REFINEMENT OF A SINGIE FACIIITY
ELECTRONIC QUEUEING SIMUIATOR
AND THE DEVELOPMENT OF AN
AUGMENTED QUEUEING
SYSTEM SIMULATOR
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Thesis Approved:


## PREFACE

This work was undertaken with the objective of developing an augmented queueing system simulator which would be useful as a classroom demonstration and teaching device and as a research tool. The system which has been designed, constructed, and tested is herein presented.

The system described is capable of generating electrical impulses at random time intervals with preset distributions and parameters, capable of using these impulses to simulate the operation of a complex queueing system, and is capable of providing both a visual and a permanent display of the data thus generated.

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## CHAPTER I

INTRODUCTION.

The purpose of this project has been to refine and improve a single station queueing simulator and to develop means by which several such units could be connected together in order to simulate a complex or augmented queueing system. The final queueing simulator was to be a refined model of a single station queueing simulator designed and constructed within the School of Industrial Engineering and Management, with the support of the Office of Engineering Research, Oklahoma State University.

A queue or waiting line is literally what the name suggests - a queue or line formed when moving units encounter an obstruction causing them to be delayed before they can proceed. It is an indication of the congestion that occurs when a flow system encounters a restriction or "bottleneck." A queue or waiting line is formed by the collection of individual units of the flow system being temporarily detained at the bottleneck. Lines formed by people waiting for seats at a motion picture theater or by automobiles waiting for traffic lights to change are familiar examples.

Queueing theory and the analysis of queves are needed in many industrial appications, such as communication and transportation networks as well as production scheduling. Telephone systems can be designed more economically when the required circuit capacity to carry traficic, with a predetermined maximum delay period is known. A similar situam tion exists in highway planning. The determination of the number of aircraft arrivals and departures a given airfield can accommodate per unit time is another example of the apo plication of queueing theory. Many problems in operations research require a fundanental knowledge of the principles of queueing theory.

It was felt that, by providing a tool whereby queueing lines could be observed and studied without having to resort to the time consuming procedure of observing industrial situations or the laboxious procedures involved in hand simulation, student interest in queueing theory could be increased within the classroom by the instructor and more students could learn to recognize situations in which queueing theory can be appilied.

The simulator will permit the student to control various statistical parameters such as the arrival and service distributions as well as the mean service time and mean arrival time, in order to experience the effect of changing these parameters on the queue. By connecting two or more simulators together. complex queueing situations may be
demonstrated to show the effect of paralleled or series services with either similar or different parameters.

The queueing simulator is quite simple in concept. It has its nucleus, two "random" time interval generators. These "random" time interval generators are connected to an add and subtract stepping relay which displays the queue length.

Augumented or complex queues can be formed by combining either the individual "random" time interval generators or the simple queueing simulators according to the desired complexity of the queue。

## CHAPTER II

## CONSTRUCTION OF RANDOM TIME INTERVAL GENERATOR SECTIONS

The single station queueing simulator is constructed from five basic sections: (1) power supply section, (2) constant impulse section, (3) queue display section, (4) and (5) two "random" time interval generator sections. Although the power supply section and the constant impulse sections are required portions of the "random" time interval generator, they are listed as separate sections due to the fact that they are shared by the two "random" time interval generators.

The power supply section provides for the power requirements of the various other sections. It provides 110 volts alternating current, 6.3 volts alternating current, and 9 volts direct current (these voltages will vary according to the line voltage to the power supply) for the other sections. This section contains the parts listed in Parts List $I$, and is wired according to Figure 1.

Leads A and B connect the power supply through the Power plug (P1-1) to an outside source of power (110 volts alternating current). Lead C provides the 110 volt

## PARTS IIST I <br> Power Supply Section

| Part <br> No. | Part <br> Description | Sapacitor | Specifications | Manafacturer |
| :--- | :--- | :--- | :--- | :--- |



Figure I. Power Supply Section
alternating current required by the outer sections. Lead D serves as a common return (ground) to the power supply for all circuits. Lead E provides alternating current stepped down by the transformer (TR-1) to about 6.3 volts. Lead $F$ provides the stepped down voltage (about 9.3 volts direct current) after it has been rectified by the silicon rectifier (SR-1) and smoothed by the power storage capacitor (C-2). The main control switch (S-1) controls the flow of power to the entire unit. The capacitor (C-1) across the power input lines serves to filter undesirable stray current and to minimize arcing at various relay contact points.

The constant impulse section provides five impulses per second to step stepping relays in the "random" time interval generators. This section contains the parts listed in Parts List 2, and is wired according to Figure 2.

Lead C provides 110 volts alternating current from the power supply to drive the motor. Lead $D$ provides a return path to the power supply for the alternating current. Lead G provides five impulses of 110 volt alternating current per second to the stepping relays in the "random" time interval generator. The cam disk (CD-1) is fitted on the shaft of the motor (M-1) so as to depress the arm of the microswitch. (MS-1). The capacitor (C-3) smooths the current flow introduced by the stepping relays in the "random" time interval generator.

## PARTS LIST 2 <br> Constant Impulse Section

| $\begin{aligned} & \text { Part } \\ & \text { No. } \\ & \hline \end{aligned}$ | Part Description | Specifications | Manufacturer | Mfg. <br> Type |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}-3$ | Capacitor | .47mfd. 600VDC | Mallory | PVC60+7 |
| CD-1 | Cam Disk | Shop made | Detail 1 |  |
| MT-1 | Motor Mount | Shop made | Detail 2 |  |
| M-1 | Motor | 60RPM 115VAC 60 cycle | Hurst | SM-60RPM |
| MS-1 | Microswiteh | SPST | Micro Switch Corp. | 6RL-2 |
| S-5 | Switch | SPST | Arrow-Hart and Hegeman | 20994-BF |



Figure 2. Constant Impulse Section

The queue display section provides a means of displaying the queue visually and a means of controlling the "random" time interval generator used to simulate services in the queueing system to prevent a service from occurring when there is no unit in the service facility. The queue display section consists of the parts listed in Parts List 3, and is wired according to Figure 3.

Lead C provides 110 volts alternating current from the power supply. Lead $D$ provides a return path to the power supply to complete various portions of the circuit. Lead H is connected to the "random" time interval generator which simulates arrivals and causes one to be added to the queue count, whenever an impulse is generated by the arrival simulator. Lead I is connected to the "random" time interval generator which simulates services and causes one to be subtracted from the queue count whenever an impulse is generated by the service simulator. Lead $J$ is connected to a control relay in the "random" time interval generator which simulates services to provide current to indicate by a light (L-21) that there is no unit in service, if no unit is in service. Lead $K$ is connected to a control relay in the "random" time interval generator which simulates services to provide current to indicate by a light that a unit is in service, if a unit is in service. Lead L provides current from a contact point of the add and subtract relay (AS-R1-1) whenever the relay wipper arm is at position zero.

## PARTS IIST 3 <br> Queue Display Section

| Part No. | $\begin{gathered} \text { Part } \\ \text { Description } \end{gathered}$ | Specifications | Manufacturer | Mifg。 Type |
| :---: | :---: | :---: | :---: | :---: |
| $A S \sim R 1-1$ | Add and Subtract Relay | 40PT 110VAC | Guardian | 1R-RAS |
| C-4, $\mathrm{C}-5$ | Capacitors | . 47 mfa . 600VDC | Mallory | PVC 6047 |
| $\frac{L-1}{20} \text { to } I-$ | Neon Light Units | Clear | Dialco | $\begin{aligned} & 5240 \times \mathrm{H} \\ & \text { Type H995 } \end{aligned}$ |
| L-21 | Neon Light Unit | Green | Dialco | $\begin{aligned} & 5240 \times H \\ & \text { Type } 932 \end{aligned}$ |
| L-22 | Neon Light Unit | Red | Dialco | $\begin{aligned} & 5240 \times H \\ & \text { Type } 991 \end{aligned}$ |
| S-2 | Switch | SPST | Arrow-Hart and Hegeman | 20994 - BF |



Figure 3. Queue Display Section

The two capacitors C-4 and C-5 smooth the current flow introduced by the add and subtract relay. One light (I-21) is lighted whenever there is no unit available to be serviced. A light ( $L$-22) is lighted whenever there is a unit in service. The other twenty lights are wired so that as a queue forms from time-to-time the number of units in the queue will be indicated by a light being lighted in a position to correspond to the number in the queue. Thus, when the system is empty, only light No. 21 will be lighted, if there is only one unit in the system (in service) only light No. 22 will be lighted, if there is a queue of length $\mathrm{x},(\mathrm{x}+1)$ in the system, light No. 22 and light No. x will be lighted. Switch (S-2) may be used to prevent the count from stepping and the lights from lighting when the simulator is used as part of an augmented system.

The two "random" time interval generator sections
serve as the source of electrical impulses which step the count. One of the "random" time generators is attached to the add side of the counter and is the arrival simulator. The other is attached to the subtract side and is the service simulaton; as these units are nearly identical, they will be discussed as one, with their differences being identified when appropriate. A "random" time interval section consists of the parts listed in Parts List 4 and is wired according to Figure 4.

Lead C is connected to the power supply section and

```
PARTS LIST 4
"Random" Time Interval Generator Section
```

| $\begin{aligned} & \text { Part } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} \text { Part } \\ \text { Description } \end{gathered}$ | Specifications | Manufacturer | Mfg。 Type |
| :---: | :---: | :---: | :---: | :---: |
| C-6 | Capacitor | 80mfd. 350VDC | Mallory | PF 128 |
| C-7 | Timing Capacitor | 50mfd。 25VDC | Mallory | TC 29 |
| C-8 | Capacitor | 10mfd. 25VDC | Mallory | TC 22 |
| C-9 | Capacitor | . 05 mfd . 400VDC | Cornell-Dublier | PM 455 |
| L-RI-1 | Latching Relay | DPDT | Guardian | 670G115 |
| PMP | Potentiometer Mounting Panel | Shop made | Detail 4 |  |
| $\begin{array}{r} P-1 \text { to } \\ P=-80 \end{array}$ | Potentiometers | 100K3 Linear Taper | Mailory | SU 41 |
| R-1 | Resistor | $10 \mathrm{~K} \Omega \mathrm{l}$ watt |  |  |
| $\mathrm{R}-2$ | Resistor | $47 \mathrm{~K} \Omega 2$ watts |  |  |
| R-3 | Resistor | 33082 watts |  |  |
| R1-1 | Relay | 10,000 ${ }^{\text {Plate Cumrent }}$ | Guardian | $\begin{aligned} & \text { 200MS } \\ & 4 \mathrm{PDT} \end{aligned}$ |

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PARTS LIST 4 (Continued)
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| $\begin{aligned} & \text { Part } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} \text { Part } \\ \text { Description } \end{gathered}$ | Specifications | Manufacturer | Mfg. Type |
| :---: | :---: | :---: | :---: | :---: |
| R1-2 | Relay | 115 VAC | Guardian | $\begin{aligned} & \text { 500MS } \\ & 4 \mathrm{PD} \mathrm{D}^{\prime} \end{aligned}$ |
| $s-3,50-4$ | Switch | SPST | Arrow-Hart and Hegeman | $20994-\mathrm{BF}$ |
| $\begin{aligned} & \mathrm{S}-\mathrm{Rl}-1 \\ & \mathrm{~S}-\mathrm{A} I-2 \end{aligned}$ | Stepping Relay | 40 Point | Guardian | IRPC |
| $S R-2, S A-3$ | Silicon Rectifier | $750 \mathrm{ma} \mathrm{600VIC}$ | Texas Instruments | IN 547 |
| T-1 | Electron Tube |  | RCA | 2 D 21 |
| $\underset{T J-3}{T J-1} \text { to }$ | Tip Jack |  | G-E Electronics | Type 33-212 |
| TS-1 | Tube Socket | 7 Pin | Elco | 235PHSPDT |



Figure 4. "RANDON" Time Interval Section
provides 110 volt alternating current. Lead $D$ is connected to the power supply section and serves as a common return (ground) for all circuits. Lead $E$ is connected to the power supply section and furnishes 6.3 volts alternating current to the tube to heat its cathode. Lead $F$ is connected to the power supply section and furnishes about 9.3 volts direct current to charge the timing capacitor (C-7) in the resistance-capacitance timing circuit. Lead $G$ is connected to the output of the constant impulse section to provide five stepping impulses per second to the resistance stepping relays. The toggle switch (S-4) is provided to permit control of the resistance stepping relays so that a fixed resistance circuit can be used if desired. Lead H (in the arrival simulator) or Lead I (in the service simulator) provides impulses to the queue display section to step the count whenever a service or an arrival occurs. A means of controlling the output of the "random" time interval generator sections to prevent them from stepping the queue count is provided in the tip jacks ( $T J-2 S$ in the service simulator, and $\mathbb{T J}-2 A$ in the arrival generator). This means can provide control either from another simulator (in an augmented queueing system) or any auxiliary device; the input must come from a source connected to the same power line as lead $A$ in the power supply section. Lead L (in the service simulator only) is connected to the service facility empty contact of the queue display section
counting relay to prevent a service from occurring when no unit is in service. In the service simulator, current from lead $I$ would close the service cotrol relay (RI-2S). When this relay is closed, contact HS allows current to flow to lead J. Lead $J$ is connected to the service facility empty light in the queue display sections. When the service control relay (Rl-2S) is open, current flows through contact HS to lead $K$. Lead $K$ is connected to the service facility occupied light (I-22) in the queue display section. Relay Rl-l provides an impulse source to other units or equipment through a tip jack (TJ-3). A means of externally stepping the queue count is provided through a tip jack (TJ-1).

