AN APPLICATION OF GAME THEORY

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PREFACE

Game theory, a mathematical model providing a method for the study of decision making in situations of conflict, has laid the foundation for a systematic and penetrating treatment of a vast range of problems in the social sciences, military operations, statistics, and industry. While game theory is generally well known to the mathematician and statistician, practical applications have been relatively meager. It is the purpose of this thesis to utilize game theory in presenting an approach, application, and solution to a problem which confronts decision makers involved in the game of football; that of strategy optimization.

Several acknowledgments are in order. First, I wish to express my indebtedness to the Ideal Cement Company, whose research fellowship at Oklahoma State University helped to make this study possible. I would also like to express my gratitude to Dr. James E. Shamblin, for his invaluable guidance and assistance in the preparation of this thesis, and to Professor Wilson J. Bentley, whose interest in both academic and non-academic matters has been an inspiration. I would like to thank Rodney L. Cleavelin, a fellow student, for his aid in the use of the 1620 computer, and Miss Velda Davis for her valuable typing of this study. Particularly, I wish to express my gratitude to my wife, Carla D. Jeter, for her confidence and patience throughout not only this study but my entire graduate program.

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CHAPTER I

INTRODUCTION: CONCEPTS AND TERMINOLOGY

One of the most vexing and persistent problems of an individual in any conflicting situation is that of out guessing his rival. If only he could calculate in advance what the competition was going to do, his planning would be far easier and more effective. Obviously, the simplest approach is applicable where experience with the behavior of a competitor makes it relatively easy to predict his strategies. Where such information is available, it is possible to choose that decision which maximizes the individual's expected gain or return after the effects of his rival's countermoves are taken into consideration. However, it is often against the competitor's interest to permit this sort of analysis in order to avoid a too obvious pattern in his decision making.

It is also possible to approach the analysis of competitive behavior by a more deductive route. Instead of asking, inductively, what can be inferred from a competitor's past behavior, one seeks to determine a rival's most profitable counterstrategy to one's own "best" moves and to formulate the appropriate defensive measures. This is the approach on which game theory has been adopted (1).

Game theory, usually considered to have originated in the late 1920's with the mathematician, J. Von Neumann, is a method for the study of decision making in situations of conflict. It deals with problems in which the individual decision maker is not in complete control of the factors influencing the outcome of the conflict. Politicians, industrialists, bandits, and bridge players are all involved in struggles which may be classified as game situations (2). The essence of a game problem is that it involves individuals or groups of individuals with different goals or objectives whose fates are interlocked.

A game is described in terms of the participants or decision makers, the rules of the game, the payoffs or outcomes of the game, the valuation that the participants assign to various payoffs, the variables that each player controls, and the information conditions that exist during the game. These elements, common to all conflicting situations, are the building blocks of game theory. They play the same role in game theory as do particles and forces in a theory of mechanics (2).

As is the case in any presentation of this nature, terminology plays an important role in clearing the path for a better understanding of game theory. It is imperative that the following terms be defined in a precise manner to avoid misrepresentation and to provide for a more lucid understanding of the subject.

<u>Game Theory</u>: A mathematical model providing a method for the study of decision making.

<u>Game</u>: A generic term incorporating conflict situations where opposing players attempt to achieve their objectives in relation to a predetermined collection of rules.

<u>Payoff</u>: A yield or result which indicates the success of a player's action.

<u>Play</u>: Any particular instance of a game.

<u>Two Person Zero-Sum Game</u>: A game involving two distinct entities where the motives of the participants are dichotomized: payoff is neither created nor destroyed, but a participant's gain is his opponent's loss.

<u>Strategy</u>: An action or group of actions which a player pursues in an effort to optimize his own payoff. The choice of strategy is based on the supposition that the opponents are of equal intelligence and each continually takes appropriate action to prevent the other from achieving his objective.

<u>Pure Strategy</u>: In repeated plays, a participant selects one and the same strategy from all alternatives open to him.

Mixed Strategy: A varied choice of strategy from one play of the game to the other.

