A SCHEME FOR MINIMAL ENERGY CONSUMPTION FOR THE TRAJECTORY CORRECTION OF AN INTERPLANETARY VEHICLE

By

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PREFACE

Considering the economics and practicability of an interplanetary mission, it may be realized that for every pound of payload placed outside the earths atmosphere, several thousand pounds of rocket booster and equipment are needed. This restricts the size of payload which can be injected into space. Nevertheless, the objective is to inject the largest possible payload. This can be achieved by eliminating all portions of the payload that are unnecessary in the attainment of a successful mission.

Since it is unlikely that a space vehicle can make an interplanetary mission without the need for corrections, a necessary amount of fuel must be carried. This leads to the determination of the minimum cost amount of fuel needed to make corrections. If too much fuel is carried, the excess fuel may be considered dead weight; a weight that could have been used to carry useful equipment. On the other hand, if the space vehicle runs out of fuel before it has reached its termination point, the mission will probably fail. This could cost loss of lives if the space vehicle is manned. Consequently, it is desirable that the space vehicle carry sufficient fuel to complete the mission with a minimum cost probability. This may be achieved by jointly minimizing the sum of the cost of carrying fuel and the risk of a mission failure.

The author expresses sincere thanks to his thesis adviser, Professor L. J. Fila, for his counsel and guidance. Also, the contributions of Dr. W. J. Fabrycky in helping the author understand the nature of the Monte Carlo method is gratefully acknowledged. Appreciation is expressed to my wife, Betty, for the many hours of typing this thesis.

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LIST OF SYMBOLS

n	Number of Simulations
μ	Mean
σ	Standard Deviation
t'	Random Normal Deviate
ΔR	Displacement Distance
Δr	Equivalent Displacement Distance
T	Total Mission Time
m	Mass of Space Vehicle
m_s	Mass of Space Vehicle Excluding Fuel
m_{pr}	Mass of Fuel at Injection
M	Momentum of Space Vehicle
t	Time From Injection Time
ΔM	Momentum Correction
ΔMT	Total Momentum Correction
L'	Characteristic Chamber Length
T_{c1}	Combustion Chamber Temperature
k	Ratio of Specific Heats
P_{c1}	Combustion Chamber Pressure
F	Thrust
R	Gas Constant
g	Acceleration of Gravity

Δm	Mass of Fuel Consumed Per Momentum Correction (Excluding Inefficiencies)
Δw	Weight of Fuel Consumed Per Momentum Correction (Excluding Inefficiencies)
Δm_L	Mass of Fuel Lost Per Momentum Correction (Excluding Inefficiencies)
$\Delta m T$	Mass of Fuel Consumed Per Momentum Correction

Subscripts:

i	Correction Reference Number (i = 1, 2, 3 ---)
z	Coordinate Perpendicular to Reference Trajectory
x	Coordinate Perpendicular to Reference Trajectory
y	Coordinate Parallel to Reference Trajectory
I	At Time of Injection into Reference Trajectory
L	At Time of Last Correction
c	Corrected Value
e	Error Value
c1	Combustion Chamber (Steady State)
M	Magnitude
D	Direction

CHAPTER I

INTRODUCTION

In analysing midcourse guidance for an interplanetary mission, it is recognized that simple ballistic trajectories will not meet with success. A study by Ehricke (1) indicates that the accuracy required of the cut off velocity at launch to result in a position error at the target planet of less than several thousand miles, appears technically unfeasible. It is therefore reasonable to expect that interplanetary space vehicles will be equipped with guidance systems which will initiate trajectory corrections in-route.

For investigating interplantary guidance problems, it is convenient to divide the flight path into three phases (i.e., launch guidance, midcourse guidance, and capture guidance). The purpose of escape guidance is to ensure that the space vehicle is at the correct position for injection into the midcourse trajectory. Midcourse guidance is to ensure a successful rendezvous with a desired target planet in space, and approach guidance is to execute a program of highly accurate maneuvers in the immediate vicinity of the planet.

There has been considerable discussion in literature on the midcourse guidance problem by Lawden (2), Ehricke (1), Fridlander-Harry (3), Coffee (4), and others. Any interplanetary trajectory that a space vehicle achieves is influenced measurably by the cutoff momentum vector at injection into the midcourse trajectory. This measurable influence occurs because any error occuring at launch, due to the large length and travel time associated with the midcourse phase, will cause a large miss distance at the target planet.

To have a successful mission, all deviations from the reference trajectory must be corrected at a future time. In practicality though, both the measurement of the deviation and the application of the correction will include errors. Therefore, it can be reasoned that a single correction will not be satisfactory and a number of corrections will be necessary to arrive at the target planet.

Purpose of Study

Considering the economics and practicability of an interplanetary mission, it may be realized that for every pound of payload placed outside the earths atmosphere, several thousand pounds of rocket booster and equipment are needed. This restricts the size of payload which can be injected into space. Nevertheless, the objective is to inject the largest possible payload. This can be achieved by eliminating all portions of the payload that are unnecessary in the attainment of a successful mission.

Since it is unlikely that a space vehicle can make an interplanetary mission without the need for corrections, a necessary amount of fuel must be carried by the space vehicle to make such corrections. This leads to the determination of the minimum cost amount of fuel needed to make corrections. If too much fuel is carried, the excess fuel may be considered dead weight; a weight that could have been used to carry useful equipment. On the other hand, if the space vehicle runs out of fuel just before it has reached its terminating point, the mission will probably fail. This could cost loss of lives if the space vehicle is manned. Consequently, it is desirable that the space vehicle carry sufficient correction fuel to complete the mission with a minimum cost probability. This may be achieved by jointly minimizing the sum of the cost of carrying fuel and the risk of a mission failure. This leads to the purpose of this study.

The purpose of this study is to define the magnitude of correction that will require the minimum amount of fuel utilized by the space vehicle to complete a successful interplanetary mission. In addition, a conceptual trade off scheme will be presented for determining the minimum cost amount of correction fuel to be provided for on board.

Plan of Development

This study considers a space vehicle proceeding on an interplanetary mission between Earth and Mars. The probable transfer orbit is a half ellipse between Earth orbit perigee and Mars orbit apogee (Figure I-1). This transfer orbit is defined as the reference trajectory from which the guidance system measures space vehicle trajectory deviations. To simplify guidance equations, the reference trajectory is considered a straight line with length equal to the half ellipse transfer orbit.

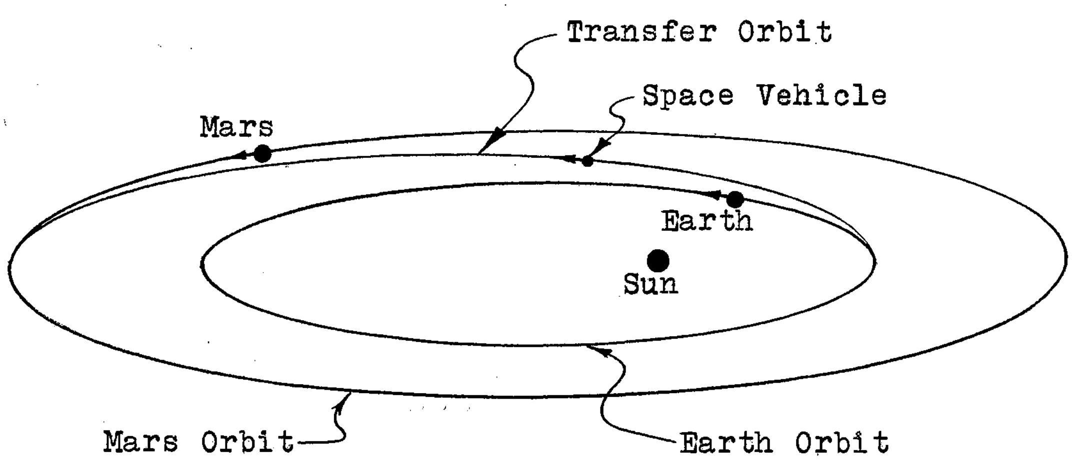


Figure I-1 Half Ellipse Transfer Orbit Between Earth and Mars Orbits.

For purposes of clarifying the development, the derivation of guidance equations is divided into two cases. Case one involves the space vehicle in two dimensions of freedom. This requires that the space vehicle have the ability to detect, measure, and correct for trajectory deviations occuring in a plane that contains the reference trajectory and a line perpendicular to the reference trajectory. In a x-y-z dimensional space, the dimensions considered are the y and z dimensions.

Case two involves the space vehicle in three dimensions of freedom. This requires that the space vehicle have the ability to detect, measure, and correct for trajectory deviations occuring in all three dimensions of space. In a three dimensional space, the dimensions considered are the x, y, and z dimensions.

The guidance equations are derived from the time, distance, and geometrical constraints of the interplanetary mission. From inevitable inaccuracies of launching, the path of the space vehicle will be observed to deviate from its precalculated reference trajectory. To correct for this deviation, the total momentum vector of the space vehicle is altered by a momentum correction. If no correction is made and the errors are allowed to go uncorrected, the space vehicle will experience a miss distance at the target planet. This miss distance is related to the magnitude, direction, and time interval between the time at which the errors occured and the time at which the space vehicle is specified to arrive at the target planet.

By applying a momentum correction in a calculated direction and at a calculated time, the miss distance at the target planet can be reduced to zero. Arbitrarily, all momentum corrections during the interplanetary mission are of constant magnitude. After each momentum correction, due to inevitable inaccuracies in the momentum correction application, the path of the space vehicle will deviate from its necessary zero miss distance trajectory and another correction will be needed to reduce the new miss distance at the target planet to zero. The space vehicle will not make a correction until the deviation from the reference trajectory is sufficient to require the specified momentum correction. All corrections are made between time of injection into midcourse phase and one day before target planet capture. After the final correction, the space vehicle is entirely under capture phase guidance and is beyond the scope of this study.

Each correction requires a specified cost of fuel depending on the magnitude of correction. All corrections made at time of injection are defined as initial cost. All corrections made during the midcourse phase of the mission are defined as midcourse cost and all corrections made at one day before target planet capture are defined as final correction cost. The total cost of fuel expended by the space vehicle to make corrections during the mission is the summation of initial cost, midcourse cost, and final cost.

The Monte Carlo technique is utilized to obtain estimates of cost of fuel consumed in making corrections during the interplanetary mission. This technique consists of the unrestricted random sampling of values from defined probability distributions and the mathematical manipulation of these values in accordance with the system logic.

Computer programs, results, and discussion of results for the two dimensional case are presented at the end of this study. Results for the three dimensional case were not obtained because the large amount of computer time needed.

CHAPTER II

TWO DIMENSIONAL GUIDANCE EQUATIONS

The objective of this chapter is to develop guidance equations for the space vehicle proceeding on an interplanetary mission. The motion of the space vehicle is constrained to two dimensions of freedom. All expressions are developed in terms of momentum.

Space Vehicle in Two Dimensions of Freedom

A space vehicle is considered which contains a guidance system with the purpose of detecting, measuring, and calculating changes in the momentum vector and trajectory of the space vehicle as they occur. The reference trajectory from which changes are measured is considered a straight line of length equal to an half ellipse transfer orbit between the Earth and Mars orbits (Figure II-1).

The guidance equations are derived from the time, distance, and geometrical constraints of the interplanetary mission. Because of inevitable inaccuracies of launching, the path of the space vehicle will be observed to deviate from its precalculated trajectory. If no correction is made for this deviation, the space vehicle will experience a miss distance at the target planet. This miss distance

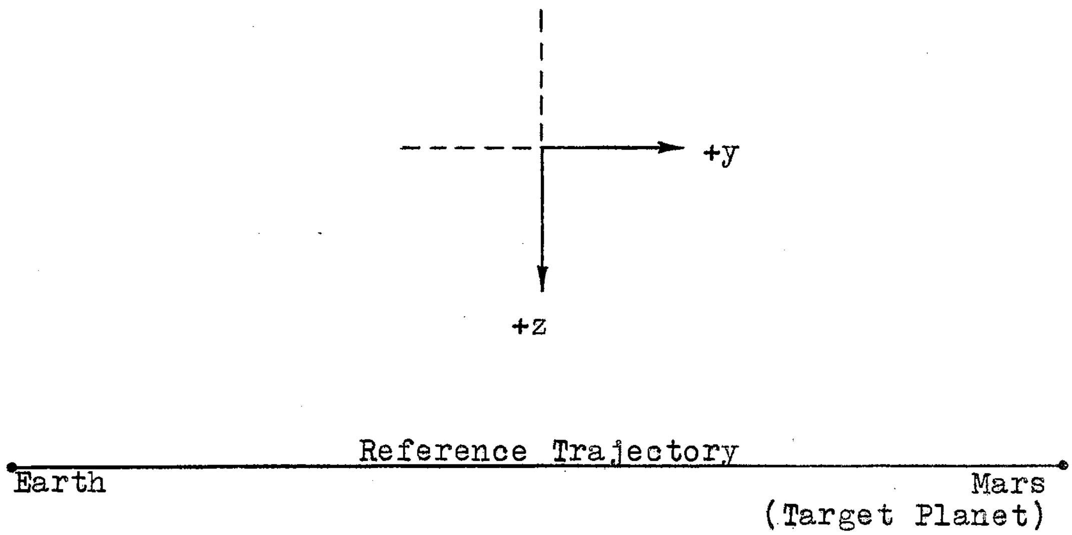


Figure II-1 Assumed Reference Trajectory

is related to the magnitude and direction of the momentum vector of the space vehicle and also to the time interval between the time at which the errors occured and the time at which the space vehicle is to arrive at the target planet. This leads to applying a specified momentum correction in a calculated direction and at a calculated time such that the miss distance at the target planet is reduced to zero. After each momentum correction, due to inevitable inaccuracies in applying the momentum correction, the path of the space vehicle will again deviate from its necessary zero miss distance trajectory and another correction will be needed.

Momentum Correction Perpendicular to Reference Trajectory in z-y Dimensions

The momentum correction in the z-y dimensions perpendicular to the reference trajectory, necessary to allow the space vehicle to converge upon the precalculated position of the target planet at time T, may be derived by considering the displacement distance at the target planet. If these errors are produced at time t_{i-1} and are allowed to continue uncorrected for the time interval $T-t_{i-1}$, there is produced at the target planet a displacement distance of amount ΔR_z. This displacement distance is the displacement

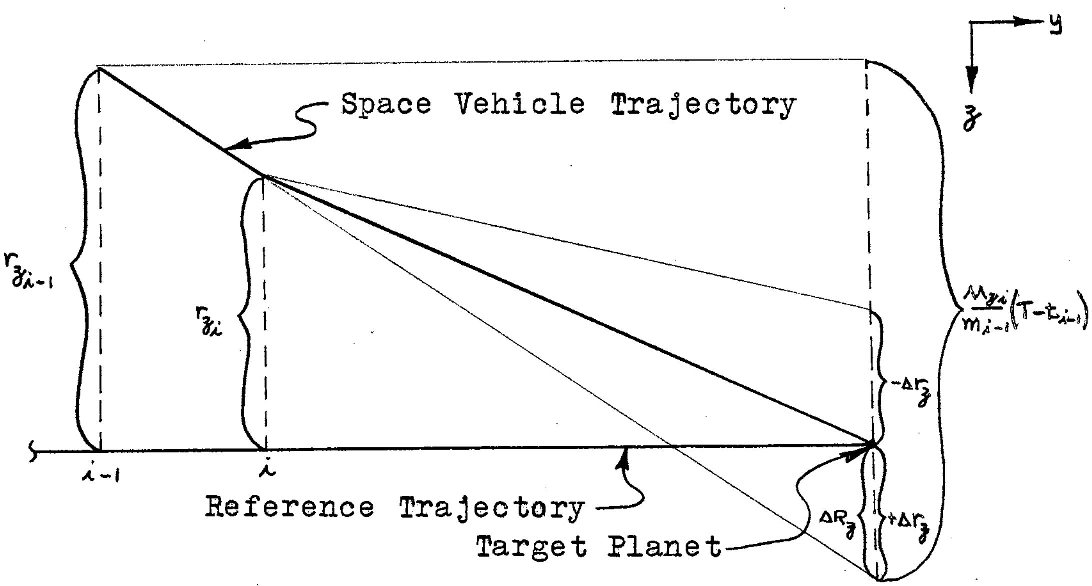


Figure II-2 Momentum Correction in z-y Dimensions

that occurs in the time interval $(T-t_{i-1})$ due to the x-component of the space vehicle momentum vector minus the position error of the space vehicle with respect to the reference trajectory at time t_{i-1}. In equation form, this displacement distance (Figure II-2) is

$$\Delta R_z = \frac{M_{z_i}}{m_{i-1}}(T-t_{i-1}) - r_{z_{i-1}}.$$

Since it is desirable to reduce this displacement distance at the target to zero, a momentum correction should be applied. This correction must be of sufficient magnitude in the z-direction so as to allow the space vehicle to converge upon the target planet from the position the vehicle occupied at time of correction. Applying this correction and assuming the correction to be applied in the z-direction, the displacement distance reduces to

$$\Delta R_z = \frac{M_{z_i}}{m_{i-1}}(T-t_{i-1}) - r_{z_{i-1}} + \frac{\Delta M_{z_i}+M_{cz_i}}{m_i}(T-t_i) - r_{z_i} = 0.$$

The several terms in the preceding equation are identified as

$$\frac{M_{z_i}}{m_{i-1}}(T-t_{i-1}) - r_{z_{i-1}} = +\Delta r_z,$$

$$\left|\frac{M_{cz_i}}{m_i}(T-t_i)\right| = \left|- r_{z_i}\right|,$$

$$\frac{\Delta M_{z_i}}{m_i}(T-t_i) = -\Delta r_z.$$

The expression for the momentum correction in the z-direction is

$$\Delta M_{z_i} = r_{z_{i-1}} \frac{m_i}{(T-t_i)} - M_{z_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i}.$$

This is the required momentum correction in the z-direction which is necessary to allow the space vehicle to converge upon the target planet.

