AN ANALYTICAL AND EXPERIMENTAL INVESTIGATION OF A MONOLITHIC TRAPEZOIDAL SHEAR PANEL SUBJECTED TO MECHANICAL LOADING

By

Everett L. Cook

Bachelor of Science in
Aeronautical Engineering
Wichita State University
Wichita, Kansas
1954

Master of Science in
Aeronautical Engineering
Wichita State University
Wichita, Kansas
1958

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
May, 1967

AN ANALYTICAL AND EXPERIMENTAL INVESTIGATION

OF A MONOLITHIC TRAPEZOIDAL SHEAR PANEL

SUBJECTED TO MECHANICAL LOADING

Thesis Approved:

R. E. Chapel
Thesis Adviser

R B Deal

Ladislaus J Fila

J M Boggs

N N Durham
Dean of the Graduate College

ACKNOWLEDGMENTS

The author wishes to express his special appreciation to his thesis adviser, Professor R. E. Chapel, for his advice and counsel throughout this study. In addition, appreciation is due Dr. J. H. Boggs, Graduate Study Committee Chairman, and Dr. R. B. Deal and Professor L. J. Fila, committee members, for their contributions.

Likewise, the author is grateful to his associates Dr. M. U. Ayres, Dr. Alvin Baker, Dr. Gordon Stone, Mr. Lawrence Bell, Mr. Robert Cates, Mr. David Ellis, Mr. Robert Hinderliter, Mr. John Levosky, and especially to Mr. Glen Warren for his dedicated efforts during the experimental program. Mrs. Aleta Gile is thanked for her assistance rendered while typing the manuscript.

The author also expresses his gratitude to the Boeing Company for the financial support and the use of their excellent digital computing facilities. Mr. Donald Strand and Mr. Howard Smith in Engineering and Mr. Willis Winder, Mr. Carl Issacs, and Mr. Phil Davis in Information Processing were all instrumental in the successful completion of this work.

Finally, the author expresses his deepest gratitude to his wife, Marie, and his children for their moral support and for their evenings and weekends.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

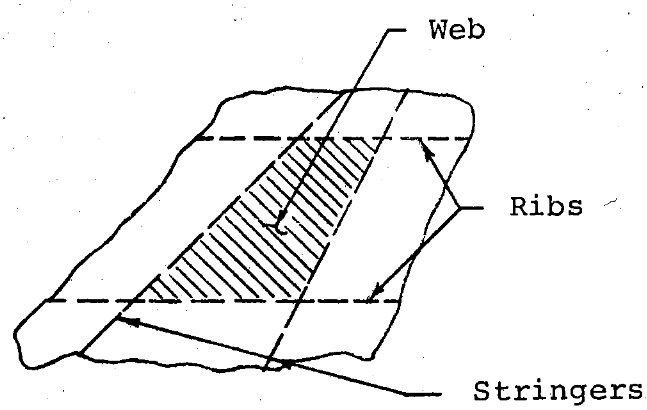
Figure | Page

CHAPTER I

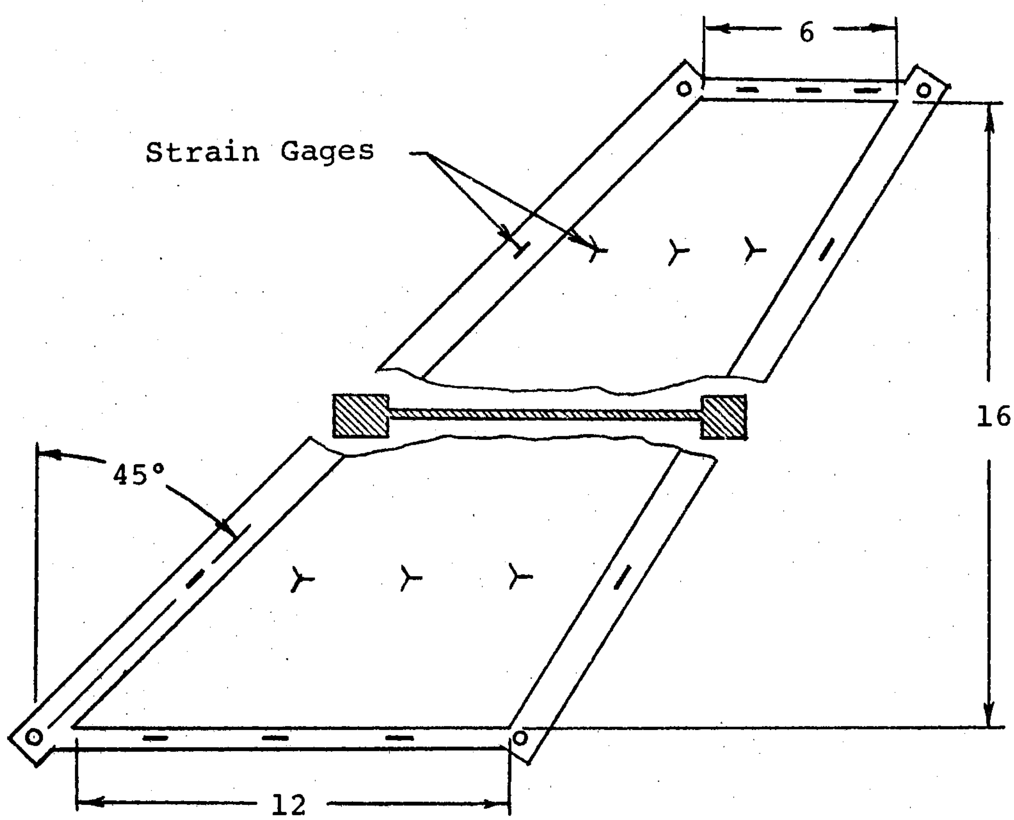
INTRODUCTION

The purpose of this document is to present the results of an analytical and experimental investigation of a monolithic trapezoidal shear panel under mechanical loading and to recommend additional investigations which would extend the field of knowledge about this important structural element. The trapezoidal shear panel was chosen because limited analytical work had been done on the stress or strain distribution in this configuration. Ayres (Reference 2) and Stone (Reference 13) have considered multiweb monolithic panels. Although Ayres has investigated the details of the stress distribution within the individual web elements, his study was limited to rectangular elements. Stone, on the other hand, has investigated a panel with symmetrical trapezoidal web elements, but the primary purpose of his investigation was extension of the force method. Figure 1 shows the geometry of the panel used in the investigation along with a sketch showing where a panel of this type is used in an aerospace structure.

The experimental investigation was intended to provide a set of data which could be used to verify not only the analytical results of this investigation, but also refined



(a) Actual Sheet-Stringer Panel



(b) Idealized Test Panel

Figure 1. Panel Geometry

analytical investigations in the future. Although the investigation described here is limited to mechanical loading, additional experimental data could be readily obtained for thermal loading to be used in verifying thermal analysis methods.

In order to obtain the most accurate experimental data possible, it was decided that the test panel should be monolithic. This type of construction eliminates the difficult to determine effects of joint and fastener friction. In addition, the panel was made symmetric with respect to the median plane of the web to minimize the effects of lateral bending due to eccentricity of loading. This symmetric, monolithic construction corresponds to the assumptions normally made in developing methods of analysis. It should, therefore, provide consistent experimental data that can be used to validate these methods. To the author's knowledge, this was the first simulated built-up structural model constructed in this manner.

The analytical investigation had a two-fold purpose:

1. To compare the two most commonly used methods of structural analysis - the displacement method and the force method - and to determine the relative accuracy of the methods for the structure being considered. The comparison is made for a series of mathematical models of the panel with various sub-element sizes. It was expected that the two methods would yield identical results for the larger sub-element sizes but that, the results would differ for the smaller sub-element sizes.
2. To determine, if possible, the optimum sub-element size for each method.

> "Optimum" is intended to mean the largest sub-element size that would yield results of the accuracy required for both analysis and design.

The most important consideration in the analytical investigation was the selection of a mathematical model, or models. In the past, the majority of the comparisons of the two methods have been based on analyses of different mathematical models of a specified structure. To insure that the comparison to be made in this investigation would be affected only by computational accuracy, the same model was used for both analyses. The model selected is the one most generally used in displacement analyses, finite elements connected only at their common nodal points. The two significant developments which were required in the analytical investigation are:

1. The derivation of a stiffness matrix for a trapezoidal plate element. Previous analyses of structures with this type of element have used four triangles joined at the centroid of the trapezoid to approximate the element stiffness. The disadvantages of the four-triangle method are that the size of the stiffness matrix is increased and the stress distribution within the plate cannot be determined.
2. The derivation of a set of self-equilibrating redundant force systems for plate elements which are not separated by stiffener elements. This development was necessary in order to be able to use the displacement-type mathematical model for the force analysis. This development not only increases the accuracy of the force method for sheet-stringer panel structures, but also makes possible the analysis of shell structures by the force method.

The two methods of matrix structural analysis - the displacement and force methods - are discussed in Chapters II and III, respectively. The details of the development of the digital computer programs are given in Appendices A and B. The experimental program is outlined in Chapter IV and the results of this program are presented in Appendix C. In Chapter V, the results of the two analyses are compared and then the analytical and experimental results are compared. The conclusions regarding the results of the investigation and the degree to which the objectives were attained are presented in Chapter VI along with recommendations for additional investigations.

CHAPTER II

THE MATRIX DISPLACEMENT METHOD

Displacement methods of structural analysis are based on the following premise: of all the geometrically compatible displacement configurations of a loaded structure, the correct configuration is the one for which every element of the structure is in equilibrium. Although it is possible to satisfy the above conditions at the infinitesimal level in simple structures, most thin-web structures are far too complex for analysis at this level. Therefore, in order to analyze structures of this type, it is necessary to subdivide the structure into a number of relatively simple elements (usually bars, beams, and plates). While equilibrium of each element is maintained, compatibility of displacements is enforced only at the assumed joints, or nodes.

Numerous methods have been developed for the analysis of structures using the procedure outlined above. The large scale digital computer has made the matrix methods the most widely accepted for complex structures. Two matrix methods are currently in use. The basic difference between the two methods is the way in which the support conditions, or constraints, are taken into account. The methods and their differences are:

1. The Direct Stiffness Method - The support conditions are ignored until the stiffness matrix of the unrestrained structure has been determined. This singular matrix is then made nonsingular by taking the support conditions into account. The nonsingular matrix can then be inverted to obtain the flexibility matrix of the structure.
2. The Indirect Stiffness Method - The constraints on the structure are considered from the beginning. When the stiffness matrix is determined it is already nonsingular and can be inverted immediately to obtain the flexibility matrix.

The direct stiffness method was chosen for the analysis presented here primarily because of the ease with which the method could be programmed for solution on a digital computer. This method was originally presented in a paper by Turner, et al (Reference 14), which remains the best available document on the method. A recent book by Martin (Reference 9) presents additional information on the analysis of framed structures; however, thin-web structures are not considered. An excellent exposition of the indirect method is presented by Pestel and Leckie in Reference 10, Chapter 10.

The direct stiffness method can be stated in terms of the two equations,

$$\{p_g\} = [k_g]\{V_g\} \quad \text{and} \tag{1}$$

$$\{f\} = [K]\{d\} \;, \tag{2}$$

where $\{p_g\}$ = the column matrix of the internal forces acting on the g^{th} element,

$\{V_g\}$ = the column matrix of the absolute displacements of the g^{th} element,

$[K_g]$ = the singular stiffness matrix of the g^{th} element,

$\{f\}$ = the column matrix of the external forces acting on the structure,

$\{d\}$ = the column matrix of the absolute displacements of the structure, and

$[K]$ = the singular stiffness matrix of the structure.

If the element stiffness matrices are available, Equation 1 can be used to determine the force-displacment relationship for each element of the structure. The force-displacement relationship for the entire structure can then be determined by combining these element stiffnesses. Since the sum of the internal forces at a node must equal the external forces, and the element displacements at a node must equal the actual displacements of the node, the stiffness of the structure [K] can be obtained by summing the element stiffnesses. Examples of this procedure are given in Reference 14.

The singular stiffness matrix [K] can readily be made nonsingular by striking out the rows and columns corresponding to the constraints on the structure. The justification for this simple procedure is given in Reference 14. An alternate method of making the stiffness matrix nonsingular is discussed in Appendix A.

The Mathematical Model

In order to apply the direct stiffness method to the experimental panel, it is necessary to replace the actual panel with a mathematical model composed of a number of discrete elements joined only at the nodes. Since the main purpose of the investigation is to determine the effect of element size on the accuracy of the predicted stresses, a total of seven models have been studied. The method of subdividing the panel for each model is illustrated in Figure 2. The subdivided panels will be identified both by the model number (1-7) and the number of rows (M) and columns (N) of nodes. The assumed support conditions and loads are shown in Figure 3.

The first model (M = N = 2) contains the minimum number of elements, four stiffener elements and one plate element. It is assumed that these elements are connected only at the nodes. Therefore the forces on each element are as shown in Figure 4. Since there is no shear tie between the stiffeners and the plate element, the load in each stiffener is constant over the length of the member. A stiffness matrix for this type of element is derived in Reference 14.

A stiffness matrix was not available, however, for a trapezoidal plate element; therefore, such a stiffness matrix was derived for this investigation. The derivation is presented in a later section.

It is obvious that the results of an analysis of Model No. 1 cannot accurately predict the stiffener stresses, and

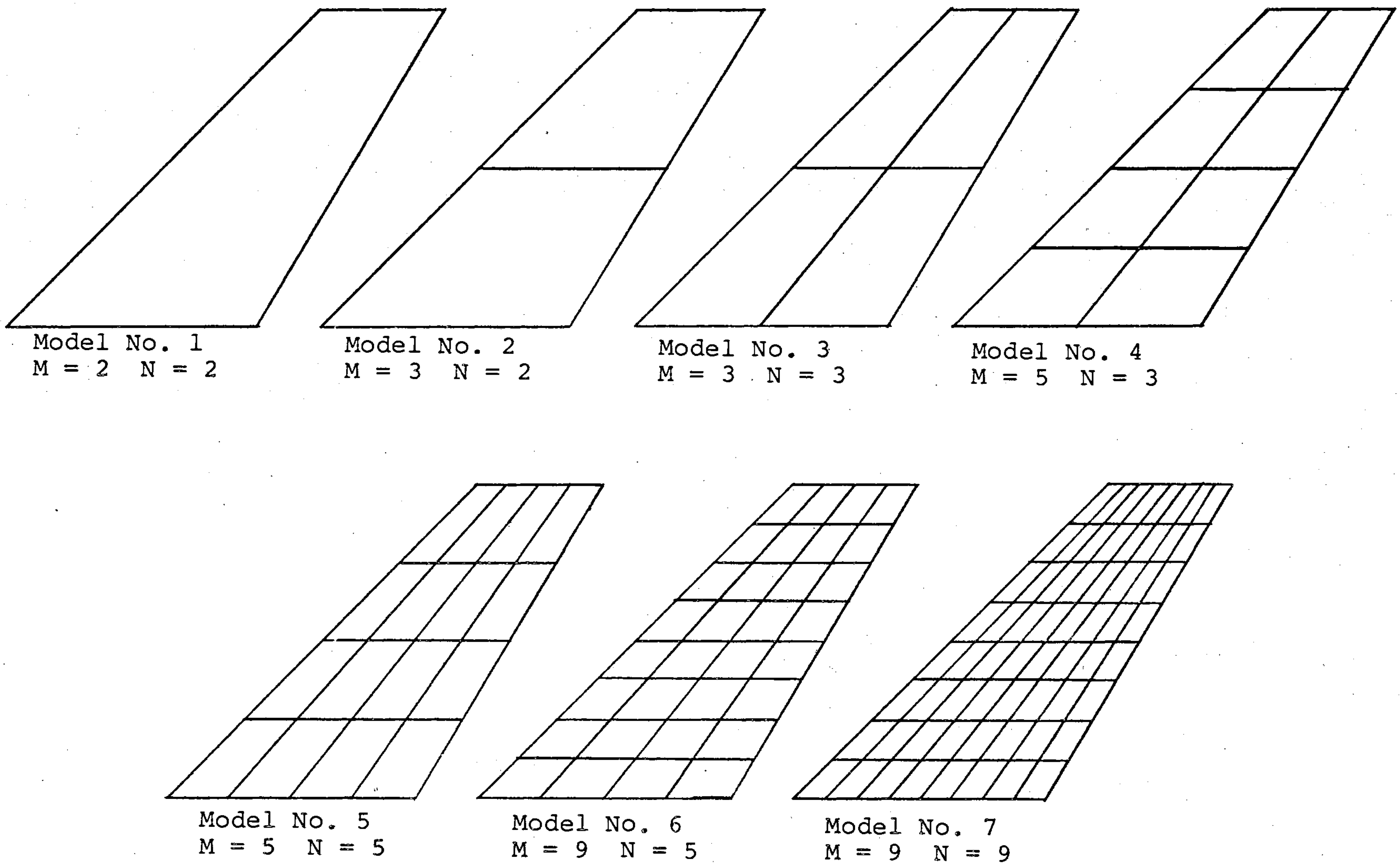


Figure 2. Panel Subdivisions for the Mathematical Models

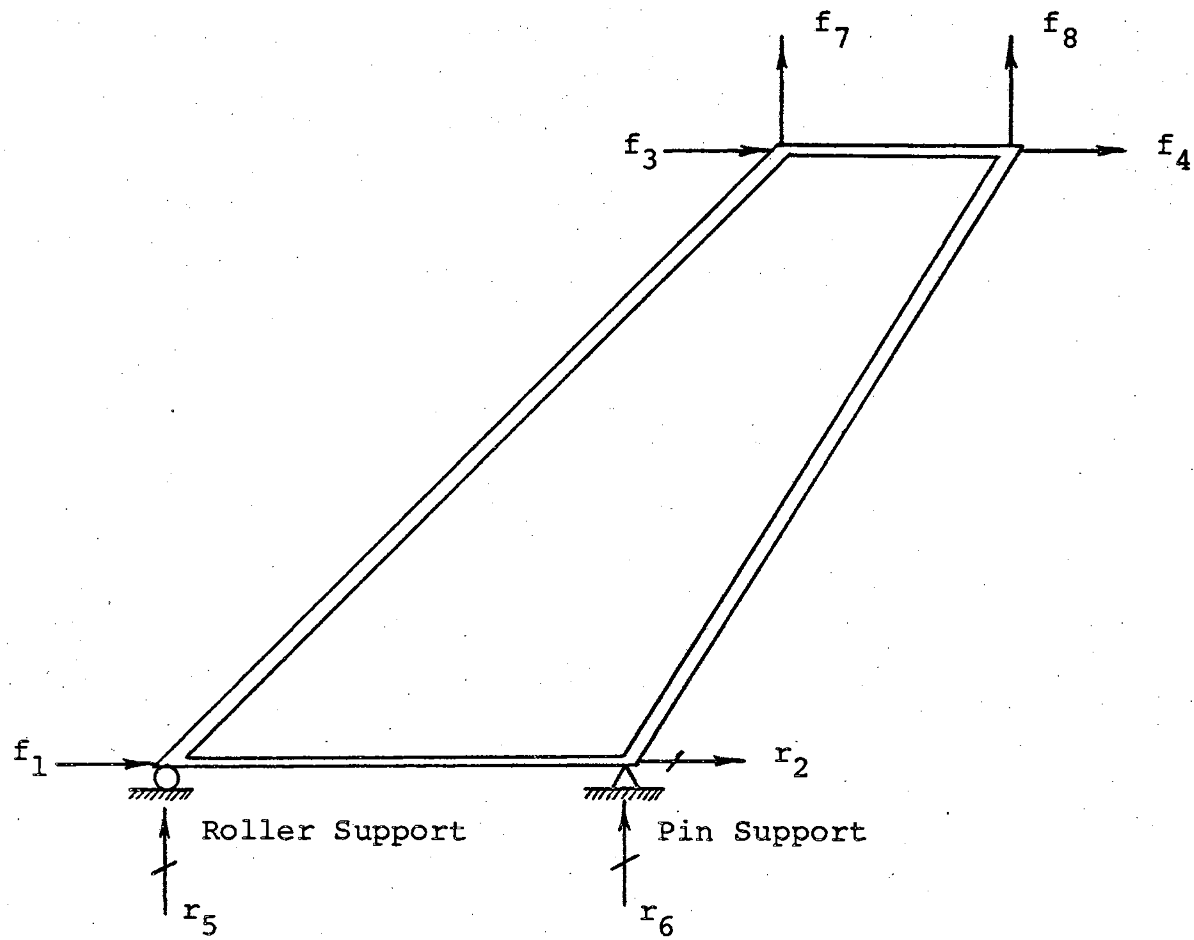


Figure 3. Assumed Support Conditions and External Loads

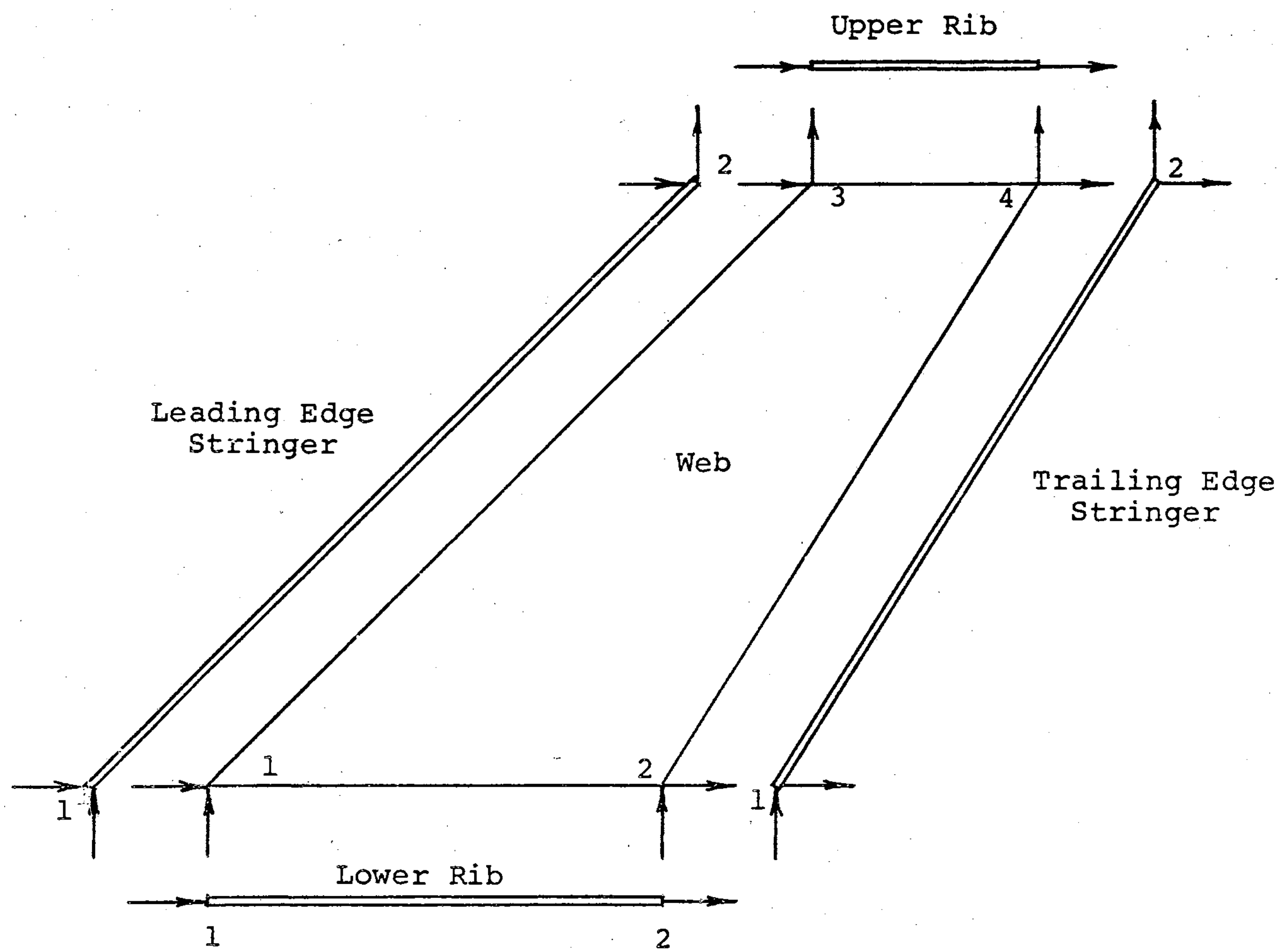


Figure 4. Internal Forces in Model No. 1

it is unlikely that the web stresses can be determined with accuracy except at points well away from the edges of the element. Therefore, Models 2 through 7 are divided into successively smaller elements in order to study the effect of element size. The smallest element size, Model No. 7, was dictated by two considerations:

1. Since there are two degrees of freedom at each node, the stiffness matrix is of order 2 x M x N. For Model No. 7 this necessitates the inversion of a 162 x 162 matrix. The largest matrix which could be inverted on the available equipment was 200 x 200; therefore, it was impractical to use smaller subelements.
2. The derivation of the stiffness matrix for a trapezoidal plate is based on the assumption of a state of plane stress, i.e., that the thickness of the plate is small in comparison to its other dimensions. A finer element breakdown would undoubtedly reduce the size of the elements to the point where this assumption would not be valid.

Trapezoidal Plate Stiffness Matrix

In order to derive a stiffness matrix for a plate element, it is necessary to assume either a stress distribution or a distribution of the displacements throughout the element. In Reference 14, a stiffness matrix for a triangular plate was derived by assuming a constant stress distribution, i.e.,

$$\sigma_{xx}(x,y) = A, \quad \sigma_{yy}(x,y) = B, \quad \text{and} \quad \tau_{xy}(x,y) = C \qquad (3)$$

Although it has not been published, the same stiffness matrix can be derived by assuming a linear displacement function of the form

$$u(x,y) = A + Bx + Cy \quad \text{and} \quad v(x,y) = E + Fx + Hy \qquad (4)$$

However, in the case of a plate with more than three nodes, different stiffness matrices result from what appear to be equivalent assumptions for the distribution of stresses or displacements. For instance, References 7 and 14 present a rectangular plate stiffness matrix based on a stress distribution described by

$$\sigma_{xx}(x,y) = Ay + B, \quad \sigma_{yy}(x,y) = Cx + D, \quad \text{and}$$
$$\tau_{xy}(x,y) = E \,. \qquad (5)$$

In Reference 1, an equivalent matrix is obtained by assuming linear displacements along the boundaries of the plate, i.e.,

$$u(x,y) = A + Bx + Cxy + Dy, \quad \text{and}$$
$$v(x,y) = E + Fx + Gxy + Hy \,. \qquad (6)$$

The resulting stiffness matrices appear different in either symbolic or numerical form for a given plate element. It has been shown, however, that for a typical sheet-stringer panel with rectangular panel elements, the most significant stresses and displacements are essentially the same when either stiffness matrix is used (Reference 4).

Another approach which has been used to derive a rectangular plate stiffness matrix (Reference 14) is to assume an assemblage of four triangular plates joined at the four corners and at the center of the rectangle. This method has also been the only available method for obtaining the stiffness of a trapezoidal plate element up until this time.

Recently (Reference 13), a flexibility matrix has been derived for a trapezoidal plate; however, this matrix includes the flexibilities of the adjoining stiffeners and cannot be used for the mathematical models shown in Figure 2.

The following derivation was developed to eliminate the necessity of treating a trapezoidal plate element as a group of triangles. There are two major advantages to this approach:

1. It eliminates two degrees of freedom for each trapezoidal plate and, consequently, reduces the size of the stiffness matrix which must be inverted. As an example of the magnitude of the reduction - a 290th order matrix would have resulted if each trapezoid of Model 7 had been broken down into four triangles.
2. It allows the calculation of the stresses at any point within the trapezoid, while the triangular break-down method yields four sets of constant stresses for the four triangular elements.

In view of the above discussion of triangular and rectangular plates, it is evident that a decision was necessary as to the type of distribution to be assumed - stress or displacement. Since the shear stress in a trapezoidal panel must vary in the direction of the taper, Equations 5 cannot be used. As will be shown later, the shear stress derived using Equations 6 vary in both the x- and y-directions; therefore, Equations 6 will be used in the following derivation.

In addition to the assumption of the displacement function, it is also assumed that the plate element is homogeneous, isotropic, and of constant thickness and that small displacement theory holds. The element geometry and nomenclature is shown in Figure 5.

Equations 6 adequately describe the displacements throughout the element; however, each of the constants is a linear function of the displacements of the nodes, ie,

$$A = A(u_1, u_2, u_3, u_4) \text{ , etc. , } \quad \text{and}$$
$$E = E(v_1, v_2, v_3, v_4) \text{ , etc.} \qquad (7)$$

In order to simplify the derivation, Equations 6 are written in matrix notation as

$$u(x,y) = [u_1\ u_2\ u_3\ u_4] \begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ A_3 & B_3 & C_3 & D_3 \\ A_4 & B_4 & C_4 & D_4 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ xy \\ y \end{bmatrix} \qquad (8)$$

and

$$v(x,y) = [v_1\ v_2\ v_3\ v_4] \begin{bmatrix} E_1 & F_1 & G_1 & H_1 \\ E_2 & F_2 & G_2 & H_2 \\ E_3 & F_3 & G_3 & H_3 \\ E_4 & F_4 & G_4 & H_4 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ xy \\ y \end{bmatrix} \qquad (9)$$

where $A = u_1A_1 + u_2A_2 + u_3A_3 + u_4A_4$, etc. , and

$$E = v_1E_1 + v_2E_2 + v_3E_3 + v_4E_4 \text{ , etc.}$$

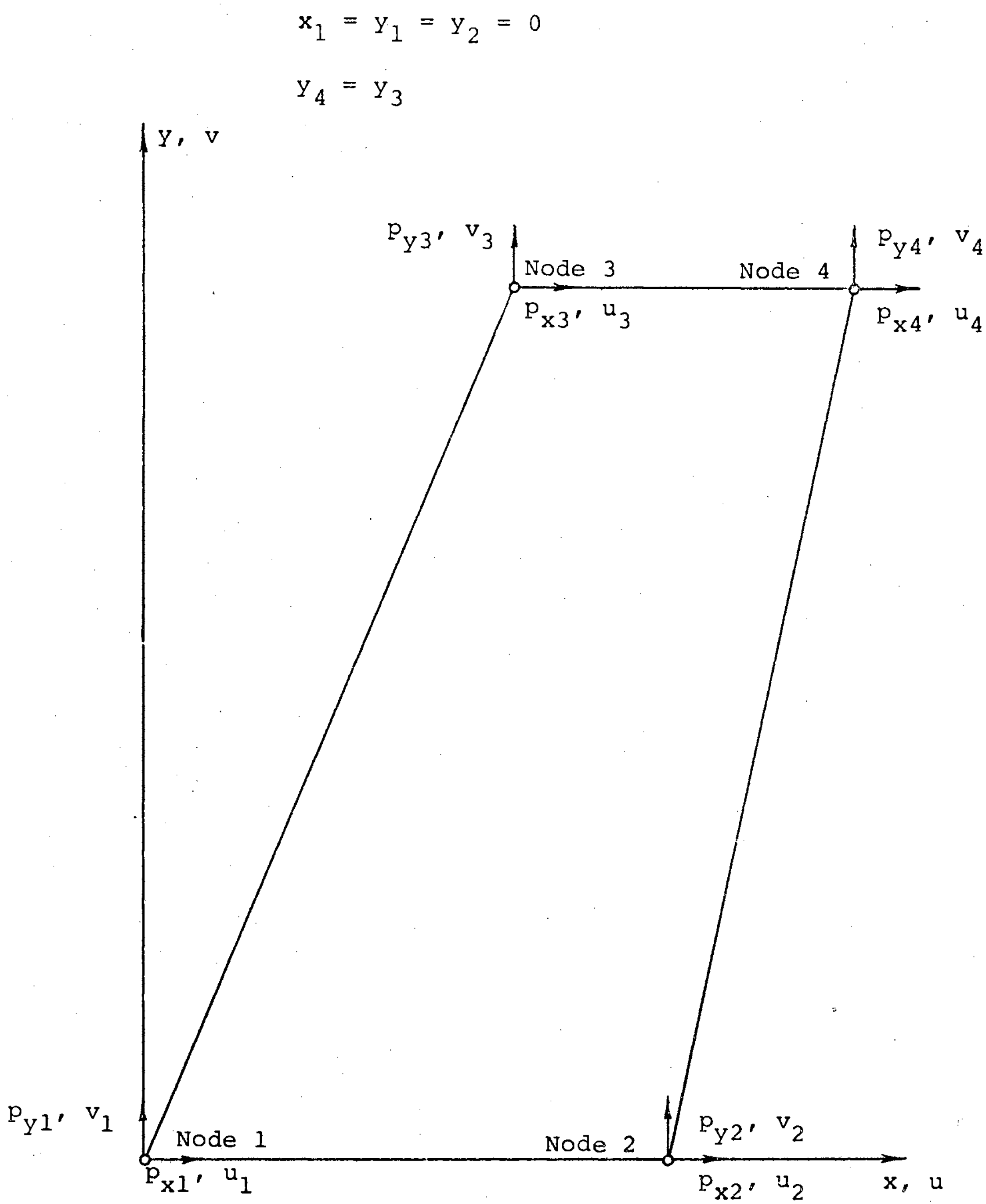


Figure 5. Web Element Geometry and Nomenclature

The coefficients in any row of Equations 8 and 9 can now be evaluated by applying a unit displacement at the corresponding node. The first row of the square matrix in Equation 8 can be evaluated by taking

$$u_1 = 1 \quad \text{and} \quad u_2 = u_3 = u_4 = 0 \ , \tag{10}$$

then

$$\begin{aligned} u(x,y) &= [A_1 \ B_1 \ C_1 \ D_1]\{1 \ x \ xy \ y\} \\ &= [1 \ x \ xy \ y]\{A_1 \ B_1 \ C_1 \ D_1\} \ . \end{aligned} \tag{11}$$

The corresponding boundary conditions are (Figure 5)

$$\begin{aligned} &u(x_1,y_1) = 1 \ , \\ &u(x_2,y_2) = 0 \ , \\ &u(x_3,y_3) = 0 \ , \quad \text{and} \\ &u(x_4,y_4) = 0 \ . \end{aligned} \tag{12}$$

Substitution of Equations 12 into Equation 11 and rearrangement in matrix notation gives

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & x_2 & 0 & 0 \\ 1 & x_3 & x_3y_3 & y_3 \\ 1 & x_4 & x_4y_3 & y_3 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \tag{13}$$

since $x_1 = y_1 = y_2 = 0$ and $y_4 = y_3$.

Equation 13 can be solved for the constants by premultiplying both sides by the inverse of the coordinate matrix with the following result

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} = \frac{1}{x_2y_3} \begin{bmatrix} x_2y_3 & 0 & 0 & 0 \\ -y_3 & y_3 & 0 & 0 \\ 1 & -1 & -\lambda & \lambda \\ -x_2 & 0 & \lambda x_4 & -\lambda x_3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{x_2y_3} \begin{bmatrix} x_2y_3 \\ -y_3 \\ 1 \\ -x_2 \end{bmatrix} \quad (14)$$

where $\lambda = x_2/(x_4 - x_3) = x_2/x_{43}$

if $x_{ij} = x_i - x_j$.

The remaining three rows of constants in Equation 8 can be readily determined from a consideration of the boundary conditions and Equations 11 through 14. For instance, if

$$u_2 = 1 \quad \text{and} \quad u_1 = u_3 = u_4 = 0 \quad (15)$$

then $u(x,y) = [1\ x\ xy\ y]\{A_2\ B_2\ C_2\ D_2\}$ (16)

where

$$\begin{aligned} u(x_1,y_1) &= 0 , \\ u(x_2,y_2) &= 1 , \\ u(x_3,y_3) &= 0 , \quad \text{and} \\ u(x_4,y_4) &= 0 . \end{aligned} \quad (17)$$

The equivalent of Equation 13 is then

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & x_2 & 0 & 0 \\ 1 & x_3 & x_3y_3 & y_3 \\ 1 & x_4 & x_4y_3 & y_3 \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{bmatrix} \quad (18)$$

It can be seen that only the two column vectors are different from those in Equation 13 so that the second row of constants in Equation 8 is equal to the second column of the coefficient matrix of Equation 14, i.e.,

$$\begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{bmatrix} = \frac{1}{x_2y_3} \begin{bmatrix} 0 \\ y_3 \\ -1 \\ 0 \end{bmatrix} \tag{19}$$

The same approach can be used to determine the remaining two rows of constants in Equation 8 so that this equation can be written as

$$u(x,y) = \frac{1}{x_2y_3} [u_1 \; u_2 \; u_3 \; u_4] \begin{bmatrix} x_2y_3 & -y_3 & 1 & -x_2 \\ 0 & y_3 & -1 & 0 \\ 0 & 0 & -\lambda & \lambda x_4 \\ 0 & 0 & -\lambda & -\lambda x_3 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ xy \\ y \end{bmatrix} \tag{20}$$

Since all of the boundary conditions for evaluating the constants in Equation 9 are identical to those used to evaluate the constants in Equation 8, Equation 9 can be written as

$$v(x,y) = \frac{1}{x_2y_3} [v_1 \; v_2 \; v_3 \; v_4] \begin{bmatrix} x_2y_3 & -y_3 & 1 & -x_2 \\ 0 & y_3 & -1 & 0 \\ 0 & 0 & -\lambda & \lambda x_4 \\ 0 & 0 & \lambda & -\lambda x_3 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ xy \\ y \end{bmatrix} \tag{21}$$

In order to simplify future calculations the following definitions are introduced

$$u(x,y) = \frac{1}{x_2 y_3} [u_1 \ u_2 \ u_3 \ u_4][J]\{1 \ x \ xy \ y\}$$

$$v(x,y) = \frac{1}{x_2 y_3} [v_1 \ v_2 \ v_3 \ v_4][J]\{1 \ x \ xy \ y\} \tag{22}$$

$$\text{where } [J] = \begin{bmatrix} x_2 y_3 & -y_3 & 1 & -x_2 \\ 0 & y_3 & -1 & 0 \\ 0 & 0 & -\lambda & \lambda x_4 \\ 0 & 0 & \lambda & -\lambda x_3 \end{bmatrix} \tag{23}$$

With these constants evaluated, it is now possible to determine stress and strain functions through the use of the equations of elasticity. For the two-dimensional case, the strain-displacement relations are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \text{and} \quad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{24}$$

Substitution of Equations 22 into these relations gives

$$\varepsilon_{xx}(x,y) = \frac{1}{x_2 y_3} [u_1 \ u_2 \ u_3 \ u_4][J]\{0 \ 1 \ y \ 0\} ,$$

$$\varepsilon_{yy}(x,y) = \frac{1}{x_2 y_3} [v_1 \ v_2 \ v_3 \ v_4][J]\{0 \ 0 \ x \ 1\} , \quad \text{and} \tag{25}$$

$$\gamma_{xy}(x,y) = \frac{1}{x_2 y_3} [u_1 \ u_2 \ u_3 \ u_4][J]\{0 \ 0 \ x \ 1\}$$

$$+ \frac{1}{x_2 y_3} [v_1 \ v_2 \ v_3 \ v_4][J]\{0 \ 1 \ y \ 0\} .$$

The stresses can be determined from the strains and the equations of plane stress. These equations can be written as

$$\sigma_{xx} = E'(\varepsilon_{xx} + \nu\, \varepsilon_{yy}) \,,$$

$$\sigma_{yy} = E'(\varepsilon_{yy} + \nu\, \varepsilon_{xx}) \,, \quad \text{and} \qquad (26)$$

$$\tau_{xy} = \frac{(1-\nu)E'}{2}\, \gamma_{xy}$$

where $E' = \dfrac{\Sigma}{1-\nu^2}$

Σ = Young's modulus

ν = Poisson's ratio .

Substitution of the strain equations into the plane stress equations yields

$$\sigma_{xx}(x,y) = \frac{E'}{x_2 y_3}\,[u_1\ u_2\ u_3\ u_4][J]\{0\ 1\ y\ 0\} + \frac{\nu E'}{x_2 y_3}\,[v_1\ v_2\ v_3\ v_4][J]\{0\ 0\ x\ 1\}$$

$$\sigma_{yy}(x,y) = \frac{E'}{x_2 y_3}\,[v_1\ v_2\ v_3\ v_4][J]\{0\ 0\ x\ 1\} + \frac{\nu E'}{x_2 y_3}\,[u_1\ u_2\ u_3\ u_4][J]\{0\ 1\ y\ 0\} \,, \quad \text{and} \qquad (27)$$

$$\tau_{xy} = \frac{(1-\nu)E'}{2x_2 y_3}\,[u_1\ u_2\ u_3\ u_4][J]\{0\ 0\ x\ 1\} + \frac{(1-\nu)E'}{2x_2 y_3}\,[v_1\ v_2\ v_3\ v_4][J]\{0\ 1\ y\ 0\} \,.$$

The stiffness coefficients can be determined from the stresses and strains by several methods. The method chosen here is the unit displacement method. The unit displacement equation is (Reference 1, p. 49)

$$1 \times k_{ij} = \int_V (\sigma_{xxi}\, \varepsilon_{xxj} + \sigma_{yyi}\, \varepsilon_{yyj} + \tau_{xyi}\, \gamma_{xyj})\, dV \ , \qquad (28)$$

$$(i,j = 1,8)$$

where k_{ij} is the force at i due to a unit displacement at j, ε_{xxj} is the strain due to a unit displacement at j, etc. The use of the equation will be demonstrated by evaluating k_{12}. The stresses are obtained from Equations 27 with $u_1 = 1$ and all other nodal displacements equal to zero. The result is

$$\sigma_{xx}\ (x,y) = \frac{E'}{x_2 y_3}\,(y - y_3) = -\frac{E'}{x_2}\,(1 - \eta) \ ,$$

$$\sigma_{yy}\ (x,y) = \frac{E'}{x_2 y_3}\,(y - y_3) = -\frac{E'}{x_2}\,(1 - \eta) \ , \text{ and} \qquad (29)$$

$$\tau_{xy}\ (x,y) = \frac{(1 - \nu)E'}{2x_2 y_3}\,(x - x_2) = -\frac{(1 - \nu)E'}{2y_3}\,(1 - \xi)$$

where $\xi = x/x_2$ and $\eta = y/y_3$.

The strains can be determined from Equations 25 with $u_2 = 1$ and all other nodal displacements equal to zero, giving

$$\varepsilon_{xx}\ (x,y) = \frac{1}{x_2 y_3}\,(y_3 - y) = \frac{1}{x_2}\,(1 - \eta) \ ,$$

$$\varepsilon_{yy}\ (x,y) = 0 \ , \quad \text{and} \qquad (30)$$

$$\gamma_{xy}\ (x,y) = \frac{-x}{x_2 y_3} = -\frac{1}{y_3}\,\xi\ .$$

Equations 29 and 30 can now be used with Equation 28 to give

$$k_{12} = E' \int_V \left[-\frac{1}{x_2^2}(1-\eta)^2 + \frac{1-\nu}{2y_3^2}(1-\xi)\ \xi\right] dV \qquad (31)$$

The differential volume can be expressed as

$$dV = t\ dx\ dy = t\ x_2 y_3\ d\xi\ d\eta \qquad (32)$$

and the limits of integration are, for y,

$$y = 0 \quad \text{to} \quad y = y_3$$
$$\qquad (33)$$
or $\quad \eta = 0 \quad \text{to} \quad \eta = 1\ ,$ and, for x,

$$x = \frac{x_3}{y_3}\,y \quad \text{to} \quad x = x_2 + \frac{x_{42}}{y_3}\,y$$
$$\qquad (34)$$
or $$\xi = \frac{x_3}{x_2}\,\eta = \rho\eta \quad \text{to} \quad \xi = 1 + \frac{x_{42}}{x_2}\,\eta = 1 + \beta\eta$$

where $\rho = \dfrac{x_3}{x_2}$ and $\beta = \dfrac{x_{42}}{x_2}$.

