A HIERARCHY OF PROCUREMENT AND INVENTORY SYSTEMS

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PREFACE

This dissertation is based on the supposition that procurement and inventory systems can be classified in a hierarchial order with the multi-item, multi-source system as its apex. It is shown that decision models can be developed to represent each system in the hierarchy. These models are manipulated mathematically to determine optimal procurement and inventory policy.

The primary objective of this dissertation is to present a hierarchy of procurement and inventory systems resulting in a generalization which embraces the multi-item, multi-source concept and yields optimal policy decisions. The secondary objective is to refine and extend procurement and inventory theory at the lower levels in the hierarchy. Chapters II through IV are devoted to the secondary objective. Chapter V is devoted to the primary objective. Chapter VI illustrates the application of the algorithm constructed for solution of multi-item, multi-source problems to the solution of problems lower in the hierarchy. The algorithm of Chapter V has been programmed for a digital computer. The computerized solution method appears in the Appendix.

Briefly, the decision situation under consideration in the multi-item, multi-source context may be described as follows. When the stock on hand and on order for each item falls to a predetermined level, action is initiated to procure a replenishment quantity from one of several sources. The objective is to determine the procurement level, the procurement quantity, and the procurement source for each item in the light of the relevant costs, and the properties of demand, lead time, replenishment rate, and restrictions on the system so that the sum of all costs associated with the procurement and inventory process is minimized. Optimal procurement and inventory policy for the probabilistic process is that policy resulting in the maximization of the probability of minimizing the sum of all costs.

The procurement and inventory systems presented in this dissertation are based on certain assumptions. These assumptions are:

(1) All systems are for the case of a single stocking point. The procurement and inventory process exists at only one echelon in the complex of supply situations.

(2) All unsatisfied demands are satisfied out of the next shipment. This is usually referred to as completely captive demand.

(3) For the development of the probabilistic systems found herein, the distributions of demand and lead time are identically and independently distributed in each time period. Thus, the parameters exhibit steady-state, invariant characteristics.

(4) Procurement and inventory processes may be deterministic or probabilistic. In the probabilistic process it is not possible to hold both the procurement quantity and the number of periods per cycle fixed as is the case with the

deterministic process. The most common probabilistic analysis of procurement and inventory systems is that in which the procurement quantity is fixed and the procurement interval is allowed to vary. This is the case treated in the investigation of probabilistic systems.

My interest in procurement and inventory theory began in 1962 as a student of Dr. M. A. Griffin at the University of Alabama. Interest in the area continued to grow through my association with Dr. W. J. Fabrycky. This dissertation was only possible through his assistance as a glance at the Bibliography indicates.

The research resulting in this dissertation was supported by a grant from the National Science Foundation (NSF GP-3000) to Dr. W. J. Fabrycky. Indebtedness is acknowledged to the Foundation for the year of financial support it provided.

A debt of gratitude is acknowledged to the staff of the Oklahoma State University Computer Center who availed themselves often. Special indebtedness is acknowledged to Mrs. Roger Eaton whose programming knowledge was often required during the year that development and testing of the computerized algorithm was in progress.

I would also like to acknowledge the contributions of Dr. J. L. Folks, Dr. R. W. Gibson, and Dr. D. A. Pierce. Each of these individuals assisted me at crucial points in the conduct of this research.

Finally, the members of my Advisory Committee, Professors W. J. Bentley, W. J. Fabrycky, J. L. Folks, T. C. Mayberry, and P. E. Torgersen, deserve special credit for guiding my doctoral program and this investigation. Thanks is due each of them for their inspiration and encouragement.

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CHAPTER I

INTRODUCTION

Progress in systems engineering and operations research is often a result of the discovery and modeling of basic relationships common to two or more separately understood systems. The end result of successful research of this nature is a unified concept which provides a higher ordered generalization about the structure of the expanded system. The research results presented in this dissertation exhibit such a higher ordered generalization for the multi-item, multi-source procurement and inventory system. The value of such a generalization results from the fact that all real world procurement and inventory systems involve both multi-item and multi-source characteristics. Such systems are an essential facet of all production and distribution operations and involve an investment representing a sizeable portion of the gross national product.

The Hierarchy of Procurement and Inventory Systems

The purpose of this treatise is the presentation of a unified hierarchy of procurement and inventory systems together with decision models for variations of each system. The hierarchy of procurement and inventory systems developed in this dissertation is presented in the following paragraphs.

A single-item, single-source (SISS) system is represented schematically in Figure 1. This system involves one item which may be procured from a specified source. The first decision model formulated for the single-item, single-source system was presented in 1915 by F. W. Harris [1]. Since then, this system has been investigated extensively. Whitin [2] gives an excellent account of the theory and application of single-item, single-source models up to about 1957. Many authors have offered further developments and refinements [3], [4], [5], [6], and [7].

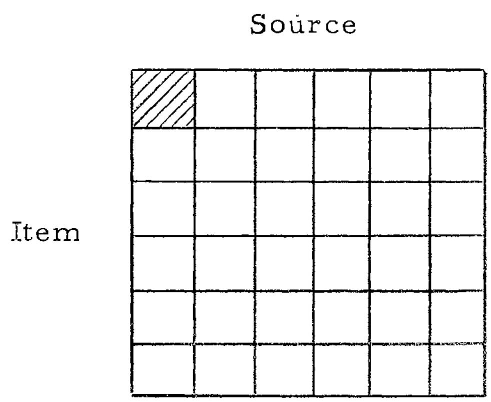


Figure 1. The SISS System in Its Hierarchial Position

Figure 2 is a schematic representation of a single-item, multi-source (SIMS) system. This system involves one item which may be procured from one of two or more sources. The single-item, multi-source concept was developed by Fabrycky [8], [9], and [10].

Application of the concept to the manufacture or purchase decision was presented by Fabrycky and Ghare [11].

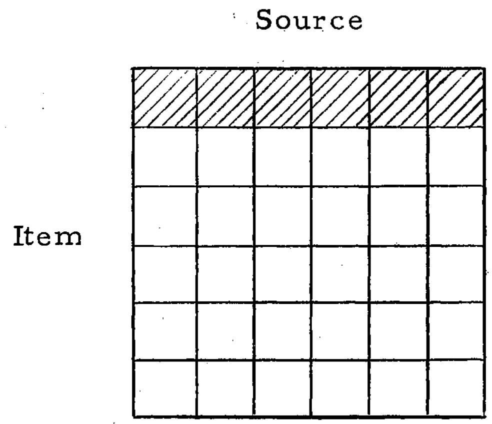


Figure 2. The SIMS System in Its Hierarchial Position

A multi-item, single-source (MISS) system is represented schematically in Figure 3. This system involves many items which may be procured from a specified source. Decision models for the multi-item, single-source system normally involve aggregate warehouse constraints and/or restrictions on set-up time or capital [3], [4], [5], [6], and [7].

The multi-item, multi-source (MIMS) system is illustrated in Figure 4. This system involves many items, each of which may be procured from one of two or more sources. The multi-item, multi-source concept was developed by Fabrycky and Banks [12]. A primary purpose of this dissertation is the presentation of a unified hierarchy of procurement and inventory systems with the multi-item, multi-source system at its apex. To facilitate identification, the acronyms SISS, SIMS, MISS, and MIMS will be adopted in the discussion that follows.

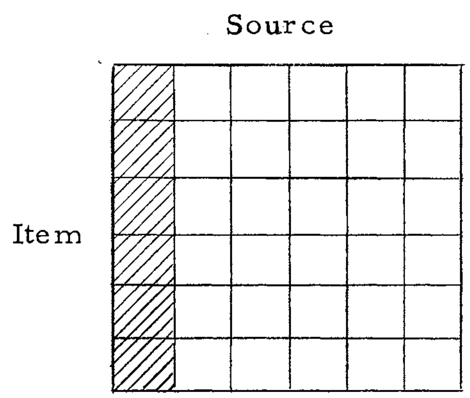


Figure 3. The MISS System in Its Hierarchial Position

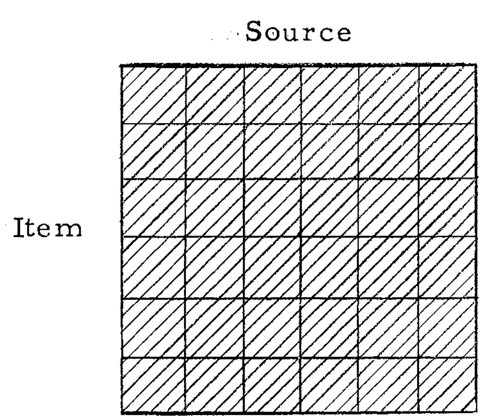


Figure 4. The MIMS System in Its Hierarchial Position

The Decision Environment

The decision environment is composed of the alternative sources for the replenishment of stock, the system and cost parameters, and the restrictions under which procurement and inventory systems must operate. Each of these components will be discussed in this section as they exist relative to the MIMS system.

Alternative Sources

A MIMS procurement and inventory system exists as a result of a demand for each item in the system. In satisfying the demand, a procurement manager finds it necessary to replenish his stock for each item periodically. One of the basic suppositions of the MIMS concept is that stock replenishment can be made by procurement from one of two or more sources.

An important facet of the procurement and inventory problem is the choice of a source from which each item should be procured so that a minimum total system cost will result. Specifically, the source choice may be one of several vendors, or one or more of several manufacturing or remanufacturing facilities, or an intrafirm transfer possibility. The system and cost parameters which serve as differentiators between these source alternatives are described in the following paragraphs together with those parameters which are source independent.

System Parameters

Demand, D, is the primary stimulus on the procurement and inventory system and the justification for its existence. It is an item dependent parameter with the dimension of units per period. The procurement and inventory system may exist to meet demand at a retail level, at a wholesale level, or at any given level in a distribution process. Demand may arise from any of these levels or the next step in a manufacturing process, the spare parts requirement of an operational weapons system, etc. The characteristics of demand, while independent of the source chosen to replenish inventories, will depend upon the nature of the demand environment.

The simplest demand pattern may be classified as deterministic. In this special case, the future demand for an item may be predicted with certainty. Demand considered in this restricted sense is only an approximation of reality. In the general case, demand may be described as a random variable, D_x, which takes on values in accordance with a specific probability distribution.

Procurement lead time, T, is the elapsed time in periods from the initiation of procurement action to the receipt of replenishment stock. It is a parameter that depends upon the item as well as the source since the characteristics of the item as well as the characteristics of the source determine the specific lead time value.

As in the case of demand, lead time that may be predicted with certainty will be classified as deterministic. This is the simplest lead time pattern and is only an approximation of reality. In its general context, lead time will be a random variable, T_x, which takes on values in accordance with a specific probability distribution.

The replenishment rate, R, reflects item and source dependency.

It has the dimension of units per period and describes the rate at which replenishment stock accumulates for each item and each source. Replenishment stock is usually received in one shipment if purchasing or intrafirm transfer action was initiated. Under this source choice, the stock on hand increases by an amount equal to the procurement quantity in an instant of time. Thus, the replenishment rate for purchasing is infinite. If the item is manufactured or remanufactured the replenishment rate will be finite due to the fact that a manufactured item accumulates as it is made.

Cost Parameters

Item cost, C_i, reflects item and source dependency and has the dimension of dollars per unit. Each vendor resides in an environment unique to his position and may be expected to price the item accordingly. For manufacturing or remanufacturing, item cost involves a summation of the costs of direct labor, direct material, and factory burden.

Procurement cost, C_p, is the summation of cost elements arising from the series of acts beginning with the initiation of procurement action and ending with the receipt of replenishment stock. The procurement cost reflects both item and source dependency and has the dimension of dollars per procurement. For the purchase alternative procurement cost involves the expenses of paper work preparation, communication, receiving, and vendor payment. Certain of these costs are dependent upon the vendor chosen. Procurement cost for manufacturing or remanufacturing will be composed of the cost elements of production planning, set-up and tear-down, scheduling, and other costs resulting from the set of acts required in the initiation of manufacturing action.

Holding cost, C_h, reflects costs that are a function of the number of units on hand and the time duration involved. It is an item dependent parameter with the dimension of dollars per unit per period. Holding cost is made up of out-of-pocket costs such as insurance, taxes, obsolescence, warehouse rental, light, heat, and maintenance. In addition, capital invested in inventories is unavailable for investment elsewhere. The rate of interest foregone represents a cost of carrying inventories. Some of these costs may depend upon the maximum inventory level. Others may depend upon the average level. Still others, like the cost of capital invested will depend upon the value of the inventory during a given interval of time.

Shortage cost, C_s, is the penalty incurred for being unable to meet a demand when it occurs. This cost parameter will not depend upon the source of replenishment stock, but will depend upon the item. Its dimension is dollars per unit short per period. The specific dollar penalty for a shortage depends upon the nature of demand and the time duration of the shortage. For example, if the demand is that of customers upon a retail establishment, the shortage cost will be due to the loss of good will and profit. In this case, shortage cost will be small relative to the cost of the item. If the demand arises from the next step in a manufacturing process, the cost of a shortage may be high relative to the cost of the item. Being unable to meet the requirement for the item may result in lost production for the duration of the shortage.

Restrictions

Normally, warehouse space is a scarce resource. It may be expressed in cubic units designated W. Each item consumes a certain amount of space which must not exceed the amount available.

Procurement and inventory policy will be derived for cases in which W is infinite and for cases in which W is finite. Optimization methods for the cases in which a warehouse restriction exists differ from the cases in which no restriction is present. Optimal policy in the face of a warehouse restriction leads to a total system cost that is greater than or equal to the total system cost when no warehouse restriction is present.

Each source has the capability of assigning only a certain maximum number of capacity units per period to the procurement manager. This will be designated H. Each unit of product procured from each source requires a certain portion of the capacity of that source. This requirement will be designated h. The total capacity consumed by all items procured from a given source must not exceed the total capacity available at that source. It will be shown that total system cost in the face of source capacity restrictions is greater than or equal to the total system cost when no source capacity restriction exists. Thus, source capacity and warehouse restrictions have the same effect on total system cost.

Contributions of This Investigation

An examination of the status of procurement and inventory theory prior to this investigation indicated that:

1. Models for systems subject to a warehouse space restriction are available for only a simple case of the MISS system.
2. The MIMS system was not formulated and no models are available for situations with multi-item, multi-source characteristics.
3. Procurement and inventory systems have not been classified into a recognizable hierarchy, although many basic inventory models are available for specific situations.

The primary objective of this dissertation is to present a hierarchy of procurement and inventory systems, resulting in a generalization which embraces the MIMS system. It will be shown that a uniform set of deterministic and probabilistic models can be developed to represent each system. These models will be manipulated to determine optimal procurement and inventory policy for the specific system under study. A secondary objective will be to define and extend procurement and inventory theory at the lower levels in the hierarchy.

A major contribution in support of the hierarchy of systems is the set of models for handling warehouse space and source capacity restrictions. Lagrangian multipliers are utilized for treating the deterministic SISS, SIMS, and MISS systems. The Lagrangian multiplier technique cannot be easily applied to the constrained deterministic MIMS system and to the probabilistic SISS, SIMS, MISS, and MIMS systems. This led to the adoption of dynamic programming as an optimization technique for these cases [14].

Finally, this treatise serves to unify and extend research at the Oklahoma State University in procurement and inventory theory [8],

[9], [10], [11]. [12], and [13]. In the chapters which follow, selected usage will be made of key paragraphs, illustrations, and examples without specific credit in all cases. Thus, the contribution of each of these to the objectives of this dissertation is hereby acknowledged.

CHAPTER II

THE SISS SYSTEM

A SISS procurement and inventory system is illustrated in Figure 5. It exists as a result of the demand stimulus, D. In satisfying this demand the procurement manager finds it necessary to replenish the stock of the item periodically. The basic supposition of the SISS concept is that replenishment can be made from a single-source only. Specifically, procurement may involve purchasing, or intrafirm transfer, or manufacturing, or remanufacturing. If the purchase alternative is involved, only one vendor is under consideration. Procurement and inventory policy for the SISS system will be that policy stating <u>when</u> to procure and <u>how much</u> to procure with the source being fixed by a prior decision. It will be the purpose of this chapter to formulate the basic concepts necessary to an understanding of the higher ordered systems.

The Deterministic SISS System

The inventory process resulting from procurement action will be either deterministic or probabilistic depending upon the nature of demand and procurement lead time. If both demand and lead time are deterministic, the resulting inventory process will be deterministic. The exhibited geometry of such a process will depend upon the procurement level, L, the procurement quantity, Q, the demand rate, D, the procurement lead time, T, and the rate of replenishment, R, as exhibited in Figure 6. In reality, the slopes D and R-D would be step functions. However, straight line approximations will be used in the geometrical interpretation of inventory processes to facilitate their mathematical description.

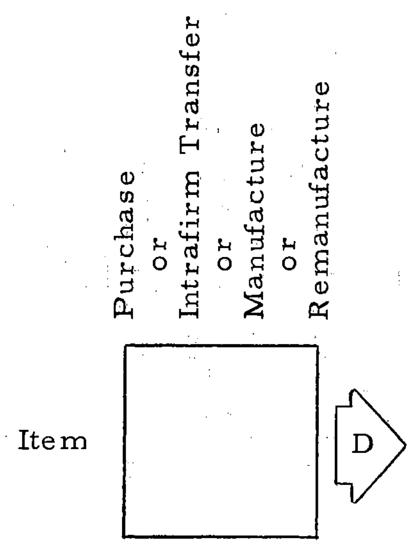


Figure 5. The SISS System

Two basic time elements are involved in Figure 6. They may be defined as follows:

(1) Period - the element of elapsed time between review of the stock position. This is usually a day but it may be any other time unit.

(2) Cycle - N, the number of periods occurring between successive procurement action.

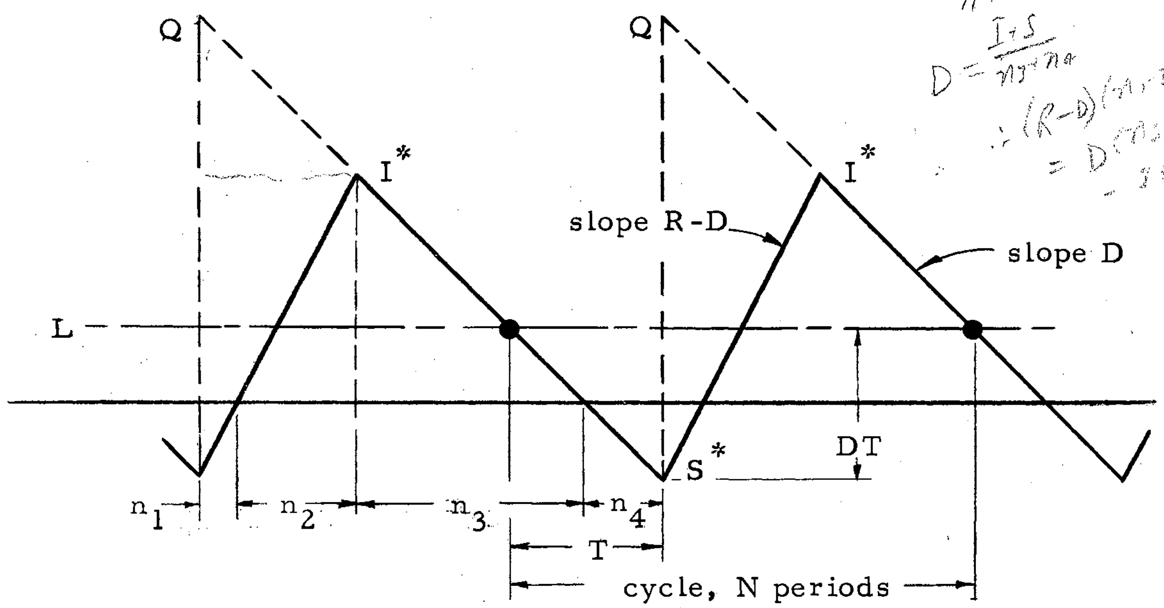


Figure 6. The Geometry of a Deterministic Inventory Process

The total system cost for the process will depend upon the exhibited geometry, the item cost, the procurement cost, the holding cost, and the shortage cost. The development of deterministic models in this treatise is based on the assumption that $D > R$ and $Q \geq \min(1, D)$.

Algebraic Relationships

From Figure 6 it is evident that the number of periods per inventory cycle is:

$$N = \frac{Q}{D} . \qquad (2.1)$$

Also, the following relationships are evident:

$$(n_1 + n_2)(R - D) = (n_3 + n_4)D \tag{2.2}$$

$$n_1 + n_2 = \frac{Q}{R} \tag{2.3}$$

and

$$n_3 + n_4 = \frac{I^* + DT - L}{D}. \tag{2.4}$$

From Equations (2.2), (2.3), and (2.4),

$$I^* = Q(1 - \frac{D}{R}) + L - DT. \tag{2.5}$$

The total number of unit periods of stock on hand during the inventory cycle I, is:

$$I = \frac{I^*}{2}(n_2 + n_3)$$

$$I = \frac{I^{*2}}{2(R-D)} + \frac{I^{*2}}{2D}$$

$$I = \frac{I^{*2}}{2}(\frac{1}{R-D} + \frac{1}{D}). \tag{2.6}$$

Substituting Equation (2.5) for I^* gives:

$$I = \frac{[Q(1 - \frac{D}{R}) + L - DT]^2}{2}(\frac{1}{R-D} + \frac{1}{D}). \tag{2.7}$$

The total number of unit periods of shortage during the cycle S, is:

$$S = \frac{S^*}{2}(n_1 + n_4)$$

$$S = \frac{S^{*2}}{2(R-D)} + \frac{S^{*2}}{2D}.$$

But, since $S^* = DT - L$:

$$S = \frac{(DT - L)^2}{2}(\frac{1}{R-D} + \frac{1}{D}). \tag{2.8}$$

Total System Cost

The total system cost per period will be a summation of the item cost per period, the procurement cost per period, the holding cost per period, and the shortage cost per period; that is:

$$TC = IC + PC + HC + SC. \qquad (2.9)$$

The item cost per period will be the product of the item cost per unit and the demand rate in units per period; that is:

$$IC = C_i D. \qquad (2.10)$$

The procurement cost per period will be the procurement cost per procurement divided by the number of periods per inventory cycle; that is:

$$PC = \frac{C_p}{N}$$

$$PC = \frac{C_p D}{Q} . \qquad (2.11)$$

Holding cost per period will be the product of the holding cost per unit per period and the average number of units on hand during the period; that is:

$$HC = \frac{C_h I}{N}$$

$$HC = \frac{C_h D}{Q} \frac{[Q(1 - \frac{D}{R}) + L - DT]^2}{2} (\frac{1}{R-D} + \frac{1}{D}).$$

Note that:

$$\frac{D}{Q}(\frac{1}{R-D} + \frac{1}{D}) = \frac{1}{Q(1 - \frac{D}{R})} . \qquad (2.12)$$

Therefore,

$$HC = \frac{C_h}{2Q(1 - \frac{D}{R})} [Q(1 - \frac{D}{R}) + L - DT]^2. \qquad (2.13)$$

Shortage cost per period will be the product of the shortage cost per unit short per period and the average number of units short during the period; that is:

$$SC = \frac{C_s S}{N}$$

$$SC = \frac{C_s D}{Q} \frac{(DT - L)^2}{2} (\frac{1}{R-D} + \frac{1}{D}).$$

Substituting Equation (2.12) gives:

$$SC = \frac{C_s(DT - L)^2}{2Q(1 - \frac{D}{R})}. \qquad (2.14)$$

The total system cost per period will be a summation of the four cost components given by Equations (2.10), (2.11), (2.13), and (2.14); that is:

$$TC = C_i D + \frac{C_p D}{Q} + \frac{C_h}{2Q(1 - \frac{D}{R})} [Q(1 - \frac{D}{R}) + L - DT]^2$$
$$+ \frac{C_s(DT - L)^2}{2Q(1 - \frac{D}{R})}. \qquad (2.15)$$

Optimal Policy for Deterministic SISS System

The minimum cost procurement level and procurement quantity may be found by setting the partial derivatives equal to zero and solving the resulting equations. Modifying Equation (2.15) gives:

$$TC = C_i D + \frac{C_p D}{Q} + \frac{C_h Q(1 - \frac{D}{R})}{2} - C_h(DT - L)$$

$$+ \frac{C_h(DT - L)^2}{2Q(1 - \frac{D}{R})} + \frac{C_s(DT - L)^2}{2Q(1 - \frac{D}{R})} . \qquad (2.16)$$

Taking the partial derivative of Equation (2. 16) with respect to Q, then with respect to $DT - L$, and setting both equal to zero gives:

$$\frac{\partial TC}{\partial Q} = - \frac{C_p D}{Q^2} + \frac{C_h(1 - \frac{D}{R})}{2} - \frac{C_h(DT - L)^2}{2Q^2(1 - \frac{D}{R})}$$

$$- \frac{C_s(DT - L)^2}{2Q^2(1 - \frac{D}{R})} = 0. \qquad (2.17)$$

$$\frac{\partial TC}{\partial (DT - L)} = - C_h + \frac{C_h(DT - L)}{Q(1 - \frac{D}{R})} + \frac{C_s(DT - L)}{Q(1 - \frac{D}{R})} = 0. \qquad (2.18)$$

Equation (2. 18) may be expressed as:

$$\frac{DT - L}{Q} = \frac{C_h(1 - \frac{D}{R})}{C_h + C_s} . \qquad (2.19)$$

Substituting Equation (2. 19) into Equation (2. 17) gives:

$$- \frac{C_p D}{Q^2} + \frac{C_h(1 - \frac{D}{R})}{2} - \frac{C_h^3(1 - \frac{D}{R})}{2(C_h + C_s)^2} - \frac{C_s C_h^2(1 - \frac{D}{R})}{2(C_h + C_s)^2} = 0$$

$$\frac{C_p D}{Q^2} = \frac{C_h C_s(1 - \frac{D}{R})}{2(C_h + C_s)}$$

$$Q = \sqrt{\frac{2C_p D (C_h + C_s)}{C_h C_s(1 - \frac{D}{R})}}$$

$$Q = \sqrt{\frac{1}{1 - \frac{D}{R}}} \sqrt{\frac{2C_pD}{C_h} + \frac{2C_pD}{C_s}} . \qquad (2.20)$$

Substituting Equation (2.20) into Equation (2.19) gives:

$$L = DT - \frac{C_h(1 - \frac{D}{R})}{C_h + C_s} \sqrt{\frac{1}{1 - \frac{D}{R}}} \sqrt{\frac{2C_pD}{C_h} + \frac{2C_pD}{C_s}}$$

$$L = DT - \sqrt{1 - \frac{D}{R}} \sqrt{\frac{2C_pD}{C_s\ (1 + \frac{C_s}{C_h})}} . \qquad (2.21)$$

Equation (2.20) and Equation (2.21) may now be substituted back into Equation (2.16) to give an expression for the minimum total system cost. After several steps:

$$TC = C_iD + \sqrt{1 - \frac{D}{R}} \sqrt{\frac{2C_pC_hC_sD}{C_h + C_s}} . \qquad (2.22)$$

Equations(2.20), (2.21) and (2.22) can be reduced to the simple economic-lot-size equations by assuming shortage cost and replenishment rate equal to infinity and lead time equal to zero. In this case, Equation (2.20) reduces to:

$$Q = \sqrt{\frac{2C_pD}{C_h}} .$$

Equation (2.21) reduces to:

$$L = 0.$$

And, Equation (2.22) reduces to:

$$TC = C_iD + \sqrt{2C_pC_h D} .$$

An Example Deterministic SISS Policy

As an example of the deterministic SISS system suppose that a procurement manager will purchase an item having the following parameters:

$D \ldots\ldots\ldots\ldots 6.00$

$R \ldots\ldots\ldots\ldots \infty$

$T \ldots\ldots\ldots\ldots 7.00$

$C_i \ldots\ldots\ldots\ldots \$34.75$

$C_p \ldots\ldots\ldots\ldots \$23.16$

$C_h \ldots\ldots\ldots\ldots \$0.30$

$C_s \ldots\ldots\ldots\ldots \$0.30$.

The minimum cost procurement quantity may be found from Equation (2.20) as:

$$Q = \sqrt{\frac{1}{1-\frac{6}{\infty}}}\ \sqrt{\frac{2(\$23.16)(6)}{\$0.30} + \frac{2(\$23.16)(6)}{\$0.30}}$$

$$Q = 43.0571.$$

The minimum cost procurement level may be found from Equation (2.21) as:

$$L = 6(7) - \sqrt{1 - \frac{6}{\infty}}\ \sqrt{\frac{2(\$23.16)(6)}{\$0.30(1+\frac{\$0.30}{\$0.30})}}$$

$$L = 20.4843.$$

The minimum total system cost may be found from Equation (2.22) as:

$$TC = \$34.75\,(6) + \sqrt{1 - \frac{6}{\infty}}\ \sqrt{\frac{2(\$23.16)(\$0.30)(\$0.30)(6)}{\$0.30 + \$0.30}}$$

$$TC = \$214.9546.$$

Values of Q and L will remain in their computed form because theoretical minimums are desired. In real world applications both Q and L would be adjusted so that each is an integer and so that the joint adjustment results in a minimum cost.

<u>Optimal Policy for Deterministic SISS System With Warehouse Restriction</u>

The single-item in the SISS system consumes a certain amount of warehouse space, w W. There exists a certain amount of total warehouse capacity, W. The maximum accumulation of inventory for the item, I^*, will consume I^*w cubic units of scarce warehouse space. Therefore, the restriction $I^*w \leq W$ must not be violated. This section will present a Lagrangian multiplier technique for finding the optimal procurement and inventory policy in the face of this restriction.

Define λ such that $\lambda < 0$ for every $W - I^*w = 0$, and $\lambda = 0$ for every $W - I^*w > 0$. Then

$$\lambda(W - I^*w) = 0. \tag{2.23}$$

Equation (2. 23) may be added to Equation (2. 15) giving:

$$TC = C_iD + \frac{C_pD}{Q} + \frac{C_h[Q(1 - \frac{D}{R}) + L - DT]^2}{2Q(1 - \frac{D}{R})} + \frac{C_s(DT - L)^2}{2Q(1 - \frac{D}{R})} + \lambda(W - I^*w). \tag{2.24}$$

Substituting Equation (2. 5) into Equation (2. 24) gives:

$$TC = C_iD + \frac{C_pD}{Q} + \frac{C_h[Q(1 - \frac{D}{R}) + L - DT]^2}{2Q(1 - \frac{D}{R})} + \frac{C_s(DT - L)^2}{2Q(1 - \frac{D}{R})} + \lambda\left\{W - [Q(1 - \frac{D}{R}) + L - DT]\ w\right\}. \tag{2.25}$$

The third term of Equation (2. 25) can be written as:

$$\frac{C_h Q(1 - \frac{D}{R})}{2} - C_h(DT - L) + \frac{C_h(DT - L)^2}{2Q(1 - \frac{D}{R})} . \tag{2. 26}$$

And, the last term can be written as:

$$-\lambda Q(1 - \frac{D}{R})w + \lambda(DT - L)w + \lambda W . \tag{2. 27}$$

Equation (2. 26) and Equation (2. 27) can now be substituted into Equation (2. 25) giving:

$$TC = C_i D + \frac{C_p D}{Q} + \frac{C_h[Q(1 - \frac{D}{R})]}{2} - C_h(DT - L) + \frac{C_h(DT - L)^2}{2Q(1 - \frac{D}{R})} + \frac{C_s(DT - L)^2}{2Q(1 - \frac{D}{R})} - \lambda Q(1 - \frac{D}{R})w + \lambda(DT - L)w + \lambda W. \tag{2. 28}$$

Taking the partial derivative of Equation (2. 28) with respect to Q, then with respect to DT - L, and setting each equal to zero gives:

$$\frac{\partial TC}{\partial Q} = - \frac{C_p D}{Q^2} + \frac{C_h(1 - \frac{D}{R})}{2} - \frac{C_h(DT - L)^2}{2Q^2(1 - \frac{D}{R})} - \frac{C_s(DT - L)^2}{2Q^2(1 - \frac{D}{R})} - \lambda(1 - \frac{D}{R})w = 0. \tag{2. 29}$$

$$\frac{\partial TC}{\partial (DT - L)} = - C_h + \frac{C_h(DT - L)}{Q(1 - \frac{D}{R})} + \frac{C_s(DT - L)}{Q(1 - \frac{D}{R})} + \lambda w = 0. \tag{2. 30}$$

Equation (2. 30) may be expressed as:

$$\frac{(DT - L)}{Q} = \frac{(C_h - \lambda w)(1 - \frac{D}{R})}{C_h + C_s} . \qquad (2.31)$$

Squaring Equation (2.31), it becomes:

$$\frac{(DT - L)^2}{Q^2} = \frac{(C_h - \lambda w)^2 (1 - \frac{D}{R})^2}{(C_h + C_s)^2} . \qquad (2.32)$$

Substituting Equation (2.32) into Equation (2.29) gives:

$$- \frac{C_p D}{Q^2} + \frac{C_h (1 - \frac{D}{R})}{2} - \frac{C_h (C_h - \lambda w)^2 (1 - \frac{D}{R})}{2(C_h + C_s)^2}$$

$$- \frac{C_s (C_h - \lambda w)^2 (1 - \frac{D}{R})}{2(C_h + C_s)^2} - \lambda (1 - \frac{D}{R}) w = 0$$

$$\frac{C_p D}{Q^2} = \frac{(1 - \frac{D}{R})(C_h C_s - \lambda^2 w^2 - 2 C_s \lambda w)}{2(C_h + C_s)}$$

$$Q = \sqrt{\frac{1}{1 - \frac{D}{R}}} \sqrt{\frac{2C_p D (C_h + C_s)}{(C_h C_s - \lambda^2 w^2 - 2C_s \lambda w)}} . \qquad (2.33)$$

Equation (2.31) may be written as:

$$L = DT - \frac{Q(C_h - \lambda w)(1 - \frac{D}{R})}{(C_h + C_s)} . \qquad (2.34)$$

Substituting Equation (2.33) into Equation (2.34) gives:

$$L = DT - (C_h - \lambda w) \sqrt{1 - \frac{D}{R}} \sqrt{\frac{2C_p D}{(C_h C_s - \lambda^2 w^2 - 2C_s \lambda w)(C_h + C_s)}} . \qquad (2.35)$$

Minimum total system cost is obtained by substituting the results of Equations (2. 33) and (2. 35) into Equation (2. 15) utilizing the given parameters and varying values of λ. This is done until the largest value of λ is found such that $I^{*}w \leq W$ where I^{*} is determined from Equation (2. 5).