Player: An autonomous decision making game participant who attempts to obtain his objective.

Payoff Matrix: A mathematical expression of a game where one player's alternatives are arranged vertically in columns, and the other player's alternatives are arranged horizontally in rows. Payoff values are assigned or computed for each intersection in the matrix. A general payoff matrix is illustrated in Figure 1.

In this game, P_1 has "m" strategy alternatives and P_2 had "n" strategy alternatives. When P_1 chooses A_1 and P_2 chooses B_1, then P_1 will receive a_{11} and P_2 will receive $-a_{11}$ (3).

Dominance: A situation where one row or column is superior to another row or column in every instance.

Value of the Game: The expected yield or payoff which will be realized by each player if he pursues an optimum strategy determined by a solution to the payoff

P_2

P_1

Strategy	B_1	B_2		B_j		B_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1j}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2j}	$\cdots$	a_{2n}
	$\vdots$	$\vdots$		$\vdots$		$\vdots$
A_i	a_{i1}	a_{i2}	$\cdots$	a_{ij}	$\cdots$	a_{in}
	$\vdots$	$\vdots$		$\vdots$		$\vdots$
A_m	a_{m1}	a_{m2}	$\cdots$	a_{mj}	$\cdots$	a_{mn}

Figure 1. General Payoff Matrix

matrix. The expected payoff will be unfavorable to that player which deviates from the optimal strategy dictated by the solution.

<u>Saddle Point</u>: A payoff value which exhibits the unique property of being the lowest value in its row and the highest value in its column. If a saddle point exists, it is the value of the game and the players' optimal strategies are pure strategies.

Mathematically sound, game theory has several primary limitations: its lack of familiarity and, consequently, its lack of application to situations where it could be of significant value, its difficulty in attempting to quantify the relationships between competitors which are often subjective in nature, and the assignment of game values to the payoff matrix which depend on just a single parameter.

As a guide to decision making, game theory is in its infancy, but it enjoys a rather vigorous and promising future if utilized in applicable situations by individuals who possess a thorough understanding of its inherent limitations and advantages.

Statement of the Problem

In an effort to apply game theory to a real-life situation, this thesis is concerned with the application of game theory to athletic endeavor; specifically, the game of football.

The game of football was chosen for game theory analysis for two primary reasons: (1) Football lends itself well to an analysis of this nature since the success of a maneuver is readily gauged in yards gained or lost and can be appropriately entered into a payoff matrix, and (2) the author's special interest in this area generated by previous experience as a participant in collegiate football.

This study will concern itself with an analysis of offensive and defensive maneuvers of a hypothetical team, team A, which endeavors to optimize its strategies in relation to those of an opponent's. The problems involved in an analysis of this nature will be the development of a payoff matrix which will realistically portray the existing relationship between the hypothetical team and its opponent, the generation of data to be used for payoff values, and a mathematical solution to the matrix to determine the value of the game and the optimum mixed strategy.

Scope of the Problem

Obviously, there are many factors which will alter and affect a team's choice of offensive and defensive maneuvers and strategies in any given instance during a football contest. Among these are: the opposing team's personnel, climate conditions, time remaining in the contest, the present score, immediate "down and yards to go" situation, and field position. It becomes readily apparent that no reasonable mathematical model could encompass all of the diverse variables present at any given instant in a football contest. For this reason, a generalized game situation will be pursued for the purposes of this analysis. The study will progress by assuming a hypothetical contest is to be played with the following conditions and restraints prevailing:

1. Climatic and field conditions are consistent.
2. Time remaining in the contest will play no role in offensive and defensive decision making.
3. The score at any particular instant is assumed to be approximately even.
4. Both offensive and defensive teams will operate under the assumption that immediate "down and yards to go" situations are favorable to the extent that standard maneuvers are involved in the analysis. This eliminates punting, field goal attempts, and "desperation" maneuvers of any sort.
5. Field position favors neither team with respect to field length nor breadth.

