction)
c) Propeller turning counter-vortex at greater than windmilling speed


Figure 31. Wind Tunnel Coordinate System
d) Propeller turning counter-vortex at less than windmilling speed
5. Post-mount with pod (full range of $\alpha$ )
a) Dummy spinner (Figure 30(a))
b) Impeller (pro- and counter-vortex) (Figure 30(c))
c) Propeller (3 speeds) (Figure 20(b))

The coordinate system adopted for the test is shown in Figure 31.

Results of the Test Program
Since the experimental results, when presented graphically, are quite bulky, only those figures of primary importance are included in the body of this paper; other results may be found in the Appendix.

## Tuft-Grid Results

In order to interpret the results of the tuft grid survey, 140 of the pictures were printed. Figure 32 shows example pictures. These pictures show the flow field in the $Y-Z$ plane at various distances downstream of the model trailing-edge (at the tip). All pictures were taken at a tunnel dynamic pressure of 10 psf. It is apparent that the position of the wingtip vortex core and the flap outboard core can be estimated.

Since the pictures were taken with the grid located at various positions, ranging from $\mathrm{x}=6$ inches to


Fiqure 32. Examples of Tuft-Grid Photographs. (a) Wing with Plain Tip, $\alpha=12^{\circ}, \mathrm{x}=12$ in.


Figure 32 (cont.). Examples of Tuft-Grid Photographs

(d) $\mathrm{N}=175 \mathrm{rps}$.

(f) $\mathrm{N}=50 \mathrm{rps}$.

Figure 32 (cont.). Examples of Tuft-Grid Photographs: Wing with propeller, CounterVortex Rotation, $a=12^{\circ}, \delta_{f}=40^{\circ}, x=48 \mathrm{in}$.

(g) Impeller Stationary

(h) N=175 rps, counter-vortex

Figure 32 (concluded). Examples of Tuft-Grid Photographs. Wing With Impeller, $\alpha=12^{\circ} ; \delta_{f}=0^{\circ}, x=48 \mathrm{in}$.
$\mathrm{x}=96$ inches, it is possible to estimate the trajectory of the trailing vortex core as it rolls-up. Figure 33 shows the trajectories of three trailing vortices in the $\mathrm{X}-\mathrm{Y}$ plane. The model configuration is the wing with the impeller installed at the tip; $\alpha=12^{\circ}$, $\delta_{f}=0^{\circ}$. With no rotation of the impeller, the vortex-span is 101 inches; counter-vortex rotation shifts the vortex outboard to a vortex-span of 107 inches; vortex-direction rotation reduces the vortex-span to 89 inches. Figure 34 shows these trajectories in the $\mathrm{X}-\mathrm{Y}$ plane and Figure 35 shows them in the Y-Z plane.

A similar graph is shown in Figure 36 which is the X-Y plane showing the trajectory of the vortex core as affected by the propeller rotation. The largest lateral shift of the trajectory was produced by the rotation of the propeller at less than windmill speed.

Figure 37 shows the position of the center of the vortex core at a distance of one-half wingspan downstream of the trailing-edge for most of the configurations tested. The wing was at an angle of attack of $12^{\circ}$ and with the flaps deflected $40^{\circ}$. The following results can be detected:
a) Adding the pod moves the trailing vortex outboard compared to the wing with a standard tip.
b) Counter-vortex rotation of the impeller and counter-vortex rotation of the





OPlain Tip
$\square$ Pod Dummy Spinner
$\diamond$ Stationary Impeller
$\triangle$ Vortex Rotating
Impeller, 175 r.p.s.
a. Counter-Vortex Rotating

Impeller, 175 r.p.s.
4-Blade Prop.
) $n=175$ r.p.s.
$n=122$ r.p.s. (Windmilling)$n=50$ r.p.s.

Reflection Plane

$$
\begin{aligned}
& x=48^{\prime \prime} \\
& \alpha=12^{\circ} \\
& s_{f}=40^{\circ}
\end{aligned}
$$

Figure 37. Position of the Center of the Vortex
Core, One-Half Span Downstream, for Various Tip Configurations

propeller (at greater than windmilling speed) moves the vortex outboard.
c) Vortex rotation of the impeller and counter-vortex rotation of the propeller at less than windmill speed causes the vortex to move inboard.
d) The tip configuration and rotor rotation have little discernible effect on the position of the flap outboard vortex.
e) In the range of rotational speeds involved, the propeller produces greater shift of the trailing vortex than does the impeller.

The effect of rotation of the two types of propellers is further illustrated in Figure 38. It can be seen that as rotation becomes more strongly a counter-vortex rotation the vortex moves outboard. As the rotation becomes more strongly vortex-directed, the trailing vortex moves inboard. The vortex tends to remain in the plane of the wing with the impeller, but there is a shift in the $z$ direction with the propeller.

## Results of the Force and Moment Measurements

The data from the balance system was reduced using the 1620 digital computer. Some of these answers were manually plotted and the remainder were plotted using a Calcomp on-line plotter controlled by the 1620.

The reflection-plane test results are compared with the full-model test in Figure 39. The complete wing test data are from reference 12. It is believed that there is satisfactory agreement of the data.

Another preliminary test was made to determine whether the proximity of the tuft grid to the wing would affect the wing lift. In run 105 , the grid was only 1 foot downstream of the wing trailing-edge at the outboard tip. In run 108, the grid was 4 feet downstream. These runs are compared in Figure 40. This plot is a computer plot. Except for a bad point near the stall, the two runs show excellent agreement. Run 102 is also plotted in Figure 40 showing the effect of changing the Reynolds Number from $1.2 \times 10^{6}$ to $6.7 \times 10^{5}$.

