AN EXPERIMENTAL INVESTIGATION OF THE NORMAL FREEZING, FRACTIONAL MELTING PROCESS AS APPLIED TO THE DESALINATION PROBLEM

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CHAPTER I

INTRODUCTION

The purpose of this study was to determine if potable water could be obtained by the freezing of saline solutions on a spherical surface. The frozen sample was separated into parts by collecting the melt on a number of watch glasses. This fractional melting technique yielded potable water from a 3.5% saline solution in a one step process. This one-step separation represented a significant improvement in the desalination of water by the freezing process. The separation process was varied by changing the sphere size, by altering the brine and sphere temperature, by use of flow and non-flow brine fields and by varying the sample melting temperatures. With the above variation of parameters the optimum process was determined and the energy requirements for the phase change were evaluated.

As stated by Strobel (1) and many others in both technical and non-technical publications, the problem of adequate water supplies is becoming one of critical importance. It has been estimated that the daily requirements of water in the United States will approach five to six hundred billion gallons daily by 1980. Reducing stream pollution, eliminating unnecessary wastage and better use of existing water supplies will serve to fulfill some of this demand. It is also apparent that the production of usable water from sea or brackish water will become necessary.

There exists a number of technically feasible solutions to the problem of converting saline water to pure water. Among these are:

1. thermal distillation,
2. solar distillation,
3. membrane processes,
4. chemical processes, and
5. freezing processes.

Separation by distillation is the oldest method of producing fresh water from salt water. Modern processes include multiple-effect systems operating as submerged-type evaporators or flash distillers, thin film evaporation systems and many others. Among the advantages of the distillation process are the ability to utilize waste heat, and the adaptability to many fuels, including nuclear energy. Disadvantages include corrosion problems and operations at high temperature levels, which increase the losses due to irreversibility in the energy transfer.

The major advantage of solar distillation is the elimination of fuel costs. The dependence on abundant solar energy restricts the number of available geographical locations for plant construction. Other disadvantages of this method include the need for large structures and the pumping of large quantities of fluids.

Membrane processes include those systems that permit the passage of ions and block water passage, and those that work in the opposite manner; that is, permit the passage of water but block ion passage. The first type is known as electrodialysis while the second is an osmotic process. Membrane processes have been applied principally to brackish water conversion.

Chemical processes, such as solvent extraction and gas hydrate processes have been tested in plants having capacities of 20,000 and 2,000 gallons per day respectively.

From the above, it is apparent that desalination may be accomplished by a variety of techniques, each having particular advantages and disadvantages. The most severe requirement, which applies to all systems, is that of economics. As indicated by Hendrickson (2), the Office of Saline Water has established the following monetary limits for water conversion processes.

Irrigation water having a salinity of 1,500 parts per million (ppm) must be produced at a cost of 12¢ per thousand gallons. Drinking water having 500 ppm salinity must be produced at a cost of 30 to 40¢ per thousand gallons. Also, plant sizes should be capable of producing from five hundred thousand to ten million gallons of water per day.

These large plant sizes have served to limit the number of economically feasible separation techniques, especially in the case of freezing processes. Research on freezing processes has been restricted to direct techniques involving refrigerant and brine in physical contact.

Freezing Processes

The advantages of the freezing process include low energy requirements, 144 Btu's per pound for the phase transformation, as compared with 1060 Btu's per pound for distillation, low temperature differences when irreversibilities occur, and reduced corrosion problems at lower temperatures. Disadvantages include:

1. difficulty of separation of pure water product from adhering brine solutions, especially when using direct techniques which produce small crystals of pure water,
2. need for complex equipment in processes operating with a phase change,
3. all of the product water must be frozen,
4. general expenses of refrigeration equipment, and
5. the necessity of using work to remove the latent heat of fusion rather than the direct application of heat, as in the distillation process.

Of these disadvantages, the first has proved to be the most troublesome. The method of fractional melting developed in this study effectively removes this difficulty and resulted in a sample which consisted of a quantity of pure water and a quantity of melt having a concentration less than the original solution.

CHAPTER II

PREVIOUS INVESTIGATIONS OF DESALINATION BY FREEZING

Indirect freezing is defined as a freezing process in which the material to be frozen and the coolant are separated by a metallic surface.

The first investigation using indirect freezing techniques for desalination was by Hendrickson (2). The study involved the use of commercial ice making machines supplied with sea water. It was impossible to obtain pure water in a one step process using this equipment. It was possible to reduce the salt concentration from 30,000 ppm to 11,000 ppm in the first stage, then to 3,200 ppm in the second stage and to less than 500 ppm in the third stage. Results of the current investigation verified these figures when the entire frozen specimen is melted and collected as a single quantity.

Bosworth, et al. (3) evaluated the costs of a two stage refrigeration system operating on the above principle. The operating costs were approximately $2.24 per thousand gallons for a plant producing 100,000 gallons per day. These authors stated that it may be possible to construct an economically competitive plant in the 500 to 2,000 gallons per day range.

Another investigation using indirect freezing was conducted by Rose and Hoover (4). The freezing was performed in beakers with the solidification directed from the outer surface inward. This experiment failed to produce pure water due to the presence of entrained brine or mother liquid among the ice crystals. The quantity of entrained liquid varied from 20 to 40% of the total sample weight.

The above studies represent the only examples of the use of indirect freezing in desalination processes. No attempt was made in these studies to determine the effect of varying the various parameters, and none of the above utilized fractional melting.

Normal Freezing and Fractional Crystallization

Normal freezing is defined as the freezing of a liquid on a given geometric surface while fractional crystallization indicates that only part of the liquid sample is frozen. Normal freezing is therefore an elementary step in the separation method known as fractional crystallization.

As indicated by Pfann (5) fractional crystallization processes have been known for hundreds of years. It was the process used by the Curies for the separation of certain rare earths and the production of radium. Its use in the past has been principally in the field of purifying metal samples rather than the desalination of water. When applied to metals the purification process becomes quite involved requiring careful separation of components and slow freezing processes. Slow freezing rates are required to prevent the formation of dendrites which will extend into the liquid and serve as a mechanism for entrapping impurities.

Tiller, et al. (6), discussed the redistribution of solute atoms during the solidification of metals by use of ordinary, normal freezing. An important parameter in normal freezing is the distribution coefficient, K, which is defined as the ratio of impurity in the solid being formed to the concentration in the original solution. It is generally assumed that this property is constant, and for a purification process the distribution coefficient must be less than unity. Actually K is not quite constant but a mean value may be used.

In Tiller's paper the following assumptions were made:

1. mass diffusion in the solid may be neglected,
2. K is a constant,
3. convective mixing in the liquid is negligible, and
4. the infinite plate configuration was used.

The physical mechanism of the process was that for an initial liquid concentration, C_b, the first solid to freeze would have a concentration KC_b. As the crystal grew the liquid concentration increased because the solute was rejected from the interface; this in turn also caused the solid concentration to increase. This process continued until steady-state conditions were reached. Fig. 1 shows the steady-state distribution, C_L, of the solute in the liquid

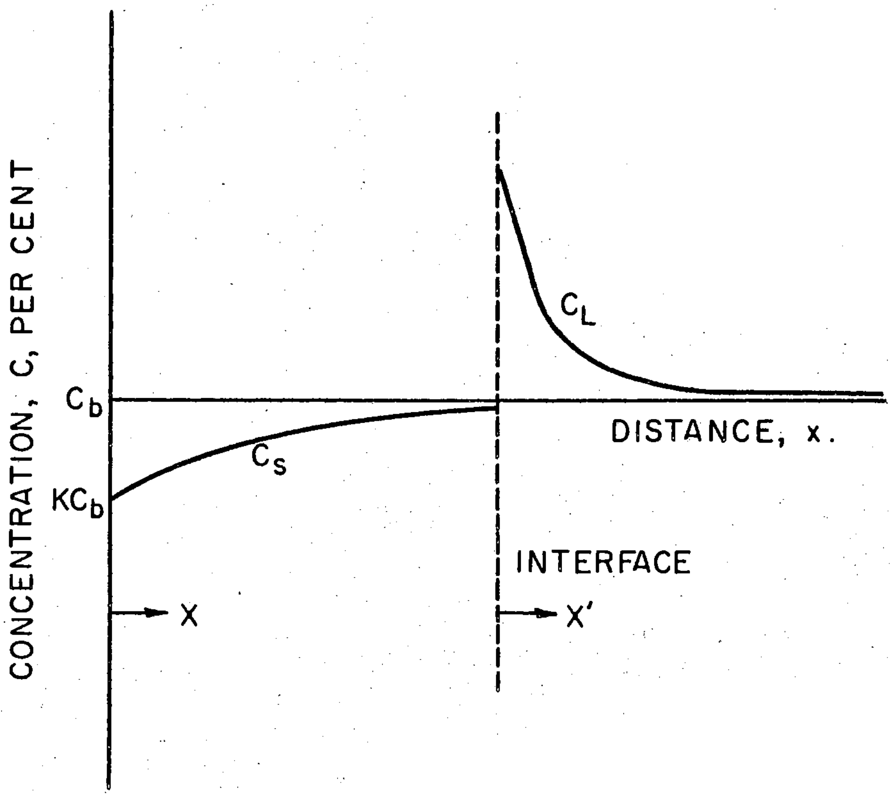


Figure 1. Distribution of Solute in the Liquid, and in the Solid C_s

in front of the freezing interface and the concentration in the solid, C_s. Two equations were developed; one for the liquid distribution:

$$C_L = C_b \left[1 + \frac{1 - K}{K} \exp(-K\frac{R'}{D} x')\right] \quad (1)$$

and one for the solid distribution:

$$C_s = C_b \left[(1 - K) \left[1 - \exp(-K\frac{R'}{D} x)\right] + K\right] \quad (2)$$

where x' is the distance from the interface and x is the distance measured from the start of the specimen, Fig. 1, D is the diffusion coefficient and R' is the freezing rate. This equation was difficult to apply since the values of the diffusion coefficient were not well established, especially for saline solutions near the freezing point.

From Equation (2) the curve of solute concentration in the solid must satisfy the following conditions:

1. The initial value of solute concentration in the solid must have a value of KC_b.

2. The concentration tends asymptotically to C_b as the freezing continues.

3. The concentration must increase continuously from KC_b to C_b.

4. The area between the concentration C_b and that of the solid C_s in Fig. 1 must equal the area between the concentrations of the liquid C_L and C_b. This is necessary for a solute balance. This statement is valid if the density of the solid and liquid phases are equal.

5. Increasing values of the distribution coefficient K will result in increased contamination of the solid.

6. As the freezing rate R' increases the concentration of solute in the solid increases. As indicated by Tiller,(6) the treatments of solute segregation are incomplete and the results of his analysis have not been completely verified.

Zone Purification Process

A modification of the normal freezing process has been applied to the purification of metals. This process consists of freezing a cylindrical sample by progressive solidification from one end of the sample to the other. The sample may then be melted in the same manner and the purification measured. Experiments were performed by the Battelle Memorial Institute (7), Himes, et al. (8), and Loss, et al. (9), which applied this process to saline solutions. These experiments verified that freezing in the direction of increasing surface area would yield purer samples. Their cylindrical section was frozen both from the inside and the outside. Freezing from the inside of the cylinder, by means of a cooled copper wire, yielded higher purity solids than freezing from the outside. The zone purification technique was found to be capable of producing pure water. When fractional melting was employed, water of high purity was obtained at the beginning of the frozen section.

In the zone process the possibility of increasing the area is restricted by the cylinder walls and small sample size. The ease of freezing and separation obtained in the normal freezing process indicated that it is more adaptable to the desalination process than the zone purification process.

CHAPTER III

EXPERIMENTAL APPARATUS

The experimental apparatus was designed for the purpose of freezing ice samples, evaluating the various parameters, fractionally separating the melt and measuring the reduction in salt concentration of the melted samples.

The experimental apparatus may be divided into the following components:

1. coolant system,
2. brine system,
3. melt sample collection and evaluation system, and
4. instrumentation.

The arrangement of these components are shown in Figures 2 and 3.

Coolant System

The purpose of the coolant system was to generate a low temperature liquid, to deliver this liquid to the inside of the sphere, and to return the coolant to the sump. Provision was also made for determination of the coolant flow rate. The generation of the low temperature coolant was accomplished by two separate techniques. The first of these employed a sodium chloride-ice mixture in the

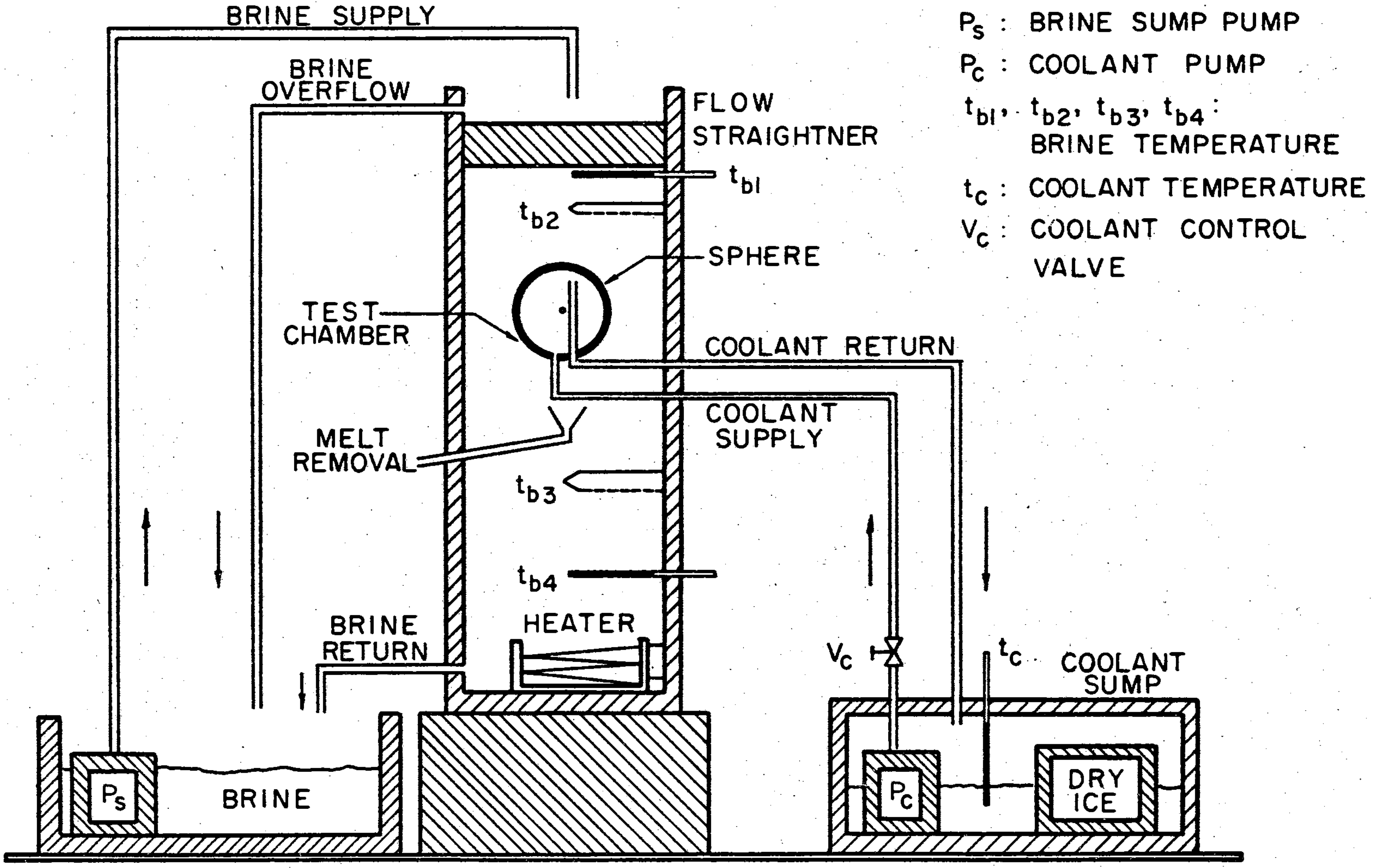


Figure 2. Experimental Apparatus

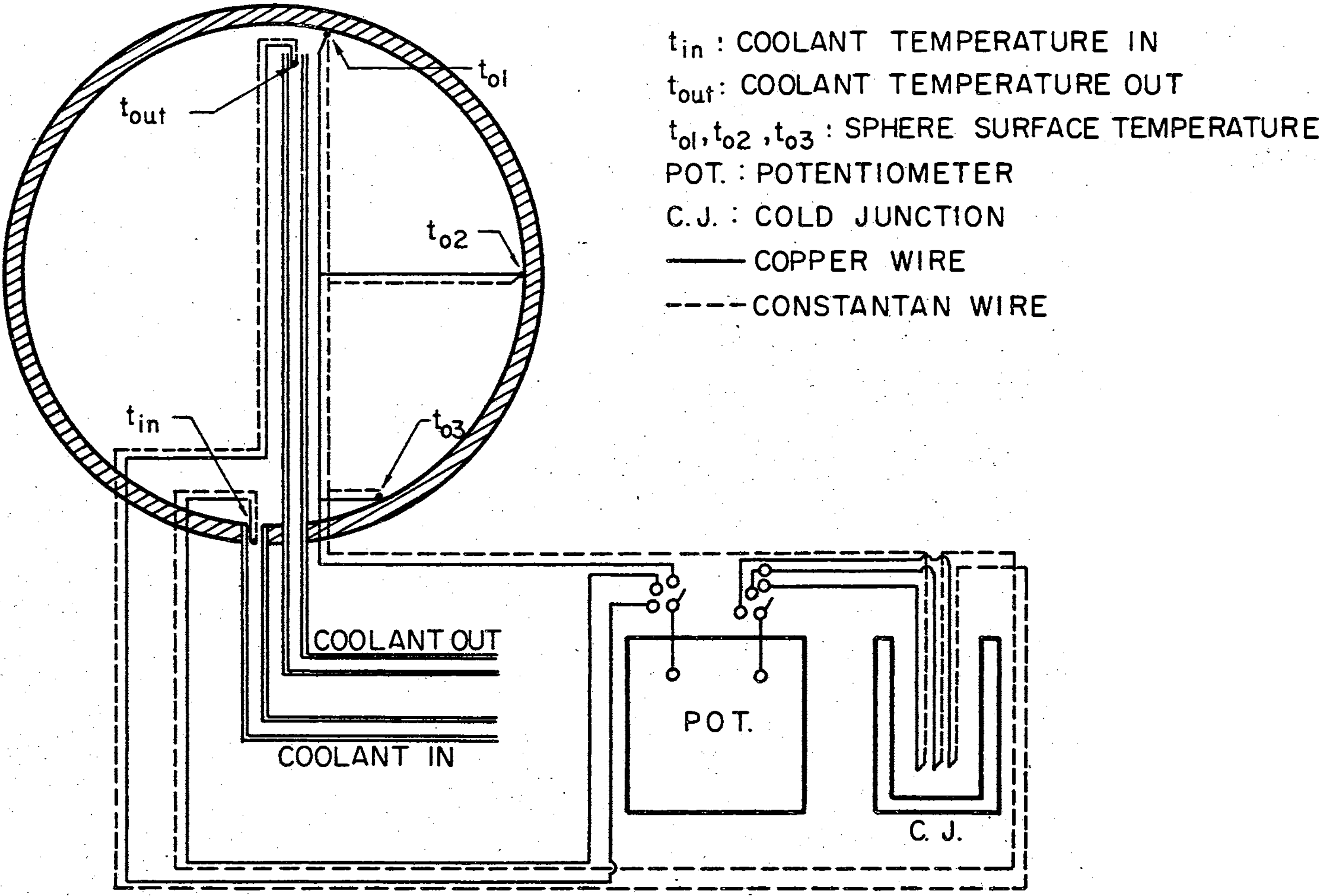


Figure 3. Temperature Instrumentation for Spherical Section

sump. Temperature control was obtained by varying the quantity of salt added to the sump. This system proved to be very stable and was capable of providing coolants with temperatures ranging from 9.9 to 22.0 $^{\circ}$F. The second system, was designed to obtain lower sump temperatures, and used methyl alcohol as a coolant. Dry ice, in a separate metal container, was used to cool the methanol. If the methanol-dry ice had been in contact, it would have been possible to reach considerably lower temperatures. Direct contact was tried in the course of the experiment but the centrifugal pump would not operate under these conditions. The methanol system was capable of supplying coolant having a temperature range from - 4.2 to 22.1 $^{\circ}$F. Temperature control was obtained by varying the quantity, or depth of methanol, supplied to the sump. Since the metal container was always full of dry ice the above procedure effectively varied the heat transfer surface and thereby the sump temperature.

A small centrifugal pump was employed to deliver the coolant to the sphere. Regulation of flow rate was obtained by means of a needle valve in the discharge line. The purpose of regulating the flow was to maintain a constant sphere surface temperature. It was necessary to supply relatively large quantities of coolant flow at the start of a run but after ten to fifteen minutes, conditions became relatively stable. Although it was later necessary to reduce the coolant flow during the run the adjustments were quite small.

The coolant flow rate was determined by measuring the time required to collect a 100 cc sample. This flow rate reading was made at ten minute intervals. The sump temperature was measured with an alcohol thermometer.

Brine System

The brine system consisted of the brine sump-pump, a straightening section, mercury thermometers, thermocouples, and a set of orifices having different diameters. The mercury thermometers were used only as a quick check of temperatures; actual temperature measurements were obtained by means of copper-constantan thermocouples located above and below the sphere. The discharge orifices were calibrated by measuring the time required to collect an 800 cc sample. Calibration runs were made at the equilibrium temperature of solutions having sodium chloride concentrations ranging from 0 to 4 per cent. No appreciable difference in the volume flow rate was observed when the sodium chloride concentration was varied.

Melt Sample Collection and Evaluation System

The melt sample collection system consisted of a funnel system, watch glasses, laboratory beam balance, weights and drying oven. The Mechanical Engineering Laboratory weights were calibrated against the precision balance located in the clean room of the laboratory. All of the weights weighed within ± 0.1 mg or ± 0.0001 grams of the value marked on the weight. The oven was used to dry the clean watch glasses and to evaporate the liquid samples. The watch glasses were weighed before the first run and after every tenth run. No change in the watch glass weight was detected.

Instrumentation System

The temperature measuring system within the sphere is shown in Fig. 3. The surface temperature of the sphere was obtained by three matched wire copper-constantan thermocouples connected in parallel. The in and out coolant temperatures were determined by means of small thermocouples inserted into the supply lines. These thermocouples and the brine thermocouples were calibrated against the mercury melting point and the steam point. For the steam point, the pressure corrected equilibrium temperature was interpolated as 210.78 °F, or 4.246 mv with the cold junction at 32 °F. The average value of five calibration runs was 4.241 with an error of .005 mv. This represents an error of - 0.2 °F. For the mercury point the correct reading would be - 1.424 mv. The average value obtained by melting the sample within a chilled vacuum bottle and plotting a temperature-time curve was - 1.422 mv with an error of + 0.1 °F. The same Leeds and Northup potentiometer, catalog No. 8686 with a least count of 0.005 mv was used for the calibration and the experimental runs. After calibration, Leeds and Northup thermocouple tables were used to evaluate the temperatures.

To determine the radius of the ice sample, a circular target was mounted on the chamber wall, Figures 4, 5 and 6. The target consisted of concentric circles with a radial increment of 0.05 inches. The target was also divided by radial lines having angular increments of 15 degrees. To prevent parallax a sighting tube was constructed which moved on the opposite chamber wall parallel to the target. This system had a least count of 0.05 inches and radial measurements were estimated to 0.025 inches.

The entire apparatus was placed in the Mechanical Engineering Laboratory cold chamber. The cold chamber maintained the surrounding air temperature within 2 ^{o}F and was operated in a temperature range from 20 ^{o}F to 35 ^{o}F.

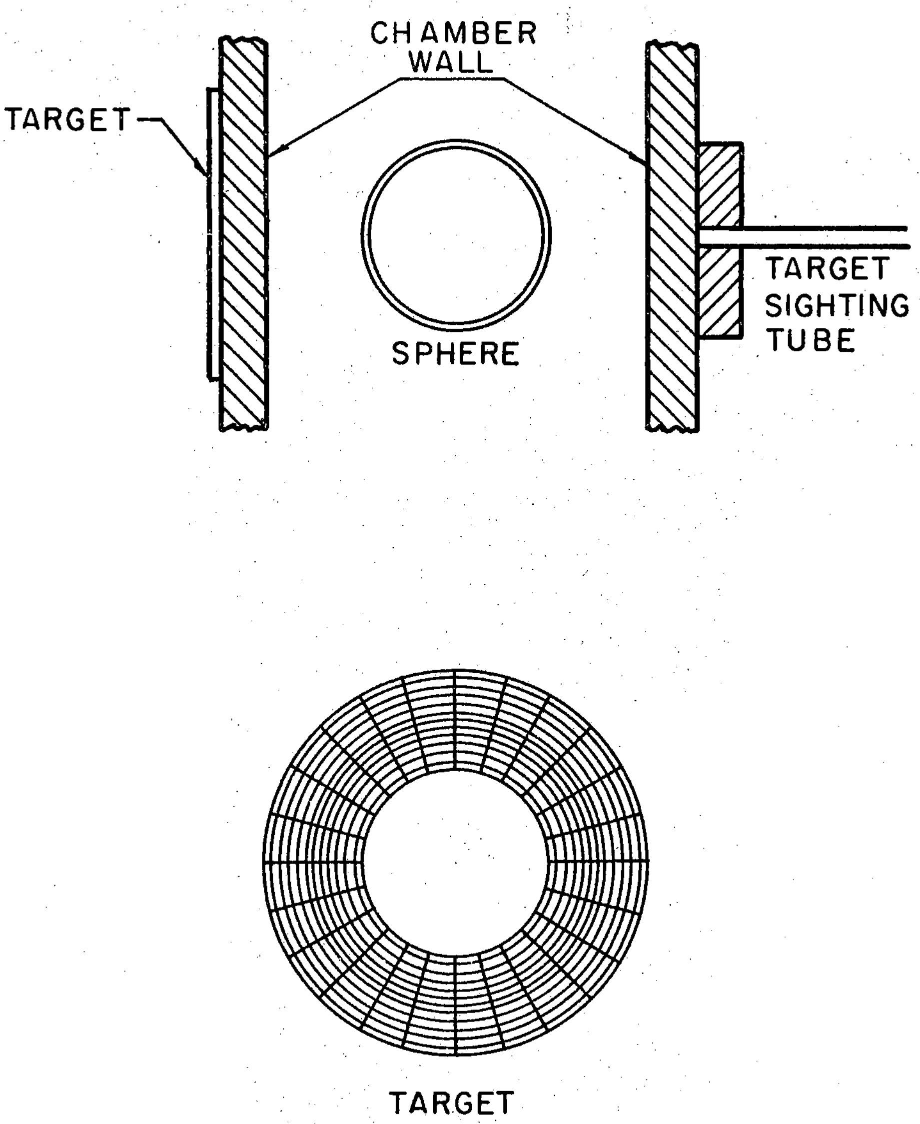
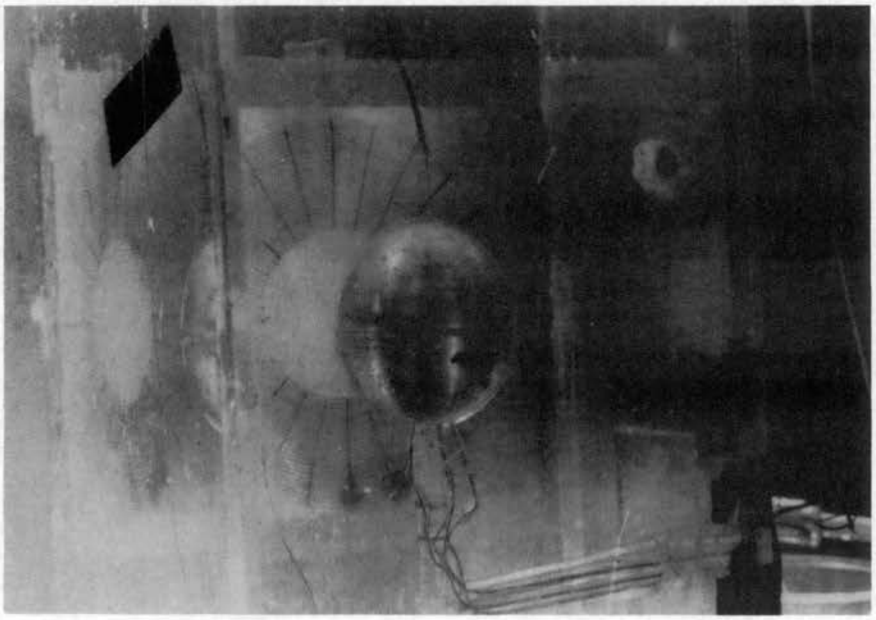
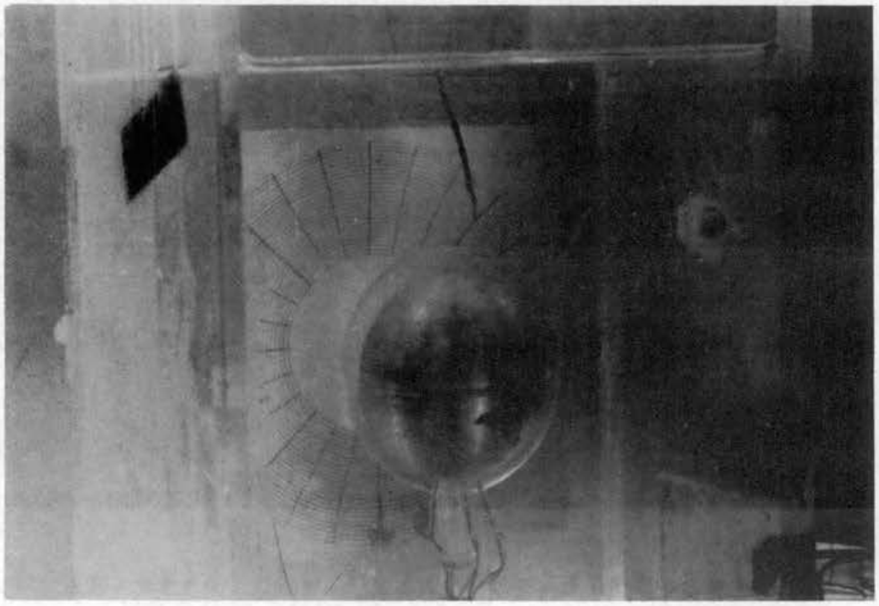


Figure 4. Radial Ice Growth Measuring System



(a)



(b)

Figure 5. Photographs of Sample Forming in Pure Water
a. Start of Freezing
b. Intermediate Sample

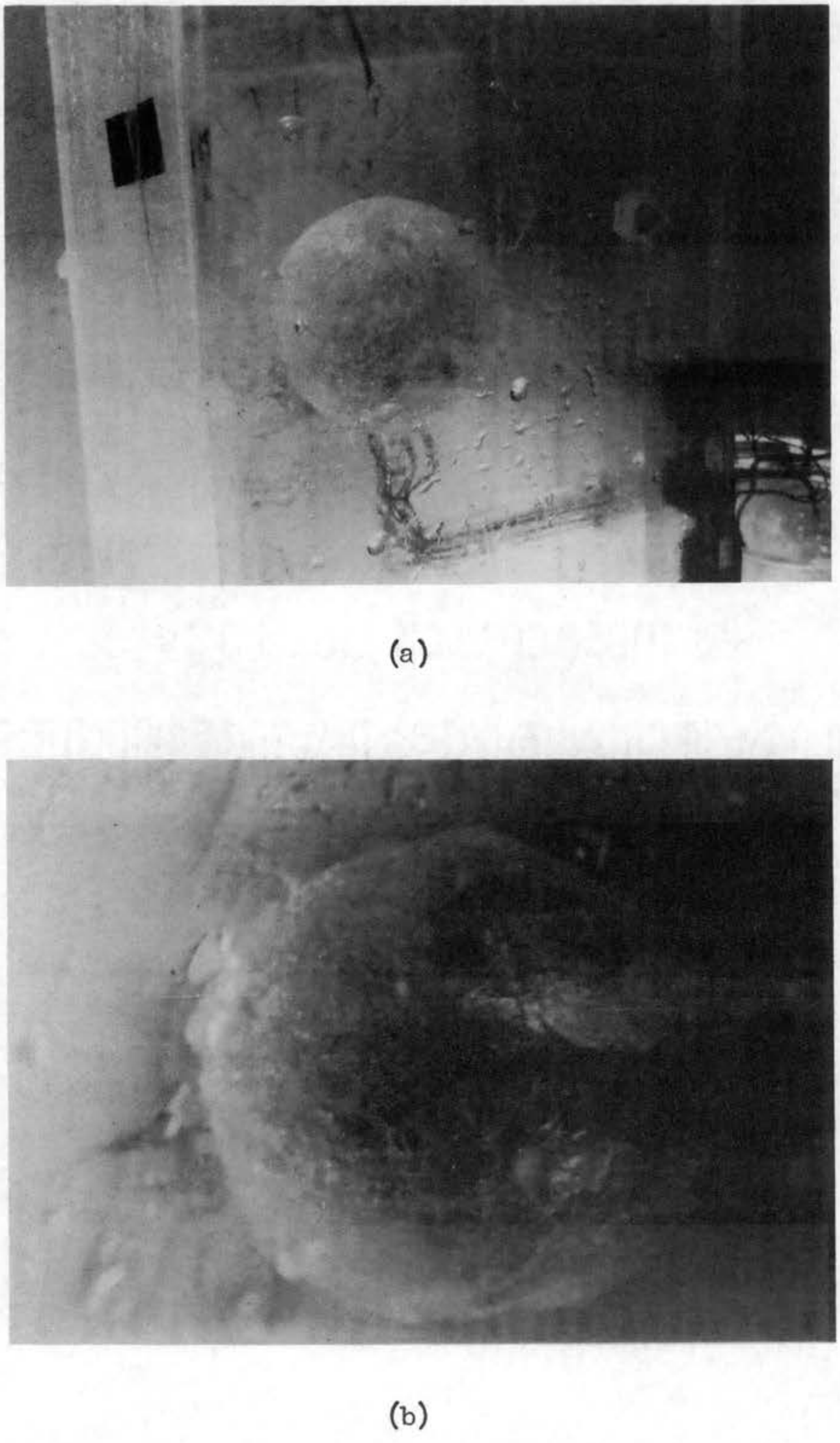


Figure 6. Photograph of Sample Frozen from 3.5% Saline Solution
- a. Sample Prepared for Melting
- b. Sample During Melting Period

CHAPTER IV

EXPERIMENTAL PROGRAM AND PROCEDURE

Experimental Program

The purpose of the experimental program was to determine the effect of various parameters on the desalination process. The parameters varied during the course of the study were:

1. brine concentration,
2. inside sphere temperature,
3. brine temperature,
4. brine velocity, and
5. sphere radius.

The brine concentration was varied from 0 to 4 per cent. Originally it was planned to run tests at specific sodium chloride concentrations of 0.0%, 0.5%, 2% and 3.5%. It was found, however, that when a sample was prepared with the correct concentration that partial freezing in the cold chamber and evaporative losses resulted in a variance of the concentration. Therefore, the brine concentration varied slightly.

The inside sphere temperature was determined by the coolant sump temperature. When the brine solution contacted the sphere, the sphere surface temperature rose rapidly. However when freezing started on the outer surface of the sphere, the sphere surface temperature could be controlled by adjusting the coolant flow rate. Therefore an exact sphere temperature was not established prior to the run, but the temperature shortly after freezing started was maintained during the test.

The brine temperature was originally established by mixing subcooled brine with warmer fluid having the same sodium chloride concentration. The cold chamber was set 5 degrees below the brine temperature and due to the heat capacity of the test apparatus no difficulty was encountered in maintaining the brine temperature at a constant value.

Constant brine velocity was easily maintained except for those runs using the smallest orifice. In subcooled brine runs, ice dendrites grew from the walls at the base of the container. These formations would separate from the walls and plug the small orifice. It was necessary to manually dislodge these particles and the net effect was to render these runs invalid for computational purposes. The two larger orifices did not plug but it is possible that ice formed within the orifice, thereby reducing their overall size.

Three sphere having radii of 0.50, 0.75 and 1.00 inches were used in the test.

In addition the following special runs were performed.

In Run 18 the brine temperature was greater than the equilibrium temperature. The equilibrium temperature was the temperature at which crystallization of the surrounding liquid began. Its specific value is a function of the brine concentration of the surrounding liquid. This run was terminated before the sample had completed its growth in order to determine the effect on concentration distribution. Runs 37, 40, 49 and 50 were melted rapidly to determine the effect of melting rate on the distribution of salt in the melted sample. The experimental program is tabulated in Table I.

Experimental Procedure

The experimental procedure was established by performing four preliminary runs in which data were collected while varying various operating conditions. These runs were not included as part of the experimental program. The following series of steps were found to yield the most consistent data.

1. The potentiometer, cold junction, pumps and other instrumentation were installed.

2. The brine temperature was established in the sump. The brine was left in the cold chamber overnight and was allowed to form a thin layer of ice. If a run with brine temperature equal to or greater than equilibrium was desired, brine, with the same concentration from outside the cold chamber, was added.

TABLE I

EXPERIMENTAL PROGRAM GROUPED BY RUN NUMBERS

	Run Numbers
Pure Water	
Temperature Conditions	
$t_e = t_b$	2, 35, 36, 41*, 45**
$t_e < t_b$	1
$t_e > t_b$	
Concentration, 0.58 - 0.75%	
Temperature Conditions	
$t_e = t_b$	5
$t_e < t_b$	6, 37, 38
$t_e > t_b$	46**
Concentration, 1.94 - 2.50%	
Temperature Conditions	
$t_e = t_b$	42*
$t_e < t_b$	10, 43*
$t_e > t_b$	40, 47**
Concentration, 3.16 - 4.12%	
Temperature conditions	
$t_e = t_b$	24, 28
$t_e < t_b$	16, 17, 25, 30, 44*, 48**, 49**
$t_e > t_b$	15, 19, 24, 26, 27, 29, 31, 34, 50*

* Initial radius of sphere, 0.75 inches

** Initial radius of sphere, 0.50 inches

All non-asterisk numbers, initial radius of sphere, 1.00 inches.

TABLE I (Continued)

Velocity Runs

Velocity range, ft/min	Concentrations, %			
	0.00	0.59-0.75	1.94-220	3.16-4.12
0.314 - 0.325		7	13	20
0.748 - 0.779	3	8	12, 24	21, 33
1.001 - 1.151	4	9	11	22, 32

Special Runs

Run 23. Velocity - 1.2715 ft/min

Run 39. Brine concentration - 2.74%

Rapid melt runs - 37, 40, 49 and 50

t_e: Brine equilibrium temperature, °F

t_b: Brine temperature, °F

3. Two samples of brine were taken to determine the exact concentration.

4. The temperature in the coolant sump was established.

5. The coolant flow through the sphere was started.

6. The inside temperature of the sphere was set to an approximate value.

7. The brine was introduced to the chamber from the bottom.

8. For non-flow runs, the pump was shut down when brine reached overflow line. For flow runs, the brine flow was switched from the bottom to the top of the test chamber.

9. Observations were made of the sphere surface and when the first indications of ice formation appeared, time readings were started.

10. Readings of temperatures and coolant flow rates were recorded at ten minute intervals. The flow rate was adjusted to maintain constant inside sphere temperature.

11. Two intermediate samples of brine were taken.

12. The final two brine samples were collected.

13. When the run was completed the brine was returned to the sump and coolant was drained from the sphere.

14. Radial measurements of ice sample were taken.

15. The temperature of cold room was decreased and sample was allowed to drain for twenty minutes. Actually, drainage was completed in a few minutes but preparations

for the succeeding steps required extra time.

16. Warm water was introduced to the chamber and the excess ice from the coolant supply lines was melted off.

17. The warm water was withdrawn from the chamber and the melted sample collecting system was purged with fresh water.

18. The cold chamber cooling system was turned off.

19. For rapid melt runs the heater was turned on.

20. Melt samples were collected and weighed immediately.

21. As samples melted, the radius, elapsed time from start of melting and chamber temperatures were measured.

22. After weighing, samples were placed in the oven for three hours at 125 ^{o}F.

23. The oven was turned off and samples left overnight.

24. The oven was heated to 200 ^{o}F and the samples heated for one hour. Typical samples are shown in Figures 7 and 8.

25. The samples were weighed.

26. The watch glasses were washed, dried and returned to oven for one hour heating period.

27. The brine concentration was altered as required for the next run and the cold box temperature was set at five degrees below the equilibrium temperature.

Elapsed time of the average run, including melt sample evaluation, was eight hours.

