THE DESIGN, CONSTRUCTION AND TESTING OF A CONCRETE HYPERBOLIC PARABOLOID SHELL INCORPORATING A PRECAST COLUMN-EDGE BEAM SYSTEM

By

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CHAPTER I

INTRODUCTION

A. The Statement of the Problem

Reinforced concrete roofs are used for buildings, churches, auditoriums, and many other well-known structures. Why shouldn't they be used for farm structures and rural buildings? Of course, concrete has been used in the construction of farm buildings! - but what about adapting precast shells?

These are some of the many queries which arose, and to answer them the following study was conducted. Because of its outstanding characteristics of:

(a) durability,

(b) adaptability,

(c) fire safety,

(d) maintainability,

reinforced concrete is a very suitable construction material. However, the cost and feasibility are rather important liabilities not to be lightly set aside. The location of the structure is also one of the major problems.

B. The Choice of Configuration

Once it was decided the project was to be a reinforced concrete structure incorporating precast units, the obvious subsequent question was, "what shape?"

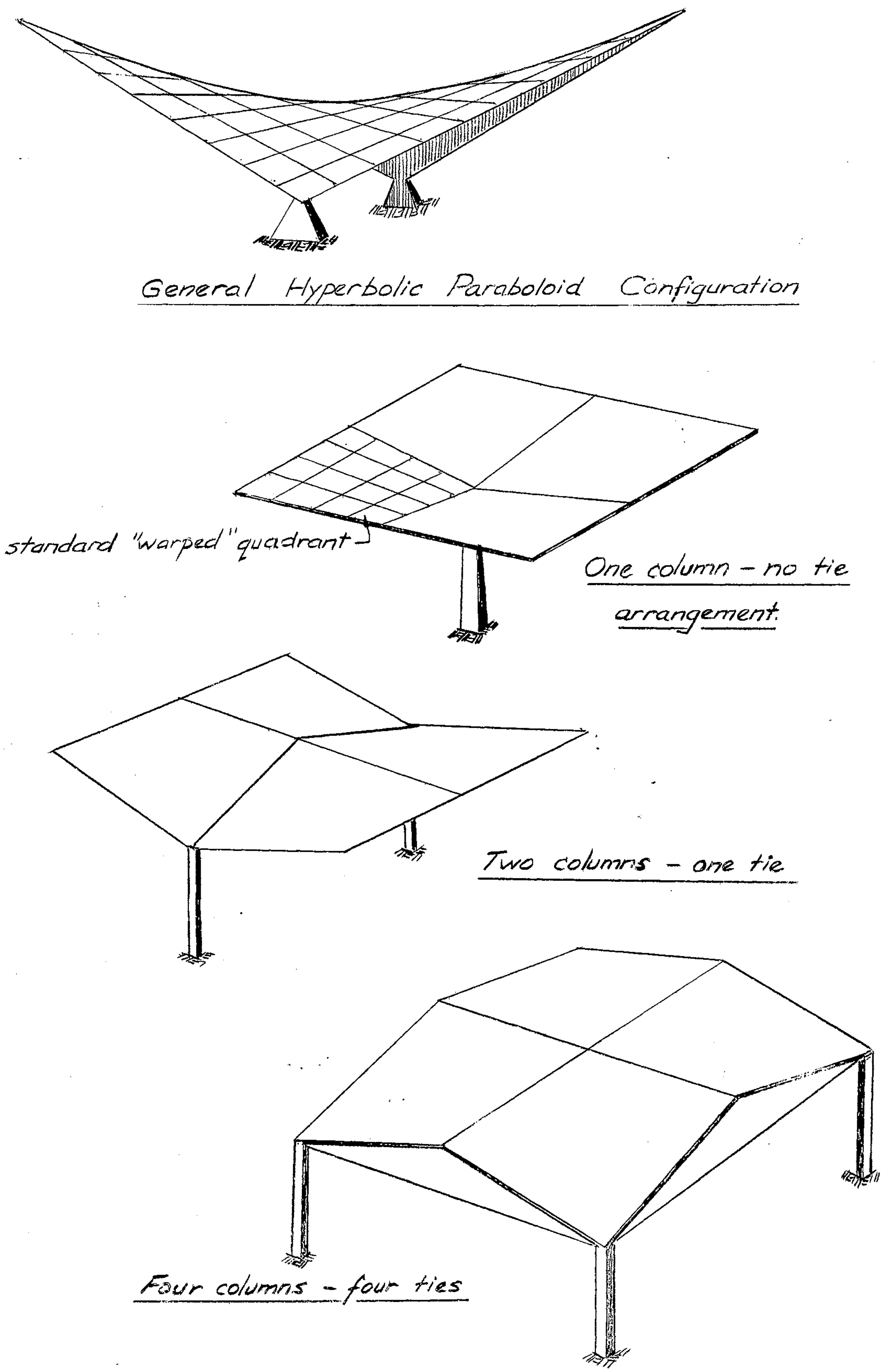


Figure 1. Various arrangements of the h-p shell.

Since we were concerned with reinforced concrete, the configuration to a large extent would be dominated by the formwork. In shell structures the formwork is usually the major expense, but if the shell or portions of it were prefabricated then there is a strong possibility that the formwork cost would be drastically reduced. Unfortunately, because shells rely almost entirely upon their monolithism for the transmission of stress, adequate connections must be provided. This is the primary drawback of all precast work.

Other questions which were considered are:

(i) What about the enclosed area?

(ii) Is it practical to construct?

(iii) Which shape requires the least material?

(iv) Can it adequately transmit the loads to the ground?

(v) Could it be suitably utilized as a farm structure?

After weighing the pros and cons it was decided to adopt the hyperbolic paraboloid shell. The main reasons are that it is a translational shell composed of straight generatrices which simplify the formwork considerably and that it requires the minimum of material to transmit stresses since it is the "ideal" shape for supporting uniformly distributed loads. An extended explanation of this configuration is given in the literature review.

The structure finally decided upon was a reinforced concrete hyperbolic paraboloid structure employing a so-called "composite construction." This "composite construction" consisted of prefabricating the columns and edge beams and subsequently casting the shell in place. (See drawing no. 1 in the Appendix.)

C. The Objectives of the Study

The objectives are:

1. To develope a design for an in situ concrete h-p shell incorporating a precast column edge beam system.

2. To study the performance of the structure and components both during construction and when subjected to various loading conditions.

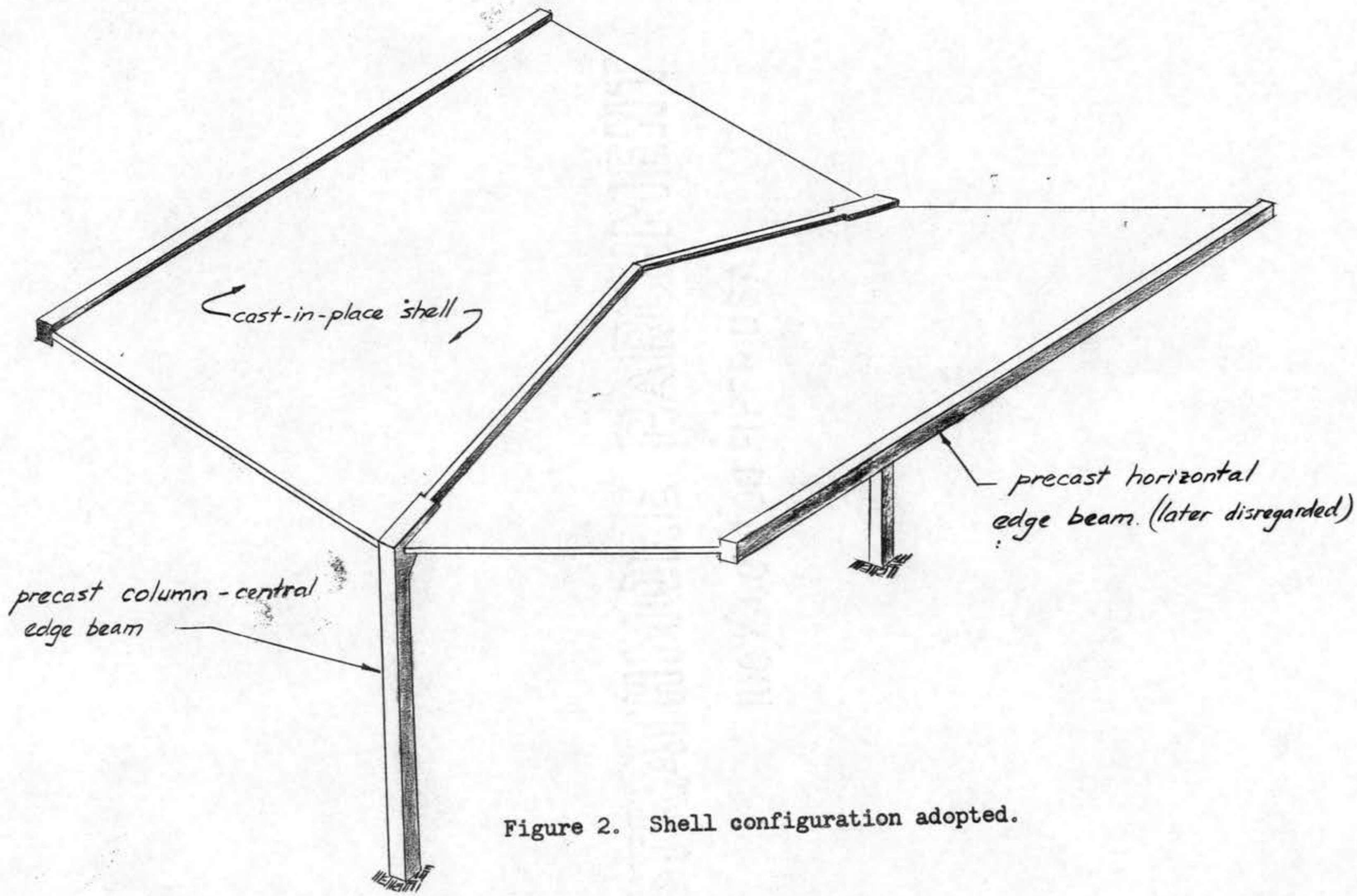


Figure 2. Shell configuration adopted.

CHAPTER II

LITERATURE REVIEW

A. Introduction

There are many types of concrete roofs, each having its own merits with regard to a particular shape of structure, location, aesthetic qualities and so on. When considering roofs for farm structures and rural buildings, basically what is required is a low cost, durable and easily maintained structure. Since reinforced concrete possesses the latter qualities, it is with the low cost characteristic we are most concerned.

This project is primarily concerned with rural structures which are both easy to construct and practical to use. A significant feature is that small contractors and rather limited equipment should be able to handle the construction of these buildings.

With this in mind an extensive initial study was made into the types of structures possible and the methods of construction.

B. Types

1. Frames

Frames or bents are quite common types of roof particularly suited to precast construction since they are moderately easy to divide into convenient sized and shaped components. It is evident that the size and shape aspect are critical in precast construction when transportation is required--especially over long distances.

For ribbed frame roofs the ribs or frames are usually spaced at 15 feet to 30 feet centers. This spacing, of course, depends upon the requirements of the structure concerned.

A precast panel system, based on the gable frame principle, was devised by Arsham Amirikian (1). In this case a thin shell is precast between two edge ribs thus forming a panel. These panels when bolted together, form a continuous roof.

Amirikian points out in his article that curved outlines of framing components are costly. To get functional and usable buildings, the preferable shape is one which results in the maximum usable floor area for a given amount of closure framing. A simple shaped frame, however, although less costly may develop comparatively lower strength and resistance than a shaped contour.

Structures making use of similar prefabricated panels may prove suitable for farm buildings. The span and height will have to be fairly large and this could cause lifting and transport difficulties. Another problem arises when the prefabrication of such elements is considered. A thin panel with tapered edge ribs, bolt holes, rebates and the like will require a fairly large precasting yard with skilled workmen. Therefore, unless, very conveniently situated, this type of structure appears to be unsuitable.

2. Folded Plates

A folded plate shell basically consists of flat "plates" or slabs of reinforced concrete configured in a suitable manner such that they form a rigid member in the direction of the span. This configuration is nearly always "sawtooth" in cross-section. The ribbing effect is necessary since the plates are very thin.

Due to its shape the folded plate shell does not present much difficulty as far as the formwork or construction is concerned. The amount of material required for a folded plate roof, however, will be betwen 1 1/2 to 2 times the amount required for an equivalent shell according to Haas (11). This may sound uneconomical, but Samuely and Whitney (25) say that folded plates have proved especially economical for large span roofs.

For the sloping "plate" in the folded plate shell, it is usually at such an angle that it forms less than 45 degrees with the horizontal. This then facilitates easy concrete placement without excessive formwork. Whitney, Anderson and Birnbaum (30) compared the folded plate construction with the ordinary joist construction and they show that it is much more economical for spans greater than 40 feet.

Folded plate construction, although suitable for prefabrication from the point of view of flat surfaces, causes difficulty in regard to the construction of the joints. Also, the precast elements will need to be fairly large to reduce the number of joints. This, then presents transport and erection headaches. Hence it appears to be beyond the capabilities of rural builders.

If, on the other hand, cast-in-place construction was used, folded plate shells should be more feasible. As already discussed, the reinforced concrete and formwork required will be in excess of that required for an equivalent shell. Therefore this will most probably prove uneconomical unless very large spans are encountered.

3. <u>Shell Arches</u>

Billig (10) devised an ingenious method of constructing economical housing in India. Hessian (burlap) was nailed between fixed arch ribs and concrete was trowelled, by hand, onto this hessian. The trowelling was done in layers so that when the second layer was applied the first layer had sufficient strength to support it and so on.

Corrugations which were essential for the structural strength, were formed automatically by the hessian which sagged under the weight of the concrete.

This type of construction would be completely unrealistic in a country like the United States since the labour costs in India are only a fraction of those in the United States.

Mensch carried out a theoretical and experimental investigation into the structural behaviour of concrete arch shells subjected to various types of loading. Pneumatic placement of concrete was employed in the construction of these arch shells.

The disadvantage of this type of construction is that there is a degree of uncertainty in the quality of the concrete. Much depends upon the skill and experience of the nozzleman and crew. Pneumatically placed concrete is more expensive per cubic foot than ordinary reinforced concrete, but less is used when wastage and strength characteristics are taken into account.

For rural structures this type of construction seems to hold many good openings. It is, however, still in its infancy and more research is needed to clarify its practical application.

4. Barrel Shells

Barrel shells are a very suitable shell to precast. Billner (4), a contractor, made full use of this quality when constructing barrel shell roofs for houses and offices in Columbia. On one set of quite simple and inexpensive forms, consisting of a few posts, stringers and a wooden deck, it was possible to cast eight panels per day. These panels were 17 feet square and 1 1/2 inches thick. The barrel shells were precast on the construction site at ground level. A "Vacuum Lifter" suspended from a crane was used to lift the shells into position.

Billner carried out a full scale test on a 31 feet by 20 feet shell. It was tested under a uniform load of 40 psf and also a concentrated load at midspan. From this test and experience Billner recommended panels which could span 80 feet longitudinally and 40 feet transversely. He suggested that they be cast in halves 20 feet by 80 feet and then the joint at the crown be made in situ with reinforcement projecting into it from each half.

The barrel shell might be suitable for farm structures in many respects but unless small components could be used, the lifting would be an expensive operation.

An alternative would be to use cast-in-place concrete. But then why use a barrel shell when an arch shell would be must simpler?

5. Hyperbolic Paraboloid Shells

A hyperbolic paraboloid shell has many distinct qualities which make it both practical and economical. Due to its double curvature, which consists of parabolas in two intersecting directions, it is the ideal shape for carrying uniformly distributed loads. Under uniform loading the shell is everywhere in shear in the direction of the generatrices (see figure 14) and the edge beams, due to this shear, are in direct tension or compression (22).

The h-p shell also has straight generatrices which simplify the formwork to a marked degree. This is a great advantage since the cost of formwork for a shell constitutes a major part of the total cost (21).

Madsen and Biggs (14) point out that the crux for economy in building many similar h-p shells was:

(a) The multiple re-use of forms.

(b) A strict schedule of operations.

The forms used by Madsen and Biggs were self-supporting and spanned between two movable devices at each end. Due to the fact that the h-p shell is made up of straight generating lines (as mentioned before), two straight trusses were used and 3/4 inch plywood was "warped" from one to the other.

Another type of formwork which proved to be economical made use of the column which had been cast previously, and aluminum trusses (7). The h-p shell in this case was the upturned umbrella type with a central column.

The aluminum trusses were placed at 10 feet centers and only required a minimum of supports. Plywood panels 3/8 inch thick was slotted to fit into the aluminum frame.

The assembly and dismantling of the formwork was rather time consuming. For this reason the contractor suggested a method of leaving the formwork in quarters and setting them on casters so that they could be used from shell to shell without dismantling component parts.

Mensch (16) developed a similar system of portable forms. These forms consisted essentially of a metal space truss with plywood as the mold former, which is made in such a way that it dismantles into convenient sized modules which are easy to handle. The main advantage of this system of forms was their reusability.

The conclusion to be gained from practical situations is that cast-in-place h-p shells are economically sound if the formwork is not unduly expensive.

Another way of reducing forming costs would be to prefabricate portion of the h-p shell and then to use it in conjunction with a minimum of formwork and supports. The most probable elements to precast would be the edge beams. These edge beams would be supported by simple frames or props, and could themselves support flat forms for casting the shell areas between them.

C. Methods of Construction

There are two main methods of constructing a reinforced concrete structure. Either the structure can be cast in position or else precast in a convenient place and then taken to the site and erected. It is often the case that a structure is made up of both in situ and precast sections.

1. In Situ Construction

As the name suggests the concrete is placed in its actual position in the structure. Various forming systems are used to mold the concrete until it hardens.

Cast in situ concrete has the great advantage of forming a monolithic structure. Therefore, its strength will not be reduced by joints and connections.

In countries where low cost labour and forming material are readily available, in situ concrete is the ideal method of construction.

2. Pneumatic Concrete Placement

This is actually a type of in situ construction but will be considered separately to distinguish it from the conventional type.

It consists essentially of forcing concrete through a hose by means of an air compressor such that it is "shot" rather than "placed" in position. There are various methods of mixing the concrete. The water can be added at the nozzle to a sand:cement mixture or else the concrete could be mixed before being forced through the nozzle. The second method is probably the most effective.

In structures with vertical or near vertical faces pneumatic placement of concrete has distinct advantages over ordinary concrete placement. The main one is that only a single layer of formwork need be provided. The water:cement ratio can be varied such that the concrete will stick and not slide down. For these reasons this is a good method for repairing concrete structures.

Pneumatic placement of concrete is the most suitable method of placing fresh concrete on steep surfaces such as are encountered in arch shells.

Another distinct advantage of this method is that the job can be stopped in an imcomplete state and then continued on the following day where it was left off. No fear need be attached to the adequacy of the bond at these construction joints according to Hession (12).

3. <u>Precast Construction</u>

In this method of construction, components of a structure are prefabricated, either in a casting yard, used exclusively for the purpose, or on site.

For greatest economy the size of the precast element should be as large as can be handled both in transit and in erection. Certain structures lend themselves to prefabrication. This is the cast particularly when a large number of repetitious elements constitute a structure.

It is often economical to combine precast with in situ construction. The in situ concrete can be placed after the erection of the precast elements and will then tend to produce continuity which so often is lacking in prefabricated structures.

D. Formwork

In any concrete structure, formwork must be used at some state or another. In situ concrete usually requires a vast quantity of falsework to support the forms in position. Precast concrete, on the other hand, needs little or no falsework.

The economy of formwork depends upon the number of uses, and the method of erecting and striking it. The crux of the economy definitely lies in the multiple re-use of the forms.

It is often economical to increase the quantity of concrete in order that the forms might be simplified or that the number of re-uses might be increased.

A very efficient method, suitable for particular types of structures, is the use of moving or "slip" forms. This is a continuous operation.

In certain cases a forming system can be placed on wheels and moved from one position to another. First, however, the concrete must be allowed to harden before they can be moved. Madsen and Biggs (14) made use of this type of formwork, which proved ideal for the repetitive nature of their task to construct forty-four identical h-p shells to form a continuous roof.

A novel approach to the formwork for a shell roof was made by Riley (24). Due to the expense of formwork and falsework for shell construction, Riley suggests the shell be prefabricated on site by making use of a mound of earth.

The earth is molded in the form of the shell and a polyethylene film is placed over it. Vertical sleeves are formed in the shell to enable precast columns to protrude up through the shell. These columns are erected before the shell is placed.

When the shell has been cast and is hardened, it is jacked up onto the columns and then wedged at the required level. Once the roof has been raised the mound of earth can easily be removed.

E. Conclusion

After considering the types of structures available and the methods of construction, it was recommended that a reinforced concrete hyperbolic paraboloid shell be designed, constructed and tested. It was also decided to use composite construction by precasting the columns and central edge beams and placing the shell and foundations in situ.

In arriving at the above conclusion, it might be worth while to indicate the major considerations.

1. What Shape?

The h-p shell was chosen because it lends itself to:

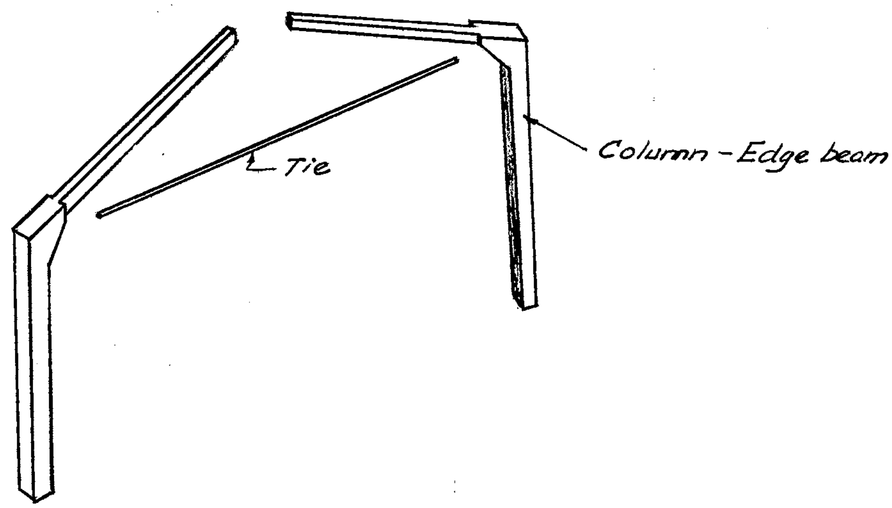


Figure 3. Prefabricated components.

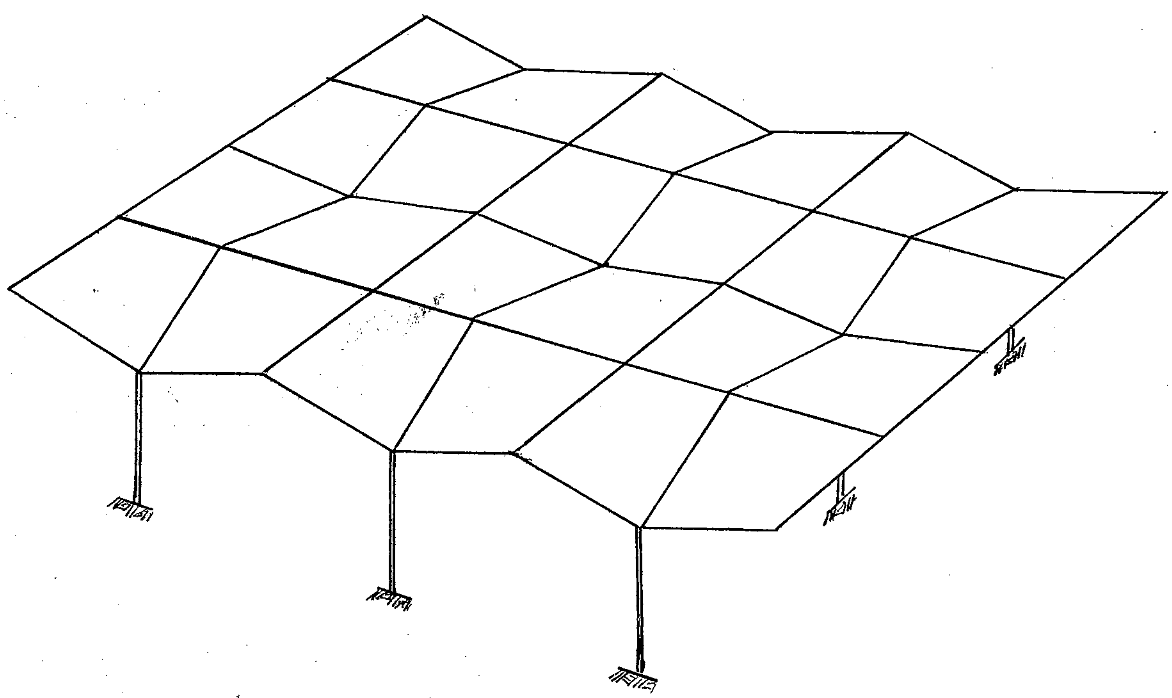


Figure 4. A typical combination of the unit shell.

(a) ideally resisting uniformly distributed loads,

(b) simple formwork composed of straight members,

(c) precast components.

An hyperbolic paraboloid shell can have one of many different arrangements. When employing the standard "warped" quadrants, they can be arranged with one, two or four columns. (See figure 1.)

For the construction of many shells the frames can be erected as for ordinary framed structures and used as a frame to support the formwork. Then the frames readjust themselves to become a column and edge beam when the shell has hardened. (See figure 4.)

There is the shear connection between the cast-in-place shell and the precast edge beam. Typical or proposed connections are illustrated in figure 5.

Connection 1, although it appears to be suitable for transmitting the shear stresses from the shell to the beam, is difficult to prefabricate--especially from the aspect of the formwork.

Type 2 is not so efficient. The shear stirrups would have to be very closely spaced for an adequate "flow" of shear stresses from the shell to the beam.

Connection 3, on the other hand, has one outstanding advantage over the other two: the shear in the shell causes no eccentric force on the edge beam. This shear is easily transferred to the beam through the shear bar and the concrete.

From the above reasoning, shear connection 3 was adopted for the test shell.

The second connection to be discussed is that which exists between two precast components. This only arises with large shells

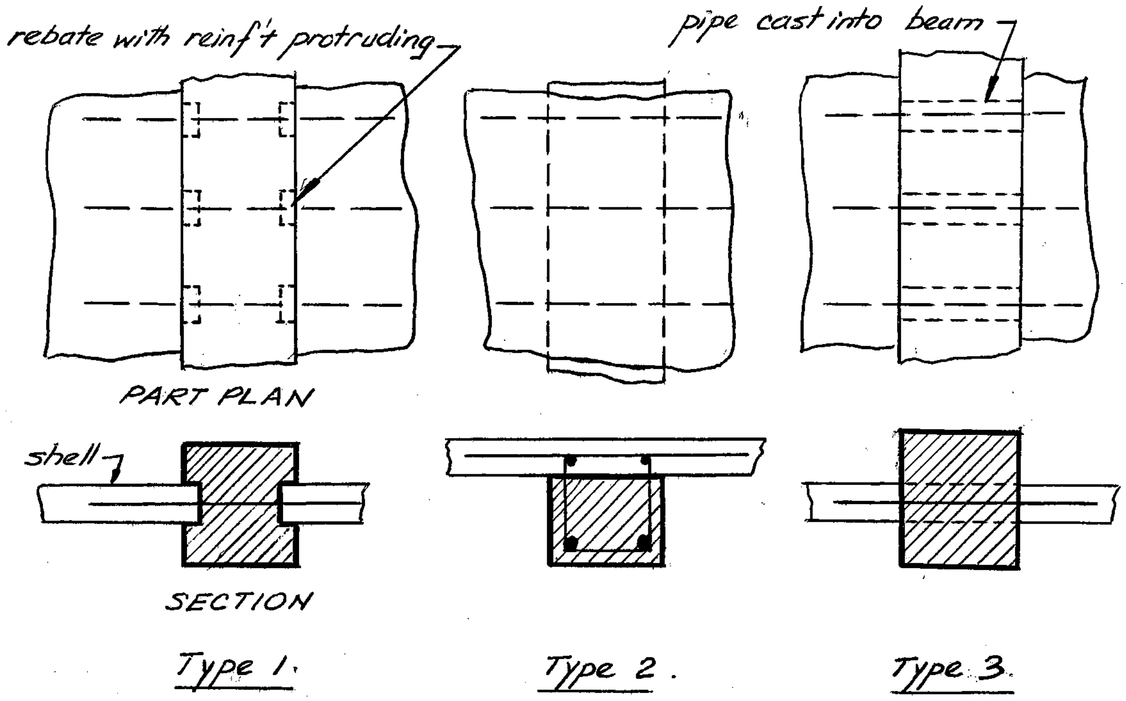


Figure 5. Shear connections between cast-in-place shell and precast edge beams.

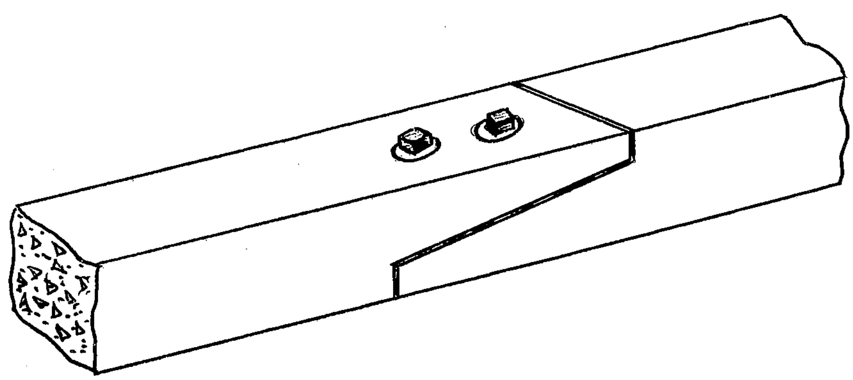


Figure 6. A bolted connection.

when the "dog leg" RSU in figure 3 is too large to transport or erect. In this case a joint is made at T (say) which must be capable of resisting initial bending stresses (if any) and a final compressive stress.

A bolted joint as in figure 6 was called for because it is a simple matter to connect during erection and it can withstand certain bending and compressive stresses.

To be certain of the capabilities of a bolted joint, compression tests were conducted on a joint such as is shown in figure 6. The reason compression will exist in the edge beam RS is that the shear stresses from the two adjacent quadrants transmit their stresses to the beam. The resulting compression varies linearly from zero at R to a maximum at S.

The results showed that for a 5 inches by 5 inches specimen the average ultimate compressive strength was about 77,000 pounds. The mode of failure was shear failure of the concrete which occurred after longitudinal cracking had developed.

For a practical sized shell of 40 feet by 40 feet by 2 1/2 inches thick having a live load of 30 lbs./sq. ft., the compression at the joint N in figure 8 would be of the order of 110,000 pounds. Therefore with a 9 inches by 6 inches edge beam this should easily be resisted. The shell which was finally chosen is shown in figure 8.

2. <u>What Supporting System Should Be Used?</u>

Now that the forms, precast components and connections are ascertained the supporting structure must be analysed.

The "rigid frame" should be able to take some of the load and the rest will be taken by a system of supports. For the interior supports ordinary props could be used. The corner supports, however, must be stable and capable of being adjusted. They should also be self-supporting and easily erected and transported.

The finally accepted supporting system shown in figure 38 made use of 2 inch by 4 inch (S4S) and double 2 inch by 6 inch (S4S) lumber props with a rigid corner supporting system. This corner supporting system was developed by a fellow research assistant (18) while engaged on a similar project.

