A QUANTITATIVE INVESTIGATION OF

POLICE PATROL FORCE

DISTRIBUTION

By

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PREFACE

The problems of the police forces of cities are many and they grow with each passing year. A very important function of any police force is that of patrolling the city for the purposes of preventing and detecting crime. Means of distribution of the patrol force in order to effectively accomplish these goals must be developed by the police force. The objective here is to investigate examples of patrol force distribution. Even though the examples may not fit particular cases, their generalization will allow application to different situations. Throughout an emphasis upon quantitative material is obvious. This is not to overlook the importance of the highly subjective process which must be present in the police patrol distribution; however, limitation of scope is necessary to maintain a reasonable length.

Upon attaining this level of education, there are many thanks to be given. I would like to express my gratefulness to all those persons who have helped me academically and personally.

Particular gratitude is given to my wife, Sandra, for the years she has worked and persevered that I might complete this thesis and accompanying degree. Also, I wish to thank my parents for the large amount they have contributed to my developing view of life, the world and people, and Mr. and Mrs. Oscar Wille for their encouragement and patience.

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NOMENCLATURE

i	index indicating a district of the division ($i = 1, 2, \ldots, n$)
j	index indicating a precinct of the division ($j = 1, 2, \ldots, m$)
a_i	time allocated to district i to perform patrolling and other duties
$\bar{a}_m$	average allocation time for a precinct in a division containing m precincts
r_i	response distance for a car in district i
$\bar{r}_j$	response distance for precinct j
R_j	maximum response distance allowable for precinct j
x_{ij}	$= \begin{cases} 0 & \text{district } i \text{ not assigned to precinct } j \\ 1 & \text{district } i \text{ is assigned to precinct } j \end{cases}$
$f(r_i)$	probability density function (pdf) of response distance of district i
$f(\bar{r}_j)$	pdf of response distance for precinct j
μ_{r_i}	mean response distance for district i
$\sigma^2_{r_i}$	variance of response distance for district i
$\bar{\mu}_{r_j}$	mean response distance for precinct j
$\bar{\sigma}^2_{r_j}$	variance of response distance for precinct j
α_j	confidence level that response distance of precinct j will not exceed $R_j (0 < \alpha_j \leq 1)$
b	total number of cars available for the precinct

f total number of men available for assignment

s_i number of men per car in district i

$$x_i = \begin{cases} 0 & \text{if car is not assigned to district } i \\ 1 & \text{if a car is assigned to district } i \end{cases}$$

CHAPTER I

INTRODUCTION

This initial chapter treats three topics, the first being a discussion of the fundamentals necessary for the understanding of the material covered in Chapters II and III. Next, an overview of literature relating to the area of research is presented. This section is not meant to be complete by any means; however, the reader should find in it necessary and adequate references so that he is able to perform his own investigation. Restriction in the literature review is due to the fact that this research is involved with only a small part of a quite large topic, the police system. Finally the objectives of this research are listed so that the extent of investigation is understood.

The Patrol Division

In the past few decades the duties of the police forces of this country have expanded greatly. Once the primary reason for the existence of police forces was the stopping of crime through apprehension and retention of suspected and judged criminals. However, the important role of preventing crime has caused the appearance of many auxiliary divisions in the police force, not directly related to divisions having administrative or operational duties. Thus, the police are now supporting sections such as youth divisions, drug education sections, and many other service groups. It is seen that the role of preventing offenses is being emphasized by those who favor the added function of social worker for the police departments.[1] However, this in no way detracts from the importance of the well established divisions of detective, patrol, administrative, traffic, etc. Among these many divisions, patrol is of most importance due to its indispensable function of patrolling the city, providing assistance when needed and preventing unlawful acts. The broadened duties of the police force have placed larger commitments upon the patrolman for he must be able to handle new situations every day. For example, narcotics is a problem that cannot be exclusively handled by the narcotics squad, because drug usage is in the street and the patrol officer must recognize its symptoms and be able to cope with the situation.[2]

It is the patrol division that is singled out for the purposes of this research. Attempts have been made to specialize the duties of the patrol force by creating separate divisions to perform certain types of jobs. One such attempt is the development of a separate traffic division. This approach allows the traffic division to handle all problems directly related to traffic, such as direction through intersections, giving of parking and other tickets, and the like. Patrol cars maintained in large numbers by these specialized functional units is not recommended. This duplication of effort may cause confusion and friction between divisions in terms of responsibilities.[3,4] Thus, a patrol division which covers the city is still the most advised. Such a force has three primary duties:[5]

1) called for services--this is the answering of calls placed by a citizen for aid, calls reported by burglary attempt devises, or calls for help from other police officers;

2) inspectional services--this is the checking of doors, windows, and stores to assure that no unlawful acts have been or are in the process of being committed; and

3) routine preventive patrol--by coverage of an area of the city, the patrol force is able to discourage crime and, in many instances, halt crime while it is happening.

It is quite noticeable in the above discussion that the only reason for existence of the patrol division, as well as all other divisions of the police department, is the distribution of a service to the public. Since this service is of a nature that the public may not always appreciate, such as the prevention of traffic violations, this service must be 'sold' by somewhat peculiar means. Thus the police force takes on the additional responsibility of having to convince people of its worth and necessity.

Make-Up of the Patrol Force

In order to accomplish the duties outlined above, the patrol division utilizes many types of patrol methods. The most common is the one-man patrol car. This method has become the most wide spread because it allows the patrolman to cover a much larger area than other methods, answer calls in a smaller amount of time, and it affords the man protection from inclement weather. Other types of patrol vehicles are the horse, bicycle, sole motorcycle, and three-wheeled motorcycle. Even though the automobile creates a much larger expense in terms of maintenance, its advantages far outweigh those of any other patrolling method.

The patrol force is by far the largest division of a police force. The patrolman makes contact with the law-abiding citizen as well as the potential and proven criminal. This police officer must be well trained to allow him to play the correct role at the correct time. Thus, much schooling and money is involved in the making of a good patrolman.

Duties and Operation of the Patrol Force

The services of the patrol force were mentioned earlier in this section. In order to perform these duties the force must be organized in an efficient manner. For this reason, the assignment of several patrol cars to a sergeant is necessary. Each of the patrol cars will be assigned certain duties in terms of preventive and inspectional patrolling. When a call for service is necessary one of the patrolmen in the vicinity of the needed service is dispatched to the scene by a dispatcher at the central police station. Since it is important that the dispatcher know which cars are available for assignment and which ones are not, each patrol car is equipped with a two-way radio. Once contacted by the dispatcher, an available car leaves his patrolling duties and proceeds to the scene of needed service. The time from contact of the patrol car to the time at which it reaches the scene is usually referred to as response time, while the distance he must travel to reach the scene is the response (travel) distance. Extensive usage of these terms is made in this research. Once the officer has completed his duties at the scene of a crime, accident, or other disturbance he reports to the dispatcher that he is proceeding on patrol and is again available for dispatching.

It is entirely likely that all cars in a certain defined area will be busy when a call for service is originated. This being the case, a queue is formed of the calls requiring service. During certain times of the day when the work load is high this queue may become quite long, thus indicating the need for additional patrol capability in the area.

Besides these duties, if a separate traffic division is not present, the patrol force will most probably be responsible for distributing traffic violation tickets, aiding stranded cars and their drivers, directing traffic at disabled lights and many other duties which tend to decrease the number of patrol cars available for assignment to calls for services.

Division of the City for Distribution

To facilitate the accomplishment of the objectives of the patrol force, a city is usually divided into small geographical areas. Several different names are used by different cities for the same level of division of the city. For example, New York City is divided into boroughs, each of which are divided into divisions. The divisions contain precincts which are composed of sectors. For sake of clarity, this paper will use the nomenclature depicted in Figure 1. It can be seen that each district, precinct and division is mutually exclusive, and that contiguous districts make up a precinct. Single districts are always contained within one precinct, as are precincts in divisions. The entire city is divided into districts which are collectively exhaustive of the responsibility area of the entire police force. The number of divisions and precincts are set and assumed independent, that is, crossing of these boundaries is only the exception, never the rule.

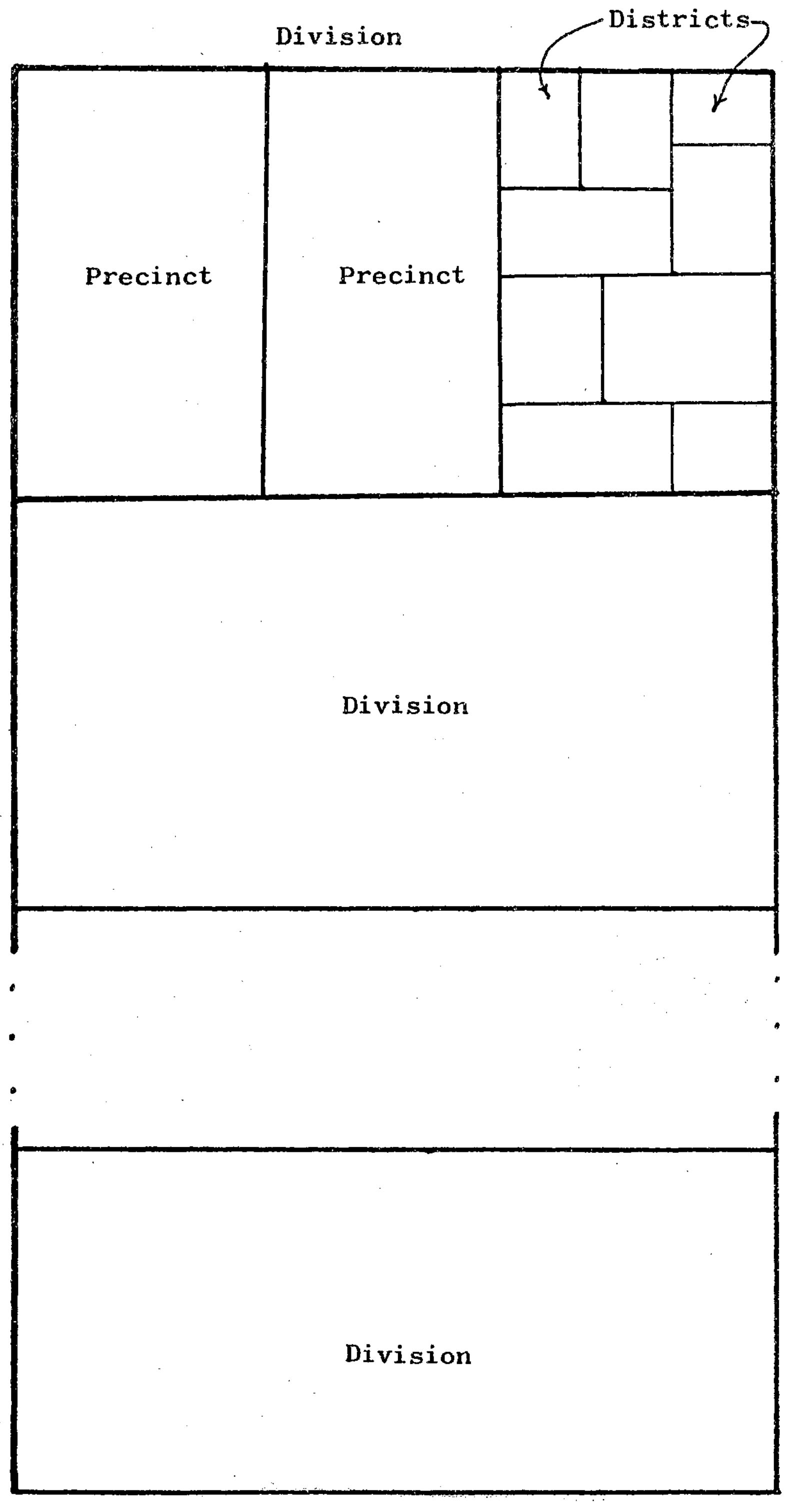


Figure 1. Breakdown of a City Into Small Geographical Areas

The actual dividing lines for districts are streets of the city. The common practice is to utilize the separation of the city into census tracts, where each district is made up of one or more contiguous tracts. This method is discussed in Chapter II.

If the police force policy of a city is to assign an officer in a car to a small section of ground, called a beat, this area will be composed of one or more contiguous districts. However, if a number of cars are assigned to an area in which calls are answered and some general patrol responsibilities are defined, the cars are assigned to a precinct. (This is a fluid or team approach to patrolling.) This being the case, call answering duties outside the precinct are rare, thus, the assumption that precincts are independent of each other for patrol and call answering duties. Larson indicates that this assumption is only rarely violated.[6]

The formulation of precincts must be accomplished with the advice of some one thoroughly familiar with the city. Travel between points may be hindered by barriers such as cliffs, rivers, expressways or railroads. These should be considered by the designers to alleviate travel problems before they are discovered by the patrolling officers in the line of duty.[7]

This research first investigates the problem of separating a division into a set number of precincts for a particular shift of the work day. Thus, it amounts to the question: "Where are the precinct boundary lines drawn so that all districts in the division are included in a limited number of precincts such that some objectives are met and certain restrictions satisfied?"

The permanency of a precinct boundary for a particular shift should be mentioned. Once boundary lines are established, changes should be made only if the crime rate and other duties in the division are altered. This stable relationship between precincts is important so that the officers can become acquainted with their area. If the work load is found to vary substantially during the same shift, then the boundaries may be altered to better distribute the patrol force, but the new distribution should differ from the previous only in that districts are removed or added to particular precincts. This simply alleviates the problem of having an officer move to a completely new area in the middle of his shift.

Once an area has been divided into precincts, shorthandedness of cars and men is a common problem. Thus for a particular precinct and shift, the best use possible must be made of a less than adequate number of patrolling facilities. This problem is also investigated in this research.

Literature Relating to Patrol Force Distribution

This section presents some of the past work performed to distribute patrols throughout a city. The increasing mobility of the patrol division has necessitated changes in the type of distribution, for example, use of precincts rather than beats. Another recent trend has been the use of one-man cars rather than two-man cars. The former has been found to be more effective and no more dangerous than the latter.

History of Patrol Distribution

As mentioned previously, past distribution of patrol, and even present distribution to a large extent, involved the assignment of an officer, either on foot or motorized, to a beat. This beat was assumed to be independent of other beats in the precinct. This technique was and still is used in order to develop within the officer and the citizens residing in the beat a 'beat identity.'[8] Larson presents a simulation program which studies precincts which are divided into independent patrolling beats.[9] By viewing the positions of the cars at different times of the shift, he found that only a few of the cars were in their respective beats due to the necessity of answering calls in other beats. Bristow argues that it is necessary to utilize the fluid or team approach based on a 'consumed time' concept to accomplish the ever expanding objectives of the patrol force.[10] Precinct assignment also has the advantage that the dispatcher is able to assign according to a 'closest car' rule, rather than attempting to keep the patrol car assignments in the correct beats. Thus, fluid or roving type patrols assigned to a precinct seem to be the trend for the future in the distribution of patrol forces.

Types of Data Available

The police force has available certain types of data which may be used in the deployment of the patrol force. Often studies are performed by outside consultants to evaluate the police force in its entirety, during which data useful in patrol distribution may be collected. Of course, one of the most common statistics kept is the crime rates in different areas of the city. Usually data of all sorts is collected by census tracts. Crimes are divided, classically, into the following:[11]

Part I and II crimes--The FBI originally classified these as the crimes to be reported to them. They include homocides, rapes, larceny, and other major crimes;

Part III incidents--These are not reported to the FBI and include lost-and-found persons, property, and the like; and

Part IV incidents--Those involving the sick, traffic accidents, suicides, and mental cases.

This division may be utilized by a police force in different forms for data collection for patrol distribution purposes.

Also of use are estimates of the time needed per shift to perform inspectional duties in a precinct. Often some sort of restriction in terms of hours of patrolling per crime will be utilized in distribution of patrols. Service data is also collected for the different crime levels considered important by the police department. This information and the patrolling responsibilities of the precinct allow some estimate of the total or consumed time per precinct needed on each shift.

Information is also collected as to how many miles of patrollable street are present in and the area of each precinct. Measures of peak traffic hours are maintained. The queue length of calls waiting for assignment of a car during different times of the day are also important. Once a car is dispatched to a particular point for service, both the time of travel and distance covered are observed. These and many other types of data are available from public records, files and reports of the police and city government. Those mentioned above are but a few, however, they are representative of those used in this research.

Manpower Calculations

It is worthwhile to mention a few of the references obtainable concerning past work accomplished in the distribution of patrols. One of the first efforts was that of O. W. Wilson (1941) at Wichita, Kansas, which distributed patrols on a proportional basis, that is, an area of a city with twenty per cent of the total crimes warranted twenty per cent of the patrol force. Wilson (1963) discusses the same method with additions concerning administration of the entire police department. Walton (1958) discusses both the need for sound distribution plans and 'proportional basis' plans employed at the time of his writing. Gourley and Bristow (1961) dedicate an entire book to the subject of patrol administration. Discussion is presented concerning modes of patrol, identification of cars on patrol, and a distribution procedure paralleling Wilson's method. Mention is also made of the different objectives which may be used in the deployment of the patrol force in a city. Bristow (1969) discusses the utilization of manpower in the patrol force in an efficient manner and gives recommendation for future distribution procedures. Larson (1969 thesis) introduces the statistical approach into patrol operations by studying the variability of response distances and the queue phenomenon of the entire patrol division.

Of the distribution procedures utilized, most are based on Wilson's technique which distributes the patrol force on a proportional basis. This method is usually referred to as the 'hazard factor' method. However, Larson (1969 (thesis), 1969 (paper)) has developed a dynamic programming procedure which finds the minimum number of patrol cars to be assigned to a precinct to meet certain restrictions. This technique discards the method of proportional distributing because of the mobility of the patrol force. As can be easily seen this area is still rich for research purposes.

Effectiveness and Statistical Measures

For a particular city a study by a consulting firm of the police force allows an overview of the effectiveness and gives hints as to areas which need improvement. Simulation models are becoming widely used in an attempt to develop better methods by which to distribute the patrol force. One of the more important aspects of patrol operation is the response distance or response time of the car. Surkis, Gordon, and Hauser (1968) have developed a simulation model which studies the entire response system of the New York City police department. Of course, many facts interrelate in the response system, such as time of day, speed of car in answering calls, layout of the city, and work load of the patrol force. However, the smaller the response time, the higher the probability of arrest. Thus, it is only reasonable that the force be adequate in an area to insure a response time less than or equal to some predetermined number which will, of course, have political and social connotations.

Larson (1969 thesis) devotes much of his study to the designing of precincts or beats in terms of their length and width and how these relate to response distance. These investigations have shown that response distance is relatively constant provided the precinct is 'reasonably compact,' that is, not extremely elongated.

The effectiveness of the patrol force is a much discussed subject. Evaluation of the patrol force is accomplished for each city individually, since the purposes of the study will vary. Qualitative discussion of the patrol force, its efficiency and role in the community, all of which contribute to its effectiveness, may be found in journals which relate to police work. Some of these are Police Journal, Police Chief, and Journal of Criminal Law, Criminology and Police Science.

