A CRITICAL EVALUATION OF SOME PILE DRIVING FORMULAS

By

MAHESH VARMA

Bachelor of Science (Engineering)
Banaras Hindu University
Varanasi, India
1951

Master of Engineering
University of Roorkee
Roorkee, India
1959

Submitted to the Faculty of the Graduate College
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Thesis Approved:

Robert L. Janes
Thesis Adviser

T. Allan Haliburton

Robert D Morrison

Edwin L Bidwell

JHBoggs
Dean of the Graduate College

PREFACE

For over a hundred years since the first of such formulas was proposed, the quest for a suitable dynamic pile formula has continued in several countries, and different persons have resolved it in different ways. As a result over a score of these formulas are presently in existence, no two of them ever showing agreement in results. Naturally, the decision as to which formula to use in a specific situation is always a difficult one to make and there exists an urgent need for a dispassionate examination of this entire question of pile driving formulas. The present study is intended to be an attempt in this direction.

The author owes his grateful thanks to several people who have helped him, directly or indirectly, in making this work fruitful. In particular he would like to thank Professor E. L. Bidwell, Dr. T. A. Haliburton, and Dr. R. A. Hultquist - members of his Advisory Committee - for their valuable suggestions and helpful attitude during the course of this work. To Dr. R. L. Janes, his Major Adviser, the author could perhaps never be sufficiently grateful. But for Dr. Janes' genial temperament and scholarly guidance at all times, the timely completion of this work would not have been possible. The help extended by Dr. Hultquist and his colleagues, Dr. R. D. Morrison and Dr. D. E. Bee, in making the statistical studies involved in this work, is very gratefully acknowledged.

The author would also like to record the consideration and encouragement he received from Professor and Mrs. R. L. Peurifoy during his present graduate work.

And last but not least, it is his great pleasure to acknowledge the sacrifice and patient suffering of his wife, Kamla, and children, Gita and Ranu, which made this work possible.

Mrs. Peggy Harrison has done a wonderful job of typing the manuscript.

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CHAPTER I

INTRODUCTION

Pile driving formulas (or simply pile formulas) occupy an important place in the science and practice of pile foundations. For over a century engineers, including some of the talents of the civil engineering profession, have been engaged in the study of the complex phenomena of pile driving and of interaction between soil and pile. Certainly, much time and energy have been expended in the study of existing pile formulas and evolution of new ones, with the result that over a score of these formulas are found to be in existence at the present time, some of them having been proposed during recent years. Also accumulated is a wealth of informative data on the several variables contributing to the results of pile formulas.

The immense popularity of these formulas may be ascribed to the design simplification on the one hand, and the ease of practical control of pile driving operations on the other, which are offered by them. However, it is well known that this simplification accrues at the cost of accuracy, and sometimes even of safety; for the results obtained through the use of most of the known formulas are often either 'too safe' or 'grossly unsafe', and the prediction of 'true' values of pile load capacity may be most aptly described as a fortuitous occurence only. In spite of this, and the fact that most of the known authorities on soil mechanics have criticized their use in design of pile foundations, the practice continues unabated and it is very unlikely that this will be replaced by rational design principles, at least in the foreseeable future.

Since the pile formulas have come to stay for an indefinite period, a critical evaluation of their merits would be of immense utility to the users of these formulas. Two specific reasons call for such a study at the present time. In the first place, the practice of pile driving has undergone great changes since the time most of these formulas were devised, and the effects of these changes on the applicability of these formulas must be known to the user. In the second, the results of the recent well-instrumented tests on piles are now available which make such a study practically feasible.

The theoretical aspects of pile formulas have been investigated in great detail since the rise of modern soil mechanics, and as stated by Cummings (4) as far back as 1940, it would seem that the mathematical approach to determine their validity is about exhausted. In this study, therefore, use of statistical techniques of evaluation has been planned, including a probabilistic approach. Hopefully, this would lead to useful information on the merits of the formulas in the context of present practice of pile driving.

CHAPTER II

THE PILE DRIVING FORMULAS

In this chapter the dynamic pile formulas selected for this study are described. These formulas are most commonly in use, and are also representative of the different theoretical approaches that form their basis, and of the assumptions which were made in the process of evolving them.

Pile formulas completely empirical in nature have been suggested, but most of the well known ones are based on varying degrees of rationality. Some formulas attempt to account for the energy losses that occur during the driving of a pile, by means of fixed coefficients incorporated in their statements, while others accomplish this by including the relative weights of pile and hammer. Still another group of formulas makes use of both fixed coefficients and pile and hammer weights, to include the effect of this energy loss. The more sophisticated ones, however, attempt to include all or some of the terms providing for impact and elastic losses during driving. The validity of the assumptions used in deriving the formulas forms part of the next chapter.

The following ten formulas were selected for this study:

1. Engineering News (Nominal Safety Factor = 6)

$$R_d = \frac{2E_n}{S + 0.1} \quad \text{(for single and double acting hammers)}$$

2. Hiley (Nominal Safety Factor = 3)

$$R_d = \frac{4E_n}{S + \frac{1}{2}(C_1 + C_2 + C_3)} \frac{W_r + e^2 W_p}{W_r + W_p}$$

3. Pacific Coast Uniform Building Code (Nominal Safety Factor = 4)

$$R_d = \frac{3E_n \frac{W_r + KW_p}{W_r + W_p}}{S + \frac{48 R_d L}{AE_L}}$$

4. Redtenbacher (Nominal Safety Factor = 3)

$$R_d = \frac{AE_L}{36L} \left[- S + \sqrt{S^2 + (12E_n \frac{W_r}{W_r + W_p}) \frac{24L}{AE_L}} \right]$$

5. Eytelwein (Nominal Safety Factor = 6)

$$R_d = \frac{2E_n}{S + 0.1 \frac{W_p}{W_r}}$$

6. Navy-McKay (Nominal Safety Factor = 6)

$$R_d = \frac{2E_n}{S(1 + 0.3 \frac{W_p}{W_r})}$$

7. Rankine (Nominal Safety Factor = 3)

$$R_d = \frac{2AE_L S}{36L} \left[\sqrt{1 + \frac{12E_n(12L)}{S^2 E_L A}} - 1 \right]$$

8. Canadian National Building Code (Nominal Safety Factor = 3)

$$R_d = \frac{4E_n \dfrac{W_r + 0.5e^2 W_p}{W_r + W_p}}{S + \dfrac{3R_d}{2A}\left[\dfrac{12L}{E_L} + 0.0001\right]}$$

9. Modified Engineering News (Nominal Safety Factor = 6)

$$R_d = \frac{2E_n}{S + 0.1}\,\frac{W_r + e^2 W_p}{W_r + W_p}$$

10. Gates (Nominal Safety Factor = 3)

$$R_d = \sqrt{E_n}\left[\log \frac{S}{10}\right]\frac{2000}{7}$$

The symbols used in the statements of the above formulas are defined as below:

R_d	=	Computed design pile load capacity, lb
E_n	=	Manufacturer's maximum rated capacity of driving hammer, ft-lb
S	=	Set or final average penetration per blow, in.
C_1	=	Temporary compression of pile cap and head, in.
$C_2 + C_3$	=	Temporary compression of pile and ground, in.
W_r	=	Weight of hammer ram, lb
e	=	Coefficient of restitution
W_p	=	Weight of pile (including driving appurtenances), lb
K	=	Coefficient analogous to restitution modulus, e^2
L	=	Total length of pile, ft

A = Net steel cross sectional area of pile, sq in.

E_L = Modulus of elasticity of steel, 30×10^6 psi

CHAPTER III

THEORETICAL BACKGROUND OF PILE DRIVING FORMULAS

Derivation

Pile driving formulas have been derived on the basic assumption that the ultimate carrying capacity of the pile is equal to the dynamic driving force on the pile. The simplest pile formula is obtained by equating the weight of the ram multiplied by the stroke to the driving resistance multiplied by the penetration of the pile tip. More elaborate formulas include terms which account for the various energy losses during driving.

The basic energy equation representing the pile driving operation which is used in derivation of the formulas may be written as:

Energy available from driving hammer = Energy loss due to impact between pile and hammer
+ Energy loss due to temporary compression of pile-soil system
+ Energy used in penetration of pile

Every rationally derived formula attempts to evaluate each of the above components of energy on the basis of certain assumptions. A background of the approach for each of the formulas selected for this study is presented here.

The following symbols are used in the discussion:

R = Resistance of pile to penetration (dynamic resistance)

W_r = Weight of hammer ram

H = Stroke of ram

W_p = Weight of pile

S = Penetration of pile per blow

e = Coefficient of restitution

C_1 = Temporary compression of pile head and cap

C_2 = Temporary compression of pile

C_3 = Temporary compression of soil under the pile tip

Before the ram strikes the pile head, it falls through a distance H and attains a velocity, say $v_1 = \sqrt{2gH}$. At the moment of striking, the pile is at rest and has a velocity, say $v_2 = 0$.

Assuming that the impact is wholly plastic (inelastic) and there is no rebound of the ram, the principle of conservation of momentum yields the equation

$$W_r v_1 + W_p v_2 = (W_r + W_p)v \tag{1}$$

where v is the common velocity of ram-pile system.

Substituting values for v_1 and v_2 and simplifying gives the value of v:

$$v = W_r \sqrt{2gH} / (W_r + W_p) \; . \tag{2}$$

Assuming further that the duration of impact is so small that the ram and pile attain this common velocity before any appreciable penetration of the pile, the energy available for penetration, if no other losses existed, would be the kinetic energy of the moving pile and ram. This energy equals

$$\left[\frac{W_r + W_p}{2g}\right]\left[\left(\frac{W_r\sqrt{2gH}}{W_r + W_p}\right)^2\right] = \frac{{W_r}^2 H}{W_r + W_p} \tag{3}$$

The energy lost in impact is the difference of the total available energy of the hammer and this useful energy, i.e.,

$$\text{Energy lost in impact} = W_r H\left(1 - \frac{W_r}{W_r + W_p}\right) = W_r H \frac{W_p}{W_r + W_p} \tag{4}$$

However, if the impact is not wholly plastic but partly elastic, the impact loss would be only $W_r H W_p (1 - e^2)/(W_r + W_p)$. The total temporary compression of pile head and cap, pile and soil under the tip = $(C_1 + C_2 + C_3)$, and the energy loss due to this compression = $\frac{1}{2}R(C_1 + C_2 + C_3)$. The energy used in actual penetration of pile = RS. Substituting the values of each of the energy components as found above in the general energy equation, the following relation is obtained:

$$W_r H = W_r H(1 - e^2)W_p/(W_r + W_p) + \frac{1}{2}R(C_1 + C_2 + C_3) + RS \tag{5}$$

This is the equation from which most of the commonly known formulas have been derived.

If impact is assumed perfectly elastic ($e^2 = 1$), and the term $\frac{1}{2}(C_1 + C_2 + C_3)$ is replaced by a constant C, the equation reduces to $R = \frac{W_r H}{S + C}$. This is the statement of the Engineering News formula. A rearrangement of the equation (5) gives the Hiley formula:

$$R = \left[\frac{W_r H}{S + \frac{1}{2}(C_1 + C_2 + C_3)}\right]\left[\frac{W_r + e^2 W_p}{W_r + W_p}\right] \tag{6}$$

The Pacific Coast Uniform Building Code formula has the same form as the Hiley formula except that the term for energy loss due to temporary compression is modified by neglecting the temporary compression of pile head and cap (C_1) and of the soil under the tip (C_3), and using twice the energy loss due to compression of the pile. This elastic compression loss equals

$$RC_2 = R(RL/AE), \tag{7}$$

where L = length of pile as driven, A = area of pile cross-section, and E = modulus of elasticity of pile. This gives the equation:

$$R = \left[\frac{W_r H}{S + (RL/AE)}\right]\left[\frac{W_r + e^2 W_p}{W_r + W_p}\right] \tag{8}$$

If the impact is considered completely inelastic, and only the loss due to compression of the pile is considered, the equation (5) reduces to the form:

$$W_r H = \frac{W_r H W_p}{W_r + W_p} + \frac{1}{2}R(RL/AE) + RS. \tag{9}$$

After transposition, the following quadratic equation is obtained:

$$R^2 \frac{L}{2AE} + RS - W_r H\left(\frac{W_r}{W_r + W_p}\right) = 0 \tag{10}$$

This, on solution, gives the Redtenbacher formula:

$$R = \frac{AE}{L}\left[-S + \sqrt{S^2 + \frac{2L}{AE}\,\frac{W_r{}^2H}{W_r + W_p}}\right]. \tag{11}$$

If the loss due to elastic compression of pile head and cap, pile and soil is neglected altogether; and the impact is assumed perfectly plastic ($e^2 = 0$), equation (5) reduces to:

$$R = \frac{W_rH}{S(1 + W_p/W_r)} \; . \tag{12}$$

This is the general form of Eytelwein formula.

In the above formula if the ratio (W_p/W_r) is modified by including an empirical coefficient, 0.3, the following equation is obtained which is the Navy-McKay formula:

$$R = \frac{W_rH}{S(1 + 0.3\,W_p/W_r)} \; . \tag{13}$$

In the Rankine formula perfectly elastic impact ($e = 1$) is assumed and the equivalent length of pile in pure friction is taken as $L/2$. No other compression losses are considered. Equation (5) reduces to the following quadratic form:

$$\frac{L}{4AE}R^2 + RS - W_rH = 0 \; . \tag{14}$$

On solution this gives:

$$R = \frac{2AES}{L}\left[\sqrt{1 + \frac{W_rHL}{AES^2}} - 1\right] \; . \tag{15}$$

The Canadian National Building Code formula can be readily derived from the Pacific Coast Uniform Building Code formula by substituting $0.5W_p$ for W_p, and replacing (RL/AE) by $\left(\frac{3RL}{2AE} + 0.0001\frac{3R}{2A}\right)$. This gives:

$$R = \left[\frac{W_r H}{S + \frac{3R}{2A}\left(\frac{L}{E} + 0.0001\right)}\right]\left[\frac{W_r + 0.5e^2 W_p}{W_r + W_p}\right]. \tag{16}$$

In the Modified Engineering News formula the component of energy loss due to elastic compression, viz., $\frac{1}{2}(C_1 + C_2 + C_3)$ is replaced by an arbitrary constant, 0.1. Thus, the following statement of equation (5) is obtained:

$$R = \left[\frac{W_r H}{S + 0.1}\right]\left[\frac{W_r + e^2 W_p}{W_r + W_p}\right]. \tag{17}$$

The Gates formula is a completely empirical formula.

Capabilities and Limitations

Each of the formulas is capable of working most effectively under certain specific situations. There is no universal pile formula which could be used under all possible conditions. It is difficult to conceive of such a thing as a universal formula due to the widely varying nature of conditions under which piles are driven. A clear understanding of the capabilities and limitations of a pile formula is, therefore, necessary before it is used in practice.

The basic assumption underlying the pile formulas, viz., that resistance to penetration of the pile under the blow of a hammer is an indication of the resistance under static load, is often not true. In a cohesionless soil or permeable fill the resistance offered to penetration of pile while being driven bears a reasonably close relationship to the resistance offered under static load, but in the case of a plastic material or saturated fine silt this assumption may lead to entirely erroneous results as the relationship between the temporary resistance to driving and permanent resistance under a static load is very uncertain. Under the effect of driving, a plastic soil undergoes remolding (fine silts are made 'quick'), with the result that resistance to penetration of the pile under the blow of the hammer will usually be much less than the true strength of the soil under static load, depending upon the sensitivity of the soil. After the pile has been driven, the material closes in against the pile and its original strength is very largely regained on account of the thixotropic process which is characterized by a rearrangement of the soil particles. Thus, time may be a very important factor in the load capacity of piles driven in plastic soils.

The soil resistance computed by pile formulas is the resistance of the strata through which the pile actually penetrates and is no indication of the strength of soil lying under the tip of the pile, though this soil may sometimes vitally affect the safety of the foundation as a whole. Again, the pile formulas do not account for any effect of negative friction which might develop after the piles are driven, if the pile cluster is surrounded by a soft unconsolidated fill or the piles have been driven through such strata.

The results of pile formulas are essentially meant for single piles whereas the load of a structure is usually supported on a group of piles. The strength of this group may not be equal to the strength of a single pile multiplied by the number of piles in the group, unless the piles are wholly end bearing, which is rarely true.

The difficulties associated with accurate determination of the parameters which are involved in computation of results from pile formulas seriously limit the accuracy of their results. The values of some of these parameters (e., C_1, C_2, etc.) which were arrived at decades ago are still in use, though the practice of pile driving has undergone substantial change during this period.

