SOLUTION OF ANISOTROPIC PLATE BY RAYLEIGH-RITZ METHOD

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Nomenclature

$D = \dfrac{Eh^3}{12(1-\mu^2)}$ Flexural Rigidity of Isotropic Plate

D_x', D_y', $D_{x'y'}'$, D_1' Rigidity Constants of Anisotropic Plate

E Young's Modulus of Isotropic Material

h Thickness of the Plate

μ Poisson's Ratio of Isotropic Plate Material

a Width of the Plate in x Direction

b Width of the Plate in y Direction

w Displacement Function in z Direction

I_1 Moment of Inertia of the Beam

A_{mn} Trigonometric Series Coefficient

α $l_{xx} = l_{yy}$ cos (x, x') or cos (y, y')

β $l_{xy} = -l_{yx}$

l_{xy} = cos (x, y')

l_{yx} = cos (y, x')

λ Center to Center Distance of Beams

K Increment in Torsional Rigidity Due to Transverse Beams

p Uniform Load Per Unit Area

x' Axes parallel to the direction of the beams.

y' Axes perpendicular to the direction of the beams.

G Modulus of Rigidity

$(w,_{xx}) = \dfrac{\partial^2 w}{\partial x^2}$

$(w,_{xy}) = \dfrac{\partial^2 w}{\partial x \partial y}$

(Nomenclature continued)

$(w,_{yy}) = \frac{\partial^2 w}{\partial y^2}$

$(w,_{x'x'}) = \frac{\partial^2 w}{\partial x'^2}$

$(w,_{x'y'}) = \frac{\partial^2 w}{\partial x' \partial y'}$

$(w,_{y'y'}) = \frac{\partial^2 w}{\partial y'^2}$

$\gamma = \frac{a}{b}$

θ = Angle Between The x and x' Axes

CHAPTER I

INTRODUCTION

1-1. General.

Anisotropic materials, unlike isotropic materials, have different elastic properties in various directions. The number of independent elastic constants involved for the analysis of a structure of isotropic material is two, namely Young's modulus and Poisson's ratio. In the case of an anisotropic material the number of independent elastic constants involved is considerably larger.

A common example of anisotropic material is wood. It has three different moduli of elasticity along three directions, longitudinal, radial and tangential. In practice we often come across anisotropic plates, such as a concrete slab reinforced with steel, or a steel plate stiffened with ribs. In both the above cases, the material used is isotropic but due to reinforcement in the former and ribs in the latter anisotropy is achieved artificially.

Gehring (1860) and Boussinesq (1879) first studied the problem of an isotropic plate. Huber (1914) [1] presented the solution of the differential equation for an anisotropic plate. The computations involved in the solution of the anisotropic plate are very tedious. In the last decade, due to the development in electronic computers, computations have been feasible.

1-2. Rayleigh-Ritz Method.

In 1877 Lord Rayleigh developed a method of obtaining an approximate value of the natural frequency of vibration of an elastic body by assuming a suitable function for the displacement. This method is used for finding an approximate value of deflection of different structures with comparatively very little effort [2]. Ritz (1908) considered an infinite series as the assumed deflection curve and developed the method of solution directly from an energy consideration without solving the differential equation, based on Rayleigh's principle. This method of assuming an infinite series for deflection curve is known as the Rayleigh-Ritz method. When finite number of terms are considered in the infinite series the solution obtained is an approximate one. This approximate solution approaches the exact value as the number of terms considered in the series is increased. Generally a few terms will yield a satisfactory solution. When all the terms of an infinite series are considered, the solution is an exact one [3].

The analysis of bending and buckling of thin isotropic plate with ribs or grooves or stiffeners can be simplified by considering this plate as an anisotropic. This can be achieved by setting up an analgous anisotropic plate equivalent to isotropic plate with ribs. The analysis of an anisotropic plate involves the solution of the following differential equation [4]

$$D_x \frac{\partial^4 w}{\partial x^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P(x, y)$$

Where D_x, D_y, D_{xy} and D_1 are the elastic rigidity constants of the anisotropic plate.

The main purpose of this thesis is to develop a method by which these elastic rigidity constants can be evaluated.

These constants can be evaluated by considering an anisotropic plate equivalent to the original isotropic plate with stiffeners. The equivalent anisotropic plate cannot be equal to the isotropic plate with stiffeners in all respects. However for the same boundary conditions and loading one can consider equal deflection or equal strain components at the corresponding points. Alternatively the strain energies may be considered equal.

If all these equalities were to give different values of the rigidity constants one cannot use them. But other investigations and experimental results show that the anisotropic plate theory is applicable to stiffened plate [7, 8, 9] provided that the stiffener spacing to plate dimension ($\lambda/b << 1$) is small to have homogenity of stiffness.

In this presentation a double trigonometric series has been assumed as a deflection curve for a rectangular plate, simply supported at the edges. The plate is assumed to be isotropic and it is considered that the plate has parallel beams λ distance apart, inclined at an angle θ to the x axis of the plate. Thus, two problems - are involved.

(1) Determination of the rigidity constants for the analgous anisotropic plate.

(2) Determination of the coefficients in the infinite series for deflection.

In this thesis methods are presented to determine the following analytically.

(1) By comparison of the strain energies of equivalent anisotropic plate and isotropic plate with stiffeners elastic rigidity constants are developed in terms of the elastic constants of the isotropic material and the geometrical and elastic properties of the stiffener.

(2) Total potential energy of the system consists of two parts: (a) strain energy, (b) potential energy due to loads. The principle of minimum energy which states that "The Potential energy is a minimum when an elastic body is in equilibrium" [3] is considered. The coefficients in the infinite series for deflection are obtained by minimizing the total potential energy of the system. As mentioned before, by this method, the solution is obtained without solving the differential equation.

A numerical example is presented with the following variation in loading.

(1) Uniformly distributed load p lbs. sq. ft. for $\gamma = a/b =$ 1, 2 and 3.

(2) Concentrated load P lbs. at $x/a = y/b = 0.25$ for $\gamma =$ 1, 2 and 3.

Computations were carried out with the help of the IBM 1620 computer.

1-3. Assumptions.

1. The thickness of the plate is small compared to its other dimensions and is constant.

2. The material of the plate is elastic, continuous and homogenous.

3. The deformations are small and they do not alter the original geometry of the plate.

4. The loads are perpendicular to the plane of the plate.

5. Stresses normal to the middle surface are negligible.

CHAPTER II

Basic Theory

2-1. Anisotropic Plate.

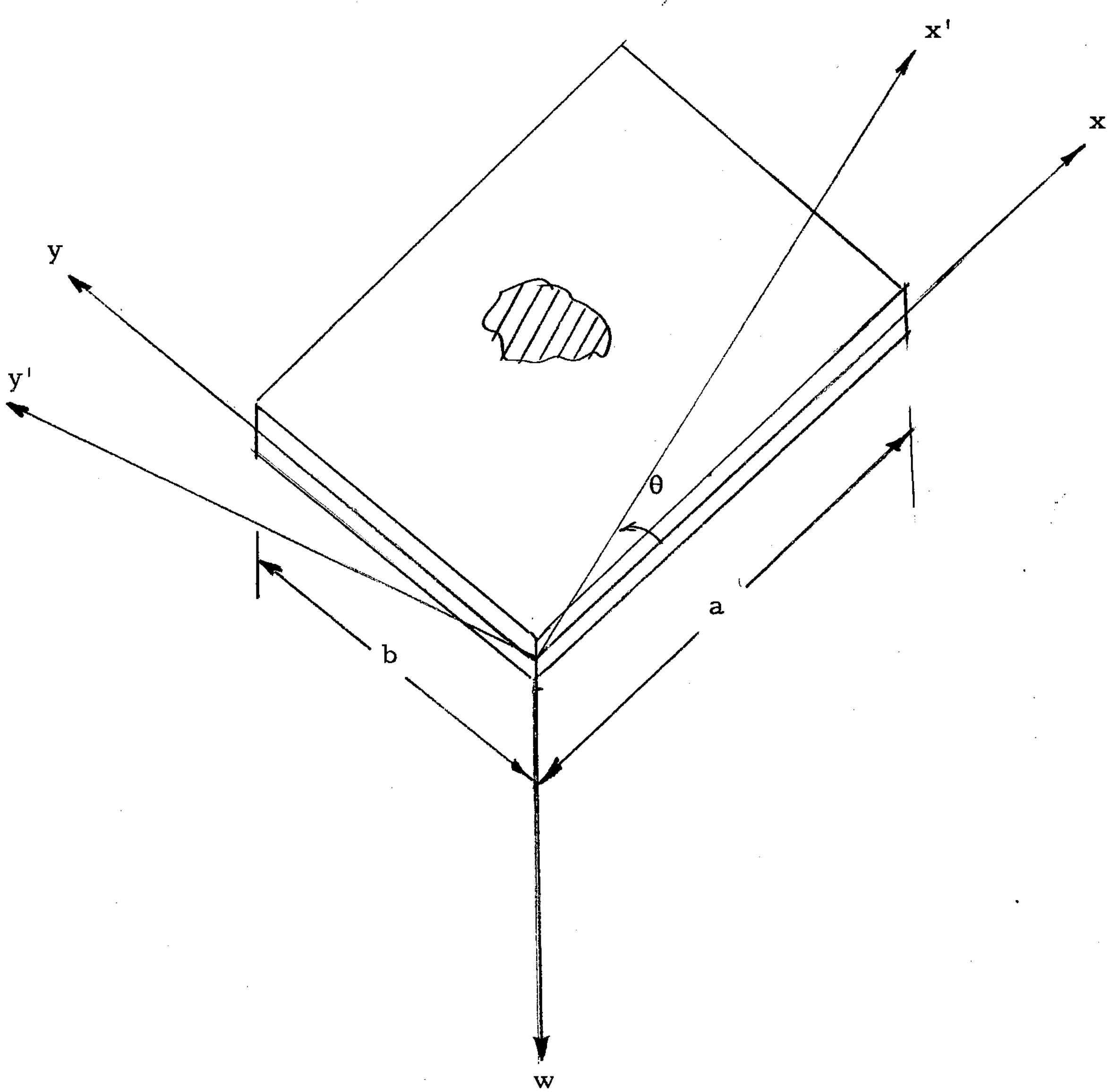

Figure 2-1. Anisotropic Plate.

