ANALYSIS OF RETICULATED STRUCTURES
BY RELAXATION

By

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NOMENCLATURE

A_x . . . . . Cross sectional area of member

A_y . . . . . Area of shear in Y direction

A_z . . . . . Area of shear in Z direction

CoS . . . . . Cofactor of S matrix

DJ . . . . . Deflection of the joint in the general system

DL . . . . . Length of member

D_x . . . . . Distance in X direction

D_y . . . . . Distance in Y direction

D_z . . . . . Distance in Z direction

E . . . . . Modulus of elasticity

Flex . . . . Flexibility matrix

G . . . . . Modulus of shear

I_x . . . . . Torsional constant

I_y . . . . . Moment of inertia about the Y axis

I_z . . . . . Moment of inertia about the Z axis

${}_{jj}K^{m}_{px}$ . . . . Stiffness element

LP (LC) . . . Load points in load condition

MBR . . . . . Member

NJ . . . . . Number of joints

NLC . . . . . Number of load conditions

NMJ . . . . . Number of members entering a joint

P . . . . . External forces on the joint

PM . . . . . Forces in the member

PNCR . . . . Increment of member forces

P_x . . . . . Force in X direction

P_y . . . . . Force in Y direction

P_z . . . . . Force in Z direction

R_1 . . . . . Real restraints

r_1 . . . . . Temporary restraints

S^{-1} or A^{-1} . Inverse of S or A matrix

SF . . . . . Stiffness of far end of a member

SJG . . . . . Stiffness of the joint in the general system

SM . . . . . Stiffness matrix

SMGS . . . . Stiffness of the member in the general system

SMMS . . . . Stiffness of the member in the member system

SN . . . . . Stiffness of near end of a member

ST . . . . . Stiffness

TSJ . . . . . Total joint stiffness

UBF . . . . . Unbalanced force

W . . . . . Rotation matrix in computer solution

WT . . . . . Transpose matrix in computer solution

Δ_x . . . . . Deflection in X direction

Δ_y . . . . . Deflection in Y direction

Δ_z . . . . . Deflection in Z direction

θ_y . . . . . Rotation about the Y axis

θ_z . . . . . Rotation about the Z axis

Σ . . . . . Summation

α . . . . . Rotation angle about the z axis

β . . . . . Rotation angle about the y axis

γ . . . . . Rotation angle about the x axis

Δ . . . . . Linear deformation

ω . . . . . Rotation matrix

θ . . . . . Angular deformation

$[\ ]$ . . . . . Square or rectangular matrix

$[\ ]^T$ . . . . Matrix transpose

$[\ ]^{-1}$ . . . . Matrix inverse

CHAPTER I

INTRODUCTION

The growing trend toward high-rise construction and the more economical use of construction materials in space frame applications have presented today's structural engineer with more complicated problems for analysis of these structures. The use of conventional methods for analysis generally requires the solution of numerous simultaneous equations. These methods are long and tedious; therefore, new and improved methods are needed for analysis of these complex structures.

In 1935 a relaxation method was introduced by Southwell (1). His concept suggested a method for calculating stresses in complex pin-jointed frameworks which eliminated the necessity for solving large numbers of simultaneous equations by using successive approximations.

When the relaxation method was first presented, the question of convergence was encountered and the first attempt to show convergence was unsuccessful. With the focus of attention centered on the problem of convergence, its close similarity in principle with the "Moment Distribution Method" of Professor Hardy Cross was not immediately recognized. Although the two methods have similar features, most engineers seem to prefer to use the "Moment Distribution Method" (1).

This study will consist of two phases. The first is the investigation of a method for extending the relaxation theory to obtain a solution for any type of reticulated structure having prismatic members. The second phase is the application of this principle to computer solutions.

1.1 The Method

The method selected in this study for the extension of the relaxation method is the distribution of deformations to find stresses in the members of reticulated structures. To apply this method to any type of reticulated structure with prismatic members, modification of the general stiffness matrices of the members must be made according to their end restraints.

1.2 Computer Application

The advent of electronic computers has made it possible to solve structural problems with many redundants in a very short time. The use of computers for solutions of structural problems is a two-part problem. First the problem must be set up and formulated in a way to permit an electronic computer solution. This part of the problem lies within the structural engineer's work. The second part of the problem requires the coding, or programming, and setting up the card system for the actual machine operation. This generally calls for the service of a trained operator familiar with the particular computer being used. This may or may not lie within the scope of the structural engineer's work.

Today, electronic computers have become a part of everyday engineering procedures, and their use is certain to increase in the future. Therefore, it is essential for the modern structural engineer to have a basic understanding of procedures used in programming for this equipment.

In the second phase of this study, a computer program based on the above described relaxation method is devised. A variety of reticulated structures having prismatic members are analyzed to show the validity of this method for particular solutions of structural problems.

CHAPTER II

THEORY AND METHOD

2.1 Stability and Redundancy

The basic relaxation theory will be extended for solutions to any reticulated structure with prismatic members. To do this, a modification to the stiffness matrix of the member is made according to the type of end restraints used. This theory is valid, provided that the structure is stable.

For the majority of structures met in practice, the question of whether or not they are stable, statically determinate, or statically indeterminate can be resolved by routine procedures (2).

Any stable truss may be viewed as an assemblage of triangles built on one basic triangle by connecting each joint with a pair of members. In the base triangle there are three members and three joints. Each added joint requires two members (3): hence, if

$$m = \text{number of members}$$

and

$$n = \text{number of joints}$$

we have $m - 3$ (added members) $= 2(n - 3)$ (twice the added joints) and

$$m = 2n - 3$$

(a) When $m = 2n - 3$, the structure is stable and determinate.

(b) Whem m is less than $2n - 3$, the structure is unstable.

(c) When m is greater than $2n - 3$, the structure is stable and indeterminate.

For space structure,

$$m - 6 = 3\,(n - 4)$$

and

$$m = 3n - 6$$

One must revise the foregoing equation when the capability of joint supports dictates. The same rules given in (a), (b), and (c) apply.

2.2 Basic Theory

When starting the relaxation procedures, all the joints of a structure are assumed to be fixed in all directions so that the loads on the joints are carried entirely by the temporary supports and there are no stresses in the members of the structure. The reactions at these temporary supports are given the name "residuals". When the relaxation procedure begins, the residuals at the joints are the external loads at the joints.

Steps of procedure:

1. Beginning in some logical order, each joint is freed or relaxed, and allowed to deform one at a

time.

2. Stresses are introduced in the members attached to the deformed joint.
3. After each relaxation, the joint is assumed to be fixed in its new position while other joints are relaxed.
4. The residuals will change at adjacent joints each time a joint is allowed to deflect.
5. This procedure is continued until the residuals are zero, or until they reach a predetermined degree of accuracy.
6. As the residuals decrease, the stresses in the members of the structure change until the members are carrying the entire load.
7. The forces or stresses in the members are a summation of the increments of force produced in the members each time the joints are relaxed.
8. The deflections at each joint are the summation of the deflections obtained each time the joint is relaxed.

The above steps of procedure describe the relaxation method. To further illustrate the relaxation method, consider the planer truss in Figure 1.

In Figure 1-a, the real restraints are shown by arrows marked R_1, R_2, and R_3. The temporary restraints or residuals are represented by arrows marked r_1 and r_2. These temporary restraints are assumed to be removed and then replaced

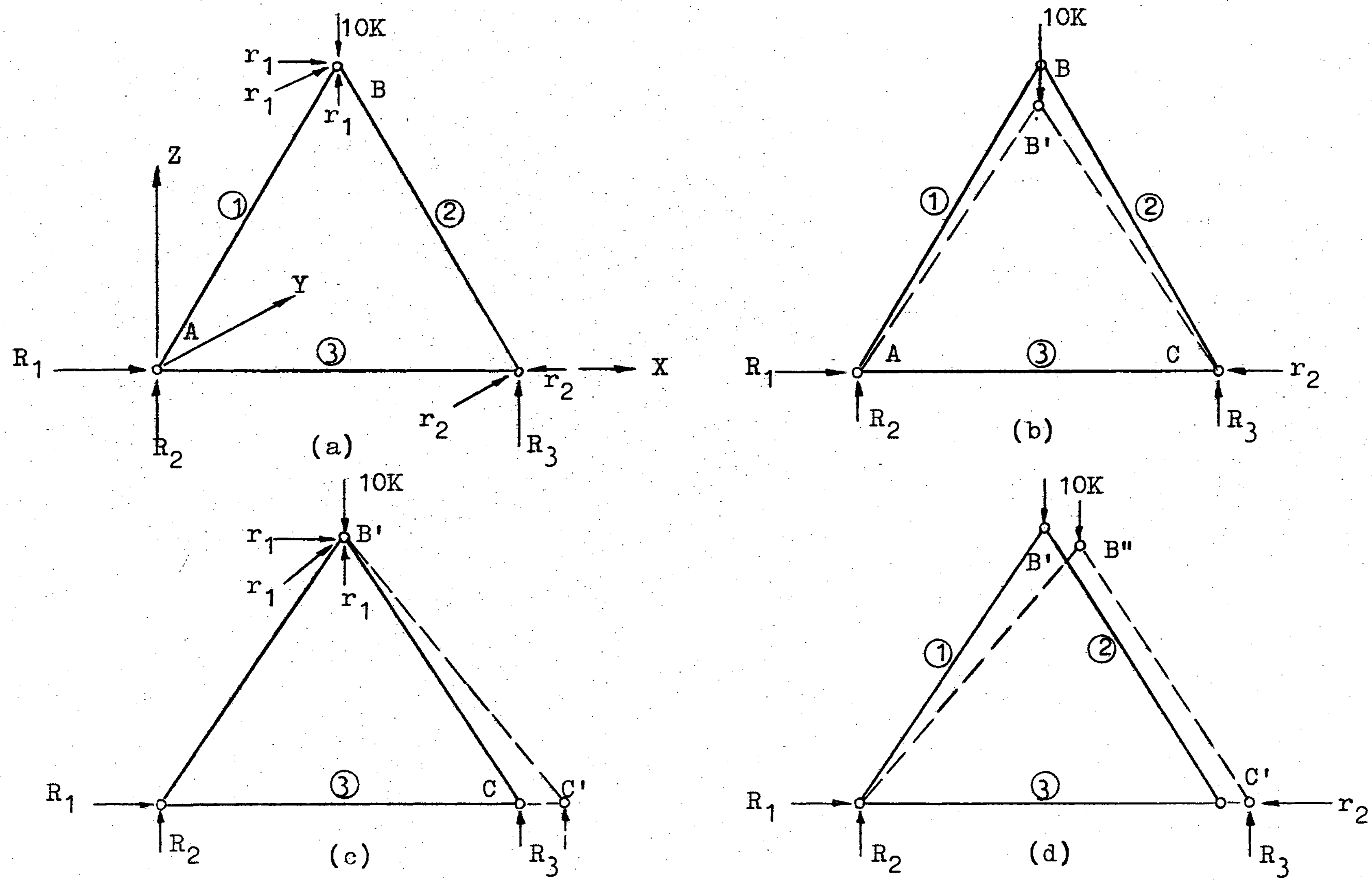


Figure 1. Illustration of the Iterative Process

each time a joint is allowed to deflect.

The structure is in the X-Z plane and will not deflect in the Y direction unless loaded in that direction. Joint A is fixed in position. Joint B has no real restraints and will translate and rotate to a balanced condition when the temporary restraints are removed. Joint C has a real restraint in the Z direction and will translate in the X direction and rotate about its Y axis when the temporary restraints are removed.

In Figure 1-a, the structure is shown with all restraints and a load of 10 Kips applied at joint B in the Z direction. All members are 200 inches long with a cross sectional area of 10.59 square inches. Releasing the residuals at joint A would not affect the structure because A has real restraints and may be neglected in the relaxation process.

Removing the temporary restraints at joint B, joint B will deflect to a new position B' (see Figure 1-b). This vertical deflection will produce equal compressive forces in members 1 and 2. Joint B is assumed to be fixed in its new position B'.

Moving to joint C and removing the temporary restraints, joint C will deflect to the right because there will be an unbalanced force in the X direction. When joint C deflects to the right, there will be a tension force produced in member 3 and the compressive force in member 2 is reduced. Joint C is now assumed to be fixed in its new position C' (see Figure 1-c).

Returning to joint B, the force in member 1 remains the same because joint A has real restraints in all directions. However, there has been a reduction of force in member 2. Taking a summation of forces at joint B will show an unbalanced force in the positive X direction and the negative Z direction. However, the unbalanced Z force has been reduced far below the 10 Kips originally placed on the joint. The restraints are again removed and joint B will deflect to some new position B" as shown in Figure 1-d.

This procedure is continued until there are no unbalanced forces at the joints. The three final deformations are a summation of the deformations obtained each time the joint is allowed to relax. The final member forces are a summation of the forces produced in the members each time the joints are allowed to relax.

A complete slide rule and computer solution to this simple pin-connected truss are presented later in this study.

In the preceding discussion, a general statement of the basic relaxation theory was presented. This was followed by a physical description of the relaxation principle. The deformation produced by an unbalanced force is a function of the total resistance of the joint.

In non-coplaner frames, usually called space frames, members, force and moment vectors need to be translated from the member system to the general system or vice-versa. To do this, an understanding of matrix algebra facilitates the applications that are necessary in structural analysis.

2.3 Matrix Algebra and Rotation Matrices

In this paper, solution of complex structures involves operations on matrices. However, the matrix operations involved in this relaxation method are not complicated.

Since the members of a complex structure may be non-coplaner, three rotation matrices are used to rotate and translate vector quantities from the member system to the general system (see Figure 2).

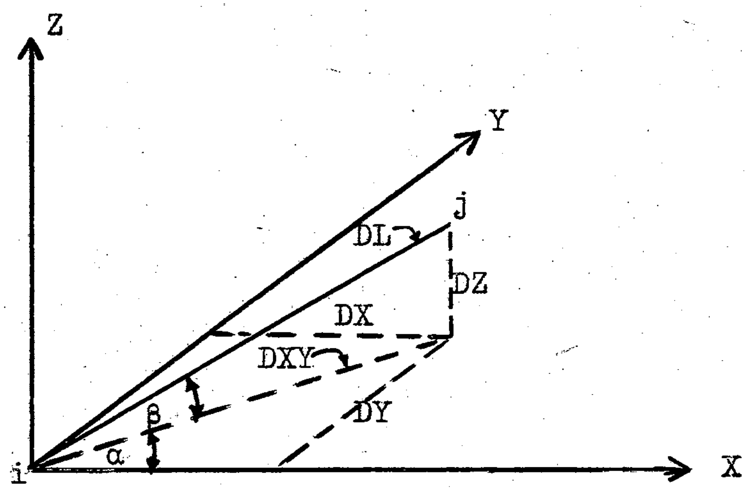
Rotation about the Z axis through angle α is called an Alpha rotation and is represented by $[\omega_\alpha]$. Similarly, rotation about the Y axis is a Beta rotation and is represented by $[\omega_\beta]$, and rotation about the X axis is a Gamma rotation and is represented by $[\omega_\gamma]$ (4). Normally two rotations are all that are required to go from the member system to the general system.

$$[{}^m\omega_3^o] = [\omega_\gamma][\omega_\beta][\omega_\alpha]$$

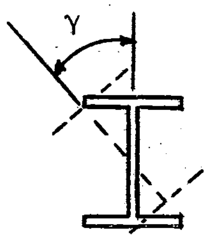
$[{}^m\omega_3^o]$ is used to rotate vector quantities from the general to the member system. The transpose of $[{}^m\omega_3^o]$ gives $[{}^o\omega_3^{m^T}]$ which rotates vector quantities from the member system to the general system.

2.4 Matrices and Structural Analysis

The distribution of deformations is similar to moment distribution. The author believes it closely resembles



General System and Member System



Looking down member ij from j to i shows a γ rotation

$$[\omega_\alpha] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ROTATION ABOUT THE Z AXIS

$$[\omega_\beta] = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

ROTATION ABOUT THE Y AXIS

$$[\omega_\gamma] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix}$$

ROTATION ABOUT THE X AXIS

$$[{}^m\omega_3^o] = [\omega_\gamma]\,[\omega_\beta]\,[\omega_\alpha]$$

ROTATION ABOUT THE X, Y, AND Z AXES

Rotates vector quantities from the general system to the member system.

