VARIABLES AFFECTING THE PRECISION OF PREDICTION FACTORS USED IN EXTENDING INCOMPLETE LACTATION RECORDS

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FACTORS USED IN EXTENDING INCOMPLETE

LACTATION RECORDS

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INTRODUCTION

The ability to predict, with reasonable precision, the anticipated 305-day production of a cow's record that is still in progress affects the success of herd culling programs. Culling is usually based either on the last available completed lactation record or on the average of all completed records. Partial records are rarely effectively utilized in the decision-making process. Extrapolation factors, used to predict completed records from partial lactation records, could help improve accuracy in selection by utilizing all available information and reducing the generation interval.

At present, selection of a bull on the performance of his offspring is delayed until several of his daughters have completed at least one record. Extrapolation of records still in progress could reduce the time required to obtain a preliminary evaluation of a sire by up to nine months.

Further, records terminated by removal of a cow from the herd before the record is completed should be extended to a complete lactation basis and used in the sire's summary to enhance the accuracy of the measure of his genetic worth.

Other possible uses of prediction factors in Dairy Herd Improvement Association (DHIA) programs include economy of testing realized by sampling less frequently and in promotional efforts designed to demonstrate the value of DHIA records.

An objective of this study was to determine the importance of variables affecting the precision of prediction factors used in extending incomplete lactation records. Factors which are thought to influence monthly records are lactation number (or age of cow), season of freshening, peak milk production and days open (parturition to last reported breeding date).

Since no studies of this nature using cows from relatively high producing herds under conditions common to California have been conducted, there is a need for undertaking this study.

LITERATURE REVIEW

Value of Extrapolated Records

VanVleck (37) and Erb et al. (5) suggested that by using extrapolation factors in extending partial records it would be possible to obtain economy of testing. However, most of the work reported (21, 24, 32, 35, 38, 40, 41, 43) involving correction factors has emphasized the need for adjustments to increase the accuracy of sire summaries.

A few workers (14, 17, 37, 39, 42) have suggested that a dairy farmer might profitably use extrapolated records in selecting cows to be retained for breeding purposes. In addition, Appleman (1) has shown how dairymen have used extrapolated records in making management decisions.

Variables Affecting the Precision of Predicting Total Milk and Fat Production

Production records of dairy cows are influenced by many environmental factors. The effects of these same variables on the relationship of total production during a lactation to production during various portions of the lactation are of primary concern in developing factors for estimating total yield for the lactation from incomplete records.

Age and Lactation Number

Eldridge and Atkeson (4) developed regression factors for estimating total yield of butterfat from one day's test for 12 different age groups. Considerable variation between age groups was observed, especially in the young cows. By regrouping the younger cows according to their lactation number, they found that the factors thus derived more accurately extended the incomplete record to a complete lactation. Erb et al. (5) found a very marked difference between young and old cows with respect to the shape of the lactation curve. They divided records into three age groups ($\leq$ 30 mo., 31-42 mo., and $\geq$ 43 mo.) to develop factors for predicting 305-day production of milk and fat from a single test. Madden and co-workers (19, 20, 21) also showed a difference in the shape of the lactation curve for first-lactation cows as compared to older cows. They concluded that separate factors were needed for records initiated at less than three years and for those started at three years and older.

Lamb (16) has reported that Kendrick and Harvey, in separate studies, separated records into three age groups roughly corresponding to first, second, and all later lactations, respectively. Differences between factors for the different age groups indicated a need for separate factors for each age group.

Fritz et al. (9) studied the influence of age and lactation number, but failed to show any influence on regression factors attributable to either of them. Lamb and McGilliard (17, 18) studied the influence of both age and lactation number on the ratio of total production in 305 days to both single and cumulative monthly production of milk and fat. Both age and lactation number were found to be important. Analyses of components of variance indicated that lactation number accounted for more of the variability in the total to part lactation relationship than did age at freshening. Even though no practical differences between the factors for age and the factors for lactation number were evident, these workers concluded that in actual practice factors based on age were the more desirable.

Searle (30) found that New Zealand monthly fat records needed to be corrected for age. Separate factors were developed for two-, three-, and four-year-old cows. VanVleck and Henderson (41) divided New York Holstein data into 60 age groups and found a significant effect of age on ratio factors for extending individual and cumulative monthly tests to a complete lactation basis, but concluded that six-month age intervals may be satisfactory and practical for use in extending records.

More recently, Lamb (16) studied the effect of 42 age groups ($<$ 20 mo, 1 month intervals from 20 mo through 59 mo, 60 mo and over) and three lactation number groups (1, 2, and all later lactations). He concluded that young cows do not produce as large a proportion of their total production for the lactation during the early months as do older cows. This observation is in general agreement with other reports (4, 5, 9, 17, 18, 19, 20, 21, 23, 30, 41).

Although the change in the shape of the lactation curve from younger to older ages was in general quite gradual, Lamb (16) reported that a distinct change in the factors for cumulative production occurred between 35 and 36 months of age for both milk and fat, and between 47 and 49 months for fat only. These were the ages roughly coinciding with the break between first and second lactations and between second and third lactations, respectively (9, 21).

<u>Season of Freshening</u>

Eldridge and Atkeson (4) and Fritz et al. (9), using methods of regression, studied the effect of season of freshening on the relationship of total to part production and found it insignificant. On the other hand, other workers (2, 8) have indicated a requirement of season of calving adjustments for completed records.

Lamb and McGilliard (17, 18) found that season of freshening exerted an influence, but to a slightly lesser degree than either lactation number or age. Fletcher et al. (6, 7) reported a need for adjustment for season of calving in extending part lactations for Jersey and Sindhi-Jersey cows. Searle (30) and VanVleck and Henderson (41) concluded that age and season of freshening should be adjusted for simultaneously if extending part-time milk and fat records.

Very little is known about the effect of season of freshening in California. Koch (15) has concluded that seasonal differences do occur in California, and that 305-day fat records will vary by as much as 15 lb, depending on season of freshening. On the other hand, Oloufa and Jones (25) found that season of freshening had no appreciable effect on fat production of cows in western Oregon.

Level of Production

Woodward (45) divided cows into three production level groups (based on completed lactation performance) and calculated persistency based on average percentage decline in milk production from one month to the next during the first eight months of the lactation. He found an average decline of 8.4, 7.5 and 6.6 per cent for low, medium and high producers, respectively. This suggests a flatter lactation curve for the higher producers than for the lower producers.

On the other hand, Lamb (16) cited a study by Harvey in which there was a slight tendency for higher producing cows to decline more rapidly in production than did the lower producing cows. He also referred to Kendrick, who found a significant difference between extension factors for low-producing and high-producing cows freshening between 31 and 44 months of age. In the more mature cows, there was only a slight difference due to level of production.

More recently, Madden (21) used quadratic regression equations and compared multiple correlation coefficients to conclude that the relationship between total and part production was similar for low-producing and high-producing cows.

Days Open

Rose et al. (29) and Smith and Legates (33, 34) found that "days open" (days not with calf) had a significant effect on persistency. The latter authors concluded that days open accounted for 7 and 5% of the variation in persistency for first and later lactation records, respectively. This was contrasted to the regression of days in the preceding dry period on 305-day records which accounted for less than 0.1% of the variation in total milk yield (34).

The effect of days open has not been considered in the development of existing prediction equations. Smith and Legates (34) developed factors for standardizing 305-day records in which days open varied. They used a base of 100 days, since it was between the mean and the mode of the data studied and a high percentage of the data required only a small correction. When conception occurred with no more than 100 days open, a 12- to 13-month calving interval was expected.

Herd

Michigan workers (9, 17) indicated that herd differences were not important in the relationship of total to part production; therefore, separate factors were not needed for each herd for extending records. More recently, VanVleck and Henderson (43) concluded that the slight additional accuracy (precision) with intra-herd regression, compared to ignoring herd effects, was not sufficiently important to justify the extra computational difficulties encountered for most practical situations. In order to use intra-herd regression coefficients in prediction equations, herd averages for total yield and for part lactations were necessary. These averages could not be obtained economically and were also likely to be estimated with considerable error (43).

Breed

Cannon et al. (2), using data from five breeds, concluded that the shapes of the lactation curves were so similar that breeds would not need to be considered separately in calculating factors for extending incomplete records. Erb et al. (5) analyzed records from three breeds and made a similar conclusion. Fletcher (7) found only slight differences between extension factors for Jerseys and those for Sindhi-Jersey crossbreds.

On the other hand, several other workers have found definite breed differences. Fritz et al. (9) studied breeds separately but made no conclusions regarding breed differences. However, a visual comparison of the extension factors presented for four breeds indicated considerable breed effect in the relationship of total to part production.

Lamb and McGilliard (17, 18) found that Holstein and Brown Swiss factors tended to be alike for milk but differed widely for fat. Guernsey and Jersey factors were similar to each other but differed from the Holstein and Brown Swiss factors for both milk and fat. A similar conclusion was reached from an inspection of the extrapolation factors published by Lamb (16) and by McDaniel et al. (23).

Frequency of Milking

Madden et al. (21) found that milking frequency was not important in extending incomplete records to a 305-day basis.

Milk and Fat

Most of the earlier developed factors for extending records were for milk only. There was some question as to whether these same factors would apply equally well to incomplete fat records. Kay and M'Candlish (13) found that the slope of the fat yield lactation curve declined less rapidly than the milk curve with the advance of lactation. More recently, Lamb and McGilliard (18), VanVleck and Henderson (41), and Ramerey and Thompson (27) concluded that separate factors were required for milk and fat.

Development of Prediction Factors

The first known attempt to predict complete lactation records from incomplete records was that reported by Yapp (46) in 1915. He found that a 7-day test early in lactation was not highly correlated with total yield for the lactation and concluded that the 7-day test was not satisfactory. Gowen and Gowen (11) reported a correlation of 0.58 between a

7-day test and 365-day milk yield. Other workers (10, 12) found the 7-day test to be of most value when it was made during the fourth or fifth month of lactation.

Single Test

Madden et al. (21) and VanVleck and Henderson (43) have shown that correlation coefficients were low when total lactation yield was estimated from single test-day results obtained in the latter months of lactation.

Cannon et al. (2) has shown that the fifth or sixth month of the lactation was the most precise estimator of total yield when only a single test was made. They obtained correlations of 0.72 and 0.91 between production for these months and 305-day yield in Iowa DHIA herds and in the Iowa State University herd, respectively. The work of Madden et al. (21) also showed the fifth month to be the most precise if only one test was to be made. However, a single test in the fourth, sixth or seventh month was almost as precise for predicting total yield during the lactation.

Searle (31) found that the fifth month of lactation was also more closely related to 305-day yield of fat ($r = 0.76$) than any other month, followed closely by a single test in the fourth or sixth month. VanVleck and Henderson (43) reported an even higher correlation (0.88 or above) between single test-day results and complete lactation yield when the tests were conducted in the fourth to the seventh month.

In all cases, the correlations between a single test extrapolated to a 305-day basis and the actual 305-day record obtained were higher during the middle months of the lactation. One might expect the highest correlation between that single month which is the largest part of the entire lactation, i.e., the month of maximum production and the actual 305-day record. However, variability also must be considered and months four through seven showed the lowest variability (21).

Cumulative Production

Lamb (16) has cited work by Harvey in which the correlations between cumulative production and total production for both milk and fat increased rapidly to 0.88 by the fourth month and 0.94 by the sixth month and then increased more slowly until the correlation at the end of 9 months was 1.00. Earlier, Kennedy and Seath (14) found that cumulative production for the first four months was at least as valuable as any single month for predicting total production for the lactation. Rendel et al. (28) found a correlation of 0.80 between 70-day yield and 305-day production. They concluded that 70-day yield could be used as a guide to early selection. Correlations of 0.90 or more between 180-day production and standard 10-month records have been reported by others (9, 16, 21, 41, 44).

