RESPONSE SPECTRA OF COUPLED ACOUSTICAL

RESONATORS TO TRANSIENT EXCITATION
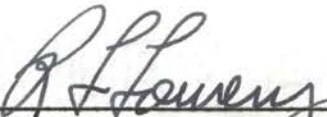
By

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Bachelor of Engineering
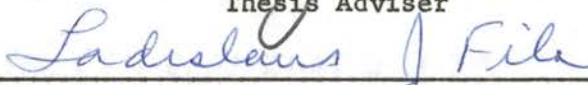University of Mysore
Mysore, India
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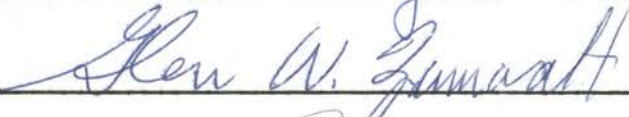Indian Institute of Science
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RESPONSE SPECTRA OF COUPLED ACOUSTICAL

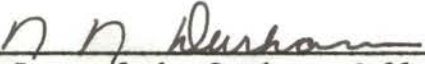RESONATORS TO TRANSIENT EXCITATION

Thesis Approved:

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Dean of the Graduate College

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LIST OF SYMBOLS

A_1, A_2 Cross sectional areas of the resonator necks

A_0 Absolute maximum value of the excitation pressure

c Velocity of sound

c_1, c_2 Viscous damping coefficients in the resonator

f_1, f_2 Uncoupled natural frequencies of the system $(2\pi p_1, \ 2\pi p_2)$

f_{12} Coupling frequency of the system $(2\pi p_{12})$

ℓ_1, ℓ_2 Lengths of the resonator necks

ℓ_{e1}, ℓ_{e2} Effective lengths of the resonator necks

p_1, p_2 Uncoupled circular natural frequencies of the system

p_{12} Coupling circular frequency of the system

p_+ Higher coupled circular natural frequency of the system

p_- Lower coupled circular natural frequency of the system

P_1, P_2 Pressures in the cavities V_1, V_2

R Radius of the resonator neck

x particle displacement

$\dot{x}, \ddot{x}$ First and second derivative of x with respect to time (particle velocity, particle acceleration)

X Volume displacement

$\dot{X}, \ddot{X}$ First and second derivative of X with respect to time (volume velocity, volume acceleration)

α Correction factor for the length of the resonator

γ Ratio of specific heats of air (c_p / c_v)

τ Period of excitation pressure

ω Circular frequency of excitation pressure $(2\pi/\tau)$

ξ_1, ξ_2 Damping factors in the system

ρ_o Mass density of the air per unit volume

CHAPTER I

INTRODUCTION

The dynamics of acoustical or mechano-acoustical systems have been
the object of the attention of engineers and scientists for the last
few years. Many structures may behave as multiple acoustical resonators
coupled together and exposed to acoustical excitation pressures.
Typical structures which constitute coupled multiple acoustical
resonators are domestic buildings with several rooms, connecting
passage ways, doors and windows. In some cases the windows, with
distributed mass and elasticity, coupled to a simple Helmholtz resonator
behave as a multiple dynamic system with mechanical and acoustical
elements coupled together. The pressure response measurements made
inside a sonic boom test house and Kinney shoe store at Oklahoma City,
Oklahoma (Figure 1), indicate that there is a considerable amount of
damping in such structures. Because of this mechanism of energy dissi-
pation, the pressure oscillations are not present for a long span of
time. This shows the need for the consideration of damping in the
response analysis of acoustic systems.

It has been a great concern to many investigators whether sonic
booms will be acceptable to people. This important factor depends
on the strength of the sonic boom and the structural response to
finite duration excitation pressures like those of sonic booms.
Whether the result of dynamic effect or not, the unintentional pressure

SONIC BOOM RECORD AT OKLAHOMA CITY - TEST HOUSE NO.1 - FLT.1 - 10 FEBRUARY 64

0.1 SEC.
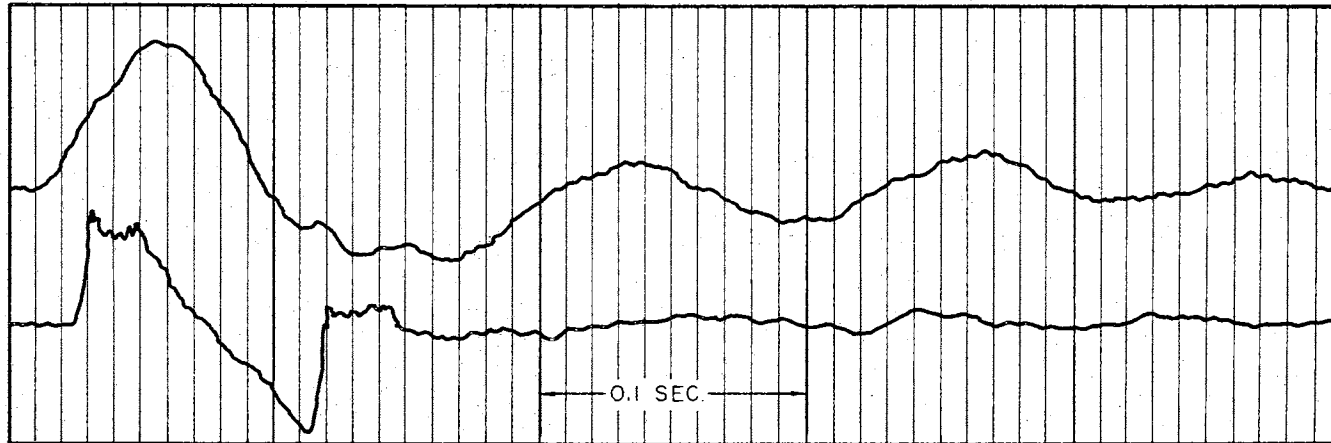
FREE PRESSURE OSCILLATIONS INSIDE KINNEY SHOE STORE AT OKLAHOMA CITY

TIME, 100MM = 1 SEC
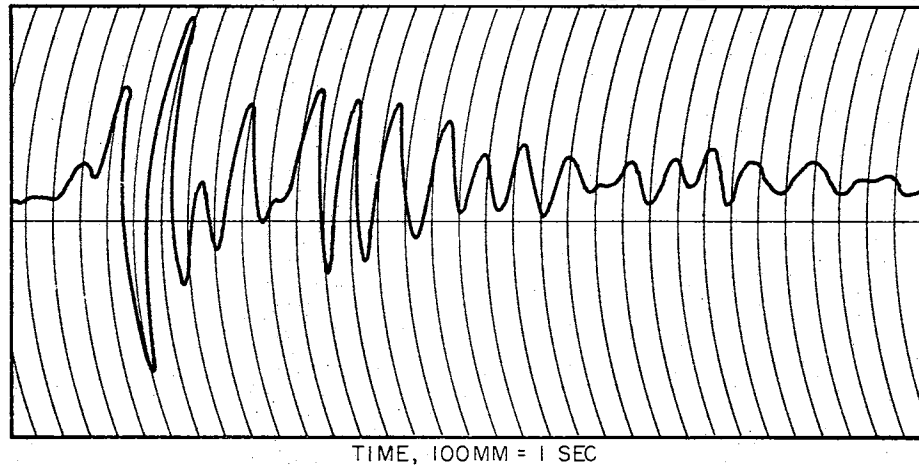
Figure 1. Pressure Records Inside Buildings

magnifications inside the buildings are the undesirable environments to be tolerated. These increased pressures can be the primary cause of damage to the structural components such as the windows and the doors in the buildings.

It is important to note that structural damage is simply an instantaneous consequence of the dynamic response of multiple acoustical resonators to transient excitation; the inadvertent discomfort of people due to pressure magnifications in buildings exposed to excitation pressures, such as blast pressures, gusts produced by storms, and noise generated by rocket launchings, is also substantial.

These considerations lead to a marked need for an understanding of the dynamic response of acoustical or mechano-acoustical systems to various types of pressures or force excitations. Recognition, isolation, and understanding of the parameters pertinent to the phenomenon of the system can provide a sound basis for design concepts.

Definition of the Problem

The total energy of a conservative system remains constant at all times. In the case of a dynamic system with multiple degrees-of-freedom this energy is distributed among the different natural modes and among various elements.

In a multiple acoustical resonator, the pressure magnification in each of the cavities or the magnification of the particle displacements at the necks depends on the type of coupling and the various system parameters. The following are the important system parameters which affect the response of acoustical networks:

1. Conductivities of various necks,

2.   Relative volumes of the cavities, and

3.   Damping in the system.

A double acoustical resonator consisting of two enclosed volumes, V_1 and V_2, and two necks, N_1 and N_2 (Figure 2), is very common among the existing structures (4). When this system is excited with transient pressures on neck N_1, the pressures inside the cavities may be higher than the excitation pressures due to the dynamic effect of the system. If there is no dissipation of energy, and the two natural frequencies are nearly equal, the pressure oscillations will exhibit the beating phenomenon due to transfer of energy from one resonator to the other.

It is necessary to investigate the factors influencing the pressure magnification and the effect of various system parameters of a coupled resonator to a finite duration and aperiodic excitation.
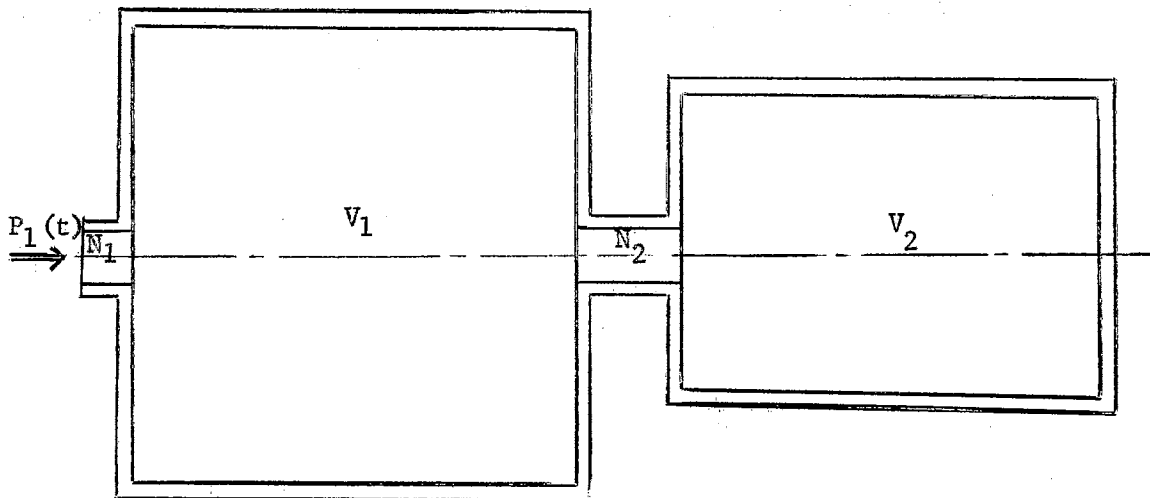
Figure 2.   Double Acoustical Resonator

Purpose and Scope of the Study

In general, any acoustical system is a continuous dynamic system.
Under certain restrictions in a case like the acoustical resonator,
it is possible to approximate the continuous system by lumping the
parameters so that the response problem is amenable to mathematical
treatment. It has been proven in the literature (3, 16) that the
lumped parameter model is a good approximation for the Helmholtz
resonator if the largest dimension is very small compared to the wave
length in the steady state case. Also, Simpson (4) has established
that the frequency limitations necessary for the lumped parameter
assumption are less severe in the case of transient response of
Helmholtz resonator than in steady state response. The lumped parameter
approximation is valid even in the case of the acoustical network shown
in Figure 3 provided that the largest dimension of any component is
small compared to the wave length at resonance.
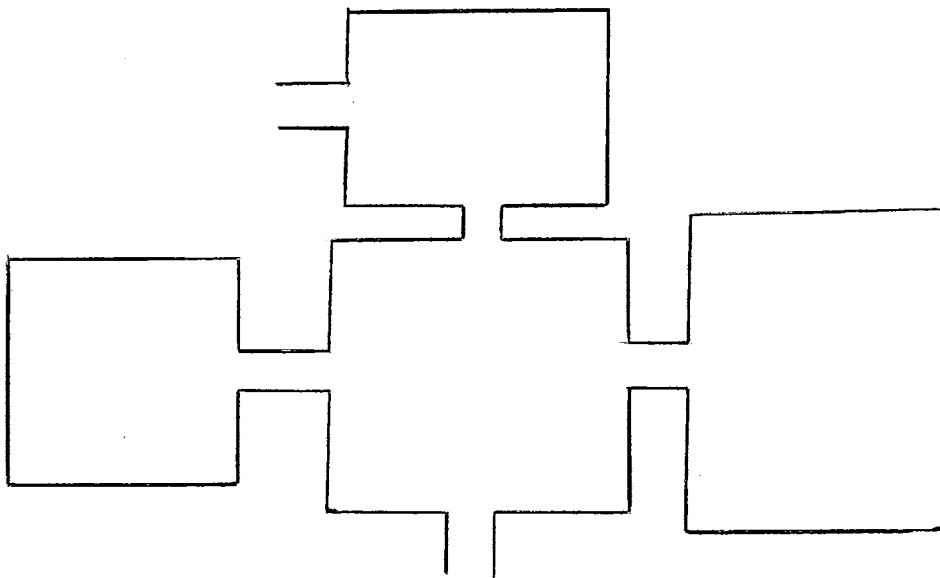
Figure 3. Acoustical Network

The purpose of this study is to investigate the pressure magnification and the response of double acoustical resonators coupled together for finite duration transient pressure excitations like N-shaped, saw tooth and sine pulse waves. The study consists of both theoretical and experimental phases. The results of this investigation are helpful in better understanding the mechanism of energy feedback in a coupled acoustical system subjected to transient excitations and thereby permitting the determination of the effect of the system parameters such as frequencies and damping on maximum pressures in the cavities. The other important application is in the area of dynamic response of various structures in excitation pressures like sonic booms, blast pressures, wind gusts, noise produced by rocket launchings, and random excitation.

The scope of the theoretical study consisted of deriving the differential equations of motion and solving for the natural frequencies of the double acoustical resonator. The transfer function of the system has been derived using the Laplace Transformation notation. Response of the system in both the frequency and time domain has been investigated for transient excitations such as finite duration pressure loadings. Numerical techniques and high speed digital computers have been used in computing and investigating the transient response. An attempt was made to use the techniques of analog computers to solve for the response. Equivalent viscous damping was assumed in the analytical treatment to approximate the various damping mechanisms in the system. The following major assumptions were made in the mathematical treatment of the problem:

1.   The wave length corresponding to the natural frequency

is 16 or more times greater than the largest dimension

of the resonator;

2. The pressure at all points inside the cavity is the

same; and

3. All the walls of the resonator are rigid.

The principal object of the experimental investigation was to substantiate the validity of the assumptions made in deriving the mathematical model. Therefore, the scope of the experimental phase of the study was to design and build a double acoustical resonator with rigid walls, satisfying frequency limitations for the lumped parameter assumption, to excite it with transient pressure pulses, and to measure the pressure response in both cavities of the resonator as a function of time. The plane wave tube in the acoustical laboratory was used as an acoustical delay line to excite the resonator with finite duration transient excitation pressures. Instrumentation was developed to produce the excitation pressure pulses and to measure and record the input pressure and pressure response in both the cavities.

CHAPTER II

PREVIOUS INVESTIGATIONS

The primary investigation has been done in the area of acoustical

networks from the stand point of musical tones and acoustical wave

filters. The acoustical resonators used by Helmholtz (16) in his

research on the quality of musical notes were almost completely

closed vessels with an aperture (Figure 4) and were used to intensify

effect of a simple tone produced in the neighborhood of the resonator.

This intensification is the result of vibrations of the air enclosed

by the resonator. Rayleigh (2) solved for the steady state response

of a vessel containing air, which communicates with the external

atmosphere by a narrow aperture or a neck. He assumed that the kinetic

energy of the air due to its velocity can be neglected except in the

vicinity of the neck or aperture. The potential energy can be calcu-

lated by treating the density of the air in the interior of the vessel

as being uniform. These assumptions are valid if the space through

which the kinetic energy is sensible is reasonably small in comparison

with the length of the wave.

The stiffness of a Helmholtz resonator depends on the volume of the

cavity and areas of the necks, and the modes of the vibration are depen-

dent on the inertia of the air in the vicinity of all the apertures.

Thus the fundamental frequency of a Helmholtz resonator with more

than one neck is different from that of the simple resonator. When two

or more interconnected vessels communicate with each other, and with external air, then the inertia of the air in a connecting passage is as great as that of the channel which communicates with the external air. By assuming lumped parameters, the system is treated as a finite, multidegree-of-freedom system. Many investigators (1, 3, 6, 7) have attempted to solve for the steady state response of multiple resonators with application to wave filters, but very little work has been done in the field of transient response.

Simpson (4) has investigated the transient response of a simple Helmholtz resonator for a sonic boom type of excitation. He established the validity of the lumped parameter description of an undamped acoustical resonator for the transient pressure excitations, and has shown theoretically that the frequency limitations necessary in the transient response of a Helmholtz resonator are less severe than those associated with the steady state response.

NECKS
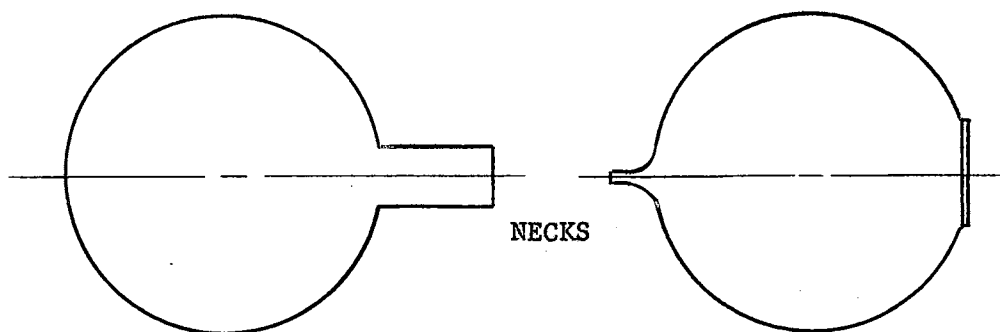
Figure 4. Simple acoustical Resonators used by
Helmholtz (Spherical Cavities)

Rayleigh (2) described briefly the method to obtain the differ-
ential equations for the particle motion of two acoustical resonators
coupled together and derived expressions for the natural frequencies
of a particular configuration. McGinnis and Albert (1) have investi-
gated an analytical method to obtain the equations of motion and
natural frequencies of different configurations of multiple Helmholtz
resonators assuming lumped parameters. They compared the theoretically
obtained frequencies with the experimental results and found them to
be in good agreement. Paris (9) in 1925 investigated the double Helm-
holtz resonator with a particular interest to the designing of a sensi-
tive acoustical receiver used to measure the sound amplitudes. He
found that increased magnification can be obtained by the use of an
acoustical system with double resonance. He demonstrated the influence
of loose coupling and tight coupling on pressure response of the
system. In his analytical work, he took into account the damping due
to viscosity and radiation.

Christian (11) examined the response of an undamped multiple res-
onator for steady state sinusoidal excitation and free vibrations. He
derived a functional relationship of coupling parameters of a double
acoustical resonator.

Olson (3) derived expressions for the frequency response spectrum
of the multiple resonators which behave as low pass, high pass, band
pass, and band elimination wave filters. He used the analogy of
electrical filters and the impedance method to derive attenuation
expressions. No explanation was given for the energy transfer from one
resonator to the other through coupling. He indicated that the simple
Helmholtz resonator can be treated as a single degree of freedom system

even for transient cases.

Ingles (21) investigated the effect of the pressure (at the equilibrium condition) on the response spectrum of Helmholtz resonator formed by two cavities and a neck. In this form of Helmholtz resonator, the variation in pressure changes both the mass of air in the neck and the volume of the air in the cavities. Thus, the resonant frequency of the system varies with the pressure. He also investigated the effect of temperature on the resonant frequency.

Morse (5) discussed a statically coupled mechanical system with two degrees-of-freedom, but mention has not been made about the transient response of coupled dynamic system.

Wein (10) examined different kinds of coupling in a two degrees-of-freedom system. He developed a relation showing the effect of various coupling parameters on free vibrations of the system.

Andrews Associates (18) in their final report on "The Studies of Structural Response to Sonic Booms" analyzed a simple oscillator for an N-wave excitation and found that the structure can be arrested from oscillations if the relationship between the natural period and duration of the pulse is at a critical value. For an N-wave, the maximax response is 2.05 times the static response.

Jacobsen and Ayre (3) have solved for the maximax response of an undamped simple mechanical oscillator for different types of transient excitation forces. The maximax response is defined as the maximum absolute displacement, velocity or acceleration of the oscillator occurring at any time as a result of the action of the forcing function. They also stated that the transient response of an undamped two degrees-of-freedom mechanical system can be solved by a classical solution of

simultaneous differential equations for particular cases; but the conditions for maximax response are not discussed.

Ormondroyd and Den Hartog (28) have given a design criterion for a vibration absorber. They discussed the conditions under which only smaller mass will have a considerable displacement when the bigger mass of the two degrees-of-freedom is driven by a steady state harmonic force. The effect of damping has also been included in their discussion.

Kryter (33) has done laboratory investigation on the relative noisiness and loudness of sonic booms having different wave fronts, the reactions of people to booms as an auditory experience, and their startling reactions to acoustic stimuli. The following statement is one of the conclusions from his investigations: "The sound heard in a house subjected to sonic booms are judged to be noisier or more unwanted than the sonic boom heard outdoors, probably because of rattles and other secondary sounds that result from vibration of the house." After this study, it appears that the pressure inside the house can be much higher for certain configurations due to dynamic acoustic effects rather than to rattles and vibrations of the house itself.

CHAPTER III

MATHEMATICAL MODEL

The response of a double acoustical resonator to transient exci-
tation, as represented by an electrical or mechanical analog, depends
on the frequency limitations, dimensions of the system, and the
mechanism of damping in the system.

Limitations of Lumped Parameters for Acoustical Resonators

The assumption of a lumped parameter model for the acoustical
system depends mainly on the critical dimension, which is defined as
the largest dimension of the system (4).

In many cases the various elements of the acoustic structures
are small in dimension compared with the wave length of sound. When
this restriction is fulfilled, the behavior of sound or pressure pulse
in the structure is analogous to the behavior of electric current in a
circuit with lumped elements or to the behavior of the displacement in
a mechanical system with lumped parameters. When the dimensions of the
elements of the structure are not small relative to the wave length,
the electric analog is a transmission line and the mechanical analog is
an elastic continuous system.

In the case of a simple Helmholtz resonator (Figure 4), the
velocity attained by the air within the volume is diminutive when
compared to the velocity attained by the air particles in the neck of

the resonator. If the neck or constriction is very small as compared to the wave length, the compressibility of air in that part is negligible. The air in the constriction has a total mass of $A\ell_e\rho_0$, where A is the cross sectional area and ℓ_e is the effective length of the neck. The effective length has to be used in the analysis to include mass of the air beyond the ends of constriction which moves along with the air in the neck. Thus, the kinetic energy is due only to the motion of the effective mass of air in the neck and the potential energy is only due to the compression and expansion of the air in the cavity of the system.

The correction factor to be added to the actual length of the neck depends on the wave length and shape of the neck. This has been derived in the literature (2, 5) by considering impedance of the open end of the tube. For a circular cross section, if the open end is fitted with a flange that is wide compared to the wave length, the approximate value of specific acoustic impedance at the open end is given by,

$$Z_\ell = \frac{\rho_0 R^2 \omega^2}{2c} + j\, \omega\, \frac{8\rho_0 R}{3\pi} \quad \text{for wave length}, \lambda > 8\pi\, R, \text{ and}$$

$$Z_\ell = \rho_0\, c + \frac{j}{\omega}\, \frac{2\rho_0 c^2}{\pi R} \quad \text{for wave length}, \lambda < 8\pi\, R;$$

where R = radius of the tube.

Open tubes having cross sectional parameters much smaller than the wave length are, therefore, nearly as successful hoarders of energy as closed tubes, for only a small percent of the stored energy can be radiated away in a period of one oscillation. From the above equations it is apparent that the reactive term is mass load equal to mass of air,

$8\rho_o R/3\pi$, for small cross sectional area of the neck. When there is no flange on the end of the tube or when the flange dimension is negligible, the reactance is reduced to some extent in magnitude changing from $8\rho_o R/3\pi$ to approximately $0.6\rho_o R$. Therefore, the correction factor, α, to be used for the effective length of the neck should be in between the limits $0.6\rho_o R$ and $8\rho_o R/3\pi$.

Consideration of Damping in Acoustical Resonators

Energy dissipation would be inevitable in any physical dynamic system. It is necessary to understand the various mechanisms of losses in order to incorporate damping in the response analysis. In acoustical systems the significant damping is due to: 1) viscosity and heat conduction, 2) anomalous gaseous absorption due to thermal relaxation, and 3) mechanical wall vibrations.

Energy dissipation due to friction in the neck is a function of the exposed area and the roughness of the neck. This dissipation can be reduced by making the surface of the neck smooth. In general, dissipation of energy due to radiation is more important when compared to that of friction in the neck, provided the resonator is so situated that it can radiate. However, when the aperture of resonator is small or the length of the neck is long enough, then the dissipation due to friction is predominant. If the walls are rigid enough, then the losses due to the vibrations of the walls can be relinquished. At low frequencies, where there is little thermal relaxation, the energy loss due to thermal relaxation can be neglected. Therefore, for a resonator with rigid walls at low frequencies, the losses that are sizeable are those due to radiation and friction in the neck.

The damping coefficient due to radiation of the resonator is derived by Crandall (17) and is given as,

$$C_R = \frac{\rho_o \omega^2}{4\pi c} \; ;$$

where

ρ_o = mass density of air/unit volume,

ω = driving frequency in radians/second,

c = velocity of sound, and

C_R = damping coefficient, energy/unit volume velocity.

Damping due to friction at the walls is given by the complex function,

$$C_F = \sqrt{\frac{\rho_o \omega \mu}{2}} \quad (1 + j) \tag{1}$$

where μ = viscosity coefficient of the fluid.

The imaginary quantity $\sqrt{\rho_o \omega \mu / 2}$ is in phase with the acceleration; therefore, it has the nature of a mass reactance. This gives the effect of virtually increasing the mass density of air which corresponds to lowering of the natural frequency of a vibrating system by loading it with added mass. However, this may be taken into consideration equally well by adding the correction factor to the length of the neck to obtain the effective length.

The equivalent viscous damping coefficient for the resonator has been derived (31) by using the criterion of equivalent energy dissipation per cycle:

$$C_{eq} = C_n \, \omega^{n-1} \, X^{n-1} \, \gamma_n \, , \tag{2}$$

where

n = the power of the velocity for which the actual damping
coefficient is proportional, and

X = the maximum amplitude.

Pressure Response of Double Acoustical Resonators

The critical dimension of the resonator is assumed to be very
small when compared to the wave length corresponding to the natural
frequencies of the system. Since the temperature gradients are
small and the process of compression and rarification are rapid, it
is reasonable to assume the process of compression and expansion in
the cavities (Figure 2) is adiabatic; therefore, the pressure and
the volume are related to each other by the relation,

$$PV^{\gamma} = constant. \tag{3}$$

Differentiation of equation (3) yields the expression for a change
in pressure as,

$$dP = -\frac{\gamma P_0}{V} dV; \tag{4}$$

where P_0 is the original pressure.

