AN INVESTIGATION OF PARAMETRIC AMPLIFICATION
OF AUDIO FREQUENCY SIGNALS

By

Carl Bluford Johnson

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

1961

Submitted to the faculty of the Graduate School
of the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1966

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Thesis Approved:

Harold Tristor
Thesis Adviser

C. M. Summers

JHBoggs
Dean of the Graduate College

PREFACE

The basic idea of parametric amplification is by no means new. However, it was the development of the solid state diode in the early 1950's that allowed the construction of a parametric amplifier which provides high gain and low noise amplification of signals in the microwave region.

In 1962 the Federal Aviation Agency began equipping radar systems with parametric amplifiers; thus FAA engineers and technicians were faced with learning the theory of operation of parametric devices. I became interested in the subject and began searching for information on parametric amplification. I found that there is no easy explanation of this amplifier. Most of the descriptions of parametric amplifiers are very complicated mathematical explanations published in technical journals. The average technician or engineer, who has been away from college for a few years, finds such articles very difficult to interpret. I decided to attempt to present an explanation of the parametric amplifier which would be useful to the technician or engineer who is suddenly faced with the installation and maintenance of parametric devices.

In about 1962 the Federal Aviation Agency began studying the possibility of transmitting digital radar information over telephone lines. One digital data transmission system design uses frequency selective circuits and amplifiers operating in the audio frequency range. It seemed that the parametric amplifier might find application in the field of data transmission; but I could find no published reports of experimental work with parametric amplifiers in this frequency range. I decided to combine the explanation of a parametric amplifier with experimental work to determine the feasibility of parametric amplification in the audio frequency range.

I would like to express my appreciation to the Federal Aviation Agency for the use of their laboratory equipment.

My thanks also goes to Professor Harold Fristoe, my major advisor, for his guidance and assistance in the preparation of this material.

And finally, my sincere appreciation goes to my wife, Nell, for her patience and understanding during the time I was preparing this thesis.

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CHAPTER I

INTRODUCTION

The name Parametric Amplifier is used to describe a particular type of electronic circuit which depends upon a nonlinear reactance for its principle of operation. This nonlinear device is used to channel useful energy from an alternating local source of power (or pump) to a useful load. Although any type of nonlinear reactance can be used, the most common is the capacitance associated with the solid state diode.

The diode parametric amplifier is finding application in many radar and communication systems because of its simplicity and low noise figure. This device can not approach the extreme low noise figure of the maser but it does not require refrigeration or permanent magnets and does permit the construction of a receiving system so sensitive that antenna noise might equal the total receiver system noise contribution.

Because the parametric amplifier has low loss and low noise characteristics at high frequencies, the majority of the reported experimental results have been in the region above 100 mc.

Planinac (1) investigated the operation of parametric amplifiers in the low power, medium radio frequency range. This thesis will extend Planinac's work into the audio frequency range.

Usually, when any new device is discovered, a large volume of literature will follow. The parametric amplifier is no exception. Many articles have appeared, especially in the technical journals and periodicals. Many of these papers deal with a single point of interest; too many assume that the reader is already familiar with the subject. This thesis will contain a basic review of parametric amplification principles which will provide an introduction to this topic. This discussion will be based on a high frequency model of the Parametric Amplifier, yet the derivations are still valid at audio frequencies.

Historical Background

Many of todays modern electronic devices are by no means based on newly acquired theory. The theoretical basis for the fuel cell, thermoelectric generator, and the parametric amplifier was developed during the last century.

Faraday (2) in 1831, presented a paper to the ROYAL SOCIETY which dealt with parametric excitation. He discussed vibrations of the particles on the surface of water in a large wine glass. His results would be very difficult to verify experimentally.

Melde (3) in 1859, performed experiments with a vibrating string driven by a tuning fork. This experiment vividly shows parametric oscillations.

Lord Rayleigh (4) later expanded the works of Faraday and Melde and showed that oscillations could be sustained in a single mechanical resonant system by the energy extracted from a source which suitably drives an energy storage element.

Hartley (5), in 1936, proposed a resonant circuit with a capacitor having movable plates. This was the first parametric amplifier which closely resembles those of today.

In 1956 Manley and Rowe (6) derived a set of general energy relations which are applicable to non-linear or time varying elements when these elements are used as frequency converters. These relations are more or less independent of circuit design and indicate the maximum power gain which a parametric device might provide.

CHAPTER II

PARAMETRIC AMPLIFIER THEORY OF OPERATION

To demonstrate the operation of a parametric amplifier, consider the electro-mechanical system depicted in Figure 1.

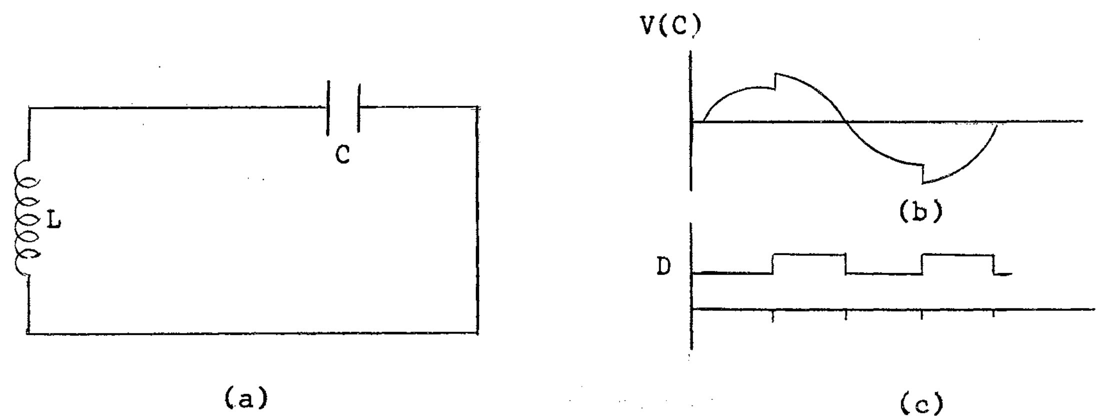


Figure 1. Simple Electromechanical System
a. Theoretical Electromechanical Amplifier
b. Voltage Across C vs. Time
c. Capacitor Plate Separation vs. Time

An analysis of this device will illustrate the mechanism whereby energy can be transfered from a "pump" to the fields of a resonant tank.

Assume that the capacitor has been charged at some previous time and that the circuit is oscillating. At the instant of time when the capacitor is fully charged, the plates are instantaneously separated a small amount. The value of capacitance is given as

$$C = \frac{\varepsilon A}{D} \tag{1}$$

where ε = dielectric constant in $\frac{\text{Farads}}{\text{Meter}}$

A = area of capacitor plates in square meters

D = separation of plates in meters

Since the plates of the capacitor are oppositely charged, work must be done to separate them. The work done increases the energy in the electric field between the capacitor plates. Since the charge on the capacitor cannot be changed instantly, the change in capacitance causes a corresponding change in voltage, $(V = Q/C)$. These changes are shown in Figure 1. A quarter of a resonant period later the capacitor charge is zero, and no force is required to restore the plates to their original position. Still another quarter period later the capacitor is again fully charged but of opposite polarity; again energy can be transferred to the circuit by pulling the plates apart. Therefore, energy at twice the resonant frequency can be pumped into the circuit and the amplitude of the oscillations will grow. In a physically realizable case the oscillations will build up to the point that the mechanical energy supplied by the pump equals the electrical energy dissipated. In an electrical parametric amplifier the mechanical pump would be replaced with a local oscillator which is also called a "pump".

Before attempting to discuss parametric amplifier circuits in detail several subjects must be reviewed. The first of these is a working definition of an amplifier.

Amplifier Definition

An amplifier is a device wherein an agent is driven by a local source of power, has motion imparted to it, and hence gains energy of motion. This agent, then by geometry or circuitry, has its movement affected, and energy is released in an external circuit by the expenditure of less energy in the input or controlling element (7). This is a classical description of an amplifier and would be strained when used to describe the quantum mechanical amplifiers.

The agent employed by the vacuum tube amplifier is the free electron. The plate supply voltage is the local source of power which imparts kinetic energy to the free electron. The control grid is the controlling element which by geometry is capable of changing the kinetic energy of the electrons by a greater amount than the energy consumed in the process.

Local Power

While the local power source may be of two classes, direct power or alternating power, the agents employed are many and varied. The type of agent used dictates the choice of local power. Direct power can be employed if the agent is free to move and this movement is unidirectional. For example: the vacuum tube amplifier utilizes direct power because the free electrons inside the vacuum tube must move from cathode to plate.

The local power source must be of the alternating type if the movement of the agent is bound or constrained at some limit. An example is the charge on the capacitance of a P-N junction such as used in the parametric amplifier.

In the case of alternating power there are two frequencies present; these are the signal frequency and the frequency of alternation of the local power supply sometimes called the local oscillator or pump. Since two frequencies are present, the modulation cross product terms are normally unavoidable, because of the nonlinearity of the modulating device. If the modulation cross product terms are absent in the alternating case then there has been no affected movement of any agent. Many modulator devices, however, having cross product terms are still devoid of amplification. Consequently the modulation process yielding cross product terms is a necessary, but not sufficient, condition for amplification in the alternating power case (8). Since modulation cross product terms are seen to be a necessary condition, a brief review of modulation theory will be presented.

Modulation

The familiar definition of modulation requires the use of the terms: modulated wave, modulating wave, and carrier. A modulated wave is defined as a wave, some parameter of which varies in accordance with the value of the modulating wave. A modulating wave is a wave which carries the specification of the message and varies the parameter of the wave that is modulated. Finally, the wave to which modulation is subsequently applied is known as the carrier. An amplitude modulated wave is a carrier wave, the amplitude factor of which is varied in accordance with a modulating wave (9).

First, assume the modulating device is so designed that the amplitude of the modulated wave is a linear function of the modulating wave. Therefore:

$$g(t) = A_m \text{ Cos } (\omega_m t + B_m) = \text{modulating wave} \quad (2)$$

$$h(t) = A_c \text{ Cos } (\omega_c t + B_c) = \text{unmodulated carrier} \quad (3)$$

$$f(t) = A \text{ Cos } (\omega_c t + B_c) = \text{modulated wave.} \quad (4)$$

Where A is the instantaneous amplitude of the modulated wave and is

$$A = A_c + k\,[A_m \text{ Cos } (\omega_m t + B_m)] \quad (5)$$

After substitution

$$f(t) = \{A_c + k\,[A_m \text{ Cos } (\omega_m t + B_m)]\} \text{ Cos } (\omega_c T + B_c) \quad (6)$$

$$f(t) = A_c \text{ Cos } (\omega_c T + B_c) + k\,a_m \text{ Cos } (\omega_m t + B_m) \text{ Cos } (\omega_c T + B_c). \quad (7)$$

Now define:

$$M = \frac{kA_m}{A_c} \text{ and } M \times 100 = \%\text{ Modulation} \quad (8)$$

$$f(t) = A_c \text{ Cos } (\omega_c T + B_c) + MA_c \text{ Cos } (\omega_m t + B_m) \text{ Cos } (\omega_c T + B_c). \quad (9)$$

Using the trigonometric identity

$$\text{Cos } A \text{ Cos } B = \frac{1}{2} \text{ Cos } (A+B) + \frac{1}{2} \text{ Cos } (A-b) \quad (10)$$

the modulated wave can be expressed as

$$f(t) = A_c \text{ Cos } (W_c T + B_c) + \frac{MA_c}{2} \text{ Cos } (\omega_c T + \omega_m T + B_c + B_m)$$

$$+ \frac{MA_c}{2} \text{ Cos } (\omega_c T - \omega_m T + B_c - B_m). \quad (11)$$

This expression then contains three terms. The first is the carrier, the second is called the "upper sideband" and, the third called the "lower sideband". The frequencies involved can be pictured as in

Figure 2.

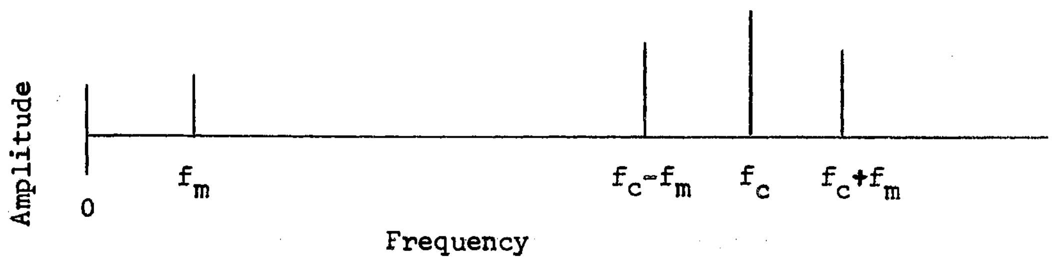


Figure 2. Spectrum of Amplitude Modulation

If the modulating wave is not a single sinusoid but a certain spectrum of frequencies, the spectrum is not inverted in the upper sideband, but is inverted in the lower sideband as is shown in Figure 3.

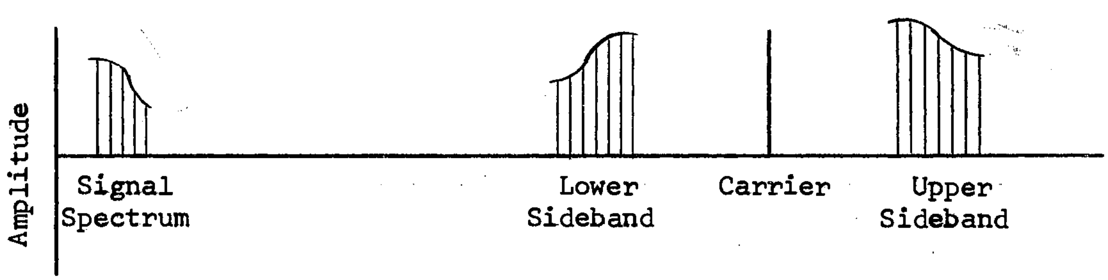


Figure 3. Inverted and Non-Inverted Spectrum

The affect of a nonlinear modulator can be demonstrated by the "square law modulator" of Figure 4. The operating characteristics of this device can be shown in the form of current as a function of voltage; therefore, it acts as a nonlinear resistance.

