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AN APPLICATION AND EVALUATION OF OPERATIONS RESEARCH TO WATER SUPPLY RESERVOIR DESIGN

CHAPTER I

INTRODUCTION

General

A problem that has recently occupied the interest of several eminent statisticians and engineers is the development of a mathematical model or other suitable algorithm for determining the design size of a water supply reservoir.

Men have been building reservoirs for a period in excess of four thousand years. Little is known of the criteria used in determining design capacity prior to the past one hundred years, but it is assumed that such criteria were no better than the criteria used since the turn of the last century. This procedure is a deterministic procedure based upon the period of lowest streamflow in the total record of streamflow available. Therefore, this procedure will not allow high percentage developments of mean flow. The procedure is called the Rippl or mass-curve procedure.

Need for Study

The increase in the per capita use of water and the large population increases within the past few years have brought the realization that we must effect higher percentage yields from streamflow for water supply purposes. The time is at hand when we must develop water resources to near maximum potential. Much has been written about the reuse of water. It would appear that near maximum potential development of available supplies would follow closely, if not precede, water reuse in priority. In addition, preferable reservoir sites are being used for low percentage yield projects. Therefore, future development of water resources will be inhibited by current developments from the point of potential reservoir sites.

The foregoing facts are the motivation for developing new and better criteria for reservoir design capacity. Several models have been presented as suitable for such purposes. Most of the mathematical work considers the storage function as a stochastic process as opposed to the deterministic approach used in the mass-curve procedure. Unfortunately, most of the theories advanced have not been applied to streamflow data. In some cases, the models have been applied to very simple discrete probability distributions. The extreme example of such a distribution is the trinomial distribution where the streamflow may assume only one of three values. Other examples that are frequently used are the Poisson distribution and the negative binomial distribution. Such distributions are not very realistic when applied to streamflow.

Some investigators have approached the problem with continuous probability distributions, assuming either normal or uniform distributions. Still others have recognized that streamflow generally follows the Pearson Type III or gamma distribution, but they developed the theory based upon the concept of an infinite dam. An infinite dam or reservoir is capable of storing any excess and supplying any deficit. Such a reservoir would indeed solve some problems.

The reason for using such assumptions as normal inflow and infinite capacity is that an exact solution for a dam of finite capacity with a gamma input is very complex. No criticism of those presentations where these simplifying assumptions are made is intended. Each contribution adds to our rather meager knowledge of storage systems and helps to understand the underlying processes. Therefore, each investigator has contributed to what is now known about storage systems with stochastic inputs and various types of outputs. There is a pressing need to evaluate the various models and underlying theories.

Purpose

The purpose of this study is to evaluate the models that have been proposed by applying them to observed streamflow data. Engineers need better methods for determining the size of a reservoir needed than those currently in use. It would be desirable that an algorithm could be solved on a desk calculator. Most government agencies and educational institutions have digital computers. Many consulting engineering firms do not have a computer and only limited access to one. This will probably change in the near future, but a model amenable to solution on a desk calculator would fill a very real need.

The emphasis in this study will be upon considerations for the design of a single reservoir, but references will be made to multiple reservoir design within a basin or total basin management where applicable. If a model can be shown to be applicable to one area, it may be applicable to other more general areas also with minor modifications.

CHAPTER II

LITERATURE SURVEY

An appreciable amount of work has been done recently by others to formulate criteria for the capacity design of water supply reservoirs. The interest in this area appears to be intensifying and doubtless more and better theories will be presented in the future. Many important concepts that impinge indirectly upon the various theories have been set forth. In the interest of clarity and the purpose pursued here, only those concepts that relate directly to the subject will be discussed.

The Rippl Method

The method of determining reservoir size that is most commonly used at present was proposed by W. Rippl (1) in 1883. Prior to 1883, the method of design was to assume a reasonable size for a supply reservoir and further assume that the reservoir was full at the beginning of the drought period. By simple addition of the estimated monthly inflow and subtraction of the estimated monthly withdrawals and losses, the calculations were made of the quantity in the reservoir at the end of each month for a period of a year.

If the calculation showed a deficiency, that is a negative quantity, the original assumed capacity was increased and the calculations repeated (1). This procedure suffered many faults that will not be discussed here as this is of historical interest only.

The Rippl procedure was far superior to the previous procedure both in the accuracy achieved and the labor necessary to determine a design capacity. The Rippl procedure consists of selecting the worst period of record from the total available record and then determining graphically the maximum deficiency that would result due to withdrawing a continuous quantity of water from a given stream. The amount of this maximum deficiency is obtained from the largest difference between a plot of cumulative streamflow versus time and cumulative withdrawal versus time. An example is shown in Figure 1. This procedure is illustrated in all texts on water supply design and is therefore used by the majority of consulting engineering firms and others engaged in water supply planning and development.

The Rippl procedure suffers the following deficiencies: 1. The analysis is based upon a particular sequence of events (streamflow) which may never occur again in the given order. 2. Almost nothing is known about the probability of failure in the design. 3. In years that streamflow is in excess of the worst period of record, a large portion of the streamflow is going unused and allowed

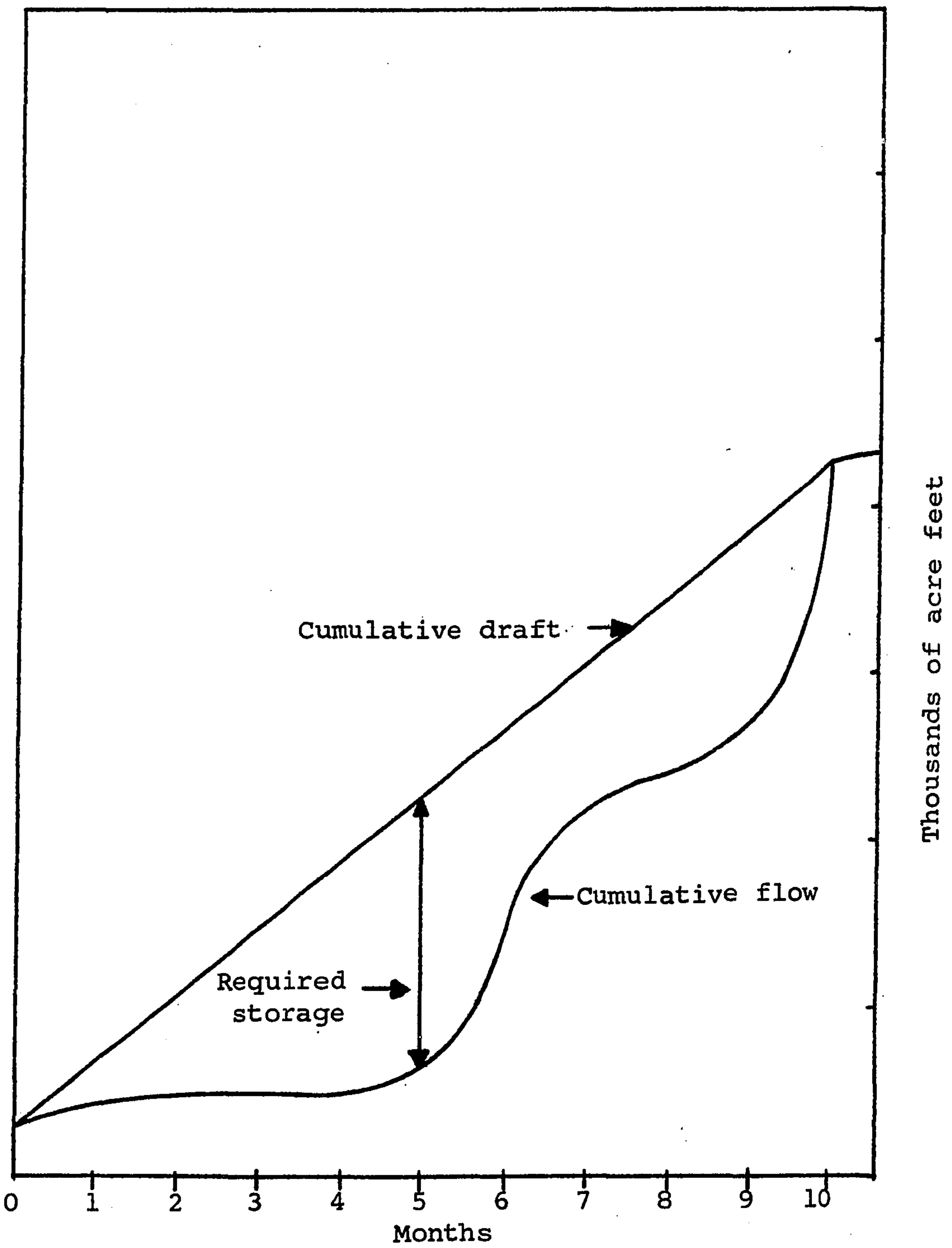

Figure 1. -- Mass diagram or Rippl procedure

to spill. This is a luxury we shall not always be able to afford. 4. The procedure was intended primarily for determining within year or seasonal storage, although it was suggested by Mr. Rippl that the consideration should not be confined to a year. The need for the determination of over-year storage requirements is evident for maximum or near maximum potential development. In some areas, it has been necessary to use a period of seven years to reevaluate the potential of reservoirs that were designed on the basis of one or two year droughts.

The Rippl procedure is still a valid procedure for the development of a small percentage, say 30 to 40 percent, of the potential of the stream. However, for a stream with large variations in flow, this procedure will not allow near maximum development.