Assuming that the air in the necks behave as masses and that in
the enclosed volumes as springs, the mechanical analog can be derived
as shown in Figure 5. If x_1 and x_2 are the displacements of masses
M_1 and M_2 (air masses at necks N_1 and N_2), respectively, then the
forces acting on each of the masses are given below:

Force on neck N_1:

$$Inertia\ force = \rho_0\ \ell_{el}\ A_1\ \frac{d^2x}{dt^2},$$
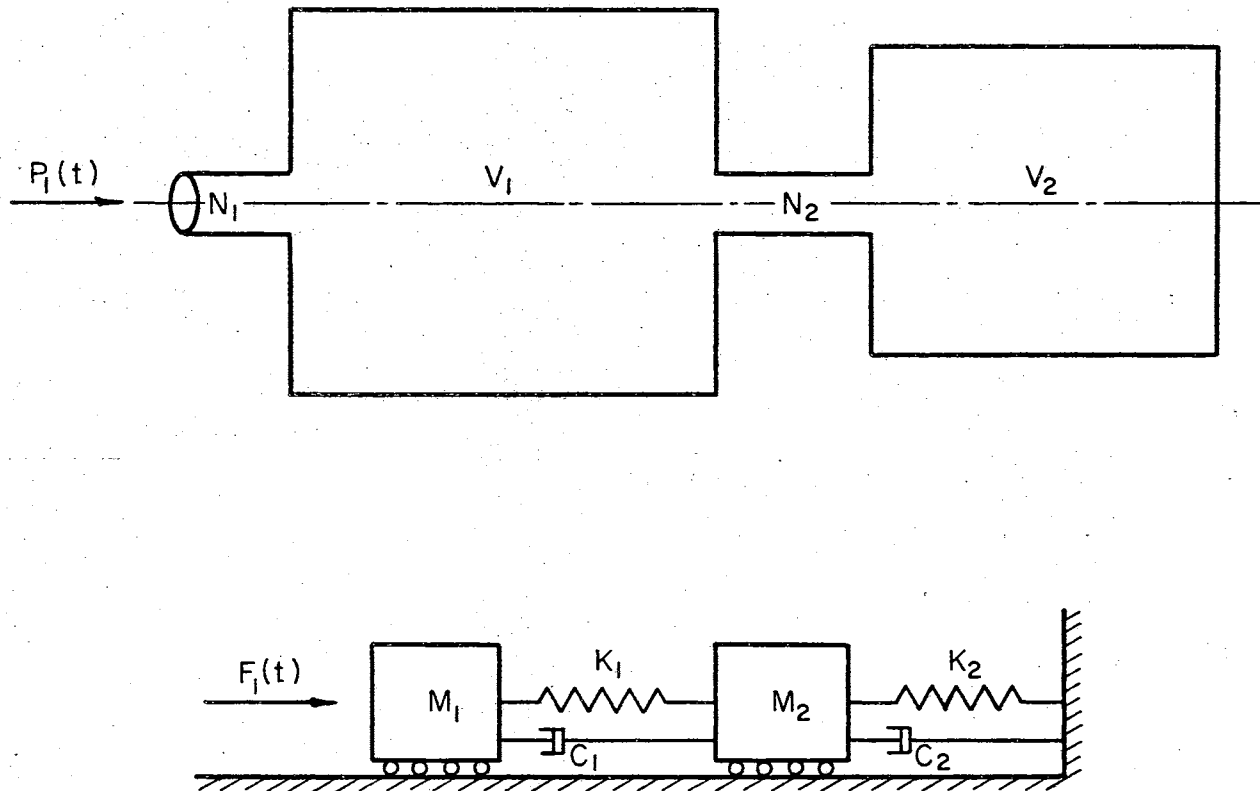
Figure 5. Equivalent Mechanical System for Double
Acoustical Resonator

Restoring force due to volume, $V_1 = -A_1 dP = \dfrac{\gamma P_o}{V_1} A_1 (A_1 x_1 - A_2 x_2)$,

Dissipating force due to damping $= c_1 A_1^2 \dfrac{dx_1}{dt}$, and

External force $= P_1 A_1$; and $\hspace{4cm}$ (5)

Force on neck N_2:

Inertia force $= \rho_o \ell_{e1} A_1 \dfrac{d^2 x}{dt^2}$,

Restoring force due to volume, $V_1 = \dfrac{\gamma P_o}{V_1} A_2 (A_2 x_2 - A_1 x_1)$,

Restoring force due to volume, $V_2 = \dfrac{\gamma P_o}{V_2} A_2 (A_2 x_2)$,

Dissipating force due to damping $= c_2 A_2^2 \dfrac{dx_2}{dt} + c_1 \left(A_2 \dfrac{dx_2}{dt} \right.$

$\left. - A_1 \dfrac{dx_1}{dt} \right) A_2$, and

External force $= P_2 A_2$. $\hspace{4cm}$ (6)

By Newtonian force balance on air masses of each of the necks, the following simultaneous differential equations of motion may be obtained,

$$\rho_o \ell_{e1} A_1 \dfrac{d^2 x_1}{dt^2} + c_1 A_1 \left[A_1 \dfrac{dx_1}{dt} - A_2 \dfrac{dx_2}{dt} \right] + \dfrac{\gamma P_o}{V_1} A_1 (A_1 x_1 - A_2 x_2)$$
$$= P_1 A_1 , \hspace{3cm} (7a)$$

and

$$\rho_o \ell_{e2} A_2 \dfrac{d^2 x_2}{dt^2} + c_1 A_2 \left[A_2 \dfrac{dx_2}{dt} - A_1 \dfrac{dx_1}{dt} \right] + c_2 A_2^2 \dfrac{dx_2}{dt}$$

$$+ \dfrac{\gamma P_o}{V_1} A_2 (A_2 x_2 - A_1 x_1) + \dfrac{\gamma P_o}{V_2} A_2^2 x_2 = P_2 A_2 . \hspace{1cm} (7b)$$

The equations of motion may be reduced to the following form by

dividing equation (7a) by A_1 and equation (7_b) by A_2 and replacing $P_o\gamma$ by its equivalent $\rho_o c^2$,

$$\frac{\rho_o \ell_{e1}}{A_1} \ddot{X}_1 + c_1 (\dot{X}_1 - \dot{X}_2) + \frac{\rho_o c^2}{V_1} (X_1 - X_2) = P_1 \quad , \text{ and}$$

(8)

$$\frac{\rho_o \ell_{e2}}{A_2} \ddot{X}_2 + c_2 \dot{X}_2 + c_1 (\dot{X}_2 - \dot{X}_1) + \frac{\rho_o c^2}{V_2} X_2 + \frac{\rho_o c^2}{V_1} (X_2 - X_1) = P_2 \; ;$$

where

$X_1 = A_1 x_1$ = volume displacement at neck N_1,

$X_2 = A_2 x_2$ = volume displacement at neck N_2,

P_1 = external pressure on neck N_1, and

P_2 = external pressure on neck N_2.

The transformation of linear simultaneous differential equations (8) into complex frequency domain by the Laplace transformation with zero initial conditions, yields the following simultaneous algebraic equations:

$$\left[\frac{\rho_o \ell_{e1}}{A_1} s^2 + c_1 s + \frac{\rho_o c^2}{V_1}\right] X_1(s) - \left[c_1 s + \frac{\rho_o c^2}{V_1}\right] X_2(s) = P_1(s) \; ,$$

and

$$\left[\frac{\rho_o \ell_{e2}}{A_2} s^2 + (c_1 + c_2) s + \frac{\rho_o c^2}{V_1} + \frac{\rho_o c^2}{V_2}\right] X_2(s) - \left[c_1 s + \frac{\rho_o c^2}{V_1}\right] \cdot$$

$$X_1(s) = P_2(s).$$

(9)

The solution of these equations gives the response of the system in complex frequency domain as,

$$X_1(s) = \frac{P_1(s) \frac{\rho_o \ell e2}{A_2} \left[s^2 + 2p_2 (\xi_2 + \xi_{12}) s + p_2^2 + p_{12}^2 \right]}{\Delta}$$

$$+ \frac{P_2(s) \frac{\rho_o \ell e1}{A_1} \left[2\xi_1 p_1 + p_1^2 \right]}{\Delta} \quad ,$$

and

$$X_2(s) = \frac{P_2(s) \frac{\rho_o \ell e1}{A_1} \left[s^2 + 2\xi_1 p_1 + p_1^2 \right]}{\Delta}$$

$$+ \frac{P_1(s) \frac{\rho_o \ell e2}{A_2} (2\xi_1 p_1 s + p_1^2)}{\Delta} \tag{10}$$

where

p_1, p_2 are the uncoupled natural frequencies,

p_{12} is the coupling frequency, and

Δ = characteristic determinant of equations (9)(Appendix A).

Response in the time domain can be obtained by factoring the complex expressions of equation (10) by partial fractions and transforming from complex frequency domain using inverse Laplace transformation.

The excitation pressure on neck N_2, $P_2(t)$ is assumed to be zero. Then equation (10) simplifies to

$$X_1(s) = \frac{P_1(s) \frac{\rho_o \ell e2}{A_2} \left[s^2 + 2p_2 (\xi_2 + \xi_{12}) s + p_2^2 + p_{12}^2 \right]}{\Delta} \quad ,$$

and

$$X_2(s) = \frac{P_1(s) \frac{\rho_o \ell e2}{A_2} \left[2\xi_1 p_1 s + p_1^2 \right]}{\Delta} \quad . \tag{11}$$

The response solution is obtained for two important types of transient excitation pressures on neck N_1: one cycle of sine wave and

a wave made of straight lines with finite duration and finite number of discontinuities.

Case 1. Response of double acoustical resonator for an excitation of one cycle of sine wave:

This excitation pressure can be illustrated graphically as in Figure 6 and expressed mathematically as,

$$P_1(t) = A_0 \sin \omega t, \qquad \text{for } 0 \leqslant t \leqslant \frac{1}{2\pi\omega} \, ;$$

$$P_1(t) = 0, \qquad \text{for } t < 0 \text{ and } t > \frac{1}{2\pi\omega} \, ; \text{ and}$$

$$P_2(t) = 0, \qquad \text{for all } t. \qquad (12)$$
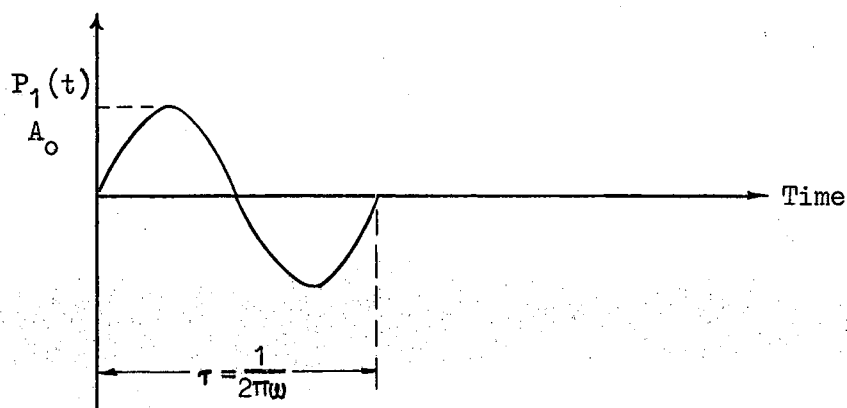
Figure 6. Excitation Pressure of Sine Pulse on Neck N_1

The volume displacement response of the system which may be obtained by inverse Laplace transformation are (Appendix A),

$$X_1(t) = \frac{A_0 \, \omega}{\dfrac{\rho_0 \ell_{e1}}{A_1}} \left\{ \left[a_1 \cos \omega t + \frac{b_1}{\omega} \sin \omega t + \frac{e^{at}}{b} (\Phi_1 \sin bt + \Phi_2 \cos bt) \right. \right.$$

$$\left. + \frac{e^{ct}}{d} (\Psi_1 \sin dt + \Psi_2 \cos dt) \right] u(t) - \left[a_1 \cos \omega(t-\tau) \right.$$

$$+ \frac{b_1}{\omega} \operatorname{Sin} \omega(t - \tau) + \frac{e^{a(t-\tau)}}{b} (\Phi_1 \operatorname{Sin} b(t - \tau) + \Phi_2 \operatorname{Cos} d(t - \tau))$$

$$+ \frac{e^{c(t-\tau)}}{d} (\Psi_1 \operatorname{Sin} d(t - \tau) + \Psi_2 \operatorname{Cos} d(t - \tau))] \ u \ (t - \tau) \Bigg\} \ ,$$

and

$$X_2(t) = \frac{A_0 \omega^2 p_1^2}{\frac{\rho_o \ell}{A_2}} \Bigg\{ [a_2 \operatorname{Cos} \omega t + \frac{b_2}{\omega} \operatorname{Sin} \omega t + \frac{e^{at}}{b} (\Phi_3 \operatorname{Sin} bt + \Phi_4 \operatorname{Cos} bt)$$

$$+ \frac{e^{ct}}{d} (\Psi_3 \operatorname{Sin} dt + \Psi_4 \operatorname{Cos} dt)] \ u \ (t) \ - \ [a_2 \operatorname{Cos} \omega(t - \tau)$$

$$+ \frac{b_2}{\omega} \operatorname{Sin} \omega(t - \tau) + \frac{e^{a(t-\tau)}}{b} (\Phi_3 \operatorname{Sin} b(t - \tau) + \Phi_4 \operatorname{Cos} b(t - \tau))$$

$$+ \frac{e^{c(t-\tau)}}{d} (\Psi_3 \operatorname{Sin} d(t - \tau) + \Psi_4 \operatorname{Cos} d(t - \tau))] u(t - \tau) \Bigg\} \ ; \quad (13)$$

where the coefficients a_1, b_1, ..., Ψ_3, Ψ_4 are given by equations (15) in Appendix A.

The pressures in the cavities may be expressed in terms of volume displacements from equations (12) as,

$$P_1 = \frac{\rho_o c^2}{V_1} (X_1 - X_2) \ , \text{ and}$$

$$P_2 = \frac{\rho_o c^2}{V_2} X_2 \ .$$

These pressures can be normalized with respect to the amplitude of the excitation pressure A_0, and can be derived as a function of uncoupled frequencies, p_1 and p_2, coupling frequency p_{12}, and the damping factors ξ_1, ξ_2 as (Appendix A),

$$\frac{P_1}{A_0} = p_1^2 \ \omega \Bigg\{ [A_x \operatorname{Cos} \omega t + B_x \operatorname{Sin} \omega t + \frac{e^{at}}{b} (C_x \operatorname{Sin} bt + D_x \operatorname{Cos} bt)$$

$$+ \frac{e^{ct}}{d} (E_x \, Sin \, dt + F_x \, Cos \, dt)] \, u(t) - [A_x \, Cos \, \omega(t - \tau)$$

$$+ B_x \, Sin \, \omega(t - \tau) + \frac{e^{a(t - \tau)}}{b} (C_x \, Sin \, b(t - \tau) + D_x \, Cos \, b(t - \tau))$$

$$+ \frac{e^{c(t - \tau)}}{d} (E_x \, Sin \, d(t - \tau) + F_x \, Cos \, d(t - \tau))] \, u(t - \tau) \Big\}$$

and

$$\frac{P_2}{A_0} = p_1{}^2 \, p_2{}^2 \, \omega^2 \Big\{ [a_2 \, Cos \, \omega t + \frac{b_2}{\omega} \, Sin \, \omega t + \frac{e^{at}}{b} (\Phi_3 \, Sin \, bt + \Phi_4 Cos \, bt)$$

$$+ \frac{e^{ct}}{d} (\Psi_3 \, Sin \, dt + \Psi_4 \, Cos \, dt)] \, u(t) - [a_2 \, Cos \, \omega(t - \tau)$$

$$+ \frac{b_2}{\omega} \, Sin \, \omega(t - \tau) + \frac{e^{a(t - \tau)}}{b} (\Phi_3 \, Sin \, b(t - \tau) + \Phi_4 Cos \, b(t - \tau))$$

$$+ \frac{e^{c(t - \tau)}}{d} (\Psi_3 \, Sin \, d(t - \tau) + \Psi_4 \, Cos \, d(t - \tau))] u(t - \tau) \Big\}; \quad (14)$$

where

$$A_x = a_1 - a_2 \, p_{12}{}^2,$$
$$B_x = \frac{1}{\omega} (b_1 - b_2 \, p_{12}{}^2),$$
$$C_x = \Phi_1 - \Phi_3 \, p_{12}{}^2,$$
$$D_x = \Phi_2 - \Phi_4 \, p_{12}{}^2,$$
$$E_x = \Psi_1 - \Psi_3 \, p_{12}{}^2, \text{ and}$$
$$F_x = \Psi_2 - \Psi_4 \, p_{12}{}^2. \quad (15)$$

In equations (12) and (14) the unit functions $u(t)$ and $u(t - \tau)$ indicate that the response expressions are valid for only $t > 0$ and $(t - \tau) > 0$, respectively, which should be taken into account in evaluating pressures in the cavities at any time.

Since from this analysis it is rather difficult to understand the effect of various system parameters on the response, the analysis of the pressure response of the undamped system subject to an excitation pressure of sine pulse will predict the parameters involved. These pressures can be written as (Appendix A),

$$\frac{P_1}{A_0} = \frac{p_1^2}{(\omega^2 - p_+^2)(\omega^2 - p_-^2)} \left\{ [a_z \sin \omega t + b_z \sin p_+ t + c_z \sin p_- t] \cdot \right.$$

$$u(t) - [a_z \sin \omega(t - \tau) + b_z \sin p_+(t - \tau) + c_z \sin p_- \cdot$$

$$\left. (t - \tau)] u(t - \tau) \right\}, \quad \text{and}$$

$$(16)$$

$$\frac{P_2}{A_0} = \frac{p_1^2 p_2^2}{(\omega^2 - p_+^2)(\omega^2 - p_-^2)} \left\{ [\sin \omega t + b_y \sin p_+ t + c_y \sin p_- t] u(t) \right.$$

$$\left. - [\sin \omega(t - \tau) + b_y \sin p_+(t - \tau) + c_y \sin p_-(t - \tau)] u(t - \tau) \right\};$$

where

$$a_z = a_x - a_y p_{12}^2,$$
$$b_z = b_x - b_y p_{12}^2, \text{ and}$$
$$c_z = c_x - c_y p_{12}^2.$$

When the frequency of driving pressure, ω, is the same as one of the coupled natural frequencies of the system, p_+ or p_-, the denominator of equation (16) will be zero, which indicates that the pressures in the cavities may be infinite. But at these conditions, the numerator also becomes zero which makes the response mathematically indeterminate and leads to the use of the L'Hospital rule to find the pressure response. Since these expressions are complex functions of the un-

coupled frequencies, p_1 and p_2, and of the coupling frequency, p_{12}, of

the system, it is more convenient to establish the response by

numerical techniques than by applying mathematical techniques to the

response expressions.

Case 2: Response of double acoustical resonator for an excitation

pressure made up of straight lines with finite number of discontinuties:

This excitation pressure can be illustrated graphically as in

Figure 7 and can be expressed mathematically as follows:

$$P_1(t) = \sum_{i=1}^{n} \left[\alpha_i + \beta_i (t - \tau_i) \right] u(t - \tau_i); \qquad (17)$$

where

    n = number of discontinuities on time axis,

    α_i = pressure jump at time, $t = \tau_i$, and

    β_i = change in slope of pressure at time, $t = \tau_i$.
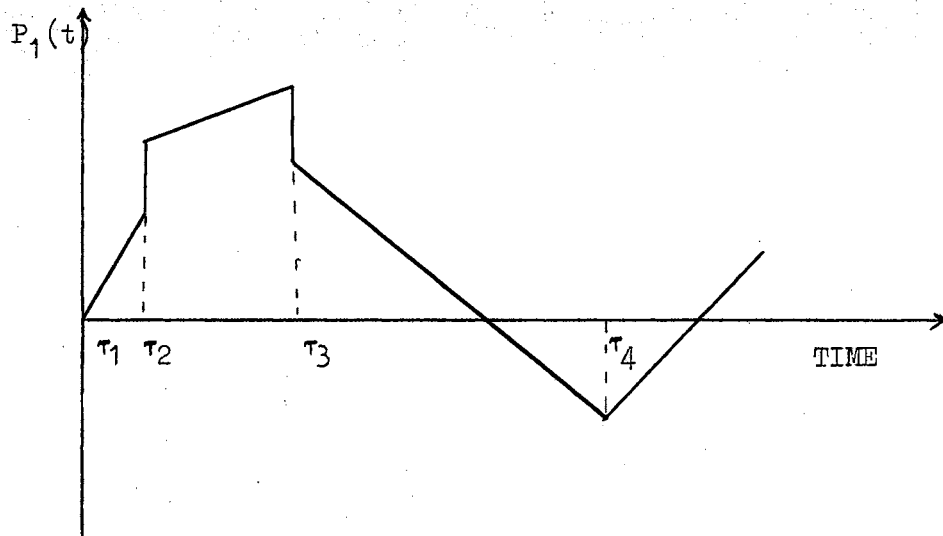
Figure 7.   Excitation Pressure Made Up of Straight
Lines

Laplace transform of $P_1(t)$ is,

$$P_1(s) = \frac{1}{s^2} \sum_{i=1}^{n} [\alpha_i s + \beta_i] e^{-\tau_i s} \quad . \tag{18}$$

The excitation pressure on neck N_2',

$$P_2(t) = 0, \text{ and hence } P_2(s) = 0.$$

The volume displacement response of the system for the excitation pressure of the type described in Figure 7 is (Appendix A),

$$X_1(t) = \frac{1}{\dfrac{\rho_o \ell_{e1}}{A_1}} \sum_{i=1}^{n} \left\{ a_{1i} + b_{1i}(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} \quad . \right.$$

$$[\Phi_{1i} \text{ Sin } b(t - \tau_i) + \Phi_{2i} \text{ Cos } b(t - \tau_i)] + \frac{e^{c(t - \tau_i)}}{d} \quad .$$

$$\left. [\Psi_{1i} \text{ Sin } d(t - \tau_i) + \Psi_{2i} \text{ Cos } d(t - \tau_i)] \right\} u(t - \tau_i) \quad , \quad \text{and}$$

$$X_2(t) = \frac{p_1^2}{\dfrac{\rho_o \ell_{e2}}{A_2}} \sum_{i=1}^{n} \left\{ a_{2i} + b_{2i}(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} \quad . \right.$$

$$[\Phi_{3i} \text{ Sin } b(t - \tau_i) + \Phi_{4i} \text{ Cos } b(t - \tau_i)] + \frac{e^{c(t - \tau_i)}}{d} \quad .$$

$$\left. [\Psi_{3i} \text{ Sin } d(t - \tau_i) + \Psi_{4i} \text{ Cos } d(t - \tau_i)] \right\} u(t - \tau_i). \tag{19}$$

The pressures in the cavities can be expressed in terms of volume displacements as follows:

$$P_1 = \frac{\rho_o c^2}{V_1} (X_1 - X_2) = p_1^2 \sum_{i=1}^{n} \left\{ a_{xi} + b_{xi}(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} \quad . \right.$$

$$[c_{xi} \; \text{Sin} \; b(t - \tau_i) + d_{xi} \; \text{Cos} \; b(t - \tau_i)] + \frac{e^{c(t - \tau_i)}}{d} \; \cdot$$

$$[e_{xi} \; \text{Sin} \; d(t - \tau_i) + f_{xi} \; \text{Cos} \; d(t - \tau_i)] \Big\} \; u(t - \tau_i), \; \text{and}$$

$$P_2 = \frac{\rho_o c^2}{V_2} X_2 = p_1^{\;2} \; p_2^{\;2} \sum_{i=1}^{n} \Big\{ a_{2i} + b_{2i}(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} \; \cdot$$

$$[\Phi_{3i} \; \text{Sin} \; b(t - \tau_i) + \Phi_{4i} \; \text{Cos} \; b(t - \tau_i)] + \frac{e^{c(t - \tau_i)}}{d} \; \cdot$$

$$[\Psi_{3i} \; \text{Sin} \; d(t - \tau_i) + \Psi_{4i} \; \text{Cos} \; d(t - \tau_i)] \Big\} \; u(t - \tau_i) \; ; \qquad (20)$$

where

$$a_{xi} = a_{1i} - a_{2i} \; p_{12}^{\;2},$$

$$b_{xi} = b_{1i} - b_{2i} \; p_{12}^{\;2},$$

$$c_{xi} = c_{1i} - c_{2i} \; p_{12}^{\;2},$$

$$d_{xi} = d_{1i} - d_{2i} \; p_{12}^{\;2},$$

$$e_{xi} = e_{1i} - e_{2i} \; p_{12}^{\;2}, \; \text{and}$$

$$f_{xi} = f_{1i} - f_{2i} \; p_{12}^{\;2} \; .$$

These response expressions can be evaluated in the time domain for given uncoupled, undamped natural frequencies, coupling frequency and the damping factors.

Transmission of Energy From One Resonator to the Other

In the case of a dynamic system with more than one degree-of-freedom or multiple oscillators tightly coupled together, the feedback energy from the driven system to the driver cannot be neglected. For

illustration, in the case of two oscillators coupled together, both oscillators are on an equal footing, each is affected by the other. In the case of an undamped system subjected to transient excitation force of finite duration, the total energy of the system computed at the end of the forced era should remain constant for the rest of the period. However, the absolute maximum displacement (pressure in acoustical system) may occur in the residual era depending on the system parameters. The absolute displacements of masses or absolute pressures in the cavities of acoustical systems depends on the type of transient excitation force or pressure, the manner by which it is applied and the system parameters. If the excitation pressure is applied on neck N_1 of the acoustical resonator (Figure 3), it is natural to expect that the pressure build up in cavity V_1 is faster when compared to that in the cavity V_2. However, the maximum pressure amplitude can be in either one of the cavities depending on the dimensions of the necks and cavities.

One can anticipate the beating phenomenon in the residual era for the system with two degrees-of-freedom. In the case of mechanical elements coupled with acoustical elements forming a dynamic system with multiple degrees-of-freedom, the beating phenomenon exists both in pressure oscillations and the displacement oscillations of mechanical masses.

When a mechanical system with two degrees-of-freedom is subjected to finite duration driving force, both masses will have initial displacements and velocities at the end of the forced era. With these initial conditions, the displacement response in the residual era is the summation of two harmonic motions with the natural frequencies of

the system. When these two frequencies are different from each other, then the combination of these two motions is not harmonic. If the amplitude of this response varies periodically, then this phenomenon is known as the beating effect.

Merits and Demerits of Analog Computer

The analog computer solution of simultaneous differential equations is of a continuous form. Accuracy depends on the various elements of the analog computer. It is rather arduous in some cases, to make the system stable for the system of undamped physical systems. The individual elements of the system can be varied very conveniently and the response can be studied. It is well suited for the solution of the differential equations for particular configuration with numerical values. To study the effect of all the possible parameters, is rather tedious, particularly to study the maximax response of the dynamic system with multiple degrees-of-freedom. For example, in the expression for the pressure response of the double acoustical resonator, for excitation pressure of one cycle of sine wave, it is necessary to study the effect of two uncoupled frequencies and the coupling frequency on the maximax response. Since the analog form of the solution is in the time domain, it is necessary to solve for different combinations of coefficients of the differential equations in order to solve for maximax response. It is time consuming to study the maximax response for all the possible variables of the system. Also one should be extremely careful in selecting the numerical values for the parameters, so that the critical point will not be missed.

CHAPTER IV

THEORETICAL RESULTS

The pressures in the cavities of the system of the multiple acouti-
cal resonator are functions of various system parameters such as un-
coupled natural frequencies, coupling frequencies, and damping factors
and the excitation pressures. The coupling frequencies are defined
as the frequencies which couple two resonators. It is not possible
to study the behavior of pressure response by the mathematical treat-
ment of equation (14), for it is an extremely complicated function of
system parameters and external pressure excitation. Also, since these
expressions are functions of several variables, it is rather difficult
to find the absolute maximum pressures in the time domain by applying
the mathematical treatment. To represent the response spectra of the
system, it is necessary to use a response surface instead of a response
curve.

Before discussing the response of double acoustical resonators
subjected to transient excitations, it is desirable to know the
minimum number of independent parameters that affect the response.
The response pressures in the cavities, when excited by a finite
duration pressure, are the functions of the conductivities of the necks,
volumes of the cavities and the period of excitation pressure. However,
from the discussion in Chapter III, it is apparent that there are five
independent parameters to represent the system, namely, p_1, p_2, p_{12},

ξ_1, and ξ_2. The two parameters, amplitude and duration, represents the transient excitation pressure. It is desirable to use normalized system parameters to present the response spectra. The frequencies may be normalized by multiplying them by the period of excitation. Pressures may be normalized by dividing by the amplitude of the excitation pressure. The choice and number of parameters necessary to represent the system response are discussed in the next section by examining the governing equations.

Governing Equations

The pressure response expressions are simpler in the case of the undamped system as compared to the damped system. Thus it is easier to understand the dynamic behavior of the system by discussing the undamped system exposed to external pressures.

The following are the expressions from Appendix A for the pressure response of an undamped double acoustical resonator in the forced era ($0 \quad t \quad \tau$) for a one cycle sine wave pressure excitation of amplitude A_o and period τ:

$$\frac{P_1}{A_o} = \frac{p_1^2}{(\omega^2 - p_+^2)(\omega^2 - p_-^2)} \left[a_z \, Sin \, \omega t + b_z \, Sin \, p_+ t \right.$$

$$\left. + c_z \, Sin \, p_- t \right] , \quad and \tag{21}$$

$$\frac{P_2}{A_o} = \frac{p_1^2 \, p_2^2}{(\omega^2 - p_+^2)(\omega^2 - p_-^2)} \left[Sin \, \omega t + b_y \, Sin \, p_+ t + c_y \, Sin \, p - t \right] ;$$

where p_+ and p_- are two undamped natural frequencies of the system and are given by equation (18) in Appendix A as,

$$p_+^2 = \frac{(p_1^2 + p_2^2 + p_{12}^2) + \sqrt{(p_1^2 - p_2^2 - p_{12}^2)^2 + 4p_1^2 p_{12}^2}}{2}$$

$$(22)$$

$$p_-^2 = \frac{(p_1^2 + p_2^2 + p_{12}^2) - \sqrt{(p_1^2 - p_2^2 - p_{12}^2)^2 + 4p_1^2 p_{12}^2}}{2}$$

From these equations it appears that the pressure response can be reduced to a function of three variables; namely, two natural frequencies and the period of the excitation pressure, but the coefficients a_z, b_z, ... c_y, are functions of p_1, p_2, p_{12} and ω, and cannot be reduced to a function of only coupled frequencies and ω. Thus, the pertinent parameters for investigating the response are the uncoupled frequencies and the type of excitation pressure. It is of interest to note that the pressure response is not an implicit function of the elements of the systems, such as the volumes of cavities and conductivities or dimensions of the necks.

