A COMPARATIVE STUDY OF SELECTED FACTORS OF MATHEMATICS ACHIEVEMENT IN HOMOGENEOUS GROUPS OF FIFTH GRADE PUPILS USING DISCOVERY

by

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PREFACE

For some time I have been interested in investigating the nature and merits of discovery as a teaching technique and their implications for teaching mathematics to pre-service elementary teachers. This study has accorded me an opportunity to realize this objective.

I am especially grateful to Dr. Vernon Troxel of the College of Education, my thesis advisor, for his counsel and continued encouragement throughout this investigation.

I wish to thank Dr. W. Ware Marsden, Director of Teacher Education, and Dr. Eugene K. McLachlan, Professor of Mathematics, for serving on the Advisory Committee.

A special debt of gratitude is due the fifth grade students, their teachers, and the school authorities of the Holden Elementary School without whose cooperation this study would not have been possible.

To all others who assisted directly or indirectly in this investigation, I express sincere appreciation.

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CHAPTER I

INTRODUCTION

Never before has there been a time when man has placed such a premium on the cultivation and efficient utilization of human resources. Many conflicting issues confront us because we do not have the manpower to resolve them, nor the procedures whereby we may forestall their occurrence. It has become the business of the educator to identify and to test practices that show promise toward these ends.

There are but few school administrators who have not felt the burden of decision-making in this area. Many proposals relative to curricular revisions in mathematics, techniques of teaching mathematics, and ability grouping procedures are competing for attention and subsequent acceptance. It is only by a continued assault on these problems through varied investigations that evidence can be forthcoming so that those in positions of leadership can render sound, productive decisions (4).

One of the most promising practices among the techniques of teaching is the discovery type learning episode. Its merits and shortcomings are freely discussed in current literature.

It is well known that, among other things, two points serve to distinguish between the learning and retention of ideas by low and high ability students of the same grade. The low ability student lacks motivation for learning and the "knack" of knowing how to learn. There are those who feel that motivation for learning and "learning how to learn" are the most valuable contributions of a discovery type episode (8:26). Perhaps we can harness these attributes at judicious points in our educational process and thus move closer to a solution of some of the problems which face society today.

Purpose of Study

The purpose of the present study is two-fold. The study was undertaken to investigate the possibility of reducing the performance gap between ability groups - by improving the performance of low ability groups - through the utilization of a discovery episode; and to analyze the nature of the discovery episode with the objective of using its meritorious aspects in teaching college classes which provide pre-service training in mathematics for elementary teachers.

It seems reasonable to assume that prospective elementary teachers are more likely to be influenced by the techniques and methods used by the college instructor teaching elementary mathematics than by the college instructor in a methods course about teaching elementary mathematics.

It is well known that once students are grouped according to ability, the gap in their levels of performance continues to widen. Perhaps the utilization of some characteristics of the discovery episode will serve to cultivate certain aptitudes of the low ability student and thereby arrest the widening of the gap maybe even narrow it somewhat.

Statement of the Problem

The problem of investigation was as follows: Is it possible to construct an environment by utilizing the characteristics of a discovery episode that will narrow the gap of performance between ability grouped fifth grade pupils in mathematics? It is assumed that this will occur if the low ability students are motivated and have experience in learning how to learn.

Hypothesis

In general, the research hypothesis was that low ability fifth grade students in mathematics would perform at the same level as high ability fifth grade students when the instructor uses a discovery type episode as a model for his presentation.

The specific null hypotheses tested were:

1. There is no significant difference in the performance of the two groups during a sequence of fifteen sessions relative to:
    a. total successes of each group per student
    b. total successes of each group per session
    c. rate of successes of each group per student
    d. rate of successes of each group per session
    e. a post-test
2. There is no significant correlation between teacher ranking and rate of successes for:
    a. the low ability group
    b. the high ability group
3. There is no correlation between results on an achievement test and rate of successes for:

a. the low ability group

b. the high ability group

c. the combined groups as a unit.

Delimitations

This investigation was a flank attack on the problems of ascertaining the nature and merit of the discovery type learning episode and of improving the performance of low ability groups in mathematics.

The delimitations of this study are as follows:

1. The study utilized two fifth grade classes from the same school system. The total number of students involved was forty-seven. This feature of the study will serve to restrict the findings and conclusions.
2. There was only one instructor, the investigator, in the teaching of these classes and the subsequent procurement of data; consequently, teacher influence in terms of personality could not be controlled or cancelled.
3. The two classes had different regularly assigned teachers; therefore, variation in the students' rapport with their teachers would have affected their adjustment to the new environment.
4. Due to the nature of this study it was not feasible to choose the two groups at random. The universe was ability-grouped classes of fifth grade students within a fifty-mile radius of Warrensburg, Missouri. The choice of a sample was dictated by the availability of time for the classes and for the experimenter to conduct the study. A random choice was made of two schools that met the necessary conditions.

Definition of Terms

There are several terms used throughout this report that may require classification for the reader. These are:

Learning Episode. This is a period of time, usually five to twenty minutes in length, during which the instructor and students participate in a discussion of one or more problems leading toward the understanding of a concept.

Discovery. This is a method of conducting a learning episode in which exposition is minimized and active student participation is maximized (a more detailed characterization of this term will occur later).

Success. This is the unit of measurement and is the correct response of a student to a given problem offered under experimental conditions.

Non-verbal Awareness. This is a level of understanding attained by a student characterized by the ability to perform successfully relative to a given concept, but unable to offer a verbalization which is intrinsically sound or coherent.

Exposition. This refers explicitly to verbalization which gives the details of a given problem.

Significant Difference or Statistically Significant. This means that a certain two quantities which are being compared differ by more than can reasonably be attributed to chance variation.

Total Successes. This is the number of successes accrued for the entire experiment.

Rate of Successes. This is the number of successes which occurred during the first thirty seconds of a data-taking episode.

CHAPTER II

REVIEW OF LITERATURE

Introduction

Investigations and literature on the topic of discovery date back many years, but interest in it as a mode of learning has increased considerably during recent years. In 1906 Young published a book on methods of teaching mathematics that encouraged teachers to use what he described as the heuristic method.

> The hueristic method is dominated by the thought that the general attitude of the pupil is to be that of a discoverer, not that of a passive recipient of knowledge. The pupil is expected in a sense to rediscover the subject, though not without profit from the fact the race had already discovered it. The pupil is a child tottering across the room, not a Stanley penetrating the heart of Africa. ....It is the function of the teacher and the text so to present things to be done, so to propose the problems to be solved that they require real discovery on the part of the pupil; that at the same time the steps are within his power, and that he attains in the end a good view of the whole subject. (50:69-70)

There was a break of approximately six years prior to 1955 during which very little research activity was evident, and afterwards a rash of publications appeared that bear directly on the topic (33). The literature reviewed in this study will be centered around that which has been written since 1955 and has dealt with discovery as a method of teaching.

The terms "discovery teaching" and "learning by discovery" have been used many different ways; consequently, it is difficult to find a clear cut definition. Characterizations in the literature vary from pure discovery in which a student must even identify the problem to be solved, to guided discovery in which a student is subjected to a variation of expository teaching. A popular description of discovery learning is in terms of the learner's goal-directed behavior (33). The amount of teacher direction or guidance offered during the learning episode serves to depict the nature of discovery as it is used in a given study. If a student is encouraged to complete a learning task with little or no assistance from the teacher, then he is learning by discovery.

An alternate characterization of discovery learning can be considered in terms of a student's role in the learning episode. The student may be a listener and have a minimal role in the development of a concept, or he may play the principal role in a learning activity which includes the assumption of responsibility regarding the conclusions drawn (8). It can be readily seen that these two characterizations are compatible and that the relative roles played by the student and the teacher represent the crucial issue in the identification of a given learning episode as discovery.

A study was included in this review if it utilized a form of discovery in either the teaching or learning process. In some studies, the learning tasks consisted of mathematical concepts, although they were not the most prevalent..

The related literature will be presented in three sections. First, a review of studies which deal with the topic of discovery at the college level; second, those studies concerned with junior and senior high school level; and third, the studies pertaining to grades five and six.

Studies Related to College

Hendrix (24) conducted a sequence of three studies in 1947 utilizing a form of discovery to investigate transfer of learning. The subjects were high school and college students, a total of forty, over the entire course of the investigation.

Three methods of teaching were used. Method I was the usual pedagogical procedure in which the generalization of a principle was stated before and after a clarifying illustration with subsequent application to several examples. Method II, referred to as unverbalized awareness, was a procedure in which the students were offered several examples representing a principle but were not encouraged - in fact were discouraged - from attempting to generalize it. In Method III the students proceeded as in Method II except they were asked to state the generalization after their performance indicated the generalization had taken place on the unverbalized level.

The learning task consisted of finding the sum of the first n consecutive odd positive integers and discovering the associated generalization.