Assume the displacement function as a double trigonometric series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

in order to satisfy the following boundary conditions.

The deflection must be zero along the simply supported edges.

$$w\,(x=0)(y=y) = 0$$
$$w\,(x=a)(y=y) = 0$$
$$w\,(x=x)(y=0) = 0$$
$$w\,(x=x)(y=b) = 0$$

Generally in Rayleigh-Ritz method, the assumed deflection function satisfies the natural geometrical boundary conditions. In this particular case it also satisfies the force boundary condition, namely the bending moments are zero along the edges

$$M_x\,(x=0)(y=y) = 0$$
$$M_x\,(x=a)(y=y) = 0$$
$$M_y\,(x=x)(y=0) = 0$$
$$M_y\,(x=x)(y=b) = 0$$

The values of the elastic rigidity constants of the anisotropic plate depend upon the geometrical configuration of stiffeners, hence it has been assumed that the properties of the analogous anisotropic plate are along $x'y'$ axes. The strain energy of the plate is given by [4]

$$= 1/2 \iint_A \Big[D_{x'}(w,_{x'x'})^2 + D_{y'}(w,_{y'y'})^2 + 2 D_{1'}(w,_{x'x'})(w,_{y'y'})$$

$$+ 4 D_{x'y'}(w,_{x'y'})^2 \Big] dA \qquad (1)$$

Transformation of co-ordinates.

As the limits of integration are in x-y co-ordinate system, it is preferable to work in x-y axis, rather than in $x'y'$ axes. Transformation is done by direction cosines.

$$x = l_{xx} x' + l_{xy} y'$$

$$y = l_{yx} x' + l_{yy} y'$$

where

$$l_{xx} = \cos(x, x')$$

$$l_{xy} = \cos(x, y')$$

$$l_{yx} = \cos(y, x')$$

$$l_{yy} = \cos(y, y')$$

$$l^2_{xx} + l^2_{xy} = 1 \text{ and } l^2_{yx} + l^2_{yy} = 1$$

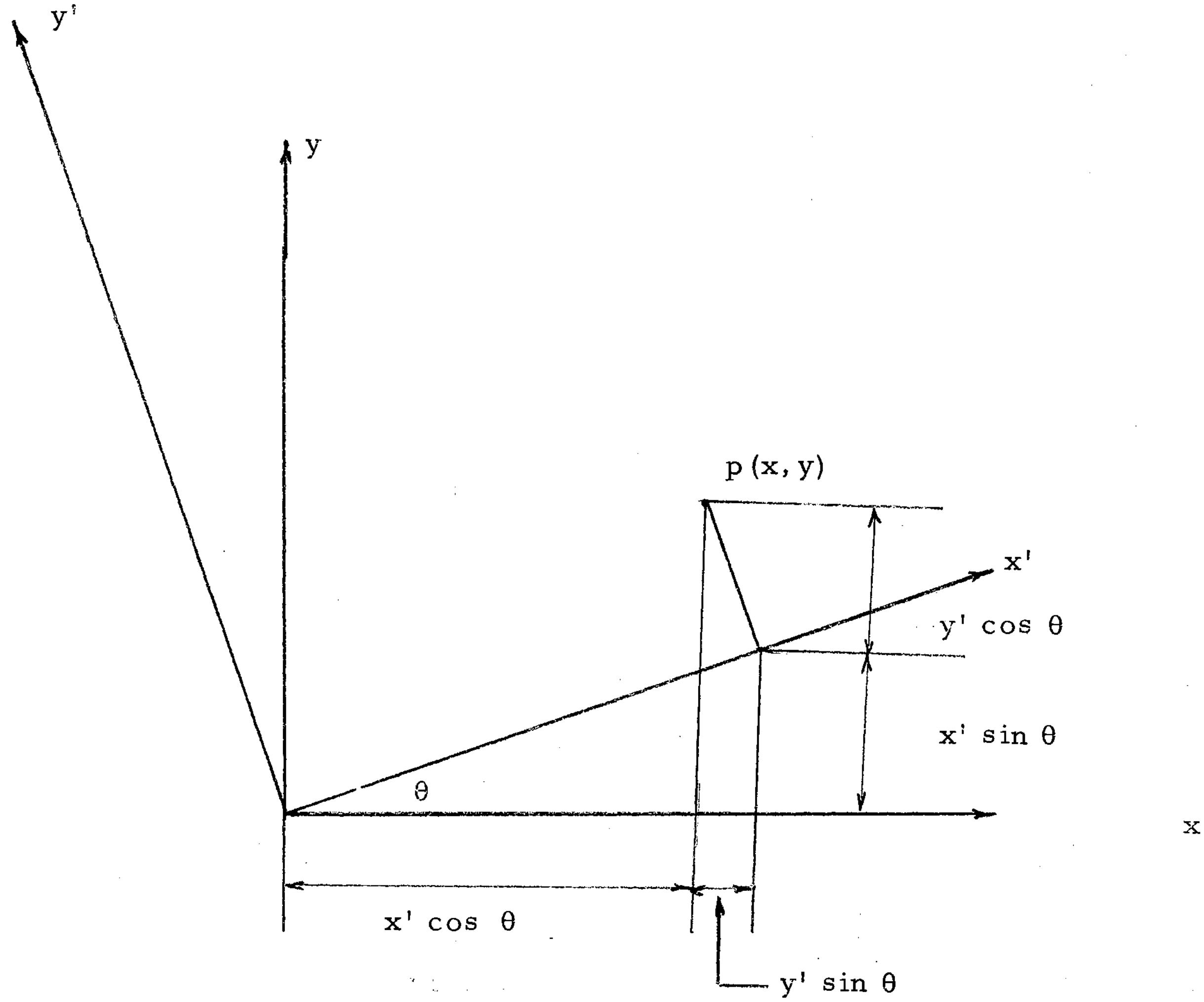

Figure 2-2. Transformation of Co-ordinates.

As w, the deflection, is a function of x and y, partial derivatives of w work out as follows.

$$\frac{\partial}{\partial x'} w(x,y) = \frac{\partial w}{\partial x} \cdot l_{xx} + \frac{\partial w}{\partial y} l_{yx}$$

$$(w,_{x'x'}) = \frac{\partial^2}{\partial x'^2} w(x,y) = \frac{\partial^2 w}{\partial x^2} l_{xx}^2 + 2\frac{\partial^2 w}{\partial x \partial y} l_{xx} l_{yx} + \frac{\partial^2 w}{\partial y^2} l_{yx}^2$$

$$(w,_{x'y'}) = \frac{\partial^2}{\partial x' \partial y'} w(x,y) = \frac{\partial^2 w}{\partial x^2} l_{xx} l_{xy} + \frac{\partial^2 w}{\partial x \partial y} l_{xx} l_{yy}$$

$$+ \frac{\partial^2 w}{\partial x \partial y} l_{yx} l_{xy} + \frac{\partial^2 w}{\partial y^2} l_{yx} l_{yy}$$

$$(w,_{y'y'}) = \frac{\partial^2}{\partial y'^2} w(x,y) = \frac{\partial^2 w}{\partial x^2} l_{xy}^2 + 2\frac{\partial^2 w}{\partial x \partial y} l_{xy} l_{yy} + \frac{\partial^2 w}{\partial y^2} l_{yy}^2$$

$$(w,_{x'x'})^2 = (w,_{xx})^2 l_{xx}^4 + 4(w,_{xy})^2 l_{xx}^2 l_{yx}^2 + (w,_{yy})^2 l_{yx}^4$$

$$+ 4(w,_{xx})(w,_{xy}) l_{xx}^3 l_{yx} + 2(w,_{xx})(w,_{yy}) l_{xx}^2 l_{yx}^2$$

$$+ 4(w,_{xy})(w,_{yy}) l_{xx} l_{yx}^3 \qquad (2)$$

$(w,_{xx})(w,_{xy})$ and $(w,_{xy})(w,_{yy})$ when integrated work out to be zero due to orthogonality.

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$\frac{\partial w}{\partial x} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Amn \frac{m\pi}{a} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

$$\frac{\partial^2 w}{\partial x^2} = (w,_{xx}) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Amn \frac{m^2 \pi^2}{a^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Squaring the above the following is obtained.

$$(w,_{xx})^2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} A_{mn} A_{rs} \pi^4 \frac{m^2}{a^2} \frac{r^2}{a^2} X$$

$$\sin \frac{m\pi x}{a} \sin \frac{r\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{s\pi y}{b}$$

Using the trigonometric identities:

$$\sin \frac{m\pi x}{a} \sin \frac{r\pi x}{a} dx = 0 \text{ for } m \neq r$$

$$= \frac{a}{2} \text{ for } m = r$$

$$\sin \frac{n\pi y}{b} \sin \frac{s\pi y}{b} dy = 0 \text{ for } n \neq s$$

$$= \frac{b}{2} \text{ for } n = s$$

and integrating the following is obtained,

$$\int_0^a \int_0^b (w,_{xx})^2 \, dx\,dy = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \frac{\pi^4}{a^4} m^4 \cdot \frac{a}{2} \cdot \frac{b}{2}$$

similarly the following expression may be obtained,

$$\int_0^a \int_0^b (w,_{yy})^2 \, dx\,dy = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \frac{\pi^4}{b^4} n^4 \cdot \frac{a}{2} \cdot \frac{b}{2}$$

Taking the second derivative of $\frac{\partial w}{\partial x}$ with respect to y and squaring the following is obtained,

$$\int_0^a \int_0^b (w,_{xy})^2 \, dx\,dy = \int_0^a \int_0^b \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} A_{mn} A_{rs} \frac{m^2 n^2}{a^2 b^2} \pi^4 \cos \frac{m\pi x}{a}$$

$$\cdot \cos \frac{r\pi x}{a} \cos \frac{n\pi y}{b} \cos \frac{s\pi y}{b} \cdot dx\,dy$$