CHAPTER II

THE DEVELOPMENT OF THE PROBLEM

Assume that a collegiate football team, team A, is busily preparing for a contest with its arch-rival, team B. During the past few contests, team A has shown signs of both offensive and defensive strength, but a study of previous game statistics and films indicate that far too many offensive and defensive strategies were poorly chosen and grossly inappropriate to the situation. In general, team A has frequently been outguessed and, consequently, many maneuvers have been directed at the stronger aspects of the opposition. The problem is critical and must be resolved before the game with team B. A thorough analysis of offensive and defensive decision making appears to be imperative if team A is to win the game.

Analysis of Offense

Primarily, team A operates offensively from a basic split-T formation with a few minor variations. Offensive play patterns may be broken down into the following basic categories:

1. Inside Running (fast developing): This category of play is utilized with little or no deception

in which the success of the play depends entirely upon speed. Included in this category are dives, "belly" series, slants, sneaks, and counterplays.

2. Inside Running (slow developing): The success of these plays depends primarily upon deception. This category includes draws and trap plays.
3. Outside Running: Speed plays an important role in the success of this category of plays. This category includes pitch-outs and running options.
4. Outside Power: The success of these plays depends upon quantity and quality of blocking. This category includes pitch-backs and wide slants.
5. Deep Pass Patterns: Very slow developing plays are included in this category of passing variations. Surge and option patterns are among the patterns in this category.
6. Short Pass Patterns: Extremely quick developing passes such as "look-ins" and hooks are included in this category.

In an effort to better prepare team A's offense, team B's defensive alignments have been scouted extensively. Scouting reports indicate that they will align defensively in the following basic formations:

1. 4-4-3. This pro-type defense features excellent pass protection and curtailment of outside running but sacrifices the inside running play.

2. 5-4-2. A fundamental split-T defense, this alignment is rather weak on passes, but strong against all types of running plays.
3. 6-3-2. This defense is rather weak in pass patterns.
4. 8-3. This short yardage defense is notoriously weak on passes, but defends well against most running plays.
5. 7-1-2-1. A defense which performs well against most running plays and certain pass patterns.
6. 6-2-3. This alignment defends well against the long pass patterns, but is often weak on outside running plays.

Further study of team B's defensive alignments reveal that they employ the same fundamental defenses that have been utilized by many other previous opponents. It would appear that an analysis of films and statistics from previous games could certainly give an indication of A's ability to move the ball against B's defense and might reveal the relative frequency which an offensive maneuver should be employed.

After an intensive study of films, reports, and statistics from prior games, the average number of yards gained or lost with a specific category of offensive maneuver employed against a specific type of defensive alignment may be compiled. This information can be summarized in a payoff matrix which tabulates what the offense will receive in actual yards gained or lost as a result of each possible combination of offensive play choice and defensive alignment choice. Arranged in a general payoff matrix, the data appears in Figure 2.

A solution to this two-person, zero-sum general payoff matrix will reveal the game value: the expected yield to be realized if an optimum strategy mix is pursued, and the relative frequency which each offensive category should be employed: the optimum strategy mix.

Analysis of Defense

Defensive strategy like offensive strategy may be analyzed effectively by utilization of game theory. Similar to offensive analysis, a quantification of the success of a specific defensive maneuver utilized against a specific offensive maneuver is necessary. Again, this is accomplished by a study of films, statistics, and all available information pertaining to previous contests where the competitors employed offensive strategy similar to that which the immediate opponent is expected to utilize. Assume that the basic defensive alignments and offensive play patterns enumerated previously and used by both teams are, therefore, applicable in defensive as well as offensive analysis.

A study of the available sources reveal the average number of yards which A's defensive team has given up or driven back the previous opposition's offense. Summarized and arranged in a general payoff matrix, the data are in

B Defense

A Offense

Strategy	(1)	(2)	(3)	(4)	(5)	(6)
(1)	6	2	0	-3	-2	2
(2)	1	0	-2	-1	0	-2
(3)	-2	2	3	0	3	2
(4)	4	-5	7	1	-4	-3
(5)	0	6	0	8	0	6
(6)	1	3	3	-4	2	0

Figure 2. Offensive Analysis Payoff Matrix

Figure 3. It will be noted that although offensive and defensive strategies are identical, the payoff values differ from those in Figure 2.

