THE DEVELOPMENT OF A METHOD TO DESIGN SYSTEMS WHICH AUGMENT MAN'S ABILITIES TO CONTROL DYNAMICAL SYSTEMS

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CHAPTER I

INTRODUCTION

The continually advancing technology of modern control theory has created a wealth of possibilities for methods to analyze and design dynamical systems. A primary result of these advanced methods has been to permit complex and efficient dynamical systems to be analytically developed. Consequently, in recent years the performance and, thus, the requirements for performance of dynamical systems has expanded. An area which has been greatly affected by this expansion has been man-machine systems. As the machine performance has expanded, man, who still remains essential as the over-all controller, has become more and more a weak link in the dynamics of the man-machine system (1). The consequence of this weakened position created by man, the controller, has resulted in a considerable effort by system engineers to determine more quantitative descriptions of the behavior of humans in control tasks - descriptions which are compatible with conventional control system design techniques so that the effect of "man in the loop" might readily be evaluated and potential problem areas avoided (2).

The application of control theory to man-machine systems was begun during World War II in working with man as a gun operator in fast acting fire control systems and as a radar operator in systems which required a target to be tracked on a radar screen. At the conclusions of the War, much of this work was terminated and research was not begun again until the 1950's. Even then the work was somewhat limited, and it was not until the 1960's that a large scale effort was made in the man-machine field. Presently, the "state of the art" has evolved by virtue of the joint efforts by engineers and psychologists into the combined discipline of man-machine systems called manual control. The major by-product of these efforts in manual control has been the conceptualization of man and the machine which he controls as one over-all man-machine system. Figure 1 shows a model of this concept (3).

The model illustrated in Figure 1 has been used extensively in man-machine system research as the basis for identifying dynamical models that will produce a response identical to that of man (Note: Man will be used throughout this thesis to designate the human as a dynamical element which effects control upon dynamical controlled elements.) for a given stimulus and controlled element. Although these attempts to identify the dynamical model of man represent the major effort with man-machine systems, other equally significant research has been conducted and generally concerned with the following: display design, the effects of controlled element and environmental variation

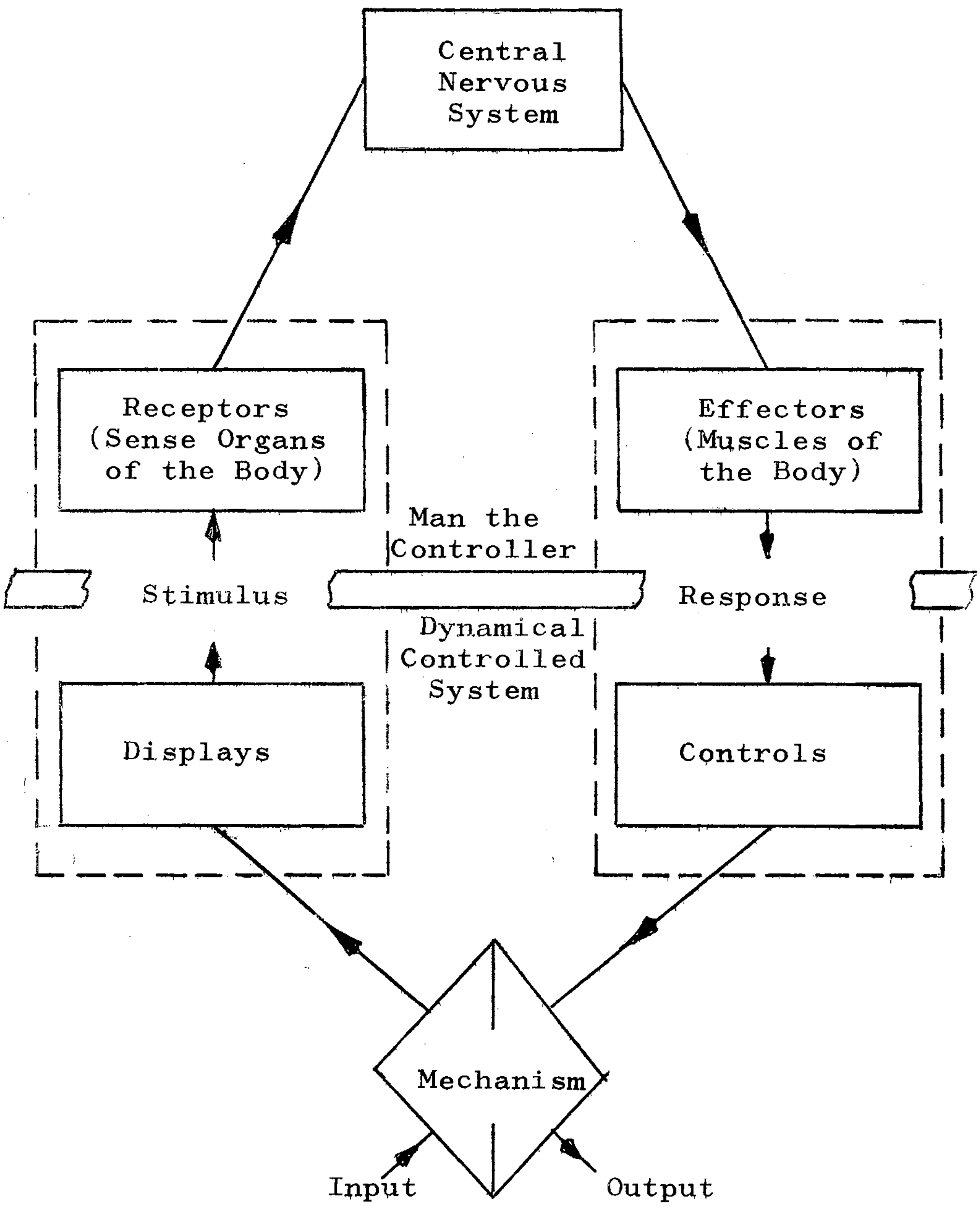


Figure 1. The Man-Machine System (3)

upon the man's dynamical model, human decision processes, and physiological modeling of the human (4).

Since man definitely poses a threat as the weak link in the man-machine system, it is obvious that his operating characteristics must be integrated by design into the total system before any type of optimum system performance can be realized. However, even with such an integration, the overall man-machine system performance can only approach the so-called optimum by constraining the machine dynamics and the system inputs to remain within the limited range imposed by man's ability to control. To have a truly effective man-machine system over a wide range of machine dynamics and system inputs, a substantial compensation must be made for man's inherent control limitations. If a system could be developed which extends the capability of man-machine systems through machine changes so as to compensate for the inherent human limitations, it would represent a definite contribution to the manual control field.

The purpose of this thesis is to propose and develop a design philosophy for a general system which can be used to augment man's ability as the controller of dynamical machines. The function of the augmentation is to supplement the perceptual capabilities of man the controller by providing him with sufficient, perceivable information such that he can generate an optimal control which causes the machine output to respond according to some predesignated criteria. The uniqueness of the proposed augmentation lies in its capability to:

1. assure that the total man-machine system will always have optimum performance according to some predesignated criteria,
2. permit man to control any machine no matter how dynamically complex it may be,
3. allow the system to be designed for any dynamical model of man,
4. recognize and compensate for the perceptual and dynamical limitations of man the controller.

Chapter II discusses how man functions as a controller and why he is a weak link in the man-machine system. The findings of Chapter II are used in Chapter III primarily to establish why it would be desirable to provide man the controller with further information through augmentation. Once this desirability is established, the past attempts to augment are reviewed and then the proposed concept for designing the augmenting system is presented. The next chapter, Chapter IV, is concerned with establishing a sound theoretical basis for the proposed design philosophy. The net result sought from this theoretical basis is to provide mathematical conditions which can be applied to the design of augmenting systems. Chapter V is an application of the design philosophy to a series of example tracking problems as found in the literature. The purpose of this chapter is to allow the reader to make an appraisal of the design philosophy. Finally, in Chapter VI, the author's conclusions regarding the design philosophy are presented and recommendations for future studies are made.

CHAPTER II

MAN THE CONTROLLER

In spite of man's inadequacy as a controller, he is a very complex and remarkable dynamical system. His powers of reason and adaptivity make him very versatile and, thus, a highly satisfactory candidate for augmentation. However, before any serious considerations can be made for augmenting him, it is first necessary to recognize how he functions as a controller and his limitations in doing such. By making this recognition, it is possible to determine more clearly why and then how he should be augmented, and, perhaps most important of all, to orient the readers' thinking towards man as something more than just a mathematically described control system.

How Man Functions as a Controller

For the purpose of illustration, man is viewed as a single channel, limited transmission capacity, information processing system. Although the single channel restriction is not entirely valid, it is used only to allow a perspective to be gained of how man functions as a controller.

Man as a control system can be broken into the three subsystems shown in Figure 2: input, central processor, and

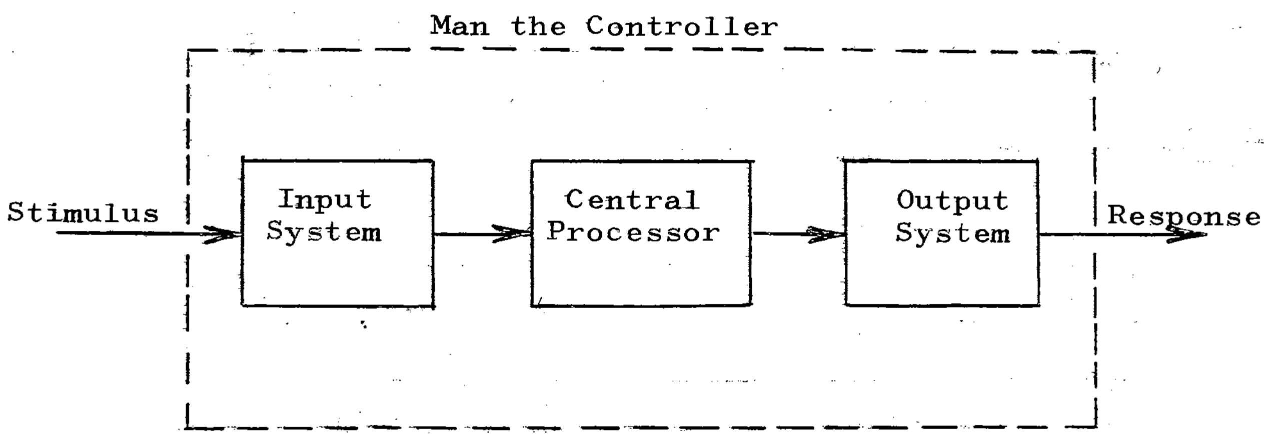


Figure 2. Simplified Block Diagram of How Man Functions as a Controller

output. Unfortunately, it is virtually impossible to isolate any one of these subsystems for individual scrutiny or even perform experiments which lead to valid inferences about their independence. However, this breakdown into subsystems does allow the important functional characteristics of the human as a controller to be examined. Each of these subsystems is examined and then an integration of them into man as a whole is considered.

Input System

The dynamic information used by man the controller is detected and measured by his senses. Although all of his senses could be directly or indirectly concerned with measuring this information, only four of them are sufficiently involved in the perception of it to warrant consideration. Specifically, these senses are: visual, auditory, cutaneous, and others which detect body position and movement.

Visual (5), (6), (7). The visual sense in the human body is naturally enough the eyes. The eyes function by taking a visual stimulus, such as a spot of light and pass it through the cornea and crystalline lens which serve to focus the light upon the retina. In turn, the retina is made up of photo receptors which through the absorbtion of light by pigment substances initiate a photochemical reaction which starts a chain of events which terminates in seeing.

The nature of man's eyes make them the most important sensual modality in aiding him to perceive information as a controller. This sense allows a much higher degree of dynamical information to be perceived than can be by the other senses. This higher perceptive ability is due primarily to the capability of vision to directly and accurately apprehend geometrical space as it extends outward from the confines of the body. Although man is unable to make an absolute measure of length, he can make a direct comparison with a reference or recall a reference from past experience. In making such references, he can often resolve gaps in angular distance on the order of two seconds of arc length (2). Additionally, at the visual level he can perceive the displacement and velocity of a transversely moving object.

Auditory (5), (7), (8). The sense receptors for the auditory modality are the ears. They function by virtue of the ear drum which transforms the stimulus energy of sound waves into mechanical motion which, in turn, is converted into fluid movement by mechanoreceptors within the ear. The pressure wave of this fluid movement then creates a traveling wave of displacement which causes receptor potentials to be generated which, in turn, trigger nerve impulses to the brain.

Since the ear functions upon sound waves, it is only logical that it is a remarkable high fidelity encoder of frequency-intensity-time patterns. However, these patterns represent a rather limited method of transmitting information when compared with the amount of information that can be transmitted visually. Still, for speech communication, frequency-intensity-time discrimination, read-in for non-directional warning signals, and as an input channel under conditions of limited visibility, the auditory modality is a definite aid in the perception of dynamical information.

Cutaneous (5), (7). The cutaneous sensual modality is the most extensive in the human body. However, it is also the most primitive. In general, the cutaneous modalities are temperature, touch, pain, and pressure; and while these modalities, or at least the receptors responsible for them, seem to be at the skin level, they are also widely distributed throughout the ligamentous structures of joints and the deep tissue planes. Thus, they not only account for sensing of external stimuli, but also for kinesthesis, i.e., body movement. In this latter role they are termed general proprioceptors and will be discussed in the next section.

The ability to sense temperature varies in different regions of the human body; however, in all parts cold is sensed more quickly than warmth. The factors affecting the stimulation are the absolute temperature beneath the skin, the rate of change of this temperature, and the total area of the surface stimulated.

The touch sensitivity of the skin varies in different parts of the body, as with temperature, and within any region there are specially sensitive areas known as touch spots. The regions of maximum touch sensitivity in man are his finger tips, lips, palm of hands, and tongue. The touch receptors, in general, are fast adapters to the stimulus.

Pain as a cutaneous sensation is somewhat different from the others in that it elicits a response by which man, or any other animal for that matter, deals with a harmful influence. The sensitivity to pain varies from individual-to-individual and, thus, is not a completely reliable stimulus.

The sensation of pressure is transmitted through the skin and, thus, affects a wide area of receptors. A valuable example of pressure sensation, although it is discontinuous, is vibration. Its value lies in its feasibility as a means of communication. For instance, man can look and hear, yet be unaware of noise, but he is almost always attentive to the stimulation of the skin. Of all the cutaneous sense modalities, vibration has the greatest number of dimensions suitable for use as items of a code delivering messages to the skin that are capable of some kind of interpretation.

Modalities for Body Position and Movement (5), (7), (9). There are two principal sets of sensual modalities which perceive body position and movement. Specifically, these modalities define functions which are described as kinesthesis and vestibular activity.

The proprioceptors of the body are responsible for the kinesthetic sensations. They are distributed widely throughout the ligamentous structures of joints and deep tissue planes. The receptors for sensing are of the same nature as those for the cutaneous sense, but the distribution is such that different groups of receptors are stimulated during various phases of movement. The immediate exciting stimulus is a compression of the receptors caused by tissue deformity during movement of a joint. During this movement, different populations of receptors will be stimulated, in turn, according to their position relative to the axis of movement. When a joint moves the rate of discharge of a group of proprioceptors increases according to the speed and degree of movement, and thereafter decreases to a steady state determined by final position. Some receptors have their maximum response at full flexion, others at full extension, or in some intermediary position.

The vestibular organs are the labyrinths which are made up of the semicircular canals and the cochleas of the inner ear. They record movements of the body as a whole relative to the environment, e.g., rotation or linear acceleration and movement of the head relative to the rest of the body. The semicircular canals are responsible for this perception while the spiral shape of the cochleas are thought to prevent movement of their fluids during angular and linear accelerations of the head.

There are three canals in the semicircular group, horizontal, superior, and posterior. Each is situated with reference to the three planes of the body; horizontal, vertical, and anteroposterior. Each has an expanded end, the ampulla, which opens into a common chamber, the utricle, which is connected indirectly with the saccule. The receptive structures are sensory cells which project into the ampulla of each canal. Each sensory cell has about 30 sensory hairs which project into narrow canals of gelatinous substance which is hinged within the ampulla. This gelatinous substance has mechanical properties that are equivalent to those of a spring loaded pendulum. The utricle and saccule also have sensory cells whose hair processes are in contact with a free floating gelatinous substance.

The stimulus for the semicircular canals is angular acceleration from the rotation of the head in the transverse, vertical, or anterposterior axis, i.e., tilting the head backwards, forwards, turning the head around, or tilting it from side-to-side. At the onset of acceleration, the fluid which fills the canals lags behind because of its inertias and so exerts a backward pressure on the hinged gelatinous substance in the ampulla causing it to swing in the opposite direction. This, in turn, causes tension and deformity in the hair processes of the sensory cells within the ampulla and, thus, creates the necessary signals to the brain. The hinged gelatinous substance has a natural period of about thirty seconds when it functions as a pendulum. Thus, when a rotation of the head is stopped, a backlog of the fluid within the canals will cause the hinged gelatinous substance to swing in the opposite direction and it will be up to thirty seconds before it returns to rest position. This is the reason for the commonly experienced sensation of rotating in the opposite direction after one turns his body round and round while standing. The persistence of a sensation with a constant velocity of rotation is due to the signals from visual and other senses.

The utricle and saccule, also a part of the vestibular organs in the ear, account for the awareness of head position when there is no movement. In this case, the free floating gelatinous substance mentioned earlier stimulates sensory cells as it obeys the laws of gravity. A tilting of the head in any direction alters the gravity pressure of this substance on both sides of the head. In addition, this substance responds to movements in linear acceleration, centrifugal and coriolis forces.

Central Processing

The central processor in man is obviously the brain. The brain is a very complex entity which has capabilities that man does not even utilize when he functions as a controller. However, assuming that man can isolate and utilize only the portion of his brain necessary for him to be a controller, his central processor can be compared quite favorably to the central processing and memory units in a modern digital computer. He has the ability to place input information in either short or long term memory and to use it as a basis for recalling past information and possibly comparing the present information with the past.

Additionally, he has the ability to logically and algebraically manipulate this information, present and past, to formulate decisions that reflect his control strategy. Although it has not been possible to break the human brain down into subcomponents for individual scrutiny and classification, the control capabilities of man as evidenced by his central processor have caused a great deal of effort to be expended in trying to mathematically model man the controller. In short, practically every type of controller available in control theory has been utilized to try to mathematically describe man. These efforts should be sufficient evidence to indicate that the human brain has a very formidable capability.

Output System

The output system is probably the most difficult to discuss in isolation. Any human output depends upon the operation of some part of man's musculature. The muscle system involves feedback at several levels, and all but the lowest interact with the central processing system. Thus, any response, whether it involves the speech musculature or an arm or foot output, is intimately tied to the operation of the system as a whole.

At the muscular level in man, the operation can be compared to a servo mechanism. A command, from the brain, is sent to the muscle of interest and an appropriate response is made. The response may be in the form of muscle position or time derivatives thereof, or it may be in the form of muscle force. Whatever the case may be, information regarding the response is fed back to the brain where the error between the desired output and the muscle response is computed and used as the basis for sending a new signal to the muscle. This process is continued until the muscle completes the desired output.

Integrated System Output

The discussions of the preceding sections have described some of the functional aspects of man the controller. Figure 3 is a diagram which has combined these functional characteristics. As an example of how these functions are integrated and allow man to control, consider the following discussion which depicts man as pilot who is presented with a disturbance only in the pitch axis and is required to make the necessary corrective maneuvers (10). Once a pitch deviation is detected, the perceived information is transmitted to the brain, which decides what kind of corrective maneuver should be applied to the controls to offset the disturbance. The brain sends the appropriate command to the motor elements of the body (muscle and supporting skeletal parts). The signal flow from sensors through the central nervous system to the motor inputs require a measurable time, the average length of which is found to be approximately constant in all normal persons. This is called the reaction time. Following perception of

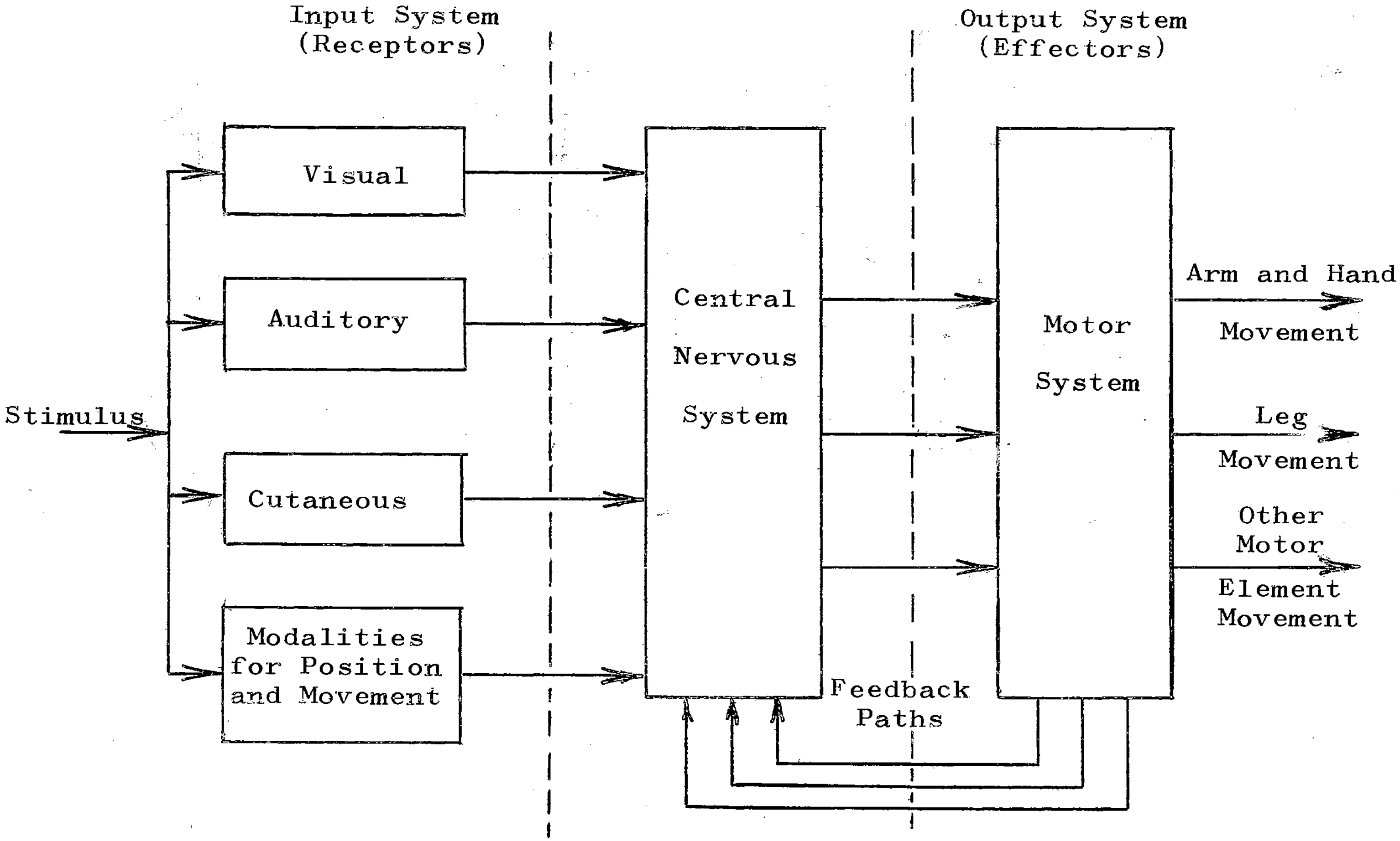


Figure 3. Integrated Block Diagram of How Man Functions as a Controller

the stimulus, the human controller apparently performs two essentially linear operations: (1) a mental computation of the stimulus, or the weighing and summing of position, rate, and acceleration to achieve a basis for the decision to act and (2) an involuntary placing of definite physical limitations on corrective hand motions, which is caused by a neuromuscular, or motor, feedback loop. Although seldom, if ever observed without a reaction time delay, mental computations do not require any conscious calculation by the pilot. The weighing, summing, and decision functions are similar to those used by the rope walker in balancing. The type of data used in mental computation can be identified with the stimulus receptors involved. The movement of the airplane with respect to the horizon is transmitted by the eyes as a signal proportional to the amplitude of the airplane displacement. Rate-of-movement information is sensed by the ear canals (viscous flow of the endolymph fluid) and the eyes (through peripheral vision). Finally, linear and angular acceleration stimulates the vestibular organs of the inner ear and the proprioceptors of body muscles, tendons, and joints.

The discussions of this section have been rather brief and, thus, somewhat limited in their segmented and integrated descriptions of man. However, as was indicated earlier, the purpose of these discussions has been only to provide some insight into how man functions as a controller and, thus, allow an understanding of the arguments to be made in the next sections regarding his inadequacy and resultant need for augmentation.

Man's Limitations as a Controller

As was stressed in the introduction, man is limited as a controller. The developments of this section begin by listing the limitations of man the controller and then close by comparing him to man as he might appear if he were an ideal optimal controller. By making such a comparison, it is possible to show why man's limitations suggest the need for augmentation.

Perceptual Limitations

Any dynamical information that man may perceive is uniquely related to a specific sensual modality. Considering only the direct perception of this information, the sensual modalities and their associated dynamical variables can be summarized as follows. At the visual level, man is limited to the perception of displacement and velocity of an external and referenced object relative to the displacement and velocity of his eyes. Acceleration and higher derivatives for such objects are not formed at the visual level. At the auditory level, man is limited to perceiving dynamical information through sound intensity, frequency, and time patterns. However, the total amount of usable dynamical information per unit time that can be perceived through this modality is very small, especially when compared to the capabilities of the visual sense. At the cutaneous level, it is difficult to assess what dynamical information is available. Certainly temperature could be perceived dynamically; however, the speed of response of the temperature senses is relatively slow. Probably the most significant method of perceiving information with the cutaneous modality is through force feedback into the pressure modalities. However, the information perceivable per unit time with the pressure modalities is much less than with hearing. At the proprioceptive and vestibular levels, the dynamical information perceived is the position and movement of the different parts of the body for the former, and the rotation or linear acceleration and movement of the head relative to the rest of the body for the latter. Unfortunately, the information perceived by the proprioceptive and vestibular senses when man is in an unnatural environment (e.g., an airplane) is often invalid and, thus, very misleading.

Central Processing Limitations

Although man certainly has limitations upon his central processing system, it is very difficult to assess them. The reason for this difficulty lies in the nature of man which makes it impossible to isolate his brain and accurately determine its capabilities. Presently, the most valid inferences about man's computational and memory capabilities have been made with regard to his total information capacity. This capacity is determined empirically by use of communications theory and is, thus, beyond the scope of this thesis. However, it will suffice to say that the reliability of man's central processing capabilities is directly related to the amount of information that must be processed.

Output Limitations

The output system of man is probably the easiest to analyze from the standpoint of limitations. The outer extremities are generally used to effect any prescribed control. These extremities can be modeled by the common lumped parameter approach and, thus, lend themselves to analysis. Unfortunately, the nature of this extremities is such that they are mechanical systems which are primarily mass with very little damping or spring resistance available (6). This mechanical nature causes the output to be limited to relatively slow response when compared to the high speed nature of the brain and central nervous system. Although the slow response of these extremities poses a definite limitation to man's ability to control, it is assumed of secondary importance in the exposition of an optimal controller. However, it is important to point out that these output limitations are accounted for in the later proposed augmentation technique.

Man as an Optimal Controller

Since man functions as a dynamical controller, the most logical measure for his dynamical capability to control is optimality. If he can generate a control which can be termed optimal according to some performance criteria, then it follows that he is sufficient as a controller. On the other hand, if his control deviates significantly from the optimal, then his sufficiency as a controller must obviously be questioned. In support of the premise that man can be judged according to his optimality, several investigators have presented evidence which indicates that man the controller strives toward a self-optimization which leads his control to be near optimal subject to inherent limitations (11), (12). However, these limitations impose sufficient restrictions upon man to cause his optimal and sub-optimal abilities to control to diverge to "not-so-optimal" to unstable for dynamical devices whose complexities exceed the relatively simple.

As a further illustration of the comparison between man and man the optimal controller, consider man involved in a simple tracking task. Figure 4 shows a scalar block diagram of man as the controller of a dynamical system for which the output, z(t), is required to track a reference input, R(t). The difference between the input and output is presented visually to man as an error, E(t), which is to be minimized. Man takes visually perceived information such as E(t) and $\dot{E}(t)$ and then utilizes these two pieces of information as the basis for formulating his control strategy.

In comparison, Figure 5 illustrates the same dynamical controlled element as was shown in Figure 4. However, the

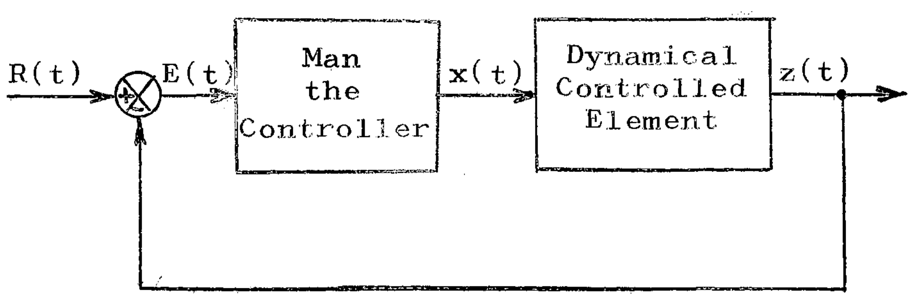


Figure 4. Scalar Block Diagram of Man as the Controller of a Tracking System

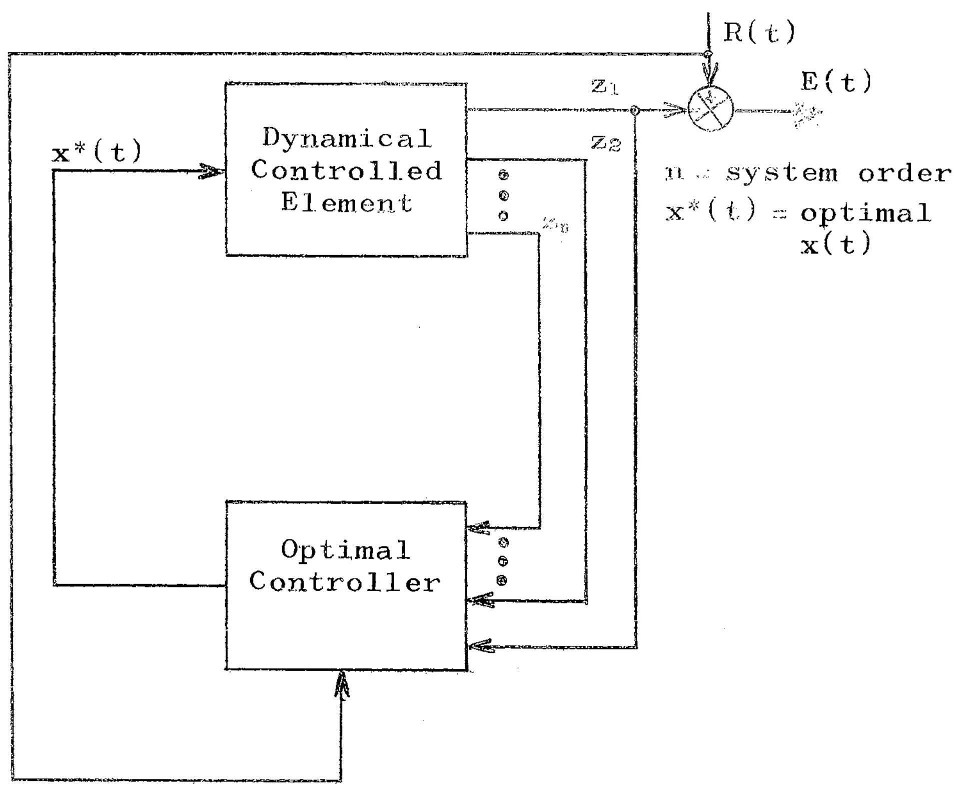


Figure 5. Block Diagram of Optimally Controlled Tracking Problem (Man Excluded)

operator has now been excluded from the diagram and the configuration for the system error has been rearranged. This diagram is used in reference (17) to describe the tracking problem for which an optimal control can be found which minimizes the system error, $E(t)$, i.e., just as man the controller strives to do. The optimal control for this tracking problem is found through use of existing optimal control methods, e.g., Pontryagin's Minimum Principle. But, in determining the control one finds that knowledge of all the dynamical controlled element states is necessary to the determination of optimal control. Figure 5 illustrates how an optimal controller might function; note that all of the dynamical controlled element states are fed into the controller.

In contrast to the optimal controller, man the optimal controller as shown in Figure 6 is limited in his formulation of optimal control. This limitation is due primarily to his lack of knowledge concerning the states of the dynamical element to be controlled. In effect, he only has indirect knowledge of two system states which he gains through his perception of $E(t)$ and $\dot{E}(t)$ (i.e., $E = R - z$ and $\dot{E} = \dot{R} - \dot{z}$, where z and $\dot{z}$ are two state variables of the dynamical element to be controlled). Additionally, if he is physically participating in the system output, $z(t)$, or the error, $E(t)$, then he might be able to perceive one more variable, $\ddot{E}(t)$, by virtue of his vestibular sense. If it can be assumed that existing optimal analysis methods are

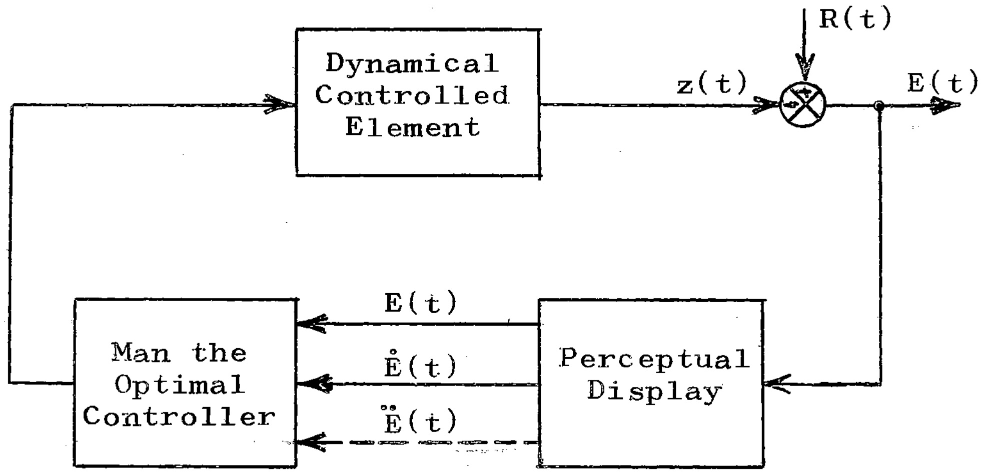


Figure 6. Block Diagram of Man as He Could be Construed to be an Optimal Controller

indicative of what information is required to specify optimal control, then it can be concluded that man is only able at best to formulate sub-optimal control because of his perceptual limitations. Thus, he is limited to the formulation of a control based upon the knowledge of only the three error states just mentioned. If the dynamical state of the system being controlled cannot be described by these variables, then he does not have total information.

CHAPTER III

FUNDAMENTAL DEVELOPMENT

The important conclusion reached in the preceding chapter was that man is limited as a controller. The logical deduction to be made from this conclusion is that man's ability to control must be augmented before he can become optimal. The pursuit of a design philosophy for a system which augments man is the purpose of this thesis. In keeping with this purpose, the objectives of this chapter are to: first, establish what characteristics are desirable in a system which augments man and can be specified as requirements for a design philosophy; next, review and discuss past attempts to design augmenting systems; and finally, present the concept to be pursued in the development of a design philosophy for a system which augments man's ability to control.

Statement of the Problem

The prime motivation behind arriving at the need for a system to augment man has been the failure of his control capabilities to compare to those of an optimal controller. It is logical to expect that a system which has the capability to augment man's control should necessarily be able to make the "augmented man" perform as an optimal controller (i.e., given a dynamical machine to control). Thus, the primary characteristic required for the augmenting system will be that it be able to assure that the total man-machine system will always have optimum performance according to some predesignated criteria.

In the earlier discussion upon the sufficiency of man the controller, it was implied that even if man had the perceptual capability to detect the necessary information to formulate an optimal control, he would lack the ability to formulate it. He quite probably would be either unable to adequately handle the required amount of information flow from his sensors, or upon receiving the information at the higher centers of his brain be unable to process all of the information which would be required to compute an optimal control strategy, or both. Since the quantity of state information required for formulation of optimal control is dependent upon the dynamical order of the machine to be controlled, it is apparent that an augmenting system should make man somewhat independent of the machine to be controlled. In other words, man should be able to control any dynamical machine without any unrealizable demands being made upon his computational ability. Stating this as a required characteristic, the augmenting system should be such that it will permit man to control any dynamical machine no matter how dynamically simple or complex it may be.

Another equally important point not mentioned earlier is that man is subject to various degrees of mental and physical taxation depending upon the dynamical complexity of the machine which he is controlling. For example, if he is controlling a machine which requires either a considerable mental or physical effort or both, then he tires very easily. The duration of his ability to produce "good" control is limited; and if it is over-extended, he then begins to perform quite poorly. Conversely, if the machine to be controlled is very simple, then man might easily be bored and become dissatisfied with his task. Thus, another required characteristic for the augmenting system is that it freely allow the designer to permit man to be as dynamically simple or complex as the situation requiring augmentation might warrant.

The next characteristic to be required is that the augmenting system be subject to analytical design. Although this requirement is not a direct benefit to man the controller, it does greatly influence the ease with which an augmenting system can be devised. Additionally, an analytical design technique is more likely to be a direct design route to the "best" augmenting system than the obvious alternative, the trial and error approach.

The final characteristic to be required of the augmenting system is that it increases man's ability to control by recognizing and compensating for his perceptual and dynamical limitations. This requirement is somewhat apart from the others in that it could easily be interpreted as the definition of the purpose for a system which augments man. However, its need is mandatory. If the perceptual and dynamical limitations of man are not compensated, then the initial purpose for augmenting man is defeated. Additionally, this compensation must be had without affecting the assumed limitations on his behavior.

In summary, the characteristics are such that they will require the augmenting system to:

1. Assure that the total man-machine system will always have optimum performance according to some predesignated criteria.
2. Permit man to control any machine no matter how dynamically complex it may be.
3. Allow the system to be designed for any dynamical model of man.
4. Be subject to analytical design.
5. Extend the control capabilities of man by recognizing and compensating for his perceptual and dynamical limitations.

These requirements will be used as design goals for the augmenting system to be proposed later in this chapter. However, before this proposal can be made, it is first beneficial to consider and discuss the attempts that have been made in the past to develop systems which extend man's capabilities to control.

Past Attempts to Extend the Capabilities of Man the Controller

Aided Control

Aided control is a method whereby man the controller can control complex machine outputs through simple responses. The main application of aided control is in tracking systems (Figure 7) where the system input (desired output) is a constant rate (ramp), constant acceleration, or some constant higher derivative term. For more simple or more complex inputs than these, aided control in contrast to unaided control generally results in poorer system performance and/or requires an increased number of control responses from the operator (3, 19, 20).

Figure 8 illustrates how an aided system functions. In the scalar block diagram of Figure 8, acceleration aided control is obtained by feeding all controlled element dynamical variables forward and algebraically summing them with the controlled element output. Aided control for dynamical machines of a lesser or higher order is obtained by feeding the machine variables forward in a similar manner. The "rule of thumb" for aiding is to feed forward variables until the number of terms fed forward exceeds by one the derivative of the input which is constant. Thus, for a step input, the number of terms fed forward should be two. The key to successful aided control lies in the proper selection of the aiding constants, i.e., K_1, K_2, and K_3 of Figure 8.

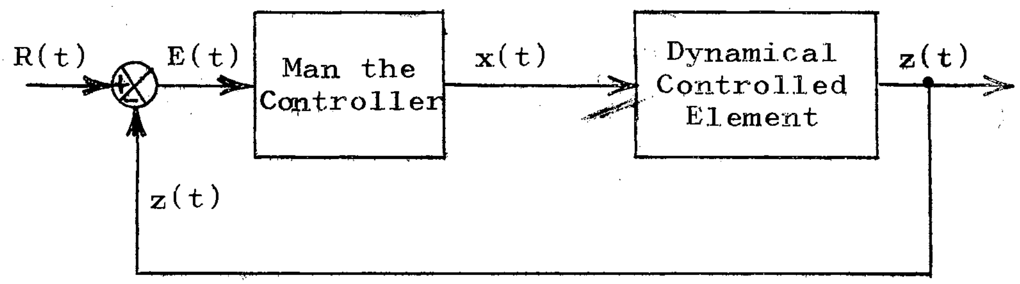


Figure 7. Scalar Block Diagram of Man as the Controller of a Tracking Problem

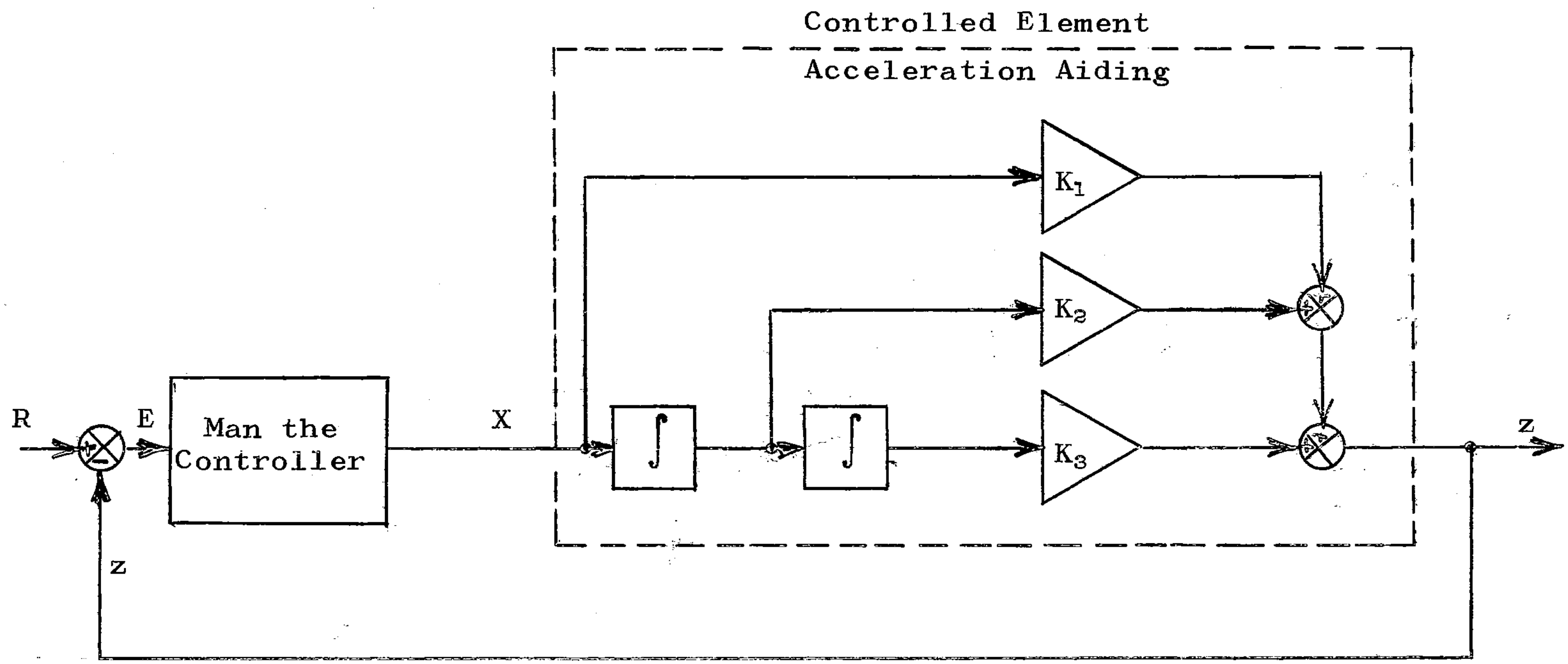


Figure 8. Block Diagram of Man the Controller Tracking With Acceleration-Aided Control (19)

Unfortunately, these constants must be confirmed experimentally for each specific situation.

