BAYES SOLUTION TO A BUSINESS DECISION PROBLEM

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Bachelor of Science

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1966

Submitted to the faculty of the
Graduate College of
Oklahoma State University
in partial fulfillment of the
requirements for the degree of
MASTER OF BUSINESS ADMINISTRATION
July, 1967

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DECISION PROBLEM

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ACKNOWLEDGMENT

The author is indebted to Dr. Wayne A. Meinhart for his suggestions, guidance, and assistance in the preparation of this report.

The author also desires to express his sincere appreciation to his wife, Betty, for her encouragement and for having typed this report.

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CHAPTER I

INTRODUCTION

In any aspect of business, one is faced with the problem of making decisions. There are several types of information available to the manager to aid him in his decision making. For example, he will always have feelings or opinions about the problem which could be described as his subjective information. Also available to him are different types of objective information such as the results of other decision problems with similarities to the problem at hand. Another desirable type of information may be the results obtained from an experiment or simulation on the problem involved.

After the decision maker has gathered all three types of information, he is faced with the problem of deciding how to combine this information into a final decision. The Bayes' solution to the decision problem provides a logical framework for working with economic losses or the utility of alternative courses of action, the prior information available to the decision maker, and formal modification of this prior information with the introduction of more current knowledge. The elements of the Bayes' solution are further defined and discussed in Chapter III.

Chapter II is a brief review of the literature on the Bayes' procedure and applications of the procedure to business problems.

Chapter IV is a further extension of the Bayes procedure to a personnel selection problem. The personnel selection was used because, to the author's knowledge, there has been relatively little work done in applying the Bayes' procedure to a problem of this type. Since personnel selection requires a great deal of subjective decision making on the part of a manager, it is felt that by applying the Bayes' procedure this subjectivity can be developed into objective statistical data.

CHAPTER II

REVIEW OF THE LITERATURE

The basic concept of a general statistical decision problem was outlined by Wald in 1939 [15]. Theoretical statistics, in recent years, has come to be dominated by the decision theory orientation, attributable mostly to Wald, which tries to unify and strengthen the whole structure of statistical theory by treating statistics as a body of methods for making wise decisions in the face of uncertainty.

Decision theory does, however, mean more than a decision orientation. Long before Wald, the idea of decision making had deeply penetrated statistics, and some writers have been guilty of inaccurate labeling of pre-Waldian statistics by liberal use of the term "decision theory" [10]. Thus "decision theory" has often appeared to be a loose catchword like "operations research" or "motivation research". Actually, decision theory is a theory of rational behavior in the face of uncertainty. Its first comprehensive presentation was given in book form by Wald in 1949, Statistical Decision Functions, [16]. In this publication the notions of risk function and minimax and Bayes' solution to decisions were studied.

Wald's mathematical model for statistical decision theory is a special case of that for game theory as introduced by von Neumann and Morgenstern [14]. Many of the results of von Neumann and Morgenstern, i.e., the reduction of games to normal form, minimax and utility theory, and much of the research stimulated by their book are of basic importance for statistical decision theory. For further discussion of the statistical decision problem in the game theory setting, the reader may refer to Blackwell and Girshirk [2], and Luce and Raiffa [7]. Although all of the works mentioned above provide excellent background into statistical decision theory, this report will be more concerned with applying these concepts to business decision problems.

Books on business statistics have often been out of touch both with statistical knowledge and with business needs. Probability and Statistics for Business Decisions, by Robert Schlaifer of the Harvard Business School, is a major exception [12]. Appearing only a little more than a decade after the pioneering work in statistical decision theory by Wald, Schlaifer's book presents an elementary account of one "school" of decision theory. Its major theme is the use of theory in making business decisions. This theme is developed by a large number of realistic though simplified examples and problems and by the author's excellent advice on the application of statistics to practice.

Schlaifer represents a school that has made fundamental changes in the structure that Wald built. Wald strengthened an earlier trend toward viewing statistical inference as the making of decisions rather than as the drawing of conclusions. He was concerned with the concept of consequences of decisions and built this concept into a scheme of viewing decisions in terms of a payoff table that relates possible acts to possible consequences. He applied the minimax principle to the actual choice of an act (making a decision). Wald's theory has had much influence on statistical theory but relatively slight influence on statistical practice.

Schlaifer departs from Wald by rejecting the minimax principle except when it approximates well the result obtained from an alternative principle and is easier to apply than the alternative. The alternative principle is that of maximizing expected income or utility. This principle, in turn, is implemented by a scheme known as Bayes' theorem.

Another book by Schlaifer, appearing in 1961, is a somewhat condensed and simplified version of his first book on decision theory [13]. This book differs in two fundamental respects from Schlaifer's first book. First, it aims at a unified treatment of classical and Bayesian statistics, whereas the earlier book relied essentially on a form of analysis which tends to conceal this unity even though it leads to the same results. Secondly, this book aims solely at teaching how to deal with samples, whereas the earlier book dealt with a wide variety of decision problems in which samples were not involved.

Another elementary book on decision theory by Chernoff and Moses also deserves high commendation [3]. It covers clearly, with self-contained explanations of the mathematics used, the subject of decision theory as it stands today, without commitment to a particular "school". It is mainly a prelude to a course or book that would go into the tools and trials of statistical applications, though it does give a general discussion of testing and estimation from the viewpoint of decision theory. An important predecessor of this book is one by Weiss [17]. However, the mathematics used are a great deal more sophisticated.

There also has been several recent articles on the various applications of the Bayesian approach to decision theory. For example, one article by Green [5] was concerned with the use of Bayesian decision theory in the selection of a "best" pricing policy for a firm in an oligopolistic industry where such factors as demand elasticity, threat of future price weakness, and potential entry of new competitors influenced the effectiveness of the firm's courses of action. F. J. Anscombe presented a talk, which was later published into an article, that was designed to illustrate the difference between the orthodox and Bayesian approaches in a marketing problem [1]. Another article by Murray and Silver made use of the Bayesian approach to analyze an inventory model that was intended to represent the problem faced in style goods merchandising, both wholesale and retail [8]. The object of this analysis was to determine a buying policy that shows the optimal action at each opportunity for each possible state of information and each possible inventory position that the vendor may face. Lastly, an article by Hirshleifer tried to convey the ease and simplicity with which the Bayes approach solves those dual fundamental weaknesses of the classical approach - what "criteria" to use in estimation problems, and how to specify the tolerable risks of error in testing problems [6].

