A COMPARISON OF TWO METHODS OF INSTRUCTION IN MATHEMATICS FOR ELEMENTARY TEACHERS

By

RAY WENDELL FLEISCHMANN

Bachelor of Science
Central Missouri State College
Warrensburg, Missouri
1962

Master of Arts
Central Missouri State College
Warrensburg, Missouri
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Thesis Approved:

W. Ware Marsden
Thesis Adviser

Robert S. Brown

Douglas B. Aichele

Gerald K. Goff

D. Durham
Dean of the Graduate College

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CHAPTER I

THE PROBLEM

Background Information

During the past twenty years many changes have taken place in the field of education and especially in relation to mathematics. One of the important changes has been the increased emphasis placed on the preparation of elementary teachers to enable them to teach the mathematics being offered at the elementary and junior high school levels today. This need for better preparation has been created by the upsurge of new ideas in the content of the total mathematics program of the public schools as well as the new methodology being employed. Today, the elementary teacher must teach more mathematics with greater meaning and understanding than at any time in history.

To meet these demands, the elementary teachers first need to improve their basic knowledge and understanding of mathematics. As an attempt to help solve this problem, much attention is now being focused on new and different methods of instruction. At Oklahoma State University in 1967, Gibbons (14) studied the relative effectiveness of three different methods of instruction used in mathematics for prospective elementary teachers. This research was the foundation for a second study by Hytche (19) which compared two of the methods used by Gibbons with two other methods. Both of these studies have pointed to the need for further research in the area of teaching methods.

Statement of the Problem

One prerequisite to improving mathematics instruction at the elementary school level is careful preparation of prospective elementary teachers. A review of the literature showed that elementary teachers need to improve their basic knowledge and understanding of mathematics (1), (7), (24), (26). Much evidence has been gathered supporting the fact that elementary teachers do not have sufficient knowledge to present todays mathematics at the elementary level. Therefore, more emphasis must be placed on finding ways to correct this problem rather than collecting more data verifying the existence of the problem. The research done by Gibbons (14) and Hytche (19) at Oklahoma State University has been conducted with this end in mind.

The purposes of Gibbons' study (14) were (1) to investigate potential ways to improve prospective elementary teachers' knowledge and understanding of elementary mathematics and (2) to investigate whether or not the mastery of this mathematics was affected by the way it was taught at the undergraduate level. The research was designed to determine whether or not undergraduate classes that were exposed to a combination of programmed learning, lecture, and discussion could achieve greater understanding in elementary mathematics than undergraduate classes that received only the lecture form of instruction. Three instructional methods were used: the Lecture-Program-Discussion (L. P. D.) method, the Program-Lecture-Discussion (P. L. D.) method, and the Lecture-Textbook (L. T.) method. (Each word used in the title of the various methods described a stage in the instructional process: "Program" indicated programmed materials were studied by the students, "Lecture" indicated lectures were given, "Discussion" indicated informal discussion was held between the instructor and students, "Textbook" indicated a traditional textbook was used.) Each of the three experimental groups was taught the content of the course Arithmetic for Elementary Teachers (Math 2413) and compared on the basis of posttest scores. The students involved in the L. P. D. method and the students in the P. L. D. method both showed a significantly greater level of achievement and understanding in mathematics than those students involved in the L. T. method. Gibbons also found that the students involved in the L. P. D. group showed a greater level of achievement and understanding in mathematics than did the students in the P. L. D. group but the level was not significantly greater.

In 1968, Hytche (19) compared the P. L. D. method and the L. P. D. method used by Gibbons with two additional methods called Program-Discussion (P. D.) and Program-

Lecture-Discussion-Quiz (P. L. D. Q.). ("Quiz" indicated that weekly quizzes were given.) Although no significant differences were found between the four groups, the P. L. D. Q. method produced the greatest average gain on the posttest over the pretest.

Based on the previous research, the purpose of this study was to investigate the relative achievement of students taught by the Program-Lecture-Discussion-Quiz method of instruction used by Hytche and a different method of instruction in mathematics for elementary teachers. A secondary purpose was to determine whether a change in the distribution of lecture and discussion time affected student understanding and achievement in mathematics for elementary teachers. The new method which was distinct from those used by Hytche was the Program-Lecture-Discussion-Test organizational scheme.

Review of Related Literature

G. Baley Price said that "the changes in mathematics in progress at the present time are so extensive, so far-reaching in their implications, and so profound that they can be described only as a revolution." (30, p. 1) These new developments are of importance but many old subjects are still highly important and need to be taught. Frequently, however, the emphasis must be placed on a different aspect of the subject, and an effort must be made to teach the subject so that the student gains a deeper understanding of it. "We must put forth whatever effort may be required to insure that the mathematics education provided by our schools is adequate for the needs of our times." (30, p. 11)

The following three components of the mathematics education adequate for our times are given by Price (30): mathematics courses with the proper mathematical content, well-qualified teachers, and counselors who will make certain that those students who have mathematical interests and abilities take at least four years of good mathematics in high school.

Of the three components listed above, this research is primarily concerned with the second--well-qualified teachers. The well-qualified teacher must know mathematics and must teach the subject with interest and enthusiasm (30). Butler and Wern (3) list two important aspects of any true profession; one of these is "signigicant knowledge." However, educators and researchers have found that many elementary teachers are not competent in the area of mathematics (24), (26).

A study conducted by Nelson and Worth (26) compared the mathematical competency of elementary teachers in the United States and Canada. It was found that the mathematical competency of prospective elementary teachers in Alberta, Canada, was higher than the mathematical competency of prospective elementary teachers in Illinois and Massachusetts. The study concluded by suggesting that those individuals who are concerned about the mathematical attainments of students in the United States might find the solution to this problem in improving the preparation program in mathematics for prospective elementary teachers.

Melson (24) concluded in a study involving forty-one elementary teachers that mathematics courses were either inadequately preparing elementary teachers or the mastery of the courses by the elementary teachers was faulty. These conclusions were based on the results of a thirty-three item test designed for grades one through six in modern elementary mathematics. This test was administered to the forty-one elementary teachers in September, 1963, and the median score was twelve correct responses. Only two of the teachers scored above 75% while twelve scored below 25%. All of these teachers had successfully completed a course in modern mathematics. It was concluded that most of the teachers involved in the study were not adequately trained in college to teach the elementary mathematics concepts which have been recommended for grades 1-6 by the National Council of Teachers of Mathematics, the state departments of public instruction, and the authors of recently published mathematics textbooks and materials.

According to Garsten (13), an elementary teacher should have a background which is both broader and deeper than the level at which the teaching is to be done. Instruction in mathematics aims at certain outcomes and objectives and the likelihood of attaining these objectives will depend on how well the instruction is planned and how well the classroom is organized.

Much has been written on the use of programmed materials as a supplement for courses and as a substitute for traditionally used textbooks in an attempt to reduce this problem. Silberman (35) reviewed fifteen studies on the use of programmed material and found that nine studies reported superior learning for the programmed material and six reported no difference. In a second summary of twelve studies where identical words were used in the "prompting" and "confirmation" procedures, Silberman found that seven studies showed superior learning for the prompted condition, two for the confirmation procedure, and three showed no difference for the two conditions. Filling in the blanks, a characteristic of the confirmation procedure, does not always produce better learning than having the blanks already filled in, a characteristic of the prompting procedure.

A study by Ripple (32) at Cornell University compared learning through programmed material with "comparable" textbook material or "conventional" instruction. The four groups tested were carefully selected sophomores enrolled in an introductory psychology course at Cornell University for the fall semester of the 1963-64 school year. One group worked through a standard programmed text with reinforcing feedback; a second group worked through the program but did not receive reinforcing feedback; another group listened to a lecture based on the programmed material; and the fourth group simply read the same material in conventional text form. A 50-item test was administered two days after the instructional period and again ten days later. The comparison of learning through programmed material and conventional instruction was based on these criterion scores. It was concluded that active involvement contributed to increased learning, reinforcement did not contribute to increased learning, retention was not improved, and individual differences were not reduced by using the programmed material.

Much research comparing learning through programmed and textbook material was reported in the literature with results substantially the same as above. Carr (4), Goodlad (15), and May (23) emphasized a lack of carefully executed experiments that unequivocally demonstrate the superiority of automated instruction, either programmed text or machine, over the usual classroom procedures.

Although studies have not always shown superiority, students can and do learn from programmed materials. Predictions based on recent research emphasize the usefulness of these devices as methods of instruction and, as better combinations are found for their use, these devices of automated instruction will be of even greater value. After reviewing much of the data available, Stolurow summarized the findings and made the following predictions about the future of auto-instructional methods and devices.