A solution to this two-person, zero-sum general payoff matrix will reveal the game value and the relative frequency which A's defensive team should employ a specific alignment against B's offense.

B Offense / Strategy	A Defense (1)	(2)	(3)	(4)	(5)	(6)
(1)	0	3	2	1	-2	1
(2)	1	-2	2	1	0	2
(3)	3	1	-1	3	0	1
(4)	2	5	0	-2	7	-5
(5)	-3	-1	4	0	4	3
(6)	-4	0	6	5	4	8

Figure 3. Defensive Analysis Payoff Matrix

CHAPTER III

DETERMINATION OF SOLUTION

Several methods exist for finding the solution to two-person, zero-sum games. These include trial and error solutions, graphical techniques, linear programming, matrix solutions, fictitious play solution, and algebraic solutions. For the purposes of this study, two methods of solution will be illustrated. The general payoff matrix relating to the offensive analysis will be solved by employing a fictitious play solution, and the general payoff matrix involved in the defensive analysis will be solved by a linear programming technique.

Before attempting to solve any payoff matrix, it is useful to take two preliminary steps. First, a check should be made for existence of a saddle point. Second, certain alternatives may be eliminated by checking for dominance, thus reducing the size of the payoff matrix (3).

Offensive Solution

After checking the matrix illustrating the relationship between A's offense and B's defense in Figure 2 (page 13) for a saddle point and dominance, the matrix is ready for solution by fictitious play. The method of finding a solution by fictitious play is based on a hypothetical series of consecutive plays of a game. In the first play, one player chooses an alternative at random, thereby forcing the other player to choose an alternative which will optimize his expected payoff in relation to the first player's initial choice. Figure 4 illustrates fictitious play of the payoff matrix associated with the offensive analysis as shown in Figure 2. The first play results in totals 2, 0, 2, -5, 6, 3 for A's offense of which 6 (starred) is the optimal payoff since $6 > 3 > 2 > 0 > -5$. The first play totals for B's defense result in 0, 6, 0, 8, 0, 6 of which one of the zero values is chosen at random as B's optimal payoff, since $0 < 6 < 8$. In succeeding plays, the payoffs are the result of total payoffs based on past plays.

It will be noted that the value of the game always lies between the highest average payoff to B's defense and the lowest average payoff to A's offense. In Figure 4, after ten iterations, the highest average payoff to B's defense occurred in fictitious play 8 and is $5/8$. The lowest payoff to A's offense occurred in play 10 and is $1\,3/5$. Hence, after ten iterations, the value of the game, U_o, is

$$.625 \leq U_o \leq 1.60.$$

By increasing the number of plays, the range within which U_o lies can be decreased to any desired accuracy. After

B Defense

A Offense

Strategy	(1)	(2)	(3)	(4)	(5)	(6)	6	4	2	2	2 4/5	2 1/3	2	1 7/8	2	1 3/5		1 8/25		1 3/50	Average Payoff Per Game
(1)	6	2	0	-3	-2	2	2	8*	6	4	1	-1	-3	-5	-7	-1		8		35	Total Payoffs Based on Plays
(2)	1	0	-2	-1	0	-2	0	1	1	1	0	0	0	0	0	1		5		12	
(3)	-2	2	3	0	3	2	2	0	3	6	6	9	12	15*	18*	16*		33*		50	
(4)	4	-5	7	1	-4	-3	-5	-1	-5	-9	-8	-12	-16	-20	-24	-20		-35		-40	
(5)	0	6	0	8	0	6	6*	6	6	6	14*	14*	14*	14	14	14		22		46	
(6)	1	3	3	-4	2	0	3	4	6*	8*	4	6	8	10	12	13		32		53*	
0	0*	6	0	8	0	6	1	2	3	4	5	6	7	8	9	10	...	25	...	50	Number of Play
-1	6	8	0	5	-2*	8	2														
0	7	11	3	1	0*	8	3														
-3/4	8	14	6	-3*	2	8	4														
2/5	8	20	6	5	2*	14	5														
1/3	8	26	6	13	2*	20	6														
2/7	8	32	6	21	2*	26	7														
5/8	6	34	9	21	5*	28	8														
4/9	4*	36	12	21	8	30	9														
1/5	2*	38	15	21	11	32	10														
							⋮														
4/5	22	90	39	20*	21	66	25														
							⋮														
19/25	38*	178	78	48	46	136	50														
Average Payoff Per Game	Total Payoffs Based On Past Plays						Number of Plays														