The investigator concluded that:

> In every case the highest transfer effects were achieved in the group taught by the unverbalized awareness procedure, Method II, and the lowest transfer effects came from the group taught by Method I, the method in which the generalization was stated first, then illustrated, then applied to new problems. The groups who learned by conscious generalization (Method III) showed up somewhere between the other two groups. Since the trends were the same in all three runs of the experiment, the probability that these rankings were due to chance is $(1/2)^3$, or 1/8. That is, results to date are significant, certainly on the 12 1/2 per cent level, and they may be even better. (24:198)

The following hypotheses emerged from this study:

> 1. For generation of transfer power, the unverbalized awareness method of learning a generalization is better than a method in which an authoritative statement of the generalization comes first.
> 2. Verbalizing a generalization immediately after discovery does not increase transfer power.
> 3. Verbalizing a generalization immediately after discovery may actually decrease transfer power. (24:198)

Hendrix does not claim resolution of the transfer of training question, but offered these results so that investigation and interpretation can proceed on a wider front.

In 1953 Craig (10) conducted a study to test the hypothesis that increased direction in discovery activities affects increases in learning without accompanying losses in retention or transfer. The sample consisted of fifty-three sophomores and juniors at the State College of Washington. The students were divided into two groups, and the treatment was characterized by directed discovery and independent discovery.

The learning tasks consisted of a series of items in which a student was asked to identify one word which did not belong to the other four in a group of five. The four words would be related in some manner to thus form the basis for a decision by the students.

> The learner's determination of the basis for a correct response including both the general formation of the relation and its application to specific items is called "discovery of established relations". (10:225)

The subjects were given three distinct learning episodes at intervals of approximately fifteen days. A test designed to measure retention was administered three days after the final learning episode; thus retention of the learnings acquired by the two groups was compared over three different intervals of time.

Craig's conclusions were:

> The group receiving the greater direction. . .learned more relations in each of the three trials. Three days after learning and seventeen days after learning, the two groups did not differ in the proportion of learned relations retained. After a total of thirty-one days, the directed group retained a greater proportion of those learned than the more independent discovery group. After a total of thirty-one days of training, the two groups improved about equally in their ability to solve problems organized on unlearned basis. (10:234)

Kersh (30) attempted to assess reasons for the superiority of learning by independent discovery in a study he conducted in 1957 using forty-eight volunteer college students. It was assumed that independent discovery learning is superior to rote learning.

> The purpose of this research was to study the process of learning tasks involving arithmetical and geometrical relationships in order to determine whether or not the superiority of the discovery and directed-discovery procedure is adequately explained in terms of 'meaningful learning' and if not, to discover a more adequate explanation. (30:282)

Meaning was defined as in the cognitive sense of understanding or organization.

The learning task of each student consisted of finding the rules for the sum of the first n consecutive odd positive integers and the sum of an arithmetic progression. The sample was divided into three groups: one, designed as the "no help" group, which received no assistance in discovering the rules; a second group, referred to as the "direct-reference" group, which was offered perceptual aids and verbal instructions focusing this group's attention on these aids; and a third group, called the "rule given" group, which was provided with the rules and practice in applying them.

The learning period began when the instructions were presented, and was terminated when each student was successful in the application of a rule to three problems in succession. One test was administered immediately after the completion of the learning period, and a second test was given to all students four to six weeks after the first test. Following each problem was a series of questions that attempted to record the thinking utilized during the problem-solving period by each student. The students were encouraged to "think aloud" and their verbalizations were recorded on magnetic tape.

Kersh's conclusions (30:290) were that (1) the directed reference group was superior to the other two groups according to the results of the first test, and (2) that the "no help" group was superior to the other two groups after the second test. In explaining the superiority of the discovery procedure, the results were inconclusive in terms of meaningful learning. The reason advanced for the superior performance of the "no help" group was that they indulged in post-experimental practice. Perhaps the discovery experience motivated the students to practice the rules after the formal learning period. In a later experiment Kersh (31) found additional evidence to support this position.

In 1958 Haslerud and Meyers (22) collaborated on a study in which they investigated the student's ability to transfer the learning of principles that were acquired by two different methods. The two methods consisted of (1) the instructor stating the principles for the student and (2) the students deriving or discovering them for themselves.

The subjects were 100 students ranging from freshmen to seniors at the University of New Hampshire. They were divided into two groups labeled as "experimental" and "control" groups with seventy-six and twenty-four students, respectively. The hypothesis was:

> ....that principles derived (or discovered) by the learner solely from concrete instances will be more readily used in a new situation that those given to him in the form of a statement of principle and an instance. (22:294)

The procedure was to administer two coding tests. The first test, consisting of twenty codes, was divided into two parts. Ten codes included specific instructions for deciphering each code and the remaining ten carried no instructions. The second test used the same twenty codes as the first test - unknown to the student - with no instructions.

There were two specific results from this study. On the first test students solved significantly more of the codes which carried instructions, however, their scores on the second test increased significantly on those codes which had formerly been discovered. The results were interpreted to mean "that independently derived principles are more transferable than those given" (22:297).

Wittrock (49) conducted a study in 1963 using 292 college students. He felt that research on learning by discovery was ambiguous or inconsistent because of the inadequate terminology used to describe the method of discovery. This study carefully avoids reference to a label depicting discovery or rote learning and resorts to phrases which describe the procedure used to promote learning.

The learning task consisted of deciphering sentences encoded according to a rule determined by the investigator. Four experimental treatments were selected as follows:

1. Rule given, answer given
2. Rule given, answer not given

3. Rule not given, answer given
4. Rule not given, answer not given

The learning period was less than two hours long and consisted of ten enciphered sentences. A test for retention and transfer was given three weeks later.

The investigator's hypotheses were:

> 1. On initial learning of specific responses, giving rules is more effective than not giving rules. Giving both answers and rules is more effective than giving either the answers or the rules. Giving neither rules nor answers is least effective.
>
> 2. On retention and/or transfer to new and similar examples, giving of rules is more effective than not giving rules. Giving of rules is more effective than giving both rules and answers or giving neither rules nor answers. (49:185)

The reported conclusions of this study strongly support the hypotheses as stated.

Moise (36) describes a variation on the theme of discovery used by Professor R. L. Moore of the University of Texas in graduate level mathematics courses. Professor Moore would present postulates, definitions, and propositions. It was the student's responsibility to ascertain which propositions were true and which were false by presenting theorems or counterexamples.

The success of this method is attested by the achievements of his students which include R. H. Bing, R. L. Wilder, and G. T. Whyburn.

Moise interprets Professor Moore's method to suggest that sheer knowledge does not play the crucial role in mathematical development that most people suppose. The student could not acquire as much knowledge under this procedure as he could from formal lectures over a comparable period of time.

> The resulting ignorance ought to be a hopeless handicap, but in fact it isn't; and the only way that I can see to resolve this paradox is to conclude that mathematics is capable of being learned as an activity, and that knowledge which is acquired in this way has a power which is out of proportion to its quantity. (36:409)

Moise suggests that a teacher has something to offer his students apart from facts: the intellectual life that he, the teacher, really leads. The method of discovery seems to hold promise toward this end.

Studies Related to High School

In 1947 Michael (35) pursued a study to ascertain the effectiveness of two methods, called inductive and deductive, of teaching selected topics in algebra to ninth grade students. The inductive method utilized exercises involving time, money, directions, and temperature in leading students to discover and learn certain fundamental principles and relationships. No statement of the rules was made by either the teacher or the pupils during the use of this method. The deductive method emphasized both an explicit statement of principles and also extensive drill and practice. The topics were signed numbers, their operations, and their applications in the solving of equations.

The sample consisted of fifteen classes which were taught by fifteen different teachers for a period of approximately nine weeks. The same test was administered as a pre-test and as a post-test. This test was designed to measure gains in three ways: computation, generalization, and attitude toward algebra and mathematics in general.

The following conclusions were reached by the investigator:

1. The inductive method produced a significant change in attitude toward algebra.
2. No significant change in attitude toward mathematics in general.
3. No significant difference in gains in computation

by either method.
4. The deductive method produced significantly better results in the area of generalization. (35:86)

Gagne and Brown (20) performed an experiment in 1960 to investigate the effects of certain variations in the programming of conceptual materials on the effectiveness of learning, as measured by performance in a problem-solving situation. The sample consisted of thirty-three boys in the ninth and tenth grades.

After learning basic concepts pertaining to a number series, the subjects were divided into three groups of eleven each. Each group went through a different learning episode which consisted of a self-instruction type program. The three treatments are characterized by the names of the groups: rule and example, guided discovery, and discovery.

The learning task consisted in finding the formula for the sum of n terms in a number series. The performance was measured in terms of the time used, the number of hints required, and a weighted time score which combined these variables.

The results of this study indicated a superiority of the guided discovery group, with the rule and example group showing the poorest performance. The investigators interpreted their results as "emphasizing 'what is learned' as opposed to 'how it is learned' for problem solving performance" (20:321).

In 1965 Price (41) reported a study on the effect of discovery teaching on the achievement of tenth grade students taking general mathematics. This study had a three-fold purpose.

