AN INVESTIGATION OF THE RELATIVE EFFECTIVENESS OF TWO METHODS OF TEACHING A LARGE SIZED UNDERGRADUATE MATHEMATICS CLASS

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TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

General Background and Need for the Study

One of the major problems facing educators today is the spiraling enrollment in our colleges and universities. United States Office of Education (51, p. 12) figures indicate that the number of students enrolled for degree-credit in colleges and universities will double between the years 1960 and 1970. From an enrollment of 3.6 million in 1960, it is projected that 7.2 million will be in institutions of higher learning by 1970. Furthermore, projections for 1977 indicate 9.7 million students, an increase of 169 per cent over the enrollment of 1960.

Reasons for this continuing increase in enrollment are suggested by Parker in the following:

> ... in spite of a wave of dissent and the student voice of protest on many campuses, it is clear that ever large numbers of young people are seeking higher education. Factors supportive of this freshman increase, despite near standstill population data, include aggressive programs for aiding, enrolling, and tutoring the economically and culturally disadvantaged, the continuation of financial aid at a high level, the affluence of many, the aversion of some to military service, social and economic pressures generally, and the escalating demands of a technical and complex society for trained manpower. (40, p. 44)

To place this situation in proper perspective, one must consider the number of teachers in colleges and universities. According to the United States Office of Education (51, p. 56) figures, there were 196,000 college teachers in 1960. This is expected to rise to 347,000 in 1970 and to 440,000 by 1977. The projected enrollment for 1977 is a 124 per cent increase over the 1960 figure. Hence, one can see that the teacher supply is not increasing at as rapid a pace as is the student population. That is, the demand for teachers is still outdistancing the supply. McNeill (34, p. 140) predicts the teacher shortage "undoubtedly will get worse."

The increased emphasis on mathematical knowledge as a prerequisite for entrance into a variety of occupations has led to a greatly increased demand for mathematics teachers at all levels of instruction. Although it may not be a completely fair or logical assumption, competence in teaching undergraduate mathematics has been equated with a doctorate in mathematics. This criterion is generally accepted and applied by administrators and mathematicians alike, and is an indication of the growing problem of staffing the mathematics departments in colleges and universities. Young (52) has estimated that there is less than one-half of one person holding a doctorate (Ph.D.) in mathematics per college or university offering an undergraduate major in mathematics. Since the larger and better universities usually have many doctorates on their staffs, other institutions must do without. This is not to say that non-doctoral degree people are not sufficiently trained to do an adequate job. But, as Perel and Vairo (41, p. 288) indicate, "there are many examples - particularly in junior colleges - of persons teaching mathematics who lack even a bachelors degree specifically in mathematics."

Consequently, if one accepts the premise that colleges and universities are interested in competent mathematics professors, then it appears that means of handling large numbers of students will depend upon methods by which fewer, not greater, numbers of teachers will be involved. Furthermore, the facts and figures underscore the need for more and better instructional methodology, including the use of media.

Four approaches to the growing problem of large student populations are suggested by Kuuitso (30). The approaches he proposes are mechanical teaching aids, the large lecture class, accelerated programs, and larger graduate schools. Of these four approaches, the latter two appear to be more dependent upon administrative decisions and, thus, are less likely to be manipulated by individual faculty members. Mechanical teaching aids and large lecture classes, however, are largely controllable by the individual instructor or his particular department.

It is quite common, as indicated by McKeachie (33), that many large colleges and universities divide class meetings between lectures and discussions. For example, a three-credit hour course might involve meeting twice a week for large group lecture and once a week in smaller discussion sessions. In institutions having graduate programs, the graduate assistant usually teaches the discussion sessions. Colleges not having graduate programs must function without this type of assistance. In the three-credit hour course described above, a class of 150 students would normally require five or six discussion sessions, thus involving seven or eight contact hours weekly for the instructor. A mechanical teaching device is a possible way of alleviating this load on the instructor and freeing him for other duties or other classes, or for providing additional preparation time.

Statement of the Problem

The purpose of this research was to investigate the relative effectiveness of two methods of teaching mathematics to students in a large sized undergraduate mathematics class. One method involved the students attending large group lecture twice a week and smaller discussion sessions once a week. These students served as the control group. The other method also involved two large group lecture meetings per week, but instead of attending a discussion session, these students used an automated teaching device consisting of an audio-tape synchronized with a student response sheet. Hence, the student-teacher contact time was reduced by one-third. In addition, American College Test (ACT) mathematics subscores were used to determine three ability levels for each method. This allowed for comparisons to be made between the experimental and control groups in regards to total students with ACT subscores, high ability students, average ability students, and low ability students.

Statement of the Hypotheses

In stating the following hypotheses, the identification of students in each group will be limited to those for whom ACT mathematics subscores are available. The principal hypotheses tested in this study were:

1. There will be no significant difference in the level of achievement of students in the experimental group when compared with the level of achievement of students in the control group at the five per cent level of significance.

2. There will be no significant difference in the level of achievement of "High Ability" students in the experimental group when compared with the level of achievement of "High Ability" students in the control group at the five per cent level of significance.

3. There will be no significant difference in the level of achievement of "Average Ability" students in the experimental group when compared with the level of achievement of "Average Ability" students in the control group at the five per cent level of significance.

4. There will be no significant difference in the level of achievement of "Low Ability" students in the experimental group when compared with the level of achievement of "Low Ability" students in the control group at the five per cent level of significance.

Limitations

The following limitations were:

1. This study was limited to the students enrolled in a general education mathematics class at a private church-related liberal arts college during the 1969-1970 school year. Consequently, any conclusions and inferences drawn may be applicable only to students in classes similar in composition.

2. Only data from those students who participated in all phases of the experiment were statistically treated.

3. The two groups realized they were part of an educational experiment, and this may have affected the results.

Assumptions

This study was designed and developed under the following assumptions:

1. More effective staff utilization is feasible and desirable.

2. Mechanical teaching devices can be structured to aid individual learning.

3. At all ability levels, students randomly assigned will have individual differences in ability to learn.

Overview

This study is composed of five chapters. Chapter I is the introductory chapter and contains sections which relate the general background and need for the study, the statement of the problem, the hypotheses of the study, limitations of the study, and assumptions of the study. Chapter II reviews the literature pertinent to the study, primarily that regarding independent study and the use of audio-tapes in teaching. Chapter III is concerned with the methodology and design of the experiment. This chapter includes descriptions of the sample, the subject, the methods of instruction, the automated instrument, and the evaluation instruments. Chapter IV summarizes the data, describes the statistical design, and presents the statistical analysis. Chapter V contains the summary, findings, conclusions, and recommendations.

CHAPTER II

REVIEW OF THE LITERATURE

Large group lecture classes, the lecture being delivered by the teacher in person or via means of television, and with or without programmed texts, have been a fairly common way of dealing with large numbers of students at the undergraduate level. In regards to class size, Hatch and Bennet (26) cite twelve studies to support the hypothesis that class size is not the critical variable in teaching effectiveness but rather that the quality of the teaching is. Two of the studies concern the teaching of mathematics at the college level. McKeachie (33) in summarizing the research on class size concludes that class size considered in relation to teaching methods is the important factor.

Often large group lecture classes are accompanied by weekly small group discussion periods where the students have the opportunity to discuss, clarify, and reinforce ideas, concepts, and problems regarding the lectures and homework assignments. Foley (18) compared the effectiveness of instruction for teachers of elementary school mathematics in large groups with small discussion groups. He found that students in the large classes compared favorably in terms of mathematics achievement and in changes in attitudes with students in regular size classes. Pethtel (42) compared televised lectures, television-discussion, and lecture-discussion methods in the teaching of mathematics for elementary teachers. He found no significant differences in achievement between the three methods. Gibbons (19) and Hytche (29) both did studies at Oklahoma State University involving large lecture classes and discussion groups. These two studies are described in a later portion of this chapter. Eckert and Neale (14) note that there have been no research projects on lecture versus discussion methods that compare with the 1957-1959 study of large group teaching procedures at Miami University. The conclusions of this study were that neither the initial learning nor the retention is significantly affected by teaching methods. Hatch and Bennet (26) in a review of the literature on the effectiveness of college teachers conclude that there is no one method that has been demonstrated to produce more or better learning than the other. McKeachie (33) argues that the appropriate teaching method depends upon what particular goals are sought.

According to McKeachie (33), research studies regarding teaching methods have been conducted over an extensive period of time. Many of these have been done on such topics as lecture method versus discussion method, distribution of lecture and discussion time, lecture versus automation, student-centered versus instructor-centered teaching, and several others. It is not the intent of the writer to make an exhaustive review of the literature involving these various teaching methods. However, due to the nature of this experiment, the writer is particularly interested in studies involving some form of independent study and the use of audio-tapes as a supplement to, or independent of, combinations of the above methods.

Independent study is a term normally used to describe programs and practices that place greater responsibility on the students for their own education. Independent study may occur in a variety of ways. Regarding independent study, Dearing states:

> ... it is important that in any discussion of independent study, the nature of the independence under consideration be made explicit. Sometimes a student may be following a course syllabus with directed readings but with little if any contact with the instructor, save for the initial setting of tasks and the final testing of the accomplishment. Sometimes the student has the continuing help of a tutor or instructor but operates independently of a standard syllabus, pursuing his intellectual interests where they take him rather than retracing an intellectual odyssey of his instructor or his instructor's instructor. Sometimes the student is freed from attending a number of his regular class meetings but is expected to cover, either on his own or in teams or groups with other students, the material that might normally have been covered in the class sessions from which he has been excused. Sometimes the student, working with films, taped lectures, programed materials, texts, and assigned readings, is expected to accomplish almost completely on his own the goals usually supported by classroom procedures of lecture and discussion. (12, p. 53)

The idea that colleges employ programs of independent study in their instructional program is not new. A 1957 report by Bonthius, Davis, and Drushal (6) described a total of 334 independent study programs in 256 institutions. Nearly all of these programs, however, were designed for honors or superior students. But, as Baskin states:

> What is new is its position as a major development in college teaching. Independent study as a concept has long been regarded as the prerogative of the superior student in honors or tutorial courses. In its growing use, it is available to all students, and, further, it is available at the beginning rather than the close of the student's college career. (2, p. 181)

Dearing concurs and further states:

> What is new in some of the more recent experimentation in the use of independent study is the incorporation of independent study as a part of the teacher's regular teaching procedures and the attempt to employ some form of independent study with all students in a particular course or program, rather than reserving these procedures for the superior or abler student only. (12, pp. 54-55)

Brown (8) provides a list of seventeen arguments in support of independent study. Furthermore, in a list of twenty-one standards of quality of teaching in colleges, item number six states (25, p. 9): "Quality may be indicated in colleges that are most successful in involving their students in independent study." That is, the more independent study techniques utilized by an institution, the higher the quality of the educational program. Dearing suggests some reasons for the current interest in development of programs of independent study:

> 1. The recognition of the waste and efficiency of a system geared to a tiny homogeneous minority, when we in America are committed to educating a vast and heterogeneous majority.
>
> 2. The necessity for accommodation to a growing shortage of fully qualified teachers.
>
> 3. The mounting evidence that mere acquisition of facts and abstract principles is far from enough to produce an educated person.
>
> 4. The growing conviction that learning is essentially an active rather than a passive process. (12, pp. 50-51)

Gruber (22) distinguishes between independent study and self-directed study. He claims that independent study refers to teaching methods that involve individual projects where students and teacher are in a one-to-one relationship; whereas, self-directed study is concerned with ways of increasing the student's responsibility for his own education. Gruber (22, p. 2) further states that investigations of self-directed study have one essential point in common: "While preserving the course system, the proportion of time devoted to formal classroom meetings is reduced."