Figure 4. Fictitious Play

twenty-five iterations, the game value, U_o, is

$$.80 \leqq U_o \leqq 1.10$$

and, after fifty iterations, the game value, U_o, is

$$.809 \leqq U_o \leqq 1.0.$$

An estimate of the optimal strategy mix can also be derived from the fictitious play solution by determining the relative frequency of which an alternative is chosen. Figure 4 illustrates that after ten iterations play category (1) was chosen 1 time out of 10, play category (3) was chosen 3 times out of 10, play category (5) was chosen 4 times out of 10, play category (6) was chosen 2 times out of 10, and play categories (2) and (4) were not utilized. Hence, the estimated relative frequency indicates that in order for the offensive team to achieve an optimal expected payoff, play category (1) should be employed at random 10 per cent of the time, play category (3) employed at random 30 per cent of the time, etc. Again, by increasing the number of play iterations, a better estimate of the optimal strategy mix may be obtained. After fifty play iterations, the optimal strategy mix indicates that play category (1) should be employed at random 16 per cent of the time; category (3), 24 per cent; category (5), 32 per cent; category (6), 28 per cent; and categories (2) and (4), 0 per cent. Any deviation from the optimal strategy mix will result in a diminished game value or expected payoff.

Defensive Solution

After checking the payoff matrix illustrating A's defense in relation to B's offense in Figure 3 (page 15) for a saddle point and dominance, the matrix may be transformed into a linear programming problem and solved for a game value and optimal strategy mix using the simplex method of linear programming.

It will be observed in Figure 3 that some of the payoff values within the matrix have negative signs. To convert the defensive analysis matrix into linear programming, a constant, 6, will be added to all payoff values in Figure 3 resulting in a payoff matrix shown in Figure 5 (4).

If the value of the game of Figure 5 is g_o, and the value of the game of Figure 3 is U_o, then the following relationship exists:

$$g_o = U_o + 6 \text{ where } g_o > 0. \qquad (1)$$

It should be noted that the optimal strategies of the players are not affected when a constant is added to all elements of the payoff matrix.

Let x_1, x_2, x_3, x_4, x_5, x_6 be the relative frequencies associated with the optimal strategy mix and the utilization of defensive alignments (1), (2), (3), (4), (5), (6), respectively. Also, let "g" be some value. The optimal relative frequency and the game value may be found if the

A Defense

B Offense

Strategy	(1)	(2)	(3)	(4)	(5)	(6)
(1)	6	9	8	7	4	7
(2)	7	4	8	7	6	8
(3)	9	7	5	9	6	7
(4)	8	11	6	4	13	1
(5)	3	5	10	6	10	9
(6)	2	6	12	11	10	14

Figure 5. Revised Defensive Analysis Payoff Matrix

following functional constraints are satisfied:

$$6x_1 + 9x_2 + 8x_3 + 7x_4 + 4x_5 + 7x_6 \geqq g \qquad (2)$$

$$7x_1 + 4x_2 + 8x_3 + 7x_4 + 6x_5 + 8x_6 \geqq g \qquad (3)$$

$$9x_1 + 7x_2 + 5x_3 + 9x_4 + 6x_5 + 7x_6 \geqq g \qquad (4)$$

$$8x_1 + 11x_2 + 6x_3 + 4x_4 + 13x_5 + x_6 \geqq g \qquad (5)$$

$$3x_1 + 5x_2 + 10x_3 + 6x_4 + 10x_5 + 9x_6 \geqq g \qquad (6)$$

$$2x_1 + 6x_2 + 12x_3 + 11x_4 + 10x_5 + 14x_6 \geqq g. \qquad (7)$$

Since x_1, x_2, x_3, x_4, x_5, and x_6 are relative frequencies, one also finds that