Hazen's Method

The first attempt to overcome some of the deficiencies of the Rippl procedure was made by Allen Hazen in 1914 (2). Hazen attacked the problem of the random variability of streamflow by constructing a Rippl diagram for each year of record from fourteen streams and computing the storage that would have been required in that year to provide assumed continuous drafts varying from 30 to 90 percent of mean annual flow. These computations resulted in a frequency distribution of estimated storage requirements, one estimate for each year of record for a stated portion of mean annual streamflow. From this frequency distribution a probability distribution of storage requirements was determined. Hazen assumed that this probability distribution was normal and developed "normal storage curves." He then suggested that the 95 percent dry year would in most cases be adequate for design, and presented tables of generalized storage requirements based upon the coefficient of variation of the annual streamflow and the percentage of mean annual flow to be developed. Hazen's work was revised and updated in 1930 by his son (3). Hazen's table of values is shown graphically in Figure 2.

Hazen's procedure is not applicable where streamflow is regulated upstream by a reservoir. The procedure is not directly applicable to irrigation storage where the pattern of draft varies markedly. However, neither of these limitations seriously affect this study where the interest is municipal water supply. Few engineers working in the field of water supply development use Hazen's criteria. The reasons why Hazen's method is not used are not known. It appears that his work in this area is not well known. In addition, the method demands a larger reservoir capacity in most cases than does the Rippl procedure.

The assumption of the normality of the distribution of storage requirements would seem to cause storage volume requirements computed by this procedure to be low. However, Fiering (4) has shown that skewness of streamflow data is

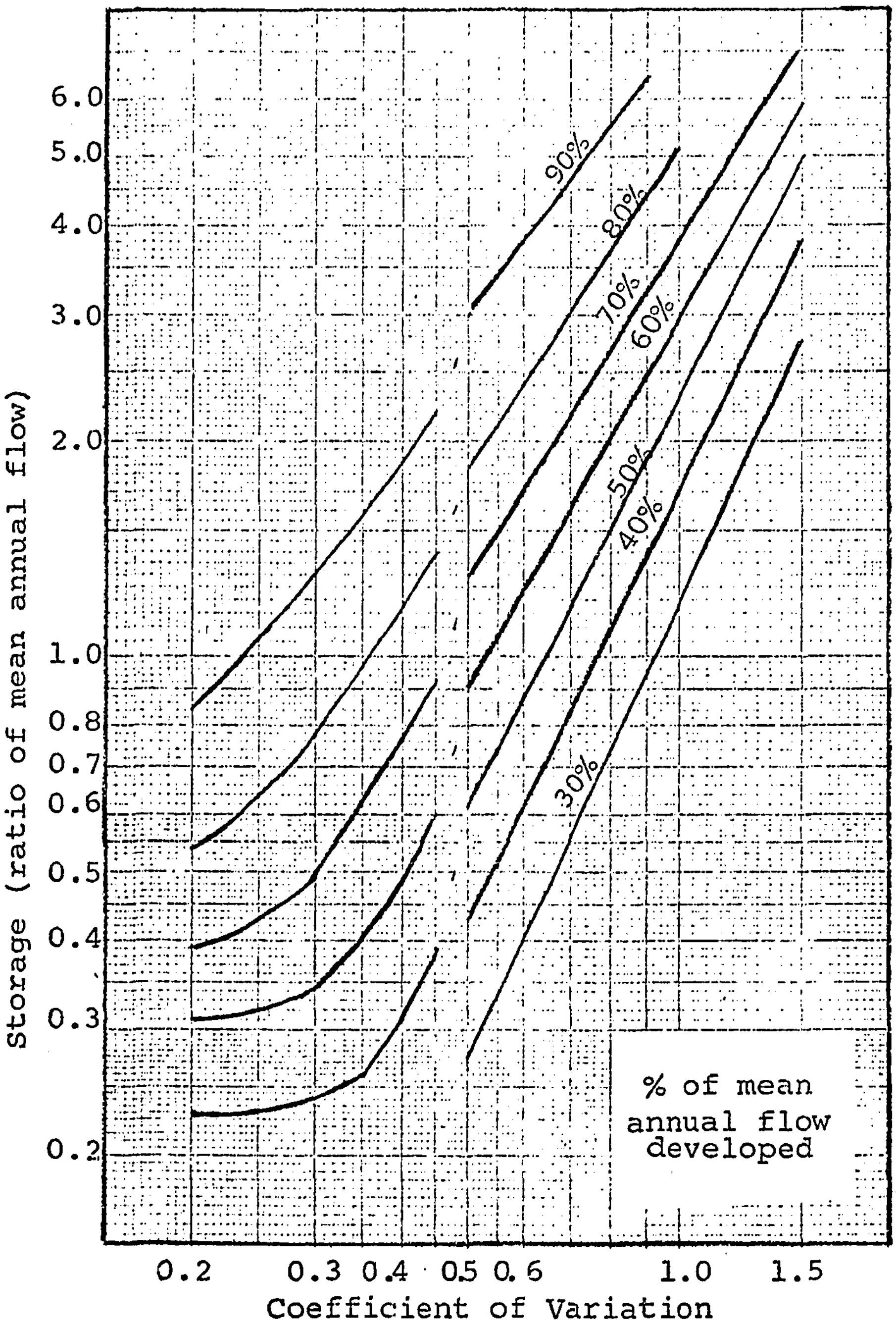

Figure 2. -- Hazen's generalized storage values

far less important than the coefficient of variation in determining the range of storage requirements. Hurst (5) reached an identical conclusion from studying several natural phenomena including the flow of the Nile river and the storage necessary to provide regulation thereof. Therefore, Hazen's criteria is acknowledged by many to provide a good first approximation to reservoir size although the streamflow data may not be normally distributed.

Moran's Model for a Dam

A probability theory of dams and storage systems was formulated by Moran in 1954 (6). The basic concept of the approach is that with a prescribed probability distribution for inflow and a prescribed release rule, an integral equation can be written for the amount of water in storage. This integral equation may then be approximated by a system of linear equations. The solution of these linear equations will provide the probability distribution of the contents of the dam. This probability distribution is the item of interest to engineers as it will reveal the probability of the dam being unable to deliver the desired draft.

Moran's original paper on this subject (6) was directed toward storage for irrigation water. First, it was assumed that water flowed in during the wet season and was stored until the dry season when it was released. Next, it was assumed the input was continuous and the release occurred once at a given time.

In a later paper (7) Moran modified the release rule to allow water to be released at shorter increments of time, for instance monthly releases instead of yearly, and presented a method of approximating a gamma distribution with a discrete distribution. Subsequently, (8, 9) it was shown that the original model could be used to approximate the situation where the input and release were both continuous, which is what occurs in a municipal water supply situation.

The model may be described as follows; Let X_t, the streamflow during time t, be independent or without serial correlation and be equal to 0,1,2,... with probabilities, $p_0, p_1, p_2, \ldots$ respectively, and let Z_t, the dam contents, be equal to 0,1,2,.....K at time t with probabilities $P_0, P_1, \ldots P_k$, and at time t+1 with probabilities P'_0, $P'_1, \ldots P'_k$, where K is the size of the reservoir. An amount of water M is taken from the reservoir and M, X_t, K, and Z_t are integral multiples of some unit.

After the reservoir has been in operation for a period of time, the probability distribution of dam contents, Z_t, will have achieved a stable distribution so that

$$P'_i = P_i \text{ for } i = 0, \ldots, K-M$$

From a recurrence relationship, Z_t will then be defined by the system of equations

$$P_0 = P_0\ (p_0+\ldots+p_m) + P_1(p_0+\ldots p_{m-1}) + \ldots P_M p_0$$

$$P_1 = P_0 p_{m+1} \qquad + P_1 p_m \qquad + \ldots P_M p_1 + P_{M+1} p_0$$

. . . .

$$P_{K-M} = P_0(p_k+\ldots) \quad + \ldots \qquad + P_{K-M}(p_m+\ldots)$$

and $P_{K-M+1} = \ldots\ldots = P_K = 0.$ ..... (2.1)

In this case $0 \leqq Z_t \leqq K\text{-}M$, and the distribution of Z_t can be found by solving the above equations. The equations can be solved in several ways. For example, write a matrix equation with the P_i on each side of the above equations as column vectors, say P_L and P_R, and let T be the matrix of coefficients on the right hand side of the equations. Then

$$P_L = TP_R.$$

Since P_R is a vector whose coefficients sum to unity, $T^n P_R$ will yield the required solution as n increases. This is most easily accomplished by successive squaring of T. T will probably need to be successively squared six or seven times to obtain the required solution (6). This is a fairly lengthy process, although not difficult.

The best method, especially for desk calculators, is to replace the final equation of (2.1) with $P_0 + P_1 + \ldots + P_{K-M} = 1$, which gives a set of non-homogenous equations, and then use the process of straightforward successive elimination of variables. This procedure should yield a solution in approximately two to three hours. One advantage of this method of solution is that solutions for smaller values of K are given by omitting one equation at a time from the system. For example, the solution for K one unit smaller than the original K is given by omitting the equation for P_{K-M-1}, the next to last equation, and setting P_{K-M-1} equal to zero in the other equations. Thus one can see how the distribution of Z_t varies with the size of the dam. This is precisely what the engineer needs to know.

Several works (10,11,12,13,14,15,16,17,18) dealing with the theory of dams and storage systems have been presented as a result of the interest created by Moran's work. It has been pointed out that the dam contents, Z_t, is a markov chain. The model has been viewed by various writers as either queuing theory, inventory theory, or a random walk. J. Gani (10) considered several storage problems including both finite and infinite dams. Kendall (11) considered only infinite dams with a gamma input distribution and developed a formula for the probability of non-exhaustion when the inflow is only slightly greater than the demand of the form

$$p \cong 1 - \exp \frac{(-2S(I-D))}{(C_v)^2 \, D^2} \tag{2.2}$$

where S is the initial storage in the reservoir, I is the annual inflow, D is the annual draft, C_v is the coefficient of variation of the input, and p is the probability of non-exhaustion. Kendall assigned little importance to the formula because the finite capacity of the dam had been ignored and Moran's model had assumed that there was no serial correlation between inflows.

