AN INVESTIGATION OF UNSTIFFENED AND STIFFENED RECTANGULAR CANTILEVERED PANELS USING ENERGY METHODS

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Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of the
requirements for
the degree of
DOCTOR OF PHILOSOPHY
July, 1967

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ACKNOWLEDGEMENTS

I wish to express my deepest appreciation to Professor L. J. Fila and Professor R. E. Chapel, who served as thesis advisers, for their guidance and financial assistance. I want to acknowledge the assistance of Dr. E. L. Harrisberger, chairman of the committee, and Dr. D. E. Boyd, committee member.

I am forever indebted to my wife, Wilmalee, without whose encouragement, sacrifices and love this work could not have been possible.

I wish to express my appreciation to my fellow graduate students, Dr. M. U. Ayres, Dr. Charles Lindbergh, Dr. Balusu Rao, and Mr. Paul Bauer, for their assistance.

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NOMENCLATURE

a	length of panel
a_{mn}, b_{rs}, A_{qp}	undetermined coefficients
b	half height of panel
E	modulus of elasticity
G	shear modulus
H	height of panel
I_p	panel moment of inertia
I_{sp}	stiffened panel moment of inertia
P	applied load
t	thickness of panel
u	deflection in x direction
v	deflection in y direction
V	strain energy
x	coordinate in horizontal direction
y	coordinate in vertical direction
ϵ_x	strain in x direction
ϵ_y	strain in y direction
σ_x	normal stress in x direction
$\sigma_{x_{sp}}$	normal stress in x direction in stiffened panel
σ_y	normal stress in y direction
τ_{xy}	shear stress

τ_{xysp}. . . . . shear stress in stiffened panel

ϕ. . . . . stress function

ϕ_{xx}. . . . . second derivative of stress function with respect to x

ϕ_{yy}. . . . . second derivative of stress function with respect to y

ϕ_{xy}. . . . . second derivative of stress function, once with respect to x, once with respect to y

μ. . . . . Poisson's ratio

γ_{xy}. . . . . shearing strain

η. . . . . non-dimensionalized co-ordinate in x direction

ξ. . . . . non-dimensionalized co-ordinate in y direction

CHAPTER I

INTRODUCTION

The prediction of stresses and deflections of airframes is a critical phase of structural analysis in the aircraft industry. Preliminary design is usually based on elementary strength of materials methods. Final design is usually based on finite element methods which require large computer programs. A need exists for methods which will yield results as accurate as finite element methods and are simple to apply.

Ritz (1) developed the energy method at the turn of the twentieth century. Because no suitable general method was available to determine the magnitude of the error in the results, a lack of confidence in the method delayed its application until Gerald Pickett (2) determined the natural frequencies of a clamped plate subjected to a lateral load. Since that time several investigators have used the energy method to determine the natural frequencies of vibration of plates with various shapes and boundary conditions. Anderson (3) determined the natural frequencies for two symmetric and two antisymmetric modes for triangular plates clamped at the base. His results pointed out that reasonable accuracy could be obtained using an eight term series to approximate deflections. Young (4) determined the natural frequencies of a square plate clamped at all edges, a square plate clamped at two adjacent edges and free along the other two edges, and a square plate clamped along one edge and free along the other three edges. A nine term series yielded exceptionally good results. Little, Stolz, and Schmerda (5) used the Ritz Method to determine the natural frequencies of a composite structure in the form of a circuit-board assembly. The results compared well with experimental data.

Investigations have been performed to determine the stresses and deflections of flat plates subjected to static transverse loads. Liessa and Niedenfuhr (6) determined the deflections of a cantilevered plate with the Ritz Method. Their results compared favorably with solutions obtained by beam theory, finite differences, and experimental methods.

Timoshenko (7) obtained a solution for the stresses and deflections of a rectangular panel subjected to a parabolic tension. Pickett (2) examined these results and found the deflections to be in excellent agreement with those determined by the Multiple Fourier Method. However, boundary stresses differed up to two percent.

The analysis of stiffened panels is a relatively recent development. Argyris (8) formulated a solution, based on the Force Method, using matrix notation for complicated structures. Turner, et al. (9) developed a method of analysis for stiffened panels based on the

Stiffness Method.

Ayres (10) investigated the stresses and deflections in stiffened rectangular panels subjected to various load conditions. At approximately the same time, Stone (11) investigated the stresses and deflections in a stiffened trapezoidal panel subjected to various load conditions. Both investigators report results which compare favorably with their experimental data.

A survey of the various methods has been conducted by Rigsby (12) in which an attempt was made to determine which method of solution should be used. This survey had applicability to stringer stresses only. The conclusion drawn was that energy methods were preferable to other methods in determining stringer stresses.

The Force Method and the Stiffness Method are the two popular methods of analysis now being used in the aircraft industry.

The Force Method is based on the premise that there are an infinite number of force systems for a given structure which will satisfy the conditions of equilibrium, but that only the correct force system will also satisfy the conditions of compatibility with regard to displacements. The structure can be idealized as webs (which sustain only shear loads) and stringers (which sustain only direct loads) as illustrated in Figure 1. This idealization requires that an "effective" stringer be used. The "effective" stringer is composed of the original stringer plus an effective area

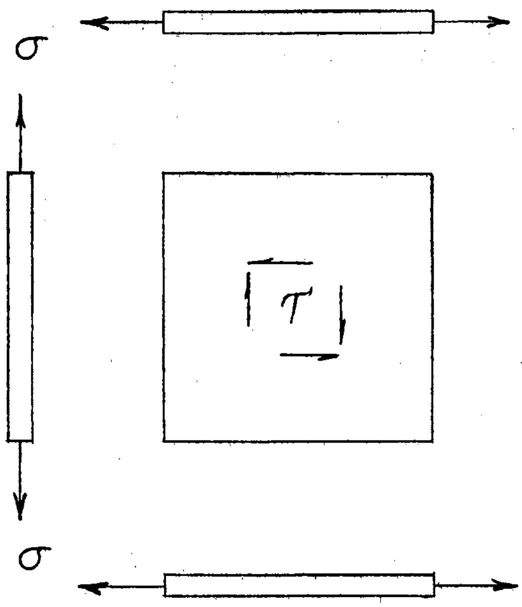

Figure 1. Force Method Assumption

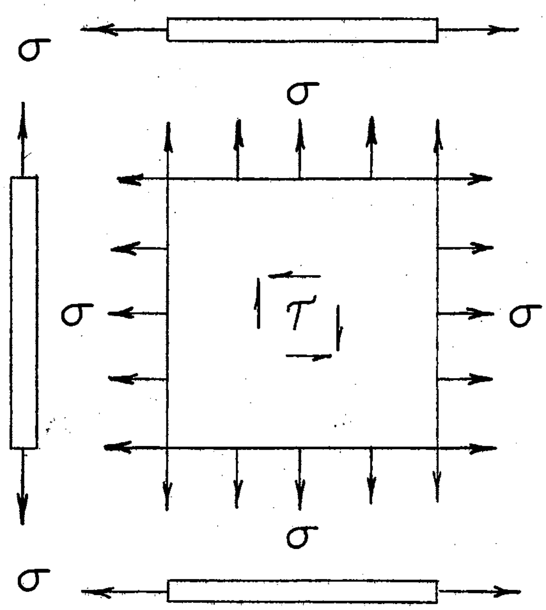

Figure 2. Stiffness Method Assumption

added because of the assumption that the web sustains only shear loads. The amount of web area added to the stringers depends on the stress levels to be encountered, the panel material, and the type of loading (13).

The unknown quantities in the Force Method are the redundant forces in the structure. The total potential energy is expressed in terms of the redundant forces and the external forces. The deformations are determined from an assumed stress-strain relationship and the kinematic relationships. Compatibility is then used to obtain a set of simultaneous equations from which the redundant forces are calculated. Additional calculations yield the displacements.

The second popular method is the Stiffness Method which requires that the structure be idealized by considering the structure to be connected only at the nodes chosen for the analysis. The forces and deflections of each member are related by an assumed stress-strain relationship. The displacements of the nodes are considered as the unknown quantities.

A further assumption is made that the webs transmit axial stresses as well as shear stresses as shown in Figure 2, however the assumed stress distribution is usually a simple one.

There are an infinite number of sets of compatible nodal displacements but only the correct one satisfies the equilibrium conditions. Once the displacements have been determined, an additional set of calculations yields the forces.

Both the Force Method and the Stiffness Method require that the elements of the structure be connected so that no discontinuities of deformation occur and so that the elements are in equilibrium with the external reactions and the forces they exert on each other.

Although the Force Method and the Stiffness Method are the two primary methods used by the aircraft industry, these methods are seriously limited in that they yield results that differ unless identical mathematical models are used for each. Identical mathematical models are not always practicable. If dissimilar mathematical models are used, the Stiffness Method yields the more satisfactory stress values and the Force Method yields the more satisfactory deflections (9).

In any finite element method, the size of the element is a critical factor. The confidence one can place in the results depends on the element size. Since many elements are usually required to obtain reasonable results, large, complicated computer programs are necessary to aid in the computations.

In order to circumvent these limitations, two other analytical methods, the Method of Timoshenko and the Rayleigh-Ritz Method, were explored in this study. The limitation of these methods is the assumption of stress or displacement functions. The more accurate the stress or displacement functions, the better the results.

The position of a vibrating system which is periodic in time may be expressed by the relationship

$$X = f(x_j)\ e^{i\omega t}$$

in which X must satisfy equilibrium conditions for the time variable forces. X must satisfy the law of conservation of energy, that is, the sum of the changes of all forms of energy must be constant with time. The function $f(x_j)$ is the shape of the deflection curve for the system: the modal shape function. The modal shape function undergoes changes only in amplitude in order to define X. In other words, $f(x_j)$ aids X in satisfying equilibrium conditions and the law of conservation of energy. For static problems, the $e^{i\omega t}$ term is equal to unity. Therefore, $f(x_j)$, being identical to X, must itself satisfy energy conservation and equilibrium. When a solution for $f(x_j)$ is readily apparent, there is no direct use of the law of conservation of energy or equilibrium; however, one or both laws are involved as an essential to the solution.

When a solution for $f(x_j)$ is not readily apparent, a solution can be obtained by properly selecting the optimum $f(x_j)$ of all conceivable functions which satisfy boundary conditions or system constraints. To aid in the proper selection of $f(x_j)$, one should first examine the Lagrangian equations. For a conservative system, a differential equation of an energy term is equivalent to the equilibrium equation. The energy equation is written as

$$L = T - U,$$

where, L = the Lagrangian function,

U = the potential energy from a fixed datum, and

T = the kinetic energy.

Both terms are positive because they represent only quadratic terms of space variables, that is, every term in T and U is positive. The logical requirement in this case is

$$T + U = \text{constant} > 0$$

which may be written as

$$T - U = -2U + C$$

or

$$U - T = -2T + C.$$

These forms indicate that a difference of the energies can be dependent on either T or U. This discussion assumes that both T and U are precisely known and that the sum of both must be a constant to satisfy the law of conservation of energy.

Both T and U may be calculated from an $f(x_j)$ which is different from the correct value. Then T and U may be expressed as

$$T = T' + \delta T, \text{ and}$$

$$U = U' + \delta U,$$

where T' and U' are the correct values. Then

$$T + U = C$$

becomes

$$T' + \delta T + U' + \delta U = C$$

or

$$T' + U' = C - \delta T - \delta U.$$

Since the arbitrary choice of T and U did not recognize how the correct value is separated from the erroneous one, δT and δU cannot vanish. However, the least value may be selected by minimizing the energy, that is,

$$d(T' + U') = -d(\delta U + \delta T) = 0.$$

In this respect the error is reduced to a minimum. The quantities δT and δU will have a least value which is constant, with the consequence,

$$T + U = C + C_1.$$

This indicates that the energy equation is in error by C_1, therefore $(T + U)$ is in error as $f(x_j)$ is in error.

It should be noticed that

$$T - U = -2U + C - (\delta U + \delta T)$$

leads to

$$d(T - U) = -2dU - d(\delta U + \delta T) = 0.$$

The preceding analysis conforms with the natural phenomenon that nature seeks an equilibrium condition which requires a minimum overall change of energy. The application of this fact does not necessarily yield the correct answer. It has been shown that an error in $f(x_j)$ introduces the error C_1 in the energy equation. This is an arbitrary error because the chosen $f(x_j)$ is arbitrary. Because of the quadratic form of T and U, both C and C_1 would be greater than zero. A more correct solution could be obtained by minimizing the error with a better choice of modal shape, i. e.,

$$\frac{\delta C_1}{\delta f(x_j)} = 0.$$

A slight variation in analysis is required when one form of energy is not a quadratic function of $f(x_j)$. As long as the conservation of energy is assumed, the sum of the positive changes must be a constant; therefore, the previous discussion applies to any system which has significant changes in energy.

In order to apply the minimizing condition,

$$d(T + U) = 0,$$

$f(x_j)$ is expressed as a function of an undefined parameter, that is, a constant factor a_i. Then the condition

$$\frac{\partial(T + U)}{\partial a_i} = 0$$

provides all the equations necessary to determine a_i. If $(T + U)$ is a homogeneous function, a_i cannot be uniquely determined, however all a_i's can be determined in terms of one a_i, say a_0. It then becomes necessary to find some additional condition to find a_0.

Thus, the Rayleigh-Ritz Method is an approximation which depends on the principle of stationary potential energy. Deflection functions are assumed in the form of a polynomial series with undetermined coefficients. This method evades the compatibility conditions, satisfying them only approximately. Although deflection equations may be obtained which are usually very accurate, no reliability can be placed on the stresses which are obtained by the differentiation of the deflection functions. Generally, the deflection functions chosen are approximations to the exact functions. Because the deflection functions are in error, the stress functions obtained by differentiating these functions will be in greater error. Therefore, when stresses are of primary interest, a method other than the Rayleigh-Ritz Method should be used. A similar method employing the principle of least work can be used. This method is usually referred to as the Method of Timoshenko. The Method of Timoshenko involves the selection of a stress function which satisfies the equilibrium and stress boundary conditions identically and compatibility approximately. If the stresses are of primary interest, the Method of Timoshenko provides a direct method of solution.

The Rayleigh-Ritz Method and the Method of Timoshenko yield identical results provided the exact deflection functions are used in the Rayleigh-Ritz solution, the exact stress function is used in the Method of Timoshenko, and an infinite number of terms is taken for both.

The stress function derived in this dissertation is approximate, as are the deflection functions. Stresses derived from the deflection functions using the Rayleigh-Ritz Method cannot be expected to compare with the stresses obtained from the Method of Timoshenko. Conversely, the deflections obtained by the integration of the stress function contain large discrepancies. This is not to say that the stresses obtained by the Method of Timoshenko are not acceptable. The stresses will be accurate because boundary conditions and equilibrium conditions are satisfied. On the other hand, the deflections obtained by the Rayleigh-Ritz Method should be a good approximation to the actual deflections.

This study was undertaken to examine the desirability of using the Method of Timoshenko and the Rayleigh-Ritz Method to calculate the stresses and deflections of cantilevered skin panels, both stiffened and unstiffened. The study is primarily theoretical; however, experimental and analytical results of other investigators are available for comparison.

The study includes stress and deflection analyses of a cantilevered panel (shown in Figure 3). The results of the cantilevered panel analyses are compared with a solution of Timoshenko (7), which was obtained using a uniform shear stress, having a resultant P, on the free end of a cantilevered panel as shown in Figure 4. The stress equations used by Timoshenko were

$$\sigma_x = \frac{Pxy}{I}, \tag{1-1}$$

$$\sigma_y = 0 \;, \tag{1-2}$$

and

$$\tau_{xy} = -\frac{P}{2I}(b^2 - y^2). \tag{1-3}$$

The deflection equations obtained by Timoshenko were

$$u = -\frac{Px^2y}{6EI} - \frac{\mu P y^3}{6EI} + \frac{Py^3}{6GI} + \left(\frac{Pa^2}{2EI} - \frac{Pb^2}{2GI}\right) y \tag{1-4}$$

and

$$v = \frac{\mu Pxy^2}{2EI} + \frac{Px^3}{6EI} - \frac{Pa^2x}{2EI} + \frac{Pa^3}{3EI} \;. \tag{1-5}$$

The analysis was then extended to a study of a stiffened cantilevered panel (shown in Figure 5) which is the same panel used by Ayres (10). The panel was chosen so that the results could be compared to the analytical and experimental results of Ayres, who used a stiffness analysis in his study.

