A HEURISTIC SUPPLY PROCEDURE FOR COVARYING MULTI-PERIOD REQUIREMENTS

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REQUIREMENTS

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PREFACE

The problems of the intermittent production facility or job shop are many and varied. This investigation is in the general area of supply for an intermittent production operation. The objective is to develop a supply procedure which does not require the assumption of independence of requirements from one period to another.

In reaching this stage of the education process, one is assisted by so many people, such as early school teachers and professors, that it would require an additional dissertation to express gratitude to all individually. I would like to thank collectively those people who are not mentioned specifically but who have helped me, academically or otherwise.

I first want to thank my wife, Virginia, for her encouragement and for cheerfully making the sacrifices which this undertaking has required. Although he is just beginning to talk, my young son, Jimmy, has also been a source of joy and encouragement in his own way. I also want to thank my parents for their part in my education and in my life; past, present, and future.

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INTRODUCTION

The research reported in this dissertation is in the general area of inventory problems related to the job shop or intermittent production systems. In such operations, items are produced to customer order and delivery to the customer is made after the required production time has elapsed. Since there is no established product, neither a finished goods inventory nor a raw material inventory is ordinarily maintained. The inventory problems of the job shop are primarily those of supplying the production process during the length of time over which a particular item is to be produced.

When a job shop accepts an order for the production of a particular item the quantity is specified. If the quantity is sufficiently large, production of this quantity is scheduled over a series of scheduling periods. There may be a correlation between requirements in one period and some other period. For example, when production is begun, the quantity actually produced in a scheduling period may be greater or less than was originally scheduled. Since there is to be a finite total production, the production of more or less than was anticipated early in the cycle has an effect on the amount of production needed in the latter part of the cycle. If such covariance exists, it should be used in making material supply decisions.

Hypothesis

The object of this research is to develop a method to determine the amount of raw material which should be made available at each period of a finite series of scheduling periods, when the demand process exhibits a covariance between the requirements in one scheduling period and the requirements in other scheduling periods. It is hypothesized that such a procedure can be developed. This problem is a special case of the inventory problem and is investigated using the methodology and terminology of inventory theory.

A material supply procedure is developed for several cost structures. The procedure allows for a positive or negative covariance between demands in various periods by treating the cumulative demand as the sum of random variables. An opportunity cost evaluation model is developed which allows a tradeoff determination of whether an established schedule should be amended when new scheduling information becomes available throughout the horizon.

The significance of the investigation is that it provides managers of intermittent production systems with a supply decision model which is not restricted to the assumption of independence of demands between periods. It also provides a tradeoff analysis which enables the supply decision maker to evaluate new requirement forecasts throughout a multi-period horizon for possible supply schedule improvements. This analysis is considered unique in that it deals with covariance between period requirements, while other studies assume independence or assume that the requirements are related to the inventory level. It is assumed in this investigation that a previously specified delivery schedule can be ammended at a cost which is less than the cost to negotiate and contract for the original order. This feature is considered unique and realistic. The features of improved information and reevaluation of decisions were not found in any of the research on this subject. This investigation recognizes and takes into account some of the information availability problems which can occur in attempting to supply future requirements. Completely accurate information may not be available at the time a decision must be made. The further ahead one tries to forecast a requirement, the less accurate that forecast is likely to be. As some scheduling periods in a horizon elapse, better information regarding the latter portion of the horizon is assumed to become available.

Background

The Importance of Inventory Study

The study of inventory and inventory theory is important for several reasons. Approximately one-third of the assets of the average

American business is invested in inventory. Inventories help to provide stable levels of sales, production, and employment. Individually for each business enterprise and collectively for the national economy, the inventory level serves as an early warning sign of potential economic difficulty.[1] In addition, many problems can be analyzed in terms of inventory theory. Personnel staffing may be considered as an inventory of a resource. The optimum number of spare machines and the amount of repair capability are other examples of situations which are amenable to analysis by inventory theory.[2]

Since inventory represents a sizable commitment of resources and is an important segment of the national economy, it is understandable why inventory has been and still remains the subject of much study. Quantitative investigations are known to have been made as early as the first decade of this century.[3] Research was conducted in inventory operations long before the term "operations research" became a recognized term for describing such an activity. Today, many investigations and publications pertain to the subject of inventory.

Inventory

Inventory is the accumulation of an idle resource. With the possible exception of the satisfaction of a miserly instinct, the accumulation in itself would have little inherent or direct advantage.

Inventories exist in order to supply a demand process. If there is no demand or anticipated future demand for a resource, then an inventory of that resource would offer more burden than benefit. Due to changes in demand, inventories can become obsolete resulting in the loss of large sums of money. An example of such a situation would be a fashion item such as clothing.

Inventory may be an accumulation of raw material to supply a production facility, it may be an in-process inventory to feed successive steps in production, or it may be a finished goods inventory accumulated to supply wholesale or retail consumer demand. The ultimate reason for all these inventories is to meet a demand which results in a sale and generates the revenue which is essential to the survival of the enterprise. If the consumer demanding process changes in any way, the amount of resource needed in any of these inventories is subject to change. Demand usually cannot be predicted exactly. There is risk associated with having an inventory since demand may cease or decrease. There is also a risk that the demand may increase and the business will be unable to meet it. Thus, the inventory decision making process is decision making under risk. The inventory level is determined to some extent by balancing the risk of the inventory level's being too low with the risk of the inventory level's being too high.

Even when there is certainty of demand, it may be desirable to build up an inventory at certain periods of time to allow the input rate to differ from the rate of demand. Manufacturing in large lots reduces the number of setups required and, consequently, the setup cost per item produced. For purchased items, purchasing in large quantities may result in reduced unit prices charged by the vendor. The purchasing of large lots may also result in economies in the shipping and receiving of materials. These and perhaps other factors may result in the total expected cost of operation being reduced by maintaining an inventory during certain periods of time.

The General Inventory Problem

Generally, inventory is an accumulation of a resource due to a nonsynchronized condition between the input rate and the output rate. It may be desirable as a protection from the risk due to the uncertainty of the output or demand, or it may be desirable due to economies in operation due to increasing the input rate at certain periods of time. To attempt to optimize inventory, one must consider both the input and output rate together. If the inventory is a finished goods inventory, the using or demanding process is not under the control of the decision maker. In such a situation the decision maker must analyze the output rate and determine the input rate which optimizes the total relevant inventory cost. Since the input rate, i.e., the production rate, influences the inventory cost, the production planning and inventory control functions are often assigned to the same unit within many organizations.

If the initial inventory level is represented by I_o, the rate of input by B, and the rate of output by E, then for any time, t, the inventory level, I, can be represented by

$$I(t) = I_o + \int_o^t (B - E)\, dt$$

if the rates are considered as being continuous. This formulation applies where the addition and removal of inventory occurs in continuous or nondiscrete units such as liquid measure. In some other cases the integral formulation is a sufficient approximation to model the actual process even though the units are discrete. In other problems the input and output occur in discrete units and a summation rather than an integration is a better model. Where the solution is to be by digital computer, the summation process will be followed. The problem can be represented by

$$I(T) = I_o + \sum_{t=1}^{T} (B_t - E_t) \quad .$$

The inventory problem to which inventory theory has addressed itself is equivalent to finding input and output functions which maximize or minimize some established measure of effectiveness, subject to certain restrictions.[4]

The usual measure of effectiveness is a maximization of profit or a minimization of the incremental operating cost relative to the inventory policy. If it is assumed that the sales revenue will remain the same regardless of inventory policy, then a minimization of cost is equivalent to a maximization of profit. An outline of most of the cost factors which would be considered in such an analysis is presented in Table I.

TABLE I

INVENTORY COST FACTORS

A. Finance (cost of capital)

B. Handling
   1. Receiving Labor
   2. Trucking
   3. Stores Labor
   4. Shipping

C. Storage
   1. Receiving Area
   2. Stores Area
   3. Shipping Area

D. Insurance
   1. Real Estate
   2. Property
   3. Industrial Compensation

E. Taxes
   1. Property
   2. Real Estate

F. Depreciation and Obsolescence
   1. Inventory Shortages
   2. Engineering or Design Changes
   3. Valuation Reduction
   4. Deterioration

G. Administration (maintaining the system)

H. Stockout
   1. Cost of Back Order
   2. Lost Sale
   3. Lost Customer

I. Setup and Order Costs

J. Item Cost (quantity discounts)

K. Opportunity Cost (loss of goodwill or future sales)

The inventory cost factors illustrated in Table I are not the only factors of importance in the analysis of an inventory problem. Non-cost factors such as lead time, lead time variation, forecasted demand, the forecast error, fixed shortage limitations, storage capacity, vendor minimums or maximums, and carload requirements may also enter into the analysis.[5] The purpose of such an analysis is to arrive at an operating policy.

An inventory policy usually consists of a set of decision rules which recommend two factors: how much to buy, and when to buy. When inventory problems become very complex, they may not be mathematically tractable. In such a situation, a great number of possible alternative policies may exist. The time and cost of evaluating such a problem may be greater than the possible savings. One restriction which may be placed upon an inventory problem is to specify the type of policy which is sought. This restriction reduces the number of alternatives to evaluate and usually provides a near optimal answer.

Classification of Inventory Problems

In addition to restricting the types of policy which may be considered in analyzing an inventory problem, several other restrictive assumptions may be made. The type of situation which is assumed to exist serves as a basis for classifying inventory problems.

One of the more general distinctions between inventory problems is that of the degree of certainty with which the parameters are known. When the parameters are assumed to be fixed or known with certainty, the problem is said to be deterministic. If the properties are not fixed or can be defined only by a probability distribution, the problem is said to be probabilistic. An actual deterministic situation exists when one analyzes past data to determine the optimum way a problem should have been handled in the past or when a definite plan is to be met. In working with consumer demand, the problem is usually probabilistic. However, if the demand variance is small or if the cost of deviation from the average is small, then deterministic analysis can be applied to make the computation and analysis simpler.

Both input and output may be treated as being

a. Continuously arriving in time

b. Arriving at discrete equidistant points in time

c. Arriving at discrete irregular points in time.

In most actual situations, condition (c) exists but condition (a) or (b) may be a sufficient approximation to provide a near optimal solution to the problem. Lead time can be assumed to be either deterministic or probabilistic.

Problems may be treated as being static or dynamic. The static problem is one in which the parameters are assumed to remain constant throughout time. This type problem would be a single period inventory situation or a multiple cycle inventory problem in which each cycle has the same parameters. A dynamic problem is one in which the parameters change with time.

Additional classifications include whether the problem considers a single item or multiple items. When a large number of products are to be stocked and the aggregate of all independent decisions violates some constraint, such as space or investment, modified algorithms must be used. Some problems deal with multistation situations in which decisions are to be made regarding the amount and timing of inventory additions at each step of processing in manufacturing an item from raw material into a finished product. The objective is to optimize the in-process inventory.[6]

Another class of problems which is far from fully investigated is the multilocation problem.[7] This area deals with determining optimum stocking policies in a group of warehouses. Decisions include many interrelationships caused by the possibility of supplying a stockout condition at one warehouse from a warehouse in a different district or from the factory. Probabilities of stockout in both of these warehouses simultaneously must be considered. An example of the types of alternatives considered in such a situation might be whether to keep moderate levels of an item in each local warehouse or to keep a small amount in each local warehouse and a large inventory in one central or "hub" warehouse.

Common Inventory Policies

Inventory policies once developed must be implemented to assure that consistent actions are taken by the diverse elements and personnel in an organization. In order to be implemented, a policy must be understandable to the persons who are to use it. The output of an inventory model, even though based on mathematical theory, will normally provide a collection of rules for how much to add to inventory and when to add it. These rules may be enlarged to include various actions to be taken depending on the environmental conditions which exist at the time of use.

There are three major types of inventory policy which are commonly used. The oldest is the fixed order system. Under such a system, the level of inventory of a particular item is monitored at each occurrence of a transaction involving that item. Whenever the level of inventory reaches a preassigned level, an order is placed for a specific amount. This amount is calculated to be the optimal lot size or "economic order quantity" (EOQ).

The second major type of inventory policy is the fixed order cycle system. Under such a system, a level, L, is set and an order is placed at regular intervals when the order date occurs. The quantity ordered is (L-I) which is the amount required to bring the inventory level, I, back up to L. The amount ordered depends upon the amount used since the last order date. The cycle length between order dates is selected so that the amount ordered will be approximately equal to the economic order quantity.

The third type of general inventory policy is a combination of the two previous policies. This type is called the (S, s) policy. Two control levels for the inventory are determined, the upper level being represented by S and the lower by s. The inventory is reviewed on a fixed cycle and if the inventory level, I, is above s, no order is placed. If the inventory level is below s, then an order (S-I) is placed for enough to bring the inventory level up to S.[8] This system has a fixed cycle for review like the fixed cycle system and a reorder level like the fixed quantity system. It differs from the fixed cycle system in that an order is not necessarily placed every cycle. It differs from the fixed order quantity system in that the order size may vary. The desired average order size is the economic order size, provided that the input rate and the output rate are approximately uniform.

Limitations of Inventory Policy

The large number of restrictions and alternatives previously mentioned suggests that exact solutions may be hard to achieve in actual situations. In seeking a policy to treat a particular situation, one must abstract a sufficient amount of detail so the solution is relevant to the problem and is near optimal.

> When mathematics is applied to the solution of inventory problems, it is necessary to describe mathematically the system to be studied. Such a description is often referred to as a mathematical model. The procedure is to construct a mathematical model of the system of interest and then to study the properties of the model. Because it is never possible to represent the real world with complete accuracy, certain approximations and simplifications must be made when constructing a mathematical model. There are many reasons for this. One is that it is essentially impossible to find out what the real world is really like. Another is that a very accurate model of the real world can become impossibly difficult to work with mathematically. A final reason is that accurate models often cannot be justified on economic grounds. Simple approximate ones will yield results which are good enough so that the additional improvement obtained from a better model is not sufficient to justify its additional cost.[9]

This quotation well illustrates the mathematical and analytical limitations and compromises.