Even though the properties of patrolling, such as response distance and speed, time to service a crime, etc., all vary, this fact has not been utilized to any great extent thus far. A forward looking approach is the work of Larson (1969) at MIT. His investigation of police response distances has introduced statistical applications into the field of police work. Possibly with time generalizations about response systems may be made and applied to better the work of the patrol force.

Objectives of This Research

This research makes no attempt to completely cover the problem of patrol distribution within a city. The aspect of chronological distribution is not discussed at all, but rather distribution is done for a particular period of time, perhaps a shift. However, the problems are of a geographical distribution nature as explained earlier in this chapter. Since many different objectives, restrictions, and types of vehicular patrols exist, after discussion of design objectives and possible restrictions in general, specific examples will be considered for automobile patrols only.

The first objective is to develop an algorithm to partition a division into a set number of precincts. The problem selected utilizes a representative objective and restrictions.

The second objective is to investigate this problem by observing the variability in distribution caused by alterations in the number of precincts and restrictions.

A final objective is to examine a representative precinct that is understaffed. The use of Pseudo-Boolean Programming for Bivalent (0, 1) Variables is made to optimize the utilization of a less than adequate number of cars in a precinct.

Both of these problems, partitioning into precincts and optimizing utilization in an understaffed precinct, are approached from an analytical rather than qualitative or simulation viewpoint. The techniques utilized for solution are not claimed to be a panacea, however, they do present the possibility of quantitative evaluation to problems usually solved exclusively by trial and error methods.

FOOTNOTES

[1]Manuel Lopez-Rey, "Defining Police-Community Relations," Police and Community Relations, ed. A. F. Brandstatter and Louis A Radalet (Beverly Hills: Glencoe Press, 1968), p. 188.

[2]Interview with Glen Stanford, Former Chief of Police, Stillwater, Oklahoma, February 19, 1970.

[3]O. W. Wilson, Police Administration (New York: McGraw-Hill, 1963), pp. 232-233.

[4]Allen P. Bristow, Effective Police Manpower Utilization (Springfield, Ill.: Charles Thomas, 1969), pp. 104-109.

[5]Wilson, p. 238.

[6]Richard Larson, Models for the Allocation of Urban Police Patrol Forces (Ph. D. dissertation published as Technical Report No. 44 by Operations Research Center at Massachusetts Institute of Technology, 1969), p. 58.

[7]Interview with Howard Hoyt, Chief of Police, Stillwater, Oklahoma, June 8, 1970.

[8]Larson, p. 52.

[9]Larson, pp. 259-268.

[10]Bristow, pp. 109-110.

[11]Wilson, p. 393.

CHAPTER II

THE PROBLEM OF PRECINCT DESIGN

This chapter discusses the problem of precinct design explained in the first chapter. A general discussion of the aspects of the problem is undertaken after which an example of a division to be divided into precincts is presented. Since precinct dimensions do not contribute heavily to varying response distances,[1] it is possible to allow virtually any precinct design except possibly very elongated ones. This being the case, restrictions upon configuration of the precinct are not imposed in this chapter.

Problem Formulation

One of the more common types of problem experienced in optimization theory is the knapsack problem.[2] This is an attempt to fit as many as possible of each of different types of articles or products into some restricted 'space' while optimizing an effectiveness function. Sometimes referred to as the space capsule problem, it includes such dilemnas as those presented by capital budgeting, optimal redundancy of units with constant reliabilities, and the like. If the effectiveness function is accompanied by several constraints, such as budget constraints for several future years in capital budgeting, the problem is called a multi-dimensional knapsack (MDK) problem.

The precinct design problem may be classified with the other knapsack problems, since some type of effectiveness function will have to be defined when separation of a division is attempted. Also present are one or more constraints imposed by the police officials, logical restrictions that a precinct consist of only contiguous districts, the restraint that each district be in only one precinct and that all districts be assigned to a precinct. Thus, the problem is classifiable as MDK.

The division may be divided into districts using the deterministic or probabilistic approach. If the deterministic is used, data collected may be averaged and this figure considered determined, for example, average allocated time or average response time for a district. Or, the data may be fitted to a density function and attempts made to solve the problem using this more realistic form.

Possible Objective Functions, Restrictions and Assumptions

The police and community officials decide the basis for distribution of patrols, that is, the objectives of the plan. There are many which may be chosen, some more important than others. The primary purpose may be to obtain an equitable distribution of work load for each precinct (or beat, if these are used) or to allocate duties so that each officer has sufficient duties to keep him busy, but not overworked. The minimizing of the response time may be important to the officials and the public; however, shortage of manpower may cause a problem here. Not as important, but worthy of consideration, may be attempts to equalize the area given each car rather than concentrate upon the respective area crime rates and other necessary duties.

To be definitely considered are the crime rates in different areas at varying times of the day. Officials must decide if all types of crime (Part I, II, III and IV) are of importance and how much each is to be considered in the distribution scheme. Gourley and Bristow present the weighting plan of Los Angeles, which includes all types of crime, radio calls and report making.[3] Of course, the more included in the plan, the more complicated and cumbersome it becomes; but, also the more accurate.

Once determined, the objectives should be reviewed periodically. Also, different objectives will probably exist for different divisions and times of day, since at certain times an area may require primarily inspection of stores, windows and doors, while others require the major portion of time devoted to answering calls. These are all problems peculiar to the individual case and must be solved in the best manner possible by the responsible officials.

Restrictions imposed may also be of many types. The designers may desire to place a minimum on the number of cars per precinct, place an upper limit on the maximum allowable response time or distance per precinct, place a maximum on the area or street miles allowable per precinct, require so many hours of patrol for each crime of certain categories per precinct and many other types of constraints. These again may vary with division and time of day.

Assumptions are always necessary to simplify the problem. For the precinct design problem, the assumptions made here are as follows:

1. Precincts are not dependent upon each other for patrolling

and call answering duties;

2. Cars assigned to a precinct may be dispatched anywhere within the precinct boundaries;

3. An objective and a set of restrictions may be formulated for the purposes of distribution of the patrol force; and

4. Response distances and times and allocation times are additive by district, that is, combining two contiguous districts to form a precinct cause allocated time for the precinct to become the sum of district allocated times, and the response distance and time are assumed to follow the extreme case and become the sum of district distances and times.

Mathematical Formulation

An objective and restrictions have been chosen in order to have a specific type of problem for investigation purposes. Operating under the previously stated assumptions districts are to be assigned to precincts so that the deviation of allocated time per precinct from the mean allocation time is minimized subject to the restriction that a car must be able to answer a call within a prescribed distance (on the average). The equalizing of work load is in keeping with Bristow's suggestion that patrols be distributed on a basis of 'consumed time' utilizing a fluid or team force.[4] This problem will be presented for both deterministic and probabilistic cases, where the response distance becomes a random variable in the latter.

Definitions used in this and the following chapter are given in the Nomenclature.

Deterministic Case:

$$\text{Minimize } C = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} a_i x_{ij} - \bar{a}_m \right)^2$$

$$\text{Subject to} \quad \sum_{i=1}^{n} r_i x_{ij} \leq R_j \qquad j = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, n$$

all contiguity constraints

$$x_{ij} = 0, 1.$$

In the above formulation $\bar{a}_m$ is the mean allocation time necessary to assign all duty time to m precincts, that is,

$$\bar{a}_m = \frac{\sum_{i=1}^{n} a_i}{m} .$$

The deviations from $\bar{a}_m$ have been squared to alleviate the problem of the negative for under average allocations; the problem now being to minimize total mean deviations squared. The constraint placed on each district, i, requires that it be assigned to only one precinct. The insertion 'all contiguity constraints' is included as a shorthand method of insuring that each precinct is composed of only contiguous districts. These constraints are many for they must include restrictions to account for two at a time violations, three at a time, etc. However, solution of the problem by an integer programming technique would require inclusion of these in their detailed form, or discarding of all violating precincts by judgemental action as solution proceeds.

The formulation is such that the upper limit of precinct response distance, R_j, may vary with precincts, however, if all precincts are designed for the same response distance $R_j = R$ for all j.

Probabilistic Case:

$$\text{Minimize } C = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} a_i x_{ij} - \bar{a}_m \right)^2$$

$$\text{Subject to } P\left(\sum_{i=1}^{n} r_i x_{ij} \leq R_j \right) \geq \alpha_j \; ; \qquad j = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, n$$

all contiguity constraints

$$x_{ij} = 0, 1 .$$

This problem has the same formulation as the deterministic except that r_i is a random variable with some pdf. Thus, the restriction of precinct response distance requires that this distance be less than or equal to R_j with a probability of at least $\alpha_j (0 < \alpha_j \leq 1)$. If the same upper distance limit and confidence level hold for all precincts, removal of the subscript j on R_j and α_j is possible.

Utilization of the shortened

$$\bar{r}_j = \sum_{i=1}^{n} r_i x_{ij}$$

is made in the remainder of this thesis.

Solution Algorithm for the Deterministic Case

Even though the precincts are assumed to be independent it is not possible to solve the problem precinct by precinct (stage-wise), for the assignment of a district to a particular precinct does not allow assignment to another precinct to be developed later in the solution which may be better in terms of the objective function. In other words, it is necessary to have an entire solution in order to compare it to some other solution.

Actually the problem may be viewed as a type of assignment problem in which districts are to be assigned to precincts, the essential difference being that a precinct may and usually must contain more than one district. This dependency in solutions necessitates the development of a solution technique which allows any precinct configuration within restrictions to be present for each precinct. A two phase algorithm may be employed for solution. The procedure with certain simplifying principles is presented here in general form.

Phase I--Precinct Generation

In order to generate a complete solution each precinct must be determined, a task accomplished by combining districts such that restrictions are not violated. Thus, for each district generate all feasible precincts that contain that district. This done for a set number of precincts, find the contribution to the objective function for each combination by calculating

$$\left(\sum_{i=1}^{n} a_i x_{ij} - \bar{a}\right)^2 ,$$

where

$$x_{ij} = \begin{cases} 0 & \text{if the district is in generated precinct} \\ 1 & \text{if district is not in generated precinct.} \end{cases}$$

In the problem considered here a feasible precinct is one that does not violate contiguity constraints and has $\bar{r}_j \leq R_j$. Also during this phase, any precinct combinations not to be included due to wishes of the designer may be omitted from consideration.

Phase II--Optimal Solutions by Precinct

Once all feasible precincts have been generated it is possible to find solutions on a precinct by precinct basis. The procedure to obtain the optimal solution(s) is outlined below.

1. Define some arbitrary order of districts, i_1, i_2, ..., i_n in which districts are to be assigned to precincts. (If district numbers are increased according to contiguous districts, the order $i_1 = 1$, $i_2 = 2$, ..., $i_n = n$ is easiest to use.)
2. Beginning with i_1 and proceeding to i_n consider all feasible precinct combinations. All including i_1 are coupled with those containing i_2, and so forth. As precincts are developed the objective function may be accumulated for comparison after all m precincts are completed. For precinct k ($k \leq m$) if i_p ($p = 1, 2, ..., n$) is the next district not assigned in a particular solution branch, a precinct may be disqualified if:
   a) it also includes any of i_1, i_2, ..., i_{p-1}, in which event precincts with i_{p+1} are considered next; or

b) the remaining availability of the resources is not sufficient to assign the not yet assigned districts to (m-k) precincts. Thus, for the problem formulated if

$$\sum_{i=1}^{n} r_i - \sum_{j=1}^{k} \bar{r}_j \leq \sum_{j=1}^{m} R_j - \sum_{j=1}^{k} R_j \qquad (1)$$

is violated, the precinct may be judged ineligible. If all R_j's are equal, the right side of (1) may be written (m-k)R. It is also important to realize that the k precincts assigned need not be the precincts j = 1, 2, ..., k, but may be any set of the m precincts with k elements in this set. For example, if k = 3, m = 7, the right side of (1) may be involved with precincts arbitrarily numbered as 1, 4 and 6. If more restrictions are present, the precincts must satisfy all constraints.

3. When the m^{th} precinct is completed, all districts must have been assigned. It is possible that all districts may be assigned before the m^{th} precinct is reached; or it is possible that some districts may remain; therefore, no solution exists for m precincts. After all districts are assigned, solutions having m precincts may be compared to discover which one has the smallest objective function.

This algorithm allows solution for a set number of precincts. It may be desirable to solve the problem for several different values of m. This would first of all entail the calculation of different sets of objective contributions in phase I since $\bar{a}_m$ varies with m. These respective contributions being determined, the solutions may be found in phase II by simply keeping the accumulated objectives separate.

Exclusion Principles

Since the contributions to the objective function are accumulated as feasible precinct combinations are added to partially determined solutions, comparison of objectives is possible as each precinct is formulated. Once a complete solution of m precincts is devised an upper bound has been placed on the objective, and when another solution of k precincts ($k \leq m$) exceeds this value the partial solution may be discarded. It has been found that the order in which districts are introduced into solution is very important in finding the optimal solution early in the branching process. Thus, when a set of feasible precincts including a particular district is being considered, first attempt solution with the precinct having the smallest objective contribution, then the second, and so forth. This procedure allows rapid deletion of non-optimal solutions once a complete solution has been formulated. (If the objective is one of maximization, a complete solution previously determined is discarded and the procedure continued with the partially determined solution to find the new objective value.)

Another type of exclusion is possible. If the restrictions are varied, say relaxed, different precinct combinations are obtained in phase I. These may better the solutions obtained in phase II; however, observation of the precincts added to the list of feasible precincts due to relaxation may shorten or completely make unnecessary the determination of solutions in phase II. Since with constraint relaxation the feasible precincts will tend to increase in number of districts in each, the added precincts will have larger objective contributions to mean deviation squared. If all added precincts have contributions larger than the final objective value of the optimal solution for the next most tightly constrained solution, the solution to the present problem will be identical to the previous, thus eliminating necessity of solution. These remarks are made for the type of problem considered and it must be borne in mind they may change for different types. This procedure is possible if the following example is considered with relaxations on the response distance restriction. An illustration is given by the associated data in the Appendix.

Example of the Deterministic Case

Consider the division presented in Figure 2, in which separation into nine districts has been accomplished arbitrarily. Table I gives allocation times and response distances for each district for a particular shift. It is assumed that the police department has compiled this data and feels that the allocation times are what is necessary to accomplish the duties of each district and that the response distances are average travel amounts to answer a call in each district.

If the nine districts are to be combined into m = 4 precincts, the deviations about $\bar{a}_4 = 1800/4 = 450$ minutes are to be minimized. Further, if the upper limit of each precinct response distance is set at R = 0.7 miles the problem may be stated as

$$\text{Minimize} \quad C = \sum_{j=1}^{4}\left(\sum_{i=1}^{9} a_i x_{ij} - 450\right)^2$$

$$\text{Subject to} \qquad \bar{r}_j \leq 0.7 \qquad\qquad j = 1, 2, 3, 4$$

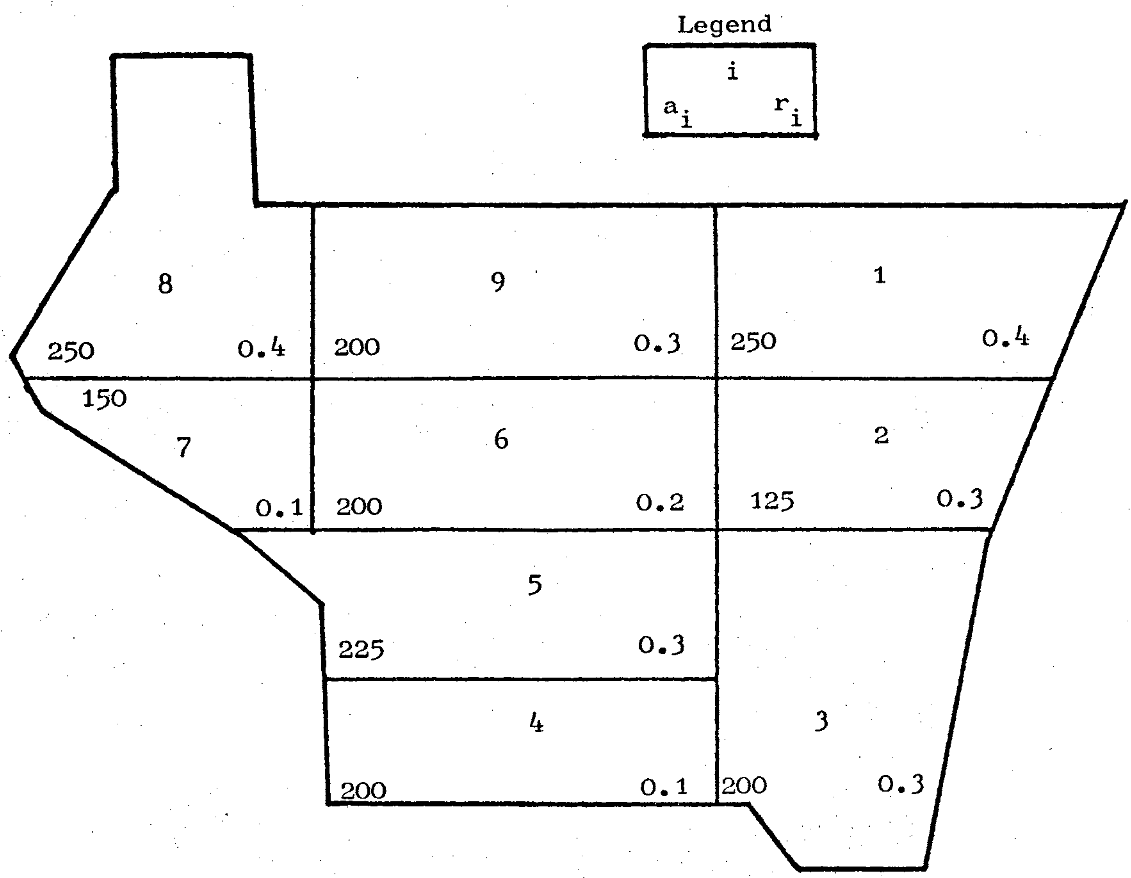


Figure 2. Example of a Division Broken Into Nine Districts

$$\sum_{j=1}^{4} x_{ij} = 1 \qquad i = 1, 2, \ldots, 7$$

all contiguity constraints

$$x_{ij} = 0, 1,$$

where the a_i and r_i are given in Table I. It will be easier to work the problem if the response distance restriction is written

$$10\bar{r}_j \leq 7.0 \qquad j = 1, 2, 3, 4$$

or

$$\bar{r}'_j \leq R'$$

and the r_i values of Table I considered in whole, not tenths of, miles.

TABLE I

ASSUMED ALLOCATION TIMES AND RESPONSE DISTANCES FOR THE EXAMPLE

District, i	Allocation Time, a_i (min.)	Response Distance, r_i (miles)
1	250	0.4
2	125	0.3
3	200	0.3
4	200	0.1
5	225	0.3
6	200	0.2
7	150	0.1
8	250	0.4
9	200	0.3

Phase I of the solution involves the generation of all feasible precincts not violating the response distance restriction. Table II gives these precincts, precinct response distances, allocation times and mean deviations squared by district.