The "random" time interval generator operates as follows. Whenever the voltage across the control capacitor (C-7) falls below a certain level, the control tube (T-1) will permit current to flow through contact $E$ of the control relay (Rl-2), if this relay is in its normally open position, to the coil of the impulse relay (Rl-l). The capacitor (C-8) across the coil of this control relay (Rl-1) smooths the current flow to insure reliable relay closure. When the impulse relay (Rl-l) closes, it permits current to flow from lead $F$ through contact $C$ to recharge the control capacitor ( $(\mathbb{C}-7)$ and cut off the flow of current through the control tube (T-I); it permits current to flow from lead $C$ through contact $D$ to $T J-3$, through
contact $B$ to lead $H$ or lead I to one of the stepping relays in the queue display section to step the queue count, and through contact $A$ to step the latching relay (L R Rl-1) to switch stepping relays and resistor circuits. When the control relay (RI-2) is open, as it normally is, current fiows from the control capacitor ( $\mathrm{C}-7$ ) through its contact $F$ to contact the latching relay ( $L$-RI-1) to be directed to one of the banks of potentiometers to be dissipated as heat to discharge the control capacitor (C-7). The current to be dissipated, which flows through contact $J$ of the latching relay (I-RI-1), will be directed by contact arm of whichever one of the two stepping relays (S-Rl-1 or S-Rl-2) that is stopped. This current will then flow through a number of the potentiometers (the number is determined by the location in which the stepping relay is stopped) to discharge the control capacitor (C-7). During the time that the current flows through one stepping relay and its corresponding potentiometers, the other stepping relay is being stepped by current from the constant impulse section which is directed to it through contact I of the latching relay ( $\mathrm{I}-\mathrm{Rl}$ - 1 ) , Whenever the latching relay ( $L \sim R 1-1$ ) is energized, it switches the control capacitor discharge current to the stepping relay which has just previously been stepping and stops this relay from stepping. It also starts stepping the stepping relay which has just previously been controlling the capacitor
discharge current. This switching action permits the system to "randomly" select the number of potentiometers to be in series in the resistance-capacitance circuit at any given time. This in turn generates the random time intervals between impulses.

The distribution of time intervals is determined by the adjustment of the potentiometers. In the long run, either of the stepping switches will stop at each of its forty positions approximately the same number of times. This provides a rectangular distribution as far as the number of potentiometers in the circuit are involved. The number of potentiometers is, of course, a discrete number.

Forty positions provide enough points to approximate a continuous distribution for most purposes. One is able to adjust the distribution of time interval lengths by adjusting the various potentiometers. If potentiometer number one is adjusted to yield some desired time interval and all other potentiometers are adjusted to zero resistance, then all time intervals will be very nearly the same. Potentiometer number one should never be set to zero resistance as this will cause a direct short across the terminals of the control capacitor ( $C-7$ ) and the direct current portion of the power supply. If all potentiometers are adjusted to the same resistance, then the difference between the time intervals generated by adjacent potentiometers will be equal. In adjusting the
potentiometers to generate a series of time intervals from different times, one must remember that the current flows through the potentiometers in series, and, therefore, that one is constructing a cumulative distribution of resistance and that the distribution of the stepping relay contacts is rectangular.

## CONSTRUCTION OF QUEUEING SIMULATOR

The components described in Chapter II may be assembled into either of two different units. With one of the ${ }^{09}$ random" time interval sections, a "random" time interval generator may be formed. This "random" time interval geno erator can be used to provide electrical impulses to trip a motion camera to make a filmed work sampling study or to indicate points for random sampling. The omission of the two sections mentioned is the only difference between the ${ }^{10}$ random ${ }^{0 n}$ time interval generator and the queueing simulator.

The queueing simulator utilizes all of the sections described in Chapter II and some additional hardware (see Parts List 5). It is wired as shown in Figure 5. It should be noted that the part number of each part in the service simulator section is followed by an $S$ and that the part numbers of each part in the arrival simulator is fols lowed by an A.

The lead (I) from the service facility empty contact (0) of the add and subtract relay ( $A S-R I-1$ ) in the queve display section is connected to the service control relay

## PARTS IIST 5 <br> Queueing Simulator Parts List

| Part No. | $\begin{gathered} \text { Part } \\ \text { Description } \end{gathered}$ | Specification | Mfg. | Mfg. No. | No. Req. for Simulator |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AS-R1-1 | ```Add and Subtract Relay``` | 40Pt. 115 VAC | Guardian | 12-RAS | 1 |
| $\underset{C-9 S}{C-1, C-9 A}$ | Capacitor | . 05 mfd . 400 VDC | CornellDublier | PM455 | 3 |
| C-2 | Capacitor Power Storage | 100mfd. 150VDC | Mallory | TC493 | 1 |
| $\begin{gathered} C-3, C-4 \\ C-5 \end{gathered}$ | Capacitor | . 47 mfd .600 VDC | Mallory | PVC6047 | 3 |
| C-6A, $\mathrm{C}-6 \mathrm{~S}$ | Capacitor | 80mfd. 350VDC | Mallory | PFI28 | 2 |
| C--7A, C-7S | Timing Capacitor | 50mfd. 25VDC | Mallory | TC 29 | 2 |
| C-8A, $\mathrm{C}-8 \mathrm{~S}$ | Capacitor | 10mfod. 25VDC | Mallory | TC 22 | 2 |
| CAB-1 | Cabinet | $\begin{aligned} & 12^{\prime \prime} \mathrm{H} \times 14^{1} / 8^{\prime \prime} \mathrm{L} \\ & \times 18^{\prime \prime} \mathrm{D} \end{aligned}$ | Bud | WA-1544 | 1 |
| CHA-1 | Chassis | $3 \mathrm{H} \times 13 \mathrm{~L} \times 17 \mathrm{D}$ | Bud | AC-420 | 1 |
| $\begin{array}{r} I-1 \text { to } \\ I-20 \end{array}$ | Neon Lamps | Clear | Dialco | $\begin{aligned} & 5240 \times \mathrm{H} \\ & \text { Type H995 } \end{aligned}$ | 20 |

PARTS LIST 5 (Continued)

| Part No. | Part <br> Description | Specification | Mfg。 | Mfg. No. | $\begin{gathered} \text { No. Req. } \\ \text { for } \\ \text { Simulator } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I-2I | Neon Lamp | Green | Dialco | $\begin{aligned} & 5240 \times H \\ & \text { Type } 932 \end{aligned}$ | 1 |
| Jume 22 | Neon Lamp | Red | Dialco | $\begin{aligned} & 5240 \times \mathrm{H} \\ & \text { Type } 991 \end{aligned}$ | 1 |
| $\mathrm{I}-\mathrm{RI} 1 .=1$ | Latching Relay | DPDT | Guardian | 6706115 | 1 |
| M-1 | Motor | 60 RPM 115600 y | Hurst | SM 60RPM | 1 |
| MS-1 | Microswrech | SPS4 | Micro Switch Corp | GRI-2 | 1 |
| $\begin{aligned} & P-1 A \text { to } \\ & P-80 A \\ & P-1 S \text { to } \\ & P=80 S \end{aligned}$ | Potentiometers | 100KR Iinear taper | Mallory | SU-41 | 160 |
| P1-1 | Power Plug | 2 wire Polarized | Cinch Jones | P302-AB | 1 |
| $\mathrm{R}-1 \mathrm{~A}, \mathrm{R}-1 \mathrm{~S}$ | Resistor | 10K81 Watt |  |  | 2 |
| $\mathrm{R}-2 \mathrm{~A}, \mathrm{R}-2 \mathrm{~S}$ | Resistor | $47 \mathrm{~K} \Omega 2 \mathrm{Watt}$ |  |  | 2 |
| $\mathrm{R}-3 \mathrm{~A}, \mathrm{R}-3 \mathrm{~S}$ | Resistor | 33082 Watt |  |  | 2 |
| $\begin{array}{r} \mathrm{RI}-1 \mathrm{~A}, \\ \mathrm{RI}-1 \mathrm{~S} \end{array}$ | Plate Relay | 10,000 $\Omega_{9} 4 \mathrm{PDT}$ | Guardian | 200MS | 2 |
| $\begin{array}{r} \mathrm{R} 1-2 \mathrm{~A} \\ \mathrm{R} 1-25 \end{array}$ | Relay | 115VAC, 4 PDT | Guardian | 500MS | 2 |

PARTS LIST 5 (Continued)

| Part No. | Description | Specification | Mfg. | Mfg. No. | $\begin{gathered} \text { No. Req. } \\ \text { for } \\ \text { Simulator } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S-1 | Switeh | 2DPT | Cutter-Hammer | 8373 K 8 | 1 |
| $\begin{aligned} & S-2, S-3 A \\ & S-3 S, S-4 A \\ & S-4 S, S-5 \end{aligned}$ | Switch | SPST | Arrow- Hart and Hegeman | 20994 mF | 6 |
| $\begin{aligned} & S R-1, S R-2 A \\ & S R-2 S, \\ & S R-3 A, \\ & S R-3 S \end{aligned}$ | Silicon Rectifier | 750 ma 600PIV | Texas Instruments | IN547 | 5 |
| $\begin{gathered} S-R 1-1 A \\ S-R 1-I S \\ S-R 1-2 A \\ S-R 1-2 S \end{gathered}$ | Stepping Relay | 40Pt. 115 VAC | Guardian | IR-PC | 4 |
| T-U, T-1S | Tube | 7 Pin | RCA | 2 D 21 | 2 |
| $\begin{aligned} & T J-1 A \text { to } \\ & M J-3 A \\ & M J-1 S \text { to } \\ & T J-3 S \end{aligned}$ | Tip Jack |  | G-E Electronics | Type 33-212 | 6 |
| TR-1 | Transformer | 6.3VAC3A | Stancor | P6466 | 1 |
| TS-1A,TS-1S | Tube Socket | 7 Pin | Elco | 235 PHSPDT | 2 |
| CD-1 | Cam Disk | See Figure 17 | Shop made |  | 1 |
| MT-1 | Motor Mount | See Figure 18 | Shop made |  | 1 |

PARTS LIST 5 (Continued)

| Part No. | Description | Specification | Mfg. | Mfg. No. | $\begin{gathered} \text { No. Req. } \\ \text { for } \\ \text { Simulator } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SB-I | Support Bracket | See Figure 20 | Shop made |  | 2 |
| $\begin{aligned} & \mathrm{PMP}-1 \mathrm{~A} \\ & \mathrm{PMP}-1 \mathrm{~S} \end{aligned}$ | Potentiometer Mounting Panel | See Figure 19 | Shop made |  | 2 |



Figure 5. Queueing Simulator
(R1-2S) so that this relay will open and prevent a service impulse from occurring whenever no unit is present for servicing. This also provides current to flow through lead (J) to light (I-21) to indicate that the service facility is not occupied. When the service control relay (RI-2S) is not closed, current flows through lead (K) to light (I-22) to indicate that the service facility is occupied.

The power plug (Pl-1) is of a polarized type to insure that inputs to the unit through the various tip jacks to either control the queue display section or either of the simulator sections matches the polarity of the current needed. It is intended that an external junction box will be constructed which would connect to a standard wall outlet (ll5AC) and provide as many leads as there are queueing simulators to be interconnected. One should note that the leads to the various tip jacks can all be followed to a common point (prong $A$ of the power plug), and that any input to these jacks should come from the same line (or a line connected to it), or that any output from these jacks (TJ-3A or 3S) should go to a line of like nature.

The line switches can be used to vary the operation of the various components of the system. The power switch (S-l) is simply the main ON-OFF switch; in the OFF position it breaks both power lines to completely isolate the unit. Switch (S-2) controls the common power line to the
add and subtract relay ( $A S \sim R 1-1$ ) and the display lights in the queue display section to deactivate the queue display section when the unit is being used to supply impulses to either another unit or to some unit requiring randomly timed impulses. The two $S-3$ switches ( $S-3 A$ and $S-3 S$ ) may be used to prevent the latching relays (I-RI-1A and I-RI-lS) from acting, and hence to prevent the normal switching of the stepping relays (S-Rl-1S, $S-R 1-2 S$, S-RI-1A, and SpRl-2A). This will permit either the service simulator section or the arrival simulator sections or both to operate as constant time interval sections. However, even with this switch open, the stepping switch not in circuit will continue to step. Switches (S-4A and $\mathrm{S}-4 \mathrm{~S}$ ) may be opened to break the common return line to prevent this stepping action. If no random impulses are needed, the constant impulse section can be turned off by opening switch S-5. The use of these various switches will permit a maximum flexibility and, at the same time, the operation of only the necessary section components.

Figure 6 shows a completed simulator mounted in its cabinet, with the queue display lights and some of the control switches on the front panel. Figure 7 shows the back of a completed simulator which has been removed from its cabinet, with its front panel removed. To be noted is the mounting of the stepping relays which are connected to the potentiometers (two are inverted and hung from one of


Figure 6. Complete Simulator<br>Mounted in<br>Cabinet


'igure 7. Back View of Simulator
Removed From Cabinet


Figure 8. Front View of Simulator
Removed From Cabinet
the support brackets), and switches, tip jacks, and power plug which are mounted on the chassis. Figure 8 shows the front of a completed simulator which has been removed from its cabinet, with its front panel removed. Note the add and subtract stepping relay which will have the leads to the queue display lights connected to it and the cam disk (with black tape on it) which is mounted on the motor shaft (motor hidden) and on the motor mount. Also, to be noted are the potentiometer mounting panels with the potentiometers mounted on them. No chassis layout is provided due to the fact that the layout is not critical. The detail drawings of the shop made parts are shown in Appendix A.

## CHAPTER IV

## OUTPUT AND USE

The primary output of the queueing simulator described previously is in the form of a visual display. This display provides a means for students to "observe" the queueing system and to acquaint themselves with it. By varying the potentiometers to alter the arrival and service parameters or by switching one random time interval generator to produce constant time intervals, the student can observe the effects on the system of various parameter changes.

Provision is made through the various tip jacks to interconnect more than one unit so that the student can observe the effects of multiple service channels or ar rival channels.

An alternate form of output is provided by the tip jacks. This output is in the form of electrical impulses from the positive (+) side of the power supply (110 VAC). This output can be taken from either the arrival simulator or the service simulator and may be utilized in various ways. It can be used to provide the input to an inventory system simulator, used directly with a recorder to record
a series of random time intervals, or it can be used to trigger a camera for work sampling studies.

If an output other than 110 VAC is desired, a lead connected directly to a power source of the desired characteristics can be substituted for the lead connected to relay contact $D$ of relay Rl-l. The desired output will then be available through TJ-3. Both 6 VAC and 6 VDC are available from the power supply section (through leads E and $F$, respectively)。

## CHAPTER V

## DATA COLLECTION AND MODIFICATIONS

Data used to evaluate the performance characteristics of the queveing simulator has been collected and recorded In several different ways, two of which have been rejected as being inadequate for this purpose.

Initially, an eight millimeter home motion picture canera was used to photograph a clock whenever an electrical impulse was generated by the random time interval generator section being studied.

This arrangement was made possible by the use of a solenoid to activate the camera. Positive film was used and specially processed without splitting so that a standard motion and time study analysis projector could be used to analyze the film. Figure 9 illustrates the film Which resulted from this procedure. The figures shows a strip of film (actual size) and a section of "blownoup" film.

