CONTACT CHATTER CHARACTERISTICS OF A LINEAR VISCOUS DAMPED CONTACT SYSTEM

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CHAPTER I

INTRODUCTION

A relay or switching device is a mechanical system in which parts of the system can exhibit resonant conditions. In particular, the contacts of the relay or switching device often have a resonant condition at which their response amplitude can become large when compared with the amplitude of the environment. During this resonant condition the contacts of the relay could have a type of failure commonly referred to as contact chatter. In essence, contact chatter is the separation of the two contact surfaces which have been forced together to complete a circuit.

One of the present methods of controlling contact chatter, when the relay is to perform in a given frequency range, is to design the relay so that its resonant conditions are not within this range. The shifting of the resonant condition is most often accomplished by adding rigidity to the relay, which is done at the cost of added weight. Not only is the weight of the relay important, but also of importance is its reliability. There can be no assurance that the relay will not be exposed to frequencies beyond those for which it was designed.

Definition of the Problem

One of the most important mechanical problems of relay design is that of contact chatter. From the testing of relays in a steady state sinusoidal vibration environment, it has been learned that contact chatter does not occur continuously in this environment but it occurs only at certain amplitudes and in certain frequency ranges. Consequently, there must be a response amplitude of the contacts below which contact chatter does not occur and above which chatter does occur. The amplitude of the response of the system at the point of initial contact chatter is defined as the separation amplitude. The separation amplitude is the finite value of the response amplitude of the contacts when the force between the two contact surfaces becomes zero.

To control the response amplitude of a contact system during resonance, the excess energy must be dissipated. This energy dissipation can be accomplished by adding damping to the system. Mechanical damping is dependent upon either displacement or velocity; therefore, there must be a response amplitude for damping to be effective.

The criteria for adding damping to a contact system is one of permitting the system to have a response amplitude so that damping is effective. The response amplitude must be controlled so that it is less than the separation amplitude. A condition in which the response amplitude is less than the separation amplitude must exist for an unlimited frequency range.

The problem then is twofold. First, the response of contact systems with damping in a sinusoidal vibration environment must be established. Second, the chatter characteristics of the damped contacts in this environment must also be determined.

The Purpose and Scope of the Study

This study consists of a theoretical investigation of an idealized preloaded contact system with lumped parameters. An experimental phase is presented to substantiate the theoretical investigation.

The mathematical model used for the theoretical study is an idealized preloaded contact system, which consists of two massless linear restoring spring forces, two springless masses, and two linear viscous dampers. The system is subjected to a steady state sinusoidal vibration environment. The equation of motion for the contact system and the equation for impending separation amplitude of the contacts are developed. Equivalent linear viscous damping coefficients are introduced to obtain approximate solutions of the equation of motion and of the equation for impending separation for various types of damping.

The scope of the theoretical study includes the development of a dimensionless number which can be used to establish the chatter characteristics of a contact system. This development of the dimensionless number, referred to as the "Baron Number," simultaneously relates the response amplitude for the system to the separation amplitude. Through the use of the Baron Number, the amplitudes of the exciting motion at which contact chatter will not occur can be determined for a given contact system.

With the use of an arbitrary set of parameters such as spring constants, masses, and damping coefficients, the chatter characteristics of the mathematical model are determined for various conditions. This enhances the study by permitting a much broader scope of the possible physical applications of adding damping to a contact system. In a like manner, the effects of varying certain parameters on the mathematical model while maintaining the others at a constant level is presented.

The purpose of this study is to further the understanding of the behavior of contact systems in a vibration environment, to establish the effects damping has on the system, and to determine the chatter characteristics of a damped contact system. An adequate understanding of the chatter characteristics of a contact system in which damping is a prevalent factor could facilitate an optimum contact system design.

Previous Work

Much work has been done in the field of contact chatter from the standpoint of rebound chatter and collision chatter. Takamura (1)[1] developed a general theory of the vibrations caused by collisions between two contact masses. Transient response of the contacts during impact closure and lift-off after opening was studied. Takei (2) analyzed the transient response of contact impact. This study considered collision of one flexible contact with rigid contact, an identical contact, and a more flexible contact. Kubokoya's (3) analysis was for armature rebound and impact on the armature back stop. It was shown that this type of rebound chattering could be minimized by increasing the operating ampere-turns and by selecting appropriate spring stiffness for the system. Takei and Takashi (4) used the phase plane delta method to analyze the transient response of contact chattering due to impact.

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[1]Numbers in parentheses refer to references of the selected bibliography.

They considered the higher modes of vibration and the energy dispersion for one and two degrees of freedom systems.

Separation criteria for a linear undamped contact system in a steady state sinusoidal environment was studied by Lowery, Riddle, and Stone (5). They studied a contact system for both one and two degrees of freedom. Also included in their study was the effect of adding viscous damping to one contact on the response of the system and preload prior to separation. No attempt was made to establish the relative amplitude for impending separation for either the undamped system or with one contact damped.

Burkhart (6) investigated the effects on a contact system of having one linear spring and one nonlinear spring. In his study he included the case for two linear springs and a qualitative analysis of how damping might affect the amplitude for impending separation. The equation of impending separation developed by Burkhart is the same as the separation equation presented in this study when it was reduced to the undamped condition. Since Burkhart did not make a theoretical study of the effects on the amplitude for impending separation with damping, no direct comparison is possible.

No known previous work in the area of contact chatter has studied the effects of damping on the amplitude for impending separation while simultaneously considering the effects of damping on the response and separation. Only Lowery (5) did a theoretical study of the effects of damping on the response. The effects of damping on the response of a single degree of freedom system, which a contact system is, has been presented in many elementary vibration text books.

In an article by Jacobsen (7), the approximate solution for the response of a single degree of freedom system influenced by damping proportional to the n^{th} power of velocity was considered by using equivalent viscous damping coefficients. The results reported in this article showed that the approximation was close for a single degree of freedom system. No known study using the theory of equivalent viscous damping coefficients to obtain an approximate solution for the amplitude of impending separation has been made; thus, there is no way to compare the results of the theory in this study.

CHAPTER II

MATHEMATICAL DERIVATIONS

The mathematical model used for the theoretical study consists of two massless linear restoring springs, two massless linear viscous dampers, and two springless masses as shown in Figure 1. The system in Figure 1

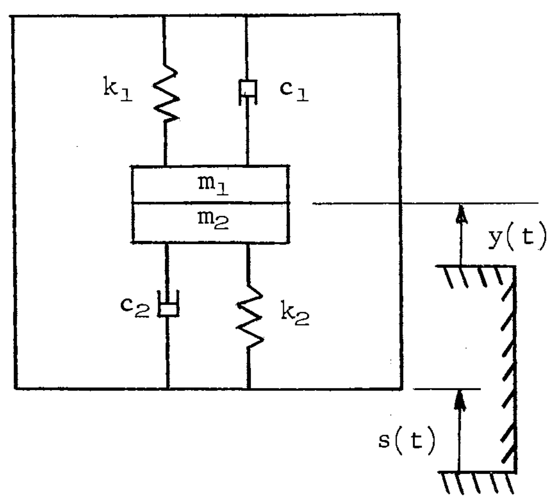


Figure 1. Idealized System with Motion Excitation.

is in the position of equilibrium so that the springs have been changed from their unstretched position. With positive displacement defined as upward, spring 1 was displaced $-\Delta_1$ by the weight of its contact and spring 2 was displaced $-\Delta_2$ by the weight of its contact from their unstretched positions. In each case

$$k_1\Delta_1 = m_1 g ,$$

and

$$k_2\Delta_2 = m_2 g \; .$$

In the remainder of this study further displacement of the springs is defined from the equilibrium position of each spring with its contact attached. Through the use of this new equilibrium position for defining displacements, the weight of the contacts need not be considered. When the two contacts are forced together spring 1 is displaced $+\delta_1$ and spring 2 is displaced $-\delta_2$ from their equilibrium positions. The dynamic forces generated by a sinusoidal excitation on the contact system as shown in Figure 1 are as follows:

$$F_{k_1}(t) = -k_1 [y(t) - s(t) + \delta_1] \; ,$$

$$F_{k_2}(t) = -k_2 [y(t) - s(t) - \delta_2] \; ,$$

$$F_{c_1}(t) = -c_1 [\dot{y}(t) - \dot{s}(t)] \; ,$$

$$F_{c_2}(t) = -c_2 [\dot{y}(t) - \dot{s}(t)] \; ,$$

$$F_{m_1}(t) = -m_1\ddot{y}(t) \; ,$$

$$F_{m_2}(t) = -m_2\ddot{y}(t) \; .$$

The sum of all the forces acting on each contact mass must equal zero; that is,

$$F_{m_1}(t) + F_{c_1}(t) + F_{k_1}(t) + F(t) = 0, \qquad (1)$$

and

$$F_{m_2}(t) + F_{c_2}(t) + F_{k_2}(t) - F(t) = 0, \qquad (2)$$

where $F(t)$ is the force between the masses which acts equally but in opposite directions on each mass. For the static condition when $y(t) = s(t) = 0$, Equations (1) and (2), upon substituting for the dynamic forces, reduce to

$$- k_1\delta_1 + F(t) = 0, \tag{3}$$

and

$$k_2\delta_2 - F(t) = 0. \tag{4}$$

The sum of Equation (3) and Equation (4) shows that $k_1\delta_1 = k_2\delta_2$ and this force, $k_i\delta_i$, is defined as the preload, F_o, of the contact system. The preload is the force between the contact masses in the static condition, $F(t) = F_o$.

The Equation of the Relative Motion

As long as the force between the masses, $F(t)$, is greater than zero, the system is a single degree of freedom system. If $F(t) = 0$ the masses could separate and move independently of each other; the system then would have two degrees of freedom. The interests of this study are the points at which $F(t) > 0$ and when $F(t)$ initially becomes zero. The equation of motion of the system under these conditions is found by taking the sum of the dynamic forces acting on the system, which must be zero:

$$\sum_{i=1}^{2} [F_{m_i}(t) + F_{c_i}(t) + F_{k_i}(t)] = 0. \tag{5}$$

Equation (5) is in terms of the masses, spring constants, and damping coefficients. Also included are the two motions $y(t)$, the absolute motion of the masses, and $s(t)$, the excitation motion.

The motion of the contact masses relative to the system's boundary is $x(t)$, which is defined by the equation

$$x(t) = y(t) - s(t). \tag{6}$$

When the sinusoidal excitation motion is of the form $s(t) = S \sin \omega t$, with S as the amplitude of the motion and ω as the circular forcing frequency, a change from the trigonometric function to complex notation can be made. From the identity $e^{j\theta} = \cos\theta + j\sin\theta$ where $j = \sqrt{-1}$, it follows that the sinusoidal function can be expressed by the imaginary part of $e^{j\theta}$. Thus, $\sin \omega t = \mathrm{Im}(e^{j\omega t})$ and the sinusoidal excitation motion is

$$s(t) = \mathrm{Im}(S\, e^{j\omega t}) . \tag{7}$$

From the Equations (5), (6), and (7) in addition to the fact that $k_1\delta_1 = k_2\delta_2$, and after substitution for the dynamic forces, the resulting equation for the relative motion of the system is

$$(m_1 + m_2)\, \ddot{x}(t) + (c_1 + c_2)\, \dot{x}(t) + (k_1 + k_2)\, x(t) = \mathrm{Im}[(m_1 + m_2)\omega^2 S\, e^{j\omega t}] . \tag{8}$$

The solution of Equation (8) is found to be (see Appendix A) in complex and trigonometric notation, respectively, as

$$x(t) = \mathrm{Im}\,[X\, e^{j(\omega t - \theta)}] , \tag{9}$$

and

$$x(t) = X \sin(\omega t - \theta), \tag{10}$$

where the amplitude of the motion, X, and the phase angle between the relative motion and exciting motion, θ, are

$$X = \frac{S\,\tau^2}{\left\{ [1 - \tau^2]^2 + \dfrac{4\,\tau^2 M(1 + C)^2 \zeta_1^2}{(1 + M)(1 + K)} \right\}^{\frac{1}{2}}}, \tag{11}$$

and

$$\theta = \arc\tan \frac{2\tau\,(1 + C)\,\zeta_1 \left[\dfrac{M}{(1 + M)(1 + K)} \right]^{\frac{1}{2}}}{1 - \tau^2}.$$

These equations are written in terms of dimensionless ratios of the system parameters. The relative magnification factor is defined as the relative amplitude over the exciting amplitude

$$\frac{X}{S} = \frac{\tau^2}{\left\{ [1 - \tau^2]^2 + \dfrac{4\,\tau^2 M(1 + C)^2 \zeta_1^2}{(1 + M)(1 + K)} \right\}^{\frac{1}{2}}}. \tag{12}$$

The Equation of the Relative Amplitude
for Impending Separation

The relative amplitude for impending separation is defined as the finite value of the relative amplitude of the response of the system when the force, F(t), between the contact masses initially becomes zero. For dynamic equilibrium to be maintained as long as $F(t) > 0$, the system is a single degree of freedom system. Consideration of the next increment of motion of the masses after F(t) has initially become zero will indicate if the two masses separated. If the force is different from zero, the contacts will remain together. If the force is still zero, separation of the contacts will occur if the response of each contact is not identical. This would only occur if all the parameters of each contact are identical. Whenever $F(t)$ remains zero for any length of time, each contact is in dynamic equilibrium independent of the other by a combination of its dynamic forces.

After substitution for the dynamic forces in Equations (1) and (2) and division by m_i the following two equations are obtained, provided neither mass is zero:

$$-\ddot{y}(t) - \frac{c_1}{m_1}[\dot{y}(t) - \dot{s}(t)] - \frac{k_1}{m_1}[y(t) - s(t) + \delta_1] + \frac{F(t)}{m_1} = 0, \qquad (13)$$

and

$$-\ddot{y}(t) - \frac{c_2}{m_2}[\dot{y}(t) - \dot{s}(t)] - \frac{k_2}{m_2}[y(t) - s(t) - \delta_2] - \frac{F(t)}{m_2} = 0. \qquad (14)$$

The difference of Equations (13) and (14), after substituting $s(t) = y(t) - s(t)$ and $F_o = k_1\delta_1 = k_2\delta_2$, yields an equation for the force between the contact masses, $F(t)$,

$$F(t)\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = F_o\left(\frac{1}{m_1} + \frac{1}{m_2}\right) + \left(\frac{c_1}{m_1} - \frac{c_2}{m_2}\right)\dot{x}(t) + \left(\frac{k_1}{m_1} - \frac{k_2}{m_2}\right)x(t). \qquad (15)$$

The finite value of the response amplitude, X, at which time the force between the contact masses, $F(t)$, initially becomes zero is defined as the relative amplitude for impending separation, $\overline{X}$. The solution of Equation (15) when $F(t)$ initially becomes zero (see Appendix B) shows that the relative amplitude for impending separation is

$$\overline{X} = \frac{F_o/k_1 \; (1 + M)}{\left\{(1 - KM)^2 + \frac{4 \, \tau^2 \, M(1 + K)(1 - CM)^2 \, \zeta_1^2}{(1 + M)}\right\}^{\frac{1}{2}}} \, , \qquad (16)$$

and the phase angle between the relative response and the sinusoidal variation of F(t) is

$$\beta = \text{arc tan} \, \frac{- 2\tau \, (1 - CM) \, \zeta_1 \left[\frac{M(1 + K)}{1 + M} \right]^{\frac{1}{2}}}{- (1 - KM)} \, . \qquad (17)$$

If the separation factor is defined as the amplitude for impending separation over the term F_o/k_1, then the separation factor is

$$\frac{\overline{X}}{F_o/k_1} = \frac{(1 + M)}{\left\{ (1 - KM)^2 + \frac{4 \, \tau^2 \, M(1 + K)(1 - CM)^2 \, \zeta_1^2}{(1 + M)} \right\}^{\frac{1}{2}}} \, . \qquad (18)$$

The sinusoidal variation of the force, F(t), between the contacts has associated with it a phase angle β, shown in Equation (17). β is the phase angle between the sinusoidal variation of x(t), the relative response, and the sinusoidal variation of F(t). The sign of β establishes whether the variation of F(t) leads or lags the response. When F(t) initially becomes zero, separation of the contacts could occur. If β is zero and F(t) becomes zero, then separation would occur at one of the peaks of the relative response. For conditions when β is not zero, separation would occur β degrees from one of the peaks of the relative response.

Approximate Solutions Using Equivalent Viscous Damping Coefficients

The introduction of equivalent viscous damping coefficients in both the equation of motion and the equation for impending separation will permit approximate solutions for various types of damping. Since in the general case no exact solution exists, the only possible solution would be an approximate one. From the theoretical standpoint the idea of equivalent viscous damping coefficients will lend itself to a qualitative analysis of the effect of various types of damping on the amplitude for impending separation.

For this study "equivalent viscous damping coefficients" are based on the criterion of equivalent dissipative work per cycle. The work per cycle of an arbitrary damper proportional to the n^{th} power of the velocity is equated to the work per cycle of a viscous damper and an equivalent viscous damping coefficient is obtained. It is assumed that the motion of the arbitrarily damped system does not vary appreciably from a sinusoidal motion and that the damping force always opposes the motion. The derivation for the equivalent viscous damping coefficient is made in Appendix C.

Equation (C-13) in Appendix C gives the equivalent viscous damping coefficient as

$$c_i = c_{n_i} \omega^{n_i - 1} X^{n_i - 1} \gamma_{n_i} . \qquad (19)$$

The relative magnification factor for the contact system with various types of damping can be found by replacing c_1 and c_2 in Equation (12) with equivalent coefficients shown in Equation (19). After Equation (12) has been factored and rearranged and two additional terms,

$$f_c = \omega_n^{n_1} S^{n_1 - 1} \gamma_{n_1} \frac{c_{n_1}}{k_1} , \qquad (20)$$

and

$$g_c = \omega_n^{n_2-n_1} S^{n_2-n_1} \frac{\gamma_{n_2}}{\gamma_{n_1}} , \qquad (21)$$

have been defined, the equation for the relative magnification factor for various types of damping, depending on the values of n_1 and n_2, is

$$\left(\frac{X}{S}\right)^{2n_1} \tau^{2n_1} f_c^{\,2} \left[\frac{1 + \frac{c_{n_2}}{c_{n_1}} \tau^{n_2-n_1} \left(\frac{X}{S}\right)^{n_2-n_1} g_c}{1 + k_2/k_1}\right]^2 +$$

$$(1 - \tau^2)^2 \left(\frac{X}{S}\right)^2 - \tau^4 = 0. \qquad (22)$$

Associated with this magnification factor is the phase angle

$$\theta = \arc\tan \frac{\tau^{n_1} \left(\frac{X}{S}\right)^{n_1-1} f_c \left[1 + \frac{c_{n_2}}{c_{n_1}} \tau^{n_2-n_1} \left(\frac{X}{S}\right)^{n_2-n_1} g_c\right]}{(1 - \tau^2)(1 + k_2/k_1)} . \qquad (23)$$

In the general case where n_i is greater than zero, Equation (22) must be solved by trial. When $n_1 = n_2 = 1$, Equation (22) reduces to Equation (12).

For various types of damping, the equation for the relative amplitude for impending separation is found by replacing c_1 and c_2 in Equation (16) by equivalent coefficients as shown in Equation (19). If Equations (20) and (21) are used in Equation (16), the result is

$$\overline{X} = \frac{F_o/k_1 \, (1 + m_1/m_2)}{\left\{\left(1 - \frac{k_2}{k_1}\frac{m_1}{m_2}\right)^2 + \tau^{2n_1}\left(\frac{X}{S}\right)^{2n_1-2} f_c^{\,2}\left[1 - \frac{c_{n_2}}{c_{n_1}}\frac{m_1}{m_2}\tau^{n_2-n_1}\left(\frac{X}{S}\right)^{n_2-n_1} g_c\right]^2\right\}^{\frac{1}{2}}} .$$

(24)

The phase angle for the force between the masses is

$$\beta = \text{arc tan} \left[\frac{\tau^{n_1} \left(\frac{X}{S}\right)^{n_1 - 1} f_c \left[1 - \frac{c_{n_2}}{c_{n_1}} \frac{m_1}{m_2} \tau^{n_2 - n_1} \left(\frac{X}{S}\right)^{n_2 - n_1} g_c \right]}{\left(1 - \frac{k_2}{k_1} \frac{m_1}{m_2}\right)} \right] . \tag{25}$$

CHAPTER III

THEORETICAL RESULTS

In the previous chapter the equations for the relative response and amplitude for impending separation were developed. The solutions of these two equations for a set of system parameters, exciting motion amplitude, and preload give the response and amplitude for impending separation. From the solutions of these equations, it can be established if separation would occur. Although this type of analysis is effective, the same procedure would have to be followed for each new set of parameters, exciting motion amplitude, and preload. In this chapter a novel theoretical method of analysis is introduced. It permits the determination of the maximum exciting motion amplitude and frequency band for a contact system where chatter will not occur. The effects of varying the system parameters on this method of analysis and the separation factor are studied.

Cause of Contact Chatter

In Appendix B it was shown that the force, $F(t)$, between the contacts is a sinusoidal varying force which oscillates about a reference position, the preload. For convenience the equation for the force between the contacts, Equation (B-5) from Appendix B, is presented now as

$$F(t) = F_o - \frac{\left\{\left(\frac{k_1}{m_1} - \frac{k_2}{m_2}\right)^2 + \omega^2\left(\frac{c_1}{m_1} - \frac{c_2}{m_2}\right)^2\right\}^{\frac{1}{2}} X \sin(\omega t - \theta + \beta)}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)},$$

where F_o is the preload and X is the amplitude of the response, $x(t)$. If a given contact system with a constant preload, constant parameters, and oscillating at a specific frequency is considered, then the force between the contacts can be written as

$$F(t) = F_o - AX \sin(\omega t - \theta + \beta), \tag{26}$$

where

$$A = \frac{\left\{\left(\frac{k_1}{m_1} - \frac{k_2}{m_2}\right)^2 + \omega^2\left(\frac{c_1}{m_1} - \frac{c_2}{m_2}\right)^2\right\}^{\frac{1}{2}}}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}.$$

Now from Equation (26) it can be seen that the amplitude of the sinusoidal varying part of $F(t)$ is AX and the oscillation is about F_o. Since A is a constant for the contact system under consideration, then X is the only variable in the amplitude of the sinusoidal part of $F(t)$. The maximum and minimum values of $F(t)$ occur when $\sin(\omega t - \theta + \beta) = \pm 1$, and these are $F(t) = F_o + AX$ and $F(t) = F_o - AX$, respectively. The response of the system, $x(t)$, is given in Equation (10) as

$$x(t) = X \sin(\omega t - \theta);$$

thus, the phase angle β appearing in Equation (26) relates the variation of $F(t)$ to the motion of $x(t)$.

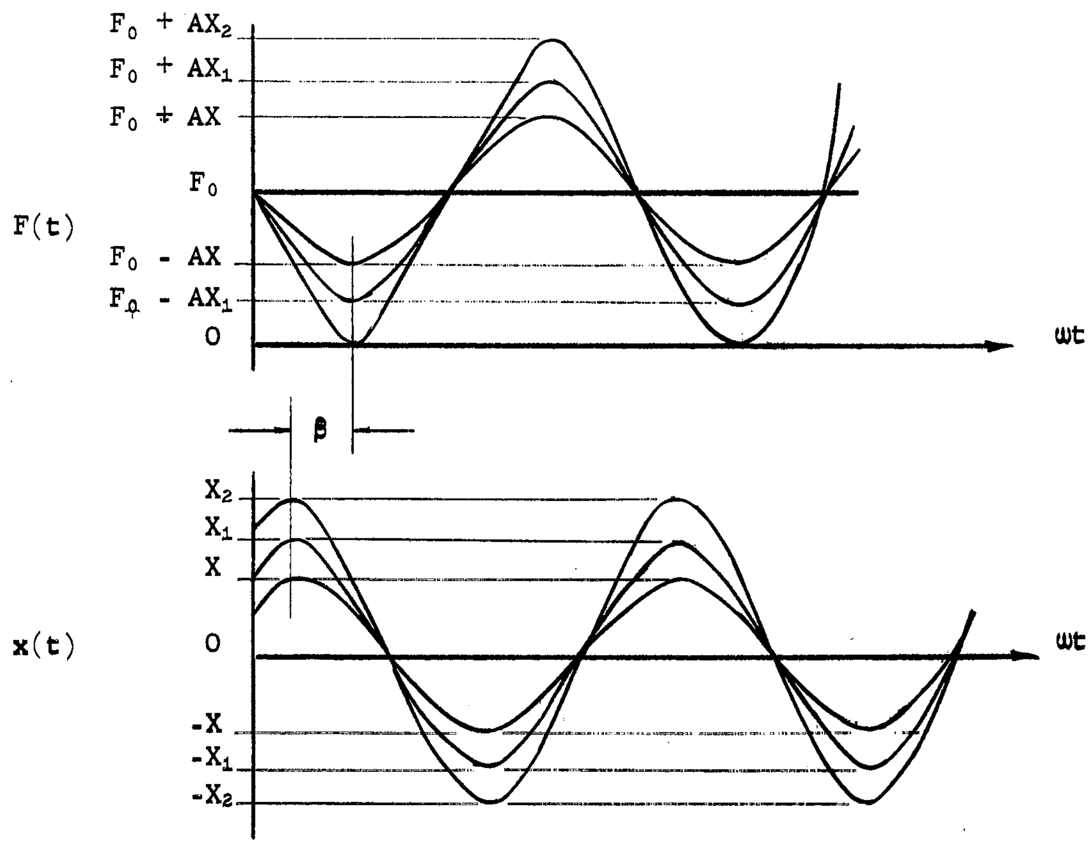


Figure 2. Motions of F(t) and x(t) Showing Phase Relation Between the Two Motions.

Figure 2 shows a sketch of a feasible variation of F(t) and its relation to x(t) for the contact system under consideration. With the use of Figure 2 as a guide, further investigation is made of the contact system. If the amplitude of the exciting motion, S, is increased from S to S_1, then the response amplitude of the system is also increased from X to X_1 [see Equation (11)]. Likewise, the amplitude of the sinusoidal varying part of F(t) is increased from AX to AX_1. Now the minimum value of F(t) is decreased from $F_o - AX$ to $F_o - AX_1$. These changes in the amplitudes of F(t) and x(t) can be seen in Figure 2. With an additional increase in the amplitude of the exciting motion, the amplitude of the response is increased from X_1 to X_2. And again the amplitude of the sinusoidal part of $F(t)$ is increased from AX_1 to AX_2. The minimum value of $F(t)$ is decreased from $F_o - AX_1$ to $F_o - AX_2$ where $F_o - AX_2 = 0$, [see Figure 2]. There is one point in each cycle of $F(t)$ where $F(t) = 0$. Whenever $F(t) = 0$, contact separation is possible. Therefore, there is one point in each cycle of $x(t)$ where contact separation is possible, but in the system under consideration this point of possible contact separation lags the peak values of $x(t)$ by β degrees as shown in Figure 2. It was previously stated that the amplitude of the response when $F(t)$ initially becomes zero would be the amplitude for impending separation, $\bar{X}$. Thus, in this example $X_2 = \bar{X}$ and the equation for $\bar{X}$ is Equation (16).

There are two important phenomena which occurred in the discussion that should be fully understood. The first is that the phase angle β has no explicit bearing on the amplitude for impending separation and only indicates in each cycle of $x(t)$ where separation is possible. The second is that contact chatter characteristics cannot be determined by merely investigating the amplitude for impending separation because of the possible amplitudes of the response. Since the contact system just considered was for one certain frequency, it is possible that at some other frequency the amplitude of the exciting motion necessary for $X = \bar{X}$ would be physically impossible to obtain. Furthermore, it is possible that only a minute exciting motion amplitude could cause $X = \bar{X}$. Therefore, to determine the chatter characteristics of a contact system, the response amplitude and amplitude for impending separation must be considered simultaneously.

Contact Separation Criteria

From Equation (16) it can be ascertained that the amplitude for impending separation is a function of the system parameters and τ, the ratio of forcing frequency to the undamped natural frequency of the system. Equation (11) indicates that the response amplitude is also a function of these same variables and the amplitude of the exciting motion. Without knowledge of the exciting motion amplitude, it is impossible to establish completely the chatter characteristics of a contact system.

With the recognition that Equation (12), the relative magnification factor, and Equation (18), the separation factor, are functions of the same variables, a new method to analyze contact systems is introduced. For chatter to occur, $X = \bar{X}$, or this can be written as

$$1 = \frac{\bar{X}}{X} .$$

The following identity

$$\frac{S}{(F_o/k_1)} = \left[\left(\frac{\bar{X}}{F_o/k_1}\right)\left(\frac{S}{X}\right)\right] \frac{X}{\bar{X}} ,$$

is introduced. When the condition for contact chatter is applied, then the result is

$$\left[\frac{S}{(F_o/k_1)}\right]_{\bar{X}=X} = \left[\left(\frac{\bar{X}}{F_o/k_1}\right)\left(\frac{1}{X/S}\right)\right] . \qquad (27)$$

The dimensionless term $S/(F_o/k_1)_{\bar{X}=X}$ is arbitrarily defined as the "Baron Number." After substitution for the separation factor and the magnification factor in Equation (27), the equation for the Baron Number is

$$\text{Baron Number} = \frac{(1+M)}{\tau^2}\left[\frac{(1-\tau^2)^2 + \dfrac{4\tau^2 M(1+C)^2 \zeta_1^2}{(1+M)(1+K)}}{(1-KM)^2 + \dfrac{4\tau^2 M(1+K)(1-CM)^2 \zeta_1^2}{(1+M)}}\right]^{\frac{1}{2}} . \qquad (28)$$

The Baron Number is a function of the ratios of the system parameters, the damping factor of contact 1, and τ. The information obtained about the chatter characteristics of a contact system by using the Baron Number is as follows:

(a) the maximum input amplitude or exciting motion amplitude for a given preload and spring constant 1 at which chatter will occur,

(b) the preload and spring constant 1 necessary to obtain a certain input amplitude without chatter,

(c) the effects on the input amplitude for chatter, for a given preload and spring constant, by varying any of the ratios of the parameters.

The equation used to determine the allowable input amplitude without contact chatter is

$$S < (F_o/k_1)(\text{Baron Number}) . \qquad (29)$$

The Baron Number should be made as large as possible in order to permit a large input amplitude before chatter occurs for a small preload. The optimum Baron Number for a contact system is infinity. Since the Baron Number is a function of τ, the frequency ratio, a useful equation for the natural frequency of the system is

$$f_n = f_1 \left\{ \frac{M(1 + K)}{1 + M} \right\}^{\frac{1}{2}} , \qquad (30)$$

where f_1 is the undamped natural frequency of contact 1. The distinction of using the Baron Number in contact system analysis is that when one contact, denoted as contact 1, is completely described, the optimum values of the mass, spring constant, and damping coefficient for the other contact can then be predicted. This is done by optimizing the Baron Number using arbitrary ratios of the masses, spring constants, and damping coefficients. From the ratios which optimized the Baron Number, the mass, spring constant, and damping coefficient of the other contact can be determined.

It has been pointed out that the chatter characteristics of a contact system cannot be determined by merely using the equation of the amplitude for impending separation, but also the response of the system must be established simultaneously. The Baron Number considers both the separation amplitude and response amplitude for each value of τ. With the use of the Baron Number, the input amplitude for contact chatter can be determined. Once the natural frequency of the system has been found, the input acceleration that a contact system can withstand without chatter is determined. This is the dynamic information in which most contact system designers are interested.

General Discussion of the Baron Number

Since the optimum value of the Baron Number is infinity for all values of τ, it can be seen from Equation (28) this condition is established if both $KM = 1.0$ and $CM = 1.0$. The term KM, which can be written as

$$KM = \left(\frac{k_2}{k_1}\right)\left(\frac{m_1}{m_2}\right) = \left(\frac{m_1}{k_1}\right)\left(\frac{k_2}{m_2}\right) = \left(\frac{f_2}{f_1}\right)^2 ,$$

is the square of the ratio of the undamped natural frequency of the two contacts. When $KM = 1.0$, the two contacts have been tuned so that they both have the same natural frequency. The term CM is the ratio of the damping frequencies and compares the transient decay of the two contacts of the system. For the condition $KM = 1.0$ and $CM = 1.0$ it is evident the damping on the system is proportioned on each contact in the same manner as the spring constants. From Equation (28) when $KM \neq 1.0$, the Baron Number is infinite at $\tau = 0.0$ and then decreases.