Walter B. Langbein (12, 13) developed a procedure from Moran's model which he called "probability routing." This procedure uses a plot of inflow versus probability on arithmetic probability paper and a plot of discharge versus storage on arithmetic paper to obtain a graph of discharge versus probability. This is an excellent technique for evaluating the capability of a reservoir already constructed. For design purposes, it suffers the deficiency that the discharge-storage relationship must be assumed in advance. To elaborate on this point, two reservoir sites will have different depth-volume relationships. A particular reservoir site is unlikely to have a depth-volume curve that is linear. The spillways on most reservoirs are designed as Ogee weirs. The formula for flow over an Ogee weir is

$$Q = CLH^{3/2}$$

where Q is the flow, C is a constant, L is the length of the weir and H is the depth of water above the weir. Therefore, to use "probability routing" for design, one would need a storage-discharge curve for each estimate of size.

Fiering (14) proposed an algorithm using queuing theory and simulation. He assumed that the inflow distribution was a truncated normal distribution and that the inflow in any year was uniform throughout the year. This is not very realistic. However, his primary purpose was to introduce economics into the models of Moran and Langbein.

Phatarfod (15) applied methods in sequential analysis to a continuous time dam model based upon Moran's discrete time model. The main objective was to derive the probability of the time at which the dam becomes empty. The finite dam of capacity K was considered by Phatarfod to have inputs that were an additive process with stationary increments and a continuous release at a unit rate except when the dam was empty. Thus the dam process was considered a random walk with barriers at $Z = 0$ and $Z = K$.

Phatarfod developed the characteristic function of the time at which the dam becomes empty for the first time before overflowing, and then the characteristic function of the time at which the dam becomes empty for the first time regardless of overflow in the meantime. He developed these characteristic functions for inputs that corresponded to two discrete probability distributions, namely the Poisson and geometric. Prabhu (16) then applied Phatarfod's analysis to a continuous input when the input distribution is gamma to obtain the probability that the dam dries up before overflowing. The equation takes the form

$$p \cong \frac{e^{2u(1-a)} - e^{2K(1-a)}}{1 - e^{2K(1-a)}} \qquad (2.3)$$

where u is the initial content of the dam, K is the design capacity and a is the mean input from the distribution of the form

$$\frac{1}{a^t \Gamma(t)} X^{t-1} e^{-X/a} \qquad (2.4)$$

The approximation is due to the fact that the nonzero real root of the equation $e^{-w} = 1 - aw$ is $w_0 = 2(1-a)$ when a is close to unity. The error introduced is quite small when a is between 0.7 and 1.4.

Kirby (18) presented three markov chain storage models for discrete time and inflow conditions. The addition to Moran's model consisted of allowing the inflows to be serially correlated.

Sequent Peak Procedure

The sequent peak procedure is a deterministic analytical procedure proposed by Thomas and Fiering (19). The cumulative difference between inflow and draft is calculated for a given period of streamflow record. As the calculations progress, peaks (local maximum) and troughs (local minimum) will occur. The maximum difference between peaks and troughs is the minimum storage necessary to prevent a deficiency in draft. It is assumed that the streamflow record will cycle in T years and two cycles or 2T years of record is needed to make the analysis.

The advantages claimed for this procedure are that the necessity for determining a value of starting storage is removed and that each person computing storage requirements will obtain the same answer as all other persons doing the same computations. The latter point is trivial, but the first point could erase some of the uncertainty that now exists in deterministic procedures. The Rippl method assumes that the reservoir is full at the beginning of a drought. At times this is a rather unsound assumption. It is stated that the sequent peak procedure is equivalent to a linear programming solution for optimal overflow or waste pattern (20).

The sequent peak procedure is open to some of the same criticism that the Rippl method receives in that it is implied that the sequence of streamflow events will be repeated during the design life of the project or that a drought of greater magnitude is unlikely to occur. These assumptions appear inherent in any deterministic analytical technique.

These comments are not intended to imply that deterministic methods should be ignored or abandoned entirely. Perhaps the most desirable algorithm would involve a combination of deterministic methods and probability. This might not satisfy the theoreticians, but could be very practical for engineering application.

Kartvelishvili (21) severely criticized the purely statistical approach to describing river flow as a totally chance event and ignoring the factors which cause the flow.

He points out that some of the factors causing flow have a stochastic character and some a deterministic character. He proposes that the runoff process should be considered as a random process and that a full solution to the regulation of rivers by reservoirs can be obtained only on the level of the theory of random processes.

Objections to probability methods are answered by Kartvelishvili (21) as follows; 1. Probability theory should not be considered as compensation for insufficient information about hydrologic processes. Such a consideration would imply that the probability would increase or decrease with the development of the science, and would lead eventually to simple confidence in the authenticity or impossibility of the studied event. This would therefore negate the objective character of probability principles, exclude probability theory from the mathematical sciences, and assign it a role in psychology. 2. Demands for proof of the accidental nature of river flow are not logical because there does not exist one fact confirming the deterministic character of flow, nor does there exist one fact refuting the accidental character of the process. 3. Chance should not be equated with unsystematicness. The fact that regularities are observed in streamflow does not mean that probability theory is inapplicable in the study of streamflow and its regulation. Regularities observed in streamflow, which some writers think contradict probability theory, can be correctly reflected only by probability methods. Laws of accidental deterministic nature, which place limits on the amount of streamflow, should always be included in a study.

Some of the thoughts that Kartvelishvili expresses are sometimes called the all encompassing term "engineering judgement." For example, it is statistically possible to have a flow several times greater than any flow ever observed, but it is not very logical. Factors that he might have mentioned that are deterministic in nature are soil type, land slope, and total amount of rainfall expected. The occurrence of the rainfall is stochastic.

Mathematical Programming

Linear Programming

The application of linear programming to both deterministic and stochastic models for water-resources design is cited by Chow (22). He gives an example for determining the design capacity when the objective function is to maximize net benefits. This is a correct procedure for a given project, but it is particularly difficult to generalize in a study such as this as to cost and benefits when so many factors involved in costs and benefits depend upon conditions that could not be determined until a specific project has been planned. Therefore, this particular model cannot be compared with other models.

The model given by Chow is confined to a duration of one year and it was assumed that there was no carry-over from year to year. Thus, the stated model is useful for illustrative purposes only. Chow states that the actual situation for the design of reservoirs is much more complicated. Thomas and Watermeyer (23) used linear programming and dam theory to formulate what they termed a stochastic sequential approach to determine optimal reservoir capacity. The objective function here also was to maximize net benefits.

A linear programming application to sizing a reservoir when the objective function is to minimize the design capacity simplifies to repetitive solution of the continuity equation for storage. Thus, the solution is analogous to the sequent peak procedure mentioned earlier and is exactly the same as a mass curve analysis of the entire streamflow record.

Dynamic Programming

Dynamic programming deals with the theory of multistage decision processes. It is applicable to problems where the consideration of time is essential and the decision sequence is important. Chow (22) cites several examples of dynamic programming application to various hydroelectric projects. The major contribution of dynamic programming, that is the decision making, is absent to a large degree in municipal water supply situations, but is very much present in hydroelectric, irrigation, and flood control projects where a decision on the amount of release must be made. In municipal water supply, the draft is nearly constant and a decision on curtailment of draft hopefully infrequent.

Therefore, dynamic programming loses much of its effectiveness when confined to reservoir capacity design.

Storage Requirement Computations

A computation of the storage requirements for various levels of streamflow regulation in the 22 major regions of the contiguous United States was made by a select committee of the United States Senate (24). Löf and Hardison (25) determined that the storage requirements given in the report of that study for high sustained-use of flows were erroneously low in all of the regions. These low storage values were caused by using linear extrapolation from low percentage yields to high percentage yields whereas the function is not linear. Therefore, they presented storage values to supersede the values determined by the select committee.

The procedure used in the computation was to plot streamflow on log-probability paper and fit one of several frequency distribution curves to the observed flow. The curves chosen were normal, log-normal or Weibull. Storage requirements were then computed by probability routing of annual discharge (Langbein, 1958). They used an assumption of constant draft rate, uniform flow within each year, and independence between years. It was further assumed that at a hypothetical delivery rate of 100 percent of mean annual flow, a seasonal storage factor of 0.4 times the mean annual flow would have to be added to the carry-over storage. Thus the total storage (seasonal plus carry-over) was obtained by adding to the carry-over storage a seasonal adjustment.

The parts of this study that were of particular interest to the writer have to do with the lower Arkansas-White River region which encompasses the area studied by the writer. The parts are: The frequency curve chosen for this region was the Weibull. A coefficient of variation value of 0.45 was taken from a map prepared by the United States Geological Survey. The practical maximum percentage of mean annual flow that could be developed for this region was seventy-eight percent for 98 percent of the time and eighty-four percent for 95 percent of the time. The practical maximum flow was defined as net flow after evaporation and was obtained by selecting a point at which a slight increase in percentage yield required much higher storage capacities. The reservoir size and depth used in the computations were determined from averages given by the Corps of Engineers and the Bureau of Reclamation for reservoirs constructed from 1954 to 1960. Evaporation losses due to the reservoir were assumed to be the difference between evaporation from the reservoir and the natural evapotranspiration loss from the same ground area with its normal vegetation.

