ANALYSIS OF LOW FLOWS BY
STATISTICAL METHODS

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CHAPTER I

INTRODUCTION

A. General

The United States Weather Bureau has defined a drought as a "lack of rainfall so great and long continued as to affect injuriously the plant and animal life of a place and to deplete water supplies both for domestic purposes and for the operation of power plants, especially in those regions where rainfall is normally sufficient for such purposes". However, the lack of rainfall in certain areas does not necessarily indicate a drought if the streamflows and groundwaters in the area are derived from rainfall in distant places.

During periods of deficient precipitation the deviation from normal conditions is greater for streamflow than for rainfall.

The quantity of moisture drawn from storage by evaporation and transpiration increases during periods of droughts. This is reflected by lower water levels in shallow and deep wells and the decrease in reservior storage. High temperatures aggravate the situation by increasing the transpiration and evaporation requirements.

According to Chow (9) the percentage of years that annual precipitation has been less than demands for evaporation and transpiration is about thirty to forty percent for the area under study, northeastern Oklahoma.

The severity of droughts may be measured by various parameters: deficiencies in rainfall and runoff, decline in soil moisture, reduction in groundwater levels, and the storage required to meet prescribed drafts or demands.

The effects of extended dry periods on reserviors in many cases, can be disastrous. Of course it is impossible to provide storage sufficient to meet low-flow hydrologic risks of great rarity. The custom is to design a reservoir for a stated risk and to add a reserve storage allowance. Extraordinary drought can be met by cutting the draft rate of the reservoir. The need for providing adequate storage cannot be overemphasized. The following study will concern the effects of droughts on streamflow characteristics.

B. Justification of This Research

The analysis of low flows from the Arkansas River and Poteau River will provide the Arkansas-Oklahoma Compact Committee with the necessary information to (1) competently make reasonable assumptions on establishing the safe or firm yield of reservoirs; and (2) make rational decisions on the appropriations of water for the states of Oklahoma and Arkansas.

Another aspect of this research was to examine the effectiveness of synthesized data for analyzing low flows of selected durations.

C. Objectives

The primary objective of this drought study was to provide the Oklahoma-Arkansas Compact Committee with reliable information about low flows of the Arkansas River and the Poteau River.

It was hoped that the methods used for analysing low flows will give some indication of flows over consecutive periods of time. In other words provide some indication of the possible magnitude of flows for different months, consecutive months, and over consecutive years.

A mathematical synthesis of streamflow sequences was examined for Station 2505 of the Arkansas River. The object of this research was to provide a hydrologically stable record, as well as, test the applicability of established methods of synthesizing data. It was hoped that the synthesized streamflow would provide data which could be analyzed in the same manner as the actual streamflow record. Another objective was to discuss and compare the results obtained from the synthesised flows and the actual flows.

A third objective was to analyze and discuss the effects of Wister Reservoir on the flows at Station 2485 of the Poteau River and to provide a method for synthesizing unregulated flow for the period of record which was affected by Wister Reservoir.

It is anticipated that this study will provide specific information about the magnitudes and probabilities of low flows for selected durations at Stations 2485 and 2505.

CHAPTER II

LITERATURE SURVEY

A. Methods of Analysing the Effects of Drought

Many hydrological phenomena are used in describing the effects of drought, some of these are, deficiencies in rainfall and runoff, decline in soil moisture, reduction in groundwater levels, and lack of storage required to meet prescribed drafts or demands (1).

Droughts are the result of cumulative deficiencies when individual rainfall records for days, months and sometimes years are insignificant. A cumulative plotting of rainfall or a mass diagram of runoff, will show the effect of extended dry periods. These curves may be constructed for the entire record or for several dry periods in order to describe the degree of drought severity. The mass-curve analysis is based on a finite period of record within which the sequence of occurrence of the events is assumed definite. However, it is highly doubtful the same sequence will occur again. Consequently, the mass-curve may be deceiving in accuracy. Analysing future runoff by stochastic characteristics may improve the method of analysing mass curves (3).

Early hydrologic practice depended on the analysis of hydrologic records to find the most severe period observed.

Reservoir design was then based on this single extreme period.

In recent years the evaluation of extreme drought severity has included estimates of the probability of occurrence of a drought of given severity and duration.

The several methods of analyzing drought frequencies and durations are based on the assumption that meteorological conditions recorded in the past will be repeated. The absence of long rainfall and runoff records, coupled with the effects of man made changes inhibit precise forecasts. However, the reliability of statistical methods has improved the forecasting of drought frequencies and storage requirements. Statistical methods were first used by Hazen to forecast drought frequencies and storage requirements in 1914 (8). In many instances, statistical methods prove to be more accurate for estimating the probable frequency or recurrence interval of a drought of stated severity than for forecasting the worst drought to be expected over a long period of years (9).

Statistical analysis as applied to hydrological engineering has a two fold purpose, (a) to estimate the future frequency or probability of hydrologic events based on information contained in hydrologic records and (b) to correlate related hydrologic variables (1). Statistical methods have been applied to rainfall and runoff variables in many reports and papers.

The simplest type of statistical analysis is a duration curve of rainfall or runoff. The weakness of this method is that it reveals nothing about the sequence of low flows nor whether the low flows occurred consecutively or randomly throughout a period of time. The analysis of low flows can be made more useful by determining the flows over a given period of consecutive days (9).

The fitting of a population of low flows to theoretical probability distributions have been investigated. Log-normal distributions, Pearson's distributions, and other distributions had been investigated for their applicability to analysis of low flows (10).

B. Sequential Generation of Hydrologic Events

The sequential generation of hydrologic information is a statistical method incorporating the Monte Carlo Method. The Monte Carlo Method refers to a process by which data is produced synthetically by some sampling technique or random number generator (4).

A Russian mathematician A. A. Markov (1856-1922) introduced the idea that the outcome of a trial depends only on the outcome of the trial immediately preceding it. This theory is known as the "Markov Chain" (2). An example of a Markov Chain model applied to hydrology is represented by the following equation:

$$X_t = rX_{t-1}(1-r)\bar{X} + S_x(1-r^2)e$$

Where

X_t = The runoff for a particular duration

X_{t-1} = The runoff for a particular duration immediately preceding X_t

$\bar{X}$ = The mean flow of a particular duration from historical records

S_x = The standard deviation of historical runoff

r = The Markov Chain coeficient

e = A random variate, normally distributed with a zero mean and unit variance

The concept of synthesis in hydrology is not new. Engineers such as Allen Hazen (8), Charles E. Sudler (13), H. A. Thomas, Jr., and Myron B. Fiering (15) have used the concept of synthesis in hydrology.

Thomas and Fiering (15) used the Markov Chain model. They applied this model to generate monthly flow by serial correlation. The following equation was used:

$$Q_{i+1} = \bar{Q}_{j+1} + B_j(Q_i - \bar{Q}_j) + S_{j+1}(1-r_j^2)^{1/2}e_i$$

Where

Q_i & Q_{i+1} = The discharges during the ith and (i+1)st month respectively

$\bar{Q}_j$ & $\bar{Q}_{j+1}$ = The mean monthly discharges during the jth and (j+1)st month respectively

B_j = The regression coefficient for estimating flow in the (j+1)st month from the flow in the jth month

S_{j+1} = The standard deviation of flow in the (j+1)st month.

r_j = The correlation coefficient between the flows of the jth and (j+1)st months

e_i = A random normal deviate with zero mean and unit variance

The statistical values will be thoroughly discussed in Chapter IV Part C.

Thomas and Fiering's method of synthesising streamflow sequences has certain advantages over the methods used by Hazen and Sudler, in that it can be used for weekly, monthly or seasonal flow. In addition it does not require the flow data to be a normal distribution and may be used with skewed distributions as well.

It is the function of mathematical synthesis to create the critical patterns of low and high events that are absent from the brief observed record of hydrological events, as runoff. Based on statistical methods, these critical patterns would be expected to be included if the actual record were as long as a mathematically generated record.

CHAPTER III

MORPHOLOGY OF THE DRAINAGE BASINS

A. Poteau River Basin

This basin, Fig. 1, of this chapter has four gaging stations located within the basin.

Station 2470 on the Poteau River has 198 square miles of drainage area, twenty-seven years of record and is located near Cauthron, Arkansas.

Station 2475 on the Fourche Maline has 121 square miles of drainage area, twenty-seven years of record, and is located near Red Oak, Oklahoma.

Station 2494 on the James Fork has 148 square miles of drainage area, seven years of record and is located near Hackett, Arkansas.

Station 2485 was selected for the drought study because of the following reasons (1) it has a large drainage area, (2) the length of record being twenty-seven years (Feb. 1939-Sept. 1965), and (3) the gaging records were reasonably good throughout the period of record. However, Wister Reservoir was constructed about 1.2 miles upstream from Station 2485, and came into effect during the water year of 1948-1949. The effects of regulated flow on the drought study will be discussed in the results, Chapter V.

The Poteau River and Fourche Maline above Wister Reservoir are largely mountainous or hilly terrain with numerous tributary streams. This characteristic divides the flood plain into many areas, each having its individual problems. The mountainous terrain is typical of most of the Poteau River Basin. Rocky, impervious soils and steep slopes of the tributary drainage areas indicate quick runoff.

Rainstorms over the basin are normally of long duration and high intensity; storms occur frequently in the spring, late fall, and winter months. The normal annual precipitation over the basin is about 44 inches (14).

B. Arkansas River Basin

Gaging Station 2505, Fig. 1, of this chapter, was selected for the drought study. This station is located below the future site of the Robert S. Kerr Lock and Dam near the Arkansas-Oklahoma border. The drainage area is 150,483 square miles, which is highly regulated, and the period of record is from October 1927, through September 1965, about 38 years of reasonably good records.

Because of the large drainage area and the length of the Arkansas River, the climatological factors vary greatly and none will be presented in this study.

The type of runoff varies from quick runoff due to steep slopes and impervious rocks of the Colorado mountains to moderate runoff in Kansas and Oklahoma.

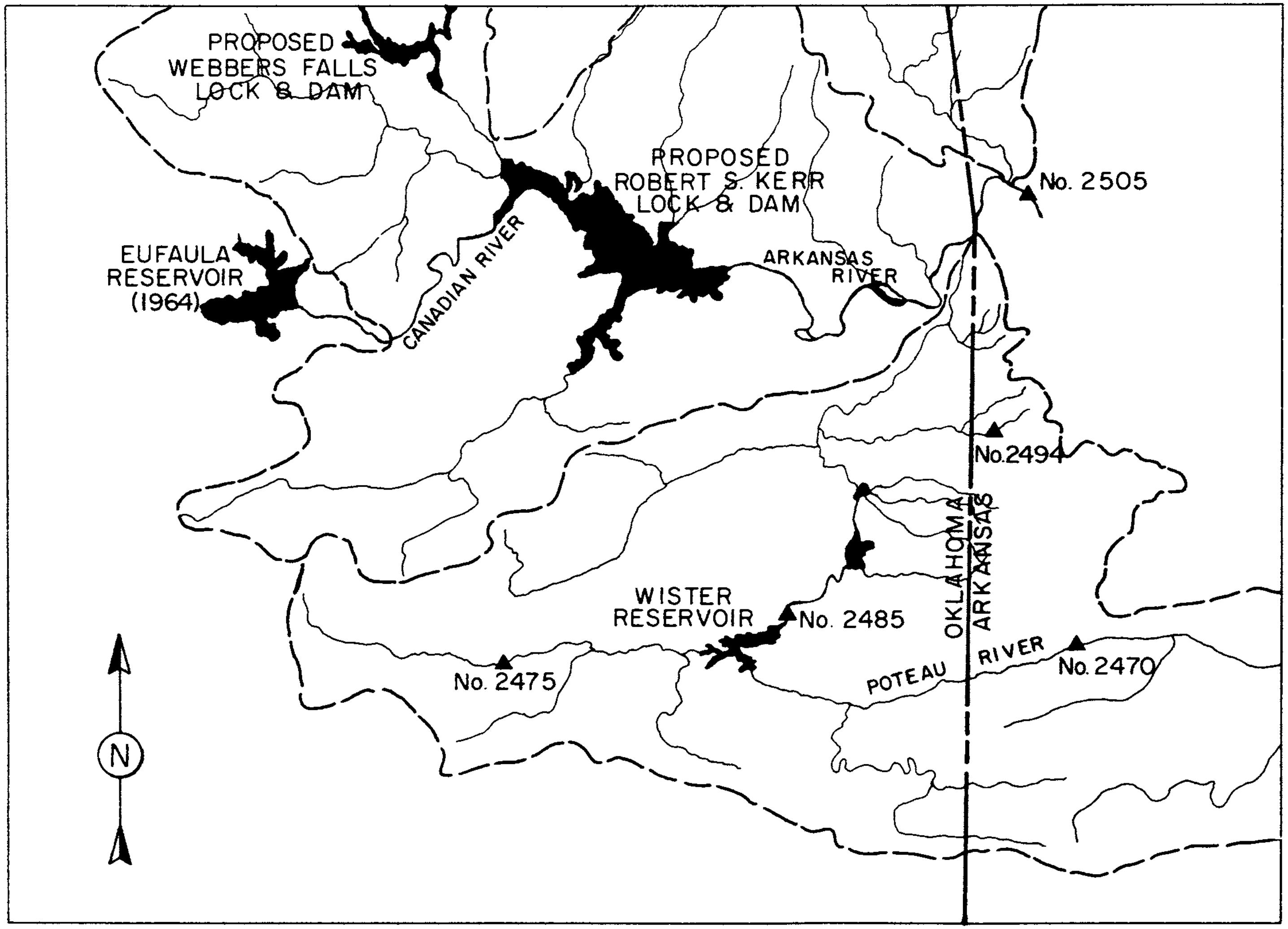


Fig. 1 Map of the Area Under Study

CHAPTER IV

THEORETICAL CONSIDERATIONS

A. Mathematical Synthesis of Streamflow Sequences

The relative brevity of existing streamflow records (almost always) impairs the precision of the final designs of river basin development when the records are analysed by simulation techniques. It is unusual for even fifty years of recorded observations to have had a stable hydrologic system. Even this long a record may lack a critical sequence of years of low or high regional runoff. If the most severe droughts or floods are not representative of the statistical population, then it is obvious that design techniques will be misrepresented (15). Therefore, with only 38 years of existing records at Station 2505, it is reasoned that the existing data is not sufficiently stable for design techniques.

A short record such as those in existence will probably not identify a true frequency of yearly or seasonal periods of unusually low or high flow. However, a reasonably short period of the magnitude representing Station 2505 will give a good approximation of the mean annual and mean seasonal flows and their variances. These variables will be used to mathematically synthesize a five hundred year monthly streamflow sequence or rather a streamflow hydrograph.

B. Method of Generating Synthetic Monthly Streamflow Sequences

Using statistical parameters, concerning the population of flows, it is possible to construct a stochastic model that will generate synthetic flow sequences for any desired length of time.

The synthesis of monthly streamflow sequences uses serial correlation of monthly flows at a given station. Twelve sets of correlation coefficients are computed for consecutive months from the observed record by the least-squares method of linear regression analysis. The method of linear regression analysis will be discussed in Chapter IV, part C. The purpose of this method is to relate the discharge during any month to the discharge of the month immediately preceding it. The term "serial correlation" connotes a month to month relationship associated with seasonal fluctuations of discharge. This relationship induces a slight year to year correlation in the synthesized flows, which correlates with those found in the observed flows.

Thomas and Fiering's method of generating streamflow sequences, Chapter II, Part B, was employed because it seemed suitable for generating periods of low flow.

C. Statistical Methods and Analysis

1. Correlation Analysis

The least square line and regression coefficient approximating the set of points (X_1, Y_1), (X_2, Y_2) ....., (X_n, Y_n) has the equation :

$$Y = A_0 + A_1 X \qquad (1)$$

Where the constants A_0 and A_1 can be determined by simultaneously solving the following equations:

$$\sum Y = A_0 N + A_1 \sum X \qquad (2)$$

$$\sum XY = A_0 \quad X + A_1 \sum X^2 \qquad (3)$$

Where X and Y are the independent and dependent variables respectively; and N is the number of pairs of observations. Equation (2) and (3) are the normal equations for the least square line.

By simultaneous substitution:

$$A_0 = \frac{(\sum Y)\ (\sum X^2) - (\sum X)\ (\sum XY)}{N \sum X^2 - (\sum X)^2} \qquad (4)$$

$$A_1 = \frac{N \sum XY - (\sum X)\ (\sum Y)}{N \sum X^2 - (\sum X)^2} \qquad (5)$$

By statistical methods it can be shown that:

$$y = \left(\frac{\sum xy}{\sum x^2} \right) x \qquad (6)$$

Where $x = X - \bar{X}$ and $y = Y - \bar{Y}$. $\bar{X}$ and $\bar{Y}$ are the arithmetic mean values of the variables. This can now be put into the form:

$$Y = \bar{Y} + B\ (X - \bar{X}) \qquad (7)$$

Where $B = \frac{\sum xy}{\sum x^2}$, B is known as the regression coefficient.

2. <u>Correlation Coefficient</u>

The total variation of Y is defined as $\sum(Y-\bar{Y})^2$, that is, the sum of the squares of the deviations of the values of Y from the mean $\bar{Y}$.

$$\sum(Y-\bar{Y})^2 = \sum(Y - Y_{est.})^2 + \sum(Y_{est.} - \bar{Y})^2 \qquad (8)$$

The first term $\sum(Y - Y_{est.})^2$ is the unexplained variation while $\sum(\text{Yest.}-\bar{Y})^2$ is the explained variation.

The coefficient of correlation is the ratio of the square root of the explained variation to the total variation. When there is zero explained variation then the total variation will be unexplained, and the ratio will be zero. The total variation will be explained, when zero unexplained variation occurs, consequently the ratio will be one. In other cases the ratio lies between zero and one. The quantity r, called the product moment correlation coefficient will be:

$$r = \mp\sqrt{\frac{\text{explained variation}}{\text{total variation}}} = \mp\sqrt{\frac{\sum(\text{Yest.}-\bar{Y})^2}{\sum(Y-\bar{Y})^2}} \qquad (9)$$

and r bounded by -1 and +1.

If a linear relationship between two variables is assumed then the correlation coefficient equation (9) becomes:

$$r = \frac{N\sum XY - (\sum X)(\sum Y)}{\sqrt{\left[N\sum X^2 - (\sum X)^2\right]\left[N\sum Y^2 - (\sum Y)^2\right]}} \qquad (10)$$

This formula, which automatically yields the proper sign of r, is called the product-moment formula and clearly shows the symmetry between X and Y.

3. <u>The Standard Error of Estimate</u>

If Yest. represents the value of Y for given values of X as estimated from $Y = A_0 + A_1X$, the measure of the scatter about the regression line of Y on X is supplied by the quantity:

$$S_{Y.X} = \sqrt{\frac{\sum(Y-\text{Yest.})^2}{N}} \qquad (11)$$

N-2

which is called the standard error of estimate of Y on X.

The standard error of estimate has properties analogous to those of the standard deviation. For example, if lines are constructed parallel to the regression line of Y on X at respective vertical distances $S_{Y.X}$, $2S_{Y.X}$, and $3\ S_{Y.X}$, one should find between these lines about 68.3%, 95.5%, 99.7% of the sample points, provided the sample population N is large enough.

By the use of correlation coefficient, equation (9) and equation (11) and the fact that the standard deviation of Y is:

$$S_Y = \sqrt{\frac{\sum(Y-\bar{Y})^2}{N-1}} \qquad (12)$$

We can find that the correlation coefficient can be written, excluding the sign, as:

$$r = \sqrt{1-\frac{S^2_{Y.X}}{S^2_Y}} \qquad \text{or} \qquad S_{Y.X} = S_Y\sqrt{1-r^2} \qquad (13)$$

For the case of linear correlation the quantity r is the same whether X or Y is considered the independent variable. From the previous explanation it could be that r is a very good measure of the linear relationship between the two variables.

4. Random Numbers

Random numbers are chosen at random from a universe which has a normal distribution and are found in tables as well as on computer tapes. Tables of random numbers are quite lengthy and none will be reproduced in this thesis. These tables may be found in many statistics books.

The random numbers used in this thesis have a mean of zero and a variance $\sigma^2=1.00$. The numbers of this universe have values near zero occurring fairly frequently, and numbers farther away from zero occurring less and less frequently.

5. Student's t Distribution

A study of sampling distributions of statistics for small samples is called the "small sampling theory." Samples of size N 30 are called small samples, where N is the number of paired observations. However, the results obtained are true for large as well as small samples.

6. Sampling Theory of Correlation

The N pairs of values (X,Y) of two variables can be thought of as a sample from a population of all possible such pairs. Since two variables are involved this is called a "bivariate population" which is assumed to be a "bivariate normal distribution."

The population coefficient of correlation, P, can be estimated by the sample correlation coefficient r. For P=0 a symmetric distribution occurs and a statistic involving students distribution can be used.

The test of Hypothesis P=0 uses the fact that the statistic:

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}} \qquad (14)$$

has student's distribution with N-2 degrees of freedom. In testing the significance of an observed correlation, the probability that such a correlation should arise, by random sampling, from an uncorrelated population was calculated. If the probability is low one can regard the correlation as significant. By using a table of student "t" values an exact test can be made.

As done with normal distributions 95%, 99% or other confidence intervals can be defined by a table of the "students t" distribution. This will enable one to estimate the population mean μ within the specified limits of confidence. For example, if $-t_{.975}$ and $t_{.975}$ are the values of t for which 2.5% of the area lies in each "tail" of a two-tailed t distribution, then 95% is the confidence interval for t; and the probability that a correlation would arise from an uncorrelated population is 0.05.