$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \pi^4 \frac{m^2 n^2}{a^2 b^2} \cdot \frac{a}{2} \cdot \frac{b}{2}$$

$$\int_0^a \int_0^b (w,_{xx})(w,_{yy}) \, dx\,dy = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \pi^4 \frac{m^2 n^2}{a^2 b^2} \frac{a}{2} \cdot \frac{b}{2}$$

let $l_{xx} = l_{yy} = \alpha$, $l_{xy} = -l_{yx} = \beta$

substituting into (2) the following is obtained

$$\int_0^a \int_0^b (w,_{x'x'})^2 \, dx\,dy = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \pi^4 \frac{ab}{4} \cdot$$

$$\left[\alpha^4 \frac{m^4}{a^4} + 6\alpha^2 \beta^2 \frac{m^2 n^2}{a^2 b^2} + \beta^4 \frac{n^4}{b^4} \right] \qquad (3)$$

$$\left[w,_{y'y'}\right]^2 = \left[(w,_{xx})\, l_{xy}^2 + 2(w,_{xy})\, l_{xy}\, l_{yy} + (w,_{yy})\, l_{yy}^2\right]^2$$

$$= (w,_{xx})^2\, l_{xy}^4 + 4(w,_{xy})^2\, l_{xy}^2\, l_{yy}^2 + (w,_{yy})^2\, l_{yy}^4$$

$$+ 4(w,_{xx})(w,_{xy})\, l_{xy}^3\, l_{yy} + 4(w,_{xy})(w,_{yy})\, l_{xy}\, l_{yy}^3$$

$$+ 2(w,_{xx})(w,_{yy})\, l_{xy}^2\, l_{yy}^2 \qquad (4)$$

substituting and integrating, the following is obtained

$$\int\int (w,_{y'y'})^2\, dx\, dy = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}^2\, \pi^4\, \frac{ab}{4} \cdot$$

$$\left[\beta^4\, \frac{m^4}{a^4} + 6\,\alpha^2\, \beta^2\, \frac{m^2 n^2}{a^2 b^2} + \alpha^4\, \frac{n^4}{b^4}\right] \qquad (5)$$

$$\left[w,_{x'x'}\right]\left[w,_{y'y'}\right] = \left[(w,_{xx})\, l_{xx}^2 + 2(w,_{xy})\, l_{xx}\, l_{yx} + (w,_{yy})\, l_{yx}^2\right] \cdot$$

$$\left[(w,_{xx})\, l_{xy}^2 + 2(w,_{xy})\, l_{xy}\, l_{yy} + (w,_{yy})\, l_{yy}^2\right]$$

$$= (w,_{xx})^2\, l_{xx}^2\, l_{xy}^2 + (w,_{xx}\, w,_{yy})(l_{xx}^2\, l_{yy}^2 + l_{xy}^2\, l_{yx}^2)$$

$$+ 2(w,_{xx})(w,_{xy})(l_{xx}^2\, l_{xy}\, l_{yy} + l_{xy}^2\, l_{xx}\, l_{yx}) + 2(w,_{yy})(w,_{xy})(l_{xx}\, l_{yx}\, l_{yy}^2 + l_{xy}\, l_{yy}\, l_{yx}^2)$$

$$+ 4(w,_{xy})^2\, l_{xx}\, l_{yx}\, l_{xy}\, l_{yy} + (w,_{yy})^2\, l_{yx}^2\, l_{yy}^2 \qquad (6)$$

substituting and integrating the following is obtained,

$$\int_0^a \int_0^b (w,_{x'x'})(w,_{y'y'})\, dx\, dy = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn}\, \pi^4 \frac{ab}{4} \bullet$$

$$\left[\left(\frac{m^4}{a^4} + \frac{n^4}{b^4} \right) \alpha^2 \beta^2 + \frac{m^2 n^2}{a^2 b^2} (-4\alpha^2 \beta^2 + \alpha^4 + \beta^4) \right] \tag{7}$$

$$\left[w,_{x'y'} \right]^2 = (w,_{xx})^2\, l^2_{xx}\, l^2_{xy} + (w,_{xy})^2 (l^2_{xx}\, l^2_{xy} + l^2_{yx}\, l^2_{yy})$$

$$+ (w,_{yy})^2\, l_{yx}\, l^2_{yy} + 2(w,_{xx})(w,_{xy})(l^2_{xx}\, l_{xy}\, l_{yy})$$

$$+ 2(w,_{xy})^2\, l_{xx}\, l_{xy}\, l_{yx}\, l_{yy}$$

$$+ 2(w,_{xy})(w,_{yy})\, l_{xy}\, l^2_{yx}\, l_{yy}$$

$$+ 2(w,_{xx})(w,_{xy})\, l_{xx}\, l^2_{xy}\, l_{yx}$$

$$+ 2(w,_{xx})(w,_{yy})\, l_{xx}\, l_{xy}\, l_{yx}\, l_{yy}$$

$$+ 2(w,_{xy})(w,_{yy})\, l_{xx}\, l_{yx}\, l^2_{yy} \tag{8}$$

Substituting and integrating the following is obtained

$$\int_0^a\int_0^b (w,_{x'y'})^2 \, dx\, dy = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A^2{}_{mn} \frac{\pi^4 ab}{4} \cdot$$

$$\left[\left(\frac{m^4}{a^4}+\frac{n^4}{b^4}\right)\alpha^2\beta^2 + \frac{m^2n^2\alpha^2\beta^2}{a^2b^2}\left(-4\alpha^2\beta^2+\alpha^4+\beta^4\right)\right] \quad (9)$$

Substituting all values in the energy expression (1), the total strain energy is obtained as follows

$$= 1/2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A^2{}_{mn} \frac{\pi^4 ab}{4}\left[D_{x'}\left\{\alpha^4\frac{m^4}{a^4} + 6\alpha^2\beta^2\frac{m^2n^2}{a^2b^2} + \beta^4\frac{n^4}{b^4}\right\}\right.$$

$$+ D_{y'}\left\{\beta^4\frac{m^4}{a^4} + 6\alpha^2\beta^2\frac{m^2n^2}{a^2b^2} + \alpha^4\frac{n^4}{b^4}\right\}$$

$$\left. + (2D_1' + 4D_{x'y'})\left\{\left(\frac{m^4}{a^4}+\frac{n^4}{b^4}\right)\alpha^2\beta^2 + \frac{m^2n^2}{a^2b^2}\left(-4\alpha^2\beta^2+\alpha^4+\beta^4\right)\right\}\right] \quad (10)$$

This can be written as

$$1/2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A^2{}_{mn} \frac{\pi^4}{4ab}\,[S] \quad (10A)$$

where

$$[S] = D_{x'} \left\{ \alpha^4 \frac{m^4}{\gamma^2} + 6\alpha^2\beta^2m^2n^2 + \beta^4n^4\gamma^2 \right\}$$

$$+ D_{y'} \left\{ \beta^4 \frac{m^4}{\gamma^2} + 6\alpha^2\beta^2m^2n^2 + \alpha^4n^4\gamma^2 \right\}$$

$$+ (2D_1' + 4D_{x'y'}) \left\{ \left(\frac{m^4}{\gamma^2} + n^4\gamma^2 \right) \alpha^2\beta^2 + m^2n^2 \left(-4\alpha^2\beta^2 + \alpha^4 + \beta^4\right) \right\}$$

The expression 10(a) represents the total strain energy of the analogous anisotropic plate in terms of elastic rigidity constants.

2-2. Isotropic Plate Stiffened By Beams at an Angle θ to X Axis.

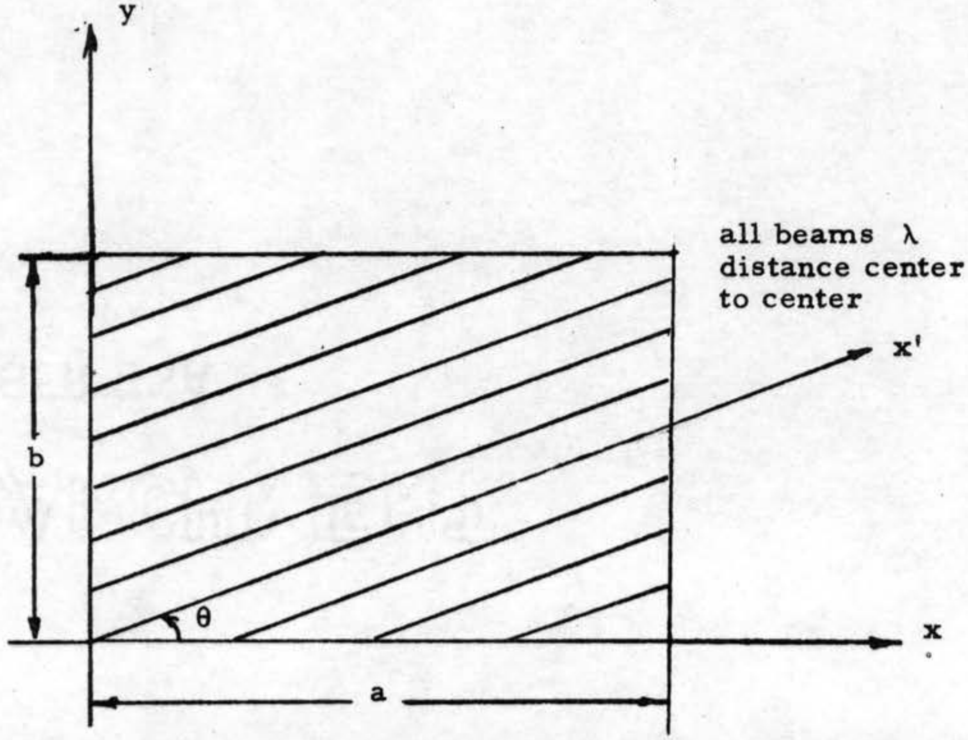

Figure 2-3. Isotropic Plate with Beams Inclined at an Angle θ to the x axis of the plate.

The total energy of the plate consists of two parts,

(1) Energy of the isotropic plate.

(2) Energy of the beams.

Consider an element of the plate. Energy of the isotropic plate element [4]

$$= D/2\left[(w,_{x'x'})^2 + (w,_{y'y'})^2 + 2\mu\,(w,_{x'x'})(w,_{y'y'}) + 2(1-\mu)(w,_{x'y'})^2\right] dx'\,dy'$$

The energy of the beam consists of two parts:

(1) Energy due to flexural bending.