Another representation for the momentum correction perpendicular to the reference trajectory is obtainable by considering the momentum error in the z-direction after the t_{i-1} correction.

With reference to Figure II-3, the z-momentum error is

$$M_{ez_i} = M_{z_i} - M_{cz_{i-1}}.$$

As a result of this error, the displacement distance at the target planet is (Figure II-4)

$$\Delta R_z = \frac{M_{ez_i}}{m_{i-1}}(T-t_{i-1}).$$

A momentum correction of sufficient magnitude is applied perpendicular to the reference trajectory at time t_i to reduce the displacement distance to zero. The condition for this is

$$\Delta R_z = \frac{M_{ez_i}}{m_{i-1}}(T-t_{i-1}) + \frac{\Delta M_{z_i}}{m_i}(T-t_i) = 0.$$

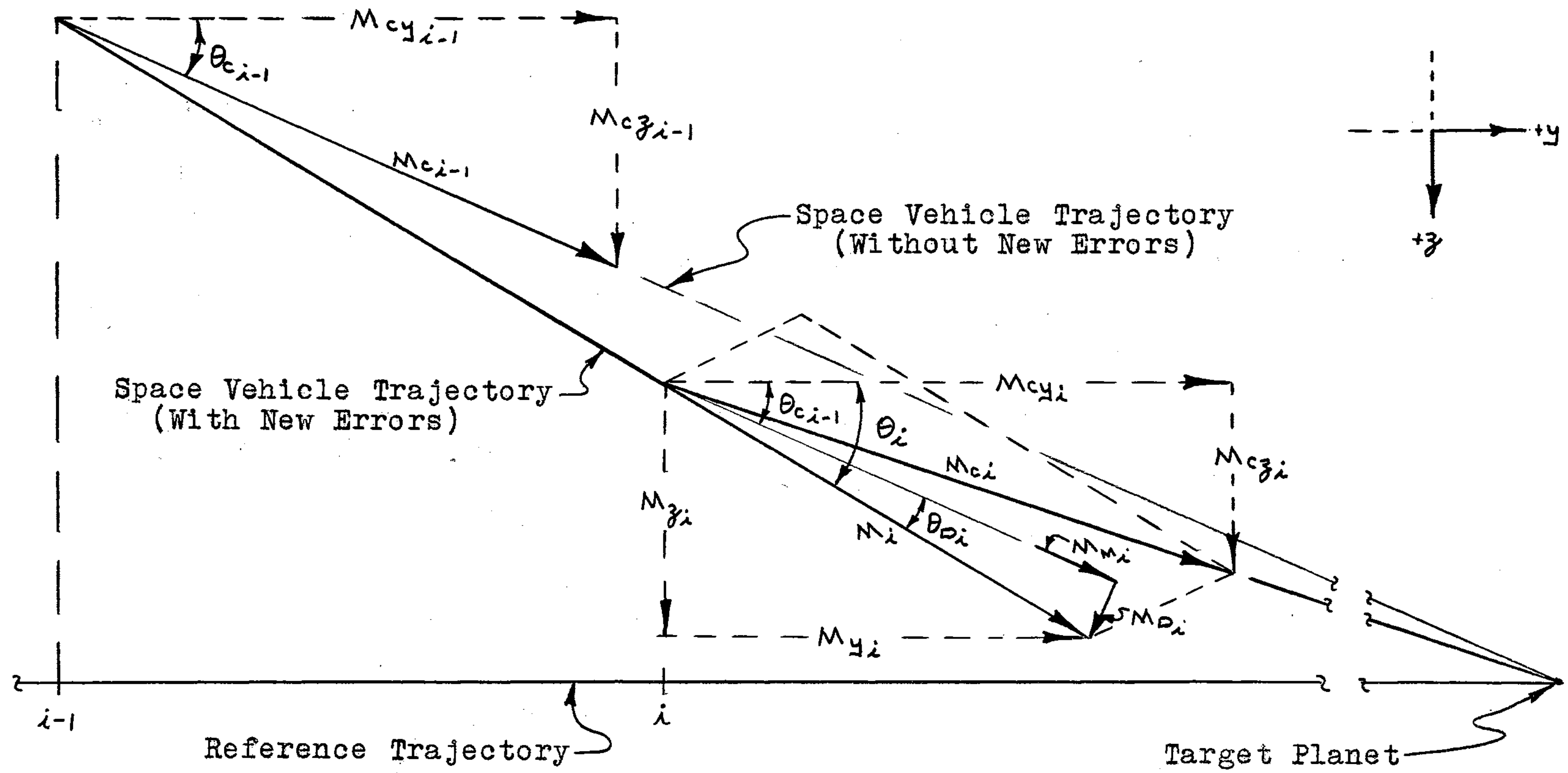


Figure II-3 Geometric Relation Between Momentum Components (z-y Dimensions)

Each portion of the above equation may be recognized as,

$$\frac{M_{ez_i}}{m_{i-1}}(T-t_{i-1}) = +\Delta r_z,$$

$$\frac{\Delta M_{z_i}}{m_{i-1}}(T-t_i) = -\Delta r_z,$$

which when summed will produce a zero displacement distance

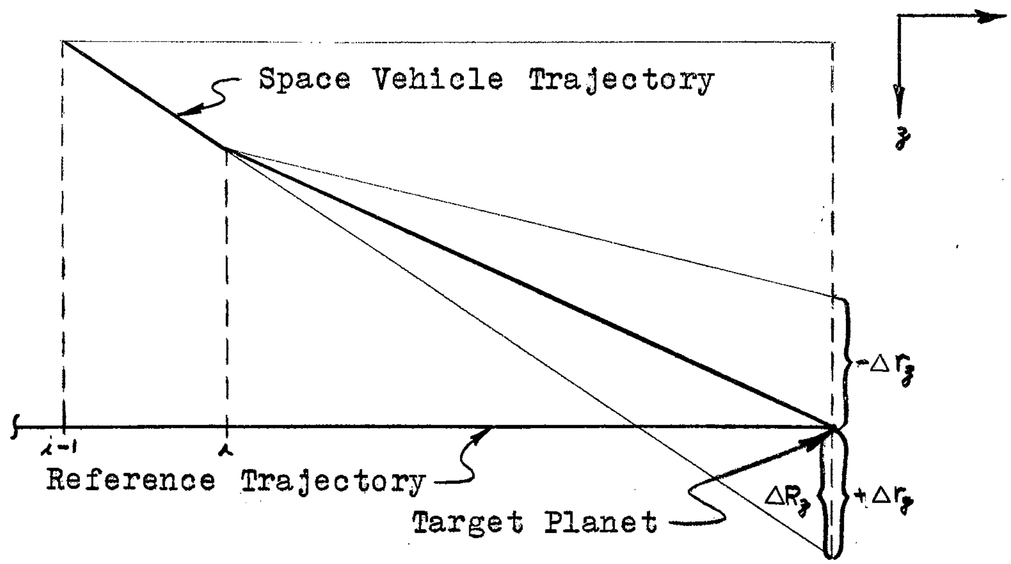


Figure II-4 Momentum Correction in z-y Dimensions

at the target planet. This leads to the solution for the required momentum correction in the z-direction as

$$\Delta M_{z_i} = - M_{ez_i}\frac{m_i}{m_{i-1}}\frac{T-t_{i-1}}{T-t_i} .$$

This is the required momentum correction in the z-direction which is necessary to allow the space vehicle to converge upon the target planet.

The equivalence of the two representations for the z-component momentum correction can be seen by equating the displacement distance expressions and analyzing the results. Methods one and two correspond to first and latter presentations respectively.

$$\Delta R_z(\text{Method 1}) = \Delta R_z(\text{Method 2})$$

$$-r_{z_{i-1}} + \frac{M_{z_i}}{m_{i-1}}(T-t_{i-1}) + \frac{\Delta M_{z_i}}{m_i}(T-t_i) = +\frac{M_{e\,z_i}}{m_{i-1}}(T-t_{i-1}) + \frac{\Delta M_{z_i}}{m_i}(T-t_i)$$

Rewrite the above equation as

$$a + b + c = d + e$$

where

$$a = - r_{z_{i-1}},$$

$$b = \frac{M_{z_i}}{m_{i-1}}(T-t_{i-1}),$$

$$c = \frac{\Delta M_{z_i}}{m_i}(T-t_i),$$

$$d = \frac{M_{e\,z_i}}{m_{i-1}}(T-t_{i-1}),$$

$$e = \frac{\Delta M_{z_i}}{m_i}(T-t_i).$$

Portions a, b, c, d, and e of the above equation are shown in Figure II-5. ΔM_{z_i} was assumed to be applied in the positive direction to conform with positive sign

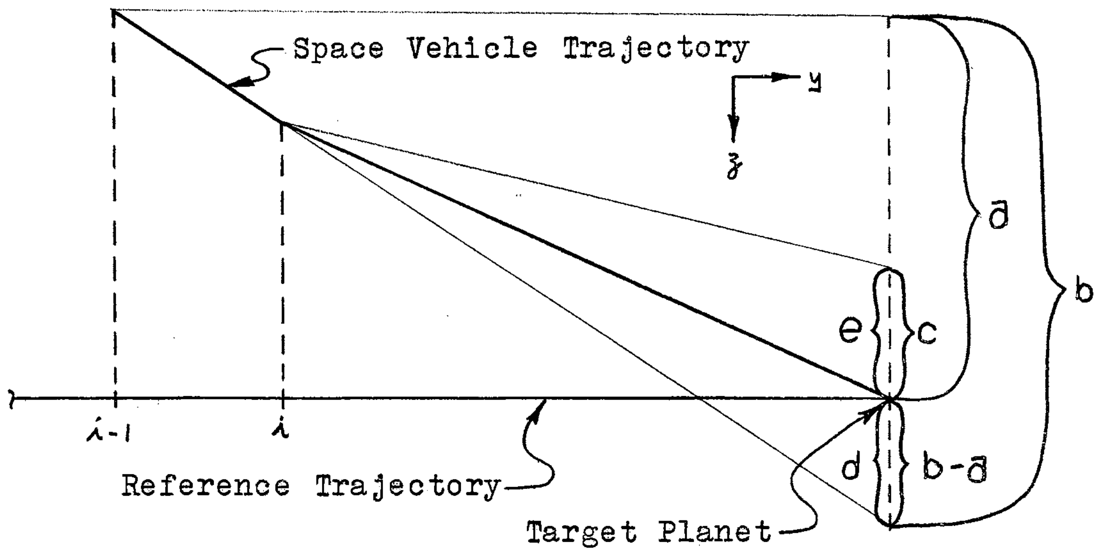


Figure II-5 Momentum Correction in z-y Dimensions

convention but will take on negative values when errors are assumed positive in direction. This can be deduced by noting c and e are shown positive in equation but negative in Figure II-5. In this study, the latter method for deriving the required momentum correction will be employed.

Momentum Correction Paralled to

Reference Trajectory in z-y Dimensions

The momentum correction parallel to the reference trajectory necessary to allow the space vehicle to converge upon the precalculated position of the target planet at time T is derived by considering the error at time t_{i-1}. If this error is left uncorrected for the time interval $T-t_{i-1}$, there is produce at the target planet a displacement distance of amount ΔR_y. This resulting displacement distance is expressed as

$$\Delta R_y = \frac{M_{ey_i}}{m_{i-1}}(T-t_{i-1}),$$

where the momentum error parallel to reference trajectory is

$$M_{ey_i} = M_{y_i} - M_{cy_{i-1}}.$$

A parallel momentum correction of sufficient magnitude is applied at time t_i to reduce the displacement distance at the target planet to zero. The condition for this is

$$\Delta R_y = \frac{M_{ey_i}}{m_{i-1}}(T-t_{i-1}) + \frac{\Delta M_{y_i}}{m_i}(T-t_i) = 0.$$

So that the parallel momentum correction becomes

$$\Delta M_{y_i} = - M_{ey_i}\, \frac{m_i}{m_{i-1}}\, \frac{T-t_{i-1}}{T-t_i}.$$

Therefore, the expression shown above is the momentum correction parallel to the reference trajectory necessary to allow the space vehicle to converge upon the target planet at time T.

When to Make Correction

In this study, the specified momentum correction (ΔM) is a fixed quantity and is held constant thoughout the mission. The space vehicle will not make a correction until the deviation from the reference trajectory is sufficient to require the specified momentum correction. This time (t_i) at which the specified momentum correction should be applied is derived from the perpendicular and parallel momentum corrections as follows:

$$\Delta M_i = \sqrt{\Delta M_{z_i}^2 + \Delta M_{y_i}^2},$$

$$\Delta M_i^2 = \Delta M_{z_i}^2 + \Delta M_{y_i}^2,$$

or

$$\Delta M_i^2 = \left[- M_{ez_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i}\right]^2 + \left[- M_{ey_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i}\right]^2,$$

$$\Delta M_i^2 = \frac{1}{(T-t_i)^2}(A^2 + B^2),$$

where

$$A = - M_{ez_i} \frac{m_i}{m_{i-1}}(T-t_{i-1}),$$

$$B = - M_{ey_i} \frac{m_i}{m_{i-1}}(T-t_{i-1}),$$

so that

$$(T-t_i)^2 = \frac{1}{\Delta M_i^2}(A^2 + B^2),$$

$$T - t_i = \frac{1}{\Delta M_i}\sqrt{(A^2 + B^2)},$$

$$t_i = T - \frac{1}{\Delta M_i}\sqrt{A^2 + B^2}.$$

This time t_i is the time that the momentum correction should be made.

The Corrected Momentum Vector

At time t_i, the correction momentum vector (ΔM) is applied by the propulsion system and the space vehicle supposedly proceeds after time t_i with the new corrected momentum vector. This corrected momentum vector is of the required magnitude and direction to allow the space vehicle to converge upon the target planet with zero displacement distance. An expression for the corrected momentum vector may be obtained by addition and substraction of the correction momentum vector (ΔM) with the momentum vector of the space vehicle that existed at time t_i before the momentum correction was applied.

With the aid of Figure II-6, the corrected momentum vector of space vehicle parallel to reference trajectory is

$$M_{cy_i} = M_{y_i} + \Delta M_{y_i},$$

and the corrected momentum vector perpendicular to reference trajectory is

$$M_{cz_i} = M_{z_i} + \Delta M_{z_i}.$$

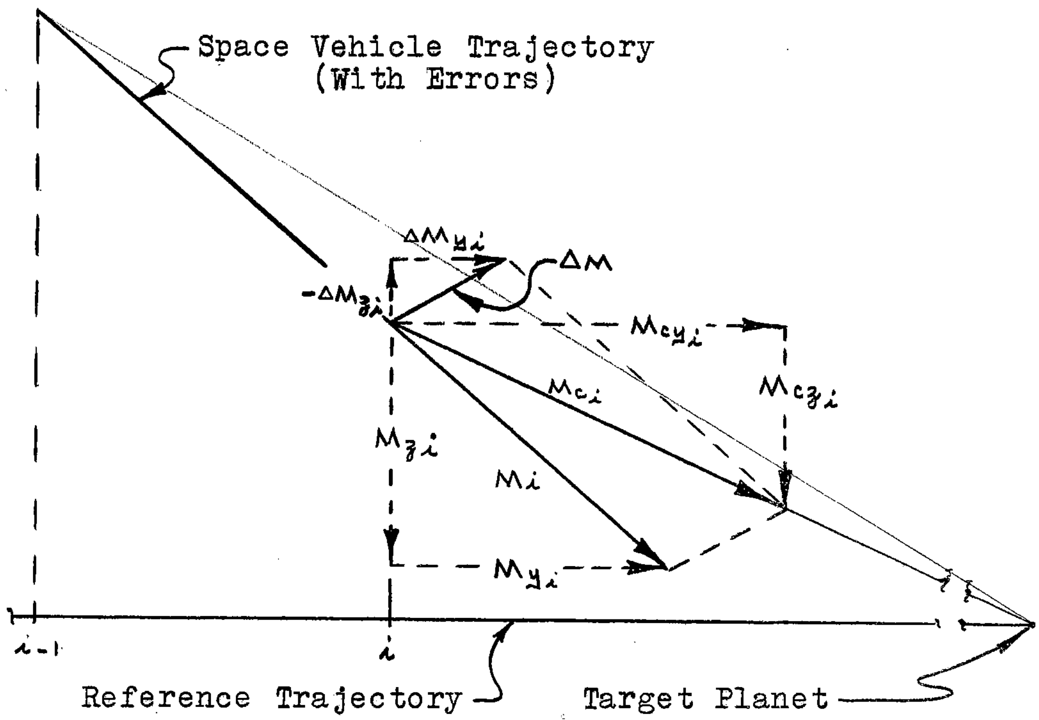


Figure II-6 Momentum Correction and Components

The total corrected momentum vector of the space vehicle after the momentum correction has been applied is,

$$M_{c_i} = \sqrt{(M_{cy_i})^2 + (M_{cz_i})^2}\,.$$

The geometric relations between momentum components are shown in Figure II-3.

CHAPTER III

THREE DIMENSIONAL GUIDANCE EQUATIONS

In this chapter, derivation of guidance equations for the space vehicle in three dimensions of freedom is undertaken. All expressions are developed in terms of momentum. In the accomplishment of this task, expressions from the previous chapter are utilized.

Space Vehicle in Three Dimensions of Freedom

In analyzing the three dimensional case, one may use the equations derived for the two dimensional case and derive the extra necessary equations to satisfy the three dimensional case. This can be achieved by reconsidering the two dimensional frame of reference (y-z dimensions) and adding a third dimension (x) as shown in Figure III-1.