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1. \qquad (8)$$

Since g_o is positive, each of the inequations can be divided by "g" without affecting the corresponding inequations for the solution sought. This results in:

$$6\,{}^{x_1}/g + 9\,{}^{x_2}/g + 8\,{}^{x_3}/g + 7\,{}^{x_4}/g + 4\,{}^{x_5}/g + 7\,{}^{x_6}/g \geqq 1 \qquad (9)$$

$$7\,{}^{x_1}/g + 4\,{}^{x_2}/g + 8\,{}^{x_3}/g + 7\,{}^{x_4}/g + 6\,{}^{x_5}/g + 8\,{}^{x_6}/g \geqq 1 \qquad (10)$$

$$9\,{}^{x_1}/g + 7\,{}^{x_2}/g + 5\,{}^{x_3}/g + 9\,{}^{x_4}/g + 6\,{}^{x_5}/g + 7\,{}^{x_6}/g \geqq 1 \qquad (11)$$

$$8\,{}^{x_1}/g + 11\,{}^{x_2}/g + 6\,{}^{x_3}/g + 4\,{}^{x_4}/g + 13\,{}^{x_5}/g + {}^{x_6}/g \geqq 1 \qquad (12)$$

$$3^{x_1/g} + 5^{x_2/g} + 10^{x_3/g} + 6^{x_4/g} + 10^{x_5/g} + 14^{x_6/g} \geqq 1 \quad (13)$$

$$2^{x_1/g} + 6^{x_2/g} + 12^{x_3/g} + 11^{x_4/g} + 10^{x_5/g} + 14^{x_6/g} \geqq 1 \quad (14)$$

$$x_1/g + x_2/g + x_3/g + x_4/g + x_5/g + x_6/g = 1/g \, . \quad (15)$$

U_1, U_2, U_3, U_4, U_5, U_6, and Z are now defined such that:

$$U_1 = x_1/g \quad (16)$$

$$U_2 = x_2/g \quad (17)$$

$$U_3 = x_3/g \quad (18)$$

$$U_4 = x_4/g \quad (19)$$

$$U_5 = x_5/g \quad (20)$$

$$U_6 = x_6/g \quad (21)$$

$$Z = 1/g \, . \quad (22)$$

Substitution yields:

$$6U_1 + 9U_2 + 8U_3 + 7U_4 + 4U_5 + 7U_6 \geqq 1 \quad (23)$$

$$7U_1 + 4U_2 + 8U_3 + 7U_4 + 6U_5 + 8U_6 \geqq 1 \quad (24)$$

$$9U_1 + 7U_2 + 5U_3 + 9U_4 + 6U_5 + 7U_6 \geqq 1 \qquad (25)$$

$$8U_1 + 11U_2 + 6U_3 + 4U_4 + 13U_5 + U_6 \geqq 1 \qquad (26)$$

$$3U_1 + 5U_2 + 10U_3 + 6U_4 + 10U_5 + 9U_6 \geqq 1 \qquad (27)$$

$$2U_1 + 6U_2 + 12U_3 + 11U_4 + 10U_5 + 14U_6 \geqq 1 \qquad (28)$$

$$U_1 + U_2 + U_3 + U_4 + U_5 + U_6 = Z.$$

A's defensive team is interested in as low a game value, g_o, as possible since it attempts to stymie B's offensive effort to gain yards. Thus, B's defense must minimize g or maximize Z.

