ANALYSIS OF A PIN JOINTED SPHERICAL DOME WITH A HEXAGONAL BASE

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PREFACE

There always has been and probably always will be a need for structures capable of supporting a roof over a large column free area. There have been developed several general systems by which this can be accomplished - trusses, arches, shells, domes, and framed domes. The object of this thesis is to investigate a specialized type of framed dome - a Pin Jointed Spherical Dome with a Hexagonal Base.

Special appreciation goes to Professor Louis O. Bass of the School of Architecture at Oklahoma State University for his basic conception of the project and his technical assistance and guidance while acting as thesis advisor.

Appreciation also should be given to Dr. Thomas Scott Dean for his editorial criticisms.

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LIST OF SYMBOLS

Algebraic Equations	Computer Program	
	D	Span of Dome.
	Y	Rise of Dome.
Rm	RM	Radius of Sphere.
R_1	R(1)	First Minor Arc radius.
L	PL	Plan Length of members.
z	Z	Height of a joint from Base Plane.
h	H	Difference in height of the ends of a member.
α	ALFA	Angle between a member and the Base Plane.
TL	TL	True length of a member.
ϕ	PHI	Internal angle of a member.
γ	GAM	Cut angle at end of a member.
S	STR	Half perimeter of a triangle.
A	ATR	Area of a triangle.
Qd	QD	Dead load at a joint.
QL	QL	Live load at a joint.
TW_D	TWD	Total transverse dead load on a member.
TW_L	TWL	Total transverse live load on a member.
BM_D	BMD	Bending moment due to dead load.
BM_L	BML	Bending moment due to live load.
V_D	VD	Shear due to dead load.

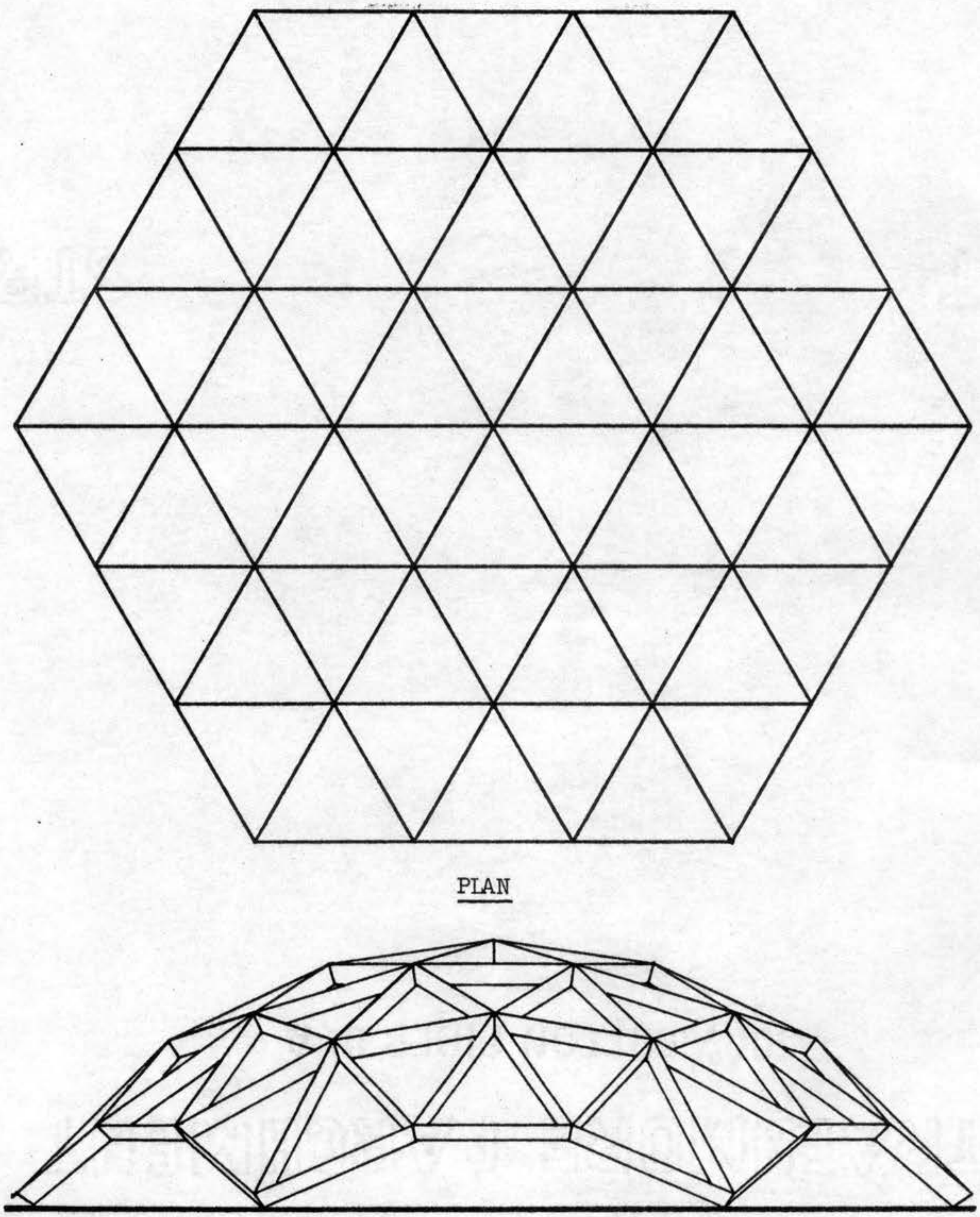
Algebraic Equations	Computer Program	
V_L	VL	Shear due to live load.
	BMTOT	Summation of bending moments.
	VTOT	Summation of shears.
	B	Summation live and dead loads at a joint.
β		Angle between a member and the "X" axis.
	SNB	Sin β
	CSB	Cos β
	A(I,J)	Matrix value.
F	F	Axial force in a member.
H_R		Horizontal reaction.
V_R		Vertical reaction.

CHAPTER I

INTRODUCTION

The purpose of this thesis is to establish a method of analysis for a pin-jointed, spherical dome with a hexagonal base. The dome that is analyzed herein is formed by intersecting three sets of seven pin-jointed arches to form a pattern of equilateral triangles in a plan projection. An illustration of this dome is shown in Figure 1. Of course, the designer is not limited to this number of arches. He may use any odd number. For a large dome it would be advisable to use more divisions to keep the size of individual members and surface facets from being too large. Also the more divisions that are used, the closer the overall shape approaches to that of a spherical section. It also follows that the more divisions are used, the more difficult becomes the mathematical analysis.

To facilitate the execution of the analysis of some examples, the solution was programed for the IBM 1620 electronic computer. The solution is very well adapted to the computer because of the various repetitious types of operations. Possibly the greatest advantage of using the computer is in solving the 12 simultaneous equations with 12 unknowns to obtain values for the axial forces in the members and the reactions. This advantage becomes even more apparent if more divisions are used. In the example domes with six members across the major



PLAN

ELEVATION

Figure 1.

THE STRUCTURE

arches, there are 12 unknowns; whereas, doubling this number would result in 35 unknowns.

The analysis of framed domes with either pinned joints or fixed joints is not new. What makes this particular dome unique is the way in which the forces and reactions are handled as a result of its geometry. In the geodesic dome, which appears very similar, all the hexagons are rigid structural units which are fastened together to form the structure. In the lamella dome, which is pin jointed, there is a system of one or more rings into which the forces terminate. In the fixed jointed framed dome, the membrane analogy governs. There are also secondary bending moments and rings in it. In the hexagonally based, pin jointed dome there are no rings, no rigid units and no induced secondary moments. The forces are handled by the arch action of a series of unstable arches, that would collapse as individuals, but acquire stability by their intersections with each other. Another difference in this dome and other domes is that the hexagonally based dome comes down to the support at only six points. This creates a scalloped appearance at the edges.

1. Analytic Approach

The analysis of the structure shall be approached by first calculating the geometry of the individual members including: their true lengths, their central angles, and their angles of inclination with the horizontal. With this information it will be possible to calculate the load areas contributing to each member which are to be utilized in determining the bending moments and shears as well as the total forces delivered to each joint. Since this is a pin-jointed structure, it is assumed that there is no joint restraint, allowing the bending moments to be computed as simple moments.

The second part of the analysis is a series of equations for each joint summing the forces in the "x", "y", and "z" directions. The equations are assembled into a matrix, the solution of which yields the axial force in each member and the horizontal and vertical reactions at the base.

In this structure there are 90 members, 37 joints, and 12 reactions; but due to the cyclo-symmetry of the geometry, there are only ten different members, six different joints, and two different reactions. This greatly simplifies the analysis. Refer to Figures 2, 3, and 4 for the designations and locations of these members and joints.

2. Assumptions and Conditions

The analysis and design of this structure is based on the following assumptions and/or conditions:

1. All the joints lie in the surface of a sphere.
2. All the joints are universal and frictionless.
3. Each member is of a homogeneous material and a constant cross section.
4. The radius of the sphere is large in comparison to the depths of the individual members.
5. The material of the members conforms to Hooke's Law, stating that stress is proportional to strain, and that all deformation and stress is within the elastic limit.
6. Effects of temperature change, displacement of supports, translation of joints, deformation of members due to lateral loads, and change in length of members due to axial

loads are all assumed to be negligible.

7. All members are straight chords from joint to joint.

CHAPTER II

THE ANALYSIS

1. Geometry of the Structure:

The structure is a spherical, hexagonally framed dome. Being spherical, all the joints lie in the surface of the sphere at a distance RM from the center of the sphere. All the members are straight lines. The dome is formed by three sets of seven hinged arches intersecting each other to form all equilateral triangles in a plan projection. The horizontal plane through the center of the sphere will be referred to as the Base Plane. The lowest points on the structure may or may not extend down to the Base Plane.

