DIFFUSED RESISTOR FREQUENCY RESPONSE

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PREFACE

The conventional technique for making resistors incorporated in integrated networks is to employ diffused strips, since this can often be accomplished with no additional processing steps. The resistance of the diffused strip is determined by the effective resistivity of the diffused layer and is isolated from the remainder of the circuit by maintaining the junction under reverse bias conditions. However, the reverse biased junction introduces a parasitic capacitance which may cause problems in high-frequency circuits and must be considered in the circuit design. The purpose of this study is to obtain analytical expressions for the frequency response of resistors formed in this manner.

Deep appreciation and gratitude are extended to my major adviser, Dr. Harold T. Fristoe, for suggestions contributed to this work and many helpful discussions throughout the study. Also, I would like to express my appreciation to Texas Instruments, Inc., Dallas, Texas, for sponsoring the project, and especially to Mr. Walter T. Matzen of Texas Instruments for his many helpful suggestions. I am deeply indebted to Gary Goodman, whose efforts brought about the preparation of the computer program, and to Louis Thomason, for subsequent additions to it. I would like also to express my appreciation to Dr. William L. Hughes, Head, School of Electrical Engineering, Oklahoma State University.

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CHAPTER I

INTRODUCTION

Various articles concerning integrated electronics have recently been presented in the literature (1, 2, 3, 4). These references consider such factors as basic theory of operation, circuit and component considerations, limitations on minimal size, and recent advancements in the field. It is believed that these publications along with the references contained therein will suffice in giving an adequate background and associated problems in the field.

This study is concerned with the analytical investigation of diffused resistive elements used in integrated circuitry. Specifically, the frequency response of such elements will be determined. The results are then presented for a general diffused element in terms of the open-circuit impedance, the short-circuit admittance, and the hybrid parameters as functions of the relative frequency. Previous work in this field either has not considered the general diffused element, or if a general element was considered, the response was not given in terms of general circuit parameters.

These resistive elements are usually formed during the same diffusion processes that create the active elements of the circuit. Figure 1-1 shows a typical cross section of a diffused layer with ohmic contacts at two points.

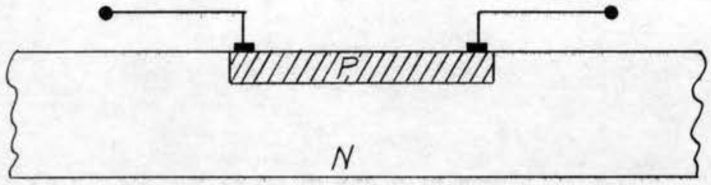

Figure 1-1. The Cross Section of a Diffused Layer Resistor.

The diffused p-type layer is isolated from the n-type substrate provided the substrate is floating relative to the diffused layer or is reverse biased. Under these conditions the diffused layer may be considered as a resistor. The value of resistance is then determined from the resistivity and geometry of the diffused layer. Also, this value of resistance will be affected by reverse leakage currents of the diode, capacitive shunting through the p-n junction depletion layer, and possible transistor action of this layer with other regions of the circuit. Since the latter effect is associated with component arrangement within a particular circuit, it will not be considered in this study.

The general approach to the problem consists of representing the diffused layer as a distributed line with the substrate acting as a common terminal. In Chapter II this analysis is considered, and the results are given in terms of the open-circuit impedance, the short-circuit admittance, and the hybrid parameters as functions of the resistance and capacitance per unit length.

Chapter III consists of a brief discussion of the two diffusion profiles to be considered in this report, the Gaussian and the complementary error function distributions. Several parameters that are needed in the capacitance calculations are then derived from this discussion.

The resistance and capacitance per unit length are determined in Chapter IV. These results are combined so that they may be readily introduced into the results of Chapter II.

In this study it is assumed that the diffusion profiles are dictated by the requirements on the active components. Thus, resistor design must be compatible with these predetermined profiles. Under these conditions it is noted in Chapter V that the only variables of the resistor response are the dimensions of the resistive element, the reverse bias of the diffused junction, and the frequency of operation. These variables are combined into a common variable and denoted as the relative frequency. The open-circuit impedance, the short-circuit admittance, and the hybrid parameters obtained in Chapter II are then presented graphically for several diffusions as functions of the relative frequency. Low and high frequency approximations to these results are also given.

Appendix D contains the computer program used for numerical evaluation of these parameters and their approximations.

CHAPTER II

FOUR-TERMINAL REPRESENTATION

As mentioned in Chapter I, the resistive element may be formed by the diffusion of an impurity layer into a background of opposite type conductivity, which for this study will be assumed homogeneous. The diffused layer is isolated from the substrate provided the junction between the substrate and diffused layer is reverse biased. The isolation is incomplete owing to leakage currents and the depletion layer capacitance of the reverse biased junction. For good quality P-N junctions in silicon, the former effect can be neglected. Although the resistivity of the substrate material may be greater than that of the diffused layer, it is noted that the cross section of the substrate is such that its effective resistance is also negligible.

Under these assumptions, an increment, Δz, of the resistive element may be approximated by a series resistance shunted by a capacitance. The differential equations describing the response of the network are derived by applying lumped concepts to the increment illustrated in Figure 2-1.

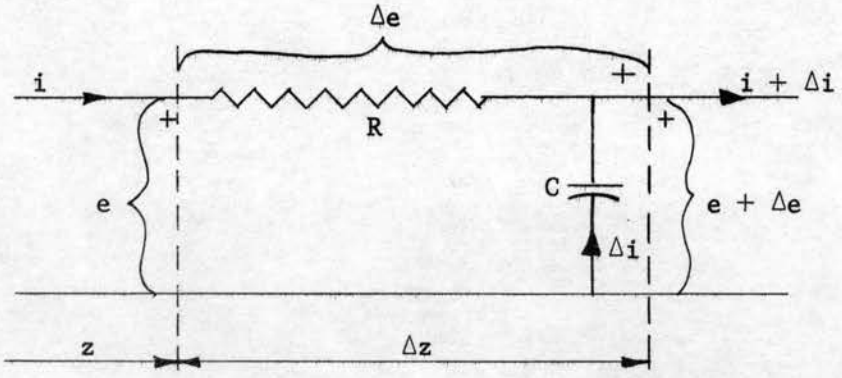

Figure 2-1. Circuit Relations for an Increment of a Diffused Resistor.

z = the distance coordinate measured from one end of the element.

Δz = an increment in the z coordinate.

e = the potential at point z.

Δe = the increment of potential experienced over the distance Δz, with reference polarity shown in Figure 2-1.

i = the current in the resistance at point z.

Δi = the increment of current flowing through the capacitance in the interval Δz.

For the response of the above network, Lepage and Seely (5) have obtained the following partial differential equations:

$$\begin{aligned} \frac{\partial^2 e(z,t)}{\partial z^2} &= RC\,\frac{\partial e(z,t)}{\partial t} \\ \frac{\partial^2 i(z,t)}{\partial z^2} &= RC\,\frac{\partial i(z,t)}{\partial t} \end{aligned} \tag{2-1}$$

where R and C represent the series resistance and shunt capacitance per unit length, respectively.

Assuming constant coefficients and sinusoidal time dependence of voltage and current, the solutions of Equations 2-1 for a resistive element of finite length L with the boundary conditions indicated by

Figure 2-2 are (5)

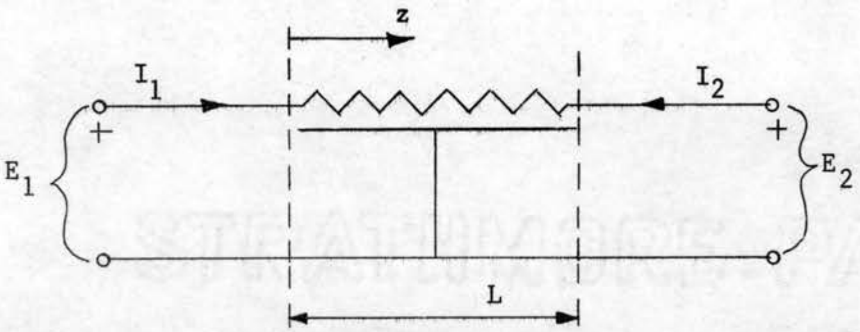

Figure 2-2. The Four-Terminal Representation of a Diffused Resistor.

$$\begin{aligned} E(x) &= E_2[\exp\gamma(L-x) + \exp-\gamma(L-x)]/2 - I_2 Z_o[\exp\gamma(L-x) - \exp-\gamma(L-x)]/2 \\ I(x) &= \frac{E_2}{Z_o}[\exp\gamma(L-x) - \exp-\gamma(L-x)]/2 - I_2[\exp\gamma(L-x) + \exp-\gamma(L-x)]/2, \end{aligned} \qquad (2\text{-}2)$$

where

$$\gamma = (j2\pi fRC)^{\frac{1}{2}} = (1 + j)(\pi fRC)^{\frac{1}{2}}$$
$$Z_o = (R/j2\pi fC)^{\frac{1}{2}} = (1 + j)^{-1}(R/\pi fC)^{\frac{1}{2}}.$$

When there is no interest in the potential and current at intermediate points along the element, it may be treated as a four-terminal network. Since $E(0) = E_1$ and $I(0) = I_1$, Equations 2-2 are in the proper form to yield the desired results. If x is set equal to zero in Equations 2-2 and writing the exponential terms as hyperbolic functions, the following equations are obtained for the four-terminal network illustrated in Figure 2-2.

$$\begin{aligned} E_1 &= (\cosh\gamma L)E_2 - Z_o(\sinh\gamma L)I_2 \\ I_1 &= \frac{1}{Z_o}(\sinh\gamma L)E_2 - (\cosh\gamma L)I_2 \end{aligned} \qquad (2\text{-}3)$$

These equations may be rewritten in matrix form as

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}, \qquad (2\text{-}4)$$

where

$$A = \cosh\gamma L$$

$$B = Z_o \sinh\gamma L$$

$$C = (1/Z_o)\sinh\gamma L$$

$$D = \cosh\gamma L.$$

There are many different sets of parameters that may be used to characterize the behavior of the network. More familiar, than those in Equations 2-4, are the open-circuit impedance, the short-circuit admittance, and the hybrid parameters. These are defined by Equations 2-5 through 2-7, respectively.

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (2\text{-}5)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \qquad (2\text{-}6)$$

$$\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix} \qquad (2\text{-}7)$$

In general, the equations of one set may be derived from those of any other set by appropriate linear transformations. Applying these transformations, the following equations result, which relate the open-circuit impedance, the short-circuit admittance, and the hybrid parameters to the hyperbolic functions which define A, B, C, and D (Appendix A).

$$z_{11} = z_{22} = 1/h_{22} = Z_o(\cosh\gamma L)/(\sinh\gamma L) \tag{2-8}$$

$$z_{12} = z_{21} = Z_o/(\sinh\gamma L) \tag{2-9}$$

$$y_{11} = y_{22} = 1/h_{11} = (\cosh\gamma L)/Z_o(\sinh\gamma L) \tag{2-10}$$

$$y_{12} = y_{21} = -1/Z_o(\sinh\gamma L) \tag{2-11}$$

$$h_{12} = -h_{21} = 1/(\cosh\gamma L) \tag{2-12}$$

As shown in Equations 2-2, γ and Z_o are complex quantities which may be written as

$$\begin{aligned} \gamma L &= (1 + j)\bar{A} \\ Z_o &= (1 + j)^{-1}(L/wt)(\bar{\rho}/\bar{A}), \end{aligned} \tag{2-13}$$

where $\bar{A} = (\pi fRCL^2)^{\frac{1}{2}}$.

Substitution of Equations 2-13 and application of the appropriate identities for hyperbolic functions with complex arguments allow Equations 2-8 through 2-12 to be written in the following form (Appendix A).

$$z_{11}/(L/wt) = z_{22}/(L/wt) = (wt/L)/h_{22} =$$

$$(\bar{\rho}/\bar{A})\left[\frac{1 + j\tanh\bar{A}\tan\bar{A}}{(\tanh\bar{A} - \tan\bar{A}) + j(\tanh\bar{A} + \tan\bar{A})}\right] \tag{2-14}$$

$$z_{12}/(L/wt) = z_{21}/(L/wt) =$$

$$(\bar{\rho}/\bar{A})\left[\frac{1}{(\sinh\bar{A}\cos\bar{A} - \cosh\bar{A}\sin\bar{A}) + j(\sinh\bar{A}\cos\bar{A} + \cosh\bar{A}\sin\bar{A})}\right] \tag{2-15}$$

$$y_{11}/(wt/L) = y_{22}/(wt/L) = (L/wt)/h_{11} =$$

$$(\bar{A}/\bar{\rho})\left[\frac{(1 - \tanh\bar{A}\tan\bar{A}) + j(1 + \tanh\bar{A}\tan\bar{A})}{\tanh\bar{A} + j\tan\bar{A}}\right] \tag{2-16}$$

$$y_{12}/(wt/L) = y_{21}/(wt/L) =$$

$$(\bar{A}/\bar{\rho})\left[\frac{-(1 + j)}{\sinh\bar{A}\cos\bar{A} + j\cosh\bar{A}\sin\bar{A}}\right] \tag{2-17}$$

$$h_{12} = -h_{21} =$$

$$\left[\frac{1}{\cosh \bar{A} \cos \bar{A} + j \sinh \bar{A} \sin \bar{A}}\right] \qquad (2\text{-}18)$$

The author has assumed in the solution of the partial differential equations that led to these results, that the resistance and capacitance per unit length are constant along the length of the resistor. However, these conditions do not exist in operative elements.

The decrease in bias due to the potential drop along the resistor will result in a reduction of the depletion region of the diffused layer. Since most of the conductivity in such diffused layers is contributed by the relatively highly doped region near the surface, the change in depletion region is assumed to have negligible effect on the value of resistance. The value of capacitance is inversely proportional to the one-third or one-half power of the bias voltage if the junction is operated in the region where the graded or step junction approximations are valid (6). This report will consider elements operated in the graded region and assume the change in bias is such that constant capacitance will be a reasonable assumption.

CHAPTER III

DIFFUSION PROFILE CONSIDERATIONS

Calculations of the argument $\bar{A}$ introduced in the preceeding chapter will depend upon the particular impurity profile used to obtain the resistive element. Two common distributions, the Gaussian and the complementary error function, are discussed briefly in this chapter. For a detailed discussion of the various diffusion profiles, one may refer to the article by Smits (7) or the text by Jost (8).

Figure 3-1 shows an assumed impurity distribution. The abscissia, x, is zero at the surface from which the diffusion takes place. N_s is the concentration of diffused donors (or acceptors) at the surface; N(x) is the net concentration at any depth x; N_b is the concentration of acceptors (or donors) originally in the material (background concentration).