An Example Deterministic SISS Policy With Warehouse Restriction

As an example of the concept just developed suppose that the SISS system of the previous section is contrained by a total warehouse space of 100 cubic units: W = 100, and that the item in the system requires 24 cubic units of space per unit. Utilizing Equations (2. 33) and (2. 35) for varying values of λ gives the results of Table I.

TABLE I

WARHOUSE SPACE CONSUMED FOR VARYING VALUES OF λ, DETERMINISTIC SISS SYSTEM

λ	L	Q	$I^{*}w$
-0.00000	20.4843	43.0571	516.3762
-0.00900	15.3037	31.0608	104.3029
-0.00910	15.2103	31.0252	101.2067
-0.00913	15.1821	31.0147	100.2798
-0.00914	15.1727	31.0113	99.9712
-0.00915	15.1633	31.0078	99.6624
-0.01000	14.3370	30.7550	73.7691

The largest value of λ for which $I^{*}w$ is within the warehouse space restriction of 100 cubic units is $\lambda = -0.00914$. The optimal procurement and inventory policy associated with this value of λ is a procurement level of 15.1727 and a procurement quantity of 31.0113. Utilizing Equation (2.15) the minimum total system cost is found to be \$216.5481. The penalty in total system cost arising due to the warehouse constraint is \$216.5481 less \$214.9546 or \$1.5935 per period.

The Probabilistic SISS System

If demand and/or lead time is probabilistic, the resulting inventory process will be probabilistic. The exhibited geometry of such a process will depend upon the procurement level, the procurement quantity, the form and parameters of the demand distribution, the form and parameters of the lead time distribution, and the rate of replenishment. The expected geometry of a particular probabilistic system having an infinite replenishment rate is exhibited in Figure 7. The m subscripts denote expected values. The expected total system cost will depend upon the expected geometry, the expected item cost, the expected procurement cost, the expected holding cost, and the expected shortage cost.

Monte Carlo Analysis of Inventory Flow

The probabilistic inventory process may be most easily described by performing a Monte Carlo analysis of inventory flow over time. This does not mean that the simulated flow exactly parallels the real world process that it patterns. The simulation never deviates from the rules, while in the real world such compliance will not occur.

Nevertheless, the results provide a useful standard against which mathematical models for the probabilistic inventory process can be checked. This section will present an example upon which the derivations of subsequent sections will be based. It will be limited to a system with an infinite replenishment rate. Finite replenishment rates for probabilistic processes will be discussed in the next section.

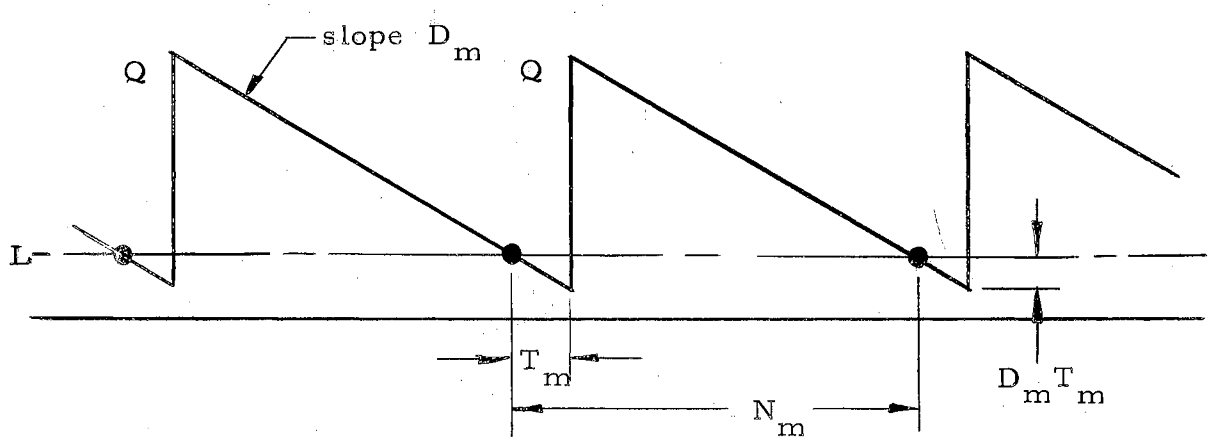


Figure 7. The Expected Geometry of a Probabilistic Inventory Process Having an Infinite Replenishment Rate

<u>Demand and lead time distributions.</u> The probabilistic inventory process usually involves both a demand distribution and a procurement lead time distribution. It is required that the form and parameters of these distributions be specified. The cumulative distributions may then be developed and used as a source of demand and lead time data needed in the analysis.

For the example under consideration, assume that demand has a

Poisson distribution with a mean of 0.6 units per period. Lead time will be assumed to have an empirical distribution with a mean of 4.3 periods. Figure 8 is an illustration of these distributions giving specific values for the random variables, together with their associated probabilities. Note that D_x and T_x are used to designate demand and lead time random variables, respectively, and that D_m and T_m are mean or expected values of the distributions.

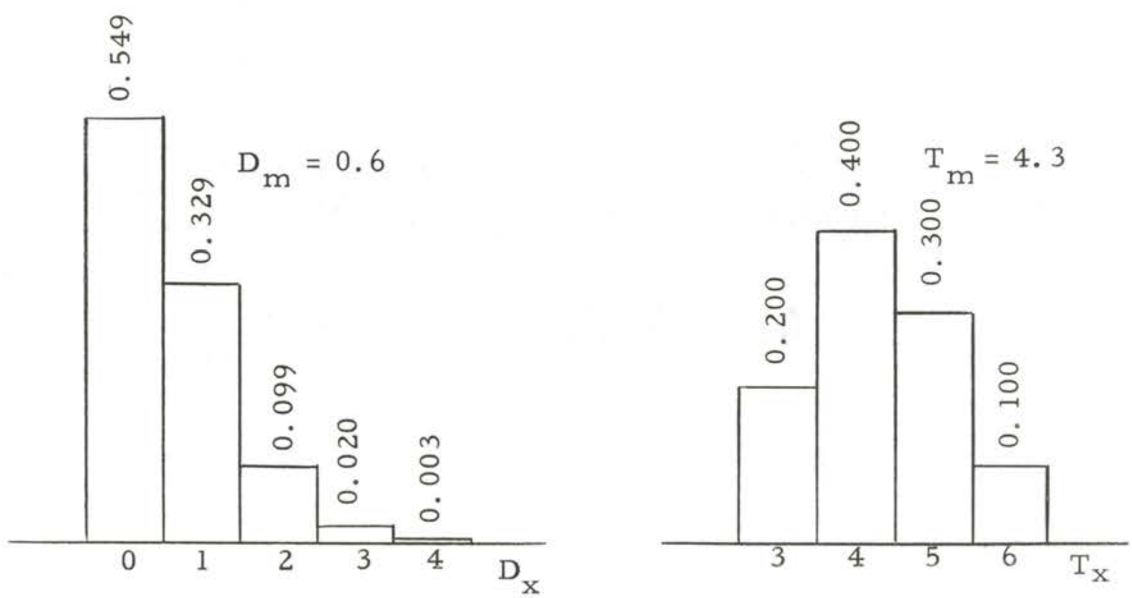


Figure 8. Demand and Lead Time Distributions for Monte Carlo Analysis

By summing the probabilities from left to right, and plotting the results, cumulative distributions may be developed. Figure 9 illustrates the cumulative distributions that result from the demand and lead time distributions of Figure 8. These are used with random rectangular variates to generate demand and lead time data for the simulated inventory flow process.

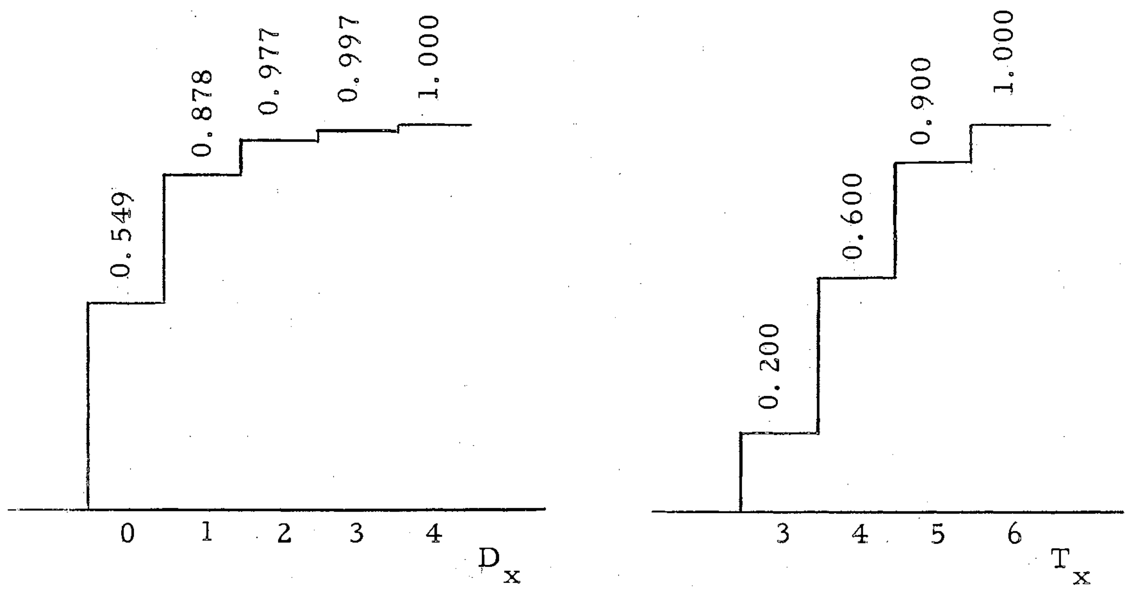


Figure 9. Cumulative Demand and Lead Time Distributions for Monte Carlo Analysis

The Monte Carlo Analysis. The inventory flow process operates in accordance with certain policies established by the decision maker. These must be obeyed by the Monte Carlo analysis. For this example, assume that the procurement level is three units and that the procurement quantity is 12 units. It will be shown later that these policies lead to a minimum total system cost for the example under consideration.

The simulation process of this example begins with the stock on hand equal to the procurement level. At the beginning of each period, the stock on hand is checked against the procurement level. If the procurement level has been reached or exceeded an order is placed for an amount equal to the procurement quantity. A value is then drawn at random from the lead time distribution and retained.

If the procurement level has not been reached, a value is drawn at random from the demand distribution. This value is subtracted from the stock on hand, resulting in a new stock level at the end of the period. Since one period has passed, 1 is subtracted from all outstanding lead time values. If a lead time value is reduced to zero, an amount equal to the procurement quantity is added to the stock on hand. The statistics for the period are calculated and the next period is considered. If a lead time value is not reduced to zero, period statistics are calculated and the next period is considered.

Output statistics for computer simulation. As the Monte Carlo analysis continues, and cycle summary data are developed, a composite picture of the probabilistic inventory process begins to develop. Table II is an abridged cycle-by-cycle summary of the simulated inventory flow performed on a digital computer for 4,000 cycles. Column 1 gives the cycle number. Column 2 gives the number of periods in the cycle, designated N_x, since it is a random variable. Column 3 gives the running average, N_m, of the individual values in column 2. Column 4 gives the total number of unit periods of stock on hand for the cycle. This is designated I_x, since it is also a random variable. Its running average, I_m, is given in column 5. Column 6 gives the

TABLE II

OUTPUT STATISTICS FOR COMPUTER SIMULATION

Cycle	N_x	N_m	I_x	I_m	S_x	S_m
1	20.000	20.000	126.000	126.000	0.000	0.000
2	27.000	23.500	189.000	157.500	0.000	0.000
3	22.000	23.000	173.000	162.666	0.000	0.000
4	11.000	20.000	50.500	134.625	0.000	0.000
5	15.000	19.000	58.000	119.300	2.000	0.400
6	13.000	18.000	97.500	115.666	0.000	0.333
7	21.000	18.428	162.500	122.357	0.000	0.285
8	23.000	19.000	170.000	128.312	0.000	0.250
9	18.000	18.888	94.500	124.555	5.000	0.777
10	27.000	19.700	220.000	134.100	0.000	0.700
11	16.000	19.363	85.000	129.636	0.000	0.636
12	15.000	19.000	61.500	123.958	4.500	0.958
13	22.000	19.230	176.000	127.961	0.000	0.884
14	18.000	19.142	111.000	126.750	0.000	0.821
15	18.000	19.066	112.000	125.766	0.000	0.766
16	6.000	18.250	5.500	118.250	15.500	1.687
17	14.000	18.000	73.000	115.588	0.500	1.617
18	24.000	18.333	153.500	117.694	0.000	1.527
19	20.000	18.421	110.000	117.289	0.500	1.473
20	19.000	18.450	133.500	118.100	0.000	1.400
-----	Cycles 21 Through 3980 Omitted	-----				
3981	19.000	19.864	132.000	131.867	0.000	0.898
3982	15.000	19.863	85.000	131.855	5.000	0.899
3983	21.000	19.863	152.000	131.860	0.000	0.899
3984	13.000	19.861	61.000	131.843	0.000	0.899
3985	19.000	19.861	125.000	131.841	0.000	0.899
3986	21.000	19.862	181.500	131.853	0.000	0.898
3987	13.000	19.860	68.000	131.837	1.500	0.899
3988	18.000	19.859	119.000	131.834	0.000	0.898
3989	25.000	19.861	141.000	131.836	1.000	0.898
3990	20.000	19.861	114.000	131.832	0.500	0.898
3991	28.000	19.863	234.500	131.858	0.000	0.898
3992	27.000	19.864	177.500	131.869	1.000	0.898
3993	13.000	19.863	62.000	131.852	0.500	0.898
3994	20.000	19.863	122.000	131.849	0.000	0.898
3995	11.000	19.861	64.000	131.832	0.000	0.897
3996	18.000	19.860	112.000	131.827	0.000	0.897
3997	18.000	19.860	143.000	131.830	0.000	0.897
3998	23.000	19.860	116.500	131.826	3.500	0.898
3999	29.000	19.863	220.500	131.848	0.000	0.897
4000	16.000	19.862	75.000	131.834	1.500	0.898

total number of unit periods of shortage for the cycle. This is a random variable and is designated S_x. Its running mean, S_m, is given in column 7.

The values for N_m, I_m, and S_m given at cycle 4,000 represent estimates of the expected values for N_x, I_x, and S_x, respectively. The relative stability of the mean values may be noted by comparing the terminal cycles with the initial cycles in Table II. Continuing the simulation beyond 4,000 cycles would contribute further to their stability.

Expressions for Expected Values

The simulation process of the previous section provides expected values for three important random variables associated with the probabilistic inventory system. These values are needed in the development of decision models for the system. However, use of the simulation method to derive expected values for even a limited number of procurement level and procurement quantity combinations is obviously unsatisfactory. Therefore, the purpose of this section will be to derive expressions that approximate N_m and I_m. A direct development for S_m will be considered in the sections which follow.

The expected inventory geometry. The expected inventory flow of a process subject to random elements would appear as in Figure 7. The geometry of the inventory process shown in Figure 7 is no different than for the deterministic system shown in Figure 6 with instantaneous replenishment. However, the orientation of Figure 7 is different from that of Figure 6. Provision is made for safety stock to absorb fluctuations in stock level from cycle-to-cycle. The need for this extra stock may be attributed to the presence of random elements.

<u>The expected number of periods per cycle.</u> Reference to Figure 7 indicates that the expected number of periods per cycle may be expressed as:

$$N_m = T_m + \frac{Q - D_m T_m}{D_m}$$

$$N_m = \frac{Q}{D_m} . \qquad (2.36)$$

The validity of this expression as a measure of the expected number of periods per cycle may be checked by reference to the simulated process. Substituting the values for Q and D_m used in the simulation results in:

$$N_m = \frac{12}{0.6} = 20.000.$$

Since the value found by simulation was 19.862, it may be concluded that Equation(2.36) gives a good means for approximating the expected number of periods per cycle for the probabilistic inventory process. Intuitive reasoning indicates that this expression yields an exact value; the discrepancy being due to the lack of complete convergence at 4,000 cycles.

<u>The expected total number of unit periods of stock.</u> Figure 7 indicates that the expected total number of unit periods of stock on hand during the cycle is the sum of two components. This may be approximated as:

$$I_m = N_m\left(\frac{Q}{2}\right) + N_m(L - D_m T_m)$$

$$I_m = \frac{Q}{D_m} \left[\frac{Q}{2} + (L - D_m T_m) \right]. \qquad (2.37)$$

The validity of Equation (2.37) as an approximation for the total number of unit periods of stock on hand for the cycle may be checked by substituting the values of Q, L, D_m, and T_m used in the simulation. This results in:

$$I_m = \frac{12}{0.6} \left[\frac{12}{2} + 3 - 0.6\,(4.3) \right] = 128.40.$$

The value found by simulation was 131.834 unit periods. The simulated result may be compared with the value found from Equation (2.37). Upon comparison, it may be concluded that Equation (2.37) yields only an approximation for the total number of unit periods of stock on hand for the cycle. This conclusion is supported by intuitive considerations and by the fact that a discrepancy of more than three unit periods is not likely to be entirely due to the lack of convergence at 4,000 cycles. The use of expected values to derive an expression for expected area yields a bias result.

The Distribution of Lead Time Demand

Expressions for the expected number of periods per inventory cycle, and for the expected number of unit periods of stock on hand for the cycle, were developed in the previous section. The derivation of an expression for the expected number of unit periods of shortage for the cycle will deviate from the procedure used there. It requires the development of the distribution of lead time demand as an important intermediate step. The paragraphs which follow will present to an exact numerical method for developing this distribution.

Lead time demand. Lead time demand is demand summed over the lead time. When both demand and lead time are random variables, lead time demand may be expressed mathematically as

$$Z_x = \sum^{T_x} D_x . \tag{2. 38}$$

This expression indicates that lead time demand is the sum of all demand over the lead time. With the distribution of D_x and T_x given, it is possible to develop the distribution of Z_x by Monte Carlo analysis. However, this method requires considerable computational effort to give a good approximation of the actual distribution. For complete generality it will be necessary to have an exact method for developing the lead time demand distribution.

Figure 10 illustrates conditional distributions of lead time demand for several specific values of lead time. When viewed as a single distribution, Figure 10 may be called a joint distribution of demand and lead time if the total probability is adjusted to unity. The probability associated with any specific value of lead time demand may then be found by summing for that value across all lead time values.

The previous qualitative description may be quantified by adopting the following notation:

$Z_x \mid T$ = lead time demand random variable given that lead time is T periods.

$f(Z_x \mid T)$ = conditional lead time demand distribution given that lead time is T periods.

The probability of $Z_x \geq Z$ for a specific lead time (conditional probability) is:

$$P(Z_x \geq Z \mid T) = \sum_{Z_x \mid T = Z}^{\infty} f(Z_x \mid T) .$$

Multiplying by $f(T_x)$ and summing over all values of T gives:

$$P(Z_x \geq Z) = \sum_{T=0}^{\infty} [f(T_x) \sum_{Z_x \mid T = Z}^{\infty} f(Z_x \mid T)] . \qquad (2.39)$$

The probability associated with each integral value of Z_x may be found from Equation (2.39). This procedure will be illustrated with an example based on the distributions of Figure 8.

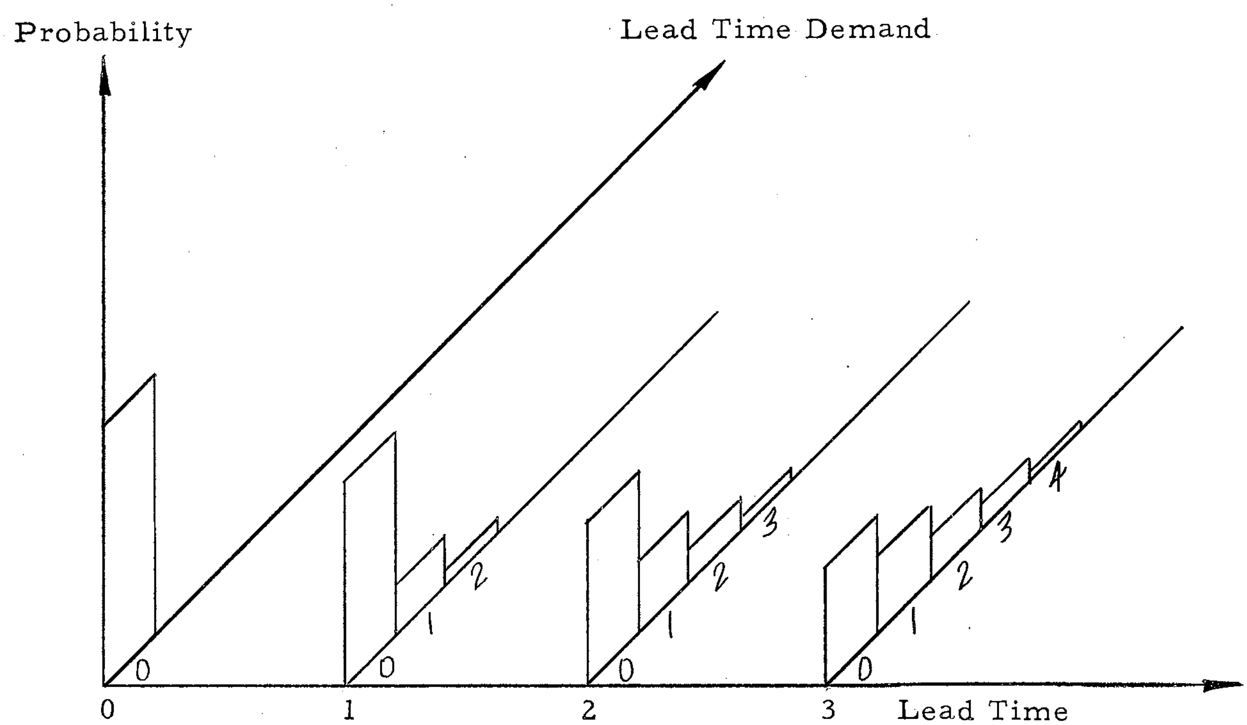


Figure 10. Joint Distributions of Demand and Lead Time

The numerical procedure presented in the example is applicable in those cases where demand has a Poisson, normal, or chi-square distribution. In selecting the conditional distributions, it is only necessary to increase the parameters of the basic demand distribution by multiplying by the specific conditional lead time value. If demand obeys some other distribution form, this method for selecting the conditional distributions does not hold. The distribution of lead time need not conform to any specific form. Any theoretical or empirical distribution may be used.

<u>Numerical development of lead time demand.</u> The computational procedure required in the development of the distribution of lead time demand may best be explained by reference to Table III. The first section is analogous to Figure 10 in that it gives the conditional distribution of lead time demand associated with each lead time value. For the case under consideration, conditional distributions are required for lead time values of 3, 4, 5, and 6. These conditional distributions are selected in accordance with the following rules:

(1) If lead time is 1 period, the basic demand distribution is the lead time demand distribution. The probabilities of each value of D_x would be associated with the respective values of Z_x under $T_x = 1$, if $T_x = 1$ were called for.

(2) Enter Z_x probabilities under $T_x = 2$, $T_x = 3, \ldots,$ associated with a demand distribution of the same form as the basic demand distribution, but with parameters increased by multiples of 2, 3, ..., etc. In Table III, this calls for Poisson probabilities for distributions with mean values of 1.8, 2.4, 3.0, and 3.6.

TABLE III

NUMERICAL DEVELOPMENT OF LEAD TIME DEMAND DISTRIBUTION

Z_x	$f(Z_x \mid T=3)$	$f(Z_x \mid T=4)$	$f(Z_x \mid T=5)$	$f(Z_x \mid T=6)$	← ADJUSTMENTS →				$P(Z_x)$
0	0.1653	0.0907	0.0498	0.0273	0.03306	0.03628	0.01494	0.00273	0.08701
1	0.2975	0.2177	0.1494	0.0984	0.05950	0.08708	0.04482	0.00984	0.20124
2	0.2678	0.2613	0.2240	0.1771	0.05356	0.10452	0.06720	0.01771	0.24299
3	0.1607	0.2090	0.2240	0.2125	0.03214	0.08360	0.06720	0.02125	0.20419
4	0.0723	0.1254	0.1680	0.1912	0.01446	0.05016	0.05040	0.01912	0.13414
5	0.0260	0.0602	0.1008	0.1377	0.00520	0.02408	0.03024	0.01377	0.07329
6	0.0078	0.0241	0.0504	0.0826	0.00156	0.00964	0.01512	0.00826	0.03458
7	0.0020	0.0083	0.0216	0.0425	0.00040	0.00332	0;00648	0.00425	0.01445
8	0.0005	0.0025	0.0081	0.0191	0.00010	0.00100	0.00243	0.00191	0.00544
9	0.0001	0.0007	0.0027	0.0076	0.00002	0.00028	0.00081	0.00076	0.00187
10		0.0002	0.0008	0.0028		0.00008	0.00024	0.00028	0.00060
11			0.0002	0.0009			0.00006	0.00009	0.00015
12			0.0001	0.0003			0.00003	0.00003	0.00004*
13				0.0001				0.00001	0.00001

*Arbitrarily reduced from 0.00006 so that $\Sigma P(Z_x) = 1.00000$

The second section of Table III involves adjustment of the total probability so it will sum to unity. The procedure is described by Equation (2.39) and is accomplished by multiplying each value of each conditional distribution by the probability of T_x taking its associated value. The result is a joint probability density function from which the lead time demand distribution may be developed.

The probability of lead time demand assuming the specific values specified as Z_x in Table III may be found by summing across all values of T_x in the second section. The results are entered under $P(Z_x)$ in the last column and make up a demand marginal distribution. This demand marginal is the required lead time demand distribution for the demand and lead time distribution of Figure 8. It is histogrammed in Figure 11.

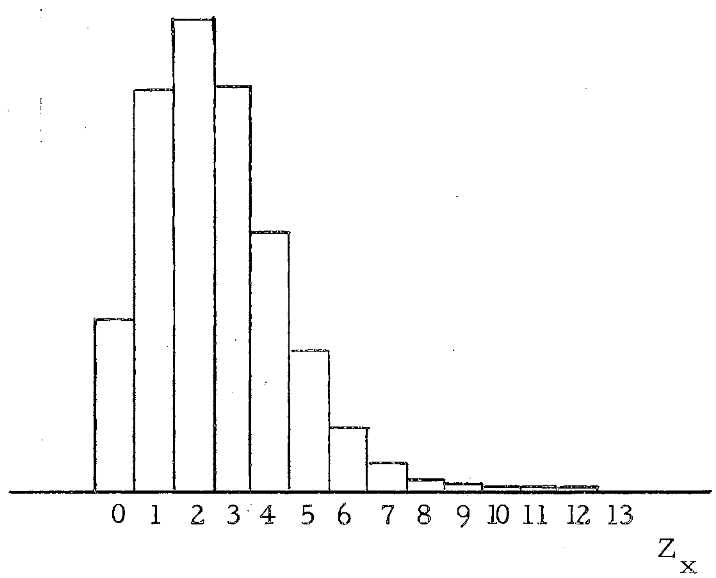


Figure 11. Distribution of Lead Time Demand

Expressions for Shortage Condition

Lead time demand is independent of the procurement level. A lead time demand distribution simply exhibits the number of demands that may occur during the lead time. The shortage conditions at the end of the inventory cycle depends jointly upon the distribution of lead time demand and the procurement level choice. In this section approximations for the probability of an empty warehouse, the probability of one or more shortages, the expected number of shortages, and the expected number of unit periods of shortage will be developed. Completion of this phase will provide the third expected value needed in the derivation of effectiveness functions for the probabilistic inventory process.

The probability of an empty warehouse. An empty warehouse will result if lead time demand is equal to or greater than the procurement level. If the lead time demand distribution is continuous, the probability of an empty warehouse at the end of the inventory cycle may be expressed as:

$$P\,[\text{empty warehouse}] = \int_{L}^{\infty} f(Z_x)\,dZ_x.$$

For the discrete lead time demand distribution of Figure 14, whose maximum is Z^*, the probability of an empty warehouse is:

$$P\,[\text{empty warehouse}] = \sum_{L}^{Z^*} f(Z_x).$$

The second column of Table IV gives the probability of an empty warehouse as a function of the procurement level.

The probability of an empty warehouse, as an expression for shortage condition, fails to give a measure of the magnitude of the shortage condition (if any) or the time duration involved. As such, it is very difficult to establish a value for shortage cost. In fact, an empty warehouse is desirable if during this period no demand occurs.

TABLE IV

SHORTAGE PROBABILITIES AS A FUNCTION OF L

L	P [empty warehouse]	P [1 or more short]
0	1.00000	0.91299
1	0.91299	0.71175
2	0.71175	0.46876
3	0.46876	0.26457
4	0.26457	0.13043
5	0.13043	0.05714
6	0.05714	0.02256
7	0.02256	0.00811
8	0.00811	0.00267
9	0.00267	0.00080
10	0.00080	0.00020
11	0.00020	0.00005
12	0.00005	0.00001
13	0.00001	0.00000

<u>The probability of one or more shortages.</u> One or more shortages will result if lead time demand is greater than the procurement level. If the lead time demand distribution is continuous, the probability of one or more shortages at the end of the inventory cycle may be

expressed as:

$$P\ [1 \text{ or more short}] = \int_{L+1}^{\infty} f(Z_x)\, d\, Z_x.$$

For the discrete lead time demand distribution of Figure 11, the probability of one or more shortages is:

$$P\ [1 \text{ or more short}] = \sum_{L+1}^{Z^*} f(Z_x).$$

The third column of Table IV gives the probability of one or more shortages as a function of the procurment level.

The probability of one or more shortages establishes with certainty the fact that a shortage condition exists. However, like the probability of an empty warehouse, it does not give a measure of the magnitude of the shortage condition or its time duration. It is, therefore, difficult to establish a value of shortage when using this measure.

