HEAT TRANSFER TO A BUBBLE-AGITATED LIQUID

By

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PREFACE

This thesis describes an effort to characterize the heat transfer between a solid surface and a bubble-agitated liquid--where the solid surface may be the container walls or a tube or coil submerged in the liquid.

As with most engineering problems, the complexity of a bubble-agitated system defies any purely mathematical analysis. Thus, an effort to predict heat transfer coefficients must rely on experiment and the intelligent use of dimensional analysis. Such has been the approach used in this study.

Of course, the experimental effort normally expended in a Masters research program is hardly extensive enough by itself to completely characterize a complex phenomena. Fortunately there have been other workers, and a survey of their findings has made my job much easier. Actually, an analysis of their work constitutes a significant part of this thesis.

A combination of their results and those of my own experiments has yielded sufficient data to develop a preliminary general correlation for heat transfer to a bubble-agitated liquid. I think the correlation should be applicable for many design purposes, but it should be used with the realization that relatively little data has gone into its construction, and that there are variables which it does not account for--such as surface tension, gas density, and gas-distributor design. There are indications, however, that these variables are not too important.

Nearly all of the literature heretofore published on bubble-agitated heat transfer has been written in a foreign language. I have made no attempt to completely translate the papers, but I think their essence has been deciphered with no misrepresentation.

A list of the nomenclature used in this thesis is presented in Appendix A. It is consistent with that used in most heat transfer texts. In presenting the equations of other authors, I have changed their nomenclature where necessary for consistency.

I wish to thank my advisor, Professor Kenneth J. Bell, for his guidance, his confidence, and his patience. I gratefully acknowledge the financial assistance of Dow Chemical Company which made this study possible.

I extend a special thanks to W. R. Penney, who helped with the photographic work, supplied the electrical heating tape, assisted in constructing some of the apparatus, and offered valuable advice and consultation throughout the course of this investigation.

My wife, Sharon, deserves special recognition for her patience and encouragement, and also for her typing of the manuscript.

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CHAPTER I

INTRODUCTION

Agitation is uniquely important as a unit operation, because very often it is essential in promoting other unit operations, such as heat and mass transfer. Conventional treatments of the subject of agitation usually confine their attention to mechanical stirring, while alternate techniques, such as "bubble agitation", are not considered. Yet, bubble agitation can be very useful, and it fits quite naturally into operations involving gas-liquid contacting.

For example, in carrying out gas-liquid reactions, an excess of the reactant gas can be circulated through the system--the rising bubbles thus providing the needed agitation. The advantages of such a technique over mechanical agitation are considerable--especially at high pressures where shaft sealing is a problem, and in column-type reactors of large length/diameter ratios, where mechanical agitation is awkward to arrange.

The proper design of reactors for such systems requires a knowledge of the heat transfer rates associated with bubble agitation. However, no general correlation for predicting the coefficients has been available, and relatively little research on bubble-agitated heat transfer has been done. There are a few correlations presented in the literature, but generally they are not well supported by consistent and extensive data. Indeed, many of them can be discounted in an almost a priori manner (as will be shown in Chapter III.)

This present study represents an effort to improve our understanding of bubble-agitated heat transfer. A major part of the work has been experimental, since the most immediate need was for more data on the subject. However, a considerable effort has gone into the compilation, study, and evaluation of previous work done in the field. This part not only supplemented the experimental work, but its presentation adds completeness to the thesis, and serves the purpose of offering an overall picture of the present "state of the art". The ultimate objective of this study, of course, has been to develop a general correlation, so that bubble-agitated systems can be designed with confidence and accuracy.

CHAPTER II

LITERATURE REVIEW

Apparently the first work of any consequence concerning bubble-agitated heat transfer was that of Novosad (1) published in 1954. The paper was written in Czech, so only a sketchy review can be given. Novosad used air to agitate water, butanol, and glycerin in a 1.5-inch diameter tube. The air was introduced through a porous disc. Unlike the other investigators, who used electrical heat, Novosad supplied heat by circulating hot water through a jacket on the vessel. Hence, the inside film coefficient had to be calculated from an over-all coefficient. Also unlike the others, Novosad's final correlation includes the Nusselt Number for natural convection which allows his equation to be extended to zero gas velocities. He also made gas holdup measurements, and used these in his final correlation for heat transfer, viz:

$$Nu = \frac{Nu_o}{1 + 30\sqrt{\Phi}} + 2.28\ Re^{0.7} Pr^{0.33}$$

where: Nu_o = Nusselt Number for natural convection (ΔT and length unspecified)

Φ = fractional gas holdup

$Re = \frac{U_s D}{\nu\sqrt{\Phi}}$

D = column diameter

U_s = superficial gas velocity

The next work published was that of Kölbel (2) on the air-water system. Kölbel used an internal electrical probe as a heat source for columns of 3.6-, 7.6-, and 11.5-inch diameter. Air was introduced through various types of perforated plates. Superficial gas velocities ranged from zero to 0.328 ft/sec. Kölbel also studied the effect of liquid viscosity using sugar solutions of various concentrations. He presented this data as a plot of h vs. U_s with viscosity, ν, as a parameter. A cross plot of this data at constant U_s, indicates that:

$$h \propto \nu^{-0.35 \text{ to } -0.40}$$

In general, Kölbel's data indicate that column diameter, liquid height, and perforated plate design do not affect the heat transfer--at least in any regular way. His final correlation for liquids with Prandtl Number equal to that of water (apparently) is:

$$Nu = 43.7\ Re^{0.22} \qquad Re>150$$

$$Nu = 22.4\ Re^{0.355} \qquad Re<150$$

where the diameter term in Nu and Re is that of the electrical heating probe--which was constant in all runs.

Kölbel later published two more papers (3,4) concerning his work with slurries of Kieselguhr, sand, and other finely divided solids in various media. For suspensions of Kieselguhr in machine oil, spindle oil, and water he offers the correlation:

$$Nu = 227.5\ Re^{0.161} Pr^{-0.038} \qquad \text{Laminar}$$

$$Nu = 454.0\ Re^{0.113} Pr^{-0.135} \qquad \text{Turbulent}$$

with no quantitative criterion being established for the laminar and turbulent regimes. The diameter term in Nu and Re is defined by:

$$d = \sqrt{(D^2 - D_H^2)(1-\Phi)}$$

where: D = column diameter

D_H = diameter of heating probe

Φ = fractional gas holdup

The Reynolds Number is then defined by:

$$Re = \frac{U_s d}{\nu \Phi}$$

which is similar to that used by Novosad.

A paper was published in English in 1962 by Fair and co-workers (5). Their work was confined to the air-water system, but they used commercial-sized equipment--an 18-inch column heated externally, and a 42-inch column heated by means of a single electrical probe disguised in a "dummy" tube bundle. Air was introduced through a 9-inch diameter perforated sparge ring. They compared their data with that obtained in a 7.6-inch column by Kölbel (2). The agreement was quite good, and it was concluded that column diameter and location of the heating surface (i.e., internal or external) do not affect the coefficient. Their results, along with Kölbel's data for water, were summarized by the following dimensional equation:

$h = 1200\ U_s^{0.22}$ $\qquad U_s > 0.005$ ft/sec

h = heat transfer coefficient, Btu/hr·ft^2·°F

U_s = superficial air velocity, ft/sec

Fair also presents gas holdup data for columns of various diameter. He concludes that, up to 18 inches, column diameter significantly affects holdup.

Later in 1962, Kast (6) published the results of his work with water, and an aqueous solution of 45% isopropanol. He used an externally-heated column of 11.3 inches in diameter, and introduced air through various sintered discs and perforated plates. He presents some sort of theoretical analysis (the paper is in German) of the liquid motion within the vessel. He compares his data with that of Kölbel for water, and presents a generalized correlation of the form:

$$St = f(ReFrPr^2)^{1/3}$$

Finally, a related work on mass transfer to a bubble-agitated liquid was published in 1964 (in English) by Yoshitome (7). It is quite an extensive work experimentally, but no general correlation is offered. In general, Yoshitome concluded that vessel diameter, liquid height, gas distributor design, and location within the vessel had little or no effect on the mass transfer coefficient.

These, then, as far as the author is aware, constitute the extent of work done on heat and mass transfer between a solid suface and a bubble-agitated liquid. While a considerable amount of actual work has been done, the state of the art has not been much advanced. Obviously, most of the work has been done with air-water systems. A thorough appraisal of the general correlations offered will be deferred until Chapter III, but a few general comments might be appropriate here.

Novosad's work would be a very important contribution but for the fact that his heat transfer data are probably not too reliable, owing to the indirect method he used in evaluating the coefficient.

The properties of an aqueous isopropanol solution, as used by Kast, are hardly different from those of water, and data obtained with this system do not contribute much more than data for pure water.

Kölbel's data are useful in elucidating some of the effects of perforated plate design, and column diameter. His work with slurries may be valuable from a practical standpoint, but probably should not be given much weight at this stage in developing a general correlation for non-slurry systems.

Most workers, including the author, have used columns of relatively small diameter. Consequently the work of Fair is especially valuable in offering data from very large diameter columns. Column diameter is probably the most inconvenient variable to study, since it involves the construction of new equipment.

Unfortunately, most of the workers did not present complete tabulations of their data and conditions of operation. As a result, it is both difficult and risky to compare the data of one with that of another. In most cases the temperature of operation is not known so that physical properties cannot be accurately evaluated.

CHAPTER III

THEORETICAL DISCUSSION

Hydrodynamically, a bubble-agitated system is tremendously complex, and to attempt a serious theoretical or mathematical analysis at this point would be highly ambitious. Rather, this Chapter will be devoted to the discussion of a few fundamental concepts and observations. Also, the general correlations presented in Chapter II will be analyzed and evaluated.

It might be well to introduce the discussion with photographs of bubble-agitated liquids taken at various superficial gas velocities. Photographs of both the water and ethylene glycol systems are shown in Figures 1 and 2 on the following pages. These should give the reader a general idea of what a bubble-agitated system looks like, even though they offer only a static glimpse of a very "dynamic" phenomenon.

Nevertheless, it can be seen that at the lower gas rates the bubbles remain fairly distinct from one another, are of a fairly uniform size, and rise in nearly single file up the column. Furthermore, at very low gas rates--say 0.001 ft/sec--the bubbles are almost spherical in shape. However, at somewhat higher rates, considerable distortion is evident, and visual observation reveals that the bubbles oscillate rather wildly as they rise. At even higher gas rates, bubbles exist throughout the cross-section of the column, considerable break-up and coalescence occurs, and it becomes impossible to isolate any representative bubble size or shape.

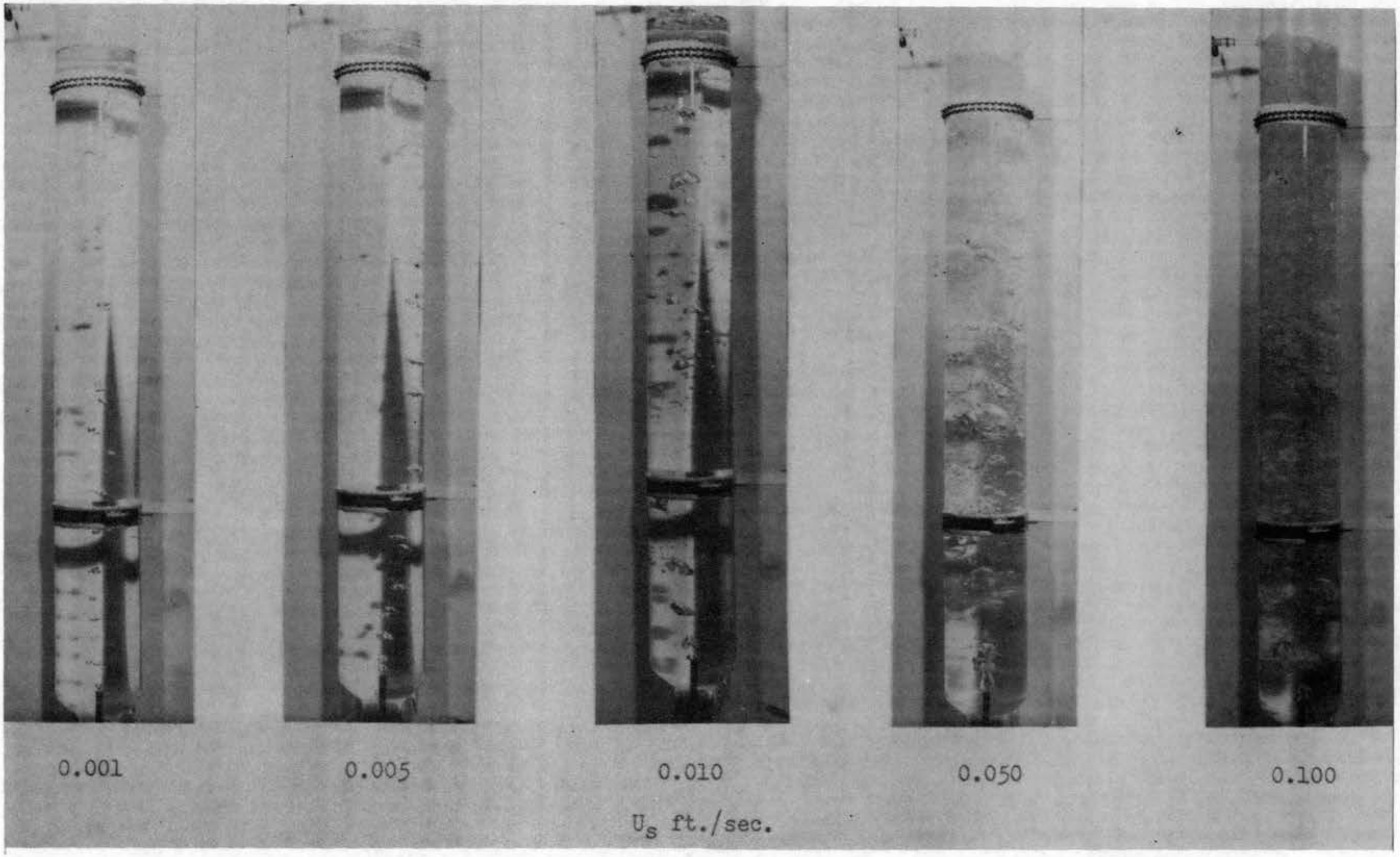
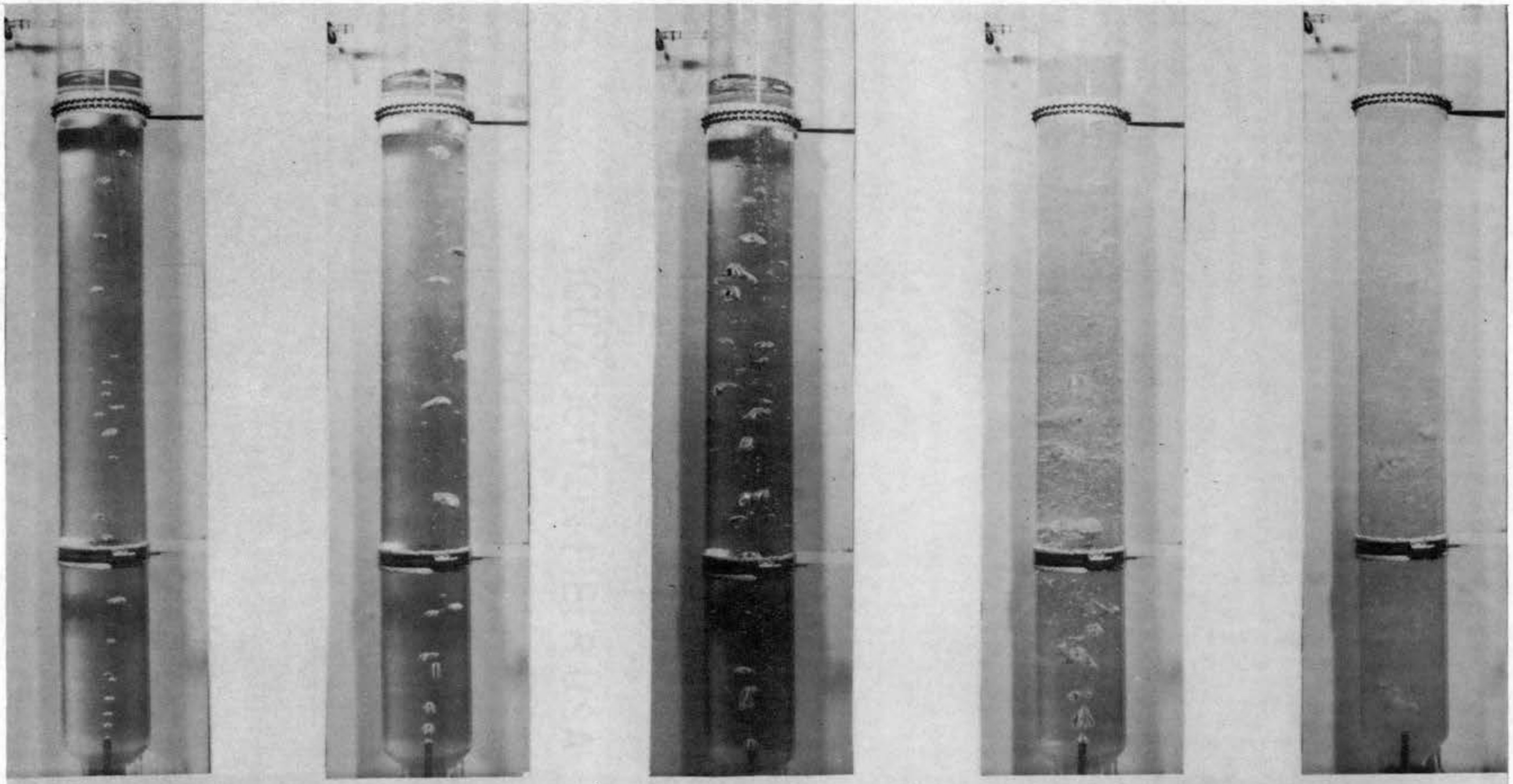


Figure 1. Photographs of Bubble-Agitated Water



0.001 0.005 0.010 0.050 0.100

U_s ft./sec.

Figure 2. Photographs of Bubble-Agitated Glycol

Even from the photographs it is evident that considerable "turbulence" is generated at the higher gas rates. On the other hand, the system is not so obviously "turbulent" at lower gas rates. Even upon actual observation of a bubble-agitated liquid at a gas velocity of say 0.005 ft/sec one is not intuitively inclined to classify the system as "turbulent"--that is, the chaos one usually associates with a "turbulent" system is not apparent.

In dealing with other hydrodynamic systems one is accustomed to using a Reynolds Number to quantitatively characterize the turbulence that exists. In this regard it will only be pointed out here that in attempting to construct a Reynolds Number for a bubble-agitated liquid one is confronted with a number of choices as to which characteristic velocity and length to use. Observation of a bubble-agitated liquid at a superficial gas velocity of say 0.1 ft/sec immediately discourages one from selecting bubble diameter as the characteristic length--not because this is necessarily the wrong length to use, but because it would be practically impossible to define.

