# MATERIAL STUDIES FOR MODEL TESTING OF CONCRETE STRUCTURAL ELEMENTS 

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## CHAPTER I

## INTRODUCTION

## General Discussion of Structural Models

Structural models are known to be valuable engineering tools, and a great amount of time and effort are involved in their construction and testing throughout the world. Model studies are carried on by large design centers and research and educational institutions in order to expand the knowledge of structures and structural materials, and to check both ideas and theory before they are put into use.

The American Concrete Institute, Portland Cement Association, and American Institute of Steel Construction are known for their work in this area, as are educational institutions such as Lehigh University, Massachusetts Institute of Technology, and others. The studies at these research centers are carefully controlled and well equipped, and the results of their investigations are commonly used as framework for building codes and are frequently used for classroom instruction. Authoritative textbooks on the principles of structural design have been written by those concerned with the building and testing of structural models.

Mr. Felix Candela of Mexico is well known for his tests
of full scale structural elements. In some instances he has built and tested whole components of his buildings rather than using a doubtful analytical design method. When he began working with the inverted umbrella in 1952, he made a full scale element in Vallejo, D. F., Mexico. Working with only some sketches and a few ideas, the structure was built and loaded, and then deflections were recorded. The measured deflections were found to be excessive and corrections were made to the design before the actual structure was built. This exaggerated use of models is not intended as a recommended procedure in this country, but illustrates that they are and can be of service to the architect and engineer.


Figure 1. Testing a Full Scale Element in Mexico (Faber, 1963)

During the summer of 1965 , the author and others conducted model studies of microconcrete. The object was to study the use of microconcrete models to predict the behavior of prototype concrete structures under static loads, and to investigate the influence of scale on the ultimate strength of compression and flexural models. To reduce the possible errors due to size effect, simply supported beams of $6^{\prime} 0^{\prime \prime}$ clear span and standard compression cylinders of $6^{\prime \prime}$ diameter were selected as prototypes. Model beams were made with clear spans of $3^{\prime} 0^{\prime \prime}, 1^{\prime} 6^{\prime \prime}$, and $0^{\prime} 9^{\prime \prime}$, and model cylinders were made with diameters of $3^{\prime \prime}, 11 / 2^{\prime \prime}$, and $3 / 4^{\prime \prime}$.

Three samples were made of each size and type of model to obtain an average value for strength of each mix design. Both Type I and Type III cements (ASTM Designation: C 15064) were used, and each mix was made from the same bag of cement in order to eliminate variations of cement. Curing was done in a moist room, and testing was carried out after 28 days of curing for Type $I$ cement, and after 8 days for Type III cement. The results of such a limited number of tests were not regarded as conclusive, but the experience of model building and testing techniques was of considerable value.

The decision was made to continue the research methods learned previously, and to investigate the possibilities of using different materials for study. It was hoped that a material (or combination of materials) having the same
physical properties as concrete could be developed, which would enable one to make models of concrete prototypes which would behave in a similar manner to that of the concrete.

## Purpose of Investigation

The plan for this thesis is to develop appropriate materials for a model system which will follow the behavior pattern of full scale concrete structural elements at failure. Such a system would reduce the time, effort, and expense that is now necessary for structural testing of full scale concrete prototypes. Although full scale tests are laborious, they are of ten necessary to determine the actual performance of structural elements before their use is allowed. An often neglected, although valid engineering tool, is the construction of a scale model of the structure. By using models, many of the actual loading conditions and strength characteristics of structures can be investigated.

In this thesis, therefore, certain model materials and techniques are investigated to determine if their behavior can be correlated to the behavior of concrete prototypes. It is hoped that, after adequate study, a close approximation can be made between model and prototype, and as a result of this and other investigations, models will have a more general and useful place in architecture and engineering.

## CHAPTER II

## METHODS AND MATERIALS OF MODEL MAKING

Webster's New Collegiate Dictionary defines a model as a miniature representation of a thing. In this case the things to be represented are test specimens used in concrete laboratory studies. The properties or qualities of these prototypes which are to be represented must be decided before any model study can begin.

The theory of structural models deals with the physical variables used in the description of the properties of the structural elements, and the values which these variables assume in the model and the prototype. Physical variables are the variables describing those properties of the element which determine its physical behavior. As a result of previous studies of concrete and microconcrete models, it is believed that sufficient background material on the behavior of concrete is available to justify this investigation.

The American Society of Testing Materials, as a result of its research experience, has proposed standard methods of tests for structural materials. These standards are widely recognized throughout the structures field, and in some instances are the only methods of testing that are allowed.


#### Abstract

It is because of the acceptability of the results of these tests, the availability of testing equipment, and the standardization of the tests, that the ASTM recommendations are being adhered to in this report.