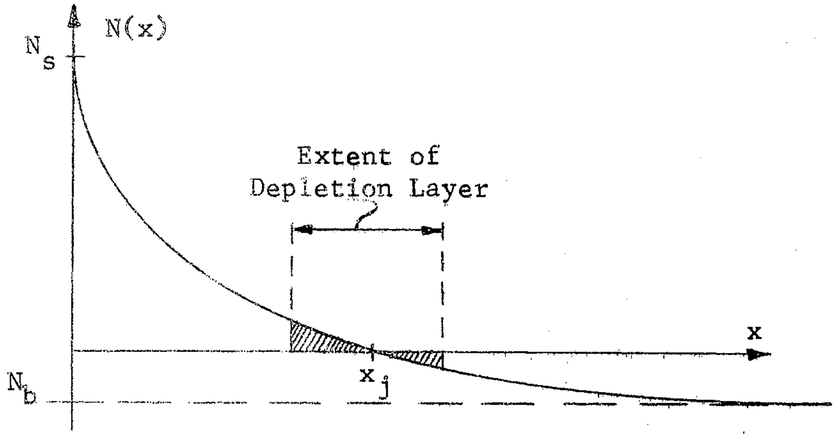

Figure 3-1. Net Impurity Distribution.

A junction is formed at a depth x_j where the impurity concentration of the diffusant equals the background concentration. The space charge region for some arbitrary applied voltage is indicated on this profile.

Mathematical forms for this distribution are obtained by solution of the diffusion equation with the appropriate boundary conditions.

Two boundary conditions are of special interest in the diffusions employed for device manufacture. The first of these occurs when a fixed amount of impurity is deposited on the surface at the beginning of the diffusion and held constant throughout the diffusion. Under these conditions, assuming the original semiconductor material contains a uniform impurity density of the opposite conductivity type, one obtains a Gaussian distribution

$$N(x) = N_s \exp-(\frac{x}{L_1})^2 - N_b, \tag{3-1}$$

where $L_1 = 2\sqrt{Dt}$; D is the diffusion constant, and t is the time of diffusion.

For this distribution, the junction depth is determined by the solution of

$$N(x_j) = N_s \exp-(x_j/L_1)^2 - N_b = 0,$$

which yields

$$x_j = L_1(\ln N_s/N_b)^{\frac{1}{2}}. \tag{3-2}$$

The slope of the impurity distribution is given by

$$\frac{dN(x)}{dx} = \frac{-2x}{L_1^2} N_s \exp-(x_j/L_1)^2.$$

The quantity a will be defined as the absolute value of the slope evaluated at the junction; thus

$$a = \left| \frac{dN(x_j)}{dx} \right| = 2x_jN_b/L_1^2, \tag{3-3}$$

or

$$a = \frac{2N_b}{L_1} (\ln N_s/N_b)^{\frac{1}{2}}. \tag{3-4}$$

The maintenance of a constant impurity concentration at the surface during the diffusion leads to the complementary error function solution of the diffusion equation, that is

$$N(x) = N_s \operatorname{erfc} (x/L_1) - N_b. \tag{3-5}$$

Several authors prefer to write Equation 3-5 in the form

$$N(x) = N_s \operatorname{erfc} (Mx/x_j) - N_b. \tag{3-6}$$

Evaluation of Equation 3-6 at the junction may be used to determine the value of M, that is

$$M = \operatorname{erfc}^{-1} (N_b/N_s), \tag{3-7}$$

where erfc^{-1} is used in the "arc function" sense. Comparison of Equations 3-5 and 3-6 yields an alternate definition for M

$$M = x_j/L_1. \tag{3-8}$$

The slope of the function is

$$\frac{dN(x)}{dx} = -(2MN_s/x_j\sqrt{\pi}) \exp-(Mx/x_j)^2.$$

Thus

$$a = [(2M/\sqrt{\pi})\ \exp-(M)^2]\ N_s/x_j \qquad (3\text{-}9)$$

Scarlett (9) has shown, by expanding the complementary error function in a Taylor's series about the normalized junction depth and approximating the n^{th} derivative of erfc(y) as $\mathrm{erfc}^{(n)}y \doteq -2y\ \mathrm{erfc}^{(n-1)}y$, that (Appendix B)

$$a \doteq 2x_jN_b/L_1^2. \qquad (3\text{-}10)$$

This approximation is identical to that obtained through exact analysis of the Gaussian distribution.

It should be noted that deviations often appear in the above relations expressing the net impurity concentration and junction depths. These deviations may include such factors as the dependence of D upon concentration, the time required to obtain the impurity at the surface at its final value, the fact that the surface is usually moving during the diffusion process, and the interaction between impurity atoms. Reynolds (10) has considered various diffusion errors affecting the calculated junction depths.

Also, the doping impurities may undergo a redistribution in the region near the surface during oxidation. Atalla and Tannenbaum (11) show the extent of the redistribution may range from a pile-up region to a depletion region near the semi-conductor-oxide interface.

CHAPTER IV

RESISTANCE AND CAPACITANCE CALCULATIONS

The resistance and capacitance per unit length for the diffused resistor will be determined in this section. These values will be determined for the Gaussian and complementary error function distributions. Using these results, the parameter $\bar{A}$ will be calculated as a function of the resistor dimensions, the bias potential, and the frequency of operation for any particular diffusion.

The series resistance is determined by its geometry and resistivity. Assuming the resistor is rectangular in shape and the length is an order of magnitude greater than the depth, Equation 4-1 is a very good approximation to the total resistance (12).

$$R_T = \bar{\rho}L/wt, \tag{4-1}$$

where $\bar{\rho}$ is the effective value of resistivity of the diffused layer; L is the distance between contacts; w is the resistor width; and t is the depth of the resistor equal to the junction depth, x_j. (Lateral spreading under the oxide mask represents a hazard to the rectangular assumption that should be noted but will be ignored in the present calculation.)

Thus, the series resistance per unit length is determined by

$$R = \bar{\rho}/wx_j. \tag{4-2}$$

Irvin (13) has calculated the average conductivity of diffused layers in silicon for Gaussian and complementary error function distributions. Tables I, II, III, and IV are taken from these calculations and show the effective resistivity of diffused layers for various values of surface and background concentrations at 300° K.

A graded junction is defined as one such that $N(x) = a(x-x_j)$ within the transition region, where a is a constant. As the voltage drop across the diffused junction is decreased the transition region becomes narrower, until N(x) may be approximated by a first-order Taylor expansion,

$$N(x) \doteq [dN(x_j)/dx](x-x_j). \tag{4-3}$$

Under such bias conditions it behaves like a graded junction.

Lawrence and Warner (14) have established the regions in which the linearly graded approximation, as opposed to the step approximation, is valid for wide ranges of voltage and diffusion profiles.

Shockley (6) has shown that the capacitance per unit area for the graded junction is given by

$$C_A = [qa\epsilon^2/12\ V_d]^{1/3}, \tag{4-4}$$

where q is the electronic charge; ϵ is the permittivity, and V_d is the potential drop across the junction.

The capacitance per unit length will be defined by

$$C = C_A\ (1 + \delta)w, \tag{4-5}$$

where w is the resistor width at the surface; δ is the shape factor which is introduced to include the effects of lateral spreading and

TABLE I

AVERAGE RESISTIVITY ($\bar{\rho}$) OF N-TYPE COMPLEMENTARY ERROR FUNCTION DIFFUSED LAYER IN SILICON (ohm-cm)

Surface Donor Concentration (cm^{-3})	Background Concentration of Acceptor Impurities (cm^{-3})						
	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
10^{21}	.0011	.0010	.00095	.00083	.00071	.00062	.00062
5×10^{20}	.0016	.0015	.0013	.0011	.0010	.00083	.00095
2×10^{20}	.0029	.0026	.0023	.0020	.0017	.0014	.0025
10^{20}	.0048	.0042	.0038	.0033	.0028	.0024	
5×10^{19}	.0083	.0077	.0067	.0055	.0045	.0045	
2×10^{19}	.017	.014	.012	.010	.010	.014	
10^{19}	.025	.022	.020	.016	.017		
5×10^{18}	.037	.032	.027	.024	.028		
2×10^{18}	.059	.050	.042	.038	.077		
10^{18}	.083	.067	.063	.059			
5×10^{17}	.12	.10	.083	.091			
2×10^{17}	.21	.17	.14	.25			
10^{17}	.33	.26	.23				
5×10^{16}	.56	.42	.42				
2×10^{16}	1.1	.83	1.3				
10^{16}	1.8	1.4					
5×10^{15}	3.2	2.9					
2×10^{15}	7.7	10.					
10^{15}	14.						

TABLE II

AVERAGE RESISTIVITY ($\bar{\rho}$) OF P-TYPE COMPLEMENTARY ERROR FUNCTION DIFFUSED LAYER IN SILICON (ohm-cm)

Surface Acceptor Concentration (cm^{-3})	Background Concentration of Donor Impurities (cm^{-3})						
	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
10^{21}	.00087	.00080	.00065	.00062	.00051	.00043	.00035
5 X 10^{20}	.0016	.0015	.0013	.0011	.00095	.00077	.00074
2 X 10^{20}	.0038	.0036	.0029	.0026	.0022	.0017	.0025
10^{20}	.0069	.0065	.0056	.0049	.0039	.0032	
5 X 10^{19}	.013	.012	.010	.0087	.0069	.0067	
2 X 10^{19}	.029	.026	.022	.019	.015	.024	
10^{19}	.050	.047	.037	.030	.028		
5 X 10^{18}	.083	.071	.062	.050	.056		
2 X 10^{18}	.145	.125	.10	.091	.17		
10^{18}	.22	.19	.16	.15			
5 X 10^{17}	.33	.29	.24	.27			
2 X 10^{17}	.59	.48	.40	.74			
10^{17}	.87	.71	.67				
5 X 10^{16}	1.4	1.1	1.2				
2 X 10^{16}	2.6	2.1	3.7				
10^{16}	4.8	3.8					
5 X 10^{15}	9.1	29.					
2 X 10^{15}	20.						
10^{15}	38.						

TABLE III

AVERAGE RESISTIVITY ($\bar{\rho}$) OF N-TYPE GAUSSIAN DIFFUSED LAYER IN SILICON (ohm-cm)

Surface Donor Concentration (cm^{-3})	Background Concentration of Acceptor Impurities (cm^{-3})						
	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
10^{21}	.00090	.00083	.00077	.00067	.00058	.00050	.00048
$5 X 10^{20}$	.0012	.0011	.0010	.00090	.00077	.00070	.00077
$2 X 10^{20}$	.0021	.0020	.0017	.0016	.0013	.0011	.0020
10^{20}	.0033	.0031	.0029	.0025	.0022	.0020	
$5 X 10^{19}$	.0059	.0050	.0048	.0040	.0037	.0036	
$2 X 10^{19}$	.012	.011	.010	.0091	.0077	.011	
10^{19}	.020	.017	.015	.013	.013		
$5 X 10^{18}$	.029	.025	.023	.020	.023		
$2 X 10^{18}$	.045	.038	.033	.030	.062		
10^{18}	.067	.055	.048	.048			
$5 X 10^{17}$	.091	.077	.067	.077			
$2 X 10^{17}$	.15	.12	.11	.20			
10^{17}	.25	.20	.17				
$5 X 10^{16}$	.40	.33	.33				
$2 X 10^{16}$	.83	.67	1.1				
10^{16}	1.3	1.2					
$5 X 10^{15}$	2.4	2.4					
$2 X 10^{15}$	5.0	7.7					
10^{15}	10.						

TABLE IV

AVERAGE RESISTIVITY ($\bar{\rho}$) OF P-TYPE GAUSSIAN DIFFUSED LAYER IN SILICON (ohm-cm)

Surface Acceptor Concentration (cm^{-3})	Background Concentration of Donor Impurities (cm^{-3})						
	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}	10^{20}
10^{21}	.00056	.00050	.00048	.00043	.00037	.00032	.00027
5×10^{20}	.0011	.00095	.00087	.00077	.00069	.00059	.00057
2×10^{20}	.0025	.0022	.0020	.0019	.0016	.0013	.0019
10^{20}	.0048	.0042	.0038	.0033	.0029	.0025	
5×10^{19}	.0091	.0077	.0071	.0065	.0056	.0051	
2×10^{19}	.020	.017	.016	.014	.012	.017	
10^{19}	.034	.030	.028	.024	.022		
5×10^{18}	.059	.050	.045	.039	.042		
2×10^{18}	.11	.091	.080	.096	.14		
10^{18}	.17	.14	.12	.11			
5×10^{17}	.24	.21	.19	.20			
2×10^{17}	.42	.37	.33	.59			
10^{17}	.67	.56	.53				
5×10^{16}	1.0	.88	.91				
2×10^{16}	1.9	1.6	2.8				
10^{16}	3.2	2.9					
5×10^{15}	6.3	5.9					
2×10^{15}	14.	20.					
10^{15}	29.						

deviation of the diffusion profile along the sides of the resistive element. For example, consider an element with no lateral spreading and identical junctions on all faces of the resistor. In this case $C = C_A(w + 2x_j)$, or $\delta = 2x_j/w$.

Substituting Equations 4-4 and 3-3 into 4-5 the capacitance for the resistive element formed by a Gaussian diffusion is obtained as

$$C = [q\epsilon^2 N_b x_j/(L_1^2\ 6\ V_d)]^{1/3}(1 + \delta)w. \qquad (4\text{-}6)$$

Substitution of Equations 4-4 and 3-9 into 4-5 yields a similar result for the capacitance for the complementary error function element,

$$C = [q\epsilon^2\ N_s M\ \exp(-M^2)/(\sqrt{\pi}\ x_j\ 6\ V_d)]^{1/3}(1 + \delta)w. \qquad (4\text{-}7)$$

As shown in Chapter II, the argument $\bar{A}$ is given by $\bar{A} = (\pi f L^2 RC)^{\frac{1}{2}}$. Introducing the appropriate terms for R and C, $\bar{A}$ may be written as

$$\bar{A}_G = [f(1 + \delta)L^2/(x_j{}^{4/3}V_d{}^{1/3})]^{\frac{1}{2}}[\bar{\rho}\pi[q\epsilon^2 N_b \ln(N_s/N_b)/6]^{1/3}]^{\frac{1}{2}} \qquad (4\text{-}8)$$

for the Gaussian diffused elements and

$$\bar{A}_C = [f(1 + \delta)L^2/x_j{}^{4/3}V_d{}^{1/3}]^{\frac{1}{2}}[\bar{\rho}\pi[q\epsilon^2 N_s M\exp(-M^2)/(6\ \sqrt{\pi}\ )]^{1/3}]^{\frac{1}{2}} \quad (4\text{-}9)$$

for complementary error function elements. (The diffusion length, L_1, has been eliminated in Equation 4-8 through the use of Equation 3-2.)

Calculations in this chapter have been based on the assumption that the resistive element is rectangular. However, odd-shaped geometries are necessarily encountered if the space available is utilized to its fullest extent. Research Triangle Institute (12) has considered several of the geometries encountered in the design of resistors. By the application of standard transformations employed in the conformal mapping of complex functions, they have calculated the effective resistance and current density for elements with right angle and rounded corners, circular strips, and elements with an abrupt change in width. These results may be used to modify the calculations of this chapter whenever these geometries are encountered.

It is also pointed out in the above mentioned report (12) that a resistor width of one mil is approaching the dimensional magnitude of the diffusion depths so that allowance for lateral spreading under the oxide mask becomes of geometrical importance. They suggest that the change in dimension can be calculated by assuming the lateral spreading to be equal to the junction depth of the diffused layer.

CHAPTER V

PRESENTATION OF RESULTS

Equations 2-14 through 2-18 will be presented graphically in this chapter as functions of the relative frequency for several resistive elements. Also, the low and high frequency approximations for these functions will be introduced.