Given two variants (X,Y) over a period of observations N, a correlation coefficient, r, can be calculated and by using equation (14) test statistic (student t) may be calculated for the correlation. By using the "student t" table, the confidence interval for the particular correlation of the two variants (X,Y) over their range of observation could be established.

The preceding statistical theory can be found in any basic statistics book. The references used for the theory can be found under the heading of selected bibiliography as references 5, 6, and 12.

D. Methods of Analyzing Low Flows

The basic data used in the analysis were low flows for selected durations.

The periods of time or durations selected for the study were seven days, thirty days, six months and one, two, three, and five years. The flows were averaged over the selected period of duration.

The flows for seven day duration consisted of the lowest possible combination of successive daily flows averaged over the seven day period for each month of the entire record. The mean monthly flow was selected as the flow for thirty day duration.

At Station 2485 the six month periods from June through November and from December through May were selected to represent durations of six months. The period from June through November had low flows; and high flows usually occurred from December through May. At Station 2505 July through December the flows were characteristically low, and January through June exhibited relatively high flows.

The mean annual flow was selected as the average flow for a duration of one water year. Durations of two, three and five year periods were studied by taking successive additions of the mean annual flows that occurred in the period of record for a particular duration. In other words, there will be N-1 flows of two year duration, N-2 flows of three year duration, and N-4 flows of five year duration, where N is the length of the record in years. For each duration the flows are ranked lowest flow, number 1, the second lowest flow, number 2, and so on, until all flows have a rank.

The recurrence interval used in plotting the low flows is computed from the formula (7):

$$\text{Recurrence interval} = \frac{N+1}{M}$$

Where N = length of record in years

M = the rank

The probability of occurrence was the percent of time the flow was less than or equal to a selected flow, and was the inverse of the recurrence interval.

The type of duration curve used in this study is a two variable plot of discharge versus recurrence interval and/or the discharge versus the probability of occurrence. The duration curves in this study were best portrayed by a smooth curve on log-normal plot.

E. Method of Synthesizing Unregulated Flow

Regulation of streamflow by a reservoir will considerably affect the characteristics of the natural flow. The synthesis of unregulated flow by reliable methods will remove the effects of regulation by a reservoir.

The records for Station 2485 show ten years of unregulated flow and seventeen years of regulated flow by Wister Reservoir. Before an effective analysis of droughts can be made, the regulating effects of the reservoir must be removed.

The following method incorporates the continuity equation. By adding the flows of Stations 2470 and 2475 for a particular month and multiplying the flow by a coefficient, the flow at Station 2485 for that same month can be synthesised. The coefficient was established by taking the average ratio of the flow at Station 2485 to the sum of the flows at Station 2470 and 2475 for each month for the ten year period of unregulated flow. Twelve monthly coefficients were determined by this procedure.

A computer program, Appendix A was used to synthesize the unregulated flow for the remaining seventeen years of record.

CHAPTER V

RESULTS

A. Poteau River Basin at Station 2485

1. Synthesis of Unregulated Flows

The historical records indicate that Wister Reservoir began regulating flow in the water year of 1948-1949. Therefore, the historical record (1938-1965) has seventeen years of regulated flow and ten years of unregulated flow. By incorporating the procedure as described in Chapter IV, part E, seventeen years of unregulated mean monthly flows were synthesized. The synthesized data was calculated using an IBM 1620 Computer (for program see Appendix A). These unregulated flows were used to analyze low flows, part A-2 of this chapter. For simplicity the historical or actual flows will be designated "regulated" flows; and the unregulated flows both actual and synthesized records will be called "unregulated" flows.

A comparison of the average mean monthly flows for the historical record and the synthesized record is presented in Table I of this chapter.

2. Analysis of Low Flows

The methods used for analysing low flows by duration curves were presented in Chapter IV, part D. The durations selected were seven days, thirty days, six months, one, two, three, and five years. Table II contains the data for the duration curve analysis from historical flow. Table III contains the necessary information for an unregulated flow-duration curve analysis.

The duration curves, Figures 2-13, show flow-duration curves for seven day and thirty day periods. The thirty day duration curves for both regulated and unregulated flows were presented together to emphasize their differences. These curves were presented on log-normal graph paper to obtain a smooth curve. Also, the duration curves for six months, one, two, three, and five year duration, Figures 14-19, were presented on probability paper to obtain a smooth curve.

TABLE I

AVERAGE OF MEAN MONTHLY FLOWS FOR EXISTING RECORD
STA. 2485 - POTEAU RIVER

MONTH	ACTUAL RECORD MEAN FLOW - cfs.	UNREGULATED RECORD MEAN FLOW - cfs.
Oct.	304	225
Nov.	661	829
Dec.	924	877
Jan.	1172	1477
Feb.	1928	1926
Mar.	1844	2141
Apr.	2062	2521
May	2259	2716
June	1052	1018
July	480	478
Aug.	340	176
Sept.	252	468

TABLE II

DATA FOR DURATION CURVE ANALYSIS
STA. 2485 - POTEAU RIVER
REGULATED FLOW

RANK	PROBABILITY %	RETURN PERIOD YEARS	DISCHARGE IN CFS. FOR INDICATED DURATIONS 1 YEAR	2 YEAR	3 YEAR	5 YEAR	6 MONTHS JUNE-NOV.	6 MONTHS DEC.-MAY
1	3.57	28.00	200	331	438	717	10	396
2	7.14	14.00	269	444	549	777	12	440
3	10.70	9.33	327	541	550	840	22	492
4	14.30	7.00	335	557	668	883	25	511
5	17.85	5.60	427	590	756	894	63	840
6	21.40	4.67	539	656	801	909	85	852
7	25.00	4.00	687	660	862	957	91	1108
8	28.60	3.50	813	966	991	961	133	1353
9	32.10	3.21	861	981	1000	968	155	1370
10	35.70	2.80	921	989	1032	984	173	1372
11	39.20	2.55	947	1068	1094	1095	188	1492
12	42.90	2.34	984	1075	1144	1104	215	1597
13	46.30	2.16	985	1128	1181	1169	257	1679
14	50.00	2.00	1101	1190	1193	1302	272	1695
15	53.50	1.87	1146	1234	1196	1318	290	1720
16	57.20	1.75	1233	1239	1234	1482	323	1822
17	60.60	1.65	1322	1279	1241	1536	358	1990
18	64.50	1.55	1438	1286	1313	1592	505	1991
19	68.00	1.47	1460	1449	1354	1688	532	2011
20	71.40	1.40	1471	1539	1816	1734	670	2190
21	75.20	1.33	1570	1610	1827	1753	869	2119
22	78.80	1.27	1611	1747	1849	1998	876	2229
23	82.00	1.22	1618	1915	1863	2044	1153	2358
24	85.50	1.17	1650	2001	1957		1296	2483
25	89.30	1.12	2085	2210	2129		1315	2650
26	92.50	1.08	2370	2369			1923	2691
27	96.40	1.04	3168				1929	4732

TABLE III

DATA FOR DURATION CURVE ANALYSIS
STA. 2485 - POTEAU RIVER
UNREGULATED FLOW

RANK	PROBABILITY %	RETURN PERIOD YEARS	DISCHARGE IN CFS. FOR INDICATED DURATIONS 1 YEAR	2 YEAR	3 YEAR	5 YEAR	6 MONTHS JUNE-NOV.	6 MONTHS DEC.-MAY
1	3.57	28.00	222	445	465	650	9	429
2	7.14	14.00	270	491	677	730	10	440
3	10.70	9.33	416	548	680	828	34	510
4	14.30	7.00	423	608	684	855	47	680
5	17.85	5.60	466	794	786	884	63	823
6	21.40	4.67	727	806	804	900	85	1142
7	25.00	4.00	759	1045	853	904	92	1252
8	28.60	3.50	825	1077	886	958	128	1455
9	32.10	3.21	934	1088	984	1008	155	1492
10	35.70	2.80	945	1090	1095	1025	173	1580
11	39.20	2.55	1049	1095	1130	1060	215	1620
12	42.90	2.34	1073	1119	1145	1090	257	1650
13	46.30	2.16	1133	1184	1234	1115	323	1679
14	50.00	2.00	1145	1187	1236	1150	374	1822
15	53.50	1.87	1164	1231	1318	1305	375	1990
16	57.20	1.75	1240	1315	1380	1348	404	1991
17	60.60	1.65	1246	1320	1423	1384	416	2060
18	64.50	1.55	1329	1347	1424	1550	507	2190
19	68.00	1.47	1581	1493	1426	1545	524	2280
20	71.40	1.40	1640	1509	1520	1695	670	2358
21	75.20	1.33	1644	1613	1590	1740	760	2483
22	78.80	1.27	1718	1653	1672	1760	995	2975
23	82.00	1.22	1773	2024	1855	2610	1000	3035
24	85.50	1.17	1912	2045	2005		1153	3080
25	89.20	1.12	2371	2218	2139		1385	3190
26	92.50	1.08	2408	2388			1755	3850
27	96.40	1.04	3195				1929	4732