Figure 2. Rotation Matrices

"Kani Moment Distribution" (5). In moment distribution, the stiffnesses of the members of a structure are the primary concern, but with the distribution of deformations, the flexibilities of the joints of the structure are used.

With this in mind, the following load-deflection relation may be given for the general structure,

$$Y_1 = a_{11}P_1 + a_{12}P_2 + \dots a_{1j}P_j + \dots a_{1n}P_n$$

$$Y_2 = a_{21}P_1 + a_{22}P_2 + \dots a_{2j}P_j + \dots a_{2n}P_n$$

$$Y_j = a_{j1}P_1 + a_{j2}P_2 + \dots a_{jj}P_j + \dots a_{jn}P_n$$

which, in matrix notation, stands for deflection at the n points of loading.

$$\begin{bmatrix} a_{11} & a_{12} & --- & a_{1n} \\ a_{21} & a_{22} & --- & a_{2n} \\ a_{n1} & a_{n2} & --- & a_{nn} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_n \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_n \end{bmatrix}$$

or

$$AP = Y \qquad (1)$$

The elements a_{ij} are called flexibility influence coefficients and are defined:

a_{ij} is the deflection of point i due to a unit load at point j, all other points being assumed unloaded (6).

The calculation of these coefficients is very lengthy and complex and is not used for computer solutions. A better and less complicated approach is to solve for P's in terms of the deflections Y. Then

$$P_1 = s_{11}Y_1 + s_{12}Y_2 + \dots s_{1n}Y_n$$

$$P_2 = s_{21}Y_1 + s_{22}Y_2 + \dots s_{2n}Y_n$$

$$P_n = s_{n1}Y_1 + s_{n2}Y_2 + \dots s_{nn}Y_n$$

which is, in matrix notation,

$$\begin{bmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & s_{22} & \dots & s_{2n} \\ s_{n1} & s_{n2} & \dots & s_{nn} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_n \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_n \end{bmatrix}$$

or

$$P = SY \qquad (2)$$

The elements s_{ij} are the stiffness coefficients and are defined as follows:

s_{ij} is the load developed at point i due to a unit deflection at point j, all other points being assumed fixed (6).

Multiplying both sides of Equation (2) by A gives:

$$AP = ASY \qquad (3)$$

which means

$$AS = 1$$

or

$$A = S^{-1}$$

$$= \frac{(\text{co } S)}{\det S}$$

or, more clearly stated, the inverse of the stiffness matrix is the flexibility matrix.

2.5 Member Stiffness

Consider the member (j,k) of Figure 3 and assume that

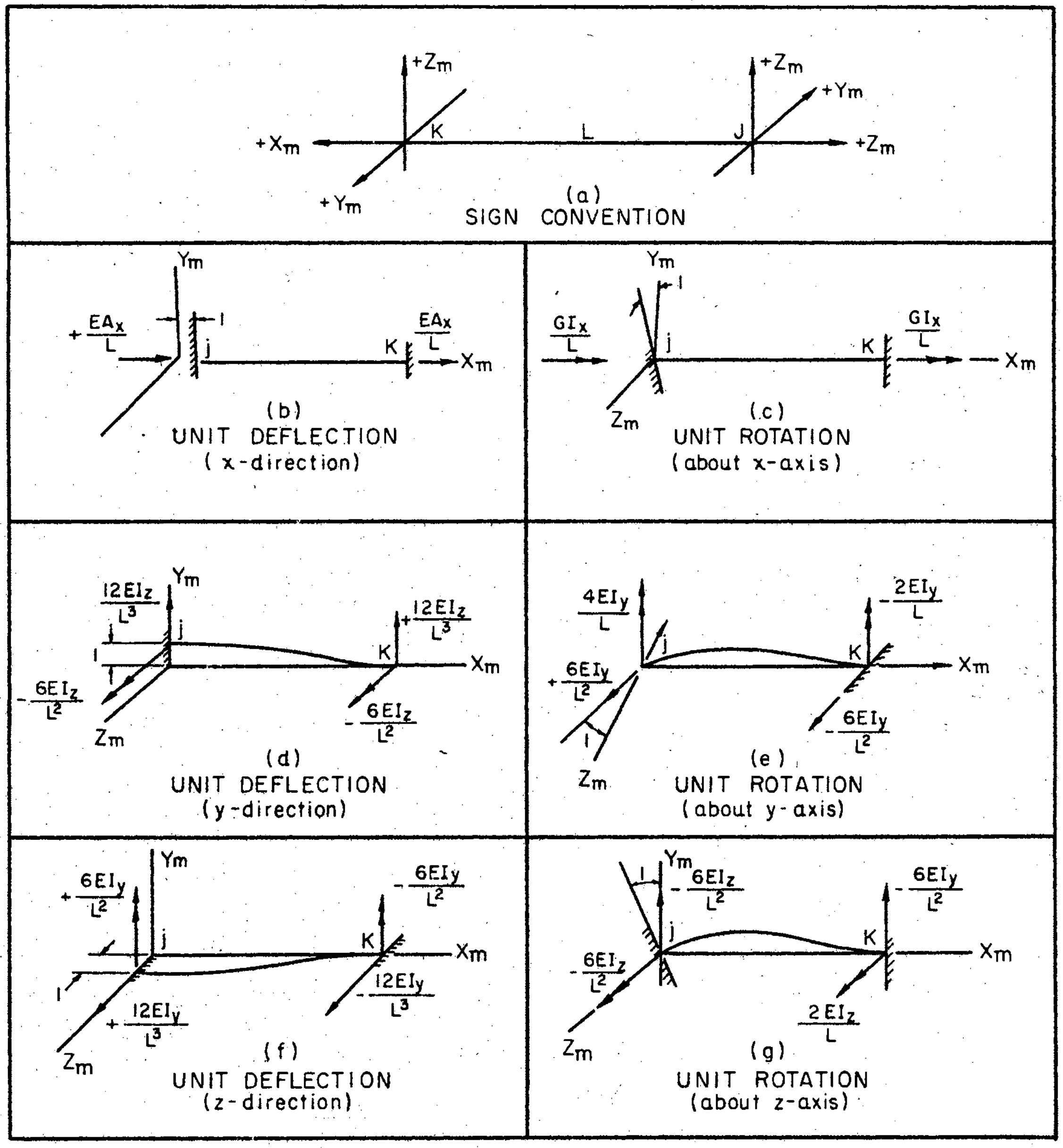


Figure 3. Member Stiffnesses and Sign Convention

this member is restrained within a structure. For each end of the member there are six possible deformations (7). Since only one joint is considered at any one time in the relaxation process, one end of the member will be considered. The member stiffnesses or restraint actions for the six possible deformations, plus the restraint action of the other end for those same displacements and rotations make up the stiffness matrix. When the joint at the k end of the member is considered, the other end deformations will be evaluated. All stiffness elements are needed when both ends of a member are fully fixed to the end joints. Later in this study the eight combinations of end restraints and their stiffnesses are presented.

In Figure 3, the six possible end displacements are shown and the member stiffnesses for the near and far ends are shown (7). These are summarized in Table I.

The matrix shown in Table II is a 6 x 12, and is a summary of the stiffness elements of a member that is fixed to the joints on both ends. SN represents the near end stiffness and SF represents the far end. When the end conditions are not fixed, many of the elements shown in Table II will be zeroes.

Under certain conditions, a modification to the stiffness matrix is necessary due to shear deformation (7). When this occurs SN(8) becomes $\frac{1}{\frac{L}{GA_y} + \frac{L^3}{12EI_z}}$ and SN(15) becomes $\frac{1}{\frac{L}{GA_z} + \frac{L^3}{12EI_y}}$.

TABLE I

MEMBER STIFFNESS MATRIX

	1	2	3	4	5	6
1	$\frac{EA_x}{L}$	0	0	0	0	0
2	0	$\frac{12EI_z}{L^3}$	0	0	0	$\frac{-6EI_z}{L^2}$
3	0	0	$\frac{12EI_y}{L^3}$	0	$\frac{6EI_y}{L^2}$	0
4	0	0	0	$\frac{GI_y}{L}$	0	0
5	0	0	$\frac{6EI_y}{L^2}$	0	$\frac{4EI_y}{L}$	0
6	0	$\frac{-6EI_z}{L^2}$	0	0	0	$\frac{4EI_z}{L}$
7	$\frac{EA_x}{L}$	0	0	0	0	0
8	0	$\frac{12EI_z}{L^3}$	0	0	0	$\frac{-6EI_z}{L^2}$
9	0	0	$\frac{-12EI_y}{L^3}$	0	$\frac{-6EI_y}{L^2}$	0
10	0	0	0	$\frac{GI_y}{L}$	0	0
11	0	0	$\frac{-6EI_y}{L^2}$	0	$\frac{-2EI_y}{L}$	0
12	0	$\frac{-6EI_z}{L^2}$	0	0	0	$\frac{2EI_z}{L}$

TABLE II

GENERAL STIFFNESS MATRIX

$$
\begin{bmatrix}
SN(1) & & & & & \\
 & SN(8) & & & & SN(12) \\
 & & SN(15) & & SN(17) & \\
 & & & SN(22) & & \\
 & & SN(27) & & SN(29) & \\
 & SN(32) & & & & SN(36) \\
SF(1) & & & & & \\
 & SF(8) & & & & SF(12) \\
 & & SF(15) & & SF(17) & \\
 & & & SF(22) & & \\
 & & SF(27) & & SF(29) & \\
 & SF(32) & & & & SF(36)
\end{bmatrix}
$$

2.6 <u>End Restraints</u>

There are many variations in end restraints for members making up a complex structure. The author will present at this time the eight conditions that one is most likely to encounter in complex structures (see Figure 4).

When a member is fixed to the joint, it is fixed against rotations and deflections. If it is slotted, it is released for translations, but fixed for rotations. And when it is pinned, it is fixed for translations, but released for rotations.

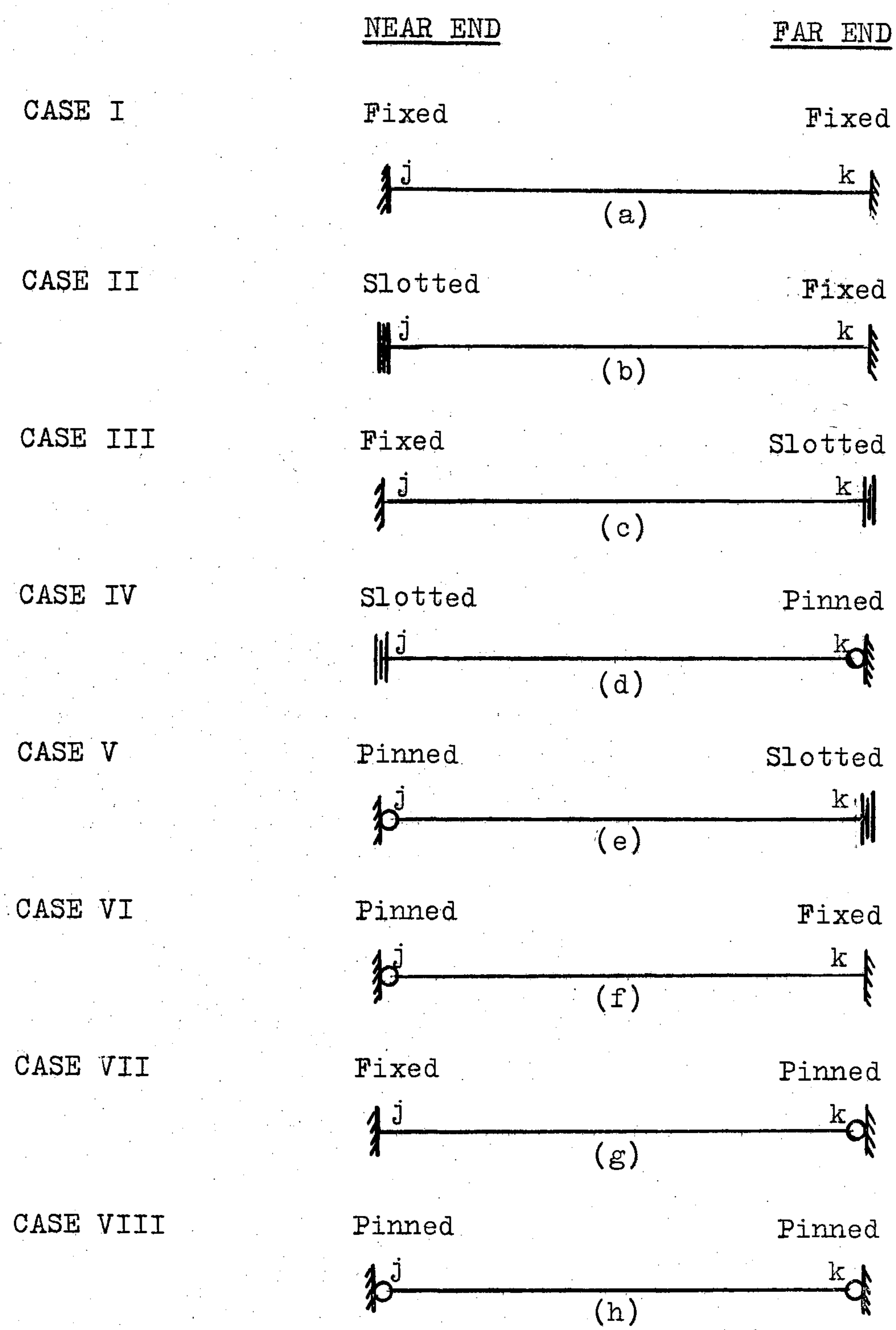


Figure 4. End Restraints

2.7 Sample Stiffnesses

Case I was considered under the member stiffness and is listed later with the summation of member stiffnesses under various end conditions.

For Case II, the member is slotted in the X, Y and Z directions on the near end for translations, but fixed for all rotations. On the far end, the member is fixed against rotations and translations (see Figure 5). There are many possible combinations of end conditions. For example, the member could be slotted in the Z direction for translation and fixed in the X or Y direction for translations. Therefore, the stiffness matrix of the member must reflect the correct conditions. The end restraints will determine the modifications that must be made to the member stiffness matrix. For a computer solution, a code must be devised that will modify the stiffness matrix of the member according to end restraints.

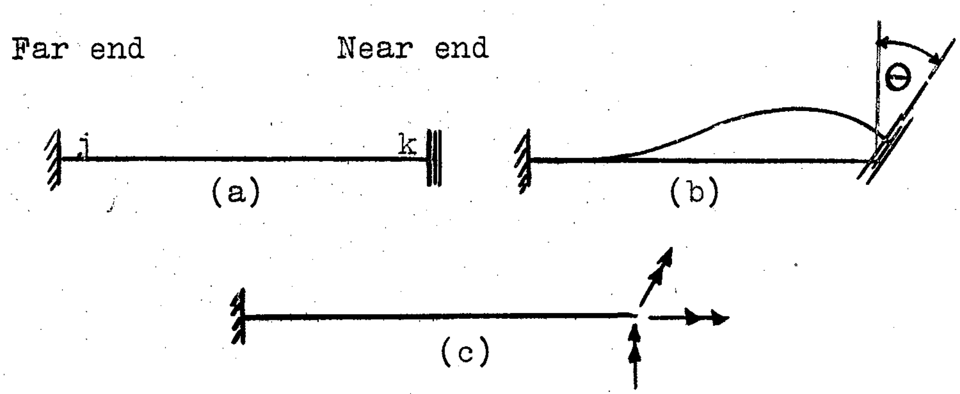


Figure 5. Member Stiffness (Case II)

Using any convenient elastic theory, the effect of rotations and translations can be computed. Joint translations have no effect on the member if the member is slotted in the directions of the translations. When the member is fixed to the joint for joint rotation, joint rotation about the X axis produces a torsional force $\frac{GI_x}{L}$ in the member. Joint rotation about the member Y axis produces a moment M_y or $\frac{EI_y \theta_y}{L}$. Joint rotation about the member Z axis produces a moment M_z or $\frac{EI_z \theta_z}{L}$.