The radius of the sphere, which is also the radius of the three (3) main arcs will be called RM. The centers of the other arcs will lie on the Base Plane at distances of $\frac{\sqrt{3}}{2}$ L, $\sqrt{3}$ L, and $\frac{3}{2}\sqrt{3}$ L from the center of the sphere, where L is the plan length of the members. The radii of the arcs will be called R_1, R_2, R_3 numbering from the center of the dome to the outer edges.

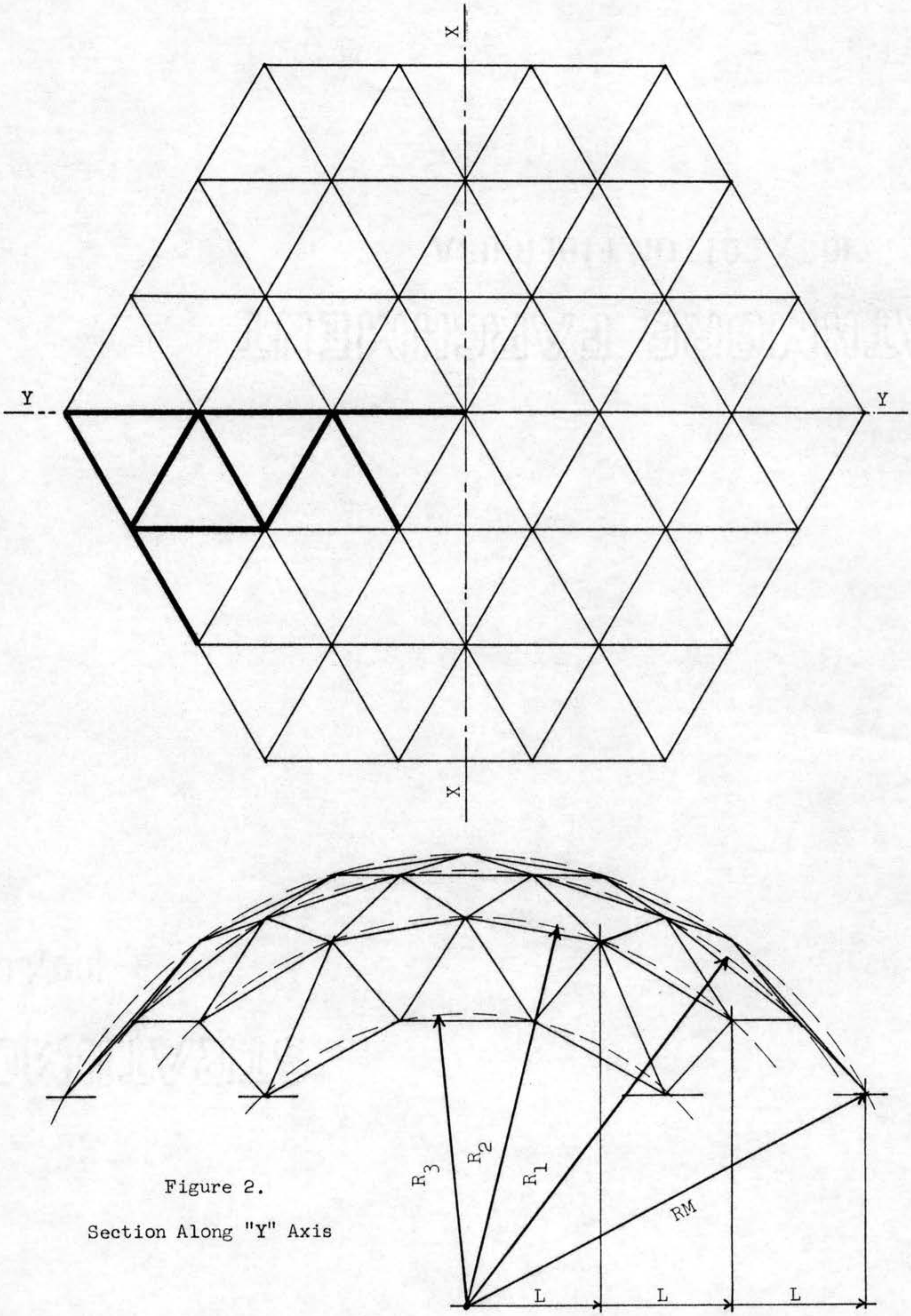


Figure 2.

Section Along "Y" Axis

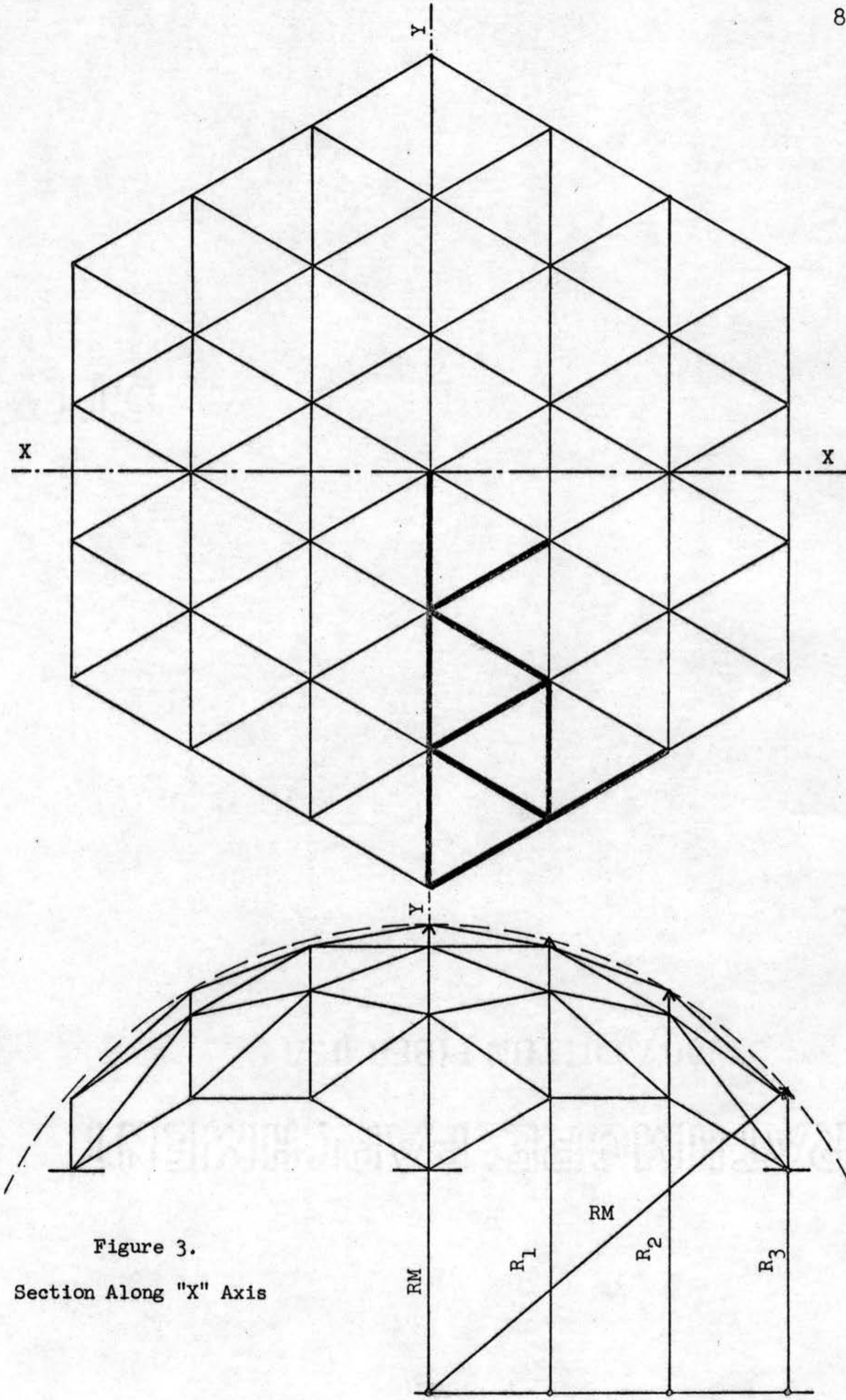


Figure 3.

