

DETERMINATION OF SUSPENDED ROOF SHAPES
USING ELECTRICAL ANALOG METHODS

By

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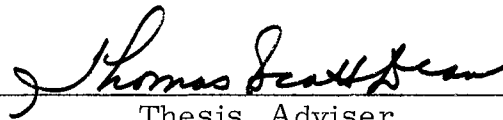
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Dean of the Graduate School

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TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
II. THE PROBLEM OF SHAPE DETERMINATION.	3
III. ELECTRICAL ANALOGS FOR SUSPENDED ROOF SYSTEMS	10
The Resistance Network Analog	11
The Conductive Sheet Analog	14
IV. SOLUTION BY ANALOG METHODS	16
Illustrative Examples	16
Summary and Conclusions	24
SELECTED BIBLIOGRAPHY	27
APPENDIX	28
Resistance Network System Components.	29
Conductive Sheet System Components.	29
Conductive Strip Voltage Divider.	33
Power Supply	35
Voltage Measuring System.	35

LIST OF ILLUSTRATIONS

Figure	Page
1. Cable Network in General Non-planar Frame	4
2. Tensile Forces Acting on a Node of the General Cable Network	6
3. Section Through Node.	6
4. Portion of Resistance Network.	12
5. Cable Net in Bent Square Frame	17
6. Solution for Example 1.	17
7. Cable Net in Bent Circular Frame.	19
8. Solution for Example 2.	19
9. Cable Net in Warped Ellipsoidal Frame.	21
10. Solution Quadrant for Example 3.	22
11. Solution Quadrant for Example 4.	23
12. Resistance Network Analog	30
13. Conductive Sheet Analog	32
14. Strip Voltage Dividers.	34
15. Power Supply	36
16. Voltage Measuring Device	36
17. Simplified Schematic of Voltage Measuring System.	38
18. Assembled Analog System	40

CHAPTER I

INTRODUCTION

Suspended roof systems occupy a new and growing position in the art and science of building. A number of buildings constructed within the past decade offer convincing proof that beauty and economy can be obtained using these systems. The structural potential of suspended roof construction will be further explored in such new buildings as the Air Force Museum, to be completed in 1966, the roof of which will span 700 feet and cover an area of eight acres (1).

While conventional numerical techniques may suffice for use in the design and analysis of roof systems of relatively simple geometry, it has been noted that some more complex three-dimensional roof structures cannot be analyzed by known mathematical methods (2). In 1963 Siev and Eidelman suggested a procedure whereby the geometry of a doubly-curved tension roof surface could be approximated by an electrical analog consisting of a resistance network (3). When developed, such an analogy approximates the relevant difference equation. Consequently, it is

limited in application to systems having fixed and determinable cable networks. The complexity of measurement increases as the fineness of the network. This concept has been understood for many years, and has been successfully employed in the solution of engineering problems in such fields as electrostatics, heat flow, wave motion, gaseous diffusion, and structural deformation analysis (4) (5).

The published work of Siev and Eidelman has been taken as a point of departure for further research into the analogy between prototype and electrically simulated roof system geometry. The author has developed mathematical relationships which govern certain such analogies, and has conducted experiments which further demonstrate applications of the analog method.

CHAPTER II

THE PROBLEM OF SHAPE DETERMINATION

A suspended roof system is characterized by the presence of main structural elements which act primarily in tension. The catalog of items used as tension elements includes chains, rods, cables, and flat bars, as well as cloth and plastic membrane-like materials. One of the simplest types of suspended roof systems consists of a series of parallel cables, suitably anchored, which span the width of a building and directly support the roof covering.

A more complex but relatively common system consists of a two-directional network of prestressed cables. The cables lie in orthogonal sets of parallel vertical planes. In designing a system of this type a basic concern is one of shape determination, that is, of finding the surface formed by the cable network. It is not always desirable or necessary that the problem be solved mathematically. The following formulation of the problem is not a prelude to a mathematical solution, but is for convenience in understanding the method of solution by analog. A somewhat more rigorous mathematical treatment has been developed by Siev and

Eidelman (3).

Figure 1 represents a two-directional cable net anchored in a general non-planar frame. The cables lie in parallel planes in each of the two directions. As is the usual case, the sets of planes are orthogonal.

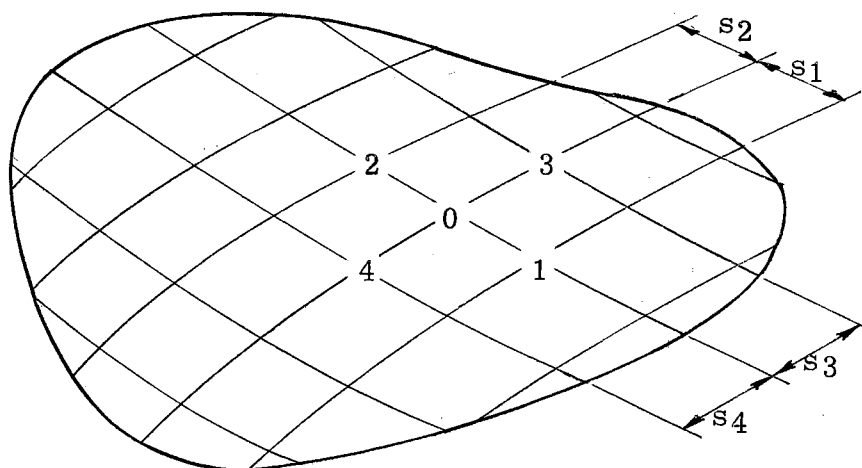


Figure 1. Cable Network in General Non-planar Frame.

In the absence of stress-relieving external forces, the cables are in tension as a result of prestressing. The tensile force is not necessarily the same in all cables. To describe the tensile forces acting in the network, any of the nodes may be taken as the origin and labeled node O, while the adjacent nodes are labeled 1, 2, 3, and 4, as shown.

Node O is acted upon by the tensile forces T_1 , T_2 , T_3 , and

T_4 , as shown in Figure 2. An additional subscript will serve to identify the x-, y-, and z- direction components of each of these forces. Thus T_1 has a horizontal component T_{1x} and a vertical component T_{1z} , and T_3 has a horizontal component T_{3y} and a vertical component T_{3z} , etc. Let the direction angles of nodes 1, 2, 3 and 4 be denoted by α_1 , α_2 , α_3 , and α_4 , respectively. Taking the elevation of node O as z_0 , let the elevations of the other nodes be denoted by z_1 , z_2 , etc. The horizontal distance between node O and node 1 is denoted by s_1 , between node O and node 2 by s_2 , etc.

From the conditions of equilibrium in the x- and y- directions, it follows that for each cable the horizontal component of force is constant over the whole length of the cable. Thus in Figure 3 it will be seen that the horizontal force components T_{1x} and T_{2x} are equal in magnitude, and may be written

$$T_{1x} = T_{2x} = T_x.$$

Similarly,

$$T_{3y} = T_{3y} = T_y.$$

The vertical components of force may be expressed in terms of the horizontal forces and the node direction angles, that is,

$$T_{1z} = T_x \tan \alpha_1,$$

$$T_{2z} = T_x \tan \alpha_2,$$

$$T_{3z} = T_y \tan \alpha_3,$$

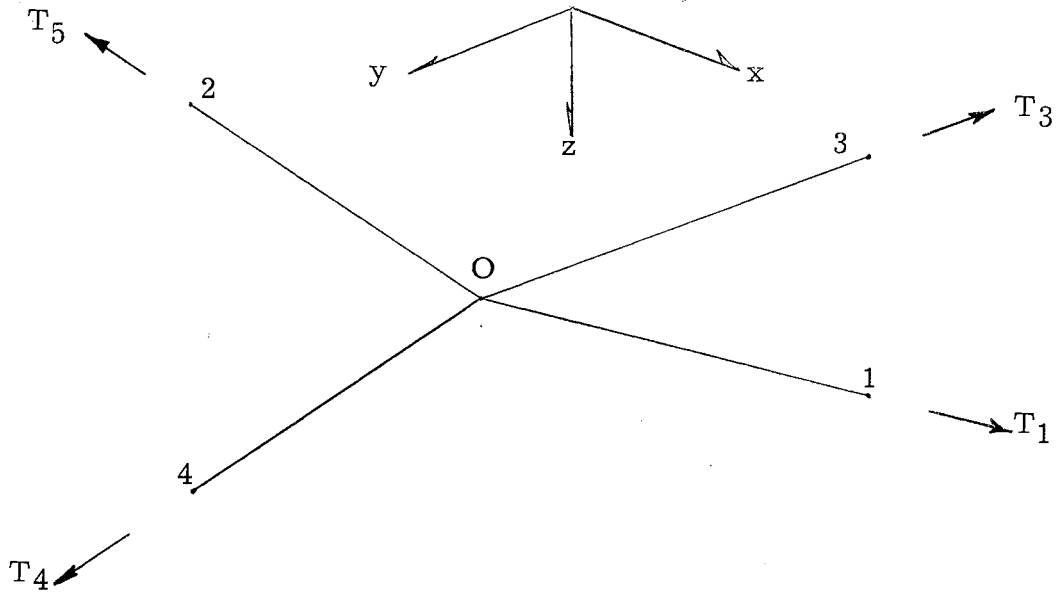


Figure 2. Tensile Forces Acting on a Node of the General Cable Network.