1. To define and categorize various aspects of discovery.
2. To prepare sample lessons which make use of the above defined dimensions of discovery.
3. To conduct an experiment using the above materials to determine the effect of discovery methods on the

> achievement and critical thinking abilities of the students so taught. (41:5)

Three classes totaling sixty-three students were selected as the sample. One class, representing the control group, was taught with the traditional textbook-lecture method. A second class, designated as the experimental group, was taught the same material as the control group, but they used discovery-type lessons which were prepared by the investigator. The third group, called the transfer group, made use of the same material as the experimental group but also used prepared transfer lessons. These transfer lessons were designed to promote critical thinking. The experimental period was one semester in length.

The following conclusions were drawn. The two groups taught mathematics by techniques designed to promote student discovery of concepts:

> 1. Showed no significant gain over the control group in achievement in mathematics . . .
> 2. Showed a greater increase in mathematical reasoning than the control group.
> 3. Showed a significant gain over the control group as measured by number series.
> 4. Showed a positive attitude change toward mathematics. The control group showed a negative change. (41:115-116)

It was also reported that students using either, or both, of the discovery and transfer materials had more interest, enthusiasm, and concern for the class. In addition they were able to cover more material than the control group in the same period of time.

Studies Related to the Intermediate Grades

Kittell (34) in a study at Pullman, Washington, divided a group of 132 sixth grade students into three groups designated as minimum direction, intermediate direction, and maximum direction. The purpose of this investigation was to study the effect of external direction during learning of certain principles on the transfer and retention of those principles.

A word task scheme was employed which necessitated that the subjects choose a word from each group of five that, according to a principle, did not belong to the group. The minimum direction group was given examples of the principles involved in the word task with directions indicating that a principle was involved. The intermediate direction group was given the same instructions and, in addition, a statement of the principle involved. The maximum direction group was told the principles involved along with answers to each of the examples.

One pre-training and three post-training tests were administered, in which the intermediate group discovered significantly more new principles than did either of the other two groups. The maximum direction group discovered more than the minimum direction group.

The conclusion was that the evidence gained indicated that furnishing learners with information promotes transfer and retention.

In 1960 Suchman (45) designed and tested a program in science employed in the intermediate elementary grades that he called "inquiry training". The primary objective of this program was to supplement the ordinary science classroom activities by giving the student a plan of operation which would help him discover causal factors of physical change through his own initiative and control, and not depend on the explanations and interpretations of others. This plan consisted of three stages, each with its own goal and peculiar set of tactics for attaining it.

The learning task consisted of problems in physics, presented by short films, which the students were to attack by gathering data, formulating hypotheses, and then asking questions of the teacher that could be answered yes or no only. The responsibility for discovering and formulating the correct hypothesis, which would lead to the solution of a given problem, was placed squarely upon the students' shoulders.

The following conclusions were suggested:

> 1. The inquiry skills of the subjects can be improved, and most students so trained became more productive in their design and use of verification and experimentation.
> 2. The chief motivating force is to "know why" something happens because the subjects had little apparent desire to improve their inquiry skills per se.
> 3. Inquiry skills cannot be successfully taught to this age group as an isolated content area. But inquiry training and abundant opportunities to attain new concepts through inquiry seem to produce increments in the understanding of content as well as a grasp of the scientific method. (45:48)

Suchman did not claim to propose inquiry training "as a new way of teaching science, but as a way of teaching basic cognitive skills that are just as important to the intellectual development of the child as reading and arithmetic" (45:168).

Scandura (43) conducted an investigation in 1961 using small groups of fifth and sixth grade students. This was actually a sequence of three studies designed to explore some variables and interrelationships which are inherent in experimental comparisons of exposition and discovery modes of instruction. Only one study, the first in the sequence, will be reported here as analysis of the other studies was basically subjective.

Two classes, each with twenty-three students, academically equivalent and judged to be above average comprised the sample. The exposition and discovery modes of instruction were used in conjunction with abstract card material. Each class met for three periods of instruction and one test period in the course of one month. Both classes were taught the same material, and the instruction continued until the performance of each class was judged approximately equal.

The proceedings were recorded on magnetic tape, and measures of specific and non-specific transfer were obtained from specially constructed tests identified as routine and novel.

The results revealed no significant difference between the two classes on the routine test, but the difference on the novel test was significant at the .01 level favoring the discovery mode of instruction. It was reported, however, that it took the discovery class longer than the exposition class, 153 minutes to 108, to reach the same level of proficiency on the routine problems during the instructional period.

The Madison Project directed by Robert B. Davis is producing materials and reporting results based on a program which uses discovery learning as its central theme. This program is one of the more widely publicized curricular reform studies in mathematics, essentially for grades two to nine. Davis in 1957 became interested in the problem of teaching mathematics to culturally deprived seventh-grade students and originated the Madison Project at Syracuse, New York (11).

The work of the Madison Project is directed at producing curriculum evaluation in actual school situations, and should not be construed as a laboratory study or an effort to prove any particular thesis (11:9). The major goal of Davis' program is the development of creative learning experiences in mathematics. Materials produced, with an accompanying suggested teaching approach, are interjected into the curriculum to supplement the regular mathematics program.

Learning by discovery in the Madison Project is accomplished through the use of informal exploratory experiences which are designed to include the following attributes:

1. The class activity is led but not dominated by the teacher.
2. Where possible, children are given success experiences.
3. The classroom atmosphere is such that "failure is non-punitive".
4. The teacher attempts to present each task to the student with reasonable clarity.
5. The teacher does not show the children how to attack the problem.
6. The teacher formulates a flexible lesson plan which allows for alternatives suggested during the lesson by the students.
7. The teacher treats the student with respect, not with condescension.
8. The teacher carefully avoids appearing omniscent; his statements nearly always suggest some measure of uncertainty in his own mind.
9. The teacher will usually accept the positive part of a student's response and ignore the negative aspect of it.
10. The teacher uses relatively informal language and an informal manner. (12:44-45)

The topics explored in this program include the arithmetic of signed numbers, plotting on a cartesian plane, linear equations, selected quadratic equations, logic and truth tables, probability and statistics, and matrices (14).

Conclusions and implications of the Madison Project include:

1. The highest 30% of suburban children can learn far more mathematics in grades 2 through 9 than has usually been thought possible. The children appear to prefer such a highly enriched mathematics curriculum to a less enriched one.
2. Mathematics can hold much more interest for many more students than has usually been realized, provided varied facets of mathematics are considered, and appropriately varied activities are included in the student's experience.
3. Mathematics appears to have educative value in helping the student learn about himself in relation to intellectual

> matters and to school in general, provided the mathematics has the appropriate relevance, vitality, and unequivocal honesty. (11:10)

Jean Piaget, a Swiss psychologist, has long been interested in children's mental development. Piaget has developed a theory of learning based "...in part on studies of how the child develops his conceptions of number and space" (1:577). This theory is extremely complex and will not be reported in detail; however, it is included because one aspect of the theory has implication for the role of discovery.

Although Piaget's work covers many years, his studies were not designed around elaborate statistical models. Most of his perceptual experiments provide very few statistics except for some means and mean deviations.

> There are generally no measures of variance, which one expects to be considerable, no tests of significance, just a categorical statement that at such and such an age children do such and such, with a few specific illustrations. (5:190)

Many of Piaget's studies were mainly observations of children's behavior during a question and answer session. Small groups, and in many cases single individuals, comprised his samples. In later years, however, the sample size has been increased considerably when dealing with upper grade students.

Be that as it may, the importance of Piaget's work is due, in part, to the fact that his conclusions parallel those reached independently by other investigators. At other times, his findings "serve to correct or supplement what psychologists with other approaches have to say" (5:190-191).

The basic tenet of Piaget's theory is that learning is a process of adaptation to environment. This notion of adaptation consists of two opposed but inseparable processes, assimilation and accomodation.

> Assimilation is the process whereby the child fits every new experience into his pre-existing mental structures. Through the functioning of these structures, he interprets his new experiences in the light of his old experiences. The process of assimilation is a kind of inertia of mental structures, a tendency of those structures to persist. However, the incorporation of new experiences into old structures inevitably modifies them. Accommodation is the process of perpetual modification of mental structures to meet the requirements of each particular experience. Accommodation is the tendency of mental structures to change under the influence of the environment. (1:577)

Piaget uses the idea of an operation in an effort to characterize knowledge.

> Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy of it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge. (38:176)

It seems reasonable to conclude that an operation is action or a set of actions - the act of doing something that modifies the object.

Piaget has identified four stages through which he feels a child passes in mental growth.

> 1. The sensory-motor stage which occurs from birth to about the age of two years.
> 2. The pre-operational stage which occurs from age two to the age of six or seven.
> 3. The concrete operations stage which occurs from age seven to the age of eleven.
> 4. The formal operations stage is above the age of eleven. It is also referred to as the stage of adult reasoning. (1:578-579)

The author of this theory does not claim that these stages necessarily occur at those specific ages as this may vary, depending on the child's environment. He does insist, however, that a child will progress through these stages in the order indicated.