Section Along "X" Axis

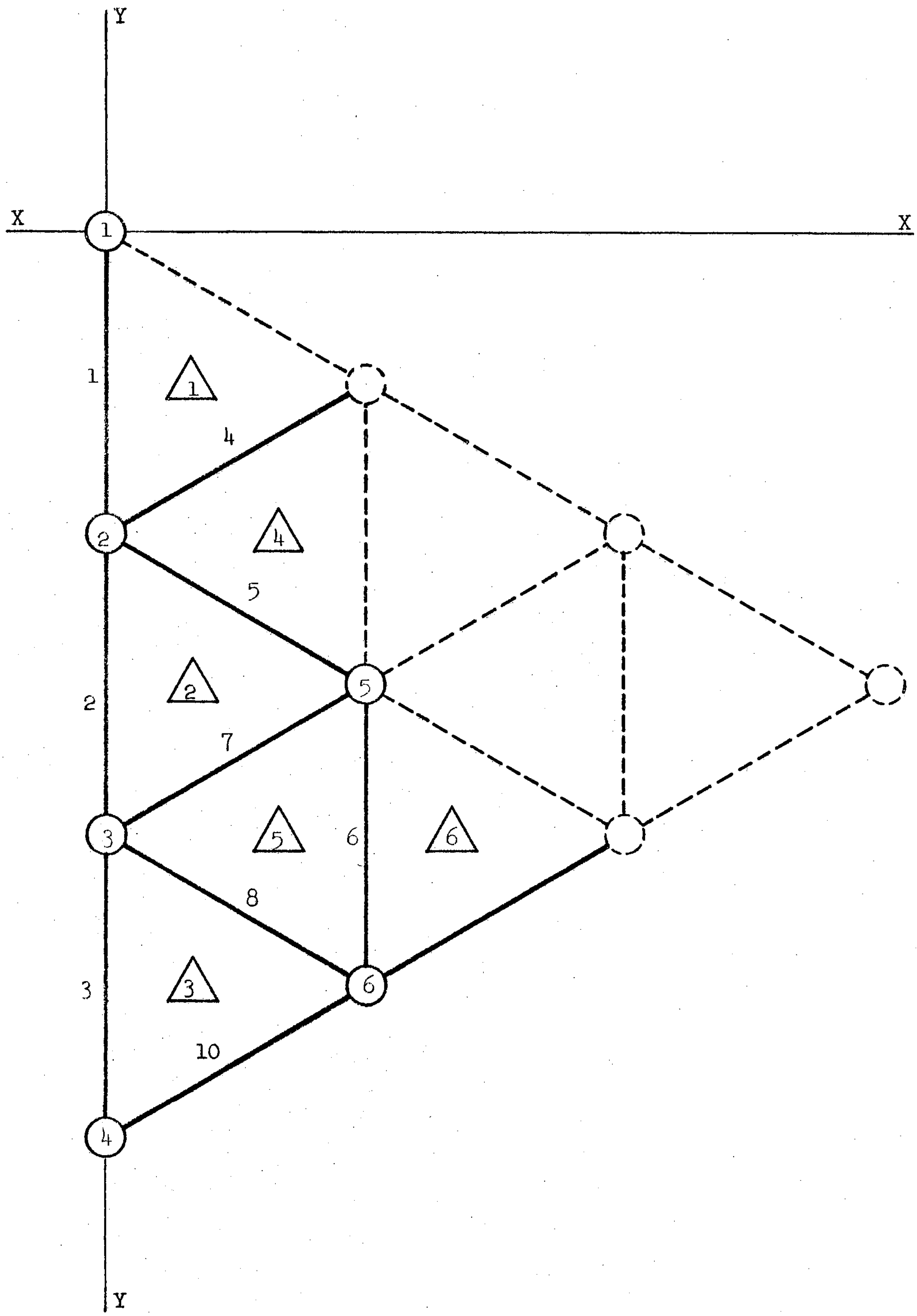


Figure 4.

Designation Numbers for Members and Joints

Calculation of Radii of Arches:

$$R_m = RM$$

$$R_1 = \sqrt{R_m^{\,2} - \left(\frac{\sqrt{3}}{2}\,L\right)^2}$$

$$R_2 = \sqrt{R_m^{\,2} - \left(2\cdot\frac{\sqrt{3}}{2}\,L\right)^2}$$

$$R_3 = \sqrt{R_m^{\,2} - \left(3\cdot\frac{\sqrt{3}}{2}\,L\right)^2}$$

$$R_i = \sqrt{R_m^{\,2} - \left(i\cdot\frac{\sqrt{3}}{2}\,L\right)^2}$$

Calculation of Heights of Joints:

$$z_i = \sqrt{R_m^{\,2} - (0\cdot L)^2}$$

$$z_2 = \sqrt{R_m^{\,2} - (1\cdot L)^2}$$

$$z_3 = \sqrt{R_m^{\,2} - (2\cdot L)^2}$$

$$z_4 = \sqrt{R_m^{\,2} - (3\cdot L)^2}$$

$$z_i = \sqrt{R_m^{\,2} - [(i - 1)L]^2}$$

$$z_5 = \sqrt{R_1^{\,2} - \left(\frac{3}{2}\cdot L\right)^2}$$

$$z_6 = \sqrt{R_1^{\,2} - \left(\frac{5}{2}\cdot L\right)^2}$$

Calculation of Height Differential of Member Ends:

$h_1 = z_1 - z_2$

$h_2 = z_2 - z_3$

$h_3 = z_3 - z_4$

$h_4 = z_2 - z_2$

$h_5 = z_2 - z_5$

$h_6 = z_5 - z_6$

$h_7 = z_5 - z_3$

$h_8 = z_3 - z_6$

$h_9 = z_6 - z_6$

$h_{10} = z_6 - z_4$

General Equations for Geometry of Members:

$$\alpha_i = \text{arc tan } \frac{h_i}{L}$$

$$TL_i = \sqrt{h_i^2 - L^2}$$

$$\phi_i = 2 \text{ arc tan } \frac{TL_i}{2\sqrt{R_m^2 - \left(\frac{TL_i}{2}\right)^2}}$$

$$\gamma_i = 90^\circ - \frac{\phi_i}{2}$$

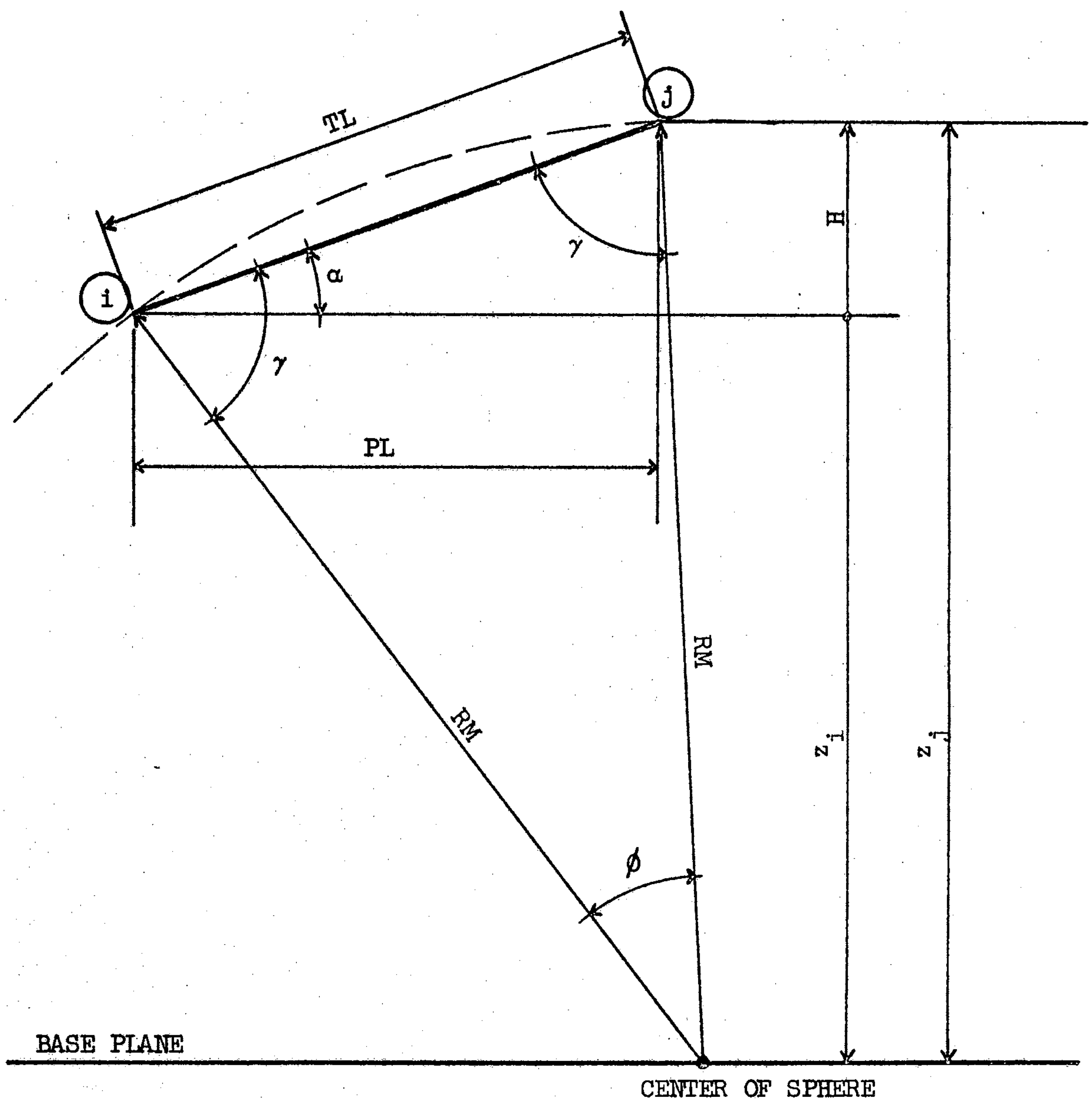


Figure 5.

Geometry of Typical Member

2. Loads

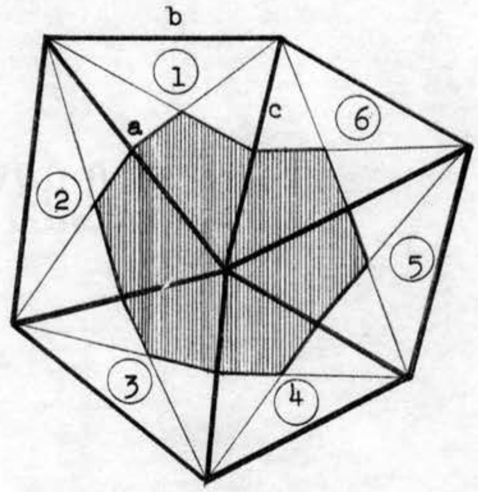


Figure 6.