> These methods and devices are here to stay. Several things will be done to acquaint teachers with the potentiality of these developments. The comparative study of live and automated teaching will stop. Future research will concern itself with discovering the important characteristics of the materials and methods. These developments will lead to a theory of teaching. Courses will be revised as a result of the new insights provided. The devices of the future will be either books (programmed or scrambled) or computer-based machines; small devices will drop out. The results of the experiments in programmed instruction suggest an impressive contribution to education; and if the right programs can be developed and combined with an economical and effective means of presentation, the application of programmed instruction will be widespread. (38, p. 526)

In a paper suggesting a direction for future research in programmed instruction, Coulson stated that programmed instruction "must be considered in perspective among other educational techniques, each having its own advantages and disadvantages for specific requirements." (5, p. 372) Reynard (31) agreed with Coulson and went on to add that future research should attempt to discover ways of combining the techniques of programmed instruction with other educational methods to "optimize instruction for different tasks and for different student characteristics."

The foregoing discussion was directed at programmed material and its contribution to learning. Other phases in the instructional process have also been studied. According to McKeachie (22) research studies regarding teaching methods have been conducted over an extensive period of time. Many of these have been done on such topics as lecture method versus discussion method, distribution of lecture and discussion time, lecture versus automation, student-centered versus instructor-centered teaching and several others.

Lifson, Rempel, and Johnson (21) conducted a study comparing groups taught by all lectures to groups where discussion meetings were substituted for one-third of the lectures. No significant difference was found between the groups. It was interesting to note, however, that more favorable student attitudes resulted from the partial discussion method.

Hovey (18) experimented at the University of Colorado with two classes of educational psychology during the spring semester of 1957. One class was taught by the traditional lecture method, three lectures per week, while the other was handled by a small-group discussion and only one lecture per week. There was a small but non-significant difference in favor of the self-directed group in the mastery of course material. Ten months after the course was completed the two groups were reassembled for posttest covering course material and measures of curiosity. The self-directed group was still slightly superior in retention of course material. On thirteen of fifteen items measuring curiosity, the self-directed group was superior. The study was repeated in a somewhat briefer form in a course in general psychology with substantially the same results.

Numerous comparisons of the lecture and discussion methods have been reported in the literature. In reference to these comparisons, McKeachie (22, p. 1127) believes that "When one is asked whether lecture is better than discussion, the appropriate counter would be, for what goals?" With regard to theories of learning, McKeachie (22, p. 1126) goes on to say that "since discussion offers the opportunity for a good deal of student activity and feedback, it could, in theory, be more effective than the lecture method in developing concepts and problem solving skills."

Much of the preceding research involved comparisons of groups of psychology students taught by different combinations of lecture and small-group discussion. Little research could be found using similar methods of instruction with students in mathematics--in particular, little was done in classes designed for prospective elementary teachers. To help fill this void, a pilot study was conducted at Oklahoma State University, Stillwater, Oklahoma during the 1968-69 school year. Two groups of students enrolled in a mathematics course for prospective elementary teachers were taught by different methods of instruction. Both groups studied the same programmed material. One group attended two lectures and one informal discussion session per week. The last fifteen minutes of the discussion period were devoted to a quiz covering concepts introduced during the two previous lectures. The other group also attended two lectures per week and one discussion session but no quizzes were given. Instead, a one-hour examination was given during the first lecture period following the conclusion of each chapter. Thus, the second group spent more time in informal discussion and less time in lectures than did the first group. The results indicated there was no significant difference in the level of achievement of the two groups.

In summary, the literature revealed at least three major points directly related to this study and mathematics education in general. First, the well-qualified teacher must know mathematics and have a background which is broader and deeper than the level at which the teaching is to be done. Unfortunately, the studies reviewed have shown that elementary teachers were not competent in the area of mathematics. Secondly, research has shown that programmed materials have contributed to increased learning when used in proper combinations with other materials and methods of instruction. Finally, several studies have concluded that substituting some informal discussion group time for lecture time tended to increase mastery of the course material as well as the retention of the material when tested several weeks after the conclusion of the course.

Hypotheses

On the basis of the findings reported in the literature and the pilot study discussed in Chapter II, two hypotheses were developed for directing the study.

The first hypothesis was structured on the total enrollment in the course:

> There is no significant difference in the level of achievement and understanding in mathematics demonstrated by students involved in the Program-Lecture-Discussion-Test organizational scheme and by students involved in the Program-Lecture-Discussion-Quiz scheme when applied to students in Mathematics 2413.

Although Mathematics 2413 was designed specifically for prospective elementary teachers, enrollment was not restricted. Each semester a small percentage of those enrolled were not majoring in elementary education. This led to a second hypothesis:

> There is no significant difference in the level of achievement and understanding in mathematics demonstrated by prospective elementary teachers involved in the Program-Lecture-Discussion-Test organizational scheme and by prospective elementary teachers involved in the Program-Lecture-Discussion-Quiz scheme.

Significance of the Study

There has been an increased emphasis on the teaching of elementary school mathematics in recent years. This has required a better preparation of elementary teachers. One approach to solving this problem has been the improvement of the techniques used to teach mathematics. Many studies have been conducted to compare various methods of instruction but very few have dealt directly with elementary teachers in mathematics classes.

The study reported here was considered significant because it compared two methods of instruction in Arithmetic for Elementary Teachers (Math 2413) to determine which method produced the greatest level of achievement and understanding by the students. In the initial planning stages of this study, a decision was made to modify the teaching of Math 2413 in accordance with significant differences found between the methods.

Hopefully, further investigation and improvement of the teaching-learning process in mathematics can be conducted as a result of questions raised during this study.

Overview

In this chapter the writer has developed the background for the problem, stated the problem and hypotheses, and reviewed the related literature.

Chapter II is the report of the pilot study used to build a rationale for the experiment. This chapter includes a description of the methods of instruction, the sample, and the analysis of the data.

Chapter III includes a description of the experimental design of the experiment. The analysis of the data is presented in Chapter IV.

In Chapter V the writer summarizes the experiment and states the conclusions and recommendations for further research.

CHAPTER II

PILOT STUDY

Statement of the Problem and Hypothesis

Prompted by the preceding discussion, an experiment was conducted at Oklahoma State University, Stillwater, Oklahoma during the 1968-1969 school year. The purpose of the pilot study was to investigate the relative effectiveness of two methods of instruction in mathematics for elementary teachers as demonstrated on a commercially produced test measuring achievement and understanding.

The literature reviewed in Chapter I indicated that substituting some informal discussion time for lecture time increased mastery of the course content but how much discussion time would be the most effective depended upon the particular class involved. Therefore, the following null hypothesis was formulated.

> There is no significant difference in the level of achievement and understanding in mathematics demonstrated by students involved in the Program-Lecture-Discussion-Test (P. L. D. T.) organizational scheme and by students involved in the Program-Lecture-Discussion-Quiz (P. L. D. Q) scheme when applied to students in Mathematics 2513.

Seven sections of Mathematics 2513 were involved in the study. Three sections taught during the fall semester were given one treatment and four sections during the spring semester were given a different treatment. The same programmed material, Basic Mathematics, A Programmed Introduction by Berg and Goff, was used with all seven sections.

The basic design of the study was pretest--treatment--posttest. The results of the experiment were analyzed on the basis of posttest scores. The .05 level was used to test for significance and since the hypothesis was not directed, a two-tailed test was employed.

Subject Matter

All students involved in the study were taught the content of Structural Concepts for Teachers (Math 2513). This was the second of a two course sequence in mathematics for elementary teachers, the first course being Arithmetic for Teachers (Math 2413). Both courses use the programmed materials mentioned above. Topics covered in Math 2413, discussed in greater detail in Chapter III, pages 28-33, included sets, whole numbers, systems of numeration, fractions, integers, the number line, and rational numbers.

The pilot study was conducted in Math 2513 and its topics were: logic and solution sets, relations and functions, intuitive geometry, measurement, and real numbers.

The unit on logic and solution sets was introduced by simple and composite "statements." Truth-values for the composite statements involving the connectives not, or, and, if-then, and if and only if were then defined through the use of truth tables. Based on set theory and the truth tables, solution sets for both simple and composite statements were developed. For statements related to number systems (natural, whole, integer, fraction, rational) intervals were introduced as one method of expressing these solution sets. In the case of more general universal sets, Venn diagrams were used as a model for interpreting the solution sets of a statement. Finally, to show some of the applications of logic, an introduction to the methods of direct and indirect proof were given.

Relations were first introduced through ordered pair notation with the reflexive, symmetric, and transitive properties illustrated for finite sets. The "divides" relation defined on integers was studied in detail and led to a discussion of such concepts as odd and even integers, sets of divisors, sets of multiples, greatest common divisors, least common multiples, prime and composite integers and the square root of certain integers. Another relation defined for integers, the "congruence" relation, was then presented to give the students an opportunity to work with a relation that was not familiar to them. The unit concluded with a study of functions as a special type of relation.

Intuitive geometry was a development of a mathematical system through intuition and reasoning rather than the deduction which is necessary for a rigorous development.

This was done primarily by definitions. After lines and subsets of lines (rays, half-lines, segments) were discussed, the concept of "curve" was introduced. This led directly into a study of the more familiar polygons called triangles and quadrilaterals. The unit concluded with descriptions of 3-space figures such as polyhedrons, prisms, pyramids, cylinders, cones, and spheres.