All of the subsequent wing tests were at a Reynolds Number of $6.7 \times 10^{5}$. This Reynolds Number resulted from a tunnel dynamic pressure of 10 pounds/sq. ft. (about 100 ft./sec. velocity) which was dictated by two requirements:
l. The behaviour of the tufts was better at less than 10 psf. dynamic pressure (also, the replacement rate for tufts was less than at higher speeds).
2. The windmilling speed of the propeller increases as speed increases. It was desirable to have a speed low enough so that the propeller could be operated at greater than, as well as less than,


Figure 39. Comparison of Reflection Plane and Complete Models of the Same Wing


Figure 40. Lift Curve, Basic Wing
windmill speed. The maximum operating speed of the propeller motor is 200 r.p.s. The results of the reduction of the balance readings are presented graphically in the following forms:

$$
\begin{array}{ll}
C_{L} \text { vs. } \alpha \text { and } C_{L} \text { vs. } C_{M_{C / 4}} & \text { (not corrected for thrust) } \\
C_{L} \text { vs. } C_{D}, \alpha, C_{M} \text { c/4 } & \text { (corrected for thrust) } \\
C_{D} \text { vs. } C_{L}^{2} & \text { (corrected for thrust) }
\end{array}
$$

The data were reduced both with and without tunnel-wall corrections. Since the results were to be correlated with the tuft-grid pictures, it was decided to use the results uncorrected for tunnel boundary. As stated previously, the correction for thrust consisted of determining the lift, drag, and pitching moment due to the thrust of the propeller and impeller as functions of angle of attack (see Figure 30). These values were then subtracted from those measured during the powered tests.

Figure 40 is an example of $C_{L}$ vs $\alpha$. The complete set is given in Appendix $B$, as are the $C_{L}$ vs $C_{M_{C / 4}}$ curves.

Figure 41 is an example of a complete wing polar: $C_{L}$ vs $C_{D}, \alpha$, and $C_{M_{C / 4}}$. The complete set of these curves are in the Appendix. The figures which resulted from powered runs have been corrected for the direct effects of thrust.

Figure 42 is an example of a $C_{D}$ vs $C_{L}{ }^{2}$ graph. The plots for the remaining runs are presented in the Appendix.


Figure 41. Wing With Plain Tip


Figure 42. Basic Wing, Standard Tip, Drag Variation

The principal results have been abstracted from these curves and are tabulated in Table V.

The most interesting of these results are shown graphically in Figures 43 and 44. In each of these figures, the abscissa is the counter-vortex rotational speed of the rotor above windmill speed. In the case of the propeller, windmill speed is about 120 r.p.s., so that the propeller curve is plotted for the range 40 r.p.s. to 175 r.p.s. (counter-vortex). In the case of the impeller, the windmill speed is zero; the negative speeds are, then, rotation in the vortex-direction and the positive speeds are rotation in the counter-vortex direction.

Figure 43 shows a very pronounced relationship between impeller or propeller speed and the effective aspect ratio. Effective aspect ratio, $A e_{w}$ is defined:

From the plots of $C_{D}$ vs $C_{L}{ }^{2}$,

$$
\frac{d C_{D}}{d C_{L}{ }^{2}}=m=\frac{1}{\pi A e_{w}}
$$

(Note: It is assumed that the graph is a straight line, i.e., $m=\frac{d C_{D}}{d C_{L}{ }^{2}}=$ constant.)

Effective aspect ratio $=A e_{W}=\frac{1}{\pi m}=0.318 \frac{\mathrm{~d} \mathrm{C}_{L}{ }^{2}}{\mathrm{dC}_{D}}$.

The relation between $C_{I_{\max }}$ and rotor speed is less obvious and less conclusive; see Figures 44 and 45.


Figure 43. Variation of Effective Aspect Ratio Caused by Rotor Speed

TABLE V
SUMMARY OF WING CHARACTERISTICS




Figure 45. Effect of Rotational Speed of Propeller on Wing Maximum Lift Coefficient

## CHAPTER IV

## ANALYSIS OF RESUITS

The principal results expected from the experimental program are:
I. Alteration of the flow field downstream of the wing. This changed flow field would be identified by changes in the downwash and by altered trajectories of the core of the trailing vortex.
2. Changes in the wing loading, resulting in changed lift-curve slope and $C_{I_{\max }}$.
3. Changes in wing drag, chiefly in the induced portion of the drag.

## Vortex Trajectories

Figures 33 through 37 show the changes in the position of the core of the trailing vortex caused by rotation of the impeller and of the propeller. The chief effect is the change in the vortex span which is apparent in Figures 33 and 36. It can be seen that the change in vortex span is not the result of bending of the flow downstream of the wing. Rather, the shift apparently occurs upstream of the
first grid position. This fact indicates that the vortex span is the result of the distribution of strength in the shed vortex sheet.

Table VI summarizes the vortex span and the effective aspect ratio for the various configurations tested.

TABLE VI
VORTEX SPAN AND EFFECTIVE ASPECT RATIO

| Configuration | Eff. Aspect Ratio $\mathrm{Ae}_{\mathrm{W}}$ | Vortex Span $b_{v}$, ft. |
| :---: | :---: | :---: |
| Dummy Spinner, No Propeller | 6.45 | 8.25 |
| Stationary Impeller | 6.32 | 8.42 |
| Impeller, Counter-Vortex Rotation, $N=175$ r.p.s. | 10.35 | 8.93 |
| $\begin{aligned} & \text { Impeller, Vortex-Rotation } \\ & \mathrm{N}=175 \mathrm{r} \cdot \mathrm{p} \cdot \mathrm{~s} . \end{aligned}$ | 5.34 | 7.42 |
| Propeller, Windmilling, $\mathrm{N}=175 \mathrm{r} . \mathrm{p} . \mathrm{s}$. | 5.73 | 8.33 |
| Propeller, Counter-Vortex Rotation, $N=175$ r.p.s. | 10.1 | 8.42 |
| Propeller, $N=50 \mathrm{r} . \mathrm{p} . \mathrm{s}$. | 4.0 | 7.42 |

In an attempt to determine a relationship between effective aspect ratio and the vortex span, these quantities are graphed in Figure 46. A curve has been fit to


Figure 46. Vortex Span and Effective Aspect Ratio for Test Configurations


Figure 47. Nomenclature for Wing Partially Submerged in Propeller Slipstream
the data; the equation of this curve is:

$$
b_{v}=-4.2+4.74\left(A e_{W}\right)-0.592\left(A e_{W}\right)^{2}+0.025\left(A e_{W}\right)^{3}
$$

The validity of this equation is suspect because of the relatively large scatter of the data. However, it can be seen that there is an increase of vortex span as the effective aspect ratio increases.