Typical Joint Load Area

Dead Load - Wd

Shown a typical joint with six (6) triangles meeting at that point. We shall assume that one-third of the load on each triangle will be delivered to the joint. The load is given in pounds per square foot.

Area of each triangle is:

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{z}(a+b+c)$, and a,b,c are sides of the triangle.

Therefore: $Q_D = \frac{W_D}{3}(A_1+A_2+A_3+A_4+A_5)$

Live Load - W_L

For live load we consider the loads acting on a horizontal projection. In a horizontal projection all the sides of a triangle would be equal to L.

$$s = \frac{1}{2}(3L) = \frac{3L}{2}$$

$$A = \sqrt{\frac{3L}{2}\left(\frac{3L}{2} - L\right)^3} = \sqrt{\frac{3L}{2}\left(\frac{L}{2}\right)^3} = \frac{L^2}{4}\sqrt{3}$$

Load on 6 triangle joint:

$$Q_L = 6 \times \frac{1}{3} \times \frac{L^2}{4}\sqrt{3} \times W_L \frac{L^2}{2}\sqrt{3}$$

For a typical 3 triangle joint:

$$Q_L = 3 \times \frac{1}{3} \times \frac{L^2}{4}\sqrt{3}\, W_L = W_L \frac{L^2}{4}\sqrt{3}$$

For a typical 2 triangle joint:

$$Q_L = 2 \times \frac{1}{3} \times \frac{L^2}{4}\sqrt{3}\, W_L = W_L \frac{L^2}{6}\sqrt{3}$$

Areas of Triangles

There are only six (6) different triangles in the structure. These may be designated by the names of the sides, such as (1, 2, 3), (4, 7, 10), etc. To simplify the designation, they will be called simply

TR_1 = TR (1, 1, 4)

TR_2 = TR (2, 5, 7)

TR_3 = TR (3, 8, 10)

TR_4 = TR (4, 5, 5)

$TR_5 \;\; = TR\ (6,\ 7,\ 8)$

$TR_6 \;\; = TR\ (6,\ 6,\ 9)$

$STR_1 \;\; = \frac{1}{2}\ (TL_1 + TL_1 + TL_4)$

$ATR_1 \;\; = \sqrt{STR_1(STR_1 - TL_1)(STR_1 - TL_1)(STR_1 - TL_4)}$

$STR_2 \;\; = \frac{1}{2}\ (TL_2 + TL_5 + TL_7)$

$ATR_2 \;\; = \sqrt{STR_2(STR_2 - TL_2)(STR_2 - TL_5)(STR_2 - TL_7)}$

$STR_3 \;\; = \frac{1}{2}\ (TL_3 + TL_8 + TL_{10})$

$ATR_3 \;\; = \sqrt{STR_3(STR_3 - TL_3)(STR_3 - TL_8)(STR_3 - TL_{10})}$

$STR_4 \;\; = \frac{1}{2}\ (TL_4 + TL_5 + TL_5)$

$ATR_4 \;\; = \sqrt{STR_4(STR_4 - TL_4)(STR_4 - TL_5)(STR_4 - TL_5)}$

$STR_5 \;\; = \frac{1}{2}\ (TL_6 + TL_7 + TL_8)$

$ATR_5 \;\; = \sqrt{STR_5(STR_5 - TL_6)(STR_5 - TL_7)(STR_5 - TL_8)}$

$STR_6 \;\; = \frac{1}{2}\ (TL_6 + TL_6 + TL_9)$

$ATR_6 \;\; = \sqrt{STR_6(STR_6 - TL_6)(STR_6 - TL_6)(STR_6 - TL_9)}$

Joint Loads - Dead

$Q_{D1} \;\; = 6(\frac{ATR}{3}1)W_D$

$Q_{D1} \;\; = 2\ ATR_1\ W_D$

$$Q_{D2} = \frac{2}{3}\,(ATR_1 + ATR_2 + ATR_4)\ W_D$$

$$Q_{D3} = \frac{2}{3}\,(ATR_2 + ATR_3 + ATR_5)\ W_D$$

$$Q_{D4} = \frac{2}{3}\,(ATR_3)\ W_D$$

$$Q_{D5} = \frac{1}{3}\,(2ATR_2 + 2ATR_5 + ATR_4 + ATR_6)\ W_D$$

$$Q_{D6} = \frac{1}{3}\,(ATR_3 + ATR_5 + ATR_6)\ W_D$$

<u>Joint Loads - Live</u>

$$Q_{L1} = \frac{L^2}{2}\sqrt{3}\ W_L$$

$$Q_{L2} = \frac{L^2}{2}\sqrt{3}\ W_L$$

$$Q_{L3} = \frac{L^2}{2}\sqrt{3}\ W_L$$

$$Q_{L4} = \frac{L^2}{6}\sqrt{3}\ W_L$$

$$Q_{L5} = \frac{L^2}{2}\sqrt{3}\ W_L$$

$$Q_{L6} = \frac{L^2}{4}\sqrt{3}\ W_L$$

In the preceding discussion, we have considered only two types of loads -- those applied to the actual surface area of the structure (W_D), and those applied to a horizontal projection of the structure (W_L). Any kind of load we might apply to the structure will be one of these two types. The loading conditions that might be considered are:

1. Dead Load
2. Snow Load
3. Drift Load
4. Wind Load

By various codes these are considered in different combinations. Since the computer program is written to consider only W_D and W_L, it will be the task of the designer to calculate which combination will deliver the heaviest loads and apply them as W_D and/or W_L. If drift loads and wind loads are considered causing one sided or antisymmetrical load conditions, it may be necessary to run the program more than once using some negative load values. This will necessitate adding or subtracting the moments, shears, and axial forces prior to the design.

3. Bending Moments and Shears

Besides the axial forces in the members caused by the arch action, bending moments and shears due to the transverse direction of the loads must be computed. Consider a typical member bounding a triangle on each side.

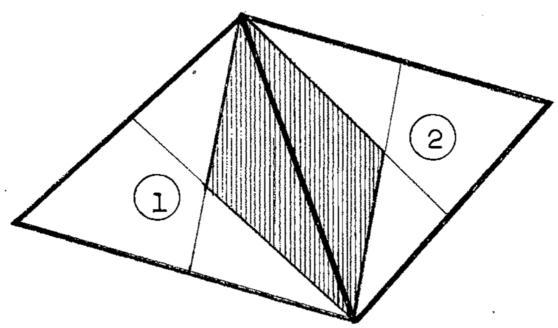


Figure 7.

Typical Load Area for
Bending Moments and Shears

From the sketch it can be concluded that the total load on the beam will be one-third of the load from each triangle. This load is increasing uniformly to approximately the center of the beam. Only if all the sides of both triangles were equal, would the load peak at the center; but for the sake of simplification it is assumed that this is the case in all instances.

If the total load to the beam is:

$$TW = \frac{1}{3}(ATR_1 + ATR_2)\,W$$

the maximum bending moment is:

$$M = \frac{TW \times L}{6} = \frac{(W \times L)}{18}(ATR_1 + ATR_2) \quad 1$$

and maximum shear is:

$$V = \frac{TW}{2} = \frac{W}{6}(ATR_1 + ATR_2) \quad 1$$

1. Manual of Steel Construction - A.I.S.C.

Dead Load Moments and Shears

$$M_{D1} = \frac{(W_D \times L)(ATR_1 + ATR_1)}{18} = \frac{(W_D \times L)ATR_1}{9}$$

$$V_{D1} = \frac{W_D \times ATR_1}{3}$$

$$M_{D2} = \frac{(W_D \times L)ATR_2}{9} \qquad V_{D2} = \frac{W_D \times ATR_2}{3}$$

$$M_{D3} = \frac{(W_D \times L)ATR_3}{9} \qquad V_{D3} = \frac{W_D \times ATR_3}{3}$$

$$M_{D4} = \frac{(W_D \times L)(ATR_1 + ATR_4)}{18} \qquad V_{D4} = \frac{W_D(ATR_1 + ATR_4)}{6}$$

$$M_{D5} = \frac{(W_D \times L)(ATR_2 + ATR_4)}{18} \qquad V_{D5} = \frac{W_D(ATR_2 + ATR_4)}{6}$$

$$M_{D6} = \frac{(W_D \times L)(ATR_5 + ATR_6)}{18} \qquad V_{D6} = \frac{W_D(ATR_5 + ATR_6)}{6}$$

$$M_{D7} = \frac{(W_D \times L)(ATR_2 + ATR_5)}{18} \qquad V_{D7} = \frac{W_D(ATR_2 + ATR_5)}{6}$$

$$M_{D8} = \frac{(W_D \times L)(ATR_3 + ATR_5)}{18} \qquad V_{D8} = \frac{W_D(ATR_3 + ATR_5)}{6}$$

$$M_{D9} = \frac{(W_D \times L)(ATR_6)}{18} \qquad V_{D9} = \frac{W_D(ATR_6)}{6}$$

$$M_{D10} = \frac{(W_D + L)(ATR_3)}{18} \qquad V_{D10} = \frac{W_D(ATR_6)}{6}$$

Live Load Moments and Shears

For live load all the areas are equal.

$$ATR = \frac{L^2}{4}\sqrt{3}$$

$$TW = W_L \times \frac{1}{3} \times 2\,\frac{L^2}{4}\sqrt{3}$$

$$M_L = \frac{L}{6} \times W_L \times \frac{1}{3} \times 2\,\frac{L^2}{4}\sqrt{3} = W_L \times \frac{L^3}{36}\sqrt{3}$$

$$V_L = W_L\,\frac{L^2}{12}\sqrt{3}$$

This same bending moment and shear will apply to all members but the ones in the exterior arch. The bending moments and shears for these members will be one half the values for the others due to the fact that they only bound one triangle each.

$$M_{L9} = M_{L10} = W_L \times \frac{L^3}{72}\sqrt{3}$$

$$V_{L9} = V_{L10} = W_L \times \frac{L^2}{24}\sqrt{3}$$

4. Joint Equations

Joint 1

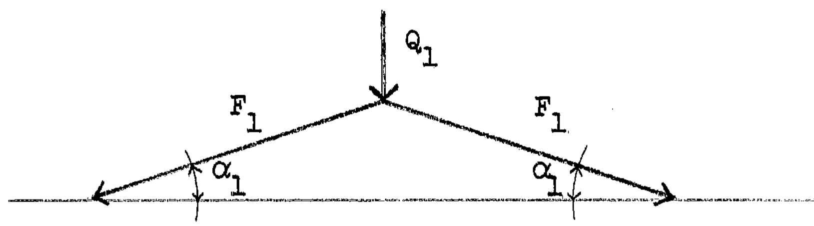


Figure 8.