TABLE I

Arrangement	Quantity of Material	Open Area Available (per shell)	Stability	Number of Ties Required
One Column	least	$\frac{ab}{2}$	poor	none
Two Column	intermediate	ab	fair	one
Four Column	most	ab	good	four

From Table I a brief comparison is readily achieved. It should be noted that only one shell is considered and when many are constructed next to one another, the stability and area would be affected, the one-column arrangement in particular.

The two-column h-p shell was selected because it has the outstanding feature of being conveniently split up into a gable frame and four warped quadrants. This factor made it ideal to precast the columns and cast the shell in place.

3. Why Precast?

To have certain parts of the structure precast and delivered

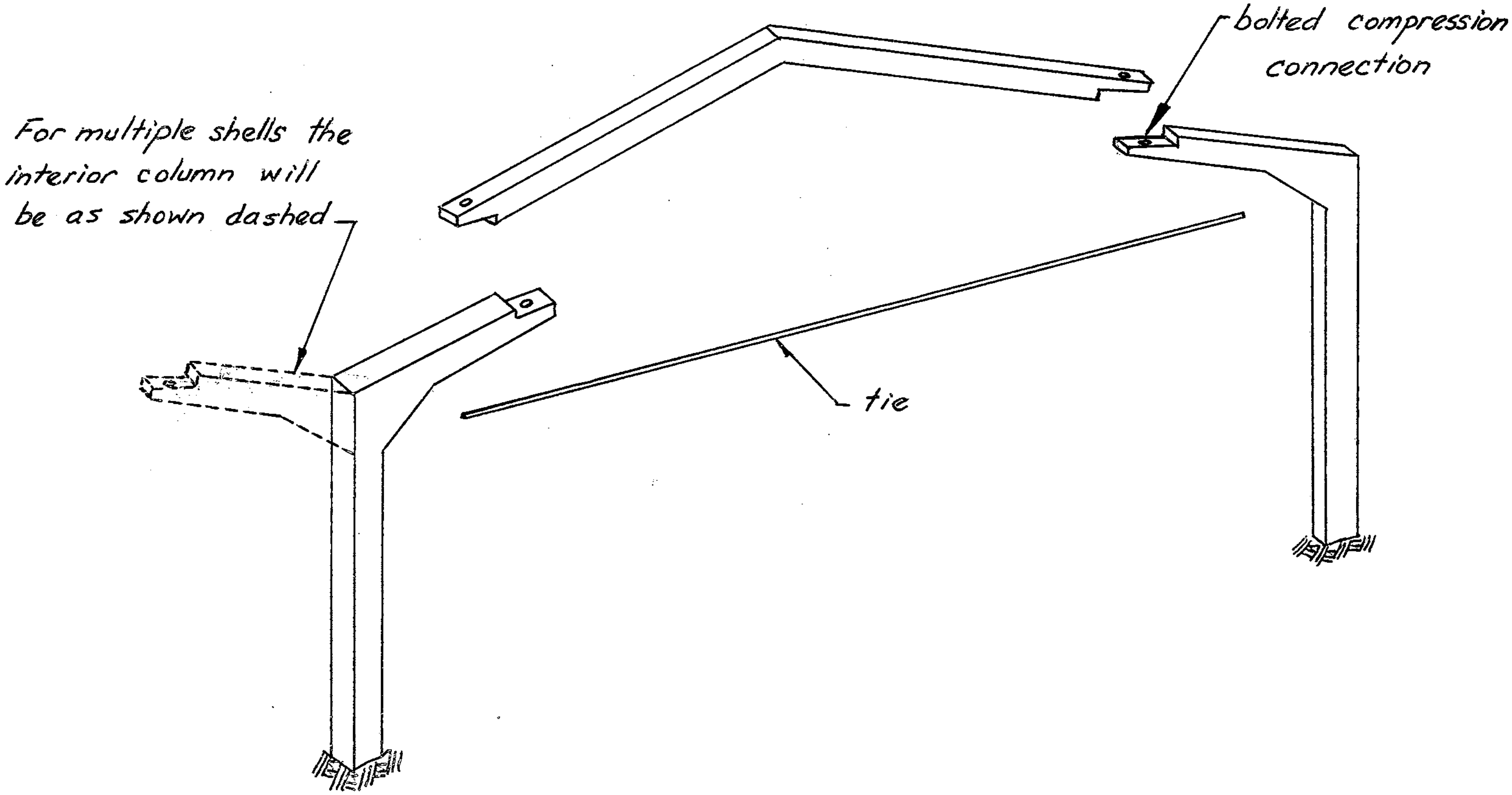


Figure 7. Precast components for a large shell roof.

to the site ready for erection, obviously results in a considerable saving in time and labour. Temporary work such as forming is much reduced, the demands on transport are less and above all effective control can be exercised under factory conditions with skilled workers. The outcome will be sound, usable units.

Of course, several principles should be borne in mind: the design of individual members should be simple, there should be as much repetition of unit shapes as possible, and the units should be as large as is consistent with the method of erection (and transportation) so as to preserve continuity and eliminate unnecessary joints.

4. What Elements Should Be Precast?

The obvious components to precast would be the columns and the edge beams. There are many factors to consider. Some questions are: What types of joints and connections are to be employed? How will the formwork and supports fit into the puzzle?

One cannot separate these topics. The prefabricated elements --the connections--and the formwork are all interdependent.

The formwork requirements shall be analyzed first. Because straight lumber could be used, the forms were designed as sheathing spanning between joists which in turn spanned between the precast edge beams. The precast edge beams need only be on two sides of each quadrant due to the fact that the joists span in one direction.

The conclusion was that the column-central edge beam and the horizontal edge beams would be the prefabricated units. (See figures 2 and 3.) This arrangement facilitated using a tied "rigid frame" which is a common structural unit ideally suited for its load carrying capabilities. Hence the formwork could be hung in some manner from the frame thus reducing the temporary supports required.

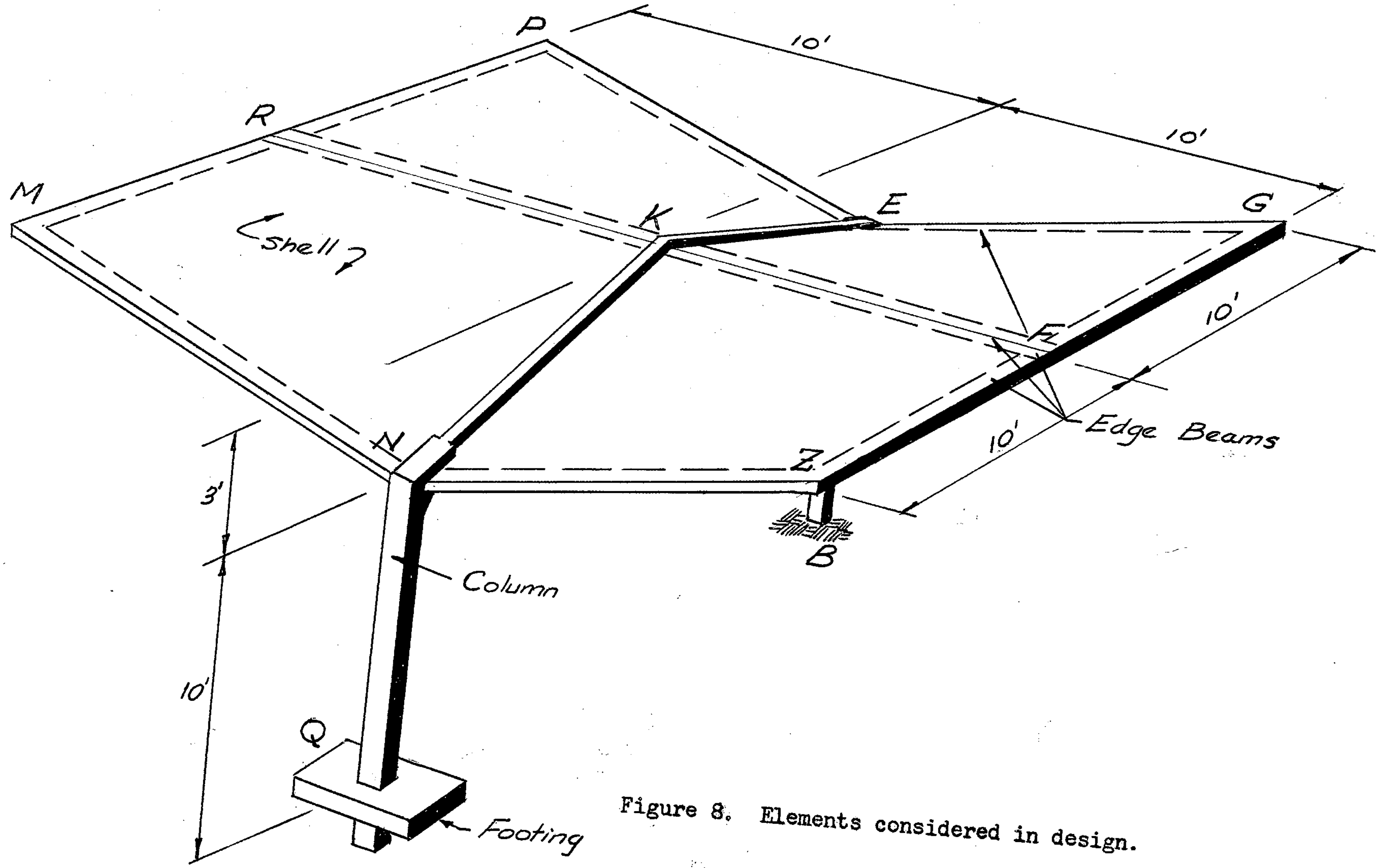


Figure 8. Elements considered in design.

CHAPTER III

THEORETICAL ANALYSIS AND DESIGN

The hyperbolic paraboloid shell basically was constituted of the following elements, each of which is considered in the design of the structure:

1. A cast-in-place shell.
2. Cast-in-place edge beams RKF, GFZ, MN and KN.
3. Precast columns (including the sloping central edge beams).
4. Cast-in-place footings.

These elements are indicated in figure 8.

A. Shell Design

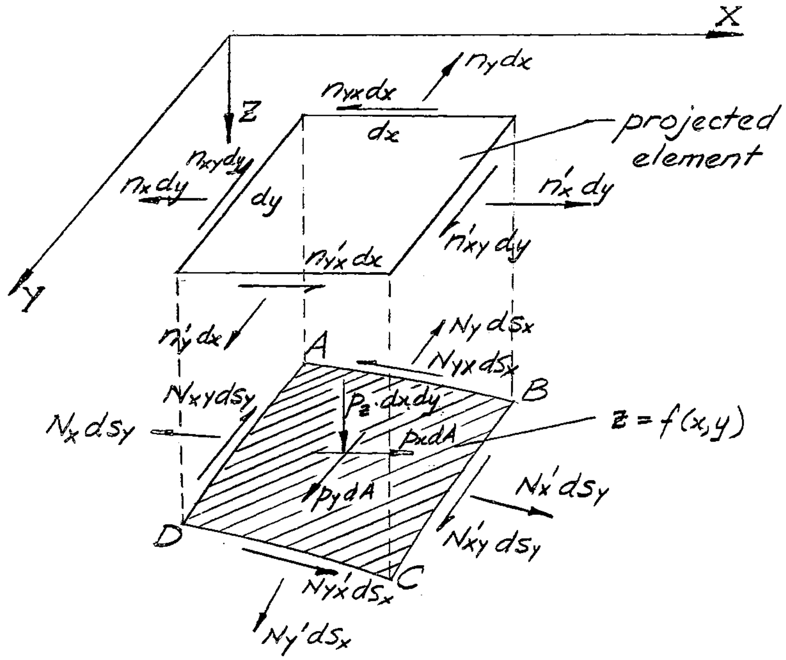
The general equations for the membrane stresses in a shell of double curvature will first be developed after which certain conditions will be applied to these equations such that they hold for the hyperbolic paraboloid.

It should be noted that the load is assumed to be uniform per square foot of projected area and also that there are no bending stresses in the shell itself. Consider the warped surface as shown in figure 10. It is termed a hyperbolic paraboloid.

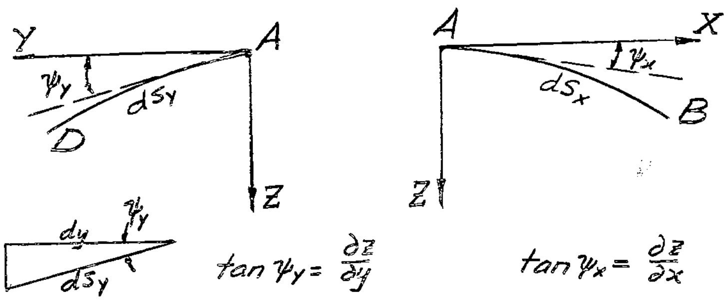
From similar triangles

$$\frac{z}{s} = \frac{y}{b}$$

$$\text{and } \frac{h}{s} = \frac{a}{x}$$



MEMBRANE FORCES ON ELEMENT



$\tan\psi_y = \frac{\partial z}{\partial y}$ $\qquad$ $\tan\psi_x = \frac{\partial z}{\partial x}$

$dy = dS_y \cdot \cos\psi_y$ $\qquad$ $dx = dS_x \cos\psi_x$

Notation:

Quantity	Unit	Meaning
$dx,\ dy$	length	projected lengths of sides
$dS_x,\ dS_y$	length	actual lengths of sides
$n_x,\ n_y$	force/unit length	membrane forces/ft on proj. element
$n_x',\ n_y'$	force/unit length	$n_x + dn_x,\ n_y + dn_y$
$n_{xy},\ n_{yx}$	force/unit length	shear forces /unit length
$N_x,\ N_y$	force/unit length	force/unit length on actual element
$N_x',\ N_y'$	force/unit length	$N_x + dN_x,\ N_y + dN_y$
$N_{xy},\ N_{yx}$	force/unit length	shear forces/unit length

$$n_x' = \left(n_x + \frac{\partial n_x}{\partial x}.dx\right)dy \qquad N_x' = \left(N_x + \frac{\partial N_x}{\partial x}.dx\right)dy$$

$$n_y' = \left(n_y + \frac{\partial n_y}{\partial y}.dy\right)dx \qquad N_y' = \left(N_y + \frac{\partial N_y}{\partial y}.dy\right)dx$$

$$n_{xy}' = \left(n_{xy} + \frac{\partial n_{xy}}{\partial x}.dx\right)dy \qquad N_{xy}' = \left(N_{xy} + \frac{\partial N_{xy}}{\partial x}.dx\right)dy$$

$$n_{yx}' = \left(n_{yx} + \frac{\partial n_{yx}}{\partial y}.dy\right)dx \qquad N_{yx}' = \left(N_{yx} + \frac{\partial N_{yx}}{\partial y}.dy\right)dx$$

$$N_x = \left(\frac{\cos\psi_y}{\cos\psi_x}\right)n_x\ ; \quad N_y = \left(\frac{\cos\psi_x}{\cos\psi_y}\right)n_y\ ; \quad N_{xy} = n_{xy}$$

EQUATIONS OF EQUILIBRIUM:

$$\Sigma F_x = 0: \quad \frac{\partial n_x}{\partial x} + \frac{\partial n_{yx}}{\partial y} + p_x = 0 \qquad -(1)$$

$$\Sigma F_y = 0: \quad \frac{\partial n_y}{\partial y} + \frac{\partial n_{xy}}{\partial x} + p_y = 0 \qquad -(2)$$

$$\Sigma F_z = 0: \quad n_x \frac{\partial^2 z}{\partial x^2} + 2n_{xy}\frac{\partial^2 z}{\partial x \partial y} + n_y \frac{\partial^2 z}{\partial y^2} = p_x \frac{\partial z}{\partial x} + p_y \frac{\partial z}{\partial y} - p_z \qquad -(3)$$

Figure 9. Membrane equations for shells of double curvature.

$$\therefore z = \frac{y}{b} \times \frac{hx}{a} = \frac{hxy}{ab}$$

$$\therefore z = kxy \qquad \text{where } k = \frac{h}{ab}$$

For this surface
$$\frac{\partial z}{\partial x} = ky \ ; \qquad \frac{\partial z}{\partial y} = kx \ ;$$

$$\frac{\partial^2 z}{\partial x^2} = 0 \ ; \qquad \frac{\partial^2 z}{\partial y^2} = 0 \ ;$$

$$\frac{\partial^2 z}{\partial x \partial y} = k$$

Hence by substitution in equations 1, 2, and 3 on the previous page also, for the vertical loading only

$$pz = p \ \text{lb/ft}^2 \text{ of projected area}$$

$$px = py = 0$$

$$\therefore 1.\ \frac{\partial nx}{\partial x} + \frac{\partial nyx}{\partial y} = 0$$

$$2.\ \frac{\partial ny}{y} + \frac{\partial nxy}{\partial x} = 0$$

$$3.\ \ 2nxy \cdot k = -p$$

$$\text{hence} \qquad nxy = \frac{-p}{2k} = \frac{-p \cdot ab}{2h}$$

$$\& \qquad nx = 0$$

$$\& \qquad ny = 0$$

$\therefore$ Forces/unit length on actual element

$$Nxy = \frac{-pab}{2h}$$

$$Nx = 0$$

$$Ny = 0$$

from figure 12 we get the following forces due to p lb/ft^2 over the entire shell:

$$C_1 = \int_o^b \frac{pab}{2h} \cdot dy = \frac{pab^2}{2h}$$

$$C_2 = \int_o^c \frac{pab}{2h} \cdot dx = \frac{pa^2b}{2h}\left(\frac{c}{a}\right) = \left(\frac{c}{a}\right)T_1$$

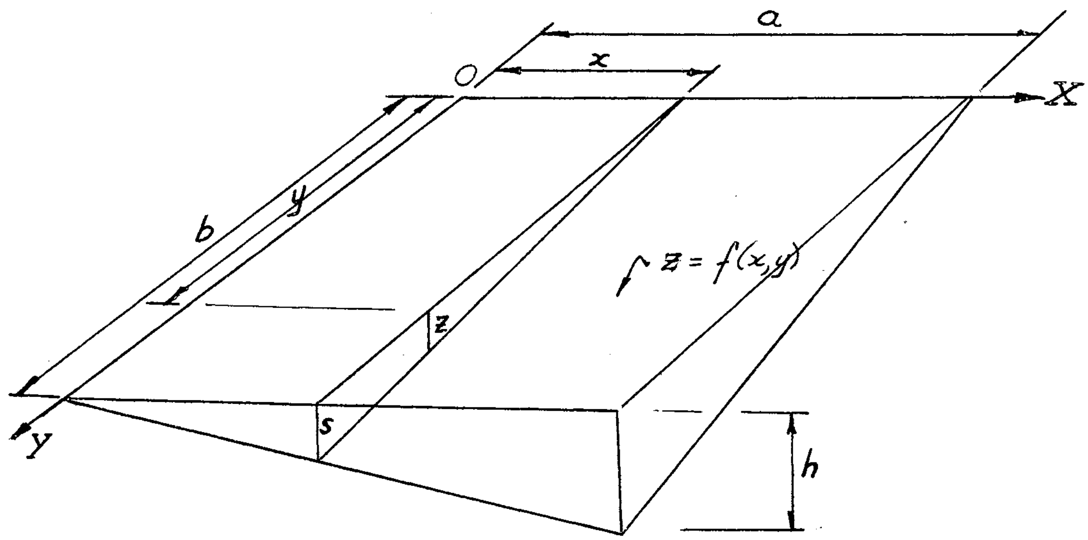


Figure 10. Warped surface.

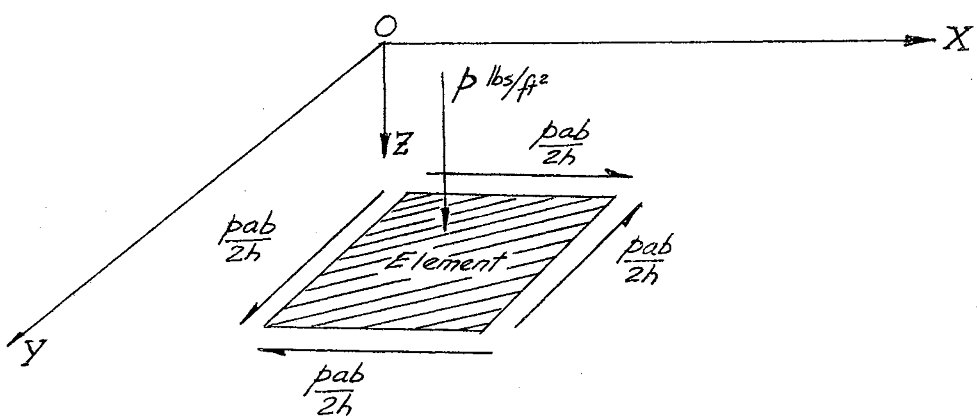


Figure 11. Membrane stresses.

$$T = C_3 \text{Cos}\theta = \left(\frac{b}{d}\right) \circ 2\int_0^d \frac{pab}{2h} \circ dy = \frac{2pab^2 d}{2hd} = \frac{pab^2}{h}$$

$$T_1 = 2\int_0^a \frac{pab}{2h} \cdot dx = \frac{2pa^2 b}{2h}$$

Next consider the actual shell under investigation.

1. Dimensions

With reference to figure 12

$a = 10$ feet

$b = 10$ feet

$c = \sqrt{100 + 9} = 10.45$ feet

$d = 10.45$ feet

$h = 3$ feet

2. Loading

Consider one square foot of shell:

wt. of shell (2½ x 150)	= 31 psf
superimposed live load	= 30 psf
edge beams etc.	= 9 psf
p	= 70 psf

3. Stresses

The shear force in the shell (membrane conditions) is

$$n_{xy} = \frac{pab}{2h} = \frac{70(10)(10)}{2(3)}$$

$$n_{xy} = 1170 \text{ plf}$$

$$\text{shear stress} = \frac{1170}{(2\tfrac{1}{2} \times 12)} = 39.1 \text{ psi} < 90 \text{ psi}$$

(Note: For concrete having $f_c' = 3750$ psi the allowable shear stress = $0.03\ f_c' = 90$ psi - ref. A. C. I. 318-56.)

The Mohr stress diagram for shear stress versus axial stress is shown with the appropriate values.

The principal planes are at $\theta = \pi/4 = 45°$ to the pure shear planes

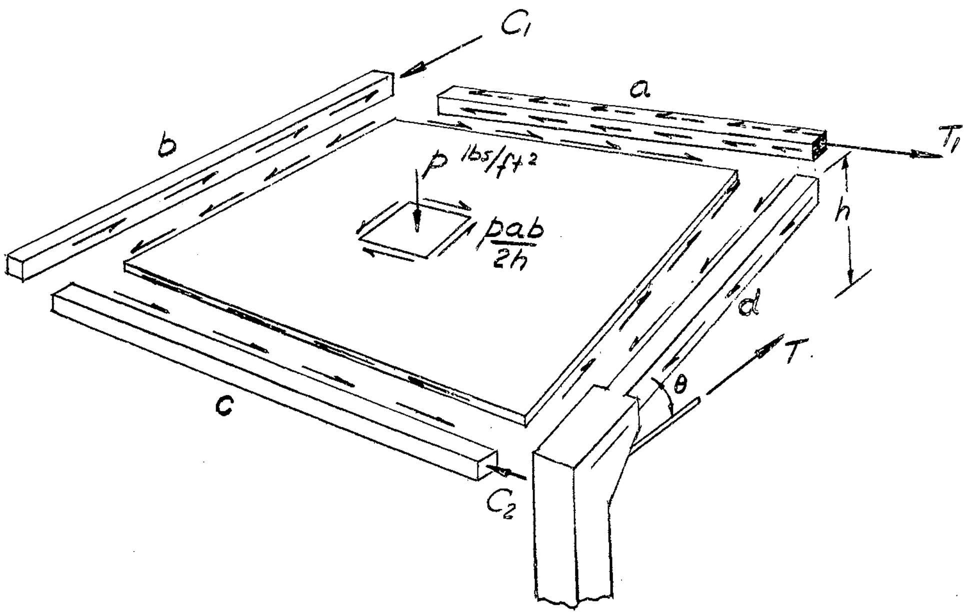


Figure 12. Shear stress distribution due to a uniform load over the entire shell.

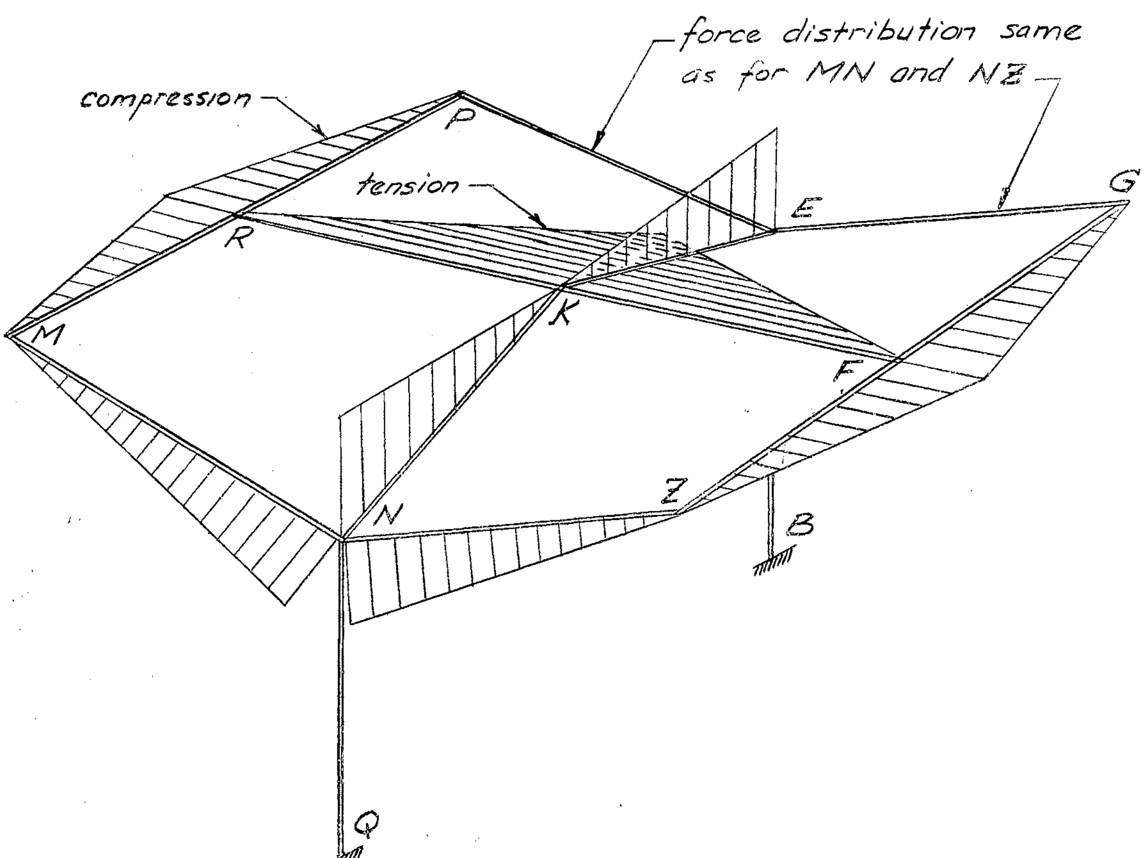


Figure 13. Distribution of forces in edge beams due to a uniform load.

(see diagram) and $F_1 = \frac{pab}{2h}$ (tensile)

$$F_2 = \frac{-pab}{2h} \text{ (compressive)}$$

These are the principal stresses.

$\therefore$ Max compressive stress in concrete $= \frac{1170}{12 \times 2\frac{1}{2}} = 39.1$ psi (quite satisfactory), and the maximum tensile stress in concrete $= \frac{1170}{12 \times 2\frac{1}{2}} = 39.1$ psi < 90 psi. Therefore theoretically no reinforcement is required. However, the concrete shall be assumed to take no tensile stress and tension reinforcement will be provided.

To resist the maximum tensile stress

$$\text{As required} = \frac{1170}{20{,}000} = 0.06 \text{ sq. ins/ft}$$

The temperature and shrinkage reinforcement according to A. C. I. - 318-56, Article 707, must be 0.25%.

$$\therefore \text{As required} = 0.0025\ (12 \times 2\tfrac{1}{2}) = 0.0625 \text{ sq. ins/ft.}$$

For convenience place reinforcement parallel to the shell edges.

Provide 1/4 inch diameter bars @ 9 inches crs. in both directions (As = 0.07 sq. ins/ft.)

Near the corners and the middle of the horizontal edge beam there tend to be secondary bending effects which need to be catered for.

Using figures 21 and 22 ref. (11) a fair idea of the critical region (due to secondary bending effects) can be assessed.

The constant:

$$\frac{ht}{ab} = \frac{3 \times 2\frac{1}{2}}{10 \times 10 \times 12} \fallingdotseq 0.006$$

From figure 22. $\frac{x}{t} = 16$

$$\text{for } fc_{max} = 100w = \frac{100 \times 70}{144} = \underline{48.6 \text{ psi}}$$

$$\therefore x = 16 \times \frac{2\frac{1}{2}}{12} = 3.33 \text{ feet}$$

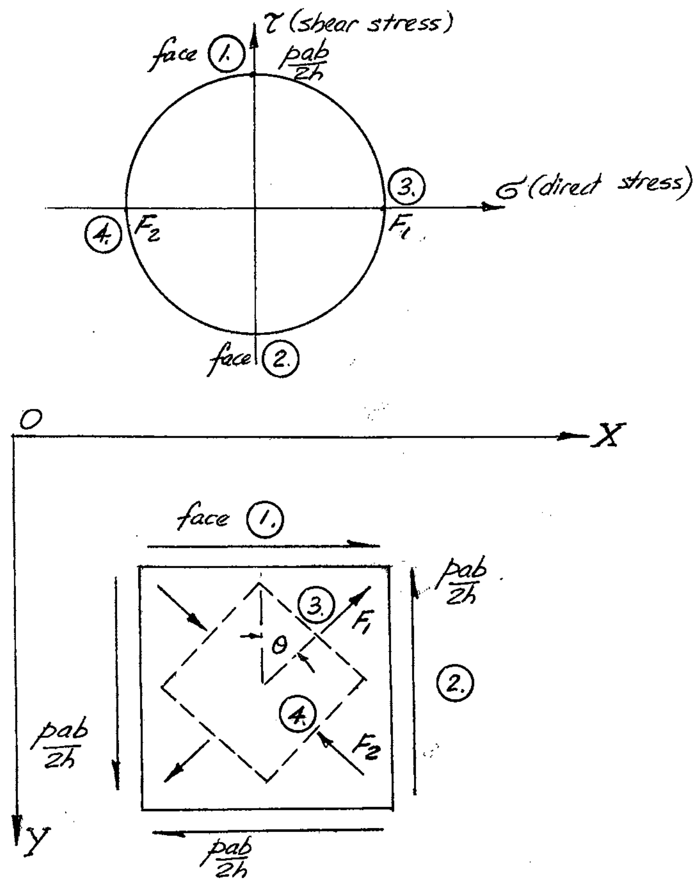


Figure 14. Orientation of principal stress planes.

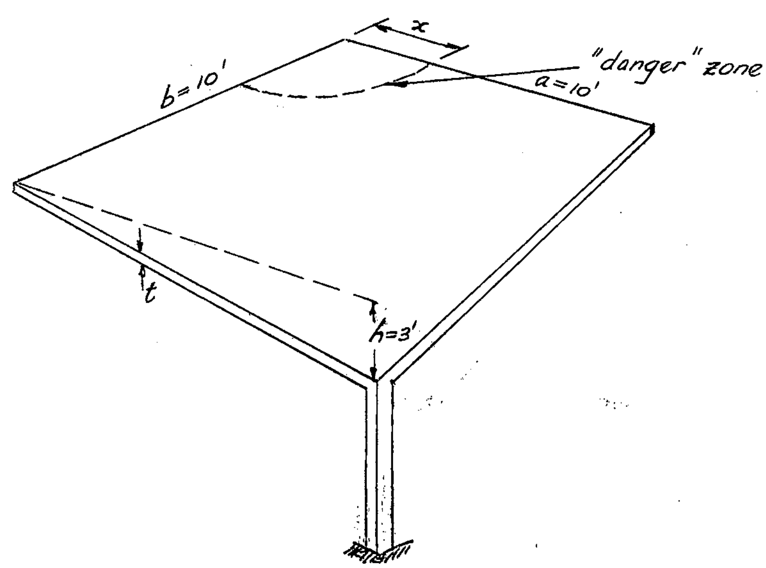


Figure 15. Shell quadrant indicating the critical region for secondary bending effects.