Equation 31 now becomes

$$k_{12} = E't\left\{\frac{1-\nu}{2m}\left[\frac{1}{2}\left(1+\beta+\frac{\beta^2-\rho^3}{3}\right) - \frac{1}{3}\left(1+\frac{3\beta}{2} + \beta^2 + \frac{\beta^3-\rho^3}{4}\right)\right] - m\left(\frac{1}{3}+\frac{\beta-\rho}{12}\right)\right\} \qquad (35)$$

where $m = y_3/x_2$.

Since there are thirty-six independent stiffness coefficients, Equation 28 must be evaluated thirty-six times. Although this could be done in the direct manner just illustrated, there is an approach which is better suited to automatic computation. The stiffness relation for the plate element, Equation 1, may be partitioned and written as

$$\left[\frac{p_x}{p_y}\right] = E' t \left[\begin{array}{cc} k_{xu} & k_{xv} \\ \hline k_{yu} & k_{yv} \end{array}\right] \left[\frac{u}{v}\right] \tag{36}$$

where $\{p_x\} = \{p_{x1}\ p_{x2}\ p_{x3}\ p_{x4}\}$,

$\{p_y\} = \{p_{y1}\ p_{y2}\ p_{y3}\ p_{y4}\}$,

$\{u\} = \{u_1\ u_2\ u_3\ u_4\}$, and

$\{v\} = \{v_1\ v_2\ v_3\ v_4\}$.

The submatrices in Equation 36 each have a component due to the direct stresses, σ_{xx} and σ_{yy}, and a component due to the shear stress, τ_{xy}. For instance, the upper left submatrix could be written as

$$\left[k_{xu}\right] = \left[k_{xu}\right]_d + \left[k_{xu}\right]_s \ . \tag{37}$$

It can be shown, however, that the shear components can be written in terms of the direct components. The resulting equations are

$$\left[k_{xu}\right] = \left[k_{xu}\right]_d + \frac{1 - \nu}{2} \left[k_{yv}\right]_d ,$$

$$\left[k_{yv}\right] = \left[k_{yv}\right]_d + \frac{1 - \nu}{2} \left[k_{xu}\right]_d , \quad \text{and} \tag{38}$$

$$\left[k_{xv}\right] = \left[k_{yu}\right]^T = \left[k_{xv}\right]_d + \frac{1 - \nu}{2\nu} \left[k_{xv}\right]_d^T$$

The three four-by-four matrices in Equations 38 can be determined from the elements of a column matrix, $\{Z\}$, defined by

$$\{Z_m\} = [A_{mn}]\{C_n\} \qquad (m=1,21;\ n=1,6) \qquad (39)$$

The matrices $[A_{mn}]$ and $\{C_n\}$ are given as Equations 41 and 42.

The equations which define the firect stiffnesses are

$$[k_{xu}]_d = \begin{bmatrix} Z_1 & -Z_1 & Z_2 & -Z_2 \\ & Z_1 & -Z_2 & Z_2 \\ & & Z_3 & -Z_3 \\ \text{Symmetric} & & & Z_3 \end{bmatrix},$$

$$[k_{xv}]_d = \begin{bmatrix} Z_4 & Z_5 & Z_6 & Z_7 \\ -Z_4 & -Z_5 & -Z_6 & -Z_7 \\ Z_8 & Z_6 & Z_{10} & Z_{11} \\ -Z_8 & -Z_9 & -Z_{10} & -Z_{11} \end{bmatrix}, \quad \text{and} \qquad (40)$$

$$[k_{yv}]_d = \begin{bmatrix} Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ & Z_{16} & Z_{17} & Z_{18} \\ & & Z_{19} & Z_{20} \\ \text{Symmetric} & & & Z_{21} \end{bmatrix}.$$

$$
[A_{mn}] = \begin{bmatrix}
m & 2m & m & 0 & 0 & 0 \\
0 & -\lambda m & -\lambda m & 0 & 0 & 0 \\
0 & 0 & \lambda^2 m & 0 & 0 & 0 \\
\nu & \nu & 0 & \nu & \nu & 0 \\
0 & 0 & 0 & -\nu & -\nu & 0 \\
-\delta\lambda\nu & -\delta\lambda\nu & 0 & -\lambda\nu & -\lambda\nu & 0 \\
\rho\lambda\nu & \rho\lambda\nu & 0 & \lambda\nu & \lambda\nu & 0 \\
0 & -\lambda\nu & 0 & 0 & -\lambda\nu & 0 \\
0 & 0 & 0 & 0 & \lambda\nu & 0 \\
0 & \delta\lambda^2\nu & 0 & 0 & \lambda^2\nu & 0 \\
0 & -\rho\lambda^2\nu & 0 & 0 & -\lambda^2\nu & 0 \\
1/m & 0 & 0 & 2/m & 0 & 1/m \\
0 & 0 & 0 & -1/m & 0 & -1/m \\
-\delta\lambda/m & 0 & 0 & -(1+\delta)\lambda/m & 0 & -\lambda/m \\
\rho\lambda/m & 0 & 0 & (1+\rho)\lambda/m & 0 & \lambda/m \\
0 & 0 & 0 & 0 & 0 & 1/m \\
0 & 0 & 0 & \delta\lambda/m & 0 & \lambda/m \\
0 & 0 & 0 & -\rho\lambda/m & 0 & \lambda/m \\
\delta^2\lambda^2/m & 0 & 0 & 2\delta\lambda^2/m & 0 & \lambda^2/m \\
-\delta\rho\lambda^2/m & 0 & 0 & -(\delta+\rho)\lambda^2/m & 0 & \lambda^2/m \\
\rho^2\lambda^2/m & 0 & 0 & 2\rho\lambda^2/m & 0 & \lambda^2/m
\end{bmatrix} \tag{41}
$$

$$C_n = \left\{\begin{matrix} 1 - \frac{\beta - \rho}{2} & -\frac{1}{2} - \frac{\beta - \rho}{3} \\ \frac{1}{3} + \frac{\beta - \rho}{4} & -\frac{1}{2} - \frac{\beta}{2} - \frac{\beta^2 - \rho^2}{6} \\ \frac{1}{4} + \frac{\beta}{3} + \frac{\beta^2 - \beta^2}{8} & \frac{1}{3} + \frac{\beta}{2} + \frac{\beta^2}{3} + \frac{\beta^3 - \rho^3}{12} \end{matrix}\right\} \qquad (42)$$

A digital computer subprogram has been written to evaluate the stiffness coefficients as defined by Equations 38 through 42. This subprogram was used in both the stiffness analysis and the force analysis to calculate the stiffness matrices of the plate elements.

The details of the matrix displacement program are given in Appendix A along with a program listing and the tabulated results. The results of the displacement method and the force method are compared in Chapter V.

CHAPTER III

THE MATRIX FORCE METHOD

Force methods of analysis are essentially the inverse of displacement methods. The force methods are based on the premise that of all the possible distributions of internal forces for which each element is in equilibrium, the correct distribution is the one which results in a geometrically compatible displacement configuration.

As in the case of the displacement method (Chapter II), the matrix force method is now an accepted method for analyzing complex structures. However, unlike the displacement method, there is only one basic matrix force method - the indirect method. A direct method is not possible because the forces acting on a structure are not completely independent. Both the applied loads and the reactions cannot be specified arbitrarily. Therefore, the constraints must be considered from the beginning. The matrix force equations used in this investigation were originally presented by Argyris and Kelsey in Reference 1; however, the nomenclature used is that of Pestel and Leckie (Reference 10).

The two basic equations of the matrix force method are

$$\{p\} = ([B_0] + [B_1][x])\{f\} \text{ , and} \qquad (43)$$

$$\{d\} = [F_d]\{f\} \;, \tag{44}$$

where $\{p\}$ = the column matrix of independent internal forces,

$\{f\}$ = the column matrix of external forces,

$[B_0]$ = the matrix of statically equivalent internal forces due to unit values of the external forces,

$[B_1]$ = the matrix of 'self-equilibrating' internal forces due to unit values of the redundants,

$[x]$ = the matrix of unit redundants,

$\{d\}$ = the column matrix of the displacements at the external forces, and

$[F_d]$ = the flexibility matrix of the assembled structure.

The equations used to determine $[x]$ and $[F_d]$ are given in Appendix B.

The Mathematical Model

In the past, the mathematical model used in a force analysis has frequently differed from the model used in a displacement analysis, especially in the analysis of sheet-stiffener type structures. For a displacement analysis the model is usually of the type described in the previous Chapter - stiffener elements and plate elements connected only at the nodes. On the other hand, the model for a force analysis has been obtained by assuming that the plate elements are in a state of pure shear while the stiffeners carry the direct loads. In order to account for the fact that the plate elements actually carry some of the direct load, a part of the plate area is added to the adjacent stiffener areas and this 'effective' stiffener area is used in the analysis. Unless the stress distribution is very nearly uniform, it is difficult to accurately determine the plate area which should be assigned to each stiffener. An even more difficult problem arises when all of the nodal lines do not coincide with the stiffener centerlines, as is the case with Models 2 through 7 (Figure 2). It then becomes necessary to assume 'effective' stiffeners along these interior nodal lines. The purpose of this "lumped parameter" type of model is to reduce the number of redundants; however, the accuracy of the results is also reduced. Since the purpose of this investigation is to compare the computational accuracy of the force and displacement methods, this type of model is not used. The models used are identical to those used in the displacement analysis (Chapter II).

Selection of Redundants

Another basic difference between the force and displacement method is that in order to use the force method the degree of redundancy of the structure is an important consideration. There is no equivalent consideration in the displacement method. As noted in the previous Chapter, the order of the matrix which must be inverted in a stiffness analysis is a function only of the number of nodes and the number of degrees of freedom at each node. On the other hand, the order of the matrix which must be inverted in a force analysis is equal to the degree of redundancy of the structure.

The degree of redundancy of a structure can be determined from the equation

$$n = \ell - e + r \tag{45}$$

where n = the degree of redundancy,

ℓ = the number of independent internal forces,

e = the number of nodal equilibrium equations, and

r = the number of reactions.

The number of independent internal forces for a given type of member must be carefully determined. For instance, in Figure 4, four forces are shown on the leading edge and trailing edge stringers; however, only one of these forces is independent. That is, given any one of the forces, the other three can be calculated using the equations of equilibrium of the element. This is true for any axially loaded element. Each trapezoidal plate element has five independent forces, since there are a total of eight forces (Figure 4) and only three available equations of equilibrium.

The number of nodal equilibrium equations is equal to the number of degrees of freedom of the nodes which is equal to the order of the 'unreduced' stiffness matrix of the structure. The number of reactions for the panel under study here is three for all seven models.

Table I shows the redundancy of each of the models along with some additional information which will be discussed below. It can be seen that Model 7 requires a larger matrix inversion for a force analysis (193 x 193) than for a displacement analysis (162 x 162). This is not true for all of the models, however. For Model 5, the matrices are approximately of equal size - 49 x 49 versus 50 x 50, and for Models 1 - 4, the flexibility matrix is smaller than the corresponding stiffness matrix.

Once the number of redundants has been determined, it is then necessary to select the internal forces which represent these redundants. For some structures, such as trusses, this is a relatively simple task. Selected elements in the structure are assumed to be 'cut' and the forces in these members are taken as the redundants. The matrix $[B_1]$ in Equation 43 can then be easily formed since each column of this matrix consists of the internal forces due to a unit value of one of the redundants. In a structure such as the one being studied here, it is difficult to devise a stable, statically determinate structure for any of the models except Model 1. The four stiffeners of Model 1 could be 'cut' and the plate element would be both stable and statically determinate for the loads and reactions shown in Figure 3.

It is not necessary to actually devise a single stable, statically determinate structure for each model. It is only necessary to find n independent sets of internal forces which are themselves in equilibrium. This fact, although

TABLE I

DEGREE OF REDUNDANCY

M	N	ℓ	n	NOA	NOB	NOC
2	2	9	4	4	0	0
3	2	16	7	6	1	0
3	3	28	13	8	4	1
5	3	52	25	12	10	3
5	5	96	49	16	24	9
9	5	184	97	24	52	21
9	9	352	193	32	112	49

M = number of rows of nodes

N = number of columns of nodes

ℓ = number of independent internal forces

n = number of redundants

NOA = number of Type A redundants

NOB = number of Type B redundants

NOC = number of Type C redundants

n = NOA + NOB + NOC

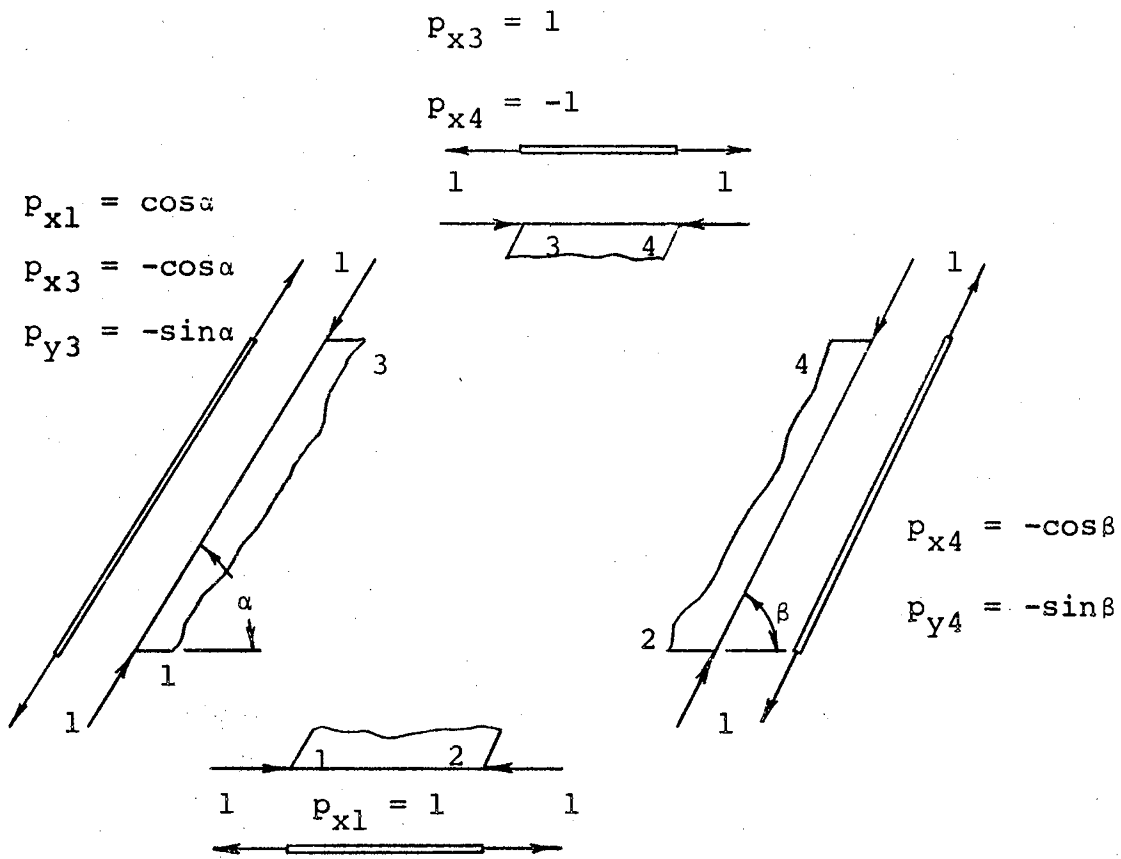
previously known, was first used on sheet-stringer type structures by Argyris (Reference 1). Argyris derived a self-equilibrating force system for the type of analysis where the stringers carry the axial load and the sheet is in pure shear (Two other systems were also developed for box type structures). This self-equilibrating system could not be used for the panel being studied here. It was possible, however, to develop a series of three systems which uniquely define the redundants. These self-equilibrating systems - called Types A, B, and C - are shown in Figures 6 and 7.

The Type A system was suggested by a similar application by Przemieniecki (Reference 11). A pair of equal and opposite unit loads are applied at the nodes of each stiffener element. These loads are reacted by unit loads at the same nodes on the adjacent web element. Since this system is in equilibrium no loads are transmitted to any of the adjoining elements.

The Type B system is equivalent to the Type A system at interior nodal lines where there are no stiffeners. Although the structure being studied here does not have any stiffeners at interior nodal lines, when there are interior stiffeners, two Type A systems are available for each stiffener element and a Type B system is not used.

The Type C system is similar to the system developed by Argyris. The major difference is that Argyris' system included interior stiffeners while the Type C system does not utilize interior stiffeners even if they are present. The

(a) Type A Redundants



(b) Type B Redundants

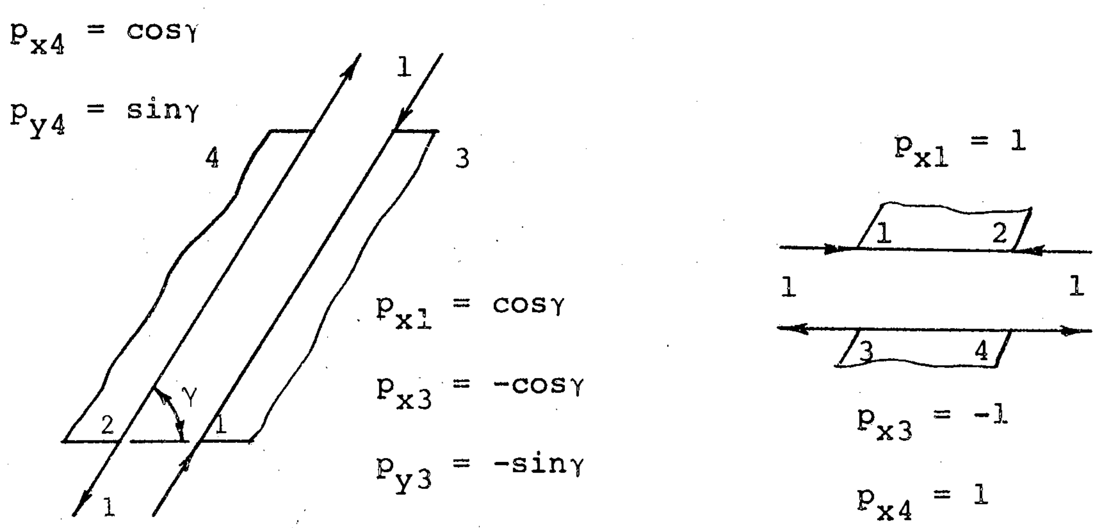


Figure 6. Type A and B Redundant Force Systems

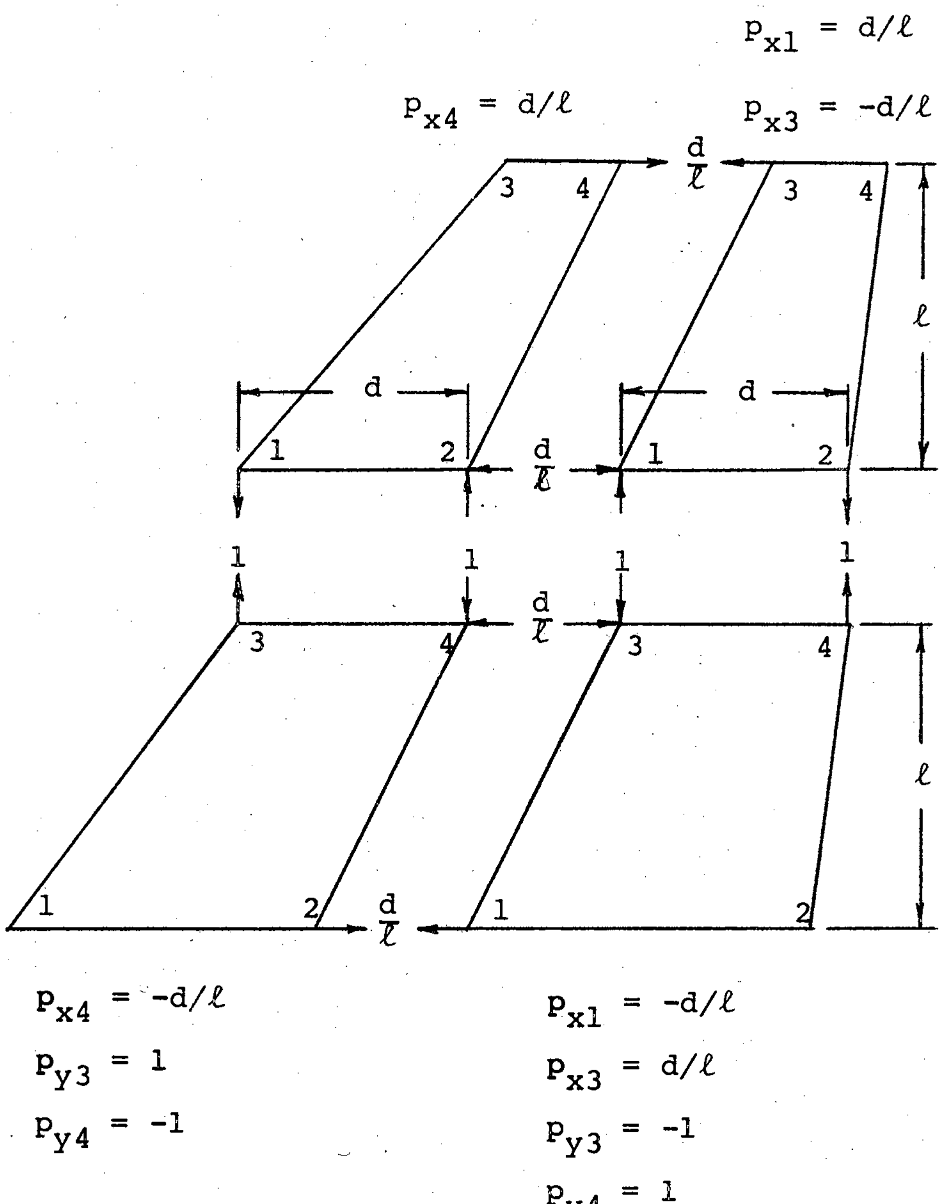


Figure 7. Type C Redundant Force System

Type A systems account for all of the redundants due to the stiffeners.

The last three columns of Table I show the number of each of the systems used for the various models. It can be seen that the total is exactly equal to the number of redundants in every case.

The $[B_0]$ matrix in Equation 43 must also be given special attention since a single stable, statically determinate structure is not used to determine $[B_1]$. Each column of $[B_0]$ represents a set of internal forces due to a unit value of one of the external loads. Argyris has shown (Reference 1) that any set of internal forces that is 'statically equivalent' to, i.e., in equilibrium with, the applied load may be used.

Since there are five external loads (Figure 3), it was necessary to select five internal force distributions from among the large number of possible ones. The selection, made on the basis of ease of programming, is as follows:

$F_1 = 1$ The load is transmitted as compression through the lower rib (Figure 3). None of the other elements are loaded.

$F_3 = 1$ The load is transmitted by the leading edge column and the lower row of web elements (Figure 8). None of the stiffeners are loaded.

$F_4 = 1$ The load is transmitted as tension by the upper rib to point 3 and then

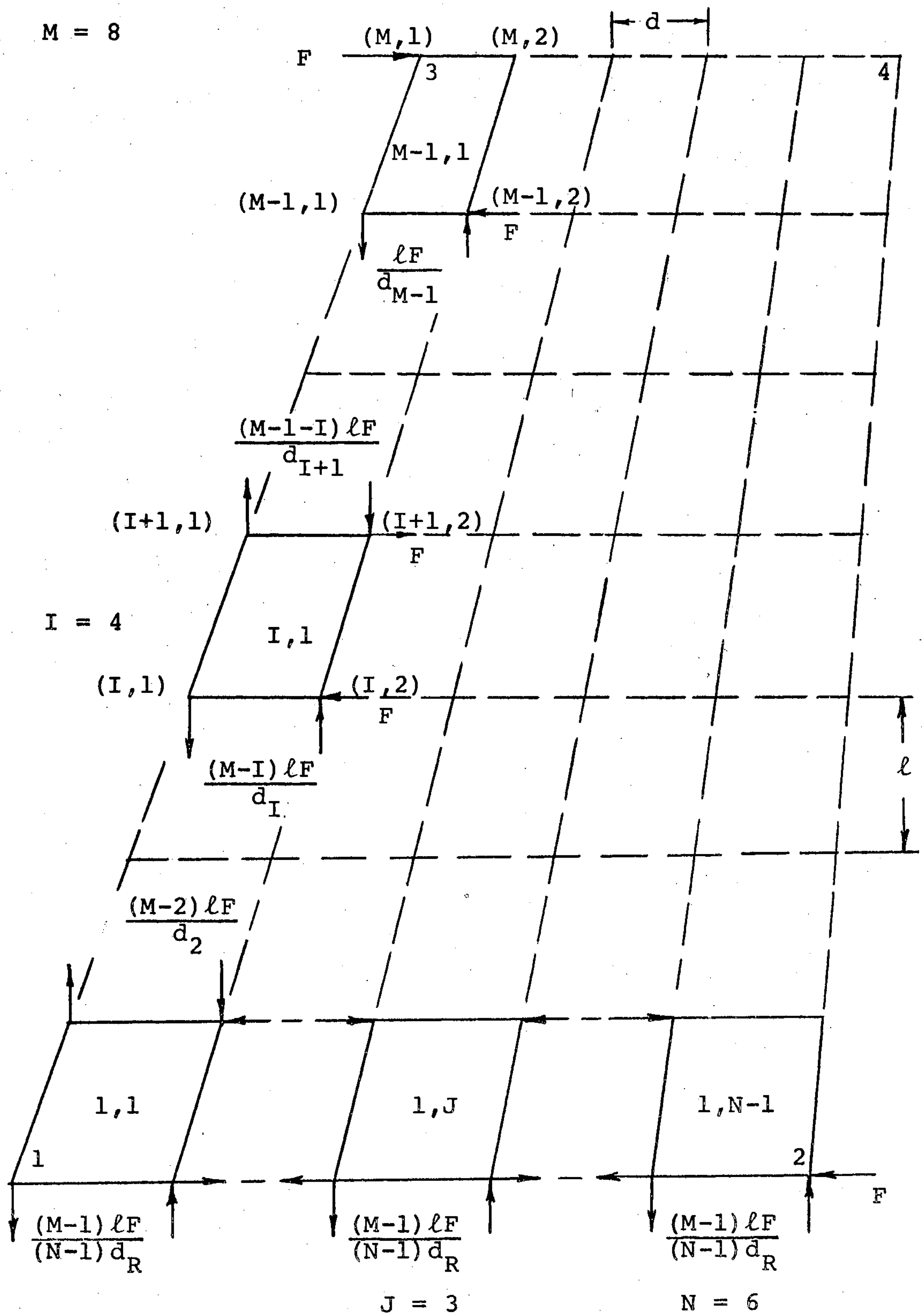


Figure 8. Statically Equivalent Structure for a Horizontal Load at Point 3

transmitted to the supports the same as F_3.

$F_7 = 1$ The load is resolved into two components, one along the leading edge stringer and one horizontal. The diagonal component is transmitted to the supports by the leading edge stringer and the lower rib and the horizontal component is transmitted the same as F_3.

$F_8 = 1$ Again the load is resolved into components. One component is transmitted directly down the trailing edge stringer to the support. The other is transmitted by the upper rib to point 3 and then through the plate elements.

Subroutine subprograms have been written to generate the $[B_1]$ and $[B_0]$ matrices using the force systems described above and are listed in Appendix B. It should be noted that these subprograms are not at all general, i.e., they were developed for the specific structure being studied. The redundant force systems developed here are general, however, and should find application in future investigations.

The displacements and stresses that are calculated using the matrix force method are tabulated in Appendix B. These results are compared in Chapter V with those determined from the matrix displacement analysis.

CHAPTER IV

THE EXPERIMENTAL PROGRAM

The three major segments of the experimental program are discussed in this Chapter. They are the design and construction of the test panel, the test fixture, and the instrumentation. The results of the experimental program are given in Appendix C.

Test Panel

As outlined in the Introduction, the test panel was a single-bay, monolithic, tapered shear web bounded by four stiffener elements (Figure 1). After the basic configuration was selected, the primary decision necessary was the selection of the material. Aluminum alloy tool and jig plate was originally projected as a suitable material due to its dimensional stability during machining. However, the low strength of this material would have resulted in a larger web thickness and larger stiffener areas than those normally found in thin-wall aerospace structures. The material finally selected was aluminum alloy 6061-T6. This alloy has good machining qualities combined with medium strendth properties.

The web of the panel has an aspect ratio of approximately

1.8 and a taper ratio of 2.0. The aspect ratio was chosen as representative of the shear panels of aerospace structures. The taper is greater than that usually found in typical structures; however, a relatively large value was used so that taper effects on the stresses would be significant. The leading edge sweepback angle of 45° was chosen so that the leading edge of the tip rib would fall aft of the trailing edge of the root rib. Thus, vertical loads at either, or both, of the upper nodes produce significant bending as well as tension in the panel.

The web thickness and the stiffener areas were determined from a parameter study using the matrix force method of analysis. The programming for this study was performed by Mr. Charles Cole of The Boeing Company and the program was run on the Boeing 7094 computer. The original parameters were chosen as: web thickness = 0.125 in., stringer area = 1.0 sq. in., and rib area = 0.5 sq. in. These were later reduced to 0.096 in., 0.929 sq. in., and 0.462, respectively, on the final test panel.

Three panels, two aluminum and one steel, were actually constructed. The first aluminum panel was made in the Boeing experimental machine shop and was used for preliminary studies. Since this panel did not have loading lugs it was suspended in the test fixture with wires. The second aluminum panel was originally machined in the Oklahoma State University Mechanical Engineering machine shop. However, since a mill was not available which would allow machining of one entire side without moving the panel, the required tolerances could not be maintained. Preliminary tests indicated that the eccentricities in the panel were too large for accurate testing; therefore, the panel was remachined in the engineering shop at Wichita State University. This remachining resulted in the final web thickness and areas given above. The steel panel was a one-inch thick plate which was milled to the proper shape and then ground flat. It was used to align the supporting structure and the loading mechanism prior to installing the final test panel.

Test Fixture

The basic test fixture used to support the panel is described in Reference 2. The fixture was subsequently modified by welding one-inch steel plate to the mounting surfaces and then machining these surfaces flat and parallel. This improvement can be seen in Figure 9.

The original base support is also described in Reference 2. This support was designed with a ball bearing carriage for the roller support which could take either up-loads or down-loads. In order to incorporate the four ball bearings, a built-up structure was necessary. As a result, the two supports could not be accurately aligned and lateral bending was induced in the test panel. The entire base support was redesigned as a two-piece structure with the movable support sliding on the fixed support (Figure 9). Although this support was much easier to align than the

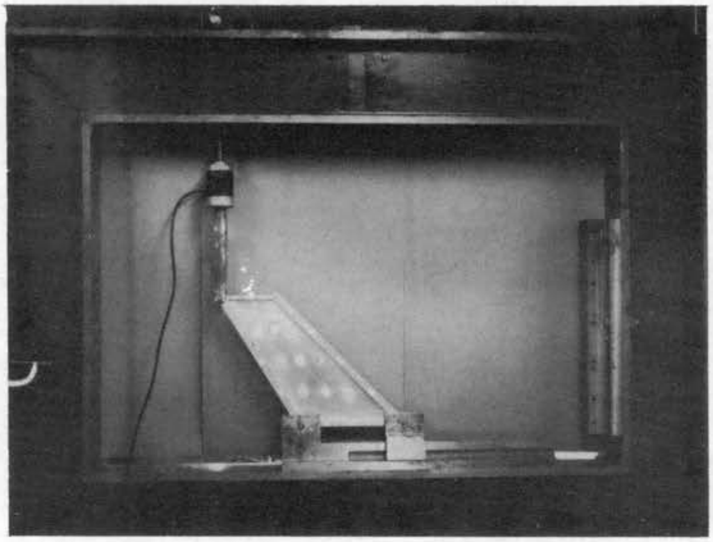


Figure 9. Final Test Panel and Loading Fixture

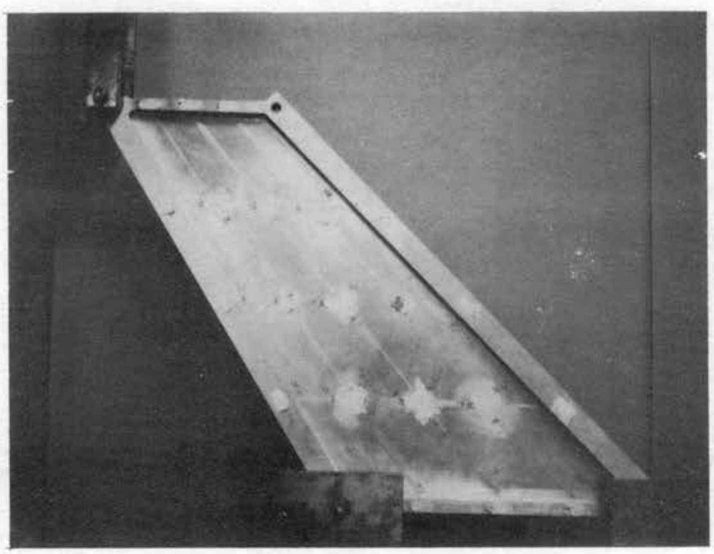


Figure 10. Close-up of Test Panel

original one, the sliding friction between the two supports was appreciable. This friction was eliminated during the tests by vibrating the movable support with a plastic mallet just prior to recording load and strain data.

The loads were applied to the panel with a dual-cylinder hydraulic jack through a 10,000-lb universal load cell. Loads were indicated on a Budd P-350 portable strain indicator. The loads were applied in nominal 500-lb increments up to 5,000 pounds and then released in the same increments. However, no attempt was made to adjust the load to an exact value. The load was increased, or decreased, to approximately the desired value and then the movable support was vibrated until no change in load occurred. The load and strains were recorded.

Instrumentation

The final test panel was instrumented with twenty-four axial strain gages and eighteen strain rosettes (Figure 10). The gages were located along the centerlines and the quarter-points of the panel. The axial gages were Budd C12-144B medium termperature gages and all but two of the rosettes were Budd C12-144D-R3Y medium temperature gages. The two outer rosettes on the horizontal centerline were Budd C12-1210-R3Y room temperature gages. In order to eliminate the effects of lateral bending of the panel, strain gages were mounted on both sides of the panel and the strain readings were averaged to obtain the strains in the median plane of the panel.

The original gages were installed with Bean BAP-1 cement. However, during the installation of the terminal strips and the wires from the terminal strips to the gage tabs, the entire set of gages was destroyed by corrosion. It was suspected that this corrosion was due to either a flux that was used during the soldering of the lead wires or a corrosive atmosphere in the furnace used to cure the cement. However, all attempts to reproduce the corrosion on practice gages were unsuccessful. The replacement gages were installed with Eastman 910 cement and, therefore, did not require curing in the furnace.

Three strain indicating systems were used to record the strains:

1. 20-Channel Budd Model A-110 System with automatic switching and balancing and printed output.
2. 10-Channel Budd Model A-110 System with automatic balancing but manual switching and recording.
3. 50-Channel Budd Model P-350 System with manual switching, balancing and recording. Two P-350 Strain Indicators were used with 5 SB-1 Switch and Balance Units. Two channels of one of the SB-1 units were not used.

All circuits were three-wire quarter-bridge circuits. The internal dummy resistors in the strain indicators were used to complete the bridge circuit.

CHAPTER V

COMPARISON OF RESULTS

The following discussion presents two independent comparisons of the results of this investigation:

1. A comparison of the nodal displacements, stiffener stresses, and web stresses as calculated by the two analysis methods - the force method and the direct stiffness method.
2. A comparison of the calculated and the measured strains in the web and the stiffeners.

Comparison of the Analytical Results

Excellent agreement was obtained between both the displacements and the stresses calculated by the two analysis methods. Since there were a total of 105 displacements (Tables III and VIII) and 2,907 stresses (Tables IV through VI and IX through XI) calculated by each method, the only feasible way to compare these values was through the use of the digital computer. A program was written for the IBM 1620 which read and compared each value obtained from the analyses. The per cent variation was calculated by means of the following equation:

$$\text{Per cent Variation} = 100\left[1 - \frac{\text{Force Value}}{\text{Displacement Value}}\right] \qquad (46)$$

The displacement value was arbitrarily chosen as the base. However, since the variations are small, no appreciable change in the magnitude of the variation would result if the second term of Equation 46 in the brackets were inverted.

The program was written so that variations which exceeded a specified per cent were punched into cards along with the actual values of the stresses or displacements. After a few trials, it was decided that only those variations equal to, or larger than, 0.5 per cent would be considered.

There was less than 0.5 per cent variation between all of the displacements. Only fourteen of the stresses differed by more than this variation, with a maximum variation of four per cent. Upon comparison of these fourteen stresses, it was discovered that their magnitudes were less than 0.02 psi per pound and that the maximum absolute variation was 0.0001 psi per pound. These stresses are identified by asterisks in the tables in Appendices A and B.

Comparison of the Analytical and Experimental Results

Since both the force and direct stiffness analyses give essentially the same analytical results, the direct stiffness method was used to calculate the strains for the comparison of analytical and experimental results. The decision to compare strains, rather than stresses, was based on two factors:

1. The measured quantities in the experimental program were the strains, not the stresses.

The strains were recorded to the nearest microstrain (1.0×10^{-6} in./in.) with an uncertainty of plus or minus 5 microstrain. This could result in errors as large as 0.03 psi/lb in calculating the normal stresses and 0.02 psi/lb in calculating the shearing stresses from strain rosette data. It can be seen from Tables IV and IX that errors of this magnitude are significant.

2. The calculation of the stresses at a point from strain rosette data requires all three strains at the point. If one gage element is inactive, the strain gage is entirely inactive for stress calculations. However, if strains are compared, any active gage elements produce usable results.

Figures 11 through 22 present both the analytical and experimental results in graphical form. The first nine of these figures present the web strains for the three loading conditions (F_3, F_8, and $F_7 + F_8$). The last three figures present the stiffener strains for these same loading conditions. Each web strain figure compares the strains along one of the horizontal rows of strain rosettes (Lower, Center, and Upper rows).

Each graph (i.e., ε_1) in the web strain figures shows analytical curves for Models 1, 3, 5, and 7. The results for the even-numbered models were omitted to improve the clarity of the graphs. Since the strain gages did not, in general, lie along the lines where the strains were calculated, it was necessary to interpolate between element centroids in order to obtain the strains along the lines of rosettes. Linear interpolation was used for these calculations. The resulting strains were connected with straight lines to form the curves in the graphs. The stiffener strains did not require interpolation; the calculated strain in each element was assumed to be the strain at the center of the element, and these strains were connected with straight lines from the curves shown in the last three figures.

It can be seen from the graphs that, in most cases, only the smallest element breakdown (Model No. 7) produced calculated strains that closely approximate the measured strains. For all three loading conditions, the calculated strains along the center row of strain rosettes are more accurate than those along the upper and lower rows. This is to be expected, since this row of rosettes is the most remote from the boundary constraints and the load points. It should also be noted that the calculated web strains are more accurate near the reentrant corners (upper right and lower left) than they are near the open corners (lower right and upper left). This is attributed to the fact that the shortest distance between the upper and lower ribs is along a line joining the open corners. This provides a direct load path from the loads to the most highly loaded support. Therefore the strain gradients in the vicinity of these corners are undoubtedly quite high and even the smallest element size used is too large to produce accurate strains in these areas.