Cumulative production from monthly tests has been shown to be a satisfactory measure of total production. M'Candlish and M'Vicar (22) found that total yield estimated from monthly samples was within 2% of actual yield determined from daily weights. Tyler and Chapman (36) determined cumulative production in a simplified manner by summing the ten monthly tests and multiplying by 30.5. The correlation between this simplified estimate and daily total was 0.99. Similarly, the correlation between the simplified estimate and standard DHIA results was 0.99.

Construction of Prediction Factors

The data used by most workers deriving prediction factors for milk and fat consisted of sequential test day records obtained once monthly in the standard DHIA program. There was some variation between workers in the requirements that had to be fulfilled before records were utilized.

Some cows, even though they remained in the herd, were not "in milk" long enough to have 10 consecutive monthly tests after freshening. The Michigan workers (9, 19, 21) and Searle (30) did not include cows with short records (less than 10 tests) in their studies. VanVleck and Henderson (40, 41, 43) and McDaniel et al. (23), on the other hand, utilized the records from those cows that went dry by giving them "zero" credit for all monthly tests between the dry date and the calculated 305th day of lactation. The literature was not clear as to the procedure used by the other workers.

VanVleck (37) further stated that the first record had to be obtained less than 50 days after the beginning of the lactation or it was considered as a later stage of lactation. If a test day record was missing from the middle of a lactation, all of the lactation was discarded. Lamb (16) used only records that conformed to the following specifications: (a) 2X milking; (b) first test day within 34 days of freshening; and (c) 10 consecutive monthly tests.

Ratio Factors

Lamb (16) computed factors for extending milk and fat records from each of 10 monthly tests separately for subgroups of various combinations of breeds, ages, lactation numbers, and seasons of freshening. Ratios of total production from 10 test days to production on each test day were averaged for all records in each subgroup. Ratio factors for cumulative test days were obtained from factors for individual test days in the following manner. The reciprocals of the factors for monthly production for the first 2 months were added and the reciprocal of the sum was the factor for extending production for the first 2 cumulative months. In general, the reciprocal of the factor for the "$\underline{i}^{th}$ + 1" month was added to the sum of the reciprocals for the first "$\underline{i}^{th}$" months and then reciprocated again to obtain the factor.

VanVleck (37) constructed ratio factors for estimating 10-month production from a single test day record for each age-season subclass by dividing the sum of the 10 monthly means of the subclass by the monthly means of the same subclass. Cumulative test day ratio factors for estimating a 10-month record were obtained in a similar manner in that the sum of the 10 monthly means of the subclass was divided by the sum of the first $\underline{i}$ monthly means, where $\underline{i}$ = 1 to 10.

Cannon et al. (2) and Madden et al. (19, 20) have also obtained prediction factors by the ratio method.

Regression Factors

Van Vleck and Henderson (37, 43) constructed prediction factors ignoring herd effects with the usual normal equations which were solved to estimate the desired regression coefficients. The right- and left-hand sides of the normal equations were made up of total sums of squares and cross-products corrected for means. The criterion for determining the accuracy (precision) of prediction was the multiple correlation coefficient. The correlation coefficient, rather than the square of the correlation coefficient, was chosen for consistency because the majority of reports which described the extension of records by regression techniques had used the correlation coefficient. The square of the correlation coefficient, however, may have been more appropriate if the only purpose was to predict total yield, since it measured the amount of variability in the complete lactation yield which was accounted for by the monthly records or by cumulative monthly records. The standard errors of the regression coefficients also were computed.

Madden et al. (19, 20, 21) and Gowen and Gowen (11) have also utilized a regression equation, whereby cumulative part production was multiplied by the regression, b, of whole or part and added to an appropriate constant, a, to estimate total production.

Comparison of Ratio and Regression

Madden et al. (21) concluded that the ratio method may underestimate total production of low-producing cows and overestimate total production of high producing cows, since the ratio method corrected only for the incompleteness of the lactation and did not take into account the incomplete repeatability of the parts of the lactation which was included in the regression method.

The choice between methods (ratio versus regression) depended on the purpose for which the method was to be used and the ease of use according to Madden et al. (21). The ratio method has appealed intuitively to many and would be easier to develop and use. The variation in the total production estimated by the ratio was more nearly like the variation in actual total production, while the total estimated by regression varied less than the actual total. The differences between estimates by the two methods were largest during the early months of lactation.

EXPERIMENTAL PROCEDURE

The main objectives of this study were to determine the importance of certain variables affecting the precision of prediction factors used in extending incomplete lactation records and to develop improved regression equations or ratio factors for predicting complete lactation records from either DHIA cumulative month records or single test-day results.

Data

Holstein cows from 19 herds located in 9 California counties were included in the data. All cows in these herds that freshened after March, 1964 and completed a ten-month lactation before April, 1966 were included. An attempt was made to select herds that would provide a sample in which the geographical distribution of cows was approximately proportional to the number of cows being tested in the DHIA program.

Herds selected for this study were chosen on the following basis: (a) DHIA records processed electronically during the entire sampling period; (b) customarily reported all known breeding dates; (c) all or nearly all cows in the herd had the current lactation number recorded; and (d) all cows in the herd were of the Holstein breed.

A total of 43,610 monthly test-day records were included in this study. Each of these records conformed to the following specifications: (a) the first test-day record obtained within 50 days after the beginning of the lactation; (b) 10 consecutive daily tests obtained at intervals of approximately one month; and (c) all tests were on 2X milking.

Each record identified the cow, the herd and the county in which the record was made, and contained information on the month and year of freshening, lactation number, the last breeding date reported during the preceding month and milk and fat production on test day. Daily milk production was recorded to the nearest one-tenth pound, and fat production was recorded to the nearest one-hundredth pound. The sum of the ten test-day records which made up a cow's record times 30.5 was defined to be a complete lactation record.

Classification of Variables

The records were classified according to lactation number (age), season of calving, peak milk production level, days open and month (stage) of lactation. Milk and fat production were studied separately. The cows were divided into 3 lactation number groups (lactation 1, 2, and 3 and over). The season classes used, determined from month of freshening, were: 1 - June through September; 2 - October through January; and 3 - February through May. Peak milk production level was determined by the higher of the first two test day milk weights. The cows were classified into one of four groups as follows: 1 - less than 55.0 lb; 2 - 55.0 through 69.9 lb; 3 - 70.0 through 84.9 lb; and 4 - 85.0 lb or more. Days open were calculated by determining the number of days in the interval between calving and the last reported breeding date. The 3 "days open" classes used in the preliminary analysis of variance of monthly means were: 1 - less than 100 days; 2 - 100 through 149 days; 3 - more than 149 days.

Requirement of 10 Test-Day Samples

Some cows do not remain in milk sufficiently long to obtain 10 consecutive monthly tests after freshening. Michigan workers (9, 19, 21) and Searle (30) did not include cows with short records in their studies. VanVleck and Henderson (40, 41, 43) and McDaniel et al. (23), on the other hand, utilized records from those cows by giving them "zero" credit for all monthly tests conducted between the dry date and the completion of the 305-day lactation period. No reasons were given for deciding on the procedure they used. In the author's opinion, preference of method depends on the intended use of the derived extrapolation factors. If the extension factors are to be used in sire summarization and are to be applied to all cows leaving the herd prior to completion of the 305-day lactation, it would seem appropriate to use factors developed from data including the short lactation records since the factors derived would be more representative of the population in which they might be applied. On the other hand, if the primary purpose of the derived prediction factors is for herd management purposes, one can conclude that excluding the short records would be the preferable procedure. Many cows in California that may be relatively good producers are culled, not because of inherent low production, but because current daily production is too low to return an immediate profit and because of an anticipated long dry period. It is not unusual to cull cows giving 30 lb of milk daily. In this case, extrapolated records could be used to demonstrate what the cow would be expected to produce should she be allowed to remain in the herd (and in milk) for the entire 10-month lactation. Thus, factors derived from a sample including only cows with

10 consecutive tests would be preferred. In addition, if cows with incomplete records are to be culled on the basis of partial records extended to a complete record basis, preference should be given to the set of factors that overestimate, rather than underestimate, the eventual 305-day record. Such a practice would tend to reduce the number of cows with acceptable producing ability being culled too early on the basis of predicted low production. For these reasons, 1,914 short duration records, which otherwise were normal, were excluded from the study. This amounted to about 30 per cent of the total.

Selection of Days Open Groups

One of the factors which probably influences the slope of the lactation curve and, thus, the prediction factor, is the number of days open (not bred). In this study, it was possible to study only the effect of a long "days open" period, since the number of cows bred and settled before 55 days post-parturition was so limited (less than 0.1 per cent). Thus, the limits on the groups studied were chosen to represent normal (ideal), moderately long, and long intervals for this trait.

Statistical Procedures

A preliminary analysis of unweighted cell means for milk and fat of 1,080 subclasses (3 lactation number groups, 3 seasons of freshening, 4 peak milk levels, 3 days open groups, and 10 months or stages of lactation) was made. Sums of squares of the monthly means were computed according to the usual balanced five-way classification analysis, where all effects were considered fixed. An approximate error mean square was obtained from the within cell variation sums of squares.

Least squares (multiple regression) analysis was used in deriving prediction factors from cumulative month milk and fat production. The model assumed was as follows:

$$Y_{ij} = \beta_0 + \beta_1 X_{1ij} + S_i + \beta_2 X_{2ij} + e_{ij}$$

where:

Y_{ij} = 305-day production for the j^{th} cow in the i^{th} season,

β_0 = a constant,

β_1 = linear regression coefficient associated with cumulative month,

X_{1ij} = cumulative month production for the $(i,j)^{th}$ cow,

S_i = effect of the i^{th} season on total milk or fat production (i = 1, 2, or 3; in order to solve the normal equations, the restriction was made that the $\Sigma S_i = 0$),

β_2 = linear regression coefficient associated with the number of days open,

X_{2ij} = number of days open for the $(i,j)^{th}$ cow, and

e_{ij} = random effect peculiar to each animal.

Using matrix notation, this model may be written as:

$$[Y] = [X] [\beta] + [E]$$

where:

Y = vector of observations,

X = observation matrix,

β = vector of coefficients to be estimated, and

E = vector of random error.

With a regression analysis program adapted for the IBM-7040 from the O.S.U. Department of Mathematics and Statistics, the normal equations

$$\left[X'X\right] \left[\hat{\beta}\right] = \left[X'Y\right]$$

were calculated and solved. Also, an analysis of variance using the Abbreviated Doolittle method was performed. From this, the reduction in total sums of squares of total yield due to regression (multiple correlation coefficient squared, R^2) and the standard error of estimate (S.E.E.) were obtained according to the methods outlined by Ostle (26).

The regression model assumed to derive prediction factors from a single month's milk and fat production was similar to that used to predict cumulative production, the only difference being that the β_1 and X_1 values were associated with single month, rather than cumulative month, production and the $\beta_2 X_2$ term was deleted. The effect of regression due to days open was not included because the population of inference, in most cases, would not have this information available.

To derive prediction factors for estimating total lactation milk or fat yield by the ratio method (either single or cumulative month), the following model was assumed:

$$Y_i = \gamma_R X_{Ri} + e_{Ri}$$

where:

$$\gamma_R = \frac{\sum_{i=1}^{N} \frac{Y_i}{X_{Ri}}}{N},$$

Y_i = 305-day production for the i^{th} cow,

X_{Ri} = either single or cumulative month production for the i^{th} cow,

Y_i/X_{Ri} = each cow's total production divided by her single or cumulative production,

N = number of observations in the subpopulation, and

e_{Ri} = random effect peculiar to each animal.

The multiple correlation coefficients squared, R^2, between total production estimated by the ratio method and actual total production were calculated.

