

INFORMATION FEEDBACK IN QUALITY CONTROL

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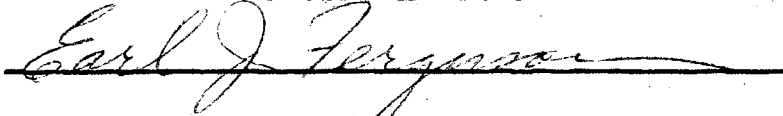
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## PREFACE

The growth of industry and the development of technology have given rise to numerous new problems in engineering and management. In quality control, the design and maintenance of an effective quality system that will meet the needs of future quality problems has become one of the major concerns. This thesis was written in an effort to review current developments on various aspects of information feedback in quality control, and to develop and present a systematic treatment of this subject.

I first became interested in this area while assisting Dr. W. E. Masing of West Germany in the Quality Control Seminar which was conducted by the Korea Productivity Center in Seoul in April, 1965. I wish to acknowledge my indebtedness to Dr. W. E. Masing who provided the basis for this development.

I also wish to express my sincere appreciation to Dr. P. E. Torgersen for his guidance and encouragement in writing this thesis.

Finally, my special thanks are due to Mr. S. Jackson who read the manuscript and gave many helpful suggestions on the correct usage of English.

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## CHAPTER I

### INTRODUCTION

#### Quality Control and Information Feedback

According to Feigenbaum, "Quality Control is an effective system for integrating the quality-development, quality maintenance, and quality-improvement efforts of the various groups in an organization so as to enable production and service at the most economical levels which allow for full customer satisfaction."<sup>1</sup>

An important term used in this definition is customer satisfaction. The customer wants: first of all, a product which meets his needs. If the product fails, then no matter how prompt or courteous the service to make good the failure, the customer has been inconvenienced. He will prefer the product which does not fail to a product which is courteously replaced. Second, the customer does want the assurance that, in the event of a product failure, he will be protected. In the event of a product failure, the customer wants an honest and prompt adjustment, not an exasperating run-around.

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<sup>1</sup>A. V. Feigenbaum, Total Quality Control, (New York: McGraw-Hill Inc.,) 1961, p. 1.



In a competitive society, the wants of the consumer (except in shortages such as those created by war) must find their way into the policies of the producers. Producers are finding ways to meet the above needs through the following measures:

1. The producer is adopting modern market research plus modern methods of quality control to insure that a better-designed product will more frequently conform to the specification.
2. The ability to produce a conforming product makes it possible to offer a quality guarantee which would otherwise be prohibitive in cost. Guarantees vary widely in their terms. There may be an offer to refund the purchase price, replace the product free, make good certain defects which appear within the guarantee period.
3. To give the customer prompt and fair adjustment of his claim, the sales-service department of the producer comes into play. This department is called upon to make many difficult judgments on claims, with the result that no one is pleased. There is only one solution to the grief of the department - to make more and more of the products conform to better and better designs.<sup>2</sup>

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<sup>2</sup>J. M. Juran, Quality Control Handbook, 2nd ed., (New York: McGraw-Hill Inc.,) 1962, Sec. I, p 23.

The term information as used here means "the process by which the form of an object of knowledge is impressed upon the comprehending mind so as to bring about the state of knowing the object."<sup>3</sup> Any dynamic process requires knowing the state of the process over the elapsed time of the process operation. This forms the basis for corrective action toward optimization of the process or realization of preset goals. The shorter the cycle time of the information feedback, the more efficient the process will be.

In quality control, three basic categories of information are necessary in order to accomplish the major tasks described in the previous paragraph. These categories may be listed as follows:

- a) Information required for the formulation of quality
- b) Information required for the realization of quality
- c) Information required for the improvement of quality.

The initial step in any industrial cycle involves the establishment of a quality goal which satisfies customer's demands and expectations. These demands and expectations, as they exist in the customer's mind, regardless of their form of existence, have to be conveyed accurately to the producer. The communication of this information may take the form of market research or receiving of the customer's order. There are other sources of information such as the

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<sup>3</sup>Webster's New International Dictionary of English Language, 2nd ed., (Springfield, Mass: G & C Merriam Co.,) 1954.

quality of competitors, and the capability of the manufacturing process which would influence the quality goals (quality of design) of the producer. All the information that can influence the design of quality will compose a set of information required for the formulation of quality.

Once this quality target is established, it is necessary for the production personnel to have the necessary information to reach the target. This information takes the form of blue prints, specifications of various elements of the product and so forth. This is the information required for the realization of quality.

The industrial process is a dynamic process. The customer's demands and expectations may change over time. The current process does not necessarily provide the best performance characteristics. It is important for the producer to strive for the continued improvement of his quality of design and quality of conformance, if he is to survive in a competitive society. The detection of a change in the customer's demands and expectations requires continued monitoring of the customer's voice. It is also important to keep abreast of recent developments in the related technological areas. This sort of information may be designated as information required for the improvement of quality.

The flow of these three basic categories of information, in and out of the organization, in an efficient manner is a prerequisite to a good quality system.

## Impact of Growth of Industry on Information Feedback

At one time the quality of a manufactured product was entirely dependent upon one or a few workers. In those days, products were manufactured according to a mental concept of the product quality, developed by the worker, based on a customer's order. Since only a limited number of people were involved in the manufacturing process, relatively accurate and quick feedback of information was possible.

Today industry is larger and more technically mature. Thousands of workers may be employed, and a highly complex product may be competing in the market. The development of specialization has brought about a system of marketing in which the producer may be several stages removed from the consumer. The designer, process engineer, operator, and inspector are all different persons in separate organizational groups. This is illustrated in Fig. 1.<sup>4</sup> This complexity makes the primitive information feedback no longer functional in present industry.

In the manufacturing process, the tolerances are becoming more and more stringent due to the rapid development of new technologies and increased demand for high product performance. Almost immediate detection and elimination of shifts in the process operating levels are essential, if excessive scrap and shut-down costs are to be avoided.

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<sup>4</sup>J. M. Juran, Quality Control Handbook, 2nd ed., (New York: McGraw-Hill Inc.,) 1962, Sec. I, p. 7.

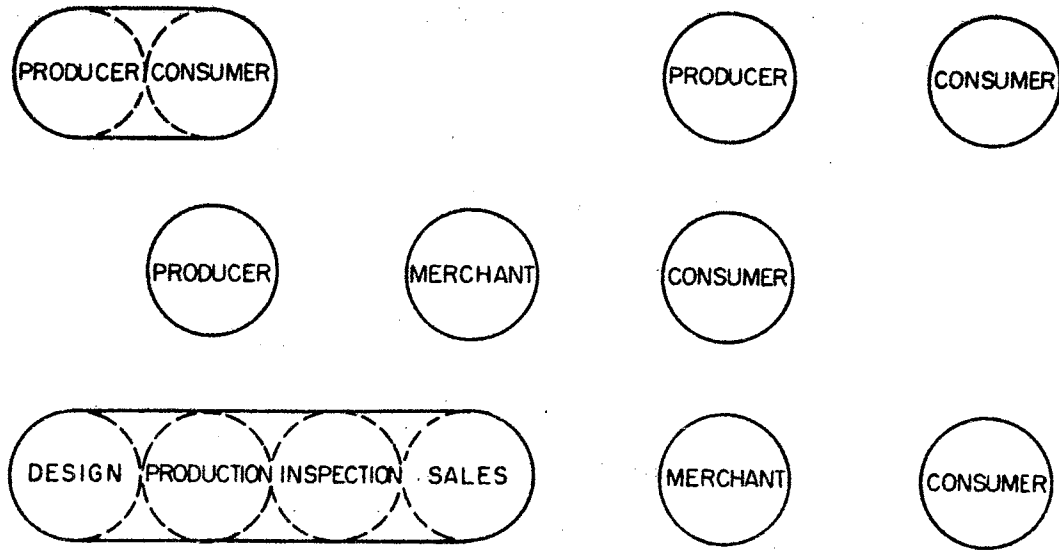


Figure 1. Growth of Complexity of Organization in Quality Function

At the same time, automation, in which rapid quality evaluation is a pivotal point, has magnified the need for mechanization of inspection and test equipment, much of which is now in the hand-tool stage.<sup>5</sup>

As a result of the growth of industry, it became necessary to devise an effective information feedback system within the organization as well as in the manufacturing process.

#### Statement of the Problem

The study of any subject matter necessitates formulation of a series of questions which clarify the issue under

<sup>5</sup>A. V. Feigenbaum, Total Quality Control, (New York: McGraw-Hill Inc.,) 1961, p. 15.

study and the tasks to be undertaken. In the study of a quality information system, the following kinds of questions must be answered:<sup>6</sup>

- a) What kinds of information are essential?
- b) How much information is needed?
- c) To what positions should information be sent?
- d) How fast must it be received to be effective?
- e) How frequently should the information be sent?
- f) In what form should it be presented to be immediately usable to serve as a basis for decision and action?

The effectiveness of the quality information system in terms of the answers to these questions should, in turn, be measured periodically to assure that it remains efficient. Such measurements must determine that:

- Paper work is kept to a minimum.
- Only usable data are being transmitted.
- Data are going to positions whose responsibilities call for its use.
- Data are adequate and are being properly applied.
- The information system is being adequately maintained.
- The information is producing effective and timely decisions for corrective action.

Industry today is so diversified that developing a generalized solution to all the questions listed is

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<sup>6</sup>Ibid., p. 145.

practically impossible. In analyzing information feedback problems, one must consider all the variable factors of information, which may differ from industry to industry, however, there is one factor possessing a common dimension to any system; this is the time factor stated under items d and e. In the following chapters, the discussion will be focused on these two items.

## CHAPTER II

### DESCRIPTION OF MODELS

The quality information system is generally composed of several interrelated sub-systems having their respective quality goals. Within and between these sub-systems, the flow of the three categories of information, discussed in Chapter I, takes place. The individual information systems required to supply the design, purchasing, and manufacturing activities may be regarded as quality information sub-systems, having as their respective goals: optimum design, supply, and manufacturing of products or components that best meet the given design. The degree of detail in breaking down a system into sub-systems, of course, depends largely on the objective of the study, the complexity involved and the size of the system.

Optimizing these sub-systems in terms of their objectives is important in the study of any system. However, like an addition of vectors in mathematics, sub-optimization does not necessarily optimize the overall system. In a quality system, effectiveness of the system<sup>1</sup> can only be evaluated in terms of the degree to which the

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<sup>1</sup>This will be discussed further on page 25.



customers' satisfaction is attained.

An important facet of the study of any system involves identifying the factors standing in the way of attaining objectives. Once the limiting factors have been identified, they may be examined to determine those which may be altered or removed to permit the attainment of the objective. Those limiting factors that may be successfully and expediently altered are called strategic factors. Effort directed to accomplishing an objective may then be applied to these strategic factors by choosing means appropriate to the situation at hand.<sup>2</sup>

As an initial step in identifying the strategic factors in the quality information system, it is necessary to understand the two essential facets of the system: first, the overall structure of the system, showing how the individual functional components (sub-systems) are interrelated, and secondly, how individual functional components operate. Amelioration of components does not necessarily improve the overall efficiency of the system when the major deficiency lies in the structure of the system. In this case, reformation of structure so as to compensate the deficit is necessary. Understanding the overall structure of the system also aids in identifying the critical components that significantly contribute to the overall efficiency. Once the

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<sup>2</sup>W. J. Fabrycky and P. E. Torgersen, Operations Economy, (New York: Prentice-Hall Inc.,) 1966, p. 8.

critical components of the system are located, understanding how these components operate is essential. The structure of the quality system and how the components operate are best described by the organizational model and the process control model respectively.

### Organizational Model<sup>3</sup>

#### 1. The basic elements in the model:

The demands and expectations of the customer are taken as the starting point for actions in the field of quality control. These customer demands and expectations are, in turn, determined by:

- the selling price of the product
- the quality characteristics of competitive products
- the level of technical development
- in the case of a specific customer, by specifications required by that customer.

These provide the basis for developing a design satisfactory to the customer. The degree to which the design meets this satisfaction, determines the quality of that design. The more the designer succeeds in satisfying the customer demands, within the given technical and economical possibilities, the better his design will be. The design has subsequently to be reproduced in the factory. The quality of production is evaluated by the extent to which the pro-

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<sup>3</sup>J. H. Enters, "The Implementation of Quality Control," Quality, Journal of EOQC, No. 1 (March 1963), pp. 10-14.

duction process conforms to the demands specified in the design. The quality of production is assessed according to the degree and consistency with which the products conform to these design demands.