Hence provide extra bars in the region from the flat corners. As an added precaution these additional bars were provided at M, R, P, G, F, & Z. (See drawing no. 2 in the Appendix.)

B. Edge Beam Design

As already calculated the shear force per unit length in the shell is $n_{xy} = 1170$ lb./ft. This has to be transferred to the edge beams as indicated in figure 12.

1. Reinforcement

(a) Edge Beam RKF (Inner Horizontal)

Max tensile force = +23,400 pounds (occurs at center of beam, at K)

$$A_s = \frac{23{,}400}{20{,}000} = 1.17 \text{ square inches}$$

$\therefore$ Furnish 4 No. 5/8 inch diameter bars (A_s = 1.24 square inches)

Next the stopping off points shall be calculated.

By direct proportion $\frac{0.62}{RJ} = \frac{1.17}{RK}$ (See figure 17.)

$$\therefore \quad \frac{0.62}{1} = \frac{1.17}{9.5}$$

$$\therefore \quad 1 = 5.02 \text{ feet, say 5 feet}$$

Anchorage length

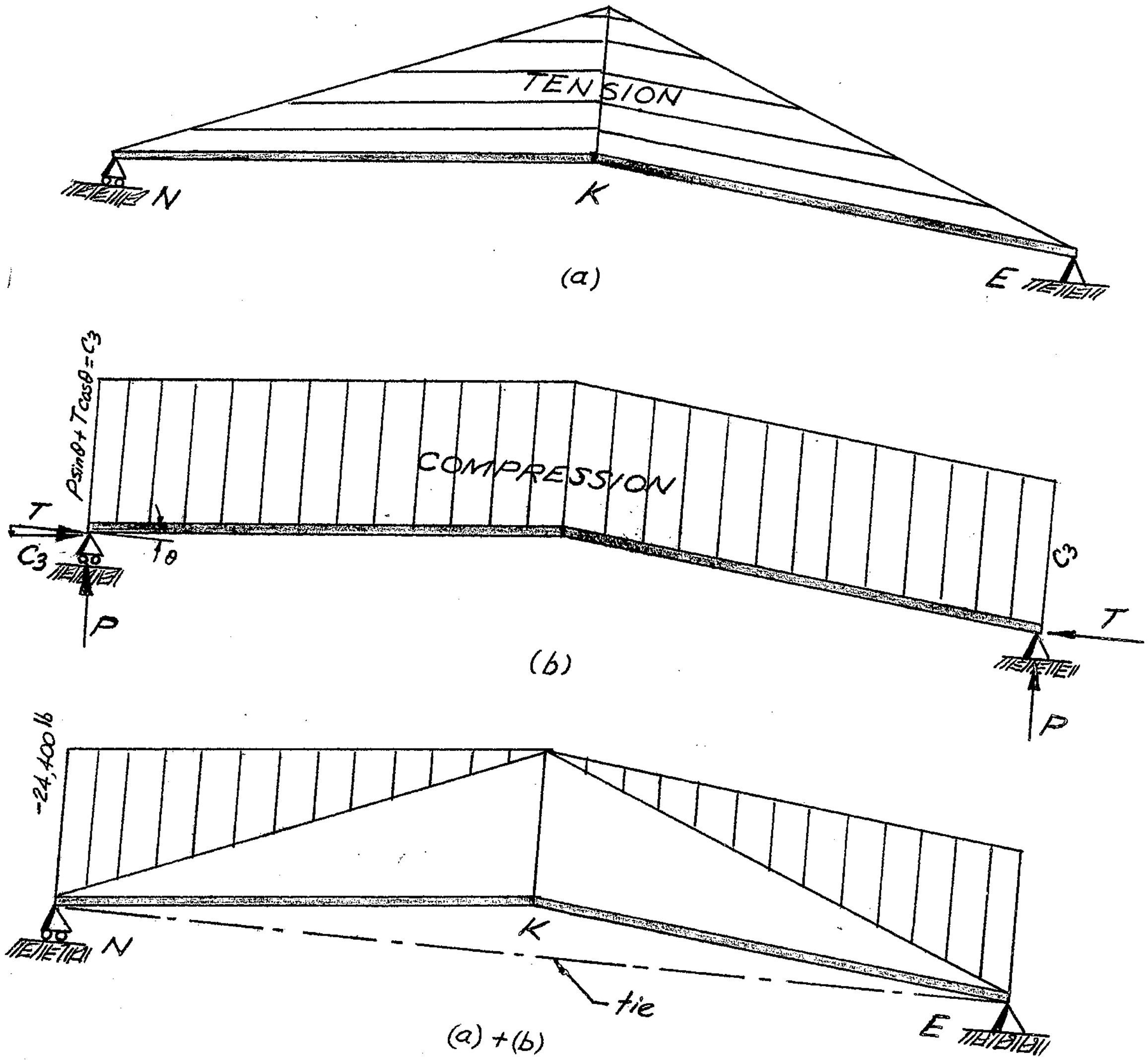
$$La = \frac{f_s D}{4u}$$

$$= \frac{10{,}500\ (0.675)}{4\left(\frac{50}{100} \times 0.07 \times 3750\right)}$$

$$La = 13.5 \text{ inches, say 15 inches}$$

(b) Edge Beams MN, PE, GE, and ZN (Outer Sloping)

Max compressive force = -12,300 pounds. Although the edge beam has a tapered section, it has been assumed to approximate a 9 inches x 3 inches column subjected to a linear varying eccentric load as shown in figure 18.



Forces in edge beams:

Edge beams MRP and ZFG: Max. compr. $C_1 = \frac{pab^2}{2h} = \frac{70(10)(10)^2}{2\times3} = -11{,}700$ lbs

Edge beams MN, PE, GE and ZN: Max. compr $C_2 = \frac{pabc}{2h} = \frac{70(10)(10)10.45}{2\times3} = -12{,}300$ lbs

Edge beam RFK: Max. tension $T = 2\left(\frac{pa^2b}{2h}\right) = \frac{70(10)^2(10)}{3} = +23{,}400$ lbs

Tension in tie: $T = \frac{pab^2}{h} = \frac{70(10)(10)^2}{3} = +23{,}400$ lbs

Edge beam NKE: Max compr $C_3 = \frac{2pabd}{2h} = \frac{70(10)(10)10.45}{3} = -24{,}400$ lbs

Figure 16. Effect of tie on force distribution in NKE and a summary of the forces in the edge beams.

Assume the eccentricity to be 4.5 inches

$$e < 2/3\ (t)$$

$$= 2/3\ (9)$$

therefore consider the edge beam similar to the design of an eccentrically loaded column with small eccentricity. Assume the uncracked section design: (See figures 19 and 20.)

$$n = 8$$

$$f_c' = 3750 \text{ psi}$$

$$f_c = 1690 \text{ psi}$$

$$Ag = 9 \times 3 = 27 \text{ square inches}$$

$$(n-1)As = (8-1)(0.4) = 2.8 \text{ square inches}$$

To locate N.A.:

$$\bar{x} = \frac{2.8\ (7.25) + 27\ (4.5)}{(27 + 2.8)}$$

$$= 4.75 \text{ inches}$$

$$N.A. = 27\ (0.25)^2 + 2.8\ (2.5)^2 + \frac{1}{12}\ (3)(9)^3$$

$$= 1.69 + 17.5 + 182$$

$$= 201 \text{ ins.}^4$$

$$\therefore f_c \text{ (tensile)} = \frac{12{,}300\ (4.5)\ 4.25}{201} = 117 \text{ psi}$$

$$f_c \text{ (compr)} = \frac{12{,}300\ (4.5)\ 4.75}{201} = 131 \text{ psi} = f_b$$

$$f_a = P/Ag = \frac{12{,}300}{27} = 455 \text{ psi}$$

$$Fa = \frac{0.8}{27}\left[27\ (0.225)\ 3750 + 20{,}000\ (0.4)\right] = 910 \text{ psi}$$

$$F_b = 0.45\ (3750) = 1690 \text{ psi}$$

$$\therefore \frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{455}{910} + \frac{131}{1690} = 0.58 < 1 \quad \text{(satisfactory)}$$

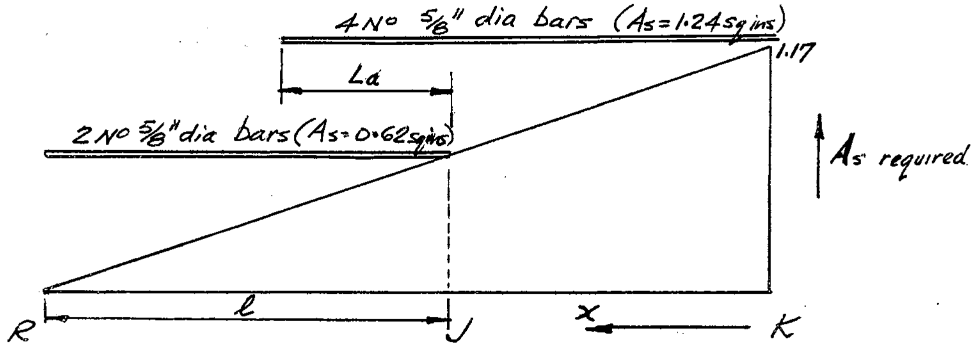


Figure 17. Reinforcement distribution in edge beam RKF.

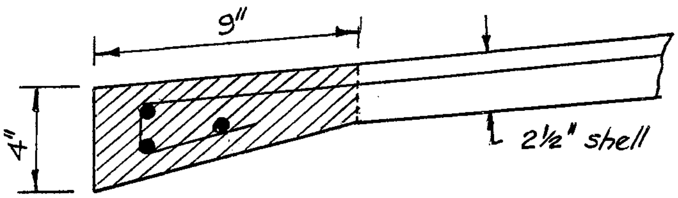


Figure 18. Edge beam section.

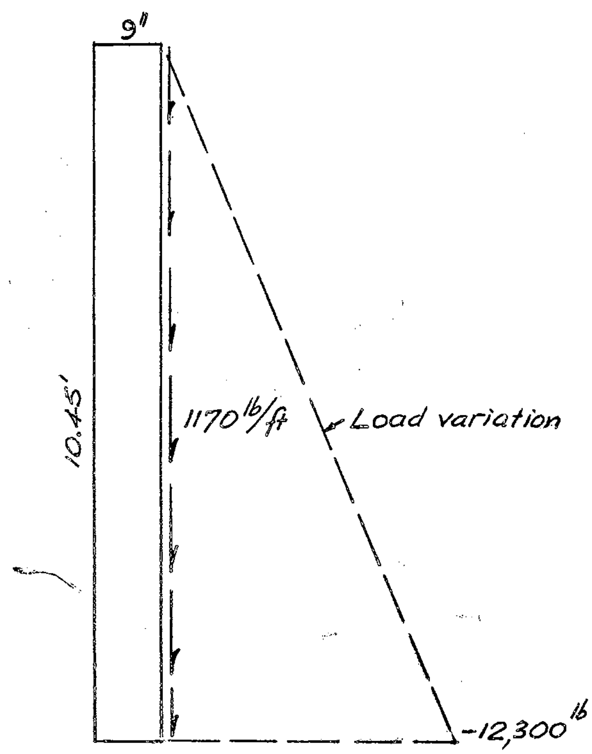


Figure 19. Load distribution in edge beam due to an eccentric shear force.

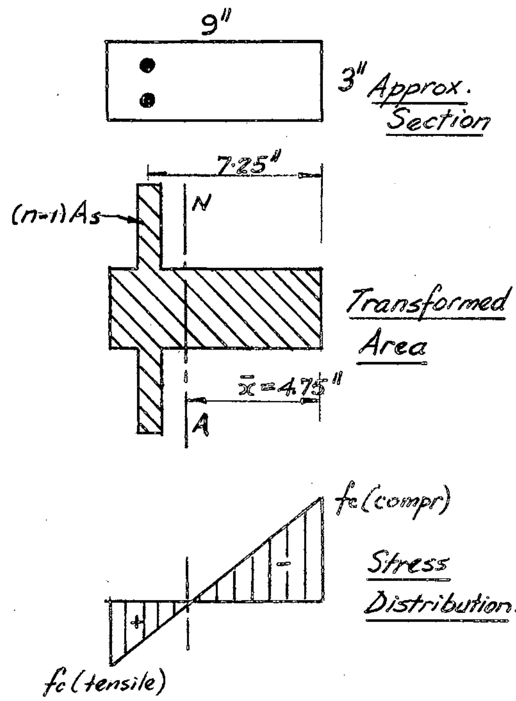


Figure 20. Variation of stress in edge beam MN, PE, GE, and ZN.

Therefore two bars are adequate; however, three 1/2 inch diameter bars were provided to assure the reinforcement to be greater than 1% and less than 4% of the gross area.

(c) Edge Beams MRP and ZFG (Outer Horizontal)

Originally it was intended to precast the horizontal edge beams and to use them to support the formwork. At a later date problems were encountered with the hanging devices to support the formwork from the precast horizontal edge beams. A practical solution was not found and since torsion would be induced by the forms on the outer edge beams when the concrete was placed and also since these edge beams would exert a considerable "line load" on the thin shell when it had hardened, they were disregarded. A temporary wooden stringer was substituted in their place to support the forms.

Here $e < 2/3\ (9); \quad = 4.5$ inches.

For this reason we assume the section to be uncracked. Consider the edge beam to have a section such as that indicated in the figure. 21.

$$fc' = 3750 \text{ psi}$$

$$fc = 1690 \text{ psi}$$

$$Ag = 27 \text{ square inches}$$

$$(n-1)As = (8-1)\ (0.31) = 2.1 \text{ square inches}$$

To locate N.A.

$$\bar{x} = \frac{(2.1)(7.2) + (2.1)(5.2) + (27)(4.5)}{(27 + 4.2)}$$

$$\underline{\bar{x} = 4.74 \text{ inches}}$$

$$I_{N.A.} = \frac{1}{12}\ (3)(9)^3 + 27(0.24)^2 + 2.1(2.46)^2$$

$$+ (2.1)(0.46)^2$$

$$= \underline{197 \text{ ins.}^4}$$

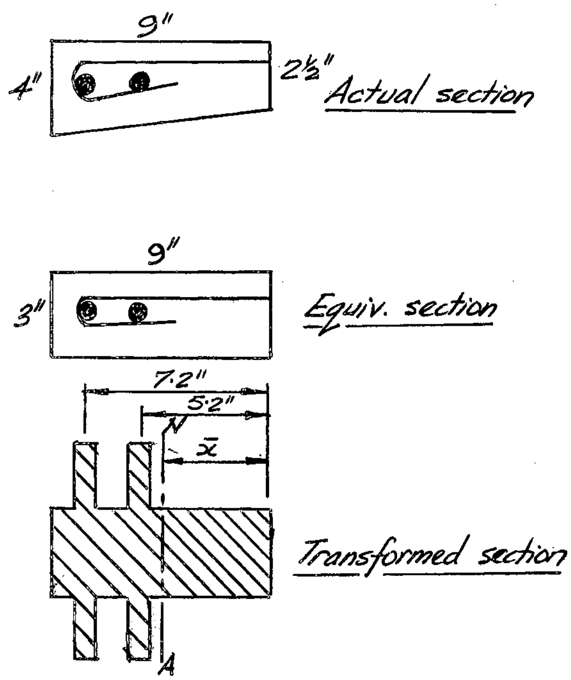


Figure 21. Transformed section for edge beams MRP and ZFG.

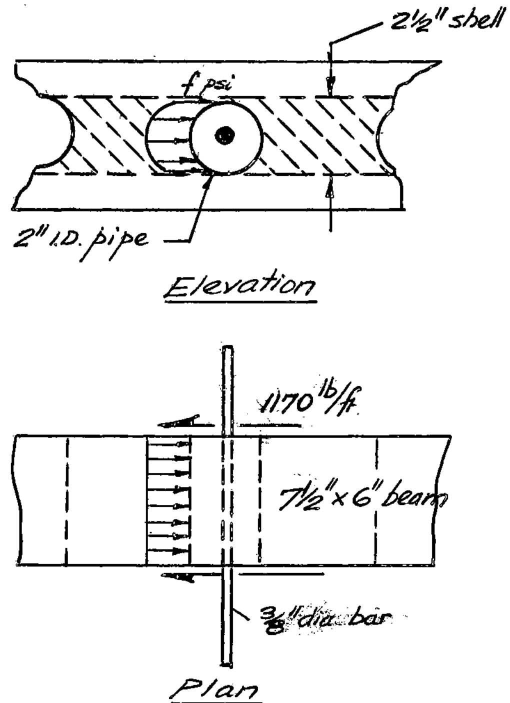


Figure 22. Shear connection between precast central edge beam and shell.

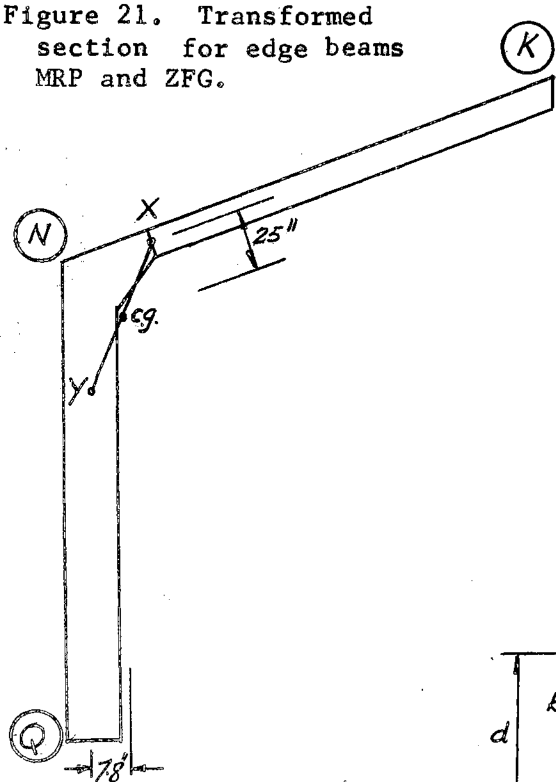


Figure 23. Lifting position.

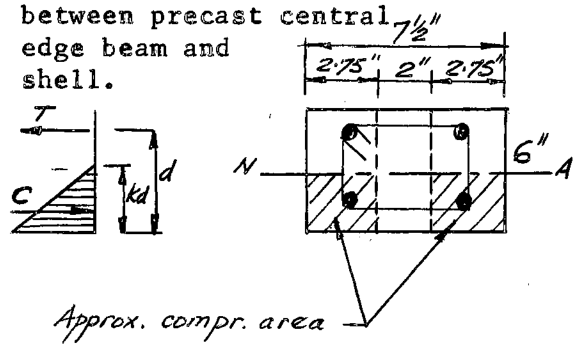


Figure 24. Edge beam NK bending in horizontal plane.

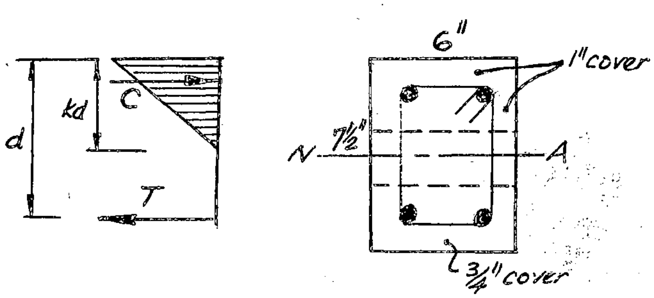


Figure 25. Edge beam NK bending in vertical plane.

$$f_c \text{ (tensile)} = \frac{11700\ (4.5)\ (4.26)}{197} = 114 \text{ psi} \doteqdot 113 \text{ psi}$$

$$f_c \text{ (compr)} = \frac{11700\ (4.5)\ (4.74)}{197} = 127 \text{ psi} = f_b$$

$$f_a = \frac{11700}{9 \times 3} = 432 \text{ psi}$$

$$F_a = \frac{0.8}{27}\left[27\ (0.225)(3750) + 20{,}000\ (0.6)\right]$$

$$= 1030 \text{ psi}$$

$$F_b = 0.45\ f_c' = 1690 \text{ psi}$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{432}{1030} + \frac{127}{1690} = 0.5 < 1.0, \text{ hence it is satisfactory}$$

(d) <u>Edge Beams KN and KE (Inner Sloping)</u>

After weighing the pros and cons for different sized beams, the finally selected cross section was a 7 1/2 ins. x 6 ins. Many important factors had to be considered since these edge beams were to be precast monolithically with the columns thus forming the so-called "rigid frame."

Since these were to be precast the lifting and transport of the units had to be looked into. Because of its non-collinear configuration torsional stresses would be introduced and had to be allowed for. At this stage in the project it was intended to use the precast "frame" to support the prefabricated formwork and also the wet concrete shell. Later, however, it proved unsatisfactory mainly due to the inadequacy of suitable hanging devices.

Finally these "beams" were to withstand the shear stresses and the result and compression due to the loaded shell. This called for suitable shear connections between the precast units and the cast-in-place shell. So, overall, these beams had to withstand:

(i) Bending in either of two planes,

(ii) Torsion,

(iii) Shear, eccentric and ordinary,

(iv) Compression.

Over and above choosing a suitable section to satisfy the above requirements, this beam had to be practical to cast from the point of view of ease and economy. Since for most of the "rigid frame's" life, it would be under its final conditions of loading with this edge beam in compression due to eccentric shear forces, the section was so chosen that it would best fulfill these requirements.

Compression

When under full load the maximum compression in the edge beam is 2(1170)10.45 = 24,400 pounds and occurs at the top of the column. (See figure 16.)

For the section as shown in the diagram:

Value of steel in compression = 0.8 (16,000) 4 (0.31)

= 15,800 pounds

Value of concrete in compression = 0.8 (0.225)(2)(6)(2.75)(3750)

= 21,300 pounds

$\therefore$ Total allowable = 37,100 pounds > 24,400 pounds which is satisfactory.

Shear

The maximum shear per foot is 2(1170) lb. = 2340 plf. Provide 2 inches I.D. pipe sections cast into the beam at 6 inch centers. Before the shell is cast a 3/8 inch diameter bar is to be inserted through each pipe having adequate anchorage length on each side of the beam. (See figure 22.)

Shear per bar $= \frac{2340}{2} = 1170$ pounds. Assuming a uniform stress distribution due to the reinforced plug bearing on the precast beam, bearing stress $= \frac{2(1170)}{2 \times 6} = 195$ psi $<$ 938 psi which is quite satisfactory.

Shearing of the "plug"

$$\text{shearing stress on concrete alone} = \frac{1170}{\pi/4\ (2)^2}$$

$$= 373 \text{ psi} > 790 \text{ psi}$$

hence must use reinforcement. For 3/8 inch diameter bar

$$\text{shear stress} = \frac{1170}{\pi/4\ (3/8)^2}$$

$$= 10{,}600 \text{ psi} < 15{,}000 \text{ psi}$$

which should be quite satisfactory since the concrete will assist it.

Lifting

The rigid frame "dog leg" is rather a clumsy-shaped object to lift and carry. For lifting bending will be assumed in two planes and torsion must be considered where it applies. A load disperser should be used to spread the two points of lift to the most suitable positions. This is because a crane and chain are to be used for lifting.

The center of gravity of the "dog leg" QNK was calculated by taking moments of the weights of the segments about two axes.

Even with the load disperser the lifting positions at X and Y were in rather dangerous areas. At X, the lifting position was approximately at the same place as the change in section. (See figure 23.)

By having two lifting points, one on each arm, torsion is eliminated.

Bending in the Horizontal Plane (See figure 24.)

This occurred during lifting. For the given section assume only tension steel.

$$n = 8$$

$$f_c' = 3750 \text{ psi}$$

$$d = 6 - (1 + 0.31) = 4.69 \text{ inches}$$

Moments about N.A.:

$$\frac{kd}{2}(2 \times 2.75)(kd) = 8(2 \times 0.31)(4.69 - kd)$$

$$\therefore kd = 2.14 \text{ inches}$$

$$(\text{M.R.}) \text{ concrete} = (4.69 - 2.14)\left[1/2(2.14)5.5(1690)\frac{1}{12}\right] = 3330 \text{ lb.ft.}$$

$$(\text{M.R.}) \text{ reinforcement} = \frac{1}{12}(4.69 - 2.14)(20{,}000)(2 \times 0.31) = 4130 \text{ lb.ft.}$$

M max for NK $= 5\left(\frac{7.5 \times 6}{144}\right)150(10) = 2350$ lb.ft. which is less than the resisting moments--hence is adequate.

<u>Bending in the Vertical Plane</u> (See figure 25.)

$$d = 7.5 - (0.75 - 0.31) = 6.44 \text{ inches}$$

Again, taking moments about the N.A.

$$(6\ kd)\frac{kd}{2} = 8(2 \times 0.31)(6.44 - kd)$$

$$kd = 2.53 \text{ inches}$$

$$(\text{M.R.}) \text{ concrete} = \left(6.44 - \frac{2.53}{3}\right)\left[\frac{1}{2}(1690)(6)(2.53)\right]\frac{1}{12} = 6150 \text{ lb.ft.}$$

$$(\text{M.R.}) \text{ reinforcement} = \left(6.44 - \frac{2.53}{3}\right)(2 \times 0.31)(20{,}000)\frac{1}{12} = 5750 \text{ lb.ft.}$$

M max for NK $= \frac{10.45}{2}(10.45)\left(\frac{7.5 \times 6}{144}\right)150 = 2580$ lb.ft. when lifted vertically to place in foundation hole. This is quite adequately catered for.

It should be noted that shear is not critical during lifting and the concrete alone can withstand the stress. However, 1/4 inch diameter ties are provided in the beam NK and KE at 12 inch centers just as an arbitrary provision for a compression member.

C. Tie NE

Due to the fully loaded shell the stresses are transmitted via the shear connections to the edge beams. (See figure 16.) When these stresses accumulate at N it is obvious that a tie is needed to withstand this large force which has a big horizontal component or else the column will have to be designed to take this large moment.

A tie is ideal for taking tensile loads and is provided as shown in the Appendix.

From figure 16 it can be clearly seen that the force in the tie

$T = 2(1170)\ 10.45 \cos\theta = 23{,}400$ pounds

As req'd $= \dfrac{23{,}400}{20{,}000} = 1.17$ square inches

$\therefore$ Provide 1 N° 1 1/4 inch diameter bar (As = 1.23 square inches)

D. Columns NQ and EB

Bending in Plane of Columns

To calculate the maximum bending moment in the plane of the two columns, consider the shell under full load. The forces are as shown in figure 26. (Bending has been neglected at "joint" N.)

Weight of column $= 10\left(\dfrac{10 \times 10}{144}\right) 150 = 1040$ pounds

Forces at N: Vert. compt. $= 2(1170)10.45\left(\dfrac{3}{10.45}\right) = 7000$ pounds

Horiz. compt. $= 2(1170)10.45\left(\dfrac{10}{10.45}\right) = 23{,}400$ pounds

For equilibrium: $\Sigma F_x = 0$: $H = (23{,}400 - T)$

$\Sigma F_y = 0$: $V = 7000 + 1040 = 8040$ pounds

$\Sigma M = 0$: $M = (23{,}400 - T)\ 10$

But T = 23,400 pounds for no movement at N. Therefore H = 0 = M.

The column is not subjected to bending in the plane of the two columns. However, should the point N be free to move outwards 1/5 inch the maximum allowable stress in bending will occur at Q.

Bending Perpendicular to Plane of Columns

For this case various loading conditions were considered and the loading with the worst effect was used in the analysis.

1. Wind

For the wind direction as in figure 27 the pressure distribution was assumed as indicated (23). Using the Beaufort Scale, $p = 0.0034v^2 =$ 15 psf for number 11 on this scale.

$M_1 = 5(0.8)(15)(20 \times 10) = 12{,}000$ lb.ft.

$M_2 = 5(0.3)(15)(20 \times 10) = 4{,}500$ lb.ft.

$\therefore$ M total = 16,500 lb.ft.

$\therefore$ M (wind) = 8250 lb.ft. in each column

2. Unsymmetrical Loading

When snow is on the leeward half of the shell, it is estimated that the snow load is only 15 psf since the wind is blowing at the same time. It seems to be valid to say that no other live load in the way of persons would be on the roof in a howling snow storm.

Using a similar approach as before, the moment in the columns

$M = 5(15)(20 \times 10) = 15{,}000$ lb.ft.

$\therefore$ M snow = 7,500 lb.ft. in each column

Combining the effect due to win and snow, M total = 8,250 + 7,500 = 15.75 k ft. in each column which is maximum.

Load in each column

shell (self wt)	= 6250 pounds
edge beams (approximately)	= 750 pounds
live load (wind)	= -750 pounds
live load (snow)	= 1500 pounds
column (self wt)	= 1040 pounds
	8790 pounds

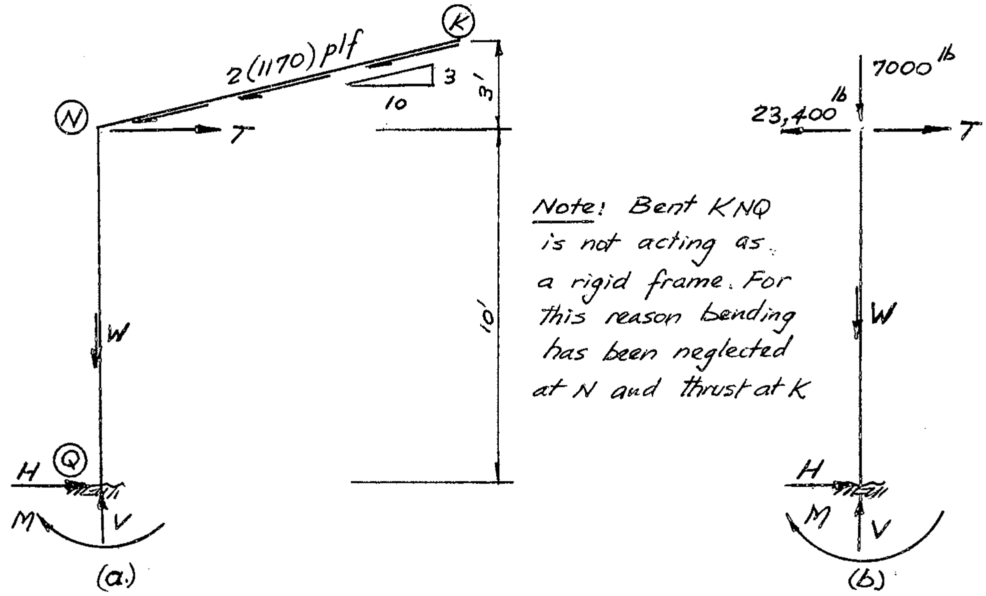


Figure 26. Forces on bents KNQ and KEB.

