DIFFERENTIAL SETTLEMENT ANALYSIS OF BUILDING FRAMES BY DIGITAL COMPUTER METHODS

By

JOHN ALAN LAMB

Bachelor of Architectural Engineering

Oklahoma State University

Stillwater, Oklahoma

1966

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF ARCHITECTURAL ENGINEERING
May, 1968

DIFFERENTIAL SETTLEMENT ANALYSIS OF

BUILDING FRAMES BY

DIGITAL COMPUTER METHODS

Thesis Approved:

R. E. Means
Thesis Adviser

Louis O. Bass

N Durham
Dean of the Graduate College

PREFACE

The general scope of this thesis is not to develop new theoretical methods of soils analysis, but to apply computer methods to existing theory. The objective of the study is to: (1) investigate the applicability and feasibility of computer solutions for differential settlement analysis, (2) develop computer programs for differential settlement analysis, (3) organize steps of procedure for the study of differential settlement. An excellent starting point for an understanding of the material contained in this study is the text Soil Mechanics and Foundations by J. V. Parcher and R. E. Means. Methods of analysis for the differential settlement of foundations that are presented in the text are the basis for the computer solutions that are presented in this study.

I would especially like to express my appreciation to Professor R. E. Means for his invaluable advice and encouragement. As an adviser, his technical advice coupled with great patience and understanding did much to lessen the difficulty of preparing this study. I would also like to thank the other members of my committee for their assistance: Professor L. O. Bass and Dr. J. V. Parcher.

I would also like to acknowledge my appreciation to the National Science Foundation for their financial support. For his aid in gaining financial support and his interest, I would also like to thank Professor F. C. Salmon.

In addition, I would like to thank Velda Davis for her typing excellence and advice.

TABLE OF CONTENTS

Chapter | Page

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

THEORY AND METHOD OF FOOTING SETTLEMENT ANALYSIS

Background Information

Structures built on compressible clay soils are likely to experience unequal foundation settlements. Unequal settlements of footing groups very often result in damage to inflexible structural and architectural materials. On normally consolidated clay soils, tolerable differential settlement is more often the governing factor in the design of footings than is adequate bearing capacity. The problem for the engineer is to predict differential settlements accurately and to design the building foundations to limit the magnitude of differential settlement.

There is no agreed upon code limits for the magnitude of allowable differential settlements. Some proposed design limits of differential settlements are presented by Sowers (1). Some representative examples of allowable differential settlements are listed in Table I. In any case, the limits of maximum differential settlement should be the maximum difference in elevation between footings that would not result in cracking or other damage to brittle building materials. The establishment of code requirements for differential settlement may only be of academic interest, because of the limited range of accuracy of a soil settlement analysis.

TABLE I

RECOMMENDED ALLOWABLE DIFFERENTIAL SETTLEMENTS

High Continuous Brick Walls	0.0005 L to 0.001 L
One Story Brick Mill Building, Wall Cracking	0.001 L to 0.002 L
Plaster Cracking	0.001 L
Reinforced Concrete Building Frame	0.0025 L to 0.004 L
Reinforced Concrete Building, Curtain Walls	0.003 L
Steel Frame, Continuous	0.002 L
Simple Steel Frame	0.005 L

Methods of Limiting Differential Settlements

For normally consolidated clay soil, differential settlement may be controlled by one of four methods (2). One method is to float the building in the soil by removal of an amount of overburden equal to the weight of the building. This is normally done by excavation for a basement. If the weight of the overburden removed were exactly equal to the weight of the building at every point on the site, there would be no change in stress on the soil beneath the building and, hence, no settlement. In actual practice a precise balance of weights would be impractical. Under these conditions, the magnitude of change in stress would be less. Therefore, the net settlement would be less, and the differential settlements would be less severe. Differential settlement can also result from the effect of a pressure heave. As the site is excavated, the unloaded soil may tend to swell at the center of the excavation. As the building load is placed on the clay soil and the soil is reloaded, some differential settlement will occur due to the recompression of the expanded soil. While the method of floating the foundation may not completely eliminate settlement, it is an effective method to limit the magnitude of settlement and to control differential settlement.

Another approach to limit differential settlement is to adjust the size of the footings for equal settlement. The settlement of a footing is influenced by both the contact pressure and the width of the footing. Increased contact pressure of a footing would cause increased settlement due to the application of greater load. Increased width of a footing with a constant contact pressure also tends to increase settlement, because the stress due to the load is carried to deeper soil layers. The adjustment of an individual footing size with a constant column load must take into account both opposite effects. When there are adjacent footings that cause overlapping pressure, the settlement of a single footing is also influenced by those footings adjacent to it. Because of the many variables, the design of a set of footings for equal settlement would be a trial and error process. It is, however, theoretically possible to design a group of footings that will have settled an equal distance at some point in time by adjusting the contact pressures. Consideration must be given to the rate of settlement. Footings of different dimensions would settle at differing rates. Therefore, it might be possible to design for equal settlement after a time period of five years. Yet there would be unequal settlement before and after that time. The approach would be difficult to apply with precision, but approximate adjustments of footing dimensions is often used to limit differential settlements. When it is known that the interior footings will settle most, the dimensions of the interior footings can be increased to limit the expected differential settlement.

A third method is to distribute the total building load over the site by means of a system of cantilevers that apply heavier loads on the exterior footings and lesser loads on the interior footings. The theory of the method is to stress the soil equally beneath each part of the structure. The interior of a structure would normally settle the most, because the interior columns for a normal uniform grid spacing would carry greater loads. By adjusting the building structure to act as a cantilever, a greater part of the total load can be carried to the exterior footings. The practical application of the procedure would involve many of the same problems of the second method. The effects of different footing widths and pressure overlaps must be considered. A completely precise balance of soil stresses at each point beneath the building would be impractical.

Another method is to increase the stiffness of the structural frame or slab to limit differential settlements. Increased stiffness of a building structure has the effect of reducing differential settlement. If a structure were infinitely stiff, differential settlements would be zero. As a general rule, the structure of a building reduces unequal settlement, yet conventional settlement analysis of foundations ignores the effect of the stiffness of the structural frame. Settlements of footings are analyzed as though each were independent and not connected structurally. The independent settlement analysis is conservative in that actual differential settlements are less than predicted unless the settlements are large enough to cause failure of the structure. The analysis of settlements considering structural stiffness is discussed in Chapter III, and the effect of structural stiffness is discussed in Chapter IV.

Computation of Settlement

If a confined layer of normally consolidated clay is subjected to additional vertical stress, the material will be compressed. The method for estimating the change in height of a normally consolidated clay layer is presented by Terzaghi (3). The computation of the settlement of a compressible clay layer is based upon a consolidation test of a soil sample, and the development of a void ratio-log pressure curve for a specific soil sample.

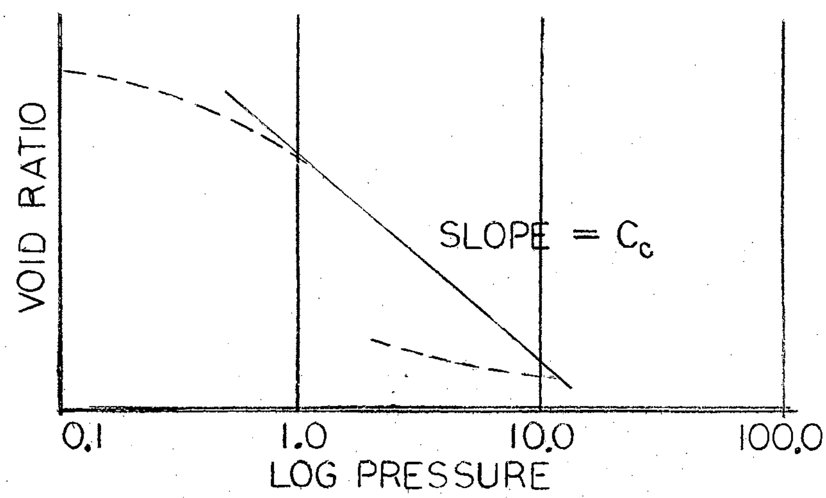


Figure 1. Typical Void Ratio-Log Pressure Curve

As presented by Terzaghi, the slope of the virgin portion of the void ratio-log pressure curve for normally consolidated clay can be approximated by a straight line relationship. The slope of the line is defined as the compression index (C_c). The development of the void ratio-log pressure curve for a soil sample makes it possible to approximate the settlement for a compressible layer as a logarithmic function of applied pressure. The change in height of a compressible clay layer can be evaluated if the average change in stress is known. The change in height of a normally consolidated clay layer of H thickness is given by the relationship:

$$\Delta H = \frac{H \times C_c}{1 + e_1} \operatorname{Log}_{10} \frac{P_1 + \Delta P}{P_1}.$$

Therefore, the settlement of an individual clay layer can be evaluated as a logarithmic function of the compression index which can be determined by laboratory tests on soil samples. The procedure for computing the settlement of a specific point on the surface of a soil mass is to sum the settlements for each of the individual soil layers beneath the point being considered.

As presented by Means (4), an alternate method to evaluate settlements due to small changes in pressure is to approximate the tangent of the void ratio-pressure curve by the secant of the curve. For a limited range of pressure change, the method has the advantage of greater convenience. The method is illustrated by Figure 2.

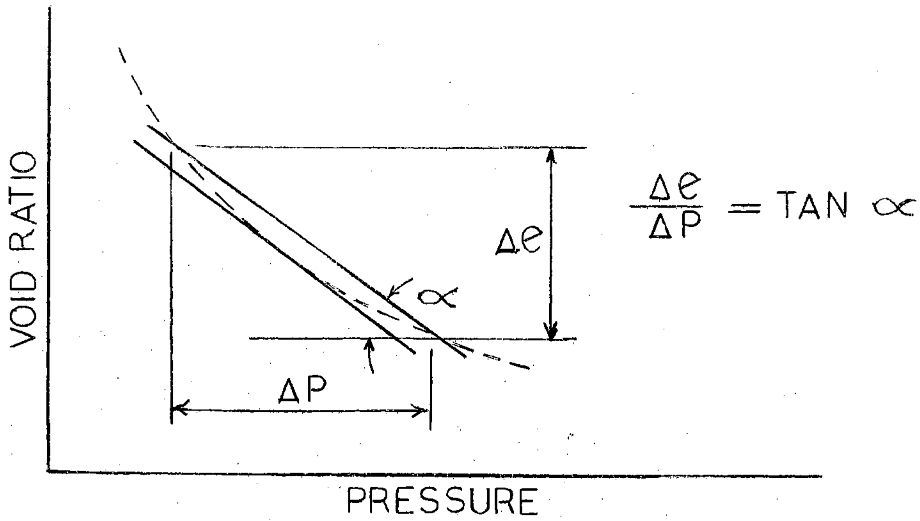


Figure 2. Void Ratio-Pressure Curve

Computation of Pressure Within a Soil Mass

When a footing load is applied to a soil mass, the compressive stress on the soil layers beneath the footing is increased. In order to calculate the change in height of a layer of normally consolidated clay, the average change in stress of the layer must first be computed. The most often used method of computing the stress change within a soil mass due to foundation loads is the method first presented by Boussinesq (5). Considering an isolated point load on the surface of a soil mass, it is possible to compute the change in stress at any point within a compressible soil layer by using the solution of Boussinesq.

$$\Delta\sigma_z = \frac{3Q}{2\pi}\,\frac{Z^3}{(r^2 + Z^2)^{5/2}}.$$

The solution assumes a semi-infinite, homogeneous, and elastic mass. The inaccuracy of these assumptions when considering a soil mass is one of the major factors that limits the accuracy of settlement computations. For some conditions of stratified soils, the solution for pressure change presented by Westergaard (6) would be more accurate.

For a footing pad with a uniform contact pressure, the stress change at a point within a soil mass is influenced by the footing dimensions as well as the total load. When the load is at a distance from the point under consideration, the effect of the distant footing's dimensions is less significant. Within the region directly under the contact area between the footing and the soil mass, the effect of the footing width on the concentration of stress is significant.

As presented by Newmark (7), an integration of the Boussinesq point load formula can be used for a more precise analysis of the stress change within the soil under a loaded contact area. In the form presented by Newmark, the pressure beneath the corner of a rectangular uniformly loaded area is found. By dividing a rectangular footing pad into four areas, the pressure beneath the center of the footing can be determined. See Figure 3. An integration of the Boussinesq formula for a circular uniformly loaded area can be applied for circular footings (8).