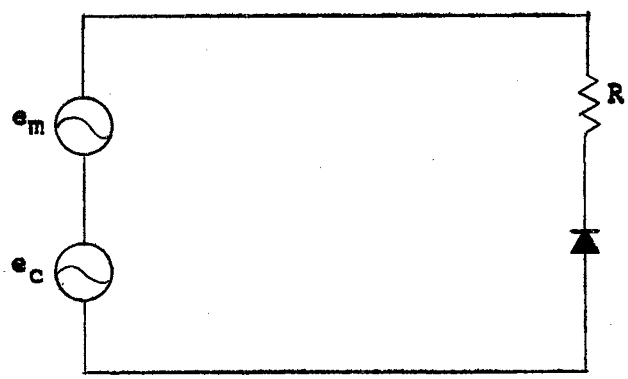


Figure 4. Square Law Modulator

Assume:

$$em = E_m \text{ Cos } \omega_m t \tag{12}$$

$$ec = E_c \text{ Cos } \omega_c t \tag{13}$$

$$e = E_m \text{ Cos } \omega_m t + E_c \text{ Cos } \omega_c t \tag{14}$$

$$i = A_1 e + A_2 e^2. \tag{15}$$

Then

$$i = A_1(E_m \text{ Cos } \omega_m t + E_c \text{ Cos } \omega_c t) + A_2(E_m \text{ Cos } \omega_m t + E_c \text{ Cos } \omega_c t)^2 \tag{16}$$

$$i = A_1 E_m \text{ Cos } \omega_m t + A_1 E_c \text{ Cos } \omega_c t + A_2 E_m^2 \text{ Cos}^2 \omega_m t + 2 E_m E_c \text{ Cos } \omega_m t \text{ Cos } \omega_c t + E_c^2 \text{ Cos}^2 \omega_c t . \tag{17}$$

The last term can be expanded by using the identities:

$$\text{Cos A Cos B} = \frac{1}{2} \text{ Cos (A+B)} + \text{Cos (A-B)} \tag{18}$$

$$\text{Cos}^2 \text{ A} = \frac{1}{2} (1 + \text{Cos 2A}). \tag{19}$$

The results are:

$$i = A_1 E_m \text{Cos } \omega_m t + A_1 E_c \text{Cos } \omega_c t + \frac{A_2}{2} E_m^2 \tag{20}$$
$$+ \frac{A_2 E_m^2}{2} \text{Cos } 2 \omega_m t + \frac{A}{2} E_m E_c \text{Cos } (\omega_m t + \omega_c t)$$
$$+ A_2 E_m E_c \text{Cos } (\omega_c t - \omega_m t) + \frac{A_2 E_c^2}{2}$$
$$+ \frac{A_2 E_c^2}{2} \text{Cos } 2 \omega_c t .$$

The current then is composed of the following components:

1. A D. C. component.
2. Components at both the carrier and the modulating frequencies.
3. Components at the upper and lower sideband frequencies.
4. Components at the 2nd harmonic of the carrier and modulating frequencies.

In general, whenever two signals of different frequencies are applied to a nonlinear device a number of new frequencies will be generated.

The diode parametric amplifier utilizes a non linear capacitance and an alternating local power source to produce amplification. As a result many new frequencies which are combinations of the signal and pump frequencies and their harmonics are produced. The amount of power developed at these various frequencies is subject to certain conservation principles. Manley and Rowe (10) first developed a set of principles for the nonlinear capacitance which relate frequency and power. Many derivations of these relations have since appeared. The derivation which follows is one given by Salzberg (11).

Manley - Rowe Relations

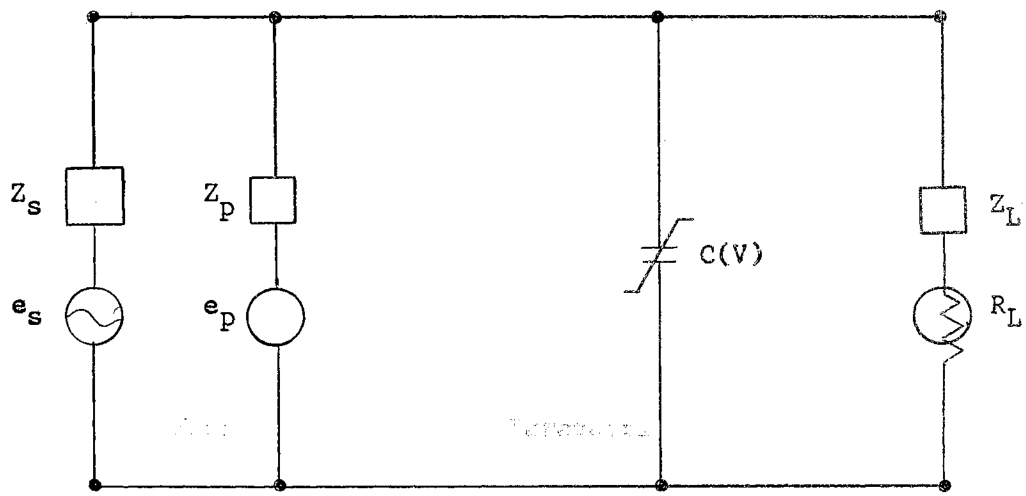


Figure 5. Theoretical Parametric Amplifier

Consider Figure 5 and make the following assumptions.

1. $E_s = E_sm \text{ Cos } \theta_s$ = Signal source

2. $E_p = E_pm \text{ Cos } \theta_s$ = Pump source

3. That the frequency f_s is incommensurable with the frequency f_p.

4. That Z_s, Z_p, and Z_L are ideal filters with the following characteristics.

   $Z_s = 0$ for frequency $f = f_s$

   $Z_s = \infty$ for all other frequencies.

   $Z_p = 0$ for frequency $f = f_p$

   $Z_p = \infty$ for all other frequencies.

$Z_L = 0$ for frequency $f = Mf_p \pm Nf_s$, where M and N are positive integers.

$Z_L = \infty$ for all other frequencies.

5. That c(V) is a lossless, nonlinear capacitance.

The filters are so arranged that any current due to E_s must flow through the nonlinear capacitor; likewise, current due to E_p must flow through this capacitor as will any current at a frequency of f_L.

The magnitude of the impedance of the nonlinear capacitance c(V) is

$$|Z| = |X_c| = \frac{1}{2\pi fc} = \frac{V_m}{I_m} \tag{21}$$

or

$$I_m = 2\pi fcV_m \tag{22}$$

where

F = frequency in cycles/sec.

C = capacitance of the nonlinear capacitance in farads.

V_m = the peak value of a sinusoidal voltage across C(V) at frequency f.

I_m = the peak value of a sinusoidal current through C(V) at frequency f.

Equation 22 can be applied using the three frequencies f_s, f_p, and f_L, and three equations result.

$$I_{sm} = 2\pi\ f_s\ C\ V_{sm} \tag{23}$$

$$I_{pm} = 2\pi\ f_p\ C\ V_{pm} \tag{24}$$

$$I_L = 2\pi\ f_L\ C\ V_{Lm}\ . \tag{25}$$

By using the relation

$$Q = CV \tag{26}$$

these equations become

$$I_{sm} = 2\pi\ f_s\ Q_s \tag{27}$$

$$I_{pm} = 2\pi\ f_p\ Q_p \tag{28}$$

$$I_{Lm} = 2\pi\ f_L\ Q_L\ . \tag{29}$$

The general definition of average real power is

$$P = V_m\ I_m\ \text{Cos}\ \theta \tag{30}$$

where the voltage and current are at the same frequency.

Combining equations 27, 28, 29, and 30, the following relations for average power absorbed or emitted by the nonlinear capacitance are obtained.

$$P_s = 2\pi\ f_s\ Q_s\ V_s\ \text{Cos}\ \theta_s \tag{31}$$

$$P_p = 2\pi\ f_p\ Q_p\ V_p\ \text{Cos}\ \theta_p \tag{32}$$

$$P_L = 2\pi\ f_L\ Q_L\ V_L\ \text{Cos}\ \theta_L\ . \tag{33}$$

The quantity $2\pi\ Q\ \ V\ \text{Cos}\ \theta$ appears in each of these and by dimensional analysis this is found to represent energy per cycle, or

$$W = 2\pi\ Q\ V\ \text{Cos}\ \theta \tag{34}$$

and

$$P_s = W_s\ f_s \text{ watts or } W_s = \frac{P_s}{f_s}\ \frac{\text{Joules}}{\text{Cycle}} \tag{35}$$

$$P_p = W_p\ f_p \text{ watts or } W_p = \frac{P_p}{f_p}\ \frac{\text{Joules}}{\text{Cycle}} \tag{36}$$

$$P_L = W_L\ f_L \text{ watts or } W_L = \frac{P_L}{f_L}\ \frac{\text{Joules}}{\text{Cycle}}\ . \tag{37}$$

Now, if the nonlinear capacitor c(V) is lossless then the sum of P_s, P_p and P_c must be zero or

$$P_s + P_p + P_L = 0 \tag{38}$$

Combining equations 35, 36, 37, and 38:

$$W_s f_s + W_p f_p + W_L \left(Mf_p \pm Nf_s\right) = 0 \tag{39}$$

$$f_s \left(W_s \pm NW_L\right) + f_p \left(W_p + MW_L\right) = 0 \tag{40}$$

where M and N take on all integral values.

Since W_L is some linear combination of f_s and f_p, the only solution to equation 40 is

$$W_s \pm NW_L = 0 \quad \text{and} \quad W_p + MW_L = 0\,. \tag{41}$$

Combining equations 35, 36, 37 and 41:

$$\frac{P_s}{f_s} \pm N \frac{P_L}{f_L} = 0 \quad \text{and} \quad \frac{P_p}{f_p} + M \frac{P_L}{f_L} = 0 \tag{42}$$

or

$$\frac{-P_L}{Mf_p \pm Nf_s} = \pm \frac{P_s}{Nf_s} \tag{43}$$

$$\frac{-P_L}{Mf_p \pm Nf_s} = \frac{P_p}{Mf_p}\,. \tag{44}$$

The negative sign indicates a power flow out of the capacitor C(V) while a positive sign indicates power flow into the capacitor.

Equations 43 and 44 are the Manley - Rowe relations which follow conservation principles in that they describe fundamental relations more or less independently of the details of excitation or of the nonlinear device used. These relations constrain the sums of real powers at various frequencies to be zero in a nonlinear reactance. Because of the nonlinearity, net power into the system at one frequency is related to net power out at other frequencies. No restrictions have been placed upon the choice of signal, pump and load frequencies; however, in practice M and N of the above equations will usually be equal to one. The quality factor, Q, of the diode and circuits must be increased if the higher harmonics are to be used.

Application of The Manley - Rowe Relations

If the parametric device of Figure 5 is designed to operate with the frequency f_L equal to the sum of the signal and pump frequencies the Manley - Rowe relations reduce to

$$\frac{P_p}{f_p} = \frac{-P_L}{f_p + f_s} \quad \text{and} \quad \frac{P_s}{f_s} = \frac{P_L}{f_p + f_s} \, . \tag{45}$$

An amplifier operating under these conditions is referred to as an "up converter" or as an "upper sideband" parametric amplifier.

If the power gain of this device is now defined as the ratio of P_L to P_s the maximum gain of the up converter could be shown by the Manley - Rowe relations to be

$$\text{Maximum Power Gains} = \frac{P_L}{P_s} = \frac{f_L}{f_s} = \frac{f_s + f_p}{f_s} \, . \tag{46}$$

These realtions also indicate that both pump and signal are supplying power, and since the power gain is finite, the up converter is a stable device.

If the up converter is made to operate in reverse, that is by putting power in at the frequency f_L and removing power at f_s, the Manley-Rowe relations indicate a down conversion gain of $\frac{f_s}{f_L}$ which is actually a loss.

If the parametric device is now changed so that it operates with the frequency f_L equal to the pump frequency minus the signal frequency the Manley-Rowe relations reduce to

$$\frac{P_p}{f_p} = \frac{-P_L}{f_p - f_s} \text{ and } \frac{P_s}{f_s} = \frac{P_L}{f_p - f_s} . \tag{47}$$

This expression indicates that if power is applied to the device at the pump frequency only, power will flow out of it at the two frequencies f_L and f_s.

This is the basis for a negative resistance amplifier, which under certain conditions would oscillate. If this negative resistance parametric amplifier is forced to operate just below the point of oscillation, very high gain could be obtained. However, much care must be exercised in the design of this device to insure stability. A discussion of stability of this device will be presented in a later portion of this paper.

Feedback

The Manley - Rowe power relations indicate that in the parametric amplifier the nonlinear device is absorbing power at the pump and signal frequencies and emitting this power at the frequency f_L. This is illustrated for the negative resistance parametric amplifier in Figure 6.

It was also shown that the negative resistance parametric amplifier might operate as an oscillator. This can be further explained by

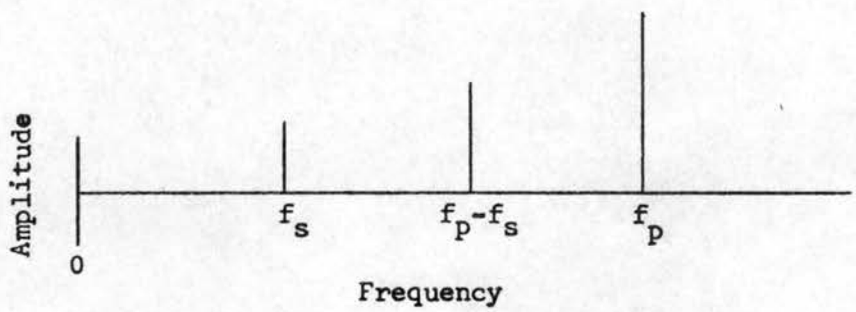


Figure 6. Negative Resistance Parametric Amplifier Frequency Spectrum

considering the amplifier as a feedback amplifier operating in the following manner; the nonlinear device acts as a modulator; it accepts energy from the pump and signal sources at the frequencies f_p and f_s and produces the new frequency $f_p - f_s$. If this signal is now allowed to combine with f_p in the modulator, new sideband frequencies are produced. This process is as follows:

First step: $f_p - f_s = f_L$ = lower sideband (48)

Second step: $f_p + f_L = f_p + f_p - f_s = 2f_p - f_s$ (49)

or: $f_p - f_L = f_p - (f_p - f_s) = f_s.$ (50)

The frequency $2f_p - f_s$ would not be allowed to build up because of the filter arrangement of the circuit. However, the second step shows a signal developed at f_s which would combine with the original signal.