More discussion on turbulence and dynamic similarity will be postponed until later in this chapter. For the present time, since heat transfer is so intimately related to fluid motion it might be well to discuss qualitatively the liquid flow patterns that exist in a bubble-agitated system.

Liquid Flow Patterns

A treatise on bubble dynamics is beyond the scope of this work. The main consideration here is to expose, in a very general way, the liquid flow patterns that exist in a bubble-agitated system. For this purpose

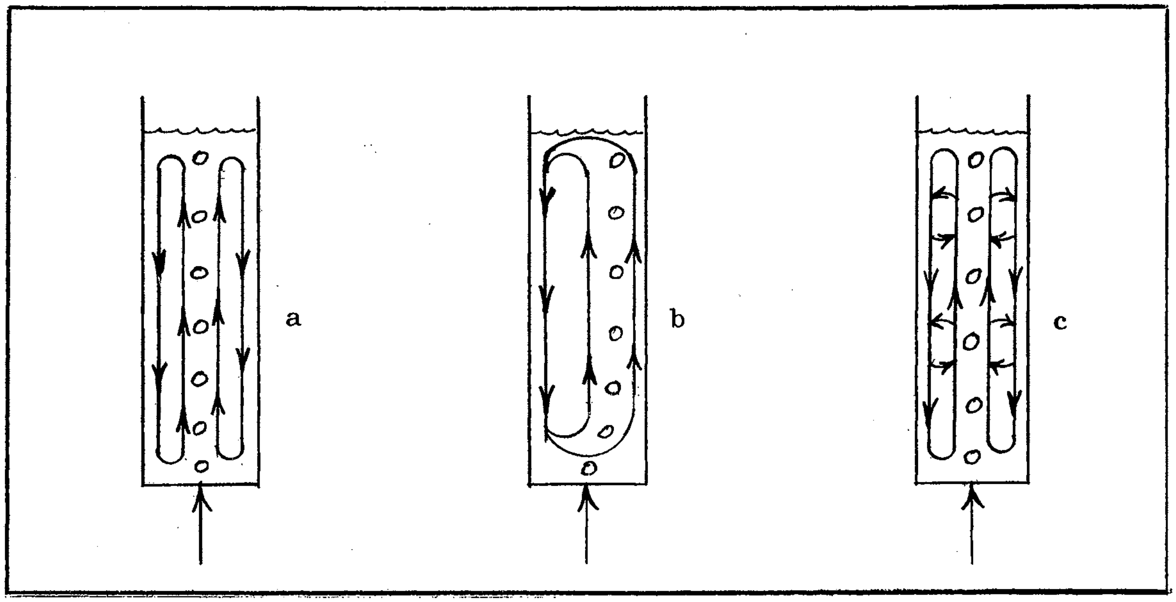


Figure 3. Liquid Flow Patterns in a Bubble-Agitated System

one need only know that, due to buoyancy forces, a bubble rises in a liquid, and that the bubble exerts some "drag" on the liquid adjacent to it. Because of this drag, a series of bubbles rising successively will effect a continuous upward flow of liquid in the vicinity of the bubble stream. Continuity then requires that an equivalent amount of liquid flow downward in regions outside the bubble stream "boundary layer". If the bubble stream rises through the center of the column, then downward flow will exist near the wall, and the flow pattern illustrated in Figure 3-a will result. However, the bubble stream does not necessarily rise through the center of the column, but changes its location in a random fashion, and sometimes rises near the vessel wall. This situation gives rise to the flow pattern shown in Figure 3-b.

It is not implied that the circulation patterns sketched in Figure 3 represent "stream lines". Almost certainly a given fluid particle will not make the complete circuit without interruption. There is appreciable short-circuiting, because the bubble stream is not a continuous thing, and a radial flow must exist all along the column height. That is, the space occupied by a bubble at any instant must be filled with liquid as soon as the bubble moves. This requires an inward flow toward the bubble stream. Likewise, an outward flow must exist in order to make room for an approaching bubble. This situation is illustrated in Figure 3-c.

Of course, the flow patterns shown in Figure 3 are highly idealized. As seen from the photographs in Figures 1 and 2, one is seldom dealing with a well-defined bubble stream. Even at moderate air rates, bubbles exist throughout the cross section of the column, and numerous so-called bubble streams exist. However, no matter how complex the circulation patterns may become, the general situation is the same--there is an upward flow in the immediate vicinity of the bubbles, and a downward flow elsewhere, with complicated radial flow patterns superimposed upon these two main streams.

Actually, an analysis of the Navier-Stokes Equations and the pertinent boundary conditions suggests that even if the bubble stream were continuous, a radial flow would exist. For example, if somehow the bubble stream in Figure 3-a could be replaced by an infinitely long cylinder rising vertically through the liquid, a radial flow would exist throughout the height of the liquid. (Note that this is not a case of "Couette" flow, because there is no net flow of liquid). Surely, this radial flow contributes significantly to the heat transfer process in a bubble-agitated system. Its existence also precludes any successful "boundary layer" type of analysis-- even in the simplest of theoretical models.

From the general discussion presented thus far, one would expect a bubble-agitated system to be a highly transient one, with large fluctuations in any instantaneous local transport coefficient. Indeed, this is the case; but the fluctuations are sufficiently rapid to allow the determination of "time-smoothed" coefficients which are well defined and reproducible. The method used to determine these time-smoothed coefficients will be discussed in Chapter V.

Heat Transfer and Dimensionless Correlations

The complex hydrodynamics associated with a bubble-agitated system obviously precludes any mathematical solution to the heat transfer problem. One must therefore rely on experiment and some kind of dimensional analysis. Yet, at this stage in the development of bubble-agitated transport theory, it would be highly ambitious to attempt the development of a completely general correlation--not only because of the complexity of the system, but also because of the limited amount of data now available on the subject. Thus, the dimensional analysis presented in this section is not intended to be perfectly general and all-encompassing. That is, there are variables associated with a bubble-agitated system which are not considered in the analysis. These variables, and the roles they play in the heat transfer process are discussed later in this Chapter.

The structure of the dimensionless groups required to describe a given situation may be deduced from the purely "operational" method of dimensional analysis, or they may be obtained by writing in dimensionless form the basic differential equations which describe the transport of momentum and energy, along with the equation which defines the heat transfer coefficient: $q = h\Delta T$. A discussion of both techniques may be found in practically any textbook on fluid dynamics and heat transfer. [For example see Bird (9), Eckert (10), Knudsen (12), McAdams (13), or

Schlichting (15).]

The latter approach suggests that any heat transfer process, which is itself describable by the differential equations, can be characterized in terms of the Nusselt, Prandtl, Reynolds, Grashof, and Froude Numbers--although all of these groups may not be required, depending upon the peculiarities of the given system. The Nusselt and Prandtl numbers are required in practically all correlations for convective heat transfer. The Reynolds, Froude, and Grashof Numbers arise each in their own special way, depending upon the types of forces involved in the hydrodynamic process.

The Reynolds Number accounts for the presence of inertial and viscous forces, the Froude Number accounts for the presence of inertial and gravitational forces, and the Grashof Number accounts for the presence of viscous and gravitational forces. Each of these groups is restricted to the recognition of only two types of forces at a time. In some instances only two types of forces are important, so that dynamic similarity can be uniquely established by a single one of these dimensionless groups--whichever one is appropriate to the forces involved. Hence, many heat transfer processes can be described by correlations of the form Nu = f(Re,Pr) or Nu = F(GrPr). However, in any system where more than two types of forces are simultaneously at play, no one of the groups mentioned above is sufficient by itself to establish dynamic similarity, and any proposed heat transfer correlation must, of course, recognize this fact.

A bubble-agitated system obviously involves inertial and viscous forces, so the Reynolds Number must enter the correlation. However, the Reynolds Number alone is not sufficient to establish dynamic similarity, because gravitational forces are also present. Indeed, gravitational forces are not only present, they are ultimately responsible for the very motion in a bubble-agitated system. Certainly then, any dimensionless correlation for bubble-agitated heat transfer must somehow include the gravitational constant, g, along with the Nusselt, Reynolds, and Prandtl Numbers. The gravitational constant can enter the correlation in either the Grashof Number or the Froude Number . The very construction of the Grashof Number limits it to the description of natural convection phenomena, and it can therefore be eliminated from consideration. One is thus led inevitably to the use of the Froudé Number, and a preliminary correlation of the form Nu = f(Re,Fr,Pr).

It is not at all apparent a priori which "characteristic" velocity and length to use in constructing the dimensionless groups of this correlation. Most previous workers have used the superficial gas velocity and the column diameter as the characteristic parameters, and it seems that these are the proper choices. The fact that U_s is the proper characteristic velocity can be deduced in a "semi-theoretical" way, and this will be discussed in the next section. The use of D as the characteristic length is, to some extent, the result of expediency. As already mentioned, it would be practically impossible to use bubble diameter, and the use of column diameter is certainly not an illogical choice. Actually, it develops that the heat transfer coefficient in a bubble-agitated system is not a fucntion of any characteristic length, so it is not really important which length one chooses.

With these choices--U_s and D--for the characteristic velocity and length respectively, the dimensionless correlation takes the form:

$$\frac{hD}{k} = f\left[\left(\frac{U_s D}{\nu}\right), \left(\frac{U_s^2}{gD}\right), \left(\frac{c_p \mu}{k}\right)\right] \qquad (1)$$

At this point one must assume some kind of functional relationship among the dimensionless groups. The most common assumption, and one which is quite successful in many cases, is that the groups are related in a logarithmically linear manner, i.e.:

$$\frac{hD}{k} = a\left(\frac{U_s D}{\nu}\right)^b \left(\frac{U_s{}^2}{gD}\right)^c \left(\frac{c_p \mu}{k}\right)^d \tag{2}$$

The validity of the assumed form can then be determined only be applying the equation to experimental data. It develops that bubble-agitated heat transfer is described quite well by such an equation.

While the above correlation was not arrived at by ordinary dimensional analysis, it can be shown that such an approach leads to exactly the same equation if it is started with the assumption that $h = f(U_s, D, g, \mu, \rho, c_p, k)$, where all physical properties are those of the liquid. If it is known at the outset that h is not a function of any characteristic length, then the dimensional analysis leads to the following equation:

$$\frac{h}{U_s \rho c_p} = a\left(\frac{U_s{}^3}{\nu g}\right)^b \left(\frac{c_p \mu}{k}\right)^c \tag{3}$$

which is equivalent to:

$$St = a(ReFr)^b Pr^c \tag{4}$$

If Equation (2) is made to satisfy the condition that D have no effect on h, then Equations (2), (3), and (4) will, of course, be equivalent. The form of the equation which one uses to correlate experimental data is a matter of convenience. Probably the most convenient form is obtained by rearranging Equation (2) into the form of a j-factor. First, in order to eliminate the effect of D on h, one must have the condition $b = c+1$. Then:

$$Nu = aRe(ReFr)^c Pr^d \tag{5}$$

and:

$$\frac{Nu}{RePr^d} = j_H^* = a(ReFr)^c \qquad (6)$$

so that a plot of j_H^* vs ReFr on logarithmic coordinates will yield a line of slope c and intercept a. The application of this correlation to the experimental results will be presented in Chapter VI.

In view of the obvious importance of gravitational forces in a bubble-agitated system, a rather simple criterion exists for evaluating the correlations offered by Novosad and Kölbel. Since neither considered gravity, neither can be fundamentally correct. This is apparent a priori, but it is also apparent from the fact that neither correlation is completely consistent with what is known to be true of bubble-agitated heat transfer. For example, neither correlation predicts that the coefficient is independent of any characteristic length. Actually, reference to their original papers will show that their data were not well correlated by their equations.

Failure to account for gravitational forces represents an oversight of an important variable. Kölbel's correlations for slurries represent an example of not only this oversight, but a physical impossibility. It will be recalled that these correlations were of the form:

$$Nu = cRe^a Pr^{-b}$$

where c represents a constant, and a and b are positive exponents. This relationship implies that the heat transfer coefficient, h, is proportional to the thermal conductivity, k, to some power greater than 1.0, and that it is an inverse function of the heat capacity, c_p. Within the framework of classical heat transfer theory, neither of these conditions is physically possible.

The correlation offered by Kast seems fundamentally correct. It is identical in form to the correlation which resulted from this present study, because it accounts for the same variables. Kast did not actually offer an equation as such, he simply used the groups St and $(ReFrPr^2)^{1/3}$ as coordinates on a logarithmic plot. There are no data points on the plot, so one cannot tell how well his results were correlated, but based on the slope of the line which was drawn, it can be deduced that his correlation reduces to:

$$Nu = 0.096\ Re^{0.77}Fr^{0.24}Pr^{0.55}$$

The constant and the exponents are not the same as in the present author's correlation, as will be seen in Chapter VI.

Despite the alleged incorrectness of Novosad's correlation, his attempt to include natural convection effects is instructive. Basically his approach was to correlate the quantity $(Nu - Nu_0)$ as a function of the Reynolds and Prandtl Numbers, where, as mentioned before, Nu_0 is the Nusselt Number for natural convection. This is probably the simplest possible approach to an inevitably complex problem; and at first it might seem an improvement over correlations which predict a zero coefficient at zero gas velocity. However, Nu_0 is a function of ΔT and a characteristic length, whereas the heat transfer in a bubble-agitated system at sufficient gas velocity is apparently a function of neither. This leads to uncertainty as to which Nu_0 to use--that is, which ΔT and which characteristic length. Therefore, such an approach cannot be completely accurate, since one can choose a number of values for Nu_0; but, at sufficient gas velocity, the actual Nusselt Number which exists is independent of the conditions used in evaluating Nu_0.

One is thus presented with a choice of two evils. If Nu_0 is included in the correlation, some error in the predicted heat transfer for a bubble-agitated system will result at all conditions different from those used in the original determination of Nu_o. If Nu_o is not included in the correlation, a large error will result at gas velocities sufficiently low for natural convection effects to be predominant.

It seems apparent that the latter is the lesser of the two evils. As will be seen later in Chapter VI, natural convection effects are apparently negligible except at extremely low gas velocities--that is, at velocities far below any which might arise in most practical situations.

As indicated before, there are variables associated with a bubble-agitated system which can not be accounted for by any correlation of the form Nu=f(Re,Fr,Pr). Natural convection effects represent one such variable, but, as just discussed, this is not a serious limitation except at extremely low gas velocities. Other variables which one would expect to have some effect are gas distributor design, surface tension, and gas density. It develops that these are all directly related to the single parameter of bubble velocity, which in turn, is directly related to the gas holdup. The following section is concerned with the discussion of these variables, and the relationships that exist among them.

Gas Holdup and Bubble Velocity

A knowledge of the gas holdup in a bubble-agitated system is useful from a practical standpoint, in that it allows one to calculate the average gas residence time. This may be important in some reacting systems. However, its immediate use in the study of bubble-agitated heat transfer lies in the fact that it allows a determination of the average bubble velocity, which, in turn reflects the roles played by other variables.

The gas holdup is denoted by the symbol Φ, and it is defined as follows:

$$\Phi = \frac{V_T - V_L}{V_T} = \frac{V_B}{V_T} \qquad (7)$$

where: V_T = total volume of aerated liquid

V_L = volume of bubble-free liquid

V_B = volume occupied by gas bubbles

For a column of constant cross-sectional area, the volumes V_T and V_L can be replaced by the corresponding heights above the gas inlet nozzle.

The gas holdup as defined above relates the bubble rise velocity, U_B, to the superficial gas velocity, U_S. The relationship is derived as follows: given a single bubble, rising at constant velocity through a liquid of height, H_T, its holdup time, t, can be written

$$t = \frac{H_T}{U_B} \qquad (8)$$

The holdup time can also be expressed as a function of the volumetric flow, $\dot{V}$, and the volume occupied by bubbles, V_B;

$$t = \frac{V_B}{\dot{V}} \qquad (9)$$

Multiplying both numerator and denominator in Equation (9) by A, the column cross-sectional area, and then eliminating t from Equation (8) and (9), we have:

$$\frac{V_B}{\dot{V}} \cdot \frac{A}{A} = \frac{V_B}{A} \cdot \frac{1}{U_S} = \frac{H_T}{U_B} \qquad (10)$$

or:

$$\frac{V_B}{H_T A} = \frac{V_B}{V_T} = \frac{U_S}{U_B} = \Phi$$

hence:

$$\Phi = \frac{U_S}{U_B} \qquad (11)$$

The only assumption involved in this derivation was that U_B was constant throughout the distance, H_T. This is not exactly true, for the bubble must decelerate (or accelerate) from orfice velocity to terminal rise velocity. This requires some distance, but if this distance is small compared to H_T, then not much error is involved.

It is a simple matter experimentally to determine Φ as a function of the superficial gas velocity, U_S. A plot of the data then will yield a line of slope $1/U_B$, so that U_B is easily determined. This is a convenient, if not exclusive, method of determining the average velocity of a complex array of bubbles--many with different velocities.

It develops (see Chapter VI) that the relationship between Φ and U_S is, for all practical purposes, linear. That is, the slope $1/U_B$ is a constant in any given system. Hence, the bubble velocity is practically independent of the superficial gas velocity. As a result, U_B can hardly be considered as the proper "characteristic" velocity of the system--useful in establishing dynamic similarity. On the other hand, it is obvious that the turbulence of the system increases with the superficial gas velocity. In fact, it can be shown that U_S is directly related to the power dissipation per unit volume. It is quite natural then to assume that U_S is the proper "characteristic" velocity to use in dimensional analysis. Experiment verifies that such is the case.

The fact that bubble velocity is essentially a constant in any "given" system does not imply, of course, that it is a "universal" constant and need not ever be considered in the analysis of bubble agitation. On the contrary, the bubble velocity can change considerably from system to system, depending upon the type of gas distributor used, the surface tension, the densities of the two phases, the liquid viscosity, and perhaps other variables.

Thus, bubble velocity is not only a variable, it is a rather unique variable in that it reflects the roles played by many _other_ variables.

That is, gas distributor design, surface tension, and gas density can affect the heat transfer only in so far as they affect bubble velocity (or, perhaps, bubble size, but the two are related). Actually, gas-distributor design can also affect the radial distribution of the bubbles, and this may have some effect on the heat transfer--especially at low gas velocities. However, with this single exception, once the effect of bubble velocity on the heat transfer is known, the effects of these other variables can be deduced.

A detailed analysis of the relationships that exist between these variables and the bubble velocity would involve a lengthy discussion of bubble dynamics, which is beyond the scope of this work. Indeed, such an analysis would be a bit premature, because presently there is not sufficient data to determine exactly what effect bubble velocity itself has on the heat transfer coefficient. However, there are indications that over a fairly wide range, changes in bubble velocity do not greatly affect the heat transfer coefficient. This conclusion is based on the fact that gas distributor design can affect bubble velocity quite strongly, but it does not greatly affect the heat transfer--as evidenced by the results of previous workers.

Thus, the fact that variations in bubble velocity, and the variables that affect it, are not accounted for in the correlation Nu = f(Re,Fr,Pr) is not a serious limitation as long as the bubble velocity stays within certain bounds. It is doubtful that these bounds can be exceeded by variations in gas distributor design, and it seems likely that they can be exceeded by changes in gas density or surface tension only in extreme cases--such as when the gas density approaches that of the liquid, or when the surface tension is given extreme values by the presence of surface active agents. In a word, these variables represent the "refinements" to be studied after a workable preliminary correlation is developed.