From Equation (27) it can been seen that the Baron Number is made up of two factors. These factors are the separation factor, Equation (18), and the magnification factor, Equation (12). Equation (27) is actually a function of the reciprocal of the magnification factor, $1/(X/S)$. The magnification factor, X/S, for a linear single degree of freedom system is a well established factor and is found in many vibration text books. Thus, it is possible to draw some general curves for the term $1/(X/S)$ appearing in Equation (27). Figure 3 shows the general shapes for the curves of $1/(X/S)$ for various values of τ and amounts of the total damping, ζ_T. From Figure 3 it can be seen that the minimum values of $1/(X/S)$ are established by the undamped system, $\zeta_T = 0.0$. At $\tau = 0.0$ the term $1/(X/S)$ is infinity; thus, the Baron Number is infinity. As τ increases from 0.0 to 0.5, the term $1/(X/S)$ decreases which indicates that the Baron Number could also decrease. For large values of τ the term $1/(X/S)$ becomes asymptotic to unity; thus, the

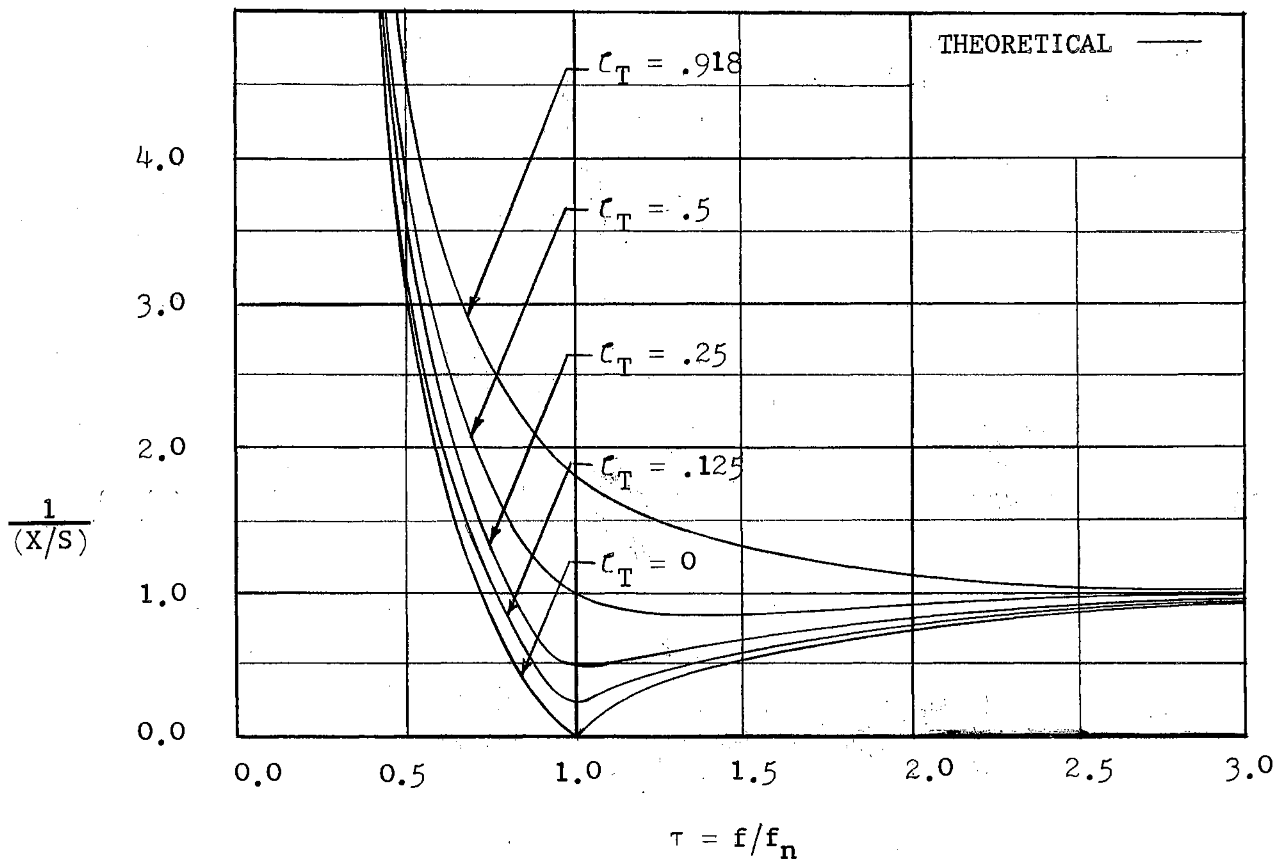


Figure 3. General Curves for the Term $1/(X/S)$.

Baron Number approaches the value of the separation factor. The term $1/(X/S)$ for the undamped system is zero at $\tau = 1.0$, which indicates the Baron Number is zero at $\tau = 1.0$ for an undamped system. It is important to note that the term $1/(X/S)$ is not appreciably influenced by the changes in the quantities K and M, but it is increased if C and ζ_1 are unequal to zero.

General Discussion of the Separation Factor

The separation factor appearing in Equation (27) is a newly developed factor and is defined in Equation (18) to be

$$\frac{\bar{X}}{F_o/k_1} = \frac{(1 + M)}{\left\{(1 - KM)^2 + \frac{4\ \tau^2\ M\ (1 + K)(1 - CM)^2\ \zeta_1{}^2}{(1 + M)}\right\}^{\frac{1}{2}}} . \qquad (31)$$

Since no general curves can be drawn for the separation factor, a study is made to establish the effects on the separation factor of varying different parameters. It was pointed out that the optimum value of the Baron Number for a contact system is infinity. From Figure 3 it is apparent that the term $1/(X/S)$ is infinity at $\tau = 0.0$ and then becomes asymptotic to unity for large values of τ. Therefore, from Equation (27) for the Baron Number to be infinity at $\tau > 0$ the value of the separation factor must be infinity. Thus, the denominator of Equation (31) must be zero for all values of τ. The denominator of Equation (31) contains two terms: the 1st term $(1 - KM)^2$ which is a constant for a given system and the 2nd term $\frac{4\tau^2\ M\ (1 + K)(1 - CM)^2\ \zeta_1{}^2}{(1 + M)}$ which is a constant times the variable τ^2. If K and M are never zero, then the 1st term is zero only when $KM = 1$. The 2nd term is zero when $CM = 1$ or when C and ζ_1 are zero. If C and ζ_1 are both zero the system is undamped. If C is zero the system has one contact damped. In general, the 1st term in the denominator, $(1 - KM)^2$, and the numerator establish the initial value of the separation factor at $\tau = 0.0$. As τ increases the separation factor is decreased from its initial value if $CM \neq 1$ and $\zeta_1 \neq 0$. If the spring constant ratio, K, or the mass ratio, M, is changed from the value where $KM = 1$, the initial value of the separation factor is decreased. The magnitude of the 1st term indicates how the separation factor is decreased if the 2nd term is not zero. If $(1 - KM)^2$ is large the separation factor has a low initial value but is decreased slowly as τ increases. For small values of $(1 - KM)^2$, the initial value of the separation factor is high but is decreased as τ increases. If the effects of the term $1/(X/S)$ and the separation factor are combined, the Baron Number, as written in Equation (27), is infinity at $\tau = 0.0$. The Baron Number then decreases as τ increases and becomes asymptotic to the separation factor when τ is large.

Variations of the Baron Number and Separation Factor

A sequence of plots of the Baron Number versus τ and the separation factor versus τ is presented to illustrate the effects of varying different parameters on the Baron Number and the separation factor. The parameter under consideration, excluding ζ_1, is varied from its value for an optimum Baron Number, for all values of τ. A parameter is varied by a constant times the optimum value of that parameter. All other parameters which are not varied, excluding ζ_1, are unity. The numerical values on each plot are not so important as the shape of the curves and their relation to each other.

Three conditions of the contact system, the undamped, one contact damped, and both contacts damped, are investigated. For the undamped system C and ζ_1 are set equal to zero. With one contact damped, C is zero and various values of ζ_1 are considered. When both contacts are damped, neither C nor ζ_1 can be zero. In every case τ is varied from 0.0 to 3.0.

Condition I. Undamped System

The effects of varying the spring constant ratio, K, from the optimum K to 0.5K, 2K, 3K, and 100K on the undamped system are shown in Figure 4 for the Baron Number and Figure 5 for the separation factor. 100K is approaching the condition of having one rigid contact and one flexible contact. It should be pointed out that in order to prevent the occurrence of the undefined number 0/0 when KM = 1.0 for the undamped condition, the value of KM = 1.0001 is used as an optimum. Figure 4 indicates that changing the spring constant ratio from the optimum K lowers the Baron Number. Also, Figure 5 shows the same effect of lowering the separation factor as K is varied further from the optimum. Since the Baron Number in Figure 4 is the product of the separation factor in Figure 5 and the term 1/(X/S) in Figure 3, the shapes of the curves of the Baron Number are the same as the undamped curve in Figure 3. Their magnitudes are varied by the same amount as the separation factor for different values of K. It is important to note that even for the optimum K the Baron Number always went to zero at $\tau = 1.0$. This exemplifies the fact that a contact system without damping always has contact separation at resonance, $\tau = 1.0$.

Figures 6 and 7 illustrate the effect of varying the mass ratio, M, on the Baron Number and separation factor, respectively. Varying the mass ratio has the same results of decreasing the Baron Number and separation factor as did varying the spring constant ratio. Comparing Figures 4 and 6 indicates that increasing the mass ratio, M, has less affect on the Baron Number than did increasing the spring constant ratio. Increasing the mass ratio, M, has less influence on the separation

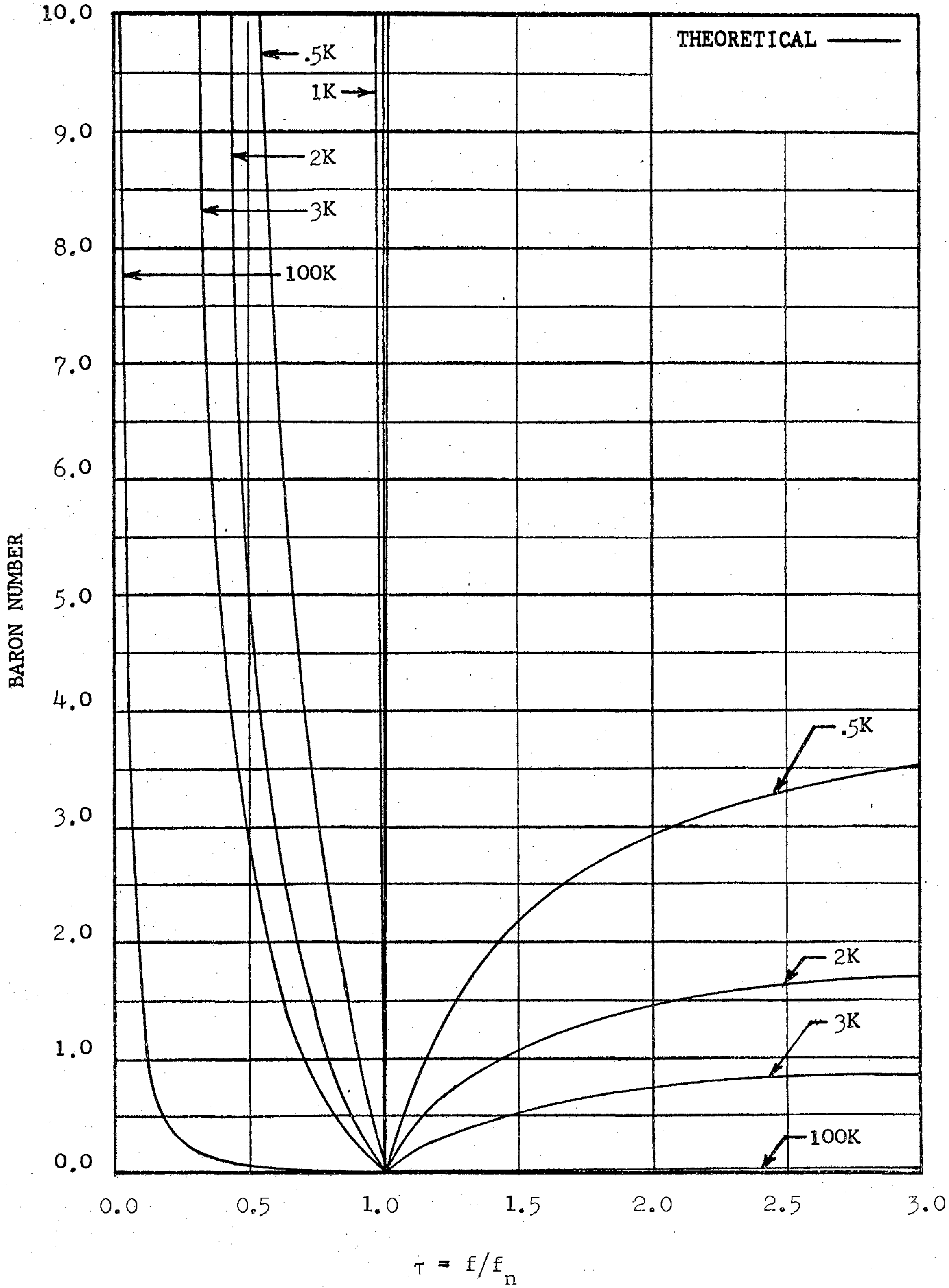


Figure 4. Baron Number for an Undamped System, $C = 0.0$, $M = 1.0001$, $\zeta_1 = 0.0$.

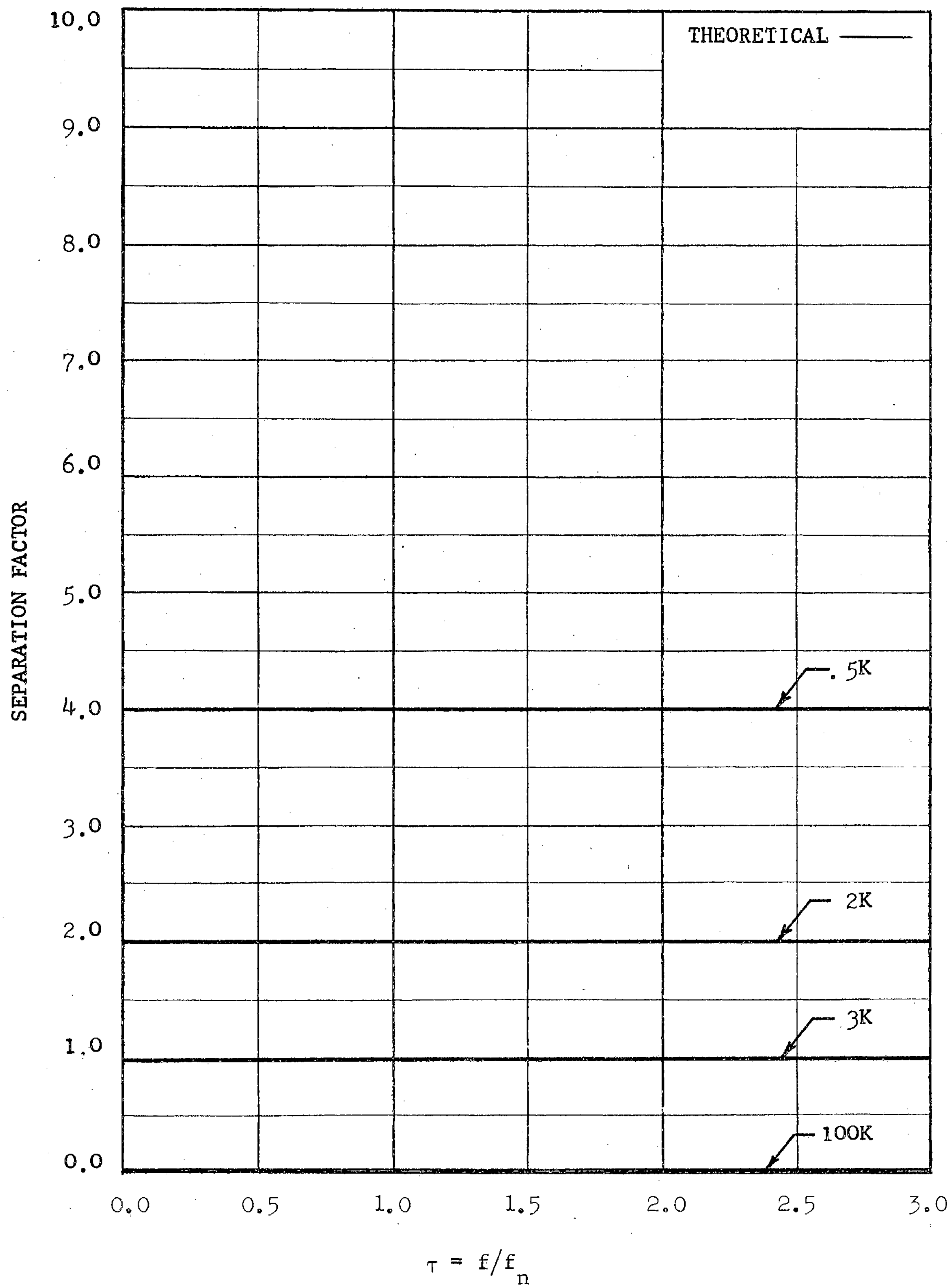


Figure 5. Separation Factor for an Undamped System, $C = 0.0$, $M = 1.0001$, $\zeta_1 = 0.0$.

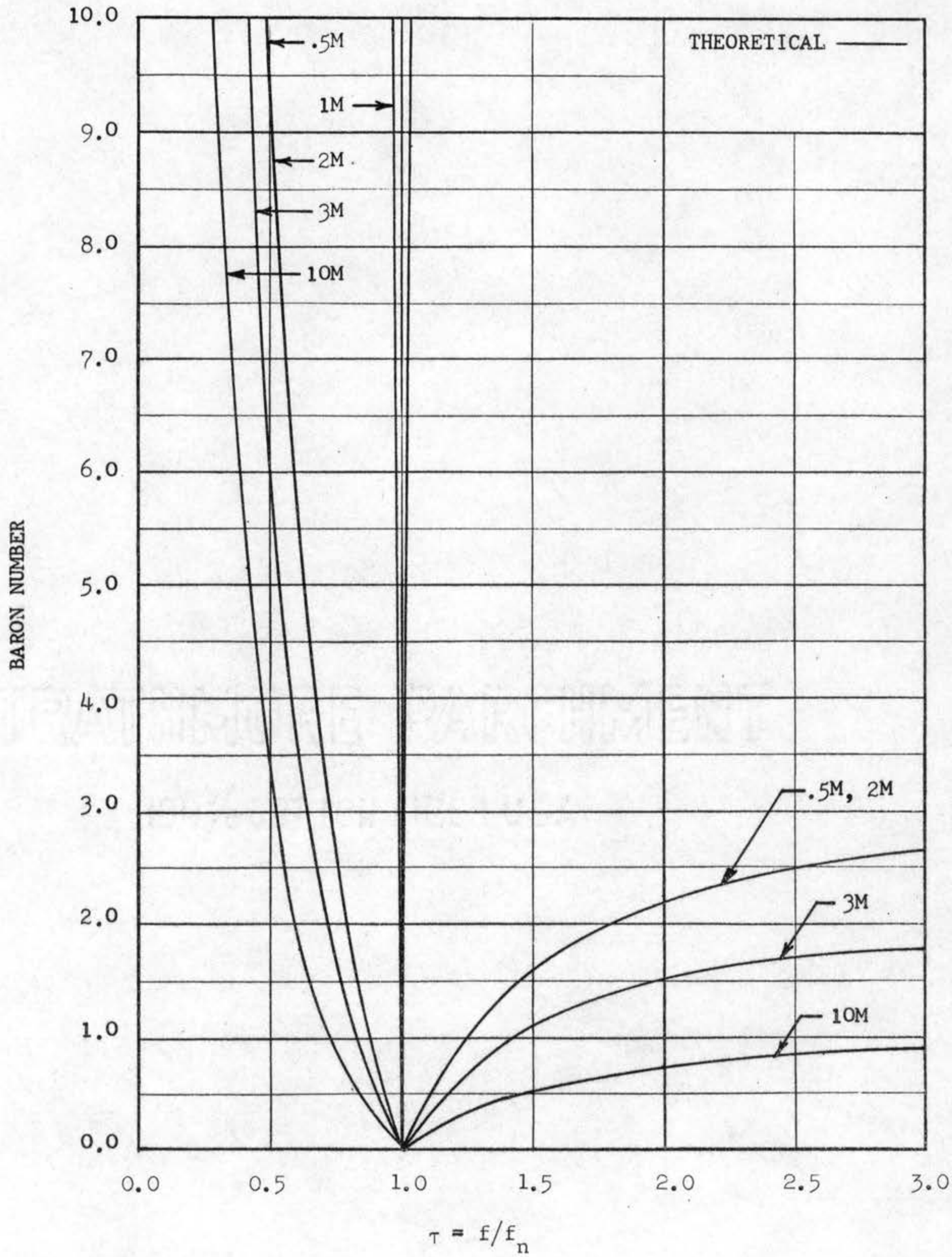


Figure 6. Baron Number for an Undamped System, C = 0.0
K = 1.0001, ζ_1 = 0.0.

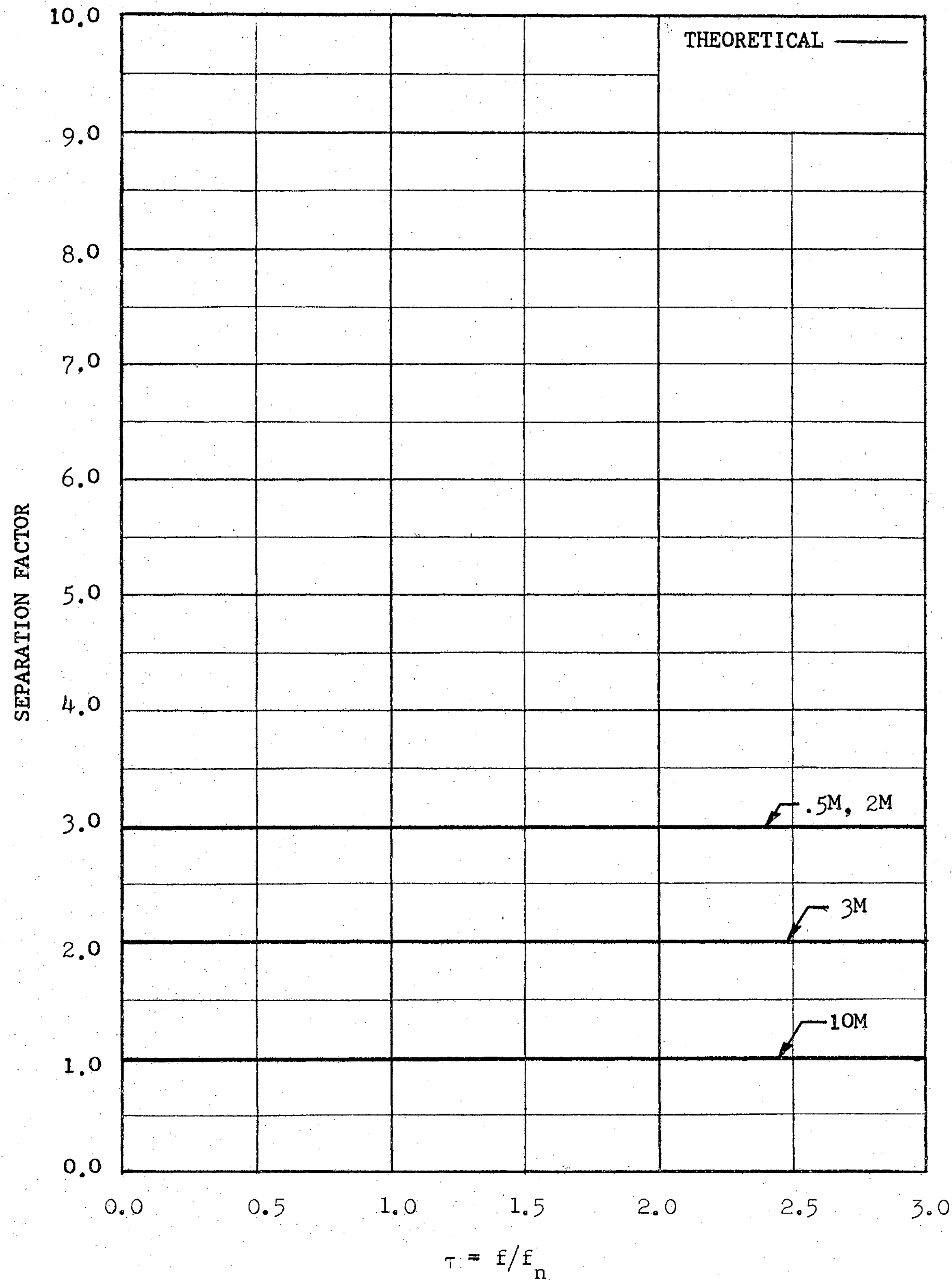


Figure 7. Separation Factor for an Undamped System, C = 0.0, K = 1.0001, $\zeta_1 = 0.0$.

factor, as can be seen by comparing Figures 5 and 7. However, decreasing the mass ratio, M, changed the separation factor more than decreasing the spring constant ratio. The term $1/(X/S)$ does not change as either K or M is varied. Thus, only the differences in the separation factors of Figures 5 and 7 cause the Baron Number in Figures 4 and 6, respectively, to be different. At $\tau = 1.0$ the Baron Number went to zero indicating that an undamped system will have contact separation at resonance.

Condition II. One Contact Damped

For the condition in which one contact is damped, CM = 0.0 and $\zeta_1 \neq 0.0$, ζ_1 is varied from 0.25 to 1.0 in increments of 0.25. For the purpose of discussion, a lightly damped contact is $\zeta_1 = 0.25$ and a heavily damped contact is $\zeta_1 = 0.75$. If the optimum condition of KM = 1.0 for the spring constant ratio and mass ratio, M, is used, Figures 8 and 9 show the effect of varying ζ_1 on the Baron Number and the separation factor. In Figure 8 the Baron Number continually decreases from $\tau = 0.0$ to $\tau = 1.0$ for all values of ζ_1. As τ further increases from 1.0, the Baron Number increases for small values of ζ_1 and then begins to decrease. For large values of ζ_1 the Baron Number always decreases when τ is greater than zero. Figure 9 indicates that the separation factor decreases for all values of τ. The rate and amount that the separation factor decreases is dependent on the value of ζ_1. When one contact is damped, the separation factor is dependent on the frequency of the exciting motion. In Figure 9 the curves for the decreasing separation factors are continuous for all values of ζ_1. From Figure 3 it can

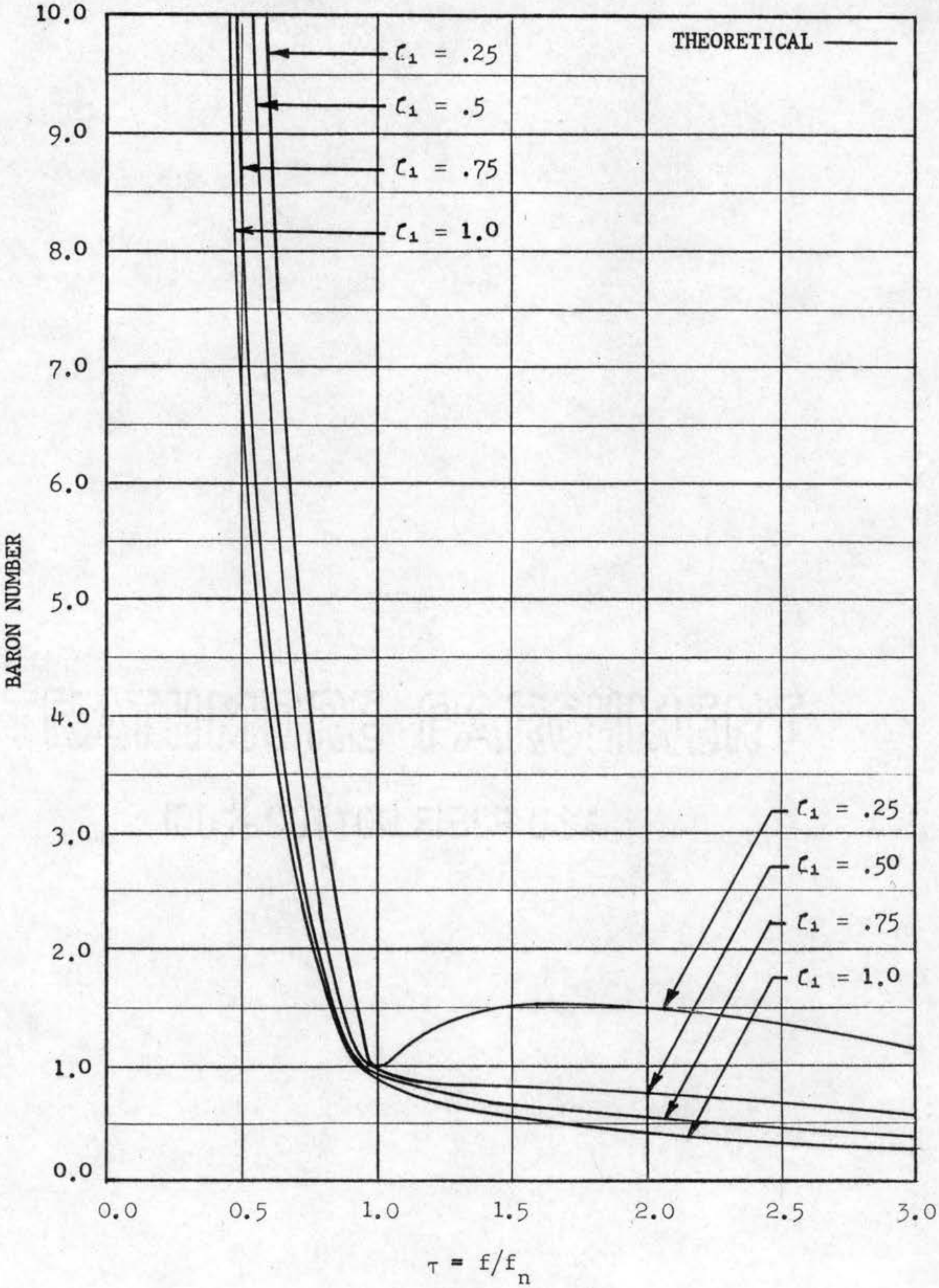


Figure 8. Baron Number for a System with One Contact Damped, C = 0.0, K = 1.0, M = 1.0.

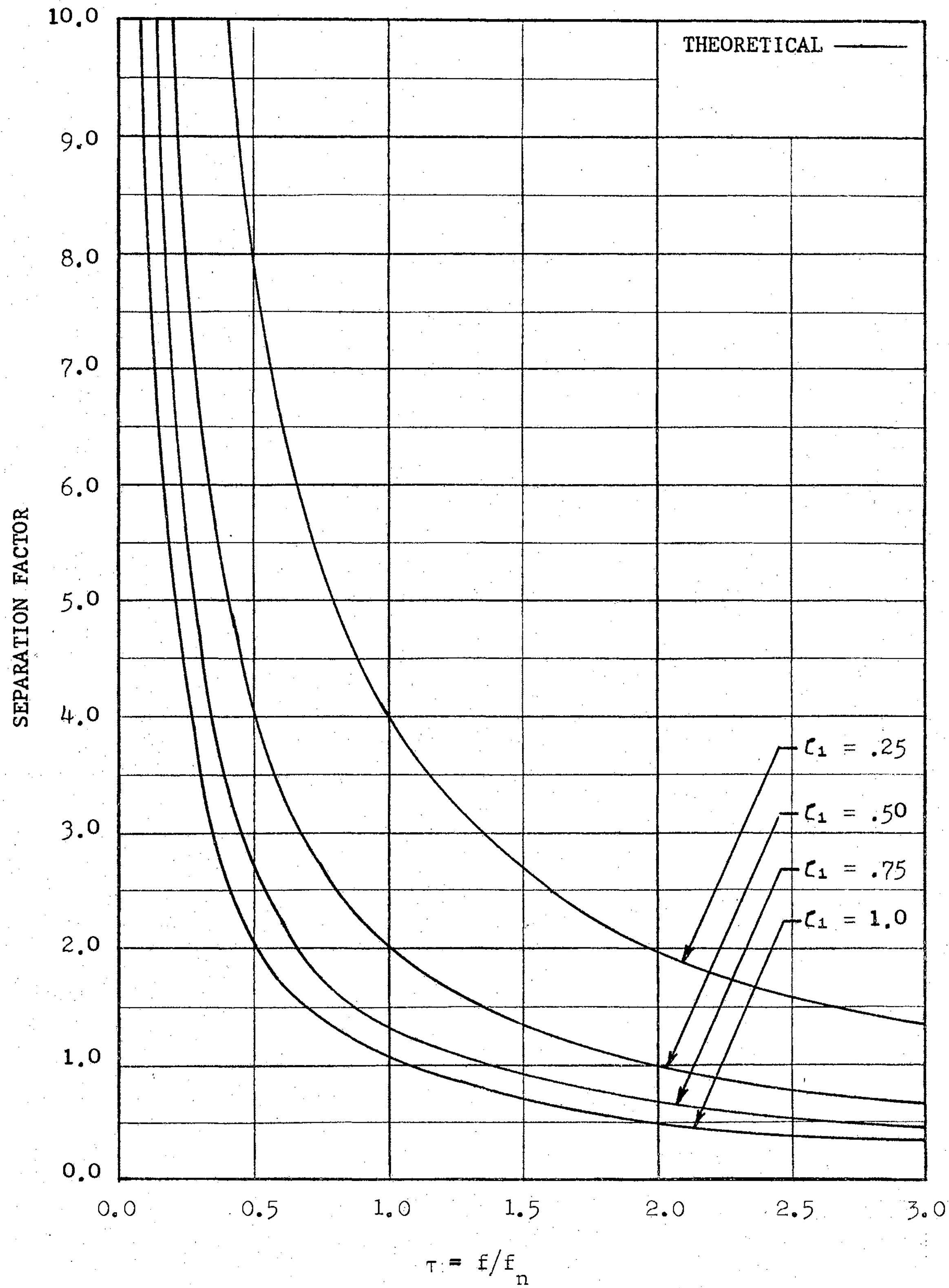


Figure 9. Separation Factor for a System with One Contact Damped, C = 0.0, K = 1.0, M = 1.0.

be seen that curves for the term $1/(X/S)$ are not smooth for a lightly damped system, but they dip at approximately $\tau = 1.0$. When the system is heavily damped, the curves for $1/(X/S)$ are continuous. Since the Baron Number is the product of the separation factor and the term $1/(X/S)$, the different values of the term $1/(X/S)$ caused the Baron Number to decrease and then increase for small values of ζ_1 as shown in Figure 8. Even though the Baron Number decreases when one contact is damped, it never is zero. Thus, there always exists an input amplitude at which separation does not occur from $\tau = 0.0$ to $\tau = 3.0$. This is not the case for the undamped system.