RESULTS AND DISCUSSION

Regression Equations for Predicting Complete Lactation Yield from Partial Records

The milk and fat means were analyzed separately. Since 50 of the 1,080 subclasses had no observations in them, the fourth peak milk level groups (high production) were not included in the five-way classification analysis. This reduction of the number of subclasses to 810 allowed for completeness of information and representation in each subclass.

The mean squares given in Table I were obtained from unweighted cell means. In order to have an approximate error mean square for testing the magnitude of the effects of the different factors, the pooled mean square among cows within cells was multiplied by the harmonic mean of the number of cows within the cells. The harmonic mean was obtained by the relation:

$$\overline{H} = \frac{\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \sum_{l=1}^{d} \sum_{m=1}^{e} \frac{1}{n_{ijklm}}}{abcde}$$

where a was the number of lactation number classes, b was the number of seasons, c was the number of peak milk level classes, d was the number of days open classes, e was the number of months, and n_{ijklm} was the subclass.

All of the main effects on milk means, and all of the main effects except days open on fat means, were significant at the 5% level. It is well known that production is different from month to month in any one

TABLE I

ANALYSIS OF VARIANCE OF MONTHLY MEANS

Source	d.f.	Milk M.S.	Milk F[b]	Fat M.S.	Fat F[b]
Lact. No. (A)	2	267.98	48.72**	29.64	31.20**
Season (B)	2	22.48	4.09*	7.07	7.44**
Peak Milk Level (C)	2	16,826.27	3,059.32**	1,084.95	1,142.05**
Days Open (D)	2	17.23	3.13*	1.21	1.27
Month (E)	9	7,840.35	1,425.52**	1,004.57	1,057.44**
AxB	4	47.88	8.71**	5.17	5.44**
AxC	4	56.45	10.26**	2.20	2.32
AxD	4	24.07	4.38**	3.58	3.77**
AxE	18	140.58	25.56**	21.28	22.40**
BxC	4	58.51	10.64**	6.05	6.37**
BxD	4	35.22	6.40**	2.23	2.35
BxE	18	18.75	3.41**	8.20	8.63**
CxD	4	5.00	0.91	1.27	1.34
CxE	18	279.71	50.86**	31.40	33.05**
DxE	18	5.78	1.05	1.18	1.24
AxBxC	8	30.24	5.50**	4.13	4.35**
AxBxD	8	23.73	4.31**	4.41	4.64**
AxBxE	36	5.46	0.99	0.93	0.98
AxCxD	8	21.50	3.91**	3.23	3.40**
AxCxE	36	3.72	0.68	0.34	0.36
AxDxE	36	4.19	0.76	0.58	0.61
BxCxD	8	37.23	6.77**	3.92	4.13**
BxCxE	36	4.32	0.79	0.76	0.80
BxDxE	36	5.32	0.97	1.20	1.26
CxDxE	36	5.53	1.01	1.12	1.18
AxBxCxD	16	29.39	5.34**	3.01	3.17**
AxBxCxE	72	2.71	0.49	0.57	0.60
AxBxDxE	72	3.00	0.55	0.54	0.57
AxCxDxE	72	2.72	0.49	0.54	0.57
BxCxDxE	72	3.61	0.66	0.45	0.47
AxBxCxDxE	144	3.55	0.65	0.62	0.65
Residual	42580	5.50[a]		0.95[a]	

[a]The residual mean squares were multiplied by the harmonic mean in order to have an approximate error mean square for hypothesis testing.

[b]F values significant at the 5% and 1% levels are indicated by * and **, respectively.

lactation. This fact is emphasized in the present study by the large F values for month of lactation. The expected age (lactation number) differences also are evident.

The large peak milk level effect was expected due to the classification scheme based on the higher of the first two months' production. Of more importance to this study, however, is the interaction between peak milk level and month which is one measure of persistency.

Even though season and days open "main" effects were statistically significant for milk, and season effects caused significant variation in monthly fat production, the comparatively small mean square values suggests that these factors contributed little to the total variation between monthly means.

Of particular importance to the objectives of this study is the influence of these "main" effects on persistency. Differences in persistency due to age, season, peak milk level and days open are the AxE, BxE, CxE, and DxE interactions, respectively. All but the days open by month of lactation interactions are statistically significant at the 1% level. Again, it appears that both age of the cow and the cow's peak milk level contribute materially to differences in persistency. The interaction of season of freshening by month of lactation, while statistically significant, is relatively insignificant in comparison to the mean square values obtained from the AxE and CxE interactions.

All the two-way interactions, except the CxD and DxE interactions, are statistically significant. The interpretation of a significant interaction mean square is the failure of the differences between levels of one factor to be the same for all levels of another factor. The lactation number by season interaction indicates that differences between yields of cows calving in different periods of the year vary with different ages at calving. Similar conclusions were made by Frick et al. (8) and VanVleck and Henderson (41).

Even though several of the three-way and four-way interactions appear to be statistically significant, as can be noted from Table I, the magnitude of the F values are generally considerably smaller than the important two-way interactions discussed previously.

The significance of the interaction mean squares indicates ratio factors or regression equations should be constructed so that the effects of lactation number, peak milk level and month can be considered simultaneously.

An example from the data of the lactation number by month interaction is graphically demonstrated in Figure 1. The shape of the lactation curve for the younger cows is much flatter than the shape of the curve for the older cows. Similarly, the peak milk level by month interaction is illustrated in Figure 2. In this example, the shape of the lactation curve for the initially lower producing cows is much flatter than the shape of the curve for the cows with high production early in lactation.

The author reasons that by accounting for the influence of high production in early lactation, a considerable amount of the herd to herd variation may be removed from the study. This reasoning is based on the hypothesis that much of the between-herd variation is the result of differences in environmental opportunity and that these differences can best be measured during periods of potentially high milk production. No attempt was made to consider herd effects in this study since the

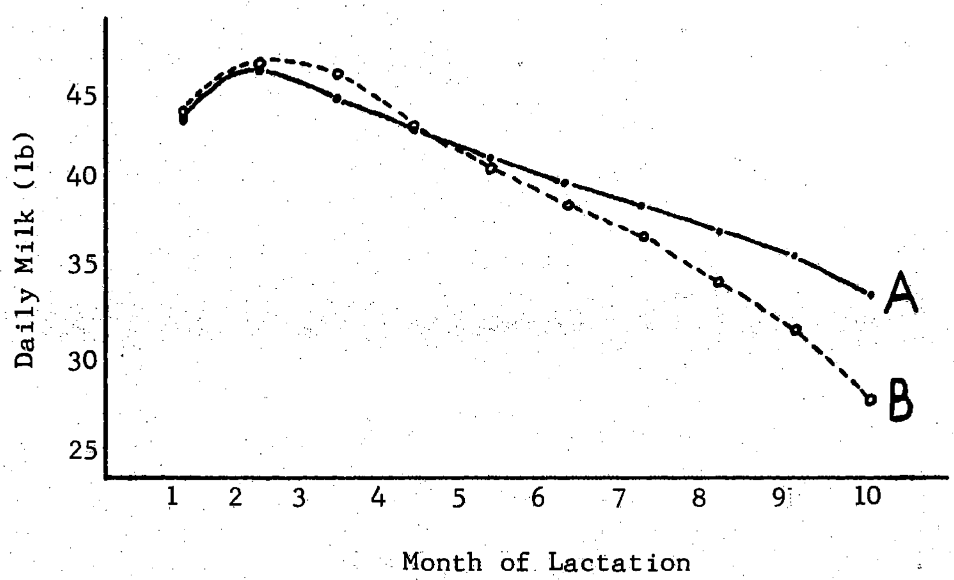

Figure 1. Example of Age by Month Interaction

(A) Cows in first lactation, low peak milk level class (< 55.0 lb).
(B) Cows in lactation number class 3, low peak milk level class.

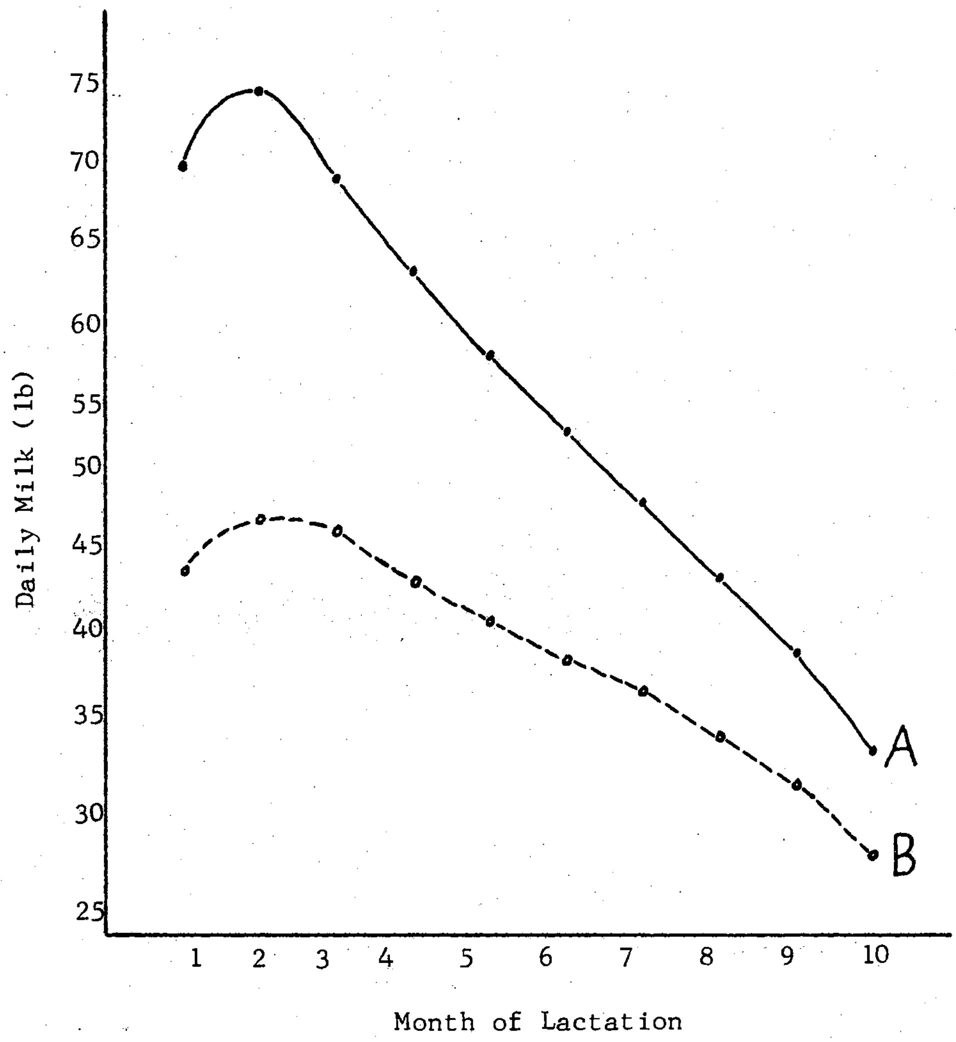

Figure 2. Example of Peak Milk Level by Month Interaction.

(A) Cows in lactation number class 3, peak milk level class 3.
(B) Cows in lactation number class 3, peak milk level class 1.

inclusion of another factor would result in a considerable increase in the number of subclasses and a corresponding decrease in number of observations in each subclass. According to VanVleck and Henderson (43), consideration of herd effects provided only slightly additional accuracy in extending incomplete records to a 305-day completed lactation.

On the basis of these results, all cows were reassigned to only twelve subpopulations. These subpopulations consisted of all possible combinations of three lactation number classes and four peak milk level classes. The number of cows, average total milk production, and average daily milk production by month of lactation for each subpopulation are presented in Table II.

Cumulative Month Regression Equations

A useful method of extrapolating records to a complete lactation basis is the extension of cumulative milk and fat records. Most, if not all, computing centers engaged in DHIA work carry forward cumulative production records rather than each monthly record in the lactation. By considering 3 lactation number (age) groups and 4 peak milk level classes for each of the cumulative months, it is necessary for such computing centers to have only 12 sets of regression equations. The regression coefficients for milk and fat associated with the lactation number, peak milk level class, and cumulative month for these 12 sets of equations are found in Tables XVI and XVII, respectively.