Another aspect of compromise is well summarized below:

> Efficient inventory-control methods can reduce but not eliminate business risk. Risk, in business as elsewhere, is essentially a measure of uncertainty concerning the future. Inventory planning and control procedures can only help the businessman assess the risk and plan a strategy, as far as production and purchasing plans are concerned, to accept it on the most favorable terms consistent with the basic policies and objectives of the business.
>
> The power of improved inventory management is limited further by the basic nature of the conflict among the objectives of a business. Better sales through improved service to customers, lower costs through smoother production operations, and lower investment needs through reduced inventories are all legitimate business aims, but they are in fundamental conflict. The best an inventory-control system can do is make the conflict evident in order to force a business decision which balances objectives, and then assure that the balance arrived at will be faithfully observed in day-to-day operations. But making decisions

> more intelligently and making action respond to them does not mean that the decisions are necessarily easier, that the basic conflicts are eliminated, or that the essential risk of the business is reduced.[10]

Here Magee brings out another highly relevant aspect of compromise and limitation. In establishing inventory policy, one is attempting to optimize several factors. Each of these factors may be under the responsibility of different organizational elements. People in each responsible area have a personal bias in the way they view the problem. In attempting to gather facts so that a model can be developed, all of the facts may not be learned. Some of the information may be distorted or even hidden, either through intent or honest difference in orientation. In addition, records may not have been kept of all the desired facts or the process may not have been in operation long enough to provide a large sample. Many factors exist which can make the actual implementation of inventory policy differ from a system based purely on theory.

Literature Related to This Research

Work on inventory problems has been conducted at many different levels. At one extreme, a considerable amount of work is concerned strictly with practical applications, while, at the other extreme, work has been done on the abstract mathematical properties of inventory models without regard to possible practical applications.[11] The former area might consist of work relating to organization and staffing, forms design, system implementation case studies, and general managerial remarks about production and inventory management. These types of remarks are of great practical importance but are not considered germane to this research. Other types of studies relate to the conceptual schemes and rationale which can be quantified to some extent. These types of studies are considered as related to this dissertation; however, it is hoped that this work will fall within the category of practicality rather than abstract theory.

A larger number of books are presented in the bibliography, but a few are mentioned here to distinguish their orientation. An extensive bibliography is given by Hanssmann (1961) who also provides a review of progress in the field of production and inventory theory. Theoretical books can be classified by the level of mathematical sophistication required. Four books which require little mathematical sophistication and are intended for practitioners are Bowman and Fetter (1961), Brown (1959), Brown (1967), and Magee (1958). Brown's 1959 book provides an extended treatment of forecasting, particularly as it relates to inventory control. The book, published by Brown in 1967, is an unusual departure from the customary textbook presentation. It contains some of the elements of a novel such as a plot and character studies in the case study presented. This treatment might provide great benefits to students by illustrating that the same problem, in actuality, is often seen from several different points of view.

Books which require a moderate degree of mathematics are Buchan and Koenigsberg (1963), Hadley and Whitin (1963), Hanssmann (1962), and Whitin (1953). Hanssmann's book provides a very extended treatment of multilevel and multi-item problems. The Whitin book is an often referenced classic.

The books with advanced treatment of the mathematical theory of inventory are primarily related to the work performed at Stanford University or performed by researchers who have studied there. The book by Arrow, Karlin, and Scarf (1958) and one edited by Scarf, Gilford, and Shelly (1963) would be classified in this category. Many articles have been published in the area of mathematical theory. An excellent review of the works in this field is provided by Inglehart (1967).

Numerous articles pertaining to inventory appear in periodicals. Some of the more common sources of these are mentioned in the following sentences. Articles dealing primarily with the theoretical and mathematical aspects of inventory may be found in Operations Research. Management Science publishes many articles which may range from management studies related to inventory to theoretical development of inventory models. The American Production and Inventory Control Society publishes a Quarterly Bulletin which contains many good inventory articles. The Naval Research Logistics Quarterly also contains a wide variety of inventory related articles as does The Journal of Industrial Engineering.

To place in context the research presented in this dissertation, a brief review of the development of related ideas would be in order. The first known publications to deal quantitatively with inventory problems occurred early in this century. The earliest development of the EOQ model was by Ford Harris of the Westinghouse Corporation in 1915. This same formula has been developed, apparently independently by many individuals since that date. One such individual was R. H. Wilson who sold an inventory scheme including this formula to many companies. Today the EOQ formula is sometimes referred to as the Wilson formula. F. E. Raymond wrote the first full-length book dealing with inventory which was published in 1931. It contained many extensions of the simple lot size formula rather than derivations of new models.[12]

During World War II, as operations research developed, attention was directed toward the stochastic aspects of inventory problems and models were developed. Arrow, Harris, and Marschak published in 1951 an article dealing with the dynamic inventory problem. This model took into account the probabilistic aspects of demand but assumed that the probability distributions for each period's demand were identical and independent from any other period.[13]

The first book in English which dealt in detail with stochastic inventory models was published in 1953 by Whitin. Shortly after this time, Bellman developed the concepts of dynamic programming and published his book (1957) on the subject. The following year, Wagner and Whitin (1958) applied the dynamic programming concept to develop a dynamic deterministic lot size algorithm.

In 1960, Samuel Karlin published a model for a probabilistic situation where the demand distributions do not have to be identical. However, one major assumption was that the demands were independent from period to period.[14]

In 1962, Karlin and Iglehart published a similar study with some covariance allowed between the demands. This covariance was assumed to be of a Markov chain type,[15] so that the demand probabilities are independent of how the system got in its current state. The problem treated in this dissertation assumes that the demand probabilities depend on the manner in which the preceding demands occurred.

FOOTNOTES

[1] Donald F. Hess, "Production Planning and Inventory Management", The Encyclopedia of Management (New York, 1963), p. 727.

[2] Fred Hanssmann, Operations Research in Production and Inventory Control (New York, 1962), pp. 100-101.

[3] Robert G. Brown, Decision Rules for Inventory Management (New York, 1967), p. 6.

[4] Hanssmann, p. 6.

[5] International Business Machines Corporation, IMPACT Application Program Bulletin (White Plains, New York, 1963), p. 1.

[6] Fred Hanssmann, "A Survey of Inventory Theory from the Operations Research Viewpoint", Progress in Operations Research, Chapter 3, edited by Russell L. Ackoff (New York, 1961).

[7] G. Hadley and T. M. Whitin, "An Inventory Transportation Model with N Locations", Multistage Inventory Models and Techniques, edited by H. E. Scarf, D. M. Gilford, and M. W. Shelby (Stanford, California, 1963), p. 116.

[8] K. Arrow, T. Harris, and J. Marschak, "Optimal Inventory Policy", Econometrica, XIX (1951), pp. 250-272.

[9] G. Hadley and T. M. Whitin, Analysis of Inventory Systems (Englewood Cliffs, 1963), p. 2.

[10] John F. Magee, Production Planning and Inventory Control (New York, 1958), p. 15.

[11] G. Hadley and T. M. Whitin, Analysis of Inventory Systems, p. 4.

[12] Ibid, p. 3.

[13] Arrow, Harris, and Marschak, p. 261.

[14] Samuel Karlin, "Dynamic Inventory Policy with Varying Stochastic Demands", Management Science, VI (1960), pp. 231-258.

[15] D. L. Iglehart and S. Karlin, "Optimal Policy for Dynamic Inventory Processes with Nonstationary Stochastic Demands", Studies in Applied Probability and Management Science, Chapter 8, edited by K. J. Arrow, S. Karlin, and H. Scarf (Stanford, California, 1962), pp. 127-147.

CHAPTER II

THE PROBLEM

This dissertation deals with a problem in the procurement of raw materials and components to meet a dynamic demand over a finite multiperiod horizon when there is covariance between the material requirements in one period and the requirements in other periods. The particular problem arises in attempting to specify the quantity to be delivered on each of a series of equidistant points in time when the actual amount needed is subject to change between the time when the order is placed and the time the delivery date occurs. Many of the factors discussed in the introduction are relevant to the proposed problem and inventory theory will be used to develop a quantitative solution to such a problem.

Since the type of inventory problem discussed here differs from the ordinary mass production, continuous demand problems usually found in textbooks, it will be described in some detail. The framework of a typical situation will be presented in addition to mathematical terminology.

Problem Environment

Many industries sell in low or moderate volume an established line of products which they do not stock in inventory to sell "off-the-shelf". Such products are typically expensive and have optional configurations which may be specified by the customer at the time of order. In such an industry, a finished goods inventory would not be desirable. Also, in such an industry, it would be expensive to maintain a raw materials inventory for some of the components. Since a finished goods inventory (which would serve as a buffer between production and demand) is not maintained, the production rate is not continuous. In addition to the intermittent demand, there is a possibility that some components may become obsolete due to design changes. Examples of this type industry are the heavy equipment, aircraft, and machine tool industries. Perhaps some types of ship building would fall in this category also.

The Schedule

The inventory management problem begins with the receipt of two kinds of information from the production planning department: the master schedule and the bill of materials. The bill of materials or "material requirement request" shows the number and type of each of the component parts in the finished product. The master schedule specifies the customer delivery date of the finished items. Usually in the material control department, this information is combined with the manufacturing or assembly time to yield calendar dates on which the raw material or parts would be needed for production.[1]

The development of a series of raw material requirements, called a requirements vector, comes from the scheduling process. When an order is received for a quantity of a product this order is broken down into the manufacturing processing time it will require in each of the work centers which must perform manufacturing operations on that product. When production time is available for the first operation the manufacturing cycle can begin, if the raw material is available. The time that the production is to begin establishes the time that there will be a requirement for raw material. If several orders which require the same raw material are received, they may be scheduled to begin in adjacent scheduling periods, resulting in a series of raw material requirements. If the available production time at some operation is not sufficient to process the entire order in one scheduling period a larger order will be divided and processed in two or more scheduling periods near that scheduling period.

The first production operation is not the only one which can result in a requirement for material. A subassembly component may be purchased and attached at any stage in the manufacturing cycle. If the same component were required in a series of adjacent scheduling periods it would also result in a multiperiod requirement.

Figure 1 shows a portion of a master schedule in which there is a series of requirements for a raw material item. There are requirements for 38, 16, and 20 part ABC raw forgings in months 3, 4, and 5, respectively.

The series of requirements may be represented by a row vector R_1, R_2, $R_3 \ldots R_N$ where N represents the number of months ahead of the present for which requirements are scheduled. The value of N depends upon the length of time until delivery dates for finished products.[2] When business is good, the scheduled production backlog may be long and in other times relatively short. Thus, the planning horizon for a particular product varies depending upon the amount of business foreseen for that product.

Similar schedules would exist for other parts required in the product line. There may be several thousand parts which fall into the category for which management does not wish to maintain an inventory. Each part schedule would differ from others depending upon the product mix sold, the amount of production to be performed on the part, the work load in the work centers at which the part is to be processed, and the point in the assembly operation at which the processed part is required in order to complete the final assembly in time to meet the promised delivery date for the completed product.

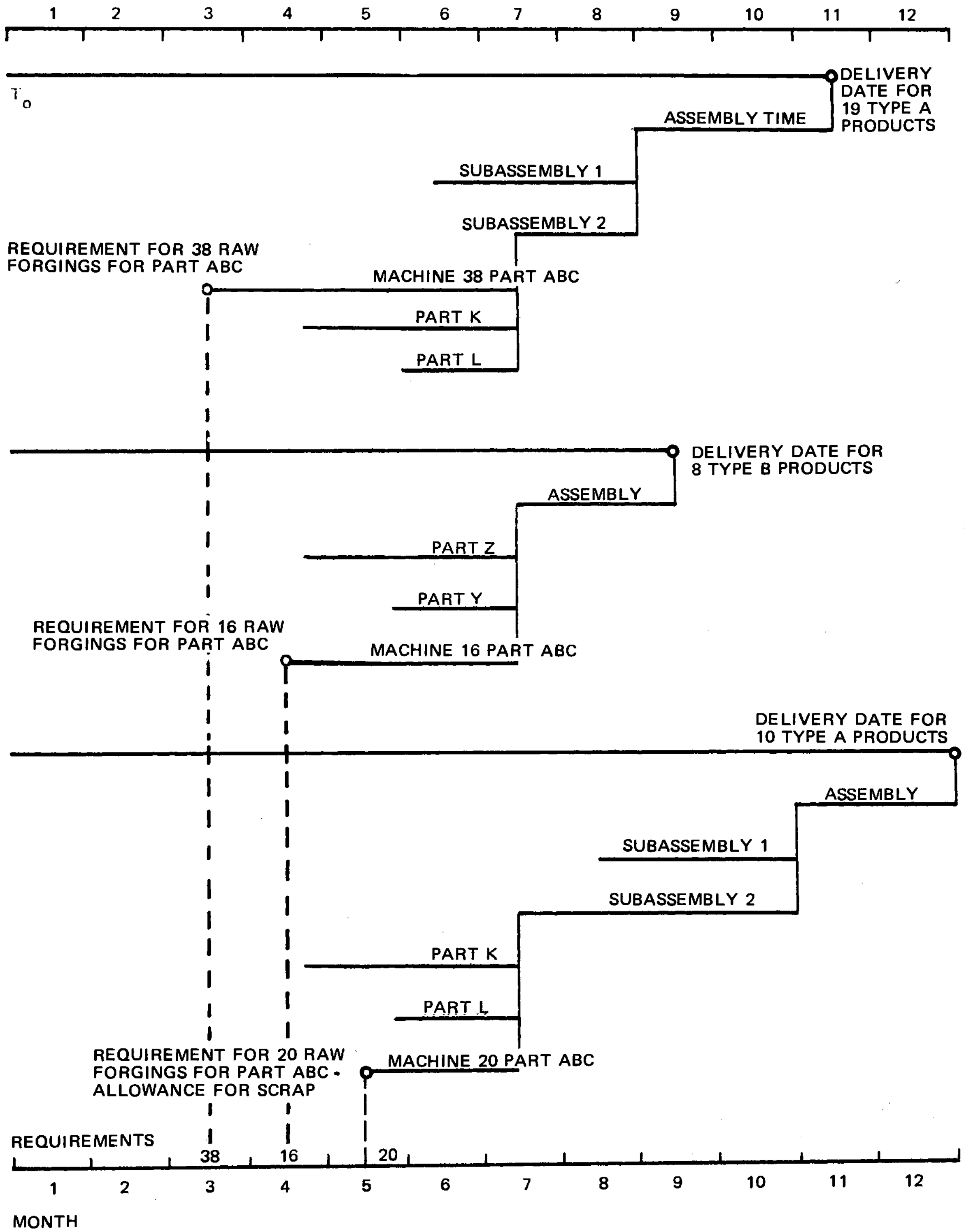

Figure 1. Part of a Master Schedule

Dynamic Nature of Requirements

Scheduling work through a facility is a complex process. Many parts must flow through a different variety of work centers. Several criteria may guide the scheduling of the production. Two major factors which it is attempted to achieve are the reduction of the time until delivery to a customer and the utilization of the available manpower and productive capacity in a balanced fashion. Uniformity of the requirements for raw material and components is not usually sought. The row vector of expected raw material requirements or "requirements vector" will usually be a series of unequal numbers. Thus the demand or use of the item is expected to be nonuniform. A situation in which the parameters change with time is said to be dynamic.[3]

Randomness

Usually a series of scheduled requirements will not remain constant through time. The actual requirement which exists when the material is delivered may differ from the expected value which was originally scheduled. Several factors may influence the actual value of the requirement for raw materials or components and make it differ from the originally scheduled value. Some orders may be cancelled or customers may establish priorities which preclude the performance of work as originally scheduled. Employee absenteeism may differ from standard and change the number of productive man-hours available. Machine breakdowns may occur which will interfere with the productive use of the manhours which are available. The rate of scrap may differ from the scrap allowance which was used in calculating the expected requirement. Customers who already own the product may order replacement parts for their equipment which would be expected to break down on a random basis. All of these chance-caused factors and more may occur so that there is a variance of the actual requirement about its expected value. When many chance-caused variables determine a value, it can be called random.[4]

Covariance

The requirements for raw material and component parts arise as a result of the production process. As the production rate increases, so will the elements in the requirements vector and vice versa. This vector can change between the time that the items must be placed on order and the time that they are to be delivered so that the elements of the requirements vector can be considered as the expected value from a series of demand probability density distributions. The accuracy with which the mean demand can be predicted will decrease, the further into the future the prediction is carried. The variance of the successive distributions should increase.[5]

The values which occur in each successive stage of the horizon will exhibit some form of covariance. The hypothesis of this research is that a procedure will be developed to supply requirements which have covariance (see page 2). Intuitive reasons may be presented to support either positive or negative covariance. To explain a negative covariance, one might reason that a company attempts to produce to a pre-established schedule. On a particular job, if the production has run ahead of schedule for some number of periods, then production facilities may be diverted to another job which is not so far ahead of schedule. Thus, if the requirement for the raw material had been greater than expected for some number of periods, P_1, then it might be expected to be lower than expected for another number of periods, P_2. The numbers P_1 and P_2 would be difficult to establish and might require many replications in order to determine them by experiment.[6]

To explain a positive covariance, one might reason as follows. If the average scrap rate has been incorrectly estimated or if business is better or worse than usual in one scheduling period, it is likely to remain that way for several periods. With a positive covariance an actual value higher than the scheduled value will be followed by one or more actual requirements which are higher than were originally scheduled for that period. This condition might be thought of as a short term bias in the forecasting system. If such a bias exists, it would be desirable to detect it.