Phase II requires the construction of precinct by precinct solutions and the accumulation of the objective function. From relation (1), since the distance remaining after k precincts ($k \leq m$) is $(m-k)R'$, it is seen that the total distance used after k precincts, that is,

$$\sum_{j=1}^{k} \bar{r}'_j$$

must be greater than the values shown below.

Number of Precincts Completed	Minimum Response Distance Assigned (miles)
1	3
2	10
3	17
4	24

An example of this for k = 2 is, when (1) becomes

$$\sum_{i=1}^{9} 10r_i - \sum_{j=1}^{2} \bar{r}'_j \leq (4-2)(7)$$

$$24 - \sum_{j=1}^{2} \bar{r}'_j \leq 14$$

$$\sum_{j=1}^{2} \bar{r}'_j \geq 10 .$$

TABLE II

ALL FEASIBLE PRECINCTS BY DISTRICT USING R' = 7

District, i	Feasible[a] Precincts Including i	Precinct[b] Response Distance, $\bar{r}'_j$ (miles)	Precinct Allocation Time (min.)	Mean Deviation Squared (min.^{2})
1	19	7	450	0
	12	7	375	5,625
	1	4	250	40,000
2	267	6	475	625
	12	7	375	5,625
	234	7	525	5,625
	23	6	325	15,625
	26	5	325	15,625
	2	3	125	105,625
3	35	6	425	625
	34	4	400	2,500
	234	7	525	5,625
	23	6	325	15,625
	345	7	625	30,625
	3	3	200	62,500
4	45	4	425	625
	34	4	400	2,500
	234	7	525	5,625
	345	7	625	30,625
	456	6	625	30,625
	4	1	200	62,500
	4567	7	775	105,625
5	35	6	425	625
	45	4	425	625
	56	5	425	625
	567	6	575	15,625
	345	7	625	30,625
	456	6	625	30,625
	5	3	225	50,625
	4567	7	775	105,625
6	267	6	475	625
	56	5	425	625
	69	5	400	2,500
	67	3	350	10,000
	679	6	550	10,000
	26	5	325	15,625
	567	6	575	15,625
	678	7	600	22,500
	456	6	625	30,625
	6	2	200	62,500
	4567	7	775	105,625
7	267	6	475	625
	78	5	400	2,500
	67	3	350	10,000
	679	6	550	10,000
	567	6	575	15,625
	678	7	600	22,500
	7	1	150	90,000
	4567	7	775	105,625
8	89	7	450	0
	78	5	400	2,500
	678	7	600	22,500
	8	4	250	40,000
9	19	7	450	0
	89	7	450	0
	69	5	400	2,500
	679	6	550	10,000
	9	3	200	62,500

[a]Punctuation omitted between districts in a precinct combination.

[b]These $\bar{r}'_j$ values are for any $j = 1, 2, 3, 4$ as assigned to a particular precinct.

Thus, after the second precinct combinations are linked to the first, the total distance accounted for must be 10.0 miles or more, otherwise the problem has no solution for four precincts utilizing the thus far assigned precincts.

With this preliminary work it is now possible to proceed with solution. Define the order of solution to be

$$i_1 = 1, \; i_2 = 2, \; \ldots, \; 1_9 = 9.$$

The solution explained in the following text is depicted in Figure 3. Precincts are indicated in circles and the branching process is continued until the first exclusion principle allows discontinuance of the solution or all districts are assigned to four precincts. On each branch is indicated the accumulated distance (in integer miles) and objective function. If a branch is terminated due to objective exclusion, an (X) is indicated after the objective value. The circled 'NO' indicates that no feasible precinct combination exists to continue the branch, thus solution is impossible. The solution procedure is now explained in detail.

1. Eligible precincts containing i_1 are 1,9; 1,2; 1; given in order of increasing mean deviation squared. Begin with 1,9 as origin of the first branch, in which case $\bar{r}'_1 = 7$.
2. Since after two precincts $\bar{r}'_1 + \bar{r}'_2$ must be ≥ 10, $\bar{r}'_3$ must be at least 3 and since $i_2 = 2$, the available second precincts are 2,6,7; 2,3,4; 2,3; 2,6; 2. Precinct 1,2 is not feasible since $i = 1$ is already assigned. Considering 2,6,7 first the distance accumulated is 13 and C = 625.

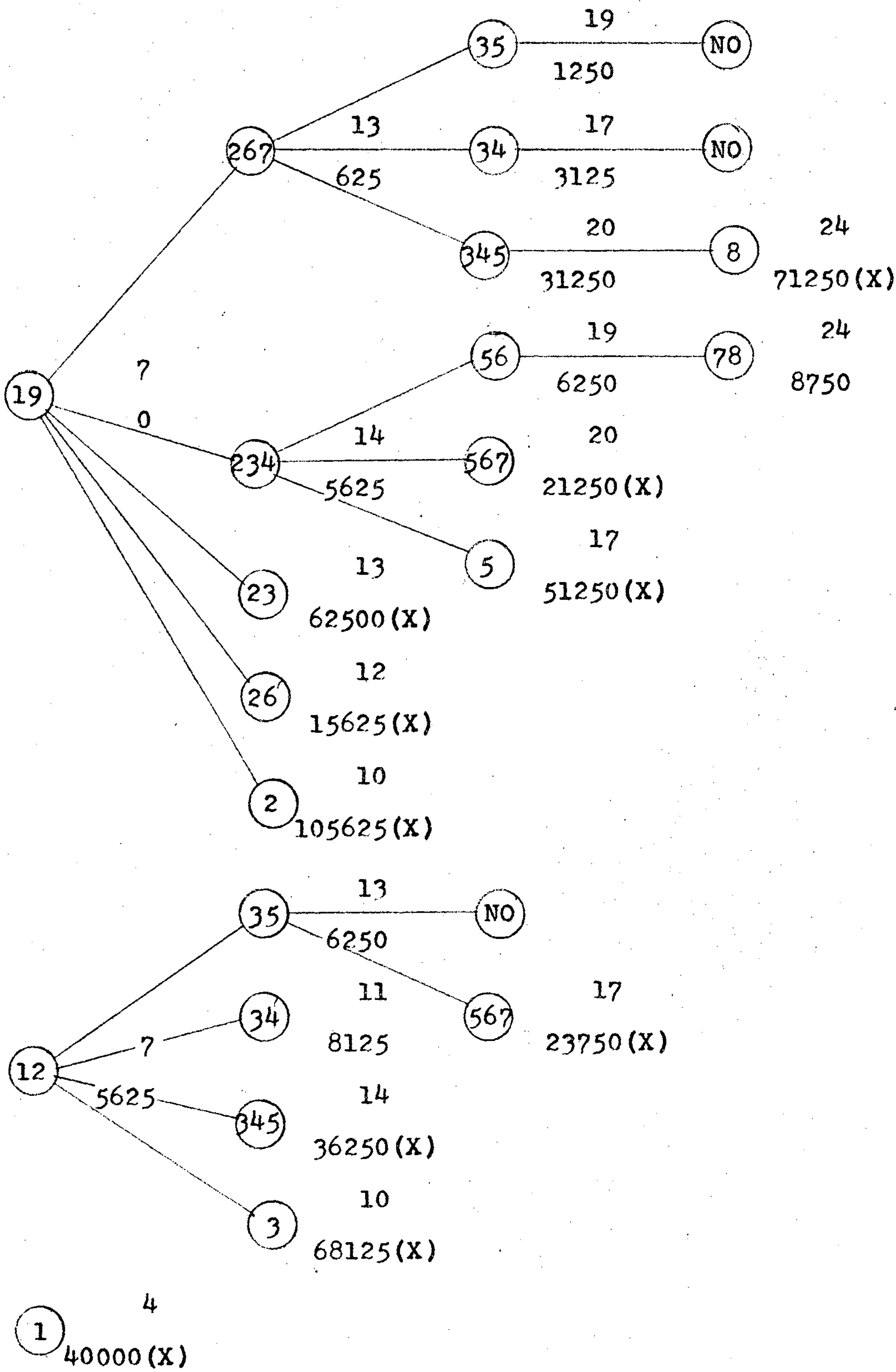


Figure 3. Solution of Example for R' = 7, m = 4

3. Continuing with the logic established, $\bar{r}_3' \geq 4$ allows precincts 3,5; 3,4 and 3,4,5 (not 3 since $10r_3 = \bar{r}_3' = 3$) to be considered. Branch 3,5 is selected since the objective contribution is smallest. Accumulated distance and C are 19 and 1250, respectively.

4. Since solution of precinct 4 is now necessary, the remaining distance and districts must be assigned by precincts eligible. Thus, $\bar{r}_4' = 5$ is needed for a precinct composed of 4,8. No such precinct exists; no solution is possible.

5. Backtracking to the last assigned precinct ($j = 2$) indicates the next feasible precinct to consider is 3,4. With this assignment (accumulated distance 17, C = 3,125) and $i_5 = 5$, again no feasible precinct exists with $\bar{r}_4' = 7$ and not including districts 1, 2, 3, or 4.

6. Consideration of 3,4,5 for $j = 3$ allows the fourth precinct to be assigned as district 8. This complete solution (1,9; 2,6,7; 3,4,5; 8) with C = 71,250 may be used for comparison with the next solution attempt.

7. The next precinct linked to 1,9 is 2,3,4 which gives an objective of 5,625. Since 5,625 < 71,250 solution is continued. Precincts available are 5,6; 5,6,7 and 5.

8. Assignment of 5,6 to $j = 3$ allows the complete solution 1,9; 2,3,4; 5,6; 7,8 (C = 8,750). Thus, the previously found solution is discarded since 8,750 < 71,250.

9. Consideration of 5,6,7 increases C to 21,250, which is larger than 8,750 and this solution attempt is terminated. Five assigned to $j = 3$ results in the same decision.

10. Feasible precincts 2,3; 2,6 and 2 linked to 1,9 all have mean deviations squared greater than 8,750 so continued search is fruitless.

11. Next consideration for precinct 1 is 1,2 ($\bar{r}_1' = 7$, C = 5,625). Feasible precincts including $i_3 = 3$ ($\bar{r}_2' \geq 3$) are 3,5; 3,4; 3,4,5; 3. Precinct 3,5 results in a 'NO' in precinct 3 since no precinct exists containing $i_4 = 4$, but not 1, 2, 3, 5 except district 4, which has $r_4 = 1 < 4$, the minimum distance needed. Other precincts accumulate so that C > 8,750 before completion.

12. District 1 used for j = 1 has mean deviation squared of 40,000 and is judged ineligible immediately.

With the above logic the solution is:

Precinct	Districts in Precinct	Precinct Allocation Time (min.)	Objective Contribution (min.^{2})
1	1,9	450	0
2	2,3,4	525	5,625
3	5,6	425	625
4	7,8	400	2,500
TOTAL	---	1800	8,750

If the same problem were worked for four precincts, but using another value of R', different precinct combinations would be generated (of course, these are already included here if R' < 7). However, if solution is attempted for R' = 7, but some other value of m different solutions may present themselves, giving alternative solutions which alter the optimal objective value. The next section elaborates upon these relations.

Effects of Altering Restrictions and Number of Precincts

The solution above causes deviations of 0, 75 above, 25 and 50 minutes below the average allocation. This is a total of 75 minutes above or below the average. Since the purpose of distribution is equitable allocation of time per precinct, m may be altered to find the value that will give the lowest total deviation. (The deviations above and below average are always equal.)

Also to be considered is precinct response distance. As R' is increased, more feasible precincts will appear. An investigation of the variability of assigned time and its deviation from average allocation time and a design constant, that is, design to allow equitable allocation about some pre-set number, is presented in the following. Response distance restriction, R' varies from four to nine miles and m from the smallest permitted value to nine precincts.

Minimizing Mean Deviations Squared

As m increases, average allocation for m precincts, $\bar{a}_m$ decreases. In the example problem above (total allocation time of 1800 minutes), $\bar{a}_m$ varies as shown.

Number Precincts m	Average Allocation, $\bar{a}_m$ (min. (hr.))
1	1800 (30.00)
2	900 (15.00)
3	600 (10.00)
4	450 (7.50)
5	360 (6.00)
6	300 (5.00)
7	257 (4.28)
8	225 (3.75)
9	200 (3.33)

The smallest possible precinct distance is $R' = 4$, since $10r_1 = 10r_8 = 4$, all other r_i being smaller. The feasible precincts for $R' = 4$ and mean deviations squared for m values of 7, 8, and 9 are given in Table III. Solution of the problems is given in Figure 4. The objective function value is accumulated at each branch for m = 7, 8, and 9 to the lower right of the precinct assignment, that is, the first number is the current C for m = 7, the next for m = 8, the last for m = 9. The enclosed objective value indicates the optimal solution for the associated number of precincts. As before, Figure 4 utilizes the indicator (X) for termination of a solution, and a dashed line indicates the solution for the m value is not possible.

The solution procedure is identical to that previously illustrated using the same order of i_j ($j = 1, 2, \ldots, 9$). Since solution is accomplished for several values of m some precedence order for number of precincts is needed. This order, being arbitrary, provides a rule for the selection of the solution branch. The order is chosen similar to that for district assignment, the numbers being defined by m_1, m_2, ..., for the total number of problems solved. Here the order used is $m_1 = 7$, $m_2 = 8$, $m_3 = 9$. Thus, in Figure 4, j = 3 the branch involving precinct combination 3 is investigated first since the objective contribution is smaller than the 3,4 combination for m = 7. This problem presents no contradictions since feasible branches always have the smaller objective value for m = 7, however, the definition of the sequence allows avoidance of problems should they occur. This is also helpful if a solution for a certain m value is not possible and branch selection involving a larger m value is necessary for another solution.

TABLE III

FEASIBLE PRECINCTS FOR EXAMPLE FOR R' = 4 AND m = 7, 8, 9

District, i	Feasible Precincts Including i	Precinct Response Distance (miles)	Precinct Allocation Time (min.)	Mean Deviations Squared		
				$m = 7$ $\bar{a}_7 = 257$	$m = 8$ $\bar{a}_8 = 225$	$m = 9$[a] $\bar{a}_9 = 200$
1	1	4	250	49	625	2,500
2	2	3	125	17,424	10,000	5,625
3	3	3	200	3,249	625	0
	34	4	400	20,449	30,625	-
4	4	1	200	3.249	625	0
	34	4	400	20,449	30,625	-
	45	4	425	28,224	40,000	-
5	5	3	225	1,024	0	625
	45	4	425	28,224	40,000	-
6	6	2	200	3,249	625	0
	67	3	350	8,649	15,625	-
7	67	3	350	8,649	15,625	-
	7	1	150	11,449	5,625	2,500
8	8	4	250	49	625	2,500
9	9	3	200	3,249	625	0

[a]No entry indicates precinct not feasible, since if m = 9, each district is a precinct.

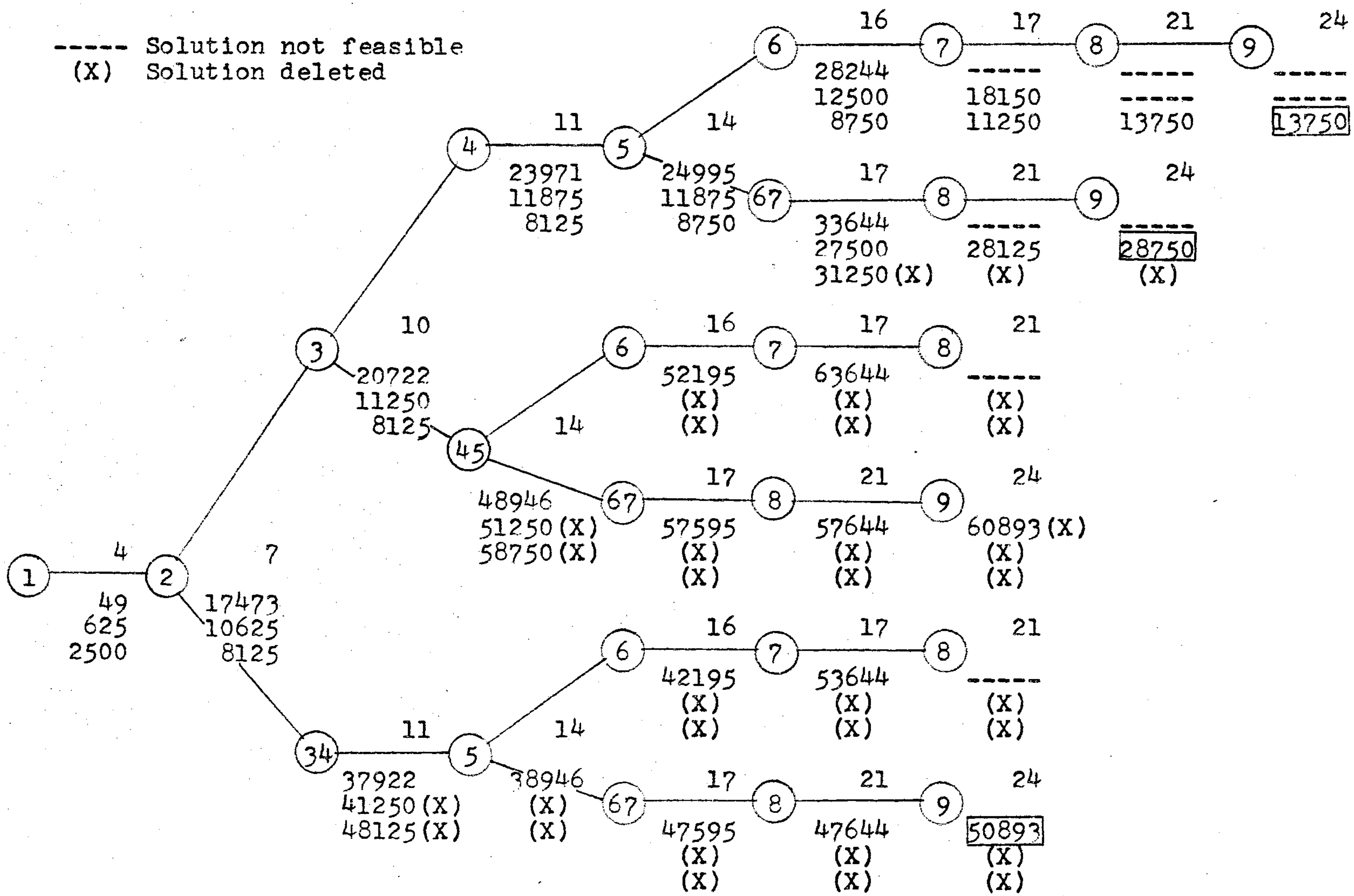


Figure 4. Solution of Example for 7, 8, and 9 Precincts Using R' = 4

It is seen that the solutions by precincts and respective time deviation from average is as shown below.

Total Number of Precincts	Solution by Precinct	Deviation From Average Above (hrs.)	Below (hrs.)
7	1;2;34;5;67;8;9	3.94	3.94
8	1;2;3;4;5;67;8;9;	2.92	2.92
9	1;2;3;4;5;6;7;8;9	2.08	2.08

The deviation in hours of total work load decreases as number of precincts increases. These values are obtained by adding the time above (or below) average for each precinct assignment in solution for m precincts.