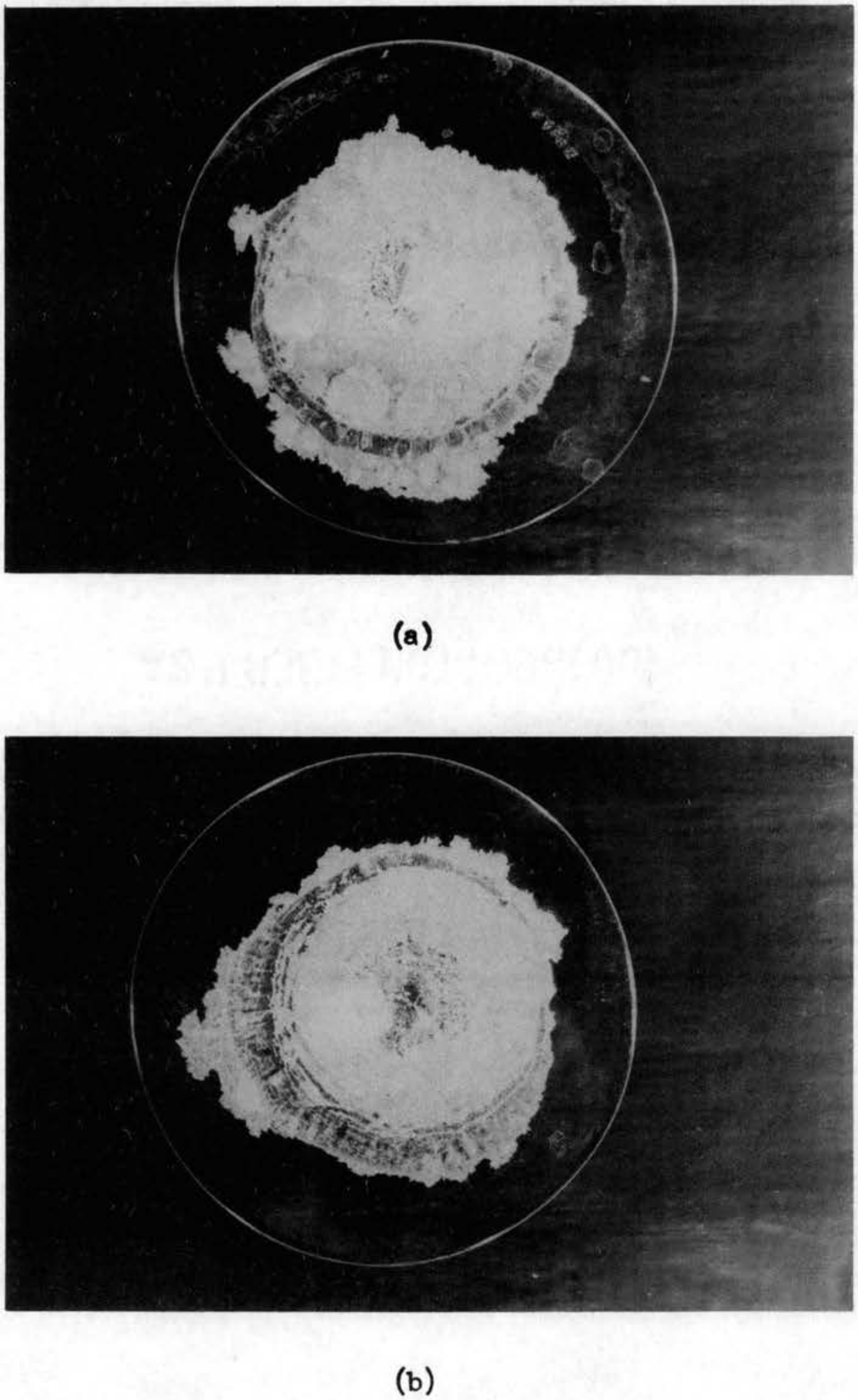
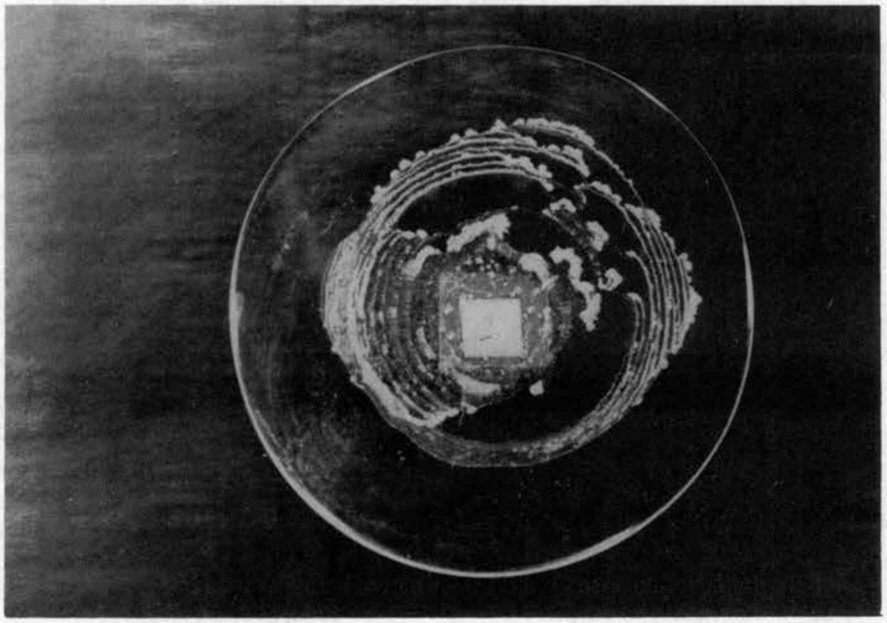
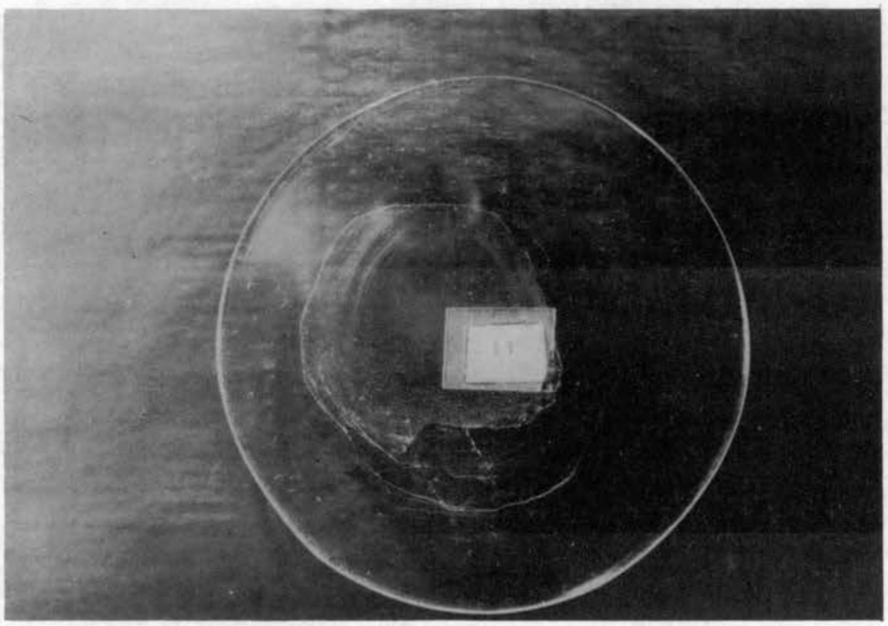


Figure 7. Solid Salt Sample Prepared for Weighing
a. Original Brine Solution. Brine Sample Weight 11 Grams. Salt Concentration 3.5%
b. Melt Sample. Brine Sample Weight 11.5 Grams. Salt Concentration 2.2%



(a)



(b)

Figure 8. Solid Salt Sample Prepared for Weighing
a. Melt Sample. Brine Sample Weight 12.1 Grams. Salt Concentration 0.8%
b. Final Melt Sample. Brine Sample Weight 12.2 Grams. Salt Concentration 0.03%

CHAPTER V

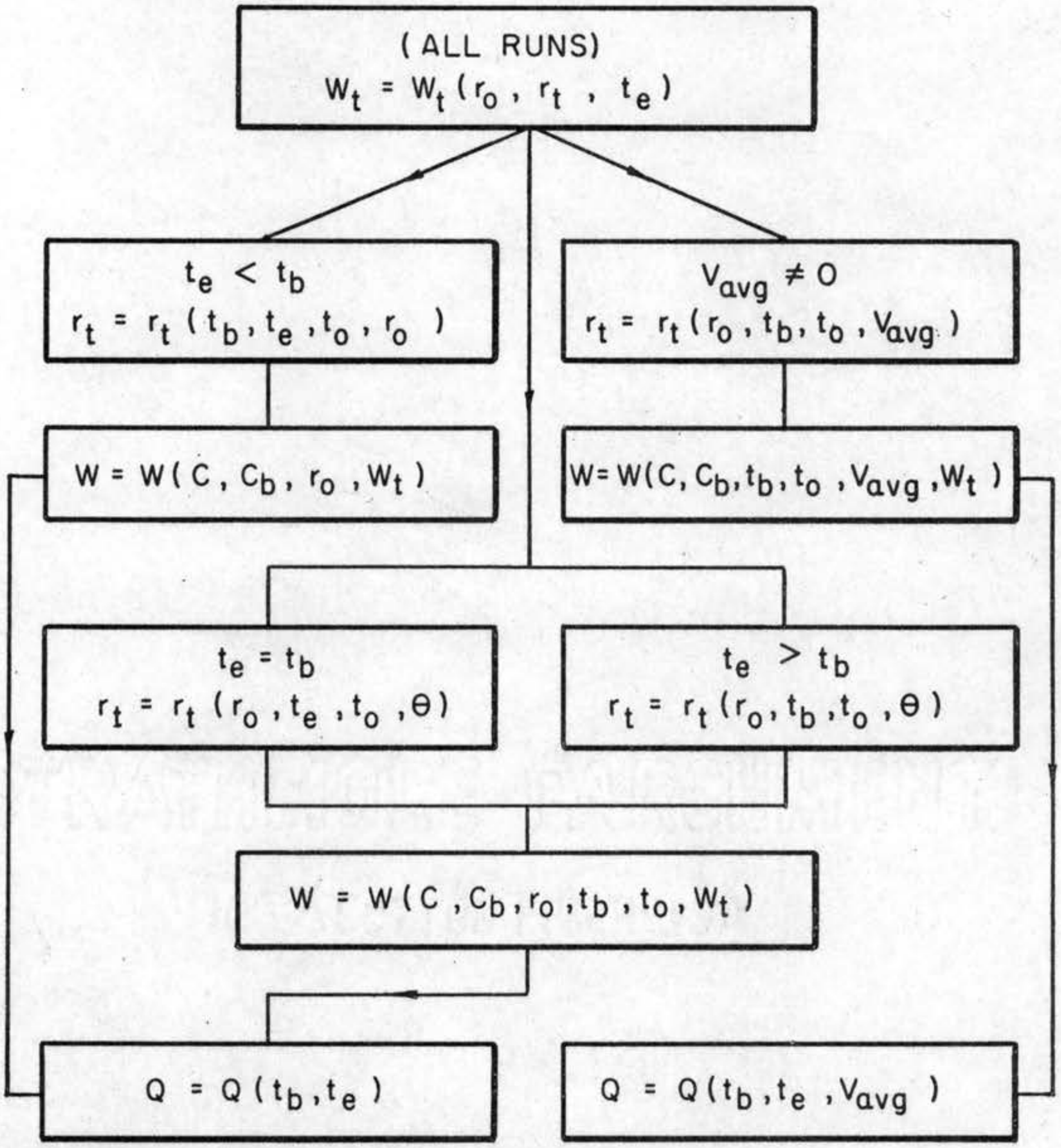
REDUCTION OF DATA

The purpose of this chapter is to outline a procedure for reducing the experimental data to a limited number of mathematical equations.

The procedure used to attain these equations is best shown by Figure 9. The problem was divided into four steps which in combination permitted the calculation of the total quantity of ice formed, the size of the ice sphere, the amount of a sample having a particular concentration and the amount of thermal energy required to freeze the sphere. These equations in turn are dependent only on such physical data as would normally be available as basic information, that is, the initial radius of the sphere, concentration of the brine, surface temperature of the sphere, temperature of the brine, velocity of the brine and time required for freezing.

As shown in Fig. 9 the steps required to present a complete solution were:

1.) An equation to establish the total weight of the final sample in terms of the initial radius, and equilibrium temperature or concentration. The equation obtained was found to be valid



SYMBOLS ARE DEFINED IN APPENDIX J.

Figure 9. Flow Sheet for Reduction of Data Equations

for all runs regardless of time, velocity or temperature relationships. The difficulty with this equation is that the inclusion of the final radius represents the use of "after the fact" information.

2.) To remove the difficulty inherent in step 1 a series of equations was derived to obtain the value of the final radius from the input data. Four separate cases were considered.

a.) The situation for which the temperature of the brine and the brine equilibrium temperature were equal. These equations were obtained from an analytical solution and exhibited an excellent degree of correlation between experimental and analytical techniques.

b.) For the case where the brine temperature was greater than the equilibrium temperature, the quantity of ice formed was finite and growth terminated when the losses by convection and conduction from the ice surface were equal to the heat conducted through the frozen section. Due to the irregular nature of the surface and general lack of information on free convection from spheres this solution is limited to the use of known parameters with an empirical determination of the exponents and numerical constants required to present a final solution.

c.) In the third case the brine temperature was less than the equilibrium temperature. Since this is a sub-cooling problem, the growth will proceed indefinitely, as it will in case a. Difficulties in theoretical analysis are severe for this situation. The solution of this problem, when the process was purely diffusional in nature, will be presented and the controlling parameters will be applied to the present problem. This technique yielded a grouping of parameters which aided in the development of empirical equations to predict the radius at a given time for this subcooled case.

d.) The fourth case required the inclusion of the brine velocity. Experimental difficulties when using the smallest orifice resulted in data which were of no value. Even for the larger orifices the results were well below the quality of the preceding sections.

The prediction of the final radius proved to be the most critical part of the experiment. Small variations in computed radii from experimental radii resulted in large errors in the weight-concentration equations.

3.) The solutions outlined in steps one and two permitted the calculation of the total weight of the final sample. Once these data were available, a formula was derived to determine the weight of the melt that had a sodium chloride concentration less than any specific value. Only three different solutions were required since the cases for brine temperature equal to, and less than, the equilibrium temperature resulted in the same equation.

The derivation of these equations for determining the weight-sodium chloride distribution was strictly empirical in nature. However in their final form, the different cases yielded similar solutions. These equations correlated the data quite well and should prove to be quite useful for future work in this field of study.

4.) The last set of equations relate the energy required to freeze a given sample to the equilibrium temperature and brine temperature. Only two equations were required, one for the velocity case and the other for the non-flow situation.

The result of these four steps is a set of equations that make it possible to examine the effects of altering various parameters, and to predict sample sizes, concentrations and energy requirements.

CHAPTER VI

SUMMARY OF DERIVED EQUATIONS

The following equations are the results of the reduction of data. Fig. 10 is a flow diagram which indicates processes yielding large and small quantities of pure water.

Total Weight Equation. Equation (37)

$$W_t = \left[0.1382\ (r_t^3 - r_o^3)\right] + 0.001667 \left[(r_t^3 - r_o^3) - 2.1500 \right] \frac{(32 - t_e)^{1.3}}{r_o}$$

All symbols used in this dissertation are defined in Appendix J. Details of the derivation of this equation are presented in Appendix C.

Final Radius Equations

Case 1. Brine temperature equal to equilibrium temperature, $t_b = t_e$. Equation (36)

$$\left[\frac{t_e - t_o}{{r_o}^2}\right] \Theta = 47.11 \left[\frac{1}{3}\left(\left(\frac{r_t}{r_o}\right)^3 - 1\right) - \frac{1}{2}\left(\left(\frac{r_t}{r_o}\right)^2 - 1\right)\right]$$

Case 2. Brine temperature greater than equilibrium temperature, $t_b > t_e$. Equation (51)

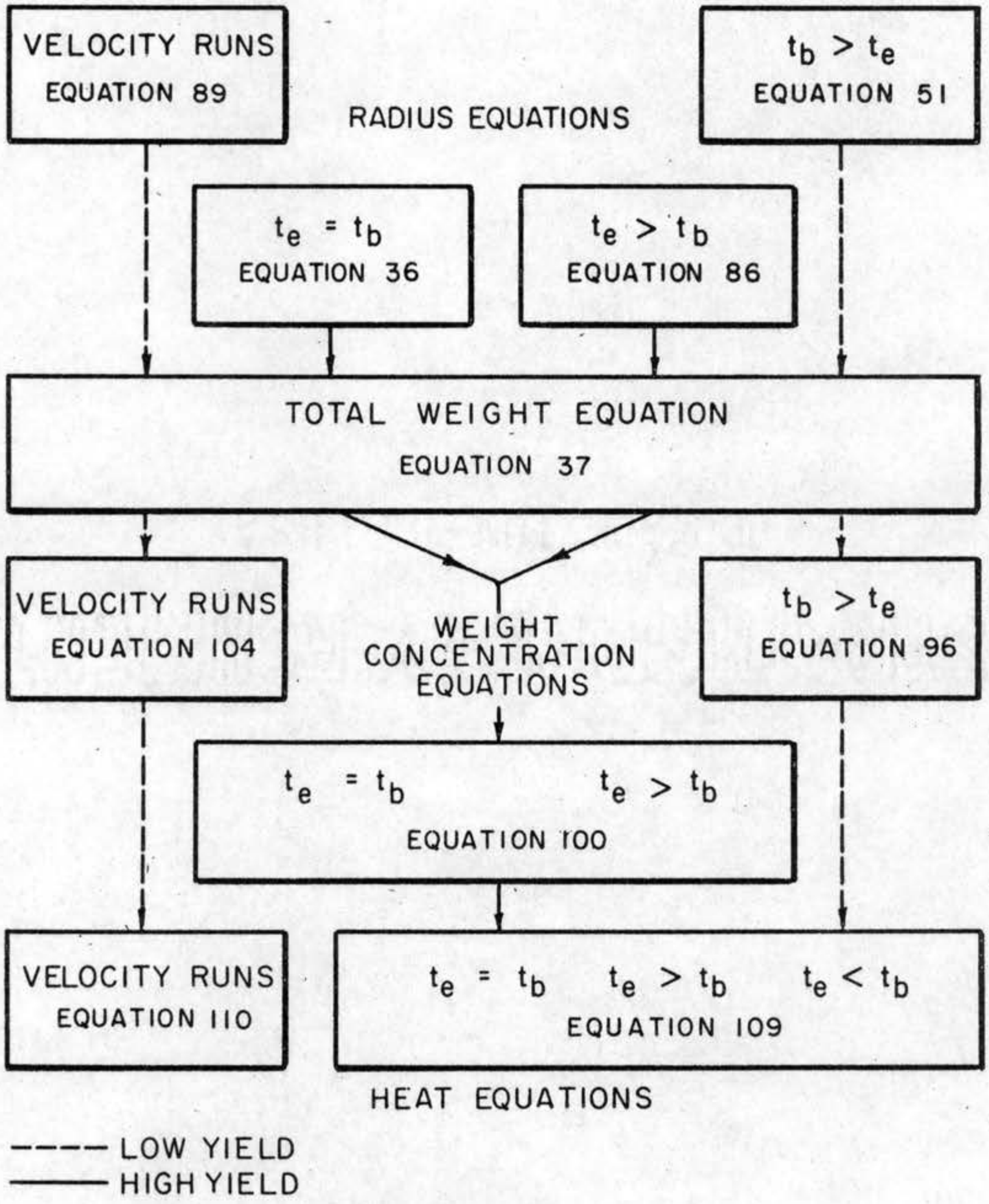


Figure 10. Reduction of Data Equations Flow Sheet

$$\frac{t_e - t_o}{\left(t_b - t_e\right)^{\frac{1}{4}}} = 7.11\left[\left(\frac{r_t}{r_o}\right) - 1\right]\left[r\right]^{7/3}$$

Case 3. Brine temperature less than the equilibrium temperature, $t_b < t_e$. Equation (86)

$$\frac{r_t - r_o}{\theta^{\frac{1}{2}}} = 0.5711\left[\log_e\left(\frac{\Delta t}{8.3467}\right)\right]$$

Case 4. Velocity runs. Equation (89)

$$\frac{r_t - r_o}{\theta^{\frac{1}{2}}} = 0.0200\left(\frac{\Delta t}{V_{avg}}\right)$$

Details of the derivation of the final radius equations are presented in Appendix D.

Weight-Concentration Equations

Case 1. Brine temperature greater than equilibrium temperature. Equation (96)

$$\frac{W}{W_t} = \left[0.85 + 0.15\left(\frac{C}{C_b}\right)\right]\left[\frac{C^{1/r_o}}{C_b}\right]^{1/3}$$

Case 2. Brine temperature equal to or less than the equilibrium temperature. Equation (100)

$$\frac{W}{W_t} = \frac{1}{2.32}\left[1 + 0.2\left(\frac{C}{C_b}\right)^2\right]\left[C^{1/3r_o}\right]\left[\frac{\Delta t^{1/3}}{C_b^{\frac{1}{2}}}\right]$$

Case 3. Velocity equations. Equation (104)

$$\frac{W}{W_t} = \frac{1}{2.10}\left[1 + 0.2\left(\frac{C}{C_b}\right)^2\right]\left[C^{(1 + 0.209V_{avg})/2.39}\right]\times$$

$$\left[\frac{\Delta t^{\frac{1}{2}}}{C_b}\right]$$

Details of the derivation of weight-concentration equations are presented in Appendix E.

Energy Equations

Non-velocity runs. Equation (109)

$$\frac{Q}{W_t} = 141\ [1 + 0.0872\ (t_b - t_e)]$$

Velocity runs. Equation (110)

$$\frac{Q}{W_t} = 120\ [1 + 1.5V_{avg}]\ [1 + 0.223\ (t_b - t_e)]$$

Details of the derivation of the energy equations are presented in Appendix F.

Average Brine Concentration

Equation (111)

$$C_{avg} = 0.342C_b$$

Details of the derivation of the average brine concentration equation are presented in Appendix G.

Range of Variables

The range of the variables covered by the equations are:

Final radius, r_t, 1.150 to 2.025 inches
Initial radius, r_o, 0.50 to 1.00 inches
Brine equilibrium temperature, t_e, 27.95 to 32.00 ^oF
Brine temperature, t_b, 27.1 to 33.7 ^oF
Sphere surface temperature, t_o, 10.1 to 28.6 ^oF

Brine concentration, C_b, 0 to 4.12%

Total temperature potential, Δt, 5.2 to 22.5 ^{o}F

Average velocity, V_{avg}, 0.00 to 1.27 ft/min

Run time, θ, 0.83 to 4.00 hours

Total sample weight, W_{xt}, 0.06 to 1.02 pounds.

Magnitude of Error

The average magnitude of the percentage error for the equations yielding the greatest amount of pure water is:

Final radius equation (36). $t_b = t_e$, 1.26%

Final radius equation (86). $t_b < t_e$, 1.21%

Total weight for $t_b = t_e$ by Equation (37), 6.53%

Total weight for $t_b < t_e$ by Equation (37), 4.69%.

Predicted sample weights for various concentration. By Equation (100).

Concentration in percent	3	2	1	.5	.1	.05	.01
Percent Error	8.96	9.61	8.21	9.57	11.75	10.14	42.14

Energy required per pound of ice. Equation (109), 3.45%.

Mean concentration equation. Equation (111), 10.25%.

CHAPTER VII

PREDICTED PERFORMANCE OF AN INDIRECT FREEZING DESALINATION PROCESS

The purpose of this chapter is to show how the melt can be reprocessed to reduce overall energy requirements for the phase change. It is recognized that a major portion of the phase change energy may be recovered by use of appropriate heat exchangers. This chapter deals only with energy requirements for the phase change, energy transferred between the ice interface and the surrounding brine, and energy transferred from the solid phase. Heat terms that have been omitted include heat required to maintain coolant temperature, heat required to maintain brine temperature, and heat that may be recovered by melting the frozen samples.

Of these three terms, the heat recovered by use of a regenerative cycle is believed to be the most important. The actual magnitude and interaction of these heat quantities will require a complete cycle analysis. This analysis will serve to determine the optimum number of stages for the freezing process and the optimum brine concentration for each stage.

The first series of calculations assume that only the accumulated sample weight having a concentration less than 0.05% was the useful yield.

Case 1.

Sphere surface temperature, $t_o = 20\ ^{o}F$

Brine temperature, $t_b = 27.3\ ^{o}F$

Brine concentration, $C_b = 3.5\%$

Initial sphere radius, $r_o = 1.00$ inch.

The brine equilibrium temperature may be calculated from Equation (11).

$$t_e = 32.0 - 1.05C_b = 28.3\ ^{o}F$$

Therefore $t_b < t_e$ and Equation (86), may be applied to the solution of the problem.

$$\frac{r - r_o}{\theta^{\frac{1}{2}}} = 0.5711 \left[\log_e \left(\frac{\Delta t}{8.3467}\right)\right]$$

where

$$\Delta t = (32 - t_b) + (32 - t_o) = 16.7\ ^{o}F.$$

By dividing the Equation (109) for Q/W_t by Equation (100) for W/W_t, Equation (3) was obtained such that

$$\frac{Q}{W} = \frac{(327.1)\ [1 + 0.0872\ (t_b - t_e)]}{[1 + 0.2\ (\frac{C}{C_b})^2]\ C^{1/3} r_o} \frac{C_b^{\frac{1}{2}}}{\Delta t^{1/3}} \qquad (3)$$

so that time does not effect the quantity of heat required per pound of yield. Fig. 11 was obtained by calculating the radius from Equation (86) and converting this to an approximate weight by means of Equation (9), or

$$r_t = r_o + 0.5711 \left[\theta^{\frac{1}{2}}\right]\left[\log_e \left(\frac{\Delta t}{8.3467}\right)\right]$$

$$W_t = 0.1382\ (r_t^3 - r_o^3).$$

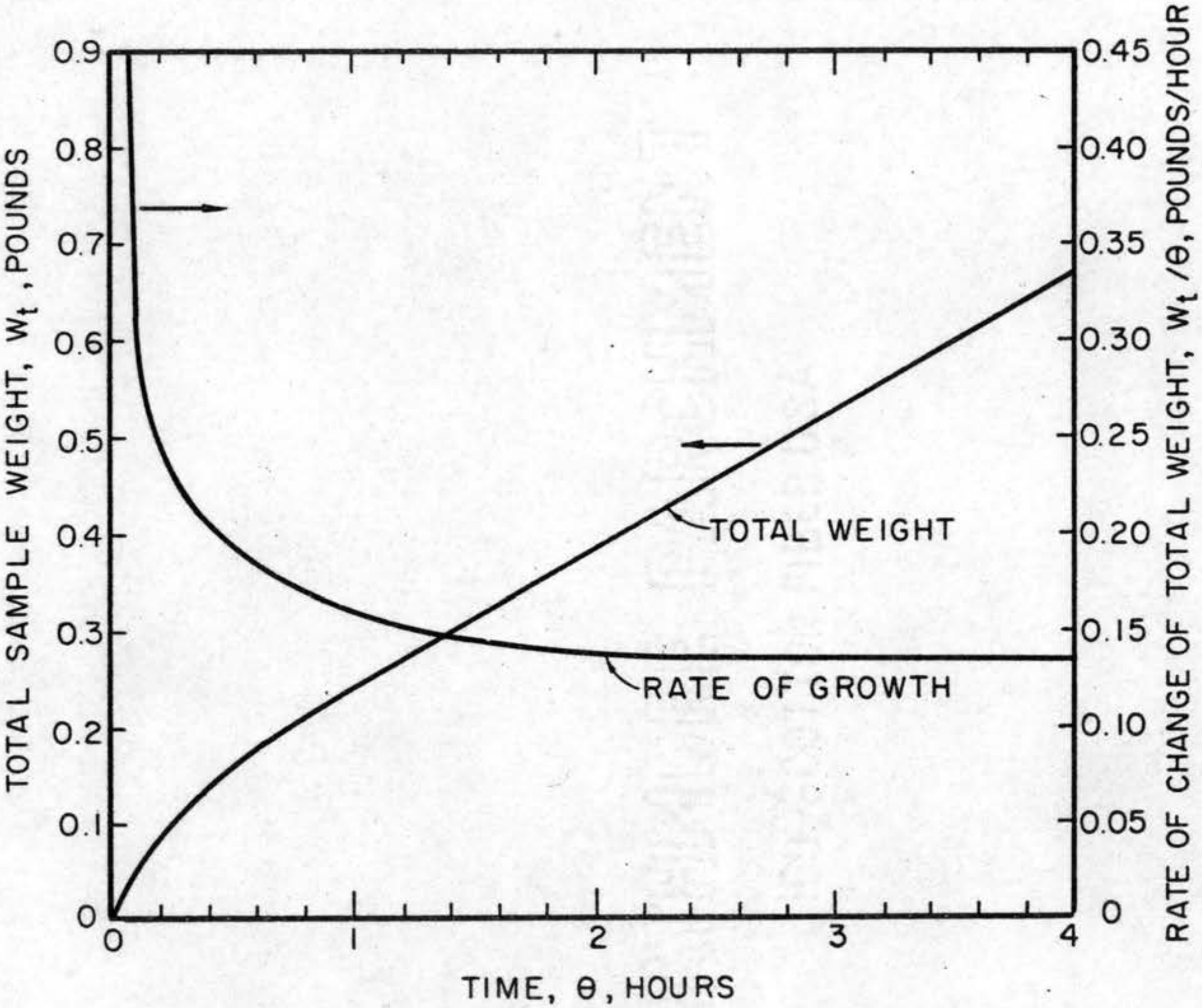


Figure 11. Total Weight and Rate of Growth From Sample Problem Data

From Fig. 11 the growth rate becomes linear with respect to time. Therefore a time θ of 1 hour was assumed which gave a value for r_t of 1.395 by means of Equation (86). Inserting the value of r_t and t_e into Equation (37) yields

$$W_t = 0.1382\ (r_t^3 - r_o^3) + \Bigg[0.001667 \times \\ [(r_t^3 - r_o^3) - 2.1500] \times \\ \frac{(32 - t_e)^{1/3}}{r_o}\Bigg] = 0.2332 \text{ lbs.}$$

From Equation (100) for C = 0.05%

$$\frac{W_{0.05}}{W_t} = \frac{1}{2.32}\ [\ 1 + 0.2\ (\frac{C}{C_b})^2\ ] \times \\ \frac{(C\,\Delta t)^{1/3}}{C_b^{\frac{1}{2}}} = 0.2168.$$

Therefore the weight of a sample having a concentration less than C = 0.05% was

$$W_{0.05} = (0.2168)(0.2332) = 0.0506 \text{ lbs.}$$

To determine the quantity of energy required Equation (109) was used

$$\frac{Q}{W_t} = 141\ [1 + 0.0872(t_b - t_e)] = 128.7 \text{ Btu/lb.}$$

The energy required per pound of pure water was

$$\frac{Q}{W_{0.05}} = \frac{(Q/W_t)}{(W_{0.05}/W_t)} = \frac{128.7}{0.2168} = 581.6 \text{ Btu}$$

which may be calculated directly by means of Equation (3).

Case 2. Case 2 is similar to Case 1, except $r_o = 0.50$ inch. By use of Equation (3)

$$\frac{Q}{W} = \frac{(327.1)\ [1 + 0.0872(t_b - t_e)]}{[1 + 0.2(C/C_b)^2]} \frac{(C_b)^{\frac{1}{2}}}{(C)^{2/3}(\Delta t)^{1/3}}$$

which for C = 0.05%, gives

$$\frac{Q}{W_{0.05}} = 1615 \text{ Btu/lb.}$$

Case 3. Case 3 is the same as Case 1 except brine temperature, $t_b = 28.3\ ^{\circ}F$. Therefore

$$t_b = t_e$$

and

$$\Delta t = (32 - t_b) + (32 - t_o) = 15.7.$$

Again using Equation (3) for C = 0.05% the energy required is

$$\frac{Q}{W_{0.05}} = 650.4 \text{ Btu/lb.}$$

Case 4. Case 4 is the same as Case 1 except brine temperature, t_b, is 29.3 $^{\circ}F$ and time must be sufficiently large to approximate steady-state conditions.

Dividing Equation (109) by Equation (96) gives

$$\frac{Q}{W} = (141) \frac{[1 + 0.0877(t_b - t_e)]}{(0.85 + 0.15\ C/C_b)} \left[\frac{C_b}{C}\right]^{1/3} \tag{4}$$

and for C = 0.05%, the energy required is calculated to be

$$\frac{Q}{W_{0.05}} = 728.6 \text{ Btu/lb.}$$

Case 5. Case 5 is the same as Case 1 except V_{avg} = 1 ft/min. Dividing Equation (110) by Equation (104) gives

$$\frac{Q}{W} = \frac{(120)(1 + 1.5V_{avg})[1 + 0.233(t_b - t_e)]}{[1 + 0.2(C/C_b)^2](C\,\frac{1 + 0.209V}{2.39})(\frac{1}{2.10})} \frac{C_b}{(\Delta t)^{\frac{1}{2}}} \tag{5}$$

and for C = 0.05%, the energy required is 1906 Btu/lb.

It will be shown how a four-stage process, Fig. 12, may be used to reduce the phase change energy requirements. Since Case 1 has the lowest value of $Q/W_{0.05}$ it will be used for this analysis. Given the values of $W_t = 0.2332$ lbs and $\Delta t = 16.68\ ^{\circ}F$, W is evaluated for various values of C, Table II, by means of Equation (100). The incremental sample weight, ΔW, is evaluated by subtracting the adjacent values of W in Table II. The value $C\Delta W$ was obtained by multiplying the values of ΔW by the average value of C, for the two steps used to determine ΔW. These terms are then summed, starting at C = 0.01%. All of the sample below C = 0.01% is assumed to have this concentration. This assumption tended to correct errors in the weight-concentration equations for salt concentration below 0.01%.

The mean concentration is now evaluated for any sample by;

$$C_{avg} = \frac{\left[C\Delta W\right]_1 - \left[C\Delta W\right]_2}{W_1 - W_2} . \tag{6}$$

For C = 0.05%, which determines the upper limit of W,

$$C_{avt} = \frac{0.00103}{0.0506} = 0.020\%.$$

Therefore, to obtain a sample having an average concentration of 0.05% an enriched portion of the melt may be added to the 0.02% sample to give the desired average value. By interpolating within Table II, for C_{avg} equals 0.05%, W equals 0.0780 lbs rather than the 0.0506 lbs calculated by Equation

TABLE II

DATA FOR THE CALCULATION OF THE AVERAGE CONCENTRATION OF A MELT SAMPLE USING SAMPLE PROBLEM INFORMATION

Concentration	Weight	$C_{avg}\Delta W$
3.2	0.2361	0.246858
3.1	0.2295	0.232053
3.0	0.2270	0.172378
2.5	0.2053	0.124903
2.0	0.1842	0.087453
1.5	0.1628	0.060703
1.0	0.1396	0.038203
0.5	0.1096	0.014203
0.4	0.1014	0.010513
0.3	0.0920	0.007223
0.2	0.0803	0.004298
0.1	0.0639	0.001838
0.09	0.0615	0.001610
0.08	0.0591	0.001406
0.07	0.0566	0.001218
0.06	0.0537	0.001030
0.05	0.0506	0.000860
0.04	0.0469	0.000694
0.03	0.0427	0.000547
0.02	0.0373	0.000412
0.01	0.0296	0.000296

(100). Q/W_t equals 128.7 Btu/lb as before, W_t equals 0.2332 lbs and Q equals 30.01 Btu. Therefore the energy required is

$$\frac{Q}{W_{0.05 - avg}} = \frac{30.01}{0.0780} = 385 \text{ Btu/lb}$$

compared to the 581.6 Btu/lb obtained previously.

The average concentration value for the remaining melt may be determined from the values in Table II by substituting into Equation (6). The subscript 2 is evaluated for the point where W equals 0.0780 lbs and from Equation (6)

$$C_{avg} = \frac{0.2419 - 0.0038}{0.2332 - 0.0780} = 1.53\%.$$

Since the salt concentration of the remaining melt is less than the mother liquid, 1.53 to 3.50%, the process will be repeated with the original concentration $C_b = 1.53\%$. Some minor changes in data will be required; the brine equilibrium temperature from Equation (11) will be:

$$t_e = 32 - 1.05(1.53) = 30.39\ ^{o}F.$$

To prevent large amounts of dendritic growth from excessive subcooling, the brine temperature will be increased to 29.4 ^{o}F. Therefore, Δt becomes 14.6 ^{o}F.

Using Equation (86) for r_t and Equation (100) for W/W_t with the new values of the variables gives:

$$W_t = 0.1748 \text{ lbs.}$$

A table, similar to Table II, was constructed from this new data. The weight of a sample having an average concentration of 0.05% was found to be:

$W_{0.05\,-\,avg} = 0.0840$ lbs.

However, the amount of brine supplied was:

$W_{t2} = (0.2332 - 0.0780) = 0.1552$ lbs

which is the difference between the original sample and the amount removed after the first process.

Therefore the yield of pure water for the second step was:

$$W_{(0.05\,-\,avg)2} = W_{(0.05\,-\,avg)} \frac{W_{t2}}{W_t}$$

$$= (0.0840) \frac{0.1552}{0.1748} = 0.0746 \text{ lbs.}$$

The quantity of heat required for this step must also be reduced by a proportional amount for step 2, Q/W_t remains at 128.7 Btu/lb since $(t_b - t_e)$ is still 1 ^oF.

$Q = (128.7)(W_t)$ Btu

and Q_2 is:

$$Q_2 = (128.7)(W_t)\frac{W_{t2}}{W_t} = 128.7\ W_{t2} = (128.7)(0.1553)$$

$$= 19.97 \text{ Btu.}$$

The results of steps 1 and 2 are

	Weight Pure Water, W	Energy Required, Q
Step 1	0.0780	30.01
Step 2	0.0746	19.97
	0.1526	49.98

and the energy per pound for the two steps

$$\frac{Q}{W} = \frac{49.98}{.1526} = 321 \text{ Btu/lb.}$$

The concentration of the sample residue is now 0.655%.

Repeating all of the calculations again gives 0.0580 lbs of pure water when 10.37 Btu of energy are supplied. This results in a total weight of 0.2106 lbs of pure water and a total energy requirement of 60.35 Btu. Therefore, the energy required for the three steps is

$$\frac{Q}{W} = \frac{60.35}{0.2106} = 287 \text{ Btu/lb.}$$

The concentration of the remaining original melt,

$$(0.2332 - 0.2106) = 0.0226 \text{ lbs,}$$

is 0.270%.

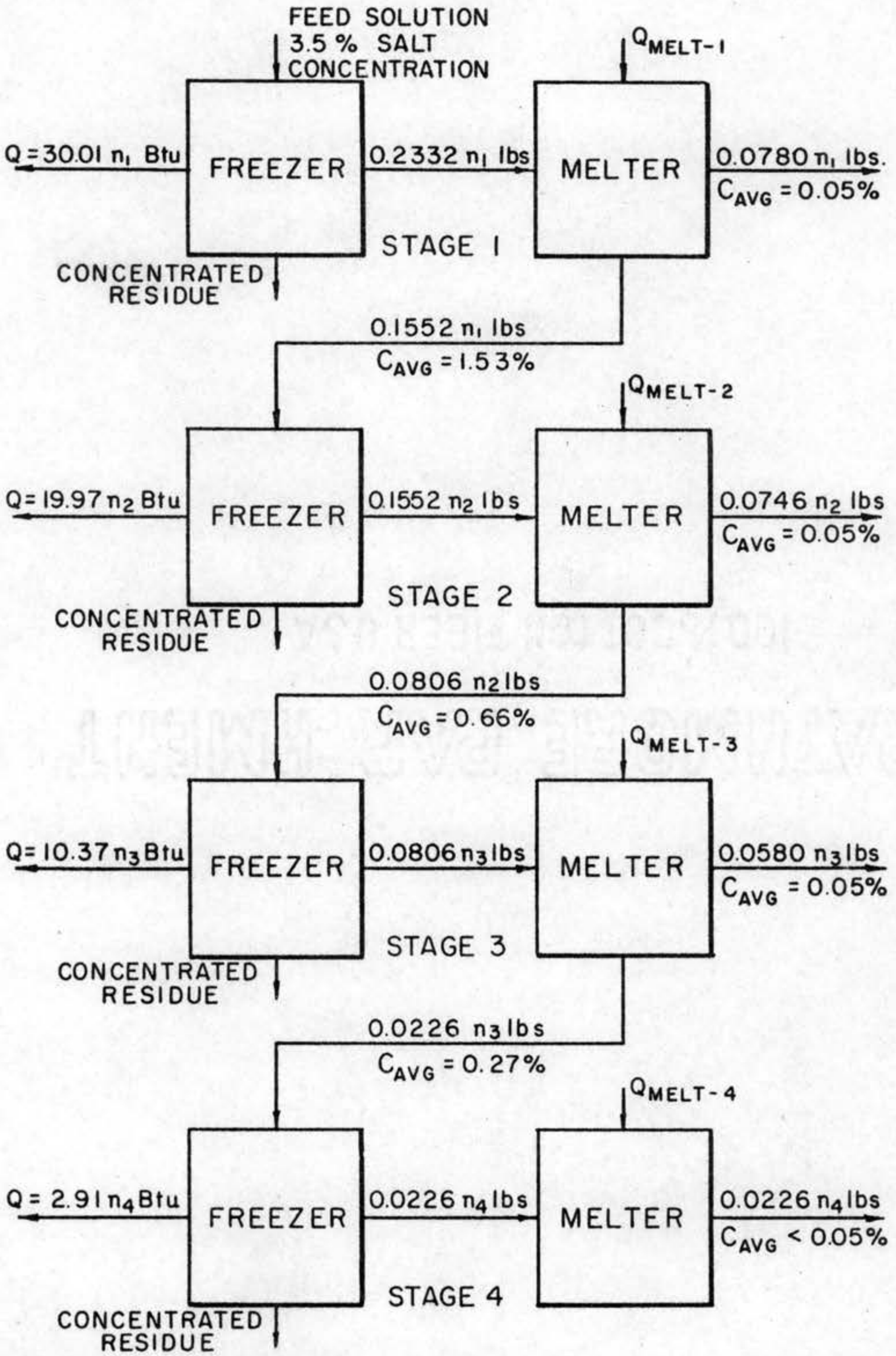
The fourth step reduced all of this material to a concentration less than 0.05%, requiring 2.91 Btu. The total sample weight is now 0.2332 lbs and the energy supplied was 63.26 Btu or

$$\frac{Q}{W} = \frac{63.26}{.2322} = 272 \text{ Btu/lb.}$$

The complete four step process is shown in Fig. (12).