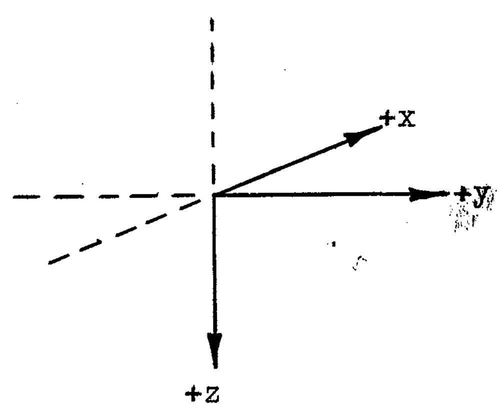


Figure III-1 Three Dimensional Frame of Reference

Instead of restricting the motions of the space vehicle to two dimensions, y and z, the space vehicle will now be free to move in three dimensional space, x, y, and z. This will allow the guidance system of the space vehicle to detect and correct for errors in all three dimensions. With this ability the simulation of the guidance complex will be more realistic which, in return, will increase the ability and validity of the model to investigate the problem at hand.

Momentum Correction Perpendicular to Reference Trajectory in x-y Dimensions

The momentum correction perpendicular to the reference trajectory (in x-y dimensions) that is necessary to allow the space vehicle to converge upon the precalculated position of the target planet at time T, may be derived by considering the displacement distance at the target planet caused by errors. If the errors are produced at time t_{i-1} and are allowed to continue uncorrected for the time interval $T-t_{i-1}$, there will be produced at the target a displacement distance of amount ΔR_x. This displacement distance can be determined from the x-momentum error after the t_{i-1} correction. The x-component of momentum error (Figure III-2) is

$$M_{ex_i} = M_{x_i} - M_{cx_{i-1}}.$$

Because of this error, the displacement distance at the target is (see Figure III-3),

$$\Delta R_x = \frac{M_{ex_i}}{m_{i-1}}(T-t_{i-1}).$$

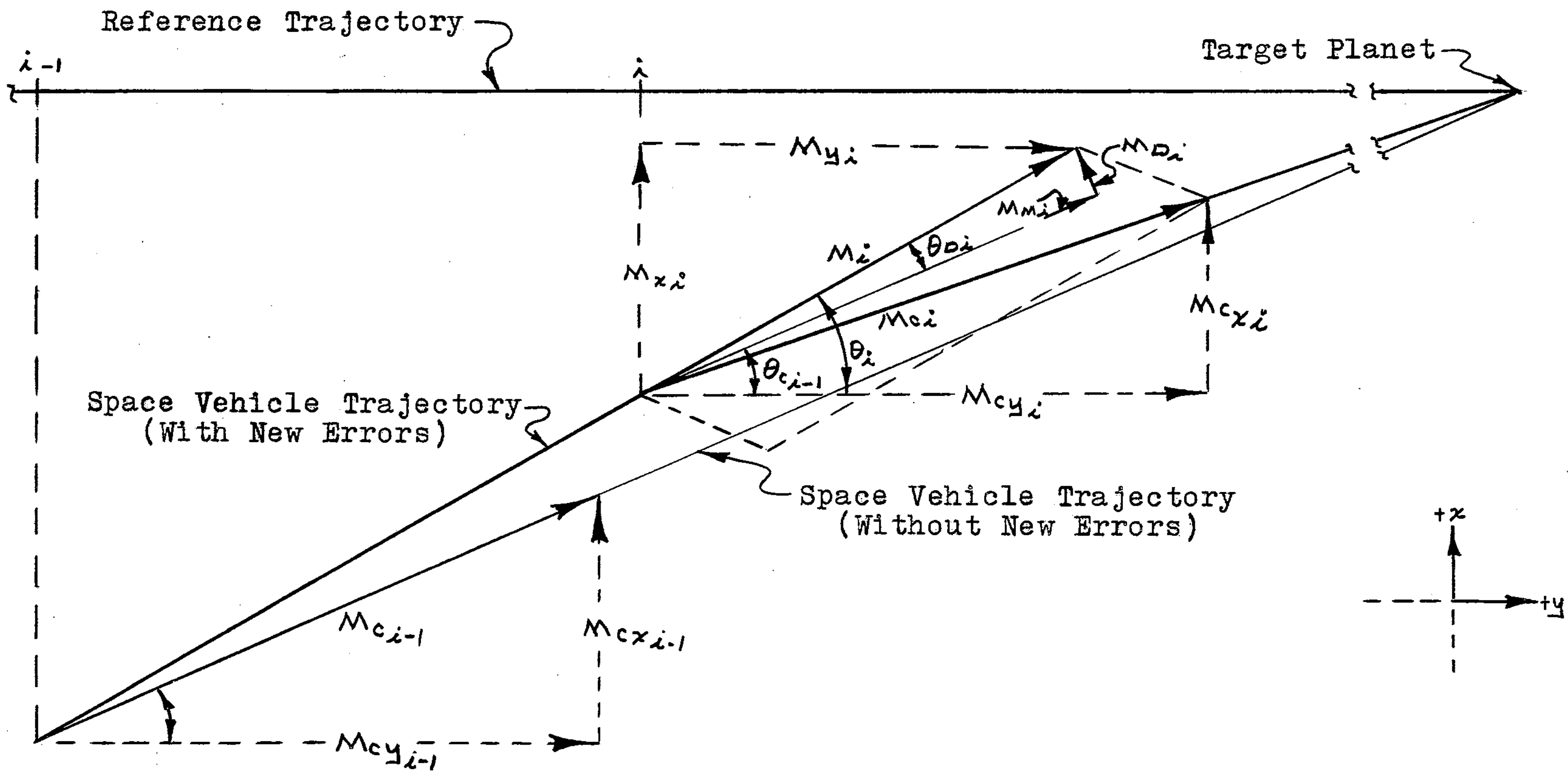


Figure III-2 Geometric Relation Between Momentum Components (x-y Dimensions)

Since it is desirable to reduce the displacement distance at the target to zero, a momentum correction may be applied at

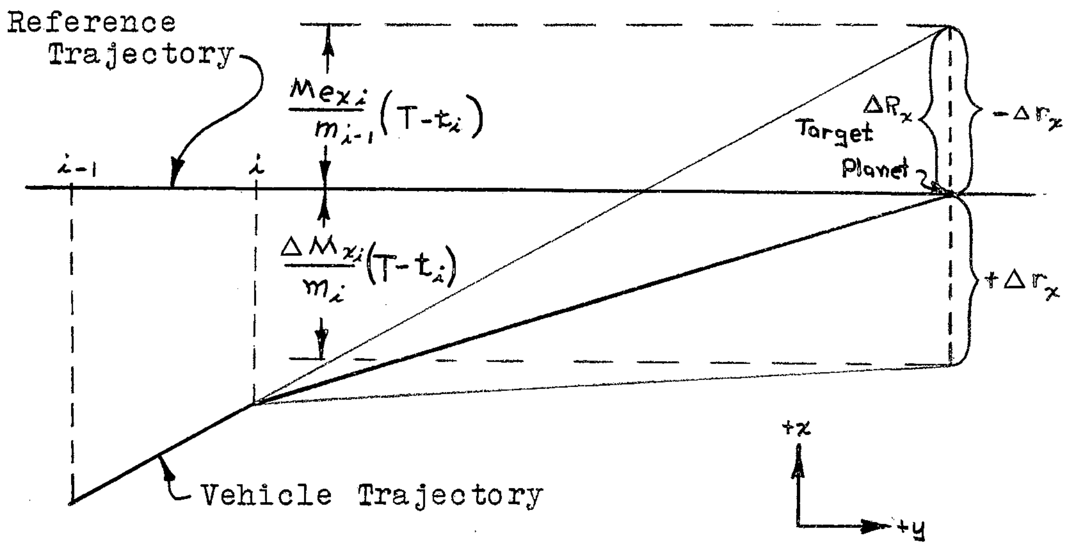


Figure III-3 Momentum Correction in x-y Dimensions

time t_i of sufficient magnitude in the x-direction so as to allow the space vehicle to converge upon the target at time T. After application of this correction and proper recognition of the sign convention, the displacement distance equation becomes

$$\Delta R_x = \frac{M_{ex_i}}{m_{i-1}}(T-t_{i-1}) + \frac{\Delta M_{x_i}}{m_i}(T-t_i) = 0.$$

Each part of the equation can be identified as

$$\frac{M_{ex_i}}{m_{i-1}}(T-t_{i-1}) = + \Delta r_x,$$

$$\frac{\Delta M_{x_i}}{m_i}(T-t_i) = -\Delta r_x,$$

where ΔM_{x_i} conforms with positive sign convention. Their sum produces a zero displacement distance at the target. From this condition, the equation yields a solution for the momentum correction,

$$\Delta M_{x_i} = - M_{ex_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i} .$$

The expression shown above is the momentum correction perpendicular to the reference trajectory (x-dimension) necessary to allow the space vehicle to converge upon the target planet at time T.

Momentum Correction Parallel to Reference Trajectory in x-y Dimensions

In correcting errors in momentum parallel to the reference trajectory, the y-component of the momentum error induced from the x-y dimensions is considered. Since the momentum error paralled to the reference trajectory in the z-y dimensions have already been considered from the two dimensional problem, it is only necessary to derive an expression for the y-component of momentum caused by the x-dimension being added, and add this result to the previously derived y-component correction (Chapter II).

The equation for the displacement distance parallel to the reference trajectory at the target planet,

$$\Delta R_y = \frac{M_{ey_i}}{m_{i-1}}(T-t_{i-1}) + \frac{\Delta M_{y_i}}{m_i}(T-t_i) = 0,$$

results in a parallel momentum correction of magnitude,

$$\Delta M_{y_i} = - M_{ey_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i} ,$$

at time t_i, to reduce the displacement distance at the target planet to zero.

The total momentum correction parallel to the reference trajectory for the space vehicle in three dimensional space, is the sum of the momentum corrections in the z-y and x-y dimensions.

$$\Delta M_{y_i}(\text{Total}) = \Delta M_{y_i}(\text{z-y plane}) + \Delta M_{y_i}(\text{x-y plane})$$

Substitution of appropriate expressions, the parallel momentum correction is

$$\Delta M_{y_i}(\text{Total}) = \left[-M_{ey_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i}\right]_{z-y} + \left[-M_{ey_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i}\right]_{x-y}$$

$$= - \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i} \left[(M_{ey_i})_{z-y} + (M_{ey_i})_{x-y}\right].$$

This is the total momentum correction parallel to the reference trajectory necessary to allow the space vehicle to converge upon the target planet at time T.

When to Make Correction

The time at which the specified momentum correction should be applied can be derived as it was for the two dimensional case. The difference is that the momentum correction in the x-dimension is considered. The total momentum correction becomes

$$\Delta M_i = \sqrt{\Delta M_{z_i}^2 + \Delta M_{y_i}^2 + \Delta M_{x_i}^2},$$

as shown in Figure III-4.

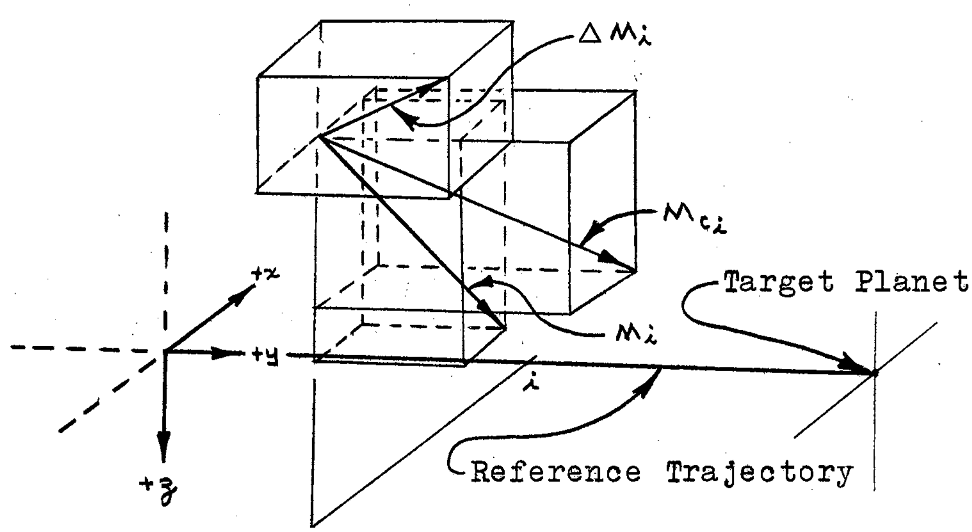


Figure III-4 Momentum Correction Vector

Substitution for ΔM_{z_i}, ΔM_{y_i}, and ΔM_{x_i} leads to the following relationships:

$$\Delta M_i^2 = \left[- M_{ex_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i} \right]^2$$

$$+ \left[- \left\{ (M_{ey_i})x\text{-}y + (M_{ey_i})z\text{-}y \right\} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i} \right]^2$$

$$+ \left[- M_{ex_i} \frac{m_i}{m_{i-1}} \frac{T-t_{i-1}}{T-t_i} \right]^2$$

$$\Delta M_i^2 = \frac{1}{(T-t_i)^2}(A^2 + B^2 + C^2),$$

where

$$A = - M_{ez_i} \frac{m_i}{m_{i-1}}(T-t_{i-1}),$$

$$B = - \left(M_{ey_i}\Big|_{x-y} + M_{ey_i}\Big|_{z-y}\right)\frac{m_i}{m_{i-1}}(T-t_{i-1}),$$

$$C = - M_{ex_i} \frac{m_i}{m_{i-1}}(T-t_{i-1}),$$

and

$$(T-t_i)^2 = \frac{1}{\Delta M_i^2}(A^2 + B^2 + C^2),$$

$$T-t_i = \frac{1}{\Delta M_i} \sqrt{A^2 + B^2 + C^2},$$

$$t_i = T - \frac{1}{\Delta M_i}\sqrt{A^2 + B^2 + C^2}.$$

Time t_i is the time that the momentum correction should be applied.

The Corrected Momentum Vector

As was explained earlier, at time t_i, the correction momentum vector (ΔM) is applied by the propulsion system and the space vehicle proceeds after time t_i with the new corrected momentum vector. This corrected momentum vector is of the required magnitude and direction to allow the space vehicle to converge upon the target planet with zero displacement distance. An expression for the corrected momentum vector may be derived by addition and substraction of the correction momentum vector with the momentum vector of the space vehicle that existed at time t_i before the correction was applied.

With the aid of Figure III-5, the x, y, and z momentum corrections are

$$M_{cy_i} = M_{y_i} + \Delta M_{y_i},$$

$$M_{cz_i} = M_{z_i} + \Delta M_{z_i},$$

$$M_{cx_i} = M_{x_i} + \Delta M_{x_i}.$$

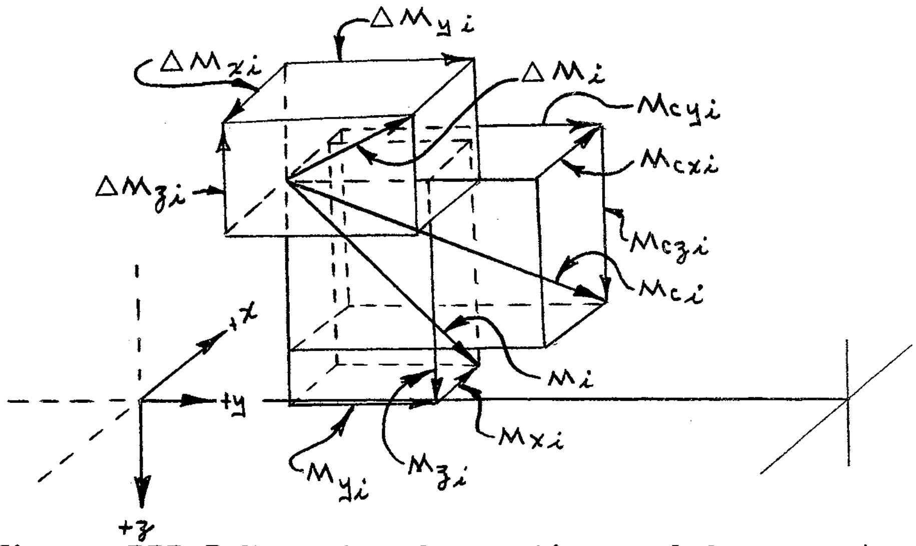


Figure III-5 Momentum Correction and Components

The total magnitude of the corrected momentum vector is

$$M_{c_i} = \sqrt{(M_{cy_i})^2 + (M_{cy_i})^2 + (M_{cx_i})^2}.$$

The geometric relations between momentum components are shown in Figure III-2 for the x-y dimensions and Figure II-3 for the z-y dimensions.

CHAPTER IV

THE SYSTEM SIMULATION

The system simulation under development can be accepted as valid only if it succeeds in accurately representing the guidance of the space vehicle on an interplanetary mission. Therefore, this chapter will assume the task of developing an accurate system simulation.

Development of System Logic

As the space vehicle proceeds on the interplanetary trajectory, the guidance system continuously senses for deviations in the trajectory of the space vehicle. When an error in the trajectory is detected, the guidance system measures the error and proceeds to compute the magnitude of correction required to allow the space vehicle to intercept the target planet. Since the magnitude of correction is held constant throughout each mission, the guidance system will compute the time (t_1) at which the deviation from the desired trajectory is sufficient to require the total magnitude of the correction. At this time (t_1), the guidance system will signal the propulsion system to make the specified momentum correction in a prescribed direction. The propulsion system will then orient itself in the prescribed direction and make the correction.

After each correction, inevitable errors will again be detected and if they are not corrected, the space vehicle will deviate from the corrected trajectory. In eliminating the new error, another correction is applied to reduce the new space vehicle miss distance at the target planet to zero. This procedure is repeated until the midcourse guidance phase is terminated. The sequence of the above procedure is illustrated in Figure IV-1.

Error Analysis

When classifying what errors are pertinent to this study, the possibilities seem unlimited. The magnitude of error sources within the guidance complex of the space vehicle depend to a large extent on the particular components involved, on the operational requirements, and on the environment.