Methods of Model Making

The standard test for compression of concrete (ASTM Designation: C 39-49) utilizes a cylinder of $6^{\prime \prime}$ diameter and 12" length, which probably is used more as a matter of convenience and acceptance than as adherence to any dimensional requirements. Sizing the cylinders for the model study was also an arbitrary matter, based primarily on convenience and availability of equipment.

Model compression cylinders were made in brass molds of $2^{\prime \prime}$ diameter and $4^{\prime \prime}$ length, being of the same shape and proportions as the standard cylinder molds. The walls of the molds were highly polished for smoothness and lightly oiled to prevent sticking. Plate glass was used for the cylinder ends to make them smooth and flat, and was also lightly oiled. Each sample was allowed 24 hours curing time before removal from the molds, whereupon it was shelved for further curing.

Specimens for flexural tests were unreinforced beams of rectangular cross section, with a depth to width ratio of about 1.5 , and length of 26 inches. Initially, the beams were $2^{\prime \prime}$ wide and $3^{\prime \prime}$ deep, but were reduced to $1 / 4^{\prime \prime}$ width
and $3^{\prime \prime}$ depth because this reduced the ultimate load considerably. The $26^{\prime \prime}$ length allowed a $24^{\prime \prime}$ clear span, a figure selected arbitrarily for convenience of handling and calculation.

Forms for the model beams were made from straight lengths of nominal $1^{\prime \prime x} 4$ " and 2 "x4" lumber. They were cut to size with a power saw, surfaced with an electric jointer, and assembled with a hammer and nails using pre-drilled nail holes. This description is given to show that the forms can be made with a minimum of shop equipment and materials. Some care and insight had to be given to the assembly of the forms so that the beams could be removed without damage to either the beams or beam forms, since the forms were used many times.

The forms were sanded and oiled before being filled, and were set on a level surface. After the material was poured and tamped, the top surface was smoothed with a trowel and a flat, wet board. The cylinders and beams (which had been poured together from the same batch mix) were removed from the forms after 24 hours and were set aside to continue curing until they were tested.

Figures 2 and 3, on the following two pages, depict the model specimens as they are removed from the forms.

## Materials

Concrete is a non-homogeneous substance formed by a


Figure 2. Model Cylinders and Mold

coalition of separate materials into a solid mass. Its constituent parts are cement, inorganic aggregate, water, and sometimes, additives which are intended to alter one or more of its properties. Materials of a cementatious nature were used in the models because it was felt that they would best simulate the properties of concrete. Such materials would be fine sand, portland cement, gypsum (Plaster of Paris), lime, and water.

A description of the materials used in the research is as follows:
A. Sand

The sand used was natural river bottom sand from Oklahoma, complying with ASTM Designation: C 33. It was checked for organic impurities and found to be within the limits specified by the ASTM. The maximum diameter was 0.0232 inches (passed number 30 seive), with a random grading.
B. Cement

Cement used was Type III (high early strength) portland cement complying with ASTM Designation:

C 150. All cement used was from one sack to prevent any variation due to differences in sacks of cement.
C. Gypsum (Plaster of Paris)

Gypsum used was a finely ground, rapid setting
plaster of commercial quality. It was a molding plaster used primarily for art work.
D. Lime

The lime used was a finely powdered material which is commonly used to neutralize high-acid soils. It is also added to masonry mortar to make it more workable.
E. Water

The water used was of potable quality, free from deleterious amounts of acids, alkalis, and organic impurities.

## Selection of Material Mixes

Determination of the proper material mix was made from a simple trial and error procedure. A few unrecorded mixes were made to determine the amount of material by weight that was necessary to fulfill the volume requirements. The first tests to be made on the material was of compression cylinders, so initially, only enough volume was made to fill the cylinder molds.

Plaster of Paris was the basic ingredient of the cylinders, with varied quantities of cement, sand, and lime added to change their properties. Different mixes were made in order to investigate the effect of various percentages of the constituents. The amounts of material by weight in grams is listed for each cylinder in Table $I$ on the following page.