(2) Energy due to torsional bending.

$$= \frac{E}{2\lambda}\left[I_1\,(w,_{x'x'})^2\,dx'\,dy'\right]$$

$$+ \frac{G}{2\lambda}\left[K(w,_{x'y'})^2\,dx'\,dy'\right]$$

substituting from previous results total energy,

$$= \frac{D}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \frac{\pi^4 ab}{4} \left[\left\{ \alpha^4 \frac{m^4}{a^4} + 6\alpha^2\beta^2 \frac{m^2n^2}{a^2b^2} + \beta^4 \frac{n^4}{b^4} \right\} \right.$$

$$+ \left\{ \beta^4 \frac{m^4}{a^4} + 6\alpha^2\beta^2 \frac{m^2n^2}{a^2b^2} + \alpha^4 \frac{n^4}{b^4} \right\}$$

$$+ 2\mu \left\{ \left(\frac{m^4}{a^4} + \frac{n^4}{b^4} \right) \alpha^2\beta^2 + \frac{m^2n^2}{a^2b^2} (-4\alpha^2\beta^2 + \alpha^4 + \beta^4) \right\}$$

$$\left. + 2(1-\mu) \left\{ \left(\frac{m^4}{a^4} + \frac{n^4}{b^4} \right) \alpha^2\beta^2 + \frac{m^2n^2}{a^2b^2} (-4\alpha^2\beta^2 + \alpha^4 + \beta^4) \right\} \right]$$

$$+ 1/2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \frac{\pi^4 ab}{4} \left[\frac{EI_1}{\lambda} \left\{ \alpha^4 \frac{m^4}{a^4} + 6\alpha^2\beta^2 \frac{m^2n^2}{a^2b^2} + \beta^4 \frac{n^4}{b^4} \right\} \right.$$

$$\left. \frac{GK}{\lambda} \left\{ \left(\frac{m^4}{a^4} + \frac{n^4}{b^4} \right) \alpha^2\beta^2 + \frac{m^2n^2}{a^2b^2} (-4\alpha^2\beta^2 + \alpha^4 + \beta^4) \right\} \right] \qquad (11)$$

Expressions (10) and (11) are for the energies of the analogous anisotropic plate and the ribbed plate. To establish equivalence between the analogous anisotropic plate and the isotropic plate with ribs these two strain energies will be considered equal. Therefore comparing these two energies, rigidity properties of analogous anisotropic plate can be determined as follows:

$$D_x' = D + \frac{EI_1}{\lambda} \tag{12}$$

$$D_y' = D \tag{13}$$

$$2D_1' + 4D_{x'y'} = 2D + \frac{GK}{\lambda} \tag{14}$$

Mr. N. J. Huffington, Jr. obtained the same properties, assuming Levy's type solution for deflection and considering the strain energy of the plate [7]. His values are as follows:

$$D_x' = D + \frac{EI_1}{\lambda}$$

$$D_y' = \frac{b^5}{30\int_a^b \frac{(by-y^2)^2}{D(y)}\, dy} .$$

$$2D_1' + 4D_{x'y'} = 2\mu D + \frac{KG}{\lambda}$$

where

$$D(y) = \frac{E[h(y)]^3}{12(1-\mu^2)}$$

h(y) is the total thickness of the plate-stiffener combination. By comparison the following can be noted:

(1) The value of D_x' is the same in both of the cases.

(2) The value of $D_y' = D$ is an approximate one. This is easier to calculate compared to tedious integral involved in Huffington's method.

(3) The equation (14) $2D_1' + 4D_{x'y'} = 2D + GK/\lambda$ is the same as that of Huffington's except that it does not have the coefficient μ in the first term of the right hand side.

For the complete analysis of the anisotropic plate it is necessary to determine the deflection surface of the plate for various types of loading. To calculate the deflection, the coefficients in the infinite series for deflection must be known. These coefficients for two different types of loadings will be considered.

2-3. Plate Loaded With Uniformly Distributed Load.

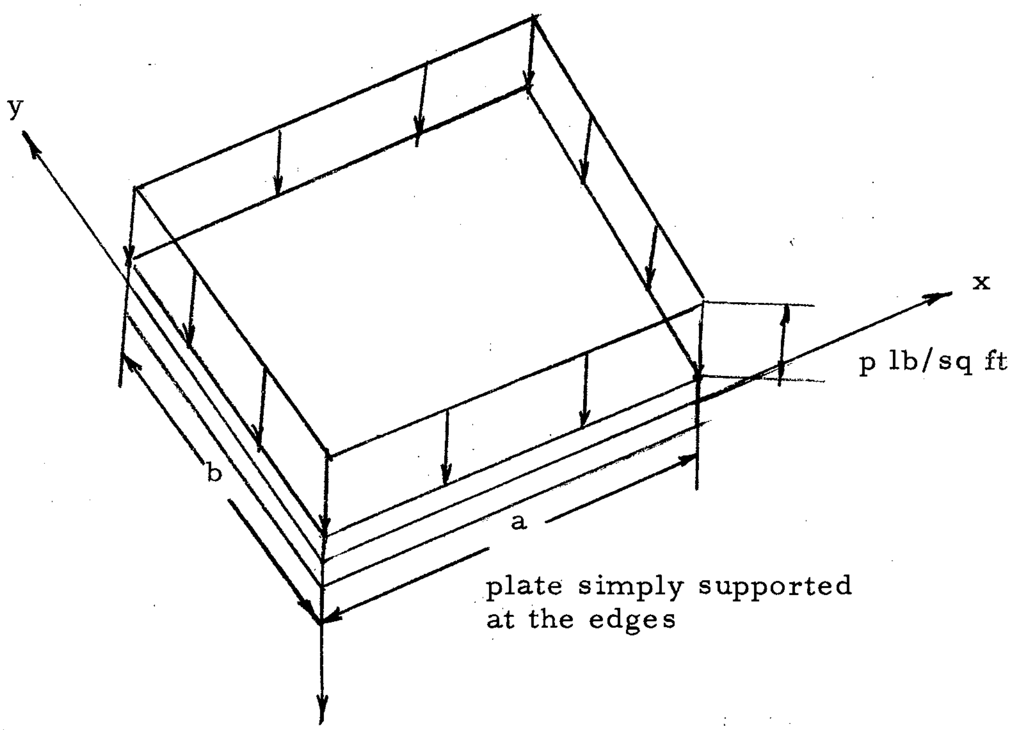

Figure 2-4. Anisotropic Plate Loaded with Distributed Load.

The potential energy of the load acting upon an element is given by

$$p \cdot w \, dx \, dy.$$

Therefore the potential energy due to the total load is

$$= \int_0^a \int_0^b p \cdot w \cdot dx \, dy$$

Substituting the value of w the following is obtained

$$= \int_0^a \int_0^b p \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dx \, dy.$$

$$= p \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \int_0^a \int_0^b \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dx \, dy$$

$$= p \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left[-\frac{a}{m\pi} \cos \frac{m\pi x}{a} \right]_0^a \left[-\frac{b}{n\pi} \cos \frac{n\pi y}{b} \right]_0^b$$

$= 0$ for $m =$ even

$n =$ even

$$= 4p \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \frac{ab}{mn\pi^2} \qquad \text{when } m = \text{odd}, \; n = \text{odd} \tag{15}$$

From equation (10) and (15), the total potential energy of the system is found to be

$$\text{III} = \text{strain energy - potential energy due to loads} \tag{16}$$

$$= 1/2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \frac{\pi^4}{4ab} [S]$$

$$- 4p \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \frac{ab}{mn\pi^2} \qquad \text{for } m = \text{odd}, \; n = \text{odd}$$

The principle of minimum energy states that "the potential energy is a minimum when an elastic body is in equilibrium" [3].

The potential energy III will be minimized with respect to each coefficient as follows

$$\frac{\partial \, III}{\partial A_{jk}} = 1/\cancel{2}\cdot\cancel{2} \; A_{jk} \, \frac{\pi^4}{4ab} \, [S] - 4p \, \frac{ab}{jk\pi^2} = 0$$

Solving for the coefficient A_{jk} yields

$$A_{jk} = \frac{4p \, \dfrac{ab}{jk\pi^2}}{\dfrac{\pi^4}{4ab} \, [S]}$$

$$= \frac{16p \, a^2 b^2}{\pi^6 \, jk \, [S]} \qquad (17)$$

2-4. Plate Loaded with Concentrated Load at $x/a = y/b = 0.25$.

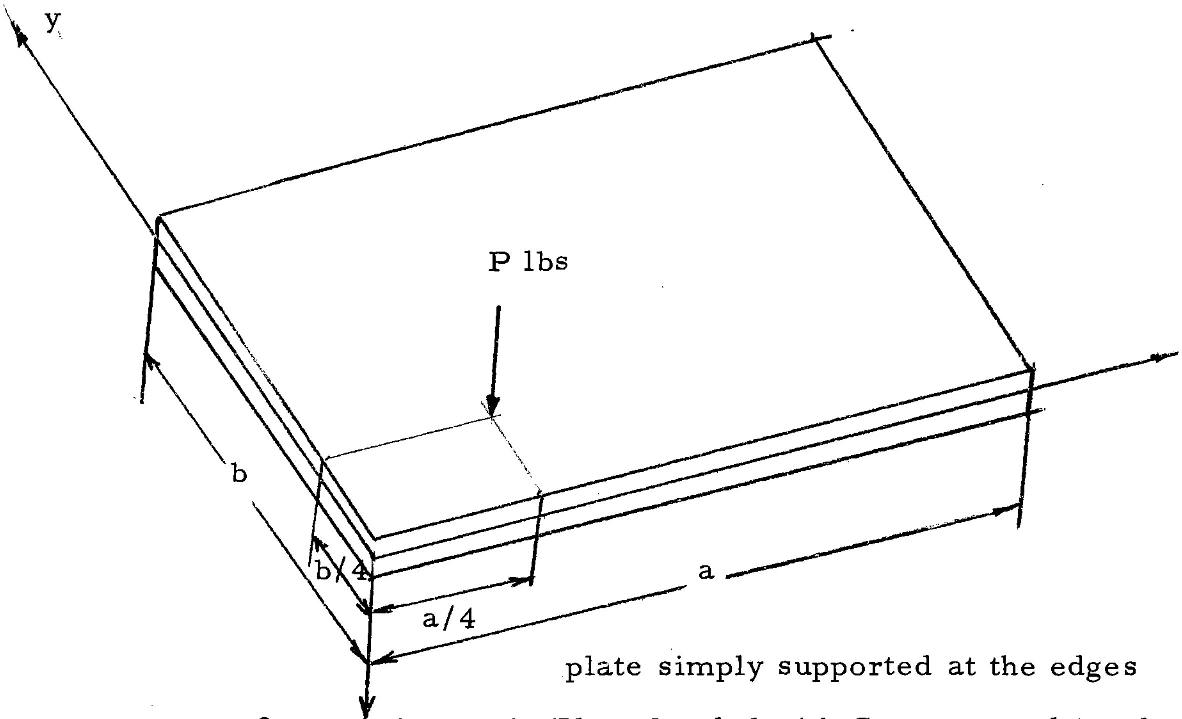

Figure 2-4. Anisotropic Plate Loaded with Concentrated Load.