Thus the elements of the stiffness matrix for the member due to Case II are:

NEAR END	FAR END
$SN(22) = \frac{GI_x}{L}$	$SF(22) = \frac{GI_x}{L}$
$SN(29) = \frac{EI_y}{L}$	$SF(29) = \frac{-EI_y}{L}$
$SN(36) = \frac{EI_z}{L}$	$SF(36) = \frac{-EI_y}{L}$

All other elements in the stiffness matrix for the given condition are zeroes.

Using any convenient elastic theory, the stiffnesses for the other six cases are computed using the same method as shown in the sample calculations for Case II.

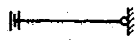
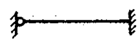
The resulting stiffnesses for all eight cases are summarized in Table III.

TABLE III

SUMMARY OF MEMBER STIFFNESSES

Selected Cases	Case I	Case II	Case III	Case IV
End Restraints	Fixed-Fixed	Slotted-Fixed	Fixed-Slotted	Pinned-Slotted
SN(1)	$\frac{A_x E}{L}$	0	0	0
SN(8)	$\frac{12EI_z}{L^3}$	0	0	0
SN(12)	$\frac{-6EI_z}{L^2}$	0	0	0
SN(15)	$\frac{12EI_y}{L^3}$	0	0	0
SN(17)	$\frac{6EI_y}{L^2}$	0	0	0
SN(22)	$\frac{GI_x}{L}$	$\frac{GI_x}{L}$	$\frac{GI_x}{L}$	0
SN(27)	$\frac{6EI_y}{L^2}$	0	0	0
SN(29)	$\frac{4EI_y}{L}$	$\frac{EI_y}{L}$	$\frac{EI_y}{L}$	0
SN(32)	$\frac{-6EI_z}{L^2}$	0	0	0
SN(36)	$\frac{4EI_z}{L}$	$\frac{EI_z}{L}$	$\frac{EI_z}{L}$	0
SF(1)	$\frac{A_x E}{L}$	0	0	0
SF(8)	$\frac{12EI_z}{L^3}$	0	0	0
SF(12)	$\frac{-6EI_z}{L^2}$	0	0	0
SF(15)	$\frac{12EI_y}{L^3}$	0	0	0
SF(17)	$\frac{-6EI_y}{L^2}$	0	0	0
SF(22)	$\frac{GI_y}{L}$	$\frac{GI_x}{L}$	$\frac{GI_x}{L}$	0
SF(27)	$\frac{-6EI_y}{L^2}$	0	0	0
SF(29)	$\frac{-2EI_y}{L}$	$\frac{-EI_y}{L}$	$\frac{-EI_y}{L}$	0
SF(32)	$\frac{-6EI_z}{L^2}$	0	0	0
SF(36)	$\frac{2EI_z}{L}$	$\frac{EI_z}{L}$	$\frac{EI_z}{L}$	0

TABLE III (Continued)

Selected Cases	Case V	CASE VI	CASE VII	CASE VIII
End Restraints	Slotted-Pinned 	Pinned-Fixed 	Fixed-Pinned 	Pinned-Pinned 
SN(1)	0	$\frac{A_xE}{L}$	$\frac{A_xE}{L}$	$\frac{A_xE}{L}$
SN(8)	0	$\frac{12EI_z}{L^3}$	$\frac{12EI_z}{L^3}$	0
SN(12)	0	0	$\frac{3EI_z}{L^2}$	0
SN(15)	0	$\frac{12EI_y}{L^3}$	$\frac{12EI_y}{L^3}$	0
SN(17)	0	0	$\frac{-3EI_y}{L^2}$	0
SN(22)	0	0	0	0
SN(27)	0	0	$\frac{-3EI_y}{L^2}$	0
SN(29)	0	0	$\frac{3EI_y}{L}$	0
SN(32)	0	0	$\frac{3EI_z}{L^2}$	0
SN(36)	0	0	$\frac{3EI_z}{L}$	0
SF(1)	0	$\frac{A_xE}{L}$	$\frac{A_xE}{L}$	$\frac{A_xE}{L}$
SF(8)	0	$\frac{12EI_z}{L^3}$	$\frac{12EI_z}{L^3}$	0
SF(12)	0	0	$\frac{-3EI_z}{L^2}$	0
SF(15)	0	$\frac{12EI_y}{L^3}$	$\frac{12EI_y}{L^3}$	0
SF(17)	0	0	$\frac{-3EI_y}{L^2}$	0
SF(22)	0	0	0	0
SF(27)	0	$\frac{3EI_y}{L^2}$	0	0
SF(29)	0	0	0	0
SF(32)	0	$\frac{-3EI_z}{L^2}$	0	0
SF(36)	0	0	0	0

2.8 Code for End Restraints

It has been shown that for a given member of a structure, there are many possible combinations of end restraints. To compensate for this in a computer program, some system must be devised that will produce the proper stiffness elements based on the end conditions of the members.

Since there are six possible displacements for each end of a member, the code system selected for this study includes six digits for each end of the member. The six digits are either ones or zeroes based on the type of connection used to secure the members to the joints. The particular combination of end code digits determines the stiffness elements that applies for both ends of the member.

For example, if a member is fixed rigidly to the joints on both ends, the code for each end is six zeroes. If the member is pinned at both ends, the code for each end is four zeroes followed by two ones. Varying the positions of the ones and zeroes of each end code, any type of end restraints can be represented by this system.

2.9 Illustrative Problem

A solution to the Planer-Truss presented earlier will now be illustrated using the relaxation method in matrix form.

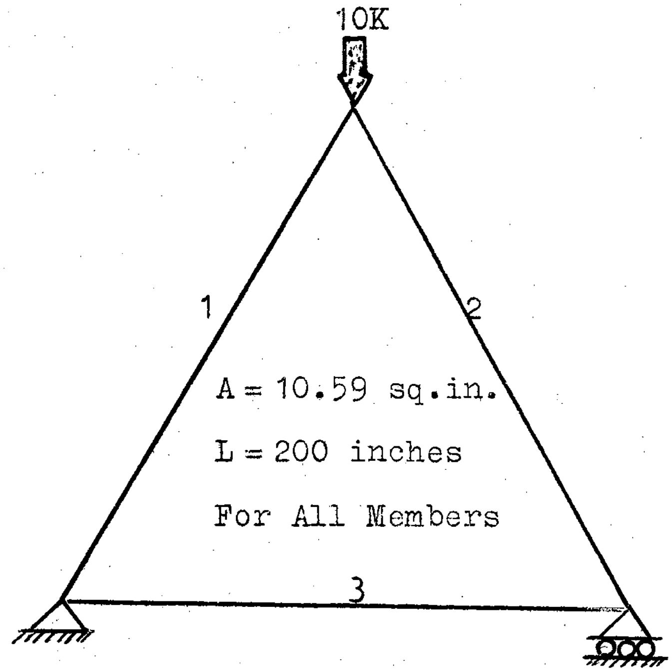


Figure 6. Planer Truss

Joint A

Has real restraints and will not deflect.

Joint B

MBR(1) $\quad \text{Cos} = \frac{100}{200} = 0.5 \qquad \text{Sin} = \frac{173.2}{200} = 0.865$

$$[\omega_\beta] = \begin{bmatrix} .5 & .865 \\ -.865 & 5 \end{bmatrix} \; ; \; [\omega_\beta]^T = \begin{bmatrix} .5 & -.865 \\ .865 & .5 \end{bmatrix}$$

$$ST = \text{Stiffness for pin connected members} = \frac{A_x E}{L} = \frac{10.59 \times 29 \times 10^6}{200}$$

$$= 1.5 \times 10^6 = \begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}$$

$[\omega_\beta]^T\,[ST][\omega_\beta] = [SMGS]$ and $\sum[SMGS][\Delta] = [P]$

$$[SMGS] = \begin{bmatrix} .5 & .865 \\ .865 & 5 \end{bmatrix}\begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} .5 & .865 \\ -.865 & .5 \end{bmatrix} = \begin{bmatrix} .5 & -.865 \\ .865 & .5 \end{bmatrix}$$

$$\begin{bmatrix} .75E6 & 1.3E6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} .375E6 & .65E6 \\ .65E6 & 1.12E6 \end{bmatrix}$$

MBR(2) Cos = −.5 Sin = .865

$$[\omega_\beta] = \begin{bmatrix} -.5 & .865 \\ -.865 & -.5 \end{bmatrix};\ [\omega_\beta]^T = \begin{bmatrix} -.5 & -.865 \\ .865 & -.5 \end{bmatrix} \text{ and } [ST] = \begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[SMGS] = \begin{bmatrix} -.5 & -.865 \\ .865 & -.5 \end{bmatrix}\begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} -.5 & .865 \\ -.865 & -.5 \end{bmatrix} = \begin{bmatrix} .375E6 & -.65E6 \\ -.65E6 & 1.12E6 \end{bmatrix}$$

Total stiffness of joint B = [TSJ] = $\sum$[SMGS]

Joint C

MBR(2) $\text{Cos} = \dfrac{200 - 100}{200} = .5$ $\text{Sin} = \dfrac{-173.2}{200} = -.865$

$$[SMGS] = \begin{bmatrix} .5 & .865 \\ -.865 & .5 \end{bmatrix}\begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} .5 & -.865 \\ .865 & .5 \end{bmatrix} = \begin{bmatrix} .375E6 & -.65E6 \\ -.65E6 & 1.12E6 \end{bmatrix}$$

MBR(3) $\text{Cos} = \dfrac{200}{200} = 1$ Sin = 0 $[ST] = \begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}$

$$[SMGS] = \begin{bmatrix} 1.0 & 0 \\ -0 & 1.0 \end{bmatrix}\begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1.0 & -0 \\ 0 & 1.0 \end{bmatrix} = \begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[TSJ] = \begin{bmatrix} .375E6 & -.65E6 \\ -.65E6 & 1.12E6 \end{bmatrix} + \begin{bmatrix} 1.5E6 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1.875E6 & -.65E6 \\ -.65E6 & 1.12E6 \end{bmatrix}$$

at joint (C) $\Delta_Z = 0$, $\therefore$ [TSJ] = [1.875E6]

[Flex] = inverse of joint stiffness

Joint (B)

$$[Flex] = \begin{bmatrix} 1.322 \times 10^{-6} & 0 \\ 0 & .446 \times 10^{-6} \end{bmatrix}$$

Joint (C)

$$[Flex] = [.534 \times 10^{-6}]$$

First Cycle Relaxation

Joint (B)

$$\begin{bmatrix} 1.322 \times 10^{-6} & 0 \\ 0 & .446 \times 10^{-6} \end{bmatrix} \begin{bmatrix} 0 \\ -10,000.0 \end{bmatrix} = \begin{bmatrix} 0 \\ -.0446 \end{bmatrix}$$

Forces in the members (PM) = 0 initially

$$MBR(1) \begin{bmatrix} .76E6 & 1.3E6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -.00446 \end{bmatrix} = \begin{bmatrix} -5800 \\ 0 \end{bmatrix} = [PNCR]$$

Force in MBR(2) same as MRB(1) [PM] = [PM] + [PNCR]

Joint (C)

$$[\omega_\beta]^T [PM] = \begin{bmatrix} -.5 & -.865 \\ .865 & -.5 \end{bmatrix} \begin{bmatrix} -5800 \\ 0 \end{bmatrix} = \begin{bmatrix} +2900 \\ -5000 \end{bmatrix} = \begin{bmatrix} P_x \\ P_z \end{bmatrix}$$

$$\Delta_Z = 0$$

$$\Delta_x = [Flex][P_x] = [.534 \times 10^{-6}][2900] = [+.00155]$$

Forces in the members (PM)

$$\text{MBR}(2)\ [\text{PNCR}] = \begin{bmatrix} .75E6 & -1.3E6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} .00155 \\ 0 \end{bmatrix} = \begin{bmatrix} 1162.0 \\ 0 \end{bmatrix}$$

[PM] = [PM] + [PNCR] = [-5800] + [1162] = [4638]

MBR(3) [PNCR] = [1.5E6][.00155] = [2325]

[PM] = [PM] + [PNCR] = [2325]

Second Cycle

Joint (B)

[UBF] $P_x = 2900 - 2319 = 581^{\#}$

$P_z = 500 - 10000 + 4020 = -980^{\#}$

$$\begin{bmatrix} 1.322 \times 10^{-6} & 0 \\ 0 & .466 \times 10^{-6} \end{bmatrix} \begin{bmatrix} 581.0 & = & .000768 \\ -980.0 & & -.000437 \end{bmatrix} = \begin{bmatrix} \Delta_x \\ \Delta_z \end{bmatrix}$$

Forces in the members

$$\text{MBR}(1)\ [\text{PNCR}] = \begin{bmatrix} .75E6 & 1.3E6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} .000768 \\ -.000437 \end{bmatrix} = \begin{bmatrix} +576.0 \\ -570.0 \end{bmatrix}$$

[PM] = [PM] + [PNCR] = [-5800]

$$\text{MBR}(2)\ [\text{PNCR}] = \begin{bmatrix} -.75E6 & 1.3E6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} .000768 \\ -.000437 \end{bmatrix} = \begin{bmatrix} -576.0 \\ -570.0 \end{bmatrix}$$

= [-1146.0]

[PM] = [PM] + [PNCR] = [-4638] + [-1146] = [-5784]

Joint (C)

$$[UBF] = \begin{bmatrix} .5 & .865 \\ -.865 & .5 \end{bmatrix} \begin{bmatrix} -5784 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix} \begin{bmatrix} 2325 \\ 0 \end{bmatrix} = [567]$$

$$\Delta_x = [.534 \times 10^{-6}]\ [567] = [.000303]$$

Forces in the members

$$MBR(2) \quad [PNCR] = \begin{bmatrix} .75E6 & -1.3E6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} .000303 \\ 0 \end{bmatrix} = [227]$$

$$[PM] = [PM] + [PNCR] = [-5784] + [227] = [-5557]$$

$$MBR(3) \quad [PNCR] = [1.5E6]\ [.000303] = [458]$$

$$[PM] = [PM] + [PNCR] = [2325] + [458] = [2783]$$

Third Cycle

Joint (B)

$$[UBF] \qquad P_x = 2900 - 2778 = 122^{\#}$$

$$P_z = -10000 + 5000 + 4820 = -180^{\#}$$

$$\begin{bmatrix} 1.322 \times 10^{-6} & 0 \\ 0 & .446 \times 10^{-6} \end{bmatrix} \begin{bmatrix} 122 \\ -180 \end{bmatrix} = \begin{bmatrix} .000162 \\ -.000081 \end{bmatrix}$$

$$PM(1) = PM(2) = -5800^{\#}$$

Joint (C)

$$[UBF] = \begin{bmatrix} .5 & .865 \\ -.865 & .5 \end{bmatrix} \begin{bmatrix} -5800 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix} \begin{bmatrix} 2783 \\ \ \end{bmatrix} = [117.0]$$

$$\Delta_x = [534 \times 10^{-6}]\ [117] = [0.000062]$$

Forces in the members (PM)

$$[PM(2)] = [-5800] + \begin{bmatrix} .75E6 & 1.3E6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} .000062 \\ 0 \end{bmatrix} = [5744]$$

$$[PM(3)] = [2783] + [1.5E6]\,[62 \times 10^{-6}] = [2876]$$

Since MBR(3) is the critical member and the force in MBR(3) is within one per cent of actual force, the relaxation process ends at three cycles.

Number of cycles = 3

Forces in the members

MBR(1) = -5800#

MBR(2) = -5784# ≅ -5800#

MBR(3) = 2876 ≅ 2900#

Joint Deflections

Joint	Δ_x	Δ_y	Δ_z
A	0.0	0.0	0.0
B	0.00093	0.0	-0.00498
C	0.001915	0.0	0.0

Later in this study a computer solution is presented and the results compared with this solution.

CHAPTER III

COMPUTER APPLICATION

3.1 Matrix Formulation for Computer Application

The stiffness of individual members entering a joint will vary according to the types of connections used to secure them to the joint. For simplicity, consider joint J in Figure 7 with three members entering the joint in a rigid condition.