Information Availability

As each period of the production duration passes, new information about the remainder of the production cycle and new estimated production schedules are developed. These new production schedules contain information regarding newly ordered products and revisions of the previously developed schedules. The requested deliveries for periods beyond the production leadtime can be revised if it is desired.

Cost Factors

Some of the cost factors mentioned in the introductory chapter of this dissertation are relevant to the problem under investigation. These factors are identified in the following sections.

Item Cost = C ($/unit)

The cost per item purchased can vary with the total quantity purchased, i.e., quantity discounts are possible. The item cost is assumed to be independent of the delivery quantities into which the total order is divided.

Order Cost = K ($/order)

The order cost is considered to be independent of the quantity purchased. It consists of the expense required to solicit quotations from various vendors, negotiate, and make a purchase specifying a delivery schedule for the quantity purchased.

Delivery Amendment Cost = A ($/amendment)

It is assumed that a previously established delivery schedule can be amended for a cost A, which is less than K.

Holding Cost = H ($/time-unit)

The holding cost pertinent to this problem are all out-of-pocket cost which result from holding one unit in inventory for one time element, such as a day. Included in this factor are storage cost components such as insurance, taxes, security, warehousing cost, and the cost of capital for the investment in inventory.

Shortage Cost = S ($/time-unit)

The shortage cost pertinent to this problem is considered to be all expenses which will result for each time element, such as a day, that a unit of raw material is not available to supply a scheduled requirement. The shortage cost may be very great since the shortage of an item may cause expenses for overtime labor, or delay the shipment of the completed assembly and result in a delay of receiving payment for the finished product, a contract penalty, or possibly result in the closing down of a production operation. The shortage cost is considered to be greater than the holding cost and perhaps much greater than the cost of the item. A shortage in this problem is not considered to result in a lost sale because the item being manufactured is already on order by a customer.

Receiving Cost = T ($/delivery)

The cost to receive and inspect a delivery is considered to be a fixed quantity.

Chapter Conclusion

This description of the problem elements and the relevant cost elements provides an understanding of the situation being investigated. In the next chapter the problem elements are analyzed to develop a method of decision making in establishing a new or amended delivery schedule.

FOOTNOTES

[1] Joseph Buchan and Ernest Koenigsberg, Scientific Inventory Management (Englewood Cliffs, 1962), p. 234.

[2] Fred Hanssmann, Operations Research in Production and Inventory Control (New York, 1962), p. 105.

[3] Ibid. p. 105.

[4] Harry M. Markowitz, Portfolio Selection-Efficient Diversification of Investments (New York, 1959), p. 38.

[5] G. Hadley and T. M. Whitin, Analysis of Inventory Systems (Englewood Cliffs, 1963), p. 351.

[6] Markowitz, p. 96.

CHAPTER III

DETERMINING THE DELIVERY SCHEDULE

Since the total amount to be ordered is known and since a delivery schedule can be amended for less than the cost to place separate orders, a minimum cost supply procedure results from ordering only once and requesting a delivery schedule which will require amending a minimum number of times. In addition, if the ordering is from an external supplier, the placing of one large order may result in a discount on the cost per item. If the ordering is from an intracompany department, the placing of one order will facilitate production planning. The original order (and any schedule amendments) should request a delivery schedule which is the best one possible for the information available at the time it is placed, to assure that it will require amending a minimum number of times. Two major questions follow:

- How is the optimal delivery schedule determined to supply a series of probabilistic requirements which have covariance?
- When should a delivery schedule be amended?

This chapter is devoted to developing analytical concepts to answer the first question. Models for determining the answer to the second question are developed for the discrete and the continuous case in the next chapter.

Throughout the analysis the focus is upon optimizing the single product situation. It is assumed that no restrictions (such as shortage of storage space or working capital) are present to prevent the implementation of an optimum delivery schedule for the aggregate of all products. This analysis is based on the arrival and usage of discrete units, and is extended to the continuous case.

The objective is to determine an optimal schedule of item deliveries which the decision-making organization should request from the supplier. To be optimal, the series of deliveries $D = \{D_1, D_2, \ldots D_N\}$ must supply a series of forecasted probabilistic requirements $R = \{R_1, R_2, \ldots R_N\}$ in a manner which will minimize the total expected cost. The total amount to be purchased is assumed to be known, since it is the sum of all the scheduled assembly requirements plus an allowance for scrap. If this amount were delivered all on the date of the first requirement ($D_2, D_3, \ldots D_N = 0$), then there would be no shortage cost but there would be a large holding cost. The material to supply the last requirement would be held until the total amount was used. The material for the next-to-last requirement would be held one period less, etc. On the other hand, if the deliveries were set equal to the expected requirements ($D_i = R_i$; $i = 1$ to N), then there would be a significant chance of stockout near the end of each scheduling period, but the holding cost would be reduced.

Optimum Probability of Meeting Requirements

It is seldom economical to carry enough inventory to meet any possible demand.[1] In the situation where inventory is not normally carried, it seldom would be advisable to request a delivery large enough to cover any possible requirement. In attempting to determine the optimum probability of meeting a probabilistic requirement, the process is obviously one of making a decision under risk. If the receipts of material are set too high, there is a risk of paying excessive holding costs. On the other hand, if the receipts of material are too low, there is a risk of cost due to stockout or shortage.

Consider a forecasted requirement vector for the probabilistic problem to which this paper is addressed. When the item procurement leadtime is λ, the forecasted requirement vector would be: R_λ, $R_{\lambda+1}$, $R_{\lambda+2}$, ... R_h, where h is the length of the planning horizon. The requirements are probabilistic, so the actual requirement $\hat{R}_\lambda$ which occurs λ periods after the estimate R_λ is not necessarily equal to R_λ. It is also true that $\hat{R}_{\lambda+1}$ may not equal $R_{\lambda+1}$, etc. The actual requirements can be considered as observable random non-negative values from a distribution of possible requirements which could occur in that period. Negative demands are not considered since they would constitute a disassembly of a completed or partially completed item and a return of the raw material item to inventory. The actual requirement which can occur can be described in terms of a probability distribution. Each level of possible requirement would have associated with it a probability of occurrence. Knowledge of these distributions at each period would aid in determining the most desirable level of delivery to request.

The probability distribution of demands at any period can be expressed as a cumulative distribution so that for each level of possible demand there is associated a probability of the demand being less than or equal to that quantity. This level can also be interpreted as a level of delivery allocation, and the cumulative probabilities are the probability of the requirement's being less than or equal to the delivery allocation. The amount of delivery which should be allocated to a period is the level which provides the optimum probability of meeting the requirement. The optimum probability of meeting the requirement at a period can be determined by incremental analysis.[2]

In its full detail incremental analysis would consist of making a whole series of decisions of whether to add one more increment to the delivery allocation for a period. When the point is reached at which it is no longer profitable to add any more units to a delivery, the optimum level has been reached. At this point the expected incremental cost is equal to the expected incremental gain. With this knowledge it is not necessary to go through a series of decisions because the optimum probability can be found by the calculations presented below. This relationship is developed on the basis of monetary value, but it can be considered on the basis of utility as is discussed in the final chapter.

Let

X^*	=	the optimum level of material to be available
$\Phi(X^*)$	=	the probability that X^* or fewer will be required
H	=	the holding cost in \$/time-unit held
S	=	the shortage cost in \$/time-unit short
E (IC)	=	the expected incremental cost, and
E (IG)	=	the expected incremental gain.

Since at the optimal point

$$E(IC) = E(IG) \quad ,$$

therefore

$$\left[1 - \Phi(X^*)\right] S = \Phi(X^*) H \quad ,$$

and

$$S - \Phi(X^*) S = \Phi(X^*) H \quad ,$$

then

$$S = \Phi(X^*) H + \Phi(X^*) S \quad ,$$

so that

$$S = \Phi(X^*)(H + S) \quad ,$$

resulting in the relationship

$$\Phi(X^*) = \frac{S}{H + S} \quad .$$ [3]

This derived formula provides a short-cut method for the computation of the optimum probability of meeting a demand. The short-cut is valid in all situations where it is true that "if the incremental profits of any given unit in the sequence is positive, the incremental profits of all earlier units are also positive".[4] An example of a situation in which the short-cut method may not hold true is a case where each delivery allocation represents a separate purchase at a quantity discount. Such a condition does not occur in the problem which is dealt with in this paper, however, because the item cost is independent of the quantities in a delivery allocation.

The optimum probability of covering the demand can be found by the analysis presented above. This probability of receipts being equal to or greater than the demand should be optimal each time one must make a tradeoff of selecting a level which will protect sufficiently but not overprotect from the cost of shortage. Thus, it is desirable to have the same probability of meeting demand at each period or stage throughout the horizon, so long as the holding cost and the shortage cost remain constant or the ratio $\frac{S}{H + S}$ remains the same.

At each stage in the horizon, i.e., the ith stage, the probability of avoiding a shortage penalty is the probability of the ending inventory, I_i, being greater than or equal to zero. I_i will be greater than or equal to zero so long as the cumulative deliveries through the ith period are greater than or equal to the cumulative requirements through the ith period. It is desirable to maintain a probability of

$\Phi(X^*)$ of meeting the cumulative requirements at each stage throughout the horizon.[5] In order to make probability statements about the cumulative requirements through a series of stages it is necessary to have a probability distribution of cumulative demand. The probability distribution of cumulative demand is obtained by constructing joint distributions for the stages through which the requirements are considered to accumulate.

The number of stages over which the requirements are considered to accumulate does not necessarily have to be the number of periods in the procurement leadtime. The procurement leadtime is the minimum length of time which it is possible for requirements to accumulate before a change in the delivery schedule can be put into effect. Joint distributions over more periods will provide the probability of cumulative demands over a longer period of time. The number of units which should be added to the cumulative receipts at each stage is equal to increment in the optimal level of cumulative receipts through that stage.

Constructing Joint Distributions

The Discrete Case

Joint distributions can be used to determine the probabilities of each possible level of cumulative requirements through more than one period. The individual stage probability distributions and the relationship between them have an effect on the joint distributions. A covariance between period demands has an effect on the probabilities at the extreme values or tails of the distributions.

The tail of the demand distribution is usually the portion which is of most interest in inventory problems. The shortage cost is normally much greater than the holding cost resulting in a ratio of (S/(H + S), i.e., the optimum probability, of 0.80 or higher. Such probability levels mean that the upper tail of the distribution is the portion which will be used in determining the level of receipts which is optimal for probabilistic requirements.[6] It is important that the underlying distributions of possible requirements and the relationship between these distributions be known so the joint distributions can be constructed over several periods.

As was discussed in the previous chapter, covariance conceivably can exist between requirements from one period to another. Examples will be used to examine the effect of covariance on the probabilities of possible cumulative requirements and to illustrate the calculations necessary to develop some of the joint distributions. The first example will consider the result of negative covariance between each stage and the stage immediately preceding it. A higher than average requirement in one stage increases the probability of a lower requirement in the following stage and vice versa. This example is simplified by assuming that the requirements at any stage are related only to the immediately preceding stage and that only three possible requirement levels can occur at each stage. The probabilities for this example are given in Table II.

TABLE II

PROBABILITIES FOR DISCRETE EXAMPLE WITH NEGATIVE COVARIANCE

Possible Requirements at Any Stage (units)		9	10	11
		Probability of Possible Requirement at Any Stage		
Requirement at Immediately Preceding Stage	0	1/3	1/3	1/3
	9	1/5	2/5	2/5
	10	1/3	1/3	1/3
	11	2/5	2/5	1/5

The maximum cumulative requirements at the second stage would be 22 units, 11 in the first period and 11 in the second. The probability of 22 units being required through the second period is $1/3 \cdot 1/5 = 1/15$ or $3/45$. The second largest requirement would be 21 units which could result from a requirement of 11 in the first period and 10 in the second or from a requirement of 10 in the first period and 11 in the second. The probability of a cumulative demand of 21 through the second stage is $(1/3 \cdot 2/5) + (1/3 \cdot 1/3) = 2/15 + 1/9 = 6/45 + 5/45 = 11/45$.

Continuing this analysis shows that the resulting cumulative demands and probabilities at the second and third stages are:

Stage 2		Stage 3	
Cumulative Demand	Probability	Cumulative Demand	Probability
22	3/45	33	27/2025
21	11/45	32	189/2025
20	17/45	31	471/2025
19	11/45	30	651/2025
18	3/45	29	471/2025
		28	189/2025
		27	27/2025 .

To demonstrate the importance of the effect of covariance on the probabilities of extreme values, the same numbers can be used in a problem with a positive covariance assumed. In this hypothetical situation a higher-than-average demand in one period will increase the probability of a high demand in the next period and a lower-than-average demand will increase the probability of a low demand in the next stage. The probabilities of such an example are shown in Table III.