It would not be practical to consider all of the diffusion profiles that might be of interest for the various applications. Thus, the only resistive elements considered numerically in this study are those formed by a Gaussian diffusion into a silicon slice of opposite conductivity type and uniform doping N_b. Also, it is assumed that the operating temperature of these elements is 300° K. A method of modification of these given results to include other profiles is then indicated.

Since the effective resistivity, the surface concentration and the background concentration are constant for a particular diffusion, Equation 4-8 may be written as

$$\bar{A}_G = F^{\frac{1}{2}} K_G^{\frac{1}{2}}, \tag{5-1}$$

where F will be denoted as the relative frequency, and K_G is a constant for a given Gaussian distribution. Analytically F and K_G are expressed by

$$F = f(1 + \delta)\, L^2/x_j^{4/3} V_d^{1/3} \tag{5-2}$$

$$K_G = \bar{\rho}\pi[q\epsilon^2 N_b \ln(N_s/N_b)/6]^{1/3}. \qquad (5\text{-}3)$$

Similarly, for the complementary error function distribution Equation 4-9 may be written as

$$\bar{A}_C = F^{\frac{1}{2}} K_C^{\frac{1}{2}}, \qquad (5\text{-}4)$$

where the relative frequency is the same as in Equation 5-2, and K_C is given by

$$K_C = \bar{\rho}\pi[q\epsilon^2 N_s M \exp(-m^2)/(6\sqrt{\pi})]^{1/3}. \qquad (5\text{-}5)$$

The following equalities are recalled from Equations 2-14 through 2-18, respectively.

$$z_{11}/(L/wt) = z_{22}/(L/wt) = (wt/L)/h_{22} \qquad (5\text{-}6)$$

$$z_{12}/(L/wt) = z_{21}/(L/wt) \qquad (5\text{-}7)$$

$$y_{11}/(wt/L) = y_{22}/(wt/L) = (L/wt)/h_{11} \qquad (5\text{-}8)$$

$$y_{12}/(wt/L) = y_{21}/(wt/L) \qquad (5\text{-}9)$$

$$h_{12} = -h_{21}. \qquad (5\text{-}10)$$

For compactness Z11, Z12, Y11, Y12, and H12 are defined as the logarithm of the magnitude of the parameters given in Equations 5-6 through 5-10, respectively. For example, Z11 is defined from Equation 5-6 as $Z11 = \log \left|z_{11}/(L/wt)\right| = \log \left|z_{22}/(L/wt)\right| = \log \left|(wt/L)/h_{22}\right|$.

PHZ11, PHZ12, PHY11, PHY12, and PHH12 are similarly defined as the respective phases. (If the above definitions are not obvious, see Equations C-1 through C-10 of Appendix C.)

It is noted in Appendix C that the following equations may be used as approximations to Equations 2-14 through 2-18 for $(FK)^{\frac{1}{2}} < 10^{-1}$.

Low Frequency Approximations:

$$Z11 = Z12 \doteq \log \bar{\rho} - \log 2 - \log (FK) \qquad (5\text{-}11)$$

$$Y11 = Y12 \doteq -\log \bar{\rho} \qquad (5\text{-}12)$$

$$H12 \doteq 0. \qquad (5\text{-}13)$$

For values of FK such that $\tanh (FK)^{\frac{1}{2}} \doteq 1$, or equivalently $\sinh (FK)^{\frac{1}{2}} \doteq \cosh (FK)^{\frac{1}{2}} \doteq \frac{1}{2} \exp (FK)^{\frac{1}{2}}$, the following approximations are given for Equations 2-14 through 2-18.

High Frequency Approximations:

$$Z11 \doteq \log \bar{\rho} - \tfrac{1}{2} \log 2 - \tfrac{1}{2} \log (FK) \qquad (5\text{-}14)$$

$$Z12 \doteq \log \bar{\rho} + \tfrac{1}{2} \log 2 - \tfrac{1}{2} \log (FK) - 0.43429\,(FK)^{\frac{1}{2}} \qquad (5\text{-}15)$$

$$Y11 \doteq -\log \bar{\rho} + \tfrac{1}{2} \log 2 + \tfrac{1}{2} \log (FK) \qquad (5\text{-}16)$$

$$Y12 \doteq -\log \bar{\rho} + \frac{3}{2} \log 2 + \tfrac{1}{2} \log (FK) - 0.43429\,(FK)^{\frac{1}{2}} \qquad (5\text{-}17)$$

$$H12 \doteq \log 2 - 0.43429\,(FK)^{\frac{1}{2}}. \qquad (5\text{-}18)$$

These approximations should be reasonably accurate for $(FK)^{\frac{1}{2}} > 10^1$.

Numerical evaluations of Equations 2-14 through 2-18 have been obtained by use of an IBM 1410 computer. The program used for the calculations is given in Appendix D. The value chosen for permitivity and the use of Tables I through IV for determining the effective resistivity require that all lengths be converted to centimeters.

Figures 5-1 through 5-3 present Equations 2-14 through 2-18 for a p-type surface concentration of 2×10^{18} atoms/cm^3 and a n-type background impurity concentration of 1×10^{17} atoms cm^3. The solid curves represent the values obtained from Equations 2-14 through 2-18. The low and high frequency approximations for this diffusion are expressed analytically by

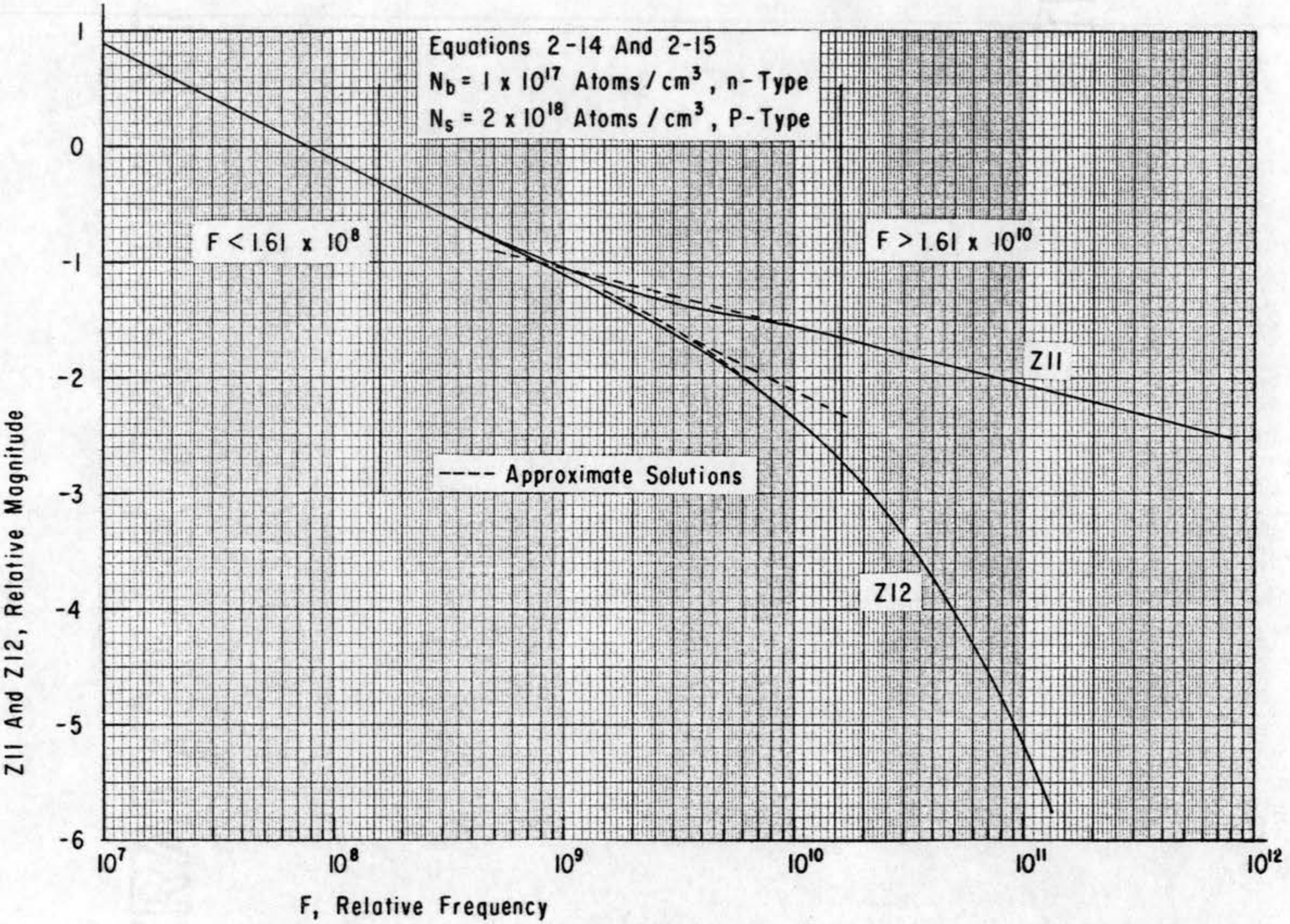

Figure 5-1. Open-Circuit Input and Transfer Impedance with Corresponding Approximations.

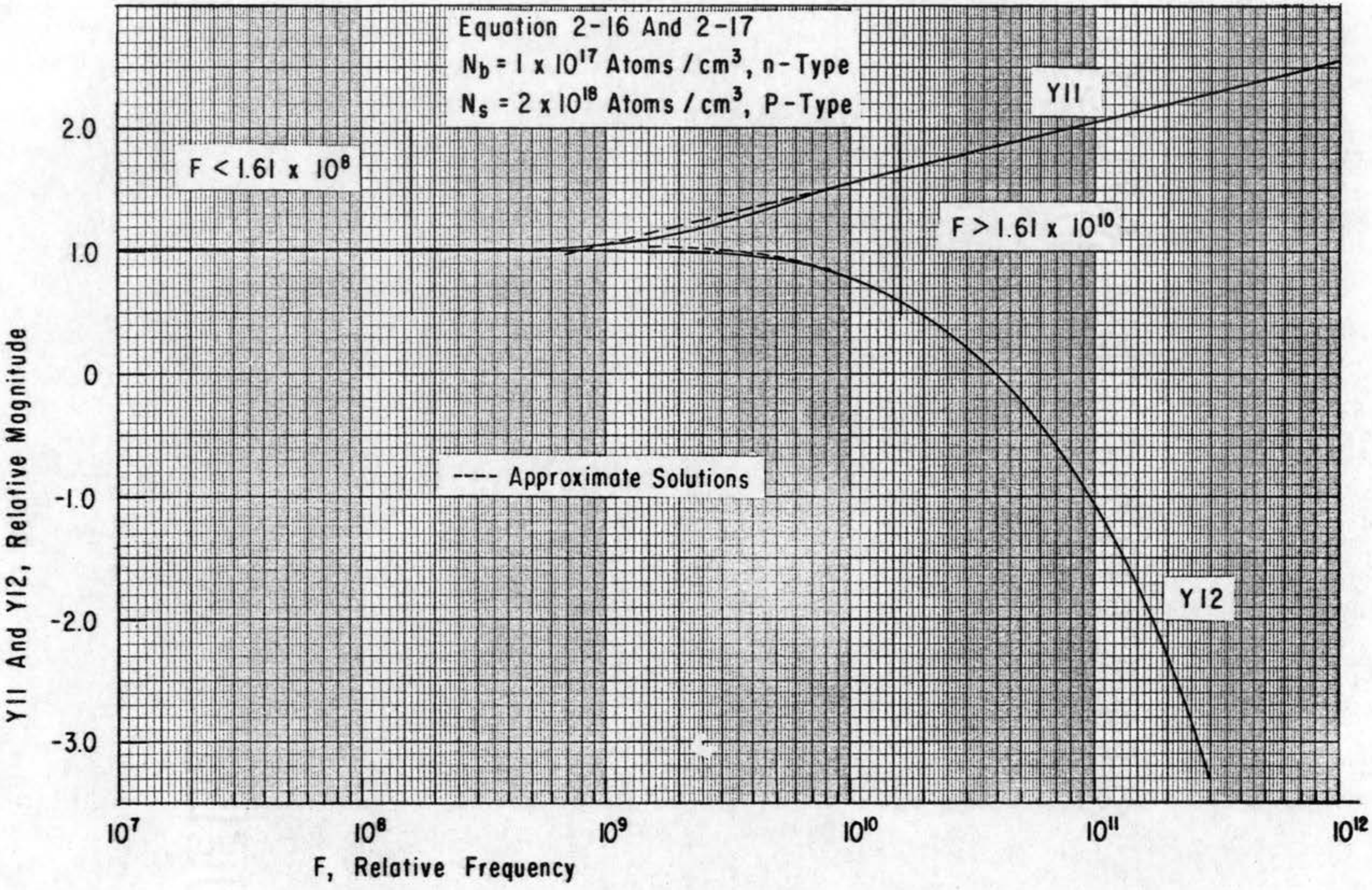

Figure 5-2. Short-Circuit Input and Transfer Admittance with Corresponding Approximations.