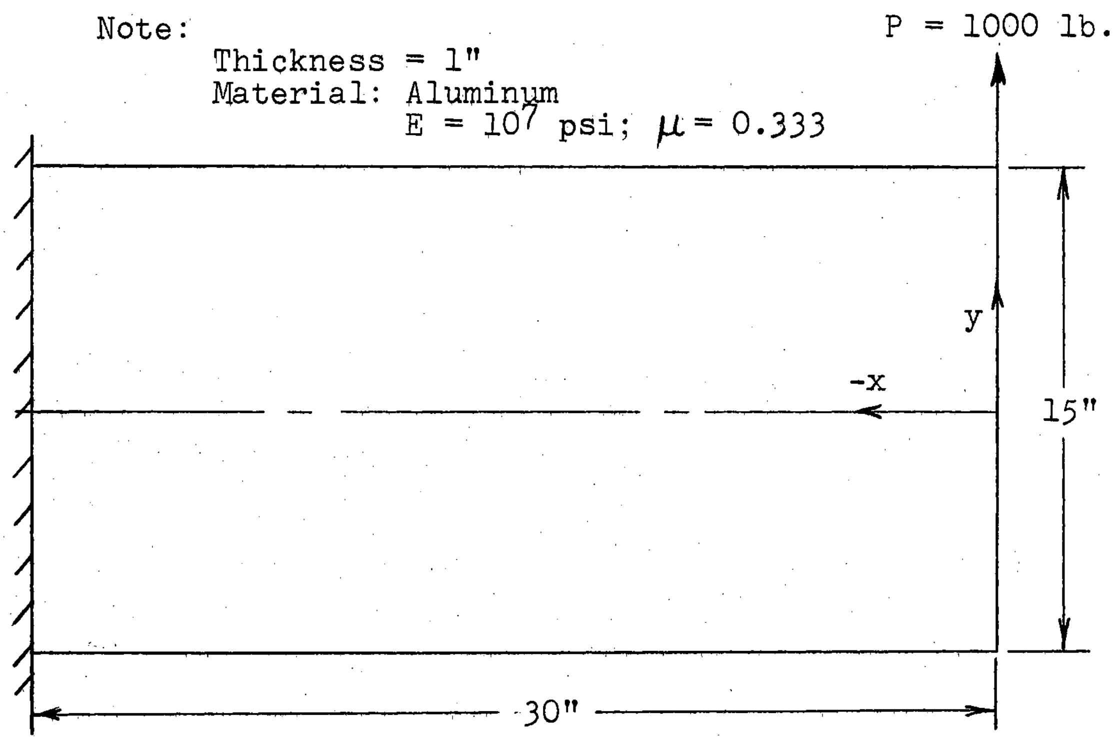

Figure 3. Unstiffened Panel Configuration

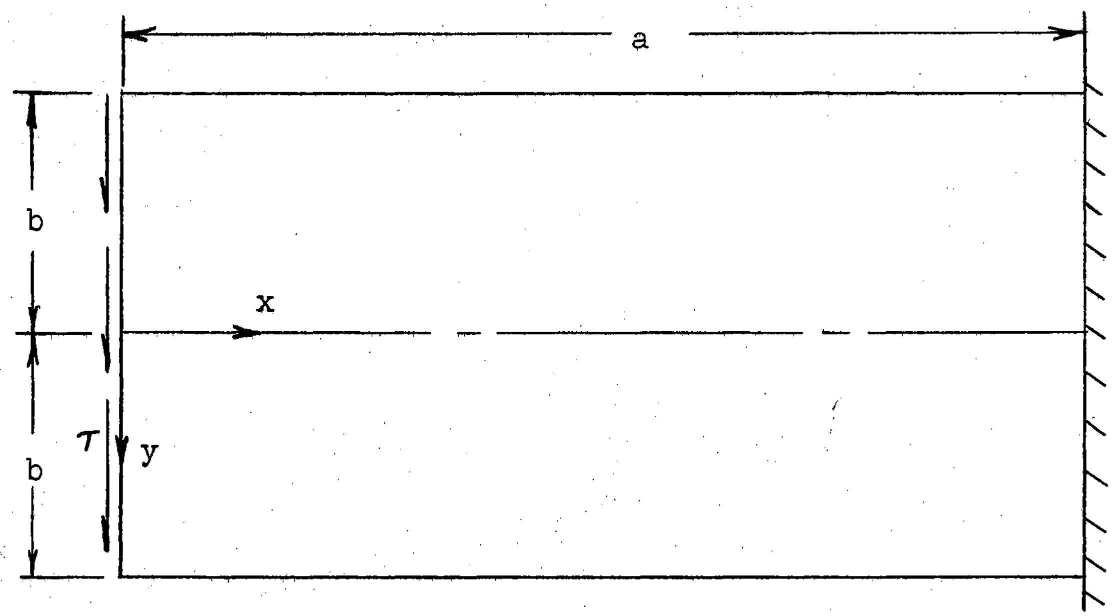

Figure 4. Assumed Loading for Timoshenko Solution

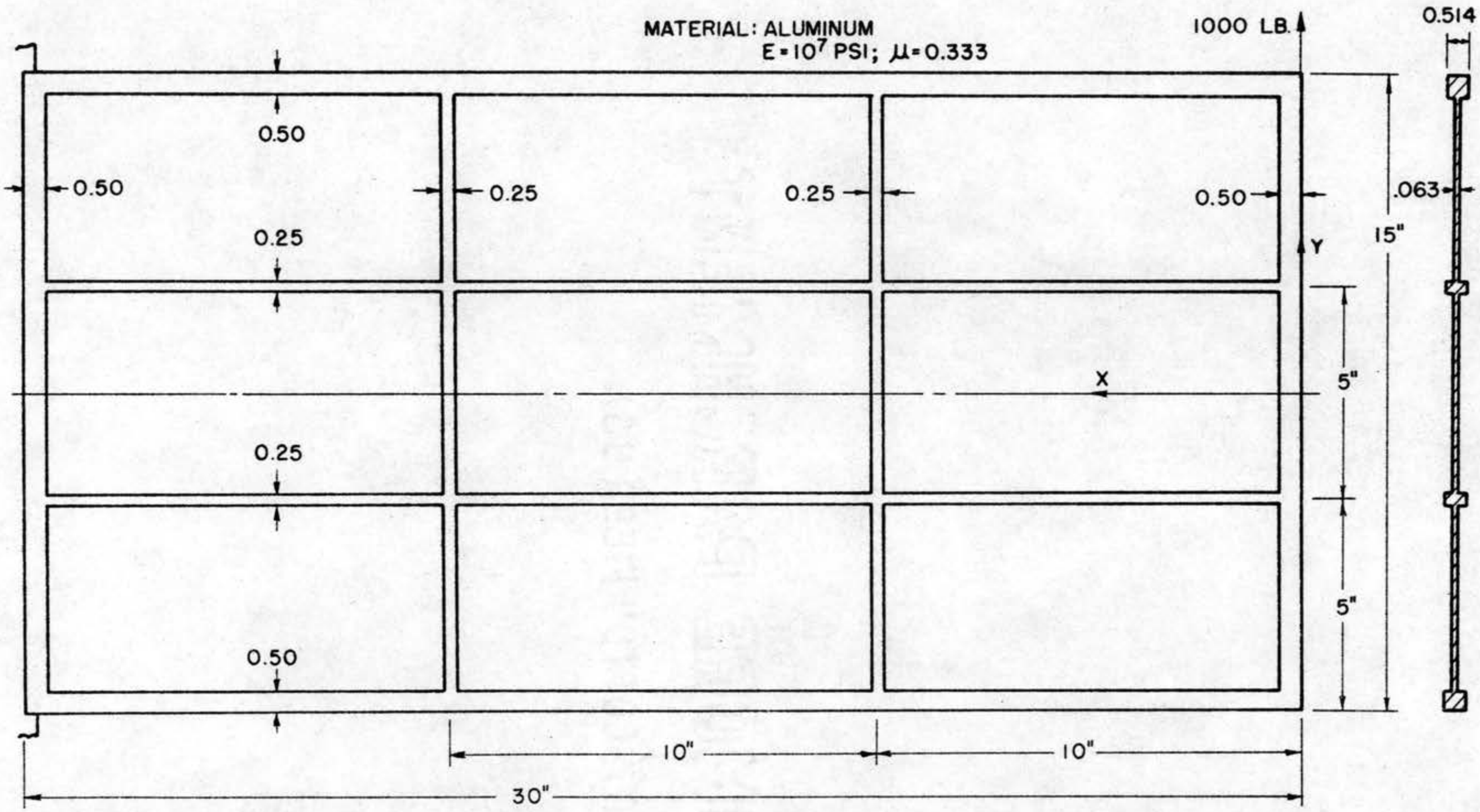

Figure 5. Stiffened Cantilevered Panel

CHAPTER II

THE DERIVATION OF A STRESS FUNCTION FOR A CANTILEVERED PANEL

The Method of Timoshenko provides a direct method of solution for the stresses in the panel. In order to implement this method a stress function is required. This chapter is concerned with the derivation of an initial stress function which satisfies all stress boundary conditions of a cantilevered panel of uniform thickness. A polynomial series containing undetermined coefficients is then added to the initial stress function in such a manner that the boundary stresses are unaffected by the series. Differentiation of the stress function yields the stresses in the panel.

In order to obtain suitable stresses and deflections, and consequently a suitable stress function, the investigation was initiated by assuming the deflection functions given in Appendix B. This approach was taken because it was believed that the Rayleigh-Ritz Method was simpler to apply and would yield accurate results. The most critical assumption in the Rayleigh-Ritz Method is the selection of the deflection functions. The effectiveness of any energy method depends upon the satisfaction of boundary conditions as closely as possible. The functions given in Appendix B were unacceptable because the only boundary conditions satisfied were the deflection conditions at the fixed support, that is, $u = 0$, $v = 0$, at $x = 0$, $y = y$. No other deflection boundary conditions were available. The selection of these functions was based on unpublished notes by Pickett (14) in which the hypothesis is put forth that certain minimum conditions are required for the effective use of the Rayleigh-Ritz Method. The minimum conditions prescribed were those of geometric boundary conditions. None of the deflection functions of Appendix B yielded satisfactory results which indicated that additional conditions were required. Since no additional conditions were available, this approach was discarded and the problem was approached from the standpoint of stresses because more boundary conditions could be prescribed.

The most critical factor in an analysis using the Method of Timoshenko is the choice of the stress function. The stress function should satisfy all stress boundary conditions and as many physical conditions as possible. However, the implementation of this requirement is not always feasible. The stress function, equation (A-1) given in Appendix A, was expressed to incorporate as many variables as possible. The mathematical manipulations became so cumbersome that the presumed advantage of simplicity desired by the use of the Method of Timoshenko was negated.

The stress expressions, equations (A-4), were then selected to determine the feasibility of using uncomplicated stress equations. However, the application of the minimization procedure showed that the coefficients, B and D, were too sensitive to round-off error. The stress function finally selected for the analysis which follows was selected after equations (A-1) and (A-4) proved to be unsatisfactory.

Formulation of an Initial Stress Function

The normal and shear stresses on an exposed surface must be zero if the surface is unloaded. If the exposed surface is loaded, the surface stresses must correspond to the applied load. These facts required that the boundary conditions be specified as follows:

(a) at $y = b$, $x = x$, $\sigma_Y = f(x)$,

(b) at $y = -b$, $x = x$, $\sigma_Y = 0$,

(c) at $x = 0$, $y = y$, $\sigma_X = 0$,

(d) at $x = 0$, $y = y$, $\tau_{XY} = 0$,

(e) at $y = b$, $x = x$, $\tau_{XY} = 0$, and

(f) at $y = -b$, $x = x$, $\tau_{XY} = 0$.

The above boundary conditions are shown in Figure 6. In addition to these boundary conditions the stress function should satisfy

(g) $\int_{-b}^{b} \sigma_x \, y \, dy$ = external moment and

(h) $\int_{-b}^{b} \tau_{xy} \, dy$ = external load

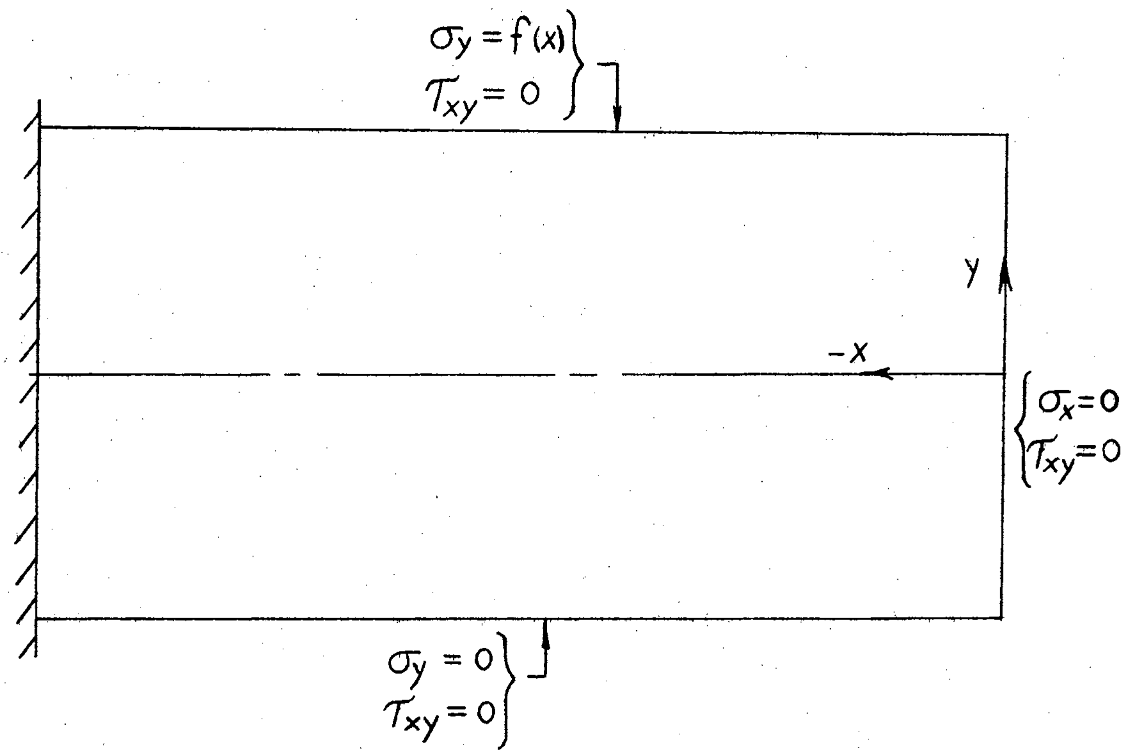

Figure 6. Stress Boundary Conditions

at any vertical cross section.

The f(x) in (a) above was determined by replacing the concentrated load at the free end of the panel with a normal stress distribution in the y direction along the upper edge of the panel. This stress distribution was selected in such a manner that at least ninety-five percent of the area under the curve was located within one half inch of the free end in order to approximate load conditions on Ayres' experimental model. The assumed loading of the panel is shown in Figure 7. The function selected was

$$f(x) = \sigma_y = \left[Ax(x+a)^2 - B(x+a)^{180}\right]\left[-4b^3\right]. \qquad (2\text{-}1)$$

The constants A and B were determined by equating the area under the σ_y curve to the load P and by equating the moment of the area of the σ_y curve to the moment of the applied load about the free end. These calculations were made in the following manner. The force due to the first term in equation (2-1) was obtained from

$$\int_0^{-a} Ax(x+a)^2(-4b^3)\,dx = -A\frac{a^4b^3}{3}\ . \qquad (2\text{-}2)$$

The force due to the second term of equation (2-1) was obtained from

$$\int_0^{-a} B(x+a)^{180}(4b^3)\,dx = -4Bb^3\,\frac{a^{181}}{181}\ . \qquad (2\text{-}3)$$

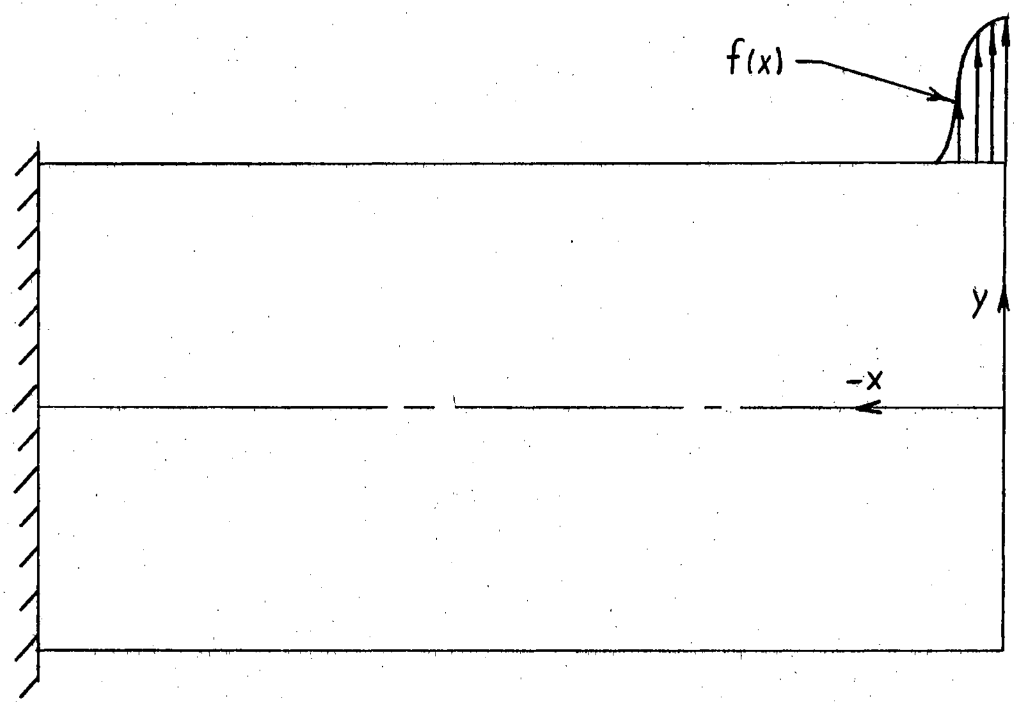

Figure 7. Assumed Loading of Panel

The moment of the first term of equation (2-1) about an axis perpendicular to the panel at x = 0, y = b was obtained from

$$\int_0^{-a} Ax(x+a)^2(-4b^3)x\,dx = 4Ab^3\frac{a^5}{30}\,. \qquad (2\text{-}4)$$