One final point is worth noting. From Equation (11) it can be seen that U_B represents an upper limit to U_S, since Φ cannot be greater than unity. At $\Phi = 1$, the column becomes completely filled with gas, and the heat transfer becomes a simple problem of forced convection to a gas in a tube. Yet, the heat transfer rate to a gas flowing at 1-2 ft/sec in a tube is considerably lower than the heat transfer to most bubble-agitated liquids. Thus, one would expect that as U_S approaches U_B, the coefficient would decrease. The data of Yoshitome suggests that even at a superficial gas velocity of about 0.3 ft/sec, the effect of U_S on the mass transfer coefficient begins to taper off. Of course this is a tremendously high gas velocity--far above what economical design might call for.

CHAPTER IV

DESCRIPTION OF APPARATUS

A flow diagram of the apparatus used in this work is presented in Figure 4. The study was conducted under steady-state conditions, in that liquid was continuously fed to a vessel supplied electrically with a definite heat flux. The bubble agitation was provided by introducing air to the bottom of the column through a single vertical nozzle. A constant liquid level was maintained in the vessel by means of a seal-leg type overflow line equipped with a vent to prevent siphoning. The heated liquid overflowed into a surge tank, from which it was pumped through a jacketed cooler into a constant head tank. The cooled liquid was then recycled through a rotameter to the heated vessel.

Temperature and flow-rate measurements determined the heat flux, and the difference in temperature between the liquid and the tube wall. Thus the heat transfer coefficient was determined.

Supplementing the flow diagram in Figure 4 is a description of each piece of apparatus. The only item requiring a more exhaustive description is the heated vessel.

The vessel was specially constructed for this project. It consists essentially of a two-foot section of copper pipe with butt-joined fiberglass ends. A drawing of the vessel, complete with construction details, is shown in Figure 5. The fiberglass was installed so as to reduce the axial heat leak from the copper section, thus allowing a more accurate

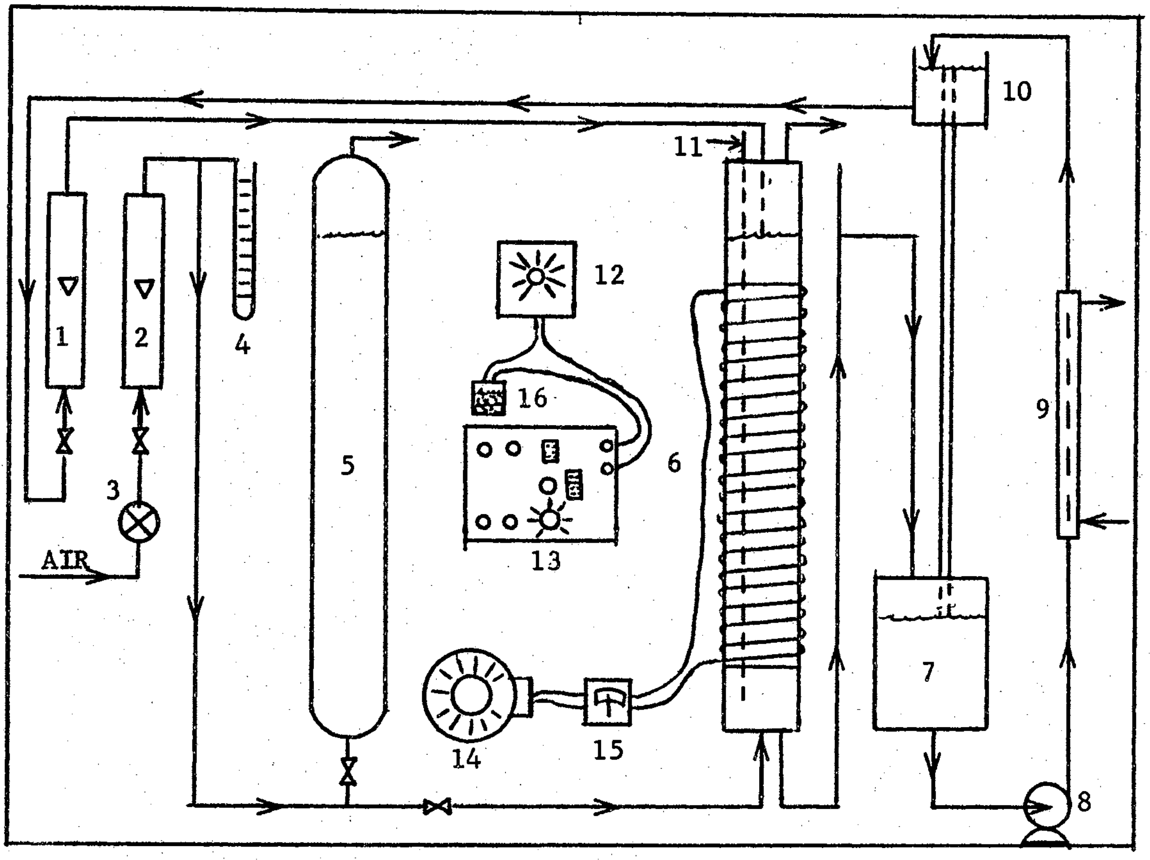


Figure 4. Apparatus for Bubble-Agitated Heat Transfer Studies

NO.	DESCRIPTION
1	Liquid feed rotameter. FP-¼"-20-G-5
2	Air feed rotameter. Same as 1
3	Air supply pressure regulator
4	Mercury manometer
5	Glass vessel for photographic and visual study, 4" ID x 42" total length
6	Heated vessel, 3.9" ID x 42" total length. 24" copper section with butt-joined fiberglass ends.
7	Recycle tank, 2 liters
8	Recycle pump, Eastern centrifugal, Model D-10.
9	Double-pipe exchanger for cooling recycle stream with tap water
10	Constant-head tank for liquid feed system
11	Probe for mounting liquid-side thermocouples, ¼" copper rod
12	Thermocouple selector switch to cold junction and potentiometer
13	Potentiometer, Leeds and Northrup Cat. No. 8687
14	Powerstat for regulating heat flux, 220 volt, 10 amp.
15	Volt-ammeter
16	Ice cold-junction

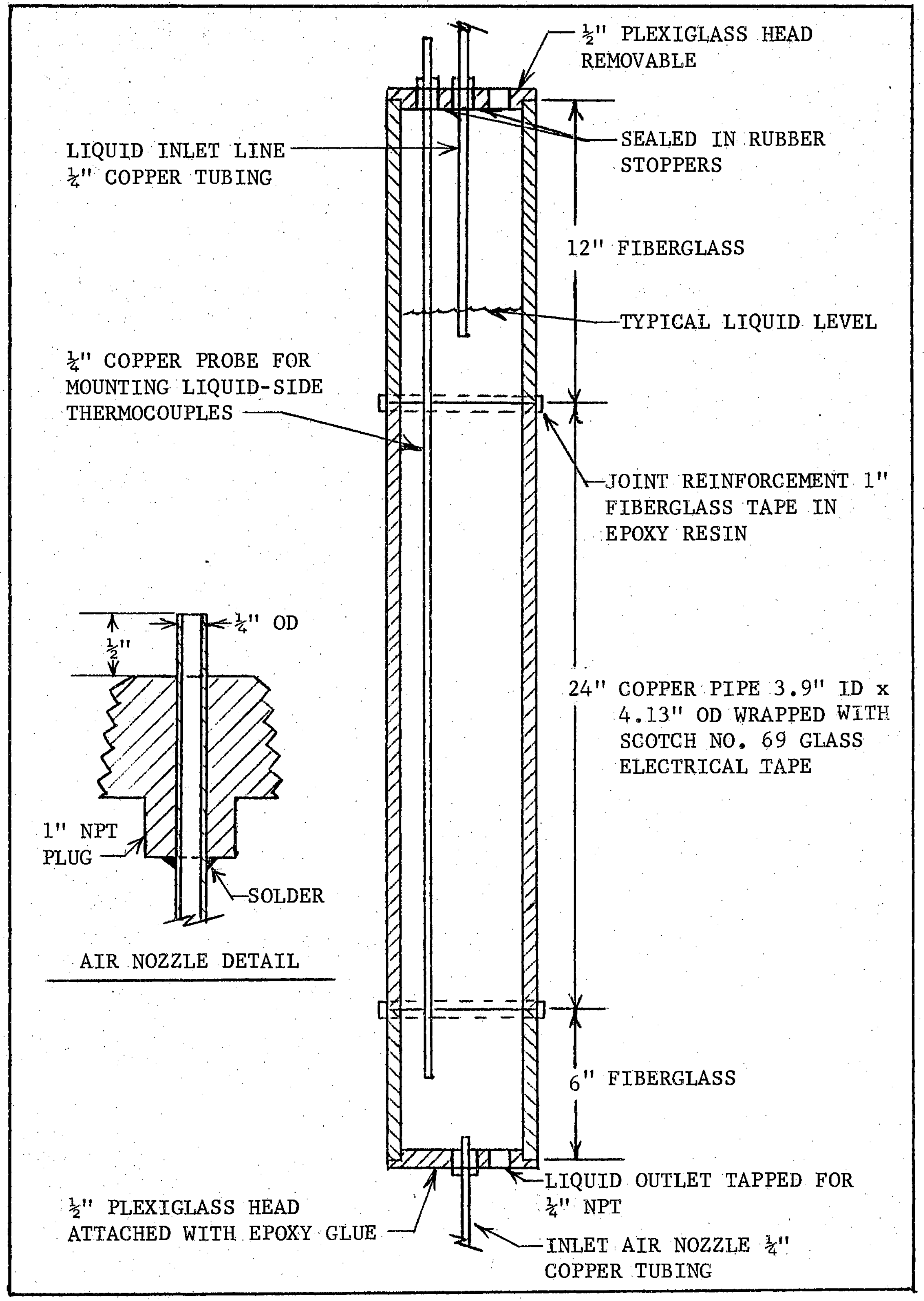


Figure 5. Construction Drawing of the Heated Vessel

determination of the heated area involved.

The copper section was wrapped with Scotch No. 69 glass tape to electrically insulate it from the heating element. The heating element was 0.5-inch by 0.002-inch chromel-A tape with a resistance of 0.531 ohms per foot. This tape was wrapped around the glass-covered copper section with a spacing of 0.25 inches between wraps, giving a total resistance of about 19 ohms. With a 10 amp powerstat this allowed a maximum power input of 1900 watts or about 3000 Btu/hr·ft^2 based on the total inside area of the copper pipe.

Nine, 30-gauge, copper-constantan thermocouples were then installed at equidistant intervals along the pipe wall. The thermocouples were inserted into 1/16-inch diameter holes bored to within about 1/64 inch of the inside pipe wall. The holes were then filled with No. 10 lead shot (two per hole), which was tapped gently with a punch, causing the lead to "flow" into every cavity. This arrangement provided excellent thermal contact. Soldering could not be used, because the fiberglass ends would have been damaged by the heat required.

To further insure against heat losses from the thermocouple junctions the wires were taped against the pipe wall for a distance of about three inches. A narrow slit was cut in the glass-tape covering, and the wires laid flat in this slit.

A photograph of the finished vessel, complete with heating element and thermocouples, is shown in Figure 6.

For measuring the liquid temperature, five copper-constantan thermocouples were mounted on a 1/4-inch copper probe. They were placed at uniform intervals over a length of 24 inches, so that when the probe was inserted into the vessel, these thermocouples were opposite five of the

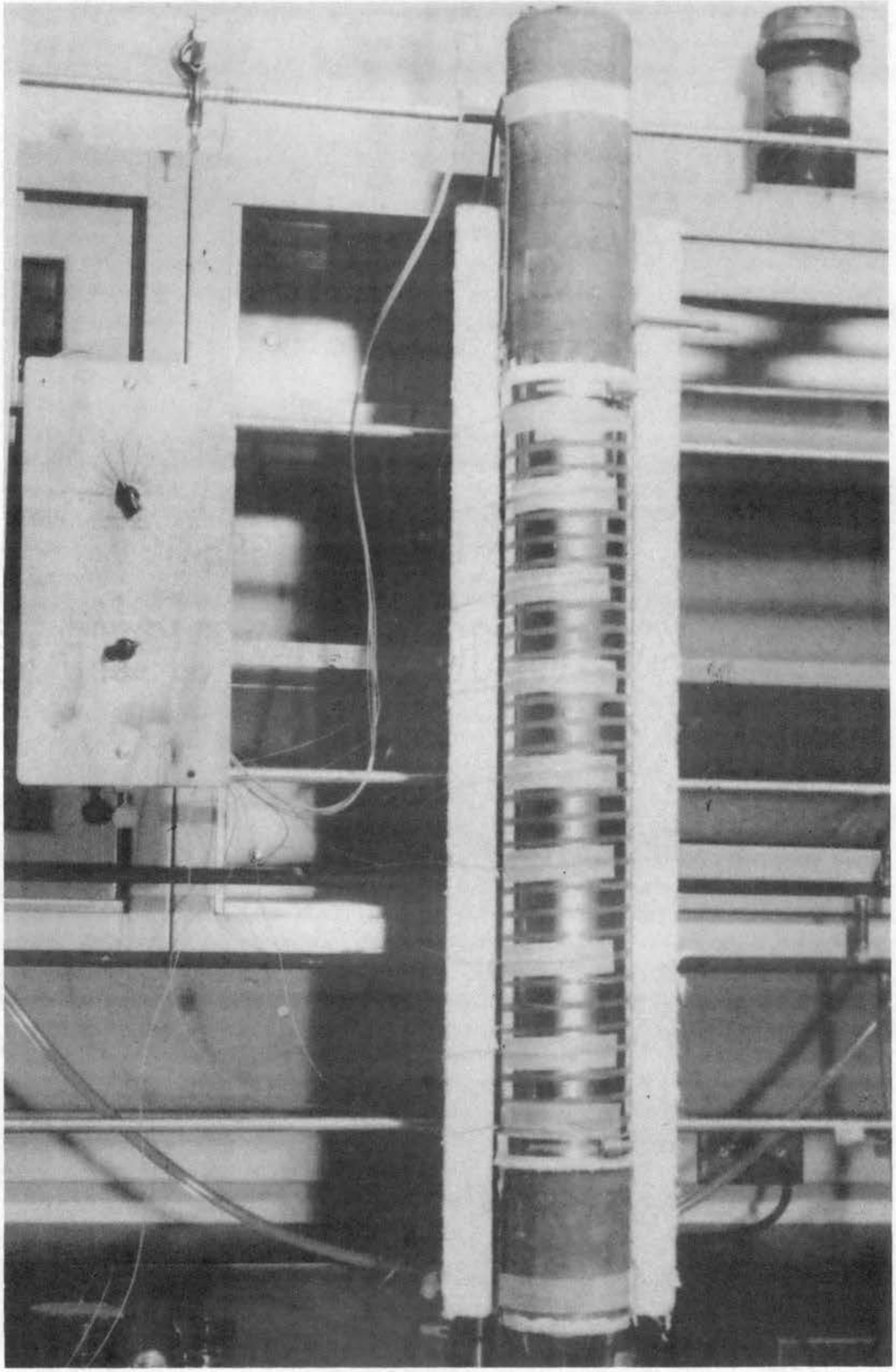


Figure 6. Photograph of the Heated Vessel

wall thermocouples. An additional thermocouple was installed in the liquid feed line for measuring the inlet temperature. All thermocouples led to a double-pole selector switch, which in turn was connected to an ice cold junction and potentiometer.

The only air nozzle used in this study was simply a piece of 1/4-inch copper tubing extending vertically into the bottom head of the vessel. Details of this arrangement are shown on the construction drawing in Figure 5.

For the benefit of anyone who may wish to copy the design of the heated vessel used in this project, it might be well to elaborate somewhat upon the construction of the fiberglass extensions. These were fabricated by the author from epoxy resin and glass cloth. Molds were provided by rolling up two large pieces of heavy paper, similar in weight to the common manila file folders. These rolls were inserted into the copper pipe as far as possible while leaving a sufficient length to extend from each end. They were then unrolled slightly in order to expand them tightly against the inside pipe wall. The molds were then taped inside and out, and a cardboard disc was fitted tightly into each end so as to maintain roundness. The molds were then covered with waxed paper to prevent the resin from sticking to them. Glass cloth and resin was then applied to the mold--in two or three applications--until a thickness about equal to that of the pipe wall was attained. Finally, the butt-joints were covered with a narrow width of heavy glass cloth which overlapped the copper about 1/4 inch. Resin was applied, and allowed to dry. The molds were then removed, and the rough ends of the fiberglass extensions were sawed off square. The result was very satisfactory.

Perhaps it should also be pointed out that the somewhat elaborate arrangement made for measuring the wall temperatures was not at all unnecessary. The method used was evolved only after a number of other schemes had failed.

No matter how accurately the thermocouples are calibrated, the temperature one ultimately measures when the system is in operation depends, of course, upon the degree of thermal contact that exists between the thermocouple junction and the pipe wall. There is always conduction to or from the thermocouple junction along the wires. The contact between the junction and the pipe wall must be such that only a negligible tempertur difference between them is required to sustain this heat flow.

A useful technique which reduces the required degree of contact, and improves the reliability of temperature measurements is to lay the first few inches of thermocouple wire (insulated of course) along the pipe wall. This insures that these first few inches will be at about the same temperature as the junction itself, hence the conduction loss or gain is minimized.

All thermocouples used in this study were prepared in a mercury welder. Near the junction, where the regular insulation had become somewhat frayed, teflon tape was wrapped around each wire so that only the junction itself was exposed when the wire was inserted into the pipe wall.

CHAPTER V

EXPERIMENTAL PROCEDURE

The discussion in this chapter is concerned with the calibration of apparatus, the general operation of the system, the method of taking data, and the treatment of the data obtained. Such topics as accuracy and precision are discussed in Chapter VI.

Calibrations

The only components requiring calibration were the thermocouples and rotameters. All thermocouples were calibrated by placing the bare junctions in boiling water. All readings fell within the range 212.0-212.3°F. The exact water temperature was not known because the barometric pressure was not known. However, the <u>absolute</u> accuracy of the thermocouples was important only in evaluating liquid physical properties, where an error of even 1.0-2.0°F could be tolerated. Determination of the heat transfer coefficient required only the measurement of temperature <u>differences</u>, and the fact that all fifteen thermocouples read within 0.3°F of each other was considered adequate precision without making corrections. The justification for this will become more apparent as the experimental technique is explained in more detail.

The air feed rotameter was calibrated under actual operating pressure with a wet test meter. The liquid feed rotameter was not calibrated until after each run, because the liquid temperature varied depending upon the performance of the recycle cooler. Thus, after each run was completed and the average liquid feed temperature was known, the rotameter was calibrated at this temperture by interrupting the regular flow cycle temporarily and collecting the liquid in a graduated cylinder for a time measured with a stop-watch. Each calibration was repeated two or three times to insure the accuracy of the measurements.