TABLE I
WEIGHT IN GRAMS OF MATERIALS FOR COMPRESSION SPECIMENS

| Cylinder <br> Numbers | Date of <br> Casting | Water | Plaster | Cement | Sand | Lime | Remarks |
| :--- | :--- | :--- | :---: | ---: | :---: | ---: | :--- |
| $1,2,3$ | $10-6-65$ | 600 | 800 | xx | xx | xx | Smooth surface |
| $4,5,6$ | $10-6-65$ | 400 | 800 | xx | xx | xx | Layer marks |
| $7,8,9$ | $10-6-65$ | 375 | 725 | xx | 250 | xx | Layer marks |
| $10,11,12$ | $10-6-65$ | 460 | 400 | 400 | 250 | xx | Soupy |
| $13,14,15$ | $10-7-65$ | 350 | 600 | xx | 250 | xx | Smooth surface |
| $16,17,18$ | $10-7-65$ | 800 | 400 | xx | xx | 400 | Crumbled taking from forms |
| $19,20,21$ | $10-7-65$ | 600 | 600 | xx | xx | 300 | Spalled |
| $22,23,24$ | $10-8-65$ | 260 | 400 | 90 | 450 | xx | Air bubbles on surface |
| $25,26,27$ | $10-9-65$ | 600 | 400 | 100 | 400 | 200 | Smooth surface |
| $28,29,30$ | $10-10-65$ | 600 | xx | 200 | 400 | 400 | Settlement during curing |
| $31,32,33$ | $11-9-65$ | 400 | 800 | xx | xx | xx | Layer marks |
| $34,35,36$ | $11-16-65$ | 375 | 600 | xx | 250 | xx | Smooth surface |
| $37,38,39$ | $11-17-65$ | 500 | 800 | xx | xx | xx | Smooth surface |

To illustrate how these figures were arrived at, calculations are shown for the mix design of cylinders 22, 23, and 24, which were made from a mixture of plaster and microconcrete, equally proportioned before pouring.

## Sample Calculations

(1) Volume of three cylinders

$$
D=\text { Diameter }=2 \text { inches }
$$

$\mathrm{L}=$ Length $=4$ inches

$$
\begin{aligned}
\text { Volume } & =D^{2}(T / 4) \mathrm{L} \\
& =2^{2}(\pi / 4) 4 \times 3 \\
& =37.7 \text { cubic inches }
\end{aligned}
$$

Make up 40 cubic inches of mix.
Forty cubic inches of mix requires 400 grams of water and 800 grams of plaster, for a total of 1200 grams of material. For a l:1 ratio of plaster and microconcrete by weight, use 200 grams of water and 400 grams of plaster, and prepare 600 grams of microconcrete.
(2) Proportions by weight of microconcrete

(3) Material by weight for cylinders 22,23 , and 24 as listed in Table I.

| Plaster | Water | Cement | Sand |
| :---: | :---: | :---: | ---: |
| 400 | 200 | 00 | 00 |
| 00 | 60 | 90 | 450 |
| $\overline{400}$ | $\overline{260}$ | $\overline{90}$ | $\overline{450}$ |

After making calculations such as these for each set of cylinders, quantities were weighed on a triple beam balance with an accuracy of one gram. Weighing of dry quantities was done in clean, dry cardboard containers, and water was weighed in pre-moistened glass bottles. Mixing vessels were also pre-moistened to eliminate any loss of mixing water.

In order to keep the compression tests as standard as possible, and to assure good results, caps of an ironitesulfur mixture were made for the cylinders. The mixture was melted in a pot over an electric coil and poured into an aluminum mold. The cylinders were put into the molten liquid (one end at a time) and allowed to remain there for three to five minutes until the solution hardened. Four hours after capping, the cylinders were tested.

## CHAPTER III

MODEL TESTING

Goals of Testing

The purpose of this study, as previously stated, is to find a material which behaves similarly to concrete. Model tests, therefore, were conducted following the recommended ASTM procedures for concrete specimen testing, and the results compared with established relationships for concrete. Compression tests on the model cylinders were performed to collect data for stress-strain patterns, and the results of flexural tests were used to compose load-deformation curves.

## Description of Apparatus for Cylinder Tests

The testing machine used for this work was the Baldwin Hydraulic Press located in the Civil Engineering laboratory. It had a capacity of 60,000 pounds, but was used in the low range with a maximum of 6,000 pounds. This range was selected for two reasons: (1) it was known that 6,000 pounds was well above the maximum load that the cylinders would support, and (2) the loads could be read to the nearest five pounds, whereas the smallest increment of load that could be read on the other scales was 200 pounds.

Load to the specimens was applied through two flat steel bearing blocks, which are part of the machine. The lower block rests on a steel platform which is completely rigid and immovable. The upper bearing block was spherically seated to allow rotation through small angles. This was used to compensate for cylinder caps which were not in parallel planes.

Deformations were measures with dial gauges that were manufactured by Soiltest, Incorporated. Movement of the dial around the face of the gauge was prompted by a helical spring, which assured an exact correspondence of dial movement to travel of the gauge shaft. They were incremented to read uni-axial movements of one-thousandth (0.001) of an inch with a total measuring capacity of one inch. This was considered sufficient and accurate enough to include all of the range of deformations.