Other authors refer to honors programs as independent study. Thus, the term independent study can include a wide variety of activities by teacher and pupils.

A three-year investigation from 1957-1959 of self-directed study in nineteen different university courses in eleven different departments by Gruber and Weitman (23) indicated that reducing the attendance at formal classes to one-third the usual number resulted in either small losses or small gains, the gains being somewhat more common than the losses. Although this may seem to be a "weak conclusion," Gruber and Weitman point out the following:

> A study of the institutional setting in which these experiments were conducted showed that by far the greater part of the student's time is spent in highly teacher-directed activities. The fact that self-directed study courses were as successful as they were takes on added impact in light of the findings that these experiments were conducted in an environment fundamentally hostile to independent intellectual work on the student's part. (24, p. 223)

Furthermore, students attending fewer lectures were more attentive, more intellectually active listeners, and thought of more questions about the subject matter during the lecture. It is also concluded that all college students, not only the most able, may profit from greater opportunity for self-directed study (23), (24).

Felder (17) conducted a study in 1963 of 520 institutions which offer four year degree programs and had enrollments of over 200. In response from 445 of these institutions, he found that 68 per cent used some type of independent study program in their undergraduate program, and that less than half of these were restricted to superior students.

He also found that 95 per cent of the colleges and universities having independent study programs allowed the students maximum freedom and there were no requirements in terms of class attendance, course assignments, or methods of study. The only requirement made by 50 per cent of the institutions was that the study proposals be approved by a specified faculty member. In 80 per cent of the colleges, students worked independently on laboratory or experimental projects.

Gruber (22) emphasizes that the research on self-directed study has produced two major findings: (i) results are indeterminant when the criteria for evaluating is the student's learning of subject matter, and (ii) results indicate a small favorable change in curiosity, critical thinking, and attitudes toward independent intellectual work.

Hovey, Gruber, and Terrel (28) experimented at the University of Colorado with two classes of educational psychology. One class was taught by a traditional lecture method of three lectures per week while the other was a self-directed group which met in small groups of five or six students twice a week and with the instructor once a week. At the end of the semester the self-directed group was slightly, but not significantly, superior to the lecture group in the mastery of course material. A retention test on these groups ten months later indicated a similar pattern. However, the experimental group did a significantly greater amount of serious reading to increase their knowledge following the course. Also, on thirteen of fifteen items measuring curiosity, the self-directed group was superior.

Beach (3) conducted a similar study in an undergraduate social psychology course at Whitworth College. The self-directed group did not attend class but met in small groups of five once weekly to study and discuss course materials. The control group met in conventional lecture-discussion classroom manner three times weekly. In an analysis of several outcomes in the total learning experience, it was found that the self-directed group averaged slightly higher in achievement and that the more social person performed better in the small self-directed group in regards to quantity and quality of study for the course, and in the amount of required and non-required reading done in conjunction with the course.

A well-executed appraisal of independent study at Antioch College from 1956-1960 is reported initially by Churchill (11) and later by Baskin (1). Churchill's study involved a comparison of lecture-discussion method with two methods of independent study -- small group and individual work -- in seven general education courses. In both independent study methods the class time was reduced by about one-half. The students were trained and given materials to guide their independent study. Churchill reported that in all courses all sections gained significantly in achieving course objectives. Sections taught by different methods did not differ in significant gain. Baskin (1) reports that data on the retention of learning two years after the courses were completed revealed no significant differences in the groups. There was no evidence that the independent study methods needed to be reserved for the superior or advanced student.

A somewhat different form of student initiative is indicated in an experiment by Rising and Pang (45). Rising, as the instructor of an introductory topology class, did not reduce the lecture time but placed his lecture notes on closed reserve in the library. The announcement was made to the class that the lecture notes were available but the students were not further urged to use them. An opinionaire administered by Pang (45) at the end of the semester indicated that 80 per cent of the students had used the lecture notes one or more times. Of those who used the lecture notes, 96 per cent found them to be helpful.

McKeachie (33) and Gruber (22) in reviews of literature on independent study both conclude that field studies conducted at Antioch College, University of Colorado, and University of Michigan seem to justify the reduction of the number of formal class meetings.

Bhushan, Jeffryes, and Nakamura (5) conducted an experiment at the Hawaii Curriculum Center at the University of Hawaii with a plane geometry class of ninth and tenth graders. The experimental group consisted of 69 students who met together twice a week for a thirty minute lecture and twice a week in groups of 23 apiece for a one hour problem solving and discussion meeting. The control group consisted of two classes, of 25 and 23 students respectively, each of which met four days per week for a one hour session of lecture, demonstration, discussion, and problem solving. Thus, the experimental group had three hours of weekly teacher contact while the control group had four. It was found that the experimental group achieved slightly higher although not significantly so. In concluding, Bhushan, Jeffryes, and Nakamura state that:

> Although we cannot definitely conclude that large-group instruction offers a method of effective presentation of mathematical concepts with a significant decrease in teacher-load, the results of this experiment certainly seem sufficiently encouraging to stimulate further experimentation and study. (5, p. 775)

According to Beckman, Janke, and Tanner:

> For many years there has been a steady increase in the size of introductory college classes. Although learning can take place in such situations, teachers must make provisions for individualizing learning experiences. Research indicates that the learning process is reinforced by the utilization of a variety of different real and vicarious learning experiences. (4, p. 241)

It is further stated that recent technological advances permit us to create a variety of learning experiences. This would include individual study booths, tape recorders, films, filmstrips, television, and programmed materials. Meserve (36) indicates that increased use of automated procedures to provide a basic education in a number of skills may be expected. May feels that:

> The student learns by multiple exposure and activity in a repeated cycle of listening, speaking, reading, problem solving, writing, getting feedback from answers and corrected problems, etc. (35, p. 445)

Gibbons (19) conducted a study at Oklahoma State University to investigate whether or not the mastery of elementary mathematics by prospective elementary teachers was affected by different teaching methods. Three different instructional methods were employed with the experimental groups: (1) Lecture Textbook (L-T) method, (2) Program Lecture Discussion (P-L-D) method, and (3) Lecture Program Discussion (L-P-D) method. The Lecture Textbook method represented the conventional method of instruction found in many colleges. Each new concept was introduced through a lecture. The assignment following each lecture consisted of solving a set of exercises from a related textbook. These problems were not collected and were not discussed unless a student requested a solution or explanation. This cycle was repeated throughout the course. The second experimental group, the Lecture

Program Discussion group, received the following method of instruction:

> Each new concept, or set of concepts, was first introduced through a lecture. The number of concepts developed in a given period varied in relation to the complexity of the given concepts. The lecture was then supplemented by a homework assignment that consisted of reading a certain number of frames from related programed materials. The concepts were then thoroughly discussed at the next class meeting. This cycle was repeated throughout the entire course. (19, pp. 20-21)

Gibbons' third experimental group, the Program Lecture Discussion group, was also a three step method of instruction. It is described as follows:

> Each new concept, or set of concepts, was first introduced through programed materials that were read prior to attending a given lecture. Again the number of concepts developed varied in relation to the complexity of the given concepts. These programed materials were then supplemented by a related lecture. The concepts were then thoroughly discussed at the next class meeting. This cycle was repeated throughout the entire course. (19, pp. 22-23)

The Lecture Program Discussion group and the Program Lecture Discussion group were found to be significantly superior to the Lecture Textbook group in achievement and understanding in mathematics. The students involved in the Lecture Program Discussion group showed a higher level of achievement and understanding than did the students in the Program Lecture Discussion group, although not significantly so.

A follow-up study was done by Hytche (29) at Oklahoma State University during the first semester of the 1966-1967 school year and during the first semester of the 1967-1968 school year. The L-P-D group and the P-L-D group were used along with two new experimental groups called (1) the Program Discussion (P-D) group and (2) the Program Lecture Discussion Quiz (P-L-D-Q) group. The Program Discussion method was a two step method of instruction. Each new concept, or set of concepts, was first introduced through programmed materials that were read prior to attending a given class. During the class the instructor encouraged group discussions. The sessions were very informal with emphasis on active participation by both student and instructor. This weekly cycle was repeated throughout the entire course. Hytche described the Program Lecture Discussion Quiz group instruction as follows:

> Each new concept or set of concepts was first introduced through programmed material prior to attending a given lecture. These programmed materials were then supplemented by a related lecture. The programmed material, related homework assignment, and lecture were discussed during the first part of the discussion session, and finally, the last ten to fifteen minutes of this session were devoted to a quiz over that portion of the material covered during the previous week. (29, pp. 25-26)

The cycle used for the P-L-D-Q group consisted of two fifty-minute organized lectures, one thirty-five to forty-minute informal discussion, and a ten to fifteen-minute quiz session. This weekly cycle was repeated throughout the course. The findings indicated no significant differences in achievement of the four groups. However, students involved in the P-L-D-Q group scored higher on the posttest than any of the other three groups, but not significantly so.

Gibbons (19) and Hytche (29) recommended that research is needed to investigate methods of instruction using much larger groups such as one hundred or more. In addition, Hytche recommended that other methods of instruction be developed and investigated with emphasis on large lecture sessions and small discussion groups.