It will be noticed, from Figures 34 and 35 , that the vortex (no power) leaves the wing at the level (in the $z$ direction) of the trailing-edge of the wing, while with power (either direction) it is lower; i.e., roughly at the level of the tip of the pod. Figures 35 and 37 indicate that the propeller produces a greater vertical shift of the vortex core than does the impeller. This result agrees with the larger changes in effective aspect ratio caused by the four-bladed propeller.

The cause of both of these effects is the axial component of propeller/impeller slipstream. This axial velocity is higher than the stream velocity. At the 12 degree angle of attack, the axial velocity has a component in the z-direction of $V_{S S} \sin 12^{\circ}$ (equals $0.208 V_{S S}$ ). This component moves the initial point of the vortex down. Also; the component will be larger for the propeller than for the impeller because the propeller has a higher slipstream velocity, $\mathrm{V}_{\mathrm{SS}}$.

## Effects of Power on Lift and Drag

It would be expected that the addition of thrust power to a wing, producing a slipstream over the wing, would result in a higher indicated lift coefficient at any given angle of attack and in a higher maximum lift coefficient. With reference to Figure 47, it may be reasoned that:

$$
\begin{aligned}
& \left(C_{L}\right)_{\text {calculated }}=\frac{L}{q_{a} S} ; \quad I=C_{L_{\text {call }}} q_{a} S \\
& L=L_{s s}+L_{\text {res }} \quad \text { where, } L_{s s} \text { is lift developed by } \\
& \text { the portion of the wing in the } \\
& \text { slipstream ( } \mathrm{S}_{\mathrm{p}} \text { ). } \mathrm{I}_{\text {res }} \text { is lift } \\
& \text { developed by the rest of the } \\
& \text { wing ( } S_{\text {res }} \text { ). } \\
& L=C_{L_{\text {actual }}} q_{S S} S_{p}+C_{L_{\text {actual }}} q_{a} S_{\text {res }} \\
& =S\left[C_{L_{\text {actual }}} q_{S S} \frac{S_{p}}{S}+C_{L_{\text {actual }}} q_{a} \frac{S-S_{p}}{S}\right] \\
& =C_{L_{\text {talc }}} q_{a} S=C_{L_{\text {actual }}} S\left[q_{S S} \frac{S_{p}}{S}+q_{a}\left(1-\frac{S_{p}}{S}\right)\right] \\
& C_{L_{\text {talc }}}=\left(C_{L}\right)_{\text {actual }}\left[1+\frac{S_{p}}{S}\left(\frac{q_{S S}}{q_{a}}-1\right)\right] .
\end{aligned}
$$

The number within the brackets is always greater than I for a thrusting propeller. The apparent $C_{L}$, ie.,
${ }^{C} L_{\text {call }}$, is larger than the $C_{L}$ actually being developed by the wing. Note that this apparent, or effective, $C_{L}$ does
not include any vertical component of the propeller thrust. As an example of this effect, data have been extracted from references 12 and 13 on the RM-9 airplane. This airplane was equipped with the same wing as the wing of this test, except that it was equipped with four engine nacelles, two on each wing. A sketch of the model is shown in Figure 48. Some effects of power are listed in Table VII.

TABLE VII
RM-9 WITH 60 DEG. FIAPS, NO TAIL

| Configuration | $C_{I}$ at $\alpha=0^{\circ}$ | $C_{I_{\max }}$ |
| :--- | :---: | :---: |
| No Propellers | 0.93 | 1.77 |
| Propellers 1 and 2 only, $T c=.82$ | 1.33 | 2.38 |
| Propellers 3 and 4 only, $\mathrm{Tc}=.82$ | 1.47 | 2.50 |
| All Propellers, Tc $\approx .82$ | 1.80 | 3.40 |
| Re $=7.3 \times 10^{5}$, Direct Thrust Effects Removed |  |  |

These data confirm the earlier statement, i.e., that the $C_{I}$ in a slipstream is higher than without a slipstream, even without including any lift component of the thrust. They also confirm the known fact that it is better to turn

the propellers in the direction opposite to the direction of the vortices being shed. Propellers 1 and 2 turn the same direction as the shed vorticity of the port wing; propellers 3 and 4 turn in the direction opposite to that of the shed vorticity of the starboard wing. The $\Delta C_{L}$ due to the propeller action is apparently only partly due to increased slipstream velocity. It is also due to the amplification or attenuation of wing lift due to interaction of the rotating propeller slipstream with the wing. Figures 44 and 45 indicate a similar effect with a single impeller or propeller mounted at the wingtip, i.e., as the counter-vortex speed of the rotor increases, the value of $\mathrm{C}_{\mathrm{L}_{\max }}$ increases. Figure 45 indicates that correcting $C_{I_{\max }}$ for thrust produces negligible change in $\mathrm{C}_{\mathrm{L}_{\text {max }}}$ 。

It is apparent that direction of rotation of the propeller, and also the spanwise position of the propeller affect the wing performance. In order to analyze these effects, additional data were abstracted from reference 54. The advantage of using these data is that both the wing and propeller used in the current test are the same as those used on the RM-9 (54). However, the propeller pitch was different ( 15.5 deg. in present test, 19 deg . on RM-9) 。

It was possible to calculate, from the tunnel tests, the thrust characteristics of the propeller and of the impeller. These are shown in Figures 49 and 50. Thrust


Figure 49. Thrust Coefficient of Impeller.