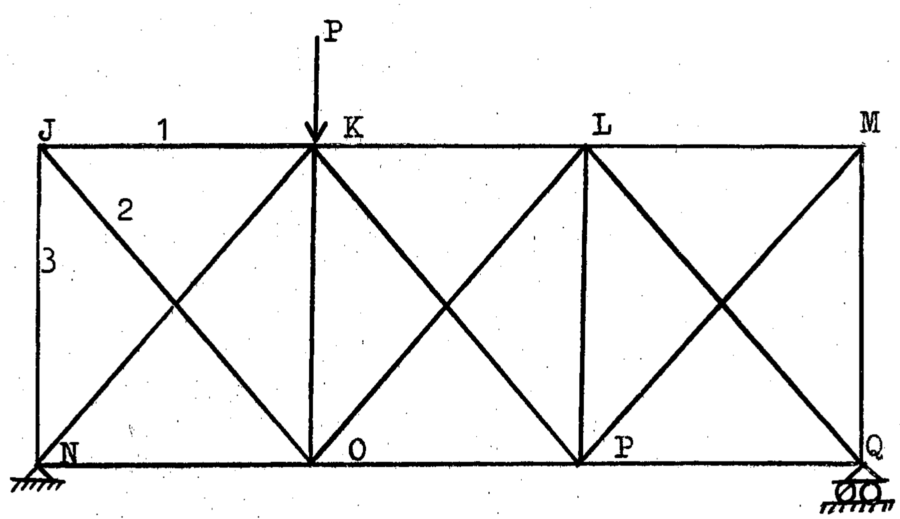


Figure 7. Illustration of Joint Stiffness

Now let [SM] represent an individual member stiffness matrix in the member system.

Then

$$[SM] = \begin{bmatrix} {}^{m}_{m}K^{m} \end{bmatrix} = \begin{bmatrix} {}_{jj}K^{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & {}_{jj}K^{m}_{py} & 0 & 0 & 0 & {}_{jj}K^{m}_{pz} \\ 0 & 0 & {}_{jj}K^{m}_{pz} & 0 & {}_{jj}K^{m}_{py} & 0 \\ 0 & 0 & 0 & {}_{jj}K^{m}_{mx} & 0 & 0 \\ 0 & 0 & {}_{jj}K^{m}_{my} & 0 & {}_{jj}K^{m}_{my} & 0 \\ 0 & {}_{jj}K^{m}_{mz} & 0 & 0 & 0 & {}_{jj}K^{m}_{mz} \end{bmatrix}$$

For rotation

$$\begin{bmatrix} {}^{m}\omega^{m}_{3} \end{bmatrix} = [\omega_{\gamma}][\omega_{\beta}][\omega_{\alpha}]$$

Let

$A = \text{Cos } \alpha$

$B = \text{Sin } \alpha$

$C = \text{Cos } \beta$

$D = \text{Sin } \beta$

$E = \text{Cos } \gamma$

$F = \text{Sin } \gamma$

Then

$$\begin{bmatrix} {}^{m}\omega^{m}_{3} \end{bmatrix} = \begin{bmatrix} \omega(1) & \omega(2) & \omega(3) \\ \omega(4) & \omega(5) & \omega(6) \\ \omega(7) & \omega(8) & \omega(9) \end{bmatrix}_{3 \times 3} \; ; \; \begin{bmatrix} {}^{o}\omega^{m}_{3} \end{bmatrix}^{T} = \begin{bmatrix} \omega(1) & \omega(4) & \omega(7) \\ \omega(2) & \omega(5) & \omega(8) \\ \omega(3) & \omega(6) & \omega(9) \end{bmatrix}$$

where

$\omega(1) = A \times B$

$\omega(2) = B \times C$

$\omega(3) = D$

$\omega(4) = (-B \times E - A \times D \times F)$

$\omega(5) = A \times E - B \times D \times F$

$\omega(6) = C \times E$

$\omega(7) = B \times F - A \times D \times E$

$\omega(8) = (-A \times F - B \times D \times E)$

$\omega(9) = C \times F$

and

$$\left[{}^{0}\omega^{m}\right]^{T} = \left[\begin{array}{c|c} {}^{0}\omega^{m\,T} & 0 \\ \hline 0 & {}^{0}\omega^{m\,T} \end{array}\right]_{6 \times 6}$$

represent the rotation matrix for rotation of vectors from the member system to the general system. And

$$\left[{}^{m}\omega^{m}\right] = \left[\begin{array}{c|c} {}^{m}\omega^{m} & 0 \\ \hline 0 & {}^{m}\omega^{m} \end{array}\right]_{6 \times 6}$$

represents the rotation matrix for rotation of vectors in the member system. Therefore,

$[SM]\,[{}^{m}\omega^{m}] = [SMMS]$ = the stiffness of the members in the member system.

And $[SMMS]\,[{}^{0}\omega^{m}]^{T} = [SMGS]$ = the stiffness of the member in the general system.

Let

$[SJG]$ = Total stiffness of the joint in the general system. And the total stiffness of joint J in Figure 7 is the total stiffness of all members entering the joint.

$$[SJG] = [SMGS_1] + [SMGS_2] + [SMGS_3]$$

Let

$$\sum [SMGS]^{-1} = [Flex_j] = [SJG]^{-1}$$

That is, the inverse of the joint stiffness will give the flexibility.

Let $PM_{m,l,k}$ = force in the members

and

$P_{j,k}$ = external force at joint J.

Taking the summation of forces at joint J will give the unbalanced forces at the joint. Or

$$\sum([\omega]^T [PM_{m,l,k}]) + [P_{j,k}] = [UBF_j]$$

where

$[UBF_j]$ = unbalanced forces on the joint under consideration.

Multiplying the unbalanced forces by the flexibility gives the deflections of the joint. That is,

$$[UBF_j] [Flex_j] = [DJ_{j,k}]$$

where

$[DJ_{j,k}]$ = joint deflections

then

$$[SMMS] [DJ_{j,k}] = [PM_{m,l,k}]$$

where

$[PM_{m,1,k}]$ = member force increments on the end entering joint J

and

$[DJ_{j,k}]$ = total joint deflections

$[PM_{m,1,k}]$ = final forces in the members.

3.2 Summary of Matrices

Let:

$$[PM_{m,1,k}] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \quad ; \quad [P_{j,k}] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{bmatrix}$$

$$[UBF_j] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \quad ; \quad [DJ_{j,k}] = \begin{bmatrix} \Delta x_j \\ \Delta y_j \\ \Delta z_j \\ \theta x_j \\ \theta y_j \\ \theta z_j \end{bmatrix}$$

Where:

PM = internal forces in the members; m is the member number, 1 = 1 or 2, indicating small or large numbered end of member; k varies from 1 to 6 where

1, 2 and 3 are x, y and z forces and 4, 5 and 6 are x, y and z moments.

P = joint external loadings, j is the joint number and k indicates forces and moments.

UBF = unbalanced forces on the joint.

DJ = deflection at a joint in the general system; j is the joint number; and k varies from 1 to 6, where 1, 2 and 3 are deflections in the x, y and z directions, and 4, 5 and 6 are rotations of the joint about the x, y and z axes.

3.3 General Procedures

The following is a brief description of the computer program.

Taking advantage of the high speed and large memory capacity of the IBM 7040 computer, the computer program is written in Fortran IV (8). There are many advantages in the use of Fortran IV, but the one most advantageous to the relaxation process is the use of subroutines which can be called at any point in the main program. This will keep the main program short and eliminates many complications.

This program is divided into two major parts. In the first part, all matrices required for the second part of the program are computed and recorded on magnetic tape. In the second part of the program, forces or loads are read into the program and the iterative process takes place.

In the beginning of the program the desired accuracy is placed in memory. The computer will continue the iterative process until that accuracy is reached. This ends the iterative process.

The results are either punched or written out. These results include the forces in all the members for both ends, the deflections and rotations of all the joints, the unbalanced forces at all joints which allow the programmer to easily assay the results. Finally, the reactions (real restraints) are recorded.

In the following pages the technique and sample calculations for each step in the computer solution are given. A flow chart of the computer program is shown in the Appendix.

3.4 The Technique

Step I — For each member entering a joint, the inter-related position of the member to the rest of the structure is related by joint cartesian co-ordinates.

Step II — The general rotation matrix for the member is computed, and this is shown under Rotation Matrix [W] in Table IV.

Step III — The transpose of the rotation matrix is then computed and is shown under Rotation Transpose Matrix [WT] in Table IV.

Step IV — From member properties read into memory, the stiffness matrix for the member is computed in the stiffness

TABLE IV

SAMPLE MATRICES (COMPUTER SOLUTION)

ROTATION MATRIX W

3

-0.65466180E 00	0.37793627E 00	0.65466182E 00	0.	0.	0.
-0.49996735E 00	-0.86604425E 00	0.	0.	0.	0.
0.56696610E 00	-0.32730953E 00	0.75592189E 00	0.	0.	0.
0.	0.	0.	-0.65466180E 00	0.37793627E 00	0.65466182E 00
0.	0.	0.	-0.49996735E 00	-0.86604425E 00	0.
0.	0.	0.	0.56696610E 00	-0.32730953E 00	0.75592189E 00

ROTATION TRANSPOSE MATRIX WT

3

-0.65466180E 00	-0.49996735E 00	0.56696610E 00	0.	0.	0.
0.37793627E 00	-0.86604425E 00	-0.32730953E 00	0.	0.	0.
0.65466182E 00	0.	0.75592189E 00	0.	0.	0.
0.	0.	0.	-0.65466180E 00	-0.49996735E 00	0.56696610E 00
0.	0.	0.	0.37793627E 00	-0.86604425E 00	-0.32730953E 00
0.	0.	0.	0.65466182E 00	0.	0.75592189E 00

STIFFNESS MATRIX SM

3

0.20124304E 07	0.	0.	0.	0.	0.
0.	0.21484892E 04	0.	0.	0.	-0.16480696E 06
0.	0.	0.38976876E 05	0.	0.33282057E 07	0.
0.	0.	0.	0.39279709E 05	0.	0.
0.	0.	0.33282057E 07	0.	0.33892366E 09	0.
0.	-0.16480696E 06	0.	0.	0.	0.16782910E 08

MYAB MATRIX SMMS

3

-0.13174613E 07	0.76057043E 06	0.13174614E 07	0.	-0.	0.
-0.10741744E 04	-0.18606867E 04	-0.	-0.93439957E 05	0.53942887E 05	-0.12458119E 06
0.22098567E 05	-0.12757503E 05	0.29463474E 05	-0.16639941E 07	-0.28823734E 07	0.
0.	0.	0.	-0.25714925E 05	0.14845226E 05	0.25714926E 05
0.18869798E 07	-0.10893534E 07	0.25158635E 07	-0.16945076E 09	-0.29352289E 09	0.
0.82398096E 05	0.14273012E 06	0.	0.95153410E 07	-0.54932064E 07	0.12686569E 08

MPYAB MATRIX SMGS

3

0.87555779E 06	-0.50421919E 06	-0.84578682E 06	-0.89671133E 06	-0.16611777E 07	0.62286525E 05
-0.50421920E 06	0.29323423E 06	0.48827275E 06	0.62556426E 06	0.89671134E 06	0.10789282E 06
-0.84578682E 06	0.48827275E 06	0.88476373E 06	-0.12578496E 07	-0.21788491E 07	0.
-0.89671134E 06	0.62556427E 06	-0.12578496E 07	0.90131556E 08	0.14362767E 09	0.71760199E 07
-0.16611777E 07	0.89671133E 06	-0.21788491E 07	0.14362767E 09	0.25600739E 09	-0.41427163E 07
0.62286525E 05	0.10789282E 06	0.	0.71760199E 07	-0.41427163E 07	0.96068898E 07

JOINT STIFFNESS

3

0.17532640E 07	0.	0.	0.	-0.34302442E 07	-0.62402344E 01
0.	0.17531180E 07	-0.49328125E 02	0.34298981E 07	-0.	-0.
0.	-0.49335938E 02	0.26542593E 07	0.12606250E 03	0.	0.
0.	0.34298981E 07	0.12606250E 03	0.51918264E 09	-0.	-0.
-0.34302442E 07	-0.	0.	-0.	0.51922982E 09	0.41512500E 03
-0.62412109E 01	-0.	0.	-0.	0.41525000E 03	0.28820727E 08

FLEX MATRIX

1

0.57783348E-06	0.	0.	-0.	0.38174038E-08	0.70127189E-13
0.	0.57788137E-06	0.10920964E-10	-0.38176819E-08	0.	0.
0.	0.10922665E-10	0.37675294E-06	-0.16363807E-12	0.	0.
-0.	-0.38176819E-08	-0.16362683E-12	0.19513253E-08	-0.	-0.
0.38174038E-08	0.	0.	-0.	0.19511487E-08	-0.27277213E-13
0.70130212E-13	0.	0.	-0.	-0.27285547E-13	0.34697251E-07

subroutine. This is seen in Table IV under Stiffness Matrix SM.

Step V — Multiplying the stiffness matrix by the rotation matrix gives the stiffness of the member in the member system and is shown in Table IV under MYAB Matrix SMMS.

Step VI — Multiplying the results obtained in Step V by the transpose matrix gives the stiffness of the member in the general system and is shown in Table IV under MPYAB Matrix SMGS.

Step VII — The first six steps are repeated for each member entering the joint and then the SMGS's are summed up for the joint which is the total stiffness of the joint and is shown in Table IV under Joint Stiffness.

Step VIII — The inverse of the joint stiffness matrix is the flexibility of the joint and is shown under Flex Matrix of Table IV.

The above steps are repeated for each joint of the structure and all information needed for the iterative process in the second part of the program is then recorded on magnetic tape.

A restatement of the relaxation process is not necessary at this time, but a complete flow chart is given in the Appendix.

3.5 Load Conditions

The calculation of displacements in a structure by means of matrix equations requires that the structure be subjected to loads acting only at the joints. In general, the actual loads on a structure do not meet this requirement. Instead, the loads may be divided into two types: loads acting on the joints, and loads acting on the members.

Loads acting on the members require special treatment. First, an equivalent joint load may be computed from the loads acting on the members. Second, all points where concentrated loads are acting on the members may be treated as additional joints. Finally, the end reactions (moments and shears) of all loaded members may be computed and applied as member forces in the member system. The type of load condition will dictate which of the above methods to use in any given condition.

CHAPTER IV

TEST PROBLEMS AND COMPUTED SOLUTIONS

The computer program was used to solve several structures, starting with the planer truss presented earlier in this study and progressing to more complex structures. A brief description of each test solution follows.

A complete solution yields the six components of force in each end of all members in a structure, the six joint deformations, and the unbalanced forces at each interior joint. The latter allows the programmer to check the solution accuracy. Finally, the restraints or reactions are found.

In some of these test problems, the unbalanced forces at interior joints are not included because of the lengthy solutions.

4.1 Test Problem One — Planer Truss

A computer solution to the planer truss is presented so the results may be compared with the slide rule solution presented earlier in this study. Results show that the two solutions do not vary more than 0.5 per cent. The slide rule solution is for three cycles versus seven cycles for the computer solution. This shows that the rate of convergence is very rapid in the early cycles and tapers off to small convergence in later cycles.

4.2 Test Problem Two — Small Space Frames

Before going to structural systems of a more complex nature, several tests were made on a tripod with both rigid and pin-connected joints. The results were checked and found to be accurate. Other variations of the pyramid shape were solved with equal success. It was found that the increase from a 6-member tripod to a 9-member space frame had no bearing on the rate of convergence.

4.3 Test Problem Three — Truss (Pin-Connected Joints)

This problem and its solution are presented for two reasons. First, it will show the results of a coplaner structure, and the results may be compared with a known solution (9). The program solves statically indeterminate as well as determinate coplaner structures with pin-connected joints, rigid connected joints, or a combination of joint conditions. The solution to this internally indeterminate truss was found to be identical to the solution found in the above reference.

4.4 Test Problem Four — Truss (Rigid Connected Joints)

The three examples given in 4.1, 4.2, and 4.3 involve only axial forces in the members. This example illustrates the capability of the program for solving secondary forces as well as primary forces. Again the results checked with a known solution (9).

It was found that the rate of convergence for this coplaner structure was slow compared to the structure given in 4.3. Since the structure is symmetrical and has a symmetrical load condition, a test was made using one half of the structure to find the stresses in the members. The only requirements for this type of operation are proper restraints where the structure is cut and an equivalent load condition.