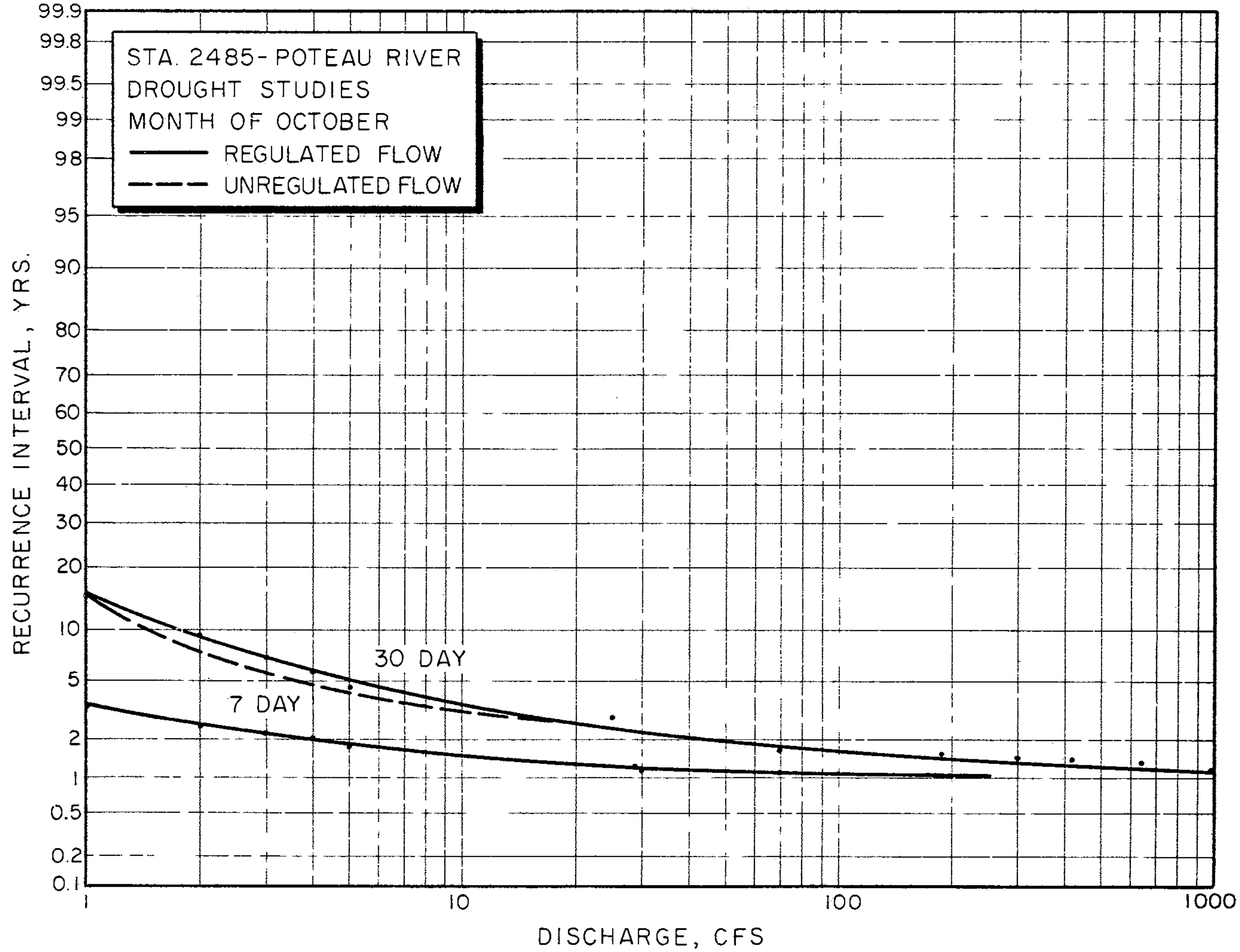


Fig. 2 Duration Curves for October at Station 2485

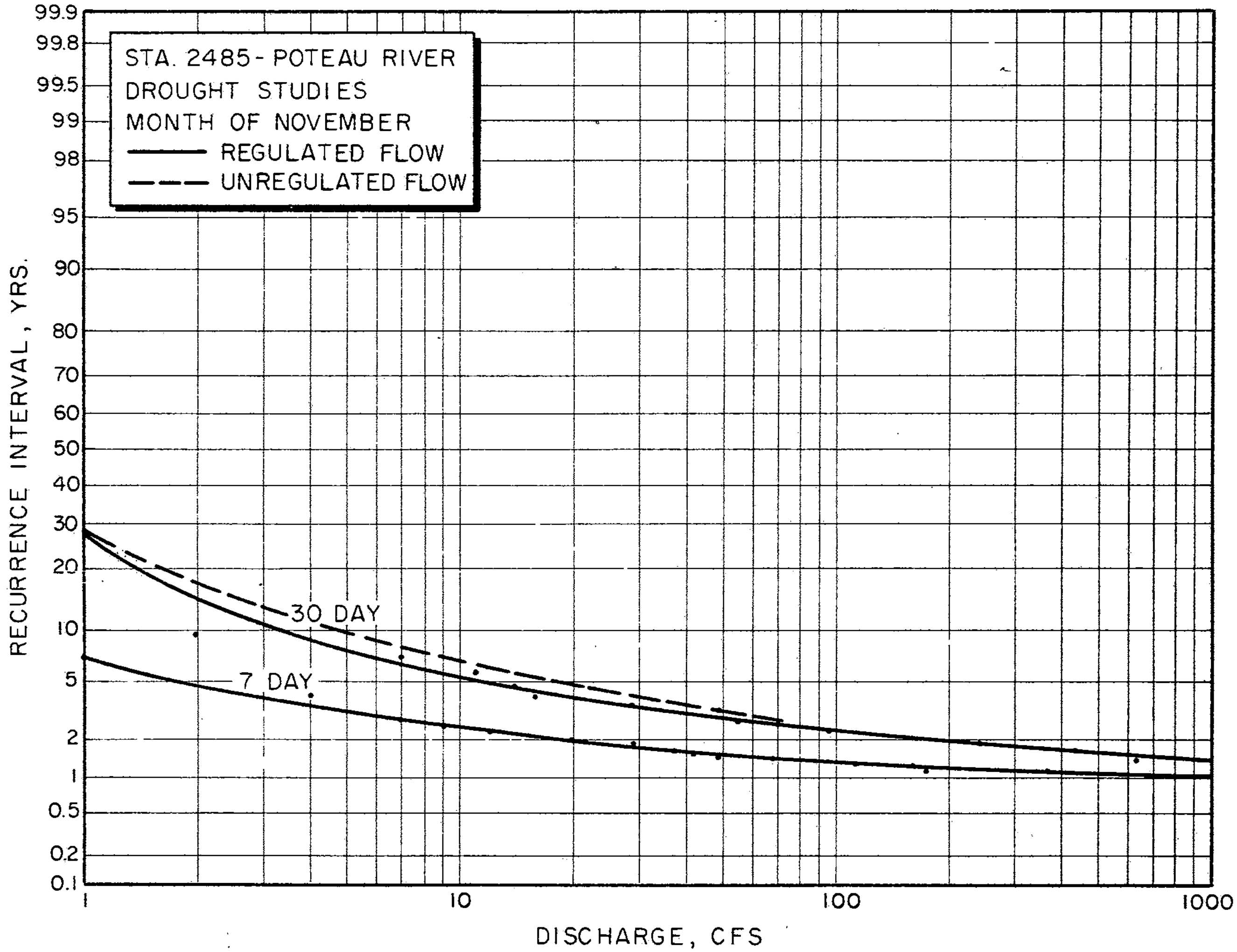


Fig. 3 Duration Curves for November at Station 2485

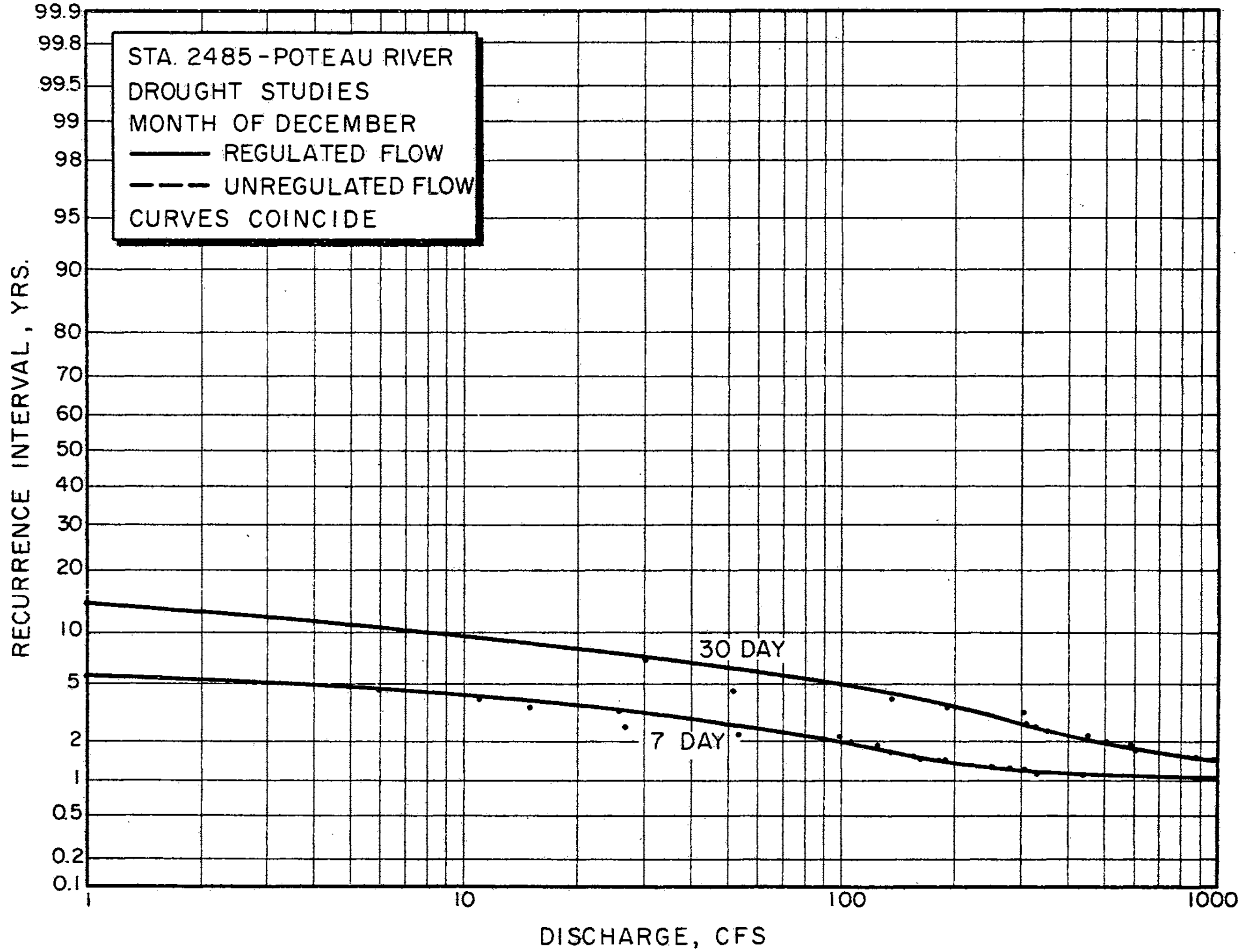


Fig. 4 Duration Curves for December at Station 2485

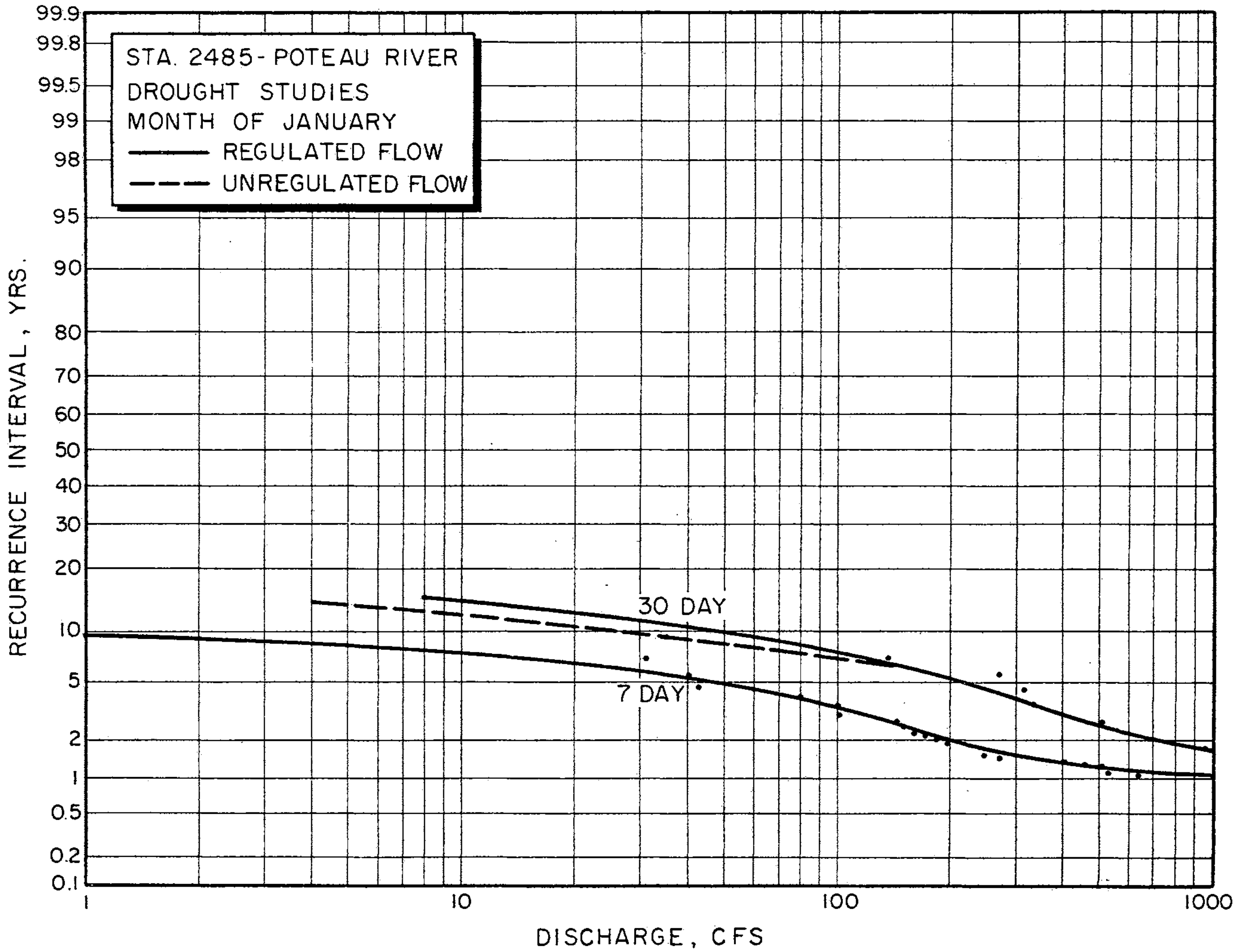


Fig. 5 Duration Curves for January at Station 2485

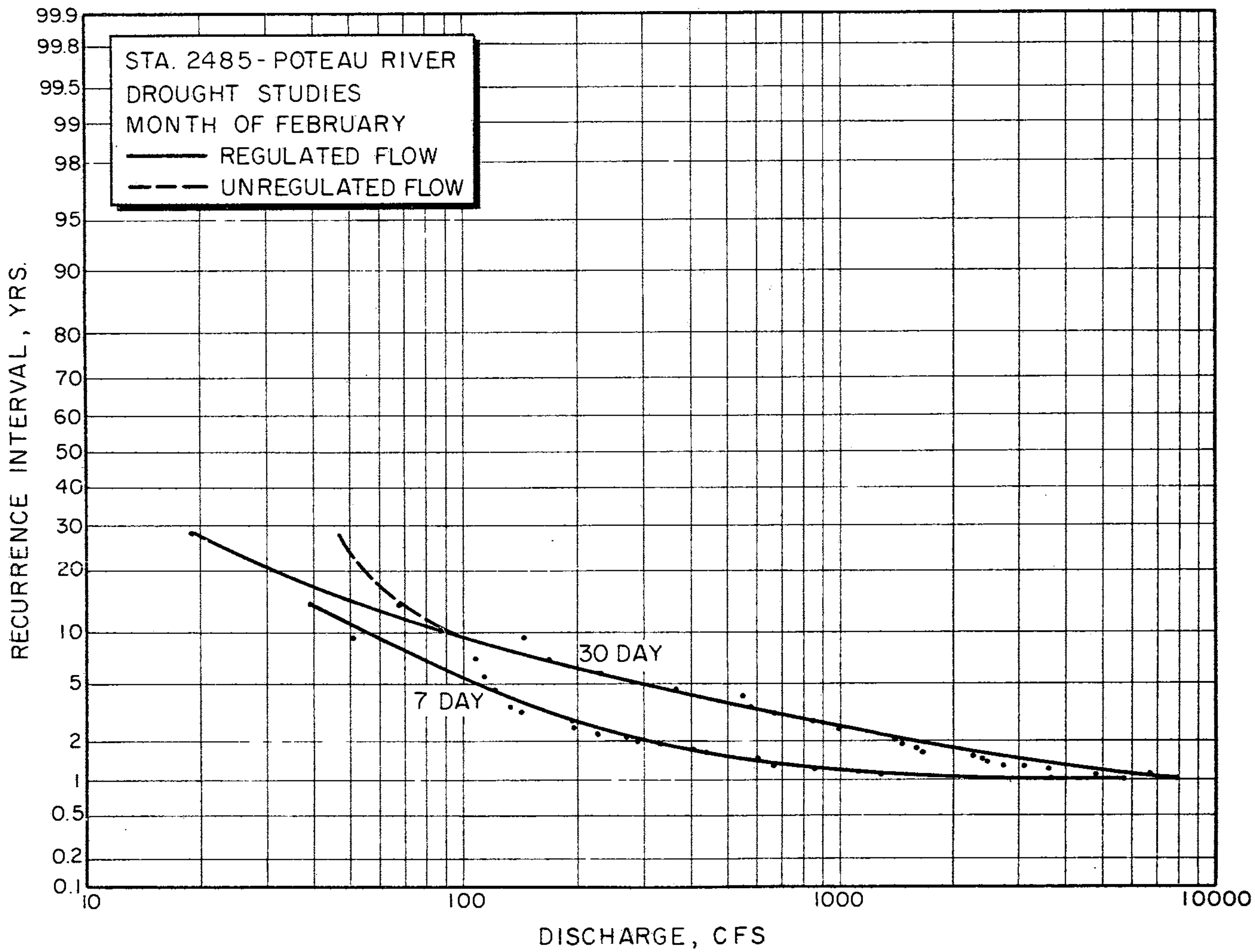


Fig. 6 Duration Curves for February at Station 2485

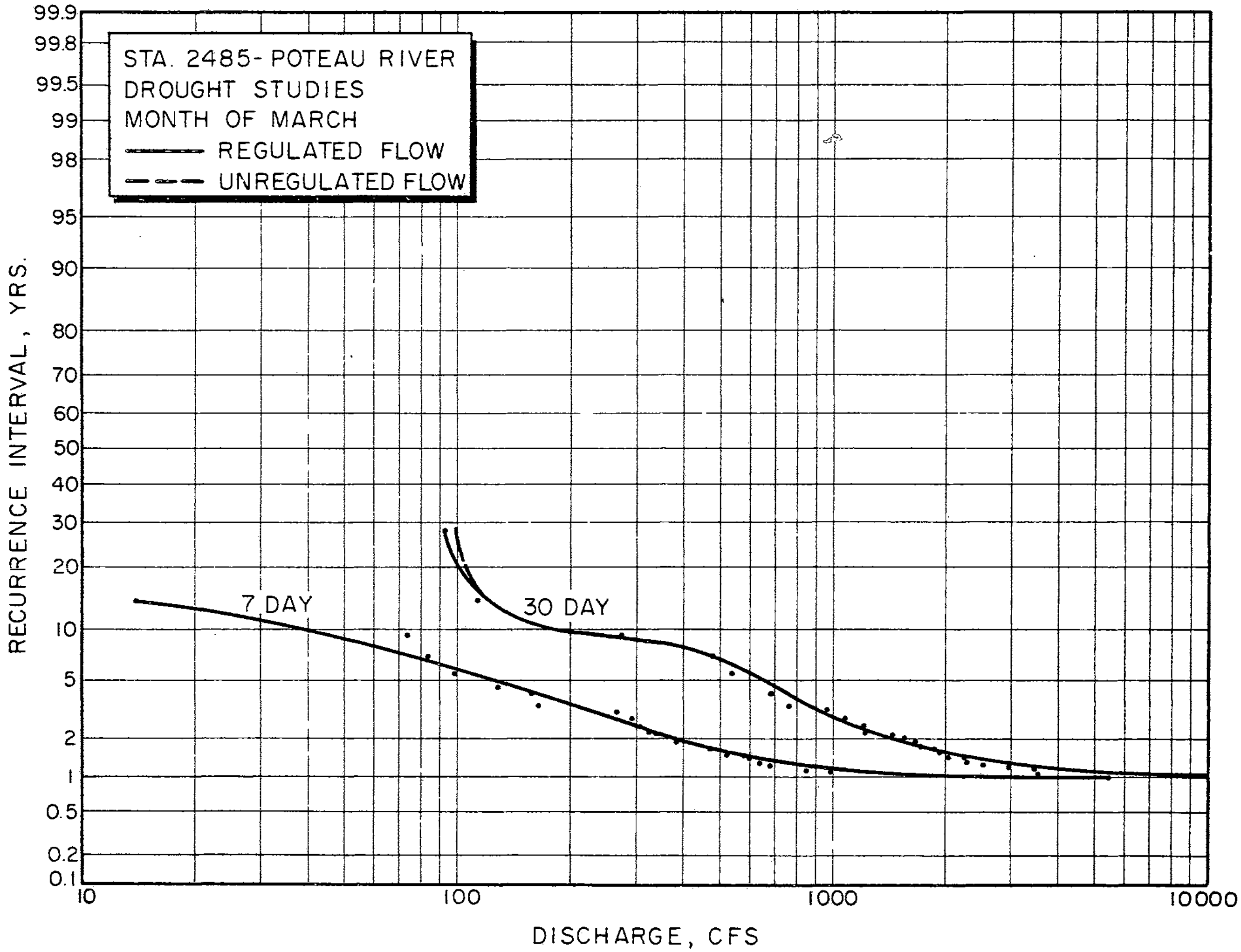


Fig. 7 Duration Curves for March at Station 2485

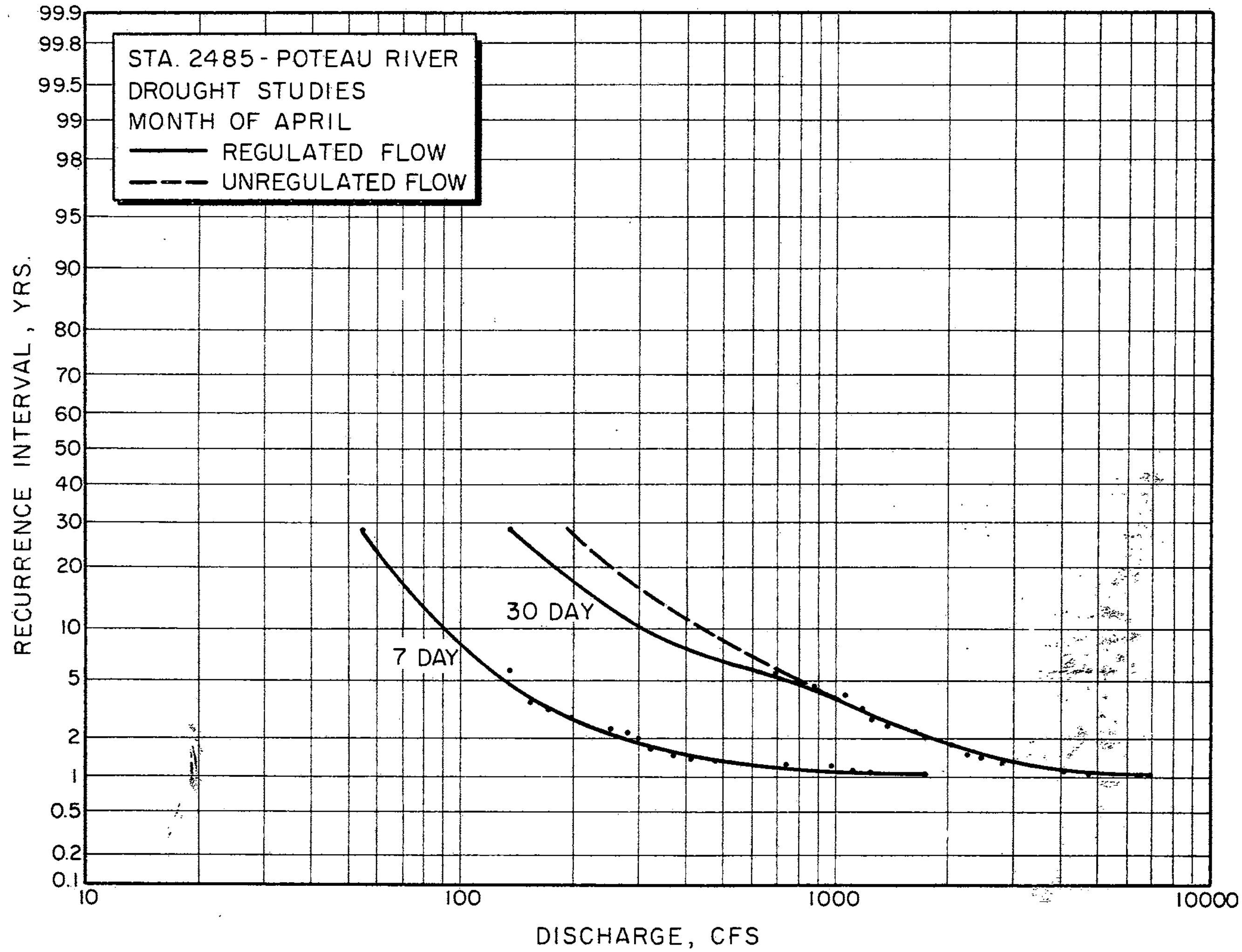


Fig. 8 Duration Curves for April at Station 2485

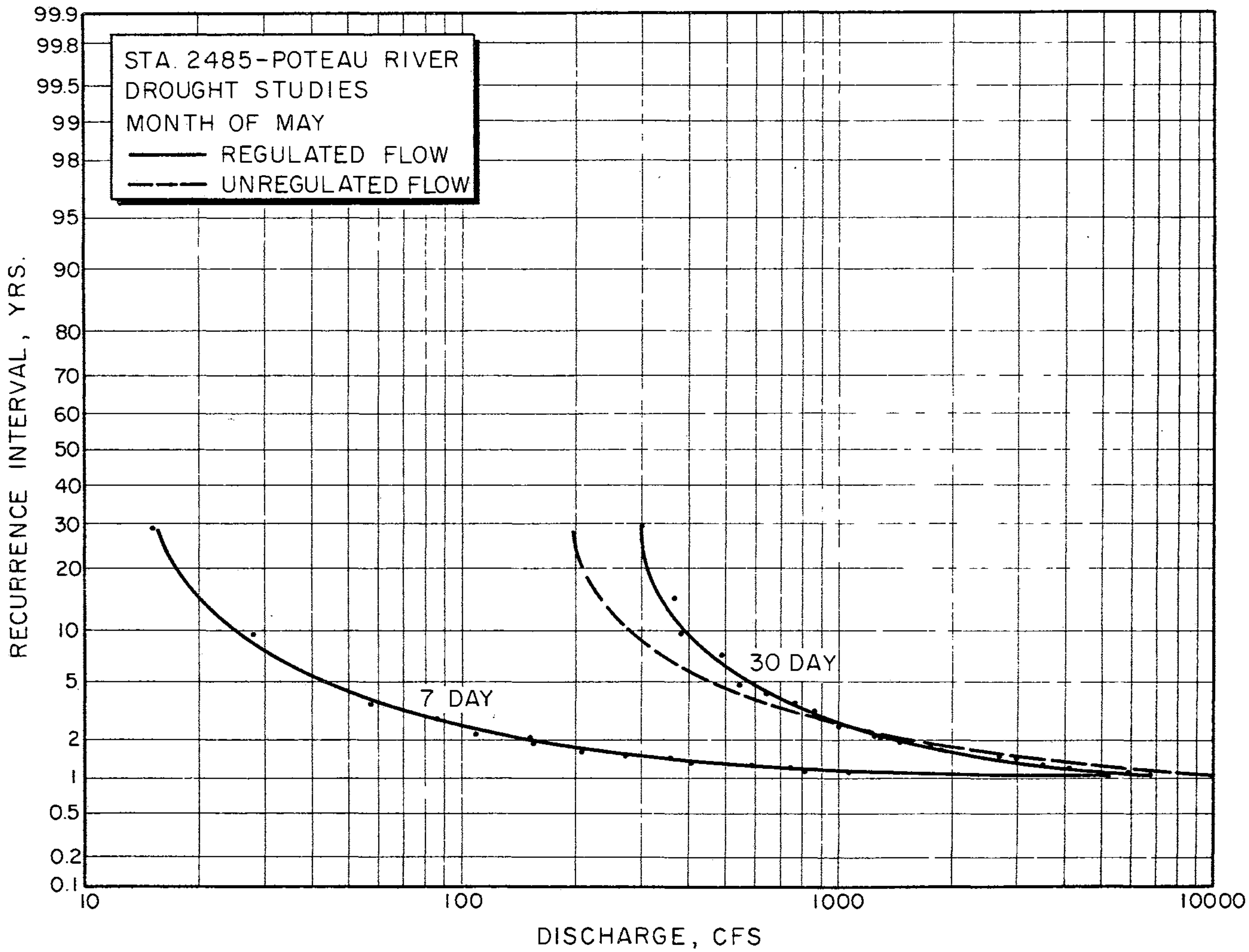


Fig. 9 Duration Curves for May at Station 2485

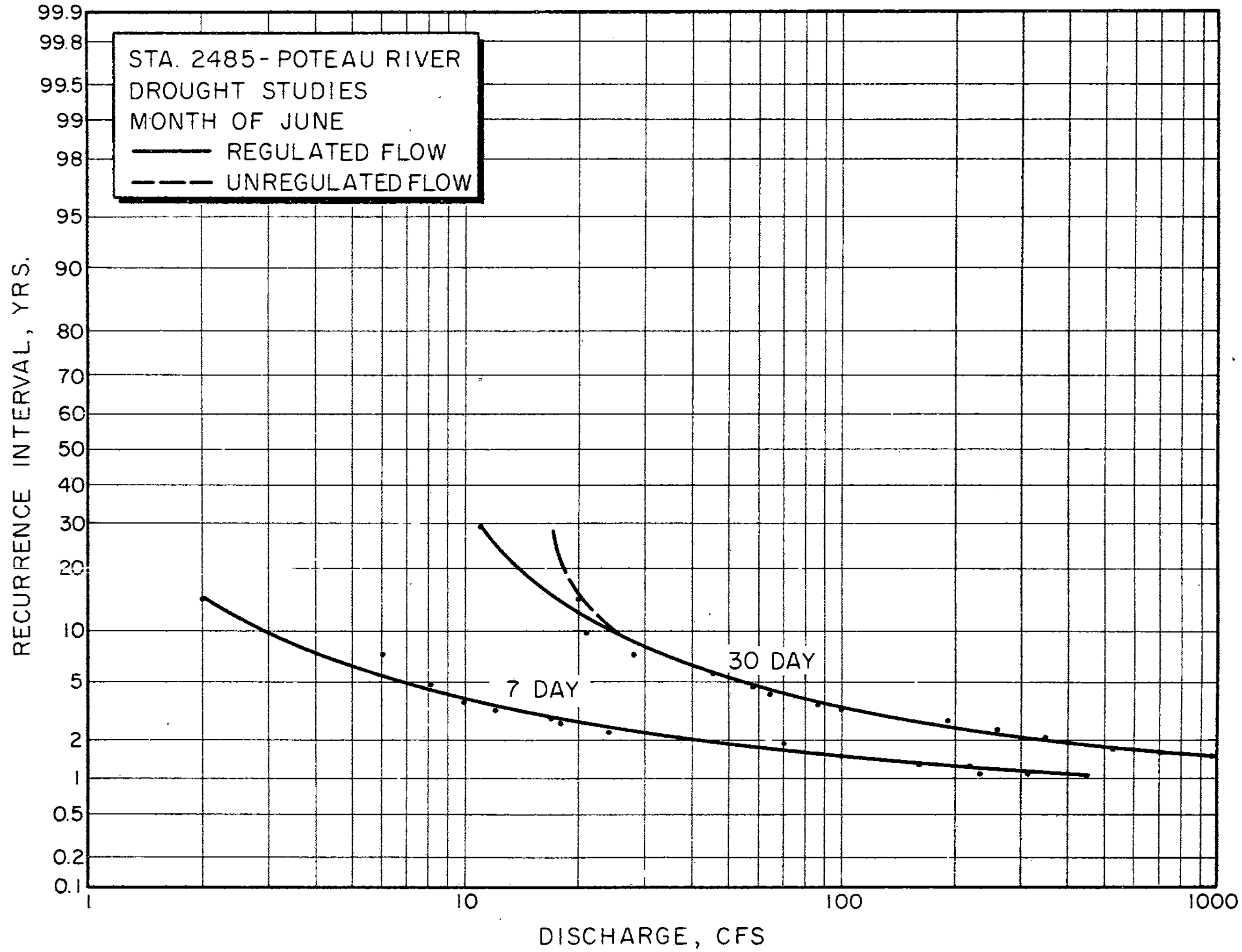


Fig. 10 Duration Curves for June at Station 2485

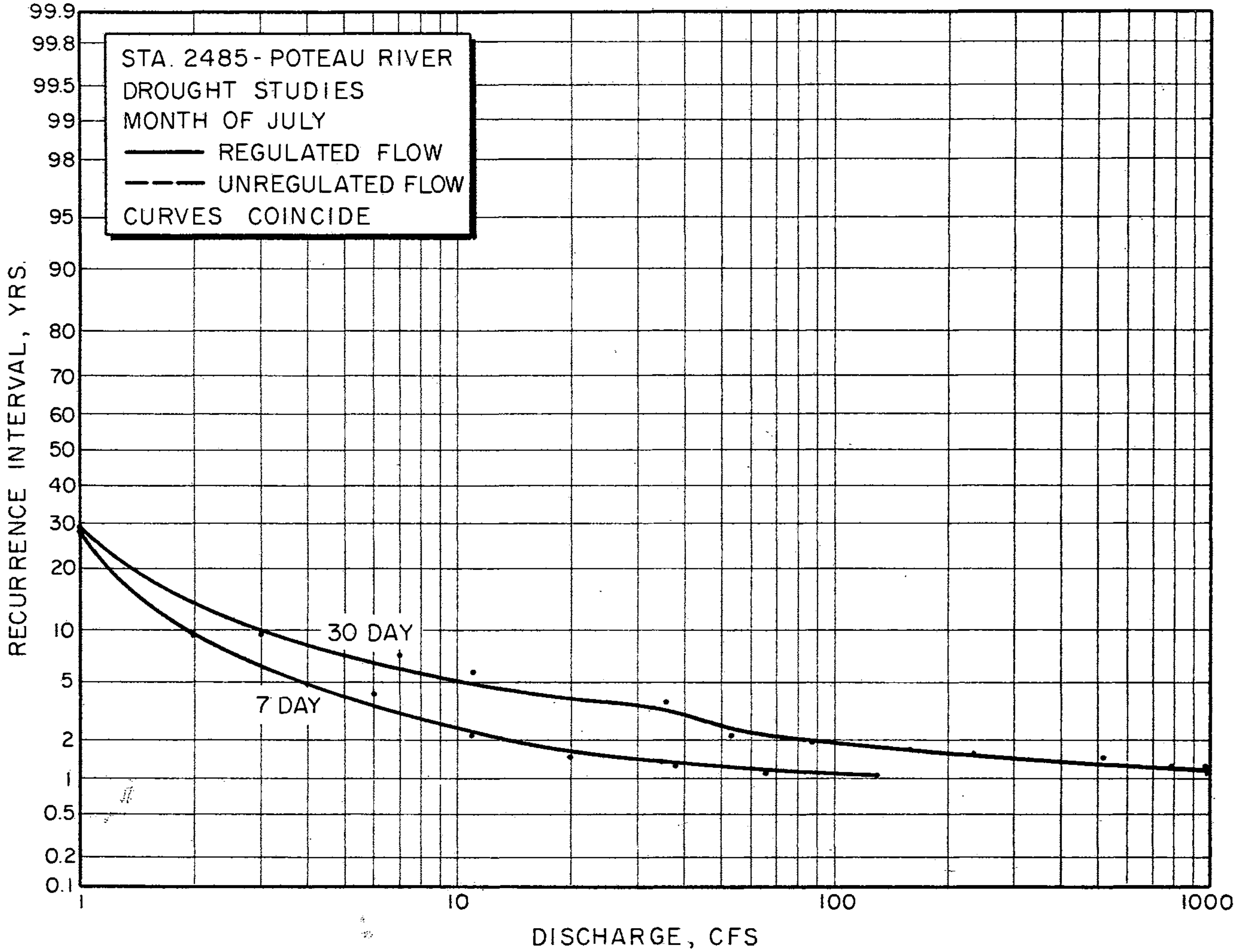


Fig.11 Duration Curves for July at Station 2485

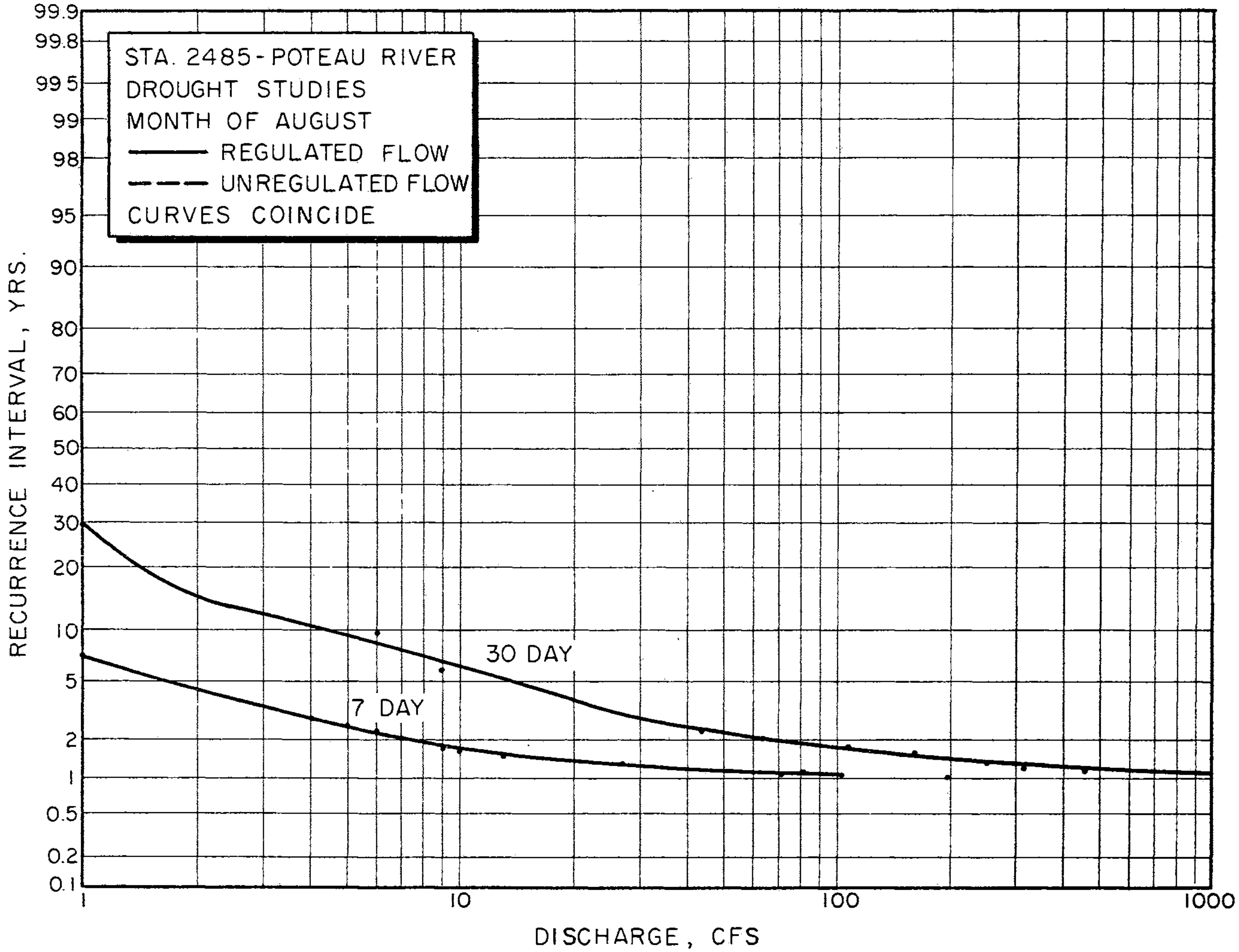


Fig. 12 Duration Curves for August at Station 2485

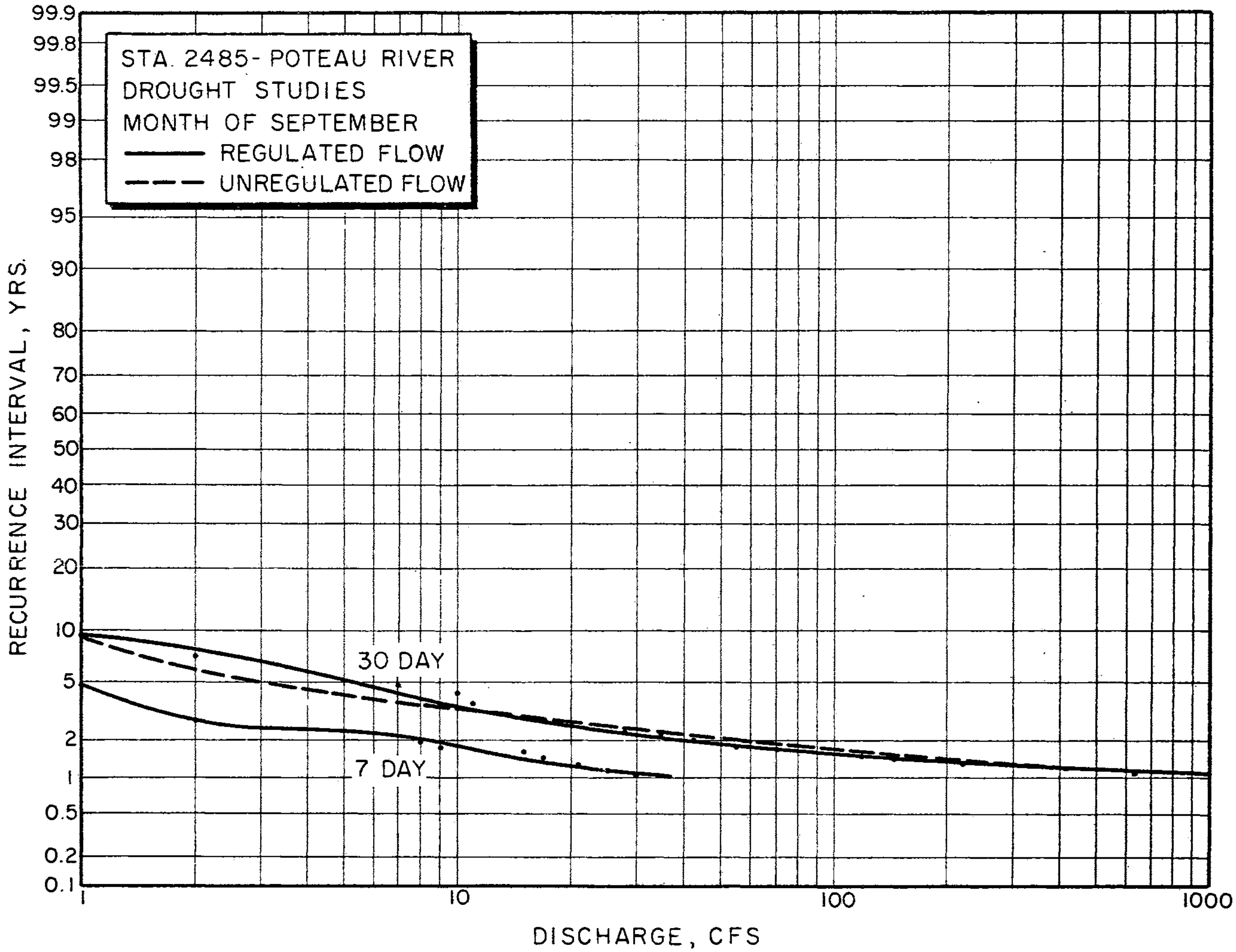


Fig. 13 Duration Curves for September at Station 2485

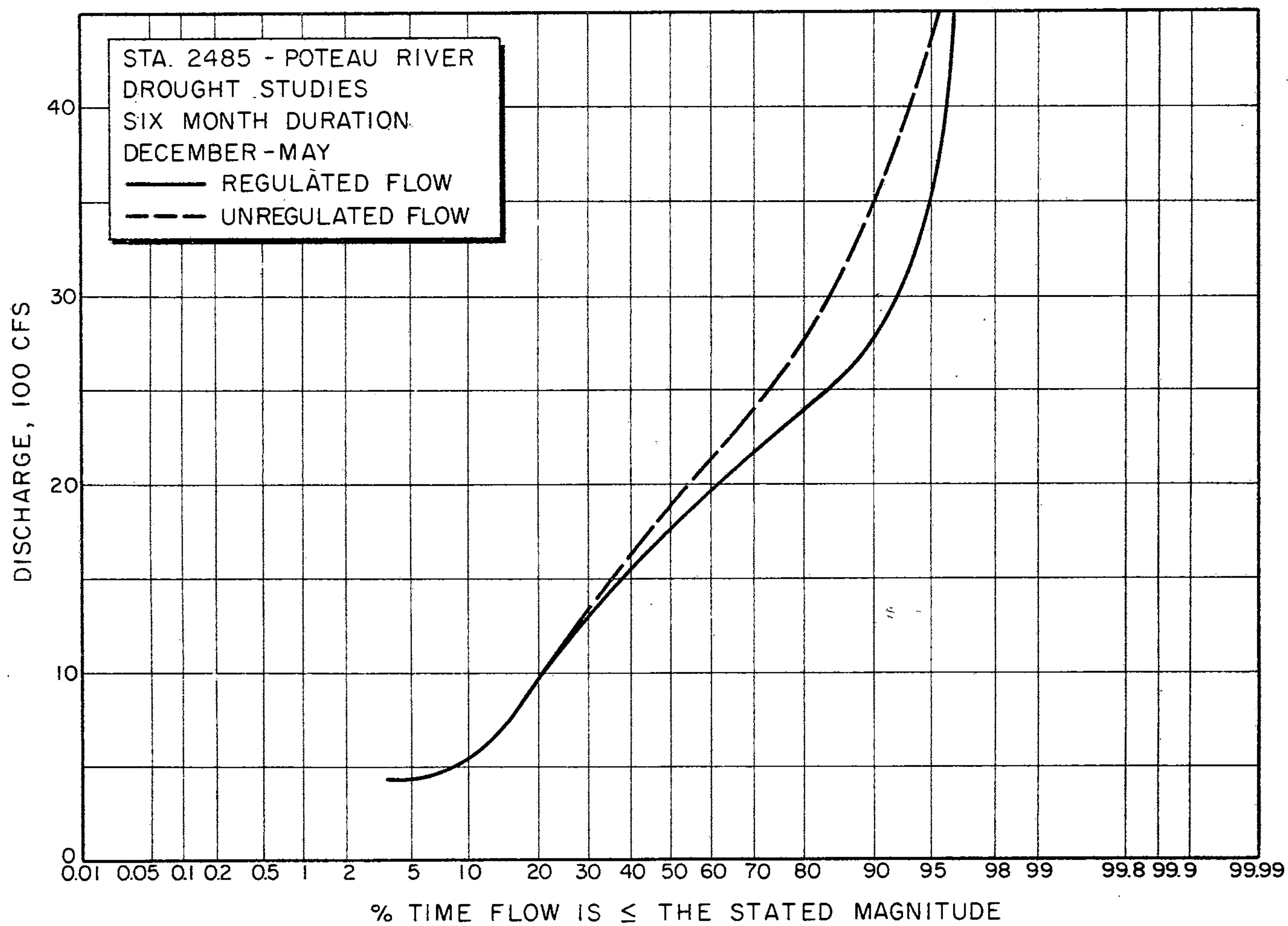


Fig. 14 Duration Curves for December - May at Station 2485

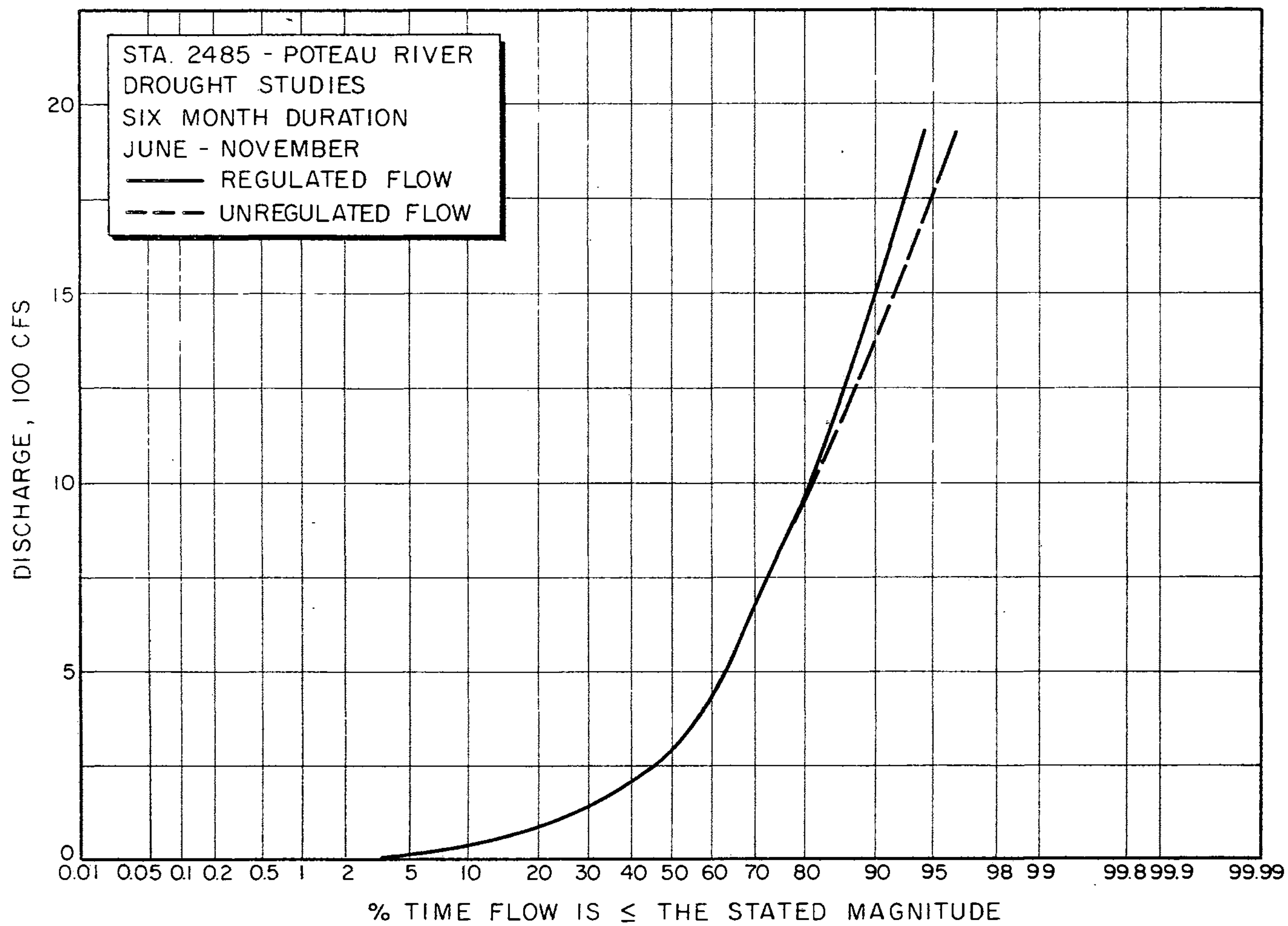


Fig. 15 Duration Curves for June - November at Station 2485

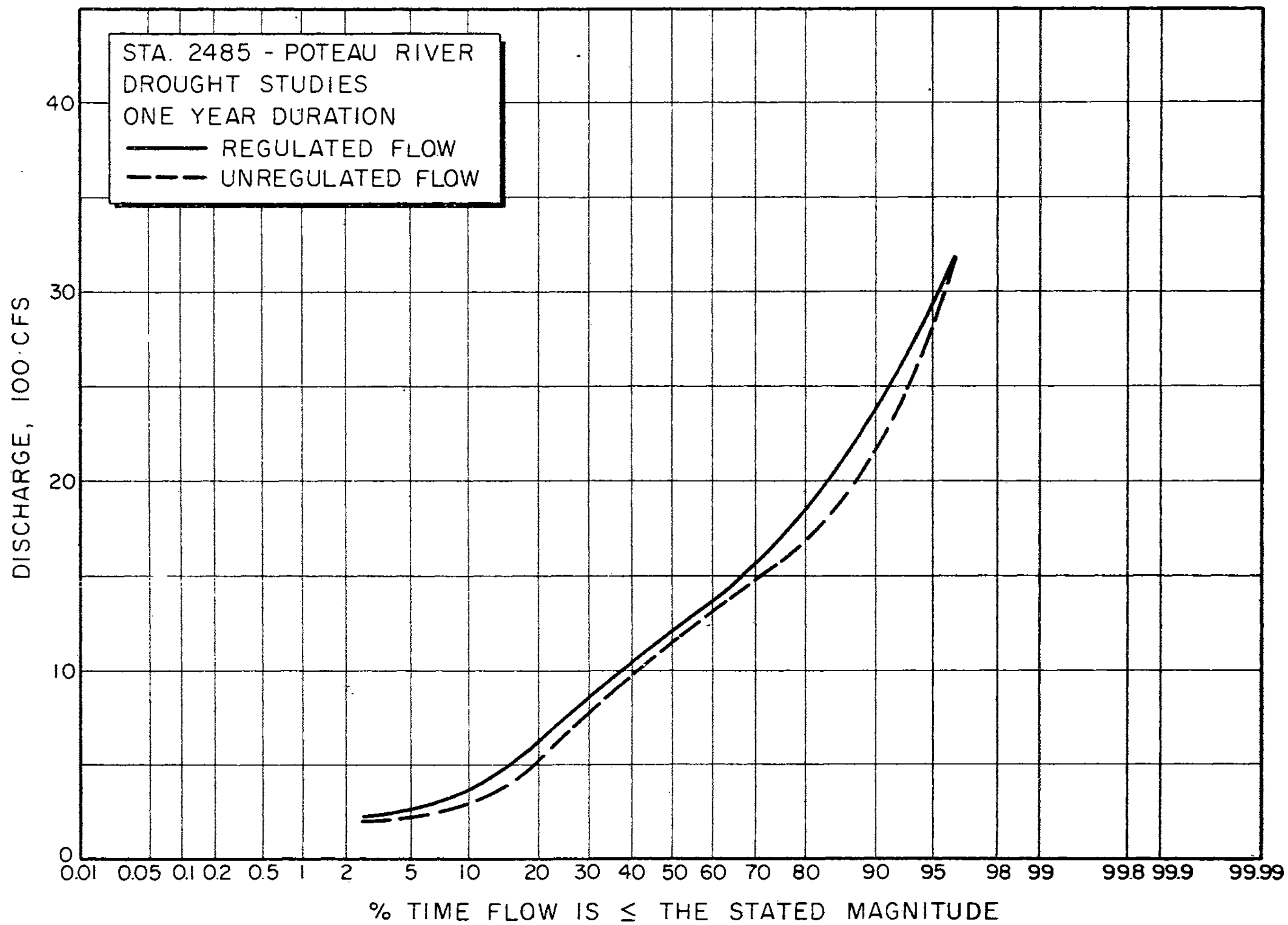


Fig. 16 Duration Curves for one year at Station 2485

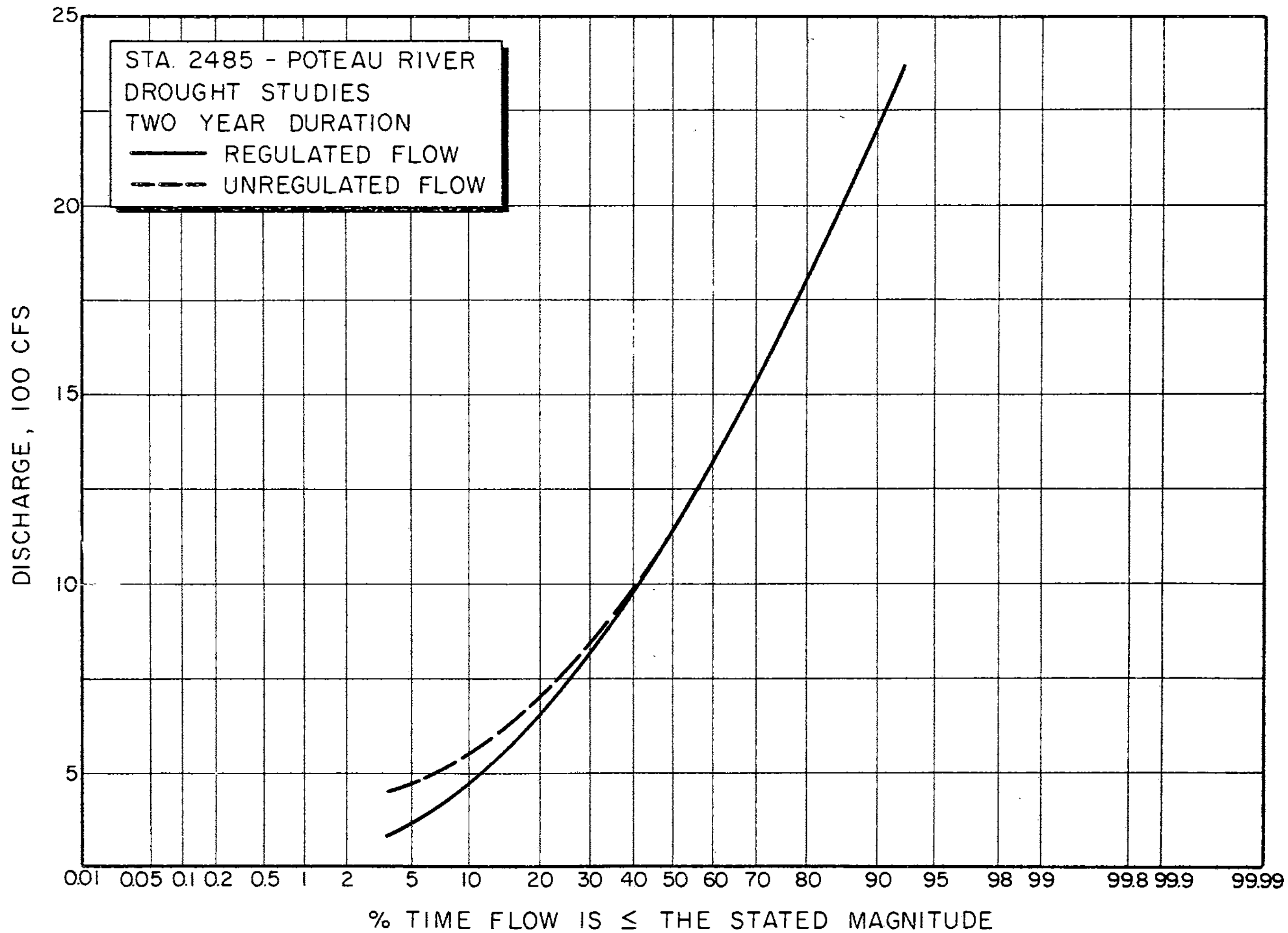


Fig. 17 Duration Curves for two years at Station 2485

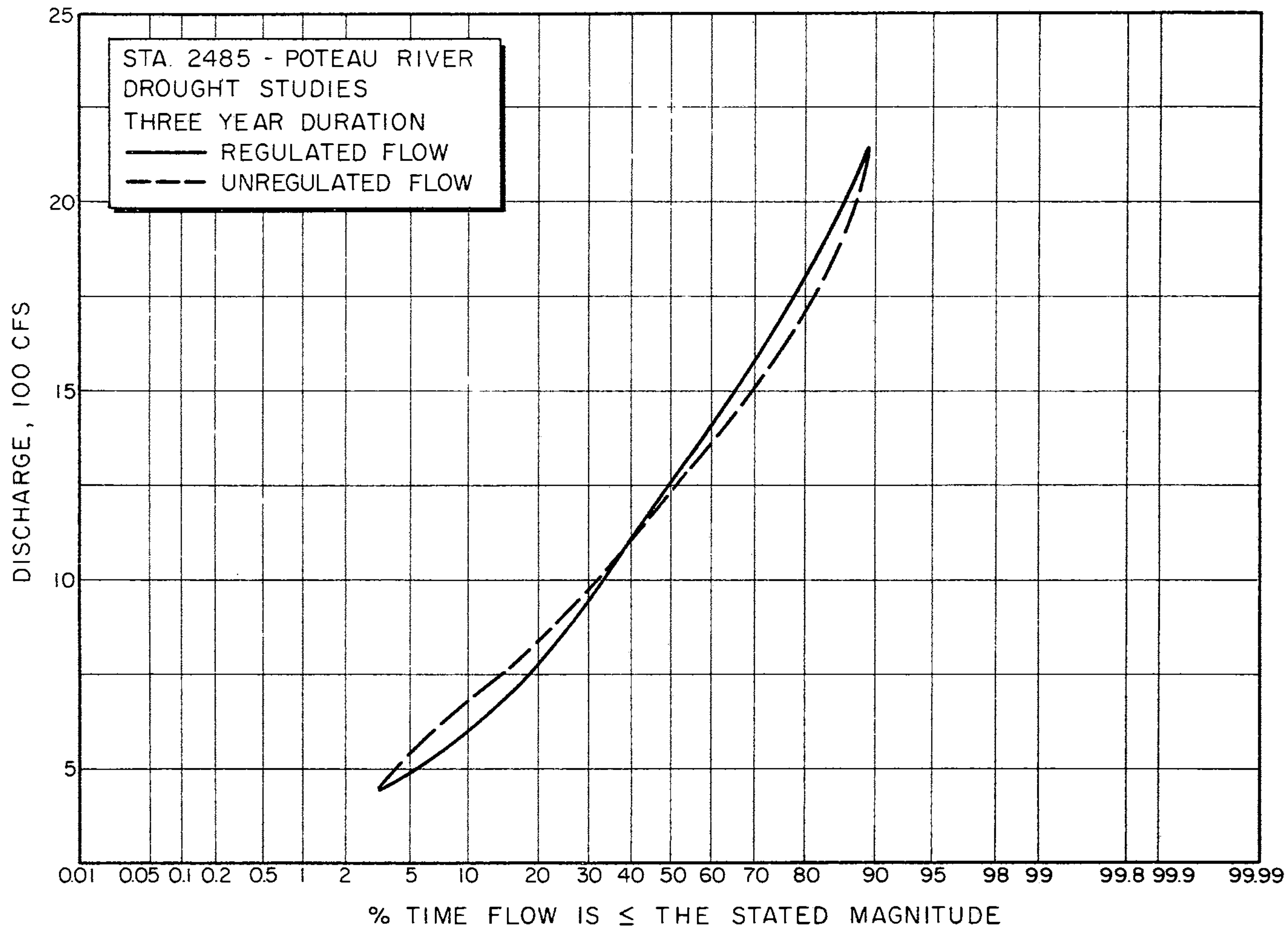


Fig. 18 Duration Curves for two years at Station 2485

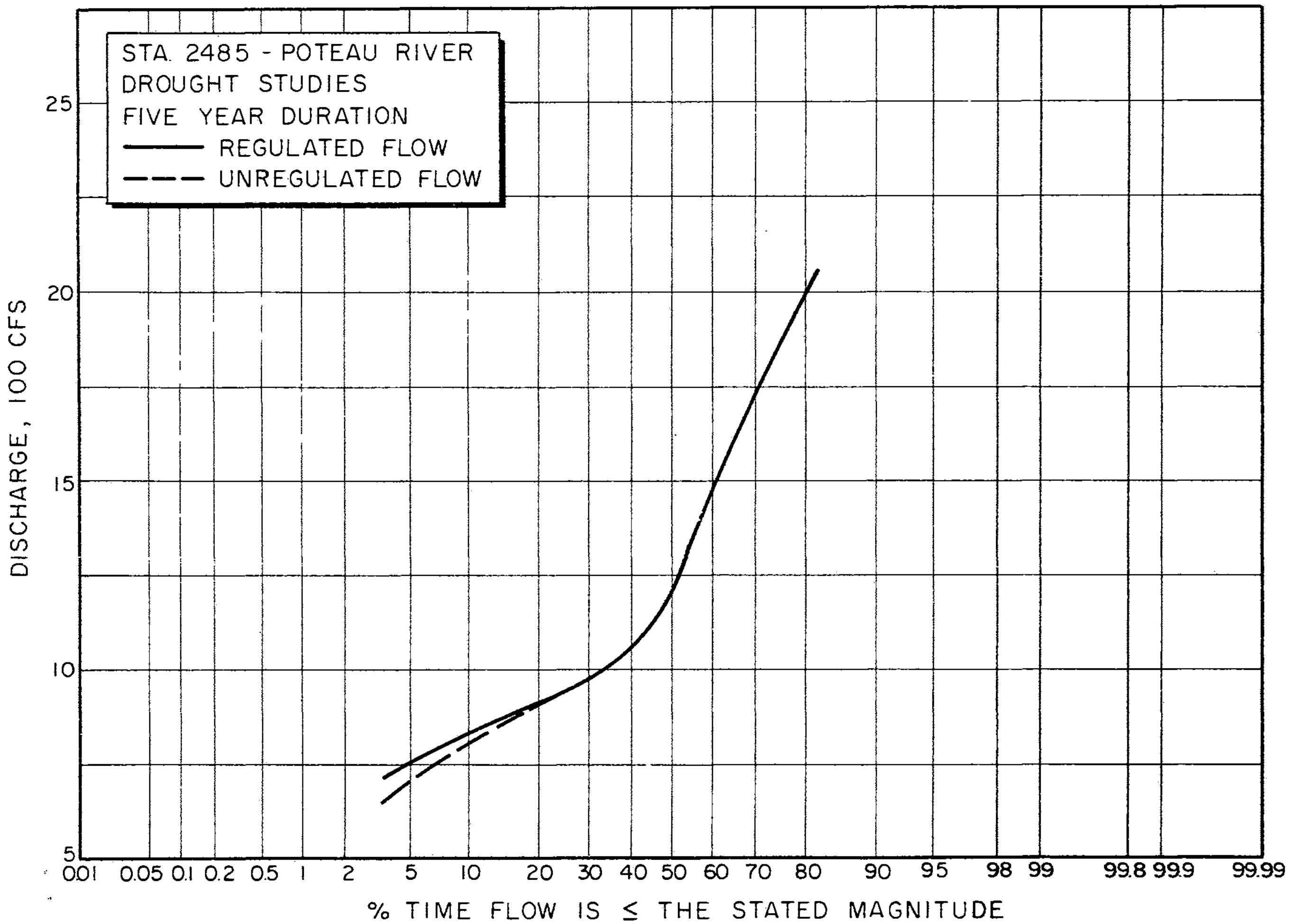


Fig. 19 Duration Curves for five years at Station 2485

B. Arkansas River Basin at Station 2505

1. Statistical Values for Mathematical Synthesis

A frequency polygon or histogram, for a ten year (1940-1950), sample from the historical record indicated a skewed distribution. This distribution was skewed at high flows. According to Thomas and Fiering (15) statistical values from a skewed distribution can be used in the mathematical synthesis of streamflow sequences. The type of distribution for each month over the period of record was slightly skewed at the higher flows; and transformed flows did not greatly reduce the skewness.

The statistical values used in Thomas and Fiering's modification of the Markov-Chain, see Chapter II, part B, are presented in Table IV of this chapter. The "P" in Table IV represents the probability that such a correlation should arise, by random sampling, from an uncorrelated population. The student's "t" value is used in determining the level of probability "P".

2. Mathematically Generated Streamflow Data

A sequential generation of streamflow was performed using Thomas and Fiering's modification (15) of the Markov-Chain. The statistics needed for this equation were reported in Table IV. Thomas and Fiering's equation was presented in Chapter II, part B; and the method used for generating synthetic monthly flow sequences was presented in Chapter IV, part B.