A corresponds to $N=175$ r.p.s. at test dynamic pressure.
$B$ corresponds to $N=122$ r.p.s. (windmilling speed) at test dynamic pressure.
C corresponds to $N=50$ r.p.s. at test dynamic pressure.
Figure 50. Thrust Coefficient of Propeller
coefficient may be defined as $C_{T}=\frac{\text { Thrust }}{\rho \bar{N}^{2} \mathcal{d}^{4}}$ or $T_{C}=\frac{\text { Thrust }}{\frac{\rho}{2} \sqrt{2}^{2} d^{2}}$, and $C_{T}=T_{C} J^{2} \cdot T_{C}$ is used in Figures 49 and 50 chiefly for comparison with values from reference 54. It will be noticed that the operational range of $J=\frac{V}{N d}$ for the impeller is considerably above that of the propeller. The curves for the propeller are extended into the negative thrust region, for this range corresponds to the rotational speeds less than windmill speed. Also in Figure 50, there is a graph of the thrust coefficient curve for the propeller at the blade angle used in the RM-9 tests.

Most of the RM-9 powered data is reported at $T_{C}=.82$ or $T_{C}=.44$. The $T_{C}=.44$ was developed at a rotational speed of approximately 188 r.p.s. This combination was the nearest to the $T_{C} \approx .65$ and $N=175 \mathrm{r} . \mathrm{p} . \mathrm{s}$. in this test.

Lift curve data was abstracted from Figures 9 and 10 of reference 54 for the RM-9 wing, body and nacelles combination with $\delta_{f}=0$. The data were corrected to the cases of single propeller operation and these corrected curves are platted in Figures 51 and 52. The effect of propeller position on the lift and drag of the wing is tabulated in Table VIII. The values of effective aspect ratio, lift-curve slope, and $D / L$ at $C_{L}=1.0$ from Table VIII are plotted in Figure 53.

Figure 53 illustrates, very well, the effects of spanwise position of the propeller on the wing

TABLE VIII
EFFECT OF PROPELIER POSITION ON WING CHARACTERISTICS

| Propeller Position | Direction of Rotation | $\alpha=\frac{\mathrm{dC}_{\mathrm{L}}}{\mathrm{~d} \mathrm{\alpha}}$ | ${ }^{\text {c }}$ D Equation | $\mathrm{Ae}_{\mathrm{W}}$ | D/L at $\mathrm{C}_{\mathrm{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Reflection Plane Wing, $\delta_{f}=0$ |  |  |  |  |  |
| No Propeller (with Pod) | None | 0.078 | $C_{D}=.013+.0493 C_{L}{ }^{2}$ | 6.45 | 0.062 |
| $\operatorname{Tip}, \frac{\mathrm{y}}{\mathrm{b} / 2}=1.0$ | * | 0.072 | $C_{D}=.032+.0795 C_{L}{ }^{2}$ | 4.0 | 0.112 |
| Tip, $\frac{y}{b / 2}=1.0$ | Counter- <br> Vortex | 0.086 | $C_{D}=.013+.0313 C_{L}{ }^{2}$ | 10.1 | 0.018 |
| RM-9 Model - Wing, Body, Nacelles, $\delta_{f}=0$ |  |  |  |  |  |
| No Propeller (with Nacelles) | None | 0.0785 | $C_{D}=.018+.0483 C_{L}{ }^{2}$ | 6.58 | 0.066 |
| No. $1, \frac{y}{b / 2}=0.66$ | Vortex | 0.0825 | $C_{D}=.02+.08 C_{L}{ }^{2}$ | 3.98 | 0.10 |
| No. 2, $\frac{y}{b / 2}=0.31$ | Vortex | 0.0825 | $C_{D}=.025+.0583 C_{L}{ }^{2}$ | 5.46 | 0.083 |
| No. 3, $\frac{y}{b / 2}=0.31$ | Counter- <br> Vortex | 0.0831 | $C_{D}=.016+.0633 C_{L}{ }^{2}$ | 5.03 | 0.079 |
| No. 4, $\frac{\mathrm{y}}{\mathrm{b} / 2}=0.66$ | Countere <br> Vortex | 0.0788 | $C_{D}=.009+.0508 C_{L}{ }^{2}$ | 6.26 | 0.060 |

*Counterovortex rotation, but at negative thrust and $N$ less than windmill speed.


Figure 51. Effect of Propeller Position on Lift Curve


Figure 52. Effect of Propeller Position


Figure 53. Effect of Propeller Position on Wing Characteristics
performance. In each case listed, there is only one propeller operating; it is operating at the spanwise position listed. All points plotted in Figure 53 correspond to $\mathbb{N}=175$ or 188 r.p.s. and $\mathbb{T}_{C}=.42$ or .65 , except the ones corresponding to the left-hand wingtip ( $\frac{\mathrm{y}}{\mathrm{b} / 2}=-1.0$ ). These points are for the propeller turning in the countervortex direction at less than windmill speed and at negative thrust (speed is about $70 \mathrm{r} . \mathrm{p} . \mathrm{s}$. less than windmill speed, and $\left.T_{C} \approx-0.8\right)$.

In considering the effect of wing lift characteristics, distinction must be made between the two effects of power:

1. Increased slipstream velocity.
2. Rotation of slipstream.

The effects of (l) were discussed above; i.e., as the slipstream velocity increases, the wing $C_{L}$ increases (at any given $\alpha$ ). The effect of (2) is that the wing is experiencing a lower angle of attack in one portion of the slipstream and a higher angle of attack in the other portion.

The effect of these two factors is sketched in Figure 54. It will be noted that experimental data from reference 15 confirms this analysis. More recent data in reference 16 indicates the same effects. The result of (2) is that the effects of the $\Delta \alpha$ and $-\Delta \alpha$ due to slipstream practically cancel for the propeller located inboard. However, when the propeller is near or at the


Figure 54. Effect of Propeller Operation Span-Ioad
tip, so that part of the slipstream is beyond the wingtip, the effects do not cancel. If the propeller at the tip is turning in the vortex-direction so that the propeller blades move down in front of the wing, there is a decrease in lift of the wing; the effect is like washout. If the wingtip propeller is turning in the counter-vortex direction, so that the propeller is moving up in front of the wing, there is an increase in the lift of the wing; the effect is like washin. For this reason, the effect of propeller location on lift is practically confined to locations near the wingtips (see $\mathrm{C}_{\mathrm{L}_{\alpha}}$ in Figure 53). The most notable effects of propeller spanwise position are those involving drag. Figure 53 shows a marked increase in effective aspect ratio as a counter-vortex turning propeller is moved toward the wingtip ( 4.6 with propeller on centerline; 9.1 at wingtip). The effective aspect ratio decreases as a pro-vortex turning propeller is moved toward the wingtip. There is a corresponding effect on drag.