General Operation

To begin operation the heated vessel and recycle tank were charged with the appropriate liquid, which, in this study, was either distilled water of refined ethylene glycol (Fisher Cat. No. E-177). The powerstat was then turned on and adjusted to give a pre-calculated heat flux. To reduce heat-up time, the recycle system was not put into operation until the liquid in the column had reached the calculated line-out temperature.

The recycle pump was then started, cooling water was admitted to the recycle exchanger, and air and liquid feeds were started to the heated column. Feed rates were adjusted to fit the desired conditions of operation.

To enhance the accuracy of the data, an effort was made to maximize the temperature difference between the vessel wall and the liquid contents. The maximum available heat flux (about 3000 Btu/hr·ft^2) was determined by the 10-amp capacity of the powerstat. However, because of other limitations this maximum power input could not always be used. In consideration of the fiberglass and plexiglass components involved, the maximum safe temperature of operation was somewhat arbitrarily established at 180-190°F. This temperature limit, along with the power input, and the heat capacity and inlet temperature of the liquid then established the required liquid feed rate. The maximum capacity of the liquid feed rotameter was 72.8 lbs/hr of water, and 46.1 lbs/hr of glycol. This water feed rate was sufficient to allow operation at the maximum heat flux without exceeding the established temperature limit. However, the maximum glycol feed rate was not (the heat capacity of glycol is only about 0.6 Btu/lb°F). Therefore, something less than the maximum flux was used in all glycol runs. Even so, it developed that the ΔT's (between vessel wall and liquid contents) involved in the glycol runs were greater than those in the water runs.

With the proper liquid feed rate and heat flux thus established, and the air rate set at the desired value, the unit was given time to line out at steady state conditions. When steady state was attained--as evidenced by a constant liquid outlet temperature--the unit was ready for the appropriate measurements to be made.

Taking of Data

With a given liquid in the system the only parameter to be varied was the superficial gas velocity. Ordinarily, each "run" involved eleven different gas velocities, ranging from zero to 0.0675 ft/sec. In one glycol run, the zero velocity was omitted. At each gas velocity, temperature, flow rate, and volt-amp measurements were made and the numbers recorded. The only measurement requiring discussion is that of temperature.

At each gas velocity, seventeen temperatures were recorded. These measurements were sufficient to establish a complete heat balance and complete temperature profiles of the liquid and the vessel wall. However, because of the transient nature of the system, there was appreciable fluctuation in the wall temperatures and the agitated liquid temperatures. In the liquid it seemed that there were rapid oscillations superimposed upon slower ones. All oscillations were highly irregular, and it was difficult to estimate an average. Fortunately, the magnitude of the fluctuations rarely exceeded ± 1.0°F, and then only near the top of the vessel where the cold liquid entered.

Rather than try to "read" averages directly from the potentiometer, the following procedure was devised for obtaining "time-smoothed" values: At each velocity all the temperatures in the system were measured once and recorded, with very little effort being made to read average values. Immediately upon completing this series of readings, the temperatures were measured again, and in many cases they were measured a third time. Then, time-smoothed temperature profiles were constructed from the arithmetic average of all the individual profiles. The effectiveness of this procedure could be seen very clearly by plotting the individual profiles along with the average profile. The smoothing effect was very obvious and quite satisfactory. Typical average temperature profiles are plotted in Appendix C.

Usually three series of readings were taken at the lower gas rates, because it was under these conditions that the fluctuations were greatest. At the higher gas rates the vigorous mixing that existed reduced the fluctuations considerably, so that only two series of readings were taken at each gas velocity.

Upon completion of the desired measurements at a given gas rate, the flow was increased to another value and all measurements repeated. The system responded quite rapidly to any change, and about fifteen minutes was ample time for steady state to be attained at the new conditions.

Three runs were made with water--two under identical conditions, and the third at a slightly higher heat flux. Actually, it was intended that all three runs be made under identical conditions, but after Run No. 2 it was discovered that the power input could be increased somewhat without overloading the powerstat. Two runs--both identical--were then made with glycol. As mentioned before, each "run" involved ten or eleven velocities.

After Run No. 2 had been completed, the liquid-side thermocouples were removed and covered with a thin wrapping of teflon tape. This damped the more rapid temperature fluctuations and made the readings easier to obtain.

Treatment of Data

The raw data existed as rotameter scale readings and emf readings from the potentiometer. Using appropriate calibrations these raw data were translated into flow rates and temperatures. Then, the "time-smoothed" temperature profiles were determined, as mentioned previously, by averaging the two or three instantaneous profiles which had been measured at each gas rate. These average profiles were then plotted and a smooth curve fitted visually to the points. A decision then had to be made as to which temperature-difference should be used in calculating heat transfer coefficients. However, as can be seen from the sample profiles presented in Appendix C, it developed that the ΔT was essentially constant throughout the height of the column--except near the ends where some axial heat loss occurred. It was decided that the ΔT in the middle of the heated section would lead to the most accurate and most conservative coefficients. Furthermore, it was judged that the heat flux at this point should be very close to the average heat flux calculated from the total inside area of the copper pipe. Thus, the heat transfer coefficients were calculated from the equation:

$$h = \frac{q}{\Delta T_M}$$

Where q is the average overall heat flux, and ΔT_M is the ΔT at the middle of the heated section. In determining ΔT_M from the plotted profiles the measurement was made between the smooth curves and not between the two temperature points. That is, no extra weight was given to the two temperature points located at the middle of the column.

Some natural-convection profiles are also shown in Appendic C. Of course the ΔT did vary with location under these conditions, and graphical integration was used to determine the average.

The heat flux was calculated from the measured temperature rise of the air and liquid streams. Liquid heat capacities were evaluated at the arithmetic average liquid temperature. Evaporation effects were incorporated into the air heat balance, assuming that the air was saturated at its outlet temperature. With glycol, evaporation effects were negligible, and the calculations were based on an average heat capacity of dry air at the average air temperature. With water, the air heat balance was based on enthalpies for moist air listed in Table 15-1 of Perry's (14).

Actually, the inlet and outlet temperatures of the liquid varied slightly with time, even under supposed steady-state conditions. Volt-amp measurements indicated that this variation was not due to changes in the heat input; it was due to slight variations in the liquid feed rate. However, the liquid feed rate was taken to be a constant for the entire run. Therefore, the heat flux calculations made at each gas velocity showed some variation (± 2 per cent maximum). Since it was suspected that the heat flux did not actually vary, the average of all the heat flux calculations was taken, and this value used as a constant for the entire run.

All pertinent data, such as flow rates, heat fluxes, time-smoothed temperatures, etc. are tabulated in Appendix B.

For the calculation of dimensionless groups, physical properties had to be evaluated. All properties were evaluated at a mean film temperature, T_F, which was taken as the average of the wall and liquid temperatures at the middle of the column. Since the physical properties of water and glycol do not vary too strongly with temperature, it might have been just as well to use the bulk liquid temperature, but the use of the mean film temperature adds a certain refinement to the procedure. Physical properties of water were taken from Table A-9 in Gröber (11), and those of glycol were taken from a Union Carbide Company publication (16). A tabulation of pertinent physical properties is presented in Appendix B.

As mentioned before, three runs were made with water and two with glycol. Each run yielded values of the heat transfer coefficient at various gas velocities. These values as well as the mean film temperatures are also tabulated in Appendix B. The final reported values of the heat transfer coefficient are averages of the values obtained in individual runs. Likewise, the mean film temperatures were averaged, so that in the end one had a single set of values of h and T_F for each liquid at each gas velocity. Based on these final values, the appropriate dimensionless groups were calculated.

There was some question as to which temperature and pressure to use in calculating the superficial gas velocities. The gas entered the column at 72°F and about 32 inches of water pressure. It left the column at 120-150°F and atmospheric pressure, saturated with the liquid. It was convenient to base the velocities on a temperature of 72°F and a pressure of one atmosphere, because these were the flow conditions upon which the rotameter calibration had been based. Fortuitously, it developed that the gas density at these conditions was very close to the gas density at the average column conditions. Thus, no extra calculations were done, and the gas velocities were based on the conditions of 72°F and 1.0 atmosphere.

The treatment of the data as described in this section is summarized by a sample calculation presented in Appendix D.

Gas Holdup Measurements

The determination of gas holdup as a function of superficial gas velocity was so simple as to hardly require discussion. The glass column was charged with the appropriate liquid. The height of the stagnant liquid level above the gas nozzle was measured and recorded. The height was then measured at various gas velocities, and the holdup calculated from the equation:

$$\Phi = \frac{H_T - H_L}{H_T}$$

where: H_T = total height of aerated liquid

H_L = height of stagnant, bubble-free liquid.

A plot of the data showed some scatter, because it is impossible to determine the exact level of an agitated liquid. However, the data were satisfactorily fitted with a straight line.

CHAPTER VI

RESULTS

The results of this study are summarized in the tables and graphs presented on the following pages. A brief description of these items will be followed by a thorough discussion of their development and their significance. As far as possible, the results are compared with those of other workers. The last part of the Chapter is devoted to an analysis of the expected accuracy of the data.

Presentation

A complete tabulation of the heat transfer results obtained in this study is presented in Table I. The table includes average heat transfer coefficients, film temperatures, and all the dimensionless groups used later in the general correlation. The coordinates of all graphs to be presented can be read from this table. A tabulation of the gas holdup measurements is presented as part of Figure 11, to be discussed later.

The relationship between heat transfer coefficient and superficial gas velocity summarizes the basic results of this study. This relationship is depicted in Figures 7, 8, and 9; Figure 9 is simply a logarithmic plot of the results shown in Figures 7 and 8. The cartesian plots emphasize the rather strong effect that bubble agitation has on the heat transfer coefficient, while the logarithmic plot is perhaps the more revealing mathematically. The dashed line in Figure 9 represents Fair's equation which was presented in Chapter 2. The fact that it does not coincide with the author's data for water will be discussed later in this Chapter.

The dimensionless correlation which best describes the results of this study may be written:

$$Nu = 0.125\ Re^{0.75} Fr^{-0.25} Pr^{0.4}$$

or:

$$\frac{hD}{k} = 0.125 \left(\frac{U_s D}{\nu}\right)^{0.75} \left(\frac{gD}{U_s^2}\right)^{0.25} \left(\frac{c_p \mu}{k}\right)^{0.4}$$

In order to determine the proper exponents, the basic hypothesis had to be made that column diameter does not affect the heat transfer coefficient. This assumption is quite well substantiated by the results of other workers, as discussed in Chapter II. For plotting, it is convenient to rearrange the equation into the form of a modified j-factor:

$$j_H^* = \frac{Nu}{RePr^{0.4}} = \frac{0.125}{(ReFr)^{1/4}} = 0.125\left(\frac{\nu g}{U_s^3}\right)^{1/4}$$

The results are shown correlated by this equation in Figure 10.

Results of the gas holdup measurements are presented in Figure 11. Also shown are the measurements of Shulman (8) and of Fair (5). Shulman's data are for water in a 4-inch column with a porous metal plate distributor. Fair's data are for water in 18- and 42-inch columns with a 9-inch diameter sparge ring. As discussed in Chapter III, the slope of these lines is $1/U_B$. Each line then represents a certain average bubble velocity, and these values of $\overline{U}_B$ are included in Figure 11.

Discussion

It is seen in Figures 7, 8 and 9 that the relationship between heat transfer coefficient and superficial gas velocity is well correlated by a single smooth curve over the entire range of gas velocities. This

TABLE I

SUMMARY OF HEAT TRANSFER RESULTS

Liquid	U_s, ft/sec	h, $\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\,{}^\circ\text{F}}$	T_F, °F	Nu	Re	ReFr	j_H^*
Water	.0	178	167	-	0	0	-
	.00159	339	161	289	119	2.88×10^{-5}	1.681
	.00295	392	160	334	219	1.82×10^{-4}	1.057
	.00434	467	160	398	332	5.80×10^{-4}	0.854
	.00780	533	161	454	583	3.39×10^{-3}	0.538
	.0145	608	160	517	1077	2.16×10^{-2}	0.331
	.0212	667	160	568	1574	6.78×10^{-2}	0.248
	.0284	698	159	594	2085	1.60×10^{-1}	0.195
	.0358	749	159	637	2630	3.24×10^{-1}	0.165
	.0513	793	158	675	3740	9.39×10^{-1}	0.123
	.0675	831	157	708	4870	2.12×10^{0}	0.0997
Glycol	.0	39.8	195	-	0	0	-
	.00159	79.5	188	182	18.5	4.48×10^{-6}	2.540
	.00295	93.4	187	214	34.1	2.84×10^{-5}	1.621
	.00434	105	188	242	50.5	9.10×10^{-5}	1.239
	.00780	127	187	292	90.3	5.26×10^{-4}	0.835
	.0145	147	186	335	167	3.35×10^{-3}	0.517
	.0212	164	186	372	242	1.04×10^{-2}	0.395
	.0284	176	185	399	321	2.47×10^{-2}	0.319
	.0358	186	185	423	406	5.00×10^{-2}	0.267
	.0513	195	185	442	581	1.46×10^{-1}	0.194
	.0675	213	185	484	762	3.31×10^{-1}	0.161