In Equations (23) to (29), a linear programming problem is represented. One has to find U_1, U_2, U_3, U_4, U_5, and U_6 which satisfy Equations (23) to (28) that maximizes Z in Equation (29). Using linear programming techniques (a simplex solution) and the facilities of the 1620 computer, the following solution is obtained:

$U_1 = .0617$ $\quad$ $U_2 = .0145$ $\quad$ $U_3 = .0530$

$U_4 = .0000$ $\quad$ $U_5 = .0015$ $\quad$ $U_6 = .0099$

$Z_u = .1406.$

Therefore, by Equations (16) through (22),

$X_1 = .4388$ $\quad$ $X_2 = .1031$ $\quad$ $X_3 = .3770$

$X_4 = .0000$ $\quad$ $X_5 = .0107$ $\quad$ $X_6 = .0704$

$g_o = 7.1124.$

From Equation (1)

$$U_o = 1.1124.$$

From this solution, it is seen that the optimal strategy mix for A's defense to utilize against B's defense is to randomly employ defensive alignment (1) 44 per cent of the time, (2) 10 per cent of the time, (3) 38 per cent of the time, (4) 0 per cent of the time, (5) 1 per cent of the time, and alignment (6) 7 per cent of the time. If A's defensive team pursues this optimal strategy mix, they may expect to give up an average value of 1.1124 yards each time B's offense attempts to move the ball.

In discussing the solution to a gaming matrix, two methods of determining the game value and optimal strategy have been discussed. Both methods have merit and are equally capable of solving a gaming matrix. However, there are certain advantages which each method may possess over the other in particular situations.

The simplex method of linear programming offers a very reliable and exact solution to the payoff matrix, but is tedious and rather complicated in comparison to a solution by fictitious play. The desirability of attempting a solution by linear programming may depend upon the knowledge and availability of a computer which can easily solve a matrix of considerable size.

A fictitious play solution, on the other hand, can be performed by one with only an elementary knowledge of fundamental arithmetic, but does not offer the accuracy which a linear programming solution affords. Also, while easy to solve by fictitious play, a large matrix may be quite tedious if carried out more than a few iterations. The same is true, however, of hand solutions to large matrices by the simplex method.

Hence, the desire for exactness, ease of solution, and availability of a computer are all factors which must be considered in the choice of solution to a gaming matrix. A considerable amount of time and effort may be saved if an initial proper choice is made.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Game theory to date is still primarily a field of pure theory. Applications have been few in number and limited in scope because of the unmanageable complexities that arise once the number of participants exceed two and the rules allow more than trivial freedom of action (3).

Game theory used as a tool to quantify the game of football is grossly inadequate in a great number of cases. Poor player morale, a muddy field, and a strong wind are obviously elements which will alter and affect the outcome of a game, but which cannot be readily accounted for in any mathematical model. These are things which limit the scope and value of any study of this nature. However, if viewed in proper perspective, game theory can be of significant value if it is considered to be a guide rather than dogma to a decision maker's actions. For example, if a solution to a payoff matrix, using previous game statistics as payoff values, instructs one to employ a certain defense only one per cent of the time, this may be an indication that the defense has been poorly coached or perhaps incorrectly carried out. A coach may note that game theory analysis reveals that a certain play should be run only five per cent of the time. This may be an indication of poor timing or incorrect execution and faking if this play is used successfully by other teams. Examples, such as these, are indicative of the "common sense" approach which should be taken toward game theory and its use.

Randomization of maneuvers, as dictated by the optimal strategy mix, is another facet of game theory which should be clearly understood and viewed in proper perspective. Obviously, the offensive and defensive signal callers cannot be expected to roll dice or carry a table of random numbers in order to achieve pure randomization of play selection. A general awareness of the game situation, the element of surprise, and inappropriateness of a maneuver are all factors which may outweigh the dictates of a random selection. Again, a common sense approach is essential if game theory is to be of value as a guide to decision making.

To one who understands the worth of game theory and yet realizes its inherent limitations, it is possible to gain insight into a multitude of situations which might not otherwise be readily apparent. It, obviously, is not the answer to all problems involving competing entities and should not be treated as such, but this fact alone does not mean that total abandonment is justified. If used rationally, it can be a most promising and powerful analytical tool for participants in a competitive field, athletic or industrial.

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