Table $V$ and Figure 43 summarize the effective aspect ratio and the induced drag due to wingtip-mounted impeller or propeller. The values tabulated and plotted were obtained by fitting straight lines to the graphs of $C_{D}$ vs $C_{L}{ }^{2}$. The equation of the line is $C_{D}=C_{D}+m C_{L}{ }^{2}$, where

$$
m=\frac{\Delta C_{D}}{\Delta C_{L}{ }^{2}}=\frac{I}{\pi A e_{W}}
$$

The change in the induced drag due to a wingtip rotor
may be explained by Figure 55. In the case of the vortexdirection rotation, the downwash due to the rotor combines with the wing downwash to produce greater downwash at the wing and an increase in the induced drag. Figure 55 also shows that a counter-vortex rotation of the rotor produces a smaller downwash at the wing, and a smaller induced drag.

In terms of the trailing vortex strength, the vortexdirection turning rotor may be called a "vortex source", while the counter-vortex turning rotor may be called a "vortex sink". ${ }^{2}$ These functions are the results of the rotating vortex sheet downstream of the propeller which are superimposed on the shed vortex sheet of the wing. The wing vortex sheet is thus attenuated or amplified. It is possible to conceive of counter-vortex rotating propellers distributed along a wing of sufficient strength to absorb completely the vortices shed by the wing, resulting in infinite effective aspect ratio.

It will be noticed, in Table $V$, that in some cases there is a decrease of parasite drag accompanying the applications of power. There may be some error in correcting for direct thrust effects. There is also some "cleaning-up" of wing flow and nacelle flow due to the propeller slipstream. Similar effects are noted in references 54 and 5. The effect is more pronounced with the

[^1]

Figure 55. Representation of Changing Pattern of Downwash Due to Propeller-Produced Tip Vortex
propeller than with the impeller (probably due to the higher slipstream velocity of the propeller).

## Effect on Aircraft Performance

In order to provide an example of the effects of wingtip propellers on aircraft performance, a hypothetical aircraft has been modified. The aircraft is an antisubmarine warfare (ASW) aircraft. It is a mid-wing, twin engine monoplane with fully retractable landing gear. The chief specifications are:

Wingspan. 80 ft.

Gross Weight 40,000 lb。

Power Loading 8.34 lb .

Wing Loading 50.2 lb .

Wing Area 796.5 sq. ft.
Aspect Ratio 8.0
Engines: 2 P. and W. R-2800-CB16
Propellers: 13 ft., 4 blades
Additional specifications are contained in reference 39 , which contains a detailed description of the design and performance of the airplane.

The drag equation for the airplane was predicted to be $C_{D}=0.0188+0.0378 C_{L}{ }^{2}$.

If the engine nacelles and propellers were moved to the wingtips with each propeller turning in the countervortex direction, it is reasonable to expect, from the results cited earlier, that the drag equation would be
altered to:

$$
C_{D} \approx 0.018+0.022 C_{L}^{2} \quad(\text { Full Throttle). }
$$

The power required, at sea-level, has been calculated using these two equations; these curves are plotted in Figure 56. In addition, a part-throttle power-required curve has been plotted in Figure 56. The basis of this curve is that the effective aspect-ratio is a function of $T_{C}$. This curve was used to determine the endurance and range improvement using the Brequet formulas. The changes in sea-level performance resulting from the wingtip propellers are listed in Table IX.

TABLE IX
PERFORMANCE COMPARISON FOR ASW AIRCRAFT

| Performance Item | AsW <br> as | With Wingtip <br> Propellers | \% Change |
| :--- | ---: | ---: | ---: |
| Velocities |  |  |  |
| Max. Vel. | 262 knots | 270.5 knots | $3.4 \%$ |
| For Max. Range | 143.8 . knots | 139 knots |  |
| For Max. Endur. | 108.2 knots | 97 knots |  |
| For Best R.C. | 142 knots | 137 knots |  |
| Best Rate of Climb | 1562 fpm. | 1750 fpm. | $12 \%$ |
| Best Angle of Climb | 8.4 deg. | 9.7 deg. | $15.4 \%$ |
| Max. Range | 1 unit | 1.165 units | $16.5 \%$ |
| Max. Endurance | 1 unit | 1.27 units | $27 \%$ |

Thrust Horsepower in Hundreds of HP


Figure 56. Power Required and Available for Original and Modified ASW Aircraft

It will be noted that this configuration is particularly suited to aircraft of this type -- long range and patrol aircraft. A $16.5 \%$ increase in range and a $27 \%$ increase in endurance are significant improvements.

The disadvantage of this design change is the problem of trim and control for one-engine-out flight. There will be a weight penalty due to the necessity of adding an oversize vertical tail. The alternative is the weight and complexity of cross shafting or some other method of insuring no inequality of power to the two propellers.

Another, and more feasible, design configuration would be to adopt a four-engine design with two engines at the wingtips and two engines inboard.

Generalized Results

In order to increase the usefulness of the results of this investigation, an attempt has been made to generalize them by examining the relationship between the strength of the vortex of the propeller and that of the wing vortex, together with the concept of the propeller acting as a vortex sink (or source) with respect to shed vortex sheet.

Each blade of a propeller sheds a vortex sheet. This helical vortex sheet combines with those of the other blades to produce a trailing vortex system consisting of two parts - a cylindrical vortex sheet encasing the slipstream and a vortex collinear with the propeller axis of rotation. The cylindrical vortex sheet may be considered
to be composed of ring vortices; the result of these vortices is that the slipstream velocity is greater than freestream velocity. The axial vortex produces the rotation of the slipstream core.