Uncoupled and Coupled Frequencies of the System

The coupled frequencies, p_+ and p_- are functions of three variables p_1, p_2, and p_{12}. For the double acoustical resonator the frequencies p_1 and p_2 are defined as the frequencies of each of the resonators when the other resonator is absent. The coupling frequency is defined as the frequency of the system when the neck N_1 is closed and the cavity V_2 is removed. The following expressions for the frequencies are obtained from equations (7),

$$p_1 = c\sqrt{\frac{A_1}{v_1 l_{e1}}},$$

$$p_2 = c\sqrt{\frac{A_2}{v_2 l_{e2}}}, \text{ and}$$

$$p_{12} = \alpha \sqrt{\frac{A_2}{v_1 l_{e2}}} \, . \tag{23}$$

These frequencies are tabulated in Table I for buildings that can be idealized as double acoustical resonators. In this analysis the combination of two rooms with two doors open and all windows closed, as shown in Figure 8, is considered. For various values of areas of the door openings, volumes of the rooms, and keeping the actual length of the necks (doors) as six inches, the frequencies are calculated.

The equations (22) can be solved for uncoupled frequencies as functions of p_+, p_- and p_{12} resulting in,

$$p_1^{\,2} = \frac{(p_+^{\,2} + p_-^{\,2} - p_{12}^{\,2}) \pm \sqrt{[p_{12}^{\,2} - (p_+^{\,2} + p_-^{\,2})]^2 - 4p_+^{\,2} p_-^{\,2}}}{2} \, ,$$

and

$$p_2^{\,2} = \frac{(p_+^{\,2} + p_-^{\,2} - p_{12}^{\,2}) \mp \sqrt{[p_{12}^{\,2} - (p_+^{\,2} + p_-^{\,2})]^2 - 4p_+^{\,2} p_-^{\,2}}}{2} \, .$$

Two combinations of p_1 and p_2 are obtained for given values of p_+, p_- and p_{12}. For each set of p_1 and p_2 values the pressure response is different and hence p_+ and p_- cannot be the pertinent system parameters to represent the response.

The pressures in the cavities were computed in time domain and plotted in Figure 9, for a short period sine pulse excitation. This illustrates the beating phenomenon of double acoustical resonator which was discussed in Chapter III.
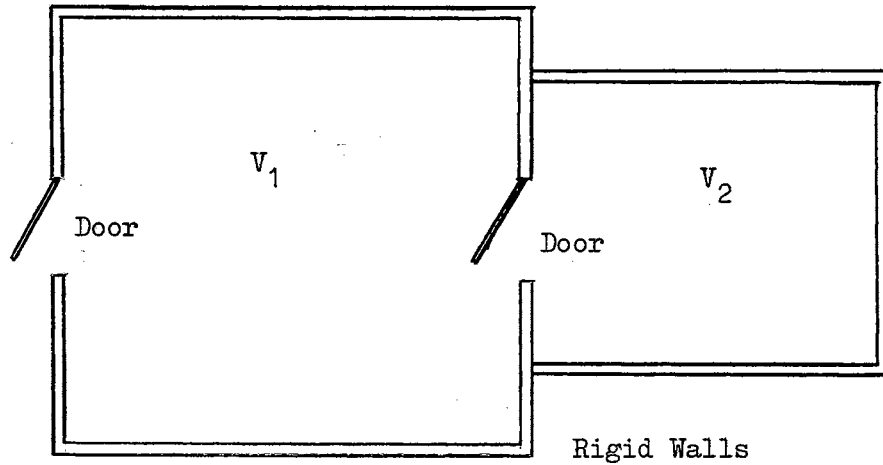
Figure 8. Building Structure That Can be Idealized as
a Double Acoustical Resonator

Damped Acoustical Resonator

As it is stated in Chapter III, incorporation of damping makes

the system more complicated to analyze. In this investigation it is

assumed that the damping due to various mechanisms can be reduced to

equivalent viscous damping which is a linear function of volume dis-

placement. The governing pressure response equation for the excitation

pressures of one cycle of sine pulse and the pressures made up of

straight lines with finite discontinuities like finite duration N-waves

and saw tooth waves are;

For sine pulse:

$$\frac{P_1}{A_o} = p_1{}^2 \omega \left\{ [A_x \cos \omega t + B_x \sin \omega t + \frac{e^{at}}{b} (C_x \sin bt + D_x \cos bt) \right.$$

$$\left. + \frac{e^{ct}}{d} (E_x \sin dt + F_x \cos dt)] u(t) - [A_x \cos \omega(t - \tau) \right.$$

TABLE I

NATURAL FREQUENCY OF TYPICAL ACOUSTIC STRUCTURES

A_1 (ft^2)	A_2 (ft^2)	V_1 (ft^3)	V_2 (ft^3)	f_1 (cps)	f_2 (cps)	f_{12} (cps)
15	15	1000	1000	11.10	11.08	11.10
30	15	1000	1000	13.40	11.08	11.10
15	30	1000	1000	11.10	13.40	13.40
30	30	1000	1000	13.40	13.40	13.40
15	15	19000	1000	2.54	11.08	2.54
30	15	19000	1000	3.08	11.08	2.54
15	30	19000	1000	2.54	13.43	3.08
30	30	19000	1000	3.08	13.43	3.08
15	15	1000	19000	11.10	2.54	11.10
30	15	1000	19000	13.43	2.54	11.10
15	30	1000	19000	11.10	3.10	13.40
30	30	1000	19000	13.40	3.10	13.40
15	15	19000	19000	2.54	2.54	2.54
30	15	19000	19000	3.08	2.54	2.54
15	30	19000	19000	2.54	3.08	3.08
30	30	19000	19000	3.08	3.08	3.08
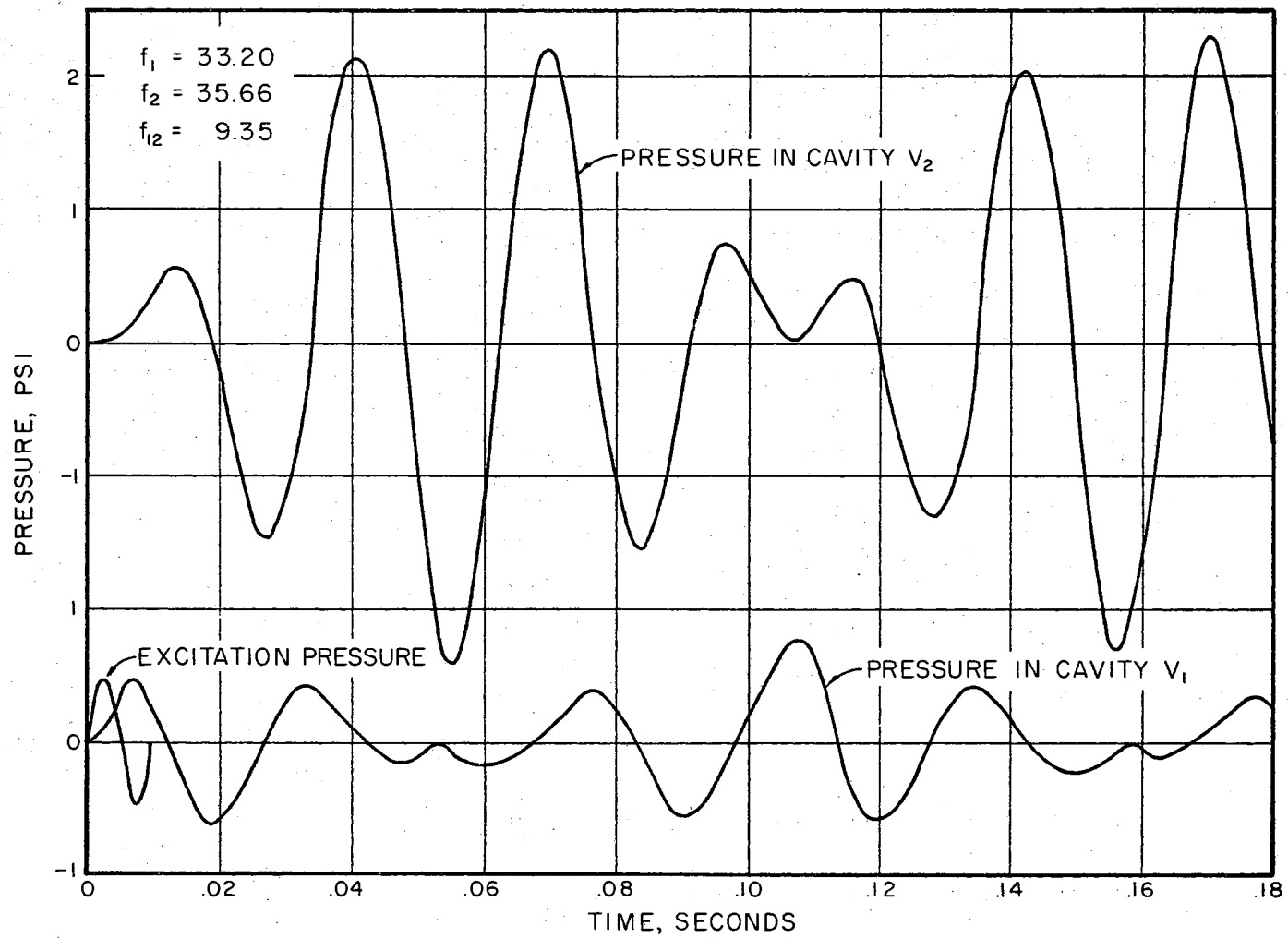
Figure 9. Pressure Beatings in a Double Acoustical Resonator

$$+ B_x \text{ Sin } \omega(t - \tau) + \frac{e^{a(t - \tau)}}{b} (C_x \text{ Sin } b \ (t - \tau) + D_x \text{Cos } b(t - \tau))$$

$$+ \frac{e^{c(t - \tau)}}{d} (E_x \text{ Sin } d(t - \tau) + F_x \text{Cos } d(t - \tau))] \ u(t - \tau) \Big\} ,$$

and

$$\frac{P_2}{A_o} = p_1^2 \ p_2^2 \ \omega \Big\{ [a_2 \text{ Cos } \omega t + \frac{b_2}{\omega} \text{ Sin } \omega t + \frac{e^{at}}{b} (\Phi_3 \text{Sin } bt + \Phi_4 \text{Cos } bt)$$

$$+ \frac{e^{ct}}{d} (\Psi_3 \text{ Sin } dt + \Psi_4 \text{ Cos } dt)] \ u(t) - [a_2 \text{ Cos } \omega(t - \tau)$$

$$+ \frac{b_2}{\omega} \text{ Sin } \omega(t - \tau) + \frac{e^{a(t - \tau)}}{b} (\Phi_3 \text{ Sin } b(t - \tau) + \Phi_4 \text{Cos } b(t - \tau))$$

$$+ \frac{e^{c(t - \tau)}}{d} (\Psi_3 \text{Sin } d(t - \tau) + \Psi_4 \text{Cos } d(t - \tau))] \ u(t - \tau) \Big\} ; \qquad (24a)$$

and

For pressures with straight lines:

$$P_1 = p_1^2 \ \sum_{i=1}^{n} \Big\{ a_{xi} + b_{xi}(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} [c_{xi} \text{Sin } b(t - \tau_i)$$

$$+ d_{xi} \text{ Cos } b(t - \tau_i)] + \frac{e^{c(t - \tau_i)}}{d} [e_{xi} \text{ Sin } d(t - \tau_i)$$

$$+ f_{xi} \text{ Cos } d(t - \tau_i)] \Big\} u(t - \tau_i),$$

and

$$P_2 = p_1^2 \ p_2^2 \ \sum_{i=1}^{n} \Big\{ a_{2i} + b_{2i}(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} [\Phi_{3i} \text{Sin } b(t - \tau_i)$$

$$+ \Phi_{4i} \text{ Cos } b(t - \tau_i)] + \frac{e^{c(t - \tau_i)}}{d} [\Psi_{3i} \text{ Sin } d(t - \tau_i)$$

$$+ \Psi_{4i} \; \mathrm{Cos} \; d(t - \tau_i)]\Big\} \; u(t - \tau_i) \quad . \qquad (24b)$$

These equations do not lend themselves to immediate conclusions as to the interactions of the system parameters due to complexity of the expressions. These expressions are computed numerically for different values of p_1, p_2, p_{12}, ξ_1, ξ_2, and period and type of excitation. The FORTRAN computer programs are written in such a way that the pressures in the cavities are computed in time domain for stated values of system parameters and the period of excitation pressures, and the printout is the maximum pressure that can occur in a given interval of time (Appendix C).

The response spectra of the transient response of the system are plotted in the frequency domain in Figures 10 to 18 for a sine pulse type excitation pressure. The system parameters used to represent responses are the frequencies of uncoupled resonators and damping factors. Since there are five system parameters in addition to the period of excitation, it is not possible to study the effect of each parameter on the response either from equations or from one representative curve. Hence, the results are presented in several figures such that the effect of two parameters can be studied in each of the figures. Figures 19a and 19b are the pressure response spectra of the same system for saw tooth and N-wave type excitations.

The frequencies are non-dimensionalized with respect to the frequencies corresponding to the period of excitation pressures. The pressures in the cavities are normalized with respect to the amplitude of the excitation pressures and thus correspond to pressure magnifications. In these figures P_1 and P_2 are the response pressures and

ξ_1 and ξ_2 are the damping factors in the cavities V_1 and V_2, respectively. The uncoupled frequencies are represented by f_1 and f_2, and coupling frequency by f_{12}. A_o and τ are the amplitude and the period of the excitation pressures.

From these spectra, it is not possible to conclude the critical circumstances under which the pressure magnification in the cavities will be maximum. However, by studying the numerical values of the maximum response for various values of frequencies and period of excitation pressures, it appears that the pressures will be magnified to a maximum possible value (maximax) whenever two natural frequencies are close to each other and also they are equal to both uncoupled frequencies and frequency of excitation. Ideally, this occurs when the two natural frequencies, p_+ and p_-, are equal; however, this cannot happen because of the presence of the coupling term in the system.

A physical explanation for these large magnifications can be given in the following way: when two natural periods are equal, the plugs of air in the necks will attempt to move with the same frequency. If the two uncoupled natural periods are equal to the natural periods, then even a small amplitude of excitation pressure with the same period will reinforce the motion of air particles in the necks, thereby increasing the displacement magnification to a large extent.

Figures 10 and 11 illustrate the effect of the frequencies, f_1 and f_2 on pressure response in the cavity V_2, for an undamped system of double acoustical resonator. In Figure 10, the maximum pressure response in V_2 to variable uncoupled frequency f_1 of cavity V_1, with selected parametric values of uncoupled frequency f_2 of cavity V_2, is

shown. In Figure 11, the maximum response pressure P_2 to variable frequency of cavity V_2, with selected parametric values of frequency f_1 of cavity V_1, is presented. In both these figures the other parameters of the system are held constant. The excitation pressure is a sine pulse with a period of 0.05 seconds, damping factors, ξ_1 and ξ_2 in the cavities are held zero, and the coupling frequency of the system, f_{12} retains a constant value of 2 cps. For each frequency f_2 in Figure 10, and f_1 in Figure 11, the maximum response curve shows the expected two humps of a two degree-of-freedom system. The family of curves is also a representative of classical dynamic absorber problem. As an illustration, as the difference between the uncoupled frequencies, f_1 and f_2 increases, the maximum response pressure P_2 diminishes. As the difference between the uncoupled frequencies and the excitation frequency increases, the humps diminish and spread farther apart. When the difference is small the merging of the two humps reinforce each other to present a maximax response condition. This is shown by drawing the dotted envelope to the larger peak values.

Figures 12 and 13 demonstrate the effect of uncoupled frequencies of the system and the coupling frequency on response pressure in cavity V_2. In Figure 12, the maximum pressure response in cavity V_2 to variable frequency f_1, with selected parametric values of coupling frequency f_{12}, is presented. The frequency of cavity V_2, is held constant at 20 cps. In Figure 13, the maximum pressure in cavity V_2, to variable uncoupled frequency, f_2, is shown for different values of coupling frequency f_{12}. The uncoupled frequency f_1, of the cavity V_1, is held constant at 20 cps. The other parameters of the system are held constant in these two figures. The damping factors are kept

at zero. The excitation pressure is a sine pulse with a period of 0.05 seconds. The maximax response pressure occurs when the frequencies of both the resonators are close together. As already stated in the earlier part of this section, it is clear from these figures that the pressure magnification in the cavity V_2, would increase as the coupling frequency f_{12}, decreased. As the difference between either of the natural periods, $1/f_1$ or $1/f_2$, and the period of excitation pressure increases, the influence of f_{12} reduces, as can be seen in these figures for larger values of f_1 or f_2.

Figure 14 is the description of the effect of coupling frequency f_{12} on the maximum response pressure in cavity V_2, for different parametric values of f_1 and f_2. The damping factors, ξ_1 and ξ_2, are kept at zero. The excitation pressure is a sine pulse with a period of 0.05 seconds. As illustrated in Figures 12 and 13, these curves also indicate that the maximum pressure in the cavity V_2 will increase as the coupling frequency decreases. It can be observed from this figure that the pressure P_2 is greater when the frequencies of two resonators are equal. This can also be seen from Figures 10 and 11.

Figures 15a and 15b are the pressure response spectra of cavity V_1 for a double acoustical resonator system. The excitation pressure is a sine pulse with a period of 0.05 seconds. In Figure 15a, the maximum response pressure in cavity V_1, with two selected parametric values of frequency f_2, is shown. The coupling frequency f_{12}, is held constant and equal to 2 cps. These curves show that the influence of frequency f_2, of the cavity V_2, on the maximum pressure P_1 in cavity V_1 is very small. In Figure 15b the maximum pressure response in cavity V_1 to variable frequency f_1, with selected values of coupling

frequency f_{12}, is shown. The frequency f_2 of the cavity V_2 is held constant at 20 cps. These curves indicate that the effect of coupling frequency on the response pressure in cavity V_1. From these two figures it can be observed that the frequency f_2 of cavity V_2, and the coupling frequency f_{12}, have very small influence on the pressure P_1 in cavity V_1 so long as the difference between the frequency f_1 and the excitation frequency corresponding to the period, τ, is large.

Figure 16a is an illustration of the effect of the frequency f_2 on the pressure magnification in cavity V_1. In this figure the variation of pressure P_2 is shown as the frequency f_2 varies for two parametric values of frequency f_1 of cavity V_1. The damping factors, ξ_1 and ξ_2 are zero and the coupling frequency f_{12} is kept constant at 2 cps. The excitation pressure is a sine pulse with a period of 0.05 seconds. From the curves in this figure it can be inferred that the variation of frequency of cavity V_2 does not affect the pressure in the cavity V_1. The significant effect of f_2 can be seen mainly when the period of excitation is very close to the natural period $1/f_2$ of cavity V_2.

In Figure 16b, the maximum pressure magnification in V_1 to the coupling frequency F_{12} is shown for two parametric values of frequency f_1 of cavity size V_1. The damping factors are zero and the frequency f_2 is held constant at 20 cps. The excitation pressure is a sine pulse with a period of 0.05 seconds. From this figure it is not possible to conclude the general effect of the system parameters on the response pressures in cavity V_1. However, from Figures 16a and 16b it can be observed that the frequency f_1 and coupling frequency f_{12} are the system parameters that affect the pressure magnification in

cavity V_1.

In Figures 17, 18a, and 18b, the maximum pressure response in cavity V_2 to frequency f_1, with the parametric values of f_2, is shown for a damped double acoustical resonator. The coupling frequency f_{12} is kept constant at 2 cps. The excitation pressure is a sine pulse with a period of 0.05 seconds. But the damping factors are different in these three figures: In Figure 17, the damping factors, ξ_1 and ξ_2 are 0.05; In Figures 18a and 18b, these damping factors are 0.1 and 0.15, respectively. In these three figures the general trend of the maximax pressure response is the same as discussed earlier and given in Figure 10. However, comparison of these figures with Figure 10 indicates the effect of damping. It can be observed from these two figures that the maximum pressure magnification in cavity V_2, may be reduced to about three by incorporating the damping factors ξ_1 and ξ_2 equal to 0.1 in the system.