An example of how to synthetically generate data by Thomas and Fiering's equation for the month of October is as follows:

$$Q_{i+1} = \bar{Q}_{j+1} + B_j(Q_i - \bar{Q}_j) + S_{j+1}(1 - r_j^{\,2})^{1/2} e_i$$

From the last two lines of Table IV the numerical values for $\bar{Q}_{j+1}$, B_j, $\bar{Q}_j$, $S_{j+1}(1-r_j^2)^{1/2}$ were taken as 24079 cfs., 0.69294, 18638 cfs., and 33989 cfs. respectively. The value Q_i, 32510 cfs. was taken from the historical record and e_i was randomly selected from a table of random normal deviates with zero mean and unit variance.

$$Q_{i+1} = 24079+.69294(32510-18638)+33989(.10)Q_{i+1} = 37090 \text{ cfs.}$$

Five hundred years of mathematically generated monthly flows are presented in Table V. In order to assure a random start the first ten years (120 synthesized monthly flows) of flow synthesis were discarded. All the negative flows were changed to zero, and the effects of changing the negative flows to zero are presented in Table VI. The number and the flow for the negative values occurring each month, and the percentage of the flow of the negative values to the total corrected flow are shown in Table VI. The corrected total flow when the negative values are changed to zero, and the corrected mean flow for each month for the five hundred years was also reported. For example, October has 76 negative values with a magnitude of 1,034,070 cfs. The percentage is 1,034,070 divided by 207,981,130 cfs. or 0.50%.

Table VII of this chapter shows the lowest synthesized flow for each month, the minimum monthly flow and minimum daily flow of the historical record for each month. Table VIII also compares the actual average mean monthly flows and the synthesized average monthly flows. The 500 years of synthesized data was calculated by the computer program in Appendix B.

TABLE IV

STATISTICAL VALUES USED IN MATHEMATICAL GENERATION OF STREAMFLOW SEQUENCES
STA. 2505 - ARKANSAS RIVER

MONTH	DISCHARGE MEAN-CFS	STD. DEV. S_j	CORR. COEF. r_j	REG. COEF. b_j	STD. ERR. OF EST. $S_{j+1}(1-r_j^2)^{1/2}$	"t" VALUE	P
Oct.	24079	37213					
			.59678	.42673	21351	4.462	<.001
Nov.	20021	26609					
			.61673	.32985	11202	4.701	<.001
Dec.	16925	14232					
			.29644	.37500	17193	1.862	.050
Jan.	19121	18003					
			.49984	.63249	19731	3.463	<.001
Feb.	23984	22781					
			.24779	.28773	25627	1.535	.14
Mar.	28705	26452					
			.58071	.99759	36994	4.280	<.001
Apr.	47054	45133					
			.28431	.38795	59042	1.779	.050
May	66958	61585					
			.49826	.41094	44038	3.448	.001
June	54147	50793					
			.39440	.28432	33640	2.575	.01
July	33507	36610					
			.62060	.31519	14414	4.749	<.001
Aug.	15672	18594					
			.53737	.51755	15102	3.823	<.001
Sept.	18638	17908					
			.34300	.69294	33989	2.191	.01
Oct.	24079	37213					

TABLE V