Cylinder Testing

Testing of the compression specimens was done, as nearly as practicable, according to the ASTM Designation: C 39-49, Standard Method of Test for Compressive Strength of Molded Concrete Cylinders. The bearing blocks were cleaned and the machine head was brought down close to the tops of the cylinders. The axis of each cylinder was aligned with the centers of the bearing blocks, and the spherically seated block was rotated to obtain uniform seating. At this point the dial gauges were put into place, one on each of two opposite
sides to account for differential deformations.
The machine head was then lowered very slowly, and dial gauge readings were taken at the first sign of movement in the load dial. These readings were recorded to give the position of the dial at what was considered to be the instant of zero load. Gauge readings were then recorded at constant increments of load until failure, at which point the final loads and corresponding gauge readings were recorded.

Figures 4 and 5 illustrate test cylinders before and after loading.


Figure 4. Model Cylinders Before Testing


Figure 5. Model Cylinders After Testing

## Description of Apparatus for Flexure Tests

Flexural tests were conducted on simply supported beams which were loaded at third points. Loads were transmitted to the beams through small lengths of steel angles which provided line bearings over the full width of the beams. Support "angles" were attached to a rigid wooden frame so that the clear spans of the beams would remain constant for all test specimens. The "angles" through which the loads were applied were attached to a smaller wooden frame which was placed on each beam.


Figure 6. Apparatus for Flexure Tests

Beam deflections were measured with the same dial gauges that were used for the compression tests. In this case, however, deflections were conveyed from the beams to the gauge through a simple lever and fulcrum system. The lever was supported at its mid-point so that a deflection of one of its ends was exactly equal to the deflection of the other end. This scheme was employed so that the gauge would not be damaged when the beam failed.

A photograph of the beam testing apparatus is shown in Figure 6 on the previous page.

Beam Testing

The beams were tested in the apparatus shown in Figure 6, with all the equipment resting on a solid surface (the laboratory floor). Loads were applied in increments of 910 grams (2.006 pounds) at intervals of thirty seconds, and deflections were recorded at every application of load. The time interval and load increment were both selected arbitrarily for convenience of testing.

Figures 7 and 8 show typical beams at failure.


Figure 7. Beam of Three Inch Depth at Failure


Figure 8. Beam of Two Inch Depth at Failure

## CHAPTER IV

## INTERPRETATION OF TEST RESULTS

An interpretation of test data must be based on the personal beliefs, judgements, and interests of the individual involved. On this basis, every interpretation, regardless of subject matter, could be distinguishable either by conclusion or processes of reasoning. A successful interpretation of the results of this study must be supported by an objective analysis of the information that was compiled, and an understanding of the laws of similitude and model behavior.

Structural Similarity

The use of scale models, and the interpretation of test results, depends wholly upon being able to discover unknown quantities by the measurement of specific quantities from which the unknowns can be determined to scale. Model design problems for structures, based on flexural similarity, is relatively simple for structures with linear properties. The question that must first be answered as a step toward accurate interpretation is, "Upon what properties are these unknowns dependent?" The answer to this question can be
found by the direct approach to similitude.
The general expression for the deflection of a point in a structure is as follows:

$$
\Delta=\mathrm{P}\left(\mathrm{C}_{1} \mathrm{~L} / \mathrm{AE}+\mathrm{C}_{2} \mathrm{AL} / \mathrm{NI}+\mathrm{C}_{3} \mathrm{~L}^{3} / \mathrm{EI}+\mathrm{C}_{4} \mathrm{~L}^{3} / \mathrm{NQ}\right)
$$

where $C_{1}, C_{2}, C_{3}$, and $C_{4}$ are appropriate constants, generally determined by the geometry of the system. The term $\mathrm{PC}_{1} \mathrm{~L} / \mathrm{AE}$ represents deflection due to axial force, $\mathrm{PC}_{2} \mathrm{AL} / \mathrm{NI}$ is deflection due to shear, $\mathrm{PC}_{3} \mathrm{~L}^{3} / \mathrm{EI}$ is deflection due to bending, and $\mathrm{PC}_{4} \mathrm{~L}^{3} / \mathrm{NQ}$ represents deflection due to torsion. These symbols and others in this chapter are defined in the following list.