The unit on measurement was divided into three major parts: measuring segments, measuring plane regions, and measuring solid regions. A careful distinction was made between the "measure" of a geometric figure and the "measurement" of the figure. The usual formulas for determining the areas of regions determined by triangles, rectangles, squares, rhombi, parallelograms, and trapezoids were derived. Similarly, formulas for finding the volume of solid regions determined by parallelepipeds, prisms, pyramids, cylinders, cones, and spheres were derived and intuitive comparisons of the volumes of certain solids were made.

The final unit on real numbers was an extension of the rational number system studied earlier and afforded the student the opportunity to review all of the properties of the number systems. Solution sets for open statements containing only one variable were found by using a model of the coordinate line. Finally, solution sets of statements having two variables were illustrated on the rectangular Cartesian coordinate system.

Methods of Instruction

Two methods of instruction were employed in the pilot study. They were (1) the Program-Lecture-Discussion-Quiz method (denoted P. L. D. Q.) and (2) the Program,Lecture-Discussion-Test method (denoted P. L. D. T.). Both methods are described in detail in Chapter III, pages 33-36. However, since the two methods appear to be similar by their titles, the pertinent differences will be pointed out.

Both of the groups were taught the same content from the programmed text and each group attended lectures and informal discussions pertaining to the lectures. The differences occurred in the testing procedures, the time allotted for actual discussion, and the number of lecture periods.

The P. L. D. T. group was given an hour examination after completion of each chapter. Because of the design of the program, this came to a total of five tests during the semester. These chapter tests were scheduled during the first lecture period immediately following the conclusion of each chapter. This meant that the number of lectures for the semester was reduced by five, or equivalently, the time spent in lectures by the P. L. D. T. group was reduced by two hundred fifty minutes during the semester. Informal discussion sessions were held for fifty minutes each week for those students desiring further explanation of the concepts presented in the lectures.

In contrast, the P. L. D. Q. group was not given an hour examination during the semester. Instead, one quiz was given each week during part of the discussion time. Thus, the discussion period was divided into two parts. The first part, approximately thirty-five minutes, was devoted to discussing the concepts covered in the preceding two lectures. During the remaining part, approximately fifteen minutes, a quiz covering these concepts was given. This fifteen minute weekly quiz resulted in a total of one hundred ninety five minutes of discussion time eliminated.

Table I summarizes the differences in the two methods of instruction.

TABLE I

TIME SPENT IN LECTURE, DISCUSSION, AND TESTING DURING A SEMESTER IN MATH 2513

Group	Distribution of Time in Minutes			
	Lecture	Discussion	Quiz	Test
P. L. D. Q.	1500	455	195	0
P. L. D. T.	1250	650	0	250

Evaluation Instruments

The three instruments used to measure the levels of achievement and understanding that resulted from the two methods of instruction were: (1) American College Test in Mathematics (A.C.T.M.), (2) The Structure of the Number System (Form A), and (3) The Structure of the Number System (Form B). A description of these instruments can be found in Chapter III, pages 37-40.

Sample

The sample for this pilot study consisted of one hundred ninety-nine students enrolled in Mathematics 2513 at Oklahoma State University, Stillwater, Oklahoma. Seventy-eight of these students were in the P. L. D. Q. group while one hundred twenty-one were in the P. L. D. T. group. Students who were repeating the course, who withdrew, or lacked scores on one of the instruments were excluded from the analysis. In the P. L. D. Q. group four students withdrew and thirty-seven were eliminated because of lack of data. In the P. L. D. T. group one student was repeating the course, two students withdrew, and fifty-three were eliminated because of lack of data.

The P. L. D. Q. group had a mean score of 19.05 on the A.C.T.M. test and a mean score of 24.86 on the pretest.

The P. L. D. T. group had a mean score of 18.05 on the A.C.T.M. test and a mean score of 23.57 on the pretest.

Analysis of the Data

A pretest--treatment--posttest design was used for this experiment. Each group was administered the pretest, The Structure of the Number System (Form A), prior to the first lecture on topics covered in the course. The posttest, The Structure of the Number System (Form B), was administered during the last week of each semester. Data from these two tests as well as scores on the A.C.T.M. test were used to test the hypothesis.

The .05 level was used to determine significance. Since the hypothesis was stated in the null form, a two-tailed test of significance was employed.

An analysis of covariance was used in comparing the two groups on the posttest results. This method was chosen because it statistically equated the means of the groups with respect to the covariates before conclusions were drawn about the treatment effects.

The data for the two experimental groups were prepared for an IBM 360 computer system at the Oklahoma State University Computing Center. A multiple analysis of covariance program was used to calculate the sum of squares, mean sum of squares, the beta coefficients and their standard errors, the adjusted treatment means and their standard errors, and the F ratio. The findings concerning these two groups (P. L. D. Q. and P. L. D. T.) are presented in Table II.

TABLE II

ANALYSIS OF COVARIANCE--PILOT STUDY

Source of Variation	df	Sum of Squares	Mean Sum of Squares	F
Total	101	3123.343		
Covariates	2	2217.782	1108.891	
Trt/Cov*	1	5.951	5.951	
Error	98	899.611	9.180	.648

*Treatments adjusted for covariates

From Table II, the calculated F value was found to be .648. The critical F value given by Steel and Torrie (37, p. 440) for the given degrees of freedom was 3.94. Since .648 is less than 3.94, this indicated that no significant difference existed between the two groups on the adjusted posttest results.

Summary

Two methods of instruction were employed in a pilot study to determine which produced the greatest level of achievement and understanding in mathematics for elementary teachers. The methods differed in the amount of time spent in lectures and informal discussion sessions. One group (P. L. D. Q.) attended two lectures per week and one discussion session with the last fifteen minutes of the discussion period devoted to a quiz covering concepts introduced in the two previous lectures. The other group (P. L. D. T.) also attended two lectures per week and one discussion period but no quizzes were given. Instead, an hour examination was given during the first lecture period following the conclusion of each unit. Thus, considering the entire semester, the P. L. D. T. group was allotted one hundred ninety-five minutes more time for informal discussion and two hundred fifty minutes less time for lectures.

The basic design of the pilot study was pretest--treatment--posttest. The pretest was administered to all subjects during the first week of each semester and the treatments (methods of instruction) were applied three times per week. A posttest was administered to all subjects during the last week of each semester.

The analysis comparing the two experimental groups disclosed the fact that no significant difference existed among the two groups on the adjusted posttest results. This finding allowed the writer to accept the hypothesis that there is no significant difference in the level of achievement and understanding in mathematics demonstrated by students involved in the Program-Lecture-Discussion-Test organizational scheme and by students involved in the Program-Lecture-Discussion-Quiz scheme when applied to students in Mathematics 2513.

CHAPTER III

THE EXPERIMENT

Introduction

The pilot study compared an established method of instruction (P. L. D. Q.) to a new method (P. L. D. T.) in Math 2513. It was found that no significant difference existed between the levels of achievement and understanding in mathematics for these two groups. Since the students in the two groups performed equally well, the same basic design was employed with students in Math 2413 where a much larger sample was drawn.

The experiment was conducted on the campus at Oklahoma State University, Stillwater, Oklahoma during the two semesters of the 1968-69 school year and during the first semester of the 1969-70 school year. The purpose of the study was to investigate the relative effectiveness of the Program-Lecture-Discussion-Quiz method of instruction and the Program-Lecture-Discussion-Test method when applied to students in Mathematics for Elementary Teachers (Math 2413).

A total of seventeen sections with five hundred twenty six students were involved in the study. Students were not restricted in enrollment and could thus select the section of their choosing.

Experimental Design

To test the hypotheses a pretest--treatment--posttest experimental design was employed. The pretest, The Structure of the Number System (Form A), was administered during the first week of each semester prior to the first lecture covering concepts to be studied in the course.

The posttest, The Structure of the Number System (Form B), was administered during the last week of each semester. The last week of each semester was used for summarizing and reviewing the material discussed during the semester. Therefore, no new concepts were introduced after the posttest had been given.

An analysis of covariance was applied to the data to compare the two groups on the posttest results. Two covariate scores, the pretest and the A.C.T. mathematics scores, were used as the control variables with only one criterion variable, the posttest scores.

Sample

The sample for this study consisted of five hundred twenty-six students enrolled in Arithmetic for Elementary Teachers (Math 2413) at Oklahoma State University, Stillwater, Oklahoma. Three hundred thirty-five on these students were in the P. L. D. Q. group while one hundred ninety-one were in the P. L. D. T. group. Students who were repeating the course, who withdrew, or who lacked scores on one of the instruments were excluded from the analysis.

Excluded from the analysis in the P. L. D. Q. group were eleven students who withdrew, four who were repeating the course, and one hundred ten for whom related data were not available. In the P. L. D. T. group fourteen students withdrew and forty-two were excluded for lack of data. As a result of these omissions, the analysis involved two hundred ten students in the P. L. D. Q. group and one hundred thirty-five in the P. L. D. T. group.