This is a form of feedback and if this "feedback" signal is of proper phase a regenerative action would occur. This is exactly the case as can be shown by using amplitude modulation theory to indicate the phase of this feedback signal.

In amplitude modulation, if the phase of the original signal at f_s is $+\theta$ radians the phase of the upper side band is $+\theta$ radians, and the phase of the lower side band is $-\theta$ radians (12).

Therefore, after step one of the feedback process, the phase of f_L is $-\theta$ radians. This signal is then combined with f_p to form a new lower sideband at f_s and again with a 180 degree phase shift; therefore, the feedback signal is in phase with the original signal and positive or regenerative feedback has occured.

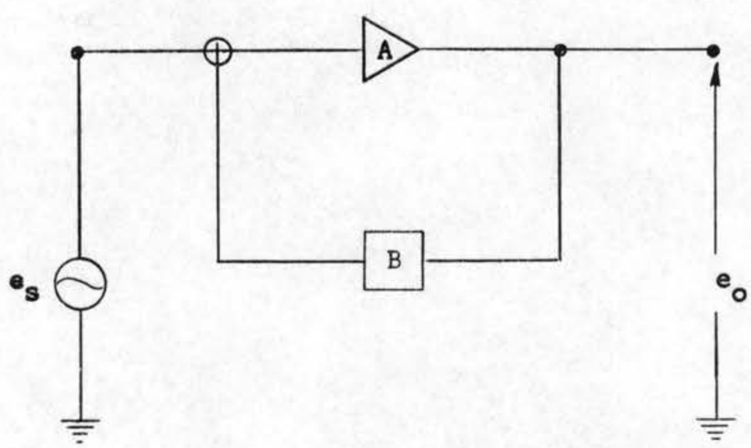


Figure 7. Conventional Feedback Amplifier

A conventional feedback amplifier can be shown as in Figure 7. The overall gain of this device has been shown to be

$$A_f = \frac{A}{1 - AB} \tag{51}$$

where A_f is overall gain with feedback.

A is gain of the amplifier without feedback.

B is the feedback factor.

If the product AB of equation 51 were equal to unity, the gain A_f would be infinite, and the device would be an oscillator. If AB is something slightly less than one the device is an amplifier with very large gain. Operation of the negative resistance parametric amplifier would be analogous to that of Figure 7. Thus, if it were operated with feedback, and overall gain properly chosen, very high gain would result.

The feedback process just described involved two steps where the phase of the signal was reversed. In this case the regenerative action is independent of the phase of the signal. However, if the frequencies of operation are so chosen that the pump frequency is twice the signal frequency, the signal and lower side band frequencies are the same. The feedback process would be a single step and a single phase reversal would occur. As a result the signal f_s at phase $+\theta$ would be adding to the lower sideband f_s and of phase $-\theta$. Therefore, the only acceptable values of θ would be 0 or 2π radians. This means that the pump and signal must be phase coherent. This is a "degenerative" negative resistance parametric amplifier. It would be difficult to incorporate into a system because of this phase coherent limitation.

An analysis of the up converter will show that it too operates as a feedback amplifier. The frequencies in this device are the signal, pump, and upper sideband. In the first step of the feedback process. the signal and pump frequencies combine to form the upper sideband

$$f_p + f_s = f_L .$$

If the phase of f_s is $+\theta$ radians, the phase of the upper sideband is $+\theta$ radians. In the second step of the feedback process the upper sideband combines with the pump frequency to form two new sidebands

$$f_p + f_L = f_p + f_p + f_s = 2\, f_p + f_s . \qquad (52)$$

$$f_p - f_L = f_p - f_p - f_s = -f_s \qquad (53)$$

The phase of $-f_s$ is now $-\theta$ radians. The negative frequency is sometimes encountered mathematically and it can be shown that the frequency $(-f_s)$ and phase $(-\theta)$ will have the same effect in a reactance as would the frequency $(+f_s)$ and phase $(+\theta)$. Positive feedback has again occured. However, the Manley - Rowe power relations indicate a finite power gain of $\frac{f_1}{f_s}$ for the up converter; therefore, the overall gain feedback factor product (AB) must have some value such that the upconverter is stable.

Parametric Amplifier Terminology

In the discussion of the Manley - Rowe relations it was indicated that the parametric amplifier may take on various forms. The various types of parametric amplifiers will now be reviewed.

The choice of the frequencies allowed to carry real power determines the type of amplifier. If the filters of Figure 5 are so chosen that the frequencies allowed are the signal, pump, and upper sideband frequencies, the device is an "up converter" parametric amplifier. Another name for this device is "non-inverting" parametric amplifier. This name refers to the fact that the upper sideband produced in a modulator is a spectrum of frequencies of the same shape as the original signal spectrum.

The maximum gain of the up convertor was shown to be the ratio of the upper sideband frequency to the signal frequency. The output of this device is taken at the upper sideband frequency.

If the parametric amplifier is operated with the frequencies of signal, pump, and lower sideband the device is a "negative resistance" parametric amplifier. Another name for this device is an "inverting amplifier. Again this name refers to the shape of the lower sideband frequency spectrum which is inverted from the original signal spectrum. This device is in effect a regenerative amplifier and very high gain can be obtained. Since the signal frequency and lower sideband frequency contain the same information, either may be used as the useful output from this amplifier.

The negative resistance parametric amplifier with output at the signal frequency must also have a circuit in which the lower sideband frequency can circulate, this circuit is referred to as the Idler circuit.

If the negative resistance amplifier is operated with the signal and lower sideband frequency being the same, it was shown that the signal and pump frequencies must be phase coherent. This is a degenerate negative resistance parametric amplifier. If the device is operated with the pump and signal frequencies incommensurable it is called a non-degenerate negative resistance parametric amplifier.

CHAPTER III

THE P. N. JUNCTION DIODE

The essential element of a parametric amplifier is a nonlinear reactance which could be either inductive or capacitive. The capacitance associated with a P - N junction is a nonlinear function of the voltage across it. The P - N junction diode provides an economical means of obtaining amplification. The Diode is small, rugged, inexpensive and should have long life. The Diode may be operated in the microwave region and closely approximates an ideal lossless nonlinear capacitor.

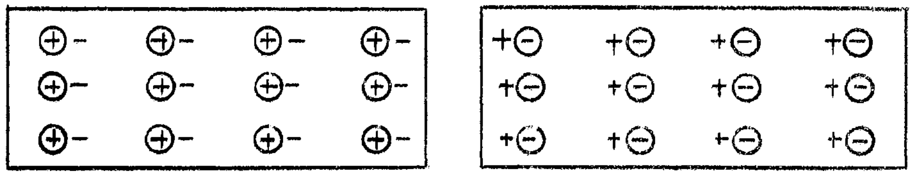
Many explanations of the formation and action of the P - N junction are available; therefore, only a brief review of this device will be given.

Consider the crystalline structure of a semiconductor such as silicon. The atoms of the crystal are covalently bonded. Now assume that at one of the lattice sites an atom of silicon is replaced by an atom of some pentavalent element such as arsenic. The arsenic atom is about the same physical size as the silicon atom. But it contains one more outer electron than does the silicon atom. With this impurity atom in the crystal there will be one electron which does not enter the convalent bonding. This electron is free to move about inside the crystal. An N type silicon material is formed by adding a controlled amount of such an impurity material to silicon. The number of impurity or donor atoms is much smaller than the number of silicon atoms but still the actual number of impurity atoms per unit volume is rather large. Each impurity atom provides one free electron to the crystal.

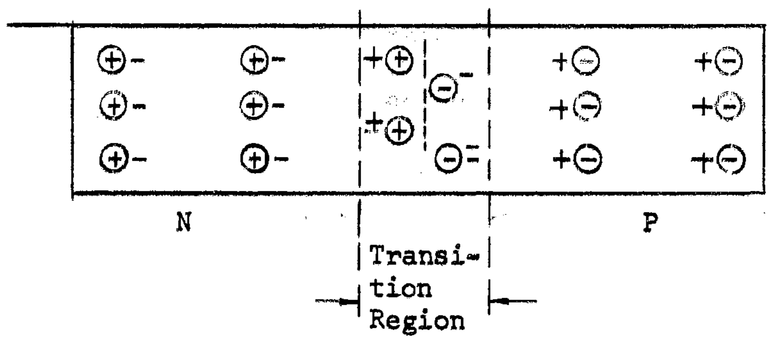
P type silicon material is formed in a similar manner. The difference being that a trivalent element such as Gallium is used as the impurity substance. This impurity atom has one less outer electron than a silicon atom; therefore, at each impurity atom there will be a covalent bond which is not complete. This represents an electron deficient state and is called a hole. In the P type material there will be one hole or positive charge carrier for each impurity atom.

Although, the N type material contains free electrons the material is electrically neutral. Likewise, the P type material contains free holes, but is also electrically neutral. This is represented in Figures 8(a) and 8(b). An encircled plus sign in the N type material indicates a donor atom and the minus sign indicates mobile electrons. The encircled minus signs in the P type material represents an acceptor atom and the plus sign mobile holes.

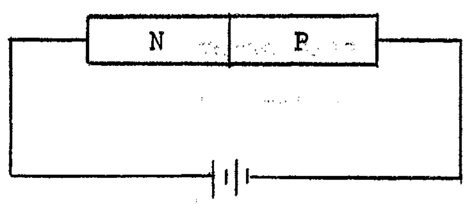
Consider a small bar of silicon so treated that one end has donor impurities added, while the other end has acceptor impurities added. Thus a P - N junction is formed. At the P - N junction free electrons will diffuse across the junction from a region of high electron density to a region of low electron density. Similarly, holes will diffuse from the P type material to the N type. This diffusion will result in the N type material becoming more positive and the P type more negative as shown in Figure 8(c). As diffusion occurs the electric field which is built up will increase to the point that any further diffusion will be retarded. An equilibrium condition will be reached where as many charge carriers cross the junction in one direction as cross it in the opposite direction. The voltage across the junction as a result of diffusion is the contact potential.



(a) N Type (b) P Type



(c) P-N Junction



(d) Reverse Bias

Figure 8. P-N Junction Formation

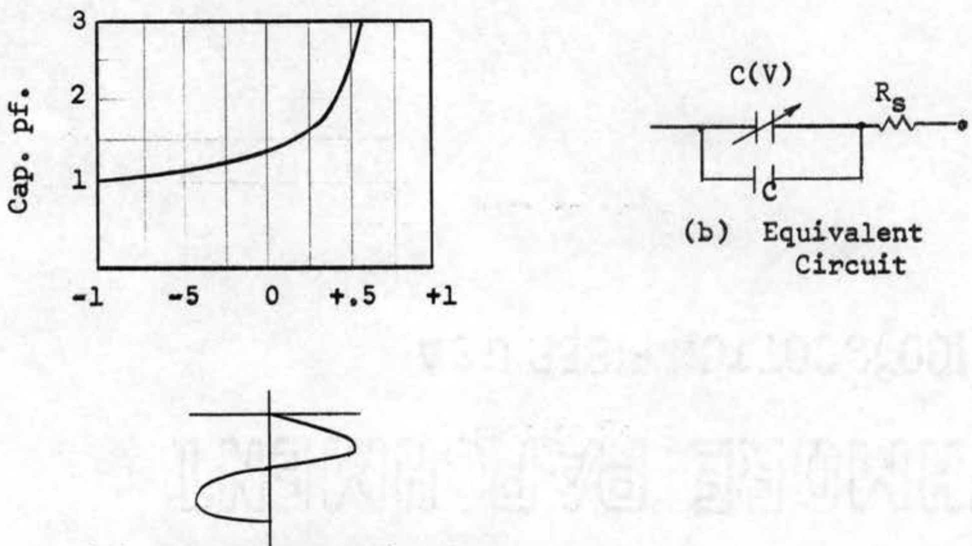
In the formation of the junction a region in the immediate vicinity of the junction became void of mobile charge carriers. This region is known as a depletion layer. Because it is void of mobile charge carriers the region may be considered as a non conducting or dielectric region bounded on either side by regions which do contain mobile charge carriers and, hence, may be considered conducting regions. This depletion layer then is similar to a parallel plate capacitor with plate separation equal to the depletion layer width.

If the P - N junction were reverse biased by an external source as in Figure 8(d), the charge distribution would change. The positive potential applied to the N material would attract the electrons to the left while the negative potential would attract holes to the right. Thus the charge distribution is changed and the depletion layer widened. This change in depletion layer width would cause a corresponding decrease in capacitance. This is analogous to making the plate separation greater.

Similarly, a forward bias will decrease the width of the depletion layer and increase the capacitance.

Thus, the capacitance associated with the P - N junction is a function of the voltage applied across the junction. This variation in capacitance results from a minute motion of holes and electrons. However, charge flow across the junction is not involved. This motion is so small that the capacitance may be changed at a very high frequency. This small motion also accounts for the low noise property of the variable capacitance diode.

Figure 9(a) shows the voltage-capacitance curve of a typical diode. It also indicates an alternating voltage applied to the P - N junction about some fixed bias point. The alternating or pump voltage is usually applied about the zero bias point and for a peak to peak amplitude of one volt may result in a 2:1 capacitance variation.



(a) Voltage-Capacitance Curve

(b) Equivalent Circuit

Figure 9. P-N Junction Capacitance

Figure 9(b) is an equivalent circuit for the variable capacitance diode. C(V) represents the varying capacitance just described, while, C represents the fixed capacitance measured with zero bias. The fixed capacitance is a function of contact area, depletion layer width, the applied bias, and type of encapsulation. R_s represents the series resistance of the diode and is due to the impedence of the bulk of the semiconductor material to the flow of majority carriers (13).