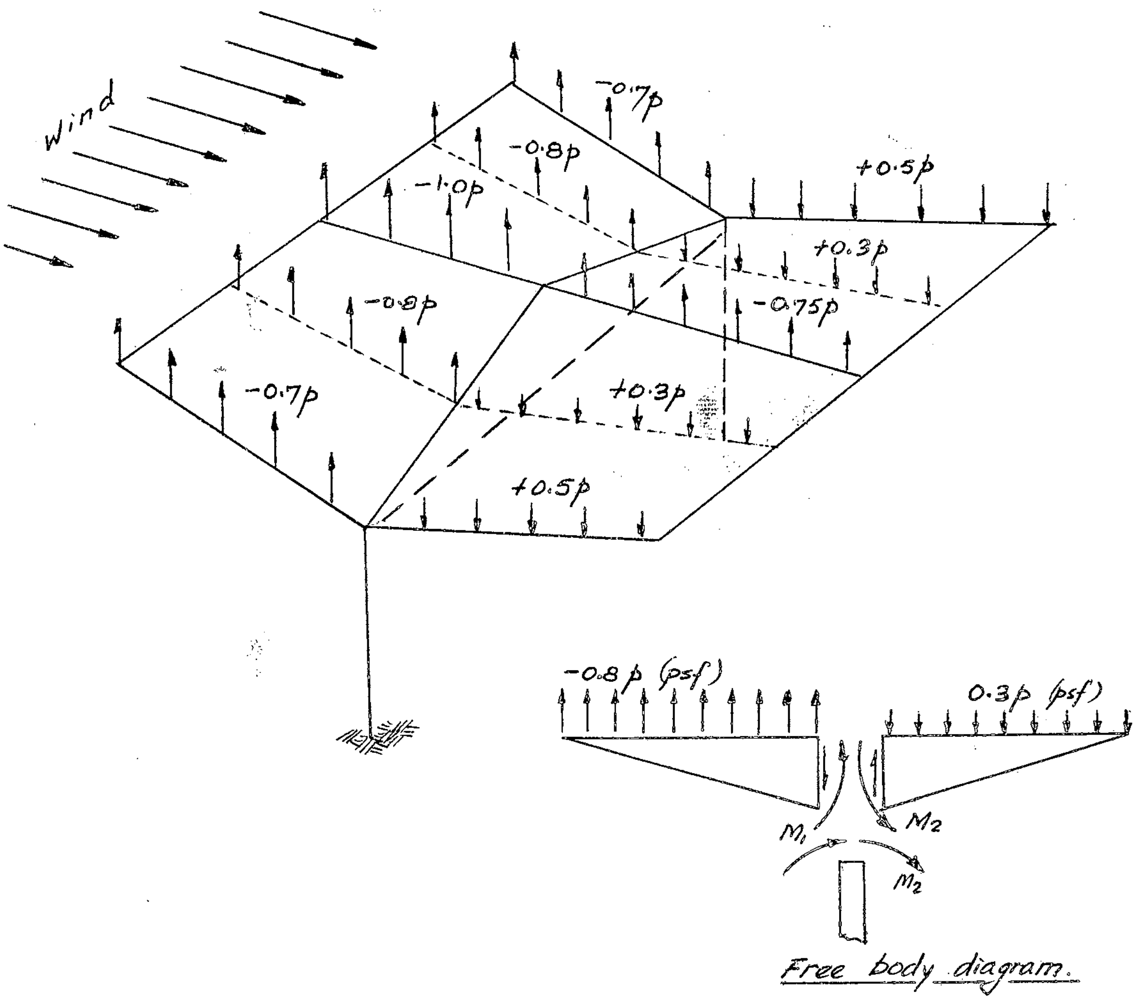


Figure 27. Moment in column due to wind loads.

3. Column Design

Employing ultimate strength techniques for columns with large eccentricities

$M = 15.75$ k ft.

$P = 8.8K$

$e' = \frac{M}{P} = 1.8$ ft. $= 21.6$ ins.

Assume a 10 inches x 10 inches column section reinforced with 4 - 1 inch diameter bars. (See figure 28.)

For $fy = 40{,}000$ psi

$fc' = 3750$ psi

$d = 10 - (1.5 + 0.25 + 0.5) = 7.75$ inches

$T = 1.58\ (40{,}000) = 63{,}100$ pounds

$C_s = 1.58\ (40{,}000) = 63{,}100$ pounds

$Z = 21.6 - 5 + 0.5a = 16.6 + 0.5a$

Taking moments about C_c gives

$$P_u \cdot Z = C_s\ (d-d')$$

$$= 63{,}100\ (7.75 - 2.25)$$

$$= 348{,}000 \text{ pounds}$$

$$\therefore P_u = \frac{348{,}000}{(16.6 + 0.5a)} \qquad \text{------(1)}$$

$$\text{also } a = \frac{Pu}{0.85\ fc'\ b} \qquad \text{------(2)}$$

Solving (1) and (2) by trial yields $a = 0.65$ inches and $Pu = 20{,}400$ pounds, therefore, for a load factor of 2

$$P_w = \frac{Pu}{2} = \underline{10{,}200 \text{ lb.} > 8{,}800 \text{ lb.}}$$

Therefore the section is satisfactory.

E. Footing Design

Try a rectangular footing 5 ft. x 2 ft. x 1 ft. as shown in

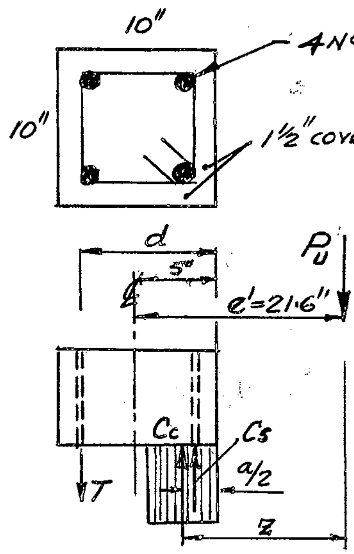


Figure 28. Column design section.

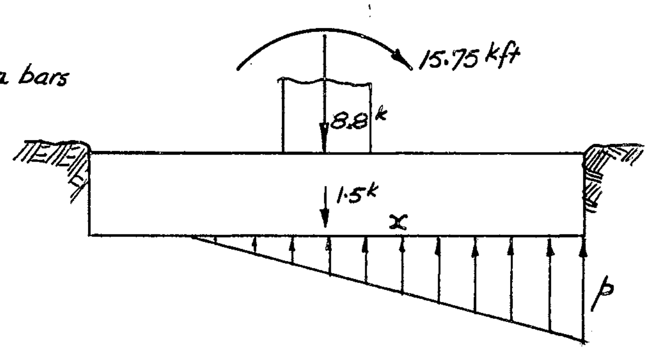


Figure 29. Base pressure distribution.

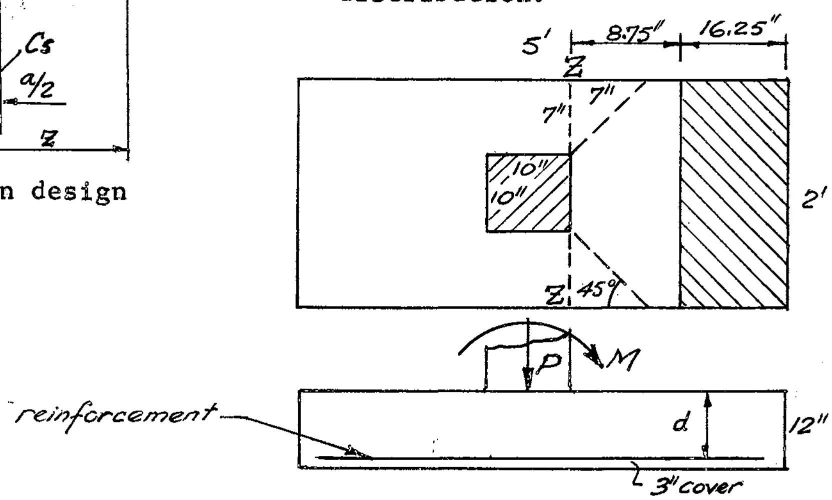


Figure 30. Footing design section.

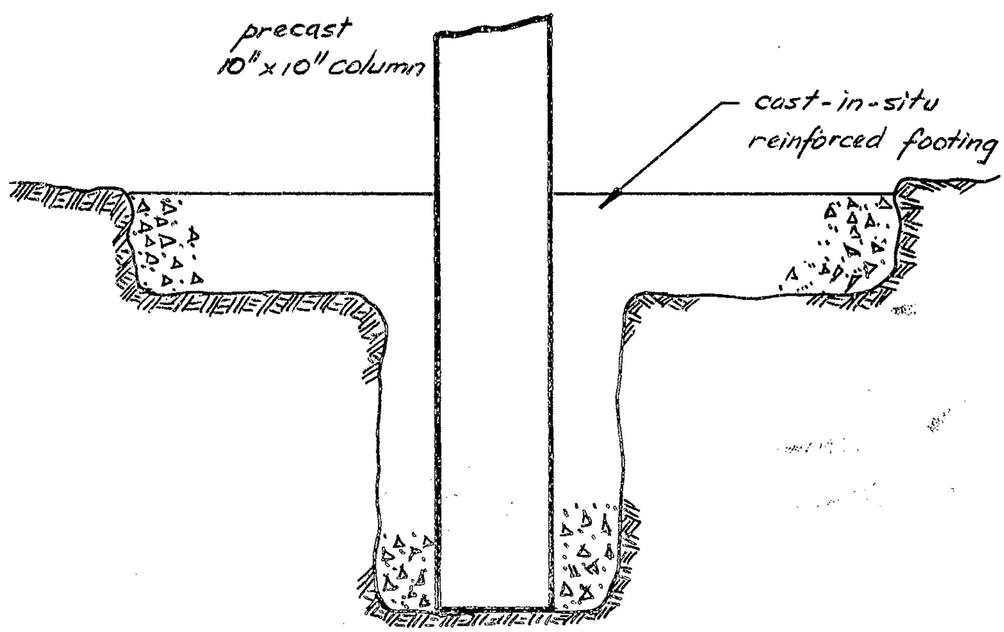


Figure 31. Elevation of column footing.

the figure 30.

weight of footing = (5 x 2 x 1) 150 = 1500 pounds

$\therefore$ $P = 8.8 + 1.5 = 10.3K$

$M = 15.75$ k ft.

$\therefore$ $e = \frac{15.75}{10.3} = 1.53$ ft.

From the soil pressure diagram 1/2 p.x.b. = P and 2/3 x P = (x-d/2 te)P

$$\therefore p = \frac{4P}{3(d - 2e)b}$$

$$\therefore p = \frac{4(10.3)}{3(5 - 3.06)2}$$

$= 3.45$ k/ft.$^2 <$ 4k/ft.^{2} which appears to be satisfactory.

$d = 12 - 3 - 0.25 = 8.75$ inches

Assume heavy pressure over shaded area approximately constant = 3.0 k/ft.

$\therefore$ Shear Force $Vr = 3\left(\frac{16.25}{12}\right)2$

$= 8,125$ lbs.

required d. $= \frac{Vr}{bjv}$

$$= \frac{8,125}{24 \times 0.88 \times 75}$$

$= 5.1$ ins. $<$ 8.75 ins. available

$$M_{zz} = 3000\left(\frac{25}{12} \times 2\right)\left(\frac{25}{2 \times 12}\right) = 156,000 \text{ lb. ins.}$$

Consider a 12 inch strip

$M = \frac{156,000}{2} = 78,000$ lb. ins./ft.

For concrete having fc' = 3000 psi, n = 10 for a balanced section.

R = 235.

$\therefore$ (M.R.) $= 235(12)(8.75)^2 = 215,000$ lb. ins. which is quite adequate.

$$As = \frac{78,000}{(0.865 \times 8.75)(20,000)} = 0.52 \text{ sq. ins./ft. use 1/2 inch}$$

diameter @ 3 1/2 centers. The reason for increasing the spacing was that the hole was roughly dug to 12 inches ± 2 inches.

In the other direction for shrinkage and temperature steel

$$p = 0.002 = \frac{As}{bt}$$

As per ft. = 0.002 (12 x 10) = 0.24 sq. ins./ft.

$$S = \frac{12 \times 0.196}{0.24}$$

= 9.8 ins. choose 1/2 in. diameter @ 9 ins. crs.

It was decided to place the precast column in a 4 feet-6 inches central hole--pour concrete around it and make this concrete monolithic with the flattened section of the footing as shown in the diagram. This would make for easier positioning of columns, cater more adequately for the overturning moment and prevent any form of punching failure.

F. Formwork Design

Consider 3/8 inch R plywood to over 2 x 6's @ 12 inch centers.

Properties

For the plywood

Area = 3.00 sq.ins.

$I = 0.05$ ins.^{4}

$w = 1.13$ lb./ft.

Allowable shear stress (rolling) = 68 psi

Allowable bending stress = 1500 psi (for wet location)

For the lumber

Area = 9.14 sq.ins.

$I_{centroid} = 24.1$ ins.^{4}

$w = 2.54$ lb./ft.

These values are obtained from the Douglas Fir Plywood Association and reference(21.)

<u>Determination of N.A.</u> (See figure 32.)

$$\Sigma M \text{ base} = A\bar{y} = A_1y_1 + A_2y_2$$

$$A_1y_1 = 9.14\ (2.81) = 25.7$$

$$A_2y_2 = \underline{3.00}\ (5.81) = \underline{17.4}$$

$$\underline{\underline{12.14}} \text{ ins.}^2 \qquad \underline{\underline{43.1}} \text{ ins.}^3$$

$$\therefore \bar{y} = \frac{43.1}{12.14} = 3.55 \text{ ins.}$$

$\therefore$ Distance to top fibre, C (top) = (5.625 + 0.375) - 3.55 = 2.45 ins.

and, C (bottom) = 3.5 ins.

<u>Moment of Inertia</u>

$$I \text{ (total)} = \left[0.05 + 3\ (2.26)^2\right] + \left[24.1 + 9.14\ (0.74)^2\right]$$
$$= 44.5 \text{ ins.}^4$$

<u>Bending Moment and Shear Force</u>

$$M(\max) = \frac{wl^2}{8} \quad \text{for simple beam}$$
$$= \frac{81(10)^2}{8}$$
$$= \underline{1010 \text{ lb.ft.}}$$

$$V(\max) = \frac{wl}{2}$$
$$= \frac{81(10)}{2}$$
$$= \underline{405 \text{ lb.}}$$

<u>Bending Stresses</u>

Top fibre: $f_t = \frac{Mc}{I} = \frac{-1010(2.45)12}{44.5} = -670 \text{ psi} < 1500 \text{ psi}$

Bottom fibre: $f_b = \frac{1010(3.55)(12)}{44.5} = +968 \text{ psi} < 1500 \text{ psi}$

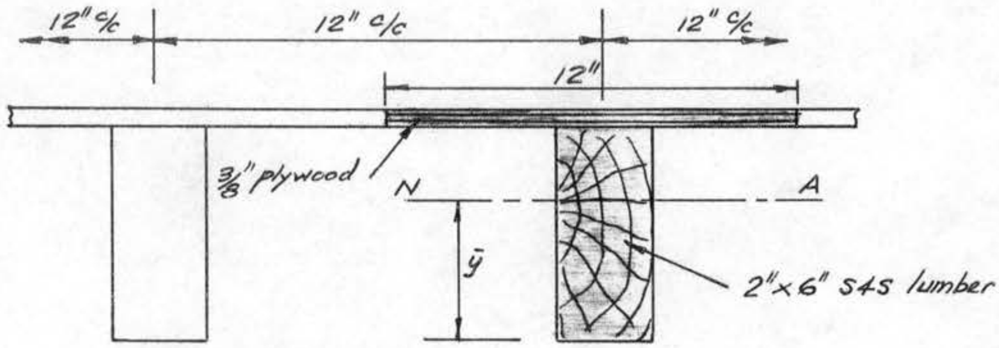


Figure 32. Typical section considered in design.

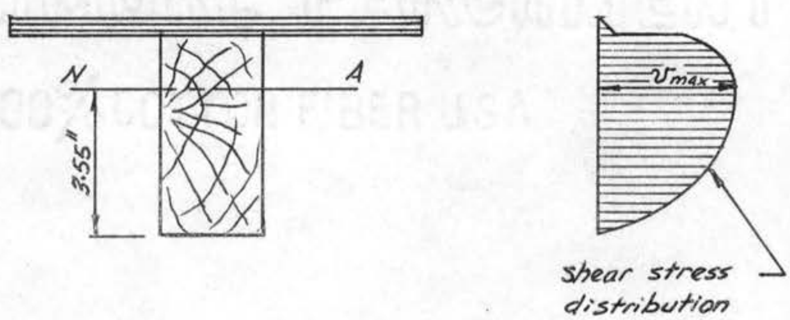


Figure 33. Design section.

The negative sign indicates compression and the positive sign tension.

Shear Stresses (See figure 33.)

(i) Rolling Shear

$Q = (2.26)(3) = 6.8$ ins.^{3}

$$= \frac{VQ}{IB}$$

$$= \frac{405(6.8)}{44.5(1.625)}$$

$= 38$ psi < 68 psi

(ii) Horizontal Shear

At the neutral axis the maximum will occur.

$Q = 2(3.55)\left(\frac{3.55}{2}\right) = 12.6$ ins.^{3}

$$v = \frac{VQ}{Ib} = \frac{405\ (12.6)}{44.5(1.625)} = 70.5 \text{ psi} < 120 \text{ psi (for lumber)}$$

Deflection

$$d_{max} = \frac{5}{384}\frac{wl^4}{EI} = \frac{5(81)(10)^4(12)^3}{384(1,600,000)44.5} = 0.25 \text{ ins} < \frac{l}{360} = 0.33 \text{ ins.}$$

CHAPTER IV

CONSTRUCTION PROCEDURE

A. Prefabrication of the Concrete Elements

1. Formwork

In the prefabrication of the column-central edge beam (rigid frame units) and the horizontal edge beams 2 x 8, 2 x 10 and 2 x 12 S4S Southern pine was used for the forms. It was decided to use heavy lumber and double headed 16^d nails to investigate the possibility of form re-use. Two inch lumber also has the great advantage of resisting deformation under the weight of wet concrete and the moisture does not cause unsightly warping.

The formwork for the rigid frame units presented little difficulty. Two identical forms were made, one for each "dog leg" or bent. (See drawing no. 1 in the Appendix.) The reason one form was not re-used was that it would have created an awkward time lag in the curing of the units. The 2 inch I.D pipe sections for the shear connection were cut 7 inches long and inserted into holes in the form lumber. (See figure 34.) Polyethylene strips were placed over the open pipe ends to prevent the wet concrete from falling into them.

Shortly after the forms were completed it was suggested that the ground clearance to the tie bar be increased by 2 feet. For this reason the four 1 inch bars, mark "a" and "b" project from the forms. (See drawing no. 2 in the Appendix.) These projecting bars subsequently

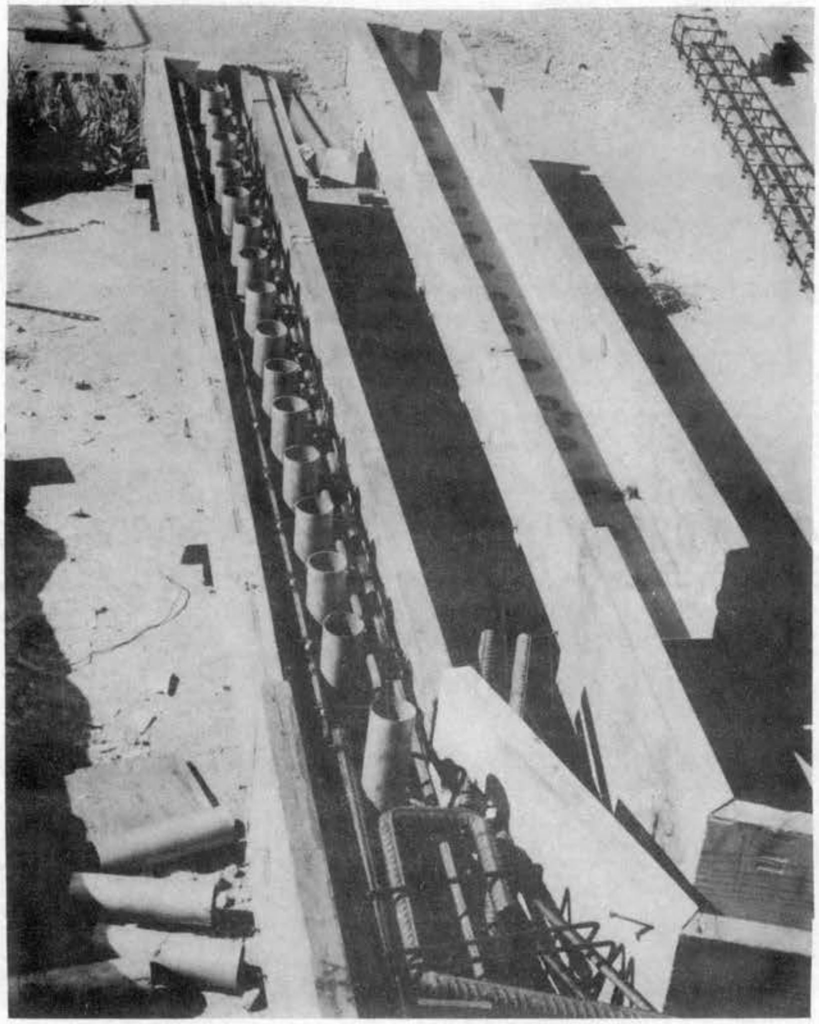


Figure 34. Formwork for precast bent showing reinforcement and pipes for shear connection

proved to get in the way--especially during the erection of the units.

The horizontal edge beams were disregarded because they were impractical and would not enhance the strength of the shell but would actually be detrimental. Originally they were planned to be 6 x 6 inches in cross section and 20 feet long. This meant that these edge beams weighed 3/8 ton and would represent a line load of approximately 40 lb/ft on the extreme edges of the shell.

2. Reinforcement (See drawing no. 2 in the Appendix.)

The reinforcement was cut and bent on a jig. However, the bars over 5/8 inch in diameter were heated before bending. In this regard the most difficulty was encountered with 1 inch diameter bars mark "b" because they had a tight bend and it was in a rather critical section with very little space to spare. Fortunately the forms were already completed and all "problem" reinforcement could be fitted. All the 1/4 inch diameter ties were spot welded as this turned out to be easier than lapping and bending. (See reinforcement bending schedules.)

Two 7/8 inch diameter bars mark "c" were butt-welded to a 3/8 inch steel plate and protruded about 12 inches from the forms. The 1 1/4 inch diameter tie bar was later welded to these bars. Because of the inadequacy of bond the plate was welded to the reinforcing bars.

Due to the fact that the rigid frame was precast the concrete cover to the reinforcement need be 1 inch for the top and side faces which are exposed to the weather, and 3/4 inch for the bottom cover which is not (20). The minimum concrete cover was used in order that the weight of the rigid frame units be kept as low as possible.

Once the steel reinforcing cages were tied together, they were dropped into the forms. Chairs, made of metal sheeting, were installed to ensure the required cover.

3. Casting

Now that the forms and reinforcement were ready, a suitable "casting yard" was prepared and the forms were positioned and shimmed such that they were both stable and horizontal.

Ready-mixed concrete with an expected 28-day strength of 3750 psi was ordered, and duly arrived within an hour. The ready-mix truck managed to maneuver close enough to the forms such that the chute could reach directly from the mixer to the forms. This saved both concrete and time.

Using a small vibrator the wet concrete "filled in" well around the reinforcement. It took three men about one hour to complete the job. At first, one manipulated the chute, one prodded, and one used the vibrator. Once the concrete was placed, the exposed surface was trowelled and wet hessian was placed over it for curing purposes. Since the weather was hot and evaporation was considerable, a sprinkler was installed in order that the hessian be kept continually damp.

B. Prefabrication of the Shell Formwork

The shell, although cast-in-place, was of such a configuration that it was decided to prefabricate the formwork. As mentioned previously, the hyperbolic paraboloid has distinct advantages as far as the formwork is concerned in that it is composed of straight generatrices. This means that straight lumber might be employed.

While the precast concrete units were curing the formwork for the shell were prefabricated in quadrants. Three-eighth inch plywood was nailed to 2 x 6 S4S beams at 12 inch centers. A jig was developed such that the plywood could be warped to the required degree. The edge beams were all 4 inches deep at the edges of each quadrant so that the forms had to be sloped to cater for them. (See drawing no. 1 in the Appendix.)

It should be noted that the quadrants were designed to span between the rigid frame and a temporary support in the position of the horizontal edge beams. (See figures 40 and 41.)

C. Site Preparation

The next step on the program was site preparation. The first job to be done was the leveling of the site, after which the accurate confines of the proposed structure were marked. A transit and tape were used for the layout. Special care was taken in the setting out of the column holes since they were important. It should be noted that a fellow research worker (18) was simultaneously engaged in building a similar shell which was constructed adjacent to the Author's shell and was to be joined at some later date. This meant that all four column holes had to be accurately positioned with respect to each other.

Following the column hole positioning an 18 inch diameter hydraulic auger on a truck was brought to the site and in no time the holes were made. To get a 20 inch diameter hole, which was desired, the auger was pused from side to side thus increasing the hole size.

A post-hole digger and shovel were brought into operation to complete the foundation hole (see drawing no. 1), which had two wings on each side of the 20 inch central hole.

D. Transportation

The site of the proposed structure was about 2 miles from the "casting yard." Hence transportation was required to get the precast units to their final position. Each "dog leg" of the rigid frame weighed about 7/8 ton, so that a 3 ton flat trailer drawn by a tractor

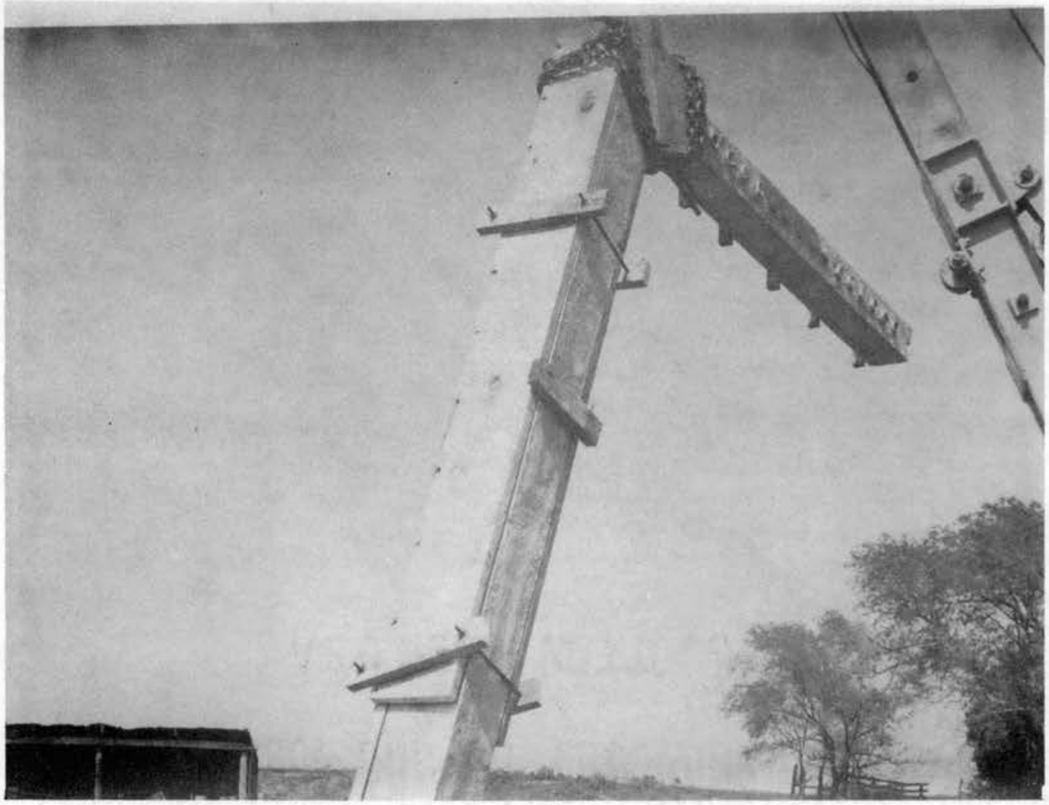


Figure 35. Lifting the precast bent during erection

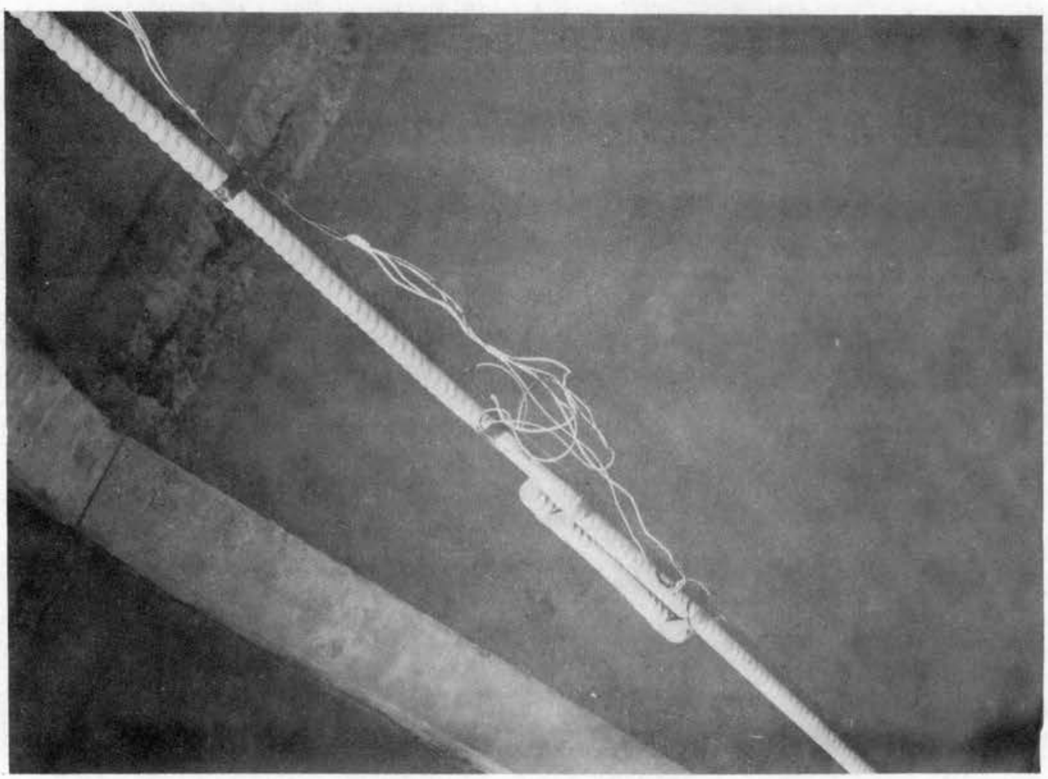


Figure 36. Tie showing strain gauges and compensating gauge

turned out to be ideal.

Once the rigid frame units were cured--(14 days moist curing and 14 days open to the atmosphere.)--a chain and tractor hoist were used to place them on the flat-bed. Care was taken to prevent undue stresses due to impact. Due to the unusual shape of the "dog leg" torsional stresses were prevalent and caution was exercised that the narrow section was well braced while lifting.

Often in precast concrete work large stresses are encountered during transportation and erection which might never be experienced again during the remaining life of the structure. This was the case here as both bending and torsional stresses in the unit resisted the "loading." When on the trailer, the units were towed to the site.