The effects on the Baron Number of varying the spring constant ratio from the condition where KM = 1.0, when one contact is lightly damped, $\zeta_1 = 0.25$, and then heavily damped $\zeta_1 = 0.75$, are shown in Figures 10 and 12. Changes in the separation factor for the two damped conditions are shown in Figures 11 and 13 for the lightly damped and heavily damped contact, respectively. As shown in Figures 11 and 13, at $\tau = 0.0$ the results of changing the spring constant ratio lowers the initial values of the separation factor, this is shown in Figure 5 for the undamped system. The added effect of damping one contact causes the separation factor to further decrease as τ increases. Figures 11 and 13 indicate that a cedrease in the value of K from KM = 1.0 causes the separation factor to be larger than that for KM = 1.0 when τ becomes large. A comparison of Figures 4, 10, and 12 for 100K shows that damping one contact had no overall effect on the Baron Number, although in Figure 4 the Baron Number did go to zero at $\tau = 1.0$.

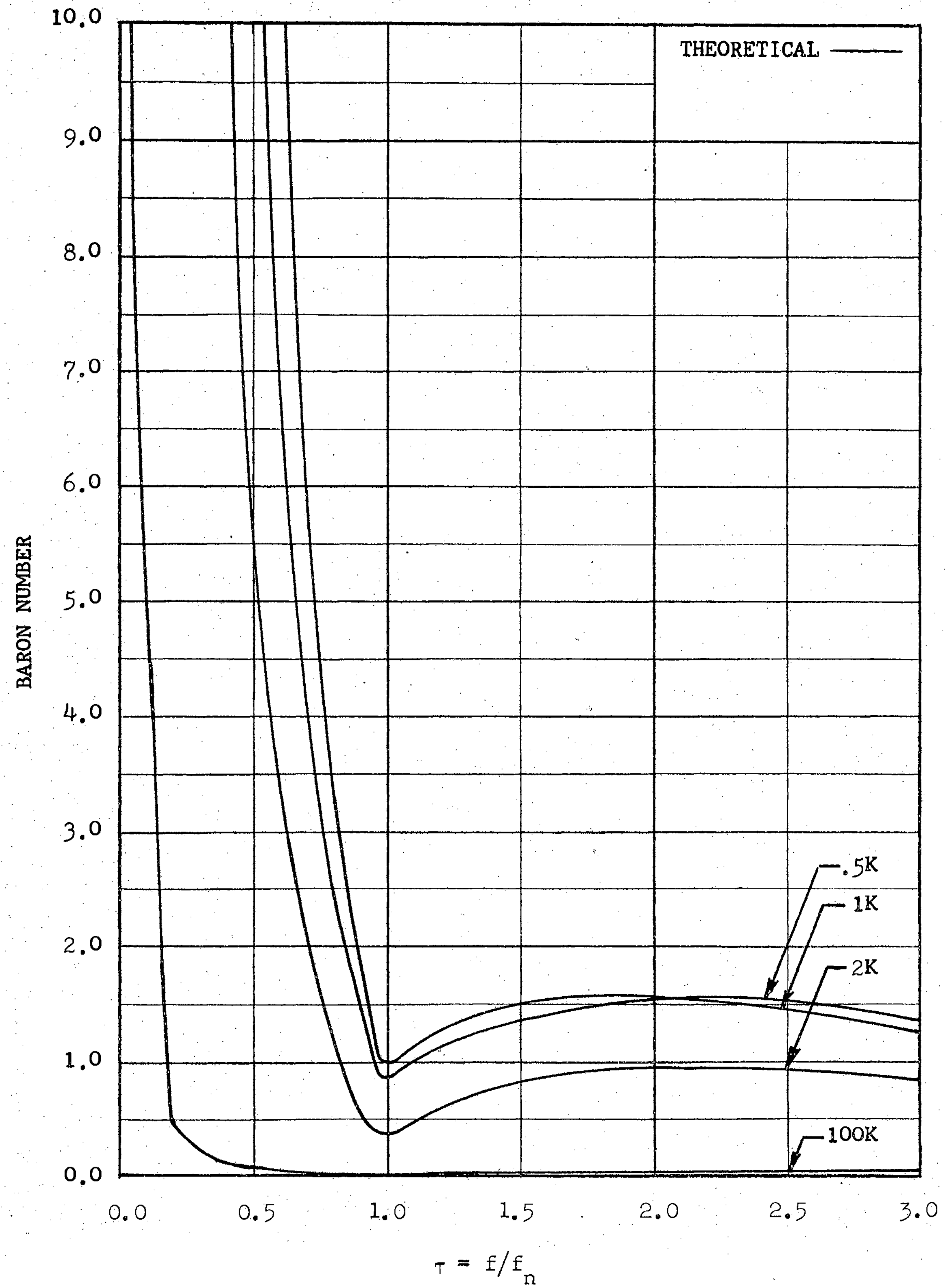


Figure 10. Baron Number for a System with One Contact Damped, $C = 0.0$, $M = 1.0$, $\zeta_1 = 0.25$.

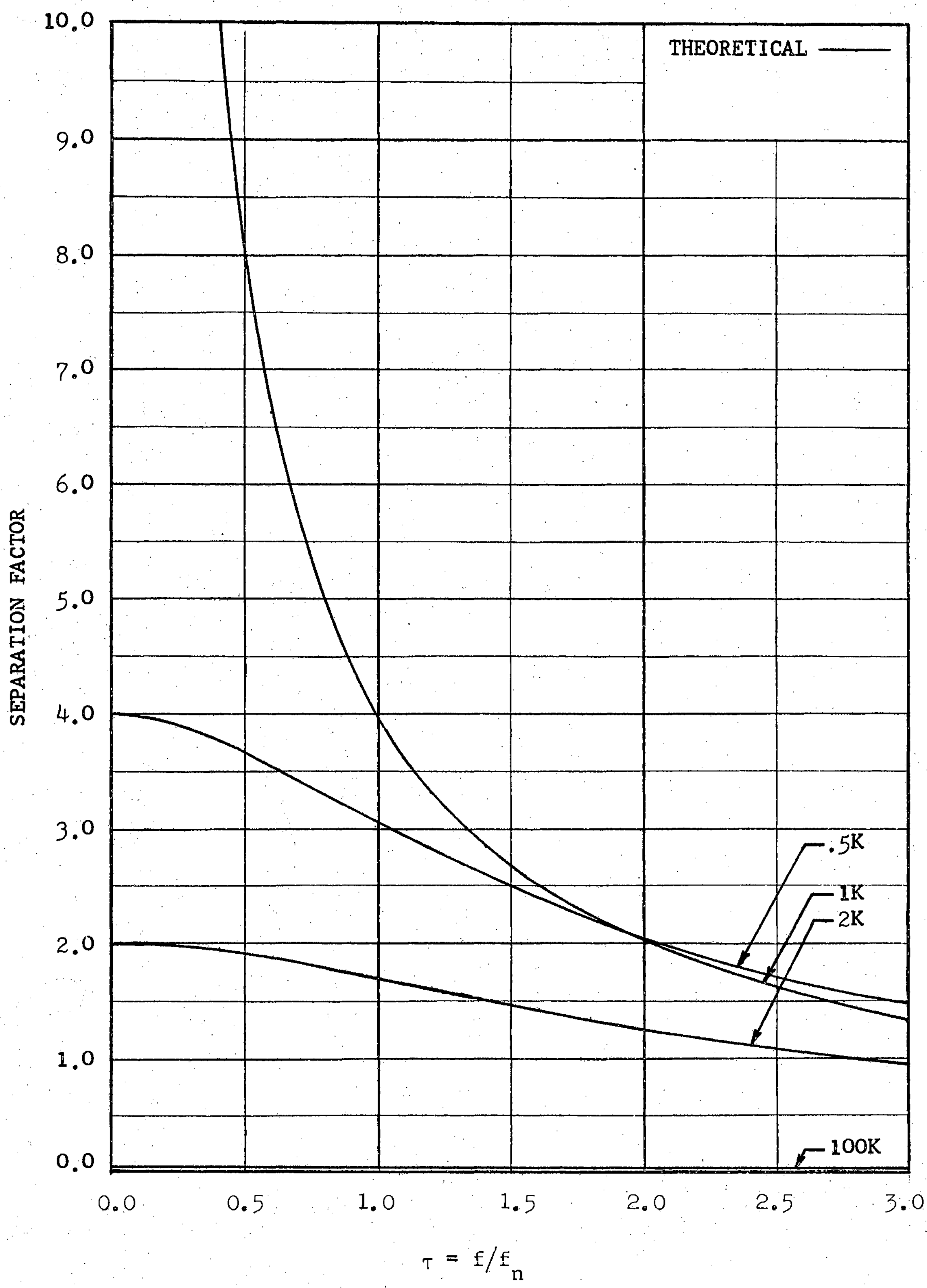


Figure 11. Separation Factor for a System with One Contact Damped, C = 0.0, M = 1.0, $\zeta_1 = 0.25$.

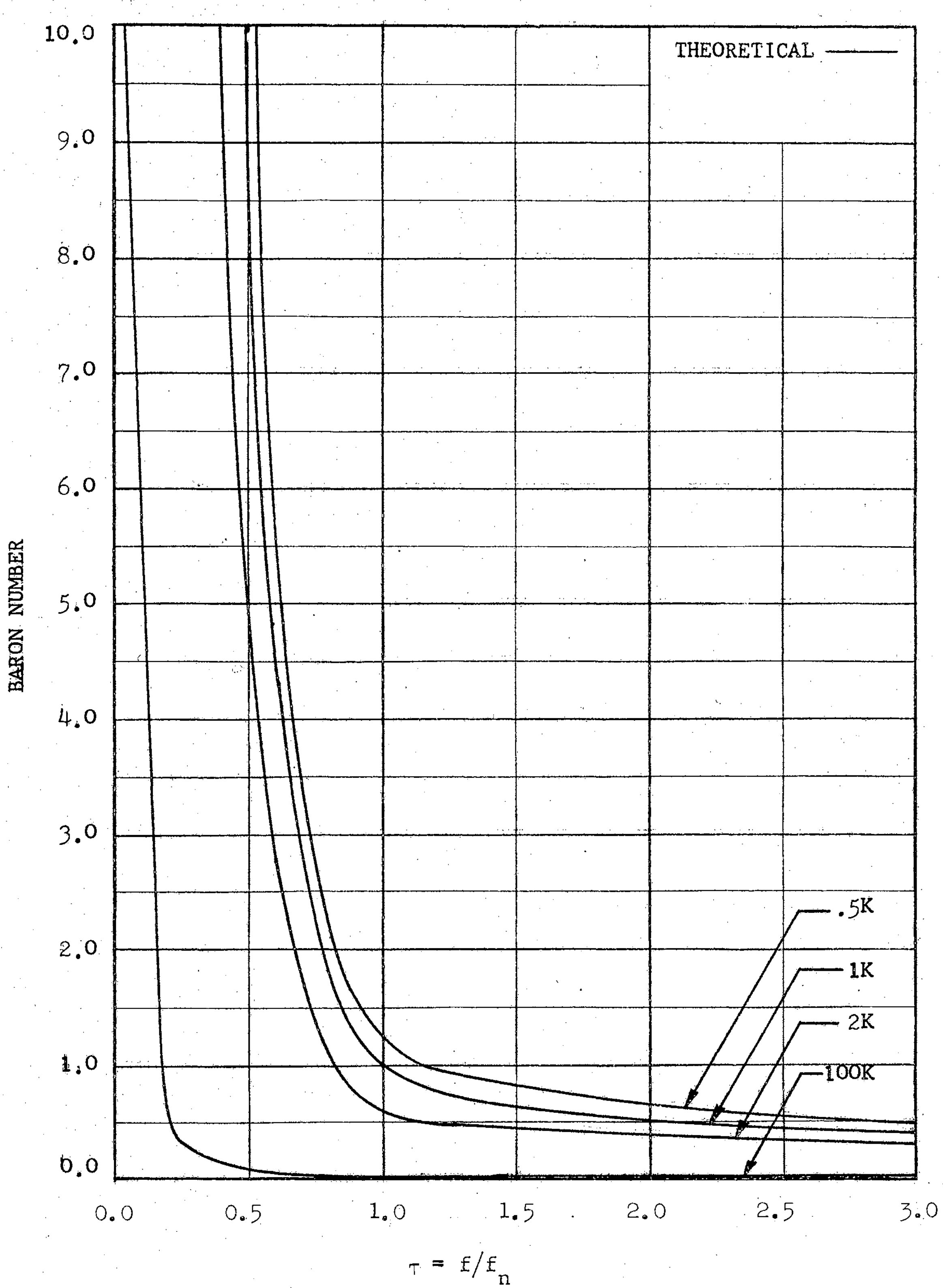


Figure 12. Baron Number for a System with One Contact Damped, C = 0.0, M = 1.0, $\zeta_1 = 0.75$.

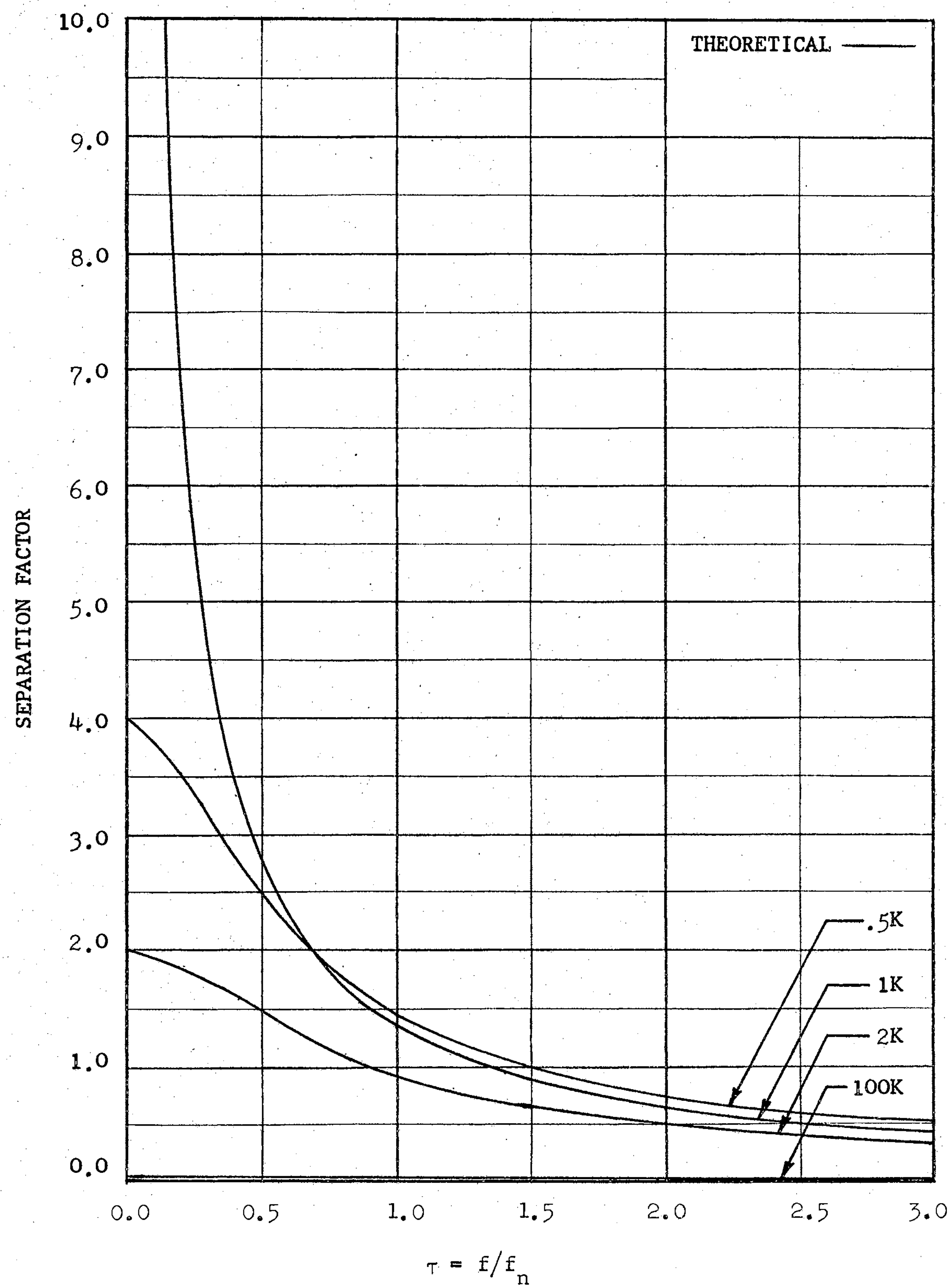


Figure 13. Separation Factor for a System with One Contact Damped, C = 0.0, M = 1.0, ζ_1 = 0.75.

The results of changing the mass ratio on the Baron Number for one contact lightly and heavily damped are shown in Figures 14 and 15, respectively. Changing the mass ratio had the same effect of reducing the initial value of the separation factor at $\tau = 0.0$ as shown in Figures 16 and 17.

A general comparison of the curves of the Baron Number for the undamped system with those of the system with one contact damped indicates the Baron Number is not zero at $\tau = 1.0$ when damping is applied. This is not the case for the undamped system. Also, when $\tau > 1.0$, the values of the Baron Number for the damped system are less than those for the undamped system. The latter observation, not as important as it may seem, is discussed in the summary of this chapter.

Condition III. Both Contacts Damped

For the contact system with both contacts damped, two cases are investigated. The first is the case in which the two damping coefficients are the same, thus producing equal damping forces, $c_1\dot{x}(t)$, on each contact. In the second case the damping coefficient on contact 1 is twice that on contact 2; thus, $c_1\dot{x}(t) = 2c_2\dot{x}(t)$ and $c_2/c_1 = 0.5$. In each case the damping factor on contact 1 is either $\zeta_1 = 0.25$ or $\zeta_1 = 0.75$. The mass and spring constant ratios are varied as before.

Figures 18 and 19 show the effect on the Baron Number of varying the spring constant ratio from the value KM = 1.0 when C = 1.0, M = 1.0, and the damping factor is $\zeta_1 = 0.25$ and $\zeta_1 = 0.75$, respectively. The separation factor is the same as that for the undamped system in Figure 5 because CM = 1.0. Since the separation factor is constant, the shape

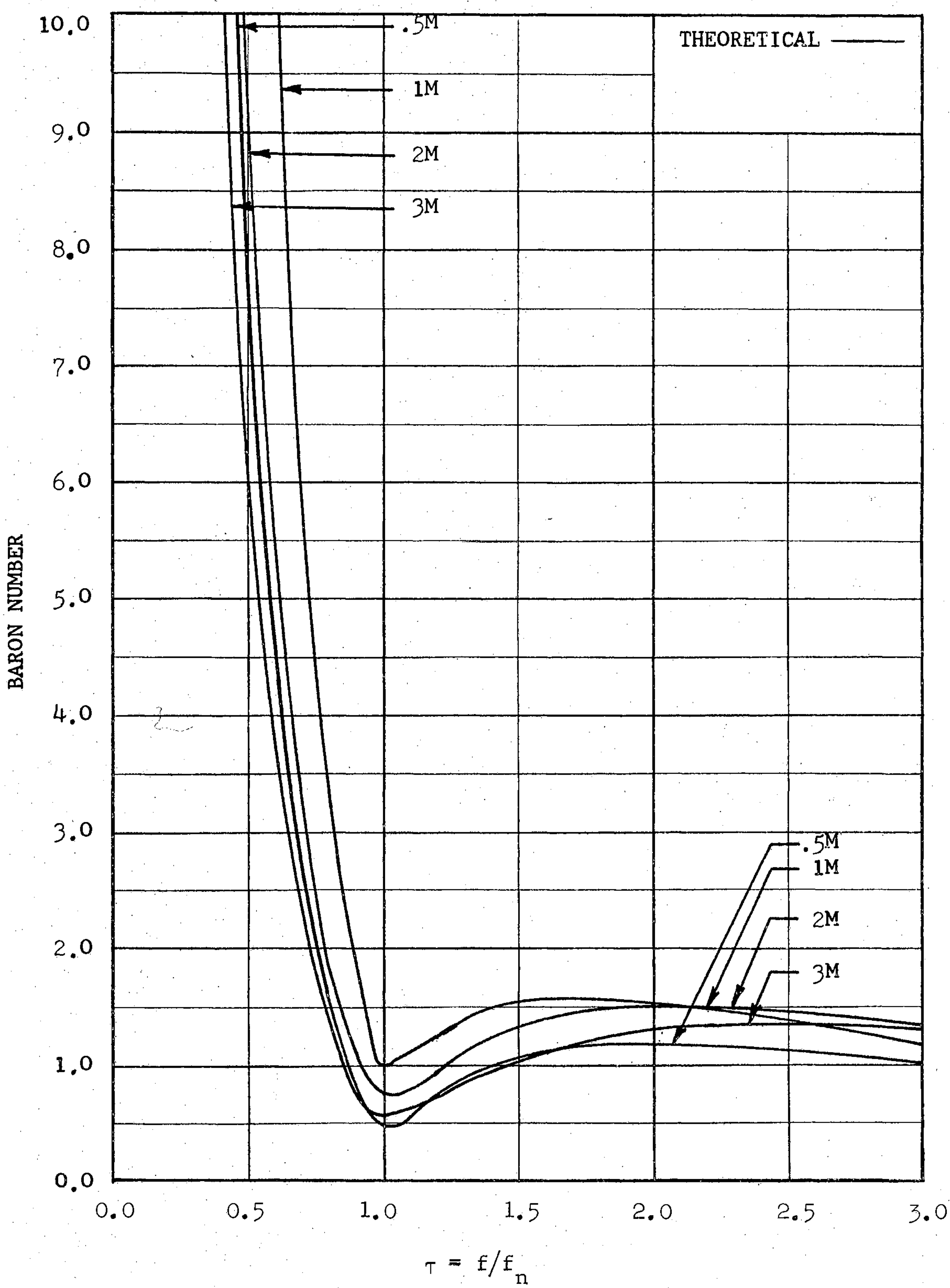


Figure 14. Baron Number for a System with One Contact Damped, C = 0.0, K = 1.0, ζ_1 = 0.25.

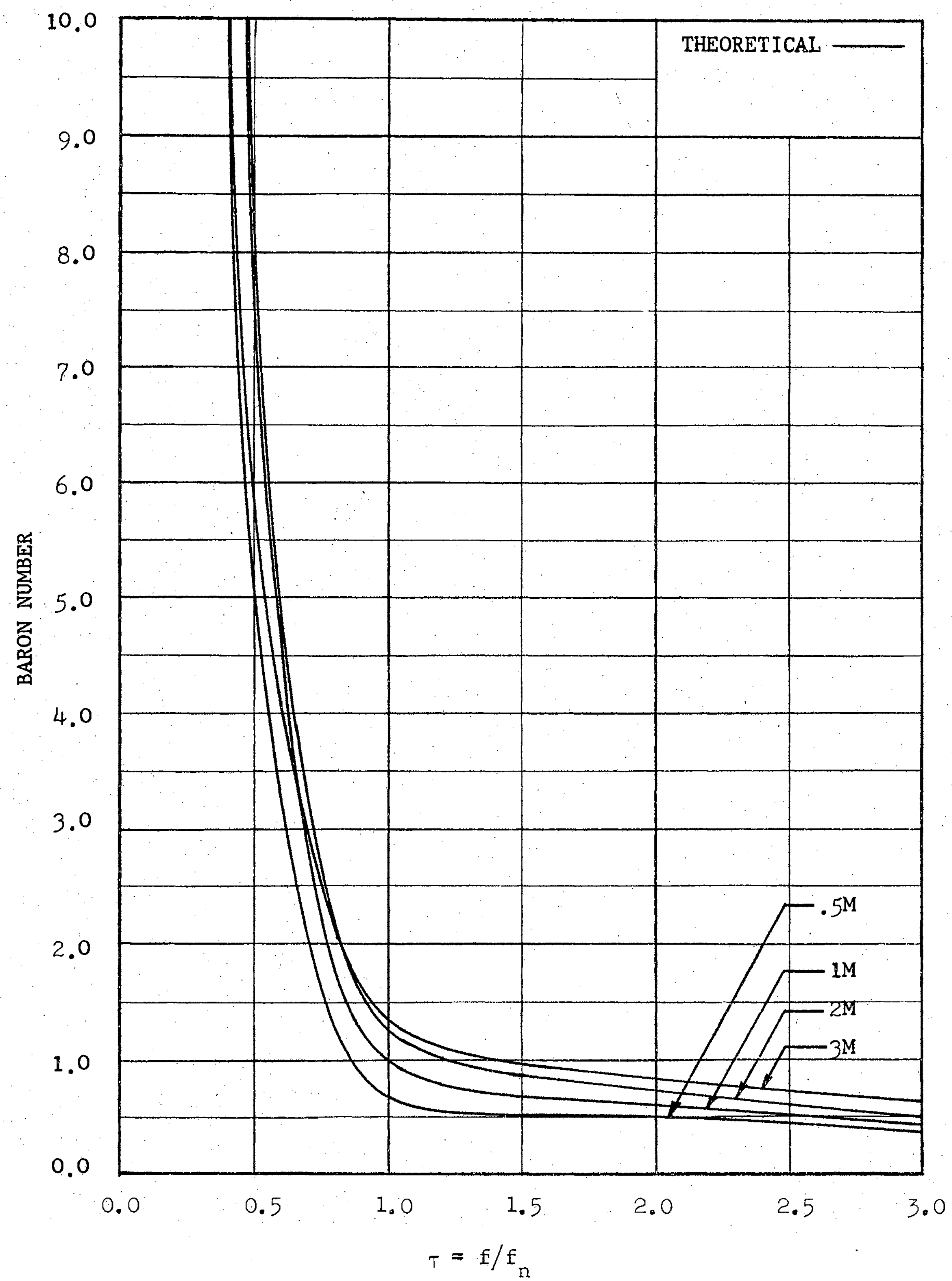


Figure 15. Baron Number for a System with One Contact Damped, C = 0.0, K = 1.0, ζ_1 = 0.75.

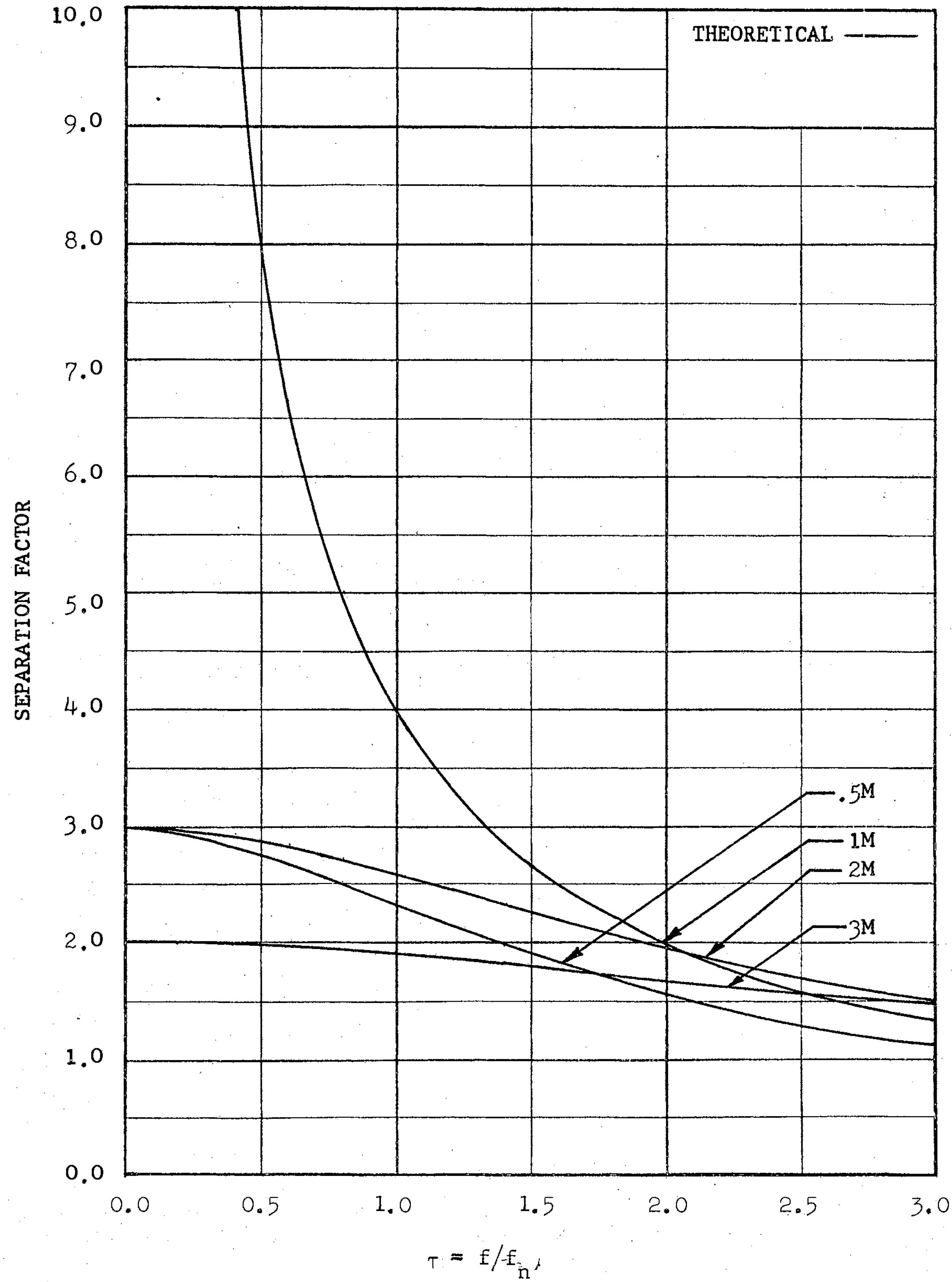


Figure 16. Separation Factor for a System with One Contact Damped, C = 0.0, K = 1.0, ζ_1 = 0.25.

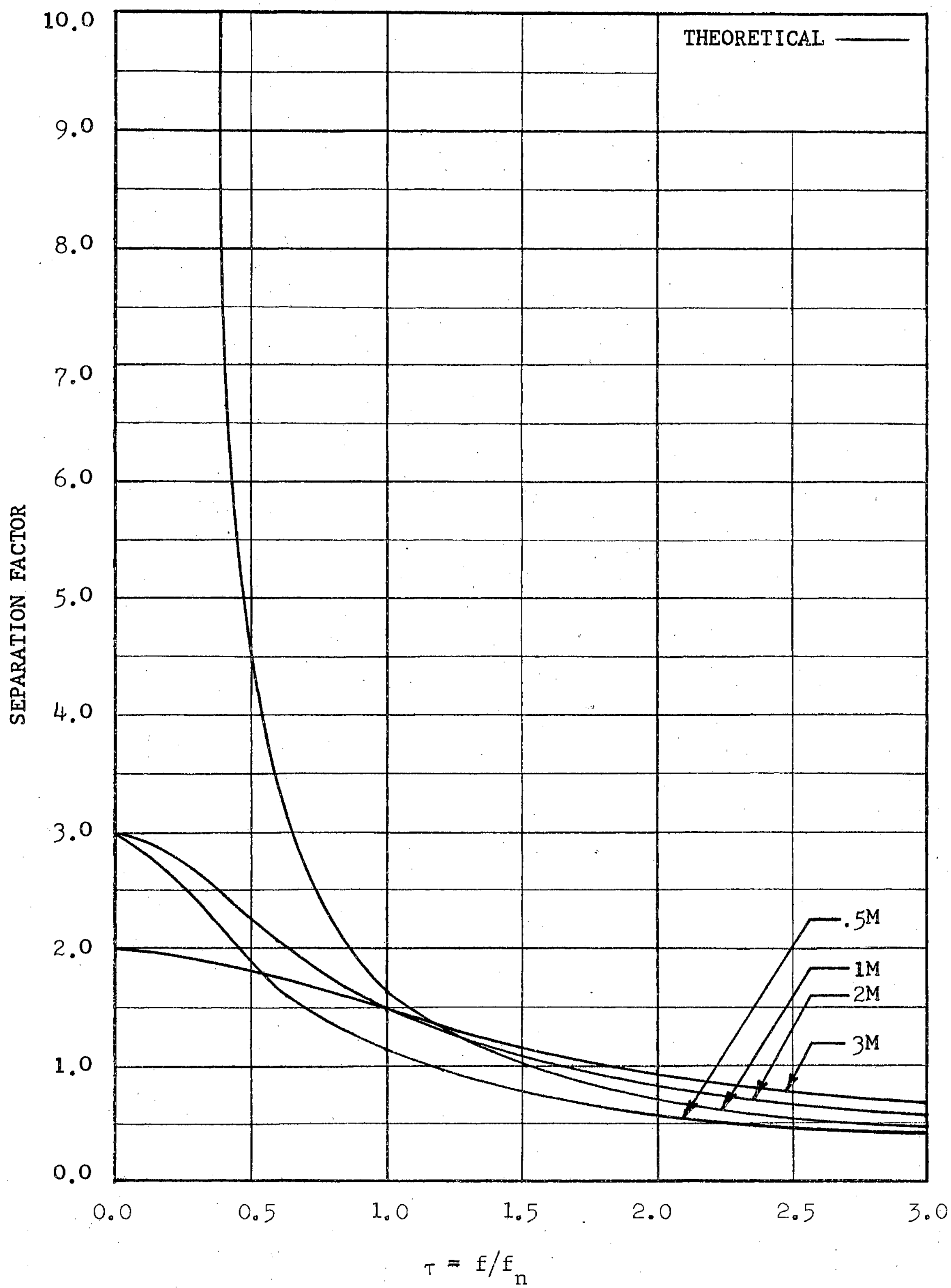


Figure 17. Separation Factor for a System with One Contact Damped, $C = 0.0$, $K = 1.0$, $\zeta_1 = 0.75$.