Figures 19a and 19b are the maximum pressure response spectra which illustrate the effect of frequencies f_1 and f_2 on the pressure in cavity V_2. The damping factors ξ_1 and ξ_2 of the cavities are held constant at 0.1. The coupling frequency in both the cases is kept at 2 cps. The variation of maximum pressure to frequency f_1 with selected parametric values of frequency f_2 of cavity V_2 is shown. In Figure 19a, the excitation pressure is a saw tooth wave with amplitude of unity and period of 0.05 seconds. In Figure 19b, the excitation pressure is an N-wave with a period of 0.05 seconds and amplitude of unity. The general trend and behavior of the curves is the same as illustrated in Figure 18a, which is the response spectra of the system for a sine pulse type excitation. Comparison of these three figures
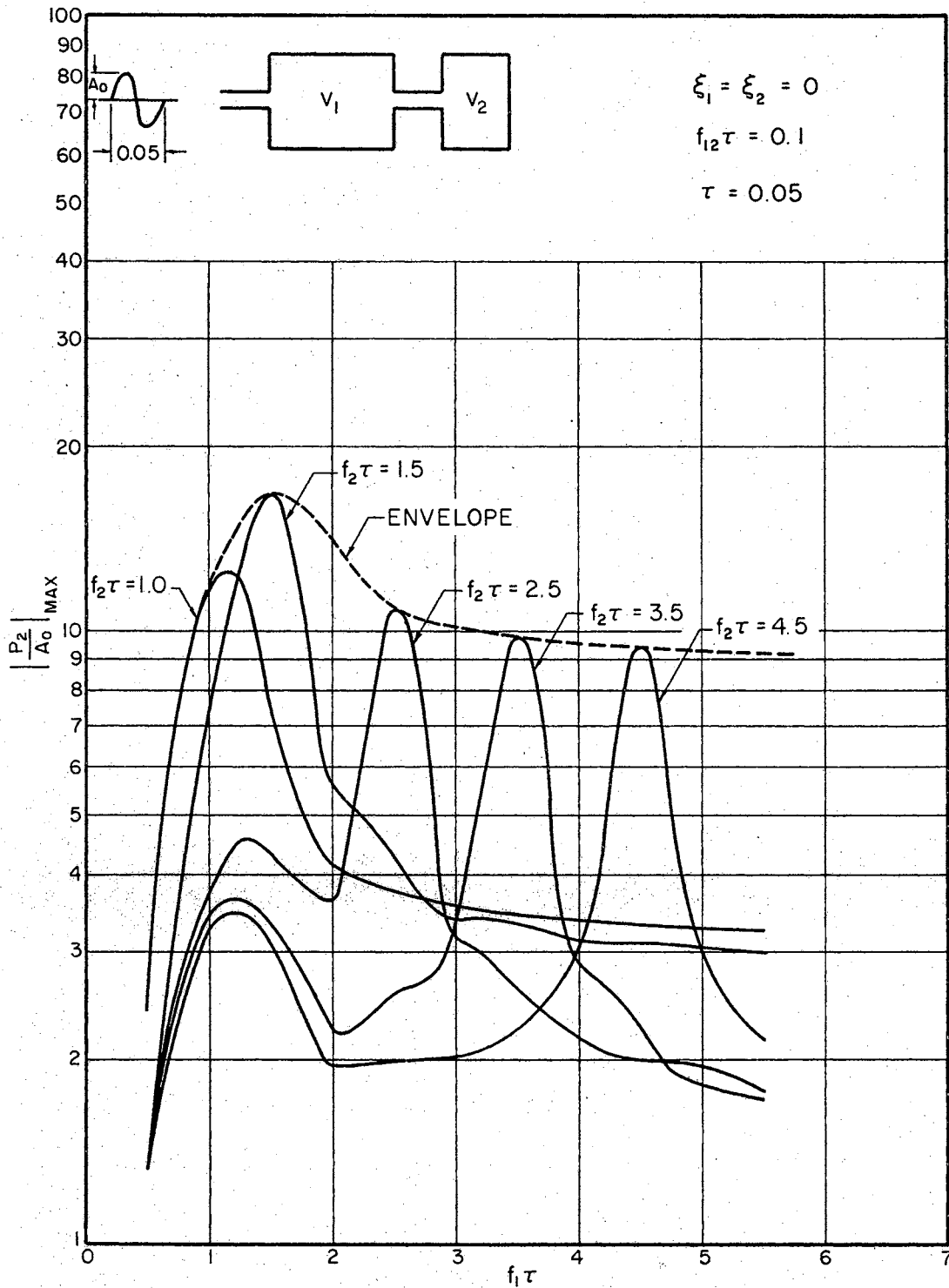
Figure 10. Pressure Response Spectra in V_2 For a Constant Coupling Frequency (Effect of f_1 on Undamped Resonator)
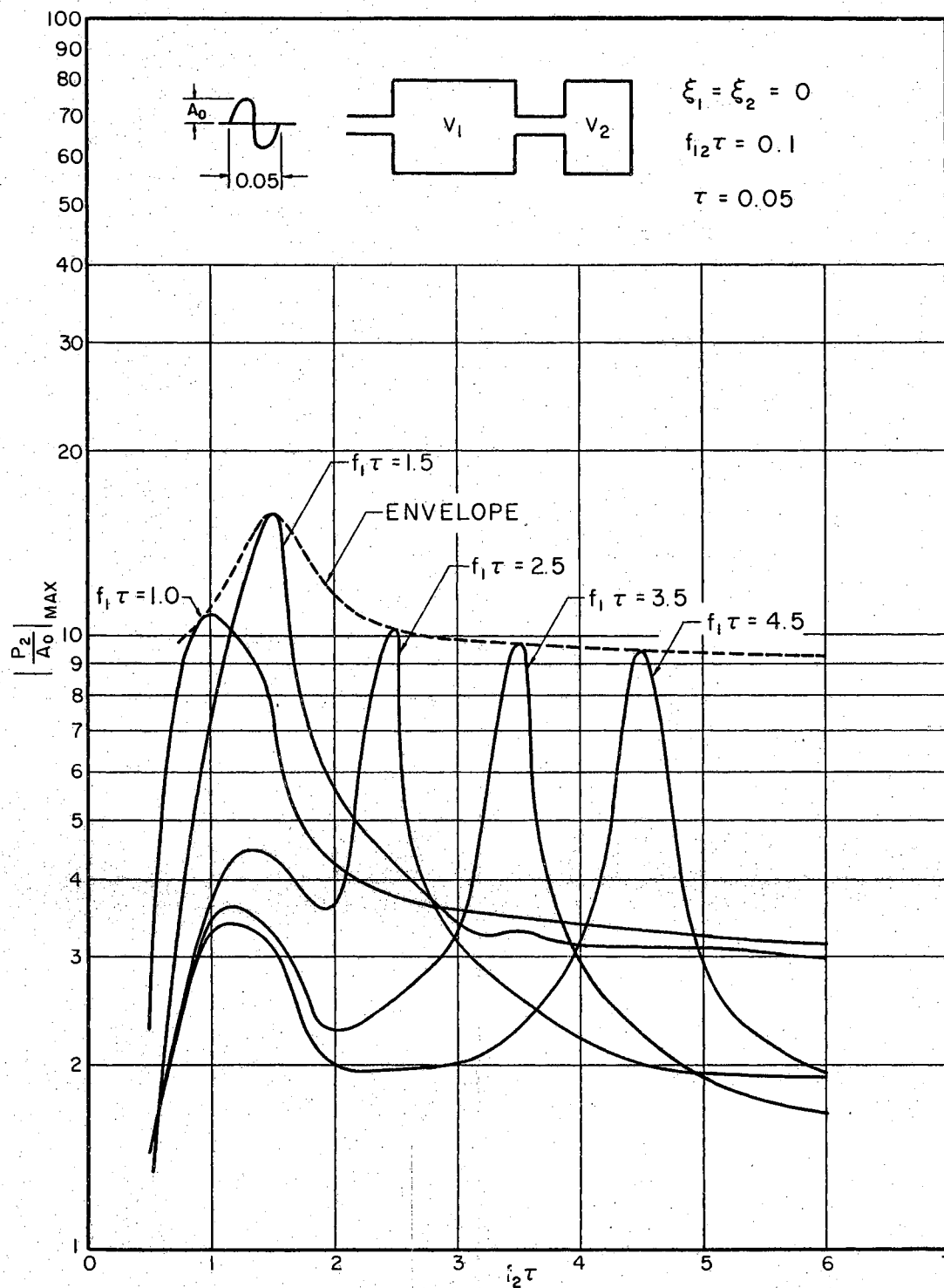
Figure 11. Pressure Response Spectra in V_2 For a Constant Coupling Frequency (Effect of f_1)
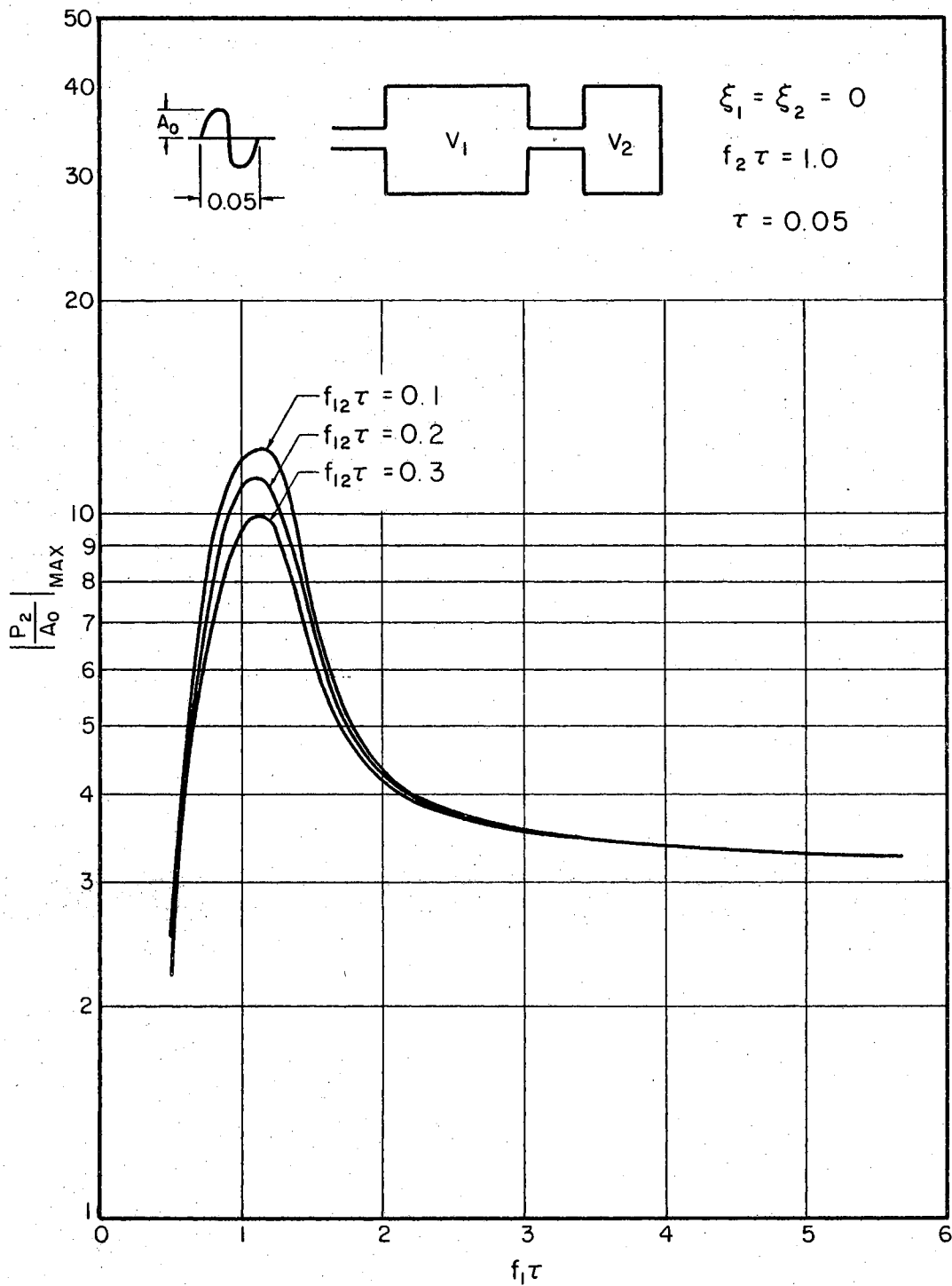
Figure 12. Pressure Response Spectra in V_2 For a Constant f_2
(Effect of f_1 and f_{12})
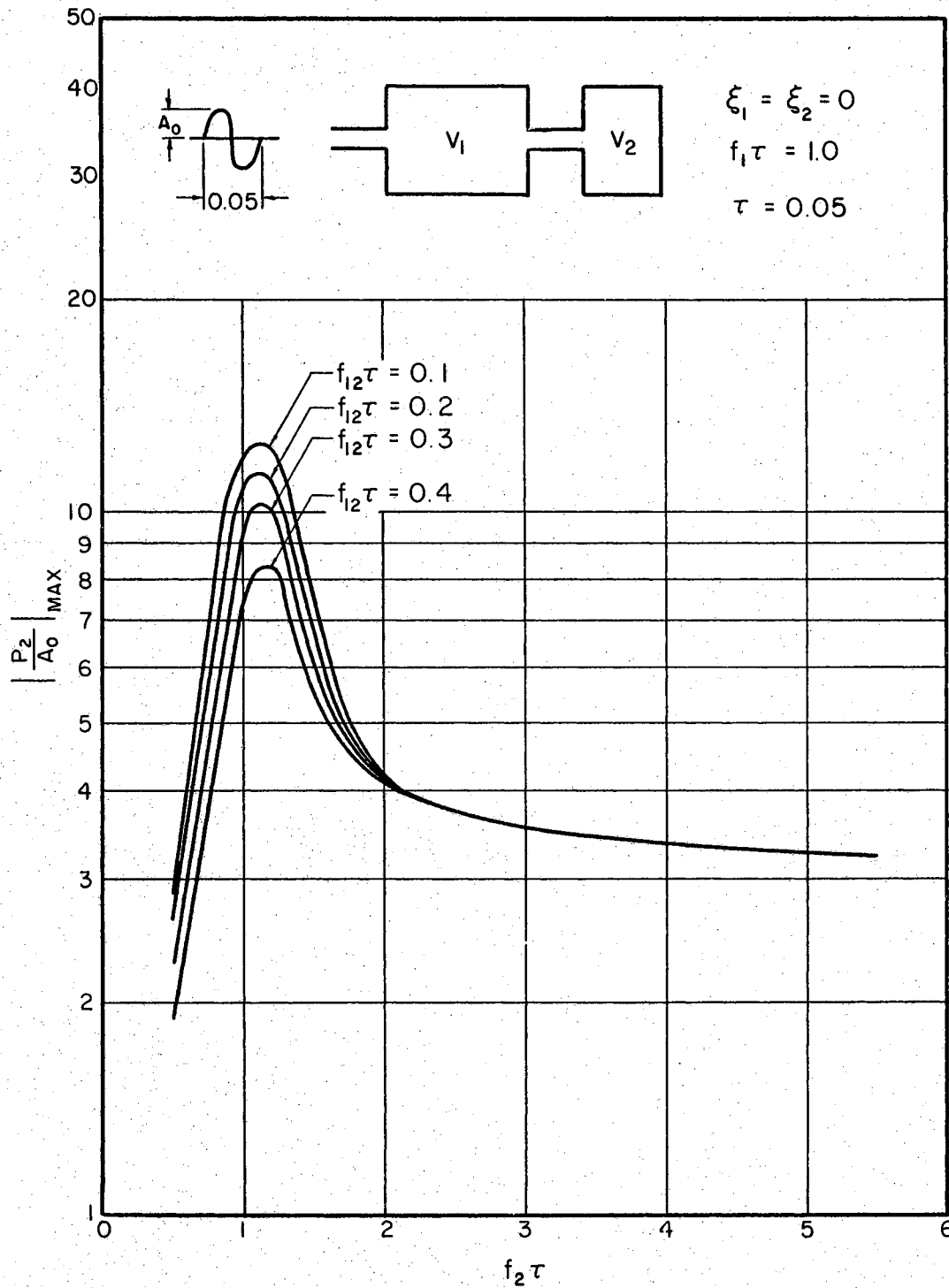
Figure 13. Pressure Response Spectra in V_2 for a Constant f_1 (Effect of f_2 and f_{12})
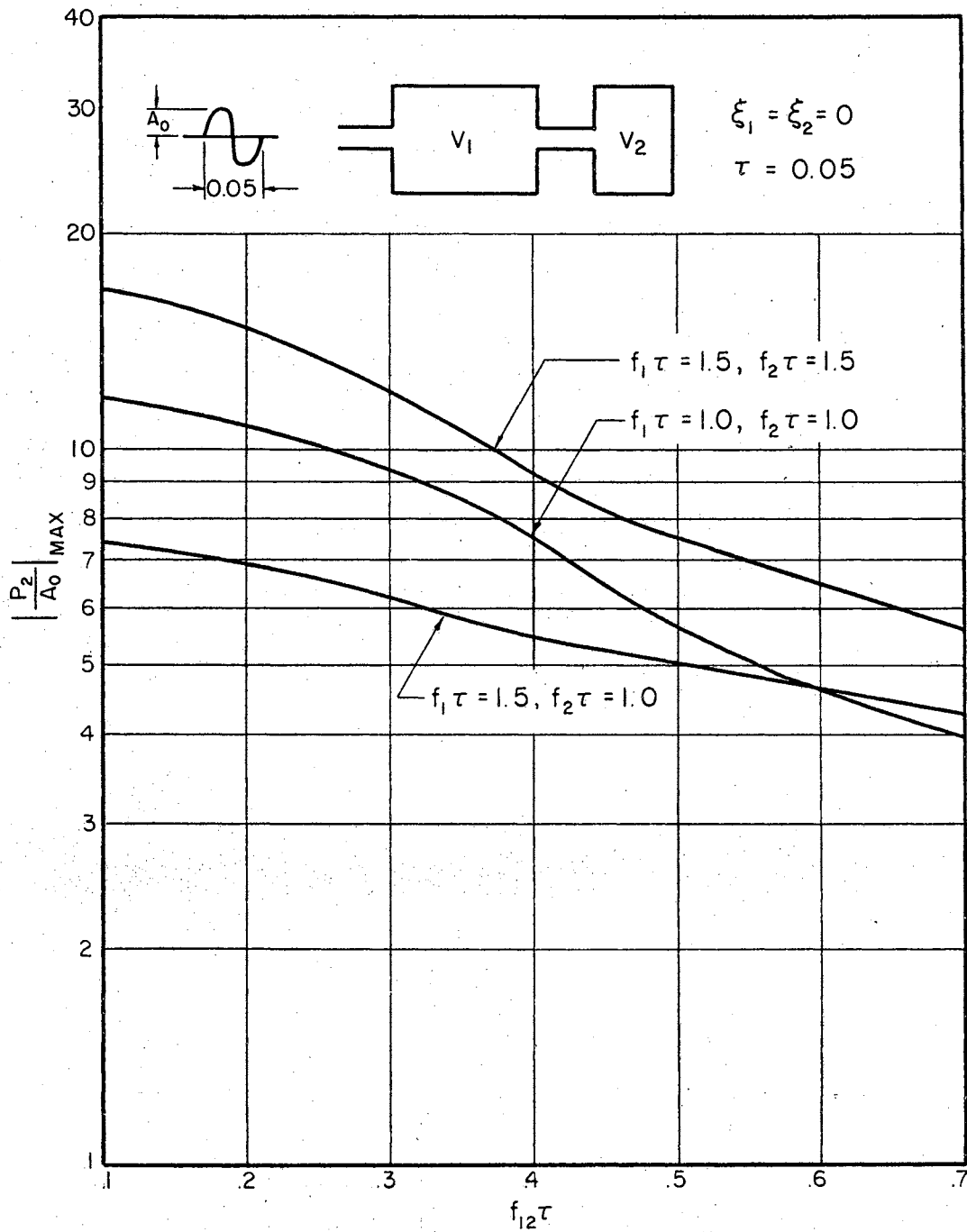
Figure 14. Pressure Response Spectra in V_2 (Effect of f_{12})
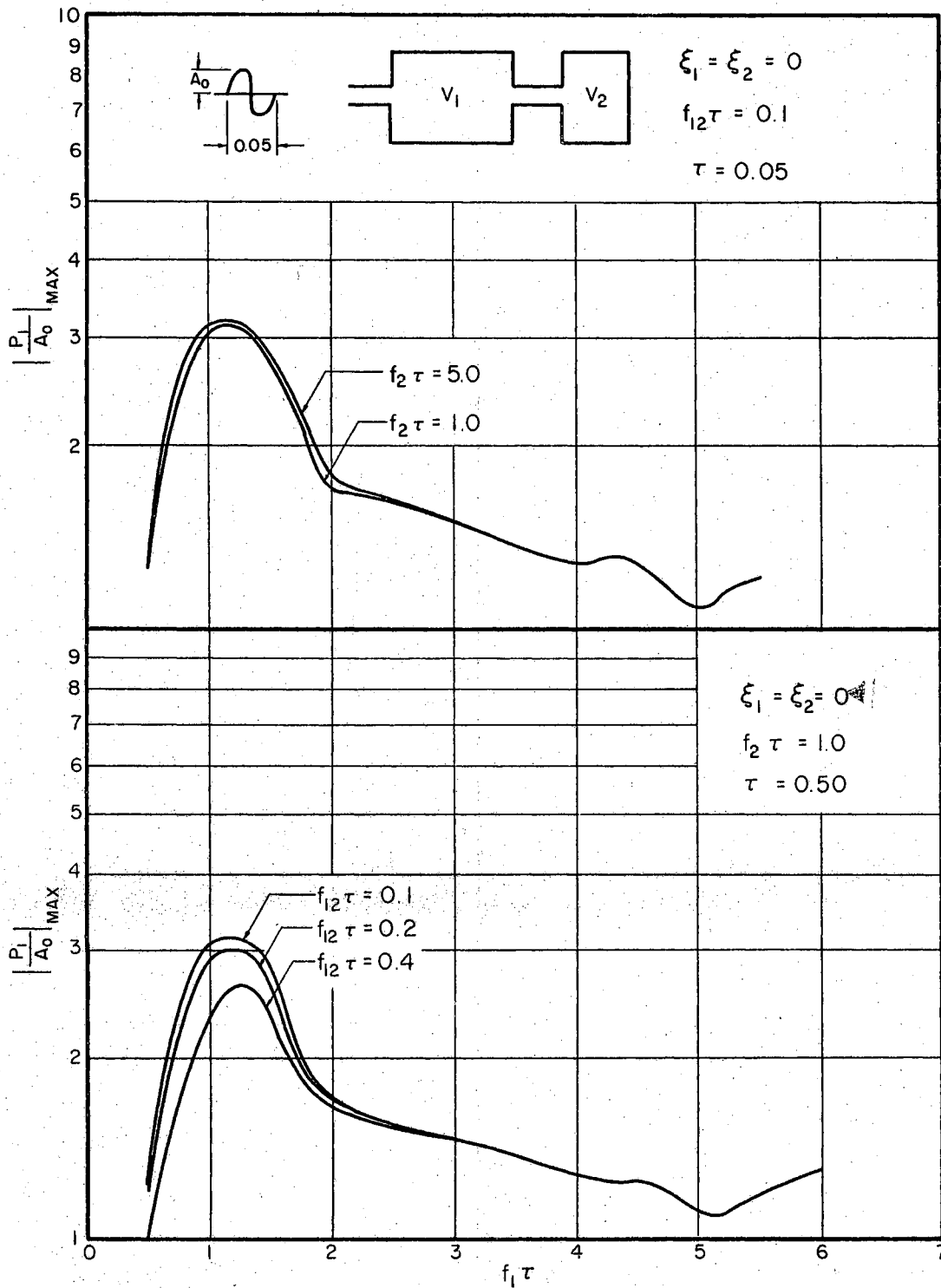
Figure 15a, 15b. Pressure Response Spectra in V_1 For Constant f_{12} and f_2 (Effect of f_1)
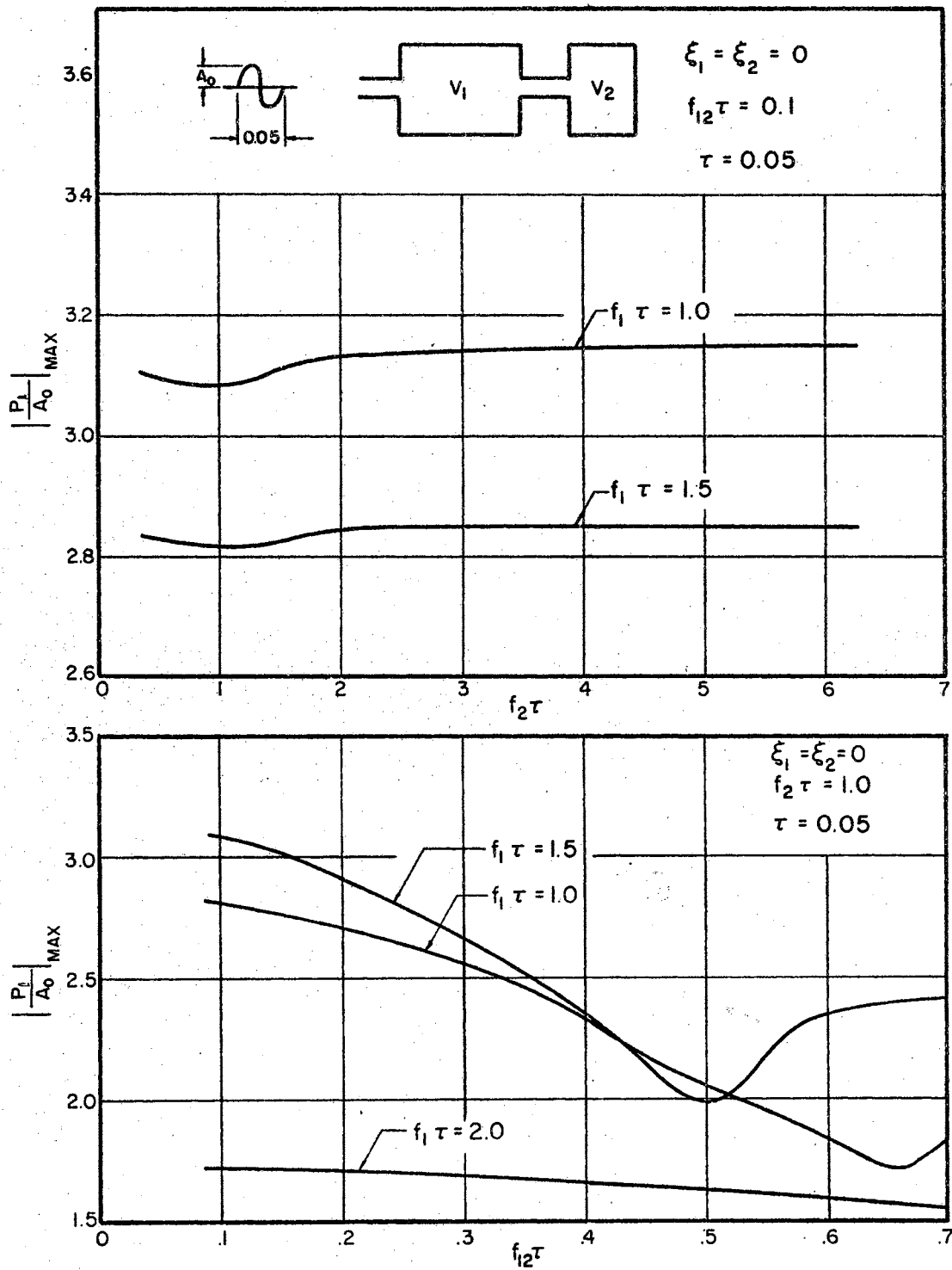
Figure 16a, 16b. Pressure Response Spectra in V_1 For Constant f_{12} and f_2 (Effect of f_2 and f_{12})
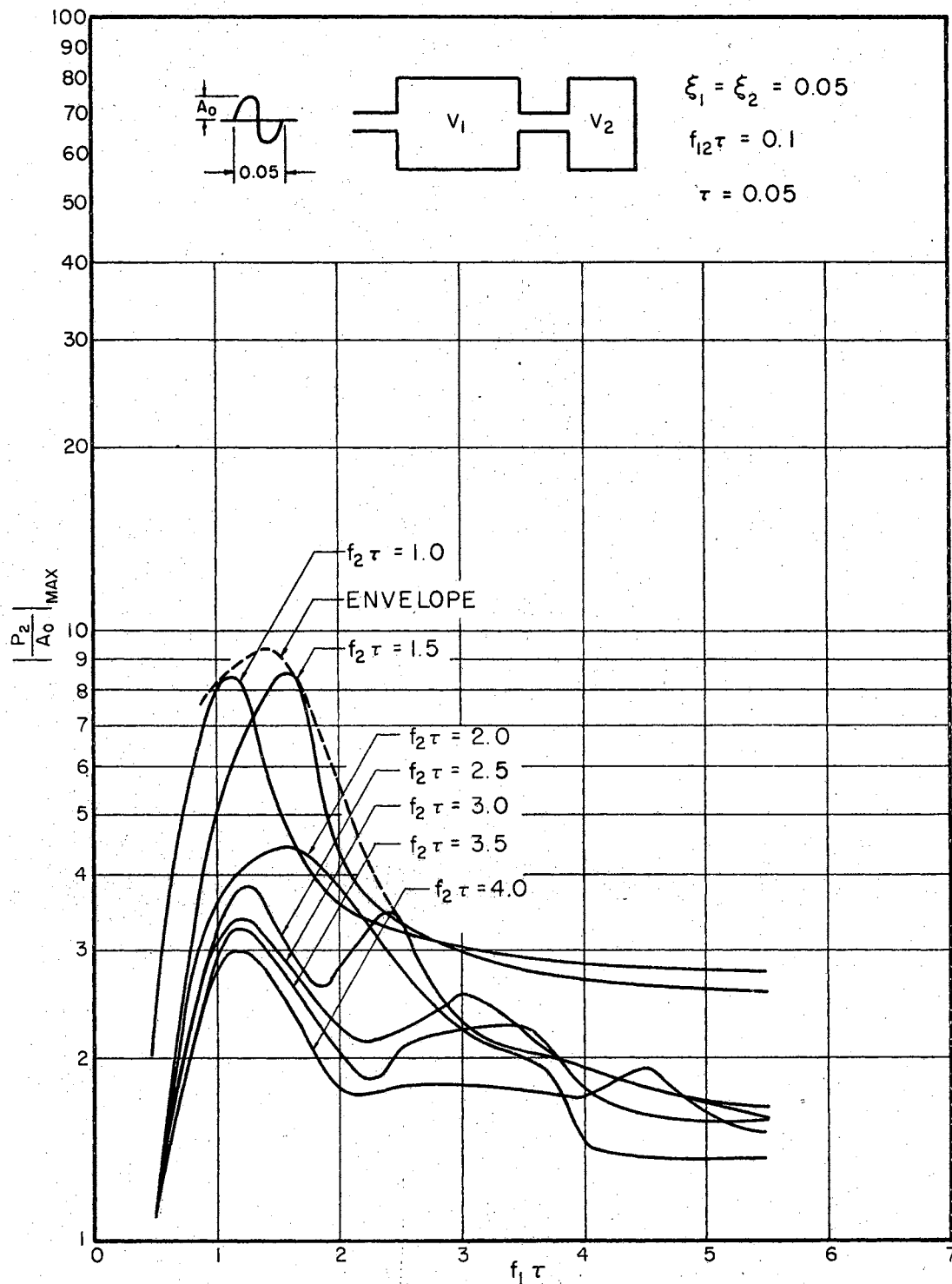
Figure 17. Pressure Response Spectra in V_2 for Constant f_{12}
(Effect of f_1 With Damping Factors, $\xi_1 = \xi = 0.05$)
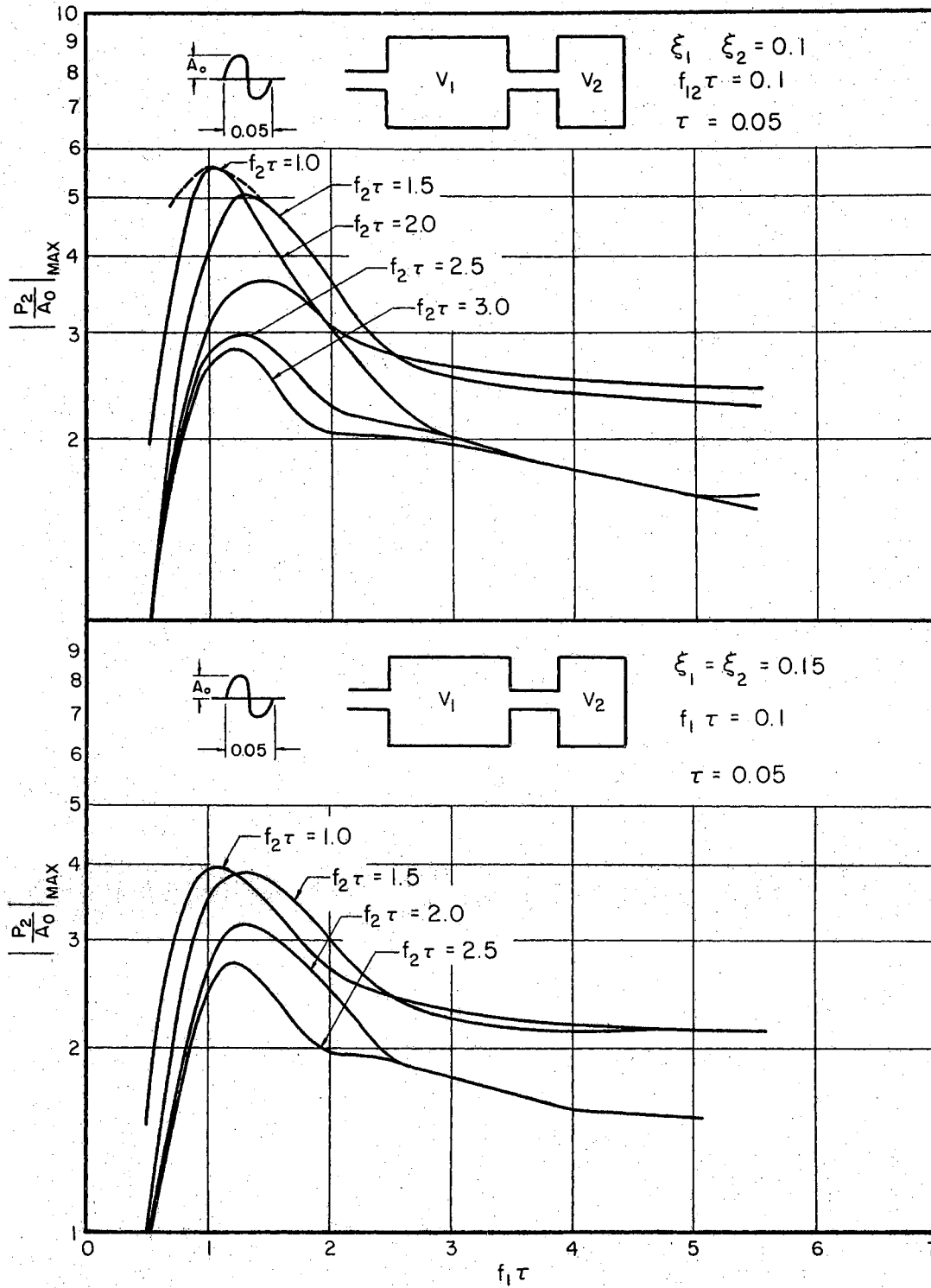
Figure 18a and 18b. Pressure Response for Constant f_{12}
(Effect of f_1 and Damping Factors)

Figure 19a and 19b. Pressure Response Spectra in V_2 for Constant f_{12} (Comparison Response to Saw Tooth and N-wave Type Excitation)
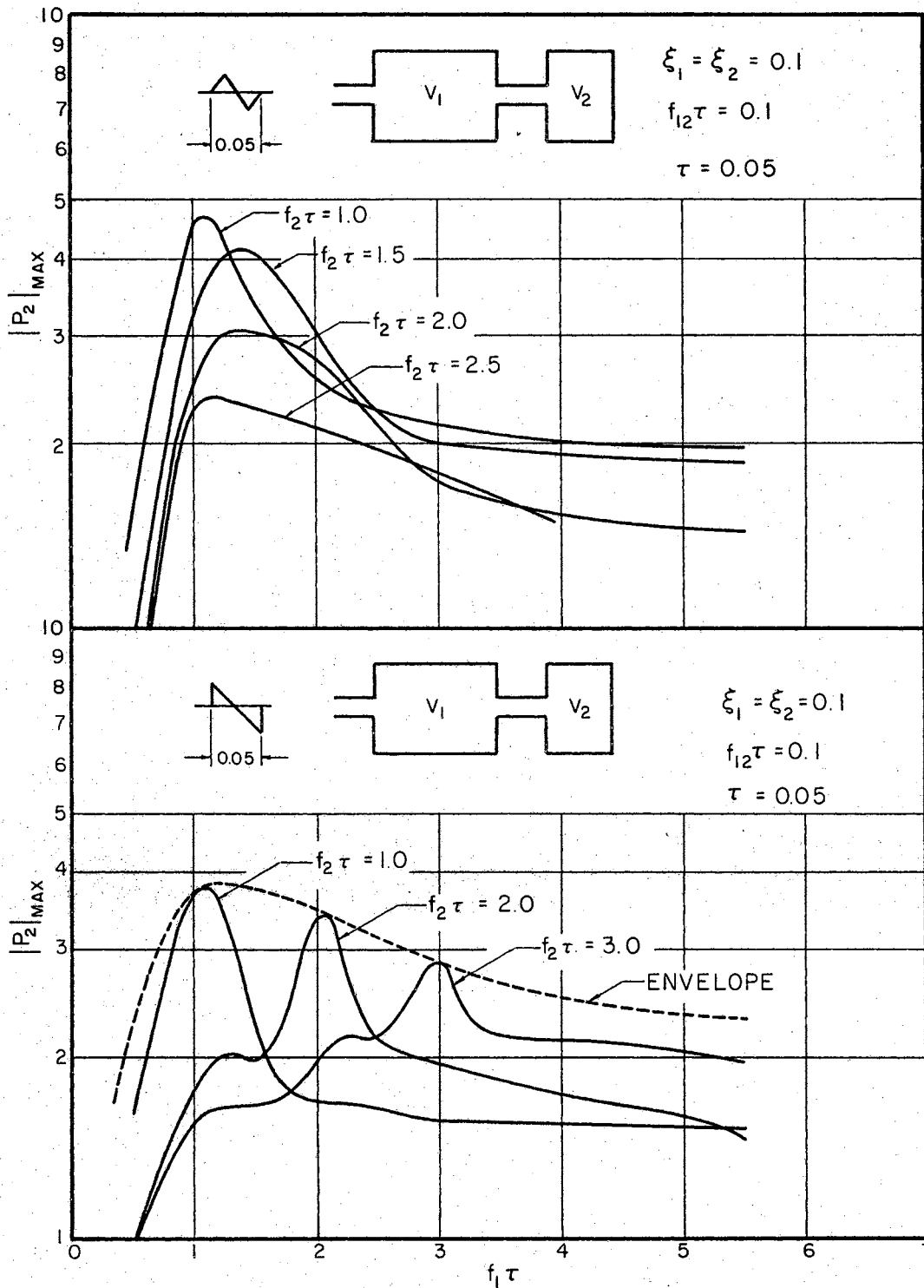
shows that the maximax pressure response will be greater for the excitation of a sine pulse, when the other parameters are held same.