Using the same data as Case 1 but with $t_o = 10\ ^oF$, the process required only two steps and resulted in a yield of 0.4731 lbs of potable water with an expenditure of 101.0 Btu of energy. This resulted in a Q/W of 214 Btu/lb. This value of t_o is outside the range for which the equations were derived so that the extrapolated results may be open to question, but it does indicate the improvement in performance with lowering sphere surface temperature.

Instead of using fractional separation the average sample could be collected by melting off the entire sample and then refreezing. Using the same set of conditions as in Case 1, C_{avg} was calculated by Equation (111) such



n_i = NUMBER OF SPHERES FROZEN IN STAGE i.

Figure 12. Four-Stage Normal Freezing, Fractional Melting Desalination Process

that

$$C_{avg} = 0.342\ C_b.$$

The first step would require 30.0 Btu as would all of the succeeding steps. After the first freezing and melting the concentration would be 1.197%. The second would yield a concentration of 0.409%, the third 0.140%, and the fourth 0.048%. Therefore, the total energy requirements are 120 Btu and Q/W would be

$$\frac{Q}{W} = \frac{120}{0.2322} = 516.7 \text{ Btu/lbs.}$$

The results from the calculations presented in this chapter are shown in Table III.

The calculations for the one step process are correct and do not need to be modified but a recycle operation process consisting of two or more steps should be recomputed for the following reason. Each time a sample is frozen and removed, the non-frozen liquid is enriched. For example, a 0.2332 lb, 3.5 per cent sample contains 0.00817 lbs of salt, the average concentration of the frozen sample would be, by means of Equation (111):

$$C_{avg} = (0.342)(3.5) = 1.20$$

having a salt content of 0.00280 lbs. Therefore the concentration of the remaining liquid is increased by 0.00537 lbs of salt. Since each freezing operation enriches the material left behind it would be necessary to discard the original solution when some particular concentration level is reached. The solutions in the 2nd, 3rd, etc., stages

TABLE III

SUMMARY OF SAMPLE PROBLEM CALCULATIONS

Process	Brine Temperature °F	Time Hours	Initial Radius Inches	$Q/W_{0.05}$ Btu/lb
	Concentration Less Than 0.05 Percent			
Single Pass	28.3	1.00	1.00	650
Single pass	29.3	----	1.00	729
Single pass	27.3	1.00	1.00	1906**
Single pass	27.3	1.00	0.50	1615
Single pass	27.3	1.00	1.00	582
	Average Concentration 0.05 Percent			
Single pass	27.3	1.00	1.00	385
Two pass	27.3	1.00*	1.00	321
Three pass	27.3	1.00*	1.00	287
Four pass	27.3	1.00*	1.00	272
Four pass	(Entire sample melted and collected as a single batch.)			517

* Time period one hour for all passes

** Velocity of brine is 1 ft/min

would also be enriched in salt content. The determination of the optimum cycle would depend on evaluating all of the energy requirements and losses for the stage freezing process.

The energy required to cool the original sample was also omitted. If the melt from each step remains at the equilibrium temperature and if the original fluid was supplied at 70 ^{o}F it would require

$$Q = mc_p(\Delta t') = (0.2332)(1)(38) = 8.5 \text{ Btu}$$

to cool each 0.2332 lb sample. For the four step process this would give an energy requirement of

$$\frac{Q}{W} = \frac{63.26 + 8.5}{.2332} = \frac{71.76}{.2332} = 309 \text{ Btu/lb.}$$

Omitted from the above calculations are the various energy losses in refrigeration equipment, pumping requirements and other material handling aspects of the problem. However, with a basic energy requirement of 309 Btu/lbs for freezing as compared to a basic requirement of 1112 Btu/lb for atmospheric boiling, with a supply temperature of 70 ^{o}F, the incentives for developing a feasible cycle are excellent.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Before the investigation several facts were known about the application of a normal freezing process to salt water conversion. It was known that separation of salt and water could be obtained but that previous efforts to obtain pure water in a one-stage process had failed. The use of fractional melting was believed to be a possibility for obtaining a quantity of pure water in one step, but the amount that could be recovered by this technique was not known. Previous investigations in this field have also neglected to evaluate the effects of the several variables on the quantity of pure water obtainable from a normal freezing process.

A series of experiments were performed in which pure water and saline solutions of various concentrations were frozen on the exterior surface of a sphere. Spheres having an external radius of 0.50, 0.75 and 1.00 inches were used. The sodium chloride concentration was varied from 0.00 to 4.12 per cent. The inside surface of the sphere temperature ranged from 10.1 to 28.6 ^{o}F while the surrounding brine temperature had values from 27.1 to 33.7 ^{o}F. Velocity runs were also performed with a maximum brine velocity of 1.27 feet per minute. The total ice frozen on the surface of the sphere had a total weight which varied from 0.06 to 1.02 pounds.

The purpose of the experimental program was to evaluate the performance of a normal freezing process on a spherical surface. In addition, fractional melting of the incremental sample was utilized to determine whether the purity of the melt varied. Equations were derived to express the effect of varying selected parameters. The results of these equations indicate the use of brine temperatures slightly below or equivalent to the equilibrium temperatures yield the greatest amount of pure water for the least energy expenditure. The equations and experiments establish that the use of smaller spheres and a moving brine solution serves to lower the percentage yield of pure water. Also in the case of velocity fields the energy requirements are increased. A sample problem was solved using the best configuration and values of the various parameters within the ranges tested in the experimental program.

Therefore the results of this experiment were:

1. The salt-concentration of a melted sample was less than the original solution when the normal freezing process was used to freeze the sample.

2. Water with a salt-content less than 500 ppm could not be obtained by normal freezing in a one stage process without fractional melting.

3. When fractional melting was utilized, approximately 30% of the sample could be recovered as water having a salt content less than 500 ppm. The energy required for the phase change was 385 Btu/lb.

4. A four-step process converted the entire original sample to potable water and the impurities were less than 500 ppm. The energy required for the phase change was 272 Btu/lb.

5. The energy required for the phase change may be further improved by using lower inside sphere temperatures.

6. Larger spheres will increase the yield.

7. The use of brine velocities is not recommended.

8. Low brine temperatures lead to the formation of dendrite plates on the sphere surface.

9. The effect of these dendritic plates on sample purity was not established.

From the above considerations it is possible to obtain a conversion from 3.5 per cent to 0.05 per cent sodium chloride concentrations with an expenditure of energy that is significantly less than the phase change energy requirements for a distillation process. From a practical standpoint the advantages of the freezing process are considerably reduced since distillation may be accomplished by a simple combustion or heat process. Freezing processes usually require the conversion of heat to work in a refrigeration cycle to cool the coolant, followed by an irreversible heat transfer to freeze the sample.

The disadvantages of the above process include the complicated material handling procedures required in a four-stage process, the extended freezing and melting times, and the inherently higher cost of refrigeration equipment as compared to distillation equipment. However, with the considerable savings of energy in the normal freezing-fractional melting process, an efficient method for salt water purification has been obtained. Further study should yield a feasible plant design especially where small quantities of potable water are required. Possible fields of application would include small units to be used in marine operations, space-craft waste recovery systems, survival gear and operation in arid areas having brine aquifiers.

CHAPTER IX

RECOMMENDATIONS FOR FUTURE STUDY

This investigation has established that an efficient normal freezing-fractional melting solution to the problem of desalination exists.

The operation of the experimental apparatus suggested that a modification should be made in the technique for handling the coolant. While the centrifugal pump used in this study proved to be a satisfactory method of supplying the sphere with coolant, lower temperature runs and larger test sections would require a more positive pumping action. For this reason it is recommended that the brine be mixed in an insulated container and pumped by air pressure into the test section. The balance of the apparatus performed satisfactorily and needs no modification.

A sequence of future experimental programs is also recommended. First, the weight-concentration experiments and equations show an increase in the percentage yield of water having a salt concentration less than 0.05 per cent when the inside sphere temperature was reduced. The use of lower coolant temperatures should therefore be attempted. Second, the yield also increased as the radii of the sphere increased. Therefore, the effect of using larger spheres should also be studied.

The spherical configuration leads to difficulties in coolant entrance and exit lines, besides being a difficult configuration to fabricate. It would increase the practicability of the process if a cylindrical system could be substituted for that of the sphere. Therefore a study of this process using a cylindrical surface is recommended.

The system of gravity draining is certainly the simplest method of fractionally melting the sample. The possibility exists that other separation techniques may result in larger and more rapid yields. The washing of the sphere with pure water may serve to increase the overall production of fresh water. This was found to be the case in large scale desalination plants when ice crystals formed by direct freezing were washed with pure water.

Another separation technique that may prove useful, especially if the cylindrical configuration maintains the yields available from the spherical system, is to rotate the frozen section and discharge the melt by centrifugal action. Different rotational speeds should alter the shape of the weight-concentration curves by separating the entrapped brine before extensive melting of the sample occurs.

Finally, when the above investigations are complete, the most efficient cycle should be evaluated. This will require a comprehensive study to determine the optimum degree of concentration build-up in the various freezing stages, the most efficient number of stages to employ, the effect of various energy losses, and the best set of operating conditions to be used.

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APPENDIX A

EXPERIMENTAL DATA

TABLE IV

EXPERIMENTAL DATA

RUN NUMBER	1	2	3	4	5	6	7
TEMPERATURES, ^{o}F							
Sphere surface, t_o	25.1	26.1	19.5	20.0	26.6	22.5	25.7
Brine, t_b	33.7	32.0	31.6	31.7	31.4	32.3	31.3
Brine equilibrium, t_e	32.00	32.00	32.00	32.00	31.38	31.43	31.36
Coolant sump, t_s	20.4	15.9	18.5	18.3	18.1	17.5	16.0
Entering coolant, t_{in}	24.0	25.4	23.6	19.2	25.6	20.2	21.9
Leaving coolant, t_{out}	25.1	26.9	25.8	20.5	27.2	22.0	23.6
Total potential, t	5.2	5.9	12.9	12.3	6.0	9.2	7.0
Melting temperature	62.0	51.2	48.9	63.3	59.9	66.6	60.7
Chamber	33.2	31.7	31.1	31.5	31.2	32.0	30.9
TIMES							
Time of run, θ, hours	2.00	2.00	3.00	4.00	0.67	2.33*	2.17
Coolant flow, θ_s, seconds	47.8	37.8	55.6	43.2	36.6	70.2	54.2
Melting time, hours	1.03	2.32	2.21	1.32	0.78	1.33	1.20
RADII, inches							
Sphere, r_o	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Ice, r_{xt}	1.400	1.600	1.550	1.475	1.300	1.500	1.450
Total weight of sample W_{xt}, grams	100.6	194.2	175.4	138.3	74.5	143.5	128.2
Brine concentration, C_b, %	0.00	0.00	0.00	0.00	0.63	0.58	0.64
Brine velocity, V_{avg}, feet per minute	0.000	0.000	0.779	1.135	0.000	0.000	0.325
Coolant, N-Sodium chloride M-Methanol	N	N	N	N	N	N	N

* Steady state time

TABLE IV (Continued)

RUN NUMBER	8	9	10	11	12	13	14
TEMPERATURES							
Sphere surface	25.3	22.1	26.4	23.9	28.6	25.2	25.5
Brine	31.4	29.9	31.4	30.0	29.4	29.2	28.9
Brine equilibrium	31.36	31.28	29.94	29.96	30.00	29.84	29.82
Coolant sump	10.8	16.0	16.0	20.1	19.0	19.2	20.1
Entering coolant	23.1	20.2	24.2	24.2	23.2	23.8	23.6
Leaving coolant	25.4	23.1	25.6	26.6	25.0	25.8	25.1
Total potential	7.3	12.0	6.2	10.1	6.0	9.6	9.6
Melting temperature	58.8	64.9	60.3	68.3	48.3	54.2	49.7
Chamber	30.9	29.2	31.3	29.8	28.7	29.0	28.6
TIMES,							
Time of run	2.25	1.58	1.00*	2.33	1.67	2.25	1.50
Coolant flow	113.1	112.3	51.8	106.9	108.4	122.4	85.6
Melting time	0.70	0.57	0.57	0.52	0.59	0.79	0.97
RADII							
Sphere	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Ice	1.275	1.250	1.250	1.250	1.200	1.300	1.300
Total sample weight	67.3	58.9	57.8	56.7	43.4	74.7	73.8
Brine concentration	0.64	0.72	2.08	2.06	2.02	2.18	2.20
Brine velocity	0.753	1.106	0.000	1.001	0.748	0.314	0.755
Type coolant	N	N	N	N	N	N	N

*Steady state time

TABLE IV (Continued)

RUN NUMBER	15	16	17	18	19	20	21
TEMPERATURES							
Sphere surface	22.1	26.2	23.2	21.4	25.4	24.0	23.7
Brine	27.8	31.5	29.5	28.3	27.7	28.1	28.2
Brine equilibrium	28.43	28.49	27.90	28.12	28.26	28.08	28.62
Coolant sump	19.0	20.6	18.1	----	18.5	17.8	18.0
Entering coolant	21.4	25.0	22.3	----	24.2	22.5	21.1
Leaving coolant	24.7	25.5	23.8	----	25.2	24.3	24.7
Total potential	14.1	6.3	11.3	----	10.9	11.9	12.1
Melting temperature	60.4	62.8	58.4	62.3	52.3	64.9	59.5
Chamber	27.2	31.5	29.4	27.8	26.9	27.8	28.0
TIMES							
Time of run	1.90	2.75*	2.00*	0.67**	1.58	1.33	2.17
Coolant flow	129.1	98.2	87.5	----	100.1	63.1	92.1
Melting time	1.28	0.26	0.77	0.35	0.48	0.65	1.29
RADII							
Sphere	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Ice	1.425	1.150	1.300	1.175	1.200	1.300	1.450
Total sample weight	119.5	27.0	71.2	33.5	39.6	71.6	127.5
Brine concentration	3.48	3.47	3.93	3.76	3.64	3.78	3.29
Brine velocity	0.000	0.000	0.000	0.000	0.000	0.314	0.768
Type coolant	N	N	N	N	N	N	N

*Steady state time
**Short run

TABLE IV (Continued)

RUN NUMBER	22	23	24	25	26	27	28
TEMPERATURES							
Sphere surface	27.1	23.3	24.9	24.6	20.2	25.8	21.4
Brine	28.1	27.2	28.2	28.9	27.1	27.2	28.2
Brine equilibrium	28.43	28.30	28.28	28.54	27.92	27.82	28.23
Coolant sump	15.7	15.5	19.5	19.6	9.9	22.0	12.4
Entering coolant	27.3	21.8	23.5	23.5	18.6	24.8	17.1
Leaving coolant	28.3	23.8	24.8	24.8	20.7	25.8	22.0
Total potential	8.8	13.5	10.9	10.5	16.7	11.0	14.4
Melting temperature	60.0	57.2	62.8	60.7	57.2	65.4	61.1
Chamber	28.2	26.9	27.9	28.6	26.7	26.9	28.0
TIMES							
Time of run	2.17	2.33	1.84	1.67*	1.50	1.67	1.67
Coolant flow	66.5	85.4	142.0	59.2	76.7	100.8	53.4
Melting time	0.40	0.76	0.37	0.85	1.14	0.41	1.87
RADII							
Sphere	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Ice	1.200	1.300	1.200	1.350	1.400	1.200	1.575
Total sample weight	40.0	71.9	38.3	84.5	107.5	41.0	187.3
Brine concentration	3.52	3.52	3.62	3.37	3.92	3.98	3.66
Brine velocity	1.101	1.2715	0.000	0.000	0.000	0.000	0.000
Type coolant	N	N	N	N	N	N	N

*Steady state time

TABLE IV (Continued)

RUN NUMBER	29	30	31	32	33	34	35
TEMPERATURES							
Sphere surface	14.7	17.1	24.5	14.0	24.0	12.8	20.0
Brine	27.4	28.1	28.2	27.9	28.0	27.7	32.0
Brine equilibrium	28.46	27.79	28.77	28.60	28.70	28.28	32.00
Coolant sump	4.8	8.4	21.2	8.0	15.7	1.7	13.5
Entering coolant	13.7	13.7	24.4	13.3	22.1	9.4	19.2
Leaving coolant	16.5	16.5	25.1	17.8	28.3	15.8	25.2
Total potential	21.9	18.8	11.3	22.1	12.0	23.5	12.0
Melting temperature	53.7	65.7	62.7	58.7	59.1	63.2	61.7
Chamber	27.4	26.2	27.9	27.5	28.6	27.2	32.0
TIMES							
Time of run	1.50	2.58*	0.83	2.58	2.50	2.50	1.16
Coolant flow	26.3	35.3	52.8	37.1	67.1	47.2	33.6
Melting time	2.68	2.14	0.28	1.82	1.98	3.92	2.34
RADII							
Sphere	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Ice	1.675	1.650	1.150	1.575	1.575	1.925	1.675
Total sample weight	237.3	226.0	26.8	183.7	187.2	405.8	231.8
Brine concentration	3.45	4.12	3.16	3.34	3.23	3.62	0.00
Brine velocity	0.000	0.000	0.000	1.151	0.782	0.000	0.000
Type coolant	M	M	N	M	M	M	M

*Steady state time

TABLE IV (Continued)

RUN NUMBER	36	37	38	39	40	41	42
TEMPERATURES							
Sphere surface	18.6	10.1	28.5	16.2	17.4	24.4	23.0
Brine	32.0	31.4	31.8	29.5	29.3	32.0	31.4
Brine equilibrium	32.00	31.26	31.34	29.21	29.49	32.00	31.42
Coolant sump	5.3	0.2	14.5	5.5	8.3	-4.2	1.2
Entering coolant	13.8	8.5	25.8	15.3	16.0	17.8	20.4
Leaving coolant	19.1	15.5	28.1	19.8	19.0	25.0	25.2
Total potential	13.4	22.5	3.7	18.3	17.3	7.6	9.6
Melting temperature	67.7	99.2	55.2	59.8	107.4	70.2	54.4
Chamber	31.9	31.0	28.9	28.9	32.0	31.1	30.5
TIMES							
Time of run	2.50	3.16*	1.16*	2.16*	2.83	1.50	1.50
Coolant flow	40.8	53.6	95.6	45.6	39.9	103.0	66.5
Melting flow	3.52	2.78	0.66	2.54	1.45	1.68	2.23
RADII							
Sphere	1.00	1.00	1.00	1.00	1.00	0.75	0.75
Ice	1.950	2.025	1.275	1.700	1.700	1.325	1.325
Total sample weight	401.7	461.3	62.8	249.4	253.5	120.6	119.4
Brine concentration	0.00	0.75	0.67	2.74	2.48	0.00	2.09
Brine velocity	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Type coolant	M	M	N	M	M	M	M

*Steady state time

TABLE IV (Continued)

RUN NUMBER	43	44	45	46	47	48	49
TEMPERATURES							
Sphere surface	20.9	21.8	18.5	14.7	14.7	13.5	11.7
Brine	30.4	29.3	32.0	30.7	29.5	29.0	28.2
Brine equilibrium	29.92	28.62	32.00	31.42	30.08	28.68	27.96
Coolant sump	6.9	10.0	10.5	8.4	7.8	9.7	5.9
Entering coolant	18.3	18.0	16.1	14.3	15.2	12.3	10.2
Leaving coolant	21.3	21.4	18.4	15.3	17.1	15.5	13.1
Total potential	12.7	12.9	13.5	18.6	19.8	21.5	24.1
Melting temperature	75.2	72.0	115.5	66.0	62.6	62.3	87.2
Chamber	28.7	32.0	30.3	29.3	28.5	27.9	27.8
TIMES							
Time of run	1.83*	1.16*	2.00	3.18	3.50	3.25*	2.83*
Coolant flow	49.0	52.0	44.4	28.4	49.4	87.8	61.9
Melting time	1.63	1.18	1.26	2.76	3.28	2.68	2.16
RADII							
Sphere	0.75	0.75	0.50	0.50	0.50	0.50	0.50
Ice	1.325	1.225	1.250	1.325	1.400	1.300	1.350
Total Sample weight	118.6	84.2	114.2	139.1	165.8	127.0	147.3
Brine concentration	2.10	3.28	0.00	0.58	1.94	3.43	3.88
Brine velocity	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Type coolant	M	M	M	M	M	M	M

*Steady state time

TABLE IV (Continued)

RUN NUMBER	50
TEMPERATURES	
Sphere surface	18.8
Brine	26.8
Brine equilibrium	27.95
Coolant sump	5.6
Entering coolant	16.1
Leaving coolant	19.1
Total potential	18.4
Melting temperature	116.2
Chamber	24.3
TIMES	
Time of run	3.17
Coolant flow	90.6
Melting time	0.42
RADII	
Sphere	0.50
Ice	1.325
Total sample weight	138.3
Brine concentration	3.89
Brine velocity	0.000
Type coolant	M

APPENDIX B

SAMPLE WEIGHTS AND SALT CONCENTRATIONS

TABLE V

EXPERIMENTAL DATA
Sample weights and salt concentrations

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 5	
1.	12.4	74.5	0.658
2.	9.6	62.1	0.437
3.	9.7	52.5	0.310
4.	10.2	42.8	0.161
5.	10.9	32.6	0.073
6.	11.4	21.7	0.018
7.	6.5	10.3	0.007
8.	3.8	3.8	0.000
		Run number 6	
1.	10.8	143.5	0.848
2.	7.5	132.7	0.639
3.	10.6	125.2	0.432
4.	8.0	114.6	0.357
5.	11.1	106.6	0.307
6.	10.3	95.5	0.215
7.	14.9	85.2	0.175
8.	13.1	70.3	0.112
9.	9.6	57.2	0.057
10.	10.2	47.6	0.038
11.	11.7	37.4	0.014
12.	9.1	25.7	0.003
13.	7.2	16.6	0.001
14.	9.4	9.4	0.000

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 7	
1.	7.5	128.2	0.682
2.	7.4	120.7	0.392
3.	8.7	113.3	0.254
4.	9.6	104.6	0.187
5.	6.6	95.0	0.127
6.	8.7	88.4	0.106
7.	9.3	79.7	0.085
8.	7.9	70.4	0.062
9.	8.2	62.5	0.041
10.	9.1	54.3	0.029
11.	7.4	45.2	0.018
12.	7.7	37.8	0.012
13.	6.5	30.1	0.008
14.	7.7	23.6	0.003
15.	9.6	15.9	0.000
16.	6.3	6.3	0.000
		Run number 8	
1.	10.9	67.3	0.662
2.	9.0	56.4	0.591
3.	10.6	47.4	0.522
4.	12.3	39.9	0.210
5.	14.5	27.6	0.106
6.	4.4	13.1	0.050
7.	3.6	8.7	0.022
8.	5.1	5.1	0.000
		Run number 9	
1.	6.6	58.9	0.682
2.	6.6	52.3	0.437
3.	7.6	45.7	0.216
4.	9.1	38.1	0.104
5.	3.4	29.0	0.067
6.	5.4	25.6	0.053
7.	4.8	20.2	0.045
8.	6.1	15.4	0.032
9.	6.6	9.3	0.016
10.	2.7	2.7	0.000

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 10	
1.	2.9	57.8	2.15
2.	6.8	54.9	1.41
3.	6.8	48.1	1.05
4.	6.5	41.3	0.792
5.	8.8	34.8	0.440
6.	7.9	26.0	0.206
7.	6.8	18.1	0.060
8.	4.0	11.3	0.017
9.	3.9	7.3	0.007
10.	3.4	3.4	0.000
		Run number 11	
1.	4.8	56.7	1.83
2.	4.5	51.9	1.47
3.	6.6	47.4	1.17
4.	6.0	40.8	0.897
5.	6.5	34.8	0.676
6.	5.8	28.3	0.448
7.	3.8	22.5	0.244
8.	5.6	18.7	0.155
9.	3.4	13.1	0.087
10.	4.6	9.7	0.039
11.	3.8	5.1	0.011
12.	1.3	1.3	0.000
		Run number 12	
1.	3.8	43.4	2.10
2.	5.4	39.6	1.85
3.	4.7	34.2	1.63
4.	5.2	29.5	1.51
5.	1.7	24.3	1.06
6.	4.6	22.6	0.762
7.	4.8	18.0	0.561
8.	5.8	13.2	0.284
9.	6.2	7.4	0.063
10.	1.2	1.2	0.023

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
Run number 13			
1.	8.4	74.7	1.92
2.	6.1	66.3	1.79
3.	6.6	60.2	1.49
4.	5.8	53.6	1.27
5.	6.3	47.8	0.924
6.	5.5	41.5	0.653
7.	3.9	36.0	0.481
8.	5.8	32.1	0.375
9.	3.9	36.3	0.239
10.	6.3	22.4	0.154
11.	5.9	16.1	0.075
12.	7.9	10.2	0.024
13.	2.3	2.3	0.000
Run number 14			
1.	9.2	73.8	2.32
2.	5.8	64.6	1.91
3.	8.8	58.8	1.40
4.	9.4	50.0	1.01
5.	7.7	40.6	0.692
6.	8.6	32.9	0.432
7.	7.8	24.3	0.218
8.	6.6	16.5	0.092
9.	7.2	9.9	0.028
10.	2.7	2.7	0.008

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
Run number 15			
1.	2.8	119.5	3.60
2.	15.2	116.7	3.39
3.	4.6	101.5	2.92
4.	4.6	96.9	2.49
5.	7.9	92.3	2.20
6.	7.4	84.4	1.82
7.	6.5	77.0	1.51
8.	5.7	70.5	1.21
9.	5.8	64.8	0.886
10.	5.4	59.0	0.727
11.	10.8	53.6	0.475
12.	9.5	42.8	0.225
13.	10.4	33.3	0.124
14.	9.9	22.9	0.048
15.	7.7	13.0	0.023
16.	5.3	5.3	0.000
Run number 16			
1.	2.1	26.9	4.33
2.	3.5	24.8	3.25
3.	5.7	21.3	1.68
4.	4.7	15.6	1.02
5.	3.6	10.9	0.396
6.	4.5	7.3	0.183
7.	2.8	2.8	0.054
Run number 17			
1.	1.7	71.2	4.00
2.	4.8	69.5	3.66
3.	6.3	64.7	3.09
4.	5.4	58.4	2.48
5.	4.5	53.0	2.00
6.	5.3	48.5	1.65
7.	5.2	43.2	1.24
8.	6.2	38.0	0.902
9.	7.7	31.8	0.532
10.	5.5	24.1	0.227
11.	4.1	18.6	0.115
12.	4.8	14.5	0.058
13.	4.8	9.7	0.032
14.	2.9	4.9	0.009
15.	2.0	2.0	0.000

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 18	
1.	2.5	33.5	3.58
2.	2.4	31.0	3.80
3.	1.5	28.6	4.20
4.	1.2	27.1	3.76
5.	1.4	25.9	3.03
6.	3.4	24.5	1.99
7.	4.9	21.1	1.55
8.	2.5	16.2	1.13
9.	2.7	13.7	0.830
10.	3.2	11.0	0.568
11.	3.0	7.8	0.309
12.	3.4	4.8	0.123
13.	1.4	1.4	0.000
		Run number 19	
1.	3.0	39.6	3.70
2.	2.9	36.6	3.37
3.	4.5	33.7	3.03
4.	3.8	29.2	2.35
5.	3.8	25.4	1.76
6.	4.1	21.6	1.15
7.	3.7	17.5	0.617
8.	3.4	13.8	0.365
9.	3.1	10.4	0.186
10.	3.2	7.3	0.090
11.	2.9	4.1	0.015
12.	1.2	1.2	0.023

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 20	
1.	3.4	71.6	3.86
2.	3.1	68.2	3.52
3.	4.8	65.1	3.28
4.	4.2	60.3	2.97
5.	3.8	56.1	2.73
6.	4.5	52.3	2.39
7.	3.9	47.8	2.05
8.	4.4	43.9	1.71
9.	3.6	39.5	1.42
10.	5.6	35.9	1.14
11.	4.9	30.3	0.706
12.	5.5	25.4	0.473
13.	6.4	19.9	0.273
14.	6.9	13.5	0.138
15.	6.6	6.6	0.035
		Run number 21	
1.	6.7	127.5	3.58
2.	4.2	120.8	3.39
3.	6.5	116.6	3.11
4.	6.1	110.1	2.85
5.	9.0	104.0	2.41
6.	9.7	95.0	2.08
7.	3.6	85.3	1.96
8.	4.3	81.7	1.65
9.	4.7	77.4	1.48
10.	3.5	72.7	1.30
11.	4.4	69.2	1.12
12.	7.6	64.8	0.980
13.	8.8	57.2	0.731
14.	9.9	48.4	0.495
15.	9.2	38.5	0.266
16.	6.3	29.3	0.176
17.	8.0	23.0	0.109
18.	11.6	15.0	0.046
19.	3.4	3.4	0.018

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 22	
1.	2.6	40.0	3.46
2.	4.3	37.4	2.98
3.	3.0	33.1	2.74
4.	3.4	30.1	2.43
5.	3.5	26.7	2.05
6.	2.8	23.2	1.67
7.	2.4	20.4	1.38
8.	4.4	18.0	1.06
9.	3.0	13.6	0.647
10.	3.9	10.6	0.363
11.	4.8	6.7	0.148
12.	1.9	1.9	0.011
		Run number 23	
1.	4.1	71.9	3.04
2.	3.2	67.8	3.74
3.	3.7	64.6	2.62
4.	3.2	60.9	2.42
5.	3.8	57.7	2.25
6.	3.3	53.9	2.06
7.	5.5	50.6	1.82
8.	4.5	45.1	1.51
9.	4.3	40.6	1.26
10.	4.7	36.3	1.02
11.	4.6	31.6	0.761
12.	5.9	27.0	0.477
13.	10.5	21.1	0.259
14.	10.6	10.6	0.104
		Run number 24	
1.	2.8	38.3	3.64
2.	2.5	35.5	3.43
3.	2.7	33.0	3.30
4.	2.4	30.3	2.59
5.	3.2	27.9	2.24
6.	3.0	24.7	1.82
7.	3.4	21.7	1.43
8.	2.9	18.3	1.00
9.	2.8	15.4	0.648
10.	3.1	12.6	0.380
11.	4.1	9.5	0.205
12.	4.0	5.4	0.072
13.	1.4	1.4	0.006

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 25	
1.	2.3	84.5	3.42
2.	2.8	82.2	3.32
3.	2.8	79.4	2.87
4.	3.1	76.6	2.66
5.	3.4	73.5	2.45
6.	4.0	70.1	2.23
7.	4.6	66.1	1.99
8.	3.1	61.5	1.71
9.	6.2	58.4	1.39
10.	4.1	52.2	1.13
11.	4.4	48.1	0.921
12.	4.8	43.7	0.632
13.	6.2	38.9	0.491
14.	7.5	32.7	0.287
15.	7.1	25.2	0.157
16.	9.1	18.1	0.067
17.	9.0	9.0	0.012
		Run number 26	
1.	0.6	107.5	4.04
2.	6.4	106.9	3.34
3.	6.0	100.5	3.33
4.	5.4	94.5	2.94
5.	5.6	89.1	2.67
6.	6.8	83.5	2.43
7.	7.0	76.7	2.08
8.	6.0	69.7	1.68
9.	4.7	63.7	1.40
10.	4.1	59.0	1.17
11.	7.0	54.9	0.929
12.	6.5	47.9	0.538
13.	7.2	41.4	0.393
14.	6.5	34.2	0.222
15.	8.4	27.7	0.140
16.	6.1	19.3	0.110
17.	9.6	13.2	0.040
18.	3.6	3.6	0.028

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 27	
1.	0.5	41.0	4.01
2.	1.6	40.5	3.83
3.	2.8	38.9	3.60
4.	3.8	36.1	3.20
5.	6.7	32.3	2.83
6.	4.9	25.6	1.74
7.	5.8	20.7	1.08
8.	6.1	14.9	0.227
9.	4.6	8.8	0.062
10.	4.2	4.2	0.005
		Run number 28	
1.	9.5	187.3	3.60
2.	8.4	177.8	3.43
3.	7.8	169.4	3.12
4.	9.9	161.6	2.75
5.	8.0	151.7	2.58
6.	9.6	143.7	2.32
7.	7.5	134.1	2.02
8.	6.8	126.6	1.78
9.	9.6	119.8	1.51
10.	8.5	110.2	1.24
11.	11.7	101.7	0.945
12.	12.1	90.0	0.654
13.	10.9	77.9	0.398
14.	9.8	67.0	0.245
15.	9.0	57.2	0.158
16.	9.6	48.2	0.092
17.	9.3	38.6	0.060
18.	9.7	29.3	0.022
19.	9.8	19.6	0.009
20.	9.8	9.8	0.000

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 29	
1.	4.6	237.2	3.96
2.	13.7	232.6	3.00
3.	10.8	218.9	2.88
4.	12.1	208.1	2.61
5.	11.5	196.0	2.28
6.	9.9	184.5	2.08
7.	13.3	174.6	1.68
8.	14.5	161.3	1.17
9.	15.6	146.8	1.06
10.	11.4	131.2	0.744
11.	14.9	119.8	0.534
12.	10.9	104.9	0.365
13.	12.6	94.0	0.228
14.	14.0	81.4	0.149
15.	14.9	67.4	0.094
16.	14.1	52.5	0.044
17.	15.5	38.4	0.028
18.	13.4	22.9	0.005
19.	9.5	9.5	0.000
		Run number 30	
1.	14.8	226.0	3.83
2.	15.1	211.2	3.33
3.	14.0	196.1	2.90
4.	17.4	182.1	2.43
5.	18.6	164.7	1.98
6.	17.3	146.1	1.49
7.	16.4	128.8	0.993
8.	16.7	112.4	0.645
9.	16.0	95.7	0.417
10.	18.7	79.7	0.247
11.	19.9	61.0	0.110
12.	18.7	41.1	0.038
13.	9.3	22.4	0.013
14.	13.1	13.1	0.010

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 31	
1.	2.7	26.8	3.02
2.	4.1	24.1	2.69
3.	3.5	20.0	1.89
4.	3.0	16.5	1.23
5.	2.7	13.5	0.748
6.	2.5	10.8	0.407
7.	2.7	8.3	0.195
8.	2.9	5.6	0.086
9.	2.7	2.7	0.029
		Run number 32	
1.	3.3	183.7	3.50
2.	9.8	180.4	2.64
3.	11.4	170.6	2.03
4.	17.3	159.2	1.59
5.	14.0	141.9	1.29
6.	11.4	127.9	1.02
7.	10.7	116.5	0.970
8.	11.8	105.8	0.706
9.	10.4	94.0	0.546
10.	8.6	83.6	0.455
11.	8.6	75.0	0.323
12.	7.8	66.4	0.255
13.	11.4	58.6	0.192
14.	10.6	47.2	0.118
15.	8.9	36.3	0.073
16.	7.5	27.7	0.065
17.	9.2	20.2	0.035
18.	11.0	11.0	0.028

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 33	
1.	5.6	187.2	3.19
2.	14.7	181.6	2.95
3.	14.9	166.9	2.54
4.	12.6	152.0	2.25
5.	16.0	139.4	1.93
6.	15.0	123.4	1.54
7.	11.6	108.4	1.21
8.	10.2	96.8	0.944
9.	11.2	86.6	0.741
10.	16.2	75.4	0.535
11.	11.2	59.2	0.295
12.	14.8	48.0	0.212
13.	10.7	33.2	0.087
14.	9.0	22.5	0.036
15.	7.3	13.5	0.025
16.	6.2	6.2	0.012
		Run number 34	
1.	14.5	405.7	3.82
2.	21.3	391.2	3.58
3.	19.3	369.9	2.68
4.	16.8	350.6	2.30
5.	23.0	333.8	2.01
6.	20.5	310.8	1.78
7.	22.7	290.3	1.51
8.	24.2	267.6	1.15
9.	23.2	243.4	0.834
10.	21.7	220.2	0.606
11.	21.1	198.5	0.464
12.	20.9	177.4	0.327
13.	21.6	156.5	0.208
14.	19.1	134.9	0.127
15.	21.0	115.8	0.086
16.	20.1	94.8	0.048
17.	20.5	74.7	0.035
18.	14.8	54.2	0.011
19.	19.3	39.4	0.007
20.	14.3	20.1	0.003
21.	5.8	5.8	0.000

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 37	
1.	12.9	461.3	1.25
2.	22.4	448.4	0.778
3.	21.8	426.0	0.653
4.	23.6	404.2	0.570
5.	23.6	380.6	0.410
6.	22.8	357.0	0.357
7.	20.3	334.2	0.310
8.	23.1	313.9	0.300
9.	21.5	290.8	0.215
10.	22.5	269.3	0.185
11.	16.8	246.8	0.165
12.	26.5	230.0	0.157
13.	21.6	203.5	0.135
14.	19.4	181.9	0.120
15.	22.3	162.5	0.115
16.	27.4	140.2	0.080
17.	23.4	112.8	0.052
18.	23.4	89.4	0.037
19.	24.7	66.0	0.008
20.	24.6	41.3	0.001
21.	16.7	16.7	0.000
		Run number 38	
1.	6.7	62.8	0.638
2.	6.4	56.1	0.535
3.	5.9	49.7	0.404
4.	8.3	43.8	0.276
5.	9.0	35.5	0.159
6.	6.5	26.5	0.083
7.	7.4	20.0	0.042
8.	6.2	12.6	0.015
9.	6.4	6.4	0.012

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 39	
1.	6.1	249.4	3.65
2.	12.2	243.3	2.97
3.	11.4	231.1	2.59
4.	13.7	219.7	2.19
5.	15.6	206.0	1.97
6.	14.8	190.4	1.64
7.	21.1	175.6	1.33
8.	16.1	154.5	0.947
9.	16.9	138.4	0.703
10.	19.5	121.5	0.468
11.	18.1	102.0	0.263
12.	18.7	83.9	0.138
13.	17.3	65.2	0.075
14.	12.1	47.9	0.038
15.	23.3	35.8	0.020
16.	12.5	12.5	0.000
		Run number 40	
1.	16.6	253.5	2.68
2.	17.8	236.9	1.92
3.	19.8	219.1	1.58
4.	18.4	199.3	1.27
5.	21.3	180.9	1.02
6.	21.5	159.6	0.864
7.	22.9	138.1	0.721
8.	21.3	115.2	0.467
9.	20.5	93.9	0.281
10.	22.7	73.4	0.153
11.	23.0	50.7	0.124
12.	18.6	27.7	0.061
13.	9.1	9.1	0.042

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 42	
1.	1.7	119.4	2.17
2.	9.9	117.7	2.07
3.	11.6	107.8	1.73
4.	13.8	96.2	1.48
5.	9.7	82.4	1.14
6.	17.2	72.7	0.885
7.	20.9	55.5	0.359
8.	14.1	34.6	0.148
9.	8.4	20.5	0.059
10.	5.9	12.1	0.023
11.	6.2	6.2	0.007
		Run number 43	
1.	1.3	118.6	2.02
2.	9.9	117.3	1.80
3.	11.5	107.4	1.59
4.	13.9	95.9	1.17
5.	9.5	82.0	1.02
6.	17.1	72.5	0.795
7.	20.8	55.4	0.371
8.	14.2	34.6	0.163
9.	8.3	20.4	0.057
10.	5.9	12.1	0.022
11.	6.2	6.2	0.008
		Run number 44	
1.	7.1	84.2	2.79
2.	12.1	77.1	2.51
3.	11.9	65.0	1.74
4.	7.0	53.1	1.23
5.	9.9	46.1	0.933
6.	20.0	36.2	0.521
7.	9.9	16.2	0.061
8.	6.3	6.3	0.037

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 46	
1.	12.6	139.1	0.522
2.	14.1	126.5	0.405
3.	16.2	112.4	0.356
4.	15.3	96.2	0.312
5.	13.1	80.9	0.248
6.	15.7	67.8	0.191
7.	13.5	52.1	0.097
8.	10.7	38.6	0.069
9.	12.1	27.9	0.040
10.	13.2	15.8	0.022
11.	2.6	2.6	0.000
		Run number 47	
1.	4.0	165.8	2.00
2.	10.0	161.8	1.20
3.	13.0	151.8	1.02
4.	8.6	138.8	0.893
5.	10.6	130.2	0.827
6.	9.2	119.6	0.734
7.	16.0	110.4	0.652
8.	12.9	94.4	0.571
9.	15.4	81.5	0.440
10.	29.9	66.1	0.339
11.	21.9	36.2	0.099
12.	10.6	14.3	0.025
13.	3.7	3.7	0.010
		Run number 48	
1.	6.5	127.0	2.82
2.	14.0	120.5	2.23
3.	12.7	106.5	1.89
4.	12.1	93.8	1.55
5.	13.0	81.7	1.24
6.	15.9	68.7	0.858
7.	11.9	52.8	0.533
8.	11.6	40.9	0.352
9.	10.7	29.3	0.195
10.	9.2	18.6	0.098
11.	6.1	9.4	0.052
12.	3.3	3.3	0.000

TABLE V (Continued)

Sample number	Sample weight grams	Accumulative sample weight grams, W_x	Salt concentration grams salt per gram of brine, %, C_x
		Run number 49	
1.	1.3	147.3	2.83
2.	10.9	146.0	2.37
3.	12.0	135.1	2.00
4.	13.2	123.1	1.65
5.	15.4	109.9	1.22
6.	14.1	94.5	1.01
7.	16.7	80.4	0.945
8.	13.7	63.7	0.708
9.	11.9	50.0	0.493
10.	12.2	38.1	0.337
11.	10.6	25.9	0.132
12.	9.4	15.3	0.076
13.	5.9	5.9	0.019
		Run number 50	
1.	3.2	138.3	3.00
2.	9.4	135.1	2.42
3.	11.9	125.7	2.02
4.	9.4	113.8	1.50
5.	30.5	104.4	1.30
6.	11.5	73.9	0.667
7.	11.6	62.4	0.521
8.	11.8	50.8	0.493
9.	11.0	39.0	0.362
10.	9.4	28.0	0.265
11.	7.8	18.6	0.152
12.	8.0	10.8	0.103
13.	2.8	2.8	0.070

APPENDIX C

DERIVATION OF TOTAL SAMPLE WEIGHT EQUATIONS

The purpose of this section is to develop an equation that may be used to calculate the final total sample weight.