An illustration of how the regression coefficients may be used in a predictive regression equation is given in the following example. This example, furthermore, serves to demonstrate how a DHIA computing

TABLE II

NUMBER OF COWS, AVERAGE 305-DAY MILK PRODUCTION, AND AVERAGE DAILY MILK PRODUCTION BY MONTH OF LACTATION FOR EACH OF TWELVE LACTATION NUMBER - PEAK MILK LEVEL GROUPS

Lact. No. Class	PML Class	No. of Cows	305-day Milk	Month of Lactation									
				1	2	3	4	5	6	7	8	9	10
			lb	pounds daily									
1	1	885	12,176	43.6	46.7	44.9	42.8	40.8	39.4	38.0	36.3	34.8	31.9
1	2	639	14,561	54.0	58.8	54.6	51.2	48.6	46.5	44.2	42.3	40.2	37.0
1	3	52	16,601	61.6	72.2	66.4	60.0	56.1	51.9	48.5	45.5	43.2	38.9
1	4	05	18,584	82.1	83.7	75.4	61.9	64.6	54.5	54.0	49.6	43.3	40.2
	Total	1581											
2	1	73	11,303	46.5	47.4	44.6	41.4	37.9	36.2	32.9	30.7	28.1	24.9
2	2	397	14,103	59.8	61.1	56.4	52.0	48.3	44.9	41.1	37.6	33.7	28.8
2	3	401	16,220	69.5	73.6	66.1	60.3	55.4	50.4	46.2	41.8	37.1	31.4
2	4	99	18,565	83.0	89.1	75.8	69.3	62.9	56.7	52.3	46.2	40.4	33.0
	Total	970											
3	1	79	11,642	43.7	47.0	46.4	42.9	40.4	37.7	36.1	32.8	29.7	25.0
3	2	402	14,454	58.6	62.0	58.5	54.0	50.4	45.8	42.2	38.5	34.6	29.3
3	3	819	16,790	70.0	75.1	69.3	63.3	57.8	52.8	48.4	43.4	38.4	32.0
3	4	510	19,602	84.0	89.9	80.5	73.1	66.7	61.0	55.7	49.7	44.3	37.8
	Total	1810											
Grand Total		4361											
Grand Average			15,201										

center can utilize these equations with little extra computation or record storage.

Assume it is wished to predict a 305-day record of a cow in her first lactation that had a peak milk level of 65.0 lb per day (PML class 2). Also assume that this cow has 7,440 lb of milk in 128 days to her credit in DHIA, that she gave 62.0 lb of milk on the last test day, that she had 100 days open, and that she freshened in March (season 3).

Before the appropriate regression equation can be used to calculate expected 305-day production, the cumulative milk (or fat) record must be adjusted so that the number of days in milk credited to the cow will have a one-to-one correspondence with the cumulative month. This adjustment can be made by multiplying the pounds of milk (or fat) produced on the last test day times minus six, the number of days required to bring about a one-to-one correspondence (see Table XVIII), and adding this amount to the cumulative record. An example is as follows:

$$\text{adjusted cum. record} = \text{cum. record} + \left(\text{adjustment factor} \times \text{daily milk or fat}\right)$$

$$7{,}068 = 7{,}440 + (-6 \times 62.0)$$

The resulting adjusted cumulative milk (or fat) record is then used in the appropriate regression equation to estimate total production in the following manner:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{S}_3 + \hat{\beta}_2 X_2$$

$$15{,}223 = 66 + (2.15 \times 7{,}068) - 151 + (1.12 \times 100)$$

where:

$\hat{Y}$ = the predicted record,

$\hat{\beta}_0$ = a constant,

$\hat{\beta}_1$ = the linear regression coefficient associated with the appropriate cumulative month,

X_1 = the adjusted cumulative record,

$\hat{S}_3$ = the effect due to the 3rd season of freshening,

$\hat{\beta}_2$ = the linear regression coefficient associated with days open, and

X_2 = days open.

The square of the correlation coefficient, R^2, was used in this study to determine the amount of variability in the complete lactation yield which was accounted for by cumulative month, season of freshening, and days open. The R^2 values obtained for milk and fat are found in Tables III and IV, respectively.

The R^2 values between total production predicted from the cumulative record and actual total production for both milk and fat increased rapidly from the first month to about 0.83 and 0.84, respectively, in the sixth month and then increased more slowly until the ninth month when 99 per cent of the variation was accounted for. In general, the R^2 values obtained for fat during the second to the fourth cumulative months were about 10 per cent higher than the corresponding R^2 values for milk. Milk and fat R^2 values were quite comparable in the latter stages of lactation as they approached unity. This indicates that fat production (determined from the sum of several monthly tests) is less variable than milk production.

In general, the R^2 values obtained were higher in those subpopulations (lactation number class x peak milk level class) in which the

TABLE III

R^2 VALUES BETWEEN TOTAL MILK PREDICTED FROM CUMULATIVE MILK RECORDS AND ACTUAL TOTAL MILK PRODUCTION

Lact. No.	PML Class	Cumulative Month							
		2	3	4	5	6	7	8	9
1	1	0.43	0.60	0.72	0.82	0.88	0.93	0.97	0.99
1	2	0.18	0.39	0.58	0.72	0.82	0.91	0.95	0.98
1	3	0.14	0.38	0.50	0.69	0.82	0.91	0.96	0.99
2	1	0.14	0.30	0.49	0.67	0.79	0.88	0.94	0.98
2	2	0.21	0.40	0.56	0.70	0.81	0.89	0.95	0.98
2	3	0.09	0.31	0.53	0.70	0.81	0.90	0.95	0.99
2	4	0.31	0.45	0.57	0.69	0.80	0.88	0.95	0.98
3	1	0.18	0.57	0.75	0.86	0.91	0.94	0.97	0.99
3	2	0.16	0.43	0.59	0.71	0.82	0.90	0.95	0.98
3	3	0.15	0.40	0.57	0.70	0.81	0.89	0.95	0.98
3	4	0.19	0.38	0.55	0.69	0.80	0.90	0.95	0.99

TABLE IV

R^2 VALUES BETWEEN TOTAL FAT PREDICTED FROM CUMULATIVE FAT RECORDS AND ACTUAL TOTAL FAT PRODUCTION

Lact. No.	PML Class	Cumulative Month							
		2	3	4	5	6	7	8	9
1	1	0.46	0.60	0.70	0.80	0.87	0.92	0.96	0.99
1	2	0.33	0.49	0.63	0.75	0.84	0.92	0.96	0.99
1	3	0.38	0.44	0.54	0.69	0.82	0.90	0.95	0.98
2	1	0.19	0.33	0.50	0.64	0.76	0.86	0.94	0.98
2	2	0.35	0.49	0.60	0.72	0.82	0.89	0.95	0.98
2	3	0.33	0.47	0.61	0.71	0.82	0.90	0.95	0.99
2	4	0.50	0.62	0.69	0.78	0.85	0.91	0.96	0.99
3	1	0.39	0.56	0.72	0.82	0.89	0.93	0.97	0.99
3	2	0.32	0.47	0.58	0.71	0.81	0.89	0.95	0.98
3	3	0.28	0.44	0.58	0.70	0.81	0.89	0.95	0.98
3	4	0.34	0.48	0.58	0.69	0.79	0.88	0.94	0.98

number of observations exceeded 400. Because of an insufficient number of observations, the author chose not to publish the prediction equation to be used on first lactation cows in PML class 4.

Standard errors of estimates (S.E.E.) for total milk production and total fat production are shown in Tables V and VI, respectively. These estimates were found by obtaining the residual sums of squares after adjusting for season and the linear effect of days open. Each estimate is used to obtain a confidence interval about predicted production.

The S.E.E. values obtained on milk decrease rapidly from about 1,700 lb in cumulative month 2 to less than 1,000 lb in cumulative month 5 and to about 200 lb in cumulative month 9. This demonstrates an inability to obtain a very precise estimate of total production when less than 5 months' production is known.

The S.E.E. values on records from cows of varying age but in the same peak milk level class were similar, but slightly smaller for the younger cows. The higher producing cows (PML class 4), on the other hand, had S.E.E. values nearly 40 per cent higher than did the lower producing cows (PML class 1). The average values, disregarding lactation number and cumulative month, for peak milk level classes 1, 2, 3, and 4 were 812, 866, 1,025, and 1,123 lb, respectively. Similar trends were observed relative to fat S.E.E. values. The average values ranged from a low of 32.5 lb for PML class 1 to a high of more than 40 lb for PML class 4.

To illustrate the variation between cows in the same lactation number and peak milk level groups, the Y (actual production) and $\hat{Y}$

TABLE V

STANDARD ERRORS OF ESTIMATES FOR PREDICTING TOTAL LACTATION MILK YIELD FROM CUMULATIVE PRODUCTION HOLDING THE VECTOR OF INDEPENDENT VARIABLES CONSTANT

Lact. No.	PML Class	Cumulative Month 2	3	4	5	6	7	8	9
1	1	1,203	1,005	842	681	543	413	293	168
1	2	1,352	1,166	969	796	631	460	328	193
1	3	1,958	1,666	1,491	1,183	893	651	423	229
2	1	1,577	1,438	1,213	981	788	587	403	235
2	2	1,608	1,402	1,204	987	794	604	421	230
2	3	1,856	1,617	1,339	1,065	843	629	427	229
2	4	1,819	1,617	1,437	1,207	977	749	510	271
3	1	1,990	1,437	1,091	829	667	523	377	205
3	2	1,773	1,457	1,238	1,052	823	624	439	238
3	3	1,889	1,584	1,339	1,116	769	676	470	258
3	4	2,111	1,844	1,569	1,301	1,043	750	506	263

TABLE VI

STANDARD ERRORS OF ESTIMATES FOR PREDICTING TOTAL LACTATION FAT YIELD FROM CUMULATIVE PRODUCTION HOLDING THE VECTOR OF INDEPENDENT VARIABLES CONSTANT

Lact. No.	PML Class	Cumulative Month 2	3	4	5	6	7	8	9
1	1	43	37	32	26	21	16	12	07
1	2	50	44	37	31	25	18	13	07
1	3	60	57	51	42	32	25	17	10
2	1	63	57	50	42	34	26	18	11
2	2	60	53	47	40	32	24	17	10
2	3	66	59	50	43	35	26	18	10
2	4	65	57	51	43	36	27	19	10
3	1	68	58	46	37	29	22	16	09
3	2	64	57	50	42	34	26	18	10
3	3	68	60	52	44	35	27	19	10
3	4	72	64	58	49	40	30	21	11

(predicted production) values for milk from two subpopulations are shown in Figure 3. The distance from the line to the observed value is a measure of the agreement between the observed value and actual production. Seventy-nine cows in lactation number class 3, peak milk level class 1, cumulative month 5 and 52 cows in lactation number class 1, peak milk level class 3, cumulative month 5 are shown. The R^2 and S.E.E. values associated with the two groups are 0.86 and 829 lb, and 0.69 and 1,183 lb, respectively. Following the pattern discussed previously, the older but lower producing cows had the higher R^2 and the lower S.E.E. values. The coefficient of variation, which is the variation expressed as a fraction of the mean, is about the same for both the high and low producers, averaging between 6.0 and 7.0 per cent.

Since one of the probable uses of extrapolated records is to assist with the culling of unprofitable cows, it is fortunate that the smaller S.E.E. values are associated with the lower producing cows; that is, the cows that are more likely to be culled. Fewer mistakes in culling will be made than would be the case if all extrapolated records had S.E.E. values of the magnitude associated with PML class 4 cows.