CHAPTER III

DEFINITION OF THE DECISION PROBLEM

The decision problem arises when a decision maker is faced with a set of alternative actions, one of which must be made, and the degree of preference for the possible decisions depends on various types of uncertainty. The two kinds of uncertainty that will be considered are the uncertainty due to the "laws of randomness" and that due to the lack of knowledge of the "states of nature".

The following example will illustrate the first of these uncertainties. If a coin is tossed, the outcome is not a determined thing and is said to be governed by the "laws of randomness". Now suppose one were offered an amount $\underline{h}$ if the coin falls heads, an amount $\underline{t}$ if it falls tails. Then there would be two possible alternatives or actions, that of playing the game or of not playing. The decision of whether or not to play should take into consideration not only the amounts $\underline{h}$ and $\underline{t}$, but also the laws of randomness governing the outcome of the toss.

The problem would be further complicated if the decision maker did not know which laws of randomness apply. For example, assume that the coin has no defects. The decision maker can then predict that heads and tails are equally likely. However, suppose the coin is bent. In this case it seems reasonable to assume that heads and tails are no longer equally likely and the decision maker is uncertain as to the laws of randomness which apply. The laws of randomness which apply will be called the "state of nature" of the system.

In the above problem the decision maker would like to know something about the state of nature before he makes a decision to play or not to play. Suppose an experiment could be performed on the coin; that is, the coin could be tossed many times. The information gained from this experiment could be used to estimate the state of nature and the decision could then be based on this estimate.

This simple example illustrates the structure of a decision problem. The decision maker is faced with the choice among a set of alternative acts such that the consequence of any actions depends upon the "states of nature". The true state is unknown to the decision maker. However, it is possible to gain information about the state by experimentation.

Let the aggregation of acts be designated by A and a particular action by a. Designate the set of states of nature by Ω and a particular state by ω. Each possible outcome of the experiment will be labeled by an X and the possible outcomes constitute the set χ. The probability of X given the state of nature ω will be denoted by $P(X|\omega)$. In order to clarify the following discussion let A have three possible actions: a_1, a_2, and a_3. Also let Ω have two possible state of nature - ω_1, ω_2, and the set χ to have two experimental outcomes X_1 and X_2. If the decision maker has a plan which tells him which act to perform for each possible outcome of the experiment, this plan will be called a decision function $d(X)$ which is a function of χ into A. For the χ and the A as defined above the possible decision functions are:

Outcomes

	X_1	X_2
d_1	a_1	a_1
d_2	a_1	a_2
d_3	a_1	a_3
d_4	a_2	a_1
d_5	a_2	a_2
d_6	a_2	a_3
d_7	a_3	a_1
d_8	a_3	a_2
d_9	a_3	a_3

Consider for example the decision function d_7. This function assigns outcome X_1 to action a_3 and outcome X_2 to action a_1.

Comparison of Decision Functions

The decision maker's objective in any statistical decision problem is to find a decision function $d(X)$ which is in some sense best or good. In order to judge the decision functions, the decision maker must have some idea of the relative merits of the different actions a for each state of nature ω. Also, assume that the loss suffered can be measured when action a is decided upon and the system is in state ω. Designate this loss by the "loss function" $\ell(a,\omega)$. For the a's and ω's above the loss functions will be as follows:

States of Nature

	ω_1	ω_2
a_1	$\ell(a_1 \mid \omega_1)$	$\ell(a_1 \mid \omega_2)$
a_2	$\ell(a_2 \mid \omega_1)$	$\ell(a_2 \mid \omega_2)$
a_3	$\ell(a_3 \mid \omega_1)$	$\ell(a_3 \mid \omega_2)$

Since the decision function $d(X)$ depends on the outcome X of the experiment, the decision maker will need to compare the decision functions for each possible outcome. A standard procedure for avoiding this situation is to examine the expected loss called the "risk", and is given by

$$R[d(X),\omega] = \Sigma\, \ell[d(X),\omega]\, P(X \mid \omega)$$

It is clear that some criterion is needed for judging a decision function "good" or "bad". A decision function $d(X)$ will be considered good if $R[d(X),\omega]$ is "small" for all states of nature ω. This is not an unreasonable criterion to judge $d(X)$ since $R[d(X),\omega]$ is the expected loss given that the system is in the state ω. However, since it is not known which state of nature is the true state, $R[d(X),\omega]$ must be considered for all possible values of ω.

To be more precise, a decision function $d_1(X)$ is said to be at least as good as $d_2(X)$ if

$$R[d_1(X),\omega] \le R[d_2(X),\omega]$$

for every $\omega_\varepsilon\Omega$; $d_1(X)$ is better than $d_2(X)$ if it is at least as good and for some $\omega_\varepsilon\Omega$.

$$R[d_1(X),\omega] < R[d_2(X),\omega].$$

A decision function $d(X)$ is called "admissible" if there is no decision function better than $d(X)$. A decision function is "inadmissible" if it is not "admissible". If a decision function $d^1(X)$ is inadmissible, that is, if there is another decision function $d^2(X)$ which is better than $d^1(X)$, then $d^1(X)$ no longer needs to be considered.

A class of decision function is "complete" if for every function outside the class there is one in the class which is better; a class is called "essentially complete" if for every decision function outside the class there is one in the class which is at least as good. A class of decision functions is said to be "minimally" complete if it is a complete class such that no proper subset is a complete class. The following simple example of a decision problem will help illustrate some of these concepts.