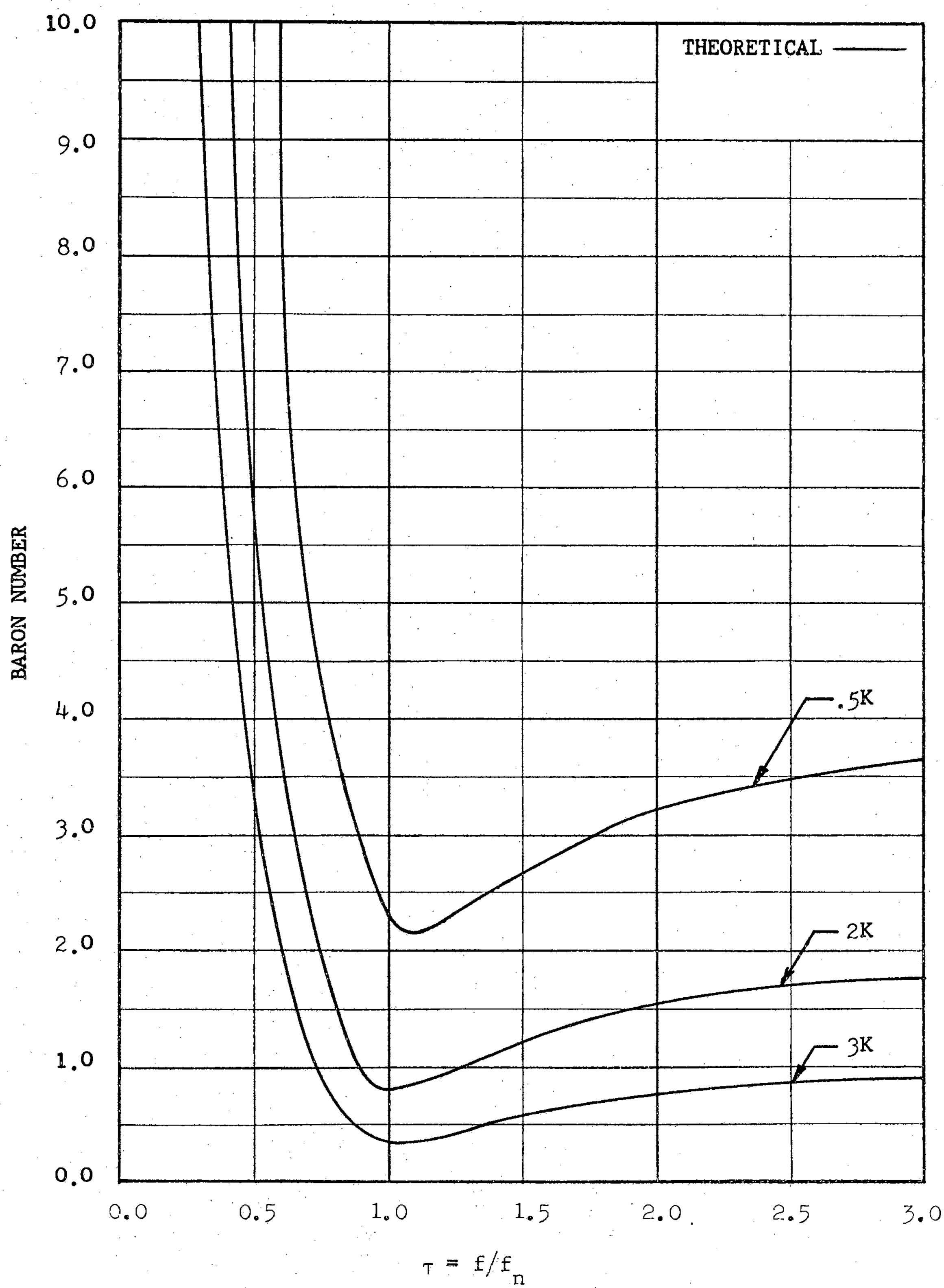


Figure 18. Baron Number for a System with Both Contacts Damped, C = 1.0, M = 1.0, ζ_1 = 0.25.

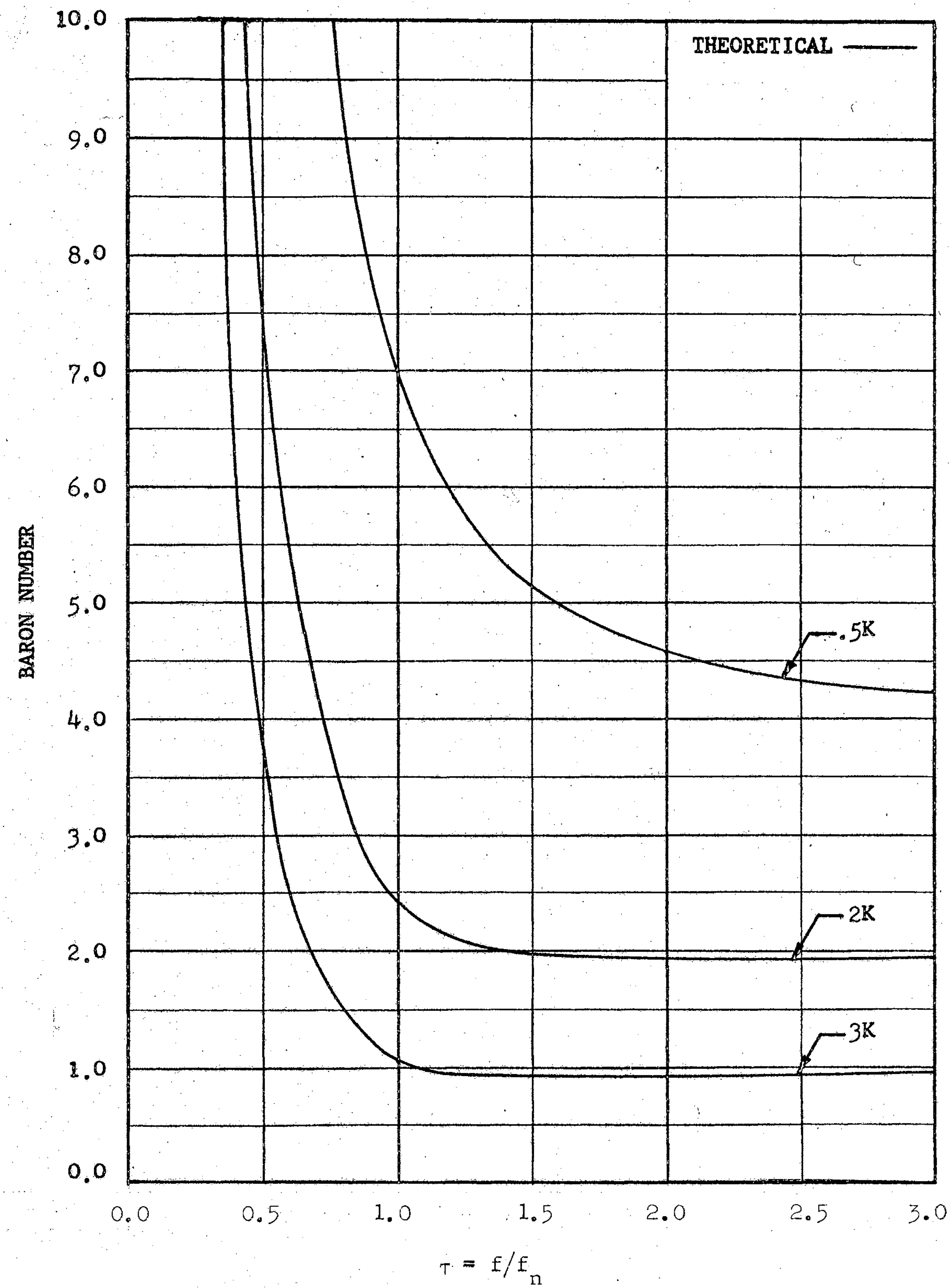


Figure 19. Baron Number for a System with Both Contacts Damped, $C = 1.0$, $M = 1.0$, $\zeta_1 = 0.75$.

of the curves of the Baron Number in Figures 18 and 19 are determined by the term $1/(X/S)$ as shown in Figure 3. When Figures 4, 18, and 19 are compared, it can be seen that by damping both contacts the Baron Number is increased above zero at $\tau = 1.0$. The amount of damping on the system determines how low the values of the Baron Number are at $\tau = 1.0$. This is shown by comparing Figures 18 and 19. In Figures 4, 18, and 19, the Baron Number for the same values of K is the same when $\tau = 3.0$. When Figures 18 and 19 are compared with Figures 10 and 12, respectively, it is apparent that the Baron Number is larger for each value of τ and K when CM = 1.0. The Baron Number is larger because the separation factor is constant when CM = 1.0. When CM = 0.0, the separation factor decreased as τ increased. Also, when CM = 1.0 both contacts are damped so that the total amount of damping on the system is increased. Thus, the term $1/(X/S)$ shown in Figure 3 has higher values than when CM = 0.0 and the total damping is less.

Figures 20 and 22 show the effect of varying the mass ratio, M, on the Baron Number when $\zeta_1 = 0.25$ and $\zeta_1 = 0.75$, respectively. The results of varying the mass ratio, M, on the separation factor is shown in Figure 21 for $\zeta_1 = 0.25$ and Figure 23 for $\zeta_1 = 0.75$. Even though C = 1.0, when M is varied then $CM \neq 1$; thus, the separation factor decreased as τ increased. With C = 1.0, varying M influenced both terms in the denominator of Equation (28) more than when C = 0.0. Therefore, the separation factor has a greater change as M is varied. This can be seen by comparing Figures 21 and 23 where C = 1.0 with Figures 16 and 17 where C = 0.0, respectively. Since the separation factor has a greater change as M is varied, likewise the Baron Number has a greater change as shown in Figures 20 and 22.

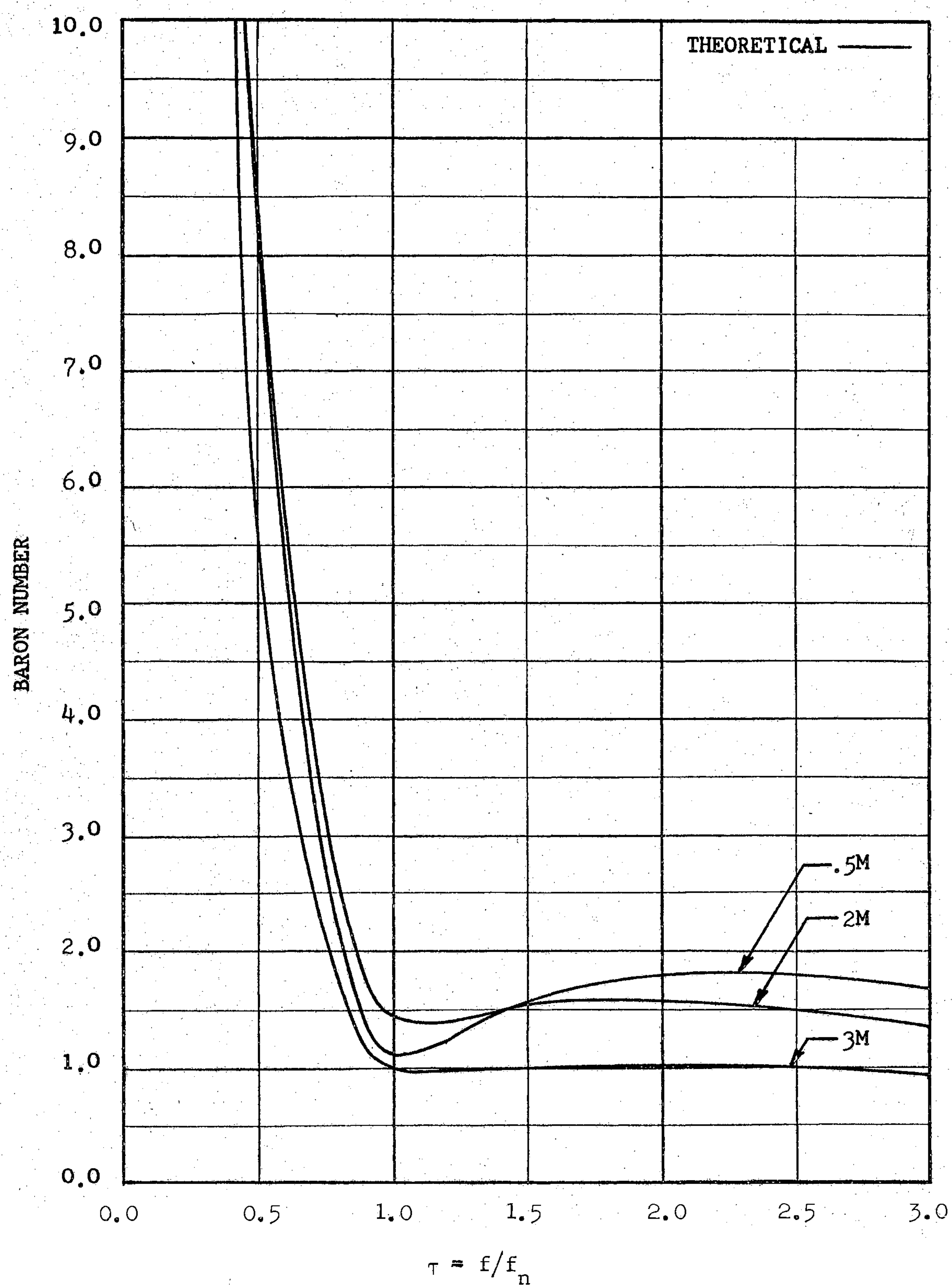


Figure 20. Baron Number for a System with Both Contacts Damped, C = 1.0, K = 1.0, $\zeta_1 = 0.25$.

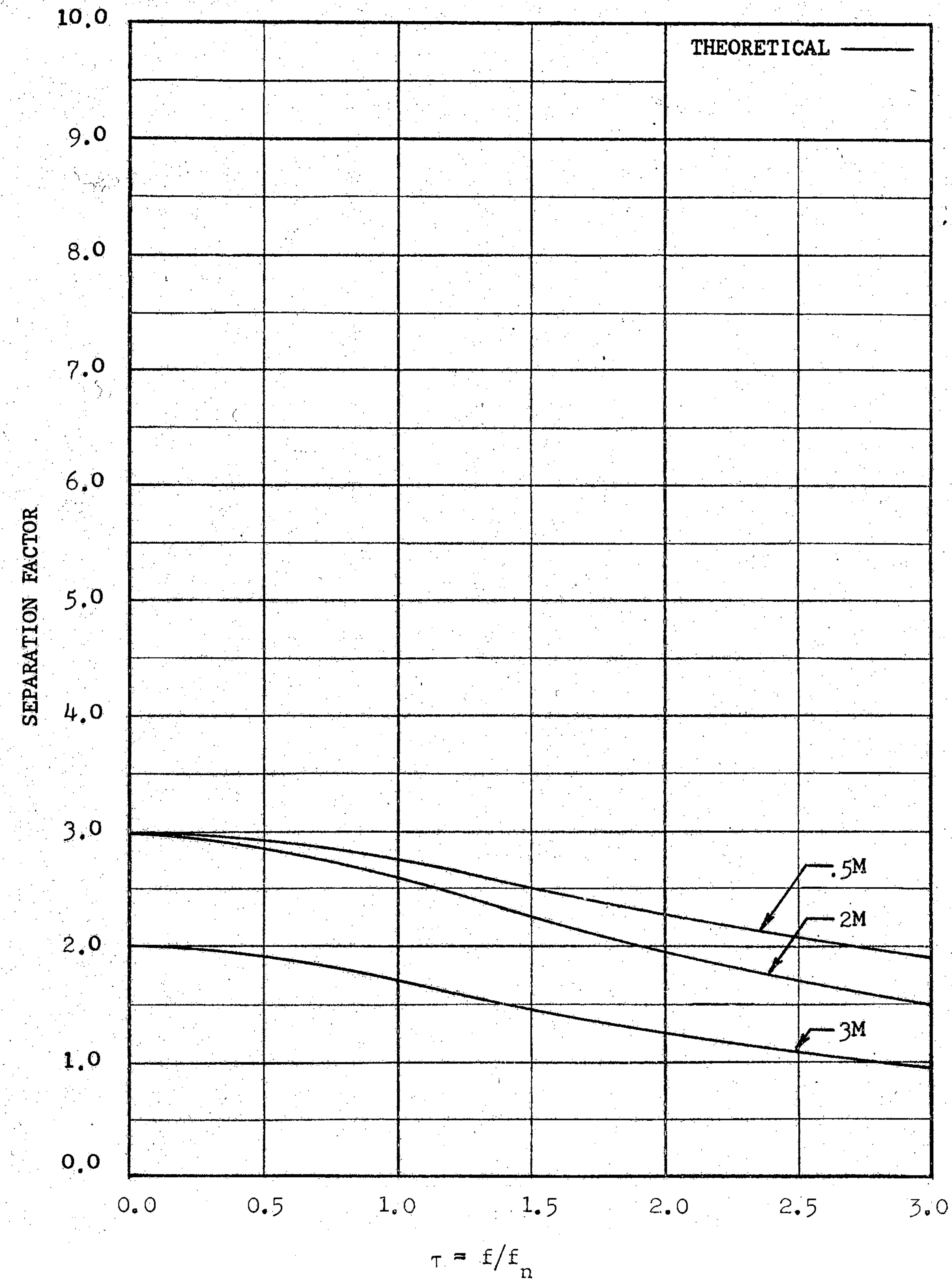


Figure 21. Separation Factor for a System with Both Contacts Damped, C = 1.0, K = 1.0, $\zeta_1 = 0.25$.

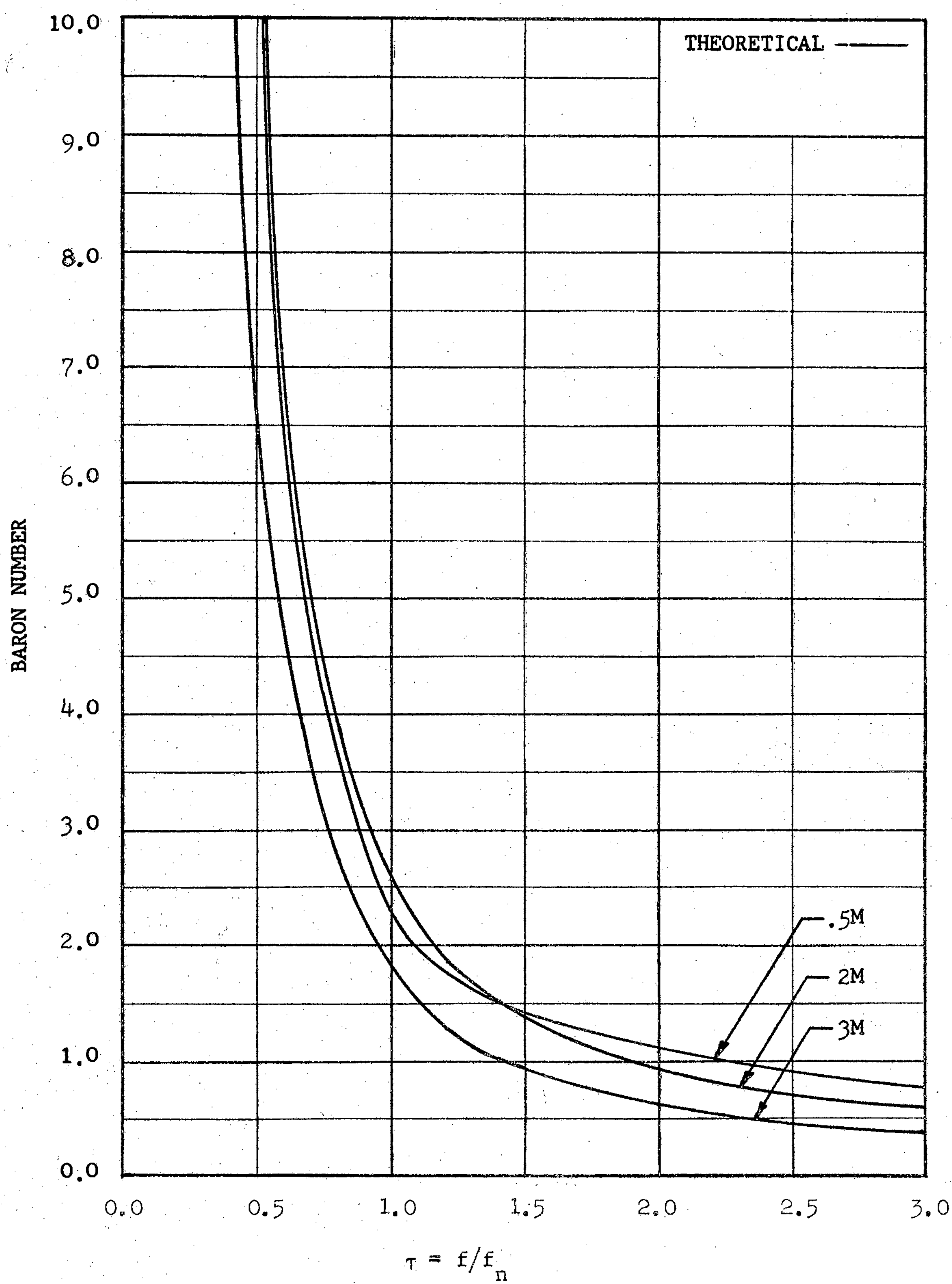


Figure 22. Baron Number for a System with Both Contacts Damped, C = 1.0, K = 1.0, ζ_1 = 0.75.

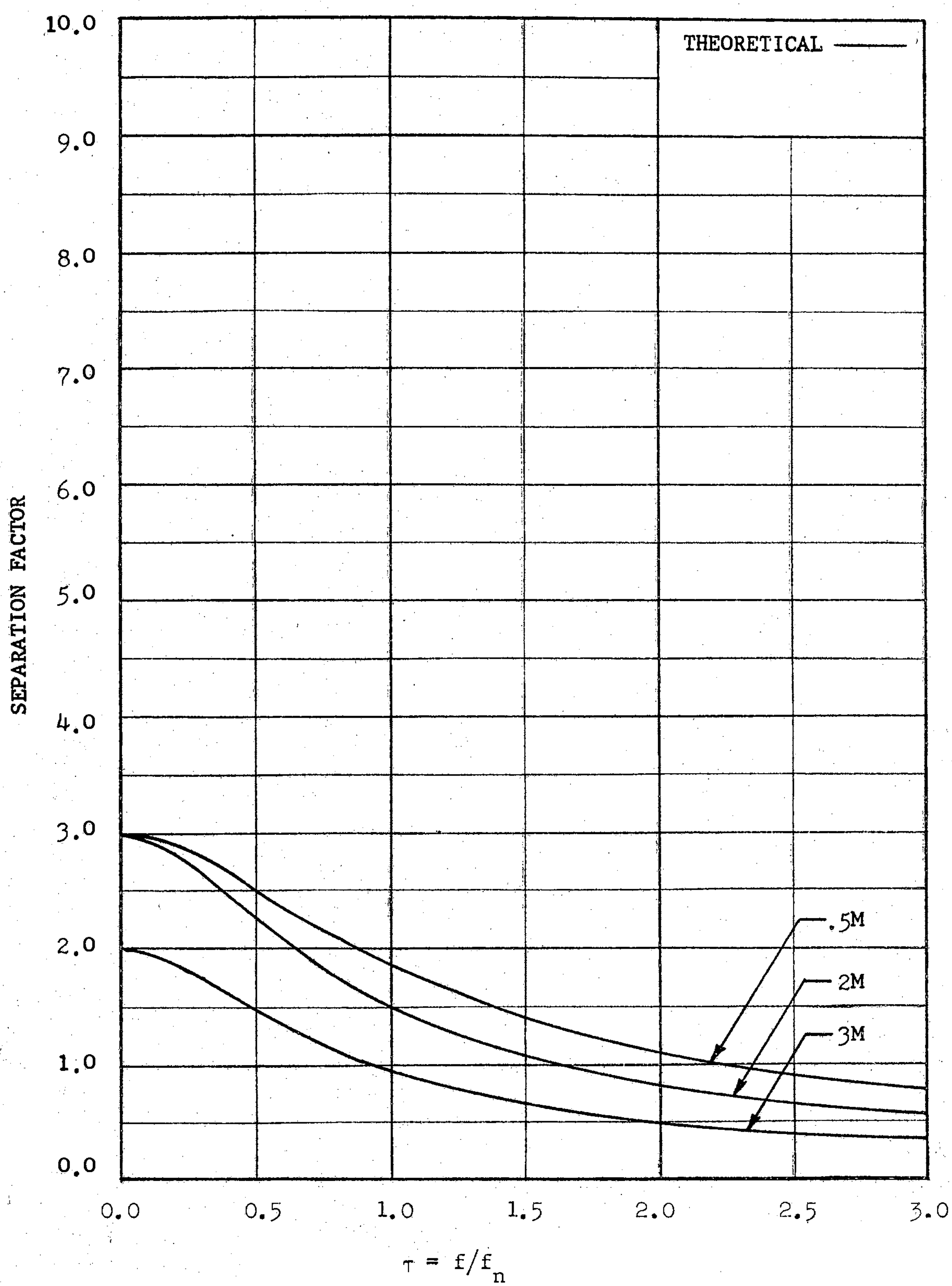


Figure 23. Separation Factor for a System with Both Contacts Damped, C = 1.0, K = 1.0, ζ_1 = 0.75.

The second case has both contacts damped and $C = 0.5$. When the value of the spring constant ratio is varied, the initial value of the separation factor at $\tau = 0.0$ is changed, as shown in Figures 11 and 13. Since $C = 0.5$ and $M = 1.0$, then $CM = 0.5$; therefore, as τ increases the separation factor decreases. The amount of the decrease is less than that shown in Figures 11 and 13 where $CM = 0.0$ because part of the term which indicates the amount of decrease is $(1 - CM)^2$. The results of varying K on the Baron Number when $C = 0.5$ is shown in Figure 24 for $\zeta_1 = 0.25$ and Figure 25 for $\zeta_1 = 0.75$. If Figure 24 is compared with Figure 10 and Figure 25 with Figure 12 at $\tau = 3.0$, it can be seen that the Baron Number is greater when $C = 0.5$ for a given value of K.

The effects of varying the mass ratio, M, on the Baron Number and the separation factor when $C = 0.5$ are similar to the case where $C = 1.0$. Variation in M causes both of the terms in the denominator of Equation (31) to change. The variation of M on the separation factor is shown in Figure 26 for $\zeta_1 = 0.25$ and in Figure 27 for $\zeta_1 = 0.75$. As seen in both Figures 26 and 27, the separation factor is constant when the mass ratio, M, increases to 2M and the term $(1 - CM)^2 = 0$. Figures 28 and 29 show the results of varying M on the Baron Number when $\zeta_1 = 0.25$ and $\zeta_1 = 0.75$, respectively. The Baron Number is seen to be largest for $\tau > 2.5$ when the value of the mass ratio, M, is 2M.

Summary

Three systems have been studied where damping is proportioned differently on each system. One system has only one contact damped, $C = 0.0$; another has both contacts damped with the damping coefficient on one

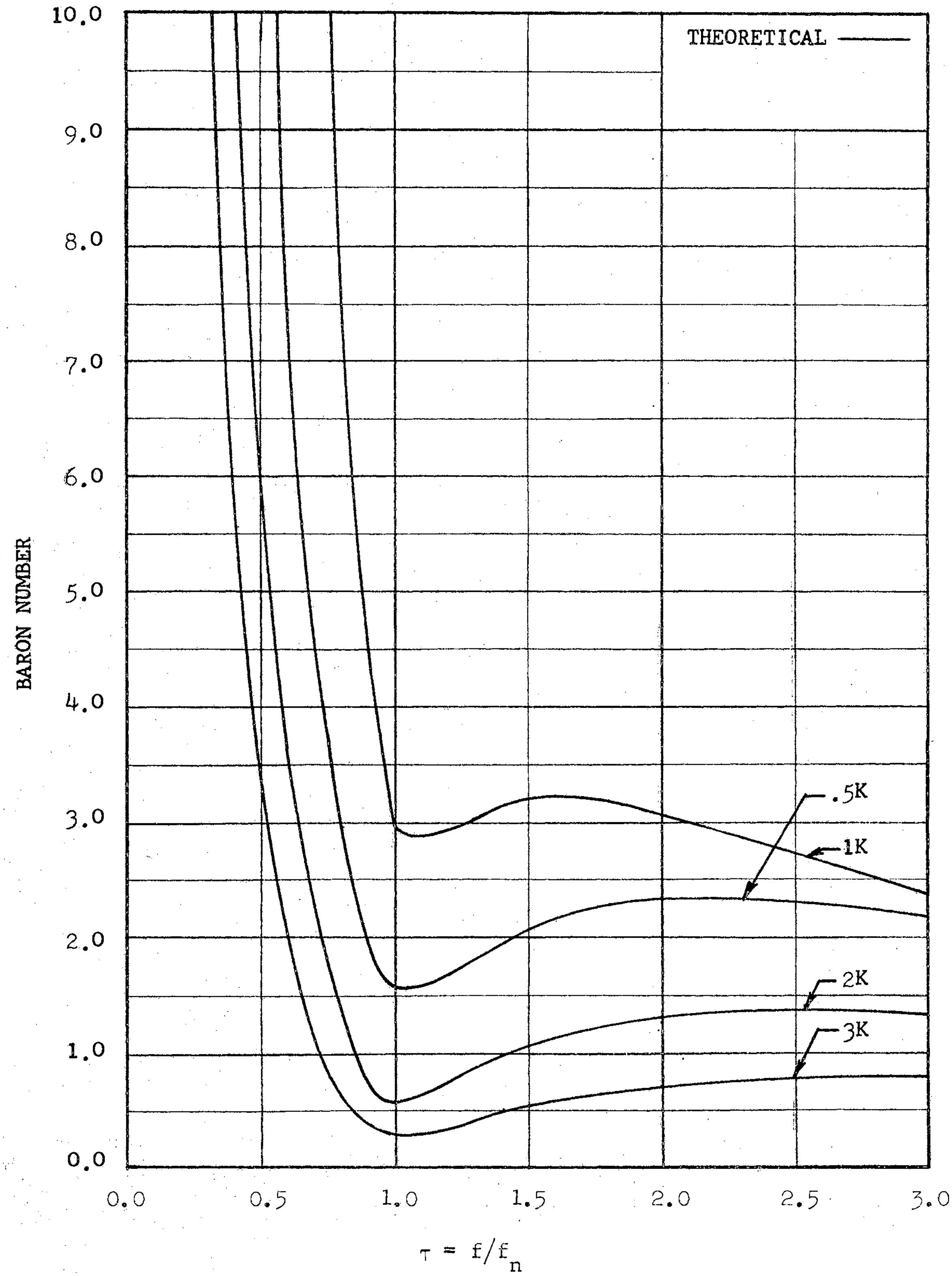


Figure 24. Baron Number for a System with Both Contacts Damped, $C = 0.5$, $M = 1.0$, $\zeta_1 = 0.25$.

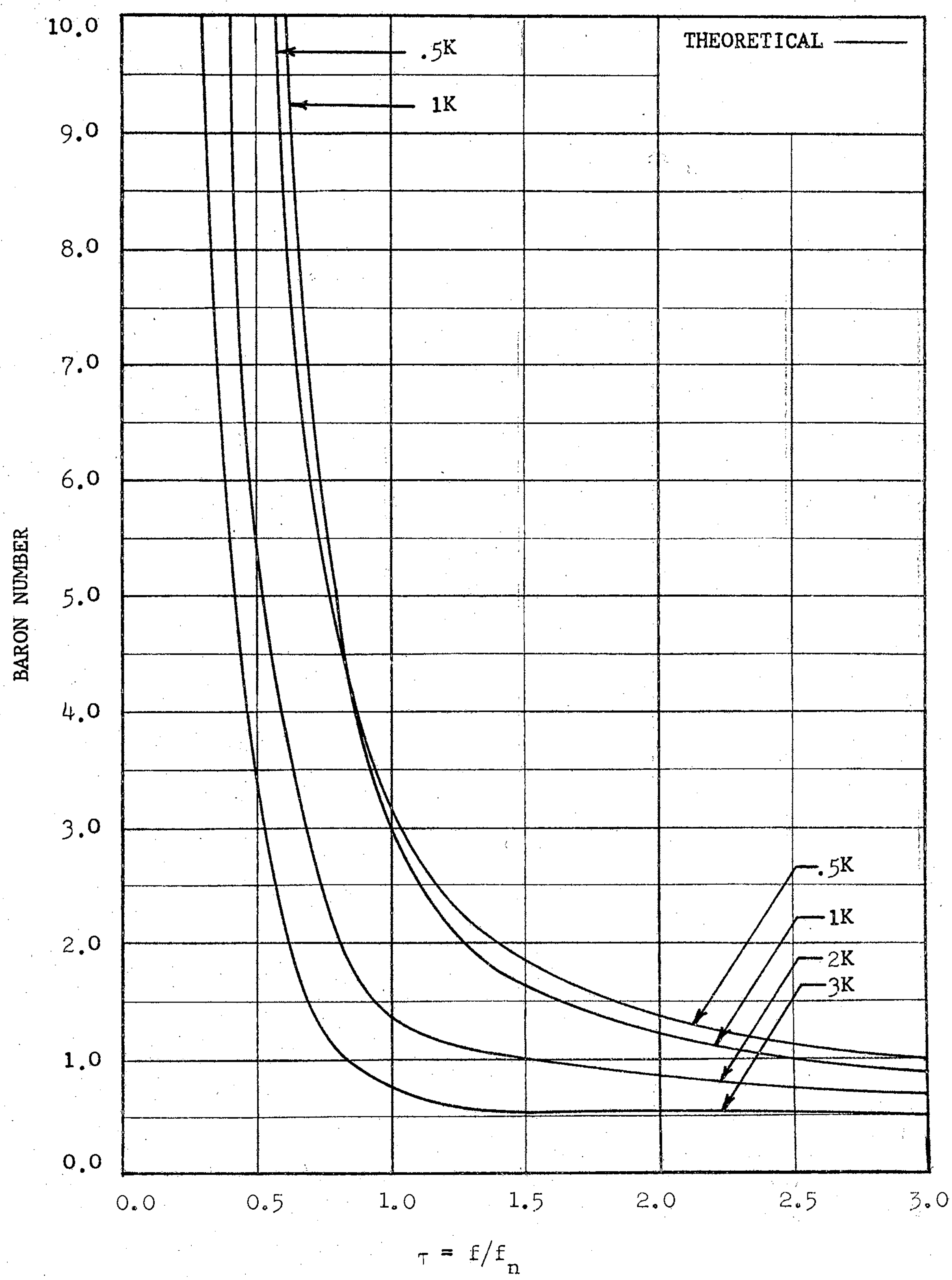


Figure 25. Baron Number for a System with Both Contacts Damped, $C = 0.5$, $M = 1.0$, $\zeta_1 = 0.75$.