List of Symbols

| $E=$ modulus of elasticity | $\mathrm{L}=$ length of member |
| :---: | :---: |
| $\mathrm{N}=$ modulus of rigidity | $\mathrm{m}=$ suffix denoting model |
| $\mathrm{A}=$ area of cross section | $\mathcal{V}=$ Poisson $^{\prime}$ S ratio |
| $\sigma=$ stress | $\begin{aligned} \mathrm{b}= & \text { width of a rectangular } \\ & \text { section } \end{aligned}$ |
| $\begin{aligned} & \mathrm{I}=\text { moment of inertia } \\ & \mathrm{Q}=\text { torsion constant } \end{aligned}$ | $\begin{aligned} & \mathrm{d}= \text { depth of a rectangular } \\ & \text { section } \end{aligned}$ |
| $C=$ constant | $\begin{aligned} \mathrm{x}= & \text { distance along a struc- } \\ & \text { tural member } \end{aligned}$ |
| $\mathrm{M}=$ bending moment | $\epsilon=$ strain |
| $\mathrm{P}=$ total applied load |  |
| If, as in the case of the | pression specimens, the |
| cts of shear, bending, and | sion of a structure are |
| igible, the deflection is | unction of $\mathrm{PC}_{1} \mathrm{~L} / \mathrm{AE}$. For |
| case of the flexural spec | the effects of axial |

force, shear, and torsion are neglected, and the deflection is a function of $\mathrm{PC}_{3} \mathrm{~L}^{3} / \mathrm{EI}$. If the ratio of the quantities L/AE of the model is equal to the ratio of those quantities of the prototype, and Poisson's ratio, $\nu$, is the same for the materials of both model and prototype, the two are said to be flexurally similar. This condition exists even when the scale of $L$ is not the same as the scale of $A$.

Two geometrically similar structures are said to be flexurally similar when their properties of deformation are similar. It should be noted that similarity implies that the relevant quantities of the two systems can be related by fixed-scale factors. Analysis of the test data is to find these fixed-scale factors, and the relationship between model and prototype properties. The data is translated into usable figures and graphs, which are compared to parallel information from concrete, and interpretations are then drawn from these comparisons of behavior.

Table II, on the following page, contains the data from compression tests of model specimens. Loads and deformations were recorded in the laboratory during testing, and stresses and strains were calculated later and recorded. Stress calculations were made by dividing the loads by the cross sectional areas of the cylinders, and strains were calculated by dividing the deformations by the original lengths of the cylinders. It was discovered that dimensional deviations in specimens of this size were immeasurable, so areas were

## TABLE II

DATA FROM COMPRESSION TESTS

| CYLINDER ONE |  |  |  | CYLINDER FOUR |  |  |  | CYLINDER SEVEN |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Load (lbs.) | Gauge (Inches) | Stress (psi) | $\begin{aligned} & \text { Strain } \\ & (i n . / i n .) \end{aligned}$ | $\begin{aligned} & \text { Load } \\ & \text { (lbs.) } \end{aligned}$ | Gauge <br> (Inches) | Stress (psi) | $\begin{aligned} & \text { Strain } \\ & \text { (in./in.. } \end{aligned}$ | $\begin{aligned} & \text { Load } \\ & \text { (lbs.) } \end{aligned}$ | Gauge (Inches) | Stress (psi) | $\begin{aligned} & \text { Strain } \\ & (i n . / i n .) \end{aligned}$ |
| 0 | 0.546 | 0 | 0 | 0 | 0.613 | 0 | 0 | 0 | 0.449 | 0 | 0 |
| 500 | 0.563 | 159.2 | 0.0043 | 500 | 0.627 | 159.2 | 0.0035 | 500 | 0.459 | 159.2 | 0.0025 |
| 1000 | 0.569 | 318.3 | 0.0053 | 1000 | 0.633 | 318.3 | 0.0050 | 1000 | 0.465 | 318.3 | 0.0040 |
| 1500 | 0.573 | 477.5 | 0.0068 | 1500 | 0.637 | 477.5 | 0.0060 | 1500 | 0.470 | 477.5 | 0.0053 |
| 2000 | 0.576 | 636.6 | 0.0075 | 2000 | 0.641 | 636.6 | 0.0070 | 2000 | 0.475 | 636.6 | 0.0065 |
| 2500 | 0.580 | 795.8 | 0.0085 | 2500 | 0.644 | 795.8 | 0.0078 | 2500 | 0.479 | 795.8 | 0.0075 |
| 3000 | 0.584 | 954.9 | 0.0095 | 3000 | 0.648 | 954.9 | 0.0088 | 3000 | 0.482 | 954.9 | 0.0083 |
| 3500 | 0.586 | 1114.1 | 0.0100 | 3500 | 0.650 | 1114.1 | 0.0093 | 3500 | 0.485 | 1114.1 | 0.0090 |
| 4000 | 0.589 | 1273.2 | 0.0108 | 4000 | 0.654 | 1273.2 | 0.0103 | 4000 | 0.489 | 1273.2 | 0.0100 |
| 4500 | 0.592 | 1432.4 | 0.0115 | 4500 | 0.659 | 1432.4 | 0.0115 | 4500 | 0.493 | 1432.4 | 0.0110 |
| 4765 | 0.595 | 1516.7 | 0.0123 | 4640 | 0.664 | 1477.0 | 0.0128 | 5000 | 0.497 | 1591.6 | 0.0120 |
|  |  |  |  |  |  |  |  | 5325 | 0.507 | 1695.0 | 0.0145 |

calculated using a 2.0 inch diameter and the length used was 4.0 inches.