Aided control has met with some success with gun fire control devices under the name of "rate aiding". A similar applicable area is radar tracking systems wherein man is required to track airplanes which are moving at a more or less constant velocity. However, with the exception of the very specific areas of application mentioned in the above discussion, aided control is not a desirable means for extending man's capability to control. If the "constant" input requirements for the aided system are not met, then aided control becomes a hindrance rather than a help.

Quickening

Quickening, unlike aiding, does not affect the system output but only changes the information displayed to man the controller. A quickened display is an attempt to simplify the man's task by providing him with a single display requiring a minimum of mental computation on his part to achieve a desired output (3, 8, 19, 20). In effect, the quickened display is supposed to tell man where to position his control. A scalar block diagram of a system which illustrates quickening is shown in Figure 9. Note that quickening is provided by algebraically summing the controlled element state variables together, once they have been acted upon by an appropriate gain, and then subtracting the sum from the system input. Once again, man is required

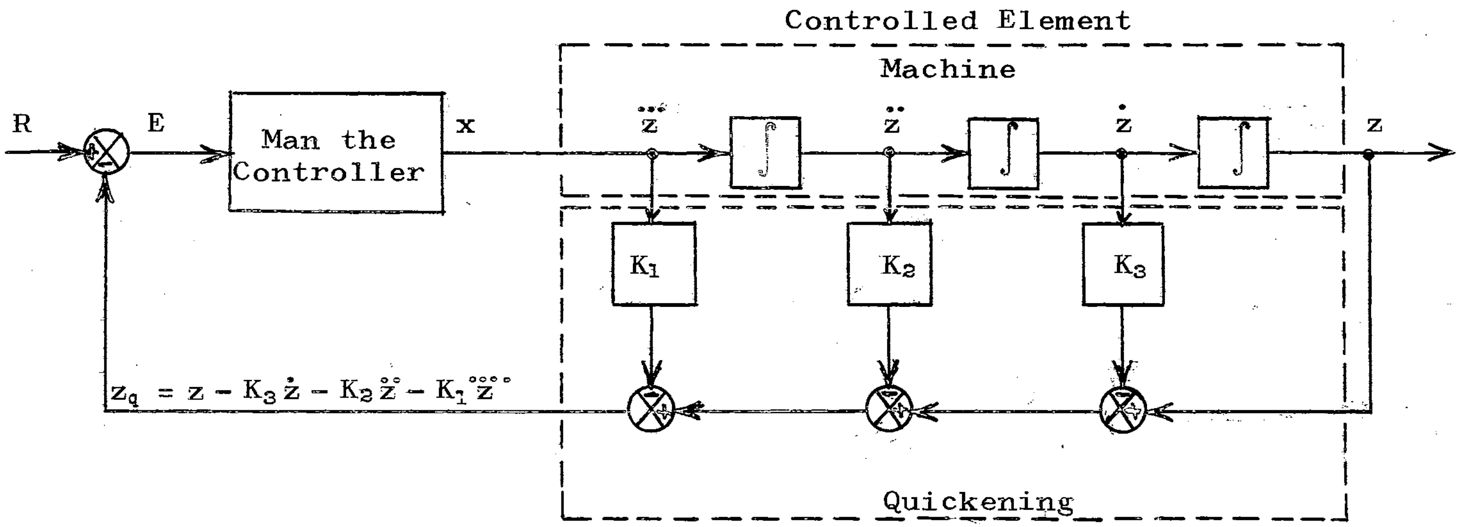


Figure 9. Scalar Block Diagram of Man the Controller Tracking With a Quickened Display (19)

to minimize the resulting error, but this time the error represents the difference between the system input and the quickened information rather than the difference between the system input and the controlled element output.

Ideally, quickening should be a very useful method for extending man's ability to control since it does in effect tell him where to position his control. If man the controller (without quickening) were to formulate the same information provided him by quickening, he would first have to have all the system state variables displayed to him and then he would have to mentally sum and weigh them according to what he feels is the best control. Consequently, the net result without quickening would be a poorer system performance primarily because of the allocation of attention that man would be required to give to perceive the unquickened information and secondly, because of the dynamical complexity with which he must operate to make use of the unquickened information.

In the laboratory situation, i.e., using analog simulators for the machine to be controlled, man has been found to give better system tracking performance with the quickened display than without it. However, the practical aspects of the quickened display are somewhat dubious. The success of the quickened display is completely dependent upon the proper selection of the weighing constants (i.e., K_1, K_2, and K_3 as shown in Figure 9). If these constants are not properly selected, then the man-machine system can easily be uncontrollable. Unfortunately, there is no simple, analytical method for selecting these constants. They must be determined empirically. In a realistic situation, e.g., piloting an airplane, there is no time for adjusting constants until the best system performance is obtained. If quickening is to be of any practical use, the display constants must be known a priori.

Optimum Filtering and Prediction

Wierwille (21) proposes a method for applying optimum filtering to man-machine tracking control systems so as to improve their performance in tracking random waveforms. The fundamental idea contained in his presentation is that of using optimum prediction and filtering to overcome time lags, rise times, and non-minimum phase characteristics in man-machine systems. Two approaches are taken. The first speculates upon a system that incorporates optimum prediction in a way which utilizes the higher-order functioning of man the controller by making available to him as much information as possible about the input waveform. This is to be accomplished by displaying the predicted input waveforms and then allowing the man to decide how he should use this information for enhancing his own tracking abilities. A block of a system suggested for this approach is shown in Figure 10. The second approach considers a technique which would maximize man's tracking ability by augmenting the display system. The fundamental proposition of this approach is to apply empirical optimization techniques to the design

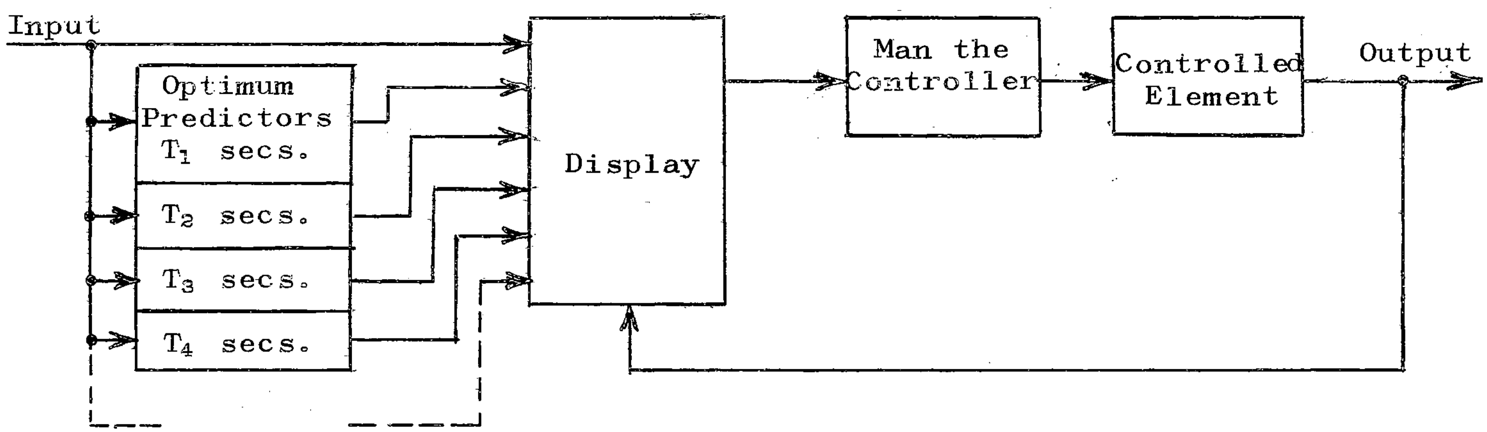


Figure 10. Block Diagram of Tracking System Using Prediction to Enhance Man the Controller's Tracking Abilities (23)

of a human augmenting display without entering the man-machine control loop and, thus, affecting man's variability. A series compensator which is proposed to accomplish the above proposition is illustrated in the block diagram of Figure 11. The compensator is in effect a filter which is to be experimentally adjusted for a test input until optimum performance is obtained; then, it is assumed that man the controller will no longer be subject to the taxing problems of variability when the actual system input is encountered.

Although both of the approaches to improving the man's tracking performance presented by Wierwille were not evaluated by him in any manner, they both appear to have definite possibilities for application. However, the application would only be helpful to man-machine systems wherein random appearing forcing function variations could be established as the main source of the man's control problems. An equally, if not more, important problem area is with the machine itself. Too many machines require control which exceeds the capabilities of man, regardless of the system input. Also, even with the aid that might be gained from optimum filtering and prediction, there are still machines for which man can provide acceptable control. However, in providing this control, he is greatly overtaxed both mentally and physically because of the amount of information which he is required to process and for which he is required to effect control.

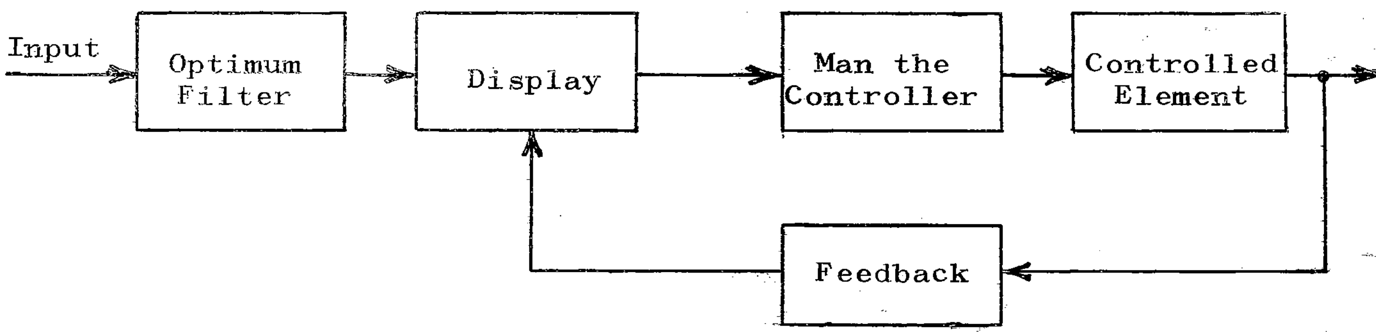


Figure 11. Block Diagram of a Feasible Optimization Configuration (23)

An Optimal Control Method for Predicting Control Characteristics and Display Requirements

Elkind (22) presents an optimal control method for predicting information display requirements, man's control characteristics, and man's instrument monitoring behavior in realistically complex aircraft systems. Primarily, the method evolves from the use of optimal control analysis to mathematically develop the general theory for an optimal feedback controller which is to parallel the operative functions of man the controller as he controls linear dynamical machines. The development is based upon: (1) the use of existing linear models of man as a controller and as an instrument monitor, (2) the assumption that man behaves in a manner which to him appears optimal, and (3) presuming that (2) is valid, the use of a quadratic cost functional which is subjectively theorized to be indicative of what man would minimize. Their purpose of developing the general theory for an optimal feedback controller, considering only the prediction of information display requirements, is to permit a sensitivity analysis of the cost functional to determine what effect a variation of the feedback gains determined from the optimal analysis will have. Since each optimal feedback gain is associated with a state variable, the importance of displaying a state variable to man can be determined from the gain sensitivity. Thus, if the cost functional is not very sensitive to changes in certain gains, then it is postulated that it is not very important to display the associated state variable to man.

Elkind applies his method to the relatively simple example of a linearized dynamical model of an XV-5A VSTOL vehicle in the hover mode with longitudinal motion only. The application indicated, with acceptable accuracy, what state variables should be exhibited to man the controller. By displaying this needed state information to man, Elkind provided him with the total amount of information that man appeared to need to formulate optimal control. The example resulted in showing that man was able to formulate a more optimal control with more state information than he was able to without it.

Elkind's work (22) makes the most optimum use of man by providing him with whatever state information he is found to need. However, man still has to perceive and process this information and then formulate whatever control he deems to be optimal. Thus, the total system is still limited to be no more optimal than man himself is. No augmentation is included to compensate for man's inability to be anything but suboptimal for many different controlled elements. Additionally, the optimization of man that is evident in Elkind's work may assure that the most effective use is made of man's control formulating capability, but there is no allowance made for the fact that he cannot physically maintain high degrees of effectiveness over long periods of time. The more complex the controlled element, the shorter is the length of time for which he can maintain this effectiveness. The method presented in this thesis not only assures that the total man-machine system will be optimal, but also makes a provision to allow man to operate at various degrees of effectiveness.

Discussion

In view of the original problem statement, the above works can now be summarized. The method of aided control is highly empirical in nature and is limited to a very narrow range of system inputs and does not have any provisions for an optimal system output (3, 19, 20). Quickening is somewhat similar to the proposed work in the summing and feedback of the system variables, but in actuality it is an empirically based method that requires gains to be adjusted until the man-machine system output is "better" (not optimal) than it was without quickening (3, 8, 19, 20). The optimum filtering and prediction of Wierwille (21) incorporates prediction into the display using the reasoning that man needs a prediction of what the system input is. Additionally, he proposes putting a filter on the system input and adjusting the filter until man's performance is optimized. However, both of these approaches require empirical adjustment and give no assurance of optimality. The work of Elkind (22) is a successful attempt to determine what individual state information man must have displayed to him. However, even with this more complete display of information, the man-machine system is still limited to being no more optimal than man himself is.

Proposed Concept

Ideally, an augmenting system should formulate man's control strategy for him and then tell him what to do. Such an augmenting system is proposed for this study. First, it is necessary to establish a major, yet fundamental assumption. This assumption is that a stationary, linear dynamical model exists for both man and the machine. The primary reasoning behind making the model linear is to facilitate the development of the proposed concept. The specification for stationarity may seem somewhat questionable in view of the fact that man is known to have a variable dynamical model which he can adjust to let him control many different dynamical elements. However, man is known to achieve model stationarity as he becomes experienced with a specific control task. In the event that man does begin to vary his dynamical model in a control task, the variation will appear in the system error which, in turn, is displayed to him. Since he is an optimizing controller, he will attempt to minimize the error and, thus, return to his origianl dynamical model (i.e., the model for which the stationarity was implied). It is conceivable that man could introduce sufficient variation into his dynamical model to cause the augmented system to function non-optimally or perhaps go unstable. If this were to happen, it would be necessary to include an updating device in the augmenting system. Such a device would continuously monitor the controller's dynamical model and account for any model variation by adjustments within the augmenting system.

A configuration which illustrates the proposed concept in the form of an augmented system is shown in Figure 12. This illustration shows the augmenting system as a device which formulates the control necessary for man to make the machine perform optimally. Once formulated, the control is compared to the man's output and the resulting error is presented as his input. Thus, he need only minimize the error by duplicating the control output of the augmenting system. He is no longer burdened with the tasks of perceiving controlled element state information and formulating the control strategy.

The uniqueness and advantages of this augmenting system lie in its capability to be analytically designed by means of optimal control analysis techniques. Furthermore, the augmented man-machine system is assured of optimal output for any specific set of machine dynamics. An existing or even hypothetical model of man can be used in the design of the augmenting system. With this capability, man can be very simple dynamically, yet control a very complex machine. Also, by specifying man's dynamical model, the augmenting system is automatically designed to compensate for whatever perceptual and dynamical limitations that the model describes the man as having. However, even with all of the

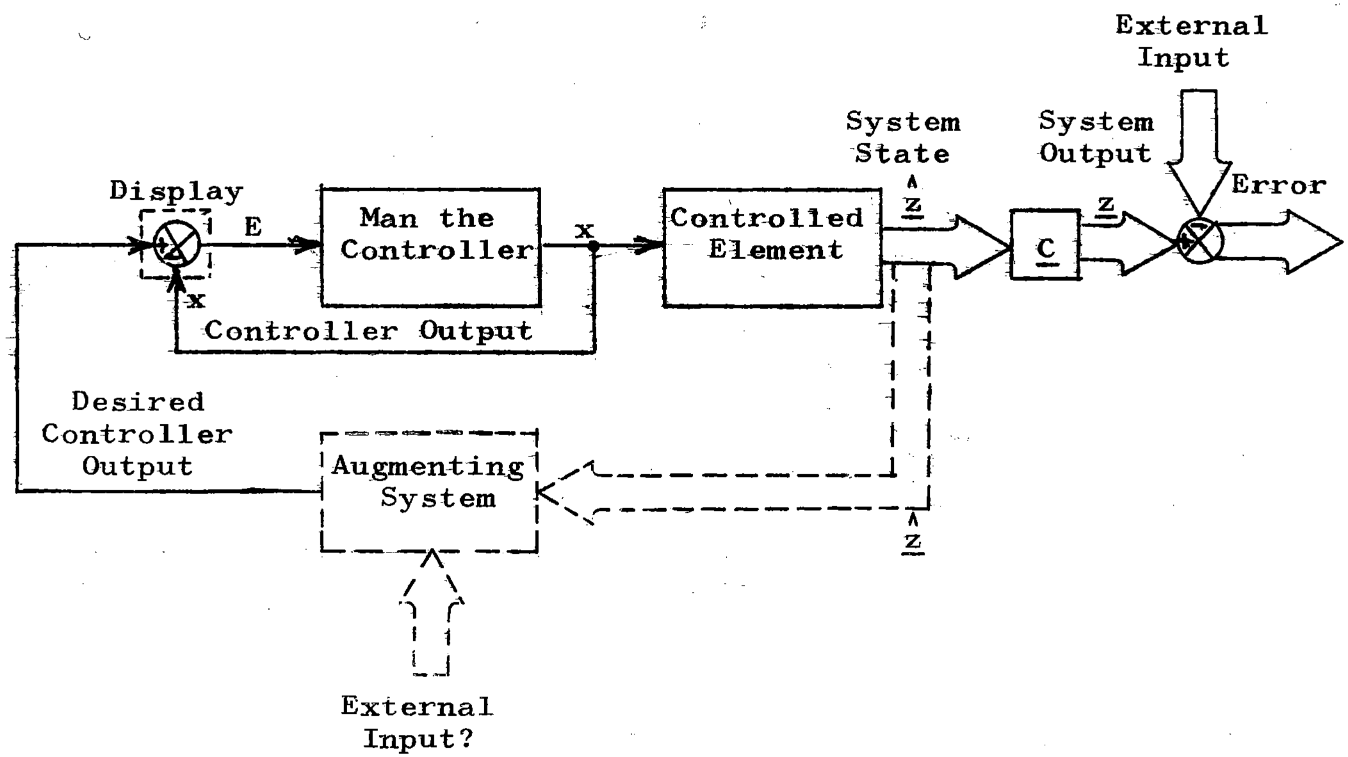


Figure 12. Single Axis Diagram of Proposed Augmented System

advantages outlined above for the operator, it is important to point out that he and his judgment are still required in the system.

The proposed augmenting system can readily satisfy the required characteristics specified earlier in this chapter. The remaining chapters are concerned with generalizing the proposed system to the extent that it will encompass the majority of man-machine systems, developing a general design philosophy for this system, and finally analytically evaluating the philosophy by application to a series of example problems.

CHAPTER IV

DESIGN PHILOSOPHY FOR THE AUGMENTING SYSTEM

The contribution from any new design technique must lie in the development of a general design philosophy to accompany it. This research is no exception. A highly satisfactory concept for a system which augments man was presented in the previous chapter. The purpose of this chapter is to take this concept and develop a general design philosophy for it.

The development must begin by first defining a general state model which describes an "augmentable" system as it would appear for the majority of man-machine systems. In conjunction with this description, a specific criteria about which the "augmentable" system performance can be extremized is investigated and then adapted into a general criteria which can be used for design. With these two fundamentals thus specified the problem for the augmenting system becomes one of producing the necessary input to the man-machine system such that the performance criteria is extremized. The obvious method of solution to this problem is optimal control analysis. Consequently, a general optimal control analysis scheme is adapted from existing methods and expressed in a form which permits identification of augmenting system requirements. Finally, by utilizing the analysis scheme, an algorithm for the design of augmenting systems is formulated.

Definition of State Model

The majority of linear man-machine systems encountered in the literature, and, thus, considered in this study, are defined as compensatory tracking systems. Figure 13 is a simple scalar block diagram of such a system. An example of this system would be an automobile driver trying to make his velocity correspond to set value on his speedometer. The definition of compensatory tracking arises from the fact that man the controller is required, through a display of error, to make the controlled output track a referenced input. Since the tracking system is to be seriously considered for augmentation in this study, several, more complex illustrations of these systems are presented below. An extended version of Figure 13 is shown in Figure 14. In this figure man has a single input but is able to effect multiple controlled element outputs. An example of this system would be an aircraft pilot trying to compensate his velocity but at the same time changing his altitude. Figure 15 depicts man as being a multiple axis controller which has multiple inputs and multiple outputs. Here, an example would be a pilot simply flying an airplane and receiving inputs or errors from his instruments. Note, however, that in Figure 15 a single human controller is regarded as having

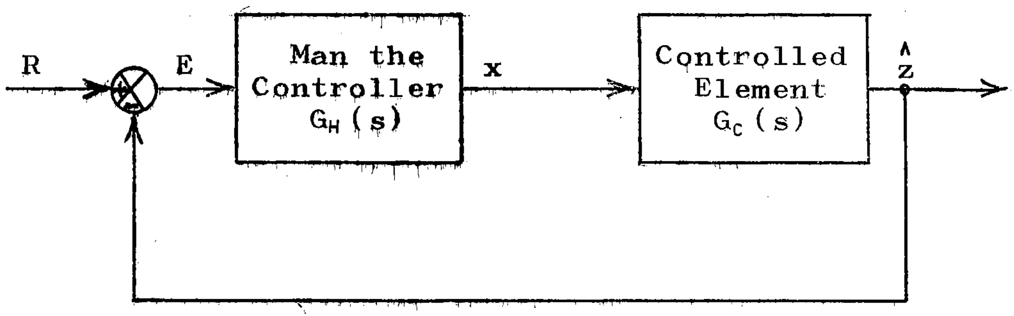


Figure 13. Single Axis Controller

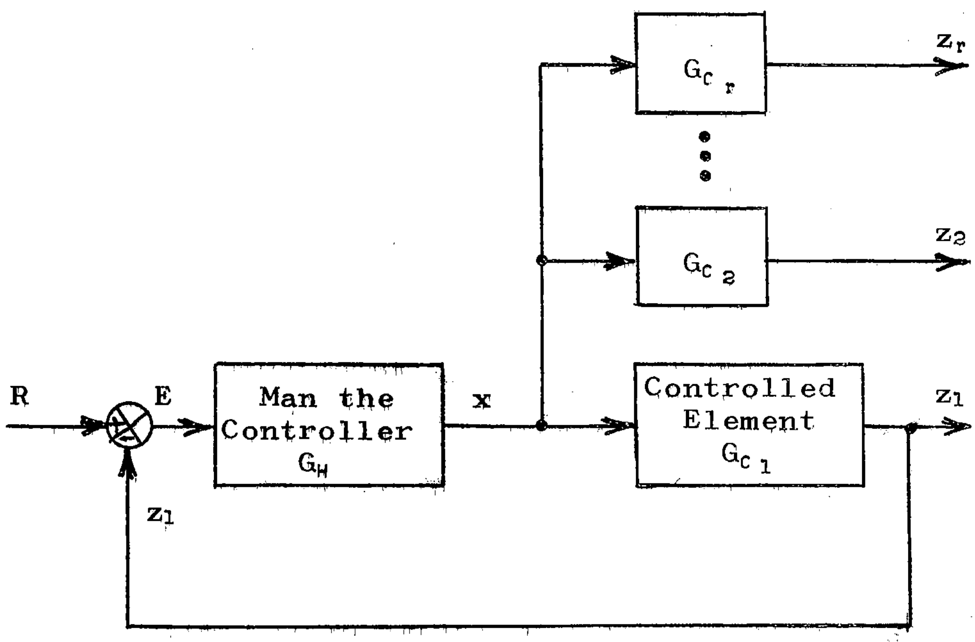


Figure 14. Single Loop Controller With Multiple Outputs

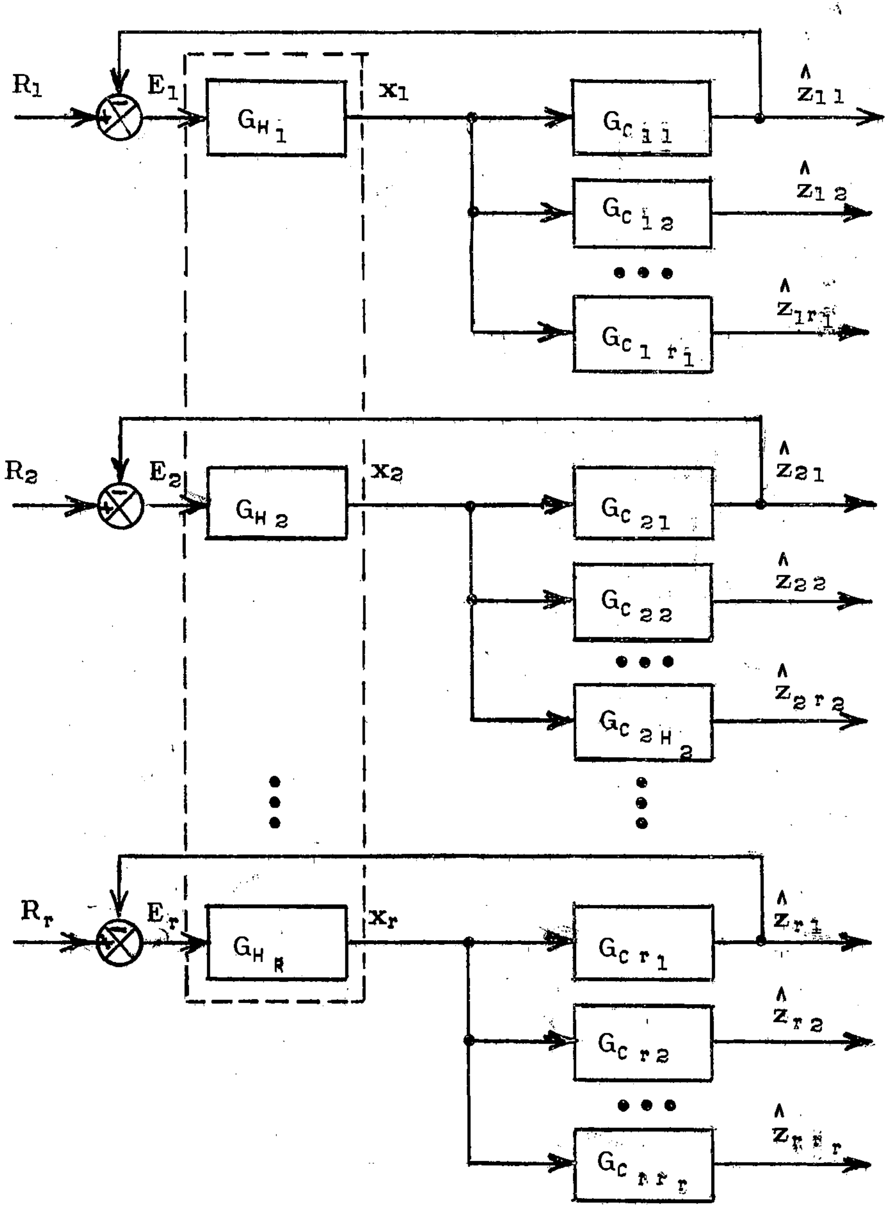


Figure 15. Multiple Single Loop Controller With Multiple Outputs in Each Axis

a specific dynamical model for each of the inputs to the system. In Figure 16 the system is multiple input-multiple output, but as opposed to Figure 15, the controlled elements for each axis are regarded as being dynamically coupled. Again, the example could be an airplane. Finally, Figure 17 is the most realistic and the most general illustration for the multiple input-multiple output controlled system because it depicts man as having internal information crossfed between the axes for which he must provide control. Also, in this latter figure, the controlled elements are shown to be dynamically coupled.

As opposed to the tracking problem, another man-machine system that is candidate for augmentation is classified as an output regulator. With this system, man the controller is required to make the magnitude of the machine output remain near zero as time increases. Fortunately for illustration purposes, this system is quite similar to the tracking system. The difference between the two arises from the fact that the output regulator system does not have a referenced input. Thus, if the tracking system has referenced input of magnitude zero, it is in effect an output regulator.

The man-machine systems discussed in the preceding paragraphs are the types which are considered for augmentation. However, in their present form these systems are not acceptable for augmentation. The purpose of the proposed augmenting system is to generate an output which is desired

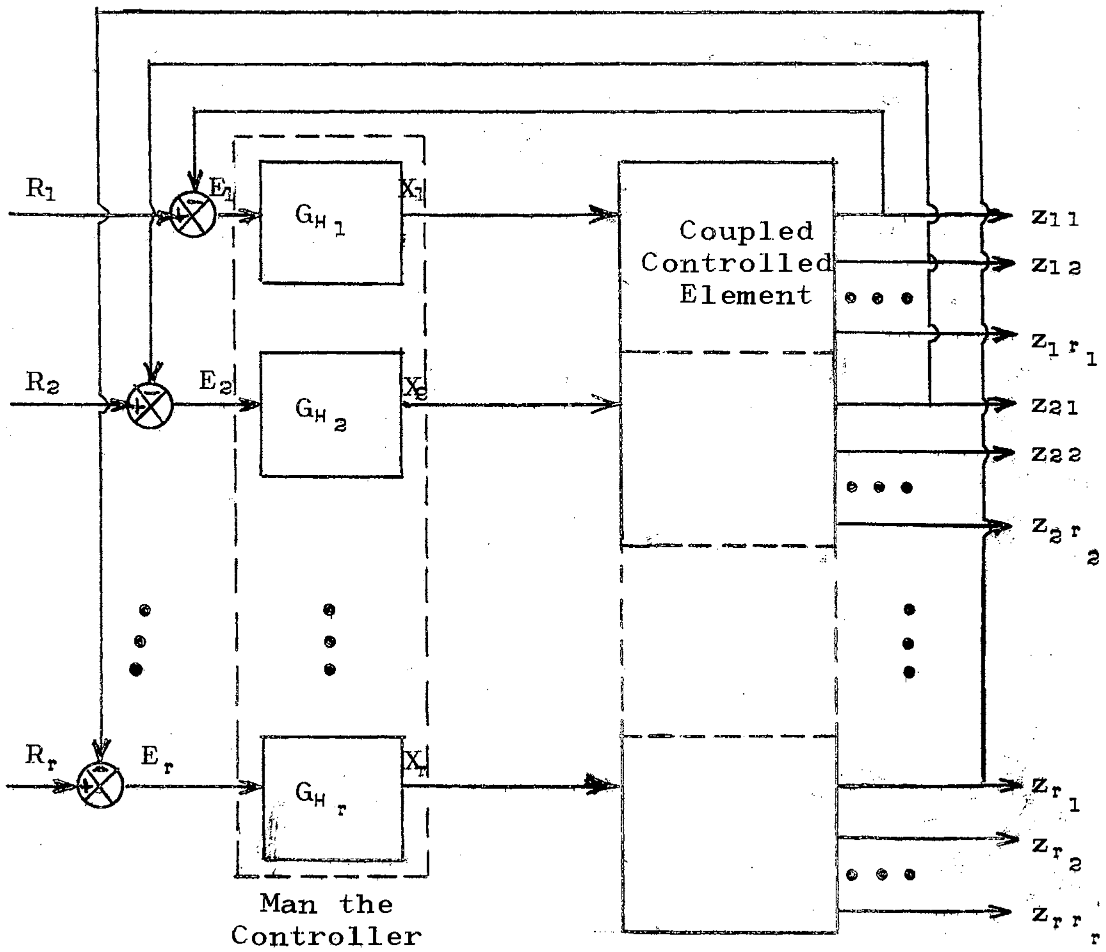


Figure 16. Multiloop Controller With Coupled Controlled Element

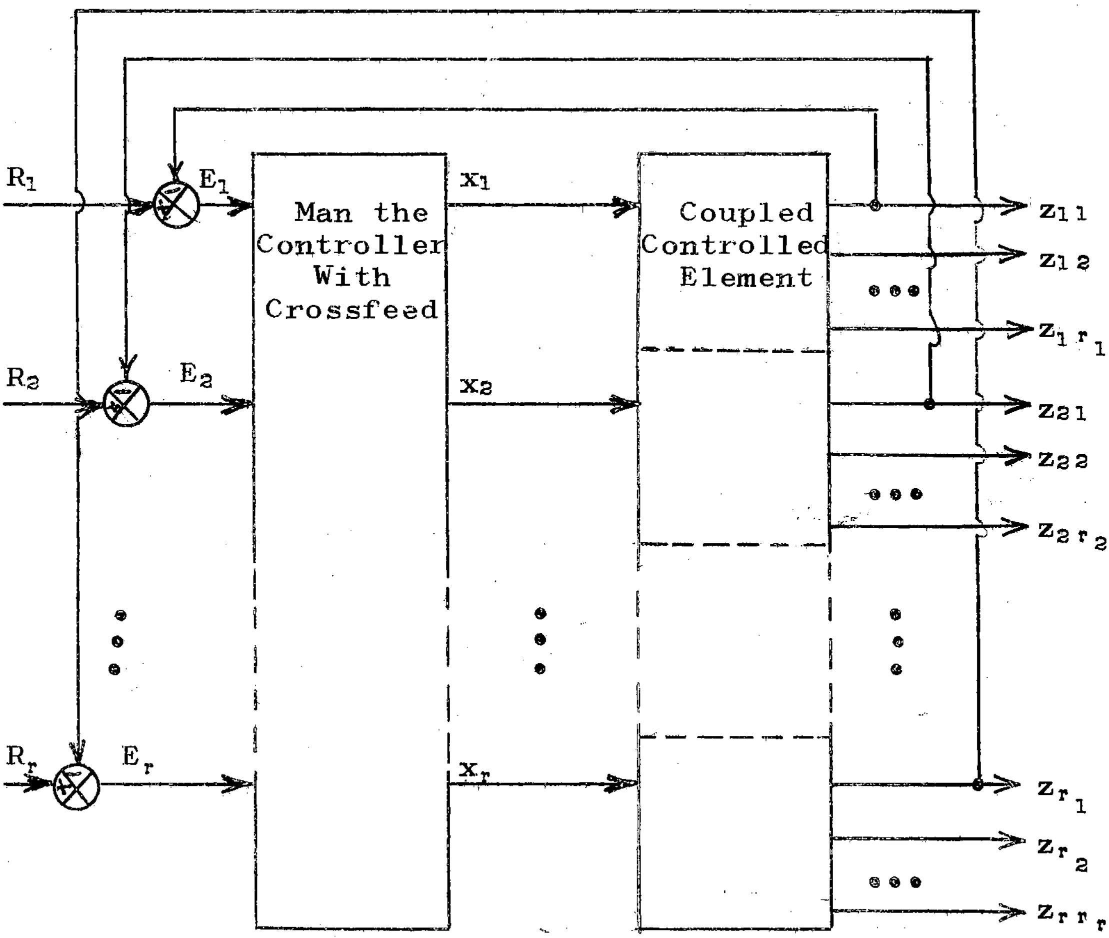


Figure 17. Crossfed Multiloop Controller With Coupled Controlled Element

from man as the controller and then compare it with his actual output. To achieve this purpose, it is necessary to make some rearrangements. The first step is to isolate both the model for man and the controlled element as shown in Figure 18. Once isolated, the man's output for each axis is fed back and algebraically subtracted from its respective input. The resultant error is then displayed to him as his input for that axis. Figure 19 illustrates this method of feeding back for the general case of a man-machine system which has crossfeed and n-axes to be controlled. This rearranged, or more specifically, this "augmentable" form for the man-machine system is the foundation about which the augmenting system is designed.

Referring again to Figure 19 the system which is diagrammed is very general because of its capability for describing the realistic, but complex, crossfeed and dynamic coupling that could occur in a man-machine system. Conversely, because of this generality, this diagram can easily describe the other, less complex systems that are of interest to this report; thus, the diagram of Figure 19 is used as the basis for defining the general state model.