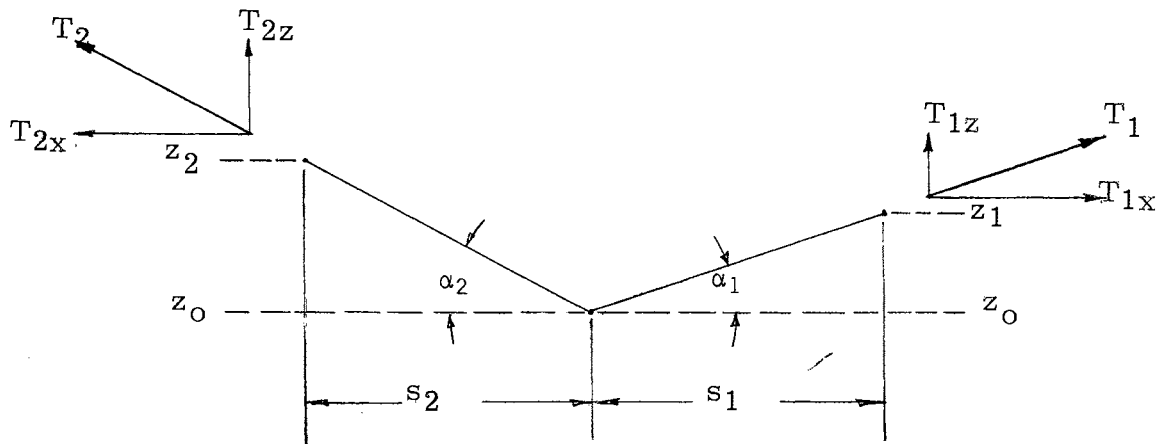


Figure 3. Section Through Node

$$T_{4z} = T_y \tan \theta_4$$

But in general, from the geometry of the system, $\tan \theta_n = (z_n - z_0)/s_n$. The vertical components of force may therefore be expressed in terms of the horizontal force components and the node elevations, that is,

$$\begin{aligned} T_{1z} &= T_x (z_1 - z_0)/s_1, \\ T_{2z} &= T_x (z_2 - z_0)/s_2, \\ T_{3z} &= T_y (z_3 - z_0)/s_3, \\ T_{4z} &= T_y (z_4 - z_0)/s_4. \end{aligned} \quad (2.1)$$

From the condition of equilibrium in the z-direction, and neglecting the weight of the cables,

$$T_{1z} + T_{2z} + T_{3z} + T_{4z} - 0 = 0. \quad (2.2)$$

Substituting Equations (2.1) in (2.2) yields the node force equilibrium equation

$$T_x (z_1 - z_0)/s_1 + T_x (z_2 - z_0)/s_2 + T_y (z_3 - z_0)/s_3 + T_y (z_4 - z_0)/s_4 = 0. \quad (2.3)$$

A node force equilibrium equation may be written for each node of the network. If the cable spacings, the height of each cable-to-frame attachment point above a reference plane, and the horizontal components of the cable tension force are fixed (known) quantities, then the only unknowns remaining are the heights of the interior nodes. The number of available equations is thus equal to the number of unknowns, and the problem becomes one of solving a large number of simultaneous linear equations. This task, while burdensome, can

be accomplished by many well known numerical methods (6). Alternatively, a solution may be obtained easily and accurately by analog methods, as will be shown in later chapters.

For the case of equal cable spacing Equation (2.3) reduces to:

$$T_x (z_1 - 2z_0 + z_2) + T_y (z_3 - 2z_0 + z_a) = 0. \quad (2.4)$$

The expressions in parentheses in Equation (2.4) are proportioned to the second central differences of functions $z(x)$ and $z(y)$. Using finite difference notation, Equation (2.4) may be written as

$$T_x (\sigma^2 z / \sigma x^2) + T_y (\sigma^2 z / \sigma y^2) = 0. \quad (2.5)$$

If the cables are closely spaced, essentially forming a continuous surface, the finite difference functions may be replaced by continuous functions to obtain the equation

$$T_x (y) (\partial^2 z / \partial x^2) + T_y (x) (\partial^2 z / \partial y^2) = 0, \quad (2.6)$$

which is the equation for the surface of the system. Here $T_x (y)$ represents the horizontal component of force per unit width in the x - direction, this force being a function of y . Similarly, $T_y (x)$ denotes a function of x .

When the tension is the same in all cables, Equation (2.6) reduces to the familiar Laplacian equation

$$(\partial^2 z / \partial x^2) + (\partial^2 z / \partial y^2) = 0. \quad (2.7)$$

If the cable tension is not equal in both directions, that is,

$$T_x (y) = C_1, \quad T_y (x) = C_2,$$

substituting in Equation (2.6) yields

$$C_1 (\partial^2 z / \partial x^2) + C_2 (\partial^2 z / \partial y^2) = 0.$$

A suitable change of variable, e. g. ,

$$\bar{x} = x (C_2 / C_1)^{1/2},$$

again yields the Laplacian equation

$$(\partial^2 z / \partial \bar{x}^2) + (\partial^2 z / \partial y^2) = 0. \quad (2.8)$$

CHAPTER III

ELECTRICAL ANALOGS FOR SUSPENDED ROOF SYSTEMS

An analog is a system in which the mathematical model corresponds to another system. When an analogy exists between two systems, information derived for one system may be directly related to the other. The fact that analogies do exist can be of considerable practical importance to the architect - engineer engaged in the design and analysis of physical systems.

A physical system may be described as an assemblage of one or more elements, in either lumped or continuous form, which responds in a known or predictable manner to some excitation. In two analogous systems there is a one-to-one correspondence between each element of the two systems and between the excitation and response of these elements and the system as a whole.

An analogy may exist between systems in the same physical category, as in the familiar case of a scale model. Analogies also may exist between systems in different physical categories, e. g., between a vibrating string and an inductance-capacitance network.

The basis for the similarity between otherwise unrelated phenomena is the existence of the fundamental principles of conservation of energy and continuity, which underlie most physical systems. These principles lead to a similarity in the form of the mathematical equations which describe analogous systems. Analogies are, in fact, ordinarily developed and demonstrated by noting the similarity between the characteristic equations of the systems.

The Resistance Network Analog

An electrical system consisting of a network of interconnected resistors is analogous to the cable network described in Chapter II. The principle of conservation of energy, which governs the static equilibrium of the cable network, and the principle of conservation of charge, which governs the current equilibrium of the electrical network, are the basis of the analogy. These principles lead to the familiar statements regarding equilibrium in the systems, that is, in the cable system the sum of the forces acting on each node is equal to zero, and in the resistor system the sum of the currents into each node is equal to zero.

Figure 4 represents a portion of an electrical network which corresponds to the portion of the cable network that was shown in Figure 2.