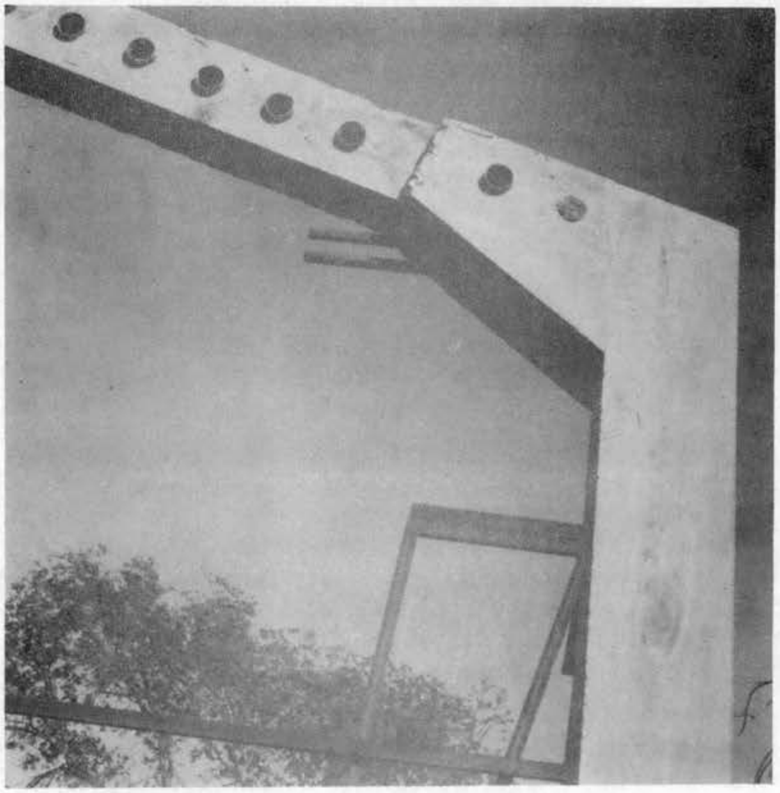
E. Erection

After arriving at the site, a small 5 ton crane lifted the units and lowered them into their respective holes. (See figure 35.)

Notice that the side forms had been removed from the beam section of each "dog leg" whereas the formwork was left on the columns and clamped so that it would furnish maximum frictional resistance. The forms were sawed off at ground level to facilitate positioning on a timber grid system to support the frame while the concrete foundation was placed.

Maneuvering the units in the hole to ensure correct elevation and position, required much patience and back-tracking. A rope attached to a tractor was tied to the lower end of the unit and was used to make sure the column was perpendicular to the ground and in the right location.

Finally, when in position the (3000 psi) ready-mix concrete was



(a)

(b)

Figure 37. Precast column-edge beam units in position

sent for and poured into the pre-dug foundations. (The reinforcement in the foundations can be seen in drawing no. 2 in the Appendix.)

A summary of the equipment and labour involved is shown in Table II.

TABLE II

UNITS	OPERATION	EQUIPMENT	LABOUR
2 Rigid Frame "Dog-Legs"	A Erection	1 - 5 ton crane 1 - tractor Hoisting tackle	* 15 man hours
	B Foundation Placement	3 Prodders 1 Shovel	3 man hours

*NOTE: The men were generally not experienced.

Using a crane with a longer boom and having experienced workers will certainly increase the efficiency in erection. Also, by using a bolted connection to a base plate (see figure 61) much time and labour would be saved. This type of bolted base is commonly used in steel buildings. The bolts can be used as "leveling screws" and when in the correct position, the lock nuts can be tightened. To finish off, the base should be grouted as shown.

F. Placement of Shell Concrete

1. Formwork

The actual formwork was prefabricated as mentioned earlier. It is with the supporting structure which we are now concerned.

Initially it was decided to make use of the rigid frame to support some of the formwork. However, a few extra props would be needed under the precast frame since it was essentially designed as a shell edge beam and it was not practical to increase its size just to withstand the temporary concrete load.

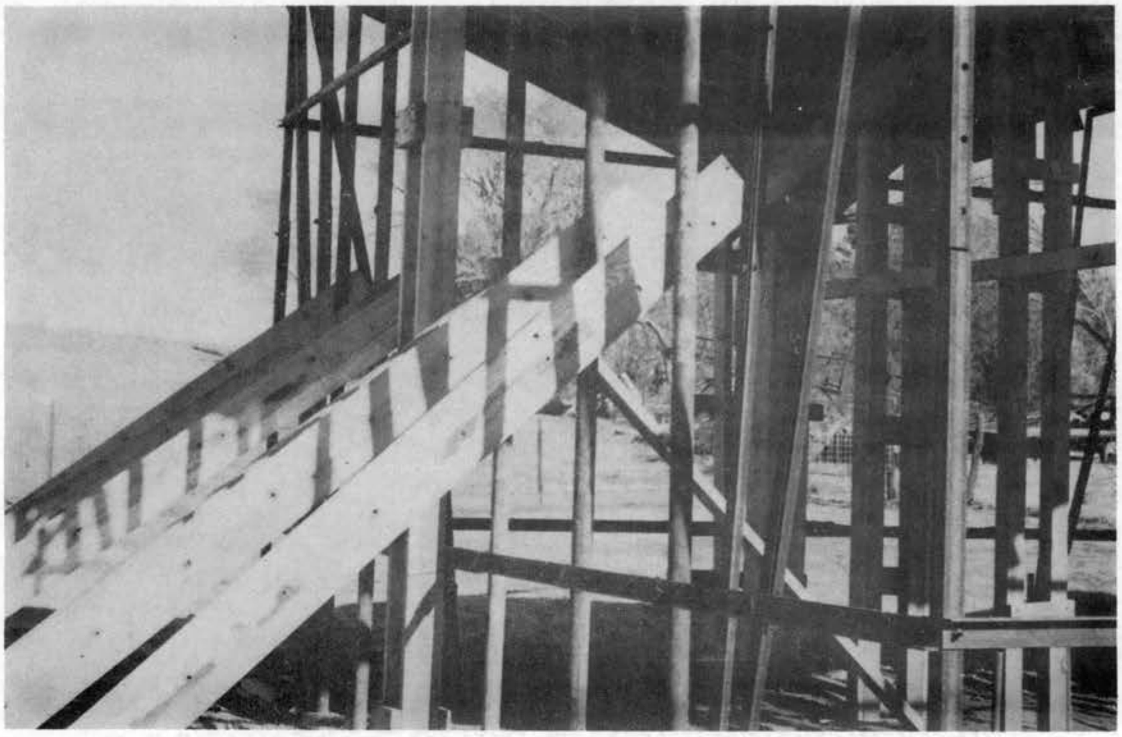


Figure 38. Formwork supports and bracing

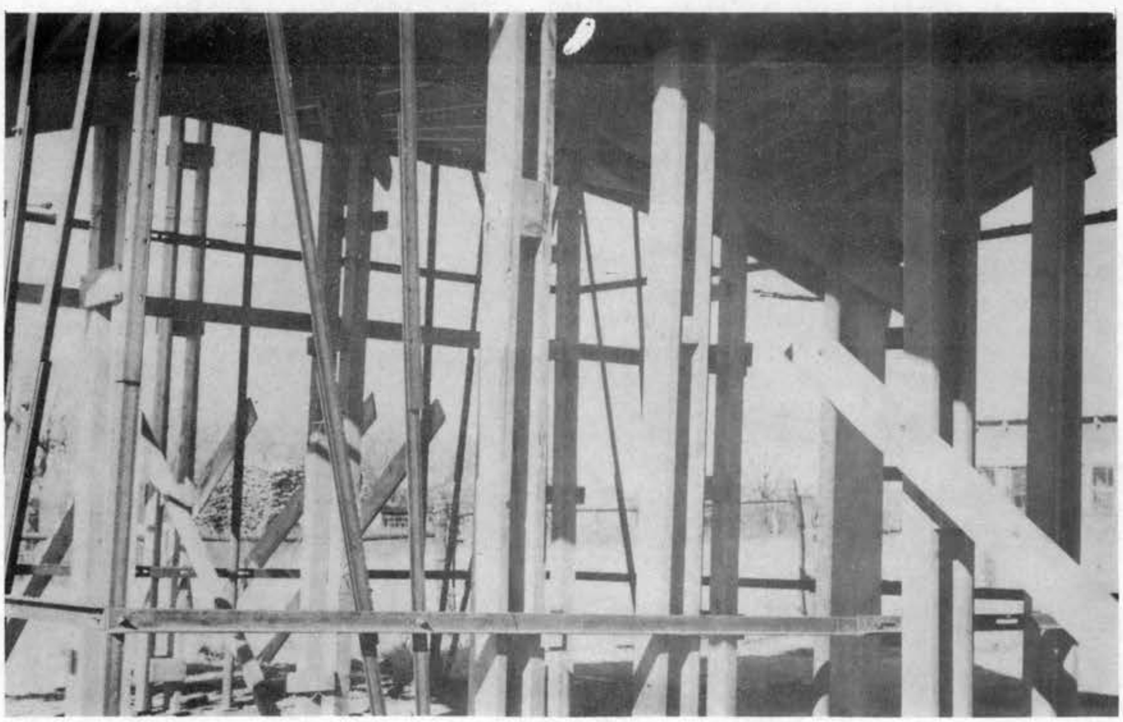


Figure 39. Formwork supports and bracing

The final solution to the support problem was to neglect the rigid frame as a support because when the hangers (which were to be used) were tried out, they entailed a lot of extra work. This extra work would be needed to "cut" the hangers free when the shell was set since they passed right through the shell concrete. To work up a suitable method of using the precast frames and a system of shores to support the forms would involve quite a study in itself. (Some of the Author's thoughts in this matter are mentioned in the conclusions and suggestions.)

Now an ordinary beam and shore arrangement was settled for. (See figure 41.) Steel frame corner supports constructed for the most part of 2 1/2 inch angles were bolted together such that they formed a rigid whole. They were developed by a fellow research worker (18). Some of the shores were 2 1/2 inch diameter steel pipe sections and the rest were double 2 x 6's. The pipe shores were braced laterally to prevent buckling since they were too slender.

The order of erection of the formwork was as follows:

1. The four frame corner supports were erected and braced.
2. The interior shores and beams were positioned alongside the rigid frame units.
3. Using a 5 ton crane the prefabricated quadrants were lifted into position.
4. The rest of the shores (the double 2 x 6's and the pipes) were shimmed into position along the two outer sides parallel to the concrete frame. This meant that the shell forms spanned about 10 feet between the interior and outer shore-beams.

A bit of difficulty arose when covering the quadrants due to the fact that the 2 inch I.D. pipe sections which were cast into the rigid frame protruded too much and got in the way. Hastily, these

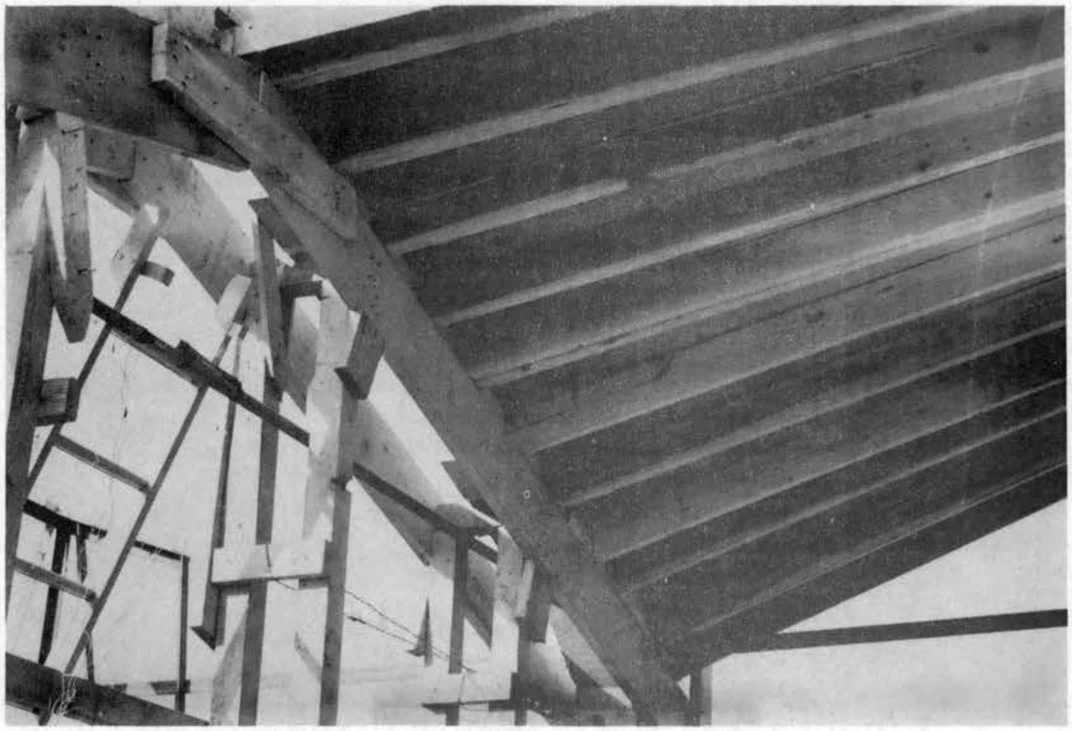


Figure 40. Prefabricated form quadrant in position

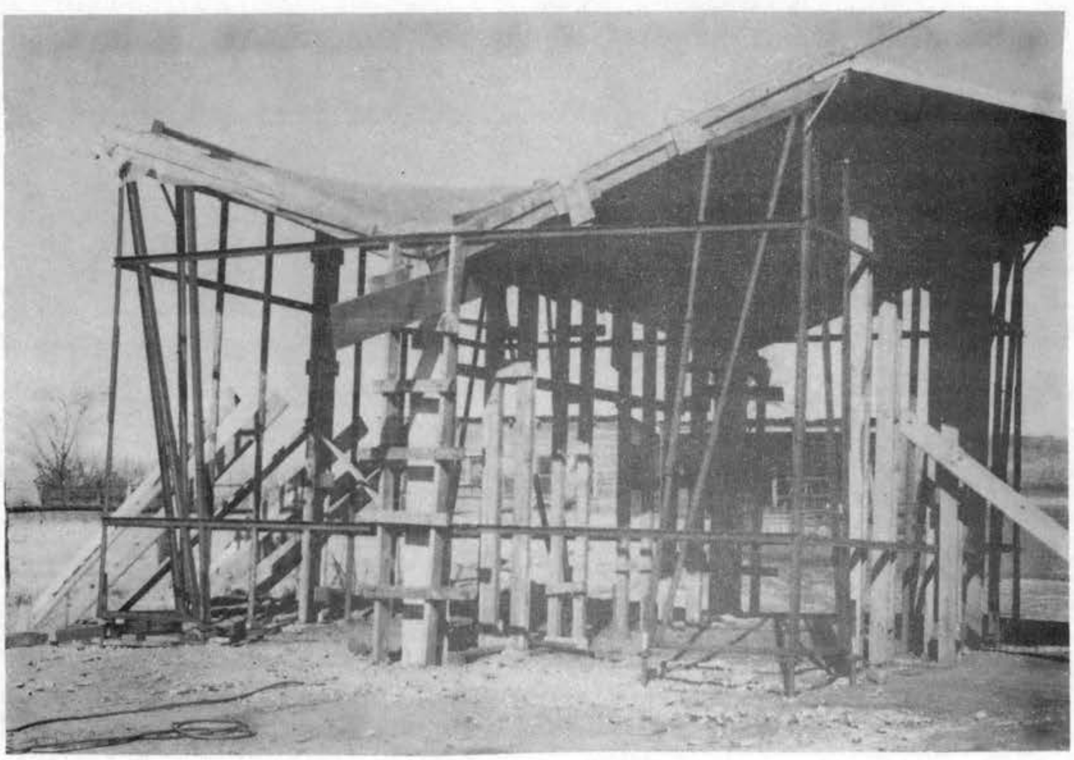


Figure 41. Formwork and supports showing corner supporting system

protrusions were cut off using a gas torch, after which everything went fairly smoothly.

2. Reinforcement (See drawing no. 2 in the Appendix.)

All the reinforcement was pre-bent in the workshop and taken out to the site for placement.

Sideboards were installed and the forms were oiled. Then the shear bars, and the edge beam reinforcement was laid in position. Finally the shell steel was positioned and tied.

The placing of the reinforcements appeared to be just about foolproof and took 3 men about 1 1/2 hours.

3. Concrete Placing

In preparation for the concrete, the steel reinforcement was chaired at regular intervals to help keep a constant cover. Also, a steel pipe was fixed between the central precast edge beam on the rigid frame and the outer side boards to be used by a screed board.

A 1/2 cubic yard bucket was used in conjunction with the 5 ton crane to get the wet concrete from the ready-mix truck to the shell.

The schedule turned out thus:

1. Run wet concrete from the ready-mix truck to the bucket-via the chute.
2. Raise the bucket to the shell level.
3. Deposit the concrete in a small pile which was then raked to distribute it evenly.
4. Screed and finish off the concrete surface.

Because the crane had a short boom and could barely reach the shell level, shovelling was required to distribute the concrete. With a longer boom this would have made for much easier distribution. Special care was taken to ensure that the concrete filled the pipe

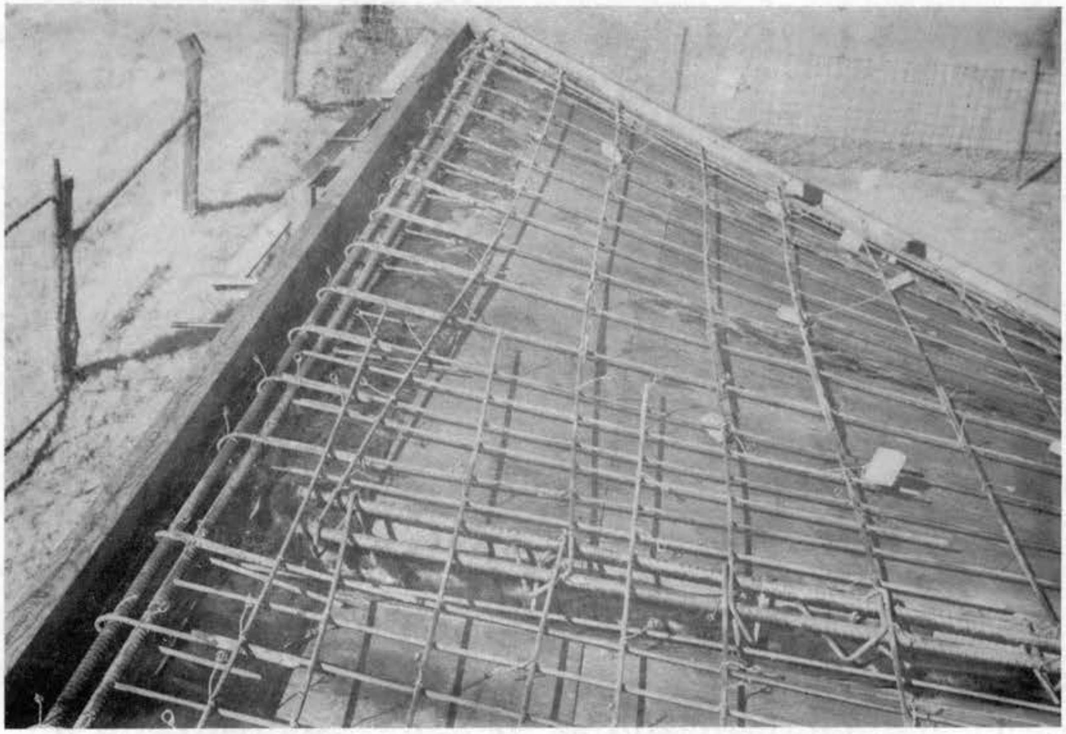


Figure 42. Shell and edge beam reinforcement

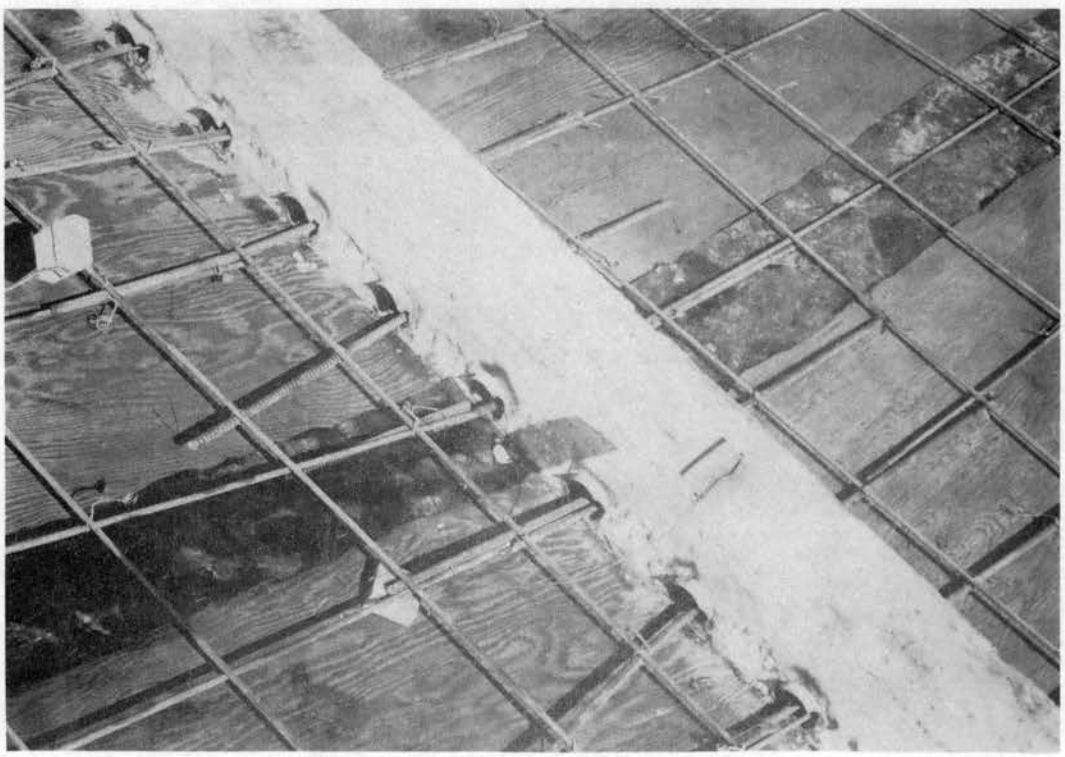


Figure 43. Reinforcement for shear connection between shell and precast edge beam

holes for the shear bars in the rigid frame. This was done by prodding with a bent bar.

Due to everyone walking on the shell reinforcement, it tended to be pushed down to touch the forms even though it had "chairs" at regular intervals. To make sure it was in the center of the shell, the reinforcement was lifted up by hand once the concrete was placed.

No vibration was used as it was not necessary. The screed board was first used as a screeder and then a tamper. This gave the satisfactory result for the concrete placement of the entire 20 feet x 20 feet shell. (See Table III.)

TABLE III

EQUIPMENT	LABOUR
1 - 5 ton crane	3 man hours
1 - 1/2 cubic yard bucket	18 man hours
concrete screeding and finishing equipment	
TOTAL	21 man hours

4. <u>Curing</u>

Damp hessian was placed over the entire shell which was in turn covered by polyethylene. Although late autumn the temperatures were above freezing during the curing period. In fact, the average temperature was about 58° F during the day and about 50° F at night.

On the night after placement a wind came up and blew the hessian off and tended to dry the concrete surface. This had the effect of producing fine drying shrinkage cracks over portions of the shell.

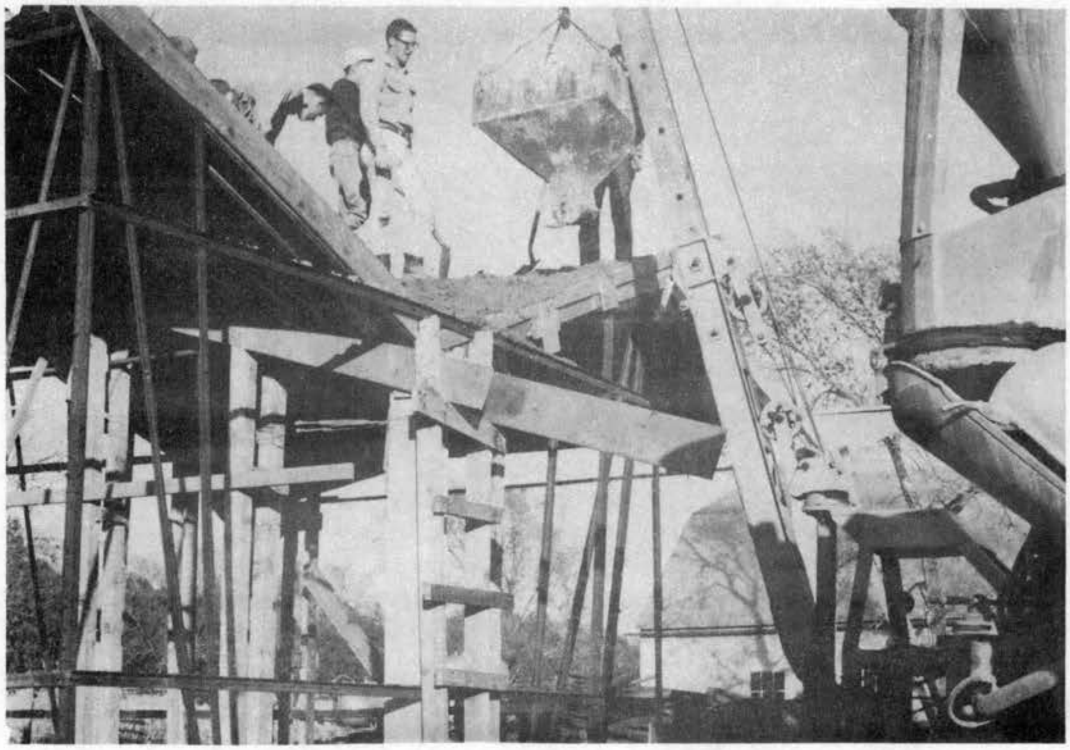


Figure 44. Placement of shell concrete

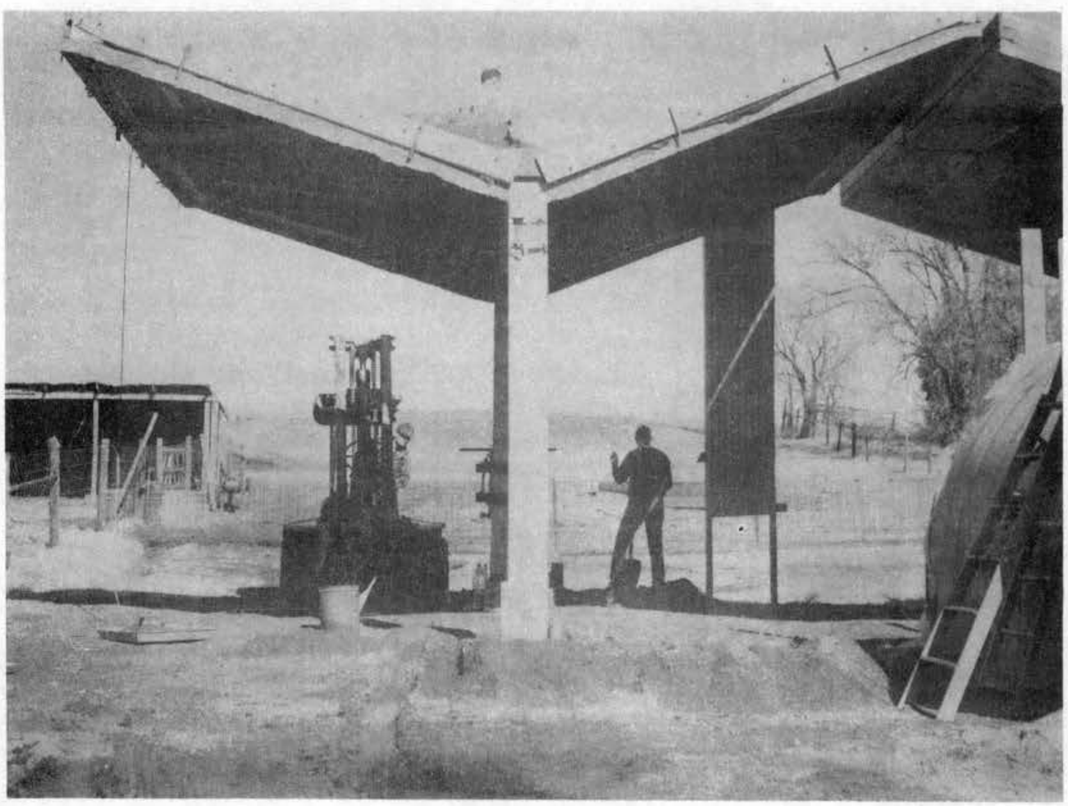


Figure 45. Unsymmetrical load of 35 psf on shell during testing

Immediately when discovered, the covering was reinstalled and precautions were taken that this did not occur again.

Every two days the hessian was wetted using a hose. The polyethylene prevented most evaporation and also had a "steaming" effect when the sun shone on it. After two weeks the covering was removed and the shell was left open to the atmosphere.

CHAPTER V

TESTING PROCEDURE

Once the shell had cured 1 x 8 side boards were attached to the perimeter of the roof by means of special concrete nails and braces to hold back the gravel which was used as the testing load. Sixteen strain gauges were attached to the concrete columns, four at the base and four at the top of each column. (See figure 46.) Each concrete strain gauge was 6 inches long and had a gauge factor of 2.13 and a resistance of 120 ohms. An additional gauge was mounted on a 3 inch concrete cylinder to act as the compensating gauge.

The strain gauges on the steel tie had been placed prior to the erection; however, during the construction one of the gauges on the tie and one on the compensating bar had been ruined and had to be replaced.

The next step was to solder the leads from the gauges to the strain gauge equipment. Here, difficulty was encountered because of cold weather.

The strain gauge equipment consisted of a direct reading Baldwin Strain Indicator with a large balancing dial and scale and a 20 channel Switch and Balanced Unit. Using this instrument the strain could be estimated to the nearest microinch per inch.

An old coffee urn was securely bolted to the column EB (see figure 47) which was to act as the reservoir for the manometer used to measure the deflection. A flexible tube was connected from the outlet of the reservoir to a glass tube which was fixed to a graduated scale on a long steel hook. A water-alcohol mixture was used in the manometer because of freezing conditions. Six "eyes" were secured to the underside of the shell--three along each horizontal side. (See figure 48.) When the deflection was wanted at any of these points, the hook end of the manometer was put in the eye and the graduated scale was read. Of course, this did not give the deflection directly--this reading from a previous reading at the same point would give the change in deflection due to certain conditions with reference to the reservoir. The graduated scale was divided into 5ths of an inch--each 5th being subdivided into ten 50ths. Readings could be made to the nearest 1/50 inch.

When all the equipment was ready for testing, a no load set of readings was taken both for the eighteen strain gauges and the seven deflection positions. The reason for the seventh deflection point is rather obvious. A mark was made on the column NQ which was to position the manometer to record the differential settlement of the columns. This point was point 4 in the tables of results.

Using a fork lift the gravel was deposited on the shell and then spread by hand. During the experiment samples were taken and weighed to determine the load equivalent for different depths of the gravel. Alternate quadrants were loaded with gravel to prevent eccentric loading at this stage. Strain and deflection readings were taken with 2 1/2 inches of gravel (20 psf) and 4 1/2 inches of gravel (35 psf). This was for a symmetrical uniform load over the entire shell.