In defining the state model, it is convenient to consider the block diagram of Figure 19 in the equivalent, but more meaningful, form of the state diagram shown in Figure 20. By utilizing this diagram, the state model is defined as:

$$\dot{\hat{\underline{z}}}(t) = \underline{\underline{A}}\,\hat{\underline{z}}(t) + \underline{B}\,\underline{x}d(t) \qquad (4\text{-}1)$$

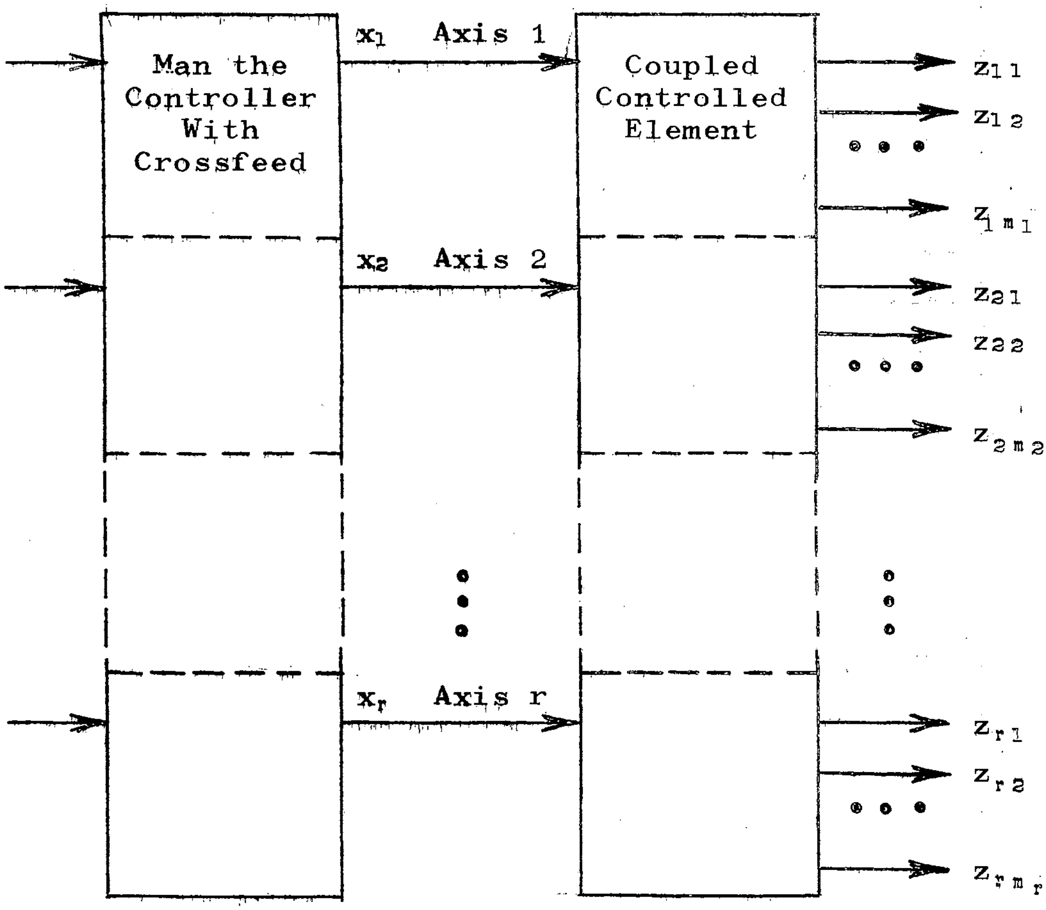


Figure 18. Isolated Controller and Controlled Element Models for n-axes Crossfed System

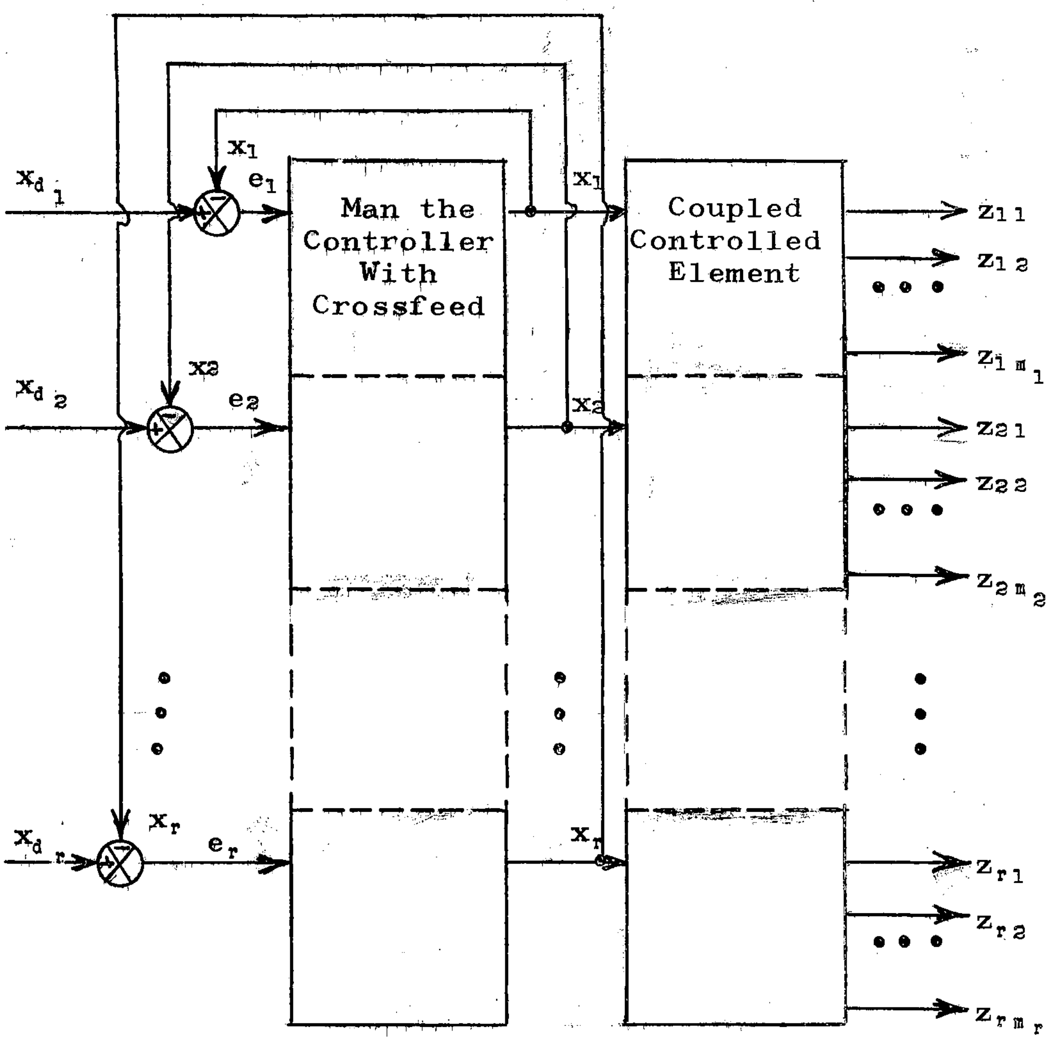


Figure 19. Isolated Controller and Controlled Element Models With Controller Output Feedback for n-axes Crossfed System

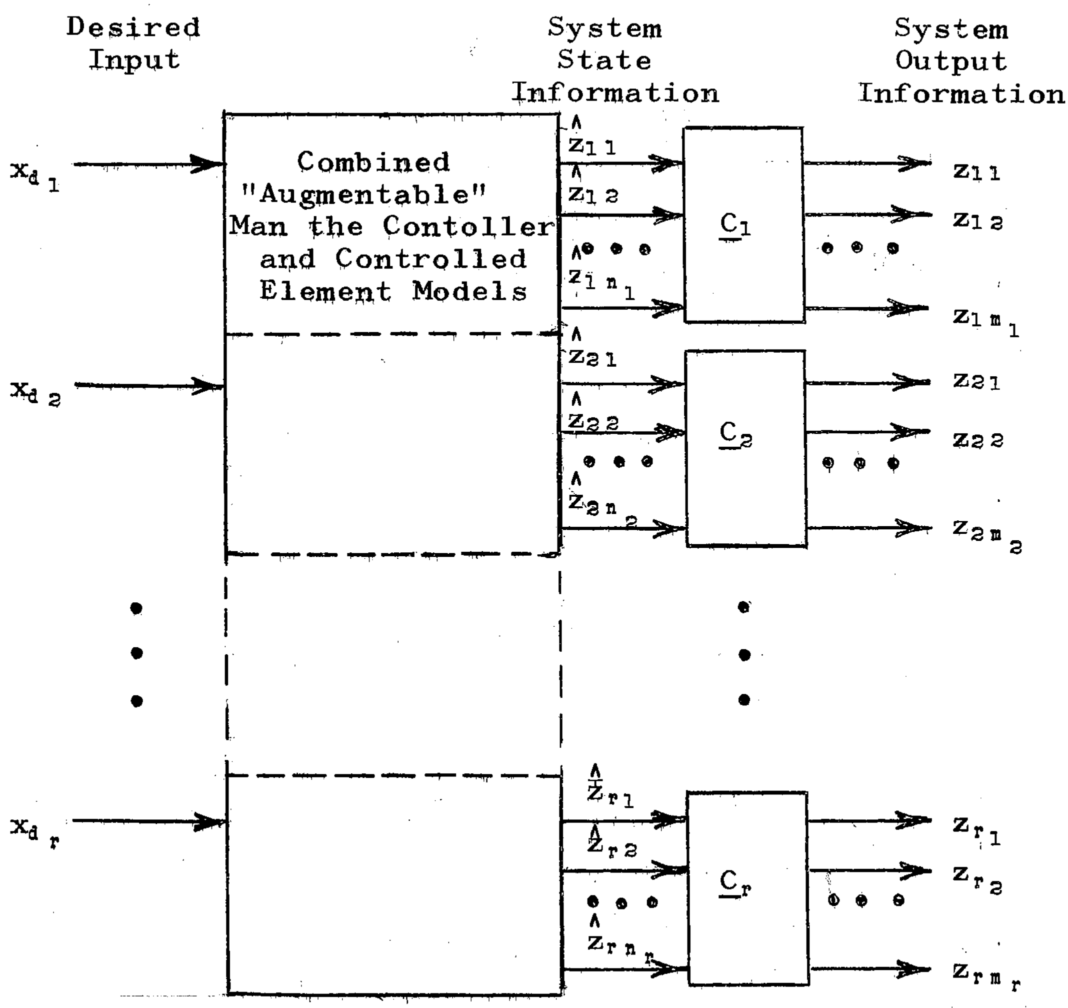


Figure 20. State Diagram of Figure 19

$$\underline{z}(t) = \underline{C}\,\underline{\hat{z}}(t) \qquad (4\text{-}2)$$

where

$\underline{z}(t)$ is a vector of dimension $(m_1 + m_2 + \ldots + m_r) = m_T$ describing the system output.

r is an integer number describing the number of axes to be controlled.

m_i $(i = 1, 2, \ldots, r)$ is an integer number defining the dimension of the output vector for each axis.

$\underline{\hat{z}}(t)$ is a vector of dimension $(n_1 + n_2 + \ldots + n_r) = n_T$ describing the system state.

n_i $(i = 1, 2, \ldots, r)$ is an integer number defining the dimension of the state vector for each axis.

$\underline{C}$ is a $(m_T \times n_T)$ coefficient matrix relating the system state to the system output.

$\underline{x}_d(t)$ is a vector of dimension r describing the desired inputs to the operator.

$\underline{A}$ is a $(n_T \times n_T)$ coefficient matrix.

$\underline{B}$ is a $(n_T \times r)$ coefficient matrix.

At this point, it can be assumed that, given a tracking or output regulator type man-machine system, it is possible to define a specific state model that defines either one of these systems as they would appear in "augmentable" form. The next logical step in the development of the general design philosophy must be to determine what performance aspects of the unaugmented man-machine system need improvement and how they can be expressed in terms of existing performance criteria.

Performance Criteria

Generally speaking, there are three main performance criteria which are commonly used in classical optimal control theory. They are: (1) quadratic, (2) minimum time, and (3) minimum fuel. Of these three, the quadratic criteria will be the primary consideration of this study. The other two criteria are equally important, but a worthwhile consideration of them is beyond the scope of this thesis.

The purpose of this section is to consider the performance requirements of the unaugmented man-machine systems and adapt to these requirements a suitable quadratic criteria which can be used later in the chapter to design the augmenting system. As mentioned earlier, there are two particular types of man-machine systems to be augmented; the tracking and output regulator systems. The improvement of the particular performance requirements of these unaugmented systems is the main concern of the augmenting system.

Specifically, the performances required from the tracking and output regulator systems are as follows: tracking - the output of the system is to follow, as closely as possible, a referenced input; output regulator - the output of the stystem is to remain as near zero as possible. Since these requirements are going to be enforced upon the "augmentable" system by the augmenting system, it is necessary to consider the system diagrammed in Figure 20 as the basis for specifying the performance criteria. (NOTE: The performance criteria will be developed concurrently for the tracking and output regulator systems.) However, a more meaningful illustration can be had if the "augmentable" system of Figure 20 is expressed as a vector block diagram and if a block diagram of the proposed augmenting system information flow is included. This augmented system is shown in Figure 21.

The augmented system of Figure 21 is basically an output regulator of the type found in reference (17). Thus, presuming the augmenting system will provide optimal control, it is quite simple to specify a quadratic criteria which when extremized will assure that the output vector of this system will remain near zero. Specifically, this criteria can be expressed as

$$J_0 = \frac{1}{2}\int_{t_0}^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle]dt \tag{4-3}$$

where $\langle , \rangle$ denotes the scalar product of the vectors $\underline{z}$ and $\underline{Q}\,\underline{z}$, (t_0, t_f) is the time interval of extremization and $\underline{Q}$ is an $(m_T \times m_T)$ positive definite constant coefficient weighting matrix.

The output requirements of the tracking system make it somewhat more difficult to specify such a criteria for the system of Figure 21. It is necessary to incorporate an

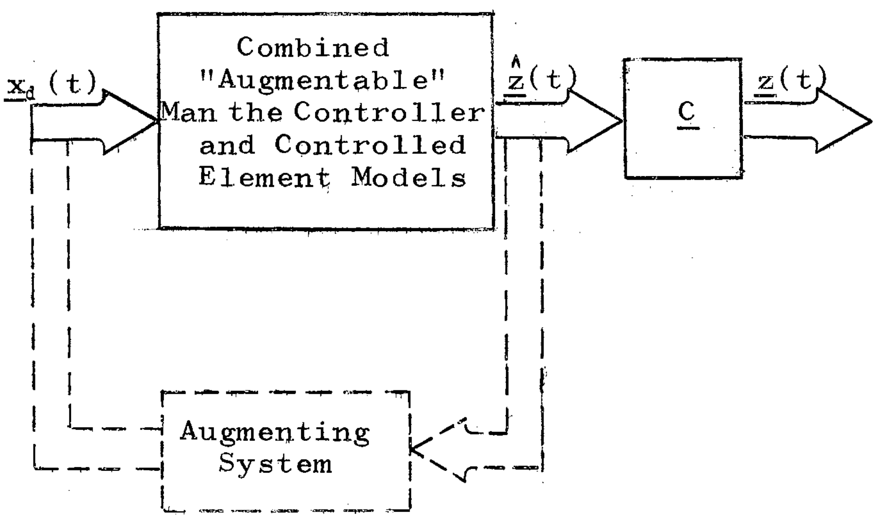


Figure 21. Vector Diagram of Augmented Output Regulator System

additional feature into the augmented system of Figure 21. The diagram of Figure 22 illustrates this added feature. This diagram illustrates the output of the augmented system as being algebraically subtracted from the referenced input (designated $\underline{R}$). The result is the error vector $\underline{E}$ (t). Obviously, if this error is extremized, it will remain near zero in magnitude. Thus, the desired output for the tracking system can be achieved. A criteria similar to Equation (4-3) can be defined as

$$J_T = \frac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle]dt. \tag{4-4}$$

It would seem that the parallel criteria of Equations (4-3) and (4-4) are sufficient to produce (using the yet to be presented results of the next section) the desired results for the two types of augmented systems. However, some further refinements are necessary. First of all, there is the practical consideration of control energy expenditure. The implications of Equations (4-3) and (4-4) are (again relying upon the yet to be presented results of the next section) that the desired control vector $\underline{x}_d$(t) will be made to go to any extreme to guarantee that the performance desired of the augmented systems will be achieved. This is not very realistic. These extremes will often require an excessive amount of control energy expenditure. In terms of the capabilities of the physical hardware for the augmenting system, this expenditure is prohibitive. Also, a more

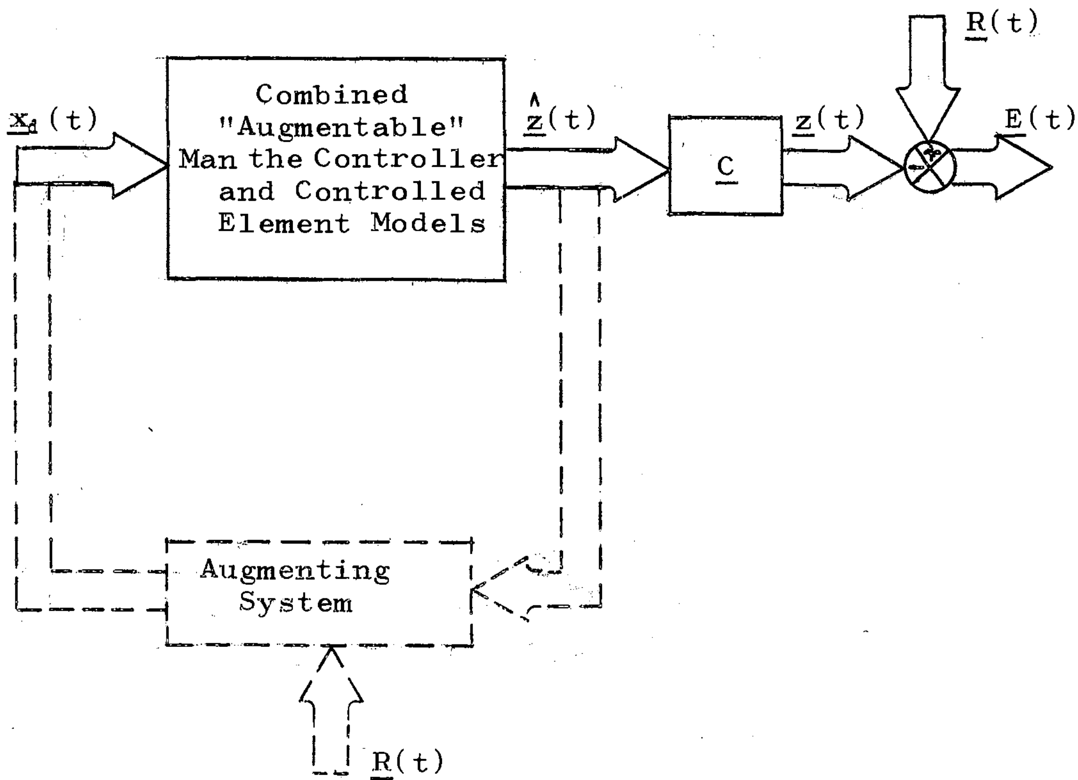


Figure 22. Vector Diagram of Augmented Tracking System

important consideration is that the human operator must duplicate the required control and would, thus, be subject to excessive physical and mental fatigue if this control energy is able to go unchecked.

To incorporate some control over the control energy expenditure, it is necessary to include another quadratic term into both of the performance criteria. To be specific, Equations (4-3) and (4-4) can be rewritten as

$$J_O = \frac{1}{2}\int_{t_o}^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \qquad (4\text{-}5)$$

and

$$J_T = \frac{1}{2}\int_{t_o}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt, \qquad (4\text{-}6)$$

respectively. Where $\underline{x}_d(t)$ is the control vector and $\underline{W}$ is an $(r \times r)$ positive definite constant coefficient weighting matrix. The inclusion of this term, as shown in Equations (4-5) and (4-6), causes the control energy to be minimized along with the output criteria, i.e., the criteria originally shown in Equations (4-3) and (4-4), when the extremization procedures to be presented in the next section are enforced.

The second refinement to be considered deals with the terminal value of the augmented system output. In particular, there may be instances that arise where the output of the augmented system is required to be "near" a specific value at the terminal time t_f. This feature is easily adapted into the augmented system by the inclusion of another term into both of the performance criteria. Specifically, Equations (4-5) and (4-6) can be rewritten as

$$J_0 = \tfrac{1}{2}\langle \underline{z}(t_f), \underline{F}\,\underline{z}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-7}$$

and

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt, \tag{4-8}$$

respectively. Where $\underline{z}(t_f)$ and $\underline{E}(t_f)$ are the desired final values of $\underline{z}(t)$ and $\underline{E}(t)$, and $\underline{F}$ is an $(m_T \times m_T)$ positive semi-definite constant coefficient matrix. If the terminal value of $\underline{z}(t)$ or $\underline{E}(t)$ is not of particular importance, then the value of $\underline{F}$ can be set equal to zero, and the remainder of the performance criteria can be relied upon to guarantee that $\underline{z}(t)$ and $\underline{E}(t)$ at time t_f are not too far from their "not so important" terminal values.

Equations (4-7) and (4-8) are the final forms of the quadratic criteria as they are utilized in this report. The next step in the development is to specify a method that can be used to produce a control which will extremize these criteria and at the same time produce sufficient information to design the augmenting system.

Derivation of Mathematical Conditions for Design of Augmenting Systems

With the state model for the "augmentable" system and the performance criteria of the last section available, it is possible to move on to the next step in the development. Specifically, this step is concerned with determining how to generate the control $\underline{x}_d(t)$ necessary to extremize the performance criteria and at the same time precipitate sufficient information to design the augmenting system.

The generation of the desired control $\underline{x}_d(t)$ and the consequent design of the augmenting system are the major sources of difficulty encountered in the augmentation. This difficulty arises because the nature of the augmenting system makes the use of optimal control theory and the solution of the complex two point boundary value problem associated with it an unavoidable necessity. Fortunately, the difficulties involved with solving the two point boundary problem are eased greatly when the systems being analyzed are linear. Additionally, the use of linear systems permits an analytical determination of the components required in the augmenting system. As might be expected, the nonlinear system provides the major source of trouble. The two point boundary value problem is difficult to solve and the components of the augmenting system must be determined mainly by non-analytical means.

Since the purpose of this research is to develop a general philosophy for the design of augmenting systems and not to become involved in the problems that evolve from optimal control analysis, the developments of this chapter are made for linear models of man and the controlled element.

The purpose of this section is to first present a method, Pontryagin's Minimum Principle, by which the performance criteria can be extremized and then specify how this method can be used to augment the output regulator and tracking systems (17). Once these specifications are made, various techniques for effecting their mathematical solution will be presented.

The Minimum Principle of Pontryagin (17)

In this study, the Minimum Principle will be used to specify in the parallel conditions necessary to determine the optimal output from the augmenting system for both the output regulator and tracking systems.

Before stating the Minimum Principle, a precise statement of the control problem is necessary. The linear dynamical system described previously by

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) + \underline{B}\,\underline{x}_d(t) \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\hat{\underline{z}}(t) \tag{4-2}$$

on the closed interval $[t_0,\ t_f]$, $t_f > t_0$ will be considered. At t_0, the initial time,

$$\hat{\underline{z}}(t_0) = \hat{\underline{z}}_0$$

is the initial state, and the final state $\hat{\underline{z}}(t_f)$ is not fixed. The functions

$$L(\underline{z}(t), \underline{x}_d(t)) \text{ and } K(\underline{z}(t))$$

for the output regulator system and the functions

$$L(\underline{E}(t), \underline{x}_d(t)) \text{ and } K(\underline{E}(t))$$

for the tracking system, defined in this study as

$$L(\underline{z}(t), \underline{x}_d(t)) = \tfrac{1}{2}\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \tfrac{1}{2}\langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle \quad (4\text{-}9)$$

$$K(\underline{z}(t)) = \tfrac{1}{2}\langle \underline{z}(t_f), \underline{F}\,\underline{z}(t_f)\rangle \quad (4\text{-}10)$$

and

$$L(\underline{E}(t), \underline{x}_d(t)) = \tfrac{1}{2}\langle \underline{E}(t), \underline{Q}\,\underline{E}(t) + \tfrac{1}{2}\langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle \quad (4\text{-}11)$$

$$K(\underline{E}(t)) = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle \quad (4\text{-}12)$$

are differentiable in $\underline{z}(t)$ and t, and $\underline{E}(t)$ and t, respectively. Furthermore, these functions describe the performance criteria given by

$$J_O = K(\underline{z}(t_f)) + \int_{t_o}^{t_f} L[\underline{z}(t), \underline{x}_d(t)]dt \quad (4\text{-}13)$$

and

$$J_T = K(\underline{E}(t_f)) + \int_{t_o}^{t_f} L[\underline{E}(t), \underline{x}_d(t)]dt. \quad (4\text{-}14)$$

The problem is to determine two different controls described by $\underline{x}_d(t)$ which minimize J_O and J_T (NOTE: J_O and J_T represent two individual problems, thus a control $\underline{x}_d(t)$ must be found to satisfy each.).

Several assumptions must also be made before a statement of the Minimum Principle can be presented. These assumptions are taken from reference (17) which is also the source for the statement of the Minimum Principle of Pontryagin. First, let the state model of Equation (4-1) be defined in the equivalent, but more general form of

$$\dot{\hat{\underline{z}}}(t) = \underline{f}(\hat{\underline{z}}(t),\underline{x}_d(t)). \tag{4-15}$$

Now, if $f_1(\hat{\underline{z}}(t),\underline{x}_d(t))$, $f_2(\hat{\underline{z}}(t),\underline{x}_d(t))$, ..., $f_{n_T}(\hat{\underline{z}}(t),\underline{x}_d(t))$ denote the components of $\underline{f}(\hat{\underline{z}}(t),\underline{x}_d(t))$, then it is assumed that the functions

$$f_i(\hat{\underline{z}}(t),\underline{x}_d(t))\ \frac{\partial f_i}{\partial \hat{\underline{z}}}(\hat{\underline{z}}(t),\underline{x}_d(t)),\ \frac{\partial f_i}{\partial t}(\hat{\underline{z}}(t),\underline{x}_d(t)),$$

$$i = 1,\ 2,\ \ldots,\ n_T$$

and the functions

$$L(\underline{z}(t),\underline{x}_d(t)),\ \frac{\partial L}{\partial \hat{\underline{z}}}(\underline{z}(t),\underline{x}_d(t)),\ \frac{\partial L}{\partial t}(\underline{z}(t),\underline{x}_d(t))$$

and

$$L(\underline{E}(t),\underline{x}_d(t)),\ \frac{\partial L}{\partial \underline{E}}(\underline{E}(t),\underline{x}_d(t)),\ \frac{\partial L}{\partial t}(\underline{E}(t),\underline{x}_d(t))$$

are continuous in the vector space containing the vectors $\hat{\underline{z}}(t)$, $\underline{x}_d(t)$, and scalar t. The terminal costs $K(\underline{z}(t))$ and $K(\underline{E}(t))$ must be independent of t, and the functions

$$K(\underline{z}(t)),\ \frac{\partial K}{\partial \hat{\underline{z}}}(\underline{z}(t)),\ \frac{\partial^2 K}{\partial \hat{\underline{z}}^2}(\underline{z}(t))$$

and

$$K(\underline{E}(t)),\ \frac{\partial K}{\partial \hat{\underline{z}}}(\underline{E}(t)),\ \frac{\partial^2 K}{\partial \hat{\underline{z}}^2}(\underline{E}(t))$$

must be continuous.

The Minimum Principle of Pontryagin can now be stated from reference (17) for the assumptions above and for the special case of terminal cost. More specifically, the Minimum Principle is stated for the two problems to be considered in this study (i.e., the output regulator and tracking systems). The output regulator problem can be formed as

$$\begin{aligned}
\dot{\hat{\underline{z}}}(t) &= \underline{f}(\hat{\underline{z}}(t), \underline{x}_d(t)) \\
\hat{\underline{z}}(t_0) &= \hat{\underline{z}}_0 \\
\underline{z}(t_0) &= \underline{z}_0 \\
\hat{\underline{z}}(t_f) &= \text{unspecified} \\
\underline{z}(t_f) &= \text{unspecified}
\end{aligned} \tag{4-16}$$

$$J_0 = K(\underline{z}(t_f)) + \int_{t_0}^{t_f} L(\underline{z}(t), \underline{x}_d(t))dt. \tag{4-17}$$

The problem is to determine the control $\underline{x}_d(t)$ which minimizes the performance criteria J_0: the control that does so is designated $\underline{x}_d^*(t)$. A set of "adjoint" variables, $\underline{p}(t)$, are introduced to play a role similar to Lagrange multipliers in differential calculus. Also, a scalar function called the Hamiltonian is defined by

$$H_0(\underline{z}(t), \hat{\underline{z}}(t), \underline{p}(t), \underline{x}_d(t)) = L(\underline{z}(t), \underline{x}_d(t)) + \langle \underline{p}(t), \underline{f}(\hat{\underline{z}}(t), \underline{x}_d(t))\rangle.$$

The Minimum Principle of Pontryagin can now be stated for the output regulator problem as follows (17):

> Let $\underline{x}_d^*(t)$ be an admissible control which drives the system of Equation (4-16) from the initial point $(\hat{\underline{z}}(t_0), t_0)$ during the time interval (t_0, t_f). Let $\hat{\underline{z}}^*(t)$ be the state trajectory and $\underline{z}^*(t)$ be the output trajectory corresponding to $\underline{x}_d^*(t)$ originating at $(\hat{\underline{z}}_0, t_0)$ and $(\underline{z}_0, t_0)$ respectively. In order that $\underline{x}_d^*(t)$ be optimal for the performance criteria (4-17), it is necessary that there exist a function $\underline{p}^*(t)$ such that:
>
> a. $\underline{p}^*(t)$ corresponds to $\underline{x}_d^*(t)$ and $\hat{\underline{z}}^*(t)$ so that $\underline{p}^*(t)$ and $\hat{\underline{z}}^*(t)$ are a solution of the canonical system
>
> $$\begin{aligned} \dot{\hat{\underline{z}}}^*(t) &= \frac{\partial H}{\partial \underline{p}}(\underline{z}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t)) \\ \dot{\underline{p}}^*(t) &= \frac{\partial H}{\partial \hat{\underline{z}}}(\underline{z}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t)) \\ \hat{\underline{z}}(t_0) &= \hat{\underline{z}}_0 \\ \underline{p}(t_f) &= \frac{\partial K}{\partial \hat{\underline{z}}}(\underline{z}^*(t_f)) \end{aligned} \qquad (4\text{-}18)$$
>
> b. The function $H(\underline{z}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t))$ has an absolute minimum as a function of $\underline{x}_d^*(t)$ at $\underline{x}_d(t) = \underline{x}_d^*(t)$ for t in $[t_0, t_f]$.
>
> c. The function $H(\underline{z}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t))$ satisfies the relations
>
> $$H(\underline{z}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t)) = -\int_{t_0}^{t_f} \frac{\partial H}{\partial t}(\underline{z}^*(\tau), \hat{\underline{z}}^*(\tau), \underline{p}^*(\tau), \underline{x}_d^*(\tau))\,d\tau,$$
>
> $$H(\underline{z}^*(t_f), \hat{\underline{z}}^*(t_f), \underline{p}^*(t_f), \underline{x}_d^*(t_f)) = 0.$$

Similarly, the tracking problem can be formed as

$$\begin{aligned} \dot{\hat{\underline{z}}}(t) &= \underline{f}(\hat{\underline{z}}(t), \underline{x}_d(t)) \\ \hat{\underline{z}}(t_0) &= \hat{\underline{z}}_0 \\ \underline{E}(t_0) &= \underline{E}_0 \\ \hat{\underline{z}}(t_f) &= \text{unspecified} \end{aligned} \qquad (4\text{-}19)$$

$$\underline{E}(t_f) = \text{unspecified}$$

$$J_T = K(\underline{E}(t_f)) + \int_{t_0}^{t_f} L(\underline{E}(t), \underline{x}_d(t))dt. \tag{4-20}$$

The problem is to determine the control $\underline{x}_d(t)$ which minimizes the performance criteria J_T: the control that does so is designated $\underline{x}_d^*(t)$. Again, a set of adjoint variables, $\underline{p}(t)$, are introduced. The Hamiltonian is defined as

$$H_T(\underline{E}(t), \hat{\underline{z}}(t), \underline{p}(t), \underline{x}_d(t)) = L(\underline{E}(t), \underline{x}_d(t)) + \langle \underline{p}(t), \underline{f}(\hat{\underline{z}}(t), \underline{x}_d(t)) \rangle.$$

The Minimum Principle of Pontryagin can now be stated for the tracking problem as follows (17):

> Let $\underline{x}_d^*(t)$ be an admissible control which drives the system of Equation (4-19) from the initial point $(\hat{\underline{z}}(t_0), t_0)$ during the time interval (t_0, t_f). Let $\hat{\underline{z}}^*(t)$ be the state trajectory and $\underline{E}^*(t)$ be the error trajectory corresponding to $\underline{x}_d^*(t)$ originating at $(\hat{\underline{z}}_0, t_0)$ and $(\underline{E}_0, t_0)$ respectively. In order that $\underline{x}_d^*(t)$ be optimal for the performance criteria (4-20), it is necessary that there exist a function $\underline{p}^*(t)$ such that:
>
> a. $\underline{p}^*(t)$ corresponds to $\underline{x}_d^*(t)$ and $\hat{\underline{z}}^*(t)$ so that $\underline{p}^*(t)$ and $\hat{\underline{z}}^*(t)$ are a solution of cannonical system
>
> $$\dot{\hat{\underline{z}}}^*(t) = \frac{\partial H}{\partial \underline{p}}(\underline{E}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t))$$
>
> $$\dot{\underline{p}}^*(t) = -\frac{\partial H}{\partial \hat{\underline{z}}}(\underline{E}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}^*_d(t))$$
>
> $$\hat{\underline{z}}(t_0) = \hat{\underline{z}}_0 \tag{4-21}$$
>
> $$\underline{p}(t_f) = \frac{\partial K}{\partial \hat{\underline{z}}}(\underline{E}^*(t_f))$$
>
> b. The function $H(\underline{E}^*(t), \hat{\underline{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t))$ has

an absolute minimum as a function of $\underline{x}_d^*(t)$ at $\underline{x}_d(t)=\underline{x}_d^*(t)$ for t in $[t_0, t_f]$.

c. The function $H(\underline{E}^*(t), \underline{z}^*(t), \underline{p}^*(t), \underline{x}^*_d(t))$ satisfies the relations

$$H(\underline{E}^*(t), \underline{\hat{z}}^*(t), \underline{p}^*(t), \underline{x}_d^*(t)) =$$

$$- \int_{t_0}^{t_f} \frac{\partial H}{\partial t} (\underline{E}^*(\tau), \underline{\hat{z}}^*(\tau), \underline{p}^*(\tau) \underline{x}_d^*(\tau)) d\tau$$

$$H(\underline{E}^*(t_f), \underline{\hat{z}}^*(t_f), \underline{p}^*(t_f), \underline{x}_d^*(t_f)) = 0.$$

With these two statements of the Minimum Principle, it is possible to continue on to the augmentation methods of the next two sections. However, before continuing, it should be pointed out that the above statements are intended only to present a usable knowledge of the Minimum Principle. More rigorous developments and proofs are available in reference (17) and other texts on modern control theory.

The Minimum Principle and the Output Regulator System

In this section, the conditions of the Minimum Principle are applied to determine the canonical equations, i.e., Equation (4-18), for the "augmentable" output regulator system of Figure 20. Once these equations are obtained, their solution is effected in a later section into the form of an optimal feedback system that can be used to design the output regulator augmenting system.

The state model for the "augmentable" output regulator system is given by

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) + \underline{B}\,\underline{x}_d(t) \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\hat{\underline{z}}(t). \tag{4-2}$$

More generally, Equation (4-1) can be written as

$$\dot{\hat{\underline{z}}}(t) = \underline{f}(\hat{\underline{z}}(t), \underline{x}_d(t)). \tag{4-15}$$

The performance criteria is given by

$$J_O = \tfrac{1}{2}\langle \underline{z}(t_f), \underline{F}\,\underline{z}(t_f)\rangle + \tfrac{1}{2}\int_{t_o}^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-7}$$

or, more generally,

$$J_O = K(\underline{z}(t_f)) + \int_{t_o}^{t_f} L[\underline{z}(t), \underline{x}_d(t)]dt. \tag{4-13}$$

Now, by assuming that an optimal control exists for any initial state, the Minimum Principle can be used to obtain the necessary conditions for optimal control. The Hamiltonian for the output regulator system is given by

$$H_O = \tfrac{1}{2}\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \tfrac{1}{2}\langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle + \langle \underline{A}\,\hat{\underline{z}}(t), \underline{p}(t)\rangle + \langle \underline{B}\,\underline{x}_d(t), \underline{p}(t)\rangle. \tag{4-22}$$

The adjoint vector $\underline{p}(t)$ is the solution of the differential equation

$$\dot{\underline{p}}(t) = -\frac{\partial H}{\partial \hat{\underline{z}}}. \tag{4-23}$$

If Equation (4-2) is substituted into Equation (4-22) to yield

$$H_0 = \tfrac{1}{2}\langle \underline{C}\,\hat{\underline{z}}(t), \underline{Q}\,\underline{C}\,\underline{z}(t)\rangle + \tfrac{1}{2}\langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle + \langle \underline{A}\,\hat{\underline{z}}(t), \underline{p}(t)\rangle +$$

$$\langle \underline{B}\,\underline{x}_d(t), \underline{p}(t)\rangle,$$

then the differential equation of Equation (4-23) can be reduced to

$$\dot{\underline{p}}(t) = -\underline{C}^T\underline{Q}\,\underline{C}\,\hat{\underline{z}}(t) - \underline{A}^T\underline{p}(t) \tag{4-25}$$

where the superscript T indicates the transpose. Additionally, from the statement of the Minimum Principle it is apparent that the following condition must hold

$$\frac{\partial H}{\partial \underline{x}_d} = 0. \tag{4-26}$$

This condition leads to

$$\frac{\partial H_0}{\partial \underline{x}_d} = \underline{W}\,\underline{x}_d(t) + \underline{B}^T\underline{p}(t) = \underline{0} \tag{4-27}$$

which, in turn, can be rearranged to give

$$\underline{x}_d(t) = -\underline{W}^{-1}\underline{B}^T\underline{p}(t). \tag{4-28}$$

Note that the earlier specified requirement that $\underline{W}$ be positive definite insures that $\underline{W}^{-1}$ exists.

The reduced canonical equations can now be obtained by first substituting Equation (4-28) into Equation (4-1) to obtain

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) - \underline{B}\,\underline{W}^{-1}\underline{B}^T\underline{p}(t). \tag{4-29}$$

A combination of Equation (4-29) with Equation (4-25), i.e.,

$$\underline{\dot{p}}(t) = -\underline{C}^T \underline{Q}\, \underline{C}\, \hat{\underline{z}}(t) - \underline{A}^T \underline{p}(t) \qquad (4\text{-}25)$$

produces the canonical equations as a set of linear homogeneous differential equations which have the boundary conditions of

$$\hat{\underline{z}}(t_0) = \hat{\underline{z}}_0 \qquad (4\text{-}30)$$

and

$$\underline{p}(t_f) = \frac{\partial K}{\partial \hat{\underline{z}}} (\underline{z}(t_f)) = \underline{C}^T \underline{F}\, \underline{C}\, \hat{\underline{z}}(t_f). \qquad (4\text{-}31)$$

Equations (4-25), (4-29), (4-30), and (4-31) represent the set of canonical equations which must be solved to obtain the optimal control $\underline{x}_d(t)$ for the output regulator problem. The solution of this problem as presented by these equations will simultaneously produce the state trajectory $\hat{\underline{z}}(t)$ and the adjoint trajectory $\underline{p}(t)$. Once the adjoint trajectory is obtained, it can be used in conjunction with Equation (4-27) to obtain the desired control $\underline{x}_d(t)$.

The function of the augmenting system should now be more apparent. Given the optimal control $\underline{x}_d(t)$ from the solution of the canonical equations, the purpose of the augmenting system is to generate this control by making use of the state information from the "augmentable" system. Unfortunately, the solution of the cannonical equations for the optimal control $\underline{x}_d(t)$ is more difficult than it appears. The solution entails solving the difficult two point boundary problem. The solution of this problem and the determination of the augmenting system requirements will be the subject of a later section. Meanwhile, the Minimum Principle and the tracking system will be covered.

The Minimum Principle and the Tracking System

In this section, the conditions of the Minimum Principle are applied to determine the canonical equations, i.e., Equation (4-21), for the "augmentable" tracking system of Figure 22. Once these equations are obtained, their solution is effected in the next section into the form of an optimal feedback system that can be used to design the tracking augmenting system.

The state model for the "augmentable" tracking system is given by

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) + \underline{B}\,\underline{x}_d(t) \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\hat{\underline{z}}(t) \tag{4-2}$$

or, more generally, Equation (4-1) can be written as

$$\dot{\hat{\underline{z}}}(t) = \underline{f}(\hat{\underline{z}}(t), \underline{x}_d(t)). \tag{4-15}$$

The performance criteria is given by

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-8}$$

or, more generally,

$$J_T = K(\underline{E}(t_f)) + \int_{t_0}^{t_f} L[\underline{E}(t), \underline{x}_d(t)]dt. \tag{4-20}$$

Again, by assuming that an optimal control exists for any initial state, the Minimum Principle can be used to obtain the necessary conditions for optimal control. The Hamiltonian for the tracking system is given by

$$H_T = \tfrac{1}{2}\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \tfrac{1}{2}\langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle + \langle \underline{A}\,\hat{\underline{z}}(t), \underline{p}(t)\rangle + \langle \underline{B}\,\underline{x}_d(t), \underline{p}(t)\rangle. \tag{4-32}$$

If the relationships $\underline{E}(t) = \underline{R}(t) - \underline{z}(t)$ and $\underline{z}(t) = \underline{C}\,\hat{\underline{z}}(t)$ are substituted into Equation (4-32), the result is

$$H_T = \tfrac{1}{2}\langle (\underline{R}(t) - \underline{C}\,\hat{\underline{z}}(t)), \underline{Q}(\underline{R}(t) - \underline{C}\,\hat{\underline{z}}(t))\rangle + \tfrac{1}{2}\langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle + \langle \underline{A}\,\hat{\underline{z}}(t), \underline{p}(t)\rangle + \langle \underline{B}\,\underline{x}_d(t), \underline{p}(t)\rangle. \tag{4-33}$$

Thus, the adjoint differential equation

$$\dot{\underline{p}}(t) = -\frac{\partial H}{\partial \hat{\underline{z}}} \tag{4-23}$$

can be evaluated as

$$\dot{\underline{p}}(t) = -\underline{C}^T \underline{Q}\,\underline{C}\,\hat{\underline{z}}(t) - \underline{A}^T \underline{p}(t) + \underline{C}^T \underline{Q}\,\underline{R}(t). \tag{4-34}$$

Additionally, from the statement of the Minimum Principle, it is apparent that the following condition must hold:

$$\frac{\partial H}{\partial \underline{x}_d} = \underline{0}. \tag{4-26}$$

This condition again leads to

$$\frac{\partial H_T}{\partial \underline{x}_d} = \underline{W}\,\underline{x}_d(t) + \underline{B}^T \underline{p}(t) = \underline{0} \tag{4-27}$$

which can be rearranged to give

$$\underline{x}_d(t) = -W^{-1}\underline{B}^T\underline{p}(t). \tag{4-28}$$

The reduced canonical equations can now be obtained by first substituting Equation (4-28) into Equation (4-1) to obtain

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) - \underline{B}\,\underline{W}^{-1}B^T\underline{p}(t). \tag{4-29}$$

A combination of Equation (4-29) with Equation (4-34), i.e.,

$$\dot{\underline{p}}(t) = -\underline{C}^T\underline{Q}\,\underline{C}\,\hat{\underline{z}}(t) - \underline{A}^T\underline{p}(t) + \underline{C}^T\underline{Q}\,\underline{R}(t) \tag{4-34}$$

produces the canonical equations for the tracking system as a set of linear homogeneous differential equations which have the boundary conditions of

$$\hat{\underline{z}}(t_0) = \underline{z}_0 \tag{4-39}$$

and

$$\underline{p}(t_f) = \frac{\partial K}{\partial \hat{\underline{z}}}(\underline{E}(t_f)) = \underline{C}^T\underline{F}\,\underline{C}\,\hat{\underline{z}}(t_f) - \underline{C}^T\underline{F}\,\underline{R}(t_f). \tag{4-36}$$

Equations (4-29), (4-34), (4-35), and (4-36) represent the set of canonical equations which must be solved to obtain the optimal control $\underline{x}_d(t)$ for the tracking problem. The next step in the development is to examine the methods of solution for the two point boundary value problems presented by the canonical equations of the output regulator and tracking systems. Once effective methods of solution are obtained, then the augmenting system can be designed to produce the desired control.

Solution of Two Point Boundary Value Problems and Determination of Augmenting System Requirements

It is at this point in the study that the earlier specified requirement for linearity in the models of man and the controlled element comes into importance. The solution of the two point boundary value problem is greatly simplified when the canonical equations are descriptive of linear systems. Additionally, the generation of the optimal control by a feedback type system, such as is implied by the augmenting systems of Figures 21 and 22, is greatly facilitated and easily implemented when the canonical system is linear.

The difficulties involved in solving the two point boundary value problem presented by the canonical equations stated earlier arise from the fact that there are no known values of the initial conditions given for the adjoint system. Thus, if classical methods of solving differential equations are to be employed to solve the canonical equations, either an analytical solution must be obtained which is independent of the numerical value of the initial conditions, or a trial and error approach must be used to solve the canonical equations for the proper trajectories by guessing at the adjoint variable initial conditions (a more systematic approach to this trial and error solution is given in reference (23)). However, a third alternative as a method of solution exists if the linear system being optimized can be provided the optimal control by a feedback system which processes and feeds back state information. Obviously, this alternative approach is of interest here. With this alternative method, it is possible to obtain a closed form solution for the canonical equations, i.e., Equations (4-29) and (4-34). The net result of such a closed form solution is to allow the augmenting system for both the output regulator and tracking systems to be explicitly designed.