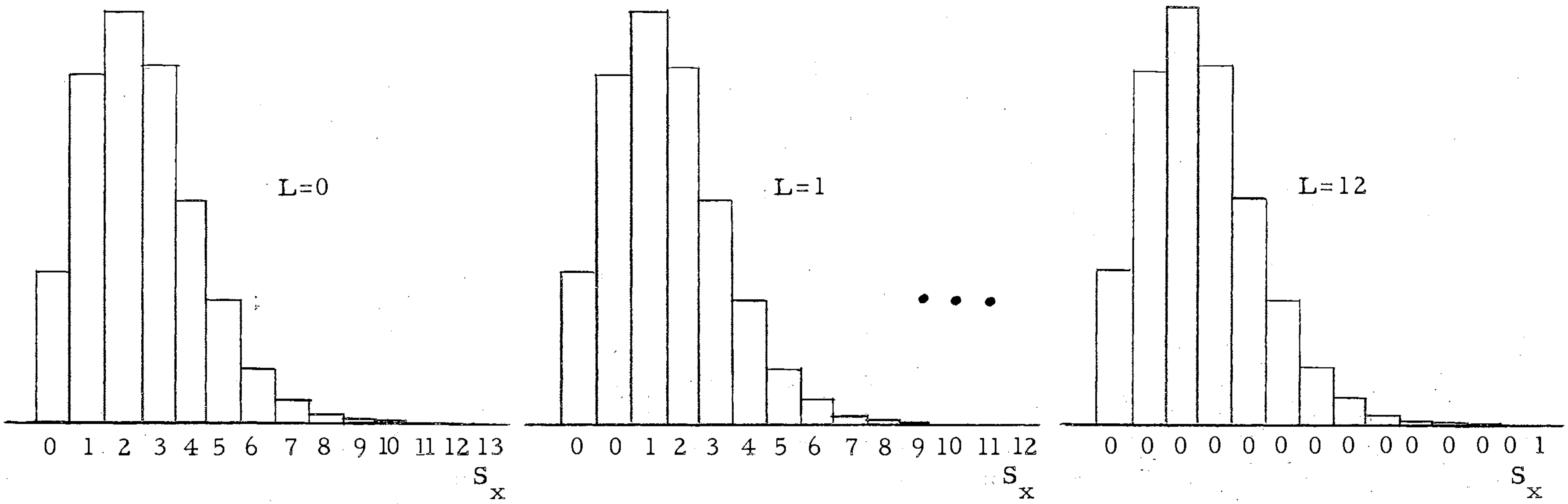
The expected number of shortages. If the lead time demand distribution is continuous, the expected number of shortages per inventory cycle may be expressed as:

$$E\ [\text{number of shortages}] = \int_{L+1}^{\infty} (Z_x - L) f(Z_x) dZ_x.$$

For the discrete lead time demand distribution of Figure 11, the expected number of shortages is:

$$E\ [\text{number of shortages}] = \sum_{L+1}^{Z^*} (Z_x - L) f(Z_x). \quad (2.40)$$

The application of Equation (2.40) is illustrated in Figure 12 and requires the development of one shortage distribution for each procurement level choice. When L = 0, the lead time demand distribution is the shortage distribution. This is verified by reasoning as



0.08701 (0)	=	0.00000
0.20124 (1)	=	0.20124
0.24299 (2)	=	0.48598
0.20419 (3)	=	0.61257
0.13414 (4)	=	0.53656
0.07329 (5)	=	0.36645
0.03458 (6)	=	0.20748
0.01445 (7)	=	0.10115
0.00544 (8)	=	0.04352
0.00187 (9)	=	0.01683
0.00060 (10)	=	0.00600
0.00015 (11)	=	0.00165
0.00004 (12)	=	0.00048
0.00001 (13)	=	0.00013
$E[S_x]$	=	2.58004

0.08701 (0)	=	0.00000
0.20124 (0)	=	0.00000
0.24299 (1)	=	0.24299
0.20419 (2)	=	0.40838
0.13414 (3)	=	0.40242
0.07329 (4)	=	0.29316
0.03458 (5)	=	0.17290
0.01445 (6)	=	0.08670
0.00544 (7)	=	0.03808
0.00187 (8)	=	0.01496
0.00060 (9)	=	0.00540
0.00015 (10)	=	0.00150
0.00004 (11)	=	0.00044
0.00001 (12)	=	0.00012
$E[S_x]$	=	1.66705

0.08701 (0)	=	0.00000
0.20124 (0)	=	0.00000
0.24299 (0)	=	0.00000
0.20419 (0)	=	0.00000
0.13414 (0)	=	0.00000
0.07329 (0)	=	0.00000
0.03458 (0)	=	0.00000
0.01445 (0)	=	0.00000
0.00544 (0)	=	0.00000
0.00187 (0)	=	0.00000
0.00060 (0)	=	0.00000
0.00015 (0)	=	0.00000
0.00004 (0)	=	0.00000
0.00001 (1)	=	0.00001
$E[S_x]$	=	0.00001

Figure 12. Development of Shortage Distributions

follows. If no demands occur during the lead time, no shortages will result; if one demand occurs, one shortage will result; if two demands occur, two shortages will result, etc. The probability of each of these events is given by the lead time demand distribution. Therefore, the expected number of shortages for $L = 0$ is the mean of the shortage distribution for that L choice. This is shown as the first phase of Figure 12.

The second phase of Figure 12 gives the shortage distribution for the case where $L = 1$. It is developed by reasoning as follows. If no demands occur during the lead time, no shortages will result; if one demand occurs, no shortages will result; if two demands occur, one shortage will result; if three demands occur, two shortages will result, etc. Again, the probability of each of these events is given by the lead time demand distribution. The mean for the resulting shortage distribution is calculated in Figure 12.

The process outlined above is continued for all values of L up to $L = Z^*$. For $L = 12$ it is evident that no shortages will occur for all values of lead time demand except 13. If lead time demand is 13, one shortage will occur. This is shown in the last phase of Figure 12. The expected value for the resulting shortage distribution is calculated as before and is found to be 0.00001. If $L = 13$, it is evident that no shortages will occur for any allowable value of lead time demand up to and including Z^*. Therefore, the expected number of shortages for this last case will be zero. The second column of Table V gives the expected number of shortages per inventory cycle as a function of the procurement level.

TABLE V

SHORTAGE EXPECTATION AS A FUNCTION OF L

L	E [shortages]	S_m
0	2.58004	5.5472
1	1.66705	2.3158
2	0.95530	0.7605
3	0.48654	0.1973
4	0.22197	0.0411
5	0.09154	0.0070
6	0.03440	0.0010
7	0.01184	0.0001
8	0.00373	0.0000
9	0.00106	0.0000
10	0.00026	0.0000
11	0.00006	0.0000
12	0.00001	0.0000
13	0.00000	0.0000

A measure of the magnitude of the shortage condition is provided by an expression for the expected number of shortages. Although the time duration involved is not specified, it is possible to establish a fairly good value of shortage cost when using this expression.

<u>The expected number of unit periods of shortage.</u> By utilizing the values for the expected number of shortages per inventory cycle, it is possible to derive an approximate expression for the expected number of unit periods of shortage. This is the value previously developed by simulation. It is an area which may be approximated as:

$$S_m = \frac{[E(S_x)]^2}{2D_m} . \qquad (2.41)$$

The third column of Table V gives specific values for S_m as a function of L. Since these values are based on the same inputs as were used in the simulation, a comparison can be made. The simulated value for S_m, given in Table II, is 0.898 unit periods. Since the procurement level was set at 3 units, this is to be compared to 0.1973 given in Table V. The discrepancy may be explained by the fact that using expected values to find an area is bias, as was the case with the expected total number of unit periods of stock. In addition, procurement action is initiated after the stock level falls below the procure-level for some cycles. The effect of this situation is to force a more severe shortage condition than the assumption that procurement action is initiated exactly on the procurement level.

The expected number of unit periods of shortage per cycle gives a measure of the magnitude and time duration of the shortage condition. As a result, the assignment of a value for shortage cost is not as difficult as for the previous expressions for shortage condition. Although the derived value for S_m does not agree with the simulated value, its deviation tends to cancel that of I_m, since total system cost models utilizing these expected values trade off costs based on their magnitudes.

Minimum Cost Policies for Numerical Lead Time Demand

By utilizing the previously derived approximations for N_m, I_m, and S_m, it is possible to develop a model that may be used to find minimum cost inventory policies. In this section an expected value model will be presented that trades off expected item cost, expected procurement cost, expected holding cost, and expected shortage cost. It will provide a means for finding the minimum cost procurement level and procurement quantity simultaneously.

Expected total system cost as a function of L and Q. When the procurement quantity is not restricted to a specific value, the expected total system cost per period will be the sum of the expected item cost per period, the expected procurement cost per period, the expected holding cost per period, and the expected shortage cost per period; that is:

$$TC_m = IC_m + PC_m + HC_m + SC_m .$$

The expected item cost per period will be the product of the item cost per unit and the expected demand rate in units per period; that is:

$$IC_m = C_i D_m .$$

The expected procurement cost per period is the procurement cost per procurement divided by the expected number of periods per inventory cycle; that is:

$$PC_m = \frac{C_p}{N_m} .$$

Substituting Equation (2. 36) for N_m gives:

$$PC_m = \frac{C_p D_m}{Q} .$$

The expected holding cost per period will be the holding cost per unit per period multiplied by the expected number of units in stock for the period; that is:

$$HC_m = \frac{C_h I_m}{N_m} .$$

Substituting Equation (2. 36) for N_m and Equation (2. 37) for I_m gives:

$$HC_m = C_h \left[\frac{Q}{2} + (L - D_m T_m) \right].$$

The expected shortage cost per period will be the shortage cost per unit short per period multiplied by the expected number of unit periods of shortage for the period; that is:

$$SC_m = \frac{C_s S_m}{N_m} .$$

Substituting Equation (2.36) for N_m gives:

$$SC_m = \frac{C_s D_m S_m}{Q} .$$

The expected total system cost per period will be a summation of the four cost components developed above, and may be expressed as:

$$TC_m = C_i D_m + \frac{C_p D_m}{Q} + C_h \left[\frac{Q}{2} + (L - D_m T_m) \right] + \frac{C_s D_m S_m}{Q} . \tag{2.42}$$

<u>An example probabilistic SISS policy for numerical lead time demand.</u> Minimization of Equation (2.42) by partial differentiation is not possible. Like Equation (2.41), it contains S_m which is only numerically related to L. As an example of the determination of the minimum cost procurement level and procurement quantity, consider the following situation. Demand and lead time are distributed as shown in Figure 8. Item cost per unit is \$15.00. Procurement cost per procurement is \$10.00. Holding cost per unit per period is \$0.09 and shortage cost per unit short per period is \$3.50. Therefore, the expected total system cost as a function of the procurement level and procurement quantity is:

$$TC_m = \$15.00\,(0.6) + \frac{\$10.00\,(0.6)}{Q} + \$.09\left[\frac{Q}{2} + (L - 2.58)\right] + \$3.50\left(\frac{0.6}{Q}\right) S_m .$$

The expected total system cost as a function of L and Q is given in Table VI. Each value is computed from the above expression with reference to Table V for values of S_m. As before, each entry is actually an expected value from a total system cost distribution. Choosing the L and Q giving a minimum expected cost is equivalent to maximizing the probability of minimizing the sum of item cost per period, procurement cost per period, holding cost per period, and shortage cost per period.

TABLE VI

EXPECTED TOTAL SYSTEM COST AS A FUNCTION OF L AND Q

L \ Q	10	11	12	13	14
0	10.983	10.867	10.779	10.711	10.659
1	10.494	10.340	10.303	10.279	10.264
2	10.158	10.133	10.121	10.118	10.121
3	10.129	10.116	10.113	10.117	10.127
4	10.187	10.176	10.175	10.182	10.193
5	10.269	10.259	10.259	10.266	10.278
6	10.358	10.348	10.348	10.355	10.367

The minimum expected cost procurement level and procurement quantity is found by inspection to be 3 and 12, respectively. These are the values that were used in the Monte Carlo analysis. They give an expected total system cost of $10.113 when used with the expressions for expected values. Any error in these values will be reflected in the expected total system cost. Using the expected values found by Monte

Carlo analysis to compute the expected total system cost gives a value of \$10.258.

A Simplified Probabilistic SISS System

The total system cost functions derived in the previous section could not be minimized by direct mathematical means. This was because the term S_m was not a mathematical function of L. This section will adopt two simplifications so that a method of finding minimum cost inventory policy mathematically for the probabilistic system may be demonstrated. Specifically, this will require that shortage cost, C_s', be based on the expected number of shortages, and that the lead time demand distribution, Z_x, be a simple function. In this case it is necessary to maintain "safety stock" to absorb lead time demand fluctuations in excess of the expected lead time demand. The geometry of the inventory process would appear as in Figure 13 if random elements were not present. The development of simplified probabilistic models in this dissertation is based on the assumption that $D_m T_m \leq L$, $D_m > R$, and $Q \geq \min(1, D_m)$.

Algebraic Relationships

From Figure 13 it is evident that the expected number of periods per inventory cycle is:

$$N_m = \frac{Q}{D_m}. \tag{2.43}$$

Also, the following relationships are evident:

$$n_1(R - D_m) = n_2(D_m) \tag{2.44}$$

$$n_1 = \frac{Q}{R} \tag{2.45}$$

$$n_2 = \frac{I^*_m + D_m T_m - L}{D_m} \tag{2.46}$$

From Equations (2.44), (2.45), and (2.46),

$$I^*_m = Q(1 - \frac{D_m}{R}) + L - D_m T_m. \tag{2.47}$$

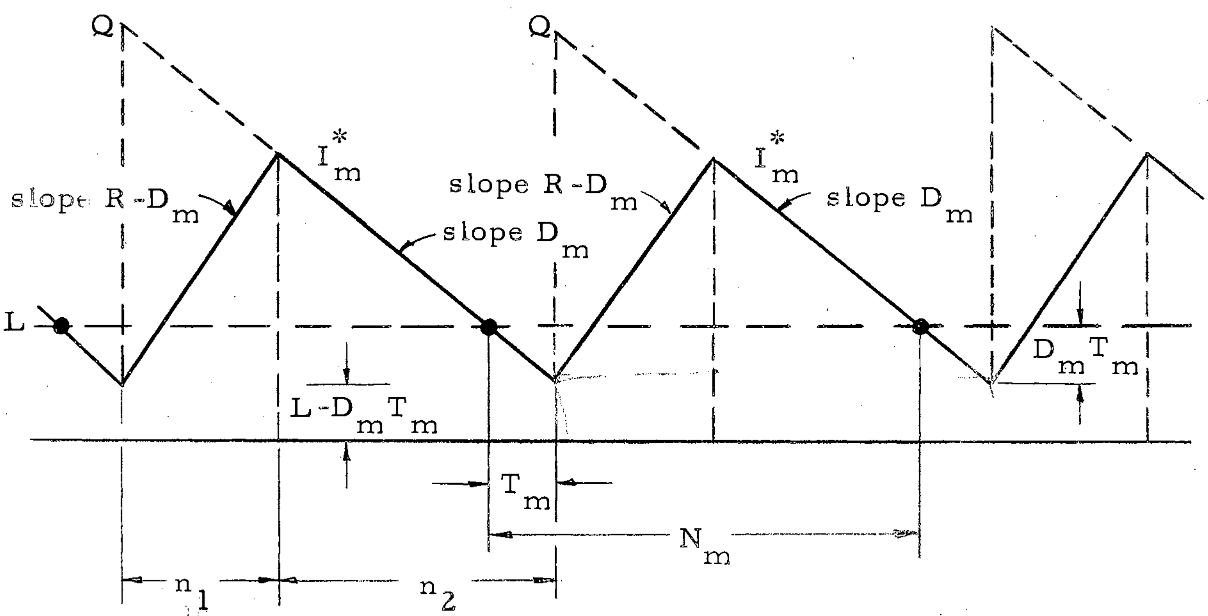


Figure 13. The Expected Geometry of a Simplified Probabilistic Inventory Process

The expected total number of unit periods of stock on hand during the inventory cycle, I_m, is:

$$I_m = \left(\frac{I^*_m - L + D_m T_m}{2}\right)\left(\frac{I^*_m - L + D_m T_m}{R - D_m}\right) + \left(\frac{I^*_m - L + D_m T_m}{2}\right)\left(\frac{I^*_m - L + D_m T_m}{D_m}\right)$$

$$+ N_m(L - D_m T_m)$$

$$I_m = \frac{(I_m^* - L + D_m T_m)^2}{2} \left(\frac{1}{D_m} + \frac{1}{R - D_m}\right) + N_m(L - D_m T_m).$$

Substituting Equation (2.47) for I_m^* gives:

$$I_m = \frac{\left[Q\left(1 - \frac{D_m}{R}\right)\right]^2}{2}\left(\frac{1}{D_m} + \frac{1}{R-D_m}\right) + N_m(L - D_m T_m). \qquad (2.48)$$

If it is assumed that lead time demand, Z_x, is distributed uniformly in the range A to A', $f(Z_x) = 1/(A' - A)$, the expected number of shortages per inventory cycle, $E(S_x)$, is:

$$E(S_x) = \int_{L+1}^{A'} (Z_x - L) f(Z_x) dZ_x$$

$$E(S_x) = \int_{L+1}^{A'} \frac{Z_x dZ_x}{A'-A} - \int_{L+1}^{A'} \frac{L}{A'-A} dZ_x$$

$$E(S_x) = \frac{1}{2(A'-A)} Z_x^2 \Big|_{L+1}^{A'} - \frac{L}{A'-A} Z_x \Big|_{L+1}^{A'}$$

$$E(S_x) = \frac{1}{2(A'-A)} A'^2 - (L+1)^2 - \frac{L}{A'-A} (A'-L-1)$$

$$E(S_x) = \frac{A'^2 - 2L(A') + L^2 - 1}{2(A'-A)}. \qquad (2.49)$$

The expected number of shortages is simplified if $A = 0$. Then,

$$E(S_x) = \frac{A'^2 - 2L(A') + L^2 - 1}{2A'}. \qquad (2.50)$$

The lead time demand random variable is the product of the demand random variable and the lead time random variable; that is:

$$Z_x = D_x T_x.$$

The expected lead time demand is:

$$E(Z_x) = E(D_x T_x).$$

Assuming independence, the expected lead time becomes:

$$E(Z_x) = E(D_x) E(T_x).$$

Taking the expected value of both sides gives:

$$Z_m = D_m T_m.$$

When $f(Z_x)$ is distributed uniformly from 0 to A',

$$Z_m = \frac{A'}{2}.$$

Thus:

$$D_m T_m = \frac{A'}{2}$$

$$T_m = \frac{A'}{2D_m} \qquad (2.51)$$

$$A' = 2D_m T_m. \qquad (2.52)$$

By specifying any two of the values in Equations (2.51) and (2.52) the third value is established.

Expected Total System Cost

The expected total system cost per period will be a summation of the expected item cost per period, the expected procurement cost per period, the expected holding cost per period, and the expected shortage cost per period; that is:

$$TC_m = IC_m + PC_m + HC_m + SC_m. \qquad (2.53)$$

The expected item cost per period will be the product of the item cost per unit and the expected demand rate in units per period; that is:

$$IC_m = C_i D_m. \qquad (2.54)$$

The expected procurement cost per period will be the procurement cost per procurement divided by the number of periods per inventory cycle; that is:

$$PC_m = \frac{C_p}{N_m}$$

$$PC_m = \frac{C_p D_m}{Q} . \qquad (2.55)$$

The expected holding cost per period will be the product of the holding cost per unit and the expected number of units on hand during the period; that is:

$$HC_m = \frac{C_h I_m}{N_m}$$

$$HC_m = \frac{C_h D_m}{Q}\left[\frac{[Q(1-\frac{D_m}{R})]^2}{2}\right](\frac{1}{R-D_m} + \frac{1}{D_m}) + C_h(L-D_m T_m).$$

Note that:

$$\frac{D_m}{Q}(\frac{1}{R-D_m} + \frac{1}{D_m}) = \frac{1}{Q(1-\frac{D_m}{R})} . \qquad (2.56)$$

Therefore,

$$HC_m = C_h\left[\frac{Q(1-\frac{D_m}{R})}{2} + L - D_m T_m\right] . \qquad (2.57)$$

The expected shortage cost per period will be the product of the shortage cost per unit short per period, C_s', and the expected number of shortages per period; that is:

$$SC_m = \frac{C_s' E(S_x)}{N_m}$$

$$SC = \frac{C_s' D_m (A'^2 - 2LA' + L^2 - 1)}{2QA'} . \tag{2.58}$$

The expected total system cost per period will be a summation of the four cost components given by Equations (2.54), (2.55), (2.57), and (2.58); that is:

$$TC_m = C_i D_m + \frac{C_p D_m}{Q} + C_h \left[\frac{Q(1 - \frac{D_m}{R})}{2} + L - D_m T_m \right]$$

$$+ \frac{C_s' D_m (A'^2 - 2LA' + L^2 - 1)}{2QA'} \tag{2.59}$$

or,

$$TC = C_i D_m + \frac{C_p D_m}{Q} + C_h \left[\frac{Q(1 - \frac{D_m}{R})}{2} + L - D_m T_m \right]$$

$$+ \frac{C_s' D_m [(A' - L)^2 - 1]}{2QA'} . \tag{2.60}$$

All terms in Equation (2.60) must be positive. For certain values of the parameters the last term can be negative. To insure that this term be positive it is required that $(A' - L)^2$ be positive, or,

$$(A' - L)^2 - 1 \geq 0.$$

Taking the positive root gives:

$$(A' - L) \geq 1.$$

Solving for L gives:

$$-L \geq 1 - A'$$

$$L \leq A' - 1.$$

Utilizing Equation (2.52) gives:

$$L \leq 2D_m T_m - 1.$$

Therefore, an upper bound on L is established in addition to the previously stated lower bound.

Optimal Policy for Simplified Probabilistic SISS System

The minimum cost procurement level and procurement quantity may be found by setting the partial derivatives equal to zero and solving the resulting equations. Modifying Equation (2.59) gives:

$$TC_m = C_i D_m + \frac{C_p D_m}{Q} + C_h \left[\frac{Q(1 - \frac{D_m}{R})}{2} + L - D_m T_m \right]$$

$$+ \frac{C_s' D_m}{Q} \left(\frac{A'}{2} - L + \frac{L^2 - 1}{2A'} \right). \tag{2.61}$$

Taking the partial derivative of Equation (2.61) with respect to Q, then with respect to L, and setting both equal to zero gives:

$$\frac{\partial TC}{\partial Q} = - \frac{C_p D_m}{Q^2} + \frac{C_h}{2} (1 - \frac{D_m}{R}) - \frac{C_s' D_m}{Q^2} [\frac{A'}{2} - L + \frac{L^2 - 1}{2A'}] = 0 . \tag{2.62}$$

$$\frac{\partial TC}{\partial L} = C_h - \frac{C_s' D_m}{Q} + \frac{L C_s' D_m}{QA'} = 0. \tag{2.63}$$

Equation (2.63) may be expressed as:

$$\frac{L C_s' D_m}{QA'} = \frac{C_s' D_m}{Q} - C_h$$

$$L = A' - \frac{C_h Q A'}{C_s' D_m} . \tag{2.64}$$

Substituting Equation (2.64) into Equation (2.62) gives:

$$- \frac{C_p D_m}{Q^2} + \frac{C_h}{2}(1 - \frac{D_m}{R}) - \frac{C_s' D_m}{Q^2} \left[-\frac{A'}{2} + \frac{C_h Q A'}{C_s' D_m} + \frac{(A' - \frac{C_h Q A'}{C_s' D_m})^2 - 1}{2A'} \right] = 0. \quad (2.65)$$

The last term may be expressed as:

$$- \left[\frac{C_h{}^2 Q^2 A'^2 - C_s'{}^2 D_m{}^2}{2A' C_s' D_m Q^2} \right].$$

And Equation (2.65) becomes:

$$- \frac{C_p D_m}{Q^2} + \frac{C_h}{2}(1 - \frac{D_m}{R}) - \left[\frac{C_h{}^2 Q^2 A'^2 - C_s'{}^2 D_m{}^2}{2A' C_s' D_m Q^2} \right] = 0.$$

Which will reduce to:

$$Q = D_m \sqrt{\frac{C_s'}{A' C_h} \left[\frac{2A' C_p - C_s'}{C_s' D_m (1 - \frac{D_m}{R}) - A' C_h} \right]}. \quad (2.66)$$

Substituting Equation (2.66) into Equation (2.64) gives:

$$L = A' - \sqrt{\frac{A' C_h}{C_s'} \left[\frac{2A' C_p - C_s'}{C_s' D_m (1 - \frac{D_m}{R}) - A' C_h} \right]}. \quad (2.67)$$

An Example Simplified Probabilistic SISS Policy

As an example of the simplified probabilistic SISS system suppose that a procurement manager will purchase an item having the following parameters:

D_m 2.00
R. ∞
T_m 4.00
A' 8.00
C_i $6.30
C_p $6.25
C_h $0.10
C_s' . . . . $4.00 .

The minimum cost procurement quantity may be found from Equation (2.66) as:

$$Q = 2\sqrt{\frac{4}{8(\$0.10)}\left[\frac{2(8)(\$6.25)-\$4.00}{\$4.00(2)(1-\frac{2}{\infty})-8(\$0.10)}\right]}$$

$$Q = 17.5021.$$

The minimum cost procurement level may be found from Equation (2.67) as:

$$L = 8 - \sqrt{\frac{8(\$0.10)}{\$4.00}\left[\frac{2(8)(\$6.25) - \$4.00}{\$4.00(2)(1-\frac{2}{\infty}-8(\$0.10)}\right]}$$

$$L = 12.4995.$$

The minimum total system cost may be found by substituting the results of Equations (2.66) and (2.67) into Equation (2.60) as:

$$TC_m = \$6.30(2) + \frac{\$6.25(2)}{17.5021} + \$0.10\left[\frac{(17.5021)(1-\frac{2}{\infty})}{2}\right.$$

$$\left. + 12.4995 - 2(4)\right] + \frac{\$4.00(2)}{17.5021}\left[\frac{8}{2} - 12.4995 + \frac{(12.4995)^2-1}{2(8)}\right]$$

$$TC_m = \$14.7998.$$

CHAPTER III

THE SIMS SYSTEM

A SIMS procurement and inventory system is illustrated in Figure 14. It exists as a result of the demand stimulus, D. In satisfying this demand the procurement manager finds it necessary to replenish the stock of the item periodically. The basic supposition of the SIMS concept allows stock replenishment to be made by procurement from one of several possible sources. Therefore, an important facet of the procurement and inventory problem involves a choice of the source that will result in a minimum total system cost. Procurement and inventory policy for the SIMS system will be that policy stating <u>when</u> to procure, <u>how much</u> to procure, and <u>from what source</u> to procure. It will be the purpose of this chapter to indicate the unified nature of procurement and inventory operations through a consideration of source dependent parameters.

The Deterministic SIMS System

An Example Deterministic SIMS Policy

As discussed in Chapter I, procurement lead time, rate of replenishment, item cost, and procurement cost are all source dependent. All other parameters remain constant for a SIMS system. This permits the use of Equations (2.20), (2, 21), and (2.22) for the solution of deterministic SIMS problems without restrictions. The procedure is to evaluate each source, selecting that source which can supply the demand at the minimum total system cost.

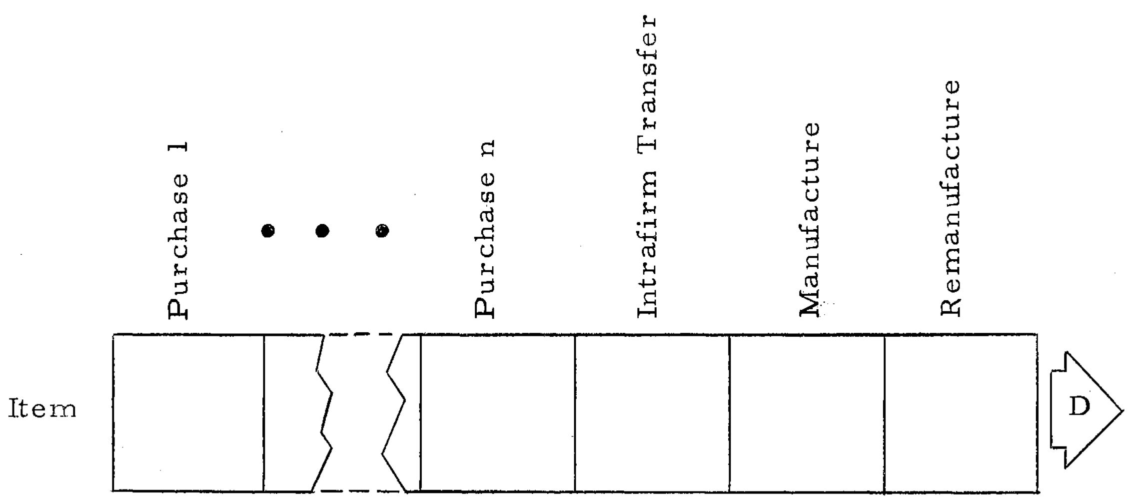


Figure 14. The SIMS System

As an example of the deterministic unrestricted SIMS system suppose that a procurement manager is experiencing a demand of 4 units per period for a certain item that may be either manufactured, or purchased from one of three vendors. Holding cost per period is \$0.24 and shortage cost per unit short per period is \$0.17. Specific values for source dependent parameters are given in Table 7.

The procurement source resulting in a minimum total system cost can be found from Equation (2.22). For the manufacturing alternative it is:

$$TC = \$19.85(4) + \sqrt{1 - \frac{4}{12}} \sqrt{\frac{2(\$17.32)(\$0.24)(\$0.17)(4)}{\$0.24 + \$0.17}}$$

$$TC = \$82.4318.$$

TABLE VII

SOURCE DEPENDENT PARAMETERS, DETERMINISTIC SIMS SYSTEM

Parameter	Manufacture	Purchase 1	Purchase 2	Purchase 3
R	12.00	∞	∞	∞
T	6.00	3.00	4.00	12.00
C_i	\$19.85	\$17.94	\$18.33	\$18.08
C_p	\$17.32	\$18.70	\$17.50	\$14.65

For the alternative designated Purchase 1, it is:

$$TC = \$17.94\ (4) + \sqrt{1 - \frac{4}{\infty}} \sqrt{\frac{2(\$18.70)(\$0.24)(\$0.17)(4)}{\$0.24 + \$0.17}}$$

$$TC = \$75.6175.$$

For the alternative designated Purchase 2, it is:

$$TC = \$18.33\ (4) + \sqrt{1 - \frac{4}{\infty}} \sqrt{\frac{2(\$17.50)(\$0.24)(\$0.17)(4)}{\$0.24 + \$0.17}}$$

$$TC = \$77.0517.$$

For the alternative designated Purchase 3, it is:

$$TC = \$18.08\ (4) + \sqrt{1 - \frac{4}{\infty}} \sqrt{\frac{2(\$14.65)(\$0.24)(\$0.17)(4)}{\$0.24 + \$0.17}}$$

$$TC = \$75.7344.$$

On the basis of this analysis, the alternative designated Purchase 1 would be chosen as the minimum cost procurement source.

The minimum cost procurement quantity for this source may be found from Equation (2.20) as:

$$Q = \sqrt{\frac{1}{1-\frac{4}{\infty}}} \sqrt{\frac{2(\$18.70)(4)}{\$0.24} + \frac{2(\$18.70)(4)}{\$0.17}}$$

$$Q = 38.7806.$$

And, the minimum cost procurement level for this source may be found from Equation (2.21) as:

$$L = 4(3) - \sqrt{1 - \frac{4}{\infty}} \sqrt{\frac{2(\$18.70)(4)}{\$0.17(1 + \frac{\$0.17}{\$0.24})}}$$

$$L = -10.6917.$$

An Example Deterministic SIMS Policy With Warehouse Restriction

As discussed in Chapter I, procurement lead time, rate of replenishment, item cost, and procurement cost are all source dependent. All other parameters remain constant for a SIMS system. This permits the use of Equations (2.33), (2.35), and (2.15) with varying values of λ for the solution of SIMS problems. The procedure is to evaluate each source, selecting the source which can supply demand at minimum total system cost subject to the restriction on scarce warehouse space.

Suppose that the SIMS system of the previous example is constrained by a total warehouse space of 100 cubic units; $W = 100$, and that each item in the system requires 24 cubic units. Utilizing Equations

(2. 33), (2. 35), and (2. 15) for varying values of λ, Tables VIII, IX, X and XI can be developed as follows:

TABLE VIII

WAREHOUSE SPACE CONSUMED AND ASSOCIATED TOTAL COSTS FOR VARYING VALUES OF λ, DETERMINISTIC SIMS SYSTEM, MANUFACTURING ALTERNATIVE

λ	L	Q	I^*_w	TC
0.00000	6.1654	45.7011	151.5942	$82.4318
-0.00300	5.7878	40.5813	106.1057	$82.4960
-0.00340	5.6933	40.0948	101.0784	$82.5121
-0.00340	5.6733	40.0017	100.0943	$82.5155
-0.00349	5.6708	39.9901	99.9718	$82.5159
-0.00350	5.6683	39.9786	99.8493	$82.5163
-0.00400	5.5362	39.4278	93.8577	$82.5388

TABLE IX

WAREHOUSE SPACE CONSUMED AND ASSOCIATED TOTAL COSTS FOR VARYING VALUES OF λ, DETERMINISTIC SIMS SYSTEM, PURCHASE 1

λ	L	Q	I^*_w	TC
0.00000	-10.6917	38.7806	192.8806	$75.6175
-0.00500	-11.8724	32.6387	105.0393	$75.8183
-0.00530	-11.9971	32.4202	100.9215	$75.8395
-0.00536	-12.0226	32.3779	100.1077	$75.8438
-0.00537	-12.0269	32.3709	99.9725	$75.8445
-0.00538	-12.0312	32.3639	99.8372	$75.8453
-0.00600	-12.3062	31.9537	91.6166	$75.8920

TABLE X

WAREHOUSE SPACE CONSUMED AND ASSOCIATED TOTAL COSTS FOR VARYING VALUES OF λ, DETERMINISTIC SIMS SYSTEM, PURCHASE 2

λ	L	Q	I^*_w	TC
0.00000	-5.9515	37.5156	186.5892	\$77.0517
-0.00500	-7.0937	31.5741	101.6132	\$77.2459
-0.00510	-7.1334	31.5024	100.2764	\$77.2526
-0.00512	-7.1414	31.4882	100.0101	\$77.2540
-0.00513	-7.1454	31.4811	99.8772	\$77.2547
-0.00514	-7.1495	31.4741	99.7442	\$77.2554
-0.00600	-7.5134	30.9115	88.6283	\$77.3172

TABLE XI

WAREHOUSE SPACE CONSUMED AND ASSOCIATED TOTAL COSTS FOR VARYING VALUES OF λ, DETERMINISTIC SIMS SYSTEM, PURCHASE 3

λ	L	Q	I^*_w	TC
0.00000	27.9152	34.3251	170.7207	\$75.7344
-0.00400	27.2067	29.6134	105.6999	\$75.8548
-0.00440	27.0781	29.3081	100.4942	\$75.8767
-0.00443	27.0681	29.2860	100.1102	\$75.8784
-0.00444	27.0648	29.2787	99.9824	\$75.8789
-0.00445	27.0614	29.2714	99.8546	\$75.8795
-0.00500	26.8702	28.8889	92.9715	\$75.9120

The optimal policy for this restricted system is associated with the source alternative designated Purchase 1. This source was selected by examining the total system cost for all sources associated with the largest value of λ for which I^*w is within the warehouse space restriction of 100 cubic units. For this source, -0.00537 is the largest value of λ for which I^*w is within the warehouse space restriction. The optimal procurement and inventory policy associated with Purchase 1 and $\lambda = -0.00537$ is a procurement level of -12.0269 and a procurement quantity of 32.3709 resulting in a minimum total system cost of $75.8445. The penalty in total system cost arising due to the warehouse constraint is $75.8445 less $75.6175 or $0.2270 per period.