TABLE III

PROBABILITIES FOR DISCRETE EXAMPLE WITH POSITIVE COVARIANCE

Possible Requirements at Any Stage (units)		9	10	11
		Probability of Possible Requirement at Any Stage		
Requirement at Immediately Preceding Stage	0	1/3	1/3	1/3
	9	2/5	2/5	1/5
	10	1/3	1/3	1/3
	11	1/5	2/5	2/5

The possible cumulative requirements which can occur at the second and third stages with the associated probability of occurrences are shown below:

Stage 2		Stage 3	
Cumulative Demand	Probability	Cumulative Demand	Probability
22	1/9	33	1/27
21	2/9	32	3/27
20	3/9	31	6/27
19	2/9	30	7/27
18	1/9	29	6/27
		28	3/27
		27	1/27

The two preceding examples illustrate the effect that covariance can have on probability levels of cumulative requirements. The upper tails of the cumulative probability distributions are of particular importance, for it is in this region that the optimum probability normally falls. A comparison of the probability distributions for the third stages reveals that the probability of a cumulative requirement of 33 is four times as great for the positive covariance example as it is for the negative covariance example. In the negative covariance example the probability of a cumulative requirement of 32 or 33 is (27 + 189)/2025 or 0.106. In the positive covariance example it is (108 + 288)/2025 or 0.195, almost twice as great. If covariance exists it can be of great importance in establishing the proper amount of material to supply for a series of scheduling periods.

The preceding examples also illustrate another point. Even though the individual distributions for each stage are definitely not normally distributed, the probability distribution for the cumulative requirements over three periods has a bell shape much like a normal curve. In an actual situation the underlying distribution at each stage would probably be more nearly normal than the simple stage distributions used in these examples.

The preceding examples do not illustrate the vast amount of computation which would be required to construct a joint distribution for a discrete problem of realistic magnitude. If there were 10 possible levels of requirements at each stage, a five stage problem would have

10^5 possible permutations of requirements through the fifth stage. It would also be possible to construct joint distributions by simulation. A computer simulation model can be used for such a problem. The model would require decision rules to select the stage probabilities at every stage but the first depending upon the requirements which occurred in the preceding stages. Such a model could provide the shape of the joint distribution without computing every possible permutation of requirements through the stages. Commonly, large distributions of numbers are represented by specifying the type of distribution and the statistical parameters of the distribution.

The Continuous Case

The normal distributions will be assumed in the following analysis of continuous distributions when a particular distribution must be used. The normal distribution can also be used to approximate a discrete distribution. As the number of possible levels of requirements increases, the values approach a smooth curve. The normal curve can be divided into intervals and the probability of the discrete values within an interval considered as the area of the normal curve within the interval.

A situation having many possible levels of requirements at each stage would be represented by a probability distribution at each stage. The probability distribution for the cumulative requirements which can occur through a series of stages can be found by combining the distributions for the stages. In constructing a probability distribution of cumulative requirements, the expected cumulative requirement level through a series of stages is equal to the sum of the expected requirements at each stage.[7] If

$\widetilde{R}_n$ = the cumulative expected requirement through the nth stage

and $\overline{R}_i$ = the expected requirement for the ith stage

then $$\widetilde{R}_n = \sum_{i=1}^{n} \overline{R}_i .$$

When covariance is present, as is assumed in this research, it should be taken into account in constructing the distributions of cumulative requirements through a series of stages. The cumulative requirements can be treated as the sum of n random variables. The variance of the cumulative demand at the nth stage can be found by:

$$\begin{aligned}\tilde{\sigma}_n^{\,2} = {} & \sigma_{11} + \sigma_{12} + \sigma_{13} + \dots + \sigma_{1n} \\ & + \sigma_{21} + \sigma_{22} + \sigma_{23} + \dots + \sigma_{2n} \\ & + \vdots \\ & + \sigma_{n1} + \sigma_{n2} + \sigma_{n3} + \dots \sigma_{nn} .\end{aligned}$$

Since $\sigma_{11} = \sigma_1^{\,2}$; $\sigma_{12} = \sigma_{21}$, etc., substitutions can be made so that the following equivalent formulation can be developed.

$$\tilde{\sigma}_n^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_n^2 + 2\sigma_{12} + 2\sigma_{13} + 2\sigma_{14} + \ldots + 2\sigma_{1n}$$
$$+ 2\sigma_{23} + 2\sigma_{24} + \ldots + 2\sigma_{2n}$$
$$+ 2\sigma_{34} + \ldots + 2\sigma_{3n},$$
etc.[8]

where

$\tilde{\sigma}_n^2$ = the variance of the distribution of cumulative requirements through the nth stage

σ_{ij} = the covariance between the ith variable and the jth variable.

This model can be used to determine the variance of the distribution of cumulative demand through each stage so that the optimal delivery allocation for each stage can be determined. The solution of an N period problem would require the values of N variances and N(N - 1)/2 covariances. If the scheduling period were in months, a six-month horizon would require six variances and 15 covariance factors. The N(N - 1) factor makes the required number of parameters grow rather rapidly as N increases. However, the covariance factors may be zero or near zero for many of the relationships possible.

Continuous Case Example

As an example of the use of this model consider the following illustration which will also be used in later portions of this paper. An order is received for a number of products which will require

190 units of raw material and the available production capacity results in a production schedule which will require four months. Assume that the probability distributions for the raw material requirements during these four periods are normal and that the following information is known:

$R_1 = 50$	$R_2 = 40$	$R_3 = 60$	$R_4 = 40$
$\sigma_1 = 6$	$\sigma_2 = 5$	$\sigma_3 = 10$	
$\rho_{12} = 0.5$	$\rho_{13} = 0.3$	$\rho_{23} = 0.4$	

where ρ_{ij} = the coefficient of correlation between the ith variable and the jth variable. No statistical analyisis will be necessary for the fourth period because the total of the four requirements is known, and any material which is not made available by the third stage must be made available for the fourth stage. Since

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} ,$$

then

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

therefore

$$\sigma_{12} = 0.5\ (5)\ (6) = 15 ,$$

$$\sigma_{13} = 0.3\ (6)\ (10) = 18 ,$$

and

$$\sigma_{23} = 0.4\ (10)\ (5) = 20 .$$

The mean and standard deviation of the cumulative distribution provide the parameters necessary to make probability statements about the cumulative demand. Assume that the optimum probability of meeting the requirements is 0.95, so that any stage, n, the optimum cumulative supply allocation, X_n^*, should be $\tilde{R}_n + 1.65\,\tilde{\sigma}_n$. The supply allocation Δ_i for the ith period is found by subtracting X_{i-1}^* from X_i^*. At the first stage

$$\Delta_1 = 50 + 1.65\ (6)$$
$$= 59.90\ .$$

At the second stage

$$\tilde{R}_2 = 50 + 40 = 90,$$

and

$$\tilde{\sigma}_2^2 = 6^2 + 5^2 + 2(15) = 91$$

therefore

$$\sigma_2 = \sqrt{91} = 9.54\ .$$

The second stage cumulative deliveries should be

$$X_2^* = \tilde{R}_2 + 1.65\ \tilde{\sigma}_2$$
$$= 90 + 1.65(9.54) = 105.74\ ,$$

and therefore

$$\Delta_2 = X_2^* - X_1^* = 105.74 - 59.90 = 45.84\ .$$

At the third stage

$$\tilde{R}_3 = 50 + 40 + 60 = 150\ ,$$

and

$$\tilde{\sigma}_3 = 6^2 + 5^2 + 10^2 + 2(15) + 2(18) + 2(20) = 267\ ,$$

therefore

$$\tilde{\sigma}_3 = \sqrt{267} = 16.34\ .$$

The third stage cumulative deliveries should be

$$X_3^* = \tilde{R}_3 + 1.65(16.34) = 176.96 ,$$

therefore

$$\Delta_3 = X_3^* - X_2^* = 176.96 - 105.74 = 71.22 .$$

The supply allocations for the first three periods are 59.90, 45.84, and 71.22, respectively. The remainder of the total requirements, 13.04 units, would be allocated to the final period.

This example can be used to demonstrate the effect that covariance can have on the cumulative requirements at the third stage. If there were no covariance (independence of the period requirements) the last three terms of the variance equation would be zero and

$$\tilde{\sigma}_3^2 = 6^2 + 5^2 + 10^2 = 161 .$$

For a situation in which the covariance is negative, the covariance causes the variance of the cumulative requirements through several stages to be less than it would be in the case of independence. This situation can be illustrated with the same numerical quantities which were used previously but with a negative sign before the covariance terms. Then

$$\begin{aligned} \tilde{\sigma}_3^2 &= 6^2 + 5^2 + 10^2 + 2(-15) + 2(-18) + 2(-20) \\ &= 36 + 25 + 100 - 30 - 36 - 40 \\ &= 65 . \end{aligned}$$

The covariance can be positive, zero, or negative. These three conditions have an effect on the range of possible values of requirements which can occur and on the level of requirements which has the optimum probability of not being exceeded. The range of values is assumed to be $\tilde{R}_n \pm 3\tilde{\sigma}_n$. Table IV summarizes these points for the third stage of the preceding example.

TABLE IV

EFFECT OF COVARIANCE ON THIRD STAGE OF EXAMPLE PROBLEM

Condition	$\tilde{\sigma}_3$	Range	0.95 Probable
+ Covariance	16.34	100.98 to 199.02	176.96
0 Covariance	12.69	111.93 to 188.07	170.94
- Covariance	8.06	125.82 to 174.18	163.30

The Probabilistic Delivery Date

In the previous analysis it was assumed that deliveries always occurred at the start of a scheduling period. Since the scheduling periods are of fixed and known length, this is equivalent to deterministic delivery date. Thus, the previous analysis determined the amount to have delivered in order to cover a probabilistic demand over a deterministic period of time.

Deliveries may not always occur on the dates when they were scheduled, but may have some probability distribution of their occurrence. The uncertainty of delivery date must be taken into account if the delivery date variations are great enough to affect the probability of stockout. In a conventional inventory situation a safety stock is maintained to be on hand for protection from stockout during the exhaustion of one delivery before the arrival of the next delivery. If the delivery date is uncertain the safety stock is increased to account for the delivery variation which may occur. In the problem under consideration, an inventory is not maintained but material is ordered when requirements exist. Since there is no permanent inventory a safety stock would not be maintained but would also be ordered when it is required. The total amount of material to be made available must be that amount which gives the optimum probability of protection from stockout with both demand rate and delivery time variations taken into account. The determination of the delivery level which offers the optimum level of protection requires a probability distribution which has both requirement variation risk and delivery date risk included to give a distribution of requirements until the next delivery.

The construction of a distribution of possible requirement levels before the next delivery and the probabilities of such can be constructed in at least three possible ways.

- Data can be collected on past occurrences of requirements between deliveries, a frequency distribution constructed from these data and a probability distribution developed.
- A distribution of requirements per time element (units/time) and a distribution of the number of time elements between deliveries (time/scheduling period) can be used to develop a distribution of requirements between deliveries.[9]
- A distribution of the average requirements per time element and a distribution of the number of time elements between deliveries can be developed. Mathematical analysis can be used to construct a distribution of requirements between deliveries.

The following analysis will determine the optimum level of delivery for a probabilistic delivery date by the latter method.

The variation of the quantity which may be required before the next delivery date arises from the variation of the average usage rate per unit of time and the variation of the length of time until the new delivery arrives. The material required until the next delivery is the product of these two variables. Time is a continuous variable, but can be measured in units so that it may be considered as being discrete. The amount of material required per unit of time can be discrete or continuous. Discrete numbers of units would be used if the item were a machined shaft or a bearing. Continuous units might be considered if the item were ounces of sulphuric acid, gallons of crude oil, or gallons of oil base in a paint blending plant.

The product of a continuous variable of time and a continuous variable of usage rate is the most conceptually difficult combination and is considered here. Let r equal the average usage rate per time element and t equal the number of time elements in a variable delivery period. Assume that both r and t independent random variables which are normally distributed. In a real problem both would have some finite range of values in which there is some measurable probability of occurrence. The product $(r \times t)$ can fall at any point in the sample space defined by an r, t coordinate system as illustrated in Figure 2.

Figure 2 pictures a bivariate normal distribution in which r and t are independent variables. The probability of a subarea of the sample space would be equal to the volume of the "hill" under the surface pictured. The total volume under the surface must total to unity.

Let α equal the complement of the optimum probability of meeting the requirement. The objective of the analysis is to find a delivery level, X^*, which the demand $(r \times t)$ will exceed with a probability of α.

$$\Phi(r \times t > X^*) = \alpha = 1 - \Phi(X^*) \quad .$$

The maximum product occurs when r is at its maximum and t is at its maximum, which is the upper right-hand corner of the sample space. At any value of r, t can vary and vice versa. Any requirement level

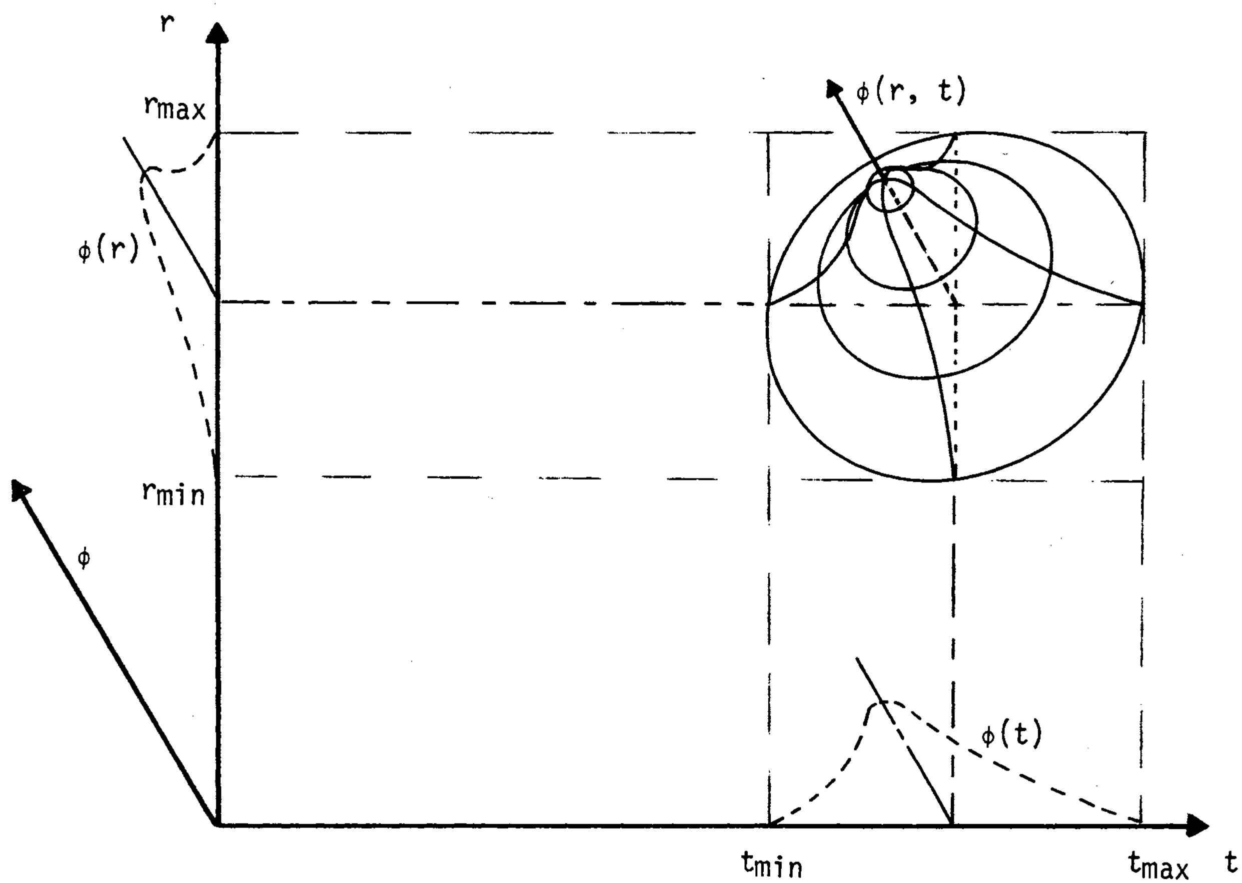

Figure 2. A Three-Dimensional View of the Probability Distribution of r, t, Pairs

can be formed by many combinations of r and t. The solution we seek is analogous to finding a surface perpendicular to the r, t plane which defines a series of constant ($r \times t$) products and cuts off the upper-right hand corner of the sample space so that a probability volume of α is removed. This value would be the optimum delivery level, X*. At this point $r \times t = X^*$ or $t = \frac{X^*}{r}$ which is the equation for a hyperbola. The problem is to find an X* value which defines a hyperbola surface that removes α of the probability volume.