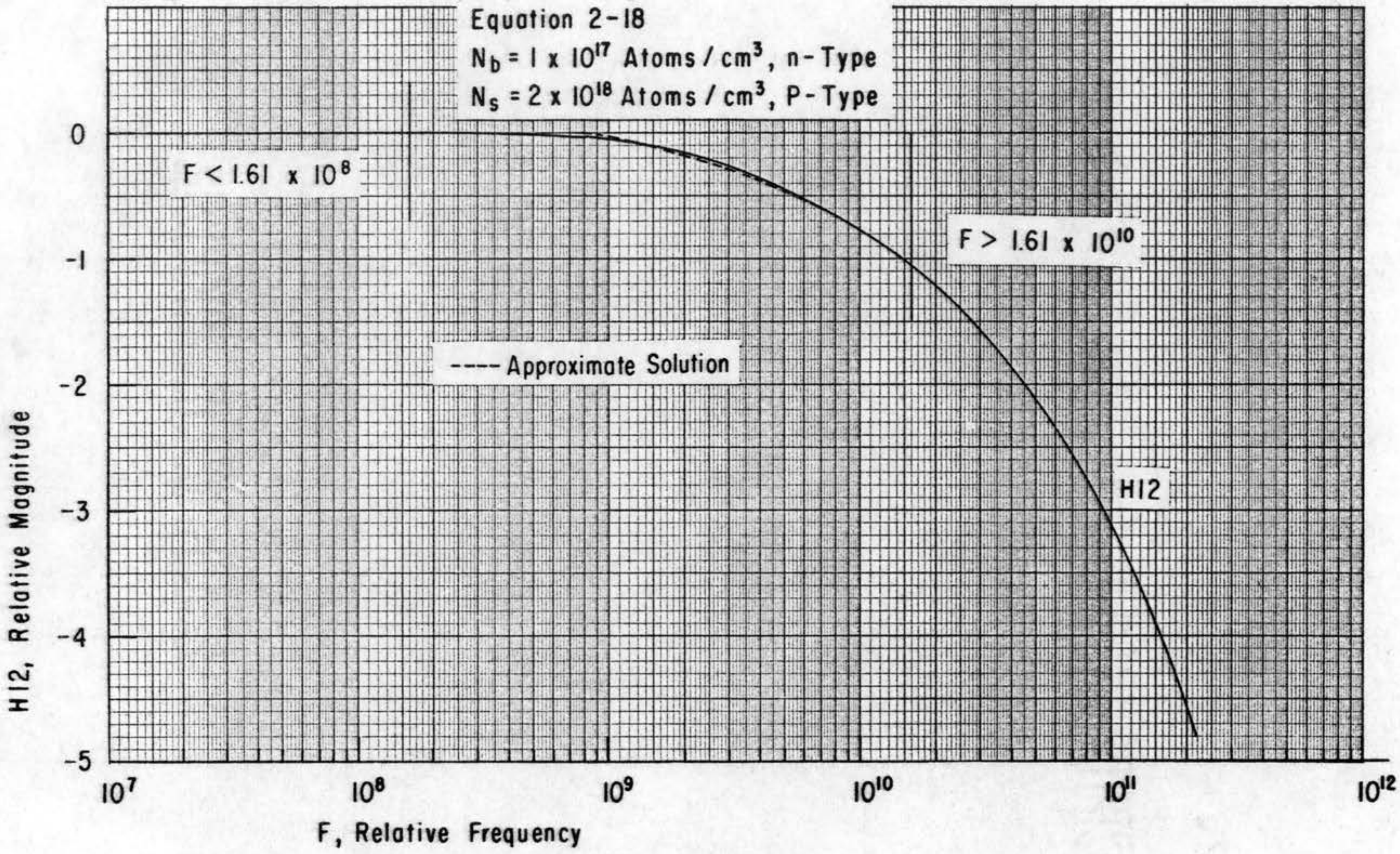

Figure 5-3. Open-Circuit Voltage Gain and Short-Circuit Current Gain with Corresponding Approximation.

$$Z11 = Z12 = 7.890 - \log F$$

$$Y11 = Y12 = 1.017$$

$$H12 = 0.0,$$

and

$$Z11 = 3.436 - \tfrac{1}{2} \log F$$

$$Z12 = 3.737 - 1.079 \times 10^{-5} (F)^{\frac{1}{2}} - \tfrac{1}{2} \log F$$

$$Y11 = -3.436 + \tfrac{1}{2} \log F$$

$$Y12 = -3.135 - 1.079 \times 10^{-5} (F)^{\frac{1}{2}} + \tfrac{1}{2} \log F$$

$$H12 = 0.3010 - 1.079 \times 10^{-5} (F)^{\frac{1}{2}},$$

respectively. These equations are shown by the dashed curves.

It was originally assumed that the low and high frequency approximations would be valid for $F < 10^{-2}/K$ and $F > 10^{2}/K$, respectively. For the case under consideration this corresponds to $F < 1.61 \times 10^{7}$ and $F > 1.61 \times 10^{11}$. However, it may be seen from the graphs that the range of each of the approximations may be extended another decade with no appreciable effect on the accuracy, that is $F < 1.61 \times 10^{8}$ and $F > 1.61 \times 10^{10}$.

The intersection of the low and high frequency approximations is in the range where the accuracy of either approximation is the poorest. Increasingly larger errors occur if either approximation is used for frequencies beyond its point of intersection with the other. If Z12, Y12, and H12 are determined by the appropriate low and high frequency approximate equations on either side of the intersections, the maximum difference between these approximate values and those calculated directly from Equations 2-14 through 2-18 will be less than 0.04. This difference corresponds to a maximum error of 10 per cent. The similar representations for Z11 and Y11 yield slightly larger deviations from the actual values, approximately 0.06 or 15 per cent. However, the oscillation of the actual values below the high frequency approximations in either case appears rather suspicious.

The magnitude and phase of Equations 2-14 through 2-18 are given in Figure 5-4 through 5-13. These graphs are presented for p-type Gaussian diffused layers into a uniform n-type background concentration of 10^{16} atoms/cm^3. Each graph contains a family of four curves corresponding to the four surface concentrations 2×10^{16}, 5×10^{16}, 2×10^{17}, and 2×10^{18} atoms/cm^3.

In the following discussion it is assumed that the approximations given by Equations 5-11 through 5-18 are sufficiently accurate to describe the response for a particular application. Then any given curve representing a parameter may be modified to represent the same parameter under different diffusion conditions. Consider two diffusions b and a; for the low frequency approximations the parameter difference if given by

$$Z11_b - Z11_a = Z12_b - Z12_a = \log \frac{\bar{\rho}_b}{\bar{\rho}_a} - \log \frac{K_b}{K_a} \qquad (5\text{-}19)$$

$$Y11_b - Y11_a = Y12_b - Y12_a = -\log \frac{\bar{\rho}_b}{\bar{\rho}_a} \qquad (5\text{-}20)$$

$$H12_b - H12_a = 0.0. \qquad (5\text{-}21)$$

For the high frequency approximations the parameter difference is given by

$$Z11_b - Z11_a = \log \frac{\bar{\rho}_b}{\bar{\rho}_a} - \tfrac{1}{2} \log \frac{K_b}{K_a} \qquad (5\text{-}22)$$

$$Z12_b - Z12_a = \log \frac{\bar{\rho}_b}{\bar{\rho}_a} - \tfrac{1}{2} \log \frac{K_b}{K_a} - 0.43429F(K_b^{\frac{1}{2}} - K_a^{\frac{1}{2}}) \qquad (5\text{-}23)$$

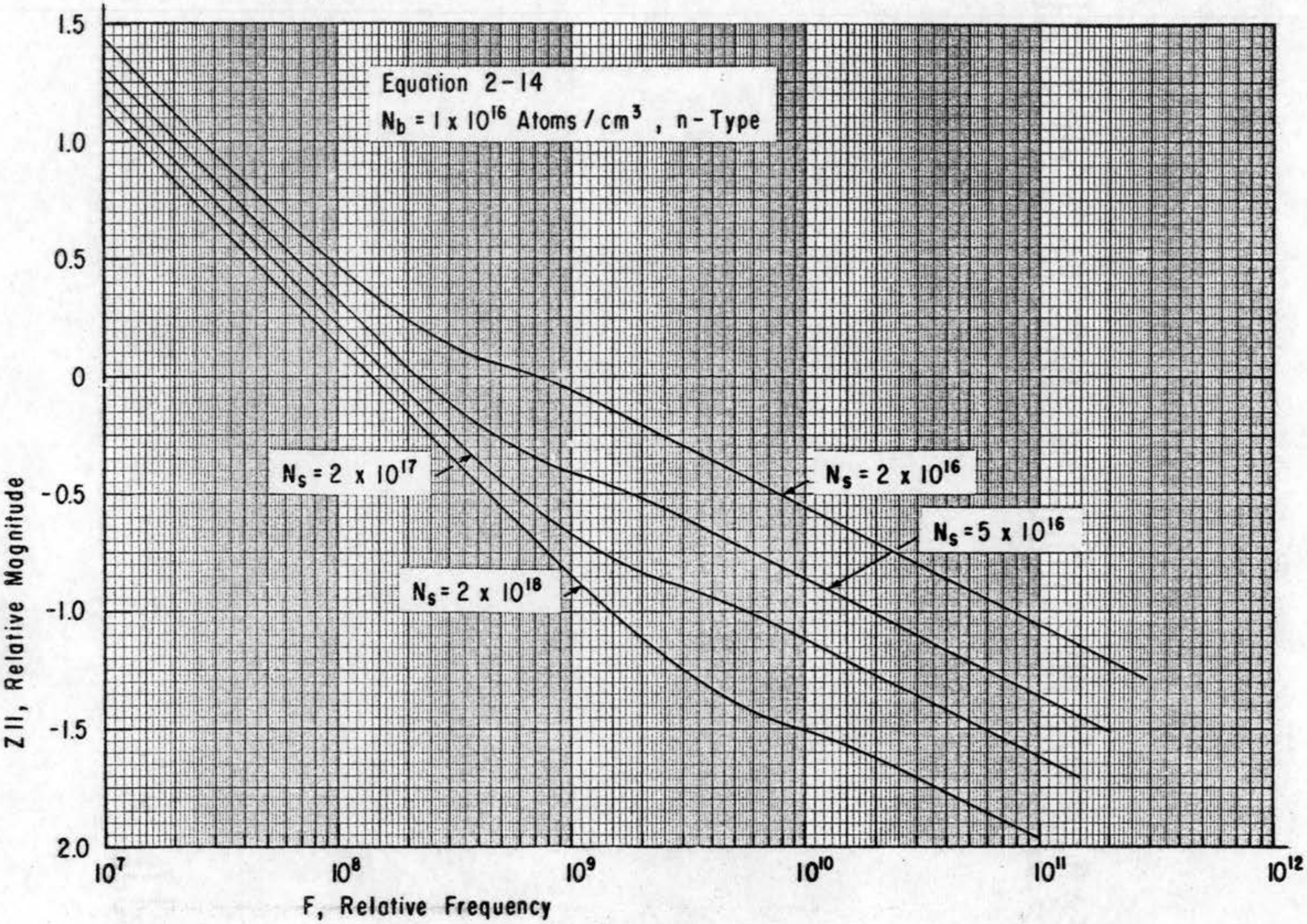

Figure 5-4. Open-Circuit Input Impedance (Magnitude).

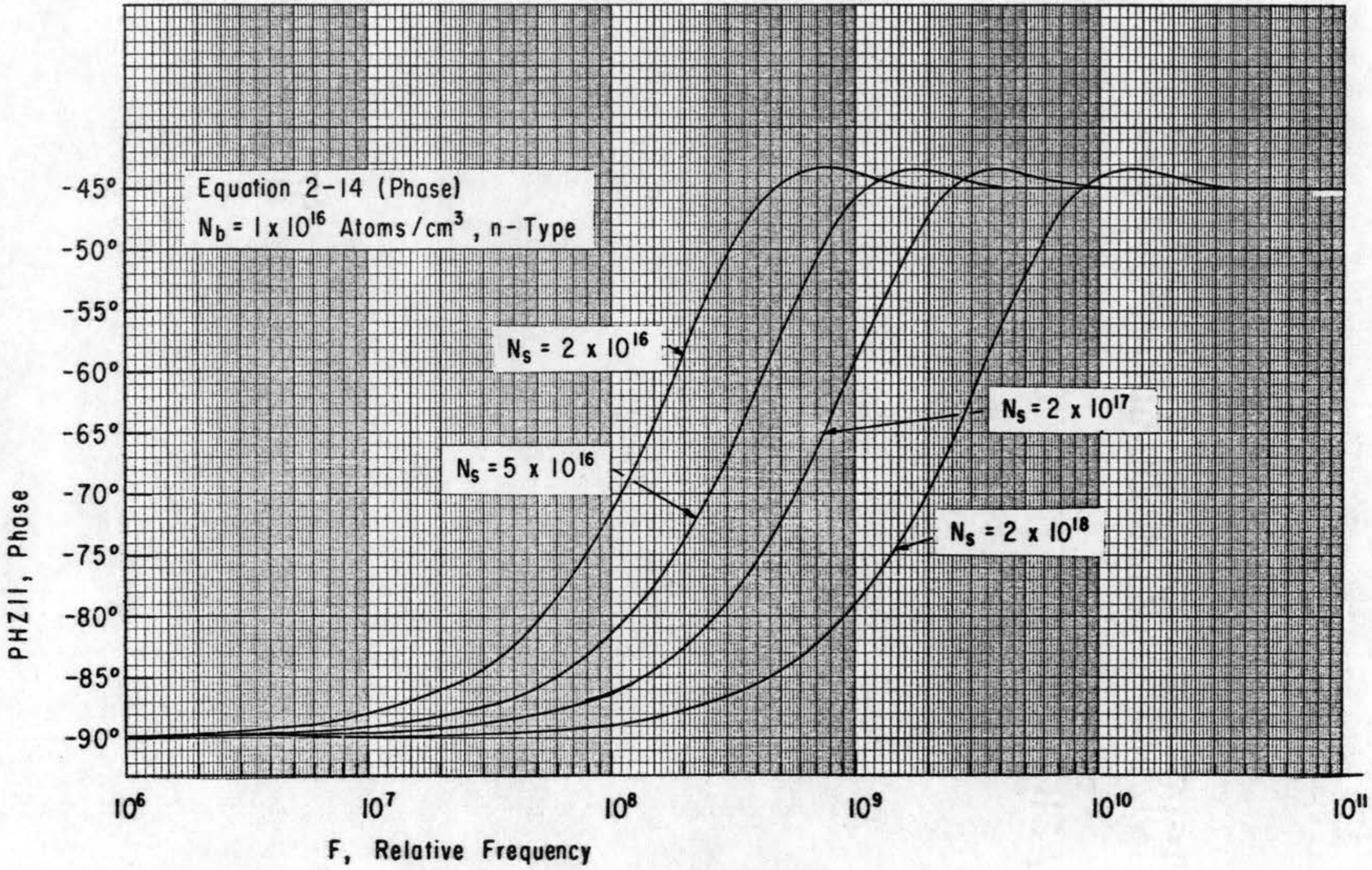

Figure 5-5. Open-Circuit Input Impedance (Phase).

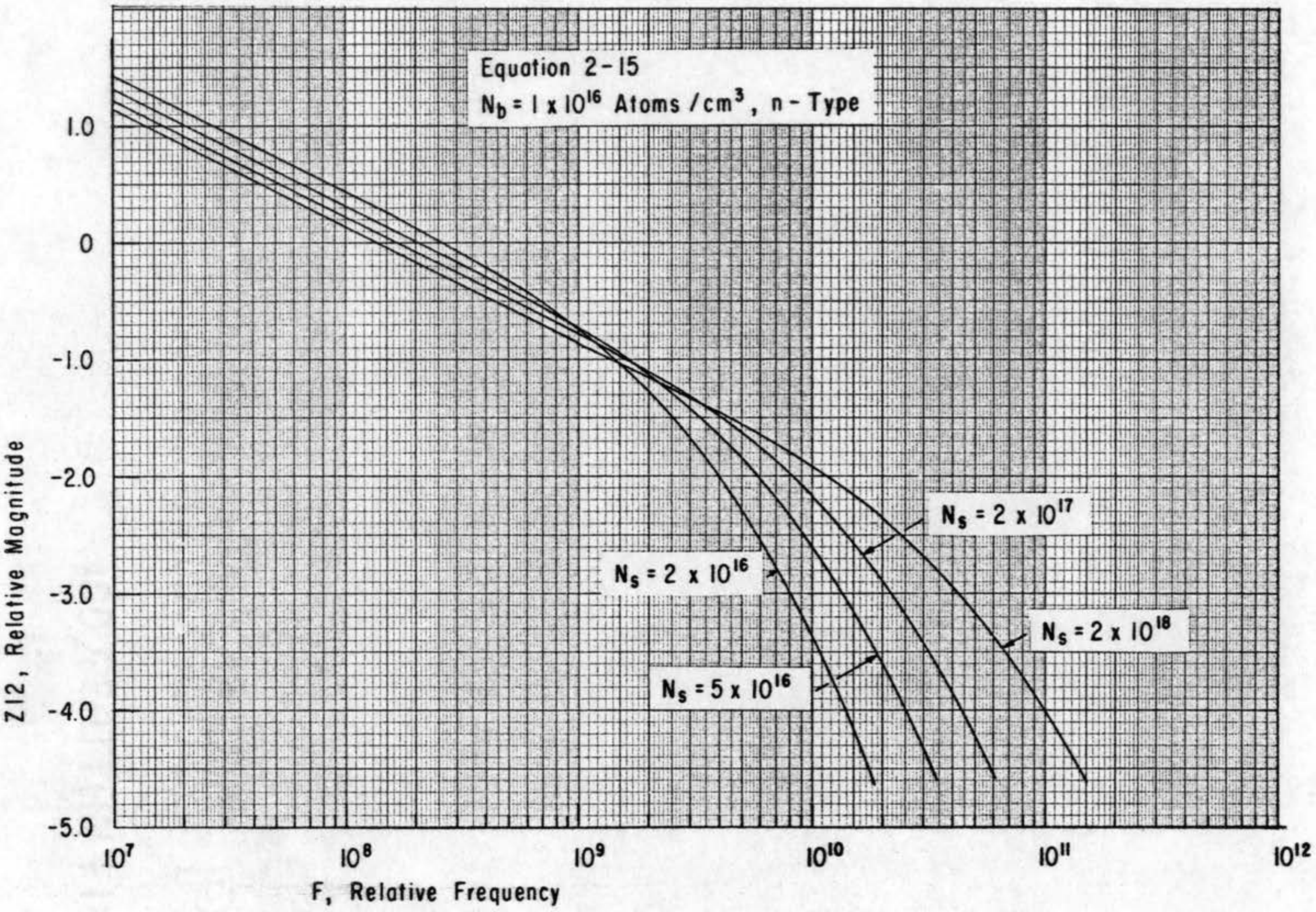

Figure 5-6. Open-Circuit Transfer Impedance (Magnitude).