The course, as indicated by the title, was designed specifically for prospective elementary teachers but enrollment was not restricted. In the P. L. D. Q. group, thirty-two students were majoring in home economics and two were in the school of Arts and Sciences. The remaining one hundred seventy-six students were majoring in elementary education. In the P. L. D. T. group, twenty-two were home economics majors, five were in the school of Arts and Sciences, and one student was majoring in business. All of the other one hundred seven students were elementary education majors.

On the A.C.T.M. test the P. L. D. Q. group had a mean score of 19.08 while the P. L. D. T. group had a mean score of 19.31. On the structure of the number system pretest the P. L. D. Q. group had a mean score of 19.43 and the P. L. D. T. group had a mean score of 19.57.

It was interesting to note that the elementary education majors in the P. L. D. Q. group had a mean score of 18.80 on the A.C.T.M. test and a mean score of 19.28 on the pretest. Both of these mean scores were slightly below the means of the entire group. In contrast, the elementary education majors in the P. L. D. T. group had a mean score of 19.45 on the A.C.T.M. test and a mean score of 19.81 on the pretest. These means were slightly above the group means for both tests. Since no obvious differences existed with respect to scheduling, meeting time, enrollment procedure, and other conditions, the use of covariance for the statistical analysis seemed appropriate.

Subject Matter

The programmed material involved in this study (Basic Mathematics, A Programmed Introduction by Goff and Berg) was selected to give the students an introduction to modern mathematics. The content was chosen to conform with the recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America for prospective and in-service elementary teachers. Seven major topics were covered: sets, whole numbers, systems of numeration, fractions, integers, the number line, and rational numbers.

Sets were introduced through simple examples familiar to most students. Descriptions of sets and elements belonging to sets were difficult using word sentences due to the amount of words required for preciseness and consequently notations were introduced. Following these basic ideas about sets, measurements of sets were defined in terms of finite, infinite, and empty. The remainder of the chapter was concerned with relationships between sets (equivalent, equal, disjoint, subset, proper subset), operations on sets (union, intersection, complement, Cartesian cross-product), and properties of set-operations (closure, commutativity, associativity, identity, and distributivity).

The unit on whole numbers was developed as a special set of elements called whole numbers which had certain properties discussed in the first unit. Number, numeral, place-value, expanded notation, order and ordinal number were all discussed and defined in this light. The four operations of addition, subtraction, multiplication, and division were introduced through the definition of the number of a set. For example, if A and B are disjoint sets, then $nA + nB = n(A \cup B)$. However, if A and B are not disjoint sets, then $nA + nB \neq n(A \cup B)$. From these two facts about the number of sets, addition of whole numbers was defined as follows: $nA + nB = n(A \cup B)$ if and only if $A \cap B = \emptyset$. Likewise, multiplication of whole numbers was defined in terms of cross-products of sets as $nA \cdot nB = n(A \times B)$. Since the operations were defined in terms of sets, the properties of these operations (closure, commutativity, associativity, identity, cancellation, distributivity) were derived directly from the similar operations described for sets. As an example, addition of whole numbers had the closure property since the operation of union had the closure property when defined on sets.

In the unit on systems of numeration the following topics were discussed: bases, place-value, expanded notation, positional notation, the four basic operations in various bases, and changing bases for numerals. Important concepts from base ten were reviewed and numerals in other bases were simplified by introducing the idea of grouping. This principle was reinforced through the use of expanded notation in presenting the operations (addition, subtraction, multiplication, and division). For example, in base five the numeral 132 expressed in expanded notation was written as $1 \cdot 10 \cdot 10 + 3 \cdot 10 + 2 \cdot 1$. In this expansion, 10 represented one base and no units, or equivalently, one group of five with no units remaining. From this expanded notation and from positional notation, addition was developed in terms of groups of bases. Again in base five, $(32 + 24)$ was presented as follows: $32 + 24 = (30 + 2) + (20 + 4) = (30 + 20) + (2 + 4) = 100 + 11 = 100 + 10 + 1 = 111$. The other operations were treated in a similar manner and after the students had gained confidence in their ability to work in other bases several algorithms were presented and explained. A review of the properties for operations defined on whole numbers in the second unit soon revealed that the operations were independent of the choice of base and were therefore true for whole numbers using these new numeration systems.

Fractions were introduced as one form of a partition of a set. Many diagrams of rectangular and circular regions were partitioned and shaded to illustrate the concept of a unit fraction and equivalent fractions. Following this intuitive approach, a fraction a/b was formally defined as an ordered pair of whole numbers with $b \neq 0$ and was used to represent a of the unit fraction 1/b. This permitted usable definitions of equivalent fractions ($a/b = c/d$ if and only if $a \cdot d = b \cdot c$) as well as definitions of the four basic operations. Since these operations were defined in terms of whole numbers, the whole number properties could be used to prove theorems for the operations of closure, commutativity, associativity, identity, multiplicative inverses, and distributivity defined on the set of fractions. As an example, the closure property of multiplication for fractions was proved in the following manner. For any two fractions a/b and c/d, $a/b \cdot c/d = a \cdot c/b \cdot d$ by the definition of multiplication of fractions. But since a,b,c, and d are whole numbers with b and d nonzero and multiplication of whole numbers has the closure property, $(a \cdot c)$ and $(b \cdot d)$ are both whole numbers with $b \cdot d \neq 0$. Therefore $a \cdot c/b \cdot d$ is an ordered pair of whole numbers and is another fraction by definition. Similarly, an order relation for fractions was defined in terms of whole numbers as follows: $a/b < c/d$ if and only if $a \cdot d < b \cdot c$. Clearly this definition related directly to material previously studied. The set of fractions was defined in a manner such that the operation of division had the closure property.

Integers were also defined as ordered pairs of whole numbers but in a way different from fractions so that the operation of subtraction would have the closure property. Two integers (a,b) and (c,d) were defined to be equivalent if and only if $a + d = b + c$. Thus $(5,2) = (7,4)$ since $5 + 4 = 2 + 7$. Addition and multiplication were defined for these ordered pairs and by applying the definition of equivalence as well as properties of the whole numbers, the properties (closure, commutativity, associativity, identity, inverse, and distributivity) of addition and multiplication for integers were proved as theorems. Then the notation was changed to allow the student to represent each ordered pair as a single number. Many properties of these signed numbers were proved by means of the ordered pairs. Subtraction and division were then defined in terms of their inverse operations of addition and multiplication.

The unit on the number line was introduced as a geometric model or representation of a set of numbers. The number line was established by drawing an arbitrary line (horizontal for convenience) having an arbitrary point as an origin and an arbitrary unit of length. With this unit, a position on the line could be determined for all of the integers. Fractions could then be located by taking the appropriate part of the unit. Its main function was to help the student gain more insight and understanding of the definitions, theorems, operations, and properties developed in the first five units. Although the model could not be used to prove statements about numbers, the properties of addition and multiplication were verified for whole numbers, fractions, and integers.

The rational numbers were defined in much the same way as fractions and integers, i.e., rational numbers were defined as ordered pairs of integers, a/b, where b was positive. Next an equivalence relation, denoted =, was defined as follows: $a/b = c/d$ if and only if $a \cdot d = b \cdot c$. These definitions were then used to define addition, subtraction, multiplication, and division. The properties of these operations (closure, commutativity, associativity, identity, inverse, and distributivity) were then proved on the basis of these definitions and analogous properties of the integers. A definition for an order relation, $<$, was given and several related theorems were proved. The unit concluded with a study of decimal representations for the rational numbers with several examples given showing how to convert from rational numbers to terminating or repeating decimals and also how to convert from terminating or repeating decimals to rational numbers.

Methods of Instruction

Two distinct methods of instruction were used in this experiment. They were (1) the Program-Lecture-Discussion-

Quiz method and (2) the Program-Lecture-Discussion-Test method.

Hytche described the Program-Lecture-Discussion-Quiz method in the following manner.