Of course, solution for $m = 9$ is identical regardless of R' since each district is a precinct. Solution for $m \leq 6$ is infeasible because $R' = 4$ does not allow enough distance per precinct to find a solution in which each precinct is composed of only contiguous districts.

A summary of optimal solutions for R' from four to nine and all precincts is given in Table IV. Also presented are deviations of precincts from average for each precinct. A listing of feasible precincts and objective contributions used to obtain these solutions is given in the Appendix. Solution is similar to that detailed for $R' = 4$.

The fact that solutions for R' values of 7, 8, and 9 are identical is due to the second type of exclusion discussed earlier. All feasible precincts that become available by virtue of an increase in R' have mean deviations squared larger than the total objective function value for the optimal solution using the next smaller R'. Thus, the same

TABLE IV

SUMMARY OF SOLUTIONS AND MEAN DEVIATIONS FOR R' VALUES AND ALL FEASIBLE m VALUES

R'	m	Solution by Precinct	Objective, C	Minutes Above (Below) Average by Precinct									Total Mean Deviation (hrs.)
				1	2	3	4	5	6	7	8	9	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
4	7	1;2;34;5; 67;8;9	50,893	(7)	(132)	143	(32)	93	(7)	(57)	-	-	3.94
	8	1;2;3;4; 5;67;8;9	28,750	25	(100)	(25)	(25)	0	125	25	(25)	-	2.92
	9	1;2;3;4; 5;6;7;8;9	13,750	50	(75)	0	0	25	0	(50)	50	0	2.08
5	6	1;26;34; 5;78;9	38,750	(50)	25	100	(75)	100	(100)	-	-	-	3.75
	7	1;26;3;4; 5;78;9	35,893	(7)	68	(57)	(57)	(32)	143	(57)	-	-	3.50
	8	1;26;3;4; 5;7;8;9	18,750	25	100	(25)	(25)	0	(75)	25	(25)	-	2.50
	9	1;2;3;4; 5;6;7;8;9	13,750	50	(75)	0	0	25	0	(50)	50	0	2.08
6	5	1;23;45; 69;78	20,750	(110)	(35)	65	40	40	-	-	-	-	2.42
	6	1;23;45; 67;8;9	33,750	(50)	25	125	50	(50)	(100)	-	-	-	3.33
	7	1;23;4; 5;67;8;9	20,893	(7)	68	(57)	(32)	93	(7)	(57)	-	-	2.67
	8[a]	1;23;4; 5;6;7;8;9	18,750	25	100	(25)	0	(25)	(75)	25	(25)	-	2.50
	9	1;2;3;4; 5;6;7;8;9	13,750	50	(75)	0	0	25	0	(50)	50	0	2.08
7 or 8 or 9	4[b]	19;234; 56;78	8,750	0	$\boxed{75}$	(25)	$\boxed{(50)}$	-	-	-	-	-	1.25
	5	1;23;45; 69;78	20,750	$\boxed{(110)}$	(35)	$\boxed{65}$	40	40	-	-	-	-	2.42
	6	1;23;45; 67;8;9	33,750	(50)	25	$\boxed{125}$	50	(50)	$\boxed{(100)}$	-	-	-	3.33
	7	1;23;4; 5;67;8;9	20,893	(7)	68	$\boxed{(57)}$	(32)	$\boxed{93}$	(7)	(57)	-	-	2.67
	8[a]	1;23;4;5; 6;7;8;9	18,750	25	$\boxed{100}$	(25)	0	(25)	$\boxed{(75)}$	25	(25)	-	2.50
	9	1;2;3;4; 5;6;7;8;9	13,750	$\boxed{50}$	$\boxed{(75)}$	0	0	25	0	(50)	$\boxed{50}$	0	2.08
9[c]	3	126;345; 789	1,250	$\boxed{(25)}$	$\boxed{25}$	0	-	-	-	-	-	-	0.42

[a]An alternative solution for m = 8, R' = 6, 7, 8 and 9 is 1;26;4;5;3;7;8;9 with other information being the same.

[b]An alternative solution for R' of 8 or 9 is 19;236;45;78 with other information being the same.

[c]Solution for 4 through 9 precincts is same as for R' of 7 or 8.

solution may be judged optimal as further searching will prove to be fruitless. The only exception in this problem is the alternative optimum for R' of 8 and 9 ($m = 4$). Here another precinct with the same contribution as for $R' = 7$ is available and another solution becomes available (see footnote to Table IV).

It is beneficial to study the distribution of time to the precincts and its deviation from the mean. Two families of curves are of particular interest in this endeavor. Figure 5 presents the family of curves for total deviation in hours of precinct time from average, in other words, a plot of column (14) in Table IV, by response distance. It is observed that once the deviation curve for $R' = 6$ is reached increases in R' only allow solution for a smaller number of precincts, but mean deviation is identical for a fixed value of m. The decrease in deviation noticed from R' of 4 to 5 to 6 is due to the availability of precincts which are able to reduce deviation, however, further addition of distance above $R' = 6$ only allows precincts to become available which have too large a contribution to cause a better solution. Also illustrative of the situation is the family of curves for the number of precincts. If deviation from average is plotted against R' for each m value, as in Figure 6, it is seen that for a particular m value deviation is constant once all precinct combinations are possible which may reduce deviation and, therefore, the objective function.

In conclusion, it is evident that if, say, six or seven precincts are desired, even if the division is laid out for a response capability of $R = 0.9$ mile, it will be possible to answer calls in a less distance, in fact in 0.6 mile. However, if a larger number of precincts are to be used and distance greatly restricted (for example, $R = 0.4$), then a

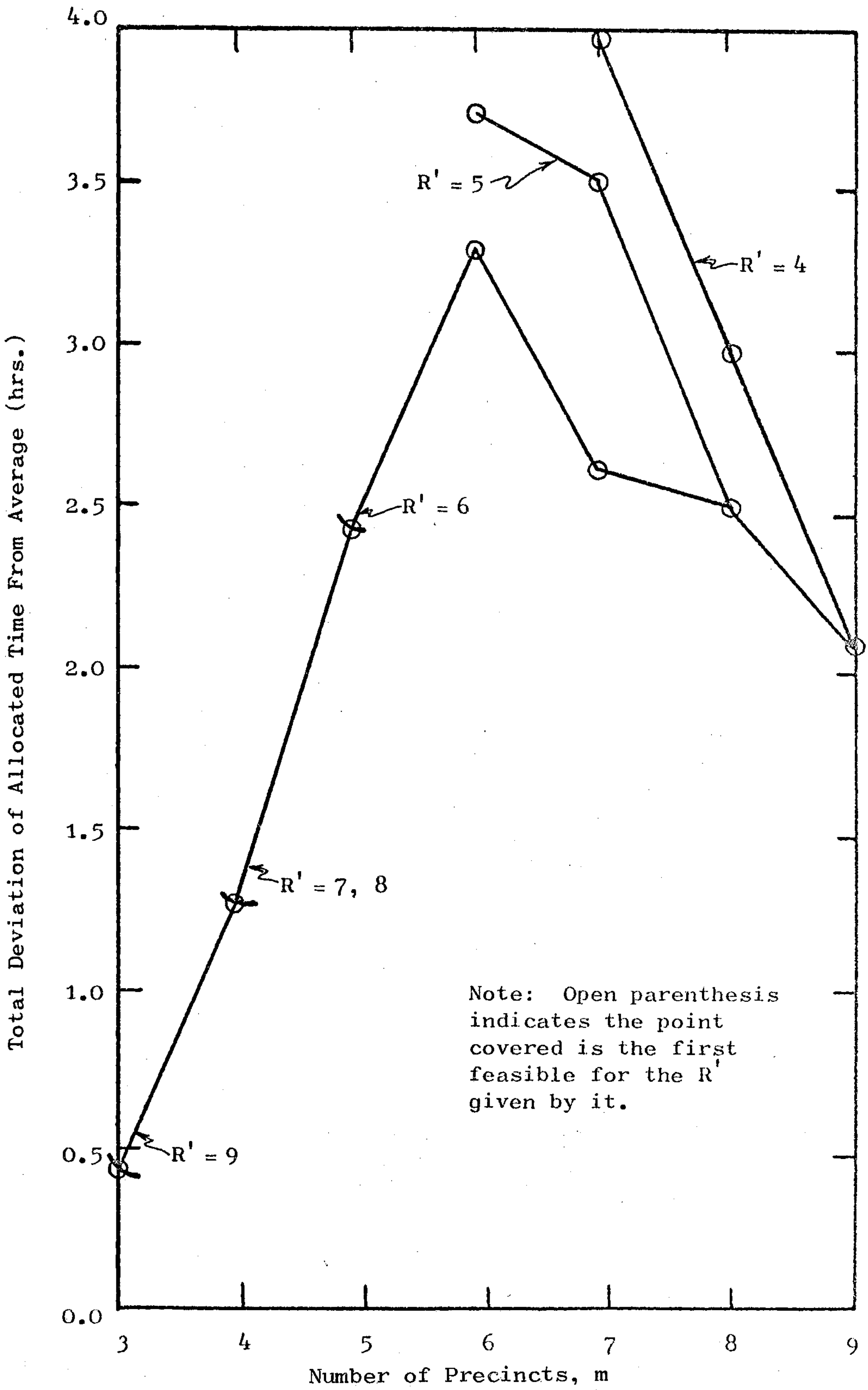


Figure 5. A Plot of Total Deviation From Average Versus Number of Precincts for Considered Values of Response Distance Restriction

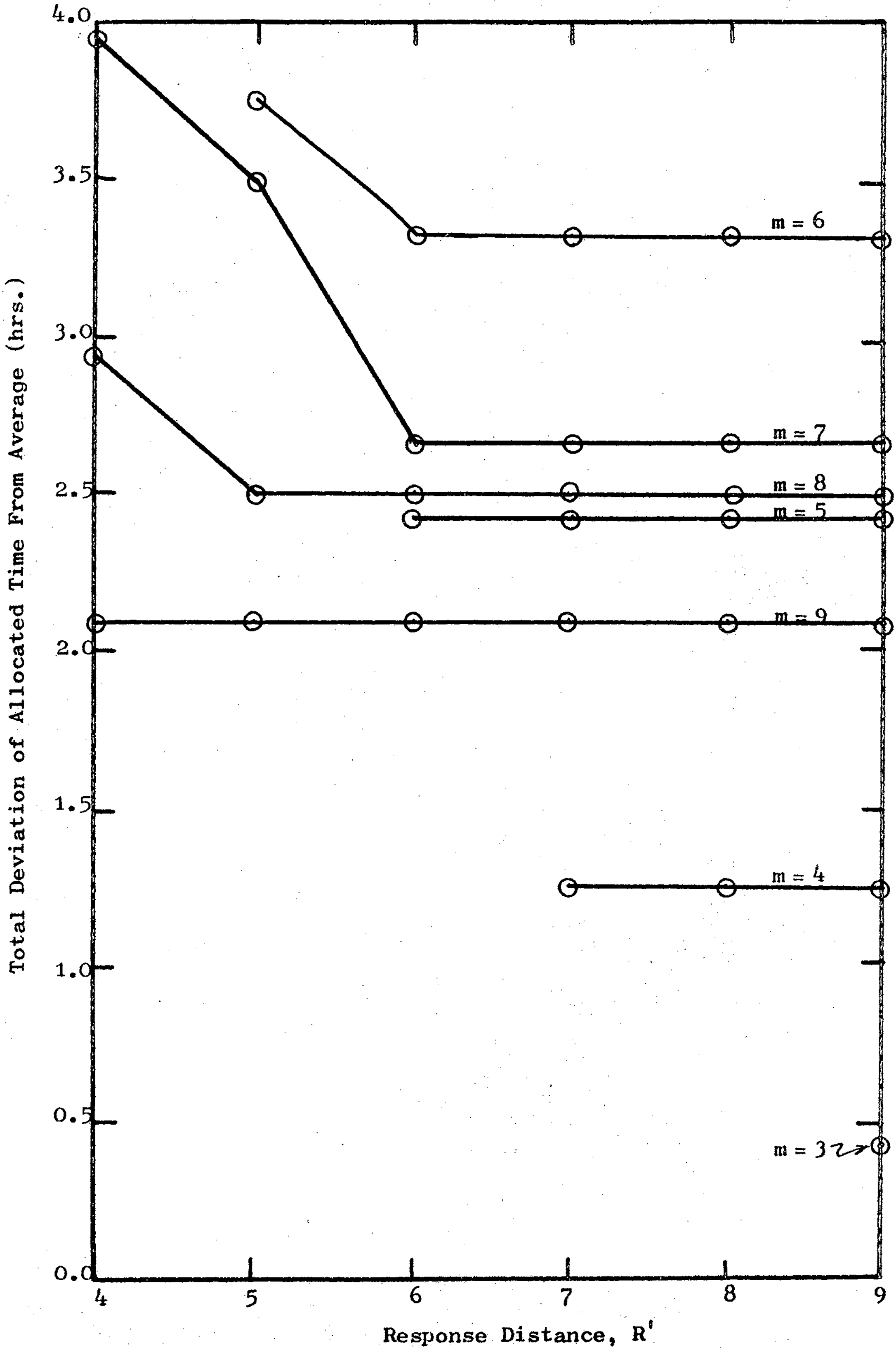


Figure 6. A Plot of Total Deviation From Average Versus Response Distance for Feasible Number of Precincts

better solution in terms of mean deviation is available if this restriction is slackened somewhat. Also, from Figure 6, notice that total deviations do not increase in the same order as m. The fact that $m = 9$ has less deviation than m values of 5, 6, 7, and 8 indicates that using smaller values of m and thus causing the precincts to become larger need not always provide automatic equity among assigned precincts.

As before, it should be borne in mind that another problem of the same type with generically different restrictions may behave quite differently.

Minimizing Deviation About a Set Value

The people in the police department may desire to design the precincts such that duty time is assigned equitably about some pre-set design constant, for example, an eight hour shift. If this is the case, feasible precincts would remain as previously derived provided the rest of the problem were unaltered. Presume that a design constant of 7.5 hours (450 minutes) is decided upon. Then the objective of the problem becomes

$$\text{Minimize } C = \sum_{j=1}^{4} \left(\sum_{i=1}^{9} a_i x_{ij} - 450 \right)^2 ,$$

while the balance of the problem remains unchanged. Since the design constant coincides with average for $m = 4$, mean deviations squared calculated in the previous section are usable in solution of this problem. If these figures are utilized results summarized in Table V are found. Notice that in all cases the solutions are identical to those minimizing mean deviations squared, however, the objective function (optimal value) is altered drastically. An interesting

TABLE V

SUMMARY OF RESULTS AND DEVIATIONS ABOUT CONSTANT (450) FOR R' AND POSSIBLE m VALUES

R'	m	Solution by Precinct	Objective' C	Minutes Above (Below) Design Constant by Precinct									Total Deviation (hrs.)	
				1	2	3	4	5	6	7	8	9	Over	Under
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
4	7	1;2;34;5; 67;8;9	311,250	(200)	(325)	(50)	(225)	(100)	(200	(250)	-	-	0	22.5
	8	1;2;3;4; 5;67;8;9	433,750	(200)	(325)	(250)	(250)	(225)	(100)	(200)	(250)	-	0	30.0
	9	1;2;3;4; 5;6;7;8;9	576,250	(200)	(325)	(250)	(250)	(225)	(250)	(300)	(200)	(250)	0	37.5
5	6	1;26;34; 5;78;9	173,750	(200)	(125)	(50)	(225)	(50)	(250)	-	-	-	0	15.0
	7	1;26;3;4; 5;78;9	296,250	(200)	(125)	(250)	(250)	(225)	(50)	(250)	-	-	0	22.5
	8	1;26;3;4; 5;7;8;9	423,750	(200)	(125)	(250)	(250)	(225)	(300)	(200)	(250)	-	0	30.0
	9	1;2;3;4; 5;6;7;8;9	576,250	(200)	(325)	(250)	(250)	(225)	(250)	(300)	(200)	(250)	0	37.5
6	5	1;23;45; 69;78	61,250	(200)	(125)	(25)	(50)	(50)	-	-	-	-	0	7.5
	6	1;23;45; 67;8;9	168,750	(200)	(125)	(25)	(100)	(200)	(250)	-	-	-	0	15.0
	7	1;23;4; 5;67;8;9	281,250	(200)	(125)	(250)	(225)	(100)	(200)	(250)	-	-	0	22.5
	8[a]	1;23;4; 5;6;7;8;9	423,750	(200)	(125)	(250)	(250)	(225)	(300)	(200)	(250)	-	0	30.0
	9	1;2;3;4; 5;6;7;8;9	576,250	(200)	(325)	(250)	(250)	(225)	(250)	(300)	(200)	(250)	0	37.5
7 or 8 or 9	4[b]	19;234; 56;78	8,750	0	75	(25)	(50)	-	-	-	-	-	1.25	1.25
	5	1;23;45; 69;78	61,250	(200)	(125)	(25)	(50)	(50)	-	-	-	-	0	7.50
	6	1;23;45; 67;8;9	168,750	(200)	(125)	(25)	(100)	(200)	(250)	-	-	-	0	15.00
	7	1;23;4; 5;67;8;9	281,250	(200)	(125)	(250)	(225)	(100)	(200)	(250)	-	-	0	22.50
	8[a]	1;23;4;5; 6;7;8;9	423,750	(200)	(125)	(250)	(250)	(225)	(300)	(200)	(250)	-	0	30.00
	9	1;2;3;4; 5;6;7;8;9	576,250	(200)	(325)	(250)	(250)	(225)	(250)	(300)	(200)	(250)	0	37.50
9[c]	3	126;345; 789	68,750	125	175	150	-	-	-	-	-	-	7.50	0

[a] Alternative solution for R' = 6, 7, 8 and 9 is 1;26;4;5;3;7;8;9.

[b] Alternative solution for 8 or 9 precincts is 19;236;45;78.

[c] Solution for 4 through 9 precincts is same as for R' of 7 or 8.

observation is that the objective function will decrease for the same m value as R' increases, but the total deviation from 7.5 hours is the same. For example, $R' = 4$, $m = 7$ has $C = 311,250$ and a deviation of zero above and 22.5 below, while $R' = 5$, $m = 7$ has $C = 296,250$ with the same deviation figures. Thus, from the viewpoint of the objective function, a better solution is obtainable but inequity in the precincts is the same.

Since the design constant of 450 minutes is employed, allocated time will deviate unequally above and below it. Figure 7 is a plot of the family of response distances for net allocated time deviation from the constant for the considered m values. The curves coincide due to the use of the design constant. It is evident that the smallest net deviation (1.25 hours above and below the constant) is found when $m = 4$ and the response distance is $R' = 7$ (same result for $R' = 8$ or 9).

Observation of Table V for deviation shows that for a particular value of m net deviation is constant and it increases as m increases. In fact, deviation varies from +7.5 hours for $m = 3$ to -37.5 hours for $m = 9$ with other m value deviations falling between these in increasing order of m. This is true since as more precincts are used each becomes smaller and objective contribution and deviation increase (deviation increase is below design constant).