Two general subsystems are recognized to be error producing in this study. These subsystems are the guidance system and the correction propulsion system (assuming the space vehicle has only the ability to make corrections by propulsion applications). When the guidance system senses an error, it proceeds to measure the error, uses the obtained measurements to compute the necessary corrections (in accordances with guidance equations and system logic) and signals the correction propulsion system to make a specified momentum correction (ΔM) in a calculated direction. The correction

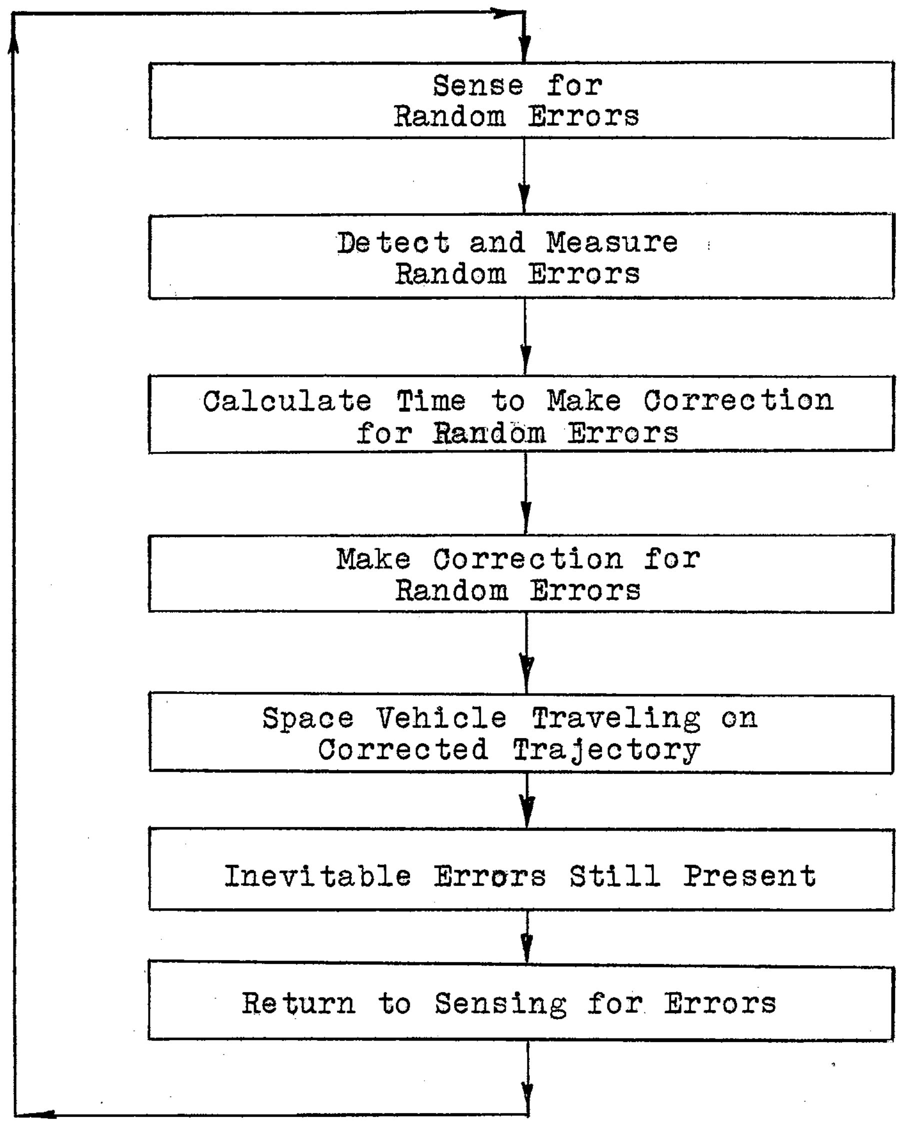


Figure IV-1 Flow Diagram Outlining System Logic propulsion system then proceeds to orient itself in the proper direction and applies the specified magnitude of correction (ΔM). If no errors are present within the system's ability to detect, measure, compute, orient, and apply the momentum correction, the true measured error will be corrected and no future corrections are necessary. All systems found in nature though, are plagued with inherent errors in the form of subsystem tolerances and therefore, the need for another correction is very probable.

Systems composed of subsystems subject to tolerances perform according to the summation of these tolerances. The result is that the over-all system performance follows some random pattern of variation according to a underlying probability distribution.

In classifying the error sources, the system may be analyzed as follows:

1. Guidance system ability to detect errors (sensitivity of gyros and accelerometers).
2. Guidance system ability to measure errors (sensitivity and calibration of gyros and accelerometers).
3. Guidance system ability to compute accurate corrections (clock errors would cause computational inaccuracies).
4. Correction propulsion system ability to orient the momentum correction in the proper direction (sensitivity and calibration

of orienting device).

5. Correction propulsion system ability to apply the correct magnitude of momentum correction (timing and starting and stoping devices).

Obviously as rapid as guidance and space vehicle technologies are progressing, the above stated error sourses would not be complete, but for this study they are satisfactory.

As illustrated in Figure IV-2, any errors induced by the systems in phases 1 and 2 are considered by phase 3 as true values of detected and measured errors. If errors are present in phase 3 also, then the output from phase 3 (considered as output of the guidance system) will contain the true errors detected and measured plus inherent system errors of phases 1, 2, and 3. This output is the input to the correction propulsion system (phases 4 and 5). Since both the orienting and momentum magnitude devices contain supposedly inherent errors, the resulting correction will contain errors from all phases of the guidance and correction propulsion systems. How much error each of the phases contributes to the resulting correction is difficult to specify. It is evident though that the resulting errors in the momentum correction involve direction and magnitude. It follows that if the error distributions for phases 1, 2, and 3 are unknown, the momentum correction may be treated as a function of an over-all directional error distribution

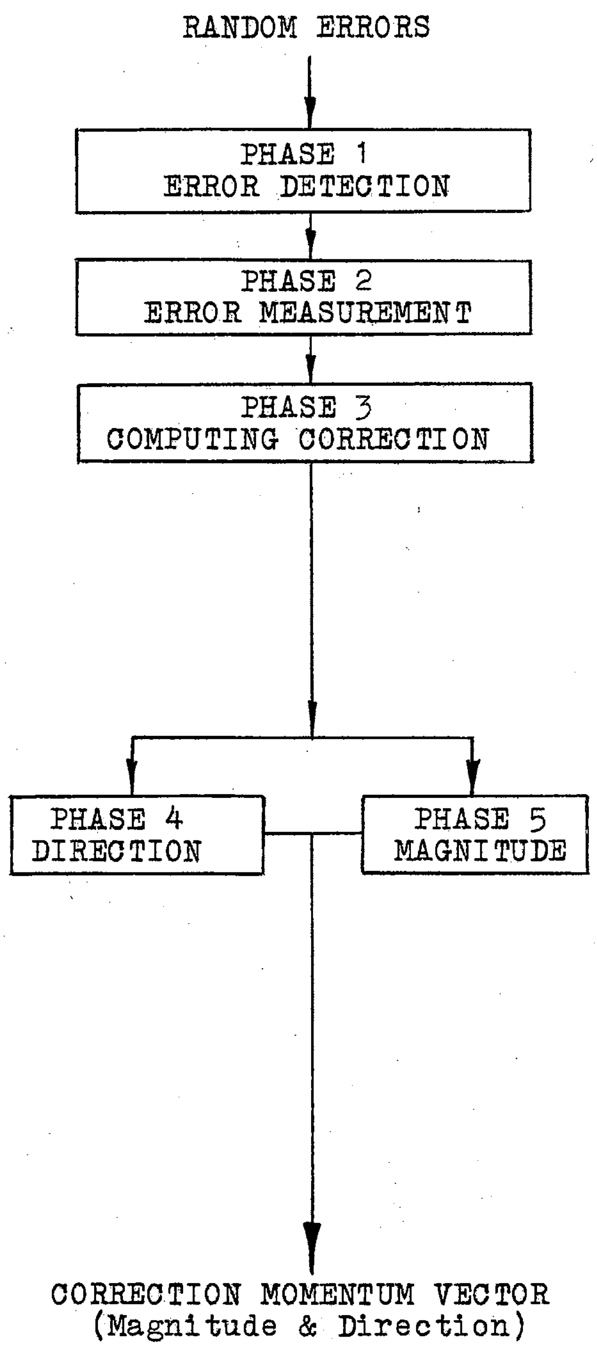


Figure IV-2 Error Flow Diagram

and an over-all correction momentum magnitude error distribution.

In defining the errors in this manner, the error distribution for the directional orienting device (phase 4) is composed of its own errors plus errors from the guidance system (phases 1, 2, and 3). Also, the error distribution of the correction momentum magnitude (phase 5) may be treated in the same manner. Then, the problem of representing the error distributions of the space vehicle subsystems will have been reduced to a simpler form, i.e., two error distributions (magnitude and direction) instead of five.

Since the normal distribution defines the occurrence of a wide variety of phenomena found in nature, it is used to define the error distributions in this study. This normal distribution may be described by the function (5)

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]},$$

and is illustrated in Figure IV-3.

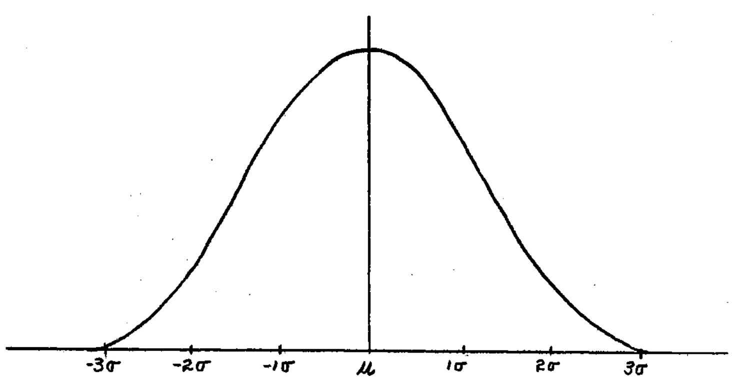


Figure IV-3 Normal Distribution

In applying the normal distribution to the distribution of errors, the mean value (μ) will correspond to zero magnitude error and the standard deviation (σ) as one third the outer most limit of the tolerance. The values $\mu \pm 3\sigma$ define the upper and lower bounds of the variation of x. Performance outside the $\pm 3\sigma$ limits is assumed negligible. This assumption may be made since the probability of such an event occurring is .003.

Analysis of Mass Reduction

The mass of the space vehicle during all phases of the flight may be related to the number of corrections made from the time of injection into the interplanetary trajectory and the magnitude of each correction. For the purpose of simplicity, the specific impulse will be considered constant throughout the time duration that the space vehicle is proceeding on the midcourse phase of the interplanetary mission.

The mass of the space vehicle at time of injection is composed of the mass of space vehicle (without correction fuel) plus the mass of correction fuel, i.e.,

$$m_r = m_s + m_{pr},$$

where

m_r = total mass of vehicle and fuel at injection,

m_s = mass of vehicle (excluding fuel), and

m_{pr} = mass of fuel at injection.

At time t_1, the mass of the vehicle may be expressed as,

$$m_i = m_{i-1} - \Delta m,$$

where m_{i-1} is the mass of the space vehicle after the correction at time t_{i-1} but before the correction at t_i and Δm is the mass of fuel expended by the propulsion system in making the correction. Δm can be related to the momentum correction by analyzing the definition of specific impulse, i.e.,

$$I_{sp} = \frac{F \text{ lbs of thurst produced during t seconds}}{\Delta w \text{ lbs of propellant consumed}},$$

or

$$I_{sp} = \frac{Ft}{\Delta w}.$$

The momentum correction (ΔM) is equal to Ft. The magnitude of the momentum change can be related to the specific impulse as

$$I_{sp} = \frac{\Delta M}{\Delta w}.$$

After rearrangement, the weight of correction fuel consumed per momentum correction is

$$\Delta w = \frac{\Delta M}{I_{sp}}.$$

The mass of fuel consumed per momentum correction is

$$\Delta m = \frac{\Delta w}{g} = \frac{\Delta M}{gI_{sp}}.$$

The mass of the space vehicle after the correction at time t_i becomes

$$m_i = m_{i-1} - \frac{\Delta M}{gI_{sp}}.$$

The above equation is true only if there are no inefficiencies present. One such inefficiency considered in this study is in starting and stopping the propulsion device. Asire (5) made such a study on inefficiencies associated with starting and stopping rockets. His results indicates that mass losses of fuel only occur during thrust termination. This mass loss is a constant for each shutdown of the rocket and a fixed cost for starting and stopping. The fuel mass loss can be expressed as,

$$\Delta m_L = \frac{L' A_t P_{c1}}{gRT_{c1}} \frac{k-1}{k+1},$$

where

L' = rocket chamber characteristic length,

A_t = rocket nozzle throat area,

$Pc1$ = steady state combustion chamber pressure,

R = gas constant,

g = acceleration of gravity,

T_{c1} = steady state combustion chamber temperature, and

k = ratio at specific heats.

The amount of momentum that can not be utilized when making the correction is,

$$\Delta M_L = \Delta m g I_{sp}.$$

If a momentum correction of ΔM is needed, the propulsion system must apply an equivalent momentum correction of,

$$\Delta MT = \Delta M + \Delta M_L,$$

that is, ΔM_L is lost due to irreversibilities and ΔM is utilized for making the required correction.

As a result of the irreveresibility, the mass of the space vehicle at time t_i is,

$$m_i = m_{i-1} - \Delta m - \Delta m_L,$$

or

$$m_i = m_{i-1} - \frac{\Delta M}{g I_{sp}} - \frac{L' A_t P_{c1}}{g R T_{c1}} \frac{k-1}{k+1},$$

where the total mass of fuel expended by the propulsion system in making the correction is,

$$\Delta m T = \frac{\Delta M}{g I_{sp}} + \frac{L' A_t P_{c1}}{g R T_{c1}} \frac{k-1}{k+1},$$

or

$$\Delta m T = \frac{\Delta MT}{g I_{sp}}.$$

With the assumption that the time duration of the momentum correction is small compared with the time between corrections, the mass history of the space vehicle can be shown as in Figure IV-4.

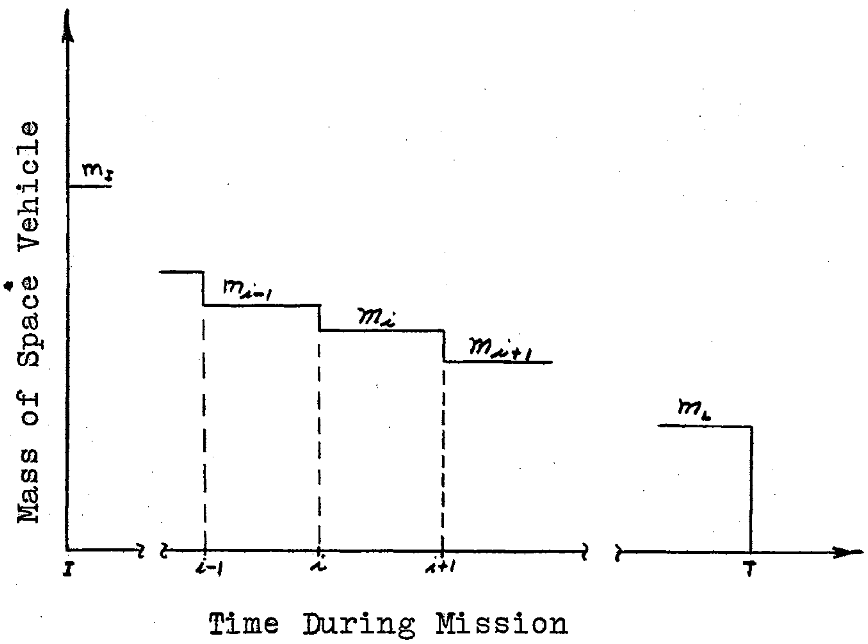


Figure IV-4 Time History of Space Vehicle Mass

Figure IV-4 is designed only to give the reader a visional insight into the manner at which the mass deductions occur at t_{i-1}, t_i, and t_{i+1}. The mass shown at time t_I is the total mass of the space vehicle (with correction fuel) at the instance just before injection. This point is emphasized because if a large amount of error is present at injection, the guidance system may require an initial correction of one or more ΔM momentum corrections to be made at time t_I, therefore the mass of the space vehicle may decrease in proportion to the initial correction. The mass m_L shown is the mass of the vehicle (without correction fuel plus the mass of correction fuel remaining after the trip to the target planet.

Development of Total Cost Function

The expected total cost of correction fuel is a function of three cost components. These cost components are the initial cost (IC), midcourse cost (MC), and final cost (FC). The expected total correction cost (TC) consumed by the space vehicle per mission is expressed by the function,

$$TC = IC + MC + FC,$$

as shown in Figure IV-5.

Initial correction cost is the cost of correction fuel

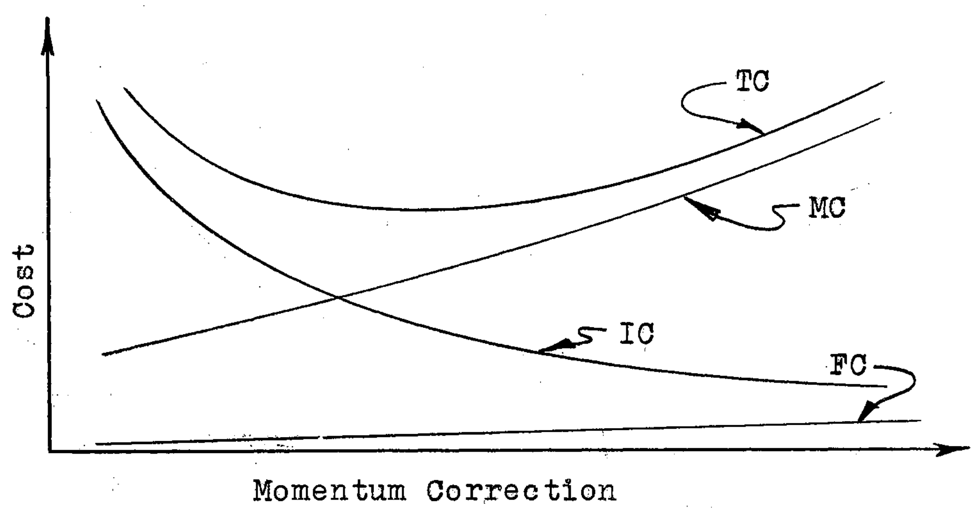


Figure IV-5 Cost Function Sketch

consumed by the space vehicle for making initial corrections at the time of injection. The space vehicle is required to make initial corrections only when the initial errors are so large that the guidance system is unable to correct for them, or when the errors encountered during the mission are so large that they can not be completely corrected for. The initial correction cost also includes all corrections which are calculated to be made at a time before the time that the last correction was made.