Output Regulator System

Returning to the canonical equations for the output regulator system (Equations (4-29) and (4-25)) for illustration, it can be assumed that the solution trajectories, $\hat{\underline{z}}(t)$ and $\underline{p}(t)$, for these equations are related by the expression

$$\underline{p}(t) = \underline{K}(t)\hat{\underline{z}}(t) \qquad (4\text{-}37)$$

where t is an element of $[t_0, t_f]$ and $\underline{K}(t)$ is a symmetric $(n_T \times n_T)$ matrix (13). The implications of Equation (4-37) can be seen more clearly if it is substituted into Equation (4-28) to yield

$$\underline{x}_d(t) = -\underline{W}^{-1}\underline{B}^T\underline{K}(t)\hat{\underline{z}}(t). \qquad (4\text{-}38)$$

For further clarity, Equation (4-38) can be implemented into Figure 21 to produce the augmented system diagram of Figure

23. The importance of the relation given by Equation (4-37) as a means to design the augmenting system should now be apparent. If a matrix $\underline{K}(t)$ can be found which satisfies Equation (4-37), then the augmenting system can be readily designed. The solution for $\underline{K}(t)$ is the subject of the proceeding paragraphs.

If Equation (4-37) is differentiated with respect to time, the following relation results:

$$\dot{\underline{p}}(t) = \dot{\underline{K}}(t)\hat{\underline{z}}(t) + \underline{K}(t)\dot{\hat{\underline{z}}}(t). \qquad (4\text{-}39)$$

Again, Equations (4-29) and (4-25) can be written as

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) - \underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{p}(t) \qquad (4\text{-}29)$$

and

$$\dot{\underline{p}}(t) = -\underline{C}^{T}\underline{Q}\,\underline{C}\,\hat{\underline{z}}(t) - \underline{A}^{T}\underline{p}(t). \qquad (4\text{-}25)$$

Now, if Equation (4-37) is substituted into Equation (4-29), the following relation is obtained:

$$\dot{\hat{\underline{z}}}(t) = (\underline{A} - \underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t))\hat{\underline{z}}(t). \qquad (4\text{-}40)$$

Similarly, the substitution of Equation (4-40) into Equation (4-39) yields

$$\dot{\underline{p}}(t) = (\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t))\hat{\underline{z}}(t) \qquad (4\text{-}41)$$

and the substitution of Equation (4-37) into Equation (4-25) yields

$$\dot{\underline{p}}(t) = -(\underline{C}^{T}\underline{Q}\,\underline{C} + \underline{A}^{T}\underline{K}(t))\hat{\underline{z}}(t). \qquad (4\text{-}42)$$

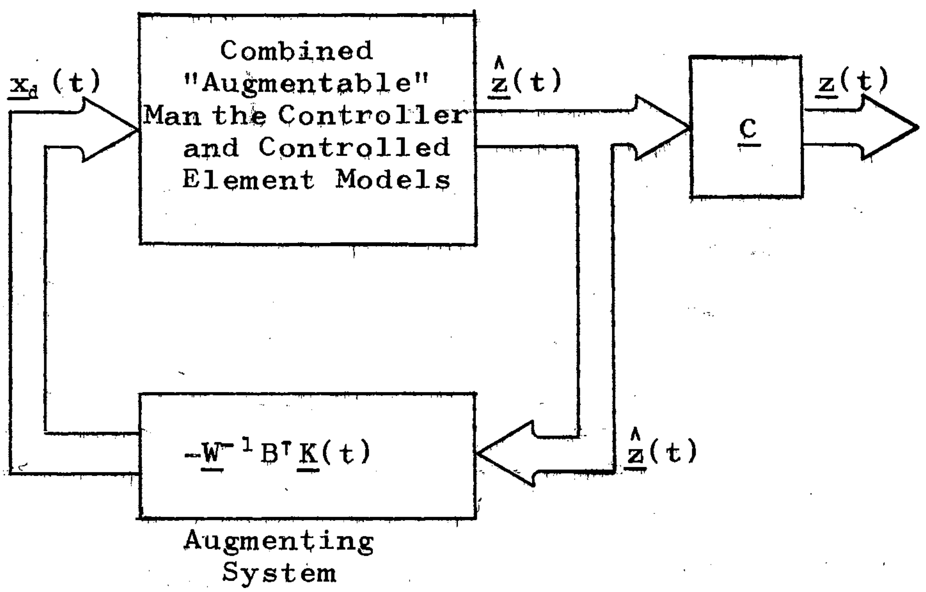


Figure 23. Vector Diagram of Augmented Output Regulator System Showing Components for Augmenting System

Finally, if Equations (4-41) and (4-42) are equated, the result is

$$(\mathring{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C})\hat{\underline{z}}(t) = \underline{0} \qquad (4\text{-}43)$$

for t, an element of $[t_0, t_f]$. Now, if the system trajectory $\hat{\underline{z}}(t)$ can be assumed to be non-trivial, then the following relation must hold:

$$\mathring{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0}. \qquad (4\text{-}44)$$

Furthermore, if Equation (4-37) is substituted into Equation (4-31), the following boundary condition can be had

$$\underline{K}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C}. \qquad (4\text{-}45)$$

The differential equation given by Equation (4-44) is in the familiar form of the matrix Riccati equation. If this equation is solved, the result is to produce the desired gain matrix $\underline{K}(t)$. Unfortunately, the computation of $\underline{K}(t)$ is not as easy as it may seem. First of all, the matrix Riccati equation is nonlinear and will, thus, for all practical purposes, require a digital computer to effect its solution (However, if the order of $\underline{K}(t)$ is small, i.e., $n_T < 3$, it is possible to obtain an analytical solution; see reference (24)). Secondly, the matrix Riccati equation, when solved will produce unstable solution trajectories unless it is integrated backwards in time. However, if the digital computer is used, it is possible to overcome these difficulties and obtain $\underline{K}(t)$.

Solution for K(t). To obtain a set of trajectory values for $\underline{K}(t)$, Equation (4-44) must, in general, be solved by utilizing a digital computer integration subroutine (Note: This type of subroutine is readily available with any science oriented digital computer.). The method of solution differs from that of an ordinary linear or non-linear equation in that the matrix Ricatti equation must be integrated from the final value at time t_f given by Equation (4-45) to the initial value of time t_0. To accomplish this feat, it is necessary to utilize a negative integration time step. However, the use of such a time step presents no computational difficulties. Once the trajectories for $\underline{K}(t)$ are obtained, then they can be utilized in conjunction with the augmenting system as shown in Figure 23 to produce the desired control $\underline{x}_d(t)$.

The solution for $\underline{K}(t)$ is a simple matter of substituting the appropriate values for the coefficient matrices into Equation (4-44) and then integrating the resulting set of first order differential equations utilizing the boundary conditions of Equation (4-45). To illustrate how this solution is obtained, a simple example is worked below. However, before considering this example, a special case for the Ricatti equation which can greatly simplify the design of the augmenting system is investigated.

Special Case for the Ricatti Equation. A special case exists (see reference (17) for further details) for which the value of $\underline{K}(t)$ in Equation (4-44) can be assumed to be constant. Specifically, when the value of the final time t_f can be assumed to be very large and the value of $\underline{K}(t_f) = \underline{0}$, then $\underline{K}(t)$ can be assumed to be equal to the constant coefficient matrix $\hat{\underline{K}}$. Consequently, the time derivative, $\dot{\underline{K}}(t)$, from Equation (4-44) becomes zero and Equation (4-44) can be written as the set of nonlinear simultaneous equations

$$-\hat{\underline{K}}\,\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\hat{\underline{K}} + \hat{\underline{K}}\,\underline{A} + \underline{A}^{T}\hat{\underline{K}} + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0}. \qquad (4\text{-}46)$$

With this special case in effect, the value of $\underline{K}(t)$ as needed in the augmenting system of Figure 23 becomes constant and the augmenting system gain is no longer subject to time variation.

Although additional discussion is needed regarding the conditions which underlie the validity of assuming a constant $\underline{K}(t)$, this discussion is postponed until the example problem is worked. The example problem will be solved for both $\underline{K}(t)$ and $\hat{\underline{K}}$ and is used to help exemplify when $\hat{\underline{K}}$ can be used with validity.

Example. As an example, consider the non-augmented system as illustrated by the scalar block diagram of Figure 13, where the input $R = 0$. The operator transfer function is assumed to have the form (23), (4)

$$G_H(s) = \frac{1}{s} \qquad (4\text{-}27)$$

and the controlled element transfer function the form (25)

$$G_c(s) = 1. \tag{4-48}$$

Now, by utilizing the transfer function information just given and the configuration shown by the block diagram of Figure 19, the "augmentable" system block diagram can be drawn as shown in Figure 24. By making use of block diagram algebra and then solving for the over-all transfer function, the "augmentable" system equation can be written as

$$\dot{\hat{z}} = -\hat{z} + x_d \, ; \quad \hat{z}(t_0) = 0. \tag{4-49}$$

Utilizing this equation, the state model as described by Equations (4-1) and (4-2) can be written as

$$[\dot{\hat{z}}] = [-1][\hat{z}] + [1]x_d \tag{4-50}$$

$$z = \hat{z} \tag{4-51}$$

where

$$\begin{aligned} \underline{A} &= [-1] \\ \underline{B} &= [1] \\ \underline{C} &= [1]. \end{aligned} \tag{4-52}$$

The performance criteria can be defined by

$$J_0 = \tfrac{1}{2}\langle \underline{z}(t_f), \underline{F}\,\underline{z}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-7}$$

where

$$\begin{aligned} \underline{F} &= [0] \\ \underline{Q} &= [1] \end{aligned} \tag{4-53}$$

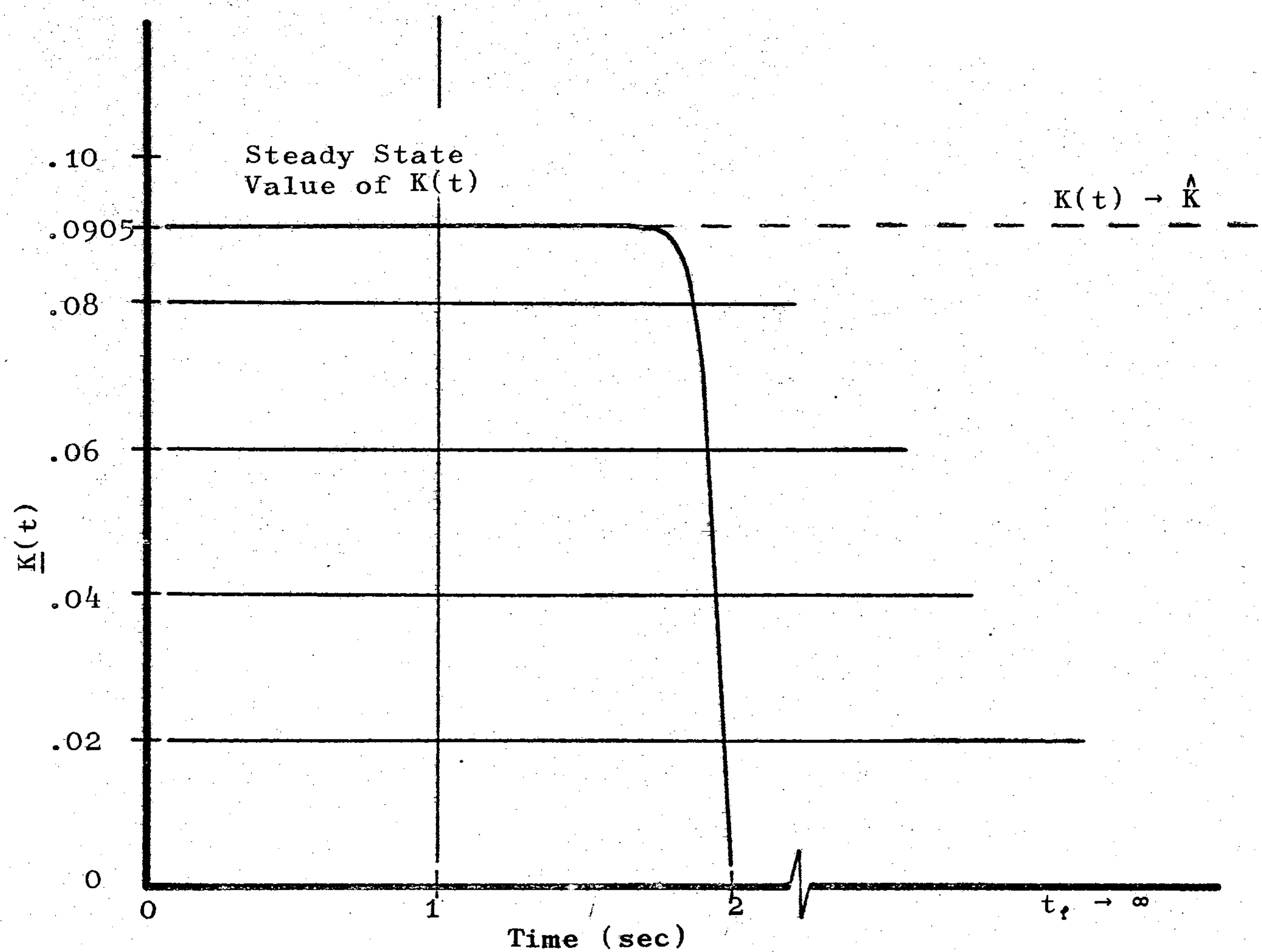


Figure 24. Solution Trajectory for K(t) for Example Output Regular Problem

$$\underline{W} = [.01].$$

The above value for the matrices can be substituted into Equation (4-44) to yield

$$\mathring{\underline{K}} + \underline{K}[-1] + [-1]\underline{K} - \underline{K}[1][.01]^{-1}[1]\underline{K} + [1][1][1] = 0 \tag{4-54}$$

or

$$\dot{K} - 2K - 100K^2 + 1 = 0. \tag{4-55}$$

with the boundary condition of

$$\underline{K}(t_f) = \underline{C}^T \underline{F}\,\underline{C} = 0. \tag{4-56}$$

If a value of $t_f = 2$ is selected, the equation defined by Equation (4-54) can be solved by backwards integration using a digital computer integration routine as mentioned earlier. The solution for $\underline{K}(t)$ obtained by using such an integration routine is shown in Figure 24.

The solution trajectory shown in Figure 24 represents the time variant gain K(t) which must be utilized in the augmenting system of Figure 23. With this gain and the appropriate values for the matrices $\underline{W}$ and $\underline{B}$ in use, the augmenting system can be made functional.

It is important to note at this point that the "steady state" portion of the trajectory shown in Figure 24. This "steady state" region is a characteristic which is generally exemplified by solutions of the matrix Ricatti equation. The implications of this characteristic are that the solutions $\underline{K}(t)$ to the matrix Ricatti equation approach a constant value after an initial (remember that the Ricatti equation is integrated backwards) transient interval. Thus, except in the transient region, the value of $\underline{K}(t)$ can be assumed constant.

The discussion of the preceding paragraph supports the earlier made simplification which allowed $\underline{K}(t)$ to be assumed equal to the constant $\underline{\hat{K}}$ for large value of t_f. The simplification stated that, if t_f is large and the final time weighting matrix, $\underline{F}$, is zero, then the variable, $\underline{\mathring{K}}(t)$, can be set equal to zero. For the example under consideration this simplification allows Equation (4-55) to be reduced to

$$100\hat{K}^2 + 2\hat{K} - 1 = 0. \tag{4-57}$$

The solution to this equation is

$$\hat{K} = .0905. \tag{4-58}$$

Note that the value indicated by Equation (4-58) corresponds to the steady state value of $\underline{K}(t)$ as indicated in Figure 24.

It should now be evident that the desired final value of the state variable $\underline{\hat{z}}(t)$ plays a significant part in determining whether or not the matrix $\underline{K}(t)$ can be considered constant. If the weighting matrix, $\underline{F}$, in the performance criteria of Equation (4-7) can be set equal to zero thus free $\underline{\hat{z}}(t)$ from achieving any specific final value, then for most systems $\underline{K}(t)$ can be taken to be a constant coefficient matrix. Thus, it is possible to determine the constant matrix value of $\underline{K}(t)$, i.e., $\underline{\hat{K}}$, for a specific system from

Equation (4-46) and then utilize this constant coefficient matrix, as opposed to the time variant matrix $\underline{K}(t)$, in the augmenting system. Obviously, the use of this constant coefficient matrix significantly simplifies the implementation of the augmenting system.

Summary. The purpose of the above discussion has been to provide a simple and direct method for deriving the augmenting system requirements for linear man-machine output regulator systems. By using the matrix Ricatti equation and its supporting mathematical conditions, the above purpose was accomplished and then illustrated through the solution of simple example problem. With this method for deriving the augmenting system requirements at hand, it is possible to progress on to the derivation of the augmenting system requirements for the man-machine tracking system, and finally, to the statement of a general algorithm which is applicable to the design of augmenting systems for both the output regulator and tracking systems.

Tracking System

The method of augmentation for the tracking system closely follows that of the output regulator system. However, an exception does arise in that the tracking system requires a referenced input to be followed. This exception is discussed in the proceeding development. Returning to the canonical equations as described by Equations (4-29) and (4-34) for illustration, it can be assumed (17) that the solution trajectories are related by the expression

$$\underline{p}(t) = \underline{K}(t)\hat{\underline{z}}(t) - \underline{g}(t) \tag{4-59}$$

where t is an element of $[t_0, t_f]$, $\underline{K}(t)$ is a symmetric $(n_T \times n_T)$, and $\underline{g}(t)$ is a column vector of dimension n_T. The implications of Equation (4-59) can be seen more clearly if it is substituted into Equation (4-38) to yield

$$\underline{x}_d(t) = -\underline{W}^{-1}\underline{B}^T(\underline{K}(t)\hat{\underline{z}}(t) - \underline{g}(t)). \tag{4-60}$$

For further clarity, Equation (4-60) can be combined with Figure 22 to produce the augmented system diagram of Figure 25. The relationship given by Equation (4-59) is equivalent to that given by Equation (4-37) with the exception of the additional term $\underline{g}(t)$. This additional term appears in Equation (4-59) because it is needed to force the augmenting system to compensate for the referenced input which the augmented system is required to track. Once the variables $\underline{K}(t)$ and $\underline{g}(t)$ are determined, then the augmenting system can readily be designed.

As was the case with the output regulator system of the previous section, the tracking system can be augmented if a matrix $\underline{K}(t)$ and additionally if a vector $\underline{g}(t)$ can be found to satisfy Equation (4-59). The solution for $\underline{K}(t)$ and $\underline{g}(t)$ will be the subject of the subsequent paragraphs.

If Equation (4-59) is differentiated with respect to time, the following relation results:

$$\dot{\underline{p}}(t) = \dot{\underline{K}}(t)\hat{\underline{z}}(t) + \underline{K}(t)\dot{\hat{\underline{z}}}(t) - \dot{\underline{g}}(t). \tag{4-61}$$

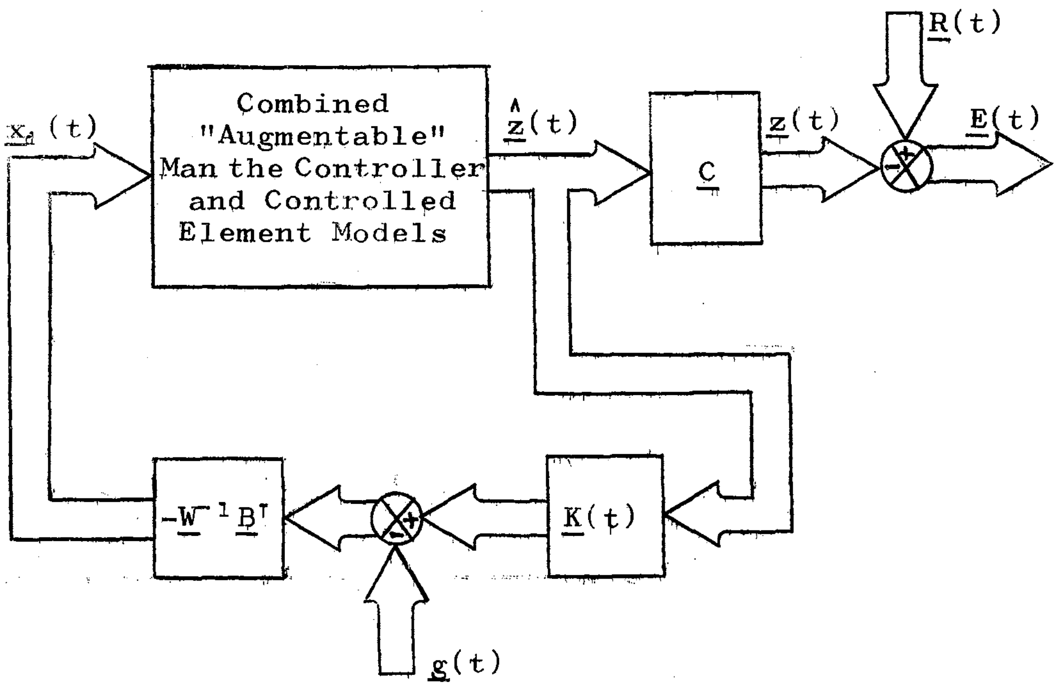


Figure 25. Vector Diagram of Augmented Tracking System Showing Components for Augmenting System

Again, Equations (4-29) and (4-34) can be written as

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) - \underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{p}(t) \tag{4-29}$$

and

$$\dot{\underline{p}}(t) = -\underline{C}^{T}\underline{Q}\,\underline{C}\,\hat{\underline{z}}(t) - \underline{A}^{T}\underline{p}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t). \tag{4-34}$$

If Equation (4-59) is substituted into Equation (4-29), the following relation is obtained:

$$\dot{\hat{\underline{z}}}(t) = (\underline{A} - \underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t)\hat{\underline{z}}(t) + \underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{g}(t). \tag{4-62}$$

Similarily, the substitution of Equation (4-62) into Equation (4-61) yields

$$\begin{aligned}\dot{\underline{p}}(t) = (\dot{\underline{K}}(t) + \underline{K}(t)\underline{A}(t) - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t))\hat{\underline{z}}(t)\\ + \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{g}(t) - \dot{\underline{g}}(t)\end{aligned} \tag{4-63}$$

and the substitution of Equation (4-59) into Equation (4-34) yields

$$\dot{\underline{p}}(t) = -(\underline{C}^{T}\underline{Q}\,\underline{C} + \underline{A}^{T}\underline{K}(t))\hat{\underline{z}}(t) + \underline{A}^{T}\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t). \tag{4-64}$$

Finally, if Equations (4-63) and (4-64) are equated, the result is

$$\begin{aligned}(\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C})\hat{\underline{z}}(t) =\\ \dot{\underline{g}}(t) - (\underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T} - \underline{A}^{T})\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t)\end{aligned} \tag{4-65}$$

for t, an element of $[t_0, t_f]$. Now, as long as an optimal solution exists, Equation (4-65) must hold for all $\hat{\underline{z}}(t)$, $\underline{R}(t)$, and t. Thus, it is possible to conclude that

$$\underline{\mathring{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0} \quad (4\text{-}66)$$

and

$$\underline{\mathring{g}}(t) - (\underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T} - \underline{A}^{T})\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t) = \underline{0}. \quad (4\text{-}67)$$

Furthermore, the boundary conditions may be derived from Equation (4-59) and Equation (4-36) as

$$\underline{p}(t_f) = \underline{K}(t_f)\underline{\hat{z}}(t_f) - \underline{g}(t_f) \quad (4\text{-}68)$$

and

$$\underline{p}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C}\,\underline{\hat{z}}(t_f) - \underline{C}^{T}\underline{F}\,\underline{R}(t_f). \quad (4\text{-}36)$$

However, since Equations (4-68) and (4-36) must hold for all $\underline{\hat{z}}(t_f)$ and $\underline{R}(t_f)$ it is possible to conclude that

$$\underline{K}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C} \quad (4\text{-}69)$$

and

$$\underline{g}(t_f) = \underline{C}^{T}\underline{F}\,\underline{R}(t_f). \quad (4\text{-}70)$$

The differential equation given by Equation (4-66) is identical to that given in the previous section by Equation (4-44). The method of solution is the same also, so the solution for $\underline{K}(t)$ is not covered again in this section. The major emphasis of the proceeding paragraphs is upon obtaining a solution, $\underline{g}(t)$, for Equation (4-67).

A point that should be made before proceeding is in regard to the a priori knowledge required of the tracking input $\underline{R}(t)$ in the backwards integration solution for $\underline{g}(t)$. Obviously, the need for this knowledge and the requirement for backwards integration pose a severe limitation upon the development of a tracking type augmenting system when sufficient information cannot be obtained to describe or predict the nature of $\underline{R}(t)$. When $\underline{R}(t)$ is deterministic, i.e., step, ramp, etc., and thus known a priori, it is possible to redefine the state variables of Equation (4-1) such that they include $\underline{R}(t)$. This inclusion reduces the tracking problem to an output regulator problem and, thus, eliminates the need to solve for $\underline{g}(t)$. However, when $\underline{R}(t)$ is non-deterministic, it is very difficult to obtain knowledge of the future values of $\underline{R}(t)$ so that they can be incorporated into the backwards integration solution for $\underline{g}(t)$. The approach that must be taken when dealing with non-deterministic inputs is to examine the statistical nature of $\underline{R}(t)$. If the statistical parameters of $\underline{R}(t)$ (An example of an $\underline{R}(t)$ possessing good statistical parameters is the wind; e.g., an airplane being perturbed from an equilibrium state by the wind.) are known, then it would be possible to predict the future values of $\underline{R}(t)$ and, thus, incorporate them into a forward integration solution for $\underline{g}(t)$. The investigation of the statistical nature of $\underline{R}(t)$ and the forward integration solution for $\underline{g}(t)$ is an area which definitely warrants further study.

Solution for $\underline{g}(t)$. As was the case in solving for $\underline{K}(t)$, the solution for $\underline{g}(t)$ must be obtained by utilizing backwards integration via a digital computer integration routine. However, as can be seen in Equation (4-67), the solution for $\underline{g}(t)$ is dependent upon the value of $\underline{K}(t)$. Thus, it is necessary to compute $\underline{K}(t)$ prior to or at least simultaneously with the solution for $\underline{g}(t)$. The solution for $\underline{g}(t)$ can be greatly simplified if the value of $\underline{K}(t)$ can be assumed to be the earlier mentioned constant $\hat{\underline{K}}$. Once the solution trajectories for $\underline{K}(t)$, or $\hat{\underline{K}}$ if possible, and $\underline{g}(t)$ are obtained, then they can be used in conjunction with the augmenting system of Figure 25 to produce the desired control $\underline{x}_d(t)$.

The solution for $\underline{g}(t)$ is a simple matter of substituting the appropriate values for the coefficient matrices into Equation (4-67) and then integrating the resulting set of first order differential equations utilizing the boundary conditions of Equation (4-70). To illustrate how this solution is obtained, the example of the previous section is adapted into the form of a tracking system and then solved.

Example. The example of the previous section is easily adapted into a tracking system (see Figure 13) if the value of the referenced input is assumed to have some value other than zero. Specifically, this example is assumed to have a referenced input of a unit step, i.e., $R = 1$. The state model for the "augmentable" system remains the same as before as defined by Equations (4-50), (4-51), and (4-52).

The performance criteria is defined by

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f}[\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-8}$$

where

$$\underline{F} = [0]$$

$$\underline{Q} = [1] \tag{4-71}$$

$$\underline{W} = [.01].$$

The above values for the matrices can be substituted into Equation (4-66) to yield the same expression as given by Equation (4-55) in the previous example. If these values for the matrices are substituted into Equation (4-67), the result is

$$\dot{g}(t) - (\underline{K}(t)[1][.01]^{-1}[1] - [-1]g(t) + [1][1][1] = 0 \tag{4-72}$$

or

$$\dot{g}(t) - (100K(t) + 1)g(t) + 1 = 0 \tag{4-73}$$

with the boundary condition of

$$g(t_f) + \underline{C}^T \underline{F}\,\underline{R}(t_f) = 0. \tag{4-74}$$

If the example of the previous section is considered and the same final time, $t_f = 2$, is used, then Equation (4-73) can be solved concurrently with Equation (4-55) by utilizing backwards integration. Although this integration is easily carried out by utilizing a digital computer, its solution is not pursued here. Instead, Equation (4-73) is solved under the assumption that the value of K(t) has the constant value as defined in the previous example. The simplification allowed by making this assumption permits g(t) to be easily determined analytically, yet it still illustrates the role that g(t) has with the augmented tracking system.

By taking the value of K(t) to be the constant

$$\hat{K} = .0905, \qquad (4\text{-}58)$$

Equation (4-73) can be written as

$$\dot{g}(t) - 10.05g(t) + 1 = 0, \qquad (4\text{-}74)$$

with the boundary condition of

$$g(t_f) = 0.$$

The solution to Equation (4-74) is obtained through backwards integration and is shown illustrated by the trajectory shown in Figure 26.

By making use of the constant value assumed for K(t) and the solution trajectory obtained for g(t), it is possible to implement the augmentation of the example tracking system. This system, when augmented, appears as shown in Figure 25.

Summary

As was indicated in the preceding developments on the output regulator and tracking systems, the purpose of this section has been to provide a method whereby the two point boundary value problem could be solved and simultaneously provide sufficient mathematical information to design the augmenting systems. This purpose was accomplished by the

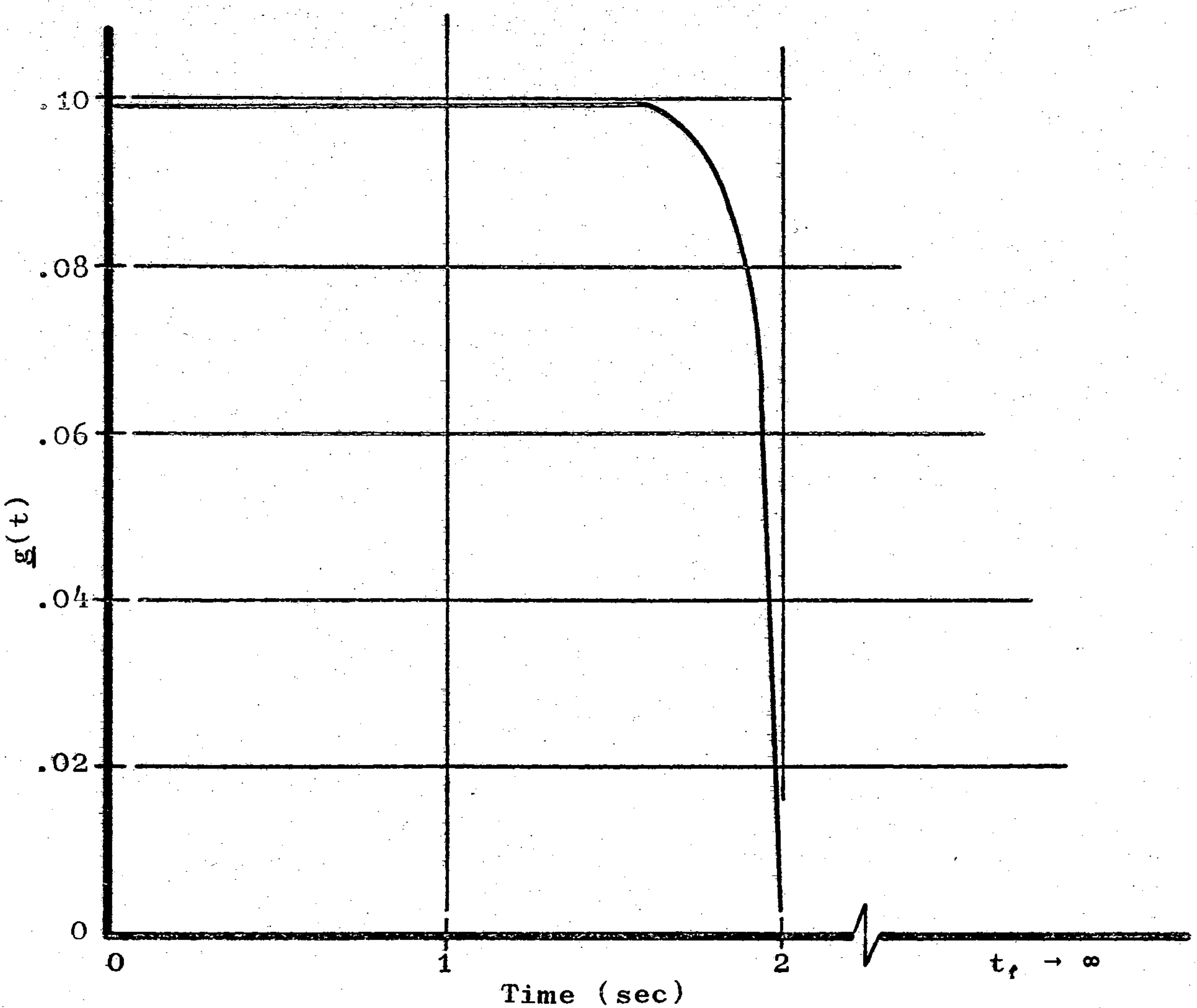


Figure 26. Solution Trajectory for $\underline{g}(t)$ for Example Tracking Problem

use of the matrix Ricatti equation for the output regulator system and by the use of the matrix Ricatti and another associated matrix equation for the tracking system. With these methods for solution of two point boundary value problem and for design of augmenting systems available, it is now possible to move on to the final development of this chapter. This development is the formulation and statement of the general algorithm for the derivation of augmenting system requirements for linear man-machine systems.

An Algorithm for the Design of Augmenting Systems

The procedures outlined in this section summarize in algorithm form the detailed discussions regarding the application of the method for augmenting system design presented in the preceding sections. The type of man-machine systems to which the following procedures are applicable are those which are adequately modeled by linear, time invariant, ordinary differential equations. In general, these procedures are oriented to the analytical design of an augmenting system.

Select the Model for Man

The initial step in the design of the augmenting system, i.e., assuming a device to be controlled is available, must be for the designer to specify the dynamic model which he desires man to have in his augmented role. Specifically, the selection of the model for man is dependent upon whatever dynamic simplicity is desired of him. If it is desired for man to be able to control a complex dynamical system with dynamic ease, then a dynamical model which depicts him as a simple controller (e.g., a dynamic model of man as he provides the control for a pure gain element in a simple tracking task) should be used as his model in the design of the augmenting system.

As was indicated, the selection of the desired model for man is the primary step in the design of the augmenting system. However, unless the desired model is known a priori it may first be necessary to gain a basic knowledge of what does and does not represent a simple model for man. Although a review which would provide such a basic knowledge is beyond the scope of this thesis, the reader can become sufficiently familiar with dynamical models of man by reviewing reference (5) and its contributing references.

Obtain the State Model for the "Augmentable" System

The next step in obtaining the augmenting system requirements is to arrange the selected model for man and the assumed existent device to be controlled into the form of the "augmentable" system as progressively shown in Figures 18, 19, 20, and 21. As the figures indicate, the arrangement into this form is a simple matter of inserting the models for man and into their respective block diagrams and then feeding back his output as shown. Once this "augmentable" form is obtained, then the resultant system can be defined by the following state model:

$$\dot{\underline{\hat{z}}}(t) = \underline{A}\,\underline{\hat{z}}(t) + \underline{B}\,\underline{x}_d(t) \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\underline{\hat{z}}(t). \tag{4-2}$$

(The vectors and matrices appearing in these equations are defined as to their function earlier in this chapter.)

Define the Augmented System

Performance Criteria

The next step in augmenting is to define the quadratic criteria by which the augmented system performance is to be extremized. Two general performance criteria are available for this extremization. One is for the output regulator man-machine system and is given by

$$J_O = \tfrac{1}{2}\langle \underline{z}(t_f), \underline{F}\,\underline{z}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt. \tag{4-7}$$

The other is for the tracking man-machine system and is given by

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt. \tag{4-8}$$

Again, the definitions for the vectors and matrices in these equations are presented earlier in this chapter.

It is apparent from Equation (4-7) and (4-8) that some discretion must be used in selecting the coefficient values for the respective weighting matrices. There are an infinite number of possibilities for selecting these coefficients for any given man-machine system to be augmented, and the magnitude of the coefficients selected often has a pronounced affect on the optimal response of the man-machine system.

Although the selection of the proper coefficients for the weighting matrices can become quite involved, there are some simplifications which can be made which are valid for the majority of optimal systems. First of all, the weighting matrices of Equations (4-6) and (4-7) can be assumed to be either diagonal matrices which individually have equal coefficients on the major diagonal, or they can be assumed to be zero matrices. Secondly, a general rule of thumb can be used which states that the regulator output or tracking error should be weighted 50 to 100 times more than the control (17). If these simplifications do not produce desirable outputs when used, then it may be necessary to experiment with different values for the matrix coefficients until desirability is obtained.

Determine the Respective Ricatti Equation

Depending upon whether the system to be augmented is either an output regulator system or a tracking system, the next step is to either determine the coefficients for the matrix Ricatti equation or for the matrix Ricatti and its associated matrix equation. If the system is an output regulator, then the matrix values from Equation (4-1), (4-2), and (4-7) are used to define the coefficients in the matrix Ricatti equation as given by

$$\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0} \qquad (4\text{-}44)$$

with the boundary conditions of

$$\underline{K}(t_f) + \underline{C}^{T}\underline{F}\,\underline{C}. \qquad (4\text{-}45)$$

Similarly, if the system to be augmented is a tracking system, then the matrix values obtained from Equations (4-1), (4-2), and (4-8) are used to define the coefficients in the matrix Ricatti equation given by

$$\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0} \qquad (4\text{-}66)$$

and the associated matrix equation given by

$$\dot{\underline{g}}(t) - (\underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T} - A^{T})\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t) = \underline{0} \qquad (4\text{-}67)$$

with the boundary conditions given by

$$\underline{K}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C} \qquad (4\text{-}69)$$

and

$$\underline{g}(t_f) = \underline{C}^{T}\underline{F}\,\underline{R}(t_f). \qquad (4\text{-}70)$$

Solve the Ricatti Equations

If the system being augmented is of the output regulator type, then the next step is to utilize the method of backwards integration as mentioned earlier to solve the matrix Ricatti equation as given by Equation (4-44) to obtain its solution trajectory, $\underline{K}(t)$.

If the system being augmented is of the tracking type, then it is necessary to utilize the method of backwards integration to simultaneously solve the matrix Ricatti equation as given by Equation (4-66) and its associated matrix equation as given by Equation (4-67) to obtain the solution trajectories $\underline{K}(t)$ and $\underline{g}(t)$.

If the final value of the system trajectory, i.e., $\underline{\hat{z}}(t_f)$ for the output regulator system and $\underline{E}(t_f)$ for the tracking system, is not significant and if the final time t_f is large, then it is possible to obtain the constant value for $\underline{K}(t)$, i.e., $\underline{\hat{K}}$, from the simultaneous nonlinear equations given by

$$-\underline{\hat{K}}\,\underline{B}\,\underline{W}^{-1}\underline{B}^T\underline{\hat{K}} + \underline{\hat{K}}\,\underline{A} + \underline{A}^T\underline{\hat{K}} + \underline{C}^T\underline{Q}\,\underline{C} = \underline{0} \qquad (4\text{-}46)$$

for both the output regulator and tracking systems.

Once the value of $\underline{\hat{K}}$ is obtained, it can be used in place of $\underline{K}(t)$ for both the output regulator and tracking systems. In the latter case, $\underline{\hat{K}}$ can be used in conjunction with Equation (4-67) to aid in the solution for $\underline{g}(t)$.

Use the Ricatti Solutions to Specify the Augmenting System Gain

By making use of the solutions obtained for either $\underline{K}(t)$ or $\underline{K}(t)$ and $\underline{g}(t)$ or $\underline{\hat{K}}$, it is possible to specify the gain required of the augmenting system to be able to generate the optimal and desired output $\underline{x}_d(t)$. If the system being augmented is of the output regulator type then, as is illustrated in Figure 23, the gain for the augmenting system is obtained by combining the time variant matrix $\underline{K}(t)$ with the constant coefficient matrices $-W^{-1}$ and $\underline{B}^T$. The result of this combination is to produce the augmenting system gain as the time variant matrix $-\underline{W}^{-1}\underline{B}^T\underline{K}(t)$. When the state variable $\hat{\underline{z}}(t)$ is interjected into the augmenting system, the optimal output $\underline{x}_d(t)$ is generated. If the Ricatti solution $\underline{K}(t)$ can be assumed to be the constant $\hat{\underline{K}}$, then the augmenting system gain can be taken to be the time invariant matrix $-\underline{W}^{-1}\underline{B}^T\hat{\underline{K}}$. This latter assumption makes the augmenting system much simplier because of the time invariance of $\underline{K}(t)$.

If the system being augmented is of the tracking type, then the augmenting system output is obtained by first, combining (see Figure 25 for illustration) the time variant matrix $\underline{K}(t)$ with the system state variable $\hat{\underline{z}}(t)$ to produce the vector $\underline{K}(t)\hat{\underline{z}}(t)$; then, subtracting the vector $\underline{g}(t)$ from the vector $\underline{K}(t)\hat{\underline{z}}(t)$; and finally, combining the resultant difference $(\underline{K}(t)\hat{\underline{z}}(t) - \underline{g}(t))$ with the matrices $-W^{-1}$ and $\underline{B}^T$ to produce $-W^{-1}\underline{B}^T(\underline{K}(t)\hat{\underline{z}}(t) - \underline{g}(t))$ as the desired augmenting system output $\underline{x}_d(t)$. Again, the augmenting system can be greatly simplified if $\underline{K}(t)$ can be taken to be the constant $\hat{\underline{K}}$.