Single Month Regression Equations

A possible use of extrapolated records is in promotional efforts designed to demonstrate the value of DHIA records. Another application might be in providing producing ability estimates of individual cows to prospective buyers of cows in herds not enrolled in a regular testing program. Either of these applications, most likely, would involve extrapolating records to a complete lactation basis from the results obtained in a single 24-hour sample.

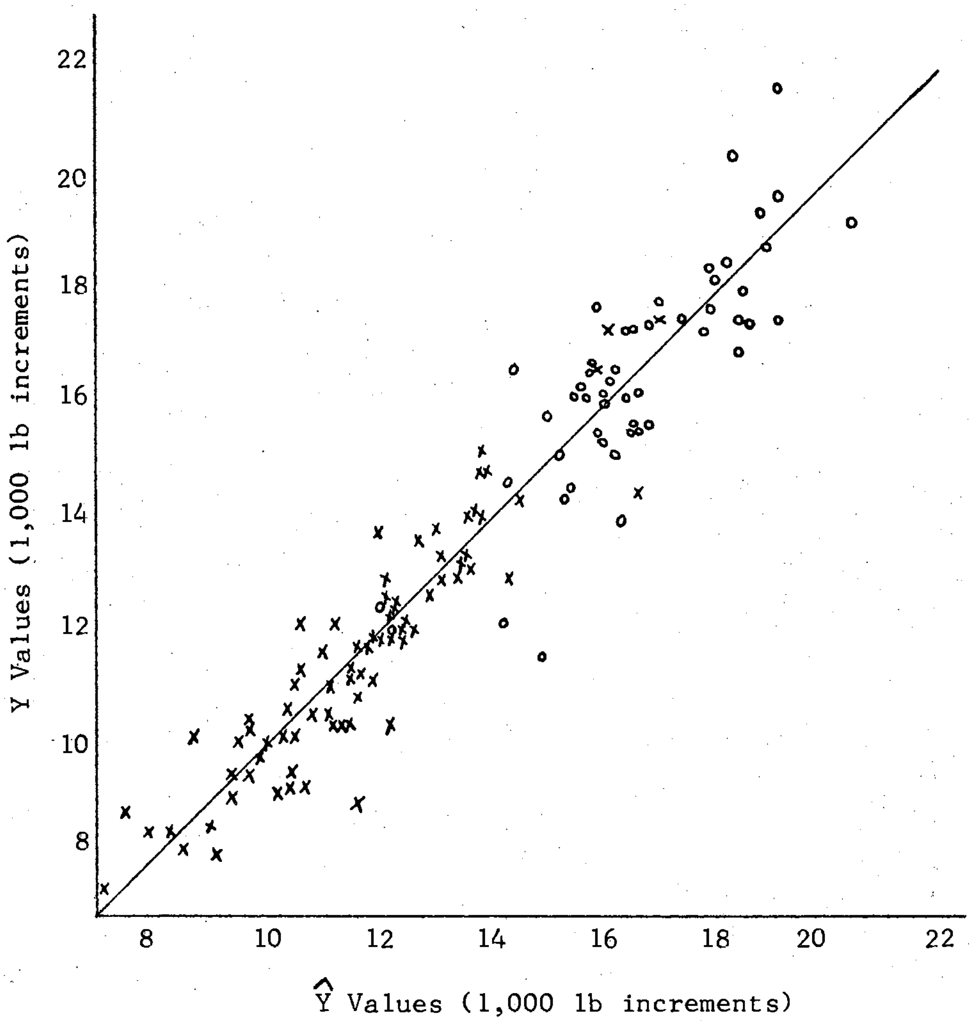

Figure 3. Example of Y vs $\hat{Y}$ Comparisons of Two Different Subpopulations

(x) Cows in lactation number class 3, peak milk level class 1, cumulative month 5.
(o) Cows in lactation number class 1, peak milk level class 3, cumulative month 5.

There are two basic differences between the regression equations developed in this study to predict complete lactation results from single tests and those that predict the total record from cumulative production. One difference is that the peak milk level cannot be determined, since only a single test-day sample is obtained from each cow, and since this sample may be obtained during any stage of lactation. The second difference is the deletion of the "days open" independent variable effect from the single month regression model. Since these equations would most likely be applicable in situations where the herd is not on test, the author assumed that the exact date of freshening or date last bred would be unavailable; thus making it impossible to determine the days open for each cow.

The regression coefficients for milk and fat associated with lactation number and month of lactation are given in Tables XIX and XX. To illustrate their use, assume the need to predict the 305-day record of a cow that produced 48.0 lb of milk when sampled in the fourth month of her first lactation, and freshened in March (season 3). The appropriate regression equation and the resulting predicted response would be as follows:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{S}_3$$

$$13{,}481 = 3{,}497 + (209)\,(48.0) - 48$$

where:

$\hat{Y}$ = the predicted record,

$\hat{\beta}_0$ = a constant,

$\hat{\beta}_1$ = the regression coefficient associated with X_1,

X_1 = the test day milk or fat record, and

$\hat{S}_3$ = the effect due to the 3rd season of freshening.

As before, the square of the correlation coefficient, R^2, was used to determine the amount of variability in the complete lactation yield which was accounted for by the stage of lactation and season of freshening. The R^2 values obtained for milk and fat are given in Table VII.

The R^2 values between total milk production predicted from a single test day weight and actual total production increased from about 0.36 in the first month to nearly 0.60 in the second month, then increased gradually until the fifth month when they approached 0.80. This was followed by a gradual decline in R^2 values during each remaining month of lactation and averaged about 0.50 in the tenth month. Similar trends were observed for fat, but at slightly lower levels, with the high R^2 values (about 0.66) occurring in the sixth month.

The only stage of lactation in which R^2 values for fat were larger than those for milk was in the first month. This is attributed to the fact that fat tests are generally higher in the first month than in later stages of lactation and that the first month's milk production is generally more variable in comparison to the succeeding month than are milk weights obtained later in the lactation period. There were no marked differences in the average R^2 value, disregarding month of lactation, between lactation number groups.

The standard errors of estimates for both milk and fat are shown in Table VIII. The S.E.E. values obtained on milk started out high in the first month (ranging from 1,634 lb in the lactation number 1 group to 2,435 lb in the older cow group), and dropped considerably in the

TABLE VII

R^2 VALUES BETWEEN TOTAL PRODUCTION PREDICTED FROM SINGLE TEST DAY RECORDS BY THE REGRESSION METHOD AND ACTUAL TOTAL PRODUCTION

Month of Lact.	Lact. No. 1		Lact. No. 2		Lact. No. 3	
	Milk	Fat	Milk	Fat	Milk	Fat
1	0.37	0.39	0.34	0.36	0.35	0.36
2	0.61	0.51	0.57	0.48	0.60	0.47
3	0.68	0.55	0.67	0.53	0.70	0.54
4	0.73	0.60	0.75	0.60	0.72	0.56
5	0.79	0.67	0.80	0.66	0.77	0.62
6	0.78	0.66	0.78	0.68	0.77	0.64
7	0.72	0.62	0.77	0.68	0.76	0.66
8	0.71	0.60	0.72	0.65	0.72	0.63
9	0.67	0.60	0.63	0.57	0.65	0.56
10	0.54	0.49	0.46	0.46	0.53	0.48

TABLE VIII

STANDARD ERROR OF ESTIMATE VALUES BETWEEN TOTAL PRODUCTION PREDICTED FROM SINGLE TEST DAY RECORDS AND ACTUAL TOTAL PRODUCTION BY LACTATION NUMBER GROUPS, FOR MILK AND FAT

Month of Lact.	Lact. No. 1		Lact. No. 2		Lact. No. 3	
	Milk	Fat	Milk	Fat	Milk	Fat
1	1,634	56	2,113	73	2,435	81
2	1,282	50	1,714	66	1,906	75
3	1,170	48	1,499	62	1,643	68
4	1,070	45	1,310	58	1,583	67
5	953	41	1,155	54	1,446	63
6	960	42	1,213	52	1,440	61
7	1,086	44	1,238	52	1,459	59
8	1,118	45	1,369	54	1,594	62
9	1,181	44	1,583	60	1,780	67
10	1,407	51	1,903	67	2,060	73

second month. There was a further reduction in S.E.E. values until either the fifth or sixth month, after which the S.E.E. again became larger.

Comparison and Application of Cumulative and Single Month Regression Equations

Most previous studies have used the multiple correlation coefficient rather than the square of the correlation coefficient as the criterion for determining the accuracy of prediction (11, 19, 20, 24, 40, 43). The observed patterns, however, are the same. When prediction of total production is made from cumulative production, the correlation becomes larger with succeeding months until near unity is obtained. When single test day results and total production are compared, correlations start out low, increase gradually until either the sixth or seventh month, then decrease.

To provide a direct comparison, the data of VanVleck and Henderson (40) was used to calculate R^2 values for both single test day and cumulative milk records. Their R^2 values are 0.69 and 0.90 for single and cumulative months, respectively. The results of this study compare quite favorably, averaging 0.75 and 0.90, respectively.

On the other hand, the R^2 values obtained by the New York workers were much higher in the early stages of lactation than were the R^2 values obtained in this study. This was expected, however, because of the homogeneity of the data in this study. This homogeneity was the result of dividing the cows into twelve subpopulations. The New York study, on the other hand, corrected for age differences and developed only one equation for each stage of lactation.

It is concluded that the use of separate regression equations, depending on each cow's peak milk level, to predict complete lactation yield from cumulative month production is desirable. The increased precision obtained, as indicated by smaller S.E.E. values, is enough to warrant the use of more equations to predict total lactation yield.

When only the results of the first month's test are known, it is impossible to properly classify each cow according to her peak milk level, since the peak level occurs more frequently at the second test. Therefore, DHIA computing centers should use the regression equations developed for the first month of lactation from single test day production (Tables XIX and XX) to predict complete lactation milk and fat yield. When extrapolating records where the cumulative total for two or more months is known, the equations in Tables XVI and XVII should be used.

Because of the large standard errors of estimates obtained, the author does question the advisability of predicting complete lactation yield before the third or fourth month of lactation. If such predictions are to be made, then certainly dairymen using this information must understand that the probability of missing the true value by 1,000 to 1,800 lb of milk or 40 to 70 lb of fat is quite high.

As discussed earlier, possible uses of records extended from single test day results are for DHIA promotional efforts or the estimating of producing ability of cows in herds not on test for use by prospect ive buyers. It appears that the relationship between predicted records and complete lactation is sufficiently high for these to be worthwhile endeavors, except when the cow is in her first, second or tenth month of lactation. At these times, the S.E.E. approaches and sometimes exceeds 2,000 lb of milk. This suggests that with such a range of potential error little confidence can be placed on the estimate.

Ratio Factors for Predicting Complete Lactation Yield from Partial Records

Regression equations derived for predicting total lactation yield from partial records have been shown to provide estimates that are slightly more precise than are estimates made by a ratio method (21). Too, regression equations are readily utilized by high speed data processing equipment currently being used by many Dairy Herd Improvement organizations. The ratio method has appealed to many because it is easier to use in the absence of electronic computers and because the derivation of the ratio factors may be explained to dairymen more easily.

Cumulative Month Ratio Factors

Factors for extending cumulative month milk and fat records to a 305-day basis by a ratio technique are presented in Tables IX and X, respectively. The ratio factor multiplied by production to date estimates 305-day production. Its use is illustrated by the same example used to illustrate cumulative month regression equations (page 30). Assuming that the cumulative milk (adjusted to 122 days) is 7,068 lb, the estimated 305-day production would be derived as follows:

$$\hat{Y} = \hat{\gamma}_R X$$

$$15{,}408 = (2.18)\ (7{,}068)$$

where:

$\hat{Y}$ = the predicted record,

$\hat{Y}_R$ = the appropriate ratio factor as defined earlier, and

X = the adjusted cumulative record.