> The P. L. D. Q. (Program-Lecture-Discussion-Quiz) method was a four step method of instruction. Each new concept or set of concepts was first introduced through programmed material prior to attending a given lecture. The number of concepts developed varied in relation to the complexity of the given concepts. These programmed materials were then supplemented by a related lecture. The programmed material, related homework assignment, and lecture were discussed during the first part of the discussion session, and finally, the last ten to fifteen minutes of this session were devoted to a quiz over that portion of the material covered during the previous week. This weekly cycle was repeated throughout the course.
>
> Each lecture was prepared in advance and included essentially the same content as was contained in the programmed material. Each lecture was presented in the following pattern: (i) a brief overview of the topics contained in the programmed materials, (ii) a structured presentation in which the individual facts and examples were put in proper perspective with regard to the total unit, and (iii) a summary that attempted to completely interrelate the lecture and the programmed materials.
>
> The discussion-quiz session was divided into two parts. The first 35 to 40 minutes of the 50-minute session were devoted to informal discussion of the material covered in the two previous lectures, the corresponding program, and related homework assignment. Discussion was carried on between groups of students, and an instructor was available for consultation with these groups; and when feasible, the instructor worked with students individually. The remaining 10 to 15 minutes of the 50-minute period were devoted to a quiz over the material covered during the preceding week.
>
> The instructors for all discussion-quiz sessions were graduate assistants pursuing the doctorate degree with an interest in the training of teachers in mathematics. The instructors

> for the lectures were regular, full-time college mathematics professors.
>
> The cycle used for this group (P. L. D. Q.) consisted of two 50-minute organized lectures, one 35 to 40 minute informal discussion, and a 10 to 15 minute quiz session. (19, p. 25)

The P. L. D. T. (Program-Lecture-Discussion-Test) method was also a four step method of instruction. Each new concept or set of concepts was first introduced through programmed material prior to attending a given lecture. Again, the number of concepts developed varied in relation to the complexity of the given concepts. These programmed materials were then supplemented by a related lecture covering the major concepts. The programmed material, homework assignment, and related lectures were then discussed during the weekly discussion session. Finally, an hour examination was given during the first lecture period following the completion of each chapter.

Each lecture was prepared in advance and included the major concepts contained in the programmed material. The lecture was a structured presentation of the definitions, theorems, properties, and ideas with several examples to clarify and illustrate their meaning. A brief summary reviewing previous material, relating it to present material, and leading to future material concluded each lecture.

The entire discussion session was designed and scheduled in a manner to permit the student to ask questions and discuss the material covered in the two previous lectures. Discussion was carried on between groups of students, the instructor and groups of students, and between the instructor and individual students. New concepts were introduced in these discussion sessions only when needed to answer questions raised by the students.

The cycle used for this group (P. L. D. T.) on each of the chapters covered consisted of one fifty minute lecture followed by one fifty minute discussion session, then two fifty minute lectures followed by one discussion session and finally a fifty minute test given during the next regularly scheduled lecture period. This cycle was repeated throughout the semester.

The subject matter and programmed material used with both groups was the same. Due to the nature of the programmed material, two weeks were required to complete each chapter. During each two week period, the P. L. D. Q. group was in lecture 200 minutes, informal discussion approximately 70 minutes, and quizzes approximately 30 minutes. In contrast, the P. L. D. T. group was in lecture 150 minutes, informal discussion 100 minutes, and tests 50 minutes. As can easily be seen, the P. L. D. T. group was in lecture 50 minutes less than the P. L. D. Q. group. However, the P. L. D. T. group gained 30 minutes of discussion time for each chapter as well as 20 minutes extra for tests.

These differences in allotment of time for the entire semester are summarized in Table III.

TABLE III

TIME SPENT IN LECTURE, DISCUSSION, AND TESTING DURING A SEMESTER IN MATH 2413

Group	Distribution of Time in Minutes			
	Lecture	Discussion	Quiz	Test
P. L. D. Q.	1500	455	195	0
P. L. D. T.	1150	650	0	350

Evaluation Instruments

Three instruments were used to measure the levels of achievement and understanding that resulted from the two methods of instruction. They were the American College Test in Mathematics (A.C.T.M.), The Structure of the Number System (Form A), and The Structure of the Number System (Form B).

The A.C.T. mathematics test was one of four parts of the A.C.T. Test developed by the American College Testing Program. It was a mathematical aptitude test considered to be a good predictor of future achievement in college mathematics (2, p. 9). The test consisted of forty multiple choice questions that sampled aptitudes related to pre-college mathematics. Shana'a studied several variables for use as placement guidelines for freshmen at the University Of Oklahoma and found that "the A.C.T.M. appears to be the best single variable for use as a placement guideline." (34, p. 85) Further, Shana'a states:

> Discriminant functions dependent on the A.C.T.M., the A.C.T.C., the high school mathematics grade point average, and the number of semesters of high school mathematics are of value in distinguishing membership in different mathematics courses at the five percent level of significance. However, they do not prove significantly better than the A.C.T.M. at this level as placement tools. (34, p. 85)

The results of this test were used as one of the covariates in the statistical analysis.

The second covariate, The Structure of the Number System (Form A) produced by the Cooperative Test Division of Educational Testing Service, was used as the pretest in this experiment. It was designed for classes in modern mathematics as an achievement test to measure understanding of the real number system up to the rational numbers. There were forty multiple choice questions covering the following concepts (10, p. 24): arithmetic judgment, properties of operations (commutative, associative, distributive laws, closure, inverses, identities), properties of integers, place value, factors, divisors, multiples, prime numbers, number lines, zero denominator, number systems, modular arithmetic, and Roman numerals.

The third instrument used in this study was The Structure of the Number System (Form B), also developed by the Cooperative Test Division of Educational Testing Service. It was also designed for classes in modern mathematics as an achievement test measuring understanding of the real number system up to the rational numbers. There were forty multiple choice questions covering the same topics as Form A described above. Form B is considered to be an alternate form of Form A (10, p. 56), and was used as the posttest in this experiment.

All of the Cooperative Mathematics tests, including The Structure of the Number System Forms A and B, were developed by the Educational Testing Service staff and written by forty-six mathematics teachers, junior high school through college. A total of forty-six pretest forms were administered to a national sample of students in May, 1960. Intensive revision was then undertaken and the revised forms were re-pretested in a national program in May, 1962. The results indicated that these tests were then appropriate for the intended populations.

These two tests were selected for use as the pretest and posttest in this experiment because they were the only commercially produced tests directly related to the objectives and content covered in the experiment. Both were measures of developed abilities, and thus content validity was of importance. The Educational Testing Service felt that content validity was insured by entrusting test construction to persons well-qualified to judge the relationship of test content to teaching objectives (10. p. 62). The reliabilities reported are measures of internal consistency, computed using the Kuder-Richardson Formula 20. Form A had a reliability coefficient of .86 with a standard error of measurement of 2.73 while Form B had a reliability coefficient of .84 with a standard error of measurement of 2.75 (10, p. 63).

An important characteristic of any test is its effectiveness in discriminating between high and low ability individuals. The distribution of biserial correlations between each item and the total test score provided a measure of this ability to discriminate. The biserial correlation for Form A was .50 with a standard deviation of .12 while the mean biserial correlation for Form B was .48 with the standard deviation of .12 (10, p. 64). Both of these were at a high level, thus indicating good discriminating power for both tests (10, p. 64). Finally, two methods were used to determine the equivalence of the two forms. First, the two forms were parallel with respect to content by the way they were constructed. Both forms covered exactly the same content and the number of questions on any one topic differed by no more than one for the two forms. Secondly, the two tests were equated to a common score scale to determine relative difficulty. The converted raw scores for Forms A and B differed by no more than two at all levels of performance and were therefore similar in difficulty (10. p. 67).

Limitations

There were several limiting factors present in this study that could place certain restrictions on the findings and conclusions. These limitations are as follows:

1. The sample for this study was restricted to students enrolled in Arithmetic for Elementary Teachers (Math 2413) at Oklahoma State University. Thus, caution should be exercised when attempting to generalize the results to other schools and different groups of students.
2. Intact groups of students were used which exhibited differences in ability and achievement on both the A.C.T.M. test and the pretest. However, an analysis of covariance was used in comparing the groups which statistically adjusted these initial differences.
3. The sample was not necessarily a representative sample of elementary education majors since it was not a random selection from all elementary education majors on the campus. However, nearly all elementary education majors enroll in this course at some time in their program.
4. The experimental groups were aware that they were part of a study and the results could have been affected by the Hawthorne effect.
5. Only two standardized tests were given to each group. The effect of taking the pretest may have affected the posttest results.
6. Finally, the experimental methods were applied during different semesters. However, the analysis of covariance corrected initial differences in the

groups but could not adjust for differences in attitudes toward the methods resulting from students involved one semester visiting with students enrolled during a later semester.

CHAPTER IV

ANALYSIS OF THE DATA

Introduction

The purpose of this chapter is to report the findings of the statistical tests used to determine the significance of the two methods of instruction as demonstrated on post-test scores. The .05 level of probability was used to judge the significance of the statistic associated with each hypothesis. Because the hypotheses were not directed, the two-tailed test of significance was employed. However, when significant differences were found, the posttest means, adjusted through covariance, were calculated to show where the difference existed.

Amalysis of Covariance--Two Groups With Combined Majors

The analysis of covariance was chosen for this experiment because intact groups of students of different sizes were used and could not be matched by identical scores on the A.C.T.M. test and the pretest on the structure of the number system without discarding a significant number from the study. This statistical tool was an extension of the analysis of variance model combined with certain features of regression analysis which statistically equated the groups with respect to the scores mentioned above before conclusions were drawn about the effect of the treatments.

The data for the two experimental groups were prepared for an IBM 360 computer system at the Oklahoma State University Computing Center. An analysis of covariance program was supplied by the computer center which calculated the sum of squares, mean sum of squares, beta coefficients and their standard errors, the adjusted treatment means and their standard errors, and the F ratio. The findings concerning the two groups (P. L. D. Q. and P. L. D. T.) are presented in Table IV.