In summary, the developments of this chapter have been concerned with providing the basic mathematical and optimal control theories that are necessary as tools in establishing the method for designing human augmenting systems. The use of these theories has permitted an analytical approach to be taken to specifying the requirements for systems which permit man to be capable of providing control to dynamical systems which will cause their output to behave in an optimal manner. This method goes far beyond those discussed in Chapter III by permitting the system designer to not only design the augmenting system analytically by having control over the dynamical requirements that will be made upon man, but also by being able to assure that the system being controlled by man will have an optimal output. Additionally, the control over man's dynamical requirements allows his perceptual and dynamical abilities to be compensated to whatever degree the designer may desire.

CHAPTER V

APPLICATION OF THE METHOD

Several example problems which illustrate the application of the method for augmentation developed in the previous chapter are presented and discussed in this chapter. The purpose of working these examples is to: show how the algorithm for designing augmenting systems is utilized; verify the importance of the optimal control theory as a basis for designing the augmenting system and at the same time verify the importance of being able to augment; and finally, illustrate the utility of the method developed for augmentation.

The examples presented were chosen primarily to illustrate that the requirements imposed upon the proposed method for augmentation were achieved. The problems include:

1. A simple single axis system which is intended to show how the algorithm of the previous chapter is applied.
2. A dynamical controlled element which man is unable to control without augmentation. Thus, a dynamical model must be chosen for man and then he must be augmented to be

able to optimally control this normally unstable system.

3. A multi-axis system in which man is required to control more than one axis. The purpose here is to show how man can be augmented in the more difficult to control multi-axis situation.
4. A VTOL aircraft which is depicted as a multi-axis system with crossfeed is required to hover when it is disturbed by a gust of wind. A model must be selected for man, and he must be augmented to be able to control the simulated aircraft with optimality.
5. A Ford Motor Company, series 755 Backhoe is discussed as a possible candidate for augmentation. This discussion is presented as a consequence of research done by Mr. G. E. Maroney and this author (see reference (26)).

Example One - Single Axis, Simple Controlled Element

The first example is intended only to show how the algorithm of the preceding chapter is utilized. The controlled element and model for man were chosen from reference (27) on the basis of the dynamical simplicity of the controlled element. It was experimentally shown that man was quite capable of exerting the control necessary to make the output of a dynamical system defined as the transfer function

$$G_C(s) = 2 \tag{5-1}$$

track a referenced input (27). Figure 27 illustrates the system configuration. The transfer function obtained for man in exerting this control was

$$G_H(s) = \frac{.93}{(1 + .14s)^2}. \tag{5-2}$$

The nature of the referenced input was to have been "random appearing" to eliminate the precognitive abilities of man. To achieve this "random appearing" nature, the input was selected (27) to be

$$R(t) = \sum_{i=1}^{10} K_i \sin w_i t \tag{5-3}$$

where

$K_j = .3142,\ j = 1, 2, \ldots, 5$

$K_k = .03142,\ K = 5, 6, \ldots, 10$

$w_i = .157, .262, .393, .602, .969, 1.49, 2.54, 4.03,$

$\quad 7.57, 13.8$ for $i = 1, 2, \ldots, 10$, respectively.

The values of these coefficients were selected to allow the input to have a root-mean-squared amplitude of 0.5 at a roll-off frequency of 1.5 rad/sec (for further details see

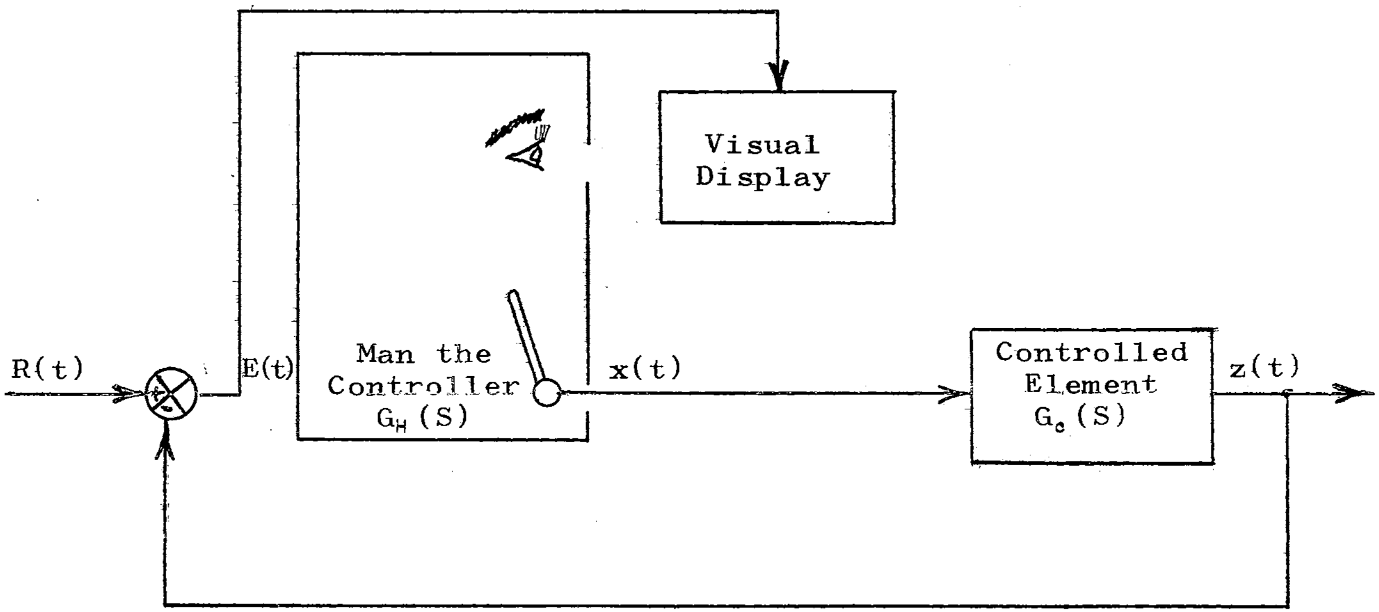


Figure 27. Simplified Block Diagram of Unaugmented System Used in Example One

reference (28)). A time trajectory of this input is shown in Figure 28.

The first step in the algorithm is to obtain the model for man. However, for this example, the model is already known and is given by Equation (5-2). The next step is to obtain the state model for the "augmentable" system. To obtain this system, it is necessary to first eliminate the feedback path shown in Figure 27 to produce the diagram illustrated in Figure 29a. Next, man's output is fedback to obtain the "augmentable" system as shown in Figure 29b. Now, the state model for this system can be obtained by first defining the transfer relationship obtained from

$$X = G_H E = G_H (x_d - x) \tag{5-4}$$

as

$$\frac{x}{x_d} = \frac{G_H}{1 + G_H} \tag{5-5}$$

and then utilizing block diagram algebra to produce the relationship

$$\frac{z}{x_d} = \frac{G_H G_C}{1+G_H} = \frac{100}{s^2 + 14.3s + 98.47} \tag{5-6}$$

which can be expressed in the time domain as

$$\ddot{z} = -14.3\dot{z} - 98.47z + 100x_d \, . \tag{5-7}$$

The state model for the "augmentable" system can be written from Equation (5-7) as

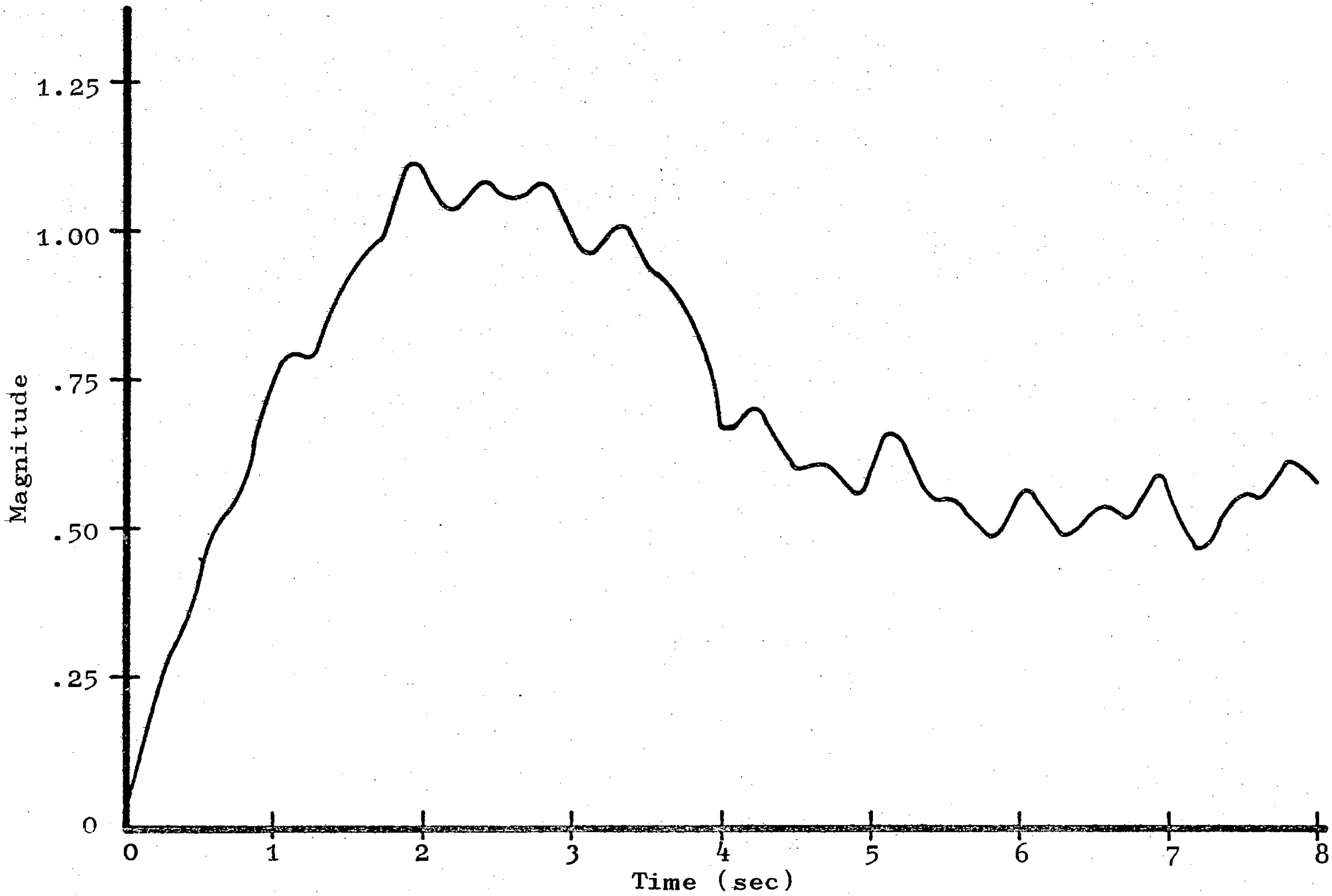
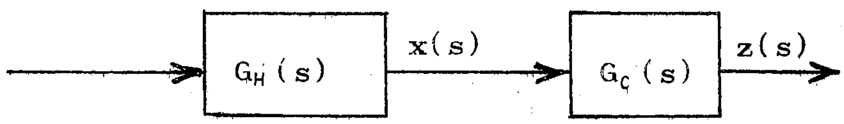
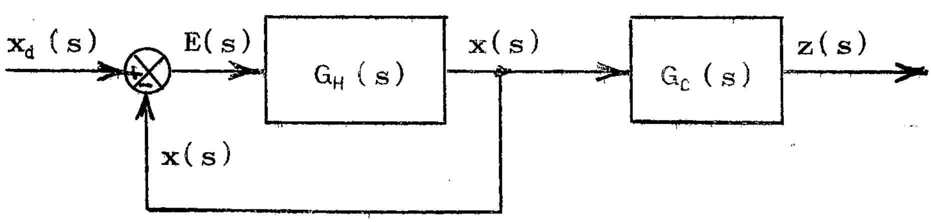


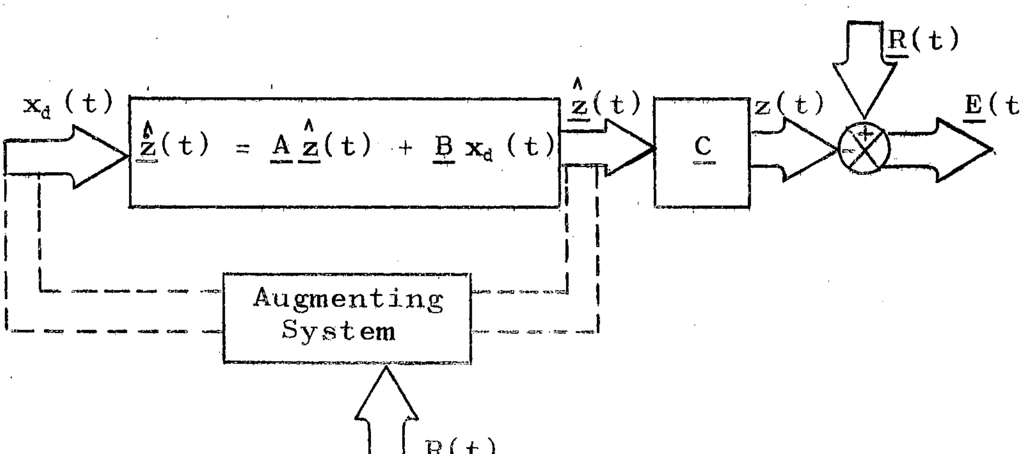
Figure 28. Input Waveform for Example One



a.) Scalar Block Diagram of Figure 27 With Feedback Path Removed



b.) Same as a.) but With Feedback Path Added Around Man the Controller



c.) Vector Diagram of Augmentable System

Figure 29. Illustration of Steps Taken to Obtain "Augmentable" System for Example One

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) + \underline{B}\,\underline{x}_d(t); \quad \hat{\underline{z}}(t_0) = \underline{0} \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\hat{\underline{z}}(t) \tag{4-2}$$

where

$$\underline{A} = \begin{bmatrix} 0 \\ -98.47 \end{bmatrix} \begin{bmatrix} 0 \\ -1.43 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ 100 \end{bmatrix} \tag{5-8}$$

$$\underline{C} = [1 \quad 0].$$

With the state model thus defined, it is possible to view the "augmentable"system as shown in Figure 29c.

The next step is to define the performance criteria for augmentation. Since the system under consideration is of the tracking type, the performance criteria can be defined from Equation (4-3) as

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-8}$$

where

$$\underline{F} = [0]$$

$$\underline{Q} = [1] \tag{5-9}$$

$$\underline{W} = [.01].$$

(Note that the weighting matrix for the error, $\underline{Q}$, is 100 times greater than the weighting matrix for the control, W.)

Given the performance index and its weighting functions, it is possible to define the Ricatti and its associated matrix equation as

$$\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0} \qquad (4\text{-}66)$$

and

$$\dot{\underline{g}}(t) - (\underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T} - \underline{A}^{T})\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t) = \underline{0} \qquad (4\text{-}67)$$

with the boundary conditions of

$$\underline{K}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C} \qquad (4\text{-}69)$$

and

$$g(t_f) = \underline{C}^{T}\underline{F}\,\underline{R}(t_f) \qquad (4\text{-}70)$$

where the weighting matrices are as defined by Equations (5-8) and (5-9). The substitution of these matrices into the above equations yields the set of first order differential equations

$$\begin{aligned} &\dot{K}_{11} - 10^{6}K_{12}^{2} - 2(98.47)K_{12} + 1 = 0 \\ &\dot{K}_{12} - 10^{6}K_{12}K_{22} + K_{11} - 14.3K_{12} - 98.47K_{22} = 0 \qquad (5\text{-}10) \\ &\dot{K}_{22} - 10^{6}K_{22}^{2} + 2(K_{12} - 14.3K_{22}) = 0 \end{aligned}$$

with the boundary conditions of

$$\begin{aligned} K_{11}(t_f) &= 0 \\ K_{12}(t_f) &= 0 \qquad (5\text{-}11) \\ K_{22}(t_f) &= 0 \end{aligned}$$

for the matrix Ricatti equation.

The associated matrix equation can be written as the set of first order differential equations

$$\dot{g}_1 = (98.47 + 10^6 K_{12})g_2 - R(t) \qquad (5\text{-}12)$$

$$\dot{g}_2 = -g_1 + (14.3 + 10^6 K_{22})g_2$$

with the boundary conditions of

$$g_1(t_f) = 0$$

$$(5\text{-}13)$$

$$g_2(t_f) = 0.$$

The next step in the procedure is to solve the Ricatti and associated matrix equations. To effect this solution, it was first necessary to choose a final time. A consideration that must be made in selecting this time is the nature of the tracking task. The nature of the referenced input is such that it could be representative of indefinite periods of tracking. As a consequence, the final time was assumed to be sufficiently large to allow $\underline{K}(t)$ to be assumed to be constant. Thus, in keeping with Equation (4-46), Equation (5-10) can be rewritten as the set of simultaneous nonlinear equations

$$10^6 \hat{K}_{12}^2 + 2(98.47)\hat{K}_{12} - 1 = 0$$

$$10^6 \hat{K}_{12}\hat{K}_{22} - \hat{K}_{11} + 14.3\hat{K}_{12} + 98.47\hat{K}_{22} = 0 \qquad (5\text{-}14)$$

$$10^6 \hat{K}_{22}^2 - 2(\hat{K}_{12} - 14.3\hat{K}_{22}) = 0$$

whose solution is

$$\hat{K}_{11} = 4.3559 \times 10^{-2}$$

$$\hat{K}_{12} = 8.9983 \times 10^{-4} \tag{5-15}$$

$$\hat{K}_{22} = 3.0302 \times 10^{-6}.$$

These values for $\underline{\hat{K}}$ make it possible to solve for $\underline{g}(t)$ without the simultaneous solution of $\underline{K}(t)$; however, some definite, finite value is needed for the final time if an exact solution is to be obtained. It is possible to assume a different value for the t_f associated with $\underline{g}(t)$ than for the t_f associated with $\underline{K}(t)$ in the situation where the latter t_f can be taken to be very large (17). Obviously, the advantage of this assumption is that it allows $\underline{K}(t)$ to be constant, yet permits a time trajectory to be found for $\underline{g}(t)$. The value of t_f was chosen such that

$$g_1(15) = 0$$
$$g_2(15) = 0. \tag{5-16}$$

This value was chosen only because it allowed a finite range of tracking to be observed. In an actually implemented application, the value of t_f for $\underline{g}(t)$ should be chosen to correspond to the desired time interval of tracking. By utilizing these final boundary conditions and backward integration upon the digital computer, the solution trajectories for $\underline{g}(t)$ were found to be as shown in Figure 30.

The next step is to use the Ricatti constants and the associated matrix equation trajectories, i.e., $\underline{\hat{K}}$ and $\underline{g}(t)$, to implement the augmenting system gain. This implementation is accomplished by using the equation

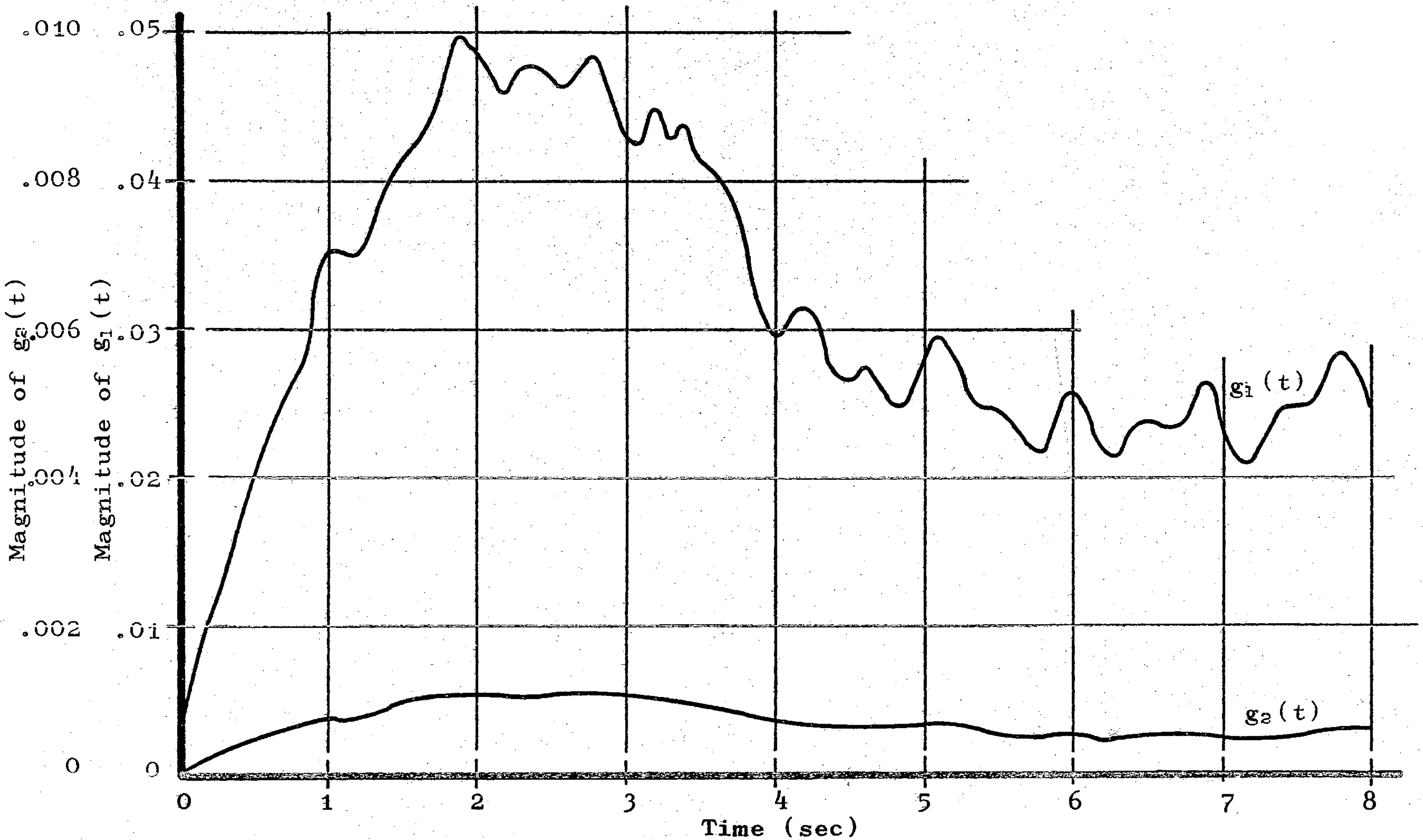


Figure 30. Solution Trajectory for $\underline{g}(t)$ - Example One

$$\underline{x}_d(t) = -\underline{W}^{-1}\underline{B}^T(\underline{K}(t)\hat{\underline{z}}(t) - \underline{g}(t)) \qquad (4\text{-}60)$$

which, with the proper values substituted in for the weighting matrices, becomes

$$\underline{x}_d(t) = -100(\hat{K}_{12}z_1(t) + \hat{K}_{22}z_2(t) - g_2(t)). \qquad (5\text{-}17)$$

This equation can be incorporated with the "augmentable" system to produce the diagram illustrated in Figure 31. This diagram depicts how the augmentation could be accomplished through algebraic operations once the constant $\underline{\hat{K}}$ and the variable $\underline{g}(t)$ are determined. The result of using this augmentation is shown in Figure 32. Also shown is the result obtained without augmentation. Note that both the augmented and non-augmented system outputs closely track the reference input even though the augmented system registers a lesser performance index. It is marginal as to whether or not augmentation is needed with a system which tracks this well on its own. However, as was pointed out at the first of this section, the system was chosen for augmentation only on the basis of its dynamical simplicity and concurrent ease of application for the augmentation procedure. The next example will deal with a system in obvious need of augmentation.

Example Two - Single Axis, Unstable Controlled Element

Example two is a clear illustration of a dynamical system which requires augmentation. In reference (27), it was

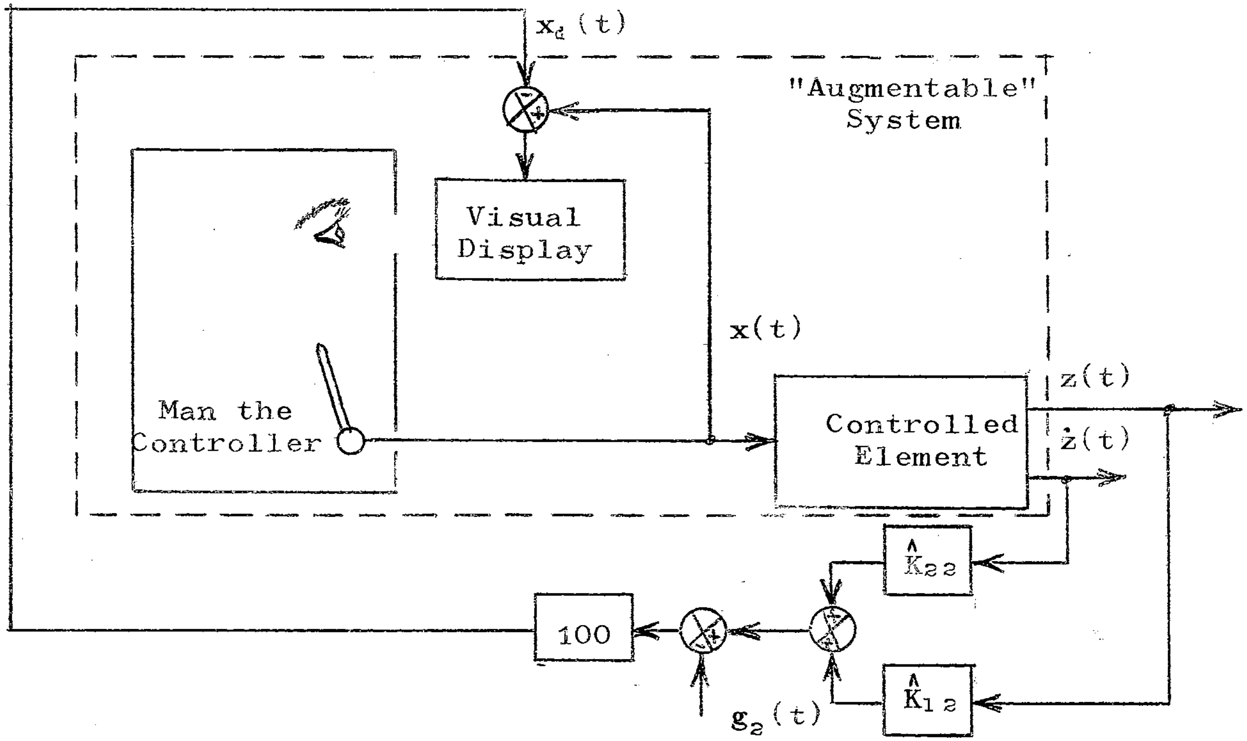


Figure 31. Block Diagram of Example One in Augmented Form

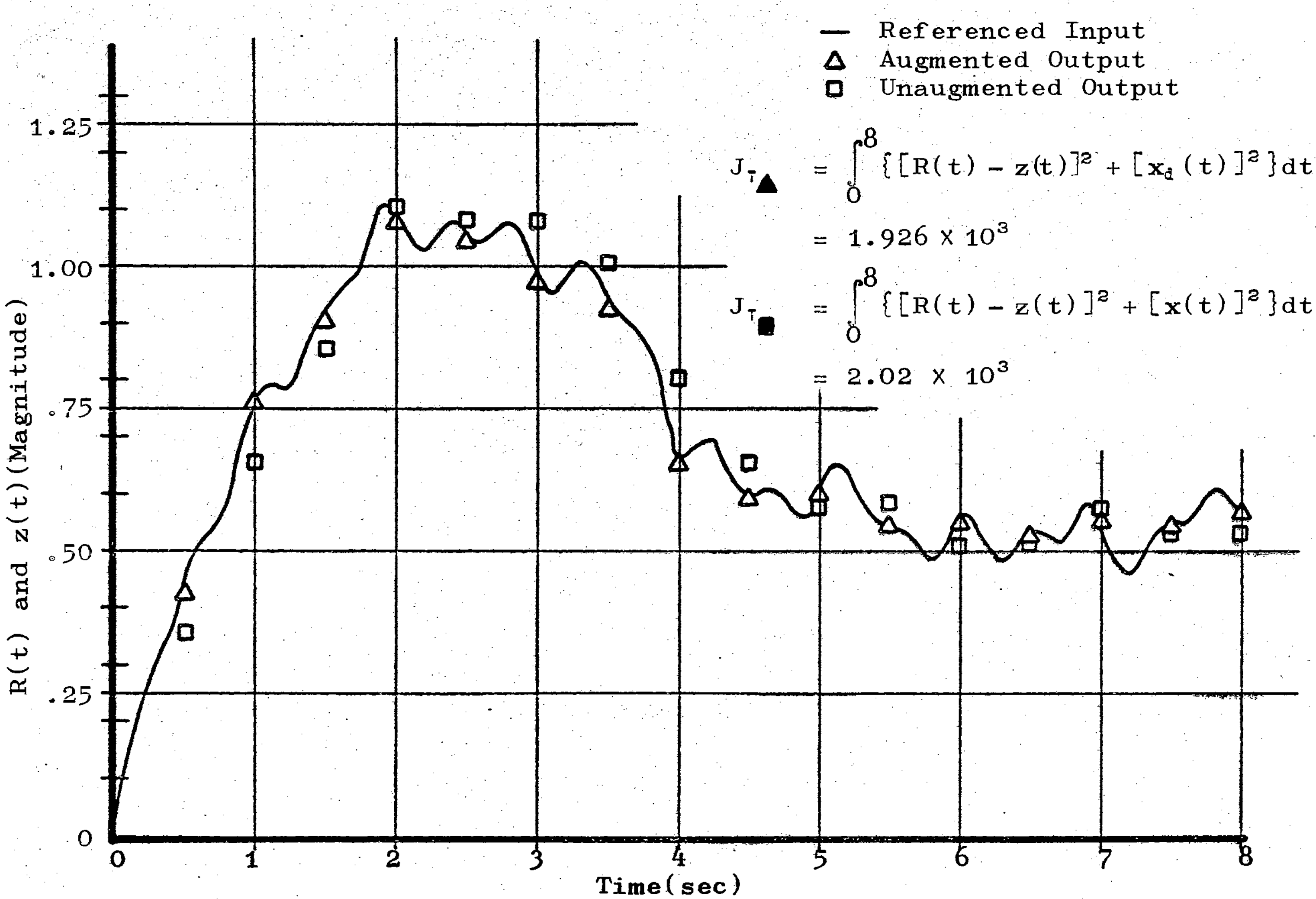


Figure 32. Augmented and Unaugmented Output for Example One

shown experimentally that man could not provide stable control for the controlled element defined in transfer function form as

$$G_C(s) = \frac{1}{s^3} \tag{5-18}$$

while tracking the referenced input defined by Equation (5-3) in Example One (This tracking situation is shown in Figure 27.). However, if it is assumed that Equation (5-18) defines a dynamical system which requires human control while tracking this input (The pitch control of a helicopter is an example of such a third order integrator, and it is known (8) to be extremely difficult to control when the pilot has only pitch information fed back to him.), then it is necessary to augment man by utilizing the method described in this research. Again, referring to the algorithm of the previous chapter, it is first necessay to select a model for man. Here, the decision must be made as to how dynamically simple or complex man should be in this application. It would seem that a relatively simple dynamical complexity would be desirable. Referring again to reference (27), a transfer function which describes man in a seemingly painless, yet comfortably challenging situation (i.e., when he is causing the output of a simple integrator to accurately track the input defined by Equation (5-3)) is defined by

$$G_H(s) = \frac{48(s + .0625)}{s(s + 25)}. \tag{5-19}$$

The "augmentable" system is again obtained by going through the steps illustrated in Figures 29a and 29b to produce the relationship

$$\frac{z}{x_d} = \frac{G_H G_C}{1 + G_H} \tag{5-6}$$

which is evaluated for this example as

$$\frac{z}{x_d} = \frac{48(s + .0625)}{s^3(s^2 + 73s + 3)}. \tag{5-20}$$

The corresponding state model as diagrammed in Figure 29c is

$$\dot{\hat{\underline{z}}}(t) = \underline{A}\,\hat{\underline{z}}(t) + \underline{B}\,\underline{x}_d(t);\ \hat{\underline{z}}(t_0) = 0 \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\hat{\underline{z}}(t) \tag{4-2}$$

where

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 & -73 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 48 \\ -3501 \end{bmatrix} \tag{5-21}$$

$$\underline{C} = [1 \quad 0 \quad 0 \quad 0 \quad 0].$$

The performance criteria can again be defined by

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-8}$$

where

$$\underline{F} = [0]$$
$$\underline{Q} = [1] \tag{5-22}$$
$$\underline{W} = [.01].$$

The Ricatti and its associated matrix equation are defined by

$$\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\,\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = 0 \tag{4-66}$$

and

$$\dot{\underline{g}}(t) - (\underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T} - \underline{A}^{T})\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t) = 0 \tag{4-67}$$

with the boundary conditions of

$$\underline{K}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C} \tag{4-69}$$

and

$$\underline{g}(t_f) = \underline{C}^{T}\underline{F}\,\underline{R}(t_f) \tag{4-70}$$

where the weighting matrices are as defined by Equations (5-21) and (5-22). The substitution of these matrices into Equations (4-66) allows the matrix Ricatti equation to be written as a set of 15 first order nonlinear differential equations (too numerous to be listed), with the boundary conditions of

$$\underline{K}(t_f) = \underline{0}. \tag{5-23}$$

The corresponding matrix differential equation can be written as a set of five first order nonlinear differential equations which have the boundary conditions of

$$\underline{g}(t_f) = 0. \tag{5-24}$$

The solution trajectories to the matrix Ricatti equation as defined by Equation (4-66) and the coefficient matrices of Equations (5-21) and (5-22) are shown in Figures 33-35. Note the "steady state" value reached by each of these trajectories after an initial transient interval. As was the case in Example One, it is logical to assume that the referenced input of Equation (5-3) would be representative of indefinite periods of tracking. Thus, the final time in $\underline{K}(t_f)$ can be assumed large (The value of t_f used in Figures 33-35 was chosen simply to illustrate the constancy of $\underline{K}(t)$.), and the values of $\underline{K}(t)$ taken to be the constants defined in Figures 33-35. If these constants are substituted for the K's given Equation (4-67), along with the coefficient matrices of Equations (5-21) and (5-22), it is possible to solve this equation for $\underline{g}(t)$ without the simultaneous solution of Equation (4-66) and the resultant time variant gain, $\underline{K}(t)$, in the augmenting system. By taking the final time for $\underline{g}(t_f) = \underline{0}$ to again be 15, it is possible to solve Equation (4-67) by backwards integration. The solution obtained for $\underline{g}(t)$ is shown in Figures 36 and 37.

The Ricatti gain, $\hat{\underline{K}}$, and associated matrix equation trajectory, $\underline{g}(t)$, can now be used to implement the augmenting system. The implementation can be accomplished through use of the equation

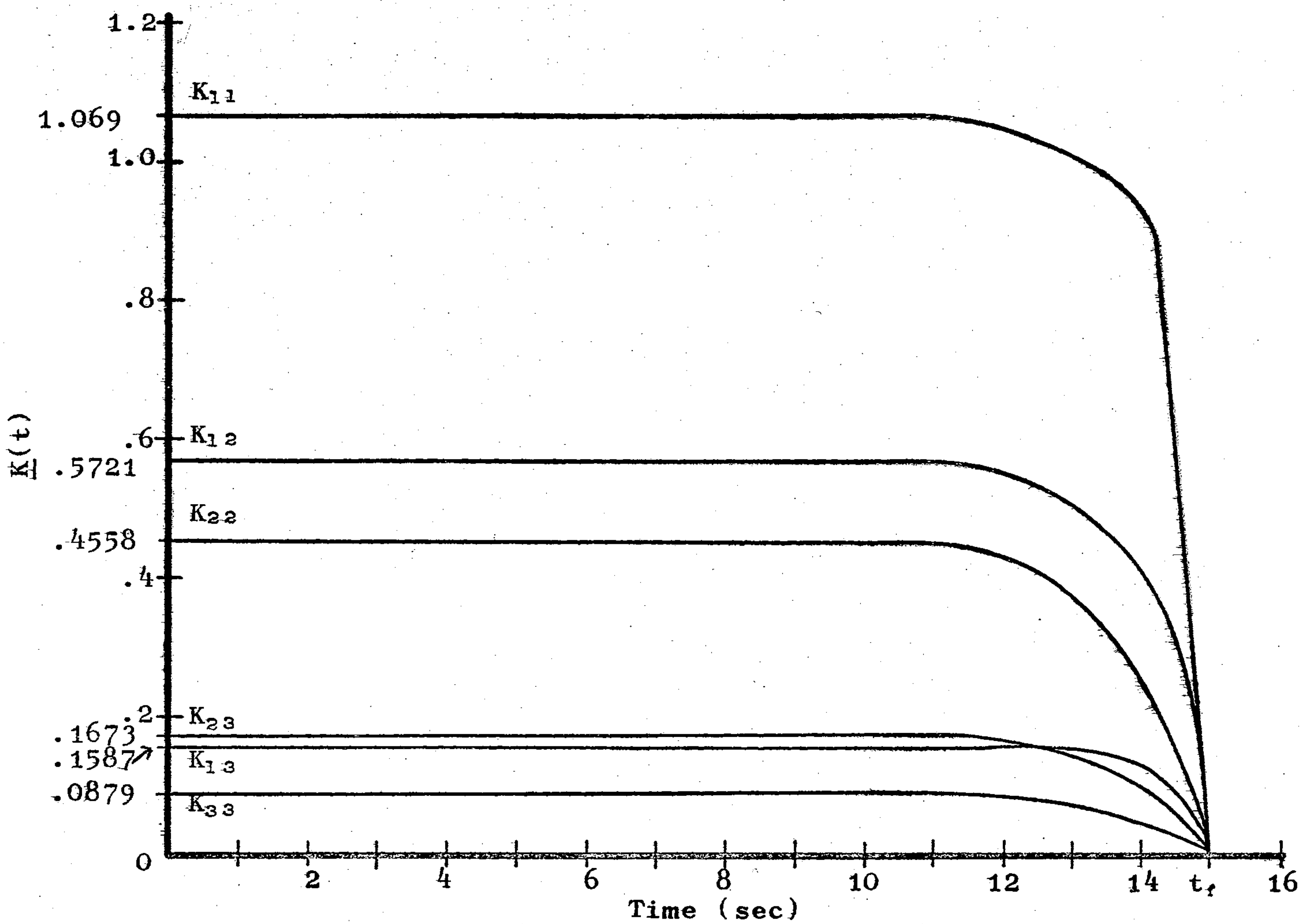


Figure 33. Solution Trajectories for Matrix Ricatti Equation of Example Two (Continued as Figures 34 and 35)

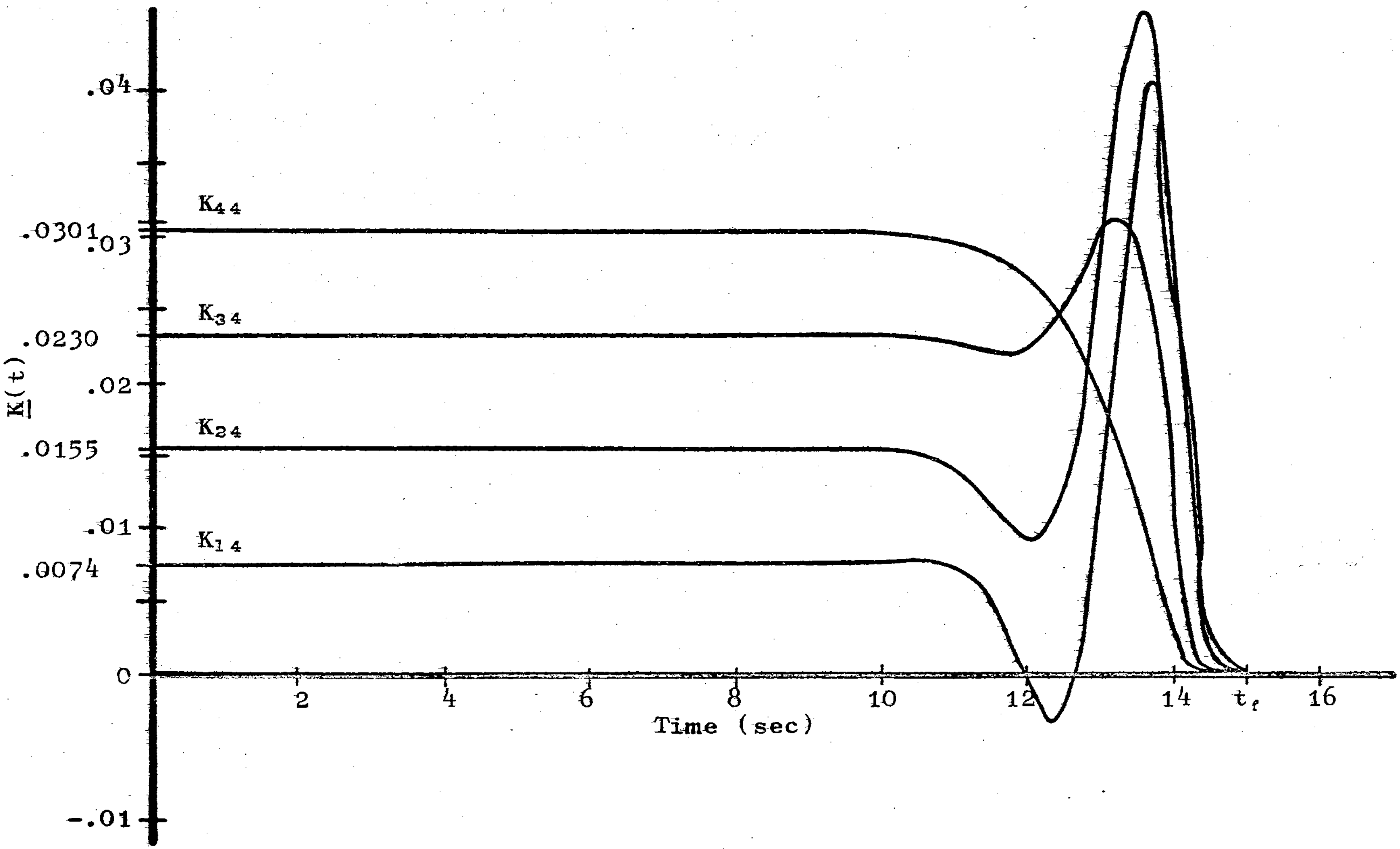


Figure 34. Solution Trajectories for Matrix Ricatti Equation of Example Two
(Continued from Figure 33)