Assuming that the initial and final sphere radii are known as well as the experimental weight, the weight of a pure ice sample, W_{it}, having the same final radius as the experimental sample may be calculated. The difference between these two quantities

$$\Delta W = W_{it} - W_{xt}$$

was expressed in terms of the initial radius r_o, final radius r_{xt}, and the brine equilibrium temperature. The equation may now be expressed as

$$W_{jt} = W_{it} + \Delta W \left[r_o, r_{xt}, t_e \right]$$

Experimental values of r_o, r_{xt}, and t_e were inserted in the functional relationship for ΔW and the results were compared with the total experimental weight.

Derivation of the Equation

If the frozen sample were pure ice without air inclusions or entrapped liquid brine, the equation for the total weight of the frozen specimen would be

$$W_t = \left[\frac{4}{3}\pi\rho\right]\left[r_t^3 - r_o^3\right]\left[\frac{1}{1728}\right] \quad r \text{ in inches.} \tag{7}$$

in which ρ is the density of the frozen material, lbs/ft^3. For a density of 57.0 lbs/ft^3, Equation (7) reduces to

$$W_t = 0.1382(r_t^3 - r_o^3) \tag{8}$$

The results of runs involving pure water are shown in Table VI. The experimental radius and total sample weight, r_{xt} and W_{xt}, were taken directly from the test data, Appendix A. The calculated weight was obtained from:

$$W_{it} = 0.1382(r_{xt}^3 - r_o^3) \tag{9}$$

where W_{it} is the weight of a sample frozen from water having radius r_{xt}. The difference column was

$$W_{it} - W_{xt}$$

while the density, ρx, was calculated from

$$\rho x = (1728)\left(\frac{3}{4}\right)(W_{xt})\frac{1}{\pi(r_{xt}^3 - r_o^3)} \tag{10}$$

The equilibrium temperature may be calculated by means of the equation

$$t_e = 32 - 1.05(C_b) \tag{11}$$

or by use of Fig. 13 from the ASHRAE Guide (10) or the Handbook of Chemistry by Lange (11) where C_b is the percentage salt concentration, pounds of salt per pound of brine, of

TABLE VI

DENSITY AND SAMPLE WEIGHT DATA FOR PURE WATER RUNS

Run Number	Experimental		Calculated Weight	Diff.	Density	Equilibrium Temperature
	Radius	Weight				
1	1.400	0.2218	0.2410	+0.0192	52.4	32.00
45	1.250	0.2518	0.2526	+0.0008	56.8	32.00
36	1.950	0.8858	0.8863	+0.0005	56.9	32.00
4	1.475	0.3050	0.3052	+0.0002	56.9	32.00
35	1.675	0.5111	0.5112	+0.0001	57.0	32.00
2	1.600	0.4282	0.4278	-0.0004	57.1	32.00
41	1.325	0.2659	0.2631	-0.0028	57.6	32.00
3	1.550	0.3868	0.3764	-0.0104	58.6	32.00

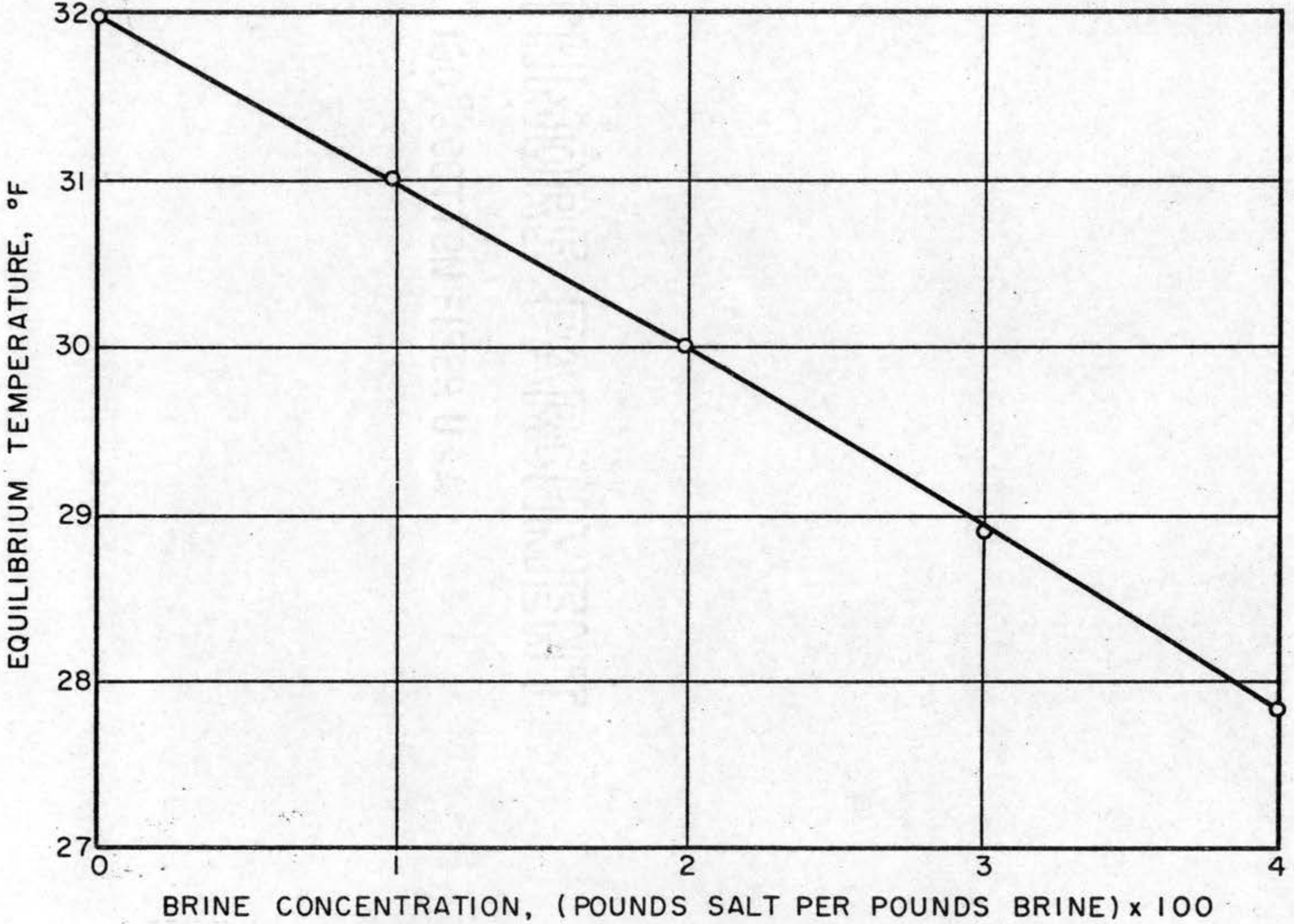

Figure 13. Equilibrium Curve for Sodium Chloride Solutions

the liquid.

The initial radius, r_o, is equal to 1.00 inches for runs 1 through 40, 0.75 inches for runs 41 through 44 and 0.50 inches for runs 45 through 50.

As shown by Table VI the difference between the experimental and calculated values of ice frozen were small. Run 1 had a positive error of 7.8% while run 3 had a negative error of 2.8%. The standard deviation of the weight difference column was 0.0050 lbs. All other runs were between these values. This was to be expected since the measurements were quite simple but the correlation did establish the accuracy of the radial measurements and sample collecting technique.

Table VII is similar to Table VI except the liquid in the test chamber was a saline solution. The table has been arranged in order of increasing density and several trends are noted. First, the samples having the lower final radius have the lower densities and secondly for a given radius an increase in sodium chloride concentration results in an increase in the difference between the calculated and experimental weight terms. The calculated weights were determined by means of Equation (9) for pure water.

Fig. 14 shows a plot of data resulting from tests conducted using a sphere of radius r_o = 1.00 inch and values of t_e between 27.4 and 28.2 °F. The linear relationship best fitting these points is given by

$$\Delta W = W_{it} - W_{xt} = -0.01104(r_{xt}^3 - r_o^3) + 0{,}02491 \qquad (12)$$

TABLE VII

DENSITY AND SAMPLE WEIGHT DATA

Run number	Experimental		Calculated weight	Diff.	Density	Equilibrium Temperature
	radius	weight				
31	1.150	0.0591	0.0720	+0.0129	46.8	28.77
16	1.150	0.0595	0.0720	+0.0125	47.2	28.49
24	1.200	0.0844	0.1006	+0.0162	47.8	28.28
18	1.175	0.0739	0.0860	+0.0121	48.9	28.12
19	1.200	0.0873	0.1006	+0.0133	49.5	28.26
22	1.200	0.0882	0.1006	+0.0124	50.0	28.43
27	1.200	0.0904	0.1006	+0.0102	51.2	27.82
25	1.350	0.1863	0.2018	+0.0155	52.6	28.54
38	1.275	0.1385	0.1482	+0.0097	53.2	31.34
11	1.250	0.1250	0.1317	+0.0062	54.1	29.96
17	1.300	0.1570	0.1654	+0.0084	54.1	27.90
44	1.225	0.1857	0.1957	+0.0100	54.1	28.62
12	1.200	0.0957	0.1006	+0.0049	54.2	30.00
20	1.300	0.1579	0.1654	+0.0075	54.4	28.08
23	1.300	0.1585	0.1654	+0.0069	54.6	28.30
6	1.500	0.3164	0.3282	+0.0118	55.0	31.43
10	1.250	0.1274	0.1317	+0.0043	55.2	29.94
48	1.300	0.2800	0.2863	+0.0063	55.8	28.68
14	1.300	0.1627	0.1654	+0.0027	56.1	29.82
26	1.400	0.2370	0.2410	+0.0040	56.1	27.92
9	1.250	0.1299	0.1317	+0.0018	56.2	31.28
21	1.450	0.2811	0.2831	+0.0020	56.6	28.62
43	1.325	0.2615	0.2631	+0.0016	56.6	29.92
5	1.300	0.1643	0.1654	+0.0011	56.6	31.38
13	1.300	0.1647	0.1654	+0.0007	56.8	29.84
7	1.450	0.2827	0.2831	+0.0004	56.9	31.36
42	1.325	0.2633	0.2631	-0.0002	57.0	31.42
8	1.275	0.1484	0.1482	-0.0002	57.1	31.36
50	1.325	0.3050	0.3041	-0.0009	57.1	27.95
15	1.425	0.2635	0.2617	-0.0018	57.4	28.43
37	2.025	1.0172	1.0092	-0.0080	57.4	31.26
49	1.350	0.3248	0.3227	-0.0021	57.4	27.96
47	1.400	0.3656	0.3619	-0.0037	57.6	30.08
29	1.675	0.5232	0.5112	-0.0121	58.3	28.46
28	1.575	0.4130	0.4017	-0.0113	58.6	28.23
33	1.575	0.4128	0.4017	-0.0111	58.6	28.70
30	1.650	0.4983	0.4825	-0.0158	58.8	27.79
40	1.700	0.5590	0.5406	-0.0184	58.9	29.49
39	1.700	0.5499	0.5407	-0.0092	59.5	29.21
34	1.925	0.8946	0.8474	-0.0472	60.2	28.28
46	1.325	0.3067	0.3041	-0.0026	62.8	31.42

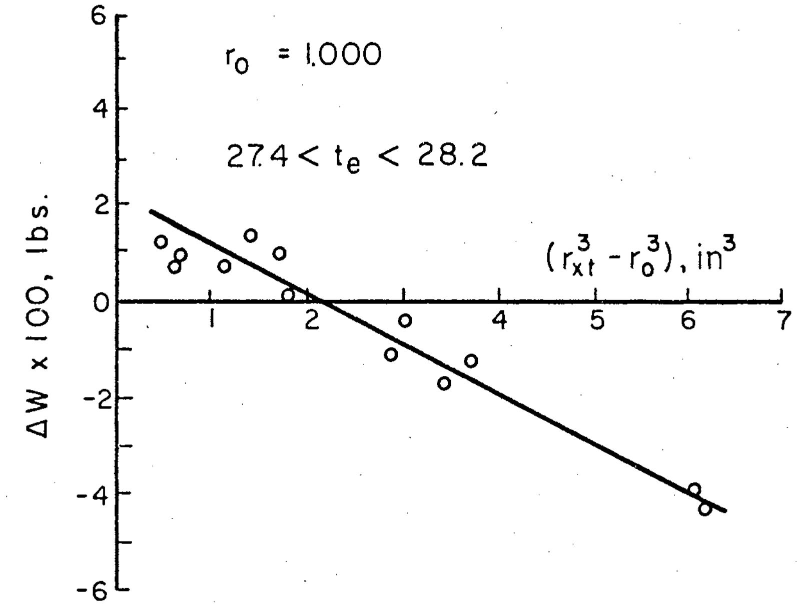

Fig. 14 Curve to Evaluate Median Point For Total Weight Equation

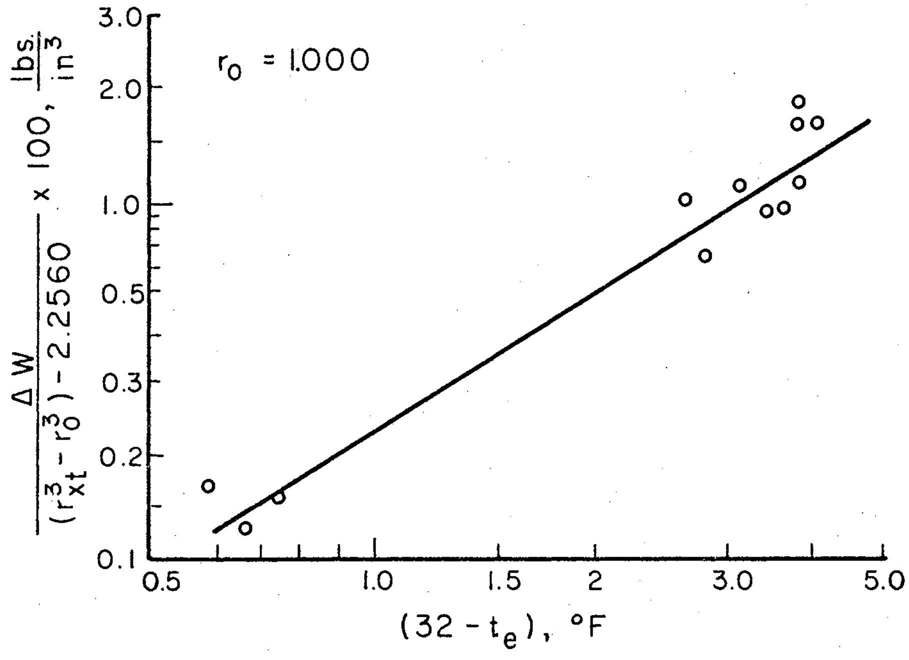

Fig. 15 Curve to Evaluate Concentration Exponent For Total Weight Equation

or

$$\Delta W = -0.01104\left[(r_{xt}^{3} - r_{o}^{3}) - 2.2560\right] \tag{13}$$

This established the point at which the difference between the calculated and experimental values change signs. Similar calculations for other solution concentrations give approximately the same value however the value 0.011043 was not constant but tended to increase with increasing concentrations or decreasing t_e.

Therefore

$$\Delta W = A_1\left[(r_{xt}^{3} - r_{o}^{3}) - 2.2560\right] f(t_e) \tag{14}$$

or

$$\frac{\Delta W}{(r_{xt}^{3} - r_{o}^{3}) - 2.2560} = A_1 f(t_e) \tag{15}$$

Fig. 15 indicates that $f(t_e)$ may be expressed as an exponential function; however, since the difference between W_{it} and W_{xt} is theoretically zero for pure water, $f(t_e)$ should be expressed as $(32 - t_e)^B$. Evaluating the slope of the line in Fig. 15, gives B = 1.21.

The equation for ΔW is now of the form

$$\Delta W = A_1\left[(r_{xt}^{3} - r_{o}^{3}) - 2.2560\right]\left[32 - t_e\right]^{1.21} \tag{16}$$

Next, the effect of varying r_o is to be ascertained, again assuming an exponential form

$$\frac{\Delta W}{\left[(r_{xt}^{3} - r_{o}^{3}) - 2.2560\right]\left[32 - t_e\right]^{1.21}} = A_2 (r_o)^{D} \tag{17}$$

Rather than using a graph to determine D, the data was grouped according to the value of r_o. The constant A_2 was eliminated for the three values of r_o and the average value of D was computed as - 1.086.

The value of the constant A_2 was obtained by evaluating

$$\Delta W = A_2\left[(r_{xt}^3 - r_o^3) - 2.2560\right]\left[32 - t_e\right]^{1.21}\left[r_o\right]^{-1.086} \quad (18)$$

for all of the runs and averaging the resultant A_2's which gave

$$A_2 = -0.002333$$

and

$$\Delta W = W_{it} - W_{xt} = -0.002333\left[(r_{xt}^3 - r_o^3) - 2.2560\right]\left[32 - t_e\right]^{1.21}\left[r_o\right]^{-1.086} \quad (19)$$

or

$$W_{xt} = W_{it} + 0.002333\left[(r_{xt}^3 - r_o^3) - 2.2560\right]\left[32 - t_e\right]\left[r_o\right]^{-1.086} \quad (20)$$

where

$$W_{it} = 0.1382(r_{xt}^3 - r_o^3) \quad (21)$$

Now let W_{jt} be the values obtained by inserting experimental values of r_{xt} and t_e into Equation (20), so that

$$W_{jt} = 0.1382(r_{xt}^3 - r_o^3) + 0.002333\left[(r_{xt}^3 - r_o^3) - 2.2560\right]\left[32 - t_e\right]^{1.21}\left[r_o\right]^{-1.086} \quad (22)$$

and evaluate the difference between W_{jt} and the experimental value W_{xt}. The differences were squared and then summed to determine the magnitude of the error. By trial and error the squared error is considerably reduced by using

$$W_{jt} = 0.1382(r_{xt}^3 - r_o^3) + 0.001667\left[(r_{xt}^3 - r_o^3) - 2.1500\right]\frac{(32-t_e)^{1.3}}{r_o} \qquad (23)$$

Table VIII, indicates the difference between the experimental values of the total weight and the calculated values. Table IX is the data applied to the velocity runs. The percentage error column was determined from

$$\left[\frac{W_{jt} - W_{xt}}{W_{jt}}\right] \times 100 \qquad (24)$$

Discussion of the Total Weight Equations

Equation (23), which applies to all the runs, was plotted in terms of the density, as determined from Equation (10), using W_{jt} as the sample weight, for various values of $(32 - t_e)$, in Fig. 16. To explain this curve, recourse must be made to visual observations of the manner in which the ice formed on the sphere. In the pure water runs the ice formation was practically transparent except for rapid growth runs in which radial air inclusions were present. For saline runs the formation was opaque and consisted of a radial needle-like formation extending from the surface of the sphere. As the run was allowed to proceed, the structure adjacent to the surface of the sphere became more compact

TABLE VIII

CALCULATED AND EXPERIMENTAL TOTAL WEIGHT DATA

Run number	Calculated weight, W_{jt}	Experimental weight, W_{xt}	Difference $W_{jt} - W_{xt}$	Per cent error
1	0.2410	0.2218	+0.0192	+7.69
2	0.4278	0.4282	−0.0004	−0.09
5	0.1645	0.1643	+0.0002	+0.12
6	0.3283	0.3164	+0.0119	+3.62
10	0.1266	0.1274	−0.0008	−0.63
15	0.2595	0.2635	−0.0040	−1.52
16	0.0581	0.0595	−0.0014	−2.41
17	0.1655	0.1570	+0.0085	+5.13
18	0.0712	0.0739	−0.0027	−3.79
19	0.0874	0.0873	+0.0001	+0.12
24	0.0875	0.0844	+0.0031	+3.54
25	0.1960	0.1863	+0.0097	+4.95
26	0.2368	0.2370	−0.0002	−0.01
27	0.0854	0.0904	−0.0050	−5.85
28	0.4088	0.4130	−0.0042	−1.32
29	0.5246	0.5232	+0.0014	+0.27
30	0.4970	0.4983	−0.0013	−0.26
31	0.0595	0.0591	+0.0004	+0.67
34	0.8841	0.8946	−0.0105	−1.18
35	0.5112	0.5111	+0.0001	−0.02
36	0.8864	0.8858	+0.0006	+0.07
37	1.0156	1.0172	−0.0016	−0.16
38	0.1470	0.1385	+0.0085	+5.78
39	0.5519	0.5499	+0.0020	+0.36
40	0.5504	0.5590	−0.0086	−1.56
41	0.2631	0.2659	−0.0028	−1.06
42	0.2628	0.2633	−0.0005	−0.19
43	0.2646	0.2615	+0.0031	+1.17
44	0.1877	0.1857	+0.0020	+1.06
45	0.2526	0.2518	+0.0008	+0.32
46	0.3052	0.3067	−0.0015	−0.49
47	0.3655	0.3656	−0.0001	−0.03
48	0.2851	0.2800	+0.0051	+1.82
49	0.3241	0.3248	−0.0007	−0.22
50	0.3051	0.3050	+0.0001	+0.03

TABLE IX

CALCULATED AND EXPERIMENTAL TOTAL WEIGHT DATA VELOCITY RUNS

Run number	Calculated weight, W_{jt}	Experimental weight, W_{xt}	Difference $W_{jt} - W_{xt}$	Per cent error
3	0.3763	0.3868	−0.0105	−2.76
4	0.3052	0.3050	+0.0002	+0.09
7	0.2831	0.2827	+0.0004	+0.14
8	0.1472	0.1484	−0.0012	−0.81
9	0.1304	0.1299	+0.0005	+0.38
11	0.1266	0.1250	+0.0016	+1.24
12	0.0948	0.0957	−0.0008	−0.84
13	0.1611	0.1647	−0.0036	−2.20
14	0.1610	0.1627	−0.0017	−1.09
20	0.1564	0.1579	−0.0015	−0.95
21	0.2822	0.2811	+0.0011	+0.39
22	0.0882	0.0882	0.0000	0.00
23	0.1569	0.1585	−0.0016	−1.03
32	0.4078	0.4051	+0.0026	+0.68
33	0.4076	0.4128	−0.0052	−1.27

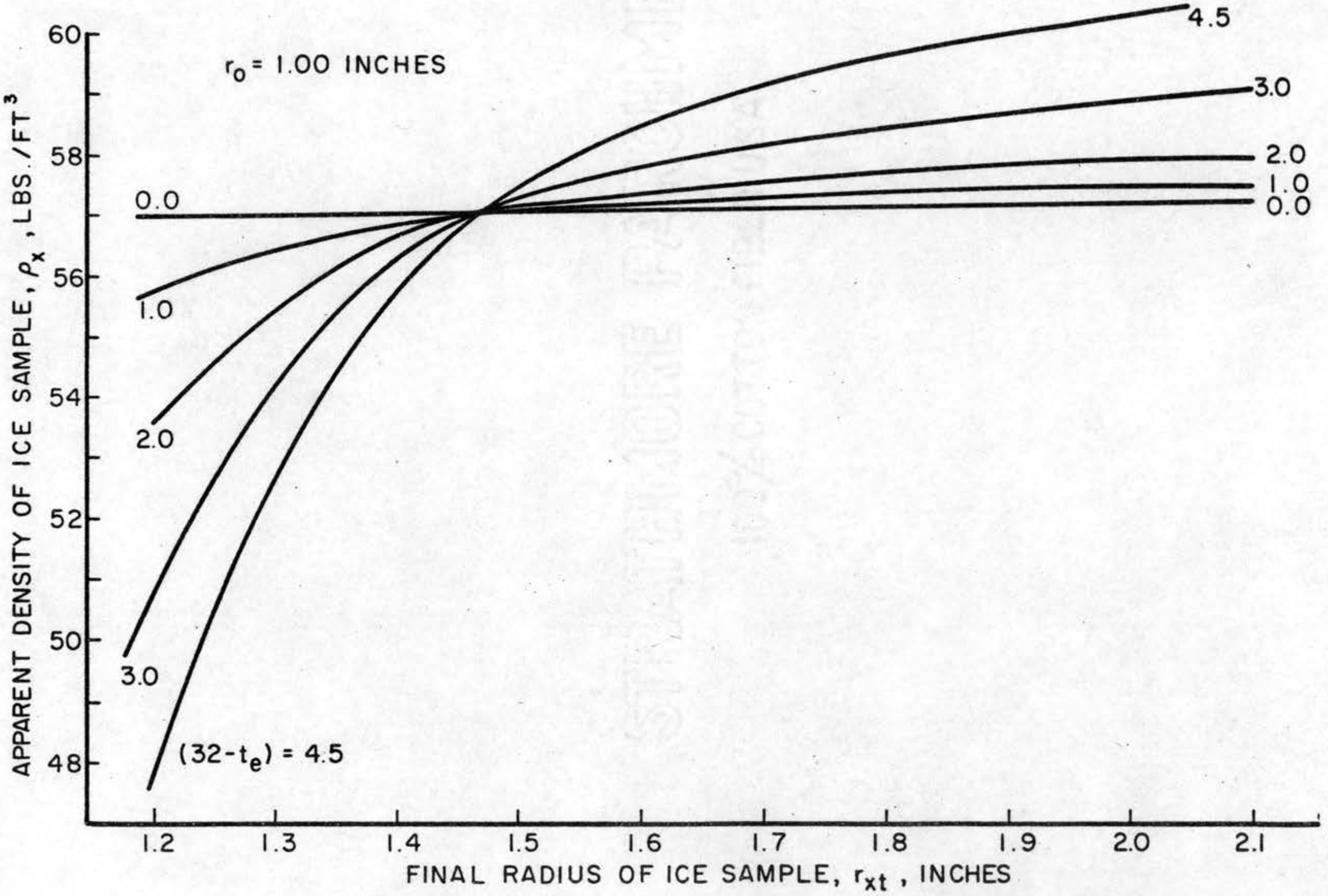

Figure 16. Density-Final Radius Curve From Total Weight Equation

but the outer surface always exhibited a needle-like appearance. For large amounts of radial growth the specimen therefore consisted of a solid core surrounded by a sponge like mass of needle growth. If the surrounding fluid temperature was sufficiently low, dendrite plates, having a random orientation, would appear at the surface and in some cases extended to the chamber wall. These formations were quite delicate, being fern like in appearance and very thin. They had no appreciable effect on the results of the experiment since drawing the fluid from the chamber invariably dislodged the plates. Therefore they did not influence the total weight calculations. Figures 5 and 6 are photographs of the ice samples formed in pure and saline water.

From the above visual description of the sample morphology, the increase in density may be explained by either one of two hypotheses:

1. The pores in the spherical section were interconnected and the brine was held in place by capillary forces.
2. The pores were not interconnected, and the sample must be melted in order to permit entrapped brine to escape.

In either case, large spherical sections will contain larger amounts of high density brine. Therefore, the density will increase with increasing radii.

As the density of the brine solution increases the density of the entrapped brine must also increase. Therefore, as the concentration increases, or as the equilibrium temperature decreases the apparent density of the spherical section must increase.

Decreasing the initial radius serves to magnify these effects so that small radial samples retain less brine and large samples retain proportionally more brine. For interconnected pores this is valid if the percentage of void spaces is proportionally larger when using a small starting radius. For pores which are not interconnected it means that starting with a small radii permits easier brine loss for small growth weight while larger samples retain more brine.

APPENDIX D

DEVIATION OF FINAL RADIUS EQUATIONS

The total weight equations permit the calculation of the final weight of entrapped brine and ice when the brine concentration, initial sphere size and final sphere radius are known. These equations, however, are of little value since no method has yet been presented to calculate the final radius of the sphere. Morphological aspects of this problem require that four separate cases be considered:

1. brine temperature equal to equilibrium temperature,
2. brine temperature greater than equilibrium temperature,
3. brine temperature less than equilibrium temperature, and
4. brine has velocity, V_{avg}, brine temperature less than or equal to equilibrium temperature.

Derivations of Equations to Evaluate Final Radius of Ice Sample When the Brine Temperature Equals the Equilibrium Temperature

The first of these in which the brine temperature and equilibrium temperature are equal may be analyzed by use of equations derived by London and Seban (12).

The solution of London and Seban is as follows:

The heat transferred by conduction from the inner surface of sphere to the outer ice surface is

$$q_1 = 4\pi k\left[\frac{t_e - t_o}{\frac{1}{r_o} - \frac{1}{r}}\right] \tag{25}$$

where

q_1 = Heat transfer by conduction, Btu/hr.

k = Thermal conductivity of frozen material, Btu/(fr ft°F),

t_o = Temperature of inside spherical surface °F.

t_e = Equilibrium temperature of surrounding liquid, °F.

Assuming zero heat capacity effects in the frozen section, all of the heat transferred through the frozen material must be used to form new ice. Therefore

$$q_2 = \rho L \frac{dV^1}{d\theta} \tag{26}$$

where

q_2 = Heat available to form new ice, Btu/hr

ρ = Density of frozen material, lbs/ft^3

V^1 = Volume, ft^3

L = Latent heat of fusion, Btu/lb.

Since

$$V^1 = \frac{4}{3}\pi r^3 \tag{27}$$

or

$$dV^1 = \left(4\pi r^2\right)dr \tag{28}$$

Equations (25) and (26) may be equated to yield

$$k\left[\frac{t_e - t_o}{\rho L}\right]d\theta = \left[\frac{r^2}{r_o} - r\right]dr \tag{29}$$

Applying the boundary conditions

$$\theta = 0 \qquad r = r_o$$

and

$$\theta = \theta \qquad r = r.$$

Equation (29) may be integrated to yield

$$K\left[\frac{t_e - t_o}{\rho L}\right]\theta = \frac{1}{3}\left[r^3 - r_o^3\right] - \frac{1}{2}\left[r^2 - r_o^2\right] \tag{30}$$

which may be simplified to

$$\frac{K}{\rho L r_o^2}\left[t_e - t_o\right]\theta = \frac{1}{3}\left[\left(\frac{r}{r_o}\right)^3 - 1\right] - \frac{1}{2}\left[\left(\frac{r}{r_o}\right)^2 - 1\right] \tag{31}$$

Introducing the dimensionless terms θ^* and r^*, where

$$\theta^* = \frac{K}{\rho L r_o^2}\left[t_e - t_o\right]\theta \tag{32}$$

and

$$r^* = \frac{r}{r_o} \tag{33}$$

gives

$$\theta^* = \frac{1}{3}\left[r^{*3} - 1\right] - \frac{1}{2}\left[r^{*2} - 1\right] \tag{34}$$

introducing the physical properties of pure ice

$$\theta^* = \frac{(1.34)(144)(t_e - t_o)\theta}{(57.0)(143.4)(r_o^2)}$$

$$= \frac{0.02361(t_e - t_o)\theta}{r_o^2} \qquad (r_o \text{ in inches}). \tag{35}$$

Table X presents the results obtained from applying the experimental data to Equations (34) and (35). These results are also plotted in Fig. 17. Equation (34) was modified to give a better data correlation. This resulted in a decrease in the calculated radius, r_{ct}, so that

$$\theta^* = 1.1143\left[\frac{1}{3}\left[r^{*3} - 1\right] - \frac{1}{2}\left[r^{*2} - 1\right]\right] \tag{36}$$

Table XI shows the results of this modification. The values of r_{ct} are used to estimate the total weight of the

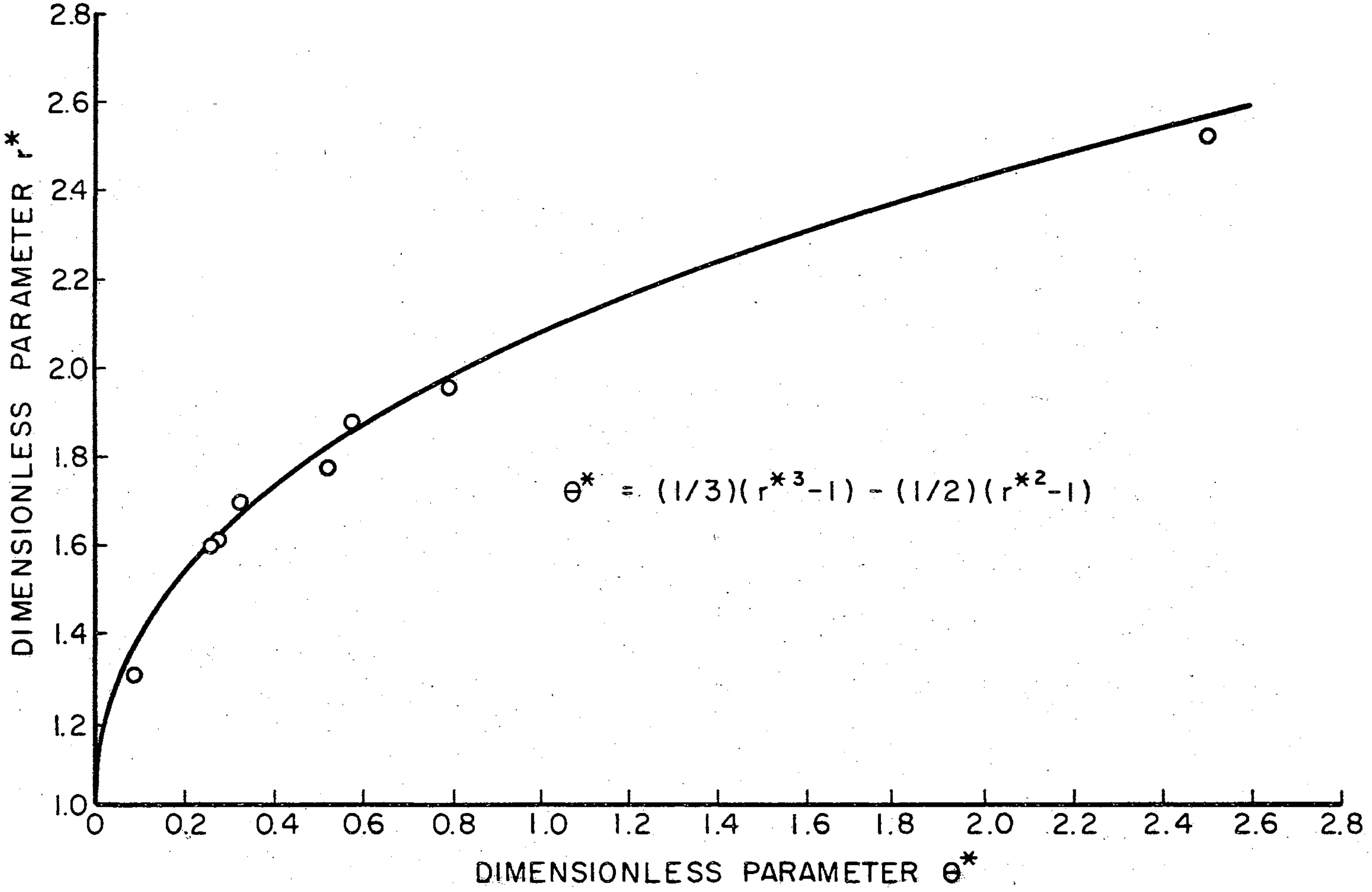

Figure 17. Equation for the Evaluation of the Ice Growth Radius When $t_e = t_b$

TABLE X

EXPERIMENTAL AND CALCULATED VALUES OF THE FINAL RADIUS FOR $t_b = t_e$ BY EQUATION 34

Run Number	t_e	t_o	θ	r_{xt}	r_{ct}	$r_{ct} - r_{xt}$	$\left(\frac{r_{ct}-r_{xt}}{r_{ct}}\right)100$
2	32.00	26.1	2.00	1.600	1.627	+0.027	+1.66
35	32.00	20.0	1.16	1.675	1.674	-0.001	-0.06
36	32.00	18.6	2.50	1.950	1.979	+0.029	+1.46
41	32.00	24.4	1.50	1.325	1.344	+0.019	+1.40
45	32.00	18.5	2.00	1.250	1.288	+0.038	+2.98
5	31.38	26.6	0.67	1.300	1.351	+0.051	+3.77
28	28.23	21.4	1.67	1.575	1.617	+0.042	+2.60
42	31.42	23.0	1.50	1.325	1.371	+0.046	+3.23

TABLE XI

MODIFIED VALUES OF THE FINAL RADIUS FOR $t_b = t_e$ BY EQUATION 36

Run Number	t_e	t_o	θ	r_{xt}	r_{ct}	$r_{ct} - r_{xt}$	$\left(\frac{r_{ct}-r_{xt}}{r_{ct}}\right)100$
2	32.00	26.1	2.00	1.600	1.593	-0.007	-0.44
35	32.00	20.0	1.16	1.675	1.638	-0.037	-2.26
36	32.00	18.6	2.50	1.950	1.919	-0.041	-1.62
41	32.00	24.4	1.50	1.325	1.312	-0.013	-0.97
45	32.00	18.5	2.00	1.250	1.250	0.000	0.00
5	31.38	26.6	0.67	1.300	1.331	+0.031	+2.33
28	28.23	21.4	1.67	1.575	1.584	+0.009	+0.57
42	31.42	23.0	1.50	1.325	1.337	+0.012	+0.90

sample from

$$W_{ct} = 0.1382(r_{ct}^3 - r_o^3) - \frac{1}{600}\left[(r_{ct}^3 - r_o^3) - 2.1500\right]\left[32 - t_e\right]^{1.3}\left[\frac{1}{r_o}\right] \quad (37)$$

The results of these calculations are shown in Table XII.

The required steps to determine the total sample were therefore

1. Θ^* was calculated by substituting experimental values of t_e, t_o, r and Θ into Equation (35).
2. r* was calculated by means of Equation (36), or by use of Fig. 17.
3. r_{ct} was calculated from $r^* = r_{ct}/r_o$.
4. W_{ct} was calculated from Equation (37).

Discussion of Final Radius Equation for Equal Brine and Equilibrium Temperatures

The major approximation introduced in the preceding derivations results from neglecting the thermal capacity of the frozen solid. As stated by London and Seban and more specifically by Cochran (13) the omission of the capacity should not result in an error larger than five percent when water is the material to be frozen. Cochran's paper includes an error curve with L* as the parameter that establishes the degree of the error, Fig. 18. The smallest value of L* in this experiment would be 15 for a t_o of 12°F; therefore, the error is probably of the order of 3 to 5%.