Some disciplines, notably foreign languages, have been making use of audio-tapes for some time. Olivero (39) reveals that tapes are also being used for independent study in such subjects as business, spelling, speech, drama, and music. The literature seems to indicate that science and mathematics are just recently beginning to use audio-tapes, either as a supplement to or in lieu of formal classroom meetings.

Postlethwait (43) developed a systems approach for a botany class at Purdue University. The systems approach involved four phases of instruction weekly: (1) one hour of large-lecture instruction, (2) one hour of small group assembly of approximately thirty students, (3) independent home study, and (4) approximately four hours of supervised study with audio-tape and laboratory materials. Regarding the latter phase, the laboratory was opened on a "library basis" from 7:30 A.M. to 10:30 P.M. each weekday. Instructions were provided by audio-tape. A single student assistant was on duty as a monitor. The student was able to fit the four hours of time into his schedule at his own convenience and could either use his skill to accelerate his laboratory work or repeat experimental procedures as often as he felt necessary for mastery. According to Postlethwait:

> The use of audiotape to program laboratory study can provide many of the advantages contributed by such devices as TV and still retain the personal contact which is so important to the student. Uniform instruction for large numbers of students can be made available on an individual basis so that the student can study at a pace and time most convenient for him. (43, p. 244)

Head and Runquist (27) developed tapes for use in a chemistry laboratory at Hamline University. The tapes were used to explain general laboratory procedures and for directions on operating various instruments. LaCava (31) taped lessons on electromagnetism for fourth grade children. The taped lessons consisted of a series of questions and directions for the performance of various science experiments.

Richason (44) developed the Audio-Visual-Tutorial (A-V-T) system as a method of independent study in geography. The A-V-T system consists of an individualized study booth, a tape player, and a 35 mm slide projector. The entire course was presented by means of taped discussions and colored slides. Students could work independently on an unscheduled basis, and could repeat any portion as often as desired. Although no statistical analysis was performed, the distribution of examination grades for the class were 16 per cent higher than the average over a ten year period under the traditional method of instruction. Erhard and Mellander (16) have adopted the A-V-T system for a physical geography class at Western Michigan University and are encouraged by preliminary findings. Syrocki, Thomas, and Fairchild (50, p. 91) adopted an A-V-T approach for a college biology class "to encourage the student to work independently, at the convenience of the student, and at the pace the student wishes."

Lorenz (32) investigated the possibility of using audio-tape recordings of live presentations to classes of college students. His study was concerned with the problem of exposing talented faculty members to increasing student enrollments, particularly in smaller colleges lacking in facilities and funds. In his study, seven live multi-media presentations were made to a control group. Audio-tape recordings of the live presentations were given to an experimental group. The same visual information seen by the control group was presented to the experimental group by student assistants. The same sequence of oral and visual information was observed by both groups. The findings were that students in the experimental group achieved as well on a criterion test as students in the control group. Lorenz concluded that small colleges with limited facilities can expose their senior staff members to large numbers of students and maintain achievement without the time consuming repetition of multiple-class presentations by using low cost educational technology and student assistants.

At present there appears to be very little adaptation of audio-tapes to mathematics classes. Christenson (10) constructed a mechanical teaching device to conduct review sessions for three pre-calculus algebra and trigonometry classes. The device consisted of a tape recorder synchronized with a slide projector whereby the students would observe a series of slides projected on a screen and accompanied by a taped voice explanation. The review sessions were held in addition to the regular class meetings, and met in the evenings approximately once per week. Half the students in each class were randomly selected to attend review sessions where the mechanical device was used, while the remaining students attended a teacher-oriented conventional type of review session. Comparisons of the two groups indicated that there were no significant differences with regards to achievement.

Meyer (37), at the University of Wisconsin-Milwaukee, developed a series of tapes for a college algebra class. Each tape covered approximately one class period with the student listening to the tape and writing the things he was instructed to. Besides the new material, most tapes included short quizzes and response items for the purpose of aiding the student in determining how he was progressing. Later on, a supplement of written material was provided to use as an addition to the tape. The students attended the taped lesson at the regular class time and additional time was made available if they wanted or needed it. In a comparison of the final examination grades of the experimental section with the final examination grades of other classes taking college algebra under the conventional lecture situation, it was found that 88 per cent of the experimental group made a grade of C or better while only 52 per cent of the students in conventional sections made a grade of C or better.

Duffey (13) used a tape recorder, earphones, a student response sheet, and a prerecorded taped lesson in a seventh grade math class to meet the needs of individual students with their variety of learning levels. He states that the student response sheet, designed to insure student reaction to the tape, is a summary of the taped lesson and contains all of the basic skills or concepts contained in the tape. Duffey reports that after a semester of using the taped lessons with 167 seventh graders, only six had marginal passes.

Robinson (46) compared the effects of tape recorded instruction on the arithmetic performance of seventh grade pupils. The study involved fourteen classes, seven experimental and seven control, along with four teachers. The experimental treatment involved the use of a series of eight tape recorded lesson segments with one lesson presentation made each week for a period of eight weeks. Each taped segment required from twelve to fifteen minutes for presentation, and a guide sheet of instructions and practice exercises accompanied each presentation. A teacher-made skill test of twenty items was administered following the lesson presentation. Pupils in the control group received the same instruction by means of traditional teaching methods (employing the use of lecture, demonstration, drill, and practice) and were administered identical tests. The findings were that seventh grade pupils who received traditional arithmetic instruction performed at a significantly higher rate than pupils who received tape instruction. In regards to ability levels as determined by I. Q. scores, there was a significant difference in the achievement of average ability students receiving traditional arithmetic instruction over those receiving taped instruction, but there were no significant differences regarding the respective high ability and low ability levels. Robinson concluded that tape recorded instruction does have value as a complementary aid to extend the effectiveness and efficiency of the classroom teacher. He indicated the need for further research which would extend over longer periods of time, involve larger samples, and include studies in other subject matter areas.

Summary

To summarize, Postlethwait gives several advantages he sees in regards to taped teaching. Some of those that particularly apply to mathematics are:

1. Emphasis is placed on student learning rather than on teaching.
2. The student paces his study according to his ability to assimilate the information. Exposure to difficult subjects is repeated as often as necessary.
3. Since the better students are not a captive audience, they can use their time effectively. Their interest is not dulled by unnecessary repetition of information already learned; instead they are free to choose activities that are more challenging and instructive.
4. The student selects a time for listening when his efficiency is at its daily peak.
5. Tapes demand attention and distractions are minimized.
6. The student feels more keenly his responsibility for his own learning. (43, p. 243)

At the same time, however, the use of teaching machines or programmed book materials in college instruction presents special problems which the instructor must take into consideration. According to Brown and Thornton:

> If programmed materials are to be used, programs will need to be found (not easy, but becoming easier) which meet one's expectations and specifications.
>
> Programs requiring nonportable machines (as opposed to book types or individual, low-cost machines which can be carried about by the user) will require special rooms; someone must be responsible, too, for loading machines, keeping them functioning, and reclaiming and analyzing residual answer sheets. Programmed book materials, now becoming more common, have the advantage of portability, flexibility, low cost, and individualization.
>
> The instructor who assigns programmed materials to students must give special consideration to changes this action is likely to require in the usual pattern of in-class and out-of-class activities. Instructors who use programmed materials, for example, sometimes find that it is possible to devote more time in class to discussion and explanation of confusions and misunderstandings arising from out-of-class study of programmed materials and less to lecturing or other instructor-presentation techniques.
>
> Instruction must be given students in how to use programmed materials and equipment. (7, p. 188)

However, even in light of these particular problems and considerations, as Postlethwait points out:

> The use of audiotape ... can provide many of the advantages contributed by such devices as TV and still retain the personal contact which is so important to the student. Uniform instruction for large numbers of students can be made available on an individual basis so that the student can study at a pace and time most convenient for him. (43, p. 244)

Finally, in regards to research, May argues:

> Vigorous experimentation should be undertaken in preparing and using a variety of special-purpose auxiliary materials - films, filmstrips, slides, transparencies, tapes, and program sequences. Experimentation in automation should concentrate on devices that maximize individualization for the single student. (35, p. 452)

Hence, the literature seems to indicate the need for research in large lecture classes in mathematics that would provide the student with the opportunity for independent and individualized study, and at the same time provide a possible method of handling large numbers of students. Further, it appeared to emphasize that research should be directed toward discovering an effective combination of educational techniques that would lead the student to maximum understanding.

CHAPTER III

THE EXPERIMENT

This was an experimental study to investigate the relative effectiveness of two methods of teaching mathematics to students in a large sized undergraduate mathematics class. One method involved the students attending large group lecture twice a week and smaller discussion sessions once a week. The other method also involved two large group lecture meetings per week, but instead of attending a discussion session, these students used an automated teaching device consisting of an audio-tape synchronized with a student response sheet.

The Sample

The sample for this study consisted of the majority of students enrolled in the Mathematics 113 class during the 1969 fall trimester at Oklahoma Christian College, Oklahoma City, Oklahoma. Most of the students involved in this research were freshmen and sophomores, although there were a few upperclassmen in the study.

The Subject

Mathematics 113 is a three-credit hour general education mathematics course entitled Fundamental Principles of Math (38). This course is commonly elected by non-science oriented majors to satisfy a three-credit hour mathematics requirement at Oklahoma Christian

College. The text for the course consisted of portions of Basic Mathematics: A Programed Introduction by Goff and Berg (20). Dr. Robert D. McMillan, Assistant Professor of Mathematics at Oklahoma Christian College, was responsible for this course.

The Procedure

At the beginning of the trimester, each student in the class was assigned to either an experimental group or a control group. The placement in these groups was made by using a table of random numbers. Before the randomization process was initiated, the writer classified the students according to their American College Test (ACT) mathematics subscores into High, Average, and Low ability groups. The Average group was defined to be the set of students whose subscores were within one-half standard deviation of the total mean of the set of subscores. Thus, if $\bar{x}$ is the mean and s the standard deviation, then the range of subscores for the Average group was $\bar{x} \pm 0.5s$. The High and Low groups were then defined to be the set of students whose subscores were, respectively, above or below the range of the Average group. Then, by a random selection, half of each of these groups were assigned to the experimental group and half to the control group. To produce assurance of random sampling, a pretest, Structure of the Number System (Form A), was administered to each group during the first week of the trimester, in September. A posttest, Structure of the Number System (Form B), was administered to each group during the last week of the trimester, in December. A detailed description of the two Number System tests (Form A and Form B) is given in a later section of this chapter entitled "Evaluation Instruments." The posttest was considered as a portion of the final examination in the course. Findings of the study were based on a comparison of posttest scores of the groups.