It should be noted that only one random time interval generator can be studied at a time. Also, this type of clock is quite difficult to read with any degree of accuracy.


These inadequacies were partially overcome by the use of a sixteen millimeter motion picture camera and by mounting a stopwatch so that it could be photographed along with the resistance circuit selection stepping relays.

The sixteen millimeter motion picture camera resulted in larger, clearer negatives, information concerning both random time interval generators, and more accurate time interval data. Furthermore, it provided information as to the identity of the resistor circuit which was responsible for each time interval. Figure 10 illustrates film exposed using this system. A camera was found to be capable of operating independently, and, thus, could be used to collect data while unattended.

This system was found to have some undesirable features along with its desirable ones. One disadvantage was that it was quite difficult to identify which section of a random time interval generator was in operation. Another was that a considerable delay was experienced in the processing of the film。 Still another was that even with the best of equipment, taking data from film is a slow and laborious process; the time intervals could not be found directly, but had to be calculated from the film data. In addition to these disadvantages, this system did not perw mit the collection of data concerning the condition of the queue which was generated.


Figure 10. Film Strip From Sixteen Millimeter Motion Picture Camera

The use of photographs to collect data to be used to study the performance characteristics was eventually rejected due to the above limitations and disadvantages.

It was eventually determined that the only data collection system which was available to the author and which would overcome the above limitations and disadvantages while providing a permanent record of the desired data was a system which would utilize a recording dynagraph. This system would provide for the determination of time interval length quite accurately, and it would provide for immediate feedback of information. The only disadvantage of this system was that it did not provide for a means for the recording of information concerning the identity of the resistor circuit which was responsible for the various time intervals. However, this information could be recorded manually directly onto the dynagraph recording paper as the time interval lengths were being recorded.

Two dynagraphs were available for use: (1) A two channel recording dynagraph - A Brush Development Company Type BL 202 - which was used for the first two tests performed when information concerning time interval lengths and their corresponding resistor circuits was required, and (2) an eight channel recording dynagraph - Offner Type $R$ - which was used for the third test performed, when information concerning queue lengths was desired in addition to the above information.

Two modifications of the queueing simulator were required before the dynagraphs could be utilized. The random time interval generator sections had to be modified so that the voltage across the timing capacitor could be recorded by the dynagraph to measure the time interval length. This modification and the parts required for it are shown in Figure ll. It should be noted that both random time interval sections were modified identically, and that this modification simply required the addition of an external plug so that the dynagraph could be attached to the system.

The second modification was made to facilitate the recording of the queue count, or rather a count of the total number in the system - in service and waiting - and was considerably more extensive than the first modification. In fact, it required removing the original queue count section and installing a new section. The new or modified queue count section is shown in Figure 12. The materials required for the modified queue count section are listed in Parts List 6 .

Two persons were required to collect data concerning the identities of the resistance circuits responsible for the various time intervals - one to read the resistance circuit identity directly from the simulator and one to record this information onto the recording paper adjacent to the time interval data recorded by the dynagraph.


Figure 1l. "RANDOM" Time Interval Section

## PARTS LIST 6

Modified Queue Count Section

| Part No. | Part Description | Specifications | Manufacturer | Mfg。 <br> Type |
| :---: | :---: | :---: | :---: | :---: |
| AS-RI-1 | Add and Subtract Relay | 40 PT IlOVAC | Guardian | IR-RAS |
| S-2,S-6 | Switch | SPST | Arrow-Hart and Hegeman | 20994-BF |
| C-4, C-5 | Capacitor | . 47 mfd 600VDC | Mallory | PVC 6047 |
| $\mathrm{C}-10, \mathrm{C}-11$ | Capacitor | . 100 mfd d 150 VDC | Sprague | TVA-1420 |
| C-12 | Capacitor | 100vtd 150v. | Sprague | TVA-1420 |
| $\mathrm{C}-13$ | Capacitor | - Imfo 200VDC | Sprague | 2TM-PIO |
| $\mathrm{C}-14$ | Capacitor | 2mfa 200VDC | Sprague | $\begin{aligned} & 118 P- \\ & 2059252 \end{aligned}$ |
| C-15 | Capacitor | 100mfd 25 VDC | Sprague | TVA-1207 |
| $\begin{gathered} S R-4, S R-5, \\ S R-6 \end{gathered}$ | Silicon Rectifier | 750 ma 600 PIV | Texas Instruments | IM547 |
| P-81 | Potentiometer | 500 ks linear taper | Mallory | SU-50 |
| R-4 | Resistor | $1 \mathrm{l} \Omega 2$ watt | IRC Carbon | SR-2 |


| $\begin{aligned} & \text { Part } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} \text { Part } \\ \text { Description } \end{gathered}$ | Specifications | Manufacturer | Mfg. Type |
| :---: | :---: | :---: | :---: | :---: |
| R-5 | Resistor | $270 \Omega 1$ watt | IRC Carbon | GBT-1 |
| R-6 | Resistor | $3.3 \mathrm{M} \Omega \frac{1}{4}$ watt | IRC Carbon | SR- $\frac{1}{4}$ |
| $\mathrm{R}-7$ to $\mathrm{R}-45$ | Resistors | 9.1kS $\frac{3}{4}$ watt | IRC Carbon | SR-7 |
| R1-3 | Relay, Plate | 10,000 8 | Guardian | 200MS, 4PDT |
| T-2 | Electron Tube |  | G.E. | 2D21 |
| $\begin{aligned} & \text { Female } \\ & \text { Plug } \end{aligned}$ | To fit recordi | vice |  |  |



Figure 12. Modified Queue Count Section

Figure 13 provides a view of the queueing simulator, Offner Dynagraph, the author and his wile prepared to make a data run. Figure 14 shows a portion of a recording made with the two channel recorder. The numbers which read from the bottom of the page identify the particular resistor circuit which was responsible for that particular time interval. The U or D reveals whether the Up or Down section was inrolved. The numbers, which read from the sides, are the lengths of the various time intervals in centimeters. This recorder was adjusted to have the recording paper pass the recording pens at a rate of one centimeter per second, so these numbers also give the lengths of the time interval in seconds. The left portion corresponds to the service section of the queueing simulator, the right to the arrival. Figure 15 shows a portion of a recording made with the Offner Dynagraph. This'recorder was operated at the rate of 05 centimeters per second.g therefore, the lengths in centimeters represent a time interval of twice the corresponding magnitude. The righthand channel records the number in the system, which will be called queue length. Each step, it will be noted, corresponds to the end of a time interval and indicates the occurrence of either a service or an arrival. It will also be noted that when the queue count is zero, the voltage recorded for the service channel does not change. This is due to the fact that the service facility


Figure 13. A Typical Data Recording Session With the Offner Dynagraph


Figure 14. A Typical Two Channel Recording

is empty and no unit is being serviced.
The numbers adjacent to the queue count recording are the queue count and the length of duration of this condition (centimeters).

## CHAPTER VI

## EVALUATION PROCEDURE AND DATA

The evaluation of the adequacy of the queueing system simulator which has been developed was divided into three phases. The first was to determine if the number of times each resistor circuit is called upon to generate a time interval is uniformly distributed (each resistor has an equal probability of being called upon for each time interval), and to detemine if a rectangular time interval distribution can be generated by adjusting each resistor to the same value.

The resistance required to yield a time interval one centimeter long on the Brush Dynagraph was determined for both the arrival section and the service section. These two values were different. Then, each resistor was ad. justed to the value determined for its section using a Simpson Model 260 Voltwohm meter. The results are shown in Figures 21, 22, 23, and 24 (Appendix B).

The purpose of the second phase was to determine if the two portions of a single section could be adjusted to produce similar distributions, and, thus, demonstrate that the distribution generated could be controlled. This phase wes
performed by adjusting a set of walues into the upper set: of resistors in the arrival section These resistors were set by a trial and error procedure to produce a desirable distribution. After setting a resistor circuit, the generator would be adjusted to produce a series of constant time intervals from this eirouit, Then, the intervals would be measured and if necessary the resistors set again. After the upper set of resistors had been adjusted thusly, the lower set was adjasted using the volt-ohm meter.

After the resistances of the lower set of resistors were adjusted as closely as possible to the upper set, the Brush Dynagraph was used to record the time interval lengths generated. Figure 25 (Appendix B) shows this set of data. The purpose of the third phase was to determine Whether predetermined distributions could be simulated and to evaluate the operation of the queue count section. This phase was designed to evaluate the entire queueing system simulator. Exponential distributions with mean times of 13.3 seconds and 10 seconds were selected for simuiation. Using these mean times, times corresponding to each 1.25 per cent increment of cumalative probability were calculated. These values were determined as follows:

1. $f(x)=\mu e^{-\mu x}$
2. $f(x)$ was integrated to yield $F(t)$

$$
F(t)=\int_{0}^{t} f^{\prime \prime}(x) d x
$$

$$
\begin{aligned}
& =\int_{0}^{t} \mu e^{-\mu x} d x \\
& =\left.\frac{\mu e^{-\mu x}}{-\mu}\right|_{0} ^{t} \\
& =1-e^{-\mu t}
\end{aligned}
$$

3. $F(t)$ was set equal to $0, .0125, .0250, .0375$, ...., .9875, 1.0000.
4. Defining $\bar{x}=\frac{1}{\mu}$ and using the value $\mu_{a}=.075$ and $\mu_{s}=.10$, the various values of $t$ were calculated. For example, $F(t)=.0125$,
$\mu=.10$

$$
\begin{aligned}
1-e^{-\mu t} & =.9875 \\
e^{\mu t} & =1.0126582 \\
\mu t & \approx .0126 \\
t & \approx .126 .
\end{aligned}
$$

The time for resistor circuit numbers 1 and 2 was selected as the time corresponding to $F(t)=.025$ or the time corresponding to the mid-point of the first five per cent of the cumulative distribution curve. Thereafter, the time interval for each resistor circuit was selected as the sequential mid-points of sections 2.5 per cent wide taken from the cumulative distribution curve. Figure 29 (Appendix C) shows these mid-point'times and the times which correspond to the edges of the sections. The resistance required to produce each of the desired times was then determined using resistance-time curves (Figure 16). Data from the previous phases, as well as supplementary data, was used to


Figure 16. Resistance-Time Curves
plot the resistance-time curves. It should be noted that the curves for the two sections differ. It is expected that the curves for all random time interval generators will differ due to manufacturing tolerances of the component parts. After the required resistance was determined for each resistance circuit, the resistors were adjusted using the ohm meter. Difficulty was experienced at this point due to the fact that the resistors were rather coarse and did not lend themselves to fine adjustments; also, some had a tendency to move slightly after they were set.

Upon completion of "setting the distributions," the queue count section was added to the circuit and the Offner dynagraph was attached. The data thus collected and analyzed is shown in Figures 26, 27, and 28. Figure 26 (Appendix B) shows data concerning the individual parts of the random time interval sections, giving the resistor circuit numbers and the time interval lengths from the corresponding resistor circuits. Figure 27 (Appendix B) shows the queue count data providing a log of the various time intervals that the queue (total number in the system) was of various lengths and the total length of time that the various lengths occurred. Figure 28 (Appendix B) shows data compiled to test the hypothesis that the number of occurrences of each resistor circuit is rectangular. It contains data concerning the number of occurrences which was collected in an extended run.

During the progress of the third phase, some effects of wear were noted in the operation of the simulator. At times, the resistance circuit stepping relays failed to step properiy, the relay to switen from one stepping switch to the other failed to operate properly, and the queue count would either fall to add with the occurrence of an arrival or else would subtract two when a service occurred. Thus, it was expected that the variance of the time distributions would be large and that the mean queue length would be low.

## CHAPTER VII

## EVALUATION OF DATA

The data for the first phase of evaluation was used to test the hypothesis that each resistor circuit would be called upon to generate time intervals the same number of times as any other, and to examine the lengths of the time intervals thus generated to determine if the author had been able to generate the desired rectangular distribution.

The data of Figures 21 to 24 was used to calculate $X^{2}$ (chi-square) values which were used as test statistics.

The chi-square critical value was determined by interpolation of values given by Snedecor (3).

$$
\begin{array}{cc}
\begin{array}{c}
\text { Degrees of } \\
\text { Freedom } \\
30
\end{array} & x^{2}(\mathrm{df}, .05) \\
39 & 43.77 \\
40 & 55.76 \\
x^{2} 39, .05 & =43.77+\frac{9}{10}(55.76-43.77) \\
x^{2} 39, .05 & =54.56 .
\end{array}
$$

The hypothesis will be accepted if the test statistics are less than this chi-square value.

Calculation of the test stotistics reveal them to be:

|  | $x^{2}$ |
| :--- | :---: |
| Arrival Up | 40.16 |
| Arrival Down | 53.88 |
| Arrival (Combinea) Section | 40.56 |
| Service Up | 47.57 |
| Service Down | 31.08 |
| Service (Combined) Section | 36.56 |

Thus, one is able to accept the hypothesis that the number of times that each resistor circuit is called upon is uniformly distributed. The time interval lengths were examined visually. It is obvious that the lengths do not agree with the circutt numbers, butit is also obvious from Figures 21 to 24 that all values which resulted from a given circuit do agree rather closely. In consideration of this visual examination, it was felt that the time in terval lengths could be controlled and, thus, a desired distribution be generated, but the setup could not be perm formed by the method used in this phase of evaluation.

The second phase of evaluation was performed to confirm that a desired distribution could be generated, as explained earlier (p. 49), oniy one random time interval generator section was used for this phase.

The hypothesis that two identical distributions could be generated using the two parts of a random time interval seetion was evaluated by comparing the means and variances of
the time intervals genereted.
Calculation of the means and variances of the data of Figure 25 reveals them to be:
Mean Vaxiance

| Upper Part | 31.35 | 156.85 |
| :--- | :--- | :--- |
| Lower Part | 31.67 | 155.88 |

No test was performed to compare these vaiues, as they are from an unknown distribution. However, it is felt that the closeness of these values does indicate that this hypothesis is correct. At least, it reassured the author to the degree that the third phase of evaluation was performed.

In the third phase of evaluation, the author attemptom ed to test the hypotheses accepted in the first phase, as well as evaluate a method of pre-estabiishing a desired distribution and simultaneously evaluate the complete system.