The 4 1/2 inches load was left on the shell and readings were

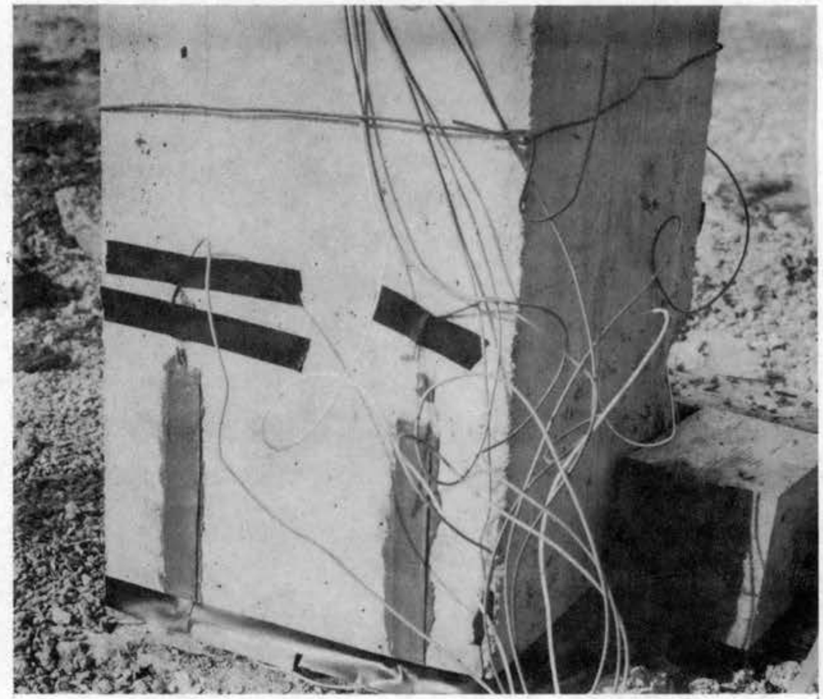


Figure 46. Strain gauges at base of column

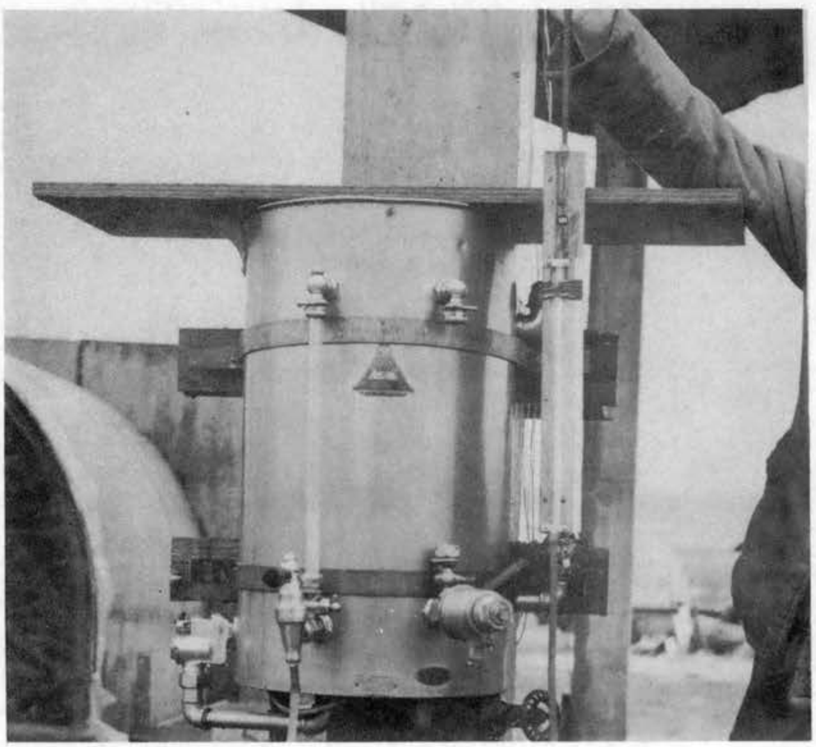


Figure 47. "Manometer" used for measuring deflections

recorded after 20 hours and 44 hours had elapsed. However, due to drizzly weather some of the gauges failed to give readings. This was most probably due to a short because they were wet. Four hours later they had dried and everything was working again. Another set of readings was taken at this time.

The load was then removed from the shell and the no load condition was recorded as a check.

Next, a similar series of tests was conducted with half the shell loaded in the same increments. (See figure 45.) Unfortunately, soon after the 4 1/2 inches gravel load was spread over the two quadrants and the readings were started, something ran amiss. The first indication that something was wrong came when the strain gauge at the bottom of one of the columns failed to respond. Upon checking it was found to be broken because the column had cracked! At this time cracking could be heard and seen in the narrow section just above the haunch in the precast bent.

Hastily the Author withdrew to watch the whole shell, pivotting about these two yielded sections, slowly and majectically keel over. (See figure 56.) This time the gravel did not need to be unloaded from the shell for it had unloaded itself!

CHAPTER VI

PRESENTATION AND EXPLANATION OF DATA

The hyperbolic paraboloid shell, when satisfactorily cured was tested as described in the previous chapter. The data obtained consists of deflection and strain measurements at suitable positions on the structure when the shell is subjected to both a uniform symmetrical load and an unsymmetrical load.

A. Deflection Measurement

The deflection readings were obtained from six points (G, F, Z, M, R, and P) along the extremities of the horizontal edge beams and one point on column NQ. These positions were renumbered point 1 to 7 in a clockwise direction. Point 4 is on column NQ. (See figure 48.)

In Table IV the manometer readings were taken for no load, 2 1/2 inches gravel load and 4 1/2 inches gravel load. It should be noted that 2 1/2 inches gravel weighs 20 psf and 4 1/2 inches weighs 35 psf. The manometer readings were all relative to a reservoir attached to column EB.

From these readings two types of deflections could be calculated. Firstly, the shell deflection relative to an imaginary reference line through the water level in the reservoir and a fixed point on column NQ can be worked out. Secondly, the relative movement of the columns can be calculated. It is interesting to point out that the column movements with respect to the ground cannot be gained from this experimental equipment since the manometer does not record any movement the columns make in unison. They could both settle 1 foot into the ground and the manometer readings would not record it!

The data in Table V is gained from Table IV. The deflection of column NQ with respect to the imaginary reference line will obviously be zero. With this in mind the roof deflections were calculated by first correcting the readings and then finding the difference between the readings for a load change. (See Table VI.)

The values in Table VI for the deflections were calculated from Table V. One unit on the manometer was equal to 0.2 inches. The results of the deflections for various load increments and also for a time change are plotted in figure 49.

To obtain the relative movement of columns EB and NQ the manometer readings for point 4 were employed (in Table V). As the column NQ moved down, the readings decreased. However, when the readings increased, it was assumed the column NQ moved up relative to column EB but not relative to the ground. This meant that column EB actually settled more than NQ. The reason is that it is assumed the columns would only move down under an increase in load. Figure 51 is a graphical representation of the differential settlement of the columns.

Similar procedures were adopted in the analysis of the data in Table VII for unsymmetrical loading conditions. The results of the corrected deflections are recorded in Table VIII and graphed in figure 50.

B. Strain Measurement

For symmetrical and unsymmetrical loading the strain was measured at eighteen positions. On each column, top and bottom, the strain was recorded at four points as indicated in figure 48. The strain gauge positions are numbered from 1 to 16 on the concrete columns and 17 and 18 on the steel tie.

The moduli of elasticity of the concrete and the steel tie were deduced from test specimens. These values multiplied by the strain give the stress at the positions considered.

The basic reason for the strain measurements was to not only find out about the actual strain but also to compare the actual stresses with theoretically expected values. In the computation of the theoretical stresses the properties of the column section are needed. These figures are shown in Table IX for both the cracked and the uncracked sections. No explanation is offered here as to the calculation procedure for these properties since it is presented in most reinforced concrete design texts (9).

Table X needs a fair amount of explanation. From the data it is evident that something unusual is occurring or the equipment is malfunctioning. When subjected to 20 psf and initially under the 35 psf load, the columns appear to be in tension! However, as time progresses the strains become compressive but continue to remain as such even when no load conditions are restored!

The steel tie strains are indicated by the readings for gauges 17 and 18. The first few readings show that the strains increase with increase in load. However, the 48 hour strain for gauge 17 is apparently compressive which is absurd. For this reason this value has been neglected when computing the tie stresses in Table XIV.

The axial strains and stresses in the columns due to the symmetrical load are laid out in Table XI. First of all, since the load is symmetrical the strains should be similar on the gauges at the bottom or top of the column. Secondly, because the strains registered are due only to the <u>change</u> in load, the values almost certainly will be identical in <u>all</u> gauges! The reason for separating the base and top strains is that apparently the location had some effect on the observed measurements.

A certain amount of time elapsed between the no load conditions, the 20 psf and 35 psf conditions. This time, although not recorded as such, was about one hour.

It is important to note that the strain when under a sustained load is in fact greater than the initial strain because of the rather unpredictable phenomena of creep. The asterisk (*) designates the values probably affected.

When the tests were carried out with an unsymmetrical load of 20 psf, the observed strains shown in Table XII are much more in agreement with the predicted values. (See Table XIII.) The column stresses are primarily due to the moment of the unsymmetrical load. Since the strain measured is due only to the 20 psf, the value is therefore relative and not absolute. A typical calculation for the theoretical stress and strain in Table XIII is as follows:

Assume the elastic equations to hold and also that the section is uncracked.

$$\text{Stress in column:} \quad S = \frac{-P}{A} \pm \frac{MC}{I} = \frac{-20(10 \times 10)}{122} \pm \frac{2000(60)(3)}{1033}$$

$$= -16 \pm 349$$

$$= -365 \text{ or } +333 \text{ psi}$$

It should be noted that the gauges were 3 inches from the column's neutral axis.

For the "cracked" section analysis the combined compressive stress at the gauge is -385 psi and the tensile stress is indeterminable because the elastic relationship between stress and strain is not proportional for a cracked section.

From the actual values in Table XIII the stress distribution is drawn for the column section in figure 52. Simple proportion was used to get the stresses at the extreme fibers as indicated in the figure.

Strain gauges 17 and 18 were on the steel tie bar. The stresses and strains for actual and theoretical conditions are compared in Table XIV for both load tests.

The strain in the tie is calculated thus:

$$\epsilon = \frac{S}{E} = \frac{T}{EA} = \frac{pab^2}{hEA}$$

$$\text{i.e.}\quad \epsilon = \frac{p(10)(10)^2}{3(30 \times 10^6)(1.27)}$$

$$\epsilon = 8.75 \text{ p micro ins/ins}$$

Due to an unsymmetrical load of 20 psf over half the shell the strain (in the tie) is assumed to be half of that for the same load over the entire shell. The tie stress is plotted in figure 53 versus the superimposed load on the shell. Both actual and theoretical relationships are drawn for comparison.

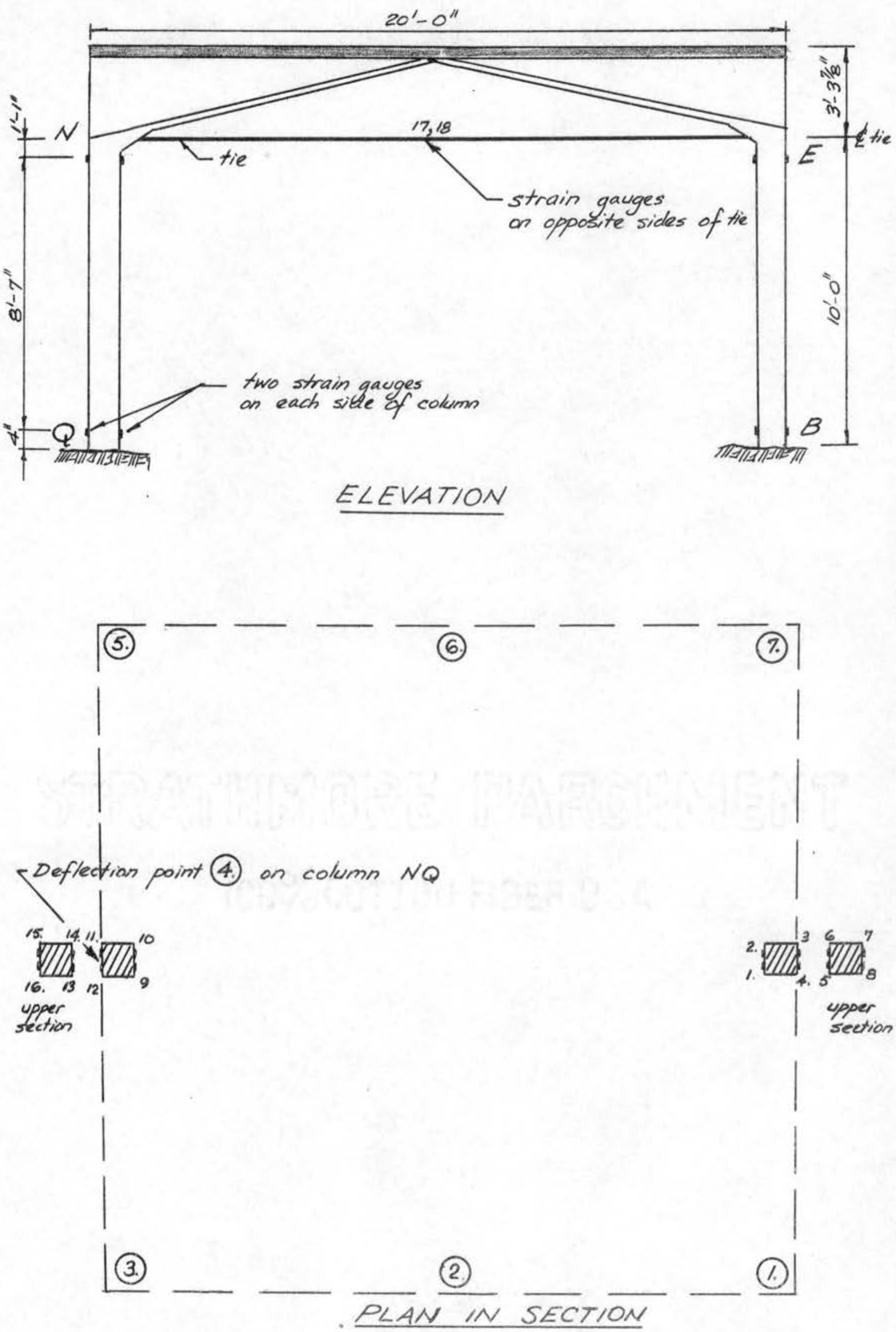


Figure 48. Positions of strain gauges and deflection points.

TABLE IV

SYMMETRICAL LOADING OVER ENTIRE SHELL

Observed Deflection x 1/5 Inch

Point No.	No Load	2 1/2 inches gravel	4 1/2 inches gravel	with 4 1/2 inches gravel 20 hrs.	44 hrs.	48 hrs.	No Load
1	24.4	24.5	24.6	22.9	20.8	24.2	23.6
2	32.3	32.5	32.9	31.1	29.4	31.4	31.4
3	28.9	28.4	29.2	27.4	25.8	28.2	28.6
4	27.1	26.7	27.7	25.7	25.2	26.3	26.7
5	38.0	37.3	37.9	37.2	35.8	38.2	37.6
6	38.9	38.0	38.2	37.9	36.6	38.7	38.1
7	34.6	33.6	33.4	32.6	31.4	33.3	32.6

TABLE V

CORRECTED DEFLECTION READINGS WITH RESPECT TO AN IMAGINARY REFERENCE LINE FOR A SYMMETRICAL LOAD

Deflection x 1/5 Inch

Load (psf)	Point	Reading w.r.t. water level	Correction	Corrected rdg w.r.t. ref line
20	1	24.5	0	24.5
	2	32.5	+0.2	32.7
	3	28.4	+0.4	28.8
	4	26.7	+0.4	27.1
	5	37.3	+0.4	37.7
	6	38.0	+0.2	38.2
	7	33.6	0	33.6
35 (0 hrs)	1	24.6	0	24.6
	2	32.9	-0.3	32.6
	3	29.2	-0.6	28.6
	4	27.7	-0.6	27.1
	5	37.9	-0.6	37.3
	6	38.2	-0.3	37.9
	7	33.4	0	33.4
35 (20 hrs)	1	22.9	+0	22.9
	2	31.1	+0.7	31.8
	3	27.4	+1.4	28.8
	4	25.7	+1.4	27.1
	5	37.2	+1.4	38.6
	6	37.9	+0.7	38.6
	7	32.6	+0	32.6
35 (44 hrs)	1	20.8	0	20.8
	2	29.4	+1.0	30.4
	3	25.8	+1.9	27.7
	4	25.2	+1.9	27.1
	5	35.8	+1.9	37.7
	6	36.6	+1.0	37.6
	7	31.4	0	31.4
35 (48 hrs)	1	24.2	0	24.2
	2	31.4	+0.4	31.8
	3	28.2	+0.8	29.0
	4	26.3	+0.8	27.1
	5	38.2	+0.8	39.0
	6	38.7	+0.4	39.1
	7	33.3	0	33.1
0 (50 hrs)	1	23.6	0	23.6
	2	31.4	+0.2	31.6
	3	28.2	+0.4	29.0
	4	26.7	+0.4	27.1
	5	37.6	+0.4	38.0
	6	38.1	+0.2	38.3
	7	32.6	0	32.6

TABLE VI

DEFLECTION (INCHES) RELATIVE TO REFERENCE LINE FOR VARIOUS LOAD CHANGES

POINT	Load Change 0-20 psf	20-35 psf	Load = 35 psf			Load Change 35-0 psf
			Time Change 0-20 hrs.	20-44 hrs.	44-48 hrs.	
1	0.02	0.02	-0.34	-0.42	+0.68	-0.12
2	0.08	-0.02	-0.16	-0.28	+0.28	-0.04
3	-0.02	-0.04	+0.04	-0.22	+0.26	0
4	0	0	0	0	0	0
5	-0.06	-0.08	+0.26	-0.18	+0.26	+0.2
6	-0.14	-0.06	+0.14	-0.20	+0.30	-0.16
7	-0.2	-0.04	-0.16	-0.24	+0.34	-0.10

TABLE VII

UNSYMMETRICAL LOADING

Observed Deflection x 1/5 Inches

Point No.	No Load	2 1/2 inches gravel (20 psf)	4 1/2 inches gravel (35 psf)
1	23.4	18.4	C
2	31.4	25.9	O
3	28.4	23.2	L
4	26.6	27.0	L
5	38.9	44.5	A
6	39.2	45.3	P
7	33.9	39.9	S
			E
			D

Remarks: The deflection was measured by use of a manometer and a graduated scale. This scale had 60 units to 1 foot, therefore each unit was 1/60 feet or 1/5 inch.

TABLE VIII

CORRECTED DEFLECTIONS WITH RESPECT TO AN IMAGINARY REFERENCE LINE FOR AN UNSYMMETRICAL LOAD CHANGE

Load (psf)	Point	Deflection x 1/5 Inches			Deflection (ins)
		w.r.t. water level	Correction	w.r.t. ref line	w.r.t. ref line
Changing	1	-6.0	0	-6.0	-1.20
from	2	-5.5	-0.2	-5.7	-1.14
	3	-5.2	-0.4	-5.6	-1.12
0 to 20	4	+0.4	-0.4	0	0
	5	+5.6	-0.4	+5.2	+1.04
	6	+6.1	-0.2	+5.9	+1.18
	7	+6.0	0	+6.0	+1.20

Remarks: 1. w.r.t. means "with respect to"

2. The reference line referred to here is an imaginary line joining fixed points on the two columns. The point on Column EB is the water level. This enables differential settlement to be recorded.

TABLE IX

PROPERTIES OF THE COLUMN SECTION

Symbol	For the section uncracked	For the section cracked
b	10 inches	10 inches
d	8 inches	8 inches
kd	5 inches	3.16 inches
jd	6.27 inches	6.55 inches
t	10 inches	10 inches
A_t	122 square inches	63.2 square inches
I_t	1033 $inches^4$	394 $inches^4$
Es	30×10^6 psi	30×10^6 psi
Ec	4.3×10^6 psi	4.3×10^6 psi
n	7	7

Definitions:

b = breadth of section
d = effective depth from compr. flange to center of tensile reinforcement
kd = neutral axis depth
jd = lever arm--note that this varies slightly depending upon the stress
t = total thickness of section in plane of bending
A_t = area of transformed section
I_t = moment of inertia of transformed section
Es = modulus of elasticity of steel reinforcement
Ec = modulus of elasticity of concrete
n = Es/Ec = modular ratio

TABLE X

SYMMETRICAL LOADING OVER ENTIRE SHELL

Observed Strain micro inches/inch

Point No.	No Load	2 1/2 inches gravel (20 psf)	4 1/2 inches gravel (35 psf)			No Load
			0 hrs.	20 hrs.	48 hrs.	
1	0	10	23	-151	-236	-255
2	0	47	43	-138	-215	-240
3	0	40	60	-108	- 97	-160
4	0	3	38	-119	-126	-148
5	0	-30	-22	-192	-190	-207
6	0	9	5	-169	-152	-200
7	0	78	8	- 50	- 40	-185
8	0	20	18	- 79	-170	-188
9	0	12	32	-111	-170	-215
10	0	31	30	-105	-188	-214
11	0	- 5	5	- 81	-220	-118
12	0	-32	- 5	-109	-294	-130
13	0	-43	-40	-179	-222	-180
14	0	-15	-30	-159	-220	-180
15	0	51	85	- 38	-202	-285
16	0	20	60	- 59	-222	-269
17	0	47	68	103	-129	- 35
18	0	95	175	190	181	- 30

Remarks:

1. The strain under 4 1/2 inches gravel after 44 hours was rather uncertain since some of the gauges were wet and it was impossible to balance the circuit. For this reason the 48 hour reading was recorded.
2. The negative sign (-) indicates compression (or shortening). The values with no sign are tensile strains.

TABLE XI

AXIAL STRAIN AND STRESS IN COLUMNS DUE TO A SYMMETRICAL UNIFORM LOAD OVER THE ENTIRE SHELL

Load psf	Time hrs.	Strain (micro ins/in)							
		Column EB		Column NQ		Average		Theoretical	
		Base	Top	Base	Top	Base	Top	Base	Top
0	0	0	0	0	0	0	0	0	0
20	0+	+25	+19	+2	+3	+14	+11	-8	-8
35	0+	+41	+2	+16	+19	+29	+11	-13	-13
	20	-129	-123	-102	-109	-116	-116	-13*	-13*
	48	-168	-138	-218	-217	-193	-178	-13*	-13*
0	50	-201	-195	-169	-229	-185	-212	-0	-0

Load psf	Time hrs.	Stress (psi)			
		Average		Theoretical	
		Base	Top	Base	Top
0	0	0	0	0	0
20	0+	+60	+47	-33	-33
35	0+	+125	+47	-58	-58
	20	-500	-500	-58	-58
	48	-833	-765	-58	-58
0	50	-796	-912	0	0

Remarks:

1. "Base" refers to the strain gauges mounted at the base of the columns.
2. The observed strains are the average of all the four gauges at the positions concerned.
3. (-) refers to compressive strains; (+) to tensile strain.
4. $E_c = 4.3 \times 10^6$ psi

TABLE XII

UNSYMMETRICAL LOADING
OBSERVED STRAIN AND CALCULATED STRESS

(Strain micro inches/inch; Stress psi)

Point No.	No Load reading	2 1/2 inches gravel (20 psf)	Strain 10^{-6} ins/ins	Stress psi
1	745	915	170	734
2	760	735	- 25	-108
3	840	800	- 40	-173
4	852	1005	153	660
5	793	943	150	648
6	800	760	- 40	-173
7	815	800	- 15	- 65
8	812	971	159	686
9	785	892	107	462
10	786	725	- 61	-263
11	882	805	- 77	-332
12	876	950	74	319
13	820	880	50	216
14	820	748	- 72	-310
15	715	652	- 63	-272
16	731	798	67	289
17	965	960	- 5	-150
18	970	1038	68	+2040

Remarks: 1. The Modulus of Elasticity for steel was taken to be $E_s = 30 \times 10^6$ psi and for concrete $E_c = 4.3 \times 10^6$ psi

2. The negative values indicate compressive strains and stresses.

TABLE XIII

STRAIN AND STRESS IN COLUMNS DUE TO
AN UNSYMMETRICAL UNIFORM LOAD OF 20 PSF

Position	Gauge No.	Strain (micro ins/ins)			Stress (psi)		
		Actual	Theoretical		Actual	Theoretical	
			Uncracked	Cracked		Uncracked	Cracked
Bottom of Column EB	1	170	77	335	734	333	---
	2	-25	-85	-90	-108	-365	-385
	3	-40	-85	-90	-173	-365	-385
	4	153	77	335	660	333	---
Top of Column EB	5	150	77	335	648	333	---
	6	-40	-85	-90	-173	-365	-385
	7	-15	-85	-90	-65	-365	-385
	8	159	77	335	686	333	---
Bottom of Column NQ	9	107	77	335	462	333	---
	10	-61	-85	-90	-263	-365	-385
	11	-77	-85	-90	-332	-365	-385
	12	74	77	335	319	333	---
Top of Column NQ	13	50	77	335	216	333	---
	14	-72	-85	-90	-310	-365	-385
	15	-63	-85	-90	-272	-365	-385
	16	67	77	335	289	333	---

Remarks: 1. The positions of the gauges on the structure can be seen in figure 48.

2. "Uncracked" designates the theoretical values calculated assuming the section to be uncracked.

TABLE XIV

STRAINS AND STRESSES IN TIE

A. <u>Due to a Symmetrical Uniform Load</u>

Superimposed Load (psf)	Time (hrs.)	Strain micro ins/ins Observed	Strain micro ins/ins Theoretical	Stress (psi) Actual	Stress (psi) Theoretical
0	0	0	0	0	0
20	0+	95	175	2750	5250
35	0+	175	307	5250	9200
	20	190	307	5700	9200
	48	211	307	6330	9200

B. <u>Due to an Unsymmetrical Load</u>

Superimposed Load (psf)	Time (hrs.)	Strain micro ins/ins Observed	Strain micro ins/ins Theoretical	Stress (psi) Actual	Stress (psi) Theoretical
0	0	0	0	0	0
20	0+	68	88	2040	2620

Remarks:

1. The stresses and strains in the table are calculated with reference to the zero superimposed load as the datum.
2. For the tie $E_s = 30 \times 10^6$ psi.
3. The above table is constructed using the values obtained from strain gauge 18 only because gauge 17 appears to be defective.
4. The readings for the strain at 48 hours were related to the no load readings at 50 hours.

CHAPTER VII

DISCUSSION OF THE RESULTS

A. Construction

From an overall standpoint the reinforced concrete hyperbolic paraboloid shell with a prefabricated column-edge beam assembly proved satisfactory. There are, however, many minor problems which need to be overcome. The next chapter is devoted to suggestions for further study in this interesting field.

The results, first of all, will be discussed from the point of view of the actual construction techniques of the structure.

1. Forms

Because of its configuration the h-p shell formwork proved very satisfactory. The prefabricated quadrants presented no major difficulty in construction. It should be borne in mind that the plywood or shell-forming surface should be made of a material which can easily be "warped."

Due to the narrowing of the precast bents just above the haunch the forms had to be made to fit. Difficulty was encountered in lowering the quadrants into position prior to the pouring of the concrete because of this change in shape as well as the fact that the "shear pipes" which were cast in the bent protruded about 1/4 inch. (See drawing 1 in the Appendix.)

For the 20 feet by 20 feet shell each prefabricated quadrant form weighed about 450 pounds and was fairly convenient to handle with normal equipment. Nevertheless, for a large shell this may present a problem and perhaps sections smaller than each quadrant could be prefabricated.

The forms for the precast column-edge beam components were very straightforward. The 2 inch lumber and 16^d double-headed nails were easy to work with. The "shear pipes" which were cast into the edge beam section were secured to the bottom of the forms by cutting 1/2 inch deep holes and hammering the pipes into these tight-fitting holes. The one disadvantage with this is that the pipes protruded from the precast units when the forms were removed. (This is mentioned above.)

2. Precast Components

Generally the precast column-edge beam units were exciting--unique--but rather inefficient and unsatisfactory. One unit, trying to fulfill so many functions is apt to be ideally suitable for one condition and hopelessly ineffective for another. However, much can be learned from them and with a certain degree of modification and experimentation these units could be highly successful for mass production, and simplicity in the construction of h-p shells for rural structures.

The two major functions of these precast bents are to temporarily support the formwork for the shell concrete and to convey the accumulated shear stresses from the shell to the columns when the completed shell is loaded. In addition, these units must be easily formed, cast, transported and erected. Each of these areas can present a headache.

In the structure concerned, the formwork was relatively simple--the reinforcement rather cramped--and the casting straightforward. With small units it was only natural that the reinforcement was cramped, but this did not create any problem.

The transportation was a much more delicate situation. The edge beam section was slender enough without the shear "holes" in it! Much care was needed that undue torsional and bending stresses were not induced while lifting and handling the bents.

The erection, also, was not too easy, especially since the units had to line up in both planes and just touch at the middle. (See figure 37.) It was unsuccessfully attempted to lift the unit and lower it, with the column vertical, into the pre-dug footing hole where it would rest on the cribbing. The unsuccessful part was to get the column vertical. Eventually, another chain was attached to the bottom of the column for the application of a horizontal force which finally did the trick. Perhaps, in future, some sort of lifting lugs may be cast into the units to ensure the column remaining vertical.

Once the footing and shell were cast, the bent transformed into an edge beam subjected to shear forces along its length and a column resisting all the vertical components of these forces in the edge beams. The precast units appeared to behave well under symmetrical loading conditions. Nevertheless the reinforcement bonding the shell to the edge beam should be increased, especially toward the column section. In addition to this it would be wise to cast wings perpendicular to the central edge beam on the top of the column. These wings would not be necessary when more than one shell is constructed and cantilever action can be catered for in a different manner. (See Failure Analysis later in this chapter.)

The completed structure was very flexible. This was particularly noticeable when walking along the edge of the shell. This flexibility was partly in the shell but mainly in the columns. A 180 pound person by moving up and down could cause the shell to deflect about 1 or 2 inches with ease. To reduce this so-called flexibility, a larger column section should be adopted. Perhaps a 12 inch x 12 inch section would be suitable. It should be mentioned that the original design called for an 8 feet and not a 10 feet column. Obviously this length would also affect it.

3. Footings

The footings appear to be quite adequate for the prevailing soil conditions. From the construction aspect the only probable difficulty to present itself would be the procurement of equipment to auger the 20 inch diameter hole 5 feet deep. Smaller equipment could be used to make a smaller hole which could later be enlarged by hand.

From the load tests the columns were observed to settle under load. This maximum settlement was of the order of 1/2 inch. To reduce this the spread section of the footing could be enlarged to, say, 6 feet by 3 feet.

4. Supports

The supporting system was not very suitable or efficient. The original idea was to devise some sort of hanging arrangement by which the prefabricated forms could be hung from the erected precast units. This did not materialize and doube 4 x 4's had to hold the forms instead.

The corner supports, used by Noyes (18) on a similar structure, were fairly practical and definitely held everything together.

The double 2 x 6 props were simple to make and easy to erect. Wedges were used to adjust elevation. The 2 1/2 inches pipe used for the same purpose were more difficult to handle and required bracing to prevent buckling.

Generally much thought and experimenting should present a practical solution. The corner supports and the hanging devices are the main problems since commercially advertised shores can be used in the other areas.

5. Tie

The tie functioned satisfactorily. It was an easy task to erect by merely welding it to protruding reinforcement.

In the case of practical structure this tie can get in the way. However, should more than one shell be constructed and/or abutments placed on the columns to take the bending, the tie could be done away with.

B. Load Tests

1. Deflections

(a) Shell (See figure 48.)

When subjected to a symmetrical uniform load the shell apparently behaved unexpectedly.

From the initial no load condition through the 20 psf to the 35 psf load the edges of the shell moved upwards with the exception of points 2 and 3. (See figure 49.) Point 7 moved up 0.24 inches and point 2 moved down 0.06 inches--these values being the extremes.