The comparison of the quality characteristics of the final product with the customer demands and expectations, as given in Fig. 2, determines his opinion of the product quality. This opinion is determined both by the characteristics of the design (which are, in principle, the same for all products of this type) and by the way in which this design is realized. As this opinion is formed when the

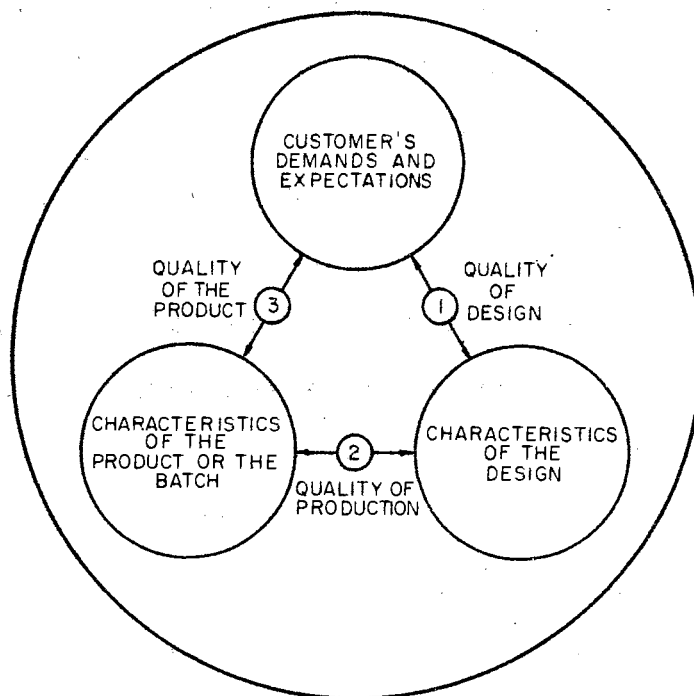


Figure 2. The Basic Elements in the Model

customer is confronted with the product, the way in which this product is offered to him (type of package and display) needs to be included in quality control.

2. The flow of information:

The first step to be taken is to systematically analyze customer demands. It will be necessary to create an information system by which these demands and expectations are brought to the attention of the design department. In cases where customers are explicitly aware of these demands, they can be investigated by a formal market research in which certain questions are asked of customers or of prospective customers. In other cases, such a direct investigation is not possible. These customer soundings should not be once-and-for-all-affairs. They have to be carried out systematically and periodically because it is important to carefully follow changes in customer opinions. Such customer inquiries vary from the creation of a customer panel to formal market research. Such market research may consist of a poll formally requesting customer opinions.

One may define production as the realization of design characteristics. The first condition, then, is that the design characteristics are known and fully understood by production management and production personnel. The production implication must be clearly specified to enable to all concerned on how the product should be made. It is also very useful to give information about why certain

characteristics are necessary. This implies creating an information system as indicated in Fig. 3, where such information is supplied by the design department to the production department. The last sector of this information system is completed when the customer is confronted with the product. In this stage, he will become aware of the quality characteristics of the product. It is part of the program of quality control to investigate these successive information systems and to improve them if necessary.

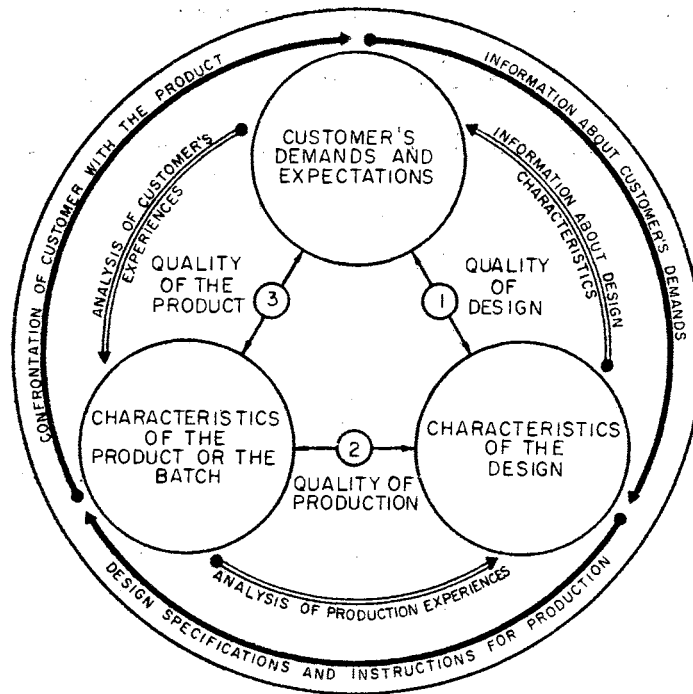


Figure 3. The Flow of Information

At the same time it will, however, be necessary to create, as part of quality control, a second information system in the opposite direction, as given in Fig. 3.

The customer's product experience will have to be analyzed systematically, and the results of this analysis will have to be brought to the attention of the production and design departments. As a result of this, the production methods or the design may have to be corrected. The production experience will have to allow the necessary corrections in design. An important element in this information system is the supply of information to the designer about process capabilities.

As the last stage, the customer will have to be informed about the quality characteristics of a certain design in order to make it clear which of his demands and expectations can be expected to be fulfilled by the design in question. The customer should understand in what respect the quality characteristics of the product are different from those of competing products. Quality control should extend its activities to the way in which such information is being given.

### Process Control Model

#### 1. Schematic Model:

The product quality, in general, is composed of product characteristics of engineering and manufacture, known as quality characteristics. These characteristics are formed under the various processes included in the industrial cycle. If we want to control the product quality, therefore, it becomes necessary to analyze and control these processes.

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Any functional component of an industry may be regarded as a process. It is a sub-system of the quality system. The process is composed of a complexity of men, machines and materials, and whose objective is to produce quality characteristics which meet the given specifications. The control procedure of a process can be illustrated by Figure 4.

The control cycle begins with the establishment of the standards in the process. These standards may be a set of specifications defining the materials at the various stages of the process, or a set of standards on operating levels of the process parameters that have to be maintained within specified limits. This is represented as a reference input in Fig. 4. What variable to control, and how much to control, form important decisions to be made at this particular stage.

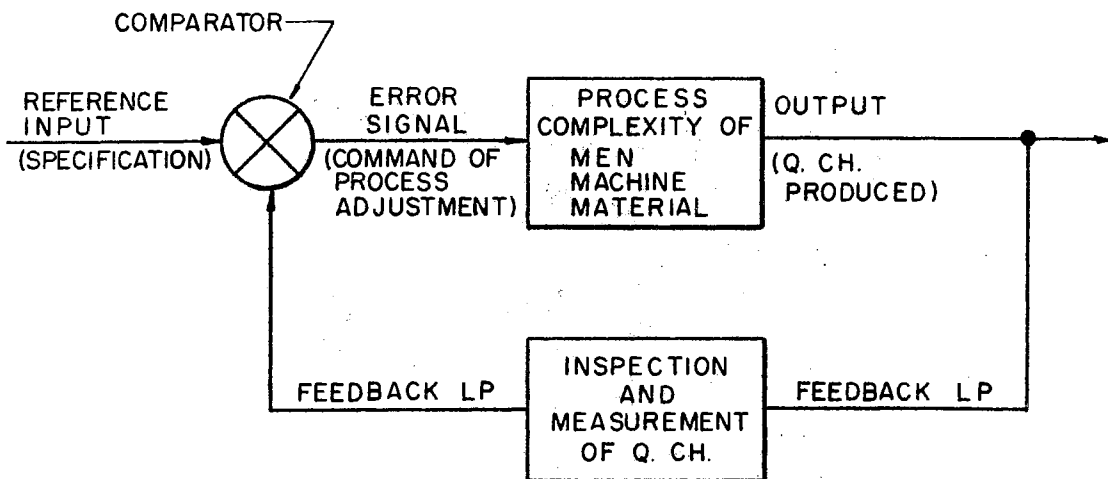


Figure 4. Schematic Representation of Process Control

The next step of the cycle is inspection and measurement (for simplicity, hereafter referred to as measurement only). In order to determine whether or not the variables under control conform to the specifications, measurement is necessary. The measurement decision encompasses the following:

- Procedures of selecting the items to be measured: random vs. systematic
- Frequency of measurement and the number of items to be measured: sampling interval and sample size
- Measuring equipment and precision of the measurement<sup>4</sup>

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<sup>4</sup>This includes the problem of deciding whether the measurement should be made on a discrete or continuous basis. When the measurement is taken as a discrete value, sometimes, it saves a great deal of time and effort. For example, in Figure 5, the original continuous scale of a yarn tensile strength meter was modified to give a discrete value of measurement. This reduces the time required for measurement, recording and the analysis that follows.

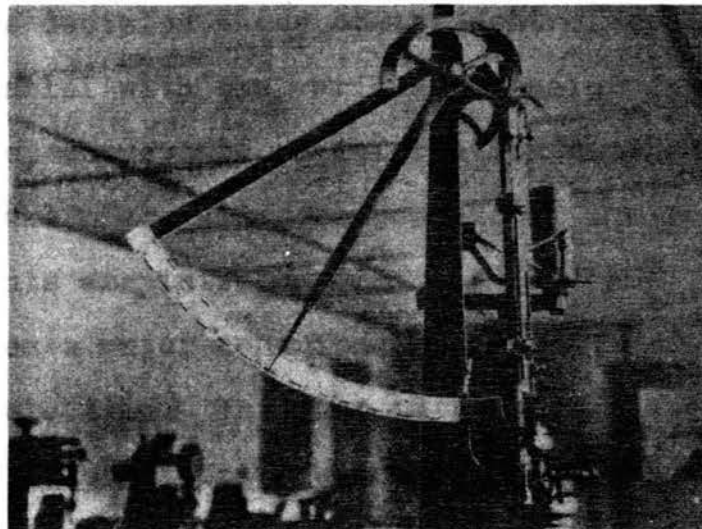


Figure 5. An Example Showing Conversion of Scale



- Methods of measurement
- Form of recording and storage of data.

Analysis and decision follow the measurement phase of the cycle. When the information concerning the process is obtained through measurement, the controller compares the information with the given standards of the process, and determines whether or not the process is under control. If he decides the process is out of control, he initiates a process adjustment (error signal in Figure 4). This information is fed back to the proper adjusting device or personnel, thereby allowing for corrective action. The cycle repeats itself throughout the life span of the process.

In many industrial situations analysis and decision on the process is based on measurement of the samples. When this is the case, it is, in a statistical sense, subject to type I and II errors. Choosing an optimum decision rule so as to minimize a loss due to the error is a common problem faced in the decision phase of the control cycle.

In parallel with the various phases of control discussed above, conveyance of information necessary for control must take place among components of the system. When human elements are interposed in the feedback loop, it sometimes becomes a major contributing factor in determining feedback cycle time. In order to reduce this cycle time, analysis of information flow and means of transmission of information must be made.

## 2. Statistical Model:

As previously stated, a process produces one or a multiple of quality characteristics of a product by taking a complex of men, machines and materials as its input. Each of these elements are subject to random variation. The first broad scale, self-explanatory, and self-supporting work in the field, was Shewart's book.<sup>5</sup> Here for the first time the stochastic nature of industrial processes was explained and "control" defined in probabilistic terms:

- a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that observed phenomenon will fall within the given limits.

The notions of "chance cause systems" and "assignable causes" were introduced and the following postulates set forth:

Postulate 1. - All chance systems of causes are not alike in the sense that they enable us to predict the future in terms of the past.

Postulate 2. - Constant systems of chance causes to exist in nature.

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<sup>5</sup>W. A. Shewhart, Economic Control of Quality of Manufactured Product, (New York: D. Van Nostrand Co.,) 1931.

Postulate 3. - Assignable causes of variation may be found and eliminated.

These postulates form the basis of current Statistical Quality Control, the name by which the quality control field is most commonly called. In essence they state that the stochastic nature of systems is due to the interaction of a multitude of factors, none of which predominates, and the effects of which are constant in a probabilistic sense. In general, one assumes that the isolation and elimination of such factors are either impossible or uneconomical. Additional factors may also affect the process forcing it out of acceptable limits. Such factors can, however, be eliminated utilizing adequately sophisticated techniques. The ultimate control activity is "maximum control" defined as:

the condition reached when the chance fluctuations in a phenomenon are produced by a constant system of a large number of chance causes in which no cause produces a predominating effect.<sup>6</sup>

In presenting the statistical model,<sup>7</sup> the following assumption will be made for the simplicity.

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<sup>6</sup> A. B. Bishop, Discrete Random Feedback Models in Industrial Quality Control, Engineering Experiment Station Bulletin 183, The Ohio State University, 1960, pp. 11-12.

<sup>7</sup> A. B. Bishop, "Automation of the Quality Control Function," Industrial Quality Control, April, 1965, Vol. 21, No. 10, pp. 509-511.

- 1) Model will be limited to simple proportional controller.<sup>8</sup>
- 2) Model is limited to discrete system.
- 3) The results of any one control action are completed before the next measurement and control action are made.