The development of intellectual capacity goes through the aforementioned stages, and each new level of development is a new coherence, a new structuring of elements which heretofore have been unsystematically related. Piaget lists four factors which contribute to this development: nervous maturation, encounters with experience, social transmission, and equilibration or auto-regulation (17:172).

While Piaget argues that the first three factors play a significant role in intellectual development, he finds each of them insufficient in itself. He concludes that an "individual's intellectual development is a process of equilibration, where the individual himself is the active motor and coordinator of his own development" (17:172). An individual learns a thing insofar as he acts upon it, transforms it and is successful in coordinating these actions and transformations. It is at this point we find the role of discovery in Piaget's theory.

Piaget makes explicit his position on discovery in the following remarks:

> The question comes up whether to teach the structure, or present the child with situations where he is active and creates the structures himself. . . The goal in education is not to increase the amount of knowledge, but increase the possibilities for a child to invent and discover. When we teach too fast, we keep the child from inventing and discovering himself . . . Teaching means creating situations where structures can be discovered; it does not mean transmitting structures which may be assimilated at nothing other than a verbal level. (17:174)

Summary

The review of the literature related to discovery teaching could best be described as equivocal and inconclusive. Some of the most striking points noted in this review were the:

1. Nature of the learning tasks
2. Lack of long-term studies
3. Variation in the methods reported as discovery
4. The wide diversity of subjects.

Of the eleven studies reported, only six used topics peculiar to an academic area. The basic assumption of the other five studies, evidently, was that discovery teaching and learning were independent of the subject. It is the opinion of the writer that while the general environment for discovery teaching is not peculiar to a given subject, there are certain features of a learning episode which would be dictated, or suggested, by the subject itself.

Perhaps the most surprising point noticed in the review was the short duration of the majority of the studies. Of the eleven studies, eight reported the time allotted the learning task was less than six hours, and, in some cases, as little as two hours. Two studies were nine weeks or longer in length. One of these, Michael's study, reported unfavorably on discovery while the other, Price's study, reported inclusive findings in achievement but reported favorably as to discovery regarding gains in reasoning and improved attitude.

If there is a finding consistent with most of the studies, it is that the discovery method is reasonably effective for what it requires the learner to do and what is practiced during the learning episode. For example, it seems reasonable that the learner may acquire more efficient means of problem solving through discovery than another process primarily because of the existing opportunity to practice certain problem solving techniques.

It is interesting to view the nature of the reported studies against the background of Piaget's theory of learning relative to the four stages of development. Some researchers seem quite willing to assume their findings are valid in any of the four stages while a given study itself utilized subjects of only one stage.

The nature of a discovery episode appears to be a function of a learner's intellectual capacity, and the different stages of development represent a categorization of this capacity. Consequently, the details of a discovery episode should be selected, in part, with the learner's stage of development in mind.

One point seems abundantly clear, sufficient evidence is not available to support a thesis that all learning should center around the discovery type episode. It seems that a more defensible approach to teaching would be to design a plan of action - a theory of instruction - which incorporates the discovery episode as an integral component utilized to accomplish specific objectives.

CHAPTER III

A PARTIAL THEORY OF INSTRUCTION

Introduction

The acts of teaching and learning are interrelated, and contain three identifiable components: an instructor, a concept, and an audience (23). The basic role of instruction is to provide a framework whereby interaction between these components can occur and produce desirable educational outcomes.

In spite of the fact that the acts of teaching and learning are interdependent, there is an inherent order of occurrence. Except in rare instances teaching must be initiated before learning will occur. It is for this reason Bruner argues that learning theories are not sufficient to deal with some of the more pressing problems in teaching today (6).

The sequence of teacher behaviors should not be random but ordered according to a well-structured plan of action. Such a plan is called a theory of instruction.

The characterization of a theory of instruction here will be restricted to the subject of mathematics and to the middle elementary grades. It is felt that the inherent nature of a discipline and the difference in human behavior at various grade levels will greatly influence the structure of certain features of such a theory.

It is assumed that any given period of instruction can be thought of as a series of learning episodes; thus, the learning episode will serve as the unit of activity around which this discussion will revolve (9:49).

The students' learning activities should be designed to allow maximum intellectual participation in the pursuit and development of a given concept. Implications of the above theme for a theory of instruction will now be considered.

Student-Teacher Relationship

The teacher should respect a student's feeling as he responds to a problem. The student should not be made to feel foolish because of an incorrect or poorly-timed response. Handling this delicate situation carefully may prevent the blocking of additional routes of investigation that a student may wish to pursue. The basic role of the teacher is to initiate and cultivate a student's participation in concept development.

The learning episode should be handled so that the student will not equate <u>truth</u> and <u>teacher</u>. The teacher will need to subordinate his role and avoid any appearance of competing with the student. A learning episode is not a setting in which the teacher should seek grandizement in the eyes of his students. It is an opportunity for the students, not the teacher, to slay dragons.

The student must be led to realize that the teacher is not the ultimate source of information, that he can acquire new understandings principally on his own efforts, and thereby he can strive toward becoming intellectually independent of the teacher.

Selection of Information to be Presented

It is well-known that students, at a given time, have previously developed skills and understandings available for application to a given problem. If a concept is offered piece-meal the student may be unable to relate it to his past experiences. Information, in the form of learning episodes, should be offered in large packets; then the students should be encouraged to break it down to bite-size, their bite-size (6:524). This will permit the student to make use of his own strengths - of which the teacher is usually unaware - in terms of past experiences, skills and understandings. This permits, even encourages, making the learning experience a personal one.

Responses and Feedback

When a student responds to a given problem, he answers what he thinks is the problem (15). If the two problems differ, then his response will probably be incorrect. If, however, the teacher will take a moment to construct the problem as it was interpreted by the student, then the student has an opportunity to capitalize on his error. A suggestion by the teacher that a student's answer has merit will give him a feeling of being successful and thus will encourage him to try again.

More often than not, a student's response is partially correct and partially incorrect. If a teacher will accept the positive portion of his answer, the student will again feel that he has made a contribution. The student will be aware that a part of his answer was ignored and will often follow suit. Incorrect responses can be quite instructive in discovery teaching, but this is not the case in lecture-recitation.

Whenever possible the teacher should avoid making a decision regarding the validity of a given response. Put the response to the test of satisfying the problem and encourage the remainder of the class to referee. As a consequence of this procedure the student, intuitively, will realize that he is not competing with the teacher in the act of learning. The learning experience will seem more like an adventure with him and the concept as the principles in it. In this setting, if a confrontation is unsuccessful, he is more likely to attack the problem again with more vigor.

Motives for Learning

The general topic of motivation is, admittedly, extremely broad and will not be considered in detail here, but it is clear that it plays a prominent role in the act of discovery which is the central theme of this partial theory of instruction (15). A basic assumption is that a student really wants to learn. If the right environment is present he will bring to bear on a given problem latent or dormant energy which, when coupled with the assistance offered by the teacher, will greatly enhance his achievement. It is basically the role of discovery to arouse and harness this ability of the student, so that with recognition of its existence and with practice the student will more nearly attain the intellectual growth of which he is capable.

The student must be convinced that he can succeed if he will try. It is possible to design discovery episodes so that every student can succeed at least on a minor scale. Through a carefully planned sequence of learning experiences, a student may find that he has reached a pinnacle which is supported by a sequence of seemingly insignificant successes.

Quite often one attempts to motivate the consideration of a topic by discussing a need for it. This represents a motivation external to the topic and is, more often than not, artificial and unconvincing. The most powerful motivating source for the topic of mathematics is in the countless number of patterns and relationships inherent in mathematics itself (15). It but remains for the teacher to devise a setting, or design a learning experience, which will reveal this to the student.

The nature of a discovery episode permits each student to view the success or failure of his fellow students. During the middle elementary grades there is a strong desire to be successful in the eyes of ones peers; thus, one can take advantage of an additional, minor but not insignificant, source of motivation.

The Sequencing of Ideas

Bruner has remarked that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (9:33). While this assertion seems reasonable, it would be nearly impossible to test it as a hypothesis; thus, it forms a basis for an assumption which has implication for teaching. It is possible to construct a sequence of learning episodes which will permit an understanding of concepts ordinarily considered beyond the capabilities of a given student.

This assumption places a premium on teacher resourcefulness. It would be folly to think that there is a right sequence (9). An acceptable sequence would depend on the kind of learning skills and attitudes that one desires to produce. In practice this means offering several particular instances, examples, or problems and encouraging the student to seek a pattern depicting an underlying regularity. The pacing of the sequence and the level of difficulty could be modified to fit the mood and capability of a given class. The pace could be stepped up to maintain the interest of a more able class, or slowed down to retain contact with a class of underachievers.

A planned sequence should be flexible enough to permit changes suggested by student responses. This would encourage additional student participation in the development of a concept and offer the student further evidence that his learning experience is on a personal basis.