The control group consisted of those students who attended large group lectures twice a week and a smaller group discussion session once a week. The large group lecture meetings were held at 3:00 P.M. on Wednesday and Friday, and were taught by Dr. McMillan. Three discussion sessions were held on Monday, at 2:00, 3:00, and 4:00 P.M., and each student attended one of these. The discussion meetings were used to aid in understanding and extending the text material and lectures. The discussion sessions were taught by the writer. The writer made a deliberate attempt to cover exactly the same material as covered by the automated teaching device, and to provide the same kinds of student activities. This was done by diligently reviewing the typed scripts that were used in constructing the audio-tapes. The students in the control group, unlike those in the experimental group, were able to interject pertinent questions.

The experimental group consisted of those students who attended the same large group lecture as students in the control group and, in lieu of attending a discussion session, used an automated teaching device located in the Learning Resources Center at Oklahoma Christian College. The automated teaching device was used to aid in understanding and extending the text material and lectures. Since the facilities in the Learning Resources Center were available from 7:00 A.M. to 11:00 P.M. daily, the students were able to use the device at any time convenient for them and could hear any particular tape as many times as desired.

The Automated Teaching Device

Preparation of a recorded passage must be done after considerable planning has taken place. Silverstone (47) recommends that all scripts for tape teaching should be written out word for word. Such scripts should be rehearsed several times prior to recording, and then taped several times. Regarding the preparation of a rough script, Duffey states:

> For lessons geared toward specific skills or concepts, step-by-step sequences were laid out to assure continuity. Questions were designed to require students to underline, circle, or write words involved in problem solving. Accomodating pauses incorporated into the script vary in length according to the degree of difficulty of the problem ... . (13, p. 56)

Silverstone (47) places tape recordings and scripts into three categories:

1. Directions - After listening to an explanation, the student is able to follow directions for completing an assignment on worksheet pages. This is the simplest kind of tape to make.

2. "Closed end" - This involves listening and participating. The listener can respond simultaneously with the teacher, take notes, or respond to a worksheet through a step-by-step procedure.

3. "Open end" - A student is given an exercise which has no formal ending. The listener will work independently to arrive at the conclusion.

For this study a series of twelve audio-tapes and student response sheets were developed by the writer and Dr. McMillan. All three types of tapes mentioned above, or combinations of the three types, were used. These tapes supplemented and extended the material of the textbook. Broad subjects covered included:

Tape Number 1: Sets
Tape Number 2: Sets
Tape Number 3: Whole Numbers
Tape Number 4: Fractions
Tape Number 5: Integers
Tape Number 6: Rational Numbers
Tape Number 7: Exponents
Tape Number 8: Logic
Tape Number 9: Logic and Solution Sets
Tape Number 10: Logic
Tape Number 11: Relations
Tape Number 12: Divisability

Books, other than the text, used to validate the content material of the tapes are listed in Appendix A (pp. 57-58).

Once the instructional program became operative, each Wednesday a student response sheet was distributed to each student in the experimental group at the conclusion of the large group lecture. Students who were absent from the lecture were able to obtain student response sheets at Dr. McMillan's office.

Each student response sheet is a written sequence of three or four pages that the student responds to in synchronization with the audio-tape. This involves filling in blanks, solving problems, and circling and selecting words in response to statements on the tape. Pauses are incorporated into the tape to allow the student time to respond. The last page of the student response sheet is a student exercise hand-in page that the student completes by answering certain questions indicated by the audio-tape. Upon completion, the student turned the student exercise hand-in page in at the control room of the Learning Resources Center. These papers were collected, graded, and returned to the student. Besides serving as a learning activity, this procedure enabled the writer to keep a record of the particular hours and days that the tapes were being used. As a sample, the dialogue, student response sheet, and student hand-in page for Tape Number 3 is included in Appendix B (pp. 59-72).

Evaluation Instruments

The main instruments for establishing a base for evaluation were two of the Cooperative Mathematics Tests of the Educational Testing Service, Princeton, New Jersey. The tests that were used were Structure of the Number System, Form A and Form B. Both are forty minute tests.

The Structure of the Number System (Form A) is an achievement test that measures understanding of the real number system up to the rational numbers. The test consisted of forty multiple choice questions that sampled the following topics: arithmetic judgment, operational properties (closure, commutative, associative, and distributive), inverses and identities, properties of integers, place values, (factors, divisors, and multiples), prime numbers, number lines, zero denominator, number systems (bases other than ten), modular arithmetic, and Roman numerals. This test was used as a pretest in the experiment.

The Structure of the Number System (Form B) is also an achievement test that measures understanding of the real number system up to the rational numbers. The test consisted of forty multiple choice questions and was used as the posttest in the experiment. Form B is considered an alternate form of Form A and, thus, covered the same topics as Form A.

The two Number System tests were prepared by the Educational Testing Service staff and some forty-six high school and college mathematics teachers throughout the United States. The tests were administered to a national sample of students in May, 1960. After analyzing the results, the tests were revised and re-pretested in a national program in May, 1962. The results from the second pretesting indicated that the tests were appropriate for the intended population.

These two tests were selected because they were the only commercially produced tests directly related to the objectives of the experiment. The writer and Dr. McMillan examined the Number System tests and compared their content with the material to be covered in Mathematics 113. From the examination and comparison, it was judged that there was a close relationship between the content of these tests and the course content of Mathematics 113.

The tests stress understanding of facts, principles, and relationships, and do not emphasize computational skills. Furthermore, the tests are measures of developed abilities, and thus their content validity is very important. Educational Testing Service feels (15, p. 62) they have insured this by entrusting test construction to persons well-qualified to judge the relationship of test content to teaching objectives. The reliabilities reported by Educational Testing Service are measures of internal consistency, computed by using the Kuder-Richardson Formula 20. The reliability of Form A was .86 with a standard error of 2.73, while the corresponding figures for Form B were .84 and 2.75 (15, p. 63). The correlation of Form A with the SCAT-Quantitative Test was .78, and that of Form B was .74. Educational Testing Service pointed out (15, p. 64) that these correlations were lower than expected but this was due to the fact that Forms A and B measure understanding with decreased emphasis on elementary mathematical operations, while the SCAT-Quantitative Test emphasizes computational skills. Form A had an item-total score discrimination correlation of .50, and that of Form B was .48. These figures indicate that the tests are effective in discriminating between high and low ability students (15, p. 64). Finally, the equivalence of these two alternate forms was very good. The converted raw scores differed by no more than two at all levels of performance. This indicated that the two forms are, in general, similar in difficulty (15, p. 67).

Another instrument, the American College Test (ACT) in Mathematics, was used to classify the students according to various ability groupings prior to the randomization process. This test is a mathematical aptitude test consisting of forty multiple choice questions. The questions emphasize mathematics reasoning ability rather than memorization of formulas, mere knowledge of techniques, or computational skill (9, p. 3).

CHAPTER IV

ANALYSIS OF THE DATA

This study was concerned with the relative achievement of students taught by two different methods in a large sized undergraduate general education mathematics class. Students in the control group attended large group lecture twice a week and smaller discussion sessions once a week. Students in the experimental group also attended two large group lecture meetings per week but used an automated teaching device instead of attending the discussion sessions. Students were randomly assigned to the groups and a pretest was given to produce further assurance of random sampling. Comparisons were made on the basis of posttest scores. Additional considerations were made relating method and mathematical ability as indicated by ACT mathematics subscores. The specific hypotheses tested in this study were:

1. There will be no significant difference in the level of achievement of students in the experimental group when compared with the level of achievement of students in the control group at the five per cent level of significance.

2. There will be no significant difference in the level of achievement of "High Ability" students in the experimental group when compared with the level of achievement of "High Ability" students in the control group at the five per cent level of significance.

3. There will be no significant difference in the level of achievement of "Average Ability" students in the experimental group when compared with the level of achievement of "Average Ability" students in the control group at the five per cent level of significance.

4. There will be no significant difference in the level of achievement of "Low Ability" students in the experimental group when compared with the level of achievement of "Low Ability" students in the control group at the five per cent level of significance.

Summary of the Data

This research was conducted with the Mathematics 113 class at Oklahoma Christian College, Oklahoma City, Oklahoma, during the fall trimester of the 1969-1970 academic year. Originally, 144 students enrolled in this class. Students who withdrew from the course, failed to take both the pretest and posttest, or for whom ACT subscores were unavailable, were eliminated from consideration in the statistical analysis. There were 26 students eliminated, leaving 118 subjects for the experiment. The data collected from these 118 students was of three types: ACT mathematics subscores, pretest scores, and posttest scores. A tabular summary of the raw data appears in Appendix C (pp. 73-76).

The ACT mathematics subscores were used to classify the students into High, Average, and Low ability groups. The Average group was defined to be the set of students whose subscores were within one-half standard deviation of the total mean of the set of subscores. The mean was 18.14 and the standard deviation was 5.81. Therefore, the range of subscores for the Average group was 16 to 21 inclusive. The High and Low groups were then defined to be the set of students whose subscores were, respectively, above or below the range of the Average group. Table I gives a breakdown of the 118 students in the experiment:

TABLE I

NUMBER OF SUBJECTS IN THE VARIOUS SUBCLASSES

Method	High	Average	Low	Total
Experimental	15	24	21	60
Control	17	24	17	58
Total	32	48	38	118

Statistical Design

The data for this experiment was analyzed by analysis of variance techniques. According to Steel and Torrie, in the analysis of variance where tests of significance are made, the basic assumptions are:

> 1. Treatment and environmental effects are additive.
>
> 2. Experimental errors are random, independently, and normally distributed about zero mean and with a common variance.
>
> The assumption of normality is not required for estimating components of variance. In practice, we are never certain that all these assumptions hold; often there is good reason to believe some are false. (49, p. 128)

The design used for this study was a two-way factorial design with a disproportionate number of observations in the subclasses. The design is represented in the following diagram. The cell numbers enclosed in parentheses indicate the number of subjects per cell.