From Table X the error was found to be well within these limits. It is noted that the saline solution runs,

TABLE XII

EXPERIMENTAL AND CALCULATED TOTAL SAMPLE WEIGHTS FOR $t_e = t_b$ BY EQUATION 37

Run Number	r_{ct}	W_{ct}	W_{xt}	$W_{ct} - W_{xt}$	$(\frac{W_{ct} - W_{xt}}{W_{ct}})100$
2	1.593	0.4203	0.4282	-0.0079	- 1.88
35	1.638	0.4691	0.5111	-0.0420	- 8.95
36	1.919	0.8382	0.8858	-0.0476	- 5.68
41	1.312	0.2537	0.2659	-0.0122	- 4.81
45	1.250	0.2525	0.2518	+0.0007	+ 0.27
5	1.331	0.1869	0.1643	+0.0226	+12.09
28	1.584	0.4187	0.4130	+0.0057	+ 1.36
42	1.337	0.2699	0.2633	+0.0066	+ 2.45

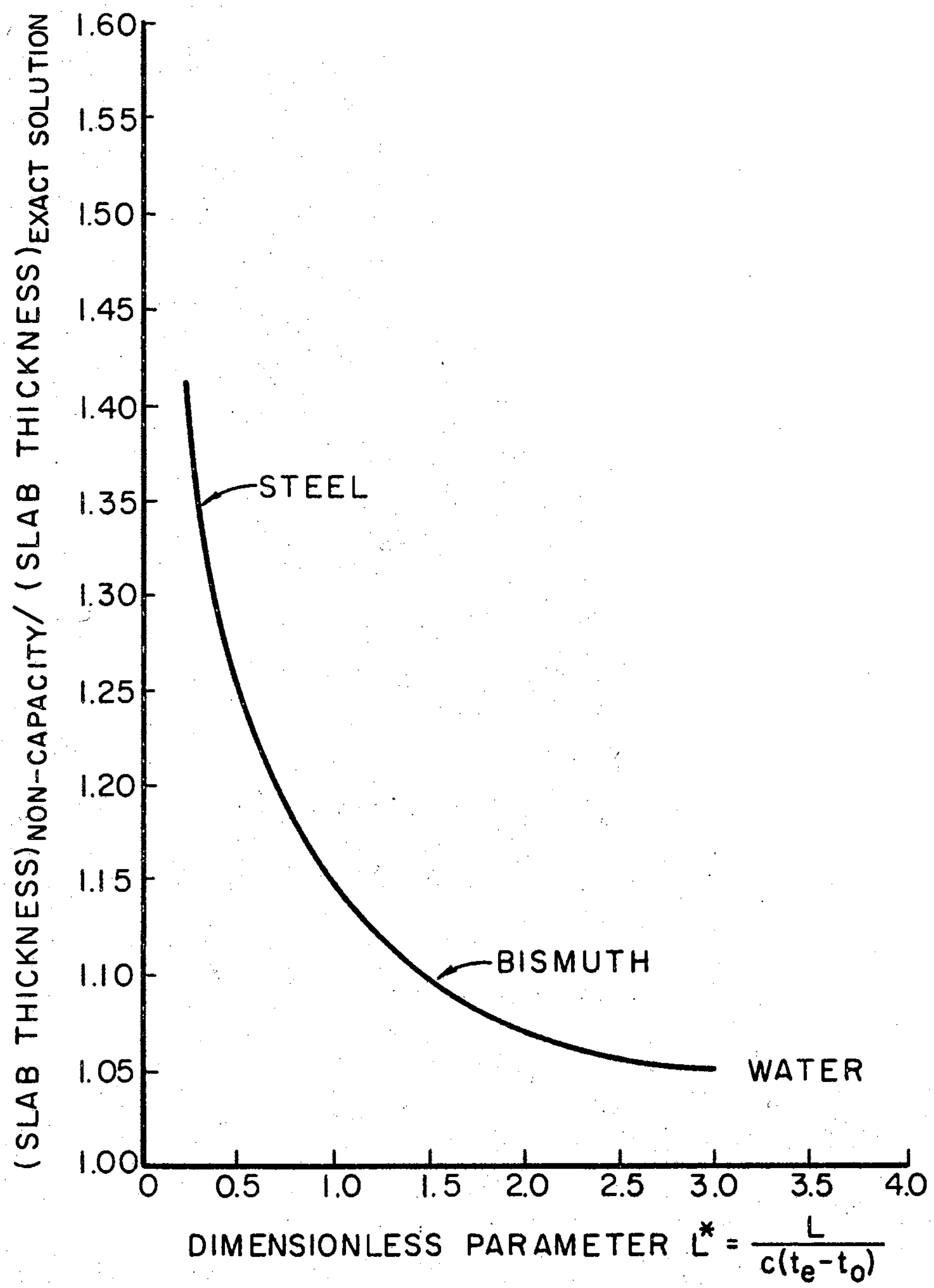

Figure 18. Slab Thickness Ratio Curve

numbers 5, 28, and 42 result in the largest errors, though these values are still less than four percent. Since the values for the physical properties used in Equation (35) were those of pure water, this represents an obvious source of errors. Finally, in saline solution runs the surface salt concentration of the sphere is not equivalent to the surrounding liquid salt concentration. Therefore the temperature at the ice interface was not equal to the temperature of the surrounding liquid. This was a basic assumption in the derivation of Equation (34).

The sign of the error was also correct since neglecting the heat capacity would result in a value of the final calculated radius larger than the experimental value.
The results of applying these equations were very satisfactory in terms of the resultant radial errors; however, relatively small errors in the radial values result in large errors in weight values which are calculated by Equation (37). For example, as shown in Table XII, run 35 has a - 2.26% radial error resulting in a - 8.95% weight error, while run 5 has a + 12.09% weight error. With the exception of these two runs the total weight error for the remaining six experiments was less than 6% in every case.

Derivation of Final Radius Equations for Brine Temperature Greater Than Equilibrium Temperature

When the brine temperature was higher than the equilibrium temperature, the sample grew to a finite size at which time steady-state conditions are attained. This means that the heat conducted through the sample was equivalent to the heat transferred at the surface, or mathematically stated:

$$q = 4\pi K\left[\frac{t_e - t_o}{\frac{1}{r_o} - \frac{1}{r_t}}\right] = hA(t_b - t_e) \tag{38}$$

where h is the convective coefficient, Btu/(hr ft^2 °F) and A the surface area of the spherical ice layer.

For the free convection problem the general expression relating the convection coefficient to the physical variables involved in the Nusselt, Grashof and Prandtl numbers is

$$Nu = Nu(Gr, Pr) \tag{39}$$

where

$$Nu = (\text{Nusselt number}) = \frac{hD}{K}$$

$$Gr = (\text{Grashof number}) = \frac{g\rho^2 D^3 (t_b - t_e)\beta}{\mu^2}$$

$$Pr = (\text{Prandtl number}) = \frac{c\mu}{K}$$

neglecting inertia effects Equation (39) may be reduced to

$$Nu = Nu(Gr\,Pr) \tag{40}$$

Assuming a logarithmic relationship for the functional relationship between the various groups, the following equation is obtained

$$\frac{hD}{K} = c_1\left[\left[\frac{g\rho^2 D^3 (t_b - t_e)\beta}{\mu^2}\right]\left[\frac{c\mu}{K}\right]\right]^{n_1} \tag{41}$$

Since the maximum temperature difference is only 1.5 °F, it is reasonable to assume that the physical properties remain constant. This is fortunate since the properties are not well established in this region.

Therefore, the equation for h may be reduced to

$$h = c_2 \left[\frac{1}{r_t}\right]\left[t_b - t_e\right]^{n_1} \left[r_t\right]^{3n_1} \tag{42}$$

Due to the needle-like surface present in the saline runs the surface area is considerably greater than $4\pi r^2$. Due to the irregular nature of the surface the following equation will be used to relate the radius and surface area

$$A = c_3 \left[r_t\right]^{n_2} \tag{43}$$

Therefore, Equation (38) may be expressed as

$$4\pi K \left[t_e - t_o\right]\left[\frac{r_o}{r_t - r_o}\right] = (c_2)\left(\frac{1}{r_t^2}\right)(t_b - t_e)^{n_1}(c_3 r_t^{n_2})(t_b - t_e)(r_t^{3n_1}) \tag{44}$$

or

$$(t_e - t_o)\left[\frac{r_o}{r_t - r_o}\right] = \left[\frac{c_2 c_3}{4\pi K}\right]\left[r_t\right]^{(3n_1 + n_2 - 2)}\left[t_b - t_e\right]^{(n_1 + 1)} \tag{45}$$

and letting

$$d = \frac{c_2 c_3}{4\pi K} \tag{46}$$

$$a = 3n_1 + n_2 - 2 \tag{47}$$

$$b = n_1 + 1 \tag{48}$$

So that

$$(t_e - t_o)\left[\frac{r_o}{r_t - r_o}\right] = d\, r_t^{a}(t_b - t_e)^{b} \tag{49}$$

The data was arranged with ascending values of $(t_e - t_o)\left(\frac{r_o}{r_t - r_o}\right)$. The data was then divided into three groups and the values of a, b, and d were determined such that

$$a = 2.45 \qquad b = 0.272 \qquad d = 3100$$

or

$$\frac{(t_e - t_o)}{(t_b - t_e)^{0.272}} = 3100 \left[\frac{r_t}{r_o} - 1\right] (r_t)^{2.45} \qquad (50)$$

Expressing r in inches and modifying the constants to obtain the best correlation with the experimental data gave

$$\frac{(t_e - t_o)}{(t_b - t_e)^{0.25}} = 7.11 \left[\frac{r_t}{r_o} - 1\right] (r_t)^{2.333} \qquad (51)$$

Figure 19 is plotted with the experimental points included, while Figure 20 presents the data in a convenient form for determining the radius r_t. Table XIII compares the calculated and experimental values of the final radius, while Table XIV compares the calculated and experimental total weights.

Discussion of Final Radius Equation for Brine Temperature Greater Than Equilibrium Temperature

The accuracy of the equation is typical of that found in non-flow freezing runs performed during this experiment. Since the variables involved are inherently stable in nature and easily measured, the results are reasonable. For this case the critical measurement was the value of $t_b - t_e$. However, the effect of this variable was greatly suppressed by the exponential power used in Equation (51). This is apparent since the value of $t_b - t_e$ varies from 0.1 to 1.7, or a factor of 17, while $(t_b - t_e)^{0.25}$ varies from 0.56 to 1.14, a factor of 2.

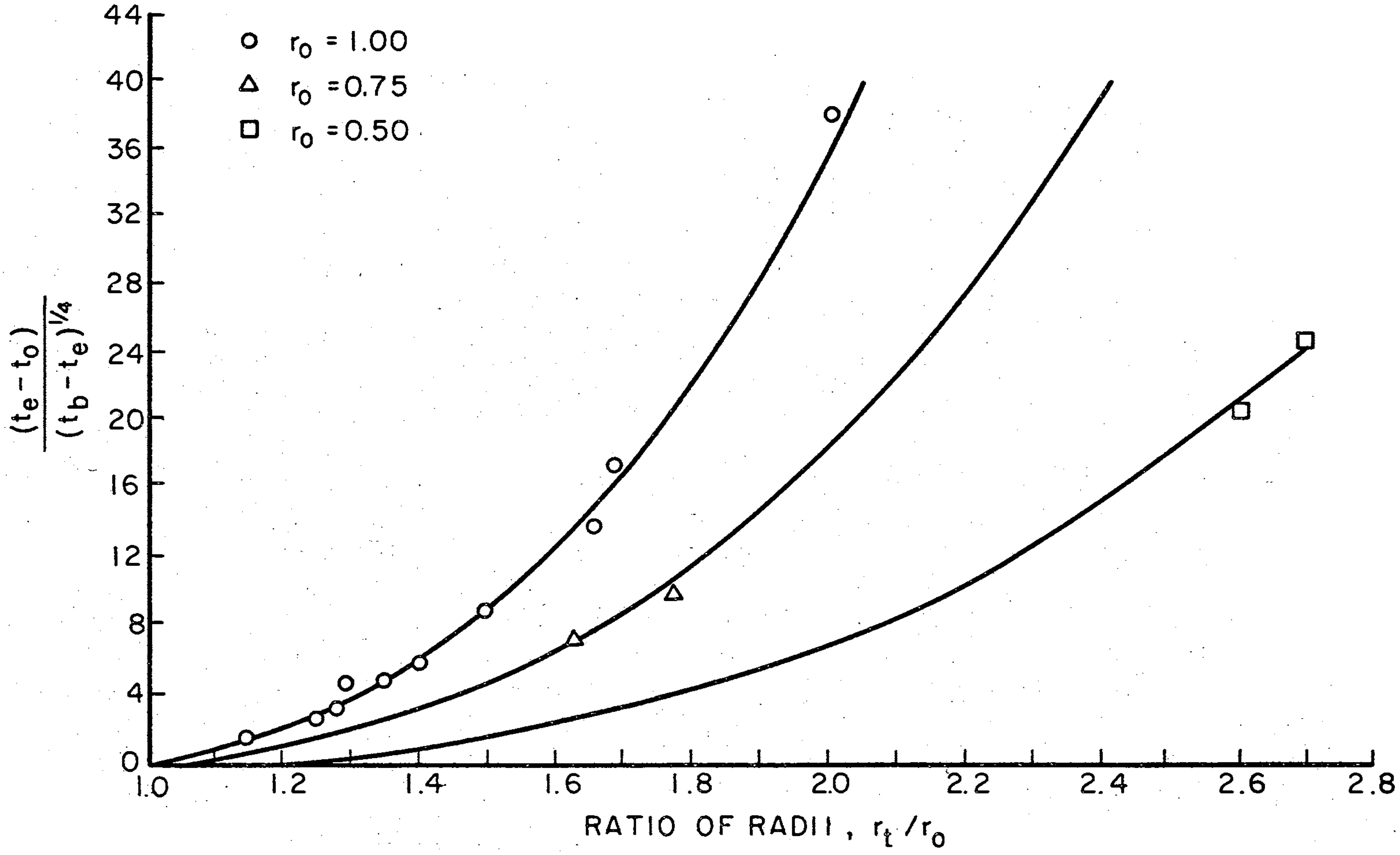

Figure 19. Ratio of Temperatures Versus Radii Ratio for $t_b > t_e$

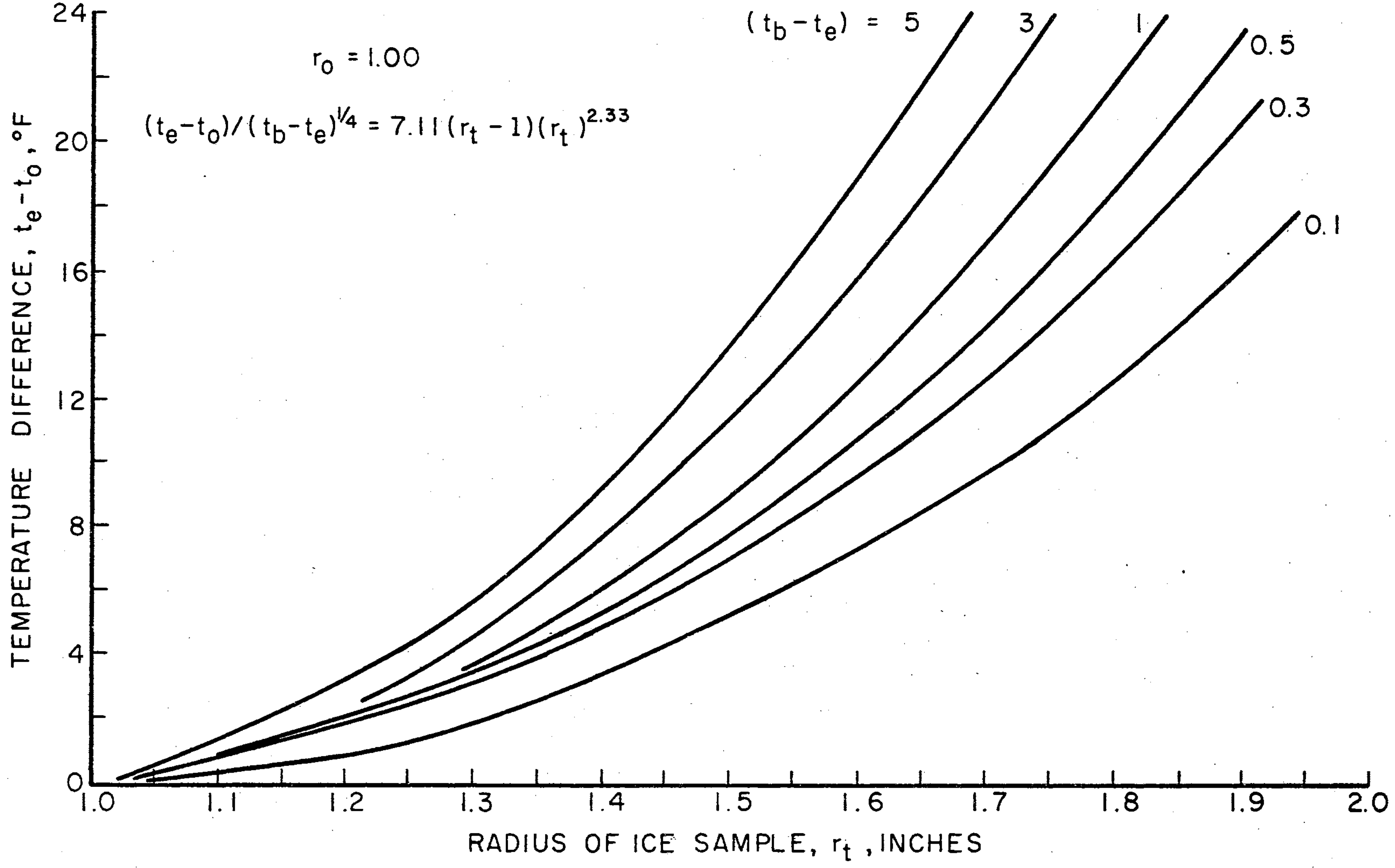

Figure 20. Ice Sample Radius for $t_b > t_e$

TABLE XIII

EXPERIMENTAL AND CALCULATED FINAL RADII VALUES OF THE FINAL RADIUS FOR $t_b > t_e$ BY EQUATION 51

Run Number	$t_e - t_o$	$t_b - t_e$	r_{xt}	r_{ct}	$\left[\frac{r_{ct} - r_{xt}}{r_{ct}}\right]100$
38	2.8	0.5	1.275	1.268	-0.58
25	3.9	0.4	1.350	1.368	+1.29
10	3.5	1.5	1.250	1.259	+0.69
16	2.3	3.0	1.150	1.169	+1.63
17	4.7	1.6	1.300	1.311	+0.86
30	10.7	0.3	1.650	1.636	-0.86
1	6.9	1.7	1.400	1.392	-0.57
6	8.9	0.9	1.500	1.499	-0.06
39	13.0	0.3	1.700	1.708	+0.46
37	21.2	0.1	2.025	2.023	-0.10
43	9.0	0.5	1.325	1.329	+0.30
44	6.8	0.7	1.225	1.231	+0.46
49	16.3	0.2	1.350	1.347	-0.23
48	15.2	0.3	1.300	1.295	-0.36

TABLE XIV

EXPERIMENTAL AND CALCULATED TOTAL SAMPLE WEIGHTS FOR $t_b > t_e$ BY EQUATION 37

Run Number	r_{ct}	W_{ct}	W_{xt}	$W_{ct} - W_{xt}$	$\left[\frac{W_{ct} - W_{xt}}{W_{ct}}\right]100$
38	1.268	0.1421	0.1385	+0.0036	+ 2.53
25	1.368	0.2102	0.1863	+0.0239	+11.37
10	1.259	0.1325	0.1274	+0.0051	+ 3.85
16	1.169	0.0689	0.0595	+0.0094	+13.64
17	1.311	0.1640	0.1570	+0.0070	+ 4.27
30	1.636	0.4800	0.4983	-0.0183	- 3.81
1	1.392	0.2346	0.2218	+0.0128	+ 5.46
6	1.499	0.3278	0.3164	+0.0114	+ 3.48
39	1.708	0.5618	0.5499	+0.0119	+ 2.12
37	2.023	1.0058	1.0172	-0.0114	- 1.13
43	1.329	0.2531	0.2615	-0.0084	- 3.32
44	1.231	0.1915	0.1857	+0.0058	+ 3.03
49	1.347	0.3189	0.3248	-0.0059	- 1.85
48	1.295	0.2814	0.2800	+0.0014	+ 0.50

By means of Equations (47) and (48) n_1 and n_2 were evaluated, and were found to be

$$n_1 = -0.75 \tag{52}$$

$$n_2 = 6.58 \tag{53}$$

For free convection calculations the value of n_1 is typically of the order + 0.25. As noted by Dunmore, Merk and Prins (14) and as shown in Fig.21, the inversion point of water causes a marked change in the shape of the Nusselt number, surrounding-temperature curve. From 3 °C to 5°C an increase in $t_b - t_e$ would cause a drop in the Nusselt number which would require a negative power since $t_b - t_e$ is greater than unity. The article by Dunmore serves to explain this difficulty and returns the expression to the form

$$Nu = 0.6(Pr\,Gr)^{\frac{1}{4}} \tag{54}$$

by correcting the thermal expansion coefficient, β. However, the range of values of $t_b - t_e$ encountered in this experiment were to the left of the inversion point which requires a positive exponent. Within the narrow range of values used in this study, it was impossible to explain the divergence of values. Further study of the convection mechanism and its stability near the inversion point would be required. As will be shown later, in an investigation devoted primarily to the desalination process the case of

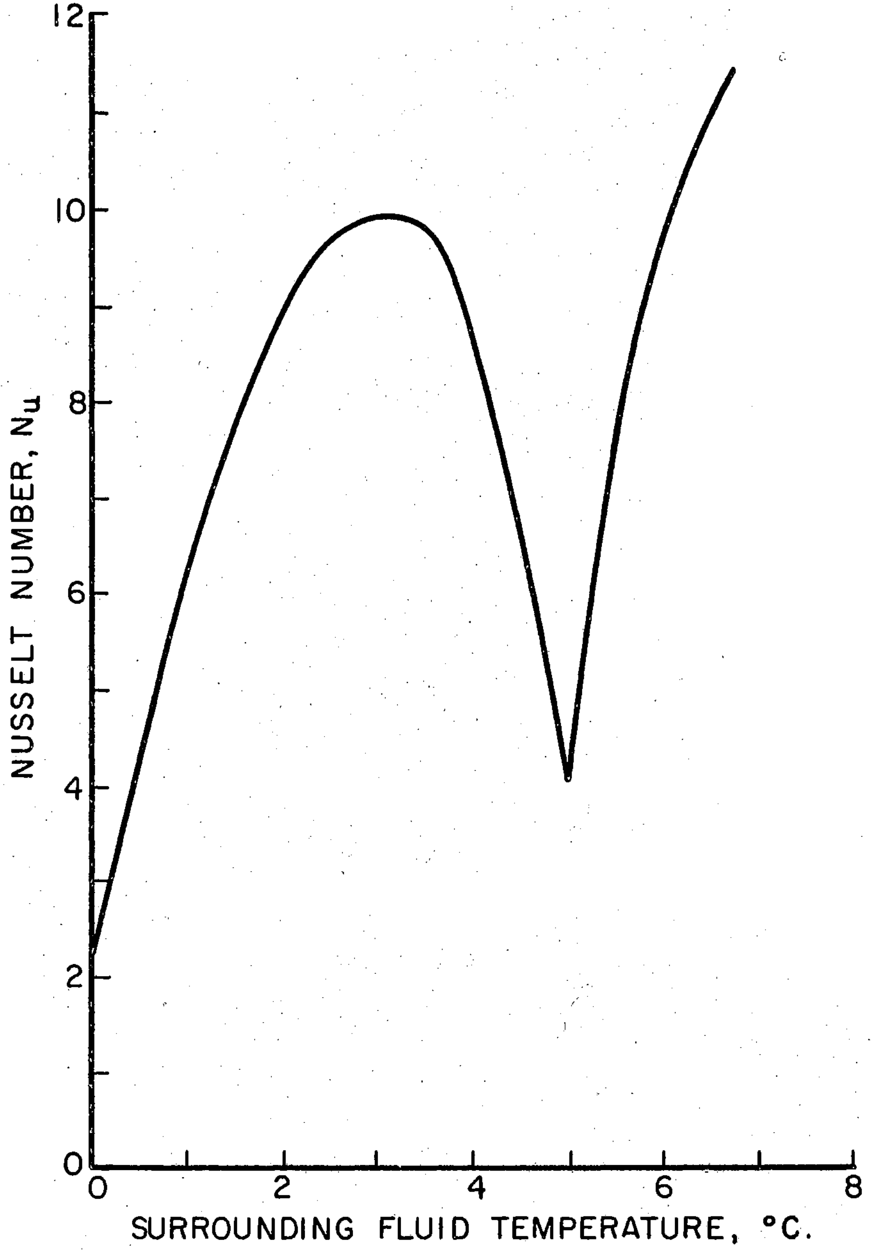

Figure 21. Convective Heat Transfer for Water-Ice

brine temperature greater than the equilibrium temperature was an inferior configuration both from an economic and salt separation standpoint.

The exponent, n_2, value of 6.58 rather than 2 is quite reasonable since the needle like surface has an exposed area considerably greater than that of a smooth surface.

The use of calculated values of the final radius to evaluate the total weight again illustrates the necessity of accurate temperature measurements. Runs 16 and 25 do not appear to have excessive radial measurement errors. However these values result in large calculated and experimental total weight differences.

Derivation of Final Radius Equation for Brine Temperature Less Than the Equilibrium Temperature

When the brine temperature is less than the equilibrium temperature the ice sample will grow indefinitely, and at a rate proportional to the degree of subcooling and to the inside temperature of the sphere. Frank (15) presents a solution to the problem of diffusionally controlled spherical growth starting from zero radius in a uniform medium. This solution, presented below, neglects convective effects and dendritic-type growth so that the times required to form a particular size sample are much greater than those obtained in the actual experiment. Since Frank's solution does not allow for these effects it can not be used to predict the results of this experimental work. However, this solution proved to be useful in selecting parameters for correlating data.

Essentially Frank's solution is a similarity transformation by means of a dimensionless reduced radius s where

$$s \equiv r D_t^{-\frac{1}{2}} \theta^{-\frac{1}{2}} \tag{55}$$

into the diffusion equation

$$\frac{\partial t}{\partial \theta} = D_t \left[\frac{\partial^2 t}{\partial r^2} + \frac{2}{r} \frac{\partial t}{\partial r} \right] \tag{56}$$

where

r is the radius, ft.

D_t is the thermal diffusivity, ft^2/hr

θ is the time, hours

t is the temperature, °F.

Taking the partial derivatives of s, the diffusion equation is reduced to

$$\frac{d^2 t}{d s^2} = -\left[\frac{s}{2} + \frac{2}{s} \right] \frac{dt}{ds} \tag{57}$$

and apply the boundary condition

$t = t_\infty$ when $s = \infty$

gives

$$t - t_\infty = A F(s) \tag{58}$$

where

$$F(s) = \left[s^{-1} e^{-\frac{s^2}{4}} - \frac{\sqrt{\pi}}{2} \left[1 - erf\left(\frac{s}{2}\right) \right] \right] \tag{59}$$

To evaluate the constant A the conditions at the growth surface are applied and for a sphere of radius R

$$R \equiv S D^{\frac{1}{2}} \theta^{1/2} \tag{60}$$

the surface conditions are

$$\begin{aligned} -4\pi R^2 \left(\frac{\partial t}{\partial r}\right)_{r=R} &= (-4\pi s^2)(D^{3/2})(\theta^{1/2})\left(\frac{\partial t}{\partial s}\right)_{s=S} \\ &= (4\pi A)(D^{3/2})(\theta^{1/2})\left[\exp\left(-\frac{S^2}{4}\right)\right] \end{aligned} \tag{61}$$

This term represents the quantity of heat crossing the spherical surface at radius R.

Equating the diffusing heat at the boundary of the growing phase to the amount of new material formed gives

$$(4\pi A)(D^{3/2})(\theta^{1/2})\left[\exp\left(-\frac{S^2}{4}\right)\right] = \frac{L}{c}\frac{dV^1}{dt} = \frac{L}{c}\frac{d}{dt}\left[\frac{4}{3}\pi R^3\right] \tag{62}$$

or

$$A = \frac{1}{2}\left(\frac{L}{c}\right)\left[\exp\left(\frac{S^2}{4}\right)\right] \tag{63}$$

Substituting the value of A into Equation (58) gives

$$t_s - t_\infty = \frac{L}{2c} S^2 \left[\exp\left(\frac{S^2}{4}\right)\right] \tag{64}$$

t_s = Surface temperature, °F

or

$$t_s - t_\infty = \frac{L}{c} f(S) \tag{65}$$

where

$$f(S) = {\frac{S}{2}}^{3}\left[\exp\left(\frac{S^2}{4}\right)\right] F(S) \tag{66}$$

The same derivation may be used for the diffusion of material across the boundary.

The equations to be solved simultaneously are

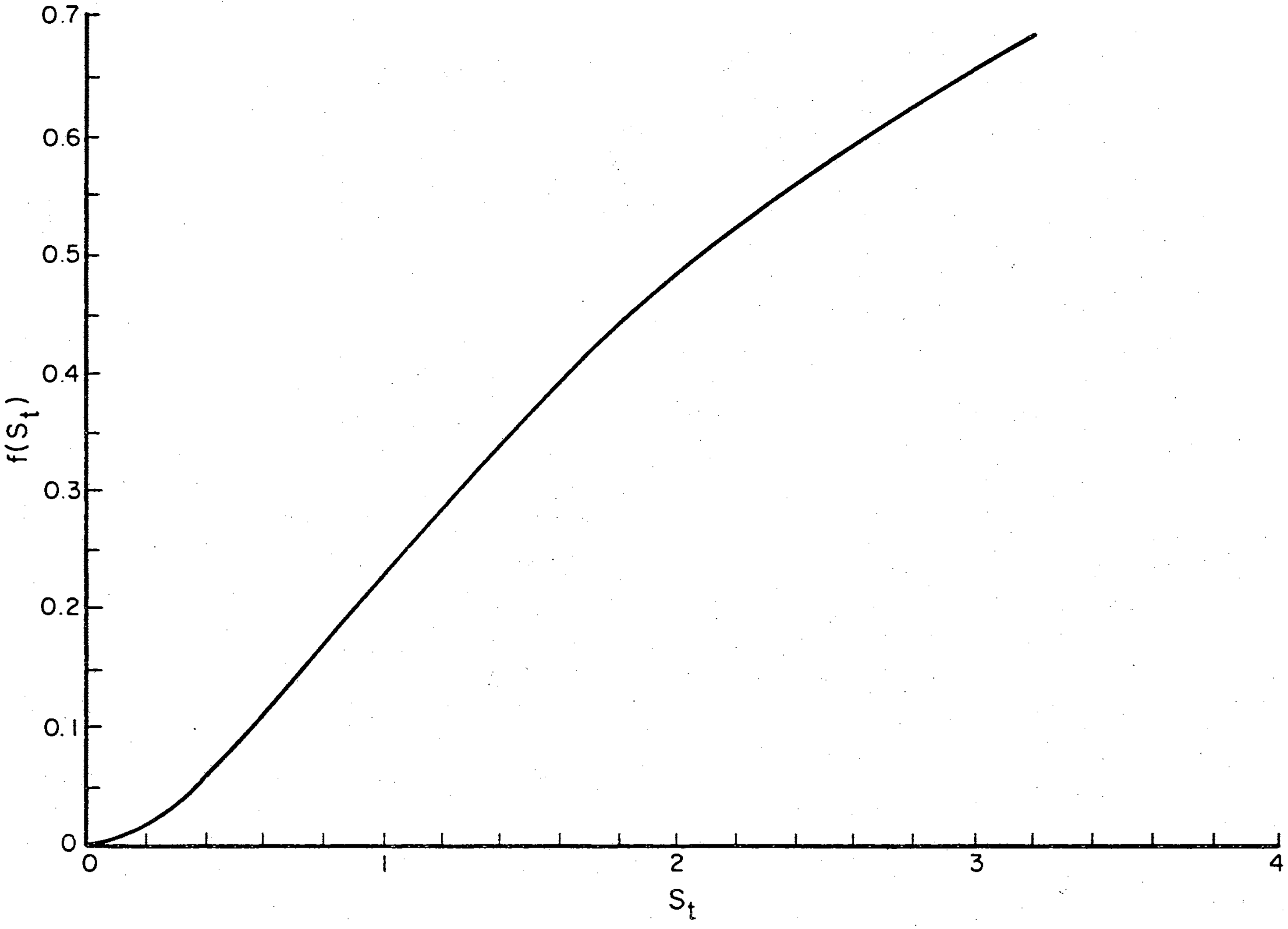

Figure 22. Curve for the Evaluation of $f(S_t)$ as Defined by Equation 66

$$t_s - t_\infty = \frac{L}{c} f(S_t) \tag{67}$$

$$C_s - C_\infty = C_s f(S_c) \tag{68}$$

$$t_s = t_m - K C_s \tag{69}$$

$$S_t D_t^{1/2} = S_c D_c^{1/2} \tag{70}$$

Equations (67) and (68) are the result of solving the thermal and material diffusion equation, where C is the concentration. Equation (69) is the depression of the melting point from its value t_m in a pure solute. Equation (70) is the result of assuming that the thermal diffusivity, D_t, and the material diffusivity, D_c, are independent constants and secondary coupling effects may be neglected. This equation was obtained by solving Equation (60) for $R\Theta^{\frac{1}{2}}$ and equating the heat and material diffusion coefficients.

Eliminating t_s and C_s from Equations (67), (68), (69) and (70) gave

$$f(S_t) = \frac{c}{L}(t_m - t_\infty) - \frac{K C_\infty}{1 - f(S_c)} \tag{71}$$

For sodium chloride solutions, from Equation (11),

$$t_s = t_m - 1.05 C_s \tag{72}$$

or K = 1.05 in the range of concentrations from zero to four percent.

Finally

$$D_t = \frac{k}{\rho c} = 5.30 \times 10^{-3}, \ \mathrm{ft}^2/\mathrm{hr}. \qquad (73)$$

and D_c, the material diffusivity, as given by Richardson (16) may be estimated as

$$D_c = 1.36 \times 10^{-5} \ \mathrm{cm}^2/\mathrm{sec} = 5.30 \times 10^{-5} \ \mathrm{ft}^2/\mathrm{hr}. \qquad (74)$$

and from Equation (70)

$$S_c = 10 S_t$$

Richardson's paper, presented in 1964, serves to illustrate the difficulty of obtaining material diffusion properties and the above estimate represents the best available value.

By substituting the values for sodium chloride into Equation (71) the following expression was obtained:

$$f(S_t) = \frac{1}{295}\left[(32 - t_\infty) - \frac{1.05\, C_\infty}{1 - f(10 S_t)}\right] \qquad (75)$$

To determine the magnitude of time required to freeze an ice sphere of a given radius for a purely diffusional process, let C = 4.0% and $t_\infty = 20$ °F. By means of Fig. 22, Equation (75) becomes

$$295 f(S_t) + \frac{4.20}{1 - f(10 S_t)} = 12 \qquad (76)$$

and solve for S_t, from which a value of 0.183 is obtained. Substituting this value of S_t into the following equation gave

$$R = S_t D_t^{1/2} \theta^{1/2} = 0.0133\, \theta^{1/2} \quad R \text{ in feet.} \qquad (77)$$

To grow a sphere having a radius of 2 inches requires 156 hours. However, assuming a starting radius of 1 inch the time required to grow a sample to a final radius of 2 inches was 116 hours. In the experimental runs the time required to grow a sample of this thickness was of the order of 2 hours. From the above analysis it is apparent that the problem was convectively controlled and the shorter times indicated that brine was entrapped.

As stated by Frank it is permissible to shift the time zero to account for initial growth but the above analysis is not valid when heat is being conducted across the ice formation. The potential for ice formation consists of two parts. These two parts were the degree of subcooling of the surrounding brine and the difference between the inside surface temperature of the sphere and the ice surface. The assumption of t_∞ equal to 20 °F was intended to approximate these conditions and to give a rough evaluation of the time required for freezing the 2 inch sphere. A strict analytical solution to this problem is not available for pure water or saline solution.

Even though the sphere growth was much too rapid for a diffusionally controlled process, it was possible that the inner core which produced the potable water was diffusionally controlled. For a two inch sample grown on a sphere of one inch radius the total sample weight was approximately one pound. As will be shown later, about twenty-five percent of a given sample is essentially pure water. If this pure water was generated by the hard inner core, it must have had a radius of 1.41 inches for an initial sphere radius of 1 inch. The above calculations indicate it would require 40 hours to form such a sample by diffusional means alone. Therefore, the growth of the inner core was not diffusionally controlled.

Visual observations also verified that the growth was not diffusional in nature, but the above analysis does indicate the parameters that should be used in correlating the experimental data. Since

$$R = S_t D^{1/2} \theta^{1/2} \tag{78}$$

the parameter $r/\theta^{\frac{1}{2}}$ should be included in the solution as well as $32 - t_b$. Also the inside sphere temperature must be included as a variable. Since t_b was related to the pure solute temperature in order to give one potential for ice formation, $32 - t_o$ was assumed to be another variable. Lastly, the concentration C_b should be included to give an equation of the form

$$\frac{r_t - r_o}{\theta^{1/2}} = f\left[(32-t_b),(32-t_o), C_b\right] \tag{79}$$

The group $(r_t - r_o)/\theta^{\frac{1}{2}}$ was plotted on cartesian and semi-log paper, Figures 23 and 24. These plots gave an excellent correlation of data when the groups $(32 - t_b)$ and $(32 - t_o)$ were added to form

$$\Delta t = (32-t_b) + (32-t_o) \tag{80}$$

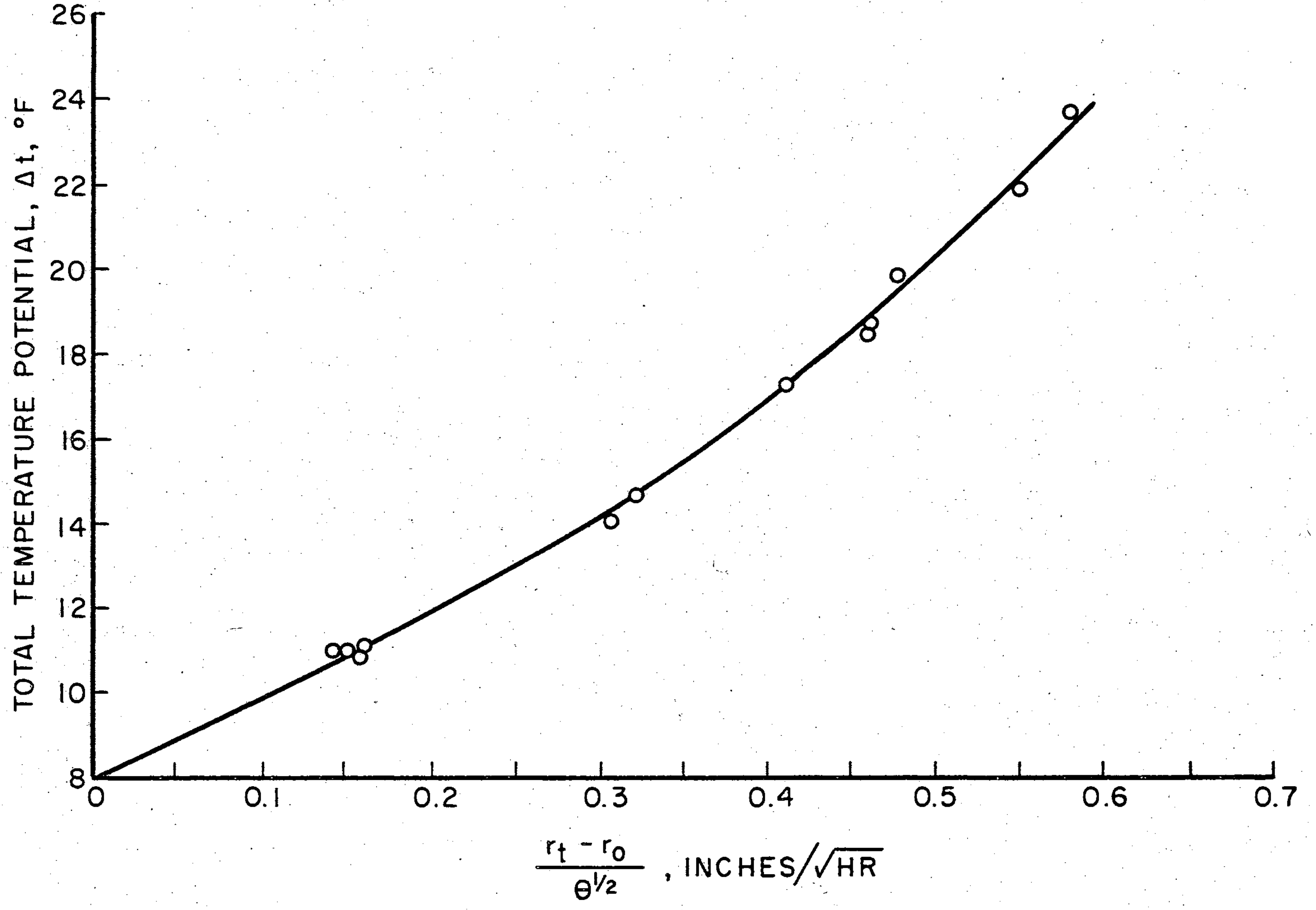

Figure 23. Total Radius Curve for $t_b < t_e$ by Equation 86

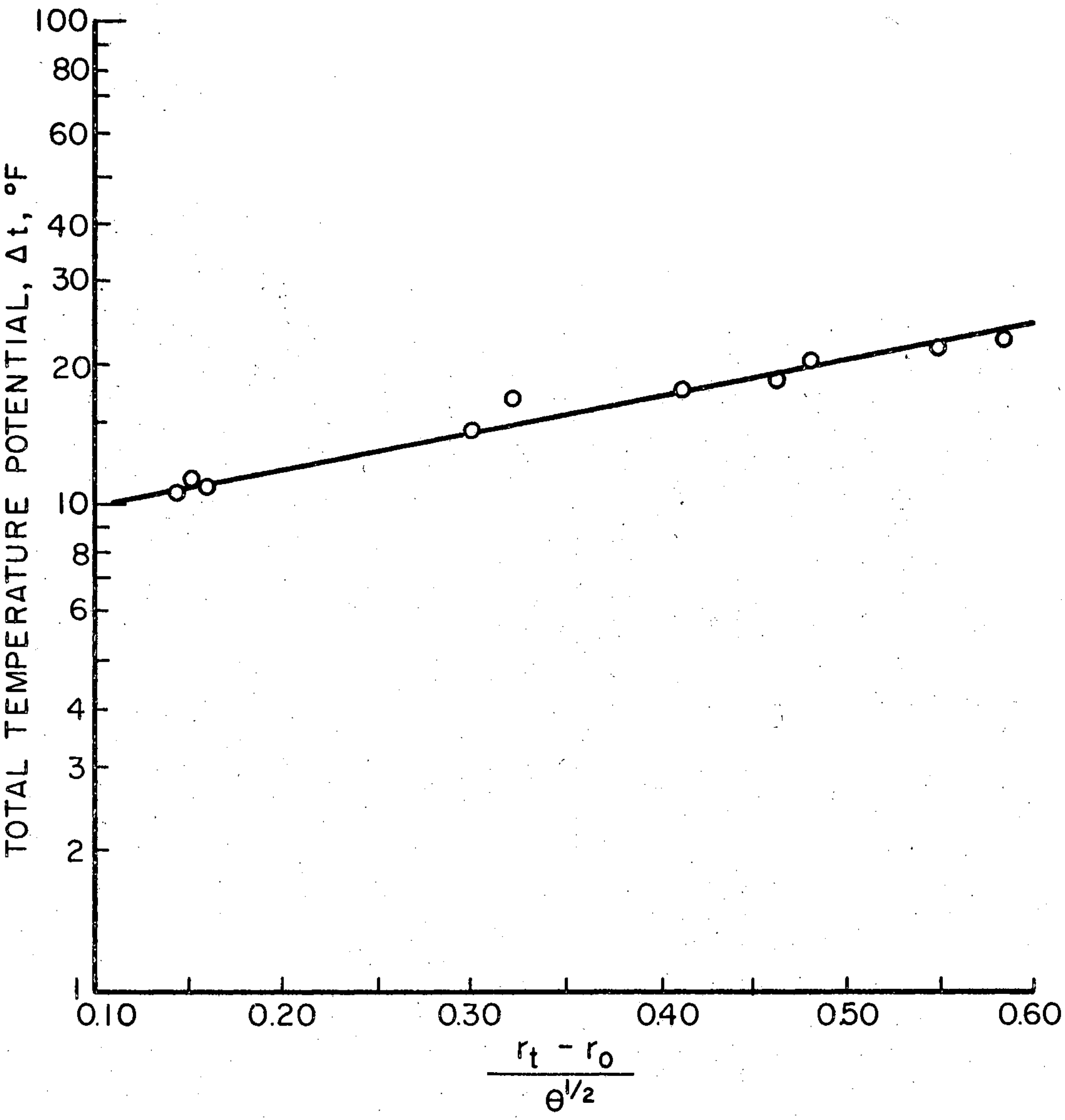

Figure 24. Curves to Evaluate Final Radius for $t_b < t_e$ by Equation 86

When these groupings were used, the variation in the concentration did not have any apparent effect on the data correlation. This was due to the narrow range of concentration values present in the test. From the semi-log plot the form of the equation for this case, where $t_b < t_e$, should be of the form

$$\Delta t = A\left[10\right]^{\left(\frac{B(r_t - r_0)}{\sqrt{\theta}}\right)} \tag{81}$$

or

$$\log(\Delta t) = (\log A) + \frac{B(r_t - r_0)}{\sqrt{\theta}} \tag{82}$$

$$\frac{r_t - r_0}{\sqrt{\theta}} = \frac{\log(\Delta t)}{B} - \frac{\log A}{B} = C + \log(\Delta t) \tag{83}$$

Grouping and solving for the constants C and D gives

$$\frac{r_t - r_0}{\sqrt{\theta}} = -1.2118 + 0.5711 \log_e(\Delta t) \tag{84}$$

where the constants were evaluated for natural logarithms and

$$\frac{r_t - r_0}{0.5711\sqrt{\theta}} = -2.1219 + \log_e(\Delta t) \tag{85}$$

$$\frac{r_t - r_0}{\sqrt{\theta}} = 0.5711 \log_e\left[\frac{\Delta t}{8.3467}\right] \tag{86}$$

The validity of the application of the parameters suggested by the analytical study to determine the final radius is shown in Table XV. For the determination of the total weight, Equation (37) was used and the results are expressed in Table XVI.