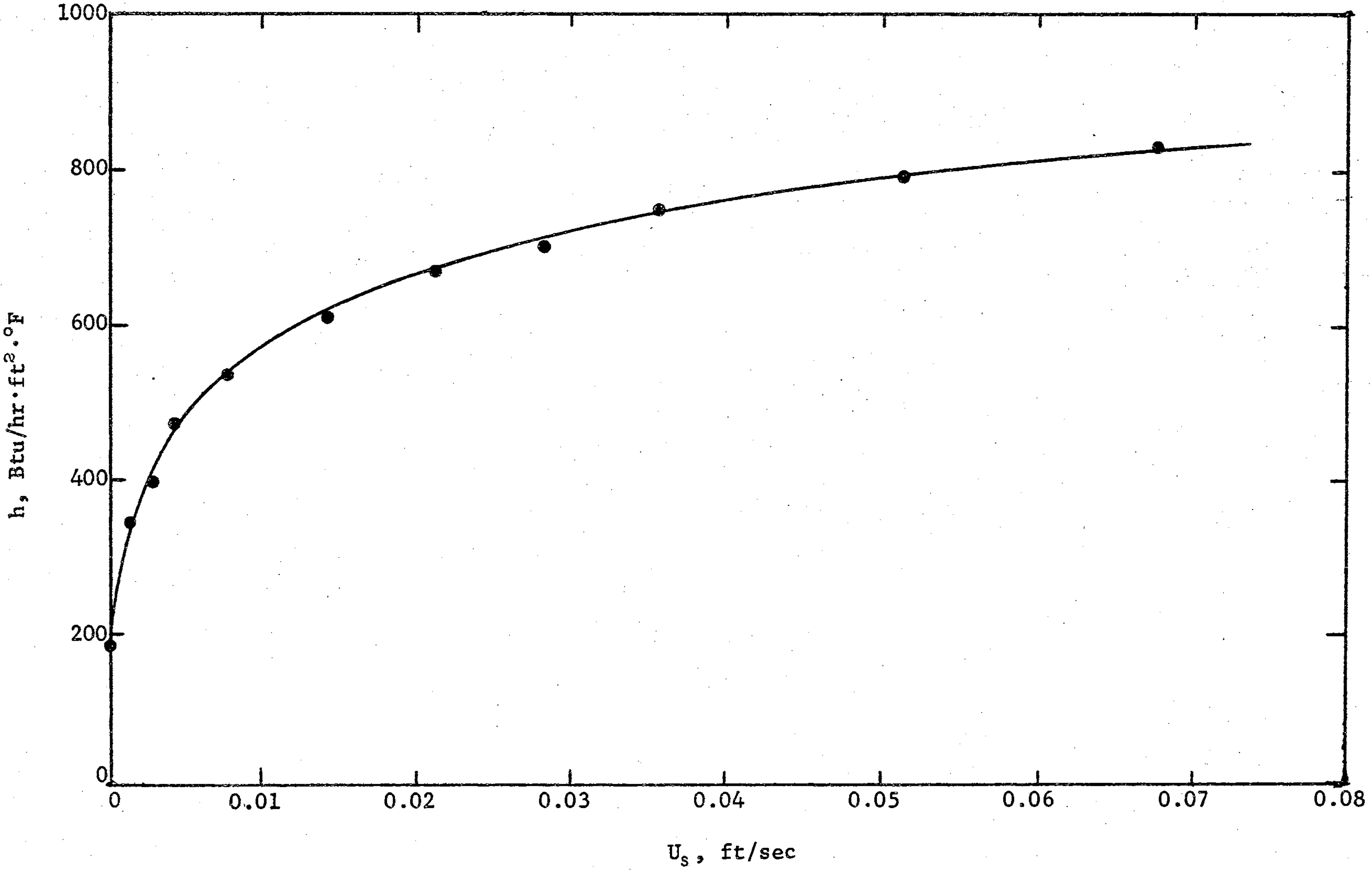


Figure 7. Effect of Bubble Agitation on the Heat Transfer Coefficient for Water

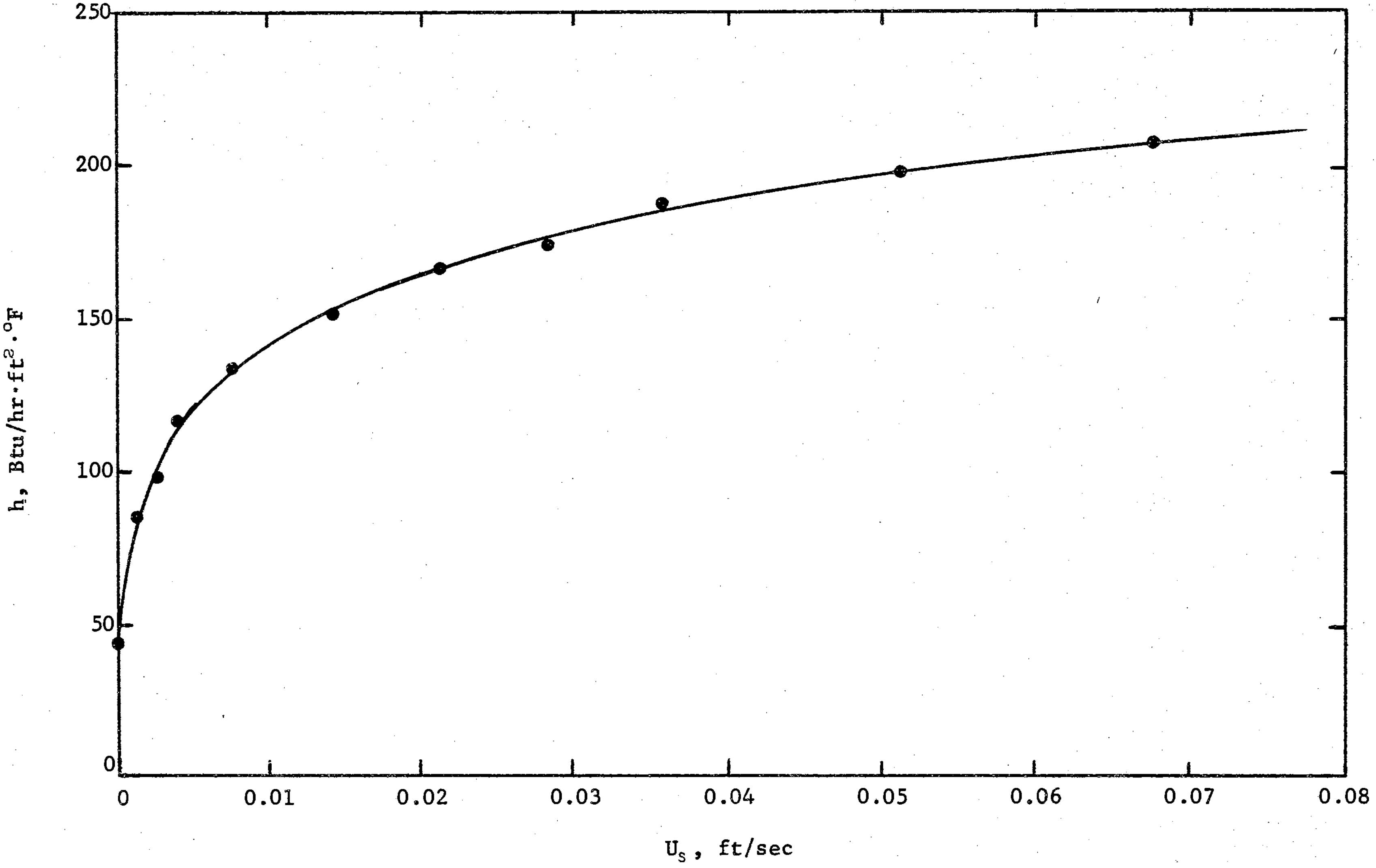


Figure 8. Effect of Bubble Agitation on the Heat Transfer Coefficient for Glycol

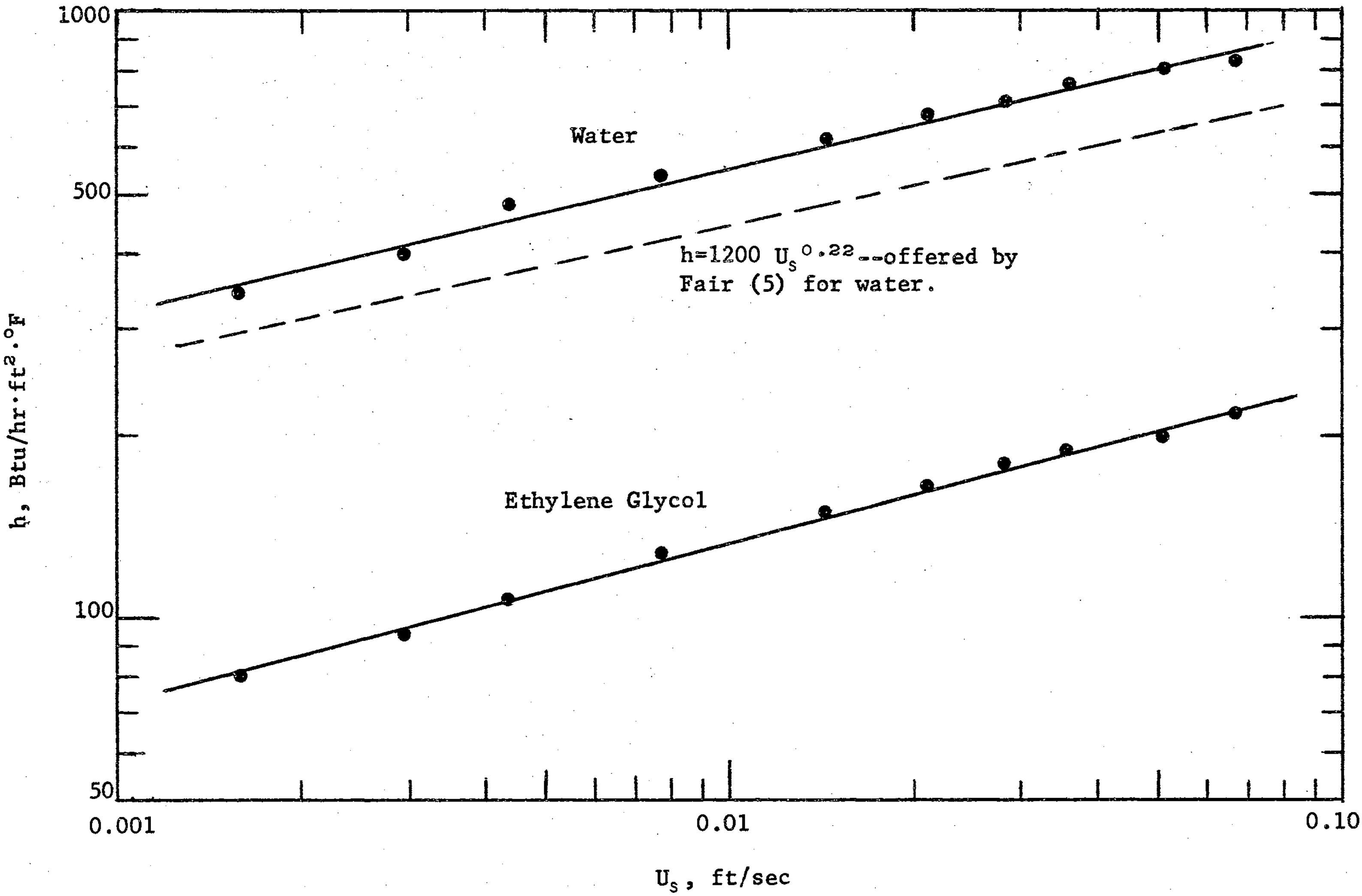


Figure 9. Heat Transfer Coefficient as a Function of Superficial Gas Velocity

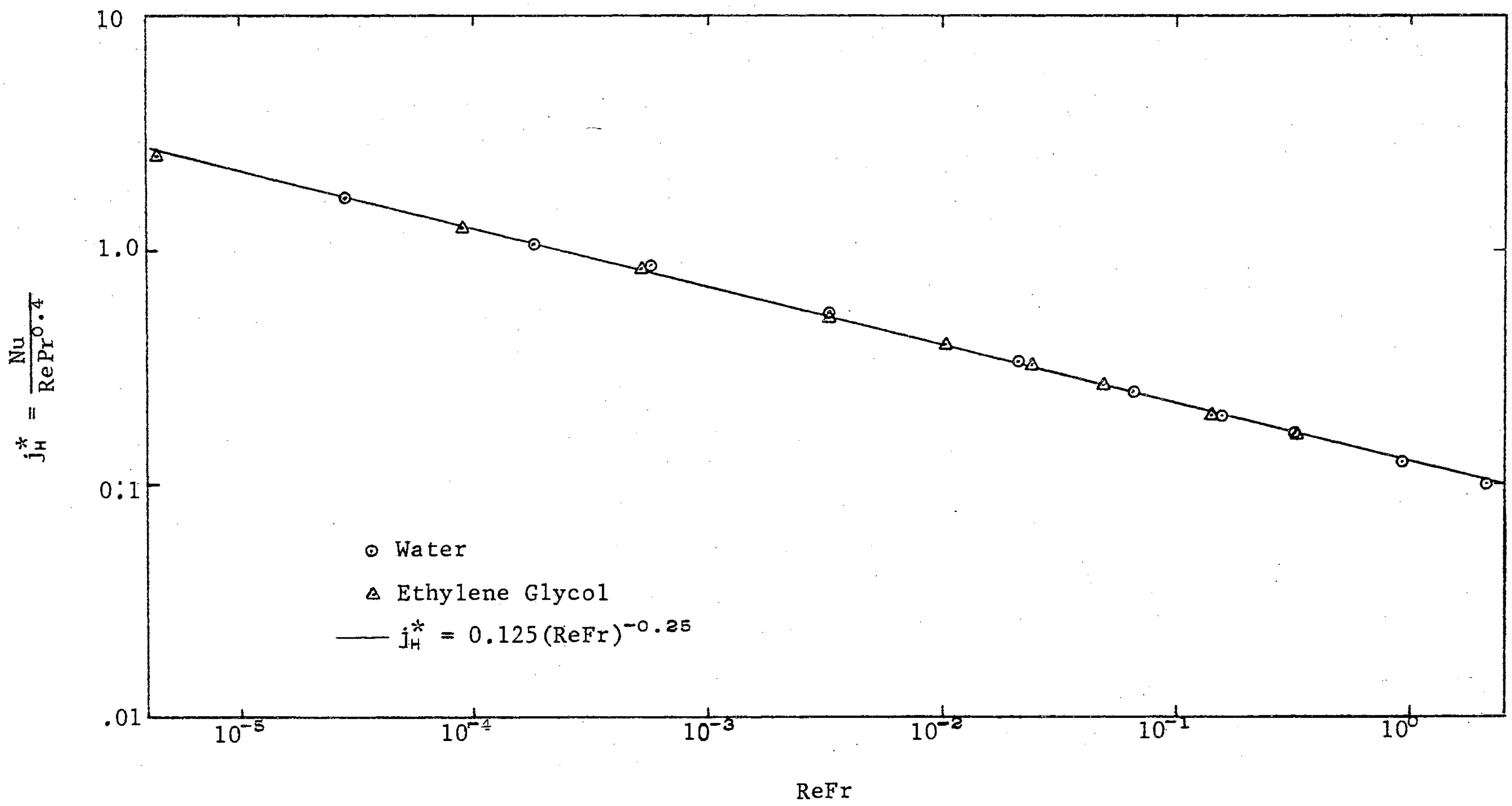
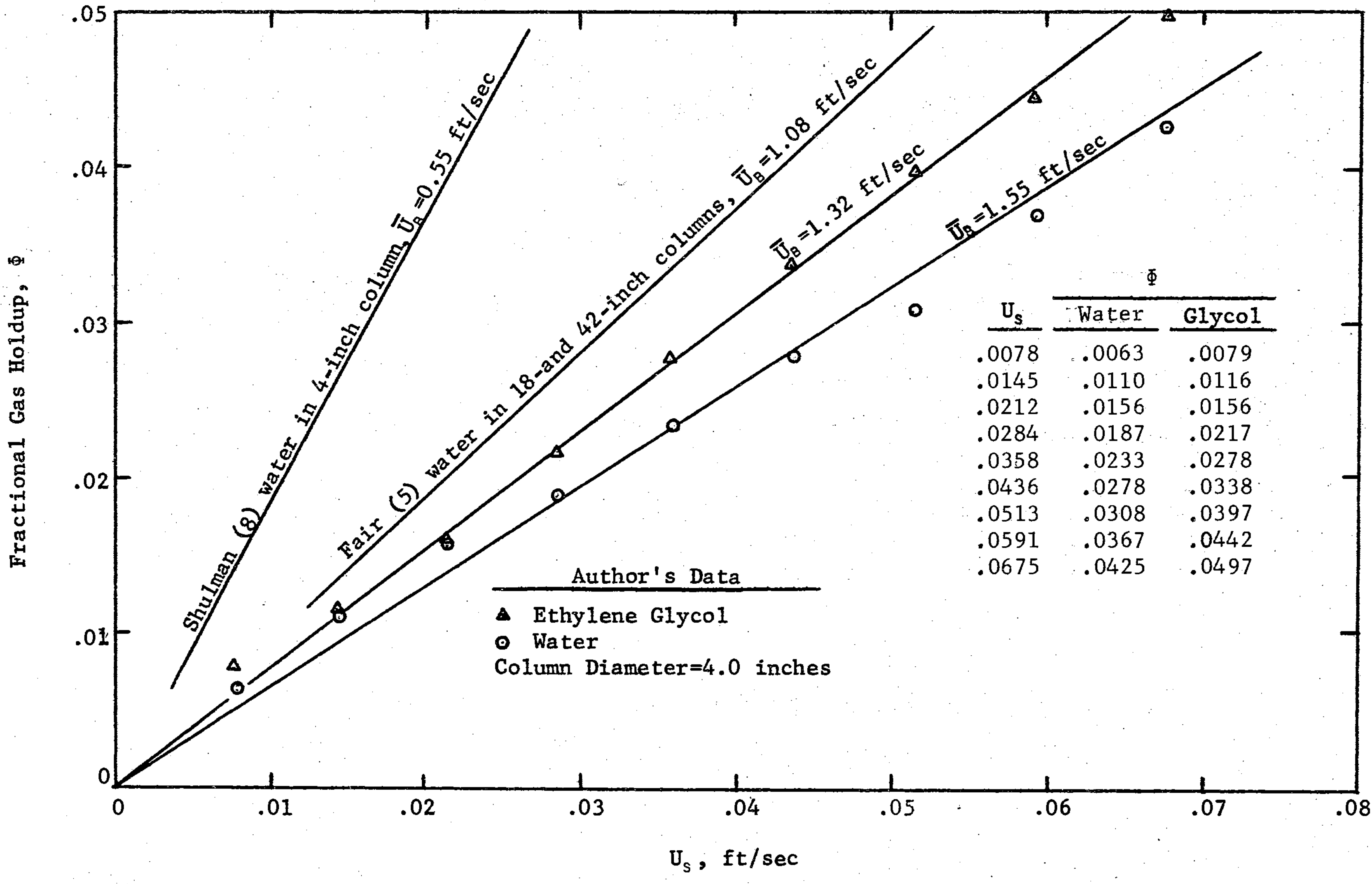


Figure 10. Dimensionless Correlation for Heat Transfer to a Bubble-Agitated Liquid



	Φ	
U_s	Water	Glycol
.0078	.0063	.0079
.0145	.0110	.0116
.0212	.0156	.0156
.0284	.0187	.0217
.0358	.0233	.0278
.0436	.0278	.0338
.0513	.0308	.0397
.0591	.0367	.0442
.0675	.0425	.0497

Figure 11. Fractional Gas Holdup as a Function of Superficial Gas Velocity

suggests that there is no fundamental change of "regime" involved when the system is considered as a whole. That is, the system cannot be classified as being "laminar" or "turbulent" such as visual observation might indicate, as discussed in Chapter III.

Of course, at zero gas velocity the heat transfer occurs soley by natural convection, while at some finite gas rate it occurs by forced convection. In this sense there is a change of "regime" involved; but the transition apparently occurs very smoothly with one effect superimposed upon the other until transition is complete. The gas velocity at which this transition is complete cannot be accurately determined from the data now available. However, it is seen in Figure 9 that the straight line apparently fits the data even down to the rather low velocity of 0.00159 ft/sec. This suggests that natural convection effects are insignificant even at this velocity. A comparison of the temperature profiles at U_s = zero and U_s = 0.00159 ft/sec. suggests the same thing. (See Appendix C).

It is emphasized that the heat transfer coefficients here presented are "averaged" or "time-smoothed". It is certain that the local coefficient varies considerably in a bubble-agitated system. Not only does the local coefficient change with time, but at any fixed instant the coefficient may vary with location along the vessel wall. Hence, the coefficients reported are both time-averaged and location-averaged. Thus, while no change of regime is apparent when the system is viewed as a whole, there may exist laminar and turbulent boundary layers at different times and points within the system.

This apparent "macroscopic" continuity suggests an interesting possibility for future work. One might devise a very simple mathematical model wherein the process is viewed as the successive rise of discrete, spherical bubbles, each obeying Stokes' Law, and each rising along a perfectly vertical path through the center of the column. Within the framework of this idealized model, one might achieve an analytical (or numerical) solution for the heat transfer at the wall. It is then conceivable that the results might be applicable to the mathematically indescribable situation which exists in a real bubble-agitated system.

The dimensionless correlation which resulted from this study has already been presented, and its application to the experimental results has been shown in Figure 10. However, its development is interesting, and perhaps instructive. First, it is reemphasized that any simple dimensional analysis which recognizes fluid properties, column diameter, superficial gas velocity, <u>and</u> the gravitational constant, g, would yield exactly this correlation--at least in form.

As shown in Chapter II, other authors, except Kast, attempted to correlate their data without the Froude Number--that is, in terms of Nu, Re, and Pr only. Proceeding in this direction one might first plot Nu vs. Re, seeking the proper exponent for Pr. Using the results of this present study, such a plot is shown in Figure 12. It is seen that, surprisingly (to the author at least), the data seems already correlated--without the Prandtl Number being considered. That is, the proper exponent for the Prandtl Number is apparently zero. Of course, such a correlation can be nothing more than an accident, resulting from the unique experimental conditions used in this particular study. In any Nusselt-type correlation a zero exponent on the Prandtl Number is just as untenable as a negative exponent--as discussed in Chapter III.

Furthermore, all data presently available indicate that the heat

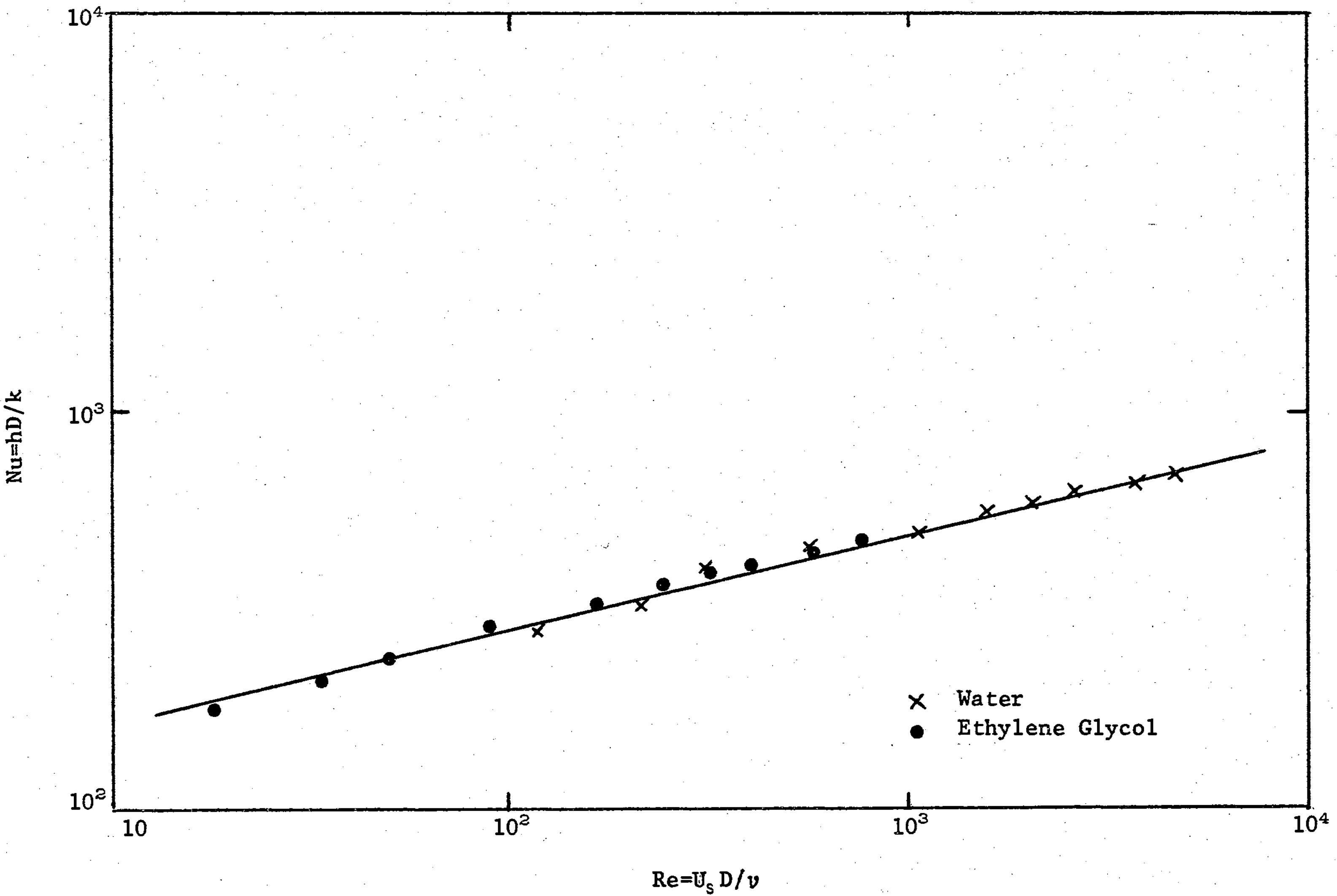


Figure 12. Accidental Correlation without the Prandtl Number

transfer is not a function of the column diameter--or, apparently of any other characteristic length. This situation requires that the exponents on Nu and Re be equal, that is, that the Re exponent be 1.0. Yet, at the same time one knows that the heat transfer coefficient is a function of the superficial gas velocity to about the 0.25-power. In addition, Kölbel's data for sugar solutions indicate that the proper exponent for ν, the kinematic viscosity, is -0.35 to -0.40. In the end, one is faced with the paradox of constructing a dimensionless correlation such that

$$\mathrm{Nu} \propto \frac{U_s{}^{0.25} D^{0.0}}{\nu^{-0.35}}$$

with the additional physical restriction that the Prandtl Number have a positive exponent. Of course, this entire analysis presupposes that the superficial gas velocity and the column diameter are the proper "characteristic" velocity and length respectively. It seems apparent that these <u>are</u> the proper parameters.