The moment of the second term of equation (2-1) about the same axis was obtained from

$$\int_0^{-a} B\,(x+a)^{180}(4b^3)\,x\,dx = \frac{4Bb^3a^{182}}{(181)(182)}\,. \qquad (2\text{-}5)$$

The sum of equations (2-2) and (2-3) must balance the load P and the sum of equations (2-4) and (2-5) must be zero. In other words,

$$4Bb^3\frac{a^{181}}{181} + A\frac{b^3a^4}{3} = P \qquad (2\text{-}6)$$

and

$$\frac{4Bb^3a^{182}}{181(182)} + \frac{4Ab^3a^5}{30} = 0\,. \qquad (2\text{-}7)$$

Equations (2-6) and (2-7) were solved simultaneously which yielded

$$A = \frac{-30P}{718\,a^4b^3}$$

and

$$B = \frac{(181)(182)P}{718\, a^{181}\, b^3}.$$

The initial stress function which satisfied all boundary conditions became

$$\phi_o = \left[A\left\{ \frac{(x+a)^5}{20} - \frac{a(x+a)^4}{12} + \frac{a^4 x}{12} + \frac{a^5}{30} \right\} - B\left\{ \frac{(x+a)^{182}}{(181)(182)} - \frac{a^{181}x}{181} - \frac{a^{182}}{(181)(182)} \right\} \right] \left[y^3 - 3b^2 y - 2b^3 \right]. \qquad (2\text{-}8)$$

Equation (2-8) was differentiated twice with respect to y to obtain the stress σ_x. The differentiation yielded

$$\sigma_x = \phi_{yy} = \left[A\left\{ \frac{(x+a)^5}{20} - \frac{a(x+a)^4}{12} + \frac{a x^4}{12} + \frac{a^5}{30} \right\} - B\left\{ \frac{(x+a)^{182}}{(181)(182)} - \frac{a^{181}x}{181} - \frac{a^{182}}{(181)(182)} \right\} \right] \left[6y \right]. \qquad (2\text{-}9)$$

Equation (2-8) was differentiated twice with respect to x to obtain the stress σ_y. The differentiation yielded

$$\sigma_y = \phi_{xx} = \left[Ax(x+a)^2 - B(x+a)^{180} \right] \left[y^3 - 3b^2 y - 2b^3 \right]. \qquad (2\text{-}10)$$

Equation (2-8) was differentiated once with respect to x and once with respect to y to obtain the stress $-\tau_{xy}$. The differentiation yielded

$$-\tau_{xy} = \phi_{xy} = \left[A \left\{ \frac{(x+a)^4}{4} - \frac{a(x+a)^3}{3} + \frac{a^4}{12} \right\} - B \left\{ \frac{(x+a)^{181}}{181} - \frac{a^{181}}{181} \right\} \right] \left[3(y^2 - b^2) \right]. \qquad (2\text{-}11)$$

Formulation of the Second and Third Stress Functions

An infinite number of stress functions exist which will satisfy stress boundary conditions. The proper function is that one which minimizes the strain energy. Because the initial stress function, ϕ_o, was not necessarily the proper stress function, it was altered by adding an infinite polynomial series which contained undetermined coefficients. The coefficients were determined by the minimization of the strain energy. The infinite series was selected so that the stresses corresponding to it vanished at the boundaries. The form of the complete stress function was

$$\phi = \phi_o + x^4 (y^2 - b^2)^2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{mn} x^m y^n. \qquad (2\text{-}12)$$

Equation (2-12) was differentiated to obtain

$$\phi_{yy} = \left[A \left\{ \frac{(x+a)^5}{20} - \frac{a(x+a)^4}{12} + \frac{a^4 x}{12} + \frac{a^5}{30} \right\} - B \left\{ \frac{(x+a)^{182}}{(181)(182)} - \frac{a^{181} x}{181} - \frac{a^{182}}{(181)(182)} \right\} \right] \left[6y \right]$$

$$+ x^4 (y^2-b^2)\sum\sum n(n-1)C_{mn}\, x^m y^{n-2}$$

$$+ 8x^4 y(y^2-b^2)\sum\sum n\, C_{mn}\, x^m y^{n-1}$$

$$+ 4x^4 (3y^2-b^2)\sum\sum C_{mn}\, x^m y^n, \qquad (2\text{-}13)$$

$$\phi_{xx} = \left[A\left\{ (x+a)^3 - a(x+a)^2 \right\} - B(x+a)^{180} \right]\left[y^3 - 3b^2y - 2b^3 \right]$$

$$+ x^4 (y^2-b^2)^2 \sum\sum m(m-1)\, C_{mn}\, x^{m-2} y^n$$

$$+ 8x^3 (y^2-b^2)^2 \sum\sum m\, C_{mn}\, x^{m-1} y^n$$

$$+ 12x^2 (y^2-b^2)^2 \sum\sum C_{mn}\, x^m y^n, \qquad (2\text{-}14)$$

and

$$\phi_{xy} = \left[A\left\{ \frac{(x+a)^4}{4} - \frac{a(x+a)^3}{3} + \frac{a^4}{12} \right\} \right.$$

$$\left. - B\left\{ \frac{(x+a)^{181}}{181} - \frac{a^{181}}{181} \right\} \right]\left[3y^2 - 3b^2 \right]$$

$$+ x^4 (y^2-b^2)^2 \sum\sum mn\, C_{mn}\, x^{m-1} y^{n-1}$$

$$+ 16x^3 y (y^2-b^2)\sum\sum C_{mn}\, x^m y^n$$

$$+ 4x^4 y (y^2-b^2)\sum\sum m\, C_{mn}\, x^{m-1} y^n$$

$$+ 4x^3 (y^2-b^2)^2 \sum\sum n\, C_{mn}\, x^m y^{n-1}. \qquad (2\text{-}15)$$

The strain energy in the plate was written as

$$V = \frac{t}{2E}\int_{x=0}^{-a}\int_{y=-b}^{b}\left[\phi_{xx}^{2} + \phi_{yy}^{2} + \phi_{xy}^{2}\right] dx\, dy. \qquad (2\text{-}16)$$

Equations (2-13), (2-14) and (2-15) were substituted into equation (2-16) to determine the completed strain energy expression as a function of the C_{mn}. This expression was minimized with respect to each C_{mn}, i. e.,

$$\frac{\partial V}{\partial C_{mn}} = 0$$

which yielded a set of independent linear equations in C_{mn}. At this point in the derivation a limitation on the number of undetermined coefficients to be evaluated was necessary in order to facilitate the solution of these equations. The number of coefficients was limited to one to obtain the second stress function, then to four to obtain the third stress function.

The use of one undetermined coefficient resulted in the equation

$$(5.6888a^9b^5 + 22.2912a^7b^7 + 46.8114a^5b^9)C_{00}$$
$$= 0.0333Pa^2b^5. \qquad (2\text{-}17)$$

The length of the panel, a, was twice the height, 2b. This relationship was substituted into equation (2-17) which yielded

$$C_{00} = 0.0046\ P/a^7.$$

C_{00} was substituted into equations (2-13) through (2-15) to obtain the stresses from the second stress function.

The use of four undetermined coefficients resulted in the equations

$$-(5.6888a^9b^5 + 22.2912a^7b^7 + 46.8114a^5b^9)C_{00}$$
$$-3.7926a^9b^5C_{01} + (46.4863a^6b^9 + 24.3809a^8b^7$$
$$+ 5.1200a^{10}b^5)C_{10} + 3.4133a^{10}b^5C_{11} =$$
$$- 0.0333\ Pa^2b^5, \qquad (2\text{-}18)$$

and

$$- 3.7926a^9b^5C_{00} -(5.1471a^9b^7 + 14.8608a^7b^9$$
$$+ 4.2556a^5b^{11}C_{01} + 3.4133a^{10}b^5C_{10} + (11.3778a^8b^9$$
$$+ 4.6324a^{10}b^7 + 5.9105a^6b^{11})C_{11} = - 0.007824\ Pa^6b^2$$
$$+ 0.000148\ Pa^2b^6 + 0.000221\ Pa^4b^4, \qquad (2\text{-}19)$$

and

$$(65.0159a^6b^9 + 15.6038a^8b^7 + 5.1200a^{10}b^5)C_{00}$$
$$+ 3.4133a^{10}b^5C_{01} - (92.8798a^7b^9 + 27.0900a^9b^7$$
$$+ 4.6545a^{11}b^5)C_{10} - 3.1030a^{11}b^5C_{11} =$$

$$0.01356\ Pa^3b^5, \tag{2-20}$$

and

$$3.4133a^{10}b^5C_{00} + (6.9892a^8b^9 + 4.6324a^{10}b^7$$
$$+ 5.9105a^6b^{11})C_{01} - 3.1030a^{11}b^5C_{10} - (8.3227a^9b^9$$
$$+ 4.2113a^{11}b^7 + 8.4436a^7b^{11})C_{11} =$$
$$- 0.000002\ Pa^3b^6 + 0.018047\ Pa^5b^4. \tag{2-21}$$

The relationship, a = 4b, was substituted into equations (2-18) through (2-21) which yielded the matrix equation

$$\left[\frac{a^7}{P}\right]\begin{bmatrix} -7.265 & -3.793 & 204.762 & 102.400 \\ -0.032 & 2.856 & 0.853 & 67.968 \\ 6.349 & 3.413 & -201.450 & -93.091 \\ 0.004 & 0.317 & 0.103 & -8.920 \end{bmatrix}\begin{bmatrix} C_{00} \\ C_{01} \\ C_{10} \\ C_{11} \end{bmatrix} = \begin{bmatrix} -0.0333 \\ -0.1250 \\ 0.0136 \\ -0.0130 \end{bmatrix}. \tag{2-22}$$

The solution of equation (2-22) yielded

$$C_{00} = 0.00883\ P/a^7,$$
$$C_{01} = 0.50708\ P/a^7,$$
$$C_{10} = -0.00019\ P/a^7, \text{ and}$$
$$C_{11} = 0.01947\ P/a^7.$$

These coefficients were substituted into equations (2-13) through (2-15) to obtain the stresses from the third stress function.

The values of the σ_x stresses are shown in Table I. The first two columns locate the point at which the σ_x stresses were calculated. Column three shows the σ_x stresses calculated from the results of Timoshenko given by equation (1-1). Column four shows the σ_x stresses calculated with the initial stress function, i. e., equation (2-9). Column five shows the σ_x stresses calculated with the second stress function, i. e., equation (2-13) with one C_{mn}. Column six shows the σ_x stresses calculated with the third stress function, i. e., equation (2-13) with four C_{mn}'s.

The values of the σ_y stresses are shown in Table II. The first two columns locate the point at which the σ_y stresses were calculated. Column three shows the σ_y stresses calculated from the results of Timoshenko as given by equation (1-2). Column four shows the σ_y stresses calculated with the initial stress function, i. e., equation (2-10). Column five shows the σ_y stresses calculated with the second stress function, i. e., equation (2-14) with one C_{mn}. Column six shows the σ_y stresses calculated with the third stress function, i. e., equation (2-14) with four C_{mn}'s.

The values of the τ_{xy} stresses are shown in Table III. The first two columns locate the point at which the τ_{xy} stresses were calculated. Column three shows the τ_{xy} stresses calculated from the results of Timoshenko as given by equation (1-3). Column four shows the τ_{xy} stresses

TABLE I

σ_x STRESSES IN PANEL

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
0	7.5	0	0	0	0
	5.0	0	0	0	0
	2.5	0	0	0	0
	0.0	0	0	0	0
	-2.5	0	0	0	0
	-5.0	0	0	0	0
	-7.5	0	0	0	0
-5.0	7.5	-133.3	-132.5	-132.5	-132.4
	5.0	- 88.9	- 88.3	- 88.3	- 88.3
	2.5	- 44.5	- 44.2	- 44.2	- 44.2
	0.0	0	0	0	0
	-2.5	44.5	44.2	44.2	44.2
	-5.0	88.9	88.3	88.3	88.3
	-7.5	133.3	132.5	132.5	132.5
-10.0	7.5	-266.7	-268.9	-268.9	-268.4
	5.0	-177.8	-179.7	-179.7	-179.8
	2.5	- 88.9	- 90.0	- 90.0	- 89.8
	0.0	0	0	0	0
	-2.5	88.9	90.0	90.0	89.8

TABLE I (Continued)

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
	-5.0	177.8	179.7	179.7	179.5
	-7.5	266.7	268.9	268.9	268.5
-15.0	7.5	-400.0	-405.8	-405.8	-404.1
	5.0	-266.7	-270.5	-270.5	-271.0
	2.5	-133.3	-135.3	-135.3	-136.0
	0.0	0	0	0	0
	-2.5	133.3	135.3	135.3	135.9
	-5.0	266.7	270.5	270.5	271.0
	-7.5	400.0	405.8	405.8	404.1
-20.0	7.5	-533.3	-531.8	-531.8	-528.9
	5.0	-355.6	-354.5	-354.5	-355.3
	2.5	-177.8	-177.3	-177.3	-178.5
	0.0	0	0	0	- 0.02
	-2.5	177.8	177.3	177.3	178.4
	-5.0	355.6	354.5	354.5	355.3
	-7.5	533.3	531.8	531.8	528.9
-25.0	7.5	-666.7	-653.5	-653.5	-652.2
	5.0	-444.4	-435.7	-435.7	-436.0
	2.5	-222.2	-217.8	-217.8	-218.4
	0.0	0	0	- 0.03	- 0.05

TABLE I (Continued)

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
	-2.5	222.2	217.8	217.8	218.3
	-5.0	444.4	435.7	435.7	436.0
	-7.5	666.7	653.5	653.6	652.4
-30.0	7.5	-800.0	-800.0	-799.9	-809.4
	5.0	-533.3	-533.3	-533.3	-530.8
	2.5	-266.7	-266.7	-266.7	-262.8
	0.0	0	0	- 0.04	- 0.06
	-2.5	266.7	266.7	266.6	262.7
	-5.0	533.3	533.3	533.3	530.9
	-7.5	800.0	800.0	800.1	809.9

TABLE II

σ_y STRESSES IN PANEL

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
0	7.5	0	6117.0	6117.0	6117.0
	5.0	0	5664.0	5664.0	5664.0
	2.5	0	4531.0	4531.0	4531.0
	0.0	0	3059.0	3059.0	3059.0
	-2.5	0	1586.0	1586.0	1586.0
	-5.0	0	453.0	453.0	453.0
	-7.5	0	0	0	0
-5.0	7.5	0	- 0.6	- 0.6	- 0.6
	5.0	0	- 0.6	- 0.6	- 0.6
	2.5	0	- 0.5	- 0.5	- 0.4
	-2.5	0	- 0.2	- 0.2	- 0.2
	-5.0	0	0	0	- 0.1
	-7.5	0	0	0	0
-10.0	7.5	0	- 0.8	- 0.8	- 0.8
	5.0	0	- 0.8	- 0.8	- 0.7
	2.5	0	- 0.6	- 0.6	- 0.5
	0.0	0	- 0.4	- 0.4	- 0.4
	-2.5	0	- 0.2	- 0.2	- 0.3
	-5.0	0	- 0.1	- 0.1	- 0.1

TABLE II (Continued)

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
	-7.5	0	0	0	0
-15.0	7.5	0	- 0.7	- 0.7	- 0.7
	5.0	0	- 0.6	- 0.6	- 0.6
	2.5	0	- 0.5	- 0.5	- 0.5
	0.0	0	- 0.4	- 0.4	- 0.4
	-2.5	0	- 0.2	- 0.2	- 0.2
	-5.0	0	- 0.1	- 0.1	- 0.1
	-7.5	0	0	0	0
-20.0	7.5	0	0	0	0
	5.0	0	0	0	- 0.2
	2.5	0	0	0	- 0.2
	0.0	0	0	0	0
	-2.5	0	0	0	0.2
	-5.0	0	0	0	0.2
	-7.5	0	0	0	0
-25.0	7.5	0	0	0	- 0.5
	5.0	0	0	0	- 0.6
	2.5	0	0	0	0
	0.0	0	0	0	0
	-2.5	0	0	0	0.7

TABLE II (Continued)

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
	-5.0	0	0	0	0.5
	-7.5	0	0	0	0
-30.0	7.5	0	0	0	0
	5.0	0	0	0	- 1.1
	2.5	0	0	0	- 1.4
	0.0	0	0	0	0
	-2.5	0	0	0	1.5
	-5.0	0	0	0	1.1
	-7.5	0	0	0	0

TABLE III

τ_{xy} STRESSES IN PANEL

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
0	7.5	0	0	0	0
	5.0	55.6	0	0	0
	2.5	88.9	0	0	0
	0.0	100.0	0	0	0
	-2.5	88.9	0	0	0
	-5.0	55.6	0	0	0
	-7.5	0	0	0	0
-5.0	7.5	0	0	0	0
	5.0	55.6	56.2	56.2	56.2
	2.5	88.9	89.9	89.9	90.0
	0.0	100.0	101.2	101.2	101.2
	-2.5	88.9	89.9	89.9	90.0
	-5.0	55.6	56.2	56.2	56.2
	-7.5	0	0	0	0
-10.0	7.5	0	0	0	0
	5.0	55.6	56.0	56.0	56.0
	2.5	88.9	89.6	89.6	89.7
	0.0	100.0	100.8	100.8	101.0
	-2.5	88.9	89.6	89.6	89.8

TABLE III (Continued)

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
	-5.0	55.6	56.0	56.0	56.1
	-7.5	0	0	0	0
-15.0	7.5	0	0	0	0
	5.0	55.6	55.8	55.8	55.9
	2.5	88.9	89.3	89.3	89.5
	0.0	100.0	100.4	100.4	100.7
	-2.5	88.9	89.3	89.3	89.6
	-5.0	55.6	55.8	55.8	56.1
	-7.5	0	0	0	0
-20.0	7.5	0	0	0	0
	5.0	55.6	55.6	55.6	56.0
	2.5	88.9	89.0	89.0	89.1
	0.0	100.0	100.1	100.1	100.2
	-2.5	88.9	89.0	89.0	89.3
	-5.0	55.6	55.6	55.6	56.2
	-7.5	0	0	0	0
-25.0	7.5	0	0	0	0
	5.0	55.6	55.6	55.6	56.3
	2.5	88.9	88.9	88.9	85.6
	0.0	100.0	100.0	100.0	99.1

TABLE III (Continued)

x	y	Timoshenko	Initial Stress Function	Second Stress Function	Third Stress Function
	-2.5	88.9	88.9	88.9	88.6
	-5.0	55.6	55.6	55.6	56.4
	-7.5	0	0	0	0
-30.0	7.5	0	0	0	0
	5.0	55.6	55.6	55.6	57.0
	2.5	88.9	88.9	88.9	87.2
	0.0	100.0	100.0	100.0	96.5
	-2.5	88.9	88.9	88.9	86.8
	-5.0	55.6	55.6	55.6	56.5
	-7.5	0	0	0	0

calculated with the initial stress function, i. e., equation (2-11). Column five shows the τ_{xy} stresses calculated with the second stress function, i. e., equation (2-15) with one C_{mn}. Column six shows the τ_{xy} stresses calculated with the third stress function, i. e., equation (2-15) with four C_{mn}'s.