The integration of the Boussinesq formula for a

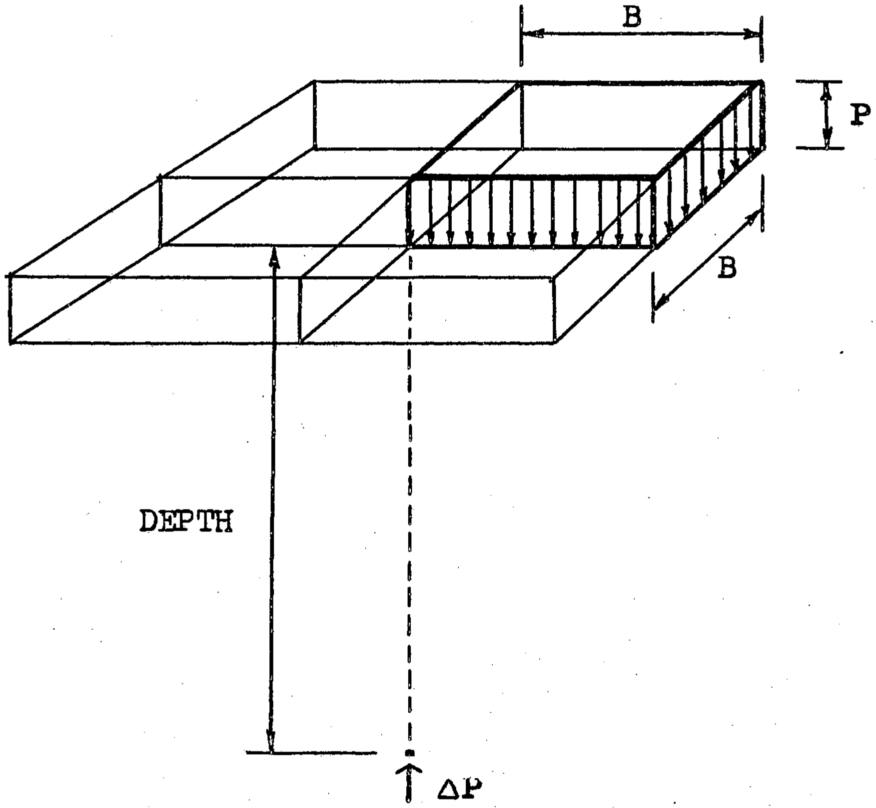


Figure 3. Change of Stress Computed by Integration of Bousinesque Point Load Equation

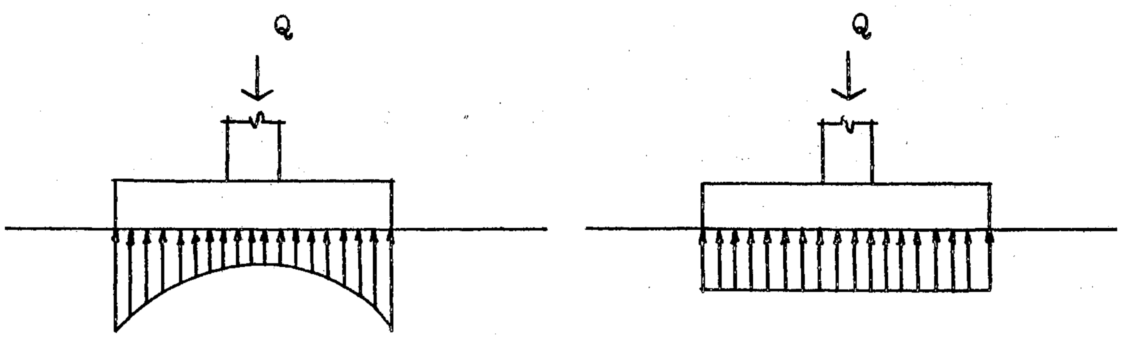


Figure 4. Typical Plots of Contact Pressure Distribution of a Stiff Footing.

footing pad assumes that the contact pressure between the footing and the soil is uniform. The assumption of a uniform contact pressure between the soil and the footing would be correct if the footing pad were completely flexible and uniformly loaded. Because of the structural stiffness of the pad, the contact pressure between the soil and the footing would be non-uniform. The actual and assumed footing contact pressures for clay are plotted in Figure 4. Because of the difficulty of precise theoretical development and because the error created is slight, the contact pressure of a footing pad is commonly assumed as uniform.

To compute the pressure at the midpoint of a compressible clay layer as caused by a group of footings, the effect of pressure overlap must be considered. Refer to Figure 5. The change of stress beneath footing B due to the loads of footings A and C can be computed with reasonable accuracy by using Boussinesq's point load formula directly. To compute the change of stress beneath footing B due to the load of footing B, the integration presented by Newmark would be more precise. By summing the pressure changes due to each cause, the net change in stress on an incremental element of soil beneath footing B can be determined. The settlement contribution of that layer for footing B could then be computed by the method presented in Section 1.3.

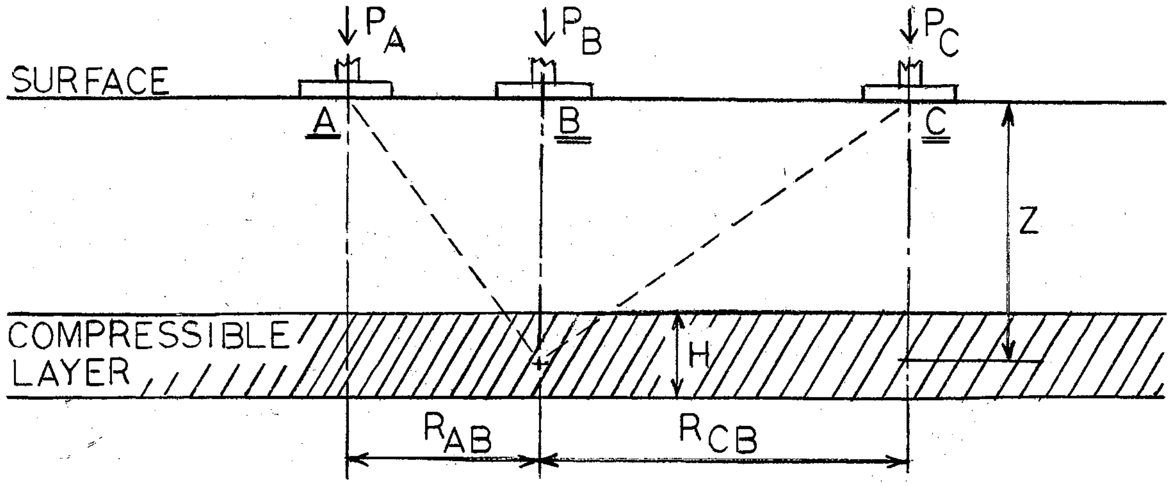


Figure 5. Effect of Pressure Overlap on Footing Settlement

Computation of Settlement for Footing Groups

A normal building foundation is made up of many footing pads. The engineer is most often interested in the differences in settlement between adjacent footings. The general procedure of an analysis for the settlement of a footing is to evaluate the change in height of each soil layer beneath the footing. The change in height of each soil layer at a position beneath the footing being considered is influenced by the stress change due to adjacent footings. The settlement of footing A is the sum of the settlements at each soil layer beneath that footing. The methods previously developed can be used to evaluate the settlement, but the volume of computation requires the organization of the work (9). The general steps of the procedure to compute the settlement of a footing group are as follows:

1. Statement of problem. The solution requires that the spacings, loads, and sizes of the footings be known. From laboratory tests on soil samples, the compression index, density, and void ratio of each soil type must be known.
2. Divide the soil profile into layers. For the purpose of calculations, the soil profile below the structure is arbitrarily divided into a number of horizontal strata. For convenience, the divisions should coincide with any natural divisions such as change in soil types, water table, and the footing base. The pressure beneath a footing changes more rapidly near the footing base. Therefore, strata divisions should be smaller near the footing base and thicker at greater depths. Sowers (10) presents a table of maximum stratum thickness for settlement analysis. See Table II. The maximum stratum thickness is determined by the footing width and the depth of the stratum. A very fine division of strata would not be significantly more accurate, and would require more cycles of computation that would increase the time and the cost of the analysis. As a minimum, all compressible layers at a depth less than the

building width should be considered.

3. Compute changes of stress. To evaluate the settlement of a footing, the average change in stress beneath that footing at each soil layer due to each footing load must be evaluated. The change in stress at the mid-point of the soil layer is assumed to be the average pressure change. At each midpoint, the stress contribution of each footing load must be computed and summed for the net stress change that is used for the settlement computation. The process must be repeated for each individual footing. The methods presented in Section 1.4 can be used. When computing the change in stress caused by an adjacent footing, the Boussinesq point load formulation can be used directly. For the effect of a footing load directly over the point being considered, the integration presented by Newmark would be used for better accuracy.
4. Calculate change in height of soil layers. To evaluate settlements, the change in height of each soil layer beneath each footing must be calculated individually. The sum of the changes in stress at each required point, as found in step 3, is used for this calculation. The method presented in Section 1.3

can be used to evaluate the change in height at each point.

5. Sum the settlements of each footing. The net settlements of each footing can be determined by adding the contribution to settlement of each soil layer.
6. Compare individual footing settlements for differential settlements. The settlements of adjacent footings can be compared numerically to determine the differential settlements. The magnitude of differential settlements indicate the amount of strain that can be expected to be placed upon building materials.

As can be readily seen, the process of calculating differential settlements when applied to a normal building would require a large volume of computation. Much of the computation could be minimized by the use of graphs. As is explained in Chapter II, the repetitive nature of the calculations makes the solution ideally suited to digital computer operations.

The degree of accuracy of the calculations when compared to actual soil behavior is very limited. Even for ideal conditions, accuracy of ten to fifty per cent would be in the expected range (11). The major reasons for the limited accuracy is that soil is never uniform or mathematically consistent in behavior as the settlement calculation must assume. The analysis also completely neglects the effect of the structure as it restrains and limits differential settlement. In this respect, the analysis is conservative, because the stiffness of the structure tends to reduce differential settlement. The effects of the stiffness of the structure on differential settlement is discussed in Chapter III and IV.

TABLE II

MAXIMUM STRATUM THICKNESS FOR SETTLEMENT ANALYSIS

Depth to Middle of Stratum in Terms of Footing Width	Maximum Thickness of Stratum or Substratum in Terms of Footing Width
1/2 B (or less)	1/3 B
B	1/2 B
2 B	2/3 B
3 B	B

FOOTNOTES

[1] G. F. Sowers, "Shallow Foundations," _Foundation Engineering_, ed. G. A. Leonards (New York, 1962), p. 597.

[2] J. V. Parcher and R. E. Means, _Soil Mechanics and Foundations_ (Columbus, 1968), p. 229.

[3] K. Terzagi , _Theoretical Soil Mechanics_ (New York, 1943), p. 290.

[4] Parcher and Means, p. 260.

[5] J. Boussinesq, _Application des Potentiels á l'Etude de Mouvement des Solides Elastiques_ (Paris, 1968).

[6] H. M. Westergaard, "A Problem of Elasticity Suggested by a Problem in Soil Mechanics: A Soft Material Reinforced by Numerous Strong Horizontal Sheets," _Mechanics of Solids: S. Timoshenko Sixtieth Anniversary Volume_, ed. S. Timoshenko (New York, 1938).

[7] N. M. Newmark, _Simplified Computation of Vertical Pressure in Elastic Foundations_ (University of Illinois Engineering Experiment Station, Circular No. 24, [Urbana, 1935]).

[8] Parcher and Means, p. 209.

[9] Ibid., p. 232.

[10] Sowers, p. 572.

[11] Ibid., p. 575.

CHAPTER II

COMPUTER APPLICATION FOR INDEPENDENT SETTLEMENT ANALYSIS

Explanation of Computer Method

Because of the large amount of computation, the only feasible method of making an independent settlemtn analysis for each footing of a structure is by application of computer methods. The high speed of a digital computer greatly reduces the time and cost of a complete analysis. In general, a computer solution allows greater flexibility in the employment of engineering theory. Because the amount or difficulty of the computation is much less important, a computer solution is more free to pursue the most applicable theory, regardless of computational difficulty. Short-cut methods need not be used at the expense of precision.