TABLE XV

EXPERIMENTAL AND CALCULATED VALUES OF THE FINAL RADIUS r_t FOR $t_b < t_e$ BY EQUATION 86

Run Number	Δt	θ	r_{ct}	r_{xt}	$r_{ct} - r_{xt}$	$(\frac{r_{ct}-r_{xt}}{r_{ct}})100$
19	10.9	1.58	1.192	1.200	-0.008	-0.67
24	10.9	1.84	1.207	1.200	+0.007	+0.16
27	11.0	1.67	1.204	1.200	+0.004	+0.33
31	11.3	0.83	1.158	1.150	+0.008	+0.69
15	14.1	1.90	1.413	1.425	-0.012	-0.85
26	16.7	1.50	1.511	1.400	+0.111	+7.35
40	17.3	2.83	1.700	1.700	0.000	0.00
50	18.4	3.17	1.304	1.325	-0.021	-1.61
46	18.6	3.18	1.316	1.325	-0.009	-0.68
47	19.8	3.50	1.423	1.400	+0.023	+1.62
29	21.9	1.50	1.674	1.675	-0.001	-0.06
34	23.5	2.50	1.934	1.925	+0.009	+0.47

TABLE XVI

EXPERIMENTAL AND CALCULATED TOTAL SAMPLE WEIGHTS FOR $t_b < t_e$ BY EQUATION 37

Run Number	r_{ct}	W_{ct}	W_{xt}	$W_{ct} - W_{xt}$	$100\left[\frac{W_{ct} - W_{xt}}{W_{ct}}\right]$
19	1.192	0.0823	0.0873	-0.0050	- 6.08
24	1.207	0.0920	0.0844	+0.0076	+ 8.26
27	1.204	0.0880	0.0904	-0.0024	- 2.73
31	1.158	0.0642	0.0591	+0.0051	+ 7.94
15	1.413	0.2488	0.2635	-0.0147	- 5.91
26	1.511	0.3415	0.2370	+0.1045	+30.60
40	1.700	0.5503	0.5590	-0.0087	- 1.58
50	1.304	0.2879	0.3050	-0.0171	- 5.94
46	1.316	0.2976	0.3067	-0.0091	- 3.06
47	1.423	0.3857	0.3656	+0.0201	+ 5.21
29	1.674	0.5233	0.5232	+0.0001	+ 0.02
34	1.934	0.9035	0.8946	+0.0089	+ 0.99

Discussion of Final Radius Equation for Brine Temperature Less Than the Equilibrium Temperature

The final radius equation for the case of the brine temperature less than the equilibrium temperature produces accurate results with the exception of Run 26. The source of this error is probably due to an error in measuring the inside sphere surface temperature. From Appendix A this temperature appears to be too high for the sump temperature present during this run.

The arrangement of the various parameters is qualitatively correct. An increase in the total potential will result in a larger final radius for a given freezing period. The sample will also increase without bound as the time increases.

Derivation of Final Radius Equation When Brine Has a Velocity

The final case involves the calculation of the total radius in the presence of a brine velocity field. To account for the fact that the test chamber has a finite cross sectional area, the velocity was corrected for the presence of the sphere at the start of a run and the presence of the sphere plus the ice at the termination of the run. The volume flows for the four orifices used during the test were

$$Q_1 = 0.05612 \text{ ft}^3/\text{min}$$

$$Q_2 = 0.1327 \text{ ft}^3/\text{min}$$

$$Q_3 = 0.1953 \text{ ft}^3/\text{min}$$

$$Q_4 = 0.2234 \text{ ft}^3/\text{min}.$$

The cross-section area of the test chamber was 0.1918 ft^2 so that the four velocities were

$$V_1 = 0.2926 \text{ ft/min}$$

$$V_2 = 0.6919 \text{ ft/min}$$

$$V_3 = 1.0182 \text{ ft/min}$$

$$V_4 = 1.1648 \text{ ft/min}.$$

The velocity, V_i, was determined by deducting the cross-sectional area of the sphere, πr_o^2, from the chamber cross-sectional area; the velocity, V_f, was evaluated in the same manner except the cross-sectional area of the ice sample was deducted from the chamber. The average velocity was then determined by

$$V_{avg} = \frac{V_n}{2} + \frac{V_i + V_f}{4} \qquad n = 1, 2, 3, 4 \tag{87}$$

where n designates the orifice used. The actual effect of these corrections results in an average 3% increase in the velocity term.

The experimental data for the brine velocity experiment are plotted in Fig. 25. This curve shows that the data may be correlated by means of the equation

$$\frac{r_t - r_o}{\sqrt{\theta}} = a\, V_{avg.}^{b}\, \Delta t^{c} \tag{88}$$

This expression may not be extended to non-flow runs.

The temperature exponent c was evaluated by determining the slope of the curves in Fig. 25 which gave a value for c

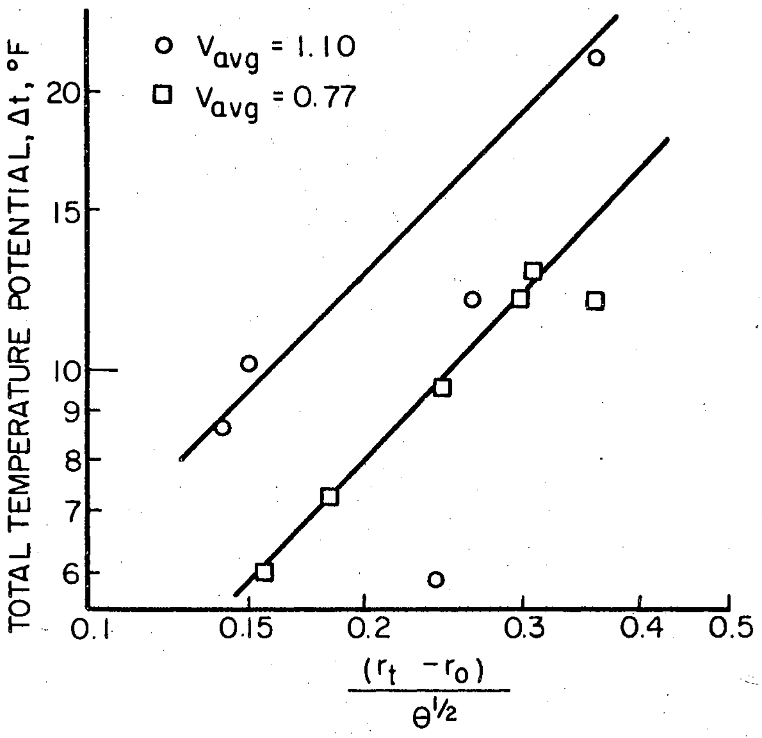

Figure 25. Curves to Evaluate Temperature Exponent

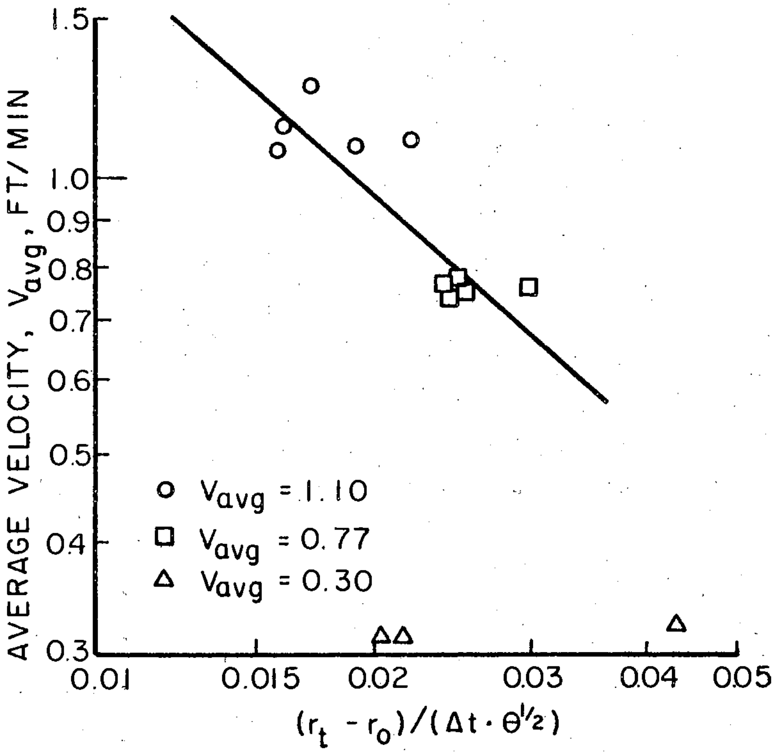

Figure 26. Curve to Evaluate Velocity Exponent

of 1.0. The velocity exponent b was approximated in the same manner from Fig. 26 giving a value for b of - 1.0. From Fig. 26 it is also apparent that the lowest velocity runs are of no practical value. The constant (a) was evaluated by repeated solutions of Equation (88) and averaging the resultant values. The final result is

$$\frac{r_t - r_0}{\sqrt{\theta}} = \frac{1}{50} \frac{\Delta t}{V_{avg.}} \qquad (89)$$

Equation (89) is plotted in Fig. 27 to illustrate the effect of the variables Δt and V_{avg}. Calculated values of the final radius and total weight are presented in Tables XVII and XVIII.

Discussion of the Velocity Equations

The series of velocity runs resulted in the largest radial and weight errors. This was expected in the case of the lowest velocities because the small orifice had a tendency to plug during the experimental runs and it was necessary to clear the orifices manually several times. By visual observation of the larger orifices the discharge appeared to be uniform. Since the non-velocity runs illustrated a high degree of radial and weight accuracy it is reasonable to assume that the difficulties encountered in these tests were due either to velocity measurement error or to the fact that the velocity runs produced samples that were not spherical in shape, as observed in Runs number 21 and 23.

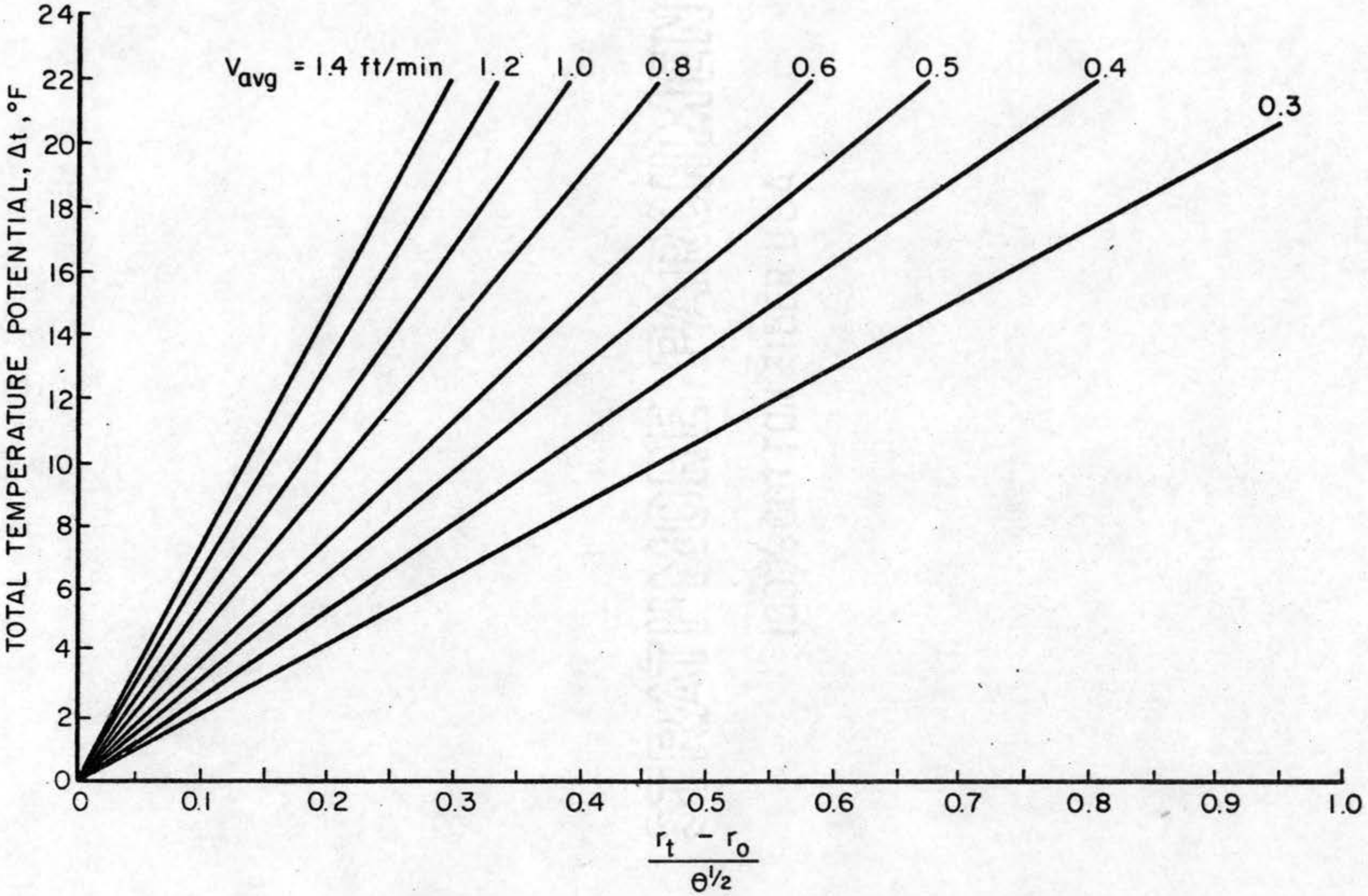

Figure 27. Total Radius Curves for Brine Velocity Runs as Calculated by Equation 89

TABLE XVII

EXPERIMENTAL AND CALCULATED VALUES OF THE FINAL RADIUS r_t FOR BRINE VELOCITY RUNS BY EQUATION 89

Run Number	V_{avg}	Δt	θ	r_{ct}	r_{xt}	$\left(\frac{r_{ct} - r_{xt}}{r_{ct}}\right) 100$
13	0.3144	9.6	2.25	1.501	1.300	+13.40
20	0.3144	11.9	1.33	1.873	1.300	+30.59
7	0.3250	7.0	2.17	1.634	1.450	+11.29
12	0.7480	6.0	1.67	1.207	1.200	+ 0.60
8	0.7534	7.3	2.25	1.290	1.275	+ 1.22
14	0.7552	9.6	1.50	1.311	1.300	+ 0.86
21	0.7684	12.1	2.17	1.374	1.450	− 5.64
33	0.7820	12.0	2.50	1.485	1.575	− 6.04
3	0.7790	12.9	3.00	1.574	1.550	+ 1.50
22	1.1007	8.8	2.17	1.236	1.200	+ 2.87
11	1.1001	10.1	2.33	1.280	1.250	+ 2.37
9	1.1059	12.0	1.58	1.273	1.250	+ 1.79
32	1.1509	22.1	2.58	1.617	1.575	+ 2.59
4	1.1346	12.3	4.00	1.432	1.475	− 3.00
23	1.2715	13.5	2.33	1.350	1.300	+ 3.72

TABLE XVIII

EXPERIMENTAL AND CALCULATED TOTAL SAMPLE WEIGHTS FOR BRINE VELOCITY RUNS BY EQUATION 37

Run Number	r_{ct}	W_{ct}	W_{xt}	$W_{ct} - W_{xt}$	$\left(\frac{W_{ct} - W_{xt}}{W_{ct}}\right) 100$
13	1.501	0.3300	0.1647	+0.1653	+50.09
20	1.873	0.8021	0.1579	+0.6442	+80.31
7	1.634	0.4662	0.2827	+0.1835	+39.36
12	1.207	0.0993	0.0957	−0.0036	− 3.62
8	1.290	0.1580	0.1484	+0.0096	+ 6.08
14	1.311	0.1693	0.1627	+0.0066	+ 3.90
21	1.374	0.2162	0.2811	−0.0649	−29.40
33	1.485	0.3129	0.4128	−0.0999	−31.93
3	1.574	0.4002	0.3868	−0.0134	− 3.34
22	1.236	0.1114	0.0882	+0.0232	+20.82
11	1.250	0.1474	0.1250	+0.0224	+15.19
9	1.273	0.1455	0.1299	+0.0156	+10.72
32	1.617	0.4370	0.4051	+0.0319	+ 7.30
4	1.211	0.2676	0.3050	−0.0374	−13.98
23	1.350	0.1960	0.1585	+0.0375	+23.02

APPENDIX E

DERIVATION OF WEIGHT-CONCENTRATION EQUATIONS

The preceding two sections present methods to calculate, first the final radius of the sphere, and second, the final sample weight. The next requirement would be the determination of the relationship between the accumulative sample weight and the salt concentration. The accumulative sample weight was adopted as a logical result of the manner in which the data was collected. Also the data correlation was considerably improved when this method was used. As the sample melted, incremental weights were collected and their concentration measured. This process was continued until the sample had completely melted. The incremental weights were then summed, starting with the sample that melted first, and the running total of these weights equaled the accumulated weight. The final value of the accumulated sample weight was also the total sample weight. It was observed that the salt concentration of the incremental weights decreased steadily as the ice sample was melted.

The zone refining and normal slab freezing processes are the closest approximations to the process used in this experiment. From the zone and normal freezing processes, methods have been developed to predict the effect of changing the parameters for the slab configuration. The significant variables in this problem were: concentration of the original brine solution, initial radius of the sphere, rate at which ice was formed, and brine velocity.

The concentration of the original brine solution obviously effected the weight-concentration distribution. As the sodium chloride concentration of the mother solution increases the weight fraction of the total sample having a concentration less than any prescribed amount must decrease. Experimentally, this was found to be true for all of the runs in this study.

The effect of the sphere radius has two effects on the weight-concentration relationship. As the initial sphere size increases there is an increased surface area available on which the freezing process may take place. This means that the sample next to the sphere surface occupies a larger volume for a given radial growth. Therefore the quantity of highly purified solid should increase. Assuming the initial frozen ice contains less entrapped brine that the adjacent liquid, the other line of reasoning states that the sodium chloride concentration of the liquid next to the freezing interface is high. A smaller starting radius will therefore permit a more rapid dilution of this liquid. For finite freezing periods it was found that the larger spheres, with their greater heat transfer surface, produced more potable water than the smaller spheres.

The effect of freezing rate was discussed previously for a slab configuration. For a slab, increasing the freezing rate increased the impurities in the solid phase, Equation (2). For a sphere, it was observed that increasing the freezing rate decreased the impurities in the solid phase.

It was expected that the introduction of a velocity field would improve the purification process by removing the highly concentratied liquid layer. The experimental runs associated with a velocity field resulted in less separation of salt and water than was obtained in comparable non-velocity runs.

Derivation of weight-concentration equations required the consideration of three separate cases:

1. brine temperature greater than the equilibrium temperature.

2. brine temperature equal to or less than the equilibrium temperature. (Equations for these two cases were originally derived separately but the exponents and constants were essentially the same so that one equation was found to offer a satisfactory solution.)

3. brine velocity runs.

Weight-Concentration Equation for Brine Temperature Greater than the Equilibrium Temperature

Fig. 28 is a plot of selected runs for the case, $t_e < t_b$. These curves indicate that the slope is not a function of the temperature or original concentration, but is affected by the initial sphere size. The weight-concentration

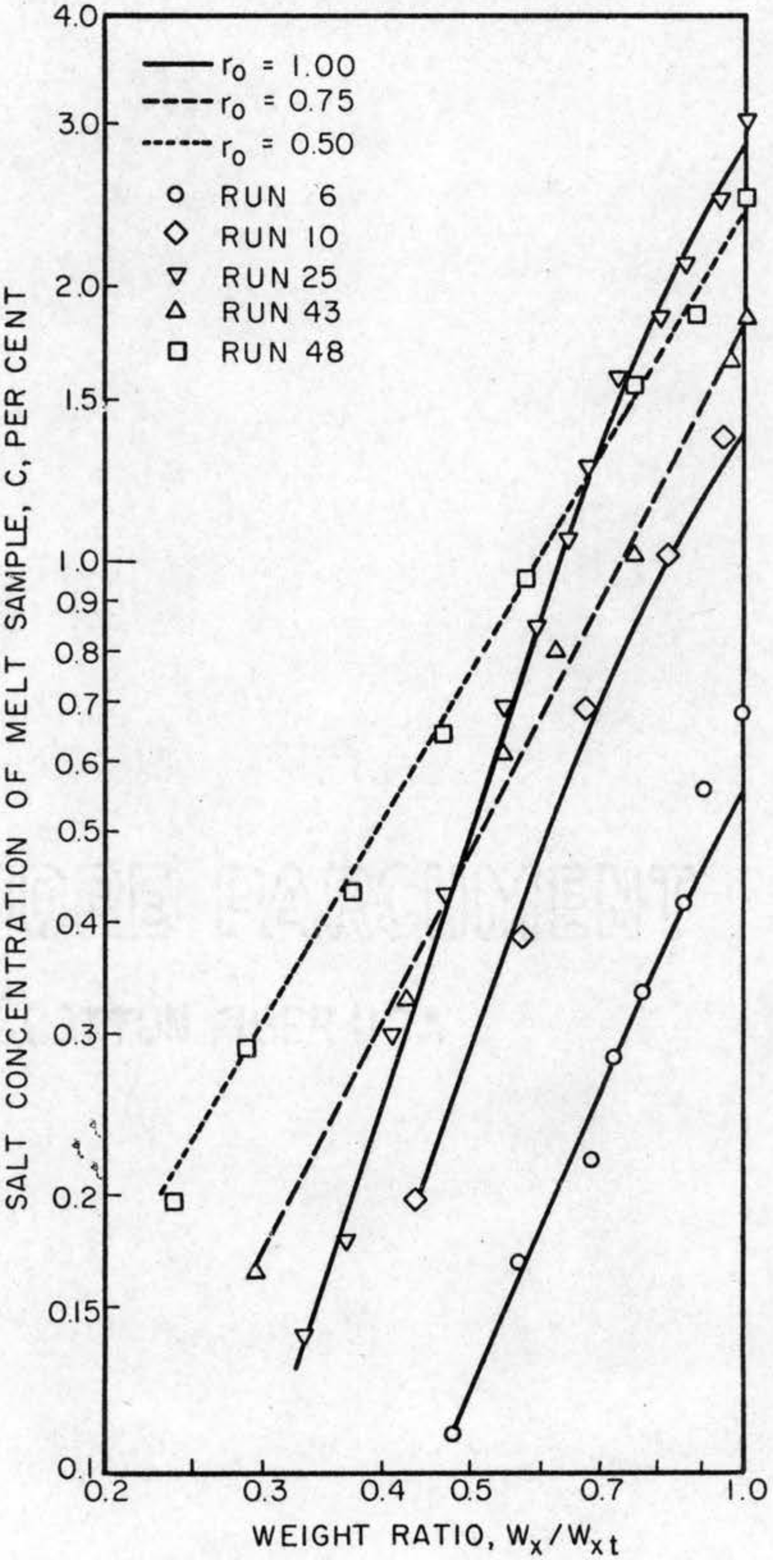

Figure 28. Experimental Weight-Concentration Data for $t_b > t_e$

equation was therefore assumed to be

$$\frac{W_x}{W_{xt}} = A_1 C_x^{n_1} \tag{90}$$

The slopes, n_1, of these curves were evaluated for all of the runs which resulted in the following average values.

r_o = 1.00 inch Slope n_1 = 0.362

r_o = 0.75 inch Slope n_1 = 0.503

r_o = 0.50 inch Slope n_1 = 0.599

Evaluating the term $n_1 r_o$:

r_o = 1.00 inch $n_1 r_o$ = 0.362

r_o = 0.75 inch $n_1 r_o$ = 0.377

r_o = 0.50 inch $n_1 r_o$ = 0.300

which has an average value of $n_1 r_o = 0.346$ or $n_1 = \frac{0.346}{r_o}$

To determine the effect of the total temperature potential the equation was re-written as

$$\frac{W_x}{W_{xt}} = A_2 [C_x]^{\frac{0.346}{r_o}} [\Delta t]^{n_2} \tag{91}$$

Runs 16, 25, 44 and 48 have approximately the same original sodium chloride concentrations. By evaluating W_x/W_{xt} for a melt concentration of $C_x = 0.1$, for these runs, the effect of varying Δt on the abscissa intersection defined by $C_x = 0.1$ may be determined. The value obtained for n_2 was approximately zero. This indicates that for the steady state runs the rate of ice formation did not have a significant effect on the weight-concentration curves.

The same procedure was followed for evaluating n_3 from the equation

$$\frac{W_x}{W_{xt}} = A_3 \left[C_x\right]^{\frac{0.346}{r_0}} \left[C_b\right]^{n_3} \tag{92}$$

Since the temperature proved to have no effect, high concentration runs (16, 17, 25, 30) and low concentration runs (6, 38) were used to evaluate n_3. The value of n_3 was found to be - 0.338.

The constant A_3 was evaluated by use of Equation (92). The values of W_x/W_{xt} at $C_x = 0.1$ were inserted in the above equation and the average value of A_3 was obtained. Therefore Equation (92) becomes

$$\frac{W_x}{W_{xt}} = 0.877 \left[C_x\right]^{\frac{0.346}{r_0}} \left[C_b\right]^{-0.338} \tag{93}$$

Since the exponents .346 and .338 are nearly the same, an accurate data correlation was obtained by

$$\frac{W_x}{W_{xt}} = 0.881 \left[\frac{C_x^{(1/r_0)}}{C_b}\right]^{\frac{1}{3}} \tag{94}$$

When this equation was applied to the experimental data the results were satisfactory for all values of W_x/W_{xt} except for those values of W_x/W_{xt} from 0.8 to 1.0. As noted in Fig. 28 the experimental values show a definite curvature at these values of W_x/W_{xt}. Here Equation (94) was modified by the substitution of

$$\left[0.85 + 0.15\left(\frac{c}{C_b}\right)\right] \tag{95}$$

for the constant 0.88, thus obtaining

$$\frac{W}{W_t} = \left[0.85 + 0.15\left(\frac{C}{C_b}\right)\right]\left[\frac{C^{(1/r_o)}}{C_b}\right]^{\frac{1}{3}} \qquad (96)$$

This term reduces the value of the concentration for higher values of C but drops out of the evaluation as C decreases.

To determine the accuracy of Equation (96), experimental values of W_x/W_{xt} were substituted for W/W_t. By using the experimental values of C_b and r_o, the calculated concentration C_j was obtained. C_j was then compared to the experimental sodium chloride concentration C_x. Values of C_x and C_j for selected runs are presented in Table XIX. The values for the remainder of the runs are available in Appendix H.

To apply Equation (96) values of W_{ct} were obtained from Table XIV for the case of $t_b > t_e$. Substituting this value for W_t in Equation (96), the value of W_c was obtained for particular values of the concentration C. Values of W_c for selected runs are presented in Table XX. The remaining runs are presented in Appendix I.

The (W_x/W_{xt}) and (W_c/W_{ct}) columns are included in Table XX to indicate the accuracy of Equation (96). The percentage error was calculated from $\left[(W_c - W_x)/W_c\right] \times 100$ and is tabulated for selected runs in Table XX. The values for the remainder of the runs is presented in Appendix I. Values of the percentage error for all of the runs, at selected values of the concentration, are shown in Table XXI.

TABLE XIX

EXPERIMENTAL AND CALCULATED VALUES OF EXPERIMENTAL CONCENTRATION AND CALCULATED CONCENTRATION BY MEANS OF EQUATION 96

$t_b > t_e$

Run Number 10

Sample Number	W_x	C_x	C_j
1	0.1274	2.15	2.08
2	0.1210	1.41	1.65
3	0.1061	1.05	1.09
4	0.0911	0.792	0.830
5	0.0767	0.440	0.510
6	0.0573	0.206	0.239
7	0.0399	0.060	0.082
8	0.0250	0.017	0.022
9	0.0161	0.007	0.007
10	0.0075	0.000	0.000

Run Number 17

Sample Number	W_x	C_x	C_j
1	0.1570	4.00	3.93
2	0.1532	3.66	3.78
3	0.1427	3.09	3.24
4	0.1288	2.48	2.52
5	0.1169	2.00	2.00
6	0.1069	1.65	1.62
7	0.0952	1.24	1.16
8	0.0838	0.902	0.838
9	0.0701	0.532	0.500
10	0.0531	0.227	0.236
11	0.0410	0.115	0.112
12	0.0320	0.058	0.054
13	0.0214	0.032	0.016
14	0.0108	0.009	0.002
15	0.0044	0.000	0.000

TABLE XIX (Continued)

Run Number 37 (Rapid Melt)

Sample Number	W_x	C_x	C_j
1	1.0172	1.25	0.750
2	0.9887	0.778	0.713
3	0.9393	0.653	0.643
4	0.8913	0.570	0.567
5	0.8392	0.410	0.510
6	0.7872	0.357	0.422
7	0.7369	0.310	0.360
8	0.6921	0.300	0.306
9	0.6412	0.215	0.247
10	0.5938	0.185	0.199
11	0.5442	0.165	0.159
12	0.5072	0.157	0.133
13	0.4487	0.135	0.094
14	0.4011	0.120	0.064
15	0.3583	0.115	0.053
16	0.3091	0.080	0.033
17	0.2487	0.052	0.018
18	0.1972	0.037	0.009
19	0.1455	0.008	0.004
20	0.0911	0.001	0.001
21	0.0368	0.000	0.000

Run Number 43

Sample Number	W_x	C_x	C_j
1	0.2615	2.02	1.89
2	0.2586	1.80	1.84
3	0.2368	1.59	1.53
4	0.2115	1.17	1.00
5	0.1808	1.02	0.924
6	0.1599	0.795	0.727
7	0.1222	0.371	0.423
8	0.0763	0.163	0.152
9	0.0452	0.057	0.050
10	0.0267	0.022	0.015
11	0.0137	0.008	0.001

TABLE XX

EXPERIMENTAL AND CALCULATED VALUES FROM WEIGHT-CONCENTRATION DATA $t_b > t_e$

C	W_x	W_c	$W_c - W_x$	% Error	$\frac{W_x}{W_{xt}}$	$\frac{W_t}{W_{ct}}$
			Run Number 10			
2.00	0.1261	0.1315	+0.0054	+ 4.1	0.990	0.992
1.00	0.1032	0.0958	-0.0074	- 7.7	0.810	0.723
0.50	0.0784	0.0730	-0.0054	- 7.4	0.615	0.551
0.10	0.0450	0.0413	-0.0037	- 9.0	0.353	0.312
0.05	0.0361	0.0327	-0.0034	- 9.4	0.283	0.247
0.01	0.0187	0.0190	+0.0003	+ 1.6	0.146	0.143
			Run Number 17			
3.00	0.1407	0.1446	+0.0039	+ 2.7	0.896	0.881
2.00	0.1169	0.1212	+0.0043	+ 3.5	0.745	0.739
1.00	0.0871	0.0942	+0.0071	+ 7.5	0.555	0.574
0.50	0.0697	0.0717	+0.0040	+ 5.6	0.431	0.437
0.10	0.0403	0.0412	+0.0009	+ 2.2	0.257	0.251
0.05	0.0295	0.0326	+0.0031	+ 9.6	0.188	0.199
0.01	0.0114	0.0190	+0.0076	+40.0	0.073	0.116
			Run Number 37			
0.50	0.8690	0.8404	+0.0286	- 3.4	0.854	0.830
0.10	0.3374	0.4503	+0.1129	+25.1	0.332	0.425
0.01	0.1031	0.2043	+0.1012	+49.5	0.101	0.202
			Run Number 43			
1.00	0.2009	0.1820	-0.0189	-10.4	0.768	0.719
0.50	0.1337	0.1278	-0.0059	- 4.6	0.511	0.508
0.10	0.0578	0.0609	+0.0031	+ 5.1	0.221	0.241
0.05	0.0412	0.0445	+0.0033	+ 7.4	0.157	0.176
0.01	0.0154	0.0217	+0.0063	+29.0	0.059	0.086

TABLE XXI

PERCENTAGE ERROR FROM EXPERIMENTAL AND CALCULATED WEIGHT CALCULATIONS FOR VARIOUS VALUES OF THE SALT CONCENTRATION IN THE MELT SAMPLE

Concentrations

Run Number	3.00	2.00	1.00	0.50	0.10	0.05	0.01
6	-----	-----	-----	+ 8.3	+ 5.6	+ 5.4	- 1.0
10	-----	+ 4.1	- 7.7	- 7.4	- 9.0	- 9.4	+ 1.6
16	+16.8	+ 9.2	+15.8	+17.6	+46.1	+60.1	+86.7
17	+ 2.7	+ 3.5	+ 7.5	+ 5.6	+ 2.2	+ 9.6	+40.0
25	+11.0	+12.0	+12.7	+10.4	+ 2.0	+23.4	+37.7
30	- 2.7	- 2.2	- 4.5	- 5.8	- 5.2	- 1.4	+48.3
38	-----	-----	-----	+ 2.5	+ 3.7	+ 8.6	- 7.5
43	-----	-----	-10.4	- 4.6	+ 5.1	+ 7.4	+29.0
44	-----	+ 7.8	+ 8.8	+ 8.0	+ 0.2	+11.0	+73.9
48	-----	+11.8	+ 3.0	- 7.8	-19.5	+ 7.9	+45.9
37	-----	-----	-----	- 3.4	+25.1	+31.4	+49.5
49	-----	+ 0.3	-12.6	+ 4.3	-46.8	-40.0	-62.5

Discussion of Weight-Concentration Equation for Brine Temperature Greater than the Equilibrium Temperature

As shown in Fig. 29 as C_b increases the percentage yield, W_x/W_{xt}, at any sodium chloride concentration, is reduced.

The effects of initial sphere radius were indicated by runs having the same original salt concentration but different starting radii. Values of W_x/W_{xt} and W_c/W_{ct} for the runs are

Run Number	r_o	C_b	W_x/W_{xt}	W_c/W_{ct}
25	1.00	3.37	18.1	20.9
44	0.75	3.28	13.9	15.1
48	0.50	3.43	7.1	7.7

for a concentration C = 0.05%. Therefore, as r_o decreases, the percentage yield from the melted sample having a concentration less than 0.05% decreases.

In runs 37 and 49 a different melting technique was adopted. Instead of letting the chamber reach its own equilibrium temperature of approximately 60 °F, a heater was used to attain temperatures of 99 °F and 87 °F respectively. Since the melt down procedure at 60 °F was tedious, requiring from one to four hours depending on the sample size, the effects of more rapid melting were explored. The melting time for Run 49 was 0.42 hours while a comparable sized sample, in Run 6, required 1.33 hours. However, higher melting rates tended to average the salt distribution in

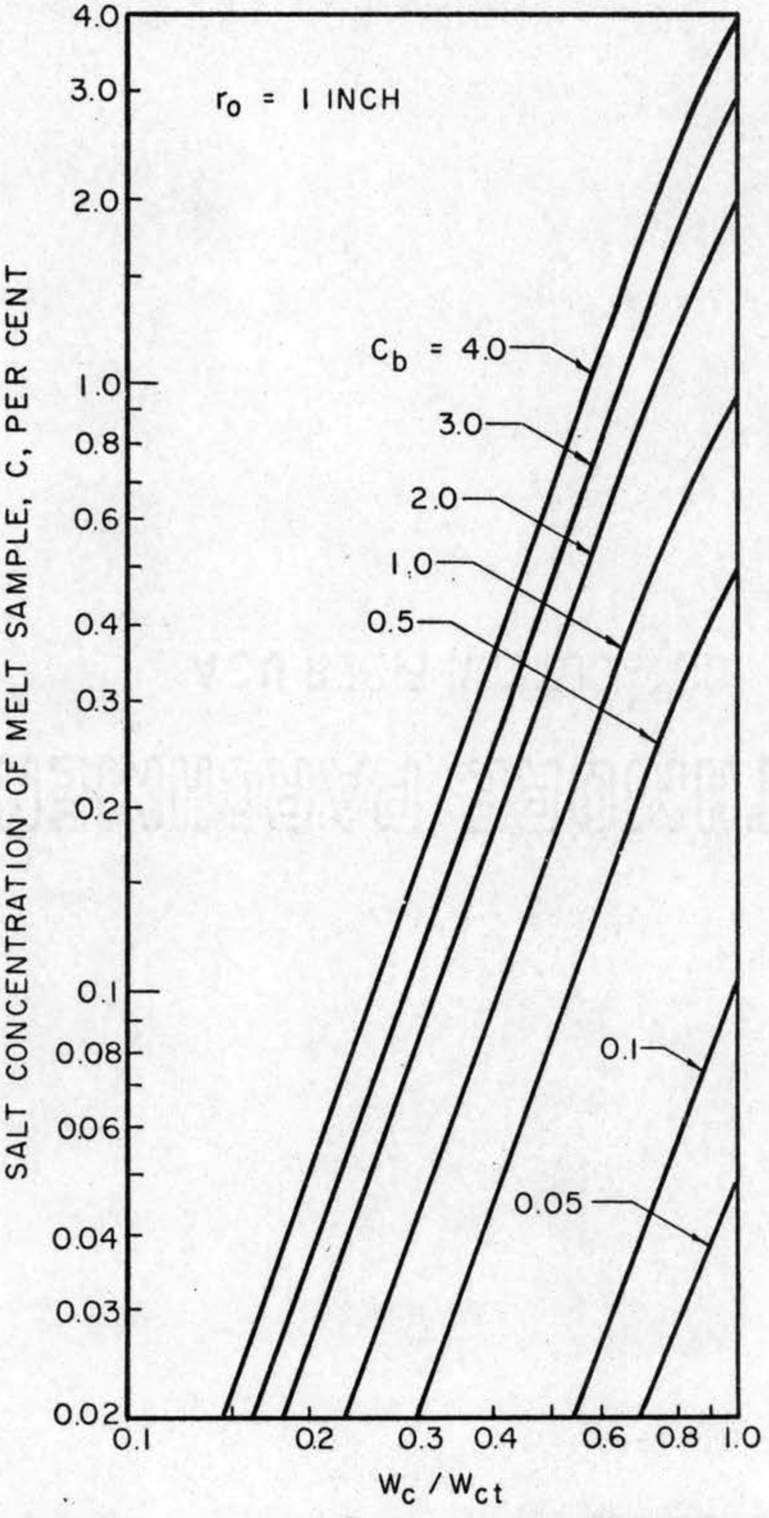

Figure 29. Calculated Weight-Concentration Data for $t_b > t_e$ by Equation 96

the melt sample. This resulted in decreased quantities of water having a sodium chloride concentration of less than 0.05 percent. This was due to non-spherical sample melting which was in turn caused by the heater location.