The midcourse correction cost (MC) is the cost of correction fuel consumed by the space vehicle for making corrections during the midcourse phase of the mission. If the errors are small and few and the specified momentum correction is large, then only a few corrections will probabily be needed and the midcourse correction cost will be small.

Final cost (FC) is the cost of fuel consumed by the space vehicle to make the last correction at time t_L. All corrections needed after this last correction are within the realm of capture phase guidance and is beyond the scope of this study.

Method of System Simulation

In obtaining estimates of the expected total cost of correction fuel utilized by the space vehicle to make corrections for trajectory deviations, the guidance complex is simulated by means of the Monte Carlo method. The Monte Carlo method consists of the unrestricted random sampling of errors from defined probability distributions and the mathematical manipulation of these errors in accordance with the system logic.

The procedure for performing the simulated sampling process involves the drawing of system errors at random from there respective probability distributions and the manipulation of the values of system errors in accordance with the system logic. The scheme utilized in developing the random errors is one developed by Fabrycky (7). The scheme utilizes the Central Limit Theorm (8) to develope random normal deviates (remembering that the errors are distributed normally) from which random errors are obtained (momentum magnitude and direction). This scheme is presented in Appendrix A.

The momentum magnitude errors and directional errors developed by the scheme above are input data for the system simulation. The system treats the errors as detected and measured errors and proceeds to calculate the time (t_1) to make correction. This is in accordance with the system logic and guidance equations. At the calculated time t_1, the momentum correction ($\triangle M$) is applied by the propulsion system. The space vehicle is now supposedly proceeding on the corrected trajectory and the amount of fuel utilized to make the correction is recorded. Due to inherent errors, new momentum magnitude and direction errors are applied to the space vehicle corrected momentum vector and the correction process is repeated.

At the time to make the last correction (t_L = 191.2

days), the space vehicle is approaching the termination of the midcourse guidance phase and may be undertaking to make a correction after the termination time (t_L). The simulated system restricts such a correction, but makes the proper portion of the correction at the termination time (t_L).

After the last correction has been applied, the total cost of fuel consumed to make all necessary corrections during the midcourse phase is computed. This resulting value is one estimate of the total correction cost (TC). Repetition of the above procedure will give many estimates of the total correction cost (TC). If a sufficient number of missions are simulated, a histogram of the TC values may be constructed. By summing the total number of total cost values (ΣTC) and dividing this sum by the total number of missions simulated (n), the mean total cost of correction fuel may be obtained, i.e.,

$$TC\ mean = \frac{\Sigma TC}{n}.$$

Defining Number of Simulations

How many simulated missions are necessary to obtain a satisfactory mean total cost value? This question may be answered by plotting the cummulative total correction cost divided by the number of missions simulated after each mission. When this value becomes stable the simulation can be terminated. Figure IV-6 illustrates that as the number

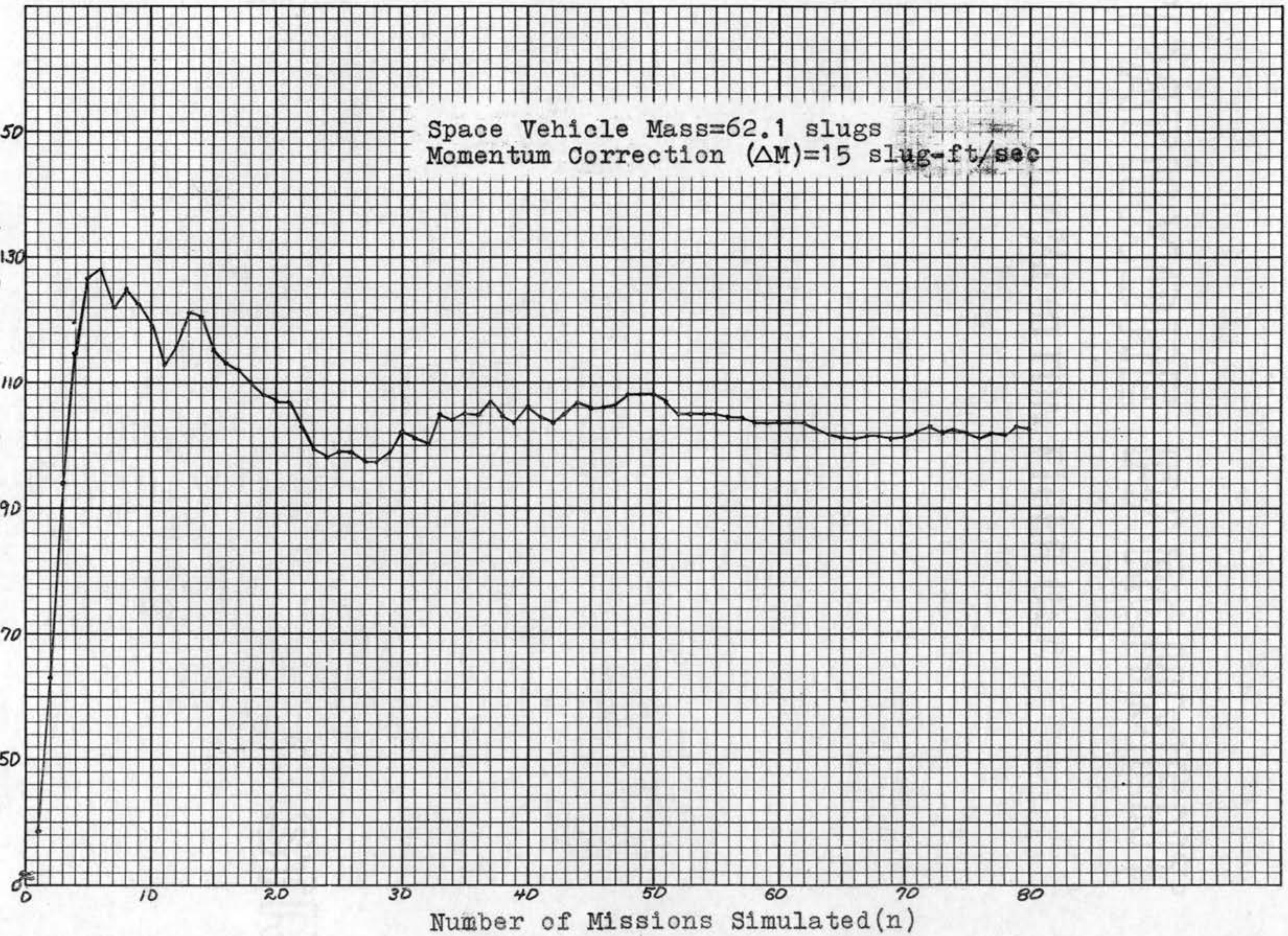


Figure IV-6 Correction Cost vs. Number of Missions Simulated of missions simulated increases, the running average total cost fluctuates above and below the mean total cost until at approximately eighty samples. At this point the fluctuation is sufficiently stable to obtain a fairly accurate estimate of the mean total correction cost.

CHAPTER V

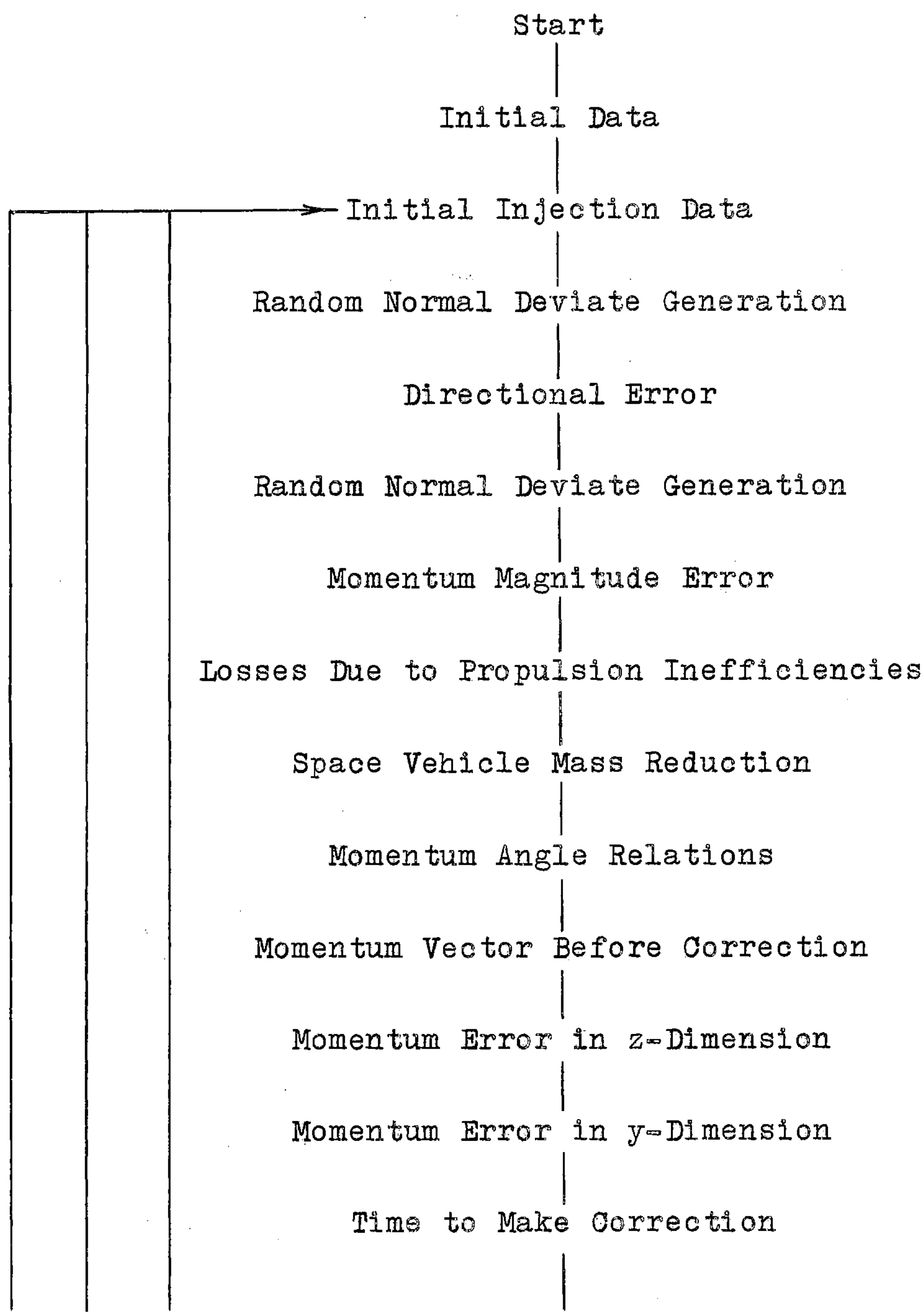
THE COMPUTER PROGRAM

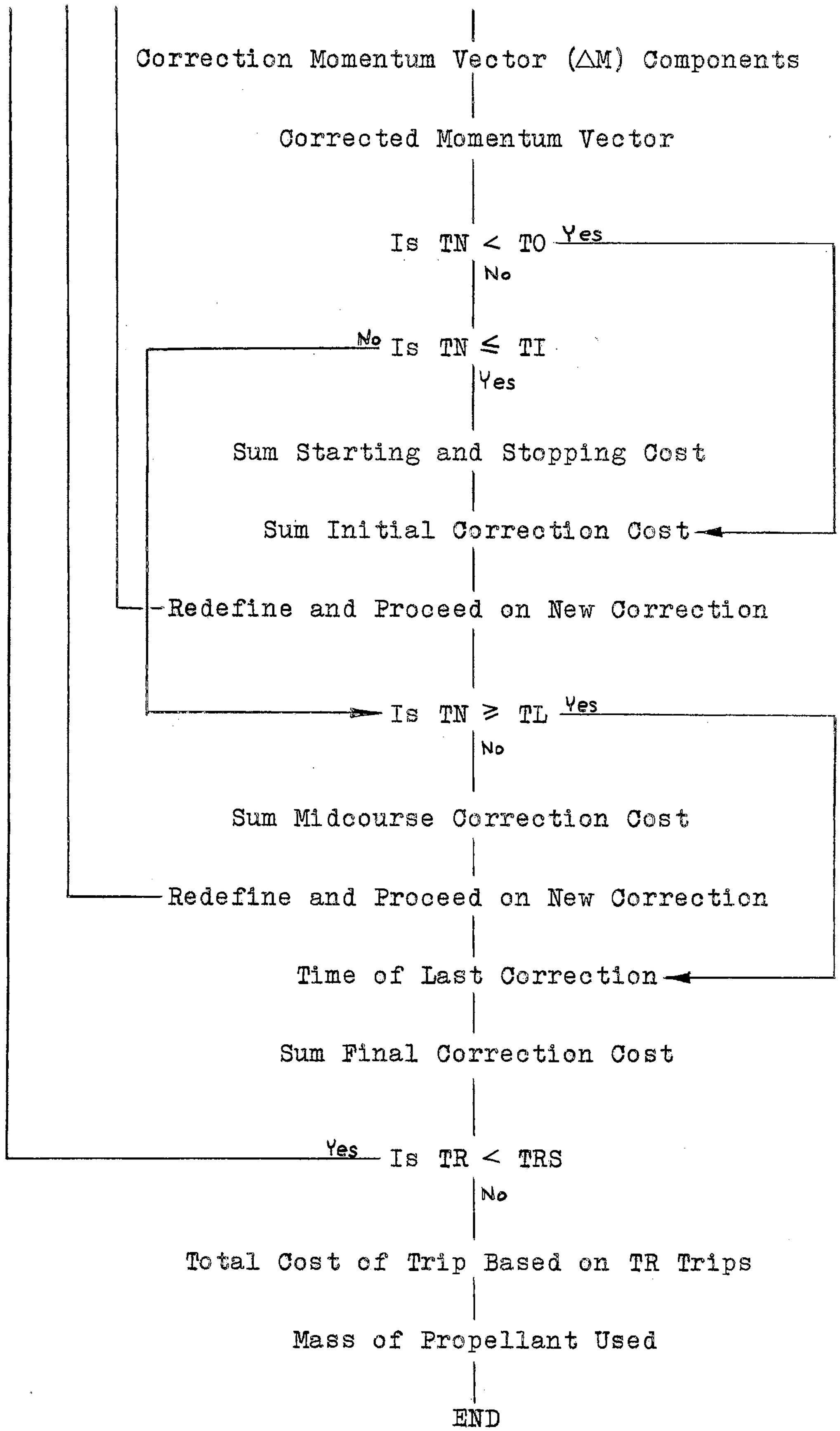
The purpose of this chapter is to present the necessary programs required to develop specific points for the cost functions of Figure IV-5. The presentation begins with the development of the general flow diagram. Since both the IBM 1620 and IBM 1410 computers are utilized in this study, Fortran programs are developed and presented for both computers. Also a table of initial input data for the computer programs is presented in the latter section of this chapter.

Development of Flow Diagram

When programs are lengthy, as they are in this study, it becomes appropriate to develop flow diagrams. This is a technique of schematically showing the steps that the computer must take to produce the answers required by the problem. Flow diagrams serve two purposes: they offer an easy notation for analysing the steps required in the solution of the problem; they offer the basic documentation in the form of a map of the program, so that anyone unfamiliar with the program can easily determine what the program does and how it does it. The more detailed the flow diagram, the easier the task of actually writing the Fortran program. The general flow diagram for the two dimensional case is developed which exhibits the sequence of major operations to be carried out.

General Flow Diagram





Correction Momentum Vector (ΔM) Components
Corrected Momentum Vector
Is TN < TO
Yes
No
No
Is TN ≤ TI
Yes
Sum Starting and Stopping Cost
Sum Initial Correction Cost
Redefine and Proceed on New Correction
Is TN ≥ TL
Yes
No
Sum Midcourse Correction Cost
Redefine and Proceed on New Correction
Time of Last Correction
Sum Final Correction Cost
Yes
Is TR < TRS
No
Total Cost of Trip Based on TR Trips
Mass of Propellant Used
END

The general flow diagram presents a clear picture of the manner and sequence of how the program is executed. This is of benefit when others, unfamiliar to the program, are interested in the manner of execution of the problem.

Development of Fortran Programs

Since both the IBM 1410 and IBM 1620 computers are utilized in this study, the flow diagram is translated into Fortran IV (PR-155) for the IBM 1410 and Fortran II (PR-108) for the IBM 1620. The computer programs for the IBM 1410 and IBM 1620 are included respectively in Appendixes B and C.

Statement number 385 instructs the computer to punch into IBM cards the momentum correction (DMT), total correction cost (TCA), and number of midcourse corrections (AAK). Statement number 391 instructs the computer to punch the final correction cost (FCA), initial correction cost (TICA), midcourse correction cost (CCA), and number of trips simulated (TR).

Program Alteration

Information on the sequence of corrections made during the midcourse phase of the mission is obtained by inserting a program instruction immediately after statement number 310. This instruction instructs the computer to punch the time t_1 at which each correction is made and the trip number.

In obtaining the running average total correction cost at the termination of each trip, statement 345 is transfered to a new position immediately following statement 391. Statement 385 is then executed at the end of each trip (i.e., the computer is instructed to punch the total correction cost).

Initial Input Data

In deciding input data values that are proper and representive of a real interplanetary guidance system, it is necessary to rely on literature to provide values (3) (9) (10). Such values that are required as initial input data for the computer programs are tabulated in Table I.