The first hypothesis tested in the third phase of evaluation was the hypothesis accepted in the first phase "each resistor circuit would be called upon to generate time intervals the same number of times as all other rem sistance circuits." Calculation of the chi-square test statistic for each of the individual parts and the two sections from the pertinent data shown in Figure 28 reveals the statistics to be:

Arrival Up
Arrival Down
Arrival (Combined) 48.05

|  | $\chi^{2}$ |
| :--- | :---: |
| Service Up | 22.63 |
| Service Down | 29.87 |
| Service (Combined) | 25.12 |

Comparison of these values with the critical value of chi square $\left[\mathrm{X}^{2}(39, .05)=54.56\right]$ determined in the first test again permits the acceptance of this hypothesis.

The second hypothesis tested in this phase of evaluetion was the hypothesis that it was possible to generate distributions determined in advance. As previously stated; the distributions selected were exponential $\left(f(t, x)=\frac{e^{-t / x}}{x}\right.$ ) with event rates of $1 / x=.075$ and .10 .

This hypothesis was tested by using maximum-
likelihood estimators as suggested by Mood (1).
The Maximum-likelihood estimators ( $\hat{\theta}_{1}, \hat{\theta}_{2}$, $\ldots, \hat{\theta}_{k}$ ) for parameters of a density function $f\left(x ; \theta_{1}, \theta_{2}, \ldots, \theta_{k}\right)$ from samples of size $n$ are for large samples approximately distributed by the multi-variate normal distribution with means $\theta_{1}, \theta_{2}, \ldots, \theta_{k}$ and with coefficient || no's || in the quadratic form when
$\sigma^{1 j}=-E\left[\frac{d^{2}}{\partial \theta_{1} d \theta_{2}}\right] \log f\left(x_{j}, \theta_{1}, \theta_{2}, \ldots, \theta_{k}\right)$. The variances and covariances of the estimators are $\left\|\frac{1}{n} \sigma_{1 j}\right\|$ where $\left\|\sigma_{1 j}\right\|=\left\|\sigma^{1}\right\|^{-1}$. In this case, $f(t, x)=\frac{1}{x} e^{-t / x}$

$$
\begin{aligned}
\log f(t, x) & =\log \frac{1}{x}-\frac{t}{x} \\
\frac{d}{d x} \log f(t, x) & =-\frac{1}{x}+\frac{t}{x^{2}} \\
\frac{d^{2}}{d x^{2}} \log f(t, x) & =\frac{1}{x^{2}}-\frac{2 t}{x^{3}}
\end{aligned}
$$

$$
\begin{aligned}
\sigma^{\prime \prime} & =-E\left[\frac{d^{2}}{d x^{2}} \log f(t, x)\right] \\
& =-E\left(\frac{1}{x^{2}}-\frac{2 t}{x^{3}}\right) \\
& =-\frac{1}{x^{2}}+\frac{2}{x^{3}} E_{(T)} \text { where } E_{(T)}=X \\
& =\frac{1}{x^{2}} .
\end{aligned}
$$

Now $\quad\left\|\sigma_{11}\right\|=\left\|\sigma^{11}\right\|^{-1}$
so

$$
\left\|\sigma_{11}\right\|=x^{2}
$$

$$
\left\|\frac{\sigma_{11}}{n}\right\|=\frac{x^{2}}{n} \text { which is the variance. }
$$

So, the standard deviation of the maximum likelihood estimate $\sigma_{\bar{x}}$ is $\frac{x}{\sqrt{n}}$. Also from Mood (I):

$$
\text { If } f\left(x_{1}, x_{2}, \ldots, x_{n}, \theta\right) \text { is the density func- }
$$ tion for a random sample of size $n$ drawn from a population with an unknown parameter $\theta$, then the maximum likelihood estimator of $\theta$ is the number $\hat{\theta}$, if it exists such that $f\left(x_{1}, x_{2}, \ldots\right.$, $\left.x_{n} ; \theta\right)>f\left(x_{1}, x_{2}, \ldots, x_{n}, \theta^{\prime}\right)$ where $\theta^{\prime}$ is any other possible value of $\theta$.

To find $x$, the likelihood estimator, $\left(\log \underset{i=1}{ } f\left(t_{i i}, x\right)\right.$ will be maximized.

$$
\begin{aligned}
L= & \log \sum_{i=1}^{n} f\left(t_{i}, x\right)=\sum_{i=1}^{n} \log f\left(t_{i}, x\right) \\
= & \sum_{i=1}^{n} \log \left(\frac{1}{x} e^{-t / x}\right) \\
= & n \log 1 / x-\frac{\sum_{i=1}^{n} t_{i}}{x} \\
& \frac{d L}{d x}=-\frac{n}{x}+\frac{\sum_{x^{2}}^{t}}{n}
\end{aligned}
$$

Setting

$$
\frac{d L}{d x}=0
$$

$$
\frac{\Sigma t_{i}}{\hat{x}}=n
$$

$$
\hat{\mathbf{x}}=\frac{\Sigma t_{i}}{\mathrm{n}} .
$$

Referring to Figure 26 for the various values of $n$, confidence limits can be found to use in evaluating the data.

|  | $n$ | $\frac{x}{x}$ | $\sigma_{\bar{x}}=\frac{x}{\sqrt{n}}$ | $x-3 \sigma_{\bar{x}}$ | $x+3 \sigma_{\bar{x}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Arrival Up | 446 | .075 | .6313 | 11.44 | 15.28 |
| Arrival Down | 447 | .075 | .6307 | 11.44 | 15.28 |
| Arrival Combined | 893 | .075 | .4462 | 11.99 | 14.67 |
| Service Up | 457 | .10 | .4677 | 8.59 | 11.40 |
| Service Down | 450 | .10 | .4715 | 8.58 | 11.41 |
| Service Combined | 907 | .10 | .3320 | 9.00 | 11.00 |
|  |  |  |  |  |  |
| Calculation of $\frac{\Sigma t_{i}}{n}$ for the relevant data of Figure 26 |  |  |  |  |  |

A

| Arrival Up | 12.350 |
| :--- | :--- |
| Arrival Down | 14.118 |
| Arrival Combined | 13.263 |
| Service Up | 10.076 |
| Service Down | 10.036 |
| Service Combined | 10.056. |

Thus, one may accept the hypothesis that it is possible to generate time interval distributions with predetermined distributions and parameters.

In passing, one also notes that the standard deviations, which for the exponential distribution should equal the mean, (shown below) of this data also falls within the above limits:

$$
\text { Standard }{ }_{\sigma} \text { Deviation }
$$

| Arrival Up | 12.06 |
| :--- | ---: |
| Arrival Down | 13.28 |
| Arrival Combined | 12.80 |
| Service Up | 8.84 |
| Service Down | 10.00 |
| Service Combined | 9.94. |

Thus, there is an indication that the standard deviations are not significantly different from the desired means, and that the researcher has succeeded in generating exponential distributions with mean times of 10 and 13.33 seconds.

Further calculations show the two sigma limits for the maximum likelihood estimator to be

|  | $\mathbf{x}-2 \sigma_{\overline{\mathbf{x}}}$ | $\mathbf{x}+2 \sigma_{\bar{x}}$ |
| :--- | :---: | :--- |
| Arrival Up | 12.07 | 14.60 |
| Arrival Down | 12.07 | 14.60 |
| Arrival Combined | 12.44 | 14.22 |
| Service Up | 9.06 | 10.94 |

$$
x-2 \sigma_{x} \quad x+2 \sigma_{x}
$$

| Service Down | 9.06 | 10.94 |
| :--- | :--- | :--- |
| Service Combined | 9.34 | 10.66 |

When the means and standard deviations are compared to these tighter limits that of the twelve values listed above only two fall beyond them.

The final portion of the third phase of evaluation involves examining the queue which was generated. The probability of a simple queue with exponential arrivals and services such as the one simulated being in any state $n$ is given by Morse (2) to be

$$
\begin{array}{r}
P_{n}=\rho^{n_{P}} P_{0} \text { where } \rho=\frac{X_{S}}{X_{p}} \\
X_{S}, X_{A}=\text { means }
\end{array}
$$

so

$$
f(n)=(1-\rho)^{\rho n} \quad P_{0}=I-\rho
$$

The maximum likelihood estimator will be found as before:

$$
\begin{aligned}
I & =\log \sum_{i=1}^{n} f\left(x_{i}, \rho\right) \\
& =\log x_{i=1}^{n}(1-\rho)^{p x_{i}} \\
& =\sum_{i=1}^{n} \log (1-\rho) p^{x_{i}} \\
& =n \log (1-\rho)+\Sigma x_{i} \log \rho
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d L}{d p}=-\frac{n}{1-p}+\frac{\Sigma x_{i}}{p} \\
& n=\text { Total number time of unit } \\
& \text { time periods. }
\end{aligned}
$$

$$
\begin{aligned}
\frac{n}{1-\hat{\rho}} & =\frac{\Sigma x_{i}}{\hat{\rho}} \\
\hat{\rho} & =\frac{\Sigma x_{i}}{n+\Sigma x_{i}}
\end{aligned}
$$

This researcher proceeds in a manner similar to his previous procedure to determine the distribution of this statistic:

$$
\begin{aligned}
\sigma^{i j} & =-E\left[\frac{d^{2}}{d \theta_{i} d \theta_{j}} \log f\left(x, \theta_{1} \theta_{2} \ldots . .\right)\right] \\
\sigma^{\prime \prime} & =-E\left[\frac{d^{2}}{d \rho^{2}} \log (1-\rho) \rho^{x}\right] \\
& =-E\left[\frac{d}{d \rho}\left(\frac{x}{\rho}-\frac{n}{1-\rho}\right)\right] \\
& =-E\left[\frac{x}{\rho^{2}}-\frac{1}{(1-\rho)^{2}}\right] \\
& =\frac{E(x)}{\rho^{2}}+\frac{1}{(1-\rho)^{2}}
\end{aligned}
$$

where

$$
\begin{aligned}
E(x) & =\sum_{i=1}^{\infty} x_{i}(1-p) p^{x_{i}} \\
& =(1-\rho)\left(p+2 p^{2}+3 p^{3}+\ldots+n p^{n}+\ldots\right. \\
E(x)\left(\frac{1}{1-p}\right) & =\rho+2 p^{2}+3 p^{3}+\ldots n p^{n}+\ldots
\end{aligned}
$$

$$
\begin{aligned}
E(x)\left(\frac{\rho}{1-\rho}\right) & =\rho^{2}+2 \rho^{3}+3 \rho^{4}+\ldots(n-1) \rho^{n}+\ldots \\
E(x) & =\rho+\rho^{2}+\rho^{3}+\ldots+\rho^{n}+\ldots \\
\rho E(x) & =\rho^{2}+\rho^{3}+\rho^{4}+\ldots \rho^{n+1} \ldots \\
E(x) & =\frac{\rho}{1-\rho}
\end{aligned}
$$

Thus

$$
\begin{aligned}
\sigma^{\prime \prime} & =\frac{\rho}{\rho^{2}(1-\rho)}+\frac{1}{(1-\rho)^{2}} \\
& =\frac{1}{\rho(1-\rho)^{2}}
\end{aligned}
$$

and

$$
\left\|\sigma_{\mathfrak{1}}\right\|=\left\|\sigma_{. \prime}^{\prime \prime}\right\|^{-1}
$$

so

$$
\sigma_{11}=\rho(1-p)^{2}
$$

so $\sigma($ standard deviation $)=\sqrt{\frac{\rho(1-\rho)^{2}}{n}}$
where $n=$ number of observations of unit length, or

$$
\sigma=(I-p) \sqrt{\frac{p}{n}}
$$

For this distribution

$$
\begin{aligned}
\rho & =\frac{.075}{.100} \text { or } \frac{10.00}{13.333}=.75 \\
\sigma_{\rho} & =(1-.75) \sqrt{\frac{.75}{5950.75}} \text { n from Figure } 27 \\
& =.00282 .
\end{aligned}
$$

The $3 \sigma$ confidence limits ( $\rho \pm 3 \sigma_{\rho}$ ) are now found to be $.75 \pm .00846$ or .74154 and .75846 , while

$$
\begin{aligned}
\hat{\rho} & =\frac{\Sigma x_{i}}{n+\Sigma x_{i}}=\frac{17055.8}{17055.8+5950.78} \\
& =.74134 .
\end{aligned}
$$

Thus, it is found that the hypothesis in this last test must be rejected. It must be noted, however, that this low value was expected due to the occasional malfunction of the worn add and subtract relay in the queue count section which was mentioned on page 54.

## CHAPTER VIII

CONCLUSIONS

It is concluded from the results of the foregoing tests, that the random time interval sections do utilize each resistor circuit approximately the same number of times in generating a set of time intervals. Furthermore, it is concluded that the lengths of the time intervals resulting from these resistor circuits can be controlled. Hence, it is concluded that this system can be used to generate time intervals from any desired distribution. This ability was demonstrated for three distributions.

Even though the attempt to simulate a queue of prea dictable characteristics failed, it is felt that this failure was due to the failure, due to wear, of a particular part rather than a failure of the total system. Therefore, it is felt that the total system has been demonstrated to be capable of simulating simple queues in its present form.

The device makes available a means of examining queues with "exodit" (other than exponential based) queues without the necessity of using digital computers as simulation devices; additionally, it can be used to simulate
many types of "Monte Carlo" problems, inventory problems, production control problems, manufacturing lines, and materials handling problems.

It has, in another study, proved useful in examining the spacing of the empty service facility occurrences due to its data printout form.

This device makes available to the classroom a means of simulating and observing and studying Monte Carlo situations without having to resort to the time consuming procedure of observing industrial situations or the laborious procedures involved in hand simulation.

The simulator will permit the student to control various statistical parameters such as the arrival and service distributions of queues, in order to experience the effects of these changes. By conmecting two or more units, complex situations may be demonstrated and studied to show or study the effects of parallel or series systems.

The random time interval units alone can be used individually to serve as the timing unit for motion pictures to be used for work sampling studies. They may also be used to time quality control samples. For that matter, they can be used whenever a series of random time intervals are desired. The only limits on the time intervals and distributions are limits imposed by the sizes of the resistors and number of resistor circuits. These limits can be overcome if necessary by the design of a special unit.