The results given in this study are not directly applicable to any specific project, nor were they intended to be. The objective was to determine generalized storage values and yields for a national overview.

CHAPTER III

STREAMFLOW DATA FOR THE STUDY

Data Selection

The input data for any study is important. The results obtained from any model, regardless of the degree of sophistication, can be no better than the data put in. Therefore, the data used herein were obtained from records of the United States Geological Survey that were described as "good." The streams chosen were selected without any intent of bias, although the criteria were that there should be no upstream regulation, drainage area should be less than 3,000 square miles, and the stream gaging station should be confined to Eastern Oklahoma or Western Arkansas.

Upstream regulation will cause man made correlation or persistence between flows. Drainage areas of less than 3,000 square miles comprise the majority of reservoirs built primarily for water supply purposes.

The streamflows were reduced to cubic feet per second per square mile (cfsm) by dividing the observed flows by the drainage area in square miles. The results of this computation (Table 1 for annual data and Table 2 for monthly data)

TABLE 1

ANNUAL STREAMFLOW DATA

Number of Years of record	Stream	$\bar{X}$	σ	Cv.	Adjusted skewness
28	Poteau River at Cauthron, Ark.	1.024	0.58	0.57	0.52
17	Lee Creek, Van Buren, Ark.	0.955	0.55	0.58	0.41
29	Fourche Maline, Red Oak, Okla.	0.973	0.59	0.61	0.63
31	Illinois River, Tahlequah, Ok.	0.863	0.47	0.54	0.25
19	Barren Fork, Eldon, Okla.	0.823	0.50	0.60	0.42
39	Buffalo River, Rush, Ark.	1.153	0.57	0.50	0.47
37	Little River, Horatio, Ark.	1.362	0.59	0.43	0.43
26	Ouachita River, Mt. Ida, Ark.	1.682	0.73	0.43	0.45
28	Strawberry River, Eve. Shade	0.889	0.48	0.54	0.47
28	Kings River, Berryville, Ark.	1.036	0.53	0.51	0.48

Notes:

$\bar{X}$ is the average annual mean daily flow in cubic feet per second per sq. mile.

σ is the standard deviation of the average annual flow, cfsm.

Cv. is the coefficient of variation of average annual flow.

Adjusted skewness is the adjusted skewness of annual flow = Pearson's definition of skewness $x(1 + \frac{8.5}{N})$.

TABLE 2

MONTHLY STREAMFLOW DATA

Number of Months of Record	Stream	$\bar{X}$	σ	Cv.	Adjusted Skewness
344	Poteau River, Cauthron, Ark.	1.05	1.59	1.51	1.33
204	Lee Creek, Van Buren, Ark.	0.95	1.41	1.49	1.33
360	Fourche Maline, Red Oak, Okla.	0.96	1.56	1.62	1.51
372	Illinois River, Tahlequah, Okla.	0.88	1.17	1.33	1.65
228	Barren Fork Creek, Eldon, Okla.	0.83	1.20	1.45	1.65
468	Buffalo River, Rush, Ark.	1.16	1.53	1.32	1.29
432	Little River, Horatio, Ark.	1.38	1.70	1.23	1.05
312	Ouachita River, Mt. Ida, Ark.	1.69	2.00	1.19	1.17
336	Strawberry River, Eve. Shade, Ark.	0.91	1.27	1.39	1.33
336	Kings River, Berryville, Ark.	1.02	1.43	1.40	1.42

Notes:

$\bar{X}$ is the mean monthly flow in cubic feet per second per square mile.

σ is the standard deviation of mean monthly flows, cfsm.

Cv. is the coefficient of variation of mean monthly flows.

Adjusted skewness is the adjusted skewness of mean monthly flows = Pearson's definition of skewness $x(1 + \frac{8.5}{N})$.

are interesting in that the means of the average annual flows and the variance of the average annual flows are indistinguishable at the 99 percent confidence level. That is, they derive from the same parent population. Therefore, the expected flows from an ungaged stream or stream section in this area could be approximated by the value of one cfsm for average annual flow with a variance of 0.32. This is significant because rarely would the best or most desirable site for a reservoir exist at the exact site of the gaging station. Consequently, an engineer could decrease or increase the expected stream discharge as he moves upstream or downstream by the change in number of square miles of drainage area contributing to discharge as a result of the move from the gage site to the reservoir site. This is significant in basin planning also. If a number of reservoirs are planned in the basin, the drainage area contributing to the first reservoir will yield one cfsm and the contribution to the second reservoir will be one cfsm on the drainage area below the first reservoir plus the release from the first reservoir, etc,.

The statistical tests used to test the means and variances for homogeneity are the Chi-square tests for homogeneity of means and Bartlett's test for homogeneity of variances.

Data Availability

The length of record of the ten streams studied varied from 19 to 39 years. Synthetic data could have been generated to provide more data. Much has been written recently about "operational hydrology" and simulation of streamflow. There seemed to be little point in this study of generating synthetic data that is statistically indistinguishable from the observed data. It may be argued that if the observed sequence is unlikely to ever occur again, then it is also unlikely that any one of a set of one thousand or more sequences generated would occur. It is claimed that the estimate of the range of the deviations in streamflow and hence the range in storage requirements can be improved by data generation. Yevdjevich (26) states, "It is claimed that the range reliability is improved (or the information is increased) by this method. It should also be noted that this claim is a point of controversy. --- Here is the essence of the controversy: Can a problem solving technique yield an increase in information? The data generation method as a technique for solving mathematical problems with stochastic variables may be compared with the numerical finite differences method for solving differential equations when both cannot be solved analytically. As the numerical finite differences method does not improve the information contained in ordinary or partial differential equations, except to produce their solutions, it may be expected that the same conclusion would be valid for the data generation method as currently used in solving stochastic problems. --- Any claim that the data generation method increases information should be subjected to a rigorous mathematical statistical analysis." Data generation has a proper role to play in the study of streamflow and streamflow regulation. The Rippl method and the sequent peak procedure can be easily applied to synthetic data. In fact, the sequent peak procedure will require either data generation or a repetition of the streamflow record, especially for situations where the streamflow record is short.

Few hydrologists would fail to concur that more data, i.e., longer streamflow records, would be desirable. The fact remains that we must begin any study with what is available to study. Streamflow records are limited in length as a consequence of history. The first official streamflow records in either Oklahoma or Arkansas were begun by the United States Weather Bureau on the White River at Newport, Arkansas in 1885.

Independency of Data

One of the first questions of interest about the observed streamflows was the degree of correlation existing between monthly flows. Linear correlation between monthly flows would affect the models and the manner of handling the data with the general result of increasing the storage capacity requirements.

Linear correlation coefficients were computed for all of the monthly data and in general there is no significant linear correlation between monthly flows. This is in contrast to reported correlation coefficients for monthly flows of from 0.17 to 0.3 (12, 26). Whether or not these reported correlation coefficients were subjected to statistical tests for significance has not been stated. Linear correlation between annual flows was not investigated in this study. However, a study by the Corps of Engineers (27) of forty-two streams throughout the country showed only two to have serial correlation between annual flows. It was further determined that the correlations existing in these two streams were due to man made influences. The terms serial correlation coefficient of lag-one, autocorrelation coefficient of lag-one, and linear correlation coefficient of lag-one are used in current literature to express the same concept. However, they could have distinctly different meanings, and do have different meanings when the lag is greater than one.

"F" tests of significance were made on all linear correlation coefficients derived from the data. The results of these computations are given in Table 3. Thirty-six correlation coefficients of 120 computed were significant at the 99 percent confidence level. Twenty-one of the thirty-six significant values occurred during periods when base flow predominates. Base flow would be expected to be correlated. The remaining fifteen correlation coefficients were generally

TABLE 3

MONTHLY CORRELATION CALCULATIONS

Poteau River at Cauthron

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01,\ 1,\ 27}$
Oct-Nov	0.163	0.74	7.72
Nov-Dec	0.608	15.88	
Dec-Jan	0.020	0.01	
Jan-Feb	0.345	3.65	
Feb-Mar	0.540	11.12	
Mar-Apr	0.073	0.14	
Apr-May	0.131	0.47	
May-June	0.401	5.20	
June-July	0.126	6.44	
July-Aug	0.786	43.64	
Aug-Sept	0.399	5.12	
Sept-Oct	-0.150	0.35	

Lee Creek at Van Buren

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01,\ 1,\ 15}$
Oct-Nov	0.199	0.62	8.68
Nov-Dec	0.531	5.91	
Dec-Jan	0.674	12.53	
Jan-Feb	0.287	1.34	
Feb-Mar	0.070	0.07	
Mar-Apr	0.300	1.48	
Apr-May	0.583	7.75	
May-June	0.669	12.14	
June-July	0.110	0.186	
July-Aug	0.730	17.16	
Aug-Sept	0.722	16.38	
Sept-Oct	-0.250	1.00	

TABLE 3.--Continued

Fourche Maline Near Red Oak

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 28}$
Oct-Nov	0.237	1.67	7.64
Nov-Dec	0.702	27.27	
Dec-Jan	0.046	0.05	
Jan-Feb	0.368	4.38	
Feb-Mar	0.402	5.40	
Mar-Apr	0.316	3.10	
Apr-May	0.073	0.15	
May-June	0.295	2.68	
June-July	-0.072	0.15	
July-Aug	0.413	5.76	
Aug-Sept	0.331	3.44	
Sept-Oct	0.103	0.31	