It is not necessary to order the level of difficulty completely in a given sequence. One should intermittently probe the students' depth of understanding by including a difficult problem, and then follow with an easier one so that those students unsuccessful with the harder problem could regain their composure and continue in the sequence. It is important that each student be challenged in depth, and still have ample opportunity to be successful.

The Nature of Rewards and Punishments

It is necessary at this point to project an image of rewards and punishments that is compatible with the climate devised within the classroom. An attempt has been made to keep decision-making on the level of the student. In order to maintain this theme one must modify the nature of rewards and punishments.

This is done by considering a student's success or failure on the basis of whether a problem is solved or not. Usually rewards and punishments are dispensed by the teacher and this tends to divert the students' efforts from the problem solving to a satisfying-the-teacher type behavior; thus, the learning initiative is removed from the student since he does not have a basis for determining right or wrong. By minimizing rewards and punishments one keeps the students' attention focused on the problem under consideration and uses his resulting success or failure as the primary source of reinforcement.

The proper environment for this role of success and failure implies that the teacher project himself out of the learning episode as soon as possible. A student's response must be allowed to meet the test of the problem and stand or fall on its own merit. The student must be convinced that the responsibility for learning is his. The role of the teacher is to devise an environment within the classroom which keeps the student ever mindful of this fact.

Summary

An attempt has been made to devise a plan of action, or theory of instruction, that would create a particular climate for learning mathematics in the middle elementary grades. The role of discovery is but one facet of this theory. It is expected that a given day's lesson in mathematics would consist of a judicious combination of discovery and expository teaching. The choice would be dictated by a consideration of the factors of time, nature of the concept, and the learning outcomes and attitudes desired.

There are many factors on which the success of this theory may hinge. Can middle elementary grade students, low and high ability, perform successfully in a discovery type learning episode? Is it possible for these students to perform successfully with topics heretofore reserved for later grades? How will the performance of a given student in a series of discovery episodes compare with his past achievement in arithmetic and his scores on a standard achievement test? How will a given low ability group, as a team, compare with a high ability group?

In recent years attention has been directed toward improving the performance of the low achiever in mathematics. The discovery type episode, which actively encourages physical and mental participation on the part of the student, may have much to contribute toward this objective (47).

These factors serve as the basis for the research hypothesis tested in this study.

CHAPTER IV

PROCEDURE

Selection of Subjects

As previously stated the purpose of this study was to investigate the possibility of using the characteristics of discovery to arrest and narrow the performance gap between ability-grouped fifth grade students. This experiment took place within the school day during the academic school year. The investigation was conducted and data were gathered during the time ordinarily allotted to the regular arithmetic class period.

The framework of this study imposed stringent demands that reduced considerably the universe of available schools; also, the study had to be conducted within reasonable driving distance of Central Missouri State College (50-mile radius).

The investigator examined each school system that utilized ability-grouping in the fifth grade. The schools having this characteristic were visited personally. The issues of participation, scheduling difficulties, and other related problems served to eliminate all but two of the schools considered. The investigator then chose one of the two remaining candidate school systems at random.

Characteristics of Subjects

The study was conducted in a rural town school system with a total enrollment of 1100 students. The predominant occupational activities of this community were agricultural in nature. The majority of the childrens' parents were either engaged in small grain farming, cattle and hog farming, or the processing activities closely allied to farming.

The school was located in a town with a population of approximately 2000 located fifty miles from Kansas City, fifteen miles from a state college, and twenty-five miles from a major air base installation.

The school system selected for the study had forty-seven fifth grade students. These students were grouped according to criteria set up by the local school authorities, and this practice was an integral part of the school's total elementary program. The curriculum of this school did not differ from the ordinary in any appreciable fashion.

Ability grouping began there in the second grade, and this had been practiced at this school for three years previous to the time this study was made. The criteria for grouping was the students' performance on the California Achievement Test and the teachers' assessment of the student's potential. It was possible for students to change groups during the year under certain circumstances; however, no student changed classes during the course of this study.

Design of Study

The study was conducted at the school two days per week - Tuesdays and Thursdays - for eight consecutive weeks during the last two months of the 1965-66 academic school year. The two fifth grade classes (designated as 5L and 5H for the purpose of this study) were located in adjacent rooms, and the periods of instruction were separated by a school recess fifteen minutes in length. The investigator taught both classes, for a period of about forty minutes each, in lieu of their regular arithmetic classes.

Their regularly assigned teacher was seated in the rear of the classroom during the instructional periods with no role to play in the study itself. Her sole concern was to watch the performance of the students relative to their regular arithmetic program. The agreement between the experimenter and the school authorities was that if the regularly assigned teacher noticed any deterioration of a normal growth pattern in the regular arithmetic classes, the study would be terminated.

Role of Student

The unit of measure was a success. The instructor would set up a discovery episode by leading the class, as a group, to begin the task of identifying an underlying concept in a graduated sequence of examples. During the next phase of the episode the class would respond individually to a related sequence of problems by writing their responses on a paper using dark crayon and holding it up for the investigator to see. If a student's response was correct, then his name was called, and thus recorded on magnetic tape that was running continuously during each period of instruction. If his response was incorrect, he was so informed and encouraged to respond anew to the same problem.

Preparation of Subjects

The preparatory activities included the administration of an achievement test, an explicit effort to establish rapport with each class, and an explanation to give them some idea of what was to be expected of them. The school, at the investigator's request, gave each student involved in the experiment the arithmetic reasoning and fundamentals section of the California Achievement Test (CAT), Form Y, which utilized 1963 norms.

The investigator visited with each class one week prior to the initial data-taking session. Rapport was established by indulging in a search for patterns that were demonstrated by utilizing a hexahexaflexagon and a multiplication table constructed by the experimenter. The students were told that a series of problems would be presented, and that they were to act like little detectives. The solution to each problem would offer a clue to the discovery of an underlying pattern which would represent an easy way of working similar but more difficult problems.

Experimental Treatment

The nature of the treatment in this study was to create a specific learning environment in the classroom by using a discovery-type learning episode with topics completely unfamiliar to the subjects. In general the environment would be characterized in terms of student and teacher behavior.

The learning experience was highly competitive among the students. At no time were the students permitted to feel that they were competing with the instructor. There was immediate reinforcement. When a student responded, he knew immediately if he was successful. Each student was very conscious of the fact that his success or failure was evident to his peers. He also knew that he would not be tested over these experiences as a part of the evaluation program of his regular arithmetic class.

The students were of the opinion that somehow these sessions weren't really arithmetic. They also knew that another class was involved. The presence of a magnetic tape recorder, used for recording responses, kept both classes aware of the fact that they were participating in some kind of an experiment.

The treatment was, insofar as it was possible, identical for each group. The role of the instructor was basically that of an impartial referee between the students and the problem at a given moment. With rare exceptions, the instructor's activities involved only the presentation of a problem and answering yes or no to a student's response to that problem. Under no circumstances would the instructor give an answer to a particular problem or reveal the underlying concept imbedded in a discovery episode.

The subjects were discouraged from telling the secret during the experimental sessions; however, there was no reason to believe that the students did not converse at other times about what they did or did not discover during the sessions.

Physical Arrangement of Classroom

The physical environment of the classroom was typical with students seated singularly at flat-topped desks with unattached chairs. The desks were arranged in four columns facing the instructor in a rectangular-shaped classroom.

The seating arrangement was changed for each successive experimental period. The students were rotated clockwise around the classroom, two seats per session. This was done to minimize any bias introduced in the recording of data. If two students would offer a response simultaneously, the instructor would ordinarily see the nearest one first, call his name first, and thus prejudice the results in relation to time. Since time was a factor in the overall design, rotating the seating arrangement permitted a random occurrence of a false time differential relative to a given student's response.

A complete record of each session's proceedings was taken via a magnetic tape recorder. It was situated near the front of the classroom in full view of the students and was operated by the investigator. A non-directional microphone was attached to a three-foot tripod located in the center of the classroom. The primary function of the recorder was to record the names of the students who made correct responses.

The Topics Utilized

The topics selected for use in this study had to meet certain specific criteria. They had to (1) be completely foreign to the students' arithmetic background, (2) represent concepts which were amenable to a discovery-type episode, (3) be close enough to the students' experience so that they could be made intuitively meaningful with a minimum of effort by the instructor, and (4) represent significant mathematical concepts.

The topics were selected from those developed by Dr. Robert B. Davis of the Madison Project (13). The investigator had attended a Summer Institute that dealt with the Madison Project Materials. The topics were:

1. Selected open sentences in one variable.

2. Nature and addition of integers.
3. Identities.
4. Graphing on a cartesian plane.
5. Selected linear equations.

There were certain physical models and immature mathematical phraseologies used in the communication with students during the study. Among these were:

1. Open sentences with one, two, or more elements in the truth set.
2. Open sentences which were true for any number.
3. Tic-tac-toe.
4. The Pet Shop Scheme.
5. What's My Rule?

The topics were not necessarily presented in the order listed above, but were intermixed to support the nature of the discovery episode utilized in the study.