A correct solution was obtained using one half of the structure, and the computer time was cut to one third of the original time required. This method is used successfully in Example Problem 4.5.

4.5 Test Problem Five — Dome (Rigid Connected Joints)

This problem demonstrates the versatility of this program for solving different types of structures. Taking full advantage of symmetry, one half of the structure with a uniform load is analyzed. The rate of convergence for this type of structure is very rapid.

For a check on the accuracy of the solution, a comparison was made with a solution obtained from a Fran solution at the Engineering Firm of Orr, Bass and Associates. The results were found to accurate and complete.

4.6 Test Problem Six — Space Frame (Rigid Connected Joints)

This problem consists of one bent of a multi-story frame. It was found that the rate of convergence for this type of structure is very rapid in the early iterative process. However, to get within five per cent accuracy requires extended computer time. For example, two test runs of 20 minutes and 2½ hours were made. It was found that the forces in the members after 20 minutes were within 200 pounds of the forces obtained from a 2½ hour computer run.

The solution to this problem was compared with a stress solution obtained from Mr. Hendren of the Department of Architecture, Oklahoma State University. The comparison shows similar results.

4.7 Test Problem Seven — Space Frame (Pin-Connected Joints)

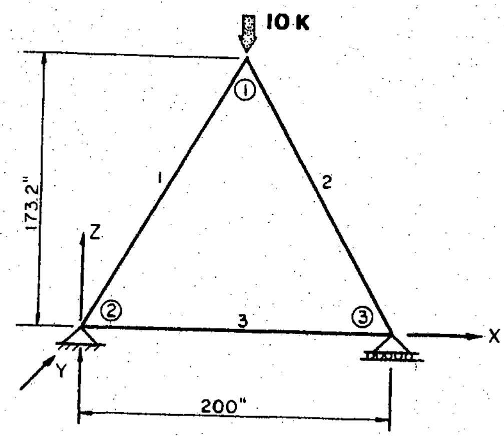
Referring to Figure 14, this structure is similar to a Unistrut frame and to other patented diagrids.

This solution is presented to demonstrate the capability of this program to solve large complex structures. From the results obtained, the number of joints in a structure has some effect on the cycle time, but the controlling factor is the number of members in the structure. If the number of members are doubled, the iteration time is doubled.

The reactions for this solution do not equal the applied load. This is due to the degree of accuracy that was predetermined. A smaller accuracy figure would produce closer results.

4.8 Computer Solutions

On the following pages, computer solutions are presented for each of the test problems selected for this study. They are listed under Figures 8, 9, 10, 11, 12, 13, and 14.



PLANER TRUSS

Non-Rigid Joints

PROPERTIES OF MEMBERS

A_x = 10.59 sq. in. for all members

E = 29×10^6 G = 12×10^6

FORCES IN THE MEMBERS

M	NN	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	1	-5773.55	0.00	0.00	0.00	0.00	0.00
1	2	-5773.55	0.00	0.00	0.00	0.00	0.00
2	1	-5773.47	0.00	0.00	0.00	0.00	0.00
2	3	-5773.47	0.00	0.00	0.00	0.00	0.00
3	2	2886.80	0.00	0.00	0.00	0.00	0.00
3	3	2886.80	0.00	0.00	0.00	0.00	0.00

JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	0.00093905	0.00000000	-0.00487960	0.00000000	0.00000000	0.00000000
2	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
3	0.00187820	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000

UNBALANCED FORCES AND MOMENTS ON INTERIOR JOINTS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	-0.04	-0.00	0.06	-0.00	-0.00	-0.00

FORCES AND MOMENTS AT REACTIONS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
2	0.04	-0.00	5000.00	-0.00	-0.00	-0.00
3	-0.00	-0.00	4999.94	-0.00	-0.00	-0.00

Figure 8. Computer Solution of a Planer Truss

STRUCTURES HAS NON-RIGID JOINTS

PROPERTIES OF MEMBERS

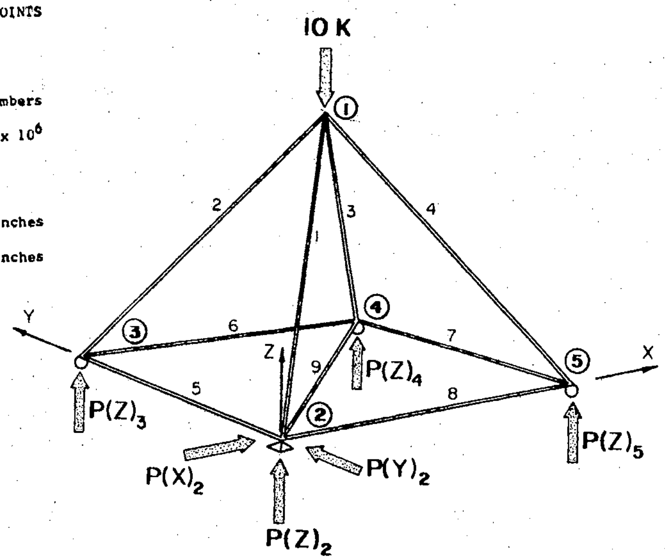
A_x = 10.59 sq. in. for all members

$E = 29 \times 10^6$ $G = 12 \times 10^6$

LENGTH OF MEMBERS

MBR #9 L = 282.8 inches

All other MBRS L = 200.0 inches



NCYCLS

8

FORCES IN THE MEMBERS

M	NN	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	1	-3987.67	0.00	0.00	0.00	0.00	0.00
1	2	-3987.67	0.00	0.00	0.00	0.00	0.00
2	1	-3083.79	0.00	0.00	0.00	0.00	0.00
2	3	-3083.79	0.00	0.00	0.00	0.00	0.00
3	1	-3987.18	0.00	0.00	0.00	0.00	0.00
3	4	-3987.18	0.00	0.00	0.00	0.00	0.00
4	1	-3083.64	0.00	0.00	0.00	0.00	0.00
4	5	-3083.64	0.00	0.00	0.00	0.00	0.00
5	2	1541.91	0.00	0.00	0.00	0.00	0.00
5	3	1541.91	0.00	0.00	0.00	0.00	0.00
6	3	1542.28	0.00	0.00	0.00	0.00	0.00
6	4	1542.28	0.00	0.00	0.00	0.00	0.00
7	4	1541.84	0.00	0.00	0.00	0.00	0.00
7	5	1541.84	0.00	0.00	0.00	0.00	0.00
8	2	1541.84	0.00	0.00	0.00	0.00	0.00
8	5	1541.84	0.00	0.00	0.00	0.00	0.00
9	2	638.28	0.00	0.00	0.00	0.00	0.00
9	4	638.28	0.00	0.00	0.00	0.00	0.00

JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	0.00000673	0.00040823	-0.00396253	0.00000000	0.00000000	0.00000000
2	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
3	-0.00098930	0.00100320	0.00000000	0.00000000	0.00000000	0.00000000
4	0.00001405	0.00081650	0.00000000	0.00000000	0.00000000	0.00000000
5	0.00100315	-0.00018665	0.00000000	0.00000000	0.00000000	0.00000000

UNBALANCED FORCES AND MOMENTS ON INTERIOR JOINTS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	-0.32	-0.16	0.02	-0.00	-0.00	-0.00

FORCES AND MOMENTS AT REACTIONS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
2	0.69	0.61	2819.67	-0.00	-0.00	-0.00
3	-0.37	0.00	2180.54	-0.00	-0.00	-0.00
4	-0.00	-0.45	2819.33	-0.00	-0.00	-0.00
5	-0.00	-0.00	2180.43	-0.00	-0.00	-0.00

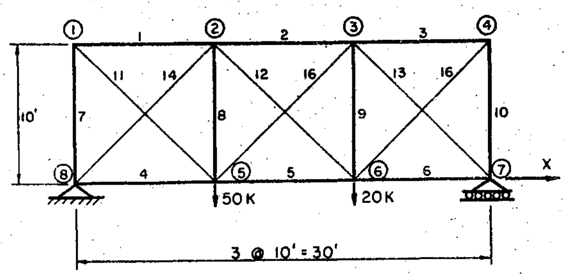
Figure 9. Computer Solution of a Small Space Frame

TRUSS HAS NON-RIGID JOINTS

PROPERTIES OF MEMBERS

A_x = 3.0 sq. in. for all members

$E = 29 \times 10^6$ $G = 12 \times 10^6$



FORCES IN THE MEMBERS

M	NN	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	1	-20132.85	0.00	0.00	0.00	0.00	0.00
1	2	-20132.85	0.00	0.00	0.00	0.00	0.00
2	2	-38702.09	0.00	0.00	0.00	0.00	0.00
2	3	-38702.09	0.00	0.00	0.00	0.00	0.00
3	3	-14097.93	0.00	0.00	0.00	0.00	0.00
3	4	-14097.93	0.00	0.00	0.00	0.00	0.00
4	5	19861.46	0.00	0.00	0.00	0.00	0.00
4	8	19861.46	0.00	0.00	0.00	0.00	0.00
5	5	31292.94	0.00	0.00	0.00	0.00	0.00
5	6	31292.94	0.00	0.00	0.00	0.00	0.00
6	6	15900.19	0.00	0.00	0.00	0.00	0.00
6	7	15900.19	0.00	0.00	0.00	0.00	0.00
7	1	-20133.59	0.00	0.00	0.00	0.00	0.00
7	8	-20133.59	0.00	0.00	0.00	0.00	0.00
8	2	21162.82	0.00	0.00	0.00	0.00	0.00
8	5	21162.82	0.00	0.00	0.00	0.00	0.00
9	3	7197.88	0.00	0.00	0.00	0.00	0.00
9	6	7197.88	0.00	0.00	0.00	0.00	0.00
10	4	-14097.93	0.00	0.00	0.00	0.00	0.00
10	7	-14097.93	0.00	0.00	0.00	0.00	0.00
11	1	28473.71	0.00	0.00	0.00	0.00	0.00
11	5	28473.71	0.00	0.00	0.00	0.00	0.00
12	2	-1832.45	0.00	0.00	0.00	0.00	0.00
12	6	-1832.45	0.00	0.00	0.00	0.00	0.00
13	3	-22486.27	0.00	0.00	0.00	0.00	0.00
13	7	-22486.27	0.00	0.00	0.00	0.00	0.00
14	2	-28094.80	0.00	0.00	0.00	0.00	0.00
14	8	-28094.80	0.00	0.00	0.00	0.00	0.00
15	3	12308.22	0.00	0.00	0.00	0.00	0.00
15	5	12308.22	0.00	0.00	0.00	0.00	0.00
16	4	19937.38	0.00	0.00	0.00	0.00	0.00
16	6	19937.38	0.00	0.00	0.00	0.00	0.00

JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	0.09642466	0.00000000	-0.02684480	0.00000000	0.00000000	0.00000000
2	0.06958084	0.00000000	-0.14450031	0.00000000	0.00000000	0.00000000
3	0.01797802	0.00000000	-0.13139154	0.00000000	0.00000000	0.00000000
4	-0.00081922	0.00000000	-0.01879723	0.00000000	0.00000000	0.00000000
5	0.02648196	0.00000000	-0.17271743	0.00000000	0.00000000	0.00000000
6	0.06820588	0.00000000	-0.14098872	0.00000000	0.00000000	0.00000000
7	0.08940615	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
8	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000

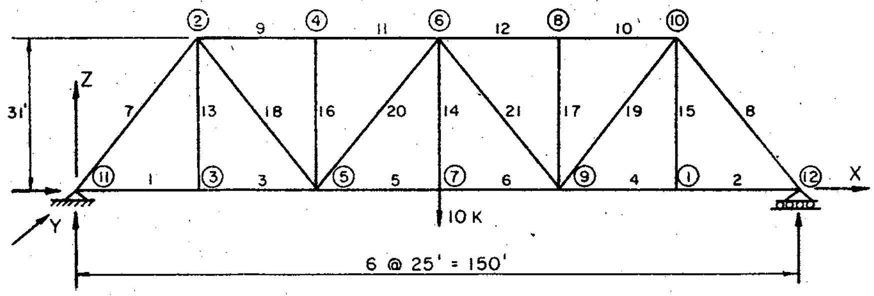
UNBALANCED FORCES AND MOMENTS ON INTERIOR JOINTS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	-1.11	-0.00	0.36	-0.00	-0.00	-0.00
2	-1.04	-0.00	1.05	-0.00	-0.00	-0.00
3	-0.75	-0.00	0.91	-0.00	-0.00	-0.00
4	-0.07	-0.00	-0.07	-0.00	-0.00	-0.00
5	-0.7[illegible]	-0.00	0.00	-0.00	-0.00	-0.00
6	-0.86	-0.00	0.00	-0.00	-0.00	-0.00

FORCES AND MOMENTS AT REACTIONS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
7	-0.00	-0.00	29998.12	-0.00	-0.00	-0.00
8	4.56	-0.00	39999.62	-0.00	-0.00	-0.00

Figure 10. Computer Solution of a Truss With Non-Rigid Joints



TRUSS HAS RIGID JOINTS

PROPERTIES OF MEMBERS

MBRS	I_y	A_x	MBRS	I_y	A_x
1,2,3,4	747.8	26.34	13,14,15	112.86	24.35
5,6	1074.0	50.22	16,17	70.62	12.20
7,8	2611.5	49.45	18,19	802.80	29.28
9,10,11,12	2385.4	43.33	20,21	858.00	32.22

$E = 30 \times 10^6$ $G = 12 \times 10^6$

FORCES IN THE MEMBERS

M	NN	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	2	4024.80	-0.00	-7.87	0.00	-1076.70	0.00
1	11	4024.80	-0.00	7.87	0.00	1285.26	0.00
2	1	4029.64	-0.00	-7.88	0.00	-1077.94	0.00
2	12	4029.64	-0.00	7.88	0.00	1284.95	0.00
3	2	4027.82	-0.00	-5.16	-0.00	-539.28	-0.00
3	5	4027.82	-0.00	5.16	-0.00	1008.48	0.00
4	1	4031.42	0.00	-5.17	-0.00	-540.74	-0.00
4	9	4031.42	0.00	5.17	-0.00	1009.43	0.00
5	5	12012.91	0.00	81.77	-0.00	8537.70	-0.00
5	7	12012.91	0.00	-81.77	-0.00	-15992.13	-0.00
6	7	12013.72	0.00	-81.76	-0.00	-15991.73	0.00
6	9	12013.72	0.00	81.76	-0.00	8537.32	-0.00
7	3	-6415.80	-0.00	3.94	0.00	597.97	0.00
7	11	-6415.80	-0.00	-3.94	0.00	-1285.26	-0.00
8	10	-6414.28	-0.00	3.93	0.00	594.36	0.00
8	12	-6414.28	-0.00	-3.93	0.00	-1284.95	-0.00
9	3	-8068.64	0.00	18.61	-0.00	3475.39	-0.00
9	4	-8068.64	0.00	-18.61	-0.00	-2108.20	0.00
10	8	-8067.79	0.00	-18.59	-0.00	-2106.78	0.00
10	10	-8067.79	0.00	18.59	-0.00	3471.54	-0.00
11	4	-8071.95	0.00	39.88	-0.00	-1398.39	-0.00
11	6	-8071.95	0.00	-39.88	-0.00	-13363.52	0.00
12	6	-8072.24	0.00	-39.88	-0.00	-13362.34	0.00
12	8	-8072.24	0.00	39.88	-0.00	-1398.46	-0.00
13	2	-13.30	0.00	-2.72	0.00	-537.24	-0.00
13	3	-13.30	0.00	2.72	0.00	473.68	-0.00
14	6	9836.47	0.00	0.00	0.00	0.06	-0.00
14	7	9836.47	0.00	-0.00	0.00	-0.07	-0.00
15	1	-13.21	-0.00	2.72	0.00	537.07	0.00
15	10	-13.21	-0.00	-2.72	0.00	-473.51	0.00
16	4	-21.10	0.00	3.93	0.00	708.95	-0.00
16	5	-21.10	0.00	-3.93	0.00	-751.79	-0.00
17	8	-21.14	-0.00	-3.93	0.00	-708.80	0.00
17	9	-21.14	-0.00	3.93	0.00	751.66	0.00
18	3	6415.41	-0.00	-11.46	-0.00	-2403.13	0.00
18	5	6415.41	-0.00	11.46	-0.00	3071.17	0.00
19	9	6413.80	-0.00	11.46	-0.00	3071.11	0.00
19	10	6413.80	-0.00	-11.46	-0.00	-2403.44	0.00
20	5	-6273.70	0.00	-7.69	0.00	-3705.71	-0.00
20	6	-6273.70	0.00	7.69	0.00	-32.72	0.00
21	6	-6271.91	0.00	7.68	0.00	-33.10	-0.00
21	9	-6271.91	0.00	-7.68	0.00	-3705.24	-0.00