The potential energy due to load is given by

$$= P \cdot w$$

$$\text{at } x = a/4$$
$$y = b/4$$

$$= P \cdot A_{mn} \sin \frac{m\pi}{4} \sin \frac{n\pi}{4}$$

$\therefore$ III = total potential energy of the system

= strain energy - potential energy due to loads.

$$III = \frac{1}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A^2_{mn} \frac{\pi^4}{4ab} \left[S \right]$$

$$- P \cdot A_{mn} \sin \frac{m\pi}{4} \cdot \sin \frac{n\pi}{4}$$

$$\frac{\partial \, III}{\partial A_{jk}} = \frac{1}{2} \, 2 \, A_{jk} \frac{\pi^4}{4ab} \left[S \right] - P \cdot \sin \frac{j\pi}{4} \sin \frac{k\pi}{4}$$

$$= 0$$

$$A_{jk} = \frac{4P \cdot ab \cdot \sin \frac{j\pi}{4} \cdot \sin \frac{k\pi}{4}}{\pi^4 \left[S \right]} \tag{18}$$

CHAPTER III

NUMERICAL EXAMPLE

3-1. Data.

To illustrate the theory discussed in the previous chapter the following example is considered. A 3/8" thick steel plate has 1/8 x 3/8" stiffeners at 1" center to center. The angle of inclination of the stiffeners (θ) with the x axis of the plate varies from 0° to 90° with an increment of 10°. Elastic rigidity constants are worked out for 3 different sizes of the plate, $\frac{a}{b} = \gamma = 1, 2$ and 3. Deflections at the various points of the plate are worked out for two types of loading

(1) Distributed Load p lbs/sq. ft.

(2) Concentrated Load P at $\frac{x}{a} = \frac{y}{b} = 0.25$

3-2. Computation of Rigidity Constants.

$$D = \frac{Eh^3}{12\,(1 - \mu^2)} \qquad E = 30 \times 10^6 \text{ lb/in}^2$$

$$= \frac{30 \times 10^6}{12(1 - .09)} \times \frac{1}{64} \qquad h = 1/4''$$

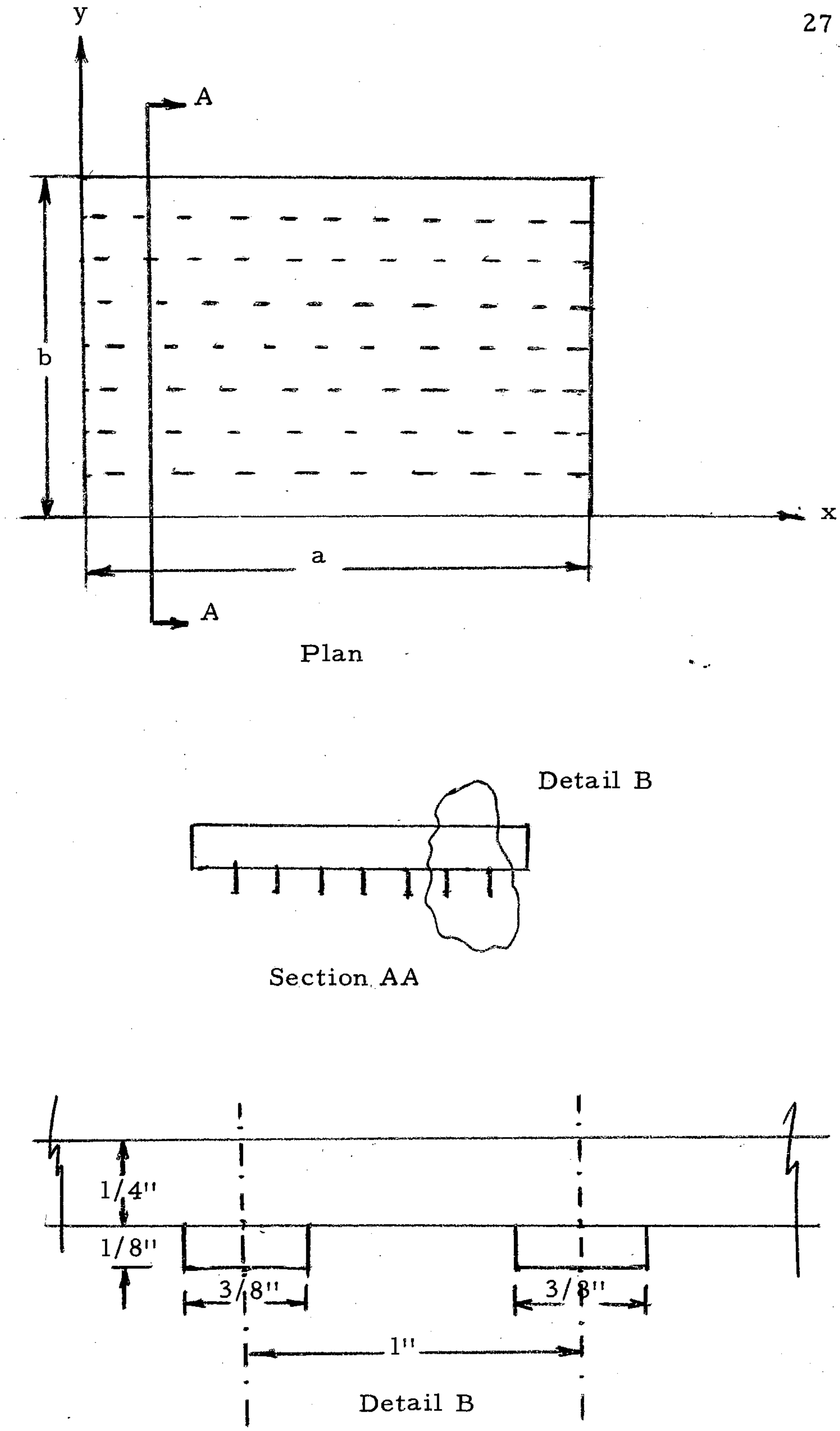

Figure 3-1. Plan, Section and Detail of Anisotropic Plate.

$= 42.92 \times 10^3$ lb in $\qquad\qquad \mu = 0.3$

$$D = 0.04292 \times 10^6 \text{ lb. in}$$

$$I_1 = \frac{bh^3}{12} + \text{Area} \times (\text{distance from } n-a)^2$$

$$= \frac{3}{8} \times \frac{1^3}{8} \times \frac{1}{12} + \frac{3}{8} \times \frac{1}{8}\left(\frac{1}{8} + \frac{1}{16}\right)^2$$

$$= 1.717 \times 10^{-3} \text{ in}^4$$

K (for whole section) [5]

$$= \beta Bh^3 \qquad\qquad \beta = 0.141$$

$$= 0.141 \times \frac{3}{8} \times \left(\frac{3}{8}\right)^3 \qquad\qquad \text{for } \frac{B}{h} = 1$$

$$= 2.786 \times 10^{-3} \text{ in}^4$$

K (for plate only)

$$= 0.196 \times \frac{3}{8}\left(\frac{1}{4}\right)^3 \qquad\qquad \beta = 0.196$$

$$= 1.148 \times 10^{-3} \text{ in}^4 \qquad\qquad \text{for } \frac{B}{h} = 1.5$$

K (increment due to beams)

$$= 1.638 \times 10^{-3} \text{ in}^4$$

$$D_x' = D + \frac{EI_1}{\lambda} \tag{12}$$

$$= .04292 \times 10^{-6} + \frac{30 \times 10^{6} \times 1.717 \times 10^{-3}}{1}$$

$$= (.04292 + .05151) \times 10^{6}$$

$$= .09443 \times 10^{6} \text{ lb in}$$

$$D_y' = D \tag{13}$$

$$= .04292 \times 10^{6} \text{ lb in}$$

$$2D_1' + 4D_{x'y'} = 2D + \frac{GK}{\lambda} \tag{14}$$

$$= 2\,(.04292 \times 10^{6}) + \frac{11.26 \times 10^{6} \times 1.638 \times 10^{-3}}{1}$$

$$= (.08584 + .01835) \times 10^{6}$$

$$= .10419 \times 10^{6} \text{ lb in}$$

3-3. Determination of Coefficients.