Joint 1 Free Body

$\Sigma Fz = 0$

$$Q_1 + 6F_1 \sin\alpha_1 = 0$$

$$6F_1 \sin\alpha_1 = -Q_1 \qquad \text{(Eq. 1.0)}$$

Joint 2

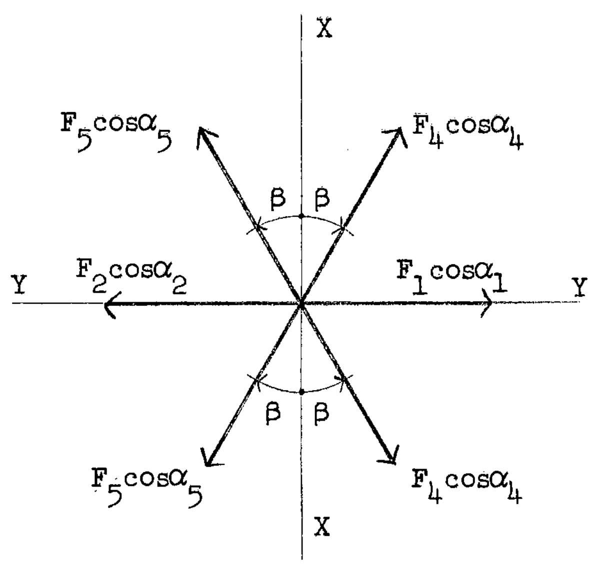


Figure 9.

Joint 2 Free Body

$\underline{\Sigma Fx = 0}$

$0 = 0$

$\underline{\Sigma Fy = 0}$

$$F_1 \cos\alpha_1 + 2F_4 \cos\alpha_4 \sin\beta - F_2 \cos\alpha_2 - 2F_5 \cos\alpha_5 \sin\beta = 0 \quad \text{(Eq. 1.1)}$$

$\underline{\Sigma Fz = 0}$

$$Q_2 - F_1 \sin\alpha_1 + F_2 \sin\alpha_2 - 2F_4 \sin\alpha_4 + 2F_5 \sin\alpha_5 = 0$$

$$-F_1 \sin\alpha_1 + F_2 \sin\alpha_2 - 2F_4 \sin\alpha_4 + 2F_5 \sin\alpha_5 = -Q_2 \qquad \text{(Eq. 1.2)}$$

<u>Joint 3</u>

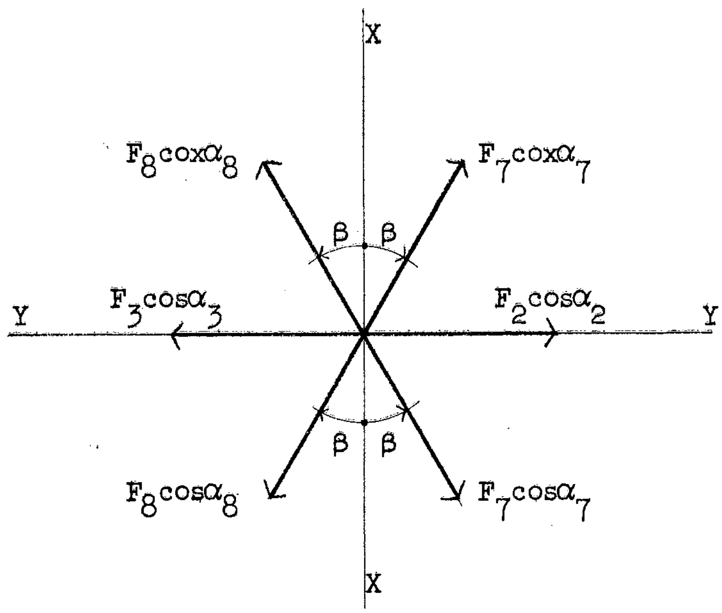


Figure 10.

Joint 3 Free Body

$\underline{\Sigma Fx = 0}$

$$0 = 0$$

$\underline{\Sigma Fy = 0}$

$$F_2 \cos\alpha_2 - F_3 \cos\alpha_3 + 2F_7 \cos\alpha_7 \sin\beta - 2F_8 \cos\alpha_8 \sin\beta = 0 \quad \text{(Eq. 1.3)}$$

$\underline{\Sigma Fz = 0}$

$$Q_3 - F_2 \sin\alpha_2 + F_3 \sin\alpha_3 - 2F_7 \sin\alpha_7 + 2F_8 \sin\alpha_8 = 0$$

$$-F_2 \sin\alpha_2 + F_3 \sin\alpha_3 - 2F_7 \sin\alpha_7 + 2F_8 \sin\alpha_8 = -Q_3 \qquad \text{(Eq. 1.4)}$$

<u>Joint 4</u>

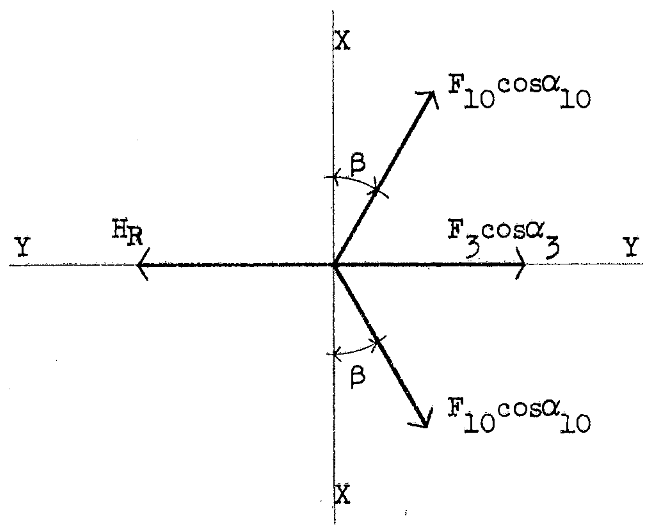


Figure 11.

Joint 4 Free Body

$\underline{\Sigma Fx = 0}$

$$0 = 0$$

$\underline{\Sigma Fy = 0}$

$$F_3 \cos\alpha_3 + 2F_{10} \cos\alpha_{10} \sin\beta - H_R = 0 \qquad \text{(Eq. 1.5)}$$

$\underline{\Sigma Fz = 0}$

$$Q_4 - F3 \sin\alpha_3 - 2F_{10} \sin\alpha_{10} + V_R = 0$$

$$-F_3 \sin\alpha_3 - 2F_{10} \sin\alpha_{10} + V_R = -Q_4 \qquad \text{(Eq. 1.6)}$$

Joint 5

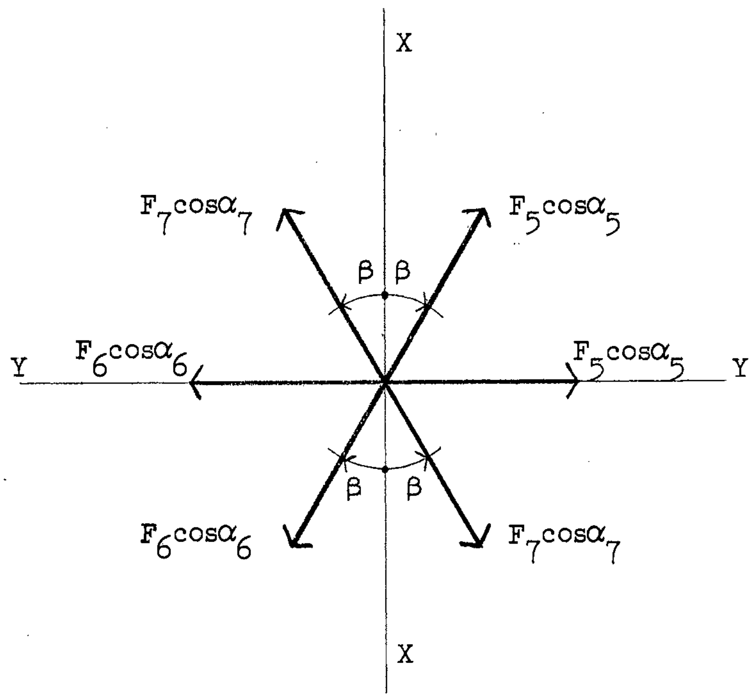


Figure 12.