Optimal Policy for a Simplified Probabilistic SIMS System

As discussed in Chapter I, procurement lead time, rate of replenishment, item cost, and procurement cost are all source dependent. All other parameters remain constant for a SIMS system. This permits the use of Equations (2.66), (2.67), and (2.60) for the solution of simplified probabilistic SIMS problems without restrictions. The procedure is to evaluate each source, selecting that source which can supply the demand at the minimum total system cost, where the minimum cost for each source is computed as in Chapter II.

As an example of the simplified probabilistic unrestricted SIMS system suppose that a procurement manager is experiencing a demand of 1.80 units per period for a certain item that may be manufactured or purchased. Holding cost per period is $0.12 and shortage cost per unit short is $3.80. Specific values for source dependent parameters are as indicated in Table XII.

TABLE XII

SOURCE DEPENDENT PARAMETERS, SIMPLIFIED PROBABILISTIC SIMS SYSTEM

Parameter	Manufacture	Purchase
R	8.00	∞
T_m	3.00	2.00
C_i	\$4.34	\$4.25
C_p	\$5.50	\$5.75

Table XIII is a display of the alternative policies and their associated minimum costs obtained by utilizing Equations (2.66), (2.67), and (2.60). On the basis of this analysis, the manufacturing alternative would be selected as the minimum cost procurement source.

TABLE XIII

ALTERNATIVE POLICIES AND ASSOCIATED MINIMUM COSTS, SIMPLIFIED PROBABILISTIC SIMS SYSTEM

Alternative	L	Q	TC_m
Purchase	7.6706	16.5161	\$9.6204
Manufacture	5.4661	13.7265	\$9.5208

CHAPTER IV

THE MISS SYSTEM

A MISS procurement and inventory system is illustrated in Figure 15. It exists as a result of the demand stimuli, D_i. In satisfying these demenads the procurement manager finds it necessary to replenish the stocks of each item periodically. The basic supposition of the MISS concept is that replenishment can be made for the aggregate of items in the system by procurement from a single-source only. Procurement may be obtained through purchase, intrafirm transfer, manufacture, or remanufacture, but only one of these is to be considered. If the purchase alternative is being examined, only one vendor is under consideration. Procurement and inventory policy for the MISS system will be that policy stating when to procure each item and how much of each item to procure with the source being fixed by prior decision. It will be the purpose of this chapter to indicate the nature of procurement and inventory operations through consideration of item dependent parameters.

The Deterministic MISS System

An Example Deterministic MISS Policy

As discussed in Chapter I, all parameters are item dependent. However, Equations (2.20), (2.21), and (2.22) can be used to solve deterministic MISS problems without restrictions. The procedure is as follows: Determine the optimal policy for each item. Realizing that the global optimum is the aggregate of the local optima, the optimal policies just determined formulate the policy of the deterministic MISS system without restrictions. The minimum total system cost is the sum of the individual minimum total costs.

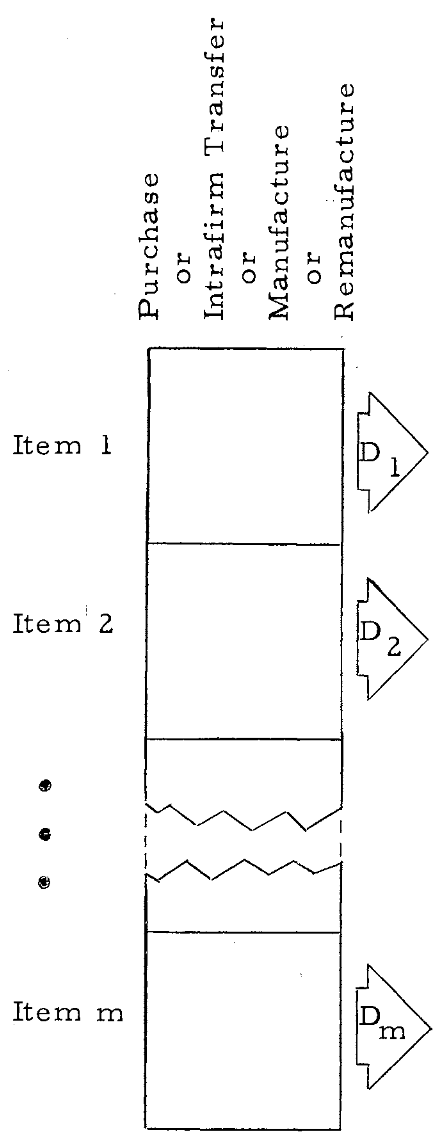


Figure 15. The MISS System

As an example of the deterministic unrestricted MISS system suppose that a procurement manager is determining the optimal policy for a system with the parameters given in Table XIV. Since R is infinite, a purchase or intrafirm transfer alternative is involved.

TABLE XIV

SYSTEM PARAMETERS, DETERMINISTIC MISS SYSTEM

Item	D	R	T	C_i	C_p	C_h	C_s
1	6.00	∞	2.00	\$30.88	\$18.30	\$0.30	\$0.30
2	4.00	∞	4.00	\$18.33	\$17.50	\$0.24	\$0.17
3	1.00	∞	1.00	\$12.00	\$15.50	\$0.12	\$0.25

Utilizing Equations (2.20), (2.21), and (2.22) Table XV is developed as follows:

TABLE XV

OPTIMAL POLICY AND ASSOCIATED MINIMUM COSTS, DETERMINISTIC MISS SYSTEM

Item	L	Q	TC
1	-7.1253	38.2737	\$191.0176
2	-5.9515	37.5156	\$ 77.0517
3	-5.3413	19.5543	\$ 13.5853

The optimal policy for the system is the aggregate of the local optimal policies. The minimum total system cost is the sum of the local minimum total costs or \$281.6546.

Optimal Policy for Deterministic MISS System With Warehouse Restriction

Each item in the MISS system consumes a certain amount of total warehouse capacity, w. The maximum accumulation of inventory for the item, I_i^*, will consume $I_i^* w_i$ cubic units of scarce warehouse capacity. Therefore, the restriction $\sum_i I_i^* w_i \leq w$ must not be violated. The subscript i will be used in this section to differentiate the items in the system. This section will present a Lagrangian multiplier technique for finding the optimal procurement and inventory policy and the minimum total sytem cost in the face of a warehouse capacity restriction.

Equation (2.15) may be modified to include item dependence such that the total system cost is given by:

$$TC = \sum_i C_{i_i} D_i + \sum \frac{C_{p_i} D_i}{Q_i} + \sum_i \frac{C_{h_i}\left[Q_i\left(1-\frac{D_i}{R_i}\right)+L_i-D_iT_i\right]^2}{2Q_i\left(1-\frac{D_i}{R_i}\right)} + \sum_i \frac{C_{s_i}(D_iT_i-L_i)^2}{2Q_i\left(1-\frac{D_i}{R_i}\right)}. \tag{4.1}$$

Define λ such that $\lambda < 0$ for every $W - \sum_i I_i^* w_i = 0$ and $\lambda = 0$ for every $W - \sum_i I_i^* w_i > 0$. Then:

$$\lambda(W - \sum_i I_i^* w_i) = 0. \tag{4.2}$$

Proceeding exactly as in Chapter II gives, after several steps:

$$TC = \sum_i C_i D_i + \sum_i \frac{C_{p_i} D_i}{Q_i} + \sum_i \frac{C_{h_i}[Q_i(1 - \frac{D_i}{R_i})]}{2}$$

$$- \sum_i C_{h_i}(D_i T_i - L_i) + \sum_i \frac{C_{h_i}(D_i T_i - L_i)^2}{2Q_i(1 - \frac{D_i}{R_i})}$$

$$+ \sum_i \frac{C_{s_i}(D_i T_i - L_i)^2}{2Q_i(1 - \frac{D_i}{R_i})} - \lambda \sum_i Q_i(1 - \frac{D_i}{R_i}) w_i$$

$$+ \lambda \sum_i (D_i T_i - L_i) w_i + \lambda W. \quad (4.3)$$

Taking the partial derivative of Equation (4.3) with respect to Q_i, then with respect to $D_i T_i - L_i$, and setting both equal to zero gives:

$$\frac{\partial TC}{\partial Q_i} = - \frac{C_{p_i} D_i}{Q_i^2} + \frac{C_{h_i}(1 - \frac{D_i}{R_i})}{2} - \frac{C_{h_i}(D_i T_i - L_i)^2}{2Q_i^2(1 - \frac{D_i}{R_i})}$$

$$- \frac{C_{s_i}(D_i T_i - L_i)^2}{2Q_i^2(1 - \frac{D_i}{R_i})} - \lambda (1 - \frac{D_i}{R_i}) w_i = 0 \quad (4.4)$$

$$\frac{\partial TC}{\partial (D_i T_i - L_i)} = -C_{h_i} + \frac{C_{h_i}(D_i T_i - L_i)}{Q_i(1 - \frac{D_i}{R_i})} + \frac{C_{s_i}(D_i T_i - L_i)}{Q_i(1 - \frac{D_i}{R_i})}$$

$$+ \lambda w_i = 0 \quad (4.5)$$

Proceeding exactly as in Chapter II gives, after several steps:

$$Q = \sqrt{\frac{1}{1 - \frac{D_i}{R_i}}} \sqrt{\frac{2C_{p_i} D_i (C_{h_i} + C_{s_i})}{C_{h_i} C_{s_i} - \lambda^2 w_i^2 - 2C_{s_i} \lambda w_i}}. \quad (4.6)$$

$$L_i = D_i T_i - (C_{h_i} - \lambda w_i)\sqrt{1 - \frac{D_i}{R_i}}\sqrt{\frac{2C_{p_i} D_i}{(C_{h_i} C_{s_i} - \lambda^2 w_i^2 - 2C_{s_i} \lambda w_i)(C_{h_i} + C_{s_i})}} \quad (4.7)$$

The procedure is to vary λ in Equations (4.6) and (4.7) until the largest value of λ is found such that $\sum_i I_i^* w_i \leq W$, where I_i^* is given by:

$$I_i^* = Q_i (1 - \frac{D_i}{R_i}) + L_i - D_i T_i. \quad (4.8)$$

Minimum total system cost is obtained by substituting the established values of Q_i and L_i, found by the procedure mentioned above, into Equation (4.1).

<u>An Example Deterministic MISS Policy With Warehouse Restriction</u>

Suppose that the MISS system of the previous example is constrained by a total warehouse space of 100 cubic units; $W = 100$. Suppose further that Item 1 requires 24 cubic units, Item 2 requires 12 cubic units, and Item 3 requires 6 cubic units. Utilizing Equations (4.6) and (4.7) for varying values of λ, and also Equation (4.8), Table XVI can be developed. From Table XVI it can be seen that the largest value of λ for which the warehouse restriction is met occurs at -0.01134.

The optimal procurement and inventory policy for this restricted MISS system is summarized in Table XVII along with the associated minimum total cost for each item. The minimum total system cost is the summation of the individual minimum total costs, or \$284.6313. The penalty in total system cost arising due to the warehouse constraint is \$284.6313 less \$281.1796 or \$2.9767 per period.

TABLE XVI

WAREHOUSE SPACE CONSUMED FOR VARYING VALUES OF λ, DETERMINISTIC MISS SYSTEM

λ	L_1	Q_1	$I_1^* w_1$	L_2	Q_2	$I_2^* w_2$	L_3	Q_3	$I_3^* w_3$	$\Sigma I_i^* w_i$
0.00000	- 7.1253	38.2737	459.0104	- 5.9515	37.5156	186.5892	-5.3413	19.5543	79.2663	725.8659
-0.01000	-12.5897	27.3384	65.5738	- 9.6568	29.2319	42.7619	-5.9373	14.2614	43.9362	152.2719
-0.01100	-13.5165	27.1615	39.0902	-10.3061	29.0049	32.2468	-6.0300	13.9859	41.7267	113.0637
-0.01130	-13.8086	27.1262	31.2317	-10.5101	28.9497	29.1363	-6.0585	13.9079	41.0883	101.4563
-0.01133	-13.8381	27.1231	30.4475	-10.5307	28.9445	28.8263	-6.0613	13.9003	41.0251	100.2989
-0.01134	-13.8480	27.1220	30.1860	-10.5376	28.9428	28.7229	-6.0623	13.8977	41.0041	99.9130
-0.01135	-13.8579	27.1210	29.9249	-10.5445	28.9411	28.6196	-6.0633	13.8951	40.9829	99.5274

TABLE XVII

OPTIMAL POLICY AND ASSOCIATED MINIMUM COSTS, DETERMINISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

Item	L	Q	TC
1	-13.8480	27.1220	$193.0344
2	-10.5376	28.9428	$ 77.8314
3	- 6.0623	13.8977	$ 13.7655

Optimal Policy for a Simplified Probabilistic MISS System

As discussed in Chapter I, all parameters are item dependent. However, Equations (2.66), (2.67) and (2.60) can be used to determine the optimal policy for a simplified probabilistic MIMS system without restrictions. The procedure is as follows: Determine the optimal policy for each item. Realizing that the global optimum is the aggregate of the local optima, the optimal policies just determined formulate the policy of the simplified probabilistic MISS system without restrictions. The minimum total system cost is the sum of the individual minimum total costs.

As an example of the simplified probabilistic unrestricted MISS system suppose that a procurement manager is determining the optimal policy for a system with the parameters indicated in Table XVIII. Since R is finite, a manufacture or remanufacture alternative is involved.

TABLE XVIII

SYSTEM PARAMETERS, SIMPLIFIED PROBABILISTIC MISS SYSTEM

Item	D_m	R	T_m	C_i	C_p	C_h	C_s
1	2.00	10.00	2.00	$7.00	$6.00	$0.10	$4.00
2	1.80	8.00	3.00	$4.34	$5.50	$0.12	$3.80

Utilizing Equations (2.66), (2.67), and (2.60) Table XIX is obtained as follows:

TABLE XIX

OPTIMAL POLICY AND ASSOCIATED MINIMUM COSTS, SIMPLIFIED PROBABILISTIC MISS SYSTEM

Item	L	Q	TC
1	6.1873	18.1265	$15.6688
2	7.6706	16.5161	$ 9.6204

The optimal policy for the system is the aggregate of the local optimal policies. The minimum total system cost is the sum of the local minimum costs or $25.2892.

CHAPTER V

THE MIMS SYSTEM

A MIMS procurement and inventory system is illustrated in Figure 16. It exists as a result of the demand stimuli, D_i. In satisfying these demands the procurement manager finds it necessary to replenish the stock of each item periodically. The basic supposition of the MIMS concept allows stock replenishment for each item to be made by procurement from one of several possible sources. The MIMS procurement and inventory system represents the highest ordered system in the hierarchy. Procurement and inventory policy for the MIMS system will be that policy stating when to procure each item, how much of each item to procure, and from what source to procure each item. It will be the purpose of this chapter to indicate the unified nature of procurement and inventory operations through a consideration of item and source dependent parameters.

The Deterministic MIMS System

An Example Deterministic MIMS Policy

As discussed in Chapter I, all parameters are either item dependent or both item and source dependent. However, Equations (2.20), (2.21), and (2.22) can be used to solve deterministic MIMS problems without restrictions. The procedure is as follows: For every item in the inventory, evaluate each source, selecting that source which can supply the demand at the minimum total cost. Realizing that the global optimum is the aggregate of the local optima, the optimal policies just determined formulate the policy of the deterministic MIMS system without restrictions. The minimum total system cost is the sum of the individual minimum total costs.

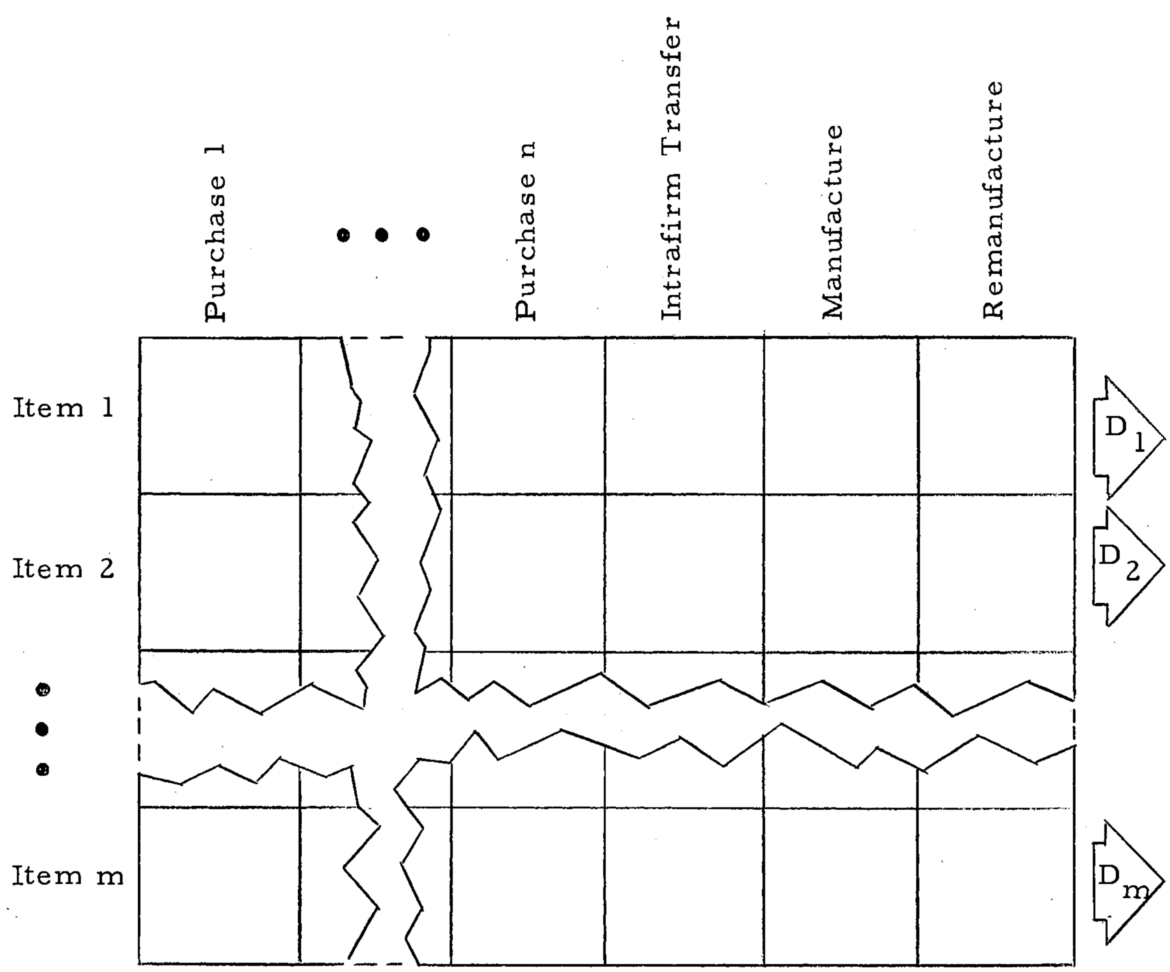


Figure 16. The MIMS System

As an example of the deterministic unrestricted MIMS system consider the determination of the minimum cost procurement and inventory policy for a system involving three items and five sources. Sources 1 and 2 are manufacturing or remanufacturing alternatives while sources 3, 4, and 5 are either vendors or intrafirm transfer possibilities. The item dependent parameters of demand, holding cost, and shortage cost are given in Table XX. Parameters that depend upon the item as well as the source are given in Table XXI. The blank cells denote that the item is not available from the source indicated.

TABLE XX

ITEM DEPENDENT PARAMETERS, DETERMINISTIC MIMS SYSTEM

Item	Demand	Holding Cost	Shortage Cost
1	6	\$0.30	\$0.30
2	4	\$0.24	\$0.17
3	1	\$0.12	\$0.25

Applying Equation (2.22) to each item and each source yields the minimum costs given in Table XXII. Inspection of these values indicates that Item 1 should be procured from Source 4, at a TC of \$191.0176, Item 2 should be procured from Source 3 at a TC of \$75.6175, and Item 3 should be procured from Source 5 at a TC of \$13.5445. These source choices result in a total system cost of \$280.1796 per period.

TABLE XXI

ITEM AND SOURCE DEPENDENT PARAMETERS, DETERMINISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3	Source 4	Source 5
			Lead Time		
1	4	-	7	2	10
2	6	-	3	4	12
3	15	3	-	1	12
			Replenishment Rate		
1	8	-	∞	∞	∞
2	12	-	∞	∞	∞
3	4	40	-	∞	∞
			Item Cost		
1	\$31.50	-	\$34.75	\$30.88	\$33.38
2	\$19.85	-	\$17.94	\$18.33	\$18.08
3	\$12.30	\$12.35	-	\$12.00	\$11.86
			Procurement Cost		
1	\$20.40	-	\$23.16	\$18.30	\$19.55
2	\$17.32	-	\$18.70	\$17.50	\$14.65
3	\$16.50	\$16.50	-	\$15.50	\$17.50

TABLE XXII

MINIMUM COST POINTS, DETERMINISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3	Source 4	Source 5
1	\$192.0298	-	\$214.9546	\$191.0176	\$206.2103
2	\$ 82.4318	-	\$ 75.6175	\$ 77.0517	\$ 75.7344
3	\$ 13.7165	\$13.9429	-	\$ 13.5853	\$ 13.5445

Application of Equation (2.21) to each item and each source results in the procurement levels given in Table XXIII. Thus, the minimum cost procurement level for Item 1 is -7.1253. The minimum cost procurement level for Item 2 is -10.6917 and the minimum cost procurement level for Item 3 is 5.2619.

TABLE XXIII

MINIMUM COST PROCUREMENT LEVELS, DETERMINISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3	Source 4	Source 5
1	13.9005	-	20.4843	-7.1253	40.2322
2	6.1654	-	-10.6917	-5.9515	27.9152
3	9.3336	-3.3719	-	-5.3413	5.2619

Applying Equation (2.20) to each item and each source results in the procurement quantities given in Table XXIV. The minimum cost procurement quantity for Item 1 is 38.2737. The minimum cost procurement quantity for Item 2 is 38.7806 and the minimum cost procurement quantity for Item 3 is 20.7776.

The optimal procurement and inventory policy for this unrestricted MIMS system is summarized in Table XXV.

<u>Optimal Policy for Deterministic MIMS System With Restrictions</u>

The i-th item in the deterministic MIMS system consumes a certain amount of scarce warehouse space, w_i. There exists a finite amount of total warehouse capacity, W. The maximum accumulation of inventory for the i-th item, I_i^*, will consume $I_i^* w_i$ cubic units of scarce warehouse space. Therefore, the restriction $\Sigma I_i^* w_i \leq W$ must not be violated. In the sections that follow, the necessary theory will be developed and a dynamic programming algorithm will be presented for finding optimal procurement and inventory policy in the face of this restriction. The source capacity constraint described in the first chapter will be considered after development and presentation of the algorithm.

TABLE XXIV

MINIMUM COST PROCUREMENT QUANTITIES, DETERMINISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3	Source 4	Source 5
1	80.7960	-	43.0571	38.2737	39.5593
2	45.7011	-	38.7806	37.5156	34.3251
3	23.2952	20.1507	-	19.5543	20.7776

TABLE XXV

OPTIMAL POLICIES, DETERMINISTIC MIMS SYSTEM

Item	L	Q	Source
1	- 7.1253	38.2737	4
2	-10.6917	38.7806	3
3	5.2619	20.7776	5

Optimal policy as a function of I^*. The objective of the dynamic programming algorithm is to find the optimal procurement and inventory policy which minimizes the function:

$$R(I_1^* w_1, I_2^* w_2, \ldots, I_K^* w_K) = g_1(I_1^* w_1) + g_2(I_2^* w_2) + \ldots + g_K(I_K^* w_K)$$

over the region $I_i^* w_i \leq 0$, $I_i^* = 0, 1, 2, \ldots, \sum_{i=1}^{K} I_i^* w_i \leq W$. Since I_i^* consumes scarce warehouse space, it is the resource which will be allocated in the dynamic programming algorithm. This necessitates the expression of TC points for each value of $I_i^* w_i$. These TC values form cost functions for the algorithm. Development of the $g_i(I_i^* w_i)$ from the cost functions is explained in the next subsection.

Tedious subscription will be avoided in the theoretical development which follows. This is possible since each cell (one item from one source) is considered on an individual basis.

Equation (2.5) may be solved for DT - L giving:

$$DT - L = Q(1 - \frac{D}{R}) - I^*. \tag{5.1}$$

Substituting Equation (2.5) and Equation (5.1) into Equation (2.15) gives:

$$TC = C_i D + \frac{C_p D}{Q} + \frac{C_h I^{*2}}{2Q(1 - \frac{D}{R})} + \frac{C_s [Q(1 - \frac{D}{R}) - I^*]^2}{2Q(1 - \frac{D}{R})}. \tag{5.2}$$

The last term of Equation (5.2) may be written as:

$$\frac{C_s Q(1 - \frac{D}{R})}{2} - C_s I^* + \frac{C_s I^{*2}}{2Q(1 - \frac{D}{R})}.$$

Equation (5.2) then becomes:

$$TC = C_i D + \frac{C_p D}{Q} + \frac{C_h I^{*2}}{2Q(1 - \frac{D}{R})} + \frac{C_s Q(1 - \frac{D}{R})}{2} - C_s I^* + \frac{C_s I^{*2}}{2Q(1 - \frac{D}{R})}. \tag{5.3}$$

Taking the partial derivative of TC with respect to Q in Equation (5.3) and setting the result equal to zero gives:

$$\frac{\partial TC}{\partial Q} = -\frac{C_p D}{Q^2} - \frac{C_h I^{*2}}{2Q^2(1-\frac{D}{R})} + \frac{C_s(1-\frac{D}{R})}{2} - \frac{C_s I^{*2}}{2Q^2(1-\frac{D}{R})} = 0. \tag{5.4}$$

Solving Equation (5.4) for Q gives:

$$-\frac{2C_p D(1-\frac{D}{R}) - C_h I^{*2} - C_s I^{*2}}{Q^2(1-\frac{D}{R})} = -C_s(1-\frac{D}{R})$$

$$Q^2 = \frac{2C_p D(1-\frac{D}{R}) + C_h I^{*2} + C_s I^{*2}}{C_s(1-\frac{D}{R})^2}$$

$$Q' = \frac{1}{1-\frac{D}{R}} \sqrt{\frac{2C_p D(1-\frac{D}{R}) + I^{*2}(C_h + C_s)}{C_s}}. \tag{5.5}$$

Solving Equation (2.5) for L gives:

$$L = I^* + DT - Q(1-\frac{D}{R})$$

or,

$$L' = I^* + DT - Q'(1-\frac{D}{R}) \tag{5.6}$$

Substituting Equation (5.5) into Equation (5.6) gives:

$$L' = I^* + DT - \sqrt{\frac{2C_p D(1-\frac{D}{R}) + I^{*2}(C_h + C_s)}{C_s}}. \tag{5.7}$$

Equation (5.5) and Equation (5.7) give the minimum cost Q and the minimum cost L as a function of I^* and other parameters. The minimum cost may be expressed as a function of I^* and other parameters by substituting the results of Equations (5.5) and (5.7) into Equation (5.2) and modifying the last term on the basis of Equation (5.1) giving:

$$TC' = C_i D + \frac{C_p D}{Q'} + \frac{C_h I^{*2}}{2Q'(1-\frac{D}{R})} + \frac{C_s(DT-L')^2}{2Q'(1-\frac{D}{R})} \, . \qquad (5.8)$$

The minimum cost value is designated TC' in Equation (5.8) to distinguish it from the minimum cost value of Equation (2.22). Likewise, the minimum cost procurement level, L', in Equations (5.6), (5.7), and (5.8) and the minimum cost procurement quantity, Q', in Equations (5.5), (5.6), and (5.8) are distinguished from the minimum cost procurement level in Equation (2.21) and the minimum cost procurement quantity in Equation (2.20) by primes.

When applying the optimizing equations, Q' is always calculated first. If $Q' < 1$ or $Q' < D$ then let $Q' = 1$ or $Q' = D$, respectively. Then, Equation (5.6) is used to calculate L'.

<u>An example deterministic MIMS policy with warehouse restriction.</u> Suppose that the MIMS system of the previous example is constrained by a total warehouse space of 100 cubic units; $W = 100$. Also suppose that Item 1 requires 24 cubic units, Item 2 requires 12 cubic units, and Item 3 requires 6 cubic units.

Application of Equations (5.5), (5.6) or (5.7), and (5.8) to the parameters of the previous example yields the TC'_{ij}, L'_{ij}, and Q'_{ij}, values of Table XXXVI. Cost values for items that cannot be procured from certain sources are given as very large values, M. The subscription in Table XXVI is explained as follows: TC'_{ij} is the minimum total cost for purchasing the i-th item from the j-th source as a function of $I_i^* w_i$. L'_{ij} and Q'_{ij} formulate the optimal policy values associated with TC'_{ij}.

TABLE XXVI

COST FUNCTIONS, DETERMINISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

$I_1^o w_1$	TC^1_{11}	L^1_{11}	Q^1_{11}	TC^1_{12}	L^1_{12}	Q^1_{12}	TC^1_{13}	L^1_{13}	Q^1_{13}	TC^1_{14}	L^1_{14}	Q^1_{14}	TC^1_{15}	L^1_{15}	Q^1_{15}
0	193.2848	9.7171	57.1314	M	M	M	217.6283	11.5723	30.4459	193.3942	-15.0473	27.0636	208.6667	32.0441	27.9726
24	193.0058	10.6473	57.4108	M	M	M	217.3381	12.5394	30.4788	193.1052	-14.0843	27.1005	208.3774	33.0083	28.0084
48	192.7680	11.4397	58.2408	M	M	M	217.0676	13.4411	30.5772	192.8384	-13.1948	27.2111	208.1095	33.9014	28.1154
72	192.5698	12.1003	59.5986	M	M	M	216.8166	14.2779	30.7404	192.5934	-12.3780	27.3945	207.8627	34.7240	28.2929
96	192.4086	12.6377	61.4491	M	M	M	216.5847	15.0509	30.9676	192.3697	-11.6325	27.6491	207.6367	35.4775	28.5395

$I_2^o w_2$	TC^1_{21}	L^1_{21}	Q^1_{21}	TC^1_{22}	L^1_{22}	Q^1_{22}	TC^1_{23}	L^1_{23}	Q^1_{23}	TC^1_{24}	L^1_{24}	Q^1_{24}	TC^1_{25}	L^1_{25}	Q^1_{25}
0	83.3627	0.6896	34.9655	M	M	M	76.8020	-17.6588	29.6707	78.1975	-12.6914	28.7029	76.7827	21.7486	26.2619
12	83.2015	1.6379	35.0430	M	M	M	76.6389	-16.6994	29.7113	78.0346	-11.7334	28.7449	76.6205	22.7027	26.3078
24	83.0577	2.4836	35.2745	M	M	M	76.4895	-15.8210	29.8329	77.8860	-10.8590	28.8706	76.4738	23.5655	26.4450
36	82.9311	3.2286	35.6570	M	M	M	76.3538	-15.0225	30.0345	77.7514	-10.0672	29.0788	76.3424	24.3383	26.6722
48	82.8210	3.8761	36.1857	M	M	M	76.2314	-14.3024	30.3145	77.6305	-9.3562	29.3679	76.2259	25.0236	26.9871
60	82.7268	4.4303	36.8544	M	M	M	76.1219	-13.6584	30.6707	77.5230	-8.7236	29.7355	76.1238	25.6242	27.3866
72	82.6476	4.8963	37.6555	M	M	M	76.0249	-13.0881	31.1005	77.4283	-8.1665	30.1786	76.0355	26.1439	27.8671
84	82.5825	5.2794	38.5808	M	M	M	75.9400	-12.5883	31.6009	77.3459	-7.6818	30.6941	75.9602	26.5867	28.4246
96	82.5304	5.5855	39.6217	M	M	M	75.8665	-12.1558	32.1687	77.2751	-7.2658	31.2783	75.8972	26.9571	29.0545

$I_3^o w_3$	TC^1_{31}	L^1_{31}	Q^1_{31}	TC^1_{32}	L^1_{32}	Q^1_{32}	TC^1_{33}	L^1_{33}	Q^1_{33}	TC^1_{34}	L^1_{34}	Q^1_{34}	TC^1_{35}	L^1_{35}	Q^1_{35}
0	14.7874	5.0501	13.2664	15.1472	-8.1888	11.4757	M	M	M	14.7837	-10.1349	11.1360	14.8178	0.1684	11.8327
6	14.5559	5.9760	13.3652	14.9136	-7.2547	11.5433	M	M	M	14.5503	-9.2012	11.2023	14.5834	1.1060	11.8951
12	14.3607	6.7569	13.6573	14.7125	-6.4503	11.7439	M	M	M	14.3494	-8.3976	11.3988	14.3797	1.9208	12.0803
18	14.1995	7.4018	14.1308	14.5422	-5.7690	12.0707	M	M	M	14.1794	-7.7178	11.7190	14.2054	2.6183	12.3829
24	14.0690	7.9239	14.7681	14.4003	-5.2012	12.5140	M	M	M	14.0379	-7.1518	12.1530	14.0583	3.2068	12.7944
30	13.9654	8.3381	15.5491	14.2838	-4.7353	13.0619	M	M	M	13.9220	-6.6880	12.6893	13.9359	3.6964	13.3049
36	13.8850	8.6598	16.4535	14.1898	-4.3592	13.7018	M	M	M	13.8285	-6.3141	13.3155	13.8355	4.0979	13.9034
42	13.8241	8.9034	17.4620	14.1152	-4.0609	14.4214	M	M	M	13.7545	-6.0181	14.0195	13.7543	4.4224	14.5790
48	13.7795	9.0816	18.5577	14.0573	-3.8293	15.2095	M	M	M	13.6971	-5.7887	14.7902	13.6900	4.6798	15.3216
54	13.7486	9.2054	19.7261	14.0136	-3.6547	16.0561	M	M	M	13.6540	-5.6162	15.6178	13.6400	4.8796	16.1219
60	13.7290	9.2837	20.9549	13.9821	-3.5284	16.9522	M	M	M	13.6230	-5.4920	16.4936	13.6025	5.0298	16.9718
66	13.7189	9.3242	22.2343	13.9608	-3.4433	17.8905	M	M	M	13.6022	-5.4088	17.4105	13.5756	5.1375	17.8641
72	13.7167	9.3330	23.5558	13.9483	-3.3931	18.8648	M	M	M	13.5901	-5.3604	18.3623	13.5577	5.2088	18.7929
78	13.7211	9.3152	24.9129	13.9432	-3.3729	19.8696	M	M	M	13.5854	-5.3418	19.3437	13.5477	5.2489	19.7530
84	13.7312	9.2748	26.3001	13.9445	-3.3781	20.9006	M	M	M	13.5871	-5.3486	20.3506	13.5445	5.2619	20.7401
90	13.7461	9.2154	27.7127	13.9513	-3.4053	21.9542	M	M	M	13.5943	-5.3772	21.3793	13.5470	5.2517	21.7504
96	13.7651	9.1395	29.1472	13.9628	-3.4514	23.0271	M	M	M	13.6061	-5.4246	22.4269	13.5546	5.2213	22.7809

The first step in finding the optimal policy for the constrained MIMS system is to develop condensed cost functions from Table XXVI. (In most instances of dynamic programming these are called return functions, but in this dissertation they will be conveniently called condensed cost functions.) These are shown in Table XXVII and are developed by searching across the TC'_{ij} entries for a specific value of $I_i^* w_i$ for a given i and seeking the minimum entry. The minimum value of TC'_{ij} together with the source for which this minimum occurs is entered in the appropriate section of Table XXVII. Symbolically, this process may be stated as:

$$g_i(I_i^* w_i) = \min_j [\underset{0 \leq I_i^* w_i \leq W}{TC'_{ij}}].$$

Each section of Table XXVII refers to an item with the source from which the minimum value of TC'_{ij} came indicated by j.