(1) Distributed Load.

$$A_{mn} = \frac{16p\, a^2 b^2}{\pi^6\, mn\, [S]} \tag{17}$$

(a) For $\gamma = \frac{a}{b} = 1$

$$A_{mn} = \frac{16\ p\ b^4}{\pi^6\ mn\ [S]}$$

$$= \frac{16\ p\ b^4}{\pi^6 \times 10^6\ mn\ [S']}$$

where $[S] = 10^6\ [S']$

$$= \left[\frac{16\ p\ b^4}{\pi^6 \times 10^6}\right] \frac{1}{mn} \quad \frac{1}{[S']} \qquad (19)$$

$$= \frac{C}{m \cdot n\ [S']} \quad \text{inches}$$

where $C = \frac{16\ pb^4}{\pi^6 \times 10^6} \times 144$

(b) For $\gamma = \frac{a}{b} = 2$

$$A_{mn} = \frac{4 \cdot C}{m \cdot n \cdot [S']} \quad \text{inches} \qquad (20)$$

(c) For $\gamma = \frac{a}{b} = 3$

$$A_{mn} = \frac{9 \cdot C}{m \cdot n\ [S']} \quad \text{inches} \qquad (21)$$

(2) Concentrated Load at $x/a = y/b = 0.25$.

$$A_{mn} = \frac{4 \cdot P \cdot \sin\frac{m\pi}{4} \cdot \sin\frac{n\pi}{4} \cdot ab}{\pi^4 \left[S\right]} \qquad (18)$$

(a) For $\gamma = \frac{a}{b} = 1$

$$A_{mn} = \frac{4 \cdot P \cdot b^2}{\pi^4} \cdot \frac{\sin\frac{m\pi}{4} \cdot \sin\frac{n\pi}{4}}{\left[S\right]} \cdot$$

$$= C' \cdot \frac{\sin\frac{m\pi}{4} \cdot \sin\frac{n\pi}{4}}{\left[S'\right]} \text{ inches} \qquad (22)$$

where $\left[S'\right] = 10^6 \left[S'\right]$

$$C' = \frac{4P\,b^2}{\pi^4 \times 10^6} \times 144$$

(b) For $\gamma = \frac{a}{b} = 2$

$$A_{mn} = \frac{C' \cdot 2 \cdot \sin\frac{m\pi}{4} \sin\frac{n\pi}{4}}{\left[S'\right]} \text{ inches} \qquad (23)$$

(c) For $\gamma = \frac{a}{b} = 3$

$$A_{mn} = \frac{C' \cdot 3 \cdot \sin\frac{m\pi}{4} \cdot \sin\frac{n\pi}{4}}{[S']} \text{ inches} \tag{24}$$

3-4. Computations.

Computations were carried out with the help of the IBM 1620 computer. The coefficients in the infinite series for deflection were computed using equations (19), (20), and (21) for distributed load, and (22), (23), and (24) for concentrated load. After determination of these coefficients, deflections starting from $\frac{x}{a} = \frac{y}{b} = 0.1$ with an increment of each value of 0.1 were computed. Various results of the deflection for different values of γ and for corresponding value of θ are shown in table 3-1 and 3-2.

The Rayleigh-Ritz method as mentioned previously, gives an exact value of the deflection if all the terms in the infinite series are considered. As it is impossible to consider all the terms, only a few terms were considered. When two consecutive terms showed very little variation, the values of the deflection were considered as sufficiently converged. For the distributed load case, m and n = 5, 7, 9 and 11 were considered. Results with m = n = 7 and 9 showed less than 3 percent variations. For concentrated load m and n = 6, 7, 8 and 9 were considered. Results with m and n = 7 and 9 showed less than 3 percent variation. Results, therefore, with m and n = 7 were considered sufficiently converged values of deflections.

The maximum value of deflection was noted in each case. Then graphs of maximum value of the deflection versus angle θ were plotted. These graphs showed that for $\gamma = 1$, a square plate, deflection is maximum for $\theta = 0°$ and $90°$ and it is minimum for $\theta = 45°$. Also from the above graphs minimum values of the maximum deflection for various γ values were noted. A separate graph showing these values was plotted. The graph showed that the maximum deflection approaches its minimum value, as θ approaches $90°$ and γ increases from 1 to 5 very rapidly for distributed load and slowly for concentrated load. This criteria confirms that beams should be along the shorter span to have the minimum value of the maximum deflection.

TABLE 3-1

DEFLECTION AT VARIOUS POINTS FOR DISTRIBUTED LOAD

Point		Deflection in Terms of C		
x/a	y/b	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
		Angle . 0°		
0.1	0.1	0.433	1.4477	2.1861
0.2	0.2	1.4969	4.9203	7.0245
0.3	0.3	2.7304	8.8350	11.9791
0.4	0.4	3.6847	11.7960	15.4576
0.5	0.5	4.0399	12.8816	16.6626
		Angle 10°		
0.1	0.1	0.4142	1.4017	2.1272
0.2	0.2	1.4308	4.7677	6.8494
0.3	0.3	2.6087	8.5657	11.7039
0.4	0.4	3.5193	11.4414	15.1232
0.5	0.5	3.8582	12.4965	16.3108
		Angle 20°		
0.1	0.1	0.3894	1.3099	2.0008
0.2	0.2	1.3442	4.4476	6.4360
0.3	0.3	2.4492	7.9800	10.9958
0.4	0.4	3.3028	10.6500	14.2134
0.5	0.5	3.6202	11.6284	15.3316

TABLE 3-1 (Continued)

x/a	y/b	γ = 1	γ = 2	γ = 3
Angle 30°				
0.1	0.1	0.368	1.2040	1.8396
0.2	0.2	1.2699	4.0735	5.8889
0.3	0.3	2.3131	7.2859	10.0266
0.4	0.4	3.1187	9.7042	12.9394
0.5	0.5	3.4182	10.5879	13.9482
Angle 40°				
0.1	0.1	0.3568	1.1102	1.679
0.2	0.2	1.2316	3.7358	5.3223
0.3	0.3	2.2440	6.6495	8.9917
0.4	0.4	3.0260	8.8289	11.5535
0.5	0.5	3.3169	9.6216	12.4326
Angle 45°				
0.1	0.1	0.3561	1.0724	1.6074
0.2	0.2	1.2296	3.5955	5.0602
0.3	0.3	2.2411	6.3797	8.5002
0.4	0.4	3.0228	8.4533	10.8855
0.5	0.5	3.3136	9.2052	11.6980

TABLE 3-1 (Continued)

Angle 50°				
x/a	y/b	γ = 1	γ = 2	γ = 3
0.1	0.1	0.3588	1.0414	1.5447
0.2	0.2	1.2398	3.4775	4.8231
0.3	0.3	2.2604	6.1475	8.0457
0.4	0.4	3.0495	8.1260	10.2609
0.5	0.5	3.3431	8.8405	11.0083
Angle 60°				
0.1	0.1	.3742	1.0003	1.4507
0.2	0.2	1.2936	3.3070	4.4394
0.3	0.3	2.3600	5.7929	7.2787
0.4	0.4	3.1853	7.6110	9.1870
0.5	0.5	3.4926	8.2614	9.8148
Angle 70°				
0.1	0.1	0.3987	.9835	1.3990
0.2	0.2	1.3788	3.214	4.1849
0.3	0.3	2.5160	5.569	6.7276
0.4	0.4	3.3963	7.2639	8.3928
0.5	0.5	3.7241	7.8626	8.923

TABLE 3-1 (Continued)

x/a	y/b	γ = 1	γ = 2	γ = 3
Angle 80°				
0.1	0.1	.4228	.9820	1.3795
0.2	0.2	1.4616	3.1759	4.0472
0.3	0.3	2.6656	5.4492	6.3978
0.4	0.4	3.5988	7.0604	7.9042
0.5	0.5	3.9460	7.6628	8.3692
Angle 90°				
0.1	0.1	.4331	.9837	1.3760
0.2	0.2	1.4970	3.1678	4.0052
0.3	0.3	2.7304	5.4127	6.2886
0.4	0.4	3.6847	6.9931	7.7395
0.5	0.5	4.04	7.5419	8.1812

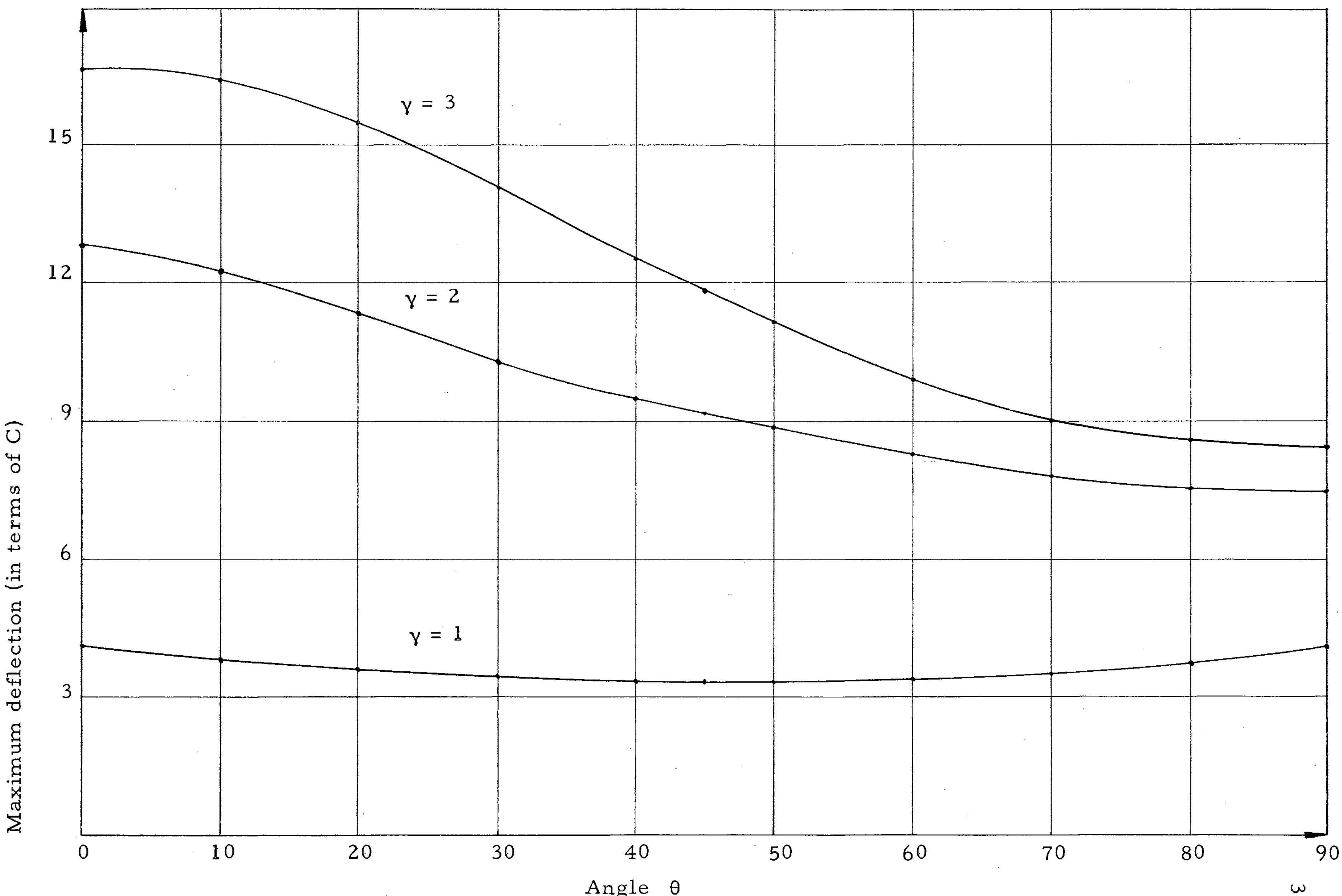

Figure 3-2. Graph showing variation of maximum deflection versus angle θ for distributed load