Nature of the Discovery Episode

Selected features of a discovery-type presentation were chosen because they offered reasonable opportunities to manipulate such factors as motivation, personal achievement, reinforcement, and a change from the normal routine of traditional teacher-student behavior in the classroom (9). The basic structure of the discovery method used is characterized by the following:

A. Mode of Operation

   1. The students' attention was oriented by the instructor toward concepts in the form of patterns.

2. A graduated sequence of problems containing the desired concept was pursued by the class as a collective activity.
3. Students were encouraged to examine each problem in the sequence in an effort to organize and structure the underlying concept using the pattern as a guide.
4. If the students reached an incorrect conclusion, the instructor interjected additional information by counterexample.

B. Essential Ingredients

1. The teacher did not state what was to be discovered.
2. There was feedback from student-to-class-to-student and from student-to-problem-to-student.
3. Exposition was minimized.
4. There was active participation by the students in the search for patterns.
5. If a student did not discover a pattern for himself, then he did not find out (i.e., within the framework of the study itself).

The discovery episode was designed to shift the responsibility for learning to the student as much as possible. This put success squarely on a personal basis; moreover, each student knew his success or failure was evident to his peer group.

The students were not informed that the experiment would involve the relative successes or failures of the two classes. The investigator and the school authorities felt they were successful in keeping this fact from the students during the conduct of the study.

Variables

The investigator taught both groups of students with no assistance from their regularly assigned teacher throughout the study. The concepts were selected before the study was under way. The time allotted to each group was identical. The investigator made a special effort to use exactly the same treatment relative to the features of a discovery episode on each class. The controlled variables were (1) the instructor, (2) concepts used, (3) method of teaching or the environmental conditions of the classroom, and (4) the time during which the treatment was applied.

The independent variables were the ability-grouping levels and the classroom environment. The dependent variables were total successes and rate of successes.

Intervening Variables

There were several possible intervening variables. Students could pursue problems which occurred during a session after school or discuss them during a regular arithmetic session. The teachers attempted to discourage this practice without dampening the ardor and enthusiasm of the students.

A recess occurred between the periods devoted to the respective classes, during which conversation between members of the two classes was possible. The investigator attempted to minimize the effect of this by changing some aspect of the problems used without changing the inherent nature or level of difficulty. A periodic check of the students' activities during recess gave no reason to be concerned in regard to the students conversing about the learning episodes.

Another intervening variable could be in the difference in the classroom environment established on a day-to-day basis by their regularly assigned teacher. Since the discovery episode attempts to generate a high level of intrinsic motivation, one could reasonably expect this variable to intervene as it would most likely differ from student to student.

Some students could have a higher aptitude for being successful in the discovery episodes. In summary, the two main categories of possible intervening variables are differences in the classes which existed prior to the study and the relative change in student behavior promoted by applying the treatment.

Collection of Data

There was a total of fifteen sessions with each of the two fifth grade classes. Frequency counts of successes, along with other data, were recorded via magnetic tape during the first fourteen sessions. The investigator constructed an examination, peculiar to the concepts pursued during the study, and administered it to each student during the fifteenth session.

Prior to conducting the study, the investigator secured from the school authorities results for each student on the California Achievement Test. This test was administered two weeks before the initial experimental session.

The regularly assigned teacher of each class ranked each student in his class relative to the student's performance in the regular arithmetic program before the study began.

The Processing of Data

The design of the study necessitated that a time interval be imposed on the tallying of successes for each student as they were taken from the tapes. The instructor arbitrarily chose a time of thirty seconds for each interval. Due to the nature of the discovery episodes and the time allotted to each problem, the investigator chose a total time of ninety seconds in which he would record successes from students for a given problem.

There was an exception to the choice of ninety seconds while the topic of identities was being pursued. While discovering identities, no time interval was imposed. A student's discovery of an identity was counted as a success response in each of the two classes.

The investigator was unable to find a timing device suitable for tallying the data in the desired intervals, so one was constructed. An ordinary electric alarm clock with a sturdy second hand was the basic component. An external six-volt circuit was set up which included an electric door bell and a pushbutton-type switch. This circuit was so designed that it was closed when the second hand of the clock passed six or twelve on the clock face; otherwise, it was open, so the tape-reader would hear a bell sounding at successive thirty-second intervals. The pushbutton switch was operated by foot so that both hands were free to operate the tape recorder and to tally the responses.

The data were transferred to a summary sheet from which they were utilized in testing the hypothesis. The examination administered during the fifteenth session was graded by hand.

Statistical Methods

The basic design used in this study was that of two independent samples. The independence arises from the fact that putting one person in the success category did not preclude another from being placed in the same category.

The measurement was the successes accumulated by each student during discovery-type learning episodes. While it was evident that the order relation existed between the numbers associated with the total successes of two given students, the investigator did not feel that equal intervals existing between the scores of two different pairs of students were necessarily significant numerically. Consequently, the level of measurement was ordinal.

> With ordinal scaling, hypotheses can be tested by using that large group of nonparametric statistical tests which are sometimes called "order statistics" or "ranking statistics". Correlation coefficients based on rankings (e.g. the Spearman r_s or the Kendall t) are appropriate. (44:25)

The investigator chose the Man-Whitney U test for part one of the hypothesis and the Spearman rank correlation coefficient for parts two and three. The corresponding parametric test, the t-test, was not used on experimental data because the investigator did not wish to make the necessary assumptions. The t-test was used, however, to test the hypothesis that the two groups were significantly different in terms of their CAT scores.

The only assumption necessary for the statistical tests chosen was that the observed scores were drawn from an underlying continuous distribution (44). Regarding the strength of the Man-Whitney test Siegel says:

This is one of the most powerful of the non-parametric tests, and it is a most useful alternative to the parametric t-test when the researcher wishes to avoid the t-test's assumptions, or when the measurement in the research is weaker than interval scaling. (44:116)

CHAPTER V

RESULTS OF THE STUDY

It was the objective of this study to investigate the nature of discovery and to identify its role in a theory of instruction. This was done by comparing the successes and the association between successes, teacher ranking, and achievement test scores for ability-grouped fifth grade students. The statistical data are presented in tabular form. The test statistics as computed by the Man-Whitney U test and Spearman rank correlation coefficient are given along with the necessary assumptions.

The t-test, as given by Ostle (37), was used to test the hypothesis that the two samples used in the experiment were not from the same population. The data for this test shown in Table I were obtained from the California Achievement Test, which was administered two weeks prior to the initial session of the experiment. The test statistic, $t = 6.03$, was found to be significant at the .01 level. The hypothesis of this study consisted of three parts: (1) comparative performances of the two groups, (2) association of rate of successes and teacher ranking, and (3) association of rate of successes and achievement test scores. The results will now be reported in terms of the hypothesis.

TABLE I

ACHIEVEMENT TEST SCORES

5-L		5-H	
Student	CAT Score	Student	CAT Score
1	6.9	24	6.0
2	4.7	25	6.9
3	6.4	26	6.4
4	5.7	27	6.3
5	5.8	28	6.7
6	6.1	29	6.5
7	2.8	30	6.2
8	5.6	31	6.8
9	5.9	32	7.2
10	4.3	33	6.4
11	4.4	34	6.9
12	5.9	35	7.6
13	6.2	36	7.5
14	3.9	37	7.4
15	6.3	38	6.8
16	6.1	39	7.8
17	6.5	40	7.5
18	5.7	41	7.1
19	5.7	42	7.6
20	6.3	43	6.8
21	3.5	44	6.6
22	5.5	45	7.7
23	6.5	46	6.3
		47	7.0

Comparative Performance of the Two Groups

The performance of the two groups was investigated in terms of successes by student and by session. The objective was to study the effect of discovery teaching on the student level and the class level.

The Man-Whitney U test was used to test the null hypothesis that two sets of scores, in terms of successes, could have been drawn from the same population. In general, the null hypothesis under the Man-Whitney U test is that two populations have the same distribution against the alternative that one is stochastically larger than the other (44).

The test statistic computed is

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

where n_1 and n_2 represented the number in samples one and two respectively, and R_1 is the sum of the ranks awarded to sample #1. As n_1 and n_2 increase in size, the sampling distribution of U rapidly approaches the normal distribution, with

$$\text{Mean} = \frac{n_1 n_2}{2}\text{, and}$$

$$\text{Standard Deviation} = \sqrt{\frac{(n_1 n_2)(n_1 + n_2 + 1)}{12}}\ .$$

When either of the sample sizes is larger than twenty, one could determine the significance of an observed value of U by

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{(n_1 n_2)(n_1 + n_2 + 1)}{12}}}\ . \quad (44{:}121)$$

If ties are present in the rankings then the variability of the rankings is affected. Corrected for ties the standard deviation becomes

$$\sqrt{\left[\frac{n_1 n_2}{N(N-1)}\right]\left[\frac{N^3 - N}{12} - \Sigma T\right]}$$

where $N = n_1 + n_2$

$T = t^3 - t$ (t is the number of observations tied for a given rank)

ΣT is the sum of the T's over all groups of tied observations. (44:124)

With the correction for ties

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\left[\frac{n_1 n_2}{N(N-1)}\right]\left[\frac{N^3 - N}{12} - \Sigma T\right]}} .$$

Table II gives a summary of the results relative to performance. The data from which the test statistics were computed are given in Tables III through VII inclusive.