A significant fact emerges from the previous discussion. In <u>any</u> correlation of the form $\mathrm{Nu} = c\mathrm{Re}^a \mathrm{Pr}^b$, the effect of velocity is <u>greater</u> than the effect of viscosity on the heat transfer coefficient. Yet, in a bubble agitated system the reverse is apparently true. Kölbel attempted to account for this situation by using a negative exponent on the Prandtl Number. Novosad attempted to account for it by using the gas holdup, Φ, in his Reynolds Number, that is:

$$\mathrm{Re} = \frac{U_S D}{\nu\sqrt{\Phi}} = \frac{U_S D U_B{}^{0.5}}{\nu U_S{}^{0.5}} = \frac{U_S{}^{0.5} U_B{}^{0.5} D}{\nu}$$

since as shown in Chapter III, $\Phi = U_S / U_B$. This maneuver allowed the Reynolds Number exponent to be doubled without changing the effect of superficial gas velocity, U_S. (As discussed in Chapter III, the bubble velocity, U_B, is practically a constant in any given system). Also, by doubling the exponent, the effect of column diameter was reduced by one-half.

Now, note the effect of introducing the Froude Number. One intuitively expects that the exponent for g should be positive, so the Froude Number is inverted and the correlation written:

$$\frac{hD}{k} = c\left(\frac{U_s D}{\nu}\right)^a \left(\frac{gD}{U_s^2}\right)^b \left(\frac{c_p \mu}{k}\right)^d$$

where a, b, and d are all positive exponents. To eliminate the effect of column diameter, one needs the condition, a + b = 1. To get an exponent of 0.25 on U_s, one needs the condition, a-2b = 0.25. Hence, a = 0.75 and b = 0.25. Actually, when this equation is written in the form of the modified j-factor, one need not restrict the exponent of U_s to 0.25. That is, a plot of j_H^* vs the product ReFr on logarithmic coordinates need not be restricted to a straight line. A little reflection about this will reveal that no matter how the exponent of U_s may change, the basic restriction that column diameter have no effect is satisfied. This is an important point because, as mentioned in Chapter III, the effect of U_s on h begins to taper off at very high gas velocities.

When the equation is written with the proper exponents inserted--as already presented earlier in this Chapter--the following relation is obtained:

$$h = 0.125 \frac{U_s^{0.25} D^{0.0} g^{0.25} c_p^{0.4} k^{0.6} \rho^{0.75}}{\nu^{0.35}}$$

This relationship not only seems physically realistic, but is consistent with all the facts presently available on bubble-agitated heat transfer. It is interesting to note that the j-factor form of this equation is very similar to Colburn's j-factor equation (based on Blasius' formula for the friction factor) for heat transfer in tubes, i.e.:

$$j_H^* = \frac{0.125}{(ReFr)^{1/4}} \qquad \text{Bubble Agitation}$$

$$j_H = \frac{0.0395}{Re^{1/4}} \qquad \text{Colburn's Equation, from (9)}$$

As already discussed the fact that this correlation does not account for variations in nozzle design, surface tension, or gas density should not be too serious a limitation as long the surface tension, or gas density do not exceed certain bounds. Actually, it probably would not be too difficult to account for these variables by introducing the Weber Number and the dimensionless group $(1-\rho_G/\rho_L)$ into the correlation. Of course, the exponents of these groups could only be determined by experiment. It may be difficult to account for variations in nozzle design, but, as already mentioned, it is doubtful that the type of gas distributor can have a very strong effect on the heat transfer--especially at high gas velocities.

It was pointed out in Chapter II that most of the other workers did not report their temperatures of operation; hence it is difficult to compare the results of this study with those of others. However, Fair noted that his 42-inch column had a 3.7 Kw heater, and that his water flow rate ranged from 10 to 13 gpm. From these figures it can be calculated that the increase in water temperature, from inlet to outlet, was only about 2.0°F. Assuming then that the feed water was near ambient temperature, one can estimate that the average water temperature was about 80°F.

The dimensional equation which Fair offered to describe both his and Kölbel's data for water was presented in Chapter II, and it is represented by the dashed line in Figure 9. It is seen that his equation gives values of h which are 20-30% lower than the author's.

The difference can perhaps be reconciled by considering the physical properties of water at 80°F and at 160°F (roughly the water temperature in the author's experiments). Using the author's dimensionless correlation, and ignoring the small change in heat capacity, the ratio of the h at 160°F to that at 80°F can be estimated, i.e.

$$\frac{h_{160}}{h_{80}} = \left(\frac{k_{160}}{k_{80}}\right)^{0.8}\left(\frac{\rho_{160}}{\rho_{80}}\right)^{0.75}\left(\frac{\mu_{80}}{\mu_{160}}\right)^{0.35}$$

Inserting the proper values for the physical constants results in $\frac{h_{160}}{h_{80}} =$ 1.35. This is slightly more adjustment than is necessary to make the lines in Figure 9 coincide, but, of course, this entire comparison is based on a rough estimation. All things considered, it seems evident that the author's data are in general agreement with Fair's.

Another means is available for estimating the validity of the experimental procedure, and that is to compare the results obtained at zero gas velocity with values predicted by natural convection correlations.

For this purpose three correlations were chosen, viz: Figure 129 in McAdams (13), Equation 14.56 in Grober (11), and the equations of Hougen presented on page 186 of McAdams. The first of these is a plot of Nu vs. GrPr based on data for vertical flat plates. The eqnation in Grober is a typical Nusselt-type general correlation. The equations of Hougen are for heat transfer to water flowing at velocities of from zero to 0.1 ft/sec in a 3-inch diameter vertical tube. Hougen presents two equations--one for upward flow and one for downward flow. He then extrapolates his results for water by means of a third equation which is a Nusselt-type dimensionless correlation.

The coefficients predicted by these correlations are summarized in Table II, along with the results of this study. It is seen that there is general agreement in the values except for those predicted for water by Equation 14.56 in Grober and Figure 129 in McAdams. The reason for the

TABLE II

COMPARISON OF NATURAL CONVECTION COEFFICIENTS

Source	h,Btu/hr·ft^2°F	
	Water	Glycol
This Study	178	39.8
Hougen - Upward Flow	172	-
Downward Flow	200	-
Generalized	-	42.8
Equation 14.56 Grober	93	43.5
Figure 129 McAdams	73	35.2

discrepancy is not known; however, it is very unlikely that the present author's data for glycol are correct while his data for water are incorrect.

Gas Holdup

It is seen in Figure 11 that a straight line satisfactorily describes the relationship between Φ and U_s. As discussed in Chapter III, this linearity verifies that the bubble rise velocity is practically independent of the superficial gas velocity. Also shown in Figure 11 are lines describing the data of Shulman (8) and of Fair (5) for water.

The line representing Shulman's data was actually taken from Fair's paper. Fair compared Shulman's data, taken in a 4-inch column, with his own data taken in columns of 18-and 42-inches diameter. He concluded that, up to 18 inches, column diameter significantly affects holdup--the holdup being greater in smaller columns. The lines in Figure 11 suggest that this is not the case. It is seen that the holdups measured by Shulman in a 4-inch column are almost three times those measured by the author in the same size column. Furthermore, Fair's data for 18- and 42-inch columns lie above the author's data.

Apparently, the effect which Fair attributed to column diameter, is actually the result of gas distributor design. The 4-inch column of Shulman had a porous metal plate distributor; the 18- and 42-inch columns of Fair had a 9-inch diameter perforated-ring distributor, and the author's 3.9-inch column had a single "sparging" tube distributor.

The effect of distributor design is understandable when one considers that, within a limited range, bubble size affects bubble velocity--smaller bubbles rising more slowly. [See general discussion in Section 18 of Perry's (14)]. At a given superficial gas velocity, a porous plate produces smaller bubbles than does a perforated ring, and any distributor produces smaller bubbles than a single "sparging" tube.

Of course, in very small columns where the bubble diameter is of the same order of magnitude as the column diameter, it is obvious that some extra "drag" would be felt by the bubbles; however, it is unlikely that this "wall effect" is significant in columns of, say, 2-inch diameter or larger.

Error Analysis

The first objective of the discussion in this section is to estimate the maximum possible error accruable in any single determination of h or U_s if the uncertainties involved in each measurement reinforced each other. Such an estimate is most easily presented schematically. Thus, in Figure 13 is shown a "flow diagram" depicting how all the individual measurements were combined to eventually produce the h vs U_s relationship. The overall uncertainty was potentially contributed to not only by the

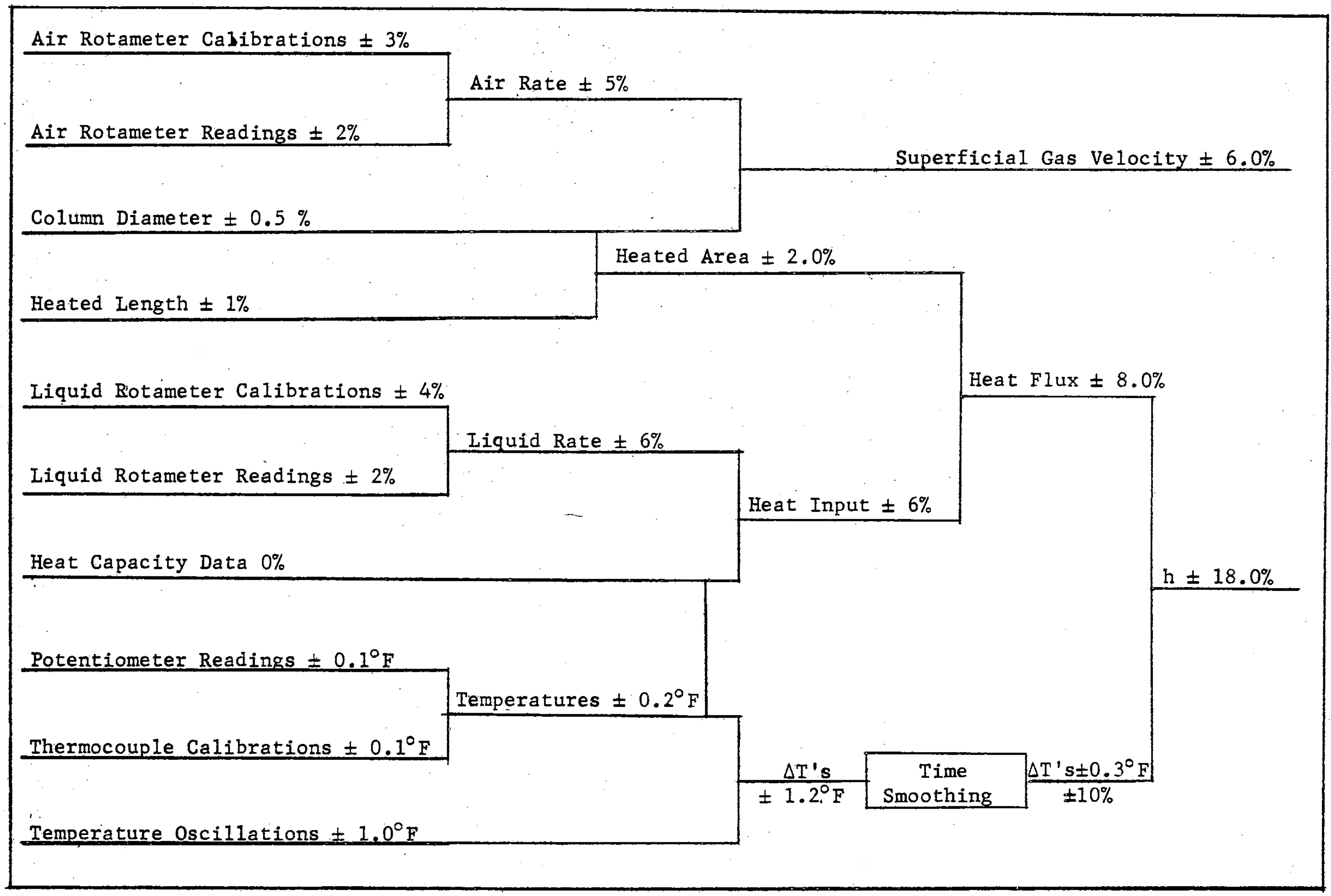


Figure 13. Estimated Maximum Accruable Error in any Single Data Point

measurements made, but by the temperature oscillations and the heat capacity data. The estimated maximum uncertainty associated with each of these factors is shown in the diagram, and they combine to give a maximum estimated error of ± 18% in h, and ± 6% in U_s.

Of course, many of the errors listed in Figure 13 are of a random nature, and thus would tend to average themselves out. Others, however are of a persistent nature, and would be reflected in every data point. Still others are "semi-persistent" in that they would not average themselves out of the results of a single run, but would tend to average out of the final results, because the final results are averages of the results obtained in individual runs. An example of this latter category is the error involved in determining the time-smoothed ΔT's.

Thus, if one considers the final h vs U_s relationship for a given liquid, persistent errors are possible in the column diameter and length measurements, the heat capacity data, and the rotameter calibrations. Then, if all random errors averaged themselves out, and only these persistent errors remained, the final results could be in error by an estimated maximum of ± 6% for h, and ± 4% for U_s.

Actually, another possible source of persistent error exists which is not listed in Figure 13. This is the assumption that the heat flux at the middle of the heated section equals the average heat flux. However, considering the homogeneity of the heat source, and the relative flatness of the wall temperature profiles at the middle of the section, it is estimated that only a negligible error is involved in this assumption.

The figures presented above represent estimates of the maximum possible errors involved. Perhaps a more realistic picture can be obtained by considering the reproducibility of the data obtained in different runs.

As mentioned before, the final reported values of h are averages of the values obtained in individual runs. Runs were repeated in order to establish the reproducibility of the measurements. Naturally, there was usually some deviation between the average value of h and the individual values. These deviations are listed in Table III. The average deviations are based on the absolute values of the individual deviations, and the overall average deviations are averages of the averages. Since only two runs were made with glycol, the average deviations are, of course, identical with the individual deviations.

It is seen that the average deviations never exceeded 7.2 per cent in the water runs, and 7.6 per cent in the glycol runs. The overall average deviations were 4.2 per cent and 2.1 per cent respectively. These figures reflect the degree of experimental precision involved. The glycol data are more consistent than the water data because of the larger ΔT's involved in the glycol runs.

The actual precision was probably somewhat better than these figures indicate. While it was intended that all runs with a given liquid be identical, there was actually some variation in mean film temperature from run to run. This variation was only 1-2°F in Runs 4 and 5, and 2-3°F in Runs 1 and 2, but Run 3 was made under conditions (see Chapter V) such that T_F was 6-8°F higher than in Runs 1 and 2. Due to the effect of temperature on the physical properties of water, the heat transfer coefficient increases with temperature. This "trend" is reflected in Table III. The deviations for Run No. 3 are nearly all positive, while those for Run No. 1 (the lowest T_F) are nearly all negative. It would have been somewhat better to convert the data to dimensionless groups before averaging, but

TABLE III

REPRODUCIBILITY OF INDIVIDUAL RUNS

	Percent Deviation of Individual Values of the Coefficient from the Average						
	Water				Glycol		
U_s, ft/sec	Run No. 1	Run No. 2	Run No. 3	Average	Run No. 4	Run No. 5	Average
0	0	-2.2	+2.2	1.5	-	-	-
.00159	0	-4.2	+4.2	2.8	-1.5	+1.5	1.5
.00295	-6.9	-2.6	+9.7	6.4	-0.9	+0.9	0.9
.00439	0	-2.2	+2.2	1.5	-2.5	+2.5	2.5
.00780	-6.8	-0.4	+5.6	4.4	-3.6	+3.6	3.6
.0145	-3.9	-3.9	-0.2	2.7	-3.7	+3.7	3.7
.0212	-7.8	+4.5	+3.1	5.1	-3.2	+3.2	3.2
.0284	-10.7	-0.1	+10.9	7.2	-5.1	+5.1	5.1
.0358	-8.3	+2.1	+6.0	5.5	+2.2	-2.2	2.2
.0513	-6.4	+1.0	+5.5	4.3	+7.6	-7.6	7.6
.0675	-1.7	-4.9	+6.5	4.4	+1.0	-1.0	1.0
Overall Average Deviation				4.2			2.1

the improvement in precision would hardly have been worth the extra calculation.

From a purely "mechanical" standpoint, the apparatus was capable of greater precision than the figures in Table III might indicate. The rotameters had a guaranteed precision of ± 2.0 per cent, and the potentiometer could detect a temperature change of $0.1^{o}F$ at least. The major part of any inprecision reflected in the final data is due to the temperature oscillations. That is, there was some "scatter" in the time-smoothed temperature profiles, so that there was some random uncertainty involved in locating the smooth curve. In the glycol runs the ΔT's were larger than those in the water runs, so the percentage uncertainty was correspondingly less. However, with a given liquid, the largest ΔT's occurred at the lowest gas rates where the oscillations were also largest, so the effects of uncertainty and ΔT-magnitude partially cancelled one another.

The generality involved in choosing the mean film temperature for evaluating physical properties can not be determined. This temperature was chosen because it is commonly used and accepted in most heat transfer work.