The workings of the program follow the same procedure as outlined in Section 1.5. The program was written in the Fortran IV language, and an IBM 7040 digital computer was used for the computation. A general flow diagram of the logic followed by the computer solution is shown in Figure 6. In Appendix A is a more complete flow diagram

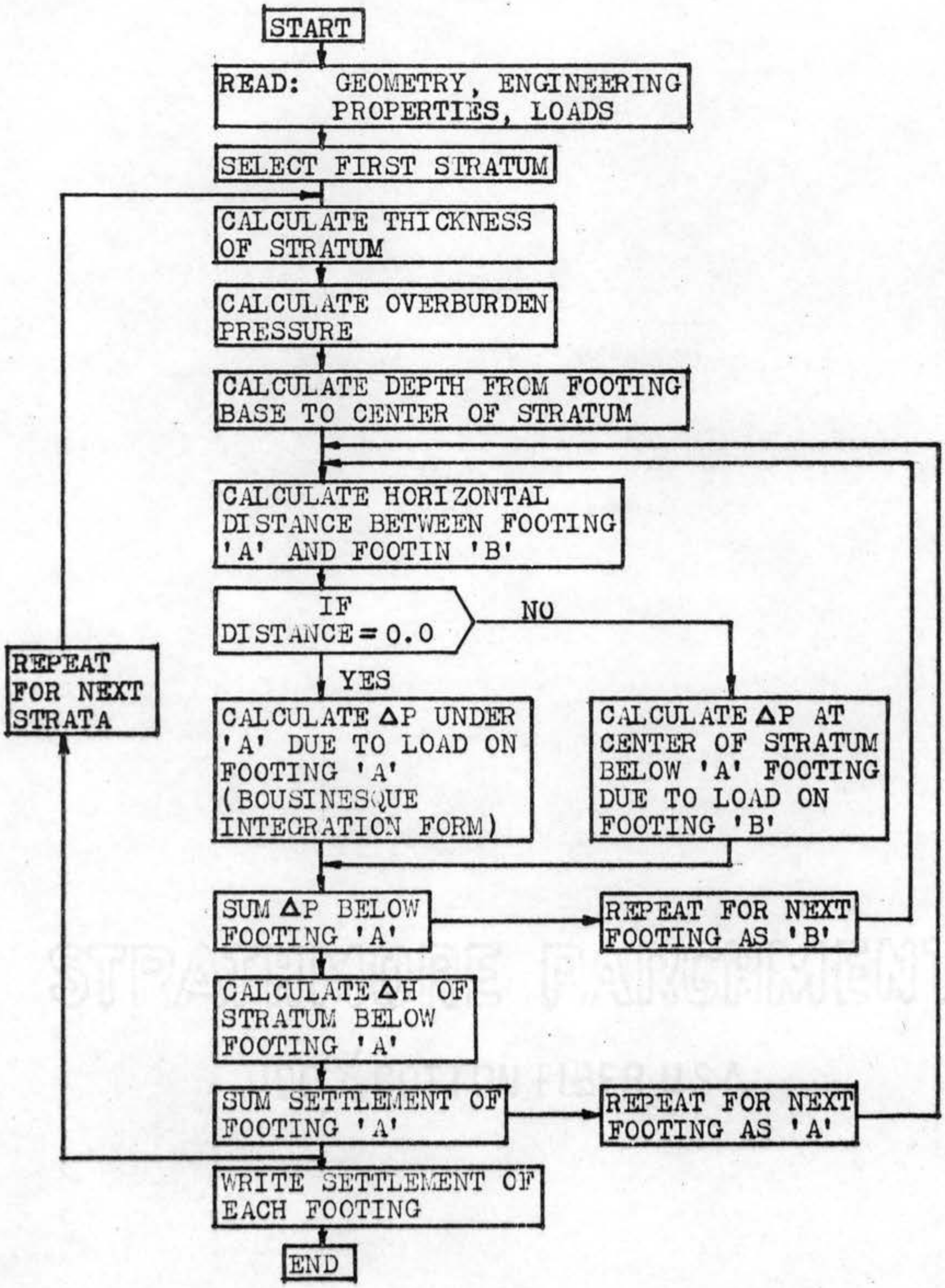


Figure 6.. General Flow Diagram of Proceedure for the Analysis of Independent Settlement of Footing Groups

of computer operations. An example problem is also included in Appendix B.

The input information for the computer solution can be divided into two classifications. The first section of input information includes the center-to-center spacing of the individual footings, the net load on each footing, and the dimensions of each footing pad. In order to organize the computation for digital computer methods, a nonuniform but rectangular coordinate system is imposed on the surface plane. For cases of irregular column spacing that do not match a grid, zero loads are introduced to fill out the grid pattern. The distances between coordinates in the 'X' and 'Y' directions are part of the input data. For settlement computations, the load considered is the dead load plus the part of the live load that would act on the structure for a long period of time. The organization of the input information for an example problem is illustrated in Appendix B.

The second section of the input data concerns the engineering properties of the soil. The soil would be divided into layers as explained in Section 1.5. By means of the input data, the strata divisions and the number of strata considered can be varied to match each individual soil profile. This is done to give a maximum of flexibility to the engineer in the judgment of specific problem requirements. For each layer, the density of the soil, the compression index, and the initial void ratio is read in as data. For a sand, rock, or gravel stratum that is incompressible, a compression index of zero can be read in the data. This will result in a zero settlement contribution for that stratum.

Applications of Computer Solution

The major application of the program is to analyze the independent differential settlement of a structure on normally consolidated clay soils. For structures that do not possess sufficient structural stiffness to significantly affect differential settlement, the solution could be applied directly. For the case of a structure that has sufficient structural stiffness to significantly reduce the differential settlement, the method presented in Chapter III would be more accurate. However, as explained in Chapter III, the solution considering structural stiffness employs the independent settlement solution.

The computer program, as a step of the operation, computes the changes in stress under each footing due to all other footings. The cause and effect of overlapping pressures is available as output information. This information could be used as a guide for the redesign of footings that have critical settlements.

By overlapping the grid systems of an existing and an adjacent new structure, the additional settlement of the existing structure as caused by the new structure could be evaluated. The settlement analysis could first be performed on the existing structure. Then, for the settlement effect caused by the newer structure, the analysis could be repeated for both structures. The additional settlement of the existing structure that is caused by the new structure load could be evaluated. The settlement of the existing structure would have already taken place, assuming that the structure had been in place for a long enough time period. The actual measured settlement of the structure compared to the computed settlement could be used as an index to the accuracy of the settlement analysis for the specific conditions of the problem.

For some cases, the soil is loaded by a distributed and flexible load. Loads of soil mounds, loads of flexible structures such as storage tanks with nonrigid bottoms, and water loads due to ponding are some common examples. The settlement of this type of load could be approximated by the computer solution. The equivalent load of a large number of closely spaced footing pads could be used as input to duplicate the effect of the distributed load. This would be, in effect, a numerical integration approach for an approximate solution.

If several stratifications of soil are encountered for one building site, the program solution could be repeated for the various soil conditions. The settlement computed for a footing that used the correct soil stratification beneath that footing would be correct and could be used. The settlements computed for that same footing when the program is repeated using another stratification would be incorrect and, therefore, would be ignored.

Evaluation of Computer Solution

The use of digital computers allows the practical use of the independent settlement analysis method. The significant advantage of the computer solution applied to a settlement analysis is the speed and ease by which direct solutions can be determined. The independent settlement analysis program for the example problem with eight footings and nine strata used less than one minute of machine time when ran on an IBM 7040 digital computer. Also, the settlement of a large number of footings can be analyzed; the limiting number of footings being determined by the memory capacity of the machine being used. It should be emphasized that the use of a computer does not necessarily provide more precise answers. The same assumptions and theory applications were made that would have been used if settlements were computed by hand methods.

The computer program developed is limited to square footing pads situated at the same elevation. The program method could be modified for more general cases that would not be so limited. However, the solution presented does illustrate the principle and the feasibility of a computer solution for the analysis of differential settlement.

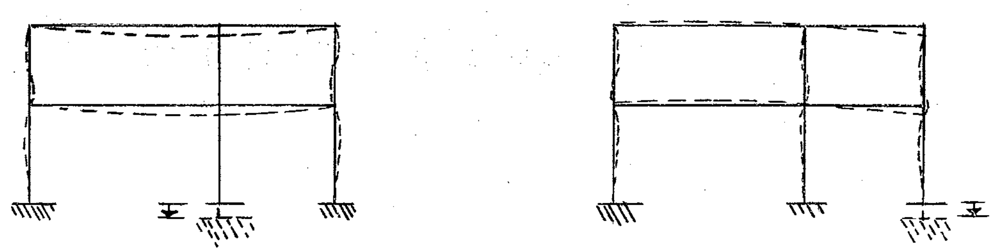
CHAPTER III

DIFFERENTIAL SETTLEMENT ANALYSIS CONSIDERING EFFECT OF STRUCTURAL STIFFNESS

Theory of Method

A conventional analysis for differential settlement ignores the effect of the structural stiffness of a building frame. The effect of the structural frame is to reduce differential settlements. Therefore, a conventional analysis is conservative in that actual differential settlements are always less than the computed differential settlements. For a more precise analysis, it would be advantageous to formulate a method of settlement analysis that considers the effect of the structural stiffness of a building frame.

Conventional analysis assumes that the column load on each footing remains constant throughout the settlement of a building. The settlement analysis is performed independent of the effect of the structural frame. This is equivalent to assuming that the building frame is completely flexible. The effect of the structural frame would be to redistribute the column loads when the frame is subjected to unequal settlements. Intuitively, it can be reasoned that upon settlement of a specific footing, some of the load carried by that footing would be transferred to other footings by the structure. The mode of this transfer can be either by a bridging or a cantilever effect. The modes of stress transfer by the structure is illustrated in Figure 7. The transfer of load by the structure during settlement would cause the loads on the footing pads to change as settlement takes place. These changes in footing loads would result in different magnitudes of settlement than calculated by assuming the initial footing load to remain constant throughout settlement.



Bridging Effect   Cantilever Effect

Figure 7. Modes of Stress Transfer by Structure Subject to Unequal Settlement

Structural materials have an elastic and, therefore, linear stress-strain relationship. In structural engineering, the assumption of linear behavior is the basis for many systems of structural analysis. Slope deflection, moment distribution, and stiffness methods are some of the more common methods. If similar elastic assumptions are accepted for a soil mass, the same general type of analysis can be used to analyze the contact forces and the subsequent settlement of a structural frame and a soil mass in combination. In order to mathematically relate the soil mass to the structure, an analogous linear relationship between an applied action and the resulting deformation must be established for the soil mass system. This can be done by developing a linear elastic relationship for the settlement of each footing due to an applied unit column load. Also, a linear relationship between the settlement of a footing and a load applied by an adjacent footing must be established.

The method can be thought of in terms of a mechanical analogy that represents the soil mass by means of springs. Refer to Figure 8. For the mechanical model of an elastic soil mass, the vertical deformation of point A due to a load applied at A would be a function of the spring constant of spring A, K_A. Likewise, the vertical deformation at A due to a vertical load at B would be a function of the spring constant K_{AB}. The vertical deformation of A due to a vertical load at C would be the vertical load times the spring constant K_{AC}. To reflect the behavior of a soil mass, the spring constant K_{AB} would be greater than K_{AC} and the spring constant K_A would be much greater than K_{AB}. The development of analogous linear relationships for a soil mass is discussed in a later section.

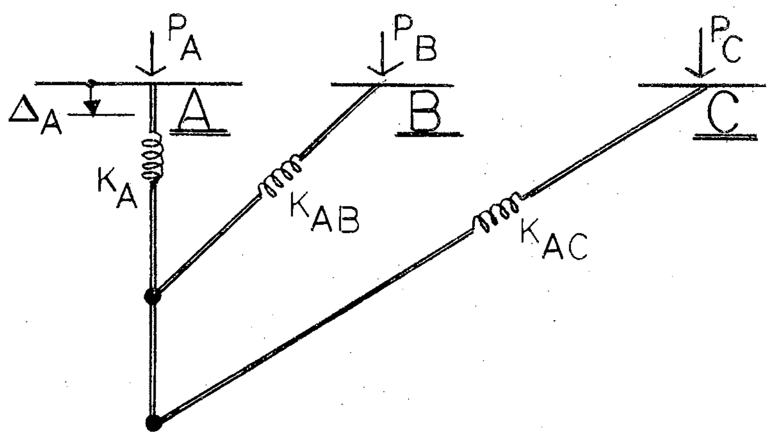


Figure 8. Mechanical Analogy of Elastic Soil for Settlement at A

An elastic structural system and an approximated elastic soil mass provide the basis for the settlement analysis. The differential settlement analysis of a structural frame resting on a soil mass is feasible by thinking of the two separate systems as a combined elastic system for which deformations and actions are linearly related. To mathematically combine the soil and the structure systems, compatibility relationships for the two systems must be established. At each individual contact point between the soil and the structure, the compressive force exchanged between the two systems must be equal at all times during and after settlement. Likewise, the settlement for each individual contact point must be the same for both the structure and the soil systems. Either set of compatibility statements can be used to relate the soil and the structure systems.