TABLE II

SUMMARY OF STATISTICS REGARDING PERFORMANCE

Hypothesis	Computed U	Computed z	Probability of z
1a	258.5	-.3727	$p > .3520$
1b	109	.19115	$p > .4207$
1c	329.5	1.1396	$p > .1271$
1d	111.5	.6203	$p > .2643$
1e	317	2.618	$p < .0045$

Parts 1a and 1b of the hypothesis (see page 3) represented a consideration of relative performance in terms of total successes. This permitted students to demonstrate effectiveness when time was not a principal factor.

TABLE III

TOTAL SUCCESSES OF EACH GROUP BY STUDENT

5-L			5-L		
Student	Score	Rank	Student	Score	Rank
1	97	40	24	47	3
2	87	30.5	25	70	12
3	104	45.5	26	65	6.5
4	91	34	27	84	25.5
5	73	13	28	90	32.5
6	66	8	29	75	15
7	81	23	30	69	10.5
8	64	4.5	31	69	10.5
9	90	32.5	32	98	41
10	20	1	33	87	30.5
11	100	42.5	34	68	9
12	64	4.5	35	92	35
13	100	42.5	36	77	17
14	80	19.5	37	81	23
15	93	36.5	38	80	19.5
16	81	23	39	104	45.5
17	65	6.5	40	80	19.5
18	75	15	41	86	28.5
19	85	27	42	86	28.5
20	93	36.5	43	96	39
21	35	2	44	80	19.5
22	95	38	45	118	47
23	103	44	46	75	15
			47	84	25.5

TABLE IV

TOTAL SUCCESSES OF EACH GROUP BY SESSION

Session	5-L		5-H	
	Score	Rank	Score	Rank
1	143	12	184	20
2	225	23	263	28
3	195	22	246	27
4	106	9	114	10
5	125	11	145	14
6	50	7	53	8
7	15	2	18	5
8	8	1	17	4
9	16	3	21	6
10	158	16	146	15
11	162	17	138	12
12	245	26	191	21
13	178	19	165	18
14	229	24	233	25

TABLE V

RATE OF SUCCESS OF EACH GROUP BY STUDENT

5-L			5-H		
Student	Score	Rank	Student	Score	Rank
1	80	45	24	29	3
2	59	34.5	25	53	28
3	70	41	26	44	15.5
4	49	9.5	27	51	25
5	37	6.5	28	68	39.5
6	50	22	29	49	19.5
7	42	12	30	37	6.5
8	31	4	31	40	9.5
9	50	22	32	68	39.5
10	13	1	33	47	17.5
11	54	29	34	50	22
12	43	3	35	58	32.5
13	58	32.5	36	47	17.5
14	41	11	37	44	15.5
15	75	44	38	52	27
16	59	34.5	39	89	47
17	39	8	40	61	36
18	34	5	41	72	43
19	43	13.5	42	66	38
20	51	31	43	63	37
21	18	2	44	51	25
22	51	25	45	86	46
23	71	42	46	40	9.5
			47	55	30

TABLE VI

RATE OF SUCCESSES OF EACH GROUP BY SESSION

Session	5-L		5-H	
	Score	Rank	Score	Rank
1	113	19	157	25
2	201	27	223	28
3	145	24	176	26
4	50	9	58	11
5	51	10	81	15
6	27	7	33	8
7	9	2	18	5
8	8	1	17	3.5
9	17	3.5	21	6
10	80	14	83	16
11	78	13	75	12
12	128	21	112	18
13	87	17	124	20
14	133	22	141	23

TABLE VII

RESULTS FROM POST-TEST

5-L			5-H		
Student	Score	Rank	Student	Score	Rank
1	28	32	24	13	11
2	6	3	25	21	25
3	34	40	26	21	25
4	7	4.5	27	33	38.5
5	12	10	28	31	35.5
6	25	30.5	29	25	30.5
9	15	14	30	15	14
10	11	9	31	17	17
11	9	6.5	32	23	27
13	24	28.5	33	18	19.5
14	15	14	34	21	25
15	30	34	35	32	37
16	17	17	36	29	33
17	14	12	37	9	6.5
18	4	2	38	24	28.5
19	20	22	40	31	35.5
20	20	22	41	35	41
21	1	1	42	36	42
22	10	8	43	20	22
			44	17	17
			45	33	38.5
			46	7	4.5
			47	18	19.5

The time factor was included in parts 1c and 1d of the hypothesis, and there was a noticeable drop in the probability that a value as large as the computed z would occur. This suggested that time was an additional handicap to low ability students even in a discovery situation.

Parts 1a and 1c represented a consideration of performance on an individual basis. Parts 1b and 1d permitted a consideration on a group level. The probabilities for z are lower on the individual basis than on the group level. This could be explained by the fact that the affect of the two extremes in each group, high and low scores, were minimized by the scores of the other students during a given session.

There was insufficient evidence to reject the null hypothesis for parts 1a, 1b, 1c, and 1d.

The post-experimental test represents a comparison between the two groups (see Table VII) in terms of simple recall. The reliability coefficient for this test was .91 as computed by Hoyt's (21:496) variation of Kuder-Richardson's formula No. 20. The computed z has a probability of occurrence which is less than .0045. This is significant at the .05 level and, therefore, the investigator rejects part 1e of the null hypothesis that there is no difference in the two groups as revealed by the post-experimental test.

Association of Rate of Successes,
Teacher Ranking, and CAT Scores

The association between the performance of the students in discovery episodes and teacher ranking, and discovery episodes and CAT scores was investigated in terms of a correlation coefficient. In general, the null hypothesis was that no correlation existed between the two sets of data under consideration and that the observed value of r_s differed from zero only by chance (44:210).

The test statistic computed is

$$r_s = \frac{\Sigma x^2 + \Sigma y^2 - \Sigma d^2}{2\sqrt{\Sigma x^2 \ \Sigma y^2}}$$

where $\Sigma x^2 = \frac{N^3 - N}{12} - \Sigma T_x$ and

$$\Sigma y^2 = \frac{N^3 - N}{12} - \Sigma T_y \, .$$

T_x represents the sum of the various values for T where $T = \frac{t^3 - t}{12}$, and t equals the number of observations tied at a given rank (44:207). A similar discussion explains the meaning of Σy^2.

Table VIII gives a summary of results relative to association. The correlation coefficients for parts 2a and 2b of the hypothesis were not high; however, they were significant at the .01 level. The rankings from which these coefficients were computed are given in Tables IX, X, and XI.

TABLE VIII

SUMMARY OF STATISTICS: ASSOCIATION

Hypothesis	Spearman's r_s
2a	.5759
2b	.5623
3a	.6969
3b	.6814
3c	.2818

The results from parts 3a and 3b suggested that the student's performance correlated closer to their scores on the CAT than to the teacher's ranking.

TABLE IX

RANKINGS: RATE OF SUCCESSES AND TEACHER

5-L				5-H			
Student	R-S	Te.	Diff.	Student	R-S	Te.	Diff.
1	1	5	4	24	24	23	1
2	5.5	17	11.5	25	11	16	5
3	4	2	2	26	19.5	19	.5
4	13	16	3	27	13.5	22	8.5
5	19	11	8	28	4.5	6	1.5
6	11.5	6	5.5	29	16	24	8
7	16	9	7	30	24	20	4
8	21	10	11	31	21.5	7	14.5
9	11.5	7	4.5	32	4.5	12	7.5
10	23	22	1	33	17.5	15	2.5
11	9	19	10	34	15	1	14
12	14.5	20	5.5	35	9	8	1
13	7	3	4	36	17.5	9	8.5
14	17	21	4	37	19.5	18	1.5
15	2	8	6	38	12	14	2
16	5.5	14	8.5	39	1	2	1
17	18	13	5	40	8	3	5
18	20	15	5	41	3	17	14
19	14.5	12	2.5	42	6	10	4
20	8	4	4	43	7	11	4
21	22	23	1	44	13.5	13	.5
22	10	18	8	45	2	4	2
23	3	1	2	46	21.5	21	.5
				47	10	5	5