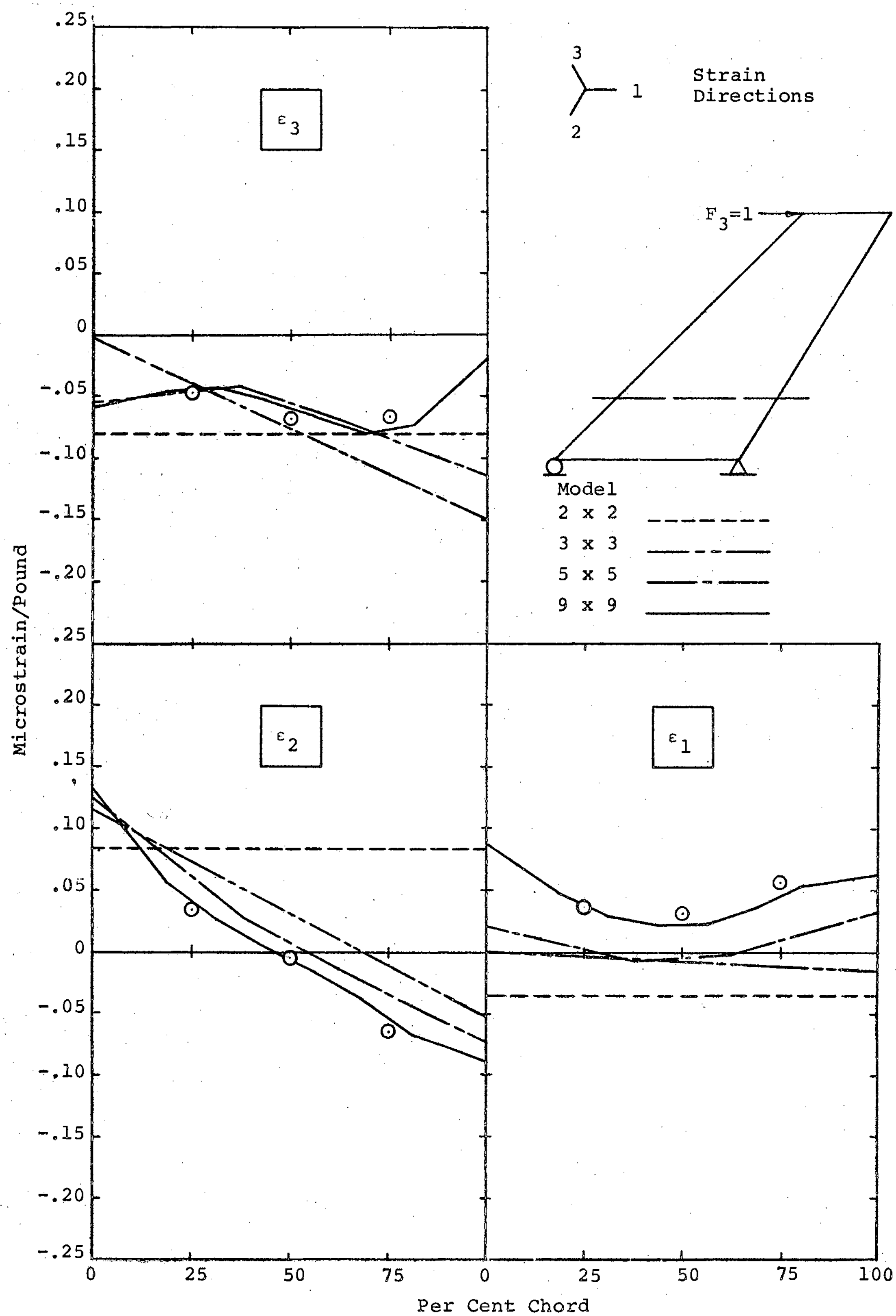


Figure 11. Web Strains - Lower Row - $F_3 = 1$

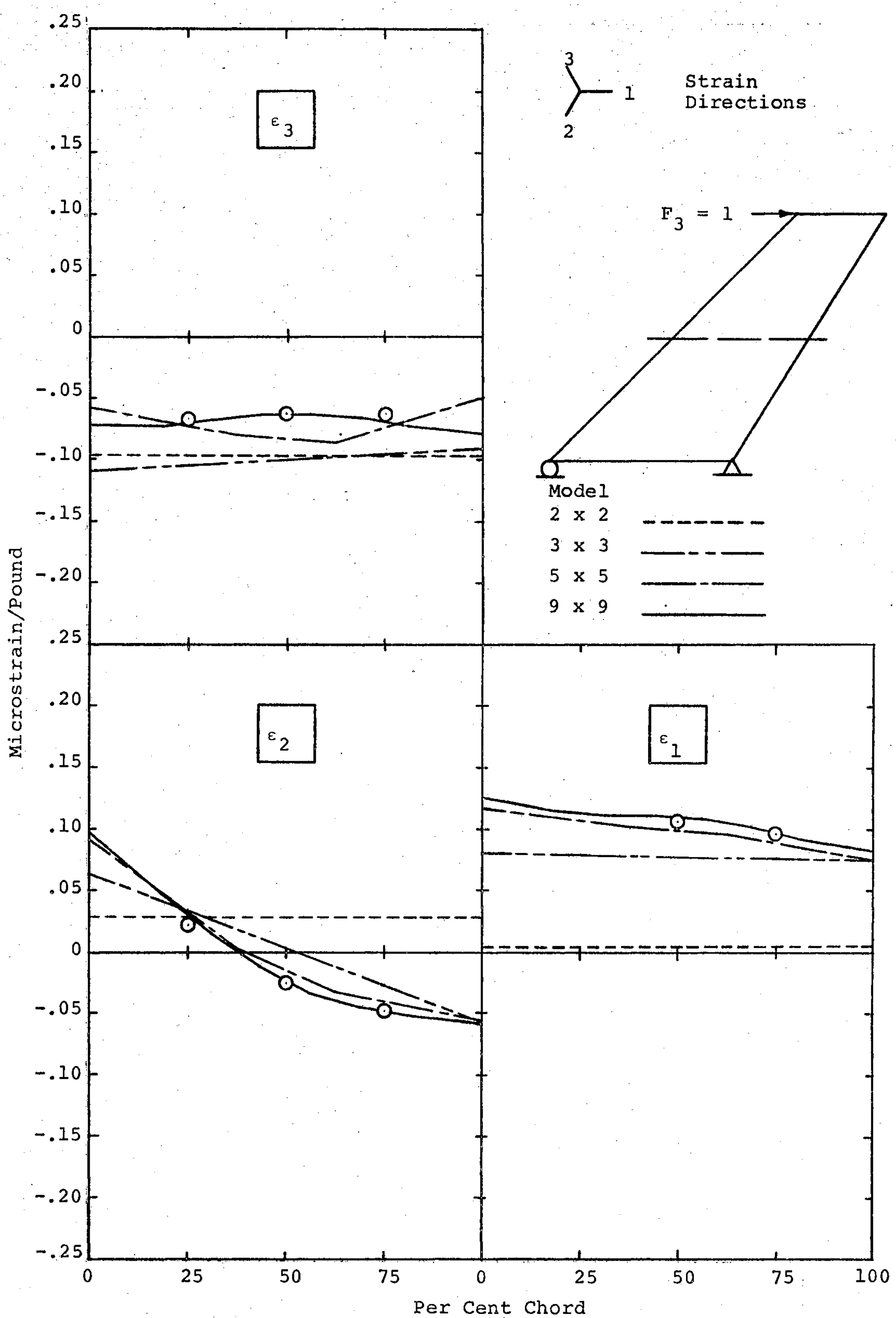


Figure 12. Web Strains - Center Row - F_3 = 1

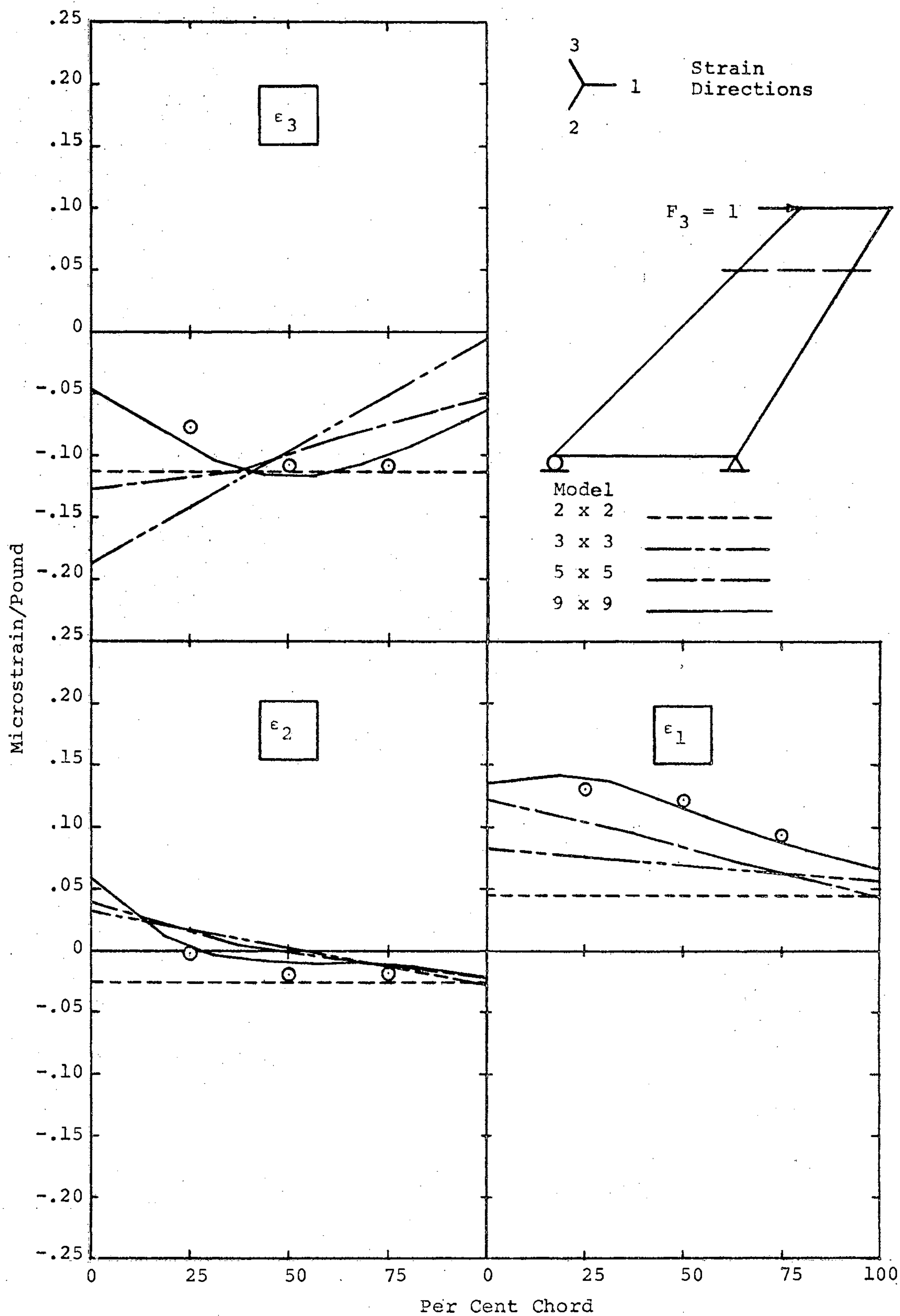


Figure 13. Web Strains - Upper Row - $F_3 = 1$

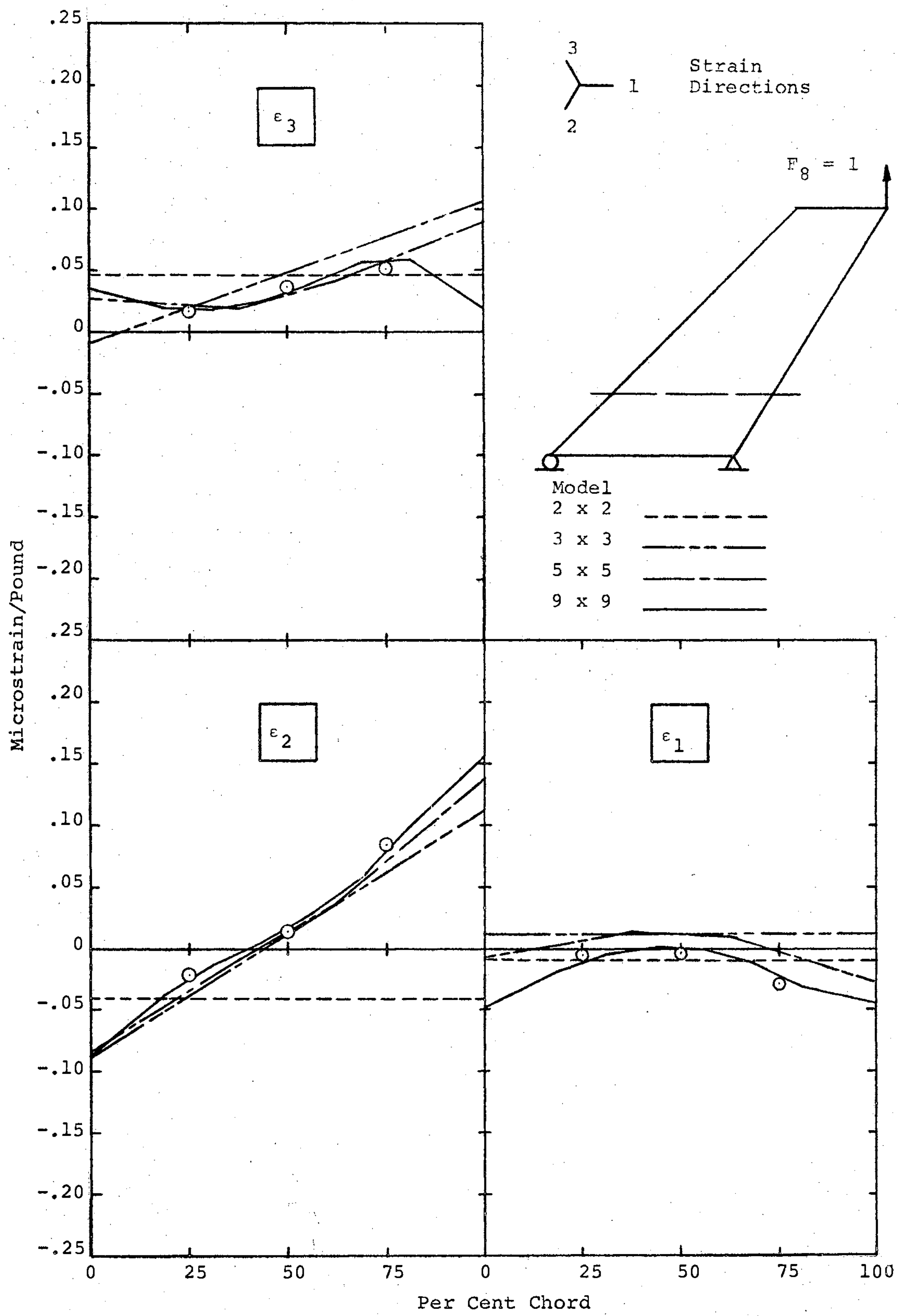


Figure 14. Web Strains - Lower Row - F_8 = 1

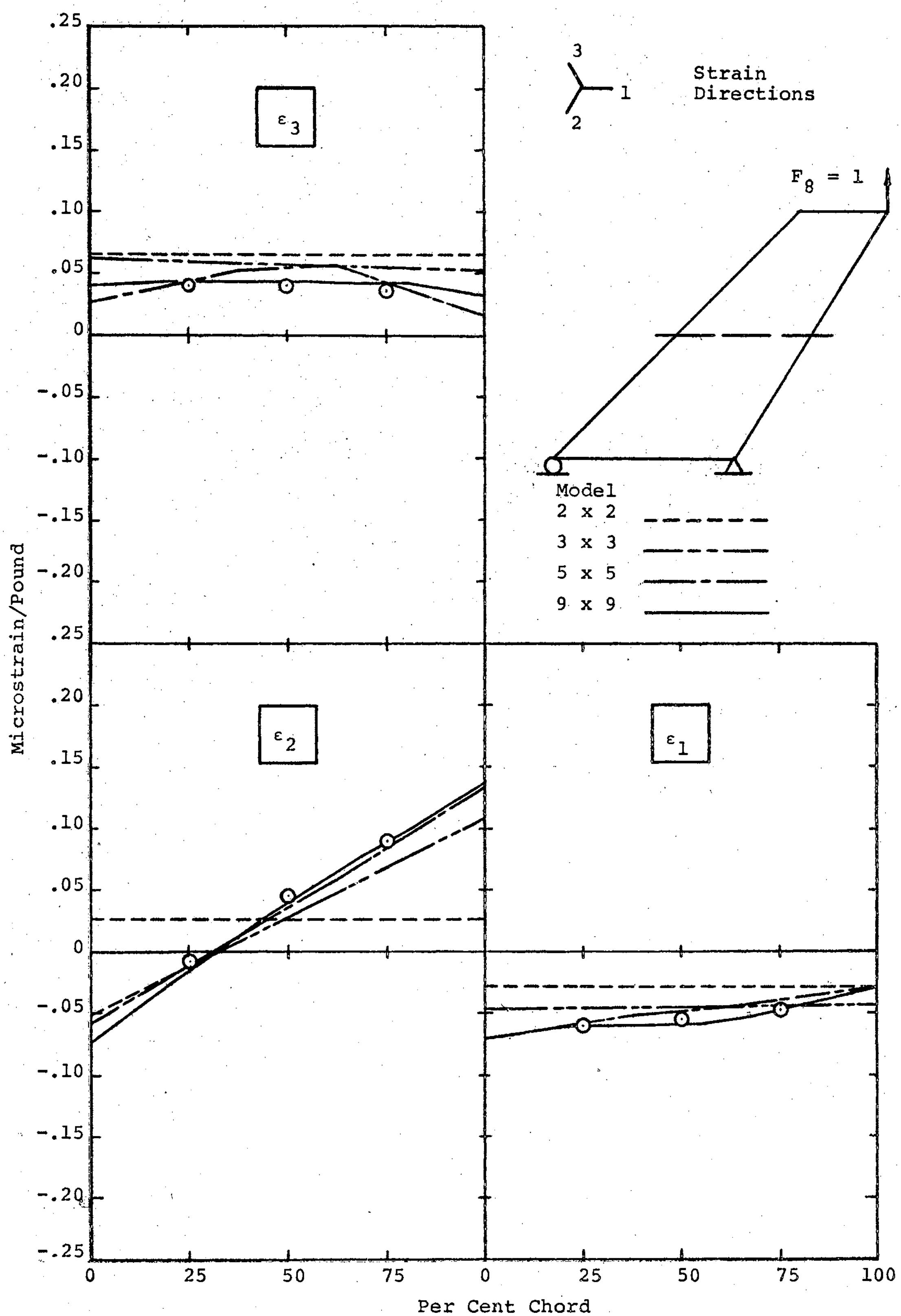


Figure 15. Web Strains - Center Row - F_8 = 1

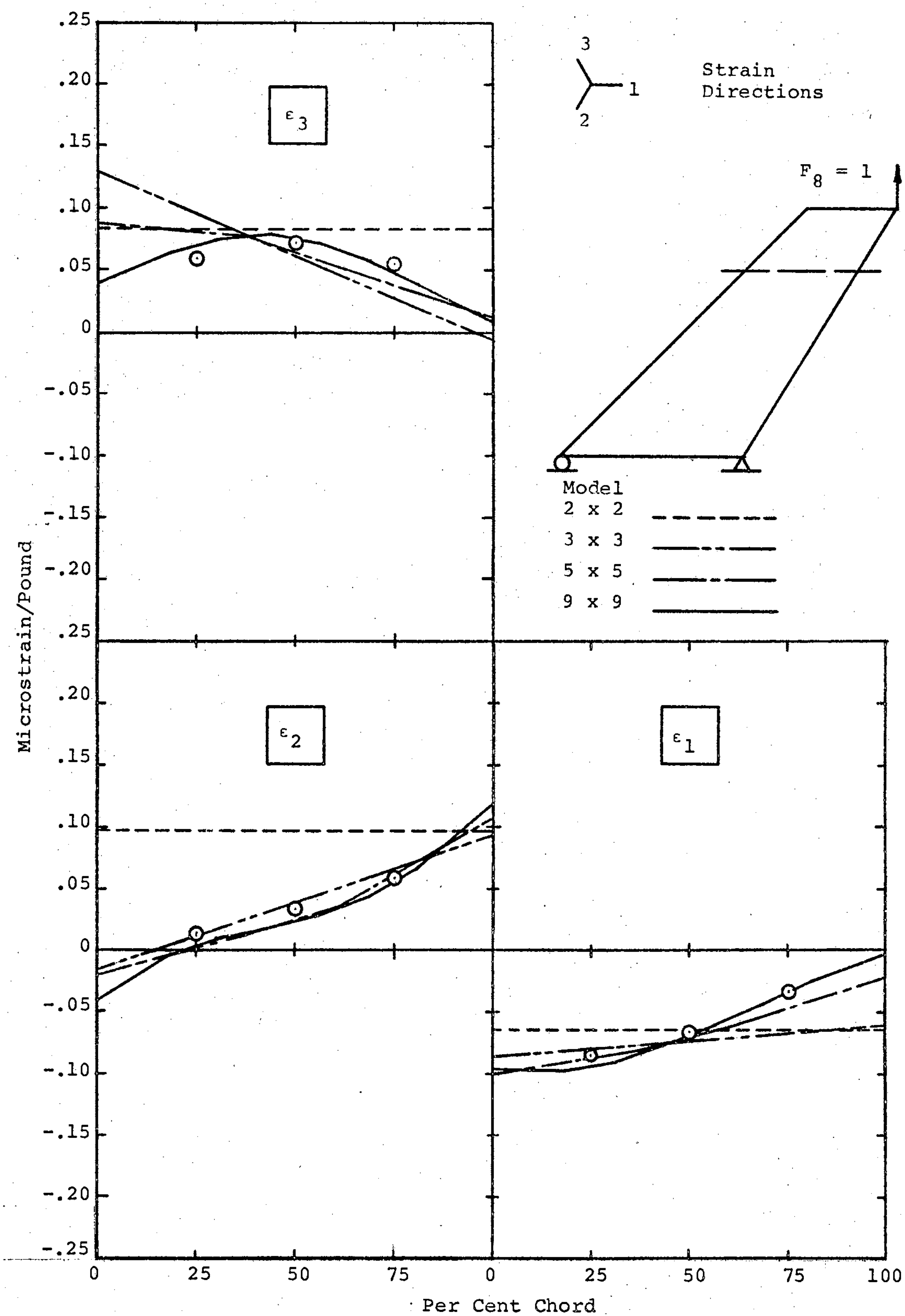


Figure 16. Web Strains - Upper Row - $F_8 = 1$

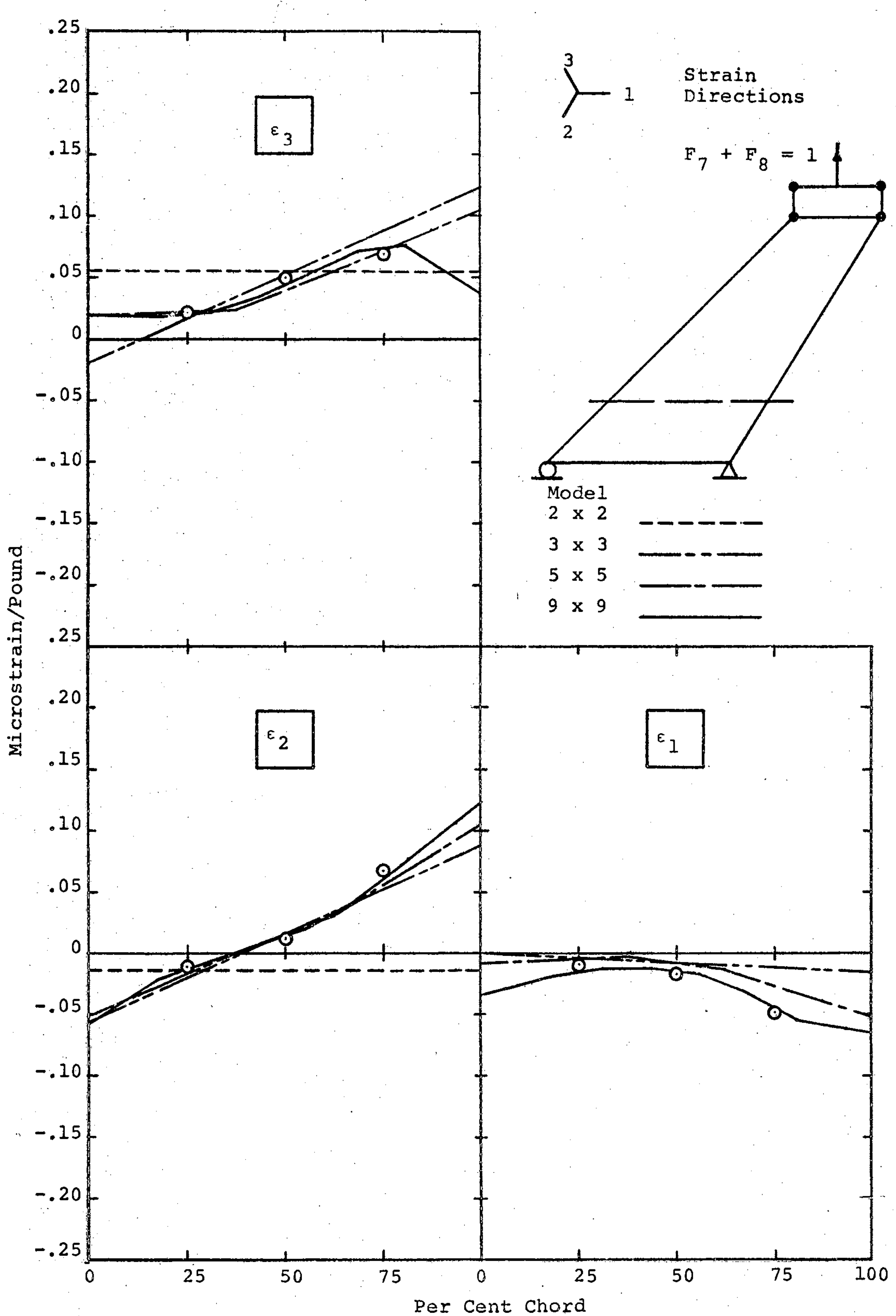


Figure 17. Web Strains - Lower Row - $F_7 + F_8 = 1$

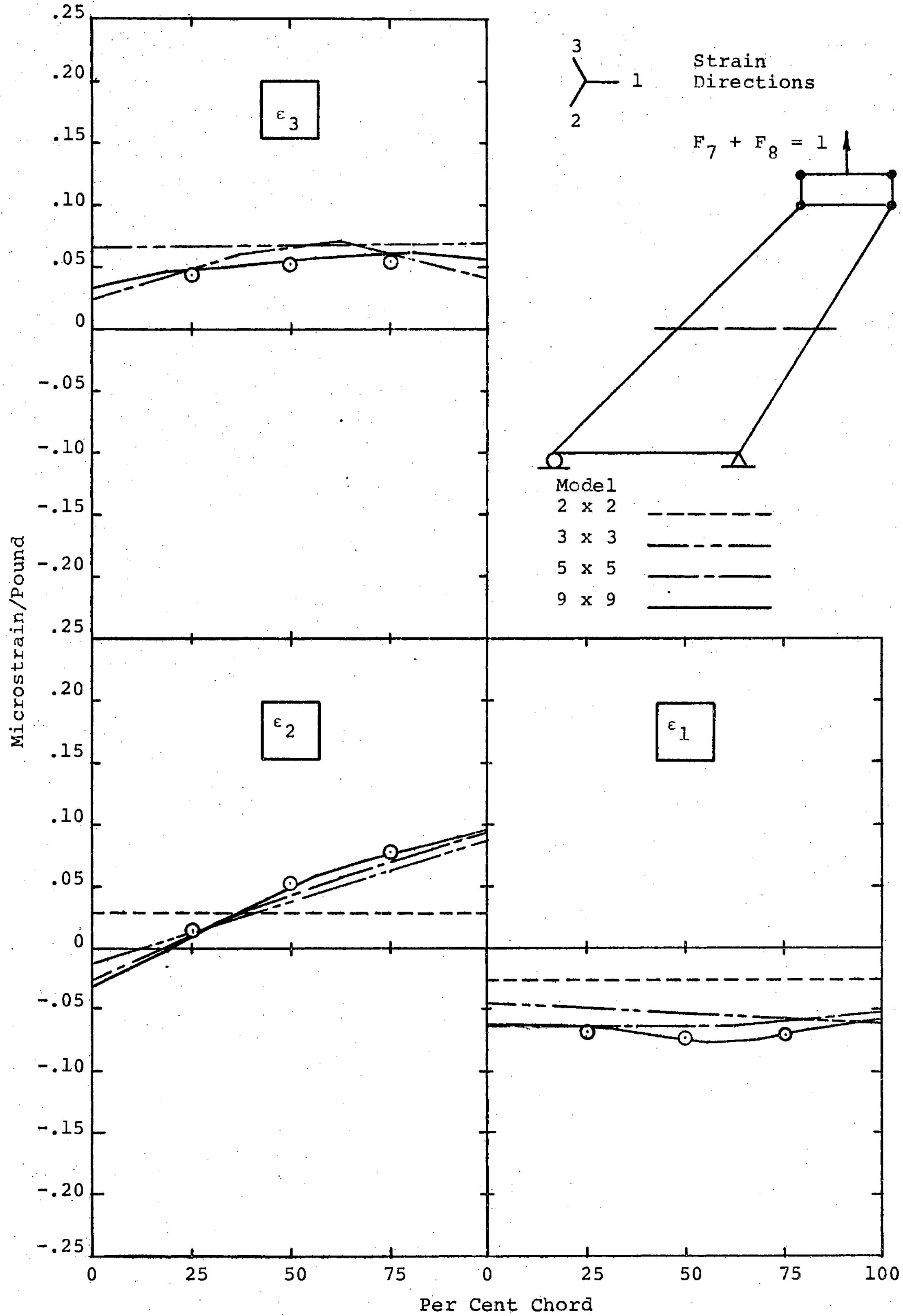


Figure 18. Web Strains - Center Row - $F_7 + F_8 = 1$

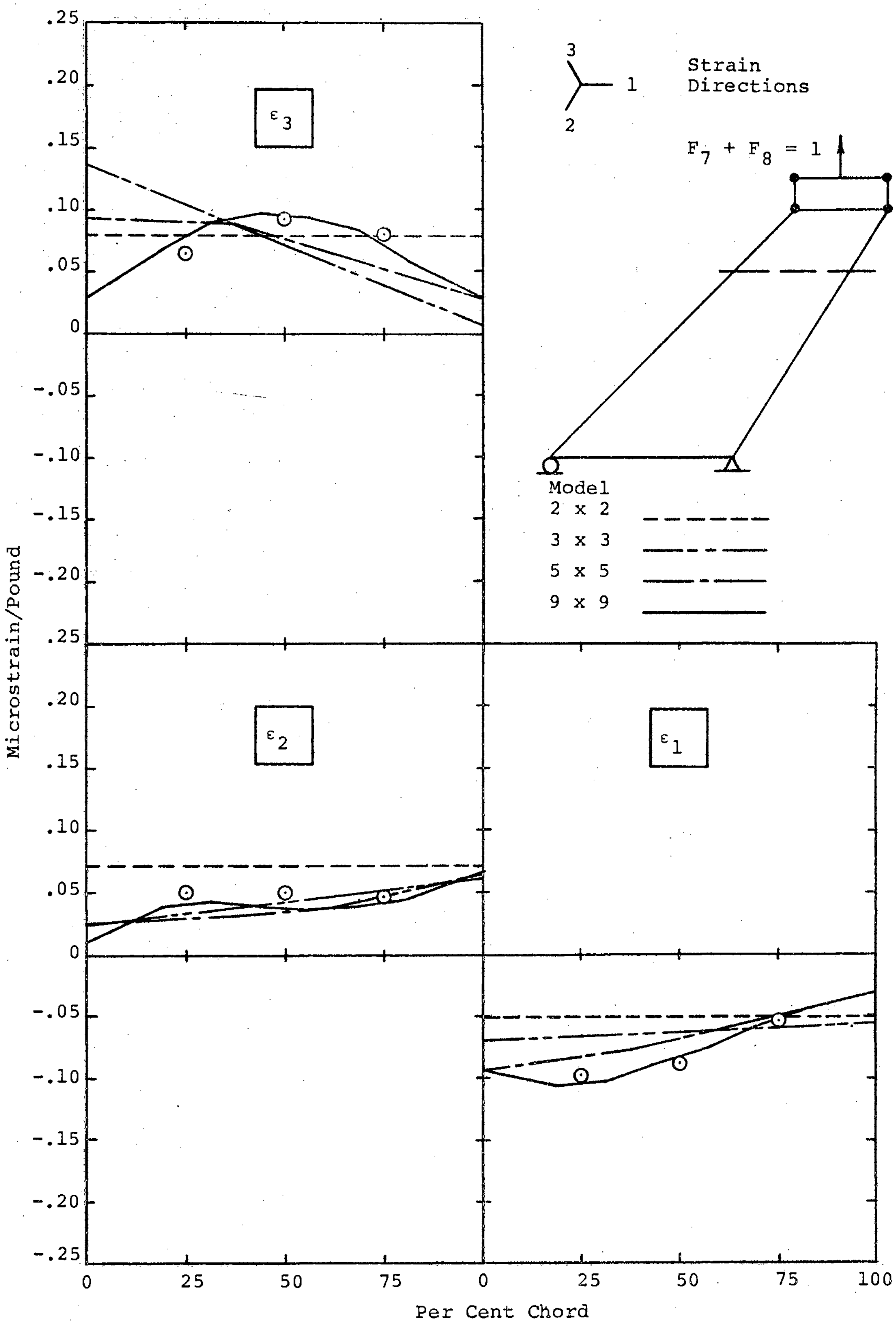


Figure 19. Web Strains - Upper Row - $F_7 + F_8 = 1$

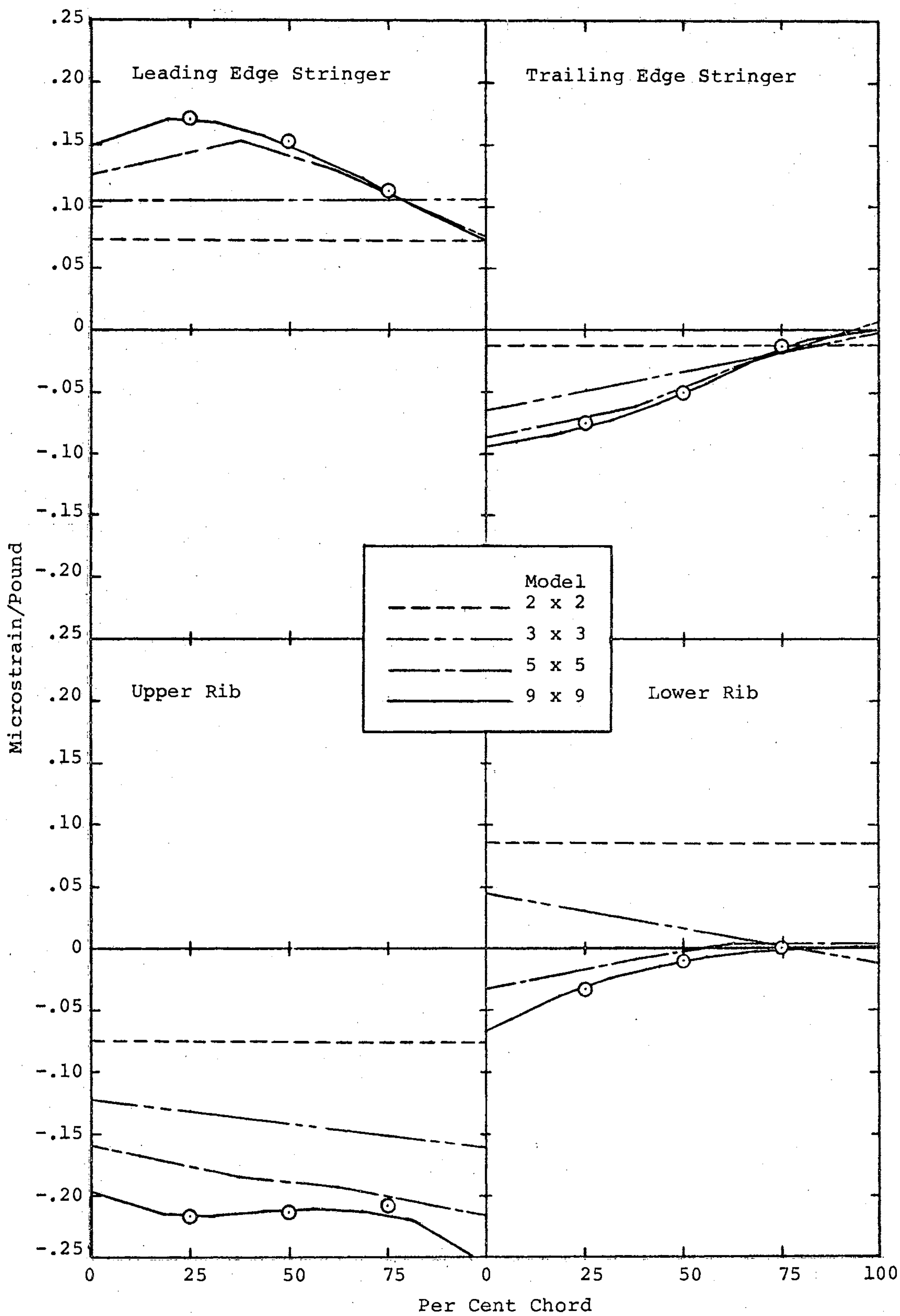


Figure 20. Stiffener Strains - $F_3 = 1$

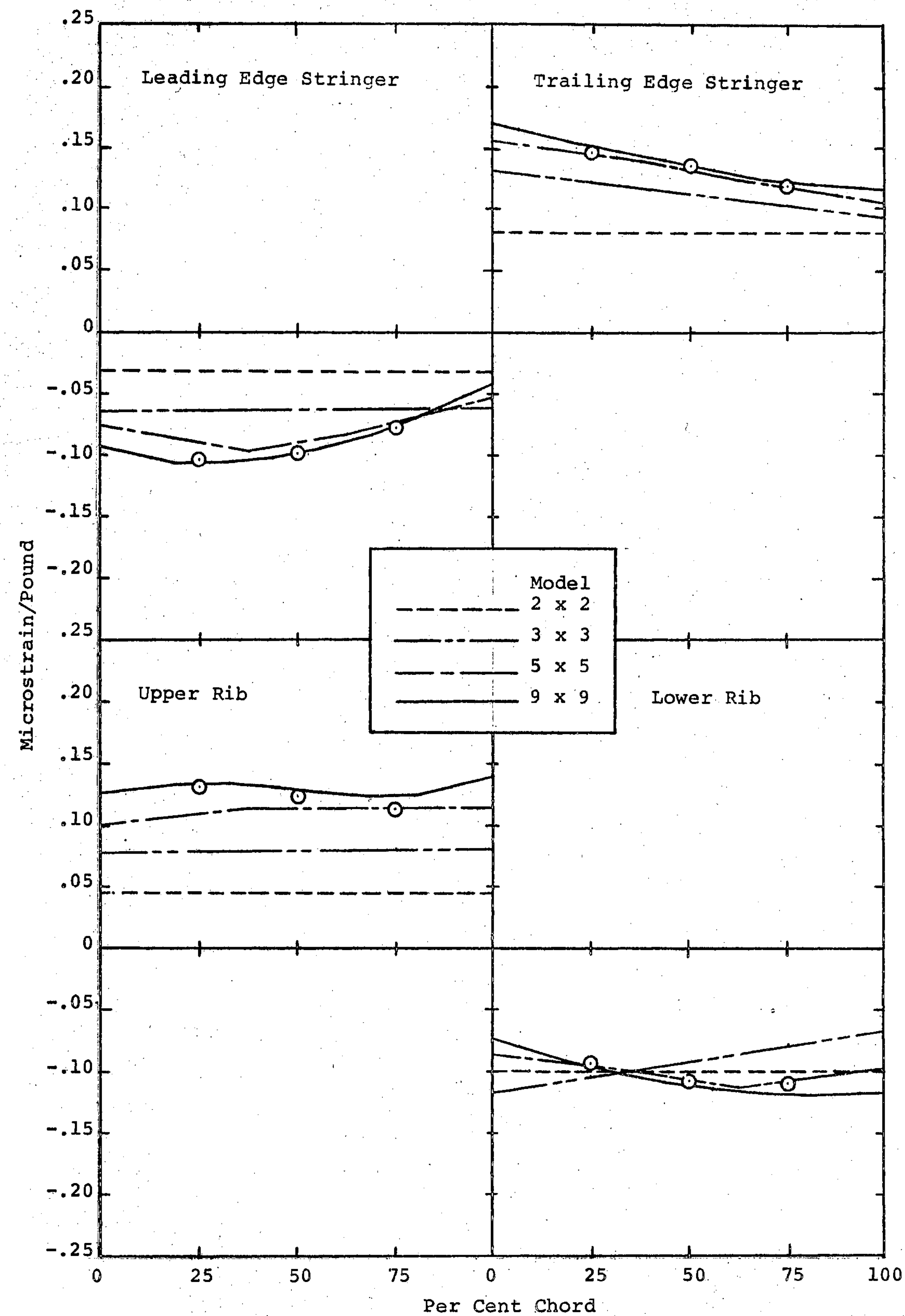


Figure 21. Stiffener Strains - $F_8 = 1$

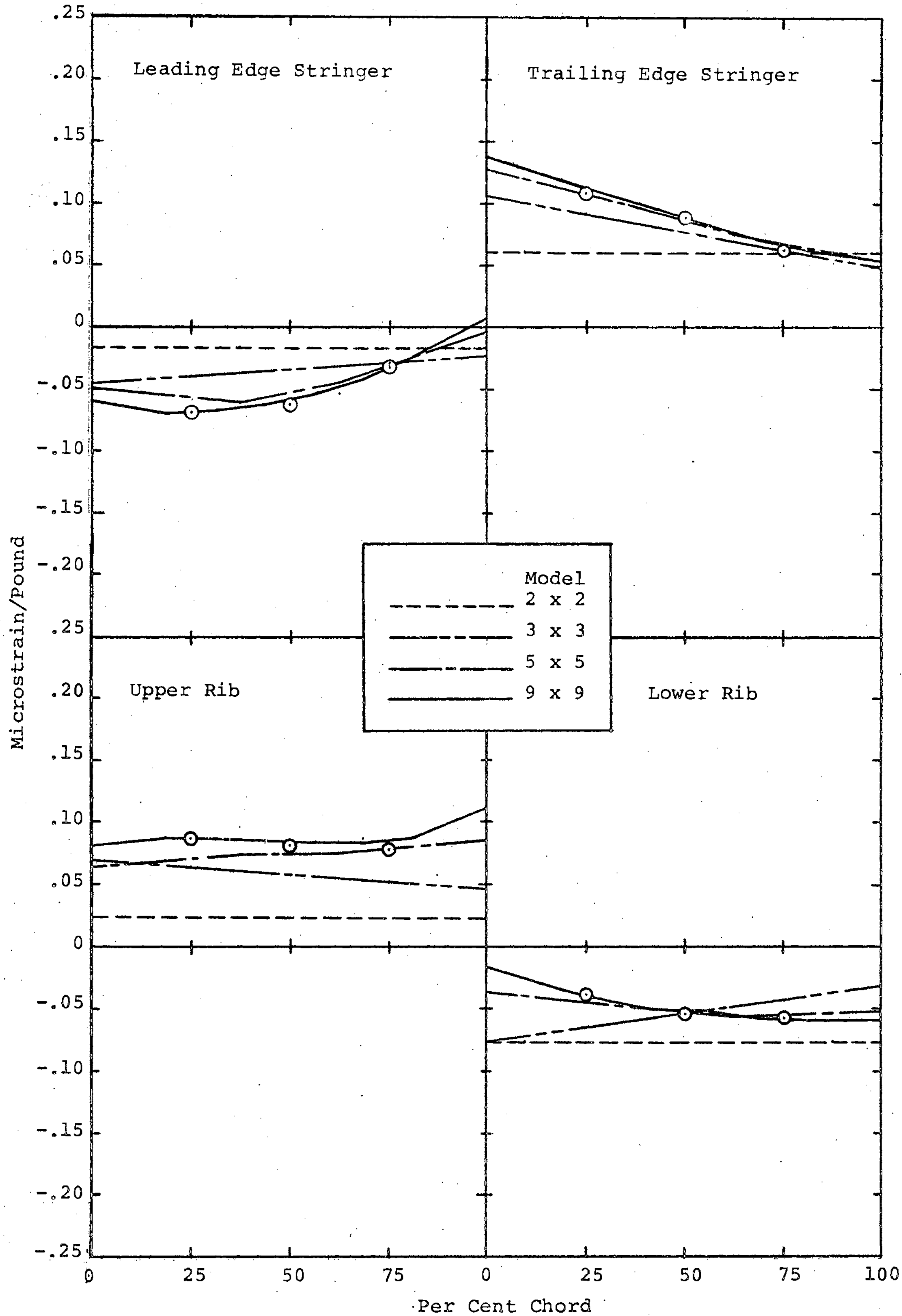


Figure 22. Stiffener Strains - $F_7 + F_8 = 1$

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The purpose and the objectives of the analytical and experimental investigation described in this dissertation are presented in the Introduction. The degree to which the objectives were attained can be summarized as follows:

1. It has been shown that the symmetrical monolithic shear panel is a valuable tool for obtaining accurate experimental results which can be used to verify new methods of analyses. The results of the study described herein, as well as the results obtained by Ayres (Reference 2) and Stone (Reference 13) have demonstrated its value in structural research.
2. The validity of the trapezoidal plate stiffness matrix derived in Chapter II has been demonstrated by the excellent agreement between the analytical and experimental results.
3. It has been shown that, provided the same mathematical model is used, essentially identical results can be obtained with either the force or displacement method. This means that the structural analyst can concentrate on selecting the most accurate mathematical model of his structure. The analysis method can then be chosen on the basis of program availability; or, if programs are available for both methods, on the basis of the ease of preparation of the input data. Analyses by both methods could also be used to verify the input data.
4. The comparison of the analytical and experimental results has shown that only the smallest sub-element size resulted in calculated strains that

are accurate enough for analysis and design. This is primarily a consequence of the small size of the test panel. Since all points in the panel are relatively close to either a load point or a free boundary, the strain gradients are large throughout the panel; therefore, small elements are required in order to obtain the desired accuracy.

5. The redundant force systems developed in Chapter III have provided a simple method for obtaining the elements of the unit redundant force matrix. The use of these force systems can be readily extended to structures with stiffeners along the interior nodal lines.