The σ_x results indicated in Table I show slight deviations from the results of Timoshenko. There is little difference in the results regardless of whether the initial stress function, second stress function, or third stress function is used. Apparently the use of the initial stress function will result in only a slight error in any calculation. The maximum difference between the initial stress function and the third stress function is 1.24 percent at the point $x = -30$, $y = -7.5$. The maximum deviation should occur somewhere along the fixed end because the fixed end is farthest removed from the boundary at which the σ_x stresses were specified in the formulation of the stress function. The maximum difference between the solution of Timoshenko and the third stress function is 2.1 percent at the point $x = -25$, $y = -7.5$.

The σ_y results indicated in Table II show a sharp deviation from the results of Timoshenko. The simplicity of the assumed loading in the analysis of Timoshenko excludes the possibility of σ_y stresses anywhere in the panel. The assumed loading in this analysis recognizes the existence of σ_y throughout the panel. The three stress functions derived in this analysis yield nearly identical σy results. The maximum value of σy naturally occurs at the point of application of the load.

The τxy results indicated in Table III differ slightly from the solution of Timoshenko. The largest deviation occurs at the free end where the load is not well defined. Once again, the results of the three stress functions fit the physical loading in this analysis more closely than Timoshenko. The free end of the panel is a free surface which can hardly support a shear stress as indicated by Timoshenko. The results of all three stress functions throughout the rest of the structure differ only slightly from Timoshenko's results. The largest deviation of 3.2% occurs at $x = -30$, $y = 0$, in the third stress function.

The σx stress distribution at several cross sections is shown in Figure 8; the σy distribution at several cross sections is shown in Figure 9; and the τxy distribution at several cross sections is shown in Figure 10.

The selected stress function resulted in stresses which are within 2% of Timoshenko's values wherever comparison is proper. Additional terms to the initial stress function change the stresses by 2.1% or less.

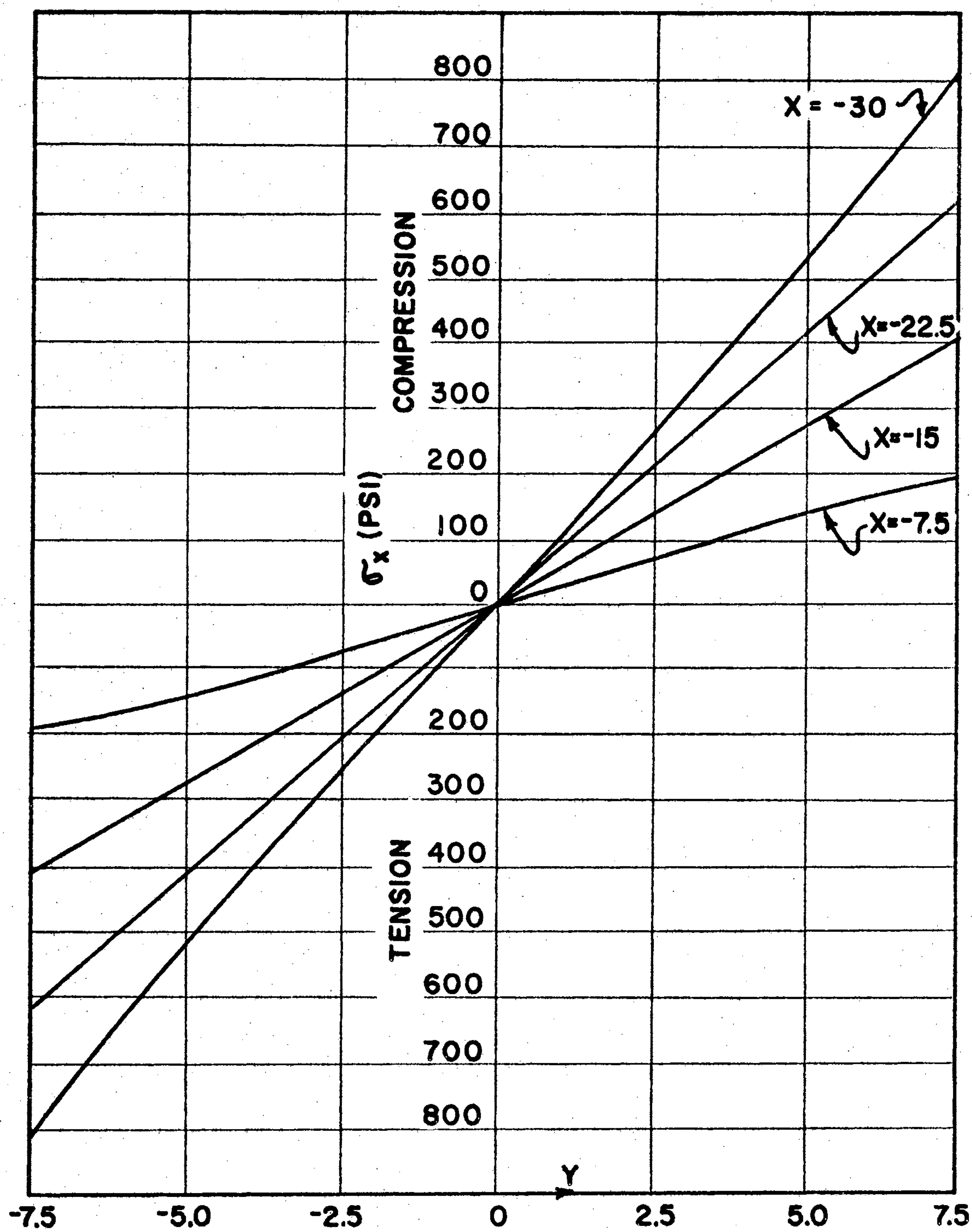

Figure 8. σx Stresses in Panel at Several Cross Sections

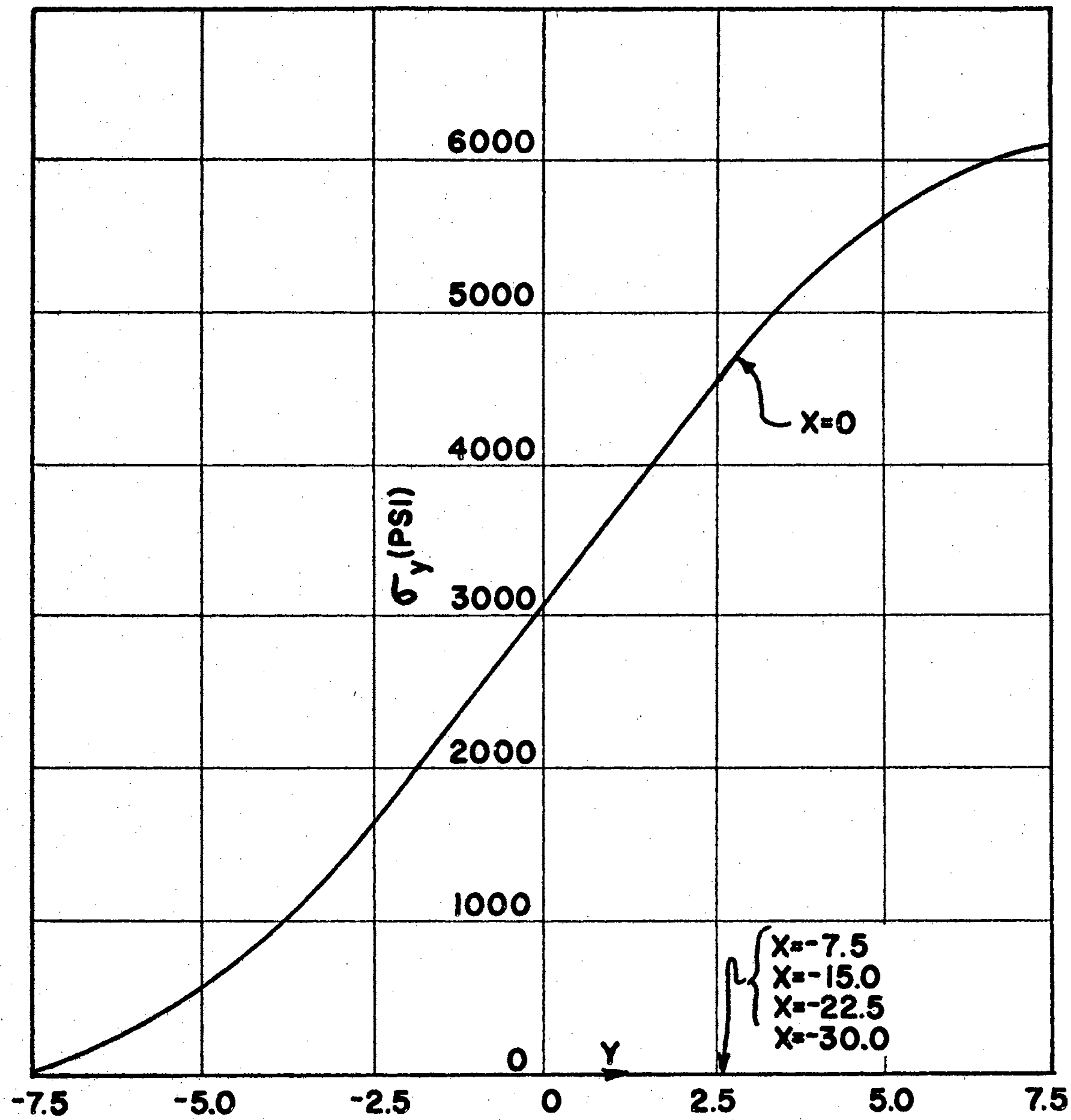

Figure 9. σ_y Stresses in Panel at Several Cross Sections

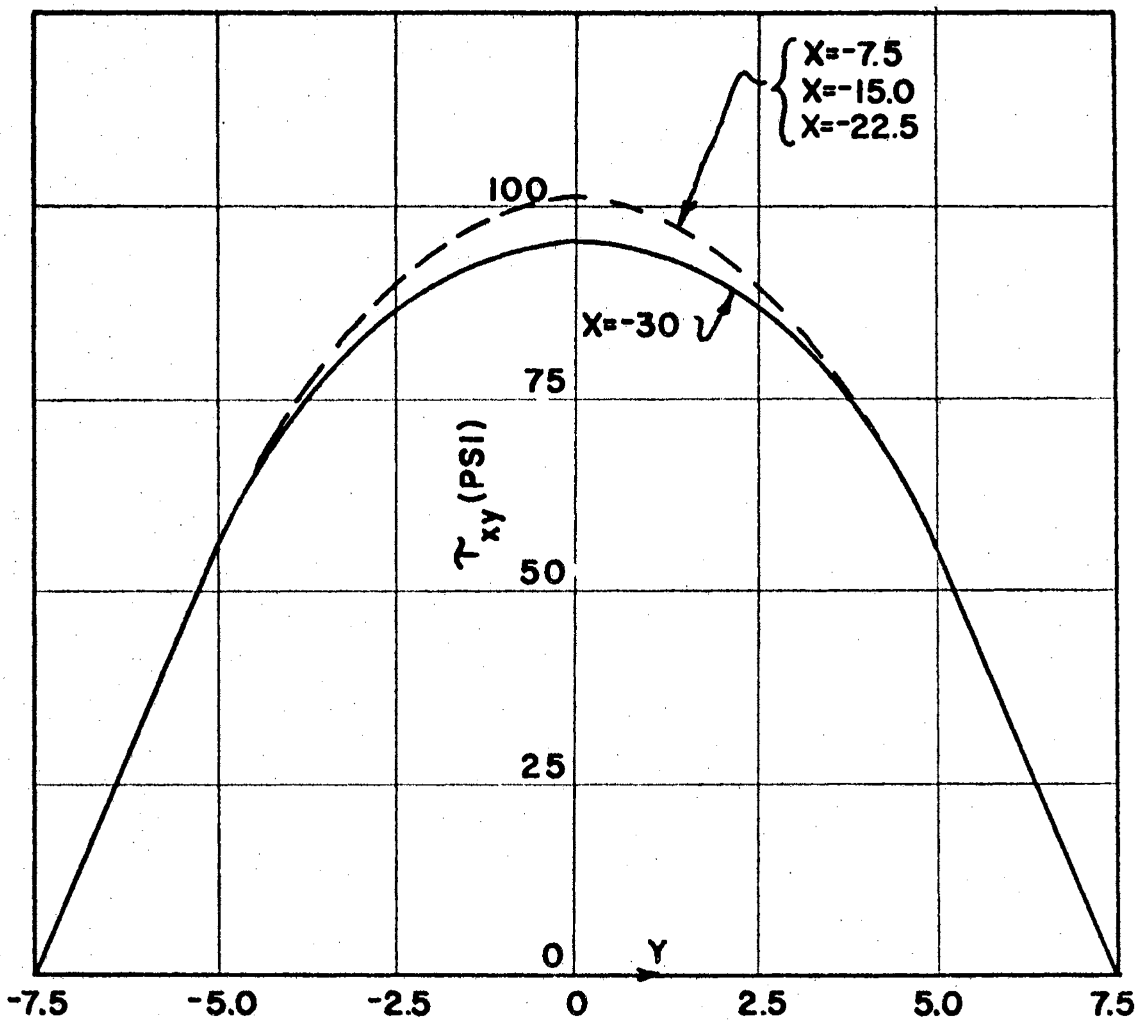

Figure 10. τ_{xy} Stresses in Panel at Several Cross Sections

CHAPTER III

THE DERIVATION OF DEFLECTION EQUATIONS FOR A CANTILEVERED PANEL

The method of Timoshenko can be extended directly to obtain the deflections, provided the stress function is exact. The stress functions chosen in Chapter II are not exact, therefore another approach was necessary to obtain the deflection equations. The Rayleigh-Ritz Method provides a direct solution for the deflections of the panel. In order to implement this method deflection equations with undetermined coefficients must be selected. The initial stress function of Chapter II was used to determine the deflection equations.

Formulation of Deflection Equations

The u and v deflections may be expressed as

$$Eu = \int \phi_{yy}\, dx - \mu \int \phi_{xx}\, dx \qquad (3\text{-}1)$$

and

$$Ev = \int \phi_{xx}\, dy - \mu \int \phi_{yy}\, dy. \qquad (3\text{-}2)$$

Substitution of equations (2-8) and (2-9) into equations (3-1) and (3-2) yielded

$$Eu = \left[A\left\{\frac{(x+a)^6}{120} - \frac{a(x+a)^5}{60} + \frac{a^4x^2}{24} + \frac{a^5x}{30}\right\}\right.$$
$$\left. - B\left\{\frac{(x+a)^{183}}{(183)(182)(181)} - \frac{a^{181}x^2}{2(181)} - \frac{a^{182}}{(182)(181)}\right\}\right]\left[6y\right]$$
$$-\mu\left[A\left\{\frac{(x+a)^4}{4} - \frac{a(x+a)^3}{3}\right\}\right.$$
$$\left. - B\,\frac{(x+a)^{181}}{181}\right]\left[y^3 - 3b^2y - 2b^3\right] + f(y) \qquad (3\text{-}3)$$

and

$$Ev = \left[A\left\{(x+a)^3 - a(x+a)^2\right\} - B(x+a)^{180}\right]\left[\frac{y^4}{4} - \frac{3b^2y^2}{2} - 2b^3y\right]$$
$$-\mu\left[A\left\{\frac{(x+a)^5}{20} - \frac{a(x+a)^4}{12} + \frac{a^4x}{12} + \frac{a^5}{30}\right\}\right.$$
$$\left. - B\left\{\frac{(x+a)^{182}}{(182)(181)} - \frac{a^{181}x}{181} - \frac{a^{182}}{(182)(181)}\right\}\right]\left[3y^2\right] + f(x). \qquad (3\text{-}4)$$

Difficulty was encountered in the attempt to determine f(y) in equation (3-3) and f(x) in equation (3-4). This difficulty was due to the inability of the stress function to satisfy compatibility. The relationship for shearing stress,

$$\frac{\tau_{xy}}{G} = \frac{du}{dy} + \frac{dv}{dx}, \qquad (3\text{-}5)$$

could not be satisfied because the stress function used in this analysis was an approximation, not an exact function. Under the classical approach with an exact stress function, equation (3-5) would be automatically satisfied. Equation (3-5) contained, not only functions of x and functions of y, but functions of the product, xy, as well. The functions f(x) and f(y) could not be determined.