Since r and t are normally distributed, they have probability density functions of

$$f(r) = \frac{1}{\sqrt{2\pi}\,\sigma_r} \exp\left[-\frac{1}{2}\frac{(r-\overline{r})^2}{\sigma_r^2}\right]$$

and

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma_t} \exp\left[-\frac{1}{2}\frac{(t-\overline{t})^2}{\sigma_t^2}\right].$$

The probability density function of r and t simultaneously is given by

$$f(r,\ t) = \frac{1}{2\pi\ \sigma_t \sigma_r} \exp\left[-\frac{1}{2}\frac{(r-\overline{r})^2}{\sigma_r^2} - \frac{1}{2}\frac{(t-\overline{t})^2}{\sigma_t^2}\right].$$

Since

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(r,\ t)\, d_t\, d_r = 1\ ,$$

the values of X* can be determined by

$$\int_{-\infty}^{\infty}\int_{\frac{X^*}{r}}^{\infty} f(r,\ t)\, d_t\, d_r = \alpha\ .$$

These equations are an abstraction of reality. In an actual application these would be some finite limits on the integrals so the probability will be represented by

$$\int_{r_{min}}^{r_{max}} \int_{\frac{X^*}{r}}^{t_{max}} \frac{1}{2\pi\ \sigma_r\ \sigma_t} \ \exp\left[-\frac{1}{2}\ \frac{(r - \overline{r})^2}{\sigma_r{}^2} \ - \frac{1}{2}\ \frac{(t - \overline{t})^2}{\sigma_t{}^2} \ \ d_t\, d_r\right] = \alpha \ .$$

Let

$$u = \frac{t - \overline{t}}{\sigma_t} \quad \text{and} \quad v = \frac{r - \overline{r}}{\sigma_r}$$

then

$$t = u\sigma_t + \overline{t} \quad \text{and} \quad r = v\sigma_r + \overline{r}$$

$$\frac{d_t}{d_u} = \sigma_t \ \text{ and } \quad \frac{d_r}{d_v} = \sigma_r$$

so

$$d_t = \sigma_t\ d_u \ \text{ and } \quad d_r = \sigma_r\ d_v \ .$$

Since

$$t = \frac{X^*}{r}$$

then

$$u\,\sigma_t + \overline{t} = \frac{X^*}{v\,\sigma_r + \overline{r}}$$

therefore

$$u = \frac{X^*}{(v\,\sigma_r + \overline{r})\ \sigma_t} - \frac{\overline{t}}{\sigma_t} \ .$$

The previous integrals can now be written as

$$\frac{1}{2}\pi \int_{\frac{r_{min} - \overline{r}}{\sigma_r}}^{\frac{r_{max} - \overline{r}}{\sigma_r}} \int_{\frac{X^*}{\sigma_t\,(v\,\sigma_r - \overline{r})} - \frac{\overline{t}}{\sigma_t}}^{\frac{t_{max} - \overline{t}}{\sigma_t}} \exp\left[-\frac{1}{2}\,(u^2 + v^2)\right] d_u\, d_v = \alpha \ .$$

The answer sought is the value X* which makes the value of the double integral equal to α. A computer program was prepared to find a solution to such equations. The program computes the maximum possible requirement over a delivery time $(\bar{r} + 3.5\ \sigma_r)(\bar{t} + 3.5\ \sigma_t)$. Twenty contours are constructed through the u, v, sample space, each describing u, v, products which would result in a constant requirement that is a specified percent of the maximum possible product. The probability of r, t pairs to the upper-right-hand side of these contours is computed and provided as output. The value of X* can be found by interpolating between these contours.

Suppose, for example, that the problem posed on page 48 had a probabilistic delivery date and the level of demand at the second stage which would be exceded only 5 percent of the time was desired. Assume that the scheduling period is one month or 21 work days in length so that the start of the third period will occur in 42 days with a standard deviation of 3 days. The interpolation program was run for this example and the data printout is shown in Figure 3. These data show 20 levels of possible requirements and the probability of exceeding each level. It is necessary to interpolate between two of these requirement levels to find the level which will be exceeded only 5 percent of the time. Graphical interpolation of the data is illustrated in Figure 4. The 0.05 level is approximately 109.75 units. For the deterministic case it was

PROBABILITY OF NORMAL PRODUCTS

PRODUCT	PROBABILITY INCREMENT	PROBABILITY OF EXCEEDING
150.66	0.000000	0.000000
146.00	0.000002	0.000003
141.34	0.000017	0.000020
136.68	0.000078	0.000098
132.02	0.000297	0.000396
127.36	0.000972	0.001369
122.70	0.002707	0.004076
118.04	0.007220	0.011296
113.38	0.015569	0.026866
108.72	0.031846	0.058712
104.06	0.059072	0.117785
99.40	0.090176	0.207962
94.74	0.127454	0.335416
90.08	0.150407	0.485824
85.42	0.157505	0.643329
80.76	0.138653	0.781983
76.10	0.101709	0.883692
71.44	0.063682	0.947374
66.78	0.033086	0.980461
62.12	0.013237	0.993698

Figure 3. Output From Integration Program Giving Probabilities of Products of Normally Distributed Variables

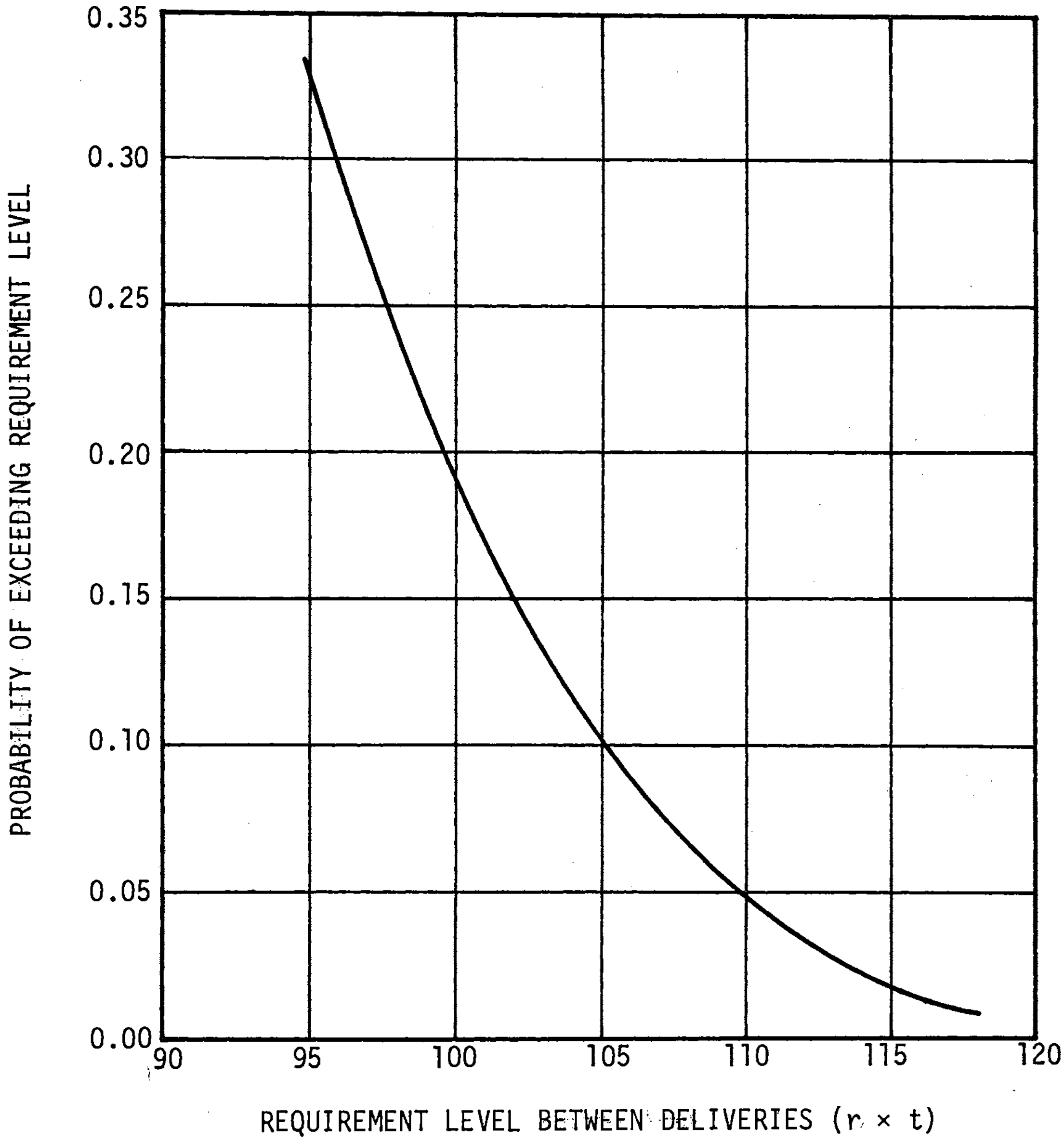

Figure 4. Interpolation Graph for Probabilistic Delivery Date Example

105.74 units, showing that more material is required to protect from the uncertainty in the delivery date.

Batching Period Allocations Into Deliveries

The delivery quantities can be determined after the period allocations have been found. It is not always desirable to have each period allocation delivered at the beginning of the period for which it is allocated. It may be more economical to have an allocation delivered in some prior period and held until the period in which it is used. This process is sometimes referred to as "batching deliveries".

It is economical to batch two allocations if the one period holding cost for the later period allocation is less than the cost of a separate delivery for that allocation. It is economical to batch three allocations if the one period holding cost for the middle allocation and the two period holding cost for the last allocation total to less than the cost of a delivery, etc. The only allocation which cannot be moved ahead in time is the first allocation. Since the items are not in stock it is necessary to have a delivery the first period. Working forward from the first delivery reveals the allocations which should be batched with the first one. When the point is reached that another delivery is economical the same type analysis is repeated to determine the allocations which should be batched into that delivery.[10]

Where the number of time elements in a scheduling period is p, the holding cost for any allocation will be equal to H times p times the allocation for every scheduling period it is held except the one in which it is used. The expected holding cost for the scheduling period in which it is used will be equal to $\left(\Delta_i + \frac{\overline{R}_i}{2}\right)(H)(p)$. The holding cost for the scheduling period in which it is used will be paid regardless of whether it is delivered at the start of that scheduling period or on some previous scheduling period. The incremental increase which can be eliminated by having a separate delivery in a period is the cost of holding it from its delivery until the start of that scheduling period in which it is used. These are the costs which must be compared to T to determine which period it should be delivered.

The continuous distribution example on page 48 will be used to illustrate the batching of allocations into deliveries. The allocations for the four-stage problem were 59.90, 45.84, 71.22, and 13.04 units, respectively. Assume a delivery cost, T, of \$50.00 and a holding cost, H, of \$1.00 per unit period. A delivery must be received in the first period since no stock of the item is maintained on hand, therefore, $D_1 \geq 59.90$. If Δ_2 were received in D_1 the incremental holding cost would be $H(\Delta_2) = \$1\ (45.84) = 45.84$.

$$45.84 < 50 \quad \text{so} \quad D_1 \geq 105.74, \ D_2 = 0.$$

If Δ_3 were delivered in D_1, it would be held 2 periods before the period in which it is used. The incremental holding cost would be

$$2(H)(\Delta_3) = 2(\$1)(71.22) = 142.44$$

$$142.44 > 50 \quad \text{so} \quad D_1 = 105.74,\ D_2 = 0\ .$$

A delivery is required in Period 3, $D_3 \geq 71.22$. If Δ_4 were delivered in D_3 it would be held one period. The incremental holding cost would be

$$(H)(\Delta_4) = 1(13.04) = 13.04$$

$$13.04 < 50 \quad \text{as} \quad D_3 = 84.26,\ D_4 = 0\ .$$

Two deliveries are optimal for the example problem under the conditions of H = 1, T = \$50. One delivery of 105.74 units the first period would supply the first two periods. A second delivery of 84.26 units at the beginning of the third period would supply the requirements during the last two periods.

The Quantity in a Purchase

In a conventional inventory problem one solves for the EOQ and orders that amount each time the reorder point is reached. In the problem proposed for this research, it is assumed that the amount to be purchased is determined at the beginning of a horizon and the problem is when it should be delivered to minimize costs. The item cost may be dependent upon the total quantity purchased but is assumed to be independent of the delivery quantities. The primary focus of the research is on the division of the total requirement into deliveries but some discussion of the amount in the total purchase is presented.

The policy of what amount to purchase when given a series of probabilistic requirements might vary from one item to another within an organization. The policy would also be expected to vary from one organization to another because it constitutes decision making under risk, and different organizations may possess different attitudes toward risk. The utility of the potential loss or gain may be different for different firms.