Validity of Some Assumptions Made in Derivation

Even the most elaborate formulas are based on several assumptions, some of which violate the laws of mechanics and constitute sources of weakness which seriously affect their accuracy.

The general expression for pile formulas developed earlier in this chapter takes account of the losses in impact and in elastic compression of the pile head and cap, pile and soil. The inclusion of these two losses of energy in the same equation is questionable since impact losses based on Newton's theory are supposed to include losses due to elastic deformation of colliding bodies. By including the elastic compression losses in addition to impact losses there is a duplication which would result in the computed values of load being on the lower side than would be normally expected. Furthermore, Newton's theory of impact was derived for impact between bodies which are not subjected to external restraint. When the ram strikes the pile head during driving, the impact is far from the idealized concept of two "free" bodies colliding. In fact, recent instrumented pile tests have revealed that pile motion under the blow of the hammer is far more complex than the motion of Newton's spheres after impact, the two ends of the pile having velocities different in magnitude and sometimes even in direction. Again, the elastic compression of the pile is obtained on the basis of stress-strain relationship which normally holds under statically applied load, whereas during the driving process the pile is subjected to rapidly repeated blows.

While there is a duplication of energy loss as pointed out above, there is at least one additional source of energy loss which is not considered by the formulas, viz., due to vibration. Under the impact of the hammer ram intense vibrations are set up in the pile and surrounding soil and the energy used up in producing these vibrations is a loss from the point of available energy for producing set.

It would be apparent from this discussion that some of the basic assumptions upon which the derivation of the pile formulas rests are open to serious objection. In fact, it is the considered opinion of some authors that pile driving operations are far more closely related to St. Venant-Boussinesq theory of longitudinal impact on rods than to the Newtonian theory of impact of spheres. A discussion on this aspect of the problem is beyond the scope of this study.

CHAPTER IV

COLLECTION AND PROCESSING OF DATA

General

The driving of piles and their behavior under actual loading conditions are complex phenomena which are affected by a large number of variables. All of these variables could not possibly be taken into account in a study such as this one, as the limitations imposed by our present knowledge of soil mechanics and availability of enough practical information on piling jobs are much too serious. The scope of this study is, therefore, restricted to include only some of the factors which are well known to affect the results of pile driving, and for which adequate data is available at present. It is hoped that this narrowing down of scope does not generally affect the comparative study of different formulas which is the purpose of this study.

Soil and Pile Types

The data collected pertain exclusively to steel piles - H and pipe sections, driven in permeable, relatively cohesionless or principally cohesionless soils. Although the information has been obtained from different sources, it is felt that these conditions represent fairly comparable conditions for this study.

The theoretical basis for pile formulas is the assumption that the calculated driving resistance will have some definite relationship to the ultimate static bearing capacity, a condition most likely to be met in gravels and coarse sands where piles deliver a significant percentage of their load in end bearing. Also, for such soils short time tests on single piles, which form the basis for the present data, are more meaningful.

The frictional resistance between sand and steel is likely to be less than between sand and sand (13), with the result that steel piles would tend to slip past the soil under load. The friction support would, therefore, be much less than support through end bearing in case of such piles. Further, the availability of well-instrumented recent test data on steel piles is another important reason why steel piles have been chosen for this study.

Data Limitations

Before proceeding with presentation and analysis of data it seems appropriate to indicate some of the inadequacies of the available information which might bear on the results of the analysis that follow in subsequent chapters. These inadequacies are unavoidable in a study based on information collected from different sources.

Pile load tests are influenced by job size, foundation conditions and time available for tests. Further, the method of applying load, i.e., whether dead load, jacking against a reaction or jacking from anchor piles, method of measurement of settlement, and in general the manner of interpretation of load tests may differ from job to job. The degree of accuracy may also vary in different tests. The load-settlement curves for piles in sand frequently do not indicate a point of complete failure and the load continues to increase somewhat as settlement increases. In such cases it may be difficult even to define a failure load.

On the other hand, the effect of individual driving technique may be appreciable and may differ from job to job. A soft renewed driving block would make a considerable difference in set and pile length. It has been said that with a good foreman and a block of wood one could get any value of pile penetration that one would like to have.

However, since the driving and test conditions remain the same for all formulas compared on the basis of a certain set of results, it is felt that the generalization about the relative behavior of different formulas would not lose much validity due to the above inadequacies of data.

Type of Information and Its Sources

The data collected comprised the following information:

1. Type of pile
2. Length of pile
3. Area of cross-section of pile
4. Weight per unit length of pile
5. Weight of driving head
6. Type of driving hammer
7. Weight of hammer ram
8. Length of stroke of ram
9. Manufacturer's rated energy of driving hammer
10. Number of blows per foot of penetration at end of driving

11. Yield load from static test.

Several organizations and individuals were contacted with a view to obtain useful and representative pile driving and test load data. These included all principal firms dealing in piles and pile driving equipment or undertaking piling contracts, Highway departments of States, universities and institutes of advanced education and research, and chief engineers of railroads.

Finally, seventy-one test results were obtained from the following sources:

1.	Highway Research Board, Special Report #36 ...	9
2.	Highway Research Board, Special Report #67 ...	24
3.	Michigan State Highway Commission Report ...	14
4.	United States Steel Corporation ...	9
5.	R. D. Chellis (Pile Foundations, 1961) ...	15
	Total	71

A summary of the data is shown in Appendix A.

Processing of Data

Using the above information, for each of the test results the ultimate carrying capacity of pile was computed according to each of the ten formulas chosen for the study, the value for factor of safety being unity in each case. For the Hiley formula, the following values of coefficients C_1, C_2, and C_3 were used as recommended by Chellis:

$C_1 = 0.1$

$C_2 = 0.006 \times \text{length of pile}$

$C_3 = 0.1$.

Where the weight of driving head for the pile was not available from the information on pile driving, a value of 1000 lb was assumed for the computations by each of the formulas.

After computing the ultimate predicted load by each formula for each set of test results, the ratio of static yield load to predicted load was computed. This ratio represents the true or built-in factor of safety in each case, and provides a measure of the efficacy of a formula under a specific situation. In an ideal case, i.e., where the formula predicts the same value of ultimate load as found from load test, this ratio would be unity.

In all, this amounted to computation of 710 theoretical results, a voluminous work, especially since some of the formulas involved the unknown, R_d, on both sides of the expression, necessitating either a quadratic or a trial-and-error solution. It was, therefore, considered proper to entrust this work to a digital computer. The IBM 1620 was utilized for this purpose. The results obtained are shown in Appendix B.

CHAPTER V

PERFORMANCE OF PILE FORMULAS AS PROBABILITY DISTRIBUTIONS

General

The theoretical results of performance of pile formulas as obtained in the preceding chapter show a great deal of variation, both for the same formula and from one formula to another. Therefore, valid generalizations about the relative effectiveness of the formulas can be made only after a study of the nature of this variation. Such a study, by application of the theory of probability, treating the theoretical results as random variables, is presented in this chapter.

The data has been assumed as random. It is believed that the plan of collection of the necessary information and in general, absence of any known systematic variation in this process of collection validate the assumption of randomness. Hopefully, this would not affect the degree of accuracy commensurate with problems encountered in pile foundations.

Frequency Distribution of Data

In order to discover the general shape of the universe the data are arranged in frequency series and histograms and frequency distribution curves are constructed for each of the ten formulas. This concentration of information in a reasonably small area enables more effective comprehension of the pattern of variation. In this connection the most important thing is how often values of various ranges have occurred in the distribution, i.e., the variation in their frequency as we progress along scale from zero. In all cases the density appears to increase until the highest value is reached after which it decreases rather slowly giving a skewed distribution curve.

The selection of interval size for the distribution function is a judgment choice for each formula. An interval too narrow would result in irregularities in the distribution associated with sampling fluctuations, while an interval too wide would cover up too much of detail needed to confidently establish the general pattern of the universe. The interval chosen in each case provides the near optimum combination of smoothness and detail.

In one case a "gap" appears towards the later part of the distribution, the value of frequency dropping and then rising again. The reason for this may be either the size limitation of the sample or an actual bimodal distribution. This situation could be overcome by increasing the length of the intervals towards the end of frequency cycle, but if this is done it would be very difficult to separate that part of the change in frequency due to change in interval length from the part due to a real change in frequency. In order to make use of the available information in the best possible manner and at the same time avoid complications in treatment, the analysis is restricted to the range of values where the distribution first gets minimum. It is hoped that this simplification will not materially affect the results of the study. It is seen from the curves that the distribution is fairly smooth in practically all cases with no lumpiness involved anywhere. This is considered rather fortunate, since a lumpy frequency distribution would be very difficult to represent with a mathematical model. The frequency distributions for the ten formulas are shown in Figs. 1-10.

Suitable Probability Distribution Function

The skewness of the frequency distributions renders the normal distribution unsuitable for this study. Furthermore, the degree of skewness is found to vary in each case requiring use of a flexible distribution function which could be made to conform to each of the distributions. The gamma function is found to meet these requirements adequately. Since the data start at zero and are always positive, two necessary conditions for applicability of gamma function, this distribution is well suited for the study.

The gamma distribution is quite flexible and describes several situations simply by changing the values of parameters occurring in the density function. The well-known Chi-square distribution is a special case of the gamma function and has immense utility in testing of hypotheses, fitness of curves and independence of treatments, and in establishing confidence intervals.

The Gamma Distribution

The gamma is a two-parameter family of distributions, the parameters being α and β, and is given by the density function:

$$f(x; \alpha, \beta) = \frac{1}{\alpha! \, \beta^{\alpha+1}} x^{\alpha} e^{-x/\beta} \quad \text{for} \quad 0<x<\infty .$$