Joint 5 Free Body

$\Sigma Fx = 0$

$$F_5 \cos\alpha_5 \cos\beta - F_6 \cos\alpha_6 \cos\beta + F_7 \cos\alpha_7 \cos\beta - F_7 \cos\alpha_7 \cos\beta = 0$$

$$F_5 \cos\alpha_5 \cos\beta - F_6 \cos\alpha_6 \cos\beta = 0 \qquad \text{(Eq. 1.7)}$$

$\Sigma Fy = 0$

$$F_5 \cos\alpha_5 + F_5 \cos\alpha_5 \sin\beta - F_6 \cos\alpha_6 - F_6 \cos\alpha_6 \sin\beta$$
$$+ F_7 \cos\alpha_7 \sin\beta - F_7 \cos\alpha_7 \sin\beta = 0$$

$$F_5 \cos\alpha_5 (1 + \sin\beta) - F_6 \cos\alpha_6 (1 + \sin\beta) = 0 \qquad \text{(Eq. 1.71)}$$

$\underline{\Sigma Fz = 0}$

$$Q_5 - 2F_5 \sin\alpha_5 + 2F_6 \sin\alpha_6 + 2F_7 \sin\alpha_7 = 0$$

$$-2F_5 \sin\alpha_5 + 2F_6 + 2F_7 \sin\alpha_7 = -Q_5 \qquad \text{(Eq. 1.8)}$$

Joint 6

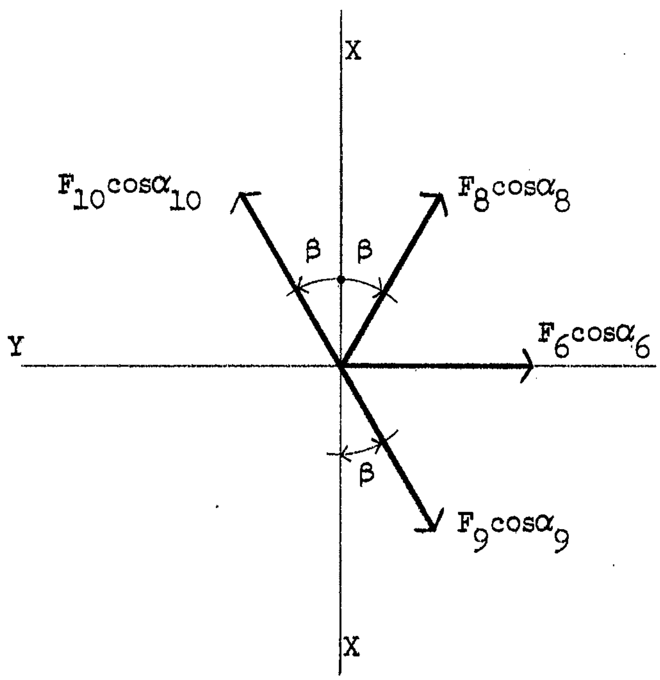


Figure 13.

Joint 6 Free Body

$\underline{\Sigma Fx = 0}$

$$F_8 \cos\alpha_8 \cos\beta - F_9 \cos\alpha_9 \cos\beta + F_{10} \cos\alpha_{10} \cos\beta = 0 \qquad \text{(Eq. 1.9)}$$

$\underline{\Sigma Fy = 0}$

$$F_6 \cos\alpha_6 + F_8 \cos\alpha_8 \sin\beta + F_9 \cos\alpha_9 \sin\beta - F_{10} \cos\alpha_{10} \sin\beta = 0 \qquad \text{(Eq. 2.0)}$$

$\underline{\Sigma Fz = 0}$

$$Q_6 - F_6 \sin\alpha_6 - F_8 \sin\alpha_8 + F_9 \sin\alpha_9 + F_{10} \sin\alpha_{10} = 0$$

$$-F \sin\alpha_6 - F_8 \sin\alpha_8 + F_9 \sin\alpha_9 + F_{10} \sin\alpha_{10} = -Q_6 \qquad \text{(Eq. 2.1)}$$

TABLE I

SUMMARY OF EQUATIONS

Eg.No	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	H_R	V_R	C
1.0	$6\sin\alpha_1$												$-Q_1$
1.1	$\cos\alpha_1$	$-\cos\alpha_2$		$2\cos\alpha_4 \sin\beta$	$-2\cos\alpha_5 \sin\beta$								0
1.2	$-\sin\alpha_1$	$\sin\alpha_2$		$-2\sin\alpha_4$	$2\sin\alpha_5$								$-Q_2$
1.3		$\cos\alpha_2$	$-\cos\alpha_3$				$2\cos\alpha_7 \sin\beta$	$-2\cos\alpha_8 \sin\beta$					0
1.4		$-\sin\alpha_2$	$\sin\alpha_3$				$-2\sin\alpha_7$	$2\sin\alpha_8$					$-Q_3$
1.5			$\cos\alpha_3$							$2\cos\alpha_{10} \sin\beta$	-1.0		0
1.6			$-\sin\alpha_3$							$-2\sin\alpha_{10}$		1.0	$-Q_4$
1.7					$\cos\alpha_5 \cos\beta$	$-\cos\alpha_6 \cos\beta$							0
1.8					$-2\sin\alpha_5$	$2\sin\alpha_6$	$2\sin\alpha_7$						$-Q_5$
1.9								$\cos\alpha_8 \cos\beta$	$-\cos\alpha_9 \cos\beta$	$\cos\alpha_{10} \cos\beta$			0
2.0						$\cos\alpha_6$		$\cos\alpha_8 \sin\beta$	$\cos\alpha_9 \sin\beta$	$-\cos\alpha_{10} \sin\beta$			0
2.1						$-\sin\alpha_6$		$-\sin\alpha_8$	$\sin\alpha_9$	$\sin\alpha_{10}$			$-Q_6$

All β's equal 30°.

TABLE II

EQUATION MATRIX

$$
\begin{array}{c}
1.0\\ 1.2\\ 1.4\\ 1.1\\ 1.7\\ 1.8\\ 1.3\\ 1.9\\ 2.0\\ 2.1\\ 1.5\\ 1.6
\end{array}
\begin{bmatrix}
6\sin\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-\sin\alpha_1 & \sin\alpha_2 & 0 & -2\sin\alpha_4 & 2\sin\alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\sin\alpha_2 & \sin\alpha_3 & 0 & 0 & 0 & -2\sin\alpha_7 & 2\sin\alpha_8 & 0 & 0 & 0 & 0\\
\cos\alpha_1 & -\cos\alpha_2 & 0 & 2\cos\alpha_4\sin\beta & -2\cos\alpha_5\sin\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cos\alpha_5\cos\beta & -\cos\alpha_6\cos\beta & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -2\sin\alpha_5 & 2\sin\alpha_6 & 2\sin\alpha_7 & 0 & 0 & 0 & 0 & 0\\
0 & \cos\alpha_2 & -\cos\alpha_3 & 0 & 0 & 0 & 2\cos\alpha_7\sin\beta & -2\cos\alpha_8\sin\beta & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\alpha_8\cos\beta & -\cos\alpha_9\cos\beta & \cos\alpha_{10}\cos\beta & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \cos\alpha_6 & 0 & \cos\alpha_8\sin\beta & \cos\alpha_9\sin\beta & -\cos\alpha_{10}\sin\beta & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\sin\alpha_6 & 0 & -\sin\alpha_8 & \sin\alpha_9 & \sin\alpha_{10} & 0 & 0\\
0 & 0 & \cos\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 2\cos\alpha_{10}\sin\beta & -1.0 & 0\\
0 & 0 & -\sin\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & -2\sin\alpha_{10} & 0 & 1.0
\end{bmatrix}
\begin{bmatrix}
F_1\\ F_2\\ F_3\\ F_4\\ F_5\\ F_6\\ F_7\\ F_8\\ F_9\\ F_{10}\\ H_R\\ V_R
\end{bmatrix}
\begin{bmatrix}
-Q_1\\ -Q_2\\ -Q_3\\ 0\\ 0\\ -Q_5\\ 0\\ 0\\ 0\\ -Q_6\\ 0\\ -Q_4
\end{bmatrix}
$$

Solving this matrix of 12 equations will yield the axial forces in the 10 members plus the horizontal and vertical reactions at the support joint. Plus values will be tension and minus values will be compression. The load constants, Q, in this matrix are of a general nature as shown. These will have to be evaluated by the equations derived on previous pages. These load constants will have different values for the various load conditions, live load, dead load, etc. These different values of the load constants may simply be inserted into the matrix to find a solution for axial forces due to the various load conditions, or the load constants may be combined to permit one solution of the matrix for the combined loading effects. In any case, for a particular dome, the body of the matrix will remain unchanged for any values assigned to the load constants.

CHAPTER III

DESIGN

In selecting the members for this dome, the designer will need the axial forces in each member due to the various loading conditions plus the bending moments and shears for each member due to the same loading conditions. It is then merely a matter of selecting the members according to the code for the particular material which has been chosen.

For detailing purposes the actual lengths of each member, TL, and the angles with which each member intersects the others, γ, were found in the solution of the geometry.

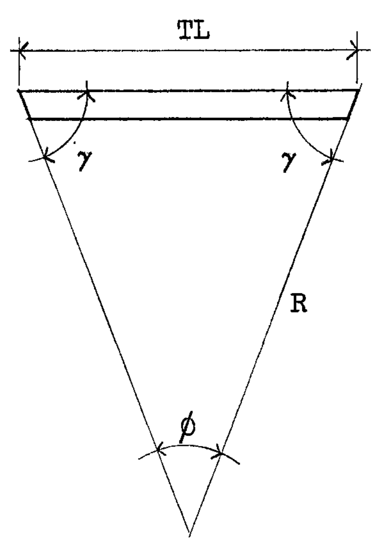


Figure 14.