TABLE IV

ANALYSIS OF COVARIANCE--COMBINED MAJORS

Source of Variation	df	Sum of Squares	Mean Sum of Squares	F
Total	344	12497.641		
Covariates	2	7152.609	3576.304	
Trt/Cov*	1	212.115	212.115	
Error	341	5132.917	15.053	14.09**

*Treatments adjusted for covariates
**Significant beyond the .001 level

The calculated F value was found to be 14.09 as shown in Table IV. The critical F value given by Steel and Torrie (37, p. 440) for the given degrees of freedom at the .05 level was 3.84. Since the calculated F was larger than 3.84, a significant difference existed between the two groups on the adjusted posttest results. In addition, the critical F at the .001 level was 10.83. This indicated that the groups were even significantly different beyond the .001 level.

Because the two groups were found to be different on the adjusted posttest results, the beta coefficients and adjusted posttest means for the groups were calculated to determine which group had the higher mean. The beta associated with the scores on the A. C. T. M. test was found to be .2761 and the beta associated with the pretest scores was .5620. Using these beta coefficients and other appropriate date from Table V, the adjusted mean score was found for each treatment. The formula used for these calculations was given by Winer (40) as follows:

$$\bar{Y}_j' = \bar{Y}_j - b_1(\bar{X}_{1_j} - \bar{X}_{1_T}) - b_2(\bar{X}_{2_j} - \bar{X}_{2_T})$$ where

j = method of instruction (P. L. D. Q., P. L. D. T.)

$\bar{Y}_j'$ = adjusted mean of posttest scores for method j

$\bar{Y}_j$ = mean of posttest scores for method j

$\bar{X}_{1_j}$ = mean of A.C.T.M. scores for method j

$\bar{X}_{1_T}$ = mean of A.C.T.M. scores for total sample

$\bar{X}_{2_j}$ = mean of pretest scores for method j

$\bar{X}_{2_T}$ = mean of pretest scores for total sample

b_1 = beta coefficient associated with A.C.T.M. scores

b_2 = beta coefficient associated with pretest scores

TABLE V

SUMS AND MEANS OF THE CRITERION AND CONTROL VARIABLES FOR TWO EXPERIMENTAL GROUPS WITH COMBINED MAJORS

		Criterion		Controls			
		Posttest		A.C.T.M.		Pretest	
Method of Instruction	n	ΣY	$\bar{Y}$	ΣX_1	$\bar{X}_1$	ΣX_2	$\bar{X}_2$
P.L.D.Q.	210	5722	27.25	4006	19.08	4080	19.43
P.L.D.T.	135	3481	25.79	2607	19.31	2642	19.57
Total	345	9203	26.68	6613	19.17	6722	19.48

From Table V and the beta coefficients given earlier, the adjusted mean of the posttest scores for the P. L. D. Q. group was found to be 27.30 while the adjusted mean of the posttest scores for the P. L. D. T. group was 25.70.

Since 27.30 was significantly higher than 25.70, the P. L. D. Q. method of instruction produced a significantly higher level of achievement and understanding in mathematics than did the P. L. D. T. method of instruction when applied to groups with combined majors.

Analysis of Covariance--Two Groups With Education Majors Only

A second point of interest in this study was to determine if different levels of achievement and understanding existed between two groups of elementary education majors when taught by the P. L. D. Q. and P. L. D. T. methods. The data were again prepared for an IBM 360 computer system at Oklahoma State University. The sums of squares, mean sum of squares, beta coefficients and their standard errors, the adjusted treatment means and their standard errors, and the *F* ratio were calculated. Table VI summarizes the findings concerning the two groups (P. L. D. Q. and P. L. D. T.) of elementary education majors.

TABLE VI

ANALYSIS OF COVARIANCE--ELEMENTARY EDUCATION MAJORS

Source of Variation	df	Sum of Squares	Mean Sum of Squares	F
Total	282	9982.953		
Covariates	2	5661.542	2830.771	
Trt/Cov*	1	159.465	159.465	
Error	279	4161.946	14.917	10.69**

*Treatments adjusted for covariates
**Significant beyond the .005 level

From Table VI, the calculated F value was found to be 10.69. The critical F value given by Steel and Torrie (37, p. 440) for the given degrees of freedom at the .05 level was 3.84. Since the calculated F was larger than 3.84, a significant difference existed between the two groups on the posttest results. In fact, the critical F at the .005 level was 7.88 which indicated that the groups were significantly different beyond the .005 level of probability.

Again there was a significant difference so it was important to determine which of the two groups had the higher mean score. The beta coefficient associated with scores on the A.C.T.M. test was calculated to be .2253 and the beta coefficient associated with the pretest scores was .6153. These coefficients as well as data from Table VII were used to determine the adjusted mean of the posttest scores for each group. The adjusted mean for the P. L. D. Q. group was 27.23 while the adjusted mean for the P. L. D. T. group was only 25.68.

TABLE VII

SUMS AND MEANS OF THE CRITERION AND CONTROL VARIABLES FOR TWO EXPERIMENTAL GROUPS WITH ELEMENTARY EDUCATION MAJORS ONLY

Method of Instruction	n	Criterion Posttest ΣY	$\bar{Y}$	Controls A.C.T.M. ΣX_1	$\bar{X}_1$	Pretest ΣX_2	$\bar{X}_2$
P.L.D.Q.	176	4761	27.05	3309	18.80	3394	19.28
P.L.D.T.	107	2779	25.97	2081	19.45	2120	19.81
Total	283	7540	26.64	5390	19.05	5514	19.48

On the basis of the calculated F value and the adjusted means of the posttest scores, the P. L. D. Q. method of instruction produced a significantly higher level of achievement and understanding in mathematics than did the P. L. D. T. method of instruction when applied to elementary education majors only.

Summary of Statistical Analysis

Included in this section is a summary of the results of the statistical analyses related to the two hypotheses of the experiment. Other conclusions and recommendations are presented in Chapter V.

The experiment was separated into two main parts. First, an analysis comparing the two experimental groups was conducted involving the total number of subjects, three hundred forty-five, without regard to their majors. Second, an analysis comparing two experimental groups with two hundred eighty-three prospective elementary education majors was conducted.

The first analysis involving the total number of students revealed that the first hypothesis should be rejected. This implied that there was a significant difference in the level of achievement and understanding in mathematics demonstrated by students involved in the Program-Lecture-Discussion-Test organizational scheme as compared with students involved in the Program-Lecture-Discussion-Quiz scheme when applied to students in Mathematics 2413. Further analysis of the data indicated that the students in the P. L. D. Q. group had the higher level of achievement.

The second analysis involving only those students majoring in elementary education revealed that the second hypothesis should also be rejected. There was a significant difference in the level of achievement and understanding in mathematics demonstrated by prospective elementary teachers involved in the Program-Lecture-Discussion-Test organizational scheme as compared with prospective elementary teachers involved in the Program-Lecture-Discussion-Quiz scheme. Again the group involved in the P. L. D. Q. scheme demonstrated the higher level of achievement.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary of the Study

The primary purpose of this research was to investigate the relative effectiveness of two experimental methods of instruction at the undergraduate level in Arithmetic for Elementary Teachers. A secondary purpose was to determine if differences in the amount of time allotted to lecture and informal discussion affected student achievement and understanding in mathematics.

Both of the experimental methods (P. L. D. Q. and P. L. D. T.) were four-step methods of instruction that employed programmed material, lectures, informal discussion, and tests. The P. L. D. Q. group spent more time in lectures and less time in discussion sessions than did the P. L. D. T. group. This was due to the fact that no hour examinations were given to the P. L. D. Q. group. Instead, weekly quizzes were given the last fifteen minutes of each weekly discussion period. In contract, the P. L. D. T. group was given a one-hour examination on each chapter during the first regularly scheduled lecture period following the conclusion of the chapter. Since weekly quizzes were eliminated, the discussion period was fifteen minutes longer each week.

Two samples were drawn for this experiment. Three hundred forty-five undergraduate students enrolled in Arithmetic for Elementary Teachers (Math 2413) at Oklahoma State University were involved in the first sample. Of these, two hundred ten were in the P. L. D. Q. group while one hundred thirty-five were in the P. L. D. T. group.

The second sample consisted of two hundred eighty-three prospective elementary teachers enrolled in Arithmetic for Elementary Teachers at Oklahoma State University. One hundred seventy-six were in the P. L. D. Q. group and the remaining one hundred seven students were in the P. L. D. T. group.

The basic design of the experiment was pretest--treatment--posttest. The pretest was administered to all subjects during the first week of each semester prior to the first lecture involving concepts relevant to the experiment. The treatments (methods of instruction) were applied three times per week throughout the semester. Finally, a posttest was administered to all subjects during the last week of each semester.

The independent variable utilized in this study was method of instruction which was categorized in two manners, P. L. D. Q. and P. L. D. T. The dependent variable, the criterion variable for the covariance regression, was the achievement of the students involved as measured by score on the posttest. Two variables used as covarient controls were score on the A.C.T.M. test and score on the pretest.