From the pressure response spectra of the double acoustical resonator to an excitation pressure of one sine pulse, it is apparent that the uncoupled natural frequency f_2 has little effect on the pressure in cavity V_1. The prime controlling parameters of this response pressure are f_1, f_{12} and the period of excitation pressure. However, the pressure in the cavity V_2 is affected by both the frequencies f_1 and f_2. The coupling frequency f_{12} has a considerable effect on the pressures in both the cavities when the normalized frequencies, $f_1 \tau$ and $f_2 \tau$, are close to unity. The damping factor of 0.1 reduces the maximax pressure magnification in the cavities from 16.7 to 5.1 as can be seen by comparing Figures 10 and 18a. From Figure 14, it appears that the pressure magnification in cavity V_2 can be as high as 18 for a system having a coupling frequency of 2 cps, uncoupled frequencies $f_1 = f_2 = 30$ cps, and damping factors $\xi_1 = \xi_2 = 0$. However, these values are not the highest possible maximax pressures, since in certain cases where the system is tuned properly by varying the system parameters and the period of excitation pressures, these pressures can be magnified theoretically to the order of about 50. Since in physical systems the damping is inevitable and the coupling is limited, the highest actual maximax magnifications would be of the order of about 15.

Mechanical System

In certain cases, an acoustical system coupled to a mechanical system can be idealized to a simple mechanical system. The physical
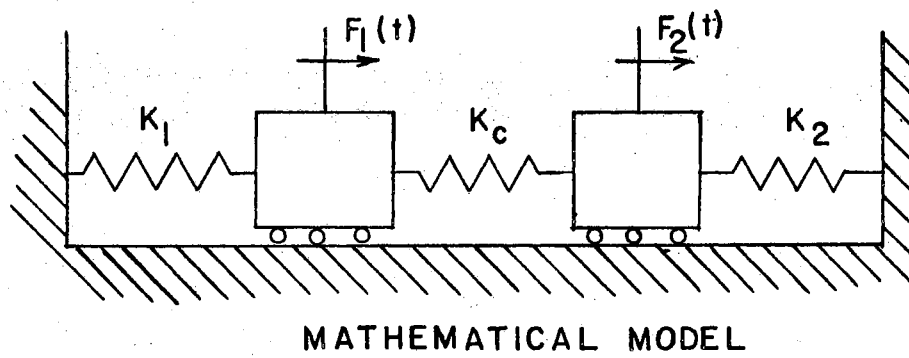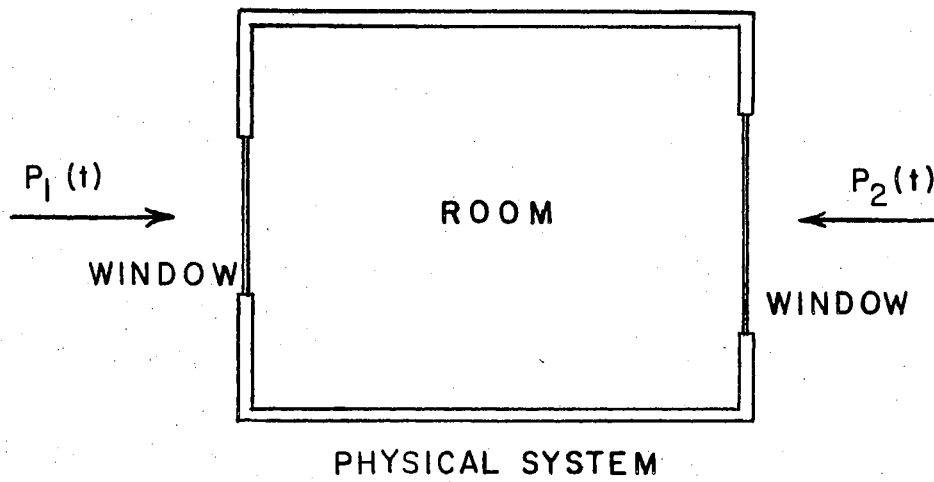
Figure 20. Mechano-acoustic System

and mathematical models of this type of system are shown in Figure 20. With the assumption of no dissipation of energy in the system, the following equations of motion may be derived:

$$M_1 \ddot{x}_1 + (k_1 + k_c) x_1 - k_c x_2 = F_1(t) \, ,$$

and

$$M_2 \ddot{x}_2 + (k_2 + k_c) x_2 - k_c x_1 = F_2(t) \, ; \tag{25}$$

where $F_1(t)$ and $F_2(t)$ are the external forces acting on masses M_1 and M_2, respectively. The response solution discussed in this chapter can be extended to this system since the differential equations (25) are of the same type as equations (8).

The displacement response spectra of the system shown in Figure 20 are plotted for a particular configuration in Figures 21 to 23. The system parameters selected for this study are that of a typical commercial building of a large size warehouse, and the sonic boom type of excitation pressure with a period of 0.135 seconds. The effect of one uncoupled frequency f_1 was studied, keeping the other two frequencies constant for three kinds of application of excitation force of one sine pulse: 1) external force only on one mass, 2) external force on both the masses in the same direction (in phase), and 3) external force on both the masses in opposite direction to each other (phase opposition). The response displacement, x_1, was normalized with respect to displacement, x_{st_1}. x_{st_1} is defined as the static displacement of mass M_1, when a static force equal to the amplitude of excitation force F_o is applied on mass M_1 while mass M_2 is held stationary. Similarly, the displacement X_2 is normalized with respect to static displacement x_{st_2}. These normalizing quantities are given by,

$$x_{st_1} = \frac{F_o}{k_1 + k_c} \; ,$$

$$x_{st_2} = \frac{F_o}{k_2 + k_c} \; .$$

In many practical cases the excitation pressures such as sonic booms and blastings would be on all external sides of buildings which can be idealized as the force acting on both the masses in opposite directions.

Figure 21 is the displacement response of the mechanical system when only one mass is excited with transient force of a sine pulse. The displacement of mass M_2, which is not excited by the external force, is much less than that of the mass M_1. Both the modes of the system are excited and the maximax of x_1 and x_2 occur at f_1 equals 8.3 and 8.8, respectively.

Figure 22 is the displacement magnifications plotted when both the masses are excited with the force of sine pulses in the opposite direction. In this case, primarily the second mode of the system is excited. The displacements x_1 and x_2 are maximum when the frequency f_1 equals 7.4 and 8.2, respectively. Figure 23 is the displacement response when both the masses are excited with forces of sine pulses in the same direction. In this case the system is excited mainly with its first mode. The maximum displacements x_1 and x_2 occur at f_1 equals 6.9 and 8.9, respectively.

When both the masses of the system are excited by the forces in the opposite direction as in Figure 22, the potential energy would be stored in all the three springs. But when the forces are applied in
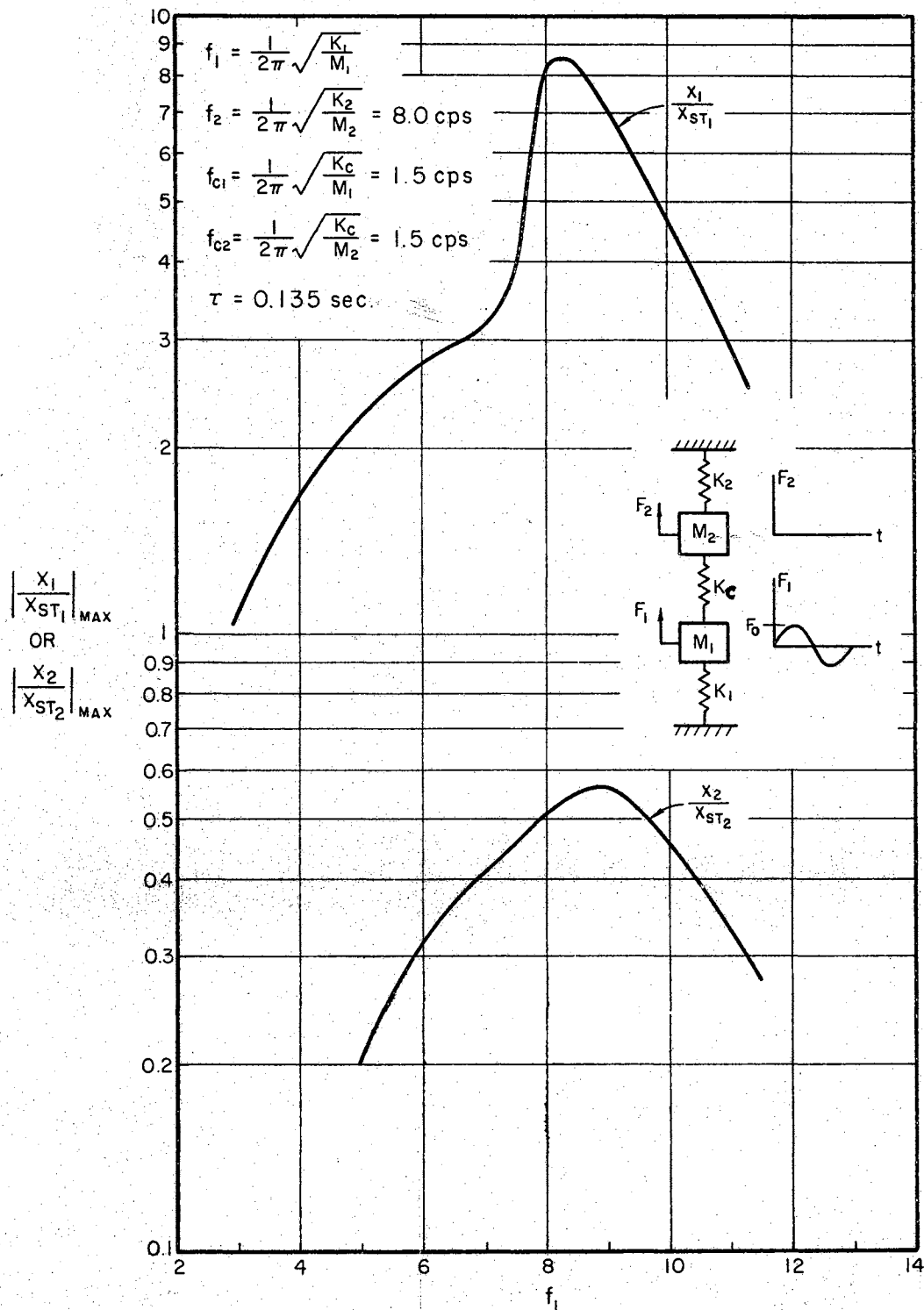
Figure 21. Displacement Response Spectra of Two
Degrees-of-Freedom System (Mass M_1 is
Excited by a Force of Sine Pulse)[1]
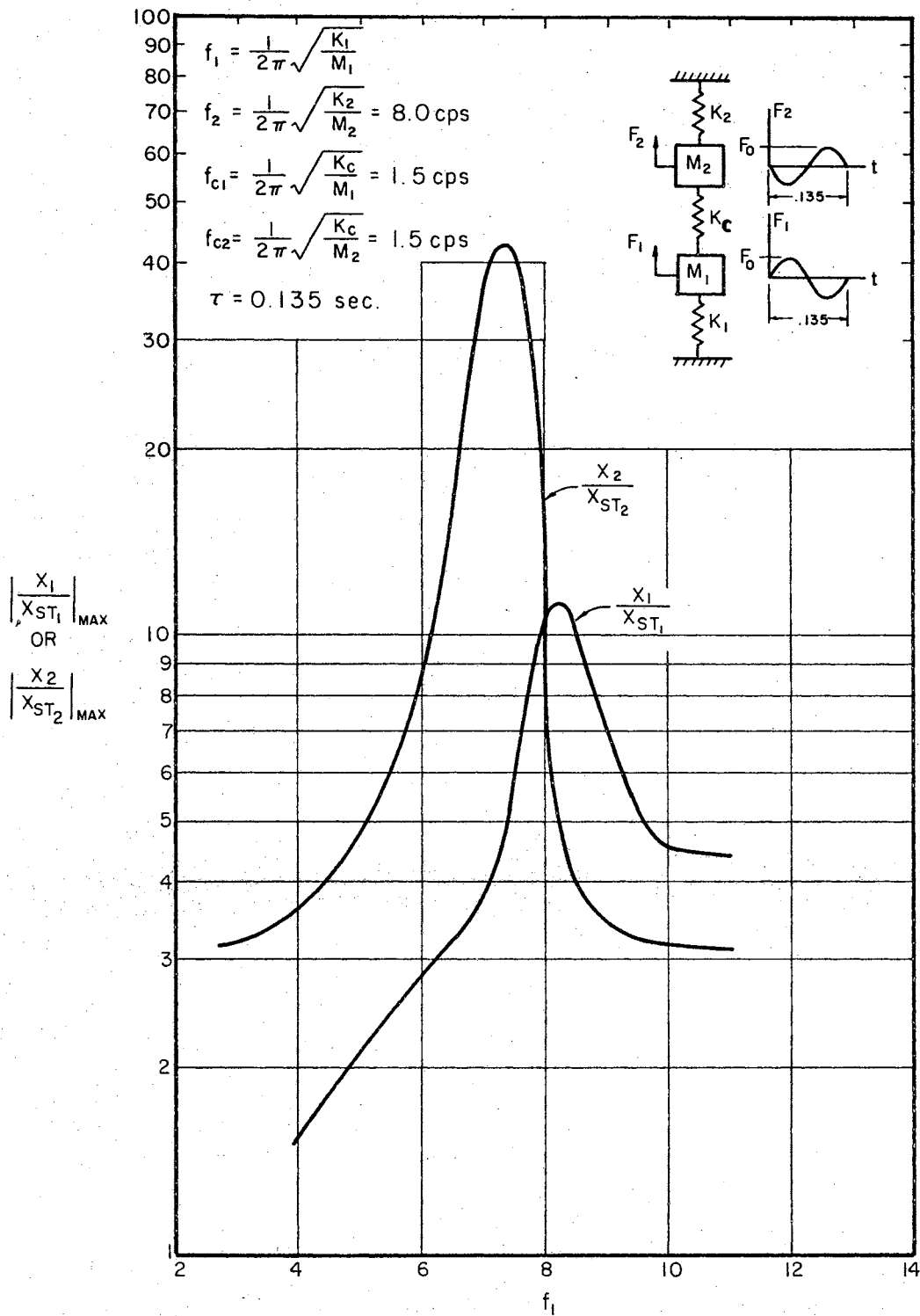
Figure 22. Displacement Response Spectra of Two
Degrees-of-Freedom System (Both the
Masses are Excited by the Forces of
Sine Pulses in the Opposite Direction)
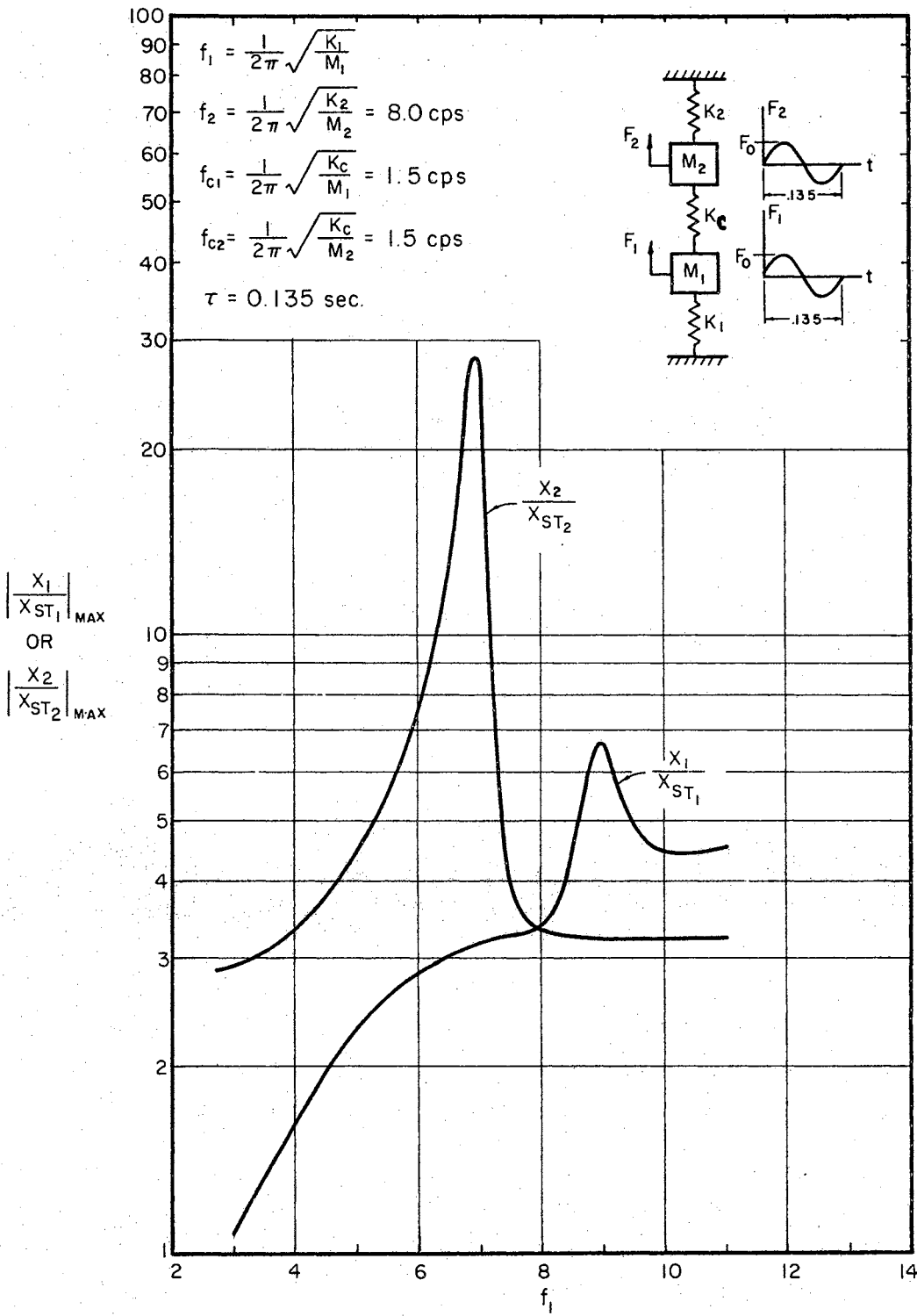
Figure 23. Displacement Response Spectra of Two
Degrees-of-freedom System (Both the
Masses are Excited by the Forces of
Sine Pulses in the Same Direction)

the same direction as in Figure 23, the potential energy would be stored

essentially in springs k_1 and k_2. Therefore, the system can store more

energy in the first case than in the second case for a given force and

in a given interval of time. The displacement magnifications are

large in the first case. In a properly tuned system, the displacements

can be large enough to damage the structures. However, the presence

of damping reduces the displacements to some extent, as discussed in

the previous section.

CHAPTER V

EXPERIMENTAL MODEL AND INSTRUMENTATION

In order to substantiate the theory established and verify the
validity of the assumptions made in the previous chapters, it was
necessary to build an experimental model of a double acoustical
resonator and devise an apparatus capable of producing appropriate
finite duration pressures whose shape, amplitude, and time duration
were within the limits which permit easy study. A small scale model
of two resonators coupled through the neck was built so that the
system parameters could be changed easily.

Description of the Model

The test resonator which was used to study the response of the
double acoustical resonator in the laboratory consisted of two cylin-
drical tubes with cylindrical necks made up of plexiglass. The two
resonators were of different sizes as shown in Figure 24. A photograph
of this assembled model is shown in Figure 25. The dimensions of the
model were dictated by the frequencies of the excitation pressures.
The diameters of the necks were mainly dictated by the coupling fre-
quency. To have a small value of coupling frequency, it was desirable
to have a smaller cross sectional area and larger length for the neck,
N_2, which coupled the two resonators. It was essential to have a large
volume for cavity V_1 and small volume for V_2 for both of the uncoupled
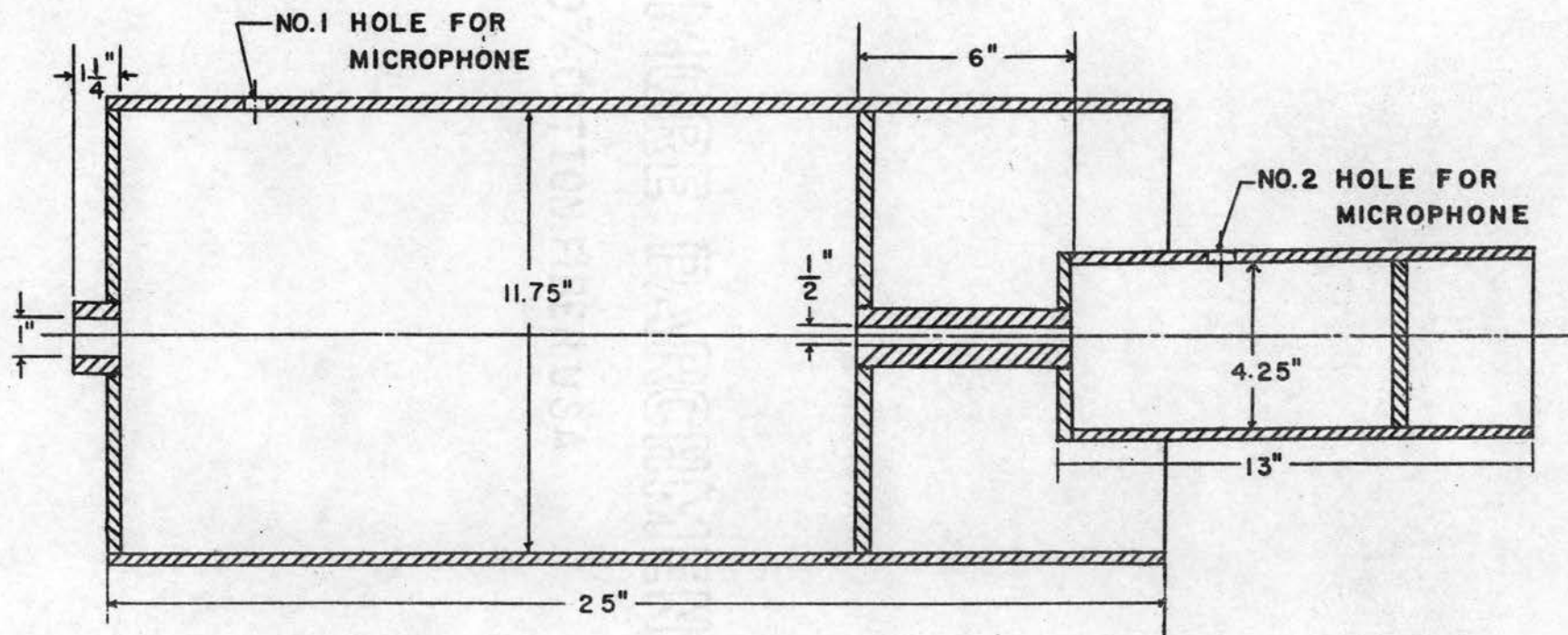
Figure 24. Configuration of Test Resonator (Double Acoustical Resonator)
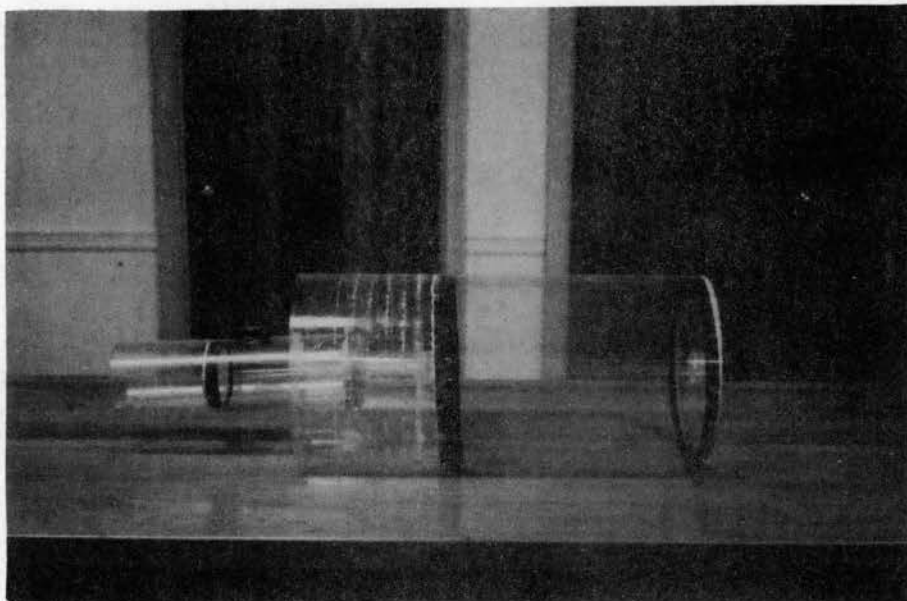
Figure 25.   Test Resonator Assembly
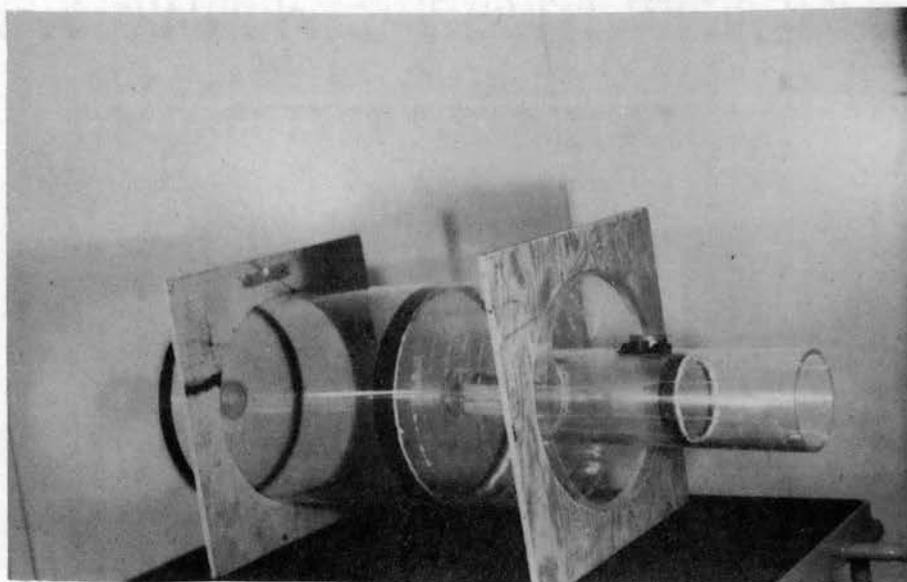
Figure 26.   Test Resonator Mounted in Wooden Frame

frequencies to be of the same order of magnitude, since the cross sectional area of neck, N_2, is small. The inside diameters of the cavities were 11.75 inches and 4.25 inches. The length of the larger resonator could be varied from zero to 24 inches and the smaller one from zero to 12 inches by varying the positions of the respective pistons. The necks were threaded to the pistons so that these could be replaced easily by the cylindrical necks of the required dimensions. By changing the volumes of the cavities and dimensions of the necks, the system parameters P_1, P_2, and P_{12} could be easily varied. Holes, measuring 5/16 of an inch, were drilled on the walls to install Altec microphones to measure the pressures inside the cavities. The cavities were sealed by using 1/16 inch felt on the circumference of the pistons. The resonators were mounted in a wooden framework as shown in Figure 26, which served two purposes, 1) as a reinforcement to make the walls rigid, and 2) as a stand for the resonator system while testing.