Although the results have extended the field of knowledge about the trapezoidal shear panel, much additional information needs to be made available. Some of the more immediate needs are:

1. The sub-element size effect on larger structures with multiple trapezoidal panels should be studied. The experimental results of Stone (Reference 13) could be used for experimental verification. In addition, the box-type structure should be investigated since it is more typical of actual construction and the boundary effects are minimized.
2. Additional trapezoidal plate stiffness matrices should be derived using other assumed displacement and stress distributions. The method used by Ayres (Reference 2) for the rectangular plate should be extended to include trapezoidal plates. The experimental results of this investigation could be used to evaluate these stiffnesses. It would also be desirable to simplify the final equations derived in Chapter II in order to reduce the computer time required to evaluate these equations.
3. The present work, and that of Ayres and Stone, should be extended to include thermal loading. Although analysis

methods are presently available there is no known experimental data.

A SELECTED BIBLIOGRAPHY

1. Argyris, J. H., and S. Kelsey. Energy Theorems and Structural Analysis. Butterworths, London, 1960.

2. Ayres, M. U. "Theoretical and Experimental Comparison of Matrix Methods for Structural Analysis." (unpub. Ph.D. thesis, Oklahoma State University, 1966).

3. Baker, A. F. "Design of a Rigid Load Frame." (unpub. M.S. report, Oklahoma State University, 1963).

4. Belirgen, H. E. "Analysis of a Stiffened Panel by Direct Stiffness Method." (unpub. M.S. thesis, Wichita State University, 1965).

5. Bruhn, E. F. Analysis and Design of Flight Vehicle Structures. Tri-State Offset Co., Cincinnati, 1965.

6. Cates, R. T. "Matrix Structural Analysis by the Method of Substructures." (unpub. M.S. report, Oklahoma State University, 1964).

7. Cook, E. L., R. E. Chapel, and W. D. Bernhart. "Comment on the Basis for Derivation of Matrices for the Direct Stiffness Method." AIAA Journal, vol. 2, p. 1161, 1964.

8. Lambert, G. E. "A Digital Computer Program Utilizing the Matrix Force and Matrix Displacment Methods of Structural Analysis." (unpub. M.S. thesis, Wichita State University, 1965).

9. Martin, H. C. Introduction to Matrix Methods of Structural Analysis. McGraw-Hill, New York, 1966.

10. Pestel, E. C., and F. A. Leckie. Matrix Methods in Elastomechanics. McGraw-Hill, New York, 1963.

11. Przemieniecki, J. S. "Matrix Structural Analysis of Substructures." AIAA Journal, vol. 1, pp. 138-147, 1963.

12. Przemieniecki, J. S. "Triangular Plate Elements in the Matrix Force Method of Structural Analysis." AIAA Journal, vol. 1, pp. 1895-1897, 1963.

13. Stone, G. C. "Stress and Displacement Analysis of Planar Stiffened Shell Structures." (unpub. Ph.D. thesis, Oklahoma State University, 1967).

14. Turner, M. J., R. W. Clough, H. C. Martin, and L. J. Topp. "Stiffness and Deflection Analysis of Complex Structures." Journal of the Aeronautical Sciences, vol. 23, pp. 805-824, 1956.

15. Wehle, L. B., Jr., and W. Lansing. "A Method for Reducing the Analysis of Complex Redundant Structures to a Routine Procedure." Journal of the Aeronautical Sciences, vol. 19, pp. 677-684, 1952.

APPENDIX A

DEVELOPMENT OF THE MATRIX DISPLACEMENT PROGRAM

In addition to the generation and inversion of the panel stiffness matrix [K], see Equation 2, it is also necessary to utilize the elements of the resulting flexibility matrix to determine the nodal displacements and the element stresses. The digital computer program written to accomplish this result is described below. The program consists of three parts:

1. Generation of the Stiffness Matrix.
2. Inversion of the Stiffness Matrix.
3. Calculation of the Displacements and Stresses.

Parts 1 and 3 (Table II) were written by this author in Fortran II for the IBM 7094 Operating System. The major elements of these two programs will be described here. Part 2, written in the SOS language was available in the Boeing Wichita program library and is not described here.

Part 1 - Generation of the Stiffness Matrix

The basic input to the program consists of the following:

1. The panel material and section properties.
2. The coordinates of the corner nodes.
3. The constraint vectors.
4. The number of panel configurations.

5. The number of rows and columns of nodes in each panel configuration, or model.

The constraint vectors are used to indicate which nodes are constrained. The vectors used for this analysis were

$$\{F_x\} = \{1\ 0\ 1\ 1\} \text{ and } \{F_y\} = \{0\ 0\ 1\ 1\} . \qquad \text{(A-1)}$$

A "one" indicates that the node is free to displace and a "zero" indicates that the node is constrained against displacement. Since the constraints are not taken into account until just prior to the matrix inversion, the use of these constraint vectors makes possible the investigation of any desired conditions of constraint.

The first two and the last basic inputs are written on the output tape for easy identification of the final results. Then several frequently used constants are calculated.

Since only the coordinates of the corner nodes are specified, it is necessary to calculate and store the coordinates of the interior nodes. These are stored in two arrays, a square array [X] for the X-coordinates and a column {Y} for the Y-coordinates.

The discussion of the generation of the 'unreduced' stiffness matrix of the panel in Chapter II implies that all of the element stiffness matrices must be developed and then combined. Although this approach is more straight-forward, it is impractical to store all of the element stiffnesses. In fact, it was necessary to store the panel stiffness matrix in a condensed form in order not to exceed the capacity of the available core storage. Therefore, as each element stiffness matrix is generated, its coefficients are added into the proper positions in the panel stiffness matrix.

The matrix equations derived in Chapter II for the stiffness of a trapezoidal plate were programmed and used to generate the plate element contributions to the panel stiffness. A relatively simple indexing system was developed to extract the nodal coordinates and to place the plate element stiffness coefficients in the panel stiffness matrix. Although this indexing system was developed for a plane structure, it could be extended for use with box-type structures. The axial element stiffness matrix derived in Reference 1 was used to calculate the stiffness coefficients of the stiffener elements.

As mentioned above, the panel stiffness matrix was stored in a condensed form in order to conserve core storage even though all computations to this point were in single-precision arithmetic. Since the stiffness matrix is symmetric, only the diagonal and upper-right-hand coefficients were stored. The storage format is shown in Figure 23.

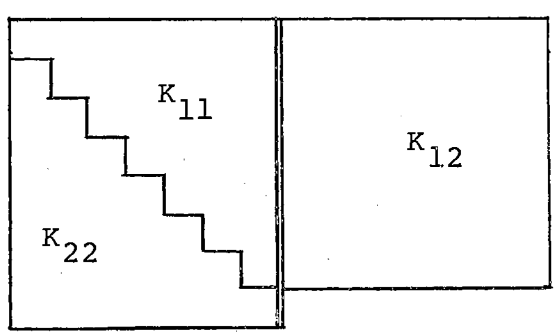


Figure 23. Storage Format for the Stiffness Matrix

Making the Stiffness Matrix Nonsingular - In order to obtain the flexibility matrix, it is necessary to make the 'unreduced' stiffness nonsingular. This can be accomplished by striking out the rows and columns corresponding to the external constraints. In the current case, this would reduce the size of the matrix by three rows and columns. It would then be necessary to expand the matrix after inversion if the indexing system previously mentioned were to be usable for determining the stresses and displacements. This approach was used in the first version of the program. However, due to the way in which the stiffness matrix is stored, the routines developed for this purpose were very lengthy. An alternate, and equally valid, procedure (suggested by Mr. Winder) was used in the later versions of the program. In this procedure all of the elements in the row and column corresponding to a constraint are set to zero except the diagonal element, which is set to one. After the inversion, the diagonal elements are set to zero.

The SOS matrix inversion program used to obtain the flexibility matrix required that the stiffness matrix be stored on magnetic tape by columns. Mr. Issacs wrote a subprogram to extract each column from the array stored as shown in Figure 23 and to write it on a scratch tape for subsequent inversion. Although the data was generated and stored as single-precision words, the actual inversion used double-precision words.

Part 3 - Calculation of the Displacements and Stresses

Due to the fact that each of the three parts of the total program are actually separate programs, the basic input data is reread for this part along with the flexibility matrix. Another subroutine, also written by Mr. Issacs, was used to enter the flexibility matrix in the format shown in Figure 23. The frequently used constants and the nodal coordinates were also recalculated.

The flexibility coefficients for the corner nodes were extracted from the panel flexibility matrix and placed in an 8x8 array. Since three rows and columns of this array are zero, this array was later reduced to a 5x5 array and only the nonzero elements are shown in the tabulated results. These results are shown in Table III.

The stresses at any point in a plate element can be determined from Equation 27. These equations relate the stresses at the point to the nodal displacements of the element, the nodal coordinates of the element, and the coordinates of the point in question. These equations can be combined into a matrix equation of the form

$$\{\sigma\} = [\Sigma]\ \{u\} \qquad \text{(A-2)}$$

The expansion of this equation is shown on the following page.

For the models with relatively large subelement sizes (Models 1 through 4), the web stresses are calculated at five points in each web element. These points are shown in

Web Stress Equation

$$\begin{bmatrix} \frac{\sigma_{xx}}{E'} \\ \frac{\sigma_{yy}}{E'} \\ \frac{\tau_{xy}}{G} \end{bmatrix} = \begin{bmatrix} -\frac{1}{x_{21}}(1-\eta) & \frac{1}{x_{21}}(1-\eta) & -\frac{\lambda}{x_{21}}\eta & \frac{\lambda}{x_{21}}\eta & -\frac{\nu}{y_{31}}(1-\xi) & -\frac{\nu}{y_{31}}\xi & \frac{\lambda_{21}}{y_{31}}(\delta-\xi) & -\frac{\lambda\nu}{y_{31}}(\rho-\xi) \\ -\frac{\nu}{x_{21}}(1-\eta) & \frac{\nu}{x_{21}}(1-\eta) & -\frac{\lambda\nu}{x_{21}}\eta & \frac{\lambda\nu}{x_{21}}\eta & -\frac{1}{y_{31}}(1-\xi) & -\frac{1}{y_{31}}\xi & \frac{\lambda}{y_{31}}(\delta-\xi) & -\frac{\lambda}{y_{31}}(\rho-\xi) \\ -\frac{1}{y_{31}}(1-\xi) & -\frac{1}{y_{31}}\xi & \frac{\lambda}{y_{31}}(\delta-\xi) & -\frac{\lambda}{y_{31}}(\rho-\xi) & -\frac{1}{x_{21}}(1-\eta) & \frac{1}{x_{21}}(1-\eta) & -\frac{\lambda}{x_{21}}\eta & \frac{\lambda}{x_{21}}\eta \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \qquad \text{(A-3)}$$

$$\frac{\tau_{xy}}{G} = \frac{2\,\tau_{xy}}{(1-\nu)E'} \qquad \xi = \frac{x-x_1}{x_{21}} \qquad \rho = \frac{x_{31}}{x_{21}} \qquad \lambda = \frac{x_{21}}{x_{43}} = \frac{1}{\delta-\rho}$$

$$x_{ij} = x_i - x_j \qquad \eta = \frac{y-y_1}{y_{31}} \qquad \delta = \frac{x_{41}}{x_{21}}$$

Stringer Stress Equation

$$\sigma = \frac{E}{L}\,[-\cos\theta \quad \cos\theta \quad -\sin\theta \quad \sin\theta]\{u_1 \; u_2 \; v_1 \; v_2\} \qquad \text{(A-4)}$$

$\theta = \alpha$ for Leading Edge Stringer and

$\theta \quad \beta$ for Trailing Edge Stringer

Rib Stress Equation

$$\sigma = \frac{E}{L}\,[-1 \quad 1]\{u_1 \quad u_2\} \qquad \text{(A-5)}$$

Figure 24. For the other models, the web stresses were calculated only at the centroids of the elements. These stresses are given in Table IV.

The stiffener stresses were calculated from equations of the form of Equation A-2. Since the stress over the length of each stiffener element is assumed constant, the equations are much simpler than the corresponding web stress equation. The expanded stiffener stress equations are shown on page 73. The rib stresses are tabulated in Table V and the stringer stresses are shown in Table VI.

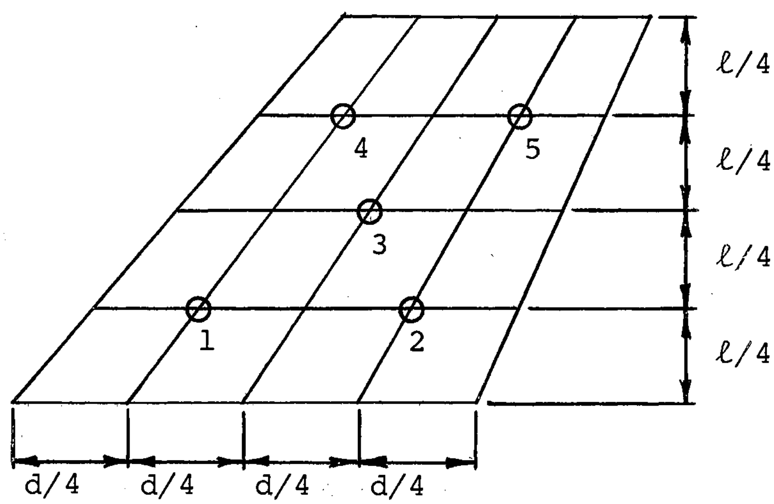


Figure 24. Location of Web Stress Points for MN≤15

Explanation of Tables

<u>Table II. Matrix Displacement Program Listing</u>

This program is listed as it was executed with two exceptions:

1. The operating system control cards have been removed.
2. The "C" in column 1 of the comment cards has not been printed.

Table III. Displacements of Corner Nodes

The displacements in this table are actually the flexibility matrices of the panel. In order to indicate the relative magnitude of these flexibilities, the computer output was multiplied by 10^6 and the results were printed with the decimal point in the proper position.

The first column in the table indicates the model number as identified in Figure 2. The second column and the last five column headings indicate the displacements and forces, respectively, as identified in Figure 3.

Table IV. Web Stresses

Since the web stresses are all of the order of one psi per pound, or less, it was only necessary to print them with the decimal point in the proper position in order to permit easy comparison.

The first column in the table indicates the model number (Figure 2). The second column identifies the element number by row and column (Figure 8). The third column indicates the points within each element at which the stresses were calculated. For Models 1 - 4, the stress points are as shown in Figure 24. For the remaining models, the stresses were determined only at the centroids of the elements. This is indicated by a "CG" in column 3. The fourth column identifies the type of stress, i.e., SXX = σ_{xx}, SYY = σ_{yy}, and SXY = τ_{xy}. The remaining columns give the stresses due to unit loads as indicated by their column heading.

Table V. Rib Stresses and Table VI. Stringer Stresses

These stresses are presented in the same manner as the web stresses with the following exceptions:

1. Since the stress is assumed constant over the length of each element, only one stress is given for each element.
2. The rib and stringer elements are numbered as shown in Figure 25.

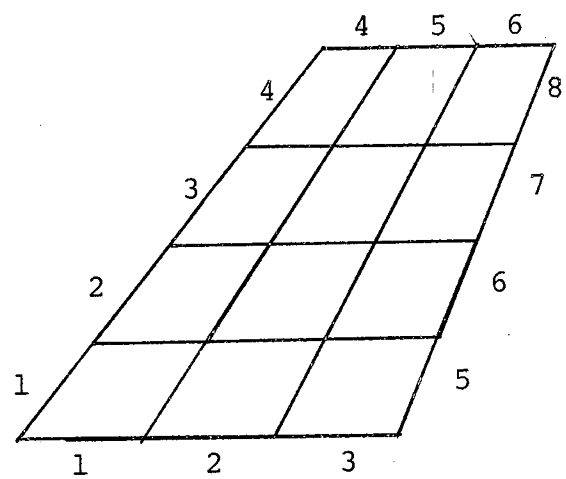


Figure 25. Rib and Stringer Numbering System

TABLE II

MATRIX DISPLACEMENT PROGRAM LISTING

```
      ISOTHERMAL DIRECT STIFFNESS ANALYSIS OF A TRAPEZOIDAL
      PLATE WITH BOUNDARY STIFFENERS - PART 1

      DIMENSION X(9,9),Y(9),A(21,6),C(6),Z(21),F(4,4),G(4,4),
     1SM11(81,82),SM12(81,82),FX(4),FY(4),H(4,4)
      COMMON SM11,SM12

      FORMAT STATEMENTS

  200 FORMAT(6E12.4)
  201 FORMAT(2I5)
  202 FORMAT(1H ,6E19.8)

      KIN=2
      KOT=3

      BASIC INPUT/OUTPUT

      CALL TESTXS(5)
      REWIND 5
      CALL ENDFIL(5)
      CALL UNLOAD(5)
    1 READ INPUT TAPE KIN,200,E,V,T,STA,RA,X1,X2,X3,X4,Y1,Y3
      WRITE OUTPUT TAPE KOT,202,E,V,T,STA,RA
      WRITE OUTPUT TAPE KOT,202,X1,X2,X3,X4,Y1,Y3
    2 READ INPUT TAPE KIN,200,(FX(I),FY(I),I=1,4)
      READ INPUT TAPE KIN,201,NOPAC
      WRITE OUTPUT TAPE KOT,201,NOPAC
      NUM=1
 2000 READ INPUT TAPE KIN,201,M,N
      WRITE OUTPUT TAPE KOT,201,M,N

      CONSTANTS

      EP=E/(1.-(V**2))
      EPT=EP*T
      GS=E/(2.*(1.+V))
      AM=M-1
      AN=N-1
      M1=M-1
      N1=N-1
      MN=M*N
      MNN=MN-N
      MN1=MN-N1
      M2N1=N*(M-2)+1
      MN4=MN-1
      MNO=MN+1
```

TABLE II (CONTINUED)

NODAL COORDINATES

```
      X(1,1)=X1
      X(1,N)=X2
      X(M,1)=X3
      X(M,N)=X4
      Y(1)=Y1
      Y(M)=Y3
      DELY=(Y3-Y1)/AM
      DO 4 I=2,M1
    4 Y(I)=Y(I-1)+DELY
      DELXR=(X2-X1)/AN
      DELXT=(X4-X3)/AN
      DO 5 J=2,N1
      X(1,J)=X(1,J-1)+DELXR
    5 X(M,J)=X(M,J-1)+DELXT
      DO 6 I=2,M1
      DO 6 J=1,N
    6 X(I,J)=X(1,J)+((X(M,J)-X(1,J))/(Y(M)-Y(1))*(Y(I)-Y(1)))
```

PLATE STIFFNESS MATRIX

```
      DO 7 I=1,MN
      DO 7 J=1,MNO
      SM11(I,J)=0.0
    7 SM12(I,J)=0.0
      DO 23 IA=1,M1
      DO 23 JA=1,N1
      XA=X(IA,JA)
      XB=X(IA,JA+1)
      XC=X(IA+1,JA)
      XD=X(IA+1,JA+1)
      YA=Y(IA)
      YC=Y(IA+1)
      AR=(YC-YA)/(XB-XA)
      D=(XD-XA)/(XB-XA)
      P=(XC-XA)/(XB-XA)
      TR=1./(D-P)
      B=D-1.
      DO 8 I=1,21
      DO 8 J=1,6
    8 A(I,J)=0.0
      A(1,1)=AR
      A(1,2)=2.*AR
      A(1,3)=AR
      A(2,2)=-TR*AR
      A(2,3)=A(2,2)
      A(3,3)=TR**2*AR
      A(4,1)=V
      A(4,2)=V
      A(4,4)=V
      A(4,5)=V
```

TABLE II (CONTINUED)

```
      A(5,4)=-V
      A(5,5)=-V
      A(6,1)=-D*TR*V
      A(6,2)=A(6,1)
      A(6,4)=-TR*V
      A(6,5)=A(6,4)
      A(7,1)=P*TR*V
      A(7,2)=A(7,1)
      A(7,4)=-A(6,4)
      A(7,5)=-A(6,4)
      A(8,2)=A(6,4)
      A(8,5)=A(6,4)
      A(9,5)=-A(6,4)
      A(10,2)=D*TR**2*V
      A(10,5)=TR**2*V
      A(11,2)=-P*TR**2*V
      A(11,5)=-A(10,5)
      A(12,1)=1./AR
      A(12,4)=2./AR
      A(12,6)=A(12,1)
      A(13,4)=-A(12,1)
      A(13,6)=-A(12,1)
      A(14,1)=-D*TR/AR
      A(14,4)=-(1.+D)*TR/AR
      A(14,6)=-TR/AR
      A(15,1)=P*TR/AR
      A(15,4)=(1.+P)*TR/AR
      A(15,6)=TR/AR
      A(16,6)=A(12,1)
      A(17,4)=-A(14,1)
      A(17,6)=TR/AR
      A(18,4)=-A(15,1)
      A(18,6)=-TR/AR
      A(19,1)=(D*TR)**2/AR
      A(19,4)=2.*D*TR**2/AR
      A(19,6)=TR**2/AR
      A(20,1)=-D*P*TR**2/AR
      A(20,4)=-(D+P)*TR**2/AR
      A(20,6)=-A(19,6)
      A(21,1)=(P*TR)**2/AR
      A(21,4)=2.*P*TR**2/AR
      A(21,6)=A(19,6)
      C(1)=1.+0.5*(B-P)
      C(2)=-0.5-(B-P)/3.
      C(3)=(1./3.)+((B-P)/4.)
      C(4)=-0.5-B/2.-(B**2-P**2)/6.
      C(5)=0.25+B/3.+(B**2-P**2)/8.
      C(6)=1./3.+B/2.+B**2/3.+(B**3-P**3)/12.
      DO 10 I=1,21
   10 Z(I)=0.0
```

TABLE II (CONTINUED)

```
   DO 11 I=1,21
   DO 11 J=1,6
11 Z(I)=Z(I)+A(I,J)*C(J)
   F(1,1)=Z(1)
   F(1,2)=-Z(1)
   F(1,3)=Z(2)
   F(1,4)=-Z(2)
   F(2,2)=Z(1)
   F(2,3)=-Z(2)
   F(2,4)=Z(2)
   F(3,3)=Z(3)
   F(3,4)=-Z(3)
   F(4,4)=Z(3)
   G(1,1)=Z(12)
   G(1,2)=Z(13)
   G(1,3)=Z(14)
   G(1,4)=Z(15)
   G(2,2)=Z(16)
   G(2,3)=Z(17)
   G(2,4)=Z(18)
   G(3,3)=Z(19)
   G(3,4)=Z(20)
   G(4,4)=Z(21)
   DO 15 I=1,4
   DO 15 J=I,4
15 H(I,J)=EPT*(F(I,J)+(1.-V)*G(I,J)/2.)
   IB=JA+N*(IA-1)
   IC=IB+1
   ID=IB+N
   IE=ID+1
   IF=IC+1
   IG=IE+1
   SM11(IB,IC)=SM11(IB,IC)+H(1,1)
   SM11(IB,IF)=SM11(IB,IF)+H(1,2)
   SM11(IB,IE)=SM11(IB,IE)+H(1,3)
   SM11(IB,IG)=SM11(IB,IG)+H(1,4)
   SM11(IC,IF)=SM11(IC,IF)+H(2,2)
   SM11(IC,IE)=SM11(IC,IE)+H(2,3)
   SM11(IC,IG)=SM11(IC,IG)+H(2,4)
   SM11(ID,IE)=SM11(ID,IE)+H(3,3)
   SM11(ID,IG)=SM11(ID,IG)+H(3,4)
   SM11(IE,IG)=SM11(IE,IG)+H(4,4)
   DO 17 I=1,4
   DO 17 J=I,4
   H(I,J)=EPT*(G(I,J)+(1.-V)*F(I,J)/2.)
17 H(J,I)=H(I,J)
   SM11(IB,IB)=SM11(IB,IB)+H(1,1)
   SM11(IC,IB)=SM11(IC,IB)+H(2,1)
   SM11(IC,IC)=SM11(IC,IC)+H(2,2)
   SM11(ID,IB)=SM11(ID,IB)+H(3,1)
   SM11(ID,IC)=SM11(ID,IC)+H(3,2)
   SM11(ID,ID)=SM11(ID,ID)+H(3,3)
```

TABLE II (CONTINUED)

```
      SM11(IE,IB)=SM11(IE,IB)+H(4,1)
      SM11(IE,IC)=SM11(IE,IC)+H(4,2)
      SM11(IE,ID)=SM11(IE,ID)+H(4,3)
      SM11(IE,IE)=SM11(IE,IE)+H(4,4)
      F(1,1)=Z(4)
      F(1,2)=Z(5)
      F(1,3)=Z(6)
      F(1,4)=Z(7)
      F(2,1)=-Z(4)
      F(2,2)=-Z(5)
      F(2,3)=-Z(6)
      F(2,4)=-Z(7)
      F(3,1)=Z(8)
      F(3,2)=Z(9)
      F(3,3)=Z(10)
      F(3,4)=Z(11)
      F(4,1)=-Z(8)
      F(4,2)=-Z(9)
      F(4,3)=-Z(10)
      F(4,4)=-Z(11)
      DO 20 I=1,4
      DO 20 J=1,4
   20 G(I,J)=F(J,I)
      DO 22 I=1,4
      DO 22 J=1,4
   22 H(I,J)=EPT*(F(I,J)+(1.-V)*G(I,J)/(2.*V))
      SM12(IB,IB)=SM12(IB,IB)+H(1,1)
      SM12(IB,IC)=SM12(IB,IC)+H(1,2)
      SM12(IB,ID)=SM12(IB,ID)+H(1,3)
      SM12(IB,IE)=SM12(IB,IE)+H(1,4)
      SM12(IC,IB)=SM12(IC,IB)+H(2,1)
      SM12(IC,IC)=SM12(IC,IC)+H(2,2)
      SM12(IC,ID)=SM12(IC,ID)+H(2,3)
      SM12(IC,IE)=SM12(IC,IE)+H(2,4)
      SM12(ID,IB)=SM12(ID,IB)+H(3,1)
      SM12(ID,IC)=SM12(ID,IC)+H(3,2)
      SM12(ID,ID)=SM12(ID,ID)+H(3,3)
      SM12(ID,IE)=SM12(ID,IE)+H(3,4)
      SM12(IE,IB)=SM12(IE,IB)+H(4,1)
      SM12(IE,IC)=SM12(IE,IC)+H(4,2)
      SM12(IE,ID)=SM12(IE,ID)+H(4,3)
   23 SM12(IE,IE)=SM12(IE,IE)+H(4,4)

      STRINGER AND RIB STIFFNESS MATRIX

      LEADING EDGE STRINGER

      DELX=X(2,1)-X(1,1)
      SL=SQRTF(DELX**2+DELY**2)
      SLA=DELX/SL
      SMU=DELY/SL
      SLA2=SLA**2*STA*E/SL
```

TABLE II (CONTINUED)

```
      SMU2=SMU**2*STA*E/SL
      SLAM=SLA*SMU*STA*E/SL
      DO 24 IA=1,M2N1,N
      IB=IA+1
      IC=IA+N
      ID=IC+1
      SM11(IA,IB)=SM11(IA,IB)+SLA2
      SM11(IA,ID)=SM11(IA,ID)-SLA2
      SM11(IC,ID)=SM11(IC,ID)+SLA2
      SM11(IA,IA)=SM11(IA,IA)+SMU2
      SM11(IC,IA)=SM11(IC,IA)-SMU2
      SM11(IC,IC)=SM11(IC,IC)+SMU2
      SM12(IA,IA)=SM12(IA,IA)+SLAM
      SM12(IA,IC)=SM12(IA,IC)-SLAM
      SM12(IC,IA)=SM12(IC,IA)-SLAM
   24 SM12(IC,IC)=SM12(IC,IC)+SLAM

      TRAILING EDGE STRINGER

      DELX=X(2,N)-X(1,N)
      SL=SQRTF(DELX**2+DELY**2)
      SLA=DELX/SL
      SMU=DELY/SL
      SLA2=SLA**2*STA*E/SL
      SMU2=SMU**2*STA*E/SL
      SLAM=SLA*SMU*STA*E/SL
      DO 25 IA=N,MNN,N
      IB=IA+1
      IC=IA+N
      ID=IC+1
      SM11(IA,IB)=SM11(IA,IB)+SLA2
      SM11(IA,ID)=SM11(IA,ID)-SLA2
      SM11(IC,ID)=SM11(IC,ID)+SLA2
      SM11(IA,IA)=SM11(IA,IA)+SMU2
      SM11(IC,IA)=SM11(IC,IA)-SMU2
      SM11(IC,IC)=SM11(IC,IC)+SMU2
      SM12(IA,IA)=SM12(IA,IA)+SLAM
      SM12(IA,IC)=SM12(IA,IC)-SLAM
      SM12(IC,IA)=SM12(IC,IA)-SLAM
   25 SM12(IC,IC)=SM12(IC,IC)+SLAM

      LOWER RIB

      SA=RA*E/DELXR
      DO 26 IA=1,N1
      IB=IA+1
      IC=IB+1
      SM11(IA,IB)=SM11(IA,IB)+SA
      SM11(IA,IC)=SM11(IA,IC)-SA
   26 SM11(IB,IC)=SM11(IB,IC)+SA
```

TABLE II (CONTINUED)

UPPER RIB

```
      SA=RA*E/DELXT
      DO 27 IA=MN1,MN4
      IB=IA+1
      IC=IB+1
      SM11(IA,IB)=SM11(IA,IB)+SA
      SM11(IA,IC)=SM11(IA,IC)-SA
   27 SM11(IB,IC)=SM11(IB,IC)+SA
```

MAKING THE STIFFNESS MATRIX NONSINGULAR

```
      IF(FX(1))30,28,30
   28 DO 29 J=1,MN
      SM11(1,J+1)=0.0
   29 SM12(1,J)=0.0
      SM11(1,2)=1.0
   30 IF(FX(2))35,31,35
   31 DO 32 I=1,N
   32 SM11(I,N+1)=0.0
      DO 33 J=N,MN
   33 SM11(N,J+1)=0.0
      SM11(N,N+1)=1.0
      DO 34 J=1,MN
   34 SM12(N,J)=0.0
   35 IF(FX(3))40,36,40
   36 DO 37 I=1,MN1
   37 SM11(I,MN1+1)=0.0
      DO 38 J=MN1,MN
   38 SM11(MN1,J+1)=0.0
      SM11(MN1,MN1+1)=1.0
      DO 39 J=1,MN
   39 SM12(MN1,J)=0.0
   40 IF(FX(4))44,41,44
   41 DO 42 I=1,MN
   42 SM11(I,MN+1)=0.0
      SM11(MN,MN+1)=1.0
      DO 43 J=1,MN
   43 SM12(MN,J)=0.0
   44 IF(FY(1))47,45,47
   45 DO 46 I=1,MN
      SM11(I,1)=0.0
   46 SM12(I,1)=0.0
      SM11(1,1)=1.0
   47 IF(FY(2))52,48,52
   48 DO 49 J=1,N
   49 SM11(N,J)=0.0
      DO 50 I=N,MN
   50 SM11(I,N)=0.0
      SM11(N,N)=1.0
      DO 51 I=1,MN
   51 SM12(I,N)=0.0
```

TABLE II (CONTINUED)

```
 52 IF(FY(3))57,53,57
 53 DO 54 J=1,MN1
 54 SM11(MN1,J)=0.0
    DO 55 I=MN1,MN
 55 SM11(I,MN1)=0.0
    SM11(MN1,MN1)=1.0
    DO 56 I=1,MN
 56 SM12(I,MN1)=0.0
 57 IF(FY(4))61,58,61
 58 DO 59 J=1,MN
 59 SM11(MN,J)=0.0
    SM11(MN,MN)=1.0
    DO 60 I=1,MN
 60 SM12(I,MN)=0.0
 61 CALL MREAR(MN)
    NUM=NUM+1
    IF(NUM-NOPAC) 2000,2000,1
    END


    SUBROUTINE MREAR(N2)
    DIMENSION A(81,82),B(81,82),TERM(170),COLSUM(170)
    COMMON A,B
1000 FORMAT(5X,6E18.8)
    ID=0
    N=N2+N2
    WRITE TAPE 5,N,N,ID
    L=N+1
    DO 10 K=1,N
 10 COLSUM(K)=0.0
    DO 210 I=1,N
    IF(I-N2) 20,20,110
 20 J=I
    M=1
    IC=J+1
    IR=1
    KS=0
 30 IF(IR+1-IC) 50,40,50
 40 KS=1
 50 TERM(M)=A(IR,IC)
    COLSUM(M)=COLSUM(M)+TERM(M)
    IF(KS) 60,60,70
 60 IR=IR+1
    GO TO 80
 70 IC=IC+1
 80 M=M+1
    IF(M-N2) 30,30,90
 90 IR=J
    DO 100 IC=1,N2
    TERM(M)=B(IR,IC)
    COLSUM(M)=COLSUM(M)+TERM(M)
```

TABLE II (CONTINUED)

```
100 M=M+1
    GO TO 190
110 J=I-N2
    M=1
    IC=J
    DO 120 IR=1,N2
    TERM(M)=B(IR,IC)
    COLSUM(M)=COLSUM(M)+TERM(M)
120 M=M+1
    IR=J
    KS=0
    IC=1
130 IF(IR-IC) 150,140,150
140 KS=1
150 TERM(M)=A(IR,IC)
    COLSUM(M)=COLSUM(M)+TERM(M)
    IF(KS) 160,160,170
160 IC=IC+1
    GO TO 180
170 IR=IR+1
180 M=M+1
    IF(M-N) 130,130,190
190 TERM(L)=0.0
    DO 200 M=1,N
200 TERM(L)=TERM(L)+TERM(M)
    WRITE TAPE 5,(TERM(M),M=1,L)
    WRITE OUTPUT TAPE 3,1000,(TERM(M),M=1,L)
210 CONTINUE
    COLSUM(L)=0.0
    DO 220 M=1,N
220 COLSUM(L)=COLSUM(L)+COLSUM(M)
    WRITE TAPE 5,(COLSUM(M),M=1,L)
    WRITE OUTPUT TAPE 3,1000,(COLSUM(M),M=1,L)
    END FILE 5
    RETURN
    END
```

TABLE II (CONTINUED)

ISOTHERMAL DIRECT STIFFNESS ANALYSIS OF A TRAPEZOIDAL PLATE WITH BOUNDARY STIFFENERS - PART 3

```
      DIMENSION X(9,9),Y(9),SM11(81,81),SM12(81,81),
     1SM22(81,81),STRES(3,8),DISP(8,8),
     2XI(5),ETA(5),SIGMA(3,8),DELTA(8,8),
      DIMENSION FX(4),FY(4),NODE(8)
      COMMON SM11,SM12,SM22

      FORMAT STATEMENTS

  200 FORMAT(6E12.4)
  201 FORMAT(2I5)
  202 FORMAT(1X,6E18.8)
  203 FORMAT(1H1)
  204 FORMAT(12H BASIC INPUT //)
  205 FORMAT(//)
  206 FORMAT(30H DISPLACEMENTS OF CORNER NODES //)
  207 FORMAT(13H WEB STRESSES //)
  208 FORMAT(19H STIFFENER STRESSES //)
  209 FORMAT(1X,E14.6,7E15.6)

      KIN=2
      KOT=3

      BASIC INPUT/OUTPUT

      CALL TESTXS(5)
      REWIND 5
      CALL UNLOAD(5)
    1 READ INPUT TAPE KIN,200,E,V,T,STA,RA
      WRITE OUTPUT TAPE KOT,203
      WRITE OUTPUT TAPE KOT,204
      WRITE OUTPUT TAPE KOT,202,E,V,T,STA,RA
      READ INPUT TAPE KIN,200,X1,X2,X3,X4,Y1,Y3
      READ INPUT TAPE KIN,200,(FX(I),FY(I),I=1,4)
      WRITE OUTPUT TAPE KOT,205
      WRITE OUTPUT TAPE KOT,202,X1,X2,X3,X4,Y1,Y3
      READ INPUT TAPE KIN,201,NOPAC
      NUM=1
    2 READ INPUT TAPE KIN,201,M,N
      WRITE OUTPUT TAPE KOT,203
      WRITE OUTPUT TAPE KOT,201,M,N

      CONSTANTS

      EP=E/(1.-(V**2))
      GS=E/(2.*(1.+V))
      AM=M-1
      AN=N-1
      M1=M-1
      N1=N-1
```

TABLE II (CONTINUED)

```
      MN=M*N
      MNN=MN-N
      MN1=MN-N1
      M2N1=N*(M-2)+1
      MN4=MN-1
      NODE(1)=1
      NODE(2)=N
      NODE(3)=MN1
      NODE(4)=MN
      NODE(5)=MN+1
      NODE(6)=MN+N
      NODE(7)=MN+MN1
      NODE(8)=MN+MN

      NODAL COORDINATES

      X(1,1)=X1
      X(1,N)=X2
      X(M,1)=X3
      X(M,N)=X4
      Y(1)=Y1
      Y(M)=Y3
      DELY=(Y3-Y1)/AM
      DO 4 I=2,M1
    4 Y(I)=Y(I-1)+DELY
      DELXR=(X2-X1)/AN
      DELXT=(X4-X3)/AN
      DO 5 J=2,N1
      X(1,J)=X(1,J-1)+DELXR
    5 X(M,J)=X(M,J-1)+DELXT
      DO 6 I=2,M1
      DO 6 J=1,N
    6 X(I,J)=X(1,J)+((X(M,J)-X(1,J))/(Y(M)-Y(1))*(Y(I)-Y(1)))

      DISPLACEMENTS OF CORNER NODES

      CALL REARR(MN)
      DO 8 I=1,4
      IF(FX(I)) 8,7,8
    7 J=NODE(I)
      SM11(J,J)=0.0
    8 CONTINUE
      DO 10 I=1,4
      IF(FY(I)) 10,9,10
    9 J=NODE(I+4)-MN
      SM22(J,J)=0.0
   10 CONTINUE
      DISP(1,1)=SM11(1,1)
      DISP(1,2)=SM11(1,N)
      DISP(1,3)=SM11(1,MN1)
      DISP(1,4)=SM11(1,MN)
      DISP(1,5)=SM12(1,1)
```

TABLE II (CONTINUED)

```
      DISP(1,6)=SM12(1,N)
      DISP(1,7)=SM12(1,MN1)
      DISP(1,8)=SM12(1,MN)
      DISP(2,2)=SM11(N,N)
      DISP(2,3)=SM11(N,MN1)
      DISP(2,4)=SM11(N,MN)
      DISP(2,5)=SM12(N,1)
      DISP(2,6)=SM12(N,N)
      DISP(2,7)=SM12(N,MN1)
      DISP(2,8)=SM12(N,MN)
      DISP(3,3)=SM11(MN1,MN1)
      DISP(3,4)=SM11(MN1,MN)
      DISP(3,5)=SM12(MN1,1)
      DISP(3,6)=SM12(MN1,N)
      DISP(3,7)=SM12(MN1,MN1)
      DISP(3,8)=SM12(MN1,MN)
      DISP(4,4)=SM11(MN,MN)
      DISP(4,5)=SM12(MN,1)
      DISP(4,6)=SM12(MN,N)
      DISP(4,7)=SM12(MN,MN1)
      DISP(4,8)=SM12(MN,MN)
      DISP(5,5)=SM22(1,1)
      DISP(5,6)=SM22(1,N)
      DISP(5,7)=SM22(1,MN1)
      DISP(5,8)=SM22(1,MN)
      DISP(6,6)=SM22(N,N)
      DISP(6,7)=SM22(N,MN1)
      DISP(6,8)=SM22(N,MN)
      DISP(7,7)=SM22(MN1,MN1)
      DISP(7,8)=SM22(MN1,MN)
      DISP(8,8)=SM22(MN,MN)
      DO 90 I=1,8
      DO 90 J=I,8
   90 DISP(J,I)=DISP(I,J)
      WRITE OUTPUT TAPE KOT,205
      WRITE OUTPUT TAPE KOT,206
      WRITE OUTPUT TAPE KOT,209,((DISP(I,J),J=1,8),I=1,8)

      WEB STRESSES

      WRITE OUTPUT TAPE KOT,203
      WRITE OUTPUT TAPE KOT,201,M,N
      WRITE OUTPUT TAPE KOT,205
      WRITE OUTPUT TAPE KOT,207
      IF(MN-15)91,91,100

      WEB STRESSES FOR MN=15, OR LESS

   91 DO 99 IA=1,M1
      DO 99 JA=1,N1
      XA=X(IA,JA)
      XB=X(IA,JA+1)
      XC=X(IA+1,JA)
```

TABLE II (CONTINUED)

```
XD=X(IA+1,JA+1)
YA=Y(IA)
YC=Y(IA+1)
D=(XD-XA)/(XB-XA)
P=(XC-XA)/(XB-XA)
TR=1./(D-P)
XBA=XB-XA
YCA=YC-YA
XI(1)=0.25*(P+0.75+0.25/TR)
XI(2)=0.25*(P+2.25+0.75/TR)
XI(3)=0.50*(P+0.5+0.5/TR)
XI(4)=0.25*(3.*P+0.25+0.75/TR)
XI(5)=0.75*(P+0.25+0.75/TR)
ETA(1)=0.25
ETA(2)=0.25
ETA(3)=0.5
ETA(4)=0.75
ETA(5)=0.75
IB=JA+N*(IA-1)
IC=IB+1
ID=IB+N
IE=ID+1
DELTA(1,1)=SM11(IB,1)
DELTA(1,2)=SM11(IB,N)
DELTA(1,3)=SM11(IB,MN1)
DELTA(1,4)=SM11(IB,MN)
DELTA(1,5)=SM12(IB,1)
DELTA(1,6)=SM12(IB,N)
DELTA(1,7)=SM12(IB,MN1)
DELTA(1,8)=SM12(IB,MN)
DELTA(2,1)=SM11(IC,1)
DELTA(2,2)=SM11(IC,N)
DELTA(2,3)=SM11(IC,MN1)
DELTA(2,4)=SM11(IC,MN)
DELTA(2,5)=SM12(IC,1)
DELTA(2,6)=SM12(IC,N)
DELTA(2,7)=SM12(IC,MN1)
DELTA(2,8)=SM12(IC,MN)
DELTA(3,1)=SM11(ID,1)
DELTA(3,2)=SM11(ID,N)
DELTA(3,3)=SM11(ID,MN1)
DELTA(3,4)=SM11(ID,MN)
DELTA(3,5)=SM12(ID,1)
DELTA(3,6)=SM12(ID,N)
DELTA(3,7)=SM12(ID,MN1)
DELTA(3,8)=SM12(ID,MN)
DELTA(4,1)=SM11(IE,1)
DELTA(4,2)=SM11(IE,N)
DELTA(4,3)=SM11(IE,MN1)
DELTA(4,4)=SM11(IE,MN)
DELTA(4,5)=SM12(IE,1)
DELTA(4,6)=SM12(IE,N)
DELTA(4,7)=SM12(IE,MN1)
```