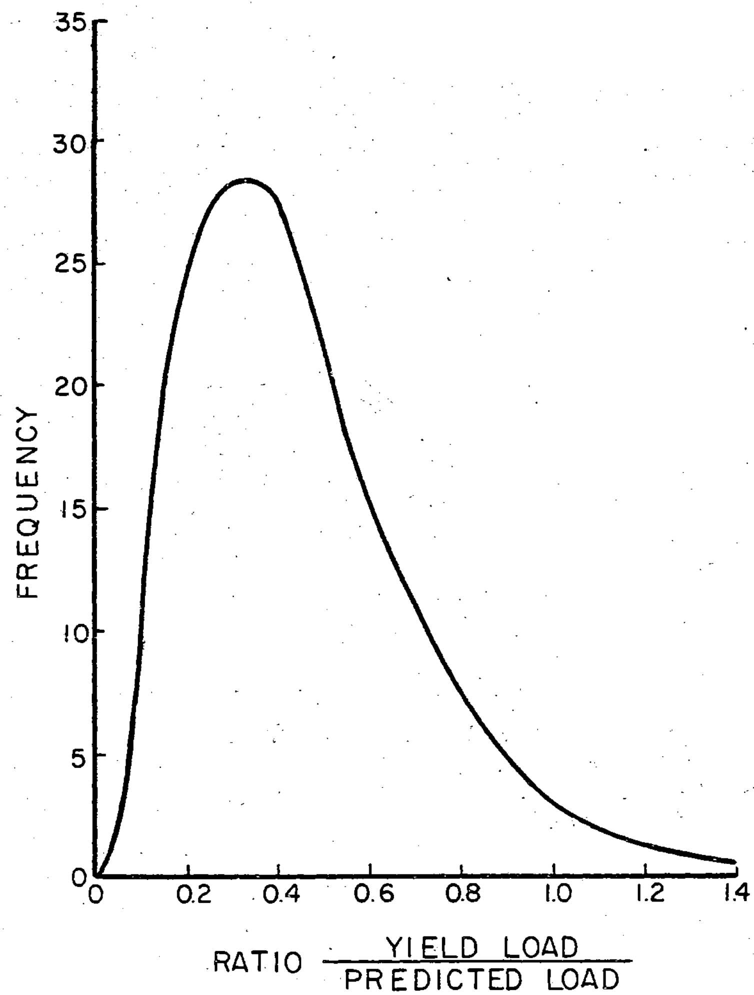

Figure 1. Frequency Distribution for Engineering News Formula

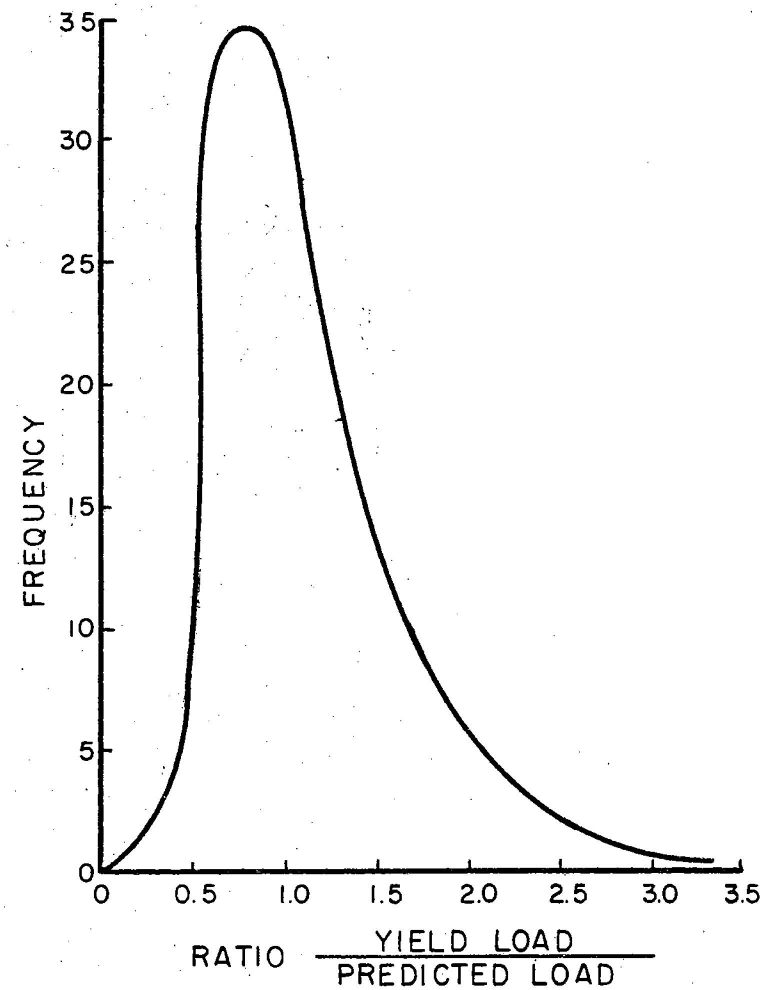

Figure 2. Frequency Distribution for Hiley Formula

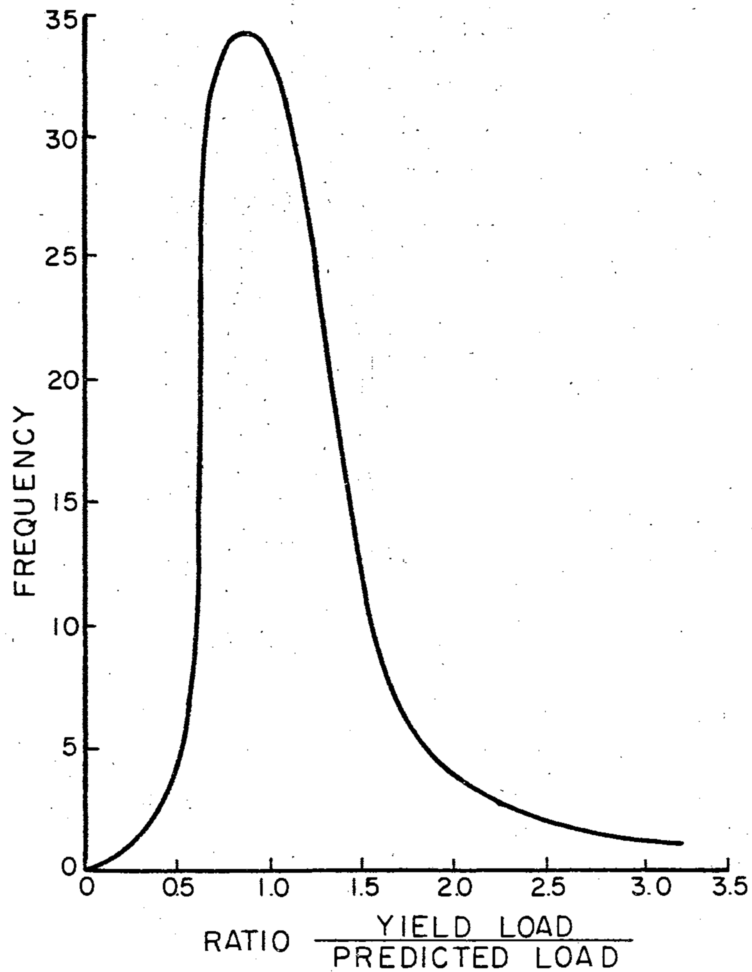

Figure 3. Frequency Distribution for Pacific Coast Formula

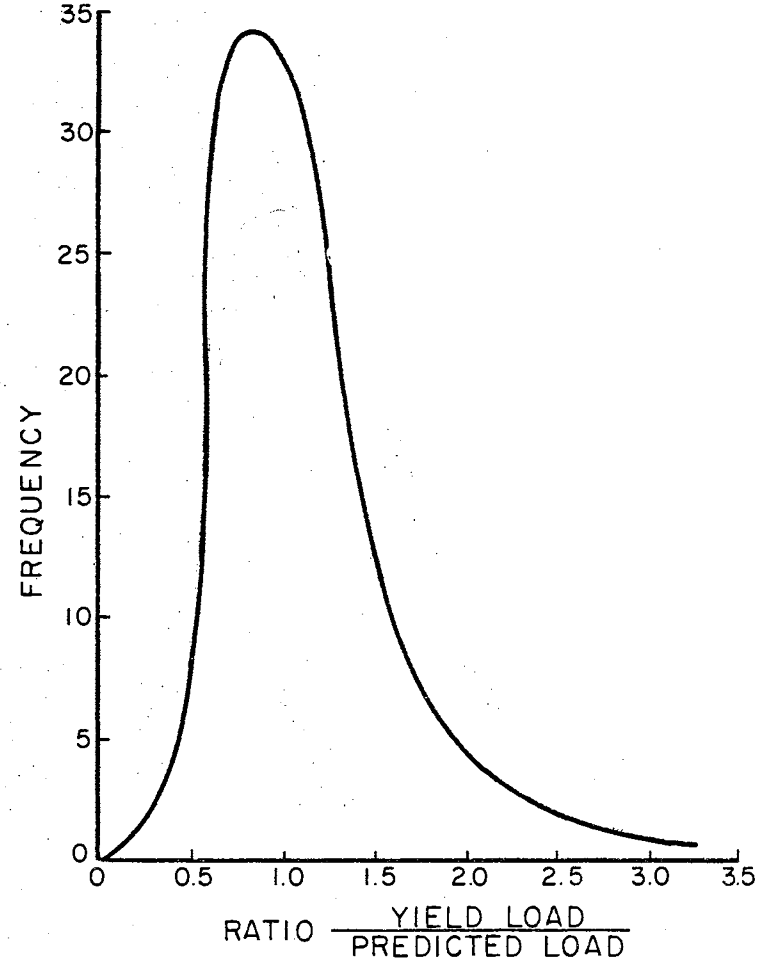

Figure 4. Frequency Distribution for Redtenbacher Formula

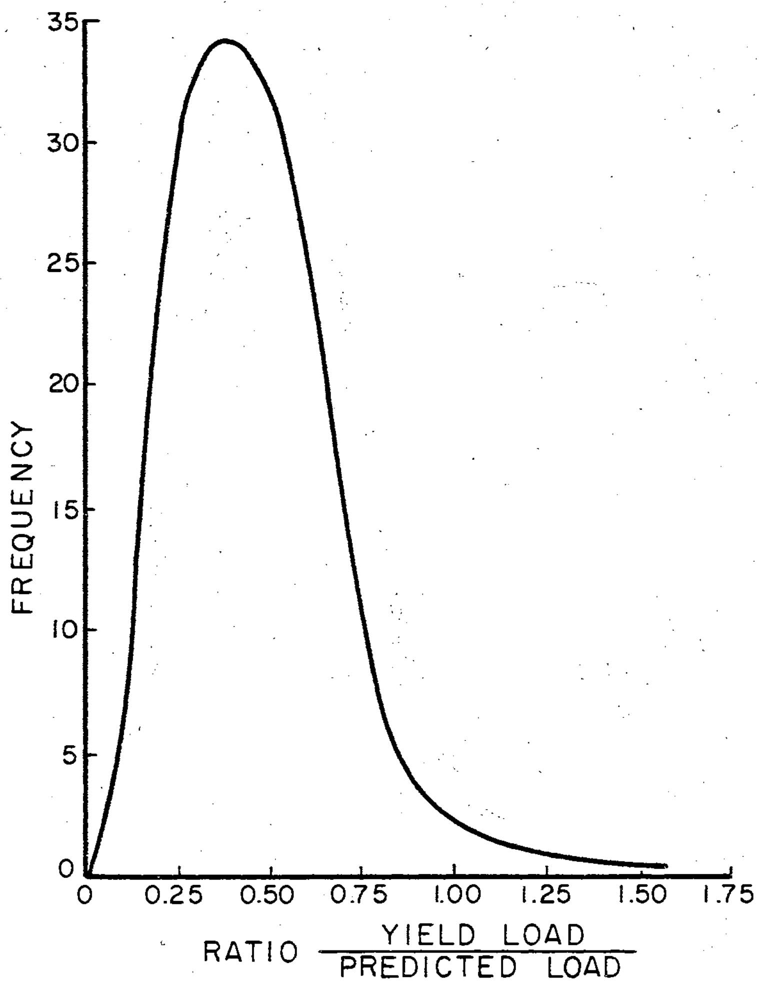

Figure 5. Frequency Distribution for Eytelwein Formula

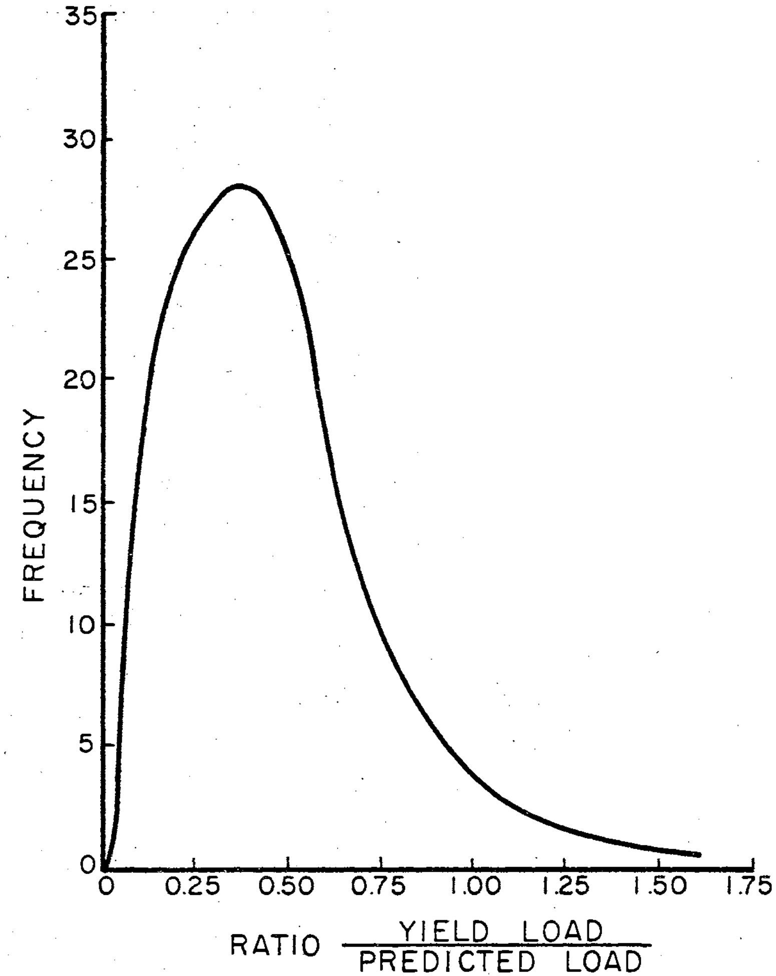

Figure 6. Frequency Distribution for Navy-McKay Formula

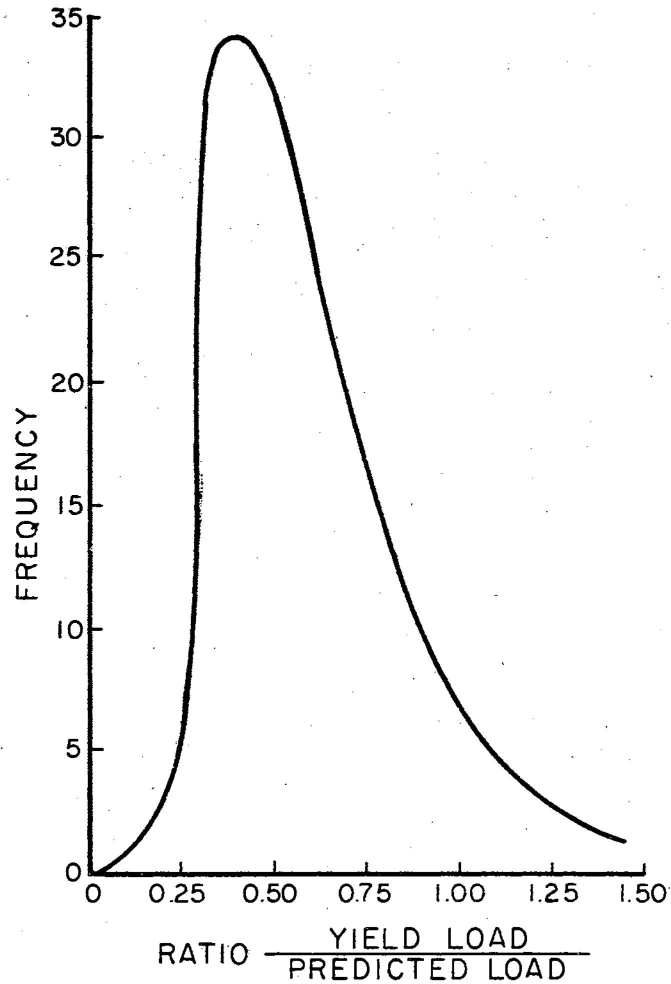

Figure 7. Frequency Distribution for Rankine Formula

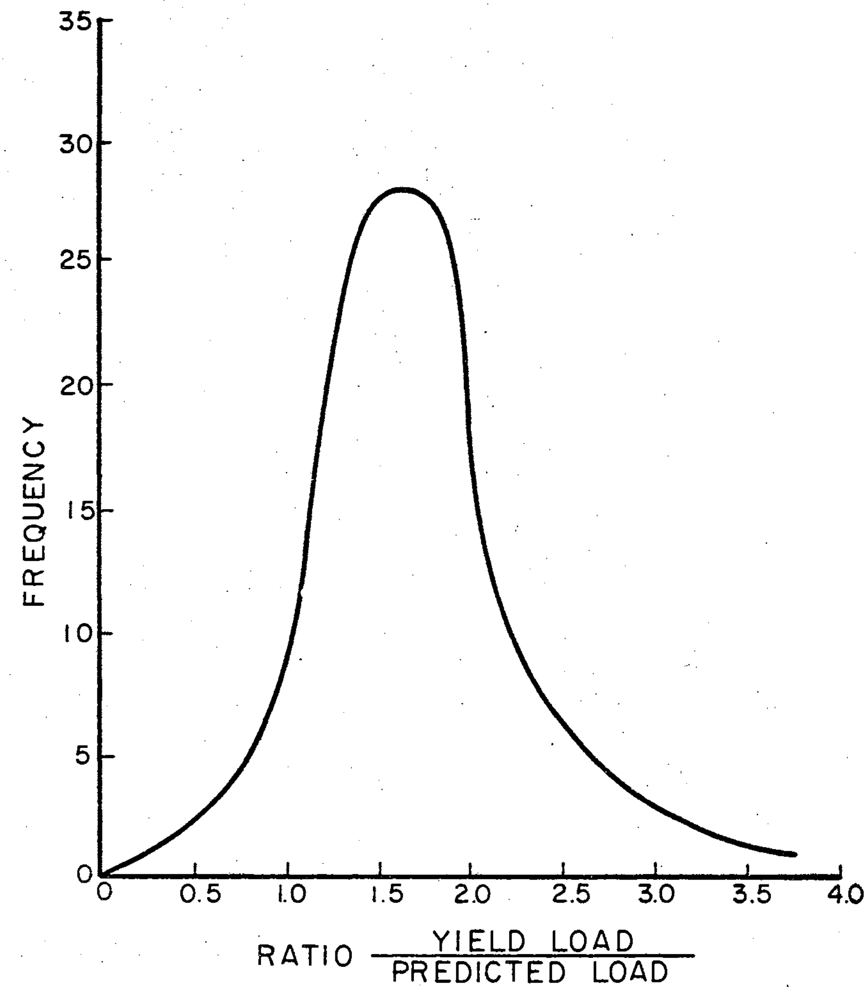

Figure 8. Frequency Distribution for Canadian National Formula

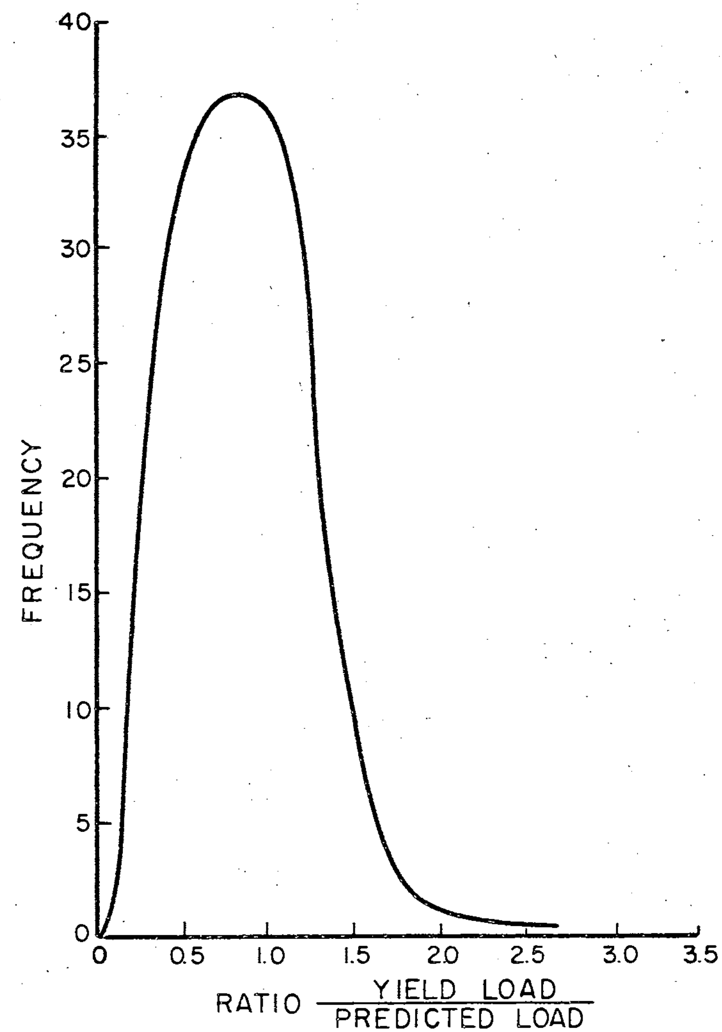

Figure 9. Frequency Distribution for Modified Engineering News Formula

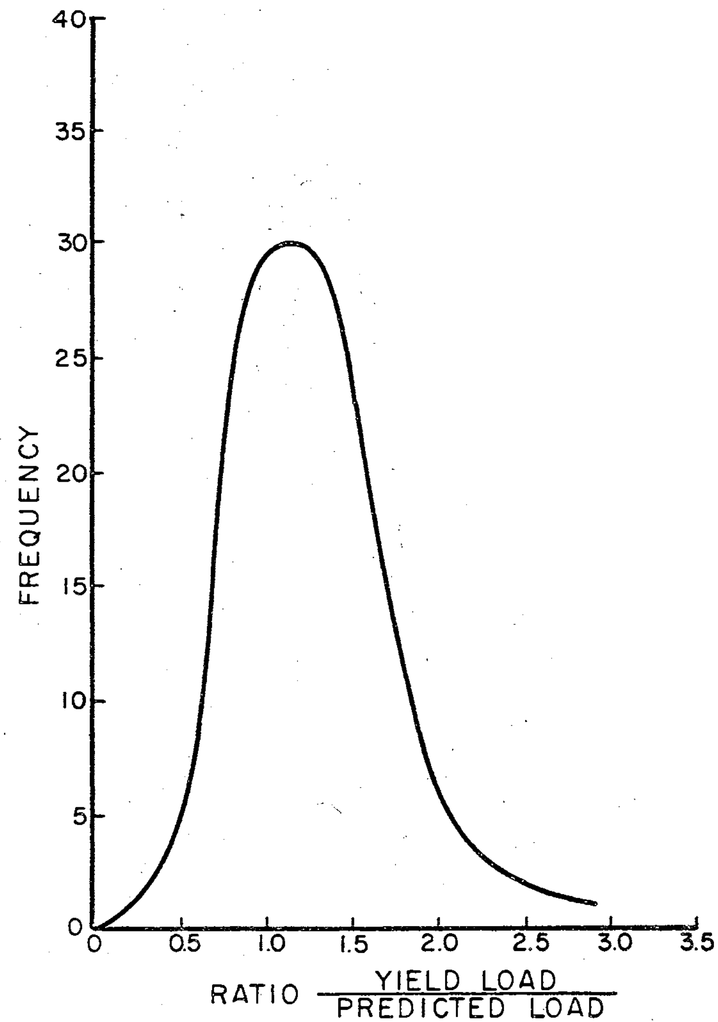

Figure 10. Frequency Distribution for Gates Formula

The only restrictions are that β must be positive and α must be greater than -1. By assigning different values to α and β, this distribution can be made to describe several practical situations. In the special case of Chi-square distribution, $\beta = 2$.