Let the states of nature be "rain tomorrow" and "no rain tomorrow" and the acts be "stay at home" (and miss work), "go out without an umbrella" (and work in a wet suit if it should rain), "go out with an umbrella". The losses are arbitrarily determined and are given by the following table:

	States of Nature	
Acts	ω_1 (Rain)	ω_2 (No rain)
a_1 (stay home)	4	4
a_2 (go, no umbrella)	5	0
a_3 (go, with umbrella)	2	5

The decision maker must listen to the weather report in order to perform the experiment. From past experience with the weather report, the decision maker is able to assign probabilities to the two outcomes of the experiment, either a forecast of rain or of no rain, given the state of nature. The frequency of responses are given by the following table:

Outcomes	States of Nature	
	ω_1 (Rain)	ω_2 (No rain)
X_1 (Rain)	0.8	0.1
X_2 (No rain)	0.2	0.9

Since there is only a finite number of acts and a finite number of experimental outcomes, the number of decision functions is finite. These decision functions are given in the table on page . The risk function can now be computed as follows:

$$\begin{aligned} R[d^4(X),\omega_1] &= \ell[d^4(x_1),\omega_1]\, P_{X|\omega_1}(x_1) \\ &\quad + \ell[d^4(x_2),\omega_1]\, P_{X|\omega_1}(x_2) \\ &= \ell[a_1,\omega_1] \cdot 0.8 + \ell[a_2,\omega_1] \cdot 0.2 \\ &= 4 \cdot 0.8 + 5 \cdot 0.2 = 3.2 + 1.0 = 4.2 \end{aligned}$$

In a similar manner $R[d(X),\omega]$ can be computed for each combination of decision function $d(X)$ and state of nature ω. The results are:

	State of Nature ω_1	ω_2
d^1	4.0	4.0
d^2	5.0	0.0
d^3	2.0	5.0
d^4	4.2	0.4
d^5	4.8	3.6
d^6	3.6	4.9
d^7	2.4	4.1
d^8	4.4	4.1
d^9	2.6	0.5

From this table it is easily seen that decision functions d^1, d^5, d^6, and d^8 are inadmissible. Decision functions d^2, d^3, d^4, d^7, and d^9 form a minimal complete class. The four decisions d^1, d^5, d^6, and d^8 have been eliminated. However, there is still no clear choice among d^2, d^3, d^4, d^7, and d^9. The problem of choosing among this set will be considered later.

The concepts of complete classes and the class of admissible decision functions will not be discussed further in this report. For theorems concerning these classes, the reader is referred to Chapter Two of Wald [16].

Decision Functions Which Minimize the Maximum Risk

One method for choosing one of the remaining decision functions is called the minimax solution. A decision function $d_o(X)$ is said to be a minimax solution of the decision problem if it minimizes the maximum of the $R[d(X),\omega]$ with respect to the state of nature ω; that is, if $d^{o}(X)$ is such that

$$\operatorname*{Max}_{\omega\in\Omega} R[d^{o}(X),\omega] \leqq \operatorname*{Max}_{\omega\in\Omega} R[d(X),\omega]$$

for all $d(X)$ under consideration. Then $d^{o}(X)$ is said to be the minimax decision function.

Consider the preceding example:

$$\text{Max } R[d^{2}(X),\omega] = 5.0$$

$$\text{Max } R[d^{3}(X),\omega] = 5.0$$

$$\text{Max } R[d^{4}(X),\omega] = 4.2$$

$$\text{Max } R[d^{7}(X),\omega] = 4.1$$

$$\text{Max } R[d^{9}(X),\omega] = 2.6$$

Thus the minimax solution is $d^{9}(X)$; that is, if rain is forecasted, go with an umbrella, and if rain is not forecasted, go without an umbrella.

In the general theory of decision functions much attention has been given to the theory of minimax solutions for two reasons:

1. A minimax solution seems to be a reasonable solution of the decision problem when the decision maker has no prior knowledge about the set of states of nature Ω;
2. The theory of minimax solutions play an important role in deriving the basic results concerning complete classes of decision functions.

Theorems concerning complete classes and minimax solutions can be found in Chapter III of Wald [16].

Objection to the Minimax Solution

There are two major objections often raised to the minimax solution. In many problems the minimax solution is too pessimistic. Consider the following example.

Suppose there is a rumor that the ABC Aircraft Corporation has landed a government contract. Assume there are three states of nature: ω_1 (no contract), ω_2 (small contract), and ω_3 (extremely large contract.) The problem is for the decision maker to decide whether to buy (a_1) or not to buy (a_2) \$500 worth of stock in the company. Also assume that at the present time the value of the company's stock is steadily decreasing, so if the decision maker invests and there is no contract with the government a portion of the investment will be lost. Suppose the gains and losses are as given in the following table:

Acts	States of Nature ω_1	ω_2	ω_3
a_1 (invest)	200	-200	-1000
a_2 (do not invest)	0	0	0

Now suppose the decision maker has a stock broker in New York. He can call on his stock broker to check into this rumor. However, this broker has been known to make mistakes. The frequency of response based on past dealings with this broker is:

Experimental Outcome	State of Nature		
	ω_1	ω_2	ω_3
x_1 (contract)	0.2	0.7	0.5
x_2 (no contract)	0.8	0.3	0.5

The possible decision functions are:

	$d^1(X)$	$d^2(X)$	$d^3(X)$	$d^4(X)$
x_1	a_1	a_1	a_2	a_2
x_2	a_1	a_2	a_1	a_2

The risk for each decision $d(X)$ and state of nature ω is:

	State of Nature		
	ω_1	ω_2	ω_3
$d^1(X)$	200	-200	-1000
$d^2(X)$	40	-140	- 500
$d^3(X)$	160	- 60	- 500
$d^4(X)$	0	0	0

It can be seen from this table that the minimax solution for the decision maker is $d^4(X)$ which is not to invest in any case. Although the decision maker would not lose anything with this solution, he would also never gain anything by following this solution. Such pessimistic decisions would not be conducive to any successful business.

The second objection often raised to the minimax solution is that it does not take into account information the decision maker may have about the set of states of nature prior to conducting the experiment. In the preceding example, suppose instead of just a rumor about the contract, the decision maker can describe his information about the set of states of nature, Ω, by a probability distribution $P(\omega)$; for example

$$P(\omega_1) = 7/10$$

$$P(\omega_2) = 2/10$$

$$P(\omega_3) = 1/10$$

This means that the decision maker knows from past experience that 7/10 of the time the state of nature ω_1 will be the true state, 2/10 of the time ω_2 will be the true state, and 1/10 of the time ω_3 will be the true state. If this is the case the decision maker can obtain the Bayes' solution to the problem, which will be discussed next.