With the sustained maximum load of 35 psf point 1 moved up

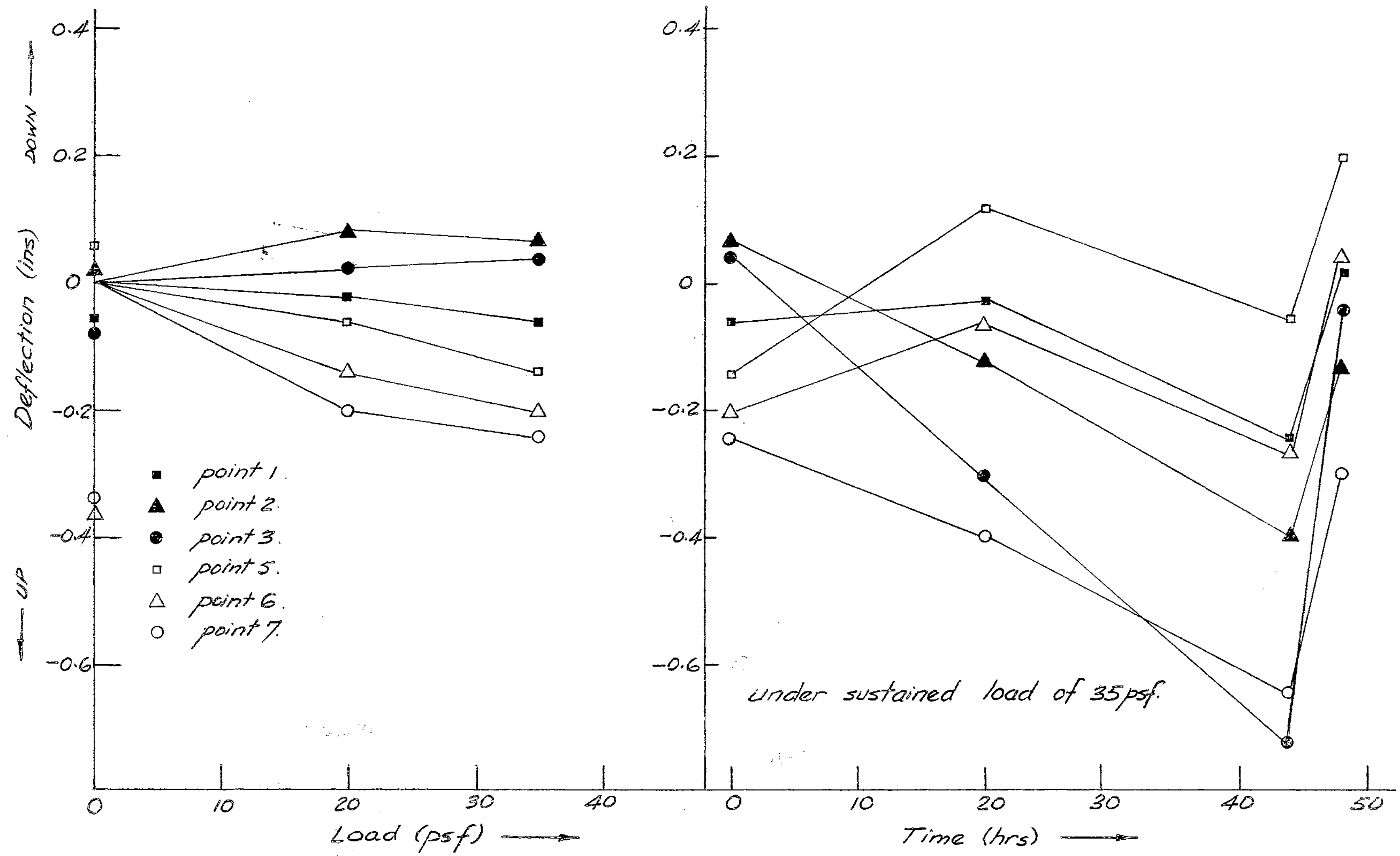


Figure 49. Shell deflection curves when under symmetrical loading conditions.

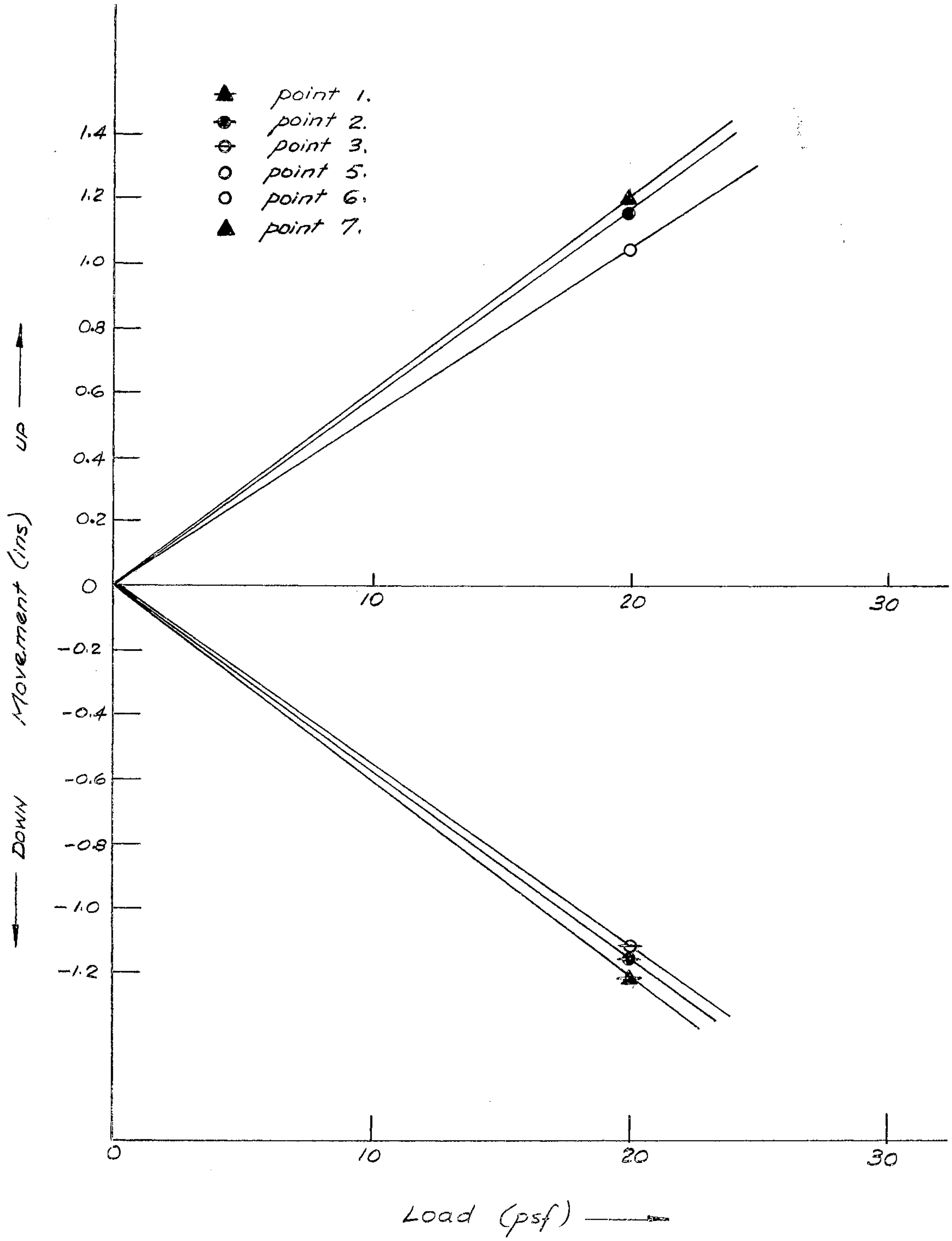


Figure 50. Load vs. Deflection when under unsymmetrical loading conditions.

0.72 inches after 44 hours and point 5 moved down 0.2 inches 48 hours after the load was applied. It is interesting to see that after being at the maximum upward deflection after 44 hours, all the edges moved downwards during the subsequent 4 hours.

A probable explanation for this is that the center of the shell sagged under the load causing the edges to move upwards. When the shell was unloaded, these edges deflected down again as the center "sprung" back.

All the shell deflections were with reference to an imaginary line passing through the water level as column EB and a fixed mark on Column NQ.

The load deflection curve for the unsymmetrical 20 psf were as expected. Points 5, 6 and 7 deflected 1.02 inches to 1.2 inches. Apparently under the loading the shell itself is not bending while the columns do all the bending. (See figure 50.)

(b) Columns

The results indicate that there is quite a good deal of differential settlement of the columns. Overall they each settled about 1/2 inch. (See figure 51.) Upon release of the load column EB moved up about 0.1 inch and column NQ did not budge.

Column NQ did not settle much for 44 hours and then apparently 0.22 inches in 4 hours. Column EB, on the other hand, behaved more as would be expected, settling about 1/2 inch over the whole 48 hours while under maximum load.

Under the cantilever load, column NQ moved down an amount 0.08 inches.

2. Stresses and Strains

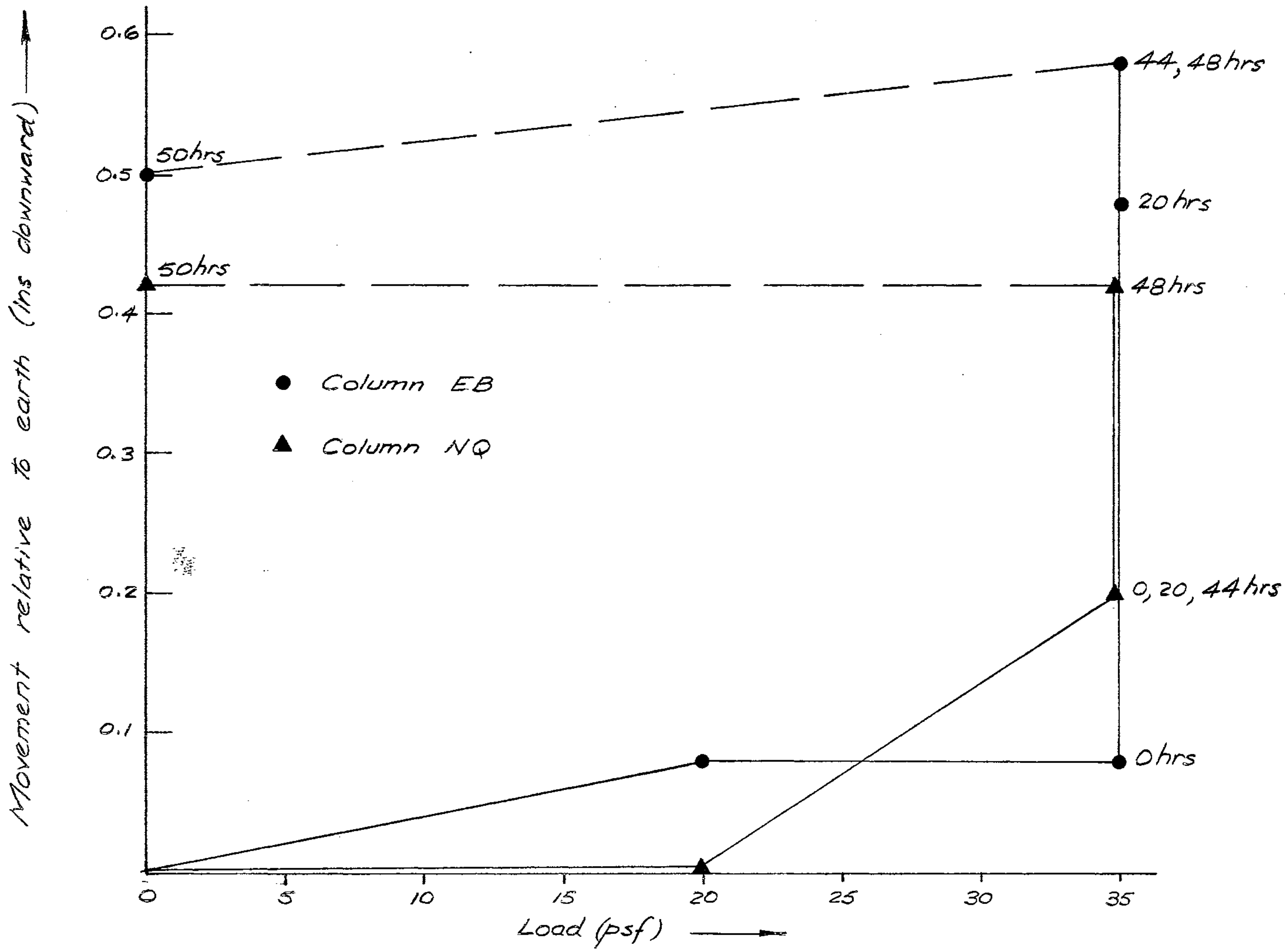


Figure 51. Load vs. Movement of columns for a symmetrical load.

(a) <u>Columns</u>

Due to a symmetrical uniform load the strain readings warrant explanation.

Initially, when the load was 20 psf (see Table X), there was apparently slight bending in the columns. This can be clearly seen by looking at the strain readings for gauges 9, 10, 11 and 12, and also 13, 14, 15 and 16 on column NQ (refer to Table X.) Gauges 9 and 10 indicate a slight tensile strain whereas 11 and 12 indicate a small compressive strain. Since these gauges are on opposite faces, this suggests bending. A similar explanation can be offered for the upper gauges 13, 14, 15 and 16. The strains in the other column are all tensile with the exception of gauge 5. Because of the smallness of these strains there could be many reasons for this occurrence. Should there have been a temperature differential of 7°F between the active and compensating gauge a strain of 50 micro ins./ins. could have resulted.

For the initial 35 psf load the gauges showed little change, but after 20 hours with the same load, all the gauges indicated large compressive strains which were much larger than expected. These large strains persisted for the 48 hour reading and even <u>increased</u> for the no load check when the load was removed.

Rain occurred during the test between 0 and 20 hours and again between 20 and 48 hours. Samples were taken of the test load gravel but little change in weight was noted. This is because the shell drained well and the gravel was not very "absorbent." This rain did cause a group of gauges to short circuit when they were read after 44 hours, but by the time the

48 hours readings were made, all functioned apparently satisfactorily.

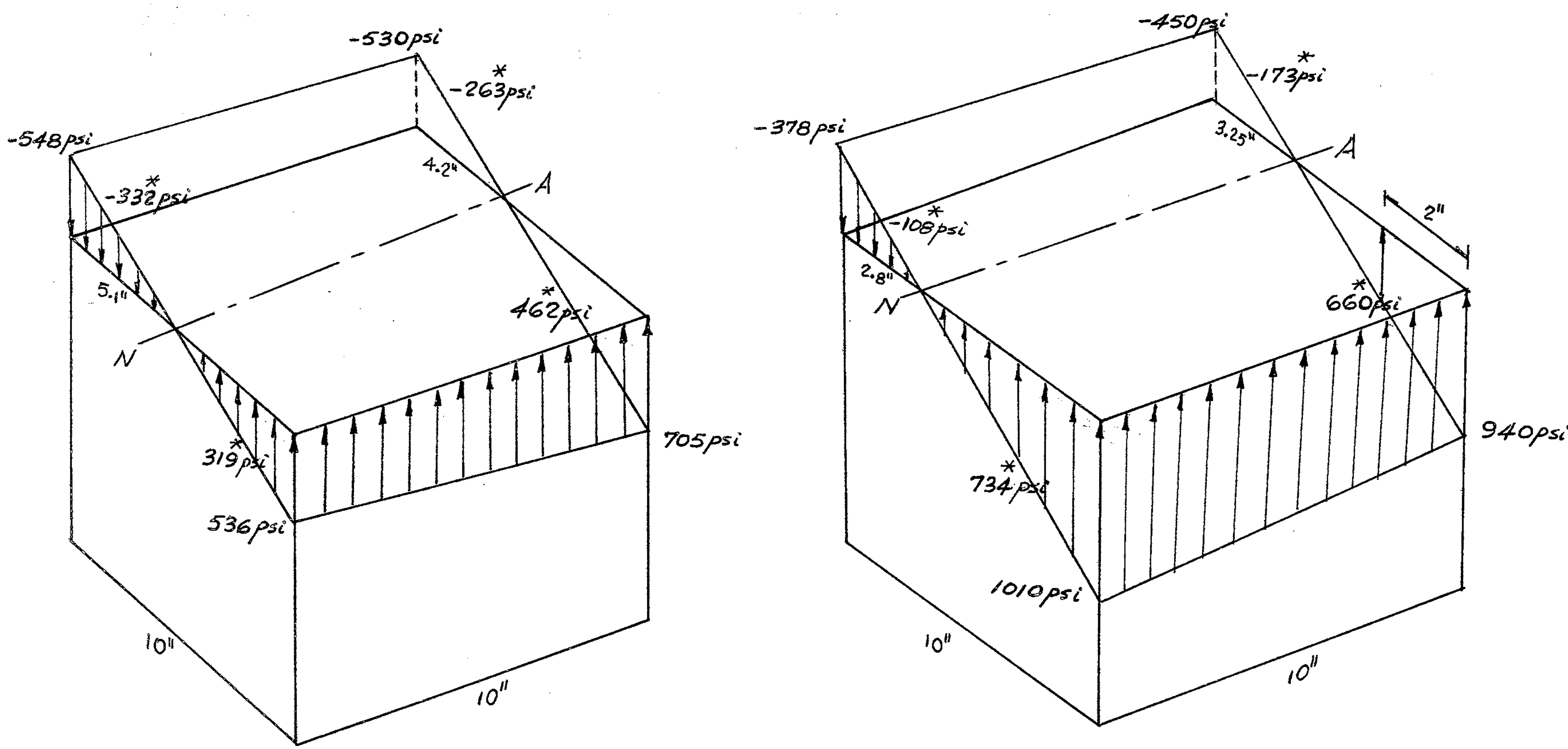
Possible explanations for these large strains are as follows: Firstly, the strain equipment was malfunctioning. This is not likely especially when the tests which were carried out at a later date proved very satisfactory.

Secondly, and more probably, the gauges themselves may have been poorly attached to the columns and the tie for that matter as well. It is important to mention that the gauges were glued in place during cold weather (40°F) which caused the resin to be almost unworkable. These conditions, also, did not encourage painstaking, thorough workmanship. Not only was the placing of the gauges done under these awkward conditions, but also, the leads were attached. Although these connections were later checked, some were probably poorly done.

Thirdly, differential settlement can induce large strains but not all the gauges would show an increase in strain as they did in this case. For this reason, this argument falls down.

The fourth thought is that the compensating gauge was not under the same temperature conditions as the active gauges. This is quite likely as the sun shone directly on certain active gauges and the compensating gauge was, for the most part, in the shade. A 20°F difference in temperature will cause a strain of 146 micro ins. per ins. which is quite an appreciable strain. This temperature difference could have occurred during the testing.

When subjected to the unsymmetrical or cantilever load of 20 psf, the results were more satisfactory. (See Table XII.) In this case the stresses at the gauge lines varied from 734 psi (tensile)



Bottom of Column NQ

Bottom of Column EB

* — Actual stresses in strain gauge planes

Figure 52. Stress distribution in columns due to an unsymmetrical uniform load of 20 psf.

to -332 psi (compressive). In all cases the compressive stresses were low compared with the theoretically expected values. The tensile stresses, however, were about double the uncracked section values.

Based on the stress distribution diagrams for the columns as in figure 52, the bending moments of the cracked section were calculated. Column EB had a BM of 3800 lb. ft. and NQ 3450 lb. ft. The theoretical value is 10,000 lb. ft.

(b) Tie

The stresses and strains in the tie were measured with the unloaded shell as the datum. This meant that the stresses were actual changes in stresses due to the superimposed load. After the 20 hour test was conducted on the symmetrically loaded shell, strain gauge 17 appears to have gone astray since from then on all the strains were recorded as compressive strains. For this reason the results for this gauge have been abandoned.

The actual values ranged from 2750 psi with 20 psf to 6330 psi due to 35 psf while the corresponding expected values were 5250 psi and 9200 psi. The actual was constantly between 50% to 70% of the theoretical. It is interesting to note that the stress increased almost directly proportional to the time the load was sustained. (See figure 53.)

For the unsymmetrical loading condition it was assumed that the strain was half of that for the corresponding symmetrically loaded case. Here the actual stress of 2040 psi agrees fairly well with the calculated stress of 2620 psi.

C. Failure Analysis

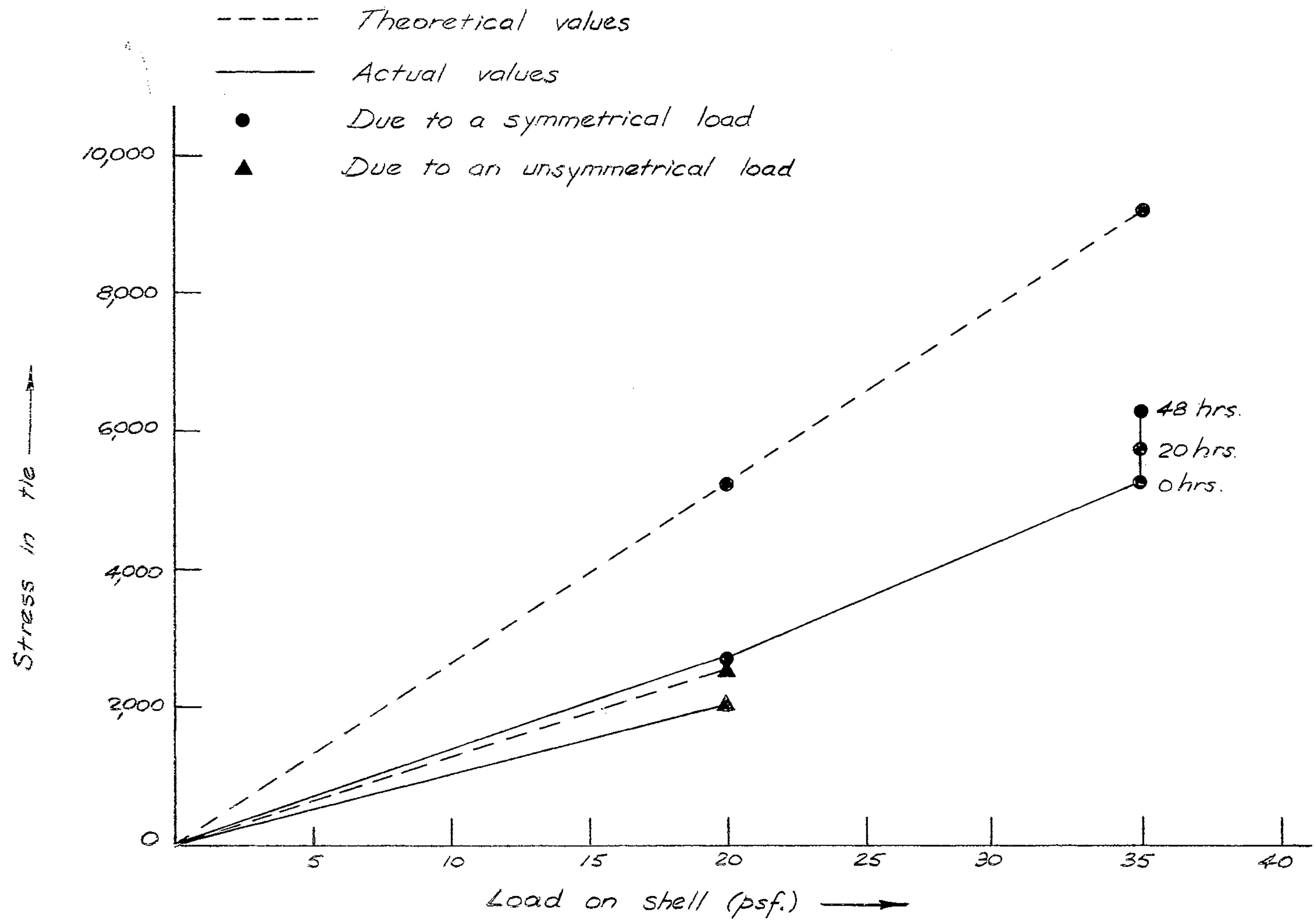


Figure 53. Stress in tie due to different loading conditions.

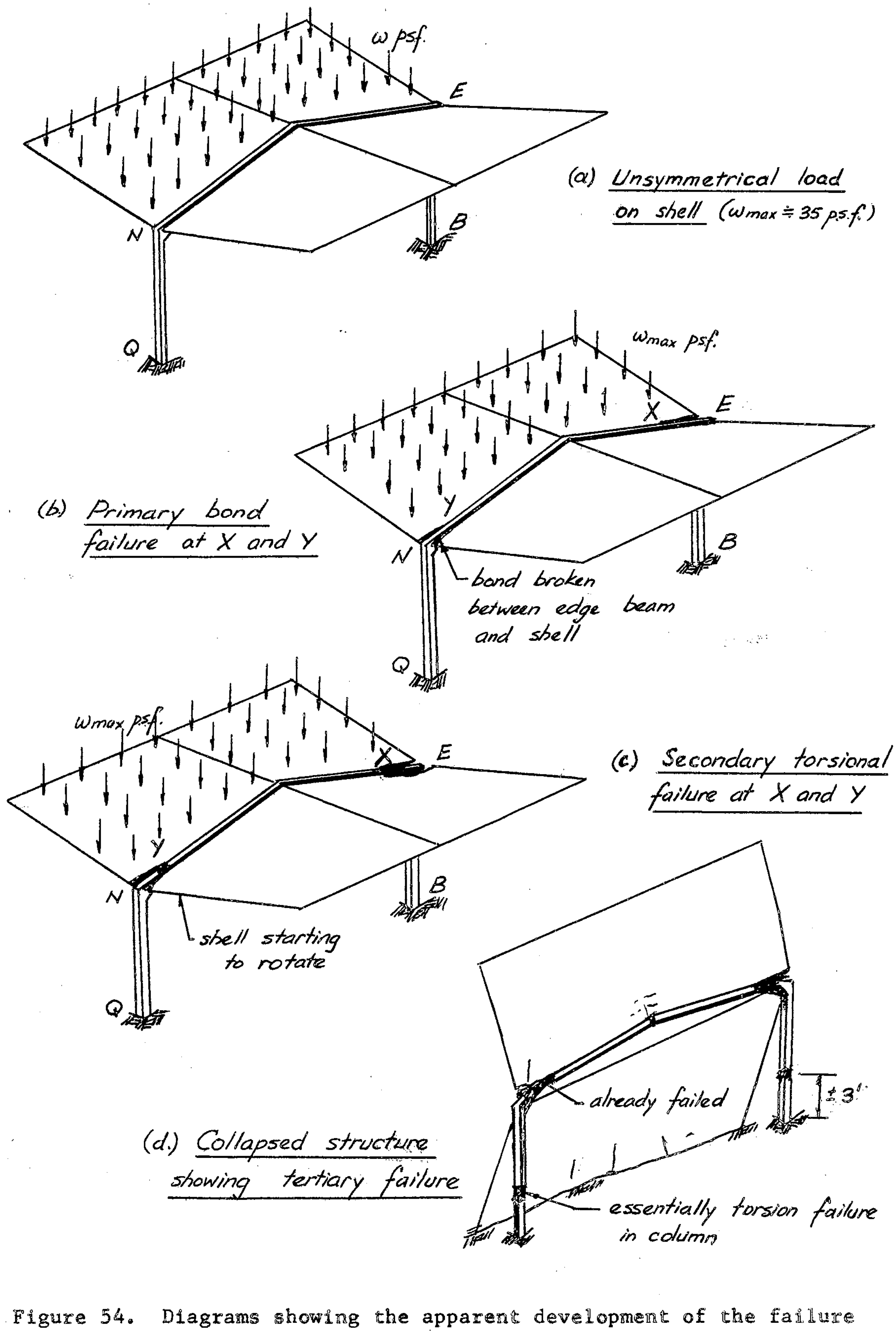


Figure 54. Diagrams showing the apparent development of the failure mechanism and final collapse.

In the discussion of the results the collapse of the shell is very important.

When subjected to an unsymmetrical load of 20 psf the shell deflected as expected. When this cantilever load was increased to 35 psf it was observed to deflect about 4 inches at the horizontal edges. Then a series of incidences took place as shown in the diagrams in figure 54 which ended up in the collapse of the structure.

The first indication was the failure of the base gauge, on column EB, to balance. Upon inspection a crack through the gauge was noticed. At this time another crack was observed in the edge beam at the Point X. (See figure 54.)

Then slowly and sluggishly the shell rotated about points X and Y where the edge beam had obviously failed in torsion. At the same time near the base of each column, diagonal tension cracks, due to torsion, superimposed upon the bending cracks and the columns failed in torsion. (See figures 54, 55, and 56.)

The collapsed structure can clearly be seen in figure 56.

In analyzing the collapse, the development of which apparently followed the subsequent stages, all within a short period of each other. (See figure 54.)

Stage 1. Primary failure was the bond failure of the shell to the edge beam along lines NY and EX.

Stage 2. Secondary failure was at points X and Y due to torsion. X and Y are the positions where the beam narrows to a 7 1/2 inches x 6 inches section.

Stage 3. Tertiary failure occurred at positions 2 to 3 feet above

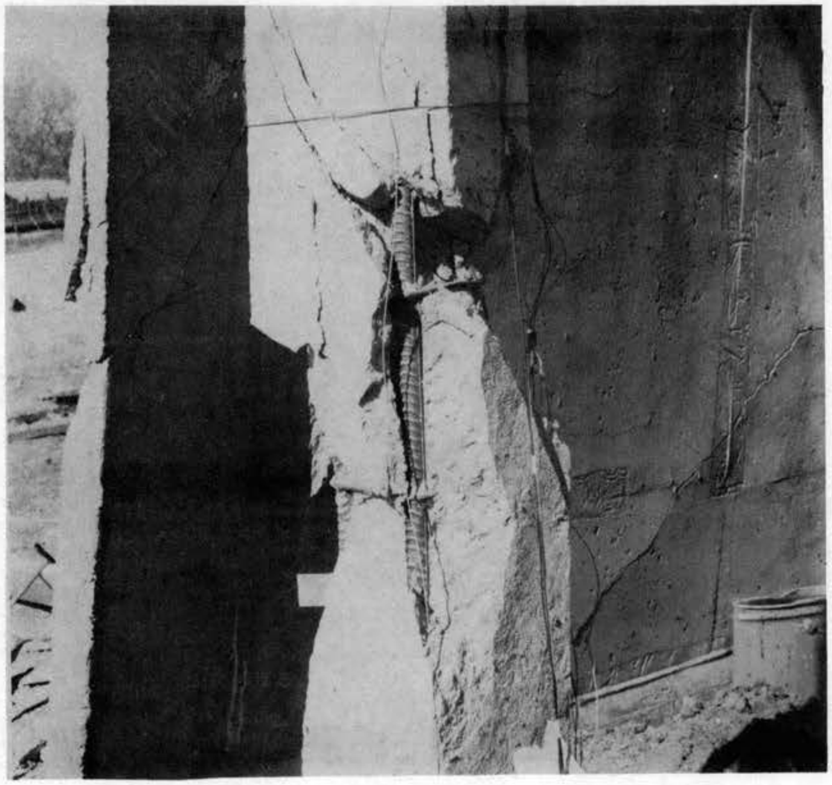


Figure 55. Failure of column NQ

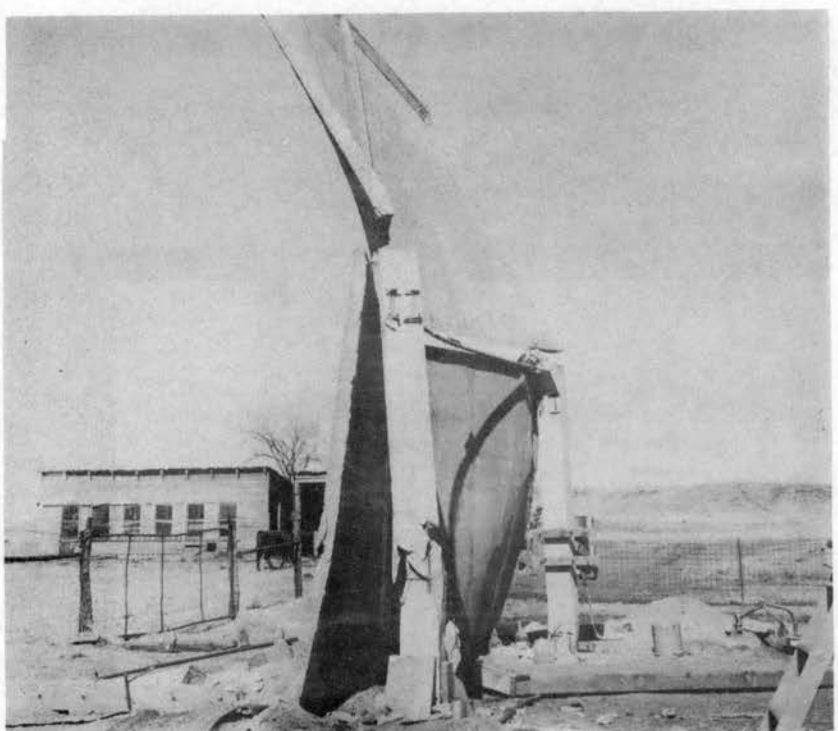


Figure 56. Collapsed h-p shell

the base of the columns. This was essentially a torsional failure of an already cracked section.