The evaluation of α and β is best done by an iterative process, but fairly accurate values for the purpose of this study can be obtained by using the following relationships:

$$\mu = \beta(\alpha + 1) \tag{18}$$

$$\sigma^2 = \beta^2(\alpha + 1) \tag{19}$$

where μ and σ^2 are the mean and variance respectively of the population. Since in this study exactness of the answer is more important than an average value, one is interested in being right as often as possible. The mode or the value that is expected to have the highest probability of occurence should, therefore, be evaluated. This is the value which has occurred most frequently in the past and is most likely to occur most often in the future.

The most frequently occurring value is obtained by differentiating the function and equating it to zero in order to solve for x.

$$\alpha x^{\alpha-1} e^{-x/\beta} + x^{\alpha}(-\frac{1}{\beta}) e^{-x/\beta} = 0 \tag{20}$$

$$e^{-x/\beta} x^{\alpha-1} (\alpha - \frac{x}{\beta}) = 0 \tag{21}$$

and finally, $x = \alpha\beta$, the most probable value.

The probability of obtaining a specific range of values would require finding the area under the curve between the limits as defined by this range. The area under the curve between limits of 0 and C is obtained by integrating the function between these limits:

$$\text{Area} = \int_0^C \frac{1}{\alpha!\,\beta^{\alpha+1}} x^{\alpha} e^{-x/\beta} dx \tag{22}$$

Putting $x = -\beta u$, $dx = -\beta du$, and for limits, $C = x = -\beta u$ or $u = -\frac{C}{\beta}$. Substituting these values,

$$\text{Area} = \frac{1}{\alpha!\,\beta^{\alpha+1}} \int_0^{-C/\beta} (-\beta)^{\alpha} u^{\alpha} e^{u} (-\beta) du$$

$$= \frac{1}{\alpha!} \int_0^{-C/\beta} (-1)^{\alpha+1} u^{\alpha} e^{u} du$$

$$= \frac{(-1)^{\alpha+1}}{\alpha!} \int_0^{-C/\beta} u^{\alpha} e^{u} du \tag{23}$$

In this form the integral can be easily solved.

Computation of Results from Frequency Distribution

For each of the formulas the estimated values of population mean, μ and population variance, σ^2 are first calculated using the standard procedure. Then, by using equations (18) and (19), estimates of parameters α and β are calculated. For the convenience of evaluating the integral in equation (23), values of α are changed to nearest integer values and values of β are recalculated using the new values of α. This operation does not materially affect the function as the parameters α and β are known to almost balance each other, maintaining the intrinsic value of the function unaffected.

The product $\alpha\beta$ gives the most probable value of the result to be expected from the formula. Using the most probable value of the ratio (yield load/predicted load) thus obtained, a suitable factor of safety for each formula is recommended. On the basis of the data used in this study it could be said that the use of this factor of safety would render safe values of load in most situations. The recommended factors are compared with those presently in use and it is found that the latter are invariably on the high side. The results are shown in Table I.

Computation of probabilities of obtaining safe values from each of the formulas, using recommended factors of safety, then follows. The estimated values of parameters μ, σ^2, α, β and the most probable value $\alpha\beta$ are recalculated on the basis of the recommended factors and the area under the curves between limits of 0 and $-C/\beta$ is evaluated using equation (23). This area is subtracted from unity to obtain the probability that the ratio (yield load/predicted load) will be greater than 1. This provides a measure of the degree of confidence that can be placed in the recommended factors of safety.

The probabilities were computed using the IBM 1620 computer. Only in evaluating the Eytelwein formula was the capacity of the machine for handling computations exceeded and thus no value could be obtained. The results are shown in Table II. A complete set of example calculations for the Engineering News formula is shown below.

TABLE I

SUMMARY OF DISTRIBUTION FUNCTION PARAMETERS

No.	Formula	μ	σ^2	α	β	Most Probable value, $\alpha\beta$	Min. F.O.S. required	Recommended F.O.S.	F.O.S. in use
1	Engineering News	0.43	0.049	3.00	0.108	0.324	3.10	4	6
2	Hiley	1.12	0.23	4.00	0.224	0.896	1.12	2	3
3	Pacific Coast	1.20	0.28	4.00	0.24	0.96	1.04	2	4
4	Redtenbacher	1.19	0.25	5.00	0.198	0.99	1.01	2	3
5	Eytelwein	0.46	0.05	43.00	0.010	0.43	2.32	3	6
6	Navy-McKay	0.43	0.08	1.00	0.215	0.215	4.65	5	6
7	Rankine	0.56	0.053	5.00	0.093	0.465	2.15	3	3
8	Canadian National	1.77	0.41	7.00	0.221	1.547	0.64	1	3
9	Mod. Engr. News	0.72	0.12	3.00	0.18	0.54	1.85	3	6
10	Gates	1.18	0.166	7.00	0.147	1.029	0.97	2	3

TABLE II

FREQUENCY DISTRIBUTION PARAMETERS AND PROBABILITY WITH RECOMMENDED FACTORS OF SAFETY

No.	Formula	Recommended F.O.S.	μ'	σ'^2	α'	β'	$\alpha'\beta'$	C/β' (C=1)	Probability
1.	Engineering News	4	1.72	0.784	3.00	0.432	1.30	2.31	0.79
2.	Hiley	2	2.24	0.920	4.00	0.448	1.79	2.23	0.92
3.	Pacific Coast	2	2.40	1.120	4.00	0.480	1.92	2.08	0.93
4.	Redtenbacher	2	2.38	1.000	5.00	0.396	1.98	2.52	0.95
5.	Eytelwein	3	1.38	0.450	43.00	0.030	1.29	33.33	-
6.	Navy-McKay	5	2.15	2.000	1.00	1.075	1.08	0.93	1.00
7.	Rankine	3	1.68	0.477	5.00	0.279	1.40	3.59	0.84
8.	Canadian National	1	1.77	0.410	7.00	0.221	1.55	4.52	0.91
9.	Mod. Engr. News	3	2.16	1.080	3.00	0.540	1.08	2.77	0.88
10.	Gates	2	2.36	0.664	7.00	0.294	1.03	6.80	0.97

Example Calculations

$\hat{\mu}$ = 30.83/71 = 0.43

$\hat{\sigma}^2$ = 3.4284/70 = 0.049

$$\hat{\beta} = \frac{\hat{\sigma}^2}{\hat{\mu}} = 0.049/0.43 = 0.114 \text{ and } \hat{\alpha} = (0.43/0.114) - 1 = 2.77$$

Rounding off values of $\hat{\alpha}$ to the nearest integer, and recalculating $\hat{\beta}$,

$\hat{\alpha}$ = 3.00

$\hat{\beta}$ = 0.43/4.00 = 0.108

and $\hat{\alpha}\hat{\beta}$ = 3 x 0.108 = 0.324

The most probable value of factor of safety should be 1/0.324 or 3.1. A factor of safety of 4 is, therefore, recommended as compared with the factor of 6 in use at present.

Recalculating values of parameters based on recommended factor of safety,

$\hat{\mu}'$ = $4\hat{\mu}$ = 4 x 0.43 = 1.72

$\hat{\sigma}'^2$ = $4^2 \times \hat{\sigma}^2$ = 16 x 0.049 = 0.784

$\hat{\alpha}'$ = $\hat{\alpha}$ = 3.00

$\hat{\beta}$ = $4\hat{\beta}$ = 4 x 0.108 = 0.432

$\hat{\alpha}'\hat{\beta}'$ = 3.00 x 0.432 = 1.30

Integrating for the area under the curve between limits of 0 and

$-C/\beta$ i.e. between 0 and $-1/0.432$,

$$\text{Area} = \frac{(-1)^3}{2!} \int_0^{-2.31} u^2 e^u du$$

$$= -\frac{1}{2}\left(u^2 e^u - 2 \int_0^{-2.31} u e^u du \right)$$

$$= -\frac{1}{2}\left[u^2 e^u - \left\{ 2(u e^u - \int_0^{-2.31} e^u du) \right\} \right]$$

$$= -\frac{1}{2}\left[u^2 e^u - (2ue^u - 2e^u) \right]_0^{-2.31}$$

$$= -\frac{e^u}{2}\left[u^2 - 2u + 2 \right]_0^{-2.31}$$

$$= 1 - 0.79 = 0.21$$

Therefore, probability of obtaining a value of 1 or more

$$= 1 - 0.21 = 0.79$$

<u>Evaluation of Formulas</u>

An examination of Table I shows that the Engineering News formula gives least variance, but its most probable value is much less than unity necessitating a high factor of safety. Next in order are the Eytelwein, Rankine, Navy-McKay and Modified Engineering News formulas. The Canadian National formula has the highest degree of variance associated with its results and gives predicted values of load which appear to be much on the low side. The

Pacific Coast, Redtenbacher and Hiley formulas show moderate degree of variance in results and give the most probable value of the ratio (yield load/predicted load) as close to unity, though still requiring a factor of safety of greater than unity. These formulas fall intermediate between the two extremes represented by the Engineering News, Eytelwein, Rankine and Navy-McKay formulas on the one hand and by the Canadian National formula on the other.

It appears that the Gates formula is generally superior to all others since the value of the ratio (yield load/predicted load) as given by this formula comes closest to unity, with a factor of safety of 1, at the same time maintaining a moderately low value of variance. Incidentally, this provides instance of the oft-quoted view that highly complicated and involved formulas are no better than simpler ones when it comes to predicting the load capacity of piles.

It is further to be seen that the factors of safety in use at the present time are generally on the high side in case of all formulas. For Hiley, Pacific Coast, Redtenbacher and Gates formulas factors of safety of 3, 4, 3 and 3 respectively are in use at present, whereas the chances are that in over ninety cases out of a hundred these formulas will predict safe values with a factor of 2. The Canadian National formula appears to predict safe values in over 90% cases using a factor of 1 as compared to the factor of 3 presently in use. The Eytelwein and Modified Engineering News formulas now employ a factor of 6, while a factor of 3 would be safe in nearly 90% of the cases.

Conclusions

The necessity of high factors of safety as recommended in case of some of the formulas might be interpreted as the consequence of inadequacy of these formulas to suitably account for all the factors involved in the pile driving process. In this respect the Hiley, Pacific Coast, and Redtenbacher formulas appear to be superior to all others. It is apparent from this study that the safety factors in use at the present time are very much on the high side for most situations, making the results too safe and at the same time rendering the design of foundations very uneconomical. For most ordinary works these high factors could be replaced by more realistic values as recommended in this study. Under extraordinary conditions, however, the use of higher factors may be justified, but in such situations elaborate soil investigation and pile tests would perhaps be economically feasible, permitting determination of a suitable factor of safety to suit the specific situation.

CHAPTER VI

DISPERSION OF RESULTS OF PILE FORMULAS

General

Of the two most important statistics employed in the study of a population, viz., the mean and the variance, the first was discussed in the preceding chapter. It is a measure of the central tendency in a population. The variance measures the dispersion or the extent to which the items cluster around or depart from the central value. A study of variances associated with the results of pile formulas is presented in this chapter.

Variable Nature of Results

It was shown in the last chapter that the mean value of the ratio (yield load/predicted load) as given by some of the formulas was fairly close to unity, while for others the ratio was found to be far removed from this value. The proximity of the mean value to one alone is not enough to justify the superiority of one formula over the other, since the degree of uniformity among the results is also a very important factor in this matter. A formula may produce results which are biased with respect to the ideal value of one, and yet it may possess the smallest relative variance as compared to other formulas. In such a case the use of the formula would be quite justified if a correction for the bias could be made. Despite the invidious connotation that usually attaches to the word "bias", the failure of a formula to give a mean value of near unity is perhaps less undesirable than a lack of uniformity in results. Undoubtedly, the best formula would have the least of both – bias and variance.

Measure for Comparing the Variances of Different Results

During the study of population distributions in the preceding chapter, values of variance for all formulas were calculated based on sample information assumed as random. These values provide estimates of the dispersion of the data as a whole and are a measure of the compactness of the population distribution. A high value of variance is indicative of a high degree of dispersion among the items of a series.

However, to compare the dispersion of two or more series, the above estimates of variance are not enough and we need a measure of relative variance, since it is known that things with large values tend to vary widely while things with small values show much smaller variation. The measure of relative variance used in this study is the coefficient of variation which is usually expressed as a percentage and is widely used for comparing variances of two or more series.

Symbolically,

$$V = \frac{s}{\overline{X}} \times 100\%, \qquad (24)$$

where s = standard deviation and $\overline{X}$ = arithmetic mean computed from the sampled data.

Using this relationship and the values of mean $(\hat{\mu} = \overline{X})$ and variance $(\hat{\sigma}^2 = s^2)$ as calculated for each formula in the preceding chapter, the coefficient of variation is computed in each case and results shown in Table III.

Inferences about Relative Variances

An examination of the tabulated values of coefficient of variation reveals the extent of relative variation among the results of different pile formulas. Arranged in the order of increasing variability, the comparative status of the formulas in this respect is at once evident from the following listing:

1. Gates
2. Canadian National
3. Rankine } same variability
4. Redtenbacher }
5. Hiley
6. Pacific Coast
7. Eytelwein } same variability
8. Modified Engineering News }
9. Engineering News
10. Navy-McKay

The Gates and Canadian National formulas show the least relative variability of results, while the Navy-McKay and Engineering News formulas are characterized by a high degree of this variability. The Gates formula, though empirical, ranks highest among the ten formulas under study. The Rankine, Pacific Coast, Redtenbacher and Hiley formulas belong practically to the same group, though Hiley and

TABLE III

VALUES OF COEFFICIENT OF VARIATION

FOR DIFFERENT FORMULAS

Formula	$\overline{X}$	s^2	$(s/\overline{X}) \times 100$
Engineering News	0.43	0.049	51%
Hiley	1.12	0.230	43%
Pacific Coast	1.20	0.280	44%
Redtenbacher	1.19	0.250	41%
Eytelwein	0.46	0.050	48%
Navy-McKay	0.43	0.080	67%
Rankine	0.56	0.053	41%
Canadian National	1.77	0.410	38%
Mod. Engr. News	0.72	0.120	48%
Gates	1.18	0.166	35%

Pacific Coast are more dispersed than others. The Eytelwein and Modified Engineering News formulas indicate high degree of variability associated with their results and rank much lower in merit in this respect. Formulas ranking high in the list could be expected to furnish more consistent results under different situations than formulas appearing towards the bottom of the list.

The above results could also be interpreted in another way. Since the coefficient of variation is representative of the extent of variance that is unaccounted for, it is indicative of the capabilities of formulas to provide for the effects of various factors which affect the actual pile driving process. In other words, the formulas showing high values of coefficient of variation seem to fail in suitably accounting for all the variables that essentially influence the load capacity of piles. The theoretical superiority of formulas with smaller value of coefficient of variation is thus vindicated. On the other hand the superiority of Gates formula demonstrates that simpler pile driving formulas, even if empirical, may be as good as the more complicated ones in actual application.