By combining compatibility effects of the structure and the elastic base material, an approximate analysis for the settlement of any structure on an elastic base is possible. For the case of a structural frame resting on an assumed elastic soil mass, the settlement of each individual structural column must equal the settlement at that corresponding point of the soil mass. Using this compatibility relationship, a set of simultaneous linear equations can be formed to approximate the settlement of each footing. The final settlement of each footing would be a function of the structural stiffness of the building frame and a function of the elastic properties of the soil mass.

Derivation of Analysis Method

The method of analysis for the settlement of a structural frame on a soil mass is presented by Parcher and Means (1). The solution can be derived by considering the soil mass system as elastic and compatible with the elastic structure system. The basic concepts of stiffness and flexibility methods, unit deformation and unit load methods, that are commonly used for structural analysis can be used to derive the method of settlement analysis (2).

For the purpose of the derivation, the following terms need to be defined in the terminology used for stiffness and flexibility solutions:

Δ_A = Deformation or settlement. Subscript denotes settlement at A.

P_A = Action or column load acting on footing. Subscript denotes action at A.

S_{BA} = Stiffness coefficient B - A. The resulting deformation at B due to a unit action at A.

F_{BA} = Flexibility coefficient B - A. The resulting deformation at B due to a unit action at A.

$\{\Delta\}$ = A matrix array of deformations. Brackets indicate a column matrix. For convenience, the column matrix is often listed in row form with brackets to indicate a column matrix.

$$\{\Delta\} = \begin{Bmatrix} \Delta_A \\ \Delta_B \\ \Delta_C \end{Bmatrix} = \{\Delta_A, \Delta_B, \Delta_C\}$$

$\{P\}$ = A column matrix array of actions.

$$\{P\} = \{P_A, P_B, P_C\}$$

$\{\Delta_L\}$ = A column matrix array of deformations or settlements due to original column loads.

$$\{\Delta_L\} = \{\Delta_{LA}, \Delta_{LB}, \Delta_{LC}\}$$

$[S]$ = A rectangular matrix array of stiffness coefficients.

$[F]$ = A rectangular matrix array of flexibility coefficients.

The first step of the derivation is to consider only the structural frame system. Refer to Figure 9. The action, column loads, at each column due to a unit deformation of one column is a stiffness coefficient. A set of stiffness coefficients can be evaluated by any of several standard methods of structural analysis. The stiffness coefficient S_{BA} is the column load at B due to a unit deformation at A. If a unit deformation of one inch is consecutively applied to each column of the example frame, a set of stiffness coefficients can be derived. The vertical deformation of a column can be related by stiffness coefficients to each column load.

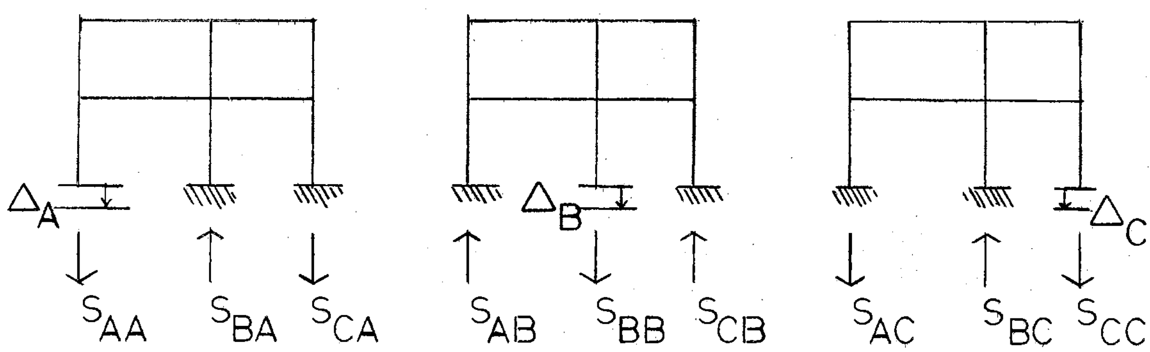


Figure 9. Stiffness Coefficients for Structure System

The complete interrelationship between column loads and settlements can be mathematically expressed in terms of stiffness coefficients. This relates corresponding settlements to column loads as a function of the stiffness coefficients $\{\Delta_A, \Delta_B, \Delta_C\}$ are the settlements that correspond with the footing loads $\{P_A, P_B, P_C\}$.

The stiffness relationship for the structural frame can be expressed as a set of simultaneous equations.

$$P_A = S_{AA}\,\Delta_A + S_{AB}\,\Delta_B + S_{AC}\,\Delta_C$$

$$P_B = S_{BA}\,\Delta_A + S_{BB}\,\Delta_B + S_{BC}\,\Delta_C$$

$$P_C = S_{CA}\,\Delta_A + S_{CB}\,\Delta_B + S_{CC}\,\Delta_C.$$

The stiffness relationship can also be stated in terms of a matrix equation.

$$\begin{Bmatrix} P_A \\ P_B \\ P_C \end{Bmatrix} = \begin{bmatrix} S_{AA} & S_{AB} & S_{AC} \\ S_{BA} & S_{BB} & S_{BC} \\ S_{CA} & S_{CB} & S_{CC} \end{bmatrix} \begin{Bmatrix} \Delta_A \\ \Delta_B \\ \Delta_C \end{Bmatrix}$$

A generalized short form of the same matrix relationship is often more convenient.

$$\{P\} = [S]\,\{\Delta\}. \tag{1}$$

A similar stiffness relationship for the soil mass system might be developed, but the solution for the actions due to a unit deformation would be a trial and error process. However, by means of an independent settlement analysis, a direct solution for the settlement of the soil mass due to a unit load is possible. A unit load of one kip can consecutively be placed on the soil mass at a position corresponding to each structure footing. The settlements would be the flexibility coefficients for the soil mass system. F_{BA} is the settlement at B due to a unit column load at A. A flexibility coefficient is the reciprocal of a stiffness coefficient.

For the soil mass system, the relationship of settlement to applied load can be expressed in matrix form by means of flexibility coefficients. This can be expressed in the general matrix form.

$$\{\Delta\} = [F]\,\{P\}. \tag{2}$$

As a footing settles, the original column load is changed by the effect of the structural frame. The final settlement of the column is reached when the footing load and the change of footing load resulting from the deformation of the structure are in a state of equilibrium. $\{\Delta_A, \Delta_B, \Delta_C\}$ are considered the final settlements, and $\{P_A, P_B, P_C\}$ are the final column loads. $\{\Delta_{LA}, \Delta_{LB}, \Delta_{LC}\}$ can be termed the settlements due to the original column loads that act on the footings before any settlements. The equilibrium conditions can be added to the matrix equation number (2). The settlements of the footings in an equilibrium state can be expressed by this relationship.

$$\{\Delta\} = [F]\,\{P\} + \{\Delta_L\}. \tag{3}$$

The actions, footing loads, in both the structure and the soil mass systems must be the same. Using this compatibility statement, the actions as evaluated by equation (1) can be substituted into the matrix equation (3). This is equivalent to stating that the settlement of the structure is equal to the settlement of the soil at each footing.

$$\{P\} = [S]\ \{\Delta\} \tag{1}$$

$$\{\Delta\} = [F]\ \{P\} + \{\Delta_L\} \tag{3}$$

$$\{\Delta\} = [F]\ \{[S]\ \{\Delta\}\} + \{\Delta_L\}. \tag{4}$$

Equation (4) may be stated in its complete matrix form and the equation may be rearranged algebraically in terms of the unknown settlements.

The following matrix is in the form required for a final solution of settlement. The term D_{AB} is equivalent to the settlement at A that would be caused by the footing loads that would result from a unit settlement at B. Each term of the stiffness matrix can be developed by placing a unit settlement in a column of the structure. The deformed structure would then be analyzed for the resulting column loads. For those column loads, the footings would be analyzed for settlement. The first column of the stiffness matrix would be the settlements due to the structure column loads that would result from the one-inch deformation of footing A.

To solve for the unknown final settlement of each footing when influenced by the structural frame, the set of simultaneous linear equations can be solved for the unknown settlements. There is one linear equation and one unknown settlement for each footing. Any of several

$$\begin{Bmatrix} \Delta_A \\ \Delta_B \\ \Delta_C \end{Bmatrix} = \begin{bmatrix} F_{AA} & F_{AB} & F_{AC} \\ F_{BA} & F_{BB} & F_{BC} \\ F_{CA} & F_{CB} & F_{CC} \end{bmatrix} \begin{Bmatrix} S_{AA}\Delta_A + S_{AB}\Delta_B + S_{AC}\Delta_C \\ S_{BA}\Delta_A + S_{BB}\Delta_B + S_{BC}\Delta_C \\ S_{CA}\Delta_A + S_{CB}\Delta_B + S_{CC}\Delta_C \end{Bmatrix} + \begin{Bmatrix} \Delta_{LA} \\ \Delta_{LB} \\ \Delta_{LC} \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta_A \\ \Delta_B \\ \Delta_C \end{Bmatrix} = \begin{Bmatrix} F_{AA}(S_{AA}\Delta_A+S_{AB}\Delta_B+S_{AC}\Delta_C)+F_{AB}(S_{BA}\Delta_A+S_{BB}\Delta_B+S_{BC}\Delta_C)+F_{AC}(S_{CA}\Delta_A+S_{CB}\Delta_B+S_{CC}\Delta_C) \\ F_{BA}(S_{AA}\Delta_A+S_{AB}\Delta_B+S_{AC}\Delta_C)+F_{BB}(S_{BA}\Delta_A+S_{BB}\Delta_B+S_{BC}\Delta_C)+F_{BC}(S_{CA}\Delta_A+S_{CB}\Delta_B+S_{CC}\Delta_C) \\ F_{CA}(S_{AA}\Delta_A+S_{AB}\Delta_B+S_{AC}\Delta_C)+F_{CB}(S_{BA}\Delta_A+S_{BB}\Delta_B+S_{BC}\Delta_C)+F_{CC}(S_{CA}\Delta_A+S_{CB}\Delta_B+S_{CC}\Delta_C) \end{Bmatrix} + \begin{Bmatrix} \Delta_{LA} \\ \Delta_{LB} \\ \Delta_{LC} \end{Bmatrix}$$

$$\Delta_A = (F_{AA}S_{AA}+F_{AB}S_{BA}+F_{AC}S_{CA})\Delta_A + (F_{AA}S_{AB}+F_{AB}S_{BB}+F_{AC}S_{CB})\Delta_B + (F_{AA}S_{AC}+F_{AB}S_{BC}+F_{AC}S_{CC})\Delta_C + \Delta_{LA}$$

$$\Delta_B = (F_{BA}S_{AA}+F_{BB}S_{BA}+F_{BC}S_{CA})\Delta_A + (F_{BA}S_{AB}+F_{BB}S_{BB}+F_{BC}S_{CB})\Delta_B + (F_{BA}S_{AC}+F_{BB}S_{BC}+F_{BC}S_{CC})\Delta_C + \Delta_{LB}$$

$$\Delta_C = (F_{CA}S_{CA}+F_{CB}S_{BA}+F_{CC}S_{CA})\Delta_A + (F_{CB}S_{AB}+F_{CB}S_{BB}+F_{CC}S_{CB})\Delta_B + (F_{CA}S_{AC}+F_{CB}S_{BC}+F_{CC}S_{CC})\Delta_C + \Delta_{LC}$$

The relationship in terms of the unknown settlements can be stated as a matrix solution. The terms of the matrix can be abbreviated into a solution matrix.

$$\begin{Bmatrix} -\Delta_{LA} \\ -\Delta_{LB} \\ -\Delta_{LC} \end{Bmatrix} = \begin{bmatrix} (F_{AA}S_{AA}-F_{AB}S_{BA}-F_{AC}S_{CA}{}^{-1}) & (F_{AA}S_{AB}-F_{AB}S_{BB}-F_{AC}S_{CB}) & (F_{AA}S_{AC}-F_{AB}S_{BC}-F_{AC}S_{CC}) \\ (F_{BA}S_{AA}-F_{BB}S_{BA}-F_{BC}S_{CA}) & (F_{BA}S_{AB}-F_{BB}S_{BB}-F_{BC}S_{CB}{}^{-1}) & (F_{AA}S_{AC}-F_{BB}S_{BC}-F_{BC}S_{CC}) \\ (F_{CA}S_{AA}-F_{CB}S_{BA}-F_{CC}S_{CA}) & (F_{CB}S_{AB}-F_{CB}S_{BB}-F_{CC}S_{CB}) & (F_{CA}S_{AC}-F_{CB}S_{BC}-F_{CC}S_{CC}{}^{-1}) \end{bmatrix} \begin{Bmatrix} \Delta_A \\ \Delta_B \\ \Delta_C \end{Bmatrix}$$

$$\begin{Bmatrix} -\Delta_{LA} \\ -\Delta_{LB} \\ -\Delta_{LC} \end{Bmatrix} = \begin{bmatrix} D_{AA} & D_{AB} & D_{AC} \\ D_{BA} & D_{BB} & D_{BC} \\ D_{CA} & D_{CB} & D_{CC} \end{bmatrix} \begin{Bmatrix} \Delta_A \\ \Delta_B \\ \Delta_C \end{Bmatrix}$$

methods for the solution of simultaneous linear equations can be used.