All things considered, it seems obvious that the data should be accurate to within ± 10 per cent or better. Without making any actual calculations it is evident from Figure 10 that the general correlation describes the data with an average precision of at least ± 5 per cent.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The heat transfer between a solid surface and a liquid can be greatly enhanced by means of bubble agitation. With water and ethylene glycol, a superficial gas velocity of 0.05 ft/sec promotes a heat transfer coefficient which is roughly four times the natural convection coefficient. There are indications that the existence of radial flow throughout the column height may be largely responsible for the high heat transfer rates.

A bubble-agitated system is a highly transient one, and the instantaneous local heat transfer coefficient fluctuates considerably. However, time-smoothed coefficients can be determined which are well defined and reproducible.

All information presently available indicates that the time-smoothed coefficient is independent of location within the vessel, vessel diameter, and liquid height, and that it is not greatly affected by gas distributor design. Natural convection effects are apparently insignificant even at the relatively low superficial gas velocity of 0.00159 ft/sec. The effects of gas density and surface tension have not been explicitly determined, but it is expected that neither of these variables is too important except in extreme cases.

In a bubble-agitated system there are three types of forces at play-inertial, viscous and gravitational (surface forces neglected). As a result, both the Reynolds Number and the Froude Number must be considered in establishing dynamic similarity. The superficial gas velocity and the column diameter are evidently the proper "characteristic" parameters to use in constructing the dimensionless groups.

The results of this study are correlated with an average precision of ± 5 per cent or better by the dimensionless equation

$$Nu = 0.125\ Re^{0.75} Fr^{-0.25} Pr^{0.4}$$

This equation is also consistent with the observations of previous workers. Although a rather limited amount of data has gone into its development, the correlation should be applicable for many design purposes.

The gas holdup in a bubble-agitated system is practically a linear function of the superficial gas velocity. The holdup is strongly affected by gas distributor design, but there is no indication that column diameter has any effect as long as the column is large compared to the bubbles. The gas holdup is directly related to the bubble velocity and the superficial gas velocity. From this relationship one can determine the average velocity of a complex array of bubbles. It develops that, in a given system, the bubble velocity is practically a constant--independent of the superficial gas velocity.

The bubble velocity represents an upper limit to the superficial gas velocity which can be achieved in any system. In most systems of practical interest this limit would never even be approached, but in liquids of extremely high viscosity, where the bubble velocity might be quite low, the power-input achievable with bubble agitation might be insufficient. Herein lies the only apparent limitation to the general use of bubble agitation.

If the superficial gas velocity is not fixed by other design variables, a velocity of about 0.05 ft/sec is recommended. This rate provides excellent agitation, and higher gas velocities will not give a proportional improvement in the heat transfer rate.

In future work, the experiments done in this study should be repeated with at least one other liquid--preferably with a viscosity greater than that of glycol. The effect of gas distributor design should be studied more thoroughly, and from this the effect of bubble velocity can be determined. Eventually it may be desirable to study the effects of surface tension and gas density, and to extend the range of superficial gas velocities used.

While a rigorous mathematical study is out of the question, some calculations based on the simplified model described in Chapter VI may be useful. At any rate, a logical mechanistic analysis would be a significant contribution.

In future experimental work, an improvement in precision could be obtained if at least two more thermocouples were installed in the liquid, and more temperature readings were taken at each gas velocity. Both of these steps would help to establish the time-smoothed temperature profiles with more certainty. Computer reduction of the data is also recommended.

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APPENDIX A

NOMENCLATURE

The nomenclature used in this thesis is presented below. Special terms which were used only in the equations of other authors are defined in Chapter II, and they are not repeated here. The definitions of the dimensionless groups shown are those used by the author. Unless specifically noted otherwise, all physical properties are those of the liquid.

A - cross sectional area of the column, ft^2.

c_P - heat capacity at constant pressure, Btu/lb°F.

D - inside column diameter, ft.

g - gravitational constant, ft/sec^2.

h - heat transfer coefficient, $Btu/hr \cdot ft^2$.

H_T - height of bubble-agitated liquid above gas distributor.

j_H^* - modified j-factor $= \frac{Nu}{RePr^{0.4}}$

k - thermal conductivity, Btu/hr·ft°F.

q - average heat flux based on total heat input, Q, and inside area of copper pipe, $Btu/hr \cdot ft^2$.

Q - total heat input, Btu/hr.

T_F - mean film temperature, °F.

ΔT - temperature difference between wall and liquid, °F.

ΔT_M - ΔT at the middle of the heated section, °F.

t - gas holdup time, sec.

U_B - bubble rise velocity, ft/sec.

$\overline{U}_B$ - average rise velocity of complex array of bubbles, ft/sec.

$\dot{V}$ - volumetric gas flow rate, ft^3/sec.

V_B - volume occupied by gas bubbles.

V_L - volume of bubble-free liquid.

V_T - total volume, $V_B + V_L$.

β - thermal expansion coefficient, $^\circ F^{-1}$

μ - dynamic viscosity, lb/ft·hr.

ν - kinematic viscosity, ft^2/sec.

Φ - fractional gas holdup, V_B/V_T.

ρ - density, lb/ft^3.

Fr - Froude Number, $U_S{}^2/gD$.

Gr - Grashof Number, $D^3 g\beta\Delta T/\nu^2$.

Nu - Nusselt Number, hD/k.

Pr - Prandtl Number, $c_p\mu/k$.

Re - Reynolds Number, $U_S D/\nu$.

St - Stanton Number, Nu/Re·Pr

APPENDIX B

TABULATED DATA AND PHYSICAL PROPERTIES

In Tables B-I through B-V are presented the averaged temperature profiles along with other pertinent data for each run. The table captions include a statement of which liquid was involved, and the values of the average liquid flow rate (W) and average heat flux (q). Included in the tables are values of Q, the calculated heat input, ΔT_M, the temperature difference at the middle of the column, and h, the heat transfer coefficient. The numbering system use to denote the location of the thermocouples is described in Appendix C.

Pertinent physical property data for water and glycol are listed in Table B-VI.

TABLE B-I

DATA FROM RUN NO.1

(Water; W=59.8 lbs/hr; q=2335 Btu/hr·ft²)

U_s, ft/sec			0	.00159	.00295	.00434	.00780	.0145	.0212	.0284	.0358	.0513	.0675
TEMPERATURES, °F	WALL	1	169.6	160.1	158.6	158.1	158.3	157.1	156.1	154.5	154.4	153.3	152.4
		2	168.8	162.3	160.7	160.3	159.6	158.1	157.4	155.8	155.5	154.6	153.6
		3	168.7	163.5	162.4	161.3	160.7	159.2	158.3	156.7	156.0	155.2	154.4
		4	170.7	164.4	162.8	161.6	161.3	159.6	158.6	156.9	156.3	155.6	154.7
		5	172.4	165.1	163.0	162.3	161.5	159.8	158.8	157.3	156.8	156.0	155.2
		6	174.8	165.4	163.9	162.9	161.7	160.3	159.3	157.5	157.2	156.4	155.5
		7	177.0	165.5	164.4	162.9	162.3	160.5	159.3	157.6	157.5	156.5	155.6
		8	177.9	166.4	164.8	163.5	162.4	160.6	159.6	158.0	157.5	156.7	155.7
		9	175.3	166.1	164.8	163.1	162.4	160.6	159.6	158.0	157.4	156.4	155.5
	LIQUID	in	80.1	79.2	79.3	78.9	78.2	78.2	77.1	77.0	77.4	78.6	79.4
		1	158.9	155.0	154.3	154.3	154.5	153.6	153.2	151.7	151.9	151.3	150.8
		3	158.0	156.6	156.0	156.4	156.0	154.7	154.5	152.8	152.4	152.2	151.6
		5	159.8	157.9	156.9	157.2	156.8	156.1	155.0	153.6	153.5	152.9	152.2
		7	161.0	158.9	157.4	158.1	158.0	156.6	156.0	154.2	154.0	153.4	152.9
		9	160.2	159.7	158.8	158.5	158.2	157.1	156.1	154.7	154.3	153.6	153.1
		out	160.2	159.7	158.8	158.5	158.2	157.1	156.1	154.7	154.3	153.6	153.1
	AIR	in	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0
		out	-	135	137	138	141	144	145	145	146	147	149
	Film, T_f		166	161	160	160	159	158	157	155	155	155	154
Q, Btu/hr			4780	4800	4750	4765	4790	4775	4800	4775	4770	4730	4760
ΔT_m, °F			13.1	6.9	6.4	5.0	4.7	4.0	3.8	3.75	3.4	3.15	2.86
h, Btu/hr·ft²			178	339	365	467	497	584	615	623	687	742	817

Table B-II

DATA FROM RUN NO. 2

(Water; W=59.8 lbs/hr; q=2370 Btu/hr·ft^2)

U_s ft/sec			0	.00159	.00295	.00434	.00780	.0145	.0121	.0284	.0358	.0513	.0675
TEMPERATURES, °F	WALL	1	167.0	157.0	154.1	155.6	157.7	157.8	157.8	157.6	157.5	157.1	155.1
		2	166.7	159.0	156.9	157.9	159.5	158.9	159.2	159.0	159.0	157.8	156.9
		3	166.1	160.2	157.9	158.7	160.4	160.0	159.6	159.6	159.3	158.4	157.5
		4	167.7	160.8	158.6	158.9	161.0	160.3	160.0	160.0	160.0	158.8	157.6
		5	168.6	161.2	159.0	159.5	161.2	160.6	160.8	160.3	160.1	159.4	157.7
		6	172.3	161.9	159.6	160.0	161.9	160.7	160.8	160.5	160.5	159.6	158.0
		7	173.3	162.2	160.2	161.0	161.7	161.5	161.3	160.5	160.6	159.7	158.4
		8	173.5	163.0	160.7	161.2	162.6	161.0	161.0	160.5	160.6	160.0	158.3
		9	171.3	163.0	160.5	160.9	162.2	160.9	160.8	160.6	160.6	159.6	158.3
		in	76.2	75.2	75.5	76.1	78.4	77.8	78.6	79.6	80.3	79.7	79.4
	LIQUID	1	155.4	151.5	150.6	152.0	155.0	154.3	155.1	155.4	155.4	154.0	153.3
		3	153.5	152.6	151.9	153.6	156.1	155.5	156.0	156.3	156.6	155.7	154.3
		5	156.6	154.2	152.8	154.3	157.1	156.8	157.2	156.8	157.1	156.4	154.8
		7	157.6	155.0	153.7	154.9	157.5	157.4	157.8	157.1	157.6	156.8	155.1
		9	156.6	155.7	154.8	156.2	158.1	157.7	158.1	157.8	157.9	156.9	155.7
		out	156.6	155.7	154.8	156.2	158.1	157.7	158.1	157.8	157.9	156.9	155.7
	AIR	in	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0
		out	-	131	133	134	141	145	147	148	148	150	151
	Film, T_f		163	158	156	157	159	159	159	159	159	158	156
Q, Btu/hr			4800	4805	4739	4794	4792	4838	4847	4817	4839	4893	4936
ΔT_m, °F			13.6	7.3	6.2	5.2	4.4	3.75	3.4	3.4	3.1	3.96	3.0
h, Btu/hr·ft^2			174	325	382	456	538	632	697	697	765	801	790

TABLE B-III

DATA FROM RUN NO. 3

(Water; W=72.8 lbs/hr; q=3095 Btu/hr·ft^2)

U_s, ft/sec			0	.00159	.00295	.00434	.00780	.0145	.0212	.0284	.0358	.0513	.0675
TEMPERATURES, °F	WALL	1	175.2	161.7	160.6	162.3	161.9	162.4	161.5	161.4	161.7	160.4	158.8
		2	174.8	165.6	164.9	164.8	164.5	164.4	163.3	163.4	163.4	162.3	160.6
		3	174.8	166.4	166.2	165.6	165.5	165.4	164.1	164.3	164.3	163.2	161.5
		4	174.8	167.6	167.1	166.6	166.2	166.3	164.4	165.2	164.5	163.4	161.6
		5	179.4	168.8	167.6	167.2	166.8	166.0	165.1	165.2	165.0	163.8	162.1
		6	182.0	170.2	167.9	168.2	167.5	167.0	165.5	165.4	165.1	164.3	162.4
		7	184.4	170.4	170.3	168.5	167.5	167.2	165.8	165.8	165.3	164.3	162.4
		8	184.6	171.5	170.0	169.4	168.2	168.2	166.0	165.8	165.4	164.6	162.4
		9	180.9	171.7	169.6	169.5	167.8	167.6	165.3	165.4	165.2	164.3	162.4
	LIQUID	in	77.2	77.3	75.5	75.5	77.3	76.8	76.8	78.5	77.7	77.0	76.4
		1	161.0	157.8	157.2	157.5	158.3	158.9	158.4	157.8	158.3	157.9	156.5
		3	161.0	158.7	159.2	159.9	159.9	160.4	159.6	160.1	160.0	159.6	157.4
		5	162.3	160.1	160.1	161.2	161.7	161.3	160.6	161.5	161.0	160.3	158.7
		7	163.2	162.1	161.7	162.0	162.2	162.4	161.4	162.2	161.8	160.6	159.1
		9	162.0	163.2	162.0	162.5	162.8	162.9	161.8	162.2	162.3	161.2	159.5
		out	162.0	163.2	162.0	162.5	162.8	162.9	161.8	162.2	162.3	161.2	159.5
	AIR	in	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0
		out	-	136	140	143	145	148	150	151	153	153	153
	Film, T_f		171	165	164	164	164	163	163	163	163	162	160
Q,Btu/hr			6170	6246	6292	6334	6237	6325	6287	6242	6389	6432	6451
ΔT_m,°F			17.0	8.75	7.2	6.5	5.5	5.1	4.5	4.0	3.9	3.7	3.5
h,Btu/hr·ft^2			182	354	430	477	563	607	688	774	794	837	885

TABLE B-IV

DATA FROM RUN NO. 4

(Glycol; W=46.1 lbs/hr; q=1091 Btu/hr·ft^2)

U_s,ft/sec			0	.00159	.00295	.00434	.00780	.0145	.0212	.0284	.0358	.0513	.0675
TEMPERATURES, °F	WALL	1	-	188.9	188.5	188.7	188.1	187.4	185.8	186.3	186.1	187.3	186.4
		2	-	191.3	190.5	190.1	189.0	188.4	187.2	187.6	187.5	188.3	187.4
		3	-	192.3	191.4	190.9	189.9	189.0	187.7	188.0	188.3	188.9	187.8
		4	-	193.3	192.0	191.5	190.2	189.2	187.8	188.3	188.3	189.2	188.1
		5	-	193.4	192.1	192.4	190.5	189.7	188.7	188.3	188.4	189.4	188.3
		6	-	194.5	192.8	192.5	190.6	189.7	188.3	188.7	188.5	189.5	188.4
		7	-	195.3	193.4	192.3	190.6	189.6	188.5	188.7	188.5	189.6	188.4
		8	-	195.5	193.7	192.9	190.9	189.5	188.7	188.6	188.6	189.6	188.4
		9	-	195.5	193.0	193.1	190.8	189.4	188.6	188.4	188.5	189.3	188.4
	LIQUID	in	-	102.7	103.2	104.3	105.9	103.1	103.0	104.8	105.6	105.8	106.2
		1	-	178.5	179.6	180.5	180.8	180.7	180.1	181.0	181.1	182.9	181.7
		3	-	179.2	180.2	180.9	181.2	181.3	180.7	181.3	181.8	183.2	182.4
		5	-	180.2	180.9	181.7	181.8	182.0	181.5	182.1	182.4	184.1	183.2
		7	-	180.9	181.7	182.3	182.4	182.4	181.7	182.4	182.9	184.4	183.5
		9	-	181.7	182.0	182.6	182.6	182.6	181.9	182.7	183.1	184.7	183.7
		out	-	181.7	182.0	182.6	182.6	182.6	181.9	182.7	183.1	184.7	183.7
	AIR	in	-	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0
		out	-	119	119	120	124	127	130	134	137	146	152
	Film, T_f		-	187	187	187	186	186	185	185	185	187	186
Q,Btu/hr			-	2240	2232	2221	2174	2256	2241	2218	2209	2255	2224
ΔT_m,°F			-	14.0	11.8	10.6	8.8	7.6	6.8	6.4	5.8	5.4	5.1
h,Btu/hr·ft^2			-	78.0	92.5	102.9	124.0	143.6	160.5	170.6	188.2	202.0	214.0

TABLE B-V

DATA FROM RUN NO. 5

(Glycol; W=46.1 lbs/hr; q=1102 Btu/hr·ft^2)

U_s,ft/sec			0	.00159	.00295	.00434	.00780	.0145	.0212	.0284	.0358	.0513	.0675
TEMPERATURES, °F	WALL	1	205.6	190.6	188.8	189.0	188.6	188.5	187.6	187.3	186.5	185.3	184.2
		2	206.7	192.5	191.0	190.7	190.0	189.7	188.9	188.3	187.9	186.5	185.4
		3	209.3	193.7	192.0	191.8	191.1	190.5	189.3	188.9	188.2	187.1	185.9
		4	212.2	194.6	192.2	192.4	191.2	190.3	189.4	189.0	188.4	187.3	186.2
		5	214.2	195.0	192.8	192.2	191.0	190.6	189.6	189.3	188.4	187.3	186.2
		6	213.3	195.9	192.8	192.3	191.5	190.8	189.8	189.5	188.8	187.6	186.4
		7	210.4	196.9	193.8	192.7	191.9	190.9	190.1	189.5	188.8	187.8	186.5
		8	207.1	197.2	194.3	193.8	192.0	190.3	190.1	189.7	188.7	187.7	186.5
		9	202.8	197.0	194.1	193.5	192.2	191.4	189.9	189.4	188.5	187.2	186.3
	LIQUID	in	101.6	104.3	104.4	104.7	104.8	103.7	104.3	104.4	103.6	102.0	102.7
		1	184.5	179.8	180.4	181.0	181.9	182.4	181.9	181.9	181.4	180.6	179.8
		3	180.4	180.4	181.0	181.8	182.5	182.9	182.6	182.5	181.9	181.0	180.5
		5	179.6	181.5	181.7	182.5	183.3	183.6	183.2	183.1	182.5	181.7	181.2
		7	179.3	182.3	182.1	182.8	183.6	184.1	183.6	183.5	183.0	182.0	181.6
		9	179.3	182.7	182.7	183.2	183.9	184.3	183.9	183.8	183.3	182.2	181.7
		out	179.3	182.7	182.7	183.2	183.9	184.3	183.9	183.8	183.3	182.2	181.7
	AIR	in	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0	72.0
		out	-	119	120	122	124	128	132	134	137	142	151
	Film, T_f		195	188	187	188	188	187	187	186	186	184	184
Q,Btu/hr			2200	2220	2220	2225	2245	2289	2267	2259	2274	2290	2269
ΔT_m,°F			27.7	13.6	11.7	10.2	8.4	7.3	6.6	6.1	6.1	5.9	5.2
h,Btu/hr·ft^2			39.8	81.0	94.2	108.0	131.3	151.0	167.0	180.8	183.8	187.0	212.0

TABLE B-VI

PHYSICAL PROPERTIES

Liquid	T, °F	$\nu \times 10^5$ ft^2/sec	k Btu/hr·ft·°F	C_p Btu/lb·°F	Pr
Water	80	0.919	0.352	0.9983	5.84
	100	0.738	0.361	0.9979	4.55
	120	0.607	0.370	0.9985	3.64
	140	0.511	0.376	0.9994	3.00
	160	0.439	0.382	1.0009	2.53
	180	0.381	0.387	1.0028	2.15
	200	0.339	0.391	1.0056	1.88
	212	0.317	0.394	1.0070	1.75
Glycol	100	9.67	0.1620	0.5887	95.0
	120	6.80	0.1575	0.6003	68.0
	140	5.10	0.1532	0.6120	51.0
	160	3.92	0.1486	0.6235	40.0
	180	3.05	0.1440	0.6353	32.2
	200	2.44	0.1395	0.6470	26.0

Water properties are taken from Grober (11). Glycol properties are taken from Union Carbide Company Booklet (16).