CHAPTER VII

CONSISTENCY OF PILE FORMULAS UNDER VARYING SITUATIONS

General

The pile driving process and the ultimate load bearing capacity of foundation piles are influenced by a number of variables such as type of pile, type of soil, length of pile etc. A "good" pile formula would adequately account for these factors and furnish consistently uniform values of the ratio (yield load/predicted load) under their varying effects. It is the purpose of this chapter to study this aspect of performance of pile formulas, and to ascertain which of them are truly "universal" in character, i.e., can be expected to furnish consistently acceptable load values in spite of changes in the variables as stated above. This is done by analyzing the results of formulas under a number of different situations using statistical techniques and drawing inferences therefrom.

Variables Included in the Study

An examination of the basic data indicates that the effects of the following factors and their interactions on performance of pile formulas could be investigated in this study:

a. Amount of set produced at end of driving (in.)

b. Type of pile

c. Length/area characteristic of pile. (ft/sq. in.)

For want of adequate data relating to the different types of hammers under varying effects of the above mentioned factors, the effect of driving hammer could not be included in the study. All data analyzed here, therefore, pertain to one type of hammer only, viz., the single acting steam hammer, which is quite popular in pile driving operations. The study is further restricted to principally non-cohesive soils.

By maintaining two of the above three factors constant and varying the third, the effect of the latter factor on the results of pile formulas could be studied. Accordingly, the following eight different situations were selected for this analysis:

1. Set range 0-0.24, H-pile, Length/Area range 0-50
2. Set range 0-0.24, Pipe pile, Length/Area range 0-50
3. Set range 0-0.24, H-pile, Length/Area range 51-100
4. Set range 0-0.24, Pipe pile, Length/Area range 51-100
5. Set range 0.25-0.49, H-pile, Length/Area range 0-50
6. Set range 0.25-0.49, H-pile, Length/Area range 51-100
7. Set range 0.25-0.49, Pipe pile, Length/Area range 51-100
8. Set range 0.50-1.00, H-pile, Length/Area range 0-50.

Non-Parametric Approach to Testing

Testing of hypothesis, which is employed in this study, forms a major area of statistical inference making. Basically one is interested in finding out if the results of a formula truly differ under the above mentioned different situations. This amounts to testing, for each formula, the null hypothesis, H_o: There is no significant difference between the results pertaining to the eight different situations. The alternative hypothesis may be stated as, H_1: There exists a significant difference between the results pertaining to the eight different situations.

An acceptance of the null hypothesis would mean that the formula gives results, i.e. ratio (yield load/predicted load), which are virtually unaffected by any differences in the factors occurring in the situations under study, and that the observed differences are merely chance variations to be expected in a random sample. On the other hand a rejection of the null hypothesis and, therefore, an acceptance of the alternative hypothesis would imply that the results under different situations are essentially different and the formula would not work satisfactorily under all situations.

The testing of hypotheses can be accomplished using either a parametric or non-parametric statistical test procedure. Parametric tests are somewhat punctilious in nature, the model for such a test specifying certain conditions which must be satisfied in order to make the test valid. These conditions pertain to the manner in which the sample of scores was drawn, the nature of the population from which the sample was drawn, and the kind of measurement. The usual parametric techniques for testing whether several independent samples come from identical populations are the analysis of variance or F-test and the Bartlett test. Each of them is, however, based on a variety of strong assumptions, an important one being that the populations are normally distributed. This assumption is difficult to justify in the present study, thus eliminating the possibility of employing any of the above test procedures here.

The model of a non-parametric statistical test (also known as distribution-free test) does not specify conditions about the parameters of the populations from which the sample was drawn. Moreover, the requirements of measurement of score are not so strong here as for parametric tests. Our data, though apparently in numerical score, has essentially the strength of ranks. Non-parametric test of significance can, therefore, be used in this analysis without the risk of sacrificing accuracy.

Some of the non-parametric tests for analyzing data from a number of independent samples are the χ^2-test, the extension of median test and the Kruskal-Wallis one-way analysis of variance by ranks. The χ^2-test is essentially a frequency test and is applicable in case of the null hypothesis that the independent samples have come from the same population or from identical populations with respect to the proportion of cases in the various categories. The extension of the median test is used to test whether the independent samples of a series could have been drawn from the same or identical populations with respect to the median. The Kruskal-Wallis test is a general test which shows whether the independent samples could have been drawn from the same continuous population. This test is more efficient because it uses more of the information in the observations and preserves the magnitude of the scores more fully than does the extension of median test. The Kruskal-Wallis test is found to have a power-efficiency of 95.5% when compared with its parametric counterpart, the F-test which is one of the most powerful statistical tests. This test is, therefore, employed in the analysis presented in this chapter.

Kruskal-Wallis Test

The procedure of this test involves the following steps:

1. All the observations for all, say k groups, are ranked in a single series assigning ranks from one to N, N being the total number of observations in all samples combined.

2. Value of a statistic, H, is computed using the following formula:

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(N+1), \tag{25}$$

where n_j = number of cases in the jth sample, and R_j = sum of ranks in the jth sample.

3. The probability associated with the computed value of H, and for degrees of freedom = k - 1 is found from tabulated values. If this probability is equal to or less than the previously set level of significance, H_o is rejected in favor of H_1.

Specimen Calculations and Results

The value of statistic H is calculated for each of the formulas and associated probability level for significance obtained from tabulated values. The results are shown in Table IV.

The tabulated values used here are taken from Nonparametric Statistics by Sidney Siegel (18), (Appendix, Table C).

A specimen of calculation for the Engineering News formula is given in Table V.

TABLE IV

SUMMARY OF RESULTS OF KRUSKAL-WALLIS TEST

Formula	H	D.O.F.	Probability level for significance
Engineering News	28.83	7	<0.001
Hiley	18.91	7	0.009
Pacific Coast	11.77	7	0.190
Redtenbacher	15.77	7	0.030
Eytelwein	31.85	7	<0.001
Navy-McKay	33.33	7	<0.001
Rankine	13.69	7	0.059
Canadian National	6.09	7	0.531
Mod. Engr. News	31.68	7	<0.001
Gates	12.62	7	0.085

TABLE V

SPECIMEN CALCULATION FOR KRUSKAL-WALLIS TEST

SITUATIONS																
	1		2		3		4		5		6		7		8	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
	0.37	18	0.23	7.5	0.18	3	0.20	6	0.69	37.5	0.64	33	0.23	7.5	1.03	42
	0.36	16.5	0.34	15	0.19	5	0.18	3	0.39	20	0.58	29	0.66	34	0.74	39
	0.36	16.5	0.18	3	0.14	1	0.26	11	0.38	19	0.50	25	0.56	27	0.59	30
	0.68	36					0.31	13.5	0.56	27	0.47	23	0.44	21.5	0.62	31
	0.31	13.5							0.56	27					0.77	40.5
	0.25	9.5							0.63	32					0.77	40.5
	0.25	9.5							0.48	24					0.67	35.0
									0.44	21.5						
									0.69	37.5						
									0.30	12						
Total		119.5		25.5		9.0		33.5		257.5		110		90.0		258
R_j^2/n_j		2040.36		216.75		27.00		280.56		6630.63		3025.0		2025.0		9509.14

N = 42, k = 8

$$\sum_{j=1}^{8} \frac{R_j^2}{n_j} = 23754.44$$

Therefore, $H = (\frac{12}{42 \times 43} \times 23754.55) - (3 \times 43)$

$= 28.83$

For H = 28.76 and degrees of freedom = 7, tabulated value of associated probability is < 0.001.

Inferences from Statistical Test

The probability level shown for each formula indicates that the null hypothesis may be rejected at that level of significance. It is customary to set in advance this level of significance based on an estimate of importance or possible practical significance of findings. The values commonly used are 0.05 and 0.01. A larger value indicates greater likelihood that type I error will be committed, i.e. that the H_o will be rejected when it is in fact true.

For this study a level of significance of 0.01 is considered satisfactory. The procedure for test is to reject the null hypothesis in favor of the alternative hypothesis if the probability of occurrence associated with the computed value of the test statistic is equal to or less than 0.01. This probability of occurrence is shown in Table IV.

It is found from the test that the null hypothesis is to be rejected in the case of Engineering News, Hiley, Eytelwein, Navy-McKay, and Modified Engineering News formulas, while in the case of other formulas there is no sufficient statistical evidence to reject the null hypothesis. It can be concluded that in the case of the above named formulas the results vary significantly under different situations and these formulas cannot be expected to be consistently valid under the varying influences of type of pile, degree of set produced at end of driving and the length/area characteristic of the pile. The load values predicted by these formulas would widely differ from the actual yield load values depending on the variables involved.

On the other hand the other five formulas exhibit varying degrees of consistency in results under changes of variables mentioned above, but there is no sufficient statistical evidence to indicate that this variability would be significant. In fact, the null hypothesis may even be accepted and it may be concluded that in the case of the following formulas the value of the ratio (yield load/predicted load) would be relatively unaffected by a change in variables during pile driving:

1. Canadian National
2. Pacific Coast
3. Gates
4. Rankine
5. Redtenbacher

CHAPTER VIII

RESULTS OF STATISTICAL TESTS

General

The statistical tests as used in this study have been aimed at answering the following specific questions concerning the behavior and usefulness of ten selected dynamic pile formulas:

1. What is the justification of using the present factors of safety with the formulas? If these factors are not appropriate, what better values could be suggested on the basis of the available data?

2. Since some amount of variance is always likely to be present when a large number of results are analyzed, what is the relative performance of the formulas in this regard? In other words, what is the degree of variability associated with the results of each of the formulas, and which of them may be expected to give more or less consistent values of the ratio (yield load/predicted load)?

3. What is the behavior of the formulas under varying practical situations? That is, which of the formulas may furnish consistent results under different conditions of driving such as pile type, pile length, and amount of set produced?

The techniques employed in this evaluation include, for each of the formulas, a study of the probability distribution of its results, computation of the coefficient of variation and the non-parametric

Kruskal-Wallis test of significance. The results of these tests are summarized in the following articles.

Study of Probability Distribution

For each formula the frequency distribution using values of ratio (yield load/predicted load) was plotted and the function analyzed assuming a gamma probability distribution. It was found that Hiley, Pacific Coast, Redtenbacher, and Gates formulas furnish this ratio with its most likely value close to unity with a factor of safety of about 1. Other formulas do not behave so well in this respect. In particular, the Engineering News and Navy-McKay formulas give much smaller values for this ratio.

This analysis demonstrated the fact that the factors of safety in use at the present time are much on the high side in case of all formulas. For instance, Hiley, Redtenbacher, Canadian National, and Gates formulas employ a factor of safety of 3, whereas in over 90% of the cases these formulas would yield safe load values with a factor of safety of 2. Accordingly, new values for these factors have been suggested for all formulas and it is believed that these represent more realistic values.

Variability in Results

The dispersion of values given by the formulas was compared on the basis of coefficient of variation computed from the results of each formula. It was found that Gates and Canadian National formulas were much superior to all others in this respect. Next in order was the group which included Hiley, Pacific Coast, Redtenbacher and Rankine formulas, while the rest of the formulas viz., Engineering News, Eytelwein, Navy-McKay and Modified Engineering News exhibited very high degree of variability in results.

Behavior under Varying Driving Conditions

The results of formulas under eight different driving conditions were tested using Kruskal-Wallis non-parametric test. It was found that the Pacific Coast, Redtenbacher, Rankine, Canadian National, and Gates formulas do not show any significant differences in results under the different conditions studied. These formulas, therefore, could be expected to furnish consistent values of the ratio (yield load/predicted load) in spite of changes in pile characteristics and set. The other formulas viz., Hiley, Engineering News, Eytelwein, Navy-McKay and Modified Engineering News show significant differences in results due to changes in driving conditions and, therefore, cannot be relied upon to maintain a uniform degree of accuracy in predicting pile capacities under varying situations.

Conclusions

Unexpected though it may seem, all the above tests have demonstrated the superiority of Gates formula in predicting load capacities of piles based on driving information. The most likely value of the ratio (yield load/predicted load) as furnished by this formula is well over unity and is also economical from the designer's point of view. At the same time the probability that the ratio would be greater than one is very high. Furthermore, the pattern of distribution is more compact around the mean for this formula than for others, and the values are consistently acceptable under varying conditions of driving.

Added to the above is the great inherent simplicity of this formula which renders it much easier in practical application than most of the other formulas. It could, therefore, be concluded that for the conditions previously specified, this formula offers an excellent answer to the long continuing search for a suitable pile driving formula, and attempts to improve upon its usability would be worthwhile. One such attempt is presented in the next chapter.

CHAPTER IX

MODIFIED PILE DRIVING FORMULA

General

As was pointed out in the last chapter, the Gates formula offers a promise to provide a suitable pile driving formula for easy practical application. If its capability to yield safe results at nearly all times could be improved, its usefulness as a convenient device for controlling pile driving operations in field would be further enhanced. An approach in this direction using regression techniques and incorporating a suitable factor of safety is presented in this chapter. A modified Gates formula has been developed and is shown to give better performance than the original formula.

Some Apparent Deficiencies in Gates Formula

An examination of the scatter diagram shown in Fig. 11, with predicted load values plotted against yield loads, indicates that in the higher range of yield load, above about 200 tons or so, the predicted values are consistently much smaller than the load test values, indicating that in this region the formula is much too safe even without a factor of safety. At the same time, in the lower ranges of yield load the points on the diagram are nearly uniformly distributed about the 45° line, and a factor of safety would perhaps

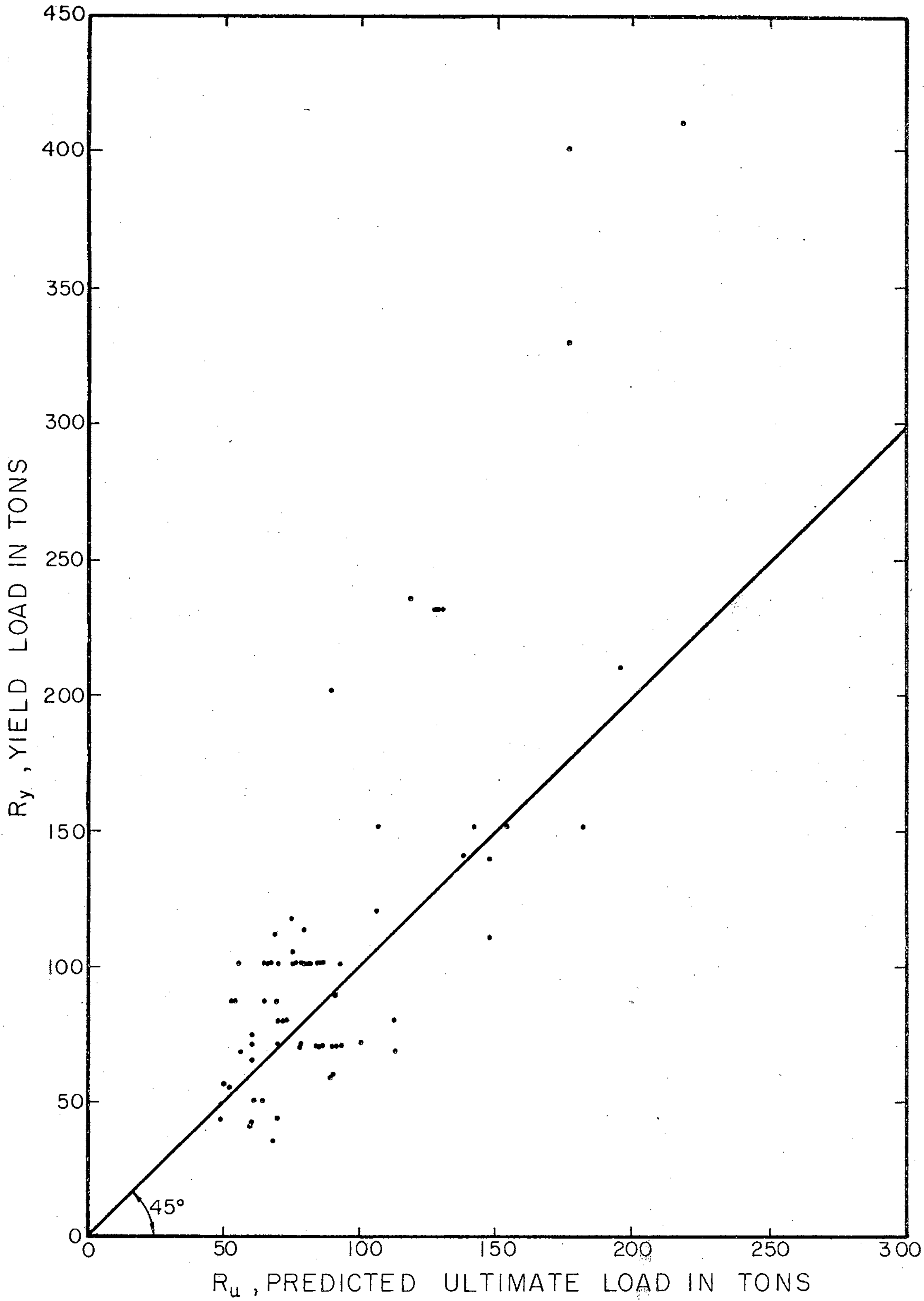

Figure 11. Behavior of Gates Formula at High Yield Loads

be desirable. However, the use of such a factor at all times would further render the use of this formula very uneconomical at high values of pile capacity.