The existing units can be improvedin several ways it is felt. Likely, the most drastic improvement would be to redesign the entire system to use solid state components rather than tubes and relays. This would overcome the component reliability problem which was encountered near the end of the testing as well as greatly reducing the noise level. A much less drastic change, which it is felt would be of considerable benefit, would be to replace relay $R 1-1$ with a single pole single throw relay of the same size and type, and use this new relay to control a 110 volt a.c. relay which would control the circuits which Rl-l now controls. This change would insure the positive action of the various circuits controlled at the present by Rl-1. Several advantages could also be gained by constructing three separate units, two random time interval generator units and a queue count unit, rather than a single system such as was constructed and is shown in the present design. This would provide the advantages of remote operation of the random time interval generators, and would also permit the operation of only those components needed at a particular time。

The process of setting up a distribution and adjusting the potentiometer to the desired values is at the present time a very laborious and time consuming procedure. It is felt that this difficulty can be partially overcome by the use of logarithmically tapered potentiometers in the resistance circuits rather than the presently used
uniformly tapered ones and then by the use of a wheatstone bridge rather than a standard ohm meter to determine the resistance. The process will still be slow and time consuming, but not as slow as with the present system.

This research and development should be continued to devise means of utilizing the present system to simulate more complex systems and to use it as a tool to investigate systems and system phenomena which cannot be readily jnvestigated by other means.

## BIBLIOGRAPHY

(1) Mood, Alexander M. Introduction to the Theory of Statistics. New York: McGraw-Hill Book Co., Inc., 1950.
(2). Morse, Philip M. Queues, Inventories, and Maintenance. New York: John Wiley and Sons, Inc., 1958.
(3) Snedecor, George Wo Statistical Methods, 5th ed. Ames, Iowa: The Iowa State College Press, 1956.

APPENDIX A

SHOP MADE PARTS


Figure 17. Cam Disk


Figure 18. Motor Mount


Figure 19. Potentiometer Mounting Bracket


Figure 20. Support Bracket

## APPENDIX B

## EVALUATTON DATA

| Resistor Circuit Number | $\begin{aligned} & \text { Time Intervals } \\ & (\mathrm{Sec}) \end{aligned}$ | Mean Time |
| :---: | :---: | :---: |
| 1 | $1,1,1,1,1,1,1,1,1,1,1$ | 1.0 |
| 2 | $2,2,2,2,2,2,2,2,2$ | 2.0 |
| 3 | 3, 3, 3, 3, | 3.0 |
| 4 | $4,4,4,4,4,4,4,4,4,4,4$ | 4.0 |
| 5 | 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5 | 5.0 |
| 6 | $6,6,6,6,6,6$ | 6.0 |
| 7 | 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7 | 7.0 |
| 8 | $8,8,8,8,8,8,8,8,8,8$ | 8.0 |
| 9 | 9, 9, 9, 9, 9, 9굴, 9, 91 | 9.2 |
| 10 | 10, 10, 10, 10, 10, 10, 10, 10 | 10.0 |
| 11 |  |  |
|  | 11, 11, 11 | 11.0 |
| 12 | $12,12,12,12,12,12,12,12,12,12,$ | 12.0 |
| 13 | 13, 13, 13, 13, 13, 13 | 13.0 |
| 14 | 14, 14, 14, 14, 14, 151 | 14.2 |
| 15 | 15, 15, 15 | 15.0 |
| 16 | $\begin{aligned} & 15,15 \frac{1}{2}, 16,16,16,16,16,15,16,15, \\ & 16,16 \end{aligned}$ | 15.7 |
| 17 | 16i $, 18,17,16,16$ | 16.7 |
| 18 | $\begin{aligned} & 17,17 \frac{1}{2}, 18,17,18 \frac{1}{2}, 17,18,17,18,18, \\ & 18,18,18 \end{aligned}$ | 17.8 |
| 19 | 18, 18, 19, 18, 19 | 18.4 |
| 20 | 19, ${ }^{\frac{1}{2}, 19,18, ~ 20, ~ 20, ~ 19, ~ 19, ~ 19 ~}$ | 19.3 |
| 21 | $\begin{aligned} & 20 \frac{1}{2}, 20,19,20,21,20 \frac{1}{2}, 20,20,20 \frac{1}{2}, \\ & 20,20,20,20,20,21 . \end{aligned}$ | 20.0 |
| 22 | 21, 21, 21, 22, 21, 21雨, 21, 21, 21, 21, |  |
|  | 21, 21, 21, 21 , 11 | 21.1 |
| 23 | ```22, 22, 22, 22, 21\frac{1}{2}, 22, 22, 22, 22, 21\frac{1}{2}, 22``` | 21.9 |
| 24 | 23, 21, 22, 23, 23, 24, 22, 23, 23, 22 $\frac{1}{2}$ | 22.7 |
| 25 | $24,23,24,24,24,24 \frac{1}{2}, 23,24,23,24,$ | 23.7 |
| 26 | 25, 24, 24, 23, 25, 24, 25, 24, 25 | 24.3 |
| 27 | $\begin{aligned} & 25,25 \frac{1}{2}, 25,25 \frac{1}{2}, 26,25,25 \frac{1}{2}, 26,25, \\ & 25,25 \end{aligned}$ | 25.3 |
| 28 | 26, 26, 27, 26 $\frac{1}{2}, 26,26$ | 26.2 |
| 29 | 27, 27, 26 $\frac{1}{2}, 25,26,27,27,27,26,27$ | 26.7 |
| 30 | 28, 28, 28, 29, 28, 28 | 28.1 |
| 31 | 28는, 28, 281 $, 29,29,30,28,29,29$ | 28.8 |
| 32 | 29费, 29, 29, 29, 30, 30, 30 | 29.5 |
| 33 | $30,31,30 \frac{1}{2}, 30,30,30 \frac{1}{2}, 30,30 \frac{1}{2}, 30,$ | 30.3 |
| 34 | 30, 31, 31, 31, 317 |  |

Figure 2l. Uniform Distribution Data - Arrival Up


| Resistor Circuit Number | $\begin{aligned} & \text { Time Intervals } \\ & (\text { Sec. }) \end{aligned}$ | $\begin{aligned} & \text { Mean } \\ & \text { Time } \end{aligned}$ |
| :---: | :---: | :---: |
| 1 | $1,1,1,1,1,1,1,1,1$ | 1.0 |
| 2 | $2,2,2,2,2,2,2,2,2$ | 2.0 |
| 3 | $3,3,3,3,3,3,3,3,3,3,3$ | 3.0 |
| 4 | $4,4,4,4,4,4,4,4,4$ | 4.0 |
| 5 | $5,5,5,5,5,5,5,5,5,5$ | 5.0 |
| 6 | $6,6,6,6,6,6,6,6,6,6$ | 6.0 |
| 7 | $7,7,7,7,7,7,7,7,7,7$ | 7.0 |
| 8 | 8，8，8，8，8，8，8，8，8 | 8.0 |
| 9 | 9，9，9，9，9，9，9 | 9.0 |
| 10 | 10，10， 10 | 10.0 |
| 11 | $11,11,11,11,11,11,11,11,11,11$, |  |
|  | ll，ll，ll， 11 | 11.0 |
| 12 | 12⿳亠丷厂彡，12，12，12，12，11， 12 | 12.0 |
| 13 | $13,13,13,13,13,13,13,13$ | 13.0 |
| 14 | 13年，13六，13－1 $, 14,14$ | 13.7 |
| 15 | $15,15,15,15,15,15,15,14 \frac{1}{2}, 15$ | 15.0 |
| 16 | $16,16,16,15 \frac{1}{2}, 16,16,16,16,16$, |  |
|  | 17，16，16，16，15 ${ }^{\text {l }}$ ，16，16 | 16.0 |
| 17 | $17,16,16,17,17,17,17,17,16 \frac{1}{2},$ | 16.7 |
| 18 | 18，17，18，17，17， 18 | 17.5 |
| 19 | 18，18，18，19， $20,18,18 \frac{1}{2}, 19,18$, |  |
| 20 | $18,19,18,18$ $19,19,19,19,19$ | 18.4 19.0 |
| 21 | 20，20，21， 20 | 20.3 |
| 22 | 21，21，20－1 ，22，21，20，21， 21 | 20.9 |
| 23 | $\text { 21- }, 22,21,22,22,22,22,23,21 \frac{1}{2},$ | 21.9 |
| 24 | $\begin{aligned} & 22,22 \frac{1}{2}, 23,22,24,23 \frac{1}{2}, 22,23,23, \\ & 22 \frac{1}{2}, 23,22,22 \end{aligned}$ | 22.5 |
| 25 | $23,24,23,24,23 \frac{1}{2}, 24,24,24,24,$ | 23.6 |
| 26 | 24，24，241 $, 24,24,24,25,24 \frac{1}{2}$ | 24.3 |
| 27 | 25，26，251 $\frac{1}{2}$ ，25，25，27，24， 26 | 25.4 |
| 28 | 26，25，26，26，26，26 ${ }^{\frac{1}{2}, 26,26,26, ~} 26$ | 26.0 |
| 29 | $26,27,25,27,26,27,27,27,26 \frac{1}{2}, 27 \frac{1}{2},$ | 26.6 |
| 30 | 28，27，27，27，28，28，28，281 $\frac{1}{2}, 28,28$ | 27.8 |
| 31 | 28，281 $\frac{1}{2}, 28 \frac{1}{2}, 28,28$ | 28.2 |
| 32 | 29，29－${ }^{\text {a }}$ ，291， $29,30,29$ | 29.3 |
| 33 | $30,30,30,30,30,30,31$ | 30.1 |
| 34 | 311， $30,31,30 \frac{1}{2}, 31,31,31,31,31,30$ | 30.8 |
| 35 | 31，31，31，31， 31 | 31.0 |

Figure 22．Uniform Distribution Data－Arrival Down

| Resistor |
| :---: |
| Circuit |
| Number | $\quad$| Time Intervals |
| :---: |
| (Sec.) |$\quad$| Mean |
| :---: |
| Time |

\(\left.$$
\begin{array}{cllll}\begin{array}{l}\text { Resistor } \\
\text { Circuit } \\
\text { Number }\end{array}
$$ \& \& Time Intervals <br>

(Sec)\end{array}\right]\)| Mean |
| :--- | :--- | :--- |
| Time |

Figure 23. Uniform Distribution Data - Service Down

| Resistor Circuit Number | Service Down | Mean Time |
| :---: | :---: | :---: |
| 3940 | 33, 33 $\frac{1}{2}, 33,33,33,33$ | 33.1 |
|  | $34,34,34,34,34,33,33,33 \frac{1}{2}, 33$, |  |
|  | $33,34,34,34,34$ | 33.7 |
|  | Figure 23. (Continued) |  |



| Resistor Circuit Number | $\begin{aligned} & \text { Time Intervals } \\ & (\mathrm{Sec}) \end{aligned}$ | $\begin{aligned} & \text { Mean } \\ & \text { Time } \end{aligned}$ |
| :---: | :---: | :---: |
| 36 | 31, 30, 31, 32, 32, 31 | 31.2 |
| 37 | 32, 31, 31 $\frac{1}{2}, 32,32,33,33$ | 32.1 |
| 38 | $33,33 \frac{1}{2}, 34,32 \frac{1}{2}, 32,33,33,33,33$, |  |
|  | 331, $33,34,33,33,33$, 34 | 33.1 |
| 39 | $33,34,33,34,33,34,34$ | 33.6 |
| 40 | $34,33 \frac{1}{2}, 35,35,37,34,34$ | 34.6 |
|  | Figure 24. (Continued) |  |

Upper Section
Resistor Circuit

No.

| 1 | 1.2, 1.2, l.2, l. 2, . . 6, 1.6 |
| :---: | :---: |
| 2 | $8,8,8,8,8,8,8$ |
| 3 | $12.8,12.8,12.8,12.8,12.8,12.8,12.4,12.8$, |
|  | $12.4, ~ 12, ~ 12.8, ~ 12.4$ |
| 4 | $12.8,12.8,12.8,12.8,12.8,12.4,12.8,12.8$, |
|  | $12.8,12.4,12.8,12.4,12.8,12.8$ |
| 5 | $17.2,16.8,16.8,16.8,16.8,16.8$ |
| 6 | $16.8,16.8,16.8,16.8,1.7 .2,16.8$ |
| 7 | $\begin{aligned} & 20,20.4,20.8,21.2,20.8,21.6,20.8,20.8, \\ & 20.8,20.8,20,20 \end{aligned}$ |
| 8 | $20.8,20.4,20.8,19.6,20.8,20.8,20.8$ |
| 9 | $20.8,21.6,20.8,20.8,20,20.8,21.2,20.8$, |
|  | $20.8,21.6,20.8,20.8$ |
| 10 | $21.6,20.8,20.8,20.8,20.8,20.8,20.4$ |
| 11 | $24.4,24.8,24.8,25.2,25.6,24.8,25.6,24.4$, |
|  | $24.8,24.8,25.2,24,25,24.8$ |
| 12 | $25.6,25.2,26,25.6,29.2,24.4,25.2,24$ |
| 13 | $25.6,24.8,25.6,25.2,24.8,24.4,24.4,25.2$, |
|  | 25.2, 24 |
| 14 | 25.2, 24.8 |
| 15 | $28.8,30,29.2,29.6,29.6,29.6,28.8,28.8$, |
|  | 28.4, 28.4, 28.8, 28, 28.8, 28, 28 |
| 16 | 29.6, 28.8, 28.8, 28.8, 28 |
| 17 | 28.8 |
| 18 | 29.6, 29.6, 28.4, 28.8, 28.8, 28, 28.4, 28.8 |
| 19 | $32.8,32,32,32,32,31.2,32,32.8,31.6,32.4$ |
| 20 | $32.8,32.4,33.2,32,34.4,31.6,32,32,30.8$ |
| 21 | $32.8,32.8,32.8,31.2,30.4,32,30$ |
| 22 | $32,30.8,32.4,31.6,31.2$ |
| 23 | $35.2,35.6,36,34.4,34$ |
| 24 | $36,34,42,34.4,34,34.8,34.8,34,35.6$ |
| 25 | $36,36,34.4,35.2,34.8,34$ |
| 26 | $36,35.2,35.6,34,34.8,35.2,34$ |
| 27 | $39.2,39.2,39.2,38,39.2$ |
| 28 | $37.6,40,39.2,38.8,37.6,38.4,37.6,38,38$ |
| 29 | $40,38.4,38.4,38.8,38,38.8$ |
| 30 | $\begin{aligned} & 39.2,39.6,40,38.8,38.4,38.4,38.4,38.4,38, \\ & 37.6,37.2 \end{aligned}$ |
| 31 | $40.8,42,43.2,41.6,42.8,42.8$ |
| 32 | $43.6,43.6,44,43.6,40.8,43.6,42,40.4,40.4$ |
| 33 | $42.8,44,42,42,41.6,42,4.1 .2,43.2,43.2$ |
| 34 | $44,44,43.6,42,41.6,42,42,42.8,42,41.6,$ |
| 35 | $44.8,48,47.6,47.2,46.4,46.4,46,46.4,45.6$ |
| 36 | $\begin{aligned} & 48,48,46,46.4,46,46.4,46.4,44.8,47.2, \\ & 45.6,46,46.8,47.2,46,45 \end{aligned}$ |