Finding the optimal procurement and inventory policy for this restricted MIMS system is now reduced to a one-dimensional allocation process of dynamic programming. The solution proceeds stagewise with the aid of recurrence relations and a functional equation technique. The cost expected from the first stage (item) if all available warehouse space is allocated to it is determined from $f_1(W) = g_1(I_1^* w_1)$. This gives:

$$f_1(0) = g_1(0) = 193.2848$$

$$f_2(24) = g_1(24) = 193.0058$$

$$f_1(48) = g_1(48) = 192.7680$$

$$f_1(72) = g_1(72) = 192.5698$$

$$f_1(96) = g_1(96) = 192.3697.$$

TABLE XXVII

CONDENSED COST FUNCTIONS, DETERMINISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

$I_i^* w_i$	$g_1(I_1^* w_1)$	j	$g_2(I_2^* w_2)$	j	$g_3(I_3^* w_3)$	j
0	193.2848	1	76.7827	5	14.7837	4
6					14.5503	4
12			76.6205	5	14.3494	4
18					14.1794	4
24	193.0058	1	76.4738	5	14.0379	4
30					13.9220	4
36			76.3424	5	13.8285	4
42					13.7543	5
48	192.7680	1	76.2259	5	13.6900	5
54					13.6400	5
60			76.1219	3	13.6025	5
66					13.5756	5
72	192.5698	1	76.0249	3	13.5577	5
78					13.5477	5
84			75.9400	3	13.5445	5
90					13.5470	5
96	192.3697	4	75.8665	3	13.5546	5

The computations for $f_1(W)$ are now complete and the results are entered in the first stage of the solution table; Table XXVIII.

From the results of $f_1(W)$, $f_2(W)$ may be computed using the recurrence relation:

$$f_K(W) = \min_{0 \le I_K^* w_K \le W} [g_K(I_K^* w_K) + f_{K-1}(W - I_K^* w_K)]. \quad (5.9)$$

When $W = 0$,

$$f_2(0) = \min_{0 \le I_2^* w_2 \le 0} [g_2(I_2^* w_2) + f_1(-I_2^* w_2)].$$

The only value of $I_2^* w_2$ that satisfies the above restriction is zero. Therefore,

$$f_2(0) = g_2(0) + f_1(0) = 76.7827 + 193.2848 = 270.0675.$$

TABLE XXVIII

SOLUTION TABLE, DETERMINISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

W	$f_1(W)$	$I_1^*w_1(W)$	$f_2(W)$	$I_2^*w_2(W)$	$f_3(W)$	$I_3^*w_3(W)$
0	193.2848	0	270.0675	0	284.8512	0
6					284.6178	6
12			269.9053	12	284.4169	12
18					284.2469	18
24	193.0058	24*	269.7586	24	284.1054	24
30					283.9895	30
36			269.6263	12	283.8960	36
42					283.8218	42
48	192.7680	48	269.4796	24*	283.7338	36
54					283.6596	42
60			269.3482	36	283.5871	36
66					283.5129	42
72	192.5698	72	269.2317	48	283.4486	48
78					283.3806	42
84			269.1104	36	283.3081	36
90					283.2339	42
96	192.3697	96	268.9939	48	283.1696	48*

When $W = 12$,

$$f_2(12) = \min_{0 \le I_2^*w_2 \le 12} [g_2(I_2^*w_2) + f_1(12 - I_2^*w_2)].$$

For values of $I_2^*w_2$ ranging from 0 to 12 this gives one feasible combination; that is:

$$f_2(12) = g_2(12) + f_1(0) = 76.6205 + 193.2848 = 269.9053.$$

When $W = 24$,

$$f_2(24) = \min_{0 \le I_2^*w_2 \le 24} [g_2(I_2^*w_2) + f_1(24 - I_2^*w_2)].$$

For values of $I_2^*w_2$ ranging from 0 to 24 this gives:

$$f_2(24) = \text{Min} \begin{bmatrix} g_2(0) + f_1(24) = 76.7827 + 193.0058 = 269.7885 \\ \\ g_2(24) + f_1(0) = 76.4738 + 193.2848 = 269.7586 \end{bmatrix}.$$

When $W = 36$,

$$f_2(36) = \min_{0 \leq I_2^* w_2 \leq 36} [g_2(I_2^* w_2) + f_1(36 - I_2^* w_2)].$$

For values of $I_2^* w_2$ ranging from 0 to 36 this gives:

$$f_2(36) = \text{Min} \begin{bmatrix} g_2(12) + f_1(24) = 76.6205 + 193.0058 = 269.6263 \\ \\ g_2(36) + f_1(0) \quad = 76.3424 + 193.2848 = 269.6272 \end{bmatrix}.$$

This process is continued until $f_2(96)$ is evaluated. The minimum value of $f_2(W)$ is identified for each value of W and entered in the second stage of Table XXVIII together with its associated value of $I_2^* w_2$.

The third stage is considered next. Using the results of $f_2(W)$, $f_3(W)$ may be computed using Equation (5.9). When $W = 0$,

$$f_3(0) = \min_{0 \leq I_3^* w_3 \leq 0} [g_3(I_3^* w_3) + f_2(I_3^* w_3)].$$

The only value of $I_3^* w_3$ that satisfies the above restriction is zero. Therefore,

$$f_3(0) = g_3(0) + f_2(0) = 14.7837 + 270.0675 = 284.8512.$$

When $W = 6$,

$$f_3(6) = \min_{0 \leq I_3^* w_3 \leq 6} [g_3(I_3^* w_3) + f_2(6 - I_3^* w_3)].$$

For values of $I_3^* w_3$ ranging from 0 to 6 this gives one feasible combination; that is:

$$f_3(6) = g_3(6) + f_2(0) = 14.5503 + 270.0675 = 284.6178.$$

When W = 12,

$$f_3(12) = \min_{0 \leq I_3^* w_3 \leq 12} [g_3(I_3^* w_3) + f_2(12 - I_3^* w_3)].$$

For values of $I_3^* w_3$ ranging from 0 to 12 this gives:

$$f_3(12) = \text{Min} \begin{bmatrix} g_3(0) + f_2(12) = 14.7837 + 269.7053 = 284.6890 \\ \\ g_3(12) + f_2(0) = 14.3494 + 270.0675 = 283.4169 \end{bmatrix}.$$

When W = 18,

$$f_3(18) = \min_{0 \leq I_3^* w_3 \leq 18} [g_3(I_3^* w_3) + f_2(18 - I_3^* w_3)].$$

For values of $I_3^* w_3$ ranging from 0 to 18 this gives:

$$f_3(18) = \text{Min} \begin{bmatrix} g_3(6) + f_2(12) = 14.5503 + 269.9053 = 284.4556 \\ \\ g_3(18) + f_2(0) = 14.1794 + 270.0675 = 284.2469 \end{bmatrix}.$$

Again, this process is continued until $f_3(96)$ is evaluated. The minimum value of $f_3(W)$ is identified for each value of W and entered in the third stage of Table XXVIII together with its associated value of $I_3^* w_3$.

Slight differences occur in the results of Table XXVIII and the Appendix offered at the conclusion of this dissertation. These slight discrepancies, the maximum of which is \$0.0001 per stage in any of the dynamic programming solutions in this investigation, is caused by the truncation of the digits four places to the right of the decimal when displayed by the computer. The hand solutions utilize the condensed cost functions displayed by the computer, truncated as above, resulting in the slight discrepancies.

Table XXVIII may now be used to find the optimal procurement and inventory policy for this constrained MIMS system. The minimum total system cost is found to be $283. 1696 per period and appears as the last entry in the third stage of Table XXVIII. Table XXVIII also indicates that 24 cubic units of warehouse space are to be allocated to Item 1, 24 cubic units to Item 2, and 48 cubic units to Item 3. These allocations of scarce space to items are indicated by asterisks and are determined by working backwards in Table XXVIII. The penalty in total system cost arising due to the warehouse constraint is $283. 1696 less $280. 1796 or $2. 9900 per period.

Reference to Table XXVII with the vector of space allocations indicates that Item 1 should be procured from Source 1, Item 2 from Source 5, and Item 3 from Source 5. Finally, reference to Table XXVI for the source established indicates that the procurement level and procurement quantity for Item 1 should be 10.6473 and 57.4108 respectively. The procurement level and procurement quantity for Item 2 should be 23.5655 and 26.4450 respectively, and for Item 3 the procurement level and procurement quantity should be 4.6798 and 15.3216 respectively.

The optimal procurement and inventory policy for this restricted MIMS system is summarized in Table XXIX. Comparison of Table XXIX and Table XXV demonstrates that the policy established for the unrestricted system in no way predicts the policy for the same system with a warehouse restriction.

TABLE XXIX

OPTIMAL POLICY, DETERMINISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

Item	L'	Q'	Source
1	10.6473	57.4108	1
2	23.5655	26.4450	5
3	4.6798	15.3216	5

An example deterministic MIMS policy with both warehouse and source capacity restrictions. The i-th item requires h_{ij} hours of scarce production time from the j-th source. There exists a certain amount of total production time available at each source, H_j. Therefore, the sum of the product of production time per unit and the number of units procured from a given source must not exceed H_j for a given j. Stated symbolically this restriction becomes

$$\sum_i h_{ij} D_i \delta_{j,j'(i)} \leq H_j$$

for every j. The symbol $\delta_{j,j'(i)}$ is defined as:

(1) $\delta_{j,j'(i)} = 0$ if the i-th item is not procured from the j-th source

(2) $\delta_{j,j'(i)} = 1$ if the i-th item is procured from the j-th source.

Suppose that the restricted MIMS system under discussion is subject to the h_{ij} and H_j values given in Table XXX. Source 3 is a vendor who has chosen not to disclose a manufacturing time or a capacity. Rather, Source 3 states that it can meet any demand schedule presented for Items 1 and 2.

TABLE XXX

SOURCE CAPACITY RESTRICTIONS, DETERMINISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3	Source 4	Source 5
1	3.43	-	0	3.50	3.82
2	1.70	-	0	1.65	1.53
3	1.08	1.12	-	1.04	1.10
H_j	22.00	5.40	-	3.60	7.10

The minimum cost allocation summarized in Table XXIX refers to the policy associated with $f_3(96)$. This policy results in the array of $\delta_{j,j'(i)}$ displayed in Table XXXI. Utilizing the information displayed in Tables XX, XXX, and XXXI the total time required from each source is as follows:

Source 1 : $(3.43)(1)(6)+(1.70)(0)(4)+(1.08)(0)(1)=20.58$

Sources 2, 3, and 4: 0

Source 5 : $(3.82)(0)(6)+(1.53)(1)(4)+(1.10)(1)(1)=7.22$.

Source 5 violates the source capacity constraint since $7.22 > 7.10$. It may be concluded that $f_3(96)$ of Table XXVIII does not yield a feasible policy.

An approach to determining a feasible policy is to try the next minimum policy until a policy is exhibited that does not violate the source capacity constraint. The next minimum policy is $f_3(90)$.

However, by tracing through the backward solution and then Table XXVII to identify the sources, it may be concluded that the array of $\delta_{j,j'(i)}$ is identical to that of Table XXXI. Thus, Source 5 again violates the source capacity constraint. Applying the procedure outlined above, the next minimum policy is $f_3(84)$ which would result in the array of $\delta_{j,j'(i)}$ offered in Table XXXII. Utilizing the information displayed in Tables XX, XXX, and XXXII the total time required for each source is as follows:

Source 1 : (3.43)(1)(6)+(1.70)(0)(4)+(1.08)(0)(1) = 20.58
Sources 2 and 3 : 0
Source 4 : (3.50)(0)(6)+(1.65)(0)(4)+(1.04)(1)(1) = 1.04
Source 5 : (3.82)(0)(6)+(1.53)(1)(4)+(1.10)(0)(1) = 6.12

The capacity of each source is sufficient to meet demand. Hence, $f_3(84)$ yields a feasible policy.

TABLE XXXI

ARRAY OF $\delta_{j,j'(i)}$ FOR $f_3(96)$

Item	Source 1	Source 2	Source 3	Source 4	Source 5
1	1	0	0	0	0
2	0	0	0	0	1
3	0	0	0	0	1

The procedure outlined above offers an approximate means of finding feasible procurement and inventory policy in the light of source capacity constraints. The total system cost for this feasible solution may be found to be $283.3081 which gives a penalty of $283.3081 less $283.1696 or $0.1385 over the system with only a warehouse constraint. Actually, this is an upper bound on the penalty incurred. The maximum per cent error is:

$$\frac{\$283.3081 - \$283.1696}{\$283.3081} \times 100 = 0.0489.$$

To find a feasible solution that yields an optimal policy in the light of source capacity constraints would require a more complex application of dynamic programming.

TABLE XXXII

ARRAY OF $\delta_{j,j'(i)}$ FOR $f_3(84)$

Item	Source 1	Source 2	Source 3	Source 4	Source 5
1	1	0	0	0	0
2	0	0	0	0	1
3	0	0	0	1	0

A Simplified Probabilistic MIMS System

An Example Simplified Probabilistic MIMS Policy

As discussed in Chapter I, all parameters are either item dependent or both item and source dependent. However, Equations, (2.66), (2.67), and (2.60) can be used to solve simplified probabilistic problems without restrictions. The procedure is as follows: For every item in the inventory, evaluate each source, selecting that source which can supply the demand at the minimum total cost. Realizing that the global optimum is the aggregate of the local optima, the optimal policies just determined formulate the policy of the simplified probabilistic MIMS system without restrictions. The minimum total system cost is the sum of the individual minimum costs.

As an example of the simplified probabilistic unrestricted MIMS system consider the determination of the minimum cost procurement and inventory policy for a system involving two items and three sources. Source 1 is a manufacturing or remanufacturing alternative while Sources 2 and 3 are either vendors or intrafirm transfer alternatives. The item dependent parameters of demand, holding cost, and shortage cost are given in Table XXXIII. Parameters that depend upon the item as well as the source are given in Table XXXIV. The blank cells denote that the item is not available from the source indicated.

TABLE XXXIII

ITEM DEPENDENT PARAMETERS, SIMPLIFIED PROBABILISTIC MIMS SYSTEM

Item	Demand	Holding Cost	Shortage Cost
1	2.0	$0.10	$4.00
2	1.8	$0.12	$3.80

TABLE XXXIV

ITEM AND SOURCE DEPENDENT PARAMETERS, SIMPLIFIED PROBABILISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3
	Lead Time		
1	2	-	4
2	3	2	-
	Replenishment Rate		
1	10	-	∞
2	8	∞	-
	Item Cost		
1	\$7.00	-	\$6.30
2	\$4.34	\$4.25	-
	Procurement Cost		
1	\$6.00	-	\$6.25
2	\$5.50	\$5.75	-

Applying Equation (2.66) to each item and each source gives the procurement quantities of Table XXXV.

TABLE XXXV

MINIMUM COST PROCUREMENT QUANTITIES, SIMPLIFIED PROBABILISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3
1	18.1265	-	17.5021
2	16.5161	13.7265	-

Application of Equation (2.67) to each item and each source results in the procurement levels given in Table XXXVI.

TABLE XXXVI

MINIMUM COST PROCUREMENT LEVELS, SIMPLIFIED PROBABILISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3
1	6.1873	-	12.4995
2	7.6706	5.4661	-

Substituting the results of Equations (2.66) and (2.67) into Equation (2.60) for each item and each source yields the minimum costs given in Table XXXVII. The optimal procurement and inventory policy for this unrestricted MIMS system are summarized in Table XXXVIII.

TABLE XXXVII

MINIMUM COST POINTS, SIMPLIFIED PROBABILISTIC MIMS SYSTEM

Item	Source 1	Source 2	Source 3
1	$15.6688	-	$14.7998
2	$ 9.6204	$9.5208	-

TABLE XXXVIII

OPTIMAL POLICY, SIMPLIFIED PROBABILISTIC MIMS SYSTEM

Item	L	Q	Source
1	12.4995	17.5021	3
2	5.4661	13.7265	2

Optimal Policy for Simplified Probabilistic MIMS System With Warehouse Restriction

The i-th item in the simplified probabilistic MIMS system consumes a certain amount of warehouse space, w_i. There exists a finite amount of total warehouse capacity, W. The maximum accumulation of inventory for the i-th item, $I^*_{m_i}$, will consume $I^*_{m_i} w_i$ cubic units of scarce warehouse space. Therefore the restriction $\sum_i I^*_{m_i} w_i$ must not be violated. In the sections that follow the necessary theory will be developed and a dynamic programming algorithm will be presented for finding optimal procurement and inventory policy in the face of this restriction.

Optimal policy as a function of I^*_m. The objective of the dynamic programming algorithm is to find the optimal procurement and inventory policy which minimizes the function:

$$R(I^*_{m_1} w_1, I^*_{m_2} w_2, \ldots, I^*_{m_K} w_K) = g_1(I^*_{m_1} w_1) + g_2(I^*_{m_2} w_2) + \ldots + g_K(I^*_{m_K} w_K)$$

over the region $I^*_{m_i} w_i \geq 0$, $I^*_{m_i} = 0, 1, 2, \ldots$, $\sum_{i=1}^{K} I^*_{m_i} w_i \leq W$. Since $I^*_{m_i}$ consumes scarce warehouse space, it is the resource which will be allocated in the dynamic programming algorithm. This necessitates the expression of TC_m points for each value of $I^*_{m_i} w_i$. These TC values form cost functions for the algorithm. Development of the $g_i(I^*_{m_i} w_i)$ from the cost functions is explained in the next subsection.

Tedious subscription will be avoided in the theoretical development which follows. This is possible since each cell (one item from one source) is considered on an individual basis.

Equation (2.47) may be solved for L giving:

$$L = I^*_m - Q(1 - \frac{D_m}{R}) + D_m T_m. \quad (5.10)$$

Substituting Equation (5.10) into Equation (2.60) gives:

$$TC_m = C_i D_m + \frac{C_p D_m}{Q} + C_h \left[I^*_m - \frac{Q(1 - \frac{D_m}{R})}{2} \right] + \frac{C_s' D_m \left\{ [A' - I^*_m + Q(1 - \frac{D_m}{R}) - D_m T_m]^2 - 1 \right\}}{2QA'}. \quad (5.11)$$

Let:

$$V_1 = C_i D_m$$
$$V_2 = C_p D_m$$
$$V_3 = C_h$$
$$V_4 = (1 - \frac{D_m}{R})$$
$$V_5 = \frac{C_s' D_m}{2A'}$$
$$V_6 = D_m T_m$$
$$X = Q$$
$$Y = L$$
$$U = I^*_m.$$

Then,

$$TC_m = V_1 + \frac{V_2}{X} + V_3[U - \frac{XV_4}{2}] + \frac{V_5}{X}[(A'-U+XV_4-V_6)^2-1]$$

$$= V_1 + \frac{V_2}{X} + V_3U - \frac{V_3V_4}{2}X + \frac{(A'-U+XV_4-V_6)^2}{\frac{X}{V_5}} - \frac{V_5}{X}. \quad (5.12)$$

Taking the partial derivative of TC_m with respect to X in Equation (5. 12) and setting the result equal to zero gives:

$$\frac{\partial TC_m}{\partial X} = -\frac{V_2}{X^2} - \frac{V_3V_4}{2} + \frac{2(A'-U+V_4X-V_6)V_4V_5}{X} - \frac{(A'-U+V_4X-V_6)^2}{X^2}$$

$$+ \frac{V_5}{X^2} = 0. \quad (5.13)$$

Equation (5. 13) subsequently reduces to:

$$\frac{-2V_2 - V_3V_4X^2 + 4V_4{}^2V_5X^2 - 2V_5(A' - U - V_6)^2 - 2V_4{}^2V_5X^2 + 2V_5}{2X^2} = 0.$$

$$X^2[-V_3V_4 + 2V_4{}^2V_5] = 2V_2 + 2V_5[(A' - U - V_6)^2 - 1]$$

$$X^2 = \frac{2V_2 + 2V_5[(A' - U - V_6)^2 - 1]}{-V_3V_4 + 2V_4{}^2V_5}$$

$$X = \sqrt{\frac{2V_2 + 2V_5[(A'-U-V_6)^2-1]}{-V_3V_4+2V_4{}^2V_5}}.$$

Returning to the original symbolism:

$$Q' = \sqrt{\frac{2C_pD_m + \frac{C_s'D_m}{A'}[(A'-I_m^*-D_mT_m)^2-1]}{\frac{C_s'D_m}{A'}(1-\frac{D_m}{R})^2 - C_h(1-\frac{D_m}{R})}}. \quad (5.14)$$

Substituting Equation (2. 51) into Equation (5. 14) gives:

$$Q' = \sqrt{\frac{2C_p D_m + \frac{C_s' D_m}{A'} [(\frac{A'}{2} - I_m^*)^2 - 1]}{\frac{C_s' D_m}{A'} (1 - \frac{D_m}{R})^2 - C_h (1 - \frac{D_m}{R})}} . \quad (5.15)$$

And, substituting Equation (2.51) into Equation (5.10) gives:

$$L' = I_m^* - Q' (1 - \frac{D_m}{R}) + \frac{A'}{2} . \quad (5.16)$$

Equation (5.15) and Equation (5.16) give the minimum cost Q and the minimum cost L as a function of I_m^* and other parameters. The expected minimum cost may be expressed as a function of I_m^* and other parameters by substituting the results of Equations (5.15) and (5.16) into Equation (2.60) as follows:

$$TC_m' = C_i D_m + \frac{C_p D_m}{Q'} + C_h \left[\frac{Q'(1 - \frac{D_m}{R})}{2} + L' - D_m T_m \right]$$

$$+ \frac{C_s' D_m [(A' - L')^2 - 1]}{2Q'(A')} . \quad (5.17)$$

The minimum cost value is designated TC' in Equation (5.17) to distinguish it from the minimum cost value without restrictions. Likewise, the minimum cost procurement level, L', in Equation (5.16) and the minimum cost procurement quantity, Q', in Equation (5.15) are distinguished from the minimum cost procurement level in Equation (2.67) and the minimum cost procurement quantity in Equation (2.66) by asterisks.

When applying the optimizing equations Q' is always calculated first. If $Q' < 1$ or $Q' < D_m$, then let $Q' = 1$ or $Q' = D_m$, respectively. If $L' < D_m T_m$, then let $L' = D_m T_m$.

<u>An example simplified probabilistic MIMS policy with warehouse restriction.</u> Suppose that the MIMS system of the previous example is constrained by a total warehouse space of 100 cubic units; $W = 100$. Also suppose that Item 1 requires 9 cubic units and that Item 2 requires 7 cubic units.

Application of Equations (5.15), (5.16), and (5.17) to the parameters of the previous example yields the $TC'_{m_{ij}}$, L'_{ij}, and Q'_{ij} values of Table XXXIX. Note that $TC'_{m_{ij}}$ is given as a function of $I^*_{m_i} w_i$ up to the maximum space available in the warehouse. Cost values for items that cannot be procured from certain sources are given as very large values, M. The subscription of Table XXXIX is explained as follows: $TC'_{m_{ij}}$ is the minimum total cost for purchasing the i-th item from the j-th source as a function of $I^*_{m_i} w_i$. L'_{ij} and Q'_{ij} formulate the optimal policy associated with $TC'_{m_{ij}}$.

The first step in finding the optimal policy for the constrained probabilistic MIMS system is to develop condensed cost functions from Table XXXIX. These are shown in Table XL and are developed by searching across the TC'_{ij} entries for a specific value of $I^*_{m_i} w_i$ for a given i and seeking the minimum entry. The minimum value of $TC'_{m_{ij}}$ together with the source for which this minimum occurs is entered in the appropriate section of Table XL. Symbolically, this process may be stated as:

$$g_i(I^*_{m_i} w_i) = \underset{j}{\text{Min}} \left[\underset{0 \leq I^*_{m_i} w_i \leq W}{TC'_{m_{ij}}} \right].$$

Each section of Table XL refers to an item with the source from which the minimum value of $TC'_{m_{ij}}$ came indicated by j.

TABLE XXXIX

COST FUNCTIONS, SIMPLIFIED PROBABILISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

$I^*_{m_1 w_1}$	$TC'_{m_{11}}$	L'_{11}	Q'_{11}	$TC'_{m_{12}}$	L'_{12}	Q'_{12}	$TC'_{m_{13}}$	L'_{13}	Q'_{13}
18	21.9000	4.0000	2.5000	M	M	M	26.8221	8.0000	2.0004
27	19.3499	4.0000	3.7500	M	M	M	22.1647	8.0000	3.0006
36	18.1000	4.0000	5.0000	M	M	M	19.8610	8.0000	4.0008
45	17.3700	4.0000	6.2500	M	M	M	18.4988	8.0000	5.0010
54	16.8884	4.4450	6.9436	M	M	M	17.6073	8.0000	6.0012
63	16.5332	4.9525	7.5592	M	M	M	16.9849	8.0000	7.0014
72	16.2733	5.3238	8.3452	M	M	M	16.5297	8.1735	7.8279
81	16.0845	5.5934	9.2582	M	M	M	16.1616	9.0941	7.9074
90	15.9480	5.7885	10.2643	M	M	M	15.8552	9.8603	8.1412
99	15.8498	5.9288	11.3389	M	M	M	15.6054	10.4850	8.5166

$I^*_{m_2 w_2}$	$TC'_{m_{21}}$	L'_{21}	Q'_{21}	$TC'_{m_{22}}$	L'_{22}	Q'_{22}	$TC'_{m_{23}}$	L'_{23}	Q'_{23}
14	15.2237	5.4000	2.5806	15.7840	3.6000	2.0003	M	M	M
21	12.8531	5.4000	3.8709	13.1727	3.6000	3.0005	M	M	M
28	11.6978	5.4000	5.1612	11.8970	3.6000	4.0007	M	M	M
35	11.0286	5.4000	6.4516	11.1556	3.6000	5.0009	M	M	M
42	10.6025	5.4000	7.7419	10.6649	4.0874	5.5135	M	M	M
49	10.3109	5.8087	8.5048	10.3101	4.5149	6.0861	M	M	M
56	10.0917	6.3997	9.0325	10.0561	4.8209	6.7802	M	M	M
63	9.9298	6.8362	9.7596	9.8754	5.0387	7.5626	M	M	M
70	9.8135	7.1499	10.6452	9.7475	5.1929	8.4085	M	M	M
77	9.7324	7.3687	11.6532	9.6579	5.3008	9.3008	M	M	M
84	9.6781	7.5150	12.7547	9.5967	5.3745	10.2272	M	M	M
91	9.6443	7.6061	13.9275	9.5569	5.4226	11.1793	M	M	M
98	9.6263	7.6548	15.1550	9.5333	5.4510	12.1511	M	M	M

TABLE XL

CONDENSED COST FUNCTIONS, SIMPLIFIED PROBABILISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

$I^*_{m_i} w_i$	$g_1(I^*_{m_1} w_1)$	j	$g_2(I^*_{m_2} w_2)$	j
14			15.2237	1
18	27.9000	1		
21			12.8531	1
27	19.3499	1		
28			11.6978	1
35			11.0286	1
36	18.1000	1		
42			10.6025	1
45	17.3700	1		
49			10.3101	2
54	16.8884	1		
56			10.0561	2
63	16.5332	1	9.8754	2
70			9.7475	2
72	16.2733	1		
77			9.6579	2
81	16.0845	1		
84			9.5967	2
90	15.8552	3		
91			9.5569	2
98			9.5333	2
99	15.6054	3		

Finding the optimal procurement and inventory policy for this restricted MIMS system is now reduced to a one-dimensional allocation process of dynamic programming. The solution proceeds stagewise with the aid of recurrence relations and a functional equation technique. The cost expected from the first stage (item) if all available warehouse space is allocated to it is determined from $f_1(W) = g_1(I^*_{m_1} w_1)$. This gives:

$$f_1(18) = g_1(18) = 21.9000$$

$$f_1(27) = g_1(27) = 19.3499$$

$$f_1(36) = g_1(36) = 18.1000$$

$$f_1(45) = g_1(45) = 17.3700$$

$$f_1(54) = g_1(54) = 16.8884$$

$$f_1(63) = g_1(63) = 16.5332$$

$$f_1(72) = g_1(72) = 16.2733$$

$$f_1(81) = g_1(81) = 16.0845$$

$$f_1(90) = g_1(90) = 15.8552$$

$$f_1(99) = g_1(99) = 15.6054.$$

The computations for $f_1(W)$ are now complete and the results are entered in the first stage of the solution table; Table XLI.

From the results of $f_1(W)$, $f_2(W)$ may be computed using the recurrence relation:

$$f_K(W) = \underset{0 \leq I^*_{m_K} w_K \leq W}{\text{Min}} [g_K(I^*_{m_K} w_K) + f_{K-1}(W - I^*_{m_K} w_K)] \qquad (5.18)$$

When $W = 32$,

$$f_2(32) = \underset{0 \leq I^*_{m_2} w_2 \leq 32}{\text{Min}} [g_2(I^*_{m_2} w_2) + f_1(32 - I^*_{m_2} w_2)].$$

For values of $I^*_{m_2} w_2$ ranging from 0 to 32 this gives one feasible combination; that is:

$$f_2(32) = g_2(14) + f_1(18) = 15.2237 + 21.9000 = 37.1237.$$

When $W = 39$,

$$f_2(39) = \underset{0 \leq I^*_{m_2} w_2 \leq 39}{\text{Min}} [g_2(I^*_{m_2} w_2) + f_1(39 - I^*_{m_2} w_2)].$$

TABLE XLI

SOLUTION TABLE, SIMPLIFIED PROBABILISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

W	$f_1(W)$	$I^*_{m_1} w_1(W)$	$f_2(W)$	$I^*_{m_2} w_2(W)$
18	21.9000	18		
27	19.3499	27		
32			37.1237	14
36	18.1000	36		
39			34.7531	21
41			34.5736	14
45	17.3700	45		
46			33.5978	28
48			32.2030	21
50			33.3237	14
53			32.9286	35
54	16.8884	54*		
55			31.0477	28
57			30.9531	21
59			32.5937	14
60			32.5025	42
62			30.3785	35
63	16.5332	63		
64			29.7978	28
66			30.2231	21
67			32.2101	49
68			32.1121	14
69			29.9524	42
71			29.1286	35
72	16.2733	72		
73			29.0678	28
74			31.9561	56
75			29.7415	21
76			29.6600	49
77			31.7569	14
78			28.7025	42
80			28.3986	35
81	16.0845	81	31.7754	63
82			28.5862	28
83			29.4061	56
84			29.3863	21
85			28.4101	49
86			31.4970	14
87			27.9725	42
88			31.6475	70
89			27.9170	35
90	15.8552	90	29.2253	63
91			28.2310	28

TABLE XLI (Continued)

W	$f_1(W)$	$I^*_{m_1} w_1(W)$	$f_2(W)$	$I^*_{m_2} w_2(W)$
92			28.1561	56
93			29.1264	21
94			27.6801	49
95			31.3082	14
96			27.4909	42*
97			29.0974	70
98			27.5618	35
99	15.6054	99	27.9754	63
100			29.9711	28

For values of $I^*_{m_2} w_2$ ranging from 0 to 39 this gives one feasible combination; that is:

$$f_2(39) = g_2(21) + f_1(18) = 12.8531 + 21.9000 = 34.7531.$$

When W = 41,

$$f_2(41) = \underset{0 \leq I^*_{m_2} w_2 \leq 41}{\text{Min}} [g_2(I^*_{m_2} w_2 + f_1(41 - I^*_{m_2} w_2)].$$

For values of $I^*_{m_2} w_2$ ranging from 0 to 41 this gives one feasible combination; that is:

$$f_2(41) = g_2(14) + f_1(27) = 15.2237 + 19.3499 = 34.5736.$$

This process is continued until $f_2(100)$ is evaluated. The minimum value of $f_2(W)$ is identified for each value of W and entered in the second stage of Table XLI together with its associated value of $I^*_{m_2} w_2$.