Further Considerations Which Equalize Work Load

Use of census tracts or other techniques to separate a division may not be well suited to the distribution of the patrol force. Separation into small districts should be done for the purpose of

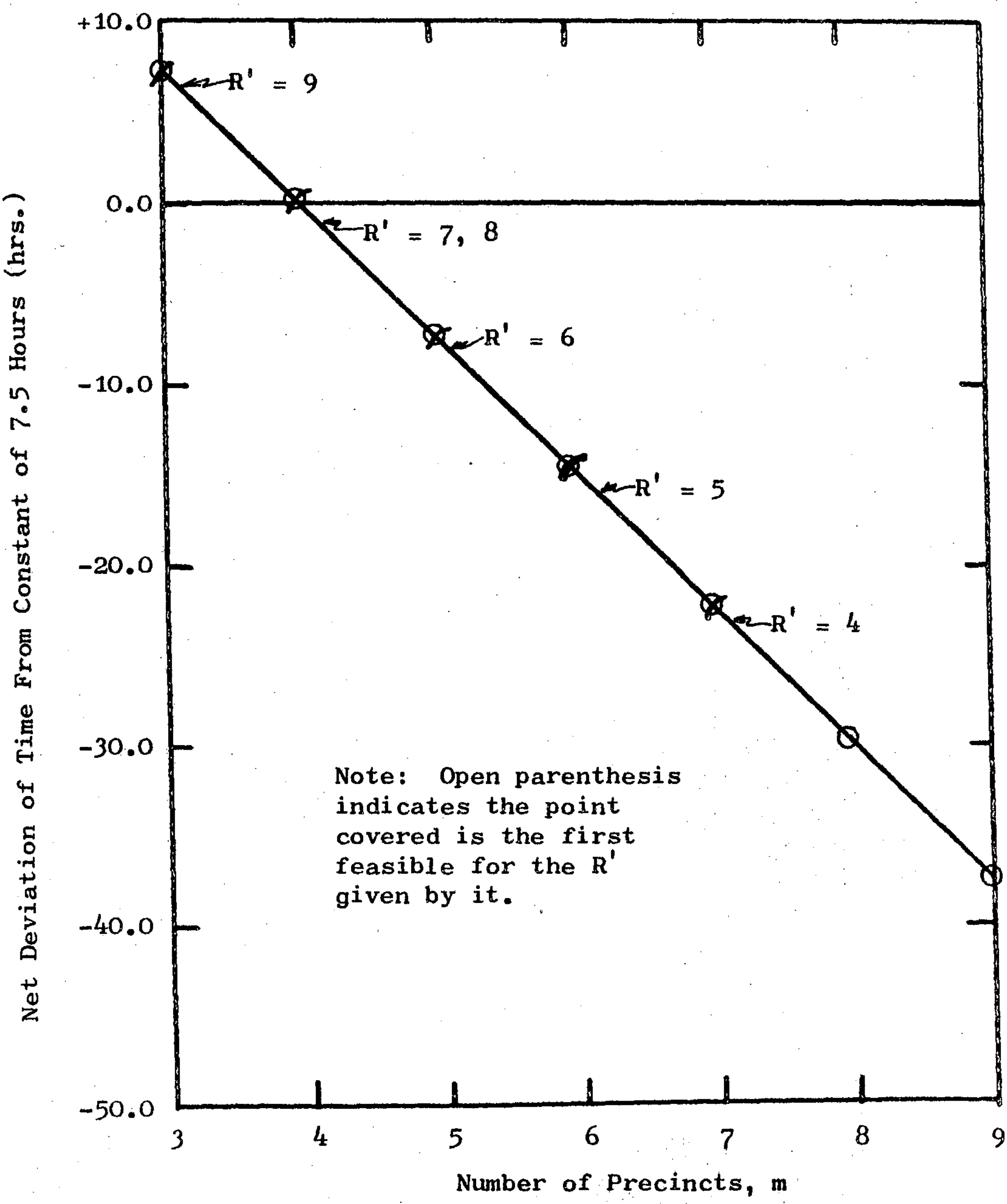


Figure 7. A Plot of Net Deviation From Design Constant Versus Number of Precincts for Considered Values of Response Distance Restriction

collecting data about the consumed or allocated time in each precinct.[5] This separation should be made to equalize allocated time per district, not to simply separate the city into small areas.

Consider briefly the problem just worked which minimizes mean deviations squared (results in Table IV). Table VI analyzes the deviations in terms of the fraction of total deviation accounted for by the precinct in the optimal solution having the largest deviation. The columns of Table VI are obtained as follows:

(1) Solution having m precincts for which analysis is made;

(2) Average allocation in hours for m precincts;

(3) Minimum total mean deviation (above or below mean) for solutions having m precincts;

(4) Total allocation time for all precincts above average. (4) = (2) + (3);

(5) Total allocation time for all precincts below average. (5) = (2) - (3);

(6) Mean deviation above average of precinct having maximum deviation. This is for solution with deviation given in (3). Value is enclosed in Table IV (minutes);

(7) Mean deviation below average of precinct having maximum deviation. Value is enclosed in Table IV (minutes);

(8) Fraction of total deviation given in (3) accounted for by the precinct with mean deviation given in (6). The value in parenthesis is the number of precincts with deviation above average; and

(9) Fraction of total deviation, (3), accounted for by mean deviation in (7). The value in parenthesis is the total

TABLE VI

ANALYSIS OF SOLUTIONS WHICH MINIMIZE MEAN DEVIATIONS SQUARED

Number Pre-cincts, m	Average Alloca-tion, $\bar{a}_m$	Minimum Total Mean Deviation (hrs.)	Total Allocation for Precincts		Maximum Deviation		Fraction of Total Deviation in Maximum Deviating Precinct	
			Above Average (hrs.)	Below Average (hrs.)	Above Average (hrs.)	Below Average (hrs.)	Above Average (hrs.)	Below Average (hrs.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3	10.00	0.42	10.42	9.58	0.42	0.42	1.00 (1)	1.00 (1)
4	7.50	1.25	8.75	6.25	1.25	0.84	1.00 (1)	0.67 (2)
5	6.00	2.42	8.42	3.58	1.08	1.83	0.45 (3)	0.76 (2)
6	5.00	3.33	8.33	1.67	2.08	1.67	0.62 (3)	0.50 (3)
7	4.28	2.67	6.95	1.61	1.56	0.95	0.59 (2)	0.36 (5)
8	3.75	2.50	6.25	1.25	1.67	1.25	0.67 (3)	0.50 (4)
9	3.33	2.08	5.41	1.25	0.84	1.25	0.40 (3)	0.61 (2)

number of precincts with deviation below average. Figure 8 indicates by m value the total deviation from mean and the value for the precinct with largest deviation. It is seen that the amount accounted for, as indicated by the fractions of Table VI, are much more than proportional. For example, when m = 8 three precincts have deviation above average and 0.67 is absorbed by the largest deviating precinct. Also for m = 8, four precincts are below average and 0.60 of total deviation is in the largest deviation below average. If this condition is not satisfactory, something must be done to better the equity of allocated time per precinct.

Restricting Mean Deviation Per Precinct

A restriction of the form

$$(1.0 - \alpha_L)\bar{a}_m \leq \sum_{i=1}^{n} a_i x_{ij} \leq (1.0 + \alpha_U)\bar{a}_m \qquad \begin{array}{l} j = 1, 2, \ldots, m \\ i = 1, 2, \ldots, n \end{array}$$

could be added, where $0 \leq \alpha_L \leq 1.0$ and $\alpha_U \geq 0$, which would prohibit any precinct combination from having an allocation time that deviated more than α_U above or α_L below the mean. If the values of $\alpha_U = \alpha_L = 0.2$ are assumed for the nine district problem of Figure 2, the following limits are placed upon precinct allocation time.

Number of Precincts, m	Precinct Average $\bar{a}_m$ (min.)	Limits on Allocation Time: Minimum (min.)	Maximum (min.)
3	600	480	720
4	450	360	540
5	360	288	432
6	300	240	360
7	257	205	309
8	225	180	270
9	200	160	240

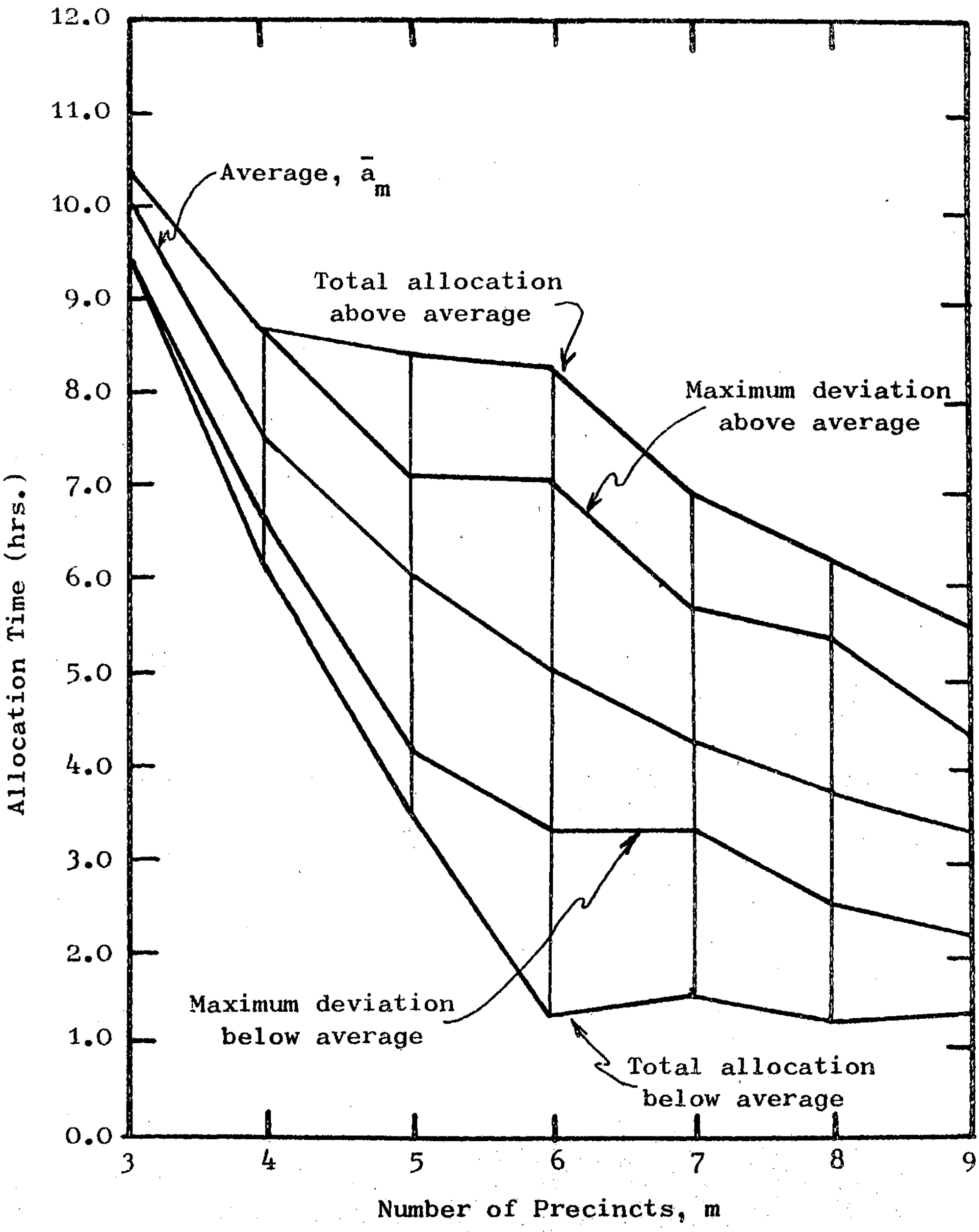


Figure 8. Comparison of Total and Maximum Precinct Mean Deviation for m Precincts

Inclusion of this restriction reduces the number of feasible precincts, number of solutions and the m and R' values for which solutions are obtainable. In most cases given a value of R', some m values may have no precinct available containing some of the districts. This is true here for R' of 4, 5, and 6. Once a district is not included in any precinct, solution is not possible. In other cases, all districts are in precincts, but solution is not possible with those available for a particular value of m. Table VII lists feasible precincts for $R' = 7$ using the restriction presented above. Solution is possible for $m = 4$ only, even though all districts are included in precincts for $m = 5$. Table VIII gives additional precincts for increases in R'. Solution is possible for $m = 4$ for R' values of 7, 8, and 9 and for $m = 3$, $R' = 9$. In all cases solutions are identical to those of Table IV.

Utilizing Districts of Approximately Equal Time

The designer of patrol distributions has available many types of data making it possible to divide the city into districts of approximately equal duty time. This done, the division may be divided into m precincts. If some sort of percentages are desired for call versus patrolling work, this may be considered when districts are organized. Patrolling time may be computed from averages estimated for time to patrol different classes of blocks--city, warehouse, residential.

The decision of how large a time each district should approximate may be made by deciding upon the smallest amount of time that can be reasonably added to or taken from a precinct. For example, if precincts are to be incremented in size by one hour intervals, the example problem

TABLE VII

FEASIBLE PRECINCTS WITH MEAN DEVIATIONS OF 20% OR LESS; R' = 7

Precinct	Response Distance, R' (miles)	Allocation Time (min.)	Mean Deviations Squared[a] m = 4 (min.^{2})	m = 5 (min.^{2})
12	7	375	5625	225
19	7	450	0	-
26	5	325	-	1225
267	6	475	625	-
23	6	325	-	1225
234	7	525	5625	-
34	4	400	2500	1600
35	6	425	625	4225
45	4	425	625	4225
56	5	425	625	4225
67	3	350	-	100
69	5	400	2500	1600
78	5	400	2500	1600
89	7	450	0	-

[a]No entry indicates the precinct is not feasible for the m value.

TABLE VIII

ADDITIONAL PRECINCTS FOR INCREASED R' WITH MEAN DEVIATIONS OF 20% OR LESS

Precinct	Allocation Time (min.)	Mean Deviations[a] Squared m = 3 (min.^{2})	m = 4 (min.^{2})
R' increased from 7 to 8			
269	525	-	5625
236	525	-	5625
R' increased from 8 to 9			
169	650	2500	-
126	575	625	-
256	550	2500	-
235	550	2500	-
2679	675	5625	-
2567	700	10000	-
2367	675	5625	-
345	625	625	-
356	625	625	-
456	625	625	-
569	625	625	-
567	575	625	-
679	550	2500	-
678	600	0	-
689	650	2500	-
789	600	0	-

[a]No entry indicates the precinct is not feasible for the m value.

would require 30 (1800/60) districts each of an allocation time of about one hour. Since certain facts may restrict the drawing of district boundaries, the precinct design problem would probably still exist, however, the mean deviations would be reduced with the much wiser choice of districts, that is, districts designed for patrol work, not another purpose. There are undoubtedly many other ways in which the districts could be designed than those in this paper, but the assumed district boundaries serve to illustrate the inequity present in optimal solutions.

Algorithm for Probabilistic Case

Consideration of the district response distances, r_i as random variables having a pdf, $f(r_i)$, a mean μ_{r_i} and variance, $\sigma^2_{r_i}$ presents the problem formulated earlier in this chapter. In order to combine districts in this case the density of $\bar{r}_j$ must be found by taking the convolution of the r_i's in the precinct, that is,

$$\bar{r}_j = \sum_{i=1}^{n} r_i x_{ij} , \qquad j = 1, 2, \ldots, m$$

where

$$x_{ij} = \begin{cases} 0 \text{ if district } i \text{ is not in precinct } j \\ 1 \text{ if the district } i \text{ is in precinct } j. \end{cases}$$

In order to satisfy the restriction

$$P\left(\sum_{i=1}^{n} \bar{r}_j \leq R \right) \geq \alpha ,$$

where the R and α are assumed the same for all j, the density of $\bar{r}_j$ such that

$$\int_{-\infty}^{R} \bar{r}_j \, d\bar{r}_j \geq \alpha \tag{2}$$

(assuming r_i's are continuous variables) is satisfied must be found. Any combination of contiguous districts, once $f(\bar{r}_j)$ is known, for which the above integral relation is not true cannot be considered a feasible precinct.

Solution of the probabilistic problem is in theory the same as that for the deterministic case. The generation of feasible precincts is accomplished according to guidelines above, however, the distribution of each r_i must be known and the convolution of them to find $f(\bar{r}_j)$ must exist and be found. This is not always easily accomplished or even possible. Phase II of the solution entails the same procedure as for the deterministic case. At any point, once a solution order of districts has been defined, if any precinct includes previously assigned districts, it is judged ineligible. Also, if assignment of a precinct would not leave enough distance remaining for the unassigned districts, the solution branch is abandoned. This would occur if for a particular precinct, k ($k \leq m$), there being (m-k) precincts remaining, it would not be possible to combine unassigned districts into (m-k) precincts such that (2) was true for each. Unfortunately, this conclusion may not always be made for all remaining precincts as in the deterministic case, since $f(\bar{r}_j)$ changes with combinations of r_i's and a 'lumping' of unassigned r_i's to obtain a density will not allow conclusions about the remaining precincts. Again, once a precinct is assigned the objective may be accumulated and the previously stated exclusion principles applied.

Example With Normality Assumption

The probabilistic response distance problem may be worked relatively easy if each r_i is assumed to have a normal distribution. Then it is possible to find the distribution of $\bar{r}_j$ by using the relationships that precinct mean response distance, $\bar{\mu}_{r_j}$, and variance, $\bar{\sigma}^2_{r_j}$, are, respectively,

$$\bar{\mu}_{r_j} = \sum_{i=1}^{n} \mu_{r_i} x_{ij} \text{ , and}$$

$$\bar{\sigma}^2_{r_j} = \sum_{i=1}^{n} \sigma^2_{r_i} x_{ij} \text{ .}$$

Larson presents in his research graphs of derived densities of response distances.[6] These densities are unimodal and 'relatively' symmetric. With this evidence and the fact that normal distributions are additive the assumption of normal r_i's is made in this section.

In order to generate feasible precincts for the normal case, the standard normal relation may be used. The following calculations indicate the final relation to be sufficient evidence that a combination of districts is or is not a feasible precinct.

$$P(\bar{r}_j \leq R) \geq \alpha$$

$$P\left(\frac{\bar{r}_j - \bar{\mu}_{r_j}}{\bar{\sigma}_{r_j}} \leq \frac{R - \bar{\mu}_{r_j}}{\bar{\sigma}_{r_j}} \right) \geq \alpha$$

$$P\left(Z_j \leq \frac{R - \bar{\mu}_{r_j}}{\bar{\sigma}_{r_j}} \right) \geq \alpha$$

or

$$Z_j \leq \frac{R - \bar{\mu}_{r_j}}{\bar{\sigma}_{r_j}}$$

$$Z_j \bar{\sigma}_{r_j} + \bar{\mu}_{r_j} \leq R \,, \qquad j = 1, 2, \ldots, m$$

where Z_j is the standard normal deviate. The last relation must be true for the precinct to be feasible. It may be written

$$Z_\alpha \bar{\sigma}_{r_j} + \bar{\mu}_{r_j} \leq R \qquad j = 1, 2, \ldots, m$$

to emphasize that Z_α is the deviate corresponding to the point, R on the normal density such that the integral over the interval $(-\infty, R)$ is equal to α.