Three commercially made tests were used in the analysis to evaluate the methods of instruction. The pretest and posttest were alternate forms of The Structure of the Number System produced by the Mathematics Tests Division of the Educational Testing Service. Form A was used as the pretest and Form B as the posttest. The third instrument used was the A.C.T.M. test produced by the American College Testing Program. The pretest scores and the A.C.T.M. scores were used as the two covariates in the experiment.

The analysis of covariance was the major statistical analysis used in comparing the two groups to determine if there was a significant difference between them. This analysis was chosen because it statistically equated the two groups on the basis of the control variables before conclusions were drawn about the treatments. Where significant differences were found, the adjusted posttest means for the groups were calculated to determine which group had the highest mean.

Conclusions

On the basis of this research and subject to the specified limitations, several conclusions seemed to be justified.

The first hypothesis stated in Chapter I was rejected. That is, the level of achievement and understanding in mathematics demonstrated by students involved in the Program-Lecture-Discussion-Quiz organizational scheme was found to be significantly higher than the level of achievement and understanding of students in the Program-Lecture-Discussion-Test scheme when applied to all students in Math 2413.

The second hypothesis stated in Chapter I was also rejected. The level of achievement and understanding in mathematics demonstrated by prospective elementary teachers in the Program-Lecture-Discussion-Quiz organizational scheme was significantly higher than the level of achievement and understanding of prospective elementary teachers in the Program-Lecture-Discussion-Test scheme.

These two conclusions were accepted as a result of the F statistic obtained from analysis of covariance and the adjusted posttest means of the two groups.

During the course of the experiment certain things were noted which, when considered along with the findings relative to the hypotheses, gave rise to conclusions of a general nature.

Since the two methods of instruction appeared to be highly similar in design, no significant difference was anticipated between the groups. According to the literature reviewed in Chapter I, if a difference was found it would likely be in favor of the group with the greater amount of discussion time. In this experiment differences were found (see Tables IV and VI) and in both cases the difference was significant beyond the .005 level of probability. However, in sharp contrast to the literature, the group involved with the smaller amount of discussion time demonstrated the higher level of achievement and understanding in mathematics (see Tables V and VII). This finding might lead the reader to believe that a decrease in discussion time produced superior achievement in the P. L. D. Q. group. Caution should be exercised in drawing such conclusions since other uncontrolled factors were observed which could also have affected the outcome of the experiment.

One such factor was the noted difference in attendance at the discussion sessions by the two groups. When weekly quizzes were given in the discussion sessions, the percentage of students attending the informal discussion was much higher than when no quizzes were given. This difference in attendance could be important for at least two reasons. First, the weekly quizzes encouraged the students to keep up-to-date with the concepts whereas the elimination of weekly quizzes permitted the students to fall behind until the end of each chapter. Second, the weekly quizzes were used to determine course grades and therefore concepts that were not clear to the students were explained and discussed weekly. When the quizzes were eliminated the students were forced to take the initiative in order to clarify material that created difficulty for them. It was the observation of the writer that many students did not assume this responsibility.

Other factors such as desire to learn, interest, and motivation could have affected the results of this experiment. It would have been very difficult to control or measure these factors although they are present in all learning situations. The weekly quizzes seemed to maintain a higher level of motivation and desire for the students than did the chapter tests which were given less often.

Finally, the preceding conclusions have pointed to a more general conclusion concerning methods of instruction. These results seemed to indicate that a method of instruction consisting of several short quizzes covering a small amount of material is significantly more effective than a method of instruction using only a limited number of tests covering larger amounts of material.

Recommendations

As a result of this study, the writer makes the following recommendations for further research.

1. A large percentage of students in Math 2513 were eliminated from the analysis used in the pilot study described in this paper. Since the findings of the pilot study were different than the findings of the study conducted in Math 2413,

further research should be carried out in Math 2513 with a larger sample using a greater percentage of the subjects.

2. Research similar to this experiment should be conducted which could evaluate the methods of instruction at various stages of development instead of only terminal behavior. Also, other methods of instruction should be investigated and related to those in this study.
3. Various sizes of groups in both lecture and discussion should be investigated to determine the best size for producing the highest level of achievement in mathematics.
4. Research should be conducted in which additional review and supplemental materials are made available for each topic covered. This could be done by developing more programmed materials related to the various concepts or by placing such information on tapes or microfilm to be used by the students.
5. Since a lack of attendance was noticed in the P. L. D. T. group, this study should be duplicated but with the requirement that all students attend discussion sessions.
6. Research similar in nature to this study should be conducted in other areas of mathematics, especially in those areas where the traditional

three lectures per week are being employed. If the methods described in this study are found to be superior in these other areas of mathematics, then similar research should be conducted in other subject matter areas.

7. This study used the A.C.T. Mathematics scores as one of the control variables to determine initial differences between the groups. The producers of this test have said that it is a good predictor of success in mathematics in college. However, it is recommended that research be conducted to determine whether the A.C.T. Mathematics scores are actually better predictors of success in modern mathematics in college than the A.C.T. Composite scores.

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APPENDIX A

INDIVIDUAL SCORES OF SUBJECTS PARTICIPATING IN THE PILOT STUDY

P. L. D. Q.			P. L. D. T.			P. L. D. T.		
ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
25	26	29	21	27	32	10	18	20
22	33	33	16	25	29	29	23	36
14	15	23	15	24	24	01	17	17
19	28	34	16	25	26	20	13	18
21	22	27	14	21	27	19	31	36
22	35	39	27	34	35	08	15	15
18	21	27	22	30	32	22	27	29
21	18	32	17	14	32	21	21	26
16	16	26	12	14	17	17	21	24
22	29	33	29	37	39	13	22	21
19	30	28	15	29	31	13	17	19
18	27	32	15	28	31	14	20	25
14	25	28	17	20	22	19	30	31
23	31	31	20	27	34	20	23	31
16	21	27	25	18	22	19	32	34
17	22	27	22	28	30	20	24	24
26	28	34	24	29	36	19	26	27
18	19	25	13	20	20	11	12	15
23	32	29	14	28	35	16	16	20
26	30	34	20	16	27	10	19	20
21	26	33	22	22	27	22	25	32
17	29	32	18	24	28	20	36	34
27	31	36	21	26	32	21	30	36
17	20	27	18	24	30	23	22	27
22	28	33	16	16	28	14	23	22
20	27	34	16	22	28	11	14	14
16	23	27	25	34	33	16	20	24
13	20	26	19	19	28	21	23	31
17	25	28	19	19	32	21	29	27
24	24	32	19	34	31	20	19	30
17	23	27	17	21	24	18	29	31
18	24	27	22	32	37	19	18	26
18	28	32	20	30	28			
15	14	18						
08	16	18						
20	25	28						
15	29	24						

APPENDIX B

INDIVIDUAL SCORES OF ALL SUBJECTS PARTICIPATING IN THE STUDY AS A COMBINED GROUP

P. L. D. Q.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
18	17	19	20	19	29	16	12	17
18	21	28	14	16	27	25	26	25
13	11	18	14	11	22	30	34	37
14	18	22	29	29	33	19	24	31
22	24	30	19	22	27	23	20	33
16	18	19	28	34	36	21	19	21
23	16	28	21	16	25	11	15	16
22	20	34	16	19	25	22	16	29
13	12	23	25	29	37	30	29	33
20	30	35	15	17	22	16	23	18
14	16	36	16	12	28	16	19	27
21	25	33	29	21	26	19	17	27
28	22	30	18	14	18	15	15	28
19	17	22	27	22	29	14	18	18
01	07	13	15	25	29	15	11	17
18	15	17	19	17	28	09	14	20
21	18	26	10	13	24	25	28	30
14	04	24	28	22	33	22	28	32
25	17	28	23	23	27	29	28	36
25	25	38	19	25	32	15	13	22
15	22	28	14	13	22	26	28	29
24	23	31	16	20	24	16	16	24
18	17	26	16	04	13	25	31	39
17	11	22	08	17	21	29	27	33
08	08	15	13	16	29	21	24	27
12	18	29	22	21	24	33	27	29
16	10	21	28	29	39	22	22	32
18	19	28	15	14	23	19	24	33
14	16	25	22	32	35	27	25	32
22	26	31	13	12	21	25	23	33
29	25	35	29	28	33	10	17	18
19	16	32	06	12	19	24	18	30
22	26	36	19	23	35	18	17	26
10	11	23	23	16	22	16	21	33
22	21	27	06	16	17	21	24	26
19	12	21	20	14	22	19	19	29
13	21	24	20	20	30	17	25	30
18	22	29	18	19	28	21	20	30
18	21	29	11	07	17	25	23	28
22	28	31	20	22	30	24	24	30
17	19	24	16	18	20	29	21	32
16	21	26	16	20	30	18	20	22
25	25	34	10	11	34	16	16	27
17	17	31	21	18	31	15	13	26
24	24	34	21	23	27	24	23	30
12	13	24	29	31	35	23	23	30
10	10	17	15	15	21	18	20	32