The frequency of operation is limited by the fixed capacitance. If the frequency is high enough the capacitance C will present essentially a short circuit to the applied signal. The upper limit on operating frequency has been shown by Uenohara (14) to be

$$f_s \text{ (max)} = \frac{1}{2\pi C R_s} \tag{54}$$

At this frequency the reactance of the variable capacitance is exactly equal to its series resistance.

The thermal noise generated by the resistance R_s is the primary source of noise in the variable capacitance diode. As the operating frequency is increased this resistance becomes a larger part of the total circuit impedence and, therefore, the noise produced in the diode increases with frequency.

Another type of diode has been developed recently which has shown much promise as a nonlinear reactance. This is the Gallium-Arsenic point contact diode. Reed (15) reports that this device is capable of amplification at frequencies approximately twice the maximum operating frequency of a P - N junction diode.

CHAPTER IV

NOISE CONSIDERATIONS

The importance of parametric amplifiers is due primarily to their low noise performance. Therefore, it is important that a discussion of noise generation be presented. A detailed analysis is beyond the scope of this paper, and only a basic discussion of noise is given so that the noise performance of electronic devices may be compared.

There are many noise sources associated with any system. One of these is due to thermal energy of electrons. Although this may be the source of only a small part of the noise contribution of a system it is easily calculated and will serve as a reference for other noise sources. Consider first the thermal noise generated by a resistor.

Any conductor contains electrons possessing kinetic energy which is proportional to the temperature of the conductor. The electrons are in random motion which causes a noise voltage to be developed. This noise voltage is randomly distributed throughout the frequency spectrum.

The resistor then may be considered as a voltage generator with a noiseless internal resistance equal to the resistor. This equivalent circuit is shown in Figure 10.

The voltage source E_s is given by

$$E_s = (4\ KTR\Delta f)^{1/2}\ \text{volts} \tag{55}$$

where

E_n is the RMS noise voltage

K is the Boltzmans constant

T is the Resistor temperature in degrees Kelvin

Δf is the interval of frequencies of interest and is limited by the band pass of the device.

Statistical analysis has shown that noise is uniformly distributed in frequency and the position of Δf in the spectrum is unimportant.

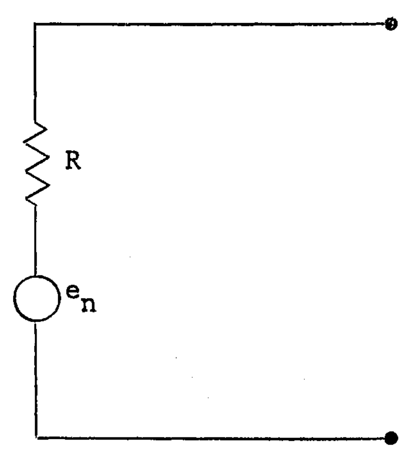


Figure 10. Noise Equivalent Circuit of a Resistance

This noise generator could deliver maximum power when connected to a matched load. This maximum noise power available is

$$N = \frac{(e_n)^2}{4R} . \tag{56}$$

Combining equations 55 and 56 gives

$$N = \frac{4KTR\Delta f}{4R} = KT\Delta f \quad (57)$$

which is independent of resistor size.

If this noise generator were connected to an ideal amplifier which introduces no additional noise and which has a rectangular band pass characteristic of width B, the maximum noise power out of the amplifier would be

$$N_0 = GKTB = \text{Maximum noise output power} \quad (58)$$

where

$$G = \frac{\text{signal power out}}{\text{signal power in}} = \text{amplifier power gain.}$$

and

$B = \Delta f$ is the band pass of the device.

The components of noise in the ouput of a two port device are only those components lying within the band pass of the device. Since the overall device band pass is a rough measure of the range of frequencies amplified, the wider the bandwidth the greater the amount of noise power in the output.

Noise Figure

The noise figure of a two port device at a specified input frequency is defined as: the ratio of the total noise power per unit bandwidth (at a corresponding output frequency) available at the output to that portion of this power engendered at the input frequency by the input termination at the standard noise temperature of 290° K. (16). The noise temperature at a pair of terminals is the absolute temperature of a resistance having an available thermal noise power per unit bandwidth equal to that available at the actual terminals. The noise figure defines the noiseness of any network and is

$$\text{Noise Figure} = F = \frac{N_o}{GKT_oB} \tag{59}$$

where N_o is the noise power out of the device and T_o is 290° K.

The noise figure of a receiver is often defined in terms of signal to noise ratio. To show this, consider Figure 11 to represent the receiver with the input termination at the standard noise temperature.

N_o is the total noise output which is the input noise amplified by the network, plus the noise generated within the network in the bandwidth B.

N is the noise generated within the network in B.

N_i is input noise in B.

G is the gain of the network in B.

N ideal is defined as the output noise of an ideal noiseless amplifier.

Then:

$$N_o = N(\text{ideal}) + N \tag{60}$$

or

$$\frac{N_o}{N(\text{ideal})} = 1 + \frac{+N}{N(\text{ideal})} = F. \tag{61}$$

Now:

$$Ni = KTB \tag{62}$$

$$N(\text{ideal}) = G\ Ni = GKTB\ . \tag{63}$$

Therefore:

$$F = \frac{N_o}{GKTB} = 1 + \frac{N}{GKTB} \tag{64}$$

$$G \equiv \frac{So}{Si} \tag{65}$$

where So is signal power output and Si is signal power input.

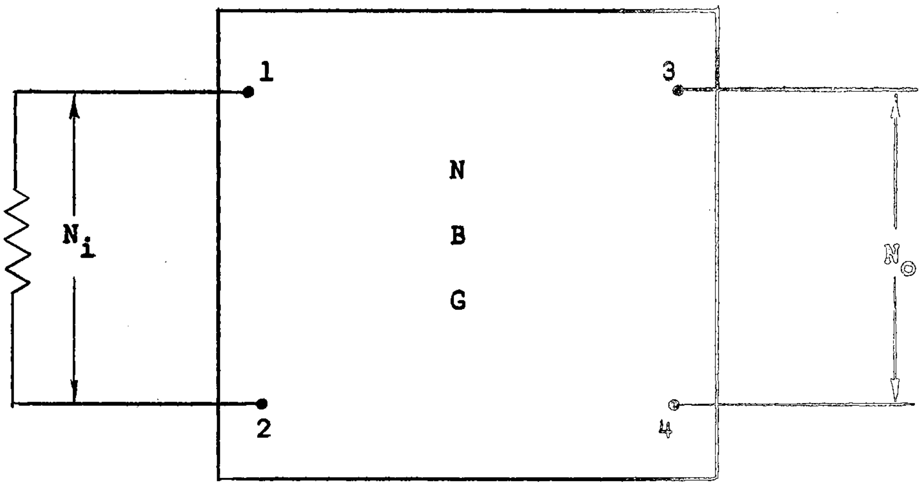


Figure 11. Receiver Representation

$$N(\text{ideal}) = G\,N \quad \text{or} \quad G = \frac{N(\text{ideal})}{Ni} \tag{66}$$

and since

$$G = \frac{So}{Si} = \frac{N(\text{ideal})}{Ni} \tag{67}$$

then

$$N(\text{ideal}) = Ni\,\frac{So}{Si} \tag{68}$$

and finally

$$F = \frac{N_o}{N(\text{ideal})} = \frac{N_o}{Ni\ So/Si} = \frac{Si/Ni}{So/No}\ . \tag{69}$$

The noise figure of the receiver is then the ratio of the input signal to noise ratio to the output signal to noise ratio at the output noise temperature is 290° K.

Noise Figure of Cascaded Stages

If several amplifiers are connected in cascade as in Figure 12, the overall noise figure can be determined.

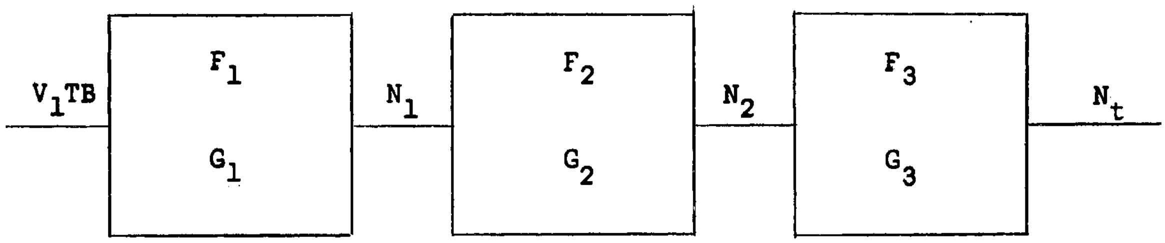


Figure 12. Cascaded Stages of Amplification

Where G_1, G_2 and G_3 are the individual stage gain, the

F_1, F_2 and F_3 are the individual stage noise figures, and

N_1, N_2 and N_3 are noise power out of each stage.

N_t is the total noise generated within the amplifiers.

Then:

$$N_t = N_1 G_2 G_3 + N_2 G_3 + N_3 .$$

$$G_t = G_1 G_2 G_3 = \text{total gain of network.}$$

$$F_t = 1 + \frac{N_t}{GTKB} = \text{overall noise figure.}$$

Therefore:

$$F_t = 1 + \frac{N_1 G_2 G_3 + N_2 G_3 + N_3}{G_1 G_2 G_3 K TB} \tag{70}$$

$$F_t = 1 + \frac{N_1}{G_1 KTB} + \frac{N_2}{G_1\ G_2\ KTB} + \frac{N_3}{G_1\ G_2\ G_3\ KTB} \tag{71}$$

or combining equations 59 and 71

$$F_t = F_1 + \frac{F_2 - 1}{G_1} \quad \frac{F_3 - 1}{G_1\ G_2}. \tag{72}$$

This derivation could be extended to a general case with N cascade stages. The noise figure would then be given by

$$F_n = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1\ G_2} + \cdots + \frac{F_n - 1}{G_1\ G_2 \cdots G_{n-1}}. \tag{73}$$

This equation indicates that the noise figure will never be improved by the addition of more stages of amplification. It should also be noted that IF the first stage gain is very high the total noise figure is approximately that of the first stage.

The noise figure can be expressed as either a ratio or in decibels, since by the defining equation

$$F_{db} = 10 \log F = \text{noise figure in decibels.} \tag{74}$$

Noise Terminology

This section is concluded by stating the definition of several terms commonly encountered in discussion of noise (17).

In any system involving an antenna and receiver many factors will affect system sensitivity. One of these is the antenna temperature which is defined as:

$$T_a = \frac{P_a}{K\Delta F} \tag{75}$$

where P_a is the available noise power at the antenna terminals, K is

Boltzmans constant and ΔF is the band of frequencies involved and determined by the band pass of the device. The noise power is partially due to antenna losses, side and back lobes, and noise received by the antenna, which depends upon antenna location.

Receiver sensitivity is defined as the available signal power at the antenna terminals when the predetection signal to noise ratio is unity. In this definition everything on the receiver side of the antenna terminals is called the receiving system.

It is often convenient to define a temperature which is called the operating noise temperature, T_{op}.

$$T_{op} = \frac{N_o}{G_a \, KB} \tag{76}$$

where N_o is the power available at the receiver output terminals, and G_a is the receiver system available gain. When the output signal to noise ratio is unity, the sensitivity is:

$$\text{Sensitivity} = K\,B\,T_{op}\ , \tag{77}$$

so that system operation is a function of T_{op}.

For linear transducers the operating temperature can be written as the effective input noise temperature of the receiver and the antenna temperature,

or:

$$T_{op} = T_o\,(F - 1) = T_a\ . \tag{78}$$

Somewhat different expressions are used in connection with parametric devices because of the nonlinear element involved.

CHAPTER V

CIRCUIT ANALYSIS

Suppose a parametric amplifier is being considered for use in an S band radar system as the first stage of amplification. From noise considerations, equation 73 indicates that this device should have very high gain and a correspondingly low noise figure.

The maximum gain for an up convertor is given by the Manley - Rowe relations as the ratio of upper sideband frequency to signal frequency. For this gain to be very large, the pump frequency must be extremely high. However, the negative resistance amplifier could be operated with very high gain and a nominal pump frequency provided steps are taken to insure stable operation. This is one reason why most parametric amplifiers designed for radar systems are of the negative resistance type. Therefore this circuit will be analysed and an equivalent circuit derived.

Small Signal Approximation

The basic theory of operation of the parametric amplifier is not based upon quantum mechanical effects. The analysis of this device will involve ordinary network analysis. Before many network theorems will apply the nonlinear element must be replaced by an equivalent linear element. One method of accomplishing this is the so called small signal approximation. This assumes the pump signal to be much larger than the input signal, and that the nonlinear capacitor may be replaced by a linear time-varying capacitance. Louisell (18) has shown the equivalence of these two elements. His argument is as follows.

Consider the charge on the nonlinear capacitor to be expanded in a Taylor series in voltage

$$q = \sum_{0}^{\infty} a_n (V - V_o)^n \tag{79}$$

where q is the charge,

V is the voltage across the capacitor,

and V_o is some reference voltage.

Now define capacitance as:

$$C(V) = \frac{dq}{dV} . \tag{80}$$

Therefore:

$$C(V) = \frac{dq}{dv} = \sum_{1}^{\infty} a_n n(V - V_o)^{n-1} . \tag{81}$$

If the voltage V varies sinusoidally as

$$V - V_o = b \cos \omega t, \tag{82}$$

Thus, C(V) can be expressed as a function of time as

$$C(V) = C(t) = \sum_{1}^{\infty} a_n n \frac{b}{2}^{n-1} \left(e^{j\omega t} + e^{-j\omega t}\right)^{n-1} \tag{83}$$

$$\equiv \sum_{-\infty}^{\infty} C_n e^{j\omega n t},$$

where C_n's are combinations of the A_n's. Therefore, the nonlinear capacitor is equivalent to a time varying capacitance. For small voltage variations the capacitance can be expressed as

$$C \approx C_o + C_1 \text{ Cos } (\omega t + \theta) \tag{84}$$

where $C_1 << C_o$.

The small signal approximation would certainly be valid where the parametric amplifier is used in a radar system with the pump signal as much as 100 db greater than the input signal.

With this approximation the pump and nonlinear capacitance may be replaced with a time varying linear capacitance.