The rationale utilized in reaching the decision of how much to buy in a multiperiod procurement will be discussed further. The assumption that the cost, K, to negotiate and consummate a purchase agreement is greater than the cost, A, to amend the agreement is considered reasonable. Models exist for placing a purchase each period. If the purchase cost were less than the exposure to risk which results from not modifying a multiperiod schedule, then it would be preferred to make a separate purchase for each period, one leadtime prior to the period.

Since it is assumed that $K > A$, it is better to be exposed to the risk of having to amend a schedule than to incur the certain expense of K in all reasonable item cost structures. Reasonable item cost structures are those in which the cost of the materials is K plus a linear function of the quantity purchased or K plus a concave function of the quantity purchased. An unreasonable item cost structure would be one in which the cost of material is K plus some convex increasing function of the quantity purchased. Only those situations classified as reasonable need to be considered. For such situations and for some conceivable subsets of the unreasonable category, the average cost per item will decrease as more and more items are purchased.

Where the average cost per item decreases with the quantity purchased, it is desirable to purchase as many items at one time as are certain to be needed. It may be wise to purchase more than the certain requirement, depending upon the amount of decrease in price per unit and the amount of risk incurred by buying more than is certain to be needed. For the situation which is the subject of this research, there is no risk involved in purchasing the full amount shown on a requirement schedule so long as no scheduled design change would require a different raw material or component. These items are essentially on order by a customer who has already ordered the product in which the material item is used. Normally such a customer agrees to pay the cost incurred if he decides to cancel his order.

The risk incurred in purchasing more than the amount on the requirement schedule is assessed in a different manner. Nonstock items, the category of parts which is the subject of this paper, are items not normally carried for inventory purposes. Purchasing more than is ordered by customers would constitute purchasing for inventory and will not be treated in detail in this paper. However, the logic of the quantity decision is generally as follows.

The elements of risk due to overbuying must be defined. Some of these elements are the probability of receiving no more orders for the item, the probability of receiving orders for the item at various periods in time, and the probability of a design change at various periods in time before the orders are received. The preceding items are examples and are not considered exhaustive. The cost of each such occurrence would be multiplied by its probability to give an expected cost of overbuying per item. The savings per item could be compared to this amount on an incremental basis to find the optimum amount of overbuy which should be accepted.

No more than the total of the requirements given in the requirement schedule is considered to be purchased. For all reasonable price structures it is considered economical to purchase that amount so long as there is no scheduled design change which would require a different component or raw material. The delivery schedules determined to be optimal are at the upper tail of the probability distribution of possible demands; yet each expected requirement is the 50-percent probability level for an unbiased forecasting scheme with no covariance between demands. It follows that enough items will not be purchased to provide a high degree of perhaps 90- to 99-percent protection from shortage in all the periods in the original planning horizon. The result of protecting to a 90- to 99-percent level in the early stages of the horizon is a shifting of a portion of the total purchase quantity to the earlier period deliveries. This shifting can be considered as analogous to the accepting of a safety stock at the beginning of a series of requirements. The safety stock level can vary each period as can the demand level. The concept of a procurement horizon, which is the length of time for which the purchased quantity lasts, is introduced here. The procurement horizon may not last so long as the planning horizon or it may last longer, depending upon the average of the actual requirements and how it compares to the average of the expected requirements as indicated on the requirement schedule at the time the procurement is made. If new requirements are forecasted in the periods beyond the procurement horizon, a new procurement will be made λ or $\lambda - 1$ periods before the end of the procurement horizon. The occasion for the beginning of a new horizon before the expiration of the old procurement horizon would be an overlap situation in which the requirement for the last period of the first procurement horizon is greater than the available quantity in that period.

A System for Delivery Scheduling

The probabilities of cumulative demands through the preceding periods determine the probability of stockout for a given receiving schedule. The nearest period in which a delivery allocation can be specified using the most current information is one leadtime in the future. It is necessary to project the joint probability distribution for demand over a minimum of λ periods. To place any order, an expense of K must be incurred λ periods before the beginning of the series of periods represented by the requirement schedule. If each period's delivery allocation were ordered λ periods before it occurred, there would be a cost of K for each period of the horizon. The total procurement related cost would be $N \cdot K$. If the requirements were deterministic, only one order would be required and all N period's delivery allocations could be specified for a total cost of K. The technique of projecting cumulative requirement distributions makes it possible to estimate the optimal delivery allocation for each stage and this delivery schedule could be requested with the initial order. If the schedule for the remainder of the horizon had to be amended each period that it is possible to make an amendment, the total procurement related cost would be at a maximum of $K + (N - 1)A$. Since $A < K$ the procurement related cost would be reduced. The greatest potential cost reduction lies in the purchasing of all the delivery allocations in one open purchase order which affords an opportunity for quantity discounts.

The best schedule which can be determined at the time of the original order should be requested. If at any stage new information which is available indicates that there is an advantage to changing the deliveries in a portion of the order λ or more periods in the future, then the change can be made. The alteration expenses would be some amount less than $(N - 1) A$ if the original forecast did not have to be changed every period. The number of times the expense A would be incurred would depend on the accuracy of the forecasting system and its ability to provide accurate predictions of the cumulative demand through a number of periods.

FOOTNOTES

[1] G. Hadley and T. M. Whitin, Analysis of Inventory Systems, (Englewood Cliffs, 1963), p. 9.

[2] Robert Schlaifer, Probability and Statistics for Business Decisions (New York, 1959), p. 69.

[3] Edward H. Bowman and Robert B. Fetter, Analysis for Production Management (Homewood, Illinois, 1957), p. 285.

[4] Schlaifer, p. 75.

[5] C. W. Churchman, R. L. Ackoff, and E. L. Arnoff, Introduction to Operations Research (New York, 1957), p. 215.

[6] Hadley and Whitin, p. 412.

[7] Harry M. Markowitz, Portfolio Selection-Efficient Diversification of Investments (New York, 1959), p. 70.

[8] Ibid, p. 93.

[9] W. J. Fabrycky and Paul E. Torgersen, Operations Economy: Industrial Applications of Operations Research (Englewood Cliffs, 1966), p. 290.

[10] Harvey M. Wagner and Thomson M. Whitin, "Dynamic Version of the Economic Lot Size Model", Management Science, V (October 1958), pp. 89-96.

CHAPTER IV

MODIFICATION WITH NEW SCHEDULE INFORMATION

The objective of an inventory management system is to develop a tool which is effective in both planning and control. One essential element in management by exception is followup to detect deviations from the plan and to initiate corrective action. The previous analysis has been directed toward the development of a technique to determine the best delivery schedule for a series of future scheduling periods, given a specified amount of information. As better information regarding the latter portion of a procurement horizon becomes available with the passage of each scheduling period, the technique can be repeated for the remaining periods in the procurement horizon. The deliveries requested for periods beyond the procurement leadtime can be amended. If the most current information indicates that a different delivery schedule is better for these periods, the schedule can be changed but will necessitate the expenditure of an amount, A. The purpose of this chapter is to develop the type of analysis required to determine the conditions under which it is beneficial to amend a delivery schedule.

One intuitively obvious fact can be surmised from the fact that more information becomes available each period. If the first period which it is possible to change is already at the optimum level, no change should be made for at least one more period. The periods which can be improved do not need to be changed at this time and advantage should be taken of the improved forecasts which become available each period. The schedule should not be amended until one leadtime before a period which can be improved to take advantage of the best information possible for determining the new schedule.

If the first period which is a candidate for amending is not at the optimum level, then there is a possible benefit from making a change at the present time. The question which logically follows is: How much benefit must be offered in order to warrant the expenditure, A, to amend the delivery schedule? In general, if the expected benefit is larger than A, a schedule change is economically warranted. Some other limit of deviation of expected cost from the optimum expected cost may be established by management who may not wish to make a change for a very small expected improvement. Theoretically, in incremental analysis the change should be made if there is any reduction in total expected cost. The expected benefit in cost can result from improvement in the first changeable period or the first plus any other changeable period. As long as the first period which is a candidate for a change (the period λ periods in the future) offers any reduction in the total expected cost, then the change is warranted if the total expected cost in all the amendable periods is beyond the established expected opportunity cost limit, which in this study is assumed to be the amount, A.

Analysis of the reduction in total expected cost in all of the amendable periods may require a great deal of computation. It may not be necessary to compute the expected benefit for improving the delivery quantity in all candidate periods. The expected benefit for each candidate period should be accumulated for all candidate periods in a delivery schedule only up to the point that the total expected benefit exceeds the cost to amend the schedule. If all periods are computed and this condition does not exist, then the schedule should not be amended. If the schedule of allocations is such that it results in the same delivery batches, the delivery schedule would not be changed.

The expected benefit which can accrue as a result of a change in the delivery schedule for a period will be called the expected opportunity cost, EOC. The total present value of the EOCs for all of the amendable periods (those remaining in a procurement horizon beyond the procurement leadtime) will be called the present opportunity cost, POC.

$$POC = \sum_{k=1}^{n} \frac{EOC_K}{(1 + i)^K}$$

The total expected incremental cost due to the inventory resulting from a delivery allocation of X will be called TEC(X).

For any level within the range of possible requirements there is some chance of the actual requirement's being higher than that level and some chance of the actual requirement's being lower than that level. For a period allocation within this range there is a possibility of shortage and a possibility of having extra material on hand to hold. The total expected cost associated with any level of allocation or cumulative allocation, X_A, will be the total of the expected holding cost and the expected shortage cost. Figure 5 (b) shows the relationship of these costs. Figure 5 depicts the probabilities and costs for a continuous distribution because this type of distribution illustrates the problem better. The same logic applies in the discrete case, but the curves would be step functions.

The Discrete Case

The expected holding cost is equal to the cost to hold an item, H, times the expected number of items which would be held for any level of X_A. The expected shortage cost is equal to the shortage cost, S, times the expected number of items short for any level of X_A. Then

$$TEC\ (X_A) = H \sum_{0}^{X_A} (X_A - X)\,\phi(X) + S \sum_{X_A+1}^{\infty} (X - X_A)\,\phi(X) \text{ .}$$ [1]

These costs are shown in Figure 5. The level of delivery allotment which results in a minimum TEC will be X*. This information can

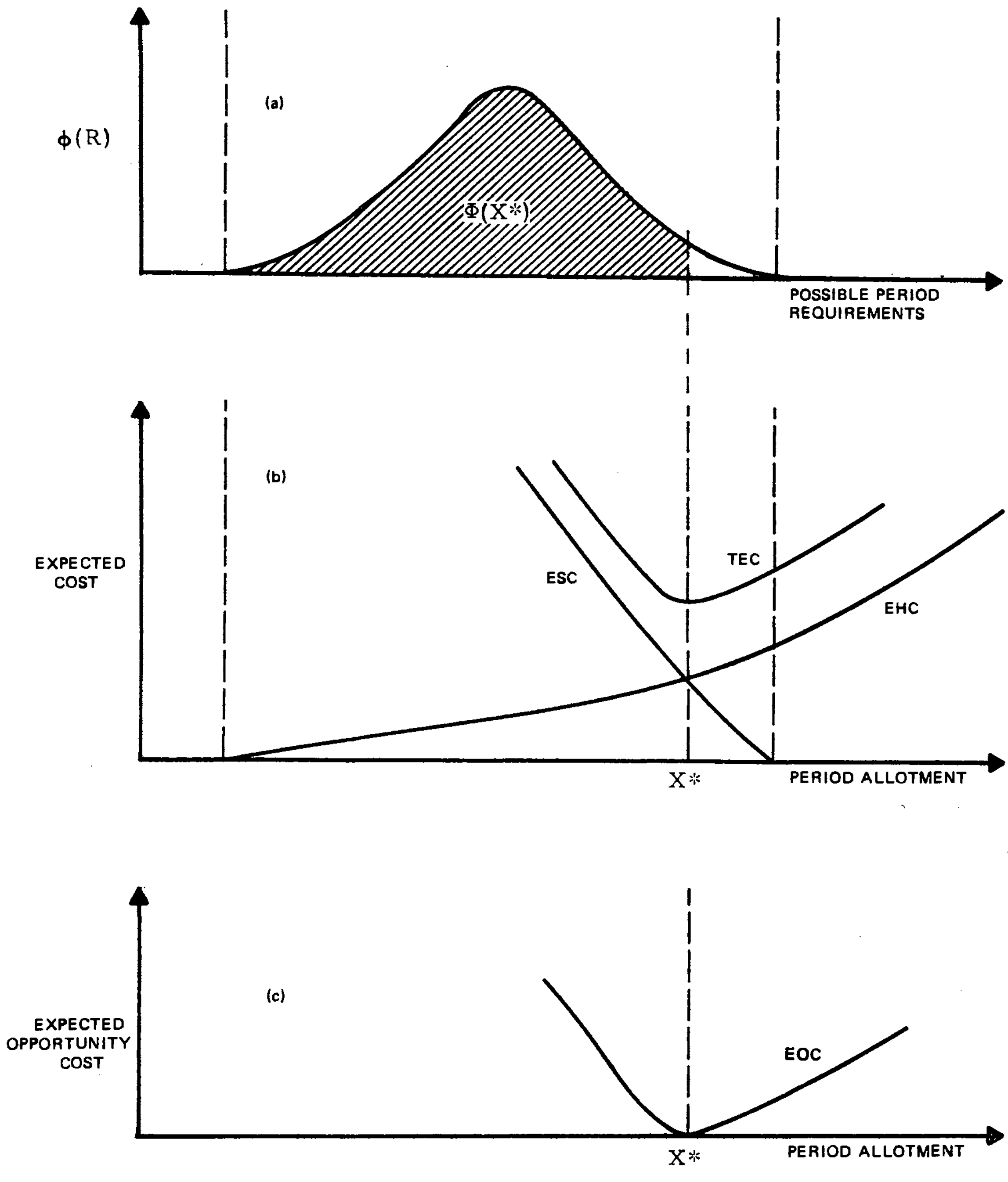

Figure 5. Relationship of Requirements, Period Allotments, and Costs for a Continuous Distribution

be used to determine the EOC for having a delivery allotment which is not at X^*. Let

X^* - the optimum delivery quantity for a period as indicated by the latest information

X_p - the presently scheduled allocation for that period.