TABLE II (CONTINUED)

```
      DELTA(4,8)=SM12(IE,MN)
      DELTA(5,1)=SM12(1,IB)
      DELTA(5,2)=SM12(N,IB)
      DELTA(5,3)=SM12(MN1,IB)
      DELTA(5,4)=SM12(MN,IB)
      DELTA(5,5)=SM22(IB,1)
      DELTA(5,6)=SM22(IB,N)
      DELTA(5,7)=SM22(IB,MN1)
      DELTA(5,8)=SM22(IB,MN)
      DELTA(6,1)=SM12(1,IC)
      DELTA(6,2)=SM12(N,IC)
      DELTA(6,3)=SM12(MN1,IC)
      DELTA(6,4)=SM12(MN,IC)
      DELTA(6,5)=SM22(IC,1)
      DELTA(6,6)=SM22(IC,N)
      DELTA(6,7)=SM22(IC,MN1)
      DELTA(6,8)=SM22(IC,MN)
      DELTA(7,1)=SM12(1,ID)
      DELTA(7,2)=SM12(N,ID)
      DELTA(7,3)=SM12(MN1,ID)
      DELTA(7,4)=SM12(MN,ID)
      DELTA(7,5)=SM22(ID,1)
      DELTA(7,6)=SM22(ID,N)
      DELTA(7,7)=SM22(ID,MN1)
      DELTA(7,8)=SM22(ID,MN)
      DELTA(8,1)=SM12(1,IE)
      DELTA(8,2)=SM12(N,IE)
      DELTA(8,3)=SM12(MN1,IE)
      DELTA(8,4)=SM12(MN,IE)
      DELTA(8,5)=SM22(IE,1)
      DELTA(8,6)=SM22(IE,N)
      DELTA(8,7)=SM22(IE,MN1)
      DELTA(8,8)=SM22(IE,MN)
      DO 99 I=1,5
      SIGMA(1,1)=-(1.-ETA(I))/XBA
      SIGMA(1,2)=-SIGMA(1,1)
      SIGMA(1,3)=-TR*ETA(I)/XBA
      SIGMA(1,4)=-SIGMA(1,3)
      SIGMA(1,5)=-V*(1.-XI(I))/YCA
      SIGMA(1,6)=-V*XI(I)/YCA
      SIGMA(1,7)=TR*V*(D-XI(I))/YCA
      SIGMA(1,8)=-TR*V*(P-XI(I))/YCA
      DO 93 J=1,4
      SIGMA(2,J)=V*SIGMA(1,J)
   93 SIGMA(2,J+4)=SIGMA(1,J+4)/V
      DO 94 J=1,4
      SIGMA(3,J)=SIGMA(2,J+4)
   94 SIGMA(3,J+4)=SIGMA(1,J)
      DO 95 J=1,2
      DO 95 K=1,8
   95 SIGMA(J,K)=EP*SIGMA(J,K)
      DO 96 J=1,8
   96 SIGMA(3,J)=GS*SIGMA(3,J)
```

TABLE II (CONTINUED)

```
      DO 97 J=1,3
      DO 97 K=1,8
   97 STRES(J,K)=0.0
      DO 98 J=1,3
      DO 98 K=1,8
      DO 98 L=1,8
   98 STRES(J,K)=STRES(J,K)+SIGMA(J,L)*DELTA(L,K)
      WRITE OUTPUT TAPE KOT,209,((STRES(J,K),K=1,8),J=1,3)
   99 WRITE OUTPUT TAPE KOT,205
      GO TO 109

      WEB STRESSES FOR MN GREATER THAN 15

  100 DO 108 IA=1,M1
      DO 108 JA=1,N1
      XA=X(IA,JA)
      XB=X(IA,JA+1)
      XC=X(IA+1,JA)
      XD=X(IA+1,JA+1)
      YA=Y(IA)
      YC=Y(IA+1)
      D=(XD-XA)/(XB-XA)
      P=(XC-XA)/(XB-XA)
      TR=1./(D-P)
      B=D-1.
      XBA=XB-XA
      YCA=YC-YA
      ETA(1)=(TR+2.)/(3.*(TR+1.))
      XI(1)=0.5*(1.+(P+B)*ETA(1))
      IB=JA+N*(IA-1)
      IC=IB+1
      ID=IB+N
      IE=ID+1
      DELTA(1,1)=SM11(IB,1)
      DELTA(1,2)=SM11(IB,N)
      DELTA(1,3)=SM11(IB,MN1)
      DELTA(1,4)=SM11(IB,MN)
      DELTA(1,5)=SM12(IB,1)
      DELTA(1,6)=SM12(IB,N)
      DELTA(1,7)=SM12(IB,MN1)
      DELTA(1,8)=SM12(IB,MN)
      DELTA(2,1)=SM11(IC,1)
      DELTA(2,2)=SM11(IC,N)
      DELTA(2,3)=SM11(IC,MN1)
      DELTA(2,4)=SM11(IC,MN)
      DELTA(2,5)=SM12(IC,1)
      DELTA(2,6)=SM12(IC,N)
      DELTA(2,7)=SM12(IC,MN1)
      DELTA(2,8)=SM12(IC,MN)
      DELTA(3,1)=SM11(ID,1)
      DELTA(3,2)=SM11(ID,N)
      DELTA(3,3)=SM11(ID,MN1)
      DELTA(3,4)=SM11(ID,MN)
```

TABLE II (CONTINUED)

```
DELTA(3,5)=SM12(ID,1)
DELTA(3,6)=SM12(ID,N)
DELTA(3,7)=SM12(ID,MN1)
DELTA(3,8)=SM12(ID,MN)
DELTA(4,1)=SM11(IE,1)
DELTA(4,2)=SM11(IE,N)
DELTA(4,3)=SM11(IE,MN1)
DELTA(4,4)=SM11(IE,MN)
DELTA(4,5)=SM12(IE,1)
DELTA(4,6)=SM12(IE,N)
DELTA(4,7)=SM12(IE,MN1)
DELTA(4,8)=SM12(IE,MN)
DELTA(5,1)=SM12(1,IB)
DELTA(5,2)=SM12(N,IB)
DELTA(5,3)=SM12(MN1,IB)
DELTA(5,4)=SM12(MN,IB)
DELTA(5,5)=SM22(IB,1)
DELTA(5,6)=SM22(IB,N)
DELTA(5,7)=SM22(IB,MN1)
DELTA(5,8)=SM22(IB,MN)
DELTA(6,1)=SM12(1,IC)
DELTA(6,2)=SM12(N,IC)
DELTA(6,3)=SM12(MN1,IC)
DELTA(6,4)=SM12(MN,IC)
DELTA(6,5)=SM22(IC,1)
DELTA(6,6)=SM22(IC,N)
DELTA(6,7)=SM22(IC,MN1)
DELTA(6,8)=SM22(IC,MN)
DELTA(7,1)=SM12(1,ID)
DELTA(7,2)=SM12(N,ID)
DELTA(7,3)=SM12(MN1,ID)
DELTA(7,4)=SM12(MN,ID)
DELTA(7,5)=SM22(ID,1)
DELTA(7,6)=SM22(ID,N)
DELTA(7,7)=SM22(ID,MN1)
DELTA(7,8)=SM22(ID,MN)
DELTA(8,1)=SM12(1,IE)
DELTA(8,2)=SM12(N,IE)
DELTA(8,3)=SM12(MN1,IE)
DELTA(8,4)=SM12(MN,IE)
DELTA(8,5)=SM22(IE,1)
DELTA(8,6)=SM22(IE,N)
DELTA(8,7)=SM22(IE,MN1)
DELTA(8,8)=SM22(IE,MN)
SIGMA(1,1)=-(1.-ETA(1))/XBA
SIGMA(1,2)=-SIGMA(1,1)
SIGMA(1,3)=-TR*ETA(1)/XBA
SIGMA(1,4)=-SIGMA(1,3)
SIGMA(1,5)=-V*(1.-XI(1))/YCA
SIGMA(1,6)=-V*XI(1)/YCA
SIGMA(1,7)=TR*V*(D-XI(1))/YCA
SIGMA(1,8)=-TR*V*(P-XI(1))/YCA
```

TABLE II (CONTINUED)

```
    DO 102 J=1,4
    SIGMA(2,J)=V*SIGMA(1,J)
102 SIGMA(2,J+4)=SIGMA(1,J+4)/V
    DO 103 J=1,4
    SIGMA(3,J)=SIGMA(2,J+4)
103 SIGMA(3,J+4)=SIGMA(1,J)
    DO 104 J=1,2
    DO 104 K=1,8
104 SIGMA(J,K)=EP*SIGMA(J,K)
    DO 105 J=1,8
105 SIGMA(3,J)=GS*SIGMA(3,J)
    DO 106 J=1,3
    DO 106 K=1,8
106 STRES(J,K)=0.0
    DO 107 J=1,3
    DO 107 K=1,8
    DO 107 L=1,8
107 STRES(J,K)=STRES(J,K)+SIGMA(J,L)*DELTA(L,K)
    WRITE OUTPUT TAPE KOT,209,((STRES(J,K),K=1,8),J=1,3)
108 WRITE OUTPUT TAPE KOT,205

    STIFFENER STRESSES

109 WRITE OUTPUT TAPE KOT,203
    WRITE OUTPUT TAPE KOT,201,M,N
    WRITE OUTPUT TAPE KOT,205
    WRITE OUTPUT TAPE KOT,208

    STRESSES IN LEADING EDGE STRINGER

    DELX=X(2,1)-X(1,1)
    SL=SQRTF(DELX**2+DELY**2)
    SIGMA(1,1)=-E*DELX/SL**2
    SIGMA(1,2)=-SIGMA(1,1)
    SIGMA(1,3)=-E*DELY/SL**2
    SIGMA(1,4)=-SIGMA(1,3)
    DO 112 I=1,M2N1,N
    IPN=I+N
    DELTA(1,1)=SM11(I,1)
    DELTA(1,2)=SM11(I,N)
    DELTA(1,3)=SM11(I,MN1)
    DELTA(1,4)=SM11(I,MN)
    DELTA(1,5)=SM12(I,1)
    DELTA(1,6)=SM12(I,N)
    DELTA(1,7)=SM12(I,MN1)
    DELTA(1,8)=SM12(I,MN)
    DELTA(2,1)=SM11(IPN,1)
    DELTA(2,2)=SM11(IPN,N)
    DELTA(2,3)=SM11(IPN,MN1)
    DELTA(2,4)=SM11(IPN,MN)
    DELTA(2,5)=SM12(IPN,1)
    DELTA(2,6)=SM12(IPN,N)
```

TABLE II (CONTINUED)

```
      DELTA(2,7)=SM12(IPN,MN1)
      DELTA(2,8)=SM12(IPN,MN)
      DELTA(3,1)=SM12(1,I)
      DELTA(3,2)=SM12(N,I)
      DELTA(3,3)=SM12(MN1,I)
      DELTA(3,4)=SM12(MN,I)
      DELTA(3,5)=SM22(I,1)
      DELTA(3,6)=SM22(I,N)
      DELTA(3,7)=SM22(I,MN1)
      DELTA(3,8)=SM22(I,MN)
      DELTA(4,1)=SM12(1,IPN)
      DELTA(4,2)=SM12(N,IPN)
      DELTA(4,3)=SM12(MN1,IPN)
      DELTA(4,4)=SM12(MN,IPN)
      DELTA(4,5)=SM22(IPN,1)
      DELTA(4,6)=SM22(IPN,N)
      DELTA(4,7)=SM22(IPN,MN1)
      DELTA(4,8)=SM22(IPN,MN)
      DO 110 K=1,8
  110 STRES(1,K)=0.0
      DO 111 K=1,8
      DO 111 L=1,4
  111 STRES(1,K)=STRES(1,K)+SIGMA(1,L)*DELTA(L,K)
      WRITE OUTPUT TAPE KOT,209,(STRES(1,K),K=1,8)
  112 WRITE OUTPUT TAPE KOT,205
      WRITE OUTPUT TAPE KOT,205

      STRESSES IN TRAILING EDGE STRINGER

      DELX=X(2,N)-X(1,N)
      SL=SQRTF(DELX**2+DELY**2)
      SIGMA(1,1)=-E*DELX/SL**2
      SIGMA(1,2)=-SIGMA(1,1)
      SIGMA(1,3)=-E*DELY/SL**2
      SIGMA(1,4)=-SIGMA(1,3)
      DO 115 I=N,MNN,N
      IPN=I+N
      DELTA(1,1)=SM11(I,1)
      DELTA(1,2)=SM11(I,N)
      DELTA(1,3)=SM11(I,MN1)
      DELTA(1,4)=SM11(I,MN)
      DELTA(1,5)=SM12(I,1)
      DELTA(1,6)=SM12(I,N)
      DELTA(1,7)=SM12(I,MN1)
      DELTA(1,8)=SM12(I,MN)
      DELTA(2,1)=SM11(IPN,1)
      DELTA(2,2)=SM11(IPN,N)
      DELTA(2,3)=SM11(IPN,MN1)
      DELTA(2,4)=SM11(IPN,MN)
      DELTA(2,5)=SM12(IPN,1)
      DELTA(2,6)=SM12(IPN,N)
      DELTA(2,7)=SM12(IPN,MN1)
```

TABLE II (CONTINUED)

```
      DELTA(2,8)=SM12(IPN,MN)
      DELTA(3,1)=SM12(1,I)
      DELTA(3,2)=SM12(N,I)
      DELTA(3,3)=SM12(MN1,I)
      DELTA(3,4)=SM12(MN,I)
      DELTA(3,5)=SM22(I,1)
      DELTA(3,6)=SM22(I,N)
      DELTA(3,7)=SM22(I,MN1)
      DELTA(3,8)=SM22(I,MN)
      DELTA(4,1)=SM12(1,IPN)
      DELTA(4,2)=SM12(N,IPN)
      DELTA(4,3)=SM12(MN1,IPN)
      DELTA(4,4)=SM12(MN,IPN)
      DELTA(4,5)=SM22(IPN,1)
      DELTA(4,6)=SM22(IPN,N)
      DELTA(4,7)=SM22(IPN,MN1)
      DELTA(4,8)=SM22(IPN,MN)
      DO 113 K=1,8
  113 STRES(1,K)=0.0
      DO 114 K=1,8
      DO 114 L=1,4
  114 STRES(1,K)=STRES(1,K)+SIGMA(1,L)*DELTA(L,K)
      WRITE OUTPUT TAPE KOT,209,(STRES(1,K),K=1,8)
  115 WRITE OUTPUT TAPE KOT,205
      WRITE OUTPUT TAPE KOT,205

      STRESSES IN LOWER RIB

      SIGMA(1,1)=-E/DELXR
      SIGMA(1,2)=+E/DELXR
      DO 118 I=1,N1
      DELTA(1,1)=SM11(I,1)
      DELTA(1,2)=SM11(I,N)
      DELTA(1,3)=SM11(I,MN1)
      DELTA(1,4)=SM11(I,MN)
      DELTA(1,5)=SM12(I,1)
      DELTA(1,6)=SM12(I,N)
      DELTA(1,7)=SM12(I,MN1)
      DELTA(1,8)=SM12(I,MN)
      DELTA(2,1)=SM11(I+1,1)
      DELTA(2,2)=SM11(I+1,N)
      DELTA(2,3)=SM11(I+1,MN1)
      DELTA(2,4)=SM11(I+1,MN)
      DELTA(2,5)=SM12(I+1,1)
      DELTA(2,6)=SM12(I+1,N)
      DELTA(2,7)=SM12(I+1,MN1)
      DELTA(2,8)=SM12(I+1,MN)
      DO 116 K=1,8
  116 STRES(1,K)=0.0
      DO 117 K=1,8
      DO 117 L=1,2
  117 STRES(1,K)=STRES(1,K)+SIGMA(1,L)*DELTA(L,K)
```

TABLE II (CONTINUED)

```
    WRITE OUTPUT TAPE KOT,209,(STRES(1,K),K=1,8)
118 WRITE OUTPUT TAPE KOT,205
    WRITE OUTPUT TAPE KOT,205

    STRESSES IN UPPER RIB

    SIGMA(1,1)=-E/DELXT
    SIGMA(1,2)=+E/DELXT
    DO 121 I=MN1,MN4
    DELTA(1,1)=SM11(I,1)
    DELTA(1,2)=SM11(I,N)
    DELTA(1,3)=SM11(I,MN1)
    DELTA(1,4)=SM11(I,MN)
    DELTA(1,5)=SM12(I,1)
    DELTA(1,6)=SM12(I,N)
    DELTA(1,7)=SM12(I,MN1)
    DELTA(1,8)=SM12(I,MN)
    DELTA(2,1)=SM11(I+1,1)
    DELTA(2,2)=SM11(I+1,N)
    DELTA(2,3)=SM11(I+1,MN1)
    DELTA(2,4)=SM11(I+1,MN)
    DELTA(2,5)=SM12(I+1,1)
    DELTA(2,6)=SM12(I+1,N)
    DELTA(2,7)=SM12(I+1,MN1)
    DELTA(2,8)=SM12(I+1,MN)
    DO 119 K=1,8
119 STRES(1,K)=0.0
    DO 120 K=1,8
    DO 120 L=1,2
120 STRES(1,K)=STRES(1,K)+SIGMA(1,L)*DELTA(L,K)
    WRITE OUTPUT TAPE KOT,209,(STRES(1,K),K=1,8)
121 WRITE OUTPUT TAPE KOT,205
    NUM=NUM+1
    IF(NUM-NOPAC) 2,2,1
    END


    SUBROUTINE REARR(N2)
    DIMENSION A(81,81),B(81,81),C(81,81),TERM(170)
    COMMON A,B,C
    N=N2+N2
    DO 60 I=1,N
    READ TAPE 5,NR,NC,NTYPE
    IF(I-N2) 10,10,40
    READ TAPE 5,(TERM(L),L=1,N)
    IR=I
 10 M=1
    DO 20 IC=1,N2
    A(IR,IC)=TERM(M)
 20 M=M+1
```

TABLE II (CONTINUED)

```
      DO 30  IC=1,N2
   30 M=M+1
      B(IR,IC)=TERM(M)
      GO TO 60
   40 IR=I-N2
      DO 50  IC=1,N2
      M=N2+1
   50 M=M+1
      C(IR,IC)=TERM(M)
   60 CONTINUE
      RETURN
      END
```

TABLE III

DISPLACEMENTS OF CORNER NODES

INCHES PER MILLION POUNDS

MOD NO	FORCE	F1	F3	F4	F7	F8
1	F1	1.3888	.9985	1.2154	-.0291	-.6057
	F3	.9985	5.1240	5.7372	-1.7008	-3.8586
	F4	1.2154	5.7372	6.9983	-2.0822	-4.5743
	F7	-.0291	-1.7008	-2.0822	1.5412	2.2442
	F8	-.6057	-3.8586	-4.5743	2.2442	4.7349
2	F1	1.5600	1.4853	1.4821	-.2945	-.7983
	F3	1.4853	7.2103	7.2773	-2.7163	-5.1704
	F4	1.4821	7.2773	8.2304	-2.8358	-5.6905
	F7	-.2945	-2.7163	-2.8358	2.2354	2.6965
	F8	-.7983	-5.1704	-5.6905	2.6965	5.8838
3	F1	1.7331	1.8883	1.8811	-.3418	-1.0720
	F3	1.8883	8.2236	8.3337	-2.8418	-5.9655
	F4	1.8811	8.3337	9.3799	-2.9734	-6.6155
	F7	-.3418	-2.8418	-2.9734	2.2746	2.8307
	F8	-1.0720	-5.9655	-6.6155	2.8307	6.7343
4	F1	1.8779	2.1979	2.1999	-.4404	-1.2753
	F3	2.1979	9.5203	9.4757	-3.4357	-6.8437
	F4	2.1999	9.4757	10.5412	-3.4387	-7.5119
	F7	-.4404	-3.4357	-3.4387	2.7682	3.1022
	F8	-1.2753	-6.8437	-7.5119	3.1022	7.5498
5	F1	2.0157	2.5335	2.5317	-.5183	-1.4970
	F3	2.5335	10.4360	10.3870	-3.7124	-7.5003
	F4	2.5317	10.3870	11.4827	-3.7034	-8.2200
	F7	-.5183	-3.7124	-3.7034	2.9390	3.3108
	F8	-1.4970	-7.5003	-8.2200	3.3108	8.1383
6	F1	2.1086	2.7236	2.7246	-.5621	-1.6134
	F3	2.7236	11.1107	10.9870	-4.0108	-7.9209
	F4	2.7246	10.9870	12.0984	-3.9433	-8.6647
	F7	-.5621	-4.0108	-3.9433	3.2064	3.4587
	F8	-1.6134	-7.9209	-8.6647	3.4587	8.5124
7	F1	2.1873	2.9076	2.9083	-.6101	-1.7345
	F3	2.9076	11.6113	11.4814	-4.1944	-8.2718
	F4	2.9083	11.4814	12.6035	-4.1182	-9.0340
	F7	-.6101	-4.1944	-4.1182	3.3345	3.5968
	F8	-1.7345	-8.2718	-9.0340	3.5968	8.8053

TABLE IV

WEB STRESSES

PSI PER POUND

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
1	1-1	1	SXX	-.6264	-.1466	-.0201	.0034	-.1141
			SYY	.2410	.6321	.7056	.3783	-.6488
			SXY	-.0619	.6025	.4811	-.1809	-.2790
		2	SXX	-.7272	-.5241	-.4560	.1264	.3215
			SYY	-.0780	-.5623	-.6738	.7674	.7298
			SXY	.1205	.8203	.8436	-.2561	-.4761
		3	SXX	-.3621	-.1055	.2535	-.0185	-.0513
			SYY	.0183	-.5009	-.5314	.7447	.6939
			SXY	.0565	.5501	.5389	-.1684	-.1607
		4	SXX	-.0102	.2638	.9062	-.1475	-.3672
			SYY	.0730	-.5956	-.5693	.7728	.8380
			SXY	.0163	.3083	.2816	-.0904	.1289
		5	SXX	-.0847	-.0150	.5842	-.0566	-.0453
			SYY	-.1626	-1.4780	-1.5884	1.0603	1.8566
			SXY	.1512	.4692	.5494	-.1460	-.0167*
2	1-1	1	SXX	-.7320	-.1091	-.1388	-.1022	-.0443
			SYY	.4165	1.4827	1.4656	-.2121	-1.1400
			SXY	-.1137	.3465	.3663	-.0551	-.1582
		2	SXX	-.9277	-.9103	-.9276	.2675	.6577
			SYY	-.2026	-1.0525	-1.0304	.9581	1.0818
			SXY	.2364	.9642	.9530	-.3516	-.4840
		3	SXX	-.5471	-.1320	-.1819	-.1025	.1817
			SYY	.0482	-.2718	-.2682	.5942	.4627
			SXY	.0851	.5360	.5364	-.1512	-.1592
		4	SXX	-.1784	.5971	.5154	-.4500	-.2512
			SYY	.2612	.3535	.3409	.3021	-.0202
			SXY	-.0446	.1456	.1558	.0308	.1455
		5	SXX	-.3501	-.1057	-.1766	-.1255	.3648
			SYY	-.2821	-1.8709	-1.8490	1.3290	1.9292
			SXY	.2625	.6876	.6706	-.2292	-.1403
	2-1	1	SXX	.0248	.6861	.9679	-.2408	-.6443
			SYY	-.0136	-.3160	-.1495	1.2694	-.2212
			SXY	.0026	.7324	.5294	-.2240	-.2769
		2	SXX	.0308	.6004	.7602	-.3903	-.2776
			SYY	.0052	-.5870	-.8067	.7965	.9390
			SXY	-.0071	.5298	.6194	-.0848	-.3201
		3	SXX	.0175	.3565	.9050	-.1919	-.3866
			SYY	-.0014	-.6280	-.6733	.9222	.7499
			SXY	-.0028	.5196	.5088	-.1754	-.1418
		4	SXX	.0046	.1056	1.0329	-.0056	-.4657
			SYY	-.0066	-.6910	-.5934	1.0096	.6551
			SXY	.0005	.4930	.4054	-.2548	.0328
		5	SXX	.0096	.0339	.8590	-.1307	-.1586
			SYY	.0091	-.9179	-1.1438	.6135	1.6269
			SXY	-.0076	.3233	.4808	-.1382	-.0032

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
3	1-1	1	SXX	-.7611	-.0494	-.1004	-.0957	.0156
			SYY	.5759	1.9918	1.8979	-.3829	-1.3816
			SXY	-.0746	.4908	.5314	-.0291	-.3079
		2	SXX	-.9457	-.6135	-.6369	.0680	.4068
			SYY	-.0080	.2065	.2000	.1355	-.1438
			SXY	.1320	.8591	.8718	-.1725	-.5027
		3	SXX	-.5677	.0383	-.0358	-.1908	.0706
			SYY	.0627	.2638	.2484	.0968	-.1469
			SXY	.0205	.4940	.5236	-.0698	-.2439
		4	SXX	-.2010	.6557	.5323	-.4398	-.2415
			SYY	.0977	.2116	.1928	.0900	-.0741
			SXY	-.0782	.1514	.1963	.0239	.0028
		5	SXX	-.3630	.1607	.0616	-.2960	.1015
			SYY	-.4147	-1.3547	-1.2968	.5449	1.0119
			SXY	.1030	.4746	.4949	-.1018	-.1680
	1-2	1	SXX	-.8579	-.7321	-.7627	.0873	.3957
			SYY	.1727	.2491	.3293	.4530	-.2234
			SXY	-.0523	.3782	.3913	-.2510	-.1830
		2	SXX	-.9662	-1.1888	-1.2254	.3156	.8426
			SYY	-.1701	-1.1961	-1.1349	1.1757	1.1909
			SXY	.1668	.7831	.7730	-.4221	-.3849
		3	SXX	-.5439	-.4010	-.4817	-.0182	.4433
			SYY	-.0274	-.9081	-.8603	1.0506	1.0264
			SXY	.0835	.4477	.4341	-.2554	-.0625
		4	SXX	-.1281	.3588	.2335	-.3382	.0713
			SYY	.0942	-.7086	-.6755	.9699	.9487
			SXY	.0137	.1372	.1186	-.0991	.2474
		5	SXX	-.2232	-.0419	-.1724	-.1378	.4635
			SYY	-.2065	-1.9767	-1.9602	1.6039	2.1897
			SXY	.2060	.4924	.4535	-.2492	.0703
	2-1	1	SXX	.1193	.8974	1.1044	-.1755	-.7031
			SYY	.0326	-.2986	-.2364	1.3669	-.0903
			SXY	.0464	.9580	.7493	-.2847	-.5110
		2	SXX	.0842	.7626	.9222	-.2210	-.4699
			SYY	-.0785	-.7254	-.8130	1.2230	.6475
			SXY	.0400	.8567	.8028	-.2375	-.5641
		3	SXX	.0581	.4676	1.0057	-.1145	-.5380
			SYY	-.1152	-.9281	-.9345	1.2198	.8154
			SXY	.0053	.7014	.6472	-.2667	-.3606
		4	SXX	.0290	.1617	1.0745	-.0118	-.5872
			SYY	-.1611	-1.1655	-1.1029	1.2048	1.0431
			SXY	-.0299	.5378	.4960	-.2921	-.1615
		5	SXX	-.0003	.0487	.9219	-.0499	-.3919
			SYY	-.2541	-1.5230	-1.5858	1.0843	1.6612
			SXY	-.0352	.4530	.5408	-.2526	-.2060

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
3	2-2	1	SXX	.1082	.8149	1.0956	-.4389	-.6540
			SYY	-.1104	-.4633	-.2285	.9034	-.0964
			SXY	-.0844	.3098	.2787	-.0814	-.0298
		2	SXX	.1590	.8416	1.0075	-.5330	-.4217
			SYY	.0502	-.3788	-.5073	.6055	.6386
			SXY	-.1178	.1702	.3067	.0127	-.0508
		3	SXX	.0806	.4913	1.0678	-.3060	-.4462
			SYY	.0432	-.4801	-.5190	.6444	.7120
			SXY	-.0743	.1781	.2276	-.0640	.1644
		4	SXX	.0063	.1432	1.1208	-.0866	-.4519
			SYY	.0492	-.5746	-.5534	.6590	.8452
			SXY	-.0335	.1747	.1507	-.1330	.3779
		5	SXX	.0489	.1656	1.0470	-.1655	-.2573
			SYY	.1839	-.5038	-.7870	.4095	1.4609
			SXY	-.0615	.0578	.1742	-.0541	.3604
4	1-1	1	SXX	-.8721	-.3535	-.3546	.0661	.2179
			SYY	.8434	2.2713	2.2674	-.4130	-1.4985
			SXY	-.1475	.4243	.4256	-.0810	-.2470
		2	SXX	-1.1940	-1.1125	-1.1130	.1975	.6947
			SYY	-.1752	-.1305	-.1324	.0029	.0103
			SXY	.1709	.8861	.8883	-.1816	-.4933
		3	SXX	-.8302	-.5228	-.5227	.0763	.3650
			SYY	.1346	.5153	.5131	-.1149	-.3825
			SXY	-.0072	.5274	.5296	-.1151	-.2772
		4	SXX	-.4760	.0443	.0449	-.0409	.0494
			SYY	.4141	1.0897	1.0874	-.2204	-.7305
			SXY	-.1759	.1824	.1847	-.0515	-.0685
		5	SXX	-.7787	-.6695	-.6682	.0827	.4978
			SYY	-.5439	-1.1693	-1.1697	.1708	.6886
			SXY	.1235	.6168	.6198	-.1461	-.3002
	1-2	1	SXX	-1.0327	-1.1210	-1.1224	.1997	.6137
			SYY	.1695	.4007	.4079	.2130	-.2478
			SXY	-.0970	.1362	.1368	-.1874	.0035
		2	SXX	-1.1973	-1.7846	-1.7870	.5861	1.2192
			SYY	-.3512	-1.6991	-1.6951	1.4355	1.6681
			SXY	.2789	.8287	.8278	-.4806	-.3745
		3	SXX	-.8038	-.9619	-.9653	.2090	.7038
			SYY	-.0993	-.9250	-.9205	1.0170	1.0361
			SXY	.1201	.4125	.4117	-.2682	-.0638
		4	SXX	-.4152	-.1588	-.1633	-.1564	.2064
			SYY	.1370	-.2133	-.2085	.6349	.4611
			SXY	-.0274	.0170*	.0161	-.0646	.2355
		5	SXX	-.5699	-.7829	-.7884	.2068	.7758
			SYY	-.3528	-2.1883	-2.1866	1.7847	2.2632
			SXY	.3262	.6682	.6660	-.3403	-.1200

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
4	2-1	1	SXX	-.1349	.7695	.7682	-.2702	-.4466
			SYY	.2804	1.3246	1.3018	-.1370	-1.0715
			SXY	-.0077	.6060	.6098	-.1005	-.3588
		2	SXX	-.3050	.1534	.1587	-.0206	.0473
			SYY	-.2577	-.6252	-.6268	.6529	.4918
			SXY	.1161	.8860	.8939	-.2817	-.5516
		3	SXX	-.1450	.5552	.5637	-.2548	-.2413
			SYY	-.1230	-.1933	-.1973	.4554	.1561
			SXY	.0283	.5973	.6071	-.1530	-.3256
		4	SXX	.0092	.9362	.9481	-.4806	-.5134
			SYY	-.0064	.1726	.1671	.2845	-.1267
			SXY	-.0552	.3180	.3298	-.0303	-.1061
		5	SXX	-.1493	.3616	.3798	-.2477	-.0527
			SYY	-.5082	-1.6455	-1.6313	1.0213	1.3311
			SXY	.0602	.5791	.5948	-.1993	-.2859
	2-2	1	SXX	-.1311	.5522	.5297	-.4785	-.2710
			SYY	-.0334	-.1812	-.1564	.6524	.1163
			SXY	-.1119	.0857	.1042	-.0489	.1118
		2	SXX	-.0990	.3237	.3032	-.3665	.0497
			SYY	.0682	-.9042	-.8732	1.0068	1.1315
			SXY	-.0866	.2152	.2095	-.1363	.0561
		3	SXX	-.0801	.5152	.4687	-.4855	-.0861
			SYY	.0521	-.6867	-.6653	.8922	.8681
			SXY	-.0802	.0949	.0959	-.0716	.1946
		4	SXX	-.0601	.6990	.6266	-.6007	-.2113
			SYY	.0394	-.4936	-.4815	.7897	.6389
			SXY	-.0729	-.0209	-.0141*	-.0098	.3311
		5	SXX	-.0301	.4859	.4153	-.4963	.0878
			SYY	.1343	-1.1678	-1.1500	1.1201	1.5856
			SXY	-.0493	.0998	.0839	-.0913	.2792
	3-1	1	SXX	.1054	1.3728	1.3357	-.6830	-.8677
			SYY	-.0075	.5089	.3511	.4932	-.4948
			SXY	.0136	.5363	.5760	-.0023	-.3522
		2	SXX	.0845	1.0454	1.0482	-.5691	-.5474
			SYY	-.0737	-.5270	-.5586	.8534	.5185
			SXY	-.0053	.5957	.6309	-.0905	-.4027
		3	SXX	.0640	1.1578	1.1504	-.6904	-.6482
			SYY	-.0729	-.3769	-.4258	.7753	.3747
			SXY	-.0124	.4409	.4947	-.0284	-.2522
		4	SXX	.0427	1.2574	1.2414	-.8073	-.7364
			SYY	-.0746	-.2674	-.3284	.7112	.2704
			SXY	-.0202	.2884	.3605	.0302	-.1037
		5	SXX	.0234	.9556	.9763	-.7024	-.4412
			SYY	-.1356	-1.2225	-1.1672	1.0433	1.2047
			SXY	-.0377	.3431	.4112	-.0510	-.1503

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
4	3-2	1	SXX	-.0517	.6562	.6675	-.4397	-.2972
			SYY	-.0387	-.8374	-.5760	1.3022	.3783
			SXY	-.0575	.3435	.3364	-.1882	-.0062
		2	SXX	-.0321	.6376	.5867	-.5577	-.0858
			SYY	.0235	-.8960	-.8318	.9289	1.0472
			SXY	-.0492	.2915	.3100	-.1136	.0386
		3	SXX	-.0259	.5781	.5708	-.4431	-.0736
			SYY	.0142	-.9042	-.7906	1.0326	.9301
			SXY	-.0420	.2936	.2789	-.1817	.1248
		4	SXX	-.0190	.5178	.5517	-.3332	-.0531
			SYY	.0073	-.9147	-.7594	1.1217	.8392
			SXY	-.0345	.2936	.2467	-.2469	.2128
		5	SXX	-.0008	.5007	.4772	-.4419	.1417
			SYY	.0647	-.9688	-.9952	.7775	1.4559
			SXY	-.0268	.2457	.2224	-.1781	.2542
	4-1	1	SXX	.0322	.6008	1.0269	-.0582	-.6989
			SYY	-.0104	-1.3302	-1.1535	2.4807	.5955
			SXY	.0051	.9786	.6801	-.3777	-.4559
		2	SXX	.0302	.7309	1.0709	-.3921	-.6141
			SYY	-.0166	-.9185	-1.0143	1.4240	.8639
			SXY	-.0025	.6814	.6694	-.1570	-.4635
		3	SXX	.0193	.2954	1.0531	-.0561	-.6295
			SYY	-.0198	-1.0757	-1.0266	1.5806	.8462
			SXY	-.0031	.7666	.6934	-.3470	-.4178
		4	SXX	.0082	-.1340	1.0373	.2644	-.6411
			SYY	-.0233	-1.2140	-1.0324	1.6883	.8409
			SXY	-.0041	.8381	.7169	-.5267	-.3724
		5	SXX	.0065	-.0159	1.0772	-.0385	-.5641
			SYY	-.0289	-.8403	-.9061	.7293	1.0845
			SXY	-.0111	.5684	.7072	-.3264	-.3793
	4-2	1	SXX	-.0113	.4422	.9873	-.2469	-.5370
			SYY	-.0066	-.3901	.0412	.6769	-.3182
			SXY	-.0152	.4101	.3937	-.1913	-.0122
		2	SXX	-.0072	.4517	.7672	-.3414	-.1639
			SYY	.0060	-.3600	-.6553	.3777	.8622
			SXY	-.0129	.2681	.5567	-.0795	-.1411
		3	SXX	-.0045	.2455	1.0342	-.1665	-.4041
			SYY	.0052	-.4291	-.4797	.4722	.6317
			SXY	-.0110	.2938	.4154	-.1462	.0767
		4	SXX	-.0015	.0397	1.2910	.0039	-.6270
			SYY	.0050	-.4969	-.3364	.5528	.4558
			SXY	-.0091	.3130	.2818	-.2078	.2886
		5	SXX	.0021	.0483	1.0912	-.0818	-.2885
			SYY	.0166	-.4695	-.9685	.2812	1.5271
			SXY	-.0070	.1841	.4296	-.1064	.1716

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
5	1-1	CG	SXX	-.8236	-.3859	-.3927	.0474	.2787
			SYY	.2073	.6390	.6385	-.1143	-.4286
			SXY	-.0223	.6088	.6099	-.1109	-.3579
	1-2	CG	SXX	-.8938	-.8293	-.8165	.1518	.5649
			SYY	.0268	.1734	.1773	.0466	-.1382
			SXY	-.0642	.2819	.2761	-.0978	-.0934
	1-3	CG	SXX	-.8693	-.9435	-.9477	.1293	.6577
			SYY	-.0769	-.2347	-.2403	.3008	.2197
			SXY	.0182	.2746	.2781	-.1958	-.0370
	1-4	CG	SXX	-.7668	-.9601	-.9779	.1630	.7173
			SYY	-.1697	-1.4324	-1.4285	1.4164	1.5590
			SXY	.1682	.5133	.5154	-.4241	-.1389
	2-1	CG	SXX	-.0295	.8195	.8491	-.2341	-.4431
			SYY	-.0964	.0815	.0918	.2087	-.1369
			SXY	.0271	.6699	.6637	-.1369	-.3910
	2-2	CG	SXX	-.1190	.6201	.5996	-.3575	-.2333
			SYY	-.1678	-.3110	-.3470	.4734	.2554
			SXY	-.0327	.3469	.3785	-.0916	-.1552
	2-3	CG	SXX	-.1031	.5135	.4652	-.4136	-.0803
			SYY	-.1458	-.9369	-.9179	1.0464	.9090
			SXY	-.0755	.2190	.2439	-.1734	-.0116
	2-4	CG	SXX	.0178	.6806	.6202	-.6224	-.1753
			SYY	.1457	-.7865	-.7265	.9829	1.0408
			SXY	-.1515	-.1043	-.1211	-.0179	.3966
	3-1	CG	SXX	.1176	1.4127	1.3537	-.7793	-.8044
			SYY	-.1354	-.1120	-.2376	.5268	.1279
			SXY	.0094	.4546	.5279	.0536	-.3187
	3-2	CG	SXX	.0764	1.0500	1.0507	-.6135	-.5176
			SYY	-.0793	-.8184	-.7693	1.1735	.6767
			SXY	-.0558	.4008	.4505	-.1357	-.2115
	3-3	CG	SXX	.0353	.8139	.8098	-.5578	-.2852
			SYY	.0353	-.8383	-.7026	1.0751	.7765
			SXY	-.0876	.2633	.2848	-.1515	.0284
	3-4	CG	SXX	-.0683	.4346	.3505	-.3783	.1423
			SYY	.0166	-.8666	-.7449	.8611	.9636
			SXY	-.0225	.2340	.1978	-.1639	.2727
	4-1	CG	SXX	.0562	.4377	1.1268	-.0862	-.7274
			SYY	-.0277	-1.2913	-1.2358	1.9850	.9542
			SXY	-.0093	.8807	.7415	-.3960	-.5086
	4-2	CG	SXX	.0199	.3556	1.1371	-.1504	-.6795
			SYY	.0022	-.8044	-.7171	1.1363	.6234
			SXY	-.0164	.6359	.6474	-.3209	-.3476
	4-3	CG	SXX	-.0271	.2291	1.0308	-.1295	-.4992
			SYY	-.0043	-.5458	-.4837	.6447	.5222
			SXY	.0012	.4331	.5538	-.2235	-.1447
	4-4	CG	SXX	.0069	.2755	1.0075	-.1903	-.3209
			SYY	.0015	-.3469	-.4340	.3262	.6364
			SXY	-.0035	.1923	.3382	-.0975	.2339

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
6	1-1	CG	SXX	-1.0783	-.8047	-.8046	.1553	.5161
			SYY	.2634	.6806	.6806	-.1354	-.4357
			SXY	-.0621	.5550	.5553	-.1022	-.3314
	1-2	CG	SXX	-1.1774	-1.3072	-1.3072	.2543	.8364
			SYY	.0378	.1924	.1919	-.0404	-.1548
			SXY	-.1479	.0667	.0679	-.0254	.0288
	1-3	CG	SXX	-1.1463	-1.4548	-1.4561	.2942	.9220
			SYY	-.0645	-.0177	-.0179*	.0265	-.0476
			SXY	-.0032	.1800	.1820	-.1701	.0173
	1-4	CG	SXX	-1.0159	-1.5011	-1.5044	.4218	1.0214
			SYY	-.3150	-1.7174	-1.7134	1.5788	1.7454
			SXY	.2868	.7582	.7567	-.6030	-.3351
	2-1	CG	SXX	-.4068	.2804	.2813	-.0550	-.1191
			SYY	.0548	.4162	.4152	-.0728	-.3230
			SXY	-.0159	.6109	.6115	-.1201	-.3567
	2-2	CG	SXX	-.5596	-.2621	-.2613	.0391	.2692
			SYY	-.1277	-.0600	-.0615	.1233	-.0326
			SXY	-.0717	.2717	.2744	-.1259	-.0942
	2-3	CG	SXX	-.5203	-.4363	-.4386	.0493	.4518
			SYY	-.2625	-.8339	-.8313	.7634	.7099
			SXY	-.0100	.3334	.3369	-.3155	-.1197
	2-4	CG	SXX	-.2054	.0953	.0859	-.3473	.1042
			SYY	.0756	-.9101	-.9020	1.0938	1.1958
			SXY	-.0841	-.0231*	-.0249	-.1205	.2548
	3-1	CG	SXX	-.0792	.7734	.7761	-.2135	-.3838
			SYY	-.0804	.2047	.2015	.0787	-.2307
			SXY	.0171	.6638	.6652	-.1441	-.3996
	3-2	CG	SXX	-.1616	.4299	.4326	-.2107	-.1050
			SYY	-.2148	-.4140	-.4164	.4992	.2802
			SXY	-.0572	.3749	.3812	-.1733	-.1856
	3-3	CG	SXX	-.1188	.4207	.4143	-.3786	-.0725
			SYY	-.1114	-.7414	-.7339	.8581	.7465
			SXY	-.1272	.1012	.1102	-.1286	.0668
	3-4	CG	SXX	-.1244	.2779	.2575	-.3942	.0637
			SYY	.0852	-.8605	-.8417	1.0130	1.1240
			SXY	-.1086	-.0900	-.0935	-.0251	.3732
	4-1	CG	SXX	.0844	1.0691	1.0767	-.4039	-.5735
			SYY	-.1472	-.0373	-.0456	.3481	-.0478
			SXY	.0115	.6402	.6441	-.1232	-.4006
	4-2	CG	SXX	.0243	.8637	.8699	-.4797	-.3960
			SYY	-.1439	-.4661	-.4706	.6940	.3877
			SXY	-.0808	.3131	.3295	-.0871	-.1376
	4-3	CG	SXX	-.0356	.6710	.6588	-.5263	-.2167
			SYY	-.0396	-.7367	-.7154	.9380	.7737
			SXY	-.1117	.1017	.1229	-.0605	.1094
	4-4	CG	SXX	-.0649	.4936	.4441	-.5102	-.0111
			SYY	.0572	-.9075	-.8634	.9995	1.1117
			SXY	-.0695	.0190	.0115	-.0597	.3543