P. L. D. Q.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
17	17	23	24	24	27	23	21	31
21	19	27	21	20	26	25	25	36
19	20	30	28	32	40	08	15	17
21	24	33	11	10	18	12	13	17
25	25	29	19	12	32	17	16	25
14	11	17	23	22	31	13	15	28
25	30	32	24	20	30	09	14	19
23	23	30	22	13	30	19	22	32
27	30	37	16	18	29	18	20	33
26	21	30	26	29	31	06	19	23
12	11	18	26	21	28	11	17	34
31	31	32	22	21	30	21	24	34
24	24	26	21	24	36	21	20	29
27	22	35	24	21	30	01	08	23
14	15	09	19	17	37	12	10	07
14	09	20	18	14	24	21	19	27
11	12	16	24	23	34	27	23	28
30	27	33	14	18	30	21	19	24
25	34	36	16	10	18	17	21	30
25	21	31	22	17	27	06	16	27
14	16	25	21	19	33	21	16	31
19	17	21	13	23	28	19	16	25
17	18	29	15	16	20	21	18	32

P. L. D. T.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
21	16	26	16	18	20	26	25	34
29	30	34	18	17	26	27	24	29
10	10	15	31	32	40	18	15	23
17	14	24	20	19	26	14	11	21
08	17	24	21	22	21	12	15	27
25	27	31	24	24	27	21	23	29
29	27	29	21	24	24	23	15	19
16	07	13	23	27	34	19	21	31
20	13	25	25	29	37	23	24	29
09	14	18	23	13	30	17	21	26
25	29	31	18	15	16	10	10	21
22	23	32	11	21	20	22	27	31
29	30	34	21	22	26	15	11	21
18	15	21	23	27	27	25	27	31
32	33	38	25	24	22	15	15	25
17	13	21	10	09	12	18	19	26
21	22	32	21	29	34	31	32	30
20	17	30	20	16	24	24	22	33
09	16	17	16	18	22	21	16	24
16	18	27	23	32	37	28	22	30
21	18	25	20	18	28	26	27	33
12	17	22	10	11	13	22	17	28
20	19	26	15	24	32	12	13	22
15	22	30	22	19	25	25	18	36
15	13	20	11	11	21	26	16	26
11	18	24	17	16	27	16	15	17
17	22	32	12	17	21	24	23	26
17	15	19	20	19	17	19	19	21
19	20	24	19	21	23	23	21	22
16	21	28	17	14	18	19	20	20
20	15	24	16	19	24	19	25	20
26	19	31	24	31	33	19	16	28
21	22	34	23	23	25	21	22	28
25	26	32	25	30	33	14	12	17
11	05	22	22	26	30	12	17	22
21	19	26	18	19	27	14	11	19
21	22	32	33	28	37	21	14	21
12	21	27	09	16	20	14	10	15
21	16	28	16	18	25	22	17	28
17	21	23	22	21	27	23	20	31
23	25	32	23	17	26	26	19	18
21	23	28	20	21	28	10	16	18
24	26	34	14	18	19	20	27	30
12	09	15	14	22	30	22	23	31
19	16	28	08	09	18	19	22	24

APPENDIX C

INDIVIDUAL SCORES OF THE ELEMENTARY EDUCATION MAJORS PARTICIPATING IN THE STUDY

P. L. D. Q.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
18	21	28	14	11	22	19	24	31
13	11	18	29	29	33	23	20	33
14	18	22	19	22	27	21	19	21
22	24	30	28	34	36	11	15	16
16	18	19	21	16	25	22	16	29
23	16	28	16	19	25	16	19	27
22	20	34	25	29	37	19	17	27
13	12	23	16	12	28	15	15	28
20	30	35	29	21	26	14	18	18
14	16	36	18	14	18	09	14	20
21	25	33	15	25	29	25	28	30
19	17	22	10	13	24	22	28	32
01	07	13	28	22	33	29	28	36
18	15	17	23	23	27	15	13	22
21	18	26	19	25	32	26	28	29
25	17	28	14	13	22	16	16	24
25	25	38	16	20	24	25	31	39
15	22	28	16	04	13	33	27	29
24	23	31	08	17	21	22	22	32
18	17	26	13	16	29	19	24	33
17	11	22	22	21	24	25	23	33
08	08	15	28	29	39	10	17	18
12	18	29	22	32	35	24	18	30
16	10	21	13	12	21	18	17	26
18	19	28	19	23	35	21	24	26
22	26	31	23	16	22	19	19	29
29	25	35	06	16	17	17	25	30
22	26	36	20	14	22	21	20	30
10	11	23	20	20	30	25	23	28
22	21	27	11	07	17	29	21	32
19	12	21	20	22	30	18	20	22
13	21	24	16	18	20	16	16	27
18	22	29	16	20	30	15	13	26
18	21	29	10	11	34	24	23	30
22	28	31	21	18	31	23	23	30
17	19	24	21	23	27	18	20	32
17	17	31	29	31	35	17	17	23
24	24	34	15	15	21	21	19	27
12	13	24	16	12	17	19	20	30
20	19	29	25	26	25	25	25	29
14	11	17	27	23	28	21	19	27
23	23	30	21	19	24	24	24	27
27	30	37	17	21	30	21	20	26
12	11	18	06	16	27	28	32	40
31	31	32	21	16	31	11	10	18
24	24	26	19	16	25	23	22	31

P. L. D. Q.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
27	22	35	15	16	20	24	20	30
14	15	09	21	18	32	22	13	30
14	09	20	12	13	17	16	18	29
11	12	16	17	16	25	26	29	31
30	27	33	13	15	28	22	21	30
25	34	36	09	14	19	24	21	30
25	21	31	19	22	32	19	17	37
14	16	25	18	20	33	18	14	24
19	17	21	06	19	23	24	23	34
17	18	29	11	17	34	14	18	30
23	21	31	21	20	29	16	10	18
25	25	36	01	08	23	13	23	28
08	15	17	12	10	07			

P. L. D. T.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
10	10	15	25	29	37	17	21	26
17	14	24	21	22	26	10	10	21
25	27	31	23	27	27	22	27	31
20	13	25	25	24	22	15	11	21
25	29	31	21	29	34	25	27	31
22	23	32	20	16	24	15	15	25
29	30	34	16	18	22	31	32	30
18	15	21	23	32	37	24	22	33
32	33	38	20	18	28	21	16	24
17	13	21	15	24	32	28	22	30
20	17	30	11	11	21	26	27	33
09	16	17	17	16	27	22	17	28
16	18	27	12	17	21	12	13	22
20	19	26	20	19	17	25	18	36
15	22	30	19	21	23	26	16	26
15	13	20	17	14	18	16	15	17
11	18	24	16	19	24	24	23	26
17	22	32	24	31	33	19	19	21
17	15	19	25	30	33	23	21	22
19	20	24	22	26	30	19	20	20
16	21	28	09	16	20	19	25	20
20	15	24	16	18	25	19	16	28
26	19	31	22	21	27	21	22	28
21	22	34	23	17	26	14	12	17
25	26	32	20	21	28	12	17	22
21	22	32	14	22	30	14	11	19
17	21	23	08	09	18	21	14	21
23	25	32	26	25	34	14	10	15
21	23	28	27	24	29	22	17	28
12	09	15	18	15	23	23	20	31
16	18	20	14	11	21	26	19	18
18	17	26	12	15	27	10	16	18
20	19	26	21	23	29	20	27	30
21	22	21	23	15	19	22	23	31
24	24	27	19	21	31	19	22	24
23	27	34	23	24	29			

VITA

Ray Wendell Fleischmann

Candidate for the Degree of

Doctor of Education

Thesis: A COMPARISON OF TWO METHODS OF INSTRUCTION IN MATHEMATICS FOR ELEMENTARY TEACHERS

Major Field: Secondary Education

Biographical:

Personal Data: Born in Slater, Missouri, October 20, 1940, the son of Mr. and Mrs. Charley W. Fleischmann.

Education: Attended grade school and high school in Slater, Missouri; graduated from Slater High School in 1958; received the Bachelor of Science in Education degree from Central Missouri State College, Warrensburg, Missouri, in May, 1962; received the Master of Arts degree from Central Missouri State College, Warrensburg, Missouri, in August, 1963, with a major in mathematics; completed the requirements for the Doctor of Education degree at Oklahoma State University in May, 1970.

Professional Experience: Employed as a teaching assistant in the mathematics department at Central Missouri State College, Warrensburg, Missouri, from 1962 through 1963; instructor and supervising teacher of mathematics in the Laboratory School at Central Missouri State College, Warrensburg, Missouri, from 1963 through 1967; teaching assistant in the mathematics department at Oklahoma State University, Stillwater, Oklahoma, from 1968 through 1970.

Professional Organizations: Missouri Council of Teachers of Mathematics, Mathematical Association of America, Phi Delta Kappa.