Bayes' Solution to the Decision Problem

Consider a situation of a medical doctor making a diagnosis of a patient's illness. If the doctor was only to take into consideration the results of the x-rays taken at the time and not the past medical history of his patient, he may have a somewhat limited diagnosis of the illness. The same principle applies in the Bayes' solution with the distribution $P(\omega)$, called the prior probability distribution on Ω, representing the probability based on the decision maker's prior information of the true state of nature. For a given $P(\omega)$, defined the "Bayes risk" for each $d(X)$ as

$$B[d(X); P(\omega)] = R[d(X), \omega_1] P(\omega_1) + R[d(X), \omega_2] P(\omega_2) + + \dots R[d(X), \omega_m] P(\omega_m)$$

if there are m ω's in Ω. Then the "Bayes' solution" is defined to be the $d(X)$ which minimizes the Bayes risk.

In the investment example in the preceding section, the prior distribution was given. The quantities $B[d(X), P(\omega)]$ are as follows:

$$B[d^1(X); P(\omega)] = (200)\ 7/10 - (200)\ 2/10 - (1000)\ 1/10 = 0$$

$$B[d^2(X); P(\omega)] = (40)\ 7/10 - (140)\ 2/10 - (500)\ 1/10 = -50$$

$$B[d^3(X); P(\omega)] = (160)\ 7/10 - (60)\ 2/10 - (500)\ 1/10 = 50$$

$$B[d^4(X); P(\omega)] = 0$$

Thus the Bayes' solution is $d^2(X)$; to invest if the broker reports a contract and not to invest if he reports no contract.

Summary

In order to apply the principles discussed in this chapter, the decision maker must be able to determine:

(1) the losses given the acts and state of nature;

(2) the distribution function or frequency of responses of the experiment;

(3) the prior probabilities or prior weights on the states of nature.

[illegible] the examples given in this chapter the losses and prior prob- [illegible] were arbitrarily determined in order to demonstrate the prob- [illegible] determination of these elements of the problem depend upon [illegible] problem. For example, in problems related to business [illegible] maker can usually ascertain the monetary loss for each [illegible] state of nature. If nc monetary loss is involved, the [illegible] must settle on some form of utility of each act, given [illegible] nature. The reader is referred to von Neumann and [illegible] or Davidson [4] for further discussion of utility [illegible] the determination of the frequency of response of the [illegible] the reader is referred to an elementary statistics book. [illegible] probabilities can be either the decision maker's subjective [illegible] states of nature prior to conducting the experiments, or [illegible] based on objective information known to the decision maker [illegible] experiment is conducted. The following chapter will attempt [illegible] further the determination of these elements of the problem [illegible]

CHAPTER IV

APPLICATION OF BAYES PROCEDURE:
PERSONNEL SELECTION

Introduction

Suppose the ABC corporation needs a decision rule in their personnel department which will establish a fixed procedure for the selection of new employees. In order to establish this procedure, the theory of the preceding chapter is applied.

There are two types of information on which the decision rule is based. First, there is the prior information about the applicant which includes such things as grade point average, graduate degree, information obtained from the interview, etc. This type of information is used as a basis for the prior probability distribution on the states of nature. The second type of information is the result of a test designed to measure the aptitude of the applicant in his particular field of interest.

Determination of the Loss Table

The problem is to determine, based on these two types of information, what salary should be offered to the applicant if the company should, in fact, make him an offer. Suppose there are three salaries which the company is willing to offer for a new employee. These salaries are:

1. \$700 per month

2. \$800 per month

3. \$900 per month.

Of course, the company does not have to make an offer to a prospective employee. Thus, the acts which this company can take are:

a_1 - do not hire

a_2 - hire at \$700 per month

a_3 - hire at \$800 per month

a_4 - hire at \$900 per month.

Assume it is possible to determine, with some degree of accuracy, the placement of each of the present employees into four levels of average monetary worth to the company. Suppose these four levels are:

ω_1 - worth \$400 per month for the first year and will lose the company \$25,000 if he remains with the company until retirement;

ω_2 - worth \$600 per month for the first year and will make his salary each year if he remains until retirement;

ω_3 - worth \$800 per month for the first year and will net the company \$25,000 if he remains with the company until retirement;

ω_4 - worth \$1,000 per month for the first year and will net the company \$50,000 if he remains with the company until retirement.

These states of nature are denoted by the ω's.

According to the employment records of the company, the personnel department finds that if an employee is in state of nature ω_1, and if he was hired at \$700 per month, there is an 80 per cent chance of his remaining with the company until retirement. If the same employee was hired at \$800 per month, there is a 90 per cent change of his remaining with the company until retirement. Table I gives the percentage of employees which remain with the company until retirement, for each act and state of nature combination.

TABLE I

PERCENTAGE OF EMPLOYEES THAT REMAIN

Acts \ States of Nature	ω_1	ω_2	ω_3	ω_4
a_1	.00	.00	.00	.00
a_2	.80	.80	.70	.50
a_3	.90	.90	.80	.70
a_4	.95	.90	.90	.80

It costs the company \$200 to interview, process, and administer the aptitutde test to each prospective employee. It costs the company an additional \$100 to hire an applicant.

Now it is possible to determine the loss function or table for these acts and states of nature. Suppose the "true" state of nature is ω_1, and a_1 is decided upon by the decision maker. Then the only loss is the \$200 required for the interviewing, processing, and administering the test. If, in fact, a_1 is the action taken, then the loss is \$200 for each state of nature. Now suppose the "true" state of nature is ω_1, and a_2 is decided upon. In this case the loss is \$200 plus \$100 for hiring cost plus 12(\$300) = \$3,600 lost the first year on the employee plus 80 per cent of $25,000 lost over the remaining period of his employment. The total loss is:

$$\$200 + \$100 + \$3,600 + \$20,000 = \$23,900 .$$

Next, suppose the "true" state of nature is ω_3, and a_2 is the action taken. The loss is:

$$\$200 + \$100 + 12(-\$100) + .7(-\$25,000) = -\$18,400$$

The minus sign indicates gain. The other losses are similarly determined. Table II gives the entire loss function.

TABLE II

LOSS TABLE
(Hundreds)

Acts \ State of Nature	ω_1	ω_2	ω_3	ω_4
a_1	2	2	2	2
a_2	239	15	-184	-283
a_3	276	27	-197	-371
a_4	301	39	-210	-409

Frequency of Response Table

In order for the company to establish a criteria for judging an applicant's performance on the aptitude test, the test was given to all the present employees. It was found that 70 per cent of those in state of nature ω_1 had a score between zero and twenty out of a possible sixty points. Thirty per cent of those in state of nature ω_1 made between twenty-one and forty on the test. Let X_1 denote the outcome of the score between zero and twenty; X_2, the outcome of the score between twenty-one and forty; X_3, the outcome of the score between forty-one and sixty. The frequencies of response for each state of nature and outcome combination are given in Table III.