Also helpful is the relation below which must be satisfied for all k (k = 1, 2, ..., m).

$$\left[\sum_{i=1}^{n} \mu_{r_i} + Z_\alpha \left(\sum_{i=1}^{n} \sigma_{r_i}^2\right)^{\frac{1}{2}}\right] - \left[\sum_{j=1}^{k} \bar{\mu}_{r_j} + Z_\alpha \bar{\sigma}_{r_j}\right] \leq (m-k)R \,. \qquad (3)$$

For the normal, as in the deterministic case, if at any k (3) is not true, the remaining response distance will not fit into that remaining available and solution may be halted for that branch. The precincts considered, j = 1, 2, ..., k, may be an arbitrary set with k elements selected from m precincts.

The same example employed thus far is again used with minor alterations. The previously given r_i values have been multiplied by ten and assumed to be means of the normal distributions. Accompanying variances are given with other necessary data in Table IX. The problem is solved using $R' = 7$ and $\alpha = 0.90$ which implies that $Z_{0.90} = 1.28$. (Since the r_i have been increased tenfold, the problem may be thought of as being involved with response time, rather than distance. Since this is merely a transformation in thought, the nomenclature is not altered from that used thus far.) With this information it is seen that any combination of contiguous districts that satisfy the relation

$$1.28\bar{\sigma}_{r_j} + \bar{\mu}_{r_j} \leq 7 \qquad j = 1, 2, \ldots, m$$

may be considered a feasible precinct. A listing of feasible precincts (not by district) and associated objective contribution for m = 5 are given in Table X. (Solution for m = 4 is not possible.) Total resource in the problem for m precincts, from the left bracketed expression of (3), is

$$24 + 1.28(1.78)^{1/2} = 25.71 .$$

Thus, if m = 5 using (3) the minimum total that

$$\sum_{j=1}^{k} \bar{\mu}_{r_j} + Z_\alpha \bar{\sigma}_{r_j}$$

can assume is as listed below.

Precinct Number, k	Minimum for $\sum_{j=1}^{k} \bar{\mu}_{r_j} + Z_\alpha \bar{\sigma}_{r_j}$ after k precincts
1	0.00 (rounded)
2	4.71
3	11.71
4	18.71
5	25.71

TABLE IX

EXAMPLE WITH NORMALLY DISTRIBUTED RESPONSE TIMES

District,	Allocation Time, a_i (min.)	Mean of Response Time, μ_{r_i} (min.)	Variance of Response Time, $\sigma^2_{r_i}$ (min.^{2})
1	250	4	0.49
2	125	3	0.25
3	200	3	0.25
4	200	1	0.04
5	225	3	0.09
6	200	2	0.04
7	150	1	0.04
8	250	4	0.49
9	200	3	0.09
TOTAL	1800	24	1.78

TABLE X

FEASIBLE PRECINCTS FOR PROBABILISTIC EXAMPLE

Precinct	$\bar{\mu}_{r_j}$	$\bar{\sigma}^2_{r_j}$	$1.28\bar{\sigma}_{r_j} + \bar{\mu}_{r_j}$	Allocation Time (min.)	Mean Deviation Squared m = 5
1	4	0.49	4.895	250	12100
2	3	0.25	3.640	125	55225
26	5	0.29	5.688	325	1225
267	6	0.33	6.735	475	13225
23	6	0.50	6.905	325	1225
3	3	0.25	3.640	200	25600
34	4	0.29	4.688	400	1600
35	6	0.34	6.747	425	4225
4	1	0.04	1.205	200	25600
45	4	0.13	4.463	425	4225
456	6	0.17	6.528	625	70225
5	3	0.09	3.384	225	18225
56	5	0.13	5.463	425	4225
567	6	0.17	6.528	575	46225
6	2	0.04	2.205	200	25600
67	3	0.08	3.363	350	100
69	5	0.13	5.463	400	1600
679	6	0.17	6.528	550	36100
7	1	0.04	1.205	150	44100
78	5	0.53	5.933	400	1600
8	4	0.49	4.895	250	12100
9	3	0.09	3.384	200	25600

Solution of the problem for m = 5 results in the same solution given in Table IV, since none of the precincts in this solution have become infeasible when the probabilistic case is considered. Notice however, that in solution the response resource quantity accumulated is not the mean now, but the expression involving the mean and standard deviation.

Observations About the Problem

One of the results of the analysis is the observation that choice of district configuration must be considered. Data collection should be performed and compilation done so that fairly equitable time allocation is possible for each precinct. Here, the breakdown of duties in a district has not been detailed, whereas actually some type of ratio of call duty time to patrol duty time would be employed.

The problem used as an example for which response distance restriction and number of precincts was changed seems to increase quite rapidly if total number of feasible precincts is counted (see Appendix). However, since the precincts are listed by district for easy solution of the problem, a precinct with four districts is listed four times. The comparison below indicates that the number of different precincts increases at a lesser rate than the exponentially growing total number of precincts listed by district.

Response Distance, R'	Number of Precincts Added	Total Number of Precincts	Number of Different Precincts Added	Total Number of Different Precincts
4	15	15	12	12
5	8	23	4	16
6	16	39	6	22
7	19	58	8	30
8	18	76	6	36
9	47	123	14	50

FOOTNOTES

[1]Richard C. Larson, Models for the Allocation of Urban Police Patrol Forces (Cambridge: Massachusetts Institute of Technology, 1969), pp. 101-104.

[2]P. C. Gilmore and R. E. Gomory, "The Theory and Computation of Knapsack Functions," Operations Research, XIV (1966), pp. 1045-1074.

[3]G. Douglas Gourley and Allen P. Bristow, Patrol Administration (Springfield, Ill.: Charles Thomas, 1961), pp. 95-97.

[4]Allen P. Bristow, Effective Police Manpower Utilization (Springfield, Ill.: Charles Thomas, 1969), pp. 109-111.

[5]Ibid., p. 108.

[6]Larson, p. 119.

CHAPTER III

PATROL CAR LOCATION FOR UNDERSTAFFED PRECINCTS

This chapter investigates the problem presented when a precinct does not have sufficient cars and/or men to adequately cover the entire precinct. It has been assumed thus far that the floating or team approach is utilized. Larson has developed a dynamic programming method to determine the minimum number of cars needed to perform the duties of the precinct.[1] Larson also discusses briefly a method called 'fixed-point-prepositioning' in which the patrolmen are stationed at a site and have primarily the duties of answering calls in that district and those surrounding it.[2] Utilization of this type of locating procedure is possible for an understaffed precinct.

The problem discussed is fairly common due to the frequency with which cars are damaged, require routine maintenance and are assigned for a time to another duty. Also, men working the precinct may become ill, quit or be reassigned.

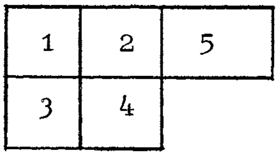
Consideration of the problem's objectives and restrictions and an example are presented in this chapter. The solution technique is explained in brief and problem solved in detail to make better understanding possible. This is the first known application of this technique to this particular field of study.

Problem Statement

This again is a multi-dimensional knapsack problem, since it is assumed some objective can be defined and several types of constraint exist. Provided the precincts have been designed to restrict response distance or time, objectives may be to minimize the response distance or response mean deviations squared. Since the precincts are divided into districts, data available about crime rates, patrol duty time and the like for a shift may be used. Objectives may be related to crimes covered or total square mileage encompassed by the prepositioned patrol car. Again, since many objectives and restrictions are possible, these are selected here and utilized as an example of the situation.

Restrictions to Insure Complete Coverage

Of all the restrictions that may be imposed certain types are of greater importance than others. One of the more desirous will probably be the ability to cover the entire precinct with call answering services using only the limited number of cars. Thus, it is necessary to develop a set of constraints to be included in the problem statement that insure this capability. For illustration purposes assume the precinct shown below is composed of five districts and calls are answered by cars as indicated. For example, a car assigned to district 1 will answer calls in districts 1, 2, and 3. If the car is occupied, that is, answering another call or performing some other duty, the call will wait; one of the drawbacks of understaffed precincts.



District	Answer Calls in
1	1, 2, 3
2	2, 4
3	1, 3
4	2, 3, 4
5	2, 5

Since at least one car must be able to answer calls in each district, a simple rearrangement of the above data allows the formulation of constraints to insure complete coverage. Rewrite the answering duties by district. That is, indicate the districts that answer calls for a district. Thus a set, A_i $(i = 1, 2, \ldots, n)$ of districts for each district is developed. For the example, A_i's are:

District, i	Calls Answered by, A_i
1	1, 3
2	1, 2, 4, 5
3	1, 3, 4
4	2, 4
5	5

If the definition

$$x_i = \begin{cases} 0 \text{ if car is not assigned to district } i \\ 1 \text{ if a car is assigned to district } i \end{cases}$$

is used, assurance of coverage is obtained by writing the restrictions

$$\begin{aligned} x_1 + x_3 &\geq 1 \\ x_1 + x_2 + x_4 + x_5 &\geq 1 \\ x_1 + x_3 + x_4 &\geq 1 \\ x_2 + x_4 &\geq 1 \\ x_5 &\geq 1 . \end{aligned} \tag{1}$$

It is seen that a car is definitely assigned to $i = 5$, this being the only way calls are answerable in that district. (It may be that initial definition of duties will not cover all districts with cars available, in which case expansion of duties is warranted.)

It is possible to delete certain of the restrictions due to repetition within them. All restrictions for coverage are of the form

$$\sum_{j \in A_i} x_j \geq 1 , \qquad i = 1, 2, \ldots, n$$

where the symbol $\in$ indicates that j is a member of the set A_i. Consider two inequalities of the above form

$$\sum_{j \in A_g} x_j \geq 1 \quad \text{and} \quad \sum_{j \in A_h} x_j \geq 1 ,$$

where g and h are integers $(1 \leq g, h \leq n)$. If A_g is a subset of A_h, $A_g \subset A_h$, the restriction involving A_h may be deleted from the constraints provided every district in the deleted constraint is included in at least one constraint in the reduced set. Inclusion in at least one constraint is necessary to insure coverage for each district. For example, number the constraints in (1) to correspond with A_i, that is, the first is number 1 for A_1, the second 2 for A_2, etc. Consider constraint 1 $(g = 1)$ and 3 $(h = 3)$. Since $A_1 \subset A_3$, deletion of constraint 3 is possible. Likewise, since $A_4 \subset A_2$, constraint 2 is omitted. In all cases notice the districts are still included in the remaining system. Resulting constraints to insure coverage are

$$x_1 + x_3 \geq 1$$
$$x_2 + x_4 \geq 1$$
$$x_5 = 1 .$$

Notice also, that since $A_5 \subset A_2$ constraint 2 could have been deleted in this fashion.

Other Considerations

Besides the choice of objective function and the necessity of coverage constraints, other considerations are possible, mostly of a constraint form. Number of cars and men available present restrictions. Supplementation of this force with scooters or footmen may be desirous. Predetermination of car assignments may cause the inclusion of logical constraints to be necessary. As before, each case is different and the idiosyncrasies of each precinct must be determined. Probably the definition of districts in which a car is to answer calls will play an important part in the degree to which available resources are used effectively.

Example Mathematical Formulation

The type of optimization problem chosen for illustrative purposes attempts to minimize a combination of response distance properties, namely, sum of response distance in the precinct and response mean deviations squared. Restrictions are established for number of cars and men available. Thus, definition of the number of men per car for each district is needed (see Nomenclature).

The problem may now be stated as

$$\text{Minimize } Z = \sum_{i=1}^{n} [r_i + (r_i - \bar{r})^2]$$

Subject to $\sum_{i=1}^{n} x_i \leq b$

$$\sum_{i=1}^{n} s_i x_i \leq f$$

coverage constraints

$$x_i = 0, 1 .$$

It should be noted that the objective is a compromise function since it may minimize neither of the components, however, it is assumed that both distance and dispersion are important. The statement 'coverage constraints' is understood to include all coverage type restrictions and $\bar{r}$ is the average response distance for the precinct under a particular scheme of assignment to districts, that is,

$$\bar{r} = \frac{\sum_{i=1}^{n} r_i x_i}{b} .$$

Solution Technique--Pseudo-Boolean Programming

This type of problem is not unlike many others in which placement of a limited number of items into a set number of places is necessary. The variables solved for are zero-one and any solution technique able to handle them will suffice for solution. However, a technique developed by Ivanescu and Rudeanu,[3] called Pseudo-Boolean Programming for Bivalent (0, 1) Variables is used since it is able to handle highly restrained problems rather easily. A brief discussion of Boolean and pseudo-Boolean functions is included here and rules for solving certain types of pseudo-Boolean problems are detailed. Since this type of problem entails use of only a small part of the capabilities of pseudo-Boolean programming, it is recommended that reference be made to the listing in the bibliography for complete details. References by Ivanescu and Rudeanu (1966) and Hammer and Rudeanu (1968) are very helpful in understanding the application of the technique.

Concept of Pseudo-Boolean Functions and Programming

A Boolean variable is one that can assume one of two values--0 or 1. It follows the negation rule, that is, if x is a Boolean variable, the following holds.

x	0	1
$\bar{x}$	1	0

This is briefly stated as $\bar{x} = 1 - x$. Two operators used are union ($\cup$) and intersection ($\cap$). If Boolean variables, x and y, are considered the operators are defined as:

	$\cup$	y = 0	y = 1
x	0	0	1
	1	1	1

	$\cap$	y = 0	y = 1
x	0	0	0
	1	0	1

A Boolean function is an algebraic relation composed of Boolean variables. The function

$$x\bar{y} \cup yx\bar{z} \cup xz$$

is a Boolean function in which x, y, and z are Boolean variables (intersection symbol omitted). The function will take on the value zero or one depending on x, y, and z values.

A pseudo-Boolean function

$$f(x_1, x_2, \ldots, x_n)$$

is a real valued function in which the variables are Boolean. One of the useful features of such a function is its ability to always be written as a polynomial linear in each of its variables. Consider the pseudo-Boolean function

$$f(x, y) = 3 + 5x - 5y + 2xy.$$

Values of the function may be defined as follows.

x	y	f(x, y)
0	0	3
0	1	-2
1	0	8
1	1	5

Therefore, if the expression considered contains n Boolean variables, there are 2^n possible values of the pseudo-Boolean function.

Pseudo-Boolean equations and inequalities may be written in the usual manner. A linear pseudo-Boolean inequality is of the form

$$a_1x_1 + a_2x_2 + \ldots + a_mx_m * c,$$

where a_i $(i = 1, 2, \ldots, n)$ and c are real numbers, x_i a Boolean variable and * an inequality operator. A pseudo-Boolean inequality is referred to as non-linear if it assumes the form

$$a_1p_1 + a_2p_2 + \ldots + a_mp_m * c,$$

where the p_i are products of Boolean variables, for example, p_3 might represent the intersection $x_1x_4\bar{x}_7$.

Since drastically different rules are involved in solving the different types of inequalities and since only linear inequalities are involved in the car location problem, solution techniques for only linear systems of equations and inequalities are covered here.

Pseudo-Boolean programming involves the finding of a solution set for the system of linear relations (equations and/or inequalities). Once the set is discovered, the objective function is evaluated for each member of the set and selection of the optimizing solution made. The determination of a variable value in one relation is carried forth into all others and entire solutions are determined in this manner. Since this carrying forth is possible, highly constrained problems may be solved quite rapidly by hand. Of course, if the problem is loosely constrained, the feasible solution set is lengthy, implying the need for computerization.

Pseudo-Boolean Programming for a System of Linear Relations

Solution of single pseudo-Boolean relations is possible, but, since systems of relations are usually the case, rules are presented for the solution of systems.

Assume there is a system of inequalities and equations that have been transformed so that the i^{th} relation is of the form $F \geq 0$ or $G = 0$, where F and G are algebraic relations. Transformation may be necessary from relations of forms involving $<$, $\leq$ or $>$ inequalities. Using the relations $\bar{x}_i = 1 - x_i$ and $x_j = 1 - \bar{x}_j$ write the i^{th} inequality in the form

$$a_1^i \tilde{x}_1 + a_2^i \tilde{x}_2 + \ldots + a_m^i \tilde{x}_m \geq d^i,$$

where $x_1, x_2, \ldots, x_m$ are variables upon which the i^{th} inequality depends, $\tilde{x}_j$ ($j = 1, 2, \ldots, m$) represents either x or $\bar{x}$, $a_1, a_2, \ldots, a_m$ are real constants in the i^{th} inequality and these follow the order

$$a_1^i \geq a_2^i \geq \ldots \geq a_m^i > 0.$$

The equations in the system are written in a similar manner. Once written in this canonical form, the relations each fall into a category usable in determining values of $\tilde{x}_j$.

The rules which allow solution of equations written in canonical form are presented in Table XI while those for inequalities are given in Table XII. (Tables are adopted from Ivanescu and Rudeanu (1966, pp. 39 and 41).) In each case the categorization should be such that all appearing variables are determined by one rule. However, often this is not possible and other avenues of approach are necessary. For selection of a relation to work on first, the following order of preference is given.

Preferential Order	Equation (Rule No.)	Inequality (Rule No.)	Level of Determination of Variables
1	1, 5 2, 6 3, 7	3 1, 4 5	determinate
2	4	2	partially determinate
3	8	6	undeterminate

It is seen that for order one, the first set of rules determines all variables in that no solutions exist, while remaining sets determine one variable. The second and third order can only simplify the relation to an extent by systematically searching possibilities in order to get

TABLE XI

RULES FOR SOLVING A SYSTEM OF LINEAR PSEUDO-BOOLEAN EQUATIONS[a]

Rule Number	Case	Conclusion	Information	
			Fixed Variables[b]	Remaining Equation
1	$d < 0$	No solution	-	-
2	$d = 0$	All fixed	$\tilde{x}_1 = \dots = \tilde{x}_m = 0$	-
3	$d \geq 0$ and $c_1 \geq \dots \geq c_p > d \geq c_{p+1} \geq \dots \geq c_m$	Part of appearing fixed	$\tilde{x}_1 = \dots = \tilde{x}_p = 0$	$\sum_{j=p+1}^{m} c_j\tilde{x}_j = d$
4	$d > 0$ and $c_1 = \dots = c_p = d > c_{p+1} \geq \dots \geq c_m$	p+1 branches	t_k: $\tilde{x}_k = 1$, $\tilde{x}_1 = \dots = \tilde{x}_{k-1} = \tilde{x}_{k+1} = \dots = \tilde{x}_m = 0$ $(k = 1, \dots, p)$	-
			t_{p+1}: $\tilde{x}_1 = \dots = \tilde{x}_p = 0$	$\sum_{j=p+1}^{m} c_j\tilde{x}_j = d$
5	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j < d$	No solution	-	-
6	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j = d$	All fixed	$\tilde{x}_1 = \dots = \tilde{x}_m = 1$	-
7	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j > d$, $\sum_{j=2}^{m} c_j < d$	One variable fixed	$\tilde{x}_1 = 1$	$\sum_{j=2}^{m} c_j\tilde{x}_j = d - c_1$
8	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j > d$, $\sum_{j=2}^{m} c_j \geq d$	Two branches	q_1: $\tilde{x}_1 = 1$	$\sum_{j=2}^{m} c_j\tilde{x}_j = d - c_1$
			q_2: $\tilde{x}_1 = 0$	$\sum_{j=2}^{m} c_j\tilde{x}_j = d$

[a]The rules given are for each equation in the system of equations and inequalities. Thus, each coefficient c_j $(j = 1, 2, \dots, m)$ and each constant d should be superscripted with an i $(i = 1, 2, \dots, n)$, where there are n equalities in the system.