Figure 25. Similar Distribution Data

| Resistor <br> Circuit <br> No. | Upper Bection (Continued) |
| :---: | :---: |
| 37 $52,49.6,50.4,50.4,50.8$ <br> 38 $50.8,52,50.8,50,50,48.8,49.2,48,50,54.4$ <br> 39 $54.4,54,52,52,52,52.8,-52$ <br> 40 $57.6,59.6,57.2,56.8,56,57.6$ |  |

Iower Section


Resistor
Circuit
No.
Time Intervals

| 30 | $\begin{aligned} & 39.6,39.6,39.6,40,40,38.8,39.2,38.4,38.8, \\ & 38.8,38.4,38.4,37.6 \end{aligned}$ |
| :---: | :---: |
| 31 | $43.2,43.6,42,41.6,42.4,41.2$ |
| 32 | $43.6,44,42,42,41.6,42,42.8,41$ |
| 33 | $\begin{aligned} & 43.6,43.6,43.6,42,42.8,42,42,42,40.4, \\ & 43.2 \end{aligned}$ |
| 34 | $41.6,42.8,42,42,43.2,42.4$ |
| 35 | $\begin{aligned} & 47.6,47.6,46,46.8,51.2,46,46,45.2,46, \\ & 44.8,45.2,47.2,45.2,45 \end{aligned}$ |
| 36 | $48.8,46,46,46.4,47.2,46.8,46.8,45.6$ |
| 37 | $51.2,49.6,48.8,48.8,50,50,54.8,48.4$ |
| 38 | $50.4,48.8,49.2,50,48.4,50,50,52,48$ |
| 39 | $52,52.4,54.8,54.4,54,54,54.4,54,52$ |
| 40 | $60.8,58,59.6,58.8,58,57.2,58,57.2,60$ |

Figure 25. (Continued)

Service Channel

|  | U |
| :---: | :---: |
| Date | 9-25-65 |
| Time | P. M. |

Resistor Circuit

No.

Time Intervals
1.
2.
3. 32.30 .382 .160
3.
4.
5.
6.
7.
8.
9.
10.

11。
12.
13.
14.
15.
16.
17.
18.
19.

20
21
$17.15,16.38,16.35$
$.65, .60 . .65$
$9.62,9.45$
7.89 1.3, 1.25, 1.2, 1.1. 1.2, 1.1
$1.6,1.55,1.62,1.6$ $6.4,5.95$
1.95, 1.78, 1.78, 1.95
$4.3,4.65,4.45$
$3.82,3.9,4.1$
.18, $18.18, .20, .20, .22, .20, .18, .20, .18, .16$

12.20, 12.15, 12.71, 12.6, 11.72, 11.6, 12.35
$.70, .65, .60, .60, .62 . .70, .62, .70, .65$,
$9.4,10.3,9.25,9.38,9.6,10.0,10.15,9.25$,
$9.5,9.78,9.9,9.95,9.6,9.76,9.85,9.65$,
$.90, .90,1.0, .90, .95, .90, .92,1.0$
$7.75,8.0,8.4,7.62,8.0,8.05,8.4,8.1,8.15$,
1.15, 1.22, 1.3. $1.25,1.23,1.18,1.25,1.2$, $6.7,6.82,6.75,6.83,6.9,6.9,6.7,6.95$,
$6.99,6.64,7.15,6.5,6.9,6.8,7.0$
$1.55,1.55,1.55,1.41,1.47,1.6,1.6,1.55$,
$1.65,1.6,1.45,1.62,1.6,1.55,1.52,1.6$,
$6.05,5.9,6.3,6.0,6.0,6.25,5.82,5.8,5.9$,
1.8, 1.95, 1.85, 1.85, 1.8, 1.82, 1.9, 1.68,
$5.3,4.95,5.35,5.4,5.27,5.32,5.1,5.35,5.6$, $4.55,5.05,5.12,5.25,5.4,5.1,5.5,5.2$
$2.15,2.12,2.26,2.21,2.55,2.12,2.5,2.35$,
$2.3,2.3,2.2,2.3,2.3,2.2,2.4$
$4.42,4.62,4.68,4.7,4.68,4.35,4.72,4.3$,
$2.55,2.52,2.65,2.69,2.7,2.64,2.68$
$3.8,4.1,4.05,4.07,4.15,4.1,3.96,4.12$,
$3.1,3.23,3.1,3.1,3.28,3.23,3.3,3.3$
$2.92,3.0,2.8,2.95,2.95,3.1,3.2,2.9,3.1$,
$3.05,3.0,2.86,2.92,3.1,3.0,3.0$
$3.42,3.22,3.72,3.45,3.41,3.4,3.4,3.6$,
3.34, 3.5

Figure 26. Exponential Distribution Data

Resistor
No.

## Time Intervals

| 22. | $3.65,3.65,3.42,3.62,3.65,3.75,4.6,4.8,$ |
| :---: | :---: |
| 23. | $\begin{aligned} & 2.75,2.8,2.9,2.82,2.9,2.7,2.8,2.7,2.85, \\ & 2.85,2.72 \end{aligned}$ |
| 24. | $\begin{aligned} & 4.35,4.12,4.4,4.15,4.18,4.32,4.25,4.4, \\ & 4.42,4.4,4.4,4.2 \end{aligned}$ |
| 25. | $2.4,2.3,2.32,2.45,2.55,2.5,2.38,2.5 .2 .5$ |
| 26. | $\begin{aligned} & 5.1,4.65,4.92,4.92,4.8,4.8,4.8,4.92,5.2, \\ & 5.15,3.72,4.82,5.35 \end{aligned}$ |
| 27. | $\begin{aligned} & 2.05,2.1,1.95,2.05,2.05,2.05,2.0,1.97, \\ & 2.12,2.1,2.0,2.1,2.0,1.85,2.0,2.05 \end{aligned}$ |
| 28. | $5.33,5.6,5.95,5.7,5.82,5.8,5.7$ |
| 29. |  |
| 30. | $6.4,6.42,6.75,6.7,7.0,6.5,6.2,6.7,6.65$ |
| 31. | $1.5,1.4,1.5,1.4,1.35,1.45,1.48,1.5,1.31$ |
| 32. | $8.0,7.52,7.85,7.8,7.5,7.5,7.5$ |
| 33. | $1.1,1.12,1.1,1.12,1.1,1.08$ |
| 34. | $\begin{aligned} & 8.75,8.86,8.95,8.85,8.68,8.92,9.0,9.07, \\ & 8.78,9.4 \end{aligned}$ |
| 35. | $\begin{aligned} & .72, .70 \cdot .78, .72, .72, .82, .80, .85, .78, .80, \\ & .72, .80, .75 \end{aligned}$ |
| 36. | $10.6,10.0,10.71,10.9,11.0,10.36,10.34,10.85$ |
| 37. | .48, . 50, . 50, . 50, .50,.50, .45,.40 |
| 38. | $\begin{aligned} & 13.55,14.2,14.07,13.25,14.32,13.63,13.45, \\ & 13.35 \end{aligned}$ |
| $\begin{aligned} & 39 . \\ & 40 . \end{aligned}$ | ```.18,.20,.22,.20,.20,.22,.20, . 20,..20,.22 22.05, 23.35, 22.4, 22.6, 23.0, 21.9, 21.92, 23.35, 21.85, 22.4, 24.1, 21.97, 23.15, 21.42, 21.9``` |

## Service Channel <br> Section Down

1. $\begin{aligned} & .20, .19, .20, .20, .20, .20, .18, .20, .20, ~ \\ & .20, .20, .20, .20, ~ .18\end{aligned}$
2. $\quad 15.58,15.95,15.82,16.82,17.14,16.4,11.7$, $16.8,16.5,16.4,17.02,17.3,17.6$
3. 

$.32, .30, .30, .30, .30, .30, .30, .30, .32, .30$,
.35 . . 35. . 30
4. 12.1, 10.95, 12.3. 11.85, 12.2, 12.12, 12.3, $12.3,12.7,12.9,12.32,11.76,12.0,11.85$, 12.0, 12.0

Figure 26. (Continued)

| 5. | .60, .60, .60, .60, . $63, .70, .70$ |
| :---: | :---: |
|  | $10.3,9.85,9.83,10.1,10.1,9.62,10.2,9.35$ |
| 7. | $.85 ., .92, .90,1.05,1.05, .85, .80, .92,1.0,$ |
| 8. | $7.8,8.5,8.2,8.8,7.97,8.0,8.1,8.42,8.05,$ |
| 9. | $1.2,1.2,1.2,1.2,1.2,1.22,1.2$ |
| 10. | $7.0,7.32,7.2,6.6,7.4,7.07,6.85,6.9$ |
| 11. | $1.62,1.55,1.52,1.55,1.58,1.62,1.45,1.51,$ |
| 12. | $5.85,6.3,6.35,5.72,5.95,5.78,5.9,6.2,$ |
| 13. | $1.8,1.67,1.8,1.75,1.8,1.58,1.9,1.8,1.8$, |
| 14 | 1.8, $5.1 .75,1.85,1.9,1.85,1.7,1.82,5.3,5.6$, |
|  | 5.32, $5.0504,5.65$, |
| 15. | $2.18,2.25,2.15,2.18,2.2,2.2$ |
|  | $4.35,4.8,4.88,4.61,4.7,4.55,$ $4.6,4.62,4.25$ |
| 17. | $\begin{aligned} & 2.52,2.62,2.45,2.6,2.6,2.65,2.8,2.7,2.5, \\ & 2.8 \end{aligned}$ |
| 18. | $3.8,4.02,3.78,3.85,4.0,4.12,3.7,4.05,$ |
| 19. | $3.15,3.1,3.05,3.1,2.98,3.05,2.35$ |
| 20. | $3.04,3.12,3.0,3.12,3.08,3.0,3.05,3.1,3.1$ |
|  | $3.3,3.5,3.6$, ${ }^{\text {a }}$, $3.4 .3 .5,3.6,3.2$ |
| 21. | $3.1,3.3,3.0,3.3 .3 .33,3.32,3.2,3.35,3.18$, |
| 22. | $3.3,3.2,3$ |
| 23 | $2.78,2.55,2.7,2.78,2.9,2.7,2.85,2.72$ |
| 24. | $4.5,4.35,4.2,4.1,4.35,4.42,4.3,4.5,4.24$, |
| 25. | $2.23,2.4,2.4,2.42,2.42,2.5,4.42,2.4,2.45$ |
| 26. | $4.4,2.5$, |
|  | $5.1,5.2,4.98$ |
| 27. | 2.0, 2.15, 2.1, 2.05, 1.95, 2.12, 1.95, 2.05 |
| 28. | $5.22,6.0,5.35,5.38,5.71,5.78,5.6,5.37,$ |
| 29. | $1.7,1.7 .1 .57,1.6,1.72,1.72,1.65,1.58,1.7$, |
|  | $1.74,1.62,1.6,1.72$ |
| 30. | $\begin{aligned} & 6.9,6.8,6.38,6.7,6.4,6.5,6.55,6.5,6.3, \\ & 7.05,6.6,6.7,6.25,6.62,6.4,6.05,5.9,6.55 \end{aligned}$ |
|  | 25. (Continued) |

## Time Intervals

31. $1.45,1.42,1.35,1.42,1.42,1.45,1.3,1.5$, $1.4,1.42,1.35,1.35$
32. $7.5,7.58,7.22,7.8$
33. 1.02, 1.1, 1.05, 1.15, 1.05, 1.05, 1.08, 1.1
$34, \quad 8.3,8.47,8.61,8.82,8.2$
34. . 80, .75, 1.15, .75,.72,.72,.80,.80,.78,
35. $\quad 10.75, .72,10.62,11.15,10.78,10.44,11.02,10.92$
36. . $45, .40, .42, .45, .45, .50, .45, .42, .50, .45$,
$.45, .45, .45$
37. $13.0,13.28,13.65,14.8,13.72,14.42,14.35$,
$13.9,13.8,14.28,13.3 .14 .35$
38. .18, .20, .20, .18, .20, .19, .20,.15,.15,.20, $.20, .20, .20, .20, .15$
39. $21.25,21.05,22.1,21.0,22.2,22.3,21.05$, $22.2,21.0,23.32,21.55,21.1,21.3,22.8,21.6$

40. . 20, .20,.19, .18, .19, .20, .19,.18, .20, .20, $2183,21.7,19.74,21.4,21.35,21.15,21.5,21.3$
$\begin{array}{ll}\text { 2. } & 21.3,21.7,19.74,21.4,21.35,21.15,21.5,21.3 \\ \text { 3. } & .40, .40, .45, .40, .38, .40, .38, .40, .40, .40,\end{array}$ $\because 40, .40$
41. $15.5,15.7,16.05,15.65,15.52,15.62,15.64$, $15.45,15.67,15.62,15.9$
42. $\quad .80, .72, .70, .75, .75, .75, .72, .70, .74, .80$
43. $12.4,12.35,12.42,12.32,12.52,12.3,12.28$, $12.61,12.5,12.55$
44. $1.2,1.1,1.1,1.1,1.08,1.1,1.08,1.1,1.11$, $1.12,1.08,1.08,1.12,1.16,1.15,1.1,1.15$ $10.2,10.58,12.6,10.45,10.33,10.6,10.5$, $10.45,10.5,10.42,10.5,10.36,10.5$
45. $1.55,1.57,1.51,1.58,1.5,1.52,1.35,1.55$, 1.5, 1.51 1.57. 1.5
46. 