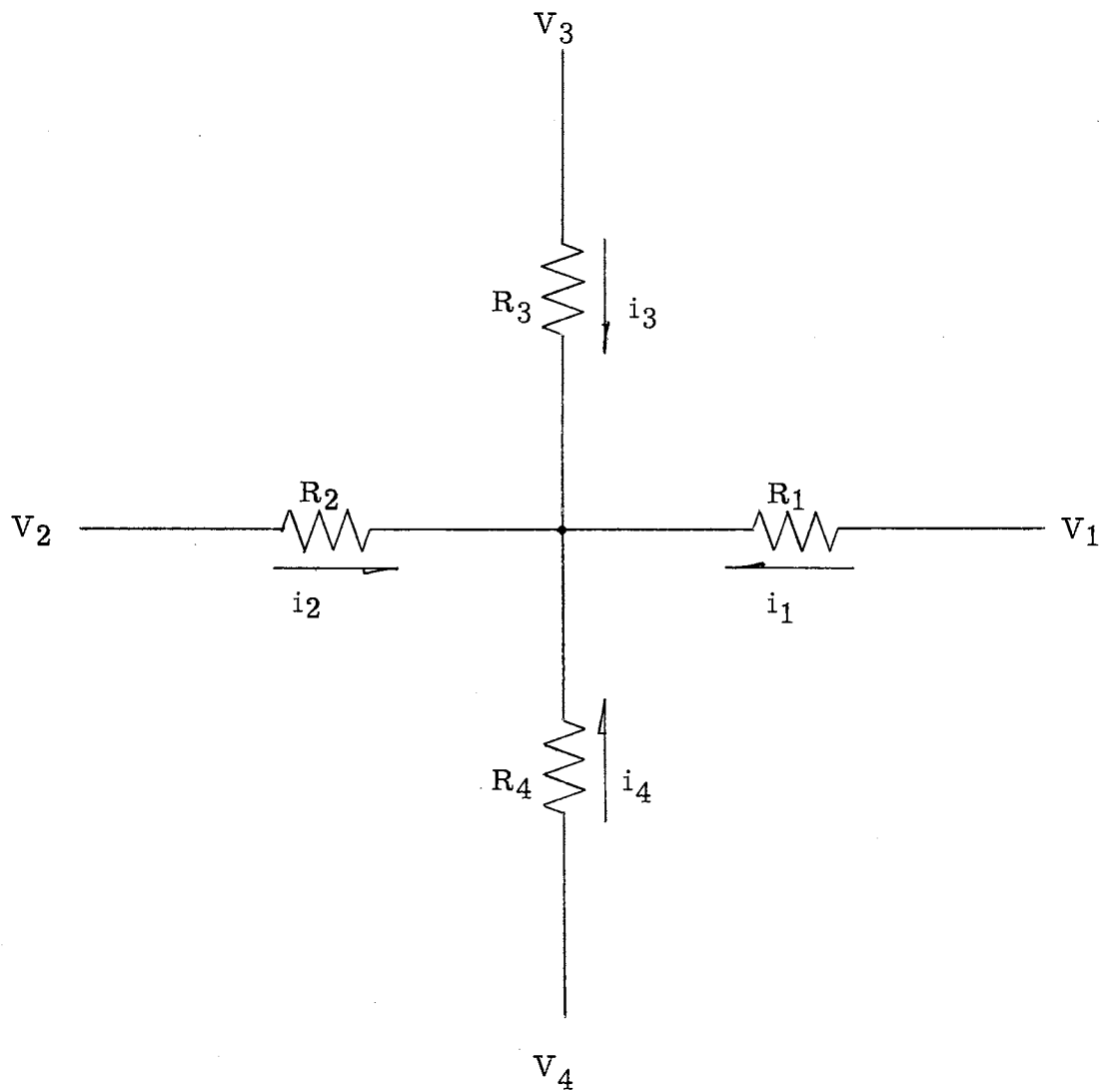


Figure 4. Portion of Resistance Network

It is assumed that all of the resistance in any branch is lumped in a single resistor, R . From Ohm's Law the current, i , in any branch is proportional to the difference in potential across the branch and is inversely proportional to the branch resistance, or in general,

$$i_j = (V_j - V_o) / R_j .$$

From Kirchoff's Current Law,

$$i_1 + i_2 + i_3 + i_4 = 0 ,$$

which may be written in terms of voltages and resistances as

$$\begin{aligned} (V_1 - V_o) / R_1 + (V_2 - V_o) / R_2 \\ + (V_3 - V_o) / R_3 + (V_4 - V_o) / R_4 = 0 . \end{aligned} \quad (3.1)$$

Comparing this with Equation (2.3) ,

$$\begin{aligned} T_x (z_1 - z_o) / s_1 + T_x (z_2 - z_o) / s_2 \\ + T_y (z_3 - z_o) / s_3 + T_y (z_4 - z_o) / s_4 = 0 , \end{aligned}$$

reveals that voltage V and height z are analogous quantities, provided that the resistance R are proportional to the cable spacings s and inversely proportional to the horizontal tension components T . The practical implication of the analogy is that an exact solution for the shape of a suspended roof cable network can be obtained by using a corresponding network of proportioned resistances. If voltages which are proportional to cable end points (i. e. , the cable -

to-frame attachment points) are applied to corresponding points on the resistance network, the resulting voltages at nodes within the resistance network will be proportional to the heights of the corresponding interior points of the cable network.

It will be noted that a resistance network, being the discretized equivalent of a continuous resistive field, may also be employed to approximate the shape of a continuous-surface suspended roof system. Here, mathematically, the resistance network represents the finite difference approximation of the Laplacian equation developed in Chapter II.

The Conductive Sheet Analog

An electrical system consisting of a sheet of uniformly resistive material is analogous to a continuous-surface suspended roof system. The basis of the analogy is, again, the fundamental physical principle of conservation. The analogy is readily demonstrated by comparing the characteristic equations of the systems.

It will be recalled from Chapter II that the Laplacian equation in two dimensions,

$$\left(\frac{\partial^2 z}{\partial x^2}\right) + \left(\frac{\partial^2 z}{\partial y^2}\right) = 0,$$

governed the shape of the continuous roof surface. It has been shown (5) (7) that for a two-dimensional continuous electrical system

the distribution of the voltage v is described by the Laplacian equation

$$\left(\frac{\partial^2 v}{\partial x^2}\right) + \left(\frac{\partial^2 v}{\partial y^2}\right) = 0 \quad (3.2)$$

Comparing Equations (2.7) and (3.2), the analogy between the elevation parameter of the roof system and the voltage parameter of the electrical system is clearly established.

In general, application of the analogy to solve a roof problem requires the use of a conductive sheet of the same shape as the plan of the roof under study. Suitable voltages are applied to the boundaries of the analog system to simulate the edge elevations of the roof system. The resulting voltages in the analog field are proportional to the elevations at corresponding points on the roof surface. The conductive sheet analog method provides an exact solution to a roof problem governed by Laplace's equation.

Having previously noted that the resistance network analog provides an approximate solution to roof surfaces governed by Laplace's Equation, it may be reasoned that the conductive sheet analog can provide an approximate solution to a roof surface governed by finite difference equations. The latter method, a definite extension of the scheme proposed by Siev and Eidelman (3), represents a significant opportunity for exploitation of the uses of conductive sheet analog systems.

CHAPTER IV

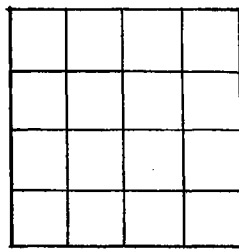
SOLUTION BY ANALOG METHODS

A number of experimental analog systems have been constructed by the author to investigate and demonstrate the solution of roof shape problems by electrical analog methods. Four such systems are described in the following examples, which have been selected as representative of a variety of roof systems.

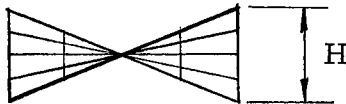
Illustrative Examples

Example 1. A hyperbolic paraboloid cable network is bounded by a frame which consists of inclined members of equal length, Figure 5. All cables are in equal tension. The cable planes are equally spaced, and the cables are interconnected at each point of intersection. The vertical height of the roof system, that is, the difference in elevation between the highest and lowest cable-to-frame attachment points, is some fixed value, H . The elevations of intermediate points on the frame are expressed in terms of H , that is, $.750H$, $.500H$, $.250H$.