TABLE III

FREQUENCY OF RESPONSE

Outcome \ State of Nature	ω_1	ω_2	ω_3	ω_4
X_1	.70	.30	.10	.00
X_2	.30	.50	.40	.20
X_3	0	.20	.50	.80
	1.00	1.00	1.00	1.00

Decision Functions

In this problem there are four acts and three possible outcomes to the experiment. In general, when there are m acts and n outcomes there are m^n possible decision functions. Thus, in this case, there are 4^3 possible decision functions or a total of sixty-four. Table IV lists the possible decision functions for this problem.

TABLE IV

POSSIBLE DECISION FUNCTIONS

Outcome \ Decision Function	$d^1(X)$	$d^2(X)$	$d^3(X)$	$d^4(X)$	$d^5(X)$	$d^6(X)$	$d^7(X)$	$d^8(X)$
X_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
X_2	a_1	a_1	a_1	a_1	a_2	a_2	a_2	a_2
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	$d^9(X)$	$d^{10}(X)$	$d^{11}(X)$	$d^{12}(X)$	$d^{13}(X)$	$d^{14}(X)$	$d^{15}(X)$	$d^{16}(X)$
X_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
X_2	a_3	a_3	a_3	a_3	a_4	a_4	a_4	a_4
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	$d^{17}(X)$	$d^{18}(X)$	$d^{19}(X)$	$d^{20}(X)$	$d^{21}(X)$	$d^{22}(X)$	$d^{23}(X)$	$d^{24}(X)$
X_1	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
X_2	a_1	a_1	a_1	a_1	a_2	a_2	a_2	a_2
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	$d^{25}(X)$	$d^{26}(X)$	$d^{27}(X)$	$d^{28}(X)$	$d^{29}(X)$	$d^{30}(X)$	$d^{31}(X)$	$d^{32}(X)$
X_1	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
X_2	a_3	a_3	a_3	a_3	a_4	a_4	a_4	a_4
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4

TABLE IV (Continued)

Outcome \ Decision Function	$d^{33}(X)$	$d^{34}(X)$	$d^{35}(X)$	$d^{36}(X)$	$d^{37}(X)$	$d^{38}(X)$	$d^{39}(X)$	$d^{40}(X)$
X_1	a_3	a_3	a_3	a_3	a_3	a_3	a_3	a_3
X_2	a_1	a_1	a_1	a_1	a_2	a_2	a_2	a_2
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4

Outcome \ Decision Function	$d^{41}(X)$	$d^{42}(X)$	$d^{43}(X)$	$d^{44}(X)$	$d^{45}(X)$	$d^{46}(X)$	$d^{47}(X)$	$d^{48}(X)$
X_1	a_3	a_3	a_3	a_3	a_3	a_3	a_3	a_3
X_2	a_3	a_3	a_3	a_3	a_4	a_4	a_4	a_4
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4

Outcome \ Decision Function	$d^{49}(X)$	$d^{50}(X)$	$d^{51}(X)$	$d^{52}(X)$	$d^{53}(X)$	$d^{54}(X)$	$d^{55}(X)$	$d^{56}(X)$
X_1	a_4	a_4	a_4	a_4	a_4	a_4	a_4	a_4
X_2	a_1	a_1	a_1	a_1	a_2	a_2	a_2	a_2
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4

Outcome \ Decision Function	$d^{57}(X)$	$d^{58}(X)$	$d^{59}(X)$	$d^{60}(X)$	$d^{61}(X)$	$d^{62}(X)$	$d^{63}(X)$	$d^{64}(X)$
X_1	a_4	a_4	a_4	a_4	a_4	a_4	a_4	a_4
X_2	a_3	a_3	a_3	a_3	a_4	a_4	a_4	a_4
X_3	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4

Comparison of the Decision Functions

At this point the risk, as described in the preceding chapter, can be found for each of these decision functions. For example,

$$\begin{aligned} R[d^1(X),\omega_1] &= \ell(a_1,\omega_1)\cdot P(X|\omega_1) + \\ &+ \ell(a_1,\omega_1)\cdot P(X_2|\omega_1) \\ &+ \ell(a_1,\omega_1)\cdot P(X_1|\omega_3) \\ &= 2(.70) + 2(.3) + 2(0) \\ &= 2.00 \end{aligned} \tag{4.1}$$

Similarly, the risk for each decision function and ω combination is calculated. These risks are given in Table V.

Now the inadmissible decision functions can be eliminated from further consideration. As defined previously, $d^K(X)$ is inadmissible if there is another decision function $d^P(X)$ such that

$$R[d^K(X),\omega] \geq R[d^P(X),\omega]$$

for all $\omega\epsilon\Omega$ and for some $\omega\epsilon\Omega$

$$R[d^K(X),\omega] > R[d^P(X),\omega] \ .$$

Take for example, $d^5(X)$ in the risks table. Here,

$$\begin{aligned} R[d^5(X),\omega_1] &= 73.1 > 2.0 = R[d^3(X),\omega_1] \\ R[d^5(X),\omega_2] &= 8.5 > 7.0 = R[d^3(X),\omega_2] \\ R[d^5(X),\omega_3] &= -72.4 > -97.5 = R[d^3(X),\omega_3] \\ R[d^5(X),\omega_4] &= -55.0 > -296.4 = R[d^3(X),\omega_4] \ . \end{aligned}$$