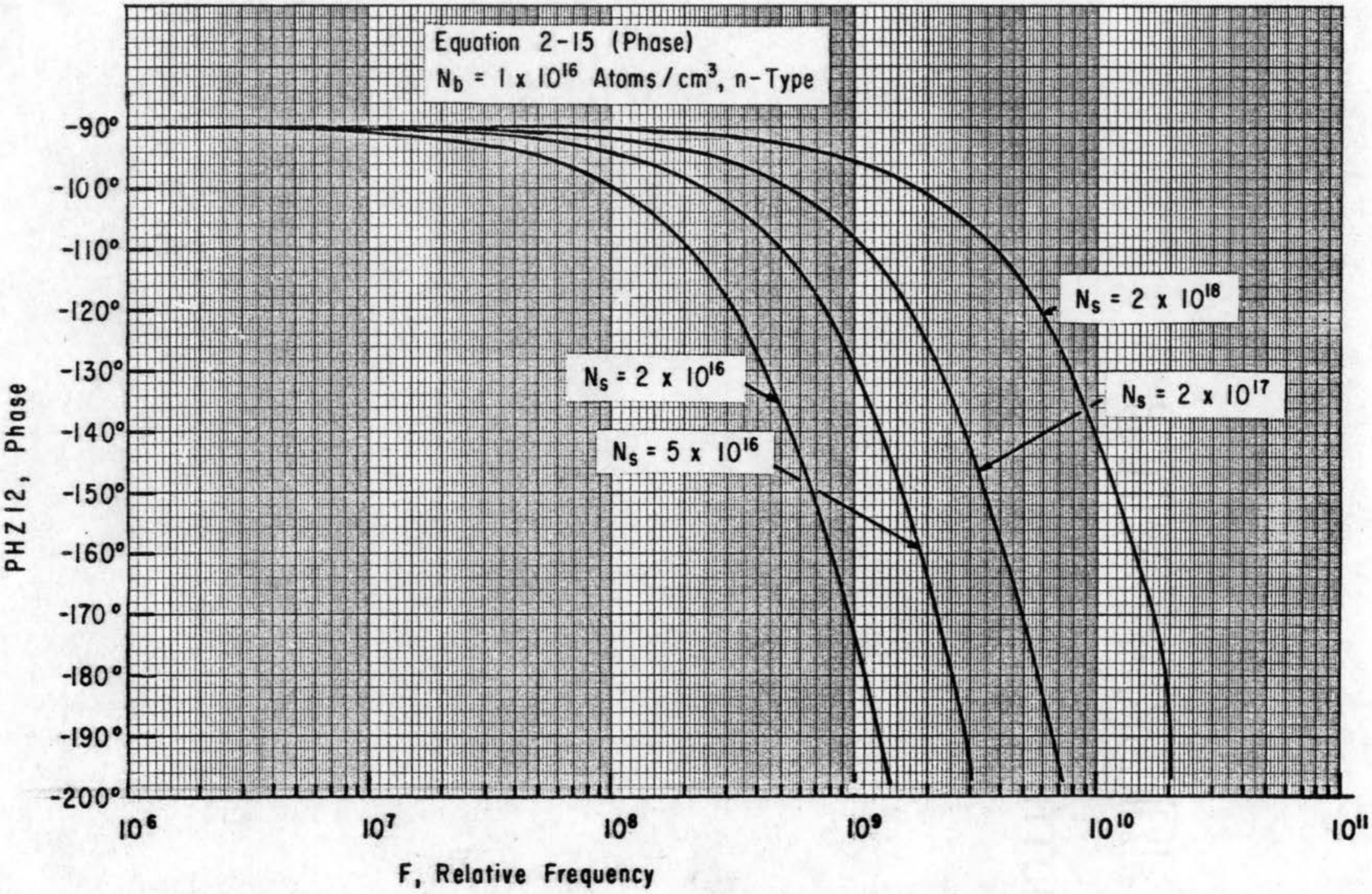

Figure 5-7. Open-Circuit Transfer Impedance (Phase).

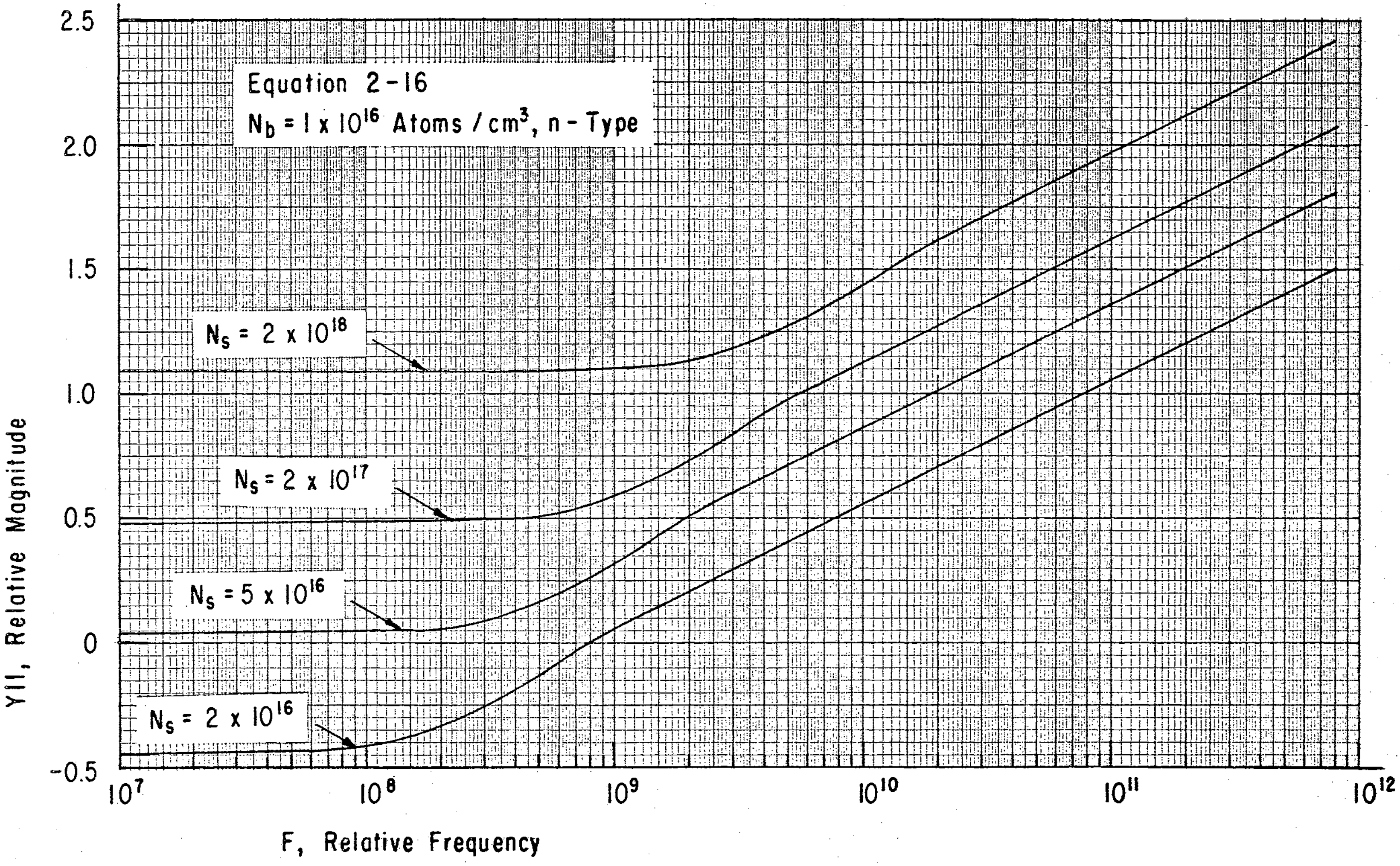

Figure 5-8. Short-Circuit Input Admittance (Magnitude).

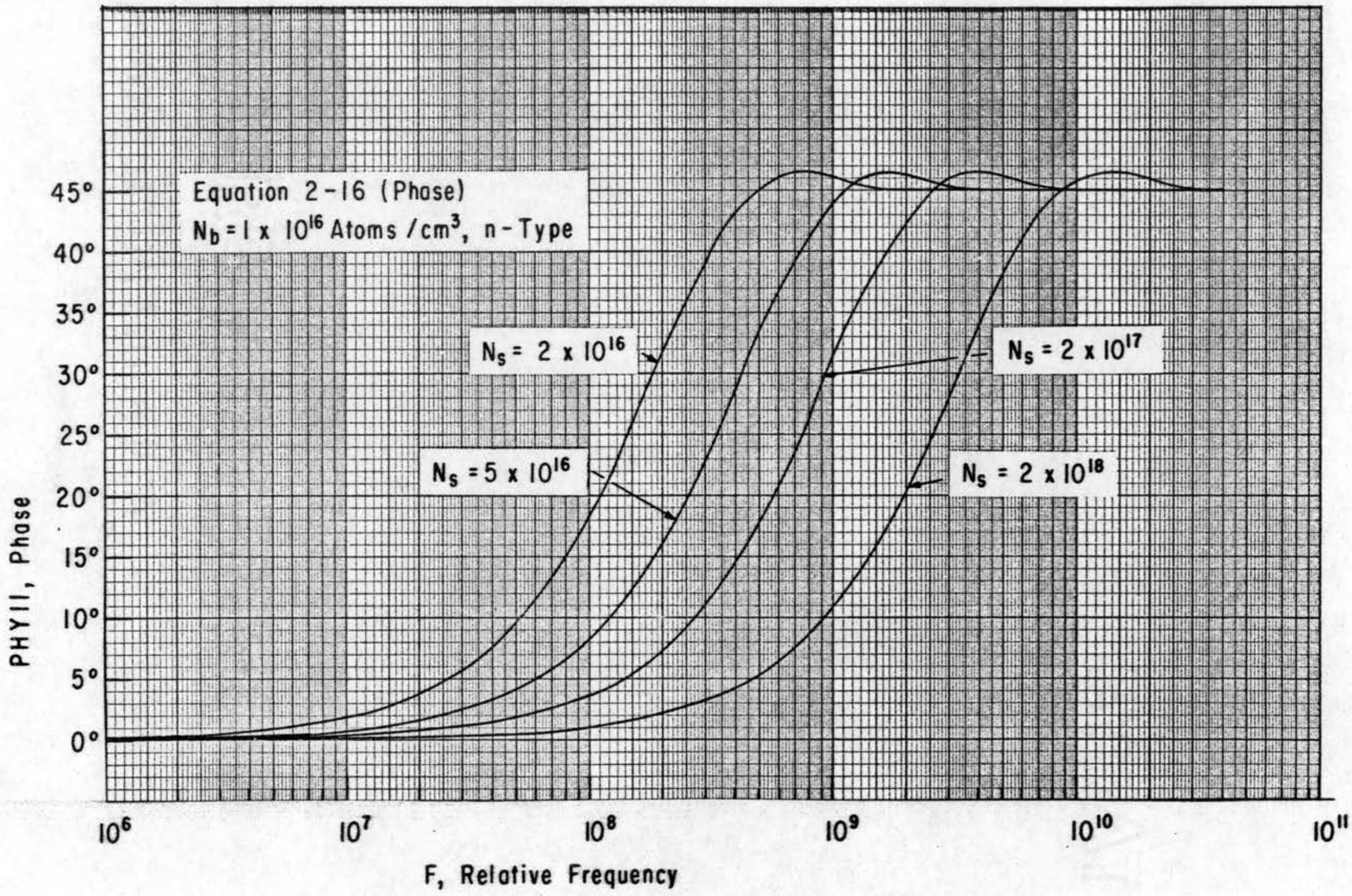

Figure 5-9. Short-Circuit Input Admittance (Phase).