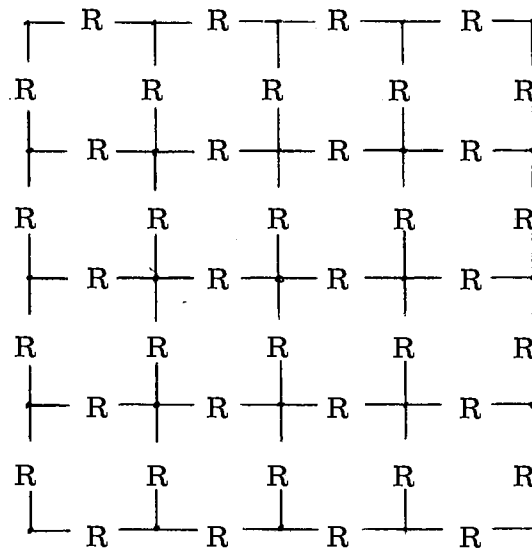
The solution to this problem was obtained using an electri-



(a) Plan



(b) Elevation



(c) Resistance Analog

Figure 5. Cable Net in Bent Square Frame

.000	.250	.500	.750	1.000
.000	.251	.502	.750	1.000
.250	.375	.500	.625	.750
.250	.375	.502	.625	.750
.500	.500	.500	.500	.500
.500	.501	.501	.501	.501
.750	.625	.500	.375	.250
.751	.625	.498	.374	.249
1.000	.750	.500	.250	.000
1.000	.751	.499	.250	.000

Height coefficient shown above each node. Voltage coefficient shown below each node.

Figure 6. Solution for Example 1.

cal analog consisting of a network of forty 1 - megohm resistors, R . A 250 v-dc power supply was the source of voltage, V , corresponding to H . A voltage divider consisting of four 1000 ohm resistors was placed in series with the power supply to obtain voltage values of 1.000 v, .750 v, .500 v, .250 v, and .000 v, corresponding to the frame edge elevations. With these voltages applied to the analog system, the resulting interior node voltages were read in terms of V corresponding to H . (A description of the voltage measuring device is contained in the Appendix.) The solution for the problem is shown in Figure 6.

Example 2. A cable net is bounded by a circular frame which consists of two inclined arcs, Figure 7. All cables are in equal tension. The cable planes are equally spaced. The cables interact at each point of intersection. Elevations of points on the frame are expressed in terms of the overall height, H , of the roof system.

The solution was obtained using an electrical analog consisting of a network of 1 - megohm resistors, R , in a grid shape which approximated the plan of the roof system frame. A 250 v-dc power supply was the source of the edge voltages. A voltage divider, consisting of a 20 inch long strip of conductive paper, was used in conjunction with the power supply to obtain the necessary edge voltage values. (A description of the voltage divider is contained in the

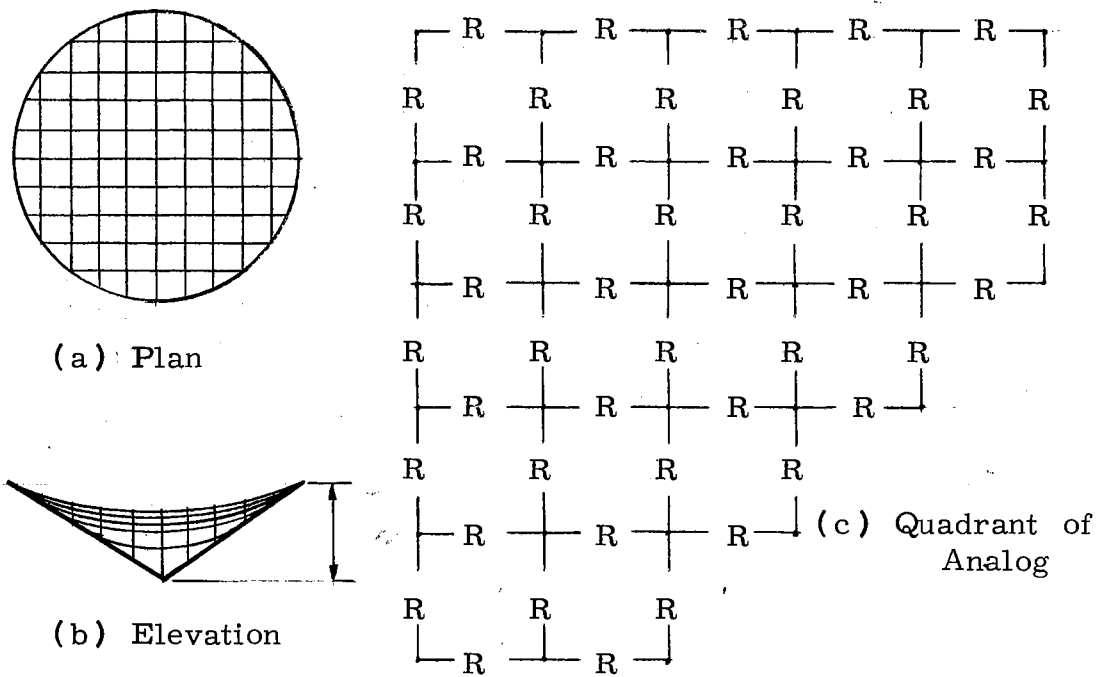


Figure 7. Cable Net in Bent Circular Frame

.626	.646	.687	.777	.880	1.000
.630	.650	.698	.770	.873	1.000
.610	.629	.682	.765	.872	.960
.616	.630	.678	.768	.858	.979
.556	.578	.639	.730	.844	.916
.557	.582	.645	.740	.837	.916
.458	.490	.568	.675	.800	
.456	.491	.582	.680	.800	
.294	.358	.467	.600		
.304	.374	.492	.606		
.000	.200	.400			
.000	.201	.408			

Height coefficient shown above each node. Voltage coefficient shown below each node.

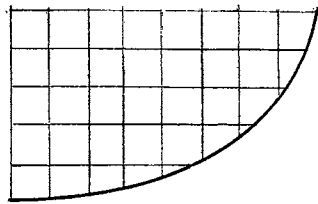
Figure 8. Solution for Example 2.

Appendix.) The analog system then yielded the solution shown in Figure 8.

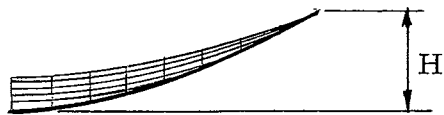
Example 3. A cable net is bounded by a frame which is elliptical in plan and parabolic in elevation, Figure 9. All cables are in equal tension. The cable planes are equally spaced. The cables interact at each point of intersection. Elevations of points on the frame are expressed in terms of the overall height, H , of the roof system.

The solution was obtained using an electrical analog consisting of a network of 1 - megohm resistors, R , in a grid which approximated the ellipsoidal plan of the roof system. It was noted that, in the vicinity of the roof frame, some node-to-frame distances were approximately one-half or one-third of the normal cable spacing. Effective resistances of one-half and one-third R used at corresponding places in the analog system in an effort to more closely approximate the shape of the roof system. Then, using the power supply and voltage divider system described in the preceding example, appropriate voltages were applied to the peripheral nodes of the analog system, and the voltages in terms of V read at the interior nodes. The solution is shown in Figure 10.

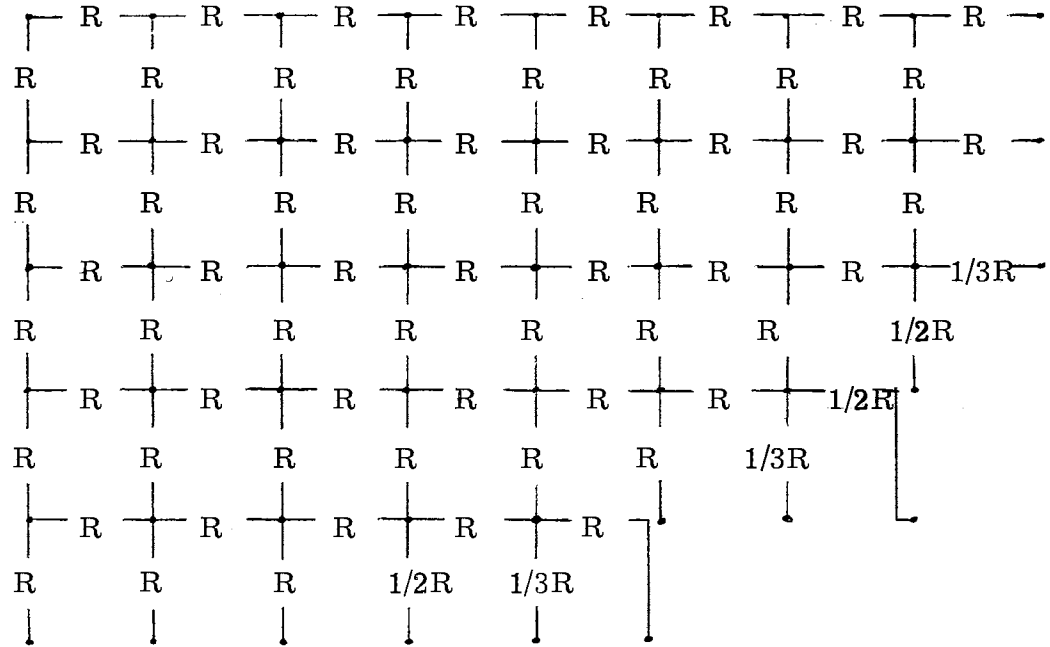
Example 4. A continuous membrane roof is bounded by the frame described in the previous examples. The tensile stresses



(a) Quarter Plan



(b) Half Elevation



(c) Quadrant of Analog

Figure 9. Cable Net in Warped Ellipsoidal Frame.