TABLE I

INITIAL INPUT DATA

Flow Diagram Symbol	Fortran Symbol	Value
t_I	T_I	0 sec-injection time
t_L	T_L	16519680 sec(191.2 days)-last correction time
m_R	RMA	Varable-mass vehicle and propellant at t_I
M_R	RMO	2914726 slug-ft/sec - reference momentum
n	TRS	80 - number of simulations
I_{SP}	SPI	352 sec - specific impulse
D	SD	.0000226 rad - directional error standard deviation
M	SM	.002 M slug-ft/sec - momentum error standard deviation
M_D	DME	0.0 - mean directional error

I (Continued)

$\bar{M}_M$	VME	0.0 - mean magnitude error
T	T	16606080. sec (192.2 days) - total trip time
g	G	32.17 ft/sec - gravity acceleration
ΔMT	DMT	Variable - momentum correction
x	X	0.572 - positive argument for IBM 1620 random number generation
L'	SL	0.5ft - characteristic length
T_{e1}	ST	5500. R - combustion chamber temperature (steady state)
k	SK	1.25 - ratio of specific heats (H_2 - O_2)
P_{c1}	SP	43200 16/ft - combustion chamber pressure
A_t	SA	.0218 ft - throat area
R	R	157.75 $\frac{ft-lb}{lb-°R}$ - gas constant (H_2 - O_2)

CHAPTER VI

RESULTS AND DISCUSSION

The primary purpose of this study was stated earlier as obtaining an optimal policy for making corrections for random deviations from an interplanetary reference trajectory with the minimum expected expenditure of correction fuel. The first part of this chapter exhibits total cost curves which define optimal momentum corrections for three sizes of space vehicles. Also contained within this chapter are discussions and results on optimal space vehicle mass, number of midcourse corrections needed, and when to make corrections. In addition, a conceptual scheme for establishing the minimum cost amount of fuel to be provided on board the space vehicle is also presented in the last section.

Total Correction Cost

In computing the total cost of fuel for making corrections, the function

$$TC = IC + MC + FC,$$

is computed for each value of momentum correction ($\triangle M$). The IC (initial cost) is the cost that the space vehicle experiences at time of injection. The MC (midcourse correction cost) is the cost substained by the space vehicle for making corrections during the midcourse phase of the trip. FC (final cost) is the cost substained by the space vehicle to make the last correction at time t_L. The minimum total cost (TC) occurs by trading between initial cost (IC), midcourse cost (MC), and final cost (FC). Since the cost of fuel is proportional to momentum, the correcting cost shown in the following figures are in terms of momentum cost. Figure VI-1 illustrates such curves.

The initial cost (IC) curve shown in Figure VI-1 is zero for large values of momentum corrections (ΔM) and increases as ΔM is made small. This may be reasoned by considering at large values of ΔM, the space vehicle is able to correct for all errors encountered, therefore no initial correction is necessary. As ΔM is made smaller, a point is reached at which the space vehicle is unable to make all corrections for errors encountered during the midcourse phase. Therefore, it must make part of the corrections at time of injection (t_I). As ΔM approaches zero, all of the error corrections tend to be made at t_I and the initial cost (IC) increases without bound.

The mid-course correction cost (MC) curve tends to increase as ΔM increases. As ΔM increases, less initial correction is needed because ΔM is becoming sufficiently large to make most of the necessary corrections during the midcourse phase of the trip. As this occurs, the midcourse cost (MC) increases. Also since the momentum correction magnitude

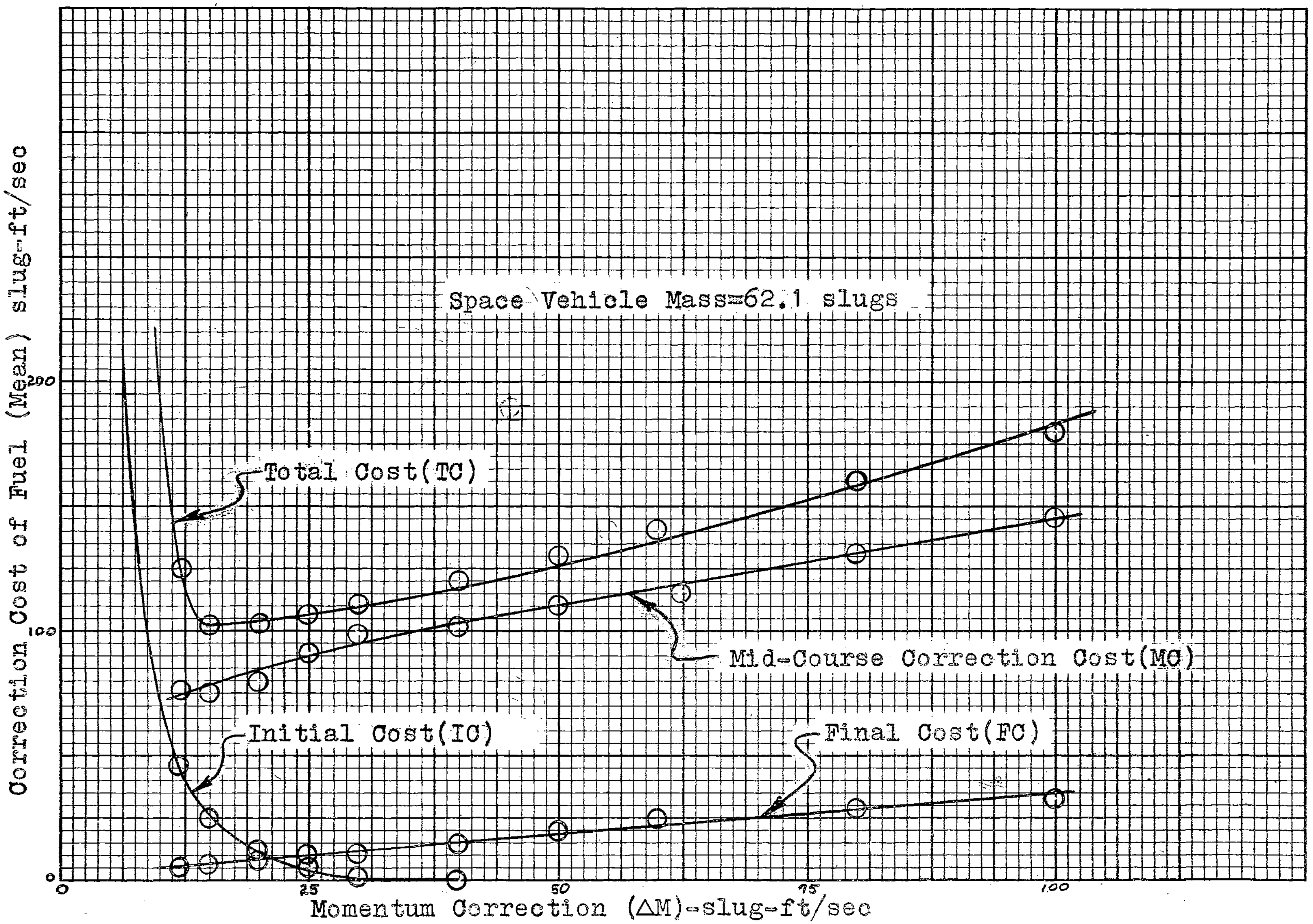


Figure VI-1 Correction Cost vs. Momentum Correction error is proportional to the size of the momentum correction, as ΔM increases, the magnitude error also increases which in return causes the midcourse cost (MC) to increase.

The final correction cost (FC) varies as ΔM varies. As seen in Figure VI-2, as ΔM increases, the final cost (FC) increases proportionally.

Figure VI-2 and Figure VI-3 shows total cost curves for two other different size space vehicles. Table II tabulates the optimal ΔM (momentum correction) values corresponding to the minimum total cost points for each size of space vehicle.

TABLE II

VEHICLE MASS VS. MOMENTUM CORRECTION TABULATION

Space Vehicle Size (Mass)	Optimal ΔM
6.21 slugs	2 slug-ft/sec
62.10 slugs	15 slug-ft/sec
621.00 slugs	200 slug-ft/sec

Optimal Size Space Vehicle

Is there an optimal sized (mass) space vehicle? Since optimal ΔM (momentum correction) values for three different size space vehicles have been found and the minimum cost for each, it is convenient to investigate to see if there is an optimal size (mass) space vehicle at some given optimal ΔM.

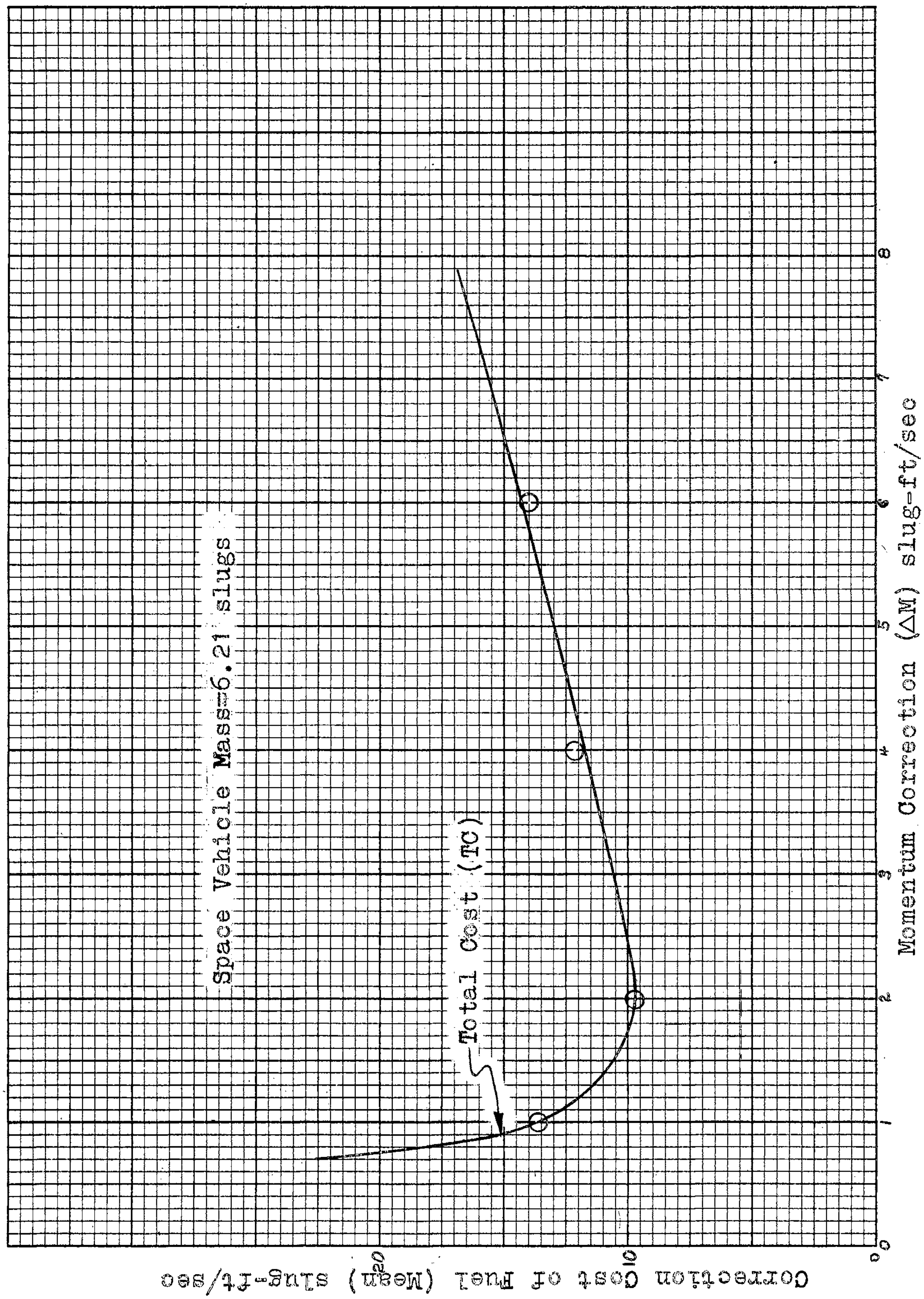
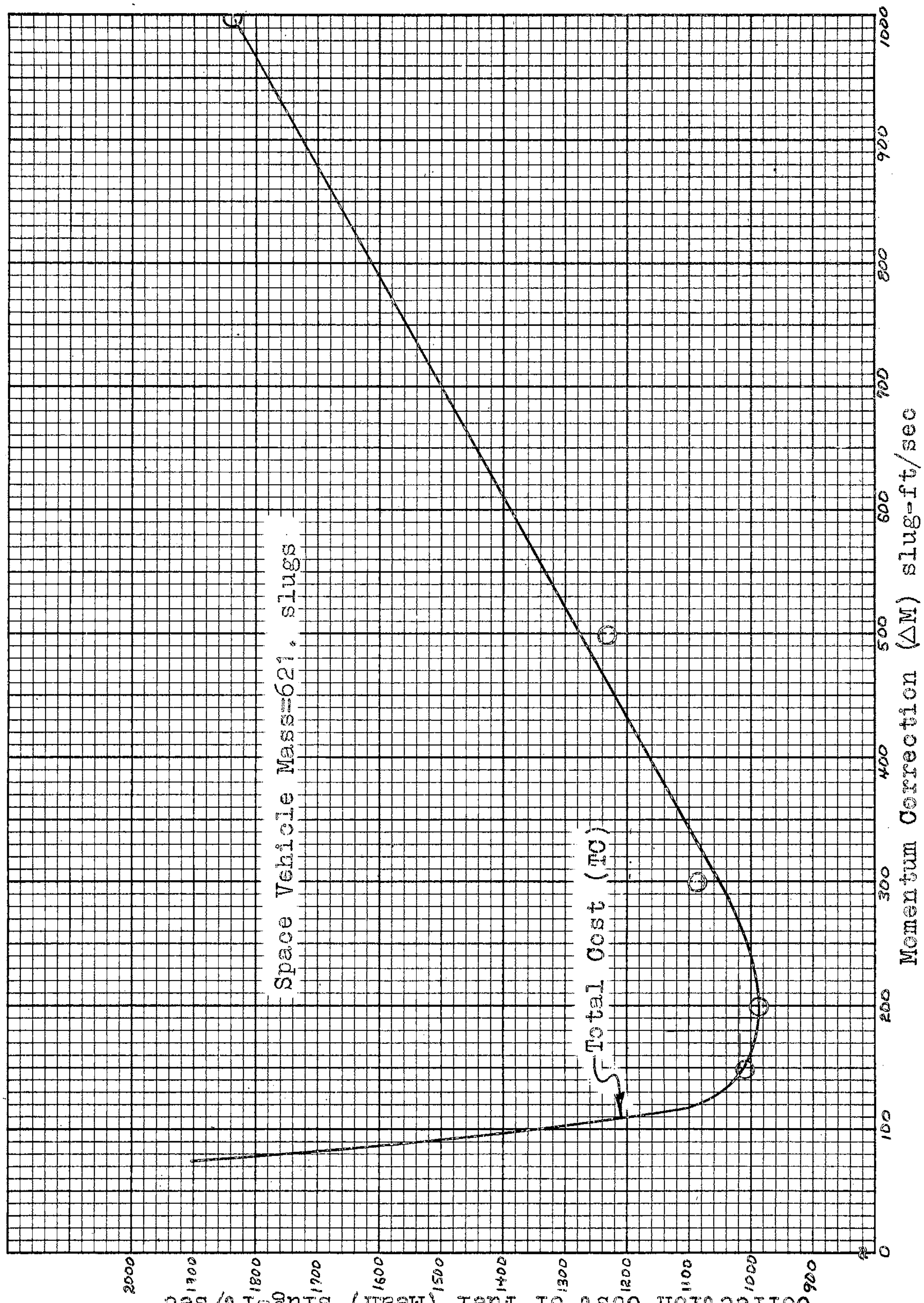


Figure VI-2 Correction Cost vs. Momentum Correction



<u>Figure VI-3</u> Correction Cost vs. Momentum Correction

By plotting space vehicle size (mass) versus optimal momentum correction (ΔM), the optimal space vehicle size may be investigated. Figure VI-4 is a plot of Table VI-1. The curve indicates that the size (mass) of space vehicle tends to zero as the optimal momentum correction size (proportional to total cost) tends toward zero. The possible conclusion to be drawn is that the cost of making corrections decreases as the size (mass) of space vehicle decreases.

Number of Corrections

The number of midcourse corrections needed during a trip are plotted in Figure VI-5 versus the size of momentum correction (ΔM). The curve indicated that as the size of momentum correction decreases, more corrections are required. Also that as ΔM approaches zero, the number of corrections needed tend to infinity. But an infinite number of corrections is umpractical, therefore the necessary corrections must be made at injection. This supports the idea of initial corrections and the initial correction cost discussed earlier.

Sequence of Corrections

Figure VI-6 illustrates the sequence of corrections versus time of correction. The sequence of corrections agree with the fit of the curve except correction T_5. Information may be gained on why T_5 deviates from the curve by computing the correction time ratios.