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
6	5-1	CG	SXX	.1141	1.1986	1.2188	-.5662	-.7052
			SYY	-.1179	-.1619	-.1848	.5780	.0885
			SXY	-.0031	.5656	.5765	-.0553	-.3603
	5-2	CG	SXX	.0442	.9869	1.0116	-.6056	-.5082
			SYY	-.0773	-.5276	-.5327	.8674	.4761
			SXY	-.0604	.3056	.3463	-.0321	-.1270
	5-3	CG	SXX	-.0091	.8605	.8256	-.6739	-.3137
			SYY	-.0108	-.8012	-.7487	1.0256	.8054
			SXY	-.0668	.1688	.2204	-.0577	.0809
	5-4	CG	SXX	-.0309	.6572	.5708	-.5928	-.0501
			SYY	.0298	-.9905	-.8873	1.0192	1.1038
			SXY	-.0355	.1703	.1504	-.1523	.2963
	6-1	CG	SXX	.0736	1.1618	1.2349	-.6190	-.7558
			SYY	-.0662	-.2690	-.3270	.7988	.2227
			SXY	-.0050	.4914	.5219	.0192	-.3261
	6-2	CG	SXX	.0288	1.1442	1.1660	-.8029	-.6322
			SYY	-.0341	-.6305	-.6392	1.0385	.5722
			SXY	-.0331	.2944	.3987	.0017*	-.1531
	6-3	CG	SXX	-.0022	.8546	.8602	-.6281	-.3109
			SYY	-.0019	-1.0080	-.8421	1.2087	.8589
			SXY	-.0316	.3550	.4177	-.2158	-.0434
	6-4	CG	SXX	-.0131	.6325	.5751	-.5018	.0074
			SYY	.0132	-.9159	-.7642	.8661	.9781
			SXY	-.0153	.2962	.2668	-.2318	.2482
	7-1	CG	SXX	.0354	1.3731	1.4513	-.9409	-.9509
			SYY	-.0292	-.3749	-.5501	1.0224	.4077
			SXY	-.0033	.4017	.4858	.0905	-.3112
	7-2	CG	SXX	.0122	.7887	1.0655	-.4688	-.5693
			SYY	-.0131	-1.1807	-.9964	1.6046	.8321
			SXY	-.0138	.6482	.7096	-.3875	-.3939
	7-3	CG	SXX	-.0009	.5145	.8085	-.3044	-.2522
			SYY	.0007	-.8752	-.6372	.9542	.6584
			SXY	-.0124	.5427	.6186	-.3787	-.1955
	7-4	CG	SXX	-.0048	.3947	.5173	-.2615	.1254
			SYY	.0051	-.5562	-.4533	.4571	.6847
			SXY	-.0057	.2910	.3193	-.1956	.2463
	8-1	CG	SXX	.0108	.0983	1.0286	.1432	-.6928
			SYY	-.0121	-1.8523	-1.6592	2.6803	1.2448
			SXY	-.0003	1.1391	.9266	-.7135	-.6526
	8-2	CG	SXX	.0039	.1027	1.1808	.0036	-.7377
			SYY	-.0022	-.6916	-.5452	.8638	.4573
			SXY	-.0053	.7838	.8155	-.5225	-.4922
	8-3	CG	SXX	-.0000	.1157	1.2223	-.0470	-.6609
			SYY	.0009	-.2893	-.2088	.2762	.2566
			SXY	-.0045	.4281	.6488	-.2587	-.2444
	8-4	CG	SXX	-.0011	.1224	1.1058	-.0671	-.3807
			SYY	.0011	-.1383	-.2949	.0882	.4890
			SXY	-.0018	.1535	.4120	-.0778	.1986

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
7	1-1	CG	SXX	-1.0205	-.6065	-.6065	.1154	.3891
			SYY	.3103	.7822	.7822	-.1491	-.4960
			SXY	-.0491	.6646	.6647	-.1242	-.4105
	1-2	CG	SXX	-1.1566	-1.0652	-1.0649	.2014	.6816
			SYY	.1099	.3403	.3402	-.0636	-.2279
			SXY	-.1959	.1775	.1776	-.0328	-.0885
	1-3	CG	SXX	-1.1982	-1.2956	-1.2959	.2432	.8290
			SYY	.0186	.1345	.1341	-.0174	-.1062
			SXY	-.2082	.0010	.0016	-.0083	.0493
	1-4	CG	SXX	-1.1973	-1.4184	-1.4180	.2718	.9059
			SYY	-.0282	.0205	.0203	.0208	-.0382
			SXY	-.1569	-.0168*	-.0159	-.0331	.0946
	1-5	CG	SXX	-1.1750	-1.4785	-1.4790	.2913	.9431
			SYY	-.0600	-.0699	-.0703	.0684	.0241
			SXY	-.0690	.0686	.0701	-.1070	.0744
	1-6	CG	SXX	-1.1333	-1.4963	-1.4978	.3110	.9563
			SYY	-.0949	-.2062	-.2059	.1698	.1409
			SXY	.0496	.2626	.2642	-.2542	-.0273
	1-7	CG	SXX	-1.0537	-1.4464	-1.4487	.3155	.9330
			SYY	-.1615	-.5754	-.5738	.4915	.5112
			SXY	.2043	.6071	.6074	-.5174	-.2591
	1-8	CG	SXX	-.9041	-1.3256	-1.3286	.3309	.9083
			SYY	-.3965	-2.2740	-2.2701	2.0746	2.3348
			SXY	.3811	1.0466	1.0444	-.8458	-.5730
	2-1	CG	SXX	-.2375	.6987	.6993	-.1401	-.3946
			SYY	.0621	.4308	.4304	-.0709	-.3164
			SXY	-.0117	.6877	.6878	-.1299	-.4207
	2-2	CG	SXX	-.4160	.2187	.2187	-.0568	-.0733
			SYY	-.0477	.1610	.1598	.0087	-.1620
			SXY	-.1012	.3608	.3615	-.0797	-.1901
	2-3	CG	SXX	-.5137	-.1037	-.1017	.0026	.1513
			SYY	-.1172	-.0390	-.0395	.1001	-.0377
			SXY	-.1130	.2208	.2216	-.0856	-.0729
	2-4	CG	SXX	-.5513	-.2961	-.2960	.0267	.3011
			SYY	-.1672	-.2207	-.2227	.2140	.0996
			SXY	-.0848	.1915	.1944	-.1339	-.0299
	2-5	CG	SXX	-.5346	-.3863	-.3876	.0279	.3920
			SYY	-.2094	-.4584	-.4579	.4054	.3152
			SXY	-.0413	.2417	.2451	-.2310	-.0487
	2-6	CG	SXX	-.4573	-.3540	-.3568	-.0186	.4068
			SYY	-.2440	-.8214	-.8178	.7380	.7034
			SXY	-.0087	.3164	.3190	-.3492	-.0980
	2-7	CG	SXX	-.3029	-.1358	-.1417	-.1704	.2886
			SYY	-.2259	-1.3550	-1.3487	1.2973	1.3788
			SXY	-.0318	.2672	.2679	-.3748	-.0533
	2-8	CG	SXX	-.0269	.4968	.4871	-.6313	-.1953
			SYY	.2211	-.8226	-.8155	1.1117	1.2651
			SXY	-.1734	-.2872	-.2900	.0423	.4798

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
7	3-1	CG	SXX	.0024	1.0032	1.0040	-.2527	-.5455
			SYY	-.0427	.3084	.3063	.0263	-.2912
			SXY	.0266	.7270	.7273	-.1381	-.4506
	3-2	CG	SXX	-.0845	.7246	.7295	-.2139	-.3417
			SYY	-.1371	.0174	.0157	.1686	-.1010
			SXY	-.0300	.5135	.5140	-.1258	-.2926
	3-3	CG	SXX	-.1342	.5444	.5445	-.2223	-.1933
			SYY	-.1955	-.2269	-.2328	.3219	.0933
			SXY	-.0576	.3764	.3817	-.1354	-.1874
	3-4	CG	SXX	-.1480	.4212	.4224	-.2358	-.0838
			SYY	-.2232	-.4919	-.4917	.5388	.3395
			SXY	-.0757	.2997	.3055	-.1784	-.1223
	3-5	CG	SXX	-.1295	.3835	.3834	-.2928	-.0355
			SYY	-.2120	-.7463	-.7424	.7813	.6382
			SXY	-.1011	.2176	.2248	-.2042	-.0511
	3-6	CG	SXX	-.0899	.4335	.4259	-.4080	-.0602
			SYY	-.1302	-.9321	-.9237	1.0099	.9499
			SXY	-.1440	.0687	.0772	-.1592	.0895
	3-7	CG	SXX	-.0615	.5084	.4897	-.5463	-.1276
			SYY	.0748	-.7925	-.7771	.9881	1.0176
			SXY	-.1822	-.1638	-.1592	-.0037	.3530
	3-8	CG	SXX	-.1626	.1771	.1501	-.3621	.1415
			SYY	.1246	-.9200	-.8993	1.0574	1.2491
			SXY	-.0833	-.1216	-.1290	.0053*	.4560
	4-1	CG	SXX	.1247	1.1727	1.1809	-.3872	-.6484
			SYY	-.1202	.1178	.1130	.2330	-.1857
			SXY	.0285	.7072	.7072	-.1224	-.4515
	4-2	CG	SXX	.0787	1.0467	1.0460	-.4298	-.5336
			SYY	-.1703	-.1402	-.1534	.3974	.0307
			SXY	-.0202	.5263	.5342	-.1031	-.3188
	4-3	CG	SXX	.0478	.8970	.9094	-.4225	-.4165
			SYY	-.1824	-.4114	-.4123	.6216	.2784
			SXY	-.0615	.4038	.4103	-.1243	-.2160
	4-4	CG	SXX	.0270	.8200	.8311	-.4762	-.3463
			SYY	-.1553	-.5867	-.5896	.7767	.5017
			SXY	-.0993	.2601	.2764	-.1043	-.0948
	4-5	CG	SXX	.0062	.7813	.7762	-.5574	-.3009
			SYY	-.0864	-.7074	-.7026	.9053	.7047
			SXY	-.1294	.1128	.1397	-.0612	.0491
	4-6	CG	SXX	-.0287	.7099	.6818	-.6015	-.2301
			SYY	.0083	-.7590	-.7330	.9657	.8477
			SXY	-.1329	.0071	.0334	-.0242	.2007
	4-7	CG	SXX	-.0855	.5218	.4721	-.5272	-.0532
			SYY	.0412	-.9101	-.8619	1.0459	1.0607
			SXY	-.0832	.0270	.0364	-.0575	.2897
	4-8	CG	SXX	-.0594	.4785	.4145	-.5144	.0248
			SYY	.0755	-.9734	-.9181	1.0162	1.2157
			SXY	-.0469	.0164*	-.0047	-.0666	.4185

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
7	5-1	CG	SXX	.1435	1.2944	1.2974	-.5741	-.7610
			SYY	-.1270	-.0404	-.0645	.4693	-.0406
			SXY	.0136	.6299	.6354	-.0580	-.4168
	5-2	CG	SXX	.1100	1.0953	1.1419	-.5017	-.6247
			SYY	-.1281	-.3154	-.3219	.7019	.2014
			SXY	-.0274	.5163	.5176	-.0822	-.3121
	5-3	CG	SXX	.0776	1.0636	1.0964	-.6087	-.5727
			SYY	-.1034	-.4105	-.4444	.7578	.3638
			SXY	-.0601	.3275	.3639	-.0103	-.1690
	5-4	CG	SXX	.0427	1.0295	1.0320	-.6957	-.5074
			SYY	-.0614	-.5793	-.5921	.9024	.5575
			SXY	-.0793	.2000	.2636	.0102	-.0503
	5-5	CG	SXX	.0049	.9408	.9120	-.7091	-.3962
			SYY	-.0185	-.7602	-.7201	1.0359	.7402
			SXY	-.0787	.1570	.2236	-.0300	.0404
	5-6	CG	SXX	-.0266	.8080	.7509	-.6579	-.2423
			SYY	.0030	-.9270	-.8351	1.1101	.9179
			SXY	-.0588	.1768	.2220	-.1027	.1153
	5-7	CG	SXX	-.0247	.7240	.6415	-.6210	-.1211
			SYY	.0290	-.9853	-.8671	1.0637	1.0383
			SXY	-.0434	.1753	.1808	-.1435	.2309
	5-8	CG	SXX	-.0322	.6034	.4985	-.5558	.0253
			SYY	.0331	-1.0055	-.9015	.9582	1.1697
			SXY	-.0205	.1703	.1256	-.1658	.3649
	6-1	CG	SXX	.0949	1.1117	1.2168	-.4924	-.7529
			SYY	-.0784	-.2286	-.2628	.7806	.1400
			SXY	.0018	.5859	.5782	-.0225	-.3869
	6-2	CG	SXX	.0696	1.2814	1.3202	-.8067	-.7999
			SYY	-.0619	-.2058	-.3284	.7079	.2456
			SXY	-.0217*	.3163	.3972	.1409	-.2200
	6-3	CG	SXX	.0428	1.2314	1.2485	-.8474	-.7135
			SYY	-.0410	-.5795	-.6158	1.0509	.5223
			SXY	-.0345	.2777	.3889	.0539	-.1729
	6-4	CG	SXX	.0179	1.0864	1.0989	-.7711	-.5624
			SYY	-.0220	-.8689	-.7940	1.2496	.7194
			SXY	-.0367	.3249	.4306	-.0958	-.1526
	6-5	CG	SXX	.0008	.9232	.9304	-.6648	-.3886
			SYY	-.0083	-1.0014	-.8440	1.2506	.8240
			SXY	-.0318	.3713	.4518	-.2126	-.1060
	6-6	CG	SXX	-.0045	.7903	.7807	-.5795	-.2238
			SYY	.0072	-.9966	-.8003	1.1120	.8639
			SXY	-.0267	.3696	.4137	-.2583	-.0013
	6-7	CG	SXX	-.0133	.6644	.6235	-.5053	-.0507
			SYY	.0112	-.9416	-.7554	.9300	.9154
			SXY	-.0160	.3335	.3322	-.2527	.1501
	6-8	CG	SXX	-.0115	.5855	.5046	-.4697	.0893
			SYY	.0129	-.8498	-.7329	.7342	1.0083
			SXY	-.0080	.2569	.2023	-.2068	.3508

TABLE IV (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
7	7-1	CG	SXX	.0415	1.6300	1.6180	-1.1781	-1.0974
			SYY	-.0310	.0073*	-.2985	.6724	.1944
			SXY	-.0011	.2564	.3677	.3098	-.2422
	7-2	CG	SXX	.0269	1.2159	1.3482	-.8073	-.8551
			SYY	-.0234	-.9537	-.9530	1.6270	.7318
			SXY	-.0088	.5198	.5987	-.1248	-.3792
	7-3	CG	SXX	.0147	.9179	1.1515	-.5678	-.6612
			SYY	-.0151	-1.1596	-1.0053	1.6496	.8157
			SXY	-.0123	.6515	.7172	-.3657	-.4261
	7-4	CG	SXX	.0066	.7111	1.0006	-.4199	-.4936
			SYY	-.0074	-1.0759	-.8529	1.3650	.7445
			SXY	-.0131	.6683	.7356	-.4474	-.3890
	7-5	CG	SXX	.0018	.5726	.8711	-.3351	-.3327
			SYY	.0002	-.9084	-.6652	1.0344	.6481
			SXY	-.0122	.6027	.6787	-.4226	-.2821
	7-6	CG	SXX	-.0036	.4741	.7317	-.2855	-.1525
			SYY	.0023	-.7515	-.5223	.7572	.5943
			SXY	-.0083	.4939	.5741	-.3449	-.1215
	7-7	CG	SXX	-.0035	.4183	.5807	-.2673	.0485
			SYY	.0043	-.6136	-.4378	.5392	.6056
			SXY	-.0055	.3591	.4224	-.2447	.0986
	7-8	CG	SXX	-.0040	.3799	.3926	-.2611	.2850
			SYY	.0040	-.4970	-.4419	.3722	.7256
			SXY	-.0026	.2216	.2254	-.1483	.3851
	8-1	CG	SXX	.0107	.1783	1.0351	.1028	-.7177
			SYY	-.0121	-2.2405	-2.0236	3.2921	1.5014
			SXY	.0007	1.2325	.9621	-.7712	-.6960
	8-2	CG	SXX	.0070	.1309	1.1107	.0518	-.7457
			SYY	-.0067	-1.3368	-1.1513	1.8699	.8761
			SXY	-.0025	1.0951	.9560	-.7500	-.6599
	8-3	CG	SXX	.0043	.1146	1.1702	.0121	-.7568
			SYY	-.0029	-.8127	-.6531	1.0596	.5228
			SXY	-.0043	.9097	.8878	-.6310	-.5718
	8-4	CG	SXX	.0021	.1141	1.2130	-.0192	-.7476
			SYY	-.0004	-.5124	-.3749	.6062	.3318
			SXY	-.0045	.7140	.7999	-.4812	-.4606
	8-5	CG	SXX	-.0004	.1176	1.2325	-.0413	-.7093
			SYY	.0002	-.3423	-.2280	.3560	.2426
			SXY	-.0035	.5282	.7133	-.3366	-.3377
	8-6	CG	SXX	-.0001	.1273	1.2286	-.0605	-.6370
			SYY	.0009	-.2411	-.1629	.2147	.2267
			SXY	-.0027	.3593	.6279	-.2121	-.1949
	8-7	CG	SXX	-.0010	.1298	1.1742	-.0707	-.5020
			SYY	.0009	-.1771	-.1785	.1319	.3016
			SXY	-.0015	.2150	.5285	-.1160	-.0031
	8-8	CG	SXX	-.0008	.1281	1.0387	-.0754	-.2648
			SYY	.0007	-.1298	-.3424	.0792	.5677
			SXY	-.0006	.0971	.3495	-.0487	.3299

TABLE V

RIB STRESSES

PSI PER POUND

MOD NO	ELEM NO	F1	F3	F4	F7	F8
1	1	-1.0372	-.7456	-.9077	.0217	.4523
	2	.3012	.8513	1.7507	-.5295	-.9934
2	1	-1.1650	-1.1092	-1.1068	.2199	.5962
	2	-.0043	.0930	1.3231	-.1659	-.7220
3	1	-1.2988	-1.3127	-1.2859	.2720	.7874
	2	-1.2898	-1.5077	-1.5237	.2385	.8137
	3	.0654	.2991	1.5449	-.2850	-1.0381
	4	-.0853	.0066*	1.3597	-.0802	-.7667
4	1	-1.4045	-1.4568	-1.4574	.2806	.8971
	2	-1.4003	-1.8258	-1.8283	.3772	1.0077
	3	.0057	-.1361	1.3497	.0174	-.9167
	4	-.0001	.0123	1.6088	-.0256	-.9384
5	1	-1.5724	-1.6827	-1.6813	.3206	1.0487
	2	-1.5040	-1.8490	-1.8530	.3371	1.1396
	3	-1.4554	-1.9434	-1.9389	.3878	1.1407
	4	-1.4895	-2.0929	-2.0893	.5027	1.1428
	5	-.0058	-.2490	1.2596	.1001	-.9048
	6	-.0102	-.0953	1.4414	.0203	-.9970
	7	.0087	.0369	1.6425	-.0426	-1.0722
	8	-.0028	.0351	1.7409	-.0280	-1.0228
6	1	-1.6603	-1.8449	-1.8449	.3476	1.1505
	2	-1.5790	-2.0249	-2.0252	.3808	1.2468
	3	-1.5052	-2.0514	-2.0525	.4080	1.2108
	4	-1.5542	-2.2146	-2.2164	.5427	1.2114
	5	.0035	-.4438	1.1981	.2567	-.8646
	6	.0018	-.1879	1.4586	.0955	-1.0273
	7	.0005	-.0520	1.6721	.0215	-1.1210
	8	.0000	-.0027	1.8431	.0009	-1.1174
7	1	-1.8003	-2.0448	-2.0448	.3854	1.2776
	2	-1.7447	-2.1532	-2.1533	.4052	1.3442
	3	-1.6657	-2.1581	-2.1580	.4052	1.3405
	4	-1.5937	-2.1304	-2.1307	.4006	1.3074
	5	-1.5437	-2.1102	-2.1106	.4059	1.2672
	6	-1.5262	-2.1253	-2.1262	.4355	1.2375
	7	-1.5529	-2.2145	-2.2159	.5165	1.2467
	8	-1.6406	-2.4347	-2.4360	.6905	1.3411
	9	.0028	-.5795	1.0936	.3773	-.7991
	10	.0022	-.3878	1.2634	.2333	-.9171
	11	.0013	-.2449	1.4071	.1341	-1.0098
	12	.0007	-.1414	1.5319	.0691	-1.0813
	13	.0005	-.0692	1.6447	.0287	-1.1354
	14	.0000	-.0248	1.7475	.0076	-1.1707
	15	.0000	-.0009	1.8442	-.0014	-1.1854
	16	-.0000	.0060	1.9293	-.0026	-1.1661

TABLE VI

STRINGER STRESSES

PSI PER POUND

MOD NO	ELEM NO	F1	F3	F4	F7	F8
1	1	-.1271	.7347	.7392	-.0395	-.3056
	2	.0671	-.1189	-.0873	.4108	.8176
2	1	-.2295	.8753	.8818	-.1590	-.5254
	2	.0057	.9481	.9116	.0461	-.4900
	3	.1143	-.3989	-.4093	.5719	1.1012
	4	-.0028	-.1433	-.0669	.2335	.9272
3	1	-.1542	1.0527	1.0587	-.1660	-.6294
	2	.0411	1.0645	1.0499	.0294	-.6207
	3	.0874	-.4928	-.5204	.6082	1.2320
	4	.0029	-.1669	-.1359	.2392	1.0338
4	1	-.2092	1.1506	1.1510	-.2179	-.7153
	2	.0220	1.4017	1.4017	-.2458	-.8924
	3	.0332	1.2204	1.2228	-.0527	-.7999
	4	.0079	.9383	.8753	.2412	-.5814
	5	.1408	-.6707	-.6727	.7662	1.3847
	6	.0262	-.5969	-.6087	.5733	1.3441
	7	.0060	-.2871	-.3269	.2647	1.1972
	8	.0004	-.0514	-.0016	.0570	1.0506
5	1	-.1324	1.3386	1.3387	-.2508	-.8345
	2	.0727	1.5219	1.5217	-.2562	-.9685
	3	.0508	1.2754	1.2798	-.0593	-.8485
	4	.0081	.9427	.8921	.2571	-.6117
	5	.1152	-.7878	-.7885	.8442	1.4991
	6	.0284	-.6106	-.6284	.5684	1.3792
	7	.0048	-.2992	-.3553	.2636	1.2384
	8	.0000	-.0601	-.0465	.0604	1.1143

TABLE VI (CONTINUED)

MOD NO	ELEM NO	F1	F3	F4	F7	F8
6	1	-.1552	1.3999	1.3999	-.2639	-.8741
	2	.0232	1.6038	1.6038	-.3033	-1.0059
	3	.0837	1.6233	1.6230	-.3024	-1.0351
	4	.0827	1.5436	1.5425	-.2543	-1.0116
	5	.0538	1.3929	1.3895	-.1436	-.9376
	6	.0262	1.2087	1.1988	.0191	-.8282
	7	.0102	1.0029	.9777	.2293	-.6882
	8	.0033	.8336	.7348	.4320	-.5251
	9	.1604	-.8259	-.8272	.9228	1.5593
	10	.0773	-.8204	-.8234	.8113	1.5304
	11	.0423	-.7216	-.7286	.6665	1.4621
	12	.0199	-.5883	-.6045	.5115	1.3884
	13	.0082	-.4126	-.4483	.3353	1.3041
	14	.0029	-.2222	-.2838	.1643	1.2271
	15	.0007	-.0754	-.1436	.0490	1.1849
	16	.0000	-.0082	.0132	.0052	1.1409
7	1	-.0916	1.5463	1.5463	-.2914	-.9662
	2	.0661	1.7040	1.7040	-.3207	-1.0694
	3	.1027	1.6686	1.6684	-.3059	-1.0632
	4	.0919	1.5646	1.5636	-.2523	-1.0240
	5	.0586	1.4069	1.4039	-.1439	-.9487
	6	.0271	1.2187	1.2087	.0171	-.8395
	7	.0093	1.0060	.9808	.2330	-.6957
	8	.0026	.8159	.7234	.4586	-.5215
	9	.1362	-.9235	-.9244	.9981	1.6504
	10	.0804	-.8388	-.8413	.8278	1.5561
	11	.0436	-.7312	-.7378	.6704	1.4759
	12	.0192	-.5936	-.6111	.5094	1.3981
	13	.0074	-.4150	-.4543	.3327	1.3133
	14	.0023	-.2270	-.2936	.1672	1.2400
	15	.0005	-.0808	-.1585	.0536	1.2046
	16	.0000	-.0108	-.0121	.0069	1.1755

APPENDIX B

DEVELOPMENT OF THE MATRIX FORCE PROGRAM

The matrix force analysis used here is based on the formulation of Pestel and Leckie (Reference 10, Chapter 9). Therefore, the basic equations of the method will not be developed here but will be taken directly from Reference 10. The basic equations are

$$[D_{11}] = [B_1]^T [F_v] [B_1] \quad \text{(B-1)}$$

$$[D_{10}] = [B_1]^T [F_v] [B_0] \quad \text{(B-2)}$$

$$[X] = -[D_{11}]^{-1} [D_{10}] \quad \text{(B-3)}$$

$$[B] = [B_0] + [B_1] [X] \quad \text{(B-4)}$$

$$\{p\} = [B] \{f\} \quad \text{(B-5)}$$

$$[F_d] = [B]^T [F_v] [B] \quad \text{(B-6)}$$

$$\{d\} = [F_d] \{f\} \quad \text{(B-7)}$$

where

$[B_1]$ = the matrix of internal forces due to unit values of the redundants (1 x n),

$[B_0]$ = the matrix of internal forces due to unit external forces (1 x m),

$[F_v]$ = the flexibility matrix of the unassembled structure. This matrix is a diagonally partioned matrix with the flexibilities of the individual structural elements as its diagonal subelements $(\ell \times \ell)$,

$[D_{11}]$ = the matrix defined by Equation B-1 $(n \times n)$,

$[D_{10}]$ = the matrix defined by Equation B-2 $(n \times m)$,

$[X]$ = the matrix of unit redundants. Each column of this matrix gives the values of the redundants due to a unit value of one of the external forces $(n \times m)$,

$[B]$ = the matrix of unit internal forces. Each column of this matrix gives the values of the internal forces due to a unit value of one of the external forces $(\ell \times m)$,

$\{f\}$ = the column matrix of external forces $(\ell \times 1)$,

$\{p\}$ = the column matrix of internal forces $(\ell \times 1)$,

$[F_d]$ = the flexibility matrix of the assembled structure. The elements of this matrix are the deflection influence coefficients of the structure $(m \times m)$,

$\{d\}$ = the column matrix of displacements $(m \times 1)$,

k = the number of elements in the structure (the number of submatrices in the unassembled structure flexibility matrix is equal to k),

ℓ = the number of independent internal forces ($\ell \leq k$),

m = the number of external forces, and

n = the number of redundants

The development of a program to solve Equations B-1 through B-7 is relatively straight-forward; however, for large structures, all of the matrices in these equations cannot be simultaneously stored in the main memory of even the largest computer available today. The simplest method of handling this problem is to store all of the large matrices on magnetic tape and to read them into main memory, usually a row or a column at a time, as they are needed. The method used here does this, in part, but two other techniques are used to minimize the number of matrices that must be stored on tape, i.e.,

1. the "Recurrence Method for Highly Redundant Systems" of Pestel and Leckie (Reference 10, Section 9-6) is used to reduce the size of the matrix $[D_{11}]$ which must be inverted, and
2. the large matrix $[F_V]$ is stored as an ℓ x 5 array instead of an ℓ x ℓ array.

The Recurrence Method

Pestel and Leckie have shown that a highly redundant system can be analyzed by subdividing the redundants and considering each subset of redundants separately. To illustrate, let us assume that the n redundants are subdivided into three sets - a, b, and c - with α, β, and γ redundants, respectively, such that

$$\alpha + \beta + \gamma = n \; . \qquad \text{(B-8)}$$

We can now proceed as though there were only α redundants and that there are $\beta + \gamma + m$ external forces and calculate the first α redundants in terms of these 'assumed' external forces. The first α redundants are then eliminated from further consideration. We next consider the b-set with β redundants and calculate these redundants as a linear combination of the $\gamma + m$ 'assumed' external forces. Finally, the c-set of redundants are considered. Since this is the last set, the 'assumed' external forces are the 'actual' external forces and all of the redundants have been eliminated.

The primary advantage of the recurrence method lies in the fact that several smaller matrices, $[D^i_{11}]$, are inverted instead of one, usually large, n x n matrix. However, in attempting to write a program using this method, two difficulties were encountered, i.e.,

1. extensive programming would be required to accommodate unequal size subsets of redundants, and
2. the storage required could become greater than that required by the direct method if the number and size of the subsets was not chosen properly.

It was discovered that not only is the programming greatly simplified, but the storage requirements are minimized if the redundants are subdivided into n sets with one redundant in each set. It is also advantageous to store and manipulate $[B_1]$ and $[B_0]$ as a single matrix $[B_{10}]$ and $[D_{11}]$ and $[D_{10}]$ as a single matrix, also called $[D_{10}]$. The recurrsion equations can then be written as

$$[D_{10}^i] = \{B_1^i\}^T [F_v] [B_{10}^i]$$

$$[X^i] = -[D_{10}^i]/D_{11}^i \qquad (i = 1, n) \qquad \text{(B-9)}$$

$$[B_{10}^{i+1}] = [B_0^i] + \{B_1^i\}[X_i]$$

where $\{B_1^i\}$ is the first column of $[B_{10}^i]$ and D_{11}^i is the first element of $[D_{10}^i]$. Each recurrsion reduces the number of columns in $[B_{10}^i]$, until after the nth recurrsion, $[B_{10}^{n+1}]$ is an ℓ x m matrix and is the unit internal force matirx [B]. Equations B-5 through B-7 can then be used to determine the internal forces and the displacements.

Equations B-9 show clearly that both storage and programming are minimized since $[D_{10}^i]$, $[X^i]$, and the matrix product $\{B_1^i\}^T [F_v]$ are all row matrices. Also, since D_{11}^i is a scalar, all requirements for matrix inversion are eliminated.

Storage of the Matrix $[F_v]$

Since the flexibility matrix for the unassembled structure can be very large, it is uneconomical to store it, either in main storage or on tape, as an ℓ x ℓ array. After it was decided to program the recurrence method for n

recurrsions with a single redundant per recurrion, it was also noticed that it would be relatively simple to obtain the product $\{B_1^i\}^T[F_v]$ if $[F_v]$ were stored as an ℓ x 5 array. The necessity for five columns is dictated by the fact that the largest element stiffness matrix is the 5 x 5 array required by a plate element.

In order to be able to obtain the product $\{B_1^i\}^T[F_v]$ for any combination and arrangement of elements in the structure, each element stiffness matrix must be right-justified in the array. For an axially loaded element, the flexibility must be entered as one row consisting of four zeros followed by the element flexibility. A beam element would require two rows each with three zeros followed by two flexibility coefficients. A rectangular or trapezoidal plate requires five rows for the twenty-five coefficients. It should be noted that the routine used to form this product in the program which follows has been simplified since the structure under study here has only two types of elements, axially loaded elements and plate elements. A general routine can be found in Reference 8.

The Matrix Force Program

A program listing is given in Table VII. The program is written in Fortran IV using the format specified for the IBM 7044/7094 Operating System. It is written as a main program with three subroutines which are used to generate the matrices $[F_v]$, $[B_1]$, and $[B_0]$. Since each subroutine is called only once, the subroutines could have been incorporated into the main program; however, this would have exceeded the maximum number (200) of statements allowed in this operating system.

The input to the program is identical to that required by the displacement program (Appendix A). The constant and nodal coordinate routines are essentially the same as in the displacement program except for some rearrangement and the addition of several constants.

The element stiffener flexibilities are generated directly, i.e.,

$$F_g = L_g/A_g\ E_g \tag{B-10}$$

and entered into the $[F_v]$ array. The web flexibilities are obtained by first generating the stiffness matrix for the element using the routine described in Chapter II and Appendix A. This matrix is then reduced from a 8 x 8 singular matrix to a 5 x 5 nonsingular matrix by striking out rows and columns 2, 5, and 6. This corresponds to the restraints shown in Figure 26. This matrix is then inverted and placed in $[F_v]$.

The B_1 and B_0 subroutines are used to generate the elements of $[B_{10}]$ and to write these elements on a magnetic tape. Each column of $[B_{10}]$ consists of the internal forces due to a unit value of either a redundant or an external force. The methods of calculating these internal forces are given in Chapter II.

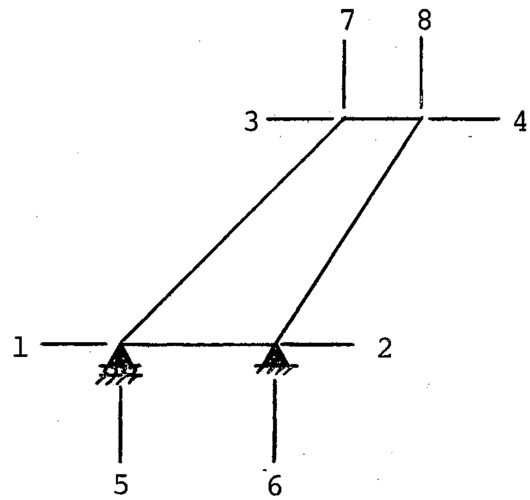


Figure 26. Plate Element Constraints

Subsequent to the generation of $[F_v]$ (in core) and $[B_{10}]$ (on magnetic tape), the recurrence method, as described previously, is used to calculate the unit internal force matrix [B]. Since only one column of $[B_{10}]$ is in core at any one time, it is necessary to read the entire array twice during each recurrsion, once to calculate $[D_{10}^{i}]$ and again to calculate $[B_{10}^{i+1}]$. It is also necessary to use two tapes in order to generate $[B_{10}^{i+1}]$ since each new $[B_{10}^{i}]$ has one less column (or tape record) than the previous one.

The flexibility matrix, the deflections due to unit external forces, is calculated using Equation B-6. It should be noted that Pestel and Leckie (Reference 10, page 255) do not present the equation for the flexibility matrix in this form. They present the equation

$$[F_d] = [B_0]^T[F_v][B] \ . \qquad \text{(B-11)}$$

It can be shown that Equations B-6 and B-11 give exactly the same results. The reason that Equation B-6 must be used here is that $[B_0]$ is destroyed in the recurrence analysis. The flexibilities are tabulated in Table VIII.

The web stresses are calculated at the same points as in the displacement analysis, i.e., for Models 1 through 4, the stresses are calculated at five points within each plate element; but for Models 5 through 7, only the stresses at the centroid are obtained. The equation for calculating the stress, however, is a modification of the stress equation used in the displacement analysis. This modification is necessary because the displacement analysis is formulated in terms of the absolute displacements of the element nodes, while the force analysis uses relative displacements, or deformations, in its formulation (See Figure 26). The modification consists of striking out columns 2, 5, and 6 in the $[\Sigma]$ matrix of Equation A-2 and replacing the displacement matrix $\{u\}$ with the deformation matrix $\{v\}$ giving

$$\{S\} = [\Sigma^1]\{v\} \qquad \text{(B-12)}$$

The web stresses are given in Table IX. The format is identical to that used in Appendix A for the displacement method web stresses.

The stiffener stresses can also be readily determined. Since the deformation is the relative displacement of the ends, measured along the length of the member, the stress is given by this equation,

$$s = (E/L)v \ , \qquad \text{(B-13)}$$

where both the stress s and the deformation v are scalars. The rib stresses are given in Table X and the stringer stresses are given in Table XI.

Explanation of Tables

Tables VII through XI are identical in format to Tables II through VI in Appendix A in order that the results of the force and displacement analyses can be compared.

TABLE VII

MATRIX FORCE PROGRAM LISTING

ISOTHERMAL MATRIX FORCE ANALYSIS OF A TRAPEZOIDAL PANEL

TYPE DECLARATION STATEMENTS

```
      DOUBLE PRECISION B10(352),FV(352,5),B1TFV(352)
      DOUBLE PRECISION D10(198),FVB(352,5),FD(5,5)
      DOUBLE PRECISION XI(5),ETA(5),SIGMA(3,5),STRES(3,5)
      DOUBLE PRECISION B(352,5)
```

DIMENSION AND COMMON STATEMENTS

```
      COMMON /PARAM/E,V,T,STA,RA,EP,EPT,GS
      COMMON /PACON/M,N,M1,N1,M2,N2,AM,AN,AM2
      COMMON /REDUN/KI,LI,MI,NI,NOA,NOB,NOC
      COMMON /DELTA/DELXR,DELXT,DELY,DELLE,DELTE
      COMMON /ARRAY/X(9,9),Y(9),B10,FV
```

FORMAT STATEMENTS

```
  300 FORMAT(2I5)
  301 FORMAT(6E12.6)
  351 FORMAT(29X   11H PARAMETERS/)
  352 FORMAT(8X,16H YOUNG*S MODULUS,E10.3,
     13X,16H POISSON*S RATIO,F6.3/)
  353 FORMAT(3X,14H WEB THICKNESS,F6.3,2X,14H STRINGER AREA,
     1F8.5,2X,9H RIB AREA,F8.5/)
  354 FORMAT(5X,3H XA,F8.3,5X,3H XB,F8.3,5X,3H XC,F8.3,
     15X,3H XD,F8.3/)
  355 FORMAT(5X,3H YA,F8.3,21X,3H YC,F8.3/)
  356 FORMAT(5X,14H ROWS OF NODES,I3,15X,
     117H COLUMNS OF NODES,I3//)
  357 FORMAT(25X,19H FLEXIBILITY MATRIX/)
  358 FORMAT(5E14.6)
  359 FORMAT(28X,13H WEB STRESSES/)
  360 FORMAT(25X,19H STIFFENER STRESSES/)
```

NUMBER OF PANEL CONFIGURATIONS

```
      READ(5,300)NOPAC
```

PANEL PARAMETERS

```
    1 READ(5,301)E,V,T,STA,RA
      READ(5,301)X1,X2,X3,X4,Y1,Y3
    2 READ(5,300)M,N
      WRITE(7,351)
      WRITE(7,352)E,V
      WRITE(7,353) T,STA,RA
      WRITE(7,354)X1,X2,X3,X4
      WRITE(7,355)Y1,Y3
      WRITE(7,356)M,N
```

TABLE VII (CONTINUED)

```
      CONSTANTS

      EP=E/(1.-(V**2))
      EPT=EP*T
      GS=E/(2.*(1.+V))
      AM=M-1
      AN=N-1
      M1=M-1
      N1=N-1
      MN=M*N
      AM2=M-2
      M2=M-2
      N2=N-2
      DELXR=(X2-X1)/AN
      DELXT=(X4-X3)/AN
      DELY=(Y3-Y1)/AM
      DELLE=SQRT((X3-X1)**2+(Y3-Y1)**2)/AM
      DELTE=SQRT((X4-X2)**2+(Y3-Y1)**2)/AM
      KI=MN+M+N-3
      LI=5*MN-3*M-3*N+1
      MI=5
      NI=3*MN-3*M-3*N+4
      NOA=2*(M1+N1)
      NOB=M2*N1+N2*M1
      NOC=M2*N2

      NODAL COORDINATES

      X(1,1)=X1
      X(1,N)=X2
      X(M,1)=X3
      X(M,N)=X4
      Y(1)=Y1
      Y(M)=Y3
      DO 3 I=2,M1
    3 Y(I)=Y(I-1)+DELY
      DO 4 J=2,N1
      X(1,J)=X(1,J-1)+DELXR
    4 X(M,J)=X(M,J-1)+DELXT
      DO 5 I=2,M1
      DO 5 J=1,N
    5 X(I,J)=X(1,J)+((X(M,J)-X(1,J))/(Y(M)-Y(1))*(Y(I)-Y(1)))

      GENERATE THE ELEMENT FLEXIBILITY MATRIX FV
      CALL FLEX

      GENERATE THE UNIT REDUNDANT MATRIX B1
      CALL B1

      GENERATE THE UNIT EXTERNAL FORCE MATRIX B0
      CALL B0
```

TABLE VII (CONTINUED)

```
   RECURSION ANALYSIS

   MINI=MI+NI
   DO 16 IR=1,NI
   REWIND 1

   READ B10(IR)
   READ(1)B10

   B1 TRANSPOSE TIMES FV
   DO 6 I=1,LI
 6 B1TFV(I)=0.0
   DO 7 I=1,NOA
 7 B1TFV(I)=B10(I)*FV(I,5)
   K=KI-NOA
   J=NOA+1
   J4=J+4
   DO 9 I=1,K
   DO 8 IA=1,5
   IAJ=IA+J-1
   DO 8 JA=J,J4
 8 B1TFV(IAJ)=B1TFV(IAJ)+B10(JA)*FV(JA,IA)
   J=J+5
 9 J4=J+4

   CALCULATION OF D10(FIRST ELEMENT IS D11)
   DO 10 I=1,MINI
10 D10(I)=0.0
   DO 11 I=1,LI
11 D10(1)=D10(1)+B1TFV(I)*B10(I)
   DO 12 J=2,MINI
   READ(1)B10
   DO 12 I=1,LI
12 D10(J)=D10(J)+B1TFV(I)*B10(I)
   REWIND 1
   REWIND 8

   CALCULATION OF REDUNDANTS (STORED IN D10)
   DO 13 I=2,MINI
13 D10(I)=-D10(I)/D10(1)

   CALCULATION OF NEW B10
   READ(1)B1TFV
   DO 15 J=2,MINI
   READ(1)B10
   DO 14 I=1,LI
14 B10(I)=B10(I)+B1TFV(I)*D10(J)
15 WRITE(8)B10
   REWIND 8
   REWIND 1
```

TABLE VII (CONTINUED)

```
   DO 85 KK=2,MINI
   READ (8) B10
85 WRITE (1) B10
   MINI=MINI-1
16 CONTINUE
   REWIND 1

   DEFORMATIONS

   DO 17 I=1,LI
   DO 17 J=1,MI
17 FVB(I,J)=0.0
   DO 21 IR=1,MI
   READ(1)B10
   DO 18 I=1,LI
18 B(I,IR)=B10(I)
   DO 19 I=1,NOA
19 FVB(I,IR)=FV(I,5)*B10(I)
   K=KI-NOA
   I=NOA+1
   I4=I+4
   DO 21 J=1,K
   DO 20 IA=I,I4
   DO 20 JA=1,5
   JAI=JA+I-1
20 FVB(IA,IR)=FVB(IA,IR)+FV(IA,JA)*B10(JAI)
   I=I+5
21 I4=I+4

   DISPLACEMENTS

   DO 22 I=1,MI
   DO 22 J=1,MI
22 FD(I,J)=0.0
   DO 23 I=1,MI
   DO 23 J=1,MI
   DO 23 K=1,LI
23 FD(I,J)=FD(I,J)+B(K,I)*FVB(K,J)
   WRITE(7,357)
   WRITE(7,358)(FD(I,1),FD(I,2),FD(I,3),FD(I,4),
  1FD(I,5),I=1,5)

   WEB STRESSES

   WRITE(7,359)
   JJ=NOA
   DO 36 IA=1,M1
   DO 36 JA=1,N1

   COORDINATES OF NODES
   XA=X(IA,JA)
   XB=X(IA,JA+1)
```