Simulation of Finite Duration Transient Pressures

The plane wave tube which was available in the Oklahoma State University Acoustic Laboratory was used as an acoustic delay line to produce transient pressures which were used as excitation pressures on the system. This apparatus had a cross sectional area of about 14 inches square and was 32 feet long. A photograph of this tube is shown in Figure 27. To have a plane wave at the test end of the tube, the length had to be much larger than either of the cross sectional dimensions and at the input end it needed to be driven in an evenly distributed manner. The first requirement was satisfied by the available dimensions of the tube, and the second requirement was accomplished by driving with a
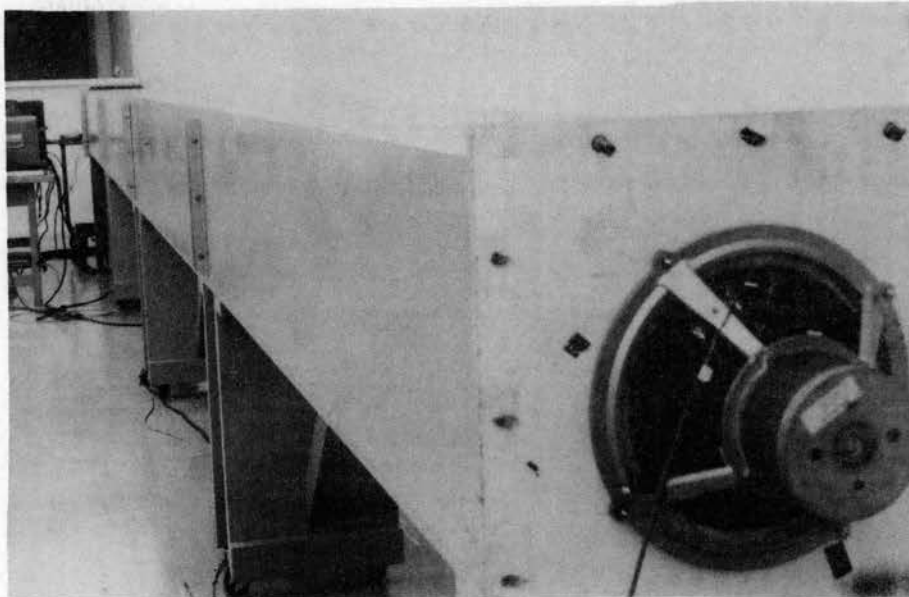
Figure 27. Plane Wave Tube
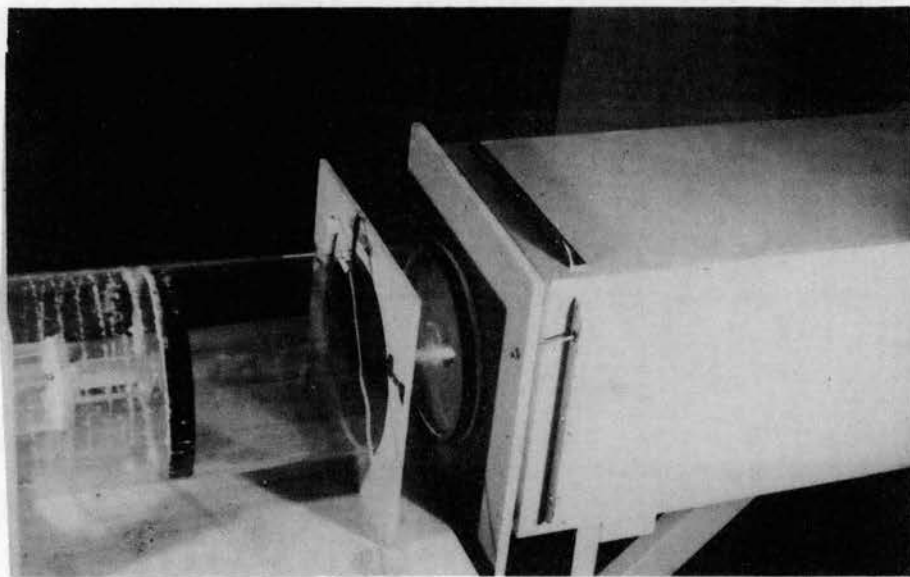
Figure 28. Resonator Arranged with
Plane Wave Tube

14 inch loud speaker, as shown in Figure 27.

A low frequency function generator which was capable of producing sine, triangular, or square wave signals from 0.02 cps to 1000 cps was used. The voltage signal from the function generator was fed to the Tone Burst generator which was a coherent gate and served to convert the signal from steady state to a transient of required duration. The signal from the gate, after being amplified, was fed to the loud speaker, as shown in the upper part of Figure 29. The pressure pulses produced by the loud speaker driven by gated signals were sent through the plane wave tube to the test end.

Testing and Instrumentation

The loud speaker produced good transients in the frequency range from 15 to 300 cycles per second. The microphone (Altec, 21 BR 150) response was flat for frequencies above 10 cycles per second. Test resonators with natural frequencies from 20 to 50 cycles per second, which were described earlier in this chapter, are compatible with these requirements. The plane wave tube was long enough so that the reflected wave took a long time to travel and to affect the response of the system.

The block diagram of the instrumentation used to measure and record the pressures is shown in Figure 29. The arrangement of the resonator with the plane wave tube is shown in Figure 28. The three factory-calibrated microphones were arranged at the end of the plane wave tube, as shown in Figure 31, to compare the sensitivities.

The excitation pressure and the response pressures in the two cavities were measured through microphones (Altec, 21 BR 150) and recorded through a C.E.C. oscillograph and also viewed on an oscillo-
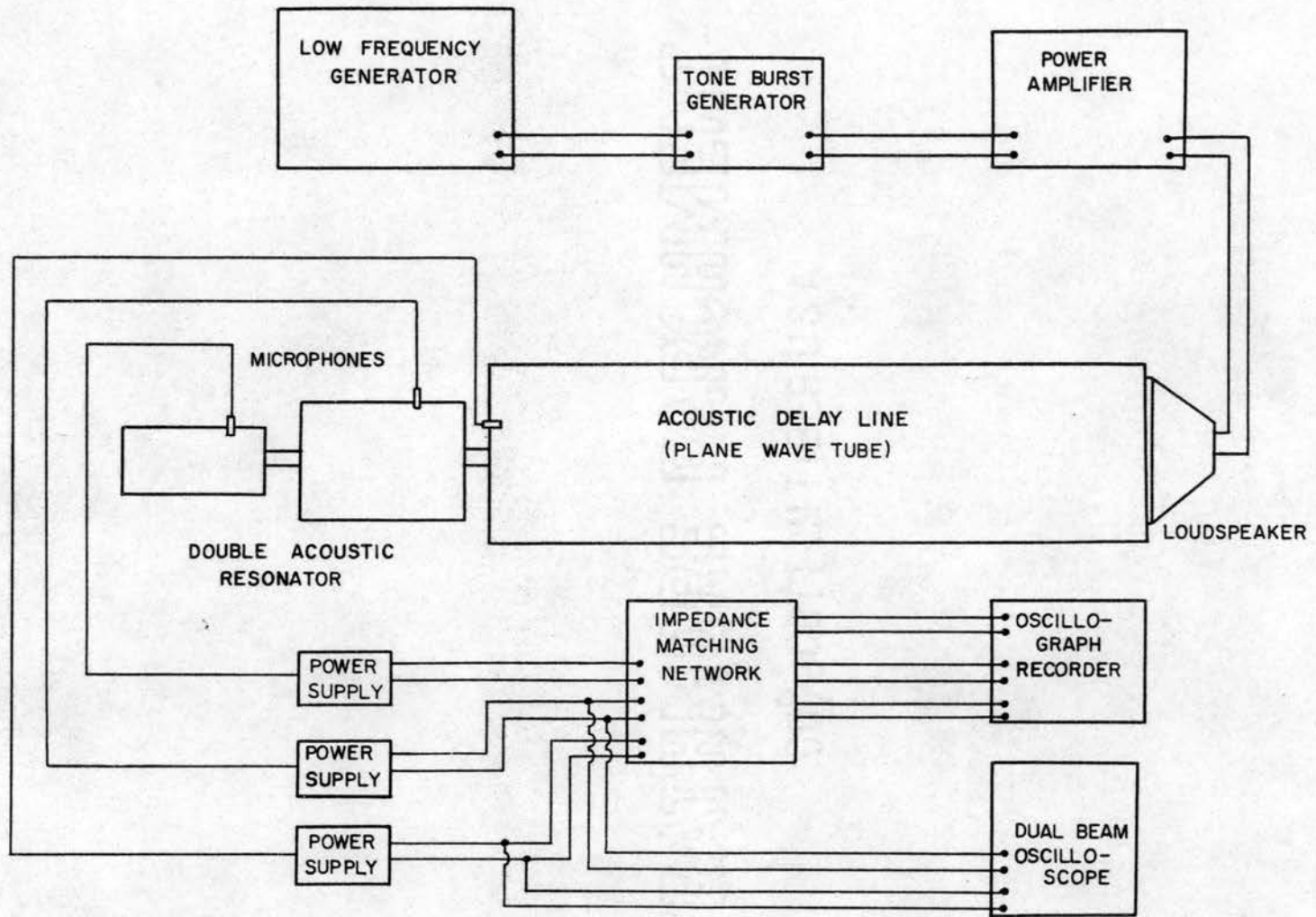
Figure 29. Block Diagram of the Instrumentation
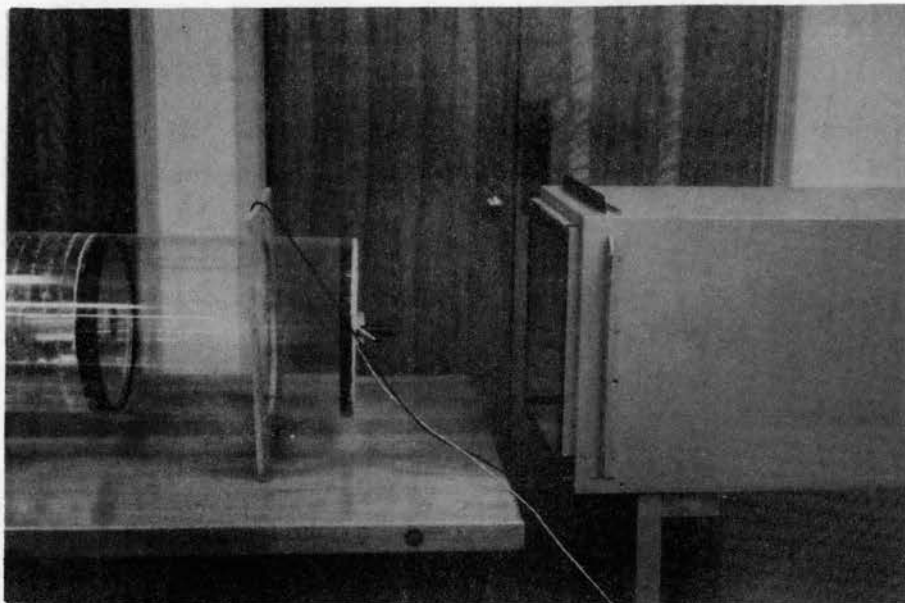
Figure 30. Arrangement of the Resonator
to Test the Effect of Loading
Due to the Impedance of the
Plane Wave Tube
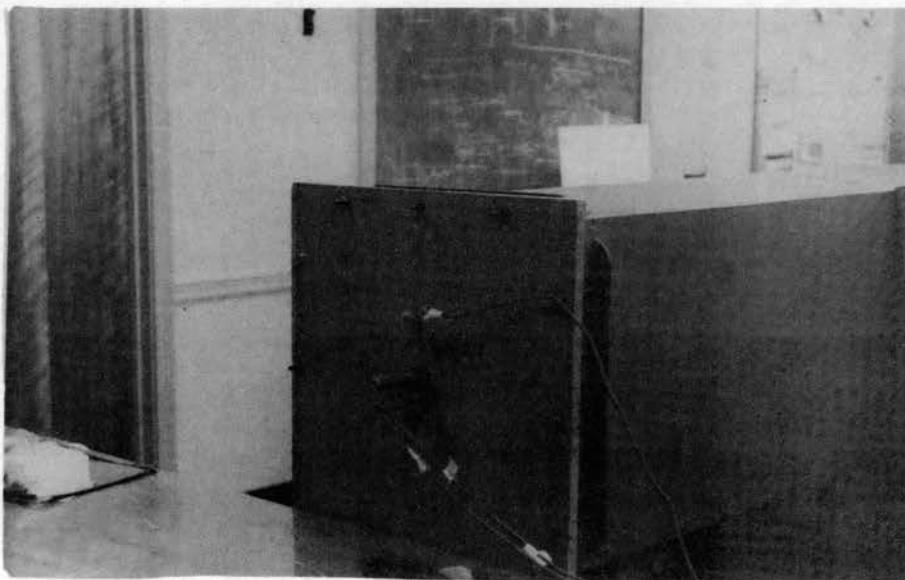
Figure 31. Arrangement of Microphones with
Plane Wave Tube for Comparative
Calibration

scope screen. Since the output impedance of the microphone power supply
was much larger than the input impedance of the oscillograph, it was
necessary to build an impedance match network. The sensitivity of the
galvanometers were adjusted through the matching network so that all
the channels had the same deflection for the same voltage signals in
the frequency range of 20 to 60 cycles per second.

The effect of impedance of the plane wave tube on the response of
the resonator was checked experimentally by measuring the response
pressures when the model was about one foot away from the test end of
the tube, as shown in Figure 30. These measurements were not much
different from the pressures measured in the cavities with the test end
closed as in Figure 28 and excited with the same type of pressure.
Since the cross-sectional area of the plane wave tube was large compared
to that of the neck, there was a little effect of the tube on the
response of the resonator. However, the impedance of the tube did
affect the excitation pressures produced by the loud speaker. Very
distinct sine pulses could be produced when the end of the tube was
closed except for the neck of the resonator.

CHAPTER VI

EXPERIMENTAL PROCEDURE AND RESULTS

The test model and the instrumentation described in Chapter V were used to test the theoretical response of the system of the double acoustical resonator in time domain. The laboratory testing also yielded a better understanding of the physical parameters and damping mechanism of the system and the general validity of the theoretical assumptions made in the analysis.

Natural Frequency and Damping Measurements

The natural frequencies of the two uncoupled resonators were measured by two methods: 1) by exciting the resonator with a short duration transient pulse and observing or recording the free relaxation pressure oscillations in the cavities; and, 2) by sweeping the steady state excitation pressure and observing the maximum pressure magnification. Both of these methods give the same values of natural frequencies. The damped natural frequency, p_d, is a function of undamped natural frequency, p, and the damping factor ξ and is given as,

$$p_d = \sqrt{1 - \xi^2}\ p\ .\tag{26}$$

Since the damping factor is of the order of 0.05, p_d and p are approximately equal.

The undamped natural frequencies were estimated from equation (23).

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The end correction for the neck was taken as 0.785 R for each end of the neck assuming an infinite baffle termination. This end correction seems to be a little high, and resulted in calculated frequencies a little low when compared with the experimentally evaluated values. For each set-up two values were taken, one while sweeping the frequency from low values to high and another while decreasing. Generally, the difference was only two to three percent. Table II gives the comparison of measured natural frequencies, when the joints of the resonator were sealed to the theoretically determined values for different volumes of the cavities.

When the joints of the resonator were not sealed well, the measured frequencies were higher than that of the theoretically estimated values. The air motion through the small holes has the effect of increasing the area of the neck. Since the natural frequency is proportional to the square root of the area of the neck, the presence of air leaks would result in higher measured frequencies.

The damping characteristics are readily evaluated from the free vibration tests. These test procedures consisted of exciting the resonator with a transient pressure pulse of short duration and observing the free pressure oscillations in the residual era. Many types of pulses were tried to excite the resonator with no control maintained over the pulse shape or period. The free pressure oscillation records always displayed the smooth decay curves of a typical simple oscillator. Figures 32 and 33 are typical traces of pressure signals obtained by exciting with a sudden impulse by waving a hand across the neck of the resonator. The damping factors measured from the logarithmic decrement of free oscillations were in the range of 0.05 to 0.065. For a smaller

TABLE II

THEORETICAL AND EXPERIMENTAL NATURAL FREQUENCIES OF RESONATORS

Cavity diameter — 11.75 inches
Neck diameter   — 1 inch
Neck length   — 1¼ inch

Length of the cavity inches	Calculated frequency cps	Experimentally determined frequency cps
21.5	27.6	27.8
20.5	28.3	28.5
19.5	29.0	30.0
18.5	29.8	30.2
17.5	30.6	30.5
16.5	31.6	32.0
15.5	32.6	33.2
14.5	33.7	34.3
13.5	34.8	35.5

Cavity diameter — 4.25 inches
Neck diameter   — 0.5 inch
Neck length   — 6 inches

Length of the cavity inches	Calculated frequency cps	Experimentally determined frequency cps
13.25	27.4	28.6
12.25	28.6	29.8
11.25	29.8	31.2
10.25	31.2	32.2
9.25	32.8	33.9
8.25	34.8	35.7
7.5	36.4	37.0
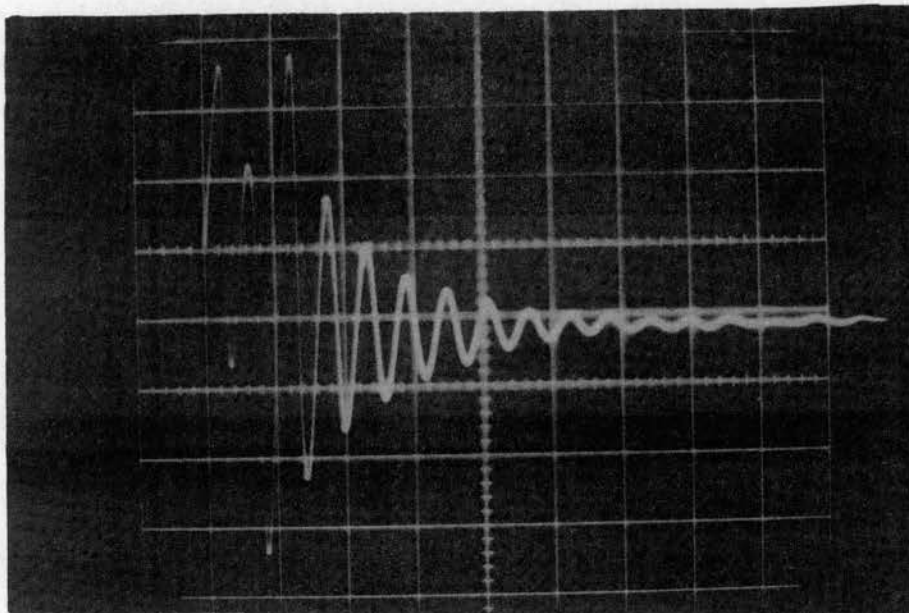
Figure 32. Free Pressure Oscillations in the Resonator
of 11.75 inches Diameter, (Neck: 1 inch
Diameter and 1.25 inches Length)
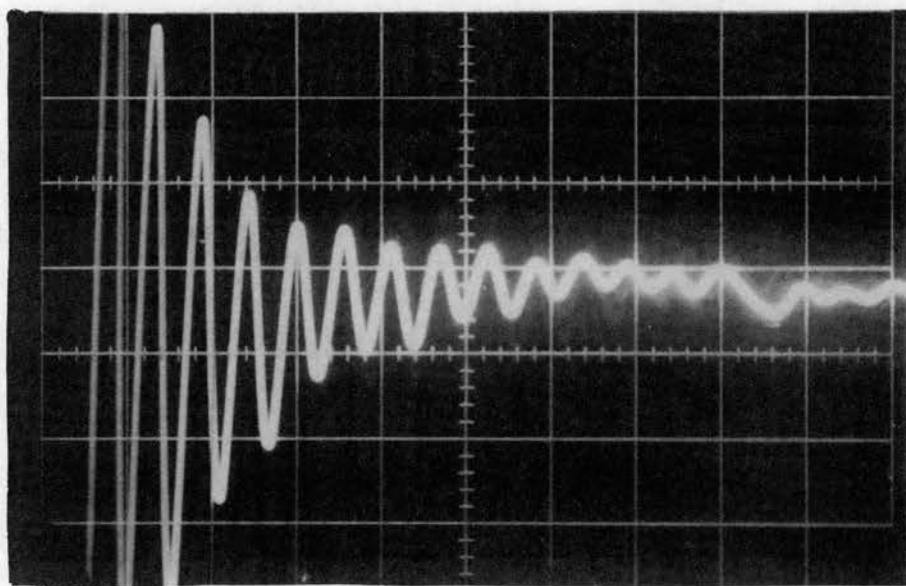
Figure 33. Free Pressure Oscillations in the Resonator
of 4.25 inches Diameter, (Neck: 0.5 inches
Diameter and 6 inches Length)

resonator with a longer neck, the damping was found to be slightly higher. Damping could also be estimated from steady state tests by measuring the amplitudes near resonance. The results were in close agreement with those obtained from the transient or free vibration tests.

These tests were conducted also to establish the mechanism of damping and the response due to higher modes. The energy dissipation could be due to the following reasons: 1) wall vibrations, 2) leakage through the felt which is used for sealing purposes, and 3) friction and reradiation through the necks. Since the 11.75 inch resonator had a wall of thickness 1/16 inch, it was suspected that wall vibrations might be the main cause of energy dissipation. These free vibration tests were conducted before and after stiffening the resonator both with circumferential and longitudinal reinforcements. The damping measurements from these tests indicated that the energy dissipation was not due to wall vibrations. To establish the damping due to leak through the felt, all the leaks were sealed well with a calking compound and tested for damping factor. It was found that the energy was not dissipated through the felt because there were no leaks even without the calking compound applied. From these tests it was inferred that the damping is only due to friction and reradiation through the necks. One could also conclude from these tests that the higher mode frequencies of the resonator were not present since the amplitude of these higher modes did not appear in the transient response oscillations, as is apparent from Figures 32 and 33.

Pressure Responses in Time Domain

The best method to establish the validity of the assumptions made

in the theoretical study is to test the model for the similated excitation pressures and compare the theoretical results with the measured values. This was done by measuring the response of the double acoustical resonator system in the time domain for the transient pressure of one sine pulse. This study was performed for several configurations of the system, all of which showed excellent agreement between the recorded traces and analytically predicted response results. These results further demonstrated the relatively minor importance of higher mode responses. The analysis was made in obtaining theoretical response curves by using an excitation pressure of one cycle of an ideal sine wave and straight line approximation. The calculated values of uncoupled natural frequencies and coupling frequency and measured values of damping factors were used to define the resonator system. The equations of motion were derived as discussed in Chapter III and solved for the response using Laplace transformation techniques and a digital computer. The pressure response in the cavities as a function of complex frequencies is given by,

$$P_1(s) = G_{A1}(s) \cdot P_A(s) + G_{B1}(s) \cdot P_B(s)$$

$$P_2(s) = G_{A2}(s) \cdot P_A(s) + G_{B2}(s) \cdot P_B(s) \tag{27}$$

where G_{A1}, G_{A2}, G_{B1} and G_{B2} are the transfer functions of the system and are defined as functions of uncoupled natural frequencies and damping factors; $P_A(s)$ and $P_B(s)$ are the external pressures applied on necks N_1 and N_2, respectively. In the present analysis there is no external pressure on neck N_2, thus, in equation (27) $P_B(s)$ is equal to zero. The correction factors for the neck length were taken as 1.57 R,

where R is the radius of the neck.

Figure 34 shows the comparison between the experimentally measured and theoretically computed pressures in cavity V_1. Also, the excitation pressure at the test end of the plane wave tube (very close to resonator neck N_1) and an ideal sine pulse are plotted in the same figure. A damping factor, $\xi_1 = \xi_2 = 0.06$, was used in the computation. There is a very close agreement between the measured and the theoretical pressures in the forced era. There is a discrepancy in the residual era, since the measured or actual external pressure was not exactly the same as that of the ideal pressure assumed in computations. Figure 35 shows the measured response and excitation pressures, computed response, and an ideal excitation pressure of sine pulse.

Figure 36 is the pressure recording inside the cavities and the excitation pressure. The trace of excitation pressure was obtained with the microphone placed close to the neck, N_1, of the resonator at the test end of the plane wave tube. The response traces were obtained with the microphones fixed through the side walls of the resonator. The amplitude of pressure in V_2 is larger than that of V_1.