Assumptions Required for Analysis

Many of the same assumptions and methods used for an independent settlement analysis are employed in a settlement analysis that considers structural stiffness. The Boussinesq methods as previously discussed can be used to evaluate the change in stress of a soil layer. Likewise, with slight modification, the same methods as previously discussed can be used to compute the change in height of a soil layer.

The derivation for settlement, as governed by the relationship between the stiffness of the soil mass and the structural frame, assumes that the soil mass is elastic. An elastic settlement as a linear function of the footing load is the basis for the linear matrix relationship. However, the settlement of a soil mass is only an approximately linear function of the applied footing load. The basic settlement equation relates the change in height of a soil layer as a logarithmic function of the change in pressure.

$$\Delta H = \frac{H \times C_L}{1 + e_1} \times \mathrm{LOG}_{10} \frac{\Delta P + P_1}{P_1}.$$

However, the solution requires that the matrix be very nearly linear. If the solution matrix is not very nearly linear, the solution for final settlements is often in error. As a general rule, the sensitivity of the solution matrix increases as more footings are considered and the size of the solution matrix becomes larger. The linear requirements and sensitivity of the matrix require that the settlement of the soil mass be approximated as a linear function of applied footing loads. This can be done by use of constants that approximate the settlement of a footing as a function of applied load. This is analogous to the spring constant example discussed previously. Refer to Figure 8. A separate constant for settlement per unit load is required for each footing size. Also, a separate constant for settlement per unit load should be developed for each distant load. A unique set of constants would be required for each problem that would present different soil stratifications.

For each size of footing, it is necessary to develop a constant for the settlement of the footing due to a load applied to that footing. To approximate an average footing load, the design contact pressure can be applied to each footing. By use of the independent settlement analysis that was discussed in Chapter II, the settlement of each footing for the design contact pressure can be computed. The settlement divided by the total load of the footing could be considered as a constant to approximate the settlement of that footing as a linear function of the load. A separate constant should be developed for each footing size by the same procedure. The approximation would be in error since the final contact pressure would not be the same as the design contact pressure. If the difference in contact pressures is not significantly large, the error introduced would be acceptable. The author has found that the solution using a constant function for the settlement per unit load does give reasonable answers for settlement despite the introduction of some error. The author has also found that if the solution matrix is formed using variable contact pressures, the final settlements computed can be wrong by as much as one hundred percent. The reason for the large magnitude of error is that the sensitive solution matrix becomes nonlinear, and a nonlinear deviation in the formation of the matrix becomes magnified in the final solution.

It is also necessary to develop a constant for the settlement of a footing due to a load on a distant footing. This would be a constant for the effect of overlapping soil pressure. The same general approach of the independent settlement analysis for the effect of distant loads can be used. An average size footing load can be applied at the required distance. The settlement of the footing divided by the average load can be used as a constant for the settlement per unit load that corresponds to the distance being considered. The procedure should be repeated for all possible combinations of distance. Since the settlement contributions of distant loads are relatively small, the magnitude of the average load assumed does not significantly affect the settlements computed.

The terms of the solution matrix are influenced by the base conditions assumed for the structural analysis. The footings can either be assumed to be fully fixed, theoretically pinned, or partially fixed. The settlement computation of a soil layer considers only vertically applied load and does not consider any effect of moment. To be consistent with the settlement computation, the force exchanged between the soil mass and the structure should only be a vertical force. Upon differential settlement of the structure, the axial loads of the columns would be redistributed by the structure. If the columns are considered to have fixed bases, part of the stress would be redistributed as moment. The axial loads of the columns are increased or decreased if the deformed structure is analyzed as a fixed base. The energy that is transferred into moment is not accounted for in the settlement analysis. In effect, the energy that is transferred into moment is lost in the process of the analysis. With a fixed base assumption, the solution matrix becomes nonlinear. All footings transmit some moment as well as axial load to the soil, even though the footings might be designed as a theoretical pinned base. Yet, for the purposes of a settlement analysis, the deformed structure should be analyzed with a theoretical pinned base. In this form, the complete reaction of the structure to deformation can be evaluated in terms of settlement.

The structural analysis can be made either with the assumption that the frame is restrained against sidesway or that the structure is free to sidesway. If the frame were restrained against lateral movement, the reaction of the structure to the unit settlement of a footing would be greater. To assume the structure to be restrained against sidesway is equivalent to increasing the stiffness of the structure. The significance of sidesway would vary with the structure being considered. When the analysis of the structure is done by computer, there is little difference in effort as to whether sidesway is or is not assumed. The choice of assumptions should best reflect the anticipated structural behavior.

The method and type of structural analysis can be varied to match the preference of the engineer and the type of structure. It is only necessary to analyze the structure to determine column loads for the case of gravity loading and for the cases of a unit settlement of each column. For a structure that acts primarily as a plane frame, the normal assumptions that are compatible with a plane frame analysis would apply. If a structural system would transfer stress three dimensionally, a space frame analysis would be more accurate.

Steps of Procedure for Analysis

The solution for the settlement of each footing involves developing the solution matrix and solving simultaneously for the unknown settlements. The number of rows and columns of the solution matrix equal the number of footings being considered. The computational effort required to solve for the settlements increases approximately in proportion to the square of the number of footings.

The first step is to calculate the footing loads for the case of no settlement. This can be done by any procedure of conventional structural analysis for the loaded frame. It should be emphasized that the loads considered should be the dead load plus that part of the live load which would be acting on the frame for a long period of time. For the structural analysis, the author used the method of Kani membent distribution (3). The computational operations were performed by an IBM 7040 computer using a computer solution developed by Seshagari (4). The method was selected because of its quick convergence that facilitates computer adaptation.

The second step is to calculate the settlements that would occur due to the footing loads when the structural stiffness of the frame is neglected. The settlements of each footing would be calculated for the footing loads that were calculated in the first step. The procedure so far corresponds with an independent settlement analysis as presented in Chapter II. The settlements form the matrix terms for settlement due to load, $\{\Delta_L\}$.

The next step is to develop the solution matrix. The first part of the solution is to form a stiffness matrix for the structural system. To do this, a unit settlement of one-inch is consecutively placed at each footing. The structure is then analyzed to evaluate the force induced in each column. For this part of the solution, the author again used the Kani moment distribution method. The analysis for a one-inch settlement of each column can be performed as one computer operation by programming the solution so that the analysis operation is repeated for each column. Appropriate fixed end moments for a one-inch settlement can be distributed to the members in accordance with member stiffness by the normal moment distribution procedure. The process would be repeated for a one-inch deformation in each column. A set of column loads will be obtained for each case of unit deformation.

For each set of column loads, the settlement of the footings can be calculated. The settlements of each set would be the settlements due to the unit deformation of the corresponding column. The settlements of each set would make up a column of the settlement matrix. The process would be repeated for each set of column loads that correspond to the unit settlement of each column. As discussed previously, the settlements should be calculated using the constants for the settlement per unit column load. This insures that the settlement matrix is linear. The form of the settlement matrix is shown in the previous section.

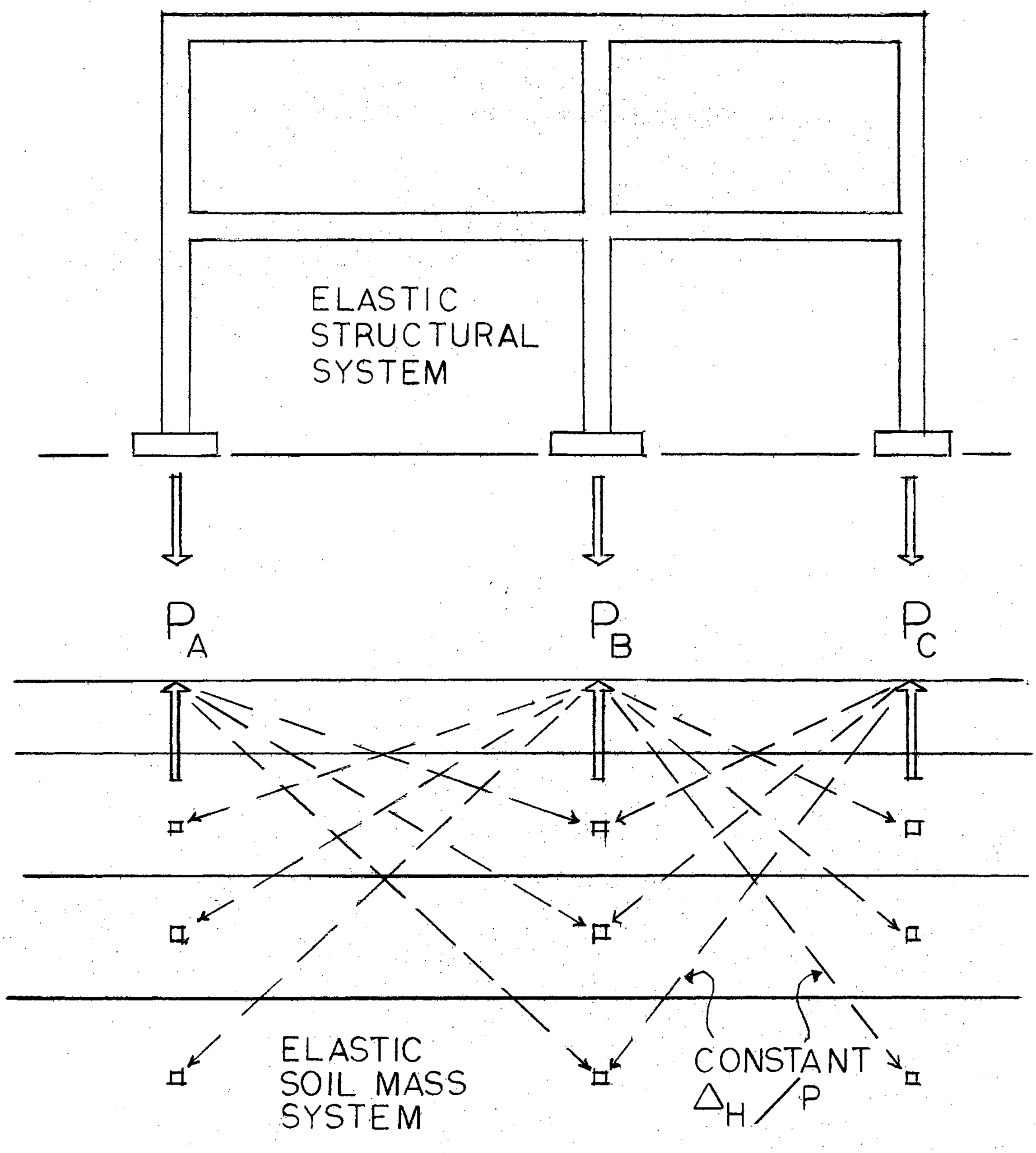


Figure 10. Combined Soil and Structure System

The last step of the solution is to solve for the settlement of each column. The terms to be solved for are the unknown final settlements, $\{\Delta\}$. This can be done by any method for the solution of simultaneous linear equations. For a set of more than three equations, the only practical approach is by computer methods. From the individual settlement of each footing, the differential settlements between footings can be determined.

There is no precise check for the accuracy of the computation, but the general accuracy of the answer can be determined. The sum of the final settlements, $\{\Delta\}$, should roughly equal the sum of the independent settlements, $\{\Delta_L\}$. This is true because the same total building load is transferred to the soil in both cases. The sums will not be precisely the same because different magnitudes of load are transferred to the soil by different size footings. An example of the check is illustrated for the example problem in Appendix C.

Evaluation of Method as Computer Solution

The settlement analysis considering structural stiffness requires a large amount of computation that would make it impractical for longhand solution. The time and expense of the calculation can be greatly reduced by computer operations. The author divided the computation into several parts for computer programming.

The first program analyzed the structure for footing loads due to gravity loads on the frame. The second program consecutively placed a settlement of one-inch at each column and analyzed the frame for the resulting column reactions. For both structural analysis programs, the author used the analysis method of Kani moment distribution. The computer time required using an IBM 7040 for the example problem that required the analysis of four separate frames was fifty seconds. Several hours are required to prepare the data and analyze the results.