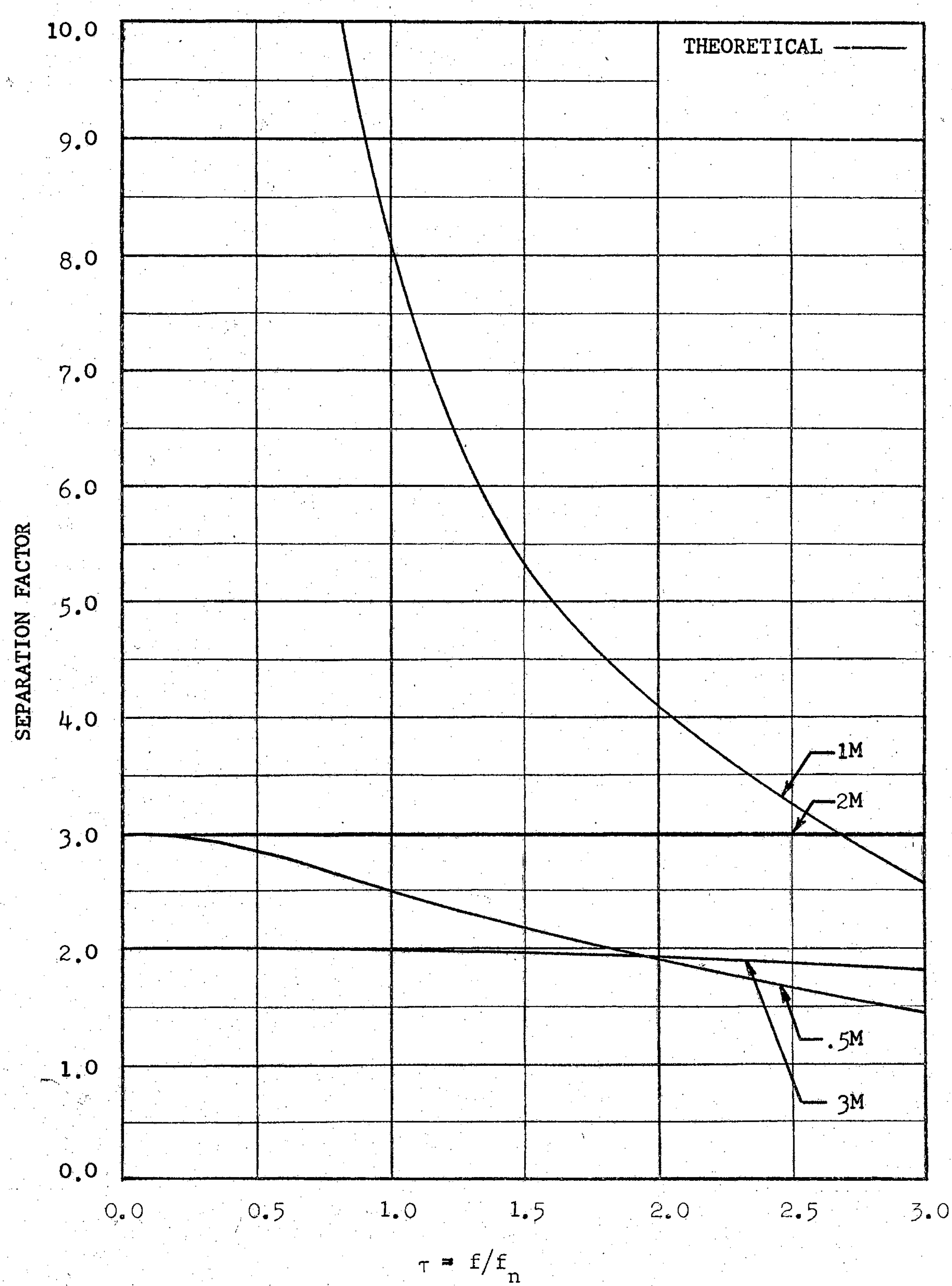


Figure 26. Separation Factor for a System with Both Contacts Damped, $C = 0.5$, $K = 1.0$, $\zeta_1 = 0.25$.

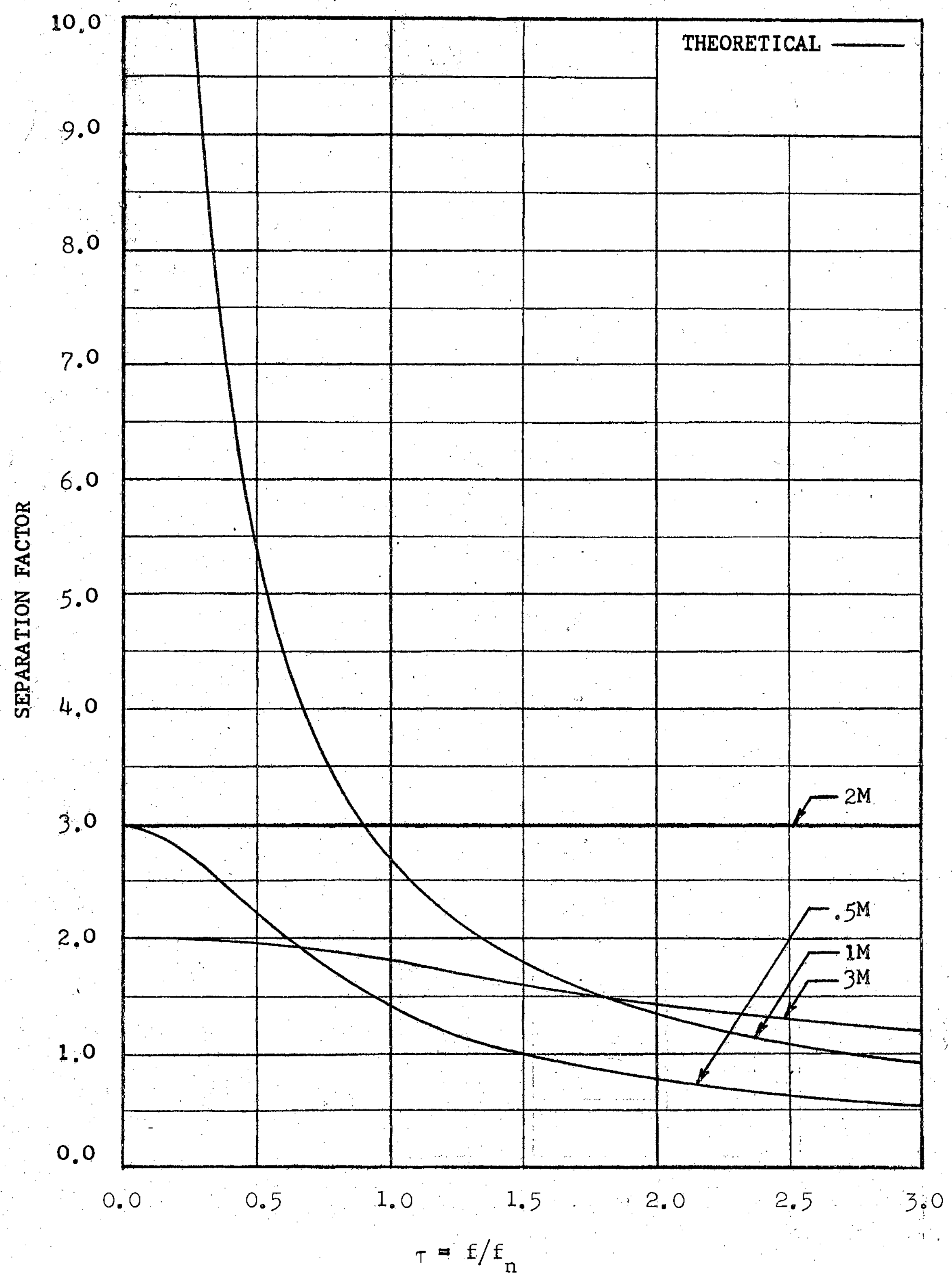


Figure 27. Separation Factor for a System with Both Contacts Damped, C = 0.5, K = 1.0, ζ_1 = 0.75.

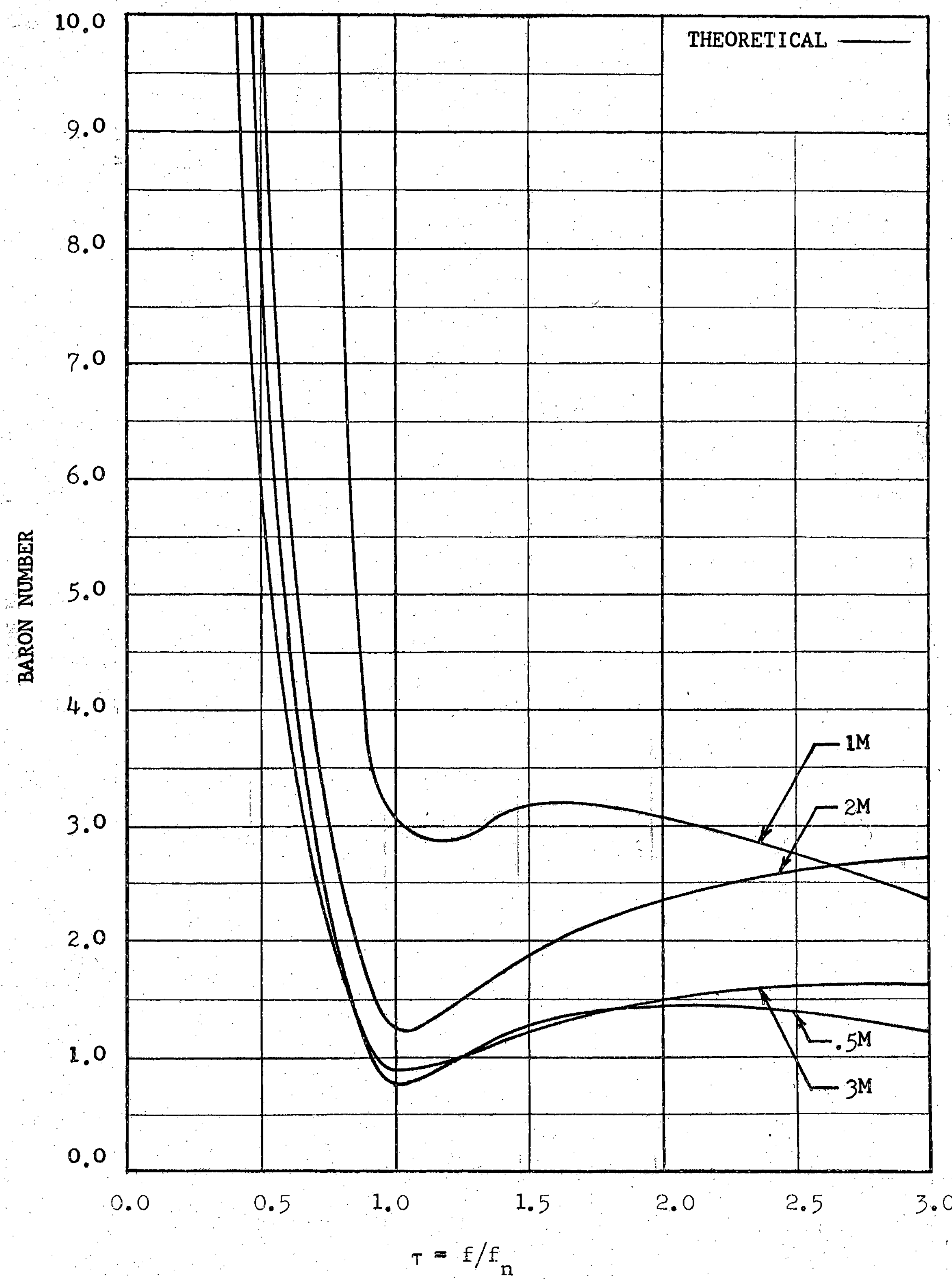


Figure 28. Baron Number for a System with Both Contacts Damped, C = 0.5, K = 1.0, ζ_1 = 0.25.

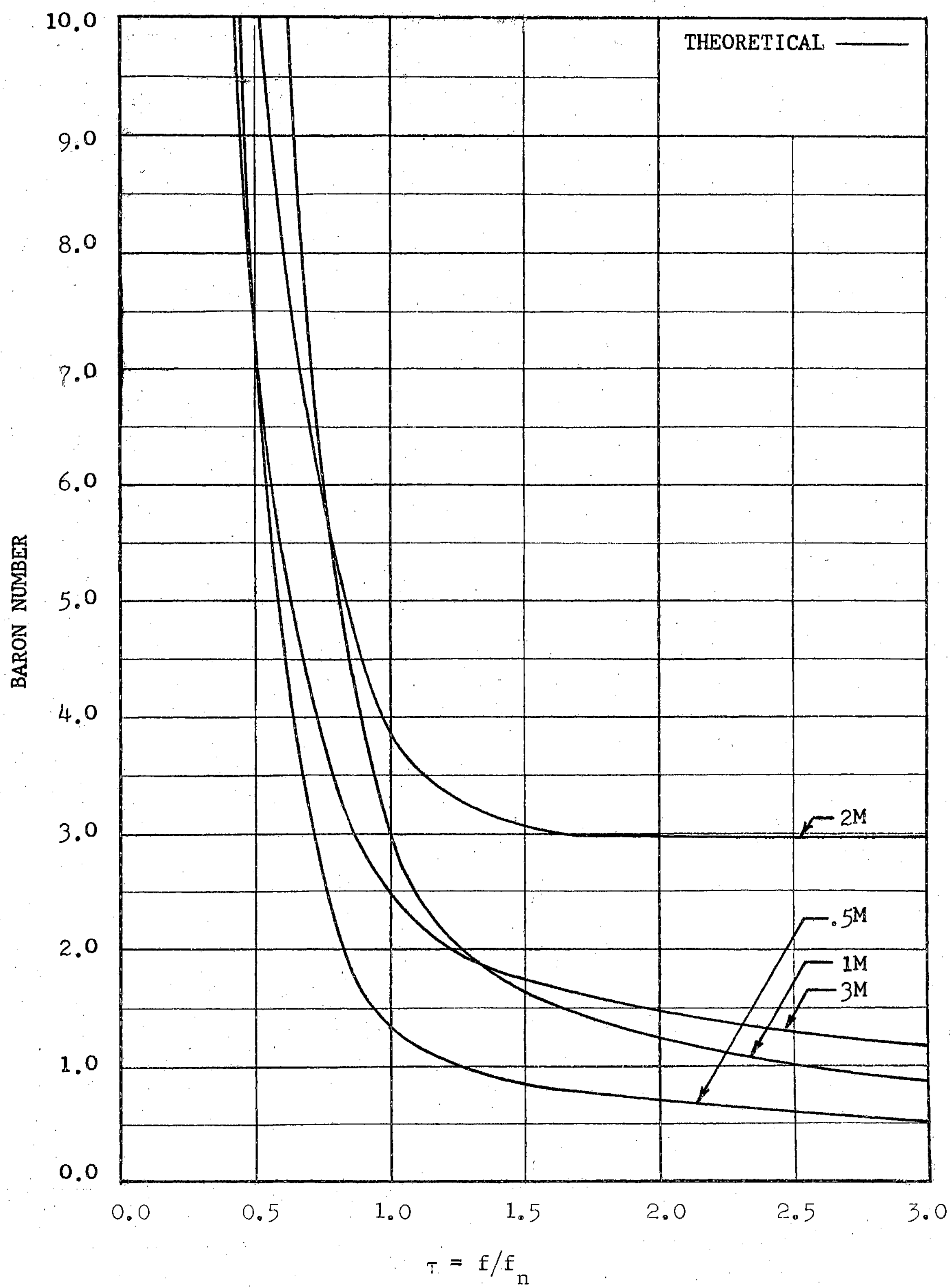


Figure 29. Baron Number for a System with Both Contacts Damped, C = 0.5, K = 1.0, ζ_1 = 0.75.

contact twice that on the other, C = 0.5; and the third system has both contacts damped with equal damping coefficients, C = 1.0. Also, an undamped system is considered. In each case the effects on the Baron Number of varying the spring constant ratio, K, and the mass ratio, M, are determined. With the use of these studies as references, general statements can be made on the contact separation criteria of these types of systems. Equation (30) should be considered for determining the undamped natural frequency of the system to obtain a complete analysis of the contact separation criteria. From the natural frequency and τ, the frequency of the exciting motion can be found from

$$f = (\tau)(f_n) , \qquad (32)$$

where f is the frequency of the exciting motion.

If the parameters of contact 1 are held constant, then any variation of the spring constant ratio, K, mass ratio, M, and damping coefficient ratio, C, are made by changing contact 2's parameters. Therefore, f_1 in Equation (30) is constant. For the undamped system Figures 4 and 6 indicate the Baron Number is always zero at $\tau = 1.0$, independent of the values of K and M. From Equation (29) at $\tau = 1.0$, it is apparent that there is no value for the input amplitude at which separation would not occur. Changing K or M would only change the exciting frequency at which separation would occur.

When contact 1 is damped, C = 0.0, Figures 10, 12, 14, and 15 all show that the Baron Number is never zero. Although varying the values of K, M, and ζ_1 did cause the Baron Number to either increase or decrease, it was never zero. Thus, from Equation (29) there always exists an input amplitude without separation. When $\zeta_1 = 0.75$ Figures 12 and 15 show that as τ increases the Baron Number decreases; thus, the input amplitude without separation is decreased. If consideration is given to the input acceleration without separation, it can be shown that the input amplitude at $\tau = 2.0$ need only be 1/4 the input amplitude at $\tau = 1.0$ to obtain the same acceleration level. Furthermore, the input amplitude at $\tau = 3.0$ need only be 1/9 the input amplitude at $\tau = 1.0$ to obtain the same acceleration level. And in no case was the Baron Number at $\tau = 3.0$ less that 1/9 its value at $\tau = 1.0$ for any damped condition.

For the two systems studied with both contacts damped, Figures 24 and 25 where $C = 0.5$ and Figures 18 and 19 where $C = 1.0$ indicate that increasing the value of K decreased the Baron Number but not as much as for the system with only one contact damped. When the mass ratio, M, is varied for the system with $C = 0.5$ and when the mass ratio, M, is 2M, the Baron Number decreases, as indicated in Figures 28 and 29, to the value of the separation factor as shown in Figures 26 and 27. Thus, when damping is applied to both contacts, the optimum value of C occurs where $CM = 1.0$ for any value of M.

Contact chatter can be delayed by properly designing damping into one or both contacts and by proper proportioning of contact masses and elasticity. The exciting motion amplitude necessary for chatter can be determined for either the damped or the undamped system by using the Baron Number.

A theoretical study of the amplitude for impending separation using equivalent viscous damping coefficients is presented in Appendix D. This study is only approximate and was not tested experimentally; therefore, it is not included in the main body of this study.

CHAPTER IV

EXPERIMENTAL MODEL AND INSTRUMENTATION

An experimental model was constructed so that the theoretical results could be substantiated by comparing them to experimental results. The design of the experimental model was predicated by many factors. The size of the model was limited by the electro-mechanical shaker system which produced the vibration environment. Yet, the model had to be large enough to permit adequate instrumentation without appreciably affecting its dynamic response.

Many different designs for a model were made and studied. The design from which the model was constructed offered the flexibility of changing the system parameters while limiting extraneous influences and effects. Spurious resonant conditions in the model were beyond the frequencies of interest, and side motion of the contact masses was minimized. The experimental model described in this chapter was the only model constructed.

Description of the Experimental Model

The model from which experimental results are obtained consisted of two helical springs, two viscous dampers, and two contact masses with guides. The contact guides supply additional mass to each contact. Figure 30 is a pictorial drawing of the model in which significant parts

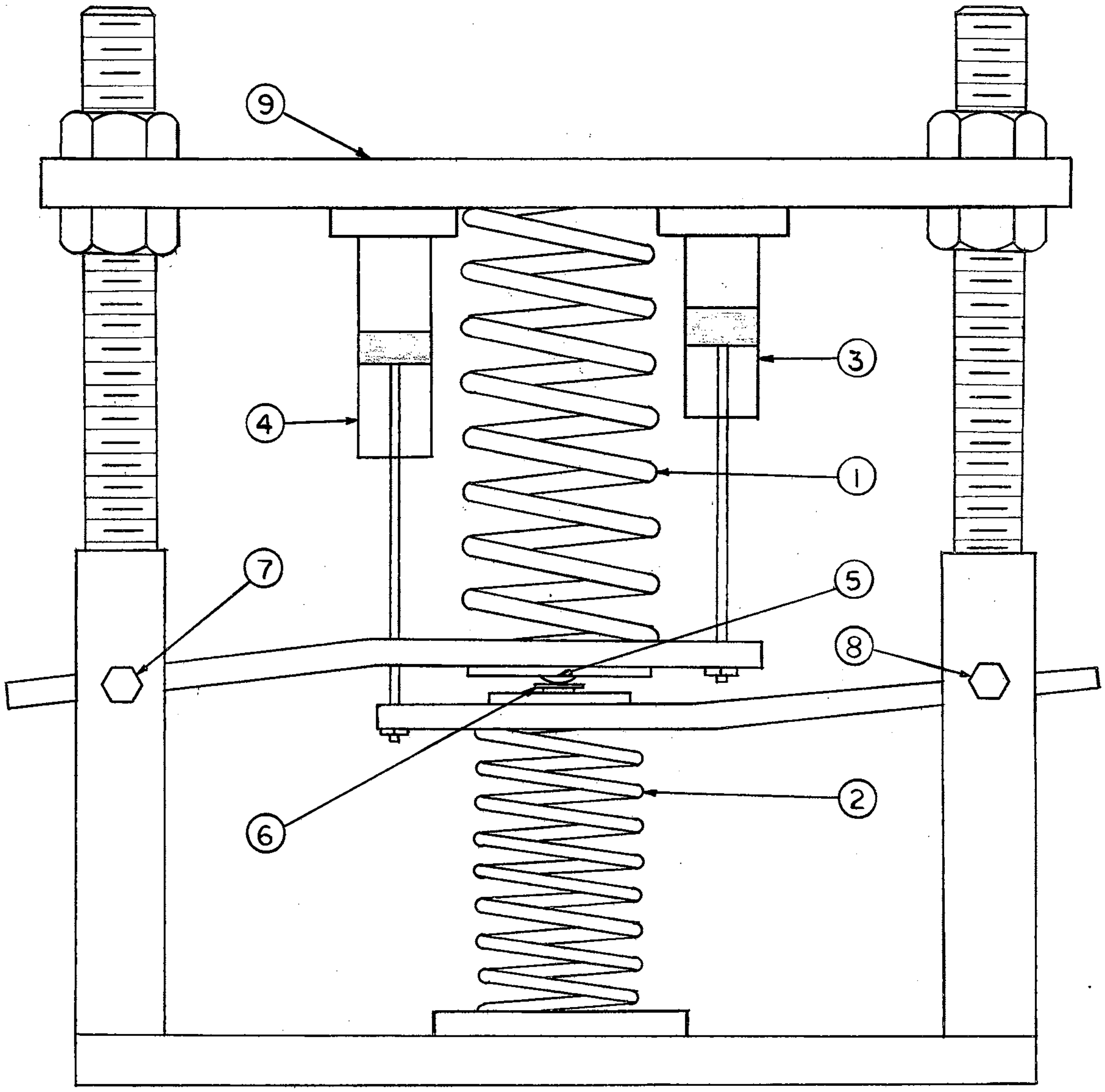


Figure 30. A Pictorial Drawing of the Model.

have been numbered and are referred to in the following description of the model.

The springs for the system, No. 1 for the upper contact and No. 2 for the lower contact, are both steel helical springs with linear restoring forces or spring constants. Dampers used on the system, No. 3 for the upper contact and No. 4 for the lower, are the type with a piston sliding in a glass cylinder with an oil film. The weight of the oil used in the cylinders along with an adjustable air leak determined the amount of damping. Friction in the dampers is negligible and the damping is assumed proportional to the first power of the velocity as indicated in the manufacturer's specifications for the dampers. The mass of each contact is a combination of the steel plate holding the contact button and the guide. A hemispherical steel button insulated from the steel plate to which it is attached is the contact for the upper contact, No. 5. The lower contact has a flat steel contact, No. 6, which is also insulated from the steel plate to which it is attached. Both contact guides are pivoted on grease-coated alignment screws approximately five inches from the contacts. Number 7 and No. 8 are the alignment screws for the upper and lower contacts, respectively. Preload for the system is varied by changing the position of the steel upper support, No. 9. The mass of the system is varied by simply adding mass to the contact guides.

Instrumentation

The instrumentation used on the experimental model is illustrated in the photograph, Figure 31, of the contact section on the model. Two accelerometers and a differential transformer are identified in Figure 31. Relative amplitude of the contacts is measured using the differential transformer which was calibrated for both static and dynamic measurements. Both of the accelerometers were calibrated with the calibration of the differential transformer for dynamic measurements. The accelerometers on the model are used, in addition to measuring accelerations, to observe the wave shape of both the input motion and contact

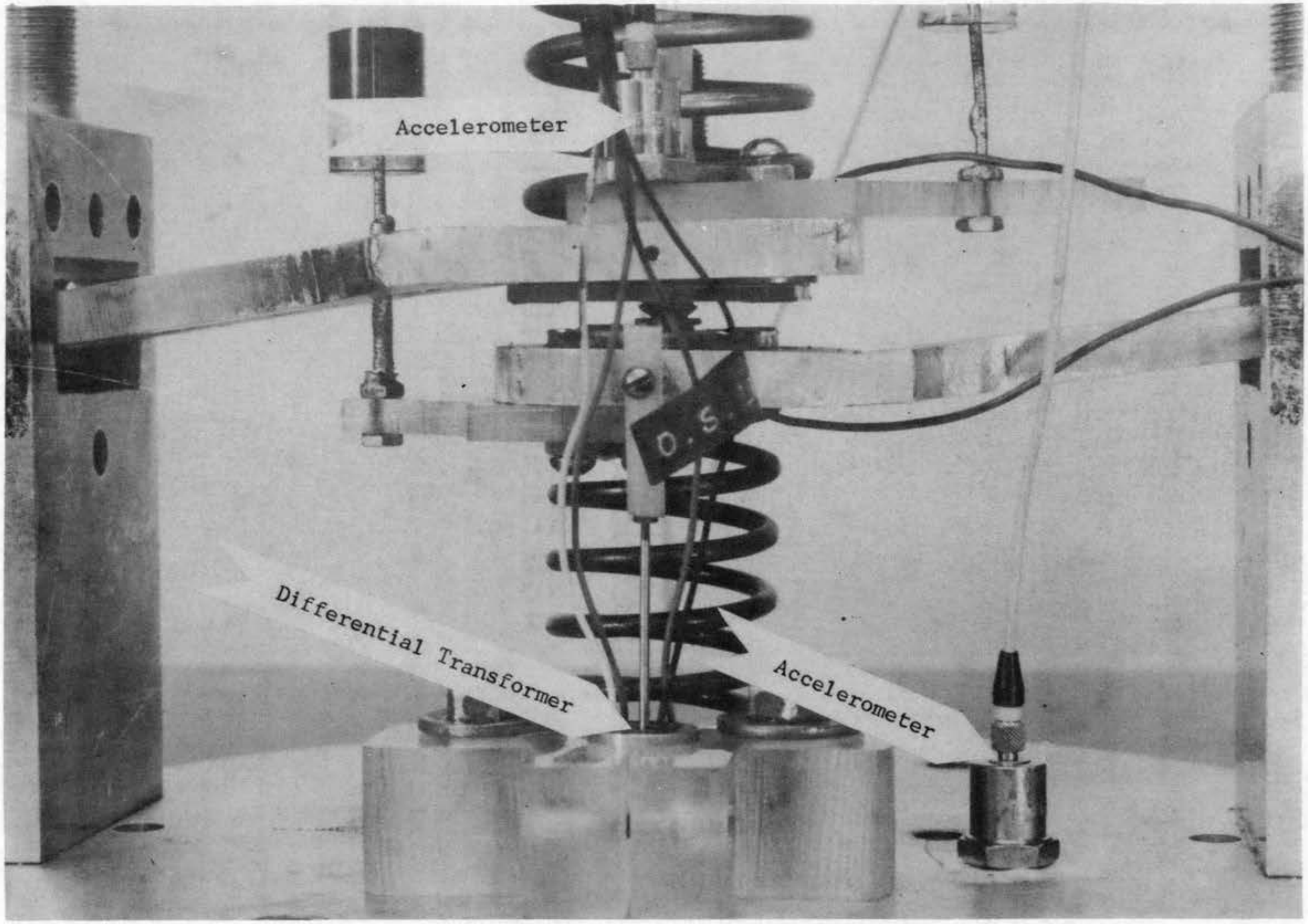


Figure 31. Instrumentation on the Experimental Model.

motion by connecting their outputs to a dual beam oscilloscope. Separation of the contacts is detected using a chatter tester set for its designed ten micro-second opening duration. Separation is considered to be occurring if the chatter tester indicated several consecutive ten micro-second openings. This type of separation detection is repeatable when successive tests on the experimental model are made.

Overall instrumentation used on the model and for collecting experimental results is shown in a block diagram of instrumentation, Figure 32. Instruments and motion sensitive devices are labeled in the block diagram.

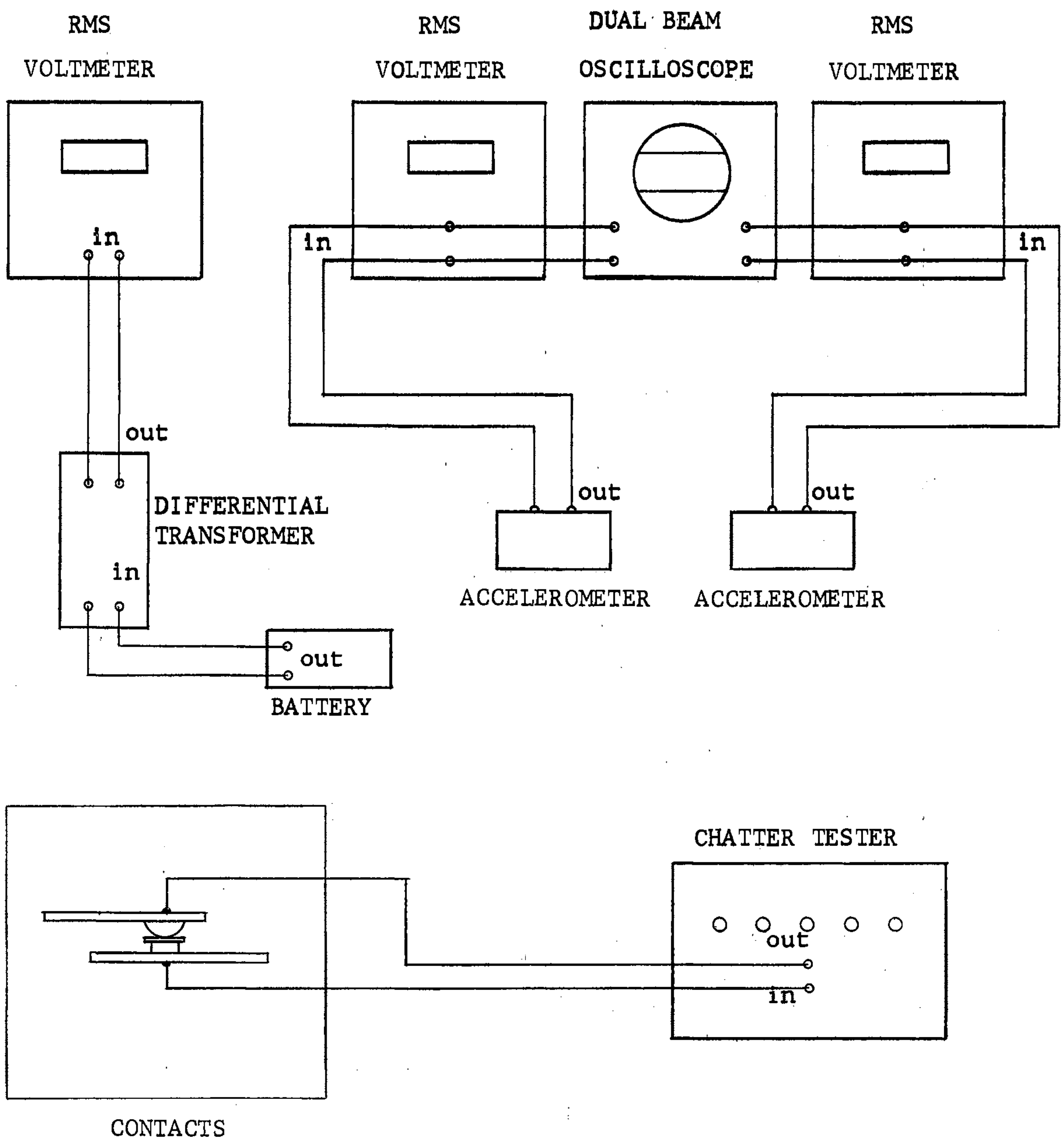


Figure 32. Block Diagram of Instrumentation.

CHAPTER V

EXPERIMENTAL PROCEDURE AND RESULTS

The validity of the equations developed is established by comparing experimental results with theoretical results. The equation for the relative response of a damped or undamped single degree of freedom system, published in numerous books on vibrations, is so well known that it has been deemed unnecessary to validate this equation. Also, the phase relation between the exciting motion and the response motion is an established relation and is not considered experimentally.