Figure 11. Computer Solution of a Truss With Rigid Joints

JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	0.00938088	0.00000000	-0.01005296	-0.00000000	-0.00003157	-0.00000000
2	0.00153034	-0.00000000	-0.01005089	0.00000000	0.00003157	-0.00000000
3	0.00918244	-0.00000000	-0.01005764	0.00000000	0.00002808	0.00000000
4	0.00731902	-0.00000000	-0.01949590	0.00000000	0.00003094	0.00000000
5	0.00306183	-0.00000000	-0.01947445	0.00000000	0.00003471	0.00000000
6	0.00545482	-0.00000000	-0.02539151	0.00000000	0.00000000	0.00000000
7	0.00545485	0.00000000	-0.03039039	0.00000000	0.00000000	0.00000000
8	0.00359056	0.00000000	-0.01949692	-0.00000000	-0.00003094	0.00000000
9	0.00784802	0.00000000	-0.01947543	-0.00000000	-0.00003470	0.00000000
10	0.00172733	0.00000000	-0.01005967	-0.00000000	-0.00002808	0.00000000
11	0.00000000	0.00000000	0.00000000	0.00000000	0.00003017	-0.00000000
12	0.01091306	0.00000000	0.00000000	-0.00000000	-0.00003018	-0.00000000

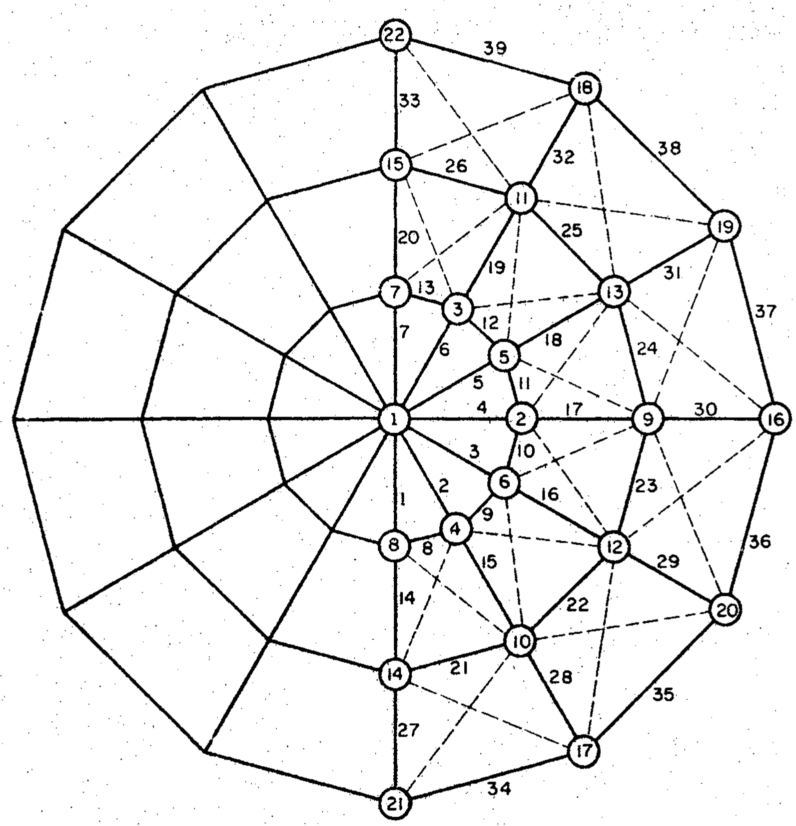
UNBALANCED FORCES AND MOMENTS ON INTERIOR JOINTS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	-0.94	-0.00	0.16	-0.00	0.13	0.00
2	-0.30	-0.00	0.27	-0.00	-0.18	-0.00
3	-0.89	0.00	0.30	-0.00	-0.61	0.00
4	-0.61	-0.00	0.17	-0.00	-0.85	-0.00
5	-0.64	-0.00	0.08	0.00	-0.54	-0.00
6	-0.84	-0.00	0.69	-0.00	-0.75	-0.00
7	-0.81	-0.00	0.00	-0.00	-0.33	-0.00
8	-0.52	0.00	0.14	-0.00	-0.48	0.00
9	-0.01	-0.00	-0.02	0.00	-0.11	-0.00
10	-0.24	0.00	0.30	-0.00	0.23	0.00

FORCES AND MOMENTS AT REACTIONS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
11	5.80	0.00	4999.54	-0.00	-0.00	-0.00
12	0.00	-0.00	4998.37	-0.00	-0.00	-0.00

Figure 11. (Concluded)



DOME

Rigid Joints

Load: Live load of 35psf

PROPERTIES OF MEMBERS

MBRS	A_x	A_y	A_z	I_x	I_y	I_z
1,7,14,20	5.3	2.99	2.31	.2495	223.2	11.1
28,29,30,31,32	11.77	7.04	4.73	.7385	515.5	26.5
27,33	5.89	3.52	2.37	.3693	257.8	13.3
34,35,36 37,38,39	20.00	10.43	9.57	1.7187	1814.5	63.8
For all other members	10.59	5.98	4.61	0.4991	446.3	22.1

Span = 150.0 Ft. Height = 108.75 Ft.

(a)

Figure 12. Computer Solution of a Dome Structure

FORCES IN THE MEMBERS

M	NN	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	1	-10162.16	0.00	147.65	0.00	31718.27	0.00
1	8	-10162.16	0.00	-147.65	0.00	-19313.42	0.00
2	1	-20260.73	-0.00	294.16	0.02	63385.95	0.61
2	4	-20260.73	-0.00	-294.16	-0.02	-38324.34	0.83
3	1	-20323.72	-0.00	295.60	0.01	63525.68	0.24
3	6	-20323.72	-0.00	-295.60	-0.01	-38654.94	0.18
4	1	-20261.79	0.00	294.56	0.00	63471.71	-0.00
4	2	-20261.79	0.00	-294.56	-0.00	-38375.78	-0.00
5	1	-20323.72	0.00	295.60	-0.01	63525.68	-0.24
5	5	-20323.72	0.00	-295.60	0.01	-38654.93	-0.18
6	1	-20260.73	0.00	294.16	-0.02	63385.94	-0.61
6	3	-20260.73	0.00	-294.16	0.02	-38324.34	-0.83
7	1	-10162.16	0.00	147.65	0.00	31718.27	0.00
7	7	-10162.16	0.00	-147.65	0.00	-19313.43	0.00
8	4	-35010.23	-0.10	-4.12	20.97	11808.88	-147.41
8	8	-35010.23	-0.10	4.12	-20.97	12655.20	165.23
9	4	-35010.44	0.10	-3.89	-21.01	11806.56	147.52
9	6	-35010.44	0.10	3.89	21.01	12498.00	-165.35
10	2	-35010.33	-0.12	-3.40	21.03	11807.44	-146.09
10	6	-35010.33	-0.12	3.40	-21.03	12498.29	166.24
11	2	-35010.33	0.12	-3.40	-21.03	11807.44	146.09
11	5	-35010.33	0.12	3.40	21.03	12498.28	-166.23
12	3	-35010.44	-0.10	-3.89	21.01	11806.56	-147.52
12	5	-35010.44	-0.10	3.89	-21.01	12497.99	165.36
13	3	-35010.23	0.10	-4.12	-20.97	11808.87	147.41
13	7	-35010.23	0.10	4.12	20.97	12655.24	-165.22
14	8	-20607.08	0.00	52.13	0.00	-16094.88	0.00
14	14	-20607.08	0.00	-52.13	0.00	-31913.37	0.00
15	4	-41169.00	0.00	103.61	0.02	-32413.72	-0.99
15	10	-41169.00	0.00	-103.61	-0.02	-63843.58	-0.13
16	6	-41213.20	0.00	103.89	0.01	-32297.77	-1.04
16	12	-41213.20	0.00	-103.89	-0.01	-63805.09	-0.45
17	2	-41170.48	0.00	103.52	-0.00	-32463.31	-0.00
17	9	-41170.48	0.00	-103.52	0.00	-63863.29	-0.00
18	5	-41213.20	-0.00	103.89	-0.01	-32297.76	1.03
18	13	-41213.20	-0.00	-103.89	0.01	-63805.09	0.45
19	3	-41169.00	-0.00	103.61	-0.02	-32413.73	0.99
19	11	-41169.00	-0.00	-103.61	0.02	-63843.58	0.13
20	7	-20607.08	0.00	52.13	0.00	-16094.87	0.00
20	15	-20607.08	0.00	-52.13	0.00	-31913.38	0.00
21	10	-24452.42	-0.05	-2.49	-30.95	-16605.65	453.97
21	14	-24452.42	-0.05	2.49	30.95	-15762.60	-437.97
22	10	-24452.37	0.04	-2.35	30.95	-16606.80	-452.45
22	12	-24452.37	0.04	2.35	-30.95	-15819.50	439.67
23	9	-24452.45	-0.05	-2.36	-30.96	-16616.75	455.12
23	12	-24452.45	-0.05	2.36	30.96	-15819.36	-437.35
24	9	-24452.45	0.05	-2.36	30.96	-16616.74	-455.12
24	13	-24452.45	0.05	2.36	-30.96	-15819.37	437.35
25	11	-24452.37	-0.04	-2.35	-30.95	-16606.80	452.46
25	13	-24452.37	-0.04	2.35	30.95	-15819.51	-439.66
26	11	-24452.42	0.05	-2.49	30.95	-16605.65	-453.96
26	15	-24452.42	0.05	2.49	-30.95	-15762.59	437.97
27	14	-31716.78	0.00	-135.19	0.00	-35564.33	0.00
27	21	-31716.78	0.00	135.19	0.00	19255.75	0.00
28	10	-63428.81	0.01	-272.07	0.01	-71547.08	-1.34
28	17	-63428.81	0.01	272.07	-0.01	38725.56	-0.91
29	12	-63433.51	0.01	-270.31	0.01	-71133.04	-1.93
29	20	-63433.51	0.01	270.31	-0.01	38528.21	-2.11
30	9	-63429.12	0.00	-272.18	-0.00	-71571.78	-0.00
30	16	-63429.12	0.00	272.18	0.00	38744.60	-0.00
31	13	-63433.51	-0.01	-270.31	-0.01	-71133.05	1.93
31	19	-63433.51	-0.01	270.31	0.01	38528.22	2.11
32	11	-63428.81	-0.01	-272.07	-0.01	-71547.08	1.34
32	18	-63428.81	-0.01	272.07	0.01	38725.56	0.91
33	15	-31716.78	0.00	-135.19	0.00	-35564.33	0.00
33	22	-31716.78	0.00	135.19	0.00	19255.76	0.00
34	17	98407.65	-0.07	-0.87	-104.94	-74411.35	16.12
34	21	98407.65	-0.07	0.87	104.94	-74006.85	15.64
35	17	98407.60	0.07	-0.80	104.96	-74411.25	-15.45
35	20	98407.60	0.07	0.80	-104.96	-74039.72	-16.62
36	16	98407.60	-0.08	-0.88	-104.98	-74447.80	19.00
36	20	98407.60	-0.08	0.88	104.98	-74038.42	18.31
37	16	98407.60	0.08	-0.88	104.98	-74447.80	-19.00
37	19	98407.60	0.08	0.88	-104.98	-74038.43	-18.31
38	18	98407.60	-0.07	-0.80	-104.96	-74411.25	15.45
38	19	98407.60	-0.07	0.80	104.96	-74039.73	16.62
39	18	98407.65	0.07	-0.87	104.94	-74411.35	-16.12
39	22	98407.65	0.07	0.87	-104.94	-74006.86	-15.64

(b)

Figure 12. (Continued)

JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	0.00000000	0.00000000	-0.33425131	-0.00000000	0.00000000	0.00000000
2	-0.03608853	0.00000000	-0.45079580	-0.00000000	0.00030942	-0.00000000
3	-0.01804252	-0.03122941	-0.45063359	-0.00026716	0.00015524	-0.00000017
4	-0.01804252	0.03122941	-0.45063358	0.00026716	0.00015524	0.00000017
5	-0.03128740	-0.01805439	-0.45055650	-0.00015289	0.00026580	-0.00000005
6	-0.03128740	0.01805439	-0.45055650	0.00015289	0.00026580	0.00000005
7	0.00000000	-0.03607477	-0.45033166	-0.00030582	0.00000000	0.00000000
8	0.00000000	0.03607477	-0.45033165	0.00030582	0.00000000	0.00000000
9	-0.04885195	-0.00000000	-0.36750456	0.00000000	-0.00087822	-0.00000000
10	-0.02443067	0.04228647	-0.36736131	-0.00076038	-0.00043858	0.00000020
11	-0.02443067	-0.04228647	-0.36736132	0.00076038	-0.00043858	-0.00000020
12	-0.04225195	0.02438355	-0.36688646	-0.00043930	-0.00076048	0.00000009
13	-0.04225195	-0.02438355	-0.36688648	0.00043930	-0.00076048	-0.00000009
14	0.00000000	0.04871046	-0.36670825	-0.00087772	0.00000000	0.00000000
15	0.00000000	-0.04871047	-0.36670828	0.00087772	0.00000000	0.00000000
16	0.14783520	0.00000000	0.00000000	0.00000000	-0.00122862	0.00000000
17	0.07387990	-0.12794884	0.00000000	-0.00106361	-0.00061405	0.00000006
18	0.07387990	0.12794886	0.00000000	0.00106361	-0.00061405	-0.00000006
19	0.12769945	0.07370937	0.00000000	0.00061316	-0.00106197	0.00000008
20	0.12769945	-0.07370936	0.00000000	-0.00061316	-0.00106197	-0.00000008
21	0.00000000	-0.14744831	0.00000000	-0.00122582	0.00000000	0.00000000
22	0.00000000	0.14744833	0.00000000	0.00122582	0.00000000	0.00000000

UNBALANCED FORCES AND MOMENTS ON INTERIOR JOINTS

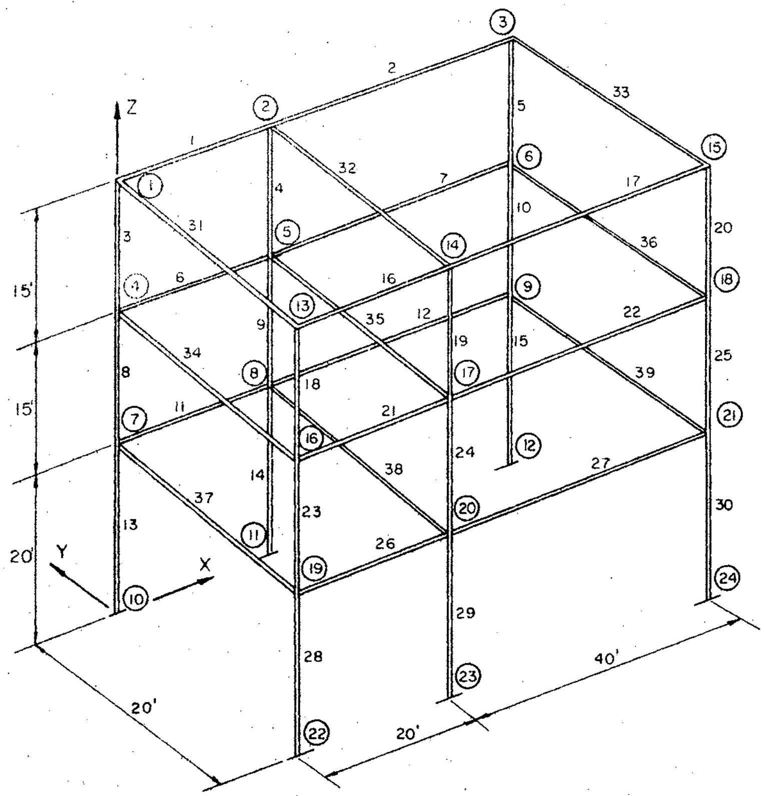
JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	75255.22	-0.00	3.44	0.00	-236887.56	-0.01
2	3.43	0.00	0.96	0.00	63.76	-0.00
3	1.89	3.25	0.79	-55.62	34.83	-0.60
4	1.89	-3.25	0.79	55.61	34.84	0.61
5	-12.73	-7.35	6.09	16.25	-28.16	-0.01
6	-12.73	7.35	6.09	-16.25	-28.15	0.01
7	33817.41	-7.50	3.11	16.76	-11867.62	3015.10
8	33817.41	7.50	3.11	-16.76	-11867.59	-3015.08
9	11.04	0.00	-7.68	0.00	16.40	-0.00
10	5.94	-10.22	-8.33	13.70	9.34	0.70