500 YEARS SYNTHETICALLY GENERATED STREAMFLOW-10 CFS.

STATION 2505

	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP
YEAR	1	2	3	4	5	6	7	8	9	10	11	12
1	1701	1229	1465	3435	4868	6117	6021	5950	5838	3741	987	3338
2	6766	3600	2695	2124	2198	0	3180	1180	5162	4014	1082	1368
3	1020	1046	713	1555	2339	5093	0	10148	3619	1928	1382	0
4	0	0	0	1669	784	8731	5709	11983	3049	2948	2467	1599
5	1689	1548	1457	3843	6518	5756	6544	0	5045	4979	5575	2485
6	2802	2133	2703	1519	3251	6423	5241	15480	4993	4610	0	1382
7	3794	1176	4006	1183	1622	2105	2351	0	0	2503	0	0
8	1890	0	0	0	0	0	0	4713	4171	3078	2472	0
9	1460	3183	2764	3936	229	1872	11368	10027	2926	3179	2195	8682
10	10079	5983	4364	1610	3666	1116	0	3932	5365	2592	586	1138
11	4092	3135	4287	6375	9762	10054	21320	8574	8411	5910	1986	7205
12	5355	1997	0	2222	6622	4918	13671	16696	15417	4690	1510	0
13	1550	3292	1107	1717	0	3264	15426	17116	8071	4605	2122	4077
14	2327	11	970	0	2927	3530	7649	6151	2137	1261	950	799
15	3344	316	1154	924	0	0	1214	8844	5850	4458	1619	0
16	1093	2118	1112	1035	691	3406	9245	11601	7851	1738	2474	0
17	0	2223	2112	4557	1144	5353	10147	15126	12009	4094	504	3081
18	3539	1021	0	1230	2816	6677	5883	9597	1050	2883	4672	1492
19	3224	2791	2080	6389	8843	7095	4913	10381	6336	2931	1411	773
20	27	723	5058	2758	0	3395	8173	4339	1597	2736	551	0
21	0	115	0	0	1489	3097	4614	2557	6160	3570	1644	0
22	0	948	0	650	4651	8917	12694	10587	1923	1932	5314	8949
23	1276	403	0	1869	2725	9285	4792	11275	12771	6643	1742	0
24	1247	0	335	2272	10	0	11264	10955	9403	3285	0	0
25	0	0	1125	0	255	0	4574	8129	2070	3893	4530	6172
26	7004	1804	0	888	0	2115	6946	0	4082	4305	1425	2031
27	4645	1905	3026	2199	6134	8774	4669	11306	11022	4784	2686	294
28	2279	2165	598	1218	0	0	2063	7897	5579	5460	3222	1375
29	1589	313	0	0	3305	759	0	8473	8727	4463	2611	8785
30	4833	2647	2176	1549	1905	0	1061	0	4233	982	2147	6780
31	6060	4316	232	0	0	0	0	0	3984	1593	4915	2302
32	3703	734	2393	3976	4219	1384	4805	5724	6101	3592	0	0
33	0	0	340	1333	3107	0	2766	12240	10141	4197	1666	8022
34	4662	4691	3015	830	5498	1272	4120	4144	4810	3441	1112	897
35	0	0	0	0	1429	5100	9919	4863	5747	5030	1672	724
36	4399	2343	531	68	2378	5644	1291	13245	4019	2136	2843	2551
37	8234	6059	4887	3010	100	0	2809	12426	15233	4512	0	4646
38	5204	1879	1625	2583	5265	5948	9247	7184	5728	4425	2357	3393
39	6177	4088	4046	3553	3872	507	0	0	0	787	893	0
40	0	829	4967	1096	1757	692	0	0	4634	3847	509	789
41	1996	1568	0	1321	6049	12863	5148	7264	0	3532	0	2012
42	3992	3401	356	1550	4285	5061	2631	7122	12335	6309	974	6910
43	6980	2545	5145	6636	5546	1344	8510	0	0	376	2612	2349
44	624	1683	0	593	5799	0	6924	14329	12011	3746	3471	1120
45	1147	2289	1714	2614	0	0	0	557	1336	2349	1403	1847
46	3451	1866	1529	0	3121	5959	6555	5017	341	2678	2852	988
47	3559	2327	4356	3562	177	0	0	0	6369	3634	4507	1114
48	0	0	0	389	8272	15470	16342	16409	6427	4693	542	3127
49	3797	3097	4625	4128	1475	3794	0	5814	0	0	543	2001
50	0	0	0	166	1939	2833	0	4072	11855	7002	2084	3533