The ratio estimate of 15,408 lb of milk is 185 lb more than that estimated by regression. There are several reasons why such differences will exist. These include:

1. Only the regression method considers the influence of season of freshening.
2. The ratio method does not consider the effect of days open while its effect is included in the regression equations.
3. Only the regression method is a least squares analysis. Using the ratio method, the estimate obtained is usually slightly biased when the sample is small and/or if the coefficient of variation of both $\bar{x}$ and $\bar{y}$ exceed 10 per cent (3).

Marked differences between ratio factors of different lactation number groups are noted in Tables IX and X, with the differences between second and third lactation groups being smaller than those between the first and second lactation groups.

The differences between PML classes within each lactation number class are almost as large as those observed between lactation number groups. It is concluded that a different relationship between partial and complete lactation exists for low and high producers in all three age groups. This conclusion is contradictory to those of Madden et al.

TABLE IX

RATIO FACTORS FOR ESTIMATING TOTAL LACTATION MILK YIELD FROM CUMULATIVE MONTHLY PRODUCTION

Lact.	PML	Ratio Factor for Cumulative Month							
No.	Class	2	3	4	5	6	7	8	9
1	1	4.44	2.96	2.24	1.82	1.55	1.35	1.20	1.09
1	2	4.24	2.85	2.18	1.79	1.52	1.33	1.19	1.08
1	3	4.08	2.72	2.09	1.72	1.48	1.30	1.18	1.08
2	1	3.99	2.69	2.06	1.70	1.46	1.29	1.16	1.07
2	2	3.84	2.62	2.02	1.67	1.44	1.27	1.15	1.07
2	3	3.73	2.54	1.97	1.62	1.41	1.26	1.15	1.06
2	4	3.55	2.46	1.92	1.60	1.39	1.24	1.14	1.06
3	1	4.24	2.78	2.12	1.73	1.48	1.30	1.17	1.07
3	2	3.94	2.65	2.03	1.67	1.44	1.27	1.15	1.06
3	3	3.80	2.57	1.98	1.64	1.42	1.26	1.15	1.06
3	4	3.71	2.53	1.96	1.63	1.41	1.26	1.15	1.06

TABLE X

RATIO FACTORS FOR ESTIMATING TOTAL LACTATION FAT YIELD FROM CUMULATIVE MONTHLY PRODUCTION

Lact.	PML	Ratio Factor for Cumulative Month							
No.	Class	2	3	4	5	6	7	8	9
1	1	4.42	2.99	2.28	1.86	1.57	1.37	1.21	1.09
1	2	4.18	2.87	2.21	1.82	1.55	1.35	1.20	1.09
1	3	3.90	2.69	2.09	1.73	1.49	1.32	1.19	1.08
2	1	4.15	2.81	2.14	1.75	1.49	1.31	1.18	1.08
2	2	3.85	2.65	2.06	1.70	1.46	1.29	1.17	1.07
2	3	3.67	2.55	2.00	1.66	1.44	1.28	1.16	1.07
2	4	3.49	2.47	1.94	1.62	1.41	1.26	1.15	1.06
3	1	4.13	2.76	2.10	1.73	1.48	1.30	1.17	1.07
3	2	3.82	2.63	2.04	1.68	1.45	1.28	1.16	1.07
3	3	3.71	2.53	1.96	1.63	1.41	1.26	1.15	1.06
3	4	3.58	2.51	1.97	1.65	1.43	1.27	1.15	1.07

(21). Lamb (16), on the other hand, cited studies by Harvey who concluded there was a slight tendency for higher producing cows to decline more rapidly in production than did the lower producing cows. He also cited studies by Kendrick who concluded that low and high producing cows under four years of age required separate extension factors. A possible explanation as to why this study showed definite differences in all age groups is the higher average production level of the cows studied. Average 305-day production of the cows in the study by Madden et al. (21) was 12,167 lb of milk, which was 3,022 lb less than that produced by the average cow in this study.

Separate ratio factors for milk and fat appear to be necessary. This is in agreement with most earlier work (18, 23, 27, 41). In this study, the derived ratio factors for milk are generally larger than those for fat in the second cumulative month, about the same after three months, and generally smaller than fat from the fifth through the eighth cumulative months. This phenomenon is attributed to the fact that fat tests are generally higher in the first month of lactation than in later stages of lactation.

The amount of variability in the 305-day yield which was accounted for by stage of lactation is indicated by the R^2 values. The milk values are summarized in Table XI and the values for fat are summarized in Table XII. The R^2 values resulting from ratio predictions range from 0 to 0.07 points below those calculated from the regression predictions. On the average, the components of the regression equations for milk accounted for 2.7, 1.8, and 1.2 per cent more of the variation than was accounted for by the ratio factors in cumulative months 2, 3 and 4, respectively. Similar trends were observed when the R^2 values for fat

TABLE XI

R^2 VALUES BETWEEN TOTAL MILK PREDICTED BY CUMULATIVE MONTH RATIO ESTIMATORS AND ACTUAL TOTAL MILK PRODUCTION

Lact.	PML	Cumulative Month							
No.	Class	2	3	4	5	6	7	8	9
1	1	0.41	0.59	0.71	0.81	0.88	0.93	0.96	0.99
1	2	0.17	0.38	0.57	0.71	0.82	0.90	0.95	0.98
1	3	0.06	0.31	0.45	0.65	0.81	0.90	0.96	0.99
2	1	0.08	0.24	0.47	0.65	0.77	0.87	0.94	0.98
2	2	0.21	0.40	0.56	0.70	0.81	0.89	0.94	0.98
2	3	0.09	0.31	0.53	0.70	0.81	0.89	0.95	0.99
2	4	0.29	0.43	0.56	0.69	0.80	0.88	0.94	0.98
3	1	0.14	0.57	0.75	0.85	0.90	0.94	0.97	0.99
3	2	0.15	0.43	0.59	0.70	0.81	0.89	0.94	0.98
3	3	0.14	0.40	0.57	0.70	0.81	0.88	0.94	0.98
3	4	0.16	0.35	0.52	0.67	0.78	0.88	0.95	0.98

TABLE XII

R^2 VALUES BETWEEN TOTAL FAT PREDICTED BY CUMULATIVE MONTH RATIO ESTIMATORS AND ACTUAL TOTAL FAT PRODUCTION

Lact.	PML	Cumulative Month							
No.	Class	2	3	4	5	6	7	8	9
1	1	0.44	0.57	0.68	0.78	0.85	0.91	0.95	0.98
1	2	0.31	0.46	0.61	0.72	0.82	0.90	0.95	0.98
1	3	0.32	0.39	0.51	0.68	0.82	0.90	0.95	0.98
2	1	0.12	0.26	0.45	0.60	0.73	0.84	0.93	0.97
2	2	0.35	0.49	0.60	0.71	0.81	0.89	0.94	0.98
2	3	0.33	0.46	0.60	0.71	0.81	0.89	0.95	0.98
2	4	0.49	0.61	0.68	0.78	0.85	0.91	0.96	0.99
3	1	0.32	0.51	0.69	0.80	0.88	0.93	0.97	0.99
3	2	0.31	0.45	0.57	0.70	0.81	0.88	0.94	0.98
3	3	0.28	0.43	0.58	0.69	0.80	0.88	0.94	0.98
3	4	0.31	0.45	0.56	0.68	0.78	0.88	0.94	0.98

were compared.

Single Month Ratio Factors

Factors for extending single month milk and fat records to a 305-day basis are given in Tables XIII and XIV, respectively. The appropriate ratio factor, given the month or stage of lactation, multiplied by daily milk or daily fat production estimates 305-day production. This set of factors differs from the regression factors for the same reasons stated in the discussion on "cumulative month ratio factors," page 42. In addition, they differ from the cumulative month ratio factors because peak milk level cannot be determined.

TABLE XIII

RATIO FACTORS FOR ESTIMATING TOTAL LACTATION MILK YIELD FROM SINGLE TEST DAY RECORDS

Lact.	Ratio Factor for Single Month									
No.	1	2	3	4	5	6	7	8	9	10
1	276	259	271	287	302	315	330	346	364	408
2	235	229	250	273	297	324	356	398	465	792
3	242	229	246	270	294	325	355	401	460	603

TABLE XIV

RATIO FACTORS FOR ESTIMATING TOTAL LACTATION FAT YIELD FROM SINGLE TEST DAY RECORDS

Lact. No.	1	2	3	4	5	6	7	8	9	10
1	261	269	285	300	312	322	330	340	351	383
2	221	243	264	285	307	329	353	389	442	595
3	219	238	261	284	306	330	358	397	447	573

An example to illustrate the use of single month ratio factors is given. If a first lactation cow produced 48.0 lb of milk when sampled in the fourth month of lactation, then the predicted total milk production would be calculated as follows:

$$\hat{Y} = \hat{\gamma}_R X$$

$$13{,}776 = (287)\ (48.0)$$

where:

$\hat{Y}$ = the predicted record,

$\hat{\gamma}_R$ = the appropriate ratio factor as defined earlier, and

X = the test day milk or fat record.

The ratio estimate of total milk yield is 295 lb more than that estimated by regression.

An inspection of Tables XIII and XIV will show a marked difference between ratio factors in the lactation number 1 and 2 groups. Smaller and somewhat inconsistent differences between lactation number classes 2 and 3 are evident. This inconsistency, or failure for one class to consistently have the higher ratio factors, is particularly noticeable in the factors derived for estimating total milk yield. This fact supports the contention of Madden et al. (19, 20, 21) that the shape of the lactation curve of second lactation cows is not materially different from that of older cows, and that they logically can be grouped together. The difference between sets of factors derived to predict total fat production is more pronounced and it is concluded that separate factors should be used to provide uniformity in the use of ratio factors.

The R^2 values between predicted production and actual production is given in Table XV. These values are never more than 0.02 units lower than the R^2 values between total production estimated by the regression method and actual production given in Table VII. This suggests that when total production is estimated from single test day records, the least squares analysis and the inclusion of season effect accounts for very little variation not already accounted for by the ratio method.

TABLE XV

R^2 VALUES BETWEEN TOTAL PRODUCTION PREDICTED FROM SINGLE TEST DAY RECORDS BY THE RATIO METHOD AND ACTUAL TOTAL PRODUCTION

Month of Lact.	Lact. No. 1		Lact. No. 2		Lact. No. 3	
	Milk	Fat	Milk	Fat	Milk	Fat
1	0.37	0.38	0.33	0.36	0.34	0.35
2	0.61	0.50	0.56	0.48	0.60	0.47
3	0.67	0.54	0.67	0.53	0.70	0.54
4	0.73	0.59	0.75	0.60	0.72	0.55
5	0.78	0.66	0.80	0.65	0.77	0.61
6	0.78	0.65	0.78	0.67	0.77	0.64
7	0.72	0.61	0.77	0.67	0.76	0.65
8	0.71	0.59	0.72	0.64	0.71	0.62
9	0.67	0.58	0.61	0.55	0.63	0.55
10	0.53	0.46	0.44	0.44	0.50	0.46

Comparison and Application of Cumulative and Single Month Ratio Factors

The most extensive set of ratio factors published to date is that of McDaniel et al. (23). More than 132,000 lactation records of Holstein cows were included in the study. Their results were generally quite similar to those found in this study in that projection factors varied substantially between milk and fat, and between age groups.

They found that factors for fat were generally higher than those for milk, especially in the later stages of lactation.

A comparison of the ratio factors in the two studies showed that the factors in this study were consistently larger than those published by the U.S.D.A. workers. This difference is attributed to the fact that the U.S.D.A. study included records of short duration, while lactations completed in less than 305 days were deleted from this study.

Based on the results of this study, it appears that enough more variation is accounted for by using the regression method to predict total yield from cumulative production that its use is warranted, particularly when high speed electronic computers are available. However, for on-the-farm use, where such equipment is not available, the ratio estimators could be utilized without sacrificing much precision.