A simple proportional control system is shown in Figure 6.

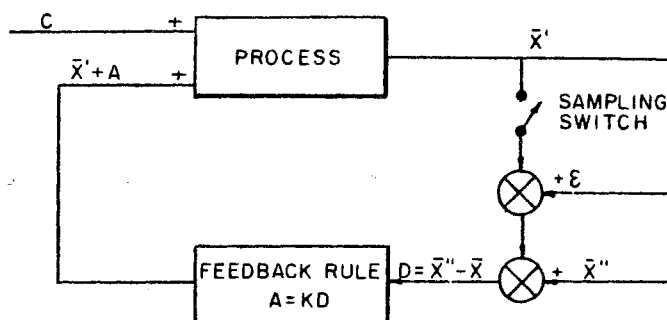


Figure 6. Idealized Discrete Simple Proportional Controller

If we let

- $\bar{X}''$ : desired process level
- $i$ : number of times switch S is closed
- $\bar{X}'_i$ : true process level when the switch S is closed for  $i$ th time
- $\bar{X}_i$ : observed process level when the switch S is closed for  $i$ th time
- $\epsilon_i$ : error associated with  $i$ th measurement

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<sup>8</sup> A device which delivers a control action to the process input which is a fixed proportion of the amount that the most recent sample average deviates from the desired level.

$A_i$ :  $i$ th process adjustment

$D_i$ :  $i$ th observed deviation

$C_i$ : net effect of all assignable causes occurring in the interval between  $(i-1)$ th and  $i$ th closing of switch S

$k$ : controller constant

then, observed process operating level at the instant S is closed for the  $i$ th time,  $\bar{X}_i$ , is given by,

$$\bar{X}_i = \bar{X}_i' + \epsilon_i . \quad (2-1)$$

And observed deviation,  $D_i$ , becomes,

$$D_i = \bar{X}'' - \bar{X}_i = \bar{X}'' - \bar{X}_i' - \epsilon_i , \quad (2-2)$$

Accordingly  $i$ th adjustment,  $A_i$ , will be,

$$\begin{aligned} A_i &= kD_i = k(\bar{X}'' - \bar{X}_i) \\ &= k\bar{X}'' - k\bar{X}_i - k\epsilon_i . \end{aligned} \quad (2-3)$$

By incorporating the effect of assignable causes into the performance of the system, we obtain,

$$\bar{X}_{i+1}' = \bar{X}_i' + A_i + C_{i+1} . \quad i=0,1,2,\dots \quad (2-4)$$

This equation states the actual operating level at instant  $(i+1)$  is equal to the actual level at the instant  $i$  plus the adjustment just made and the net effect of all assignable causes which occurred since the last measurement was made.

For the simplicity, let  $\bar{X}'' = 0$ , then,

$$D_i = -\bar{X}_i , \quad (2-5)$$

$$A_i = -k\bar{X}_i = -k\bar{X}_i' - k\epsilon_i , \quad (2-6)$$

And, from (2-4) and (2-6),

$$\begin{aligned}\bar{X}'_{i+1} &= \bar{X}'_i - k\bar{X}'_i - k\epsilon_i + C_{i+1} \\ &= (1-k)\bar{X}'_i - k\epsilon_i + C_{i+1},\end{aligned}\quad (2-7)$$

with initial condition  $\bar{X}'_0 = C_0$ ,

Equation (2-7) is a first order linear difference equation, which can be solved by standard difference equation procedure to obtain,

$$\bar{X}'_i = \sum_{n=0}^i C_n (1-k)^{i-n} - k \sum_{n=0}^{i-1} \epsilon_n (1-k)^{i-1-n} \quad (2-8)^9$$

Equation (2-8) shows that the actual level of process operation at any given time is equal to the weighted sum of all assignable causes from the set-up conditions to the present and all previous measurement error. The weighting functions are successive integral powers of the quantity  $(1-k)$ . The set-up error is weighted by  $(1-k)^i$ . The error associated with the initial measurement (time 0) is weighted by  $k(1-k)^{i-1}$ . From this it is apparent that if  $\bar{X}'_i$  is going to remain reasonably close to the desired level  $\bar{X}'' = 0$  as  $i$  increases,  $(1-k)$  had better be less than one in absolute value. Otherwise, the effects of early assignable causes and error would cause  $\bar{X}'_i$  to become infinite. Thus with

$$|1-k| < 1, \quad (2-9)$$

the constant  $k$  is limited to the range

$$0 < k < 2. \quad (2-10)$$

This is the range of  $k$  which permits the process operating

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<sup>9</sup>This equation may be proved in following manner. Starting with  $\bar{X}'_0 = C_0$ , then successively apply (2-7) with  $i=0, 1, 2, \dots$

level to remain finite as time goes to infinity.

In many practical situations, it is almost impossible to derive a probability density function depicting the situation given. In such a condition, the expected value (mean),  $E(\bar{X}_i)$ , and variance,  $\sigma_{\bar{X}_i}^2$ , will suffice. These are given by, respectively,

$$E(\bar{X}_i) = \sum_{n=0}^i E(C_n)(1-k)^{i-n} - k \sum_{n=0}^{i-1} E(\epsilon_n)(1-k)^{i-1-n}, \quad (2-11)$$

and

$$\sigma_{\bar{X}_i}^2 = \sum_{n=0}^i \sigma_{C_n}^2 (1-k)^{2(i-n)} + k^2 \sum_{n=0}^{i-1} \sigma_{\epsilon_n}^2 (1-k)^{2(i-1-n)}, \quad (2-12)$$

since  $\bar{X}_i$  is a linear combination of the  $C_n$ 's and  $\epsilon_n$ 's.

Assuming furthermore measurement error to be unbiased; i.e.,

$$E(\epsilon_n) = 0 \quad n = 0, 1, 2, \dots \quad (2-13)$$

and the variances of both the assignable cause and error to be independent of time; i.e.,

$$\sigma_{C_n}^2 = \sigma_c^2, \quad (2-14)$$

and

$$\sigma_{\epsilon_n}^2 = \sigma_{\epsilon}^2, \quad (2-15)$$

Equations (2-11) and (2-12) can be simplified to yield

$$E(\bar{X}_i) = \sum_{n=0}^i E(C_n)(1-k)^{i-n}, \quad (2-16)$$

and

$$\begin{aligned}
 \sigma_{\bar{X}_i}^2 &= \sigma_c^2 \sum_{n=0}^i (1-k)^{2n} + k^2 \sigma_\varepsilon^2 \sum_{n=0}^{i-1} (1-k)^{2n} \\
 &= \sigma_c^2 \frac{1 - (1-k)^{2(i+1)}}{k(2-k)} \\
 &\quad + k^2 \sigma_\varepsilon^2 \frac{1 - (1-k)^{2i}}{k(2-k)} .
 \end{aligned} \tag{2-17}$$

The steady-state performance is found from the limit approach by  $\bar{X}_i$  as  $i$  increase indefinitely. Let  $M$  and  $V$  represent the steady-state mean and variance, respectively, of  $\bar{X}_i$ . Then,

$$M = \lim_{i \rightarrow \infty} E(\bar{X}_i) = \lim_{i \rightarrow \infty} \sum_{n=0}^i E(C_n) \cdot (1-k)^{i-n} \tag{2-18}$$

and

$$V = \frac{\sigma_c^2 + k^2 \sigma_\varepsilon^2}{k(2-k)} . \tag{2-19}$$

All these derivations assume  $k$  as given by (2-10).

### Effectiveness of Quality System

One measure of effectiveness which may be used in quality control is the total quality cost. According to this approach, all the costs associated with the quality of a product are classified into three categories: prevention, appraisal and failure cost, and then summed to obtain the total operating quality cost. These elements of operating quality cost have different characteristics. When the cost associated with prevention activities is in the relatively low bracket, an increased expenditure for pre-



vention may bring about significantly reduced failure and appraisal costs and thereby reduce operating quality cost. Reduction in operating quality cost continues up to the point when the increment of expenditure on prevention balances itself with the reduction in the cost associated with appraisal and failure of the product. The point which yields in minimum operating quality cost is considered to be optimal in the sense of quality cost.

This measure, however, has one drawback; it does not truly reflect the degree of the customer's satisfaction. The minimization of operating quality cost may increase the customer's satisfaction by indirect means, but it does not insure complete satisfaction. Many other factors may influence customer satisfaction. Some of the major factors are:

- Design characteristics of the product
- Manufactured characteristics of the product
- Reliability of the product
- Price of the product.

If a yardstick of effectiveness could be developed by incorporating these factors, it would provide an improved means of measuring the effectiveness of the quality system. The difficulty in solving this kind of problem is that of combining numbers, each obtained from several scales of measurement, each of relatively different significance, and each not necessarily independent of the others, into one number whose magnitude is indicative of the combination. The

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Desirability Function<sup>10</sup> can be applied to provide a mathematical solution to this problem.

### Desirability Scale

In the desirability function this combination is effected by transformation of the measured properties to the desirability scale. If by some means, the several properties could be measured in consistent units, or, even better, could be expressed as numbers on a dimensionless scale, then the arithmetic operations intended to combine these measures become feasible. To perform this transformation of scales, it is necessary to discover a scale common to all properties, and to which some physical significance may be attached. This scale is referred to as the "desirability scale", and will be abbreviated as the scale of "d". Overall Desirability, "D", will be developed later in this section.

A useful range of the "d" scale is between 0.00 and 1.00. A scale value,  $d$ , = 0.00, corresponds to a completely undesirable level of the property in question, (i.e., so poor that the product is completely unacceptable for the intended use), and  $d = 1.00$  corresponds to a completely acceptable level of property (i.e., an improvement in the

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<sup>10</sup>Desirability Function discussed in this section is adopted from Edwin C. Harrington, Jr., "The Desirability Function," Industrial Quality Control, April, 1965, Vol. 21, No. 10, pp. 494-497.

<sup>11</sup>Overall Desirability can be referred to as an Effectiveness of Quality System.

property would serve no useful purpose). Intermediate values on the desirability scale are identified in Table I.<sup>12</sup>

TABLE I  
DESIRABILITY SCALE FOR "d"

Scale of d	Quality equivalents of the scale of "d" description
1.00	Represents the ultimate in "satisfaction" or quality, and improvement beyond this point would have no appreciable value.
1.00 - 0.80	Acceptable and excellent. Represents unusual quality, or performance, well beyond anything commercially available.
0.80 - 0.63	Acceptable and good. Represents an improvement over the best commercial quality, the latter having the value of 0.63.
0.63 - 0.40	Acceptable but poor. Quality is acceptable to the specification limits, but improvement is desired and products are likely to lose out to competition.
0.40 - 0.30	Borderline. If specification exists, some of the product would lie outside of these specifications. (If quality lies exactly on the specification maximum or minimum, its d should be $0.36788 = 1/e$ ).
0.30 - 0.00	Unacceptable. Products of this quality would lead to failure of the project.
0.00	Completely unacceptable.

<sup>12</sup>From mathematical standpoint, it is convenient to assign a desirability value of 0.37 to any property at its specification value, maximum or minimum, assuming that realistic specification limits exist for this property. The number 0.37 is approximately  $1/e$  (0.36788), where e is the base of the natural logarithms. A second such useful landmark is the value of a property corresponding to the best commercial quality (existing or anticipated), for which a desirability value of 0.63 ( $= 1 - 1/e$ ) is appropriate.

### Transformation of Properties to "d"

The simplest sort of transformation is possible when there exist lower and upper specification limits: these limits being the sole and unalterable criteria of quality. Outside these limits the value of  $d$  is 0.00 and within, the value of  $d$  is 1.00. This situation is shown in Fig. 7a.

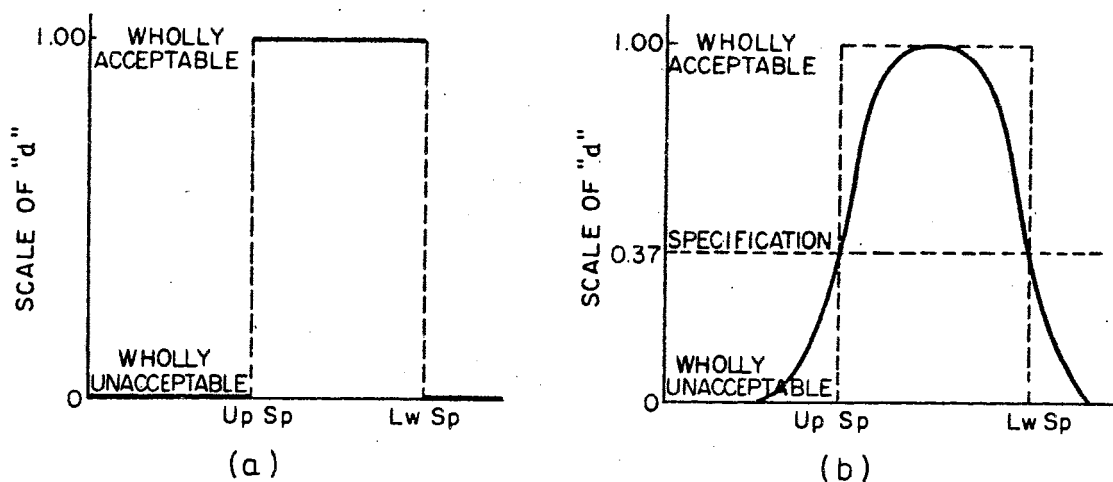


Figure 7. Graphical Illustration of Relationship Between  $Y_i$  and "d" (Two Sided Specification)

However, in many industrial situations, due to the inherent process variability and testing imprecision, it is quite impossible to separate borderline quality into two unequivocal groups, the acceptable and the unacceptable product. The effect of these considerations is to smooth the discontinuities of Fig. 7a as shown in Fig. 7b.