Illinois River at Tahlequah

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 29}$
Oct-Nov	0.621	18.25	7.60
Nov-Dec	0.754	38.25	
Dec-Jan	0.173	0.89	
Jan-Feb	0.250	1.94	
Feb-Mar	0.365	4.47	
Mar-Apr	0.580	14.76	
Apr-May	0.122	0.44	
May-June	0.4999	9.66	
June-July	0.207	1.29	
July-Aug	0.394	5.34	
Aug-Sept	0.360	4.32	
Sept-Oct	0.034	0.033	

TABLE 3.--Continued

Barren Fork at Eldon

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 17}$
Oct-Nov	0.836	39.57	8.40
Nov-Dec	0.717	18.01	
Dec-Jan	0.336	2.17	
Jan-Feb	0.508	5.93	
Feb-Mar	0.328	2.05	
Mar-Apr	0.121	0.25	
Apr-May	0.528	6.57	
May-June	0.619	10.55	
June-July	0.073	0.09	
July-Aug	0.828	37.14	
Aug-Sept	0.591	9.14	
Sept-Oct	-0.104	0.19	

Buffalo River Near Rush

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 37}$
Oct-Nov	0.374	6.02	7.38
Nov-Dec	0.543	15.52	
Dec-Jan	0.139	0.731	
Jan-Feb	0.415	7.73	
Feb-Mar	0.091	0.306	
Mar-Apr	0.614	22.50	
Apr-May	0.281	3.16	
May-June	0.481	7.83	
June-July	0.374	6.03	
July-Aug	0.367	5.76	
Aug-Sept	0.551	16.12	
Sept-Oct	-0.037	0.052	

TABLE 3.--Continued

Little River Near Horatio

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 34}$
Oct-Nov	-0.023	0.017	7.44
Nov-Dec	0.460	9.12	
Dec-Jan	0.255	2.37	
Jan-Feb	0.599	15.49	
Feb-Mar	0.183	1.19	
Mar-Apr	0.339	4.43	
Apr-May	0.175	1.08	
May-June	0.443	8.33	
June-July	0.198	1.38	
July-Aug	0.296	3.28	
Aug-Sept	0.811	65.35	
Sept-Oct	0.118	9.482	

Ouachita River Near Mt. Ida, Ark.

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 24}$
Oct-Nov	0.071	0.120	7.82
Nov-Dec	0.204	1.046	
Dec-Jan	0.261	1.760	
Jan-Feb	0.376	3.945	
Feb-Mar	0.523	9.017	
Mar-Apr	0.166	0.677	
Apr-May	0.038	0.036	
May-June	0.344	3.333	
June-July	0.059	0.084	
July-Aug	0.310	2.548	
Aug-Sept	0.874	77.752	
Sept-Oct	0.199	0.989	

TABLE 3.--Continued

Strawberry River Near Evening Shade

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 26}$
Oct-Nov	0.392	4.71	7.72
Nov-Dec	0.684	22.91	
Dec-Jan	0.256	1.82	
Jan-Feb	0.627	16.88	
Feb-Mar	0.269	2.03	
Mar-Apr	0.294	2.94	
Apr-May	0.234	1.51	
May-June	0.038	0.038	
June-July	0.216	1.28	
July-Aug	0.608	15.29	
Aug-Sept	0.133	0.465	
Sept-Oct	0.013	0.004	

Kings River Near Berryville

Month	Computed Correlation Coefficient	Calculated "F"-Value	Significant "F"-Value $F_{0.01, 1, 26}$
Oct-Nov	0.328	3.12	7.72
Nov-Dec	0.824	55.04	
Dec-Jan	0.070	0.128	
Jan-Feb	0.511	9.19	
Feb-Mar	0.292	2.43	
Mar-Apr	0.560	11.92	
Apr-May	0.300	2.58	
May-June	0.337	3.33	
June-July	0.223	1.37	
July-Aug	0.524	9.87	
Aug-Sept	0.703	25.43	
Sept-Oct	-0.142	0.539	

less than 0.5. This gives a coefficient of determination of 0.25 or less, which means that 75 per cent or more of the variation in the data is unexplained and only 25 per cent is explained by the influence of one months flow upon another. In the case of linear correlation coefficients of 0.2, the coefficient of determination is 0.04, and the coefficient of alienation is 0.96. Therefore, the streamflow in this study is uncorrelated.

Mean monthly flows and average annual mean daily flows in the area of study follow a gamma or Pearson Type III frequency distribution.

CHAPTER IV

APPLICATION OF THE ALGORITHMS

The algorithms discussed in Chapter II were applied to the data discussed in Chapter III. The order of application was the Rippl method, the Hazen method, Moran's model, and the sequent peak procedure, and the applications will be discussed in that order. Comparisons will be made for a draft of 80 percent of mean annual flow.

The Rippl Method

In applying the Rippl method to streamflow data, the first requirement is to determine the worst period of record. That is, the period of record that will demand the largest size reservoir to sustain a given draft. This worst period might be a short period with very low flows or no flow in some months, or it might be a period of long duration with reduced flows. It has already been pointed out that in many cases this method will not allow high percentage draft rates.

The worst period of record of approximately a year in length, although not necessarily confined to a year, for most of the streams studied occurred during some part of the water year 1963. That is, there were periods of reduced flow between June 1962 and October 1964. The consideration was to find the combination of continuous months during this period that gave the worst conditions. Then the cumulative flows or mass curve was plotted and the maximum draft obtainable, if less than 80 percent, was determined and plotted on the mass curve. The maximum ordinate between draft and the mass curve gives the required storage volume to sustain that said draft.

Hazen's Method

Hazen's method is very simple to apply once the computations have been made to determine the mean, the standard deviation, and the coefficient of variation of the annual mean streamflow data. Then Figure 2 is entered with the coefficient of variation as abcissa. Progression is then made upward to the percentage of mean annual flow to be developed. The ratio of storage to mean annual flow is then read from the ordinate. The storage volume obtained by this procedure was determined by Hazen to be adequate 95 percent of the time. Thus, one would expect that the storage would be inadequate on an average of one year in twenty.

Moran's Model

The variables to be inserted in Moran's Model are the reservoir size, K, the draft, M, and an approximation of the probability distribution of the streamflow volumes. Since K is the object of interest, an approximation of a reasonable size was made using Hazen's Method. The value so obtained was then rounded off to a multiple of the mean annual streamflow. Specific examples of these values will be given later. The most important variable to be determined was the probability distribution of the streamflow or the input to the reservoir.

Moran (7) approximated the gamma distribution,

$\frac{1}{B^{a+1}\,\Gamma(a+1)}\, e^{-x/B}\, x^{a}$ (4.1) by a discrete probability distribution where the probabilities were proportional to $(i + 1)^{t}B^{i}$ and suggested that such could be fitted by equating moments. However, this often gives fractional results which are not easily managed. Moran also gave a numerical example (6) where the input followed a gamma distribution and B and a chosen so that the distribution was a Chi-square with three degrees of freedom. The values were taken from Pearson's Table of the Incomplete Gamma function. This table may be found in most handbooks of mathematical functions.

There are other ways to approximate the streamflow distribution. One approach used by the writer was to take the average mean and average variance from the streamflow parameters and construct an average gamma distribution. The mean of a gamma distribution is given by $B(a + 1)$ and the variance is given by $B^2(a + 1)$ as a result of the first and second moments about the mean of a distribution. Furthermore, if a is an integer, it may be shown by successive integeration by parts that the cumulative distribution, $F(x)$, is given by

$$F(x) = 1 - [1 + \frac{x}{B} + \frac{1}{2}(\frac{x}{B})^2 + --- (\frac{x}{B})^a] \, e^{-x/B} \qquad (4.2)$$

when x is greater than zero and $F(x) = 0$ when x is less than or equal to zero. If a is not an integer the cumulative distribution, $F(x)$, must be evaluated by numerical methods (28), but in this case would be more easily determined from tables of the incomplete gamma function. The parameter a is a shape factor in this distribution and B is a scale factor. Thus, changing the value of a will change the shape and changing the value of B will only change the scale.

Since the average mean from the streamflow data was approximately unity and the average variance was approximately 0.33 we have: $B(a + 1) = 1.0$ and $B^2(a + 1) = 0.33$. When we divide the last by the first we obtain a equal to 2 and B equal to 0.33. When these values, $a = 2$ and $B = 0.33$, are inserted in equation (4.2), and x is varied in increments of 0.1 or 0.2, points are obtained on an average cumulative frequency distribution that approximates the cumulative gamma distribution of the streamflow data. The algebraic difference between these points on the cumulative frequency distribution comprises a frequency histogram that approximates the gamma distribution. The usual procedure in applied engineering statistics is to construct a continuous probability distribution from a histogram. The reverse procedure is used here of constructing a frequency histogram from a continuous frequency distribution. The points so obtained were used as probabilities in equations (2.1).

A much simpler technique is to plot a fitted Pearson Type III curve to the streamflow data and select points at even increments of streamflow volume from the fitted curve and use these values as the probabilities of streamflow in equations (2.1). The fitted curve may be constructed quite easily from a table of values of the area under the Pearson Type III curve such as the one shown graphically in Figure 3. Example curves are shown in Figure 4. It is not necessary to plot the streamflow data on the fitted curve unless it is desired to get a visual estimate of the goodness of fit of the fitted curve. This technique was applied to the streams under study using streamflow increments of 0.1 cfsm.

The Sequent Peak Procedure

Beginning with a streamflow record of length T at a proposed reservoir site and the desired draft for the periods comprising T, Feiring (20) gives the following eleven steps for applying the sequent peak procedure.