TABLE V

RISKS TABLE

State of Nature \ Decision Function	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8
ω_1	2.0	2.0	2.0	2.0	73.1	73.1	73.1	73.1
ω_2	2.0	4.6	7.0	9.4	8.5	11.1	13.5	15.9
ω_3	2.0	-91.0	-97.5	-104.0	-72.4	-165.4	-171.9	-178.4
ω_4	2.0	-226.0	-296.4	-326.8	-55.0	-283.0	-353.4	-383.8
	d^9	d^{10}	d^{11}	d^{12}	d^{13}	d^{14}	d^{15}	d^{16}
ω_1	84.2	84.2	84.2	84.2	91.7	91.7	91.7	91.7
ω_2	14.5	17.1	19.5	21.9	20.5	23.1	25.2	28.7
ω_3	-77.6	-170.6	-177.1	-183.6	-82.8	-175.8	-192.3	-188.8
ω_4	-72.6	-300.6	-371.0	-401.4	-80.2	-268.2	-378.6	-409.0
	d^{17}	d^{18}	d^{19}	d^{20}	d^{21}	d^{22}	d^{23}	d^{24}
ω_1	167.9	167.9	167.9	167.9	239.0	239.0	239.0	239.0
ω_2	5.9	8.5	10.9	13.3	12.4	15.0	17.4	19.8
ω_3	-17.2	-109.6	-116.1	-122.6	-91.0	-184.0	-190.5	-197.0
ω_4	2.0	-226.0	-296.4	-326.8	-55.0	-283.0	-353.4	-383.8
	d^{25}	d^{26}	d^{27}	d^{28}	d^{29}	d^{30}	d^{31}	d^{32}
ω_1	243.1	243.1	243.1	243.1	257.6	257.6	257.6	257.6
ω_2	18.4	21.0	23.4	25.8	24.2	27.0	30.4	31.8
ω_3	-96.2	-189.2	-195.7	-202.2	-101.4	-194.4	-200.9	-207.4
ω_4	-72.6	-300.6	-371.0	-401.4	-80.2	-308.2	-378.6	-409.0

TABLE V (Continued)

State of Nature \ Decision Function	d^{33}	d^{34}	d^{35}	d^{36}	d^{37}	d^{38}	d^{39}	d^{40}
ω_1	193.8	193.8	193.8	193.8	264.9	264.9	264.9	264.9
ω_2	9.5	12.1	14.5	16.9	16.0	18.6	21.0	23.4
ω_3	-17.9	-110.9	-126.4	-123.9	-92.3	-185.3	-191.8	-198.3
ω_4	2.0	-226.0	-296.4	-326.8	-55.0	-283.0	-353.4	-383.8
	d^{41}	d^{42}	d^{43}	d^{44}	d^{45}	d^{46}	d^{46}	d^{48}
ω_1	276.0	276.0	276.0	276.0	313.6	313.6	313.6	313.6
ω_2	22.0	11.1	27.0	29.4	28.0	30.6	33.0	35.4
ω_3	-97.5	-190.5	-197.0	-203.5	-34.3	-127.3	-202.2	-208.7
ω_4	-73.2	-300.6	-371.0	-401.4	-80.2	-308.2	-378.6	-409.0
	d^{49}	d^{50}	d^{51}	d^{52}	d^{53}	d^{54}	d^{55}	d^{56}
ω_1	211.3	211.3	211.3	211.3	282.4	282.4	282.4	282.4
ω_2	13.1	15.7	18.1	205.	19.6	22.2	24.6	27.0
ω_3	-19.2	-112.2	-118.7	-185.2	-93.6	-186.6	-193.1	-199.6
ω_4	2.0	-226.0	-296.4	-326.8	-55.0	-283.0	-353.4	383.8
	d^{57}	d^{58}	d^{59}	d^{60}	d^{61}	d^{62}	d^{63}	d^{64}
ω_1	293.5	293.5	293.5	293.5	301.0	301.0	301.0	301.0
ω_2	25.6	32.7	30.6	33.0	31.6	34.2	36.6	39.0
ω_3	-98.8	-191.8	-198.3	-204.8	-104.0	-197.0	-203.5	-210.0
ω_4	-72.2	-300.6	-371.0	-401.4	-80.2	-308.2	-378.6	-409.0

Thus, $d^5(X)$ is inadmissible. After eliminating the inadmissible decision functions, the remaining decision functions are given in Table VI.

At this point in the analysis the minimax solution can be ascertained. In order to do this the maximum risk over the states of nature is found. Then the decision function $d^K(X)$ with the minimum-maximum risk is the minimax solution. In this case the minimax solution is $d^1(X)$. This function indicates not to hire no matter what score is made on the test. The reason for this outcome is the manner in which the problem was constructed. None of the prior information, such as grade point, interview, etc., was considered in obtaining the solution.

Consideration of Prior Information

The personnel department decided that the following information should be included in the establishment of a prior probability distribution:

1. grade point average,
2. graduate degree,
3. interview,
4. references,
5. experience,
6. extracurricular activities,
7. military status.

At this point, consider the grade point average. The personnel department finds that of the employees with grade point averages between

TABLE VI

RISKS OF ADMISSIBLE DECISION FUNCTIONS

State of Nature \ Decision Function	d^1	d^2	d^3	d^4	d^6	d^7	d^8	d^{12}
ω_1	2.0	2.0	2.0	2.0	73.1	73.1	73.1	84.2
ω_2	2.0	4.6	7.0	9.4	11.1	13.5	15.9	21.9
ω_3	2.0	-91.0	-97.5	-104.0	-165.4	-171.9	-178.4	-183.6
ω_4	2.0	-226.0	-296.4	-326.8	-283.0	-353.4	-383.8	-401.4
	d^{15}	d^{16}	d^{18}	d^{19}	d^{20}	d^{22}	d^{23}	d^{24}
ω_1	91.7	91.7	167.9	167.9	167.9	239.0	239.0	239.0
ω_2	25.2	28.7	8.5	10.9	13.3	15.0	17.4	19.8
ω_3	-192.3	-188.8	-109.6	-116.1	-122.6	-184.0	-190.5	-197.0
ω_4	-378.6	-409.0	-226.0	-296.4	-326.8	-283.0	-353.4	-383.8
	d^{28}	d^{32}	d^{39}	d^{40}	d^{42}	d^{44}	d^{48}	d^{52}
ω_1	243.1	257.6	264.9	264.9	276.0	276.0	313.6	211.3
ω_2	25.8	31.8	21.0	23.4	11.1	29.4	35.4	20.5
ω_3	-202.2	-207.4	-191.8	-198.3	-190.5	-203.5	-208.7	-185.2
ω_4	-401.4	-409.0	-353.4	-383.8	-300.6	-401.4	-409.0	-326.8
	d^{64}							
ω_1	301.0							
ω_2	39.0							
ω_3	-210.0							
ω_4	-409.0							

2.0 and 2.3, on a 4.0 basis, 40 per cent are in state of nature ω_1, 55 per cent in state of nature ω_2, and 5 per cent in state of nature ω_3. Table VII gives the percentage breakdown for each state of nature and grade point interval combination.