The angular velocity, according to Konig (26), is $2 \mathrm{a}^{\prime} \Omega$, where $\Omega=2 \pi \mathrm{~N}$ and $\mathrm{a}^{\prime}=\frac{8}{\pi^{2}} \frac{{ }^{C_{Q}}}{1+2} \frac{\mathrm{~V}}{\mathrm{Nd}}$. The maximum tangential velocity is $\mathrm{V}_{\mathrm{t}_{\max }}=\frac{16}{\pi} \frac{\mathrm{C}_{Q}}{1+2} \mathrm{~V}$. Measurements indicate that, instead of this maximum tangential velocity occurring at the propeller tip, it occurs at $r / R=0.4$. $V_{t}$ falls to approximately zero at $r / R=1$.

There is, then, a trailing vortex system superimposed on the wing trailing vortex system. Schaffer (44) has shown that the vortices will combine to strengthen the trailing vortex (if they have the same sense) or to decrease the strength (if they have opposite sense).

For a given diameter, the vortex strength of a propeller will be proportional to the blade lift and, therefore, to the propeller thrust.

$$
T=T_{C} \frac{\rho}{2} V^{2} d^{2}
$$

circulation (i.e., vortex strength of the wing) is proportional to the wing lift.

$$
\begin{gathered}
I=C_{L} \frac{\rho}{2} V^{2} S \\
\frac{\Gamma_{\text {prop }}}{\Gamma_{\text {Wing }}}=\frac{T}{L}=\frac{T_{C} P / 2 V^{2} d^{2}}{C_{L} P / 2 V^{2} d^{2}}=\frac{T_{C} d^{2}}{C_{L} S}
\end{gathered}
$$

This ratio has been used to examine $\Delta C_{D} / C_{L}$ and $\Delta C_{D} / C_{D}$. Figure 57 shows the dependence of $\Delta C_{D} / C_{D}$ on $\frac{T_{C} \mathrm{~d}^{2}}{\mathrm{C}_{\mathrm{L}} \mathrm{S} / 2}$. There appears to be a functional relation and a single curve has been fit to the data.

Data for the impeller is also plotted in Figure 57. As would be expected, these data do not fit the propeller data for the reason that the rotation of the slipstream of the impeller is a direct function of $N$, rather than of thrust.
$\mathrm{T}_{\mathrm{C}} \mathrm{d}^{2}$ Figure 58 shows the relation between $\Delta \mathrm{C}_{\mathrm{L}} / \mathrm{C}_{\mathrm{L}}$ and $\frac{{ }_{C} C^{\mathrm{C}^{2}}}{{ }_{\mathrm{L}} \mathrm{S} / 2}$. It is not clear that there is as strong dependence of $\Delta C_{L}$ on the parameter. It does appear that $\Delta C_{L}$ is roughly proportional to $\frac{T_{C} d^{2}}{S}$.

It is felt that the strong dependence of span efficiency factor, $e=\frac{A}{A}$, on the positioning of the propeller on the wing, as shown in Figure 53, is a general result. Although the wing is common to all tests, the nacelle shapes are all different, and some tests are with fullwing and fuselage and others are from a reflection-plane wing test without fuselage. The agreement of the values is good.

A similar parameter, $\frac{\Delta C_{D_{i}}}{C_{L}{ }^{2}}$, has been evbluated for various values of $N \frac{d}{b}$, where $N$ is the difference between rotor speed and windmilling speed. It was found that $\left(\Delta C_{D_{i}} / C_{L}{ }^{2}\right)$ varies with the cube root of ( $N \frac{d}{b}$ ). This relation is shown graphically in Figure 59.


Figure 57. Correlation of Drag Coefficient Increment Due to Wingtip-


Figure 58. Correlation of Lift Coefficient Increment Due to Wingtip Propellers


Figure 59. Effect of Propeller Speed and Size Ratio on Induced Drag Coefficient Increment

## CHAPTER V

## SUMMARY AND CONCLUSIONS

The traditional method of simulating higher aspect ratio has been examined; i.e., the use of end-plates. A new method, that of using mechanical rotors at the wingtips, has been proposed and tested. The principal conclusions are:

1. End-plates are not effective in obtaining the desired effect; i.e., increasing effective aspect ratio and decreasing induced drag.
2. The use of propellers mounted at the wingtips can produce simultaneous lift increase and drag decrease. The fractional changes in both drag coefficient and lift coefficient are functions of $\frac{T_{C} d^{2}}{C_{L} S}$.

The detailed results of the investigation include the following:

1. End-plates increase the effective aspect ratio of the wing. They function by shifting the wing-loading (more lift
toward the tip) and by decreasing the downwash.
2. For STOL aircraft, in which operation at high $C_{L}$ is important, end-plates are effective in decreasing take-off distance but they do not decrease landing distance. In some cases their use will increase landing distance.
3. It is more effective to use the same airfoil area as an extension of the wingspan than it is to use it in the form of end-plates.
4. Energy may be employed to affect the wing's flow field and lift distribution by mounting the aircraft's propellers at the wingtips.
5. Use of a rotor turning in the direction opposite to that of the wing's trailing vortex shifts the core of the trailing vortex outboard and downward.
6. Use of a pro-vortex turning rotor (or counter-vortex propeller turning at less than windmill speed) moves the core of the trailing vortex inboard.
7. Wingtip configuration and/or rotor rotation have little effect on the position of the vortex trailing from the outboard
end of a deflected flap.
8. There is a mutual dependence between vortex span and effective aspect ratio. As one increases, the other does also. Actual functional relation is not yet clearly defined.
9. Effect of power on $C_{L}$ is mainly due to higher slipstream velocity for inboard propellers. However, for propellers at the wingtips, the effect is chiefly due to the altered angle of attack in, and adjacent to, the slipstream as well as to increased slipstream dynamic pressure.
10. A counter-vortex turning propeller decreases wing drag (mainly, induced drag); a vortex-turning propeller increases drag. The rotor at the tip serves as a trailing vortex sink (counter-vortex) or source (vortex).
11. The effectiveness of the propeller in affecting the lift and drag increases as it is moved outboard toward the wingtip.
12. Counter-vortex propellers at the wingtips improve an aircraft's climb and cruise performance compared to conventional positioning.