FORCES AND MOMENTS AT REACTIONS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
11	5.94	10.22	-8.33	-13.70	9.34	-0.70
12	-17.47	10.09	14.89	0.32	0.57	-0.01
13	-17.47	-10.09	14.89	-0.32	0.56	0.01
14	23619.54	10.36	7.65	0.30	13505.05	7275.90
15	23619.54	-10.36	7.65	-0.30	13505.04	-7275.89
16	4.91	-0.00	52528.01	0.00	4.77	0.00
17	2.62	-4.40	52527.94	3.85	2.73	-0.05
18	2.62	4.40	52527.94	-3.85	2.74	0.05
19	-0.00	0.00	52532.65	0.00	0.00	0.00
20	0.00	0.00	52532.65	0.00	0.00	0.00
21	-95054.51	0.00	25792.35	0.00	71457.97	15.64
22	-95054.50	0.00	25792.35	0.00	71457.98	-15.64

(c)

Figure 12. (Concluded)



SPACE FRAME

RIGID JOINTS

PROPERTIES OF MEMBERS

MBRS	A_x	I_x	I_y	I_z
1,16	7.65	0.38	243.0	8.3
2,17	13.24	1.19	583.0	30.5
6,21,34,35,36	9.12	0.50	373.0	11.6
7,12,22,27	18.23	1.97	1327.0	53.1
11,26	10.59	0.59	446.0	22.6
4,5,19,20	16.5	375.0	296.0	305.0
9,10,24,25	22.0	500.0	392.0	413.0
14,15,29,30	28.0	625.0	466.0	600.0
31,32,33	4.86	0.114	105.3	2.99
37,38,39	14.71	1.34	800.6	37.2
3,18	9.0	128.0	106.0	106.0
8,23	12.5	176.0	142.0	158.0
13,28	20.0	272.0	205.0	314.0

$A_y = A_z = A_x(.25)$ for all members.

$E = 30 \times 10^6$ $G = 12 \times 10^6$

Load 700 lbs./ft.

Loaded members: 1, 2, 6, 7, 11, 12, 16, 17, 21, 22, 26, & 27

Figure 13. Computer Solution of A Bent of a Multi-Story Frame

FORCES IN THE MEMBERS

M	NN	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
1	1	-2432.74	-0.00	4577.01	-0.00	235992.12	0.46
1	2	-2432.74	-0.00	9414.89	-0.00	816539.09	0.21
2	2	-6257.93	-0.00	15052.26	0.00	1201589.48	0.59
2	3	-6257.93	-0.00	12931.62	0.00	692639.48	0.80
3	1	-4576.84	0.00	2376.40	-0.17	235959.71	-0.19
3	4	-4576.84	0.00	-2376.40	-0.17	-191793.18	-0.20
4	2	-24467.17	0.00	3775.48	-0.36	384957.92	-0.48
4	5	-24467.17	0.00	-3775.48	-0.36	-294628.23	-0.36
5	3	-12931.79	0.00	-6258.97	-0.40	-692730.98	-0.36
5	6	-12931.79	0.00	6258.97	-0.40	433882.25	-0.24
6	4	318.18	-0.01	5718.81	-0.00	371987.22	0.82
6	5	318.18	-0.01	8273.10	-0.00	678504.16	0.46
7	5	1473.66	-0.01	14705.07	0.00	1183312.86	1.32
7	6	1473.66	-0.01	13278.81	0.00	841013.72	1.68
8	4	-10295.47	0.00	1997.87	-0.05	180151.98	-0.30
8	7	-10295.47	0.00	-1997.87	-0.05	-179463.98	-0.35
9	5	-47445.36	0.01	2558.95	-0.60	210073.46	-1.11
9	8	-47445.36	0.01	-2558.95	-0.60	-250538.92	-0.85
10	6	-26210.79	0.01	-4786.54	-0.73	-407237.71	-0.91
10	9	-26210.79	0.01	4786.54	-0.73	454341.73	-0.73
11	7	1463.86	-0.01	4983.14	-0.00	294067.80	1.82
11	8	1463.86	-0.01	9008.77	-0.00	777144.94	1.19
12	8	3470.56	-0.01	14780.08	0.00	1148558.66	1.68
12	9	3470.56	-0.01	13203.80	0.00	770255.30	2.07
13	7	-15278.56	0.00	477.42	-0.00	114580.46	-0.57
13	10	-15278.56	0.00	-477.42	-0.00	-0.00	0.00
14	8	-71234.18	0.02	503.45	-0.00	120827.54	-4.77
14	11	-71234.18	0.02	-503.45	-0.00	-0.00	0.00
15	9	-39414.62	0.02	-1316.45	-0.00	-315948.60	-3.64
15	12	-39414.62	0.02	1316.45	-0.00	-0.00	0.00
16	13	-2432.75	0.00	4577.01	-0.00	235991.99	-0.24
16	14	-2432.75	0.00	9414.90	-0.00	816539.26	-0.19
17	14	-6257.94	0.00	15052.26	0.00	1201589.09	0.04
17	15	-6257.94	0.00	12931.62	0.00	692639.55	-0.12
18	13	-4576.84	0.00	2376.40	0.14	235959.57	-0.22
18	16	-4576.84	0.00	-2376.40	0.14	-191793.01	-0.23
19	14	-24467.17	0.00	3775.48	0.09	384957.34	-0.48
19	17	-24467.17	0.00	-3775.48	0.09	-294627.67	-0.36
20	15	-12931.79	0.00	-6258.97	0.09	-692731.07	-0.34
20	18	-12931.79	0.00	6258.97	0.09	433882.60	-0.22
21	16	318.17	0.00	5718.81	-0.00	371987.07	-0.43
21	17	318.17	0.00	8273.10	-0.00	678504.47	-0.31
22	17	1473.65	0.00	14705.07	0.00	1183312.50	-0.01
22	18	1473.65	0.00	13278.81	0.00	841014.16	-0.30
23	16	-10295.45	0.00	1997.87	0.16	180151.99	-0.35
23	19	-10295.45	0.00	-1997.87	0.16	-179463.73	-0.41
24	17	-47445.33	0.01	2558.95	0.15	210073.33	-1.10
24	20	-47445.33	0.01	-2558.95	0.15	-250538.49	-0.84
25	18	-26210.76	0.01	-4786.55	0.21	-407237.81	-0.88
25	21	-26210.76	0.01	4786.55	0.21	454342.39	-0.68
26	19	1463.85	0.01	4983.14	-0.00	294068.54	-0.93
26	20	1463.85	0.01	9008.76	-0.00	777144.36	-0.66
27	20	3470.55	0.00	14780.08	0.00	1148559.67	-0.13
27	21	3470.55	0.00	13203.79	0.00	770254.10	-0.43
28	19	-15278.53	0.00	477.42	0.00	114581.45	-0.61
28	22	-15278.53	0.00	-477.42	0.00	-0.00	0.00
29	20	-71234.06	0.02	503.46	-0.00	120829.56	-4.76
29	23	-71234.06	0.02	-503.46	-0.00	-0.00	0.00
30	21	-39414.52	0.02	-1316.45	0.00	-315946.75	-3.61
30	24	-39414.52	0.02	1316.45	0.00	-0.00	0.00
31	1	0.00	-0.00	0.00	-0.00	0.23	0.22
31	13	0.00	-0.00	-0.00	-0.00	-0.22	0.10
32	2	0.01	-0.00	0.00	-0.00	0.45	0.07
32	14	0.01	-0.00	-0.00	-0.00	-0.45	0.06
33	3	0.00	-0.00	0.00	-0.00	0.33	0.11
33	15	0.00	-0.00	-0.00	-0.00	-0.34	0.03
34	4	-0.00	-0.01	0.00	-0.00	0.58	0.98
34	16	-0.00	-0.01	-0.00	-0.00	-0.56	0.41
35	5	0.01	-0.00	0.01	-0.00	1.42	0.37
35	17	0.01	-0.00	-0.01	-0.00	-1.43	0.27
36	6	0.01	-0.00	0.01	-0.00	1.12	0.56
36	18	0.01	-0.00	-0.01	-0.00	-1.12	0.18
37	7	-0.01	-0.02	0.01	0.00	1.11	3.08
37	19	-0.01	-0.02	-0.01	0.00	-1.01	1.09
38	8	0.01	-0.01	0.05	0.00	5.57	1.47
38	20	0.01	-0.01	-0.05	0.00	-5.58	0.94
39	9	0.01	-0.01	0.04	0.00	4.27	2.14
39	21	0.01	-0.01	-0.04	0.00	-4.31	0.65

Figure 13. (Continued)

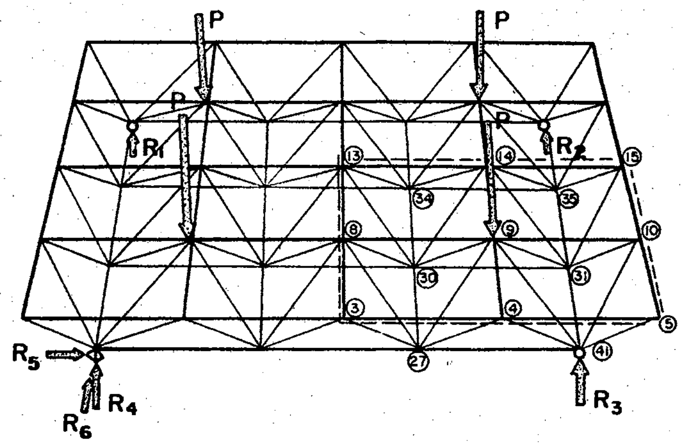
JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	-0.08753962	0.00000402	-0.01410451	-0.00000000	0.00225973	0.00000015
2	-0.09006716	0.00001223	-0.04218948	-0.00000001	0.00115976	0.00000003
3	-0.09753935	0.00000939	-0.02311222	-0.00000000	-0.00356722	0.00000008
4	-0.01860529	0.00000315	-0.01105326	-0.00000000	0.00100974	0.00000017
5	-0.01832557	0.00001080	-0.03329228	-0.00000001	0.00027127	0.00000004
6	-0.01703004	0.00000831	-0.01840973	-0.00000001	-0.00102115	0.00000010
7	0.00566225	0.00000228	-0.00611142	-0.00000000	0.00099668	0.00000017
8	0.00676706	0.00000881	-0.02035263	-0.00000001	0.00056521	0.00000006
9	0.00981810	0.00000672	-0.01126133	-0.00000001	-0.00136331	0.00000012
10	0.00000000	0.00000000	0.00000000	-0.00000001	-0.00046295	0.00000017
11	0.00000000	0.00000000	0.00000000	-0.00000005	-0.00024031	0.00000006
12	0.00000000	0.00000000	0.00000000	-0.00000004	0.00074302	0.00000012
13	-0.08754188	0.00000402	-0.01410449	-0.00000000	0.00225973	-0.00000001
14	-0.09006942	0.00001222	-0.04218945	-0.00000001	0.00115976	0.00000001
15	-0.09754161	0.00000939	-0.02311218	-0.00000000	-0.00356722	-0.00000003
16	-0.01860792	0.00000315	-0.01105324	-0.00000000	0.00100974	-0.00000003
17	-0.01832820	0.00001080	-0.03329225	-0.00000001	0.00027127	0.00000001
18	-0.01703268	0.00000830	-0.01840969	-0.00000001	-0.00102115	-0.00000003
19	0.00565964	0.00000228	-0.00611141	-0.00000000	0.00099667	-0.00000004
20	0.00676444	0.00000880	-0.02035260	-0.00000001	0.00056521	0.00000000
21	0.00981548	0.00000672	-0.01126130	-0.00000001	-0.00136331	-0.00000004
22	0.00000000	0.00000000	0.00000000	-0.00000001	-0.00046296	-0.00000004
23	0.00000000	0.00000000	0.00000000	-0.00000005	-0.00024033	0.00000000
24	0.00000000	0.00000000	0.00000000	-0.00000004	0.00074300	-0.00000004

FORCES AND MOMENTS AT REACTIONS

JOINT	P(X)	P(Y)	P(Z)	M(X)	M(Y)	M(Z)
10	-477.65	0.00	-15278.56	0.00	-37.48	-0.00
11	-503.91	0.02	-71234.18	0.01	-73.49	-0.00
12	1316.01	0.02	-39414.62	0.00	-70.08	-0.00
22	-477.66	0.00	-15278.53	0.00	-37.48	0.00
23	-503.92	0.02	-71234.06	0.01	-73.50	-0.00
24	1316.01	0.02	-39414.52	0.00	-70.09	0.00

Figure 13. (Concluded)



SPACE FRAME

Non-Rigid Joints

PROPERTIES OF MEMBERS

Area of all members = 10.59 sq. in.

Length of all members = 200 inches

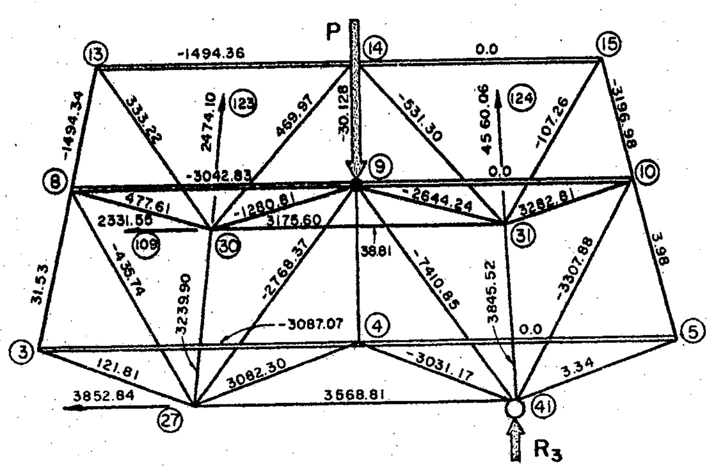
$E = 29 \times 10^6$ $G = 12 \times 10^6$

Loads: P = 10Kips

See blow up below for forces in the members

$R_1 = 10074.64^\#$ $R_2 = 9991.65^\#$ $R_3 = 10074.64^\#$

$R_4 = 9925.46^\#$ $R_5 = -170.70^\#$ $R_6 = 170.70^\#$



(a)