Negative Resistance Parametric Amplifier

Figure 13 represents a basic parametric amplifier which appears as two resonant tank circuits coupled by a time varying capacitance which represents the nonlinear capacitance and pump circuit. The analysis is as follows.

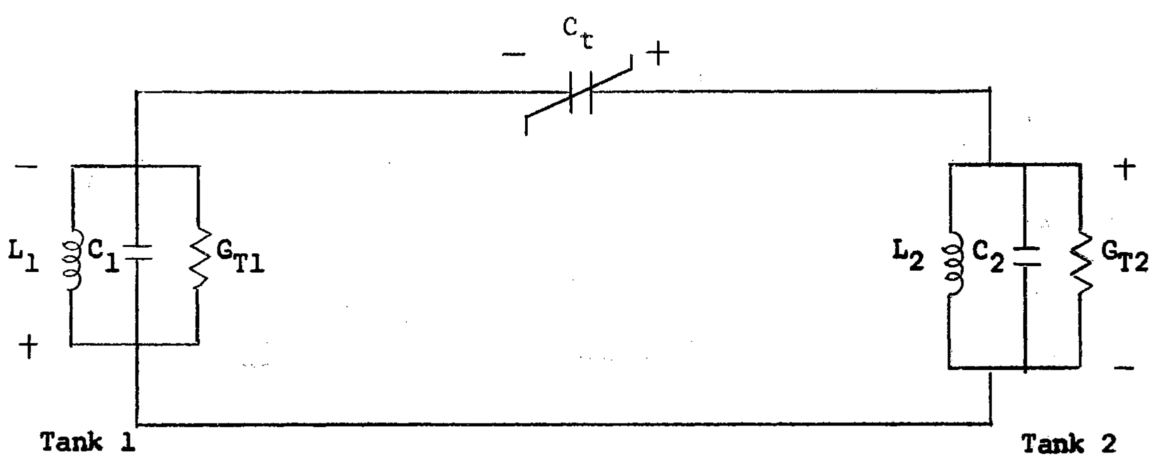


Figure 13. Basic Parametric Amplifier

Make the following assumptions:

1. That tank 1 is resonant at the signal frequency ω_1.

2. That tank 2 is resonant at the idler frequency ω_2, and the tank circuits are essentially short circuits for any frequency except their respective resonant frequency.

3. That f_3 is the pump frequency.

4. That $\omega_3 = \omega_1 + \omega_2$ (85)

and 5. That $C_t = C_3 \text{ Sin } (\omega_3 t + \theta_3)$, or n=3 in equation 81. (86)

Where C_3 is one term of the Fourier series representation of junction capacitance as a function of the voltage across the junction. Further assume that a voltage exists across the tank circuits and is given by

$$v_1 = V_1 \text{ Sin } (\omega_1 t + \theta_1) \tag{87}$$

and

$$v_2 = V_2 \text{ Sin } (\omega_2 t + \theta_2). \tag{88}$$

The voltage across the capacitor C(t) would be:

$$V_c = V_1 \text{ Sin } (\omega_1 t + \theta_1) + V_2 \text{ Sin } (\omega_2 t + \theta_2) \tag{89}$$

and by convention this is positive if the right hand plate is positive with respect to the left hand plate.

The charge on the capacitor would be $q \pm C_t V_c$ and the current flowing out of the capacitor to the right is

$$i = \frac{dq}{dt} = \frac{d(C_t V_c)}{dt} \tag{90}$$

combining equations 86, 89, and 90 gives

$$i = \frac{d}{dt}[C_3 \text{Sin}(\omega_3 t+\theta_3)][V_1 \text{Sin}(\omega_1 t+\theta_1) + V_2 \text{Sin}(\omega_2 t+\theta_2)]. \tag{91}$$

By using the exponential definition of the sine of a function the current becomes:

$$i = \frac{d}{dt}\left[-\frac{jC_3}{2}\right]\left[e^{j(\omega_3 t+\theta_3)} - e^{-j(\omega_3 t+\theta_3)}\right]\left[-\frac{jV_1}{2}\left(e^{j(\omega_1 t+\theta_1)} - e^{-j(\omega_1 t+\theta_1)}\right)\right.$$

$$\left. - \frac{jV_2}{2}\left(e^{j(\omega_2 t+\theta_2)} - e^{-j(\omega_2 t+\theta_2)}\right)\right] \tag{92}$$

$$i = \frac{jC_3}{2}\frac{d}{dt}\left\{-\frac{jV_1}{2}\left[e^{j(\omega_3 t+\theta_3+\omega_1 t+\theta_1)} - e^{j(\omega_3 t+\theta_3-\omega_1 t-\theta_1)}\right]\right.$$

$$-\frac{jV_2}{2}\left[e^{j(\omega_3 t+\theta_3+\omega_2 t+\theta_2)} - e^{j(\omega_3 t+\theta_3-\omega_2 t-\theta_2)}\right]$$

$$+\frac{jV_1}{2}\left[e^{-j(\omega_3 t+\theta_3-\omega_1 t-\theta_1)} - e^{-j(\omega_3 t+\theta_3+\omega_1 t+\theta_1)}\right]$$

$$\left.+\frac{jV_2}{2}\left[e^{-j(\omega_3 t+\theta_3-\omega_2 t-\theta_2)} - e^{-j(\omega_3 t+\theta_3+\omega_2 t+\theta_2)}\right]\right\}. \tag{93}$$

If this were rearranged it would indicate that modulation action has occured and that the current has components at the frequencies

$$f_3 \pm f_1 \quad \text{and} \quad f_3 \pm f_2.$$

The component at $f_3 - f_2$ is at the signal frequency of:

$$f_1 = f_3 - f_2. \tag{94}$$

This component is

$$i_1 = -\frac{jC_3}{2}\frac{d}{dt}\frac{jV_2}{2}\left[e^{j(\omega_3 t+\theta_3-\omega_2 t-\theta_2)} - e^{-j(\omega_3 t+\theta_3-\omega_2 t-\theta_2)}\right] \tag{95}$$

$$i_1 = \frac{C_3 V_2}{4}\left[-j\omega_1 e^{j(\omega_1 t+\theta_3-\theta_2)} + j\omega_1 e^{-j(\omega_1 t+\theta_3-\theta_2)}\right] \tag{96}$$

$$i_1 = -\frac{jC_3 V_2 \omega_1}{2}\,\text{Sin}\,(\omega_1 t+\theta_3-\theta_2).$$

Similarly the component of current at the idler frequency $f_2=f_3-f_1$ is

$$i_2 = -\frac{j\omega_2 C_3}{2} V_1 \operatorname{Sin}(\omega_2 t+\theta_3-\theta_1). \tag{97}$$

The voltage across tank 1 is the result of the current component at f_1. The admittance presented across tank 1 by the remainder of the circuit is shown in Figure 14 and is

$$Y_{\omega 1} = \frac{\text{Complex } i_1}{\text{Complex } v_1} \tag{98}$$

and

$$Y_{\omega 1} = -\frac{\frac{j\omega_1 C_3 V_1}{2} \operatorname{Sin}(\omega_1 t+\theta_3-\theta_2)}{V_1 \operatorname{Sin}(\omega_1 t+\theta_1)} \tag{99}$$

$$= -\frac{j\omega_1 C_3 V_2}{2V_1} e^{j(\theta_3-\theta_2-\theta_1)}. \tag{100}$$

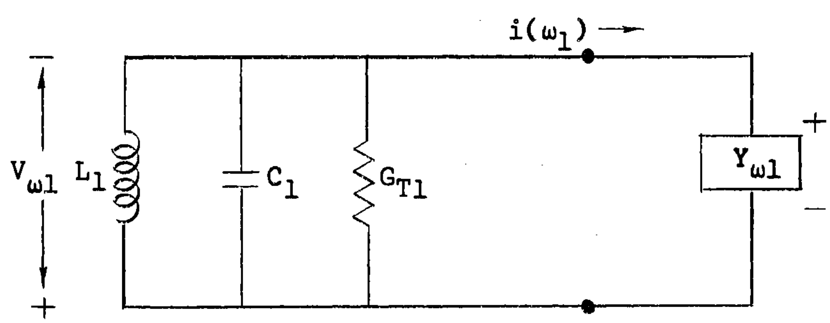


Figure 14. Partial Equivalent Circuit for Parametric Amplifier

This admittance presented to tank 1 is proportional to the amplitude of V_2, the voltage in the idler circuit. To examine V_2 and its relationship to V_1 consider the equivalent circuit of Figure 15.

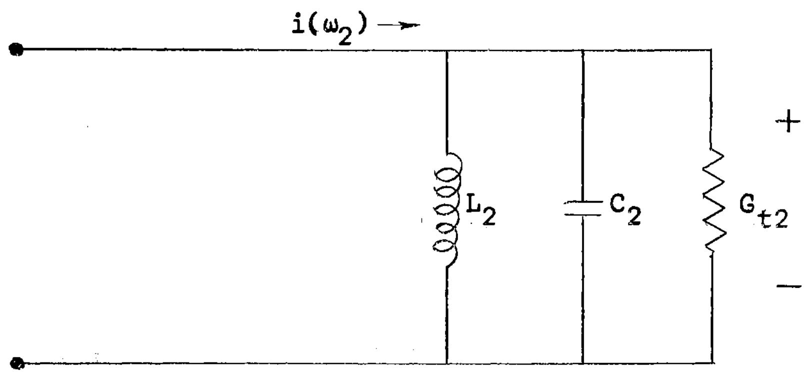


Figure 15. Idler Tank Equivalent Circuit

Let Y_2 be the admittance of tank 2 where

$$Y_2 = G_{t2} + j\omega_2 C_2 - j\frac{1}{\omega_2 L_2} \tag{101}$$

Also:

$$Y_2 = \frac{\text{Complex } i_2}{\text{Complex } v_2} \tag{102}$$

or:

$$Y_2 = \frac{\frac{-j\omega_2 C_3 V_1}{2}\operatorname{Sin}(\omega_2 t + \theta_3 - \theta_1)}{V_2 \operatorname{Sin}(\omega_2 t + \theta_2)} \tag{103}$$

or:

$$Y_2 = -\frac{j\omega_2 C_3 V_1}{2V_2}\, e^{j(\theta_3 - \theta_1 - \theta_2)} \tag{104}$$

and

$$V_2 = -\frac{j\omega_2 C_3 V_1}{2Y_2}\, e^{j(\theta_3 - \theta_1 - \theta_2)} \, . \tag{105}$$

By substituting the complex conjugate of the expression for $\underline{Y_2}$ into equation 99,

$$Y_{\omega 1} = - \frac{\omega_1 \omega_2 C_3^2}{4Y_2^*} \tag{106}$$

and

$$Y_2^* = \text{Complex conjugate of } Y_2.$$

Notice that $Y_{\omega 1}$ is a function of the capacitor variation C_3.

The above equations are general in that they were independent of how the voltages v_1 and v_2 were produced. They apply if the circuit is used as either an oscillator or an amplifier. Figure 16 shows an equivalent circuit for the amplifier.

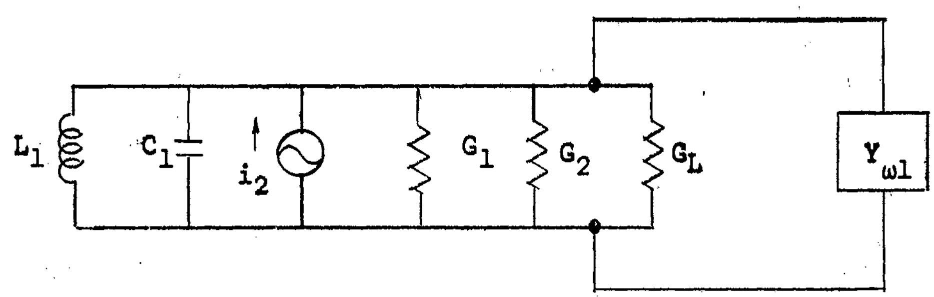


Figure 16. Parametric Amplifier Equivalent Circuit

If the signal frequency is exactly the resonant frequency of tank 1, $Y_{\omega 1}$ is a negative real number and represents a negative conductance. If the value of $|Y_{\omega 1}|$ is exactly equal to G_{t2} the circuit acts as an oscillator. The same would be true for tank 2; therefore, the pumping of the variable capacitor C_t sustains oscillations in both circuits.

If a signal generator were coupled into tank 1, tuned to f_1, an output load coupled to tank 1, and C_t adjusted so that oscillations are not sustained the signal is responsible for the voltage v_1 and the modulating action of the capacitor C_t will result in the voltage v_2 being developed across tank 2.

If the applied signal is not exactly at the resonant frequency of tank 1 the equivalent circuit must be changed.

To show the changes the following definitions are used:

Q_1 = Quality factor of tank 1,

Q_2 = Quality factor of tank 2,

$$\omega_1 = \Omega_1 + \Delta\omega_1 \tag{107}$$

and

$$d = \frac{\Delta\omega_1}{\Omega_1}, \tag{108}$$

where ω_1 = applied frequency and Ω_1 = resonant frequency of tank 1, and $\Delta\omega_1 = |\omega_1 - \Omega_1|$.

Also, for tank circuit 2

$$\omega_2 = \Omega_2 - \Delta\omega_1$$

where Ω_2 = resonant frequency of tank 2, and the admittance of tank 2 becomes:

$$Y_2 = G_{t2}\,[1 - j2d\,(\Omega_1/\Omega_2)Q_2] \tag{109}$$

and

$$Y_{\omega 1} = \frac{\omega_1\omega_2 C_3^2}{4G_{t2}[1+j2d(\Omega_1/\Omega_2)Q_2]} \tag{110}$$

which is no longer a pure real number.

Now define:

$$G_{t1} = G_g + G_L + G_1 \tag{111}$$

where G_{t1} is the total conductances of the amplifying circuits.

G_g is the generator internal resistance,

G_L is the load conductance, and

G_1 is a circuit loss conductance.

Therefore, an equivalent circuit is that of Figure 17.