TABLE X

RANKINGS: RATE OF SUCCESSES AND ACHIEVEMENT TEST

5-L				5-H			
Student	R-S	CAT	Diff.	Student	R-S	CAT	Diff.
1	1	1	0	24	24	24	0
2	5.5	19	13.5	25	11	11.5	.5
3	4	2	2	26	19.5	19.5	0
4	13	14	1	27	13.5	21.5	8
5	19	12	7	28	4.5	16	11.5
6	11.5	8	6.5	29	16	18	2
7	16	13	3	30	24	23	1
8	21	17	4	31	21.5	14	7.5
9	11.5	10	1.5	32	4.5	8	3.5
10	23	21	2	33	17.5	19.5	2
11	9	20	11	34	15	11.5	5.5
12	14.5	11	3.5	35	9	3.5	5.5
13	7	7	0	36	17.5	5.5	12
14	17	22	5	37	19.5	7	12.5
15	2	6	4	38	12	14	2
16	5.5	9	3.5	39	1	1	0
17	18	4	14	40	8	5.5	2.5
18	20	15	5	41	3	9	6
19	14.5	16	1.5	42	6	3.5	2.5
20	8	5	3	43	7	14	7
21	22	23	1	44	13.5	17	3.5
22	10	18	8	45	2	2	0
23	3	3	0	46	21.5	21.5	0
				47	10	10	0

TABLE XI

RANKINGS: SUCCESSES AND ACHIEVEMENT TESTS

FOR THE GROUPS AS A UNIT

Student	Treatment	CAT	Diff.	Student	Treatment	CAT	Diff.
1	8	12	4	24	45	33	12
2	17.5	43	25.5	25	36	12	24
3	2.5	23	20.5	26	41.5	23	18.5
4	14	39	25	27	22.5	26.5	4
5	35	36.5	1.5	28	15.5	17	1.5
6	40	31.5	8.5	29	33	20	13
7	25	36.5	11.5	30	37.5	29.5	8
8	43.5	41	2.5	31	37.5	15	22.5
9	15.5	34.5	19	32	7	8	1
10	47	45	2	33	17.5	23	5.5
11	5.5	44	38.5	34	39	12	27
12	43.5	34.5	9	35	13	5.5	25.5
13	5.5	29.5	24	36	31	5.5	25.5
14	28.5	46	17.5	37	25	7	18
15	11.5	26.5	15	38	28.5	15	13.5
16	25	31.5	6.5	39	2.5	1	1.5
17	41.5	20	21.5	40	28.5	5.5	23
18	33	39	6	41	19.5	9	10.5
19	21	39	18	42	19.5	3.5	16
20	11.5	26.5	15	43	9	15	6
21	46	47	1	44	28.5	18	10.5
22	10	42	32	45	1	2	1
23	4	20	16	46	33	26.5	6.5
				47	22.5	10	12.5

The null hypothesis relative to parts 2a, 2b, 3a, and 3b was rejected and the investigator concluded that an association existed between successes in a discovery episode, teacher ranking, and CAT scores for both groups.

The correlation coefficient for part 3c was .2818 which was not significant at the .05 level. This means the successes from the two groups combined as a unit did not correlate with their CAT scores. This apparent contradiction with the findings of parts 3a and 3b could be explained, in part, by the overlap of the students' CAT scores in the two groups. Since there were criteria for grouping other than the CAT scores, one could find students in the low ability group whose CAT scores were higher than the lowest student's CAT score in the high ability group. This would serve to rearrange the rankings existing in parts 3a and 3b and, subsequently, to give rise to a different correlation coefficient.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Review of the Study

This study was undertaken primarily to investigate the nature of discovery and to gain evidence of its merit as a teaching technique. Secondary considerations were to utilize the characteristics of a discovery episode in an attempt to reduce the performance gap between ability groups and to explore the possibility of incorporating its features in the teaching of pre-service elementary teachers.

The specific hypothesis to be tested was that no significant difference existed between the performance of ability-grouped fifth graders and no correlation existed between their performance, past achievement in arithmetic, and CAT scores.

The investigator taught two ability-grouped fifth grade classes twice a week for a total of fifteen sessions. The method of discovery and topics used for both groups were the same. The treatment was in the form of an unusual classroom environment generated by the use of a discovery-type learning episode. Each episode consisted of a pattern exhibited by a series of problems. Exposition was minimized so that the student would have ample opportunity to participate in the discovery of a concept. The learning experience was that of non-verbal awareness as the students were not permitted to verbalize a discovery.

The unit of measurement for the treatment was a success, and the data were collected on magnetic tape. The hypothesis was tested by using the Man-Whitney U test and the Spearman rank correlation coefficient.

Conclusions of the Study

On the basis of this research and subject to the specified limitations, the findings of the study seemed to justify the following conclusions:

1. Hypothesis 1a and 1c, that there is no significant difference in the two groups when one considers successes on a student basis could not be rejected. Apparently the low-ability student could compete on an equal basis in terms of total successes or rate of successes. It should be noted, however, that the probability for the computed z of part 1a (see Table II) was approximately three times that of part 1c. It was concluded that the low-ability students competed more favorably in terms of total successes.
2. Hypothesis 1b and 1d, that there is no significant difference in the two groups when one considered successes in terms of class sessions could not be rejected. Once again it was found that the probability for the computed z was higher for total successes than it was for the rate of successes.
3. Hypothesis 1e, that there is no significant difference in the two groups as measured by a post-experimental test which measured simple recall was rejected. In spite of the fact that the low-ability group competed favorably with the

high-ability group in terms of successes, they evidently did not retain the understandings as well.

4. Hypothesis 2a and 2b, that there is no correlation between past performance in arithmetic as indicated by teacher rankings and performance in discovery episodes was rejected.
5. Hypothesis 3a and 3b, that there is no correlation between the results on an achievement test and performance in discovery episodes was rejected.
6. Hypothesis 3c, that there is no correlation between the results on an achievement test and performance in discovery episodes when considering the two groups combined as a unit could not be rejected at the .05 level.

During the course of the experiment the investigator noted things which, when considered along with the findings relative to the hypothesis, gave rise to conclusions of a general nature.

The low-ability students could perform equally well as the high-ability students during the discovery episodes, but apparently did not learn as much. The high-ability students seemed to be more curious as to why a certain response was correct, whereas the low-ability group appeared to be content with the fact that the response was correct, and desired to move on to the next problem. Several times a student from the high-ability group was heared to say, "Why won't my answer work?" This remark, or an equivalent one, did not occur in the low-ability group.

This same factor seemed to manifest itself in terms of motivation. Each group appeared to be equally motivated by the discovery episodes to perform, in the sense that they showed about the same amount of activity when confronted with a challenge. The effort expended by the low-ability group, however, was mostly to find an acceptable answer to the problem at hand, whereas the other group exhibited the ability to divide their efforts between seeking a solution and investigating its implication for the underlying pattern.

In other words, the low-ability group seemed to lose sight of the long range objective - to ascertain the concept depicted by the underlying pattern. This explains, in part, why the two groups appeared to be equally successful in terms of performance in discovery episodes but were significantly different in terms of what they had actually learned.

The procedure by which the students revealed their responses to the investigator made it possible for them to get a response from other than their own efforts. This, to be sure, would tend to bias the results somewhat. It appeared that periodic appeals by the investigator were effective in reducing this activity to a minimum.

It was of interest to note that this activity was more prevalent in the low-ability group. These students had the same strong desire to be successful as the other group, but being unable at times to find a correct response by themselves they would resort to other means. There is little doubt that the high level of anxiety generated by the discovery episode contributed to this state of affairs.

The performance in discovery episodes seems to correlate well with past achievement in arithmetic. This suggests that one could include discovery episodes along with traditional classroom practices and still have a coherent activity.

In summary, the conclusions were that students, low and high ability alike, could be successful in discovery episodes, but the high ability students would learn more. There were significant correlations between performances in discovery episodes, past achievement in arithmetic, and CAT scores.

Recommendations

The investigator offers the following recommendations in regard to the use of the discovery technique and further exploration of the nature and merits of discovery.

1. The discovery technique should be an integral part of a teacher's plan of instruction and utilized to accomplish specific objectives. Among these are generating student interest and leading a student to realize that the responsibility for learning belongs to him.
2. There are various forms of discovery teaching. Among these are nonverbal awareness and different degrees of guided discovery. A teacher should select the form of discovery which offers the best chance of achieving the objective under consideration.
3. On the basis of this study, it appears that a form of discovery that uses verbalization might be more effective for low-ability groups than the nonverbal awareness form used herein. This is supported by the fact that those students either did not actually learn much during the episodes or failed to retain it, in view of their performance on the post-experimental test. A new study incorporating this change might be valuable.

4. The best way for an interested teacher to learn about the discovery technique, after preliminary study, is to actually try it. There are certain features, such as the feel of a class as they pursue a concept and the nature of the derived motivation, that are extremely difficult to communicate to others.
5. A teacher should consider using discovery if he wishes to develop among his students a more constructive attitude toward mathematics. While this factor was not an integral part of this study, the level of interest and motivation achieved would undoubtedly make a contribution toward that objective.
6. It would be advisable to have a second person, preferably their regularly assigned teacher, to assist in a study of this type to identify and record unusual events which occur. There were many incidents that were lost because the investigator was unable to record them.
7. The discovery technique should be considered seriously for the teaching of mathematics and methods-of-teaching mathematics courses in the pre-service training of elementary teachers. The investigator believes that prospective teachers should experience the same feelings of frustration and excitement of success that the elementary student does. This will permit acceptance or rejection of the discovery technique primarily on the basis of its own merit.

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