1. Calculate $X_i - D_i$, inflow minus draft, for all $i = 1, 2, \ldots, 2T$ and calculate the net cumulative inflow $\sum_{i=1}^{t} (X_i - D_i)$ for $t = 1, 2, \ldots, 2T$.
2. Locate the first peak (local maximum), P_1, in the cumulative net inflow.
3. Locate the "sequent" peak, P_2, which is the next peak of greater magnitude than the first.

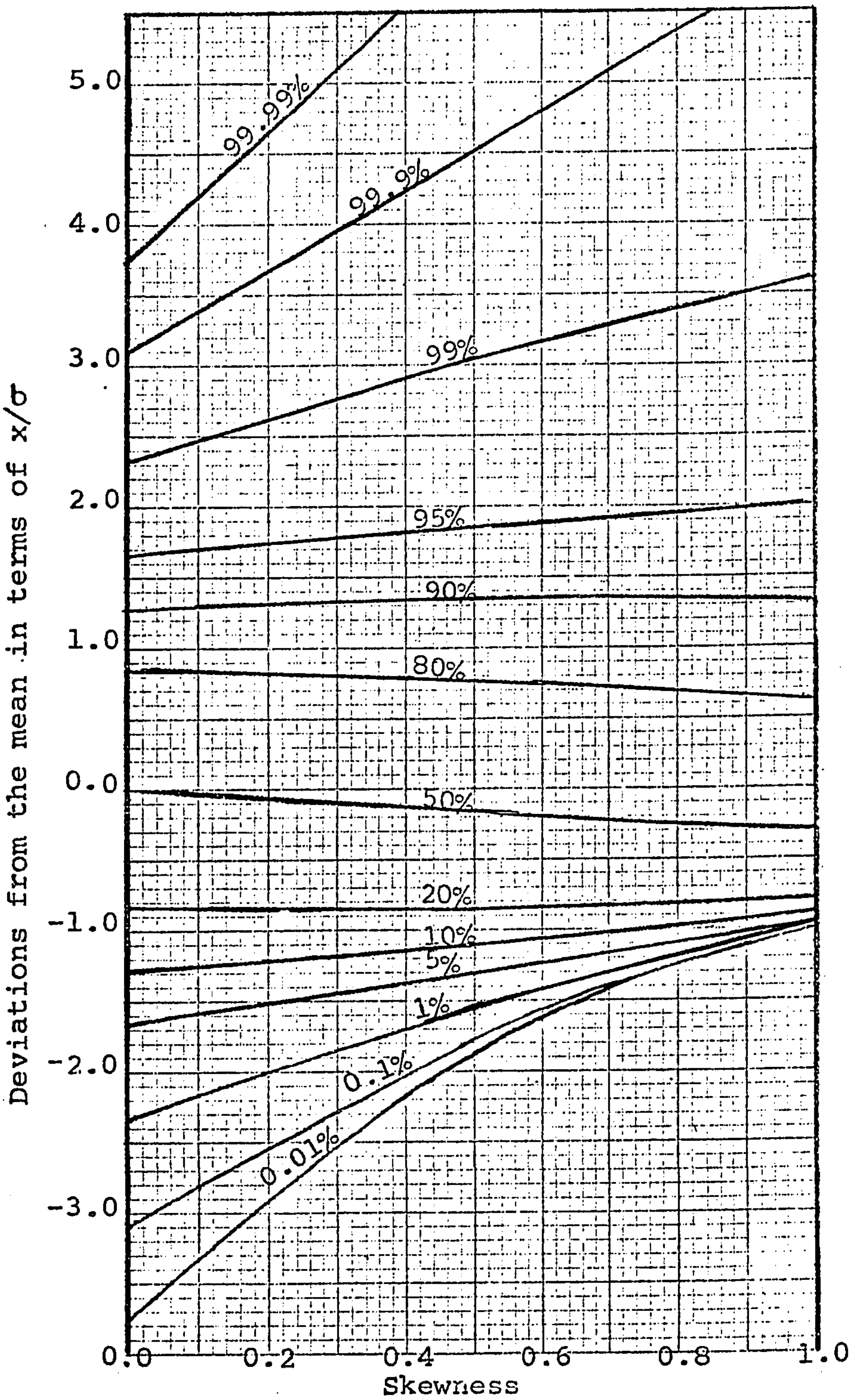

Figure 3. -- Areas of Pearson's Type III curves

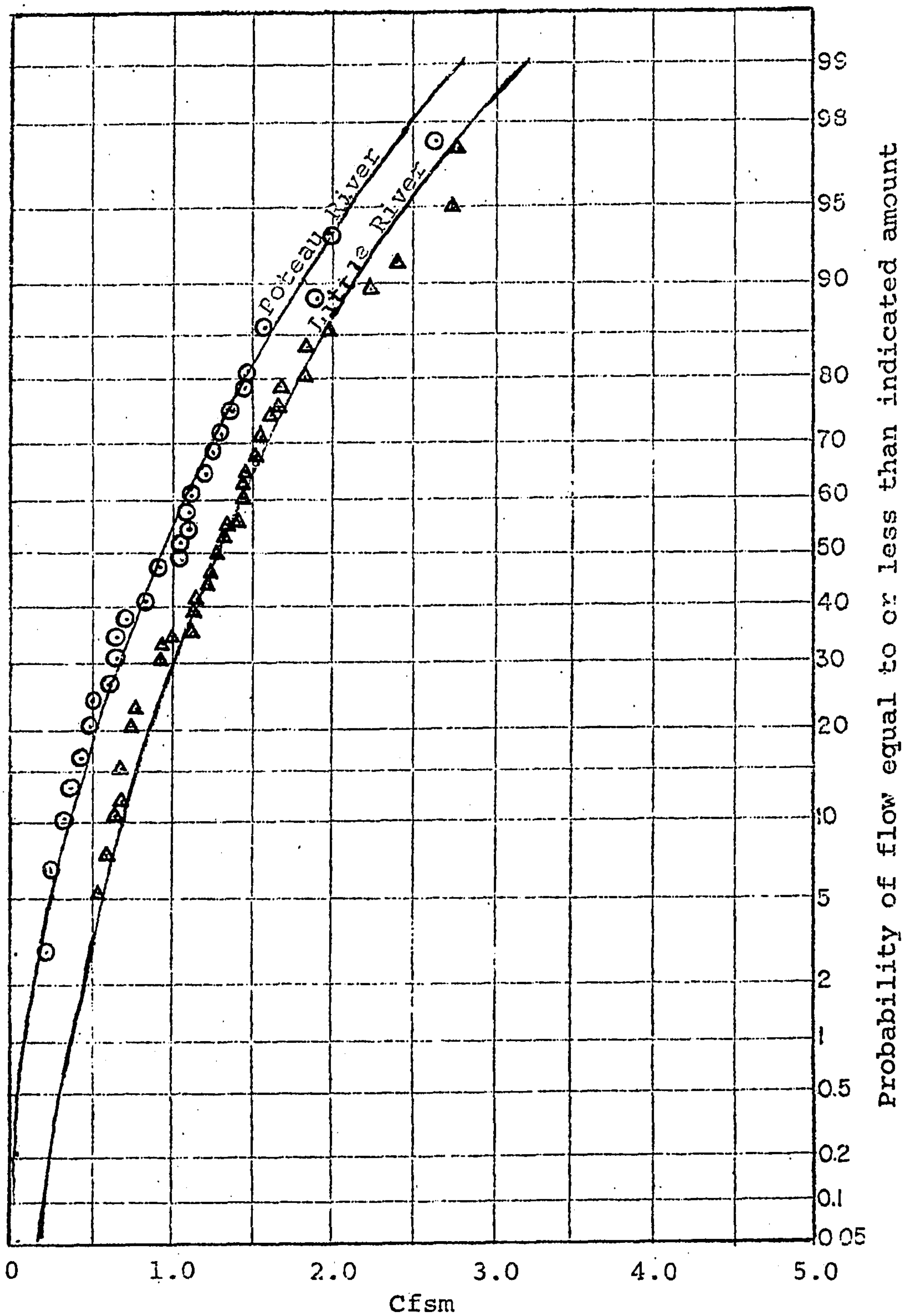

Figure 4. -- Fitted Pearson Type III curves for two streams

4. Between this pair of peaks find the lowest trough (local minimum), T_1, and calculate $P_1 - T_1$.
5. Starting with P_2, find the next sequent peak, P_3, that has magnitude greater than P_2.
6. Find the lowest trough, T_2, between P_2 and P_3 and calculate $P_2 - T_2$.
7. Starting with P_3, find P_4 and T_3; calculate $P_3 - T_3$.
8. Repeat this procedure for all sequent peaks in the series for 2T periods.
9. The required reservoir size is the maximum $(P_i - T_i) = P_m - T_m$.
10. The reservoir will be empty following the occurrence of the minimum trough, T_m, between the pair of sequent peaks P_m and $P_m + 1$ where m identifies the peak and trough associated with the maximum difference, $P_i - T_i$.
11. The storage, S_i, at the end of the i^{th} period and the waste flow during this season may be calculated from the continuity relations:

$$S_i = \text{Min}\,[S_m, (S_{i-1} + X_i - D_i)]$$

$$W_i = \text{Max}\ 0, [(X_i - D_i - (S_m - S_{i+1})]$$

This procedure was applied to monthly streamflow data from the streams under study. The draft rate was assumed to be 80 percent of the mean calculated from the existing entire record.

CHAPTER V

RESULTS, DISCUSSION AND CONCLUSIONS

Results

The Rippl Method

The results of analyses using the Rippl method are given in the following table.