The crux of the matter seems to lie in the lack of bond reinforcement in the shell near the column. Had this been adequate the torsion would never have been induced, because the columns are only subjected to bending. The collapse would then not have taken place. The load of 35 psf was 18% in excess of the design load, but this should have easily been withstood when considering the safety factors involved.

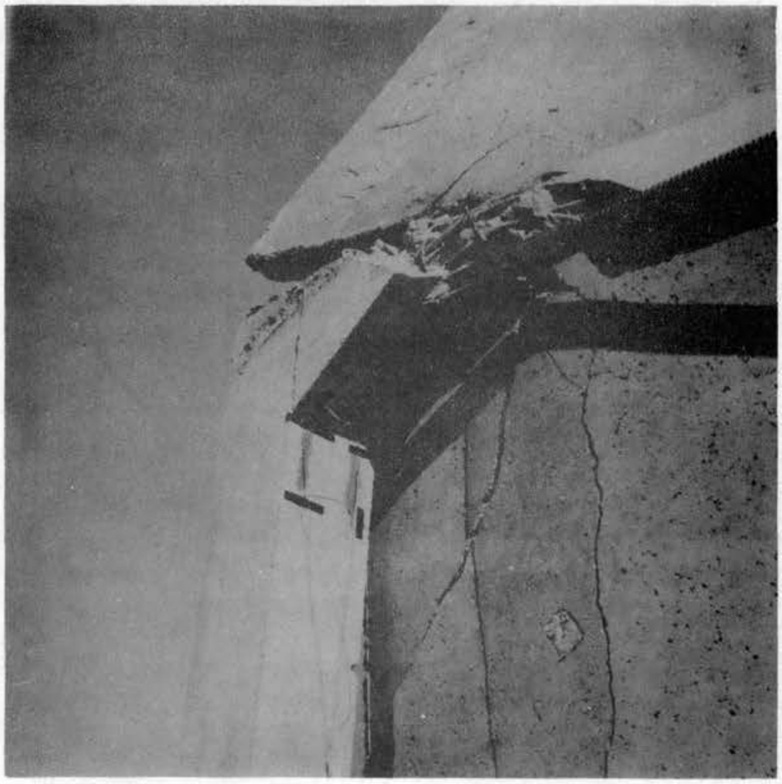


Figure 57. Torsional and bond failure near haunch

Figure 58. Failure of column EB

CHAPTER VIII

SUMMARY AND CONCLUSIONS

A study was conducted on a 20 feet by 20 feet hyperbolic paraboloid shell to evaluate the feasibility of a prefabricated column-edge beam system and a cast-in-place shell for rural structures.

The project was subdivided into the following phases:

A. Design

B. Fabrication of precast units and formwork

C. Erection of prefabricated components and placing of shell

D. Testing of structure

A. Design

A comprehensive design was made of:

(1) A cast-in-place shell

(2) Cast-in-place edge beams

(3) Precast columns and sloping central edge-beams

(4) Cast-in-place footings

(5) Formwork

B. Fabrication of Precast Units and Formwork

The column-edge beam bent was precast using lumber forms and double-headed nails. At about the same time each quadrant of the shell formwork was prefabricated on a jig made especially for the purpose.

C. Erection

First the precast bents were erected and the bases poured to form a type of rigid frame. The tie was welded in place and the corner supports positioned. Next, the prefabricated forms were lifted into position and a system of props was inserted. Finally, the shell concrete was placed and left to cure.

D. Testing of the Structure

Once cured, a load was placed on the structure and the deflections were measured at six points on the extremities of the shell and one point on a column, all relative to a reservoir attached to the other column.

The strains were recorded for sixteen concrete strain gauges, four at the bottom and four at the top of each column and also two gauges on the steel tie. The strains were measured directly by an electrical strain-gauge indicator.

The test load was first placed symmetrically over the entire shell. First 20 psf was placed and then 35 psf and readings were taken at 0, 20 and 48 hours with the sustained maximum load. The shell was then tested with an unsymmetrical load of 20 psf. Finally, when 35 psf was eccentrically loaded the structure collapsed.

RESULTS AND CONCLUSIONS

The following conclusions were gained from the study:

(1) The structure is feasible for use in rural buildings. Certain problems need still be eradicated but are comparatively small.

(2) The precast column-edge beam units behaved satisfactorily

both from the standpoint of construction and erection. These units were weak in torsion and loading and lifting must be such that excessive torsional stresses do not arise.

(3) The prefabricated forms turned out well. They were both functional to construct and use. However, a suitable supporting system for these forms need still be evolved.

(4) The footings were strong and adequately transmitted the loads to the ground. Here, again, a change could be made for the better. (See Chapter IX.)

(5) Cast-in-place shells present no difficulty once the supporting forms are erected. The shear connection between the shell and precast edge beams behaved well but was difficult to work with since the shear pipes were small.

(6) When subjected to a symmetrical load of 35 psf, the shell extremities (horizontal sides) displaced upwards a small amount. The maximum was 0.7 inches under a load of 35 psf.

(7) Under symmetrical loading conditions of 20 psf, the shell deflected 1.2 inches. The one edge moved down while the opposite moved upwards as anticipated.

(8) Due to symmetrical load the columns underwent differential settlement, the maximum being about 1/2 inch when withstanding 35 psf.

(9) The tie resisted 50% to 70% of the expected horizontal force at the top of the column.

(10) The bond of the shear bars between the shell and the precast bents is important to ensure monolithism.

CHAPTER IX

SUGGESTIONS FOR FURTHER STUDY

The suggestions put forward at this time are due to the culmination of ideas (in connection with) obstacles encountered while engaged in this study. Of course, many of the difficulties will never be waylayed, but with further study they should be somewhat alleviated.

This semi-precast hyperbolic paraboloid shell arrangement has great potentials. Initial teething troubles are natural, and once they are ironed out, this type of structure should prove practical and desirable.

A. Precast Units

The first suggestions for further study apply to the precast units. Having to perform satisfactorily various functions, these units require a lot of study.

The shear connection employing reinforcement bars protruding through pipes cast in the edge beam apparently proved satisfactory. The cantilever load, on the other hand, required more than just a shear connection to the precast unit. More pipes plus longer reinforcement to ensure bond might do the trick. A further suggestion is to increase the pipe size for ease of working.

A suggestion is to deepen the haunch and leave a fairly large hole in the bent such that a couple of bars 1/2 inch diameter or greater may be pushed through and cast into the shell when it is placed. An idea is pictured in figure 59. The main reinforcement in the column should be easily fitted without much harassment due to these holes. To adequately cater for this additional reinforcement the shell edge beams would be deepened at the column.

What about using a steel rigid frame with a tapered I-section? (See figure 61.) The shear bars are welded to the web and extend into the shell as shown in section A-A. For bending perpendicular to the plane of the columns, tapered steel plates are welded to the column web should this be in excess of the allowable.

One of the main advantages of this frame is that it is light, portable and easy to erect. It also has good potentials as far as supporting the formwork is concerned. (See section A-A, figure 61.) If the I-section is sufficiently deep the forms could be shimmed up to bear on the bottom flange. Should the section be too shallow a notch could be sawed in the form. To decenter the forms the shims are removed and the forms are lowered (this may take some horizontal movement first). A thin layer of concrete is placed over the steel beam section to prevent corrosion.

B. <u>Forms</u>

Because of its configuration the hyperbolic paraboloid is the ideal shape to resist uniformly distributed loads. To make sure of this property the formwork could be designed to take the shear stresses in the same manner the final shell does. This then should mean that the formwork would only need two supports!

The Dow Chemical product, "styrofoam", has been used quite extensively for formwork and could well be used in this shell. It

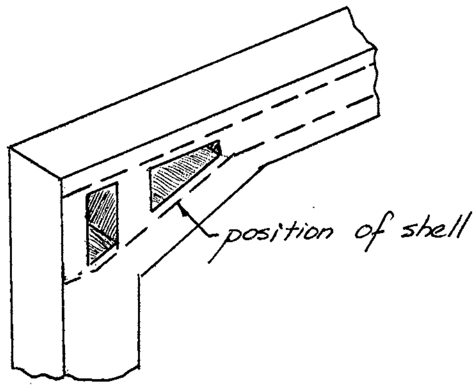


Figure 59. Holes in precast bent for bond to shell.

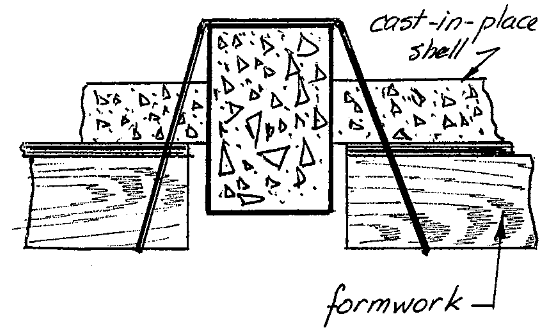
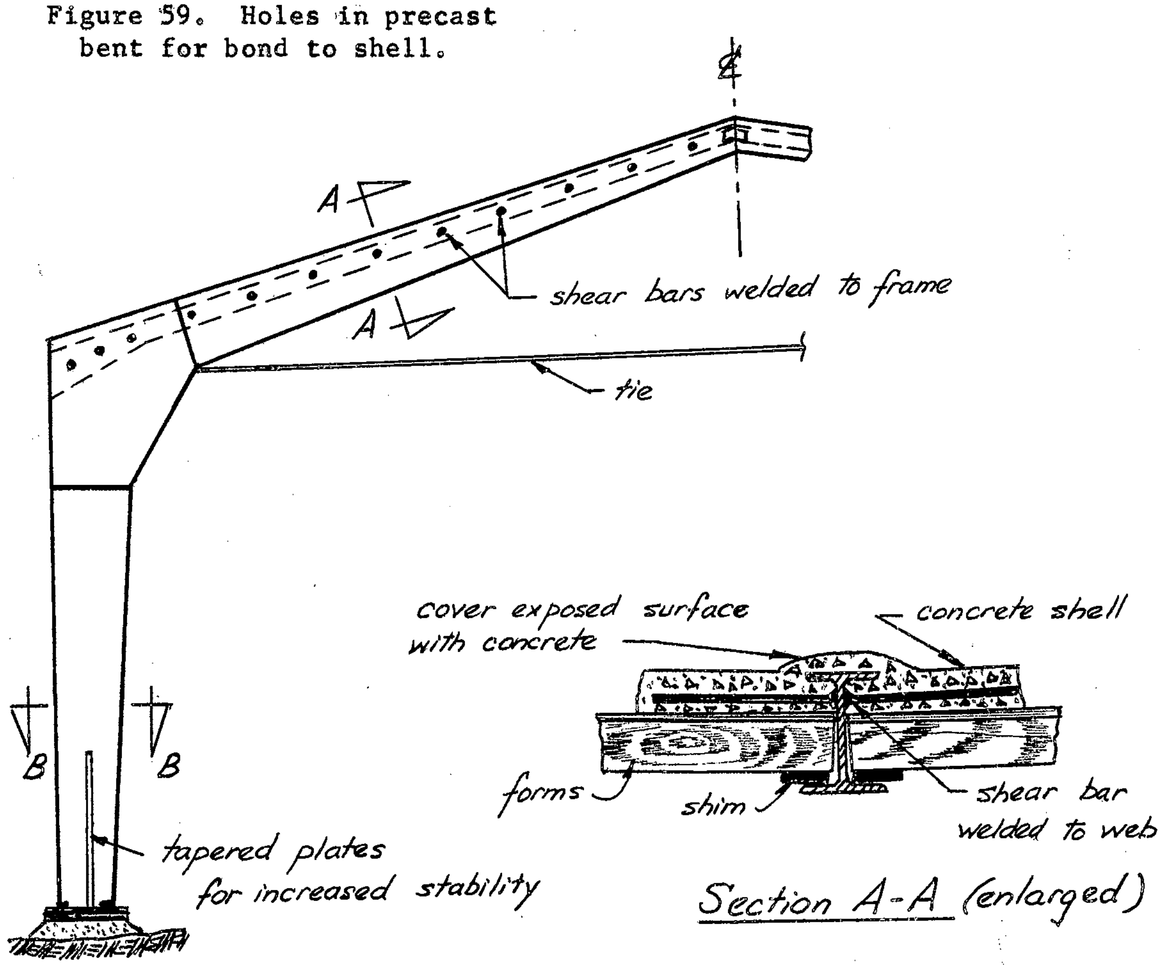


Figure 60. A type of hanger for forms.



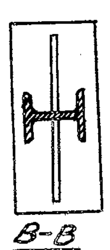


Figure 61. Steel rigid frame unit to be used in conjunction with a cast-in-place shell.

is usually bonded to the shell and left in place as an insulator on the inside of the final structure. This is suitable where needed but can afford additional expense since the forms of styrofoam cannot be reused.

An idea worth pursuing is that of using earth as the mould-former. A hole, the shape of the precast bent could be dug in the ground-smoothed and covered with a film of polyethylene--the reinforcement then is placed in position and the concrete poured. This was tried by the Author while constructing concrete benches--it proved highly satisfactory. This method is, of course, limited to precast units.

Figure 60 shows a type of hanging device to hold the prefabricated forms from the precast bents. After the shell has been poured these hangers should be cut close to the shell.

C. Base of Column

The final functioning of the fixed base or footing of the h-p shell constructed, seemed to be very satisfactory. However, to eliminate erection and positioning headaches a bolted base would have been much more practical. The bolted base consists essentially of a plate cast into the bottom of the column. Four bolts are cast into the footing itself. With the use of eight nuts the column can be "levelled" on the base in a similar manner to a transit. Grout is then placed under the plate as a bearing pad. This is commonly employed in construction nowadays.

D. Other

One other suggestion is that bar-chairs be used to support the shell reinforcement while placing the concrete. The wooden blocks used were horribly ineffective. It should be noted that the reason they were used was that the bar-chairs were not available when needed.

A vibrator, fixed to a screedboard, should be a great asset. The long cylindrical hose vibrator is not suitable under these circumstances. In the tested shell hand compaction and screeding proved satisfactory, but a "screedboard" vibrator would have helped a great deal.

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APPENDIX A

SUPPLEMENTARY TABLES AND DRAWINGS

TABLE A-I

28-DAY
CONCRETE STRENGTH

1. <u>Precast Units</u>

Sample No.	Ult. Stress
1	3630 psi
2	6000 psi
3	4860 psi
4	4560 psi
5	6600 psi
6	4200 psi

2. <u>Cast-in-place Shell</u>

Mix No.	Sample No.	Ult. Stress
I	1	5100
	2	2920*
	3	5650
II	1	6100
	2	6310
	3	poor specimen neglected

*poor seating caused local failure.

Value of the Modulus of Elasticity for Concrete

The A. S. C. E. - A. C. I. Joint Committee on Recommended Practice for Prestressed Concrete recommends:

$$E_c = 1,800,000 = 500\ f_c'$$

(a) For Precast Units

The average 38 day strength $f_c' = 4980$ psi

$$E_c = 1,800,000 + 500\ (4980)$$

$$\underline{E_c = 4,290,000 \text{ psi}}$$

(b) For Shell

$$f_c' = 5000 \text{ psi (approx.)}$$

$$E_c = 1,800,000 + 500\ (5000)$$

$$\underline{E_c = 4,300,000 \text{ psi}}$$

Value of the Modulus of Elasticity for the Steel Reinforcement

From a series of tensile tests run by Noyes (18) the stress-strain curve was plotted and the modulus of elasticity calculated. The results indicated $E_s = 30.48 \times 10^6$ psi. However, in this study E_s was taken to be 30×10^6 psi which is the usual specified value for mild steel.

TABLE A-II

1. SHELL REINFORCEMENT

Position	No.	Mk.	Dia. Inches	Length	Bending
central	2	a_s	5/8	18 ft. - 9 ins.	Straight
edge beam	2	b_s	5/8	12 ft. - 9 ins.	Straight
sloping edge beam	12	c_s	1/2	9 ft. - 4 ins.	Straight
horizontal edge beam	4	d_s	5/8	19 ft. - 9 ins.	Straight
shell @ 9 ins. crs.	16	e_s	1/4	11 ft. - 6 ins.	x o/a; 1½" 6"; Bar e_s: x = 10'-2"
	16	f_s	1/4	11 ft. - 8 ins.	Bar f_s: x = 10'-4"
	16	g_s	1/4	12 ft. - 0 ins.	Bar g_s: x = 10'-7"
shell @ 9 ins. crs.	24	j_s	1/4	10 ft. - 8 ins.	1½" 10'-0" o/a
	24	k_s	1/4	10 ft. - 6 ins.	1½" 9'-8" o/a
shell corners	12	q	1/4	2 ft. - 0 ins.	Straight
	20	r	1/4	3 ft. - 0 ins.	Straight
	2	s	1/4	6 ft. - 6 ins.	9" 2'-0"
	6	t	1/4	6 ft. - 0 ins.	Straight
shear connectors @ 6 ins. crs.	30	u	3/8	2 ft. - 6 ins.	Straight
	2	v	3/8	3 ft. - 0 ins.	Straight
shell corners	8	w	3/8	2 ft. - 6 ins.	1'-3" 1'-3"

A-II (CONTINUED)

2. PRECAST BENT REINFORCEMENT

2 No. Thus

Position	No.	Mk.	Dia. Inches	Length	Bending
	2	a	1	15 ft. - 9 ins.	13'-11" 107.3° 1'-10'
	2	b	1	16 ft. - 0 ins.	13'-11" 11"o/a 6"o/a
	2	c	7/8	2 ft. - 8 ins.	Straight
@ 9 ins. crs.	2	d	1/4	2 ft. - 3 ins.	spot weld 5 7/8 6 3/8" int. dims.
	2	e	5/8	12 ft. - 6 ins.	10'-2"o/a
	2	f	5/8	12 ft. - 0 ins.	9'-9"o/a 107.3°
@ 6 ins. crs.	16	g	1/4	1 ft. - 8 ins.	spot weld 5 1/4 3 3/8" int dims.
	1	h_1	3/8	3 ft. - 1 ins.	3" 2'-5" R 127.5° R bend to clear.
	1	h_2	3/8	3 ft. - 1 ins.	same as above with RR bent in opposite direction
	1	i	1/4	2 ft. - 4 ins.	6½" x
	1	j	1/4	2 ft. - 9 ins.	
@ 6 ins. crs.	1	k	1/4	3 ft. - 0 ins.	
	1	l	1/4	3 ft. - 3 ins.	

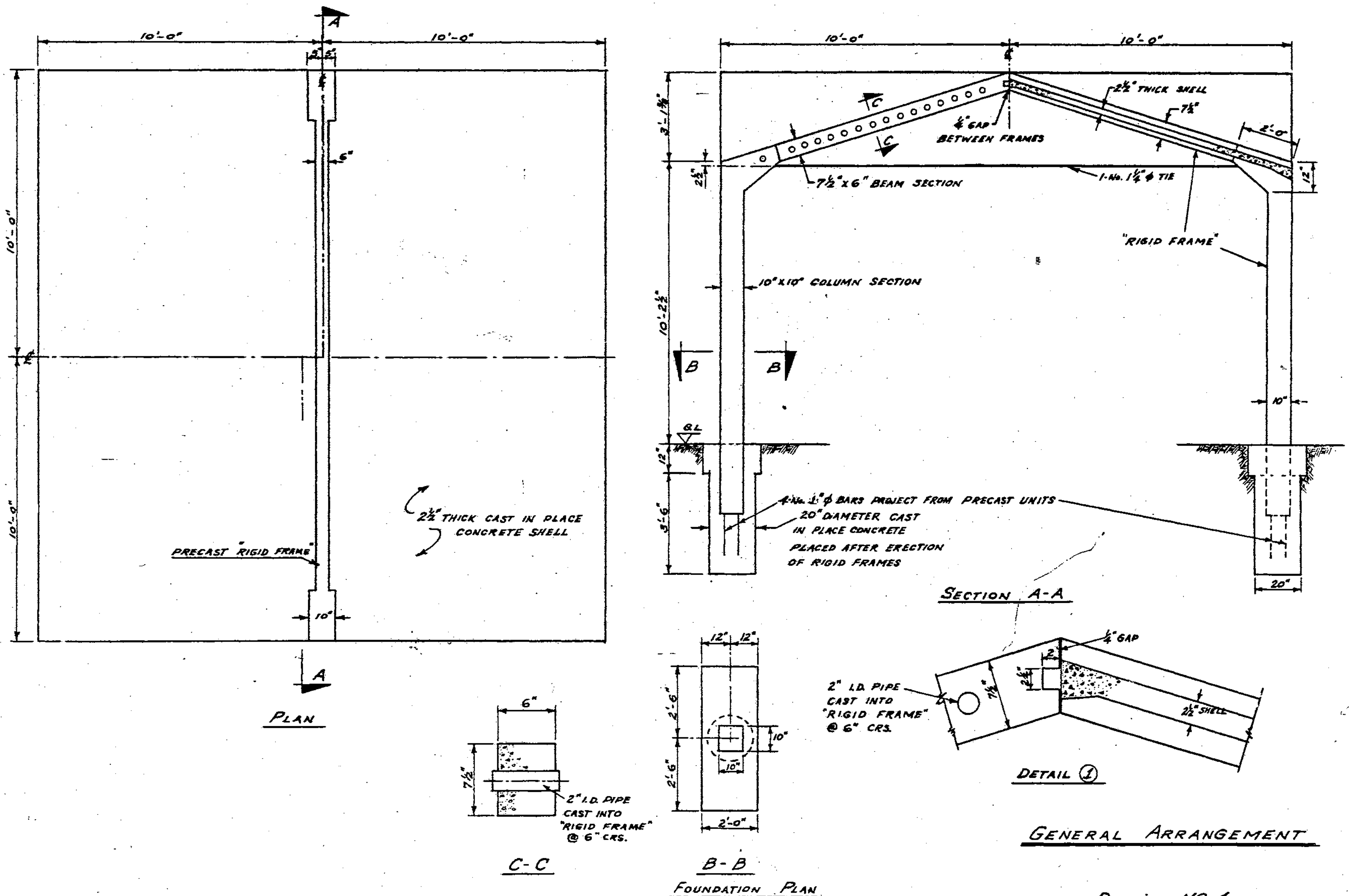
Bar	x
i	6¼"
j	8¾"
k	10½"
l	12"

A-II (CONTINUED)

3. FOOTING REINFORCEMENT

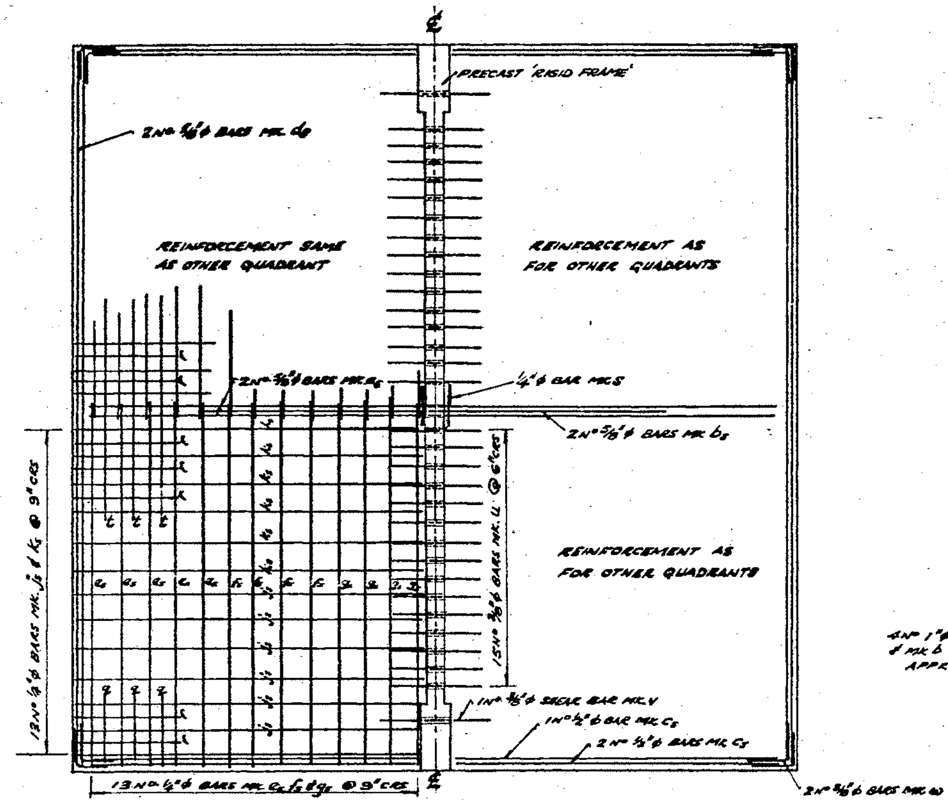
2 No. Thus

Position	No.	Mk.	Dia. Inches	Length	Bending
@ 9 ins. crs.	6	m	1/2	1 ft. - 6 ins.	Straight
@ 3 1/2 ins. crs.	4	n	1/2	4 ft. - 6 ins.	Straight
@ 3 1/2 ins. crs.	4	p	1/2	2 ft. - 6 ins.	1'-6"

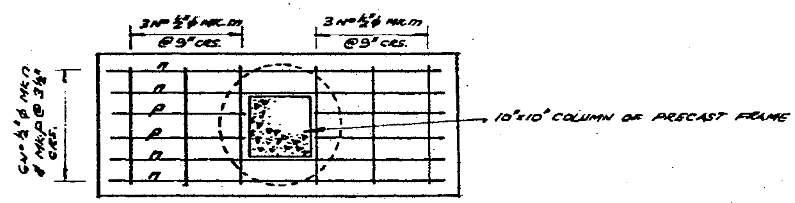


GENERAL ARRANGEMENT

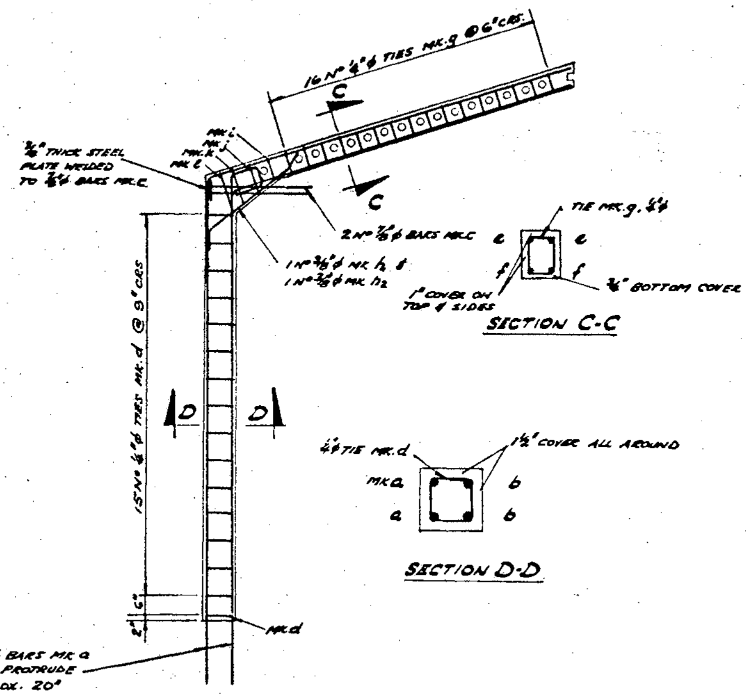
Drawing No. 1



SHELL REINFORCEMENT



CAST IN PLACE FOOTING - 2 No. THUS



PRECAST BENT FOR RIGID FRAME

2 No. THUS

REINFORCEMENT DETAILS

Drawing No. 2.

TABLE A-III

SUMMARY OF REINFORCEMENT

Position	Dia. Inches	Total Length	wt/ft plf	weight lbs.	
shell	5/8	142 ft. - 0 ins.	1.043	148.0	
	1/2	112 ft. - 0 ins.	0.668	74.8	
	3/8	101 ft. - 0 ins.	0.376	38.0	
	1/4	1204 ft. - 0 ins.	0.167	201.0	461.8 lbs.
2 precast bents	1	127 ft. - 0 ins.	2.670	340.0	
	7/8	10 ft. - 8 ins.	2.044	21.8	
	5/8	98 ft. - 0 ins.	1.043	102.5	
	3/8	12 ft. - 4 ins.	0.376	4.7	
	1/4	86 ft. - 0 ins.	0.167	14.3	483.3 lbs.
2 footings	1/2	74 ft. - 0 ins.	0.668	49.5	49.5 lbs.
1 tie	1 1/4	20 ft. - 0 ins.	4.303	95.0	95.0 lbs.
				TOTAL	1089.6 lbs.

Cost @ $0.097/lb = $106

TABLE A-IV

SUMMARY OF CONCRETE

Position	Volume	Volume Cubic Yards
Shell and cast-in-place edge beams	$4\left[\frac{2\frac{1}{2}}{12}\left(\frac{10+10\cdot 45}{2}\right)^2\right] + 100\left[\frac{1}{2}\frac{(9 \times 1\frac{1}{2})}{144}\right] =$ 88 cu. ft. + 4.8 = 92.8 cu. ft.	3.43 cubic yards
Precast Bents	$2\left[12.5\left(\frac{10 \times 10}{144}\right) + 10.5\left(\frac{7.5 \times 6}{144}\right)\right] =$ 24 cu. ft.	0.89 cubic yards
Footings	$2\left[\pi/4(20)^2\,\frac{(3.5)}{144} + (1 \times 2 \times 5)\right.$ $\left.- 2.5\left(\frac{10 \times 10}{144}\right)\right] =$ 31.8 cu. ft.	1.18 cubic yards
	Cost $15.50 per cubic yard Total	5.5 cubic yards
	Total Cost	$85.20

VITA

Martin R. Page

Candidate for the Degree of

Master of Science

Thesis: THE DESIGN, CONSTRUCTION AND TESTING OF A CONCRETE HYPERBOLIC PARABOLOID SHELL INCORPORATING A PRECAST COLUMN-EDGE BEAM SYSTEM

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Grahamstown, South Africa, April 11, 1940, the son of Rodney G. H. and Barbara I. Page.

Education: Graduated from Roosevelt High School, Johannesburg, South Africa, in 1957; attended the University of the Witwatersrand from January 1958 to November 1961 receiving a Bachelor of Science Degree in Civil Engineering at this time; studied at Oklahoma State University from August 1962 to January 1964 and completed the requirements for the Master of Science in August 1965.

Experience: Spent three consecutive two month holidays working in a drawing office, surveying and in a consulting engineering concern; worked as a Civil Engineer in a Concrete Contracting Firm in London, England, for eight months; was employed as a graduate research assistant for the Agricultural Engineering Department and Portland Cement Association, Oklahoma State University, for one year; instructed in general engineering at LeTourneau College for eighteen months.

Organizations: Member of the American Concrete Institute.