SUMMARY AND CONCLUSIONS

A total of 43,610 monthly test-day records from 4,361 Holstein cows in 19 California herds were studied to: (a) determine the importance of certain factors affecting the precision of prediction factors used in extending incomplete lactation records; and (b) develop improved regression equations or ratio factors for predicting complete lactation records from either DHIA cumulative month records or single test-day results.

The separate analyses of monthly means for both milk and fat indicated statistical differences because of lactation number (age), season of freshening, level of peak milk production, days open (not bred), month or stage of lactation, and interactions between these effects. The large interactions of lactation number by month and peak milk level by month, which indicated differences in the shape of the lactation curves, resulted in the development of separate prediction factors for cows of three different age groups and four different levels of peak milk production.

A least squares (multiple regression) analysis was used to derive extrapolation factors to predict completed records from partial lactation milk and fat records. Multiple correlation coefficients squared (R^2) were used to measure the amount of variation accounted for by the variables included in the equations. These were cumulative production, season of freshening, and days open. Predicted records, based on six months cumulative production accounted for about 83 and 84 per cent of the variation in total milk and fat, respectively, and increased with each additional month until near unity was obtained. Such records appear quite satisfactory as a tool in culling and herd management. Because of the large standard errors of estimates obtained, the advisability of predicting complete lactation yield before the third month of lactation is questionable.

When only a single monthly test is obtained, as might be the case in DHIA promotional programs or when special tests are conducted for the benefit of prospective buyers, the relationship between the predicted record and the complete lactation record is lowered considerably, but still high enough (R^2 values ranging from 0.50 to 0.75) between the third and ninth months to be useful. When only a single test is obtained in either the first, second or last month of lactation, the standard error of estimate may exceed 2,000 lb of milk, suggesting such a range of potential error that little confidence can be placed on the estimate.

Regression equations for predicting total yield from partial records were shown to be slightly more precise than estimates made by the ratio method. Even though regression equations are readily utilized by high speed data processing equipment being used by many Dairy Herd Improvement organizations, the ratio method has appealed to many because of its simplicity. Thus, ratio factors were developed and compared to the regression estimators. The regression method accounted for up to seven per cent more variation than did the ratio estimators, with the larger differences occurring early in lactation.

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A P P E N D I X

TABLE XVI

REGRESSION FACTORS FOR ESTIMATING TOTAL LACTATION MILK YIELD FROM CUMULATIVE PRODUCTION

Lact. No.	PML Class	Cum. Month	β_0	β_1	Season 1	Season 2	Season 3	β_2	S.E.E.	R^2
1	1	2	3,433	3.09	12	162	-174	1.68	1,203	0.43
1	1	3	1,366	2.56	65	109	-174	1.86	1,005	0.60
1	1	4	388	2.13	101	35	-136	1.71	842	0.72
1	1	5	- 177	1.82	121	- 33	- 88	1.53	681	0.82
1	1	6	- 396	1.57	109	- 69	- 40	1.42	543	0.88
1	1	7	- 458	1.38	84	- 80	- 04	1.06	413	0.93
1	1	8	- 426	1.23	60	- 75	15	0.97	293	0.97
1	1	9	- 292	1.10	26	- 44	18	0.67	168	0.99
1	2	2	2,984	2.45	149	75	-224	1.01	1,352	0.18
1	2	3	1,947	2.45	156	49	-205	1.12	1,166	0.39
1	2	4	66	2.15	181	- 30	-151	1.12	969	0.58
1	2	5	- 668	1.85	163	- 71	- 92	1.40	796	0.72
1	2	6	-1,063	1.62	146	- 92	- 24	1.27	631	0.82
1	2	7	-1,106	1.42	121	- 93	- 28	1.25	460	0.91
1	2	8	- 901	1.26	89	- 85	- 04	1.07	328	0.95
1	2	9	- 552	1.12	49	- 50	01	0.66	193	0.98
1	3	2	9,733	1.78	708	93	-801	1.90	1,958	0.14
1	3	3	1,060	2.58	698	- 12	-686	-0.42	1,666	0.38
1	3	4	- 996	2.22	574	39	-613	0.38	1,491	0.50
1	3	5	-4,489	2.18	417	35	-452	1.08	1,183	0.69
1	3	6	-4,361	1.86	287	- 19	-268	1.29	893	0.82
1	3	7	-3,591	1.58	141	46	-187	1.35	651	0.91
1	3	8	-2,566	1.35	52	24	- 76	1.02	423	0.96
1	3	9	-1,155	1.15	29	- 11	- 18	0.21	229	0.99
2	1	2	7,632	1.31	578	-395	-183	-1.71	1,577	0.14
2	1	3	3,856	1.79	455	-301	-154	-1.71	1,438	0.30
2	1	4	701	1.96	320	-181	-139	-1.92	1,213	0.49
2	1	5	- 750	1.84	253	-123	-130	-1.94	981	0.67
2	1	6	-1,125	1.62	209	-142	- 67	-1.63	788	0.79
2	1	7	-1,427	1.47	188	-127	- 61	-1.61	587	0.88
2	1	8	-1,052	1.29	123	-116	- 07	-1.22	403	0.94
2	1	9	- 587	1.13	68	- 64	- 04	-0.45	235	0.98
2	2	2	3,619	2.80	-132	125	07	1.37	1,608	0.21
2	2	3	563	2.49	- 56	113	- 57	0.75	1,402	0.40
2	2	4	- 621	2.10	38	56	- 94	0.51	1,204	0.56
2	2	5	-1,456	1.84	60	24	- 84	0.32	987	0.70
2	2	6	-1,587	1.60	81	11	- 92	0.29	794	0.81
2	2	7	-1,551	1.41	111	- 37	- 74	0.32	604	0.89
2	2	8	-1,191	1.25	102	- 59	- 43	0.39	421	0.95
2	2	9	- 702	1.12	55	- 42	- 13	0.34	230	0.98

TABLE XVI (Continued)

Lact. No.	PML Class	Cum. Month	β_0	β_1	Season 1	Season 2	Season 3	β_2	S.E.E.	R^2
2	3	2	7,973	1.85	-226	160	66	1.21	1,856	0.09
2	3	3	- 204	2.55	- 97	124	- 27	0.97	1,617	0.31
2	3	4	-3,799	2.43	19	84	-103	0.37	1,339	0.53
2	3	5	-4,670	2.10	106	- 08	- 98	0.42	1,065	0.70
2	3	6	-4,313	1.79	155	- 64	- 91	0.71	843	0.81
2	3	7	-3,587	1.53	170	- 84	- 86	1.03	629	0.90
2	3	8	-2,521	1.32	152	- 97	- 55	0.97	427	0.95
2	3	9	-1,296	1.14	76	- 57	- 19	0.59	229	0.99
2	4	2	7,439	2.05	-455	249	206	2.01	1,819	0.31
2	4	3	4,366	1.82	-380	214	166	2.80	1,617	0.45
2	4	4	2,685	1.61	-225	107	118	2.11	1,437	0.57
2	4	5	1,010	1.50	- 72	53	19	0.90	1,207	0.69
2	4	6	- 221	1.40	04	10	- 14	0.79	977	0.80
2	4	7	-1,067	1.31	83	- 35	- 48	0.82	749	0.88
2	4	8	-1,210	1.21	91	- 61	- 30	0.77	510	0.95
2	4	9	- 709	1.09	33	- 20	- 13	0.48	271	0.98
3	1	2	4,902	2.41	169	-556	387	0.82	1,990	0.18
3	1	3	-1,089	3.06	147	-252	105	0.33	1,437	0.57
3	1	4	-1,439	2.38	113	- 85	- 28	0.02	1,091	0.75
3	1	5	-1,345	1.92	139	22	-161	0.52	829	0.86
3	1	6	-1,087	1.60	143	12	-155	0.66	667	0.91
3	1	7	- 846	1.38	136	- 07	-129	0.58	523	0.94
3	1	8	- 594	1.22	112	- 35	- 77	0.37	377	0.97
3	1	9	- 518	1.11	91	- 54	- 37	0.40	205	0.99
3	2	2	5,030	2.41	28	- 24	- 04	4.13	1,773	0.16
3	2	3	- 645	2.69	61	- 32	- 29	2.96	1,457	0.43
3	2	4	-1,665	2.23	87	- 26	- 61	1.96	1,238	0.59
3	2	5	-1,753	1.86	125	- 12	-113	1.02	1,052	0.71
3	2	6	-2,086	1.63	181	- 34	-147	1.01	823	0.82
3	2	7	-1,858	1.42	202	- 71	-131	1.07	624	0.90
3	2	8	-1,391	1.26	159	- 67	- 92	0.98	439	0.95
3	2	9	- 845	1.12	91	- 45	- 46	0.70	238	0.98
3	3	2	5,969	2.36	-138	86	52	2.67	1,889	0.15
3	3	3	- 762	2.64	- 12	51	- 39	2.28	1,584	0.40
3	3	4	-2,508	2.25	67	61	-128	1.85	1,339	0.57
3	3	5	-3,311	1.94	151	18	-169	1.47	1,116	0.70
3	3	6	-3,140	1.66	178	- 10	-168	1.59	769	0.81
3	3	7	-2,623	1.44	195	- 40	-155	1.53	676	0.89
3	3	8	-1,959	1.27	172	- 66	-106	1.38	470	0.95
3	3	9	-1,116	1.12	96	- 52	- 44	0.96	258	0.98

TABLE XVI (Continued)

Lact. No.	PML Class	Cum. Month	β_0	β_1	Season 1	Season 2	Season 3	β_2	S.E.E.	R^2
3	4	2	8,409	1.95	51	123	-174	6.08	2,111	0.19
3	4	3	2,710	2.08	90	156	-246	5.67	1,844	0.38
3	4	4	- 780	1.97	157	103	-260	5.37	1,569	0.55
3	4	5	-2,491	1.79	259	30	-289	4.62	1,301	0.69
3	4	6	-2,974	1.59	276	- 10	-266	3.87	1,043	0.80
3	4	7	-3,053	1.43	258	- 54	-204	3.25	750	0.90
3	4	8	-2,360	1.27	206	- 75	-131	2.45	506	0.95
3	4	9	-1,406	1.13	117	- 65	- 52	1.49	263	0.99

TABLE XVII

REGRESSION FACTORS FOR ESTIMATING TOTAL LACTATION FAT YIELD FROM CUMULATIVE PRODUCTION

Lact. No.	PML Class	Cum. Month	β_0	β_1	Season 1	Season 2	Season 3	β_2	S.E.E.	R^2
1	1	2	180	2.59	11	-09	-02	0.06	43	0.46
1	1	3	117	2.16	11	-11	00	0.06	37	0.60
1	1	4	71	1.89	09	-12	03	0.05	32	0.70
1	1	5	39	1.67	07	-13	06	0.04	26	0.80
1	1	6	20	1.49	05	-11	06	0.03	21	0.87
1	1	7	04	1.34	02	-09	07	0.02	16	0.92
1	1	8	-03	1.21	01	-06	05	0.02	12	0.96
1	1	9	-07	1.10	00	-03	03	0.02	7	0.99
1	2	2	248	2.19	14	-11	-03	0.00	50	0.33
1	2	3	156	1.98	12	-14	02	0.03	44	0.49
1	2	4	96	1.79	11	-15	04	0.02	37	0.63
1	2	5	47	1.64	08	-15	07	0.02	31	0.75
1	2	6	17	1.49	05	-13	08	0.01	25	0.84
1	2	7	-01	1.35	03	-10	07	0.01	18	0.92
1	2	8	-12	1.23	02	-07	05	0.02	13	0.96
1	2	9	-10	1.11	01	-03	02	0.01	7	0.99
1	3	2	249	2.29	22	-01	-23	0.03	60	0.38
1	3	3	194	1.85	20	00	-20	0.07	57	0.44
1	3	4	108	1.76	19	-01	-18	0.09	51	0.54
1	3	5	22	1.72	12	-01	-11	0.11	42	0.69
1	3	6	-40	1.62	04	-01	-03	0.07	32	0.82
1	3	7	-40	1.42	01	00	-01	0.04	25	0.90
1	3	8	-28	1.25	-01	01	00	0.02	17	0.95
1	3	9	-12	1.10	00	-01	01	0.00	10	0.98
2	1	2	305	1.48	15	-26	11	-0.14	63	0.19
2	1	3	206	1.60	15	-27	12	-0.13	57	0.33
2	1	4	110	1.66	13	-22	09	-0.12	50	0.50
2	1	5	40	1.64	11	-19	08	-0.11	42	0.64
2	1	6	-01	1.52	09	-16	07	-0.09	34	0.76
2	1	7	-27	1.42	08	-12	04	-0.09	26	0.86
2	1	8	-27	1.27	06	-08	02	-0.06	18	0.94
2	1	9	-18	1.12	04	-04	00	-0.02	11	0.98
2	2	2	222	2.17	04	-01	-03	0.04	60	0.35
2	2	3	127	1.97	04	-06	02	0.05	53	0.49
2	2	4	72	1.75	04	-07	03	0.04	47	0.60
2	2	5	26	1.61	03	-08	05	0.02	40	0.72