		Method	
		Experimental	Control
Ability Groups	High	(15)	(17)
	Average	(24)	(24)
	Low	(21)	(17)

A comparison between columns relates total students with ACT subscores in each method. A comparison between rows would relate the total students in both methods with high, average, and low ACT subscores. Row comparisons were not pursued since the hypotheses to be examined were concerned with methods. The statistical test for interaction allowed for comparing three pairs: the two High groups, the two Average groups, and the two Low groups.

For this design, the mathematical description of an observation with interaction is given by

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk} \quad \begin{cases} k = 1,2,\ldots,n_{ij} \\ i = 1,2,3 \\ j = 1,2 \end{cases}$$

where y_{ijk} is the kth observation in the ij-th cell; μ, α_i, β_j are unknown parameters; and e_{ijk} are random variables. The ij-th cell contains n_{ij} observations.

The model when interaction is not present is given by

$$y_{ijk} = \mu + \alpha_i + \beta_j + e_{ijk}.$$

Snedecor and Cochran suggest the following approach for a correct analysis of a two-way classification with unequal numbers:

> 1. First, test for interactions.
>
> 2a. If interactions appear negligible, this means that an additive model
>
> $$y_{ijk} = \mu + \alpha_i + \beta_j + e_{ijk}$$
>
> is a satisfactory fit.
>
> 2b. If interactions are substantial, examine the row effects separately in each column, and vice versa, with a view to understanding the nature of the interactions and writing a summary of the results. The overall row and column effects become of less interest, since the effect of each factor depends on the level of the other factor. (48, p. 473)

Snedecor and Cochran (48, p. 472), Graybill (21, p. 287), and Steel and Torrie (49, p. 252) all give excellent discussions of the analysis of disproportionate data.

Steel and Torrie describe the procedure for testing for interaction in a two-way classification as follows:

> First, we include interaction with the other sources of variation called for by the model, then we estimate all the components, that is, the α's, β's, and $(\alpha\beta)$'s, in such a way that the residual sum of squares is a minimum. Thus we obtain the reduction in the total sum of squares attributable to all sources of variation including interaction. Secondly, we drop interaction out of the model and again proceed to minimize the residual sum of squares. Now we obtain the reduction in the total sum of squares attributable to all sources of variation other than interaction. Finally, the difference between the two reductions gives the additional reduction attributable to interaction or interaction adjusted for other effects. (49, p. 256)

The statistical test for interaction allowed the writer to compare three pairs: the two High Ability groups, the two Average Ability groups, and the two Low Ability groups. The procedure used to complete the analysis depended upon whether or not there was evidence of interaction. According to Steel and Torrie:

> If interaction is nonsignificant, it is concluded that the factors under consideration act independently of each other; the simple effects of a factor are the same for all levels of the other factors, within chance variation as measured by experimental error. . . . the simple effects are equal to the corresponding main effects and a main effect, in a factorial experiment, is estimated as accurately as if the entire experiment had been devoted to that factor. (49, p. 199)

All tests for interaction in this experiment revealed nonsignificance. Consequently, as indicated earlier, the model

$$y_{ijk} = \mu + \alpha_i + \beta_j + e_{ijk}$$

is appropriate. Several procedures were available, had interaction been significant. Snedecor and Cochran (48, p. 474) point out that "when interactions are large, this fact may be obvious by inspection, or can sometimes be verified by one or two *t*-tests." Another procedure would have been to apply the method of weighted squares of means as described in Steel and Torrie (49, p. 265).

Treatment of the Data

The raw data in Appendix C were prepared for the Oklahoma State University Computing Center so calculation could be performed on the computer. Graybill (21, p. 302) suggests that the Doolittle technique be used in computing the data in a two-way classification with unequal numbers in the subclasses. A program entitled "Abbreviated Doolittle" was adapted to this experiment.

F ratios for interaction and methods were calculated by dividing the appropriate mean square by the mean square of the error. These

F ratios were used to establish the equivalence between groups and to test the hypotheses.

The .05 level of significance was arbitrarily selected for this research since it is the common level accepted for research in the behavioral sciences.

Equivalence of Groups

The equivalence of the total experimental and control groups, and of the respective High, Average, and Low ability subgroups, were established by two-way analysis of variance based on the ACT subscores (Table II and Table III). The F value for interaction (Table II) was .07. The F ratio for 2 and 112 degrees of freedom at the .05 level of significance is 3.09. The F value for methods (Table III)

TABLE II

ANALYSIS OF VARIANCE FOR INTERACTION BASED ON ACT SUBSCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	117	3943.83		
ACT Levels and Methods (unadj)	3	3025.49		
Interaction (adj)	2	1.12	.56	.07
Error	112	917.22	8.19	

TABLE III

ANALYSIS OF VARIANCE FOR METHODS BASED ON ACT SUBSCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	117	3943.83		
ACT Levels (unadj)	2	3025.05		
Methods (adj)	1	.44	.44	.05
Error	114	918.34	8.06	

was .05. The F ratio for 1 and 114 degrees of freedom at the .05 level of significance is 3.94. Since the F value for methods was not significant, this indicates that the total experimental and control groups were equivalent with respect to ACT subscores. Since the F value for interaction was not significant, the pairs of groups, High Experimental and High Control, Average Experimental and Average Control, and Low Experimental and Low Control, have the same relationship to methods as the total experimental and control groups. Therefore, the two High, the two Average, and the two Low groups in the two methods are respectively equivalent with respect to ACT subscores.

An analysis of variance was computed on pretest scores (Table IV and Table V) to provide additional substance to the assumption of equivalent groups resulting from random assignment of students to methods. The F value for interaction (Table IV) was .29. The F ratio for 2 and 112 degrees of freedom at the .05 level of significance

TABLE IV

ANALYSIS OF VARIANCE FOR INTERACTION BASED ON PRETEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	117	3808.55		
ACT Levels and Methods (unadj)	3	1783.98		
Interaction (adj)	2	10.55	5.28	.29
Error	112	2014.02	17.98	

TABLE V

ANALYSIS OF VARIANCE FOR METHODS BASED ON PRETEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	117	3808.55		
ACT Levels (unadj)	2	1780.52		
Methods (adj)	1	3.46	3.46	.19
Error	114	2024.57	17.76	

is 3.09. The F value for methods (Table V) was .19. The F ratio for 1 and 114 degrees of freedom at the .05 level of significance is 3.94. Since the F value for methods was not significant, this indicates that the total experimental and control groups were equivalent with respect

to pretest scores. Since the F value for interaction was not significant, the pairs of groups, High Experimental and High Control, Average Experimental and Average Control, and Low Experimental and Low Control, have the same relationship to methods as the total experimental and control groups. Therefore, the two High, the two Average, and the two Low groups in the two methods are respectively equivalent with respect to pretest scores. Hence, the randomization procedure resulted in equivalent groups.

Testing the Hypotheses

An analysis of variance was computed on posttest scores (Table VI and Table VII) to test the four hypotheses. The F value for interaction (Table VI) was .06. The F ratio for 2 and 112 degrees of freedom at the .05 level of significance is 3.09. The F value for

TABLE VI

ANALYSIS OF VARIANCE FOR INTERACTION
BASED ON POSTTEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	117	5424.11		
ACT Levels and Methods (unadj)	3	2458.17		
Interaction (adj)	2	3.21	1.61	.06
Error	112	2962.73	26.45	

TABLE VII

ANALYSIS OF VARIANCE FOR METHODS
BASED ON POSTTEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	117	5424.11		
ACT Levels (unadj)	2	2412.70		
Methods (adj)	1	45.47	45.47	1.75
Error	114	2965.94	26.02	

methods (Table VII) was 1.75. The F ratio for 1 and 114 degrees of freedom at the .05 level of significance is 3.94. The F value for methods was not significant, thus indicating no significant difference between the total experimental and control groups. Therefore, the first hypothesis was accepted. Since the F value for interaction was not significant, the pairs of groups, High Experimental and High Control, Average Experimental and Average Control, and Low Experimental and Low Control, have the same relationship to methods as the total experimental and control groups. Therefore, there were no significant differences between the two High, the two Average, and the two Low groups in the two methods. Consequently, the second, third, and fourth hypotheses were accepted.

Summary

Four specific hypotheses were tested in this study. Each hypotheses was related to the comparative effectiveness of two methods of teaching mathematics. The analysis indicated that neither method is statistically preferred to the other in regards to achievement. Also, the analysis relating mathematical ability and methods showed there were no significant differences between the corresponding ability groups in the two methods.

Additional Considerations

The following points, though not included in the statistical analysis or related to the hypotheses, may be of interest to the reader. Attendance records were kept for both the experimental (tapes) and control (discussion) groups. The records showed that the average tape usage was 88 per cent and the average discussion attendance was 92 per cent. However, these figures may be somewhat misleading inasmuch as three people contributed 36 per cent of the total non-usage of tapes and three people contributed 34 per cent of the total absences from discussion. Attendance records regarding the two weekly large group lectures indicated that the average attendance for those in the taped group was 93 per cent while it was 94 per cent for those in the discussion groups.

An examination of the days and times that tapes were being used revealed the following information. Weekday usage constituted 96 per cent of the total tape usage, with Monday (27 per cent) and Tuesday (24 per cent) being the days most frequently cited. In regards to time of day of tape usage, afternoon usage (noon to 6:00 P.M.) constituted 67 per cent of the total. Morning usage made up 9 per cent and evening usage comprised 24 per cent. One contributing factor to the low percentage of morning usage would be the fact that the 10:00 A.M. hour each day is devoted to chapel services. These figures seem to indicate that the students were quite heterogeneous in regards to days and times selected for listening to the tapes. However, it should be kept in mind that these figures are merely averages over the whole taped group and should not be extended to any individual subjects.

As a matter of personal interest, during the last week of the trimester the writer asked the students in each group to complete an opinionated questionaire regarding the course. Students were asked not to use their names on the survey. Following are the results of a few responses. In the taped group, 84 per cent felt that their understanding of mathematics had increased; 86 per cent of the discussion group responded likewise. Forty-four per cent of both the taped and discussion groups indicated a positive change in attitude toward mathematics as a result of this course; 9 per cent of the taped group and 8 per cent of the discussion group indicated their attitude toward mathematics had been affected unfavorably as a result of this course.