APPENDIX C

TYPICAL TEMPERATURE PROFILES

Plotted on the following pages are samples of the averaged temperature profiles, as listed in Appendix B. The wall thermocouples were numbered 1 through 9 starting at the top of the heated section. The liquid side thermocouples were numbered 1, 3, 5, 7 and 9. The thermocouples were then arranged so that those having the same number were opposite one another--i.e., located in the same horizontal plane.

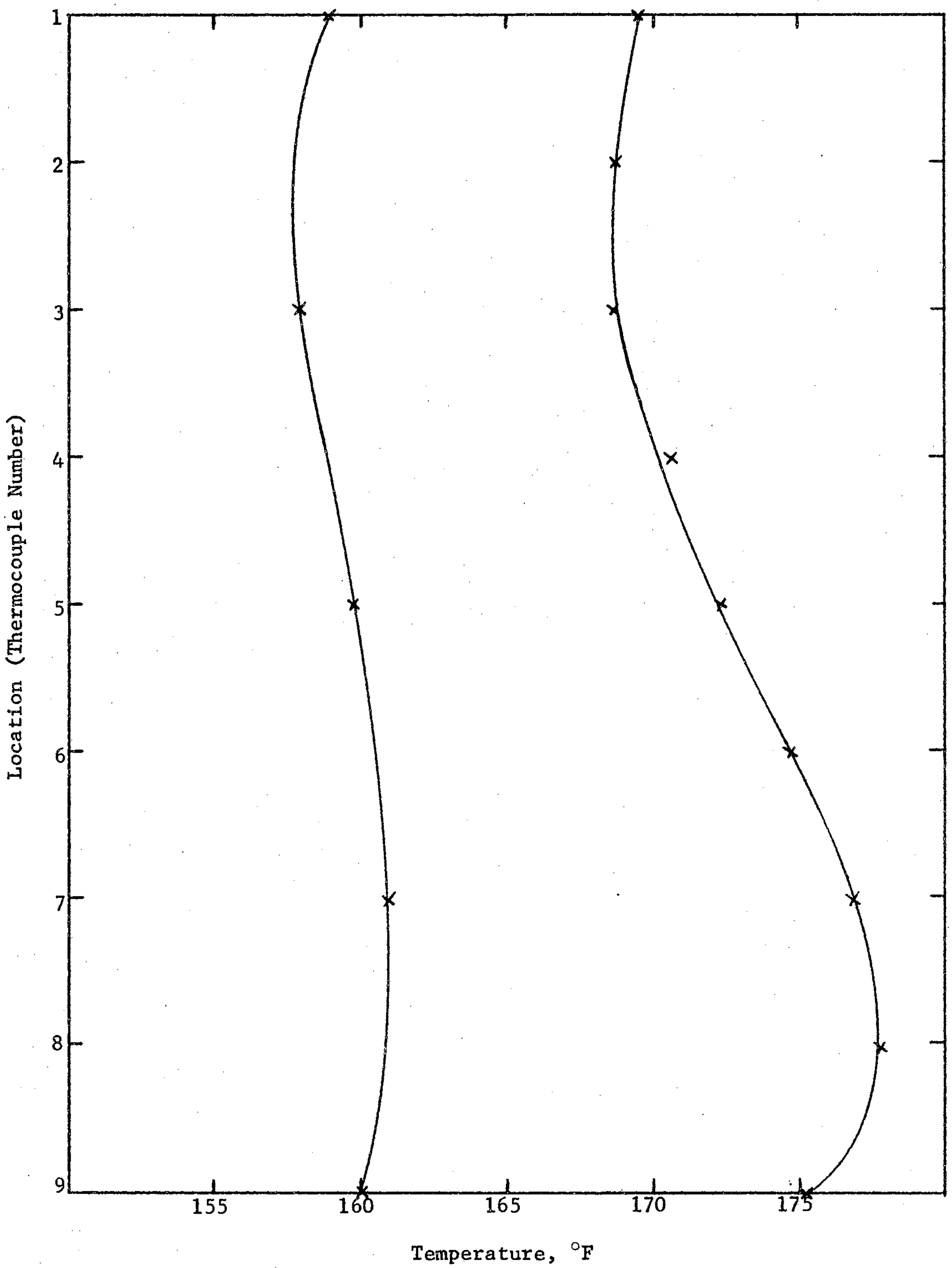

Figure C-1. Sample Profile from Run No. 1 with Water. U_s=0.0 ft/sec

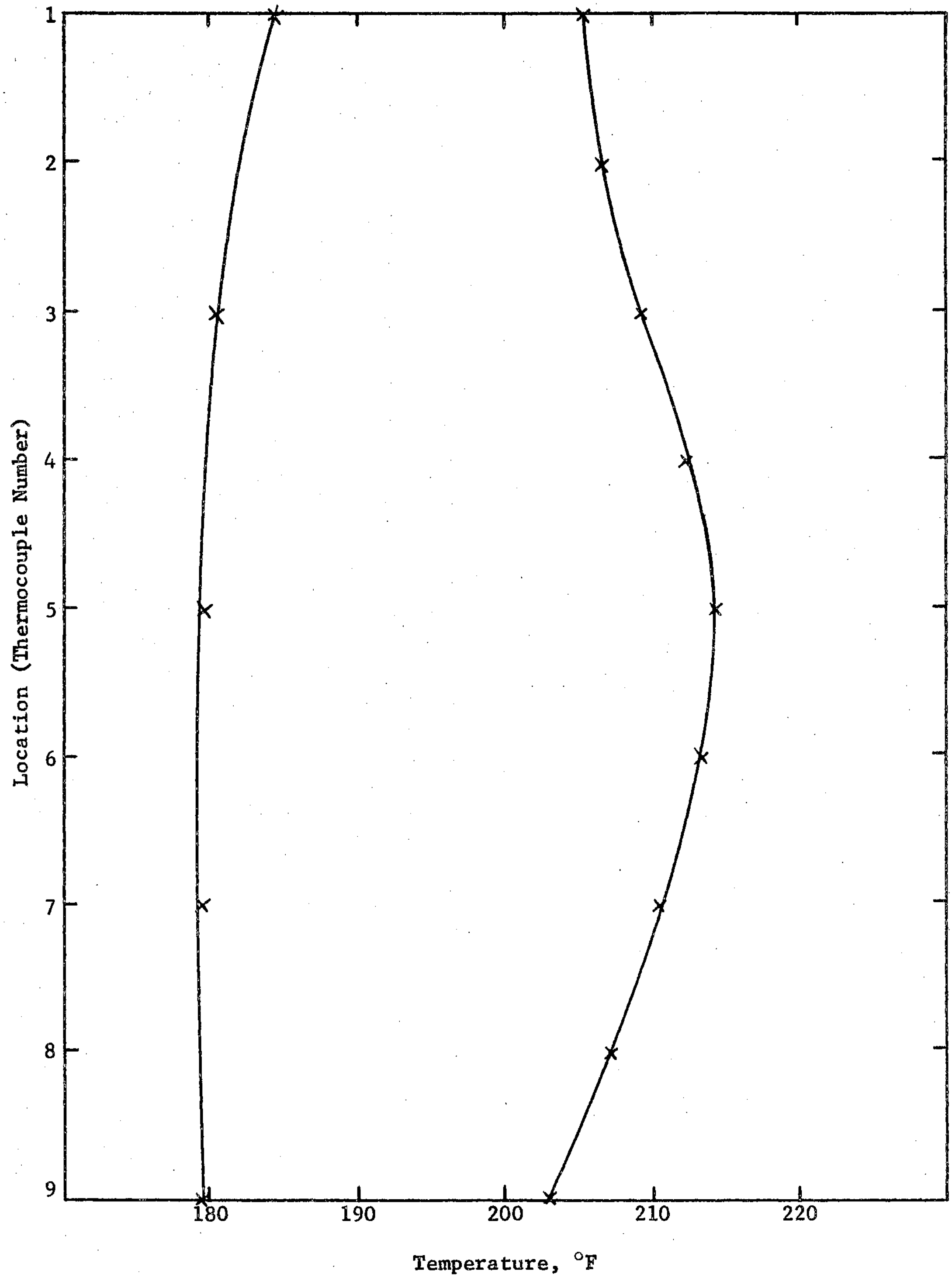

Figure C-2. Sample Profile from Run No. 5 with Glycol. U_s=0.0 ft/sec

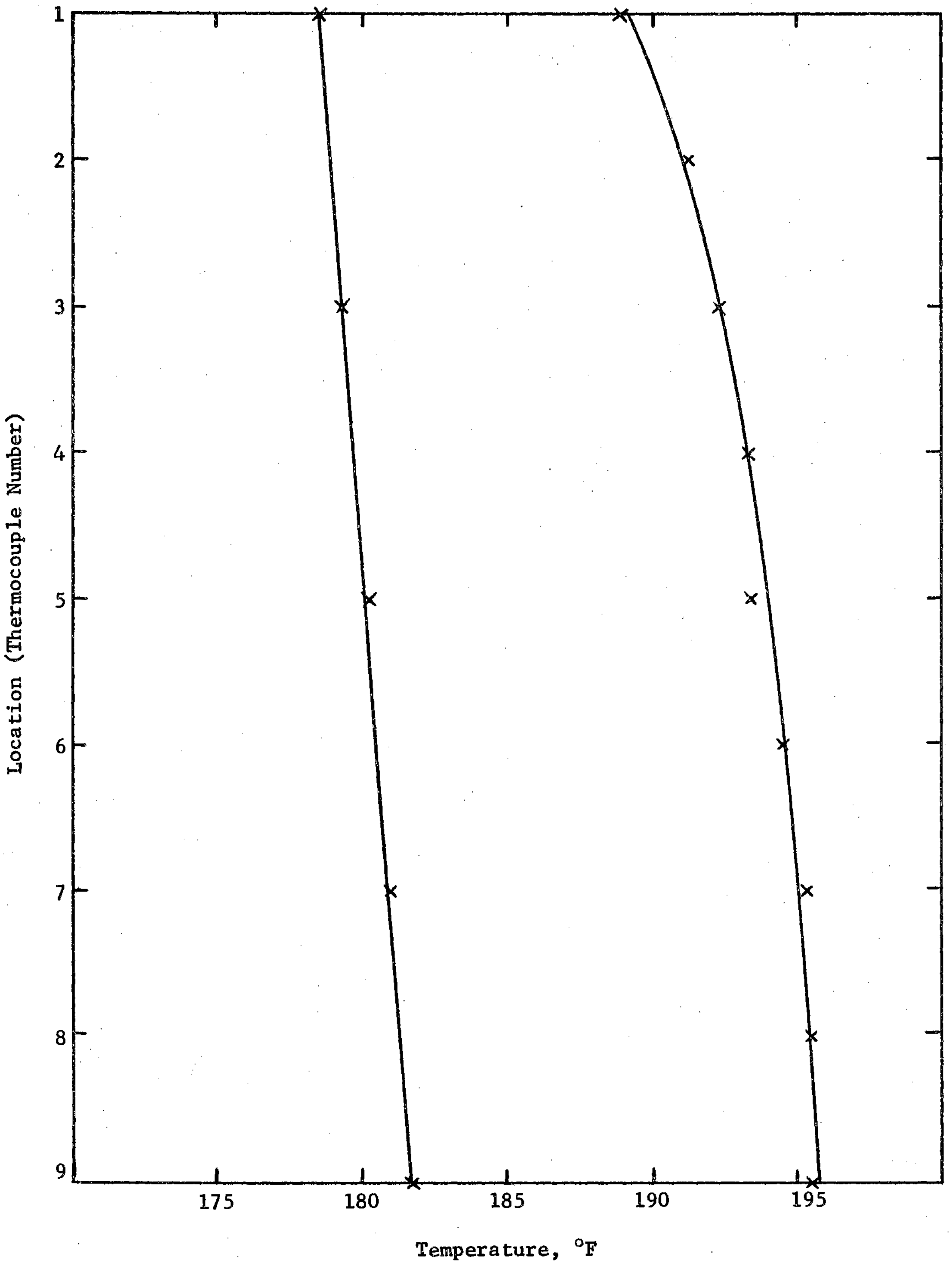

Figure C-3. Sample Profile from Run No. 4 with Glycol. U_s=0.00159 ft/sec

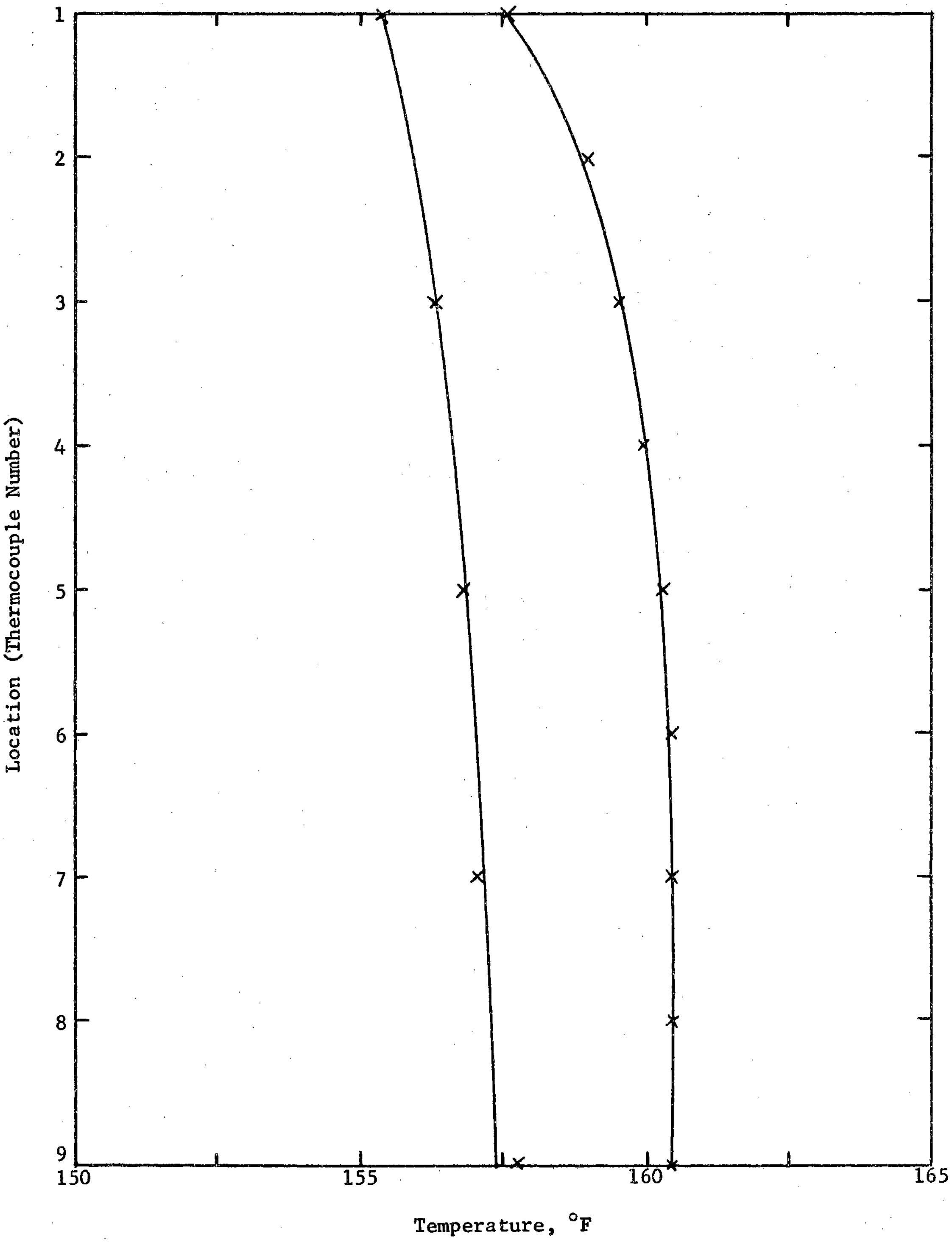

Figure C-4. Sample Profile from Run No. 2 with Water. U_s=0.0284 ft/sec

APPENDIX D

SAMPLE CALCULATION

The method of treatment of the data will be summarized by a sample calculation based on the data obtained in Run No. 2 for water at a superficial gas velocity of 0.0284 ft/sec. The time-smoothed temperature profile for these conditions is plotted in Figure C-4. The water flow rate was set at a constant value of 59.8 lbs/hr for the entire run. The calculation proceeds as follows:

water inlet temperature = 79.6°F
water outlet temperature = 157.8°F
average water temperature = 118.7°F
at 119°F, $C_p = 0.998$ Btu/lb.°F

heat input to water = (0.998)(59.8)(157.8-79.6)
= 4670 Btu/hr

air inlet temperature = 72°F
air outlet temperature = 147°F

From Table 15-1 of Perry's (14):

enthalpy of saturated air at 147°F = 252 Btu/lb
enthalpy of dry air at 72°F = 18 Btu/lb

air flow rate = 0.628 lbs/hr

heat input to air = (0.628)(252 - 18)
= 147 Btu/hr

then:

total heat input = Q = 4670 + 147
= 4817 Btu/hr

This same calculation was made at each gas velocity, and the average of all the Q's thus obtained was used to calculate the heat flux.

average Q = 4840 Btu/hr

inside diameter of copper pipe = 0.325 ft
length of copper pipe = 2.0 ft
heated area = $\pi(0.325)(2.0)$
= 2.042 ft^2

then:

average heat flux = q = 4840/2.042
q = 2370 Btu/hr·ft^2

Now, from Figure C-4, the ΔT is measured at the middle of the heated section (thermocouple location No. 5), and found to be:

$$\Delta T_M = 3.4°F$$

hence:

$$h = \frac{q}{\Delta T_M} = \frac{2370}{3.4}$$

$$= \underline{697 \text{ Btu/hr·ft}^2 °F}$$

Also from Figure C-2, the mean film temperature is estimated to be:

$$T_F \approx 159°F$$

The final reported value of h for water at a superficial gas velocity of 0.0284 ft/sec is the average of the values obtained in Runs 1, 2, and 3. By a procedure identical to the one just demonstrated it is found that:

from Run No. 1: h = 623 Btu/hr·ft^2°F

at T_F = 155°F

from Run No. 3: h = 774 Btu/hr·ft^2°F

at T_F = 163°F

Averaging the three values of h and T_F, we obtain the final result:

h = 698 Btu/hr·ft^2°F

at T_F = 159°F

and U_S = 0.0284 ft/sec

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