TABLE 4

RESERVOIR SIZE DETERMINED BY THE RIPPL METHOD

Duration months	Stream	Fraction of annual mean flow developed	Storage requirement acre-ft
14	Poteau	0.27	11,900
24	Lee	0.443	41,000
12	Fourche Maline	0.32	7,500
26	Illinois	0.467	83,200
26	Barren Fork	0.27	10,700
26	Buffalo	0.435	131,500
12	Little River	0.565	336,000
13	Ouachita	0.47	47,500
24	Strawberry	0.63	20,500
24	Kings	0.475	176,000

Column one gives the duration of low flow in months and column three shows the percentage of mean annual flow that could be developed and sustained during that period by a reservoir of the volume given in column four. Evaporation and other losses are included in draft. Therefore, when the Rippl method is applied to these streams, only two could be developed to more than fifty percent of the mean annual flow. The larger part of the water resource is allowed to spill and be lost to the user at the site of the reservoir. This is the major deficiency of the Rippl method.

The Hazen Method

The results of the application of Hazen's method to the streams under study are given in Table 5.

TABLE 5

RESERVOIR SIZE DETERMINED BY HAZEN'S METHOD

Stream	Fraction of annual mean flow developed	Storage requirement acre-ft
Poteau	0.8	324,920
Lee	0.8	675,460
Fourche Maline	0.8	209,900
Illinois	0.8	1,224,040
Barren Fork	0.8	447,400
Buffalo	0.8	1,680,425
Little River	0.8	3,443,179
Ouachita	0.8	648,278
Strawberry	0.8	304,450
Kings	0.8	761,068

Moran's Model

The output from Moran's model is a set of probabilities. $P_0, P_1, \ldots, P_{K-M}$. P_0 is the probability of the reservoir being dry, P_1 is the probability of the reservoir containing one tenth of the mean annual flow, P_2 is the probability of the reservoir containing two tenths of the mean annual flow, etc. The probability of primary concern is the probability of the reservoir going dry. It was observed that there is an orderly relationship between P_0 and reservoir size over the range of sizes considered. The size, K, was varied from 1.0 to 2.2 cfsm per year for a draft rate of 0.8 cfsm, and from 0.9 to 2.1 cfsm per year for a draft rate of 0.7 cfsm for each stream. Thus, there was a constant draft rate from each stream. P_0, the probability of the reservoir being dry, versus the size, K, is shown in Figure 5 for a draft of 0.8 cfsm and in Figure 6 for a draft of 0.7 cfsm. The equation for the relationship between P_0 and K is:

$$\begin{aligned} \ln P_0 &= a - bK \\ \ln b &= c(\ln \text{mean}) + d \\ \ln a &= f(\ln \text{mean}) + g. \end{aligned} \tag{5.1}$$

The least-square regression values of c,d,f, and g are 3.49, 0.862,4.85, and 0.071 respectively for a draft of 0.8 cfsm and 2.83,1.34,3.55, and 0.671 respectively for a draft of 0.7 cfsm. The coefficients of determination, R^2, for both draft rates are 0.97 for c and d, and 0.94 for f and g.

When these constant draft rates are divided by the mean annual flow rates of the streams, the results are the percentages of mean annual flow taken from the stream. These results are shown in Table 6.

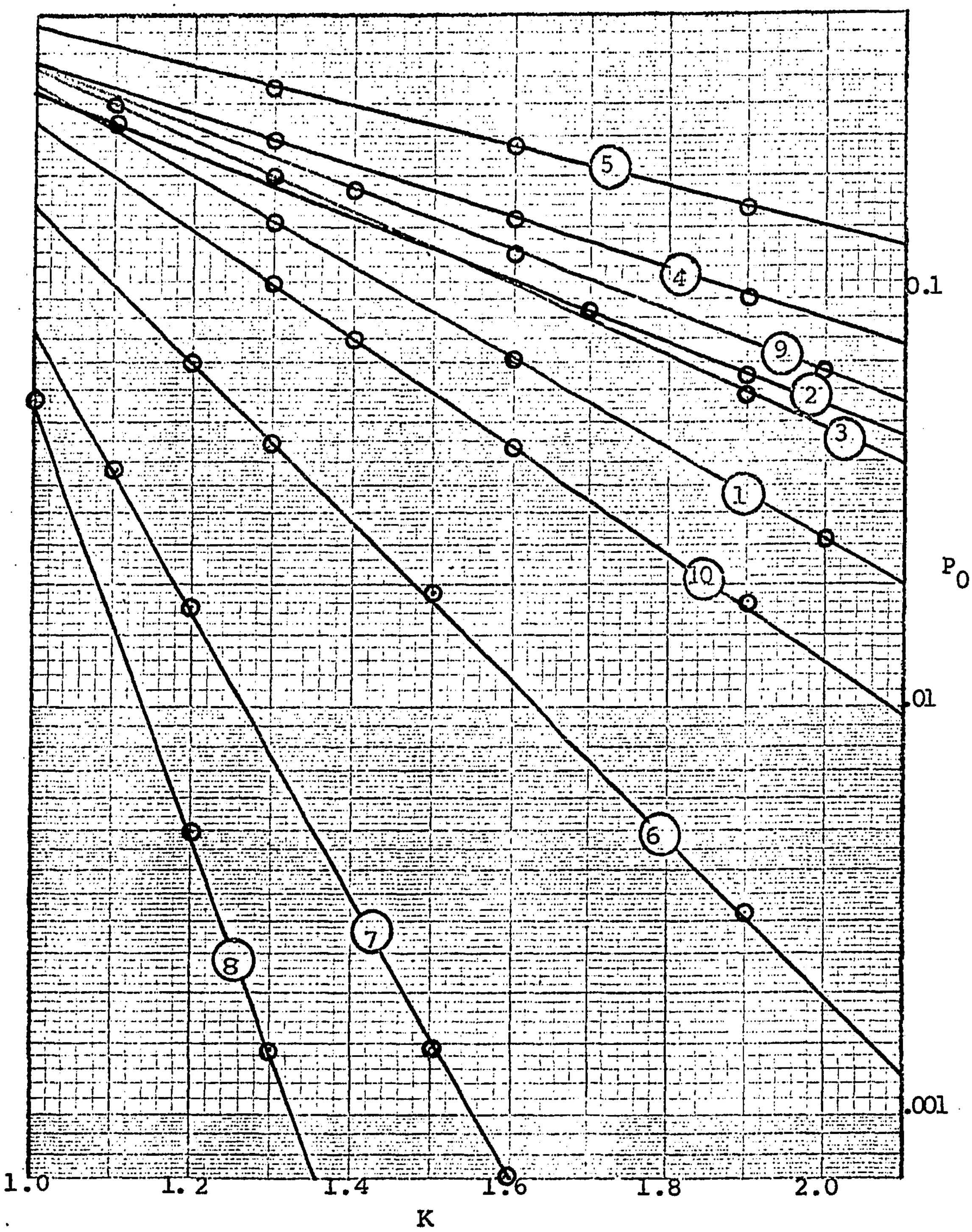

Figure 5. -- P_0 versus K for a draft of 0.8 cfsm
1. Poteau 2. Lee 3. Fourche Maline 4. Illinois 5. Barren Fork 6. Buffalo 7. Little River 8. Ouachita 9. Strawberry 10. Kings