The third program used was similar to the independent settlement program presented in Chapter II, except that constants for the settlements per unit footing load were used. The program computed the settlement of the footings due to loads that were calculated by the first two programs. The computer time required using an IBM 7040 to analyze the settlement of six footings three times was thirty-two seconds. Approximately three thousand units of memory core were required. Neither the time nor core memory requirements pose a significant problem for a computer solution.

The fourth step of a computer solution is to solve for the final settlements. This requires the solution of the simultaneous equations that make up the solution matrix equation. The only feasible approach for the solution is by use of computer methods. There are several standard computer solutions available to solve for the unknowns of a set of simultaneous linear equations. The solution matrix always must have nonzero terms for the diagonal. For each row, the diagonal term is also the largest in magnitude. Therefore, the solution does not present any unique problems for a matrix solution. The number of footings that could be considered would probably not be limited by the memory units required for a settlement analysis but by the number of simultaneous equations that can be solved. The number would depend on the memory capacity of the computer being used for the simultaneous solution. The size and character of the solution matrix for a normal sized building would not be beyond the memory capacity of an average sized computer.

As an engineering material soil is inconsistent and not subject to precise analysis. An independent settlement analysis can have an expected range of accuracy of ten to fifty percent. The analysis for settlement that considers structural stiffness should be more accurate, because the additional parameter of structural stiffness is considered. The analysis should still be conservative, because only the structural materials are considered to resist differential settlement. As is common with structural analysis, nonstructural masonry, panels, sheathing, etc. are not considered. However, these materials that are present in almost all buildings do act in a structural manner to resist deformation. The actual degree of accuracy would be difficult to estimate without comparison of computed solutions to the behavior of real structural frames.

The solution matrix was found to be increasingly sensitive with size. For a solution involving as few as six footings, a slight error in the formation of the solution matrix can be greatly magnified in the final answer. A set of two simultaneous linear equations with only two unknowns can be represented graphically by a two dimensional plot of two straight lines. The solution for the unknown values common to both equations would be the coordinates for the point of intersection of the two lines. If the plots of the two lines are nearly parallel, the position of the point of intersection is greatly influenced by a slight difference in the slopes of the two lines. In order to be able to graphically illustrate the solution of a sample problem in terms of a two dimensional plot, a sample problem of five footings with symmetry in two directions was devised. For the case of the special problem, there are only two unknown settlements due to the symmetry. The linear plot of the two solution equations is illustrated by Figure 11. The two equations for the sample problem illustrate the general characteristic of a set of simultaneous linear equations for which the solution is sensitive. An error or inaccuracy in the formation of the solution equations, would be magnified in the final answer.

The settlement of a soil layer involves the summation of small increments of stress change. The settlement of a

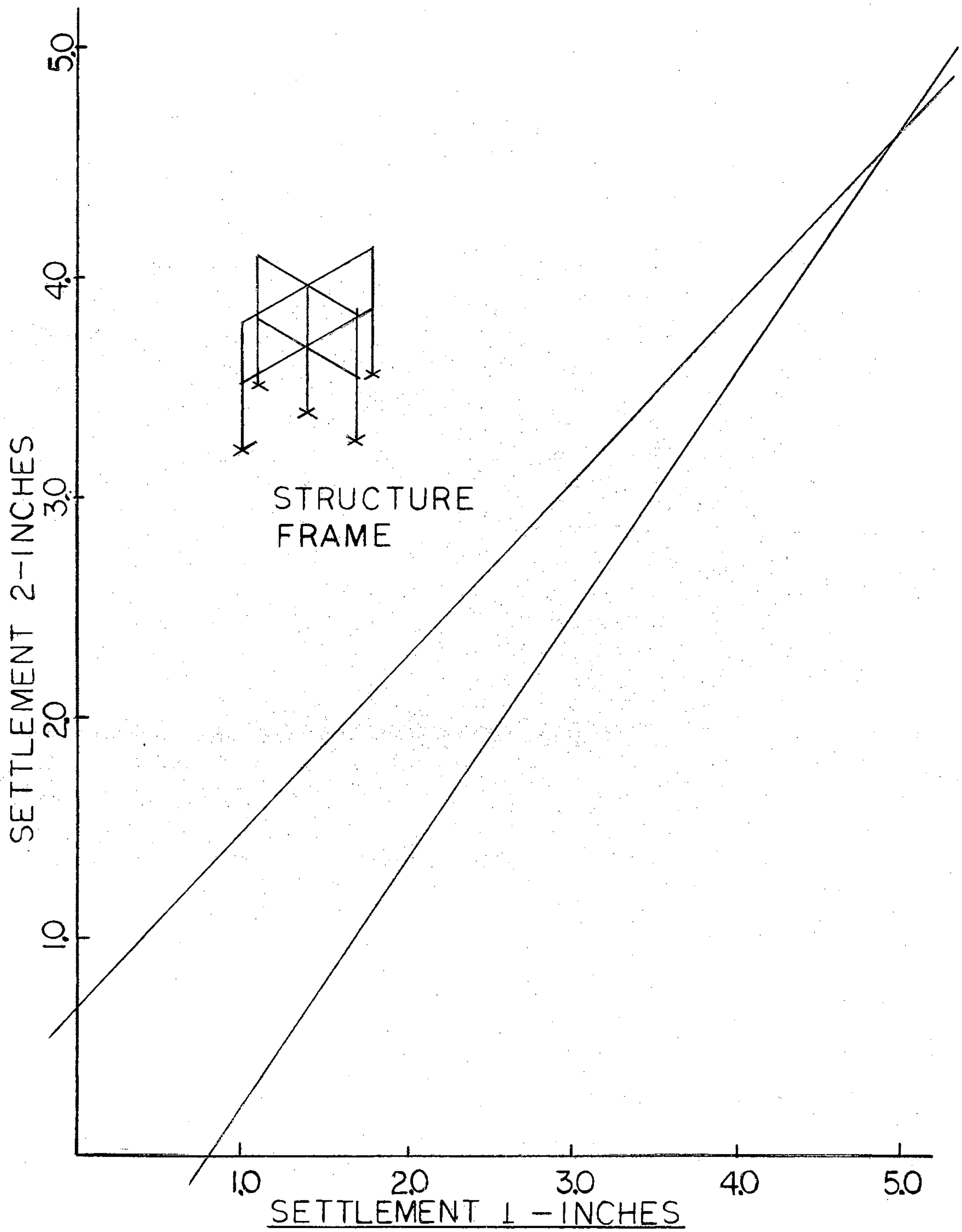


Figure 11. Plot of Two Solution Equations for an Example Problem

specific footing involves the summation of small increments of settlement for each layer. For computer operations, the addition and subtraction of small values can in some cases cause the buildup of error due to the automatic truncation operations of computer calculations. This is especially possible when two numbers of almost equal value are subtracted. The difference of the two numbers may be beyond the number of accurate digits carried in the calculation. The sensitivity of the solution coupled with the possibility of truncation error should be considered by the programmer. The possibility of truncation error can be minimized by avoiding as much as possible the addition and subtraction of many small terms. This is effectively done by using a constant term for the settlement per unit load.

From a theoretical standpoint, it would be possible to predict the stress of structural materials by the same analysis for the settlement of structural footings. However, there is a major difference in the accuracy of soil and structural analysis. This is reflected by the different safety factors required. The safety factor for steel design is approximately 1.6. For soil settlement a safety factor of 3.0 to 10.0 is often required. For this reason, the settlement analysis should not be used directly to predict structure stress. The settlement analysis could be used to predict structure stress by using appropriate safety factors that correspond to the soil analysis. Another approach would be to perform the settlement analysis using worst case assumptions for the soil. This should give the maximum possible value for differential settlement. On the basis of the worst possible settlement, a conservative analysis of structure stress due to differential settlement could be performed. An analysis for structure stress due to settlement should not be necessary if the differential settlements are within the range of allowable differential settlements. See Table I.

It can be concluded that an analysis for settlement considering structural stiffness does provide a reasonable method for the engineer to predict differential settlement. The analysis does require the use of a high speed electronic computer. The computer time required is not excessive, and the use of a digital computer frees the engineer from much tedious calculations. The cost of a computer analysis for the settlement of each separate footing when considering the effect of structural stiffness should cost less than a more approximate analysis that could be performed without the use of a digital computer. However, the program solution for settlement can still be flexible enough to allow for variable engineering judgments.

FOOTNOTES

[1]Parcher and Means, p. 263.

[2]James M. Gere and William Weaver, Jr., Analysis of Framed Structures (New York, 1965), pp. 41-134.

[3]Gaspar Kani, Analysis of Multistory Frames (New York, 1957), pp. 7-52.

[4]Seshagiri Natesan, "Design of Tall Buildings by Use of A Simulator" (unpub. thesis, Oklahoma State University, 1966), p. 73.

CHAPTER IV

EFFECT OF STRUCTURAL STIFFNESS ON DIFFERENTIAL SETTLEMENTS

Introduction

One of the possible solutions to limit excessive differential settlements is to increase the stiffness of the structural frame. This would be done by the selection of heavier structural sections. It is known that heavier structure sections would reduce differential settlements, but the relationship between structural stiffness and differential settlements is not known. The design engineer needs to know whether the increase of structural stiffness is an economically feasible method to limit differential settlement. The engineer also needs to know how much the stiffness of the frame must be increased in order to satisfactorily limit differential settlements. The objective of this chapter is to propose a procedure by which an engineer may study the effect of increased stiffness on the settlement of a particular building frame. It is also the objective of this chapter to draw some general conclusions concerning the relationship of stiffness to differential settlement.

Procedure for a Comparative Study

For the sample problem of Appendix C, the structural stiffness was varied by increments of twenty percent. This is equivalent to varying the column loads that are used to form the solution matrix by increments of twenty percent. For each set of variable stiffness, the settlements were computed by the settlement program presented in Chapter III. The differential settlements between footings one and three were plotted as a function of the percentage of the structure stiffness. The differential settlement between footings one and three was selected because it was the maximum differential settlement of the example problem. Refer to Figure 12.

The plot helps to illustrate the effect of decreased structural stiffness on differential settlement. A similar plot could be made for any structure, and on the basis of such a plot the engineer could determine if increasing or decreasing structural stiffness would be a feasible solution to differential settlement. For the example problem, it can be concluded that increasing the stiffness of the frame does not significantly limit differential settlement. Therefore, if it were desired to further limit differential settlement of the example frame, other alternatives would probably be more practical.

The conclusions reached for the example frame would not necessarily apply to all frames. The designed stiffness of a frame and the soil conditions of each site would

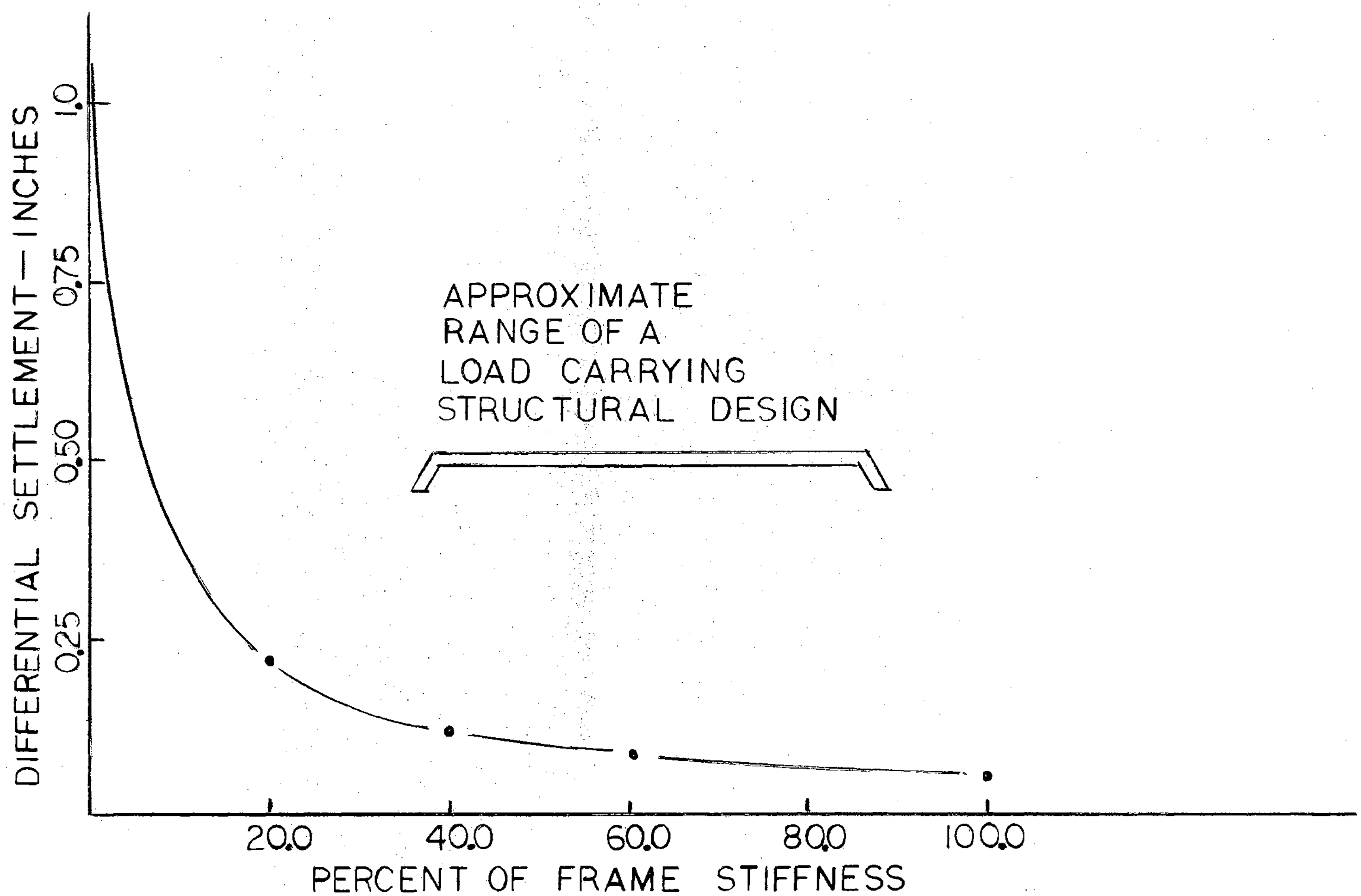


Figure 12. Differential Settlements Between Columns 1 and 3 Plotted as a Function of Relative Structural Stiffness