These pressures were compared with the theoretically calculated pressure responses in the cavities. Figure 37 gives the comparison between the measured pressures in the cavities and the computed values by assuming the excitation pressure as a sine pulse. The agreement between theory and experiment is excellent in the forced era. In the residual era they deviate. This is because of the difference between the actual and ideal pressure pulses as shown in Figure 37. Figure 38 is the computed response to an idealized wave of straight lines. These results are compared with the measured values. The trend of the
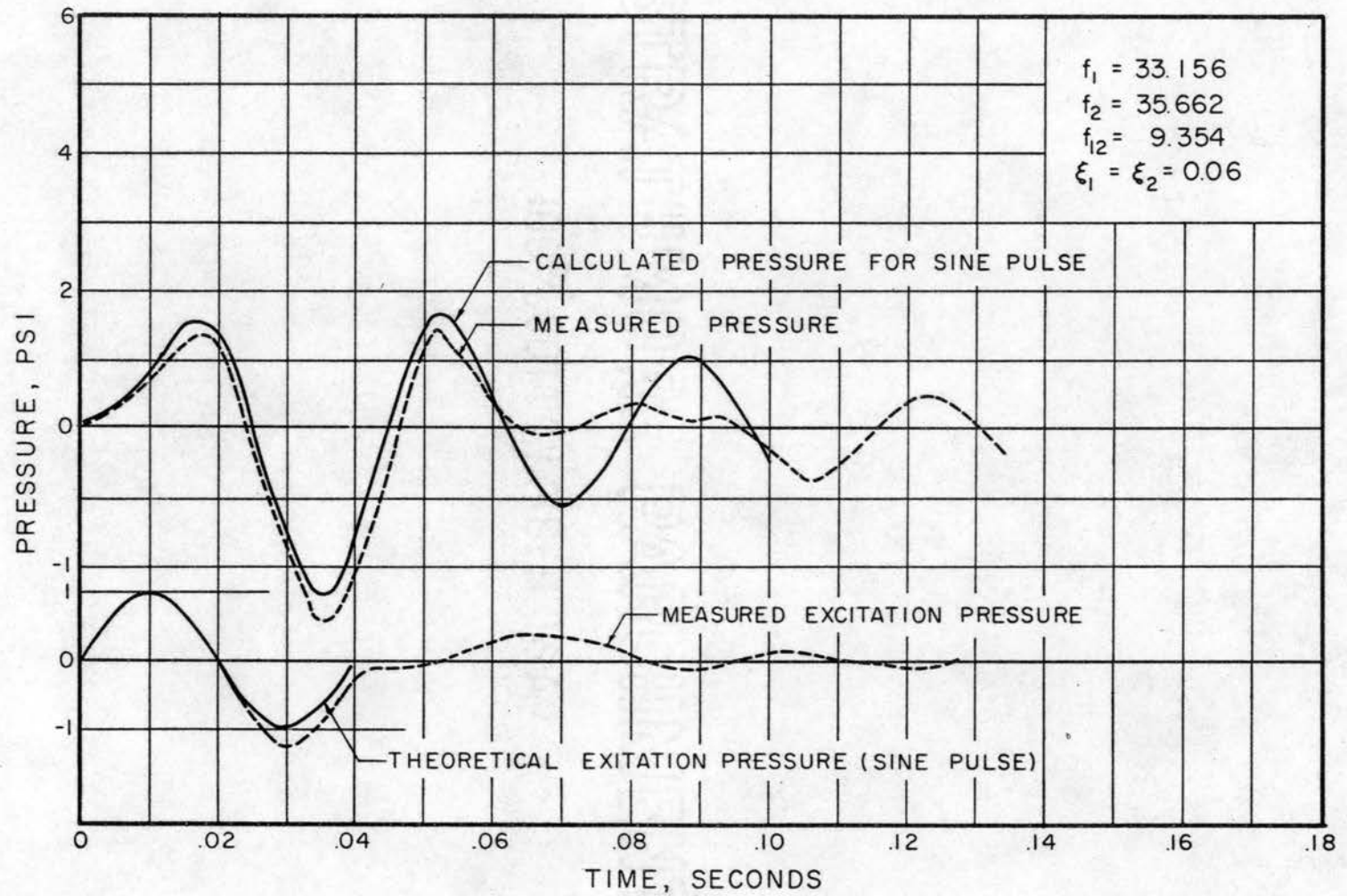
Figure 34. Pressure Response in Cavity V_1
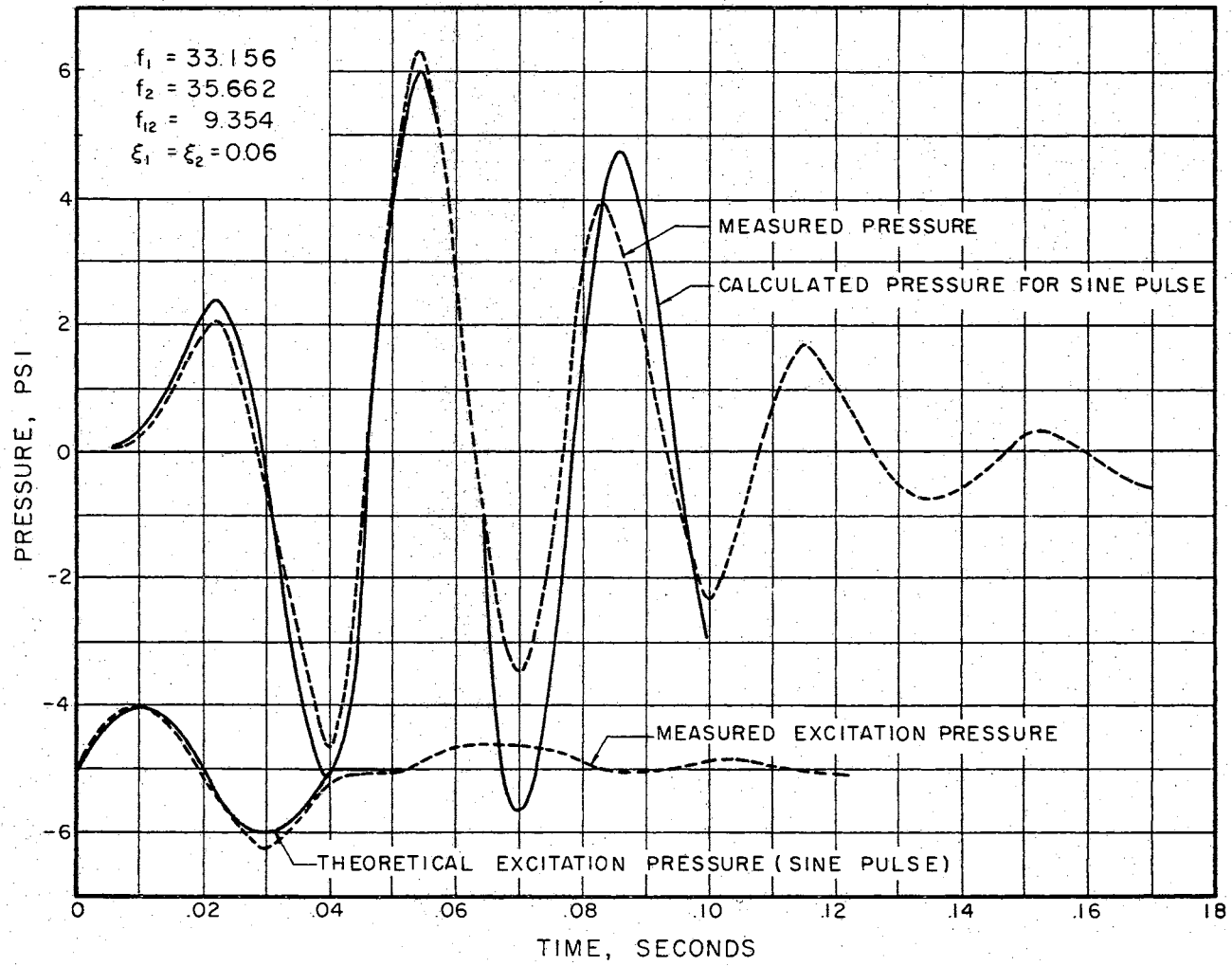
Figure 35. Pressure Response in Cavity V_2
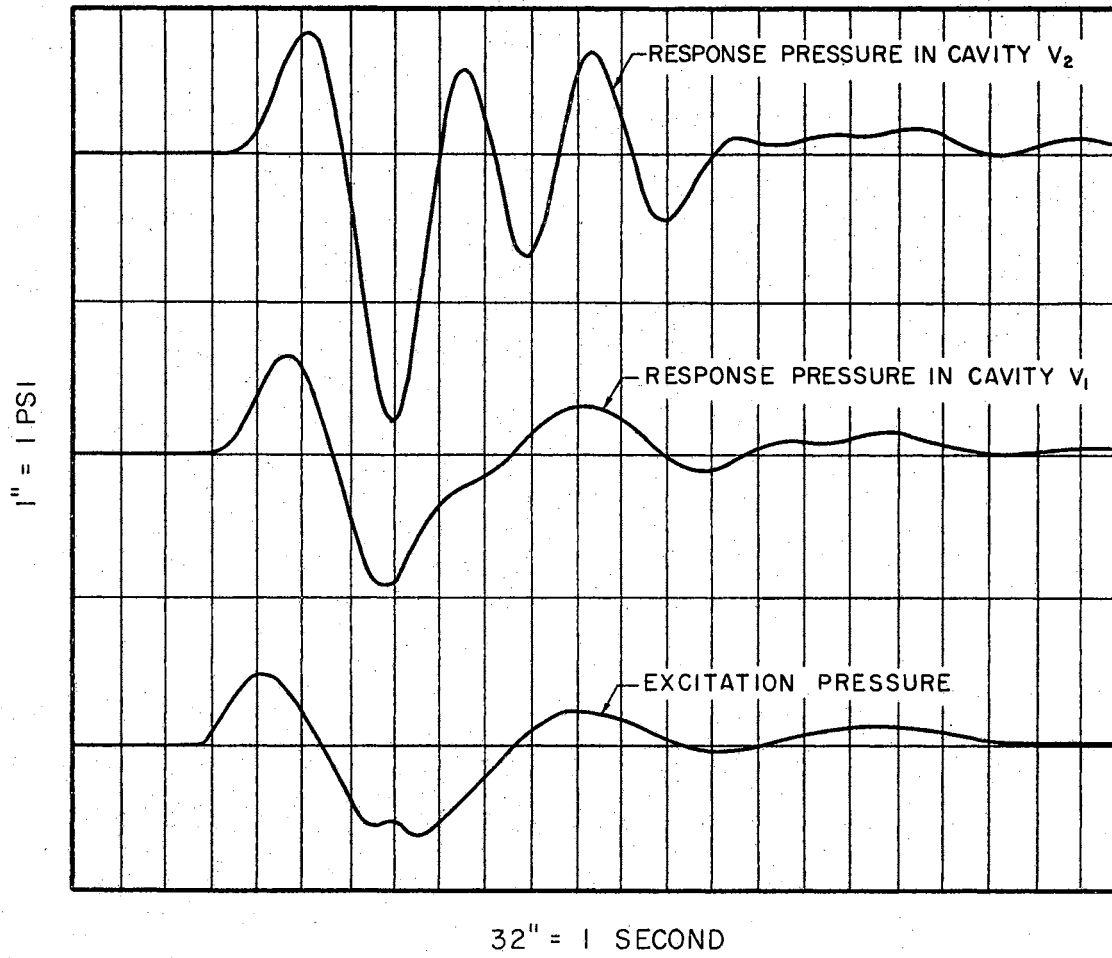
Figure 36. Time Response Measurements
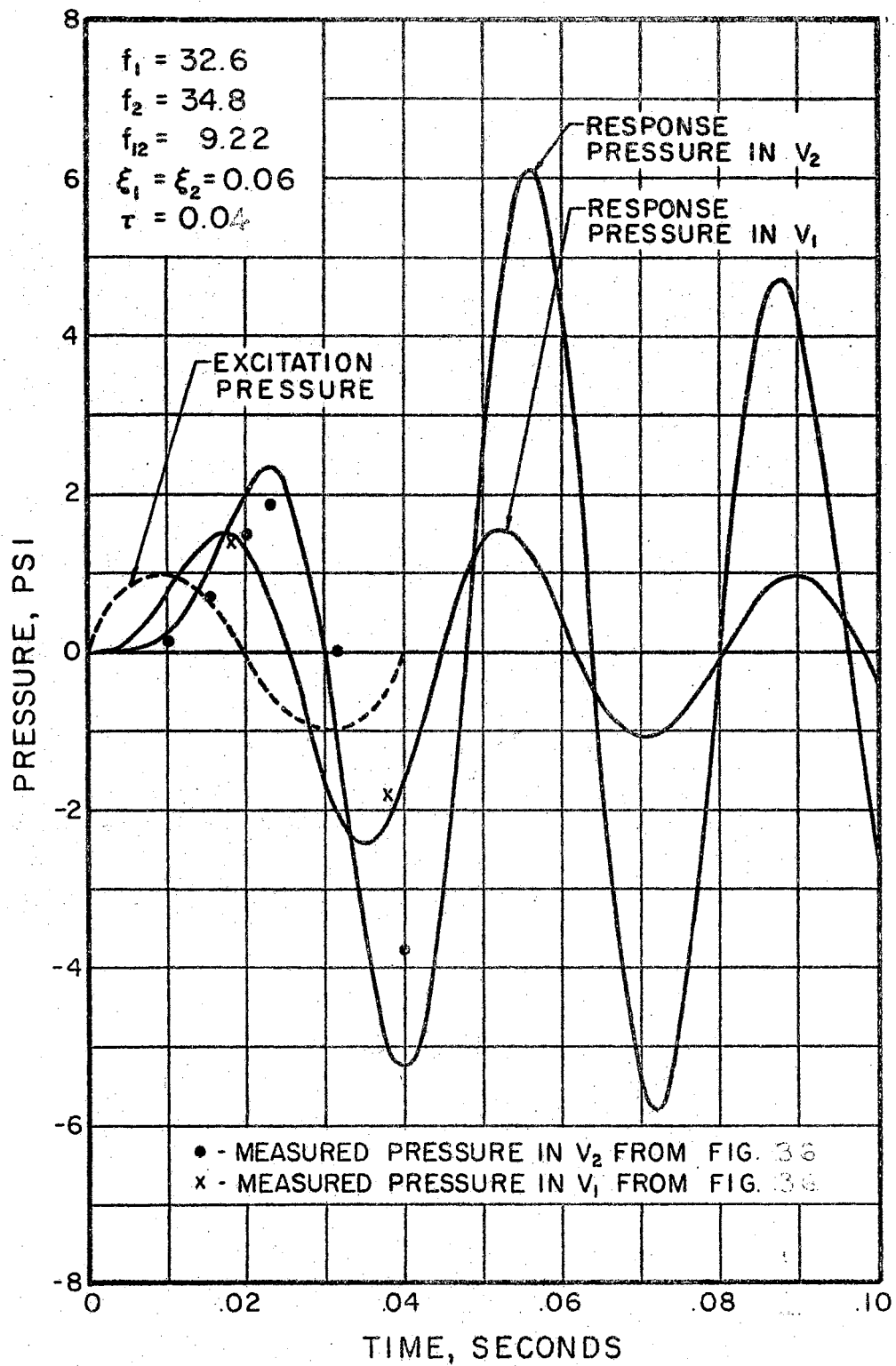
Figure 37. Response Pressures From Idealized Sine Pulse
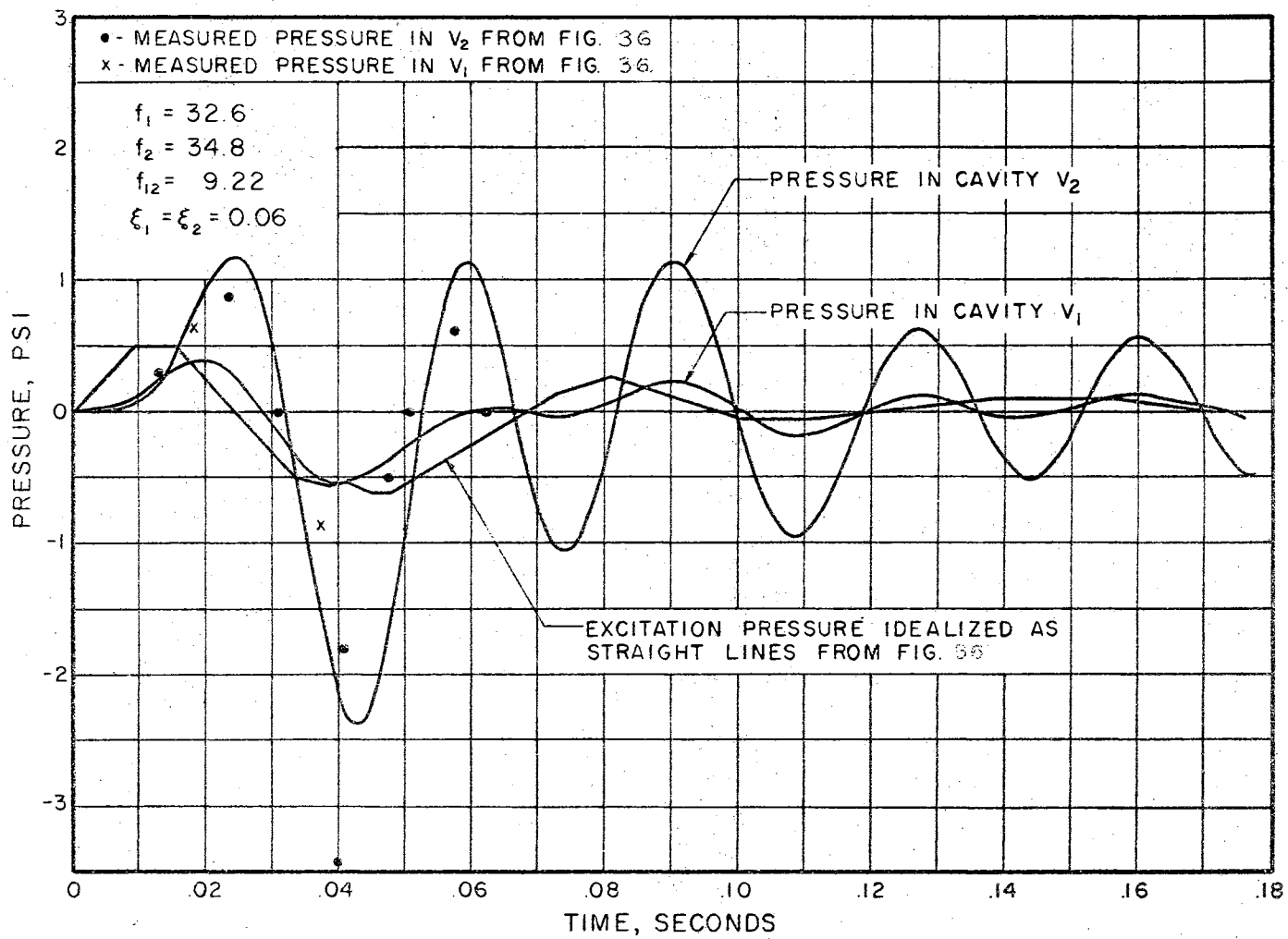
Figure 38. Response Pressures by Straight Line Approximation

responses of both calculated and measured pressures are the same, but the absolute values are not in agreement. This may be due to the shape of the measured excitation pressure being quite different from the idealized excitation pressure.

The study of this particular time response indicates that the assumptions made in the mathematical model are valid. The higher modes are not important in the response analysis. Thus, the results obtained in the theoretical analysis may be accepted.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The conclusions drawn as a consequence of this investigation may be divided into two groups depending on the phase of the investigation.

I.   The following deductions have been attained from the experimental investigation:

1.   The lumped parameter representation of coupled acoustical resonators is adequate for the response analysis.

2.   The presence of air leaks will have a large effect on the natural frequencies and on the response of the system.

3.   In actual physical systems such as buildings, it is rather difficult to estimate the amount of leakage and damping; and thus, it is necessary to use the measured estimates of natural frequencies and damping factors in the response analysis.

4.   The assumptions made in the derivation of the mathematical model are valid.

5.   The damping of a resonator with rigid walls is due probably to the friction and re-radiation at the necks.

6.   In a physical system the higher mode response is not important, since the small amount of damping present in the system reduces the amplitude of higher modes in a short time.

II. From the theoretical analysis and response spectra, the following conclusions have been attained:

1. The pressure magnification of a double acoustical resonator can be as large as 50 when the undamped system is tuned properly and the coupling frequency is very small. However, since in a physical system the damping is inevitable and the coupling frequency is limited by the elements of the system, the pressures can only be magnified to the order of about 15.

2. The pressure oscillations associated with acoustic response in the structure may possibly act as a secondary source driving other properly tuned elements of the system to large amplitudes.

3. Mathematical modeling by use of lumped parameters is an excellent method, which is amenable for the transient response analysis.

4. If the walls are rigid such that there is no contribution of response due to wall vibrations, the response can be predicted analytically within four percent, by using measured values for damping.

5. From this analysis, it is possible to predict the response of double acoustical resonators to any type of finite duration transient excitation pressures which can either be approximated by straight lines or sine pulses.

6. The damage to the acoustic structures exposed to transient pressures can be reduced by incorporating damping. A damping factor of about 0.1 reduces the maximax pressures by about three times.

7.   The two uncoupled frequencies of the system are parameters which affect the response of the cavity of the resonator which is excited by external pressure.

8.   The uncoupled natural frequency related to the resonator excited with external pressure is the parameter which mainly affects the response of this cavity.

9.   Pressure magnifications can be reduced to about unity by designing the structures in such a way that the natural frequencies are apart from each other and also apart from the frequency of excitation pressure.

Suggestions for Future Study

It is recommended that further study be conducted in the area of dynamic response of mechano-acoustic structures in transient conditions. It appears that the transient response of the acoustical systems by lumped parameter representation is more fruitful; however, in certain cases where lumped parameters may not fulfill the requirements, it would be advantageous to explore the continuous systems.

Specifically the following recommendations are made for further study in this area:

1.   The conclusions stated concerning the mechanism of damping in the resonator are primarily based on experimental tests. It would be useful if additional effort would be put on mathematical analysis. It appears that the damping due to various mechanisms can be separated and analyzed as a function of system parameters.

2. The influence of non-rigid wall vibrations on the response of the system should be investigated. These wall vibrations might cause dissipation of energy in the system.

3. It is necessary to determine the influence of higher mode response for larger resonators where the critical dimension is not less than 16 times the wave length corresponding to the natural frequency. From the experimental study of this investigation, it seems that this restriction is not strictly necessary.

4. There are many acoustical structures which may not be idealized to the system of the double acoustical resonator. Hence, it is worthwhile to analyze the multiple coupled resonators with three or more degrees of freedom.

5. The study of the response of continuous mechanical systems such as windows and ceilings coupled to an acoustical resonator needs to be investigated.

6. The correction factors for neck lengths of Helmholtz resonator, as a function of dimensions of the necks and of the flanges at the ends of the neck needs to be examined.

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APPENDIX A

RESPONSE SOLUTION OF THE DOUBLE ACOUSTICAL RESONATOR

The equations of motion for the system of double acoustical reso-
nator shown in Figure 2 are derived in complex frequency domain in
Chapter III as,

$$\left[\frac{\rho_o \ell_{e1}}{A_1} s^2 + c_1 s + \frac{\rho_o c^2}{V_1}\right] X_1(s) - \left[c_1 s + \frac{\rho_o c^2}{V_1}\right] X_2(s) = P_1(s) ,$$

$$\left[\frac{\rho_o \ell_{e2}}{A_2} s^2 + (c_1 + c_2) s + \frac{\rho_o c^2}{V_1} + \frac{\rho_o c^2}{V_2}\right] X_2(s) - \left[c_1 s + \frac{\rho_o c^2}{V_1}\right] \cdot$$

$$X_1(s) = P_2(s). \tag{1}$$

The characteristic determinant of this equation is

$$\Delta = \begin{vmatrix} \dfrac{\rho_o \ell_{ei}}{A_1} s^2 + c_1 s + \dfrac{\rho_o c^2}{V_1} & -\left[c_1 s + \dfrac{\rho_o c^2}{V_1}\right] \\[4mm] -\left[c_1 s + \dfrac{\rho_o c^2}{V_1}\right] & \dfrac{\rho_o \ell_{e2}}{A_2} s^2 + (c_1 + c_2)s - \rho_o c^2 \dfrac{1}{V_1} + \dfrac{1}{V_2} \end{vmatrix} . \tag{2}$$

This equation can be simplified to

$$\Delta = \frac{\rho_o^2 \ell_{ei} \ell_{e2}}{A_1 A_2} s^4 + 2(p_2 \xi_2 + p_1 \xi_1 + p_2 \xi_{12}) s^3 + (p_1^2 + p_2^2$$

$$+ p_{12}^2 + 4p_1p_2\xi_1\xi_2) \ s^2 + 2p_1p_2(\xi_1p_2 + \xi_2p_1)s + p_1^2p_2^2 \ ;(3)$$

where the damping factors ξ_1, ξ_2 and ξ_{12} are defined as,

$$\xi_1 = \frac{c_1}{2M_1p_1} \ ,$$

$$\xi_2 = \frac{c_2}{2M_2p_2} \ , \ \text{and}$$

$$\xi_{12} = \frac{c_1}{2M_2p_2} = \xi_1 \frac{p_{12}^2}{p_1^2p_2^2} \ .$$

Solution of equation (1) for volume displacements, $X_1(s)$ and $X_2(s)$, are,

$$X_1(s) = \frac{P_1(s) \frac{\rho_o\ell e2}{A_2} \left[s^2 + 2p_2(\xi_2 + \xi_{12})s + p_2^2 + p_{12}^2 \right]}{\Delta}$$

$$\frac{+ \ P_2(s) \frac{\rho_o\ell e1}{A_1} \left[2\xi_1p_1s + p_1^2 \right]}{\Delta} \ , \ \text{and}$$

$$X_2(s) = \frac{P_2(s) \frac{\rho_o\ell e1}{A_1} \left[s^2 + 2\xi_1p_1 + p_1^2 \right]}{\Delta}$$

$$\frac{+ \ P_1(s) \frac{\rho_o\ell e1}{A_1} p_1^2 \left[\frac{2\xi_1}{p_1} s + 1 \right]}{\Delta} \ . \tag{4}$$

For the excitation pressures of one cycle of sine wave (Figure 6) on neck, N_1, and no excitation on neck, N_2, the equation (4) can be reduced to,

$$X_1(s) = \frac{A_o\omega\left[s^2 + g_1 s + g_o\right]\left[1 - e^{-\tau s}\right]}{\frac{\rho_o \ell e1}{A_1}\,(s^2 + \omega^2)\left[s^4 + B_3 s^3 + B_2 s^2 + B_1 s + B_o\right]}, \text{ and}$$

$$(5)$$

$$X_2(s) = \frac{A_o\omega\, p_1^2\left[h_1 s + 1\right]\left[1 - e^{-\tau s}\right]}{\frac{\rho_o \ell e2}{A_2}\,(s^2 + \omega^2)\left[s^4 + B_3 s^3 + B_2 s^2 + B_1 s + B_o\right]};$$

where

$$g_1 = 2p_2\,(\xi_2 + \xi_{12}),$$
$$g_o = p_2^2 + p_{12}^2,$$
$$B_3 = 2\,p_1\xi_1 + p_2\xi_2 + p_2\xi_{12},$$
$$B_2 = p_1^2 + p_2^2 + p_{12}^2 + 4p_1 p_2\xi_1\xi_2,$$
$$B_1 = 2p_1 p_2\,[\xi_1 P_2 + \xi_2 P_1],$$
$$B_o = p_1^2 p_2^2, \text{ and}$$
$$h_1 = \frac{2\xi_1}{p_1}.$$

$$(5a)$$

Equation (5) may be transformed into time domain from the complex frequency domain by the technique of partial fractions. It is necessary to find the roots of the characteristic equation (3). The characteristic equation,

$$s^4 + B_3 s^3 + B_2 s^2 + B_1 s + B_o,$$

$$(7)$$

is a fourth degree, linear polynomial equation. For a stable system, the real parts of the roots of this equation should be negative. With this assumption, the equation can have one of the following three combinations of the roots:

1. All four of the roots may be complex with negative real part,

2.   All of the roots may be real negative numbers, and

3.   Two roots may be complex and two may be real negative numbers.
If the system has to have at least some roots with pure real numbers,
the response of the system is not oscillatory, but it is overdamped.
Assuming that the system has an oscillatory motion for which the roots
of the characteristic equation must be of the form a $\pm$ jb where a and
b are real numbers and j $= \sqrt{-1}$.

If the roots of the polynomial equation are a $\pm$ jb and c $\pm$ jd,
the following identity can be written:

$$
\begin{aligned}
s^4 + B_3 s^3 + B_2 s^2 + B_1 s + B_0 &= [s - (a + jb)][s - (a - jb)] \\
&\quad [s - (c + jd)][s - (c - jd)] \quad\quad (8) \\
&= [(s-a)^2 + b^2][(s-c)^2 + d^2] \ .
\end{aligned}
$$

Equation (5) can be reduced to the following form by utilizing the
identity (8),

$$
X_1(s) = \frac{A_0 \, \omega}{\dfrac{\rho_0 \ell_{e1}}{A_1}} \; \frac{[s^2 + g_1 s + g_0][1 - e^{-\tau s}]}{(s^2 + \omega^2)[(s - a)^2 + b^2][(s - c)^2 + d^2]} \ , \text{ and}
$$

$$
(9)
$$

$$
X_2(s) = \frac{A_0 \, \omega}{\dfrac{\rho_0 \ell_{e2}}{A_2}} \, p_1^2 \; \frac{(h_1 s + 1)(1 - e^{-\tau s})}{(s^2 + \omega^2)[(s - a)^2 + b^2][(s - c)^2 + d^2]} \ .
$$

The following expressions may be obtained by factoring the right
hand side of equation (9) in terms of partial fractions, so that
each term can be reduced to known form:

$$
\frac{s^2 + g_1 s + g_0}{(s^2 + \omega^2)[(s - a)^2 + b^2][(s - c)^2 + d^2]} = \frac{a_1 s + b_1}{s^2 + \omega^2}
$$

$$+ \frac{c_1 s + d_1}{(s - a)^2 + b^2} + \frac{e_1 s + f_1}{(s - c)^2 + d^2} \text{ , and}$$

$$\frac{h_1 s + 1}{(s^2 + \omega^2)[(s - a)^2 + b^2][(s - c)^2 + d^2]} = \frac{a_2 s + b_2}{s^2 + \omega^2}$$

$$+ \frac{c_2 s + d_2}{(s - a)^2 + b^2} + \frac{e_2 s + f_2}{(s - c)^2 + d^2} \text{ .} \tag{10}$$

When the right side of equation (10) is reduced to a common denominator and the numerators of both sides of the equations are equated, the following will result:

$$s^2 + g_1 s + g_0 = (a_1 s + b_1)[(s - a)^2 + b^2][(s - c)^2 + d^2]$$

$$+ (c_1 s + d_1)[s^2 + \omega^2][(s - c)^2 + d^2]$$

$$+ (e_1 s + f_1)[s^2 + \omega^2][(s - a)^2 + b^2] \text{ , and} \tag{11}$$

$$h_1 s + h_0 = (a_2 s + b_2)[(s - a)^2 + b^2][(s - c)^2 + d^2]$$

$$+ (e_2 s + d_2)(s^2 + \omega^2)[(s - c)^2 + d^2]$$

$$+ (e_2 s + f_2)(s^2 + \omega^2)[(s - a)^2 + b^2] \text{ .}$$

The equations (11) can be simplified and the like powers of s on both sides are equated to obtain the two sets of simultaneous equations (12) and (13) written in matrix form.

Equations (5) can be rewritten in the following form, by utilizing equations (9) and (11) and the solution of equations

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
2(a+c) & -1 & 2c & -1 & 2a & -1 \\
\begin{matrix}(a+c)^2+b^2\\+d^2+2ac\end{matrix} & -2(a+c) & c^2+d^2+\omega^2 & -2c & a^2+b^2+\omega^2 & -2a \\
\begin{matrix}2[a(c^2+d^2)\\+c(a^2+b^2)]\end{matrix} & \begin{matrix}-[(a+c)^2\\+b^2+d^2\\+2ac]\end{matrix} & 2\omega^2 c & \begin{matrix}-(c^2+d^2\\+\omega^2)\end{matrix} & 2\omega^2 a & \begin{matrix}-(a^2+b^2\\+\omega^2)\end{matrix} \\
\begin{matrix}(a^2+b^2)\cdot\\(c^2+d^2)\end{matrix} & \begin{matrix}-2[a(c^2+d^2)\\+c(a^2+b^2)]\end{matrix} & \omega^2(c^2+d^2) & -2c\omega^2 & \omega^2(a^2+b^2) & -2a\omega^2 \\
0 & \begin{matrix}(a^2+b^2)\cdot\\(c^2+d^2)\end{matrix} & 0 & \omega^2(c^2+d^2) & 0 & \omega^2(a^2+b^2)
\end{bmatrix}
\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \\ f_1 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ g_1 \\ g_0 \end{bmatrix}
\qquad (12)
$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\[4pt] 2(a+c) & -1 & 2c & -1 & 2a & -1 \\[4pt] \begin{array}{l}(a+c)^2+b^2\\+d^2+2ac\end{array} & -2(a+c) & c^2+d^2+\omega^2 & -2c & a^2+b^2+\omega^2 & -2a \\[12pt] \begin{array}{l}2[a(c^2+d^2)\\+c(a^2+b^2)]\end{array} & \begin{array}{l}-[(a+c)^2\\+b^2+d^2\\+2ac]\end{array} & 2\omega^2c & \begin{array}{l}-(c^2+d^2\\+\omega^2)\end{array} & 2\omega^2a & \begin{array}{l}-(a^2+b^2\\+\omega^2)\end{array} \\[18pt] \begin{array}{l}(a^2+b^2)\cdot\\(c^2+d^2)\end{array} & \begin{array}{l}-2[a(c^2+d^2)\\+c(a^2+b^2)]\end{array} & \omega^2(c^2+d^2) & -2c\omega^2 & \omega^2(a^2+b^2) & -2a\omega^2 \\[14pt] 0 & \begin{array}{l}(a^2+b^2)\cdot\\(c^2+d^2)\end{array} & 0 & \omega^2(c^2+d^2) & 0 & \omega^2(a^2+b^2) \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \\ e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h_1 \\ 1 \end{bmatrix}$$

(13)

(12) and (13):

$$X_1(s) = \frac{A_o \omega}{\frac{\rho_o \ell_{e1}}{A_1}} \left[\frac{a_1 s + b_1}{s^2 + \omega^2} + \frac{c_1 s + d_1}{(s - a)^2 + b^2} + \frac{e_1 s + f_1}{(s - c)^2 + d^2} \right] \cdot$$

$$(1 - e^{-Ts}) \text{ , and} \tag{14}$$

$$X_2(s) = \frac{A_o \omega}{\frac{\rho_o \ell_{e2}}{A_2}} \left[\frac{a_2 s + b_2}{s^2 + \omega^2} + \frac{c_2 s + d_2}{(s - a)^2 + b^2} + \frac{e_2 s + f_2}{(s - c)^2 + d^2} \right] \cdot$$

$$(1 - e^{-Ts}) \text{ .}$$

Inverse Laplace transformation of functions of the type,

$$\frac{c_1 s + d_1}{(s - a)^2 + b^2}$$

is discussed below.