TABLE VII (CONTINUED)

```
      XC=X(IA+1,JA)
      XD=X(IA+1,JA+1)
      YA=Y(IA)
      YC=Y(IA+1)
      D=(XD-XA)/(XB-XA)
      P=(XC-XA)/(XB-XA)
      TR=1./(D-P)
      BX=D-1.
      XBA=XB-XA
      YCA=YC-YA
      IF(MN-15)24,24,25

      COORDINATES OF STRESS POINTS(MN=15, OR LESS)
   24 NO=5
      XI(1)=0.25*(P+0.75+0.25/TR)
      XI(2)=0.25*(P+2.25+0.75/TR)
      XI(3)=0.50*(P+0.5+0.5/TR)
      XI(4)=0.25*(3.*P+0.25+0.75/TR)
      XI(5)=0.75*(P+0.25+0.75/TR)
      ETA(1)=0.25
      ETA(2)=0.25
      ETA(3)=0.5
      ETA(4)=0.75
      ETA(5)=0.75
      GO TO 26

      COORDINATES OF STRESS POINTS(MN GREATER THAN 15)
   25 NO=1
      ETA(1)=(TR+2.)/(3.*(TR+1.))
      XI(1)=0.5*(1.+(P+BX)*ETA(1))

      SIGMA MATRIX
   26 DO 35 I=1,NO
      SIGMA(1,1)=-(1.-ETA(I))/XBA
      SIGMA(1,2)=-TR*ETA(I)/XBA
      SIGMA(1,3)=-SIGMA(1,2)
      SIGMA(1,4)=TR*V*(D-XI(I))/YCA
      SIGMA(1,5)=-TR*V*(P-XI(I))/YCA
      DO 27 J=1,3
   27 SIGMA(2,J)=V*SIGMA(1,J)
      DO 28 J=4,5
   28 SIGMA(2,J)=SIGMA(1,J)/V
      SIGMA(3,1)=-(1.-XI(I))/YCA
      DO 29 J=2,3
   29 SIGMA(3,J)=SIGMA(2,J+2)
      DO 30 J=4,5
   30 SIGMA(3,J)=SIGMA(1,J-2)
      DO 31 J=1,2
      DO 31 K=1,5
   31 SIGMA(J,K)=EP*SIGMA(J,K)
      DO 32 J=1,5
   32 SIGMA(3,J)=GS*SIGMA(3,J)
```

TABLE VII (CONTINUED)

```
      STRESSES
      DO 33 J=1,3
      DO 33 K=1,5
   33 STRES(J,K)=0.0
      DO 34 J=1,3
      DO 34 K=1,5
      DO 34 L=1,5
      LL=L+JJ
   34 STRES(J,K)=STRES(J,K)+SIGMA(J,L)*FVB(LL,K)
   35 WRITE(7,358)(STRES(J,1),STRES(J,2),STRES(J,3),
     1STRES(J,4),STRES(J,5),J=1,3)
      JJ=JJ+5
   36 CONTINUE

      STIFFENER STRESSES

      DO 37 I=1,N1
      J=I+N1
      DO 37 K=1,5
      FVB(I,K)=FVB(I,K)*E/DELXR
   37 FVB(J,K)=FVB(J,K)*E/DELXT
      DO 38 I=1,M1
      J=I+2*N1
      K=I+2*N1+M1
      DO 38 L=1,5
      FVB(J,L)=FVB(J,L)*E/DELLE
   38 FVB(K,L)=FVB(K,L)*E/DELTE
      WRITE(7,360)
      WRITE(7,358)(FVB(I,1),FVB(I,2),FVB(I,3),FVB(I,4),
     1FVB(I,5),I=1,NOA)

      NOPAC=NOPAC-1
      IF(NOPAC)39,39,2
   39 STOP
      END


      SUBROUTINE FLEX

      ISOTHERMAL MATRIX FORCE ANALYSIS OF A TRAPEZOIDAL PANEL
      SUBROUTINE TO GENERATE THE ELEMENT FLEXIBILITY MATRIX

      TYPE DECLARATION STATEMENTS

      DOUBLE PRECISION B10(352),FV(352,5)

      DIMENSION AND COMMON STATEMENTS

      COMMON /PARAM/E,V,T,STA,RA,EP,EPT,GS
      COMMON /PACON/M,N,M1,N1,M2,N2,AM,AN,AM2
      COMMON /REDUN/KI,LI,MI,NI,NOA,NOB,NOC
```

TABLE VII (CONTINUED)

```
      COMMON /DELTA/DELXR,DELXT,DELY,DELLE,DELTE
      COMMON /ARRAY/X(9,9),Y(9),B10,FV
      DIMENSION A(21,6),C(6),Z(21),F(4,4),G(4,4),H(8,8)

      STIFFENER FLEXIBILITIES

    1 DO 2 I=1,NOA
      DO 2 J=1,5
    2 FV(I,J)=0.0

      RIBS
      RFL=DELXR/(RA*E)
      RFU=DELXT/(RA*E)
      DO 3 I=1,N1
      J=N1+I
      FV(I,5)=RFL
    3 FV(J,5)=RFU

      STRINGERS
      STFLE=DELLE/(STA*E)
      STFTE=DELTE/(STA*E)
      K=2*N1+1
      L=2*N1+M1
      DO 4 I=K,L
      J=M1+I
      FV(I,5)=STFLE
    4 FV(J,5)=STFTE

      WEB FLEXIBILITIES

      KA=NOA
      DO 26 IA=1,M1
      DO 26 JA=1,N1
      XA=X(IA,JA)
      XB=X(IA,JA+1)
      XC=X(IA+1,JA)
      XD=X(IA+1,JA+1)
      YA=Y(IA)
      YC=Y(IA+1)
      AR=(YC-YA)/(XB-XA)
      D=(XD-XA)/(XB-XA)
      P=(XC-XA)/(XB-XA)
      TR=1./(D-P)
      B=D-1.
      DO 5 I=1,21
      DO 5 J=1,6
    5 A(I,J)=0.0
      A(1,1)=AR
      A(1,2)=2.*AR
      A(1,3)=AR
      A(2,2)=-TR*AR
      A(2,3)=A(2,2)
```

TABLE VII (CONTINUED)

```
A(3,3)=TR**2*AR
A(4,1)=V
A(4,2)=V
A(4,4)=V
A(4,5)=V
A(5,4)=-V
A(5,5)=-V
A(6,1)=-D*TR*V
A(6,2)=A(6,1)
A(6,4)=-TR*V
A(6,5)=A(6,4)
A(7,1)=P*TR*V
A(7,2)=A(7,1)
A(7,4)=-A(6,4)
A(7,5)=-A(6,4)
A(8,2)=A(6,4)
A(8,5)=A(6,4)
A(9,5)=-A(6,4)
A(10,2)=D*TR**2*V
A(10,5)=TR**2*V
A(11,2)=-P*TR**2*V
A(11,5)=-A(10,5)
A(12,1)=1./AR
A(12,4)=2./AR
A(12,6)=A(12,1)
A(13,4)=-A(12,1)
A(13,6)=-A(12,1)
A(14,1)=-D*TR/AR
A(14,4)=-(1.+D)*TR/AR
A(14,6)=-TR/AR
A(15,1)=P*TR/AR
A(15,4)=(1.+P)*TR/AR
A(15,6)=TR/AR
A(16,6)=A(12,1)
A(17,4)=-A(14,1)
A(17,6)=TR/AR
A(18,4)=-A(15,1)
A(18,6)=-TR/AR
A(19,1)=(D*TR)**2/AR
A(19,4)=2.*D*TR**2/AR
A(19,6)=TR**2/AR
A(20,1)=-D*P*TR**2/AR
A(20,4)=-(D+P)*TR**2/AR
A(20,6)=-A(19,6)
A(21,1)=(P*TR)**2/AR
A(21,4)=2.*P*TR**2/AR
A(21,6)=A(19,6)
C(1)=1.+0.5*(B-P)
C(2)=-0.5-(B-P)/3.
C(3)=(1./3.)+((B-P)/4.)
C(4)=-0.5-B/2.-(B**2-P**2)/6.
C(5)=0.25+B/3.+(B**2-P**2)/8.
```

TABLE VII (CONTINUED)

```
      C(6)=1./3.+B/2.+B**2/3.+(B**3-P**3)/12.
      DO 6 I=1,21
    6 Z(I)=0.0
      DO 7 I=1,21
      DO 7 J=1,6
    7 Z(I)=Z(I)+A(I,J)*C(J)
      F(1,1)=Z(1)
      F(1,2)=-Z(1)
      F(1,3)=Z(2)
      F(1,4)=-Z(2)
      F(2,2)=Z(1)
      F(2,3)=-Z(2)
      F(2,4)=Z(2)
      F(3,3)=Z(3)
      F(3,4)=-Z(3)
      F(4,4)=Z(3)
      G(1,1)=Z(12)
      G(1,2)=Z(13)
      G(1,3)=Z(14)
      G(1,4)=Z(15)
      G(2,2)=Z(16)
      G(2,3)=Z(17)
      G(2,4)=Z(18)
      G(3,3)=Z(19)
      G(3,4)=Z(20)
      G(4,4)=Z(21)
      DO 8 I=1,4
      DO 8 J=I,4
    8 H(I,J)=EPT*(F(I,J)+(1.-V)*G(I,J)/2.)
      DO 9 I=1,4
      DO 9 J=I,4
    9 H(I+4,J+4)=EPT*(G(I,J)+(1.-V)*F(I,J)/2.)
      F(1,1)=Z(4)
      F(1,2)=Z(5)
      F(1,3)=Z(6)
      F(1,4)=Z(7)
      F(2,1)=-Z(4)
      F(2,2)=-Z(5)
      F(2,3)=-Z(6)
      F(2,4)=-Z(7)
      F(3,1)=Z(8)
      F(3,2)=Z(9)
      F(3,3)=Z(10)
      F(3,4)=Z(11)
      F(4,1)=-Z(8)
      F(4,2)=-Z(9)
      F(4,3)=-Z(10)
      F(4,4)=-Z(11)
      DO 10 I=1,4
      DO 10 J=1,4
   10 G(I,J)=F(J,I)
```

TABLE VII (CONTINUED)

```
   DO 11 I=1,4
   DO 11 J=1,4
11 H(I,J+4)=EPT*(F(I,J)+(1.-V)*G(I,J)/(2.*V))
   DO 12 I=1,8
   DO 12 J=I,8
12 H(J,I)=H(I,J)

   MAKING THE WEB STIFFNESS MATRIX NONSINGULAR

   DO 13 I=1,8
   DO 13 J=3,8
13 H(I,J-1)=H(I,J)
   DO 14 I=1,8
   DO 14 J=6,7
14 H(I,J-2)=H(I,J)
   DO 15 I=3,8
   DO 15 J=1,5
15 H(I-1,J)=H(I,J)
   DO 16 I=6,7
   DO 16 J=1,5
16 H(I-2,J)=H(I,J)

   INVERTING THE WEB STIFFNESS MATRIX

   H(1,6)=1.0
   DO 17 I=1,5
17 H(I+1,6)=0.0
   DO 24 K=1,5
   DO 18 J=1,5
18 H(6,J)=H(1,J+1)/H(1,1)
   DO 23 I=2,5
   IF(5-K-I+2)19,19,20
19 CONST=H(I,1)
   GO TO 21
20 CONST=-H(I,1)
21 JCOL=I-1
   DO 22 J=JCOL,5
22 H(I-1,J)=H(I,J+1)+CONST*H(6,J)
   DO 23 J=1,JCOL
23 H(I,J)=H(J,I)
24 H(5,5)=H(6,5)

   TRANSMITTING THE WEB FLEXIBILITY TO FV

   DO 25 I=1,5
   KA=KA+1
   DO 25 J=1,5
25 FV(KA,J)=H(I,J)

26 CONTINUE
   RETURN
   END
```

TABLE VII (CONTINUED)

```
      SUBROUTINE B1

      ISOTHERMAL MATRIX FORCE ANALYSIS OF A TRAPEZOIDAL PANEL
      SUBROUTINE TO GENERATE THE UNIT REDUNDANT MATRIX

      TYPE DECLARATION STATEMENTS

      DOUBLE PRECISION B10(352),FV(352,5)

      DIMENSION AND COMMON STATEMENTS


      COMMON /PACON/M,N,M1,N1,M2,N2,AM,AN,AM2
      COMMON /REDUN/KI,LI,MI,NI,NOA,NOB,NOC
      COMMON /DELTA/DELXR,DELXT,DELY,DELLE,DELTE
      COMMON /ARRAY/X(9,9),Y(9),B10,FV

      REWIND 1

      TYPE A REDUNDANTS

      LOWER RIB
    1 DO 3 I=1,N1
      DO 2 IJ=1,LI
    2 B10(IJ)=0.0
      K=I
      B10(K)=1.0
      K=NOA+5*(I-1)+1
      B10(K)=1.0
    3 WRITE(1)B10

      UPPER RIB
      DO 5 I=1,N1
      DO 4 IJ=1,LI
    4 B10(IJ)=0.0
      K=N1+I
      B10(K)=1.0
      K=NOA+5*(N1*M2)+5*(I-1)+2
      B10(K)=1.0
      B10(K+1)=-1.0
    5 WRITE(1)B10

      LEADING EDGE STRINGER
      ALPHA=ATAN((Y(M)-Y(1))/(X(M,1)-X(1,1)))
      DO 7 I=1,M1
      DO 6 IJ=1,LI
    6 B10(IJ)=0.0
      K=2*N1+I
      B10(K)=1.0
      K=NOA+5*N1*(I-1)+1
      B10(K)=COS(ALPHA)
      B10(K+1)=-B10(K)
```

TABLE VII (CONTINUED)

```
      B10(K+3)=-SIN(ALPHA)
    7 WRITE(1)B10

      TRAILING EDGE STRINGER
      BETA=ATAN((Y(M)-Y(1))/(X(M,N)-X(1,N)))
      DO 9 I=1,M1
      DO 8 IJ=1,LI
    8 B10(IJ)=0.0
      K=2*N1+M1+I
      B10(K)=1.0
      K=NOA+5*N1*I-2
      B10(K)=-COS(BETA)
      B10(K+2)=-SIN(BETA)
    9 WRITE(1)B10

      TYPE B REDUNDANTS

      IF(NOB)22,22,10

      HORIZONTAL
   10 IF(M2)14,14,11

   11 J=N1*M2
      DO 13 I=1,J
      DO 12 IJ=1,LI
   12 B10(IJ)=0.0
      K=NOA+5*(I-1)+2
      B10(K)=-1.0
      B10(K+1)=1.0
      K=NOA+5*N1+5*(I-1)+1
      B10(K)=1.0
   13 WRITE(1)B10

      DIAGONAL
   14 IF(N2)18,18,15

   15 DO 17 I=1,N2
      DO 17 J=1,M1
      GAMMA=ATAN((Y(M)-Y(1))/(X(M,I+1)-X(1,I+1)))
      DO 16 IJ=1,LI
   16 B10(IJ)=0.0
      K=NOA+5*(I-1)+5*N1*(J-1)+3
      B10(K)=COS(GAMMA)
      B10(K+2)=SIN(GAMMA)
      B10(K+3)=B10(K)
      B10(K+4)=-B10(K)
      B10(K+6)=-B10(K+2)
   17 WRITE(1)B10
```

TABLE VII (CONTINUED)

```
   TYPE C REDUNDANTS

18 IF(NOC)22,22,19

19 DO 21 I=1,N2
   DO 21 J=1,M2
   DOL=(X(J+1,2)-X(J+1,1))/DELY
   DO 20 IJ=1,LI
20 B10(IJ)=0.0
   K=NOA+5*(I-1)+5*N1*(J-1)+3
   B10(K)=-DOL
   B10(K+1)=1.0
   B10(K+2)=-1.0
   B10(K+3)=-DOL
   B10(K+4)=DOL
   B10(K+6)=-1.0
   B10(K+7)=1.0
   K=NOA+5*(I-1)+5*N1*J+3
   B10(K)=DOL
   B10(K+3)=DOL
   B10(K+4)=-DOL
21 WRITE(1)B10

22 RETURN
   END


   SUBROUTINE B0

   ISOTHERMAL MATRIX FORCE ANALYSIS OF A TRAPEZOIDAL PANEL
   SUBROUTINE TO GENERATE THE UNIT EXTERNAL FORCE MATRIX

   TYPE DECLARATION STATEMENTS
   DOUBLE PRECISION B10(352),FV(352,5)

   DIMENSION AND COMMON STATEMENTS

   COMMON /PACON/M,N,M1,N1,M2,N2,AM,AN,AM2
   COMMON /REDUN/KI,LI,MI,NI,NOA,NOB,NOC
   COMMON /DELTA/DELXR,DELXT,DELY,DELLE,DELTE
   COMMON /ARRAY/X(9,9),Y(9),B10,FV

   F1=1

 1 DO 2 IJ=1,LI
 2 B10(IJ)=0.0
   DO 3 K=1,N1
 3 B10(K)=-1.0
   WRITE(1)B10
```

TABLE VII (CONTINUED)

```
      F3=1

      NUMB=0
      CONST=1.0
      DO 4 IJ=1,LI
    4 B10(IJ)=0.0
    5 IF(M+N-5)12,11,6
    6 IF(M+N-6)9,9,7

      LEADING EDGE COLUMN OF WEB ELEMENTS
    7 DO 8 I=2,M2
      K=NOA+5*N1*(I-1)+3
      B10(K)=1.0*CONST
      AMI=M1-I
      B10(K+1)=((AMI*DELY)/(X(I+1,2)-X(I+1,1)))*CONST
    8 B10(K+2)=-B10(K+1)

      LOWER ROW OF WEB ELEMENTS
    9 DO 10 J=2,N1
      K=NOA+5*(J-1)+1
      ANJ=N-J
      B10(K)=(1.0-(ANJ*AM/AN))*CONST
      B10(K+1)=(ANJ*AM/AN)*CONST
      ANJ1=ANJ-1.0
   10 B10(K+2)=-(ANJ1*AM/AN)*CONST

      WEB ELEMENT 1,1
   11 K=NOA+3
      B10(K)=-(AM2-(AM/AN))*CONST
      B10(K+1)=(AM2*DELY)/(X(2,2)-X(2,1))*CONST
      B10(K+2)=-B10(K+1)

      ELEMENT M-1,1
   12 K=NOA+5*M2*N1+2
      B10(K)=1.0*CONST
      WRITE(1)B10

      NUMB=NUMB+1
      GO TO(13,16,20,24),NUMB

      F4=1

   13 CONST=1.0
      DO 14 IJ=1,LI
   14 B10(IJ)=0.0

      UPPER RIB
      DO 15 I=1,N1
      K=N1+I
   15 B10(K)=1.0
      GO TO 5
```

TABLE VII (CONTINUED)

```
      F7=1

   16 ALPHA=ATAN((Y(M)-Y(1))/(X(M,1)-X(1,1)))
      CONST=-COS(ALPHA)/SIN(ALPHA)
      DO 17 IJ=1,LI
   17 B10(IJ)=0.0

      LOWER RIB
      DO 18 K=1,N1
   18 B10(K)=CONST

      LEADING EDGE STRINGER
      DO 19 I=1,M1
      K=2*N1+I
   19 B10(K)=1.0/SIN(ALPHA)
      GO TO 5

      F8=1

   20 BETA=ATAN((Y(M)-Y(1))/(X(M,N)-X(1,N)))
      CONST=-COS(BETA)/SIN(BETA)
      DO 21 IJ=1,LI
   21 B10(IJ)=0.0

      UPPER RIB
      DO 22 I=1,N1
      K=N1+I
   22 B10(K)=CONST

      TRAILING EDGE STRINGER
      DO 23 I=1,M1
      K=M1+2*N1+I
   23 B10(K)=1.0/SIN(BETA)
      GO TO 5

   24 RETURN
      END
```

TABLE VIII

DISPLACEMENTS OF CORNER NODES

INCHES PER MILLION POUNDS

MOD NO	FORCE	F1	F3	F4	F7	F8
1	F1	1.3888	.9984	1.2154	-.0291	-.6057
	F3	.9984	5.1241	5.7374	-1.7009	-3.8589
	F4	1.2154	5.7374	6.9985	-2.0824	-4.5745
	F7	-.0291	-1.7009	-2.0824	1.5413	2.2443
	F8	-.6057	-3.8589	-4.5745	2.2443	4.7353
2	F1	1.5600	1.4853	1.4821	-.2945	-.7984
	F3	1.4853	7.2106	7.2776	-2.7165	-5.1708
	F4	1.4821	7.2776	8.2307	-2.8361	-5.6909
	F7	-.2945	-2.7165	-2.8361	2.2355	2.6967
	F8	-.7984	-5.1708	-5.6909	2.6967	5.8844
3	F1	1.7330	1.8883	1.8811	-.3418	-1.0720
	F3	1.8883	8.2240	8.3341	-2.8421	-5.9660
	F4	1.8811	8.3341	9.3802	-2.9737	-6.6161
	F7	-.3418	-2.8421	-2.9737	2.2747	2.8310
	F8	-1.0720	-5.9660	-6.6161	2.8310	6.7349
4	F1	1.8778	2.1979	2.1999	-.4405	-1.2754
	F3	2.1979	9.5208	9.4762	-3.4361	-6.8444
	F4	2.1999	9.4762	10.5417	-3.4391	-7.5126
	F7	-.4405	-3.4361	-3.4391	2.7684	3.1026
	F8	-1.2754	-6.8444	-7.5126	3.1026	7.5506
5	F1	2.0156	2.5335	2.5317	-.5184	-1.4971
	F3	2.5335	10.4365	10.3875	-3.7128	-7.5009
	F4	2.5317	10.3875	11.4832	-3.7038	-8.2207
	F7	-.5184	-3.7128	-3.7038	2.9392	3.3113
	F8	-1.4971	-7.5009	-8.2207	3.3113	8.1391
6	F1	2.1085	2.7236	2.7246	-.5622	-1.6134
	F3	2.7236	11.1111	10.9875	-4.0112	-7.9217
	F4	2.7246	10.9875	12.0989	-3.9437	-8.6655
	F7	-.5622	-4.0112	-3.9437	3.2066	3.4592
	F8	-1.6134	-7.9217	-8.6655	3.4592	8.5133
7	F1	2.1872	2.9076	2.9083	-.6102	-1.7345
	F3	2.9076	11.6118	11.4819	-4.1948	-8.2725
	F4	2.9083	11.4819	12.6040	-4.1187	-9.0348
	F7	-.6102	-4.1948	-4.1187	3.3348	3.5972
	F8	-1.7345	-8.2725	-9.0348	3.5972	8.8062

TABLE IX

WEB STRESSES

PSI PER POUND

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
1	1-1	1	SXX	-.6264	-.1466	-.0201	.0034	-.1141
			SYY	.2410	.6321	.7056	.3783	-.6489
			SXY	-.0619	.6025	.4811	-.1809	-.2790
		2	SXX	-.7272	-.5241	-.4560	.1264	.3215
			SYY	-.0780	-.5623	-.6738	.7674	.7298
			SXY	.1205	.8203	.8436	-.2561	-.4762
		3	SXX	-.3621	-.1055	.2535	-.0185	-.0513
			SYY	.0183	-.5010	-.5315	.7447	.6939
			SXY	.0565	.5501	.5389	-.1684	-.1607
		4	SXX	-.0102	.2638	.9062	-.1475	-.3672
			SYY	.0730	-.5956	-.5693	.7729	.8381
			SXY	.0163	.3083	.2816	-.0904	.1289
		5	SXX	-.0847	-.0150	.5842	-.0566	-.0453
			SYY	-.1626	-1.4781	-1.5885	1.0604	1.8567
			SXY	.1512	.4692	.5494	-.1460	-.0166*
2	1-1	1	SXX	-.7320	-.1091	-.1388	-.1022	-.0443
			SYY	.4165	1.4828	1.4657	-.2122	-1.1401
			SXY	-.1137	.3465	.3663	-.0551	-.1582
		2	SXX	-.9277	-.9103	-.9276	.2675	.6577
			SYY	-.2026	-1.0526	-1.0304	.9581	1.0819
			SXY	.2364	.9642	.9530	-.3516	-.4841
		3	SXX	-.5471	-.1320	-.1819	-.1026	.1817
			SYY	.0482	-.2718	-.2682	.5943	.4627
			SXY	.0851	.5360	.5364	-.1513	-.1592
		4	SXX	-.1784	.5971	.5154	-.4500	-.2512
			SYY	.2612	.3535	.3409	.3021	-.0202
			SXY	-.0446	.1456	.1558	.0308	.1455
		5	SXX	-.3501	-.1057	-.1766	-.1255	.3648
			SYY	-.2821	-1.8710	-1.8491	1.3291	1.9294
			SXY	.2625	.6876	.6706	-.2293	-.1403
	2-1	1	SXX	.0248	.6861	.9679	-.2409	-.6443
			SYY	-.0136	-.3161	-.1495	1.2694	-.2212
			SXY	.0026	.7324	.5294	-.2240	-.2770
		2	SXX	.0308	.6005	.7603	-.3903	-.2777
			SYY	.0052	-.5870	-.8067	.7965	.9390
			SXY	-.0071	.5298	.6194	-.0848	-.3201
		3	SXX	.0175	.3565	.9050	-.1919	-.3866
			SYY	-.0014	-.6280	-.6734	.9223	.7500
			SXY	-.0028	.5196	.5087	-.1754	-.1418
		4	SXX	.0046	.1056	1.0329	-.0056	-.4658
			SYY	-.0066	-.6910	-.5934	1.0096	.6551
			SXY	.0005	.4930	.4054	-.2548	.0328
		5	SXX	.0096	.0339	.8590	-.1307	-.1587
			SYY	.0091	-.9180	-1.1438	.6135	1.6270
			SXY	-.0076	.3233	.4808	-.1382	-.0032

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
3	1-1	1	SXX	-.7611	-.0494	-.1003	-.0957	.0156
			SYY	.5759	1.9918	1.8980	-.3830	-1.3817
			SXY	-.0746	.4908	.5314	-.0291	-.3079
		2	SXX	-.9457	-.6135	-.6369	.0681	.4068
			SYY	-.0080	.2066	.2000	.1355	-.1438
			SXY	.1319	.8591	.8718	-.1725	-.5027
		3	SXX	-.5677	.0383	-.0358	-.1908	.0706
			SYY	.0627	.2638	.2484	.0968	-.1468
			SXY	.0205	.4940	.5236	-.0698	-.2439
		4	SXX	-.2010	.6557	.5324	-.4398	-.2416
			SYY	.0976	.2115	.1928	.0900	-.0740
			SXY	-.0782	.1514	.1963	.0239	.0028
		5	SXX	-.3630	.1607	.0616	-.2960	.1015
			SYY	-.4147	-1.3548	-1.2969	.5450	1.0120
			SXY	.1030	.4746	.4949	-.1018	-.1680
	1-2	1	SXX	-.8579	-.7321	-.7627	.0873	.3957
			SYY	.1727	.2492	.3294	.4530	-.2234
			SXY	-.0523	.3782	.3913	-.2510	-.1830
		2	SXX	-.9662	-1.1888	-1.2254	.3156	.8427
			SYY	-.1701	-1.1961	-1.1349	1.1757	1.1909
			SXY	.1668	.7831	.7730	-.4221	-.3849
		3	SXX	-.5439	-.4010	-.4817	-.0182	.4433
			SYY	-.0274	-.9081	-.8603	1.0507	1.0265
			SXY	.0835	.4477	.4341	-.2554	-.0625
		4	SXX	-.1281	.3588	.2335	-.3382	.0713
			SYY	.0942	-.7087	-.6755	.9699	.9488
			SXY	.0137	.1372	.1186	-.0991	.2474
		5	SXX	-.2232	-.0419	-.1724	-.1378	.4635
			SYY	-.2065	-1.9769	-1.9604	1.6040	2.1898
			SXY	.2060	.4924	.4535	-.2492	.0703
	2-1	1	SXX	.1193	.8975	1.1045	-.1756	-.7032
			SYY	.0326	-.2986	-.2364	1.3669	-.0903
			SXY	.0464	.9580	.7493	-.2847	-.5110
		2	SXX	.0842	.7626	.9223	-.2210	-.4700
			SXY	.0400	.8567	.8028	-.2375	-.5642
			SYY	-.0785	-.7254	-.8130	1.2231	.6476
		3	SXX	.0581	.4676	1.0058	-.1146	-.5381
			SYY	-.1152	-.9281	-.9346	1.2198	.8154
			SXY	.0053	.7014	.6472	-.2667	-.3607
		4	SXX	.0290	.1617	1.0745	-.0118	-.5873
			SYY	-.1611	-1.1655	-1.1030	1.2048	1.0432
			SXY	-.0299	.5378	.4960	-.2921	-.1615
		5	SXX	-.0003	.0487	.9219	-.0499	-.3919
			SYY	-.2541	-1.5230	-1.5859	1.0844	1.6614
			SXY	-.0352	.4530	.5408	-.2526	-.2060

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
3	2-2	1	SXX	.1082	.8149	1.0957	-.4389	-.6540
			SYY	-.1104	-.4634	-.2285	.9034	-.0964
			SXY	-.0844	.3098	.2787	-.0814	-.0298
		2	SXX	.1590	.8416	1.0076	-.5330	-.4217
			SYY	.0502	-.3788	-.5073	.6055	.6386
			SXY	-.1178	.1702	.3067	.0127	-.0508
		3	SXX	.0806	.4913	1.0678	-.3060	-.4462
			SYY	.0432	-.4802	-.5191	.6444	.7121
			SXY	-.0743	.1781	.2276	-.0639	.1644
		4	SXX	.0063	.1432	1.1208	-.0866	-.4519
			SYY	.0492	-.5746	-.5535	.6590	.8452
			SXY	-.0335	.1747	.1507	-.1330	.3779
		5	SXX	.0489	.1656	1.0470	-.1655	-.2573
			SYY	.1839	-.5038	-.7870	.4095	1.4610
			SXY	-.0615	.0578	.1741	-.0541	.3604
4	1-1	1	SXX	-.8721	-.3535	-.3546	.0661	.2179
			SYY	.8434	2.2714	2.2676	-.4131	-1.4987
			SXY	-.1475	.4243	.4256	-.0810	-.2470
		2	SXX	-1.1940	-1.1125	-1.1131	.1976	.6948
			SYY	-.1751	-.1305	-.1324	.0029	.0103
			SXY	.1709	.8862	.8883	-.1816	-.4933
		3	SXX	-.8302	-.5228	-.5227	.0764	.3650
			SYY	.1346	.5153	.5131	-.1149	-.3825
			SXY	-.0072	.5274	.5296	-.1151	-.2773
		4	SXX	-.4760	.0443	.0450	-.0409	.0493
			SYY	.4141	1.0897	1.0874	-.2204	-.7305
			SXY	-.1759	.1824	.1847	-.0515	-.0685
		5	SXX	-.7787	-.6695	-.6683	.0827	.4979
			SYY	-.5439	-1.1694	-1.1698	.1708	.6888
			SXY	.1235	.6168	.6198	-.1461	-.3002
	1-2	1	SXX	-1.0327	-1.1211	-1.1224	.1998	.6137
			SYY	.1695	.4008	.4079	.2130	-.2479
			SXY	-.0970	.1362	.1368	-.1874	.0035
		2	SXX	-1.1973	-1.7847	-1.7871	.5861	1.2192
			SYY	-.3512	-1.6992	-1.6952	1.4356	1.6682
			SXY	.2789	.8287	.8278	-.4806	-.3746
		3	SXX	-.8038	-.9619	-.9653	.2091	.7038
			SYY	-.0993	-.9250	-.9206	1.0171	1.0362
			SXY	.1201	.4125	.4117	-.2682	-.0638
		4	SXX	-.4151	-.1588	-.1633	-.1564	.2064
			SYY	.1370	-.2133	-.2085	.6350	.4611
			SXY	-.0274	.0169*	.0161	-.0645	.2356
		5	SXX	-.5699	-.7829	-.7884	.2068	.7759
			SYY	-.3528	-2.1885	-2.1867	1.7848	2.2634
			SXY	.3262	.6682	.6660	-.3403	-.1200

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
4	2-1	1	SXX	-.1349	.7696	.7683	-.2703	-.4467
			SYY	.2804	1.3246	1.3018	-.1371	-1.0716
			SXY	-.0077	.6060	.6098	-.1005	-.3589
		2	SXX	-.3049	.1534	.1588	-.0206	.0473
			SYY	-.2576	-.6252	-.6269	.6530	.4919
			SXY	.1161	.8860	.8940	-.2818	-.5516
		3	SXX	-.1450	.5552	.5638	-.2548	-.2413
			SYY	-.1230	-.1934	-.1973	.4554	.1562
			SXY	.0283	.5973	.6071	-.1530	-.3256
		4	SXX	.0092	.9362	.9482	-.4806	-.5134
			SYY	-.0064	.1726	.1670	.2845	-.1267
			SXY	-.0552	.3180	.3298	-.0303	-.1061
		5	SXX	-.1493	.3617	.3798	-.2478	-.0527
			SYY	-.5082	-1.6456	-1.6314	1.0214	1.3312
			SXY	.0602	.5791	.5948	-.1993	-.2859
	2-2	1	SXX	-.1311	.5522	.5298	-.4785	-.2710
			SYY	-.0334	-.1812	-.1564	.6524	.1163
			SXY	-.1119	.0857	.1042	-.0489	.1118
		2	SXX	-.0989	.3237	.3033	-.3665	.0497
			SYY	.0683	-.9042	-.8732	1.0068	1.1315
			SXY	-.0866	.2152	.2095	-.1363	.0562
		3	SXX	-.0801	.5152	.4688	-.4855	-.0862
			SYY	.0521	-.6867	-.6653	.8923	.8681
			SXY	-.0802	.0949	.0959	-.0716	.1946
		4	SXX	-.0601	.6990	.6266	-.6007	-.2113
			SYY	.0394	-.4936	-.4816	.7897	.6390
			SXY	-.0729	-.0209	-.0142*	-.0098	.3312
		5	SXX	-.0301	.4860	.4154	-.4963	.0877
			SYY	.1343	-1.1679	-1.1500	1.1201	1.5857
			SXY	-.0493	.0998	.0839	-.0913	.2793
	3-1	1	SXX	.1054	1.3729	1.3358	-.6830	-.8677
			SYY	-.0075	.5090	.3511	.4932	-.4948
			SXY	.0136	.5364	.5760	-.0023	-.3522
		2	SXX	.0845	1.0455	1.0483	-.5692	-.5475
			SYY	-.0737	-.5270	-.5586	.8535	.5186
			SXY	-.0053	.5957	.6309	-.0905	-.4027
		3	SXX	.0640	1.1579	1.1505	-.6905	-.6482
			SYY	-.0729	-.3770	-.4258	.7753	.3747
			SXY	-.0124	.4409	.4947	-.0284	-.2522
		4	SXX	.0427	1.2575	1.2414	-.8073	-.7365
			SYY	-.0746	-.2674	-.3285	.7112	.2705
			SXY	-.0202	.2884	.3605	.0302	-.1037
		5	SXX	.0234	.9556	.9763	-.7024	-.4412
			SYY	-.1356	-1.2226	-1.1673	1.0434	1.2048
			SXY	-.0377	.3431	.4112	-.0510	-.1503

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
4	3-2	1	SXX	-.0517	.6562	.6676	-.4398	-.2972
			SYY	-.0387	-.8374	-.5760	1.3022	.3783
			SXY	-.0575	.3435	.3364	-.1882	-.0062
		2	SXX	-.0321	.6377	.5867	-.5577	-.0858
			SYY	.0235	-.8960	-.8319	.9289	1.0472
			SXY	-.0492	.2915	.3100	-.1135	.0386
		3	SXX	-.0259	.5781	.5708	-.4432	-.0736
			SYY	.0142	-.9043	-.7906	1.0326	.9302
			SXY	-.0420	.2936	.2789	-.1817	.1248
		4	SXX	-.0190	.5178	.5518	-.3332	-.0531
			SYY	.0073	-.9148	-.7594	1.1217	.8392
			SXY	-.0345	.2936	.2467	-.2469	.2128
		5	SXX	-.0008	.5008	.4772	-.4420	.1417
			SYY	.0647	-.9688	-.9953	.7775	1.4560
			SXY	-.0268	.2457	.2224	-.1781	.2542
	4-1	1	SXX	.0322	.6008	1.0269	-.0582	-.6989
			SYY	-.0104	-1.3302	-1.1535	2.4807	.5955
			SXY	.0051	.9786	.6801	-.3777	-.4559
		2	SXX	.0302	.7309	1.0709	-.3922	-.6141
			SYY	-.0166	-.9185	-1.0144	1.4240	.8640
			SXY	-.0025	.6814	.6694	-.1570	-.4635
		3	SXX	.0193	.2954	1.0531	-.0561	-.6296
			SYY	-.0198	-1.0758	-1.0266	1.5806	.8463
			SXY	-.0031	.7666	.6934	-.3470	-.4178
		4	SXX	.0082	-.1340	1.0373	.2644	-.6411
			SYY	-.0233	-1.2140	-1.0325	1.6883	.8410
			SXY	-.0041	.8381	.7169	-.5267	-.3724
		5	SXX	.0065	-.0159	1.0772	-.0386	-.5641
			SYY	-.0289	-.8403	-.9062	.7293	1.0846
			SXY	-.0111	.5684	.7072	-.3264	-.3793
	4-2	1	SXX	-.0113	.4423	.9873	-.2469	-.5370
			SYY	-.0066	-.3901	.0412	.6769	-.3182
			SXY	-.0152	.4101	.3937	-.1912	-.0122
		2	SXX	-.0072	.4518	.7672	-.3414	-.1640
			SYY	.0060	-.3600	-.6553	.3777	.8622
			SXY	-.0129	.2681	.5566	-.0795	-.1411
		3	SXX	-.0045	.2455	1.0342	-.1665	-.4041
			SYY	.0052	-.4291	-.4798	.4722	.6317
			SXY	-.0110	.2938	.4154	-.1462	.0767
		4	SXX	-.0015	.0397	1.2910	.0039	-.6271
			SYY	.0050	-.4969	-.3364	.5528	.4558
			SXY	-.0091	.3130	.2817	-.2078	.2886
		5	SXX	.0021	.0483	1.0912	-.0818	-.2885
			SYY	.0166	-.4696	-.9686	.2812	1.5271
			SXY	-.0070	.1841	.4296	-.1064	.1717

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
5	1-1	CG	SXX	-.8236	-.3859	-.3927	.0474	.2788
			SYY	.2073	.6390	.6385	-.1143	-.4286
			SXY	-.0223	.6088	.6099	-.1109	-.3580
	1-2	CG	SXX	-.8938	-.8293	-.8165	.1518	.5649
			SYY	.0268	.1734	.1773	.0466	-.1382
			SXY	-.0642	.2820	.2761	-.0978	-.0934
	1-3	CG	SXX	-.8693	-.9436	-.9477	.1294	.6578
			SYY	-.0769	-.2347	-.2403	.3008	.2198
			SXY	.0182	.2746	.2781	-.1958	-.0370
	1-4	CG	SXX	-.7668	-.9602	-.9779	.1630	.7173
			SYY	-.1697	-1.4325	-1.4286	1.4165	1.5591
			SXY	.1682	.5133	.5154	-.4241	-.1389
	2-1	CG	SXX	-.0294	.8195	.8491	-.2342	-.4432
			SYY	-.0964	.0815	.0917	.2087	-.1369
			SXY	.0271	.6700	.6637	-.1369	-.3910
	2-2	CG	SXX	-.1190	.6201	.5996	-.3575	-.2333
			SYY	-.1678	-.3110	-.3470	.4735	.2555
			SXY	-.0327	.3469	.3785	-.0916	-.1553
	2-3	CG	SXX	-.1031	.5135	.4653	-.4136	-.0804
			SYY	-.1458	-.9369	-.9180	1.0465	.9090
			SXY	-.0755	.2189	.2439	-.1734	-.0116
	2-4	CG	SXX	.0178	.6806	.6202	-.6224	-.1754
			SYY	.1457	-.7865	-.7265	.9829	1.0408
			SXY	-.1515	-.1043	-.1212	-.0179	.3966
	3-1	CG	SXX	.1176	1.4127	1.3537	-.7794	-.8045
			SYY	-.1354	-.1120	-.2376	.5269	.1280
			SXY	.0094	.4546	.5279	.0536	-.3187
	3-2	CG	SXX	.0764	1.0500	1.0507	-.6135	-.5177
			SYY	-.0793	-.8184	-.7693	1.1736	.6768
			SXY	-.0558	.4008	.4505	-.1357	-.2115
	3-3	CG	SXX	.0353	.8139	.8098	-.5579	-.2852
			SYY	.0353	-.8383	-.7027	1.0751	.7766
			SXY	-.0876	.2633	.2848	-.1515	.0284
	3-4	CG	SXX	-.0683	.4346	.3505	-.3783	.1423
			SYY	.0166	-.8667	-.7449	.8611	.9637
			SXY	-.0225	.2340	.1978	-.1638	.2727
	4-1	CG	SXX	.0562	.4377	1.1268	-.0863	-.7275
			SYY	-.0277	-1.2913	-1.2358	1.9851	.9542
			SXY	-.0093	.8807	.7415	-.3960	-.5086
	4-2	CG	SXX	.0199	.3556	1.1372	-.1504	-.6795
			SYY	.0022	-.8044	-.7171	1.1363	.6235
			SXY	-.0164	.6359	.6474	-.3209	-.3476
	4-3	CG	SXX	-.0271	.2291	1.0308	-.1295	-.4993
			SYY	-.0043	-.5459	-.4838	.6447	.5222
			SXY	.0012	.4331	.5537	-.2235	-.1447
	4-4	CG	SXX	.0069	.2755	1.0075	-.1903	-.3209
			SYY	.0015	-.3469	-.4340	.3262	.6364
			SXY	-.0035	.1923	.3381	-.0975	.2339