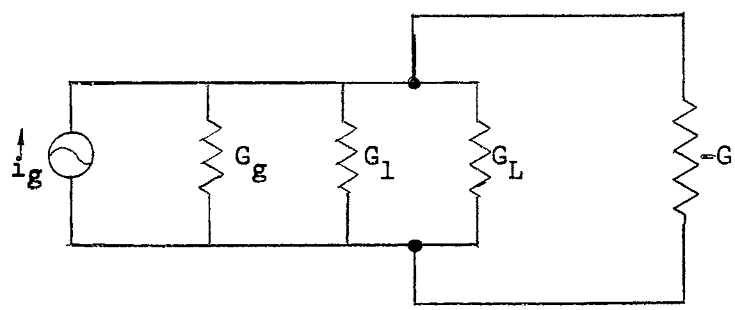


Figure 17. Equivalent Circuit of Ideal Parametric Amplifier

From the equivalent circuit of Figure 17 the gain of the amplifier might be calculated. The gain is defined as the ratio of power dissipated in the load conductance to power available from the generator.

$$\text{Power gain} = \frac{[ig^2/(G_{t1}-G)^2]G_L}{[ig_L^2/4\ G_g]} = \frac{4\ G_g G_L}{(G_{t1}-G)^2}\ . \tag{112}$$

If $G \approx G_{t1}$ very high gain is available. For this condition to exist the capacitance requirement is

$$C_3 = 2\ (G_{t1}\ G_2\ /\ \omega_1\omega_2)^{1/2}.$$

The above power gain is valid only if the input frequency is exactly that of the resonant frequency of tank 1. If the input frequency varies the gain equation is more complicated. Hefner and Wade (19) have given the gain for this case and also equations for bandwidth and noise figure for this amplifier. These relations are as follows:

$$\text{Power gain} = \frac{4\ G_g G_L}{\left\{G_{t1} - \dfrac{G}{1+[2dQ_2(\Omega_1/\Omega_2)]^2}\right\}^2 + 4d^2\left\{G_{t1}Q_1 + \dfrac{G(\Omega_1/\Omega_2)Q_2}{1+[2dQ_2(\Omega_1/\Omega_2)]^2}\right\}^2} \tag{113}$$

$$\text{Gain Bandwidth product (3db point)} = G\ B = \frac{1}{Q_2}\frac{\Omega_1}{\Omega_1}\ 2\left(\frac{G_g\ G_L}{G\ G}\right)^{1/2}. \tag{114}$$

$$\text{Noise Figure} = F = \frac{1}{\text{Power Gain}} \cdot \frac{1}{KTB} N_o = \frac{1}{4KTB}\frac{(G_{t1} - G)^2}{G_g\ G_L} N_o \tag{115}$$

The output noise N_o could be caused by the following:

1. Thermal noise at ω_1 in tank 1.
2. Thermal noise at ω_2 in tank 2.
3. Noise current at ω_1 emanating from C_t.
4. Noise current at ω_2 emanating from C_t.
5. Changes in the value of the variable capacitor C_t due to noise fluctuations at ω_3.
6. Changes in the value of the variable capacitor C_t due to noise fluctuations at 2ω.
7. Changes in the value of the variable capacitor C_t due to noise fluctuations at $2\omega_2$.

8. Changes in the value of the variable capacitor

C_t due to noise fluctuations at $(\omega_1 - \omega_2)$.

One of the most important of these is the thermal noise originating in the idling circuit which has been found to be approximately the ratio of ω_1 to ω_2. By making the idling frequency large with respect to the signal frequency this term can be minimized.

Amplifier Performance With Circulators

The negative resistance amplifier is normally operated with high gain and is just below the point where oscillations would occur. Therefore some method of making this a stable amplifier must be used. One commonly used method is to employ a ferrite circulator.

A ferrite circulator is a lossless three-port device and is best described in terms of waves. A wave entering port 1 of Figure 18 is transmitted without loss to port 2 only, a wave entering port 2 is transmitted without loss to port 3 only, and a wave entering port 3 is transmitted without loss to port 1 only. Since the circulator is lossless it should not affect the minimum noise figure.

The ideal ferrite circulator is a device which will pass energy from one port to another in one direction only. This device routes energy from one point to another but never absorbs energy.

By action of the circulator the signal to be amplified is introduced at terminal 1 and guided to terminal 2 where it enters the parametric amplifier and is amplified. This amplified signal together with the idler signal re-emerge at terminal 2 and are guided to terminal 3. The filter will allow the amplified signal to pass to the output termination

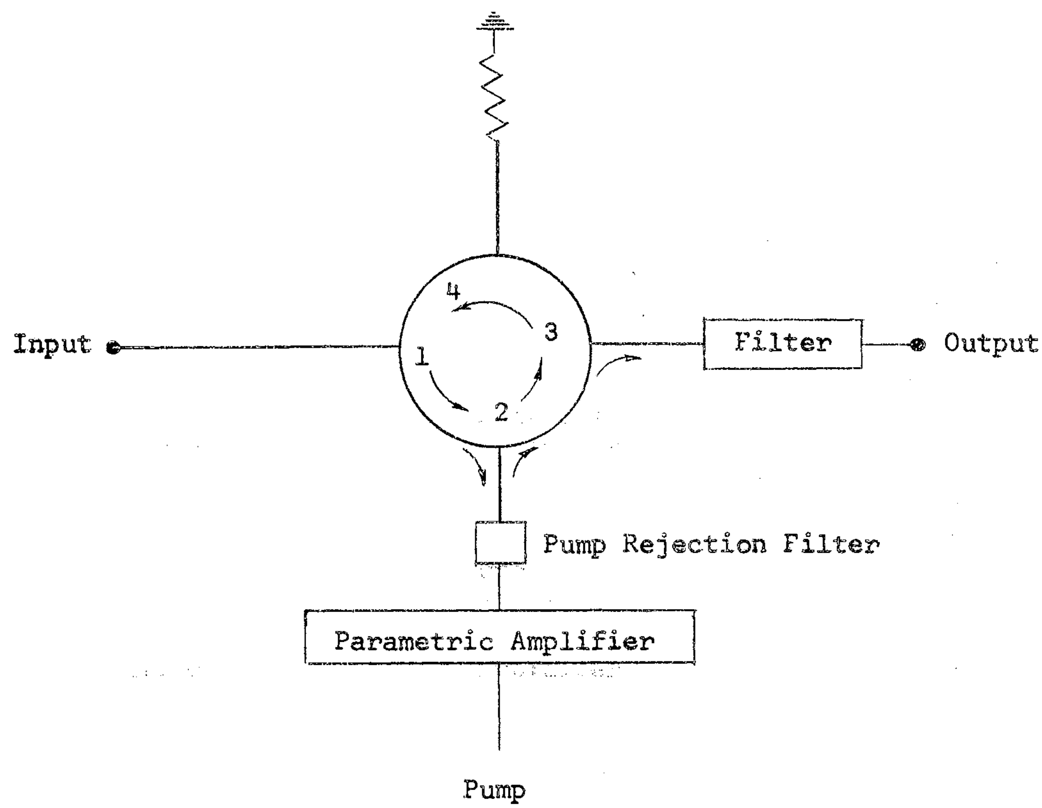


Figure 18. Parametric Amplifier with Circulator

while the idler signal will be totally reflected by this filter and routed to terminal 4 where it is terminated. The pump rejection filter prevents the pump signal from entering terminal 2 of the circulator. This arrangement allows the amplifier to always be terminated in a matched load because regardless of changes in load or generator resistances there are no reflections back through the circulator.

The circulator is not an ideal device; there is some loss between terminals and there is some reflection of power. However, practical devices appraoch the ideal and by artificially cooling the circulator a low noise figure for the negative resistance amplifier is available.

Conclusions

The analysis of the negative resistance parametric amplifier may be summarized as follows:

1. The gain of this device is proportional to the Q of the idling circuit, and to the variation of the variable reactance. To increase gain either of these could be increased.
2. The bandwidth is inversely proportional to the Q of the idling circuit, and to the voltage gain, and directly proportional to the ratio of idling frequency to amplifying frequency. This amplifier is essentially a narrow-band device.
3. One of the most important sources of noise is the thermal noise from the idling tank. This can be minimized by choosing a large ratio of idling frequency to amplifying frequency or by artificially cooling the tank circuitry.
4. A large ratio of idling to amplifying frequency will improve both the noise figure and the gain-band width product.
5. The use of a circulator will improve the stability of this amplifier.
6. The pump source should supply a signal stable both in frequency and voltage.
7. The derivations in this chapter are based upon certain approximations; therefore, the results are not exact. However, these results are satisfactory for most circuit analysis.
8. No mention of limitations upon the choice of pump frequency has been made. The conditions where pumping can be accomplished

are related to the Mathieu equation. A complete analysis of pumping has been made by Hooper (20).

Airborne Instruments Laboratory (21) has made a study and experimental investigation of the use of semiconductor diodes as low noise amplifiers, convertors, and harmonic or sub-harmonic generators.

This report indicates that optimum performance is obtained by using a ferrite circulator in the parametric amplifier.

CHAPTER VI

PARAMETRIC DIODE FIGURE OF MERIT

A diode figure of merit has been proposed by Uhlir, (22), Robinson (23), and Mortenson (24). The figure of merit derived by Mortenson is directly applicable in comparing diodes of various types and to optimize their respective design for low noise amplifier use. It can be employed by the circuit designer to predict an amplifiers noise figure performance including the effects of choice of bias, pump swing, signal frequency and pump frequency for a given diode. This figure of merit is based upon amplifier noise performance. Mortensons derivation of this figure of merit is as follows.

Derivation of Figure of Merit

Figure 13, which represents a parametric amplifier using an ideal lossless diode, can be modified as shown in Figure 19 to include any diode loss. The parallel equivalent circuit of the diode is used. The result of including a diode conductance is that the diode conductance evaluated at the signal frequency, G_{D1}, in effect, appears across the signal tank, while that evaluated at the idler frequency, G_{D2}, in effect appears across the idler tank.

The noise figure for such a parametric amplifier could be expressed as in equation 116, provided that Q of the diode is greater than four at the signal and idler frequencies.

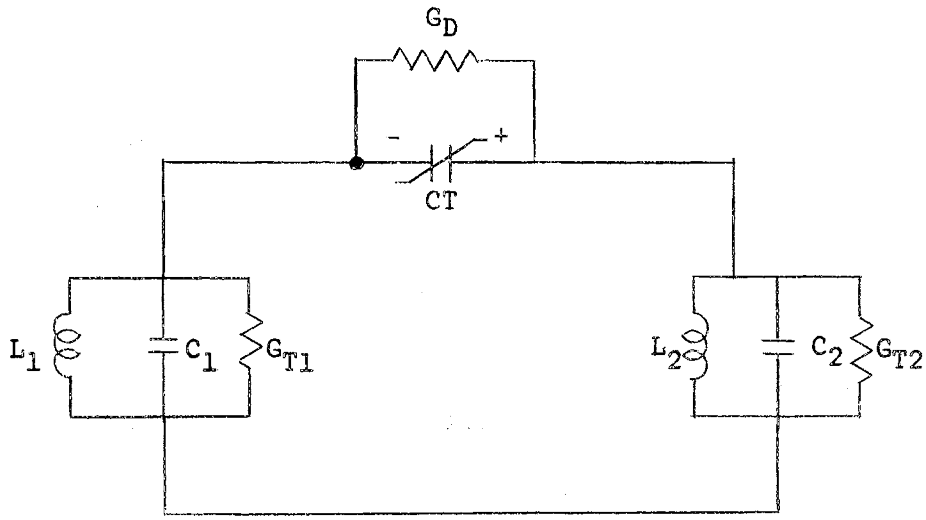


Figure 19. Parametric Amplifier Using a Lossy Diode

$$F \approx 1 + G_1/G_g + \left[\frac{G(\omega_1)}{G_g}\right] + \frac{\omega_1}{\omega_2} + \frac{G_{D1}}{G_g} \qquad (116)$$

where

G_1 = unloaded conductance of the signal tank.

G_g = the source conductance in shunt with the signal tank.

$G(\omega_1)$ = the negative conductance produced in shunt the signal tank.

G_{D1} = the diode conductance at the signal frequency under pumped conditions.

ω_1 = the angular signal frequency.

ω_2 = the angular idler frequency.

The effect of G_{D2} on the noise figure of the amplifier is the reduction of $G(\omega_1)$ as shown by equation 106 which could be written in a more general form as equation 117.

$$G(\omega_1) = C_n^2\ (\omega_1)\ (\omega_2)\ /\ 4G_{t2}\ . \qquad (117)$$

where

C_n = appropriate harmonic time dependent capacitance.

G_{t2} = total conductance in shunt with the idler tank.

or

$G_{t2} = G_2 + G_{D2}$.

Under pumped conditions the capacitance C_n, can be represented by a Fourier series such as equation 118.

$$C(t) = C_o + 2C_1 \operatorname{Cos} \omega_3 t + 2C_2 \operatorname{Cos} 2\omega_3 t + \cdots + \qquad (118)$$

This is assuming that C(t) is an even function. The Fourier coefficients can be determined from the plot of capacitance versus voltage by graphical analysis or by the following expression, assuming the pump voltage to be a cosine function of time.

$$a_n = \frac{2}{\pi} \int_o^{\pi} \frac{\operatorname{Cos} n\theta \, d\theta}{(1 + \gamma \operatorname{Cos} \theta)^m} \quad n = 1, 2, 3, \cdots \qquad (119)$$

where

a_n = the normalized Fourier coefficients.

$\gamma = V_m/Vo'$ = the relative voltage swing.

V_m = applied pump voltage amplitude.

$Vo' = Vo + \theta_m$ = sum of the reverse bias, Vo, and internal voltage θ_m.

m = the exponent of the C versus V law.

or

m = 1/2 for abrupt junction or 1/3 for graded junction.

θ = integration factor.

This coefficient, a_n, depends upon the C-V law exponent, m, the relative voltage swing, γ, but not on the absolute values of Vo' or V_m.

The capacitance C_n of equation 117 can be expressed in terms of the normalized coefficient, a_n, and the value of junction capacitance at the chosen bias point, C_o. Thus equation 117 can be written as

$$G(\omega_1) = C_t^2\ \omega_1\omega_2/4\ Gt_2 = (a_nC_o)^2\ \omega_1\omega_2/4\ Gt_2. \qquad (120)$$

a_n then is a particular harmonic of the pump signal.