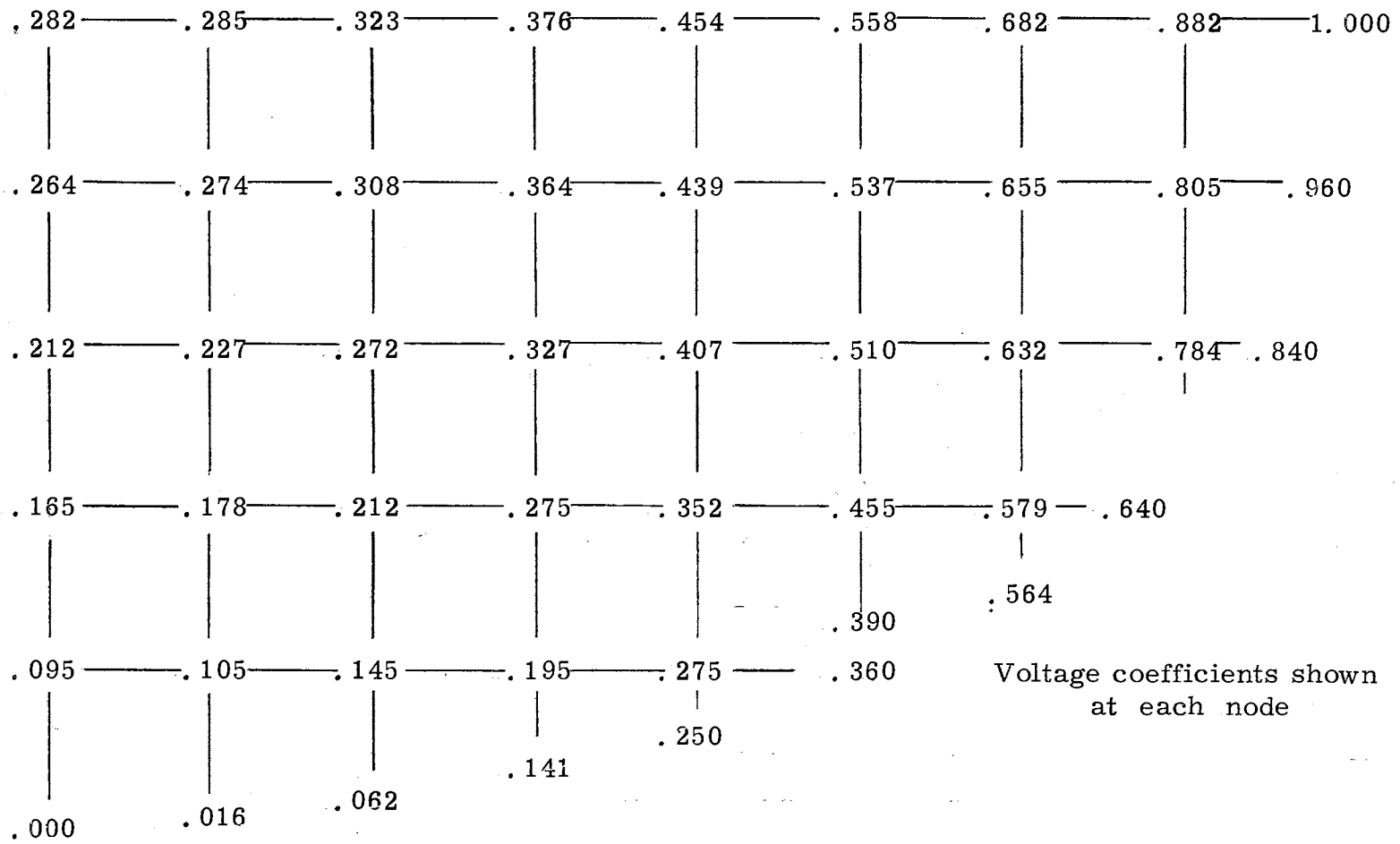


Figure 10. Solution Quadrant for Example 3.

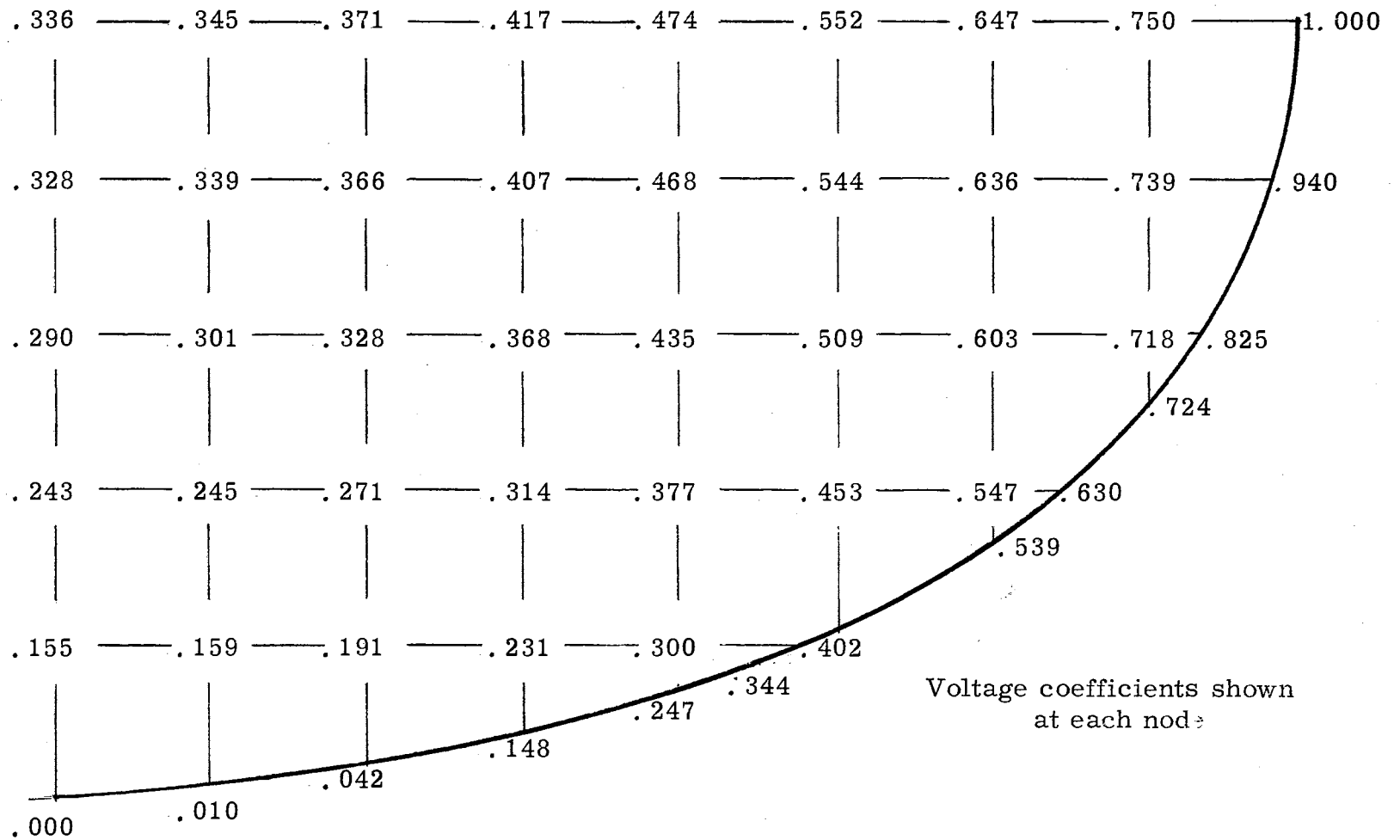


Figure 11. Solution Quadrant for Example 4.

per unit width are equal in the two principle directions of the roof.