Table XLI may now be used to find the optimal procurement and inventory policy for this constrained probabilistic MIMS system. The minimum expected total system cost is found to be $27.4909 per period and is noted with an asterisk in column 2. Table XLI also indicates that 54 cubic units of warehouse space are to be allocated to Item 1 and 42 cubic units are to be allocated to Item 2. The penalty in expected total system cost arising due to the warehouse constraint is $27.4909 less $24.3206 or $3.1703 per period.

Reference to Table XLI with the vector of space allocations indicates that Item 1 should be procured from Source 1 and that Item 2 should be procured from Source 1. Finally, reference to Table XXXIX with the sources established indicates that the procurement level and procurement quantity for Item 1 should be 4.4450 and 6.9436 respectively. The procurement level and procurement quantity for Item 2 should be 5.4000 and 7.7419 respectively.

The optimal procurement and inventory policy for this restricted MIMS system is summarized in Table XLII.

TABLE XLII

OPTIMAL POLICY, SIMPLIFIED PROBABILISTIC MIMS SYSTEM WITH WAREHOUSE RESTRICTION

Item	L'	Q'	Source
1	4.4450	6.9436	1
2	5.4000	7.7419	1

CHAPTER VI

REDUCTION TO LOWER ORDERED SYSTEMS

The deterministic and probabilistic MIMS systems presented in the previous chapter can be reduced to lower ordered systems. Optimal procurement and inventory policy for the lower ordered systems can be found by the computational schemes presented. Specifically, this chapter will indicate how the previous algorithms can be used to determine procurement and inventory policy for a MISS system, a SIMS system, and a SISS system. Both the constrained and the unconstrained versions of these systems will be presented.

Reduction of the Deterministic MIMS System

Reduction to the Deterministic MISS System

Suppose that the three items of the deterministic MIMS System described in Chapter V can only be procured from Source 4 and that the parameters indicated for that source apply. When the MISS system is not constrained, the minimum cost points found in Table XXII can be used to find the total system cost. This total system cost is $191.0176 + $77.0517 + $13.5853 = $281.6546. The minimum cost procurement levels given in Table XXIII and the minimum cost procurement quantities given in Table XXIV are applicable. These are summarized in Table XLIII.

TABLE XLIII

OPTIMAL POLICY, REDUCTION TO DETERMINISTIC MISS SYSTEM

Item	L	Q
1	-7.1253	38.2737
2	-5.9515	37.5156
3	-5.3413	19.5543

When the warehouse space is finite the solution may be found by dynamic programming. Assume, as before, that the warehouse space is 100 cubic units and that the cubic units of space required by Item 1, Item 2, and Item 3 are 24, 12, and 6 respectively.

The condensed cost functions for this situation may be derived from Table XXVI by reference to Source 4. These are exhibited in Table XLIV. As before, each section refers to an item with the source indicated by j.

The cost expected from the first stage if all available warehouse space is allocated to it is determined from $f_1(W)=g_1(I_1^* w_1)$. This gives:

$$f_1(0) = g_1(0) = 193.3942$$

$$f_1(24) = g_1(24) = 193.1052$$

$$f_1(48) = g_1(48) = 192.8384$$

$$f_1(72) = g_1(72) = 192.5934$$

$$f_1(96) = g_1(96) = 192.3697.$$

The computations for $f_1(W)$ are now complete and the results are entered in the first stage of the solution table; Table XLV.

TABLE XLIV

CONDENSED COST FUNCTIONS, REDUCTION TO DETERMINISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

$I^*_i w_i$	$g_1(I^*_1 w_1)$	j	$g_2(I^*_2 w_2)$	j	$g_3(I^*_3 w_3)$	j
0	193.3942	4	78.1975	4	14.7837	4
6					14.5503	4
12			78.0346	4	14.3494	4
18					14.1794	4
24	193.1052	4	77.8860	4	14.0379	4
30					13.9220	4
36			77.7514	4	13.8285	4
42					13.7545	4
48	192.8384	4	77.6305	4	13.6971	4
54					13.6540	4
60			77.5230	4	13.6230	4
66					13.6022	4
72	192.5934	4	77.4283	4	13.5901	4
78					13.5854	4
84			77.3459	4	13.5871	4
90					13.5943	4
96	192.3697	4	77.2751	4	13.6061	4

TABLE XLV

SOLUTION TABLE, REDUCTION TO DETERMINISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

W	$f_1(W)$	$I^*_1 w_1(W)$	$f_2(W)$	$I^*_2 w_2(W)$	$f_3(W)$	$I^*_3 w_3(W)$
0	193.3942	0	271.5917	0	286.3754	0
6					286.1420	6
12			271.4288	12	285.9411	12
18					285.7711	18
24	193.1052	24*	271.2802	24	285.6296	24
30					285.5137	30
36			271.1398	12	285.4202	36
42					285.3462	42
48	192.8384	48	270.9912	24	285.2573	36
54					285.1833	42
60			270.8576	36*	285.1087	36
66					285.0347	42
72	192.5934	72	270.7244	24	284.9683	36
78					284.8942	42
84			270.5898	36	284.8197	36
90					284.7457	42
96	192.3697	96	270.4689	48	284.6851	36*

From the results of $f_1(W)$, $f_2(W)$ may be computed using the recurrence relation given by Equation (5.10). When $W = 0$,

$$f_2(0) = \min_{0 \le I_2^* w_2 \le 0} [g_2(I_2^* w_2) + f_1(-I_2^* w_2)].$$

The only value of $I_2^* w_2$ that satisfies the above restriction is zero. Therefore,

$$f_2(0) = g_2(0) = 78.1975 + 193.3942 = 271.5917.$$

When $W = 12$,

$$f_2(12) = \min_{0 \le I_2^* w_2 \le 12} [g_2(I_2^* w_2) + f_1(12 - I_2^* w_2)].$$

For values of $I_2^* w_2$ ranging from 0 to 12 this gives one feasible combination; that is:

$$f_2(12) = g_2(12) + f_1(0) = 78.0346 + 193.3942 = 271.4288.$$

When $W = 24$,

$$f_2(24) = \min_{0 \le I_2^* w_2 \le 24} [g_2(I_2^* w_2) + f_1(24 - I_2^* w_2)].$$

For values of $I_2^* w_2$ ranging from 0 to 24 this gives:

$$f_2(24) = \text{Min} \begin{bmatrix} g_2(0) + f_1(24) = 78.1975 + 193.1052 = 271.3027 \\ \\ g_2(24) + f_1(0) = 77.8860 + 193.3942 = 271.2802 \end{bmatrix}.$$

This process is continued until $f_2(96)$ is evaluated. The minimum value of $f_2(W)$ is identified for each value of W and entered in the second stage of Table XLV together with its associated value of $I_2^* w_2$.

The third stage is considered next. Using the results of $f_2(W)$, $f_3(W)$ may be computed using Equation (5.10). When $W = 0$,

$$f_3(0) = \underset{0 \leq I_3^* w_3 \leq 0}{\text{Min}} [g_3(I_3^* w_3) + f_2(-I_3^* w_3)]$$

The only value that satisfies the above restriction is zero. Therefore,

$$f_3(0) = g_3(0) + f_2(0) = 14.7837 + 271.5917 = 286.3754.$$

When W = 6,

$$f_3(6) = \underset{0 \leq I_3^* w_3 \leq 6}{\text{Min}} [g_3(I_3^* w_3) + f_2(6 - I_3^* w_3)].$$

For values of $I_3^* w_3$ ranging from 0 to 6 this gives one feasible combination; that is:

$$f_3(6) = g_3(6) + f_2(0) = 14.5503 + 271.5917 = 286.1420.$$

When W = 12,

$$f_3(12) = \underset{0 \leq I_3^* w_3 \leq 12}{\text{Min}} [g_3(I_3^* w_3) + f_2(12 - I_3^* w_3)].$$

For values of $I_3^* w_3$ ranging from 0 to 12 this gives:

$$f_3(12) = \text{Min} \begin{bmatrix} g_3(0)+f_2(12)=14.7837+271.4288=286.2125 \\ \\ g_3(12)+f_2(0)=14.3494+271.5917=285.9411 \end{bmatrix}.$$

Again, this process is continued until $f_3(96)$ is evaluated. The minimum value of $f_3(W)$ is identified for each value of W and entered in the third stage of Table XLV together with its associated value of $I_3^* w_3$.

Table XLV may now be used to find the optimal procurement and inventory policy for this constrained MISS system. The minimum total system cost is found to be $284.6851 per period and appears as the last entry in the third stage of Table XLV. Table XLV also indicates that 24 cubic units of warehouse space are to be allocated to Item 1, 36 cubic units to Item 2, and 36 cubic units to Item 3. These allocations of scarce warehouse space to items are indicated by asterisks and are determined by working backwards in Table XLV. Reference to Table XXVI with the vector of space allocations indicates that the procurement level and procurement quantity for Item 1 should be -14.0843 and 27.1005 respectively. The procurement level and procurement quantity for Item 2 should be -10.0672 and 29.0788 respectively, and for Item 3 the procurement level and procurement quantity should be -6.3141 and 13.3155 respectively.

The optimal procurement and inventory policy for this restricted MISS system is summarized in Table XLVI. The penalty in total system cost arising due to the warehouse constraint is \$284.6851 less \$281.6546 or \$3.0505 per period. Comparison of the optimal policy utilizing the dynamic programming algorithm discussed above and the optimal policy utilizing the Lagrangian multiplier technique resulting in Table XVII indicates strong agreement between the two methods. The total system cost associated with Table XVII is slightly lower than that associated with Table XLVI only because the latter is restricted to integral values of I_i^*.

TABLE XLVI

OPTIMAL POLICY, REDUCTION TO DETERMINISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

Item	L	Q
1	-14.0843	27.1005
2	-10.0672	29.0788
3	- 6.3141	13.3155

Reduction to the Deterministic SIMS System

Suppose that a single-item inventory system with several sources of replenishment stock exists. As an example, suppose that the item is Item 2 of the previous chapter. When this SIMS system is not constrained the minimum cost points found in Table XXII can be used to find the minimum total system cost. This total system cost is \$75.6175 since it is the minimum of the minimums. Thus, Source 3 would be chosen as the minimum cost source. The minimum cost procurement level for the item is -10.6917 and the minimum cost procurement quantity is 38.7806. These are found in Tables XXIII and XXIV respectively.

When the warehouse space is finite, the solution may be found by dynamic programming. Assume, as before, that the warehouse space is 100 cubic units and that the item requires 12 cubic units of space.

The first step in finding the optimal procurement and inventory policy for this constrained SIMS sytem is to develop condensed cost functions from Table XXVI. These are shown in Table XLVII and are developed by searching across a specific value of $I_2^* w_2$ and selecting the minimum value of TC'_{2j} together with the source for which this minimum occurs. Symbolically, this process may be stated as:

$$g_2(I_2^* w_2) = \min_{0 \leq I_2^* w_2 \leq W} [TC'_{2j}].$$

This SIMS system is now solved as single stage dynamic programming process. It is not necessary to set up a solution table. Inspection of $g_2(I_2^* w_2)$ in Table XLVII establishes Source 3 as the minimum cost source of replenishment stock. The total sytem cost is given in

Table XLVII as \$75.8665. Reference to Table XXVI establishes the procurement level and procurement quantity at -12.1558 and 32.1687 respectively. The penalty in total system cost due to the warehouse constraint is \$75.8665 less \$75.6175 or \$0.2490 per period. Again, close agreement is indicated between the optimal policy and the associated total cost of the method discussed above and the Lagrangian multiplier technique resulting in Table IX.

TABLE XLVII

CONDENSED COST FUNCTIONS, REDUCTION TO DETERMINISTIC SIMS SYSTEM WITH WAREHOUSE RESTRICTION

$I_2^*w_2$	$g_2(I_2^*w_2)$	j
0	76.7827	5
12	76.6205	5
24	76.4738	5
36	76.3424	5
48	76.2259	5
60	76.1219	3
72	76.0249	3
84	75.9400	3
96	75.8665	3

Reduction to the Deterministic SISS System

Suppose that a single-item inventory system with a single source of replenishment stock exists. As an example, suppose that the item is Item 1 of the previous chapter and that it may be procured only from Source 3. When this SISS system is not constrained the minimum cost point found in Table XXII is the total system cost of

\$214.9546. The minimum cost procurement level and the minimum cost procurement quantity is 20.4843 and 43.0571 respectively. These are found in Table XXIII and Table XXIV.

When the warehouse space is finite, the solution may be found as a trivial case of dynamic programming. Assume that the warehouse space is 100 cubic units and that the item requires 24 cubic units of space as was established previously.

The first step in finding the optimal procurement and inventory policy for this constrained SISS system is to obtain condensed cost functions from Table XXVI. These are shown in Table XLVIII and are developed by transferring the values of TC'_{13} for all values of $I_1^* w_1$. This SISS system is now solved as a single stage dynamic programming process. It is not necessary to set up a solution table. Inspection of TC'_{13} in Table XLVIII indicates that the minimum total system cost will be \$216.5847 giving a penalty for the constraint on warehouse space of \$216.5847 less \$214.9546 or \$1.6301. The minimum cost procurement level and procurement quantity are exhibited in Table XXVI as 15.0509 and 30.9676 respectively. Once again, close agreement is indicated between the optimal policy and associated total system cost of the method discussed above and the Lagrangian multiplier technique resulting in Table I.

Reduction of the Simplified Probabilistic MIMS System

Reduction to the Simplified Probabilistic MISS System

Suppose that the two items of the simplified probabilistic MIMS system described in Chapter V can only be procured from Source 1 and that the parameters indicated for that source apply. When the MISS system is not constrained, the minimum cost points found in Table XXXVII may be used to find the expected total system cost. This expected total system cost is $15.6688 + $9.6204 = $25.2892. The minimum cost procurement levels given in Table XXXVI and the minimum cost procurement quantities given in Table XXXV are applicable. These are summarized in Table XLIX.

TABLE XLVIII

CONDENSED COST FUNCTIONS, REDUCTION TO DETERMINISTIC SISS SYSTEM WITH WAREHOUSE RESTRICTION

$I_1^* w_1$	TC'_{13}
0	217.6283
24	217.3381
48	217.0676
72	216.8166
96	216.5847

TABLE XLIX

OPTIMAL POLICY, REDUCTION TO SIMPLIFIED PROBABILISTIC MISS SYSTEM

Item	L	Q
1	6.1873	18.1265
2	7.6706	16.5161

When the warehouse space is finite, the solution may be found by dynamic programming. Assume, as in the previous chapter, that the warehouse space is 100 cubic units and that the cubic units of space required by Item 1 is 9 cubic units and by Item 2 is 7 cubic units.

The condensed cost functions for this situation may be developed from Table XXXIX by reference to Source 1. These are exhibited in Table L. As before, each section refers to an item with the source indicated by j.

TABLE L

CONDENSED COST FUNCTIONS, REDUCTION TO SIMPLIFIED PROBABILISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

$I^*_{m_i} w_i$	$g_1(I^*_{m_1} w_1)$	j	$g_2(I^*_{m_2} w_2)$	j
14			15.2237	1
18	21.9000	1		
21			12.8531	1
27	19.3499	1		
28			11.6978	1
35			11.0286	1
36	18.1000	1		
42			10.6025	1
45	17.3700	1		
49			10.3109	1
54	16.8884	1		
56			10.0917	1
63	16.5332	1	9.9298	1
70			9.8135	1
72	16.2733	1		
77			9.7324	1
81	16.0845	1		
84			9.6781	1
90	15.9480	1		
91			9.6443	1
98			9.6263	1
99	15.8498	1		

The cost expected from the first stage if all available warehouse space is allocated to it is determined from $f_1(W)=g_1(I^*_{m_1} w_1)$. This gives:

$$f_1(18) = g_1(18) = 21.9000$$
$$f_1(27) = g_1(27) = 19.3499$$
$$f_1(36) = g_1(36) = 18.1000$$
$$f_1(45) = g_1(45) = 17.3700$$
$$f_1(54) = g_1(54) = 16.8884$$
$$f_1(63) = g_1(63) = 16.5332$$
$$f_1(72) = g_1(72) = 16.2733$$
$$f_1(81) = g_1(81) = 16.0845$$
$$f_1(90) = g_1(90) = 15.9480$$
$$f_1(99) = g_1(99) = 15.8498.$$

The computations for $f_1(W)$ are now complete and the results are entered in the first stage of the solution table; Table LI.

From the results of $f_1(W)$, $f_2(W)$ may be computed using the recurrence relation given by Equation (5.18). When $W = 32$,

$$f_2(32) = \min_{0 \leq I^*_{m_2} w_2 \leq 32} [g_2(I^*_{m_2} w_2) + f_1(32 - I^*_{m_2} w_2)].$$

For values of $I^*_{m_2} w_2$ ranging from 0 to 32 this gives one feasible combination; that is:

$$f_3(32) = g_2(14) + f_1(18) = 15.2237 + 21.9000 = 37.1237.$$

When $W = 39$,

$$f_2(39) = \min_{0 \leq I^*_{m_2} w_2 \leq 39} [g_2(I^*_{m_2} w_2) + f_1(39 - I^*_{m_2} w_2)].$$

TABLE LI

SOLUTION TABLE, REDUCTION TO SIMPLIFIED PROBABILISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

W	$f_1(W)$	$I^*_{m_1} w_1(W)$	$f_2(W)$	$I^*_{m_2} w_2(W)$
18	21.9000	18		
27	19.3499	27		
32			37.1237	14
36	18.1000			
39			34.7531	21
41			34.5736	14
45	17.3700	45		
46			33.5978	28
48			32.2030	21
50			33.3237	14
53			32.9286	35
54	16.8884	54*		
55			31.0477	28
57			30.9531	21
59			32.5937	14
60			32.5025	42
62			30.3785	35
63	16.5332	63		
64			29.7978	28
66			30.2231	21
67			32.2109	49
68			32.1121	14
69			29.9524	42
71			29.1286	35
72	16.2733	72		
73			29.0678	28
74			31.9917	56
75			29.7415	21
76			29.6608	49
77			31.7569	14
78			28.7025	42
80			28.3986	35
81	16.0845	81	31.8298	63
82			28.5862	28
83			29.4416	56
84			29.3863	21
85			28.4109	49
86			31.4970	14
87			27.9725	42
88			31.7135	70
89			27.9170	35
90	15.9480	90	29.2797	63
91			28.2310	28

TABLE LI (Continued)

W	$f_1(W)$	$I^*_{m_1} w_1(W)$	$f_2(W)$	$I^*_{m_2} w_2(W)$
92			28.1917	56
93			29.1264	21
94			27.6809	49
95			31.3082	14
96			27.4909	42*
97			29.1634	70
98			27.5618	35
99	15.8498	99	28.0298	63
100			27.9711	28

For values of $I^*_{m_2} w_2$ ranging from 0 to 39 this gives one feasible combination; that is:

$$f_2(39) = g_2(21) + f_1(18) = 12.8531 + 21.9000 = 34.7531.$$

When W = 41,

$$f_2(41) = \min_{0 \le I^*_{m_2} w_2 \le 41} [g_2(I^*_{m_2} w_2) + f_1(41 - I^*_{m_2} w_2)].$$

For values of $I^*_{m_2} w_2$ ranging from 0 to 41 this gives one feasible combination; that is:

$$f_2(41) = g_2(14) + f_1(27) = 15.2237 + 19.3499 = 34.5736.$$

This process is continued until $f_2(100)$ is evaluated. The minimum value of $f_2(W)$ is identified for each value of W and entered in the second stage of Table LI together with its associated value of $I^*_{m_2} w_2$.

Table LI may now be used to find the optimal procurement and inventory policy for this constrained probabilistic MISS system. The minimum expected total system cost is found to be $27.4909 per period in the second stage of Table LI. Table LI also indicates that 54 cubic units of warehouse space are to be allocated to Item 1 and 42 cubic units to Item 2. These allocations of scarce warehouse space are indicated by asterisks. Reference to Table XXXIX with these allocations indicates that the procurement level and procurement quantity for Item 1 should be 4.4450 and 6.9436 respectively. The procurement level and procurement quantity for Item 2 should be 5.4000 and 7.7419 respectively.

The optimal procurement and inventory policy for this restricted MISS system is summarized in Table LII. The penalty in expected total system cost arising due to the warehouse constraint is $27.4909 less $25.2892 or $2.2017 per period.

TABLE LII

OPTIMAL POLICY, REDUCTION TO SIMPLIFIED PROBABILISTIC MISS SYSTEM WITH WAREHOUSE RESTRICTION

Item	L'	Q'
1	4.4450	6.9436
2	5.4000	7.7419

Reduction to the Simplified Probabilistic SIMS System

Suppose that a single-item inventory system with several sources of replenishment stock exists. Specifically, suppose that the item is Item 2 of the previous chapter. When this probabilistic SIMS system is not constrained the minimum cost points found in Table XXXVII may be used to find the expected total system cost. This expected total system cost is \$9.5208 since it is the minimum of the minimums. Thus, Source 2 would be chosen as the minimum cost source. The minimum cost procurement level for the item is 5.4661 and the minimum cost procurement quantity is 13.7265. These are found in Tables XXXVI and XXXV respectively.

When the warehouse space is finite the solution may be found by dynamic programming. Assume, as before, that warehouse space is 100 cubic units and that the item requires 7 cubic units of space.

The first step in finding the optimal procurement and inventory policy for this constrained SIMS system is to develop condensed cost functions from Table XXXIX. These are shown in Table LIII and are developed by searching across a specific value of $I^*_{m_2} w_2$ and selecting the minimum value of $TC'_{m_{2j}}$ together with the source for which this minimum occurs. Symbolically, this process may be stated as:

$$g_2(I^*_{m_2} w_2) = \underset{0 \le I^*_{m_2} w_2 \le W}{\text{Min}} [TC'_{m_{2j}}].$$

This SIMS system is now solved as a single stage dynamic programming process. It is not necessary to set up a solution table. Inspection of $g_2(I^*_{m_2} w_2)$ in Table LIII establishes Source 2 as the minimum cost source of replenishment stock. The expected total system cost is given in Table LIII as \$9.5333. Reference to Table XXXIX establishes the procurement level and procurement quantity at 5.4510 and 12.1511 respectively. The penalty in expected total system cost due to the warehouse constraint is \$9.5333 less \$9.5208 or \$0.0125 per period.

TABLE LIII

CONDENSED COST FUNCTIONS, REDUCTION TO SIMPLIFIED PROBABILISTIC SIMS SYSTEM WITH WAREHOUSE RESTRICTION

$I^*_{m_2} w_2$	$g_2(I^*_{m_2} w_2)$	j
14	15.2237	1
21	12.8531	1
28	11.6978	1
35	11.0286	1
42	10.6025	1
49	10.3101	2
56	10.0561	2
63	9.8754	2
70	9.7475	2
77	9.6579	2
84	9.5967	2
91	9.5569	2
98	9.5333	2

Reduction to the Simplified Probabilistic SISS System

Suppose that a single-item inventory system with a single-source of replenishment stock exists. Specifically, suppose that the item is Item 1 of the previous chapter and that it may be procured only from Source 3. When this probabilistic SISS system is not constrained the minimum cost point found in Table XXXVII is the expected total system cost of \$14.7998. The minimum cost procurement level and the minimum cost procurement quantity is 12.4995 and 17.5021 respectively. These are found in Table XXXVI and Table XXXV.

When the warehouse space is finite, the solution may be found as a trivial case of dynamic programming. Assume that the warehouse space is 100 cubic units and that the item requires 9 cubic units of space as was established previously.

The first step in finding the optimal procurement and inventory policy for this constrained SISS system is to obtain condensed cost functions from Table XXXIX. These are shown in Table LIV and are developed by transferring the values of $TC'_{m_{13}}$ for all values of $I^*_{m_1 w_1}$. The SISS system is now solved as a single stage dynamic programming process. It is not necessary to set up a solution table. Inspection of $TC'_{m_{13}}$ in Table LIV indicates that the minimum expected total system cost will be $15.6054 giving an expected penalty for the constraint on warehouse space of $15.6054 less $14.7998 or $0.8056 per period. The minimum cost procurement level and procurement quantity are exhibited in Table XXXIX to be 10.4850 and 8.5166 respectively.

TABLE LIV

CONDENSED COST FUNCTIONS, REDUCTION TO SIMPLIFIED PROBABILISTIC SISS SYSTEM WITH WAREHOUSE RESTRICTION

$I^*_{m_1 w_1}$	$TC'_{m_{13}}$
18	26.8221
27	22.1647
36	19.8610
45	18.4988
54	17.6073
63	16.9849
72	16.5297
81	16.1616
90	15.8552
99	15.6054

CHAPTER VII

SUMMARY AND CONCLUSIONS

A unified concept of procurement and inventory theory was presented in this dissertation through the establisment of a hierarchy of procurement and inventory systems. Fundamentals of the SISS, SIMS, and MISS systems were presented as prerequisites to the development of the MIMS system. This concluding chapter will be composed of three sections. The first summarizes the material in this treatise by reviewing the content of each chapter. The second gives a critical analysis of the methods for deriving procurement and inventory policy as presented herein. Proposals for further study are listed in the last section.

Summary

Each system in the hierarchy was represented schematically in Chapter I. Literature was cited to indicate the state of development to date. The decision environment was described in the context of the MIMS system. Finally, the contributions of this treatise were outlined.

The SISS system was developed in Chapter II. Models were formulated and applied to the unrestricted and restricted deterministic system and to the unrestricted probabilistic system. The material in this chapter provided a basis for the chapters which followed.

Chapters III and IV were devoted to the intermediate systems in the hierarchy; the SIMS and the MISS systems. Both the deterministic and the probabilistic aspects of these systems were treated. As in Chapter II, Lagrangian multipliers were used to find the optimal procurement and inventory policy for the restricted deterministic systems.

Chapter V presented the MIMS system in its deterministic and probabilistic form. Previously derived models were used to find the optimal procurement and inventory policy for the unrestricted system. The restricted system was optimized by the use of dynamic programming. Since the MIMS system is the most general in the hierarchy, this chapter concluded the hierarchial development.

Reduction of the MIMS system to the lower ordered MISS, SIMS, and SISS systems was presented in Chapter VI. The results from the reduced systems utilizing dynamic programming agree with those from the same systems optimized with the aid of Lagrangian multipliers.

All examples presented in Chapters V and VI were developed from the computer program in the Appendix. The program develops the condensed cost functions for the system under investigation. It then processes the condensed cost functions by dynamic programming yielding a solution table for the problem.

Conclusions

The methods employed to optimize the procurement and inventory systems presented in this dissertation were general in their simultaneous approach to the determination of the minimum cost procurement and inventory policy. The analysis of determinstic systems was further generalized by holding the number of simplifying assumptions to a minimum.

A closed mathematical solution was possible for the unrestricted probabilistic systems by assuming that lead time demand had a uniform distribution. It is possible to find the optimal procurement and inventory policy mathematically for other distributions of lead time demand. However, the optimizing equations become quite cumbersome for most of the common distributions.

The dynamic programming algorithm and the Lagrangian multiplier technique yielded nearly identical results in the development of optimal policy for systems subject to a warehouse restriction. The independent agreement exhibited indicates the validity of each approach.

The method for simultaneously dealing with a warehouse and a source capacity restriction was crude. A more favorable method to determine the optimal policy for the doubly restricted system would be to treat the situation as a multidimensional allocation problem of dynamic programming. Although this is difficult, it may be possible to convert the multidimensional formulation through use of Lagrangian multipliers yielding a decomposition of complex processes into simpler parts.

In its present form, the computerized dynamic programming algorithm requires an excess of computer time. Solution of a restricted MIMS problem, comparable to the first example in Chapter V, took approximately 12 minutes on the IBM 1410. This time increases as the size of the problem, especially the number of items, increases. Much of the time was involved in compiling the program, a routine which could be eliminated by converting the source deck to an object deck and placing the contents of the object deck on magnetic tape. Unnecessary printouts could also be eliminated resulting in a more efficient program. Utilization of a larger computer would eliminate the necessity of phasing the program and placing the intermediate calculations on magnetic tape. There is a possible future for the computerized algorithm in the solution of real world problems if the above changes are considered.

A general criticism retarding the application of advanced procurement and inventory theory is that managerial techniques presently in use do not and cannot obtain the information necessary for the optimization of a complex system. Individuals in managerial positions state that the time and expense required for collecting and digesting the required information more than offsets the returns. However, with the use of modern electronic computers in the control of inventory, the collection of the input parameters and distributions should become more prevalent. The availability of models, such as those presented in this treatise, will provide an incentive to collect input data. In any event, the explanation of basic procurement and inventory phenomena provided by these models should prove more useful in the routine management of procurement and inventory systems.

Proposals for Future Study

This investigation resulted in the vertical generalization of procurement and inventory systems. The proposals for future study listed below recommend horizontal refinements which will aid in the application of the models presented to real-world procurement and inventory management problems.

(1) Use a substitution parameter in the models developed for systems subject to a warehouse restriction. A substitution parameter allows the utilization of space allocated to one item to meet the space requirements of another item.

(2) Study the sensitivity of using the MIMS system rather than some lower ordered system to demonstrate the value of the concept. The study may indicate that savings gained from using MIMS are negligible or non-existent.

(3) Include a backorder parameter to allow loss of all or part of the demand when a shortage condition occurs.

(4) Split the holding cost into two components. The first component would consider the fixed portion of holding cost and the second would consider the variable portion.

(5) Derive an expression for shortage cost in the simplified probabilistic system which will result in an extension of the solution region of the model.

(6) Derive models representing the probabilistic systems for other possible lead time demand distributions.

(7) Determine optimal policies for all systems subject to the simultaneous consideration of a warehouse and a source capacity restriction.

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APPENDIX

SOLUTION OF PROCUREMENT AND INVENTORY PROBLEMS BY IBM 1410

This appendix will be presented in the following manner: An introductory section will describe the capability of the computer program and give a program listing. A section describing the input data, concluded by an example will follow. The last section will describe the output data and give solutions to the problems forwarded in Chapters V and VI.