The difficulty was circumvented in the following manner:

1. u was completely defined by imposing the boundary condition that u was zero everywhere along the support.
2. v was determined by means of various expressions for f(x) in the form of polynomials with undetermined coefficients.
3. The undetermined coefficients were evaluated with the Rayleigh-Ritz procedure.

The boundary condition,

$$u = 0 \text{ at } x = -a,\ y = y,$$

was applied and the expression for u became

$$Eu = \left[A\left\{ \frac{(x+a)^6}{120} - \frac{a(x+a)^5}{60} + \frac{a^4x^2}{24} + \frac{a^5x}{30} \right\} - B\left\{ \frac{(x+a)^{183}}{(183)(182)(181)} - \frac{a^{181}x^2}{2(181)} - \frac{a^{182}x}{(182)(181)} \right\} \right] \left[6y \right]$$
$$- \mu\left[A\left\{ \frac{(x+a)^4}{4} - \frac{a(x+a)^3}{3} \right\} - B\,\frac{(x+a)^{181}}{181} \right] \left[y^3 - 3b^2y - 2b^3 \right]$$

$$-\left[\frac{Aa^6}{120}+\frac{90\,B\,a^{183}}{(181)(182)}\right]\left[6y\right]. \qquad (3\text{-}6)$$

The deflection of a cantilevered beam can be closely approximated by a cubic equation therefore the f(x) in equation (3-4) was replaced with the expression

$$f(x) = A_o + A_2(x+a)^2 + A_3(x+a)^3$$

in which each A coefficient was undetermined. A_0 was evaluated by imposing the condition that the average v deflection at the support was zero, i. e.

$$\int_{-b}^{b} v\Big]_{x=-a} dy = 0.$$

The procedure yielded A_0 = 8550. The strain energy for the plate was written in the form

$$V = \frac{t}{2}\iint\left\{\frac{E}{1-\mu^2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + 2\mu\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2\right] + \frac{E}{2(1+\mu)}\left[\left(\frac{\partial u}{\partial y}\right)^2 + 2\,\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \left(\frac{\partial v}{\partial x}\right)\right]\right\} dx\,dy. \qquad (3\text{-}7)$$

The potential energy of the external load was expressed as

$$PE = \int \left(\sigma_y\, v\right)\Big|_{y=b} t\,dx\,. \qquad (3\text{-}8)$$

Equations (3-4) and (3-6) were substituted into equation

(3-7). Equation (2-8), evaluated at the upper edge, and equation (3-4), also evaluated at the upper edge, were substituted into equation (3-8). The resulting expression was substituted into the Rayleigh-Ritz condition

$$\frac{\partial}{\partial A_n}\left(V - PE\right) = 0. \qquad (3\text{-}9)$$

This yielded two simultaneous equations,

$$0 = 25.689 - A_2 - 33.75A_3 \qquad (3\text{-}10)$$

and

$$0 = 21.435 - A_2 - 36.00A_3, \qquad (3\text{-}11)$$

which in turn yielded

$$A_2 = 89.477$$

and

$$A_3 = -1.89.$$

A_2 and A_3 were substituted into equation (3-4) to determine the v-deflections. These results, along with the evaluation of the u-deflections, are compared with the results of Timoshenko in Table IV.

TABLE IV

DEFLECTIONS OF CANTILEVERED PANEL

		u		v		
x	y	Timoshenko	Rayleigh-Ritz	Timoshenko	Rayleigh-Ritz	Adjusted
0.0	7.5	-0.00104	-0.00126	0.00320	0.00755	0.00660
	0.0	0.0	-0.00002	0.00320	0.00380	0.00300
	-7.5	0.00104	0.00120	0.00320	0.00295	0.00200
-15.0	7.5	-0.00074	-0.00117	0.00105	0.00243	0.00148
	0.0	0.0	0.0	0.00100	0.00223	0.00138
	-7.5	0.00074	0.00117	0.00105	0.00243	0.00148
-30.0	7.5	-0.00016	0.0	0.00100	0.00096	0.0
	0.0	0.0	0.0	0.0	0.00086	0.0
	-7.5	0.00016	0.0	0.00100	0.00096	0.0

Table IV also contains a column of adjusted values for the v-deflections. The v-deflections at the support should be zero, however this condition could not be obtained from the assumed stress function. Although an average v-deflection of zero at the fixed support was imposed, and other attempts to help v vanish did not succeed, the v-deflections at the fixed support were assumed to be the reference null values.

The results indicated in Table IV compare favorably with the results of Timoshenko. The differences are caused primarily by the difference in loading used in this investigation and that used by Timoshenko and by failure to satisfy compatibility precisely. As pointed out in Chapter II, the application of a shear load on the free end of the panel does not accurately approximate the actual loading, therefore the results of Timoshenko do not show a difference in v-deflections between the upper free edge and the lower free edge. With the assumed loading of this study a difference in v-deflections between the two edges does exist. The v-deflection at the center of the free end does compare favorably with the value given by Timoshenko. It should also be noted that the average value of the v-deflections at the free end compares with the value given by Timoshenko. The u-deflections differ in a similar manner. Those given by the methods used in this investigation are slightly larger than those given by Timoshenko. Deflections at several cross sections are shown in Figures 11 and 12.

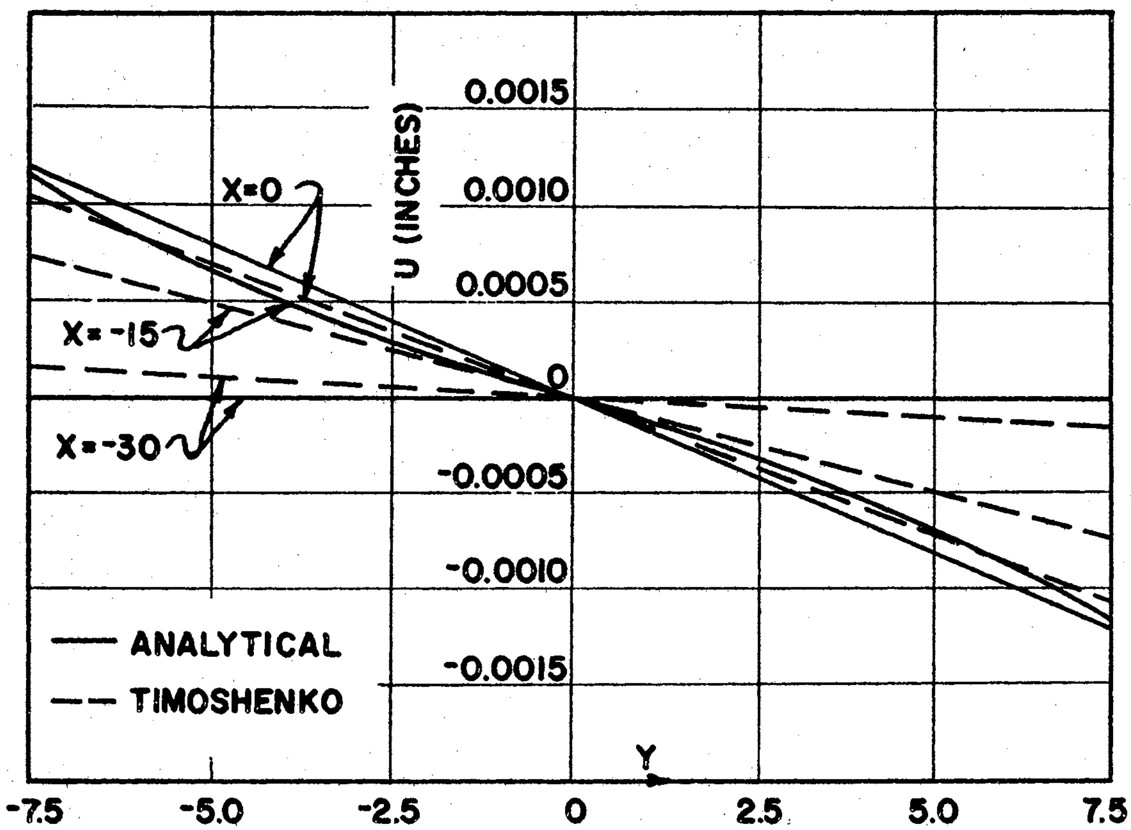

Figure 11. u-deflections of Panel at Several Cross Sections

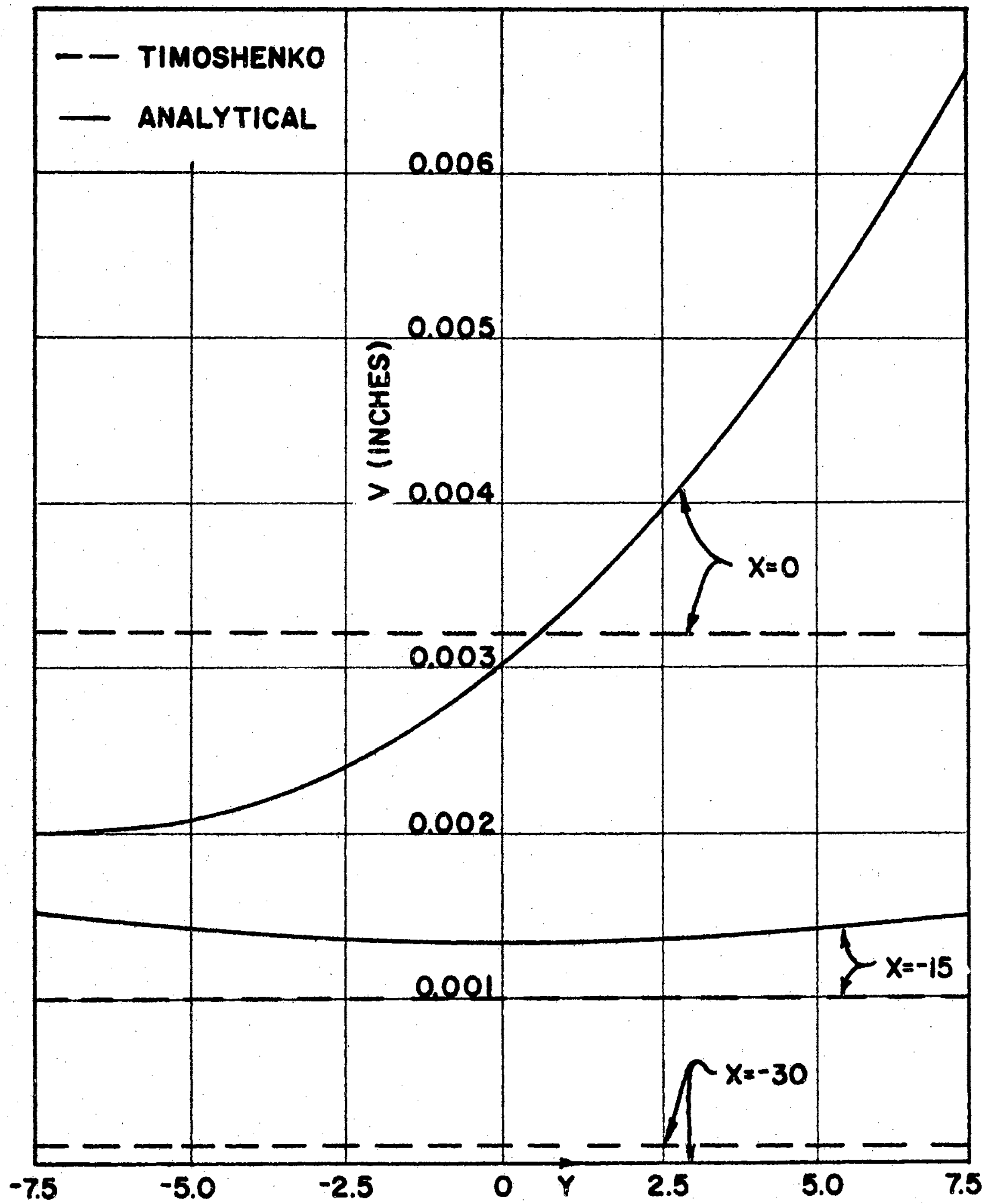

Figure 12. v-deflections of Panel at Several Cross Sections

CHAPTER IV

THE DERIVATION OF STRESSES IN A CANTILEVERED STIFFENED PANEL

The method of Chapter II was extended to a stiffened panel by evaluating the effects of a change in cross section on various parameters of the physical configuration. The initial assumption in this investigation was that the problem was one of plane stress for which the significant parameters are the thickness, the moment of inertia, and the first moment of area about the neutral axis.

σ_x Stresses

The nature of the applied load suggested that the most significant parameter in the determination of the σ_x stresses was the moment of inertia because bending was a dominant feature. The σ_x stresses in the stiffened panel were determined from the equation

$$\sigma_{x_{sp}} = \frac{I_p}{I_{sp}} \sigma_{x_p} . \qquad (4-1)$$

This procedure resulted in an approximation to the σ_x stresses in the stiffened panel.

The analytical values shown in Table V were calculated

TABLE V

σ_x STRESSES IN STIFFENED PANEL

x	y	Analytical	Ayres' Experimental	Ayres' Theoretical
0.0	7.5	0	-	-
	5.0	0	-	-
	2.5	0	-	-
	0.0	0	-	-
	-2.5	0	-	-
	-5.0	0	-	-
	-7.5	0	-	-
- 5.0	7.5	- 829	-1050	-1050
	5.0	- 553	- 178	-
	2.5	- 274	50	- 300
	0.0	0	339	-
	-2.5	274	300	450
	-5.0	553	488	-
	-7.5	829	850	650
-10.0	7.5	-1682	-	-
	5.0	-1126	-	-
	2.5	- 563	-	-
	0.0	0	-	-
	-2.5	563	-	-

TABLE V (Continued)

x	y	Analytical	Ayres' Experimental	Ayres' Theoretical
	-5.0	1126	-	-
	-7.5	1682	-	-
-15.0	7.5	-2531	-2050	-2400
	5.0	-1697	-1464	-1625
	2.5	- 851	- 650	- 725
	0.0	- 0.04	112	50
	-2.5	851	750	875
	-5.0	1697	1518	1600
	-7.5	2531	2050	2400
-20.0	7.5	-3313	-	-
	5.0	-2225	-	-
	2.5	-1118	-	-
	0.0	- 0.12	-	-
	-2.5	1117	-	-
	-5.0	2225	-	-
	-7.5	3313	-	-
-25.0	7.5	-4085	-3200	-3950
	5.0	-2731	-2425	-
	2.5	-1368	-1150	-1075
	0.0	- 0.34	- 80	-
	-2.5	1367	950	1075

TABLE V (Continued)

x	y	Analytical	Ayres' Experimental	Ayres' Theoretical
	-5.0	2731	2430	-
	-7.5	4085	3650	3950
-30.0	7.5	-5072	-	-
	5.0	-3324	-	-
	2.5	-1646	-	-
	0.0	- 0.75	-	-
	-2.5	1645	-	-
	-5.0	3325	-	-
	-7.5	5072	-	-

from equation (4-1). The numerical values of the moments of inertia were $I_{sp} = 44.9$ in^4 and $I_p = 281.25$ in^4. Ayres' experimental values were obtained by calculating the stresses based on axial strain gage readings at the points shown. Ayres' theoretical values were obtained with a stiffness analysis.

Generally, the analytical σx values of this investigation compare favorably with Ayres' theoretical values. However, Ayres' experimental values vary greatly from the values obtained in this analysis. The largest difference between the analytical values and Ayres' theoretical values occurs at a cross section five inches from the free end. The reason for this difference will be discussed later. These results are shown in Figure 13.