Typical Member

CHAPTER IV

COMPUTER PROGRAM

The program for the solution of this dome was written in the IBM language Fortran Without Format. It is recommended that if other domes of this same sort with more members are programmed, one of the computer languages with format be used. This would give the programmer more possibilities within the program itself as well as having the answers come out in a more organized form with titles. Despite the disadvantages of the Fortran Without Format, it did permit the problem to be solved and the geometry computed much quicker than it possibly could be done by conventional methods.

There are twenty-three (23) data cards that must be entered with the program to solve the problem. The last twenty-two (22) are cards which give: 1) the combinations of sides with which to figure the areas of the triangles, 2) the combinations of triangles common to a joint for figuring joint loads, and 3) the combinations of triangles common to a member for figuring bending moments and shears. These cards will be common to all domes of this sort with the same number of arches. Therefore, the first data card is the only unique one. On it is to be punched the values for: 1) the diameter of the dome, 2) the rise of the dome, 3) the dead load, and 4) the live load. The values must be punched in that order with a space between each.

When the program is executed on the computer, the answers will be punched on cards. The first 10 cards will give the geometry of the individual members. They will give the member number, the actual length from joint center to joint center, the central angle, and the cut angles at the ends, in that order. The second 10 cards will have the member number, the total bending moment, and the total shear, and in that order.

Next will be punched 157 cards. These cards will be the coefficients and constants for solution of the matrix for the axial forces. These cards will then be entered as data into a general solution for a matrix. The answer cards from this solution will provide the member number along with its axial force. Number 11 will be the horizontal reaction, and Number 12 will be the vertical reaction.

The actual program is listed below with the pertinent items designated by the notes to the right:

```
      THESIS PINNED JOINTED HEXAGONAL FRAMED DOME
      DIMENSION R(2),Z(6),H(10),ALFA(10),TL(10),PHI(10),GAM(10),ATR(7)
      DIMENSION QD(6),BMD(10),VD(10),QL(6),BML(10),VL(10)
      DIMENSION B(12),A(12,12)
1     READ, D, Y, WD, WL
      PL = D/6.0
      RM = ((D*D)/(8.0*Y))+(Y/2.0)
      RMS = RM*RM
      PLS = PL*PL
      R(1) = SQR(RMS-((1.732051*PL*0.5)**2))
      Z(1) = RM
      Z(2) = SQR(RMS-PLS)
      Z(3) = SQR(RMS-(4.0*PLS))
      Z(4) = SQR(RMS-(9.0*PLS))
      Z(5) = SQR((R(1)*R(1))-((1.5*PL)**2))
      Z(6) = SQR((R(1)*R(1))-((2.5*PL)**2))
      H(1) = Z(1)-Z(2)
      H(2) = Z(2)-Z(3)
      H(3) = Z(3)-Z(4)
      H(4) = 0.0
      H(5) = Z(2)-Z(5)
      H(6) = Z(5)-Z(6)
      H(7) = Z(5)-Z(3)
      H(8) = Z(3)-Z(6)
      H(9) = 0.0
      H(10) = Z(6)-Z(4)
      DO 45 I = 1,10
      ALFA(I) = ATN(H(I)/PL)
      TL(I) = SQR((H(I)*H(I))+PLS)
      PHI(I) = 2.0*ATN(TL(I)/(2.0*SQR(RMS-(0.25*TL(I)*TL(I)))))
      PHI(I) = PHI(I)*57.29578
      GAM(I) = 90.0-(PHI(I)/2.0)
45    PUNCH, I, TL(I), PHI(I), GAM(I)<---------(Geometry Answers - 10 cards)
47    DO 50 N = 1,6
      READ, J, K, L
      STR = (TL(J)+TL(K)+TL(L))*0.5
50    ATR(N) = SQR(STR*(STR-TL(J))*(STR-TL(K))*(STR-TL(L)))
      ATR(7) = 0.0
      DO 55 N = 1,6
      READ, I, J, K, L, M, NN
55    QD(N) = WD*((ATR(I)+ATR(J)+ATR(K)+ATR(L)+ATR(M)+ATR(NN))/3.0)
      DO 60 N = 1,10
      READ, I, J
      TWD = WD*(ATR(I)+ATR(J))/3.0
      BMD(N) = (TWD*TL(N))/6.0
60    VD(N) = TWD*0.5
      QL(1) = WL*1.732051*0.5*PLS
      QL(2) = QL(1)
      QL(3) = QL(1)
      QL(4) = QL(1)/3.0
      QL(5) = QL(1)
      QL(6) = QL(1)/2.0
      BML(1) = (WL*1.732051*PLS*PL)/36.0
      DO 65 I = 2,8
```

```
65     BML(I) = BML(1)
       BML(9) =BML(1)/2.0
       BML(10) = BML(9)
       VL(1) = (WL*1.732051*PLS)/12.0
       DO 70 I= 2,8
70     VL(I) = VL(1)
       VL(9) = VL(1)/2.0
       VL(10) = VL(9)
       DO 80 N = 1,10
       BMTOT = BMD(N)+BML(N)
       VTOT = VD(N)+VL(N)
80     PUNCH, N, BMTOT, VTOT  <------  (Bending Moments and Shears
                                                10 Cards)
       DO 100 I = 1,12
       B(I) = 0.0
       DO 100 J = 1,12
100    A(I,J) = 0.0
       SNB = 0.50000
       CSB = 0.86603
       A(1,1) = 6.0*SIN(ALFA(1))
       A(2,1) = -SIN(ALFA(1))
       A(2,2) = SIN(ALFA(2))
       A(2,4) = -2.0*SIN(ALFA(4))
       A(2,5) = 2.0*SIN(ALFA(5))
       A(3,2) = -SIN(ALFA(2))
       A(3,3) = SIN(ALFA(3))
       A(3,7) = -2.0*SIN(ALFA(7))
       A(3,8) = 2.0*SIN(ALFA(8))
       A(4,1) = COS(ALFA(1))
       A(4,2) = -COS(ALFA(2))
       A(4,4) = 2.0*COS(ALFA(4))*SNB
       A(4,5) = -2.0*COS(ALFA(5))*SNB
       A(5,5) = COS(ALFA(5))*CSB
       A(5,6) = -COS(ALFA(6))*CSB
       A(6,5) = -2.0*SIN(ALFA(5))
       A(6,6) = 2.0*SIN(ALFA(6))
       A(6,7) = 2.0*SIN(ALFA(7))
       A(7,2) = COS(ALFA(2))
       A(7,3) = -COS(ALFA(3))
       A(7,7) = 2.0*COS(ALFA(7))*SNB
       A(7,8) = -2.0*COS(ALFA(8))*SNB
       A(8,8) = COS(ALFA(8))*CSB
       A(8,9) = -COS(ALFA(9))*CSB
       A(8,10) = COS(ALFA(10))*CSB
       A(9,6) = COS(ALFA(6))
       A(9,8) = COS(ALFA(8))*SNB
       A(9,9) = COS(ALFA(9))*SNB
       A(9,10) = -COS(ALFA(10))*SNB
       A(10,6) = -SIN(ALFA(6))
       A(10,8) = -SIN(ALFA(8))
       A(10,9) = SIN(ALFA(9))
       A(10,10) = SIN(ALFA(10))
       A(11,3) = COS(ALFA(3))
       A(11,10) = 2.0*COS(ALFA(10))*SNB
       A(11,11) = -1.0
```

```
      A(12,3) = -SIN(ALFA(3))
      A(12,10) = -2.0*SIN(ALFA(10))
      A(12,12)= 1.0
      B(1) = -(QD(1)+QL(1))
      B(2) = -(QD(2)+QL(2))
      B(3) = -(QD(3)+QL(3))
      B(6) = -(QD(5)+QL(5))
      B(10) = -(QD(6)+QL(6))
      B(12) = -(QD(4)+QL(4))
      K = 12
      PUNCH, K
      DO 110 I = 1,12
      DO 110 J = 1,12
110   PUNCH, A(I,J)
      DO 120 I = 1,12
120   PUNCH, B(I)
      GO TO 1
      END
```

(Coefficients and Constants for Matrix Solution 157 Cards)

Data Cards

```
90.00 15.00 15.00 30.00
1  1  4
2  5  7
3  8  10
4  5  5
6  7  8
6  6  9
1  1  1  1  1  1
1  1  2  2  4  4
2  2  3  3  5  5
3  3  7  7  7  7
2  2  4  6  5  5
3  5  6  7  7  7
1  1
2  2
3  3
1  4
2  4
5  6
2  5
3  5
6  7
3  7
```

(Typical Data)

(Side Combinations for Triangle Areas)

(Triangle Combinations for Joint Loads)

(Identical for All Data)

(Triangle Combinations for Moments and Shears)

CHAPTER V

SAMPLE PROBLEMS

Three (3) sample problems were run on the computer. All of the domes had a span of 90 feet, but they differed in heights at the center. The values for the rises were taken as 15 feet, 30 feet, and 45 feet. The dead loads were taken as 15, 16, and 17 pounds per square foot respectively. This was to take into account a built-up roof, a wood deck, and a steel structure. The live load used was a snow load of 30 pounds per square foot. The results of the three problems were as follows:

Problem No. 1.

Span = 90'; Rise = 15'; W_D = 15 psf; W_L = 30 psf.