Introduction

The program discussed in this appendix will process both deterministic and simplified probabilistic procurement and inventory problems. The maximum dimension for any problem is 5 items, 5 sources, and 100 cubic units of warehouse space. The program is easily expanded to accomodate larger problems if an increased core storage is available to the user. This expansion is accomplished by changing the limits of the dimension statements and rewriting some of the format statements along with appropriate modifications of the input data. The program is self contained in that it not only generates the condensed cost functions, but solves the allocation problem it creates. The condensed cost functions are computed by utilizing the equations developed in Chapter V and the allocation problem is solved by the dynamic programming algorithm also appearing in that chapter. The program is in three phases. The beginning of each phase can be identified in the listing given below by the C in the left margin of the first statement. Written in FORTRAN IV, the program is as follows:

```
      MON$$     JOB  252340022       JERRY BANKS
      MON$$     ASGN MGO,A2
      MON$$     ASGN MJB,A3
      MON$$     MODE GO,TEST
      MON$$     EXEQ FORTRAN,,,,,,,PH1
C     PHASE 1
      DIMENSION D(5),R(5,5),CP(5,5),CH(5),CS(5),T(5,5),CI(5,5),
     1PLOPT(5,5),PQOPT(5,5),TCOPT(5,5),W(5)
1     FORMAT(5F10.4)
2     FORMAT(2I10)
3     FORMAT(/)
4     FORMAT(///)
5     FORMAT(10X, 20HDETERMINISTIC SYSTEM,I5,1X,7HITEM(S),I5,1X,9HSOURCE
     1(S)//)
6     FORMAT(10X, 20HPROBABILISTIC SYSTEM,I5,1X,7HITEM(S),I5,1X,9HSOURCE
     1(S)//)
7     FORMAT(//)
11    FORMAT(6H ITEM ,I1,5X,5F10.4)
14    FORMAT(10X,9HITEM COST/ 12X,50H  SOURCE 1  SOURCE 2  SOURCE 3  SOU
     1RCE 4  SOURCE 5)
15    FORMAT(10X,16HPROCUREMENT COST/ 12X,50H  SOURCE 1  SOURCE 2  SOURC
     1E 3  SOURCE 4  SOURCE 5)
16    FORMAT(10X,12HHOLDING COST)
17    FORMAT(10X,13HSHORTAGE COST)
18    FORMAT(10X,6HDEMAND)
19    FORMAT(10X,21HRATE OF REPLENISHMENT/12X,50H  SOURCE 1  SOURCE 2  S
     1OURCE 3  SOURCE 4  SOURCE 5)
20    FORMAT(10X,9HLEAD TIME/ 12X,50H  SOURCE 1  SOURCE 2  SOURCE 3  SOU
     1RCE 4  SOURCE 5)
21    FORMAT(10X,21HTOTAL WAREHOUSE SPACE/15X,F10.4//)
22    FORMAT(10X,38HSPACE REQUIREMENT FOR INDIVIDUAL ITEMS)
26    FORMAT(F10.4)
134   FORMAT(10X,35HMINIMUM COST PROCUREMENT QUANTITIES/12X,50H  SOURCE
     11  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5)
```

```
135   FORMAT(10X,31HMINIMUM COST PROCUREMENT LEVELS    /12X,50H  SOURCE
     11  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5)
136   FORMAT(10X,30HASSOCIATED MINIMUM TOTAL COSTS/12X,50H  SOURCE 1  SO
     1URCE 2  SOURCE 3  SOURCE 4  SOURCE 5)
137   FORMAT(51H POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS//)
150   FORMAT(6H ITEM ,I1,5X,F10.4)
153   FORMAT(1H1)
      READ(1,2)M,N
      READ(1,1)((CI(I,J),J=1,N),I=1,M)
      READ(1,1)((CP(I,J),J=1,N),I=1,M)
      READ(1,1)(CH(I),I=1,M)
      READ(1,1)(CS(I),I=1,M)
      READ(1,1)(D(I),I=1,M)
      READ(1,1)((R(I,J),J=1,N),I=1,M)
      READ(1,1)((T(I,J),J=1,N),I=1,M)
      READ(1,26) SPACE
      READ(1,1) (W(I),I=1,M)
      READ(1,26) TYPE
      WRITE(3,153)
      IF(TYPE.EQ.0.0) GO TO 34
      WRITE(3,6) M,N
      GO TO 37
34    WRITE(3,5) M,N
37    WRITE(3,14)
      DO 501 I=1,M
501   WRITE(3,11)I,(CI(I,J),J=1,N)
      WRITE(3,3)
      WRITE(3,15)
      DO 502 I=1,M
502   WRITE(3,11)I,(CP(I,J),J=1,N)
      WRITE(3,3)
      WRITE(3,16)
      DO 508 I=1,M
508   WRITE(3,150) I,CH(I)
```

```
      WRITE(3,3)
      WRITE(3,17)
      DO 509 I=1,M
509   WRITE(3,150) I,CS(I)
      WRITE(3,3)
      WRITE(3,18)
      DO 510 I=1,M
510   WRITE(3,150) I,D(I)
      WRITE(3,3)
      WRITE(3,19)
      DO 503 I=1,M
503   WRITE(3,11)I,(R(I,J),J=1,N)
      WRITE(3,3)
      WRITE(3,20)
      DO 504 I=1,M
504   WRITE(3,11)I,(T(I,J),J=1,N)
      WRITE(3,3)
      WRITE(3,21) SPACE
      WRITE(3,22)
      DO 511 I=1,M
511   WRITE(3,150) I,W(I)
      WRITE(3,4)
      IF(TYPE.NE.1.0) GO TO 200
      WRITE(3,137)
      DO 141 I=1,M
      DO 141 J=1,N
      IF(D(I).GE.R(I,J)) GO TO 139
      AF2=1.0-D(I)/R(I,J)
      APRIM=2.0*D(I)*T(I,J)
      PQOPA=(2.0*APRIM*CP(I,J))-CS(I)
      PQOPB=(CS(I)*D(I)*AF2)-APRIM*CH(I)
      PQOPC=CS(I)/(APRIM*CH(I))
      PQOPT(I,J)=D(I)*SQRT((PQOPA/PQOPB)*PQOPC)
      IF(PQOPT(I,J).GE.D(I)) GO TO 164
```

```
      PQOPT(I,J)=5555.5
      PLOPT(I,J)=5555.5
      TCOPT(I,J)=5555.5
      GO TO 141
164   IF(PQOPT(I,J).GE.1.0) GO TO 163
      PQOPT(I,J)=6666.6
      PLOPT(I,J)=6666.6
      TCOPT(I,J)=6666.6
      GO TO 141
163   TD=T(I,J)*D(I)
      PLOPT(I,J)=APRIM*(1.0-((CH(I)*PQOPT(I,J))/(CS(I)*D(I))))
      IF(PLOPT(I,J).GE.TD) GO TO 170
      PQOPT(I,J)=7777.7
      PLOPT(I,J)=7777.7
      TCOPT(I,J)=7777.7
      GO TO 141
170   TD2=2.0*TD-1.0
      IF(PLOPT(I,J).LE.TD2) GOTO165
      PLOPT(I,J)=3333.3
      PQOPT(I,J)=3333.3
      TCOPT(I,J)=3333.3
      GO TO 141
165   AF=1.0-D(I)/R(I,J)
      GEOM=PQOPT(I,J)*AF+PLOPT(I,J)-TD
      IF(GEOM.GE.0.0) GO TO 166
      PQOPT(I,J)= 4444.4
      PLOPT(I,J)= 4444.4
      TCOPT(I,J)= 4444.4
      GO TO 141
166   CIAP=CI(I,J)*D(I)
      CPAP=(CP(I,J)*D(I))/PQOPT(I,J)
      CHCA=(PQOPT(I,J)*AF)/2.0
      CHCB=PLOPT(I,J)-(D(I)*T(I,J))
      CHCP=CH(I)*(CHCA+CHCB)
```

```
      CSAQ=((APRIM-PLOPT(I,J))**2)-1.0
      CSAR=(CS(I)*D(I))/(2.0*PQOPT(I,J)*APRIM)
      CSAP=CSAR*CSAQ
147   TCOPT(I,J)=CIAP+CPAP+CHCP+CSAP
      GO TO 141
139   PQOPT(I,J)=9999.9
      PLOPT(I,J)=9999.9
      TCOPT(I,J)=9999.9
141   CONTINUE
      GO TO 145
200   WRITE(3,137)
      DO 140 I=1,M
      DO 140 J=1,N
      IF(D(I).GE.R(I,J)) GO TO 138
      FACTR=SQRT(1.0-D(I)/R(I,J))
      PQOPT(I,J)=(1.0/FACTR)*SQRT(((2.0*CP(I,J)*D(I))/CH(I))+((2.0*CP(I,
     1J)*D(I))/CS(I)))
      PLOPT(I,J)=D(I)*T(I,J)-FACTR*SQRT((2.0*CP(I,J)*D(I))/(CS(I)*(1.0+
     1CS(I)/CH(I))))
      TCOPT(I,J)=CI(I,J)*D(I)+FACTR*SQRT((2.0*CP(I,J)*CH(I)*CS(I)*D(I))/
     1(CH(I)+CS(I)))
      IF(PQOPT(I,J).GE.D(I)) GO TO 210
      PQOPT(I,J)=5555.5
      PLOPT(I,J)=5555.5
      TCOPT(I,J)=5555.5
      GO TO 140
210   IF(PQOPT(I,J).GE.1.0) GO TO 140
      PQOPT(I,J)=6666.6
      PLOPT(I,J)=6666.6
      TCOPT(I,J)=6666.6
      GO TO 140
138   PQOPT(I,J)=9999.9
      PLOPT(I,J)=9999.9
      TCOPT(I,J)=9999.9
```

```
140    CONTINUE
145    WRITE(3,134)
       DO 505 I=1,M
505    WRITE(3,11) I,(PQOPT(I,J),J=1,N)
       WRITE(3,3)
       WRITE(3,135)
       DO 506  I=1,M
506    WRITE(3,11) I,(PLOPT(I,J),J=1,N)
       WRITE(3,3)
       WRITE(3,136)
       DO 507 I=1,M
507    WRITE(3,11) I,(TCOPT(I,J),J=1,N)
       WRITE(3,4)
       CALL NEXTPH
       END
      MON$$      EXEQ FORTRAN,,,,,,,PH2
C      PHASE 2
       DIMENSION D(5),R(5,5),CP(5,5),CH(5),CS(5),T(5,5),CI(5,5),SUBTC(101
      1),JS(101),W(5),SUBPL(101),SUBPQ(101)
1      FORMAT(5F10.4)
2      FORMAT(2I10)
3      FORMAT(///)
4      FORMAT(3I10)
9      FORMAT(4I10,F14.4,5X,16HSUB-STAGE POLICY//)
11     FORMAT(6H ITEM ,I1,5X,5F10.4)
25     FORMAT(I2,3I7,3F12.4/)
26     FORMAT(F10.4)
27     FORMAT(23X,3F12.4)
28     FORMAT(2F14.4)
151    FORMAT(5X,47HDEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS//)
152    FORMAT(1X,24HNO OPTIMAL POLICY EXISTS/)
       REWIND 4
       READ(1,2)M,N
       READ(1,1)((CI(I,J),J=1,N),I=1,M)
```

```
      READ(1,1)((CP(I,J),J=1,N),I=1,M)
      READ(1,1)(CH(I),I=1,M)
      READ(1,1)(CS(I),I=1,M)
      READ(1,1)(D(I),I=1,M)
      READ(1,1)((R(I,J),J=1,N),I=1,M)
      READ(1,1)((T(I,J),J=1,N),I=1,M)
      READ(1,26) SPACE
      READ(1,1) (W(I), I=1,M)
      READ(1,4) J10,J25,J27
      READ(1,26) TYPE
      IF(J10.EQ.1) GO TO 33
      WRITE(3,151)
33    KSP=SPACE
      DO 50 I=1,M
      MOST=SPACE/W(I)
      IWI=W(I)
      LEFT=KSP-(MOST*IWI )+1
      IMAX=(SPACE/W(I))+1.0
      DO 51 K=1,IMAX
      KK=K-1
      UNITS=KK
      JS(K)=1
      DO 52 J=1,N
      IF(D(I).LT.R(I,J)) GO TO 59
      TRY=9999.9
      PLA=9999.9
      PQQ=9999.9
      GO TO 57
59    AF=1.0-(D(I)/R(I,J))
      IF(TYPE.EQ.1.0) GO TO 39
      PQQA=1.0/AF
      PQQB=2.0*CP(I,J)*D(I)*AF
      PQQC=(UNITS**2)*(CH(I)+CS(I))
      PQQD=(PQQB+PQQC)/CS(I)
```

```
      PQQ=PQQA*(SQRT(PQQD))
      IF( PQQ.GE.D(I)) GO TO 40
      PQQ=D(I)
      IF(PQQ.LT.1.0) PQQ=1.0
40    PLA=UNITS+(D(I)*T(I,J))-(PQQ*AF)
      CIA=CI(I,J)*D(I)
      CPA=(CP(I,J)*D(I))/PQQ
      CHC=(CH(I)*(UNITS**2))/(2.0*PQQ*AF)
      CSA = (CS(I)*(PLA   -D(I)*T(I,J))**2)/(2.0*PQQ*AF)
      GO TO 36
39    A=2.0*T(I,J)*D(I)
      CHECK=(((CS(I)*D(I))/A)*((1.0-D(I)/R(I,J))**2))-(CH(I)*(1.0-D(I)/
     1R(I,J)))
      IF(CHECK.LT.0.0) GO TO 38
      PQQF=2.0*CP(I,J)*D(I)
      PQQG=(CS(I)*D(I))/A
      PQQH=(((A/2.0)-UNITS)**2)-1.0
      PQQI=PQQF+(PQQG*PQQH)
      PQQJ=PQQG*(AF**2)
      PQQK=CH(I)*AF
      PQQL=PQQJ-PQQK
      PQQM=PQQI/PQQL
      PQQ=SQRT(PQQM)
      IF(PQQ.GE.D(I)) GO TO 41
38    PQQ=D(I)
      IF(PQQ.LT.1.0) PQQ=1.0
41    PLA=UNITS+(A/2.0)-(PQQ*AF)
      TD=T(I,J)*D(I)
      IF(PLA.GE.TD) GO TO 170
      PLA=TD
      PQQ=(UNITS+(A/2.0)-PLA)/AF
170   TD2=2.0*TD-1.0
      IF(PLA.LE.TD2) GO TO 175
      TRY=9999.9
```

```
      PQQ=9999.9
      PLA=9999.9
      GO TO 57
175   CIA=CI(I,J)*D(I)
      CPA=(CP(I,J)*D(I))/PQQ
      CHC=(((PQQ*AF)/2.0)+PLA-(A/2.0))*CH(I)
      CSA=((CS(I)*D(I))/(2.0*PQQ*A))*(((A-PLA)**2)-1.0)
36    TRY=CIA+CPA+CSA+CHC
      IF(PQQ.GE.D(I)) GO TO 568
      TRY=9999.9
      PQQ=9999.9
      PLA=9999.9
568   IF(PQQ.GE.1.0) GO TO 57
      TRY=9999.9
      PQQ=9999.9
      PLA=9999.9
57    IF(J27.EQ.0) GO TO 571
      WRITE(3,27)PQQ,PLA,TRY
571   IF(J.NE.1) GO TO 45
      SUBTC(K)=TRY
      SUBPQ(K)=PQQ
      SUBPL(K)=PLA
      GO TO 52
45    IF(TRY.GE.SUBTC(K)) GO TO 52
      SUBTC(K)=TRY
      SUBPQ(K)=PQQ
      SUBPL(K)=PLA
      JS(K)=J
52    CONTINUE
      IF(J25.EQ.0) GO TO 55
      KKIWI=KK*IWI
      IF(SUBTC(K).LT.9999.9) GO TO 573
      WRITE(3,152)
      GO TO 55
```

```
573   WRITE(3,25) I,JS(K),KK,KKIWI,SUBPQ(K),SUBPL(K),SUBTC(K)
55    IW=0
      DO 70 NALW=1,IWI
      IF(KK.EQ.MOST) GO TO 72
      IW=(KK*IWI)+NALW-1
      IXXI=KK*IWI
      IF(IW.NE.IXXI) GO TO 300
      GO TO 305
300   SUBTC(K)=9999.9
305   WRITE(4) I,IW,JS(K),KK,SUBPQ(K),SUBPL(K),SUBTC(K)
      IF(J10.EQ.0) GO TO 70
      IF(SUBTC(K).GE.9999.9) GO TO 70
      WRITE(3,25) I,JS(K),KK,IW,SUBPQ(K),SUBPL(K),SUBTC(K)
      GO TO 70
72    DO 73 NADD=1,LEFT
      IW=(KK*IWI)+NADD-1
      IXXI=KK*IWI
      IF(IW.NE.IXXI) GO TO 400
      GO TO 405
400   SUBTC(K)=9999.9
405   WRITE(4) I,IW,JS(K),KK,SUBPQ(K),SUBPL(K),SUBTC(K)
      IF(J10.EQ.0) GO TO 73
      IF(SUBTC(K).GE.9999.9) GO TO 73
      WRITE(3,25) I,JS(K),KK,IW,SUBPQ(K),SUBPL(K),SUBTC(K)
73    CONTINUE
      GO TO 50
70    CONTINUE
51    CONTINUE
50    CONTINUE
      END FILE 4
      REWIND 4
      CALL NEXTPH
      END
     MON$$     EXEQ FORTRAN,,,,,,,PH3
```

```
C     PHASE 3
      DIMENSION FN(101),GNM1(101),FNM1(101),STC(101),IPI(101)
11    FORMAT(  /2X,52HSPACE UTILIZED     TOTAL COST   ALLOCATION    STAG
     1E ,I1)
23    FORMAT(///10X,36HDYNAMIC PROGRAMMING SOLUTION FOLLOWS//)
24    FORMAT(2X,53HSPACE UTILIZED     TOTAL COST   ALLOCATION    STAGE 1
     1)
26    FORMAT(F10.4)
29    FORMAT(I10,F20.4,I10)
      READ(1,26) SPACE
      WRITE(3,23)
      WRITE(3,24)
      KSPAS=SPACE+1.0
75    DO 76 KKKX=1,KSPAS
      READ(4) I,IW,JSJ,KK,SBPQ,SBPL,SBTC
      STC(KKKX)=SBTC
      IF(I.NE.1) GO TO 78
      FNM1(KKKX)=STC(KKKX)
      IPI(KKKX)=IW
      IF(FNM1(KKKX).GE.9999.9) GO TO 76
      WRITE(3,29) IW,FNM1(KKKX),IPI(KKKX)
      GO TO 76
78    GNM1(KKKX)=STC(KKKX)
76    CONTINUE
      IF(I.EQ.1) GO TO 75
      DO 95 KKKX=1,KSPAS
      IF(KKKX.NE.1) GO TO 77
      WRITE(3,11) I
77    ALPHA=9999.9
      GAMMA=0.0
      DO 90 IWF=1,KKKX
      IS=KKKX-IWF+1
      BETA=GNM1(IWF)+FNM1(IS)
      IF(BETA.LT.ALPHA) GO TO 79
```

```
      GO TO 90
79    ALPHA=BETA
      GAMMA=IWF-1
90    CONTINUE
      FN(KKKX)=ALPHA
      IPI(KKKX)=GAMMA
      IF(FN(KKKX).GE.9999.9) GO TO 95
      IWX=KKKX-1
      WRITE(3,29) IWX,FN(KKKX),IPI(KKKX)
95    CONTINUE
      DO 100 KKKX=1,KSPAS
      FNM1(KKKX)=FN(KKKX)
      FN(KKKX)=0.0
      IPI(KKKX)=0.0
100   CONTINUE
      GO TO 75
      END
     MON$$      EXEQ LINKLOAD
                PHASEMATHPROB
                CALL PH1
                PHASE
                BASE1PH1
                CALL PH2
                PHASE
                BASE1PH2
                CALL PH3
     MON$$      EXEQ MATHPROB,MJB
```

Input Data

Input is via standard punch cards. It may be divided into 12 sections, each of which is explained below:

Section 1: M and N. The symbol M refers to the number of items and N refers to the number of sources. There may be from 1 to 5 items and 1 to 5 sources. The value M is placed in column 10 and the value N is placed in column 20, both on the same data card and in fixed point notation.

Section 2: CI(I, J). The symbol CI(I, J) is analogous to C_i as explained in Chapter I. The values are entered by item row wise for each source, 5 values per card in floating point notation. The fields are 1-10, 11-20, 21-30, 31-40, and 41-50. Each value may be entered anywhere in the field. There may be a maximum of 4 digits after the decimal point for each value. Any item which cannot be obtained from a particular source is given the dummy value 8888.8 as indicated in Table A1.

Section 3: CP(I, J). The symbol CP(I, J) is analogous to C_p as explained in Chapter I. The input of these values follows the form of Section 2. Any value which cannot be procured from a particular source is given the dummy value 0.0.

Section 4: CH(I). The symbol CH(I) is analogous to C_h as explained in Chapter I. These values are entered by item row wise with up to 5 values on a single card in floating point notation. The number of entries on the card is identical to the value M. The fields are 1-10, 11-20, 21-30, 31-40, and 41-50.

TABLE A 1

SUMMARY OF OUTPUT SIGNALS AND THEIR CAUSES

Output Section	Problem Type	Signal	Cause
1	Both	C_i = 8888.8	Source does not produce item
2	Deterministic	Q = 5555.5 L = 5555.5 TC = 5555.5	$Q < D$
		Q = 6666.6 L = 6666.6 TC = 6666.6	$Q < 1$
		Q = 9999.9 L = 9999.9 TC = 9999.9	$D \geq R$
	Probabilistic	Q = 3333.3 L = 3333.3 TC_m = 3333.3	$L_m > (2D_m T_m - 1)$
		Q = 4444.4 L = 4444.4 TC_m = 4444.4	$I^*_m < 0$
		Q = 5555.5 L = 5555.5 TC_m = 5555.5	$Q < D_m$
		Q = 6666.6 L = 6666.6 TC_m = 6666.6	$Q < 1$
		Q = 7777.7 L = 7777.7 TC_m = 7777.7	$L < D_m T_m$
		Q = 9999.9 L = 9999.9 TC_m = 9999.9	$D_m \geq R$
3	Deterministic	Q' = 9999.9 L' = 9999.9 TC' = 9999.9	$D \leq R$, or Q'<D & it is impossible to let Q'=D, or Q'<1 & it is impossible to let Q'=1.
	Probabilistic	Q' = 9999.9 L' = 9999.9 TC'_m = 9999.9	$L > (2D_m T_m - 1)$, or $D_m \geq R$, or Q'<D & it is impossible to let Q'=D_m, or Q'<1 & it is impossible to let Q'=1.

Each value may be entered anywhere in the field. There may be a maximum of 4 digits after the decimal point for each value.

Section 5: CS(I). The symbol CS(I) is analogous to C_s in the deterministic case or C_s' in the probabilistic case, both of which are explained in Chapter I. The input format for these values follows that of Section 4.

Section 6: D(I). The symbol D(I) is analogous to D in the deterministic case or D_m in the probabilistic case, both of which are explained in Chapter I. The input for these values follows that of Section 4.

Section 7: R (I, J). The symbol R(I, J) is analogous to R as explained in Chapter I. The input for these values follows that of Section 2. Any item which cannot be otained from a particular source is given the dummy value of 0.0 and any item which can be obtained at an infinite replenishment rate is given the dummy value 9999.9.

Section 8: T(I, J). The symbol T(I, J) is analogous to T in the deterministic case or T_m in the probabilistic case, both of which are explained in Chapter I. The input of these values follows that of Section 2. Any item which cannot be obtained from a particular source is given the value 0.0.

Section 9: SPACE. The symbol SPACE is analogous to W as explained in Chapter I. This value is entered anywhere in columns 1-10. It should be entered as a floating point value whose maximum is less than or equal to 100.0, Although floating point notation is used, the entry should be an integer value.

Section 10: W(I). The symbol W(I) is analogous to w as explained in Chapter I. The input of these values follows that of Section 2. Although floating point notation is used, these entries should be integer values.

Section 11: J10, J25, and J27. These symbols are utilized to govern the output of the development of the condensed cost functions. The values of the symbols are zero or one in columns 10, 20, and 30. The use of the set 0, 1, 1 or the set 1, 0, 0 is acceptable. The result of using either set will be discussed later.

Section 12: TYPE. The symbol TYPE refers to the type of problem being considered. If the problem is deterministic, the value 0.0 should be placed in the field 1-10. If the problem is probabilistic, the value 1.0 should be placed anywhere in the field 1-10.

The program is written in three phases. This necessitates certain values being read into memory more than once. The input data should be ordered, section after section, in the following manner: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 9. The input data for the first example problem presented in Chapter V are displayed in Figure A1 as they appeared on the data cards.

	3	5		
31.50	8888.8	34.75	30.88	33.38
19.85	8888.8	17.94	18.33	18.08
12.30	12.35	8888.8	12.0	11.86
20.40	.000	23.16	18.30	19.55
17.32	.00	18.70	17.50	14.65
16.50	16.05	.00	15.50	17.50
.30	.24	.12		
.300	.17	.25		
6.0	4.0	1.0		
8.0	0.0	9999.9	9999.9	9999.9
12.0	0.0	9999.9	9999.9	9999.9
4.0	40.0	.00	9999.9	9999.9
4.0	.00	7.0	2.0	10.0
6.0	.00	3.0	4.0	12.0
15.00	3.00	.00	1.0	12.0
100.0				
24.0	12.0	6.0		
0.0				
	3	5		
31.50	8888.8	34.75	30.88	33.38
19.85	8888.8	17.94	18.33	18.08
12.30	12.35	8888.8	12.0	11.86
20.40	.000	23.16	18.30	19.55
17.32	.00	18.70	17.50	14.65
16.50	16.05	.00	15.50	17.50
.30	.24	.12		
.300	.17	.25		
6.0	4.0	1.0		
8.0	0.0	9999.9	9999.9	9999.9
12.0	0.0	9999.9	9999.9	9999.9
4.0	40.0	.00	9999.9	9999.9
4.0	.00	7.0	2.0	10.0
6.0	.00	3.0	4.0	12.0
15.00	3.00	.00	1.0	12.0
100.0				
24.0	12.0	6.0		
	0	1	1	
0.0				
100.0				

Figure A1. Input Data for Deterministic MIMS Example

Output

Output is via the standard print feature of the computer and may be divided into 4 sections as follows:

Section 1: Input Data Printout.

Section 2: Optimal Policy Without Constraint. These values are applicable to the situation in which there is no constraint on warehouse space. This section is also useful in calculating the penalty imposed by adhering to a warehouse constraint. The program has many checks to disallow any unfavorable situation. If an unfavorable situation arises and it cannot be corrected, a signal will be given. These signals and their causes are given in Table A 1.

Section 3: Condensed Cost Functions. As mentioned previously, the format of this section is governed by certain input data. The input 0, 1, 1 (J10, J25, J27) in columns 10, 20, and 30 will result in a printout of each Q', L', and TC', for a particular item and I^* value for all sources. The condensed cost functions are then displayed along with associated pertinent data. The output will be presented in the following order: Item Number, Source Selected, Maximum Units in Inventory, Space Required to Warehouse These Units, Optimal Procurement Quantity, Optimal Procurement Level, and Associated Total Cost. The program has many checks to disallow any unfavorable situation. If an unfavorable situation arises, and it cannot be corrected, a signal will be given. These signals and their causes are given in Table A 1.

The input 1, 0, 0 (J10, J25, J27) in columns 10, 20, and 30 will result in a display of the condensed cost functions and associated pertinent data only. The output will be displayed as discussed above. Again, the signals shown in Table A1 will be given in the case of an unfavorable situation.

Section 4: Dynamic Programming Solution. The dynamic programming solution is given in stages. The user selects the minimizing value in the last stage and performs the backward solution as in Chapters V and VI. For purposes of illustration, the output data pertaining to the solution of the examples in Chapters V and VI are presented below.

DETERMINISTIC SYSTEM 3 ITEM(S) 5 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	31.5000	8888.8000	34.7500	30.8800	33.3800
ITEM 2	19.8500	8888.8000	17.9400	18.3300	18.0800
ITEM 3	12.3000	12.3500	8888.8000	12.0000	11.8600

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	20.4000	.0000	23.1600	18.3000	19.5500
ITEM 2	17.3200	.0000	18.7000	17.5000	14.6500
ITEM 3	16.5000	16.0500	.0000	15.5000	17.5000

HOLDING COST

ITEM 1	.3000
ITEM 2	.2400
ITEM 3	.1200

SHORTAGE COST

ITEM 1	.3000
ITEM 2	.1700
ITEM 3	.2500

DEMAND

ITEM 1	6.0000
ITEM 2	4.0000
ITEM 3	1.0000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	8.0000	.0000	9999.9000	9999.9000	9999.9000
ITEM 2	12.0000	.0000	9999.9000	9999.9000	9999.9000
ITEM 3	4.0000	40.0000	.0000	9999.9000	9999.9000

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	4.0000	.0000	7.0000	2.0000	10.0000
ITEM 2	6.0000	.0000	3.0000	4.0000	12.0000
ITEM 3	15.0000	3.0000	.0000	1.0000	12.0000

TOTAL WAREHOUSE SPACE
100.0000

SPACE REQUIREMENT FOR INDIVIDUAL ITEMS

ITEM 1	24.0000
ITEM 2	12.0000
ITEM 3	6.0000

POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

MINIMUM COST PROCUREMENT QUANTITIES

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	80.7960	9999.9000	43.0571	38.2737	39.5593
ITEM 2	45.7011	9999.9000	38.7806	37.5156	34.3251
ITEM 3	23.2952	20.1507	9999.9000	19.5543	20.7776

MINIMUM COST PROCUREMENT LEVELS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	13.9005	9999.9000	20.4843	-7.1253	40.2322
ITEM 2	6.1654	9999.9000	-10.6917	-5.9515	27.9152
ITEM 3	9.3336	-3.3719	9999.9000	-5.3413	5.2619

ASSOCIATED MINIMUM TOTAL COSTS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	192.0298	9999.9000	214.9546	191.0176	206.2103
ITEM 2	82.4318	9999.9000	75.6175	77.0517	75.7344
ITEM 3	13.7165	13.9429	9999.9000	13.5853	13.5445

DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

				57.1314	9.7171	193.2848
				9999.9000	9999.9000	9999.9000
				30.4459	11.5723	217.6283
				27.0636	-15.0473	193.3942
				27.9726	32.0441	208.6667
1	1	0	0	57.1314	9.7171	193.2848
				57.4108	10.6473	193.0058
				9999.9000	9999.9000	9999.9000
				30.4788	12.5394	217.3381
				27.1005	-14.0843	193.1052
				28.0084	33.0083	208.3774
1	1	1	24	57.4108	10.6473	193.0058
				58.2408	11.4397	192.7680
				9999.9000	9999.9000	9999.9000
				30.5772	13.4411	217.0676
				27.2111	-13.1948	192.8384
				28.1154	33.9014	208.1095
1	1	2	48	58.2408	11.4397	192.7680
				59.5986	12.1003	192.5698
				9999.9000	9999.9000	9999.9000
				30.7404	14.2779	216.8166
				27.3945	-12.3780	192.5934
				28.2929	34.7240	207.8627
1	1	3	72	59.5986	12.1003	192.5698
				61.4491	12.6377	192.4086
				9999.9000	9999.9000	9999.9000
				30.9676	15.0509	216.5847

				27.6491	-11.6325	192.3697
				28.5395	35.4775	207.6367
1	4	4	96	27.6491	-11.6325	192.3697
				34.9655	.6896	83.3627
				9999.9000	9999.9000	9999.9000
				29.6707	-17.6588	76.8020
				28.7029	-12.6914	78.1975
				26.2619	21.7486	76.7827
2	5	0	0	26.2619	21.7486	76.7827
				35.0430	1.6379	83.2015
				9999.9000	9999.9000	9999.9000
				29.7113	-16.6994	76.6389
				28.7449	-11.7334	78.0346
				26.3078	22.7027	76.6205
2	5	1	12	26.3078	22.7027	76.6205
				35.2745	2.4836	83.0577
				9999.9000	9999.9000	9999.9000
				29.8329	-15.8210	76.4895
				28.8706	-10.8590	77.8860
				26.4450	23.5655	76.4738
2	5	2	24	26.4450	23.5655	76.4738
				35.6570	3.2286	82.9311
				9999.9000	9999.9000	9999.9000
				30.0345	-15.0225	76.3538
				29.0788	-10.0672	77.7514
				26.6722	24.3383	76.3424
2	5	3	36	26.6722	24.3383	76.3424
				36.1857	3.8761	82.8210
				9999.9000	9999.9000	9999.9000
				30.3145	-14.3024	76.2314
				29.3679	-9.3562	77.6305
				26.9871	25.0236	76.2259
2	5	4	48	26.9871	25.0236	76.2259
				36.8544	4.4303	82.7268
				9999.9000	9999.9000	9999.9000
				30.6707	-13.6584	76.1219
				29.7355	-8.7236	77.5230
				27.3866	25.6242	76.1238
2	3	5	60	30.6707	-13.6584	76.1219
				37.6555	4.8963	82.6476
				9999.9000	9999.9000	9999.9000
				31.1005	-13.0881	76.0249
				30.1786	-8.1665	77.4283
				27.8671	26.1439	76.0355
2	3	6	72	31.1005	-13.0881	76.0249
				38.5808	5.2794	82.5825

				9999.9000	9999.9000	9999.9000
				31.6009	-12.5883	75.9400
				30.6941	-7.6818	77.3459
				28.4246	26.5867	75.9602
2	3	7	84	31.6009	-12.5883	75.9400
				39.6217	5.5855	82.5304
				9999.9000	9999.9000	9999.9000
				32.1687	-12.1558	75.8665
				31.2783	-7.2658	77.2751
				29.0545	26.9571	75.8972
2	3	8	96	32.1687	-12.1558	75.8665
				13.2664	5.0501	14.7874
				11.4757	-8.1888	15.1472
				9999.9000	9999.9000	9999.9000
				11.1360	-10.1349	14.7837
				11.8327	.1684	14.8178
3	4	0	0	11.1360	-10.1349	14.7837
				13.3652	5.9760	14.5559
				11.5433	-7.2547	14.9136
				9999.9000	9999.9000	9999.9000
				11.2023	-9.2012	14.5503
				11.8951	1.1060	14.5834
3	4	1	6	11.2023	-9.2012	14.5503
				13.6573	6.7569	14.3607
				11.7439	-6.4503	14.7125
				9999.9000	9999.9000	9999.9000
				11.3988	-8.3976	14.3494
				12.0803	1.9208	14.3797
3	4	2	12	11.3988	-8.3976	14.3494
				14.1308	7.4018	14.1995
				12.0707	-5.7690	14.5422
				9999.9000	9999.9000	9999.9000
				11.7190	-7.7178	14.1794
				12.3829	2.6183	14.2054
3	4	3	18	11.7190	-7.7178	14.1794
				14.7681	7.9239	14.0690
				12.5140	-5.2012	14.4003
				9999.9000	9999.9000	9999.9000
				12.1530	-7.1518	14.0379
				12.7944	3.2068	14.0583
3	4	4	24	12.1530	-7.1518	14.0379
				15.5491	8.3381	13.9654
				13.0619	-4.7353	14.2838
				9999.9000	9999.9000	9999.9000
				12.6893	-6.6880	13.9220
				13.3049	3.6964	13.9359
3	4	5	30	12.6893	-6.6880	13.9220