make each problem unique. By application of the method, the general effects on settlement of increasing or decreasing structural stiffness can be determined. For any specific frame, the engineer could evaluate the benefits of increasing frame stiffness by plotting several adjacent points on the differential settlement--relative stiffness curve.

The method of varying the stiffness by a percentage is a helpful tool to study a particular problem in general terms. However, the percentage increases of stiffness does not consider the possibility of selectively stiffening the frame. The frame could be stiffened only at specific locations by heavier members, X bracing, shear walls, etc. The engineer could intuitively stiffen the frame at certain locations and analyze the settlement for that particular solution. The reduction of differential settlement could be compared directly to the increased weight and cost of the structure. Because the settlement analysis does not require extensive effort, a guided trial and error approach would be completely feasible.

General Conclusions

The differential settlement--relative stiffness plot does indicate the differences between considering the stiffness of the structure frame and an analysis that is independent of the structure frame stiffness. The zero percent stiffness would correspond to the independent settlement. The consideration of some structural stiffness by the analysis does present considerable difference in differential settlement. This indicates that the built in safety factor of an independent settlement is very conservative.

For the example problem, it would not be of advantage to increase structural stiffness. The example frame is relatively stiff. But, if the frame were even fifty percent as stiff, the differential settlement would not be greatly reduced by increased stiffness. For the range of stiffness required by the structure to carry the gravity loads, there would appear to be a minimal advantage to increasing the structural stiffness. More studies of example problems would be required in order to make more positive conclusions that would apply in general terms to all structural frames. However, if the example frame can be considered as typical, it can be concluded that when a structure is designed to carry gravity loads a further increase in frame stiffness would not greatly limit differential settlement. Similar studies of many structural frames with the objective of studying the general effects of increased stiffness on differential settlement would be a good area for future study.

BIBLIOGRAPHY

Gere, James W., and William Weaver, Jr. *Analysis of Framed Structures.* New York: D. Van Nostrand, 1965.

Kani, Gasper. *Analysis of Multistory Frames.* New York: F. Ungar, 1957.

Leonards, G. A. *Foundation Engineering.* New York: McGraw-Hill, 1962.

Means, R. E., and J. V. Parcher. *Physical Properties of Soils.* Columbus: Merrill, 1964.

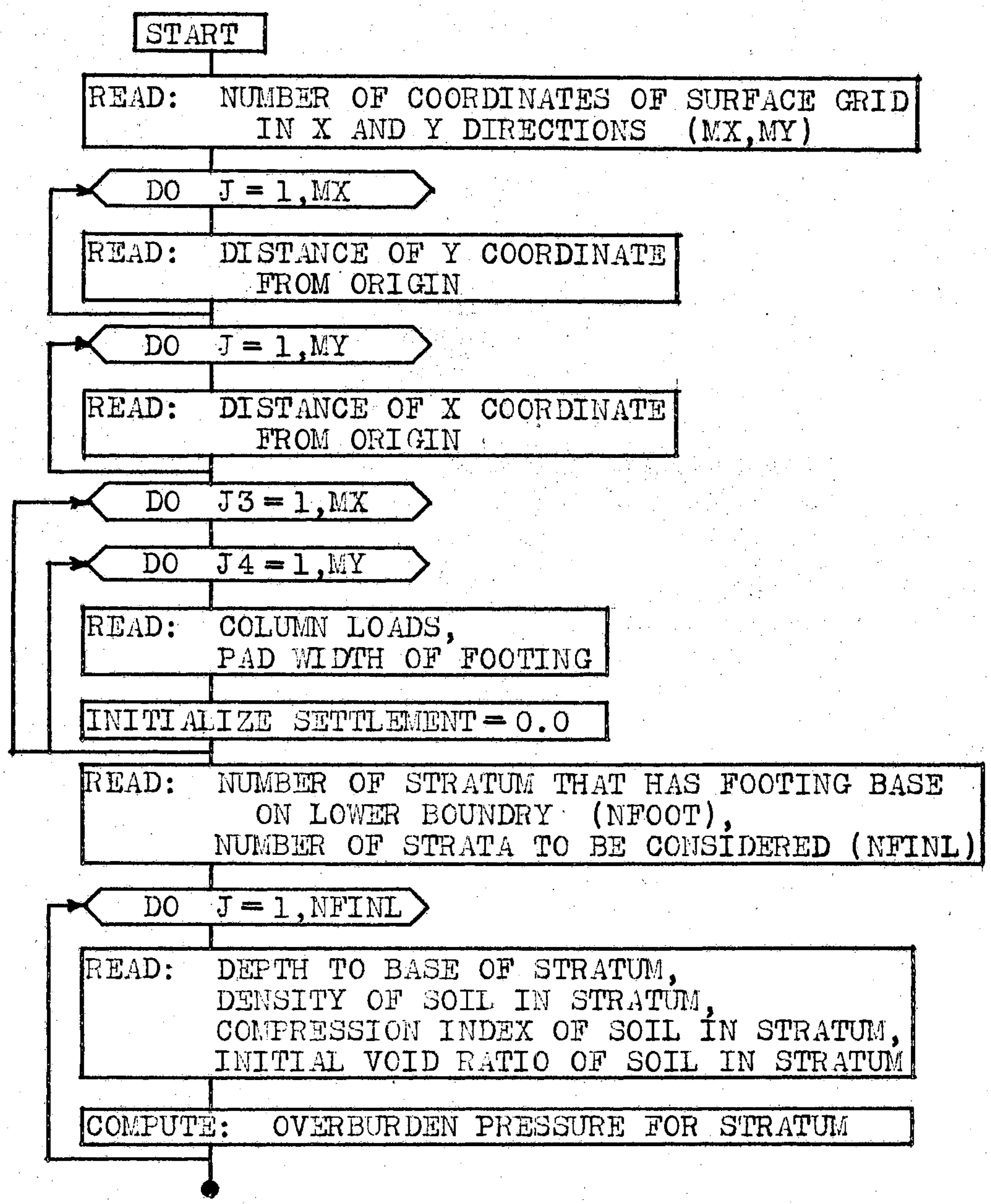
Natesan, Seshagiri. "Design of Tall Buildings by Use of a Simulator." (unpub. M. S. thesis, Oklahoma State University, 1966).

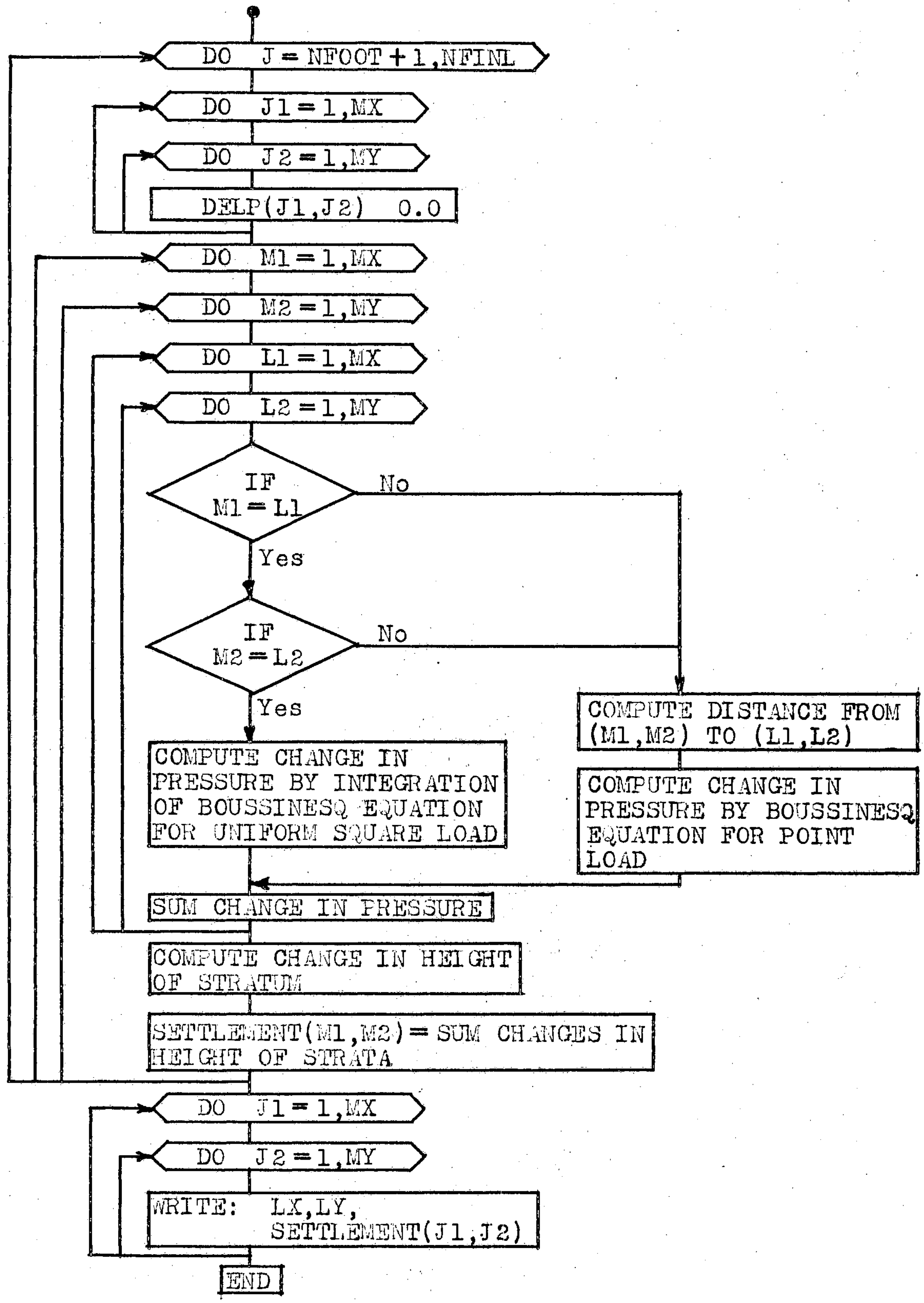
Parcher, J. V., and R. E. Means. *Soil Mechanics and Foundations.* Columbus: Merrill, 1968.

Peck, Ralph B., Walter E. Hanson, and Thomas H. Thornburn. *Foundation Engineering.* New York: Wiley, 1966.