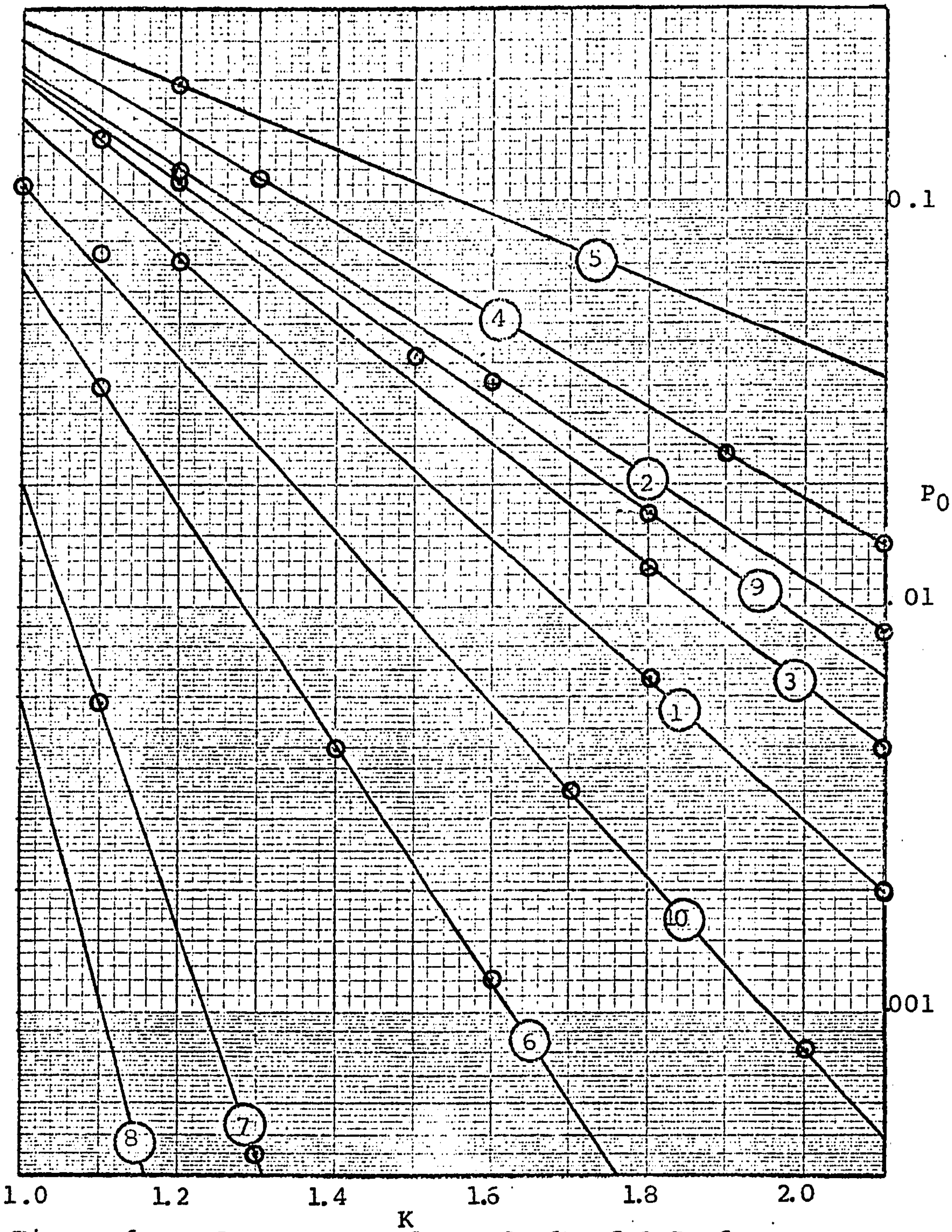

Figure 6. -- P_0 versus K for a draft of 0.7 cfsm
1. Poteau 2. Lee 3. Fourche Maline 4. Illinois 5. Barren Fork 6. Buffalo 7. Little River 8. Ouachita 9. Strawberry 10. Kings

TABLE 6

STREAMFLOW PARAMETERS USED IN MORAN'S MODEL

Stream	Draft rate cfsm	Per cent of mean flow
Poteau	0.8	78.2
Lee	0.8	83.8
Fourche Maline	0.8	82.4
Illinois	0.8	92.7
Barren Fork	0.8	97.2
Buffalo	0.8	69.3
Little River	0.8	58.7
Ouachita	0.8	47.5
Strawberry	0.8	90.3
Kings	0.8	77.3
Poteau	0.7	68.3
Lee	0.7	73.4
Fourche Maline	0.7	72.0
Illinois	0.7	81.1
Barren Fork	0.7	85.0
Buffalo	0.7	60.5
Little River	0.7	51.3
Ouachita	0.7	41.5
Strawberry	0.7	78.7
Kings	0.7	67.6

A regression of the percentages of mean annual flow shown in Table 6 versus the slopes and intercepts of the lines in Figures 5 and 6 yielded the following equations;

$$\ln \text{slope} = 0.159 - 3.30 \ln(\text{percent draft})$$

$$\ln \text{intercept} = -0.868 - 4.34 \ln(\text{percent draft}).$$

The percent draft is expressed as a decimal in these equations. The coefficient of determination was 0.966 for the first equation and 0.931 for the second equation. The percentage of draft in Table 6 varies from a low of 41.5 to a high of 97.2. Therefore, there are three equations which give results comparable to Moran's model for drafts greater than 37 percent of mean annual flow. These three equations are;

$$\begin{aligned} \ln P_0 &= n - sK \\ \ln s &= 0.159 - 3.30 \ \ln(\text{percent draft}) \\ \ln n &= -0.868 - 4.34 \ln(\text{percent draft}). \end{aligned} \qquad (5.2)$$

The percent draft is expressed as a decimal in these equations.

These results are considered significant because it is much easier to solve these three equations than it is to solve the K-M simultaneous equations of Moran's model. Table 7 gives the size of reservoir determined by these equations for a draft of 80 percent of the mean annual flow with a probability of being dry of 0.05. Example calculations are given in the Appendix.

TABLE 7

RESERVOIR SIZE DETERMINED BY EQUATIONS (5.2)

Stream	Reservoir size acre-ft
Poteau	248,200
Lee	495,000
Fourche Maline	144,000
Illinois	1,008,000
Barren Fork	306,000
Buffalo	1,530,000
Little River	4,450,000
Ouachita	836,000
Strawberry	245,500
Kings	668,000

The Sequent Peak Procedure

The sequent peak procedure yielded the results in Table 8.

TABLE 8

RESERVOIR SIZE DETERMINED BY THE SEQUENT PEAK PROCEDURE

Stream	Reservoir size acre-ft	Cycle length (months) / Total record (months)
Poteau	100,500	69/348
Lee	174,500	69/204
Fourche Maline	84,200	104/360
Illinois	436,000	56/357
Barren Fork	128,200	66/228
Buffalo	615,000	78/468
Little River	1,380,000	67/432
Ouachita	267,000	103/312
Strawberry	82,500	88/336
Kings	305,000	80/336

Column three in Table 8 gives the time in months from the peak to the trough, P_m to T_m, over the total number of months in the record. Table 9 gives a summary of the results.

Discussion

The Rippl method will not allow high percentage draft rates from a stream with a sizable variation in the stream flow rate. This method should not be used on such streams to determine the maximum amount, or "firm yield" that a stream will provide.

Hazen's method will generally specify a reservoir size that is too large because the storage volumes are

TABLE 9

SUMMARY OF RESULTS

	Storage Requirements (acre-ft)			
Stream	Rippl Method	Hazen's Method	Sequent Peak	Equations (5.2)
Poteau	11,900	324,920	100,500	248,200
Lee	41,000	675,460	174,500	495,000
Fourche Maline	7,500	209,900	84,200	144,000
Illinois	83,200	1,224,040	436,000	1,008,000
Barren Fork	10,700	447,400	128,200	306,000
Buffalo	131,500	1,680,425	615,000	1,530,000
Little River	336,000	3,443,179	1,380,000	4,450,000
Ouachita	47,500	648,278	267,000	836,000
Strawberry	20,500	304,450	82,500	245,500
Kings	176,000	761,068	305,000	668,000

Notes:

All storage requirements based upon a draft of 80 percent of mean flow except the Rippl Method. The maximum allowable draft by the Rippl Method is given in Table 4. Hazen's Method and Equations (5.2) calculated for a probability of going dry, P_0, of 0.05.

assumed to be normally distributed. In addition, Hazen's method is based upon the 95 per cent dry year with no provisions for changing to a recurrence interval of other than once in twenty years. However, Hazen's method is superior to the Rippl method for developing water supplies to near maximum potential.

Moran's model gives results that are larger than either the Rippl method or the sequent peak procedure, but generally less than Hazen's method. There are two streams in this study, Ouachita and Litter rivers, where the Hazen method yields a smaller size than Moran's model or the equations approximating Moran's model. These streams have relatively large annual flows with resulting coefficients of variation that are relatively smaller. Since Hazen's method is based upon the coefficient of variation, the method specifies a smaller size than Moran's model.

The sequent peak procedure will allow high percentage developments of the mean annual flow of a stream. However, it suffers the inflexibility of the fixed recurrence interval of the record of streamflow. Specifically, column three of Table 8 shows the fixed ratio of the length of time of reduced flows to the total record length. If these values are converted to percentages, they are quite high. If it may be implied that this would be the return interval of the drought, the procedure determines a reservoir size that is too small for adequate design purposes in this area of study. When applied to streams in the eastern part of the United States, where the coefficient of variation is small relative to the coefficient of variation found in the record of flow of most western streams, the procedure would be more satisfactory than it would be in the west or southwest part of the country. Therefore, the locality determines to a large extent whether or not this procedure would be useful.

Conclusions

Moran's model is superior to the other algorithms here considered. The input to the reservoir is allowed to conform to a gamma distribution which characterizes most streamflow. The three equations, (5.2), are a good approximation to Moran's model. A suggested procedure for design of a reservoir for water supply purposes is to use the equations to determine the size required for the probability of going dry and the percentage of mean annual flow desired. A capacity equal to an additional time requirement, say thirty days of draft, could be added to the size determined by the equations if the concept of going dry is disconcerting to the designer.

Future studies should include the general applicability of the equations (5.2). Also, the theoretical relationship between the size of the reservoir and the probability of the reservoir going dry when the input to the reservoir follows a Pearson Type III distribution should be developed by those especially skilled in mathematics and statistics. The preliminary work has been done by Phatarfod (15) and Prabhu (16).

Moran's model or the equations (5.2) should be used in determining required reservoir size in order to obtain maximum utilization of our water resources. The concept of withdrawals of 80 percent should not disturb potential users downstream from a reservoir site if there is no out-of-basin transfer of water. The potential downstream user will receive 20 percent of the original flow plus, in time, approximately 70 percent of the amount withdrawn by the upstream user.

APPENDIX

EXAMPLE CALCULATIONS FOR DESIGN SIZE USING EQUATIONS (5.2)

Information needed for the Buffalo River near Rush, Arkansas:

Drainage area = 1091 square miles

Annual mean flow = 1.153 cfsm

Percent draft = 80 percent

Probability of being dry = 0.05

$$\ln P_0 = n - sK$$

$$\ln s = 0.159 - 3.30 \ln (\text{percent draft})$$

$$\ln n = -0.868 - 4.34 \ln (\text{percent draft})$$

Calculations:

$$\ln s = 0.159 - 3.30(-0.223) = 0.895$$

$$s = 2.44$$

$$\ln n = -0.868 - 4.34(-0.223) = 0.100$$

$$n = 1.105$$

$$\ln P_0 = \ln(0.05) = -3.00 = 1.106 - 2.44K$$

$$K = 1.68$$

$$\text{Storage (acre-ft)} = 1.68(1091)(1.9835)(365)(1.153)$$

$$= 1{,}530{,}000 \text{ acre-ft.}$$

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