Run 18 was also a variation from the usual operating procedure. This run was terminated before steady-state conditions were reached in order to determine if:

1. pure solid was being formed exclusively from the very beginning of a run or

2. whether longer freezing periods were required. The value of W_x/W_{xt}, percent yield, for C = 0.05% in this run was less than 10% while the expected value would be at least 15%. This indicates the necessity of attaining steady state conditions before melting the sample.

From Table XXI the percentage errors at C = 0.05%, for calculated and experimental weights, as determined by Equation (96), are less than 12% except for Runs 16, 25, 37 and 49. Runs 16 and 25 are in error due to the erroneous values of W_{ct} as noted in Table XIV. The errors in runs 37 and 49 were due to the high melting rate. The columns W_x/W_{xt} and W_c/W_{ct} are in close agreement, indicating that the weight-concentration equation, Equation (96), is correct.

The percentage error values for C = 0.01% are in error for two reasons; first the experimental data scatters below concentrations of approximately 0.03%. Second, the derived equations can not be extrapolated to such low values of C. For C less than 0.05% all of the water may be considered as potable.

Weight-Concentration Equation for Brine Temperature Equal to or Less than the Equilibrium Temperature

From Fig. 30 it is apparent that the general form of the equation for this case is similar to that of the preceding case and will have the form of Equation (90). The average value for the slopes are

$r_o = 1.00$ inch	$n_1 = 0.358$
$r_o = 0.75$ inch	$n_1 = 0.460$
$r_o = 0.50$ inch	$n_1 = 0.715$

and

$r_o = 1.00$ inch	$n_1 r_o = 0.358$
$r_o = 0.75$ inch	$n_1 r_o = 0.345$
$r_o = 0.50$ inch	$n_1 r_o = 0.357.$

The average value of $(n_1 r_o)$ is 0.355.

Now, writing

$$\frac{W_x}{W_{xt}} = A_2 \left[C_x\right]^{\frac{0.355}{r_o}} \left[\Delta t\right]^{n_2} \tag{97}$$

and by use of Runs (19, 24, 34) and (15, 29) which have the same brine concentration, the value of the exponent n_2 was determined by substituting values of W_x/W_{xt} at $C_x = 0.1$ into Equation (97). The calculated value of n_2 is 0.328. Equation (97) was re-written as

$$\frac{W_x}{W_{xt}} = A_3 \left[C_x\right]^{\frac{0.355}{r_o}} \left[\Delta t\right]^{0.328} \left[C_b\right]^{n_3} \tag{98}$$

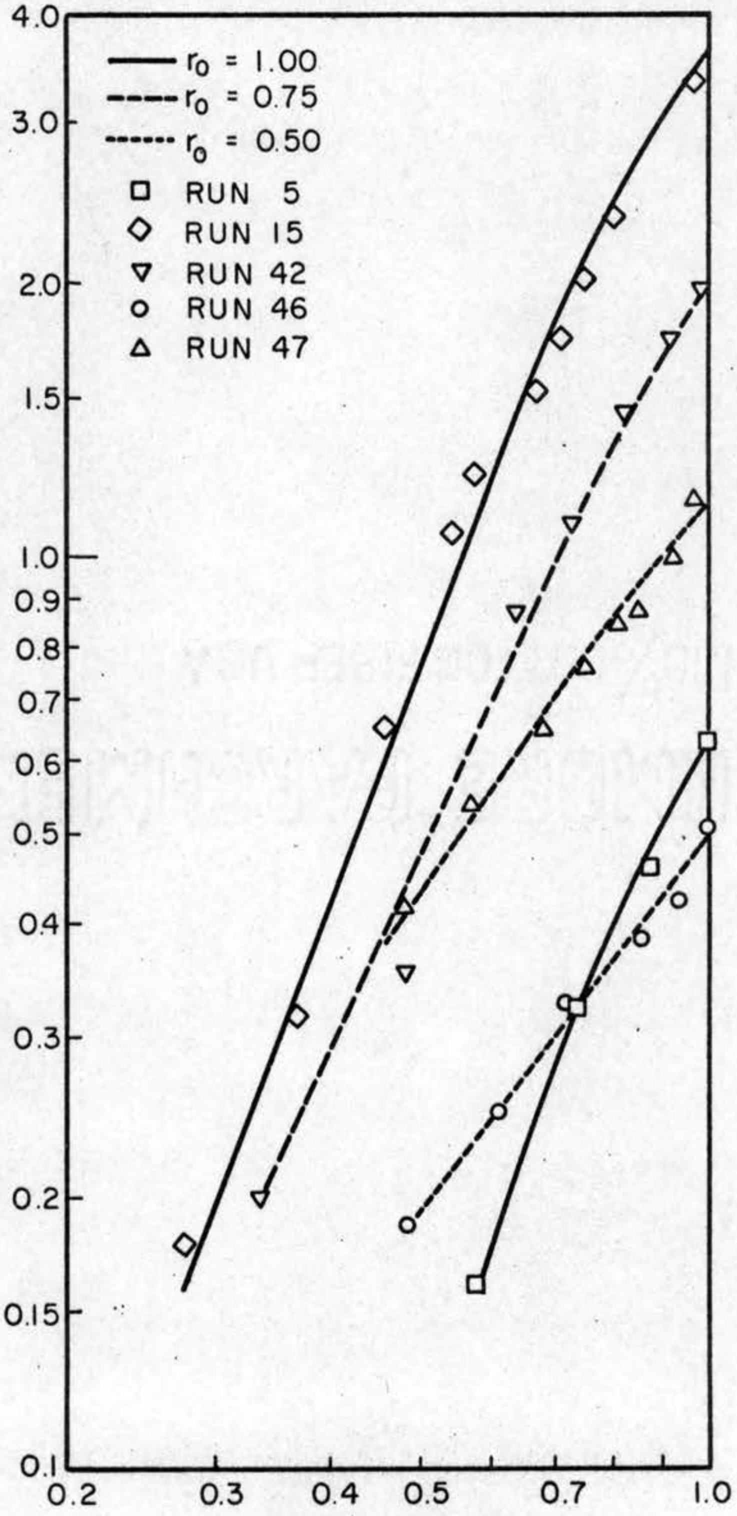

Figure 30. Experimental Weight-Concentration Data for $t_b = t_e$ and $t_b < t_e$

By selecting runs with widely divergent values of C_b, the exponent n_3 was evaluated. This gave a value of n_3 - 0.519.

Again the value of the constant A_3 was evaluated by means of Equation (98) for each run and was averaged. This gave an A_3 of 1/1.94. The equation is now represented by

$$\frac{W}{W_t} = \frac{1}{1.94}[C]^{\frac{0.353}{r_o}}[\Delta t]^{0.328}[C_b]^{-0.519} \qquad (99)$$

Again it was necessary to modify the equation for large values of W/W_t. In this case it was required that the concentration term degenerate more rapidly than in Equation (96). The final equation to predict weight-concentration relationship for $t_b = t_e$ or $t_b < t_e$ became

$$\frac{W}{W_t} = \frac{1}{2.32}\left[1+0.2\left(\frac{c}{c_b}\right)^2\right][C]^{\frac{1}{3r_o}}\left[\frac{(\Delta t)^{\frac{1}{3}}}{(C_b)^{1/2}}\right] \qquad (100)$$

Discussion of Weight-Concentration Equation for Brine Temperature Equal to or Less Than the Equilibrium Temperature

The major difference between Equation (100) and Equation (96), obtained in the preceding section, is the introduction of the (Δt) term and the change in the exponent for C_b. When $t_b > t_e$ the ice sample grew to a finite size, and the sodium chloride concentration gradient disappeared and ice formation ceased. Therefore, the rate of ice formation had little effect on the amount of brine entrapped. However, when $t_b = t_e$ or $t_b < t_e$ the sample grew steadily and the rate of growth was important in establishing the variations of sodium chloride concentration in the solid.

The reason for the decrease in the brine concentration exponent n_3, when comparing Equation (100) to Equation (96), is not definitely known. When the weight-concentration equation was calculated for the velocity runs in the next section the brine concentration exponent decreased still farther. A possible reason for this decrease in the brine concentration exponent was that with a continuously moving interface, the brine concentration of the mother liquid had an increased effect on the salt concentration of the liquid nearest the interface. This in turn increases the salt concentration in the solid.

The following two runs illustrate the effect of changing the temperature on the experimental and calculated percentage yield:

Run Number	C_b	Δt	W_x/W_{xt}	W_c/W_{ct}
15	3.48	14.1	19.4	20.6
29	3.45	21.9	22.9	23.9

for C = 0.05%. In the case of $t_b > t_e$, Run 25, which had a slightly lower brine concentration, the values of W_x/W_{xt} and W_c/W_{ct} were respectively

$$W_x/W_{xt} = 18.1$$

and

$$W_c/W_{ct} = 20.9$$

for

$$C = 0.05\%$$

Therefore Run 29 had an increased experimental yield of 4.8% over Run 25. Table XXII lists values of C_x and C_j for selected runs with the omitted runs presented in Appendix H. Table XXIII includes selected run values of W_x, W_c, percent error, W_x/W_{xt} and W_c/W_{ct}. Table XXIV gives the percentage error for all runs at various concentrations C. From Table XXIV the largest errors at $C = 0.05\%$ occur in runs 26, 31, 40 and 50. The errors in Runs 26 and 31 are due to errors in calculating W_{ct}, while those in 40 and 50 are caused by the use of high melt rates.

Weight-Concentration Equation for Brine Velocity Runs

The experimental data for the brine velocity runs is plotted in Fig. 31. The slope of the curves again varies, but since all the runs were performed with a radius $r_o = 1$ inch, some other property must be responsible for this variation. For the non-flow runs the slope of the weight-concentration curves was found to be independent of the temperature difference and initial brine concentration. Assuming that the slope of the weight-concentration equations for flow runs were also independent of these terms, the variation in slope must be caused by the fluid velocity.

TABLE XXII

EXPERIMENTAL AND CALCULATED VALUES OF EXPERIMENTAL CONCENTRATION AND CALCULATED CONCENTRATION BY MEANS OF EQUATION 100

$t_b = t_e$ OR $t_b < t_e$

Run Number 15

Sample Number	W_x	C_x	C_j
1	0.2635	3.60	3.42
2	0.2573	3.39	3.28
3	0.2238	2.92	2.54
4	0.2137	2.49	2.38
5	0.2035	2.20	2.12
6	0.1871	1.82	1.72
7	0.1698	1.51	1.39
8	0.1554	1.21	1.10
9	0.1429	0.886	0.887
10	0.1303	0.727	0.686
11	0.1182	0.475	0.513
12	0.0944	0.225	0.267
13	0.0734	0.124	0.149
14	0.0505	0.048	0.041
15	0.0288	0.023	-----
16	0.0117	0.000	-----

Run Number 28

Sample Number	W_x	C_x	C_j
1	0.4130	3.60	3.60
2	0.3943	3.43	3.36
3	0.3735	3.12	3.04
4	0.3550	2.75	2.61
5	0.3345	2.58	2.45
6	0.3169	2.32	2.21
7	0.2957	2.02	1.90
8	0.2791	1.78	1.66
9	0.2641	1.51	1.45
10	0.2430	1.24	1.16
11	0.2242	0.945	0.938
12	0.1984	0.654	0.671
13	0.1718	0.398	0.437
14	0.1477	0.245	0.288
15	0.1263	0.158	0.177
16	0.1063	0.092	0.103
17	0.0851	0.060	0.050
18	0.0647	0.022	0.028
19	0.0432	0.009	-----
20	0.0216	0.000	-----

Run Number 31

Sample Number	W_x	C_x	C_j
1	0.0591	3.02	3.40
2	0.0531	2.69	2.89
3	0.0441	1.89	2.04
4	0.0364	1.23	1.30
5	0.0298	0.748	0.775
6	0.0240	0.407	0.420
7	0.0183	0.195	0.187
8	0.0123	0.086	0.058
9	0.0060	0.029	0.027

Run Number 47

Sample Number	W_x	C_x	C_j
1	0.3656	2.00	1.17
2	0.3568	1.20	1.14
3	0.3347	1.02	1.06
4	0.3060	0.893	0.942
5	0.2871	0.827	0.874
6	0.2637	0.734	0.776
7	0.2434	0.652	0.694
8	0.2082	0.571	0.558
9	0.1797	0.440	0.450
10	0.1458	0.339	0.341
11	0.0800	0.099	0.099
12	0.0315	0.025	0.037
13	0.0082	0.010	0.015

TABLE XXIII

EXPERIMENTAL AND CALCULATED VALUES FROM WEIGHT-CONCENTRATION DATA

$t_b = t_e$ OR $t_b < t_e$

C	W_x	W_c	$W_c - W_x$	% Error	$\frac{W_x}{W_{xt}}$	$\frac{W_c}{W_{ct}}$
			Run Number 5			
3.00	0.2295	0.2300	+0.0005	+ 0.2	0.871	0.924
2.00	0.1943	0.1865	+0.0078	− 4.2	0.737	0.750
1.00	0.1473	0.1414	−0.0061	− 4.3	0.559	0.567
0.50	0.1175	0.1106	−0.0069	− 6.2	0.446	0.445
0.10	0.0662	0.0645	−0.0017	− 2.6	0.251	0.259
0.05	0.0512	0.0511	−0.0001	− 0.2	0.194	0.206
0.01	0.0187	0.0299	+0.0112	+37.4	0.071	0.120
			Run Number 28			
3.00	0.3676	0.3714	+0.0038	+ 1.0	0.089	0.898
2.00	0.2944	0.3030	+0.0086	+ 2.8	0.713	0.732
1.00	0.2280	0.2302	+0.0022	+ 1.0	0.552	0.556
0.50	0.1824	0.1806	−0.0018	− 1.0	0.441	0.437
0.10	0.1087	0.1053	−0.0034	− 3.2	0.263	0.254
0.05	0.0798	0.0835	+0.0037	+ 4.4	0.193	0.202
0.01	0.0447	0.0488	+0.0041	+ 8.4	0.108	0.118
			Run Number 31			
3.00	0.0589	0.0594	+0.0005	+ 0.8	0.996	0.926
2.00	0.0454	0.0475	+0.0021	+ 4.4	0.768	0.740
1.00	0.0333	0.0356	+0.0023	+ 6.5	0.563	0.555
0.50	0.0261	0.0278	+0.0017	+ 6.1	0.442	0.434
0.10	0.0132	0.0162	+0.0030	+18.5	0.223	0.253
0.05	0.0084	0.0129	+0.0045	+34.9	0.142	0.200
0.01	0.0020	0.0075	+0.0055	+73.3	0.034	0.117
			Run Number 47			
1.00	0.3568	0.3400	−0.0168	− 4.9	0.976	0.881
0.50	0.1935	0.1676	−0.0259	−15.4	0.529	0.534
0.10	0.0803	0.0695	−0.0108	−15.5	0.220	0.180
0.05	0.0478	0.0438	−0.0040	− 9.1	0.131	0.114
0.01	0.0082	0.0150	+0.0068	+45.3	0.022	0.039

TABLE XXIV

PERCENTAGE ERROR FROM EXPERIMENTAL AND CALCULATED WEIGHT CALCULATIONS FOR VARIOUS VALUES OF THE SALT CONCENTRATION IN THE MELT SAMPLE

	Concentrations						
Run Number	3.00	2.00	1.00	0.50	0.10	0.05	0.01
5	———	———	———	+12.8	+12.4	+12.1	+24.9
15	+ 0.2	- 4.2	- 4.3	- 6.2	- 2.6	- 0.2	+37.4
19	- 9.5	- 7.6	- 8.1	- 2.7	+11.0	+19.1	+99.9
24	+ 6.9	-10.2	- 9.8	- 9.5	-10.3	-10.0	-11.1
26	+31.3	+32.0	+35.2	+33.9	+33.8	+36.7	+70.8
27	-13.0	- 7.1	- 4.0	- 9.8	-14.8	-10.3	-11.0
28	+ 1.0	+ 2.8	+ 1.0	- 1.0	- 3.2	+ 4.4	+ 8.4
29	———	+11.9	+ 8.6	+ 5.5	+ 3.7	+ 4.3	+20.7
31	+ 0.8	+ 4.4	+ 6.5	+ 6.1	+18.5	+34.9	+73.3
34	———	+ 6.3	+ 5.2	+ 3.6	+ 0.9	+ 2.0	+11.7
42	———	———	+ 2.7	- 8.3	+ 1.2	+ 9.6	+25.1
46	———	———	———	———	-24.9	-15.8	+ 7.8
47	———	———	- 4.9	-15.5	- 9.1	- 9.1	+45.3
40	———	———	+ 3.8	+15.6	+49.7	+74.3	+94.4
50	———	- 2.9	-20.9	-10.8	+35.5	+79.7	+87.8

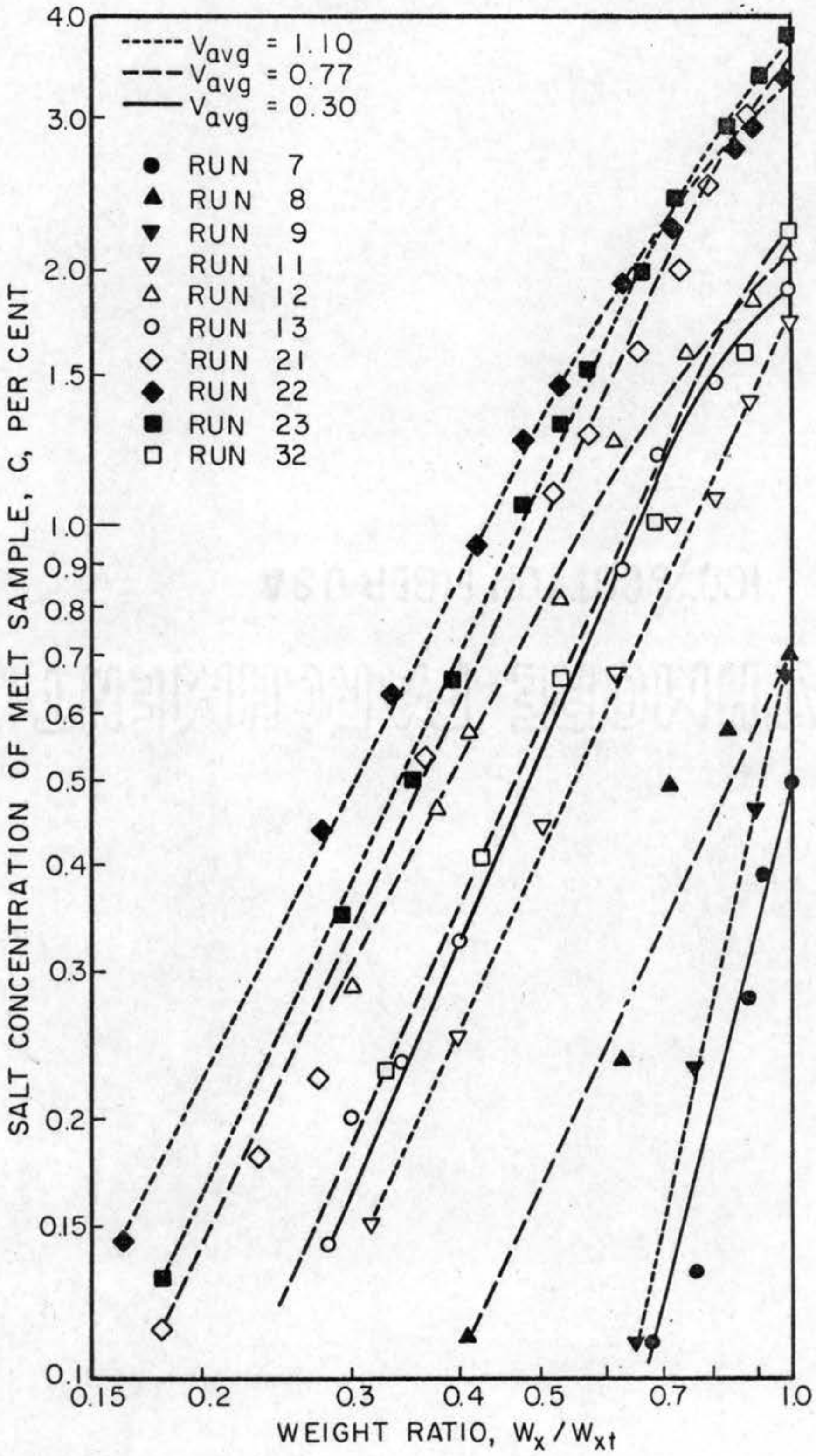

Figure 31. Experimental Weight-Concentration Data for Velocity Runs

Writing

$$\frac{W_x}{W_{xt}} = A_1\left[C_x\right]^{n_1} \tag{101}$$

the values of the slope n_1 were found to be

$n_1 = 0.493$ for $V_{avg} = 0.763$

$n_1 = 0.521$ for $V_{avg} = 1.181.$

By assuming a linear relationship

$$n_1 = a + b\,(V_{avg}) \tag{102}$$

and solving for a and b, n_1 may be written as

$$n_1 = 0.442\left[1 + 0.151\, V_{avg}\right] \tag{103}$$

The values for the temperature difference exponent, brine concentration exponent and equation constants were evaluated in the same manner as in the preceding sections. The solution is now

$$\frac{W}{W_t} \doteq 0.430\left[C\right]^{0.418(1+0.209 V_{avg})}\left[1 + 0.2\left(\frac{C}{C_b}\right)^2\right]\left[\frac{(\Delta t)^{1/2}}{C_b}\right] \tag{104}$$

It should be noted that the data for $V_{avg} = 0.3$ ft/min was omitted in deriving the equation. The reason for this was the inability to find a set of exponents for the temperature difference and brine concentration that would fit all three cases. The low velocity data were obviously in error when the terminal radius and total weight were calculated. This prohibited the extension of these data to weight-concentration calculations. Therefore, there was no loss of information by omitting these runs from the weight-concentration derivations.

Table XXV presents values of C_x and C_j for selected runs while Table XXVI presents weight-concentration data, also for selected runs. The bulk of the data may be found in Appendix H and Appendix I, respectively. Table XXVII lists the value of the percentage error, $\left[(W_{ct} - W_x)/W_c\right] \times 100$, for all runs at selected concentrations.

Discussion of Weight-Concentration Equation for Brine Velocity Runs

Table XXVI indicates the values of W_x/W_{xt} and W_c/W_{ct} are in agreement. The low velocity runs (13, 20 and 7) which were not included in the equation derivation were predictable by Equation (104). However, Table XXVII, which lists the percentage error, shows that the ability to predict the amount of sample having a particular concentration from Equation (104) was limited to runs (12, 14 and 32). It was therefore concluded that while the weight-concentration equations are certainly valid, the use of the calculated total weights to estimate the quantity of melt having a specific concentration was invalid. In summary, W_c/W_{ct} is predictable while W_c is not predictable.

The effect of the terms in Equation (104) have been discussed with the exception of the velocity variation. The

TABLE XXV

EXPERIMENTAL AND CALCULATED VALUES OF CONCENTRATIONS BY MEANS OF EQUATION 104

Run Number 20

Sample Number	W_x	C_x	C_j
1	0.1579	3.86	4.08
2	0.1504	3.52	3.81
3	0.1435	3.28	3.59
4	0.1330	2.97	3.25
5	0.1237	2.73	2.94
6	0.1153	2.39	2.62
7	0.1053	2.05	2.25
8	0.0968	1.71	1.87
9	0.0871	1.42	1.58
10	0.0792	1.14	1.30
11	0.0668	0.706	0.828
12	0.0560	0.473	0.625
13	0.0439	0.273	0.369
14	0.0298	0.138	0.157
15	0.0146	0.035	0.031

Run Number 21

Sample Number	W_x	C_x	C_j
1	0.2811	3.58	3.04
2	0.2663	3.39	2.80
3	0.2571	3.11	2.68
4	0.2428	2.85	2.46
5	0.2293	2.41	2.26
6	0.2095	2.08	1.95
7	0.1881	1.96	1.63
8	0.1801	1.65	1.52
9	0.1707	1.48	1.37
10	0.1603	1.30	1.22
11	0.1526	1.12	1.06
12	0.1429	0.980	0.970
13	0.1261	0.731	0.755
14	0.1067	0.495	0.568
15	0.0849	0.266	0.332
16	0.0646	0.176	0.176
17	0.0507	0.109	0.112
18	0.0331	0.046	0.045
19	0.0075	0.018	0.014

Run Number 32

Sample Number	W_x	C_x	C_j
1	0.4051	3.50	1.92
2	0.3978	2.64	1.87
3	0.3762	2.03	1.70
4	0.3512	1.59	1.52
5	0.3129	1.29	1.25
6	0.2820	1.02	1.04
7	0.2569	0.970	0.880
8	0.2333	0.706	0.758
9	0.2073	0.546	0.590
10	0.1843	0.455	0.478
11	0.1654	0.323	0.384
12	0.1464	0.255	0.311
13	0.1292	0.192	0.246
14	0.1041	0.118	0.163
15	0.0807	0.073	0.098
16	0.0611	0.065	0.058
17	0.0445	0.035	0.034
18	0.0242	0.028	0.010

TABLE XXVI

EXPERIMENTAL AND CALCULATED VALUES FROM WEIGHT-CONCENTRATION DATA FOR VELOCITY RUNS

C	W_x	W_c	$W_c - W_x$	% Error	$\frac{W_x}{W_{xt}}$	$\frac{W_c}{W_{ct}}$
Run Number 20						
3.00	0.1341	0.6411	+0.5070	+79.1	0.849	0.799
2.00	0.1041	0.5020	+0.3979	+79.3	0.659	0.626
1.00	0.0752	0.3540	+0.2788	+78.8	0.476	0.441
0.50	0.0576	0.2565	+0.1989	+77.5	0.365	0.320
0.10	0.0240	0.1296	+0.1056	+81.4	0.152	0.162
0.05	0.0168	0.0919	+0.0751	+81.7	0.106	0.115
0.01	0.0042	0.0449	+0.0407	+90.6	0.026	0.056
Run Number 21		Velocity 0.7684 ft/min				
2.00	0.1951	0.1663	-0.0288	-17.3	0.694	0.769
1.00	0.1442	0.1110	-0.0322	-34.1	0.513	0.514
0.50	0.1062	0.0803	-0.0259	-32.2	0.378	0.371
0.10	0.0483	0.0357	-0.0126	-35.3	0.172	0.165
0.05	0.0342	0.0256	-0.0086	-33.6	0.122	0.118
0.01	0.0104	0.0118	+0.0014	+11.8	0.037	0.054
Run Number 32						
1.00	0.2721	0.2986	+0.0265	+ 8.9	0.672	0.683
0.50	0.1962	0.2063	+0.0101	+ 4.9	0.460	0.472
0.10	0.0948	0.0898	-0.0050	- 5.6	0.234	0.206
0.05	0.0530	0.0628	+0.0098	+15.6	0.131	0.144
0.01	0.0196	0.0264	+0.0158	+39.8	0.026	0.061

TABLE XXVII

PERCENTAGE ERROR FROM EXPERIMENTAL AND CALCULATED WEIGHT CALCULATIONS FOR VARIOUS VALUES OF THE SALT CONCENTRATION IN THE MELT SAMPLE

	Concentration, Percent						
Run Number	3.00	2.00	1.00	0.50	0.10	0.05	0.01
13	-----	-----	+53.5	+49.6	+52.6	+52.3	+57.2
20	+79.1	+79.3	+78.8	+77.5	+81.4	+81.7	+90.6
7	-----	-----	-----	-----	+50.7	+46.7	+43.6
12	-----	+ 5.3	+12.1	+10.6	+ 2.7	- 5.9	+82.2
8	-----	-----	-----	-----	+47.6	+61.5	+57.3
14	-----	-----	+ 7.3	+ 5.3	- 0.5	- 0.7	+39.0
33	-----	-28.0	-34.4	-41.2	-46.0	-33.7	+33.5
21	-----	-17.3	-34.1	-32.2	-35.3	-33.6	+11.8
22	+ 8.3	+15.0	+15.6	+15.5	+19.7	+26.0	+ 7.5
11	-----	-----	+ 9.9	+ 9.2	- 0.6	- 6.4	-13.0
9	-----	-----	-----	-----	+15.0	+22.6	+46.9
32	-----	-----	+ 8.9	+ 4.9	- 5.6	+15.6	+39.8
23	-----	+21.0	+20.1	+15.0	+23.4	+44.9	+75.0

results for runs having approximately the same values of brine concentration and temperature differences are shown below:

Run Number	V_{avg}	C_b	Δt	W_x/W_{xt}	W_c/W_{ct}
21	.77	3.29	12.1	12.2	11.8
23	1.27	3.52	13.5	7.2	10.6

The decrease in the experimental yield, W_x/W_{xt}, for C = 0.05%, with velocity increase is apparent. The reason was that the higher the velocity, the smaller the sample, and in all cases small samples result in poor purification.

Also the yields were from 10 to 15 percent below the values obtained for runs with comparable properties but a stagnant field. Therefore the use of a brine velocity was found to be of no value for desalination purposes.

APPENDIX F

DERIVATION OF ENERGY EQUATIONS

To evaluate the quantity of heat required to produce a given sample the following equation was used

$$Q = mc\left[t_{out} - t_{in}\right] \tag{105}$$

where

Q = Energy required to freeze sample, Btu

m = Weight of coolant used, lbs

c = Specific heat of coolant, Btu/lb °F

t_{in} = Entering coolant temperature, °F

t_{out} = Leaving coolant temperature, °F.

The weight of coolant was measured by determining the time required for a flow of 100 cc collected at regular intervals during the experiment. Therefore the quantity of coolant flow is given by:

$$m = \frac{\rho \theta V_c}{\theta_s} \tag{106}$$

where

V_c = Volume of coolant, 100 cc
θ_s = Time required to collect 100 cc, seconds
θ = Time of run, hours
ρ = Density of coolant, lbs/ft^3

correcting units

$$m = 793.31\,(S.G.)\left[\frac{\theta}{\theta_s}\right] \qquad (107)$$

where

(S.G.) = Specific gravity of coolant

Therefore Equation (105) becomes

$$Q = 793.31\,(S.G.)\left[\frac{\theta}{\theta_s}\right](c)\left[t_{out} - t_{in}\right] \qquad (108)$$

The values of (S.G.) and (c) are calculated by two separate techniques depending on whether the coolant is brine or methanol. For brine the properties are evaluated from data presented in the brine section of the ASHRAE Guide (10). To evaluate specific values of c and S.G. the equilibrium temperature in the brine sump was used. For practical purposes constant values of c = 0.87 Btu/lb °F and S.G. = 1.08 could have been used with less than a 2% error in either term. Actually, the error would be considerably less than this amount since as S.G. increased, c decreased, and the product remains almost constant.

For the methanol calculations, values for S.G. and c were obtained from the International Critical Tables (17). The average of the methanol coolant temperatures into and out of the sphere was used to evaluate these properties. Again a value of c = 0.55 Btu/lb °F could be used for all runs with less than a 1% error and a constant value of S.G. of 0.82 would be accurate within 2%.

Since the primary interest was in the quantity of energy required to freeze a given sample the term Q/W_{xt} was plotted in Fig. 32 from calculations presented in Table XXVIII. Three separate cases are listed in this table; brine temperature, equal to, greater than, or less than the equilibrium temperature. The fourth case involves energy requirements in velocity runs which are not included in Table XXVIII.

From Fig. 32 two equations are obtained, the first

$$\frac{Q}{W_t} = 141\left[1 + 0.0872(t_b - t_e)\right] \tag{109}$$

for non-flow runs, and

$$\frac{Q}{W_t} = 120\left[1 + 1.5V_{avg}\right]\left[1 + 0.223(t_b - t_e)\right] \tag{110}$$

for velocity runs. $\frac{Q}{W_{xt}}$ is compared to $\frac{Q}{W_{ct}}$, obtained by the use of the above equations, in Table XXVIII.

Discussions of Energy Requirement Equations

To evaluate the energy required to freeze the samples more accurate temperature measurements will be required than were present in this test. For $t_b = t_e$ it is noted that

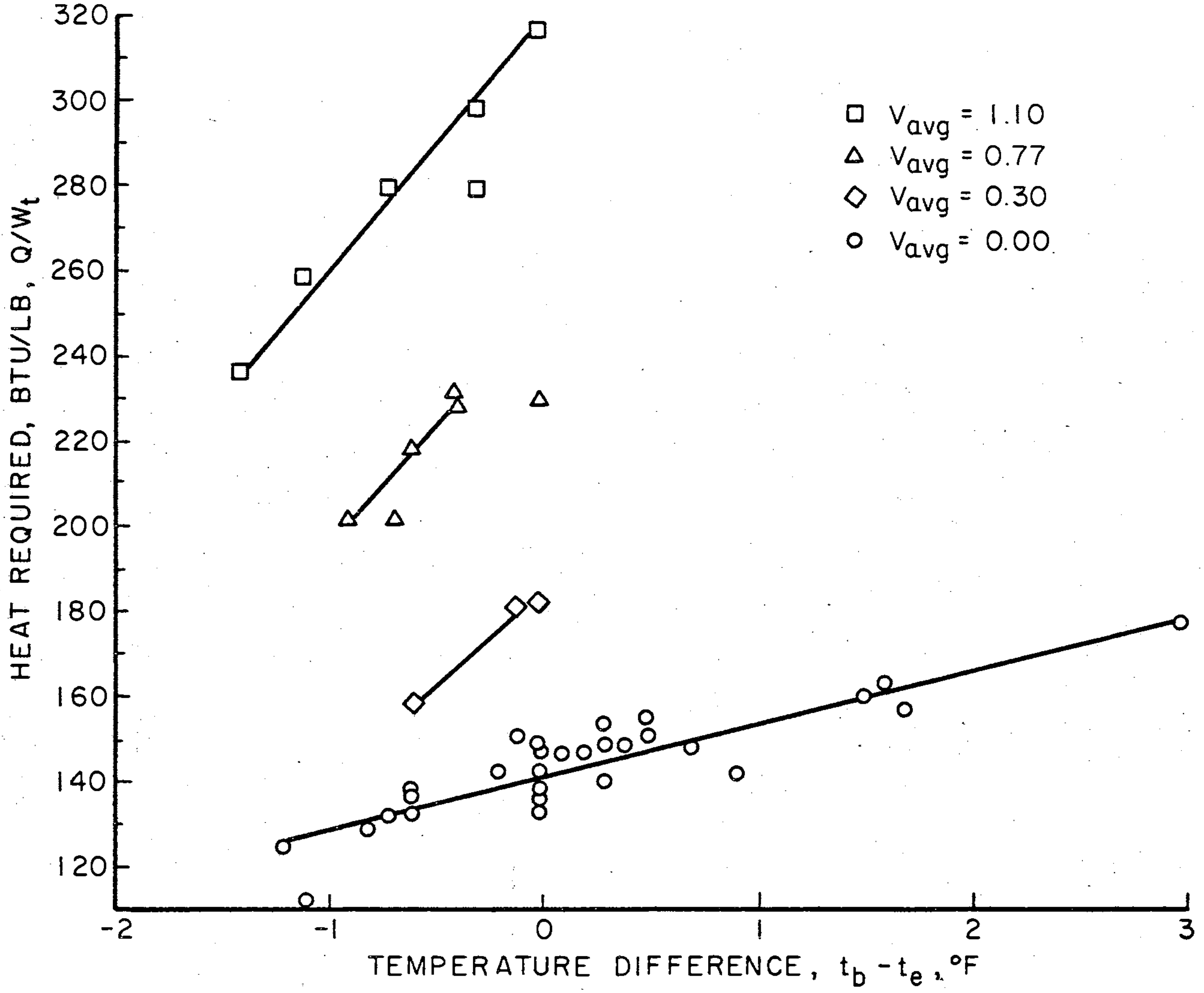

Figure 32. Heat Required Per Pound of Sample Frozen

TABLE XXVIII

EXPERIMENTAL AND CALCULATED VALUES OF THE HEAT REQUIRED TO FREEZE AN ICE SAMPLE

$t_b = t_e$

Run Number	Q_x Btu	$\frac{Q_x}{W_{xt}}$ Btu/lb	$\frac{Q_c}{W_{ct}}$ Btu/lb	Percentage Error
36	117.4	132.5	141	+ 5.7
28	55.5	134.3	141	+ 5.0
2	59.3	138.5	141	+ 2.2
41	38.0	142.8	141	- 1.4
35	75.0	146.7	141	- 4.2
42	39.2	149.0	141	- 5.7
45	37.5	149.0	141	- 5.7
		$t_b > t_e$		
37	149.3	146.8	142.2	- 3.5
49	47.9	147.4	143.4	- 2.8
30	74.2	148.9	144.7	- 2.8
39	77.2	140.4	144.7	+ 2.8
48	42.9	153.1	144.7	- 5.5
25	27.6	148.3	145.9	- 2.7
38	20.9	150.7	147.1	- 2.7
43	40.6	155.1	147.1	- 5.4
44	27.4	147.8	149.6	+ 1.3
6	45.0	142.2	152.1	+ 6.6
10	20.2	159.0	159.4	0.0
17	25.7	163.9	160.7	- 1.8
1	34.7	156.6	161.9	+ 3.1
16	10.6	177.6	177.9	0.0
		$t_b < t_e$		
24	12.7	150.7	139.8	- 7.9
40	79.7	142.6	138.5	- 2.9
15	36.5	138.7	133.6	- 3.7
19	11.8	135.9	133.6	- 1.5
27	12.5	138.8	133.6	- 3.7
31	8.3	139.9	133.6	- 4.5
34	122.0	136.4	133.6	- 2.2
47	48.8	133.4	133.6	0.0
46	40.4	131.9	132.4	0.0
26	30.6	129.2	131.2	+ 1.5
29	57.8	110.6	127.5	+13.3
50	38.0	124.6	121.1	- 3.3

Q/W_t varies from 132.5 to 149.0 Btu/lb with a mean value of 141.5 Btu/lb. Since these runs are for pure water the value should be 144.3 Btu/lb, and if heat capacity effects were included the value would be higher. One reason that the mean value was low was the location of the coolant thermocouples. With the thermocouples placed at the sphere entrance and in the coolant exit line, the freezing obtained from the exposed coolant lines was not accounted for in the calculations. While the mean value was reasonable the spread on both sides was large. The two higher velocity runs also had points with relatively large errors.

In spite of these shortcomings several factors of importance are readily seen from the equations or curves. As noted in the weight-concentration section, the use of a velocity field results in poor yields of pure water. In this section it was found that the formation of these inferior samples required the expenditure of excessive thermal energy.

As expected when the brine temperature was less than the equilibrium temperature a smaller quantity of energy was required to freeze the sample. Since this was also the direction of increasing yields the advantages are obvious. Two precautions should be noted in the above. The possibility exists that if the brine temperature is too low a rapid dendrite growth occurs which may reduce the quantity of pure water obtained. This factor was not investigated in this study and the actual results are not known. Second, energy must be removed to cool the entire brine sample and the total coolant requirements may exceed those for higher brine temperature tests.