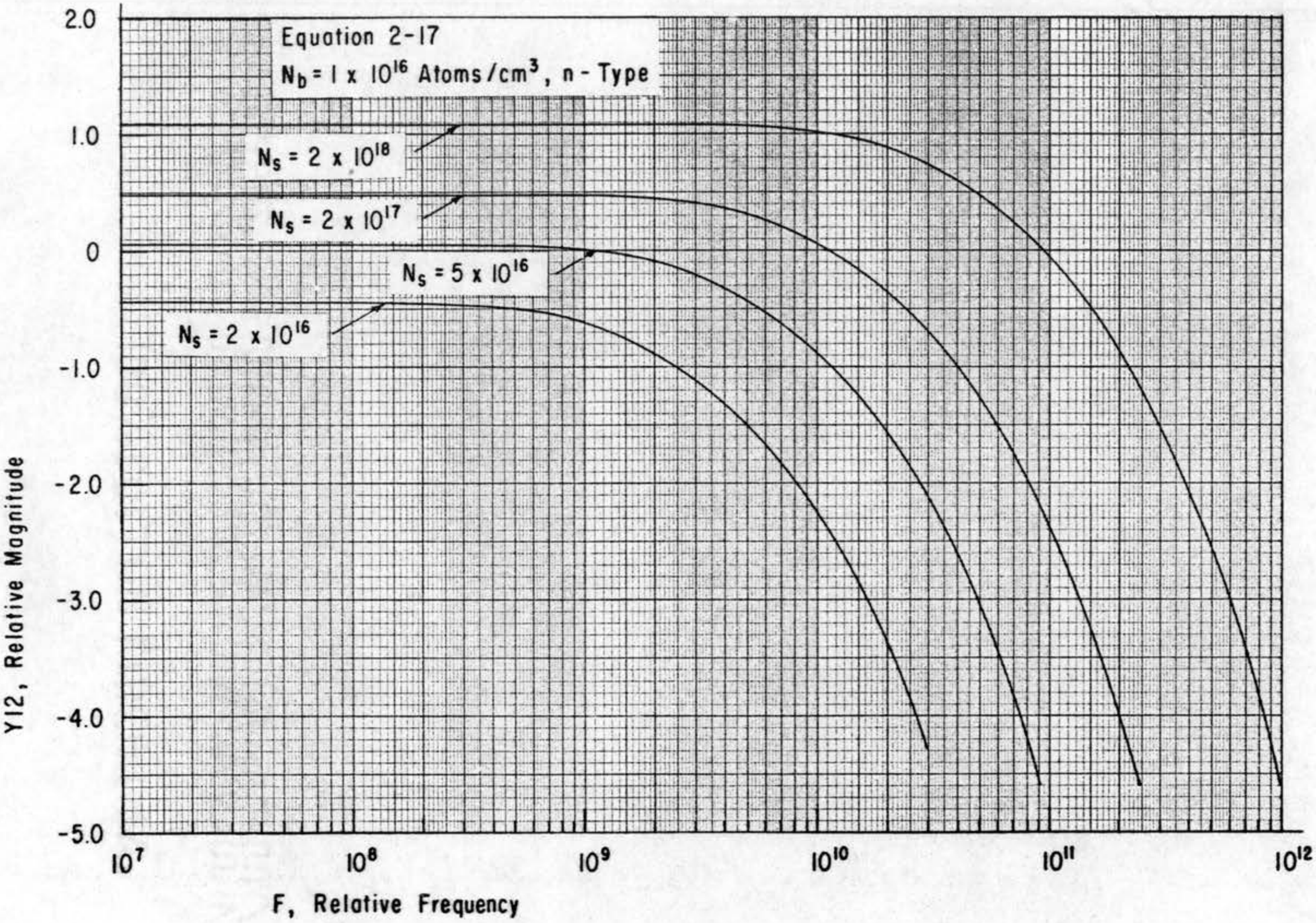

Figure 5-10. Short-Circuit Transfer Admittance (Magnitude).

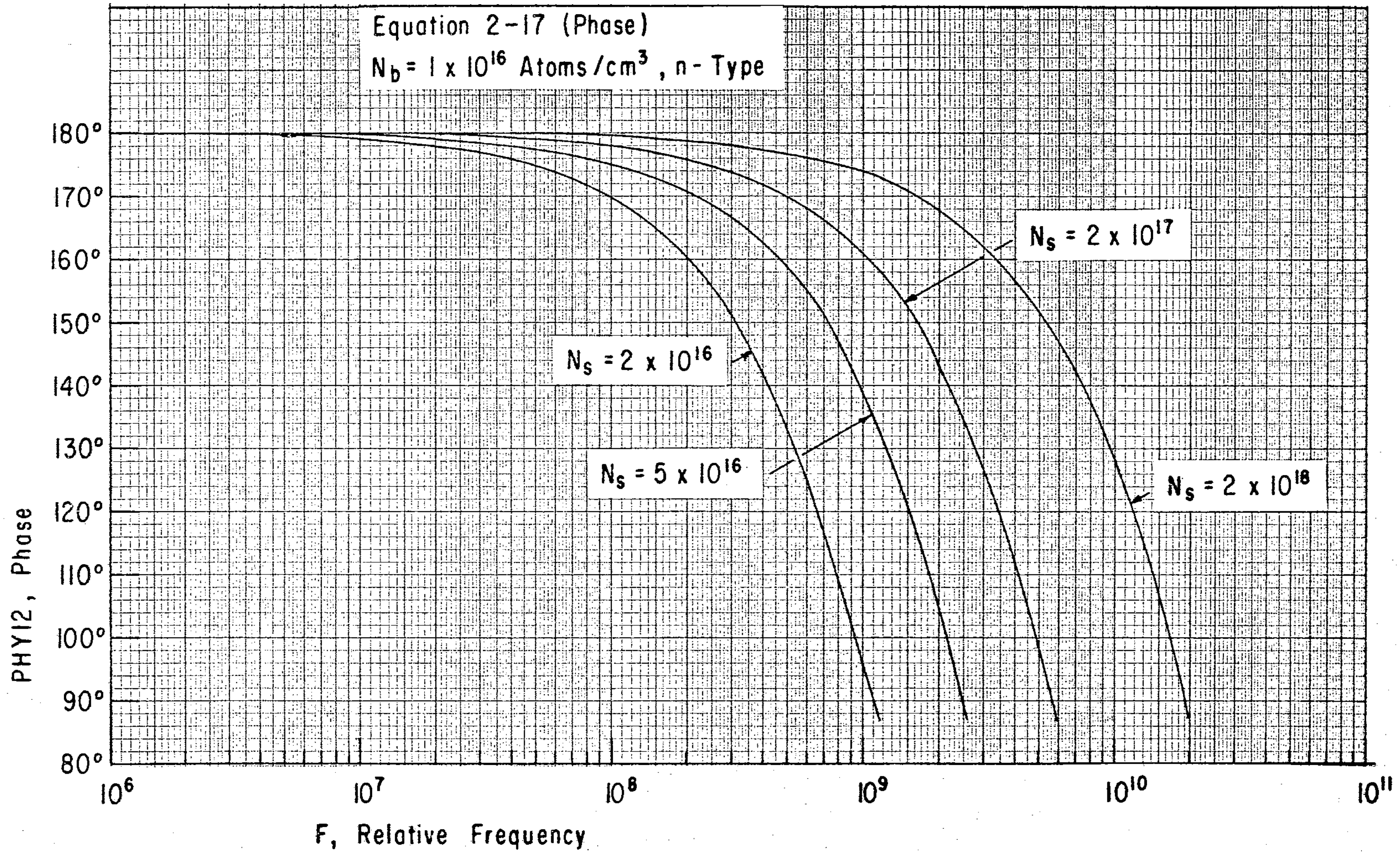

Figure 5-11. Short-Circuit Transfer Admittance (Phase).

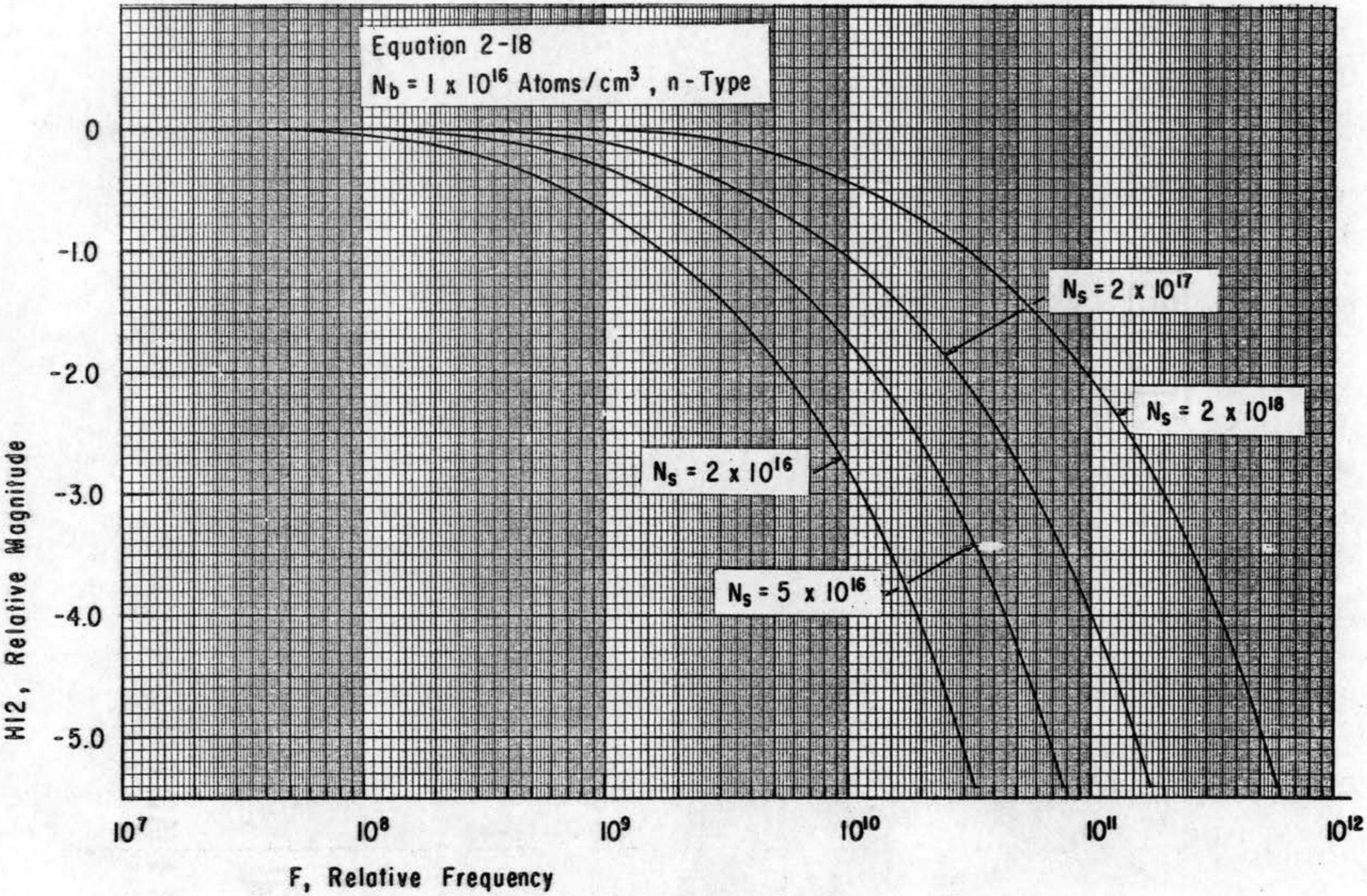

Figure 5-12. Open-Circuit Voltage Gain and Short-Circuit Current Gain (Magnitude).

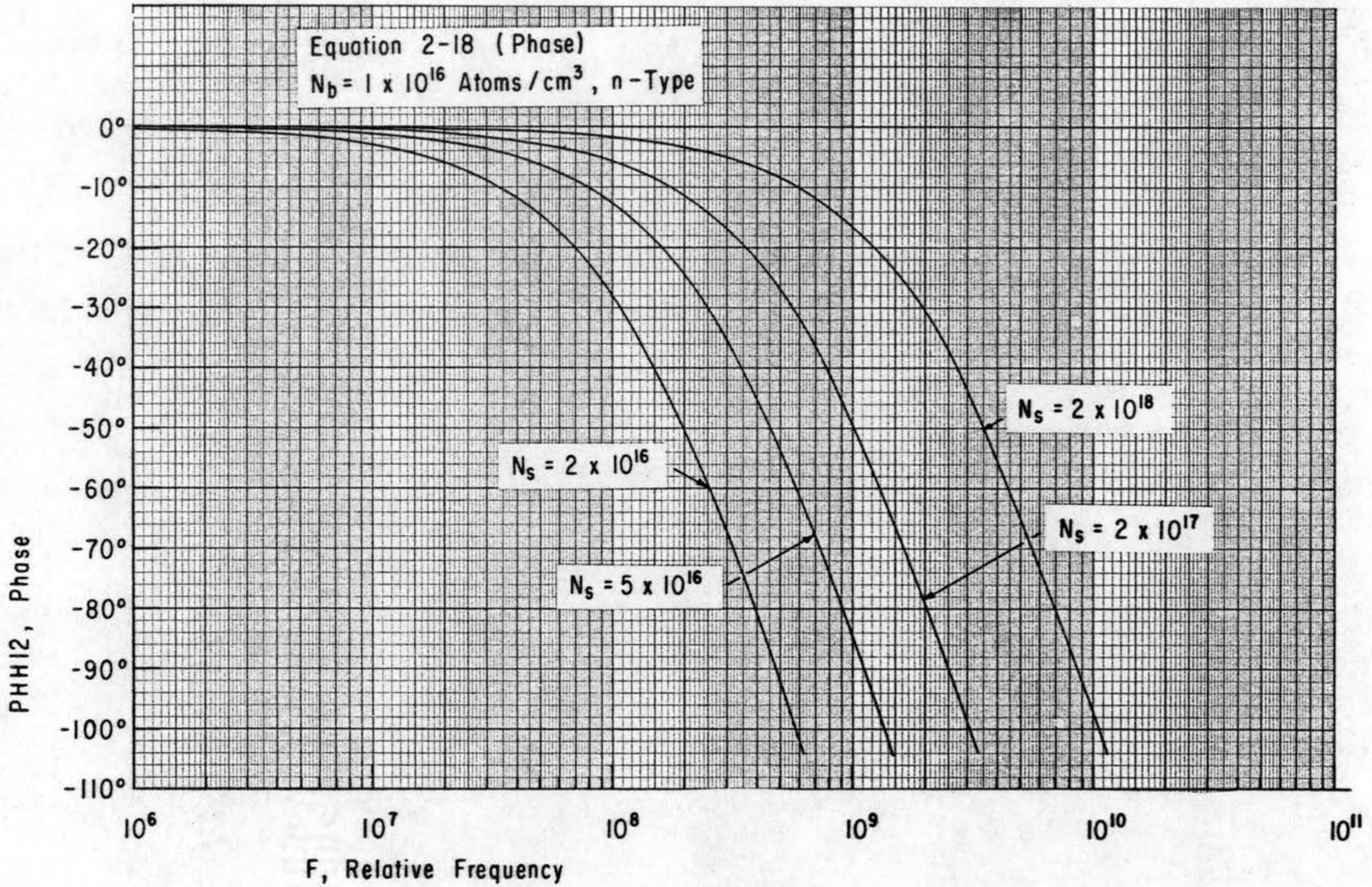

Figure 5-13. Open-Circuit Voltage Gain and Short-Circuit Current Gain (Phase).

$$Y11_b - Y11_a = -\log \frac{\bar{\rho}_b}{\bar{\rho}_a} + \tfrac{1}{2} \log \frac{K_b}{K_a} \qquad (5\text{-}24)$$

$$Y12_b - Y12_a = -\log \frac{\bar{\rho}_b}{\bar{\rho}_a} + \tfrac{1}{2} \log \frac{K_b}{K_a} - 0.43429F(K_b^{\frac{1}{2}} - K_a^{\frac{1}{2}}) \qquad (5\text{-}25)$$

$$H12_b - H12_a = -0.43429F(K_b^{\frac{1}{2}} - K_a^{\frac{1}{2}}), \qquad (5\text{-}26)$$

where $\bar{\rho}_b$ and $\bar{\rho}_a$ are to be determined from Tables I through IV; K_b and K_a are to be determined from Equation 5-3 or 5-5 depending upon the type of diffusion under consideration.

In conclusion, the primary results of this study are the parameters described in Equations 2-14 through 2-18 when expressed as functions of the relative frequency. With a sufficient knowledge of the diffusion profile, these five parameters will describe the response of a resistive element in any configuration. However, it is first recalled that the value of the effective resistivity calculated by Irvin (13) requires that the element be operated at 300° K. Secondly, it is required that the element be operated such that its junction remains in the linear graded region. [Calculations obtained from Lawrence and Warner (14) would be extremely valuable for such determinations.] Also, the change in bias along the length of the element must be such that the linearization of Equations 2-1 is justifiable. The shape factor δ was introduced so that the designer could empirically allow for lateral spreading and the increased capacitance near the semiconductor-oxide interface.