A solution was obtained using a conductive-sheet analog. The analog system was constructed of Teledetos paper, a material having a resistivity of approximately 1500 ohms per square. The paper was cut to the ellipsoidal shape of the roof plan. Using the 250 v-dc power supply, and a voltage divider consisting of a number of fixed resistors, the continuously varying frame elevation was approximated by a series of fixed voltages applied at discrete intervals along the boundary of the sheet. The solution is shown in Figure 11.

Summary and Conclusions

Calculation of the shape of roof structures by numerical methods ordinarily involves an amount of computational effort that varies with the geometrical complexity of the roof surface. The amount of this effort can be relatively high in cases of roof surfaces of double curvature. When mathematical techniques are inadequate or impracticable, some alternative method of solution must be employed. Solution by electrical analog, which has been widely used in other areas of engineering analysis and design, has recently been recommended for use in determining the shape of roof systems formed by prestressed cable networks (3). This suggestion was

taken as the basis for further engineering research, with the objectives of (1) establishing a mathematical basis for the correspondence between suspended roof systems and analogous electrical systems, and (2) developing practical analog systems for the solution of roof shape problems.

Characteristic node equations were developed for a general network of counteracting cables in orthogonal planes, and for a network of electrical resistance elements. These equations demonstrated the analogy between the systems. A correspondence also was found between the equations for a continuous roof surface and a continuous conductive sheet analog system.

The resistance network analog systems proved to be simple, reliable, and convenient. In cases where the shape of the prototype could be simulated exactly using a standard pattern of resistors of equal magnitude, a relatively high degree of accuracy was obtained; the measured coefficients of voltage were consistently within 0.2% of the true values. When a curved boundary was approximated with a standard resistance network, as in Example 2, the measured coefficients were within 2.5% of the calculated values. The conductive strip voltage divider was of considerable value, as it yielded more accurate voltage values than could be obtained with a system of fixed resistors.

The experimental conductive sheet analog systems successfully

simulated the shape of prototype roof systems. However, less accurate results were obtained than with the resistance network systems.

In general it is considered that analog systems can be a practical and effective means of roof shape determination. The method may also be a value in the analysis and design of other structural systems, and is deserving of further investigation and experimentation.

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APPENDIX

APPENDIX

COMPONENTS OF EXPERIMENTAL SYSTEMS

Resistance Network System Components

A non-conductive plastic panel served as a mounting board for the network of 1-megohm wirewound resistors. Terminal lugs were bolted to the panel to form a rectangular array of 480 possible node points. Individual snap-type connectors were soldered to each resistor lead, facilitating mounting and interconnecting of the resistors. The number of resistors used in the construction of individual analog systems ranged from 40 to 278, the quantity varying with the complexity of the problem and the accuracy desired. A typical assembly of resistance network system components is shown in Figure 12.

Conductive Sheet System Components

The conductive sheet analog systems were constructed of a material which is marketed under the trade name of Teledeltos, and which in recent years has found great popularity as a conductive sheet analog. The material consists of a carbon-filled paper base with a lacquer backing and a thin aluminum facing.

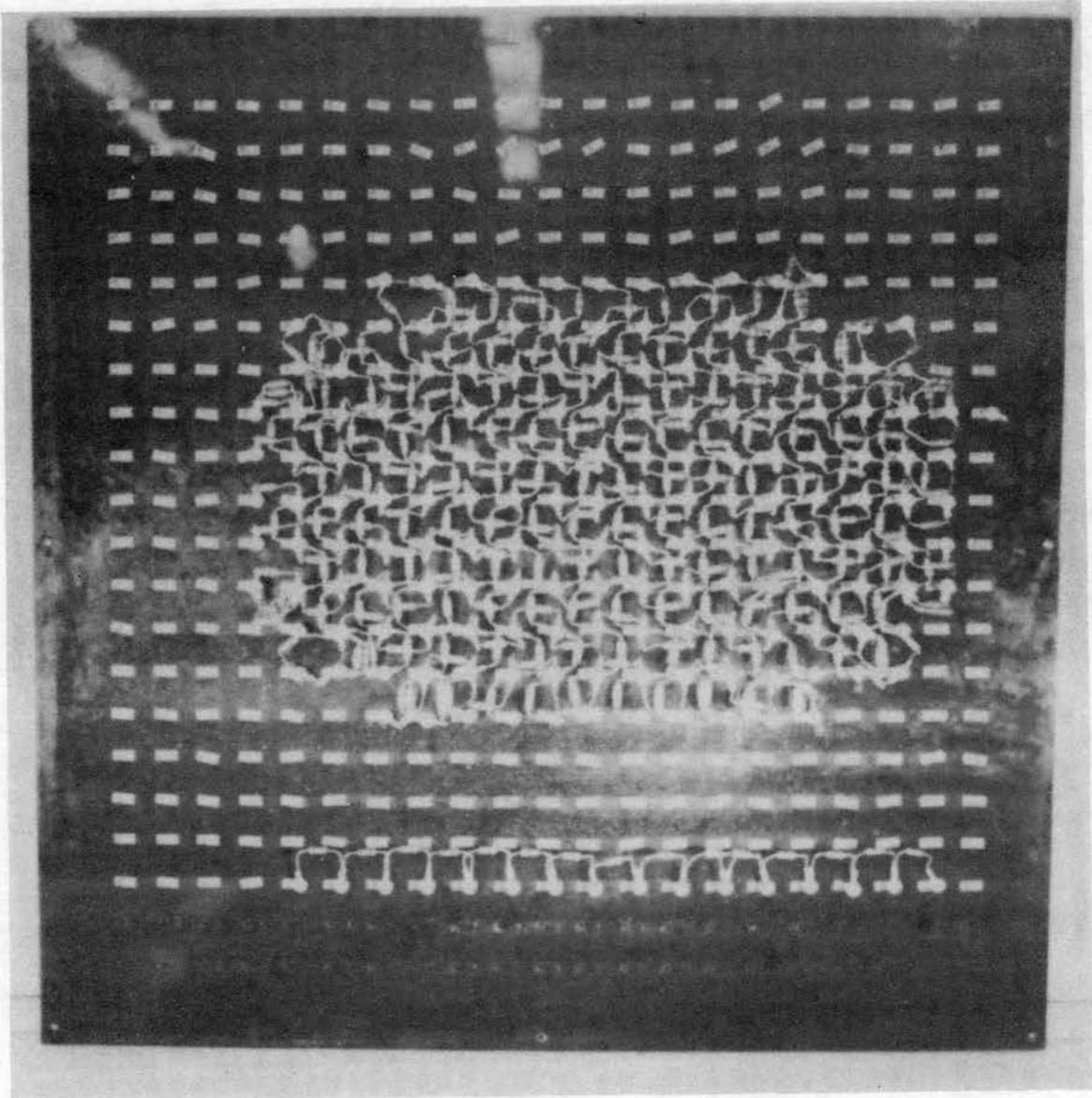


Figure 12. Resistance Network Analog

The paper was cut to the desired shape, and marked with grid lines to facilitate identification of points on the surface. A non-conducting fibrous panel served as a mounting board for the paper. Common push pins were used for the dual function of fastening both the paper and the boundary node terminals to the mounting board. A typical assembly of conductive sheet system components is shown in Figure 13.

As the Teledeltos paper available for analog construction was of unknown resistivity, a series of tests was conducted to evaluate this characteristic. The test specimens consisted of rectangular pieces of material cut to arbitrarily selected sizes. One-half inch wide silver point electrodes, having negligible resistance, were applied to two opposing edges of each specimen. A standard Simpson Model 260 volt-ohm-milliammeter was then used to measure the resistance between the electrodes of each specimen. The resistivity of each specimen was then calculated, being taken as the product of the measured resistivity and the ratio of the effective length (distance between electrodes) to the width of the specimen. The approximate average value of resistivity to current flow parallel to the roll axis of the paper was found to be 1300 ohms per square. Perpendicular to the roll axis, the value was 1475 ohms per square.