TABLE 3-2

DEFLECTION AT VARIOUS POINTS FOR CONCENTRATED LOAD

Point		Deflection in Terms of C		
x/a	y/b	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
		Angle 0°		
0.1	0.4	0.9872	2.1612	2.4893
0.25	0.25	2.2677	4.0721	4.6651
0.3	0.6	1.4144	3.463	4.1385
0.4	0.8	.7427	1.7961	1.8404
0.5	0.5	1.9190	2.9520	2.1967
0.6	0.9	.3461	0.6059	.3706
0.7	0.3	1.3460	1.1959	.4993
0.8	0.7	0.6082	0.6945	.2481
0.9	0.2	0.3494	0.2406	.0621
		Angle 10°		
0.1	0.4	.9424	2.0817	2.4131
0.25	0.25	2.1738	3.9313	4.5214
0.3	0.6	1.3446	3.3339	4.0075
0.4	0.8	.7037	1.7321	1.7913
0.5	0.5	1.8309	2.8681	2.1645
0.6	0.9	.3277	0.5895	.3689
0.7	0.3	1.2939	1.1769	.5076
0.8	0.7	0.5778	0.6835	.2553
0.9	0.2	0.3370	0.2388	0.0651

TABLE 3-2 (Continued)

Angle 20°				
x/a	y/b	γ = 1	γ = 2	γ = 3
0.1	0.4	0.8922	1.9452	2.2460
0.25	0.25	2.0493	3.685	4.2402
.3	0.6	1.2760	3.1163	3.7501
0.4	0.8	0.6679	1.6094	1.6705
0.5	0.5	1.7155	2.6595	2.0322
0.6	0.9	.3080	0.5430	0.3494
0.7	.3	1.2041	1.0962	0.4960
0.8	0.7	0.537	0.6320	0.2551
0.9	0.2	0.3133	0.2234	.0682

Angle 30°				
0.1	0.4	0.8548	1.8015	2.0458
0.25	0.25	1.9387	3.417	3.9087
0.3	0.6	1.2342	2.8921	3.4519
0.4	0.8	0.6485	1.4762	1.5165
0.5	0.5	1.6187	2.4025	1.8259
0.6	0.9	0.2949	0.4851	.3130
0.7	0.3	1.1110	0.9767	.4491
0.8	0.7	0.5032	0.5581	.2341
0.9	0.2	0.2870	0.1982	.0634

TABLE 3-2 (Continued)

Angle 40°				
x/a	y/b	γ = 1	γ = 2	γ = 3
0.1	0.4	0.8445	1.6887	1.8585
0.25	0.25	1.8769	3.1964	3.6045
0.3	0.6	1.2382	2.7227	2.1849
0.4	0.8	0.6552	1.3673	1.3631
0.5	0.5	1.5717	2.1564	1.5826
0.6	0.9	0.2935	0.4293	0.2655
0.7	0.3	1.0389	0.8414	0.3692
0.8	0.7	0.4867	0.4774	0.1924
0.9	0.2	0.2647	0.1671	.0499
Angle 45°				
0.1	0.4	0.8519	1.6498	1.7776
0.25	0.25	1.8703	3.1162	3.4787
0.3	0.6	1.2607	2.6677	3.0763
0.4	0.8	0.6701	1.3264	1.2931
0.5	0.5	1.5712	2.0462	1.4576
0.6	0.9	0.2978	0.4037	0.2398
0.7	0.3	1.0142	0.7728	0.3220
0.8	0.7	0.4862	0.4369	0.1668
0.9	0.2	0.2561	0.1506	0.0411

TABLE 3-2 (Continued)

Angle 50°				
x/a	y/b	γ = 1	γ = 2	γ = 3
0.1	0.4	0.8683	1.6236	1.7067
0.25	0.25	1.8817	2.6579	3.3753
0.3	0.6	1.2978	2.6339	2.9879
0.4	0.8	0.6929	1.2949	1.2293
0.5	0.5	1.5865	1.9460	1.3351
0.6	0.9	0.3056	0.3800	0.2140
0.7	0.3	0.9974	0.7052	0.2732
0.8	0.7	0.4907	0.3968	0.1402
0.9	0.2	0.2492	0.1339	.0319
Angle 60°				
0.1	0.4	0.9275	1.6074	1.5944
0.25	0.25	1.9559	3.0140	3.2390
0.3	0.6	1.4134	2.6259	2.8720
0.4	0.8	0.7615	1.2568	1.1214
0.5	0.5	1.6597	1.7735	1.1062
0.6	0.9	0.3305	0.3357	0.1645
0.7	0.3	0.9850	0.5761	0.1799
0.8	0.7	0.5120	0.3183	0.0902
0.9	0.2	0.2401	0.1014	0.0145

TABLE 3-2 (Continued)

Angle 70°				
x/a	y/b	γ = 1	γ = 2	γ = 3
0.1	0.4	1.0125	1.6283	1.5160
0.25	0.25	2.0823	3.0449	3.1877
0.3	0.6	1.5685	2.6789	2.8276
0.4	0.8	0.8509	1.2438	1.0401
0.5	0.5	1.7705	1.6355	0.9135
0.6	0.9	0.3628	0.2967	0.1228
0.7	0.3	0.9920	0.4625	0.1043
0.8	0.7	0.5420	0.2469	0.0521
0.9	0.2	0.2358	.0727	0.0125

Angle 80°				
0.1	0.4	1.0949	1.6616	1.4659
0.25	0.25	2.2111	3.107	3.1885
0.3	0.6	1.7142	2.7499	2.8265
0.4	0.8	0.9331	1.2433	0.9868
0.5	0.5	1.8751	1.5379	0.7788
0.6	0.9	0.3914	0.2669	0.0945
0.7	0.3	1.0039	0.3797	0.0561
0.8	0.7	0.5672	0.1942	0.0303
0.9	0.2	0.2336	.0523	-.0639

TABLE 3-2 (Continued)

Angle 90°				
x/a	y/b	γ = 1	γ = 2	γ = 3
0.1	0.4	1.1306	1.6776	1.4478
0.25	0.25	2.2677	3.1387	3.1968
0.3	0.6	1.7764	2.7833	2.8326
0.4	0.8	0.9677	1.2447	0.9677
0.5	0.5	1.9190	1.5012	0.7293
0.6	0.9	0.4030	0.2553	0.0844
0.7	0.3	1.0092	0.3487	0.0398
0.8	0.7	0.5768	0.1744	0.0236
0.9	0.2	0.2329	0.0445	-.0878

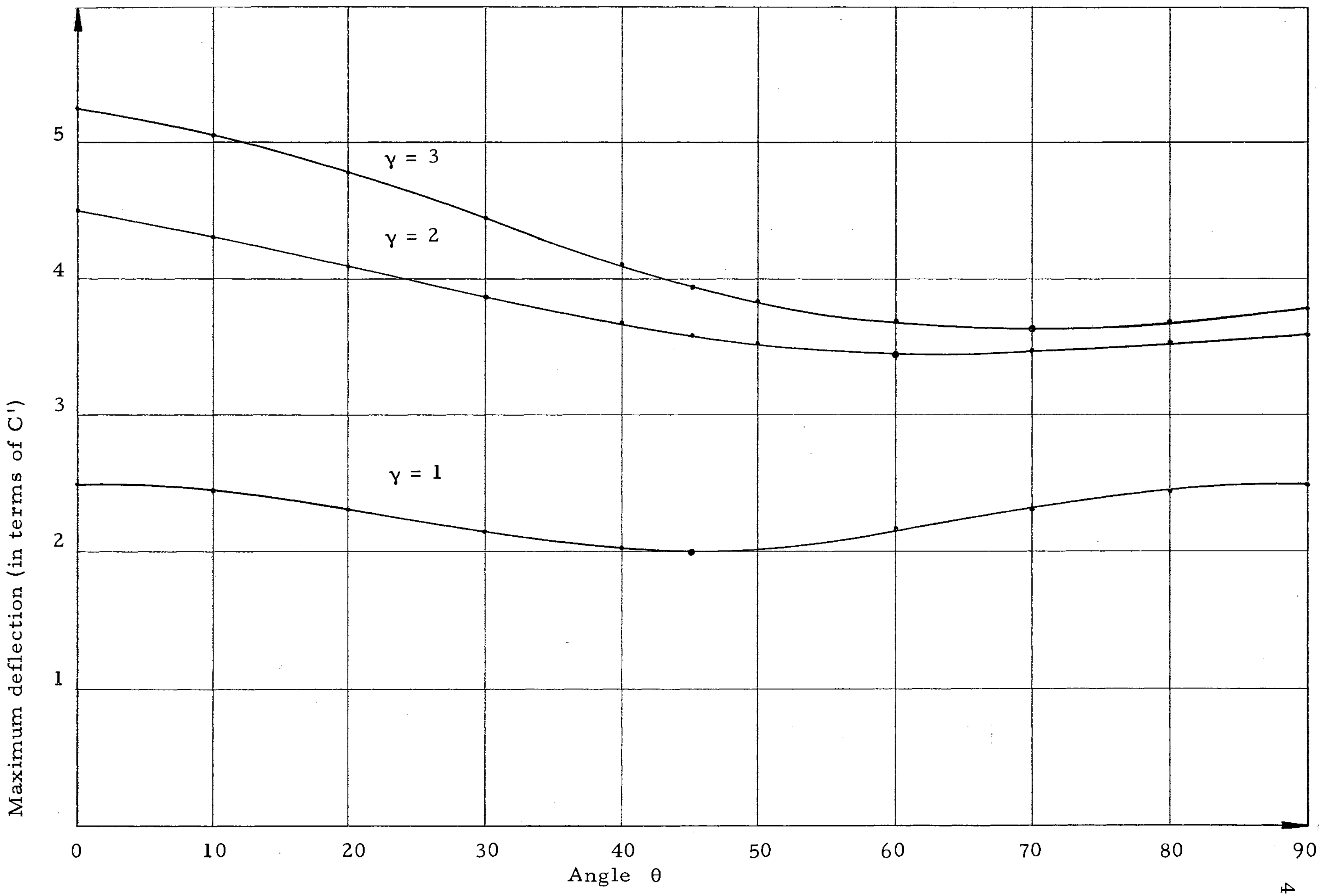

Figure 3-3. Graph showing variation of maximum deflection versus angle θ for concentrated load