The following recommendations are made for further
work in this field:

1. It would be desirable to obtain more data on the relation between vortex span and effective aspect ratio. Additional experimental work using a rotating or non-rotating vorticity meter is needed.
2. It would be interesting to see whether a wingtip jet, with variable amounts of vorticity, would have similar effects to those found with the propellers.
3. Additional data with wings of different aspect ratio, taper ratio, and $\mathrm{d} / \mathrm{b}$ would help establish the relation between $\frac{\Delta C_{D}}{C_{D}}$ and $\frac{T_{C} d^{2}}{C_{L} S}$.

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## APPENDIX

## RESULTS OF EXPERIMENTAL PROGRAM

This appendix presents results of the experimental program additional to those presented in Chapters III and IV.

## Tuft Grid Survey

In order to interpret the results of the tuft grid survey, 140 of the pictures were printed. Examples are shown in Figures 60 and 61. Figure 60 illustrates how clearly the position of both the tip vortex and the flap outboard vortex may be located.

Figure 61 illustrates the tracing of the vortex trajectory. In this figure the impeller is mounted on the wing. In the left column of pictures (a, $d, g, j$, and $m$ ), the impeller is turning at $175 \mathrm{r} . \mathrm{p} . \mathrm{s}$. in the vortex direction. In the center column of pictures (b, e, $h, k$, and n), the impeller is stationary. In the right column of pictures ( $c, f, i, l$, and 0 ) the impeller is turning at 175 r.p.s. in the counter-vortex direction.

## Performance Calculations

The power required for the ASW airplane, with and
without modification, was calculatedusing the 1620 computer. The results are listed in Tables $X$ and XI. Table XII tabulates the significant values for power required for part-throttle operation. The improvement in induced drag is a function of power. Thus, in level flight, it is a function of flight speed (e.g., at $V=100 \mathrm{ft} . / \mathrm{sec} ., C_{D_{i}} \approx .026 \mathrm{C}_{\mathrm{L}}{ }^{2} ;$ at $\mathrm{V}=300 \mathrm{ft} . / \mathrm{sec} .$, $\left.C_{D_{i}} \approx .029 C_{L}{ }^{2}\right)$. The computer program for the parabolic drag airplane was used repeatedly at various values of $m$ to obtain the part-throttle power required curve.

Figure 85 is the sea-level climb performance curve for the aircraft.

Balance Measurements

The balance data taken during the test program were reduced using the 1620 computer. The reduced data in coefficient form were read-out in two forms -- corrected for tunnel boundary and uncorrected. Since changes due to power were of primary interest (rather than absolute values) and since the changes were to be coordinated with flow field data, it was decided to use the uncorrected answers throughout.

The answers were also corrected to remove the apparent lift, drag, and pitching moment due to thrust. The graphical presentation of the coefficients is done two ways. Most of the answers uncorrected for thurst were plotted using the 1620 computer and a Calcomp
plotter. In addition, the data which were manually corrected for thrust were manually plotted.
table X

POWER REQUIRED. ASW AIRPLANE

| $C D=.0188+.037$ <br> SEA-LEVEL ALTITUDE |  | AREA $=796.5 S Q . F T$. <br> 40.000 LBS. GROSS WEIGHT |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| VEL | VEL | LIFT COEF | DRAG |  | PWR REQ |
| FT/SEC | KNOTS |  | POUNDS |  | HP |
| 50.00 | 29.6 | 16.901 | 25600.13 |  | 2327.28 |
| 75.00 | 44.4 | 7.511 | 11458.17 |  | 1562.47 |
| 100.00 | 59.2 | 4.225 | 6566.87 |  | 1193.97 |
| 125.00 | 74.0 | 2.704 | 4366.97 |  | 992.49 |
| 150.00 | 88.8 | 1.877 | 3239.94 |  | 883.62 |
| 175.00 | 103.6 | 1.379 | 2631.20 |  | 837.20 |
| 200.00 | 118.4 | 1.056 | 2309.10 |  | 839.67 |
| 225.00 | 133.3 | . 834 | 2162.97 |  | 884.85 |
| 250.00 | 148.1 | . 676 | 2134.52 |  | 970.24 |
| 275.00 | 162.9 | . 558 | 2190.70 |  | 1095.35 |
| 300.00 | 177.7 | .469 | 2311.59 |  | 1260.86 |
| 325.00 | 192.5 | . 400 | 2484.65 |  | 1468.20 |
| 350.00 | 207.3 | . 344 | 2701.65 |  | 1719.23 |
| 375.00 | 222.1 | . 300 | 2957.00 |  | 2016.13 |
| 400.00 | 236.9 | . 264 | 3246.80 |  | 2361.30 |
| 425.00 | 251.8 | . 233 | 3568. 26 |  | 2757.29 |
| 450.00 | 266.6 | . 208 | 3919.36 |  | 3206.75 |
| 475.00 | 281.4 | .187 | 4298.57 |  | 371 ®.40 |
| 500.00 | 296.2 | . 169 | 4704.76 |  | 4277.05 |
| 525.00 | 311.0 | . 153 | 5137.05 |  | 4903.54 |
| 550.00 | 325.8 | -139 | 5594.74 |  | = 5594.74 |
| 575.00 | 340.6 | . 127 | 6077.31 |  | 6353.55 |
| 600.00 | $355 \cdot 4$ | -117 | $6584 \cdot 33$ |  | $7182 \cdot 90$ |

TABLE XI
POWER REQUIRED, MODIFIED ASW AIRPLANE


TABLE XII
PART-THROTTLE POWER, MODIFIED ASW AIRPLANE



Figure 60. Basic Wing With Plane Tip, $\alpha=12 \bullet, \delta_{f}=40^{\circ}, x=12 \mathrm{M}$


Figure 61. Impeller, $N=175 \mathrm{r} . \mathrm{p} . \mathrm{s} ., \alpha=12, \delta_{f}=0$


Figure 61. (Continued)


Figure 62. $C_{L}$ Vs $\alpha$, Basic Wing


109 -- Dummy Spinner.
124 -- Stationary Propeller, 2 Blades Aligned with the Wing.
125 -- Stationary Propeller, Blades $45^{\circ}$ to Wing.
Figure 63. Dummy Spinner


Wing, Pod, and Impeller, $\delta_{f}=0$.
A No Rotation -- Runs 114 and 123.