In Fig. 7b the values of the property being considered are represented on the horizontal scale, and the equivalent

values of "d" are obtained by reference to the vertical scale. Mathematical transformation from the measurement of the property to the scale of "d" is accomplished by the basic equation:

$$d = e^{-(|Y'|)^n}, \quad (2-20)$$

where e is the logarithmic constant e, 2.71828...

n is a positive number ( $0 < n < \infty$ ), not necessarily integral.

Y' is a linear transformation of the property variable, Y, such that  $Y' = -1$  when Y is equal to the lower specification limit,  $Y_{\min}$  and  $Y' = +1$  when Y is equal to the upper specification limit,  $Y_{\max}$ .

|Y'| is the absolute value of Y'.

Any particular value of Y, identified as  $Y_i$ , may be transformed to the corresponding  $Y'_i$ , by the relation:

$$Y'_i = \frac{2Y_i - (Y_{\max} + Y_{\min})}{Y_{\max} - Y_{\min}} \quad (2-21)$$

Equation (2-20) represents a family of curves, all of which

- a) asymptotically approach  $d=0$  as the absolute value of Y', |Y'|, exceeds 1.0
- b) pass through  $d=1/e=0.37$  when the absolute value of Y' equals 1.00 (this is one reason for selecting  $d=0.37$  to represent the specification value)
- c) pass through  $d=1.00$  at the midpoint between the upper and lower specification limits.

The exponent, n of Equation (2-20) determines the slope of the curve, and as n becomes large, the curve approaches the limiting case of  $d=0$  outside the specification limits and  $d=1.00$  within these limits. For any given desirability curve corresponding to Equation (2-20), n may be calculated

by selecting a value of  $d$ , finding its corresponding  $|Y'|$ , and substituting in the equation,

$$n = \frac{\ln \ln 1/d}{\ln |Y'|} \quad (2-22)$$

In the case of a one-sided specification another form of the exponential is convenient, a special form of the Gompertz growth curve:

$$d = e^{-(e^{-Y'})} \quad (2-23)$$

which is illustrated in Fig. 8.

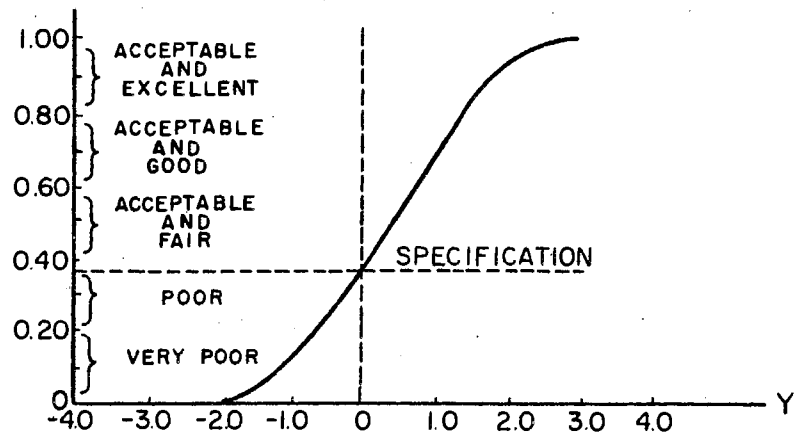


Figure 8. Graphical Illustration of Relationship Between  $Y_i$  and "d" (One Sided Specification)

In this equation the slope is determined by the scaling of  $Y$  onto  $Y'$ ; the exponent,  $n$ , of Equation (2-20) is not required. This scaling is accomplished by selecting two values of the measured property,  $Y$ , and assigning to them

desirability values according to Table I. These two desirability values are transformed to their equivalent in  $Y'$  either graphically (using Figure 8) or by the equation,

$$Y' = -[\ln(-\ln d)] \quad (2-24)$$

From these paired values of  $Y$  and  $Y'$ , the linear transformation equation of the form

$$Y' = b_0 + b_1 Y \quad (2-25)$$

is easily derived by calculating the two constants,  $b_0$  and  $b_1$ , from the two equations which result from substituting these paired values in Equation (2-25). Although one might conceive of many alternate forms of Equations (2-20) and (2-23), these exponential equations are convenient to use and are usually entirely adequate for the purpose of transforming measured properties to the desirability scale. Convenience arises from the fact that the only arithmetic operation involved is "table look-up" in a table of the exponential function.

### Overall Desirability

Having transformed the several measures of quality of properties to the dimensionless scale of "d", it is possible to combine these "d's" by any arithmetic operations to develop the overall desirability,  $D$ , of the product. A basic premise in the step is: if any one property is so poor that the product is not suitable to the application, the product will not be acceptable, regardless of the remaining properties. It is true that customer reaction to a product is to

a large extent dependent upon the less desirable properties of the product because these properties possess potential trouble.

The mathematical model analogous to these psychological reaction is the geometric mean of the component "d's",

$$D = \sqrt[n]{d_1 d_2 \dots d_n} \quad (2-26)$$

If any  $d_i$  is zero, the associated  $D$  will also be zero in this equation. Furthermore,  $D$  is strongly weighted by the smaller  $d$ 's.

This  $D$  can be equated to the effectiveness of the quality system when  $d_i$  represents factors influencing the effectiveness of the system.

Because of the subjective procedures utilized, the establishment of the relationship between each property  $Y_i$  and the corresponding "d" scale is the critical step in developing overall desirability. However, it is important to realize that any other measure of quality, interpreted as a value, is also subjective. A conventional method for establishing this relation is simply sketching the landmarks and connecting these points with a continuous curve. If the judgement of several persons is involved, develop individual relationships, then seek a compromise representing group judgement.



## CHAPTER III

### ECONOMY OF INFORMATION FEEDBACK

When a process goes out of control, one of two decisions must be made: either to locate and remove the assignable cause of the change in population parameter, or to adjust the level of one or more input variables to compensate fully or partially for the apparent changes in output parameter. The former case involves investigation and possible shutdown of the process. In the latter case, this is not necessary. The following discussion will be focused on the assumption of the latter case.

An economic evaluation of the information feedback in process control reduces to the selection of an alternative which minimizes total cost function<sup>1</sup> given by,

$$C_T = C_D + C_F + C_V + C_I + C_A \quad (3-1)$$

where  $C_T$  : total cost

$C_D$  : cost of defective, scrap and rework

$C_F$  : fixed cost of sampling

$C_V$  : variable cost of sampling

$C_I$  : cost of interpretation and decision making

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<sup>1</sup>N. N. Barish and N. Hauser, "Economic Design for Control Decision," The Journal of Industrial Engineering, May-June, 1963, Vol. XIV, No. 3, pp. 125-132.

$C_A$  : cost of process adjustment, all in unit time basis. All the cost elements of the total cost function, except the cost of defective,  $C_D$ , can be estimated within limits of reasonable accuracy. However, in estimating the cost of defective,  $C_D$ , it is necessary to understand the pattern of defective occurrence.

### The Patterns of Defective Occurrence

The defectives occur in a different manner depending on the nature of the process and characteristics of the assignable cause. The simplest pattern of defective occurrence can be observed in a shop where characteristics of the products are determined by the process chosen. Those characteristics which are determined by the process chosen will not vary from product to product, once the process has been correctly set-up. The punch press in a metal sheet fabrication process is a good example of this sort. When a set-up error is committed, almost a constant rate (in a probabilistic sense) of a defectives turn out until a proper corrective action is taken.

The second pattern takes place when there is a shift in the process mean. This is illustrated in Fig. 9. When an assignable cause arrives, the process mean starts to deviate until the shift is detected, and a proper corrective action is completed. The pattern of deviation is dependent upon the nature of the assignable cause and the characteristics of the process. The output from lathe, shaper, or

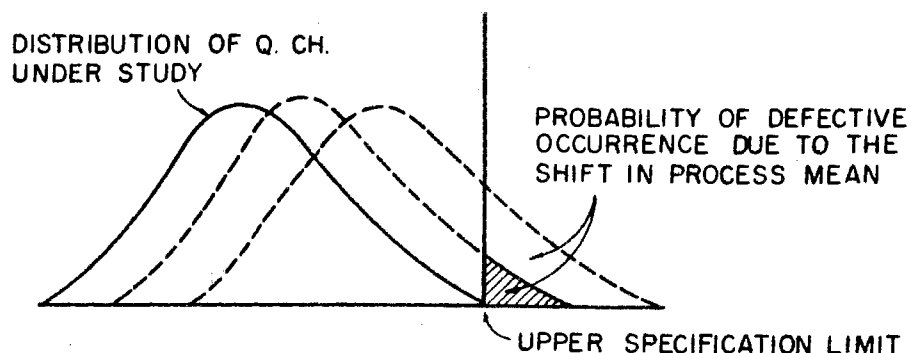


Figure 9. The Conceptual Illustration of Shift in Process Mean

other forming operations; concentration of reagents or catalysts in chemical processes; maintenance of temperature, pressure, humidity, etc., can be listed as process having this type of problem. Generally, a simple process adjustment may effectively counteract this sort of assignable cause.

The third pattern takes place when the variance of the process tends to increase due to the assignable cause as illustrated in Fig. 10. This phenomenon occurs due to

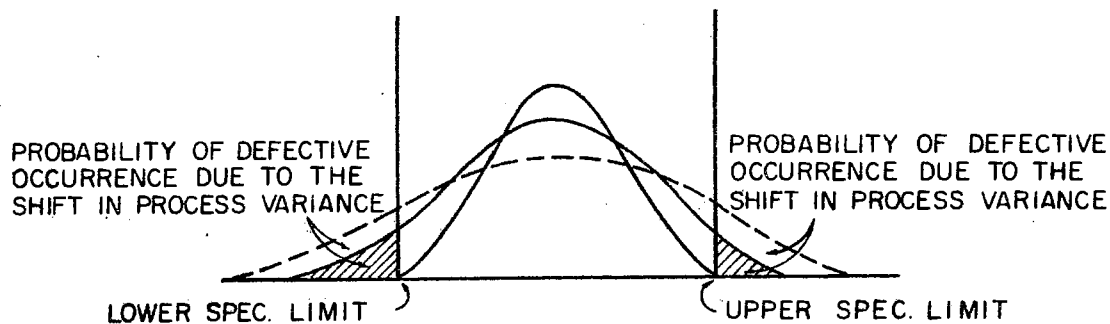


Figure 10. Conceptual Illustration of Shift in Process Variance

improper materials being fed to the process or to malfunctioning of the process equipment. In this case, shutdown of the process to locate and remove the assignable cause may be required.

All the other cases of defective occurrence, not covered above, can be described as one of any combination of the patterns shown above.

In estimating the cost of defective,  $C_D$ , in terms of analytical method, at least, the following items must be known within limits of accuracy, commensurable with the confidence level of the answer desired:

- a) Theoretical distribution of the process
- b) Arrival pattern and magnitude of assignable causes expressed as a mathematical model
- c) Expected value of feedback cycle,<sup>2</sup> under a given decision rule.

It is previously mentioned that to estimate the cost of defective,  $C_D$ , the patterns of defective occurrence must be known. However, with the above observation, one comes to the following conclusion: in most cases, to describe the items listed above, in terms of a mathematical model, is practically impossible. Furthermore, manipulating the model to obtain the expected cost of a defective is too complicated and time consuming even if it is possible at all.

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<sup>2</sup>The term, feedback cycle, refers to an amount of time elapsed between the moments when a process starts to turn out defectives due to an assignable cause, and when this has been detected and corrective action completed.

### An Approach to the Problem

An approach to the solution may be found by observing Equation (3-1). When feedback cycle is taken as a variable factor, the cost elements to the right of equal sign of the equation can be divided into two groups, namely cost of defective,  $C_D$ , and the cost associated with feedback.<sup>3</sup> Regardless of the patterns of defective occurrence previously shown, a possible means of reducing the cost of defective is to shorten the feedback cycle. This can be done by increasing the feedback cost. The problem, then, is to compress the feedback cycle, through increase of feedback cost, until it reaches the minimum point in Equation (3-1). An illustration of this concept is given in Fig. 11.