In regard to the approximate equations one may conclude that for any given diffusion there is a frequency below which the low frequency approximations to Equations 2-14 through 2-18 yield excellent results and a frequency two decades higher above which the high frequency approximations yield negligible error. If a parameter is to be represented by the approximate equations over the entire frequency range, the maximum error will be less than 15 per cent and will occur in the region of the intersection of the approximate solutions.

A graph of a parameter for a particular diffusion may be modified to represent the same parameter under different diffusion conditions by use of Equations 5-19 through 5-26.

As an example of how these results may be used, consider the following problem. Suppose that it is desired to maintain the input impedance of a resistive element at 1 x 10^4 ohms with 20 per cent tolerance up to ten megacycles. The element is to be operated under a.c. (alternating current) short-circuit output conditions. Furthermore, suppose that the active elements of the circuit require that this resistor be formed by the diffusion of surface concentration of 2 x 10^{17} atoms/cm^3, p-type, into a uniform background of 1 x 10^{16} atoms/cm^3, n-type.

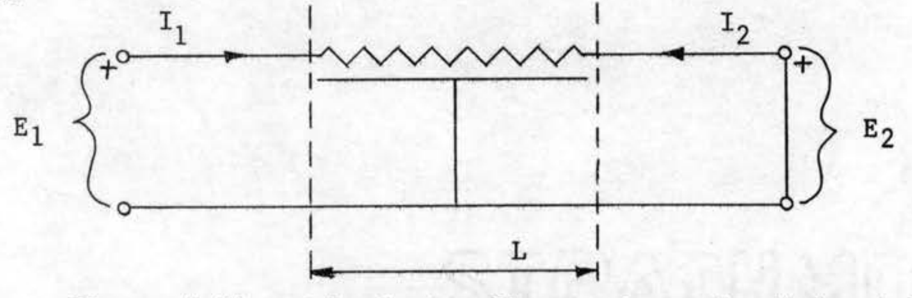

Figure 5-14. A Resistive Element Operating Under A.C. Short-Circuit Output Conditions.

Assume that it is known that the junction depth is 0.33 x 10^{-3} cm, where this value is obtained through measurement or is calculated with the knowledge of the diffusion length used for this particular diffusion.

With the above information, one may proceed as follows. It is seen from Equations 2-7 that the parameter under consideration is h_{11}, that is

$$h_{11} = \left.\frac{E_1}{I_1}\right|_{E_2 = 0} .$$

From Equation 5-8 it is recalled that

$$Y11 = \log \left| y_{11}/(wt/L) \right| = \log \left| (L/wt)/h_{11} \right| .$$

Therefore, the graph to be used is given in Figure 5-8. The low frequency range indicates that $Y11 = .481$ which corresponds to

$$\frac{h_{11}}{(L/wt)} = 0.33.$$

Substituting the junction depth for t and 1×10^4 ohms for h_{11} yields the requirement that

$$\frac{L}{w} = 10.$$

If the diffusion is such that the resistive element may be assumed rectangular, δ may be approximated as

$$\delta = \frac{2x_j}{w} = \frac{0.66 \times 10^{-3}}{w} .$$

Substitution of δ, (L/w) and x_j into the expression for the relative frequency yields

$$F = f(L^2 + 0.66 \times 10^{-2}L)\ 0.439 \times 10^5/{V_d}^{1/3}.$$

At this point it is noted in Figure 5-8 that Y11 = .551 when F = 8×10^8. This corresponds to a decrease of 0.07 or 18 per cent in h_{11}. This value of F will be taken as the relative frequency corresponding to the actual frequency of 10 megacycles. With these substitutions for the frequency and relative frequency the above equation becomes

$$L^2 + 0.66 \times 10^{-2}L - 0.1825 \times 10^{-2}V_d{}^{1/3} = 0.$$

If it is desired that the reverse bias on the resitive element be 4 volts, the positive root of the above equation yields

$$L = 5.06 \times 10^{-2} \text{ cm},$$

which is approximately 20 mils. Thus, a resistive element with L = 20 mils and w = 2 mils will satisfy the impedance magnitude requirements for the predetermined diffusion.

This study has resulted in a systematic procedure for predicting the resultant resistance and the corresponding frequency characteristics of resistors designed as an integral part of a diffused microelectronic circuit. The procedure of this study can be summarized as follows. The resistive element was represented as a distributed line, and the resulting partial differential equations were determined. These equations were solved under the assumption of constant resistance and capacitance per unit length along the element and the results given in terms of the open-circuit impedance, the short-circuit admittance, and the hybrid parameters. The argument of the above functions was denoted as $\bar{\lambda}$ and was seen to be a function of the resistance and capacitance per unit length. The diffusion profiles to be considered were then limited to those obtained through Gaussian and complementary error function diffusions, and the impurity concentration gradient at the junction depth was determined for both diffusion profiles.

The resistance per unit length was obtained by assuming a rectangular element and using the values of effective resistivity of diffused layers calculated by Irvin (13). The capacitance per unit length was obtained from the expression derived by Shockley (6) for the capacitance per unit area of linear graded junctions, substitution of the impurity concentration gradients previously determined, and multiplication by the factor $(1 + \delta)w$, where δ was denoted as the shape factor and w was the resistor width at the surface.

The values determined for resistance and capacitance per unit length were then combined in the argument $\bar{A}$. $\bar{A}$ was rewritten as $\bar{A} = F^{\frac{1}{2}} K^{\frac{1}{2}}$; where F was denoted as the relative frequency which was a function of the resistor dimensions, the bias voltage on the junction, and the frequency of operation; K was a combination of constants and parameters depending only on the diffusion profile. With these results the three sets of circuit parameters mentioned above were plotted as functions of the relative frequency for various fixed diffusions. Asymptotic expressions were then derived for the low and high relative frequency response of the three sets of circuit parameters.

Throughout this study the author has indicated that the frequency response of diffused resistive elements was of prime interest. The preceeding example has indicated one manner in which the results of this study may be helpful in the design of resistive elements. These results are also immediately applicable to problems where phase is of major importance and may be modified to describe the response of resistive elements operating in series or parallel connection or under load conditions other than open- and short-circuit.

The results obtained here should not be confined to the study of resistive elements. As indicated in Figures 2-1 and 2-2, they should provide useful information in the investigation of distributed R-C networks and capacitors formed by solid-state diffusion.

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APPENDIX A

PARAMETER RELATIONSHIPS

The (A, B, C, D), the open-circuit impedance, the short-circuit admittance and the hybrid parameters are defined by Equations 2-4 through 2-7.

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix} \tag{2-4}$$

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2-5}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{2-6}$$

$$\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix} \tag{2-7}$$

The following equations are given by Guillemin (15), which relate the latter three sets of parameters to the (A, B, C, D) parameters.

$$z_{11} = \frac{A}{B} = Z_o \frac{\cosh\gamma L}{\sinh\gamma L}$$

$$z_{12} = z_{21} = \frac{1}{C} = \frac{Z_o}{\sinh\gamma L}$$

$$z_{22} = \frac{D}{C} = Z_o \frac{\cosh\gamma L}{\sinh\gamma L}$$

$$y_{11} = \frac{D}{B} = \frac{\cosh\gamma L}{Z_o \sinh\gamma L}$$

$$y_{12} = y_{21} = \frac{-1}{B} = \frac{-1}{Z_o \sinh\gamma L}$$

$$y_{22} = \frac{A}{B} = \frac{\cosh\gamma L}{Z_o \sinh\gamma L}$$

$$h_{11} = \frac{B}{D} = \frac{1}{y_{11}}$$

$$h_{12} = -h_{21} = \frac{1}{D} = \frac{1}{\cosh\gamma L}$$

$$h_{22} = \frac{C}{D} = \frac{1}{z_{22}}$$

Since several of the parameters are either identical or reciproical, the above equations may be written as

$$z_{11} = z_{22} = \frac{1}{h_{22}} = Z_o \frac{\cosh\gamma L}{\sinh\gamma L} \tag{2-8}$$

$$z_{12} = z_{21} = \frac{Z_o}{\sinh\gamma L} \tag{2-9}$$

$$y_{11} = y_{22} = \frac{1}{h_{11}} = \frac{\cosh\gamma L}{Z_o \sinh\gamma L} \tag{2-10}$$

$$y_{12} = y_{21} = \frac{-1}{Z_o \sinh\gamma L} \tag{2-11}$$

$$h_{12} = -h_{21} = \frac{1}{\cosh\gamma L} \tag{2-12}$$

From Equation 2-2 it is recalled that

$$\gamma = (1 + j)\,(\pi fRC)^{\frac{1}{2}}$$

$$Z_o = (1 + j)^{-1}(R/\pi fC)^{\frac{1}{2}} = R(1 + j)^{-1}(\pi fRC)^{-\frac{1}{2}}.$$

Letting $\bar{A} = (\pi fRCL^2)^{\frac{1}{2}}$ in the two above equations and noting that $R = \bar{\rho}/wt$, the expression for γL and Z_o become

$$\gamma L = (1 + j)\,\bar{A}$$
$$Z_o = (1 + j)^{-1}(L/wt)(\bar{\rho}/\bar{A}). \qquad (2\text{-}13)$$

Substituting Equation 2-13 into Equations 2-8 through 2-12 yields:

$$z_{11} = z_{22} = 1/h_{22}$$
$$= (L/wt)(\bar{\rho}/\bar{A})(1 + j)^{-1}\,\frac{\cosh(1 + j)\bar{A}}{\sinh(1 + j)\bar{A}} \qquad (2\text{-}8)'$$

$$z_{12} = z_{21} = (L/wt)(\bar{\rho}/\bar{A})(1 + j)^{-1}\,\frac{1}{\sinh(1 + j)\bar{A}} \qquad (2\text{-}9)'$$

$$y_{11} = y_{22} = 1/h_{11} = (wt/L)(\bar{A}/\bar{\rho})(1+j)\,\frac{\cosh(1+j)\bar{A}}{\sinh(1+j)\bar{A}} \qquad (2\text{-}10)'$$

$$y_{12} = y_{21} = -(wt/L)(\bar{A}/\bar{\rho})(1+j)\,\frac{1}{\sinh(1+j)\bar{A}} \qquad (2\text{-}11)'$$

$$h_{12} = -h_{21} = \frac{1}{\cosh(1+j)\bar{A}} \qquad (2\text{-}12)'$$

The relations between hyperbolic functions with complex arguments and functions with real arguments are given by Churchill (16) to be

$$\sinh\,(1 + j)\,\bar{A} = \sinh\bar{A}\cos\bar{A} + j\cosh\bar{A}\sin\bar{A}$$
$$\cosh\,(1 + j)\,\bar{A} = \cosh\bar{A}\cos\bar{A} + j\sinh\bar{A}\sin\bar{A}.$$

Substituting these expressions into 2-8' through 2-12' and simplyfing establishes Equations 2-14 through 2-18.

$$z_{11}/(L/wt) = z_{22}/(L/wt) = (wt/L)/h_{22} =$$
$$= (\bar{\rho}/\bar{A})\,\left[\frac{1 + j\tanh\bar{A}\tan\bar{A}}{(\tanh\bar{A} - \tan\bar{A}) + j(\tanh\bar{A} + \tan\bar{A})}\right] \qquad (2\text{-}14)$$

$$z_{12}/(L/wt) = z_{21}/(L/wt) =$$

$$= (\bar{\rho}/\bar{A})\left[\frac{1}{(\sinh\bar{A}\cos\bar{A} - \cosh\bar{A}\sin\bar{A}) + j(\mathrm{Sinh}\bar{A}\cos\bar{A} + \cosh\bar{A}\sin\bar{A})}\right] \quad (2\text{-}15)$$

$$y_{11}/(wt/L) = y_{22}/(wt/L) = (L/wt)/h_{11} =$$

$$= (\bar{A}/\bar{\rho})\left[\frac{(1-\tanh\bar{A}\tan\bar{A}) + j(1 + \tanh\bar{A}\tan\bar{A})}{\tanh\bar{A} + j\tan\bar{A}}\right] \quad (2\text{-}16)$$

$$y_{12}/(wt/L) = y_{21}/(wt/L) =$$

$$= (\bar{A}/\bar{\rho})\left[\frac{-(1 + j)}{\sinh\bar{A}\cos\bar{A} + j\cosh\bar{A}\sin\bar{A}}\right] \quad (2\text{-}17)$$

$$h_{12} = -h_{21} = \frac{1}{\cosh\bar{A}\cos\bar{A} + j\sinh\bar{A}\sin\bar{A}}. \quad (2\text{-}18)$$

APPENDIX B

APPROXIMATION TO THE ERROR FUNCTION

The complementary error function distribution may be written in terms of the normalized distance $y = x/L_1$.

$$N(y) = N_s \operatorname{erfc}(y) - N_b \tag{B-1}$$

Erfc(y) may be expanded, in a Taylor's series about y_j, the normalized junction depth, as

$$\operatorname{erfc}(y) = \operatorname{erfc}(y_j) + \sum_{n=1}^{\infty} \operatorname{erfc}^{(n)} y_j \frac{(y-y_j)^n}{n!}. \tag{B-2}$$

It can be shown to a first approximation (17):

$$\operatorname{erfc}^{(n)} y \doteq -2y \operatorname{erfc}^{(n-1)} y. \tag{B-3}$$

Thus, $\operatorname{erfc}^{(n)} y_j \doteq (-2y_j)^n \operatorname{erfc}(y_j)$, and from (2):

$$\operatorname{erfc}(y) \doteq \operatorname{erfc}(y_j) \left[1 + \Sigma \frac{[-2y_j(y-y_j)]^n}{n!}\right]$$

$$= \frac{N_b}{N_s} \exp\,[-2y_j(y-y_j)]. \tag{B-4}$$

The net impurity density can then be written, from 1,

$$N(y) \doteq N_b\,[\exp\,[-2y_j(y-y_j)]-1]. \tag{B-5}$$

The slope of the impurity distribution is given by

$$\frac{d\ N(x)}{dx} = \frac{1}{L_1}\frac{d\ N(y)}{dy} \doteq \frac{N_b}{L_1}\left[-2y_j \exp\left[-2y_j(y-y_j)\right]\right]. \qquad \text{(B-6)}$$

Evaluating at the junction,

$$a = \left|\frac{1}{L_1}\frac{d\ N(y)}{dy}\right|_{y=y_j} \doteq \frac{N_b}{L_1}(2y_j) = \frac{2x_j}{L_1^2} N_B. \qquad \text{(B-7)}$$

APPENDIX C

LOW AND HIGH FREQUENCY APPROXIMATIONS

The following parameters are defined.