The general noise figure expression for the parametric amplifier might be more useful if the effect of gain on noise considerations is determined. Consider equation 112 which is rewritten as

$$\text{power gain} = 4G_gG_L/(G_{t1}-G(\omega_1))^2 \qquad (121)$$

where G_L = load conductance in shunt with the signal tank.

$Gt_1 = G_1+G_L+GD_1$ = total conductance in shunt with the the signal tank if no circulator is used.

Equation 73 indicates that the parametric amplifier should have high gain so that any post-amplifier noise becomes insignificant. Equation 112 indicates that for high gain G_t must be approximately equal to $G(\omega_1)$. Also

$$G_g \gg G_1 + G_{D1} + G_L \text{ (with no circulator)} . \qquad (122)$$

Therefore,

$$G_g \approx G_{t1} \approx G(\omega_1) . \qquad (123)$$

Further assume

$G_{D2} \gg G_2$ (generally true at least for the narrow to moderate bandwidth idler circuit) .

Using the above approximations, the noise figure expression becomes

$$F \approx 1 + G_1/G_g + f_1/f_2 + \frac{4\, G_{D1}\, G_{D2}}{a_n^2 C_o^2 \omega_1 \omega_2} . \tag{125}$$

The effect of G_2 can be minimized by proper choice of G_g.

From high gain noise figure considerations

$$\frac{G_{D1}}{\omega_1 C_o} \approx \omega_1\, a_o^2\, C_o\, R_D \tag{126}$$

and

$$\frac{G_{D2}}{\omega_2 C_o} \approx \omega_2\, a_o^2\, C_o\, R_D \tag{127}$$

where R_D is the series equivalent diode resistance.

Substituting equations 126 and 127 into equation 125 gives

$$F \approx 1 + G_1/G_g + [f_1/f_2 + f_1 f_2/f_D^2] . \tag{128}$$

where

$$f_D \equiv \left(\frac{a_n}{4}\right) a_o^2\, C_o\, R_D . \tag{129}$$

f_D is thus defined as the diode figure of merit, and has the units of frequency.

The high gain noise figure is restricted in the sense that f_2 must be chosen to exceed a certain fraction of f_D, if amplification is to exist and oscillations are not to exist. This restriction results from the fact that as a_n/a_o^2 is increased to increase f_D and lower the noise figure, G_g must be increased to prevent oscillations. There is, however, a limit to increasing G_g if any amplification is to exist, namely

$$B_1/G_{t1} \geq 1 \tag{130}$$

where

$$G_{t1} \approx G_g \approx G(\omega_1) \tag{131}$$

and

$$B_1 \approx \omega_1 a_o C_o . \tag{132}$$

From equations 117 and 129,

$$f_2 \geq (a_n/2a_o)f_D . \tag{133}$$

For an example, assume various values for γ. The following conditions on f_2 must be met for fundamental pumping (using $a_n = a_1$):

$$\text{if } \gamma = 0.7 \text{ then } f_2 \geq 0.21 f_D$$
$$\gamma = 0.8 \text{ then } f_2 \geq 0.26 f_D$$
$$\gamma = 0.9 \text{ then } f_2 \geq 0.33 f_D$$
$$\gamma = 1.0 \text{ then } f_2 \geq 0.5 f_D .$$

Should f_2 be chosen less than this limiting fraction of f_D, then γ, the relative pump swing, must be reduced so that a_1/a_o is reduced, and a state of amplification, not oscillation, exists. Under these conditions the full potential of a given diode is not utilized and a poorer noise figure than possible will exist.

To minimize the part of equation 125 within the brackets f_2 should be chosen so that the idler frequency is equal to the figure of merit or

$$f_2 = f_D$$

thus

$$f_{min} = 1 + G_1/G_g + 2f_1/f_D . \tag{134}$$

Since f_D can be evaluated for any voltage dependent capacitor from its C-V relationship, series resistance, chosen bias and relative pump swing, this figure of merit should be useful in comparing capacitors of various types as well as optimizing their respective designs. Since this figure of merit is directly involved in the choice of idler frequency and in conjunction with the choice of signal and idler frequencies it determines the amplifier noise figure.

Figure of Merit in Terms of Diode Design Parameters

The following expressions for junction capacitance and diode resistance, derived by Mortenson (25), can be substituted into equation 129.

(a) $$C_o = \frac{\varepsilon A_j^2 eN_d}{2(\theta+V_o)^2} \text{ farads .} \tag{135}$$

and

(b) $$R_D = t/e\mu_n N_d \propto A_j \text{ ohms .} \tag{136}$$

where

ε = the dielectric constant

e = the electronic charge

N_d = the base doping concentration (N type)

A_j = the junction area

t = the base thickness

μ_n = the electron mobility

$\propto$ = the ratio of effective base cross sectional area to junction area.

Combining equations 129, 135 and 136 for an abrupt junction diode gives

$$f_D = \frac{\sqrt{2}}{4\pi}\left(\frac{e}{\varepsilon}\right)^{1/2} \frac{\propto}{t} \mu_n N_d^{1/2} (\theta_o + V_o)^{1/2} \frac{a_n \gamma}{a_o^2 \gamma} . \tag{137}$$

Equation 137 could then be used to compare diodes of different types or to study the effects of impurity concentration in various diodes.

Capacitance Measurements

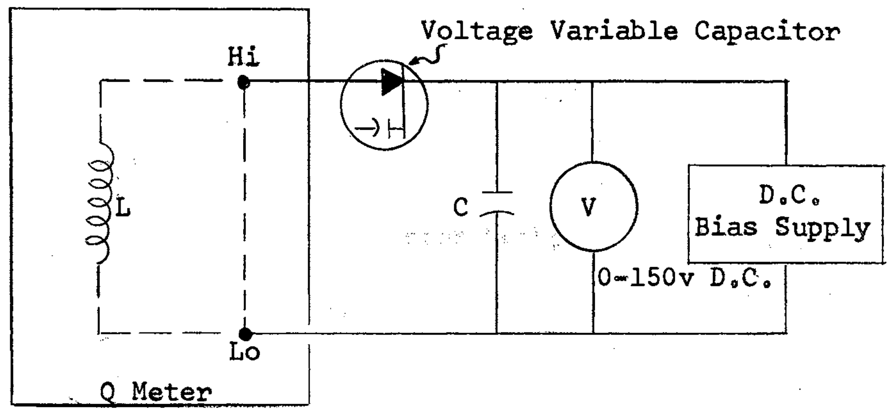
Once the capacitance-voltage relationship for a diode is known, the Fourier coefficients used in the figure of merit can be determined by a graphical method. A digital computer would be very useful in the computations involved in this graphical procedure. The Fourier coefficients can then be used to determine the diode figure of merit.

Figure 20 shows several methods of making a measurement of diode capacitance (26).

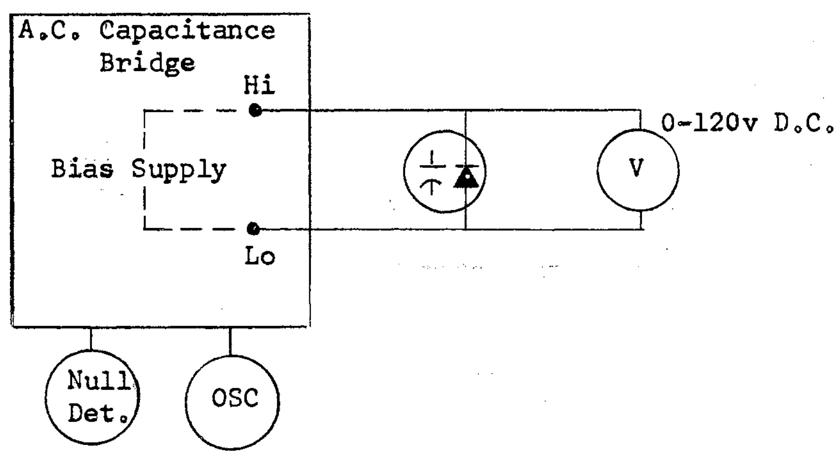
The Q meter technique shown in Figure 20A can measure capacitance up to 460 pf. The frequency range of the A.C. bridge method of Figure 20B is extended by using an external oscillator and null detector. Capacitances from 1 to 1000 pf can be accurately determined by this method. The R-X meter method is directly applicable up to 20 pf capacitance.

Limitations on the Evaluation of the Figure of Merit

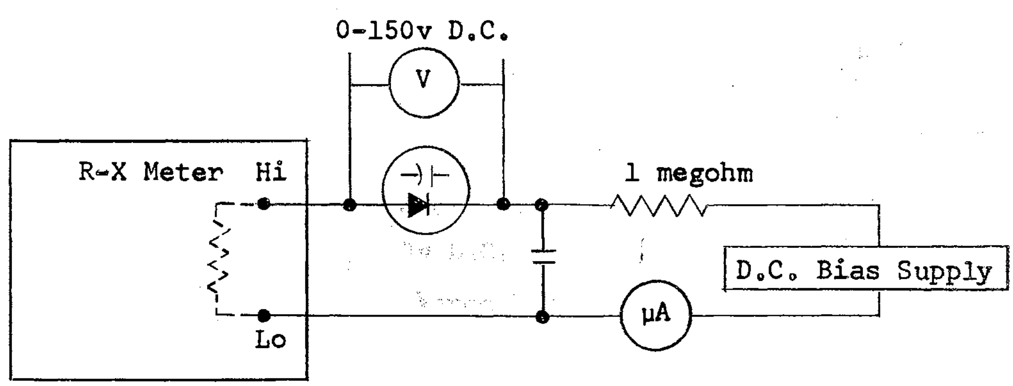
Mortenson (27) points out that the actual diode deviates from the assumed model in several ways. First, the diode, particularly those of double alloyed type, have a small base thickness and are lightly doped and will have a base thickness which is voltage dependent and is thus a function of the bias applied to the diode. The diode resistance, R_D, then is dependent upon the applied bias. Secondly, surface effects appear as additional resistive and capacitive elements in shunt with the diode, thereby making the figure of merit a function of frequency.



(a) Q Meter Method



(b) A.C. Bridge Method



(c) R-X Meter Method

Figure 20. Capacitance Measurement

Thirdly, the packaging of the diode has been neglected.

Another problem is the definition of reverse voltage breakdown and forward conduction. There seems to be little agreement as to how these points are defined. A general rule is to define these points as the voltage at which the respective forward or reverse current reaches one microamp. Robinson (28) discusses this problem in detail.

Conclusions

The diode figure of merit can be calculated from experimentally derived data. At microwave frequencies this would probably involve building a degenerate parametric amplifier in order to determine the diode characteristics. An approximate method of obtaining this data would be to measure the diode capacitance with a Q meter and calculate the resistance from equation 136. The digital computer could then be used to compute the Fourier coefficients and diode figure of merit.

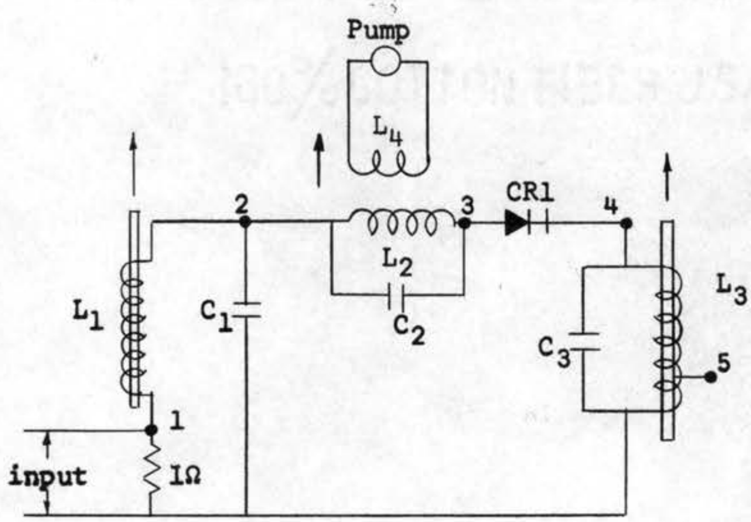
CHAPTER VII

EXPERIMENTAL PARAMETRIC AMPLIFIER

Very little experimental work has been reported on parametric amplifiers operating in the audio frequency range. Planinac (29) reported successful operation of such an amplifier in the medium radio frequency range. A variation of the circuit discussed by Planinac is investigated in this study.

Circuit Configuration

Figure 21 shows the parametric amplifier constructed for this study.



$L_1 = L_3 = 5\text{-}20$ Mh

$L_2 \approx 10$ Mh

$C_1 \approx 0.05$ uf

$C_2 \approx 0.005$ uf

$C_3 \approx 0.005$ uf

Figure 21. Experimental Parametric Amplifier

The major difficulty encountered in constructing the amplifier was obtaining inductors with sufficiently high Q. Those chosen were supplied by a commercial manufacturer and were of rather low Q, but were relatively inexpensive.

Polystyrene capacitors were used in the circuit. The tank circuits constructed from these components had a Q of approximately 25 at a frequency of 10 K.C.

The frequencies chosen for investigation were basically as follows:

Signal frequency = 10 kc;

Pump frequency = 130 kc;

Idler frequency = difference of 130 kc and 10 kc

= 120 kc.

Various types of diodes and transistors were tested in this circuit. The best results were obtained from a 750 ma "top hat" silicon rectifier. The capacitance of this diode was measured at a frequency of 1 kc. This capacitance was found to vary from approximately 50 picofarad, under reverse bias, to 60 pf, at zero external bias, to 1000 pf, under forward bias conditions.

Preliminary adjustment of the tank circuits was made before applying both signal and pump power to the circuit. Final adjustment of pump power, signal frequency, pump frequency, and idler tank tuning was made by individually adjusting these parameters for peak stable signal at point 2 of the circuit shown in Figure 21.

Measurements

The basic measuring device used in this experimental work was a Tektronix 545A oscilloscope with a type LA preamp. The signal source was a Hewlett Packard Model 201B audio oscillator. The pump source was a General Radio Type 1210-C R-C oscillator. Frequency measurements were taken directly from the signal generators. Ample warm up time was allowed before data was recorded.