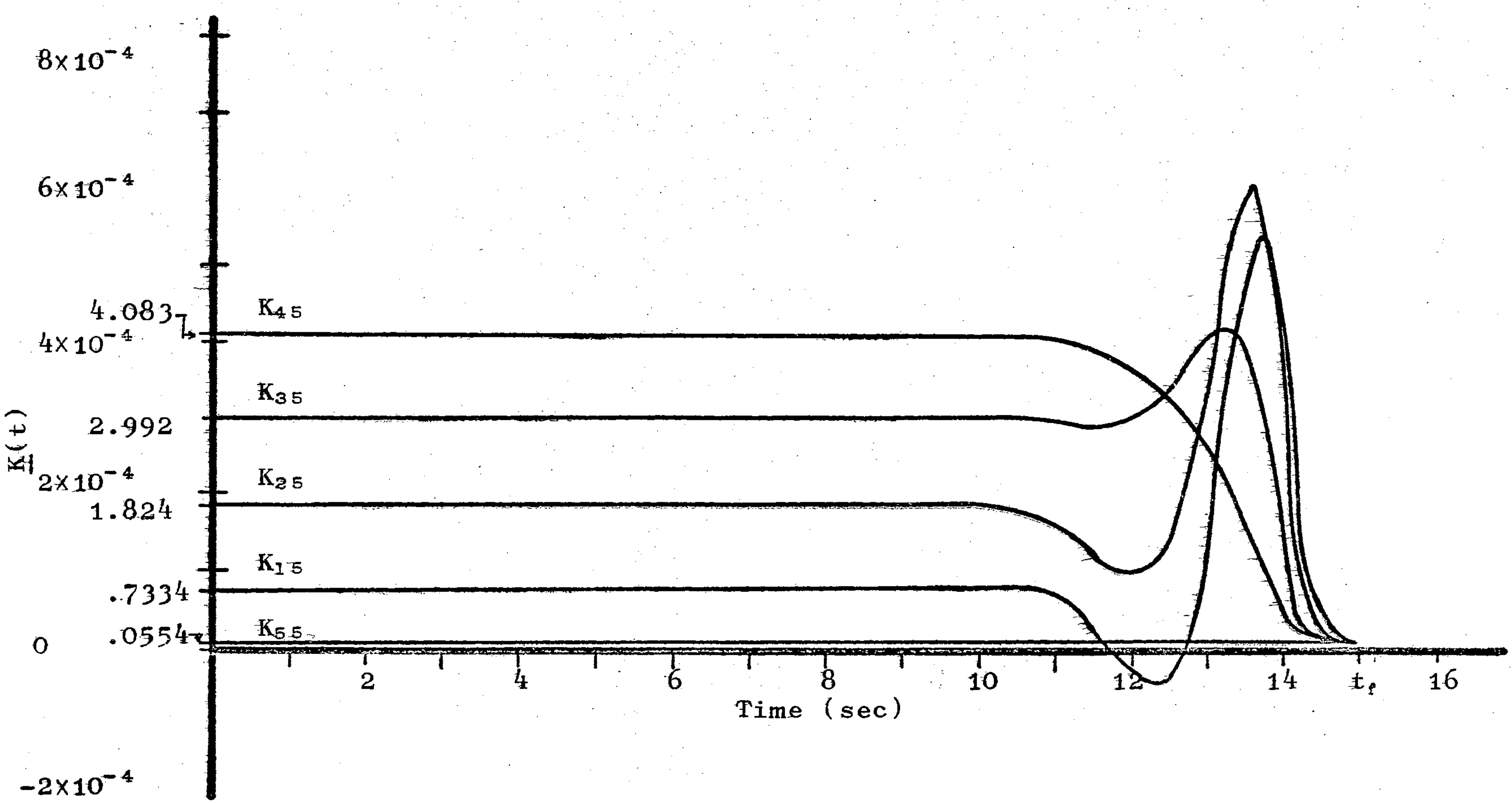


Figure 35. Solution Trajectories for Matrix Ricatti Equation of Example Two (Continued from Figure 33)

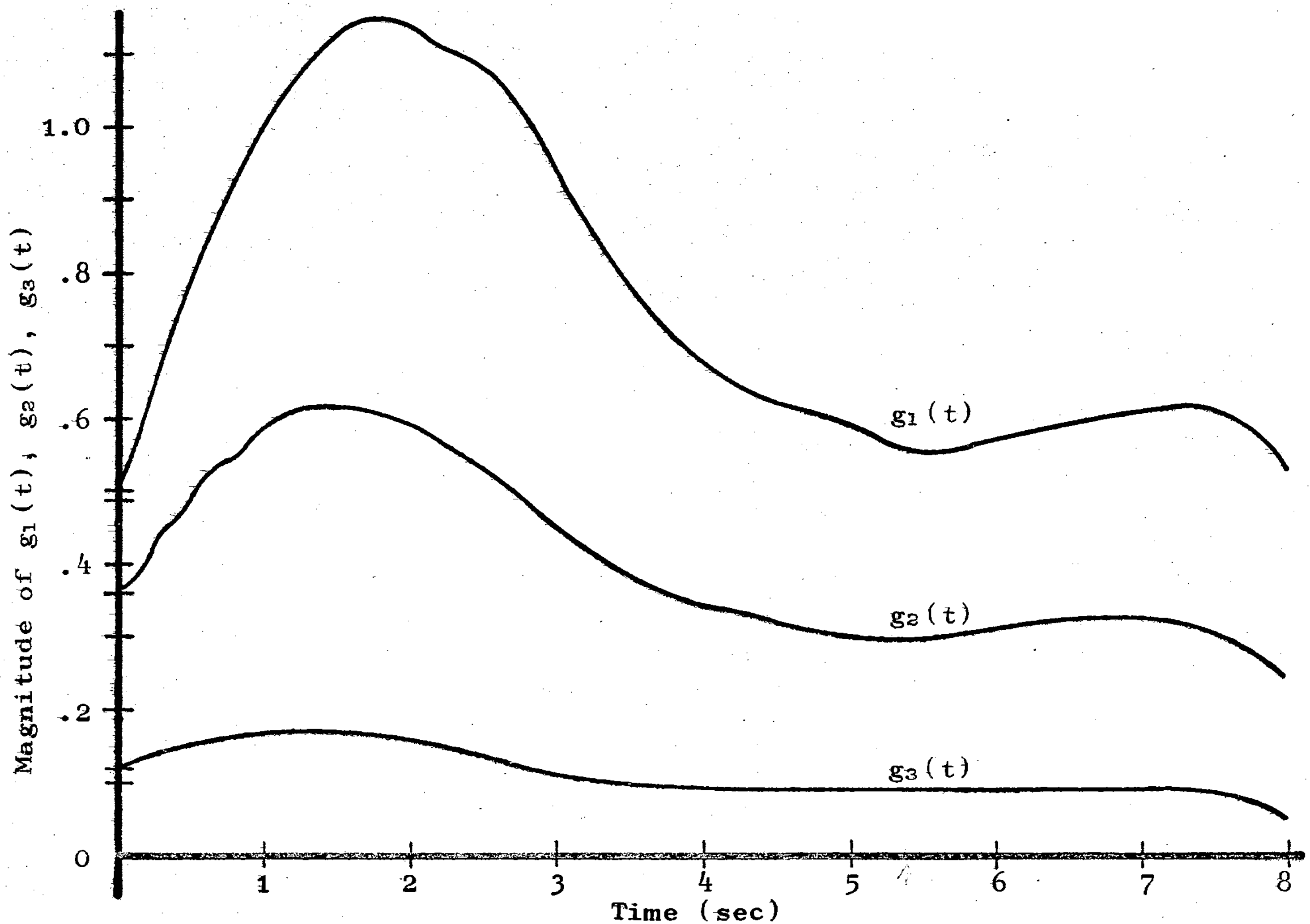


Figure 36. Solution Trajectory for $\underline{G}(t)$ – Example Two
(Continued as Figure 37)

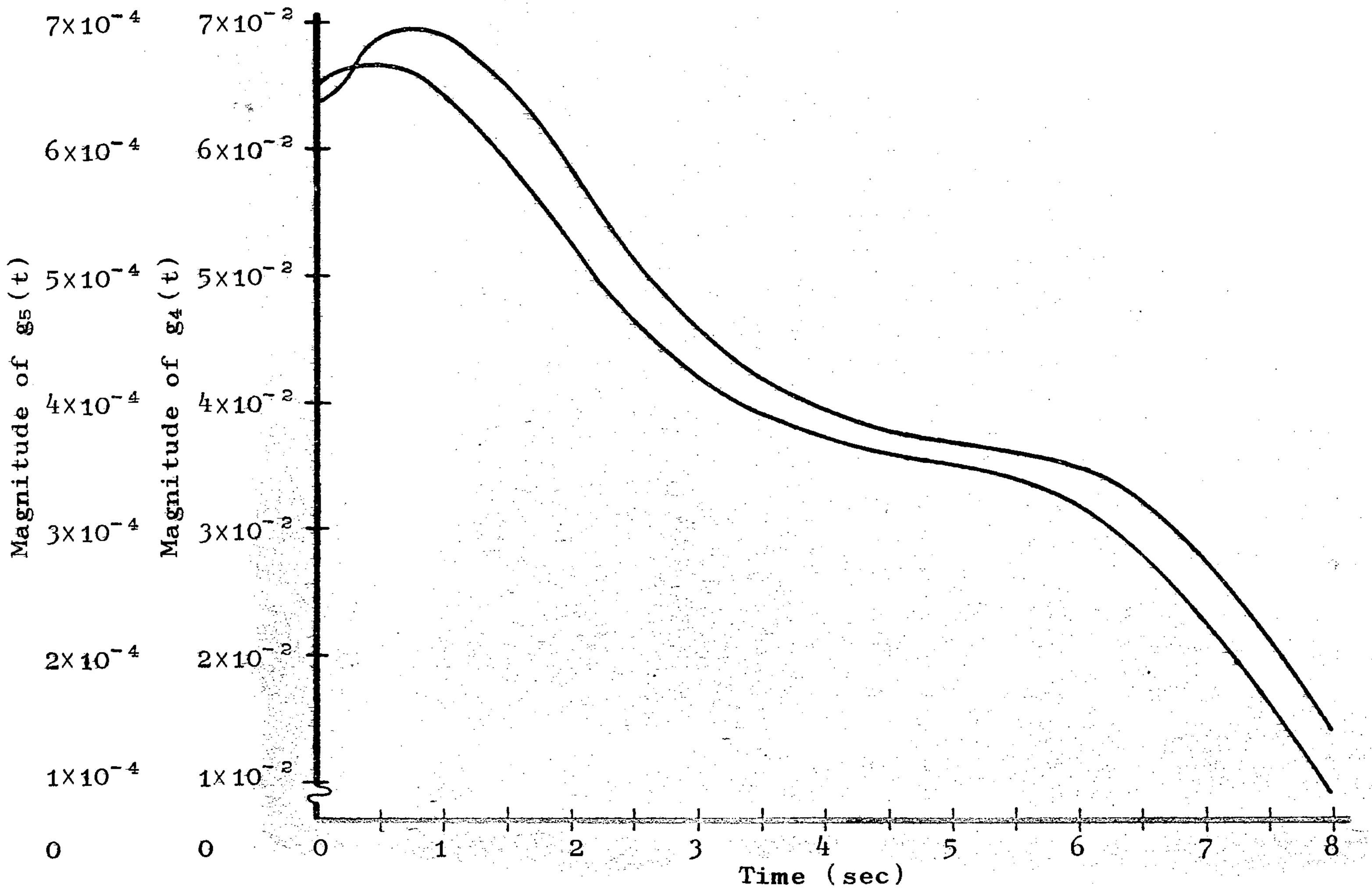


Figure 37. Solution Trajectory for $\underline{G}(t)$ - Example Two
(Continued from Figure 36)

$$\underline{x}_d(t) = -\underline{W}^{-1}\underline{B}^T(\underline{K}(t)\hat{\underline{z}}(t) - \underline{g}(t)) \tag{4-60}$$

which by substitution becomes

$$\begin{aligned} x_d(t) = -\{4800[&\hat{K}_{14}\hat{z}_1(t) + \hat{K}_{24}\hat{z}_2(t) + \hat{K}_{34}\hat{z}_3(t) + K_{44}\hat{z}_4(t) \\ &+ \hat{K}_{45}\hat{z}_5(t) - g_4(t)] - 350,100[\hat{K}_{15}\hat{z}_1(t) + \hat{K}_{25}\hat{z}_2(t) \\ &+ \hat{K}_{35}\hat{z}_3(t) + \hat{K}_{45}(t)\hat{z}_4(t) + \hat{K}_{55}(t)\hat{z}_5(t) - g_5(t)]\}. \end{aligned} \tag{5-25}$$

This equation can be incorporated with the "augmentable" system of Figure 29c to produce the system diagrammed in Figure 38. This diagram depicts how the augmentation could be accomplished through algebraic operations once $\hat{\underline{K}}$ and $\underline{g}(t)$ are determined.

The result of augmenting Example Two as diagrammed in Figure 38 is illustrated in Figure 39. It should be noted that there is an unaugmented output for man shown for comparison in Figure 39 because man alone was unable to provide stable control. However, note how well man is able to control, with dynamical simplicity, the tracking when augmentation is utilized. This example serves as a practical illustration of the need for and usefullness of augmentation.

Example Three - Two Axes Without Crossfeed

The third example for augmentation is a system which requires man to attend to and control two axes at one time. A multi-axis system such as this is a prime candidate for

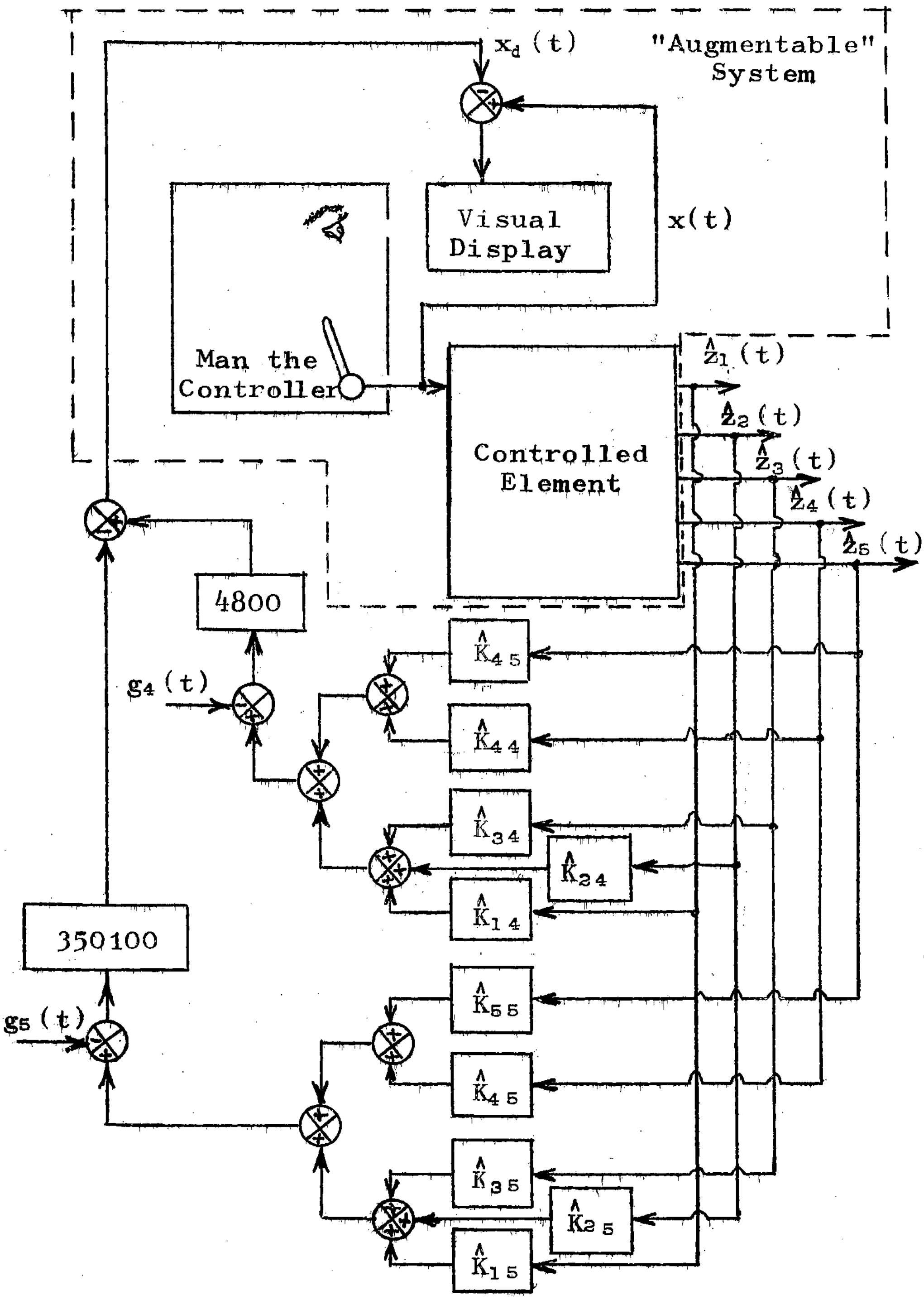


Figure 38. Block Diagram of Example Two in Augmented Form

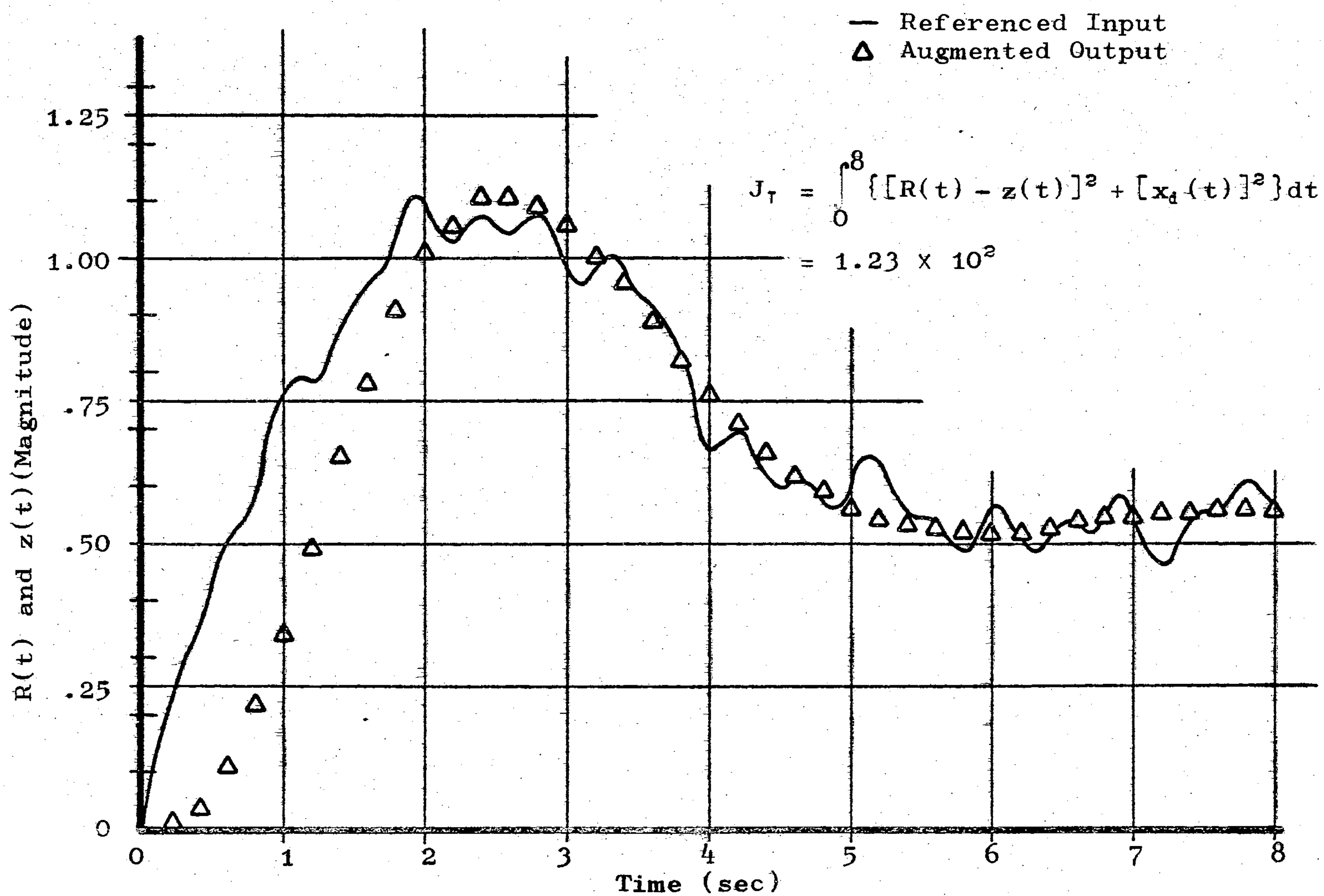


Figure 39. Augmented Output for Example Two

augmentation because the man's capability to control decreases as the number of axes being controlled increases. A system which could be described by two such axes would be the pitch and roll control in an aircraft.

The system to be augmented is diagrammed in Figure 40. The controlled elements are defined in transfer function form as (29)

$$G_{C_1}(s) = \frac{2}{s(s+1)} \tag{5-26}$$

and

$$G_{C_2}(s) = \frac{2}{s(s+1)} \tag{5-27}$$

and the inputs to both axes are defined by Equation (5-3). The models for man (man is described by a model for each axis he controls) are chosen to be the same as those exhibited by him when he was controlling the same system without augmentation. The reason that these models are chosen is to provide a comparison between the augmented and unaugmented systems. Although such a choice leads to a less dramatic example than was encountered in Example Two, it illustrates that augmentation allows man to provide better control with augmentation than he could without it. The models of man are defined in transfer function form as

$$G_{H_1}(s) = \frac{45.5(s+.714)}{(s+5)^2} \tag{5-28}$$

and

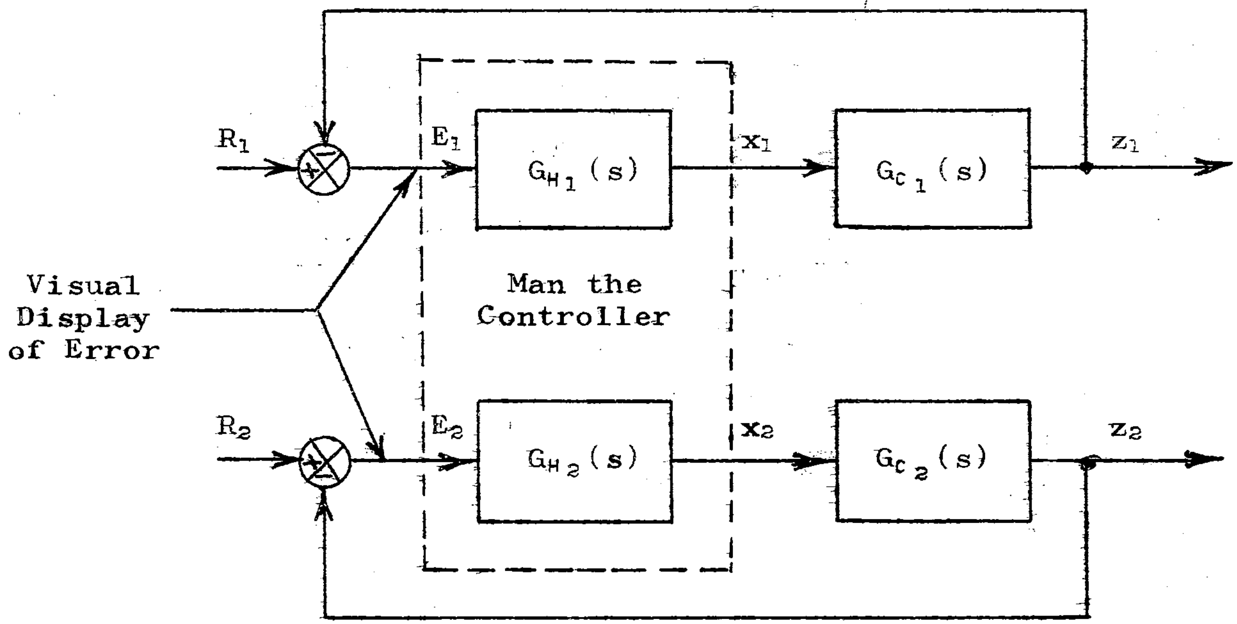


Figure 40. Block Diagram of Unaugmented System Used in Example Three

$$G_{H2}(s) = \frac{50(s+1)}{(s+5)^2}. \tag{5-29}$$

With the models for man and the controlled element defined, the next step is to obtain the state model for the "augmentable" system. As was the case with the single axis systems of the two previous examples, it is first necessary to eliminate the feedback paths shown in Figure 40 to produce the diagram shown in Figure 41a. Next, man's output is fed back to obtain the "augmentable" system as shown in Figure 41b. The state model for this sytem is obtained by utilizing the earlier defined relationship

$$\frac{z}{x_d} = \frac{G_H \ G_C}{1 + G_H} \tag{5-6}$$

to produce

$$\frac{z_1}{x_{d1}} = \frac{91(s+.714)}{s^4 + 56.5s^3 + 112.987s^2 + 57.487s} \tag{5-30}$$

and

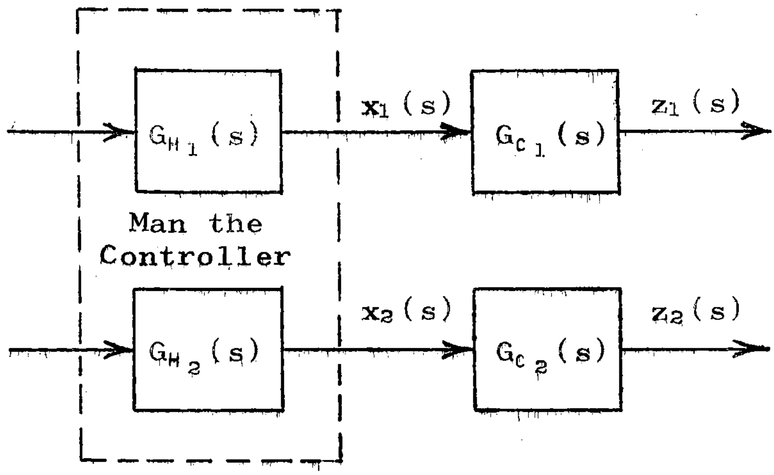
$$\frac{z_2}{x_{d2}} = \frac{100(s+1)}{s^4 + 61s^3 + 135s^2 + 75s} \tag{5-31}$$

The corresponding state model as diagrammed in Figure 29c is

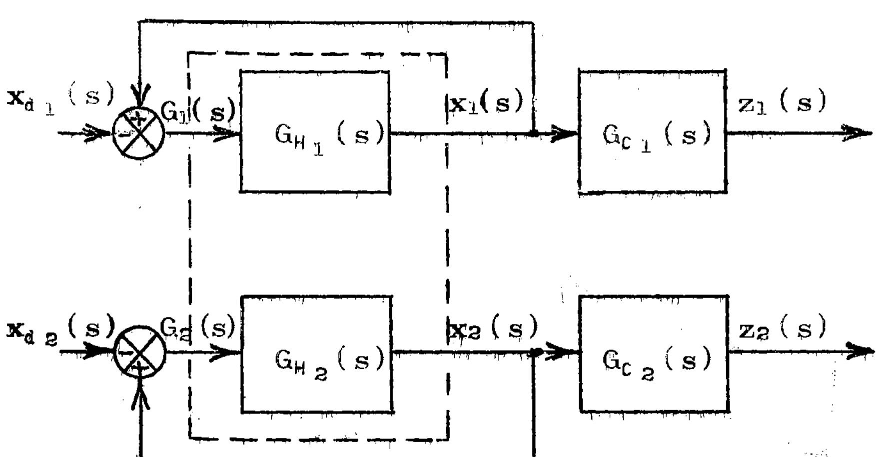
$$\underline{\dot{\hat{z}}}(t) = \underline{A}\,\underline{\hat{z}}(t) + \underline{B}\,\underline{x_d}(t); \ \underline{\hat{z}}(t_0) = \underline{0}. \tag{4-1}$$

$$\underline{z}(t) = \underline{C}\,\underline{\hat{z}}(t) \tag{4-2}$$

where



a.) Multi-Axis Block Diagram of Figure 40 With Feedback Path Removed



b.) Same as a.) but With Feedback Paths Added Around Man the Controller

Figure 41. Illustration of Steps Taken to Obtain "Augmentable" System for Example Three

$$\underline{A} = \left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & & & & \\ 0 & 0 & 1 & 0 & & 0 & & \\ 0 & 0 & 0 & 1 & & & & \\ 0 & -57.487 & -112.987 & -56.5 & & & & \\ \hline & & & & 0 & 1 & 0 & 0 \\ & 0 & & & 0 & 0 & 1 & 0 \\ & & & & 0 & 0 & 0 & 1 \\ & & & & 0 & -75 & -135 & -61 \end{array}\right]$$

$$\underline{B} = \left[\begin{array}{c|c} 0 & \\ 0 & 0 \\ 91 & \\ -5076.53 & \\ \hline & 0 \\ 0 & 0 \\ & 100 \\ & -6000 \end{array}\right] \tag{5-32}$$

$$\underline{C} = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & & 0 & & \\ \hline & 0 & & & 1 & 0 & 0 & 0 \end{array}\right]$$

The performance criteria is again defined by

$$J_T = \tfrac{1}{2}\langle \underline{E}(t_f), \underline{F}\,\underline{E}(t_f)\rangle + \tfrac{1}{2}\int_{t_0}^{t_f} [\langle \underline{E}(t), \underline{Q}\,\underline{E}(t)\rangle + \langle \underline{x}_d(t), \underline{W}\,\underline{x}_d(t)\rangle]dt \tag{4-8}$$

where

$$\underline{E} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5-33}$$

$$\underline{W} = \begin{bmatrix} .01 & 0 \\ 0 & .01 \end{bmatrix}.$$

The Ricatti and its associated matrix equation are defined by

$$\underline{\dot{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T}\underline{K}(t) + \underline{A}^{T}\underline{K}(t) + \underline{C}^{T}\underline{Q}\,\underline{C} = \underline{0} \qquad (4\text{-}66)$$

and

$$\underline{\dot{g}}(t) - (\underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^{T} - \underline{A}^{T})\,\underline{g}(t) + \underline{C}^{T}\underline{Q}\,\underline{R}(t) = \underline{0} \qquad (4\text{-}67)$$

with the boundary conditions of

$$\underline{K}(t_f) = \underline{C}^{T}\underline{F}\,\underline{C} \qquad (4\text{-}69)$$

and

$$\underline{g}(t_f) = \underline{C}^{T}\underline{F}\,\underline{R}(t_f) \qquad (4\text{-}70)$$

where the weighting matrices are as defined by Equations (5-32) and (5-33).

At this point, it is convenient to make some simplifications before presenting the set of first order nonlinear differential equations which made up the Ricatti equation. Since the two axis system being investigated has no crossfeed present, it is possible to break the Ricatti equation into two lesser ordered Ricatti equations (The Ricatti equation as it now stands represents an 8×8 matrix, or in short, 64 first order differential equations which can be reduced to 36 because of symmetry. However, since there is no crossfeed, it is possible to break the 8×8 matrix Ricatti equation into two 4×4 matrix equations. This simplification presents only a total of 32 first order differential equations whose number can be reduced to 20 because of symmetry of the Ricatti matrix.). These two equations and their associated matrix equations are still defined by

Equations (4-66), (4-67), (4-69), and (4-70); however, the coefficient matrices are redefined as

$$\underline{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -57.487 & -112.987 & -56.5 \end{bmatrix}$$

$$\underline{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 91 \\ -5076.53 \end{bmatrix} \qquad (5\text{-}34)$$

$$\underline{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

for axis one, and as

$$\underline{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -75 & -135 & -61 \end{bmatrix} \qquad (5\text{-}35)$$

$$\underline{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 100 \\ -60000 \end{bmatrix}$$

$$\underline{C}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

and axis two, and as

$$\underline{F}_{1,2} = [0]$$

$$\underline{Q}_{1,2} = [1] \qquad (5\text{-}36)$$

$$\underline{W}_{1,2} = [.01]$$

for both axes.

By making use of the matrices of Equations (5-34), (5-35), and (5-36), it is possible to write Equation (4-66) as two sets (one set for each axis) of ten first order non-linear differential equations with the boundary condition for both axes given by

$$\underline{K}(t_f) = \underline{0}. \tag{5-37}$$

The corresponding matrix differential equation can be written, using Equation (4-67) and Equations (5-34)-(5-36), as two sets (one set for each axis) of four first order non-linear differential equations with the boundary conditions for both axes defined by

$$\underline{g}(t_f) = \underline{0}. \tag{5-38}$$

The solution trajectories for both sets of the Ricatti equations are shown in Figures 42-45 for axes one and two, respectively. Again, note the "steady state" value reached by each of these trajectories after an initial transient interval. As was the case in Example Two, the tracking will be assumed to exist over extended time intervals and the value of $\underline{K}(t)$ thus taken to be constant. The substitution of the constant values of $\underline{K}(t)$, i.e., $\hat{\underline{K}}$, into Equation (4-67) along with coefficient matrices of Equations (5-34)-(5-36) allows the solution trajectories for $\underline{g}(t)$ to be solved for by backwards integration and plotted as shown in Figures 46 and 47 for axes one and two, respectively.

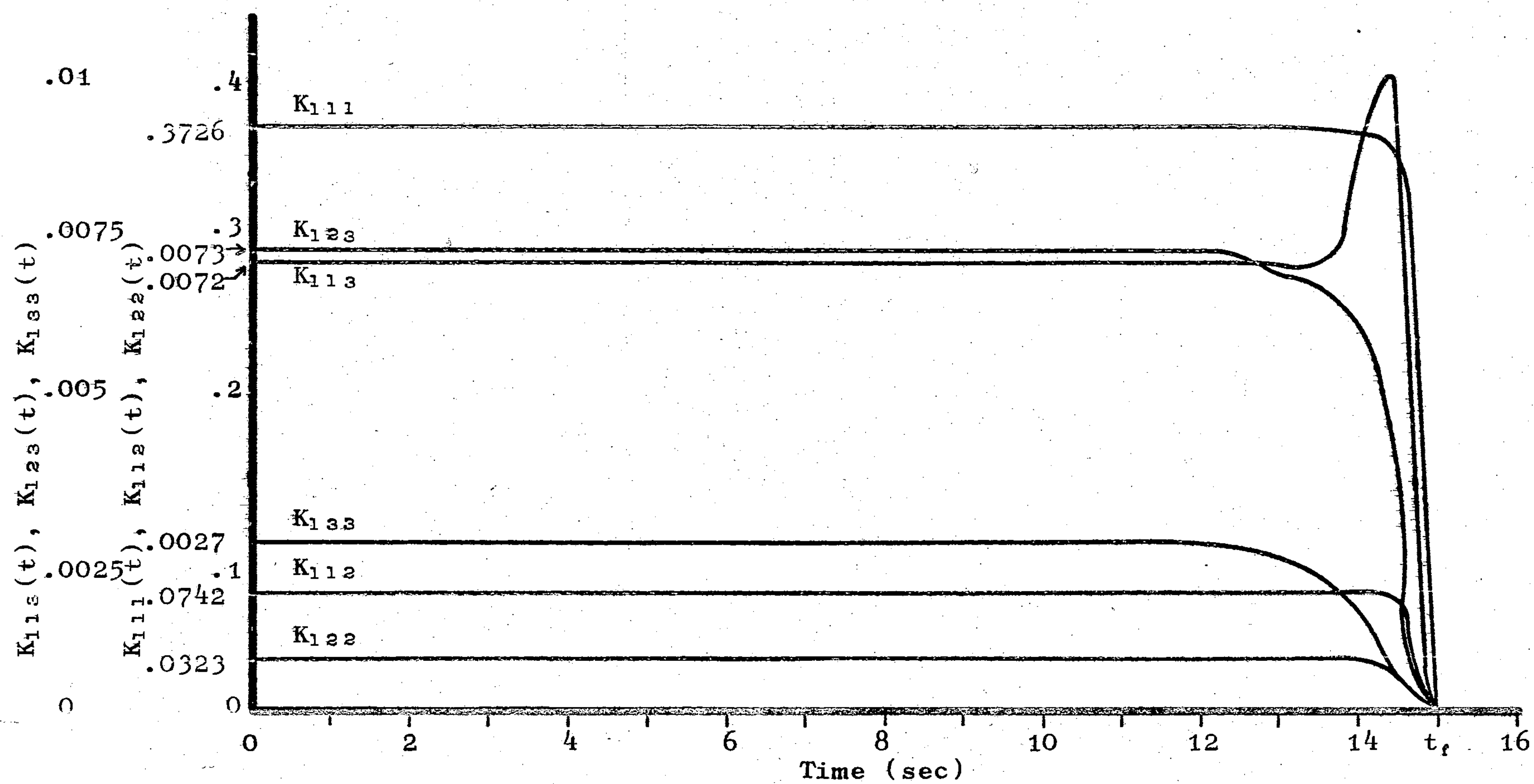


Figure 42. Solution Trajectories for Matrix Ricatti Equation of Example Three - Axis One (Continued as Figure 43)

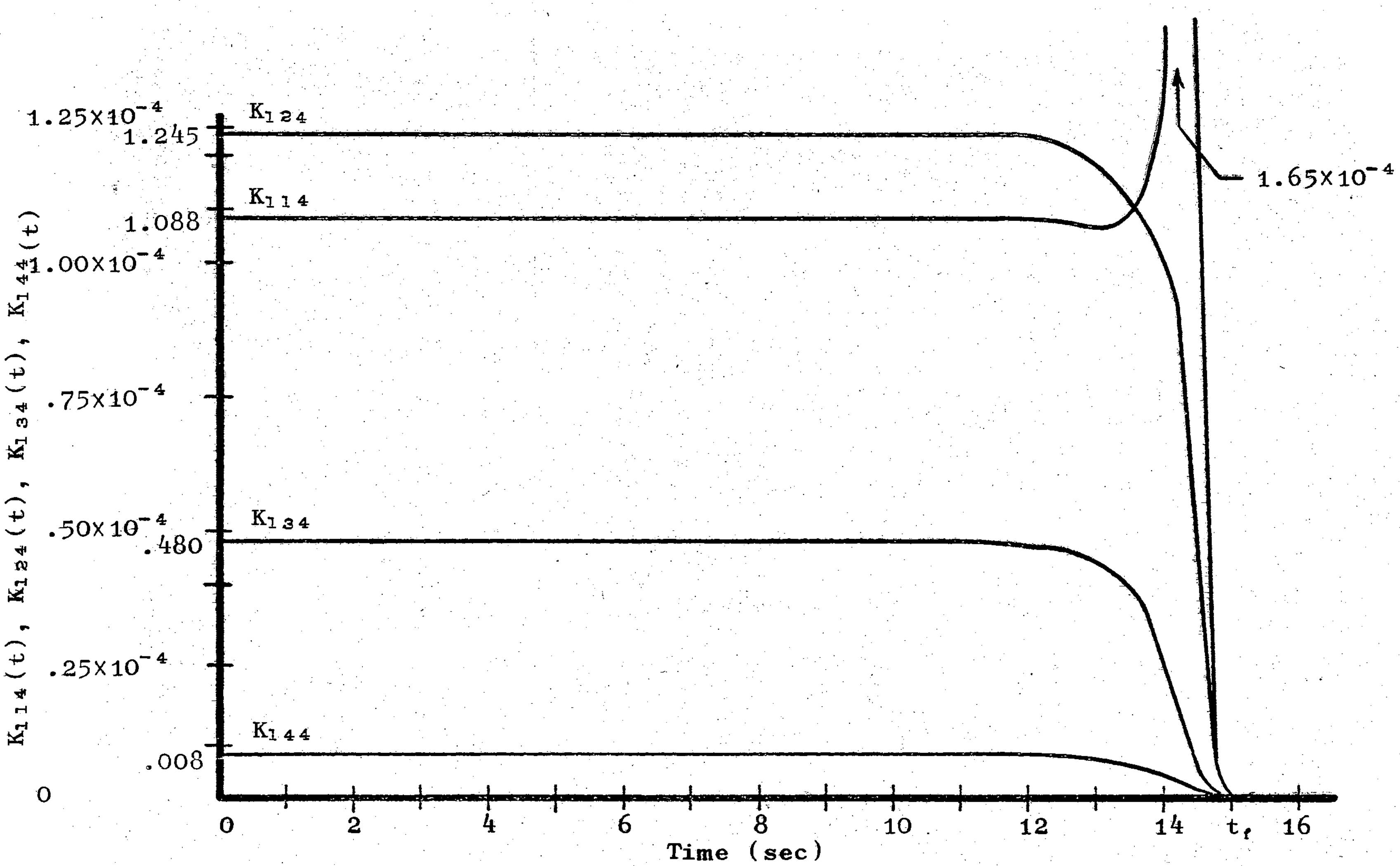


Figure 43. Solution Trajectories for Matrix Ricatti Equation of Example Three - Axis One (Continued from Figure 42)