				16.4535	8.6598	13.8850
				13.7018	-4.3592	14.1898
				9999.9000	9999.9000	9999.9000
				13.3155	-6.3141	13.8285
				13.9034	4.0979	13.8355
3	4	6	36	13.3155	-6.3141	13.8285
				17.4620	8.9034	13.8241
				14.4214	-4.0609	14.1152
				9999.9000	9999.9000	9999.9000
				14.0195	-6.0181	13.7545
				14.5790	4.4224	13.7543
3	5	7	42	14.5790	4.4224	13.7543
				18.5577	9.0816	13.7795
				15.2095	-3.8293	14.0573
				9999.9000	9999.9000	9999.9000
				14.7902	-5.7887	13.6971
				15.3216	4.6798	13.6900
3	5	8	48	15.3216	4.6798	13.6900
				19.7261	9.2054	13.7486
				16.0561	-3.6547	14.0136
				9999.9000	9999.9000	9999.9000
				15.6178	-5.6162	13.6540
				16.1219	4.8796	13.6400
3	5	9	54	16.1219	4.8796	13.6400
				20.9549	9.2837	13.7290
				16.9522	-3.5284	13.9821
				9999.9000	9999.9000	9999.9000
				16.4936	-5.4920	13.6230
				16.9718	5.0298	13.6025
3	5	10	60	16.9718	5.0298	13.6025
				22.2343	9.3242	13.7189
				17.8905	-3.4433	13.9608
				9999.9000	9999.9000	9999.9000
				17.4105	-5.4088	13.6022
				17.8641	5.1375	13.5756
3	5	11	66	17.8641	5.1375	13.5756
				23.5558	9.3330	13.7167
				18.8648	-3.3931	13.9483
				9999.9000	9999.9000	9999.9000
				18.3623	-5.3604	13.5901
				18.7929	5.2088	13.5577
3	5	12	72	18.7929	5.2088	13.5577
				24.9129	9.3152	13.7211
				19.8696	-3.3729	13.9432
				9999.9000	9999.9000	9999.9000
				19.3437	-5.3418	13.5854
				19.7530	5.2489	13.5477

3	5	13	78	19.7530	5.2489	13.5477
				26.3001	9.2748	13.7312
				20.9006	-3.3781	13.9445
				9999.9000	9999.9000	9999.9000
				20.3506	-5.3486	13.5871
				20.7401	5.2619	13.5445
3	5	14	84	20.7401	5.2619	13.5445
				27.7127	9.2154	13.7461
				21.9542	-3.4053	13.9513
				9999.9000	9999.9000	9999.9000
				21.3793	-5.3772	13.5943
				21.7504	5.2517	13.5470
3	5	15	90	21.7504	5.2517	13.5470
				29.1472	9.1395	13.7651
				23.0271	-3.4514	13.9628
				9999.9000	9999.9000	9999.9000
				22.4269	-5.4246	13.6061
				22.7809	5.2213	13.5546
3	5	16	96	22.7809	5.2213	13.5546

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
0	193.2848	0	
24	193.0058	24	
48	192.7680	48	
72	192.5698	72	
96	192.3697	96	

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 2
0	270.0675	0	
12	269.9053	12	
24	269.7587	24	
36	269.6263	12	
48	269.4796	24	
60	269.3482	36	
72	269.2317	48	
84	269.1105	36	
96	268.9940	48	

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 3
0	284.8513	0	
6	284.6178	6	
12	284.4170	12	
18	284.2470	18	
24	284.1055	24	
30	283.9896	30	
36	283.8961	36	
42	283.8219	42	
48	283.7339	36	

54	283.6597	42
60	283.5872	36
66	283.5131	42
72	283.4487	48
78	283.3807	42
84	283.3082	36
90	283.2340	42
96	283.1696	48

DETERMINISTIC SYSTEM 3 ITEM(S) 1 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	30.8800				
ITEM 2	18.3300				
ITEM 3	12.0000				

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	18.3000				
ITEM 2	17.5000				
ITEM 3	15.5000				

HOLDING COST

ITEM 1	.3000
ITEM 2	.2400
ITEM 3	.1200

SHORTAGE COST

ITEM 1	.3000
ITEM 2	.1700
ITEM 3	.2500

DEMAND

ITEM 1	6.0000
ITEM 2	4.0000
ITEM 3	1.0000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	9999.9000				
ITEM 2	9999.9000				
ITEM 3	9999.9000				

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	2.0000				
ITEM 2	4.0000				
ITEM 3	1.0000				

TOTAL WAREHOUSE SPACE
100.0000

SPACE REQUIREMENT FOR INDIVIDUAL ITEMS

ITEM 1	24.0000
ITEM 2	12.0000
ITEM 3	6.0000

POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

MINIMUM COST PROCUREMENT QUANTITIES

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	38.2737				
ITEM 2	37.5156				
ITEM 3	19.5543				

MINIMUM COST PROCUREMENT LEVELS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	-7.1253				
ITEM 2	-5.9515				
ITEM 3	-5.3413				

ASSOCIATED MINIMUM TOTAL COSTS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	191.0176				
ITEM 2	77.0517				
ITEM 3	13.5853				

DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

				27.0636	-15.0473	193.3942
1	1	0	0	27.0636	-15.0473	193.3942
				27.1005	-14.0843	193.1052
1	1	1	24	27.1005	-14.0843	193.1052
				27.2111	-13.1948	192.8384
1	1	2	48	27.2111	-13.1948	192.8384
				27.3945	-12.3780	192.5934
1	1	3	72	27.3945	-12.3780	192.5934
				27.6491	-11.6325	192.3697
1	1	4	96	27.6491	-11.6325	192.3697
				28.7029	-12.6914	78.1975
2	1	0	0	28.7029	-12.6914	78.1975
				28.7449	-11.7334	78.0346
2	1	1	12	28.7449	-11.7334	78.0346
				28.8706	-10.8590	77.8860

2	1	2	24	28.8706	-10.8590	77.8860
				29.0788	-10.0672	77.7514
2	1	3	36	29.0788	-10.0672	77.7514
				29.3679	-9.3562	77.6305
2	1	4	48	29.3679	-9.3562	77.6305
				29.7355	-8.7236	77.5230
2	1	5	60	29.7355	-8.7236	77.5230
				30.1786	-8.1665	77.4283
2	1	6	72	30.1786	-8.1665	77.4283
				30.6941	-7.6818	77.3459
2	1	7	84	30.6941	-7.6818	77.3459
				31.2783	-7.2658	77.2751
2	1	8	96	31.2783	-7.2658	77.2751
				11.1360	-10.1349	14.7837
3	1	0	0	11.1360	-10.1349	14.7837
				11.2023	-9.2012	14.5503
3	1	1	6	11.2023	-9.2012	14.5503
				11.3988	-8.3976	14.3494
3	1	2	12	11.3988	-8.3976	14.3494
				11.7190	-7.7178	14.1794
3	1	3	18	11.7190	-7.7178	14.1794
				12.1530	-7.1518	14.0379
3	1	4	24	12.1530	-7.1518	14.0379
				12.6893	-6.6880	13.9220
3	1	5	30	12.6893	-6.6880	13.9220
				13.3155	-6.3141	13.8285
3	1	6	36	13.3155	-6.3141	13.8285
				14.0195	-6.0181	13.7545
3	1	7	42	14.0195	-6.0181	13.7545
				14.7902	-5.7887	13.6971
3	1	8	48	14.7902	-5.7887	13.6971
				15.6178	-5.6162	13.6540
3	1	9	54	15.6178	-5.6162	13.6540
				16.4936	-5.4920	13.6230
3	1	10	60	16.4936	-5.4920	13.6230
				17.4105	-5.4088	13.6022

3	1	11	66	17.4105	-5.4088	13.6022
				18.3623	-5.3604	13.5901
3	1	12	72	18.3623	-5.3604	13.5901
				19.3437	-5.3418	13.5854
3	1	13	78	19.3437	-5.3418	13.5854
				20.3506	-5.3486	13.5871
3	1	14	84	20.3506	-5.3486	13.5871
				21.3793	-5.3772	13.5943
3	1	15	90	21.3793	-5.3772	13.5943
				22.4269	-5.4246	13.6061
3	1	16	96	22.4269	-5.4246	13.6061

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
0	193.3942	0	
24	193.1052	24	
48	192.8384	48	
72	192.5934	72	
96	192.3697	96	

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 2
0	271.5917	0	
12	271.4288	12	
24	271.2802	24	
36	271.1399	12	
48	270.9913	24	
60	270.8567	36	
72	270.7244	24	
84	270.5898	36	
96	270.4690	48	

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 3
0	286.3754	0	
6	286.1420	6	
12	285.9411	12	
18	285.7712	18	
24	285.6297	24	
30	285.5137	30	
36	285.4202	36	
42	285.3462	42	
48	285.2574	36	
54	285.1834	42	
60	285.1087	36	
66	285.0347	42	
72	284.9685	36	

78	284.8944	42
84	284.8198	36
90	284.7458	42
96	284.6852	36

DETERMINISTIC SYSTEM 1 ITEM(S) 5 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	19.8500	8888.8000	17.9400	18.3300	18.0800

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	17.3200	.0000	18.7000	17.5000	14.6500

HOLDING COST

ITEM 1 .2400

SHORTAGE COST

ITEM 1 .1700

DEMAND

ITEM 1 4.0000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	12.0000	.0000	9999.9000	9999.9000	9999.9000

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.0000	.0000	3.0000	4.0000	12.0000

TOTAL WAREHOUSE SPACE

100.0000

SPACE REQUIREMENT FOR INDIVIDUAL ITEMS

ITEM 1 12.0000

POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

MINIMUM COST PROCUREMENT QUANTITIES

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	45.7011	9999.9000	38.7806	37.5156	34.3251

MINIMUM COST PROCUREMENT LEVELS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.1654	9999.9000	-10.6917	-5.9515	27.9152

ASSOCIATED MINIMUM TOTAL COSTS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	82.4318	9999.9000	75.6175	77.0517	75.7344

DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

				34.9655	.6896	83.3627
				9999.9000	9999.9000	9999.9000
				29.6707	-17.6588	76.8020
				28.7029	-12.6914	78.1975
				26.2619	21.7486	76.7827
1	5	0	0	26.2619	21.7486	76.7827
				35.0430	1.6379	83.2015
				9999.9000	9999.9000	9999.9000
				29.7113	-16.6994	76.6389
				28.7449	-11.7334	78.0346
				26.3078	22.7027	76.6205
1	5	1	12	26.3078	22.7027	76.6205
				35.2745	2.4836	83.0577
				9999.9000	9999.9000	9999.9000
				29.8329	-15.8210	76.4895
				28.8706	-10.8590	77.8860
				26.4450	23.5655	76.4738
1	5	2	24	26.4450	23.5655	76.4738
				35.6570	3.2286	82.9311
				9999.9000	9999.9000	9999.9000
				30.0345	-15.0225	76.3538
				29.0788	-10.0672	77.7514
				26.6722	24.3383	76.3424
1	5	3	36	26.6722	24.3383	76.3424
				36.1857	3.8761	82.8210
				9999.9000	9999.9000	9999.9000
				30.3145	-14.3024	76.2314
				29.3679	-9.3562	77.6305
				26.9871	25.0236	76.2259
1	5	4	48	26.9871	25.0236	76.2259
				36.8544	4.4303	82.7268
				9999.9000	9999.9000	9999.9000
				30.6707	-13.6584	76.1219
				29.7355	-8.7236	77.5230
				27.3866	25.6242	76.1238
1	3	5	60	30.6707	-13.6584	76.1219
				37.6555	4.8963	82.6476
				9999.9000	9999.9000	9999.9000
				31.1005	-13.0881	76.0249
				30.1786	-8.1665	77.4283
				27.8671	26.1439	76.0355
1	3	6	72	31.1005	-13.0881	76.0249
				38.5808	5.2794	82.5825
				9999.9000	9999.9000	9999.9000
				31.6009	-12.5883	75.9400

				30.6941	-7.6818	77.3459
				28.4246	26.5867	75.9602
1	3	7	84	31.6009	-12.5883	75.9400
				39.6217	5.5855	82.5304
				9999.9000	9999.9000	9999.9000
				32.1687	-12.1558	75.8665
				31.2783	-7.2658	77.2751
				29.0545	26.9571	75.8972
1	3	8	96	32.1687	-12.1558	75.8665

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
0	76.7827	0	
12	76.6205	12	
24	76.4738	24	
36	76.3424	36	
48	76.2259	48	
60	76.1219	60	
72	76.0249	72	
84	75.9400	84	
96	75.8665	96	

DETERMINISTIC SYSTEM 1 ITEM(S) 1 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	34.7500				

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	23.1600				

HOLDING COST

ITEM 1 .3000

SHORTAGE COST

ITEM 1 .3000

DEMAND

ITEM 1 6.0000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	9999.9000				

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	7.0000				

TOTAL WAREHOUSE SPACE
100.0000

SPACE REQUIREMENT FOR INDIVIDUAL ITEMS

ITEM 1 24.0000

POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

MINIMUM COST PROCUREMENT QUANTITIES

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	43.0571				

MINIMUM COST PROCUREMENT LEVELS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	20.4843				

ASSOCIATED MINIMUM TOTAL COSTS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	214.9546				

DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

				30.4459	11.5723	217.6283
1	1	0	0	30.4459	11.5723	217.6283
				30.4788	12.5394	217.3381
1	1	1	24	30.4788	12.5394	217.3381
				30.5772	13.4411	217.0676
1	1	2	48	30.5772	13.4411	217.0676
				30.7404	14.2779	216.8166
1	1	3	72	30.7404	14.2779	216.8166
				30.9676	15.0509	216.5847
1	1	4	96	30.9676	15.0509	216.5847

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
0	217.6283	0	
24	217.3381	24	
48	217.0676	48	
72	216.8166	72	
96	216.5847	96	

PROBABILISTIC SYSTEM 2 ITEM(S) 3 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	7.0000	8888.8000	6.3000		
ITEM 2	4.3400	4.2500	8888.8000		

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.0000	.0000	6.2500		
ITEM 2	5.5000	5.7500	.0000		

HOLDING COST

ITEM 1	.1000
ITEM 2	.1200

SHORTAGE COST

ITEM 1	4.0000
ITEM 2	3.8000

DEMAND

ITEM 1	2.0000
ITEM 2	1.8000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	10.0000	.0000	9999.9000		
ITEM 2	8.0000	9999.9000	.0000		

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	2.0000	.0000	4.0000		
ITEM 2	3.0000	2.0000	.0000		

TOTAL WAREHOUSE SPACE

100.0000

SPACE REQUIREMENT FOR INDIVIDUAL ITEMS

ITEM 1	9.0000
ITEM 2	7.0000

POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

MINIMUM COST PROCUREMENT QUANTITIES

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	18.1265	9999.9000	17.5021		
ITEM 2	16.5161	13.7265	9999.9000		

MINIMUM COST PROCUREMENT LEVELS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.1873	9999.9000	12.4995		
ITEM 2	7.6706	5.4661	9999.9000		

ASSOCIATED MINIMUM TOTAL COSTS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	15.6688	9999.9000	14.7998		
ITEM 2	9.6204	9.5208	9999.9000		

DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000

NO OPTIMAL POLICY EXISTS

				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000

NO OPTIMAL POLICY EXISTS

				2.5000	4.0000	21.9000
				9999.9000	9999.9000	9999.9000
				2.0004	8.0000	26.8221
1	1	2	18	2.5000	4.0000	21.9000
				3.7500	4.0000	19.3499
				9999.9000	9999.9000	9999.9000
				3.0006	8.0000	22.1647
1	1	3	27	3.7500	4.0000	19.3499
				5.0000	4.0000	18.1000
				9999.9000	9999.9000	9999.9000
				4.0008	8.0000	19.8610
1	1	4	36	5.0000	4.0000	18.1000
				6.2500	4.0000	17.3700
				9999.9000	9999.9000	9999.9000
				5.0010	8.0000	18.4988
1	1	5	45	6.2500	4.0000	17.3700
				6.9436	4.4450	16.8884
				9999.9000	9999.9000	9999.9000
				6.0012	8.0000	17.6073
1	1	6	54	6.9436	4.4450	16.8884
				7.5592	4.9525	16.5332
				9999.9000	9999.9000	9999.9000
				7.0014	8.0000	16.9849
1	1	7	63	7.5592	4.9525	16.5332
				8.3452	5.3238	16.2733
				9999.9000	9999.9000	9999.9000
				7.8279	8.1735	16.5297
1	1	8	72	8.3452	5.3238	16.2733
				9.2582	5.5934	16.0845

				9999.9000	9999.9000	9999.9000
				7.9074	9.0941	16.1616
1	1	9	81	9.2582	5.5934	16.0845
				10.2643	5.7885	15.9480
				9999.9000	9999.9000	9999.9000
				8.1412	9.8603	15.8552
1	3	10	90	8.1412	9.8603	15.8552
				11.3389	5.9288	15.8498
				9999.9000	9999.9000	9999.9000
				8.5166	10.4850	15.6054
1	3	11	99	8.5166	10.4850	15.6054
				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000
NO OPTIMAL POLICY EXISTS						
				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000
				9999.9000	9999.9000	9999.9000
NO OPTIMAL POLICY EXISTS						
				2.5806	5.4000	15.2237
				2.0003	3.6000	15.7840
				9999.9000	9999.9000	9999.9000
2	1	2	14	2.5806	5.4000	15.2237
				3.8709	5.4000	12.8531
				3.0005	3.6000	13.1727
				9999.9000	9999.9000	9999.9000
2	1	3	21	3.8709	5.4000	12.8531
				5.1612	5.4000	11.6978
				4.0007	3.6000	11.8970
				9999.9000	9999.9000	9999.9000
2	1	4	28	5.1612	5.4000	11.6978
				6.4516	5.4000	11.0286
				5.0009	3.6000	11.1556
				9999.9000	9999.9000	9999.9000
2	1	5	35	6.4516	5.4000	11.0286
				7.7419	5.4000	10.6025
				5.5135	4.0874	10.6649
				9999.9000	9999.9000	9999.9000
2	1	6	42	7.7419	5.4000	10.6025
				8.5048	5.8087	10.3109
				6.0861	4.5149	10.3101
				9999.9000	9999.9000	9999.9000
2	2	7	49	6.0861	4.5149	10.3101

				9.0325	6.3997	10.0917
				6.7802	4.8209	10.0561
				9999.9000	9999.9000	9999.9000
2	2	8	56	6.7802	4.8209	10.0561
				9.7596	6.8362	9.9298
				7.5626	5.0387	9.8754
				9999.9000	9999.9000	9999.9000
2	2	9	63	7.5626	5.0387	9.8754
				10.6452	7.1499	9.8135
				8.4085	5.1929	9.7475
				9999.9000	9999.9000	9999.9000
2	2	10	70	8.4085	5.1929	9.7475
				11.6532	7.3687	9.7324
				9.3008	5.3008	9.6579
				9999.9000	9999.9000	9999.9000
2	2	11	77	9.3008	5.3008	9.6579
				12.7547	7.5150	9.6781
				10.2272	5.3745	9.5967
				9999.9000	9999.9000	9999.9000
2	2	12	84	10.2272	5.3745	9.5967
				13.9275	7.6061	9.6443
				11.1793	5.4226	9.5569
				9999.9000	9999.9000	9999.9000
2	2	13	91	11.1793	5.4226	9.5569
				15.1550	7.6548	9.6263
				12.1511	5.4510	9.5333
				9999.9000	9999.9000	9999.9000
2	2	14	98	12.1511	5.4510	9.5333

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
18	21.9000	18	
27	19.3499	27	
36	18.1000	36	
45	17.3700	45	
54	16.8884	54	
63	16.5332	63	
72	16.2733	72	
81	16.0845	81	
90	15.8552	90	
99	15.6054	99	

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 2
32	37.1237	14	
39	34.7531	21	

41	34.5737	14
46	33.5978	28
48	32.2031	21
50	33.3237	14
53	32.9286	35
55	31.0478	28
57	30.9531	21
59	32.5937	14
60	32.5025	42
62	30.3786	35
64	29.7978	28
66	30.2231	21
67	32.2101	49
68	32.1121	14
69	29.9525	42
71	29.1286	35
73	29.0678	28
74	31.9561	56
75	29.7415	21
76	29.6601	49
77	31.7569	14
78	28.7025	42
80	28.3986	35
81	31.7754	63
82	28.5863	28
83	29.4061	56
84	29.3863	21
85	28.4101	49
86	31.4970	14
87	27.9725	42
88	31.6475	70
89	27.9171	35
90	29.2254	63
91	28.2310	28
92	28.1561	56
93	29.1264	21
94	27.6801	49
95	31.3083	14
96	27.4910	42
97	29.0975	70
98	27.5618	35
99	27.9754	63
100	27.9711	28

PROBABILISTIC SYSTEM   2 ITEM(S)   1 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	7.0000				
ITEM 2	4.3400				

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.0000				
ITEM 2	5.5000				

HOLDING COST

ITEM 1	.1000
ITEM 2	.1200

SHORTAGE COST

ITEM 1	4.0000
ITEM 2	3.8000

DEMAND

ITEM 1	2.0000
ITEM 2	1.8000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	10.0000				
ITEM 2	8.0000				

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	2.0000				
ITEM 2	3.0000				

TOTAL WAREHOUSE SPACE
100.0000

SPACE REQUIREMENT FOR INDIVIDUAL ITEMS

ITEM 1	9.0000
ITEM 2	7.0000

POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

MINIMUM COST PROCUREMENT QUANTITIES

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	18.1265				
ITEM 2	16.5161				

MINIMUM COST PROCUREMENT LEVELS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.1873				
ITEM 2	7.6706				

ASSOCIATED MINIMUM TOTAL COSTS

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	15.6688				
ITEM 2	9.6204				

DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

				9999.9000	9999.9000	9999.9000
NO OPTIMAL POLICY EXISTS						
				9999.9000	9999.9000	9999.9000
NO OPTIMAL POLICY EXISTS						
				2.5000	4.0000	21.9000
1	1	2	18	2.5000	4.0000	21.9000
				3.7500	4.0000	19.3499
1	1	3	27	3.7500	4.0000	19.3499
				5.0000	4.0000	18.1000
1	1	4	36	5.0000	4.0000	18.1000
				6.2500	4.0000	17.3700
1	1	5	45	6.2500	4.0000	17.3700
				6.9436	4.4450	16.8884
1	1	6	54	6.9436	4.4450	16.8884
				7.5592	4.9525	16.5332
1	1	7	63	7.5592	4.9525	16.5332
				8.3452	5.3238	16.2733
1	1	8	72	8.3452	5.3238	16.2733
				9.2582	5.5934	16.0845
1	1	9	81	9.2582	5.5934	16.0845
				10.2643	5.7885	15.9480
1	1	10	90	10.2643	5.7885	15.9480
				11.3389	5.9288	15.8498
1	1	11	99	11.3389	5.9288	15.8498
				9999.9000	9999.9000	9999.9000
NO OPTIMAL POLICY EXISTS						
				9999.9000	9999.9000	9999.9000
NO OPTIMAL POLICY EXISTS						
				2.5806	5.4000	15.2237
2	1	2	14	2.5806	5.4000	15.2237
				3.8709	5.4000	12.8531
2	1	3	21	3.8709	5.4000	12.8531
				5.1612	5.4000	11.6978
2	1	4	28	5.1612	5.4000	11.6978

				6.4516	5.4000	11.0286
2	1	5	35	6.4516	5.4000	11.0286
				7.7419	5.4000	10.6025
2	1	6	42	7.7419	5.4000	10.6025
				8.5048	5.8087	10.3109
2	1	7	49	8.5048	5.8087	10.3109
				9.0325	6.3997	10.0917
2	1	8	56	9.0325	6.3997	10.0917
				9.7596	6.8362	9.9298
2	1	9	63	9.7596	6.8362	9.9298
				10.6452	7.1499	9.8135
2	1	10	70	10.6452	7.1499	9.8135
				11.6532	7.3687	9.7324
2	1	11	77	11.6532	7.3687	9.7324
				12.7547	7.5150	9.6781
2	1	12	84	12.7547	7.5150	9.6781
				13.9275	7.6061	9.6443
2	1	13	91	13.9275	7.6061	9.6443
				15.1550	7.6548	9.6263
2	1	14	98	15.1550	7.6548	9.6263

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
18	21.9000	18	
27	19.3499	27	
36	18.1000	36	
45	17.3700	45	
54	16.8884	54	
63	16.5332	63	
72	16.2733	72	
81	16.0845	81	
90	15.9480	90	
99	15.8498	99	

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 2
32	37.1237	14	
39	34.7531	21	
41	34.5737	14	
46	33.5978	28	
48	32.2031	21	
50	33.3237	14	
53	32.9286	35	
55	31.0478	28	

57	30.9531	21
59	32.5937	14
60	32.5025	42
62	30.3786	35
64	29.7978	28
66	30.2231	21
67	32.2109	49
68	32.1121	14
69	29.9525	42
71	29.1286	35
73	29.0678	28
74	31.9917	56
75	29.7415	21
76	29.6609	49
77	31.7569	14
78	28.7025	42
80	28.3986	35
81	31.8298	63
82	28.5863	28
83	29.4417	56
84	29.3863	21
85	28.4109	49
86	31.4970	14
87	27.9725	42
88	31.7135	70
89	27.9171	35
90	29.2798	63
91	28.2310	28
92	28.1917	56
93	29.1264	21
94	27.6809	49
95	31.3083	14
96	27.4910	42
97	29.1635	70
98	27.5618	35
99	28.0298	63
100	27.9711	28

PROBABILISTIC SYSTEM 1 ITEM(S) 3 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	4.3400	4.2500	8888.8000		

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	5.5000	5.7500	.0000		

HOLDING COST

ITEM 1	.1200

```
         SHORTAGE COST
ITEM 1         3.8000

         DEMAND
ITEM 1         1.8000

         RATE OF REPLENISHMENT
             SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         8.0000 9999.9000     .0000

         LEAD TIME
             SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         3.0000    2.0000     .0000

         TOTAL WAREHOUSE SPACE
              100.0000

         SPACE REQUIREMENT FOR INDIVIDUAL ITEMS
ITEM 1         7.0000


POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

         MINIMUM COST PROCUREMENT QUANTITIES
             SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1        16.5161   13.7265 9999.9000

         MINIMUM COST PROCUREMENT LEVELS
             SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         7.6706    5.4661 9999.9000

         ASSOCIATED MINIMUM TOTAL COSTS
             SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         9.6204    9.5208 9999.9000


    DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

                                    9999.9000   9999.9000   9999.9000
                                    9999.9000   9999.9000   9999.9000
                                    9999.9000   9999.9000   9999.9000
NO OPTIMAL POLICY EXISTS

                                    9999.9000   9999.9000   9999.9000
                                    9999.9000   9999.9000   9999.9000
                                    9999.9000   9999.9000   9999.9000
NO OPTIMAL POLICY EXISTS

                                       2.5806      5.4000     15.2237
                                       2.0003      3.6000     15.7840
                                    9999.9000   9999.9000   9999.9000
1      1      2      14                2.5806      5.4000     15.2237

                                       3.8709      5.4000     12.8531
```

				3.0005	3.6000	13.1727
				9999.9000	9999.9000	9999.9000
1	1	3	21	3.8709	5.4000	12.8531
				5.1612	5.4000	11.6978
				4.0007	3.6000	11.8970
				9999.9000	9999.9000	9999.9000
1	1	4	28	5.1612	5.4000	11.6978
				6.4516	5.4000	11.0286
				5.0009	3.6000	11.1556
				9999.9000	9999.9000	9999.9000
1	1	5	35	6.4516	5.4000	11.0286
				7.7419	5.4000	10.6025
				5.5135	4.0874	10.6649
				9999.9000	9999.9000	9999.9000
1	1	6	42	7.7419	5.4000	10.6025
				8.5048	5.8087	10.3109
				6.0861	4.5149	10.3101
				9999.9000	9999.9000	9999.9000
1	2	7	49	6.0861	4.5149	10.3101
				9.0325	6.3997	10.0917
				6.7802	4.8209	10.0561
				9999.9000	9999.9000	9999.9000
1	2	8	56	6.7802	4.8209	10.0561
				9.7596	6.8362	9.9298
				7.5626	5.0387	9.8754
				9999.9000	9999.9000	9999.9000
1	2	9	63	7.5626	5.0387	9.8754
				10.6452	7.1499	9.8135
				8.4085	5.1929	9.7475
				9999.9000	9999.9000	9999.9000
1	2	10	70	8.4085	5.1929	9.7475
				11.6532	7.3687	9.7324
				9.3008	5.3008	9.6579
				9999.9000	9999.9000	9999.9000
1	2	11	77	9.3008	5.3008	9.6579
				12.7547	7.5150	9.6781
				10.2272	5.3745	9.5967
				9999.9000	9999.9000	9999.9000
1	2	12	84	10.2272	5.3745	9.5967
				13.9275	7.6061	9.6443
				11.1793	5.4226	9.5569
				9999.9000	9999.9000	9999.9000
1	2	13	91	11.1793	5.4226	9.5569

				15.1550	7.6548	9.6263
				12.1511	5.4510	9.5333
				9999.9000	9999.9000	9999.9000
1	2	14	98	12.1511	5.4510	9.5333

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
14	15.2237	14	
21	12.8531	21	
28	11.6978	28	
35	11.0286	35	
42	10.6025	42	
49	10.3101	49	
56	10.0561	56	
63	9.8754	63	
70	9.7475	70	
77	9.6579	77	
84	9.5967	84	
91	9.5569	91	
98	9.5333	98	

PROBABILISTIC SYSTEM 1 ITEM(S) 1 SOURCE(S)

ITEM COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.3000				

PROCUREMENT COST

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	6.2500				

HOLDING COST

ITEM 1 .1000

SHORTAGE COST

ITEM 1 4.0000

DEMAND

ITEM 1 2.0000

RATE OF REPLENISHMENT

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	9999.9000				

LEAD TIME

	SOURCE 1	SOURCE 2	SOURCE 3	SOURCE 4	SOURCE 5
ITEM 1	4.0000				

TOTAL WAREHOUSE SPACE

100.0000

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          SPACE REQUIREMENT FOR INDIVIDUAL ITEMS
ITEM 1         9.0000


POLICY DEVELOPMENT FOR UNRESTRICTED SYSTEM FOLLOWS

          MINIMUM COST PROCUREMENT QUANTITIES
              SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         17.5021

          MINIMUM COST PROCUREMENT LEVELS
              SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         12.4995

          ASSOCIATED MINIMUM TOTAL COSTS
              SOURCE 1  SOURCE 2  SOURCE 3  SOURCE 4  SOURCE 5
ITEM 1         14.7998


    DEVELOPMENT OF CONDENSED COST FUNCTIONS FOLLOWS

                                         9999.9000    9999.9000    9999.9000
NO OPTIMAL POLICY EXISTS

                                         9999.9000    9999.9000    9999.9000
NO OPTIMAL POLICY EXISTS

                                            2.0004       8.0000      26.8221
1       1       2       18                  2.0004       8.0000      26.8221

                                            3.0006       8.0000      22.1647
1       1       3       27                  3.0006       8.0000      22.1647

                                            4.0008       8.0000      19.8610
1       1       4       36                  4.0008       8.0000      19.8610

                                            5.0010       8.0000      18.4988
1       1       5       45                  5.0010       8.0000      18.4988

                                            6.0012       8.0000      17.6073
1       1       6       54                  6.0012       8.0000      17.6073

                                            7.0014       8.0000      16.9849
1       1       7       63                  7.0014       8.0000      16.9849

                                            7.8279       8.1735      16.5297
1       1       8       72                  7.8279       8.1735      16.5297

                                            7.9074       9.0941      16.1616
1       1       9       81                  7.9074       9.0941      16.1616

                                            8.1412       9.8603      15.8552
1       1      10       90                  8.1412       9.8603      15.8552
```

				8.5166	10.4850	15.6054
1	1	11	99	8.5166	10.4850	15.6054

DYNAMIC PROGRAMMING SOLUTION FOLLOWS

SPACE UTILIZED	TOTAL COST	ALLOCATION	STAGE 1
18	26.8221	18	
27	22.1647	27	
36	19.8610	36	
45	18.4988	45	
54	17.6073	54	
63	16.9849	63	
72	16.5297	72	
81	16.1616	81	
90	15.8552	90	
99	15.6054	99	

VITA

Jerry Banks

Candidate for the Degree of

Doctor of Philosophy

Thesis: A HIERARCHY OF PROCUREMENT AND INVENTORY SYSTEMS

Major Field: Engineering

Biographical:

Personal Data: Born in Birmingham, Alabama, April 23, 1939, the son of Jake and Rose Friedman Banks.

Education: Graduated from Huntsville High School, Huntsville, Alabama, in May 1957; received the degree of Bachelor of Science in Industrial Engineering from the University of Alabama in May, 1961; received the degree of Master of Science in Industrial Engineering from the University of Alabama in May, 1963; completed the requirements for the degree of Doctor of Philosophy in September, 1965.

Professional Experience: Graduate Research Assistant, Oklahoma State University, June, 1964 to June, 1965; Member, Operations Research Group, Oklahoma State University, January, 1964 to June, 1964; Graduate Assistant, Oklahoma State University, September, 1963 to January, 1964; Engineer, Brown Engineering Company, part-time since August, 1963; Graduate Research Assistant, University of Alabama, September, 1962 to June, 1963; Industrial Engineer, Kennedy Space Flight Center, June, 1963 to September, 1963; Industrial Engineer, Marshall Space Flight Center, June, 1962 to September, 1962.