## Sample Calculations of Stress and Strain

Calculations are made for cylinder 7.

$$
\sigma=\text { stress (pounds per square inch) }
$$

$$
\mathrm{P}=\text { load (pounds) }
$$

$\mathrm{A}=$ area (square inches)
$\mathcal{E}=$ strain (inches per inch)
$\Delta=$ vertical deformation (inches)
$\mathrm{L}=$ original length (inches)

Stress $=\frac{\text { Load }}{\text { Area }}, \quad \sigma=\frac{\mathrm{P}}{\mathrm{A}}$
$\mathrm{A}=\mathrm{D}^{2}(\pi / 4)=(2 \text { inches })^{2} \frac{(3.1416)}{4}$
$A=3.1416$ square inches

$$
\sigma=\frac{5,325 \text { pounds }}{3.1416 \mathrm{sq} \cdot \mathrm{in} .}=1,695 \mathrm{psi}
$$

Strain $=\frac{\text { Vertical deformation }}{\text { Original length }}, \epsilon=\frac{\triangle}{\Sigma}$

$$
\epsilon=\frac{0.0516 \text { inch }}{4.0 \text { inches }}=0.0129
$$

Stress-strain curves were plotted from these equations for each of the cylinders tested. Those curves which were best related to the stress-strain curves for concrete were then multiplied by constant factors and replotted to the same scale as the concrete curves. From these replotted specimen
curves the materials which seemed to best simulate the behavior of concrete were selected for further study (i.e., flexure tests).

When the first stress-strain curves were made, each specimen showed a curious condition between stresses of 0 to 500 psi (see Figure 9 on the following page) which was not in accordance with more familiar data of this type. This seemed at first to be an error of data recording, but was decided to be a result of the testing procedure. Stresses were calculated by assuming that the applied forces were distributed over the entire area, while this was not the case, initially. The two cylinder caps of each specimen were not in parallel planes since they were made individually, and the vertically moving machine head did not come into immediate contact with the full crosswsectional area. After the caps became parallel by crushing, a smooth, straight line stress-strain curve could be drawn which better represented the behavior of the material.

Analysis of the curves showed that although some of the specimens attained stresses in the range of fifty per cent of that for concrete, the modulus of elasticity was normally in the range of ten per cent of that for concrete. This meant that models of the same strength as concrete would deform about five times as much as concrete under an equivalent load. Conversely, models of the strengths that were used here would require only about one-tenth of the load that


Figure 9. Model Stress-Strain Curves Demonstrating Initial Curvature
would be necessary to test concrete elements of the same size.
Some of the material mixes were found to be too deficient in compressive strength and/or cohesion to be of any value, the most notable of these being the cylinders made with lime. No test data is available for these cylinders because they either separated upon removal from the molds or else crumbled immediately upon application of load. Those specimens which were made by mixing gypsum and cement tested well enough, but were not used in flexure tests because of the disadvantages of mixing and the amount of time that was required for curing.

Figures 11 and 12 , on the following two pages, illustrate stress-strain curves of model cylinders, corrected to zero coordinates, plotted along with a stress-strain curve for 3000 psi concrete. They are plotted in this manner to permit a better comparison of curve shapes. Figure 10 represents


Figure 10. Comparison of Concrete and Model Stress-Strain Curves


Figure 11. Model Stress-Strain Curves


Figure 12. Model Stress-Strain Curves
a selected curve from Figures 11 and 12 which has had its ordinates changed by constant factors and then replotted. This procedure is exemplified by the following illustration. The concrete curve has a maximum stress of 3000 psi with a corresponding strain of 0.002 inches per inch and the model curve has a maximum stress of 1695 psi, with an accompanying strain of 0.0145 inches per inch. To superimpose the model curve on the concrete curve, all of the model stresses are multiplied by the ratio $3000 / 1695$, and model strains are multiplied by the ratio 0.0020/0.0145. Figures 13, 14, and 15 , on pages 35,36 , and 37 , are superpositions of curves from three different material mixes upon the selected standard concrete stress-strain curve.