Figure 64. Impeller, $\delta_{f}=0^{\circ}$


Wing with Pod, $\quad \delta_{f}=0$.

$$
\begin{aligned}
& 109 \text { - Dummy Spinner } \\
& 126 \text { - Propeller, } N=119 \text { r.p.s. (windmillingl } \\
& 129-4 \quad, N=175 \text { r.p.s. } \\
& 130-4 \quad, \quad N=50 \text { r.p.s. }
\end{aligned}
$$

Figure 65. Propeller, $\delta_{f}=0^{\circ}$


107 -- Wing with Standard Tip.
110 -- Wing, Pod, Dummy Spinner. $\delta_{f}=40^{\circ}$
Figure 66. Basic Wing and Pod, $\delta_{f}=40^{\circ}$


Figure 67. $C_{L}$ vs $\alpha$, Impeller, Vortex Rotation


Wing, Pod, and Impeller, $\delta_{f}=40^{\circ}$, Counter-Vortex
A - No Rotation
$B-N=50$ r.p.s.
$C-N=50$ r.p.s.
$D-N=175$ r.p.s.
Figure 68. $\mathrm{C}_{\mathrm{L}}$ vs $\alpha$, Impeller, Counter-Vortex


Figure 69。 $C_{L}$ vs $\alpha$, Propeller, $\delta_{f}=40^{\circ}$



A - No Rotation: Runs 114, 123.
B - Vortex-Direction Rotation, $N=50 r p s .(117)$
$C$ - Vortex-Dir. Rotation, $N=175$ rps. (117).
D - Counter-Vortex Rotation, $N=50$ rps. (123)
E - Counter-Vortex Rotation, $N=175 \mathrm{rps}$.
Figure 71. $C_{I}$ vs $C_{M_{C / 4}}$, Impeller, $\delta_{f}=0^{\circ}$


109 -- Dummy Spinner
126 -- Windmilling Propeller, $N=119$ r.p.s.
129 -- Propeller, $N=175$ r.p.s.
130 -- " $N=50$ r.p.s.
Figure 72. $C_{L}$ vs $C_{M_{C / 4}}$, Propeller, $\delta_{f}=0^{\circ}$


107 -- Wing with Standard Tip.
110 -- Wing with Dummy Pod.
Figure 73. $\mathrm{C}_{\mathrm{L}}$ VS $\mathrm{C}_{\mathrm{M} / 4}$. Basic Wing and
Pod, $\delta_{f}=40^{\circ}$


Figure 74. $\mathrm{C}_{\mathrm{L}}$ vs $\mathrm{C}_{\mathrm{M}}^{\mathrm{c} / 4}$, , Impeller, Vortex
Rotation


Wing, Pod, and Impeller.
Figure 75. $\mathrm{C}_{\mathrm{L}}$ vs $\mathrm{C}_{\mathrm{M}_{\mathrm{c} / 4}}$, Impeller, Counter-Vortex


Wing, Pod. Propeller.
$\delta_{f}=40^{\circ}$
110: Dummy Sinner
13la: Windmilling Propeller, $N=120$.
131b: $N=175$ r.p.s.
131c: $N=100$ r.p.s.

Figure 76. $C_{L}$ vS $C_{M_{c / 4}}$, Propeller, $\delta_{f}=40^{\circ}$


Figure 77. Wing Characteristics, Basic Wing


Figure 78. Wing Characteristics, Wing With Pod


Figure 79. Wing Characteristics, Impeller, corrected for Thrust


Figure 80. Wing Characteristics, Propeller, Corrected for Thrust


Basic Wing, Standard Tip, $\delta_{f}=0^{\circ}$. 105 \& 108: Eff. $\mathrm{Re}=6.7 \times 10^{5}, \mathrm{C}_{\mathrm{D}}=.012+.0477 \mathrm{C}_{\mathrm{L}}^{2}$ 102: Eff. $\operatorname{Re}=1.2 \times 10^{6}, C_{D}=.014+.047 C_{L}^{2}$

Figure 81(a). Drag Polar, Basic Wing


Wing, Pod, Dummy Spinner, $\delta_{f}=0^{\circ}$.

$$
C_{D}=.013+.0493 C_{L}^{2}
$$

Figure 81(b). Drag Polar, Dummy Spinner


Figure 82. Drag Polar, Impeller Corrected for Thrust


Figure 83. Drag Polar, Propeller Corrected for Thrust


Figure 84. Drag Polar, Dummy Spinner


Figure 85. Climb Performance of ASW Airplane

VITA
Melvin H. Snyder, Jr. Candidate for the Degree of

Doctor of Philosophy

Thesis: EFFECTS OF A WINGTIP-MOUNTED PROPEILER ON WING LIFT, INDUCED DRAG, AND SHED VORTEX PATTERN

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[^0]:    ${ }^{l_{\text {Alexander }}}$ Lippisch. Kenneth Razak, in 1965, reported to the writer a conversation on the subject which he had with the famous designer of the German rocket propelled fighter. At that time (about 1956) Lippisch was engaged in O.N.R. sponsored research in V/STOL aircraft.

[^1]:    2"Vortex source" and "vortex sink" were used in this sense by Maurice Roy, Director of the O.N.E.R.A. See his article in reference 51.