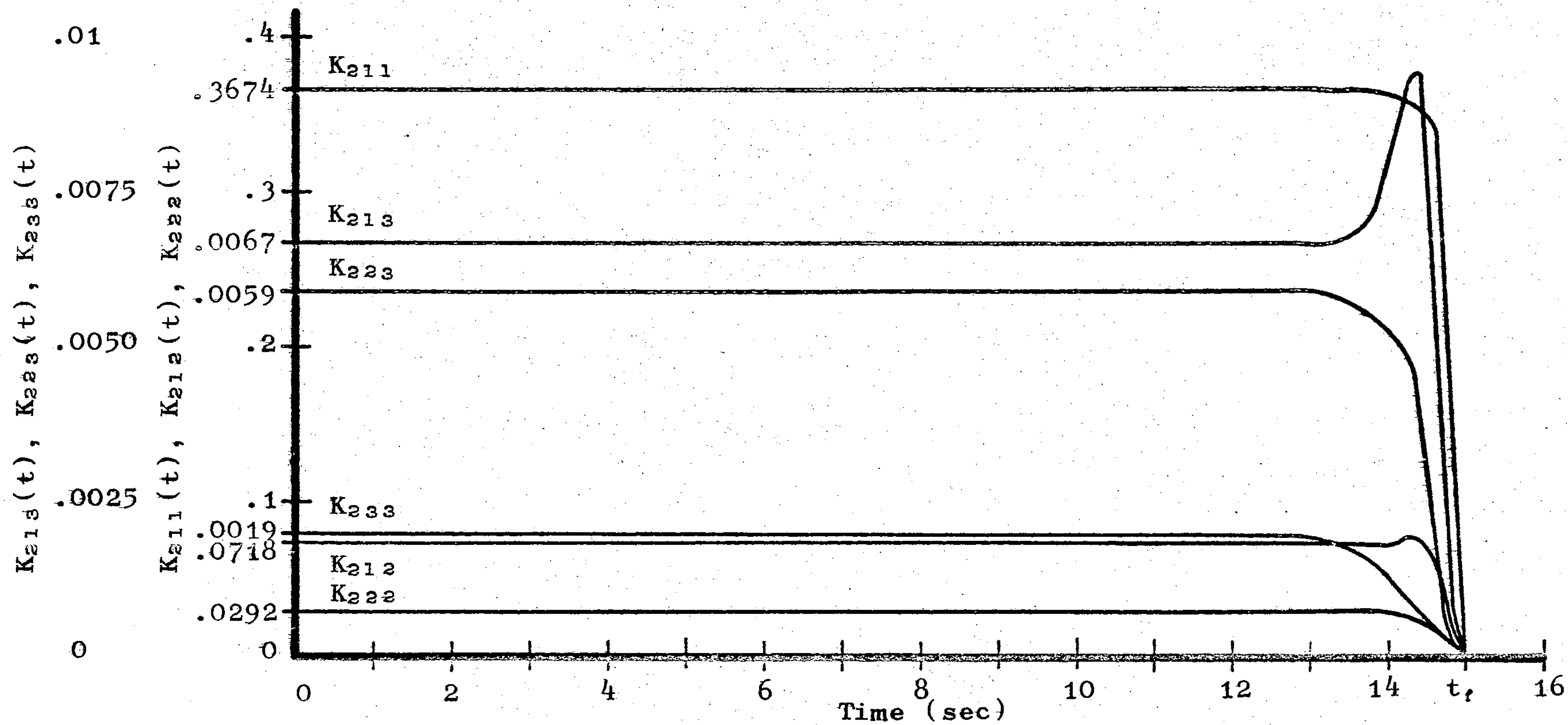


Figure 44. Solution Trajectories for Matrix Ricatti Equation of Example Three - Axis Two (Continued as Figure 45)

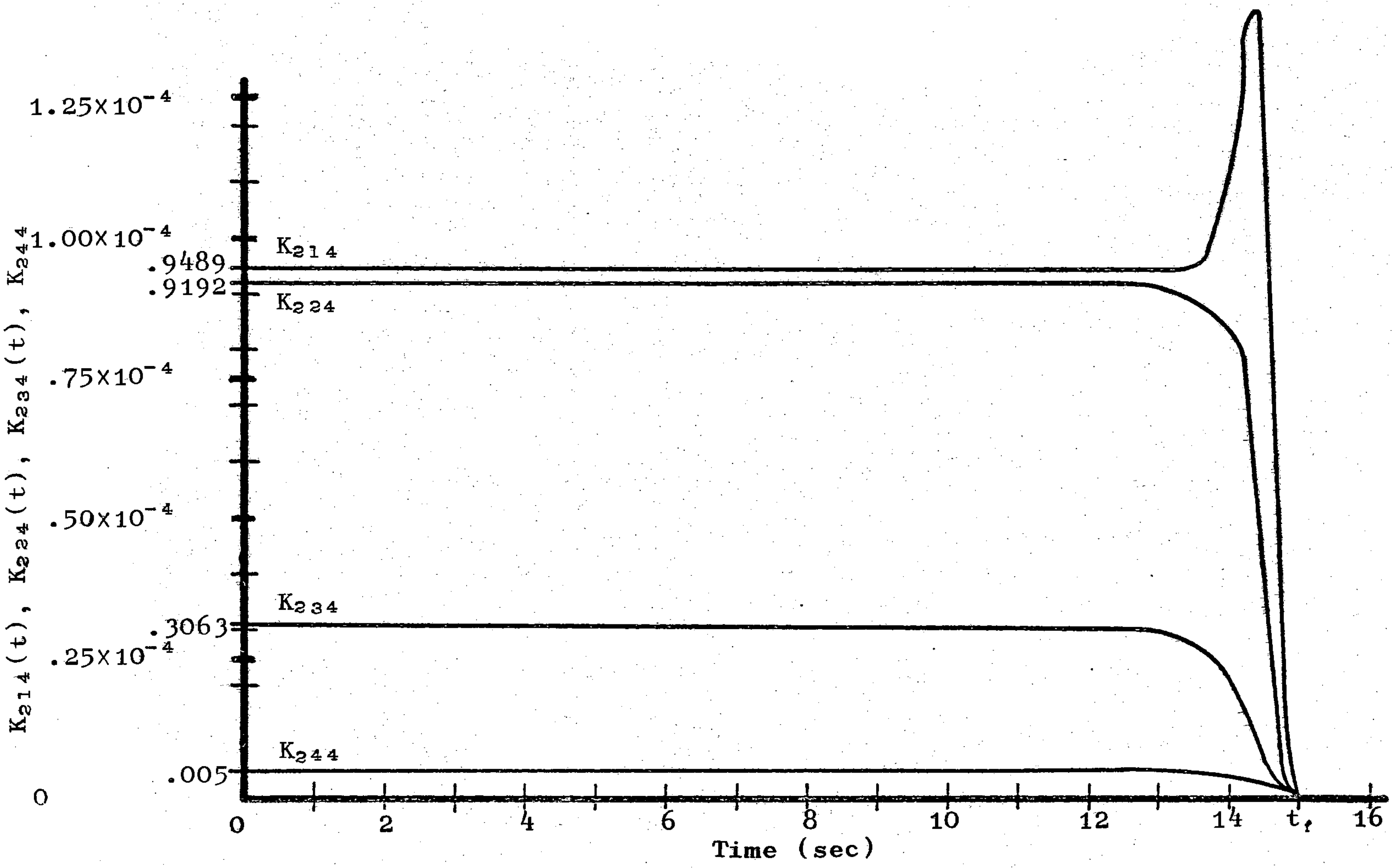


Figure 45. Solution Trajectories for Matrix Ricatti Equation of Example Three - Axis Two (Continued from Figure 44)

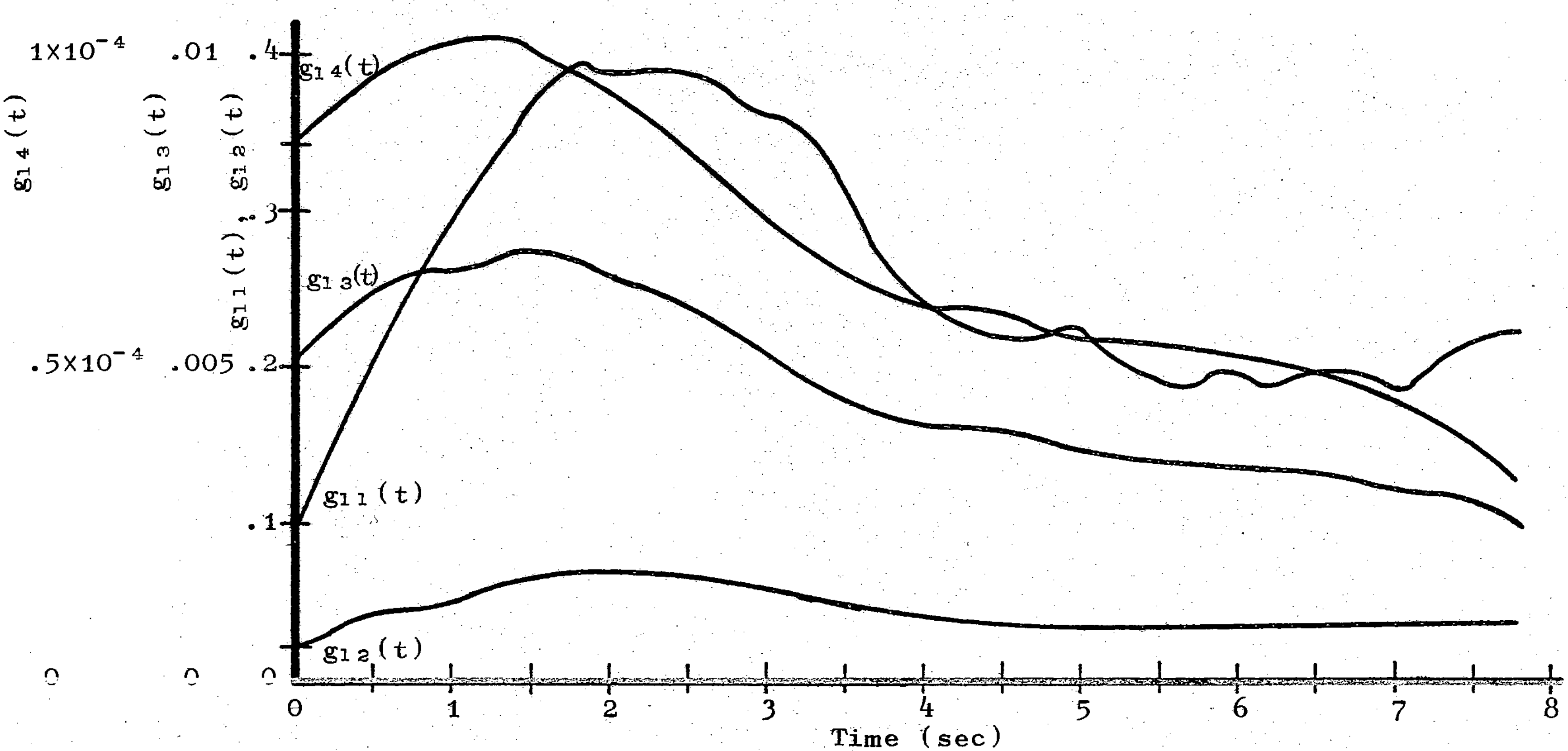


Figure 46. Solution Trajectories for $\underline{g}(t)$ - Example Three, Axis One

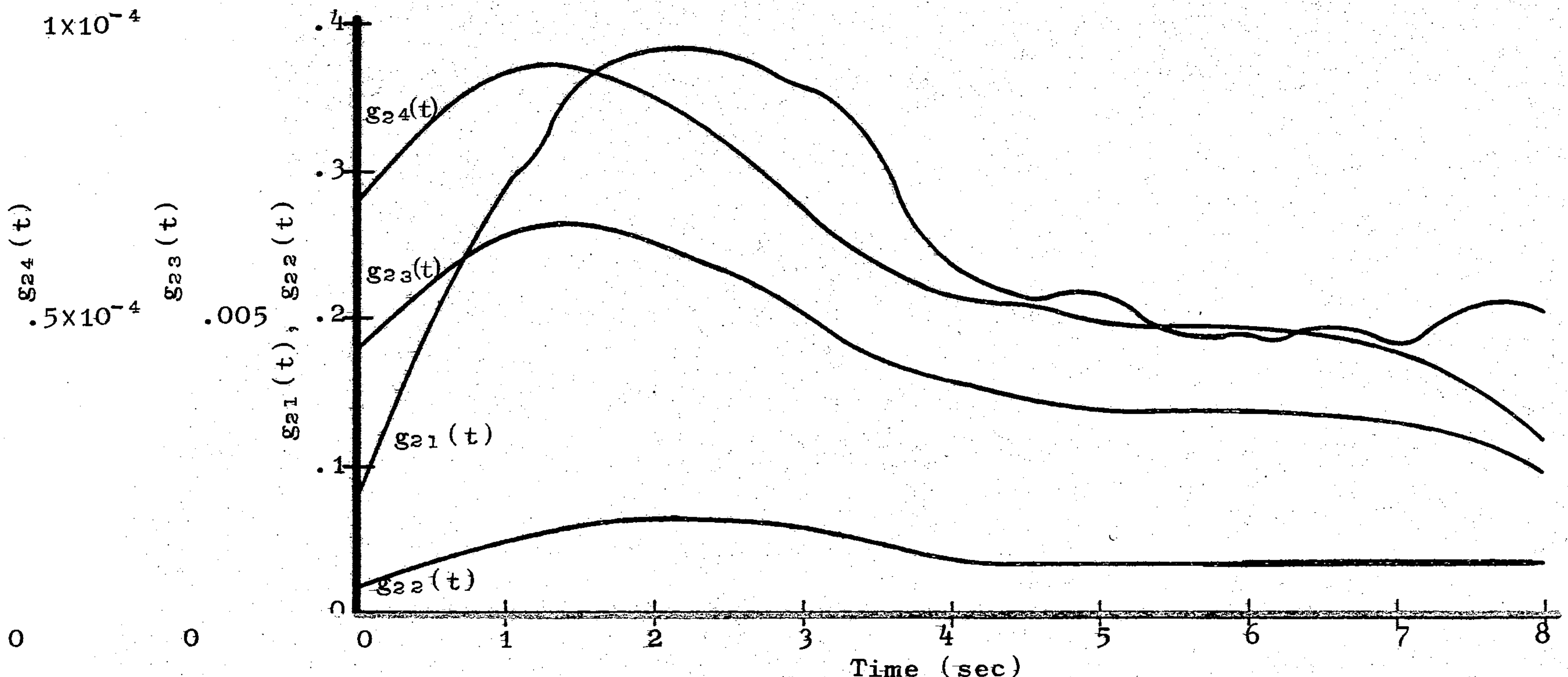


Figure 47. Solution Trajectories for $\underline{g}(t)$ - Example Three, Axis Two

The gains for the Ricatti equations, $\underline{\hat{K}_1}$ and $\underline{\hat{K}_2}$, and the associated matrix equation trajectories, $\underline{g}_1(t)$ and $\underline{g}_2(t)$, can now be used to implement the augmenting system. Again, the implementation is accomplished by using the equation

$$\underline{x}_d(t) = -W^{-1}\underline{B}^T(\underline{K}(t)\underline{\hat{z}}(t) - \underline{g}(t)) \qquad (4\text{-}60)$$

which with the proper values substituted in for the weighting matrices becomes

$$\begin{aligned} x_{d1}(t) = -100\{ & 91[\hat{K}_{113}\hat{z}_{11}(t) + \hat{K}_{123}\hat{z}_{12}(t) + \hat{K}_{133}\hat{z}_{13}(t) \\ & + \hat{K}_{134}\hat{z}_{14}(t) - g_{13}(t)] - 5076.53\,[\hat{K}_{114}\hat{z}_{11}(t) \\ & + \hat{K}_{124}\hat{z}_{12}(t) + \hat{K}_{134}\hat{z}_{13}(t) + \hat{K}_{144}\hat{z}_{14}(t) - g_{14}(t)]\} \end{aligned} \qquad (5\text{-}39)$$

for axis one and

$$\begin{aligned} x_{d2}(t) = -100\{ & 100[\hat{K}_{213}\hat{z}_{21}(t) + \hat{K}_{223}\hat{z}_{22}(t) + \hat{K}_{233}\hat{z}_{23}(t) \\ & + \hat{K}_{234}\hat{z}_{24}(t) - g_{23}(t)] - 6000[\hat{K}_{214}\hat{z}_{21}(t) \\ & + \hat{K}_{224}\hat{z}_{22}(t) + \hat{K}_{234}\hat{z}_{23}(t) + \hat{K}_{244}\hat{z}_{24}(t) - g_{24}(t)]\} \end{aligned} \qquad (5\text{-}40)$$

for axis two.

These equations can be incorporated with the "augmentable" system of Figure 29c to produce the system diagrammed in Figure 48. This diagram depicts how augmentation could be accomplished through algebraic operations once $\underline{\hat{K}}$ ang $\underline{g}(t)$ are determined.

The result of augmenting the system of Example Three is

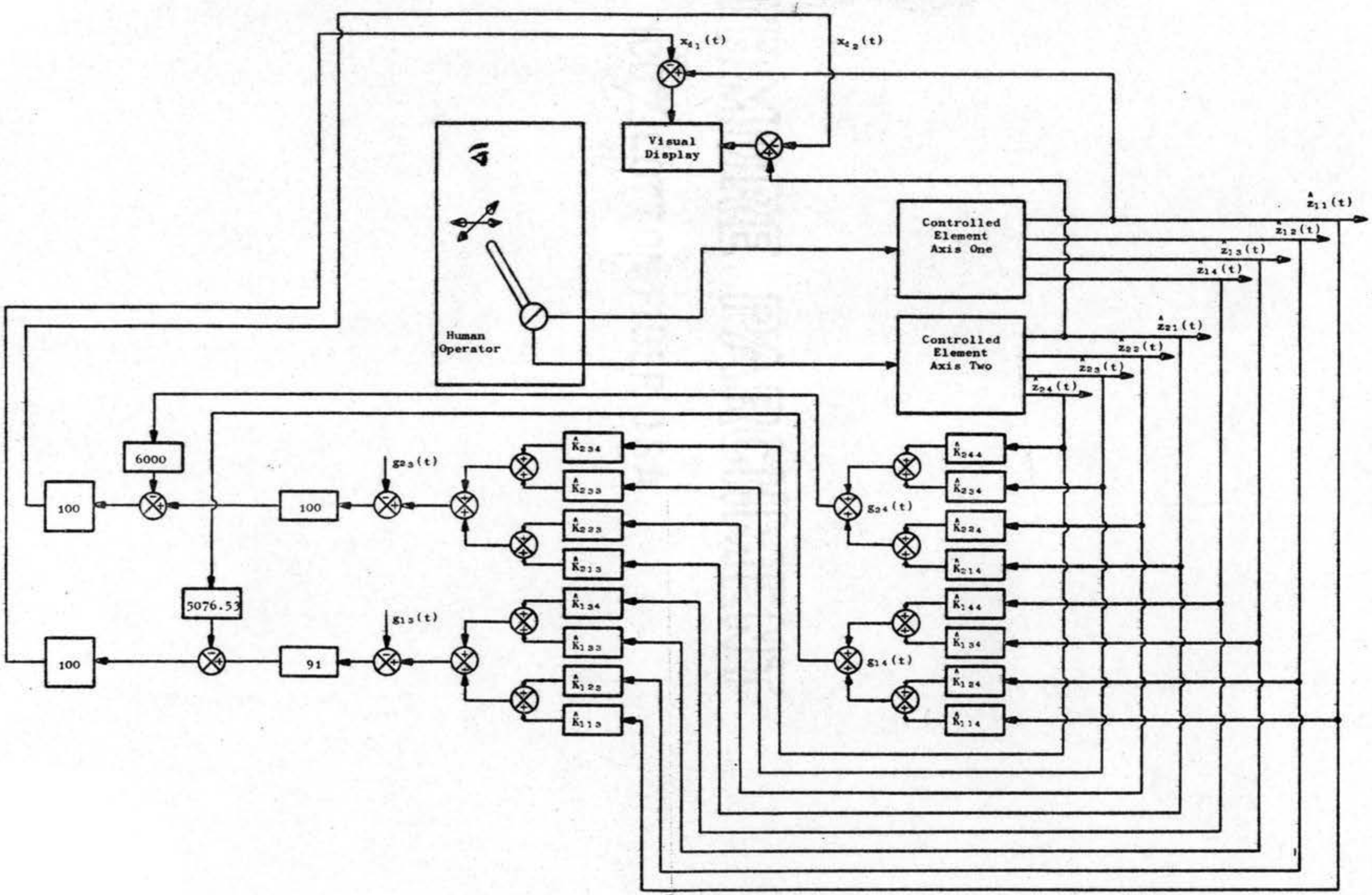


Figure 48. Block Diagram of Example Three in Augmented Form

shown in Figures 49 and 50. Note the marked improvement in the augmented over the unaugmented system as evidenced by the performance indices. A point that should be made in regard to Example Three is that although man's unaugmented control appeared to exhibit what might be considered in a cursory examination to be acceptable tracking (Figure 49 and 50), and perhaps not require augmentation if this were a real system, the example has shown that a multi-axis system can be augmented and provide optimal tracking. The reason that a more illustrative multi-axis system (i.e., a system for which man could only provide very poor control) was not chosen is that there has been very little research conducted in the past to obtain unaugmented multi-axis operator and controlled element models. Thus, only a very finite population of these systems were available for application of the augmentation procedure. Although several controlled element models could have been generated and mated with a model of man for an extreme illustration of the capability of the augmenting procedure, it was felt to be in the best interests of this research to utilize for example application only models which have been realistically obtained.

Example Four - Multi-Axis With Crossfeed

The fourth example is a vertical take off and landing aircraft, designated XV-5A, which is required to hover at a finite altitude when suddenly disturbed by a gust of wind.

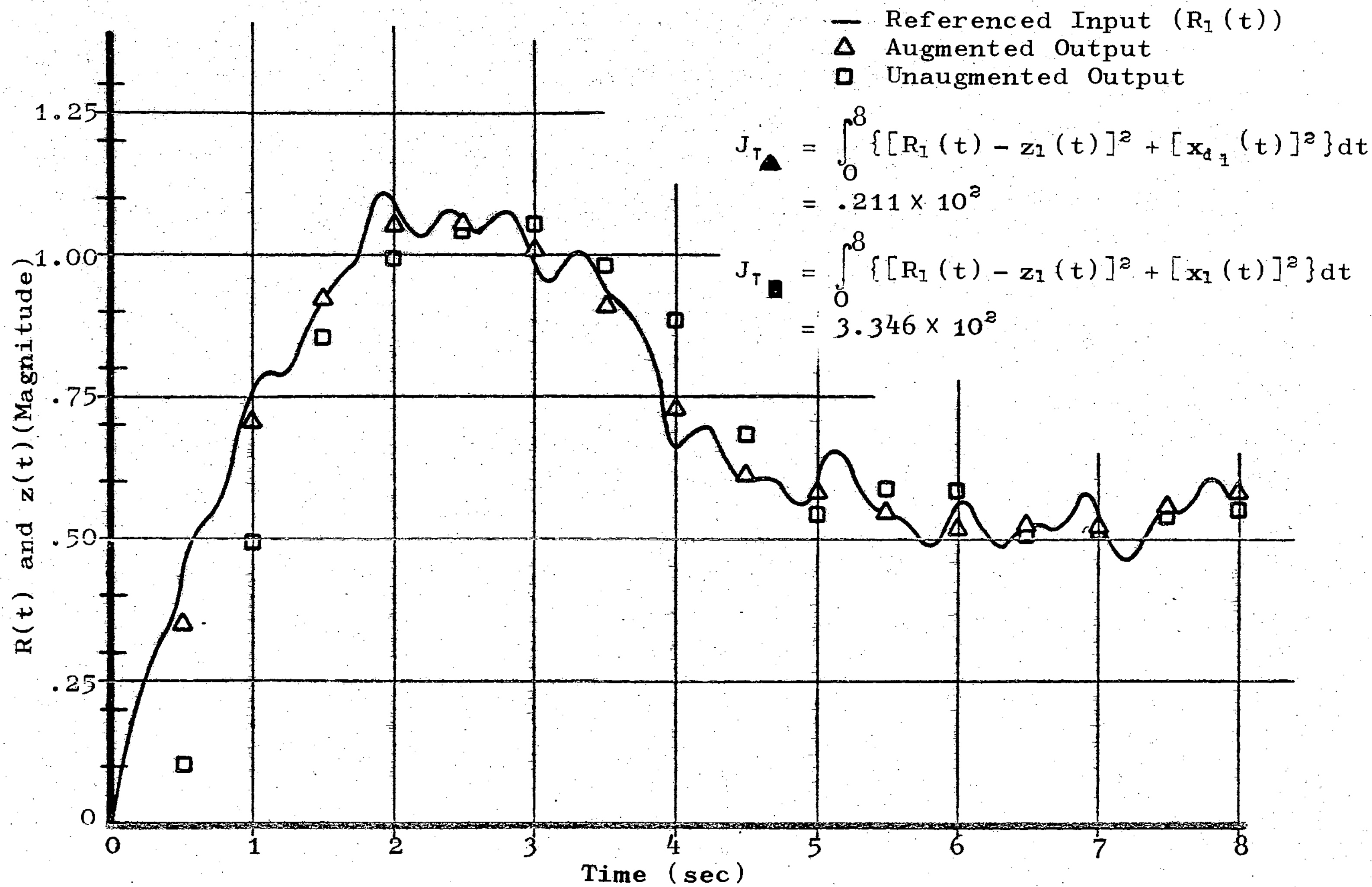


Figure 49. Augmented and Unaugmented Output for Example Three - Axis One

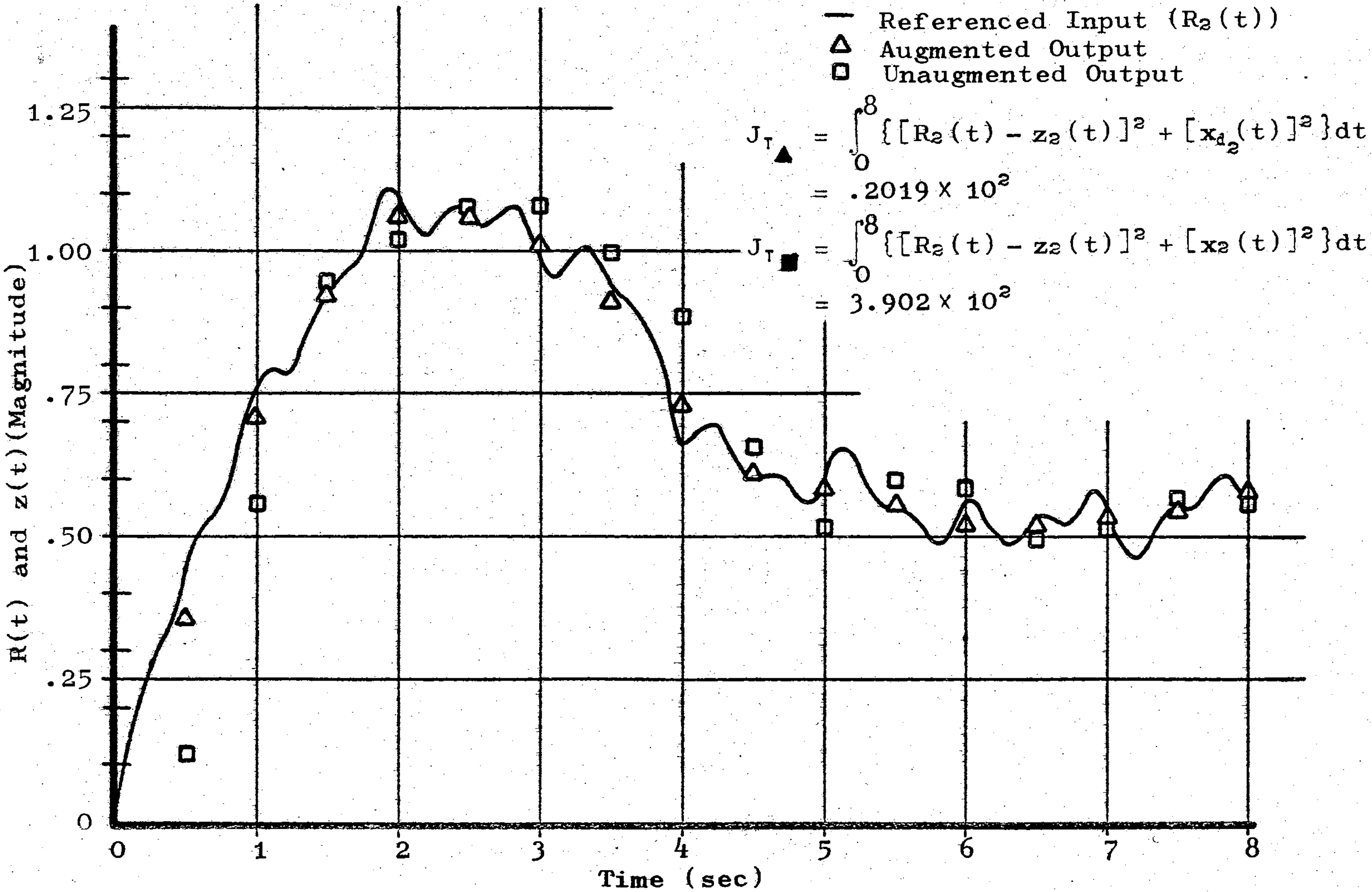


Figure 50. Augmented and Unaugmented Output for Example Three - Axis Two

The simplified equations of motion for this aircraft are given by (22):

$$\begin{bmatrix} \dot{U} \\ \dot{x} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_U & 0 & 0 & -q \\ 1 & 0 & 0 & 0 \\ M_U & 0 & Mq & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ X \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_{N_f} \\ 0 \end{bmatrix} \delta_\theta + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} R \qquad (5\text{-}41)$$

where

u = translational velocity along x-axis

x = displacement along x-axis

q = angular velocity about x-axis

θ = pitch angle at operating point $\theta_0 = 0$

R = disturbance (taken to be a step input of wind)

δ_θ = control input from pilot.

Since the aircraft is to hover, the problem becomes one of minimizing the deviation of the variables u, x, q, and θ from their initial conditions of zero; i.e., assuming the aircraft is hovering initially. Thus, the augmentation must be performed for the output regulator system. However, in order to utilize Equation (5-41) in the "augmentable" system and consider the latter an output regulator, it is necessary to redefine the variables. Since R is a step input, the following statements can be made:

$$W_1 = U$$

$$W_2 = X$$

$$W_3 = q$$

$$W_4 = \theta - {}^{R}/g.$$

The substitution of these new variables back into Equation (5-41) results in

$$\begin{bmatrix} \dot{W}_1 \\ \dot{W}_2 \\ \dot{W}_3 \\ \dot{W}_4 \end{bmatrix} = \begin{bmatrix} X_U & 0 & 0 & -g \\ 1 & 0 & 0 & 0 \\ M_U & 0 & Mq & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_{NF} \\ 0 \end{bmatrix} \delta_\theta . \tag{5-42}$$

Again, it is necessary to select a model to represent man in the augmented system. Let it be assumed that it is desirable for man to be able to control the aircraft with the same ease by which he would control a simple gain element that requires only dynamical information that is perceived through the visual mode. The model of man utilized in Example One depicts such a control situation and will be used again here. This model is given by

$$G_H(s) = \frac{47.4}{(s + 7.15)^2}. \tag{5-2}$$

To incorporate this transfer function into the "augmentable" system, it is necessary to feed back man's output as indicated in the transfer relationship of Equation (5-5). The use of this relation results in

$$\frac{\delta_\theta}{\delta_d} = \frac{47.4}{s^2 + 14.30s + 51.12} \tag{5-43}$$

where

δ_θ = the actual output from man

δ_d = the desired output from man.

This transfer function can be written in state model form as

$$\begin{bmatrix} \dot{\delta}_{\theta_1} \\ \dot{\delta}_{\theta_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} \delta_{\theta_1} \\ \delta_{\theta_2} \end{bmatrix} + \begin{bmatrix} 0 \\ K \end{bmatrix} \delta_d . \qquad (5\text{-}44)$$

where

a = 14.30

b = 51.12

and

K = 47.4.

If the variables, δ_{θ_1} and δ_{θ_2}, are redefined as W_5 and W_6, respectively, and the state models of Equations (5-42) and (5-44) incorporated, then the state model for the "augmentable" system, as illustrated in Figure 29c, can be defined as

$$\begin{bmatrix} \dot{W}_1 \\ \dot{W}_2 \\ \dot{W}_3 \\ \dot{W}_4 \\ \dot{W}_5 \\ \dot{W}_6 \end{bmatrix} = \begin{bmatrix} z_U & 0 & 0 & -g & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ M_U & 0 & M_q & 0 & M_{NF} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -b & -a \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K \end{bmatrix} \delta_d \qquad (5\text{-}45)$$

$$\underline{z} = \begin{bmatrix} U \\ x \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \end{bmatrix} \tag{5-46}$$

where the coefficient matrices of Equation (5-45) correspond to $\underline{A}$ and $\underline{B}$ of Equation (4-1) and the coefficient matrix of Equation (5-46) corresponds to $\underline{C}$ of Equation (4-2).

The performance criteria for the output regulator system is given by

$$J_0 = \tfrac{1}{2}\langle \underline{z}(t_f), \underline{F}\,\underline{z}(t_f)\rangle + \int_0^{t_f} [\langle \underline{z}(t), \underline{Q}\,\underline{z}(t)\rangle + \langle \underline{\delta_d}(t), \underline{W}\,\underline{\delta_d}(t)\rangle]dt \tag{4-7}$$

where

$$\underline{F} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5-47}$$

$$\underline{W} = [.01].$$

The Ricatti equation is defined by

$$\dot{\underline{K}}(t) + \underline{K}(t)\underline{A} - \underline{K}(t)\underline{B}\,\underline{W}^{-1}\underline{B}^T\underline{K}(t) + \underline{A}^T\underline{K}(t) + \underline{C}^T\underline{Q}\,\underline{C} = \underline{0} \tag{4-44}$$

with the boundary condition of

$$\underline{K}(t_f) = \underline{C}^T \underline{F}\, \underline{C} = \underline{0}. \qquad (4\text{-}45)$$

The substitution of $\underline{A}$, $\underline{B}$, and $\underline{C}$ from Equations (5-45) and (5-46) and $\underline{F}$, $\underline{Q}$, and $\underline{W}$ from Equation (5-47) into Equation (4-44) yields a set of 21 nonlinear first order differential equations with the boundary conditions of Equation (4-45). The solution trajectories for this set of equations are shown in Figures 51-53. Again, the value of t_f was chosen to indicate the "steady state" nature of $\underline{K}(t)$.

The output regulator nature of Example Four makes the reasoning for the assumption that $\underline{K}(t)$ is constant somewhat different than it was for the indefinite tracking periods encountered in the previous examples. However, the fact remains that the optimal control is dependent upon the value of $\underline{K}(t)$ in the constant region. Thus, it will suffice to assume that $\underline{K}(t)$ is the constant $\underline{\hat{K}}$. The augmentation is implemented by using the equation

$$\underline{\delta}_{\theta_d}(t) = -\underline{W}^{-1}\underline{B}^T(\underline{K}(t)\underline{W}(t)), \qquad (4\text{-}58)$$

which, with the proper values substituted in for $\underline{W}$, $\underline{B}$, and $\underline{K}(t)$ becomes

$$\delta_{\theta_d}(t) = -4740[\hat{K}_{16}W_1(t) + \hat{K}_{26}W_2(t) + \hat{K}_{36}W_3(t) \qquad (5\text{-}48)$$
$$+ \hat{K}_{46}W_4(t) + \hat{K}_{56}W_5(t) + \hat{K}_{66}W_6(t)].$$

This equation can be incorporated with the "augmentable" system described by Equation (5-45) to produce the augmented

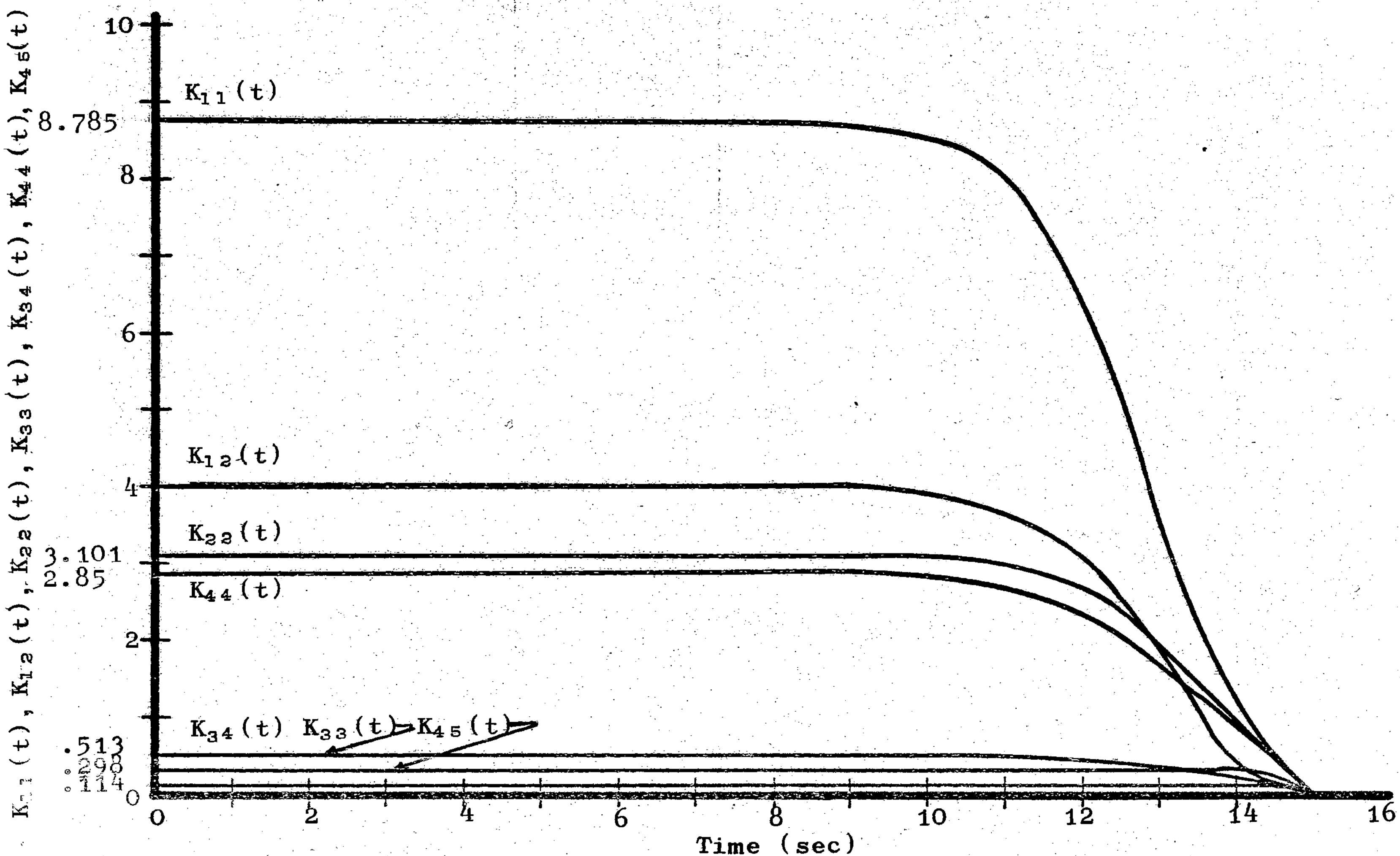


Figure 51. Solution Trajectories for Matrix Ricatti Equation of Example Four (Continued as Figures 52 and 53)

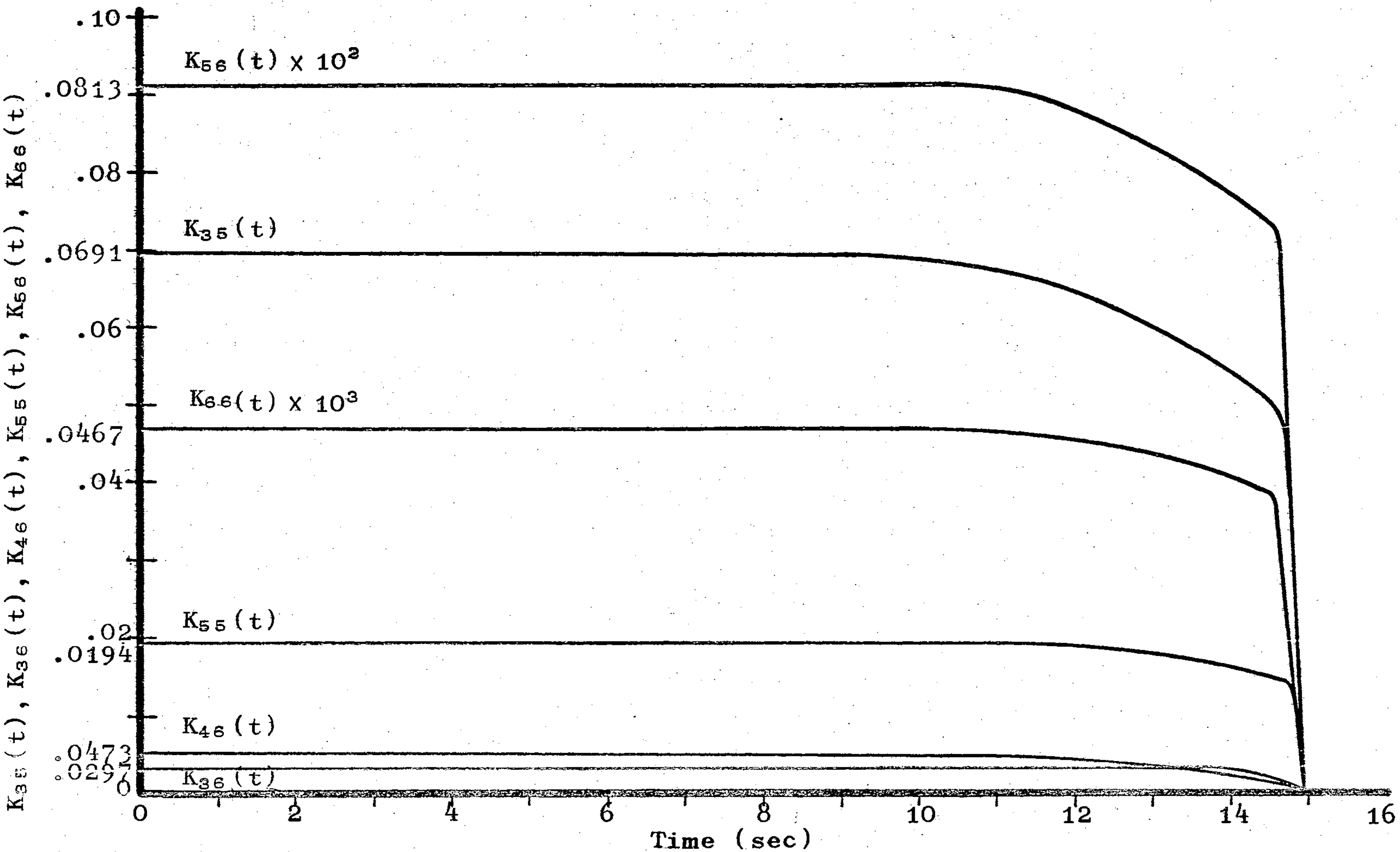


Figure 52. Solution Trajectories for Matrix Ricatti Equation of Example Four (Continued from Figure 51)

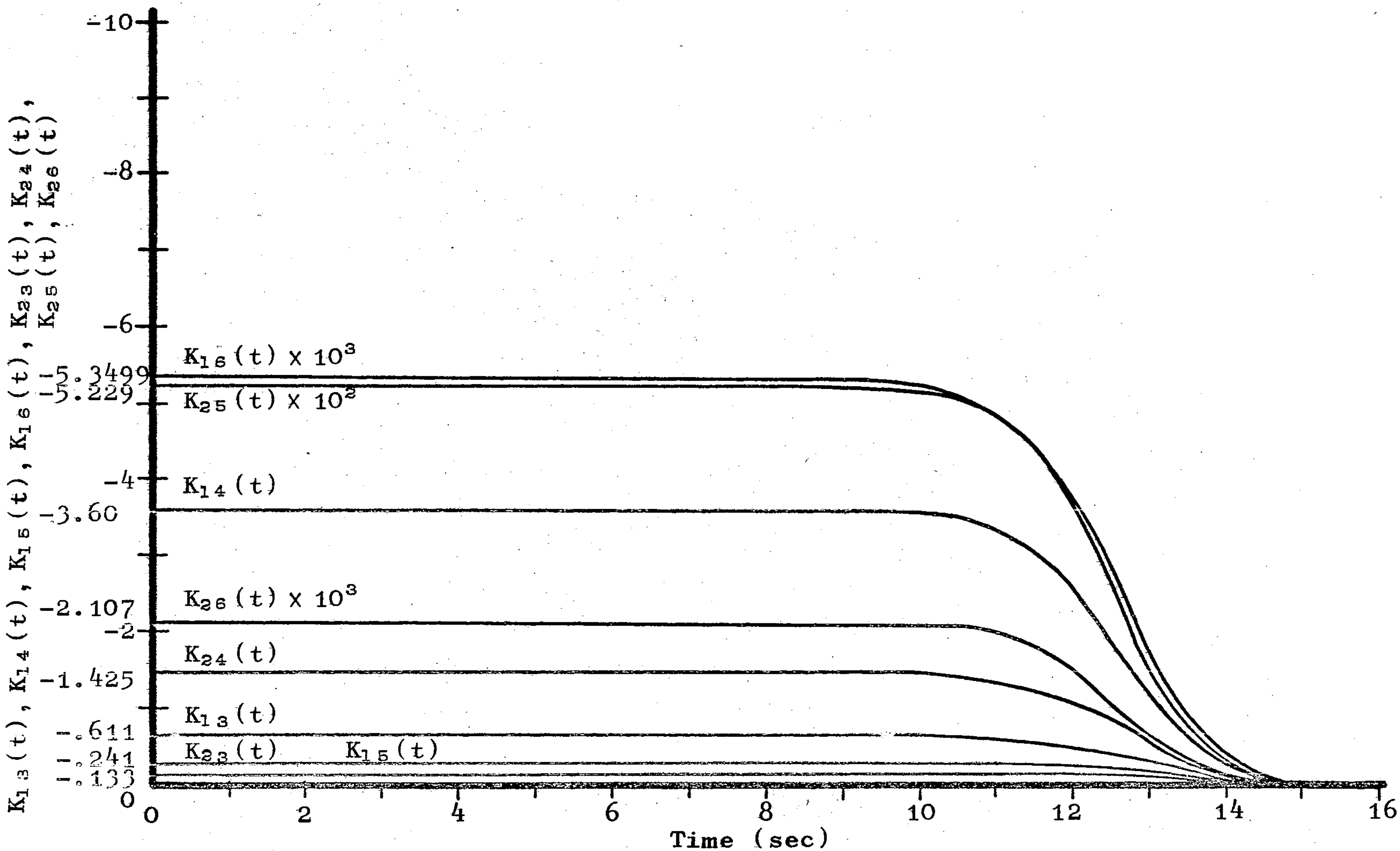


Figure 53. Solution Trajectories for Matrix Ricatti Equation of Example Four (Continued from Figure 51)

system shown in Figure 54.