APPENDIX A

SPECIFIC FLOW DIAGRAM OF INDEPENDENT SETTLEMENT COMPUTATION




DO J = NFOOT + 1, NFINL
DO J1 = 1, MX
DO J2 = 1, MY
DELP(J1,J2) 0.0
DO M1 = 1, MX
DO M2 = 1, MY
DO L1 = 1, MX
DO L2 = 1, MY
IF M1 = L1
No
Yes
IF M2 = L2
No
Yes
COMPUTE CHANGE IN PRESSURE BY INTEGRATION OF BOUSSINESQ EQUATION FOR UNIFORM SQUARE LOAD
COMPUTE DISTANCE FROM (M1,M2) TO (L1,L2)
COMPUTE CHANGE IN PRESSURE BY BOUSSINESQ EQUATION FOR POINT LOAD
SUM CHANGE IN PRESSURE
COMPUTE CHANGE IN HEIGHT OF STRATUM
SETTLEMENT(M1,M2) = SUM CHANGES IN HEIGHT OF STRATA
DO J1 = 1, MX
DO J2 = 1, MY
WRITE: LX,LY, SETTLEMENT(J1,J2)
END

APPENDIX B

EXAMPLE PROBLEM OF INDEPENDENT SETTLEMENT ANALYSIS

Problem Statement: Refer to Figures 13 and 14 for a description of the building footing plan and the soil stratification. The answer sought is the final differential settlement that would occur between adjacent footings.

Problem Assumptions:

(1) Normal settlement theory is used. Refer to Chapter I.

(2) The settlement of each footing is considered to be independent of the settlements of adjacent footings.

Organization and Solution: The computation for the problem was performed by an IBM 7040 computer. The flow diagram of computer operations is given in Appendix A. The input data and results for the settlement of each footing is given in Table III.

Results: Final differential settlements can be evaluated by comparing the independent settlements of adjacent footings.

footing 1 to 2 ...... 0.18 inches

footing 1 to 3 ...... 1.15 inches

footing 2 to 4 ...... 0.97 inches

DEPTH SCALE	SOIL TYPE	SOIL DIVISIONS AND STRATUM NUMBERS	DEPTH E(_)	DENSITY DEN(_)	CONSOLIDATION INDEX (C_c(_))	INITIAL VOID RATIO E(_)
	TOP SOIL	1	4 ft.	115.0	1.0	1.0
5 ft. 10	SOFT CLAY	2 Base of Footing	12	115.0	1.0	1.0
15		3 Water Table	15	115.0	0.41	1.065
20		4 Division Line	20	52.6	0.41	1.033
25		5	26	52.6	0.41	1.008
	SAND	6	28	60.0	0.0	1.0
30 35	SILTY CLAY	7 Division Line	35	63.6	0.22	1.511
40 45		8 = NFINL	45	63.6	0.22	1.491
	ROCK					

Data values not used in calculation. May read any value except zero.

Zero value of C_c(_). Assumes no compression of sand layer.

Figure 13. Physical Description of Input Data for Sample Problem

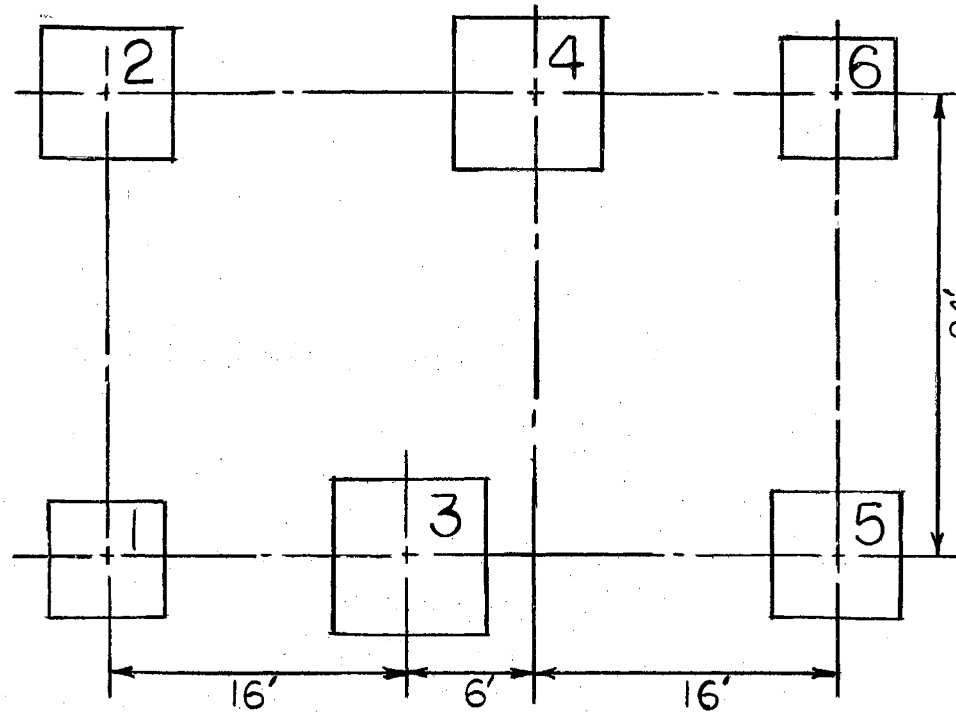


Figure 14. Plan View of Footings for Sample Problem

TABLE III

ORGANIZATION OF SAMPLE PROBLEM SOLUTION

Footing Number	Total Load	Square Dimensions	Final Independent Settlement
1	63,400.0 lb.	5.5 ft.	5.32 in.
2	82,900.0	6.5	5.50
3	125,900.0	8.0	6.47
4	125,900.0	8.0	6.47
5	82,900.0	6.5	5.50
6	63,400.0	5.5	5.32

APPENDIX C

SAMPLE SETTLEMENT PROBLEM CONSIDERING STRUCTURAL STIFFNESS

Problem Statement: The same group of footings and loads as used for the settlement analysis of Appendix B are used for the settlement analysis of Appendix C. The additional effect on settlement of the structural frame is considered. The geometry and moments of inertia for the frame are shown in Figure 15. As before, the objective of the solution is to evaluate structural stiffness.

Method of Solution: The solution is organized according to the procedures listed in Chapter III. The steps of the solution are illustrated by Tables IV and V. For the final answers the solution matrix can be formulated directly from Table V. As a check on the accuracy of the calculations, the sum of the independent settlements were compared to the sum of the settlements for the case of considering structural stiffness.

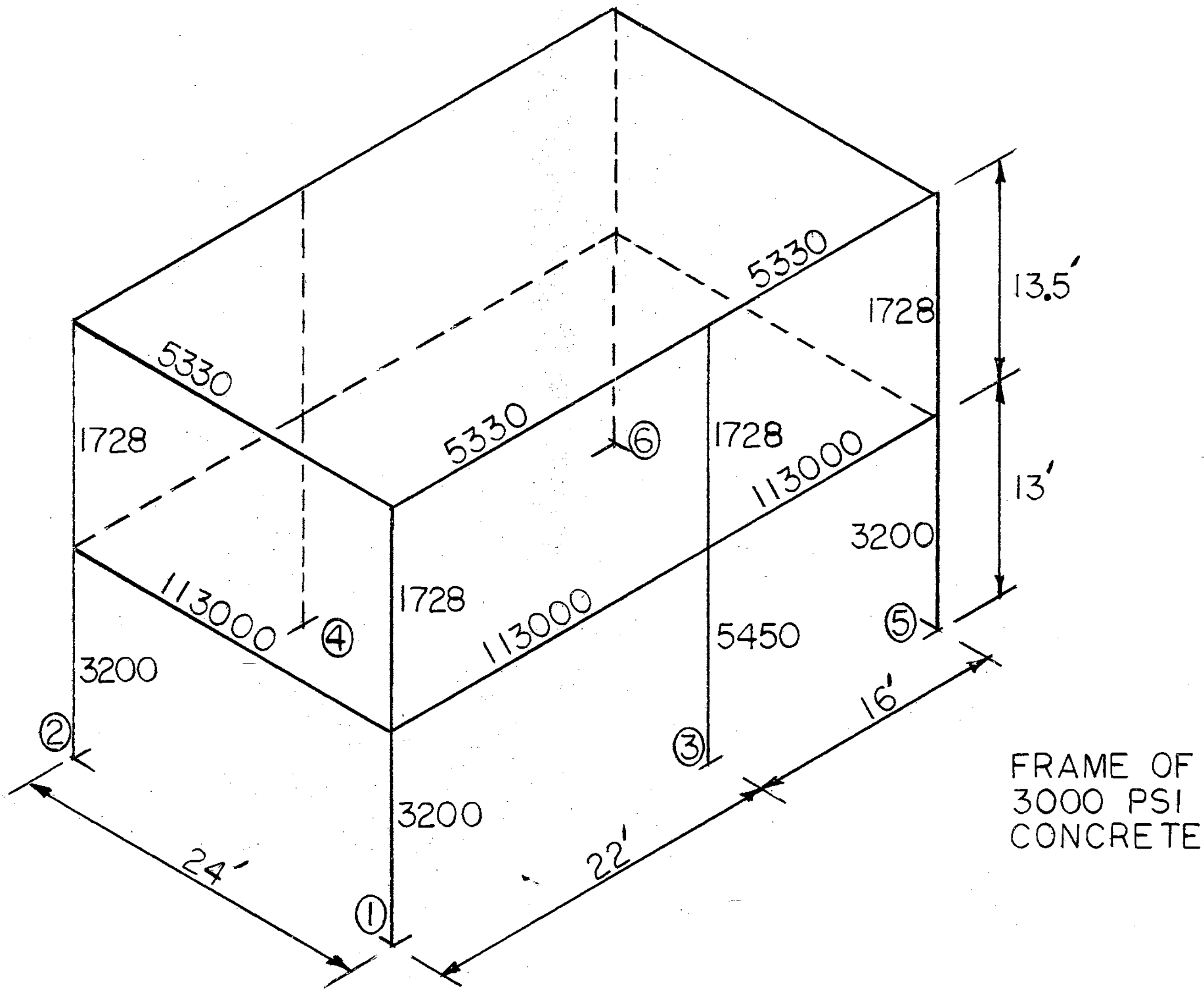


Figure 15. Geometry and Moments of Inertia for Sample Problem Frame

TABLE IV

ORGANIZATION OF COMPUTATION FOR DIFFERENTIAL SETTLEMENTS CONSIDERING STRUCTURAL STIFFNESS

Footing Number	Column Loads For Case of No Settlement (lbs.)	Column Reactions Due to a One-Inch Deformation at Column:					
		No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	63,400.	-100,915.	11,444.	137,903.	0	-48,431.	0
2	82,900.	11,444.	-57,743.	0	94,731.	0	48,431.
3	125,900.	137,902.	0	-234,634.	0	94,730.	0
4	125,900.	0	94,730.	0	-232,634.	0	137,902.
5	82,900.	-48,431.	0	94,731.	0	-57,743.	11,444.
6	63,400.	0	-48,431.	0	137,903.	11,444.	-100,915.

TABLE V

FORMATION OF EXAMPLE PROBLEM SOLUTION MATRIX

Footing Number	Settlement Due to Column Loads (in.)	Settlements Produced by Reaction Loads From Table IV at:					
		No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	-5.318	-7.757	.891	10.537	.0129	-3.680	-.0033
2	-5.503	.719	-3.644	.00994	5.941	.00032	-3.027
3	-6.470	6.586	.0435	-11.151	-.070	4.542	.0479
4	-6.470	.0479	4.542	.0698	-11.151	.0435	6.586
5	-5.503	-3.027	.00032	5.941	.00994	-3.644	.719
6	-5.318	- .0033	-3.680	.0129	9.312	.891	-8.757

Solution: The settlements of each footing are as follows:

Footing Number	Independent Settlement Solution	Solution Considering Stiffness
1	5.32 inches	5.84 inches
2	5.50	5.82
3	6.47	5.88
4	6.47	5.88
5	5.50	5.82
6	5.32	5.83
	35.58	35.08 Sums.

The final differential settlements when considering the effect of the structural frame are:

Footing 1 to 2 ...... 0.02 inches

Footing 1 to 3 ...... 0.04 inches

Footing 2 to 4 ...... 0.06 inches.

VITA

John Alan Lamb

Candidate for the Degree of

Master of Architectural Engineering

Thesis: DIFFERENTIAL SETTLEMENT ANALYSIS OF BUILDING FRAMES BY DIGITAL COMPUTER METHODS

Major Field: Architectural Engineering

Biographical:

Personal Data: Born in Pampa, Texas, January 3, 1942, the son of Mr. and Mrs. Fred I. Lamb.

Education: Graduated from Muskogee Central High School, Muskogee, Oklahoma, in May 1960; attended Northeastern State College, Tahlequah, Oklahoma, in 1962 and 1963; received Bachelor of Architectural Engineering from Oklahoma State University in May 1966; completed requirements for the Master of Architectural Engineering degree at Oklahoma State University, in May 1968, as a National Science Fellow.

Professional Experience: Engineer in Training, Johns Engineering Co., Summer 1964; Construction work, Manhattan Construction Co., Summer 1965; graduate research assistant, School of Architecture, Oklahoma State University, 1966-67.