Parameters appearing in the theoretical equations are determined from the experimental model described in Chapter IV. Once the parameters on the model are ascertained, they are substituted in the theoretical equations. With the use of a digital computer, theoretical values are calculated for various value of τ, the ratio of the exciting frequency to the undamped natural frequency of the system.

Measurement of Model Parameters and Preload

The parameters associated with the experimental model, required for the solution of the various theoretical equations, are k_1, k_2/k_1, m_1/m_2, c_1, c_2/c_1, and F_o. This list of parameters, excluding F_o, is merely the physical properties of one contact and its comparison with the other.

Spring constants k_1 and k_2 are found by measuring the deflection of the springs under various loads. The loads are applied by using weights, and deflections are obtained using the differential transformer. The ratio of the differential transformer outputs for equal loads on each spring is used as the ratio of the spring constants.

Undamped natural frequencies of the two contacts, f_1 and f_2, and that of the combined contacts, f_{12}, are the frequencies of the exciting motion when maximum response is obtained with a minimum exciting motion. The frequency f_{12} is also the undamped natural frequency f_n. The natural frequencies, f_1 and f_2, along with the spring constant ratio are employed to find the mass ratio from the relation

$$\frac{m_1}{m_2} = \left(\frac{f_2}{f_1}\right)^2 \frac{k_1}{k_2} .$$

Values of the mass ratio, m_1/m_2, and of the spring constant ratio, k_2/k_1, are checked by comparing the values obtained as indicated with the values found by the relations

$$\frac{m_1}{m_2} = \frac{(f_2/f_{12})^2 - 1}{1 - (f_1/f_{12})^2} ,$$

and

$$\frac{k_2}{k_1} = \frac{(f_{12}/f_1)^2 - 1}{1 - (f_{12}/f_2)^2} .$$

Values of the mass and spring constant ratios from these two independent methods of solution are comparable in every case.

The force, F_m, required to separate the contacts in the static condition, is measured in order to determine the preload, F_o, of the system. The preload is then calculated from the relationship between the measured force, F_m, and the spring constant ratio which is

$$F_o = \frac{F_m}{1 + k_2/k_1} .$$

This equation for F_o is valid when F_m is the force applied to contact 2 until separation occurs. If F_m is the force applied to contact 1, then

$$F_o = \frac{F_m}{1 + k_1/k_2} .$$

When damping is applied to the system, the values of the damping coefficients, c_1 and c_2, for each contact are determined. The method for determining the damping coefficient of a contact is to measure the response amplitude and exciting motion amplitude of the damped contact at the undamped natural frequency; the damping coefficient is then calculated by the following equation:

$$c_i = \frac{k_i}{\left(\frac{X}{S}\right)_i \omega_i} .$$

c_i is the damping coefficient, k_i the spring constant, ω_i the undamped circular natural frequency, and $(X/S)_i$ the ratio of the response amplitude to the exciting motion amplitude for the contact.

Dynamic Response Measurements of the Model

The amplitude of the sinusoidal excitation or input motion is measured by applying an accelerometer to the base of the model. Output voltage from the accelerometer is read on a RMS voltmeter and converted to acceleration or displacement. An accelerometer is attached to the upper contact [see Figure 31], so that its response could be measured when the contacts are held apart. Output of this accelerometer is measured in the same manner as the output of the accelerometer on the base of the model. The accelerometer on the upper contact is used to determine its undamped natural frequency and its damped or undamped response. The differential transformer is attached to the lower contact [see Figure 31]. Although the output of the differential transformer is a d.c. voltage, when the lower contact vibrates sinusoidally, the output of the differential transformer becomes a sinusoidal varying d.c. voltage. Thus, in a sinusoidal environment the differential transformer output is read on a RMS voltmeter and converted to displacement using its calibration factor. One use of the differential transformer is to determine the undamped natural frequency of the lower contact and its damped or undamped response when the contacts are held apart.

The differential transformer is also used to measure the relative amplitude of impending contact separation. Again, the output of the differential transformer is a varying d.c. voltage and is read on a RMS voltmeter. The amplitude of impending separation is determined by exciting the system at a certain frequency and increasing the exciting motion amplitude until contact chatter is detected.

The phase relation between the location where $F(t) = 0.0$ and the relative response amplitude is determined by visually observing the response and chatter on a dual beam oscilloscope. The d.c. voltage across the contacts and the output of the differential transformer are connected to the oscilloscope. Prior to chatter, one beam on the scope is a straight line for the d.c. voltage across the contacts. The other beam is the sinusoidal wave for the relative response from the differential transformer at a certain frequency. When the exciting motion amplitude is increased to the point of impending separation, the scope beam for the voltage across the contacts has intermittent changes. The location of these intermittent voltage changes across the contact, relative to the peaks of the sinusoidal wave, indicates the phase difference between the response amplitude and the location where $F(t) = 0.0$.

Experimental Results

Experimental tests are made on the model for essentially three different conditions: undamped, one contact damped, and both contacts damped. In each case the experimental results are compared with those predicted by the theoretical equations. No effort is made to explain the scatter of data points for successive tests on the experimental model for a given set of parameters. When consideration is given to instrument readability and the possible error associated with repeating the same frequencies and vibration environment the scatter of data is not excessive. A general explanation for the deviation of the data points from the theoretical values, when the system is damped, is that the measured damping coefficients or the type of damping substituted into the theoretical equation is in error. Nevertheless, the trend of the measured values follows that of the theoretically predicted values. It was found that if the amount of damping is increased by merely tightening the adjustable air leak on the damper, instead of simultaneously changing the weight of the oil used to form the oil film, the damped natural frequency does not change from the undamped natural frequency. The condition, where the damped natural frequency does not change from the undamped natural frequency, is not observed when the adjustable air leak is loose or moderately tight. This indicates that if the adjustable air leak is tight the compressibility of the air is adding elasticity to the system. Furthermore, forcing the air through the small orifice could cause the damping to be a combination of viscous damping and damping proportional to the square of the velocity. In Appendix D the approximate solution of the normalized separation amplitude indicates that for damping proportional to the square of the velocity, $n = 2$, the normalized separation amplitude at large values of τ is less than that for the viscous damping, $n = 1$. These results suggest that if the data points are below the predicted values at large values of τ then the damping is not completely viscous as assumed. If the data points are consistently above or below the predicted values, then not only is there possible error in the measured value of the damping coefficients, but there may be error in the measured values of other parameters associated with the contact system.

Experimental Variations of the Preload

For the undamped condition the equation of the amplitude for impending separation indicates the separation amplitude is independent of the frequency and directly proportional to the preload. Figures 33 and 34 show the results of two of the tests made on the undamped model. In both tests the spring constant ratio equaled 1.2 and the mass ratio, M,

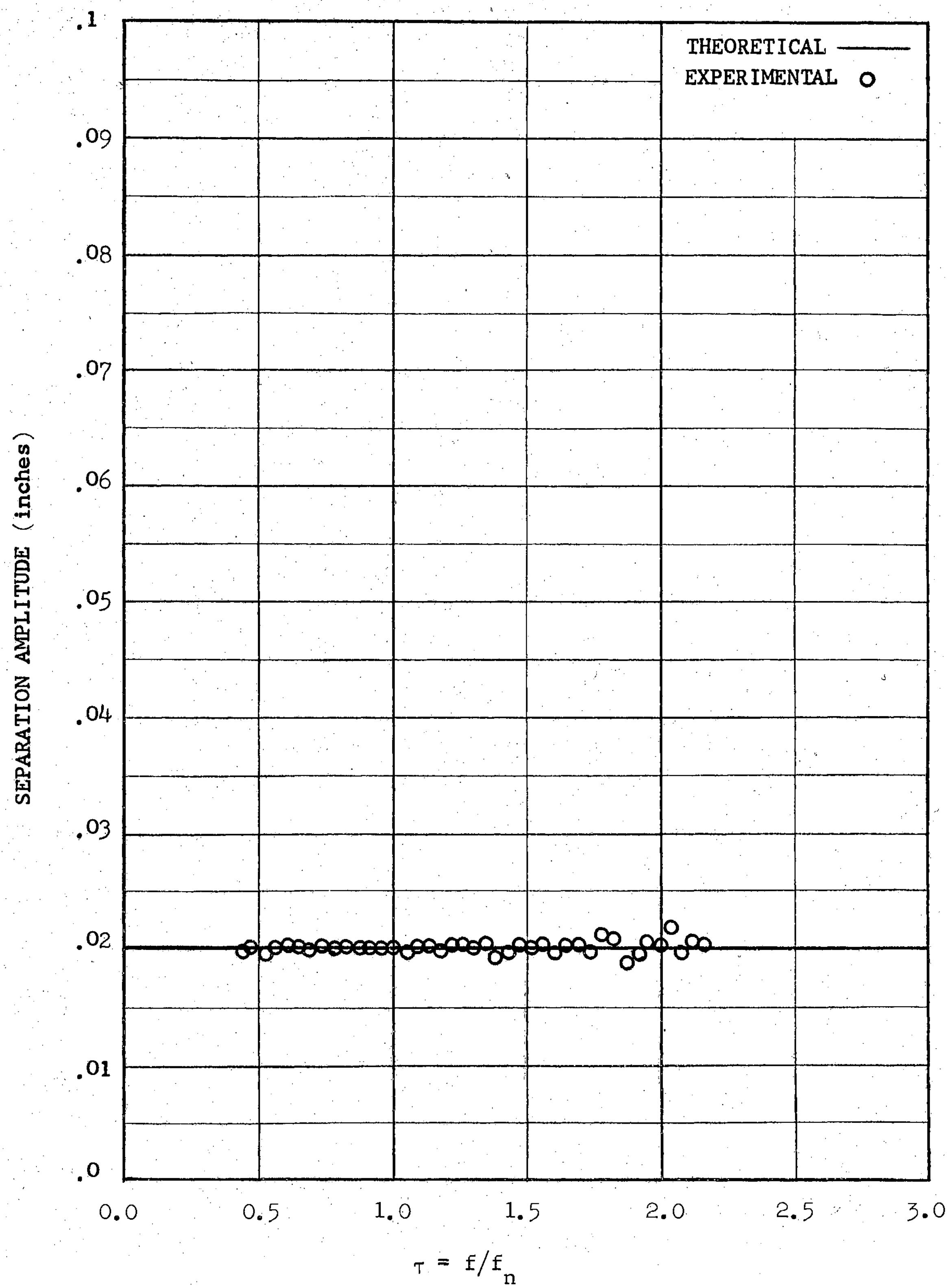


Figure 33. Amplitude for Impending Separation, C = 0.0, K = 1.2, M = 1.57, ζ_1 = 0.0, F_o = 0.23 lbs.

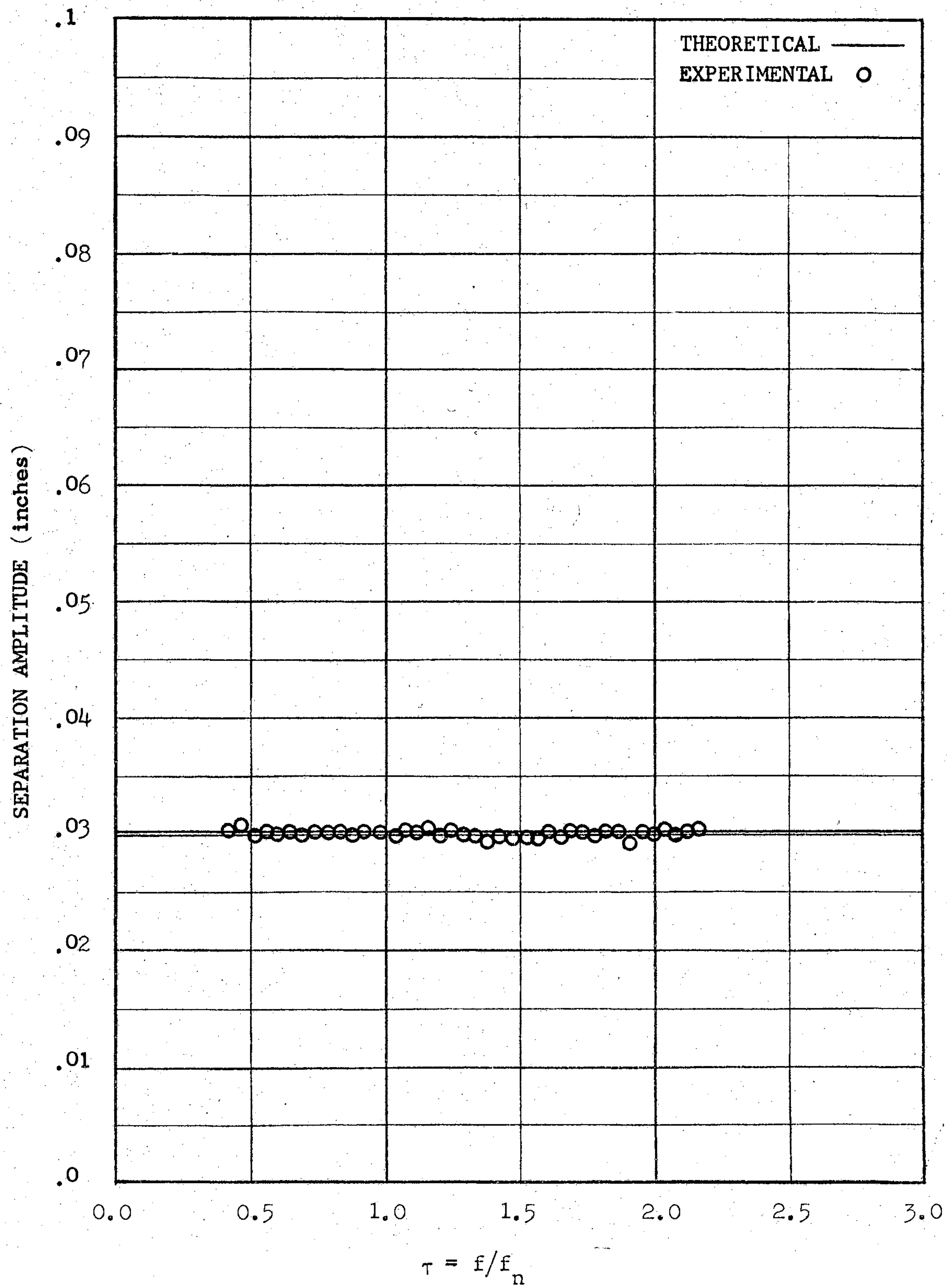


Figure 34. Amplitude for Impending Separation, $C = 0.0$, $K = 1.2$, $M = 1.57$, $\zeta_1 = 0.0$, $F_o = 0.34$ lbs.

equaled 1.57. Only the preload is changed. The natural frequency of the system is 23 cps.

With one contact damped the amplitude for impending separation is a linear function of the preload but is no longer independent of the frequency. Also, the separation amplitude is a function of the amount of damping on the one contact and is decreased as the value of τ increases. The effects of damping one contact are considered in more detail later and only the influence of the preload and frequency on the one damped contact are of interest here. Figures 35 and 36 show the results of two tests on the model when $k_2/k_1 = .806$ and $m_1/m_2 = .716$. The damping coefficient of the damper and preload are varied in each test. At $\tau = 0.0$ the effects of different preloads are best illustrated.

When both contacts are damped, the separation amplitude is still a linear function of the preload but may or may not be a function of the frequency depending on how the damping is proportioned on the system. Effects of damping both contacts are considered in more detail later and only the influence of the preload is of interest here. Figures 37 and 38 show the results of two test on the model when $k_2/k_1 = .806$ and $m_1/m_2 = .716$. The preload is varied in each case along with the amount of damping on the system.

From the results of Figures 33 through 38 it can be seen that increasing the preload increases the initial value of the separation factor. The effect of adding damping to the system causes the separation factor to change as τ increases if the damping is not proportioned correctly on the system.

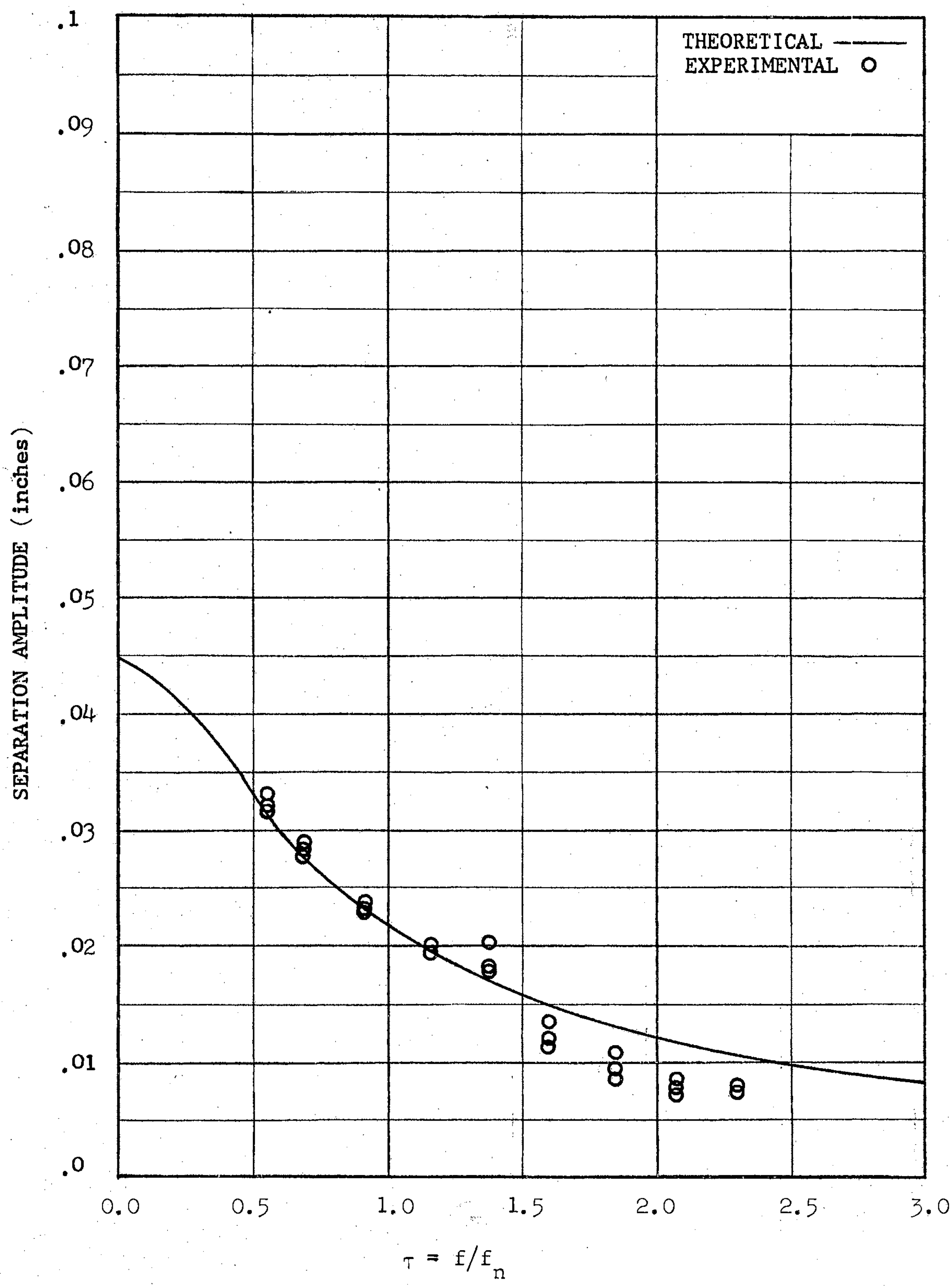


Figure 35. Amplitude for Impending Separation, C = 0.0, K = 0.806, M = 0.716, ζ_1 = 0.45, F_o = 0.46 lbs.

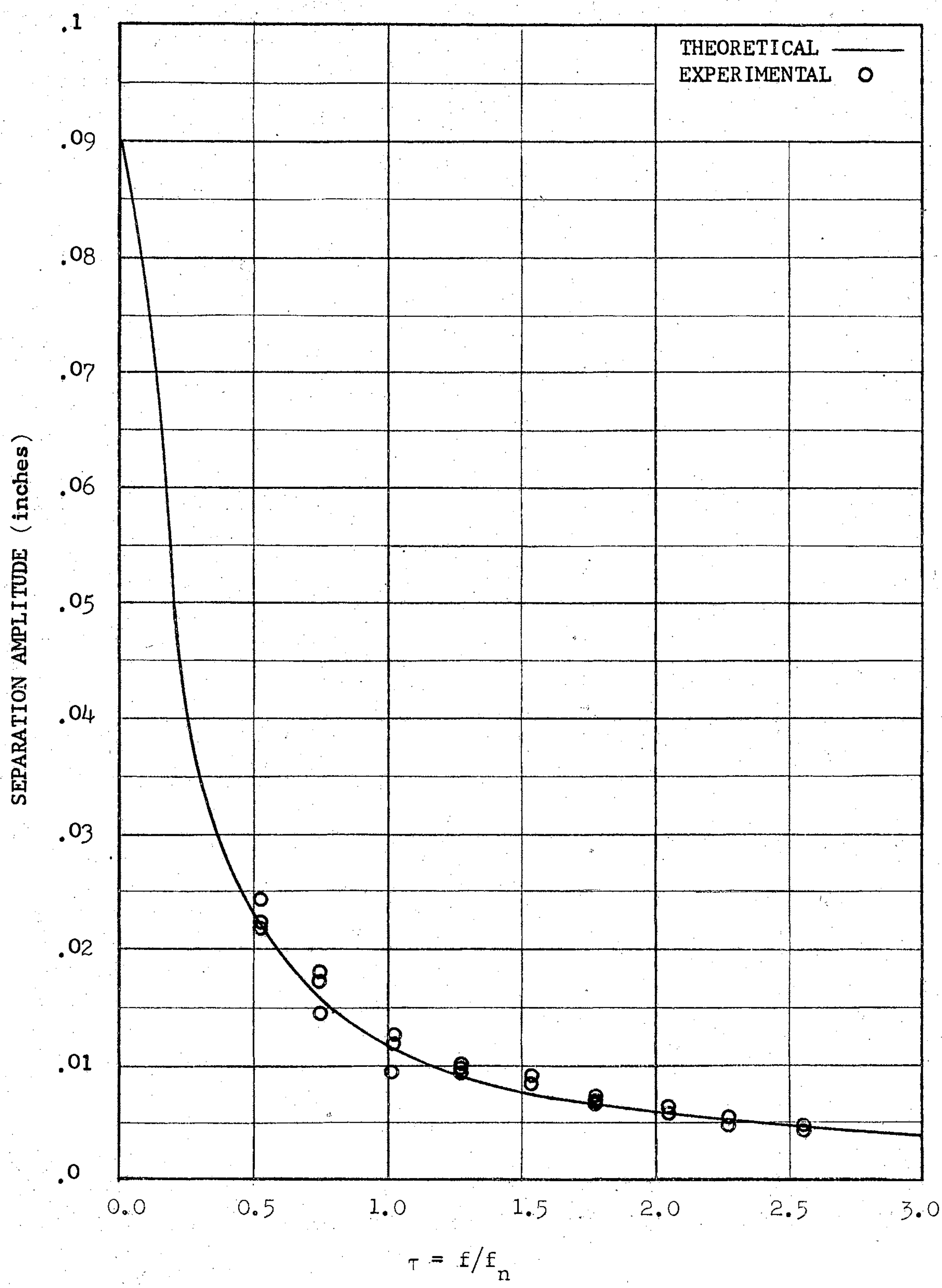


Figure 36. Amplitude for Impending Separation, C = 0.0, $K = 0.806$, $M = 0.716$, $\zeta_1 = 2.12$, $F_o = 0.893$ lbs.

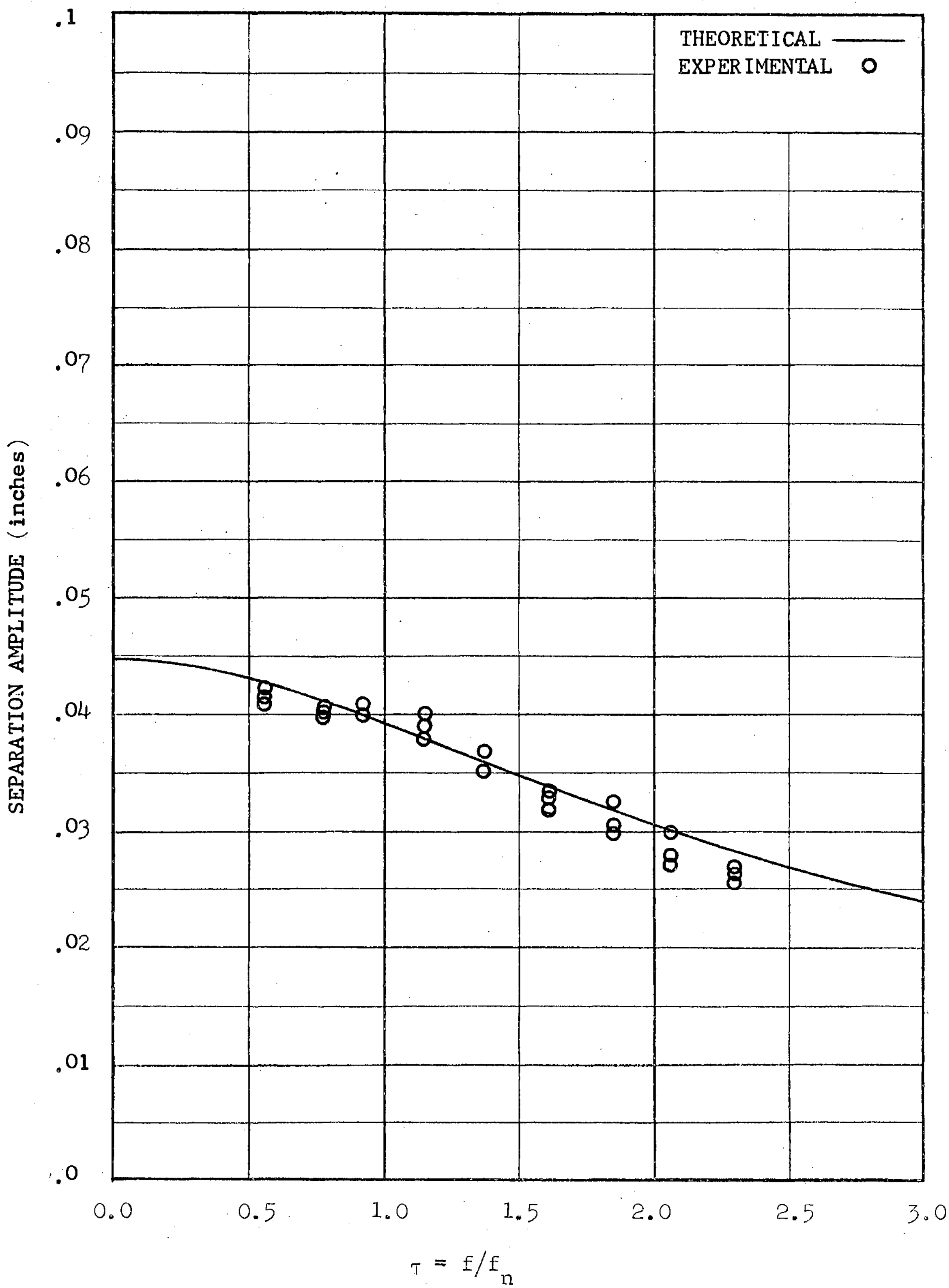


Figure 37. Amplitude for Impending Separation, $C = 2.85$, $K = 0.806$, $M = 0.716$, $\zeta_1 = 0.393$, $F_o = 0.46$ lbs.

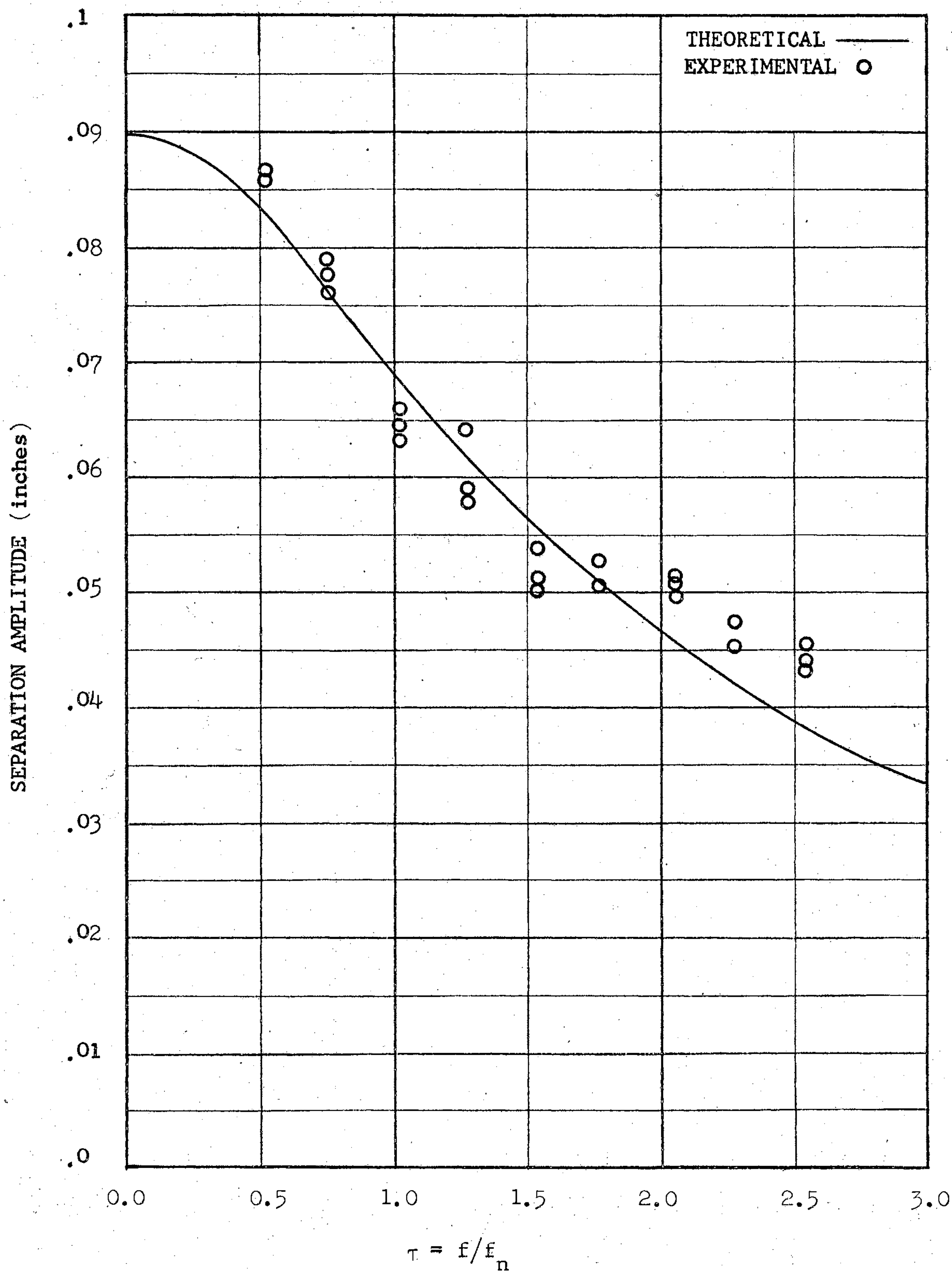


Figure 38. Amplitude for Impending Separation, $C = 0.358$, $K = 0.806$, $M = 0.716$, $\zeta_1 = 0.31$, $F_o = 0.893$ lbs.

Experimental Variations of the Damping

Tests were made on the model with various amounts of damping in order to more clearly illustrate the effects of added damping to one contact on the separation amplitude. Since preloads, mass ratios, M, and spring constant ratios are not held constant for each test, the results are normalized. Results of tests are normalized by dividing the separation amplitude at each value of τ by the value of the separation amplitude at $\tau = 0.0$. This makes the normalized separation amplitude start at unity for $\tau = 0.0$ and vary accordingly to $\overline{X}/\overline{X}_{\tau = 0.0}$. Figure 39 shows the plot of six normalized tests for which the amount of damping is different for each test. It can be seen that when one contact is damped, the separation amplitude decreases as τ increases. The greater the amount of damping, the more the separation amplitude decreases.