TABLE V, Continued

51	3559	2821	1410	0	0	6833	4307	9983	7492	4784	860	0
52	1746	3904	1796	0	4622	7481	11924	921	5447	2153	923	0
53	3519	2707	4468	2041	0	0	0	6968	4766	121	3142	110
54	72	1571	4311	674	1442	0	0	0	353	375	1752	7423
55	7513	3146	550	2767	751	0	2807	2299	1698	2613	0	0
56	0	524	0	0	0	2089	9315	12523	6871	4457	859	7443
57	2676	1084	2940	2292	198	0	1732	4418	2641	1660	2989	535
58	308	1187	0	0	0	2439	0	2393	9236	5584	0	672
59	3966	1395	982	4256	1952	5846	2715	4940	5060	4316	2624	0
60	0	1769	0	351	0	0	5015	4871	3137	5660	4338	9139
61	6049	929	444	0	4817	2328	16780	14279	9968	4349	1138	0
62	3759	2502	1433	48	1958	0	8817	10316	10506	6116	2467	2158
63	3417	1703	3371	2096	2690	8178	6311	12184	8593	3855	2193	0
64	753	2342	914	1724	0	2472	0	6417	2976	4205	971	3685
65	4654	1793	3671	0	758	0	0	3450	2589	3315	1415	1380
66	0	2075	1093	3654	3051	5982	10445	10327	12585	2916	1732	3053
67	528	484	4435	3043	2791	14261	4958	3688	7558	3180	0	0
68	872	2132	2306	1763	4512	0	5654	5823	6547	1790	1763	5284
69	5657	2951	3783	6471	6263	8849	1037	10097	6922	5050	0	0
70	547	1902	3472	2416	1495	0	8435	11264	10380	1779	980	4024
71	2104	858	3901	5392	1454	756	1878	8816	6204	4077	1592	1987
72	2612	1890	863	2599	0	5146	10282	5776	6569	2818	2368	2008
73	225	1265	0	1663	0	304	2481	3838	3742	3672	1203	12911
74	8014	4530	4468	5037	4109	8584	0	10179	6291	4844	860	6042
75	6339	3458	1288	5291	4782	8858	0	3325	2391	1359	0	0
76	393	2063	1817	5279	5665	4521	8031	8242	5811	4560	1058	1439
77	4243	1177	1693	3947	1088	2965	3516	7947	4935	2284	674	430
78	0	1215	2059	0	5167	3997	6336	8932	5347	1465	0	0
79	609	1400	2675	724	0	0	2057	2812	4537	4176	2181	3986
80	3956	2421	5451	5230	3510	1001	2705	2467	2483	169	3871	2414
81	3551	3682	1883	3566	7057	7401	0	0	5806	4573	2986	2514
82	4408	4545	0	914	2965	5699	8889	20548	9188	5342	1292	3405
83	2572	3245	0	3500	1487	5437	8148	1419	3146	0	1711	5920
84	8540	4169	3369	2294	1079	546	0	0	299	2614	1161	0
85	764	793	1188	3455	1826	559	0	9117	8672	1239	0	2355
86	1056	962	3175	2252	6314	5972	0	5361	7663	4924	1286	1958
87	4673	4026	1231	2767	4208	8371	6688	11324	6663	3258	2521	1670
88	2344	1291	158	271	1124	1237	4032	1788	1097	4213	1164	0
89	1278	1868	4018	5710	493	6525	3029	11913	10007	2734	3197	527
90	3312	829	928	4423	0	736	7193	8384	10250	3912	2758	0
91	0	516	0	0	0	1524	14507	12885	5054	4070	489	0
92	3581	2523	0	2380	0	486	3152	5918	8738	3506	683	6791
93	5459	2338	3683	1458	1787	3294	1299	8261	6091	5831	957	3799
94	6596	2048	2205	5123	2318	5552	0	5832	3802	4497	1758	1757
95	4080	1872	1012	852	2280	1873	1679	7133	10508	5745	2356	1615
96	3060	2207	2928	2977	5650	8997	2276	9400	4216	2493	3669	3818
97	2148	2300	3534	6982	577	0	5294	9886	817	2232	2655	2749
98	4987	1449	1044	4282	5428	3904	115	7011	581	2147	1412	4638
99	5104	4859	2141	3929	3386	2455	10233	21358	5796	4080	2066	4369
100	5018	5132	6920	5425	2495	4543	195	0	3195	407	43	0

TABLE V, Continued

101	742	2076	1636	7584	6375	7804	7983	13304	7367	2659	1927	1794
102	3713	1280	0	0	1707	0	8031	7076	8440	3077	1301	888
103	3332	2893	4332	5115	0	0	0	3173	4992	3259	1338	6889
104	4597	1992	2266	0	4952	9540	0	10110	10917	3410	2031	4202
105	1846	924	1025	1416	1839	0	374	8123	9204	3897	1491	3665
106	1058	1873	2480	6263	4506	4229	9409	8123	9890	5851	4109	0
107	2717	4595	5091	5495	3951	5748	11864	5399	2345	3332	1484	2321
108	4031	3120	1898	6850	3369	1081	7321	6565	0	0	1689	1978
109	3753	1687	3850	3652	5666	10131	0	3084	0	0	1057	4229
110	2979	1672	2347	5491	4215	12955	13223	10908	8853	2900	3537	3263
111	5047	2149	0	1054	671	226	0	4708	6736	6140	2627	9329
112	4733	2108	2605	1159	4356	10465	3742	4970	0	1997	3587	2406
113	510	2459	671	1900	4526	6718	0	5402	6261	4857	0	0
114	609	1402	759	2161	4754	4064	4221	9834	1545	0	1823	3463
115	607	978	0	0	4348	12488	12036	8515	12024	4745	1549	0
116	3543	2800	1603	3833	6293	10497	0	0	3868	4772	3002	4235
117	2967	2027	1658	2429	3288	6141	2304	6581	211	681	1770	1710
118	5356	5308	4860	5222	6856	13068	1927	13396	453	3177	697	8077
119	2563	2083	0	0	2130	1318	5467	4605	1878	4341	3548	9240
120	3724	2412	1228	1007	0	0	0	3634	7059	3358	886	1497
121	6344	2041	4756	5477	5732	5803	14469	12557	12796	4091	2918	7497
122	4414	641	1405	3909	6913	2861	12129	18080	12098	4242	3856	1124
123	7050	1771	1051	1501	1243	0	1448	3057	526	2459	1429	1134
124	1630	1303	579	741	2310	0	0	2204	7046	1564	2224	0
125	0	1270	320	5168	4618	9460	11106	12602	5929	3684	0	0
126	3295	2742	321	1042	2150	3769	0	4951	422	1075	1759	4227
127	6120	4681	1823	0	577	0	0	0	4822	749	1899	0
128	1667	454	3683	4118	1647	4707	0	7644	0	0	374	1373
129	4960	1549	0	657	479	6446	10048	9347	3197	5094	2274	0
130	2361	1944	3930	3430	1891	2258	0	2056	5415	410	969	0
131	1925	2329	3695	3460	0	0	9256	3369	4555	2054	1658	5728
132	5377	3290	1860	5035	345	2904	7506	13198	16931	5530	2223	0
133	0	1880	3930	3738	1584	0	29	2835	6354	4791	267	0
134	292	1562	2481	7532	2126	5133	0	1943	2685	3349	1946	5746
135	3067	1998	1841	3892	4820	6868	6632	14073	10827	2719	222	5253
136	4000	2283	1656	271	0	0	0	0	0	317	0	655
137	1741	1338	197	4186	997	2848	1882	9833	6046	5227	1306	0
138	265	1833	0	6001	2786	2875	0	1626	3529	3439	1729	0
139	2468	1711	5882	1712	2995	3841	3752	6965	1020	1477	1343	0
140	0	0	0	0	0	0	0	0	2032	3393	2066	5934
141	5720	3517	1952	4930	6727	4764	0	6278	7043	5321	3635	6550
142	6119	2620	0	2731	3813	2294	0	10048	6188	4201	3054	9572
143	3808	3760	3630	4397	704	8832	1156	148	2547	1023	2412	3955
144	1866	1994	4511	6396	4894	4252	0	1326	1449	3058	1333	3386
145	0	1795	0	0	3106	4516	3960	7914	10899	3402	1748	2480
146	6144	3569	3083	6012	1463	378	10660	6380	4787	5014	58	8730
147	6385	3947	4133	4873	543	0	2477	9344	9995	3115	1627	34
148	4688	2815	5552	7892	5769	1662	9710	4191	5259	4255	3999	8711
149	5471	2908	2868	2574	3009	533	0	544	2495	3793	1036	0
150	1502	2363	0	1884	4624	4806	10320	12477	11291	5284	1688	2511

TABLE V, Continued

151	1345	333	0	0	1308	0	0	4398	3060	2948	3002	5964
152	5146	5225	506	3491	1833	1772	0	7314	0	3683	3500	3551
153	3188	2639	1990	3117	1443	7412	20657	16683	13061	7006	0	1204
154	1314	1235	2741	3871	2135	2590	12689	12396	4494	4668	2461	0
155	2781	2105	0	0	1119	2832	5336	10147	4104	3358	3688	3778
156	2878	1696	2308	1192	288	3392	5659	4789	3884	5091	1026	1875
157	2949	1559	0	1090	113	3388	36	8846	11272	5002	4261	1969
158	3041	469	252	0	2834	6839	6636	4313	5381	3444	0	3104
159	1840	1544	4768	3819	5239	10141	4744	6785	0	0	3657	4255
160	3489	1816	3240	1714	0	0	0	3242	8971	4527	0	4308
161	467	1638	3010	0	2333	5398	0	6139	5148	5143	5072	6459
162	671	3181	3677	0	2767	0	0	7292	7441	3570	2871	0
163	0	0	1642	0	1649	2473	13246	15253	5228	2780	1345	0
164	0	2189	3234	4568	5323	2775	0	2879	2977	2400	2147	4216
165	2530	2667	2070	437	4818	4344	0	7142	5823	3766	4432	4822
166	3689	3423	4096	2066	2852	0	4792	16138	3353	4294	2222	6902
167	2540	1320	1641	4837	2684	0	7827	4066	3552	1295	390	112
168	835	1375	2602	2442	4297	3414	14574	7598	3941	4112	2533	5719
169	6795	3140	2208	2875	2909	0	2798	3336	8269	3900	0	0
170	501	2192	2524	631	1373	2769	8140	14073	905	635	2203	6422
171	7631	3176	382	987	0	6058	5442	14223	5562	2673	1856	1632
172	3203	2355	3511	3106	0	2711	10504	11278	6836	3455	2062	0
173	418	1668	1474	313	1361	3457	8367	8890	5743	3957	3733	4233
174	0	309	2538	0	0	0	844	6985	851	1392	0	0
175	5331	4342	2804	3309	6204	4950	3534	6713	4088	4415	0	3041
176	6415	3118	0	54	1602	265	2745	9080	1339	2053	2424	6735
177	3608	1835	0	1352	3603	0	0	4744	6252	2379	4757	651
178	6009	3998	3650	3693	6028	7148	6480	11323	9034	7295	2108	31
179	375	786	2074	760	5808	5944	1868	4190	2628	2466	2074	2138
180	0	2878	5167	4061	0	0	0	0	1560	1764	3499	3079
181	2759	1762	669	540	2133	5302	3663	6840	8877	7004	39	3772
182	790	3765	943	62	1410	1020	8790	11875	6506	5776	3550	2717
183	5245	4163	2738	166	3651	3626	11226	15756	8834	3768	1228	466
184	4236	1213	0	1366	4520	3284	0	79	7768	2529	0	543
185	0	0	2986	2392	5726	0	5859	4009	6526	905	799	2598
186	6100	3184	4309	4245	1356	2566	13918	5550	8210	5575	434	397
187	729	2996	489	1439	1126	0	0	0	0	208	0	5150
188	3283	3418	0	0	0	0	6983	5297	6136	2426	2678	2823
189	4721	5834	684	255	0	4402	4176	13399	8336	5986	920	12088
190	8113	4692	2273	1276	203	2073	9207	11608	0	0	1187	3206
191	3335	3341	3560	1735	3475	3322	4562	13521	3070	3189	1218	1313
192	3213	2131	3223	4393	3381	7474	0	13354	3779	2656	1761	0
193	7722	4096	3837	7242	5645	3138	3283	4423	5995	3030	63	0
194	0	1847	3572	510	2639	0	6508	7969	7591	3152	0	445
195	0	1538	67	10	4794	5574	8487	0	1565	0	0	1433
196	3974	1385	0	0	0	1251	6304	9845	6276	4777	608	7317
197	4526	3145	2998	1816	3090	1500	0	9799	83	2220	2550	0
198	1067	2508	603	2549	0	0	0	0	9087	4879	3969	244
199	720	1765	2303	1562	2036	246	13380	15908	11721	1959	324	0
200	0	2518	1430	2566	0	0	4797	12905	7137	3658	1475	2203

TABLE V, Continued

201	4133	4131	3697	16	4601	5594	5551	1828	5233	2306	0	4198
202	1386	3452	91	1933	7746	3487	3578	11179	10444	3043	1489	2191
203	5906	2357	1760	1567	4331	4720	0	4434	11665	6240	2799	1494
204	2285	2143	0	4562	4360	3005	7658	6630	4125	4714	23	2848
205	1718	2633	2133	0	7728	7047	6863	6575	2705	1847	838	487
206	3051	541	2816	2594	3326	704	0	0	0	1116	0	2959
207	2434	2559	4714	4064	2509	3775	5318	9193	5428	4696	816	0
208	3249	2446	1277	0	0	0	0	1338	4212	4182	3343	7636
209	4800	2829	2038	273	4796	4241	0	3732	5716	4566	0	0
210	1706	1545	1580	3853	2964	2752	12992	16752	9381	5842	2161	0
211	1336	2373	1323	3616	0	0	0	3064	6289	2388	0	2869
212	74	3117	0	1697	3748	949	5954	6664	5366	1953	2514	3214
213	3644	1980	2671	2808	2997	6090	10467	10634	8312	5754	690	3915
214	4339	2210	1561	4701	692	4447	15325	15619	7459	5444	2641	2633
215	154	1740	4004	2692	4651	8842	8325	10599	10278	6160	1767	571
216	1413	1432	1925	2542	0	0	8375	7163	12524	5508	626	4036
217	3126	1089	2895	273	0	2214	0	4297	1925	1283	1467	3483
218	10115	5634	1309	1578	3208	3274	1921	0	0	1001	1202	495
219	2169	535	5320	4894	0	0	3624	9720	0	0	3363	2017
220	1484	2934	0	0	2861	1737	11970	13928	13802	7371	1043	0
221	1950	1584	2103	1013	2784	0	16857	23617	9908	5297	1074	5703
222	4195	2931	3453	5399	5782	10688	0	8144	3435	1930	1268	4266
223	4063	2810	2273	1900	3844	1838	1560	3143	5409	3270	1425	2811
224	2743	3515	3662	6336	1305	5982	3336	12934	10651	3078	23	2501
225	264	625	522	4707	663	1182	9279	3422	2232	2726	395	6447
226	3023	3030	1188	2611	2272	2663	0	5248	12506	3698	2550	143
227	0	2439	0	0	3770	5259	11340	8865	9526	4718	3524	4626
228	6531	3972	481	581	5025	3138	10089	11477	8030	4313	2388	0
229	6189	1346	655	3688	5418	9283	0	1972	2449	2373	1574	6857
230	1794	673	838	1171	5354	9587	14202	13170	6112	3539	1705	0
231	0	1296	2916	0	1424	0	11342	9123	7751	4693	4328	5239
232	3957	418	1595	4146	156	4348	12670	17329	5958	2477	0	0
233	0	462	4407	2593	6258	13402	0	9680	3803	2678	2353	0
234	0	830	0	1912	0	3433	13265	10565	1637	2861	2078	5072
235	494	2602	1829	2104	617	0	13974	3549	1332	1253	307	1612
236	3854	1426	811	1078	2458	3150	6345	11250	5067	2854	0	1821
237	2166	1926	727	0	2965	4105	0	0	6773	1820	1745	6553
238	3670	639	1945	0	4105	7514	4415	4223	4228	3069	1952	0
239	0	2222	0	1152	0	2428	13424	15416	11133	3981	0	3266
240	0	962	750	2809	4809	2857	1909	6168	4129	5128	2904	0
241	51	4285	5120	4458	1161	6320	9055	15696	13433	6022	3976	0
242	0	0	2761	2416	3544	5394	2448	5603	4427	2520	1503	9068
243	7109	3790	0	1666	0	0	0	274	6035	3888	4397	8989
244	6138	2218	3118	1135	2930	408	1236	3787	7016	5832	2324	0
245	0	1181	1970	2616	8518	2417	12498	9385	4825	2057	3510	6527
246	4038	2717	3164	1666	4152	1300	7731	13512	4304	3842	782	0
247	3735	739	2162	419	6439	9478	698	7590	4541	2470	0	1220
248	26	1007	3265	4607	7638	7420	20103	18600	9931	6706	2270	294
249	2396	3251	2173	2355	1092	3773	5041	9136	7965	1333	1172	2590
250	5593	2727	3223	2073	5278	3969	6839	0	4659	5004	2648	4151

TABLE V, Continued

251	5217	2844	2429	0	5529	8727	0	5063	457	5337	0	0
252	0	481	370	1723	0	3050	6749	9957	7882	0	2930	3678
253	855	0	0	1929	2716	2869	0	3782	1879	3667	990	5512
254	9087	4692	3532	4593	3341	6856	10888	5610	879	1521	0	4547
255	2520	1286	2386	6351	2190	8247	3409	7033	761	3710	1857	3022
256	597	1815	3318	3444	1316	0	0	0	844	1474	0	0
257	3782	4272	2470	5217	2656	3908	721	6154	0	660	0	0
258	0	0	1428	2750	1361	975	2835	7008	1223	1037	437	0
259	1749	1411	0	3261	2577	2585	4627	4605	3715	2823	2773	2766
260	2811	1497	1353	2087	4521	3407	2046	2683	0	5445	1164	0
261	205	739	0	565	4011	4646	1270	0	5550	3094	2182	3059
262	4046	3253	0	0	2289	5829	2853	7813	6852	3624	0	1510
263	3412	1999	4618	4013	1693	0	3981	4520	4711	5711	0	4531
264	4616	3246	0	0	0	1482	8426	9853	2325	4688	3211	1227
265	0	0	396	3959	5602	5951	1514	2917	767	2788	1584	5655
266	3432	1421	0	1778	5555	529	11398	6280	8582	4175	2112	4877
267	3444	2688	5010	5065	1503	115	809	0	0	0	3042	2268
268	1748	1170	3096	3742	4341	6044	0	3780	4552	3081	0	0
269	0	1525	0	1989	0	0	0	4067	5391	2753	552	0
270	1507	1460	3988	1720	3092	7224	6008	16844	4008	3406	1619	0
271	0	1142	1441	0	5759	5319	2554	4510	4683	3872	1921	1139
272	2668	2463	1189	805	587	0	0	0	8171	3759	1689	0
273	0	0	1957	3842	7179	7051	0	0	429	2477	0	2937
274	5434	1062	0	0	5473	0	4651	10383	10973	2405	2104	4392
275	6322	2426	1680	3023	7762	14166	19111	15138	10964	3497	3713	6981
276	4802	3986	0	882	3733	927	0	12441	2392	5035	2084	3689
277	4190	2047	1416	4134	3080	0	18968	11576	10168	2956	3832	5395
278	5881	2726	4796	3137	5284	10258	8085	13723	8203	4064	3152	6359
279	4669	1877	1658	5352	3619	0	8518	3262	80	839	1483	9649
280	5404	3448	6203	2826	12271	13874	7593	0	713	3689	2918	10031
281	1962	725	0	0	3430	6160	2589	3897	8256	3660	0	3307
282	902	4627	1127	121	1797	695	0	4576	3603	0	3769	6103
283	3046	1739	539	2097	0	0	12805	5467	7118	2948	2055	5131
284	320	1227	2909	6545	0	3690	12881	4659	8666	2729	0	324
285	0	938	728	49	0	0	9441	8266	3480	3214	4443	745
286	4215	3366	2699	2425	4007	2001	14064	6481	5308	4788	2949	3466
287	2127	1336	0	1847	1163	0	3356	6892	2779	4205	1225	4909
288	2770	3084	0	1159	1404	5	3457	1205	4541	3429	2335	4456
289	5122	3360	13	2034	0	0	16898	4925	6195	2540	1089	5324
290	3769	2501	2529	2547	2583	0	0	741	6607	2949	1748	0
291	0	3757	0	71	3019	3233	1691	1375	1697	340	0	0
292	1865	2748	4066	6504	0	0	4232	10421	7353	5104	3728	5093
293	4527	1421	5771	3552	5624	863	920	6031	7722	4819	1007	0
294	1068	3095	330	0	2006	0	20142	7862	1360	1890	3468	2841
295	2751	672	4256	3676	5677	2398	1082	10254	9849	6185	0	2585
296	129	907	2580	4540	0	8206	21561	18953	12156	4017	3514	2077
297	2881	6014	1610	5633	7721	8051	1252	5938	1527	2099	1259	2264
298	2354	3723	6457	5014	2591	8755	10013	10639	6770	3093	2415	1102
299	0	1133	679	419	3991	3184	8641	17497	17799	8684	2880	2424
300	3623	2582	2170	0	0	0	0	0	2825	3338	2781	0