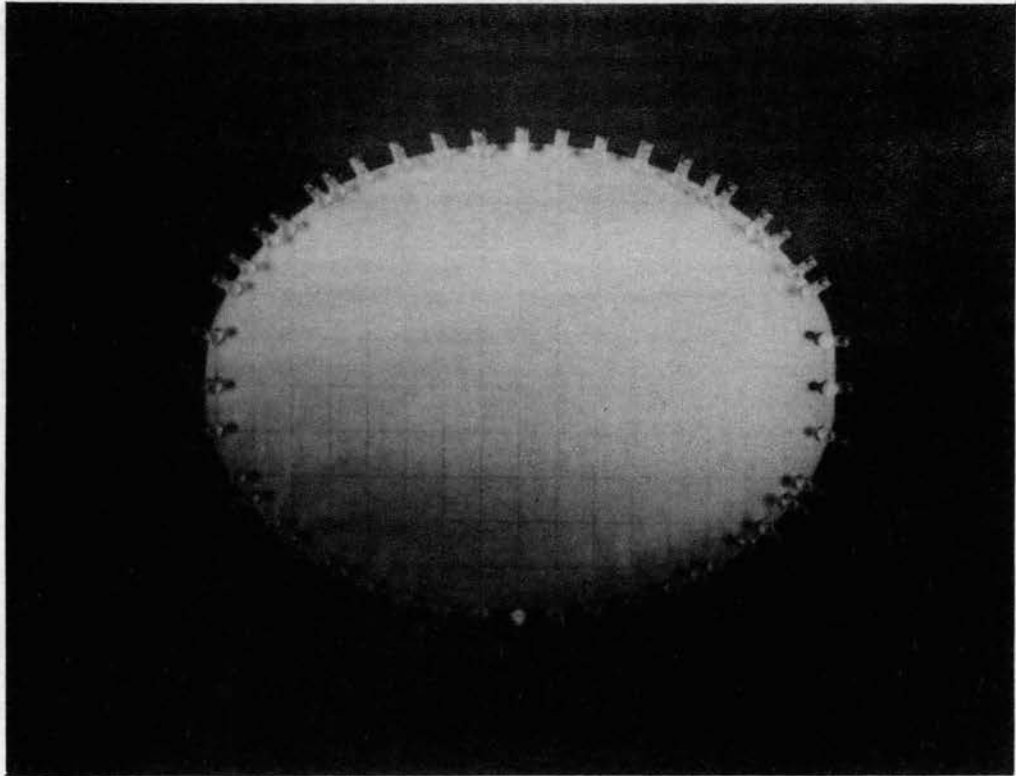


Figure 13. Conductive Sheet Analog

Conductive Strip Voltage Divider

A voltage divider constructed of conductive paper was devised and successfully employed with the resistance network analog systems. The device consists of a strip of Teledeltos paper cemented to a non-conductive backing board. The strip is one inch wide and twenty-one inches long, and has a one-half inch wide silver point electrode at each end. Terminals at the ends of the strip provide contact points for the input voltage. Other terminals are located along the length of the strip to provide taps for the output voltages. The terminals are secured in slots which run lengthwise on both sides of the conductive strip. The number of terminals can be changed at will, and their location can be adjusted as desired to yield accurate output voltages.

The device described above has an overall resistance of 42,000 ohms. A similar device was constructed having a resistance of only 260 ohms. The dimensions of the conductive paper strip, in the latter case, was five and one-half inches by thirty inches. A one-quarter inch wide silver point electrode was applied on the long edges. The paper was then pleated to form a stack five and one-half inches long and one inch wide. This element was then attached to a backing board, for use in the same manner as the previously constructed divider.

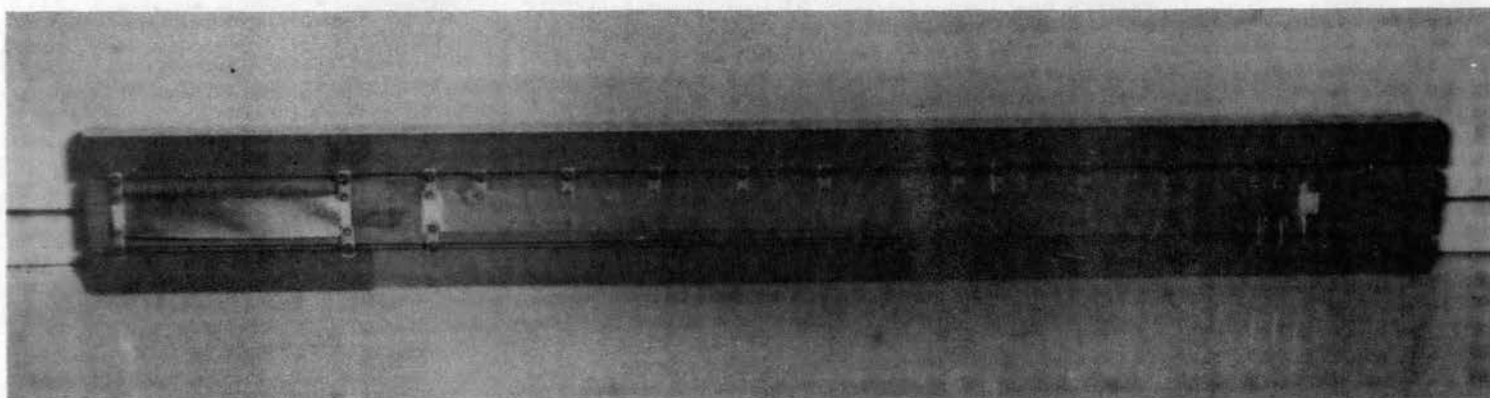


Figure 14. Strip Voltage Dividers.

The two dividers are shown in Figure 14.

Power Supply

The source of the excitation voltages of the analog systems was the direct current power supply shown in Figure 15. The voltage values which could be obtained by using various combinations of the output jacks ranged from 42 volts to 320 volts, with 8 fixed intermediate values.

Voltage Measuring System

The device shown in Figure 16 was used to sense the electrical potential at points of interest in the analog system.

The components of the voltage measuring system included a precision 10-turn potentiometer (variable resistor), a direct current galvanometer, and a standard test probe. Additional resistors in the circuit assured a high input impedance so that the device would not itself affect the distribution of potential in the analog.

For use with analog system, the device is connected as shown in Figure 17, with the output voltage of the system power supply applied across the end terminals of the potentiometer. With the probe at a point where potential is to be measured, the potentiometer knob is adjusted to produce a null current reading

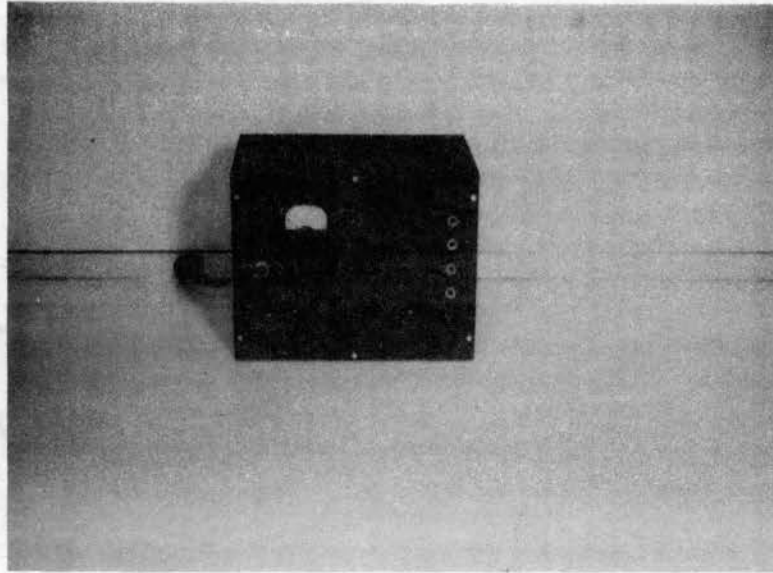


Figure 15. Power Supply

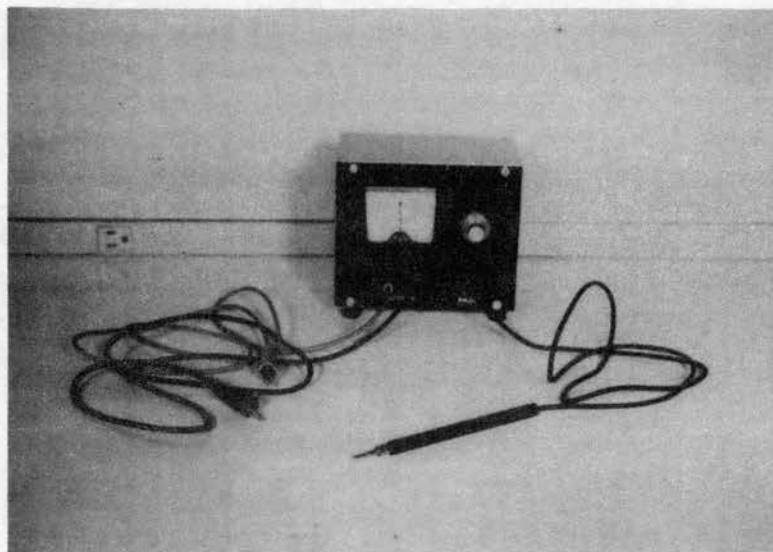


Figure 16. Voltage Measuring Device.

on the galvanometer dial, thus indicating that the voltage applied through the potentiometer balances the voltage sensed by the probe.