A function $\Phi(s)$ is defined as,

$$\Phi(s) = c_1 s + d_1,$$

then

$$\Phi(a + jb) = c_1(a + jb) + d.$$

This identity may be reduced to the following form:

$$\Phi_1 + j\Phi_2 = c_1 a + d_1 + jc_1 b.$$

The real and imaginary parts of both sides are equated to obtain the

following equations:

$$\Phi_1 = c_1 a + d_1 \text{ , and}$$

$$\Phi_2 = c_1 b \text{ .}$$

Solution of these two equations yield the following expressions,

$$c_1 = \frac{\Phi_2}{b} \text{ , and}$$

$$d_1 = \Phi_1 - \frac{a}{b} \Phi_2.$$

Hence

$$\frac{c_1 s + d_1}{(s-a)^2 + b^2} = \frac{\frac{\Phi_2}{b} s + (\Phi_1 - \frac{a}{b}\Phi_2)}{(s-a)^2 + b^2} = \frac{\Phi_2}{b} \frac{(s-a)}{(s-a)^2 + b^2}$$

$$+ \frac{\Phi_1}{b} \frac{b}{(s-a)^2 + b^2} .$$

The inverse transformation is as follows:

$$L^{-1} \left[\frac{c_1 s + d_1}{(s-a)^2 + b^2} \right] = e^{at} \left[\frac{1}{b} \sqrt{\Phi_1^2 + \Phi_2^2} \; Sin(bt + \theta) \right] ,$$

where

$$\tan \; \theta = \frac{\Phi_2}{\Phi_1} \; .$$

Equations (14) may be transformed to time domain and written as,

$$X_1(t) = \frac{A_o \omega}{\frac{\rho_o \ell_{e1}}{A_1}} \Biggl\{ [a_1 \; Cos \; \omega t + \frac{b_1}{\omega} \; Sin \; \omega t + \frac{e^{at}}{b} \sqrt{\Phi_1^2 + \Phi_2^2} \; . $$

$$Sin \; (bt - \theta_1)) + \frac{e^{ct}}{d} (\sqrt{\Psi_1^2 + \Psi_2^2} \; Sin \; (dt - \theta_2))]u(t)$$

$$- [a_1 \; Cos \; \omega(t - \tau) + \frac{b_1}{\omega} \; Sin \; \omega(t - \tau) + \frac{e^{a(t-\tau)}}{b} \; .$$

$$(\sqrt{\Phi_1^2 + \Phi_2^2} \; Sin \; (b(t - \tau) - \theta_1)) + \frac{e^{c(t-\tau)}}{d} (\sqrt{\Psi_1^2 + \Psi_2^2} \; .$$

$$\text{Sin } (d(t - \tau) - \theta_2))]u(t - \tau)\Bigg\} \quad , \text{ and} \qquad (15)$$

$$X_2(t) = \frac{\dfrac{A_o \omega}{\dfrac{\rho_o \ell e2}{A_2}}}{\Bigg\{[a_2 \text{ Cos } \omega t + \frac{b_2}{\omega} \text{ Sin } \omega t + \frac{e^{at}}{b} (\sqrt{\Phi_3{}^2 + \Phi_4{}^2} \cdot}$$

$$\text{Sin } (bt - \theta_3)) + \frac{e^{ct}}{d} (\sqrt{\Psi_3{}^2 + \Psi_4{}^2} \text{ Sin } (dt - \theta_4))] \, u(t)$$

$$- [a_2 \text{ Cos } \omega(t - \tau) + \frac{b_2}{\omega} \text{ Sin } \omega(t - \tau) + \frac{e^{a(t - \tau)}}{b} \cdot$$

$$(\sqrt{\Phi_3{}^2 + \Phi_4{}^2} \text{ Sin } (b(t - \tau) - \theta_3))] + \frac{e^{ct}}{d} (\sqrt{\Psi_3{}^2 + \Psi_4{}^2} \cdot$$

$$\text{Sin } (d(t - \tau) - \Phi_4))] \, u(t - \tau)\Bigg\} \quad ;$$

where

$$\theta_1 = \tan^{-1} \frac{\Phi_2}{\Phi_1} \, ,$$

$$\theta_2 = \tan^{-1} \frac{\Psi_2}{\Psi_1} \, ,$$

$$\theta_3 = \tan^{-1} \frac{\Phi_4}{\Phi_3} \, ,$$

$$\theta_4 = \tan^{-1} \frac{\Psi_4}{\Psi_3} \, ,$$

$$\Phi_3 = c_2 a + d_2 \, ,$$

$$\Phi_3 = c_2 b \, ,$$

$$\Psi_1 = e_1 c + f_1 \, ,$$

$$\Psi_2 = e_1 d \, ,$$

$$\Psi_3 = e_2 c + f_2, \text{ and}$$

$$\Psi_4 = e_2 d \, .$$

The volume displacements $X_1(t)$ and $X_2(t\cdot)$ at necks N_1 and N_2, respectively, can be evaluated using numerical techniques.

If there is no energy dissipation (damping) in the system of double acoustical resonator, the characteristic determinant may be simplified to

$$\Delta = \frac{\rho_o{}^2 \ell_{e1} \ell_{e2}}{A_1 A_2} \left[(s^2 + P_+{}^2)(s^2 + P_-{}^2) \right]; \tag{17}$$

where p_+ and p_- are two undamped, coupled natural frequencies of the system and are given by

$$p_+{}^2 = \frac{(p_1{}^2 + p_2{}^2 + p_{12}{}^2) + \sqrt{(p_1{}^2 - p_2{}^2 - p_{12}{}^2)^2 + 4\, p_1{}^2 p_{12}{}^2}}{2},$$

and
$$\tag{18}$$

$$p_-{}^2 = \frac{(p_1{}^2 + p_2{}^2 + p_{12}{}^2) - \sqrt{(p_1{}^2 - p_2{}^2 - p_{12}{}^2)^2 + 4\, p_1{}^2 p_{12}{}^2}}{2}.$$

The volume displacement response for excitation pressure of one cycle of sine wave on neck N_1 of the system can be expressed as a function of natural frequencies, P_+ and P_-, as follows:

$$X_1(s) = \frac{A_o \omega}{\frac{\rho_o \ell_{e1}}{A_1}} \frac{(s^2 + p_2{}^2 + p_{12}{}^2)(1 - e^{-\tau s})}{(s^2 + \omega^2)(s^2 + p_+{}^2)(s^2 + p_-{}^2)} \quad , \text{ and}$$

$$X_2(s) = \frac{A_o \omega}{\frac{\rho_o{}^2 \ell_{e1} \ell_{e2}}{A_1 A_2}} \frac{(i - e^{-\tau s})}{(s^2 + \omega^2)(s^2 + p_+{}^2)(s^2 + p_-{}^2)} \quad . \tag{19}$$

The expressions in equations (19) may be factored by partial fractions and can be simplified to

$$X_1(s) = \frac{A_o \omega}{\frac{\rho_o{}^2 \ell_{e1}}{A_1}} \left[\frac{a_1 s + b_1}{s^2 + \omega^2} + \frac{c_1 s + d_1}{s^2 + p_+{}^2} + \frac{e_1 s + f_1}{s^2 + p_-{}^2} \right] (1 - e^{-\tau s}) \quad ,$$

and

$$X_2(s) = \frac{A_o \omega}{\frac{\rho_o{}^2 \ell_{e1} \ell_{e2}}{A_1 A_2}} \left[\frac{a_2 s + b_2}{s^2 + \omega^2} + \frac{c_2 s + d_2}{s^2 + p_+{}^2} + \frac{e_2 s + f_2}{s^2 + p_-{}^2} \right] (1 - e^{-\tau s}); \tag{20}$$

where

$$a_1 = c_1 = e_1 = a_2 = c_2 = e_2 = 0,$$

$$b_1 = \frac{p_2^2 + p_{12}^2 - \omega^2}{(\omega^2 - p_+^2)(\omega^2 - p_-^2)} \quad ,$$

$$d_1 = \frac{p_2^2 + p_{12}^2 + p_+^2}{(\omega^2 - p_+^2)(p_-^2 - p_+^2)} \quad ,$$

$$f_1 = \frac{(\omega^2 - p_+^2)(\omega^2 - p_-^2)(p_-^2 - p_+^2) - (p_1^4 - p_+^4)(p_2^2 + p_{12}^2 + \omega^2)}{(\omega^4 - p_+^4)(\omega^2 - p_-^2)(p_-^2 - p_+^2)}$$

$$- \frac{(\omega^4 - p_-^4)(p_2^2 + p_{12}^2 - p_1^2)}{(\omega^4 - p_+^4)(\omega^2 - p_-^2)(p_-^2 - p_+^2)} \quad ,$$

$$b_2 = \frac{1}{(p_+^2 - \omega^2)(p_-^2 - \omega^2)} \quad ,$$

$$f_2 = \frac{(\omega^4 - p_+^4)}{(p_+^4 - \omega^4)(p_-^4 - \omega^4)(p_+^2 - p_-^2)} \quad , \text{ and}$$

$$d_2 = \frac{1}{(p_+^2 - \omega^2)(p_+^2 - p_-^2)} \quad .$$

Transformation of equation (20) into time domain yields

$$X_1(t) = \frac{A_o}{\frac{\rho_o \ell_{e1}}{A_1}(\omega^2 - p_+^2)(\omega^2 - p_-^2)} \left\{ [a_x \sin \omega t + b_x \sin p_+ t \right.$$

$$+ c_x \sin p_- t]u(t) - [a_x \sin \omega(t - \tau) + b_x \sin p_+(t - \tau)$$

$$\left. + c_x \sin p_-(t - \tau)]u(t - \tau) \right\} , \text{ and}$$

$$X_2(t) = \frac{A_o \dfrac{\rho_o c^2}{V_1}}{\dfrac{\rho_o{}^2 \ell_{e1}\ell_{e2}}{A_1 A_2}\,(\omega^2 - p_+{}^2)(\omega^2 - p_-{}^2)}\left\{[\text{Sin } \omega t + b_y \text{ Sin } p_+ t \right.$$

$$+ c_y \text{ Sin } p_- t]u(t) - [\text{Sin } \omega(t - \tau) + b_y \text{ Sin } p_+(t - \tau)$$

$$\left. + c_y \text{ Sin } p_-(t - \tau)]\, u(t - \tau)\right\} ; \tag{21}$$

where the coefficients are defined as

$$a_x = p_2{}^2 + p_{12}{}^2 - \omega^2 \quad ,$$

$$b_x = \frac{(p_2{}^2 + p_{12}{}^2 - p_+{}^2)(\omega^2 - p_-{}^2)\,\omega}{(p_-{}^2 - p_+{}^2)\,p_+} \quad ,$$

$$c_x = \frac{(\omega^2 - p_+{}^2)(\omega^2 - p_-{}^2)(p_-{}^2 - p_+{}^2) - (p_2{}^2 + p_{12}{}^2 - \omega^2)(p_-{}^4 - p_+{}^4)}{(\omega^2 + p_+{}^2)(p_-{}^2 - p_+{}^2)p_-}$$

$$- \frac{(\omega^4 - p_-{}^4)(p_2{}^2 + p_{12}{}^2 - p_+{}^2)\,\omega}{(\omega^2 + p_+{}^2)(p_-{}^2 - p_+{}^2)p_-} \quad ,$$

$$b_y = \frac{(\omega^2 - p_-{}^2)\,\omega}{(p_-{}^2 - p_+{}^2)\,p_+} \quad , \quad \text{and}$$

$$c_y = \frac{(\omega^4 - p_+{}^4)\,\omega}{(p_+{}^2 - p_-{}^2)(\omega^2 + p_+{}^2)p_-} \quad .$$

Because of absence of the damping in the system the volume displacement equations (21) are very much simpler as compared to the equations (15). From these equations, the effect of undamped natural frequencies on the response can be evaluated.

Excitation Pressure Made up of Straight Lines on Neck N_1

and No Pressure on Neck N_2

The excitation pressure $P_1(t)$ is defined in Chapter III as

$$P_1(t) = \sum_{i=1}^{n} [\alpha_i + \beta_i(t - \tau_i)] \, u(t - \tau_i) \qquad (22)$$

where

n = number of discontinuities of $P_1(t)$ on time axis,

α_i = jump in pressure, $P_1(t)$ at time, $t = \tau_i$,

β_i = change in slope of pressure at time, $t = \tau_i$, and

$u(t - \tau_i)$ = unit step function such that $P(t)$ multiplied by $u(t - \tau_i)$

will have a value of zero for t τ_i and $P(t)$ in the region

t τ_i .

The Laplace transform of $P_1(t)$ is

$$P_1(s) = \frac{1}{s^2} \sum_{i=1}^{n} [\alpha_i s + \beta_i] \, e^{-\tau_i s} \ . \qquad (23)$$

The excitation pressure on neck N_2,

$$P_2(t) = 0 \ , \qquad \text{and hence} \qquad P_2(s) = 0 \ .$$

The expressions for the volume displacement response of the system may be reduced to the following form after substitution of equation (23) in equation (4):

$$X_1(s) = \frac{(s^2 + g_1 s + g_0) \sum\limits_{i=1}^{n} (\alpha_i s + \beta_i) \, e^{-\tau_i s}}{\dfrac{\rho_o \ell_{e1}}{A_1} s^2 [s^4 + B_3 s^3 + B_2 s^2 + B_1 s + B_0]} \quad , \text{ and}$$

$$X_2(s) = \frac{p_1^{\,2} (h_1 s + 1) \sum\limits_{i=1}^{n} (\alpha_i s + \beta_i) \, e^{-\tau_i s}}{\dfrac{\rho_o \ell_{e2}}{A_1} s^2 [s^4 + B_3 s^3 + B_2 s^2 + B_1 s + B_0]} \quad ; \qquad (24)$$

where the coefficient of s in the denominator is defined by the equation (5a).

For the stable oscillatory response of the system, equation (24) can be reduced to the following form after utilizing equation (8)

$$X_1(s) = \frac{(s^2 + g_1 s + g_0) \sum_{i=1}^{n} (\alpha_i s + \beta_i) e^{-\tau_i s}}{\frac{\rho_o \ell e1}{A_1} s^2 [(s-a)^2 + b^2][(s-c)^2 + d^2]} \quad , \text{ and}$$

$$X_2(s) = \frac{p_1^2 (h_1 s + 1) \sum_{i=1}^{n} (\alpha_i s + \beta_i) e^{-\tau_i s}}{\frac{\rho_o \ell e2}{A_2} s^2 [(s-a)^2 + b^2][(s-c)^2 + d^2]} \quad ; \qquad (25)$$

where $(a \pm jb)$ and $(c \pm jd)$ are the roots of polynomial equation (8).

These equations can be expanded in terms of the partial fractions as follows:

$$\frac{(s^2 + g_1 s + g_0)(\alpha_i s + \alpha_i)}{s^2[(s-a)^2 + b^2][(s-c)^2 + d^2]} = \frac{a_i s + b_i}{s^2} + \frac{c_i s + d_i}{(s-a)^2 + b^2}$$

$$+ \frac{e_i s + f_i}{(s-c)^2 + d^2} \quad , \quad \text{and}$$

$$\frac{(h_1 s + 1)(\alpha_i s + \beta_i)}{s^2[(s-a)^2 + b^2][(s-c)^2 + d^2]} = \frac{A_i s + B_i}{s^2} + \frac{C_i s + D_i}{(s-a)^2 + b^2}$$

$$+ \frac{E_i s + F_i}{(s-c)^2 + d^2} \quad . \qquad (26)$$

The right side of this equation is reduced to a common denominator, and the like powers of s on both sides of the equation are

equated to obtain the following two sets of simultaneous matrix equations, (27) and (28).

Solution of these equations for the co-efficients is utilized along with equation (26) to reduce the response equation into the following form:

$$X_1(s) = \frac{1}{\frac{\rho_o \ell_{e1}}{A_1}} \sum_{i=1}^{n} \left[\frac{a_i s + b_i}{s^2} + \frac{c_i s + d_i}{(s-a)^2 + b^2} + \frac{e_i s + f_i}{(s-c)^2 + d^2} \right] e^{-\tau_i s} \quad , \text{ and}$$

$$X_2(s) + \frac{p_1^2}{\frac{\rho_o \ell_{e2}}{A_2}} \sum_{i=1}^{n} \left[\frac{A_i s + B_i}{s^2} + \frac{C_i s + D_i}{(s-a)^2 + b^2} + \frac{E_i s + F_i}{(s-c)^2 + d^2} \right] e^{-\tau_i s} \quad . \quad (29)$$

Transformation of the equations (29) into time domain by inverse transformation yields the following equations:

$$X_1(t) = \frac{1}{\frac{\rho_o \ell_{e1}}{A_1}} \left\{ a_i + b_i(t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} \right.$$

$$\left[\Phi_2 \cos b(t - \tau_i) + \Phi_1 \sin b(t - \tau_i) \right.$$

$$\left. + \frac{e^{c(t - \tau_i)}}{d} \left[\Psi_2 \cos d(t - \tau_i) + \Psi_1 \sin d(t - \tau_i) \right] \right\} \quad , \text{ and}$$

$$X_2(t) = \frac{p_1^2}{\frac{\rho_o \ell_{e2}}{A_2}} \left\{ A_i + B_i \ (t - \tau_i) + \frac{e^{a(t - \tau_i)}}{b} \right.$$

$$\left[\Phi_3 \ \text{Sin} \ b(t - \tau_i) + \Phi_4 \ \text{Cos} \ b(t - \tau_i) \right]$$

$$+ \frac{e^{c(t - \tau_i)}}{d} \left[\Psi_3 \ \text{Sin} \ d(t - \tau_i) + \Psi_4 \text{Cos} \ d(t - \tau_i) \right] \right\} \ ; (30)$$

where

$$\Phi_1 = c_i a + d_i \ ,$$

$$\Phi_2 = c_i b \ ,$$

$$\Psi_1 = e_i c + f_i \ ,$$

$$\Psi_2 = e_i d \ ,$$

$$\Phi_3 = C_i a + D_i \ ,$$

$$\Phi_4 = C_i b \ ,$$

$$\Psi_3 = E_i c + F_i \ , \ \text{and}$$

$$\Psi_4 = E_i d \ .$$

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\[2mm]
2(a+c) & -1 & 2c & 1 & -2a & 1 \\[2mm]
\begin{array}{l}a^2+b^2+c^2\\+d^2+4ac\end{array} & -2(a+c) & d^2 & -2c & b^2 & -2a \\[2mm]
\begin{array}{l}2[a(c^2+d^2)\\+c(a^2+b^2)]\end{array} & \begin{array}{l}-[a^2+b^2+c^2\\+d^2+4ac]\end{array} & 0 & -d^2 & 0 & -b^2 \\[2mm]
\begin{array}{l}(a^2+b^2)\cdot\\(c^2+d^2)\end{array} & \begin{array}{l}-2[a(c^2+d^2)\\+c(a^2+b^2)]\end{array} & 0 & 0 & 0 & 0 \\[2mm]
0 & \begin{array}{l}(a^2+b^2)\cdot\\(c^2+d^2)\end{array} & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
a_i \\[2mm] b_i \\[2mm] c_i \\[2mm] d_i \\[2mm] e_i \\[2mm] f_i
\end{bmatrix}
=
\begin{bmatrix}
0 \\[2mm] 0 \\[2mm] \alpha_i \\[2mm] a_i\alpha_i+\beta_i \\[2mm] a_o\alpha_i+a_i\beta_i \\[2mm] a_o\beta_i
\end{bmatrix}
\tag{27}
$$

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
2(a+c) & 1 & -2c & -1 & 2a & 1 \\
\begin{array}{c}a^2+b^2+c^2\\+\,d^2+4ac\end{array} & -2(a+c) & -d^2 & -2c & b^2 & -2a \\
\begin{array}{c}-2[a(c^2+d^2)\\+\,c(a^2+b^2)]\end{array} & \begin{array}{c}a^2+b^2+c^2\\+\,d^2+4ac\end{array} & 0 & d^2 & 0 & b^2 \\
\begin{array}{c}(a^2+b^2)\cdot\\(c^2+d^2)\end{array} & \begin{array}{c}-2[a(c^2+d^2)\\+\,c(a^2+b^2)]\end{array} & 0 & 0 & 0 & 0 \\
0 & \begin{array}{c}(a^2+b^2)\cdot\\(c^2+d^2)\end{array} & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
A_i \\ B_i \\ C_i \\ D_i \\ E_i \\ F_i
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ -c_1\alpha_i \\ c_1\beta_i+\alpha_i \\ \beta_i
\end{bmatrix}
\qquad (28)
$$

APPENDIX B

CALIBRATIONS AND LIST OF MAJOR INSTRUMENTATION

Three Altec condenser microphones, Tektronic dual beam oscil-
loscope, and C.E.C. oscillograph were used to measure and record the
pressures. All the microphones were calibrated at the factory and
the sensitivities are given by manufacturers as -54.5 DB (reference
1 volt per dyne per centimeter) for the frequencies of 10 to 4000
cycles per second. Conversion of these sensitivities into pressures
gives 1.095 psf/volt. A test was conducted by arranging all the
three microphones at the test end of the plane wave tube as shown in
Figure 31 and measuring the responses for steady state pressures.
It was found that there was a slight difference in the sensitivities.
Table III shows the relative sensitivities at various frequencies.
However, while recording in the oscillograph the sensitivities of all
the three channels were compensated by adjusting the matching network
so that all the response traces are to the same scale. They were not
calibrated for the absolute sensitivities, since that was not
necessary for this investigation.

The amplitude calibration was checked with the internal square
wave calibrator of the scope and found to be satisfactory. In short,
the calibrations of the recording instrumentation was satisfactory
for the relative pressure response measurements.

TABLE III

RELATIVE SENSITIVITIES OF MICROPHONES

Frequency cps	Microphone #1	Microphone #2	Microphone #3
20	0.50	0.51	0.50
30	0.49	0.52	0.50
40	0.54	0.58	0.57
50	0.56	0.56	0.55
60	0.45	0.48	0.47
70	0.62	0.66	0.63
80	0.38	0.40	0.39
90	0.81	0.87	0.84
100	0.34	0.37	0.36
110	0.71	0.76	0.73
120	0.37	0.40	0.40
130	0.31	0.34	0.32
140	0.62	0.65	0.66
150	0.30	0.31	0.30
160	0.75	0.80	0.77
170	0.28	0.31	0.30
180	0.35	0.36	0.36
190	0.34	0.35	0.35
200	0.24	0.25	0.24

List of Major Instrumentation

Microphone System: Model 21BR150 condenser microphones; 165A bases;

    Model 526B power supply; manufacturer – Altec Lansing Corporation.

Dual Beam Oscilloscope: Model 502; manufacturer – Tektronix; serial

    number – 022893.

Low Frequency Function Generator: Model 202A; manufacturer –

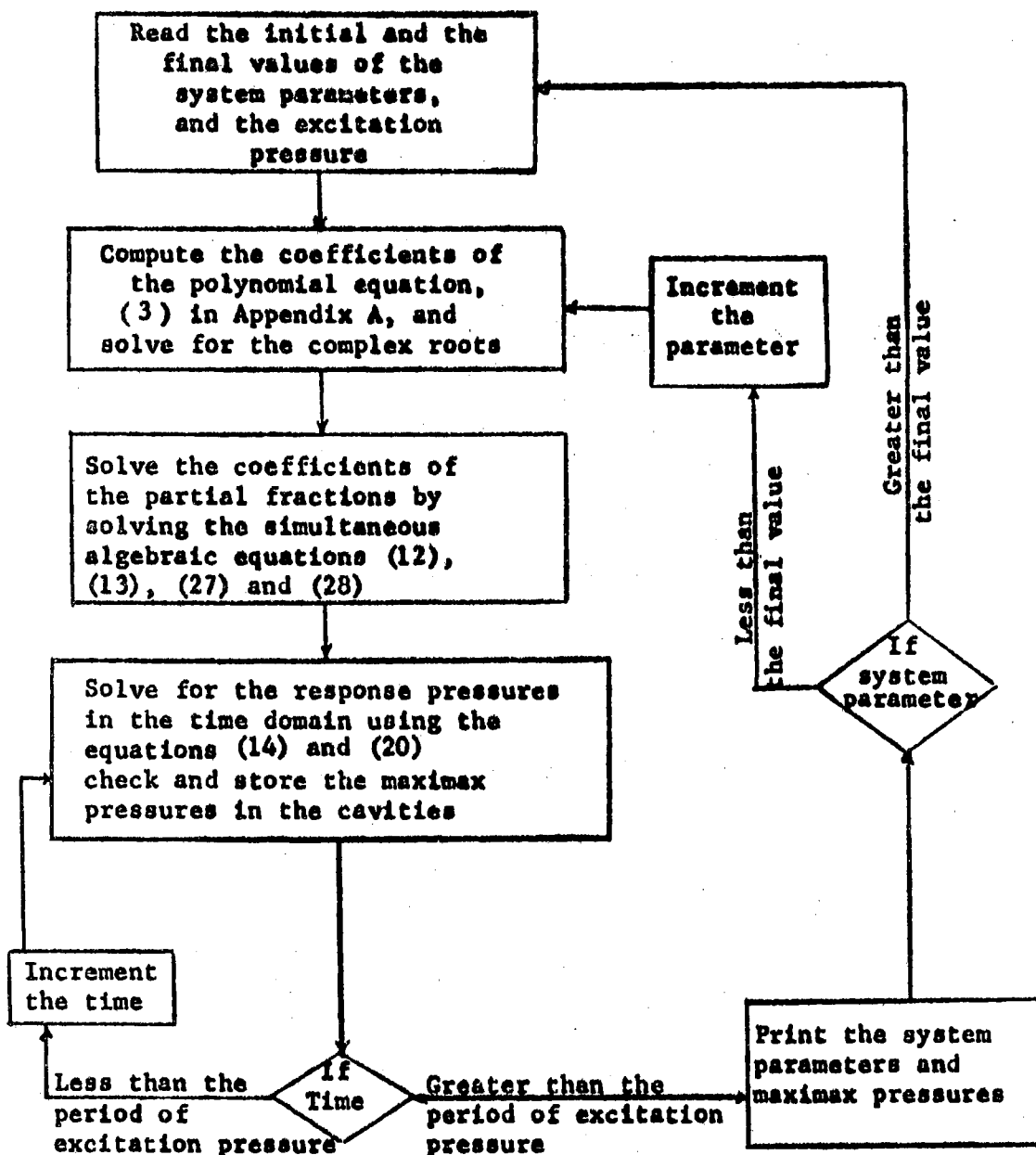   Hewlett-Packard; serial number – 037-09559.

Tone Burst Generator: Type 1396-A; manufacturer – General Radio

    Company; serial number 354.

Power Amplifier: Model MC75; manufacturer – McIntosh.

Oscillograph: Model 5-124; manufacturer – Consolidated Electrodynamics

    Corporation; serial number 6307.

APPENDIX C

FLOW DIAGRAM OF COMPUTER PROGRAM FOR THE MAXIMAX RESPONSE

OF DOUBLE ACOUSTICAL RESONATOR

```
┌─────────────────────────┐
│   Read the initial and  │
│   the final values of   │◄──────────────────────┐
│   the system parameters,│                        │
│   and the excitation    │                        │
│   pressure              │                        │
└───────────┬─────────────┘                        │
            │                                       │
            ▼                                       │
┌─────────────────────────┐   ┌──────────────┐     │
│  Compute the coefficients│  │  Increment   │     │ Greater than
│  of the polynomial       │◄─│    the       │     │ the final value
│  equation, (3) in        │  │  parameter   │     │
│  Appendix A, and solve   │  └──────┬───────┘     │
│  for the complex roots   │         │             │
└───────────┬─────────────┘         │             │
            │                        │             │
            ▼                        │ Less than   │
┌─────────────────────────┐         │ the final   │
│  Solve the coefficients  │        │ value       │
│  of the partial fractions│        │             │
│  by solving the          │        │         ◄──┐ │
│  simultaneous algebraic  │        │        ╱If ╲│
│  equations (12), (13),   │        └───────╱system╲
│  (27) and (28)           │                ╲parameter╱
└───────────┬─────────────┘                 ╲   ╱
            │                                  │
            ▼                                  │
┌─────────────────────────┐                    │
│  Solve for the response  │                    │
│  pressures in the time   │                    │
│  domain using the        │                    │
│  equations (14) and (20) │                    │
│  check and store the     │                    │
│  maximax pressures in the│                    │
│  cavities                │                    │
└───────────┬─────────────┘                    ▼
            │                         ┌──────────────────┐
┌───────────┐                         │  Print the system│
│ Increment │                         │  parameters and  │
│ the time  │                         │  maximax pressures│
└─────┬─────┘                         └──────────────────┘
      │   Less than the      ╱If ╲   Greater than the
      └───  period of       ╱Time ╲  period of excitation
            excitation      ╲     ╱  pressure
            pressure         ╲   ╱
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VITA

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