TABLE V, Continued

301	2118	1845	2857	3781	4390	10177	12177	6332	5953	3070	2421	2910
302	7682	5997	3327	6136	1848	1288	6577	7559	6622	3379	2590	1086
303	1514	1431	1656	4125	5937	8693	5983	5841	7875	2969	2164	3167
304	5077	5829	3714	3296	1512	5465	5263	12591	6586	5547	1381	2916
305	2329	230	1012	0	0	0	0	4375	7595	4614	6	3851
306	3305	1200	1823	1627	977	1855	0	12003	12392	5530	3923	6179
307	7252	3883	4553	2078	0	0	9540	10455	7100	4159	1274	0
308	3428	1505	1824	554	0	0	735	2139	7767	4191	0	0
309	4964	3574	2535	1116	448	5410	8025	6258	10091	4874	0	0
310	2466	242	3259	2475	2669	4	11980	10498	7250	277	1654	1278
311	3177	1580	2404	3240	2568	4544	12519	18181	11214	3877	2203	4895
312	6080	3247	2929	1656	1914	4265	14756	14899	3911	1711	0	259
313	0	355	6037	8635	2142	0	0	0	325	2284	880	3973
314	1646	646	2055	4501	4128	3866	2648	0	2763	2512	308	1582
315	1301	342	1161	458	2608	4182	950	0	1850	3528	4117	6973
316	5284	2565	0	1834	12	7530	8537	2250	7235	2971	1042	4870
317	7175	3245	3407	5297	5292	7362	7210	13966	9090	5111	2205	6500
318	6375	1092	3696	3876	2911	6566	716	9069	4219	3396	1100	0
319	2457	1665	864	1858	0	0	3700	1525	10354	8138	3164	7025
320	834	328	691	1730	1949	5693	11284	4772	3239	3910	0	3907
321	3335	1769	4027	1687	0	452	4654	8328	1530	3959	568	2004
322	3301	315	687	0	892	0	292	13596	5877	1591	4592	7998
323	3155	2809	3581	6335	861	1320	467	13206	7669	6252	1617	3324
324	0	874	0	2425	4753	10562	2715	2060	8780	4298	3786	8078
325	5331	4940	1710	867	459	0	6456	12472	10095	4427	443	5034
326	8336	2348	0	0	172	991	19560	2694	6298	2134	4365	4682
327	4256	1309	2075	6016	4474	438	5505	9231	9041	3846	590	2854
328	0	1574	2248	6134	5984	4489	5508	9315	8070	5831	4011	11726
329	4520	2617	756	0	620	4190	0	7260	8157	2712	1372	0
330	0	2014	3421	2138	0	4209	6517	11805	4397	0	1593	5278
331	4430	3940	2293	0	1484	0	0	8683	9489	4568	116	0
332	1469	2951	4113	2600	2470	2838	5035	8136	8132	5189	308	1044
333	2578	1487	2012	0	1519	6558	3902	12514	9143	4642	1964	2848
334	842	397	0	0	2354	7015	10136	13476	8364	2715	3521	11583
335	2786	738	3347	5214	6886	7097	0	8464	8014	4959	2917	4221
336	5115	1164	0	0	0	0	7466	6428	6183	4301	1695	3536
337	7185	3979	861	531	467	258	0	0	2369	1072	1860	0
338	1092	1733	189	2898	2126	5486	155	10590	8365	5165	4451	7709
339	3214	2555	2266	60	8196	2769	0	1638	4649	3705	0	564
340	1391	4	1049	3367	7830	10381	15999	7566	9425	6261	1926	2280
341	2363	1295	16	1196	5484	5100	6692	6766	4360	1339	0	0
342	5641	2047	2156	3011	1685	0	18343	8221	3926	0	0	6431
343	7222	4459	3757	3142	1279	4442	5582	17655	12297	4607	475	0
344	3606	1865	431	2593	1997	4459	7877	12405	14294	7131	2667	6781
345	3235	3260	924	2592	3354	7920	0	0	4669	4466	1528	5950
346	7748	3279	995	1923	3946	0	2377	0	4785	4325	3303	0
347	1645	3267	4675	5768	5464	5713	0	7427	12208	7366	2814	5476
348	3422	1973	2133	2372	0	0	1980	0	4838	4414	4630	3550
349	3654	2410	0	0	2793	3997	5657	6108	2347	2004	1398	1839
350	1282	2075	2372	4990	7992	13298	3821	10432	7125	4775	51	2375

TABLE V, Continued

351	4924	4758	65	1219	1498	714	0	862	9829	3227	2932	4411
352	4593	3096	420	869	0	366	0	2881	10029	3609	0	2226
353	1323	1344	0	0	2148	4927	4604	4457	6836	3187	1671	332
354	2647	4398	4066	3290	0	7525	14166	16623	5652	3140	2665	5336
355	5033	2371	559	302	826	6781	4615	9646	8598	7197	0	0
356	0	1289	2612	998	4110	2491	725	2707	1748	0	2747	901
357	5086	4461	901	2036	154	958	0	0	9855	7287	189	0
358	1149	814	330	0	0	5537	0	3106	788	1787	27	0
359	1525	2735	1460	5086	2549	5441	1015	7849	1902	1190	894	7792
360	3513	1309	3782	4395	0	0	0	3680	3201	721	1078	0
361	1648	2420	1653	3339	5507	1257	0	6613	4431	4350	0	0
362	0	0	0	828	0	3957	1750	4030	3813	4631	2631	3998
363	231	3665	3769	2034	2493	6077	5363	6091	313	690	2912	26
364	82	0	1656	2942	0	0	2400	6785	1591	723	1528	0
365	1467	903	0	0	630	247	1411	8773	6468	4525	2406	7192
366	5942	2192	4019	1499	5648	7233	6751	4583	7489	2286	166	0
367	586	0	660	2175	5841	9554	20506	7822	7071	628	1739	0
368	0	0	0	383	0	0	1822	8890	656	2179	3586	6277
369	2858	0	0	2047	0	0	4463	10758	10725	2905	2218	144
370	5520	1276	3272	6259	4690	5034	5056	7093	6484	607	0	0
371	1580	899	43	1860	2406	8433	6999	5389	7153	6469	771	2887
372	569	0	0	1307	2086	0	11601	2724	8540	5185	1990	4086
373	3054	2944	2482	1150	2294	2462	9638	9690	10762	6723	5094	0
374	1490	2261	1600	2699	4241	0	0	0	0	1241	2951	710
375	4729	2913	1674	585	1798	6146	12229	14087	11362	5462	0	3949
376	5224	2059	37	1549	0	0	0	12444	6120	1061	3080	0
377	1879	731	2784	2983	0	6545	10817	9517	6430	4706	879	2459
378	0	840	2483	1208	952	691	0	1561	2458	4373	2946	4916
379	2636	1328	0	6253	5573	3163	1951	10754	3478	931	0	1110
380	0	404	0	0	0	0	8224	12006	7350	4934	3295	5018
381	3259	2714	1835	1139	0	0	0	274	1156	5325	1409	0
382	0	2272	2529	3247	0	0	1102	1146	5459	2545	34	5383
383	3609	3857	1304	2777	9283	8372	8403	13325	7595	3670	1707	2157
384	4025	2034	3686	4533	6186	10831	19175	17466	10504	6817	750	2021
385	0	610	0	0	0	8897	10096	13364	11625	5323	24	1037
386	1939	0	947	91	3841	7256	7818	5386	7677	3717	1045	0
387	0	2012	511	4149	2610	0	0	3650	6435	1923	4141	2146
388	3598	2388	728	1770	0	0	3631	1200	3276	3052	1836	0
389	3706	5672	1610	4896	6767	5316	8358	11689	5590	1083	2120	5349
390	4293	0	0	2021	835	0	11080	8271	7576	3245	0	0
391	278	173	422	284	2587	6580	12035	7458	5823	4598	2930	2765
392	10	131	2160	119	3307	0	7716	8270	8140	4120	4174	0
393	0	818	2851	2489	1542	2216	12079	18410	7100	3737	2555	0
394	697	2672	2901	0	0	0	4857	3417	6436	2388	1256	3983
395	1460	92	0	1	122	0	0	9217	4203	3351	0	531
396	4059	2133	2455	2325	3666	581	2696	9474	6074	3264	476	3804
397	5162	2483	1246	1320	3893	2712	6325	8979	6669	5794	0	0
398	0	1107	0	1029	1220	2641	15583	5383	5547	0	1412	0
399	0	732	3196	4519	4030	10648	8131	5307	3825	2599	1315	6089
400	6387	4350	4046	485	0	0	4212	10664	2670	2353	2392	6904

TABLE V, Continued

401	0	0	964	0	4998	0	9888	6249	3600	3102	718	775
402	3292	2698	415	2189	2931	978	3450	2976	996	477	1478	4893
403	4996	4042	2887	7442	7030	10684	3655	9599	6654	6551	1754	7747
404	1066	1525	4394	4450	7016	7050	16903	14214	10733	4137	895	2698
405	2598	1787	2046	5306	4912	6180	12306	10784	195	3344	2765	0
406	1141	555	4350	6227	3808	675	5080	9329	7534	3918	2867	5269
407	3255	1988	1937	0	2498	3180	5482	11656	5439	2114	1511	6459
408	3516	2415	3859	2536	3819	1860	10429	8329	5886	3116	874	1426
409	2413	4392	1779	2631	1620	6227	7265	51	7969	3605	2129	0
410	530	1593	0	0	5770	1071	6803	5299	4852	2211	1093	4234
411	3164	0	0	2907	4857	5143	3431	9114	5165	3980	282	0
412	0	0	0	1190	0	0	0	0	6258	3918	1454	7660
413	5197	3997	5405	2920	1869	0	0	999	3831	1445	110	0
414	1307	1879	1389	197	0	0	0	13335	5485	4257	2385	7814
415	7433	3299	3146	408	3234	8514	5300	1485	1649	1264	0	721
416	284	0	1166	0	0	364	4109	6395	7121	3449	3423	651
417	0	374	900	0	1748	5218	6741	13591	3940	4766	2114	281
418	0	621	0	187	2374	0	5468	17273	9878	4081	5431	5413
419	846	159	0	0	3154	0	0	10792	4813	3778	1712	0
420	0	177	194	4117	364	2296	2018	0	1825	1088	0	1326
421	1721	1480	1581	612	2696	6835	6181	10460	0	1659	3863	4279
422	4214	3873	3005	1358	7082	4283	14176	11090	6505	2228	2502	4542
423	7337	4646	1785	3624	0	2031	10583	10311	6186	848	1163	0
424	2671	2684	1634	4322	6649	3565	3439	10568	5841	2407	1167	1296
425	3231	1421	1188	3592	778	1038	8535	13092	13292	5830	1119	5139
426	0	3441	1365	0	0	0	0	0	0	0	553	0
427	0	0	0	217	4110	5536	0	6504	5470	4742	2363	1760
428	2370	1415	1830	5824	2387	0	3767	9331	3964	1093	2593	7412
429	4066	4985	1250	0	4359	0	4064	4887	2249	970	2382	4026
430	4965	1732	269	1411	0	5670	4144	10408	5810	4698	2088	0
431	0	1901	2828	206	6676	8379	0	3713	6592	4840	0	0
432	0	1227	15	1514	0	0	0	5951	8967	6346	1520	2452
433	2135	2655	233	2092	1834	4252	7032	6062	5002	6514	2	7504
434	5184	4031	1973	2935	1402	4657	6180	955	0	3109	3166	4077
435	2172	0	2582	780	1321	5018	11447	10429	5000	2132	3477	0
436	3745	1860	2229	0	1888	0	829	4802	7945	1949	0	0
437	0	0	0	612	5499	4940	7720	6208	5341	2575	0	1857
438	5735	3169	1536	0	4610	0	8951	4169	7440	4382	0	2239
439	0	2795	0	1302	3385	11265	14281	5506	9689	2294	191	6557
440	3189	1931	160	0	4847	5681	7949	5915	5059	1525	1726	7573
441	4481	3322	840	4487	8912	11358	0	2613	7898	5489	2605	5190
442	4807	737	1808	2795	7647	8476	7064	11891	5970	2570	1990	3762
443	1989	1860	0	1822	4525	4881	1557	7216	6086	4072	2323	3426
444	1172	1360	2650	995	730	0	10580	12850	8973	1934	2390	6642
445	4812	1863	468	1284	1039	6743	12832	11864	9719	2624	0	3748
446	2665	3658	2451	2729	4657	9910	17580	19311	0	1254	977	551
447	102	1374	0	1516	1842	5098	4497	15649	7939	5670	2923	7048
448	3811	2488	1158	883	2748	7126	6106	8973	1472	2722	0	0
449	0	809	198	0	0	502	8745	3364	5061	5690	2629	0
450	1324	2197	2107	0	350	6668	0	5065	12789	5371	1561	3320

TABLE V, Continued

451	4140	1960	3250	3037	98	4156	5259	8643	10391	4382	2868	4029
452	215	1286	2696	2651	0	0	0	3814	0	1072	1058	2637
453	1872	725	747	4162	0	0	4141	0	4315	2113	0	0
454	2113	3993	751	1334	4566	10122	13634	8267	7749	5626	2667	3689
455	2795	2261	1514	2190	1763	2370	0	5896	3095	3229	587	677
456	4218	2064	2440	4048	9160	9249	10210	10146	10714	6002	1383	0
457	3151	3288	651	3469	4244	0	1987	0	2425	4078	1293	0
458	2319	3278	1978	1015	0	0	2933	1137	3471	1920	1813	1329
459	0	0	0	0	1550	2929	3138	10835	9215	2861	1571	6543
460	4402	1218	0	786	6501	10818	13717	1671	6291	2859	1780	2720
461	3112	2239	3242	3793	7012	10549	13700	15063	6571	4952	2848	4445
462	6134	3847	2347	3178	7248	8537	12609	5082	10586	4333	2383	3614
463	1624	2587	1253	1681	4390	1586	0	0	5366	1618	0	0
464	0	189	3055	2585	3770	4801	2167	5912	5060	4522	2633	3151
465	2273	2168	2034	3666	2999	3537	13930	9740	8512	5018	3886	5803
466	3506	3003	2852	4128	4419	3811	16201	10357	4985	855	795	0
467	385	2810	2432	5151	467	0	5235	12087	5423	2979	2293	0
468	1199	3269	562	1736	2229	7905	12227	4031	6791	4755	4159	601
469	3024	1519	2856	3419	0	4283	0	5502	7796	4970	1967	3370
470	2054	1659	2296	1766	2714	10093	4486	3106	1952	0	3037	620
471	1670	2315	0	1329	0	0	3975	13746	0	2333	0	3557
472	4310	2858	623	2649	4599	0	0	0	9089	5924	1653	1402
473	2757	3366	3309	2892	6103	5481	4231	3679	2236	1986	1214	940
474	941	1377	0	2724	3435	10496	2906	7647	2731	3566	5394	4453
475	6915	4707	0	168	4802	5768	4405	3765	1690	1270	1902	3518
476	1716	1008	569	1873	3232	7310	15947	8410	4057	3998	3123	293
477	1448	3190	3715	3520	6742	7141	8689	9004	3007	2323	0	2666
478	5	0	508	0	1191	0	0	7067	4237	3007	2822	6518
479	5122	2215	1900	301	0	1878	5869	7426	1894	1190	1595	1912
480	2936	1467	1664	468	3984	6226	0	6923	5975	2798	3709	2021
481	20	263	0	2100	7044	9491	9257	12553	4412	4596	1047	0
482	1044	3340	1836	1440	1339	0	8385	1779	2706	1682	1273	4524
483	5862	751	0	1465	1971	0	2252	0	9990	6212	296	392
484	2294	0	0	1497	0	0	6687	10953	9510	3827	2214	5070
485	0	2213	2566	0	435	0	0	5744	9616	2138	2746	8082
486	2540	993	1627	1327	6036	0	0	0	2029	1868	901	0
487	3596	3656	134	0	0	1239	14973	11552	11071	3938	1340	3058
488	7114	4130	1464	1251	0	0	0	0	5530	4425	3835	215
489	161	646	0	0	0	3018	5903	12445	7285	6275	2929	1730
490	6661	2939	3302	2667	1470	2266	4927	0	4293	3275	0	0
491	1501	2885	3723	3409	1790	6038	6911	15323	5447	3291	1409	4959
492	2830	3046	2859	5109	4195	4347	23304	13328	4241	2886	4406	0
493	1920	851	0	1054	0	4567	0	10729	3365	4230	2046	1514
494	312	1379	2672	1718	4612	0	268	3868	3868	3475	89	2114
495	0	1001	1308	5300	7783	8703	1761	13974	11420	5208	1493	4744
496	5232	3615	1995	0	3825	7390	1072	7345	4555	2551	2893	4277
497	0	0	1425	3453	2056	4671	13557	14552	8757	4096	1421	3620
498	8144	4798	4701	0	3480	7344	11286	10733	6690	1705	394	0
499	1812	769	903	5188	2748	0	0	0	0	0	0	3096
500	5499	1059	1038	3564	365	0	10364	8529	2928	4313	1047	2464

TABLE VI

EFFECTS OF NEGATIVE FLOW ON SYNTHESIZED DATA

STA. 2505 - ARKANSAS RIVER

MONTH	NUMBER	NEGATIVE FLOW - 10 CFS.	PERCENTAGE	CORRECTED FLOW - 10 CFS.	MEAN FLOW - 10 CFS.
Oct.	76	103407	0.50	1394623	2789
Nov.	35	21418	0.10	1038641	2077
Dec.	98	84551	0.41	913368	1827
Jan.	82	109219	0.53	1138955	2278
Feb.	97	155423	0.75	1372429	2745
March	133	380539	1.84	1816481	3633
April	124	481537	2.33	2696137	5392
May	53	112866	0.55	3642843	7286
June	28	41818	0.20	2856975	5714
July	20	17095	0.09	1695662	3391
Aug.	76	61123	0.30	864190	1728
Sept.	130	336414	1.63	1367810	2736
TOTALS	952	1905404	9.23	20798113	

TABLE VII

COMPARISON OF MINIMUM FLOWS FROM THE SYNTHESIZED AND ACTUAL FLOWS AT STATION 2505 - ARKANSAS RIVER

MONTH	SYNTHESIZED FLOW - CFS YEAR	SYNTHESIZED FLOW - CFS MIN. MONTHLY	ACTUAL FLOW - CFS MIN. MONTHLY	ACTUAL FLOW - CFS MIN. DAILY
Oct.	478	50	492	306
Nov.	340	40	1262	602
Dec.	289	130	1421	695
Jan.	395	10	1194	559
Feb.	24	100	2328	1260
Mar.	288	40	2401	1360
April	133	290	3185	1280
May	409	510	7450	2280
June	279	800	5353	2440
July	53	1210	1571	795
Aug.	432	20	658	245
Sept.	178	310	742	418

TABLE VIII

COMPARISON OF AVERAGE SYNTHESIZED AND ACTUAL FLOWS AT STATION 2505 - ARKANSAS RIVER

MONTH	AVERAGE SYNTHESIZED FLOW - CFS.	AVERAGE ACTUAL FLOW - CFS.
Oct.	27890	24079
Nov.	20770	20021
Dec.	18270	16925
Jan.	22780	19125
Feb.	27450	23984
May	36330	28705
Apr.	53920	47054
May	72860	66958
June	57140	54147
July	33910	33507
Aug.	17280	15672
Sept.	27360	18638

3. Analysis of Low Flows

The methods used for analyzing low flows were presented in Chapter IV, part D. These same methods were used to analyze low flows from the historical record and the synthesized data.

Duration curves of thirty days, six months, one, two, three, and five years were analyzed. A thirty day duration curve was not determined for the synthesized data because the monthly flows of zero magnitude were not considered as characteristic of the actual flow.

The five hundred years of synthesized data was divided into five - 100 year periods. Each of these one hundred year periods was analyzed for low flows. A computer program was devised to rank each flow, find its return period and probability of occurrence. The IBM 7040 computer was used in this operation. Table IX typifies the computer output, program is in Appendix C.

Table X contains the necessary data for analyzing low observed flows by duration curves. The thirty day duration curves, Figures 20-31, were presented on logarithmic normal paper to obtain a smooth curve. The duration curves, Figures 32-37, for six months, one, two, three, and five years were presented on probability paper. The hydrograph, Figure 38, in this chapter compares the synthesized flows with the actual (observed) flows of one year duration.

TABLE IX

SAMPLE OF COMPUTER OUTPUT FOR DURATION CURVE ANALYSIS OF SYNTHESIZED FLOWS AT STATION 2505 - ARKANSAS RIVER

FOR THE 5TH 100 YEARS
DISCHARGES OF ONE YEAR DURATION
10 cfs.