[b]Subscripts on determined variables are for variables in canonical form.

TABLE XII

RULES FOR SOLVING A SYSTEM OF LINEAR PSEUDO-BOOLEAN INEQUALITIES[a]

Rule Number	Case	Conclusion	Information: Fixed Variables[b]	Information: Remaining Inequality
1	$d \leq 0$	Redundant inequality	-	-
2	$d > 0$, $c_1 \geq \ldots \geq c_p \geq d > c_{p+1} \geq \ldots \geq c_m$	p+1 branches	t_k: $\tilde{x}_1 = \ldots = \tilde{x}_{k-1} = 0$, $\tilde{x}_k = 1$ $(k=1,\ldots,p)$	-
			t_{p+1}: $\tilde{x}_1 = \ldots = \tilde{x}_p = 0$	$\sum_{j=p+1}^{m} c_j \tilde{x}_j \geq d$
3	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j < d$	No solution	-	-
4	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j = d$	All fixed	$\tilde{x}_1 = \ldots = \tilde{x}_m = 1$	-
5	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j > d$, $\sum_{j=2}^{m} c_j < d$	One fixed	$\tilde{x}_1 = 1$	$\sum_{j=2}^{m} c_j \tilde{x}_j \geq d - c_1$
6	$d > 0$, $c_j < d$ (all j), $\sum_{j=1}^{m} c_j > d$, $\sum_{j=2}^{m} c_j \geq d$	Two branches	q_1: $\tilde{x}_1 = 1$	$\sum_{j=2}^{m} c_j \tilde{x}_j \geq d - c_1$
			q_2: $\tilde{x}_1 = 0$	$\sum_{j=2}^{m} c_j \tilde{x}_j \geq d$

[a]The rules are given for each inequality in the system. Thus, each coefficient c_j $(j = 1, 2, \ldots, m)$ and each constant d should be superscripted with an $i^j (i = 1, 2, \ldots, n)$ where there are n inequalities in the system.

[b]Subscripts on determined variables are for variables in canonical form.

the relation into order one. Notice, as indicated in footnotes to the tables, the rules are for a relation in the system. By using the rules and reducing the system one variable at a time, feasible solutions are generated.

Importance of the Objective Function

Since pseudo-Boolean programming operates upon a system of relations, the objective is not important until the set of solutions is obtained. For this reason the form of the objective, provided it is written in terms of zero-one variables, is immaterial. (Hammer (February, 1969) does present a procedure for maximizing only.) Thus, restrictions being unchanged, the problem may be altered in content or even sense, minimize to maximize. This relation allows solution of problems in which the variables are in the exponent, if they are summed and each given a value of zero or one. Adaptation of the technique to many types of problems is foreseeable, some of which are indicated in Ivanescu (1965), Hammer (February, 1969; March, 1969) and Hammer and Shlifer (1969).

An Example and Its Solution

Assume that for a particular shift a precinct, such as the one shown in Figure 9, is understaffed. The precinct is composed of seven districts and usually requires seven cars, however, only three cars are available on a particular day. It is decided to place the three cars in districts and allow them to answer calls in that and defined neighboring districts. Since this situation occurs frequently the patrol force has determined the number of men needed per car, the set of districts A_i in

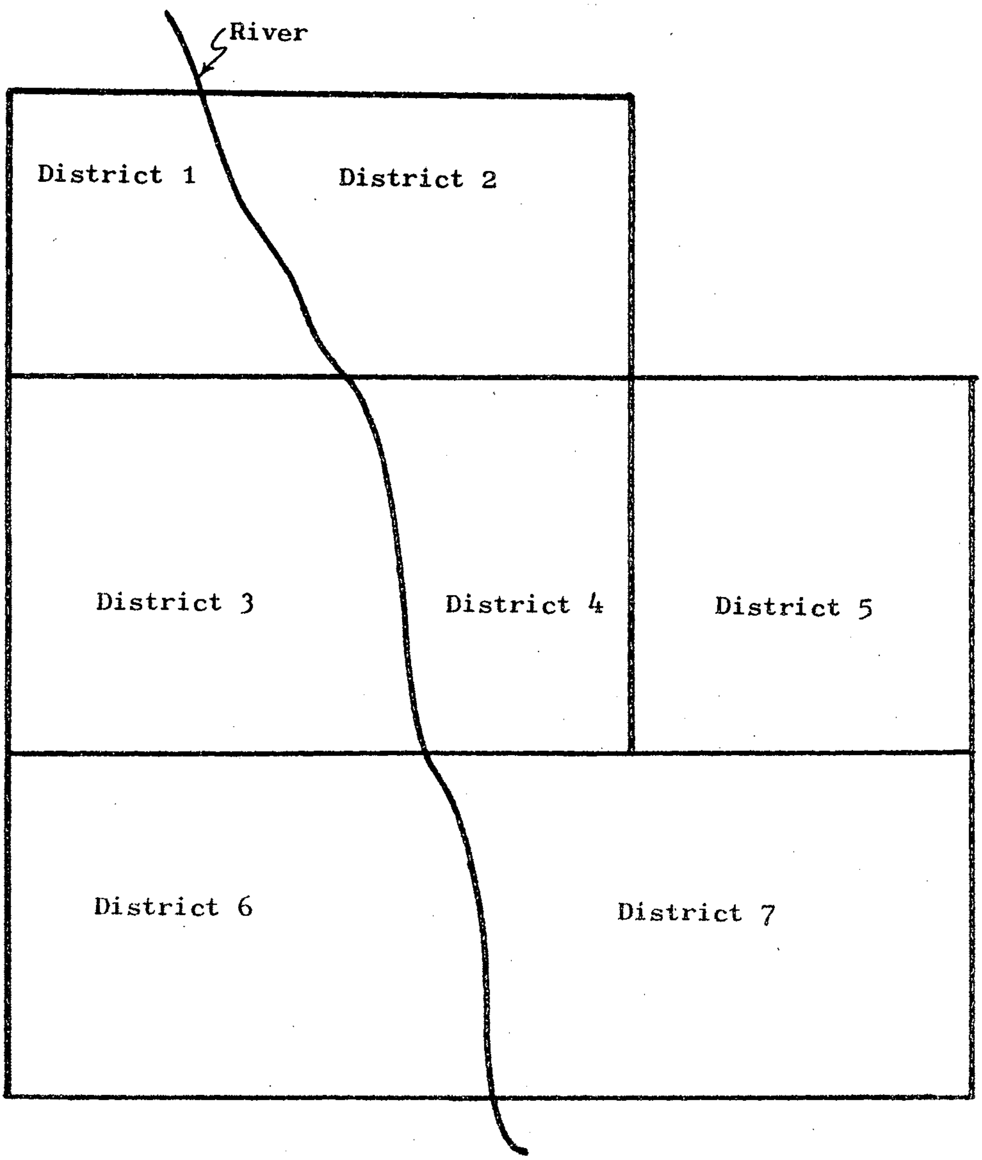


Figure 9. An Understaffed Precinct Composed of Seven Districts

which a car will answer calls, average response distance a car must travel if it is placed in district i and answers calls in A_i (i = 1, 2, ..., 7). If five men are available for the three cars, both men and car restrictions are utilized and

$$b = 3,$$

$$f = 5.$$

Additional information about the precinct is given below.

District	Patrolmen per car, s_i	Answer Calls In	Calls Answered by, A_i
1	1	1, 2, 3	1, 2
2	1	1, 2, 4	1, 2
3	2	3, 4, 6	1, 3, 4
4	2	3, 4, 5, 7	2, 3, 4, 5
5	2	4, 5, 7	4, 5, 7
6	1	6, 7	3, 6, 7
7	1	5, 6, 7	4, 5, 6, 7

From the A_i given it is possible to develop seven inequalities insuring complete coverage, however, since $A_1 \subset A_2$ and $A_5 \subset A_7$, the inequalities corresponding to A_2 and A_7 may be deleted, resulting in the system (2).

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ x_1 + x_3 + x_4 &\geq 1 \\ x_2 + x_3 + x_4 + x_5 &\geq 1 \\ x_4 + x_5 + x_7 &\geq 1 \\ x_3 + x_6 + x_7 &\geq 1 \end{aligned} \tag{2}$$

Remaining is the development of the pseudo-Boolean relations for the response distance in each district. Since this value depends on the assignment configuration of cars, r_i relations are generated from data for response distances under each configuration. Thusly, response distance information given below is assumed and relations below each set of data obtained by substituting values of x_i. When $x_i = 0$ for the district considered this implies calls are answered by another district's car, since no car is assigned to district i; therefore, $r_i = 0$.

x_1	x_2	x_3	r_1 (miles)
1	0	0	1.3
1	0	1	0.8
1	1	0	0.7
1	1	1	0.5
0	-	-	0.0

x_2	x_1	x_4	r_2 (miles)
1	0	0	1.5
1	0	1	0.7
1	1	0	1.0
1	1	1	0.5
0	-	-	0.0

$r_1 = 1.3x_1 - 0.5x_1x_3 - 0.6x_1x_2 + 0.3x_1x_2x_3.$

$r_2 = 1.5x_2 - 0.5x_1x_2 - 0.8x_2x_4 + 0.3x_1x_2x_4.$

x_3	x_4	x_6	r_3 (miles)
1	0	0	2.0
1	0	1	1.3
1	1	0	1.1
1	1	1	0.8
0	-	-	0.0

x_4	x_3	x_5	x_7	r_4 (miles)
1	0	0	0	2.2
1	1	0	0	1.3
1	0	1	0	1.1
1	0	0	1	1.2
1	1	1	0	0.8
1	1	0	1	0.7
1	0	1	1	0.8
1	1	1	1	0.5
0	-	-	-	0.0

$r_3 = 2.0x_3 - 0.7x_3x_6 - 0.9x_3x_4 + 0.4x_3x_4x_6.$

$r_4 = 2.2x_4 - 0.9x_3x_4 - 1.1x_4x_5 - 1.0x_4x_7 + 0.6x_3x_4x_5 + 0.4x_3x_4x_7 +$
$0.7x_4x_5x_7 - 0.4x_3x_4x_5x_7.$

x_5	x_4	x_7	r_5 (miles)
1	0	0	1.0
1	0	1	0.7
1	1	0	0.8
1	1	1	0.5
0	-	-	0.0

x_6	x_7	r_6 (miles)
1	0	0.8
1	1	0.6
0	-	0.0

x_7	x_5	x_6	r_7 (miles)
1	0	0	1.2
1	0	1	0.9
1	1	0	0.9
1	1	1	0.7
0	-	-	0.0

$r_5 = 1.0x_5 - 0.3x_5x_7 - 0.2x_4x_5.$

$r_6 = 0.8x_6 - 0.2x_6x_7.$

$r_7 = 1.2x_7 - 0.3x_6x_7 - 0.3x_5x_7 + 0.1x_5x_6x_7.$

Adding the car and men restrictions, the problem may be stated as:

$$\text{Minimize } Z = \sum_{i=1}^{7} [r_i + (r_i - \bar{r})^2]$$

$$\text{Subject to} \quad \sum_{i=1}^{7} x_i \leq 3 \tag{3}$$

$$\sum_{i=1}^{7} s_i x_i \leq 5 \tag{4}$$

$$x_1 + x_2 \geq 1 \tag{5}$$

$$x_1 + x_3 + x_4 \geq 1 \tag{6}$$

$$x_2 + x_3 + x_4 + x_5 \geq 1 \tag{7}$$

$$x_4 + x_5 + x_7 \geq 1 \tag{8}$$

$$x_3 + x_6 + x_7 \geq 1 \tag{9}$$

$$x_i = 0,\ 1 \qquad i = 1, 2, \ldots, 7$$

Rewriting the system in the canonical form alters (3) and (4) to the form (using $x_i = 1 - \bar{x}$)

$$\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4 + \bar{x}_5 + \bar{x}_6 + \bar{x}_7 \geq 4 \tag{3}$$

$$2\bar{x}_3 + 2\bar{x}_4 + 2\bar{x}_5 + \bar{x}_1 + \bar{x}_2 + \bar{x}_6 + \bar{x}_7 \geq 5. \tag{4}$$

The remaining inequalities are the same. Solution using the rules of Table XII is now summarized. (Rule numbers from Table XII are given in [].)

No relation is determinate, however, (5) follows [2] with $p = 2$, thus there are three branches:

t_1) $x_1 = 1$;

t_2) $x_1 = 0$, $x_2 = 1$; and

t_3) $x_1 = x_2 = 0$.

Immediately t_3 is discarded since this substitution into (5) implies $0 \geq 1$, which is impossible. For the alternative t_1 ($\bar{x}_1 = 0$) the system reduces to

$$\bar{x}_2 + \bar{x}_3 + \bar{x}_4 + \bar{x}_5 + \bar{x}_6 + \bar{x}_7 \geq 4 \tag{3}$$

$$2\bar{x}_3 + 2\bar{x}_4 + 2\bar{x}_5 + \bar{x}_2 + \bar{x}_6 + \bar{x}_7 \geq 5 \tag{4}$$

$$x_2 \geq 0 \tag{5}$$

$$x_3 + x_4 \geq 0 \tag{6}$$

$$x_2 + x_3 + x_4 + x_5 \geq 1 \tag{7}$$

$$x_4 + x_5 + x_7 \geq 1 \tag{8}$$

$$x_3 + x_6 + x_7 \geq 1. \tag{9}$$

Inequalities (5) and (6) may be deleted by [1]. None of these are determinate; however, by [2], (8) gives $p = 3$ and:

t_1') $x_4 = 1$;

t_2') $x_4 = 0$, $x_5 = 1$;

t_3') $x_4 = x_5 = 0$, $x_7 = 1$; and

t_4') $x_4 = x_5 = x_7 = 0$ (deleted, since in (8), $0 \geq 1$).

For t_1' $(\bar{x}_1 = \bar{x}_4 = 0)$, the system is

$$\bar{x}_2 + \bar{x}_3 + \bar{x}_5 + \bar{x}_6 + \bar{x}_7 \geq 4 \tag{3}$$

$$2\bar{x}_3 + 2\bar{x}_5 + \bar{x}_2 + \bar{x}_6 + \bar{x}_7 \geq 5 \tag{4}$$

$$x_3 + x_6 + x_7 \geq 1, \tag{9}$$

where (7) and (8) were deleted by [1]. Again no relation is determinate, but (9) follows [2] ($p = 3$) and gives:

t_1'') $x_3 = 1$;

t_2'') $x_3 = 0$, $x_6 = 1$;

t_3'') $x_3 = x_6 = 0$, $x_7 = 1$; and

t_4'') $x_3 = x_6 = x_7 = 0$ (deleted, since in (9), $0 \geq 1$).

For the alternative t_1'' $(\bar{x}_1 = \bar{x}_3 = \bar{x}_4 = 0)$, the system is

$$\bar{x}_2 + \bar{x}_5 + \bar{x}_6 + \bar{x}_7 \geq 4 \tag{3}$$

$$2\bar{x}_5 + \bar{x}_2 + \bar{x}_6 + \bar{x}_7 \geq 5, \tag{4}$$

where (9) is omitted due to [1]. By [4], (3) is determinate and $\bar{x}_2 = \bar{x}_5 = \bar{x}_6 = \bar{x}_7 = 1$. The system is satisfied and a solution obtained by backtracking the x_i values. Solution is $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$, $x_5 = x_6 = x_7 = 0$. (Here after solutions will be indicated by (1011000) where the entries are values of $x_1, x_2, \ldots, x_7$.)

The next alternative is t_2'' $(x_1 = x_4 = x_6 = 1,\ x_3 = 0)$ for which substitution in the system prior to this alternative results in

$$\bar{x}_2 + \bar{x}_5 + \bar{x}_7 \geq 3 \qquad (3)$$

$$2\bar{x}_5 + \bar{x}_2 + \bar{x}_7 \geq 3, \qquad (4)$$

where (9) is deleted by [1]. For (3), by [4], $\bar{x}_2 = \bar{x}_5 = \bar{x}_7 = 1$ and another solution is (1001010).

Consideration of t_3'' $(x_1 = x_4 = x_7 = 1,\ x_3 = x_6 = 0)$ gives a similar situation with the system appearing as

$$\bar{x}_2 + \bar{x}_5 \geq 2 \qquad (3)$$

$$2\bar{x}_5 + \bar{x}_2 \geq 2. \qquad (4)$$

By [4], (3) is determinate and a solution is (1001001).

Proceeding to alternative t_2' $(x_1 = x_5 = 1,\ x_4 = 0)$ causes the system to become

$$\bar{x}_2 + \bar{x}_3 + \bar{x}_6 + \bar{x}_7 \geq 3 \qquad (3)$$

$$2\bar{x}_3 + \bar{x}_2 + \bar{x}_6 + \bar{x}_7 \geq 3 \qquad (4)$$

$$x_3 + x_6 + x_7 \geq 1, \qquad (9)$$

where (7) and (8) are deleted by [1]. None are determinate, but by [2] for (9), p = 3, and:

t_1''') $x_3 = 1$;
t_2''') $x_3 = 0,\ x_6 = 1$;
t_3''') $x_3 = x_6 = 0,\ x_7 = 1$; and
t_4''') $x_3 = x_6 = x_7 = 0$ (deleted, since in (9), $0 \geq 1$).

Alternative t_1''' reduces the system to

$$\bar{x}_2 + \bar{x}_6 + \bar{x}_7 \geq 3, \qquad (3 \text{ or } 4)$$

which by [4] is determinate, resulting in a solution (1010100). The branch t_2''' results in

$$\bar{x}_2 + \bar{x}_7 \geq 2 \qquad (3)$$

$$\bar{x}_2 + \bar{x}_7 \geq 1. \qquad (4)$$

Since (3) is dominant and by [4], $\bar{x}_2 = \bar{x}_7 = 1$, giving the solution (1000110). Branch t_3''' leaves only

$$\bar{x}_2 \geq 1, \qquad (3)$$

(4) being deleted by [1]. Solution here is (1000101).

Consideration of the alternative t_3' ($x_1 = x_7 = 1$, $x_4 = x_5 = 0$) alters the system to

$$\bar{x}_2 + \bar{x}_3 + \bar{x}_6 \geq 2 \qquad (3)$$

$$2\bar{x}_3 + \bar{x}_2 + \bar{x}_6 \geq 1 \qquad (4)$$

$$x_2 + x_3 \geq 1, \qquad (7)$$

where (8) and (9) are omitted due to [1]. According to [2], (7) gives p = 2 and the branches:

t_1^4) $x_2 = 1$;
t_2^4) $x_2 = 0$, $x_3 = 1$; and
t_3^4) $x_2 = x_3 = 0$ (deleted, since in (7), $0 \geq 1$).