Experimental Results

This experimental work shows that a parametric amplifier operating in the audio frequency range is feasible. The experimental model provided stable amplification for a narrow band of frequencies.

This circuit seemed to operate satisfactorily without external biasing. Planinac discussed the operation of such a self biased amplifier, and explained that the self bias of the diode was developed as a result of hole storage in the diode.

The pump power was adjusted so that maximum gain without oscillations was obtained. If the pump power exceeded a certain level the circuit would act as a generator, converting power at the pump frequency to power at the input and idler frequencies, even with no input signal applied.

Table 1 shows the amplitude of signals at various points in the circuit of Figure 21. The waveform observed at point 2 was essentially at the input frequency with a small amount of pump signal also present. The pump signal was more apparent when the pump was tuned "off frequency". A higher Q tank circuit would have reduced this pump signal component. The signal at point 3 appeared to be the pump signal modulated by the input signal. The values recorded in the table are peak to peak values and thus represent the total signal applied to the diode. The signal at point 4 appeared to contain components of input, pump, and idler frequencies. The idler frequency was the predominant frequency.

TABLE I

EXPERIMENTAL DATA

POINT IN CIRCUIT						
1	2	2'	3	4	A = 2/1	A' = 2'/1
0.001	0.2	0.03	2.1	3.4	200	6.7
0.0015	0.4	0.05	2.2	3.6	266	8.0
0.004	0.6	0.08	2.2	3.6	150	7.5
0.01	0.8	0.24	2.6	3.7	80	3.3
0.03	1.0	0.54	2.6	3.9	33	1.9

The amplitude of signals is recorded in peak to peak volts.

Frequency of input signal = 11.4 kc.

Frequency of pump source = 132 kc.

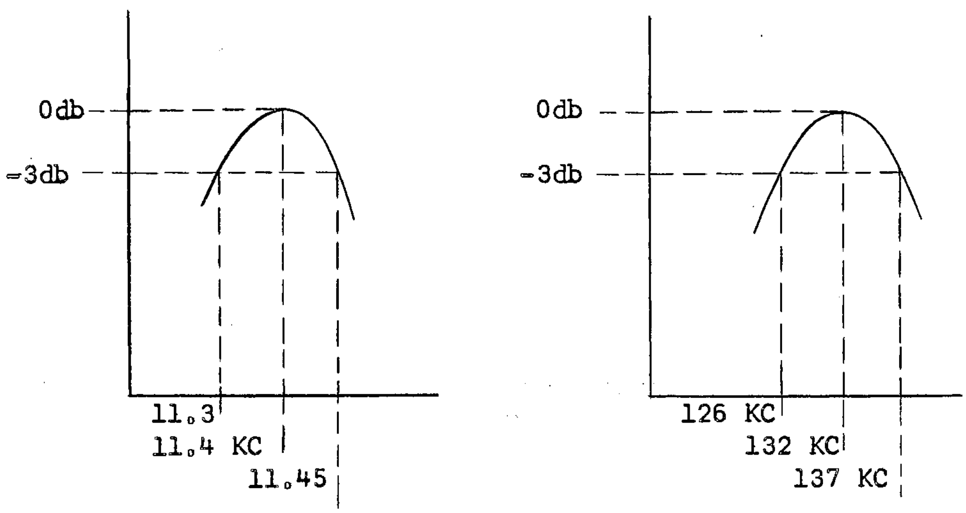
Idler circuit tuned for difference of pump and input frequencies.

The data of Table 1 indicates that the gain of the amplifier is greater for low amplitude signals. The tabulated results were obtained with no noticeable distortion in the signal at point 2 of Figure 21. If the input signal exceeded the values listed in table, the waveform taken at point 2 indicated considerable distortion. For linear operation the input signal must be small with respect to the pump signal.

The column of table labeled 2' indicates the signal present at point 2 of the circuit with the pump power reduced to zero.

The column labeled A indicates the ratio of signal amplitude at point 2 to input signal amplitude. The ratio of the signal amplitude at point 2 with zero pump power supplied, to the input signal amplitude is recorded in the column labeled A'. The values listed for A' are probably more indicative of parametric amplification than those values of A.

The experimental amplifier was a narrow band device. Figures 22A and 22B show the relative amplitude of the signal at point 2 of Figure 21 as the pump and source frequencies were varied.



Pump Frequency = 132KC   Input Frequency = 11.4KC

Figure 22. Relative Gain Bandwidth of Experimental Amplifier

Interpretation of Results

The results obtained from this experimental work indicates that the operation of a neagive resistance parametric amplifier in the audio frequency range is feasible. This circuit provides stable amplification with the output at either the input frequency or at a higher frequency. This circuit could find applications where frequency separation or shift is required.

CHAPTER VIII

SUMMARY

Synopsis

The parametric amplifier is a low noise device which uses the mixing properties of a non-linear reactance to accomplish amplification. The underlying theory of operation of this device is not new; however, the method of obtaining amplification by this means was not available until the P-N Junction diode was developed. Only recently has the parametric amplifier found widespread use in such a system as radar.

The non-degenerate negative resistance parametric amplifier is basically two resonant tank circuits coupled by a nonlinear capacitor. One tank circuit is tuned to the input signal frequency, the other to the idler frequency. This idler frequency is the difference of the frequency of the local power supply and the input signal. These two frequencies are incommensurable. The coupling capacitor is a nonlinear function of the voltage applied across it. This voltage is primarily that supplied by the pump. The nonlinear element then produces new frequencies which are combinations of the pump and input signal frequencies and their harmonics. By proper circuit design the lower sideband, or idler frequency, is allowed to develop across the tank circuit which is resonant at that frequency. The input signal frequency will cause a voltage across only the input tank circuitry. All other frequencies produced by the nonlinear element are essentially short circuited. The Manley-Rowe relations indicate the amount of power available at the various frequencies. These relations indicate that the negative resistance parametric amplifier is a regenerative feedback amplifier and if it is operated below the point of oscillation very high power gain is available.

In making an analysis of this circuit the nonlinear capacitor may be replaced by a linear time varying capacitance. This allows all circuit theorems to be used. The result is that an equivalent circuit may be derived. The analysis of this equivalent circuit indicates that to increase the power gain, the Q of the idler circuit or the variation in the variable capacitance must be increased. The bandwidth is inversely proportional to this Q and to the voltage gain. The parametric amplifier is normally a narrow bandwidth device when used as a high gain amplifier.

One of the most important sources of noise is the thermal noise generated in the idler circuit. This noise source can be reduced by choosing the idler frequency to be large compared to the input signal frequency. Another method of reducing this noise source is to artifically cool the idler circuitry.

The use of a circulator allows near optimum performance and increases the stability of the device.

The parametric amplifier, using a circulator to connect the various circuits, provides a low noise high gain device which is very useful as the first stage of amplification in a receiving system. The noise figure for such a receiver is approximately that of the parametric amplifier itself.

The diode figure of merit can be computed from experimental data, and should be useful to the circuit designer in the comparison of various diodes and in optimizing the design of the parametric amplifier.

An experimental negative resistance parametric amplifier was investigated in this study. The experimental model operated quite satisfactorily as a narrow band amplifier in the audio frequency range. The Q of the various tank circuits was not as high as desirable. Thus, the separation of the various frequencies in the amplifier was not as great as might be necessary. The Q of these circuits affected the gain of the amplifier.

Several diodes and transistors were tested in the experimental circuit. Diodes of the power rectifier type provided the best circuit operation. This is probably due to the very large variation of the junction capacitance.

Suggestions for Future Investigations

The possibility of constructing M derived band pass filters for use in amplifiers in the audio frequency range could be investigated. These filters should allow the construction of amplifiers with greater gain and band pass.

It is possible that a parametric amplifier could be used to an advantage in the transfer of digital data over telephone circuits. One existing system transfers digital data over phone lines by designating ones and zeros by different frequencies. Since band pass filters are used in the existing equipment, and often it is desirable to shift to a higher frequency after the signal has been detected and decoded, the parametric amplifier might prove to be an economical means of accomplishing gain and frequency separation and shift.

SELECTED BIBLIOGRAPHY

1. Planinac, J. W. "An Investigation of Junction Diode Parametric Amplifiers at Medium Radio Frequencies." (unpub. M. S. thesis, Oklahoma State University, 1960).

2. Faraday, M. "On a Peculiar Class of Acoustical Figures, and Certain Forms Assumed by a Group of Particles Upon Vibrating Elastic Surfaces." Phil. Trans. Roy. Soc. Vol. 109. (1831) 299-318.

3. Melde, F. "Uber Erregung Stehender Wellen Eines Faden-Formeigen Korpers." Annalen der Physic und Chemie. Vol. 109. (1859) 193-215.

4. Lord Rayliegh. "On the Crispations of Fluid Resting Upon a Vibrating Support." Philosophical Magazine. Vol. 16. (1883) 50-53.

5. Hartley, R. V. L. "Oscillations in a System With Nonlinear Reactance." The Bell System Technical Journal. Vol. 15. (1936) 424-440.

6. Manley, J. M. and H. E. Rowe. "Some General Properties of Nonlinear Elements - Part I - General Energy Relations." Proc. IRE. Vol. 44. (July, 1956) 904-913.

7. Sarbacher, R. I. Encyclopedic Dictionary of Electronics and Nuclear Engineering. New Jersey: Prentice Hall, 1959, 47.

8. Hooper, E. T., Jr. The Nature of Microwave Parametric Amplification. United States Ordinance Report AD 290775 (Cameron Station, Alexandria, Virginia).

9. Black, H. S. Modulation Theory. New York: Van Nostrand, 1953, 125.

10. Uenohara, M. "Noise Considerations of the Variable Capacitance Parametric Amplifier." Proc. IRE. Vol. 48. (Feb., 1960) 169-179.

11. Salzberg, B. "Masers and Reactance Amplifier - Basic Power Relations." Proc. IRE. Vol. 45, No. 11. (Nov., 1957) 1544-1545.

12. Black, H. S. Modulation Theory. New York: Van Nostrand, 1953, 125-145.

13. Reed, E. D. "Diode Parametric Amplifiers - Principles and Experiments Part I." Semiconductor Products. Vol. 4, No. 1 (Jan. 1961) 25-30.

14. Uenohara, M. "Noise Considerations of the Variable Capacitance Parametric Amplifier." Proc. IRE. Vol. 48. (Feb., 1960) 169-179.

15. Reed, E. D. "Diode Parametric Amplifiers - Principles and Experiments Part I." Semiconductor Products. Vol. 4, No. 1. (Jan., 1961) 25-30.

16. "Definition of Terms," IRE Dictionary of Electronics Terms & Symbols (IRE, 1961) 96.

17. Blackwell, L. A. and K. L. Kotzebue. Semiconductor Diode Parametric Amplifiers. New Jersey: Prentice Hall, 1961.

18. Louisell, W. H. Coupled Mode and Parametric Electronics. New York: John Wiley & Sons, 1960, 109.

19. Hefner, H. and G. Wade. "Gain Bandwidth and Noise Characteristics of the Variable Parameter Amplifier." Journal of Applied Sciences. Vol. 29, No. 9. (Sept., 1958) 1321-31.

20. Sarbacher, R. I. Encyclopedic Dictionary of Electronics and Nuclear Engineering. New Jersey: Prentice Hall, 1959, 47.

21. Final Technical Report on Application of Semiconductor Diodes to Low Noise Amplifiers, Harmonic Generators and Fast Acting TR Switches. Airborne Instruments Laboratory Report No. 4589-1 (Deer Park Long Island, New York) 1961.

22. Uhlir, A., Jr. "The Potential of Semiconductor Diodes in High Frequency Communications." Proc. IRE. Vol. 46, No. 6 (June, 1958) 1099.

23. Robinson, B. J. and J. T. DeJager. "Optimum Performance of Parametric Diodes at S-Band." The Institute of Electrical Engineers Paper No. 3849 E. (May, 1962).

24. Mortenson, K. E. "Parametric Diode Figure of Merit and Optimization." Journal of Applied Physics. Vol. 31, No. 7. (July, 1960) 1207-12.

25. Mortenson, K. E. "Parametric Diode Figure of Merit and Optimization." Journal of Applied Physics. Vol. 31, No. 7. (July, 1960) 1207-12.

26. Chiffey, F. P. and J. L. Gurley. "Three Ways to Measure Varacters of the Future." Electronics Engineering Manual. Vol. X. New York: McGraw Hill, 125-127.

27. Mortenson, K. E. "Parametric Diode Figure of Merit and Optimization." Journal of Applied Physics. Vol. 31, No. 7. (July, 1960) 1207-12.

28. Planinac, J. W. "An Investigation of Junction Diode Parametric Amplifiers at Medium Radio Frequencies." (unpub. M. S. thesis, Oklahoma State University, 1960).

VITA

Carl Bluford Johnson

Candidate for the Degree of

Master of Science

Thesis: AN INVESTIGATION OF PARAMETRIC AMPLIFICATION OF AUDIO FREQUENCY SIGNALS

Major Field: Electrical Engineering

Biographical:

Personal Data: Born near Altus, Oklahoma, June 5, 1930, the son of Carl E. and Myrtle May Johnson.

Education: Attended grade and high school at Humphreys, Oklahoma; graduated from Humphreys Public High School in 1948; received the Bachelor of Science degree from the Oklahoma State University, with a major in Electrical Engineering, in May, 1961.

Professional experience: Employed by the General Telephone Company from 1950 until 1952. Served with the United States Army from 1952 until 1954, then was reemployed by the General Telephone Company until 1958. While with the Telephone Company the duties included the installation and repair of telephone equipment, and the preparation of engineering plans for installation and rehabilitation of telephone plant facilities. Employed by the Federal Aviation Agency from May, 1961 until the present as an Electronics Engineer, serving as an Instructor in the Radar Branch of the Federal Aviation Agency Academy, Oklahoma City, Oklahoma. Registered as a professional Engineer in the State of Oklahoma in May, 1965. Is a member of the Oklahoma Society of Professional Engineers, Sigma Tau and Eta Kappa Nu Engineering Societies.