It appears that the basic data utilized by Gates to evolve his formula largely pertained to lower ultimate load values than those covered in this study. The data analyzed here includes some of the latest pile load tests with pile capacities as high as 470 tons. In fact, only about 10% of the results pertain to values of fifty tons or below, the rest being above this value and nearly 60% relating to capacities of 100 tons or above. This may be one of the principal causes for inadequacy of this formula in the high load region.

In some other respects too, the data used by Gates (6) appear to be appreciably different. For example, he did not include piles driven by diesel and differential hammers, or by the heavier single acting hammers such as the Vulcan OR type. Most of the data related to drop hammers of various sizes and types. Again, the conditions of driving included in his work were relatively "soft" as compared to those of this study. The value of set ranged to as high a value as 4.44 inches in his data while in this study the maximum value of set is restricted to one inch. There also appears to be a great deal of difference between the two studies as regards pile types and characteristics. Whereas only steel piles are included in the present investigation, Gates has utilized data on timber, steel, and reinforced concrete piles. Nearly 60% of the piles used in this study were fifty feet or over in length and almost 70% weighed forty pounds per lineal foot or more. This information in respect of piles used by Gates is not available; however, it is likely to be very different.

These limitations of data would have necessarily affected the form of the relationship evolved by Gates. It is believed that the information on which the present study is based is much more representative of present trends in pile driving than the data used by Gates, and a modification of his formula in the light of the present study would constitute a useful contribution to the subject.

Curve Fitting Using Method of Least Squares

To determine a suitable functional relationship between the test loads and the results predicted by the Gates formula (with f. o. s. = 1), the method of least squares using simple linear regression has been used in this study. The computations are presented below:

The two normal equations may be written as

$$(\Sigma x)b_o + (\Sigma x^2)b_1 = \Sigma xy \qquad (26)$$

$$(n)b_o + (\Sigma x)b_1 = \Sigma y \qquad (27)$$

where,

x = Load as predicted by existing formula,

y = Yield load value corresponding to the predicted load,

n = Number of observations, and

b_o, b_1 are constants to be ascertained.

The following values are computed from the data to be used in the two normal equations:

$$\Sigma x = 6,479$$

$$\Sigma x^2 = 688,971$$

$$\Sigma xy = 845,319.43$$

$$\Sigma y = 7,886$$

$$n = 71$$

Substituting these values and simplifying yields the following equations:

$$b_o + 106.34\, b_1 = 130.47$$

and

$$b_o + 91.25\, b_1 = 111.07 .$$

Upon solving these equations the following values are obtained:

$$b_o = -5.73 \quad \text{and} \quad b_1 = 1.28 .$$

Thus, the regression equation is found to be:

$$y = 1.28x - 5.73 . \tag{28}$$

However, for the sake of brevity, and also since the last term is small especially in the high load range, this term may be dropped altogether leaving the simplified form of the equation as $y = 1.28x$. This modifies the present Gates formula to the following form:

$$R_u' = 1.28 \left[\frac{3}{7} \sqrt{E_n}\ \text{ABS}(\log \frac{S}{10})\right]$$

$$= 0.55 \sqrt{E_n}\ \text{ABS}(\log \frac{S}{10}) \tag{29}$$

where R_u' is the ultimate predicted load (f.o.s. = 1) as found from the modified formula.

The values for R_u' were computed for the data and are shown in Table VI. The new scatter diagram using these values is shown in Fig. 12.

Performance of Modified Gates Formula and Use of Factor of Safety

A comparison of the two scatter diagrams (Fig. 11 and Fig. 12) shows that in its modified form the formula can be expected to give more realistic values of pile bearing capacity in higher ranges than could be expected from the original formula. Also, the predicted values would be quite safe without any factor of safety being used.

However, the result of this modification in the lower range of load (below about 200 tons) is to shift most of the points on the diagram below the 45° line, indicating the need for a suitable factor of safety in this region. An examination of the predicted values suggests that a factor of safety of two would be quite adequate in this region of lower values, while no such factor is necessary above load capacities of 200 tons.

Based on this idea the predicted values shown in Fig. 12 are revised by dividing those values corresponding to yield loads below 200 tons, by two; and keeping the other values unchanged. These new values constitute safe design loads and are shown in Table VII. The corresponding scatter diagram is shown in Fig. 13.

An examination of this diagram shows that predicted values are now safe in practically all cases, while these are no more 'too safe' in the region of high yield loads. Thus, the suggested modification

TABLE VI

RESULTS OF MODIFIED GATES FORMULA

No.	Yield Load Tons	Pred. Load, R_u' Tons	No.	Yield Load Tons	Pred. Load, R_u' Tons	No.	Yield Load Tons	Pred. Load, R_u' Tons
1	85.0	67.4	28	100.0	100.4	55	410.0	279.6
2	100.0	82.4	29	100.0	92.3	56	330.0	227.5
3	85.0	91.6	30	150.0	196.4	57	46.0	89.3
4	77.0	91.6	31	100.0	117.6	58	120.0	134.9
5	105.0	95.2	32	100.0	105.4	59	150.0	137.9
6	115.0	98.8	33	100.0	108.8	60	71.0	103.7
7	50.0	82.4	34	70.0	128.3	61	56.0	64.1
8	100.0	102.6	35	35.0	86.9	62	45.0	77.0
9	120.0	94.3	36	230.0	162.0	63	44.5	64.1
10	50.0	76.9	37	235.0	149.9	64	55.0	68.6
11	72.0	74.4	38	61.0	115.7	65	67.5	72.1
12	85.0	69.1	39	70.0	109.7	66	45.0	75.9
13	60.0	113.3	40	70.0	108.1	67	79.0	75.9
14	100.0	69.1	41	70.0	105.9	68	67.0	77.0
15	70.0	86.9	42	70.0	116.9	69	89.0	82.6
16	80.0	94.3	43	70.0	118.6	70	113.0	87.1
17	80.0	91.2	44	70.0	112.1	71	88.0	117.7
18	200.0	113.6	45	70.0	99.3			
19	67.0	144.9	46	230.0	162.9			
20	80.0	144.9	47	230.0	165.1			
21	100.0	109.2	48	138.0	191.2			
22	100.0	110.9	49	150.0	238.5			
23	100.0	110.9	50	110.0	190.4			
24	100.0	82.4	51	140.0	176.6			
25	100.0	88.9	52	210.0	250.8			
26	100.0	83.9	53	150.0	179.0			
27	100.0	97.9	54	400.0	224.8			

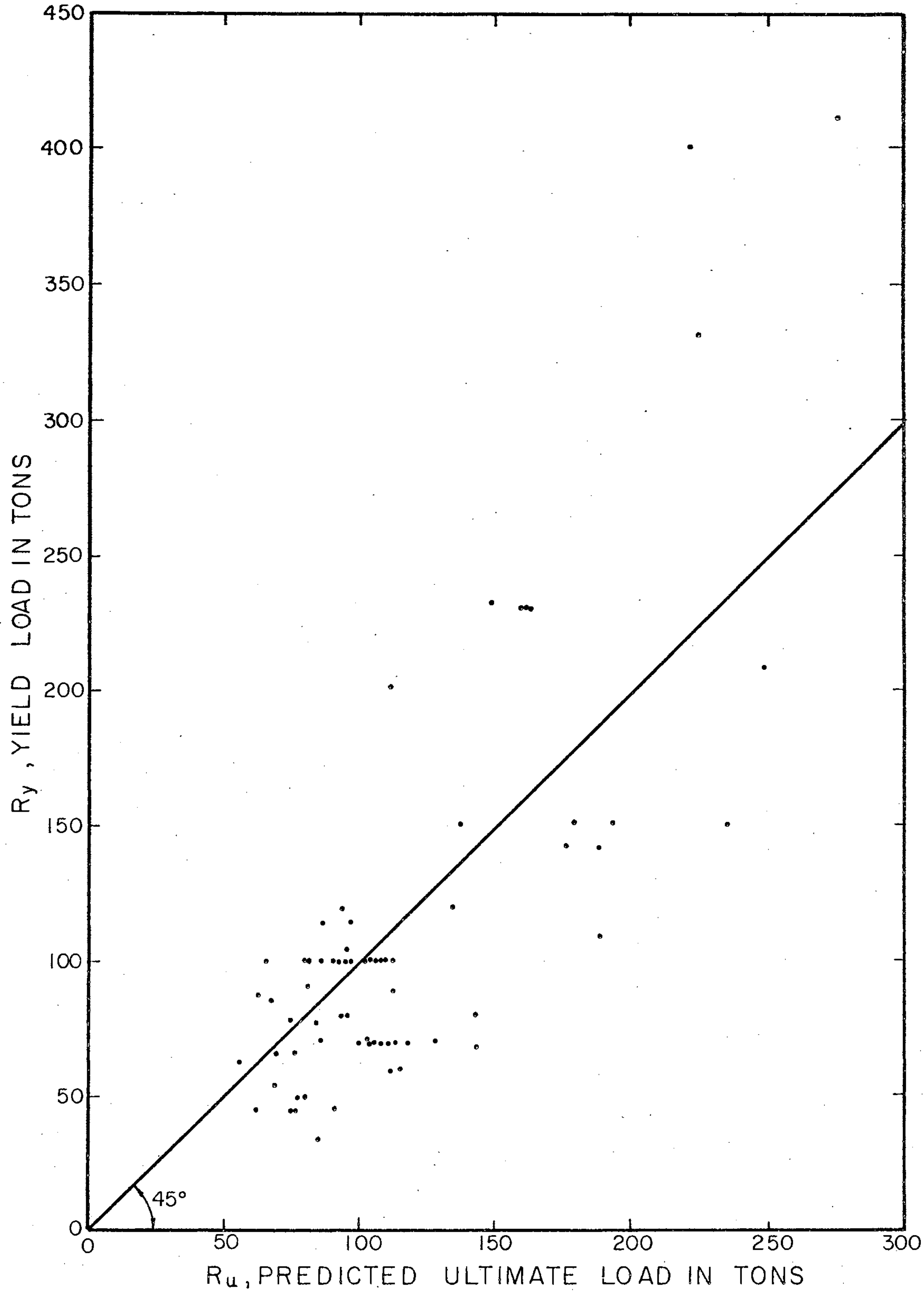

Figure 12. Behavior of Modified Gates Formula Without F.O.S.

TABLE VII

RESULTS OF MODIFIED GATES FORMULA WITH SUGGESTED F.O.S.

No.	Yield Load Tons	Pred. Load, R_d Tons	No.	Yield Load Tons	Pred. Load, R_d Tons	No.	Yield Load Tons	Pred. Load, R_d Tons
1	85.0	33.7	26	100.0	41.9	51	140.0	88.3
2	100.0	41.2	27	100.0	48.9	52	210.0	250.8
3	85.0	45.8	28	100.0	50.2	53	150.0	89.5
4	77.0	45.8	29	100.0	46.2	54	400.0	224.8
5	105.0	47.6	30	150.0	98.2	55	410.0	279.6
6	115.0	49.4	31	100.0	58.8	56	330.0	227.5
7	50.0	41.2	32	100.0	52.7	57	46.0	44.7
8	100.0	51.3	33	100.0	54.4	58	120.0	67.5
9	120.0	47.2	34	70.0	64.2	59	150.0	68.9
10	50.0	38.5	35	35.0	43.5	60	71.0	51.9
11	72.0	37.2	36	230.0	162.0	61	56.0	32.1
12	85.0	34.6	37	235.0	149.9	62	45.0	38.5
13	60.0	56.7	38	61.0	57.8	63	44.5	32.1
14	100.0	34.6	39	70.0	54.9	64	55.0	34.3
15	70.0	43.5	40	70.0	54.1	65	67.5	36.1
16	80.0	47.2	41	70.0	52.9	66	45.0	37.9
17	80.0	45.6	42	70.0	58.5	67	79.0	37.9
18	200.0	113.6	43	70.0	59.3	68	67.0	39.0
19	67.0	72.5	44	70.0	56.1	69	89.0	41.3
20	80.0	72.5	45	70.0	49.7	70	113.0	43.6
21	100.0	54.6	46	230.0	162.9	71	88.0	58.9
22	100.0	55.5	47	230.0	165.1			
23	100.0	55.5	48	138.0	95.6			
24	100.0	41.2	49	150.0	119.2			
25	100.0	44.5	50	110.0	95.2			

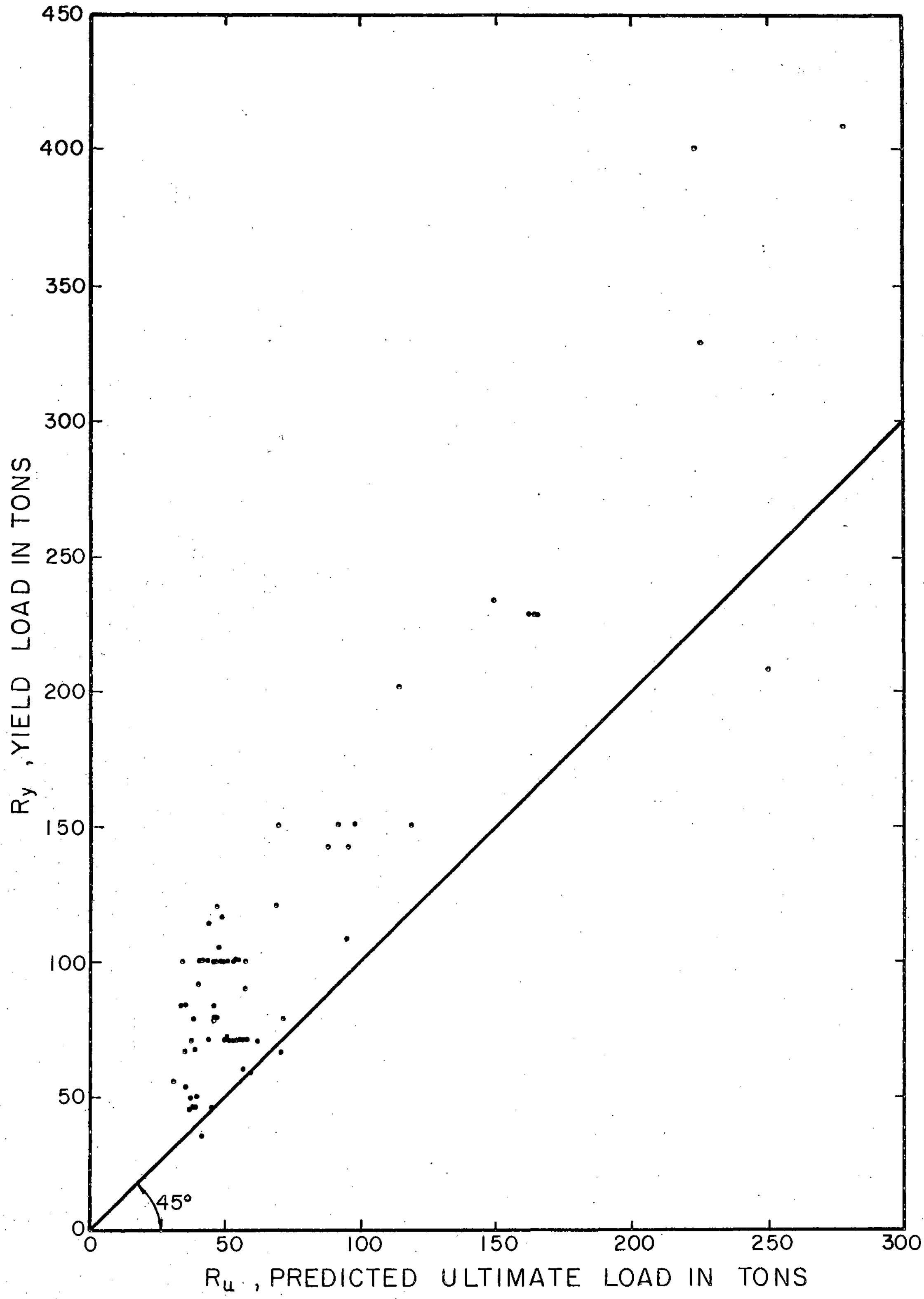

Figure 13. Behavior of Modified Gates Formula with Suggested F.O.S. in Lower Range of Yield Load

renders the formula capable of predicting pile bearing capacities which are both safe and economical from the design point of view, at nearly all times. This is further vindicated by statistical tests applied to the above results as presented in the next article.