TABLE VII

GRADE POINT AVERAGE

Grade Point \ State of Nature	ω_1	ω_2	ω_3	ω_4
2.0 - 2.3	.40	.55	.05	.00
2.3^+- 2.6	.20	.65	.15	.00
2.6^+- 2.9	.20	.55	.20	.05
2.9^+- 3.2	.10	.50	.30	.10
3.2^+- 3.5	.05	.45	.40	.10
3.5^+- 4.0	.00	.30	.50	.20

Similarly, Tables VIII through XIII give the percentage breakdowns for graduate degree, interview, references, experience, extracurricular activities, and military status, respectively.

TABLE VIII

GRADUATE DEGREE

Degree \ State of Nature	ω_1	ω_2	ω_3	ω_4
M.S., M.B.A., MA., L.L.B., etc.	.10	.40	.30	.20
Ph.D., D.B.A., etc.	.00	.10	.50	.40

TABLE IX

INTERVIEW

Interview \ State of Nature	ω_1	ω_2	ω_3	ω_4
Below Average	.30	.60	.10	.00
Average	.20	.50	.20	.10
Above Average	.10	.20	.40	.30

TABLE X

REFERENCES

Reference \ State of Nature	ω_1	ω_2	ω_3	ω_4
Below Average	.40	.50	.10	.00
Average	.20	.30	.30	.20
Above Average	.10	.20	.50	.20

TABLE XI

EXPERIENCE

Experience \ State of Nature	ω_1	ω_2	ω_3	ω_4
No Experience	.30	.50	.10	.10
Experience Not in Field	.20	.30	.30	.20
Experience in Field	.10	.20	.50	.20

TABLE XII

EXTRACURRICULAR ACTIVITIES

Activities \ State of Nature	ω_1	ω_2	ω_3	ω_4
Below Average	.40	.40	.20	.00
Average	.20	.30	.40	.10
Above Average	.10	.20	.40	.30

TABLE XIII

MILITARY STATUS

Obligation \ State of Nature	ω_1	ω_2	ω_3	ω_4
Unfulfilled	.20	.60	.10	.10
Fulfilled	.20	.20	.40	.20

Each of the categories described above are given points according to their relative merit. Suppose it is decided that the grade point average is worth 50 points, graduate degree 40 points, interview 40 points, references 15 points, experience 5 points, extracurricular activities 5 points, and military status 5 points. At this point an example will best illustrate the use of the preceding tables in obtaining a prior distribution.

Construction of the Prior Probability Distribution

Suppose J. Q. Student has sent in an application, resumé, and has been interviewed. According to the information from these sources he has a grade point average of 2.3, a B.S. degree, a below average interview, below average references, no work experience, below average activities, and has not fulfilled his military obligation.

In considering his grade point average,

$$.40(50) = 20 \text{ points}$$

are assigned to state of nature ω_1;

$$.55(50) = 27.5 \text{ points}$$

are assigned to state of nature ω_2;

$$.05(50) = 2.5 \text{ points}$$

are assigned to state of nature ω_3; and

$$.00(50) = 0 \text{ points}$$

are assigned to state of nature ω_4. Points are assigned to the states of nature for each of the other classifications in a similar manner. The results of these tabulations are given in Table XIV.

TABLE XIV

PRIOR WEIGHTS ON STATES OF NATURE

Prior Consideration \ State of Nature	ω_1	ω_2	ω_3	ω_4
GPA	20.0	27.5	2.5	0.0
Interview	6.0	12.0	2.0	0.0
References	6.0	7.5	1.5	0.0
Experience	1.5	2.5	0.5	0.5
Activities	2.0	2.0	1.0	0.0
MiL. Status	1.0	3.0	0.5	0.5
Total	36.5	54.5	8.0	1.0

There are a total of one hundred points in this table. Thus the prior probabilities on the states of nature are:

$$
\begin{aligned}
P(\omega_1) &= \frac{36.5}{100} = .365 \\
P(\omega_2) &= \frac{54.5}{100} = .545 \\
P(\omega_3) &= \frac{8.0}{100} = .08 \\
P(\omega_4) &= \frac{1.0}{100} = .01
\end{aligned}
\tag{4.2}
$$

Now the Bayes risk for $d^K(X)$ is

$$
\begin{aligned}
B[d^K(X);P(\omega)] &= R[d^K(X),\omega_1]\,P(\omega_1) \\
&+ R[d^K(X),\omega_2]\,P(\omega_2) \\
&+ R[d^K(X),\omega_3]\,P(\omega_3) \\
&+ R[d^K(X),\omega_4]\,P(\omega_4)
\end{aligned}
\tag{4.3}
$$

For the prior probabilities given in equation (4.2), the best decision function is $d^2(X)$. The Bayes risk for this decision function is

$$B[d^2(X);\ P(\omega)] = -\ 6.960.$$

For comparison, some of the other Bayes risks are:

$$
\begin{aligned}
B[d^3(X);\ P(\omega)] &= -\ 6.219 \\
B[d^4(X);\ P(\omega)] &= -\ 5.735 \\
B[d^{12}(X);\ P(\omega)] &= 23.567.
\end{aligned}
$$

Therefore, for J. Q. Student the best decision rule is not to hire if he makes between zero and forty on the test and to hire at $700 per month if he makes between forty-one and sixty on the test.

Suppose Joe Average submits an application to the company. Joe has a grade point average of 2.5, a M.B.A. degree, average interview, average references, work experience not related to his field, average activities, and he has not fulfilled his military obligation. In this case there are 140 total points. It is found that 24 points are in ω_1, 69 points in ω_2, 32 points in ω_3, and 15 points in ω_4. Thus the prior probabilities on the states of nature for this applicant are:

$$\begin{aligned} P(\omega_1) &= \frac{24.0}{140} \cong .17 \\ P(\omega_2) &= \frac{69.0}{140} \cong .49 \\ P(\omega_3) &= \frac{32.0}{140} \cong .23 \\ P(\omega_4) &= \frac{15.0}{140} \cong .11 \end{aligned} \tag{4.4}$$

The best decision function for Mr. Average is $d^8(X)$. The Bayes risk for this decision function is:

$$B[d^8(X);\ P(\omega)] = -\ 63.032$$

For comparison, some of the other Bayes risks for this application are:

$$\begin{aligned} B[d^4(X);\ P(\omega)] &= -\ 54.922 \\ B[d^6(X);\ P(\omega)] &= -\ 51.206 \\ B[d^7(X);\ P(\omega)] &= -\ 59.396 \\ B[d^{12}(X);\ P(\omega)] &= -\ 61.337. \end{aligned}$$

Therefore, the best decision rule for Joe Average is not to hire if he scores between zero and twenty on the test, hire at \$700 per month if he scores between twenty-one and forty on the test, and hire at $900 per month if he scores between forty-one and sixty on the test.