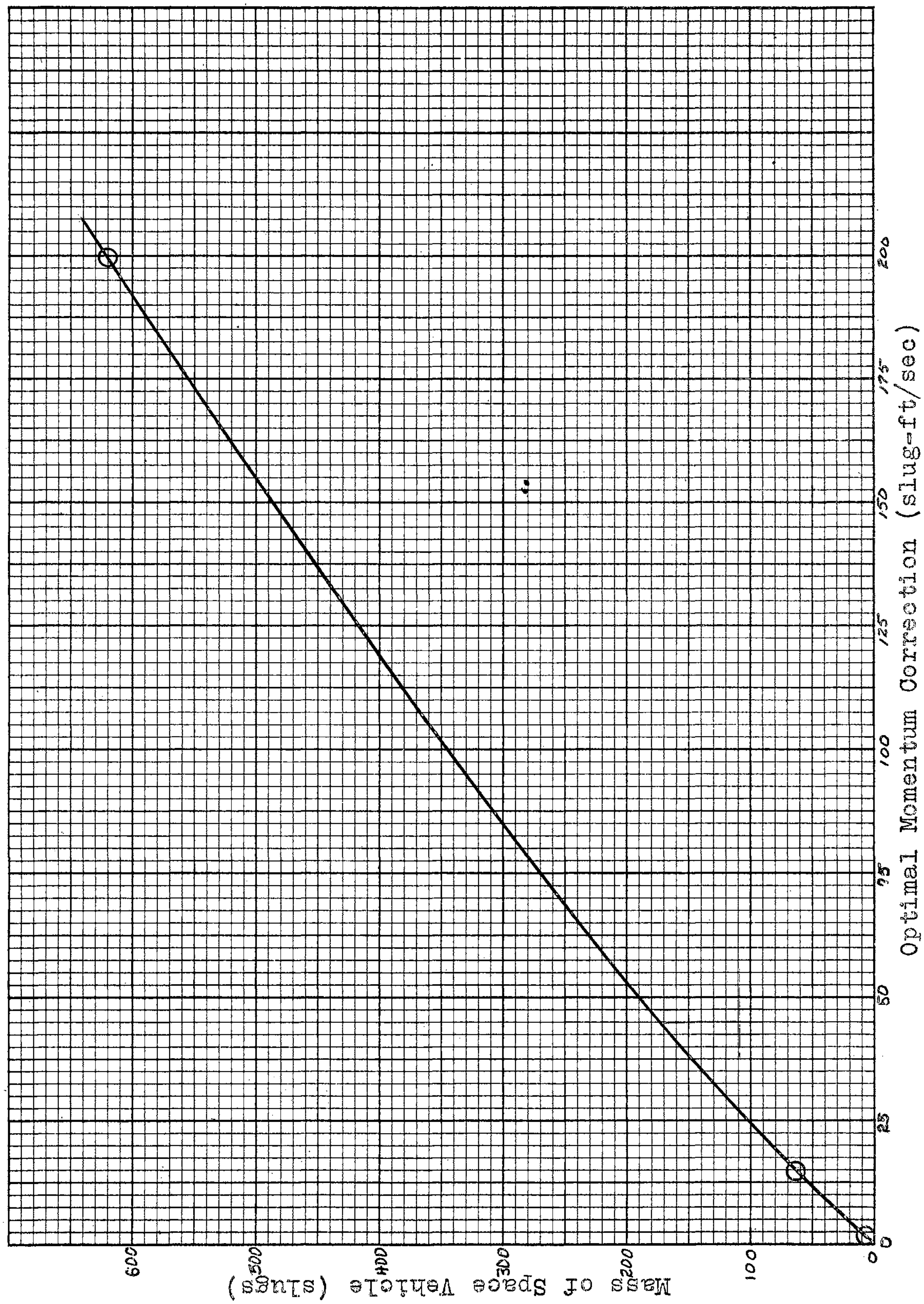


Figure VI-4 Mass of Vehicle vs. Optimal Momentum Correction

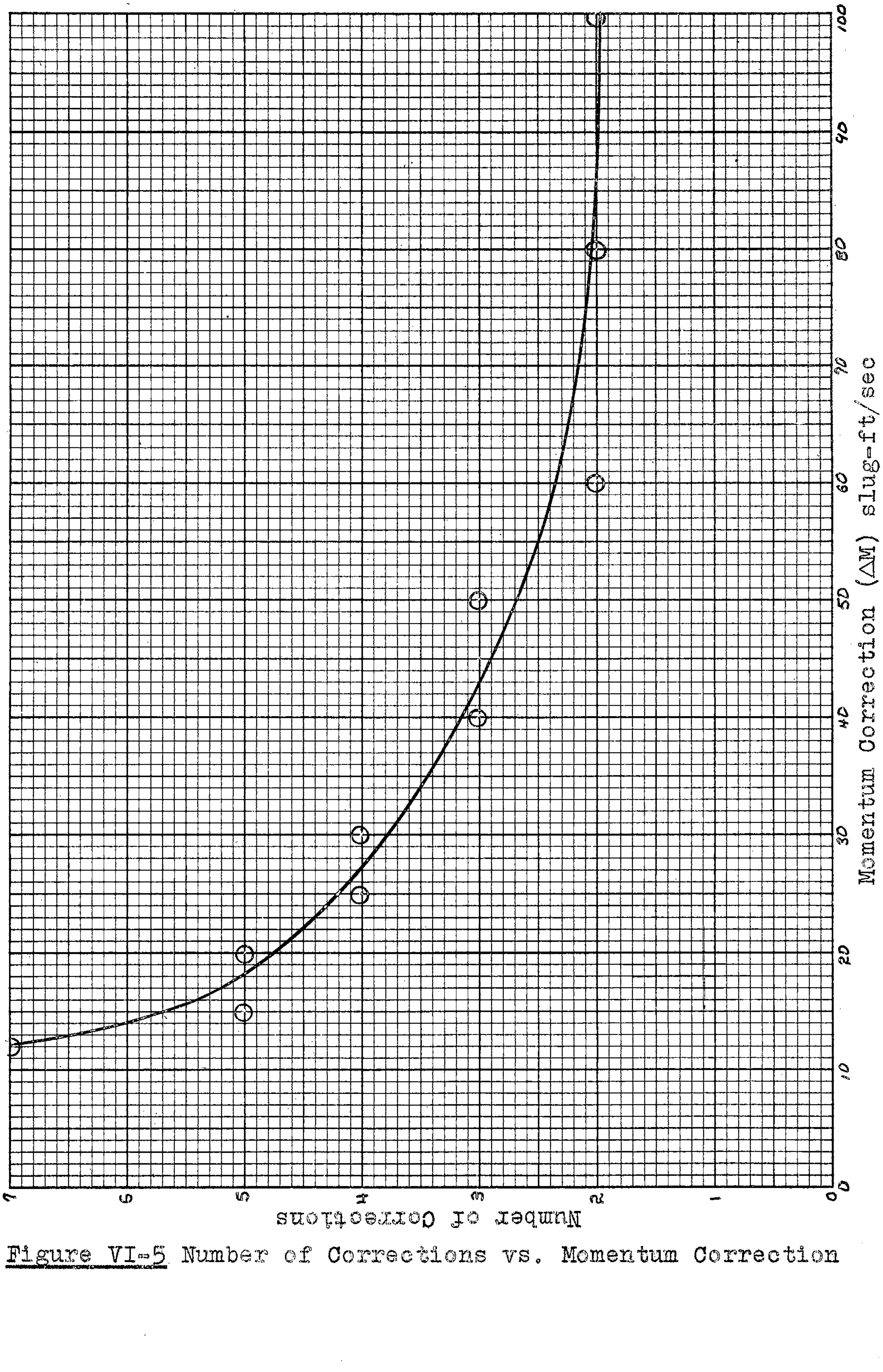


Figure VI-5 Number of Corrections vs. Momentum Correction

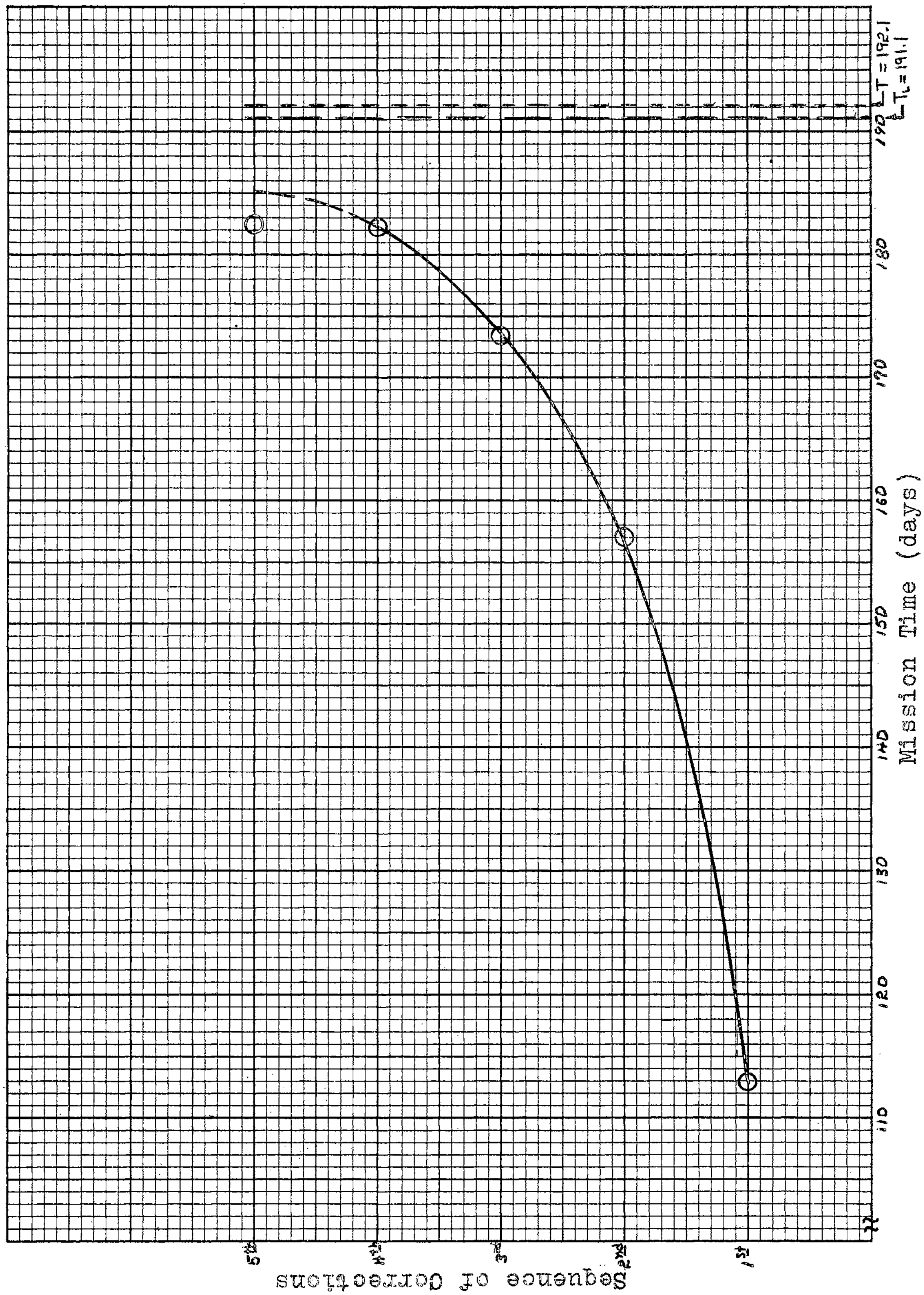


Figure VI-6 Sequence of Corrections vs. Mission Time

$$\frac{T_1}{T_I} = \frac{113.1}{0} = \infty$$

$$\frac{T_2}{T_1} = \frac{157.1}{113.1} = 1.39$$

$$\frac{T_3}{T_2} = \frac{173.5}{157.1} = 1.11$$

$$\frac{T_4}{T_3} = \frac{182.3}{173.3} = 1.05$$

$$\frac{T_5}{T_4} = \frac{182.39}{182.31} = 1.00$$

Figure VI-8 shows a plot of the above ratios versus ratio sequence. The curve indicates that the ratio T_5/T_4 is not far from its true value. If $T_5 = 190$, then

$$\frac{T_5}{T_4} = \frac{190.0}{182.8} = 1.042$$

It can be concluded that the ratio T_5/T_4 is not very sensitive and is not a good value on which to base a prediction.

A more valid reason for T_5's displacement may be obtained by considering that all of the T_5 values from the eighty simulations are not present. This could come about when the time of last correction (T_L) destroys the upper end of the T_5 distribution. Figure VI-7 illustrates that only the lower portion of the T_5 distribution is averaged, therefore without

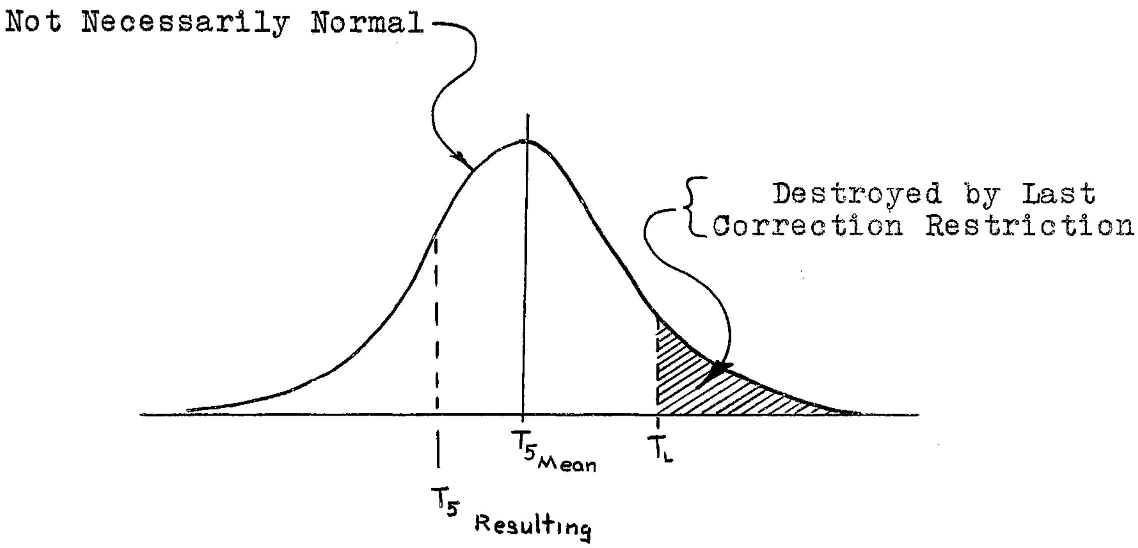
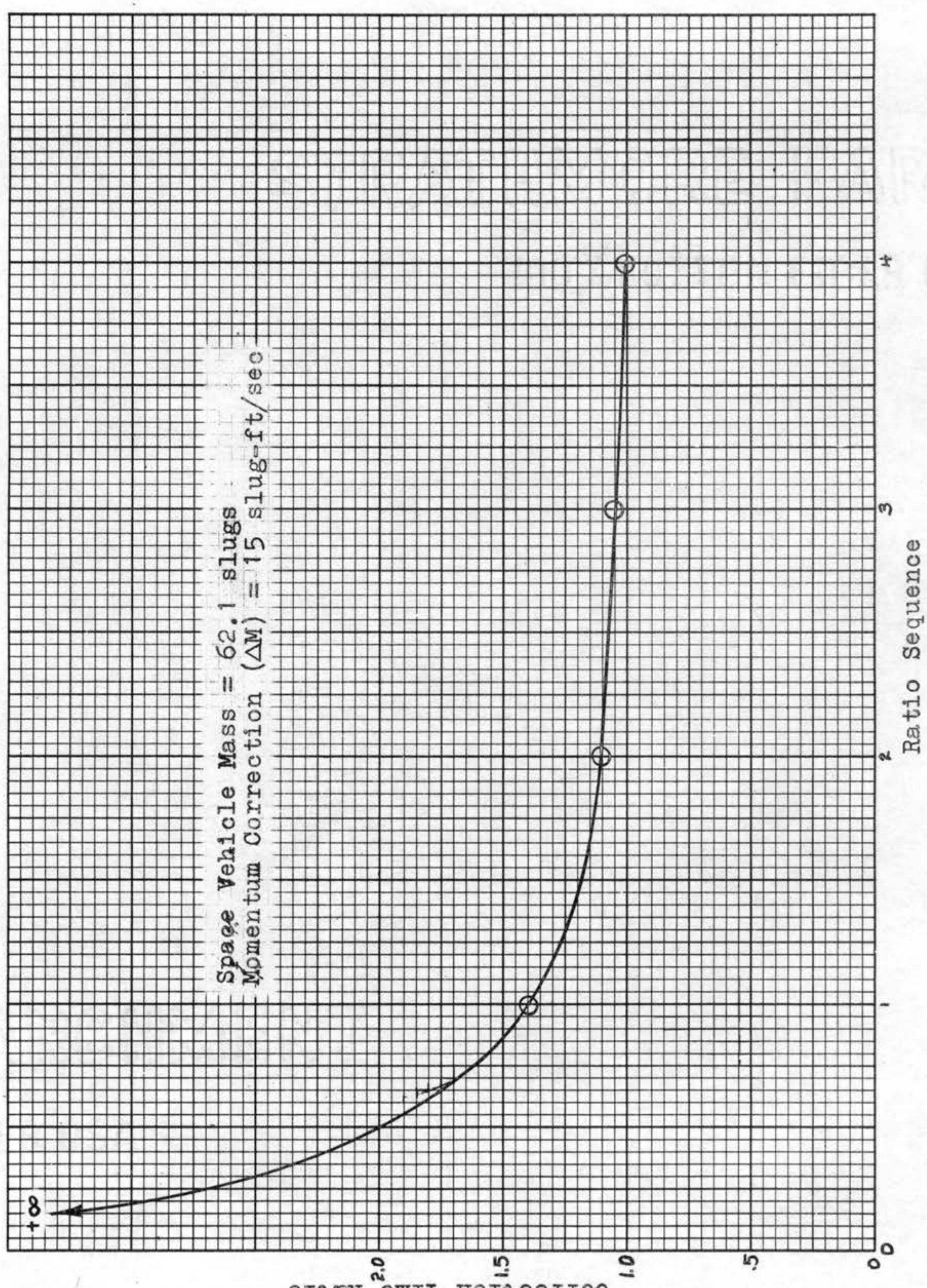


Figure VI-7 T_5 Distribution

the presence of the upper portion of the distribution, T_5 resulted in being a low value.