Statistical Tests Applied to Modified Results

These tests are identically the same as used earlier in this investigation and include the probability study associated with prediction of safe values, evaluation of coefficient of variation to test the dispersion of results, and the Kruskal-Wallis test to test for consistency of results under varying situations.

Using the ratio (yield load/predicted load) based on modified predicted load values, the sample mean and sample variance are first determined and then values of parameters $\hat{\alpha}$ and $\hat{\beta}$ are computed assuming a gamma distribution function. The following results are obtained:

$$\bar{X} = 1.67$$

$$S^2 = 0.22$$

$$\hat{\alpha} = 13$$

$$\hat{\beta} = 0.119.$$

Based on these values, the most likely value for the ratio was found to be 1.55 and the probability that the ratio would be greater than unity was computed as 95%, indicating that nearly always the predicted value of capacity would be safe for the pile. The great improvement in the capability of the formula over its original form is, thus, clearly visible.

To test the degree of variation among results, coefficient of variation was computed as (0.47/1.67) x 100 = 28%. This value is lower than the earlier value, indicating a further improvement in the pattern of variability as a result of the suggested modification, this variability being the lowest among the formulas under study.

For the Kruskal-Wallis test of significance the parameter H was found to be 8.1. The level of significance for this value of H and for degrees of freedom equal seven, was found from the tabulated values as 0.326. This shows that at the preassigned level of significance of 0.01 the null hypothesis of no significant difference between results under different pile driving conditions, cannot be rejected. That is, the formula still maintains its consistency of predicting acceptable results under varying situations.

Conclusions

The tests described in the foregoing article demonstrate that the modification of the formula as developed in this chapter greatly improves its performance and versatility. This improvement results in the capability to furnish results which are both safe and economical, and enhances the degree of certainty that these results would be safe at nearly all times. The variance characteristic and its ability to predict consistently accurate values despite changes in driving conditions are improved as a result of the suggested modification.

Since the conditions of this study are fairly representative of the modern trends in pile driving, it is believed that the proposed modification of the formula would prove of great practical value in the present-day pile foundation work.

CHAPTER X

SUMMARY OF INVESTIGATION AND SUGGESTIONS FOR FURTHER STUDY

Summary and Conclusions

A critical evaluation of ten popular pile driving formulas using statistical methods has been presented in this study. These formulas are:

1. Engineering News
2. Hiley
3. Pacific Coast Uniform Building Code
4. Redtenbacher
5. Eytelwein
6. Navy-McKay
7. Rankine
8. Canadian National Building Code
9. Modified Engineering News
10. Gates .

The study is confined to two types of piles (steel H-pile and steel pipe pile) and to one type of soil (non-cohesive and sandy or gravelly). The data which form the basis for the investigation are mostly of recent origin and have been obtained from various sources. It is believed that these data are fairly representative of the present practice in pile driving.

A total of seventy-one test results were examined. These included a wide range of driving hammers, pile characteristics, failure loads and sets. The hammers included are single-acting, differential and double acting, diesel and drop hammers. Pile lengths vary from 18 ft. to 160 ft., with nearly 60% of these being 50 ft. or longer in length. Pile weights vary from 23.09 lb/lft to 117 lb/lft with only 30% being lower than 40 lb/lft. Yield load values range from 35.0 tons to 400 tons, and about 90% are above the 50 ton mark. Sets vary from 0.006 in. to 1 in.

The statistical methods used in this work include - a probability study of the results of the formulas, test for the degree of variability among the results and the non-parametric test of significance to ascertain the uniformity in results under different driving conditions.

The gamma probability distribution was found to be suitable for these data, and the necessary parameters for the distribution function were computed for each of the formulas. These parameters were then used in determination of probability that the ratio (yield load/predicted load) would be greater than unity. The results showed that Hiley, Pacific Coast, Redtenbacher, Canadian National, and Gates formulas are capable of predicting safe pile capacities over 90% of the time with a factor of safety of 2. The Eytelwein, Rankine and Modified Engineering News formulas need a factor of safety of 3; Engineering News formula, a factor of 4; and Navy-McKay formula, a factor of at least 5. This analysis also demonstrated the fact that in case of all formulas the factors of safety in use at the present time are much on the high side.

The test for dispersion of results was made by computing the value of coefficient of variation for each formula. It was found that variance associated with results of Canadian National and Gates formulas was much less than for other formulas. In particular, the Engineering News, Eytelwein, Navy-McKay, and Modified Engineering News formulas showed very high degrees of variability in results.

Since the probability function was considered non-normal, the usual statistical tests of significance could not be employed in this study. Recourse was had, therefore, to a non-parametric test and the Kruskal-Wallis test was used to study the behavior of the formulas under different situations. The test showed that in the case of Pacific Coast, Redtenbacher, Rankine, Canadian National, and Gates formulas there was no significant difference between the results under varying pile driving conditions, while the results of other formulas were significantly affected by a change in these conditions.

All of the above tests indicated the superiority of the Gates formula in predicting pile bearing capacities on the basis of driving data. Further study of this formula was, therefore, made with a view to suggest a modification to improve its workability, and a modified form was evolved using simple linear regression. This formula when used with a factor of safety of two for loads under about 200 tons, and without any factor of safety at other loads, appeared to give optimum results which were quite safe and also economical from design point of view. It is believed that for the specific conditions of this study this modified Gates formula offers the best answer yet known to the quest for a suitable dynamic formula.

Suggestions for Further Study

In a work of this nature the need for a large amount of basic data cannot be over-emphasized. On the other hand, this study was based upon a limited number of test results only, in an attempt to do the best from what was available. It, therefore, appears necessary that further information on pile tests should be collected and processed in the manner as outlined in this investigation so that a more dependable relationship between pile capacity and driving data may be developed.

It is suggested that similar studies be conducted for data on different kinds of soils, piles, and ranges of set and failure load. These would lead to valuable information not only for evolving a better dynamic formula, but also for our understanding of the complex phenomenon of soil-pile interaction.

It may be possible to eliminate the need of a factor of safety for specific values of yield load as proposed here, or to provide a uniform factor of safety for all values of loads by including the failure load on the right hand side of the formula on the lines of the Pacific Coast and Canadian National formulas. This would need a large number of correlations at varying failure loads, and may ultimately result in a more accurate formula though at the cost of sacrifice of simplicity of the present form.

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APPENDIX A

SUMMARY OF PILE DRIVING DATA

No.	Type of pile	Length of pile, ft.	X-Section area, sq. in.	Weight per ft. lbs.	Weight of driving head, lbs.	Type of driving hammer	Weight of hammer ram, lbs.	Stroke of ram in.	Manuf. rated energy ft. lbs.	No. of blows per ft.	Yield load, tons
1	2	3	4	5	6	7	8	9	10	11	12
1	10H42	44.0	12.35	42.00	1000.0	Vul-1	5000.0	36.0	15000	12	85.0
2	10H42	44.0	12.35	42.00	1000.0	Vul-1	5000.0	36.0	15000	20	100.0
3	12H53	40.0	15.58	53.00	1000.0	MT-10B3	3000.0	19.0	13100	34	85.0
4	12H53	40.0	15.58	53.00	1000.0	MT-10B3	3000.0	19.0	13100	34	77.0
5	14H89	82.0	26.19	89.00	1000.0	Vul-1	5000.0	36.0	15000	31	105.0
6	Armco	44.0	6.80	23.09	1000.0	Vul-1	5000.0	36.0	15000	35	115.0
7	Armco	30.0	8.55	29.06	1000.0	Vul-1	5000.0	36.0	15000	20	50.0
8	Armco	55.0	9.63	32.74	1000.0	Vul-1	5000.0	36.0	15000	40	100.0
9	Armco	44.0	6.80	23.09	1000.0	Vul-1	5000.0	36.0	15000	30	120.0
10	10H42	60.0	12.35	42.00	1000.0	ST-D.A.	950.0	16.0	6750	60	50.0
11	10H42	20.0	12.35	42.00	1000.0	Gravity	2640.0	60.0	13200	18	72.0
12	10H42	20.0	12.35	42.00	1000.0	Gravity	3150.0	60.0	15750	12	85.0
13	10H42	114.0	12.35	42.00	1000.0	ST-D.A.	950.0	16.0	6750	384	60.0
14	10H42	60.0	12.35	42.00	1000.0	Gravity	3150.0	60.0	15750	12	100.0
15	10H42	40.0	12.35	42.00	1000.0	Gravity	3000.0	60.0	15000	24	70.0
16	10H42	30.0	12.35	42.00	1000.0	Gravity	3000.0	60.0	15000	30	80.0
17	10H42	30.0	12.35	42.00	1000.0	Gravity	3000.0	60.0	15000	27	80.0
18	12H53	50.0	15.58	53.00	670.0	Vul-1	5000.0	36.0	15000	58	200.0
19	12H53	30.0	15.58	53.00	1000.0	ST-D.A.	950.0	16.0	6750	1920	67.0
20	12H53	40.0	15.58	53.00	1000.0	ST-D.A.	950.0	16.0	6750	1920	80.0
21	12H53	50.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	50	100.0
22	12H53	55.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	53	100.0
23	12H53	47.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	53	100.0
24	12H53	65.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	20	100.0
25	12H53	75.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	25	100.0

1	2	3	4	5	6	7	8	9	10	11	12
26	12H53	75.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	21	100.0
27	12H53	85.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	34	100.0
28	12H53	95.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	37	100.0
29	12H53	84.0	15.58	53.00	1000.0	Vul-1	5000.0	36.0	15000	28	100.0
30	Pipe	50.0	21.22	82.77	1000.0	Vul-1	5000.0	36.0	15000	980	150.0
31	Pipe	39.0	7.32	33.38	1000.0	Gravity	3160.0	60.0	15800	60	100.0
32	Pipe	29.0	7.32	33.38	1000.0	Gravity	3160.0	60.0	15800	40	100.0
33	Pipe	18.0	7.32	33.38	1000.0	Vul-2	3000.0	29.0	7250	250	100.0
34	Pipe	60.0	8.50	29.00	2910.0	Vul-1	5000.0	36.0	15000	96	70.0
35	Pipe	60.0	8.50	29.00	2910.0	Vul-1	5000.0	36.0	15000	24	35.0
36	Pipe	133.5	9.23	31.38	4505.0	Delmag	4850.0	98.0	39700	36	230.0
37	Pipe	134.0	9.23	31.38	4050.0	Vul-80	8000.0	16.0	24450	66	235.0
38	Pipe	58.3	8.50	29.00	4910.0	Vul-1	5000.0	36.0	15000	62	61.0
39	Pipe	80.0	9.23	31.38	4505.0	Delmag	4850.0	98.0	39700	12	70.0
40	Pipe	80.0	9.23	31.38	4050.0	Vul-80	8000.0	16.0	24450	21	70.0
41	Pipe	80.0	9.23	31.38	4050.0	Vul-80	8000.0	16.0	24450	20	70.0
42	Pipe	80.0	9.23	31.38	6160.0	MT-D	4000.0	96.0	32000	18	70.0
43	Pipe	80.0	9.23	31.38	6160.0	MT-D	4000.0	96.0	32000	19	70.0
44	Pipe	80.0	9.23	31.38	4470.0	LB-D	5070.0	43.0	30000	18	70.0
45	Pipe	80.0	9.23	31.38	4470.0	LB-D	5070.0	43.0	30000	13	70.0
46	Pipe	160.0	9.23	31.38	4050.0	Vul-80	8000.0	16.0	24450	93	230.0
47	Pipe	160.0	9.23	31.38	4505.0	Delmag	4850.0	98.0	39700	38	230.0
48	10H42	125.0	12.35	42.00	930.0	Vul-OR	9300.0	39.0	30225	119	138.0
49	10H42	125.0	12.35	42.00	930.0	Vul-OR	9300.0	39.0	30225	372	150.0
50	10H42	125.0	12.35	42.00	930.0	Vul-OR	9300.0	39.0	30225	117	110.0

1	2	3	4	5	6	7	8	9	10	11	12
51	12H53	125.0	15.58	53.00	930.0	Vul-OR	9300.0	39.0	30225	84	140.0
52	12H53	125.0	15.58	53.00	930.0	Vul-OR	9300.0	39.0	30225	500	210.0
53	12H53	125.0	15.58	53.00	930.0	Vul-OR	9300.0	39.0	30225	89	150.0
54	14H117	125.0	34.44	117.00	930.0	Vul-OR	9300.0	39.0	30225	268	400.0
55	14H117	125.0	34.44	117.00	930.0	Vul-OR	9300.0	39.0	30225	999	410.0
56	14H117	125.0	34.44	117.00	930.0	Vul-OR	9300.0	39.0	30225	286	330.0
57	Pipe	50.0	14.58	49.56	600.0	Vul-2	3000.0	29.0	7250	96	46.0
58	Pipe	80.0	9.82	33.38	1080.0	Vul-1	5000.0	36.0	15000	120	120.0
59	Pipe	80.0	9.82	33.38	690.0	Vul-1	5000.0	36.0	15000	133	150.0
60	10H42	46.0	12.35	42.00	750.0	Vul-1	5000.0	36.0	15000	41	71.0
61	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	15	56.0
62	10H60	29.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	26	45.0
63	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	15	44.5
64	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	19	55.0
65	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	21	67.5
66	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	25	45.0
67	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	25	79.0
68	10H60	29.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	26	67.0
69	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	33	89.0
70	10H60	39.3	18.20	60.00	900.0	ST-S.A.	3000.0	36.0	10800	40	113.0
71	12H65	99.2	19.11	65.00	750.0	ST-D.A.	3625.0	20.0	22080	33	88.0

Note: The following symbols have been used for driving hammers:

Vul-1	Vulcan-1	Vul-80	Vulcan-80
MT-10B3	McKiernan-Terry 10B3	MT-D	McKiernan-Terry, Diesel
ST-S.A.	Steam, Single-Acting	LB-D	Link-Belt, Diesel
Vul-2	Vulcan-2	Vul-OR	Vulcan-OR
Delmag	Delmag, Diesel	ST-D.A.	Steam, Double-Acting

APPENDIX B

RESULTS FROM PILE DRIVING FORMULAS

		ENGR. NEWS		HILEY		PACIFIC COAST		REDTENBACHER		EYTELWEIN	
No.	Yield load tons R_y	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u
1	2	3	4	5	6	7	8	9	10	11	12
1	85.0	81.8	1.03	51.9	1.63	56.4	1.50	53.2	1.59	85.1	0.99
2	100.0	128.5	0.77	76.8	1.30	79.3	1.26	80.2	1.24	136.9	0.72
3	85.0	173.5	0.48	81.4	1.04	90.2	0.94	87.0	0.97	172.0	0.49
4	77.0	173.5	0.44	81.4	0.94	90.2	0.85	87.0	0.88	172.0	0.44
5	105.0	184.7	0.56	61.6	1.70	81.1	1.29	71.0	1.47	162.7	0.64
6	115.0	203.2	0.56	120.6	0.95	88.2	1.30	104.5	1.09	234.8	0.48
7	50.0	128.5	0.38	89.1	0.56	85.3	0.58	90.1	0.55	141.1	0.35
8	100.0	225.0	0.44	113.6	0.87	91.5	1.09	106.2	0.94	252.7	0.39
9	120.0	180.0	0.66	109.7	1.09	84.3	1.42	98.0	1.22	204.3	0.58
10	50.0	135.0	0.37	31.3	1.59	44.4	1.12	32.6	1.53	70.9	0.70
11	72.0	103.3	0.69	64.4	1.11	72.1	0.99	65.8	1.09	107.5	0.66
12	85.0	85.9	0.98	57.5	1.47	63.1	1.34	57.5	1.47	89.2	0.95
13	60.0	308.5	0.19	26.9	2.22	42.1	1.42	35.3	1.69	63.2	0.94
14	100.0	85.9	1.16	42.7	2.33	48.1	2.07	41.3	2.42	85.0	1.17
15	70.0	150.0	0.46	77.9	0.89	81.7	0.85	78.9	0.88	152.7	0.45
16	80.0	180.0	0.44	100.2	0.79	101.9	0.78	102.7	0.77	189.3	0.42
17	80.0	165.3	0.48	93.2	0.85	96.5	0.82	95.5	0.83	173.1	0.46
18	200.0	293.2	0.68	134.2	1.48	121.5	1.64	139.9	1.42	329.3	0.60
19	67.0	381.1	0.17	85.9	0.77	106.9	0.62	114.8	0.58	145.2	0.46
20	80.0	381.1	0.20	69.5	1.14	90.0	0.88	92.9	0.86	121.0	0.66
21	100.0	264.7	0.37	121.8	0.82	114.9	0.86	128.4	0.77	287.5	0.34
22	100.0	275.7	0.36	119.0	0.84	111.5	0.89	125.1	0.79	295.3	0.33
23	100.0	275.7	0.36	129.4	0.77	120.4	0.83	135.7	0.73	303.8	0.32
24	100.0	128.5	0.77	62.8	1.59	69.8	1.43	66.9	1.49	130.6	0.76
25	100.0	155.1	0.64	67.3	1.48	73.7	1.35	72.7	1.37	155.3	0.64