The total expected incremental cost for a period can be found by

$$TEC\ (X_p) = H \sum_{X=0}^{X_p} (X_p - X)\ \phi(X) + S \sum_{X=X_p+1}^{\infty} (X - X_p)\ \phi(X) \ .$$

If $X^* \neq X_p$, then a lesser TEC would result at X^* which can be found by

$$TEC\ (X^*) = H \sum_{X=0}^{X^*} (X^* - X)\ \phi(X) + S \sum_{X=X^*+1}^{\infty} (X - X^*)\ \phi(X) \ .$$

The EOC due to having the delivery quantity set at X_p rather than at the X^* indicated by the latest information, is the difference between the two preceding equations.

$$\begin{aligned} EOC &= TEC\ (X_p) - TEC\ (X^*) \\ &= H \sum_{X=0}^{X_p} (X_p - X)\ \phi(X) + S \sum_{X=X_p+1}^{\infty} (X - X_p)\ \phi(X) \\ &\quad - H \sum_{X=0}^{X^*} (X^* - X)\ \phi(X) - S \sum_{X=X^*+1}^{\infty} (X - X^*)\ \phi(X) \end{aligned}$$

$$EOC = H\left[X_p \sum_{o}^{X_p} \phi(X) - \sum_{X=0}^{X_p} X \cdot \phi(X) - X^* \sum_{X=0}^{X^*} \phi(X) + \sum_{X=0}^{X^*} X \cdot \phi(X)\right]$$

$$+ S\left[\sum_{X=X_p+1}^{\infty} X \cdot \phi(X) - X_p \sum_{X=X_p+1}^{\infty} \phi(X) - \sum_{X=X^*+1}^{\infty} X \cdot \phi(X) + X^* \sum_{X=X^*+1}^{\infty} \phi(X)\right] .$$

The EOC for any period is the total expected cost for the set allocation level less the minimum possible expected cost, i.e., TEC(X^*). Figure 4 (c) shows the EOC for the problem illustrated in the figures above it.

To determine if a schedule should be changed, it is necessary to find whether POC > A. This determination can be made by beginning with any period which is a candidate for a change, computing EOC, and adding the EOC for another candidate period. If all periods are added and POC ≤ A, the schedule should not be changed. At any point that POC > A, the computation can be stopped. To reduce the number of periods which would be calculated in the latter situation, it would be advisable to begin with the period which appears to offer the highest EOC. Since the shortage cost, S, is usually much greater than the holding cost, H, the period in which X_p is less than

X^* by the greatest amount should offer the highest EOC. The periods should be taken in descending order of the amount ($X^* - X_p$) where $X^* > X_p$; then the periods in which $X_p > X^*$ whould be taken in descending order of the quantity ($X_p - X^*$). This type of logic can be performed by an electronic computer or manually for very simple problems.

The amount, A, would be spent at the current time to amend the schedule but the expected savings which result from the amendment will occur at some future date. If the cost of capical is sufficiently high or if the time until an expected savings, i.e., the EOC, is very long, the present value of the expected savings may be significantly less than the expected savings. The POC, which is the sum of the present values of the expected savings, should be compared to the amount A to see if the cost to amend a schedule should be spent. If $POC \leq A$ the schedule should not be amended. If $POC > A$ the schedule should be amended to request the optimal deliveries for the latest requirement information.

The model for determining the ECO for a discrete distribution will be illustrated using the discrete joint distribution developed for the positive covariance example in Chapter III. Consider the third stage of this example (from page 44) for which the requirements and probabilities are repeated in Table V.

TABLE V

PROBABILITIES OF DISCRETE EXAMPLE FOR EOC

Requirement (X)	Probability ϕ(X)		Cumulative Probability Φ(X)
	Fractional	Decimal	
33	27/2025	0.013	1.000
32	189/2025	0.093	0.987
31	471/2025	0.233	0.894
30	651/2025	0.322	0.661
29	471/2025	0.233	0.339
28	189/2025	0.093	0.106
27	27/2025	0.013	0.013

Assume that this distribution was developed using the most recent information and that at some prior time the cumulative allocation at this period had been set at $30 = X_p$. With H = \$5 and S = \$95, the optimum probability of meeting the requirement is $\Phi(X^*) = 95/95 + 5 = 0.95$. In order to provide a 0.95 assurance of supplying the cumulative probabilistic requirements through this stage the cumulative allocations should be $32 = X^*$.

The discrete case EOC formula is used to find the expected opportunity cost for having the allocation set at 30 when the optimal allocation is 32. This amount is discounted to its present value for comparison with the cost to amend.

$$X_p = 30 \quad \text{and} \quad X^* = 32 \quad \text{so}$$

$$\text{EOC} = H\left[30\sum_{0}^{30}\phi(X) - \sum_{0}^{30} X\cdot\phi(X) - 32\sum_{0}^{32}\phi(X) + \sum_{0}^{32} X\cdot\phi(X)\right]$$

$$+S\left[\sum_{31}^{\infty} X\cdot\phi(X) - 30\sum_{31}^{\infty}\phi(X) - \sum_{33}^{\infty} X\cdot\phi(X) + 32\sum_{33}^{\infty}\phi(X)\right]$$

$$= H\,[30(0.661) - 27(0.013) - 28(0.093) - 29(0.233)$$

$$30(0.322) - 32(0.987) + 27(0.013) + 28(0.093)$$

$$+ 29(0.233) + 30(0.322) + 31(0.233) + 32(0.093)]$$

$$+S\,[31(0.233) + 32(0.093) + 33(0.013) - 30(0.339)$$

$$-33(0.013) + 32(0.013)]$$

$$= H\,[19.830 - 31.584 + 7.223 + 2.976]$$

$$+S\,[7.223 + 2.976 - 10.170 + 0.416]$$

$$= \$5\,[-1.555] + \$95[0.445] = -\$7.775 + \$42.275 = \$34.50 \quad .$$

There would be an expected savings of \$34.50 in the third period if the cumulative allocations at that period were increased from 30 units to 32 units. The time of this expected savings is three schedule periods in the future. For a schedule period of one month and a cost of capital of 2 percent per month the present value of the expected saving would be

$$POC = \frac{1}{(1.02)^3} (\$34.50)$$

$$= \frac{\$34.50}{1.0612}$$

$$= \$32.51 \quad .$$

Assuming this period is the only one which can be amended and that if the procurement leadtime is 3 months so that it must be amended this far ahead if it is to be amended, it is desirable to amend the allocation for this period if it can be done for any amount less than \$32.51. If savings were offered in other periods, the present value of the expected savings in those periods should also be considered in making the amendment decision.

The Continuous Case

The preceding model for EOC was based on a discrete probability distribution for the possible cumulative requirements. The same type of analysis can be performed for a continuous distribution. In the continuous situation it is still true that

$$EOC = TEC(X_p) - TEC(X^*) \quad .$$

For continuous distributions the TECs would be formulated with integrals instead of the summations used previously.

Then,

$$EOC = H\int_{-\infty}^{X_p} (X_p - X)\,\phi(X)\,dX + S\int_{X_p}^{\infty} (X - X_p)\,\phi(X)\,dX$$

$$- H\int_{-\infty}^{X^*} (X^* - X)\,\phi(X)\,dX - S\int_{X^*}^{\infty} (X - X^*)\,\phi X\,dX \quad ,^2$$

which can be put into the form

$$EOC = H\left[X_p\int_{-\infty}^{X_p} \phi(X)\,dX - X^*\int_{-\infty}^{X^*} \phi(X)\,dX\right]$$

$$+ S\left[X^*\int_{X^*}^{\infty} \phi(X)\,dX - X_p\int_{X_p}^{\infty} \phi(X)\,dX\right] + (H + S)\int_{X_p}^{X^*} X\,\phi(X)\,dX \quad .$$

Assume that the requirement distributions are normal. The values of all of these integrals except the last can be found in a table of normal probabilities. The last term of the equation can be put into a form that has one term which can be obtained from the normal table and another term which can be integrated.

Let

$$I_A = \frac{H + S}{\sqrt{2\pi}\,\sigma}\int_{X_p}^{X^*} X \exp\left[-\frac{(X - \overline{X})^2}{2\sigma^2}\,dX\right] \quad ,$$

which is the last term if a normal probability is used, and let

$$\text{let} \qquad z = \frac{X - \overline{X}}{\sqrt{2}\,\sigma} \quad , \qquad \text{then} \qquad X = \overline{X} + \sqrt{2}\,\sigma z$$

and

$$dX = \sqrt{2}\,\sigma\, dz \quad .$$

$$I_A = \frac{H + S}{\sqrt{2\pi}\ \sigma} \int_{\frac{X_p - \overline{X}}{\sqrt{2}\ \sigma}}^{\frac{X^* - \overline{X}}{\sqrt{2}\ \sigma}} (\overline{X} + \sqrt{2}\ \sigma\ z) \exp\,(-z^2)\ \sqrt{2}\ \sigma dz$$

$$= \frac{(H + S)\,\overline{X}}{\sqrt{\pi}} \int_{\frac{X_p - \overline{X}}{\sqrt{2}\ \sigma}}^{\frac{X^* - \overline{X}}{\sqrt{2}\ \sigma}} \exp\,(-z^2)\,dz + \frac{(H + S)\ \sqrt{2}\ \sigma}{\sqrt{\pi}} \int_{\frac{X_p - \overline{X}}{\sqrt{2}\ \sigma}}^{\frac{X^* - \overline{X}}{\sqrt{2}\ \sigma}} z \exp\,(-z^2)\ dz$$

$$= \frac{(H + S)\,\overline{X}}{\sqrt{\pi}} \int_{\frac{X_p - \overline{X}}{\sqrt{2}\ \sigma}}^{\frac{X^* - \overline{X}}{\sqrt{2}\ \sigma}} \exp\,(-z^2)\,dz + \frac{(H + S)\sqrt{2}\ \sigma}{\sqrt{\pi}} \left[-\frac{1}{2} \exp(-z^2) \right]_{\frac{X_p - \overline{X}}{\sqrt{2}\ \sigma}}^{\frac{X^* - \overline{X}}{\sqrt{2}\ \sigma}} \quad .$$

Let the second term of the above equation $= I_B$ and $\frac{\omega}{\sqrt{2}} = z$. Then

$\omega = \sqrt{2}\ z$

$$I = \frac{(H + S)\ \overline{X}}{\sqrt{2\,\pi}} \int_{\frac{\sqrt{2}\ (X_p - \overline{X})}{\sqrt{2}\ \sigma}}^{\frac{\sqrt{2}\ (X^* - \overline{X})}{\sqrt{2}\ \sigma}} \exp\left(-\frac{\omega^2}{2}\right) d\omega + I_B$$

$$= (H + S)\,\overline{X} \int_{\frac{X_p - \overline{X}}{\sigma}}^{\frac{X^* - \overline{X}}{\sigma}} \frac{1}{\sqrt{2\,\pi}} \exp\left(-\frac{\omega^2}{2}\right) d\omega + I_B \quad .$$

The first integral is now the area under a normal probability curve from X_p to X^*.

The total EOC equation for the normally distributed probability of requirements can be written as

$$EOC = H\left\{X_p\left[N(X_p) - N(-\infty)\right] - X^*\left[N(X^*) - N(-\infty)\right]\right\}$$
$$+ S\left\{X^*\left[N(\infty) - N(X^*)\right] - X_p\left[N(\infty) - N(X_p)\right]\right\}$$
$$+ H + S\left\{\overline{X}\left[N(X^*) - N(X_p)\right]\right.$$
$$\left. + \frac{\sigma}{\sqrt{2\,\pi}}\left[\exp\left(-\frac{X_p - \overline{X}}{\sqrt{2}\,\sigma}\right)^2 - \exp\left(-\frac{X^* - \overline{X}}{\sqrt{2}\,\sigma}\right)^2\right]\right\}$$

where N(X) means the cumulative probability under the normal curve up to X.

The above equation can be reduced to

$$EOC = H\left[X_p N(X_p) - X^* N(X^*)\right] + S\left\{X^*\left[1 - N(X^*)\right] - X_p\left[1 - N(X_p)\right]\right\}$$
$$+ (H + S)\left\{\overline{X}\left[N(X^*) - N(X_p)\right] + \frac{\sigma}{\sqrt{2}}\left[\exp\left(-\frac{X_p - \overline{X}}{\sqrt{2}\,\sigma}\right)^2 - \exp\left(-\frac{X^* - X}{\sqrt{2}\,\sigma}\right)^2\right]\right\}$$

As an example of the use of this model, assume for the continuous distribution example introduced on page 48, that the holding cost and receiving cost were such that the deliveries were set equal to the period allocations of 59.90, 45.84, 71.22, and 13.04 units, respectively. Let H = \$5, S = \$95, A = \$15, and assume that a first period requirement of 57.50 units has taken place, leaving three periods in the procurement horizon. If the procurement leadtime is 1 month or less the last two period allocations can be amended. Since no new orders for the item have been received, the total requirement is still 190 units. The last period EOC need not be considered because a shortage or overage in the next-to-last period will be corrected in the last period. The only period for which the EOC must be calculated is the second period of the current problem.

Assume that the latest scheduling information gives the following estimates of the requirements for the remaining three periods:

$$R_1 = 45 \qquad R_2 = 55 \qquad R_3 = 32.5$$

$$\sigma_1 = 5 \qquad \sigma_2 = 7 \qquad \sigma_3 = 8 \quad ,$$

and the coefficients of correlation between the period requirements still remain $\rho_{12} = 0.5$, $\rho_{13} = 0.3$, and $\rho_{23} = 0.4$. Now,

$$\sigma_{ij} = \rho_{ij}(\sigma_i)(\sigma_j)$$

so

$$\sigma_{12} = 0.5\ (5)(7) = 17.5$$

$$\sigma_{13} = 0.3\ (5)(8) = 12$$

and

$$\sigma_{23} = 0.4\ (7)(8) = 22.4 \quad .$$

For the new information

$$\tilde{R}_2 = 45 + 55 = 100$$

$$\tilde{\sigma}_2^{\,2} = 5^2 + 7^2 + 2(17.5) = 109$$

$$\sigma_2 = \sqrt{109} = 10.44$$

and

$$\Phi(X^*) = \frac{95}{95 + 5} = 0.95$$

so

$$X^* = 100 + 1.65(10.44) = 117.23.$$

The first delivery was 59.90 units and the first period requirement was 57.50 units, leaving 2.40 units on hand. The second delivery of 45.84 units now arrives making 48.24 units on hand. If it is not amended, the third delivery will be 71.22 units making the cumulative deliveries at the second stage of the new problem equal to 119.46 units. So

$$X_p = 119.46$$

$$X^* = 117.23$$

$$\overline{X} = 100$$

and

$$\sigma = 10.44 \quad ,$$

and we wish to find EOC.

$$z_{X_p} = \frac{19.46}{10.44} = 1.86$$

and from a table of normal probabilities $\Phi(X_p) = 0.9686$.

$$z_{X^*} = 1.65 \text{ and } \Phi(X^*) = 0.950 \quad .$$

$$\begin{aligned} EOC &= \$5[119.46(0.9686) - 117.23(0.950)] \\ &\quad + \$95[117.23(0.050) - 119.46(0.0314)] \\ &\quad + \$100\ 100(0.950 - 0.9686) + \frac{10.44}{\sqrt{2\pi}} \exp\left(-\frac{1.86}{\sqrt{2}}\right) - \exp\left(-\frac{1.65}{\sqrt{2}}\right) \\ &= \$21.70 + \$200.54 - \$186.00 - \$17.91 = \$18.33 \quad . \end{aligned}$$

The expected savings of \$18.33 will occur one month in the future. If the cost of capital is 2 percent per month, the present value of this saving is \$17.99. Since A = \$15, and 17.99 > 15 the schedule should be amended. The new delivery for the next-to-last period should be

(117.23 - 48.24) or 68.99 units, a reduction of 2.23 units. The delivery for the last period should be increased by 2.23 units to 15.27 units.