When both contacts are damped, the amount of damping does not affect the separation amplitude so much as the way the damping is proportioned on each contact. Again, the results to be compared have been normalized as indicated before. The total amount of damping on the system is the sum of c_1 and c_2. Figure 40 shows the results of two different tests on the model where the damping is proportioned on each contact in different amounts. The test with the largest amount of total damping has the highest normalized amplitude for separation compared with the test with less total damping. In either case the normalized separation amplitude is larger than that for a system with the same amount of damping on only one contact. Figures 41 and 42 show the results of three tests where damping is the only parameter changed. The preload for the results shown

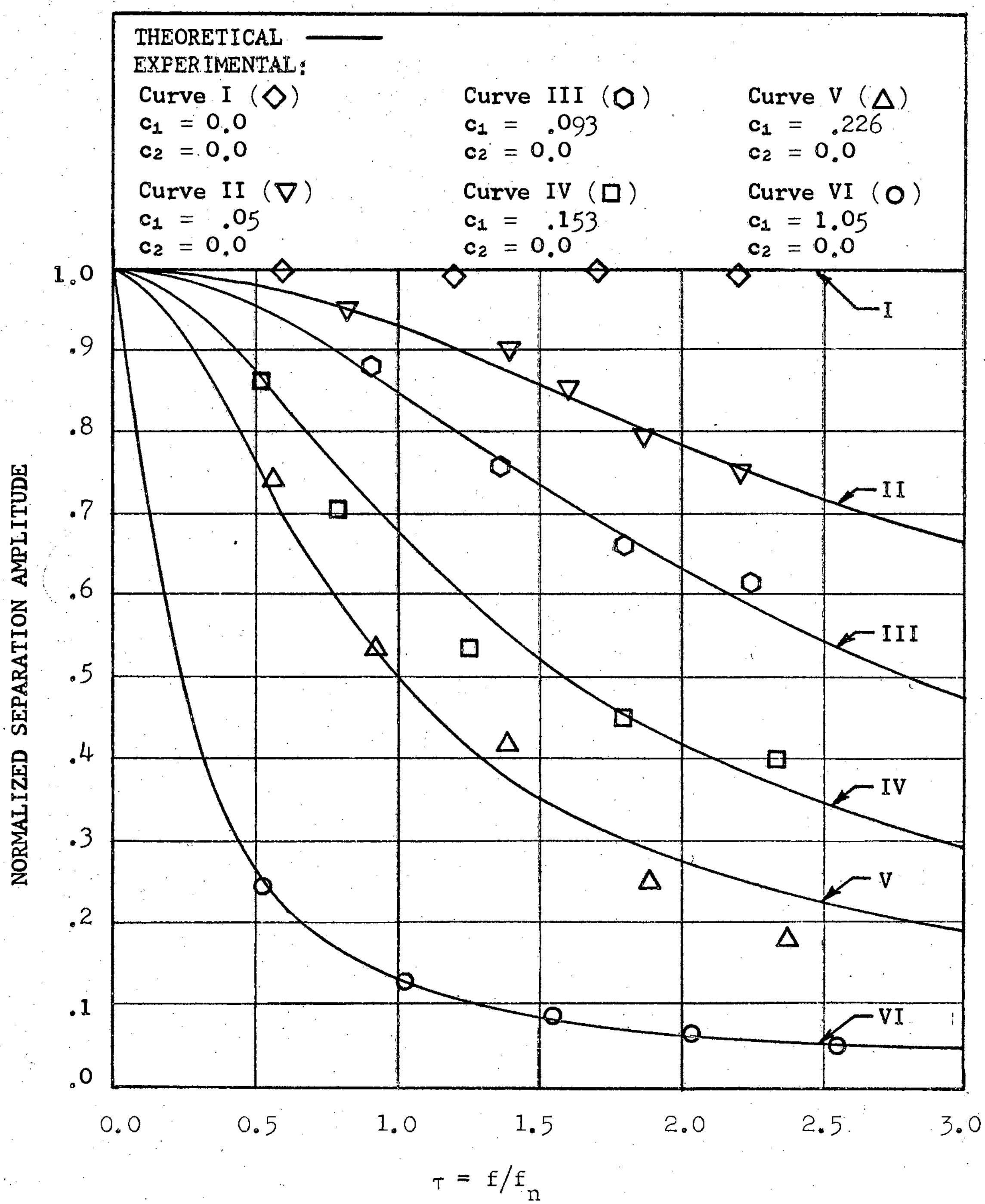


Figure 39. Normalized Amplitudes for Impending Separation.

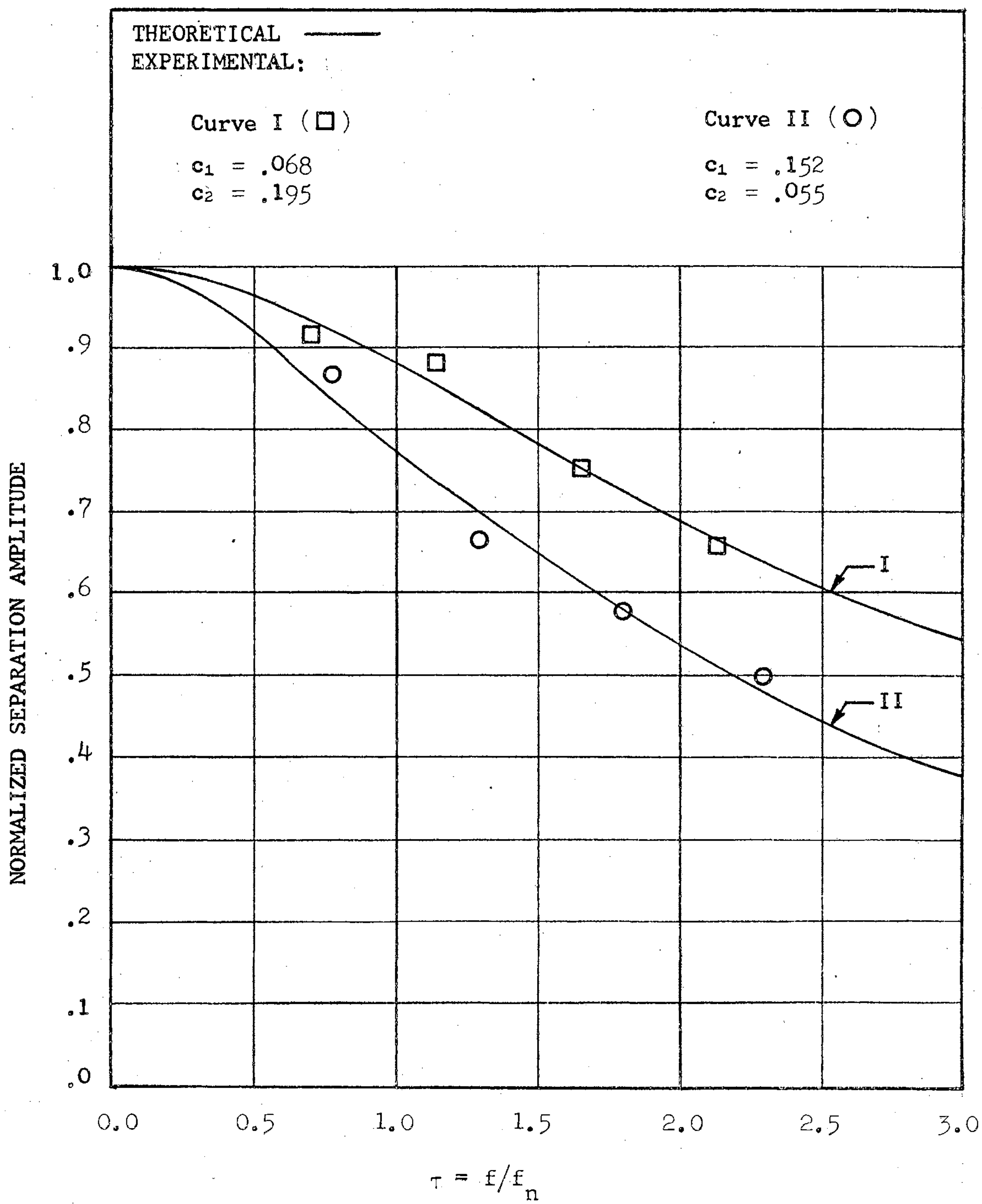


Figure 40. Normalized Amplitude for Impending Separation.

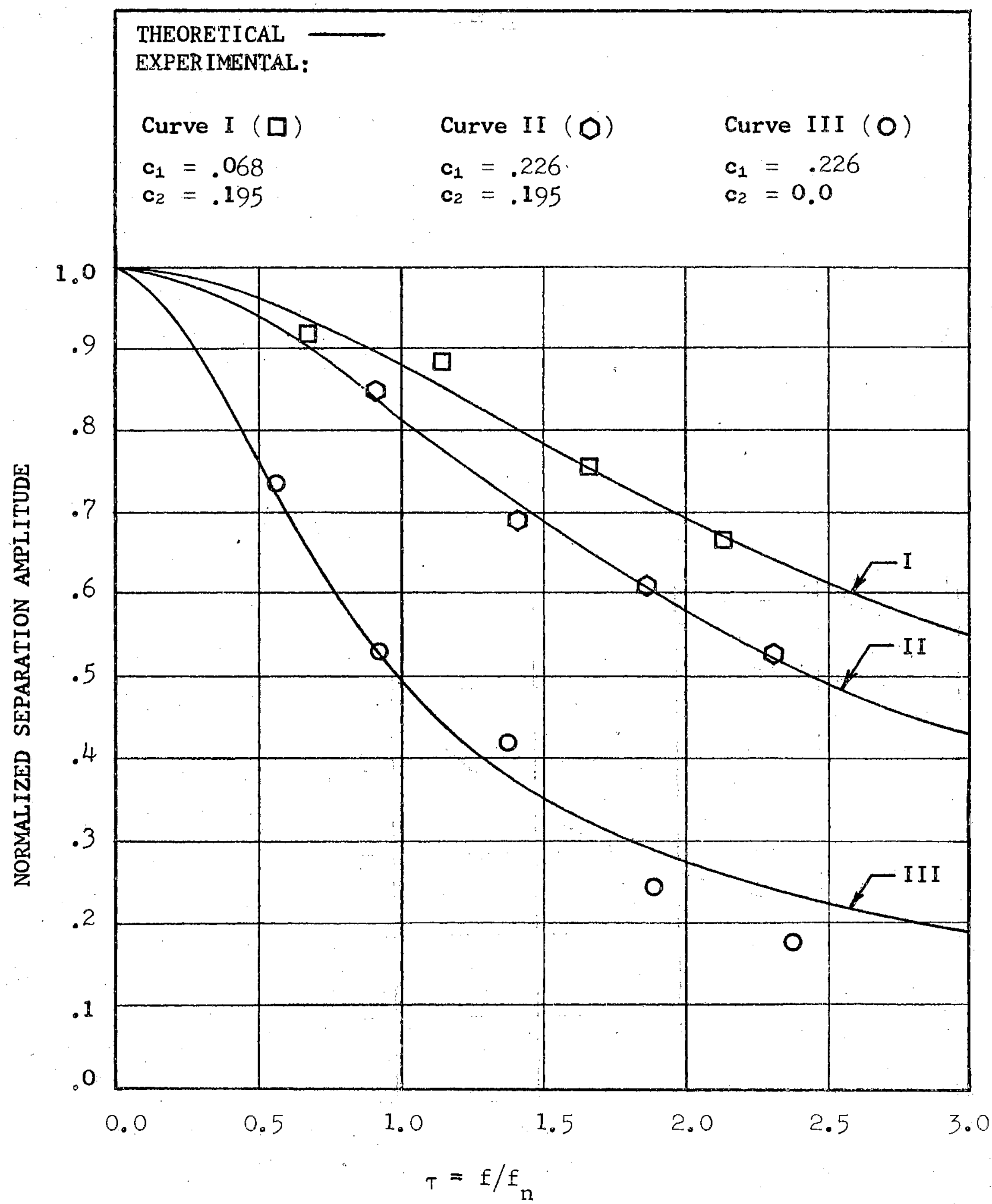


Figure 41. Normalized Amplitude for Impending Separation.

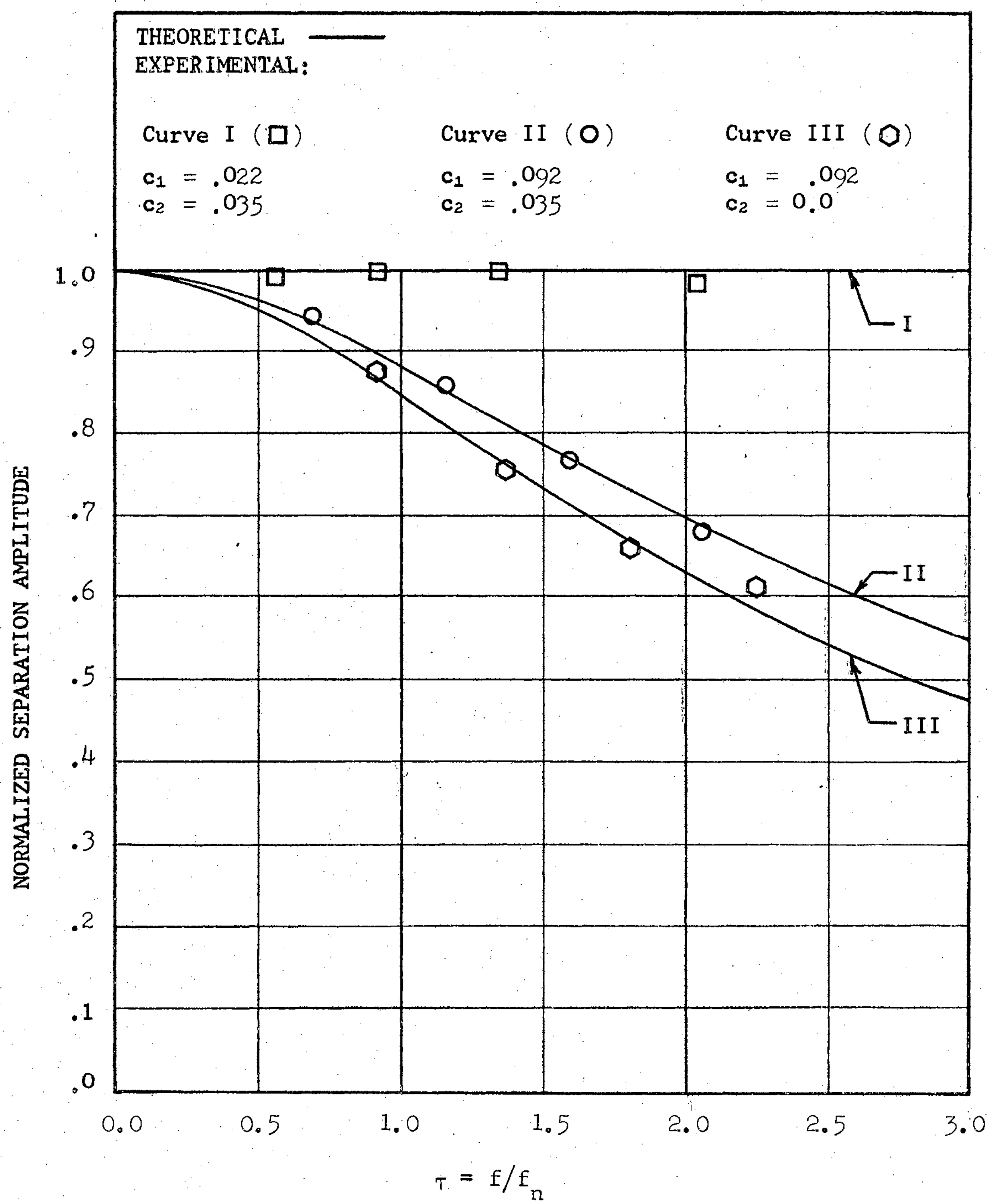
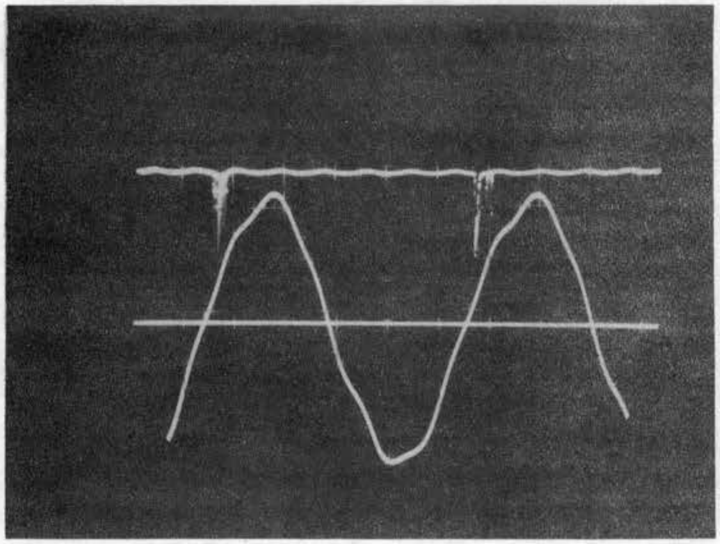


Figure 42. Normalized Amplitude for Impending Separation.

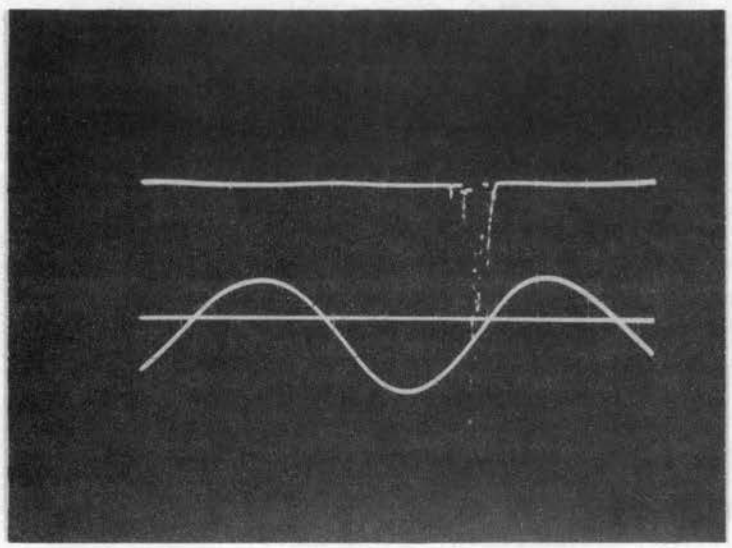
in Figure 41 is different from those shown in Figure 42. In both figures the system is investigated with only one damper, c_1, applied. Then the damper, c_2, is applied to the system. In each case it is seen that adding the second damper increased the normalized separation amplitude even though the total damping is greater. Next the amount of damping, c_1, is reduced for each experimental setup. In Figure 41 again a small increase is seen in the normalized separation amplitude. In Figure 42 an optimum proportionment between the two dampers is found and the normalized separation amplitude is independent of the frequency. The optimum proportionment of damping in Figure 42 is found experimentally but can be determined theoretically by making $CM = 1.0$.

Experimental Phase Relations

The phase relation between the response amplitude and the force between the masses, indicated in Chapter II, is checked experimentally. Even though only an approximate measurement of the phase angle is made experimentally, its existence is clearly shown in Figures 43 and 44. Figure 43 shows results for the model with one damper applied; the upper photograph is for $\tau = 0.45$ and the lower photograph is for $\tau = 1.59$. The theoretically predicted phase angle for $\tau = 0.45$ and $\tau = 1.59$ is $\beta = -37°$ and $\beta = -68°$, respectively. The experimentally measured value at $\tau = 0.45$ is $\beta = -43°$. The value at $\tau = 1.59$ is $\beta = -73°$. The negative sign indicates the separation lags the response amplitude, which is the case in Figure 43. Figure 44 shows the results for the system with two dampers. One damper is the same as in Figure 43, and an additional damper is added to the system. It can be seen in Figure 44

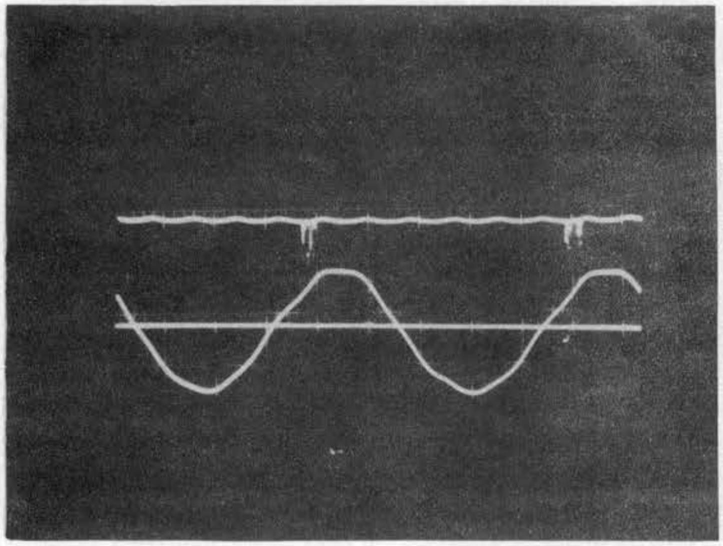


Upper Photograph, $\tau = 0.45$.

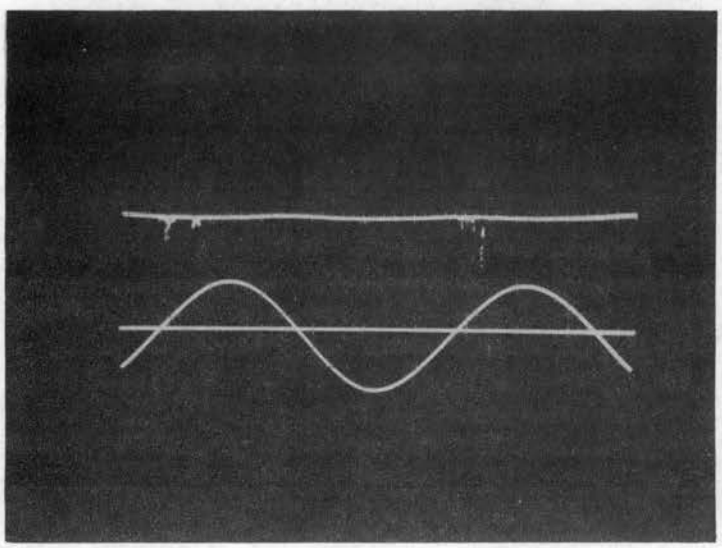


Lower Photograph, $\tau = 1.59$.

Figure 43. Phase Relation for Contact Separation, One Contact Damped.



Upper Photograph, $\tau = 0.45$.



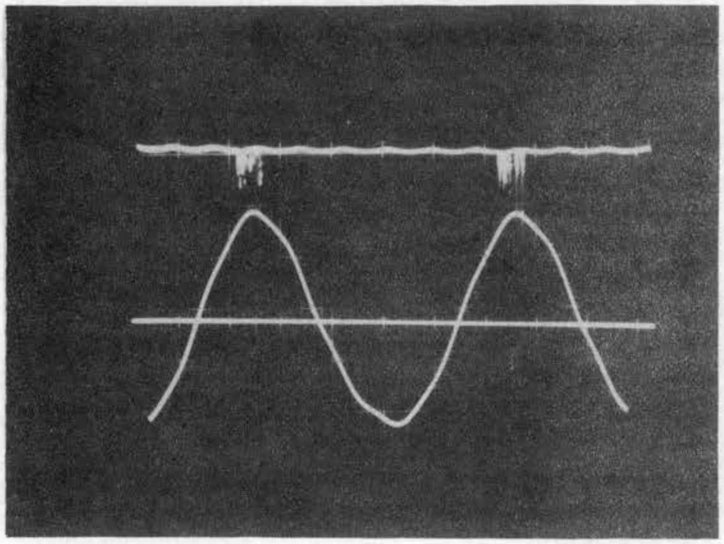
Lower Photograph, $\tau = 1.59$.

Figure 44. Phase Relation for Contact Separation, Both Contacts Damped.

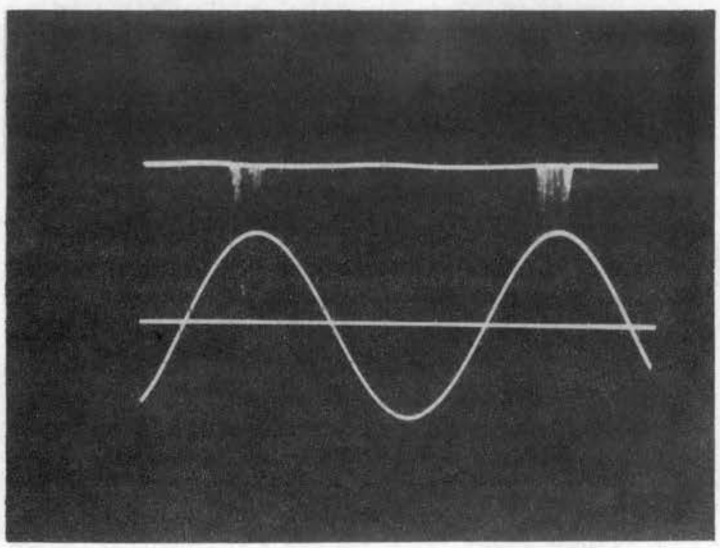
that the phase angles for $\tau = 0.45$ and $\tau = 1.59$ are less than those for the same values of τ in Figure 43. The predicted values of the phase angles with two dampers are $\beta = -24°$ when $\tau = 0.45$ and $\beta = -56°$ when $\tau = 1.59$. The experimental measured values are $\beta = -28°$ when $\tau = 0.45$, and $\beta = -52°$ when $\tau = 1.59$. For the undamped case shown in Figure 45, the phase angle for all the values of τ should be zero. In the upper photograph where $\tau = 0.45$, the phase angle is close to zero. In the lower photograph where $\tau = 1.59$, there exists a small phase difference.

Experimental Baron Number

A new parameter, the Baron Number, was introduced in Chapter III which relates the response amplitude and impending separation amplitude in a dimensionless parameter for various values of τ. The validity of the Baron Number is checked experimentally by comparing experimental points from the model with the curve of the Baron Number versus τ for a given set of model parameters. Figures 46, 47, 48, and 49 show the comparison of the Baron Number and experimental points for four different tests on the model. The correspondence of the experimental points with the curves of the Baron Number is good. This indicates the contact chatter characteristics can satisfactorily be determined by using the Baron Number.



Upper Photograph, $\tau = 0.45$.



Lower Photograph, $\tau = 1.59$.

Figure 45. Phase Relation for Contact Separation, Undamped.

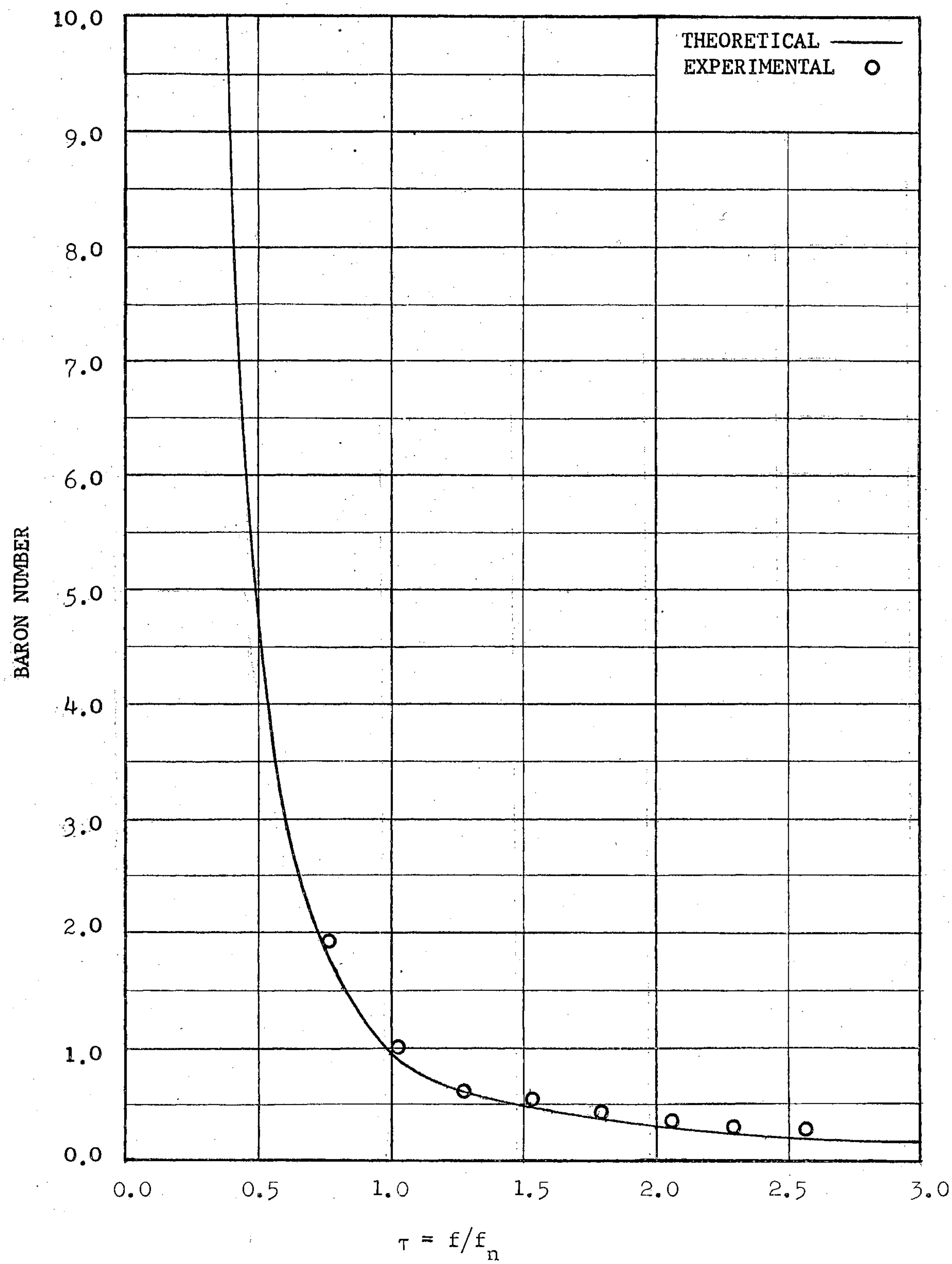


Figure 46. Experimental Baron Number for a System with One Contact Damped, C = 0.0, K = 0.806, M = 0.716, ζ_1 = 2.12.

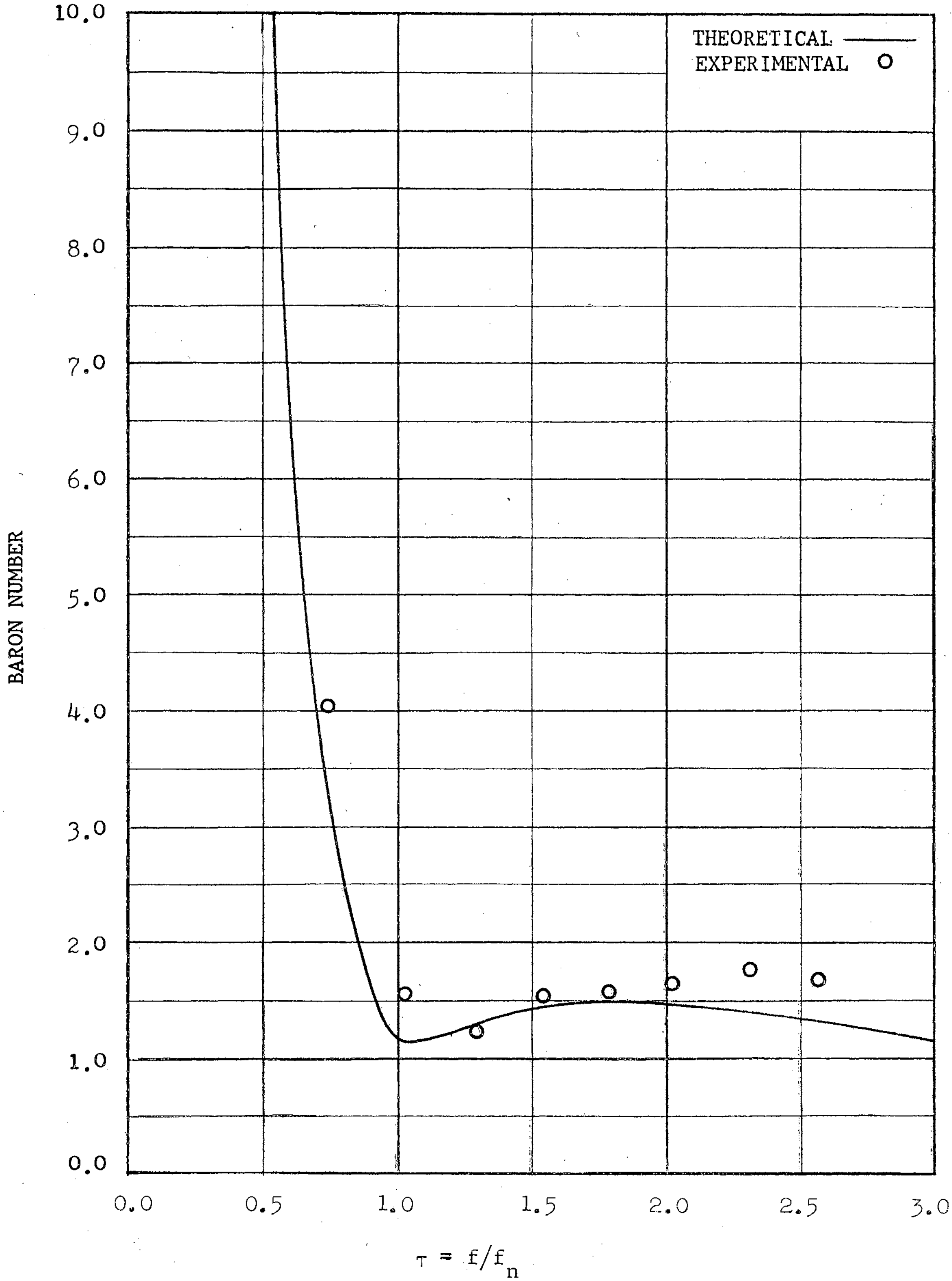


Figure 47. Experimental Baron Number for a System with Both Contacts Damped, C = 0.358, K = 0.806, M = 0.716, ζ_1 = 0.31.