1	2	3	4	5	6	7	8	9	10	11	12
26	100.0	134.0	0.74	60.4	1.65	67.6	1.47	64.7	1.54	134.1	0.74
27	100.0	198.7	0.50	73.9	1.35	78.5	1.27	80.9	1.23	194.3	0.51
28	100.0	212.1	0.47	71.5	1.39	76.2	1.31	78.9	1.26	202.2	0.49
29	100.0	170.2	0.58	67.3	1.48	73.4	1.36	73.3	1.36	167.4	0.59
30	150.0	801.8	0.18	204.4	0.73	168.8	0.88	210.6	0.71	782.5	0.19
31	100.0	316.0	0.31	150.9	0.66	102.0	0.97	120.2	0.83	347.4	0.28
32	100.0	237.0	0.42	135.0	0.74	106.1	0.94	119.4	0.83	261.6	0.38
33	100.0	293.9	0.34	155.5	0.64	116.2	0.86	147.1	0.67	429.1	0.23
34	70.0	400.0	0.18	136.8	0.51	90.4	0.77	108.2	0.65	412.8	0.17
35	35.0	150.0	0.23	71.0	0.49	65.9	0.53	67.5	0.52	151.7	0.23
36	230.0	549.6	0.42	139.4	1.65	89.9	2.55	95.9	2.40	464.6	0.49
37	235.0	520.5	0.45	127.6	1.84	80.9	2.90	96.9	2.42	514.7	0.46
38	61.0	307.8	0.20	105.2	0.58	81.0	0.75	88.9	0.69	277.5	0.22
39	70.0	216.5	0.32	93.9	0.75	83.8	0.83	76.8	0.91	208.0	0.34
40	70.0	223.7	0.31	104.9	0.67	84.8	0.82	92.1	0.76	230.0	0.30
41	70.0	213.1	0.33	101.2	0.69	83.3	0.84	89.6	0.78	218.8	0.32
42	70.0	250.4	0.28	86.6	0.81	77.5	0.90	67.3	1.04	217.3	0.32
43	70.0	264.8	0.26	90.3	0.77	79.4	0.88	69.8	1.00	228.0	0.31
44	70.0	234.7	0.30	96.2	0.73	82.2	0.85	80.1	0.87	223.7	0.31
45	70.0	178.3	0.39	77.5	0.90	72.2	0.97	66.4	1.05	171.9	0.41
46	230.0	642.8	0.36	119.3	1.93	75.2	3.06	90.7	2.53	607.2	0.38
47	230.0	577.4	0.40	125.8	1.83	82.3	2.79	87.4	2.63	468.0	0.49
48	138.0	902.9	0.15	214.6	0.64	119.1	1.15	152.0	0.90	1084.0	0.12
49	150.0	1371.1	0.10	243.6	0.61	123.2	1.21	160.1	0.93	1837.2	0.08
50	110.0	895.2	0.12	214.0	0.51	119.0	0.92	151.8	0.72	1072.9	0.10

1	2	3	4	5	6	7	8	9	10	11	12
51	140.0	746.7	0.18	188.5	0.74	126.2	1.10	155.7	0.89	809.2	0.17
52	210.0	1462.5	0.14	233.5	0.89	135.0	1.55	172.8	1.21	1723.2	0.12
53	150.0	772.2	0.19	191.0	0.78	126.8	1.18	156.8	0.95	839.3	0.17
54	400.0	1252.6	0.31	174.7	2.28	174.5	2.29	201.3	1.98	855.2	0.46
55	410.0	1619.0	0.25	186.5	2.19	179.9	2.27	212.0	1.93	1011.6	0.40
56	330.0	1277.4	0.25	175.7	1.87	174.9	1.88	202.2	1.63	866.8	0.38
57	46.0	193.3	0.23	69.1	0.66	78.9	0.58	87.5	0.52	191.1	0.24
58	120.0	450.0	0.26	134.6	0.89	89.4	1.34	111.2	1.07	514.2	0.23
59	150.0	473.1	0.31	142.1	1.05	91.5	1.63	115.4	1.29	571.6	0.26
60	71.0	231.2	0.30	123.1	0.57	108.4	0.65	123.7	0.57	262.5	0.27
61	56.0	75.3	0.74	38.7	1.44	46.9	1.19	39.1	1.43	74.6	0.75
62	45.0	117.5	0.38	63.4	0.70	76.3	0.58	69.3	0.64	120.0	0.37
63	44.5	75.3	0.59	38.7	1.14	46.9	0.94	39.1	1.13	74.6	0.59
64	55.0	88.5	0.62	44.6	1.23	54.4	1.01	46.2	1.18	87.5	0.62
65	67.5	99.6	0.67	49.3	1.36	60.3	1.11	52.1	1.29	98.3	0.68
66	45.0	113.5	0.39	55.0	0.81	67.3	0.66	59.5	0.75	111.8	0.40
67	79.0	113.5	0.69	55.0	1.43	67.3	1.17	59.5	1.32	111.8	0.70
68	67.0	117.5	0.56	63.4	1.05	76.3	0.87	69.3	0.96	120.0	0.55
69	89.0	140.7	0.63	65.5	1.35	79.3	1.12	73.3	1.21	138.1	0.64
70	113.0	162.0	0.69	73.1	1.54	87.5	1.29	83.4	1.35	158.5	0.71
71	88.0	285.7	0.30	81.7	1.07	90.0	0.97	82.8	1.06	235.6	0.37

		NAVY-MCKAY		RANKINE		CANADIAN NATL.		MOD. ENGR. NEWS		GATES	
No.	Yield load tons R_y	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u	Predicted load, tons R_u	Ratio R_y/R_u
1	2	3	4	5	6	7	8	9	10	11	12
1	85.0	76.8	1.10	84.8	1.00	43.0	1.97	58.1	1.46	52.5	1.61
2	100.0	128.1	0.78	129.9	0.76	54.2	1.84	91.3	1.09	64.2	1.55
3	85.0	169.7	0.50	177.0	0.48	55.5	1.53	102.9	0.82	71.3	1.19
4	77.0	169.7	0.45	177.0	0.43	55.5	1.38	102.9	0.74	71.3	1.07
5	105.0	155.2	0.67	180.0	0.58	58.0	1.80	92.8	1.13	74.2	1.41
6	115.0	234.1	0.49	162.6	0.70	53.0	2.16	156.6	0.73	76.9	1.49
7	50.0	134.8	0.37	130.1	0.38	52.6	0.94	100.6	0.49	64.2	0.77
8	100.0	256.8	0.38	178.5	0.55	58.5	1.70	160.5	0.62	80.0	1.24
9	120.0	200.7	0.59	151.1	0.79	51.6	2.32	138.7	0.86	73.4	1.63
10	50.0	95.8	0.52	125.7	0.39	25.7	1.93	50.2	0.99	59.8	0.83
11	72.0	98.2	0.73	112.6	0.63	46.5	1.54	69.4	1.03	57.9	1.24
12	85.0	80.4	1.05	91.7	0.92	45.2	1.87	60.6	1.40	53.8	1.57
13	60.0	458.3	0.13	139.8	0.42	26.6	2.24	97.1	0.61	88.3	0.67
14	100.0	70.7	1.41	87.1	1.14	36.3	2.74	49.7	2.00	53.8	1.85
15	70.0	141.9	0.49	150.6	0.46	52.2	1.33	93.5	0.74	68.3	1.02
16	80.0	183.5	0.43	183.9	0.43	59.0	1.35	118.3	0.67	73.4	1.08
17	80.0	165.1	0.48	170.6	0.46	57.3	1.39	108.6	0.73	71.0	1.12
18	200.0	362.7	0.55	246.4	0.81	74.1	2.69	199.9	1.00	88.5	2.25
19	67.0	3564.5	0.01	316.2	0.21	43.4	1.54	158.7	0.42	112.9	0.59
20	80.0	3264.0	0.02	274.8	0.29	40.7	1.96	148.1	0.54	112.9	0.70
21	100.0	307.6	0.32	231.5	0.43	71.0	1.40	175.6	0.56	85.1	1.17
22	100.0	321.8	0.31	231.0	0.43	70.1	1.42	179.1	0.55	86.4	1.15
23	100.0	328.6	0.30	241.7	0.41	72.6	1.37	185.3	0.53	86.4	1.15
24	100.0	118.4	0.84	127.4	0.78	51.2	1.95	80.3	1.24	64.2	1.55
25	100.0	144.3	0.69	145.2	0.68	53.5	1.86	93.4	1.07	69.2	1.44

1	2	3	4	5	6	7	8	9	10	11	12
26	100.0	121.2	0.82	129.3	0.77	50.1	1.99	80.7	1.23	65.3	1.53
27	100.0	191.6	0.52	167.8	0.59	56.6	1.76	115.6	0.86	76.3	1.31
28	100.0	203.7	0.49	169.4	0.59	55.8	1.79	119.5	0.83	78.2	1.27
29	100.0	158.2	0.63	151.9	0.65	53.7	1.86	99.4	1.00	71.8	1.39
30	150.0	5617.9	0.02	424.2	0.35	92.0	1.63	477.7	0.31	153.0	0.98
31	100.0	388.9	0.25	218.8	0.45	55.0	1.81	209.7	0.47	91.6	1.09
32	100.0	266.2	0.37	205.0	0.48	54.7	1.82	164.4	0.60	82.1	1.21
33	100.0	781.1	0.12	252.5	0.39	43.5	2.29	212.3	0.47	84.7	1.18
34	70.0	562.9	0.12	212.0	0.33	55.0	1.27	246.2	0.28	100.0	0.69
35	35.0	140.7	0.25	131.3	0.27	44.4	0.78	92.3	0.38	68.3	0.51
36	230.0	464.6	0.49	235.0	0.98	68.4	3.35	268.2	0.86	126.2	1.82
37	235.0	616.1	0.38	195.6	1.20	63.9	3.67	309.7	0.76	116.7	2.04
38	61.0	335.2	0.18	195.4	0.31	48.8	1.25	168.1	0.36	90.1	0.68
39	70.0	166.1	0.42	181.2	0.39	60.0	1.17	114.4	0.61	85.4	0.82
40	70.0	211.9	0.33	171.8	0.41	61.0	1.15	143.3	0.49	84.2	0.83
41	70.0	200.1	0.35	167.1	0.42	60.2	1.16	136.5	0.51	82.5	0.85
42	70.0	174.5	0.40	192.0	0.36	52.2	1.34	113.7	0.62	90.2	0.78
43	70.0	186.1	0.38	198.2	0.35	53.2	1.31	120.3	0.58	92.4	0.76
44	70.0	191.0	0.37	182.9	0.38	57.7	1.21	126.3	0.55	87.4	0.80
45	70.0	140.1	0.50	153.2	0.46	52.1	1.34	95.9	0.73	77.3	0.90
46	230.0	853.8	0.27	188.0	1.22	62.1	3.69	370.4	0.62	126.9	1.81
47	230.0	479.6	0.48	220.8	1.04	65.3	3.52	272.3	0.84	128.6	1.79
48	138.0	1499.4	0.09	275.4	0.50	93.7	1.47	615.4	0.22	148.9	0.92
49	150.0	4687.3	0.03	291.4	0.51	96.5	1.55	934.6	0.16	185.8	0.80
50	110.0	1474.2	0.07	275.0	0.39	93.7	1.17	610.2	0.18	148.3	0.74

1	2	3	4	5	6	7	8	9	10	11	12
51	140.0	1020.6	0.13	294.6	0.47	98.8	1.41	479.8	0.29	137.6	1.01
52	210.0	6075.5	0.03	328.7	0.63	104.8	2.00	939.7	0.22	195.4	1.07
53	150.0	1081.4	0.13	296.7	0.50	99.2	1.51	496.1	0.30	139.5	1.07
54	400.0	2696.9	0.14	469.9	0.85	129.9	3.07	627.4	0.63	175.2	2.28
55	410.0	10053.0	0.04	491.6	0.83	133.6	3.06	810.9	0.50	217.8	1.88
56	330.0	2878.0	0.11	471.7	0.69	130.2	2.53	639.8	0.51	177.3	1.86
57	46.0	266.0	0.17	176.6	0.26	46.6	0.98	115.2	0.39	69.5	0.66
58	120.0	734.6	0.16	206.3	0.58	60.5	1.98	296.1	0.40	105.0	1.14
59	150.0	830.1	0.18	208.9	0.71	62.1	2.41	321.4	0.46	107.4	1.39
60	71.0	268.1	0.26	204.0	0.34	66.5	1.06	166.8	0.42	80.8	0.87
61	56.0	64.3	0.87	81.5	0.68	35.0	1.59	44.0	1.27	49.9	1.12
62	45.0	113.4	0.39	131.3	0.34	49.5	0.90	73.5	0.61	60.0	0.74
63	44.5	64.3	0.69	81.5	0.54	35.0	1.26	44.0	1.00	49.9	0.89
64	55.0	77.3	0.71	96.2	0.57	39.1	1.40	51.7	1.06	53.4	1.02
65	67.5	88.7	0.76	108.4	0.62	42.1	1.60	58.2	1.15	56.1	1.20
66	45.0	103.8	0.43	123.6	0.36	45.4	0.99	66.4	0.67	59.1	0.76
67	79.0	103.8	0.76	123.6	0.63	45.4	1.73	66.4	1.18	59.1	1.33
68	67.0	113.4	0.59	131.3	0.51	49.5	1.35	73.5	0.91	60.0	1.11
69	89.0	135.6	0.65	152.0	0.58	50.5	1.76	82.3	1.08	64.3	1.38
70	113.0	162.9	0.69	172.9	0.65	53.7	2.10	94.7	1.19	67.9	1.66
71	88.0	228.3	0.38	222.7	0.39	64.0	1.37	134.1	0.65	91.7	0.95

VITA

Mahesh Varma

Candidate for the Degree of

Doctor of Philosophy

Thesis: A CRITICAL EVALUATION OF SOME PILE DRIVING FORMULAS

Major Field: Civil Engineering

Biographical:

Personal Data: Born August 10, 1928, in Najibabad, U. P., India, son of Shri and Shrimati Mohan Lal Varma.

Education: Graduated from Herbert Intermediate College, Kota, India with High School Diploma in 1945 and Intermediate (Science) Diploma in 1947; passed the Final Examination for degree of Bachelor of Science (Engineering) in 1951, and received the degree from Banaras Hindu University, India, in 1953; attended University of Roorkee, India for graduate work and received the degree of Master of Engineering in 1959; completed the requirements for Doctor of Philosophy degree from Oklahoma State University in July, 1966.

Professional Experience: Joined Birla Bros., India in Sept., 1951, and served as Assistant Engineer on construction and erection of industrial plants until Jan., 1953; entered service of Panjab State Government, India in Feb., 1953, and served as executive officer on construction of multi-purpose projects till July, 1961; joined Panjab Engineering College, Chandigarh, India, on deputation as Associate Professor and taught Construction Engineering and Power Plant Engineering till July, 1964; served as graduate assistant employed on grading work and institutional research, School of Civil Engineering, Oklahoma State University from Sept., 1964, to May, 1966, and as research assistant from June, 1966, to July, 1966.

Professional Organizations: Associate Member of American Society of Civil Engineers, Associate Member of Institute of Engineers (India), Member of American Society for Engineering Education.

Honor Societies: Sigma Tau, Chi Epsilon.