In regards to a question as to whether the course had provided enough individual attention, 36 per cent of the taped group felt that it had, 42 per cent said sometimes, and 22 per cent said no; 47 per cent of the discussion group felt that there had been enough individual attention, 29 per cent said sometimes, and 24 per cent said no. Thirty-five per cent of those in the taped group felt that the use of tapes as an instructional device in college courses was being overemphasized; 65 per cent did not feel this way; 69 per cent were satisfied at having been selected to attend taped sessions rather than discussion sessions. Fifty-nine per cent of those in the discussion group felt that the use of tapes was being overemphasized; 41 per cent did not feel this way; 90 per cent were satisfied at having been selected to attend discussion sessions rather than taped sessions.

As a consequence of having had this course, would you select another math course that used these same methods of instruction? In response to this question, 31 per cent of the taped group said they would, 39 per cent were undecided, and 30 per cent said no. In responding to the same question, 37 per cent of the discussion group said they would select another math course using discussion methods, 29 per cent were undecided, and 33 per cent said no.

Mathematics 113 was also offered during the spring trimester at Oklahoma Christian College. During this time the entire class was taught by using tapes and hence might be considered as a replication of the experimental method. Out of 119 students originally enrolled in this class, ACT mathematics subscores, pretest scores, and posttest scores were available for 96 students. A tabular summary of the raw data appears in Appendix D (pp. 77-79). Using the same range of ACT subscores to define ability levels as previously, 25 students were classified as "High Ability," 42 as "Average Ability," and 29 as "Low Ability." Analyses of variance were computed comparing this group with the experimental and control groups of the fall trimester in regards to ACT subscores, pretest scores, and posttest scores. These analyses appear in Appendix E (pp. 80-83). The $\underline{F}$ values obtained from these analyses are summarized in Table VIII.

TABLE VIII

SUMMARY OF F VALUES FOR INTERACTION AND METHODS BASED ON ACT SUBSCORES, PRETEST SCORES, AND POSTTEST SCORES IN COMPARING FALL EXPERIMENTAL, FALL CONTROL, AND SPRING TRIMESTER GROUPS

	ACT	Pretest	Posttest
Interaction	.33	.19	.25
Methods	.73	.12	1.39

The _F_ ratio for 4 and 205 degrees of freedom at the .05 level of significance is 2.26 while the _F_ ratio for 2 and 209 degrees of freedom is 3.04. Consequently, the ACT _F_ values for interaction and methods of .33 and .73 and the pretest _F_ values for interaction and methods of .19 and .12 indicate that the three groups were respectively equivalent to one another. The posttest _F_ values for interaction and methods of .25 and 1.39 indicate that there were no significant differences in the level of achievement among any of the three groups.

CHAPTER V

SUMMARY, FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

The purposes of this chapter are to summarize the study, to present the findings of the study, to draw conclusions based on the findings, and to make recommendations for further research pertinent to the use of audio-tapes as a means of mathematics instruction.

Summary

The purpose of this research was to investigate the relative effectiveness of two methods of teaching mathematics to students in a large sized undergraduate mathematics class. Further attention was focused on assessing the achievement of students who possessed different ability levels.

One method of instruction involved the students attending large group lectures twice a week and a smaller discussion session once a week. These students served as the control group. The other method of instruction also involved two large group lecture meetings per week, but instead of attending a discussion session, these students used an automated teaching device consisting of an audio-tape synchronized with a student response sheet. A series of twelve audio-tapes and student response sheets were prepared for this study. One tape per week was presented for a period of twelve weeks. Discussion sessions of the same content were presented to the classes in the control group.

The sample for this study consisted of 118 students enrolled in the Mathematics 113 class during the 1969 fall trimester at Oklahoma Christian College, Oklahoma City, Oklahoma. The students were randomly assigned to either the control group or the experimental group. A pretest was given to assure the equivalence of groups.

The main instruments for establishing a base for evaluation were three commercially made tests. Two of these were Cooperative Mathematics Tests of the Educational Testing Service, Princeton, New Jersey. Form A of _Structure of the Number System_ was used as a pretest while Form B was used as a posttest. The American College Test (ACT) mathematics subscores were used to classify the students into High, Average, and Low ability groups.

The statistical analysis consisted of analysis of variance based on the posttest scores. The research design was a two-way factorial design with a disproportionate number of observations in the subclasses. _F_ ratios were used to test the four hypotheses at the five per cent level of significance.

Findings

On the basis of the statistical analysis the following findings concerning the hypotheses can be stated:

1. Hypothesis 1, that there is no significant difference in the level of achievement of the students in the experimental group as compared with the level of achievement of the students in the control group, at the five per cent level of significance.

2. Hypothesis 2, that there is no significant difference in the level of achievement of "High Ability" students in the experimental

group as compared with the level of achievement of "High Ability" students in the control group at the five per cent level of significance.

3. Hypothesis 3, that there is no significant difference in the level of achievement of "Average Ability" students in the experimental group as compared with the level of achievement of "Average Ability" students in the control group at the five per cent level of significance.

4. Hypothesis 4, that there is no significant difference in the level of achievement of "Low Ability" students in the experimental group as compared with the level of achievement of "Low Ability" students in the control group at the five per cent level of significance.

Conclusions

Within the assumptions and limitations imposed on this experiment, the collected data seem to warrant the following conclusions:

1. The audio-tape and discussion methods were equally effective with regards to mathematical achievement.

2. With reference to specific ability levels, the audio-tape and discussion methods were equally effective with regards to mathematical achievement.

3. Audio-taped instruction can be used to assist in the teaching of mathematics with satisfactory results in mathematics classes with characteristics similar to those described in this study. Consequently, an automated instrument such as developed in this study can provide for more effective and efficient staff utilization.

4. Both methods of instruction allowed their respective groups to raise the level of their mathematical achievement and understanding. This was indicated by examining group pretest and posttest mean scores. However, the reader is cautioned not to extend this result to individual subjects as each method of instruction contained subjects that had either zero gainscores or negative gainscores.

Recommendations

As a result of this study, the writer makes the following recommendations for further research:

1. To repeat this experiment in other large sized undergraduate mathematics courses.

2. To repeat this experiment on different campuses and different educational environments.

3. Inasmuch as this study was concerned only with the relative merits of two methods in producing mathematics achievement, it is recommended that studies be undertaken to determine the effects on attitude of pupils when taped instruction is utilized in mathematics courses.

4. Research should be conducted to determine significant factors that influence students' attitudes toward different teaching approaches in mathematics.

5. Studies involving other combinations of lecture, discussion, and tapes. For example, a weekly sequence of one lecture, one discussion, and one tape might be considered.

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APPENDIX A

TEXTBOOKS USED IN VALIDATING CONTENT OF TAPES

Textbooks Used in Validating Content of Tapes

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APPENDIX B

DIALOGUE, STUDENT RESPONSE SHEET, AND STUDENT HAND-IN PAGE FOR TAPE NUMBER 3: WHOLE NUMBERS

Tape Number 3 - Dialogue

Whole Numbers

This is tape number 3 of Mathematics 113. Before listening to this tape you should have completed the assigned sections in your textbook for Chapter 2. The concepts of number and set are very closely related as has been pointed out in the previous tapes. We shall begin our discussion today with the idea of a number. Pause for just a moment and mentally try to think how you would answer the question, "What is a number?" (Pause) You have probably found this to be a very difficult concept to explain. Our text explains the concept of number as a property associated with a given set and all sets equivalent to the given set. For example, consider the set of tires for an automobile and the set consisting of your arms and legs. These two sets are equivalent sets and they have something in common. What is this common something? Certainly it is not the elements since in one set the elements are tires, and in the other, arms or legs. The common something we are talking about here is the idea of "four." That is, "four" is a property associated with each of the sets. Any set which is equivalent to the given sets also has this concept of "four" associated with it. Thus, we see that the idea of "four" or the idea of "fourness" is not an easy concept to explain; it is just an idea which cannot be defined in specific terms and only exists in our mind.

Item Number 1: A whole number is an abstract idea. (Pause) Even though we cannot define a number in specific terms, most of us have mastered the concept and have a feeling for what numbers are. Since numbers are properties associated with sets, and since sets may have no elements, one element, two elements, and so on, then we define the set of whole numbers as the numbers zero, one, two, three, four, and so on. This set is sometimes denoted by the letter W. We need to notice that since sets cannot have a negative amount of elments, then there are no negative numbers in the set of whole numbers. That is, the whole numbers are always zero or positive numbers.

Item Number 2: Counting is the process of determining the number property of a given set. (Pause) Thus, if I asked for the amount of dollars in your billfold, you would know that I meant the number property of the set of dollars in your billfold. In some way you could arrive at this number and tell me. The process or way that you arrived at this number is called counting.

Item Number 3a: In the blank beside Item 3a, write down the number "four." (Pause) Now, if you filled in the blank with the symbol for the idea "four," you are incorrect. Why, you ask? We have just explained that a number is an abstract idea which cannot be expressed in specific terms. Thus, you cannot write down the idea "four." You can, however, write down a symbol for the idea "four," or a number name for the idea "four," and this is probably what you did.

Item Number 3b: Symbols for numbers are called numerals. (Pause) We have ten different symbols in the Hindu-Arabic system to represent numbers. In the blank in Item Number 3c, write down the remaining number symbols in the Hindu-Arabic system. (Pause) The number symbols in the Hindu-Arabic system are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. We have many more numbers than we have symbols for numbers. Yet, in the Hindu-Arabic system, we have a unique representation for each whole number. For example, consider the number forty-six. By using a combination of the numerals 4 and 6, we are able to write down a symbol for forty-six without having to invent new symbols. This is accomplished by using a device known as place value.

In Item Number 4a, fill in the blank with the numeral for the number twenty-three. (Pause) Notice that I said numeral for twenty-three and not just the number twenty-three.

Item Number 4b: 23 (equals/does not equal) 32. Circle the correct response. (Pause) You should have circled the phrase "does not equal." That is, 23 does not equal 32. Thus, we see that whenever we interchange places with the 2 and 3, we get a different numeral.

Item Number 4c: The numeral 23 means 2 tens and 3 units, whereas the numeral 32 means 3 tens and 2 units. Therefore, when we interchange places, we are interchanging the number of tens and units we have, which, in most cases, would cause us to have a different number. This is what we mean when we say that our number system has place value. That is, where we write the symbol determines the value of the symbol.