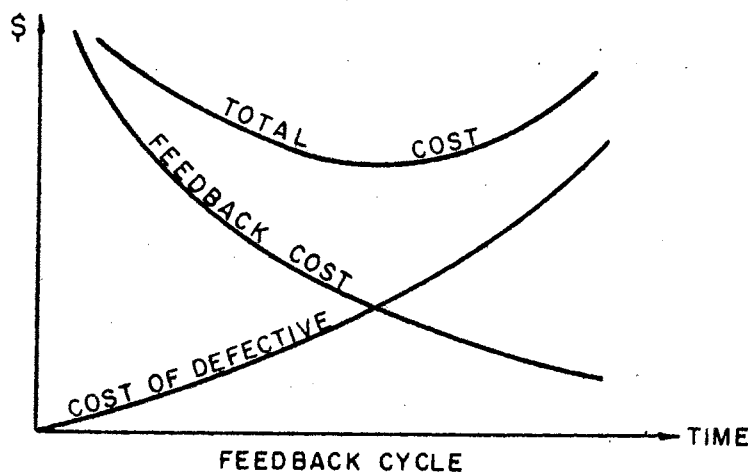


Figure 11. Conceptual Illustration  
of Economy of Infor-  
mation Feedback

<sup>3</sup>The cost associated with feedback refers to the sum,  $C_F + C_V + C_I + C_A$ , and will hereafter be referred as feedback cost.

There are many ways by which the compression of the feedback cycle can be achieved. By looking at Table II, one may choose any variable factor under control to compress the cycle; however, the increment of the feedback cost associated with the compression will depend upon the choice of the method to be employed. Accordingly, an important notion in this procedure is to achieve a compression by the change of variables that have minimum cost increment.

With the advent of the high-speed computers, Monte Carlo simulation has been increasingly important in recent years as a research tool and method of solving industrial and managerial systems problems. Since the technique does not assume theoretical distribution, difficulties of formulating mathematical model, discussed in Sec. 1 of this chapter, can be avoided by the use of empirical distributions, provided the empirical study can be done and the study is reasonably accurate.

#### Hauser's Simulation<sup>4</sup>

The original intention of Hauser, in running the Monte Carlo Simulation, was to investigate various combinations of control chart design with an objective of minimizing the

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<sup>4</sup>N. Hauser, "Economic Design of Control Charts for Process Adjustment," (Unpublished Doctor's dissertation, New York University, 1962), and

N. N. Barish and N. Hauser, "Economic Design for Control Decision," The Journal of Industrial Engineering, March-June, 1963, Vol. XIV, No. 3, pp. 125-132.

TABLE II  
COMPONENTS OF FEEDBACK CYCLE

Phases of Feedback Cycle	Contents of the Phase	Variable Factors Under Control	Remarks
Detection	<ol style="list-style-type: none"> <li>1) Sampling</li> <li>2) Inspection and Measurement</li> <li>3) Interpretation and Analysis of Data</li> </ol>	<ol style="list-style-type: none"> <li>1) Sampling Procedure and Method</li> <li>2) Method of Inspection and Measurement, Equipment Employed</li> <li>3) Method Used in Interpretation and Analysis of Data</li> <li>4) Decision Rule</li> </ol>	<p>When a decision is based on sample the decision is subject to both type I and type II error and this will influence the feedback cycle.</p>
Flow of Information	<ol style="list-style-type: none"> <li>1) Handling of Materials</li> <li>2) Flow of Paper</li> <li>3) Communication</li> </ol>	<ol style="list-style-type: none"> <li>1) Systems of Information Flow</li> <li>2) Equipment and Procedure Employed</li> </ol>	
Corrective Action		<ol style="list-style-type: none"> <li>1) Means of Adjustment</li> </ol>	

total cost function given by Equation (3-1). The design variables used in the simulation were:

- a) Decision rule and process estimator.
- b) Sample size.
- c) Sampling interval.

This may be interpreted as a varying feedback cycle, by changing variables of decision phase, listed in Table II, to minimize the total cost function.

The model used in this simulation is given below.

$$X_t = \mu'' + \sum_{k=1}^t \delta_k + \sum_{j=1}^t \rho_j + \epsilon_t$$

where

- $X_t$  : Process level reading at time  $t$
- $\mu''$  : Desired process mean
- $\delta_k$  : Random variable denoting assignable cause magnitude ( $\delta_k=0$  unless an assignable cause arrival is indicated at time  $k$ )
- $\rho_j$  : Change introduced by adjusting device at time  $j$
- $\epsilon_t$  : Random variable representing the effect of change cause of variation.

In running this simulation, the following assumptions were made:

- a) The measuring device provides information with no significant variability, and the adjusting mechanism is perfectly calibrated.
- b) Any variation caused by measurement and adjustment variability are reflected in the error of



the adjustment.

- c) No time delay between process adjustment and reaction of the process is assumed.
- d) The process variation produced by chance causes are assumed to follow a standard normal distribution, i.e.,  $E(\epsilon) = 0$ ,  $\text{Var}(\epsilon) = 1$ .
- e) The number of arrivals of assignable causes during any time interval of duration  $t$  is a Poisson Variable with mean  $\lambda$ . For a small time interval,  $h$ , the probability of exactly one arrival is approximately  $\lambda h$ .<sup>5</sup>
- f) The magnitude of an assignable cause,  $\delta$ , is a random variable with distribution  $f(\delta)$ , having a mean and standard deviation of 0 and  $K$  respectively.<sup>6</sup>
- g) Assuming stochastic independence, the cumulative effect of assignable cause will follow a Compound Poisson Distribution.<sup>7</sup> If let  $f(\delta)$  be normal, this distribution is described by the parameters  $\lambda h$  and  $K$ .

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<sup>5</sup>E. Parzen, Modern Probability Theory and Its Application, (New York: John Wiley & Sons, Inc.), 1960, p. 252.

<sup>6</sup> $K$  is defined as  $K = \frac{\sigma_\delta}{\sigma_\epsilon}$ .

<sup>7</sup>W. Feller, An Introduction to Probability Theory and Its Application, 2nd Ed., (New York: John Wiley & Sons, Inc.) 1957, p. 270.

- h) Normal distributions of assignable causes, having probability 0.01 of arrival, during a given time interval were used. The respective standard deviations were 1, 2, 5, and 10 times that of the chance cause population. (i.e.,  $K=1,2,5$  and 10)
- i) Process with two levels of capability to meet specifications were considered:
- 1) Process with  $3\sigma$  specification limits
  - 2) Process with  $1\sigma$  specification limits.

#### Decision Rule and Process Estimator

- 1)  $1\sigma_{\bar{X}}$  Decision Rule:  
If a sample mean falls outside  $\mu'' \pm 1\sigma_{\bar{X}}$ , introduce an adjustment equal and opposite to the indicated deviation as measured by the last sample mean.
- 2)  $2\sigma_{\bar{X}}$  Decision Rule:  
If a sample mean falls outside  $\mu'' \pm 2\sigma_{\bar{X}}$ , proceed as in 1) above.
- 3)  $3\sigma_{\bar{X}}$  Decision Rule:  
If a sample mean falls outside  $\mu'' \pm 3\sigma_{\bar{X}}$ , proceed as in 1) above.
- 4) Runs Decision Rule:
  - a. If a sample mean falls outside  $\mu'' \pm 3\sigma_{\bar{X}}$ , proceed as in 1) above.
  - b. If two of the last three sample means fall within  $\mu'' + 2\sigma_{\bar{X}}$  and  $\mu'' + 3\sigma_{\bar{X}}$  or within

$\mu'' - 2\sigma_{\bar{X}}$  and  $\mu'' - 3\sigma_{\bar{X}}$  introduce an adjustment equal and opposite to the indicated deviation as measured by a weighted average of the last two or three sample means.

- c. If four of the last five sample means fall within  $\mu'' + \sigma_{\bar{X}}$  and  $\mu'' + 3\sigma_{\bar{X}}$  or  $\mu'' - \sigma_{\bar{X}}$  and  $\mu'' - 3\sigma_{\bar{X}}$  introduce an adjustment equal and opposite to the indicated deviation as measured by the weighted average of the last four or five sample means. The weighted average of the last  $n$  sample means is obtained by

$$\hat{\mu} = \frac{\sum_{i=1}^n (i\bar{X}_{n-i+1})}{\sum_{j=1}^n j}$$

where  $\bar{X}_1$  is the most recent sample mean,  $\bar{X}_2$  is the previous sample mean, and  $\bar{X}_n$  is the most remote sample mean. If, as a result of the last sample value, more than one of the above (a, b, c) apply, the first listed is used.

5) Geometric Decision Rule:

If the geometric mean  $z_t$ , defined as  $z_t = r\bar{X}_t + (1-r)z_{t-1}$  (where  $0 \leq r \leq 1$  and  $\bar{X}_t$  is the sample mean at time  $t$ ) exceeds the desired process mean by more than  $[3\sqrt{r/(2-r)}]\sigma_{\bar{X}}$  introduce an adjustment equal and opposite to the indicated deviation as measured by the last sample mean. After each

adjustment,  $z_t$  is changed to zero. In the simulation  $r=0.4$  was taken. In this rule, all previous readings back to the last adjustment are taken into account in determining whether to adjust, but the magnitude of the adjustment is based upon reading of last samples only.

For each decision rule, the sample size was varied, keeping the sample size-sampling interval ratio constant, until a minimum cost was reached. This process was repeated at different size-interval ratios until the one combination of sample size and sampling interval giving minimum cost was found.

#### Cost Parameters

1) Cost of Defectives:

$c_d$  is defined as the cost of each per cent defective produced during one time interval. It depends on the length of the time interval; the rate of production during that period; the proportion of defectives scrapped, repaired, and passed; and the cost of scrapping, repairing, and overlooking each defective piece.  $C_D$ , the cost of defectives per unit time, is given by  $C_D = c_d F$ , where  $F$  is the expected per cent defective product.

2) Fixed Sampling Cost:

$c_f$  is defined as the fixed cost of obtaining one sample regardless of its size. It depends on the

time and skill required to obtain it and on whether the process must be interrupted.  $C_F$ , the fixed cost of sampling per unit time, is given by

$C_F = c_f/H$ , where  $H$  is the interval between samples.

3) Variable Sampling Cost:

$c_v$  is defined as the variable cost of measuring each piece in the sample. It depends on the time, skill, and equipment required, as well as the damage caused to each piece in testing.  $C_V$ , the the variable sampling cost per unit time, is given by  $C_V = c_v N/H$ , where  $N$  is the sample size.

4) Cost of Interpretation and Decision:

$c_i$  is defined as the cost of interpreting each sample, that is, whether to adjust and, if so, by what amount. It depends on the complexity of the decision rule and deviation estimates used.  $C_I$ , the interpretation cost per unit time, is given by  $C_I = c_i/H$ .

5) Cost of Adjustments:

$c_a$  is defined as the cost of making one adjustment. It depends on the time, skill, and equipment required, as well as the number of defectives caused by each adjustment.  $C_A$ , the adjustment cost per unit time, is given by  $C_A = c_a A$ , where  $A$  is the expected number of adjustments per unit time.

## 6) Total Cost:

$C_T$ , the total cost per unit time, is given by

$$C_T = C_D + C_F + C_V + C_I + C_A,$$

or equivalently

$$C_T = c_d F + (1/H)(c_f + c_v N + c_i) + c_a A.$$

The dimension of this total cost can be reduced without affecting the generality of any obtained solution by dividing each term by  $c_d$ . If let  $c'_j$  represent  $c_j/c_d$  for  $j = \{f, i, v, a\}$  and

$$C'_T = C_T/c_d, \text{ then,}$$

$$C'_T = F + (c'_j + c'_i)(1/H) + c'_v N/H + c'_a A.$$

With these parameter definitions, models were tested a number of different cost configurations. Twelve such configurations are shown in Figures 12 through 15.