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
6	1-1	CG	SXX	-1.0783	-.8048	-.8046	.1554	.5162
			SYY	.2634	.6807	.6806	-.1354	-.4358
			SXY	-.0621	.5551	.5553	-.1022	-.3314
	1-2	CG	SXX	-1.1774	-1.3073	-1.3073	.2544	.8365
			SYY	.0378	.1924	.1919	-.0404	-.1548
			SXY	-.1479	.0667	.0679	-.0254	.0288
	1-3	CG	SXX	-1.1463	-1.4548	-1.4562	.2942	.9220
			SYY	-.0645	-.0177	-.0180*	.0265	-.0476
			SXY	-.0032	.1800	.1820	-.1701	.0173
	1-4	CG	SXX	-1.0158	-1.5012	-1.5044	.4219	1.0215
			SYY	-.3150	-1.7174	-1.7135	1.5788	1.7455
			SXY	.2868	.7582	.7567	-.6030	-.3351
	2-1	CG	SXX	-.4067	.2805	.2813	-.0551	-.1191
			SYY	.0548	.4162	.4152	-.0728	-.3230
			SXY	-.0159	.6109	.6115	-.1202	-.3567
	2-2	CG	SXX	-.5596	-.2621	-.2613	.0391	.2692
			SYY	-.1277	-.0600	-.0616	.1233	-.0326
			SXY	-.0717	.2717	.2744	-.1259	-.0943
	2-3	CG	SXX	-.5202	-.4363	-.4386	.0493	.4518
			SYY	-.2624	-.8339	-.8313	.7634	.7100
			SXY	-.0100	.3334	.3369	-.3155	-.1197
	2-4	CG	SXX	-.2054	.0953	.0859	-.3473	.1042
			SYY	.0756	-.9101	-.9020	1.0938	1.1958
			SXY	-.0841	-.0232*	-.0249	-.1205	.2548
	3-1	CG	SXX	-.0792	.7735	.7762	-.2135	-.3838
			SYY	-.0804	.2047	.2015	.0787	-.2306
			SXY	.0171	.6638	.6652	-.1441	-.3996
	3-2	CG	SXX	-.1616	.4300	.4326	-.2107	-.1050
			SYY	-.2148	-.4141	-.4164	.4993	.2803
			SXY	-.0572	.3749	.3812	-.1733	-.1856
	3-3	CG	SXX	-.1188	.4207	.4144	-.3786	-.0726
			SYY	-.1114	-.7414	-.7339	.8581	.7466
			SXY	-.1272	.1012	.1102	-.1285	.0668
	3-4	CG	SXX	-.1244	.2780	.2575	-.3942	.0637
			SYY	.0852	-.8605	-.8417	1.0130	1.1241
			SXY	-.1086	-.0900	-.0935	-.0251	.3733
	4-1	CG	SXX	.0844	1.0691	1.0768	-.4039	-.5736
			SYY	-.1472	-.0373	-.0456	.3482	-.0478
			SXY	.0115	.6402	.6441	-.1233	-.4006
	4-2	CG	SXX	.0243	.8638	.8699	-.4797	-.3960
			SYY	-.1439	-.4661	-.4706	.6940	.3878
			SXY	-.0808	.3131	.3295	-.0871	-.1376
	4-3	CG	SXX	-.0356	.6710	.6588	-.5264	-.2167
			SYY	-.0396	-.7368	-.7154	.9381	.7738
			SXY	-.1117	.1017	.1229	-.0605	.1095
	4-4	CG	SXX	-.0649	.4936	.4441	-.5102	-.0111
			SYY	.0572	-.9075	-.8634	.9995	1.1117
			SXY	-.0695	.0190	.0115	-.0596	.3544

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
6	5-1	CG	SXX	.1141	1.1987	1.2189	-.5663	-.7052
			SYY	-.1179	-.1619	-.1848	.5780	.0885
			SXY	-.0031	.5656	.5765	-.0553	-.3603
	5-2	CG	SXX	.0442	.9870	1.0116	-.6056	-.5082
			SYY	-.0773	-.5276	-.5327	.8674	.4762
			SXY	-.0604	.3056	.3463	-.0321	-.1270
	5-3	CG	SXX	-.0091	.8605	.8256	-.6739	-.3138
			SYY	-.0108	-.8012	-.7488	1.0256	.8055
			SXY	-.0667	.1688	.2204	-.0577	.0809
	5-4	CG	SXX	-.0308	.6572	.5709	-.5928	-.0501
			SYY	.0298	-.9906	-.8873	1.0192	1.1038
			SXY	-.0355	.1703	.1504	-.1522	.2964
	6-1	CG	SXX	.0736	1.1618	1.2350	-.6191	-.7559
			SYY	-.0662	-.2690	-.3270	.7989	.2227
			SXY	-.0050	.4914	.5219	.0192	-.3261
	6-2	CG	SXX	.0288	1.1442	1.1660	-.8030	-.6323
			SYY	-.0341	-.6305	-.6392	1.0385	.5723
			SXY	-.0331	.2944	.3987	.0018*	-.1531
	6-3	CG	SXX	-.0022	.8547	.8602	-.6282	-.3109
			SYY	-.0019	-1.0081	-.8421	1.2087	.8589
			SXY	-.0316	.3550	.4177	-.2158	-.0434
	6-4	CG	SXX	-.0131	.6325	.5751	-.5018	.0074
			SYY	.0132	-.9159	-.7642	.8662	.9782
			SXY	-.0153	.2962	.2667	-.2318	.2482
	7-1	CG	SXX	.0354	1.3732	1.4513	-.9410	-.9509
			SYY	-.0292	-.3749	-.5501	1.0224	.4077
			SXY	-.0033	.4017	.4858	.0905	-.3112
	7-2	CG	SXX	.0122	.7887	1.0655	-.4689	-.5694
			SYY	-.0131	-1.1807	-.9964	1.6046	.8321
			SXY	-.0138	.6482	.7096	-.3875	-.3939
	7-3	CG	SXX	-.0009	.5145	.8085	-.3044	-.2522
			SYY	.0007	-.8752	-.6373	.9543	.6585
			SXY	-.0124	.5426	.6186	-.3787	-.1955
	7-4	CG	SXX	-.0048	.3947	.5173	-.2615	.1254
			SYY	.0051	-.5563	-.4533	.4571	.6847
			SXY	-.0057	.2910	.3192	-.1956	.2463
	8-1	CG	SXX	.0108	.0983	1.0286	.1432	-.6928
			SYY	-.0121	-1.8523	-1.6592	2.6804	1.2449
			SXY	-.0003	1.1391	.9266	-.7136	-.6527
	8-2	CG	SXX	.0039	.1027	1.1808	.0036	-.7377
			SYY	-.0022	-.6916	-.5452	.8638	.4573
			SXY	-.0053	.7838	.8155	-.5225	-.4923
	8-3	CG	SXX	-.0000	.1157	1.2223	-.0470	-.6610
			SYY	.0009	-.2893	-.2088	.2762	.2566
			SXY	-.0045	.4281	.6488	-.2587	-.2444
	8-4	CG	SXX	-.0011	.1224	1.1058	-.0672	-.3807
			SYY	.0011	-.1383	-.2949	.0882	.4890
			SXY	-.0018	.1535	.4120	-.0778	.1986

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
7	1-1	CG	SXX	-1.0205	-.6065	-.6065	.1154	.3892
			SYY	.3103	.7822	.7822	-.1492	-.4960
			SXY	-.0491	.6647	.6647	-.1242	-.4105
	1-2	CG	SXX	-1.1565	-1.0652	-1.0649	.2014	.6816
			SYY	.1099	.3403	.3402	-.0637	-.2280
			SXY	-.1959	.1775	.1776	-.0328	-.0886
	1-3	CG	SXX	-1.1982	-1.2956	-1.2959	.2433	.8290
			SYY	.0186	.1345	.1341	-.0174	-.1062
			SXY	-.2082	.0010	.0016	-.0083	.0493
	1-4	CG	SXX	-1.1973	-1.4185	-1.4180	.2718	.9059
			SYY	-.0282	.0205	.0203	.0208	-.0382
			SXY	-.1569	-.0167*	-.0159	-.0331	.0946
	1-5	CG	SXX	-1.1750	-1.4785	-1.4790	.2914	.9432
			SYY	-.0600	-.0699	-.0703	.0684	.0241
			SXY	-.0690	.0686	.0701	-.1070	.0744
	1-6	CG	SXX	-1.1333	-1.4964	-1.4979	.3111	.9563
			SYY	-.0949	-.2062	-.2059	.1699	.1409
			SXY	.0496	.2626	.2642	-.2542	-.0273
	1-7	CG	SXX	-1.0536	-1.4465	-1.4488	.3155	.9331
			SYY	-.1615	-.5754	-.5739	.4916	.5112
			SXY	.2043	.6071	.6074	-.5174	-.2592
	1-8	CG	SXX	-.9041	-1.3256	-1.3286	.3310	.9084
			SYY	-.3965	-2.2741	-2.2702	2.0747	2.3349
			SXY	.3811	1.0465	1.0444	-.8458	-.5730
	2-1	CG	SXX	-.2375	.6988	.6993	-.1402	-.3946
			SYY	.0621	.4308	.4304	-.0709	-.3165
			SXY	-.0117	.6877	.6878	-.1299	-.4208
	2-2	CG	SXX	-.4160	.2187	.2187	-.0568	-.0733
			SYY	-.0477	.1610	.1598	.0087	-.1620
			SXY	-.1012	.3608	.3615	-.0797	-.1901
	2-3	CG	SXX	-.5137	-.1037	-.1017	.0026	.1513
			SYY	-.1172	-.0390	-.0395	.1001	-.0377
			SXY	-.1130	.2208	.2216	-.0856	-.0729
	2-4	CG	SXX	-.5513	-.2961	-.2960	.0267	.3011
			SYY	-.1672	-.2207	-.2227	.2141	.0996
			SXY	-.0848	.1915	.1944	-.1339	-.0300
	2-5	CG	SXX	-.5346	-.3863	-.3876	.0279	.3920
			SYY	-.2094	-.4585	-.4579	.4054	.3152
			SXY	-.0413	.2417	.2451	-.2310	-.0487
	2-6	CG	SXX	-.4572	-.3540	-.3568	-.0186	.4069
			SYY	-.2440	-.8215	-.8179	.7381	.7035
			SXY	-.0087	.3163	.3190	-.3492	-.0980
	2-7	CG	SXX	-.3029	-.1358	-.1417	-.1704	.2886
			SYY	-.2259	-1.3550	-1.3488	1.2973	1.3788
			SXY	-.0318	.2671	.2679	-.3748	-.0532
	2-8	CG	SXX	-.0269	.4968	.4871	-.6313	-.1953
			SYY	.2211	-.8226	-.8155	1.1117	1.2652
			SXY	-.1734	-.2873	-.2901	.0424	.4798

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
	3-1	CG	SXX	.0024	1.0032	1.0041	-.2527	-.5456
			SYY	-.0427	.3084	.3063	.0263	-.2912
			SXY	.0266	.7270	.7274	-.1381	-.4506
	3-2	CG	SXX	-.0845	.7246	.7296	-.2140	-.3417
			SYY	-.1371	.0174	.0157	.1686	-.1010
			SXY	-.0300	.5135	.5140	-.1258	-.2926
	3-3	CG	SXX	-.1342	.5444	.5445	-.2224	-.1933
			SYY	-.1955	-.2270	-.2328	.3220	.0934
			SXY	-.0576	.3764	.3817	-.1354	-.1874
	3-4	CG	SXX	-.1480	.4212	.4225	-.2358	-.0839
			SYY	-.2232	-.4919	-.4918	.5388	.3396
			SXY	-.0757	.2997	.3055	-.1784	-.1223
	3-5	CG	SXX	-.1295	.3835	.3835	-.2929	-.0356
			SYY	-.2120	-.7464	-.7424	.7813	.6382
			SXY	-.1011	.2175	.2248	-.2042	-.0511
	3-6	CG	SXX	-.0899	.4335	.4259	-.4080	-.0602
			SYY	-.1302	-.9321	-.9237	1.0099	.9499
			SXY	-.1440	.0687	.0771	-.1592	.0895
	3-7	CG	SXX	-.0615	.5084	.4897	-.5463	-.1276
			SYY	.0748	-.7925	-.7771	.9881	1.0176
			SXY	-.1822	-.1638	-.1592	-.0037	.3531
	3-8	CG	SXX	-.1626	.1771	.1501	-.3621	.1415
			SYY	.1246	-.9201	-.8994	1.0575	1.2491
			SXY	-.0832	-.1216	-.1290	.0054*	.4561
	4-1	CG	SXX	.1247	1.1728	1.1809	-.3873	-.6485
			SYY	-.1202	.1178	.1130	.2330	-.1857
			SXY	.0285	.7072	.7072	-.1224	-.4515
	4-2	CG	SXX	.0787	1.0467	1.0460	-.4299	-.5337
			SYY	-.1703	-.1402	-.1534	.3975	.0308
			SXY	-.0202	.5263	.5342	-.1031	-.3188
	4-3	CG	SXX	.0478	.8971	.9095	-.4225	-.4166
			SYY	-.1824	-.4114	-.4124	.6216	.2784
			SXY	-.0615	.4039	.4103	-.1243	-.2160
	4-4	CG	SXX	.0270	.8200	.8312	-.4763	-.3464
			SYY	-.1552	-.5867	-.5896	.7767	.5018
			SXY	-.0993	.2601	.2764	-.1043	-.0948
	4-5	CG	SXX	.0062	.7814	.7762	-.5574	-.3009
			SYY	-.0863	-.7074	-.7027	.9054	.7047
			SXY	-.1294	.1128	.1397	-.0611	.0491
	4-6	CG	SXX	-.0287	.7100	.6818	-.6015	-.2302
			SYY	.0083	-.7590	-.7331	.9657	.8478
			SXY	-.1329	.0071	.0334	-.0241	.2008
	4-7	CG	SXX	-.0855	.5218	.4721	-.5272	-.0532
			SYY	.0412	-.9102	-.8619	1.0459	1.0608
			SXY	-.0832	.0270	.0364	-.0575	.2898
	4-8	CG	SXX	-.0594	.4785	.4145	-.5145	.0248
			SYY	.0755	-.9735	-.9181	1.0162	1.2158
			SXY	-.0469	.0163*	-.0047	-.0666	.4186

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
	5-1	CG	SXX	.1435	1.2944	1.2974	-.5742	-.7611
			SYY	-.1270	-.0404	-.0646	.4693	-.0406
			SXY	.0136	.6299	.6354	-.0580	-.4169
	5-2	CG	SXX	.1100	1.0953	1.1419	-.5018	-.6248
			SYY	-.1281	-.3154	-.3219	.7019	.2014
			SXY	-.0274	.5163	.5177	-.0822	-.3122
	5-3	CG	SXX	.0776	1.0636	1.0964	-.6087	-.5728
			SYY	-.1034	-.4105	-.4445	.7579	.3638
			SXY	-.0601	.3275	.3639	-.0103	-.1690
	5-4	CG	SXX	.0427	1.0295	1.0320	-.6958	-.5075
			SYY	-.0614	-.5793	-.5921	.9024	.5576
			SXY	-.0793	.2000	.2636	.0102	-.0503
	5-5	CG	SXX	.0049	.9408	.9120	-.7091	-.3962
			SYY	-.0185	-.7602	-.7201	1.0359	.7403
			SXY	-.0787	.1570	.2236	-.0299	.0404
	5-6	CG	SXX	-.0266	.8080	.7510	-.6579	-.2423
			SYY	.0030	-.9271	-.8352	1.1102	.9179
			SXY	-.0588	.1768	.2220	-.1027	.1153
	5-7	CG	SXX	-.0247	.7241	.6415	-.6210	-.1212
			SYY	.0290	-.9854	-.8671	1.0637	1.0384
			SXY	-.0434	.1753	.1808	-.1434	.2310
	5-8	CG	SXX	-.0322	.6034	.4985	-.5558	.0253
			SYY	.0331	-1.0056	-.9015	.9582	1.1698
			SXY	-.0205	.1703	.1255	-.1658	.3649
	6-1	CG	SXX	.0949	1.1117	1.2168	-.4924	-.7529
			SYY	-.0784	-.2286	-.2628	.7806	.1400
			SXY	.0018	.5859	.5782	-.0225	-.3869
	6-2	CG	SXX	.0696	1.2814	1.3202	-.8067	-.8000
			SYY	-.0619	-.2058	-.3284	.7079	.2456
			SXY	-.0216*	.3163	.3972	.1409	-.2200
	6-3	CG	SXX	.0428	1.2314	1.2485	-.8474	-.7136
			SYY	-.0410	-.5795	-.6158	1.0509	.5223
			SXY	-.0345	.2777	.3889	.0539	-.1729
	6-4	CG	SXX	.0179	1.0864	1.0989	-.7711	-.5625
			SYY	-.0220	-.8689	-.7940	1.2496	.7194
			SXY	-.0366	.3248	.4306	-.0958	-.1526
	6-5	CG	SXX	.0008	.9232	.9304	-.6648	-.3886
			SYY	-.0083	-1.0014	-.8441	1.2506	.8241
			SXY	-.0318	.3713	.4518	-.2125	-.1060
	6-6	CG	SXX	-.0045	.7903	.7808	-.5795	-.2238
			SYY	.0072	-.9967	-.8004	1.1121	.8640
			SXY	-.0267	.3695	.4136	-.2583	-.0013
	6-7	CG	SXX	-.0133	.6644	.6235	-.5054	-.0507
			SYY	.0112	-.9416	-.7554	.9300	.9154
			SXY	-.0160	.3335	.3322	-.2526	.1501
	6-8	CG	SXX	-.0115	.5855	.5046	-.4697	.0893
			SYY	.0129	-.8498	-.7329	.7342	1.0084
			SXY	-.0080	.2569	.2023	-.2068	.3508

TABLE IX (CONTINUED)

MOD NO	ELEM NO	PT	STR	F1	F3	F4	F7	F8
	7-1	CG	SXX	.0415	1.6300	1.6180	-1.1781	-1.0974
			SYY	-.0310	.0072*	-.2985	.6724	.1945
			SXY	-.0011	.2564	.3677	.3098	-.2422
	7-2	CG	SXX	.0269	1.2159	1.3482	-.8074	-.8551
			SYY	-.0234	-.9537	-.9530	1.6270	.7318
			SXY	-.0088	.5198	.5987	-.1248	-.3792
	7-3	CG	SXX	.0147	.9179	1.1515	-.5679	-.6612
			SYY	-.0151	-1.1596	-1.0053	1.6497	.8158
			SXY	-.0123	.6515	.7172	-.3657	-.4261
	7-4	CG	SXX	.0066	.7111	1.0006	-.4199	-.4936
			SYY	-.0074	-1.0759	-.8530	1.3650	.7445
			SXY	-.0131	.6683	.7356	-.4474	-.3890
	7-5	CG	SXX	.0018	.5726	.8711	-.3352	-.3327
			SYY	.0002	-.9084	-.6652	1.0345	.6481
			SXY	-.0122	.6026	.6787	-.4226	-.2821
	7-6	CG	SXX	-.0036	.4741	.7317	-.2855	-.1525
			SYY	.0023	-.7515	-.5223	.7572	.5943
			SXY	-.0083	.4938	.5741	-.3449	-.1215
	7-7	CG	SXX	-.0035	.4183	.5808	-.2673	.0485
			SYY	.0043	-.6136	-.4378	.5392	.6056
			SXY	-.0055	.3591	.4224	-.2447	.0986
	7-8	CG	SXX	-.0040	.3799	.3926	-.2611	.2850
			SYY	.0040	-.4970	-.4419	.3723	.7256
			SXY	-.0026	.2216	.2254	-.1483	.3851
	8-1	CG	SXX	.0107	.1783	1.0351	.1027	-.7178
			SYY	-.0121	-2.2405	-2.0236	3.2921	1.5014
			SXY	.0007	1.2325	.9621	-.7712	-.6960
	8-2	CG	SXX	.0070	.1309	1.1107	.0518	-.7457
			SYY	-.0067	-1.3368	-1.1513	1.8699	.8761
			SXY	-.0025	1.0951	.9560	-.7500	-.6599
	8-3	CG	SXX	.0043	.1146	1.1703	.0121	-.7568
			SYY	-.0029	-.8127	-.6531	1.0596	.5228
			SXY	-.0043	.9097	.8878	-.6310	-.5718
	8-4	CG	SXX	.0021	.1142	1.2130	-.0192	-.7477
			SYY	-.0004	-.5124	-.3749	.6062	.3318
			SXY	-.0045	.7140	.7999	-.4811	-.4606
	8-5	CG	SXX	-.0004	.1176	1.2325	-.0413	-.7094
			SYY	.0002	-.3423	-.2280	.3560	.2426
			SXY	-.0035	.5282	.7133	-.3366	-.3377
	8-6	CG	SXX	-.0001	.1274	1.2286	-.0605	-.6371
			SYY	.0009	-.2411	-.1629	.2147	.2267
			SXY	-.0027	.3593	.6279	-.2121	-.1949
	8-7	CG	SXX	-.0010	.1298	1.1742	-.0707	-.5020
			SYY	.0009	-.1771	-.1785	.1319	.3017
			SXY	-.0015	.2150	.5285	-.1160	-.0031
	8-8	CG	SXX	-.0008	.1281	1.0387	-.0754	-.2648
			SYY	.0007	-.1298	-.3424	.0792	.5677
			SXY	-.0006	.0971	.3495	-.0487	.3300

TABLE X

RIB STRESSES

PSI PER POUND

MOD NO	ELEM NO	F1	F3	F4	F7	F8
1	1	-1.0371	-.7456	-.9077	.0217	.4523
	2	.3012	.8513	1.7508	-.5295	-.9935
2	1	-1.1650	-1.1092	-1.1069	.2199	.5962
	2	-.0043	.0930	1.3231	-.1659	-.7220
3	1	-1.2988	-1.3127	-1.2859	.2720	.7875
	2	-1.2898	-1.5077	-1.5237	.2386	.8137
	3	.0654	.2991	1.5449	-.2850	-1.0381
	4	-.0853	.0065*	1.3597	-.0802	-.7668
4	1	-1.4046	-1.4569	-1.4575	.2806	.8972
	2	-1.4003	-1.8259	-1.8283	.3773	1.0078
	3	.0057	-.1361	1.3497	.0174	-.9167
	4	-.0001	.0123	1.6088	-.0256	-.9385
5	1	-1.5724	-1.6827	-1.6814	.3207	1.0488
	2	-1.5040	-1.8490	-1.8531	.3372	1.1397
	3	-1.4554	-1.9435	-1.9390	.3878	1.1408
	4	-1.4895	-2.0930	-2.0894	.5028	1.1429
	5	-.0058	-.2490	1.2596	.1001	-.9049
	6	-.0102	-.0953	1.4414	.0203	-.9971
	7	.0087	.0369	1.6425	-.0426	-1.0722
	8	-.0028	.0351	1.7409	-.0280	-1.0229
6	1	-1.6603	-1.8450	-1.8450	.3476	1.1505
	2	-1.5790	-2.0250	-2.0252	.3808	1.2469
	3	-1.5052	-2.0514	-2.0525	.4080	1.2109
	4	-1.5542	-2.2147	-2.2165	.5428	1.2115
	5	.0035	-.4438	1.1981	.2567	-.8646
	6	.0018	-.1879	1.4586	.0955	-1.0274
	7	.0005	-.0520	1.6721	.0215	-1.1211
	8	.0000	-.0027	1.8431	.0009	-1.1174
7	1	-1.8003	-2.0448	-2.0448	.3854	1.2776
	2	-1.7447	-2.1533	-2.1533	.4053	1.3443
	3	-1.6657	-2.1581	-2.1581	.4053	1.3406
	4	-1.5937	-2.1305	-2.1308	.4006	1.3075
	5	-1.5437	-2.1102	-2.1106	.4059	1.2673
	6	-1.5262	-2.1254	-2.1262	.4356	1.2376
	7	-1.5529	-2.2146	-2.2159	.5166	1.2468
	8	-1.6405	-2.4347	-2.4360	.6905	1.3412
	9	.0028	-.5794	1.0936	.3773	-.7992
	10	.0022	-.3878	1.2634	.2333	-.9171
	11	.0013	-.2449	1.4071	.1341	-1.0098
	12	.0007	-.1414	1.5319	.0690	-1.0814
	13	.0005	-.0692	1.6447	.0287	-1.1354
	14	.0000	-.0248	1.7475	.0076	-1.1708
	15	.0000	-.0009	1.8442	-.0014	-1.1855
	16	-.0000	.0060	1.9293	-.0026	-1.1661

TABLE XI

STRINGER STRESSES

PSI PER POUND

MOD NO	ELEM NO	F1	F3	F4	F7	F8
1	1	-.1271	.7347	.7392	-.0395	-.3056
	2	.0671	-.1189	-.0872	.4108	.8176
2	1	-.2295	.8753	.8818	-.1590	-.5254
	2	.0057	.9481	.9117	.0460	-.4900
	3	.1143	-.3989	-.4093	.5719	1.1012
	4	-.0028	-.1433	-.0669	.2335	.9272
3	1	-.1542	1.0527	1.0587	-.1660	-.6295
	2	.0410	1.0645	1.0499	.0294	-.6207
	3	.0874	-.4928	-.5204	.6082	1.2320
	4	.0029	-.1669	-.1359	.2392	1.0338
4	1	-.2092	1.1507	1.1510	-.2179	-.7154
	2	.0220	1.4017	1.4018	-.2459	-.8925
	3	.0332	1.2204	1.2229	-.0527	-.8000
	4	.0079	.9383	.8753	.2412	-.5814
	5	.1408	-.6707	-.6728	.7662	1.3847
	6	.0262	-.5969	-.6087	.5733	1.3441
	7	.0060	-.2871	-.3269	.2647	1.1973
	8	.0004	-.0514	-.0016	.0570	1.0506
5	1	-.1324	1.3387	1.3388	-.2508	-.8345
	2	.0727	1.5219	1.5218	-.2562	-.9686
	3	.0508	1.2755	1.2798	-.0593	-.8486
	4	.0081	.9428	.8921	.2571	-.6118
	5	.1152	-.7879	-.7885	.8442	1.4991
	6	.0284	-.6106	-.6284	.5684	1.3793
	7	.0048	-.2992	-.3553	.2636	1.2384
	8	.0000	-.0601	-.0465	.0604	1.1143

TABLE XI (CONTINUED)

MOD NO	ELEM NO	F1	F3	F4	F7	F8
6	1	-.1552	1.3999	1.4000	-.2640	-.8742
	2	.0232	1.6038	1.6038	-.3034	-1.0060
	3	.0837	1.6233	1.6230	-.3024	-1.0352
	4	.0827	1.5437	1.5425	-.2544	-1.0116
	5	.0538	1.3929	1.3895	-.1436	-.9377
	6	.0262	1.2087	1.1988	.0191	-.8282
	7	.0102	1.0029	.9777	.2293	-.6883
	8	.0033	.8336	.7348	.4320	-.5251
	9	.1604	-.8260	-.8272	.9229	1.5594
	10	.0773	-.8204	-.8234	.8113	1.5304
	11	.0423	-.7216	-.7287	.6665	1.4621
	12	.0199	-.5884	-.6046	.5115	1.3884
	13	.0082	-.4126	-.4483	.3353	1.3042
	14	.0029	-.2222	-.2838	.1643	1.2271
	15	.0007	-.0754	-.1436	.0490	1.1849
	16	.0000	-.0082	.0132	.0052	1.1409
7	1	-.0916	1.5464	1.5464	-.2914	-.9663
	2	.0661	1.7041	1.7040	-.3207	-1.0695
	3	.1027	1.6687	1.6684	-.3059	-1.0633
	4	.0919	1.5646	1.5636	-.2523	-1.0240
	5	.0586	1.4069	1.4039	-.1440	-.9488
	6	.0271	1.2187	1.2087	.0171	-.8395
	7	.0093	1.0061	.9808	.2329	-.6957
	8	.0026	.8159	.7234	.4586	-.5215
	9	.1362	-.9235	-.9244	.9981	1.6504
	10	.0804	-.8388	-.8413	.8278	1.5562
	11	.0436	-.7312	-.7379	.6705	1.4760
	12	.0192	-.5936	-.6111	.5094	1.3982
	13	.0074	-.4150	-.4543	.3327	1.3133
	14	.0023	-.2270	-.2936	.1672	1.2400
	15	.0005	-.0808	-.1585	.0536	1.2046
	16	.0000	-.0108	-.0121	.0069	1.1755

APPENDIX C

EXPERIMENTAL RESULTS

The experimental results of this investigation consist of the strain per unit load at the seventy-eight strain gages for the three loading conditions $F_3 = 1$, $F_8 = 1$, and $F_7 + F_8 = 1$ (Figure 3). The average strain per unit load for the back-to-back strain gages was also calculated.

As indicated in Chapter V, the loads and strains were recorded at load increments of approximately 500 pounds up to a maximum load of 5,000 pounds. The strain per unit load was calculated for each gage by determining the slope of the least-square straight line through a plot of strain versus load. The slopes were determined analytically using a program written for the IBM 1620. The average strain per unit load for the back-to-back gages was calculated using the individual gage readings rather than averaging the back-to-back strain readings or averaging the slopes of the lines for the back-to-back gages.

The strains per unit load are tabulated in Tables XII through XIV. The strain gage numbering system is shown in Figure 27. All even-numbered gages are on one side of the panel while all odd numbered gages are on the opposite side, It can be seen from the data, that although lateral bending

Odd-numbered gages are
on the opposite side.

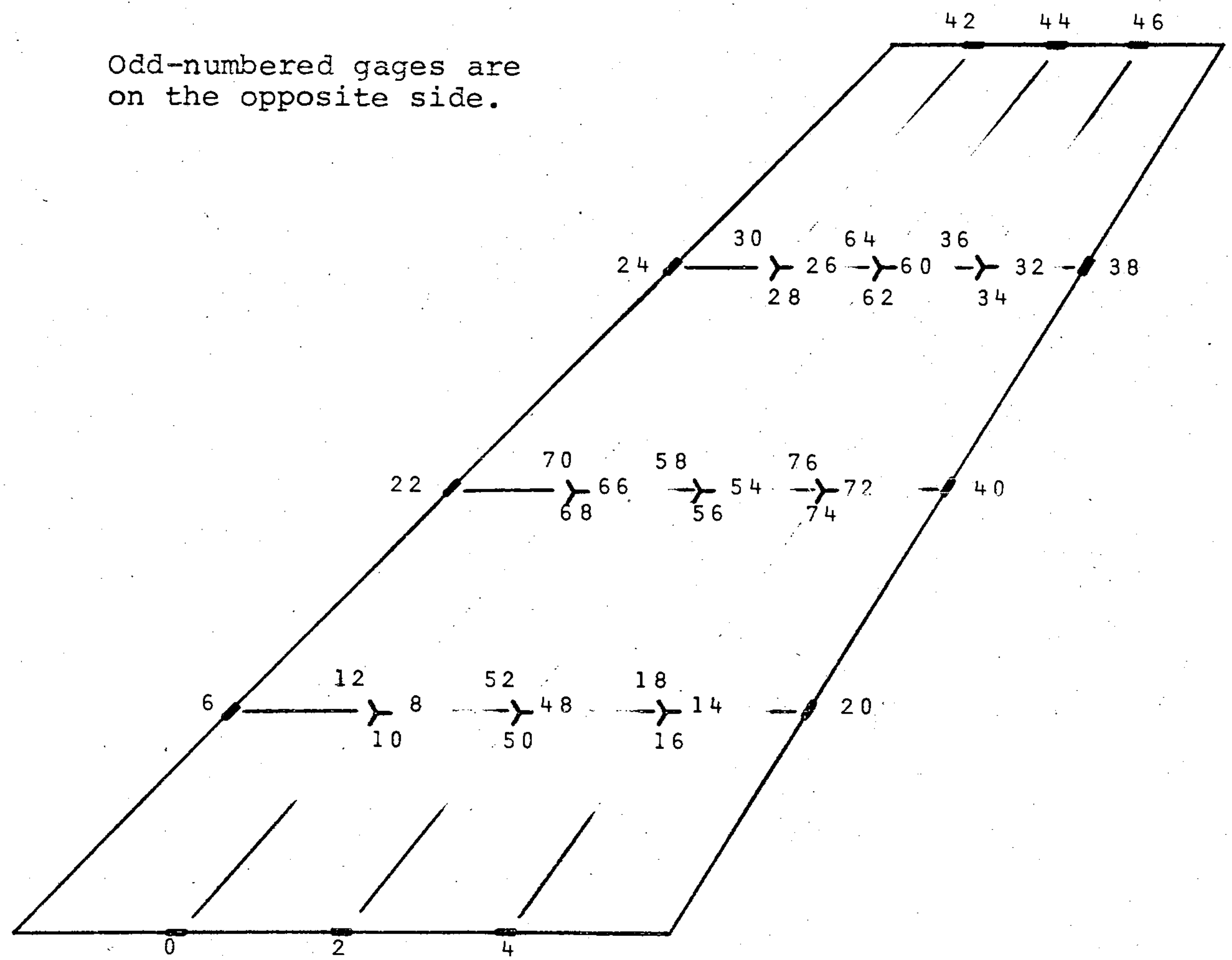


Figure 27. Strain Gage Numbering System

was present, the strains could be reproduced on subsequent runs, with a few exceptions. In Table XIV, the data for the last three rosettes is missing for Run 5-22 because these three rosettes had not been installed at the time of the run. In Table XII, the data for Gages 66-67 is missing because Gage 67 had been damaged beyond repair prior to these runs. Due to the lateral bending, only the average strain per unit load can be compared with the analytical results; therefore, the data for Gage 66 is not included.

TABLE XII

STRAIN PER UNIT LOAD - F3

GAGE NUMBER	RUN NO 6-25	RUN NO 6-26	GAGE NUMBER	RUN NO 6-25	RUN NO 6-26
0	-0.218	-0.218	20	-0.075	-0.074
1	-0.218	-0.217	21	-0.075	-0.076
	-0.218	-0.217		-0.075	-0.075
2	-0.213	-0.213	22	0.156	0.116
3	-0.209	-0.212	23	0.147	0.147
	-0.211	-0.213		0.152	0.132
4	-0.212	-0.211	24	0.110	0.111
5	-0.207	-0.207	25	0.115	0.114
	-0.209	-0.209		0.113	0.112
6	0.178	0.178	26	0.129	0.129
7	0.161	0.161	27	0.131	0.131
	0.170	0.170		0.130	0.130
8	0.036	0.036	28	-0.002	-0.001
9	0.036	0.037	29	-0.002	-0.002
	0.036	0.036		-0.002	-0.002
10	0.034	0.034	30	-0.076	-0.076
11	0.033	0.033	31	-0.079	-0.078
	0.033	0.033		-0.077	-0.077
12	-0.046	-0.046	32	0.093	0.093
13	-0.048	-0.048	33	0.092	0.092
	-0.047	-0.047		0.093	0.093
14	0.061	0.056	34	-0.020	-0.020
15	0.058	0.058	35	-0.018	-0.018
	0.060	0.057		-0.019	-0.019
16	-0.065	-0.064	36	-0.106	-0.105
17	-0.067	-0.067	37	-0.110	-0.111
	-0.066	-0.065		-0.108	-0.108
18	-0.067	-0.067	38	-0.012	-0.012
19	-0.066	-0.067	39	-0.013	-0.014
	-0.067	-0.067		-0.013	-0.013

TABLE XII (CONTINUED)

GAGE NUMBER	RUN NO 6-25	RUN NO 6-26	GAGE NUMBER	RUN NO 6-25	RUN NO 6-26
40	-0.050	-0.050	60	0.120	0.121
41	-0.051	-0.052	61	0.121	0.121
	-0.051	-0.051		0.121	0.121
42	-0.034	-0.034	62	-0.021	-0.021
43	-0.035	-0.035	63	-0.019	-0.019
	-0.034	-0.034		-0.020	-0.020
44	-0.008	-0.008	64	-0.104	-0.104
45	-0.011	-0.011	65	-0.111	-0.111
	-0.010	-0.010		-0.107	-0.108
46	0.002	0.002	66		
47	-0.002	-0.003	67		
	0.000	-0.001			
48	0.031	0.032	68	0.022	0.022
49	0.032	0.032	69	0.020	0.020
	0.032	0.032		0.021	0.021
50	-0.003	-0.003	70	-0.069	-0.066
51	-0.006	-0.006	71	-0.064	-0.064
	-0.004	-0.005		-0.067	-0.065
52	-0.065	-0.065	72	0.095	0.095
53	-0.071	-0.071	73	0.097	0.097
	-0.068	-0.068		0.096	0.096
54	0.105	0.105	74	-0.045	-0.045
55	0.107	0.107	75	-0.048	-0.048
	0.106	0.106		-0.047	-0.047
56	-0.025	-0.025	76	-0.063	-0.063
57	-0.025	-0.025	77	-0.061	-0.061
	-0.025	-0.025		-0.062	-0.062
58	-0.061	-0.061			
59	-0.061	-0.062			
	-0.061	-0.062			

TABLE XIII

STRAIN PER UNIT LOAD - F8

GAGE NUMBER	RUN NO 6-11	RUN NO 6-12	GAGE NUMBER	RUN NO 6-11	RUN NO 6-12
0	0.128	0.129	20	0.142	0.142
1	0.134	0.133	21	0.151	0.151
	0.131	0.131		0.147	0.146
2	0.124	0.123	22	-0.102	-0.102
3	0.123	0.123	23	-0.097	-0.096
	0.123	0.123		-0.099	-0.099
4	0.116	0.115	24	-0.078	-0.078
5	0.111	0.111	25	-0.077	-0.076
	0.113	0.113		-0.078	-0.077
6	-0.105	-0.105	26	-0.085	-0.085
7	-0.101	-0.101	27	-0.085	-0.085
	-0.103	-0.103		-0.085	-0.085
8	-0.006	-0.006	28	0.014	0.014
9	-0.006	-0.006	29	0.014	0.014
	-0.006	-0.006		0.014	0.014
10	-0.021	-0.021	30	0.059	0.059
11	-0.020	-0.021	31	0.060	0.059
	-0.020	-0.021		0.059	0.059
12	0.018	0.017	32	-0.034	-0.034
13	0.017	0.017	33	-0.033	-0.033
	0.018	0.017		-0.033	-0.033
14	-0.028	-0.028	34	0.060	0.060
15	-0.029	-0.028	35	0.057	0.057
	-0.028	-0.028		0.059	0.059
16	0.084	0.084	36	0.054	0.054
17	0.085	0.085	37	0.056	0.056
	0.085	0.084		0.055	0.055
18	0.052	0.051	38	0.119	0.118
19	0.057	0.051	39	0.118	0.117
	0.055	0.051		0.118	0.118

TABLE XIII (CONTINUED)

GAGE NUMBER	RUN NO 6-11	RUN NO 6-12	GAGE NUMBER	RUN NO 6-11	RUN NO 6-12
40	0.133	0.133	60	-0.066	-0.066
41	0.137	0.137	61	-0.066	-0.066
	0.135	0.135		-0.066	-0.066
42	-0.094	-0.094	62	0.035	0.035
43	-0.091	-0.091	63	0.034	0.034
	-0.092	-0.093		0.034	0.034
44	-0.108	-0.105	64	0.071	0.071
45	-0.106	-0.103	65	0.073	0.073
	-0.107	-0.104		0.072	0.072
46	-0.110	-0.110	66	-0.060	-0.060
47	-0.109	-0.109	67	-0.059	-0.059
	-0.110	-0.109		-0.060	-0.060
48	-0.003	-0.003	68	-0.007	-0.007
49	-0.003	-0.003	69	-0.006	-0.006
	-0.003	-0.003		-0.007	-0.007
50	0.013	0.013	70	0.041	0.041
51	0.015	0.015	71	0.039	0.039
	0.014	0.014		0.040	0.040
52	0.036	0.036	72	-0.047	-0.049
53	0.039	0.038	73	-0.045	-0.046
	0.038	0.037		-0.046	-0.048
54	-0.056	-0.056	74	0.089	0.089
55	-0.055	-0.055	75	0.091	0.091
	-0.055	-0.055		0.090	0.090
56	0.046	0.046	76	0.040	0.037
57	0.047	0.046	77	0.034	0.034
	0.046	0.046		0.037	0.036
58	0.040	0.040			
59	0.040	0.040			
	0.040	0.040			

TABLE XIV

STRAIN PER UNIT LOAD - F7 + F8

GAGE NUMBER	RUN NO 5-22	RUN NO 6-04	GAGE NUMBER	RUN NO 5-22	RUN NO 6-04
0	0.085	0.083	20	0.113	0.110
1	0.094	0.091	21	0.110	0.107
	0.090	0.087		0.112	0.109
2	0.086	0.083	22	-0.062	-0.061
3	0.083	0.082	23	-0.051	-0.063
	0.085	0.082		-0.055	-0.062
4	0.089	0.083	24	-0.033	-0.033
5	0.073	0.073	25	-0.036	-0.034
	0.081	0.078		-0.034	-0.034
6	-0.069	-0.067	26	-0.098	-0.098
7	-0.073	-0.070	27	-0.099	-0.099
	-0.071	-0.069		-0.099	-0.098
8	-0.010	-0.009	28	0.051	0.050
9	-0.010	-0.009	29	0.049	0.049
	-0.010	-0.009		0.050	0.050
10	-0.011	-0.011	30	0.063	0.062
11	-0.011	-0.011	31	0.065	0.065
	-0.011	-0.011		0.064	0.064
12	0.018	0.019	32	-0.054	-0.053
13	0.021	0.021	33	-0.053	-0.053
	0.020	0.020		-0.053	-0.053
14	-0.047	-0.047	34	0.047	0.047
15	-0.049	-0.049	35	0.045	0.044
	-0.048	-0.048		0.046	0.045
16	0.067	0.067	36	0.079	0.079
17	0.068	0.068	37	0.080	0.080
	0.067	0.068		0.080	0.080
18	0.069	0.069	38	0.061	0.061
19	0.068	0.067	39	0.066	0.066
	0.069	0.068		0.063	0.062

TABLE XIV (CONTINUED)

GAGE NUMBER	RUN NO 5-22	RUN NO 6-04	GAGE NUMBER	RUN NO 5-22	RUN NO 6-04
40	0.089	0.088	60		-0.086
41	0.089	0.088	61		-0.087
	0.089	0.088			-0.087
42	-0.035	-0.035	62		0.051
43	-0.042	-0.041	63		0.050
	-0.039	-0.038			0.050
44	-0.052	-0.052	64		0.090
45	-0.057	-0.055	65		0.093
	-0.055	-0.054			0.092
46	-0.054	-0.055	66		-0.068
47	-0.060	-0.058	67		-0.066
	-0.057	-0.057			-0.067
48	-0.017	-0.017	68		0.015
49	-0.018	-0.017	69		0.015
	-0.017	-0.017			0.015
50	0.012	0.011	70		0.044
51	0.012	0.010	71		0.043
	0.012	0.011			0.043
52	0.047	0.047	72		-0.070
53	0.050	0.050	73		-0.071
	0.049	0.048			-0.070
54	-0.071	-0.071	74		0.077
55	-0.073	-0.073	75		0.078
	-0.072	-0.072			0.077
56	0.052	0.053	76		0.056
57	0.053	0.053	77		0.052
	0.052	0.053			0.054
58	0.052	0.051			
59	0.052	0.052			
	0.052	0.052			

VITA

Everett L. Cook

Candidate for the Degree of

Doctor of Philosophy

Thesis: AN ANALYTICAL AND EXPERIMENTAL INVESTIGATION OF A MONOLITHIC TRAPEZOIDAL SHEAR PANEL SUBJECTED TO MECHANICAL LOADING

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born at Mounds, Oklahoma, September 9, 1927, the son of Peter Everett and Nona Mae Cook.

Education: Attended grade schools in Skiatook, Tulsa, Drumright, and Miami, Oklahoma and Baxter Springs, Kansas; graduated from Miami High School in 1945; received Associate in Arts degree from Spartan College in June, 1950; received Bachelor of Science in Aeronautical Engineering from Wichita State University in May, 1954; received Master of Science in Aeronautical Engineering from Wichita State University in May, 1958; completed requirements for the Doctor of Philosophy degree in May, 1967.

Professional Experience: Designer, Technical Training Aids, Inc., Tulsa, Oklahoma, 1950; Instructor in Aeronautical Engineering, Spartan College, 1951; Stress Analyst, Beech Aircraft Corporation, 1951 to 1953; Associate Professor of Aeronautical Engineering, Wichita State University, 1953 to present; Director of Digital Computing, Wichita State University, 1964 to present.

Professional and Honorary Organizations: Tau Beta Pi and Sigma Gamma Tau.

Honors and Awards: National Science Foundation Science Faculty Fellowship, 1962-1963.