Item Number 4d gives us an example of an old Egyptian system which does not use place value. Let "cap" be a symbol for ten and "one" be a symbol for one. Then twenty-three equals "cap-cap-one-one-one" or "cap-one-one-one-cap" or "cap-one-one-cap-one" and so forth. Thus, the symbols can be written in any order and still stand for the same number. There are several disadvantages to this type of system, such as repeating symbols over and over again, and we are fortunate that our number system has place value. Place value in the Hindu-Arabic system simply means that the place of the numeral tells us how many units, tens, hundreds, thousands, ten thousands, etc. we have. Consider another example. In the blank beside Item 4e, write the numeral 3456. (Pause) The symbols mean that we have 6 units, 5 tens, 4 hundreds, and 3 thousands.

Let us continue now with some additional properties of the whole numbers.

Item Number 5a: An operation * is said to be closed (Pause) in a set S if for every two elements of S, say a and b, a * b is an element of S. (Pause) In other words, whenever we take any two elements in S and perform the operation on them, we get back an element of S. Consider the following examples of what we mean for a set to be closed under a given operation.

Item Number 5b: The set W of whole numbers is closed under the operation +. That is, the sum of two whole numbers is a whole number. For specific examples, notice Item Number 5c: $11 + 15 = 26$, and $72,512 + 765 = 73,277$.

Item Number 5d: Thus, if a and b are whole numbers, then a + b is a whole number. We sometimes say that the operation + has the closure property on the set W. That is, if $x \in W$, $y \in W$, then $x + y$ ($\in$, $\notin$) W. Circle the correct response. (Pause) The correct answer is that x + y is an element of W.

Item Number 5e: The operation $\cdot$ (does/does not) have the closure property on the set W of whole numbers. That is, whenever we multiply any two whole numbers, do we get back another whole number? Circle the correct response. (Pause) The operation $\cdot$ does have the closure property on the set W of whole numbers. For specific examples, notice Item Number 5f: $3 \cdot 5 = 15$ and $176 \cdot 432 = 76,032$.

For an example of an operation which is not closed on the set W of whole numbers, consider the operation -. Consider the following examples in Item Number 5g: $9 - 3 = 6$ since $9 = 3 + 6$; $3467 - 67 = 3400$ since $3467 = 67 + 3400$; $25 - 52 = ?$ since there does not exist a whole number b such that $25 = 52 + b$; $3 - 9 = ?$ since there does not exist a whole number a such that $3 = 9 + a$. Hence, we see that while we can subtract two whole numbers and get back another whole number in some cases, this cannot be done for all whole numbers. For example, $3 - 9$ does not exist in the set of whole numbers. Therefore, in Item Number 5h, the operation - (does/does not) have the closure property on the set W. Circle the correct response. (Pause) Of course, the answer is that the operation - does not have the closure property on the set W. Notice how important the set W becomes when we talk about the closure property. From algebra you may have been taught that the answer to $3 - 9$ is -6. But -6 is not a whole number from our definition of what is meant by whole numbers.

Let us now recall two properties about sets which also apply to whole numbers.

Item Number 6a: The statement $A \cup B = B \cup A$ is an example of the commutative property of union of sets. (Pause) That is, the order in which we union the two sets, A and B, does not make any difference in the end result.

Item Number 6b: The statement $A \cup (B \cup C) = (A \cup B) \cup C$ is an example of the associative property of the union of sets. (Pause) That is, it does not make any difference how we group the sets in taking their union.

Now let us consider the whole numbers in regards to the commutative and associative properties of addition. To say that the whole numbers are commutative with respect to the operation of addition would mean that the order in which we add the whole numbers does not make any difference as far as the sum is concerned. For specific examples, notice that $3 + 2 = 2 + 3$ and $7 + 12 = 12 + 7$. The same sum is obtained regardless of the order in which we add the numbers.

Item Number 7a: The whole numbers (are/are not) commutative with respect to the operation +. Circle the correct response. (Pause) The whole numbers are commutative with respect to the operation +. Thus, if a and b are whole numbers, then $a + b = b + a$. Fill in the blank in item number 7a with the statement $a + b = b + a$. (Pause) This is the commutative property of addition of whole numbers.

Item Number 7b: The whole numbers (are/are not) commutative with respect to the operation $\cdot$. Circle the correct response. (Pause) The whole numbers are commutative with respect to the operation $\cdot$. That is, the same results are obtained regardless of the order in which the multiplication is carried out. Fill in the blank of item number 7b with the statement $a \cdot b = b \cdot a$. This is the commutative property of multiplication of whole numbers.

There are two special whole numbers which have the property of being identities for the operations of + and $\cdot$. We define an identity element in Item Number 8a: The element b is said to be an identity for the operation * if $b * a = a * b = a$ for all elements a. (Pause) That is, b is said to be an identity if whenever we perform the operation on any element of our set, say a, we always get a back again. Consider the set W of whole numbers.

Item Number 8b: 0 is the identity for $+$ since $0 + a = a + 0 = a$ for all whole numbers a. (Pause) Therefore, whenever we add zero to a, we always get a back again. Now, in the blank beside Item Number 8c, write down the identity for multiplication of whole numbers. (Pause) You should have written down the whole number such that for any number a, this number times a will give a back again. This number is 1.

Item Number 8d: Therefore, 1 is the identity for the operation $\cdot$ in the set W. (Pause) Notice that when we change our operation, this changes the identity element for the operation. Consequently, 0 is not an identity for both the operations $+$ and $\cdot$.

There is one additional property of whole numbers which we need to discuss. This is the distributive property.

Fill in the reasons in the blanks of item number 9 as we go from step to step.

Item Number 9a: $3\cdot(4 + 5) = 3\cdot9$ because addition of whole numbers is closed. Thus, in the first blank of item number 9a, you should write "closure under +." (Pause) Now, $3\cdot9 = 27$ because of closure under $\cdot$. (Pause) Therefore, 3 times 4 plus 5 is 27.

Item Number 9b: Next consider $3\cdot4 + 3\cdot5$. This is equal to $12 + 15$ because of closure under $\cdot$. (Pause) But, $12 + 15 = 27$ because of closure under $+$. (Pause) Therefore, in Item Number 9c, we see that $3\cdot(4 + 5) = 3\cdot4 + 3\cdot5$. This illustrates the distributive property of multiplication over addition. This means that we can add 4 and 5 and then multiply by 3, or we can multiply 3 by 4 and 3 by 5 and then add to achieve the same result.

Item Number 9d: The operation $\cdot$ is said to distribute over the operation $+$ for whole numbers if $a\cdot(b + c) = a\cdot b + a\cdot c$. In this case the whole numbers are said to have the distributive property. (Pause) In Item Number 10 we now sum up the properties of the whole numbers which we have discussed. The numbered answers on the right are not in their proper order to go with the lettered statements on the left. In the blank beside each statement, you should place the correct numeral which tells the name of the property represented by the statement. You have three minutes to do this. (Pause)

The correct answers are as follows. Item 10a. $a + b$ is a whole number. The answer is number 7, closure under +. Item 10b. $a \cdot b$ is a whole number. The answer is number 9, closure under $\cdot$. Item 10c. $a + b = b + a$. The answer is number 1, commutative under +. Item 10d. $a \cdot b = b \cdot a$. The answer is number 2, commutative under $\cdot$. Item 10e. $a + (b + c) = (a + b) + c$. The answer is number 4, associative under +. Item 10f: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. The answer is number 5, associative under $\cdot$. Item 10g. $a \cdot (b + c) = a \cdot b + a \cdot c$. The answer is number 3, distributive property. Item 10h. $a + 0 = a$. The answer is number 8, identity for +. Item 10i. $a \cdot 1 = a$. The answer is number 6, identity for $\cdot$.

Remember, this list is a handy reference for the basic properties of the whole numbers.

This concludes the formal portion of tape number 3. You should now detach the last page of the student response sheet and begin answering the questions as they are presented to you. This is to be handed in at room 303 of the Learning Center. You may keep the remaining portion of the student response sheet. (Pause)

The first six questions are true and false questions. Each question will be repeated once, and you should write "T" or "F" in the appropriate place. Question 1: The symbol for zero is a whole number. True or false. (Pause, Repeat, Pause)

Question 2: In the numeral 222, each of the 2's has the same meaning. True or false. (Pause, Repeat, Pause)

Question 3: The whole numbers are closed under the operation of division. That is, when we divide any two whole numbers, we always get another whole number back again. True or false. (Pause, Repeat, Pause)

Question 4: The identity element for multiplication of whole numbers is zero. True or false. (Pause, Repeat, Pause)

Question 5: The whole numbers are associative with respect to the operation of multiplication. True or false. (Pause, Repeat, Pause)

Question 6: The statements $0/5 = 0$, $5/0 = 0$, and $0/0 = 0$ are all correct statements. True or false. (Pause, Repeat, Pause)

For questions 7-10, write the letter of the property justifying the given statements from the list of possible answers.

After you have completed this, turn in the student hand-in sheet to room 303.

This concludes tape number 3.

Tape Number 3 - Student Response Sheet

1. A whole number is an abstract ________________.

   The whole numbers are the set $W = \{0,1,2,3,4,5,\ldots\}$

2. Counting is the process of determining the ________________ property of a given set.

3. (a) ______________

   (b) Symbols for numbers are called ________________.

   (c) The number symbols in the Hindu-Arabic system are 0,1,________________________.

4. (a) ______________

   (b) 23 (equals/does not equal) 32.

   (c) The numeral 23 means 2 tens and 3 units, whereas the numeral 32 means _____ tens and _____ units.

   (d) Let ∩ be a symbol for 10 and 1 be a symbol for 1. Then 23 = ∩∩111 = ∩111∩ = ∩11∩1, etc.

   (e) ______________

5. (a) An operation * is said to be _________________ in a set S if for every two elements of S, say a and b, a * b is an __________________ of S.

   (b) The set W of whole numbers is closed under the operation +.

   (c) 11 + 15 = 26,   72,512 + 765 = 73,277

   (d) Thus, if a and b are whole numbers, then a + b is a whole number. We sometimes say that the operation + has the closure property on the set W. That is, if $x \in W$, $y \in W$, then $x + y$ ($\in$, $\notin$) W.

(e) The operation • (does/does not) have the closure property on the set W of whole numbers.

(f) 3·5 = 15,   176·432 = 76,032

(g) 9 - 3 = ________ since 9 = 3 + 6.

3467 - 67 = ________ since 3467 = 67 + 3400.

25 - 52 = ? since there does not exist a whole number b such that 25 = 52 + b.