### Results of the Simulation

Tables showing the number of adjustments and fraction of defective output for processes with 3 $\sigma$  and with 1 $\sigma$  specification limits when the various decision rules and process estimators are used. Process parameter values of  $P=0.01$  and  $K=1,2,5$  and 10 are used with various sample sizes for different ratios of sample size to sampling interval. Figures 12 and 13 show the relative economy of the selected decision rules for the twelve selected cost configurations for various  $K$  under the processes with 3 $\sigma$  and 1 $\sigma$  specification limits respectively. In each case, the value for the

GOOD PROCESS CAPABILITY - 3  $\sigma$  SPECIFICATION LIMITS

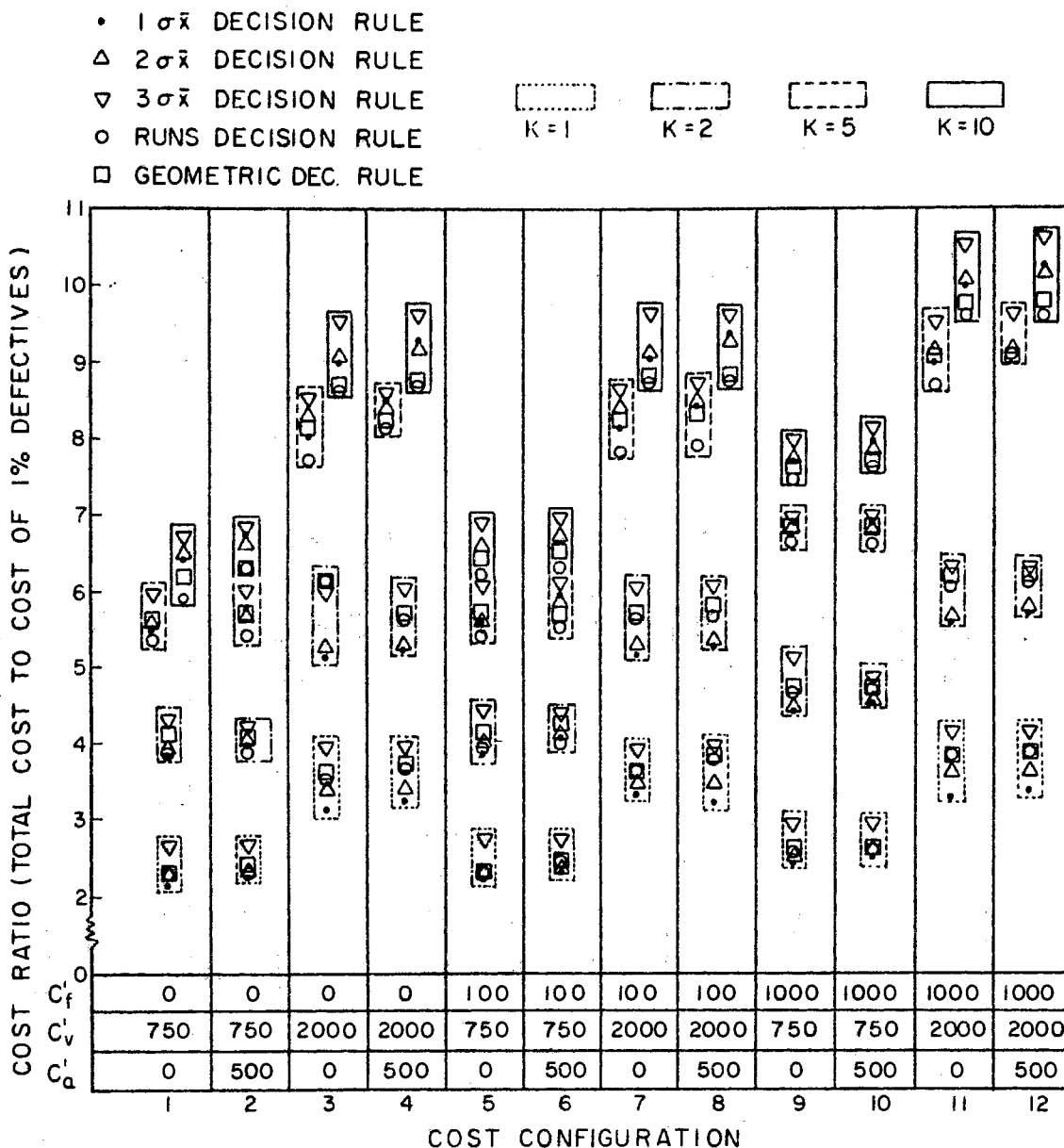


Figure 12. Comparative Economy of Decision Rule (a)<sup>8</sup>

<sup>8</sup>Figure 12, 13, 14 and 15 are adopted from:  
 N. N. Barish and N. Hauser, "Economic Design for Control  
 Decisions," The Journal of Industrial Engineering, May-  
 June, 1963, Vol. XIV, No. 3, pp. 129-132.

SELECTED DECISION RULES  
 POOR PROCESS CAPABILITY -  $1\sigma$  SPECIFICATIONS LIMITS

- $1\sigma\bar{x}$  ..... DEC. RULE
- △  $2\sigma\bar{x}$  ..... DEC. RULE
- ▽  $3\sigma\bar{x}$  ..... DEC. RULE
- GEOMETRIC DEC. RULE
- RUNS ..... DEC. RULE

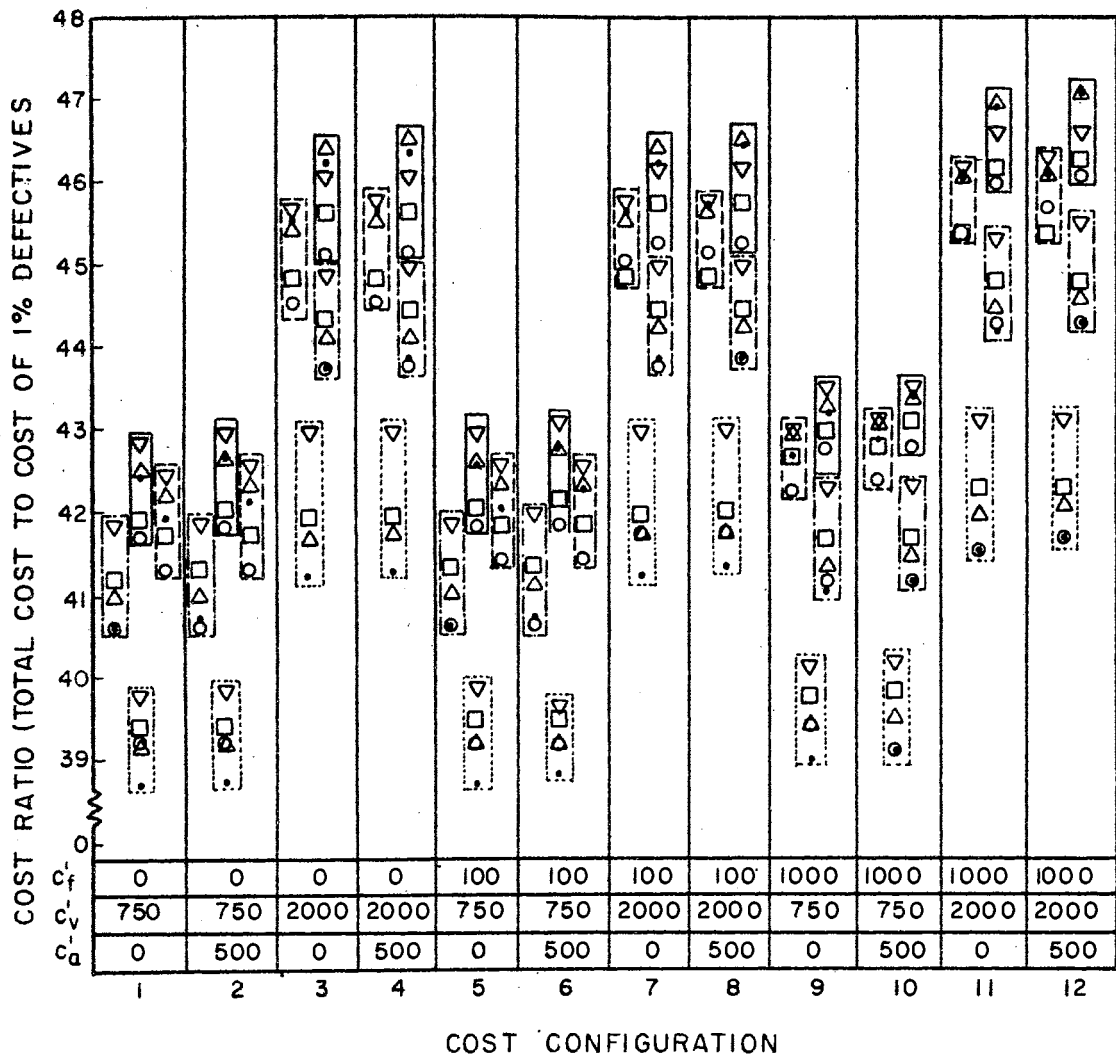
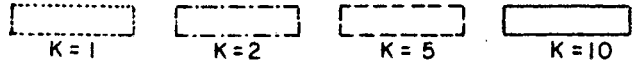


Figure 13. Comparative Economy of Decision Rule (b)



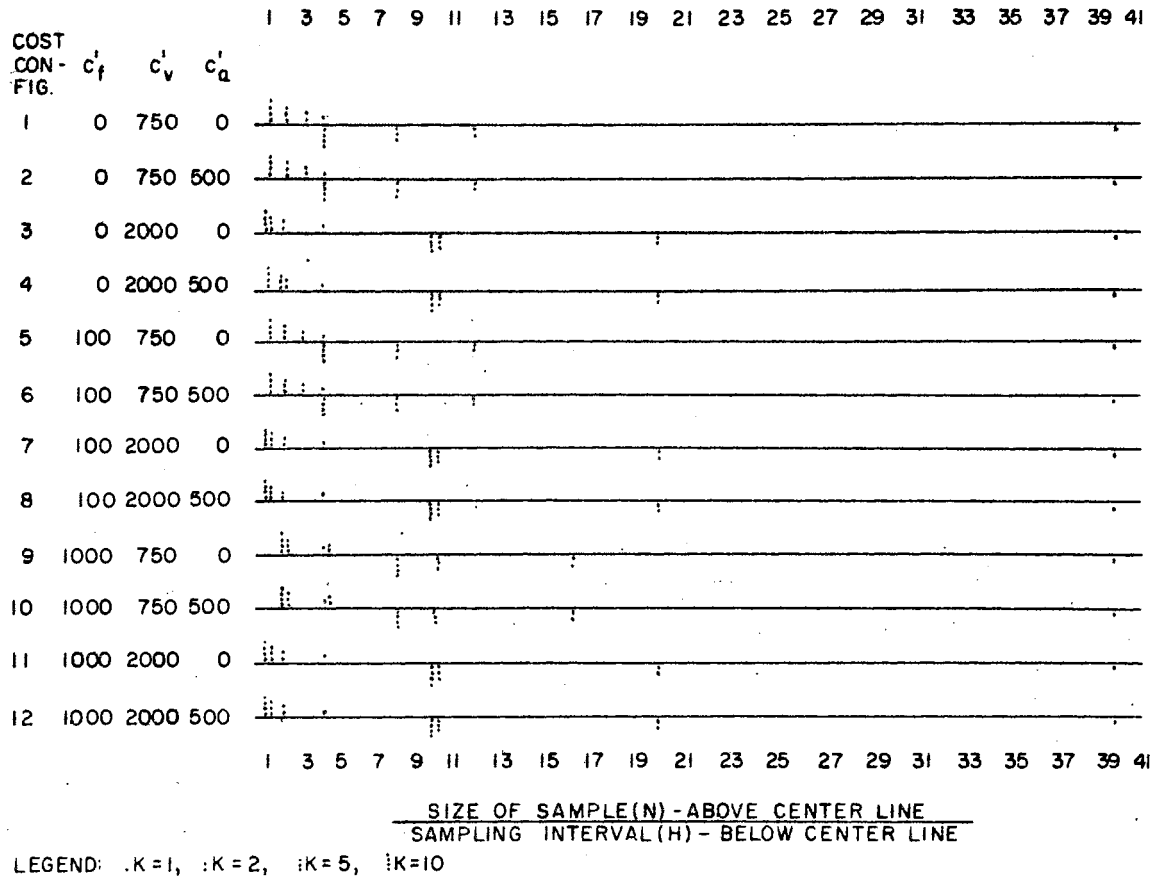
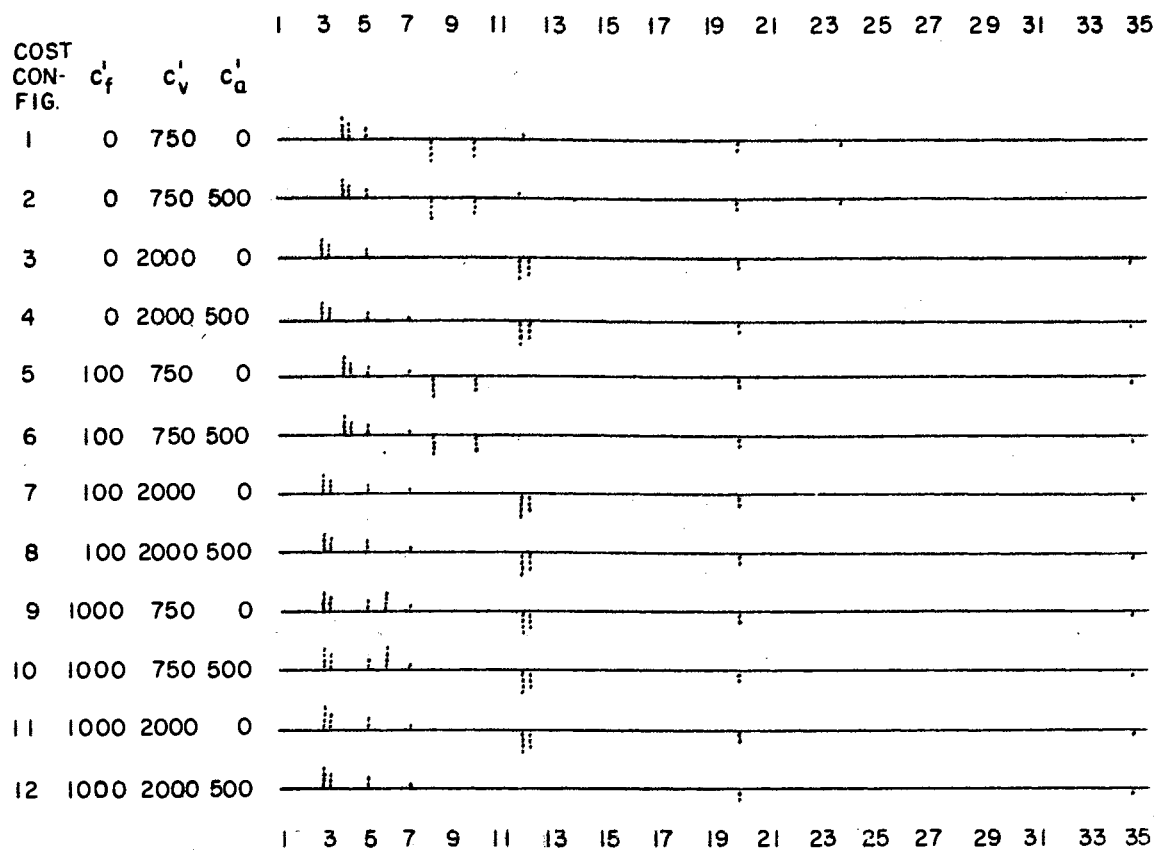