APPENDIX G

AVERAGE SALT CONCENTRATION OF MELTED SAMPLE

One way of evaluating the advantage of the fractional method of melting, used in this work, was to determine the average sodium chloride concentrations that would be obtained from melting the entire sample simultaneously. The average concentration was calculated by summing the product of the incremental sample weight and its concentration. The above sum was divided by the entire sample weight to obtain the average concentration, C_{avg}. These values are presented in Table XXIX. This data is also plotted in Fig. 33 and the equation

$$C_{avg} = 0.342 C_b \tag{111}$$

was derived to express the relationship between C_{avg} and C_b. The accuracy of this equation is indicated in Table XXV where the percentage error is

$$\left[\frac{C_{e\text{-}avg} - C_{x\text{-}avg}}{C_{e\text{-}avg}}\right] \times 100 \tag{112}$$

Equation (111) verifies the statement that it is impossible to obtain pure water from the ordinary one step freezing process. If the entire sample were melted without fractional separation a 3.5 percent sample would be reduced

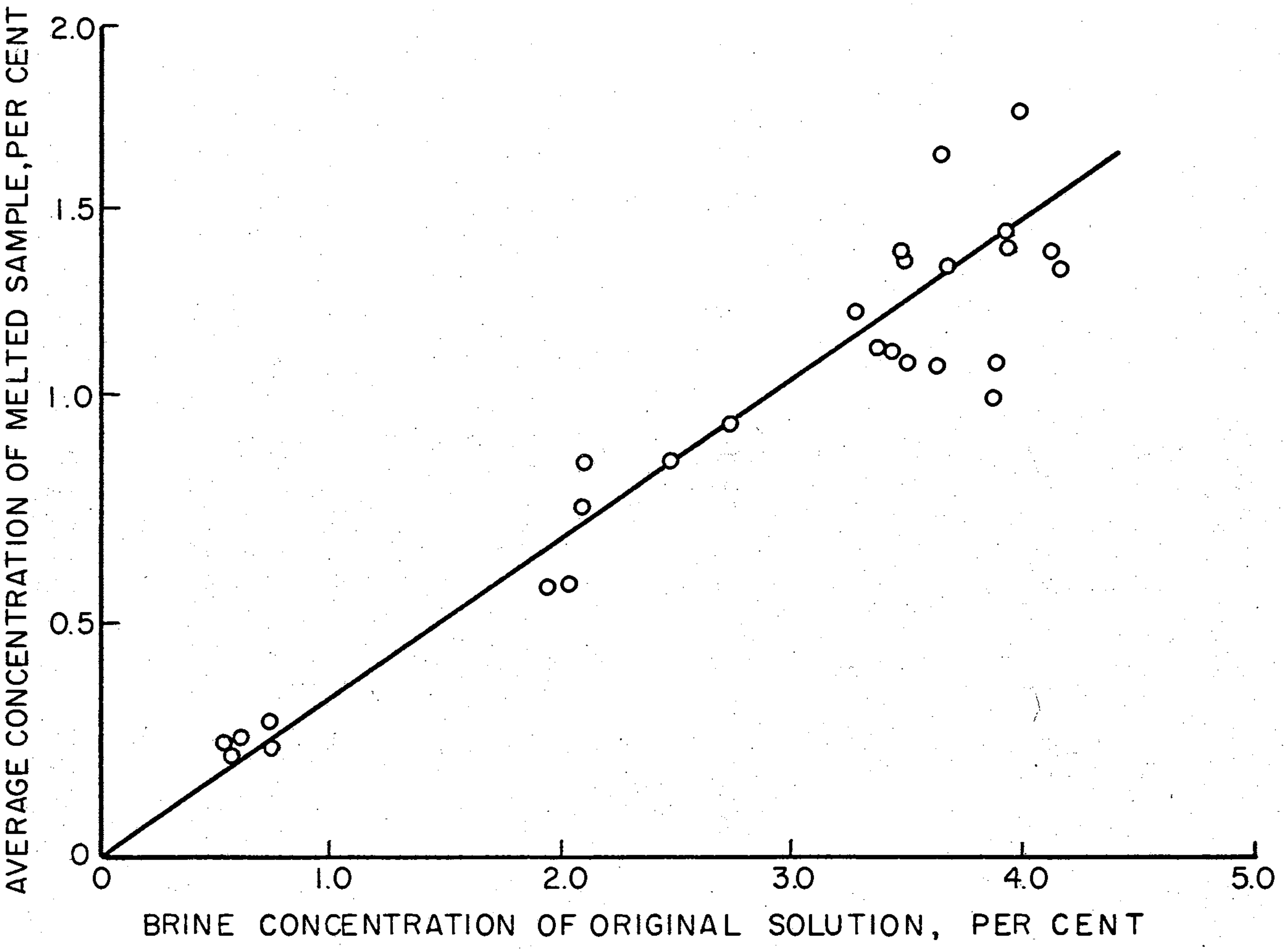

Figure 33. Brine Concentration of Original Solution Versus Average Concentration

TABLE XXIX

AVERAGE CONCENTRATION AS CALCULATED BY MEANS OF EQUATION 111

Run Number	C_b	C_{x-avg}	C_{c-avg}	Percentage Error
6	0.58	0.225	0.198	-13.6
46	0.58	0.229	0.198	-15.7
5	0.63	0.242	0.215	-12.6
38	0.67	0.235	0.229	- 2.6
37	0.75	0.229	0.256	+10.2
47	1.94	0.564	0.663	+14.9
10	2.08	0.590	0.711	+17.0
43	2.09	0.750	0.715	- 4.9
40	2.48	0.843	0.848	+ 0.6
49	2.74	0.934	0.937	+ 0.3
31	3.16	1.246	1.081	-15.3
44	3.28	1.188	1.122	- 5.9
25	3.37	1.095	1.153	- 5.0
48	3.43	1.069	1.173	- 8.9
29	3.45	1.065	1.180	-10.5
16	3.47	1.380	1.187	+16.3
15	3.48	1.275	1.190	+ 7.1
34	3.62	1.051	1.238	+15.1
24	3.62	1.524	1.238	-23.1
19	3.64	1.508	1.245	-21.1
28	3.66	1.274	1.252	- 1.8
49	3.88	0.991	1.327	+25.3
50	3.89	0.956	1.330	+28.1
26	3.92	1.337	1.341	+ 0.3
17	3.93	1.312	1.344	+ 2.3
27	3.98	1.605	1.361	-17.9

to approximately 1.2 percent sodium chloride concentration in the first step. By fractional separation, as much as 30% of the total weight was drawn off as water with an impurity content of less than 500 ppm.

The advantage in collecting the entire sample at one time is that higher chamber temperatures may be used, avoiding the necessity of long melting periods.

APPENDIX H

EXPERIMENTAL AND CALCULATED VALUES OF SALT CONCENTRATION

TABLE XXX

EXPERIMENTAL AND CALCULATED VALUES OF C_x AND C_j BY MEANS OF EQUATION 100 FOR $t_b = t_e$ AND $t_b < t_e$

Run number 5

Sample Number	W_x	C_x	C_j
1	0.1643	0.658	0.774
2	0.1369	0.437	0.501
3	0.1157	0.310	0.330
4	0.0944	0.161	0.181
5	0.0719	0.073	0.086
6	0.0478	0.018	0.024
7	0.0229	0.007	-----
8	0.0084	0.000	-----

Run Number 19

Sample Number	W_x	C_x	C_j
1	0.0873	3.70	4.00
2	0.0807	3.37	3.62
3	0.0743	3.03	3.18
4	0.0644	2.35	2.45
5	0.0560	1.76	1.82
6	0.0478	1.15	1.23
7	0.0386	0.617	0.632
8	0.0304	0.365	0.343
9	0.0229	0.186	0.150
10	0.0161	0.090	0.050
11	0.0090	0.015	-----
12	0.0026	0.023	-----

Run Number 24

Sample Number	W_x	C_x	C_j
1	0.0844	3.63	3.88
2	0.0783	3.43	3.52
3	0.0728	3.30	3.14
4	0.0668	2.59	2.71
5	0.0615	2.24	2.27
6	0.0545	1.82	1.74
7	0.0478	1.43	1.30
8	0.0404	1.00	0.791
9	0.0340	0.648	0.490
10	0.0278	0.380	0.275
11	0.0209	0.205	0.133
12	0.0119	0.072	0.021
13	0.0031	0.006	-----

Run Number 26

Sample Number	W_x	C_x	C_j
1	0.2370	4.04	3.77
2	0.2357	3.34	3.74
3	0.2216	3.33	3.40
4	0.2084	2.94	3.02
5	0.1965	2.67	2.70
6	0.1841	2.43	2.41
7	0.1691	2.08	1.96
8	0.1537	1.68	1.54
9	0.1405	1.40	1.22
10	0.1303	1.17	1.00
11	0.1210	0.929	0.809
12	0.1056	0.538	0.552
13	0.0913	0.393	0.361
14	0.0754	0.222	0.200
15	0.0611	0.140	0.118
16	0.0426	0.110	0.036
17	0.0291	0.040	0.012
18	0.0079	0.028	-----

TABLE XXX (Continued)

Run Number 27

Sample Number	W_x	C_x	C_j
1	0.0904	4.01	------
2	0.0893	3.83	------
3	0.0858	3.60	------
4	0.0798	3.20	3.80
5	0.0714	2.83	3.14
6	0.0564	1.74	1.91
7	0.0456	1.08	1.12
8	0.0328	0.227	0.439
9	0.0194	0.062	0.090
10	0.0093	0.005	0.001

Run Number 29

Sample Number	W_x	C_x	C_j
1	0.5232	3.96	2.83
2	0.5131	3.00	2.73
3	0.4827	2.88	2.48
4	0.4589	2.61	2.22
5	0.4322	2.28	1.95
6	0.4068	2.08	1.69
7	0.3850	1.68	1.53
8	0.3557	1.17	1.22
9	0.3235	1.06	0.952
10	0.2908	0.744	0.696
11	0.2642	0.534	0.534
12	0.2324	0.365	0.392
13	0.2073	0.228	0.264
14	0.1795	0.149	0.171
15	0.1486	0.094	0.095
16	0.1158	0.044	0.040
17	0.0847	0.028	0.018
18	0.0505	0.005	------
19	0.0209	0.000	------

Run Number 34

Sample Number	W_x	C_x	C_j
1	0.8946	3.82	2.69
2	0.8626	3.58	2.52
3	0.8156	2.68	2.25
4	0.7731	2.30	2.00
5	0.7360	2.01	1.78
6	0.6853	1.78	1.50
7	0.6401	1.51	1.26
8	0.5900	1.15	1.01
9	0.5367	0.834	0.784
10	0.4855	0.606	0.586
11	0.4377	0.464	0.434
12	0.3912	0.327	0.314
13	0.3451	0.208	0.212
14	0.2975	0.127	0.139
15	0.2553	0.086	0.085
16	0.2090	0.048	0.046
17	0.1647	0.035	0.023
18	0.1195	0.011	0.010
19	0.0869	0.007	------
20	0.0443	0.003	------
21	0.0128	0.000	------

Run Number 40 (Rapid Melt)

Sample Number	W_x	C_x	C_j
1	0.5590	2.68	1.99
2	0.5524	1.92	1.77
3	0.4831	1.58	1.49
4	0.4394	1.27	1.21
5	0.3988	1.02	0.958
6	0.3519	0.864	0.692
7	0.3045	0.721	0.461
8	0.2540	0.467	0.276
9	0.2070	0.281	0.137
10	0.1618	0.153	0.069
11	0.1118	0.124	0.023
12	0.0611	0.061	------
13	0.0201	0.042	------

TABLE XXX (Continued)

Run Number 42

Sample Number	W_x	C_x	C_j
1	0.2633	2.17	2.20
2	0.2595	2.07	2.09
3	0.2377	1.73	1.69
4	0.2121	1.48	1.47
5	0.1817	1.14	1.11
6	0.1603	0.885	0.863
7	0.1224	0.359	0.396
8	0.0763	0.148	0.129
9	0.0452	0.059	0.053
10	0.0267	0.023	0.016
11	0.0137	0.007	-----

Run Number 46

Sample Number	W_x	C_x	C_j
1	0.3656	0.522	0.452
2	0.2789	0.405	0.407
3	0.2478	0.356	0.354
4	0.2121	0.312	0.290
5	0.1784	0.248	0.230
6	0.1495	0.191	0.164
7	0.1149	0.097	0.097
8	0.0851	0.069	0.079
9	0.0615	0.040	0.049
10	0.0348	0.022	0.029
11	0.0057	0.000	-----

Run Number 50 (Rapid Melt)

Sample Number	W_x	C_x	C_j
1	0.3050	3.00	2.10
2	0.2979	2.42	2.03
3	0.2772	2.02	1.84
4	0.2509	1.50	1.62
5	0.2302	1.30	1.44
6	0.1629	0.667	0.884
7	0.1376	0.521	0.690
8	0.1120	0.493	0.512
9	0.0860	0.362	0.350
10	0.0617	0.265	0.212
11	0.0419	0.152	0.096
12	0.0238	0.103	0.050
13	0.0062	0.070	-----

TABLE XXXI

EXPERIMENTAL AND CALCULATED VALUES OF C_x AND C_j BY MEANS OF EQUATION 96
$t_b > t_e$

Run Number 6

Sample Number	W_x	C_x	C_j
1	0.3164	0.848	0.580
2	0.2926	0.639	0.494
3	0.2761	0.432	0.436
4	0.2527	0.357	0.351
5	0.2351	0.307	0.299
6	0.2106	0.215	0.220
7	0.1879	0.175	0.164
8	0.1550	0.112	0.091
9	0.1261	0.057	0.058
10	0.1049	0.038	0.042
11	0.0825	0.014	0.016
12	0.0567	0.003	0.005
13	0.0366	0.001	0.001
14	0.0207	0.000	0.000

Run Number 16

Sample Number	W_x	C_x	C_j
1	0.0595	4.33	3.47
2	0.0546	3.25	3.02
3	0.0470	1.68	2.02
4	0.0344	1.02	0.905
5	0.0240	0.396	0.354
6	0.0161	0.183	0.110
7	0.0062	0.054	0.006

Run Number 25

Sample Number	W_x	C_x	C_j
1	0.1863	3.42	3.37
2	0.1812	3.32	3.24
3	0.1751	2.87	3.00
4	0.1689	2.66	2.74
5	0.1621	2.45	2.52
6	0.1546	2.23	2.24
7	0.1458	1.99	1.96
8	0.1356	1.71	1.62
9	0.1288	1.39	1.42
10	0.1152	1.13	1.05
11	0.1061	0.921	0.842
12	0.0964	0.632	0.626
13	0.0858	0.491	0.472
14	0.0721	0.287	0.295
15	0.0556	0.157	0.142
16	0.0399	0.067	0.057
17	0.0198	0.012	0.007

Run Number 30

Sample Number	W_x	C_x	C_j
1	0.4983	3.83	3.94
2	0.4656	3.33	3.38
3	0.4323	2.90	2.92
4	0.4015	2.43	2.40
5	0.3631	1.98	1.89
6	0.3221	1.49	1.37
7	0.2841	0.993	0.970
8	0.2479	0.645	0.672
9	0.2111	0.417	0.436
10	0.1758	0.247	0.260
11	0.1344	0.110	0.120
12	0.0907	0.038	0.038
13	0.0495	0.013	0.007
14	0.0289	0.010	0.002

TABLE XXXI (Continued)

Run Number 38

Sample Number	W_x	C_x	C_j
1	0.1385	0.638	0.670
2	0.1237	0.535	0.535
3	0.1096	0.404	0.408
4	0.0968	0.276	0.276
5	0.0783	0.159	0.164
6	0.0584	0.083	0.076
7	0.0441	0.042	0.033
8	0.0278	0.015	0.009
9	0.0141	0.012	0.001

Run Number 39

Sample Number	W_x	C_x	C_j
1	0.5499	3.65	2.74
2	0.5365	2.97	2.63
3	0.5098	2.59	2.35
4	0.4844	2.19	2.11
5	0.4542	1.97	1.81
6	0.4198	1.64	1.49
7	0.3872	1.33	1.22
8	0.3407	0.947	0.865
9	0.3052	0.703	0.644
10	0.2679	0.468	0.458
11	0.2249	0.263	0.282
12	0.1850	0.138	0.162
13	0.1438	0.075	0.078
14	0.1056	0.038	0.032
15	0.0789	0.020	0.014
16	0.0276	0.000	0.001

Run Number 44

Sample Number	W_x	C_x	C_j
1	0.1857	2.79	2.64
2	0.1700	2.51	2.58
3	0.1433	1.74	1.64
4	0.1171	1.23	1.08
5	0.1017	0.933	0.842
6	0.0798	0.521	0.512
7	0.0357	0.061	0.086
8	0.0139	0.037	0.010

Run Number 48

Sample Number	W_x	C_x	C_j
1	0.2800	2.82	2.26
2	0.2657	2.23	1.91
3	0.2348	1.89	1.62
4	0.2068	1.55	1.37
5	0.1801	1.24	1.20
6	0.1513	0.858	0.892
7	0.1164	0.533	0.624
8	0.0902	0.352	0.427
9	0.0646	0.195	0.168
10	0.0410	0.098	0.131
11	0.0207	0.052	0.080
12	0.0073	0.000	0.010

TABLE XXXI (Continued)

Run Number 49 (Rapid Melt)

Sample Number	W_x	C_x	C_j
1	0.3248	2.83	2.32
2	0.3219	2.37	2.14
3	0.2979	2.00	1.97
4	0.2714	1.65	1.73
5	0.2423	1.22	1.44
6	0.2084	1.01	1.16
7	0.1773	0.945	0.952
8	0.1405	0.708	0.701
9	0.1102	0.493	0.521
10	0.0840	0.337	0.372
11	0.0571	0.132	0.105
12	0.0337	0.076	0.052
13	0.0130	0.019	0.010

TABLE XXXII

EXPERIMENTAL AND CALCULATED VALUES OF C_x AND C_j BY MEANS OF EQUATION 104, FOR VELOCITY RUNS

Run Number 7

Sample Number	W_x	C_x	C_j
1	0.2827	0.682	0.219
2	0.2661	0.392	0.191
3	0.2498	0.254	0.169
4	0.2306	0.187	0.146
5	0.2095	0.127	0.112
6	0.1949	0.106	0.094
7	0.1757	0.085	0.077
8	0.1552	0.062	0.058
9	0.1378	0.041	0.044
10	0.1197	0.029	0.032
11	0.0997	0.018	0.021
12	0.0833	0.012	0.014
13	0.0664	0.008	-----
14	0.0520	0.003	-----
15	0.0351	0.000	-----
16	0.0139	0.000	-----

Run Number 8

Sample Number	W_x	C_x	C_j
1	0.1484	0.662	0.219
2	0.1244	0.591	0.123
3	0.1045	0.522	0.109
4	0.0880	0.210	0.075
5	0.0609	0.106	0.035
6	0.0289	0.050	-----
7	0.0192	0.022	-----
8	0.0112	0.000	-----

Run Number 9

Sample Number	W_x	C_x	C_j
1	0.1299	0.682	0.290
2	0.1153	0.437	0.148
3	0.1008	0.216	0.083
4	0.0840	0.104	0.049
5	0.0639	0.067	0.039
6	0.0564	0.053	0.024
7	0.0445	0.045	0.015
8	0.0340	0.032	-----
9	0.0205	0.016	-----
10	0.0060	0.000	-----

Run Number 11

Sample Number	W_x	C_x	C_j
1	0.1250	1.83	1.51
2	0.1144	1.47	1.32
3	0.1045	1.17	1.14
4	0.0900	0.897	0.902
5	0.0767	0.676	0.678
6	0.0624	0.448	0.469
7	0.0496	0.244	0.307
8	0.0412	0.155	0.213
9	0.0289	0.087	0.107
10	0.0214	0.039	0.059
11	0.0112	0.011	0.017
12	0.0029	0.000	-----

TABLE XXXII (Continued)

Run Number 12

Sample Number	W_x	C_x	C_j
1	0.0957	2.10	2.35
2	0.0873	1.85	2.13
3	0.0754	1.63	1.74
4	0.0650	1.51	1.42
5	0.0536	1.06	1.02
6	0.0498	0.762	0.747
7	0.0397	0.561	0.581
8	0.0291	0.284	0.306
9	0.0163	0.063	0.092
10	0.0026	0.023	0.041

Run Number 13

Sample Number	W_x	C_x	C_j
1	0.1647	1.92	1.81
2	0.1462	1.79	1.51
3	0.1327	1.49	1.28
4	0.1182	1.27	1.04
5	0.1054	0.924	0.764
6	0.0915	0.653	0.622
7	0.0794	0.481	0.466
8	0.0708	0.375	0.364
9	0.0580	0.239	0.236
10	0.0483	0.154	0.158
11	0.0355	0.075	0.079
12	0.0225	0.024	0.027
13	0.0051	0.000	————

Run Number 14

Sample Number	W_x	C_x	C_j
1	0.1627	2.32	1.80
2	0.1424	1.91	1.35
3	0.1297	1.40	1.27
4	0.1102	1.01	0.941
5	0.0895	0.692	0.636
6	0.0725	0.432	0.412
7	0.0538	0.218	0.226
8	0.0364	0.092	0.098
9	0.0218	0.028	0.033
10	0.0060	0.008	————

Run Number 22

Sample Number	W_x	C_x	C_j
1	0.0882	3.46	3.85
2	0.0825	2.98	3.56
3	0.0730	2.74	3.07
4	0.0664	2.43	2.74
5	0.0589	2.05	2.28
6	0.0512	1.67	1.83
7	0.0449	1.38	1.47
8	0.0397	1.06	1.19
9	0.0300	0.647	0.711
10	0.0234	0.363	0.443
11	0.0147	0.148	0.178
12	0.0042	0.011	0.016

TABLE XXXII (Continued)

Run Number 23

Sample Number	W_x	C_x	C_j
1	0.1585	3.04	3.12
2	0.1495	2.74	2.91
3	0.1424	2.62	2.73
4	0.1343	2.42	2.49
5	0.1272	2.25	2.32
6	0.1188	2.06	2.11
7	0.1116	1.82	1.89
8	0.0994	1.51	1.56
9	0.0895	1.26	1.31
10	0.0800	1.02	1.08
11	0.0697	0.761	0.839
12	0.0595	0.477	0.542
13	0.0465	0.259	0.396
14.	0.0235	0.104	0.125

Run Number 33

Sample Number	W_x	C_x	C_j
1	0.4128	3.19	2.86
2	0.4004	2.95	2.74
3	0.3678	2.54	2.45
4	0.3352	2.25	2.12
5	0.3074	1.93	1.85
6	0.2721	1.54	1.50
7	0.2390	1.21	1.17
8	0.2134	0.944	0.933
9	0.1910	0.741	0.749
10	0.1663	0.535	0.563
11	0.1305	0.295	0.347
12	0.1058	0.212	0.223
13	0.0732	0.087	0.100
14	0.0496	0.036	0.044
15	0.0298	0.025	0.016
16	0.0137	0.012	—

APPENDIX I

SAMPLE WEIGHTS FOR SELECTED VALUES OF SALT CONCENTRATION

TABLE XXXIII

EXPERIMENTAL AND CALCULATED VALUES FROM WEIGHT-CONCENTRATION DATA FOR

$t_b = t_e$ $t_b < t_e$

C	W_x	W_c	$W_c - W_x$	% Error	W_x/W_{xt}	W_c/W_{ct}
Run Number 5						
0.50	0.1437	0.1648	+0.0211	+ 12.8	0.875	0.882
0.10	0.0787	0.0899	+0.0112	+ 12.4	0.479	0.481
0.05	0.0598	0.0680	+0.0082	+ 12.1	0.364	0.364
0.01	0.0298	0.0397	+0.0099	+ 24.9	0.181	0.213
Run Number 19						
3.00	0.0739	0.0675	-0.0064	- 9.5	0.846	0.820
2.00	0.0593	0.0551	-0.0042	- 7.6	0.679	0.669
1.00	0.0452	0.0418	-0.0034	- 8.1	0.518	0.508
0.50	0.0337	0.0328	-0.0009	- 2.7	0.386	0.399
0.10	0.0170	0.0191	+0.0021	+ 11.0	0.195	0.233
0.05	0.0123	0.0152	+0.0029	+ 19.1	0.141	0.184
0.01	0.0000	0.0088	+0.0088	+100.0	0.000	0.108
Run Number 24						
3.00	0.0703	0.0755	+0.0052	+ 6.9	0.833	0.820
2.00	0.0679	0.0616	-0.0063	- 10.2	0.805	0.669
1.00	0.0514	0.0468	-0.0046	- 9.8	0.609	0.508
0.50	0.0402	0.0367	-0.0035	- 9.5	0.476	0.399
0.10	0.0236	0.0214	-0.0022	- 10.3	0.280	0.233
0.05	0.0187	0.0170	-0.0017	- 10.0	0.221	0.184
0.01	0.0110	0.0099	-0.0011	- 11.1	0.130	0.108
Run Number 26						
3.00	0.2103	0.3061	+0.0958	+ 31.3	0.887	0.896
2.00	0.1713	0.2519	+0.0806	+ 32.0	0.718	0.738
1.00	0.1246	0.1925	+0.0679	+ 35.2	0.526	0.564
0.50	0.1000	0.1512	+0.0512	+ 42.2	0.422	0.443
0.10	0.0584	0.0882	+0.0298	+ 33.8	0.246	0.258
0.05	0.0443	0.0700	+0.0257	+ 36.7	0.187	0.205
0.01	0.0117	0.0400	+0.0283	+ 70.8	0.049	0.120

TABLE XXXIII (Continued)

C	W_x	W_c	$W_c - W_x$	% Error	W_x/W_{xt}	W_c/W_{ct}
Run Number 27						
3.00	0.0767	0.0679	-0.0088	- 13.0	0.848	0.772
2.00	0.0600	0.0560	-0.0040	- 7.1	0.664	0.636
1.00	0.0445	0.0428	-0.0017	- 4.0	0.492	0.487
0.50	0.0369	0.0336	-0.0033	- 9.8	0.408	0.381
0.10	0.0225	0.0196	-0.0029	- 14.8	0.249	0.223
0.05	0.0172	0.0156	-0.0016	- 10.3	0.190	0.177
0.01	0.0101	0.0091	-0.0010	- 11.0	0.112	0.103
Run Number 34						
2.00	0.7338	0.7832	+0.0500	+ 6.3	0.820	0.867
1.00	0.5649	0.5950	+0.0310	+ 5.2	0.631	0.659
0.50	0.4501	0.4670	+0.0169	+ 3.6	0.503	0.517
0.10	0.2697	0.2721	+0.0024	+ 0.9	0.301	0.301
0.05	0.2115	0.2159	+0.0044	+ 2.0	0.236	0.239
0.01	0.1114	0.1262	+0.0148	+ 11.7	0.124	0.140
Run Number 40						
1.00	0.3929	0.4085	+0.0156	+ 3.8	0.703	0.731
0.50	0.2672	0.3165	+0.0493	+ 15.6	0.478	0.566
0.10	0.0924	0.1837	+0.0913	+ 49.7	0.165	0.329
0.05	0.0375	0.1457	+0.1082	+ 74.3	0.067	0.261
0.01	0.0048	0.0852	+0.0804	+ 94.4	0.009	0.152
Run Number 42						
1.00	0.1702	0.1749	+0.0047	+ 2.7	0.646	0.663
0.50	0.1325	0.1224	-0.0101	- 8.3	0.503	0.464
0.10	0.0595	0.0602	+0.0007	+ 1.2	0.226	0.228
0.05	0.0406	0.0449	+0.0043	+ 9.6	0.154	0.170
0.01	0.0161	0.0215	+0.0054	+ 25.1	0.061	0.082
Run Number 46						
0.10	0.1160	0.0929	-0.0231	- 24.9	0.378	0.312
0.05	0.0697	0.0602	-0.0095	- 15.8	0.227	0.202
0.01	0.0190	0.0206	+0.0016	+ 7.8	0.062	0.070
Run Number 50						
2.00	0.2763	0.2684	-0.0079	- 2.9	0.906	0.933
1.00	0.1969	0.1628	-0.0341	- 20.9	0.646	0.565
0.50	0.1125	0.1015	-0.0110	- 10.8	0.369	0.353
0.10	0.0223	0.0346	+0.0123	+ 35.5	0.073	0.120
0.05	0.0044	0.0217	+0.0173	+ 79.7	0.014	0.076
0.01	0.0009	0.0074	+0.0065	+ 87.8	0.003	0.026

TABLE XXXIV

EXPERIMENTAL AND CALCULATED VALUES FROM WEIGHT-CONCENTRATION DATA

$t_b > t_e$

C	W_x	W_c	$W_c - W_x$	% Error	W_x/W_{xt}	W_c/W_{ct}
Run Number 6						
0.50	0.2800	0.3054	+0.0254	+ 8.3	0.885	0.951
0.10	0.1508	0.1598	+0.0090	+ 5.6	0.476	0.487
0.05	0.1184	0.1252	+0.0068	+ 5.4	0.374	0.381
0.01	0.0722	0.0722	-0.0007	- 1.0	0.230	0.220
Run Number 16						
3.00	0.0535	0.0643	+0.0108	+16.8	0.895	0.933
2.00	0.0485	0.0534	+0.0049	+ 9.2	0.814	0.775
1.00	0.0342	0.0406	+0.0064	+15.8	0.574	0.589
0.50	0.0257	0.0312	+0.0055	+17.6	0.432	0.453
0.10	0.0097	0.1805	+0.0083	+46.1	0.163	0.262
0.05	0.0057	0.1438	+0.0086	+60.1	0.096	0.207
0.01	0.0011	0.0083	+0.0072	+86.7	0.018	0.121
Run Number 25						
3.00	0.1771	0.1989	+0.0218	+11.0	0.951	0.946
2.00	0.1462	0.1661	+0.0199	+12.0	0.785	0.790
1.00	0.1096	0.1256	+0.0160	+12.7	0.588	0.597
0.50	0.0869	0.0970	+0.0101	+10.4	0.466	0.462
0.10	0.0545	0.0556	+0.0011	+ 2.0	0.293	0.265
0.05	0.0337	0.0440	+0.0103	+23.4	0.181	0.209
0.01	0.0165	0.0257	+0.0097	+37.7	0.089	0.122
Run Number 30						
3.00	0.4289	0.4178	-0.0111	- 2.7	0.900	0.870
2.00	0.3557	0.3480	-0.0077	- 2.2	0.747	0.725
1.00	0.2776	0.2656	-0.0120	- 4.5	0.583	0.533
0.50	0.2183	0.2063	-0.0120	- 5.8	0.438	0.430
0.10	0.1250	0.1188	-0.0062	- 5.2	0.262	0.247
0.05	0.0953	0.0940	-0.0013	- 1.4	0.200	0.196
0.01	0.0280	0.0547	+0.0267	+48.4	0.059	0.114
Run Number 38						
0.50	0.1208	0.1239	+0.0031	+ 2.5	0.872	0.872
0.10	0.0628	0.0657	+0.0024	+ 3.7	0.454	0.462
0.05	0.0470	0.0514	+0.0044	+ 8.6	0.339	0.362
0.01	0.0116	0.0298	+0.0182	+61.0	0.114	0.131

TABLE XXXIV (Continued)

C	W_x	W_c	$W_c - W_x$	% Error	W_x/W_{xt}	W_c/W_{ct}
Run Number 39						
2.00	0.4584	0.4952	+0.0268	+15.5	0.816	0.864
1.00	0.3468	0.3633	+0.0165	+ 4.5	0.617	0.647
0.50	0.2712	0.2794	+0.0082	+ 2.9	0.483	0.497
0.10	0.1601	0.1597	-0.0004	- 0.2	0.285	0.284
0.05	0.1180	0.1261	+0.0081	+ 6.4	0.210	0.224
0.01	0.0642	0.0736	+0.0094	+12.8	0.114	0.131
Run Number 44						
2.00	0.1523	0.1651	+0.0128	+ 7.6	0.820	0.862
1.00	0.1052	0.1154	+0.0102	+ 8.8	0.567	0.603
0.50	0.0761	0.0827	+0.0066	+ 8.0	0.410	0.432
0.10	0.0395	0.0396	+0.0001	+ 0.2	0.213	0.207
0.05	0.0258	0.0290	+0.0032	+11.0	0.139	0.151
0.01	0.0037	0.0142	+0.0105	+73.9	0.020	0.074
Run Number 48						
2.00	0.2448	0.2776	+0.0328	+11.8	0.874	0.986
1.00	0.1618	0.1688	+0.0050	+ 3.0	0.578	0.592
0.50	0.1105	0.1025	-0.0080	- 7.8	0.394	0.364
0.10	0.0410	0.0343	-0.0067	-19.5	0.146	0.122
0.05	0.0198	0.0215	+0.0017	+ 7.9	0.071	0.077
0.01	0.0040	0.0074	+0.0034	+45.9	0.014	0.026
Run Number 49 (Rapid Melt)						
2.00	0.2979	0.2987	+0.0008	+ 0.3	0.917	0.937
1.00	0.2031	0.1804	-0.0227	-12.6	0.625	0.565
0.50	0.1064	0.1112	+0.0048	+ 4.3	0.328	0.349
0.10	0.0549	0.0374	-0.0175	-46.8	0.169	0.117
0.05	0.0329	0.0235	-0.0094	-40.0	0.101	0.074
0.01	0.0130	0.0080	-0.0050	-62.5	0.040	0.025

TABLE XXXV

EXPERIMENTAL AND CALCULATED VALUES FROM WEIGHT-CONCENTRATION DATA FOR VELOCITY RUNS

C	W_x	W_c	$W_c - W_x$	% Error	W_x/W_{xt}	W_c/W_{ct}
Run Number 7						
0.10	0.1894	0.3846	+0.1952	+50.7	0.670	0.825
0.05	0.1453	0.2727	+0.1274	+46.7	0.514	0.585
0.01	0.0747	0.0578	+0.0578	+43.6	0.264	0.284
Run Number 8						
0.10	0.0573	0.1093	+0.0520	+47.6	0.386	0.692
0.05	0.0288	0.0749	+0.0461	+61.5	0.194	0.474
0.01	0.0147	0.0345	+0.0198	+57.3	0.099	0.218
Run Number 9						
0.10	0.0818	0.0962	+0.0144	+15.0	0.630	0.661
0.05	0.0520	0.0672	+0.0152	+22.6	0.400	0.462
0.01	0.0150	0.0283	+0.0133	+46.9	0.116	0.194
Run Number 11						
1.00	0.0953	0.1058	+0.0105	+ 9.9	0.762	0.718
0.50	0.0654	0.0720	+0.0066	+ 9.2	0.523	0.489
0.10	0.0313	0.0311	-0.0002	- 0.6	0.250	0.211
0.05	0.0232	0.0218	-0.0014	- 6.4	0.186	0.148
0.01	0.0104	0.0092	-0.0012	-13.0	0.083	0.062
Run Number 12						
2.00	0.0924	0.0976	+0.0052	+ 5.3	0.965	0.983
1.00	0.0529	0.0602	+0.0073	+12.1	0.553	0.606
0.50	0.0362	0.0405	+0.0043	+10.6	0.378	0.408
0.10	0.0183	0.0188	+0.0005	+ 2.7	0.191	0.190
0.05	0.0143	0.0135	-0.0008	- 5.9	0.149	0.136
0.01	0.0011	0.0062	+0.0051	+82.2	0.011	0.063
Run Number 13						
1.00	0.1083	0.2331	+0.1248	+53.5	0.657	0.706
0.50	0.0394	0.0831	+0.0437	+52.6	0.239	0.252
0.05	0.0291	0.0589	+0.0308	+52.3	0.177	0.178
0.01	0.0123	0.0288	+0.0165	+57.2	0.075	0.087

TABLE XXXV (Continued)

C	W_x	W_c	$W_c - W_x$	% Error	W_x/W_{xt}	W_c/W_{ct}
Run Number 14						
1.00	0.1098	0.1184	+0.0086	+ 7.3	0.675	0.700
0.50	0.0766	0.0809	+0.0043	+ 5.3	0.471	0.478
0.10	0.0375	0.0373	−0.0002	− 0.5	0.230	0.220
0.05	0.0269	0.0267	−0.0002	− 0.7	0.165	0.158
0.01	0.0075	0.0123	+0.0048	+39.0	0.046	0.073
Run Number 22						
3.00	0.0827	0.0902	+0.0075	+ 8.3	0.938	0.810
2.00	0.0578	0.0680	+0.0102	+15.0	0.655	0.621
1.00	0.0384	0.0455	+0.0071	+15.6	0.435	0.408
0.50	0.0266	0.0315	+0.0049	+15.5	0.302	0.283
0.10	0.0110	0.0137	+0.0027	+19.7	0.125	0.123
0.05	0.0071	0.0096	+0.0025	+26.0	0.081	0.086
0.01	0.0037	0.0040	+0.0003	+ 7.5	0.042	0.036
Run Number 23						
2.00	0.1171	0.1483	+0.0312	+21.0	0.739	0.757
1.00	0.0792	0.0991	+0.0199	+20.1	0.500	0.506
0.50	0.0583	0.0299	+0.0070	+23.4	0.144	0.153
0.05	0.0114	0.0207	+0.0093	+44.9	0.072	0.106
0.01	0.0022	0.0088	+0.0066	+75.0	0.014	0.045
Run Number 33						
2.00	0.3136	0.2449	−0.0687	−28.0	0.760	0.782
1.00	0.2192	0.1631	−0.0561	−34.4	0.531	0.521
0.50	0.1599	0.1132	−0.0467	−41.2	0.387	0.362
0.10	0.0767	0.0525	−0.0242	−46.0	0.186	0.168
0.05	0.0503	0.0376	−0.0127	−33.7	0.122	0.120
0.01	0.0115	0.0173	+0.0058	+33.5	0.028	0.055

APPENDIX J

LIST OF SYMBOLS

TABLE XXXVI

LIST OF SYMBOLS

a	Constant in equation derivations
A	Surface area, ft^2
A_n	Constants in weight-concentration equation, $n = 1, 2, 3$
b	Constant in equation derivation
c	Specific heat, Btu/lbm °F, Constant in equation derivation
c_n	Constant in equation derivation, $n = 1, 2, 3$
C	Sodium chloride concentration, (lb salt/lb brine) x 100 per cent
d	Constant in equation derivation
D	Diameter, ft
D	Diffusion coefficient, ft^2/hr
g	Gravitational constant, ft/sec^2
Gr	Grashof number $g\rho^2D^3\ (t_e - t_o)/\mu^2$
h	Convection coefficient, Btu/(hr ft^2 °F)
k	Thermal conductivity, Btu/(hr ft °F)
K	Distribution coefficient
L	Latent heat of fusion, Btu/lb
L^*	Dimensionless group, $L/\ c(t_e - t_o)$
m	Weight of coolant, lbs
n_n	Exponents in weight concentration equations, subscript $n = 1, 2, 3$
Nu	Nusselt number, hD/k
Pr	Prandt number, $c\mu/k$
q	Heat transfer $t_e < t_b$, Btu/hr
q_1	Heat transfer by conduction, $t_e = t_b$, Btu/hr

TABLE XXXVI (Continued)

q_2	Heat to form ice, $t_e = t_b$, Btu/hr
Q_n	Volume rate of brine flow, ft^3/min
Q	Heat removed by coolant, Btu
r	Sphere radius, inches
r^*	Dimensionless radii, r/r_o
R'	Advance of freezing front, ft/hr
R	Sphere radius to ice surface, feet
s	Dimensionless group, $rD^{-\frac{1}{2}}\theta^{-\frac{1}{2}}$
S	Dimensionless Group, $RD^{-\frac{1}{2}}\theta^{-\frac{1}{2}}$
S.G.	Specific gravity
t	Temperature, °F
V^1	Volume, ft^3
V_n	Velocity for chamber cross-sectional area, ft/min
V_i	Velocity corrected for projected area of sphere, ft/min
V_f	Velocity corrected for projected area of frozen sphere, ft/min
V_{avg}	Average velocity, ft/min
W	Accumulative sample weight, lbs
x	Distance from start of slab freezing, ft
x'	Distance from interface of slab freezing, ft

Greek Symbols

β	Thermal expansion coefficient, $^{\circ}F^{-1}$
Δt	Total temperature potential $(32 - t_b) + (32 - t_o)$, °F
θ	Time of run, hours
θ_s	Time to collect 100 cc coolant, seconds

TABLE XXXVI (Continued)

θ^*	Dimensionless group, $(k/Lr_o^2)(t_e - t_o)\theta$
μ	Absolute viscosity, lbm/(hr-ft)
ρ	Density, lbm/ft^3

Subscripts

c	Final calculated value, concentration in diffusion equation
e	Value of property at equilibrium conditions
i	Properties for pure ice
in	Coolant property entering sphere
j	Result of intermediate calculation
o	Property on inner surface of sphere
out	Coolant property leaving sphere
s	Property at ice-liquid interface
t	Property for total sample, temperature in diffusion equation
x	Experimental data
∞	Property evaluated at large distance from interface

VITA

Robert Kingsley Will

Candidate for the Degree of

Doctor of Philosophy

Thesis: AN EXPERIMENTAL INVESTIGATION OF THE NORMAL FREEZING, FRACTIONAL MELTING PROCESS APPLIED TO THE DESALINATION PROBLEM

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