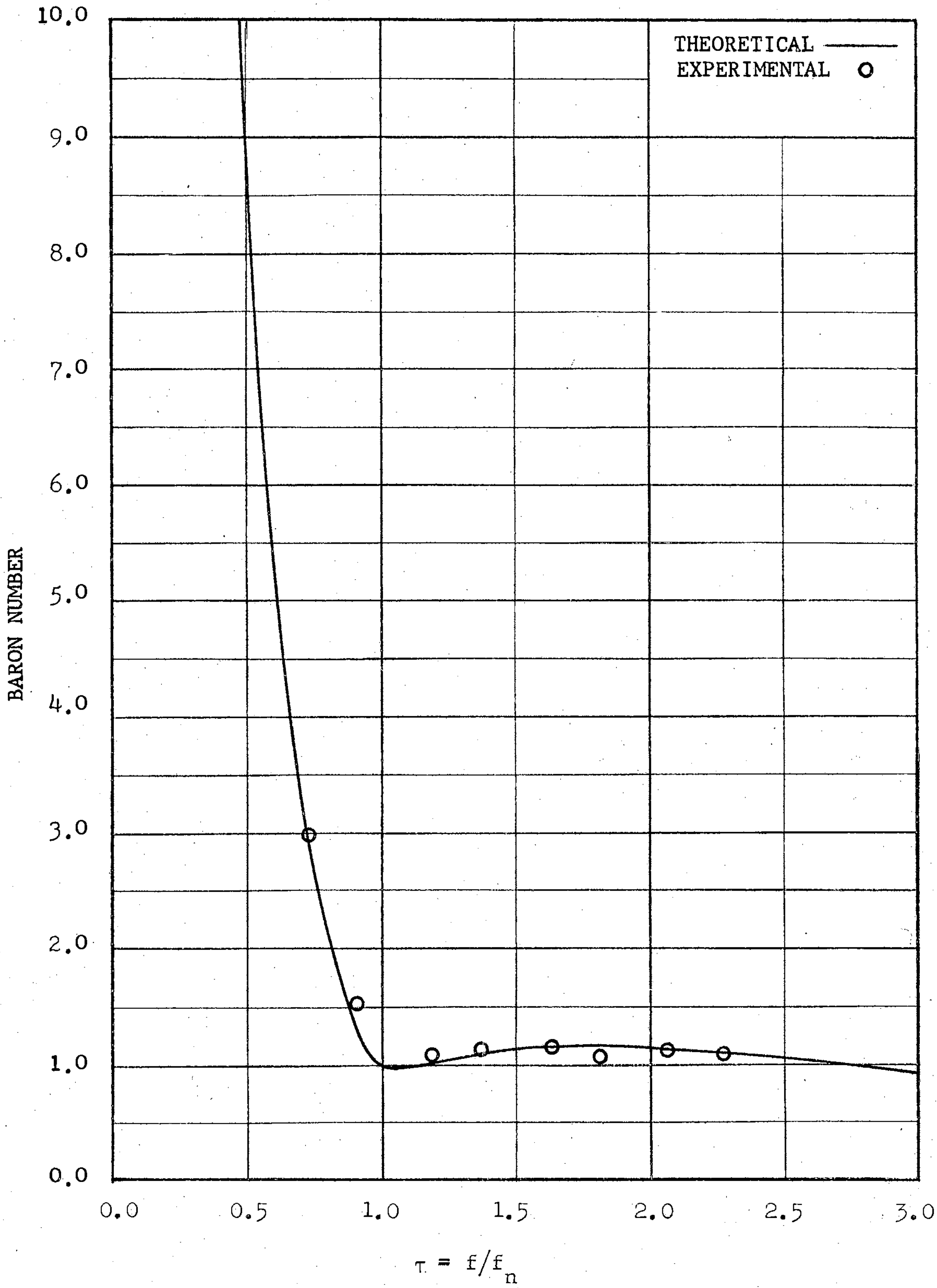


Figure 48. Experimental Baron Number for a System with One Contact Damped, C = 0.0, K = 1.0, M = 1.85, ζ_1 = 0.375.

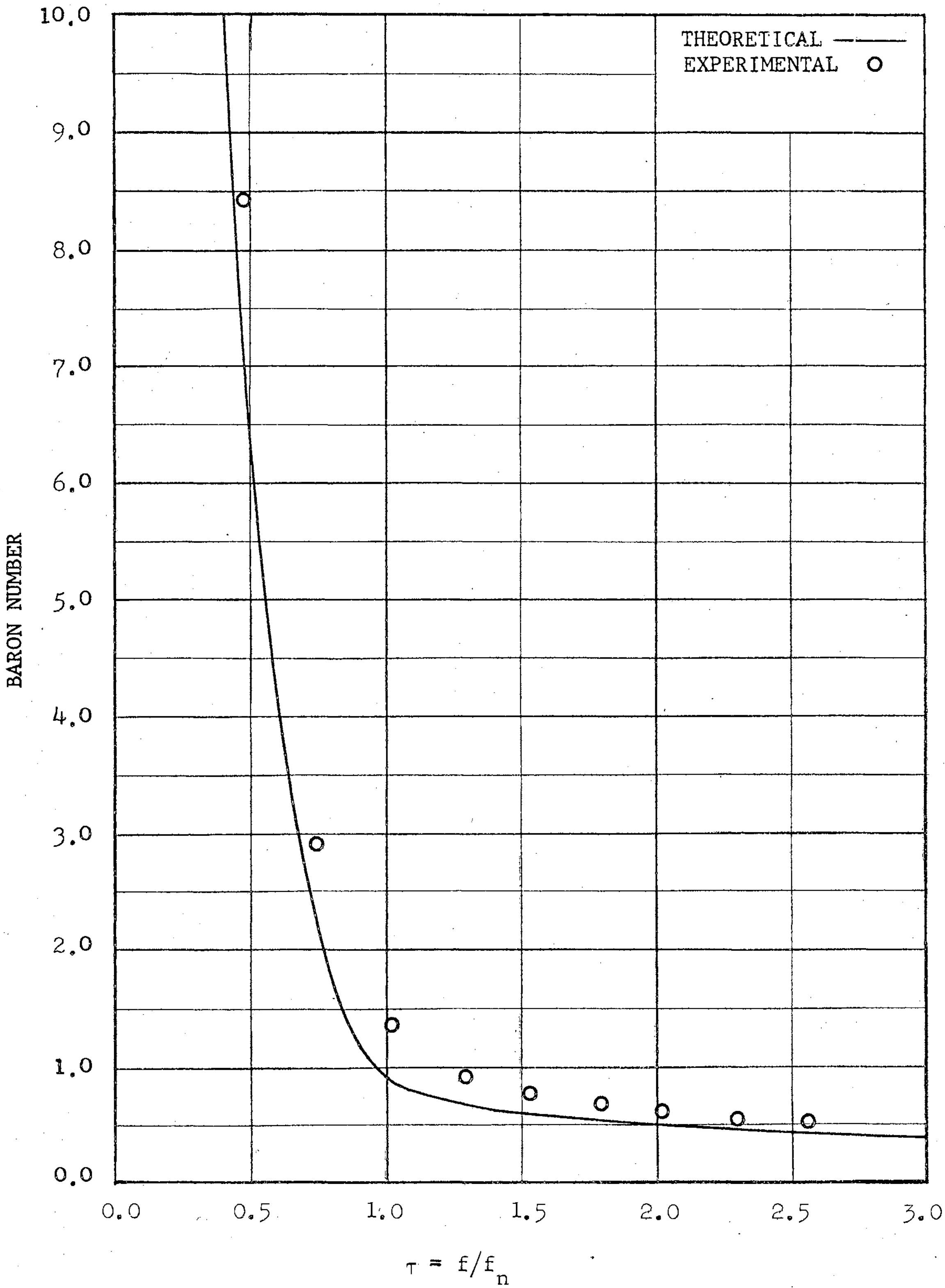


Figure 49. Experimental Baron Number for a System with One Contact Damped, $C = 0.0$, $K = 0.806$, $M = 0.716$, $\zeta_1 = 0.796$.

CHAPTER VI

CONCLUSIONS

The following conclusions about the contact chatter characteristics of a linear viscous damped contact system subjected to a steady state sinusoidal vibration environment are made on the basis of this study:

1. Contact separation of a preloaded set of contacts will occur when the force between the contacts is zero.

2. Initial contact separation will occur only once in each cycle of the contacts sinusoidal oscillation. The location of the separation within each cycle of oscillation depends on how the damping is proportioned between the two contacts.

3. The amplitude for impending contact separation is determined by the parameters of the contact system and preload.

4. The amplitude for impending contact separation is independent of damping if the damping coefficient ratio, c_2/c_1, is equal to the mass ratio, m_2/m_1.

5. The application of damping to a contact system with one rigid and one flexible contact does not affect the amplitude for impending separation, since the separation amplitude is zero for this type of configuration.

6. The Baron Number can be used to establish the maximum amplitude of the exciting motion without separation.

7. The Baron Number, along with the undamped natural frequency of the system, can be used to establish the maximum exciting acceleration for nonseparation of the contacts.

8. The Baron Number can be used to establish the preload necessary for a contact system to have no separation for a given amplitude of the exciting motion.

9. The Baron Number, along with the undamped natural frequency of the system, can be used to establish the preload necessary for a contact system to have no separation for a given exciting acceleration.

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APPENDIX A

SOLUTION OF THE EQUATION OF RELATIVE MOTION

The equation of relative motion of the contact masses to the system boundary is developed in Chapter II, Mathematical Derivation, Equation (8). The equation is

$$(m_1 + m_2)\,\ddot{x}(t) + (c_1 + c_2)\,\dot{x}(t) + (k_1 + k_2)\,x(t) =$$

$$\text{Im}[(m_1 + m_2)\,\omega^2 S\, e^{j\omega t}] \,. \qquad \text{(A-1)}$$

For the solution of this equation the following assumptions are made:

(a) Neither mass is equal to zero.

(b) The force between the masses, $F(t)$, is greater than zero.

(c) The coefficients of functions of $x(t)$ are constant.

The fact that the excitation is sinusoidal, $s(t) = S \sin \omega t$, suggests that the particular solution or steady state solution of the equation of relative motion has a probable sinusoidal solution. Thus, the general sinusoidal solution of $x(t) = X \sin(\omega t + \varphi)$ is assumed where X is the peak value of the response and φ is the phase angle between the excitation and response. In complex notation the assumed solution and its derivatives are:

$$x(t) = \text{Im}[X\, e^{j(\omega t + \varphi)}] \,,$$

$$\dot{x}(t) = \text{Im}[j\omega X e^{j(\omega t + \varphi)}] ,$$

$$\ddot{x}(t) = \text{Im}[-\omega^2 X e^{j(\omega t + \varphi)}] . \tag{A-2}$$

Substitution of Equation (A-2) into Equation (A-1) results in the following equation:

$$\text{Im}\left[X e^{j(\omega t + \varphi)} \left\{- (m_1 + m_2)\omega^2 + (c_1 + c_2)j\omega + (k_1 + k_2)\right\}\right] =$$

$$\text{Im}[(m_1 + m_2) \omega^2 S e^{j\omega t}] . \tag{A-3}$$

When Equation (A-3) is divided by $(m_1 + m_2)$ and the terms

$$\omega_n^2 = \frac{k_1 + k_2}{m_1 + m_2} ,$$

and

$$\tau = \omega/\omega_n ,$$

are introduced, Equation (A-3) reduces to

$$\text{Im}\left[\omega_n^2 - \omega^2 + \left(\frac{c_1 + c_2}{m_1 + m_2}\right) j\omega\right] X e^{j(\omega t + \varphi)} = \text{Im}(\omega^2 S e^{j\omega t}) , \tag{A-4}$$

where ω_n is the system circular natural frequency and τ is the ratio of the exciting frequency to the undamped natural frequency of the system.

When ω_n is factored out of Equation (A-4) and the equation is rearranged, the equation has the following form:

$$\text{Im}[X e^{j(\omega t + \varphi)}] = \text{Im}\left[\frac{\tau^2 S e^{j(\omega t)}}{(1 - \tau^2) + j\left[\omega \frac{c_1}{k_1}\left(\frac{1 + c_2/c_1}{1 + k_2/k_1}\right)\right]}\right] . \tag{A-5}$$

Equation (A-5) can be put in a more convenient form by changing the right-hand side to the modulus and an argument of the complex number as follows:

$$\mathrm{Im}[X\ e^{j(\omega t + \varphi)}] = \mathrm{Im}\left[\frac{\tau^2\ S\ e^{j(\omega t - \theta)}}{\left\{(1 - \tau^2)^2 + \left[\omega \frac{c_1}{k_1}\left(\frac{1 + c_2/c_1}{1 + k_2/k_1}\right)\right]^2\right\}^{\frac{1}{2}}}\right], \quad \text{(A-6)}$$

where

$$\theta = \arctan \frac{\omega \frac{c_1}{k_1}\left(\frac{1 + c_2/c_1}{1 + k_2/k_1}\right)}{1 - \tau^2} .$$

By definition two complex numbers which are equal must have the same modulus and argument. Therefore,

$$X = \frac{\tau^2\ S}{\left\{(1 - \tau^2)^2 + \left[\omega \frac{c_1}{k_1}\left(\frac{1 + c_2/c_1}{1 + k_2/k_1}\right)\right]^2\right\}^{\frac{1}{2}}} , \quad \text{(A-7)}$$

and

$$\varphi = -\theta .$$

If the ratios of k_2/k_1, c_2/c_1, and m_1/m_2 are represented by dimensionless ratios as $K = k_2/k_1$, $C = c_2/c_1$, and $M = m_1/m_2$, and if the damping factor of contact 1 is defined as

$$\zeta_1 = \frac{c_1}{2\sqrt{k_1 m_1}} ,$$

then the second term in the denominator of Equation (A-7) can be written as

$$\left[\omega \frac{c_1}{k_1}\left(\frac{1 + c_2/c_1}{1 + k_2/k_1}\right)\right]^2 = \frac{4\ \tau^2\ M\ (1 + C)^2\ \zeta_1^{\ 2}}{(1 + M)(1 + K)} .$$

With substitution of these relations in Equation (A-7), the modulus of the complex number, which is the amplitude of the response, is

$$X = \frac{S\,\tau^2}{\left\{(1 - \tau^2)^2 + \frac{4\,\tau^2\,M\,(1 + C)^2\,\zeta_1^{\,2}}{(1 + M)(1 + K)}\right\}^{\frac{1}{2}}}, \qquad \text{(A-8)}$$

and the argument of the complex number, which is the phase angle between the response and the exciting motion, is

$$\theta = \text{arc tan}\;\frac{2\,\tau\,(1 + C)\,\zeta_1\left[\frac{M}{(1 + M)(1 + K)}\right]^{\frac{1}{2}}}{1 - \tau^2}. \qquad \text{(A-9)}$$

The resulting solution of the equation for the relative motion of the system in complex and trigonometric notation is

$$x(t) = \text{Im}[X\,e^{j(\omega t - \theta)}]\,,$$

and (A-10)

$$x(t) = X \sin\,(\omega t - \theta)\,,$$

where X is the amplitude of the motion and is always positive and θ is the phase angle between the exciting motion and the relative response of the system.

APPENDIX B

SOLUTION OF THE EQUATION FOR THE RELATIVE AMPLITUDE FOR IMPENDING SEPARATION

The equation for the force, $F(t)$, between the contact is shown in Chapter II, Mathematical Derivations, in Equation (14) as

$$\left(\frac{c_1}{m_1} - \frac{c_2}{m_2}\right)\dot{x}(t) + \left(\frac{k_1}{m_1} - \frac{k_2}{m_2}\right)x(t) + F_o\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = F(t)\left(\frac{1}{m_1} + \frac{1}{m_2}\right). \tag{B-1}$$

The relative amplitude of the response of the system for $F(t) > 0$ is found in Appendix A, shown in Equation (A-10), to be

$$x(t) = X \sin(\omega t - \theta) , \tag{B-2}$$

thus

$$\dot{x}(t) = \omega X \cos(\omega t - \theta) , \tag{B-3}$$

where X is the amplitude of the relative response and θ is the phase angle between the exciting motion and the relative response.

Substitution of Equations (B-2) and (B-3) into Equation (B-1) yields the following equation:

$$F_o\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - \left\{-\left(\frac{c_1}{m_1} - \frac{c_2}{m_2}\right)\omega X \cos(\omega t - \theta) - \left(\frac{k_1}{m_1} - \frac{k_2}{m_2}\right) X \sin(\omega t - \theta)\right\} = F(t)\left(\frac{1}{m_1} + \frac{1}{m_2}\right). \tag{B-4}$$

Since the coefficients of the sine and cosine terms in Equation (B-4) are amplitudes of perpendicular vectors, the terms inside the brackets can be written as

$$- \left(\frac{c_1}{m_1} - \frac{c_2}{m_2} \right) \omega X \cos (\omega t - \theta) - \left(\frac{k_1}{m_1} - \frac{k_2}{m_2} \right) X \sin (\omega t - \theta) =$$

$$\left[\left(\frac{k_1}{m_1} - \frac{k_2}{m_2} \right)^2 + \omega^2 \left(\frac{c_1}{m_1} - \frac{c_2}{m_2} \right)^2 \right]^{\frac{1}{2}} X \sin (\omega t - \theta + \beta) ,$$

where

$$\beta = \arc \tan \frac{- \omega \left(\frac{c_1}{m_1} - \frac{c_2}{m_2} \right)}{- \left(\frac{k_1}{m_1} - \frac{k_2}{m_2} \right)} .$$

The equation for the force, $F(t)$, between the contacts is now written in the following form:

$$F(t) = F_o - \frac{\left[\left(\frac{k_1}{m_1} - \frac{k_2}{m_2} \right)^2 + \omega^2 \left(\frac{c_1}{m_1} - \frac{c_2}{m_2} \right)^2 \right]^{\frac{1}{2}} X \sin (\omega t - \theta + \beta)}{\left(\frac{1}{m_1} + \frac{1}{m_2} \right)} . \tag{B-5}$$

From Equation (B-5) it can be seen that β is the phase angle between the force, $F(t)$, and the relative response, $x(t)$. For the contacts to separate, the force, $F(t)$, must be zero. The amplitude of the response when the force, $F(t)$, initially becomes zero is defined as the amplitude for impending separation, $\bar{X}$. Equation (B-5) indicates that the minimum response amplitude for $F(t)$ to initially equal zero will occur when $\sin (\omega t - \theta + \beta) = 1$. The solution of Equation (B-5) for the amplitude for impending separation when $F(t)$ is initially zero and $\sin (\omega t - \theta + \beta) = 1$ results in

$$\bar{X} = \frac{F_o\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{\left[\left(\frac{k_1}{m_1} - \frac{k_2}{m_2}\right)^2 + \omega^2\left(\frac{c_1}{m_1} - \frac{c_2}{m_2}\right)^2\right]^{\frac{1}{2}}} . \qquad \text{(B-6)}$$

This equation can be factored and rearranged so that

$$\bar{X} = \frac{F_o/k_1\ (1 + m_1/m_2)}{\left[\left(1 - \frac{k_2}{k_1}\frac{m_1}{m_2}\right)^2 + \omega^2\left(\frac{c_1}{k_1}\right)^2\left(1 - \frac{c_2}{c_1}\frac{m_1}{m_2}\right)^2\right]^{\frac{1}{2}}} .$$

The substitutions $K = k_2/k_1$, $C = c_2/c_1$, $M = m_1/m_2$ and the change in the product $\omega^2(c_1/k_1)^2$ to $4\ \tau^2\ M(1 + K)\ \zeta_1{}^2/(1 + M)$ are made. ζ_1 is the damping factor for contact 1 and τ is the ratio of the exciting frequency undamped natural frequency. Thus, the final form of the equation for the amplitude for impending separation is

$$\bar{X} = \frac{F_o/k_1\ (1 + M)}{\left[(1 - KM)^2 + \frac{4\ \tau^2\ M(1 + K)(1 - CM)^2\ \zeta_1{}^2}{(1 + M)}\right]^{\frac{1}{2}}} . \qquad \text{(B-7)}$$

The phase angle β, when $F(t)$ initially becomes zero, locates the position of $F(t) = 0$ relative to the response amplitude and is

$$\beta = \text{arc tan}\ \frac{-\ 2\tau\ (1 - CM)\left[\frac{M\ (1 + K)}{1 + M}\right]^{\frac{1}{2}}}{-\ (1 - KM)} . \qquad \text{(B-8)}$$

APPENDIX C

EQUIVALENT VISCOUS DAMPING COEFFICIENTS

Equivalent viscous damping coefficients are derived in this study by using the criterion of equivalent energy dissipation per cycle. The energy used for this derivation is the work done by the damping force in one cycle. The work done by a linear viscous damper proportional to the 1^{st} power of the velocity in a steady state sinusoidal environment is equated to the work done by a damper proportional to the n^{th} power of the velocity in the same environment.

The force of a linear viscous damper is

$$F_c(t) = c\dot{x}(t) , \qquad (C\text{-}1)$$

where $x(t) = X \sin(\omega t - \theta)$. The work of the damping force is

$$\text{Work} = F_c(t)\ dx . \qquad (C\text{-}2)$$

The displacement, dx, is replaced by the following equation

$$dx = dx \frac{dt}{dt} = \dot{x}(t)\ dt . \qquad (C\text{-}3)$$

After substitution for the damping force, $F_c(t)$, and the displacement, dx, in Equation (C-2) and after integration over four times a quarter of a cycle, beginning at $t = \theta/\omega$ and ending at $t = \frac{\pi}{2\omega} + \frac{\theta}{\omega}$, the work per cycle is

$$\text{Work/cycle} = 4 \int_{\frac{\theta}{\omega}}^{\frac{\pi}{2\omega}+\frac{\theta}{\omega}} c[\omega X \cos(\omega t - \theta)]^2 \, dt \,. \qquad \text{(C-4)}$$

If the variable of integration is changed by letting $u = \omega t - \theta$ so that $dt = \frac{du}{\omega}$, then the limits of the definite integral are changed to $u = 0$ at $t = \theta/\omega$ and $u = \pi/2$ at $t = \pi/2\omega + \theta/\omega$. If the above substitutions are made, Equation (C-4) becomes

$$\text{Work/cycle} = 4\, c\omega\, X^2 \int_0^{\frac{\pi}{2}} \cos^2 u \, du \,. \qquad \text{(C-5)}$$

The solution of Equation (C-5) from the table of integrals is

$$\text{Work/cycle} = \pi\, c\omega\, X^2 \,, \qquad \text{(C-6)}$$

which is the work of a viscous damping force over one cycle.

The force of an arbitrary damper proportional to the n^{th} power of the velocity is

$$F_{c_n}(t) = c_n\, \dot{x}(t)^n \,. \qquad \text{(C-7)}$$

Work per cycle for this damping force can be evaluated by using Equations (C-7) and (C-3) in Equation (C-2). Thus, the work is

$$\text{Work/cycle} = 4 \int_{\frac{\theta}{\omega}}^{\frac{\pi}{2\omega}+\frac{\theta}{\omega}} c_n[\omega X \cos(\omega t - \theta)]^{n+1} \, dt \,. \qquad \text{(C-8)}$$

If the same change of variables is made as before, the work of an arbitrary damper is found from the solution of the integral

$$\text{Work/cycle} = 4\, c_n\, \omega^n\, X^{n+1} \int_0^{\frac{\pi}{2}} \cos u^{n+1}\, du\,. \qquad \text{(C-9)}$$

From the table of integrals the solution of the integral in Equation (C-9) for $n > -1$ has the general form of

$$\int_0^{\frac{\pi}{2}} \cos u^{n+1}\, du = \frac{1}{2}\sqrt{\pi}\, \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)}\,, \qquad \text{(C-10)}$$

where $\Gamma(n)$ is defined as the gamma function. Thus, the work of an arbitrary damper proportional to the n^{th} power of the velocity is

$$\text{Work/ cycle} = 2\, c_n\, \omega^n\, X^{n+1} \sqrt{\pi}\, \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)}\,. \qquad \text{(C-11)}$$

After combination of Equations (C-6) and (C-11), the result is

$$\pi c \omega X^2 = 2 c_n\, \omega^n\, X^{n+1} \sqrt{\pi}\, \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)}\,. \qquad \text{(C-12)}$$

If $\gamma_n = \frac{2}{\sqrt{\pi}} \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)}$ and Equation (C-12) is rearranged, an equivalent viscous damping coefficient for the damping proportional to the n^{th} power of the velocity is

$$c = c_n\, \omega^{n-1}\, X^{n-1}\, \gamma_n\,. \qquad \text{(C-13)}$$

APPENDIX D

THEORETICAL RESULTS USING EQUIVALENT VISCOUS DAMPING COEFFICIENTS

Jacobsen (7) established that the use of equivalent viscous damping coefficients to determine the response of a single degree of freedom system is in good agreement with experimental results. The theory of equivalent viscous damping coefficients is applied to determine the amplitude for impending separation. The idea is that, if the theory gives a good approximation for the response of a system, the possibility exists that it may give a good approximation for the amplitude for impending separation.

To illustrate the influence of changing the exponent, n, of the velocity-damping term on the magnification factor, three curves are drawn in Figure 50. The curves are drawn to give the same magnification factor at $\tau = 1.0$. Figure 51 shows the normalized amplitudes for impending separation using the same values of the exponents as shown in Figure 50. When $n = 1.0$ the equations for these approximate solutions reduce to the exact solution, so that in Figures 50 and 51 when $n = 1.0$, the curve is exact.

Figure 51 indicates that for values of $\tau < 1.0$ the curves for $n = 1.5$ and $n = 2.0$ are above the curve for $n = 1.0$ and differ in shape. For $\tau > 1.0$ the curves follow the same shape but are lower for

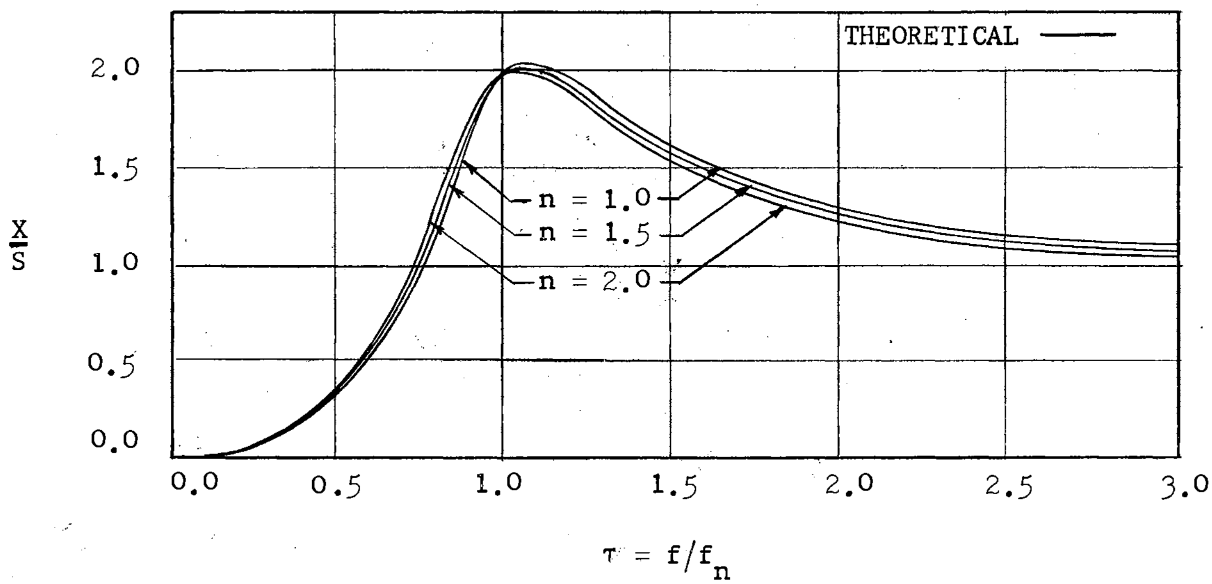

Figure 50. Magnification Factor for Various Values of n.

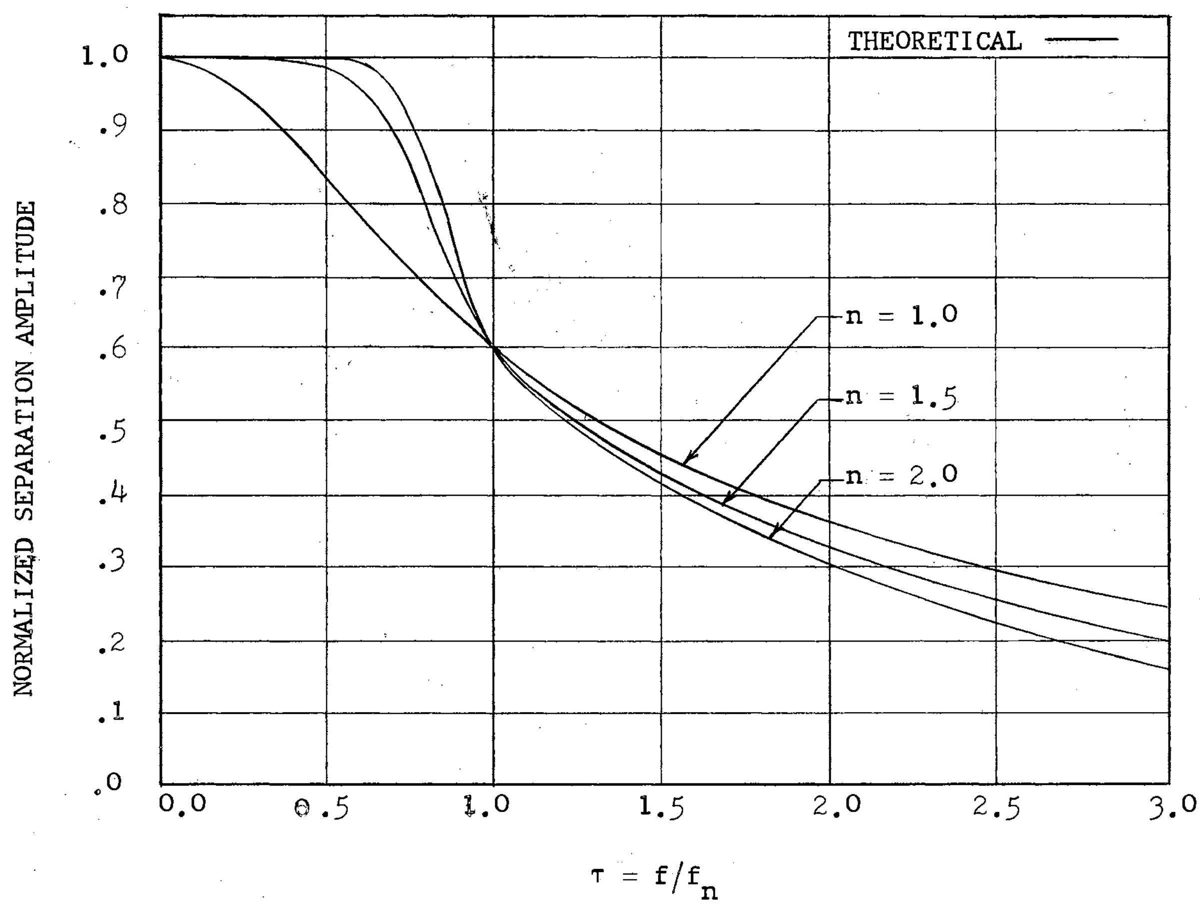

Figure 51. Normalized Amplitude for Impending Separation.

increasing values of n. Since there is no appreciable decrease of either curve from the curve for $n = 1.0$ it may be that this type of analysis would give a reasonable approximation of the amplitude for impending separation.

APPENDIX E

LIST OF MAJOR SYMBOLS

C	Damping coefficient ratio c_2/c_1.
c_1	Linear damping coefficient for viscous damper 1.
c_2	Linear damping coefficient for viscous damper 2.
$F(t)$	Force between the contact masses as a function of time.
$F_{c_1}(t)$	Damping force exerted by viscous damper 1.
$F_{c_2}(t)$	Damping force exerted by viscous damper 2.
$F_{k_1}(t)$	Restoring force exerted by spring 1.
$F_{k_2}(t)$	Restoring force exerted by spring 2.
$F_{m_1}(t)$	Inertial force of mass 1.
$F_{m_2}(t)$	Inertial force of mass 2.
F_o	Static force between the contact masses, the preload.
f	The frequency of the exciting motion.
f_1	The undamped natural frequency of contact 1.
f_2	The undamped natural frequency of contact 2.
f_{12}, f_n	The undamped natural frequency of the contact system.
g	Gravitational acceleration.
i	General subscript which is either 1 or 2.
K	Spring constant ratio k_2/k_1.

k_1	Linear spring constant for spring 1.
k_2	Linear spring constant for spring 2.
M	Mass ratio m_1/m_2.
m_1	Mass of contact 1.
m_2	Mass of contact 2.
n	General exponent with values from 0.5 to 3.0.
S	Amplitude of the exciting motion.
s(t)	Exciting motion of the system.
X	Amplitude of the relative response.
$\bar{X}$	Relative amplitude for impending contact separation.
x(t)	Relative response of the system.
y(t)	Absolute response of the system.
β	Phase angle between the relative response and the sinusoidal variation of the force between the contacts.
Δ_1	Static deflection of spring 1 from the weight of its contact.
Δ_2	Static deflection of spring 2 from the weight of its contact.
δ_1	Static deflection of spring 1 from the preload.
δ_2	Static deflection of spring 2 from the preload.
ζ_1	Damping factor for contact 1.
ζ_T	Damping factor for the contact system.
θ	Phase angle between the exciting motion and the relative response.
τ	Ratio of the exciting frequency and the system's undamped natural frequency, f/f_n.
ω	The circular frequency of the exciting motion.
ω_n	The undamped circular natural frequency of the system.

APPENDIX F

LIST OF MAJOR INSTRUMENTATION

Accelerometers--Model 5D41; Manufacturer, Clevita; Serial Nos. 7093 and 7060.

Air Damping Dashpots--Model 303; Manufacturer, Electric Regulator Corporation; Serial No. 85-11-1.

Audio Oscillator--Model 200 AB; Manufacturer, Hewlett-Packard; Serial No. 130-13888.

Chatter Tester--Model D959501; Manufacturer, Sandia Corporation; Serial No. 811-60.

DC Nullvoltmeter--Model 413A; Manufacturer, Hewlett-Packard; Serial No. 139-00188.

Dual Beam Oscilloscope--Model 502; Manufacturer, Tektronix; Serial No. 006852.

Linear Differential Transformer--Model 7DCDT-050; Manufacturer, Sanborn; Serial No. FG.

True Root-Mean-Square Voltmeter--Model No. 320; Manufacturer, Ballantine Laboratories, Inc.; Serial No. 3900.

Vibration Test Equipment--Model T112031; Manufacturer, MB Electronics; Serial No. 121.

Vibration Meter: Model N550; Manufacturer, MB Electronics.

Sine Random Generator: Model N670; Manufacturer, MB Electronics.

Control Equipment: Model T251; Manufacturer, MB Electronics.

Shaker: Model C-10; Manufacturer, MB Electronics.

VITA

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