Figure 14. Optimum Sample Sizes and Sampling Intervals for Runs Decision Rule for the Process with 3  $\sigma$  Specification Limits



LEGEND: :K=1, :K=2, :K=5, :K=10

Figure 15. Optimum Sample Size and Sampling Interval for Runs Decision Rule for the Process with 1  $\sigma$  Specification Limits

optimum combination of sample size and sampling interval is plotted. Based on these Figures, one may conclude the following:

- a) Under the process with 3 $\sigma$  specification limits, when  $K=1$ , the spread in total costs increases as the variable sampling cost increases, and the  $1\sigma_{\bar{x}}$  decision rule is consistently more economical.
- b) As  $K$  becomes larger, the runs and geometric decision rules, which use information from more than one sample, become more economical. The runs decision rule appears to be superior to the geometric rule for the given values of  $r(=0.4)$ .
- c) The cost of adjustment has little effect on the relative economy of the decision rules.
- d) Under none of the conditions tested, is the popular  $3\sigma_{\bar{x}}$  decision rule most economical (in most cases being the least economical of the rules).
- e) When the process capability is poor, there is greater sensitivity to alternative decision rules than when the process capability is good.

Figures 14 and 15 show the optimum sample size and sampling interval based on the runs decision rule. The runs decision rule is used since the decision rule is shown to be optimal for a majority of the parameter values considered in Fig. 12.

The Figures 14 and 15 provide the following information:

- a) As the relative magnitude of assignable cause,  $K$ , increases the economical sample size decreases and

the economical sampling interval shortens.

- b) When variable sampling costs are high, the economic sample size tends to be small.
- c) When the process capability is poor, larger samples are generally more economical than when the process capability is good. Except when  $K=1$ , the optimum sampling interval is also large (sample taken less frequently).

The result of the simulation so far discussed is informative. However, it is important to realize that the result is applicable only when the assumptions previously stated hold true in the reality. The important implication of this discussion is that it provides strong evidence that the simulation technique may be effectively employed in approaching the feedback problems.

## CHAPTER IV

### APPLICATION OF THE WORK STUDY CONCEPT

The economy of information feedback discussed in Chapter III was confined to a process where a form of quantitative model could be developed for the study. However, in quality control a large part of the information feedback process is dependent upon human elements, and the formulation of a quantitative model, in general, is extremely difficult and complicated.

A more general solution may be found by applying the concept of work study; information feedback as used in this context is work involved in creation of information about the process and communication and execution of corrective action with an objective of controlling the process. The information feedback, then, can be treated as an operation which allows the following steps of analysis.<sup>1</sup>

- (1) Choose a process to be studied.
- (2) Consider eliminating entire feedback procedure.
- (3) Failing this, break the feedback cycle down by:
  - a. Listing the work elements.

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<sup>1</sup>E. M. Barry, "Work Simplification Applied to Inspection," Industrial Quality Control, May, 1959, Vol. XV, No. 11, pp. 56, 58.  
And *ibid.*, June, 1959, Vol. XV, No. 12, pp. 19, 20.

- b. Listing the equipment used.
  - c. Preparing flow diagrams and work place sketches.
  - d. Discussing the feedback procedure with the personnel involved.
  - e. Analysis.
- (4) Formulate a proposed procedure based on the study.
- (5) Install the proposed method, and follow up.

### Components of Feedback Cycle<sup>2</sup>

The time to complete one cycle of information feedback is as shown in Figure 16, and the explanation of the work contents in the figure is given below.

A. The basic work content of information feedback

The basic work content is an irreducible amount of work required to perform a given task, i.e., information feedback. This includes all the essential components in the detection, flow of information and corrective action phases of feedback cycle given in Table II of Chapter III. This is an ideal situation, which can never occur in practice; nevertheless, the irreducible amount of work content can be set as a goal.

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<sup>2</sup>Sections 1 and 2 of this chapter is based on: International Labor Office, Introduction to Work Study, (Geneva, 1962), pp. 15-33.

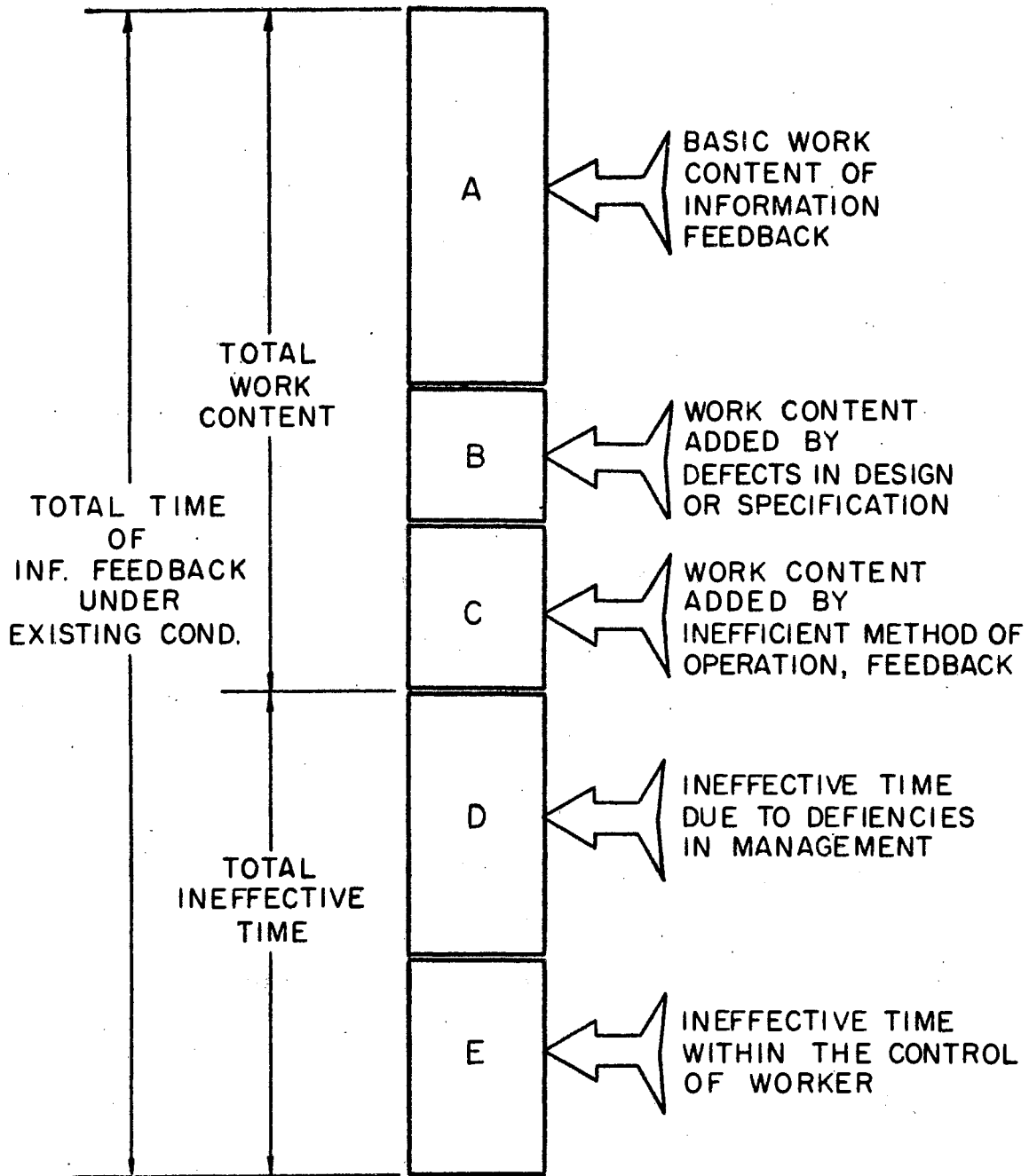


Figure 16. Components of Feedback Cycle

B. The work content added by defects in design or specification

There are several ways by which work content in information feedback can be added due to defects in design or specifications. Some of the important causes are shown in Figure 17.

- (1) Improper Design: Improper design of product or components may affect the information feedback in two ways:
  - a. A poorly designed product generally requires more processes than might be required for a better design.
  - b. Defectives found in any one process may be caused by poorly designed components in a previous process.

In both cases, unnecessary feedback would be created.

- (2) Lack of Standardization: When there is an excessive variety of products or a lack of standardization, it would be difficult to implement a stable information feedback system. This would result in an excessive amount of work in information feedback.
- (3) Improper Quality Standard: Incorrect quality standard, whether too high or too low, may increase work content. In engineering practice, close tolerances require extra machining and closer control of the process. When the



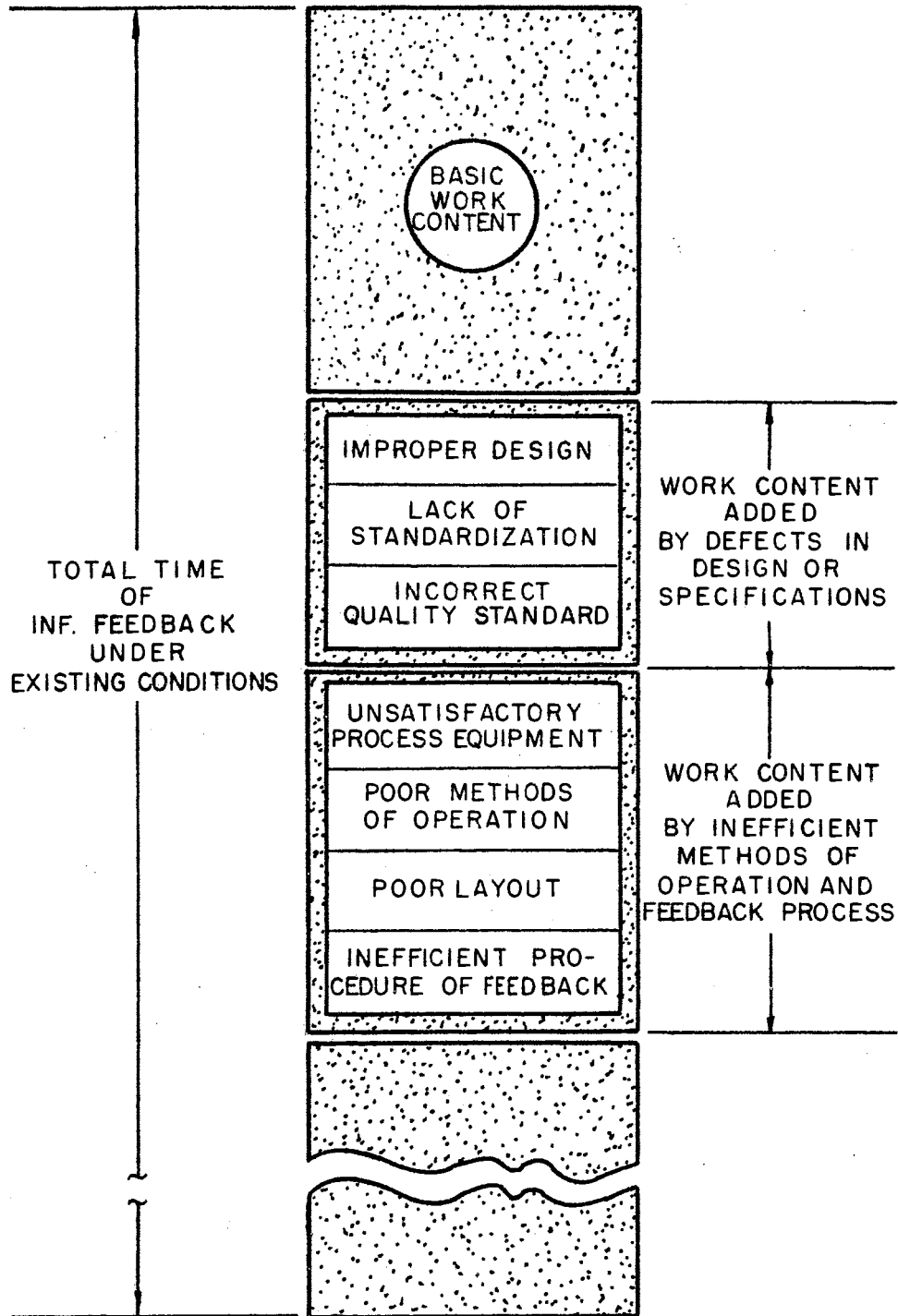


Figure 17. Ineffective Components of Feedback Cycle (a)