Alternative t_1^4 reduces the system to

$$\bar{x}_3 + \bar{x}_6 \geq 2 \qquad (3)$$

$$2\bar{x}_3 + \bar{x}_6 \geq 1, \qquad (4)$$

which causes (3) to be determinate by [4] giving the solution (1100001).

Finally t_2^4 leaves only the inequality

$$\bar{x}_6 \geq 1 \tag{3}$$

giving the solution (1010001).

Consideration of the alternative t_2 in the original system will generate an additional five solutions. Thus a total of 13 solutions are considered for assignment configurations. Determination of the best plan is accomplished by evaluating the objective function, Z, for each one and choosing the smallest value. Table XIII presents the response distances, their averages based on b = 3 in all cases and mean deviations squared for the districts which are assigned cars according to the solutions (all feasible solutions are obtainable by compliance with the footnote to the table). The objective function is determined for each solution and the lowest value (3.0267) indicates cars are to be placed in districts 1, 2, and 7. This solution utilizes all cars, but only three men, allowing assignment of the remaining two elsewhere.

(Solution of this type of problem by pseudo-Boolean methods is longer than many due to the uniform smallness of the coefficients and constants in the system of inequalities.)

Effects of Prepositioning in Understaffed Precincts

This method of optimization of effectiveness of a limited number of cars and men allows coverage of the precinct for calls, however, if the effected shift is a relatively busy one, the queue of waiting calls and waiting time per call may become quite long and necessitate outside help. Also, since the car is assigned to a particular district and has primarily call answering duties, routine and inspectional patrol will

TABLE XIII

OBJECTIVE FUNCTION DETERMINATION FOR ASSIGNMENT IN UNDERSTAFFED PRECINCT

District Response Distance[a]							$\bar{r}$	District Mean Deviations Squared							Objective, Z
r_1	r_2	r_3	r_4	r_5	r_6	r_7		1	2	3	4	5	6	7	
0.8		1.1	1.3				1.07	0.0729		0.0009	0.0529				3.3267
1.3			2.2		0.8		1.43	0.0169			0.5929		0.3969		5.3067
1.3			1.2			1.2	1.23	0.0049			0.0009			0.0009	3.7067
0.8		2.0		1.0			1.26	0.2116		0.5476		0.0676			4.6268
1.3				1.0	0.8		1.03	0.0729				0.0009	0.0529		3.2267
1.3				0.7		0.9	0.97	0.1089				0.0729		0.0049	3.0867
0.7	1.0					1.2	0.97	0.0729	0.0009					0.0529	3.0267
0.8		2.0				1.2	1.33	0.2809		0.4489				0.0169	4.7467
	0.7	1.1	1.3				1.03		0.1089	0.0049	0.0729				3.2867
	0.7		2.2		0.8		1.23		0.2809		0.9409		0.1849		5.1067
	0.7		1.2			1.2	1.03		0.1089		0.0289			0.0289	3.2667
	1.5	2.0		1.0			1.50		0	0.2500		0.2500			5.0000
	1.5	2.0				1.2	1.57		0.0049	0.1849				0.1369	5.0267

[a]Feasible solutions may be observed by substituting a one in the r_i columns with an entry and a zero in those with no entry.

greatly suffer from this type of assignment. These facts must be considered before such a plan is implemented.

FOOTNOTES

[1]Richard C. Larson, _Models for the Allocation of Urban Police Patrol Forces_ (Cambridge: Massachusetts Institute of Technology, 1969), pp. 188-224.

[2]Ibid., p. 54.

[3]P. L. Ivanescu and S. Rudeanu, _Lecture Notes in Mathematics_, Vol. XXIII: _Pseudo-Boolean Methods for Bivalent Programming_ (Berlin: Springer-Verlag, 1966), pp. 7-52.

CHAPTER IV

RELATED PROBLEMS AND CONCLUSIONS

Problems related to those discussed here are many due to their classification as MDK problems, however, some interesting relations to the precinct design problem (Chapter II) are investigated. This chapter also presents a summary of the more important findings of the work detailed in the previous chapters. Suggestions for future research in the area are also made.

Problems Related to Precinct Design

The precinct design problem is a member of a type of resource allocation problem which attempts to partition a resource into a set number of pieces according to certain restrictions while optimizing some function. Two general types of related problems are distinguishable and discussed here.

The first type of resource allocation that may be distinguished involves the partitioning of an area such that distance from some particular point, object or building is minimized. Examples of this may be:

1. Division of a city or rural area into a number of school districts. It is vital that each section of ground be included in a school district and that each district be composed of contiguous sections such that distance traveled to the school be minimized. Yeates (1968,

pp. 107-112) discusses this problem and solves an example problem using the transportation algorithm in which supply is school children and demand is the capacity of each school in the area.

2. Placement of ambulance stations in a city. Again, the partitioned city must be designed such that distance from the station to the scene of an accident is minimized.
3. Design of the area of responsibility for existing fire stations or the placement of new stations in developing areas.
4. Placement of shopping centers in urban areas to provide convenience by minimizing distance from homes to the center.
5. Placement of facilities of all sorts in cities, counties, states, or countries.

In all of the above descriptions, concentration of interest upon a number of stationary locations is obvious. These locations may be occupied by a school, fire station or shopping center. If the location is allowed to move about, problems similar to the patrol precinct design problem are presented. Interest may still be upon distance from this moving position; however, other factors such as population of the area or volume of sales in an area may become more important. A few descriptions of this type of problem are given below. These are in addition to the precinct design problem which attempts to minimize the distance between a patrolling car and an incident requiring attention by the patrolman.

1. Partitioning of a state into political districts for a number of representatives. Garfinkel (1969) presents this problem in mathematical form and develops an analytical solution technique to optimally divide a state into a set number of political districts such that population mean deviations squared are minimized.
2. Division of a country into a number of management areas in which interests of the company are overseen. Since sales promotion, customer services and many other endeavors of the company must be managed by an individual, the size and design of the area for which he is responsible should be determined in some orderly fashion.
3. Internal Revenue reporting district determination used for the collection and investigation of tax returns. This again is a population problem; however, attention is directed to serving the taxpayers of an area as quickly and accurately as possible.

In all of the above descriptions, it is seen that no particular point in an area is of greater importance than any other, but the entire area must be covered with whatever service is in question. The precinct design problem fits into the latter class of problems since both the service, patrolling officers, and the incidents, calls for service and inspectional duties, are constantly moving throughout the area of responsibility.

Summary of Results

This research is directed to the investigation of representative examples of the distribution of police patrol forces in a city. Problems are set up so that the use of analytical techniques for their solution is possible.

The problem of precinct design requires the development of a solution technique to design a set number of precincts in a division. The solution minimizes the mean deviations squared for allocation time per precinct such that calls can be answered within a certain travel distance of a patrol car. The solution algorithm is illustrated for the case in which all data is assumed to be determined and the case in which the response distance is allowed to become a random variable.

The effect upon distribution is studied for alterations in the number of precincts and altering maximum allowable precinct response distance. This effect is studied for the objectives of minimization about mean allocation time per precinct and about a preset design constant. In the first case, it is shown that after a certain amount of relaxation of response distance, the solutions are all identical; that is, distribution of patrol forces are the same. This fact is accounted for by the increased size in precincts when distance restrictions are relaxed, thus causing the mean deviations squared to increase greatly. Consideration is also made of the inequity in work load in the optimal solutions. Suggestions such as limits on precinct allocation time and equalized district allocation time are made to alleviate this inherent problem of inequity.

If the precinct design problem is solved so that deviations about a design constant are minimized, a best solution exists for some number of precincts and response distance restriction in that inequity is the smallest.

Location of a less than adequate number of patrol cars in an existing precinct is also investigated. By allowing the patrol cars to be placed in specific districts of the precinct and answer calls in that and other defined districts, the ability to answer calls throughout the precinct is maintained, however, routine and inspectional patrol suffers under such a design. This problem is formulated and solved according to Pseudo-Boolean Programming, which is explained in detail so that the reader is able to understand the solution procedure.

Restrictions utilized in the problems may not allow solution. This being the case, relaxation of these constraints or increased resources are necessary. Also needed is an understanding of the effects caused by the relaxation of the constraints.

Future Research Suggestions

The areas of police science and other public service systems are available for much research by quantitative analysis. There are large amounts of data in these systems; however, the formulation of problems in mathematical terms that may be tackled by analytical techniques is lacking. In most cases the definition of specific objectives and restrictions are difficult.

With regard to the research presented in this thesis, application of the suggested techniques should be made to actual police situations. However, extension of the problems here is possible as is investigation of new problems in police patrol work. A few areas that are worthy of further research are discussed below.

1. Precincts have been designed by combining predefined sections of ground. Investigation into ways in which

the precinct may be 'built'up' by accumulating time and still not violate certain restrictions should be performed. If the boundaries of the precinct are completely flexibile, inequity in work load between precincts would not be a problem, since the boundary line could be designated as the street which allocates a desired amount of time to a precinct. Such a method would require much accumulation and manipulation of data so that accurate times could be utilized in the design of precincts.

2. This research utilizes the fluid or team approach in that precincts rather than beats are designed within the division. The trend to precinct patrols is mentioned, however, actual verification of the benefits of precincts over beats is not yet presented. Research concerning the advantages and disadvantages of both should be performed.

3. The use of statistics in police patrol work is hardly noticeable. Studies performed to discover the probability distributions of response times, times to service different classes of calls for service and studies of the statistical relation between apprehension rate and short response distance are necessary. Research in this area would hopefully allow some generalizations about response systems and patrol operations to be made. This would help cities 'know' that certain observations are to be expected when they study

their city's patrol operations for the purpose of bettering service to the citizen.

4. Use of the independence assumption is made throughout this thesis. Even though this assumption is rarely violated in precinct patrol work, the understaffed precinct, when operating under this assumption, must suffer in terms of service to the citizens and businesses in the precinct. Research is needed into procedures that may be used so that precincts might cooperate to maintain patrolling and call answering duties when one or more precincts are understaffed. Study to learn the consequences of making neighboring precincts into one so that mutual cooperation between the patrol officers should be accomplished. Or the redesign of precinct boundaries for understaffed conditions may allow better service to be given.

Analytical solutions to problems in this thesis or any of those discussed above cannot be expected to be completely foolproof answers, however, the solution coupled with wise judgemental action will allow a better patrol force to become a reality.

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APPENDIX

FEASIBLE PRECINCTS

This appendix presents the feasible precincts for the example problem of Chapter II, deterministic case. Table XIV details precincts, response distance, $\bar{r}_j$, precinct allocation time and mean deviations squared for all m values in which these precincts are used for solution when R' = 4. Table XV presents the same data for only those precincts that become available when R' is increased. Lack of entry for an m value indicates the precinct is not feasible for solution. For example, if m = 8 only one or two districts may be combined to form a precinct.

Notice that if the problem of Chapter II is solved utilizing a design constant of 7.5 hours (450 minutes), mean deviations squared for m = 4 are to be used. If this figure had been different, other calculations would be needed.

A brief illustration of the second exclusion principle of Chapter II may be made here. Reference to Table IV shows that for R' = 7, m = 5, C = 10,750. Table XV indicates that if R' is increased to 8, all added precincts (m = 5) have contributions larger than C, thus the solution already found is optimal again. This exclusion is possible since solution for R' = 7 was obtained before attempting solution for the larger R' value.

TABLE XIV

SUMMARY OF PRECINCTS AND OBJECTIVE CONTRIBUTIONS FOR R' = 4

i	Pre-cinct	$\bar{r}_j$	Pre-cinct Time	Mean Deviations Squared						
				m = 3	m = 4	m = 5	m = 6	m = 7	m = 8	m = 9
1	1	4	250	122500	40000	12100	2500	49	625	2500
2	2	3	125	225625	105625	55225	30625	17424	10000	5625
3	34	4	400	40000	2500	1600	10000	20449	30625	-
	3	3	200	160000	62500	25600	10000	3249	625	0
4	45	4	425	30625	625	4225	15625	28224	40000	-
	34	4	400	40000	2500	2600	10000	20449	30625	-
	4	1	200	160000	62500	25600	10000	3249	625	0
5	45	4	425	30625	625	4225	15625	28224	40000	-
	5	3	225	140625	50625	18225	5625	1024	0	625
6	67	3	350	62500	10000	100	2500	8649	15625	-
	6	2	200	160000	62500	25600	10000	3249	625	0
7	67	3	350	62500	10000	100	2500	8649	15625	-
	7	1	150	202500	90000	44100	22500	11449	5625	2500
8	8	4	250	122500	40000	12100	2500	49	625	2500
9	9	3	200	160000	62500	25600	10000	3249	625	0

TABLE XV

ADDITIONAL PRECINCTS AVAILABLE IF R' IS INCREASED

i	Pre-cinct	Pre-cinct Time	Mean Deviations Squared					
			m = 3	m = 4	m = 5	m = 6	m = 7	m = 8[a]
			R' increased from 4 to 5					
2	26	325	75625	15625	1225	625	4649	10000
5	56	425	30625	625	4225	15625	28224	40000
6	56	425	30625	625	4225	15625	28224	40000
	69	400	40000	2500	1600	10000	20449	30625
	26	325	75625	15625	1225	625	4649	10000
7	78	400	40000	2500	1600	10000	20449	30625
8	78	400	40000	2500	1600	10000	20449	30625
9	69	400	40000	2500	1600	10000	20449	30625
			R' increased from 5 to 6					
2	267	475	15625	625	13225	30625	47524	-
	23	325	75625	15625	1225	625	4624	10000
3	35	425	30625	625	4225	15625	28224	40000
	23	325	75625	15625	1225	625	4624	10000
4	456	625	625	30625	70225	105625	135424	-
5	456	625	625	30625	70225	105625	135424	-
	567	575	625	15625	46225	75625	101124	-
	35	425	30625	625	4225	15625	28224	40000
6	456	625	625	30625	70225	105625	135424	-
	567	575	625	15625	46225	75625	101124	-
	679	550	2500	10000	36100	62500	85849	-
	267	475	15625	625	13225	30625	47524	-
7	567	575	625	15625	46225	75625	101124	-
	679	550	2500	10000	36100	62500	85849	-
	267	475	15625	625	13225	30625	47524	-
9	679	550	2500	10000	36100	62500	85849	-

TABLE XV (Continued)

i	Pre-cinct	Pre-cinct Time	Mean Deviations Squared					
			m = 3	m = 4	m = 5	m = 6	m = 7	m = 8[a]
			R' increased from 6 to 7					
1	19	450	22500	0	8100	2500	49	625
	12	375	50625	5625	225	5625	13924	22500
2	234	525	5625	5625	27225	50625	71824	-
	12	375	50625	5625	225	5625	13924	22500
3	345	625	625	30625	70225	105625	135424	-
	234	525	5625	5625	27225	50625	71824	-
4	345	625	625	30625	70225	105625	135424	-
	234	525	5625	5625	27225	50625	71824	-
	4567	775	30625	105625	172225	225625	-	-
5	345	625	625	30625	70225	105625	135424	-
	4567	775	30625	105625	172225	225625	-	-
6	678	600	0	22500	57600	90000	117649	-
	4567	775	30625	105625	172225	225625	-	-
7	678	600	0	22500	57600	90000	117649	-
	4567	775	30625	105625	172225	225625	-	-
8	678	600	0	22500	57600	90000	117649	-
	89	450	22500	0	8100	22500	37249	50625
9	19	450	22500	0	8100	2500	49	625
	89	450	22500	0	8100	22500	37249	50625
			R' increased from 7 to 8					
2	256	550	2500	10000	36100	62500	85849	
	236	525	5625	5625	27225	50625	71824	
	269	525	5625	5625	27225	50625	71824	
3	356	625	625	30625	70225	105625	135424	
	236	525	5625	5625	27225	50625	71824	
5	569	625	625	30625	70225	105625	135424	
	356	625	625	30625	70225	105625	135424	
	256	550	2500	10000	36100	62500	85849	
6	569	625	625	30625	70225	105625	135424	
	356	625	625	30625	70225	105625	135424	
	256	550	2500	10000	36100	62500	85849	
	269	525	5625	5625	27225	50625	71824	
	236	525	5625	5625	27225	50625	71824	

TABLE XV (Continued)

i	Precinct	Precinct Time	Mean Deviations Squared m = 3	m = 4	m = 5	m = 6	m = 7
7	789	600	0	22500	57600	90000	117649
8	789	600	0	22500	57600	90000	117649
9	789	600	0	22500	57600	90000	117649
	569	625	625	30625	70225	105625	135424
	269	525	5625	5625	27225	50625	71824
R' increased from 8 to 9							
1	126	575	625	15625	46225	75625	101124
	169	650	2500	40000	84100	122500	154449
2	235	550	2500	10000	36100	62500	85849
	2679	675	5625	50625	99225	140625	-
	2367	675	5625	50625	99225	140625	-
	2567	700	10000	62500	115600	160000	-
	2346	725	15625	75625	133225	180625	-
	2456	750	22500	90000	152100	202500	-
3	345	625	625	30625	70225	205625	135424
	2367	675	5625	50625	99225	140625	-
	2346	725	15625	75625	133225	180625	-
	3567	775	30625	105625	172225	225625	-
	3456	825	50625	140625	216225	275625	-
4	2346	725	15625	75625	133225	180625	-
	2456	750	22500	90000	152100	202500	-
	3456	825	50625	140625	216225	275625	-
	4569	825	50625	140625	216225	275625	-
5	235	550	2500	10000	36100	62500	85849
	2567	700	10000	62500	115600	160000	-
	2456	750	22500	90000	152100	202500	-
	5679	775	30625	105625	172225	225625	-
	3567	775	30625	105625	172225	225625	-
	4569	825	50625	140625	216225	275625	-
	3456	825	50625	140625	216225	275625	-
6	126	575	625	15625	46225	75625	101124
	689	650	2500	40000	84100	122500	154449
	169	650	2500	40000	84100	122500	154449
	2367	675	5625	50625	99225	140625	-
	2679	675	5625	50625	99225	140625	-
	2567	700	10000	62500	115600	160000	-
	2346	725	15625	75625	133225	180625	-
	2456	750	22500	90000	152100	202500	-

TABLE XV (Continued)

i	Precinct	Precinct Time	Mean Deviations Squared				
			m = 3	m = 4	m = 5	m = 6	m = 7
	3567	775	30625	105625	172225	225625	-
6	5679	775	30625	105625	172225	225625	-
	4569	825	50625	140625	216225	275625	-
	3456	825	50625	140625	216225	275625	-
	2367	675	5625	50625	99225	140625	-
	2679	675	5625	50625	99225	140625	-
7	2567	700	10000	62500	115600	160000	-
	5679	775	30625	105625	172225	225625	-
	3567	775	30625	105625	172225	225625	-
8	689	650	2500	40000	84100	122500	154449
	169	650	2500	40000	84100	122500	154449
	689	650	2500	40000	84100	122500	154449
9	2679	675	5625	50625	99225	140625	-
	5679	775	30625	105625	172225	225625	-
	4569	825	50625	105625	172225	225625	-

[a]No feasible precincts are added for R' = 8 or 9.

VITA

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