Tables III and IV, following Figure 15, contain the data from flexural tests of model specimens. Loads and deformations were recorded for each specimen during testing, and stresses and strains were entered into the table after calculations were made. The following example illustrates the methods of calculation.


$$
\begin{aligned}
& \mathrm{M}_{\mathrm{L} / 3}<\mathrm{x}<2 \mathrm{~L} / 3=(\mathrm{P} / 2)(\mathrm{L} / 3)=\mathrm{PL} / 6=1.33 \mathrm{P} \text { inch lbs. } \\
& \mathrm{S}=\mathrm{bd}^{2} / 6 ; \mathrm{b}=1.25^{\prime \prime}, \mathrm{d}=2.00^{\prime \prime}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{S}=1.25 \times 4 / 6=5 / 6=0.833 \mathrm{in.}^{3} \\
& \sigma=\mathrm{M} / \mathrm{S}=1.33 \mathrm{P} \text { in. } 1 \mathrm{bs} . / 0.833 \mathrm{in}^{3}=1.60 \mathrm{P} \mathrm{psi} \\
& \Delta=\frac{\mathrm{P}(\mathrm{~L} / 3) 3 \mathrm{~L}^{2}-4(1 / 3)^{2}}{24 \mathrm{EI}}=490.67 \mathrm{P} / \mathrm{EI} \text { inches } \\
& \mathrm{I}=\mathrm{bd}^{3} / 12=1.25 \times 2^{3} / 12=10 / 12=0.833 \mathrm{in.} \\
& \mathrm{E}=490.67 \mathrm{P} / \Delta \mathrm{I}=588.80 \mathrm{P} / \Delta \\
& \epsilon=\mathrm{F} / \mathrm{E}=\frac{1.60 \mathrm{P}}{588.80 \mathrm{P} / \Delta}=0.00272 \Delta
\end{aligned}
$$

Load-deformation and stress-strain curves were plotted from these equations (see Figures 16 and 17 on pages 40 and 41) for each beam that was tested. From these curves it was found that the modulus of elasticity for the plaster beams in flexure is about one half of the modulus of elasticity of concrete in flexure. This property of the material makes it very useful in model studies, since it is a factor in addition to linear scale which allows reduced loads for model testing.


Figure 13. Superposition of Model Stress-Strain Curves Upon Concrete Stress-Strain Curve


Figure 14. Superposition of Model Stress-Strain Curves Upon Concrete Stress-Strain Curve


Figure 15. Superposition of Model Stress-Strain Curves Upon Concrete Stress-Strain Curve

TABLE III
DATA FROM FLEXURAL TESTS ON 2" x $3^{\prime \prime}$ SPECIMENS

BEAM ONE (14 Days)
BEAM TWO (14 Days)

| Load <br> (1bs) | Gauge (Inches) | Stress (psi) | $\begin{aligned} & \text { Strain } \\ & \times 10^{-6} \end{aligned}$ | Load <br> (1bs) | Gauge (Inches) | Stress (psi) | Strain $\times 10^{-6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.0780 | 0 | 0 | 0 | 0.1100 | 0 | 0 |
| 16.9 | 0.0750 | 7.51 | 12.24 | 16.9 | 0.1080 | 7.51 | 8.16 |
| 33.8 | 0.0720 | 15.02 | 24.48 | 33.8 | 0.1052 | 15.02 | 19.58 |
| 50.7 | 0.0705 | 22.53 | 30.60 | 50.7 | 0.1036 | 22.53 | 26.11 |
| 67.6 | 0.0688 | 30.04 | 37.54 | 67.6 | 0.1010 | 30.04 | 36.72 |
| 84.5 | 0.0672 | 37.56 | 44.06 | 84.5 | 0.0990 | 37.56 | 44.88 |
| 101.4 | 0.0655 | 45.07 | 51.00 | 101.4 | 0.0975 | 45.07 | 51.00 |
| 118.3 | 0.0642 | 52.58 | 56.30 | 118.3 | 0.0952 | 52.58 | 60.38 |
| 135.2 | 0.0630 | 60.09 | 61.20 | 135.2 | 0.0940 | 60.09 | 65.28 |
| 152.1 | 0.0603 | 67.60 | 72.22 | 152.1 | 0.0925 | 67.60 | 71.40 |
| 169.0 | 0.0588 | 75.11 | 78.34 | 169.0 | 0.0912 | 75.11 | 76.70 |
| 185.9 | 0.0578 | 82.62 | 82.42 | 185.9 | 0.0900 | 82.62 | 81.60 |
| 202.8 | 0.0565 | 90.13 | 87.72 | 202.8 | 0.0870 | 90.13 | 93.84 |
| 219.7 | 0.0555 | 97.64 | 91.80 | 211.3 | 0.0852 | 93.91 | 101. 18 |
| 236.6 | 0.0543 | 105.16 | 96.70 | 219.8 | 0.0845 | 97.69 | 104.04 |
| 253.5 | 0.0530 | 112.67 | 102.00 | 228.3 | 0.0838 | 101.47 | 106.90 |
| 270.4 | 0.0512 | 120.18 | 109.34 | 236.8 | 0.0832 | 105.24 | 109.34 |
| 279.2 | 0.0490 | 124.09 | 118.32 | 241.0 | 0.0828 | 107.11 | 110.98 |
| 288.0 | 0.0482 | 127.99 | 121.58 | 245.2 | 0.0824 | 108.98 | 112.61 |
| 296.8 | 0.0475 | 131.91 | 124.44 | 249.4 | 0.0810 | 110.84 | 118.32 |
| 305.6 | 0.0470 | 135.82 | 126.48 | 253.6 | 0.0810 | 112.71 | 118.32 |
| 314.4 | 0.0463 | 139.73 | 129.34 | 257.8 | 0.0808 | 114.58 | 119.14 |
| 323.2 | 0.0456 | 143.64 | 132.19 |  |  |  |  |
| 332.0 | 0.0449 | 147.56 | 135.05 |  |  |  |  |