A comparison of the σx stresses at a vertical cross section through the middle of the panel is shown in Figure 14. The section was chosen so that the values from the beam equation,

$$\sigma_x = \frac{My}{I}, \qquad (4\text{-}2)$$

could be included. The Principle of St. Venant should apply at this cross section, therefore the results obtained from equation (4-2) should closely approximate the true values. The analytical results of this investigation compare very well with the values obtained from equation (4-2). The theoretical values of Ayres are less everywhere

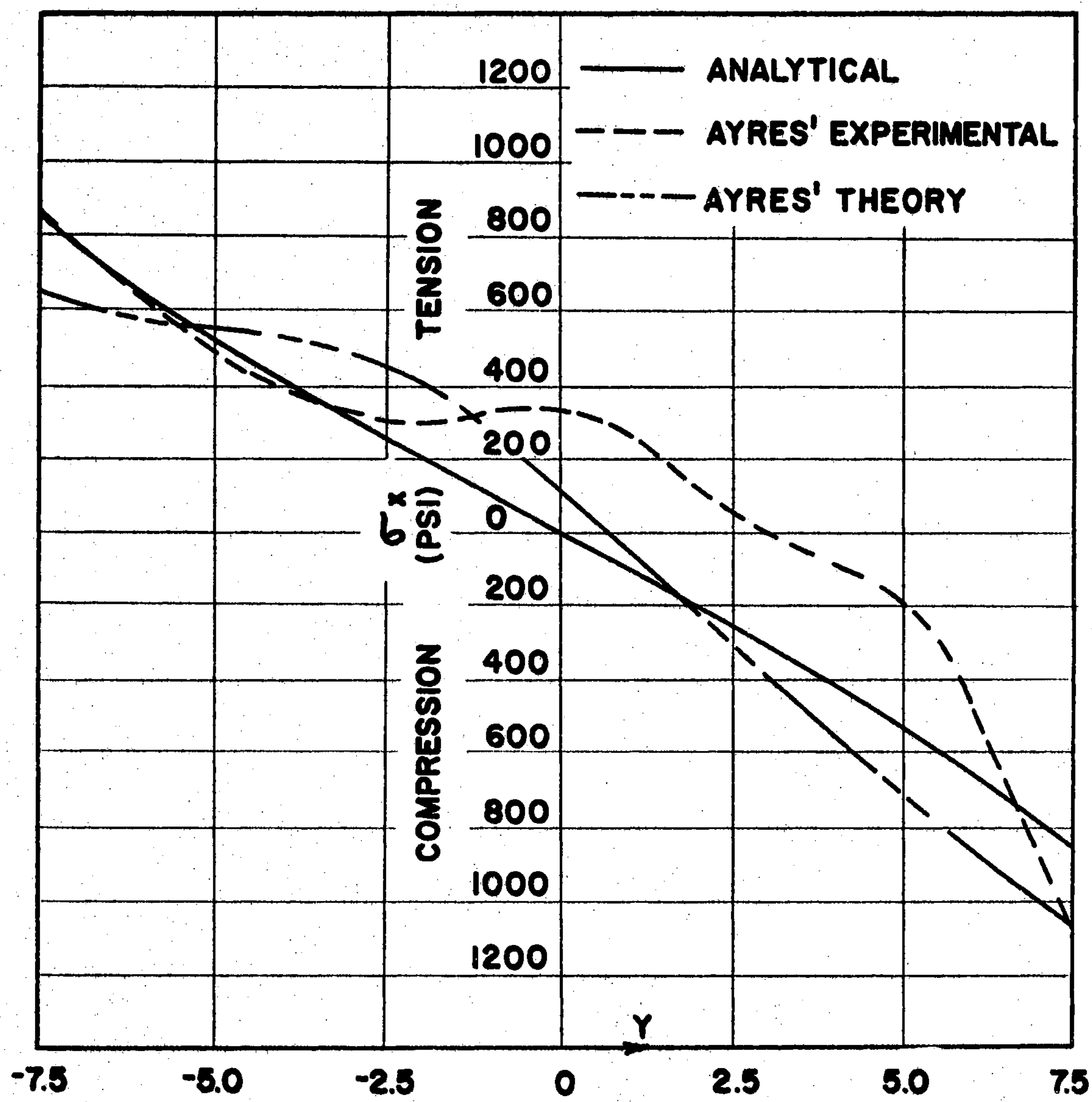

Figure 13. Comparison of σx Stresses at a Section 5 Inches from the Free End

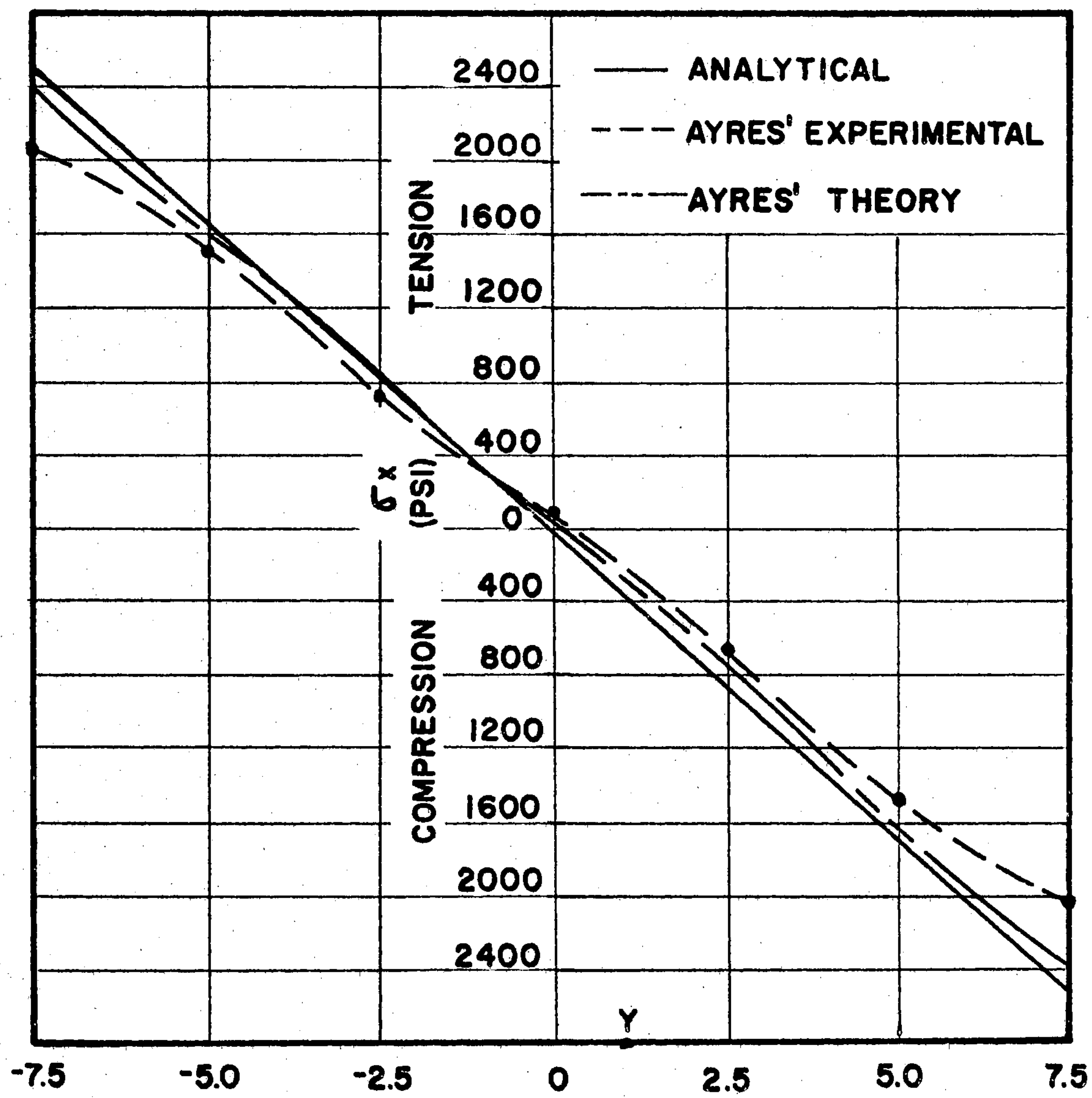

Figure 14. Comparison of σx Stresses at a Section 15 Inches from the Free End

by approximately 5%; Ayres' experimental values are less by approximately 18%. The resisting mament on the cross section, calculated from the results of this investigation, was within 5% of the actual moment. The moment calculated from the experimental results of Ayres was 18% less than the actual moment. These observations indicate that the experimental results were in error because moment equilibrium is not satisfied.

A comparison of the values obtained in this analysis and the theoretical and experimental values of Ayres at a section twenty-five inches from the free end is shown in Figure 15. Again, the results of this analysis compare more favorably with Ayres' theoretical values than with Ayres' experimental values.

Several factors could be responsible for the apparent error in Ayres' experimental results. The experimental stresses were obtained by calculations based on surface strain readings. The surface strains developed in the panel, particularly in the stringers, do not represent the true strains over the entire thickness. A variation of strain may exist with the maximum value occurring at the mid-point of the section. A photoelastic analysis of a "T" section beam subjected to pure bending was performed by Shah (15), whose results show that the axial surface stresses in the large portion of the "T" are approximately 7% less than the axial stresses at the center of the section.

A slight error could have been incurred because of an

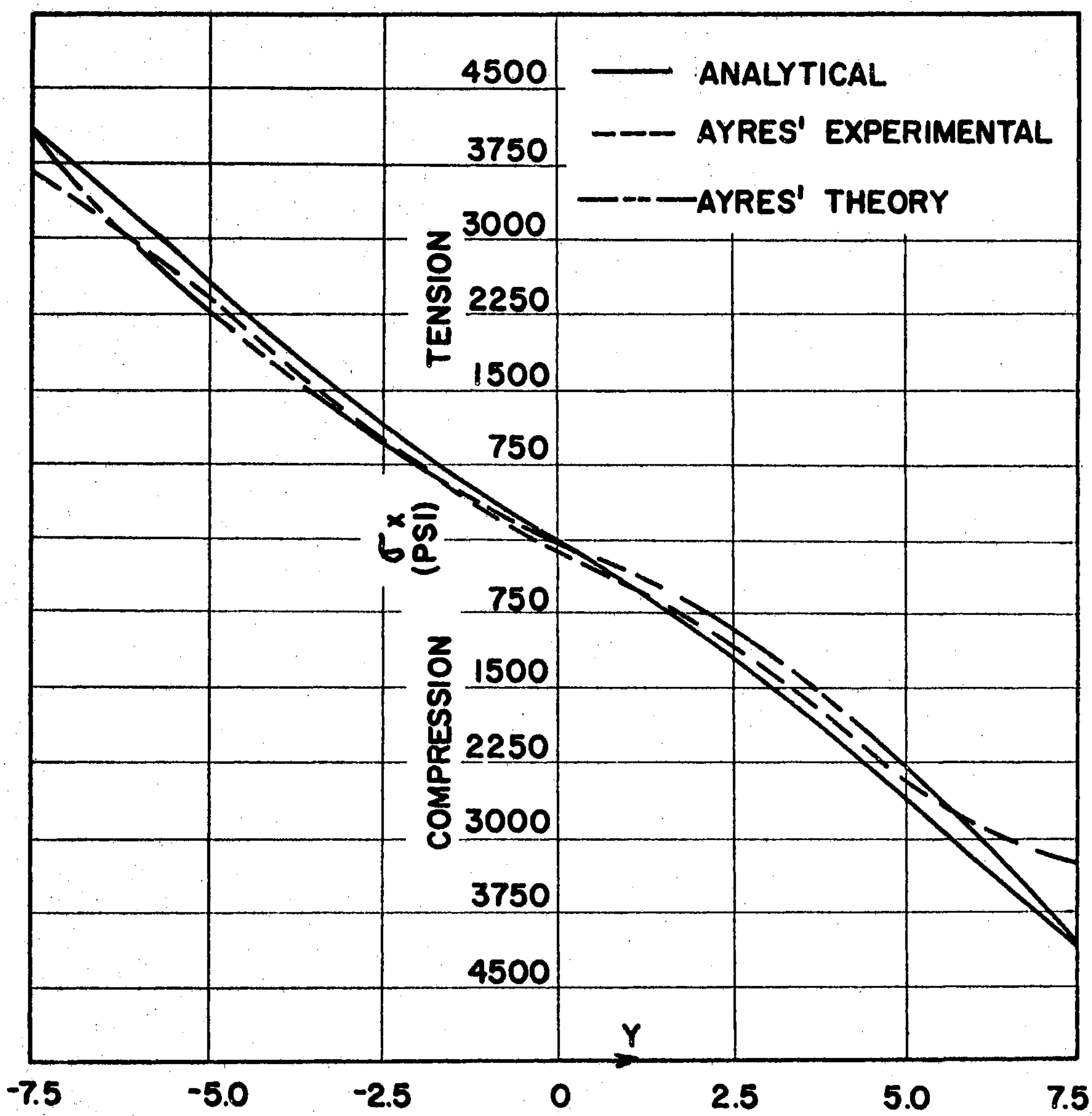

Figure 15. Comparison of σ_x Stresses at a Section 25 Inches from the Free End

error in the applied load. A serious error in this respect is improbable however, because the load was applied through a calibrated strain gage load cell.

The difference between the results of this investigation and the theoretical results of Ayres could partly result because of the approximations used in each analysis. The Stiffness Method is a finite element approximation and, as such, the number of elements has a direct bearing on the accuracy of the results. The Method of Timoshenko is also an approximation. The accuracy of the stress function, hence the stress values, depends on the number of terms used in the approximation.

τ_{xy} Stresses

The shear stress in a cantilevered beam is evaluated from the equation

$$\tau = \frac{VQ}{It} .$$

The significant parameters are I, the moment of inertia of the cross section, Q, the first moment of the area above the point at which the stress is to be evaluated, and t, the thickness at the point at which the shear stress is evaluated.

The shear stresses in the unstiffened panel were extended to the stiffened panel by means of the equation

$$\tau_{xy_{sp}} = K\,\tau_{xy_p}, \tag{4-3}$$

where k was determined from the equation,

$$K = \left(\frac{Q}{It}\right)_{sp} \bigg/ \left(\frac{Q}{It}\right)_{p} .$$

This procedure resulted in an approximation to the τxy stresses in the stiffened panel. The value of k varies with y at any vertical cross section. Values of k for various points on a vertical cross section are shown in Table VI.

The results of equation (4-3) and the experimental data of Ayres' are shown in Table VII. The data of Ayres were obtained by calculating the stresses based on strain rosette values at the points shown. A large difference between the analytical τxy values and Ayres' experimental values exists at every cross section.

A comparison of the τxy stresses at a section five inches from the free end is shown in Figure 16. There appears to be no correlation between the data at all.

A comparison of the analytical values and Ayres' experimental values at a section fifteen inches from the free end is shown in Figure 17. Once again, no correlation is evident.

A comparison of the shear stresses at a point 17.5 inches from the free end of the panel is shown in Table VIII and in Figure 18. The results of this investigation compare

TABLE VI

k VALUES

y	Vertical Sections at x=0,-10,-20,-30	Vertical Sections at all other x's
± 6.25	6.88	29.16
± 5.0	6.88	17.35
± 3.75	6.88	14.43
± 2.5	6.88	1.76
± 1.25	6.88	13.65
0.0	6.88	13.44

TABLE VII

τ_{xy} STRESSES IN STIFFENED PANEL

x	y	Analytical	Ayres' Experimental
0.0	7.5	0	-
	5.0	0	-
	2.5	0	-
	0.0	0	-
	-2.5	0	-
	-5.0	0	-
	-7.5	0	-
- 5.0	7.5	0	-
	5.0	975	599
	2.5	158	-
	0.0	1361	1065
	-2.5	158	-
	-5.0	975	751
	-7.5	0	-
-10.0	7.5	0	-
	5.0	385	-
	2.5	617	-
	0.0	695	-
	-2.5	618	-
	-5.0	386	-

TABLE VII (Continued)

x	y	Analytical	Ayres' Experimental
	-7.5	0	-
-15.0	7.5	0	-
	5.0	970	726
	2.5	157	-
	0.0	1354	1070
	-2.5	157	-
	-5.0	973	871
	-7.5	0	-
-20.0	7.5	0	-
	5.0	385	-
	2.5	613	-
	0.0	689	-
	-2.5	614	-
	-5.0	386	-
	-7.5	0	-
-25.0	7.5	0	-
	5.0	977	751
	2.5	150	-
	0.0	1332	981
	-2.5	156	-
	-5.0	979	915

TABLE VII (Continued)

x	y	Analytical	Ayres' Experimental
	-7.5	0	-
-30.0	7.5	0	-
	5.0	392	-
	2.5	600	-
	0.0	664	-
	-2.5	597	-
	-5.0	389	-
	-7.5	0	-