$9.18,9.35,9.15,9.06,9.0,9.0,9.75,9.1$, 9.15.9.11
11. 1.51. 2.0, 2.05, 1.95. 2.07. 1.95, 2.0, 1.8, 2.0, $2.0,2.05,2.0,1.98,1.85,2.0,2.0$
12. $8.0,7.85,7.9,8.0,7.9,7.82,7.95,7.85,7.96$, $7.88,7.92,8.02$

Figure 26. (Continued)

Resistor Circuit No. Time Intervals
13. $2.41,2.38,2.4,2.44,1.82,2.38,2.41,2.4$,
14. 6.9, 6.9.7.05, 6.9,7.0.7.01, 7.08, 6.98, 6.92,
$6.96,7.0,7.0$,
15. $2.85,2.81,2.9$
16. $\quad 6.14,6.1,6.1,6.4,6.05,6.05,5.95$
17. $3.41,3.45,3.41,3.4,3.4,3.45,3.45,3.4$, $3.45,3.48,3.5$
18. $5.3 .5 .3 .5 .24,5.35,5.5,5.48,5.21,5.35$,
19. $4.05,4.0,3.9,4.08,4.0,3.97,4.02,4.05$,
20. $4.71,4.6,4.72,4.62,4.65,4.65,4.55,4.62$,
$4.68,4.74,4.72$
21.
22.
$43,4.28,4.3$
$4.3,4.32,4.4,4.32$
$5.0,5.09,4.9,4.82,5.0,5.08,4.91,5.01,5.0$,
$5.0,5.0,5.1$
$\begin{array}{ll}\text { 23. } & 3.69,3.74,3.61,3.7,3.69,3.61,3.7 \\ 24 . & 5.7,5.72,5.85,5.7,5.71,5.7,5.75,5.75,\end{array}$
$5.65,5.7,5.7,5.7,5.7,5.8$
25. $3.17,3.25,3.2,3.15,3.12,3.1,3.18,3.04$,
$3.15,3.2,3.15,3.1$
26. $6.38,6.55,6.4,6.58,6.56,6.6,6.55,6.53$,
$6.38,6.31,6.53,6.5$
27. $\quad 2.65,2.61,2.58,2.62,2.65,2.62,2.54,2.6$,
28. $7.25,7.65,7.4,7.41,7.4,7.5,7.43,7.48$, $7.22,7.48$
29. 2.21, 2.15, 2.2, 2.11, 2.2, 2.17, 2.2. 2.2, 2.16, $2.15,2.2,2.18,2.18,2.21,2.2$
30. $8.67,8.41,8.35,8.35,8.45,8.4,8.4,8.55$, $8.45,8.22,8.5,8.5,8.55$
31. $1.7,1.7,1.75,1.7,1.77,1.7,1.72,1.74,1.69$, $1.75,1.65,1.75,1.65,1.68,1.74$
32. $9.58,9.52,9.83,9.55,9.4,9.78,9.6$
33. $1.3,1.28,1.3,1.37,1.3,1.3,1.37,1.35,1.3$, 1.28, 1.31
34. $11.3,11.4,11.42,11.45,11.4,11.31,11.32$, 11.45, 11.4
35. .91,.92,.90,.91, .95,.90,.91,.90,.92,.92, .95
36. $13.35,13.85,14.0,14.05,13.81,14.25,14.5$, 13.85
37. . 55,. 55,.60,.55,.50,. 55, . 55,. 40, . 52
38. $17.72,17.6,17.81,17.8,17.75,17.6,17.9$, $17.75,17.5,18.0,17.42,17.8$
39. .20,.20,.19..19, .19, .19, .18, .20, .20, . 20 40. $27.4,27.3,28.0,28.2,30.0,27.4,27.4,27.8$, $28.08,27.85,27.02,28.4,27.5,28.35$

## Arrival Channel <br> Section Down

1. .19, .18, .20, .20, .20, .17, .18, .18, .20, .19,
2. $22.0,22.05,22.52,22.0,21.91,21.91,21.95$,
$21.95,21.95,22.05,22.08,21.91,22.2,22.42$
3. . $40, .40, .40, .38, .42, .42, .41, .41, .40$,
4. $\quad: 38.95, .416 .25,16.12,16.0,16.2,16.12,16.12$, $16.16,16.0,16.0,16.35$
 $12.75,13.0,12.95$
5. 1.09. 1.07, 1.1. 1.05, 1.07, 1.1, 1.1, 1.1, 1.1, 1.1, 1.06
6. $10.8,10.7,10.95,10.78,10.92,10.81,11.0$,
7. $1.85,1.5,1.51,1.5,1.45,1.5,1.45,1.5,1.42$
8. $9.4,9.7,8.8,9.2,9.5,9.2,9.21,9.3,9.36$, $8.45,9.45,9.3$
9. $1.98,1.95,1.95,191,1.91,1.97,2.0,2.0,1.95$,
1.97, 2.0 $8.07,8.0,8.1,8.0,8.1,8.07,8.0,8.06,8.15$,
10. $\quad 8.07,8.0,8.1,8.0,8.1,8.07,8.0,8.06,8.15$,
$8.0,7.95,8.0,8.0,8.02,8.05,8.1,7.9,8.01$, 8.1
11. $2.31,2.42,2.31,2.4,2.37,2.35,2.38,2.4$, $2.35,2.32,2.31,2.38$
12. $7.1,6.9,7.1,7.1,6.81,7.01,6.95,7.1,6.9$, $7.05,7.05,7.05,6.9$
13. $2.85,2.9,2.78,2.82,2.9,2.82,2.8,2.8,2.85$, 2.85, 2.85
14. $5.99,5.92,6.08,6.05,5.97,6.05,6.05,6.05$
15. $\quad 3.4,3.4,3.51,3.37,3.35,3.35,3.38,3.37$
16. $5.3,5.35,5.2,5.2,5.2,5.22,5.32,5.29,5.21$,
17. $\quad \begin{aligned} & 3.95,3.85,3.9,4.05,4.05,3.9,3.9,4.0,3.92, \\ & 3.97,4.0,3.92,4.0\end{aligned}$

Figure 26. (Continued)

Time Intervals

| 20. | $\begin{aligned} & 4.6,4.65,4.6,4.68,4.68,4.5,4.58,4.6,4.57, \\ & 4.62,4.51,4.55,4.58,4.62,4.6,3.98,4.82 \end{aligned}$ |
| :---: | :---: |
| 21. | $4.21,4.3 .4 .3,4.3,4.2,4.3,4.2,4.32$ |
| 22. | $4.95,4.92,4.9,4.5,4.95,4.95,5.0,4.96$ |
| 23. | $3.5,3.65,3.69,3.7,3.7,3.62$ |
| 24. | $\begin{aligned} & 5.78,5.69,5.68,5.62,5.65,5.75,5.7,5.72, \\ & 5.85 \end{aligned}$ |
| 25. | $3.1,3.1,3.1,3.1,3.1,3.12,3.1$ |
| 26. | $\begin{aligned} & 6.5,6.5,6.55,6.6,6.5,6.58,6.68,6.53,6.6, \\ & 6.55,6.32,6.55 \end{aligned}$ |
| 27. | $\begin{aligned} & 2.6,2.6,2.57,2.54,2.55,2.58,2.58,2.46, \\ & 2.6,2.51 \end{aligned}$ |
| 28. | $7.5,7.48,7.5,7.55,7.25,7.6,7.45,7.55,7.4,$ |
| 29. | $\begin{aligned} & 2.2,2.17,2.2,2.14,2.13,2.12,2.2,2.35,2.28, \\ & 2.15,2.13,2.18,2.34 \end{aligned}$ |
| 30. | $\begin{aligned} & 8.57,8.4,8.45,8.72,8.43,8.51,8.51,8.6, \\ & 8.45,8.6,8.45,8.51 \end{aligned}$ |
| 31. | $1.71,1.7,1.65,1.64,1.8,1.95,1.72,1.7,1.65$ |
| 32. | $\begin{aligned} & 10.1,9.64,10.2,10.15,10.0,10.02,10.0, \\ & 10.19,10.05 .10 .2,10.15 \end{aligned}$ |
| 33. | $1.21,1.3,1.26,1.3,1.3,1.26,1.28,1.28,1.31$ |
| 34. | $\begin{aligned} & 11.9,11.7,12.0,11.89,11.9,11.6,11.85,11.05, \\ & 11.91 \end{aligned}$ |
| 35. | $\begin{aligned} & 13.9, .90, .89, .92, .90, .90, .98, .95, .95, \\ & .92, .91, .95, .90 \end{aligned}$ |
| 36. | $\begin{aligned} & 14.0,14.47,14.05,14.5,14.35,14.48,14.6, \\ & 14.68,14.25 \end{aligned}$ |
| 37. | $\begin{aligned} & .55, .60, .55, .56, .60, .55, .58, .60, .60, .58, \\ & .60, .60 \end{aligned}$ |
| 38. | $\begin{aligned} & 18.6,18.1,18.45,18.3,18.3,18.45,19.0,18.05, \\ & 18.6,18.57,18.46,18.31,18.4,18.7,18.4,18.4, \end{aligned}$ |
| 39. | $18.78,18.8,8.61, \ldots$ |
| 40. | $\begin{aligned} & 28.71,28.95,28.9,29.3,28.65,28.52,29.0, \\ & 29.85,28.79,28.57,29.1,28.72,28.81,28.95, \\ & 29.6 \end{aligned}$ |

Figure 26. (Continued)

## Time Interval

0. $\quad 5.4,5.7,1.2,1.1,2.3,11.1,1.8,5.65,1.25$,
$6.9,16.7,4.9, .40,4.2,5.3,4.8,6.6,2.7,6.5$,
$19.2, .40,1.2, .80,3.4,3.1,2.1,2.85,2.1$, $.50, .10,9.6,2.2, .20 .0,14.7,16.5,2.1,23.0$, $3.1,13.0, .80 .3 .5,2.1, .30,6.7,9.9,2.8, .90$, $13.6,9.1,5.1,1.4,4.9,4.4,8.9,5.4,6.3,1.3$, $.40,4.5,2.3,2.5,10.5,3.9,3.85,17.0,12.6$, $5.0,2.3,10.0, .60,11.6,1.4,3.1,2.3, .40$, $2.4, .60,5.5,7.3,1.7,2.15,22.2,3.6,1.2$, $19.8,4.3 .5 .9,11.2,6.6,5.5,1.9,7.4,6.7$, 8.9. 3.9, 11.9. 25.2, 1.2, . 30, 13.9, 5.4. 9.4, $4.6,8.0,5.4,2.1,3.0,1.7, .70,3.0,14.3,8.0$, 5.3. .20, 9.9. .8. 20.7, 2.2, 1.8, 3.7, 1.0, 1.7, $.10, .40,2.9,20.3,6.3,6.4,8.2,18.3,26.7$, $11.1,3.3,3.1,3.9,8.2,6.9, .70,3.1, .20,5.2$, $8.6,3.9,20.7,4.7,10.2,3.7, .20,7.1,4.4$, $13.0,1.3,5.5,3.9,20.6,9.2,11.2,26.4,4.0$, $6.0,11.8, .10,1.2, .40,11.2,4.8,2.8,18.5$, $2.7,2.3,1.6,14.9,11.2,1.0,14.5,20.3,3.7$. $7.8, .70,4.6, .80,6.8,5.3,16.9,2.7,4.1, .60$, $7.7,6.8,20.7, .30,2.2, .70,18.4,1.6,14.7$, $6.3,1.4,2.9, .20, .20,8.8,2.1,5.4,9 ., 1.5$, $3.8,4.8,2.2,17.7,28.5,10.6,7.4$
1。
1.1, 1.4, 3.1, 1.7, 1.15, 3.15, 4.4, 2.1, 9.4,
$1.5, .20,7.25,1.85,4.95,1.7, .40,4.5,1.9$, $.90 . .90,1.0,2.3 . .50,2.2,1.95, .20,5.7,1.3$, $.70,3.4,6.9,8.1,3.1,1.1,2.25,4.1,1.5,2.0$, $1.6,3.1,1.0,1.1,1.9,4.0, .40,8.4,4.3,5.35$, 5.6, 1.95, 1.3. . $40,1.2, .20,1.0,2.1,1.7,1.75$, $.70, .80,3.4,1.7,1.6, .20,1.4, .10, .60,2.5$, $.30,3.9,1.1,2.6, .60, .30, .50,5.2,3.40,13.4$,
$.68,1.1,1.6,9.7,1.7,4.3, .45,1.25,2.3,3.8$, $1.2,2.3, .90,3.1,1.9,1.2,1.8,4.3, .70, .80$, $5.7,2.5,5.4,3.1,2.0,5.8, .20, .40,1.4,1.3$, $.90,4.85,5.8,1.85,1.0,3.25, .20,3.7,2.4$, $2.1,6.4, .50, .80,1.6, .60, .70,2.2, .70,2.2$, $3.35,5.8,1.9,1.65,3.6,3.75,1.1, .50,2.7$, $4.4,1.2,3.7, .20,1.1,1.2,6.9, .70,8.0,4.6$, $2.2,5.8,17.3, .80, .90,3.6,3.5,1.7, .80$, $5.3,5.4,11.7,1.5,2.1,1.4, .20,3.2,1.8,4.0$, $1.4,2.9,1.5,2.0,3.8,26.8,1.4,15.8,1.7$,