TABLE IV
DATA FROM FKEXURAL TESTS ON 1.25" x 2" SPECIMENS

BEAM SEVEN (2 Days)

| Load | Gauge | Stress | Strain | Load | Gauge | Stress | Strain |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1bs) | (Inches) | (psi) | $\times 10^{-6}$ | (1bs) | (Inches) | (psi) | $\times 10^{-6}$ |


| 0 | 0.0785 | 0 | 0 | 0 | 0.4118 | 0 | 0 |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.25 | 0.0765 | 6.80 | 5.44 | 4.25 | 0.4080 | 6.80 | 10.24 |
| 8.50 | 0.0748 | 13.60 | 10.06 | 8.50 | 0.4062 | 13.60 | 15.13 |
| 12.75 | 0.0700 | 20.40 | 23.12 | 12.75 | 0.4025 | 20.40 | 25.20 |
| 17.00 | 0.0668 | 27.20 | 31.82 | 17.00 | 0.3991 | 27.20 | 34.44 |
| 21.25 | 0.0620 | 34.00 | 44.88 | 21.25 | 0.3960 | 34.00 | 42.88 |
| 25.50 | 0.0570 | 40.80 | 58.48 | 25.50 | 0.3938 | 40.80 | 48.86 |
| 29.75 | 0.0510 | 47.60 | 74.80 | 29.75 | 0.3910 | 47.60 | 56.48 |
| 34.00 | 0.0395 | 54.40 | 106.80 | 34.00 | 0.3880 | 54.40 | 64.64 |
| 38.25 | 0.0348 | 61.20 | 118.86 | 38.25 | 0.3850 | 61.20 | 72.80 |
| 42.50 | 0.0290 | 68.00 | 134.64 |  |  |  |  |
| 46.75 | 0.0247 | 74.80 | 146.34 |  |  |  |  |



Figure 16. Load-Deflection Curves of Model Beams


Figure 17. Stress-Strain Curves of Model Beams

## CHAPTER V

## SUMMARY AND CONCLUSIONS

Results of this study demonstrate that mixtures of gypsum and water, with or without added sand, will make a satisfactory model material. Tests of both compression and flexural specimens revealed linear stress-strain patterns which were related to concrete by fixed-scale factors. It is the existence of these fixed-scale factors, according to the laws of similitude, which enable this model material to simulate concrete.

There are several qualities and properties of gypsum mixes which make them beneficial to the structural model user. Among these is the ease with which the mixes are made, the necessary equipment being only mixing and weighing containers and a weighing scale. One testing advantage is that low ultimate strengths are available and practicable so that large testing loads are not necessary. Another practical advantage is that deflections under load are large compared to concrete, so that data recording is easier and errors of scale are less apt to occur.

It should be cautioned that the age of the material is very important in testing. Prior to four days of curing at
room conditions of temperature and humidity, creep seriously hindered any data recording. During flexure tests on specimens that had cured for forty eight hours, the indicator of the dial gauge that was measuring deflections never stopped moving, even when loads were applied at sixty second intervals. For this reason, most of the testing was done on the fifth day of curing, when creep was no longer a problem.

## Suggestions for Future Study

Model researchers in this area of study would be aided considerably by having data available to them on all of the characteristics of plaster. Tables of strength versus water/ gypsum ratios for specific periods of curing would reduce the time and energy required to complete a model study. This task could be easily performed, but would require several months to amass the quantity of data necessary to obtain dependable average results.

Another field of model research which needs development is the use of models to simulate the behavior of reinforced concrete structures. This development would require study of a reinforcing material to be used in conjunction with plaster material mixes. A complete understanding of both materials acting independently would be necessary before they can be combined to act as a unit.

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[^0]:    Submitted to the faculty of the Graduate College of the Oklahoma State University
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