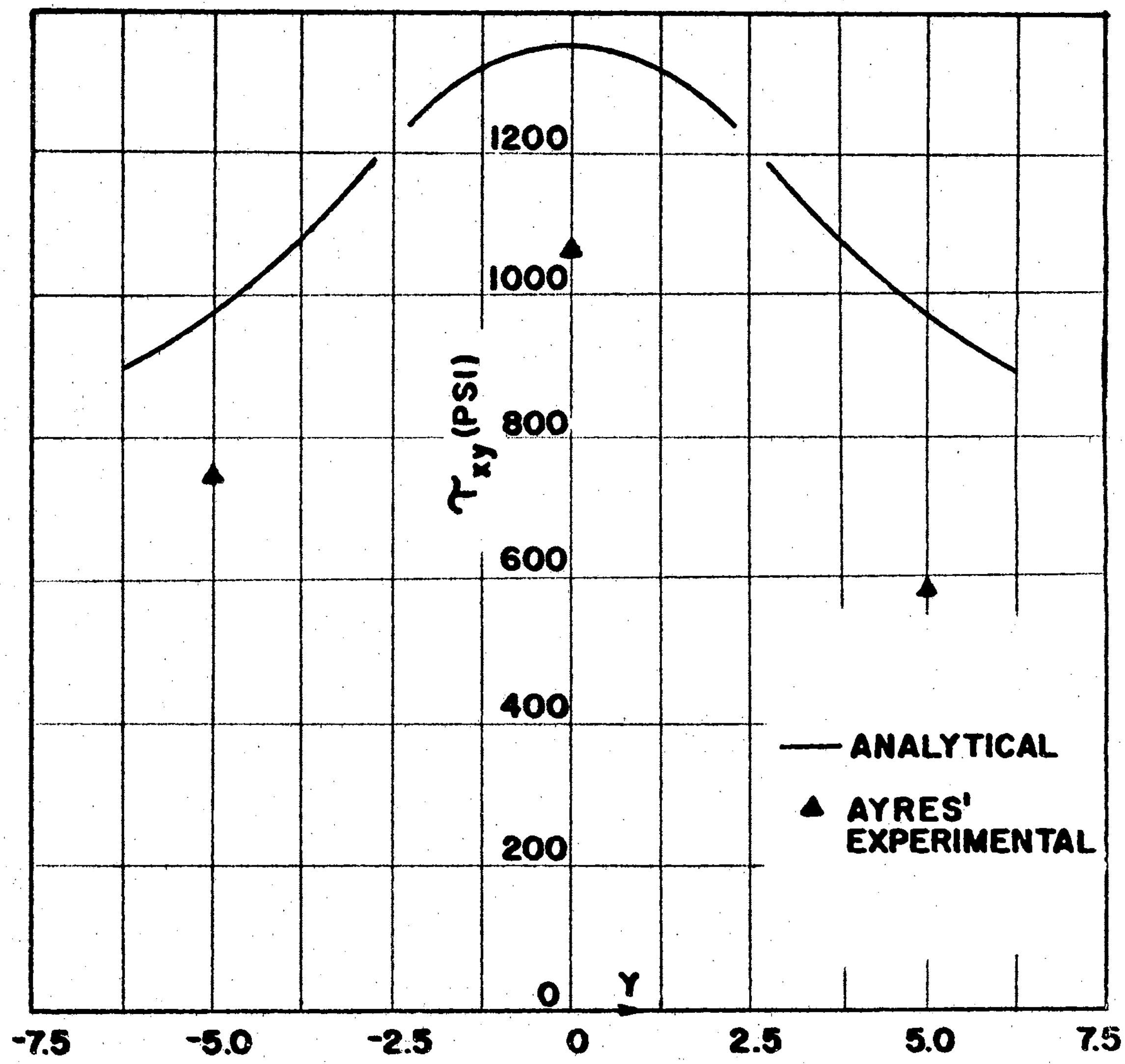

Figure 16. Comparison of τ_{xy} Stresses at a Section 5 Inches from the Free End

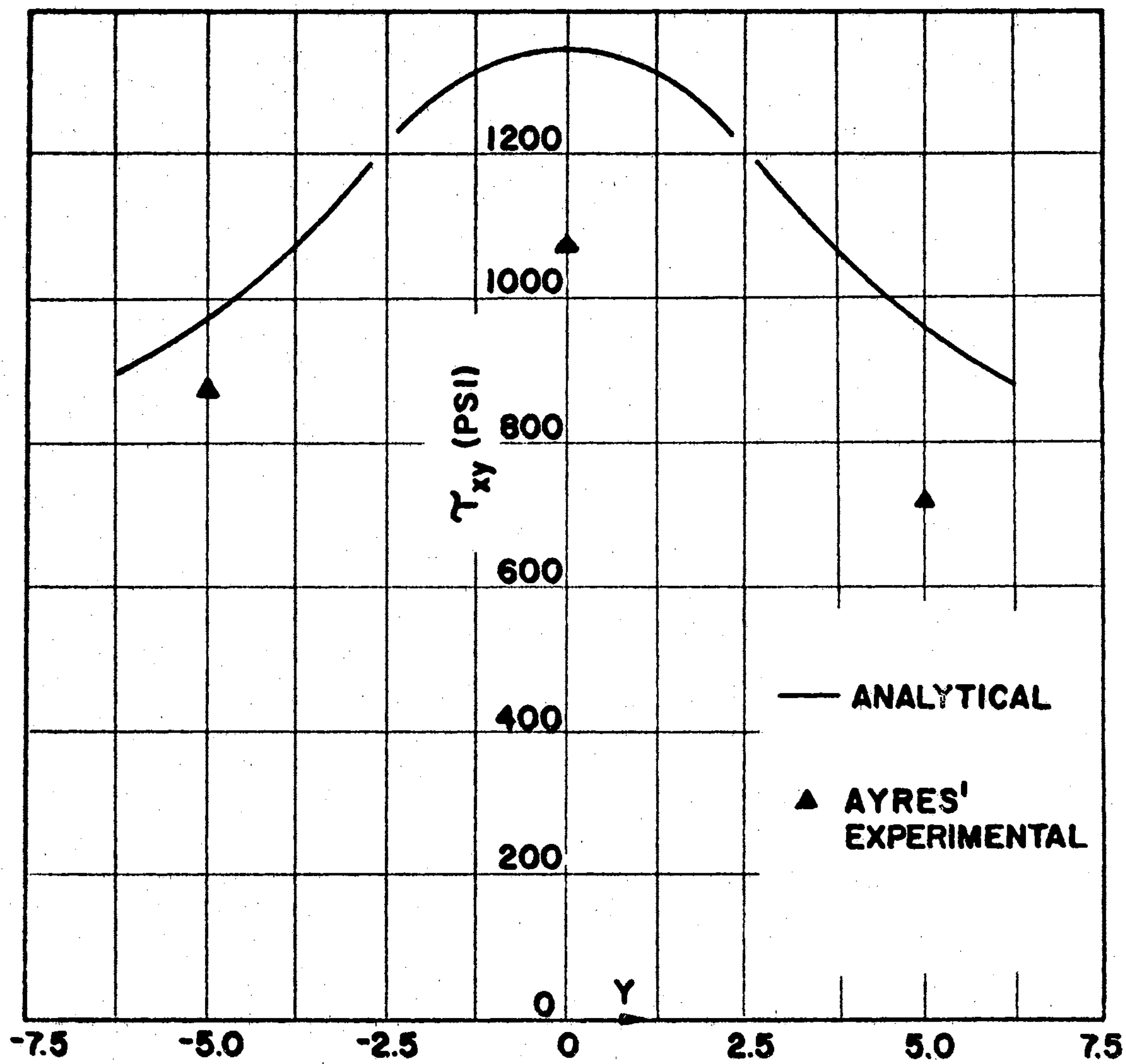

Figure 17. Comparison of τ_{xy} Stresses at a Section 15 Inches from the Free End

TABLE VIII

COMPARISON OF τ_{xy} STRESSES AT A CROSS-SECTION

17.5 INCHES FROM THE FREE END

y	Ayres' Experimental	Ayres' Theoretical	Analytical
6.25	750	-	892
5.0	800	1020	970
3.75	860	-	1085
1.25	940	-	1331
0.0	960	1320	1348
-1.25	980	-	1331
-3.75	900	-	1085
-5.0	800	960	970
-6.25	760	-	892

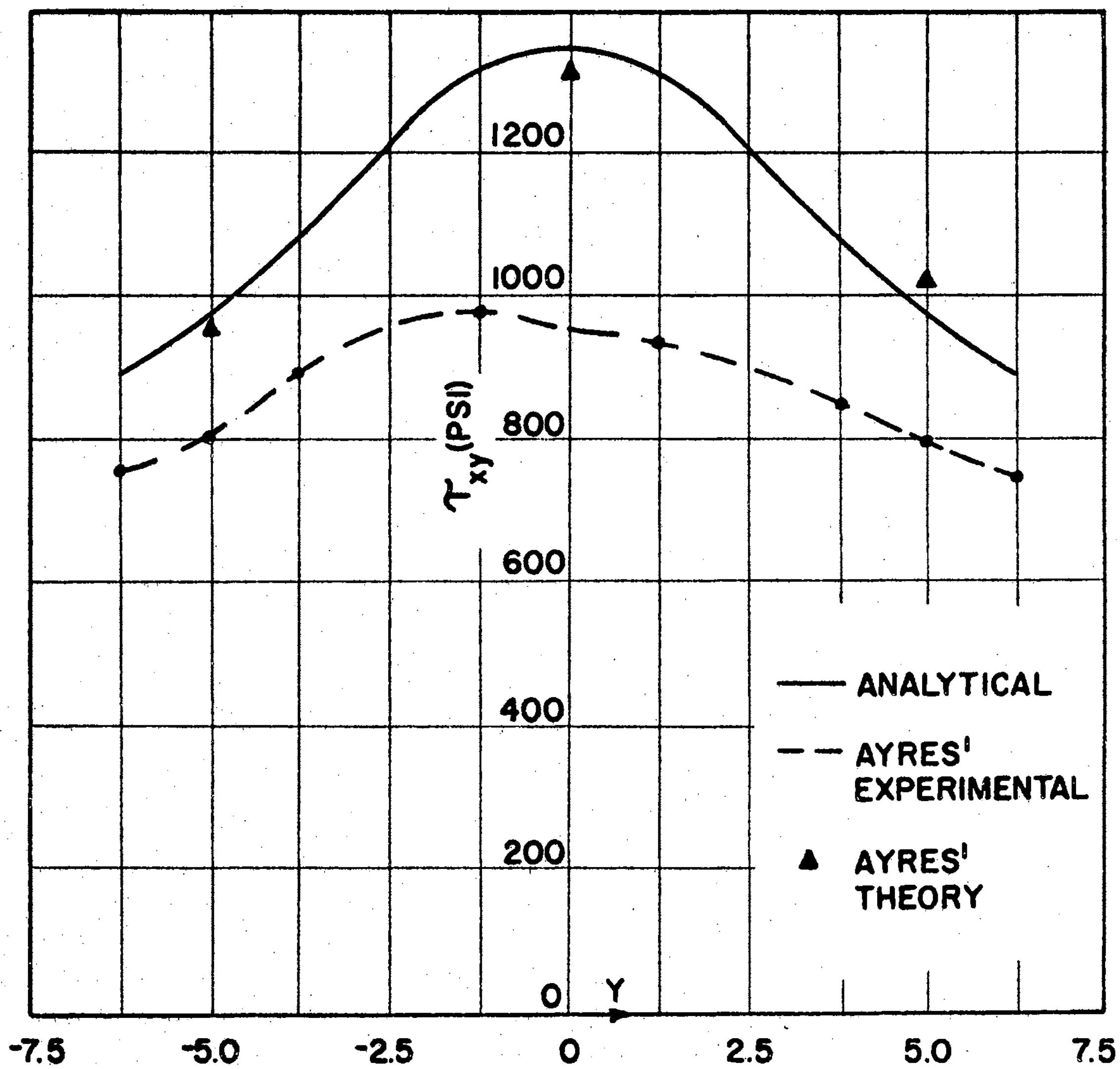

Figure 18. Comparison of τ_{xy} Stresses at a Section 17.5 Inches from the Free End

favorably with the theoretical values of Ayres. The largest difference between these values is 5%. The experimental values differ by as much as 43% with the analytical values. Experimental data were not reported for the stringers, therefore an evaluation of the results in impossible. The summation of the shear forces on any vertical cross section should balance the external load. The analytical data are within 5% of this requirement. The summation of the shear forces based on experimental data cannot balance the external load because all data points are less than those obtained in this analysis. This evidence seems to indicate that the experimental data are in error. Possible reasons for the error have been discussed previously. A comparison of the analytical and experimental τ_{xy} values at a section twenty-five inches from the free end is shown in Figure 19. The difference in all values is extremely large.

The salient features of the previous discussion are:

1. Analytical results differed from experimental results by 18% for σ_x stresses and by 43% for τ_{xy} stresses. Experimental results do not satisfy the fundamental equilibrium requirements.
2. Analytical results compare favorably with elementary theory.
3. Analytical results compare favorably with Ayres' theoretical analysis.
4. Analytical results satisfy equilibrium more closely than Ayres' experimental results.

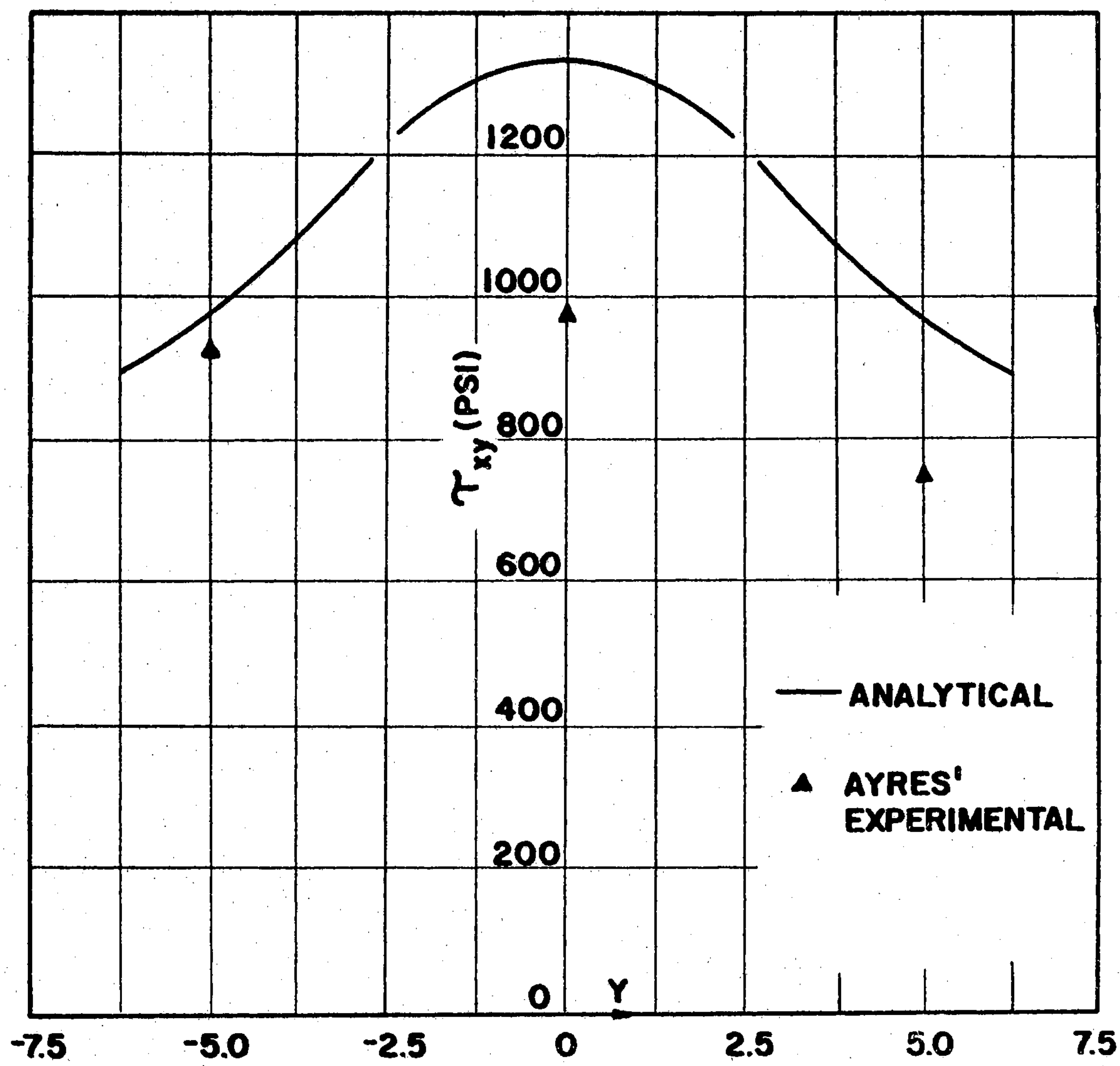

Figure 19. Comparison of τ_{xy} Stresses at a Section 25 Inches from the Free End

CHAPTER V

THE DERIVATION OF DEFLECTIONS FOR A CANTILEVERED STIFFENED PANEL

The deflections of a panel depend upon several parameters of the panel. These parameters are the moment of inertia of the cross section, the modulus of elasticity, and the shearing modulus. The unstiffened panel was of the same material as the stiffened panel, therefore the most significant parameter is the cross sectional moment of inertia. The method of Chapter III was extended to a stiffened panel by evaluating the effect of a change in cross section on the deflections.

Formulation of Deflection Equations

The deflections of the stiffened panel were determined from the equations

$$u_{sp} = \frac{I_p}{I_{sp}} u_p \qquad (5\text{-}1)$$

and

$$v_{sp} = \frac{I_p}{I_{sp}} v_p \; . \qquad (5\text{-}2)$$

This procedure resulted in an approximation to the deflections of the stiffened panel.

The results of equations (5-1) and (5-2) are shown in Table IX. The numerical values of the moments of inertia were $I_{sp} = 44.9\ in^4$ and $I_p = 281.25\ in^4$. Experimental data were available from a study by Ayres for a few selected points. A comparison of the v-deflections along the top stringer is shown in Figure 20. No experimental data for u-deflections were reported by Ayres, therefore a comparison was impossible.

The results shown in Figure 20 indicate a variance in the v-deflections of Ayres' data with the results of this investigation. The deflection of a cantilevered beam may be accurately represented in the form of a cubic equation. A cantilevered panel should exhibit similar characteristics. Ayres' data appears to represent a straight line. A deflection curve of this type should be exhibited by a panel in pure shear, however the loading on the panel in this investigation induced shear and moment at every section. The experimental curve is defined by only three data points. An error in one data point could markedly change the shape of the curve, therefore the accuracy of such a curve could be questioned. The analytical results shown in Figure 20 appear to exhibit the trend expected because of the nature of the physical loading. An evaluation of the accuracy of the results is difficult because the error involved in the Rayleigh-Ritz Method cannot be evaluated.