The measure of voltage is obtained by reading the calibrated knob of the potentiometer. The dial yields accurate readings to the nearest 1000th of the total voltage applied across the potentiometer. Figure 18 shows the voltage measuring being used in a typical analog setup.

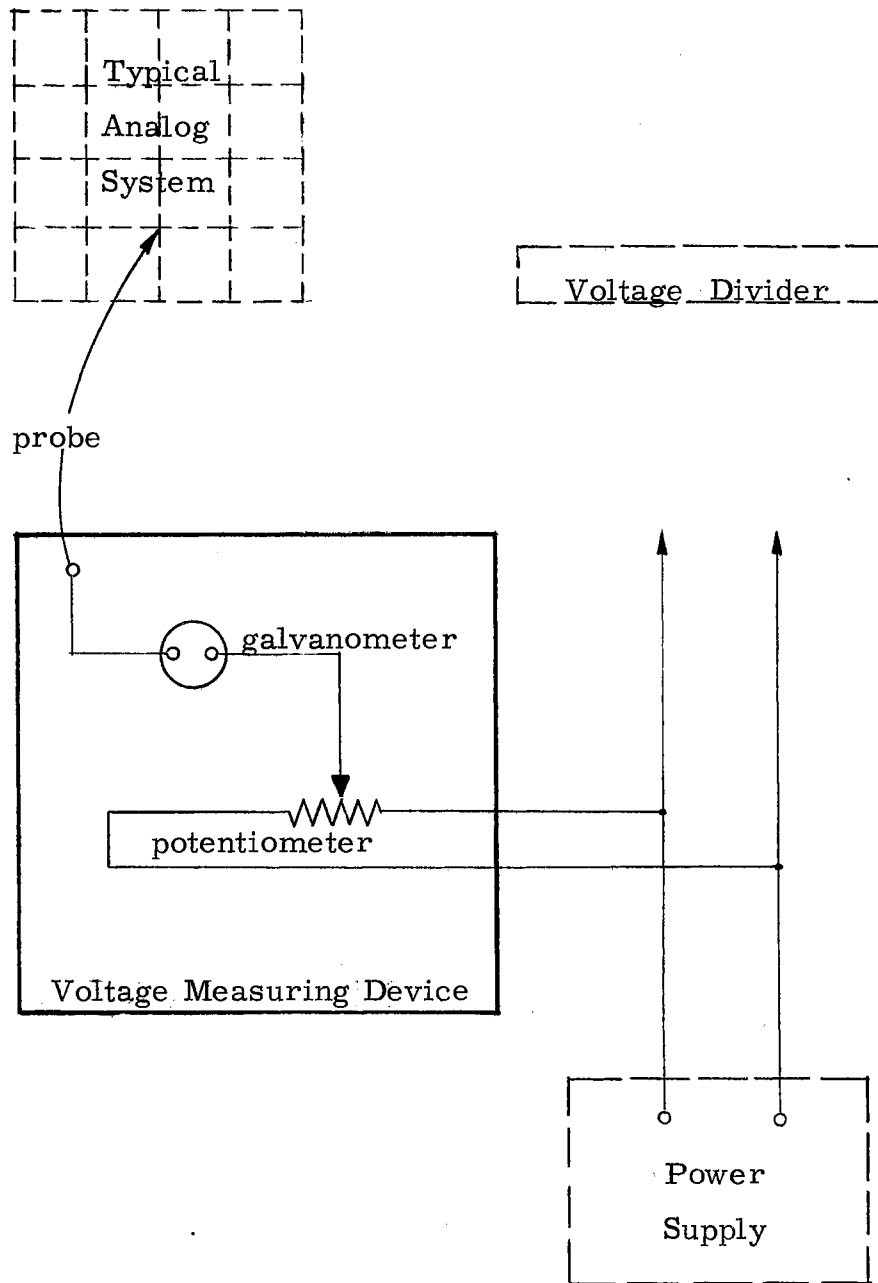


Figure 17. Simplified Schematic of Voltage Measuring System.

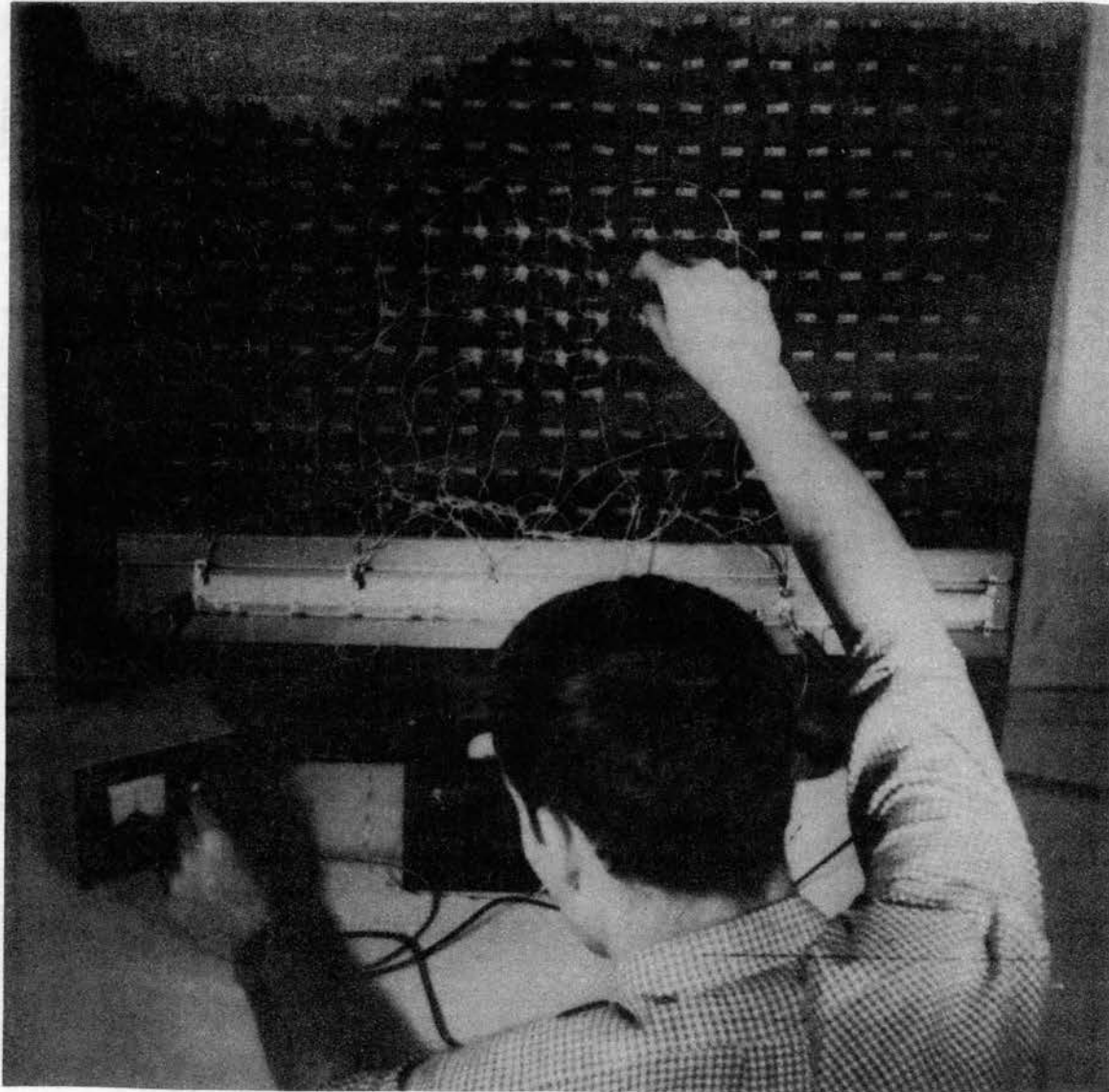


Figure 18. Assembled Analog System.

VITA

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