TABLE XVII (Continued)

Lact. No.	PML Class	Cum. Month	β_0	β_1	Season 1	Season 2	Season 3	β_2	S.E.E.	R^2
2	2	6	-07	1.48	02	-07	05	0.01	32	0.82
2	2	7	-29	1.36	02	-07	05	0.01	24	0.89
2	2	8	-27	1.23	02	-05	03	0.00	17	0.95
2	2	9	-21	1.11	01	-03	02	0.01	10	0.98
2	3	2	221	2.22	02	-05	03	0.05	66	0.33
2	3	3	119	2.01	04	-07	03	0.04	59	0.47
2	3	4	36	1.86	06	-10	04	0.02	50	0.61
2	3	5	-07	1.68	06	-11	05	0.01	43	0.71
2	3	6	-44	1.55	05	-10	05	0.00	35	0.82
2	3	7	-59	1.40	04	-08	04	0.01	26	0.90
2	3	8	-54	1.26	04	-06	02	0.01	18	0.95
2	3	9	-32	1.12	02	-03	01	0.01	10	0.99
2	4	2	245	2.04	02	-03	01	0.12	65	0.50
2	4	3	169	1.75	-01	-03	04	0.10	57	0.62
2	4	4	113	1.57	02	-07	05	0.07	51	0.69
2	4	5	58	1.47	02	-05	03	0.01	43	0.78
2	4	6	20	1.37	03	-06	03	0.00	36	0.85
2	4	7	06	1.25	02	-04	02	0.00	27	0.91
2	4	8	-07	1.16	02	-03	01	0.00	19	0.96
2	4	9	-11	1.08	01	-01	00	0.00	10	0.99
3	1	2	217	2.12	27	-18	-09	0.08	68	0.39
3	1	3	117	2.02	25	-18	-07	0.06	58	0.56
3	1	4	28	1.95	18	-15	-04	0.03	46	0.72
3	1	5	-08	1.73	14	-10	-04	0.05	37	0.82
3	1	6	-32	1.56	10	-09	-01	0.04	29	0.89
3	1	7	-30	1.37	07	-07	00	0.02	22	0.93
3	1	8	-27	1.24	04	-04	00	0.01	16	0.97
3	1	9	-19	1.11	03	-04	01	0.01	9	0.99
3	2	2	253	1.93	07	-08	01	0.06	64	0.32
3	2	3	170	1.75	08	-12	04	0.05	57	0.47
3	2	4	110	1.59	07	-11	04	0.03	50	0.58
3	2	5	65	1.47	06	-11	05	0.01	42	0.71
3	2	6	24	1.38	05	-09	04	0.01	34	0.81
3	2	7	-05	1.29	05	-07	02	0.01	26	0.89
3	2	8	-16	1.19	04	-05	01	0.02	18	0.95
3	2	9	-16	1.09	02	-02	00	0.02	10	0.98
3	3	2	283	1.88	02	-06	04	0.06	68	0.28
3	3	3	176	1.77	03	-09	06	0.05	60	0.44
3	3	4	87	1.68	04	-10	06	0.04	52	0.58
3	3	5	24	1.58	05	-10	05	0.03	44	0.70

TABLE XVII (Continued)

Lact. No.	PML Class	Cum. Month	β_0	β_1	Season 1	Season 2	Season 3	β_2	S.E.E.	R^2
3	3	6	-14	1.45	05	-08	02	0.03	35	0.81
3	3	7	-33	1.34	05	-07	02	0.02	27	0.89
3	3	8	-36	1.22	05	-05	00	0.02	19	0.95
3	3	9	-26	1.11	03	-03	00	0.02	10	0.98
3	4	2	308	1.78	10	-05	-05	0.22	72	0.34
3	4	3	207	1.63	10	-07	-03	0.21	64	0.48
3	4	4	115	1.56	09	-07	-02	0.19	58	0.58
3	4	5	49	1.47	09	-08	-01	0.15	49	0.69
3	4	6	05	1.38	08	-07	-01	0.12	40	0.79
3	4	7	-28	1.30	06	-06	00	0.09	30	0.88
3	4	8	-34	1.20	04	-04	00	0.06	21	0.94
3	4	9	-31	1.10	03	-03	00	0.04	11	0.98

TABLE XVIII

FACTORS FOR ADJUSTING DAYS IN MILK TO CUMULATIVE MONTH[a]

Adjust.	Cumulative Month										Adjust.
Factor	1	2	3	4	5	6	7	8	9	10	Factor
	Days in Milk										
15	15	46	76	107	137	168	198	229	259	290	15
14	6	7	7	8	8	9	9	230	260	1	14
13	7	8	8	9	9	170	200	1	1	2	13
12	8	9	9	110	140	1	1	2	2	3	12
11	9	50	80	1	1	2	2	3	3	4	11
10	20	1	1	2	2	3	3	4	4	5	10
9	1	2	2	3	3	4	4	5	5	6	9
8	2	3	3	4	4	5	5	6	6	7	8
7	3	4	4	5	5	6	6	7	7	8	7
6	4	5	5	6	6	7	7	8	8	9	6
5	5	6	6	7	7	8	8	9	9	300	5
4	6	7	7	8	8	9	9	240	270	1	4
3	7	8	8	9	9	180	210	1	1	2	3
2	8	9	9	120	150	1	1	2	2	3	2
1	9	60	90	1	1	2	2	3	3	4	1
0	30	1	1	2	2	3	3	4	4	305	0
-1	1	2	2	3	3	4	4	5	5		-0
-2	2	3	3	4	4	5	5	6	6		-2
-3	3	4	4	5	5	6	6	7	7		-3
-4	4	5	5	6	6	7	7	8	8		-4
-5	5	6	6	7	7	8	8	9	9		-5
-6	6	7	7	8	8	9	9	250	280		-6
-7	7	8	8	9	9	190	220	1	1		-7
-8	8	9	9	130	160	1	1	2	2		-8
-9	9	70	100	1	1	2	2	3	3		-9
-10	40	1	1	2	2	3	3	4	4		-10
-11	1	2	2	3	3	4	4	5	5		-11
-12	2	3	3	4	4	5	5	6	6		-12
-13	3	4	4	5	5	6	6	7	7		-13
-14	4	75	5	136	6	197	7	258	8		-14
-15	45	-	106	-	167	-	228	-	289		-15

[a]Locate the appropriate "days in milk" in the body of the table. The heading at the top of the column is the correct cumulative month. The left- or right-hand side number on the same line is the adjustment factor to use.

TABLE XIX

REGRESSION FACTORS FOR ESTIMATING TOTAL LACTATION MILK YIELD FROM SINGLE TEST DAY PRODUCTION

Lact. No.	Month of Lact.	β_0	β_1	Season 1	Season 2	Season 3	S.E.E.	R^2
1	1	6,622	136	67	151	-218	1,634	0.37
1	2	3,914	181	115	74	-189	1,282	0.61
1	3	3,901	190	171	- 03	-168	1,170	0.68
1	4	3,497	209	195	-147	- 48	1,070	0.73
1	5	3,462	221	119	-172	53	953	0.79
1	6	3,560	228	43	-135	92	960	0.78
1	7	4,489	215	- 42	- 37	79	1,086	0.72
1	8	4,661	221	- 84	10	74	1,118	0.71
1	9	5,254	216	-175	178	- 03	1,181	0.67
1	10	7,206	179	-170	275	-105	1,407	0.54
2	1	7,310	120	-271	143	128	2,113	0.34
2	2	4,577	158	-167	107	60	1,714	0.57
2	3	3,502	191	- 50	98	- 48	1,499	0.67
2	4	3,517	208	78	- 16	- 62	1,310	0.75
2	5	3,810	220	47	- 87	40	1,155	0.80
2	6	4,302	229	- 04	- 73	77	1,213	0.78
2	7	5,211	229	04	-117	113	1,238	0.77
2	8	6,490	220	-222	- 75	297	1,369	0.72
2	9	8,065	201	-515	177	338	1,583	0.63
2	10	10,257	165	-617	301	316	1,903	0.46
3	1	8,189	121	-184	- 13	197	2,435	0.35
3	2	4,472	166	- 50	62	- 12	1,906	0.60
3	3	3,568	192	- 09	96	- 87	1,643	0.70
3	4	3,948	204	41	35	- 76	1,583	0.72
3	5	3,737	226	187	- 80	-107	1,446	0.77
3	6	5,146	221	64	-123	59	1,440	0.77
3	7	5,780	228	- 15	-211	226	1,459	0.76
3	8	7,010	225	-278	-168	446	1,594	0.72
3	9	8,525	213	-579	- 11	590	1,780	0.65
3	10	10,969	178	-787	233	554	2,060	0.53

TABLE XX

REGRESSION FACTORS FOR ESTIMATING TOTAL LACTATION FAT YIELD FROM SINGLE TEST DAY PRODUCTION

Lact. No.	Month of Lact.	β_0	β_1	Season 1	Season 2	Season 3	S.E.E.	R^2
1	1	287	106	12	-06	-06	56	0.39
1	2	215	149	11	-10	-01	50	0.51
1	3	211	160	06	-09	03	48	0.55
1	4	189	182	03	-07	04	45	0.60
1	5	174	199	-01	-05	06	41	0.67
1	6	181	200	-05	01	04	42	0.66
1	7	202	192	-07	08	-01	44	0.62
1	8	203	197	-02	11	-09	45	0.60
1	9	217	193	-02	15	-13	44	0.60
1	10	281	159	01	13	-14	51	0.49
2	1	314	93	-01	00	01	73	0.36
2	2	245	133	00	-04	04	66	0.48
2	3	213	160	-08	01	07	62	0.53
2	4	184	188	-04	-01	05	58	0.60
2	5	194	196	-11	03	08	54	0.66
2	6	184	217	-12	08	04	52	0.68
2	7	222	207	-09	10	-01	52	0.68
2	8	260	199	-10	11	-01	54	0.65
2	9	311	180	-18	18	00	60	0.57
2	10	381	148	-18	19	-01	67	0.46
3	1	340	93	01	-07	06	81	0.36
3	2	265	131	04	-08	04	75	0.47
3	3	238	156	-02	-06	08	68	0.54
3	4	226	175	-05	00	05	67	0.56
3	5	224	190	-05	00	05	63	0.62
3	6	243	193	-08	06	02	61	0.64
3	7	250	205	-07	06	01	59	0.66
3	8	292	197	-10	09	01	62	0.63
3	9	336	186	-15	09	06	67	0.56
3	10	412	157	-20	14	06	73	0.48

VITA

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Candidate for the Degree of

Doctor of Philosophy

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