Now suppose Bill P. Brain sends an application to the company and was interviewed by a company representative. This information revealed that Mr. Brain has a grade point average of 3.1, a M.B.A. degree, above average interview, average references, work experience related to his field, average activities, and he has fulfilled his military obligation. In this case there are 140 total points again. It is found that 16.5 points are in ω_1, 52.5 are in ω_2, 46.0 are in ω_3, 25.0 are in ω_4. Thus the prior probabilities on the states of nature for this applicant are:

$$\begin{aligned} P(\omega_1) &= \frac{16.5}{140} \cong .12 \\ P(\omega_2) &= \frac{52.5}{140} \cong .37 \\ P(\omega_3) &= \frac{46.0}{140} \cong .33 \\ P(\omega_4) &= \frac{25.0}{140} \cong .18 \end{aligned} \tag{4.5}$$

The best decision function for Mr. Brain then is $d^{12}(X)$. The Bayes risk for this decision function is

$$B[d^{12}(X);\ P(\omega)] = -\ 114.633$$

For comparison, some of the other Bayes risks are:

$$\begin{aligned} B[d^{8}(X);\ P(\omega)] &= -\ 114.301 \\ B[d^{15}(X);\ P(\omega)] &= -\ 111.279 \\ B[d^{16}(X);\ P(\omega)] &= -\ 114.301 \\ B[d^{28}(X);\ P(\omega)] &= -\ 100.160 \end{aligned}$$

Therefore, the best decision rule for Bill Brain is not to hire if he makes between zero and twenty on the test, hire at $800 per month if he makes between twenty-one and forty on the test, and hire at $900 per month if he makes between forty-one and sixty on the test.

Summary

It is clear that in this problem, though it is more realistic than the examples given in Chapter III, several simplifying assumptions are necessary. The most prominent is the assumption that each of the present employees of this company can be placed in one of the states of nature as they were defined. In order to make this problem more realistic, perhaps more states of nature should have been defined. Also, the monetary values assigned to the states of nature would need to be adjusted for price level changes.

One thing which was not considered is the fact that not all applicants will accept if the company makes then an unattractive offer. This problem could have been alleviated by defining further acts which include the possibility of increasing the amount of an offer in case the first offer is rejected. However, increasing the number of acts greatly increased the number of decision functions. For example, suppose in this case there had been ten acts. This would have increased the number of decision functions to

$$10^3 = 1000 .$$

If a computer were available to the decision maker, many decision functions could have been handled with ease. However, for this report, a small number of decision functions is necessary.

Even with these limitations, this application presents a logical procedure for weighing and combining the information available. It also provides a method whereby both the subjective and objective information is considered while eliminating subjectivity in the procedure.

CHAPTER V

SUMMARY

The problem facing any decision maker is that of constructing a procedure which will take into consideration all available information. The Bayes' procedure provides a logical framework for working with economic losses or the utility of alternative courses of action, the prior information available to the decision maker, and formal modification of this prior information with the introduction of more current knowledge.

The basic problem was described, decision functions were defined, and the application of the Bayes' procedure was outlined. The minimax solution to the decision problem was also discussed; however, it was concluded that when economic losses are involved this solution tends to be too conservative for business applications. A realistic, though somewhat academic, application of the Bayes' procedure was constructed.

Although this report deals with the Bayes' procedure, the Bayes' formula per se did not appear either in the theory or the application. The formula was used, but in a somewhat disguised manner. Suppose there are k states of nature,

$$\omega_1, \omega_2, \ldots, \omega_k;$$

and n possible outcomes to the experiment,

$$X_1, X_2, \ldots, X_n .$$

The risk was defined as

$$R[d(X),\ \omega j] = \sum_{i=1}^{n} \ell[d(X_i),\ \omega_j]\ P[X_i|\omega_j]$$

where $\ell[d(X_i),\ \omega_j]$ is the loss incurred when X_i is observed and the true state of nature is ω_j, and where $P[X_i|\omega_j]$ is the probability that X_i is observed when the true state of nature is ω_j. The Bayes risk is then

$$B[d(X);\ P(\omega)] = P(\omega_1) \sum_{i=1}^{n} \ell[d(X_i),\ \omega_1]\ P[X_i|\omega_1]$$

$$+ P(\omega_2) \sum_{i=1}^{n} \ell[d(X_i),\omega_2]\ P[X_i|\omega_2] + \ldots +$$

$$P(\omega_k) \sum_{i=1}^{n} \ell[d(X_i),\omega_k]\ P[X_i|\omega_k] =$$

$$\sum_{i=1}^{n} \ell[d(X_i),\omega_1] \left\{P[\omega_1]\ P[X_i|\omega_1]\right\} +$$

$$\sum_{i=1}^{n} \ell[d(X_i),\omega_2] \left\{P[\omega_2]\ P[X_i|\omega_2]\right\} + \ldots +$$

$$\sum_{i=1}^{n} \ell[d(X_i),\omega_k] \left\{P[\omega_k]\ P[X_i|\omega_k]\right\} =$$

$$\sum_{i=1}^{n} \ell[d(X_i),\omega_1]\ C_i\ P[\omega_1|X_i] +$$

$$\sum_{i=1}^{n} \ell[d(X_i),\omega_2]\ C_i\ P[\omega_2|X_i] + \ldots +$$

$$\sum_{i=1}^{n} \ell[d(X_i),\omega_k]\ C_i\ P[\omega_k|X_i]\ .$$

where

$$P[\omega_j|X_i] = \frac{P[\omega_j]\, P[X_i|\omega_j]}{C_i} \tag{5.1}$$

and

$$C_i = \sum_{j=1}^{k} P[\omega_j]\, P[X_i|\omega_j] \, .$$

Equation (5.1) is Bayes' formula. Thus, Bayes' formula was used; but, as stated before, was somewhat disguised.

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