The result of this augmentation is shown as the aircraft state variables as plotted in Figures 55 through 58. Since there is no experimental, unaugmented data for man the controller to compare with these trajectories, it is impossible to make any specific comments about how well the augmented system enabled man to perform. However, the fact remains that the augmentation did allow man to be depicted as the simple system defined by Equation (5-2) and enable him to control the example aircraft through the apparently optimal and, thus, correct responses illustrated in Figures 55 through 58.

Example Five - Ford Series 755 Backhoe

The research done upon the backhoe was performed under contract to the Tractor Division of Ford Motor Company (26). The purpose of this research was to obtain dynamical models of the man while he controlled the backhoe. The intended use of these models is to evolve the backhoe towards a better interface with man and more optimal operation. The research reported in this thesis is also sponsored by Ford, and is intended to be a means by which man can be made to make the backhoe operate more optimally. Although the operator modeling work on the backhoe was too involved to allow sufficient progress to be made in the first contract year to permit any serious consideration to be given to augmentation,

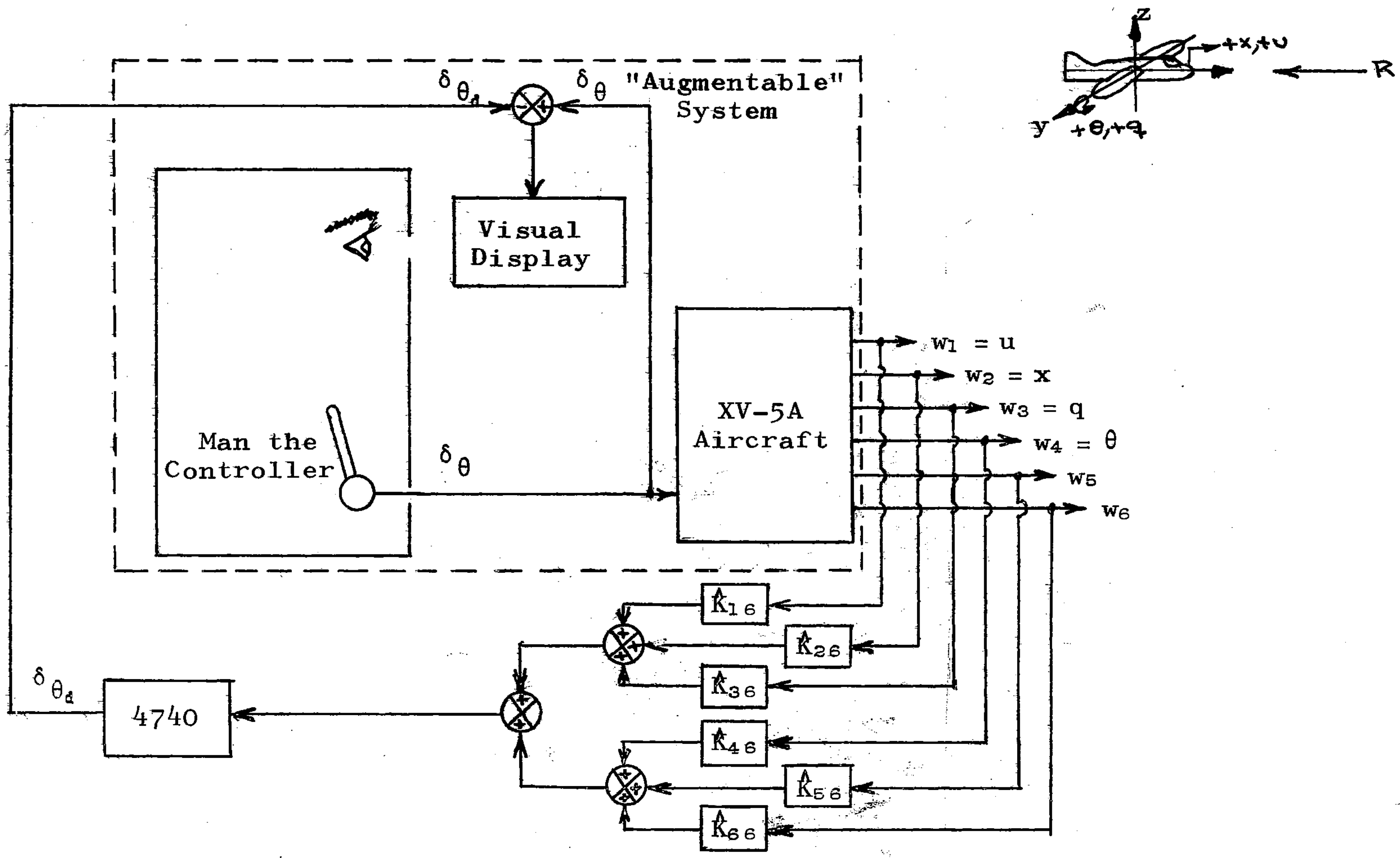


Figure 54. Block Diagram of Example Four in Augmented Form

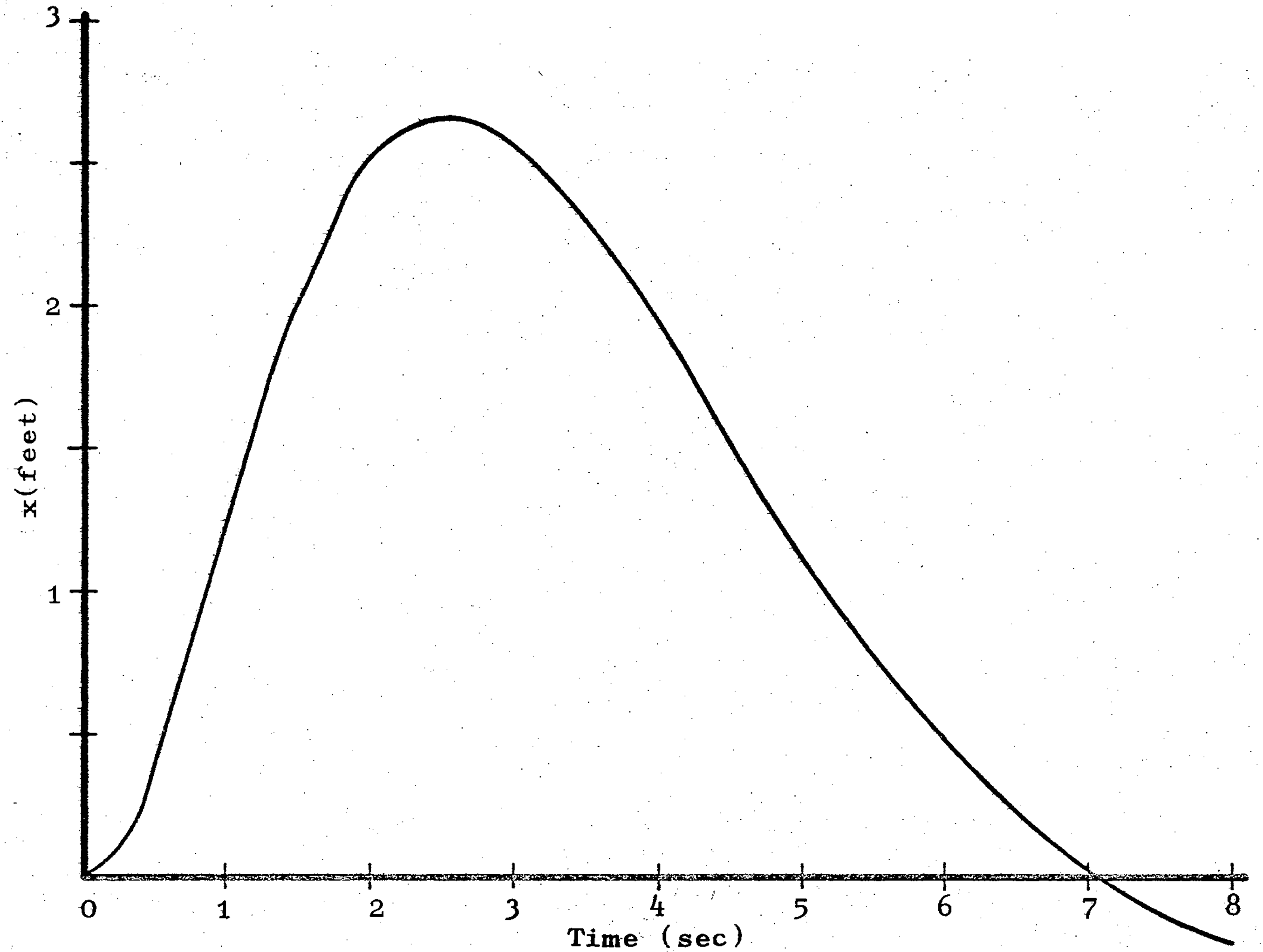


Figure 55. Displacement Output for Augmented System of Example Four

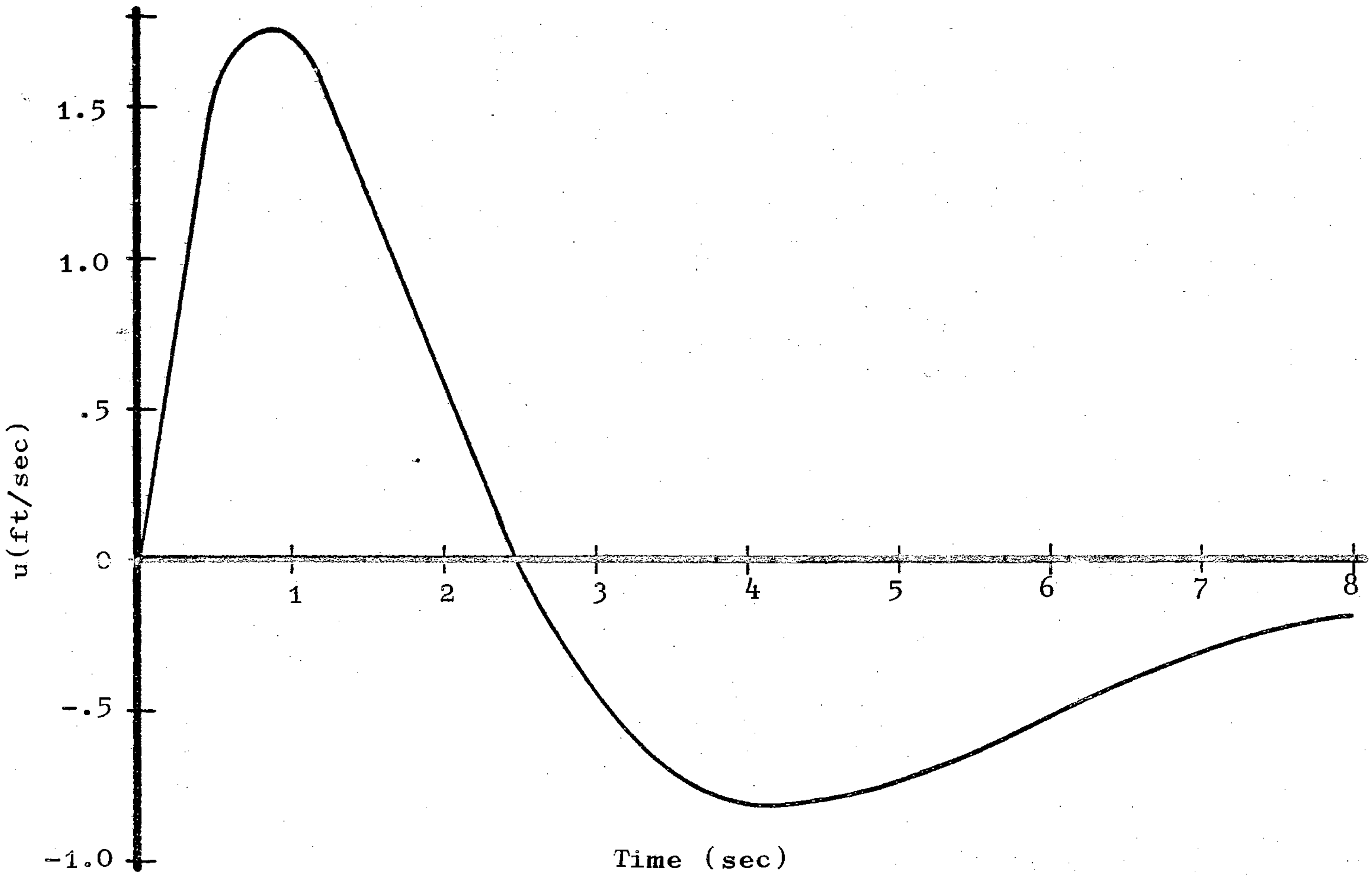


Figure 56. Translational Velocity Output for Augmented System of Example Four

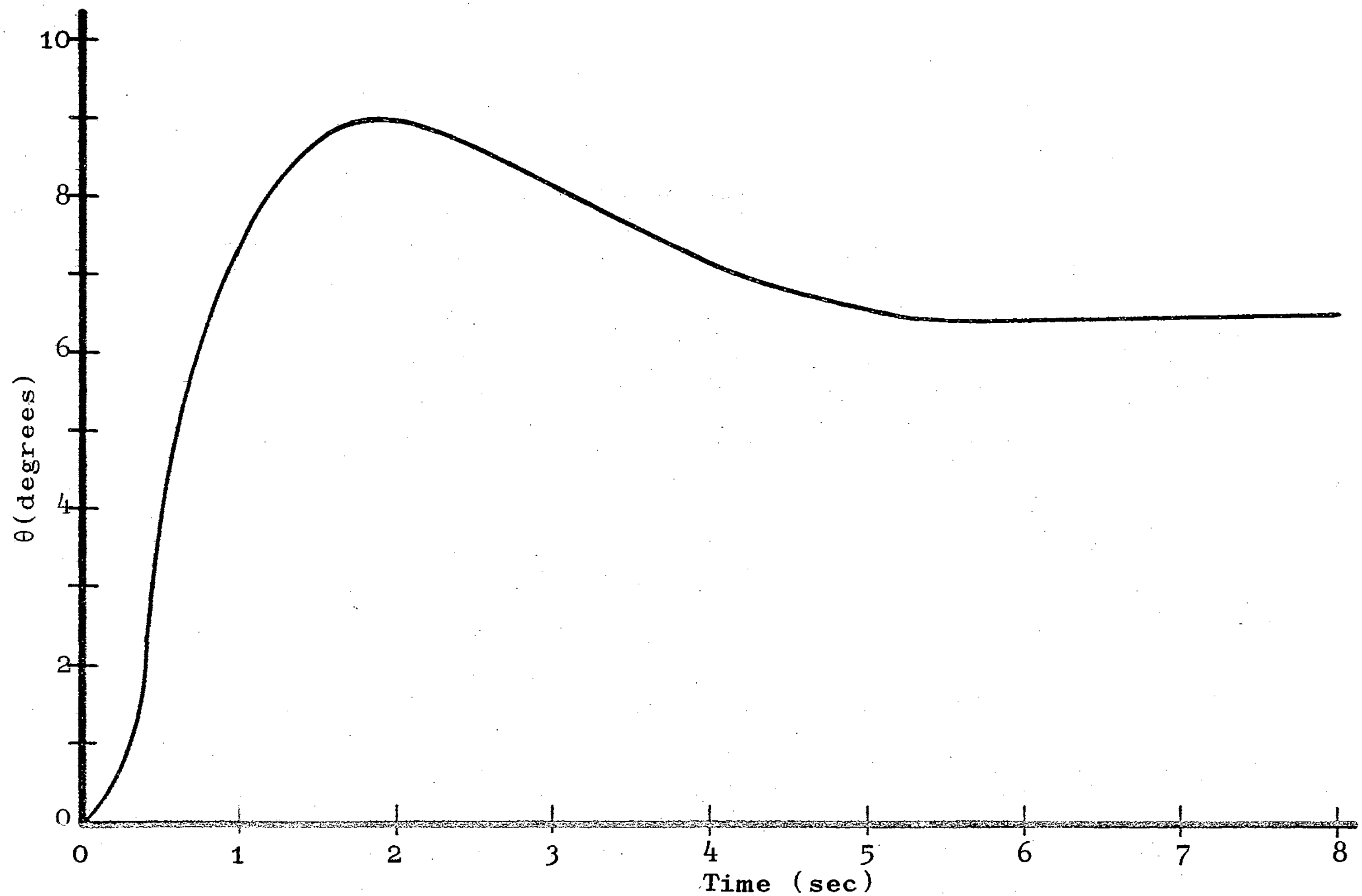


Figure 57. Pitch Output for Augmented System of Example Four

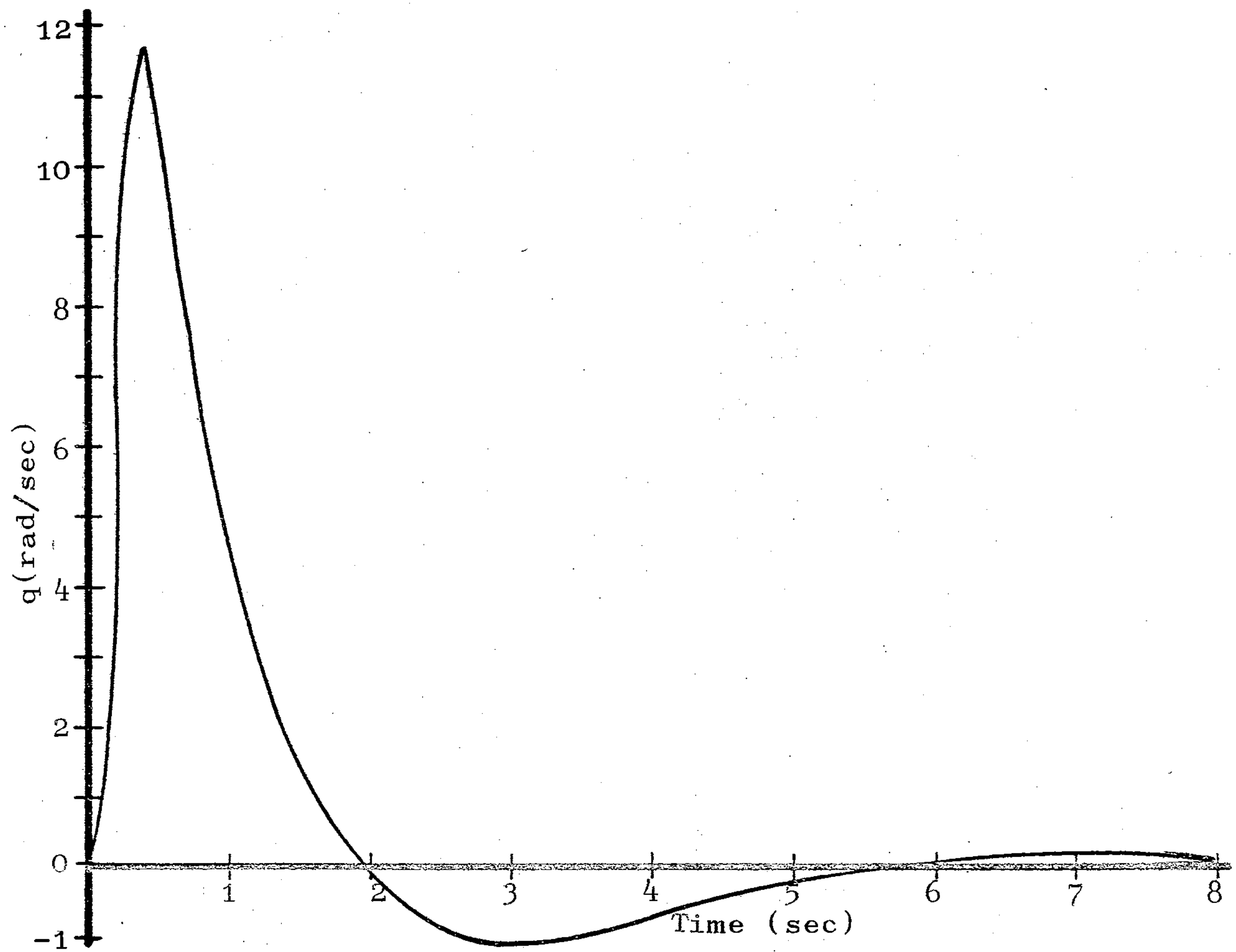
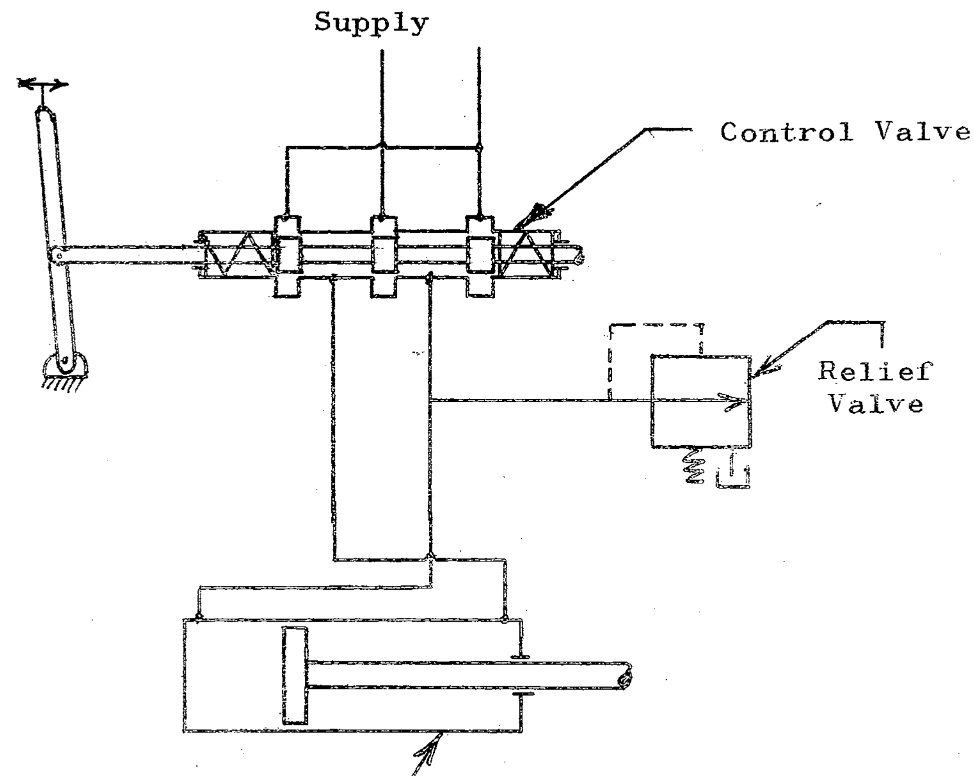


Figure 58. Angular Velocity Output for Augmented System of Example Four

there was sufficient experience gained to allow some valid inferences to be made. These inferences are concerned with why the backhoe needs augmentation and how this augmentation could be physically implemented.

The basic hydraulic circuit by which the operator controls the backhoe output is shown by the circuit diagram of Figure 59. The physical configuration for the backhoe is shown in Figure 60. As is indicated in this figure, the backhoe operator has four variables which he must control. To better understand how he must control them, consider the following description of the dig cycle operational procedure in which the backhoe is involved when it is used to dig a trench (30):

1. The operator places the bucket in the proper digging attitude.
2. He actuates the lever which causes the crowd cylinder to extend. This extension continues until the bucket encounters sufficient load to create a pressure overload and a consequent relief valve opening. At this point, the extension of the crowd cylinder stops because of the power loss through the relief valve.
3. With the above overload on the crowd circuit, the next step is for the operator to actuate the lever which causes the curl cylinder to extend. The result of this curl cylinder extension produces one of two possible states:



NOTE: This generalized circuit is utilized in the crowd, curl, and lift circuits on the backhoe. The swing circuit differs in that it has two single acting output cylinders as opposed to the one double acting as shown.

Figure 59. Simplified Hydraulic Circuit Diagram for Backhoe Output

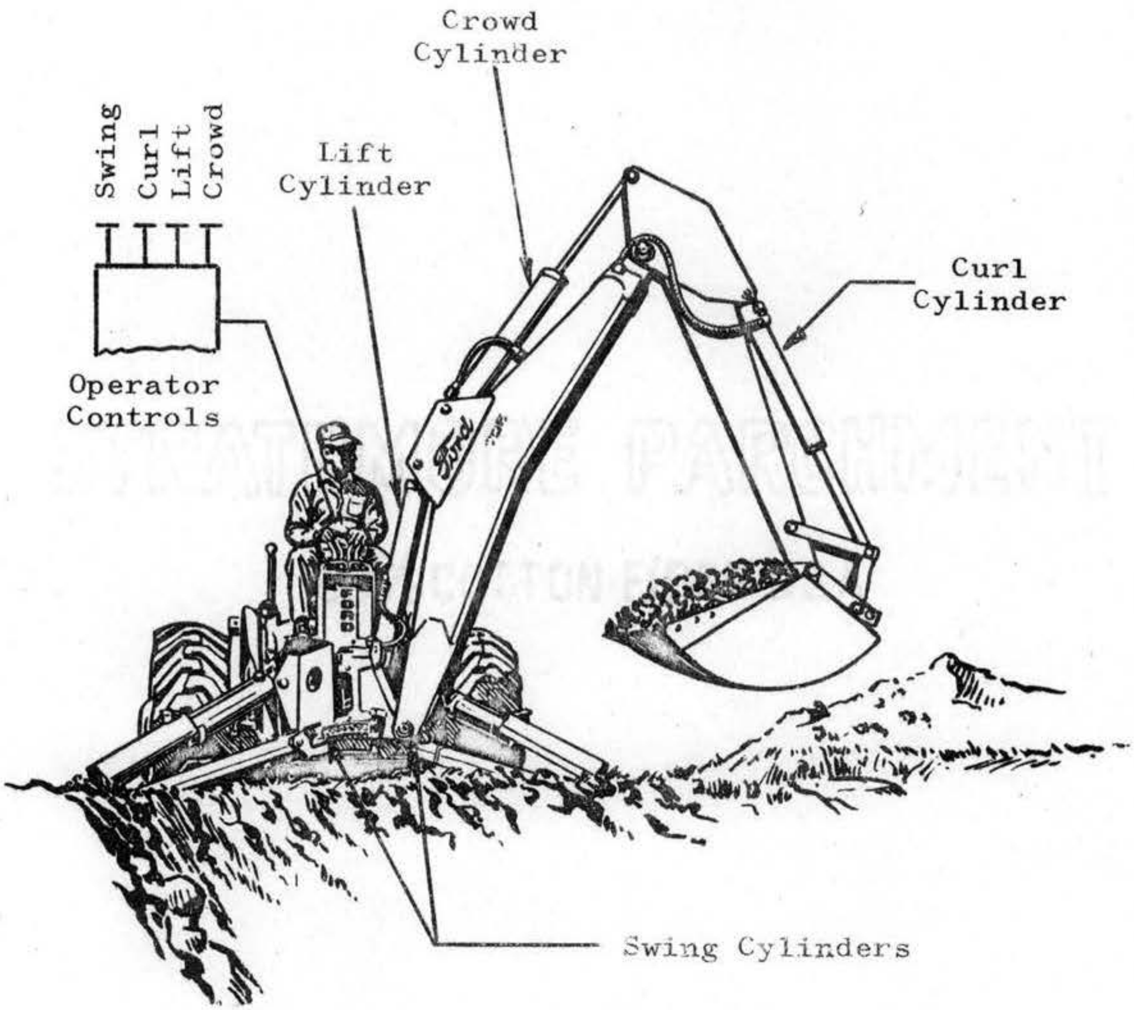


Figure 60. Side View of Backhoe Showing the Four Actuation Models

a. Extension of the curl may relieve the overload condition on the crowd. If so, the operator should stop the curl motion and return to procedure 2.

b. The curl cylinder may also encounter an overload condition without relieving the crowd overload.

4. When he encounters an overload on both the crowd and curl cylinders, he should actuate the lever which extends the lift cylinder. The result of this lift cylinder extension produces one of two possible states:

   a. Extension of the lift may relieve the overload condition on either the crowd or curl. If the overload condition is relieved, and the bucket is not full, he should revert back to statement 2 or 3.

   b. Extension of the lift may reveal a full bucket. If so, the operator should continue lifting until the bucket is out of the hole and then terminate the digging operation.

After completion of the dig cycle, the operator must contend with swinging the bucket to the dump pile and then dumping it, and with returning the bucket to the proper position to begin the next dig cycle once it has been dumped. Although these procedures are not as involved as the dig cycle, they do require the operator to manipulate all four system variables. This manipulation results from the need to have the bucket in the dumping position in the time it takes to swing it from the hole to the dump pile. Similarly, the need exists to have the bucket in the proper digging attitude in the time it takes to swing it from the dump pile back to the hole.

The conclusions reached from the above discussion of backhoe operation and from experience are that the operator is unable to control the four channels of the backhoe sufficiently enough to make it perform in an optimal manner. He can make the backhoe perform in what might be an optimal manner for short time intervals; however, when controlling in this manner he must closely monitor all the system variables that he can possibly perceive to the point that he is very heavily taxed both physically and mentally.

The indications are, given some design improvements in the backhoe control valves and in the means of actuating these valves, that the backhoe could be easily and reasonably augmented by the procedure of this report. The augmentation could be applied to any one (partial augmentation) or all of the cycles discussed above. An example of how it might be applied can be seen in the dig cycle.

The backhoe system for the dig cycle can be interconnected with an operator as shown in the block diagram of Figure 61. Note that each actuation system has several output states. These states not only represent the position,

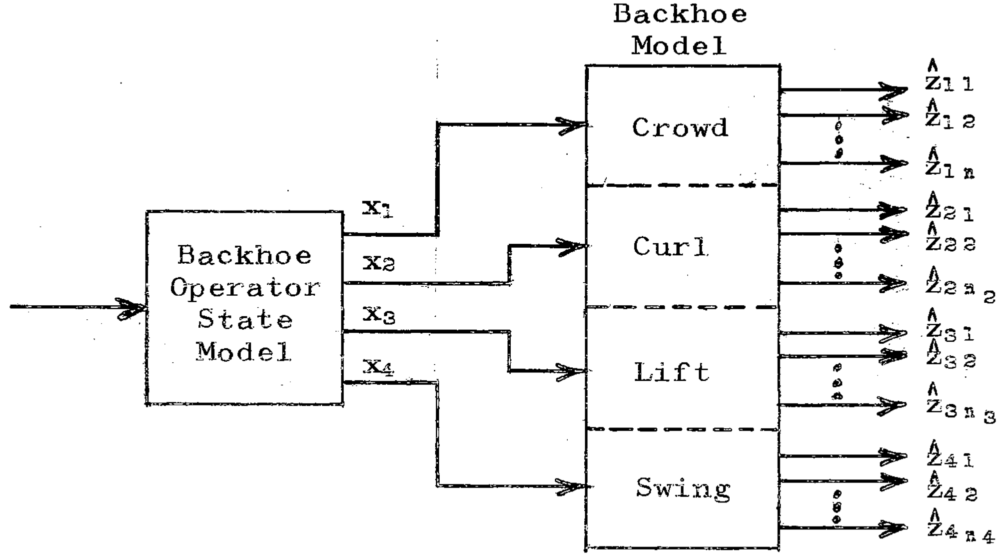


Figure 61. Block Diagram of Operator and Backhoe

velocity, etc., of the cylinder rod, but also the hydraulic pressure and flow variables inside the cylinder. To augment this sytem, assuming that the system of Figure 61 is converted to "augmentable" form and the state model is defined, it is necessary to define the criteria of performance. Referring back to the earlier discussion on the dig cycle, a probable criteria could be based upon the requirement that the movement of the bucket and the pressure in the cylinders would have some relationship for which the bucket could obtain a full load with a minimum expenditure of energy from the backhoe. If such a criteria was defined, it would be possible to augment the backhoe by designing a system which would provide the operator with sufficient information to control the dig cycle in an optimum manner. Although a visual display as utilized in the previous augmented examples might not be too practical upon the backhoe, the necessary information for optimal control could be provided to the operator through force feedback into his pressure modalities at the control levers.

As was mentioned earlier, there is insufficient backhoe operator-model data to allow any considerations for augmentation more serious than the example above to be made. However, a few more general comments can be advanced. First of all, the backhoe system would in reality require a nonlinear model to describe it. With such a model, it would be necessary to abort the matrix Ricatti approach utilized in the proposed method and revert to another method for solving the two point boundary value problem (23). Once the problem was solved, then the information required by the operator could be provided to him by an empirically designed augmenting system which also functions upon the augmented system output states. Secondly, another approach to augmenting the example system could be to use a time optimal performance criteria. This criteria could be word stated as obtaining a full bucket of dirt in minimum time. Finally, there are still many facets of the backhoe which would lend themselves to the proposed augmentation procedure. These facets could be considered individually as isolated backhoe improvements or taken as a whole to comprise an improved total backhoe system.

Summary

The illustrative problems used in this chapter were not chosen to illustrate the breadth of problems for which the proposed method of augmentation is applicable. Rather, they were chosen to show that the earlier specified requirements upon the method of augmentation have been satisfied. These requirements were that the augmenting system be able to:

1. Assure that the total man-machine system will always have optimum performance according to some predesignated criteria.
2. Permit man to control any machine no matter how dynamically complex it may be.
3. Allow the designer to depict man with any

dynamical model.

4. Be subject to analytical design.

5. Extend the control capabilities of man by recognizing and compensating for his perceptual and dynamical limitations.

The satisfaction of these requirements are evidence of the significiance of having a method which enables man to be augmented such that his somewhat limited capabilities as a controller can be greatly extended.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The general method presented in this thesis permits the analytical design of optimum man-machine systems which compensate for man's limitations as a controller of complex dynamical processes. The contribution of this method lies in the capability it has to permit man to control systems which he heretofore has been able to control only with great difficulty or not control at all.

It has been shown that the method is a significant improvement over the past attempts to augment man as a controller of complex dynamical devices. The improvement is due primarily to the fact that the man-machine system is assured of optimal performance with the method of this thesis, whereas with the past attempts it was not. In addition, the method presented allows an analytical approach to be taken in designing an augmenting system which makes full compensation for the perceptual and dynamical limitations of man. The advantage of having such a method is evidenced by the fact that a valuable tool which can be put to practical use in the design of human augmenting systems is now available.

There are a number of limitations to the method which suggest natural extensions for future study. These limitations are as follows:

1. The method as presented is limited to linear systems even though, as was indicated earlier, the optimal control theory that has been presented is applicable to nonlinear systems. However, to extend the method to include these systems will require that the augmenting system be implemented by using some means other than the direct analytical approach of the matrix Ricatti equation. The natural course to pursue here would be to solve the two point boundary value problem and then empirically design an augmenting system which will generate the desired operator control as a function of the augmented system state variables.
2. The need to determine the optimal control information, i.e., $\underline{K}(t)$ and $\underline{g}(t)$, prior to augmentation implies the need to have knowledge of the future states of the augmented system. This futuristic need could be eased and possibly overcome through use of an adjoint system in conjunction with a high-speed digital computer to provide the augmenting system output information on an instantaneous basis. A more realistic approach might be to statistically examine the tracking and perturbation

requirements of the systems to be augmented and then statistically predict the data that is needed a priori.

3. The need to directly obtain state variable information from the system being augmented to feed into the augmenting system may present itself as a limitation. This state variable information is often not available for direct measurement and, thus, may have to be approximated by mathematical operations upon the system output.

4. It is conceivable that the operator may introduce variation into his dynamical model as prescribed by augmentation procedure. In the event that this variation becomes evident and does present itself as a limitation upon the proposed augmentation procedure, it will be necessary to include an "updater" upon the actual operator model such that the augmenting system can be made to account for the variation.

Additionally, there are several areas which do not arise as limitations, yet are desirable for future study. These areas are:

1. Time optimal problem - The area of "bang-bang" control is of interest in the field of man-machine systems when the operator must control

the on-off nature of a system (A specific example would be a space module which depends upon rocket thrust for control.). A specific consideration within this area could be to provide the operator with switching curves that had been determined with his dynamics in mind.

2. Ford Backhoe - Another area of application is in the design of sequential circuitry for the automatic backhoe (30). The need for an augmented display of information to the operator (Example Five) parallels a similar need for optimum switching signals in the sequential circuitry of the automatic backhoe. The method of this thesis could possibly be extended to determining how the switching signals could be optimally augmented.

3. Human interest levels - An investigation of how the human's interest varies with his control task could be undertaken to provide augmented systems that not only guarantee optimal control, but also keep the human operator's interest level high.

4. Task difficulty adjustments - It would be in the interest of the human operator to provide him with a means by which he could adjust his dynamical requirement in an augmented system

which requires him to provide repetitious control. Thus, he could overt boredom by making the system a challenge to control.

5. Multi-input models - A logical extension of the method would be to obtain operator models based upon inputs from sensual modalities other than just vision. This would allow the more realistic, multi-input operator to be augmented.

Although the developments of this thesis certainly represent an advancement in the field of man-machine systems, a further indication of its significance is the impetus that it provides for further investigation. The areas recommended above for future study lend themselves to further investigation, and the method developed within this thesis is a means by which their exploration can be effected.

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