FOOTNOTES

[1] C. W. Churchman, R. L. Ackoff, and E. L. Arnoff, Introduction to Operations Research (New York, 1957), p. 209.

[2] Ibid, p. 212.

CHAPTER V

SUMMARY AND CONCLUSIONS

This research is directed primarily toward the development of a procedure for managing the procurement and inventory aspects of multi-period materials requirements which may involve covariance between the probabilistic requirements in various periods. The particular problem defined for study involves items which experience intermittent demand and are not normally stocked. The item cost is assumed to be independent of the quantity in a delivery. Forecasts are available for the multi-period horizon and the accuracy of the forecasts decreases as it represents a period more distant into the future. It is also assumed that a new forecast is provided each scheduling period and that it is possible to amend the portion of a delivery schedule which is beyond one procurement leadtime in the future.

Such a situation would probably be found in a moderate volume job shop. Many elements of the environment contributing to the existence of the problem are presented and analyzed. A rationale is developed for determining the optimum probability of covering the cumulative requirements at each stage of the horizon. A technique is presented for constructing the probability of various levels of cumulative requirements throughout the horizon. The optimum receiving policy is found by selecting the level of delivery allocation which would provide the optimum probability of meeting the cumulative requirements at each stage and batching these amounts into the most economical shipment sizes.

As a procurement and inventory management tool, this algorithm includes provisions for both planning and control. Monitoring of changes in the forecasted requirement schedule and the increased accuracy of later forecasts allow the recalculation of new optimal delivery schedules for a portion of the horizon which can be revised. A model is developed for computing the expected opportunity cost which would result from not revising the delivery schedule to its new optimal values. The schedule should be revised if the expected opportunity cost is greater than some established limit of deviation such as the cost to revise the delivery schedule.

The procedure for normally distributed requirement probabilities can be summarized into the following steps:

1. Determine R, the requirement vector, from the master schedule.
2. $\tilde{R}_n = \sum_{i=1}^{n} \overline{R}_i$
3. $\tilde{\sigma}_n^2 = \sigma_1^2 + \sigma_2^2 + \ldots \sigma_n^2 + 2\sigma_{12} + 2\sigma_{13} + \ldots + 2\sigma_{1n}$
   $+ 2\sigma_{23} + \ldots + 2\sigma_{2n}$, etc.

4. $\Phi(X^*) = \frac{S}{H + S}$

5. Determine z_{X^*}, $X_n^* = R_n + z_{X^*}\sigma_n$

6. $\Delta n = X_n^* - X_{n-1}^*$

7. Batch Δ's into the delivery vector D. If $H\Delta_2 < T$, $D_1 \geq \Delta_1 + \Delta_2$. If $2H\Delta_3 < T$, $D_1 \geq \Delta_1 + \Delta_2 + \Delta_3$, etc. Repeat this logic considering each allocation which requires a new delivery as the first allocation.

8. With new scheduling information repeat steps 1 through 5.

9. If $X_p \neq X^*$ determine EOC and sum present values for POC.

10. If POC > A, repeat steps 6 and 7 with new X^*'s and amend D.

11. If POC $\leq$ A retain the same delivery schedule.

It is concluded that a supply procedure can be developed which does not require the assumption of independence of requirements from one schedule period to another. To apply the methods developed in this study some estimates of the variance and covariance of the period requirements are required. The procedure can be applied even though there may be insufficient empirical data to construct the variance and covariance factors for the item under consideration.

The probabilistic nature of the analysis results from the uncertainty of the schedules provided by the forecasting and scheduling system. Raw material supply decisions for items with insufficient historical data can be determined by using the data from similar items which are produced by similar operations and are scheduled by the same scheduling system. Such items would be exposed to the same type of schedule variations.

Situations in which there is no similar part would require the estimation of the probability distribution of cumulative requirements at the various stages or an estimation of the individual period distributions and the covariance relationships between them. This approach would be similar to PERT (Program Evaluation and Review Technique) in that a knowledgeable person provides probability beliefs for the possible events. The use of this procedure would provide the optimum delivery schedule for the assumed conditions.

Future Investigation

It was assumed in this investigation that variance and covariance factors could be obtained. Implicit in this assumption is the availability of some historical data for developing the factors or estimates of these factors. No assumptions were made of the weights assigned to historical data. Future research might investigate the feasibility of assigning higher weights to more recent data in determining the covariance of requirements from one period to another.

If covariance is present it might be used in predicting future requirements in a procurement horizon from actual requirements as they occur earlier in the same horizon.

In this paper the optimum probability of meeting a requirement was determined on the basis of expected monetary value. The concept of utility has been used in some areas and inventory problems represent an area for further research in the application of this concept.

Inventory decisions quite often involve opportunity costs and loss of goodwill or the possible loss of a customer. Businesses in various competitive situations have varying utilities for customers. A business which has a short term overcapacity will value customer accounts much more highly than a business which is working at a stable capacity, even though the amount of profit involved may be equal in both cases. Utility adds a dimension which measures the individual's or company's attitude toward risk and may differ for two companies facing the same situation. Utility profiles for companies and people are subject to alteration or revision periodically to reflect the changes in attitude which may occur in a dynamic business environment. The expected momentary value figures do not provide this feature.

The analysis in this dissertation uses the distribution of cumulative raw material requirements to determine the amount of raw material which should be supplied at each stage of a time horizon. This analysis can be extended to cumulative requirements for any resource over a time horizon where there is a cost associated with having too much resource and a cost associated with having too little resource at each stage. The funds required for the production of large construction projects, the development of a business, or a government project may be considered as such a resource. In arranging funding over an extended period of time a question arises regarding how much money should be obtained each period. If too much money is obtained there will be idle funds on which interest must be paid. If too little money is obtained there may be costs for items such as higher interest for short term loans, loss of trade discounts available for early payment, or delay of the project. In such a problem the total resources needed for the project may be considered as a probabilistic amount rather than a deterministic amount, as was assumed in this study.

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APPENDIX

APPENDIX

ALTERNATE METHOD

If it is assumed that the relative accuracy of forecasts is the same for any size forecasted requirement, an alternate method can be used for developing probabilities for the cumulative requirements. This method does not require the use of the covariance factors. If the covariance factors are not required for any other purposes or if not enough data are available to develop these factors, this alternative method might be used.

The element that is desired in order to reach a delivery schedule decision is the ratio of cumulative expected requirements, which is exceeded no more than $[1 - \Phi(X^*)]$ portion of the times. This information at each stage is sufficient to provide a desired delivery at each stage. Rather than analyze empirical data to determine correlation coefficients and use these coefficients to construct new cumulative demand probability distributions, one could use the following method. Historical data are analyzed to determine the ratio of cumulative actual requirements to cumulative expected requirements at each stage. For each stage a frequency distribution is established and a probability distribution constructed. From this it is possible to select the ratio of expected requirements which has been exceeded no more than $[1 - \Phi(X^*)]$ portion of the times. An example computer printout of a model incorporating this method is included in this appendix.

The following pages are an example of actual empirical data from a source which prefers to remain anonymous. The part number is fictitious. This example illustrates several points related to this dissertation. The observations are presented on two bases. The simple ratios represent the ratio of the actual requirement which occurred to the requirement which was forecasted to occur. The cumulative ratios represent the ratio of the total actual requirements through the stage to the total requirements which were forecasted to occur between the time from one leadtime in the future through the subject period.

The average ratios are all less than 1.0 which indicates that there may be some bias in the forecasting system. The variances of the simple ratios increase as the observations represent more distant periods of time, as was discussed in this paper. The variances of the cumulative ratios decrease as they represent the sum of more and more periods, indicating a central limit effect. The 0.9500 protection ratio is based on the cumulative average as presented in Chapter III.

CUMULATIVE RATIOS FOR PART NUMBER XYZ-123

THE DESIRED LEVEL OF PROTECTION IS .9500

LEADTIME IS 2 PERIOD(S)

HORIZON IS 6 PERIODS BEYOND LEADTIME

HISTORY MATRIX

73.	80.	72.	72.	72.	72.	40.	68.
80.	72.	72.	72.	72.	40.	68.	60.
72.	42.	64.	64.	32.	60.	56.	56.
80.	64.	64.	32.	60.	56.	56.	52.
84.	64.	32.	60.	56.	56.	52.	40.
80.	32.	71.	56.	56.	52.	40.	32.
32.	71.	30.	56.	52.	40.	32.	30.
71.	30.	56.	52.	40.	32.	30.	41.
30.	56.	52.	42.	33.	31.	41.	40.
56.	41.	42.	33.	31.	41.	40.	44.
41.	40.	33.	31.	41.	40.	44.	44.
40.	33.	31.	41.	40.	44.	44.	46.
55.	34.	41.	40.	44.	44.	46.	50.
34.	42.	40.	44.	44.	46.	51.	56.
42.	40.	44.	44.	46.	51.	56.	62.
43.	44.	45.	46.	51.	56.	62.	67.
21.	46.	46.	51.	56.	62.	67.	70.
46.	31.	49.	56.	62.	67.	70.	74.
31.	49.	56.	62.	67.	70.	74.	73.
49.	54.	62.	66.	70.	74.	73.	76.
53.	60.	66.	70.	74.	73.	76.	75.
29.	56.	68.	74.	73.	76.	75.	70.
57.	71.	76.	74.	76.	75.	70.	73.
71.	76.	74.	76.	75.	70.	73.	73.

STAGE 1

SIMPLE RATIOS	CUMULATIVE RATIOS
1.0000	1.0000
1.1111	1.1111
1.3125	1.3125
1.2500	1.2500
1.0000	1.0000
1.0000	1.0000
1.0000	1.0000
1.0000	1.0000
0.7885	0.7885
0.9524	0.9524
1.6667	1.6667
1.0968	1.0968
1.0244	1.0244
1.0750	1.0750
0.4773	0.4773
1.0222	1.0222
0.6739	0.6739
1.0000	1.0000
0.9464	0.9464
0.4677	0.4677
0.8636	0.8636
1.0441	1.0441
SIM AVE = 0.9897	CUM AVE = 0.9897
SIM VAR = 0.0620	CUM VAR = 0.0620

THE .9500 PROTECTION RATIO IS 1.667 SAMPLE SIZE = 22

STAGE 2

SIMPLE RATIOS	CUMULATIVE RATIOS
1.1111	1.0556
1.1667	1.1389
1.2500	1.2812
1.0000	1.1667
1.1833	1.1196
0.5357	0.7953
1.0000	1.0000
0.7885	0.8981
0.9524	0.8617
1.6667	1.2667
1.0968	1.3906
1.0244	1.0556
1.0750	1.0494
0.4773	0.7619
1.0455	0.7614
0.6739	0.8462
0.9608	0.8247
0.9464	0.9714
0.4677	0.6949
0.8636	0.6719
1.0143	0.9412
SIM AVE = 0.9667	CUM AVE = 0.9787
SIM VAR = 0.0726	CUM VAR = 0.0385

THE .9500 PROTECTION RATIO IS 1.391   SAMPLE SIZE = 21

STAGE 3

SIMPLE RATIOS	CUMULATIVE RATIOS
1.1667	1.0926
1.1111	1.1296
1.0000	1.2250
1.1833	1.1731
0.5357	0.8986
1.0000	0.8579
0.7885	0.9203
1.0000	0.9257
1.6667	1.0709
1.0968	1.2170
1.0244	1.2476
1.0750	1.0625
0.4773	0.8480
1.0455	0.8594
0.6739	0.7313
0.9608	0.8873
0.9464	0.8693
0.4677	0.7844
0.8507	0.7514
1.0143	0.7929
SIM AVE = 0.9542	CUM AVE = 0.9672
SIM VAR = 0.0722	CUM VAR = 0.0270

THE .9500 PROTECTION RATIO IS 1.248 SAMPLE SIZE = 20

STAGE 4

SIMPLE RATIOS	CUMULATIVE RATIOS
1.1111	1.0972
0.8000	1.0781
1.1833	1.2136
0.5357	1.0047
1.0000	0.9265
0.7885	0.8426
1.0000	0.9382
1.7187	1.0667
1.0968	1.0759
1.0244	1.1633
1.0750	1.2000
0.4773	0.8974
1.0455	0.8994
0.6739	0.8103
0.9608	0.7946
0.9464	0.9040
0.4677	0.7535
0.8507	0.8034
1.0143	0.8235
SIM AVE = 0.9353	CUM AVE = 0.9628
SIM VAR = 0.0785	CUM VAR = 0.0202

THE .9500 PROTECTION RATIO IS 1.214   SAMPLE SIZE = 19

STAGE 5

SIMPLE RATIOS	CUMULATIVE RATIOS
0.8000	1.0610
1.0441	1.0710
0.5357	1.0761
1.0000	1.0037
0.7885	0.8984
1.0000	0.8655
1.7187	1.0571
1.1333	1.0762
1.0244	1.0653
1.0750	1.1444
0.4773	1.0317
1.0455	0.9300
0.6739	0.8512
0.9608	0.8444
0.9464	0.8299
0.4677	0.8000
0.8507	0.7766
1.0143	0.8520
SIM AVE = 0.9198	CUM AVE = 0.9575
SIM VAR = 0.0788	CUM VAR = 0.0132

THE .9500 PROTECTION RATIO IS 1.144 SAMPLE SIZE = 18

STAGE 6

SIMPLE RATIOS	CUMULATIVE RATIOS
1.0441	1.0581
0.5000	0.9818
1.0000	1.0633
0.7885	0.9688
1.0000	0.9122
1.7187	0.9544
1.1333	1.0667
1.0244	1.0677
1.0750	1.0669
0.4773	1.0173
1.0455	1.0343
0.6739	0.8821
0.9800	0.8755
0.9464	0.8648
0.4677	0.7558
0.8507	0.8104
1.0143	0.8239

SIM AVE = 0.9259 CUM AVE = 0.9532

SIM VAR = 0.0844 CUM VAR = 0.0100

THE .9500 PROTECTION RATIO IS 1.068 SAMPLE SIZE = 17

VITA

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