tolerances are too low, it may cause difficulties in process control for subsequent operations.

C. The work content added due to inefficient methods of operation and the feedback procedure

Inefficient methods of operation and the procedure of information feedback may add to the work content of feedback in the following manners, as shown in Fig. 17.

- (1) Unsatisfactory Process Equipment: When the precision of machine and tools employed for a process is incompatible with the given design of the product or component, a greater effort is required in process control.
- (2) Poor Methods of Operation: When the method used in the process deviates from normal procedure, it may lower the precision of the process; thereby increasing the effort of process control.
- (3) Poor Layout: When a large part of the feedback process is performed by human elements, a bad layout of the process may add ineffective time due to wasted movements.
- (4) Inefficient Procedure of Feedback: The procedure employed to obtain the information feedback is the most important factor in determining cycle time.

(See Table II, Chapter III)

D. Ineffective time derived from deficiencies in management

Ineffective time is often created due to deficiencies in

management. Figure 18 illustrates detail.

- (1) Excessive Product Variety: When a marketing policy demands an excessive variety of products, it dictates short runs of each type. This conflicts with stabilization of the feedback system in the manufacturing process.
- (2) Design Changes: When management fails to insure that designs meet customers' requirements, design changes are generally brought about in an effort to meet the demand. These changes will create unnecessary feedback throughout manufacturing processes.
- (3) Inefficient Quality Policy: When the existing quality policy is not properly implemented, it may cause confusion in executing a quality program, adding ineffective time to feedback activities.
- (4) Machine Breakdown: Machine tools and other major manufacturing equipment inevitably will wear under constant use, the resulting loose bearings and worn pins may cause a process to go out of control.
- (5) Improper Raw Material: Improper raw material due to either a bad planning or a poor acceptance practice may be the cause for the difficulties in controlling the process.

E. Ineffective time due to the operator

It is the operator who performs the important operations affecting product quality.

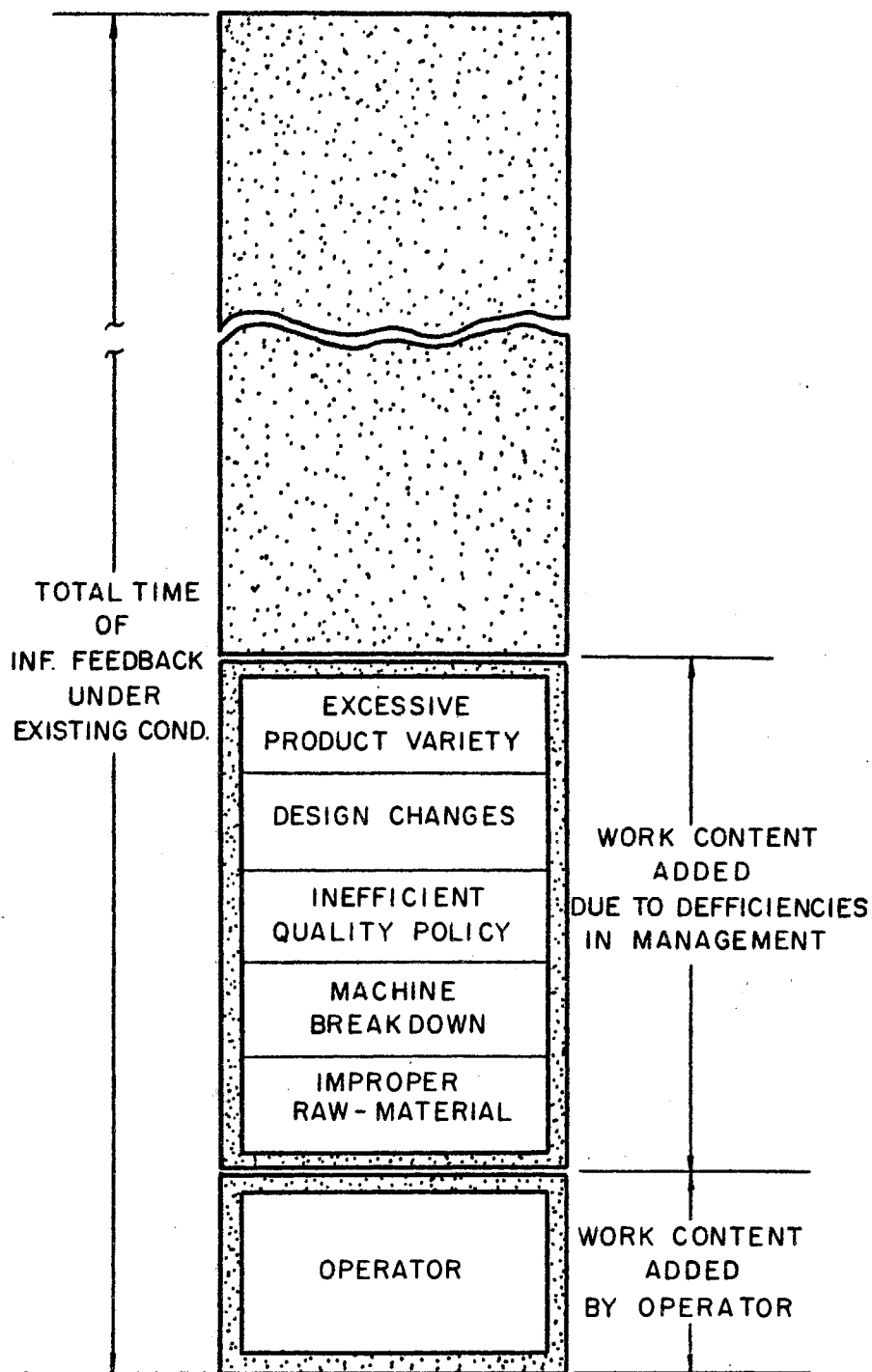


Figure 18. Ineffective Components of Feedback Cycle (b)

He also forms a part of an information feedback system, and there are numerous ways in which he can add ineffective time to the feedback cycle.

#### Reduction of Ineffective Time

In the previous section, the cause and the nature of ineffective time in the feedback cycle were discussed. If all the causes could be identified and eliminated by some means, then the total time of information feedback would be reduced to the minimum amount, i.e., the irreducible amount of work to perform the information feedback; the approach may take the form of the procedure outlined on pages 54-55.

Obviously, the elimination of the ineffective time in feedback cycle requires an effective coordination between the various functional components within an organization. The following are the categories of ineffective time stated in section 2 of this chapter, and their probable areas where a solution may be sought.

- A. Ineffective time due to defects in design or specification may be eliminated by:
  - (1) Product Development
  - (2) Specialization and Standardization
  - (3) Market and Product Research
- B. Ineffective time due to methods of operation and procedure of feedback may be eliminated by:
  - (1) Process Planning
  - (2) Process Research

- (3) Methods Study
  - (4) A better procedure of feedback.
- C. Ineffective time due to deficiencies in management may be eliminated by:
- (1) Marketing and Specialization
  - (2) Product Development
  - (3) Quality Planning
  - (4) Preventive Maintenance
  - (5) Incoming Materials Control.
- D. Ineffective time due to the operator may be eliminated by improved operator training.

#### Reduction of the Basic Work Content

When all the ineffective components of the feedback cycle have been eliminated, only the basic work content will remain in the feedback cycle. Accordingly, when a situation calls for further reduction in feedback cycle, a prospective means of accomplishing this task is to mechanize partial or entire process control functions. Mechanization becomes even more significant when one considers the rapid development of new technologies and increased demand for high product performance. Quality Control Programs, in many industries, are changing the entire concept of testing and inspecting, and equipment associated with the control of quality. That is, "today's devices must control the process by not only measuring characteristics, but also by analyzing functional data and making decisions as well. This analysis

and decision-making must be both accurate and timely. Under this broadened concept, the equipment of Quality Control can attain its maximum usefulness by furnishing pertinent information, not just measurement."<sup>3</sup>

The degree to which the feedback process leans toward equipment for the control of processes, rather than toward people and procedures, may be determined on the basis of several considerations.

The first consideration, and one of the more important, is that of economics: to establish the balance between the cost of accomplishing specific functions automatically as compared with performing them manually. Although the economic consideration is important, other criteria must go beyond that point. In many high-speed processes, the human being cannot observe, decide, and adjust rapidly and accurately enough to prevent the manufacture of large amounts of nonconforming product. When this is the case, operator adjustment must be replaced by fully automatic equipment control.

Another consideration on which a decision for fully automatic equipment control should be based is the matter of safety to operating personnel. Greater safety might be assured not only through closer control of hazardous processes but also by removing the operator from hazardous

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<sup>3</sup>Bernard Sussman, "Quality Information Equipment," Industrial Quality Control, July 1964, Vol. XXI, No. 1, pp 10-11.

locations, e.g., those subject to radiation, high heat, or explosions.<sup>4</sup>

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<sup>4</sup>A. V. Feigenbaum, Total Quality Control, (New York: McGraw-Hill Inc.,) 1961, pp. 178, 179.

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## CHAPTER V

### SUMMARY AND CONCLUSIONS

The primary objective of a quality control system is the prevention of defectives. This objective is realized through immediate corrective action, taken on the basis of information feedback. In addition, organization is needed for data collection, analysis and estimation of the nature of future quality problems.

It has been the author's intention, in writing this thesis, to review current developments on various aspects of information feedback in quality control, and to develop and present a systematic treatment of the subject based on these theories. The study has been conducted through the following phases:

- (1) The formulation of the problem was based on the study of various facets of information feedback as a means of attaining a quality objective.
- (2) Following this formulation stage, a model depicting organizational and process control aspects of quality control were described. Discussion of a means of measuring the effectiveness of the quality system was also included.
- (3) Based on these models, a quantitative approach.

toward economizing the information feedback process was examined.

- (4) Improvement of the effectiveness of the information feedback was considered from the view point of work study.

There are several factors determining the effectiveness of the quality information system; however, only the time factor was considered herein because of the variations among industries.

Having observed the information feedback problems throughout the foregoing chapters, one comes to the following conclusion.

The rate of growth of present industry is constantly demanding a better quality information system that will meet the needs of future quality problems. Therefore, designing, maintaining and improving a quality information system has become one of the key factors in determining the success of a quality program. The two major areas which these tasks involve are organization and process control.

Specialization on the part of industries involved in production and distribution and increased consumer demand have brought about the need for a better coordination. This need can be satisfied by proper maintenance of a standards and specifications systems.

In the area of process control, an almost instantaneous feedback of information is required if excessive scrap and shut-down costs are to be avoided. The Monte Carlo Simu-

lation may be employed as a means of approaching this problem; however, it leaves the problem of formulating a model, which represents reality, within the limits of reasonable accuracy. Also, there is another facet of process control that needs to be considered. Modern process control demands equipment that measures a quality rapidly and accurately during the manufacturing cycle. Accordingly, there is a need for support from quality information equipment engineering in developing a quality program. This is the area where strong coordination between quality control and engineering function is required.

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