TABLE IX

u AND v DEFLECTIONS OF

STIFFENED PANEL

x	y	u	v
0.0	7.5	-0.00793	0.0413
	3.75	-0.00392	0.0279
	0.0	-0.00012	0.0188
	-3.75	0.00373	0.0136
	-7.5	0.00753	0.0125
-15.0	7.5	-0.00735	0.0093
	3.75	-0.00282	0.0085
	0.0	0.0	0.0085
	-3.75	0.00282	0.0085
	-7.5	0.00735	0.0093
-30.0	7.5	0.0	0.0
	3.75	0.0	0.0
	0.0	0.0	0.0
	-3.75	0.0	0.0
	-7.5	0.0	0.0

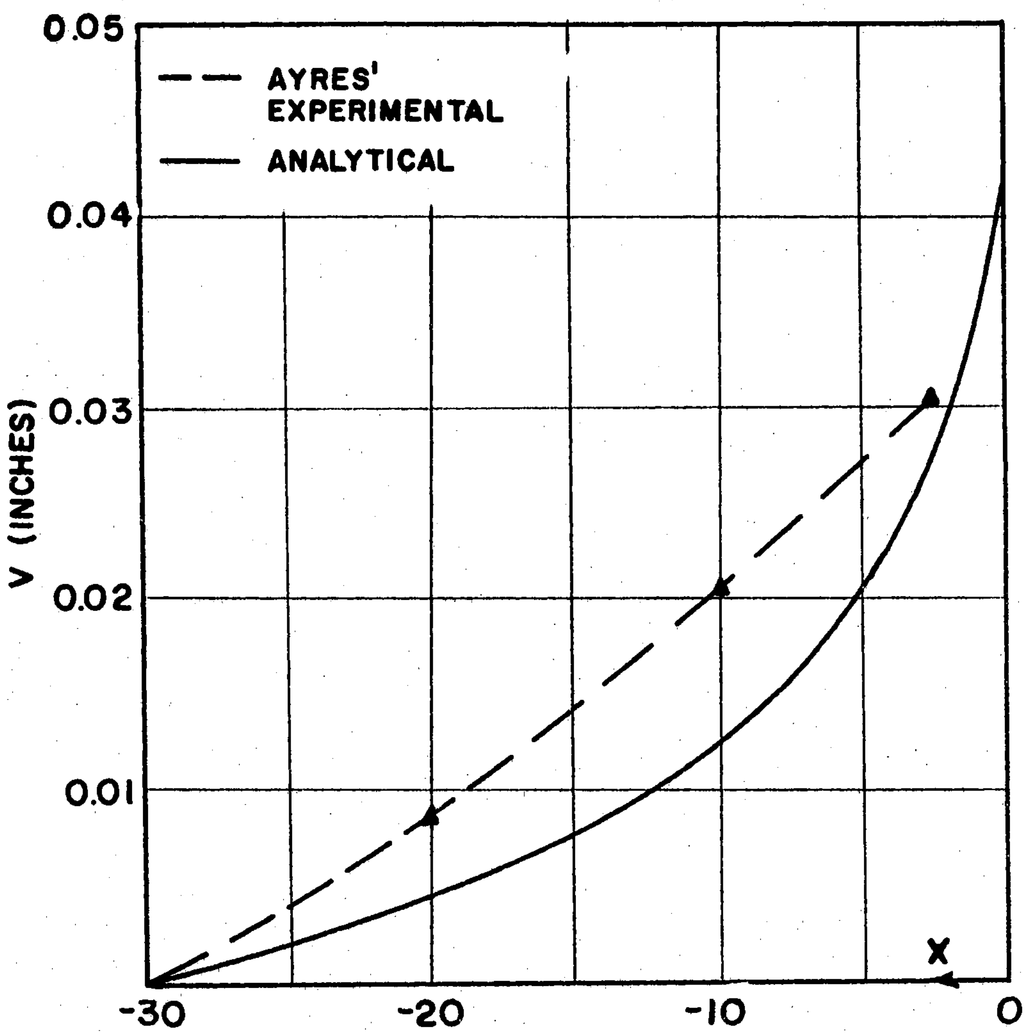

Figure 20. Comparison of v-deflections Along the Upper Stringer

Several factors were evident which would tend to cause the analytical results and Ayres' experimental results to differ. The mathematical model and the physical model differed in the manner in which the fixed end of the panel was represented. Four bolts were used to clamp the panel in place. Whether this method of clamping actually represents a fixed support is subject to debate. Indeed, Ayres reported some rotation of the panel at the support. This rotation was subtracted from the readings of the dial indicators to determine the deflections. The accuracy of the dial indicators is also subject to question.

Some error is inherent in the mathematical model because of the manner in which the concentrated load was represented by a distributed load, however the error should be very slight.

The analytical results shown in Figure 20 exhibit the trends expected with the type of loading of this analysis. Large strains should be evident at the applied load and decrease rapidly as the fixed support is approached. Ayres' experimental results do not appear to yield trends which are consistent with the physical loading.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The resulting σ_x and τ_{xy} stresses obtained with the Method of Timoshenko differ by as much as 18% and 43% respectively from Ayres' experimental values. However, Ayres' experimental values for σ_x stresses did not satisfy moment equilibrium and his τ_{xy} stresses did not satisfy force equilibrium. The σ_x and τ_{xy} stresses obtained with the Method of Timoshenko compare within 5% of Ayres' theoretical values which were obtained with a stiffness analysis. Ayres' theoretical σ_x values and the σ_x values obtained with the Method of Timoshenko both satisfy moment equilibrium within 5%. Ayres' theoretical τ_{xy} values and the τ_{xy} values obtained with the Method of Timoshenko both satisfy force equilibrium within 5%. From these results it can be concluded that the Method of Timoshenko can be successfully applied to a cantilevered stiffened panel.

The Rayleigh-Ritz Method produced deflections for the cantilevered stiffened panel. A valid evaluation of these deflections was impossible because of the lack of sufficient experimental data. The available experimental data were of questionable reliability because of the manner in which they were obtained. The magnitude of the deflections and the deflection characteristics of this study were reasonably realistic. The conclusion drawn from these results is that the Rayleigh-Ritz Method can be successfully applied to a cantilevered stiffened panel.

There are no specific guidelines to indicate the precise distance from the applied load at which St. Venant's Principle may be invoked. The results of the analysis indicate that St. Venant's Principle may be applied in the central portion of the panel; however, there is no evidence to indicate that such is the case near the applied load. The results of this investigation apply near the applied load as well as in other sections of the panel.

Both solutions, for stresses and deflections, have been checked with the classical Timoshenko analysis of an unstiffened cantilever problem. The resulting σx stresses were within 1.24% of those obtained from the Timoshenko analysis. The resulting σy stresses deviate sharply from the results of Timoshenko, but this deviation arises because of the difference in the description of the applied load. The resulting τxy stresses were within 3.2% of the results of the Timoshenko analysis. Therefore, it is concluded that the Method of Timoshenko and the Rayleigh-Ritz Method can be successfully applied to an unstiffened cantilevered panel.

The difficulties encountered in this investigation were the result of the selection of the stress functions and the deflection functions. These difficulties have been discussed in Chapter II. The functions should satisfy as many boundary conditions and physical conditions as possible, yet be expressed in a simple form. Otherwise the mathematical manipulations become extremely cumbersome, if not impossible. The investigation which was begun using the deflection functions in Appendix B with the Rayleigh-Ritz Method yielded unsatisfactory results because it was impossible to define sufficient boundary conditions. The Method of Timoshenko was then used with the stress function of Appendix A. The nature of this particular function resulted in unwieldy expressions in attempting to minimize the strain energy.

Thus, it is now apparent that the Method of Timoshenko and the Rayleigh-Ritz Method, subject to the restrictions given above, fulfills the need for a method which will yield results as accurate as finite element methods. This method could be applied to other areas such as:

1. An investigation of the effects of various aspect ratios on the stress and deflection values of a rectangular stiffened panel.
2. An investigation of the stresses and deflections of skin panels of various geometric shapes and various load conditions. The Method of Neou (16) should be examined before selecting a stress function. It is a simplified procedure for reducing stress functions expressed as doubly in-

finite power series to desired polynomial forms on the basis of compatibility and boundary conditions. The Method of Neou was not directly applicable to this analysis.

3. An investigation of skin panels with cutout sections using the methods outlined in this analysis. A serious problem could arise in the selection of a stress function which will satisfy boundary conditions, including those of the cut-out section.

4. A photoelastic analysis of skin panels of various geometric shapes and load conditions. Such analyses would be invaluable in order to corroborate the results of previous investigations.

5. A thorough similitude study of the parameters of panels with various cross sections.

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APPENDIX A

THE USE OF POLYNOMIAL EXPRESSIONS TO APPROXIMATE STRESS FUNCTIONS

The Method of Timoshenko will yield excellent results if the stress function completely describes physical conditions. The stress function selected in this appendix was written in general terms to take advantage of this fact. The polynomial expression used to approximate the stress function for the unstiffened panel was

$$\phi = A_{qp}\eta^{2q+2}\xi^{2p+1}(1-\xi^2)^2 a^{2q+2} b^{2p+1} \tag{A-1}$$

with the coordinate axes shown in Figure 21. Equation (A-1) was differentiated to obtain σx, σy, and τxy. These expressions were substituted into the stress-strain relationships

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{1}{E}(\sigma_x - \mu\sigma_y) \tag{A-2}$$

and

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{1}{E}(\sigma_y - \mu\sigma_x) . \tag{A-3}$$

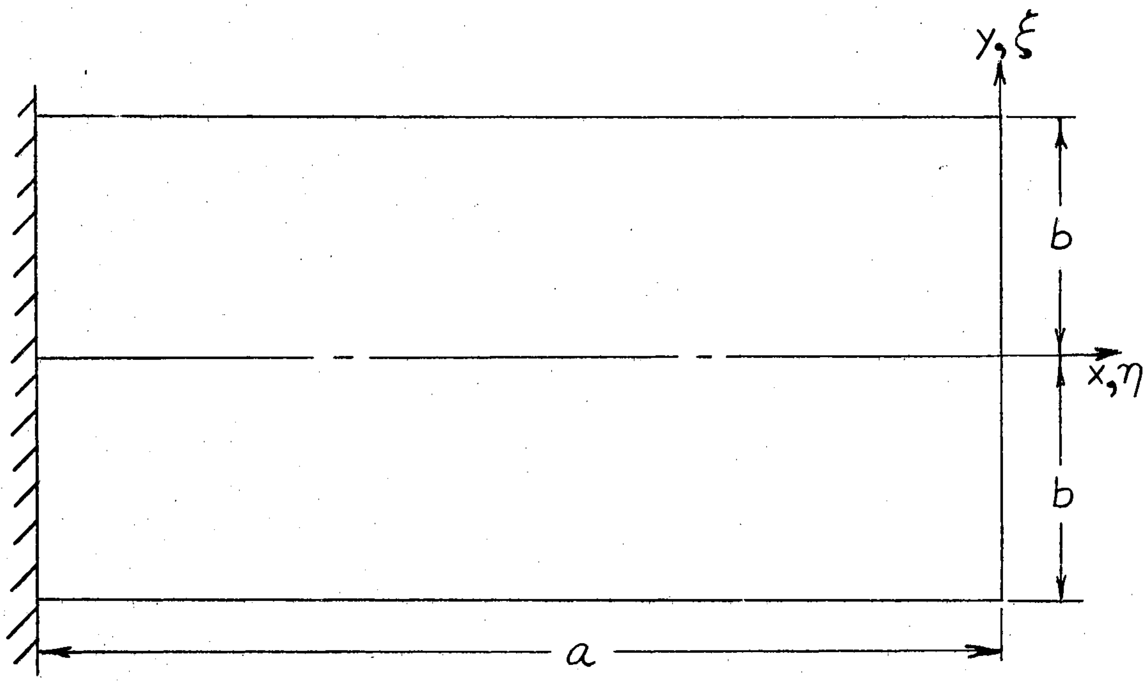

Figure 21. Coordinate Axes for Stress Function

Equation (A-2) was integrated to obtain the expression for u which contained a function of y, f(y). Equation (A-3) was integrated to obtain an expression for v which contained a function of x, g(x). These functions, f(y) and g(x), were evaluated by applying the boundary conditions on the expressions for u and v. The potential energy of the applied load was calculated using the v-deflection equation. The strain energy in the panel, based on stresses, was calculated. At this point the equations became too unwieldy to be of any further use. This stress function was discarded.

A second approximation was attempted by defining individual stresses with undetermined coefficients. The assumed stresses were

$$\sigma_x = 6Dxy,$$

$$\sigma_y = 6B(x+a)(y+b),$$

and

$$\tau_{xy} = 3D(b^2-y^2) - 3B(x+a)^2.$$

The procedure was the same as that used in the previous approximation. No satisfactory values for the coefficients B and D were obtained, therefore these equations were discarded.

APPENDIX B

THE USE OF TRIGONOMETRIC EXPRESSIONS TO APPROXIMATE DEFLECTION EQUATIONS

The Rayleigh-Ritz Method will yield exceptionally accurate results if the deflection functions closely approximate physical conditions. The fixed conditions at the support were the only boundary conditions available.

Several trigonometric expressions were used to approximate the deflection equations for the stiffened panel. These deflection equations were

(a) $$u = \sum \sum a_{mn} \sin \frac{m\pi x}{2B} \cos \frac{n\pi y}{2H} ,$$

$$v = \sum \sum b_{rs} \sin \frac{r\pi x}{2B} \cos \frac{s\pi y}{2H} ;$$

(b) $$u = \sum \sum a_{mn} \sin \frac{2m\pi x}{B} \cos \frac{2n\pi y}{H} ,$$

$$v = \sum \sum b_{rs} \sin \frac{2r\pi x}{B} \cos \frac{2s\pi y}{H} ;$$

(c) $$u = \sum \sum a_{mn} \sin \frac{m\pi x}{4B} \cos \frac{n\pi y}{4H} ,$$

$$v = \sum \sum b_{rs} \sin \frac{r\pi x}{4B} \cos \frac{s\pi y}{4H} ;$$

and

$$\text{(d)} \qquad u = \sum\sum a_{mn}\left(1-\cos\frac{m\pi x}{4B}\right)\cos\frac{n\pi y}{4H},$$

$$v = \sum\sum b_{rs}\left(1-\cos\frac{r\pi x}{4B}\right)\cos\frac{s\pi y}{4H}.$$

The reference axes used in this appendix are shown in Figure 22.

The deflection equations given above were applied in the following manner:

1. The strain energy, based on strains, was written for the web section of the stiffened panel.
2. The strain energy of bending and the axial strain energy was written for the stringers.
3. The potential energy of the applied load was written.
4. The total energy of the system was minimized with respect to the undetermined coefficients in the deflection equations which yielded a set of linear simultaneous equations from which the coefficients were evaluated.

The calculations were performed on a digital computer using as many as thirty-nine coefficients. The deflections were from ten to three hundred times too small. No further calculations were considered.

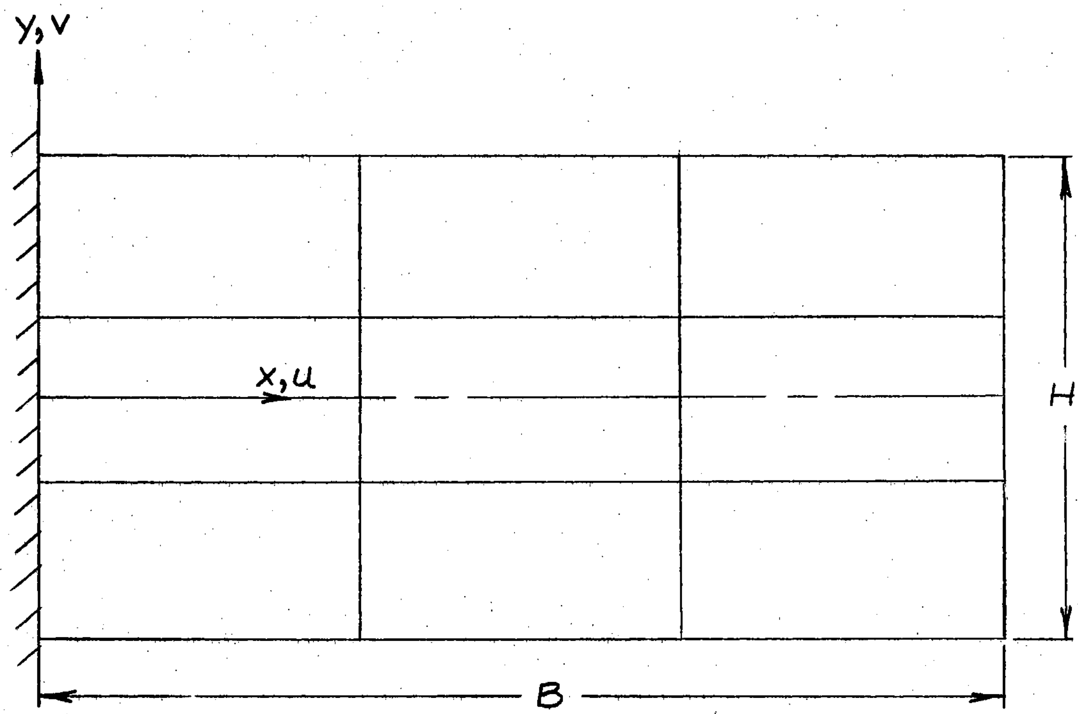

Figure 22. Coordinate Axes for Deflection Functions

VITA

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