Figure 27. Queue Length Data



Figure 27. (Continued)
5. 2.2, 1.8, .20, 7.0, 6.5, 1.5, 4.75,.70, 2.3. .70, $6.5,1.5,4.75, .70,2.3, .70,3.1, .60, .15,1.4$,
$1.5,1.9,1.4,1.1,2.2, .40, .70, .60,2.9, .60$, $1.0,1.7,040,10.5,1.7,5.0,1.0,1.1,1.3,3.6$, $1.5,1.1,3.7, .30, .30, .80,2.1,3.6, .20, .20$, $11.2,1.5,4.3,1.4, .90,1.2, .30, .50,4.1, .60$, 1.1, 2.4, 11.8, 2.5, .40, 5.7, 1.0, 3.8, . 50, 3.2, $3.7,2.9,3.7,2.6, .10,4.1,1.2,2.0, .60,1.2$, 2.7, 2.1, 4.2, 1.3, 1.1. 7.4, 1.8, 10.4, .60, 10.4, $.60,10.4, .80,3.0,2.2,2.1, .20, .30, .80,4.4$, $6.9,1.3,5.5,10.5,2.9,1.6,1.0,5.3,1.6,2.6$
6. $1.7,2.4,6.9,6.25,2.25,1.0,5.3,2.9,10.8$,
 $3.8,2.0,9.5, .75,8.1, .40,7.7, .20,3.0,4.7$,
1.7, 2.95, 1.7, 1.1, 2.9. 2.45, 3.9, 4.3, 1.8,
$.90,2.0,1.7,1.8,13.8,6.1,6.7,1.2, .80, .50$,
$.70,3.1,3.1,1.2,3.0, .80, .90, .30,3.5,1.0$,
$1.4,10.3, .20,1.0, .90, .90,2.6, .10, .20,2.7$,
$.60,5.2,12.7, .80,11.0, .60,1.4,3.5, .30$,
.20, $20,30,5,1.0,12.0, .60,15.9,1.1,3.8$,
8.6, 3.1
7. $9.4,3.4,1.7,6.1,5.9,5.3,1.75,4.65, .50, .40$,
$.40, .70,04,1.1,3.5,6.7,1.4,12.3,7.5,3.5$,
$3.7,7.9,2.1, .20,1.4, .50, .80,5.7, .50,1.1$,
$3.3,5.4,5.2,3.6,2.6,3.8, .20,2.4, .40,2.6$,
$2.4, ~ 20, ~ .60, ~ 1.8, ~ 2.4, ~ 1.3, ~ 1.0, ~ 4.3, ~ . ~ 30, ~ 7.5, ~$
$8.6,2.8, ~ 2.4, ~ 11.8, ~ 4.7, ~ 2.2, ~ 1.0, ~ 1.9, ~ 2.4, ~ 5.1, ~$
2.2, 2.0, 3.8, 6.2, 2.3. 7.8, 1.6, 2.0, .70, . 50
8. $\quad 6.1,2.0, .90,060,4.1, .60,1.5,2.9,2.6,5.9$, $.20,2.65,2.1,1.4,1.2,2.5,3.15,1.6,4.5$,
1.8, 3.0, .90, 2.0, 1.6, 6.1. $4.7,2.4,1.0, .30$,
$.20,3.0,2.8, .30,4.4,1.8,8.4$
9. $1.35,1.7,4.5, .60, .60,2.2,1.2, .20,5.8, .20$, .30, 2.6, .80, 5.9, 1.6, 8.5,.60, 2.7,.20, 3.4, 3.4. 3.1. 4.1
10. $1.6, .80, .20,6.4,7.9,2.7,4.4,1.2,3.5,2.6$, .20
11. $1.3,1.5,2.4,1.1, .90, .30, .50,4.6$
12. . 40, .70, 4.3, 8.1, 1.6, .80, 1.2,. . 20
13. $3.2,5.4,2.65, .80, .80,7.75,3.7, .80,1.2$
14. $10.9,1.6, .90,3.9,1.4,2.0,2.0,6.7,6.1$, $2.4,1.8,3.5, .30$
15. 2.9,7.0, 2.7.2.5.3.2, 1.1. . $40, .90,2.2,1.6$, 10.4

Figure 27. (Continued)


| Resistor Circuit No. | Up SectionDate |  |  | val s Tota | Down Section |  |  | Total <br> Down <br> Section | Total Arrival Section |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Tote |  |  |  |  |  |
|  | 9-2 | 9-10 | 9-25 |  | 9-2 | 9-10 | 9-25 |  |  |
| 1. | 3 | 12 | 11 | 26 | 5 | 9 | 11 | 25 | 51 |
| 2. | 1 | 11 | 10 | 22 | 4 | 8 | 10 | 22 | 44 |
| 3. | 4 | 13 | 12 | 29 | 4 | 17 | 11 | 32 | 61 |
| 4. | 6 | 9 | 8 | 22 | 1 | 12 | 12 | 25 | 47 |
| 5. | 5 | 11 | 10 | 26 | 7 | 3 | 9 | 19 | 45 |
| 6. | 5 | 10 | 11 | 24 | 3 | 7 | 13 | 23 | 47 |
| 7. | 5 | 14 | 17 | 36 | 9 | 10 | 11 | 30 | 66 |
| 8. | 5 | 14 | 9 | 24. | 2 | 14 | 9 | 25 | 49 |
| 9. | 14 | 18 | 12 | 4.4 | 6 | 11 | 9 | 26 | 68 |
| 10. | 9 | - 9 | 15 | 33 | 2 | 12 | 9 | 23 | 56 |
| 11. | 5 | 14 | 15 | 34 | 3 | 14 | 12 | 29 | 63 |
| 12. | 5 | 7 | 15 | 26 | 4 | 17 | 13 | 34 | 50 |
| 13. | 5 | 10 | 11 | 26 | 6 | 7 | 12 | 25 | 51 |
| 14. | 3 | 9 | 11 | 23 | 4 | 12 | 10 | 24 | 47 |
| 15. | 1 | 12 | 3 | 16 | 3 | 9 | 11 | 23 | 39 |
| 16. | 6 | 12 | 12 | 28 | 3 | 12 | 7 | 22 | 50 |
| 17. | 5 | 13 | 11 | 29 | 2 | 10 | 8 | 20 | 49 |
| 18. | 1 | 11 | 7 | 19 | 8 | 8 | 7 | 23 | 42 |
| 19. | 6 | 13 | 14 | 33 | 8 | 11 | 13 | 32 | 65 |
| 20. | 5 | 11 | 13 | 29 | 4 | 11 | 8 | 23 | 52 |
| 21. | 8 | 7 | 11 | 26 | 2 | 13 | 17 | 32 | 58 |
| 22. | 3 | 14 | 12 | 29 | 5 | 13 | 8 | 26 | 55 |
| 23. | 3 | 12 | 11 | 26 | 3 | 9 | 12 | 24 | 50 |
| 24. | 11 | 12 | 14 | 37 | 4 | 6 | 9 | 19 | 56 |
| 25. | 6 | 7 | 7 | 20 | 8 | 10 | 8 | 26 | 46 |
| 26. | 3 | 11 | 12 | 26 | 3 | 12 | 21 | 36 | 62 |

Figure 28. Compiled Data - Number of Occurrences of Resistor Circuits


| Resistor |  | Secti |  | Total |  | Down Sec |  | Total | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 9-2 | 9-10 | 9-25 |  | 9-2 | 9-10 | 9-25 |  |  |
| 7. | 3 | 14 | 8 | 25 | 6 | 9 | 10 | 25 | 50 |
| 8. | 6 | 12 | 6 | 24 | 7 | 16 | 8 | 32 | 56 |
| 9. | 6 | 12 | 14 | 32 | 3 | 6 | 7 | 16 | 48 |
| 10. | 5 | 7 | 9 | 21 | 3 | 13 | 12 | 28 | 49 |
| 11. | 5 | 14 | 20 | 39 | 2 | 12 | 11 | 25 | 64 |
| 12. | 7 | 4 | 14 | 25 | 3 | 16 | 13 | 32 | 57 |
| 13. | 2 | 12 | 12 | 26 | 6 | 10 | 16 | 32 | 58 |
| 14. | 4 | 6 | 16 | 26 | 3 | 14 | 11 | 28 | 54 |
| 15. | 6 | 8 | 15 | 29 | 5 | 7 | 6 | 18 | 47 |
| 16. | 6 | 16 | - 9 | 31 | 2 | 14 | 13 | 29 | 60 |
| 17. | 4 | 13 | 7 | 24 | 3 | . 4 | 10 | 17 | 41 |
| 18. | 2 | 12 | 11 | 25 | 3 | 9 | 8 | 20 | 45 |
| 19. | 5 | 8 | 16 | 29 | 3 | 10 | 17 | 30 | 59 |
| 20. | 7 | 13 | 9 | 29 | 8 | 10 | 17 | 35 | 64 |
| 21. | 3 | 12 | 10 | 25 | 2 | 11 | 12 | 25 | 50 |
| 22. | 3 | 9 | 9 | 21 | 9 | 15 | 7 | 31 | 52 |
| 23. | 9 | 13 | 11 | 33 | 3 | 11 | 11 | 25 | 58 |
| 24. | 3 | 10 | 12 | 25 | 4 | 8 | 14 | 26 | 51 |
| 25. | 7 | 14 | 11 | 32 | 2 | 10 | 12 | 24 | 56 |
| 26. | 5 | 9 | 13 | 27 | 3 | 8 | 11 | 22 | 49 |
| 27. | 3 | 11 | 17 | 31 | 4 | 14 | 14 | 32 | 63 |
| 28. | 6 | 13 | 7 | 26 | 3 | 12 | 11 | 26 | 52 |
| 29. | 6 | 11 | 11 | 26 | 4 | 10 | 13 | 27 | 55 |
| 30. | 1 | 14 | 12 | 27 | 5 |  | 18 | 23 | 60 |
| 31. | 8 | 12 | 15 | 35 | 7 | 14 | 8 | 29 | 64 |
| 32. | 4 | 8 | 7 | 19 | 8 | 12 | 4 | 24 | 43 |



# APPENDIX C CUMULATIVE DISTRIBUTION VALUES 

| $F_{(t)}$ |  | Exponential Times (sec)Service Dist. Arrival Dist. |  |  |  | Resistor Circuit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Class | Class | Class | Class | Class | Class |  |
| Limit | Mid-Point | Limit | Mid-Point | Limit | Mid-Point | Number |
| . 0000 |  | 0 |  | 0 |  |  |
|  | . 0250 |  | . 253 |  | . 337 | $1,(2=\infty)$ |
| . 0500 |  | . 513 |  | . 684 |  |  |
|  | . 0625 |  | . 645 |  | . 860 | 3 |
| . 0750 |  | . 780 |  | 1.040 |  |  |
|  | . 0875 |  | . 915 |  | 1.220 | 4 |
| . 1000 |  | 1.045 |  | 1.393 |  |  |
|  | . 1125 |  | 1.194 |  | 1.592 | 5 |
| . 1250 |  | 1.335 |  | 1.780 |  |  |
|  | .1375 |  | 1.479 |  | 1.972 | 6 |
| . 1500 |  | 1.625 |  | 2.167 |  |  |
|  | . 1625 |  | 1.773 |  | 2.364 | 7 |
| . 1750 |  | 1.924 |  | 2.565 |  | 8 |
| . 2000 | . 1825 | 2.231 | 2.076 | 2.975 | 2.768 | 8 |
|  | . 2125 |  | 2.389 |  | 3.185 | 9 |
| . 2250 |  | 2.549 |  | 3.399 |  |  |
|  | .2375 |  | 2.712 |  | 3.616 | 10 |
| . 2500 |  | 2.877 |  | 3.836 |  |  |
|  | . 2625 |  | 3.113 |  | 4.151 | 11 |
| . 2750 | . 2875 | 3.216 | 3.390 | 4.288 | 4.520 | 12 |
| . 3000 |  | 3.567 |  | 4.756 |  |  |
|  | . 3125 |  | 3.747 |  | 4.996 | 13 |
| - 3250 |  | 3.930 |  | 5.240 |  |  |
| . 3500 | . 3375 | 4.3808 | 4.117 | 744 | 5.489 | 14 |
|  | . 3625 |  | 4.502 |  | 6.003 | 15 |
| . 3750 |  | 4.700 |  | 6.267 |  |  |
|  | . 3875 |  | 4.902 |  | 6.536 | 16 |
| . 4000 |  | 5.108 |  | 6.811 |  |  |
|  | . 4125 |  | 5.319 |  | 7.092 | 17 |
| . 4250 | . 4375 | 5.534 | 5.754 | 7.379 | 7.672 | 18 |
| . 4500 |  | 5.978 |  | 7.971 |  |  |
|  | . 4625 |  | 6.208 |  | 8.277 | 19 |
| . 4720 |  | 6.444 |  | 8.592 |  |  |
|  | . 4875 |  | 6.684 |  | 8.912 | 20 |
| . 5000 |  | 6.931 |  | 9.241 |  |  |
|  | . 5125 |  | 7.185 |  | 9.580 | 2.1 |
| . 5250 |  | 7.444 |  | 9.925 |  |  |
|  | . 5375 |  | 7.711 |  | 10.281 | 22 |
| . 5500 |  | 7.985 |  | 10.647 |  |  |
|  | . 5625 |  | 8.267 |  | 11.023 | 23 |

Figure 29. Cumulative Distribution Values

Exponential Times (sec) Service Dist. Arrival Dist.

| $F_{(t)}$ |  | $\mu_{s}=.10$ |  | $\mu_{\mathrm{a}}=.075$ |  | Resistor <br> Circuit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Class | Class | Class | Class | Class | Class |  |
| Limit | Mid-Point | Limıt | Mid-Point | Limit | Mid-Point | Number |
| . 5750 |  | 8.557 |  | 11.409 |  |  |
|  | . 5875 |  | 8.855 |  | 11.807 | 24 |
| . 6000 |  | 9.163 |  | 12.217 |  |  |
|  | . 6125 |  | 9.480 |  | 12.640 | 25 |
| . 6250 |  | 9.808 |  | 13.077 |  |  |
|  | . 6375 |  | 10.147 |  | 13.529 | 26 |
| . 6500 | . 6625 | 10.498 | 10.862 | 13.997 | 14.483 | 27 |
| . 6750 |  | 11.239 |  | 14.985 |  |  |
|  | . 6875 |  | 11.632 |  | 15.509 | 28 |
| . 7000 |  | 12.040 |  | 16.053 |  |  |
|  | . 7125 |  | 12.465 |  | 16.620 | 29 |
| . 7250 |  | 12.910 |  | 17.213 |  |  |
|  | - 7375 |  | 13.375 |  | 17.833 | 30 |
| - 7500 | . 7625 | 13.863 | 14.376 | 18.484 | 19.668 | 31 |
| . 7750 |  | 14.917 |  | 19.889 |  |  |
|  | . 7875 |  | 15.488 |  | 20.651 | 32 |
| . 8000 |  | 16.094 |  | 21.459 |  |  |
|  | . 8125 |  | 16.740 |  | 22.320 | 33 |
| . 8250 | 837 | 17.430 | 18.17 | 23.240 | 24.228 | 34 |
| . 8500 |  | 18.971 |  | 25.295 |  | 3 |
|  | . 8625 |  | 19.841 |  | 26.455 | 35 |
| . 8750 |  | 20.794 |  | 27.725 |  |  |
|  | . 8875 |  | 21.848 |  | 29.131 | 36 |
| . 9000 |  | 23.026 |  | 30.701 |  |  |
|  | . 9125 |  | 24.361 |  | 32.481 | 37 |
| . 9250 | . 9375 | 25.803 | 27.726 | 34.537 | 36.968 | 38 |
| . 9500 |  | 29.957 |  | 39.943 |  |  |
|  | . 9675 |  | 32.834 |  | 43.778 | 39 |
| . 9750 |  | 36.885 |  | 49.185 |  |  |
| 1.0000 | . 9875 | $\infty$ | 43.820 | $\infty$ | 58.427 | 40 |

Figure 29. (Continued)

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