Optimizing Correction Fuel Payload

The results of the first part of this chapter established the magnitude of momentum correction that the space vehicle should make to minimize the mean total cost of fuel consumed in making corrections. If this mean total cost of fuel (point A in Figure VI-9) is provided aboard the space vehicle, the probability of success of the mission will be 50% and the probability of failure will be 50%. If the probability of success for the mission is to be 99%, the amount of fuel



<u>Figure VI-8</u> Correction Time Ratio vs. Ratio Sequence

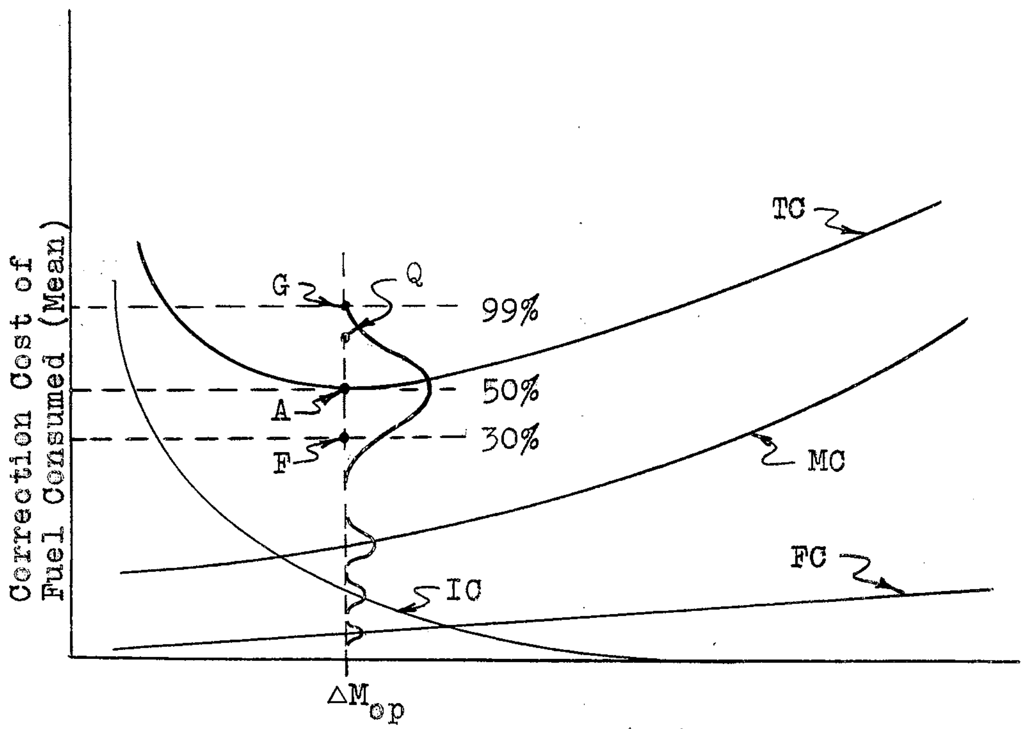


Figure VI-9 Cost Function Sketch

corresponding to point G should be provided aboard the space vehicle.

The over-all cost, evaluated in terms of dollars, may be found by developing curves for the cost of carrying fuel and the cost of a mission failure (Figure VI-10). The cost (in terms of dollars) of carrying fuel involves evaluating the cost of rocket booster power, fuel cells, equipment, and amount of instruments carried, for different amounts of fuel carried. The cost of a mission failure involves evaluating the cost (in terms of dollars) of loosing men, space vehicle, equipment, instruments, and benefitial return data, for different amounts of fuel carried. A minimum over-all cost

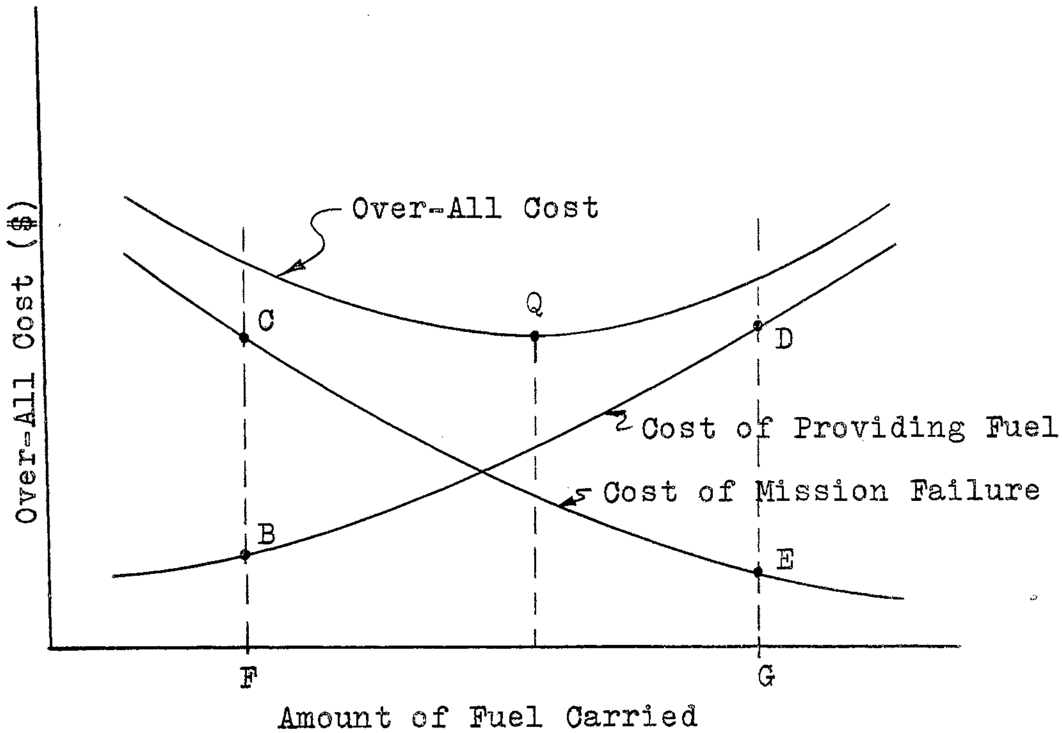


Figure VI-10 Over-All Cost Sketch

may be found by trading between the cost of carrying fuel and the cost of a mission failure.

Referring to Figure VI-10, assume that location F is equal to the 30% point in Figure VI-9. If this amount of fuel is provided aboard the space vehicle, the cost of carrying this amount of fuel will be low and the cost of a mission failure will be high (70%). This is shown by points B and C in Figure VI-10. On the other hand, assume location G is equal to the 99% point in Figure VI-9. If this amount of fuel is provided aboard the space vehicle, the cost of carrying the fuel will be high and the cost of a mission failure will be low (1%). This is shown by points D and E in Figure VI-10.

There exists an optimum value of the amount of fuel carried by the space vehicle which minimizes the sum of the cost of fuel carried and the cost of a mission failure. This optimum point (Q) is illustrated in Figure VI-10.

Assume that point Q corresponds to the 80% point in Figure VI-9. If this amount of fuel is provided aboard the space vehicle, the probability of a mission success will be 80% and the probability of a mission failure will be 20%. But, at this probability of mission failure (20%), the over-all cost will be a minimum (point Q in Figure VI-10). A possible desired location for the point Q is at point G. This will give a probability of mission success greater than 99%, a probability of mission failure less than 1%, and a minimum over-all cost.

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APPENDIX A

RANDOM NORMAL DEVIATE GENERATION

For an generated population of random numbers, x, in the range 0 to 1,

$$f(x) = \frac{1}{b-a} = 1,$$

where $a \leq x \leq b$; $a = 0$ and $b = 1$.
The first moment about the origin; the mean,

$$\mu_1' = \int_{R_X} x\, f(x)\, dx,$$

$$= \int_0^1 x\, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} = \mu.$$

The second moment about the origin,

$$\mu_2' = \int_{R_X} x^2\, f(x)\, dx,$$

$$= \int_0^1 x^2\, dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}.$$

And the second moment about the mean; the variance,

$$\mu_2 = \mu_2' - (\mu_1')^2,$$

$$\mu_2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12} = \sigma^2.$$

From the derivation of the central limit theorm (8), the random variable,

$$t = \frac{(\bar{x} - \mu)\sqrt{n}}{\sigma},$$

has a distribution that approaches the standard normal distribution as n becomes infinite. Since,

$$\bar{x} = \frac{\Sigma x}{n}, \quad \mu = \frac{1}{2} \text{ and } \sigma^2 = \frac{1}{12},$$

$$t = \frac{\left(\frac{\Sigma x}{n} - \frac{1}{2}\right) \sqrt{n}}{\sqrt{\frac{1}{12}}}.$$

Choosing n = 30, as a value that will give a very good approximation, results in,

$$t = \frac{\Sigma x \sqrt{30}\ \sqrt{12}}{30} - \frac{\sqrt{30}\ \sqrt{12}}{2},$$

$$t = \Sigma x(.632411) - (9.486164).$$

That the expected value of t is zero, $E(t) = 0$, may be shown as follows:

$$E(x) = \mu = \frac{1}{2},$$

for $n = 30$,

$$E(\Sigma x) = 30\left(\frac{1}{2}\right) = 15,$$

$E(t) = E(\Sigma x)(C_1) - (C_2)$ from above,

$$E(t) = 15(.632411) - (9.486164) = 0.$$

APPENDIX B

FORTRAN IV PROGRAM FOR IBM 1410 ELECTRONIC COMPUTER

```
      MON$$     JOB  252740033 GERALD KNORI
      MON$$     ASGN MGO,A2
      MON$$     ASGN MJB,A3
      MON$$     ASGN MW1,A4
      MON$$     ASGN MW2,A5
      MON$$     MODE GO,TEST
      MON$$     EXEQ FORTRAN,,,15,,PROGRAM10
6       FORMAT(F3.0,2X,E14.0,1X,E12.0,F8.0,F4.0,F5.0,E12.0)
10      FORMAT(E9.0,F6.0,F3.0,F3.0,2X,F7.0,F4.0,F6.0,F5.0,
           F7.0,F6.0,F7.0)
25      FORMAT(F7.0,F7.0)
228     FORMAT(1X,E14.8,5X,E14.8,E14.8,E14.8)
390     FORMAT(1X,E14.8,E14.8,F6.0,E14.8,E14.8,F6.0)
392     FORMAT(1X,E14.8,E14.8,E14.8,E14.8,F6.0,F6.0)
5       READ(1,6)TI,TL,RMO,RMA,TRS,SPI,T
8       READ(1,10)SD,SM,DME,VME,G,SL,ST,SK,SP,SA,R
15      DO 395 M=1,10
20      READ(1,25)DMT,X
30      SSC=0.0
35      TR=0.0
40      CI=0.0
45      CC=0.0
50      AK=0.0
51      SS=0.0
55      FC=0.0
56      Q=0.0
58      L=1234567893
65      TO=TI
70      ZCMO=0.0
75      WMAO=RMA
80      YCMO=RMO
85      CMO=RMO
86      GO TO 94
89      WMAO=WMAN
90      ZCMO=ZCMN
91      YCMO=YCMN
92      CMO=CMN
93      TO=TN
94      SY=0.0
95      DO 105 J=1,30
100     L=L*L
101     L=L/100000
102     Y=L/100
103     Y=Y/100000000.
105     SY=SY+Y
110     C1=SQRT(12.0/30.)
115     C2=SQRT(3.0*30.)
120     T1=SY*C1-C2
125     EDN=(T1*SD)/SQRT(30.)+DME
130     SY=0.0
135     DO 145 J=1,30
140     L=L*L
141     L=L/100000
```

```
142    Y=L/100
143    Y=Y/100000000.
145    SY=SY+Y
150    C1=SQRT(12.0/30.)
155    C2=SQRT(3.0*30.)
160    T2=SY*C1-C2
166    WMASS=((SL*SA*SP)/(G*R*ST))*((SK-1.0)/SK+1.0))
167    DMSS=WMASS*G*SPI
168    DM=DMT-DMSS
169    EMN=(T2*SM*DM)/SQRT(30.)+VME
170    WMAN=WMAO-DM/(G*SPI)
175    CDO=ATAN(ZCMO/YCMO)
180    DN=CDO+EDN
185    VMN=(CMO+EMN)/COS(EDN)
190    ZVMN=VMN*SIN(DN)
195    YVMN=VMN*COS(DN)
200    ZEMN=ZVMN-ZCMO
205    YEMN=YVMN-YCMO
210    A=-ZEMN*(WMAN/WMAO)*(T-TO)
215    B=-YEMN*(WMAN/WMAO)*(T-TO)
220    TN=T-(SQRT(A*A+B*B))/DM
225    ZDMN=A/(T-TN)
226    TRR=TR+1.0
227    WRITE(1,228)TRR,A,B,C
230    YDMN=B/(T-TN)
235    ZCMN=ZDMN+ZVMN
240    YCMN=YDMN+YVMN
245    CMN=SWRT(ZCMN*ZCMN+YCMN*YCMN)
250    Q=Q+1.0
284    IF(TN.LT.TO)GO TO 293
285    IF(TN.LE.TI)GO TO 295
286    SSC=SSC+DMSS
290    AK=AK+1.0
291    GO TO 305
293    SS=SS+1.0
295    CI=CI+DM
300    GO TO 89
305    IF(TN.GE.TL)GO TO 320
310    CC=CC+DM
315    GO TO 89
320    TN=TL
325    DMF=(SQRT(A*A+B*B))/(T-TN)
335    FC=FC+DMF
340    TR=TR+1.0
345    IF(TR.LT.TRS)GO TO 65
350    TICA=CI/TR
355    AAK=AK/TR
356    QQ=Q/TR
357    SSS=SS/TR
360    CCA=CC/TR
365    FCA=FC/TR
370    SSCA=SSC/TR
375    TCA=TICA+CCA+FCA+SSCA
```

```
380    WPUA=TCA/(G*SPI)
385    WRITE(2,390)DMT,TCA,AAK,
391    WRITE(2,392)FCA,TICA,CCA,TR
395    CONTINUE
400    END
      MON$$        EXEQ LINKLOAD
                   PHASEPROGRAM100
                   CALL PROGRAM10
      MON$$        EXEQ PROGRAM100,MJB
```

APPENDIX C

FORTRAN II PROGRAM FOR IBM 1620

ELECTRONIC COMPUTER

```
5       READ 6,TI,TL,RMO,RMA,TRS,SPI,T
6       FORMAT(F3.0,2X,E14.0,1X,E12.0,F8.0,F4.0,F5.0,E12.0)
8       READ 10,SD,SM,DME,VME,G,SL,ST,SK,SP,SA,R
10      FORMAT(E9.0,F6.0,F3.0,F3.0,2X,F7.0,F4.0,F6.0,F5.0,
           F7.0,F6.0,F7.0)
15      DO 395 M=1,10
20      READ 25,DMT,X
25      FORMAT(F7.0,F7.0)
30      SSC=0.0
35      TR=0.0
40      CI=0.0
45      CC=0.0
50      AK=0.0
51      SS=0.0
55      FC=0.0
56      Q=0.0
65      TO=TI
70      ZCMO=0.0
75      WMAO=RMA
80      YCMO=RMO
85      CMO=RMO
86      GO TO 94
89      WMAO=WMAN
90      ZCMO=ZCMN
91      YCMO=YCMN
92      CMO=CMN
93      TO=TN
94      SY=0.0
95      DO 105 J=1,30
100     Y=RAN(X)
105     SY=SY+Y
110     C1=SQR(12.0/30.)
115     C2=SQR(3.0*30.)
120     T1=SY*C1-C2
125     EDN=(T1*SD)/SQR(30.)+DME
130     SY=0.0
135     DO 145 J=1,30
140     Y=RAN(X)
145     SY=SY+Y
150     C1=SQR(12.0/30.)
155     C2=SQR(3.0*30.)
160     T2=SY*C1-C2
166     WMASS=((SL*SA*SP)/(G*R*ST))*((SK-1.0)/(SK+1.0))
167     DMSS=WMASS*G*SPI
168     DM=DMT-DMSS
169     EMN=(T2*SM*DM)/SQR(30.)+VME
170     WMAN=WMAO-DM/(G*SPI)
175     CDO=ATN(ZCMO/YCMO)
180     DN=CDO+EDN
185     VMN=(CMO+EMN)/COS(EDN)
190     ZVMN=VMN*SIN(DN)
195     YVMN=VMN*COS(DN)
200     ZEMN=ZVMN-ZCMO
```

```
205     YEMN=YVMN-YCMO
210     A=-ZEMN*(WMAN/WMAO)*(T-TO)
215     B=-YEMN*(WMAN/WMAO)*(T-TO)
220     TN=T-(SQR(A*A+B*B))/DM
225     ZDMN=A/(T-TN)
230     YDMN=B/(T-TN)
235     ZCMN=ZDMN+ZVMN
240     YCMN=YDMN-YVMN
245     CMN=SQR(ZCMN*ZCMN-YCMN*YCMN)
250     Q=Q+1.0
284     IF(TN-TO)293,285,285
285     IF(TN-TI)295,295,286
286     SSC=SSC+DMSS
290     AK=AK+1.0
291     GO TO 305
293     SS=SS-1.0
295     CI=CI+DM
300     GO TO 89
305     IF(TN-TL)310,320,320
310     CC=CC+DM
315     GO TO 89
320     TN=TL
325     DMF=(SQR(A*A+B*B))/(T-TN)
335     FC=FC+DMF
340     TR=TR+1.0
345     IF(TR-TRS)65,350,350
350     TICA=CI/TR
355     AAK=AK/TR
356     QQ=Q/TR
360     CCA=CC/TR
365     FCA=FC/TR
370     SSCA=SSC/TR
375     TCA=TICA+CCA+FCA+SSCA
380     WPUA=TCA/(G*SPI)
385     PUNCH 390,DMT,TCA,AAK,
390     FORMAT(E14.8,E14.8,F6.0,E14.8,E14.8,F6.0)
391     PUNCH 392,FCA,TICA,CCA,TR
392     FORMAT(E14.8,E14.8,E14.8,E14.8,F6.0,F6.0)
395     CONTINUE
400     END
```

VITA

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