$$\text{Z11} = \log \left| z_{11}/(L/wt) \right| \tag{C-1}$$

$$\text{PHZ11} = \text{the phase of } z_{11} \tag{C-2}$$

$$\text{Z12} = \log \left| z_{12}/(L/wt) \right| \tag{C-3}$$

$$\text{PHZ12} = \text{the phase of } z_{12} \tag{C-4}$$

$$\text{Y11} = \log \left| y_{11}/(wt/L) \right| \tag{C-5}$$

$$\text{PHY11} = \text{the phase of } y_{11} \tag{C-6}$$

$$\text{Y12} = \log \left| y_{12}/(wt/L) \right| \tag{C-7}$$

$$\text{PHY12} = \text{the phase of } y_{12} \tag{C-8}$$

$$\text{H12} = \log \left| h_{12} \right| \tag{C-9}$$

$$\text{PHH12} = \text{the phase of } h_{12}. \tag{C-10}$$

The following relations are easily derived from Equations 2-14 through 2-18.

$$\begin{aligned}
\text{Z11} &= \log (\bar{\rho}/\bar{A}) + \tfrac{1}{2} \log (1 + \tanh^2 \bar{A} \tan \bar{A}) \\
&\quad - \tfrac{1}{2} \log 2(\tanh^2 \bar{A} + \tan^2 \bar{A}) \\
\text{PHZ11} &= \tan^{-1} [\tanh \bar{A} \tan \bar{A}] \\
&\quad - \tan^{-1} [(\tanh \bar{A} + \tan \bar{A})/(\tanh \bar{A} - \tan \bar{A})] \\
\text{Z12} &= \log (\bar{\rho}/\bar{A}) - \tfrac{1}{2} \log 2(\sinh^2 \bar{A} \cos^2 \bar{A} + \cosh^2 \bar{A} \sin^2 \bar{A}) \\
&= \log (\bar{\rho}/\bar{A}) - \tfrac{1}{2} \log 2(\cosh^2 \bar{A} - \cos^2 \bar{A})
\end{aligned}$$

$$\mathrm{PHZ12} = -\tan^{-1}\left[(\tanh\bar{A} + \tan\bar{A})/(\tanh\bar{A} - \tan\bar{A})\right]$$

$$\mathrm{Y11} = \log(\bar{A}/\bar{\rho}) + \tfrac{1}{2}\log 2(1 + \tanh^2\bar{A}\tan^2\bar{A})$$
$$- \tfrac{1}{2}\log(\tanh^2\bar{A} + \tan^2\bar{A})$$

$$\mathrm{PHY11} = \tan^{-1}\left[(1 + \tanh\bar{A}\tan\bar{A})/(1 - \tanh\bar{A}\tan\bar{A})\right]$$
$$- \tan^{-1}(\tan\bar{A}/\tanh\bar{A})$$

$$\mathrm{Y12} = \log(\bar{A}/\bar{\rho}) + \tfrac{1}{2}\log 2$$
$$- \tfrac{1}{2}\log(\sinh^2\bar{A}\cos^2\bar{A} + \cosh^2\bar{A}\sin^2\bar{A})$$
$$= \log(\bar{A}/\bar{\rho}) + \tfrac{1}{2}\log 2 - \tfrac{1}{2}\log(\cosh^2\bar{A} - \cos^2\bar{A})$$

$$\mathrm{PHY12} = 5\pi/4 - \tan^{-1}(\tan\bar{A}/\tanh\bar{A})$$

$$\mathrm{H12} = -\tfrac{1}{2}\log(\cosh^2\bar{A}\cos^2\bar{A} + \sinh^2\bar{A}\sin^2\bar{A})$$

$$\mathrm{PHH12} = -\tan^{-1}(\tan\bar{A}\tanh\bar{A}).$$

For sufficiently small values of $\bar{A}$, that is $\bar{A} < 10^{-1}$, one may assume

$$\tanh\bar{A} \doteq \tan\bar{A} \doteq \bar{A}$$

and $$\cosh^2\bar{A} - \cos^2\bar{A} \doteq 2(\bar{A})^2.$$

By introducing these approximations and neglecting the higher order terms of $\bar{A}$ when compared with unity, the above magnitudes may be approximated as

$$\mathrm{Z11} = \mathrm{Z12} \doteq \log\bar{\rho} - \log 2 - \log(\bar{A})^2 \qquad \text{(C-11)}$$

$$\mathrm{Y11} = \mathrm{Y12} \doteq -\log\bar{\rho} \qquad \text{(C-12)}$$

$$\mathrm{H12} \doteq 0. \qquad \text{(C-13)}$$

For large values of $\bar{A}$, that is $\bar{A} > 10^1$, one may assume

$$\tanh\bar{A} \doteq 1$$

and $$\sinh\bar{A} \doteq \cosh\bar{A} \doteq \tfrac{1}{2}\exp(\bar{A}).$$

These approximations yield

$$Z11 \doteq \log \bar{\rho} - \tfrac{1}{2} \log 2 - \log \bar{A} \tag{C-14}$$

$$Z12 \doteq \log \bar{\rho} + \tfrac{1}{2} \log 2 - \log \bar{A} - 0.43429\ \bar{A} \tag{C-15}$$

$$Y11 \doteq - \log \bar{\rho} + \tfrac{1}{2} \log 2 + \log \bar{A} \tag{C-16}$$

$$Y12 \doteq - \log \bar{\rho} + 3/2 \log 2 + \log \bar{A} - 0.43429\ \bar{A} \tag{C-17}$$

$$H12 \doteq \log 2 - 0.43429\ \bar{A}. \tag{C-18}$$

APPENDIX D

PROGRAM FOR EVALUATION OF PARAMETERS

Calculation of Equations 2-14 through 2-18 were performed on an IBM 1410 computer. The program used for these computations is given in this appendix.

The program was designed to calculate the value of the relative frequency, F, for which $\bar{A} = 0.1$, then return to the beginning of the decade of which this F belongs to start calculations. Equations 2-14 through 2-18 were then calculated for F = 1.0, 1.5, 2.0, 3.0, 4.0, 6.0, 8.0 of this decade and the following five decades.

The program outputs are: the value of F when $\bar{A} = 0.1$; the low and high frequency approximation for $\log \left| z_{11}/(L/wt) \right|$ and $\log \left| z_{12}/(L/wt) \right|$; the low and high frequency approximation for $\log \left| y_{11}/(wt/L) \right|$ and $\log \left| y_{12}/(wt/L) \right|$; the low and high frequency approximations for $\log \left| h_{12} \right|$; and the parameters defined in Equations 1 through 10 of Appendix C versus F.

The following program variables are defined.

CB	=	N_b	A	=	$\bar{A}$
CS	=	N_s	PHI	=	π
P	=	$\bar{\rho}$	Q	=	q
X	=	F	EE	=	ϵ

```
 MON$$      JOB  251540029,DIFFUSED RESISTOR FREQUENCY RESPONSE
 MON$$      ASGN MJB,A2
 MON$$      ASGN MGO,A3
 MON$$      ASGN MW1,A4
 MON$$      ASGN MW2,A5
 MON$$      MODE GO,TEST
 MON$$      EXEQ FORTRAN,SOF,SIU,08,03,,,P1
 1 FORMAT(I4,2E8.1,F10.5)
 2 FORMAT(61X,10HRUN NUMBER,I4)
 3 FORMAT(46X,3HNB=,E8.1,4X,3HNS=,E8.1,4X,2HP=,F10.5)
 4 FORMAT(56X,2HX=,E10.3,8H WHEN A=,F3.1)
 5 FORMAT (/20H LOG Z11 = LOG Z12 =,E10.4,6H-LOG X,14X,9HLOG Z11 =,E1
  10.4,12H - 0.5 LOG X,/,50X,9HLOG Z12 =,E10.4,3H - ,E10.4,17H X**0.5
  2-0.5 LOG X)
 6 FORMAT (/20H LOG Y11 = LOG Y12 =,E10.4,20X,9HLOG Y11 =,E10.4,12H +
  1 0.5 LOG X,/,50X,9HLOG Y12 =,E10.4,3H - ,E10.4,17H X**0.5+0.5 LOG
  2X)
 7 FORMAT (/14H LOG H12 = 0.0,36X,9HLOG H12 =,E10.4,3H - ,E10.4,7H X*
  1*0.5)
 8 FORMAT(/7X,1HX,10X,3HZ11,8X,5HPHZ11,8X,3HZ12,8X,5HPHZ12,8X,3HY11,8
  1X,5HPHY11,8X,3HY12,8X,5HPHY12,8X,3HH12,8X,5HPHH12)
 9 FORMAT(1H ,11(E10.3,2X))
10 FORMAT(1H ,E10.3)
11 FORMAT(/1H ,E15.8)
20 CB=0.0
   CS=0.0
   P=0.0
   Y=0.0
   X=0.0
   A=0.0
   Y1=0.0
   Y2=0.0
   Y3=0.0
   Z=0.0
   Z11X=0.0
   READ(1,1) DATA,CB,CS,P
   CALL PAGE
   WRITE(3,2)DATA
   WRITE(3,3)CB,CS,P
   PHI=3.1415927
   Q=1.602E-19
   EE=1.035918E-12
   C=0.43429
   Y1=ALOG(CS/CB)
   Y2=(CB*Q*(EE**2.0))/6.0
   Y3=(Y1*Y2)**0.33333333
   Y4=(P*PHI*Y3)**0.5
   X=(0.1/Y4)**2.0
   A=0.1
   WRITE(3,4)X,A
   Z=P/(2.0*(Y4**2))
   Z11X=C*ALOG(Z)
   Z11XX=0.5*C*ALOG(Z*P)
   Z12XX=0.5*C*ALOG(Z*P*4.0)
   COEF=C*Y4
   WRITE(3,5)Z11X,Z11XX,Z12XX,COEF
   Y11X=(-C)*ALOG(P)
   Y11XX=0.5*C*ALOG(1.0/(Z*P))
   Y12XX=0.5*C*ALOG(4.0/(Z*P))
   WRITE(3,6)Y11X,Y11XX,Y12XX,COEF
   H12XX=C*ALOG(2.0)
   WRITE(3,7)H12XX,COEF
```

```
XLOG=C*ALOG(X)
XE=AINT(XLOG)
X=10.0**XE
WRITE(3,11)X
X=X*1.000001
WRITE(3,11)X
WRITE(3,8)
X=(4.0*X)/5.0
DO 21 J=1,6
DO 21 I=1,7
IF(I.EQ.1) X=(5.0*X)/4.0
IF(I.EQ.2) X=1.5*X
IF(I.EQ.3) X=(4.0*X)/3.0
IF(I.EQ.4) X=1.5*X
IF(I.EQ.5) X=(4.0*X)/3.0
IF(I.EQ.6) X=1.5*X
IF(I.EQ.7) X=(4.0*X)/3.0
A=(X**0.5)*Y4
SINHA=(EXP(A)-EXP(-A))/2.0
COSHA=SINHA+EXP(-A)
TANHA=SINHA/COSHA
SINA=SIN(A)
COSA=COS(A)
D=2.0*PHI
E=0.00001
R=AMOD(A,D)
R1=R-(PHI/2.0)
R1=ABS(R1)
IF(R1.LT.E)GO TO 22
R1=R-(1.5*PHI)
R1=ABS(R1)
IF(R1.LT.E)GO TO 22
TANA=SINA/COSA
F=C*ALOG(P/A)
G=C*ALOG(1.0+(TANHA**2.0)*(TANA*TANA))
H=(TANHA*TANHA)+(TANA*TANA)
U=(COSHA*COSHA)-(COSA*COSA)
V=(COSHA*COSHA)+(COSA*COSA)-1.0
Z11=F+0.5*G-0.5*C*ALOG(2.0*H)
Z12=F-0.5*C*ALOG(2.0*U)
Y11=-F+0.5*C*ALOG(2.0)+0.5*G-0.5*C*ALOG(H)
Y12=-F+0.5*C*ALOG(2.0)-0.5*C*ALOG(U)
H12=(-0.5)*C*ALOG(V)
W=180.0/PHI
T=1.0
Y=TANHA*TANA
CALL ARCT(T,Y,PH)
PH1=PH
T=TANHA-TANA
Y=TANHA+TANA
CALL ARCT(T,Y,PH)
PHZ11=(PH1-PH)*W
T=SINHA*COSA-COSHA*SINA
Y=SINHA*COSA+COSHA*SINA
CALL ARCT(T,Y,PH)
PHZ12=(-PH)*W
T=1.0-TANHA*TANA
Y=1.0+TANHA*TANA
CALL ARCT(T,Y,PH)
PH1=PH
T=TANHA
Y=TANA
```

```
      CALL ARCT(T,Y,PH)
      PHY11=(PH1-PH)*W
      T=SINHA*COSA
      Y=COSHA*SINA
      CALL ARCT(T,Y,PH)
      PHY12=(1.25*PHI-PH)*W
      T=COSHA*COSA
      Y=SINHA*SINA
      CALL ARCT(T,Y,PH)
      PHH12=-W*PH
      WRITE(3,9)X,Z11,PHZ11,Z12,PHZ12,Y11,PHY11,Y12,PHY12,H12,PHH12
   21 CONTINUE
      GO TO 20
   22 WRITE(3,10)X
      GO TO 21
      END
     MON$$      EXEQ FORTRAN,SOF,SIU,08,03,,,
      SUBROUTINE PAGE
    1 FORMAT(1H1)
      WRITE(3,1)
      RETURN
      END
     MON$$      EXEQ FORTRAN,SOF,SIU,08,03,,,
      SUBROUTINE ARCT(T,Y,PH)
      PHI=3.1415927
      IF(T.EQ.0.0)GO TO 1
      IF(T.LT.0.0)PH=PHI+ATAN(Y/T)
      IF(T.GT.0.0)PH=ATAN(Y/T)
    2 RETURN
    1 IF(Y.LT.0.0)PH=-(PHI/2.0)
      IF(Y.GT.0.0)PH=PHI/2.0
      GO TO 2
      END
     MON$$      EXEQ LINKLOAD
                PHASEPROB1
                CALL P1
     MON$$      EXEQ PROB1,MJB
35+1.0E+16+2.0E+1600002.80000
36+1.0E+16+5.0E+1600000.91000
37+1.0E+16+1.0E+1700000.53000
38+1.0E+16+2.0E+1700000.33000
39+1.0E+16+5.0E+1700000.19000
40+1.0E+16+1.0E+1800000.12000
41+1.0E+16+2.0E+1800000.08000
42+1.0E+16+5.0E+1800000.04500
50+1.0E+17+2.0E+1700000.59000
51+1.0E+17+5.0E+1700000.20000
52+1.0E+17+1.0E+1800000.11000
53+1.0E+17+2.0E+1800000.09600
54+1.0E+17+5.0E+1800000.03900
55+1.0E+17+1.0E+1900000.02400
56+1.0E+17+2.0E+1900000.01400
57+1.0E+17+5.0E+1900000.00650
89+1.0E+15+1.0E+1600001.20000
90+1.0E+15+2.0E+1600000.67000
91+1.0E+15+5.0E+1600000.33000
92+1.0E+15+1.0E+1700000.20000
93+1.0E+15+2.0E+1700000.12000
94+1.0E+15+5.0E+1700000.07700
95+1.0E+15+1.0E+1800000.05500
96+1.0E+15+2.0E+1800000.03800
97+1.0E+15+5.0E+1800000.02500
```

VITA

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Candidate for the Degree of

Master of Science

Thesis: DIFFUSED RESISTOR FREQUENCY RESPONSE

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Biographical:

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