3 - 9 = ? since there does not exist a whole number a such that 3 = 9 + a.

(h) The operation - (does/does not) have the closure property on the set W.

6. (a) A ∪ B = B ∪ A is an example of the ____________________ property of union of sets.

(b) A ∪ (B ∪ C) = (A ∪ B) ∪ C is an example of the ________________________ property of union of sets.

7. (a) The whole numbers (are/are not) commutative with respect to the operation +. _________________________

(b) The whole numbers (are/are not) commutative with respect to the operation •. _________________________

8. (a) The element b is said to be an identity for the operation * if b * a = a * b = a for __________ elements a.

(b) 0 is the __________________ for + since 0 + a = a + 0 = a for all whole numbers a.

(c) _____________

(d) Therefore, ________ is the identity for the operation • in the set W.

9. (a) $3\cdot(4 + 5) = 3\cdot 9$ because ______________________________

$= 27$ because ______________________________

(b) $3\cdot 4 + 3\cdot 5 = 12 + 15$ because ______________________________

$= 27$ because ______________________________

(c) Therefore, $3\cdot(4 + 5) = 3\cdot 4 + 3\cdot 5$

(d) The operation $\cdot$ is said to distribute over the operation + for whole numbers if $a\cdot(b + c) = a\cdot b + a\cdot c$. In this case the whole numbers are said to have the __________________ property.

10. For any whole numbers a,b,c:

____(a) $a + b$ is a whole number

____(b) $a\cdot b$ is a whole number

____(c) $a + b = b + a$

____(d) $a\cdot b = b\cdot a$

____(e) $a + (b + c) = (a + b) + c$

____(f) $a\cdot(b\cdot c) = (a\cdot b)\cdot c$

____(g) $a\cdot(b + c) = a\cdot b + a\cdot c$

____(h) $a + 0 = a$

____(i) $a\cdot 1 = a$

Possible Answers

1. Commutative under +
2. Commutative under $\cdot$
3. Distributive property
4. Associative under +
5. Associative under $\cdot$
6. Identity for $\cdot$
7. Closure under +
8. Identity for +
9. Closure under $\cdot$

Tape Number 3

Student Hand-In Page

Name ____________________

Date ____________________

Time ____________________

After filling out the information requested on this page, turn in to the control room of the Learning Center. This is Room 303.

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______ 0/5 = 0, 5/0 = 0, 0/0 = 0

7. ______ 3 + (5 + 6) = (3 + 5) + 6

8. ______ 5 + 0 = 0 + 5 = 5

9. ______ 2·(3 + 5) = 2·3 + 2·5

10. ______ 3·1 = 1·3 = 3

Possible Answers for Numbers 7-10:

(a) Commutative property for addition
(b) Commutative property for multiplication
(c) Associative property for addition
(d) Associative property for multiplication
(e) Distributive property
(f) Identity element for addition
(g) Identity element for multiplication

APPENDIX C

INDIVIDUAL SCORES OF SUBJECTS PARTICIPATING IN THE STUDY

Student	Group	ACT Subscore	Pretest	Posttest
1	E	22	22	28
2	E	24	31	35
3	E	28	29	38
4	E	22	22	25
5	E	25	26	34
6	E	23	25	23
7	E	30	26	21
8	E	22	15	25
9	E	23	21	30
10	E	23	25	33
11	E	22	24	36
12	E	25	23	32
13	E	32	28	35
14	E	22	23	34
15	E	28	23	36
16	C	23	23	32
17	C	24	20	28
18	C	22	29	23
19	C	27	30	36
20	C	26	32	39
21	C	23	24	28
22	C	25	24	33
23	C	22	17	27
24	C	22	14	23
25	C	24	35	35
26	C	31	25	39
27	C	24	28	31
28	C	22	24	30
29	C	32	35	39
30	C	22	22	29
31	C	28	22	36
32	C	28	23	38
33	E	20	17	21
34	E	19	22	21
35	E	20	18	25
36	E	18	21	24
37	E	21	23	24
38	E	20	15	29
39	E	16	12	10
40	E	16	16	18
41	E	19	15	27
42	E	17	12	17
43	E	19	18	23
44	E	17	22	31
45	E	17	20	25
46	E	21	22	30
47	E	19	25	27
48	E	19	15	19
49	E	18	19	27
50	E	18	20	28

Student	Group	ACT Subscore	Pretest	Posttest
51	E	19	20	30
52	E	21	23	24
53	E	16	14	26
54	E	17	15	31
55	E	18	22	30
56	E	21	18	26
57	C	19	24	29
58	C	18	18	31
59	C	21	13	22
60	C	19	23	31
61	C	21	20	27
62	C	21	15	21
63	C	17	9	15
64	C	20	25	32
65	C	17	18	27
66	C	18	21	30
67	C	18	17	26
68	C	19	18	27
69	C	21	23	27
70	C	16	17	22
71	C	16	17	23
72	C	18	19	25
73	C	16	17	17
74	C	20	13	23
75	C	21	17	29
76	C	21	16	31
77	C	20	21	30
78	C	17	13	26
79	C	18	21	34
80	C	20	20	27
81	E	10	9	12
82	E	15	10	15
83	E	14	15	28
84	E	10	7	8
85	E	12	15	20
86	E	10	21	23
87	E	15	20	28
88	E	8	15	22
89	E	4	14	16
90	E	12	9	13
91	E	15	21	22
92	E	15	19	26
93	E	14	17	19
94	E	14	17	22
95	E	15	13	21
96	E	13	19	25
97	E	14	16	22
98	E	6	12	18
99	E	15	13	19
100	E	5	6	12

Student	Group	ACT Subscore	Pretest	Posttest
101	E	12	12	17
102	C	15	9	11
103	C	12	12	17
104	C	14	11	18
105	C	12	13	15
106	C	1	21	23
107	C	15	19	30
108	C	12	21	21
109	C	10	13	22
110	C	15	15	24
111	C	13	22	26
112	C	15	14	20
113	C	13	14	27
114	C	12	14	16
115	C	12	14	20
116	C	9	13	18
117	C	13	18	24
118	C	5	13	13

APPENDIX D

INDIVIDUAL SCORES OF STUDENTS ENROLLED IN MATHEMATICS 113 DURING THE SPRING TRIMESTER

Student	ACT Subscore	Pretest	Posttest
1	31	30	36
2	23	25	32
3	25	22	33
4	22	28	30
5	27	24	33
6	22	26	30
7	31	28	35
8	26	29	34
9	22	20	27
10	23	21	26
11	25	20	34
12	22	21	25
13	24	17	29
14	24	28	30
15	22	31	32
16	22	28	31
17	24	22	26
18	23	20	22
19	22	23	37
20	22	29	37
21	24	24	33
22	27	25	37
23	26	29	33
24	25	26	33
25	25	26	34
26	20	21	27
27	20	16	26
28	21	25	34
29	16	17	26
30	19	16	24
31	18	16	32
32	19	29	30
33	18	18	21
34	21	13	21
35	20	31	35
36	18	22	28
37	20	6	17
38	19	20	28
39	21	19	19
40	17	22	26
41	16	11	14
42	21	12	23
43	18	17	26
44	16	14	24
45	18	18	19
46	18	13	13
47	19	28	26
48	16	19	15
49	18	24	25
50	20	17	24

Student	ACT Subscore	Pretest	Posttest
51	21	18	16
52	19	16	20
53	18	18	18
54	16	16	26
55	20	17	21
56	18	21	28
57	18	14	25
58	21	19	27
59	17	18	18
60	20	16	28
61	18	18	23
62	19	19	28
63	18	20	29
64	19	18	26
65	18	15	28
66	19	21	27
67	19	16	24
68	13	10	18
69	10	18	21
70	9	18	27
71	12	16	15
72	12	22	30
73	12	15	22
74	12	9	11
75	10	11	18
76	6	14	22
77	10	6	13
78	11	8	16
79	12	14	22
80	15	16	21
81	15	16	19
82	1	12	25
83	14	16	28
84	12	14	17
85	5	11	15
86	14	16	23
87	13	22	16
88	8	12	17
89	10	16	17
90	1	11	20
91	11	13	24
92	11	14	17
93	15	11	20
94	8	18	17
95	15	15	19
96	15	18	21

APPENDIX E

ANALYSES OF VARIANCE TABLES COMPARING FALL EXPERIMENTAL, FALL CONTROL, AND SPRING TRIMESTER GROUPS

TABLE IX

ANALYSIS OF VARIANCE FOR INTERACTION BASED ON ACT SUBSCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	213	7147.83		
ACT Levels and Methods (unadj)	4	5570.51		
Interaction (adj)	4	10.05	2.51	.33
Error	205	1567.27	7.65	

TABLE X

ANALYSIS OF VARIANCE FOR METHODS BASED ON ACT SUBSCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	213	7147.83		
ACT Levels (unadj)	2	5559.53		
Methods (adj)	2	10.98	5.49	.73
Error	209	1577.32	7.55	

TABLE XI

ANALYSIS OF VARIANCE FOR INTERACTION BASED ON PRETEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	213	7002.36		
ACT Levels and Methods (unadj)	4	3332.75		
Interaction (adj)	4	13.72	3.43	.19
Error	205	3655.89	17.83	

TABLE XII

ANALYSIS OF VARIANCE FOR METHODS BASED ON PRETEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	213	7002.36		
ACT Levels (unadj)	2	3328.63		
Methods (adj)	2	4.12	2.06	.12
Error	209	3669.61	17.56	

TABLE XIII

ANALYSIS OF VARIANCE FOR INTERACTION BASED ON POSTTEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	213	9361.08		
ACT Levels and Methods (unadj)	4	4372.37		
Interaction (adj)	4	23.78	5.95	.25
Error	205	4964.93	24.22	

TABLE XIV

ANALYSIS OF VARIANCE FOR METHODS BASED ON POSTTEST SCORES

Source of Variation	df	Sum of Squares	Mean Squares	F
Total	213	9361.08		
ACT Levels (unadj)	2	4306.12		
Methods (adj)	2	66.25	33.13	1.39
Error	209	4988.71	23.87	

VITA

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Eddie Joe Brown

Candidate for the Degree of

Doctor of Education

Thesis: AN INVESTIGATION OF THE RELATIVE EFFECTIVENESS OF TWO METHODS OF TEACHING A LARGE SIZED UNDERGRADUATE MATHEMATICS CLASS

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