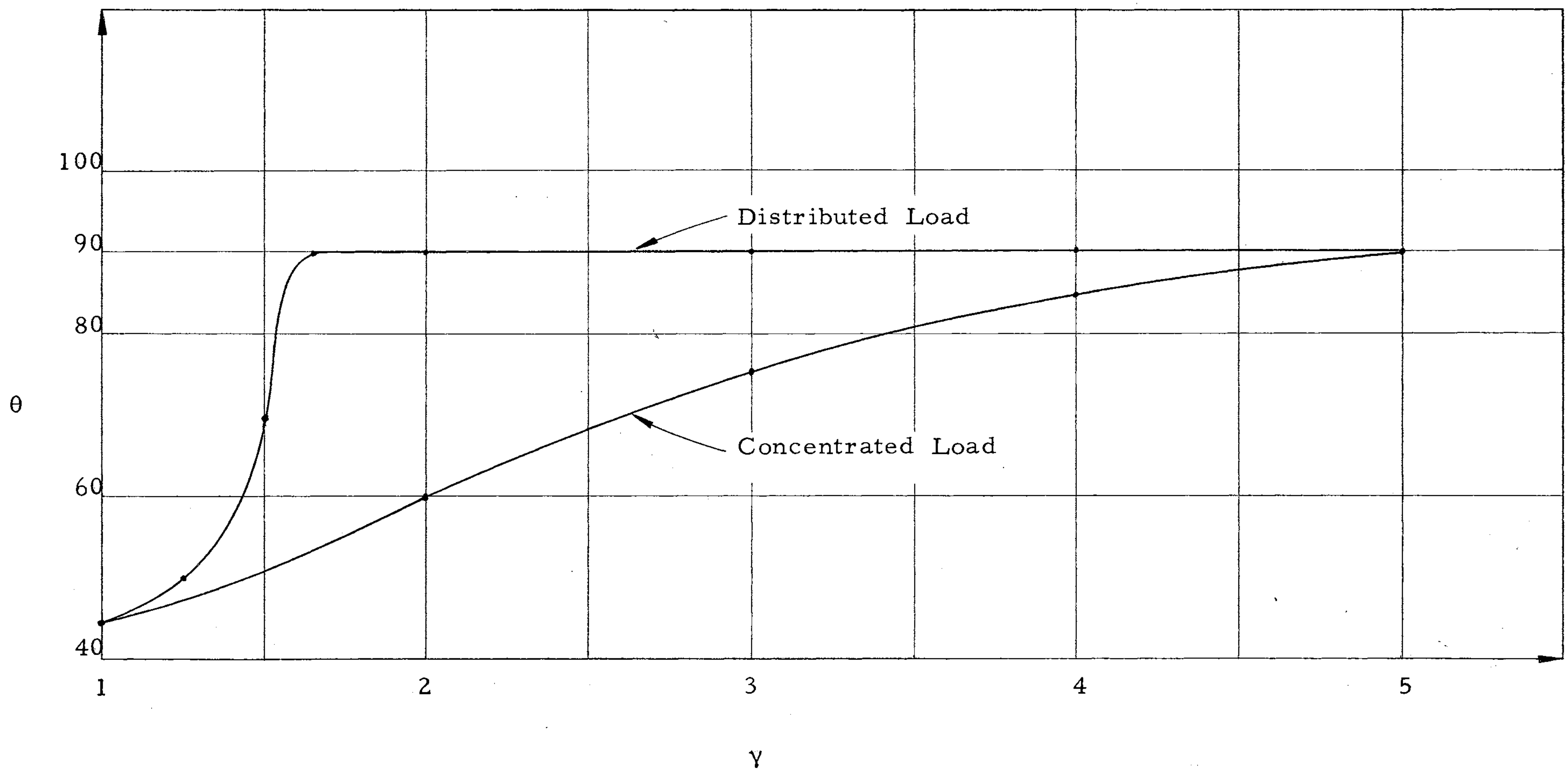

Figure 3-4. Graph showing variation of θ versus γ for minimum value of maximum deflection

CHAPTER IV

CONCLUSIONS AND SUMMARY

From the discussion in the previous chapters it can be concluded that the isotropic plate with stiffeners can be considered equivalent to an anisotropic plate. The elastic rigidity constants derived are, in general, in good agreement with the theoretical and experimental results derived by others. The value of $D_{y'} = D$ is an approximate one compared to values obtained by others.

From the numerical example the following conclusions may be drawn

(1) For $\gamma = 1$, a square plate, deflection is maximum for $\theta = 0^\circ$ and 90° and deflection is minimum for $\theta = 45^\circ$.

(2) The maximum deflection approaches its minimum value, as θ approaches 90° and γ increases from 1 to 5, very rapidly for distributed load and slowly for concentrated load. This criteria confirms that the beams should be along the shorter span to have the minimum value of the maximum deflection.

<u>SUMMARY</u>

An isotropic plate simply supported at the edges, with beams inclined at an angle θ to the x axis of the plate can be analyzed by the method presented in this thesis. This plate can be considered as an anisotropic plate. The rigidity constants can be found in terms of elastic properties and geometrical configuration of the isotropic plate and the stiffners. The coefficients in the infinite series for deflection are obtained by minimizing the total potential energy of the system. This method can be applied to any other kind of general loading. Once the deflection is known the complete analysis of the plate is possible. With the help of the computer it is always possible to check if the series has converged, or if a few more terms are required.

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APPENDIX

IBM FORTRAN (WITHOUT FORMAT) PROGRAM

1 DISTRIBUTED LOAD

```
C     CALCULATION OF COEFFICIENTS
1     READ, C, R, DX, DY, DI
      READ, IF, JF, ALP, BET
      DIMENSION P ( 15,15 )
      PUNCH, R, ALP, BET
      DO 10 I = 1,  IF, 2
      DO 10 J = 1, JF, 2
      FI = I
      FJ = J
      FI2 = I * I
      FJ2  = J * J
      FI4 = FI ** 4
      FJ4 = FJ ** 4
      ALP2 = ALP * ALP
      BET2 = BET * BET
      ALP4 = ALP ** 4
      BET4 = BET ** 4
      R2 = R * R
      T = 6.0 * ALP2 * BET2 * FI2 * FJ2
      PT = DX * (((ALP4 * FI4)/R2) + T + (BET4 * FJ4 * R2))
      PT = PT + DY * (((BET * FI4)/R2) + T + (ALP4 * FJ4 * R2))
      PT = PT + DI * ((FI4/R2) + (FJ4 * R2)) * (ALP2 * BET2)
```

```
      PT = PT + DI * (FI2 * FJ2) * (-4.0 * ALP2 * BET2 + ALP4 + BET4)
      P(I, J) = C/(PT* FI * FJ)
      PUNCH, P (I, J)
10    CONTINUE
C     CALCULATION OF DEFLECTION
      READ, IF, JF, CMIN, DMIN
      READ, CDELT, DDELT, CMAX, DMAX
      C = CMIN
11    D = DMIN
18    SUM = 0.
20    DO 40 I = 1, IF, 2
30    DO 40 J = 1, JF, 2
      FI = I
      FJ = J
      SUM = SUM + P(I, J) * SIN(FI * 3.1416 * C) * SIN(FJ * 3.1416 * D)
40    CONTINUE
      PUNCH, SUM, C, D,
      D = D + DDELT
      IF (D-DMAX) 18, 18, 60
60    C = C + CDELT
      IF (C-CMAX) 11, 11, 80
80    PAUSE
      GO TO 1
      END
```

2 CONCENTRATED LOAD

```
C     CALCULATION OF COEFFICIENTS
1     READ, C, R, DX, DY, DI
      READ, IF, JF, ALP, BET
      DIMENSION P (10,10)
      PUNCH, R, ALP, BET
      DO 10 I = 1, IF, 1
      DO 10 J = 1, JF, 1
      FI = I
      FJ = J
      FI2 = I * I
      FJ2 = J * J
      FI4 = FI ** 4
      FJ4 = FJ ** 4
      ALP2 = ALP * ALP
      BET2 = BET * BET
      ALP4 = ALP ** 4
      BET4 = BET ** 4
      R2 = R * R
      T = 6 . 0 * ALP2 * BET2 * FI2 * FJ2
      PT = DX * (((ALP4 * FI4)/R2) + † + (BET4 * FJ4 * R2))
      PT = PT + DY * (((BET * FI4)/R2) + † + (ALP4 * FJ4 * R2))
      PT = PT + DI * ((FI4/R2) + (FJ4 * R2)) * (ALP2 * BET2)
      PT = PT + DI * (FI2 * FJ2) * (-4. 0 * ALP2 * BET2 + ALP4 + BET4)
      P(I, J) = (C * SIN(FI * 3. 1416/4. ) * SIN(FJ * 3. 1416/4. ))/PT
```

```
      PUNCH, P(I,J)
10    CONTINUE
C     CALCULATION OF DEFLECTION
      READ, IF, JF, CMIN, DMIN
      READ, CDELT, DDELT, CMAX, DMAX
      C = CMIN
11    D = DMIN
18    SUM = 0.
20    DO 40 I = 1, IF, 1
30    DO 40 J = 1, JF, 1
      FI = I
      FJ = J
      SUM = SUM + P(I,J) * SIN (FI * 3.1416 * C) * SIN (FJ * 3.1416 * D)
40    CONTINUE
      PUNCH, SUM, C, D
      D = D + DDELT
      IF (D-DMAX) 18, 18, 60
60    C = C + CDELT
      IF(C-CMAX) 11, 11, 80
80    PAUSE
      GO TO 1
      END
```

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