Member No.	True Length (feet)	PHI (degrees)	GAM (degrees)	Bending Moment (lb.-ft.)	Shears (lbs.)	Axial Force (lbs.)
1	15.076344	11.536958	84.231521	7336.0858	1464.7209	-14573.020
2	15.732928	12.041218	83.979391	7555.0459	1486.0052	-17868.898
3	17.359829	13.291717	83.354142	8137.5913	1538.7193	-51326.954
4	14.999999	11.478340	84.260830	7350.1659	1470.0332	-14347.678
5	15.322768	11.726186	84.136907	7457.8596	1480.6753	-12064.578
6	16.435118	12.580762	83.709619	7846.4923	1517.3412	-12940.400
7	15.086969	11.545116	84.227442	7510.2286	1499.0034	-15592.743
8	15.842983	12.125764	83.937118	7781.6531	1525.3604	12474.174
9	14.999999	11.478340	84.260830	3806.7020	761.34043	-12287.140

Problem No. 1. continued.

Member No.	True Length (feet)	PHI (degrees)	GAM (degrees)	Bending Moment (lb.-ft.)	Shears (lbs.)	Axial Force (lbs.)
10	15.435241	11.812564	84.093718	3887.7429	769.35968	-24796.798
11						-68447.340
12						-40608.595

Problem No. 2.

Span = 90'; Rise = 30'; W_D = 16 psf; W_L = 30 psf.

Member No.	True Length	PHI	GAM	Bending Moment	Shear	Axial Force
1	15.185304	17.920210	81.039895	7544.8003	1502.4354	-9646.6577
2	16.980684	20.059658	79.970171	8211.1613	1564.3199	-10855.199
3	24.741613	29.400258	75.299871	11940.448	1831.4243	-34552.788
4	14.999999	17.699764	81.150118	7588.6301	1517.7260	-8259.9650
5	15.854578	18.716967	80.641517	7906.9618	1548.6683	-8667.0563
6	19.808962	23.444649	78.277676	9608.6804	1691.7247	-10828.757
7	15.263505	18.013260	80.993370	8157.0234	1620.0602	-3214.7755
8	18.091347	21.386666	79.306667	9571.1259	1753.6124	9889.8060
9	14.999999	17.699764	81.150118	4269.1221	853.82445	-2402.5323
10	17.788462	21.024501	79.487750	4976.9139	915.71218	-12573.390
11						-31550.605
12						-44658.570

Problem No. 3.

Span = 90 '; Rise = 30'; W_D = 17 psf; W_L = 30 psf.

Member No.	True Length	PHI	GAM	Bending Moment	Shear	Axial Force
1	15.219177	19.471218	80.264391	7726.6203	1537.1002	-9089.7841
2	17.434164	22.339092	78.830454	8613.0456	1618.1269	-8470.1680
3	36.742344	48.189684	65.905158	21594.783	2339.7378	-31734.248

Problem No. 3. continued.

Member No.	True Length	PHI	GAM	Bending Moment	Shear	Axial Force
4	14.999999	19.188133	80.405934	7785.0201	1557.0040	-7148.7620
5	16.040840	20.533539	79.733231	8203.8176	1597.5174	-9432.0999
6	21.590605	27.760759	76.119621	10911.174	183.5021	-12695.392
7	15.337810	19.624492	80.187754	8608.2889	1705.1970	3223.1740
8	19.415847	24.916908	77.541546	11936.974	2066.0024	11416.621
9	14.999999	19.188133	80.405934	4586.8431	917.36865	3269.9012
10	25.980760	33.557308	73.221346	8348.3074	1169.8688	-9613.1908
11						-18505.633
12						-49347.020

For the sake of comparison of the size of the members to be used in the three different domes, a wide flange section is chosen and checked against the beam-column inneraction formula:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

where:

f_a = computed axial stress

F_a = allowable axial stress

f_b = computed compressive bending stress

F_b = allowable compressive bending stress

The allowable stresses used are those given in the Manual of Steel Construction, AISC, Sixth Edition, for A36 steel. Assume the members are laterally supported along their entire lengths by the roof deck. The section chosen is an 8 WF17; A = 5.00 in^2; S = 14.1 in^3. Member No. 3 is chosen in each dome, because the values for the moments and axial forces are the largest.

	Problem 1	Problem 2	Problem 3
Length	17.36 ft.	24.74 ft.	36.74 ft.
Moment	97.7 kip-in.	143.3 kip-in.	259 kip-in.
Axial Force	51.33 kip.	34.55 kip.	31.73 kip.
$f_a = \frac{P}{A} =$	10.28 ksi	6.92 ksi	6.34 ksi
$F_a =$	21.56 ksi	21.56 ksi	21.56 ksi
$\frac{f_a}{F_a} =$	0.477	0.321	0.248
$f_b = \frac{M}{S} =$	6.92 ksi	10.17 ksi	18.36 ksi
$F_b =$	24.00ksi	24.00 ksi	24.00 ski
$\frac{f_b}{F_b} =$	0.289	0.424	0.765
$\frac{f_a}{F_a} + \frac{f_b}{F_b} =$	$\underline{0.766}$	$\underline{0.745}$	$\underline{1.013}$

CHAPTER VI

SUMMARY AND CONCLUSIONS

Because of the fact that the structure is statically determinant, the analysis has proven to be reasonably simple. It is only complicated by the multitude of cumbersome arithmetical calculations, especially the matrix solution. The use of the electronic computer to do these calculations makes this a feasible approach to spanning an area with a column free structure.

Nearly any kind of structural member could be used in the fabrication of this dome - steel rolled sections, aluminum sections, pipe, solid timber sections, or glue-laminated sections. Even precast concrete could be used, but the connections might prove to be an almost insurmountable problem. In the examples a steel section was chosen; and it can be noted that for a given span, there will be very little difference in the total stresses in members regardless of the rise. This is due to the fact that as the rise is increased, the bending moments increase and the axial forces decrease.

A small scaled, wire model was made of a dome of this type. From the model, it was learned that there is an unstable condition at the edge arches. Rotation could occur in several of the members but without effecting the stability of the structure as a whole. This is illustrated in the Figure 15. To stabilize these members some other members must be applied to the exterior joints to prevent their movement. In an actual application this could easily and readily be handled with window mullions or framing members for the exterior walls or the walls themselves. Cables could even be utilized for this purpose. In the analysis of the structure it was assumed that these stabilizing members were non load bearing. Therefore the dome itself will be designed to carry the loads within its own frame, but it will need the stabilizing members to hold the edge alignment.

l. <u>Recommendations for Further Study</u>. In the writing of this thesis several questions have arisen that might form the basis for some future research. The answers to some of these could only be obtained by building a large scale model and subjecting it to tests.

1. What would be the maximum feasible span and rise for a dome of this nature? At what span should the number of divisions be increased?
2. In a large scale, accurately built model, is the instability at the edge arches as prevalent as in the small model?
3. One-sided or antisymmetrical loadings, such as those for wind and drift, will have what effect upon the structure?
4. A large single concentrated load at any joint will have what effect upon the structure?
5. What will be the effect of secondary moments and torsion caused by joints not being universal and frictionless as they were assumed in the analysis?

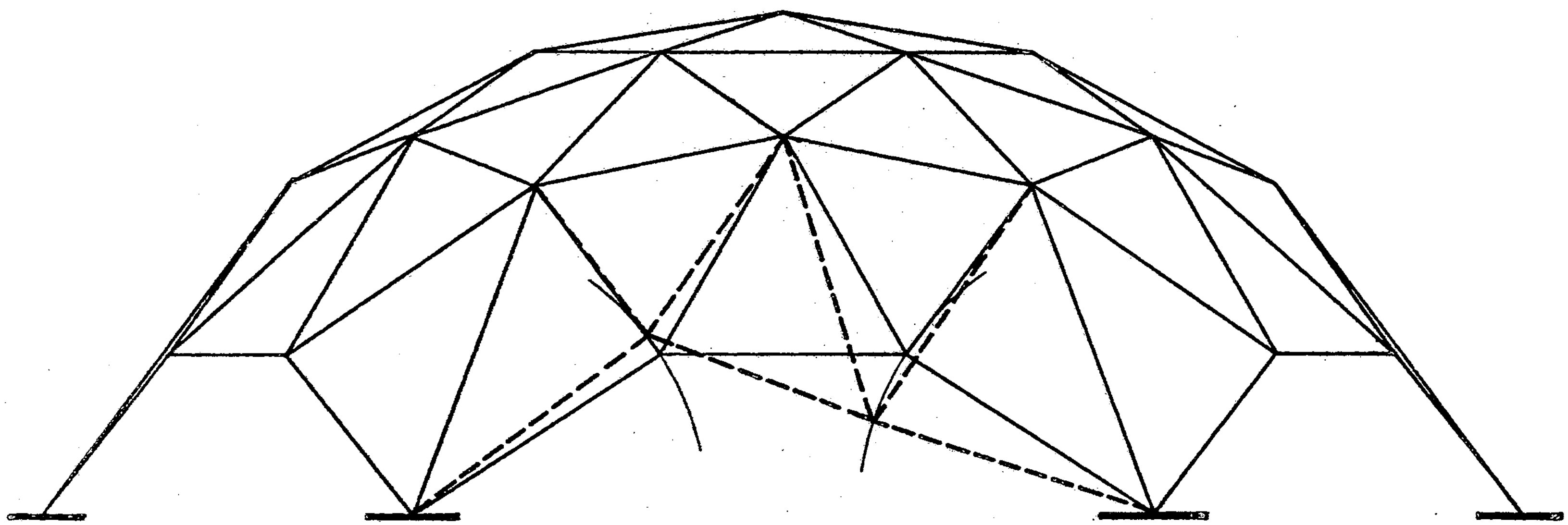


Figure 15.

Edge Movement

VITA

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Master of Architectural Engineering

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