Figure 14. Computer Solution of a Diagrid

JOINT DEFLECTIONS

JOINT	DEF(X)	DEF(Y)	DEF(Z)	ROT(X)	ROT(Y)	ROT(Z)
1	0.00464838	0.00464838	0.00657405	0.00000000	0.00000000	0.00000000
2	0.00466789	0.00488302	-0.00268875	0.00000000	0.00000000	0.00000000
3	0.00269540	0.00339716	-0.00943567	0.00000000	0.00000000	0.00000000
4	0.00068693	0.00400405	-0.00269183	0.00000000	0.00000000	0.00000000
5	0.00068693	0.00405767	0.00843297	0.00000000	0.00000000	0.00000000
6	0.00488303	0.00466789	-0.00268875	0.00000000	0.00000000	0.00000000
7	0.00489825	0.00489825	-0.01450964	0.00000000	0.00000000	0.00000000
8	0.00289469	0.00341768	-0.01577714	0.00000000	0.00000000	0.00000000
9	0.00091497	0.00402930	-0.01506741	0.00000000	0.00000000	0.00000000
10	0.00091497	0.00408361	-0.00335833	0.00000000	0.00000000	0.00000000
11	0.00339716	0.00269540	-0.00943567	0.00000000	0.00000000	0.00000000
12	0.00341768	0.00289469	-0.01577714	0.00000000	0.00000000	0.00000000
13	0.00244264	0.00244264	-0.01854287	0.00000000	0.00000000	0.00000000
14	0.00147038	0.00206947	-0.01633108	0.00000000	0.00000000	0.00000000
15	0.00147038	0.00200359	-C.01046271	0.00000000	0.00000000	0.00000000
16	0.00400405	0.00068693	-0.00269183	0.00000000	0.00000000	0.00000000
17	0.00402930	0.00091497	-0.01506741	0.00000000	0.00000000	0.00000000
18	0.00206947	0.00147038	-0.01633108	0.00000000	0.00000000	0.00000000
19	0.00010206	0.00010206	-0.01529396	0.00000000	0.00000000	0.00000000
20	0.00010206	-0.00010370	-0.00309218	0.00000000	0.00000000	0.00000000
21	0.00405767	0.00068693	0.00843297	0.00000000	0.00000000	0.00000000
22	0.00408361	0.00091497	-0.00335833	0.00000000	0.00000000	0.00000000
23	0.00200359	0.00147038	-0.01046271	0.00000000	0.00000000	0.00000000
24	-0.00010370	0.00010206	-0.00309218	0.00000000	0.00000000	0.00000000
25	-0.00010370	-0.00010370	0.00851523	0.00000000	0.00000000	0.00000000
26	0.00172335	-0.00028518	-0.01129938	0.00000000	0.00000000	0.00000000
27	0.00423008	-0.00081341	-0.01143997	0.00000000	0.00000000	0.00000000
28	-0.00028518	0.00172335	-0.01129938	0.00000000	0.00000000	0.00000000
29	0.00172792	0.00172792	-0.01720483	0.00000000	0.00000000	0.00000000
30	0.00324487	0.00129453	-0.01747032	0.00000000	0.00000000	0.00000000
31	0.00531097	0.00050350	-0.01201906	0.00000000	0.00000000	0.00000000
32	-0.00081341	0.00423008	-0.01143997	0.00000000	0.00000000	0.00000000
33	0.00129453	0.00324487	-0.01747032	0.00000000	0.00000000	0.00000000
34	0.00290423	0.00290423	-0.01766411	0.00000000	0.00000000	0.00000000
35	0.00502206	0.00347036	-0.01201532	0.00000000	0.00000000	0.00000000
36	0.00050350	0.00531097	-0.01201906	0.00000000	0.00000000	0.00000000
37	0.00347036	0.00502206	-0.01201532	0.00000000	0.00000000	0.00000000
38	0.00594359	0.00594359	0.00000000	0.00000000	0.00000000	0.00000000
39	-0.00199847	0.00655201	0.00000000	0.00000000	0.00000000	0.00000000
40	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
41	0.00655201	-0.00199847	0.00000000	0.00000000	0.00000000	0.00000000

(b)

Figure 14. (Concluded)

CHAPTER V

SUMMARY AND CONCLUSIONS

A variety of computer programs for solution of structural problems are in use today. The majority of these programs are for a particular solution to one problem. As stated earlier, this relaxation method can be used to compute solutions for reticulated structures with prismatic members and the work simplified with computer aids.

The major problem encountered in this study was the selection of a routine to give the proper inverse for matrices with zeroes along the diagonal. This presented no problem if the elements of the row and column containing the common zero element along the diagonal were all zeroes. When this occurs the matrix is reduced (the columns and rows containing the zero elements are eliminated). However, under certain conditions it was found that the elements on the diagonal contained zeroes, but the elements of the rows and columns containing the zero element along the diagonal were not all zeroes. This presented a special case for finding the inverse of a matrix with zero elements along the diagonal, which could not be reduced to a smaller matrix.

This problem was solved by a special routine that rearranged the elements of the matrix according to their absolute value, interchanged rows and columns, and took the inverse of the matrix. It then replaced the rows, columns, and elements to their proper location. Although the solution is not a true inverse, it satisfies the requirements of this study.

With the inversion problem solved, many example problems were solved and compared with other known solutions. By using a variety of structures, the versatility of the program was demonstrated.

From the results obtained in this study, it is concluded that the relaxation method is practical for solutions of any reticulated structure with prismatic members. The rate of convergence is very rapid in the early cycles of the relaxation process. However, it decreases very rapidly as the member forces approach their true values.

The major advantage of this method is its adaptation to computer solutions, plus the ability to solve a variety of structural problems using the same computer program.

The author would now like to propose two areas for the extension of this study. The first area projected is an investigation for applying this relaxation method to obtain solutions to plate and shell structures. It is also proposed that the use of magnetic tapes in the present computer program be eliminated by placing computed values that are used in the second part of the program in core memory.

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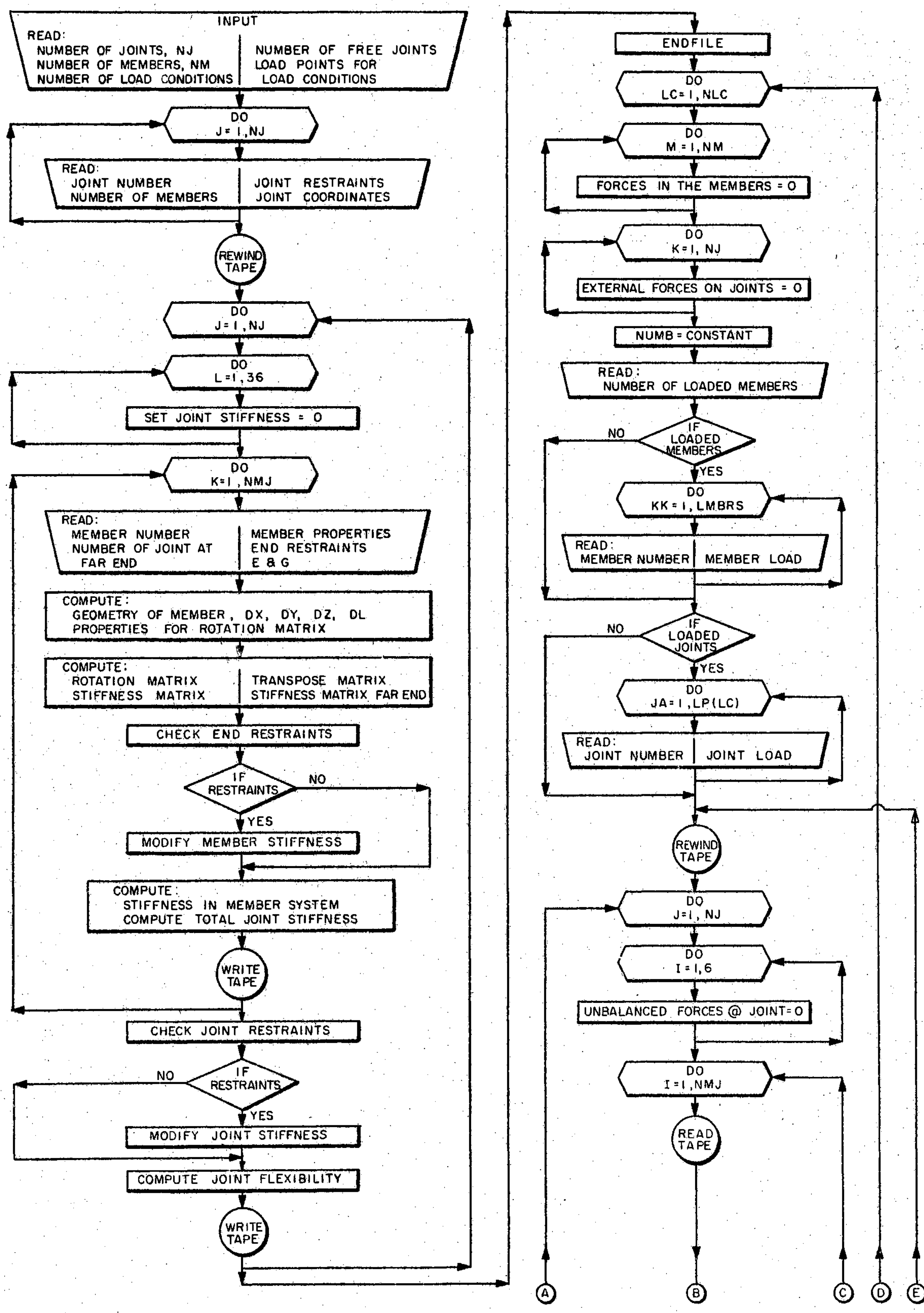
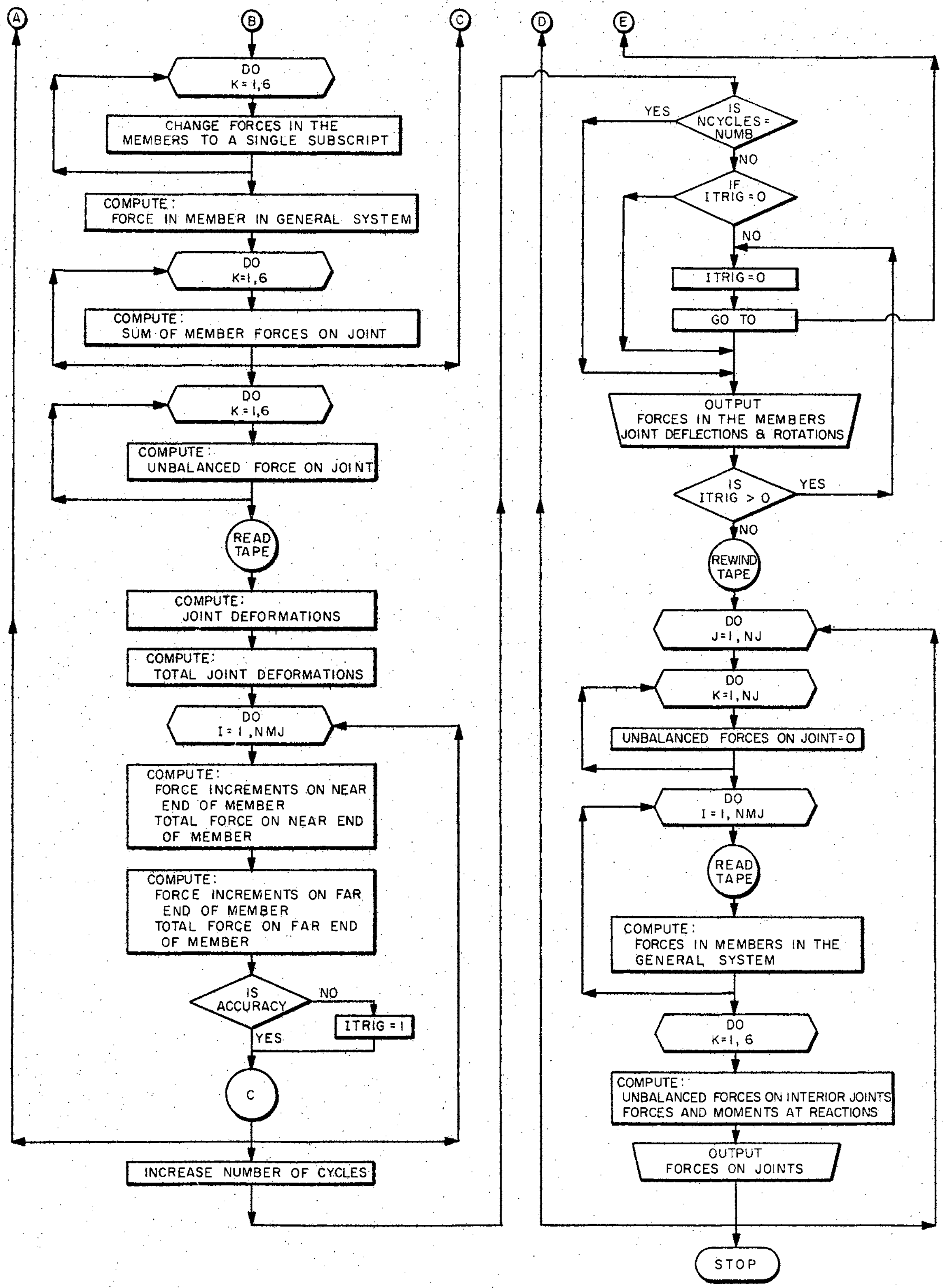
APPENDIX

COMPUTER PROGRAM FLOW DIAGRAM


INPUT
READ:
NUMBER OF JOINTS, NJ
NUMBER OF MEMBERS, NM
NUMBER OF LOAD CONDITIONS
NUMBER OF FREE JOINTS
LOAD POINTS FOR LOAD CONDITIONS
DO J = 1, NJ
READ:
JOINT NUMBER
NUMBER OF MEMBERS
JOINT RESTRAINTS
JOINT COORDINATES
REWIND TAPE
DO J = 1, NJ
DO L = 1, 36
SET JOINT STIFFNESS = 0
DO K = 1, NMJ
READ:
MEMBER NUMBER
NUMBER OF JOINT AT FAR END
MEMBER PROPERTIES
END RESTRAINTS
E & G
COMPUTE:
GEOMETRY OF MEMBER, DX, DY, DZ, DL
PROPERTIES FOR ROTATION MATRIX
COMPUTE:
ROTATION MATRIX
STIFFNESS MATRIX
TRANSPOSE MATRIX
STIFFNESS MATRIX FAR END
CHECK END RESTRAINTS
IF RESTRAINTS
NO
YES
MODIFY MEMBER STIFFNESS
COMPUTE:
STIFFNESS IN MEMBER SYSTEM
COMPUTE TOTAL JOINT STIFFNESS
WRITE TAPE
CHECK JOINT RESTRAINTS
IF RESTRAINTS
NO
YES
MODIFY JOINT STIFFNESS
COMPUTE JOINT FLEXIBILITY
WRITE TAPE
ENDFILE
DO LC = 1, NLC
DO M = 1, NM
FORCES IN THE MEMBERS = 0
DO K = 1, NJ
EXTERNAL FORCES ON JOINTS = 0
NUMB = CONSTANT
READ:
NUMBER OF LOADED MEMBERS
IF LOADED MEMBERS
NO
YES
DO KK = 1, LMBRS
READ:
MEMBER NUMBER
MEMBER LOAD
IF LOADED JOINTS
NO
YES
DO JA = 1, LP(LC)
READ:
JOINT NUMBER
JOINT LOAD
REWIND TAPE
DO J = 1, NJ
DO I = 1, 6
UNBALANCED FORCES @ JOINT = 0
DO I = 1, NMJ
READ TAPE
A
B
C
D
E


A
B
C
D
E
DO
K=1,6
CHANGE FORCES IN THE MEMBERS TO A SINGLE SUBSCRIPT
COMPUTE: FORCE IN MEMBER IN GENERAL SYSTEM
DO
K=1,6
COMPUTE: SUM OF MEMBER FORCES ON JOINT
DO
K=1,6
COMPUTE: UNBALANCED FORCE ON JOINT
READ TAPE
COMPUTE: JOINT DEFORMATIONS
COMPUTE: TOTAL JOINT DEFORMATIONS
DO
I=1,NMJ
COMPUTE: FORCE INCREMENTS ON NEAR END OF MEMBER TOTAL FORCE ON NEAR END OF MEMBER
COMPUTE: FORCE INCREMENTS ON FAR END OF MEMBER TOTAL FORCE ON FAR END OF MEMBER
IS ACCURACY
NO
ITRIG=1
YES
C
INCREASE NUMBER OF CYCLES
IS NCYCLES=NUMB
YES
NO
IF ITRIG=0
NO
ITRIG=0
GO TO
OUTPUT FORCES IN THE MEMBERS JOINT DEFLECTIONS & ROTATIONS
IS ITRIG > 0
YES
NO
REWIND TAPE
DO
J=1,NJ
DO
K=1,NJ
UNBALANCED FORCES ON JOINT=0
DO
I=1,NMJ
READ TAPE
COMPUTE: FORCES IN MEMBERS IN THE GENERAL SYSTEM
DO
K=1,6
COMPUTE: UNBALANCED FORCES ON INTERIOR JOINTS FORCES AND MOMENTS AT REACTIONS
OUTPUT FORCES ON JOINTS
STOP

VITA

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Professional Organizations: Phi Kappa Phi (National Honorary Society).