RANK	MAGNITUDE	TP	P	YEAR
1	446.58333206	101.00000000	0.00990099	426
2	1117.08332825	50.50000000	0.01980198	420
3	1209.66665649	33.66666651	0.02970297	499
4	1285.75000000	25.25000000	0.03960396	452
5	1443.41665649	20.19999981	0.04950495	486
6	1506.25000000	16.83333325	0.05940594	453
7	1675.41665649	14.42857134	0.06930693	463
8	1706.66665649	12.62500000	0.07920792	412
9	1766.08332825	11.22222221	0.08910891	458
10	2031.25000000	10.09999990	0.09900990	494
11	2048.83331299	9.18181813	0.10891089	457
12	2103.91665649	8.41666663	0.11881188	436
13	2104.50000000	7.76923072	0.12871287	419
14	2112.91665649	7.21428567	0.13861386	478
15	2147.75000000	6.73333329	0.14851485	413
16	2198.08331299	6.31250000	0.15841584	455
17	2231.08331299	5.94117641	0.16831683	402
18	2246.83331299	5.61111110	0.17821782	416
19	2249.83331299	5.31578946	0.18811881	449
20	2330.33331299	5.04999995	0.19801980	488
21	2332.66665649	4.80952376	0.20792079	432
22	2410.41665649	4.59090906	0.21782178	471
23	2432.58331299	4.39130431	0.22772277	483
24	2445.66665649	4.20833331	0.23762376	482
25	2523.00000000	4.03999996	0.24752475	493
26	2524.50000000	3.88461536	0.25742574	401
27	2558.50000000	3.74074072	0.26732673	427
28	2608.50000000	3.60714284	0.27722772	479
29	2650.00000000	3.48275861	0.28712871	490
30	2758.91665649	3.36666664	0.29702970	472
31	2769.83331299	3.25806451	0.30693069	429
32	2788.00000000	3.15625000	0.31683168	410
33	2795.00000000	3.06060603	0.32673267	485
34	2815.25000000	2.97058821	0.33663366	470
35	2896.00000000	2.88571426	0.34653465	437
36	2927.91665649	2.80555555	0.35643564	431
37	3037.75000000	2.72972971	0.36633663	415
38	3123.91665649	2.65789473	0.37623762	448
39	3139.08331299	2.58974358	0.38613861	434
40	3153.75000000	2.52499998	0.39603961	464
41	3170.25000000	2.46341461	0.40594060	411
42	3170.66665649	2.40476188	0.41584159	414
43	3180.91665649	2.34883720	0.42574257	480
44	3182.83331299	2.29545453	0.43564356	473
45	3220.16665649	2.24444443	0.44554456	459
46	3225.50000000	2.19565216	0.45544555	469
47	3242.50000000	2.14893615	0.46534654	475
48	3271.83331299	2.10416666	0.47524752	467
49	3306.08331299	2.06122446	0.48514852	417
50	3313.08331299	2.01999998	0.49504951	443

TABLE IX, Continued

RANK	MAGNITUDE	TP	P	YEAR
51	3340.08331299	1.98039214	0.50495049	409
52	3366.00000000	1.94230768	0.51485149	489
53	3396.00000000	1.90566038	0.52475248	450
54	3430.83331299	1.87037036	0.53465346	500
55	3432.91665649	1.83636363	0.54455446	430
56	3447.25000000	1.80357142	0.55445544	421
57	3498.83331299	1.77192982	0.56435643	428
58	3504.33331299	1.74137931	0.57425743	484
59	3519.25000000	1.71186440	0.58415841	438
60	3696.50000000	1.68333332	0.59405941	435
61	3729.16665649	1.65573770	0.60396039	496
62	3776.41665649	1.62903225	0.61386138	433
63	3793.25000000	1.60317460	0.62376238	407
64	3796.25000000	1.57812500	0.63366336	440
65	3805.83331299	1.55384615	0.64356435	474
66	3853.58331299	1.53030302	0.65346535	424
67	4005.41665649	1.50746268	0.66336633	408
68	4042.83331299	1.48529410	0.67326733	423
69	4122.00000000	1.46376811	0.68316831	468
70	4189.66662598	1.44285713	0.69306931	444
71	4227.16662598	1.42253521	0.70297030	418
72	4229.41662598	1.40277778	0.71287128	406
73	4231.91662598	1.38356164	0.72277228	481
74	4287.08331299	1.36486486	0.73267327	477
75	4294.66662598	1.34666666	0.74257425	476
76	4351.08331299	1.32894737	0.75247525	451
77	4351.91662598	1.31168830	0.76237624	405
78	4396.91662598	1.29487179	0.77227723	460
79	4471.50000000	1.27848101	0.78217822	447
80	4546.41662598	1.26249999	0.79207921	487
81	4576.00000000	1.24691357	0.80198020	466
82	4723.83331299	1.23170730	0.81188119	491
83	4749.66662598	1.21686746	0.82178218	445
84	4766.25000000	1.20238094	0.83168317	441
85	4772.08331299	1.18823528	0.84158416	439
86	4800.66662598	1.17441860	0.85148515	497
87	4854.58331299	1.16091953	0.86138614	425
88	4939.58331299	1.14772727	0.87128713	498
89	4959.75000000	1.13483146	0.88118812	442
90	5224.58331299	1.12222221	0.89108911	495
91	5297.16662598	1.10989010	0.90099010	465
92	5375.91662598	1.09782608	0.91089109	454
93	5404.83331299	1.08602150	0.92079208	422
94	5478.58331299	1.07446808	0.93069308	446
95	5802.83331299	1.06315789	0.94059406	456
96	5824.83331299	1.05208333	0.95049505	462
97	5879.25000000	1.04123710	0.96039604	492
98	6086.75000000	1.03061223	0.97029704	403
99	6256.75000000	1.02020201	0.98019803	404
100	6460.50000000	1.00999999	0.99009901	461

TABLE X

DATA FOR DURATION CURVE ANALYSIS
STA. 2505 - ARKANSAS RIVER
ACTUAL FLOW

Rank	PROBABILITY %	RETURN PERIOD YEARS	DISCHARGE IN CFS. FOR INDICATED DURATIONS 1 YEAR	2 YEAR	3 YEAR	5 YEAR	6 MONTHS AUG.-JAN.	6 MONTHS FEB.-JULY
1	2.56	39.00	5965	9062	9002	13394	1688	5431
2	5.12	19.50	7523	9402	11304	17389	2060	7776
3	7.69	13.00	7737	10368	14681	19070	3267	9454
4	10.25	9.75	8474	10657	15894	20000	3613	10667
5	12.82	7.80	10600	10707	16475	21371	4221	11058
6	15.38	6.50	11890	15743	16732	21393	5418	12498
7	17.95	5.57	12800	17250	18090	23572	5899	12604
8	20.49	4.88	12840	18120	20889	24393	7250	17797
9	23.09	4.33	12940	18350	21615	24510	7469	20668
10	25.64	3.90	13000	19622	22285	25402	7827	22104
11	28.16	3.55	14900	19845	22308	25808	8723	22131
12	30.76	3.25	19600	21590	22710	26004	9552	26800
13	33.33	3.00	21750	23540	24399	26227	9623	27435
14	35.84	2.79	23750	26350	24522	26248	11081	27345
15	38.46	2.60	23900	28465	27918	27149	13309	27435
16	40.98	2.44	24350	28600	28265	27460	14220	30180
17	43.59	2.39	26750	30548	28334	29551	14607	30335
18	46.08	2.17	28800	30755	28486	30055	15533	31510
19	48.78	2.05	31800	31210	29858	31460	15733	33042
20	51.28	1.95	32580	33740	31860	32858	16461	38981
21	53.76	1.86	33520	34850	34509	33131	16575	44748
22	56.76	1.77	33790	36225	35429	34071	16910	44772
23	58.82	1.70	34080	36360	36056	34490	19551	51397
24	61.34	1.63	34910	36785	37069	37348	19637	51957
25	64.10	1.56	35450	36905	37488	37715	19749	52997
26	66.67	1.50	38660	40045	37729	38837	20657	56007
27	69.44	1.44	41620	42000	38679	38884	20709	57117
28	71.94	1.39	43400	42730	39188	40764	21410	57182
29	74.62	1.34	43900	43400	40164	41266	23727	57930
30	76.92	1.30	45180	43700	42748	41556	26941	59573
31	79.36	1.26	45960	43790	42983	43096	30077	59821
32	81.96	1.22	49620	45290	43811	43878	32140	72050
33	84.74	1.18	50100	45730	44679	48798	34171	74398
34	86.95	1.15	50480	46000	48490	49196	34560	75107
35	90.09	1.11	52060	48365	48685		50183	77483
36	92.09	1.08	55130	48525	49253		52503	82783
37	95.23	1.05	62940	57815			54479	102970
38	97.08	1.03	65250				64747	103055

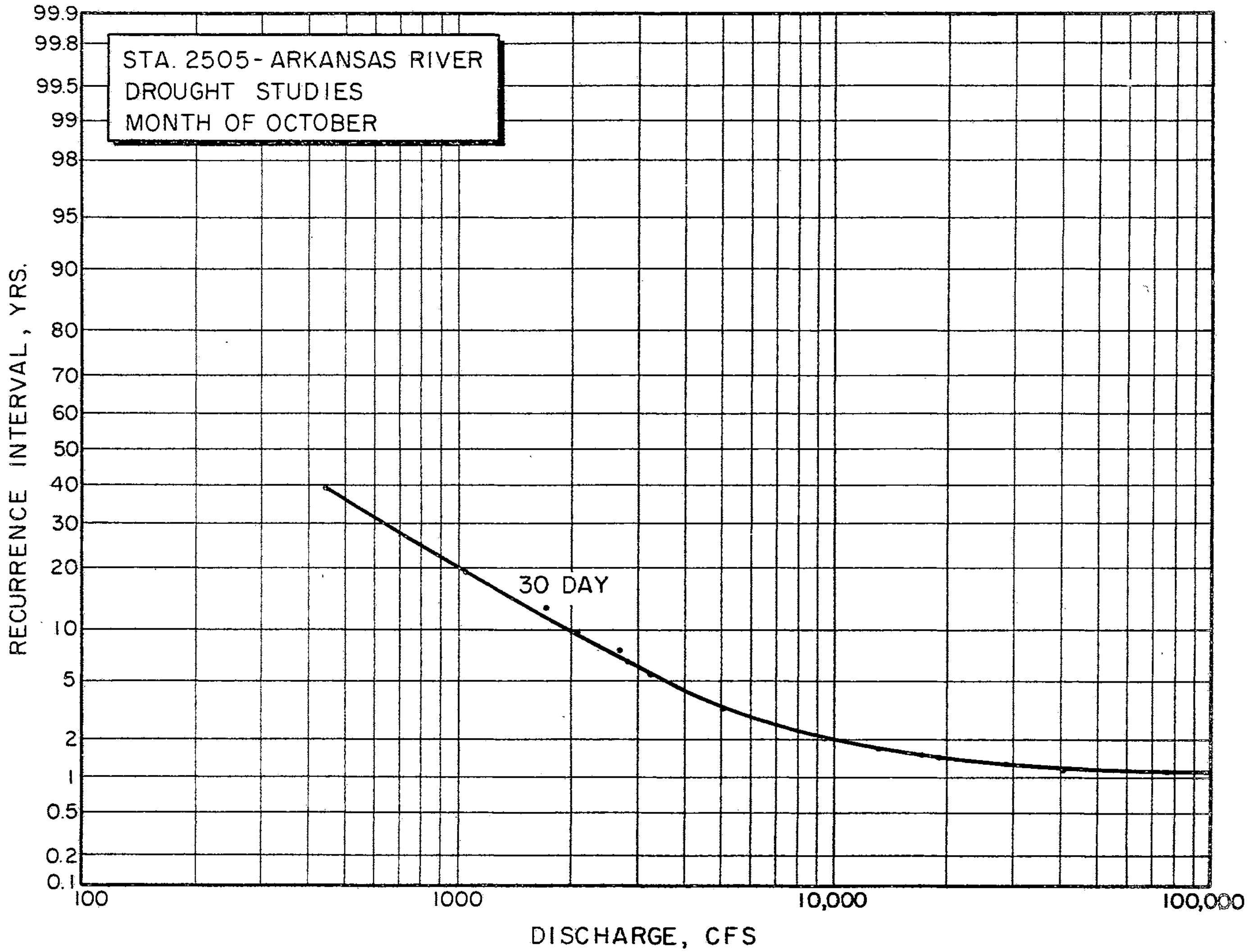


Fig. 20 Duration Curves for October at Station 2505

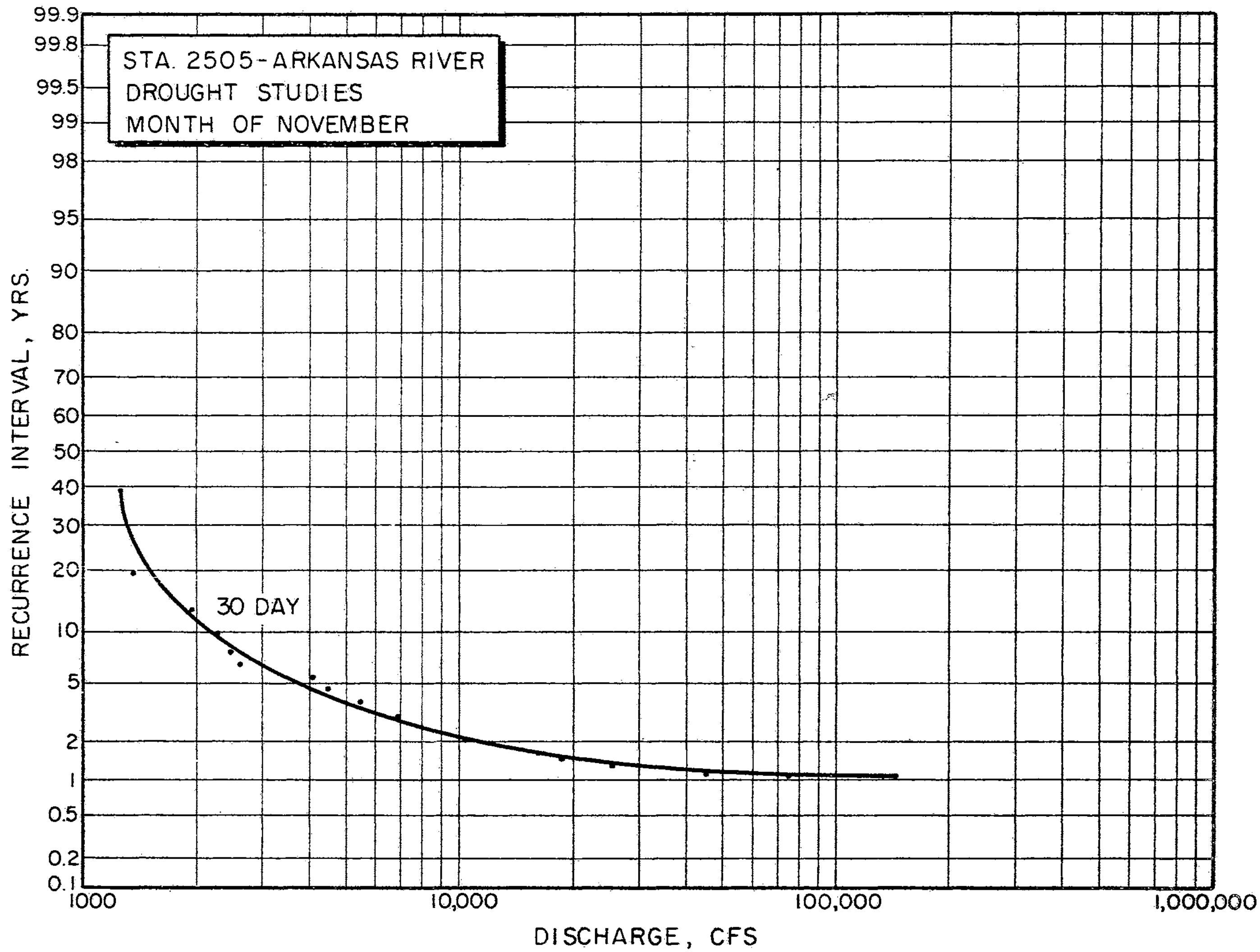


Fig. 21 Duration Curves for November at Station 2505

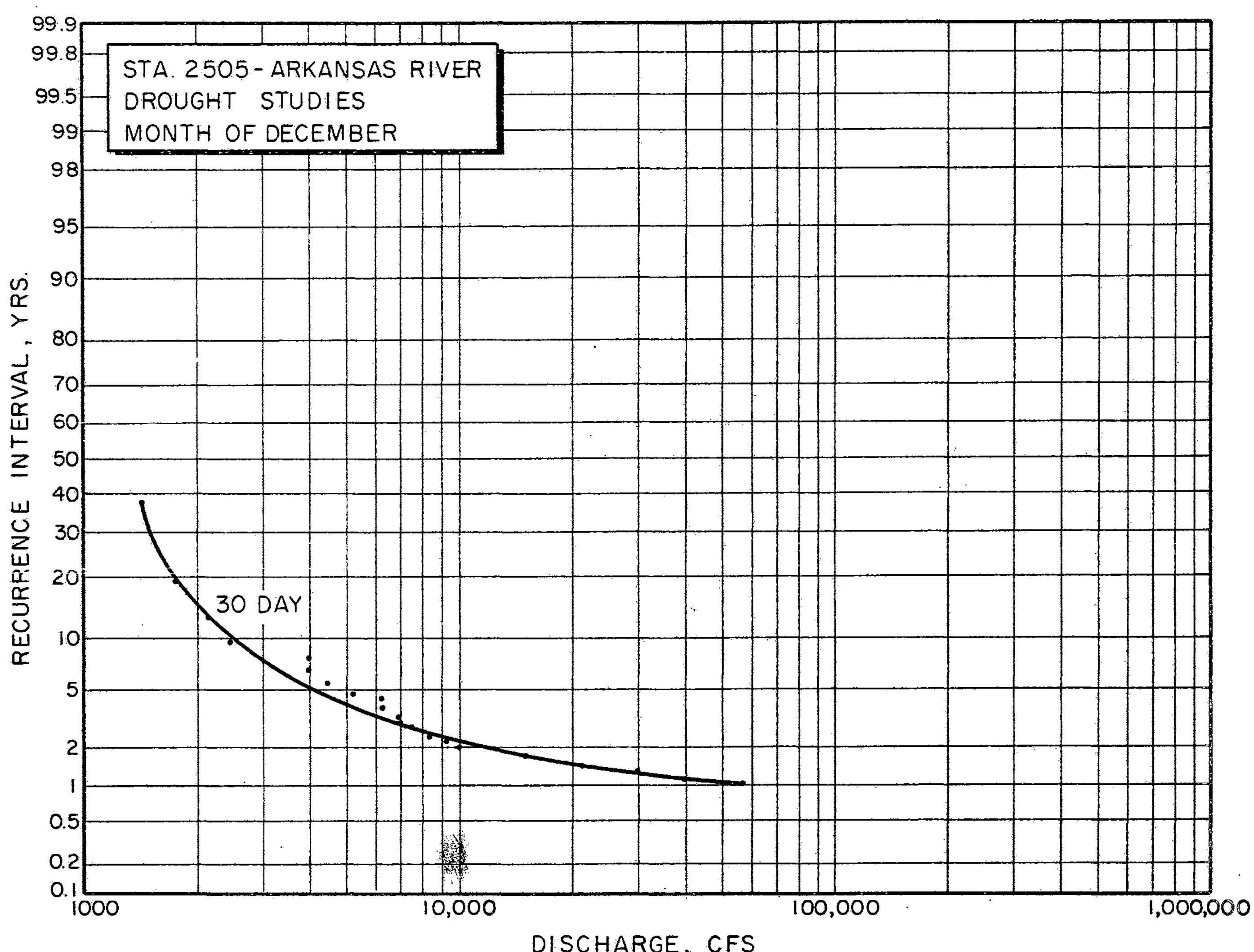


Fig. 22 Duration Curves for December at Station 2505

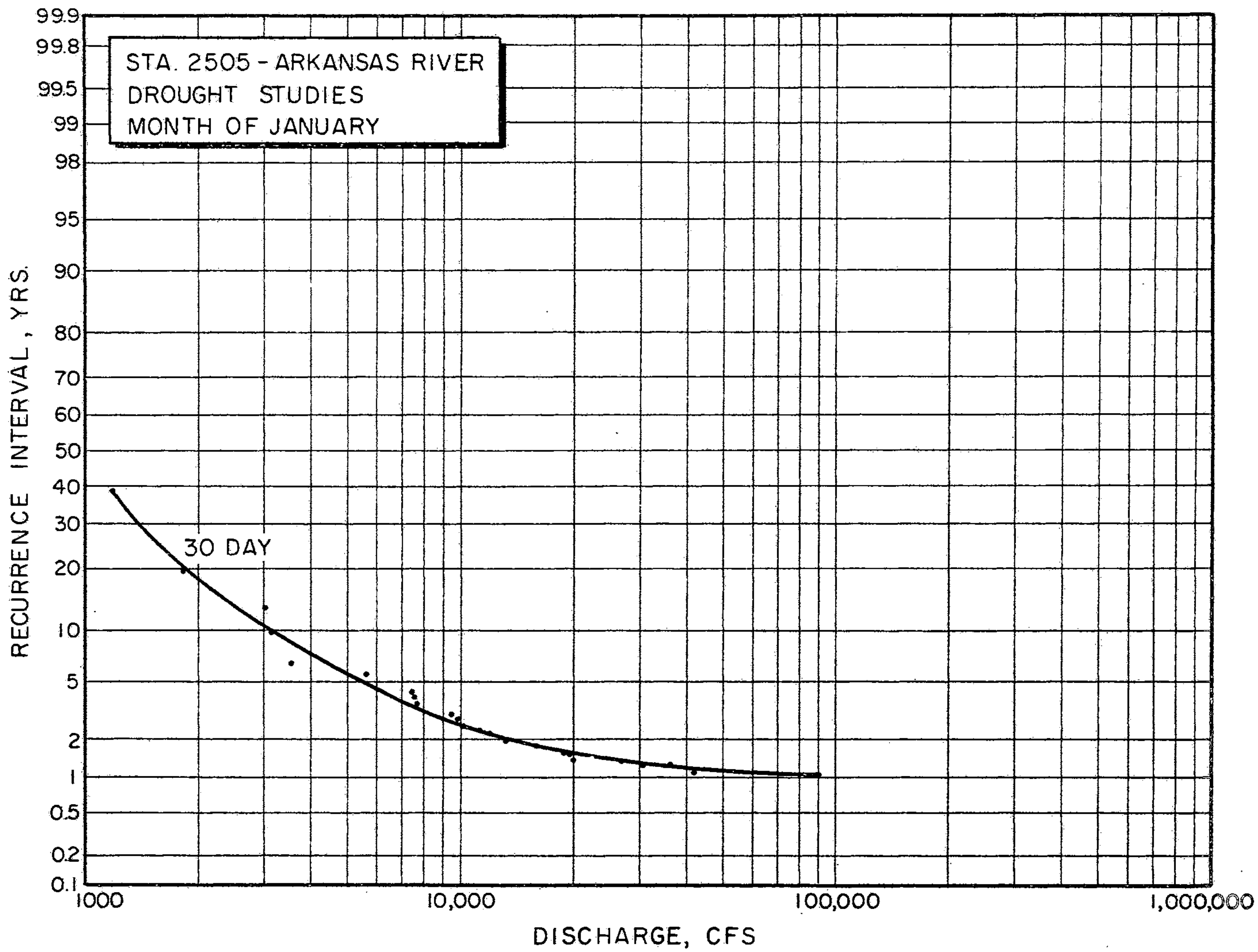


Fig. 23 Duration Curves for January at Station 2505

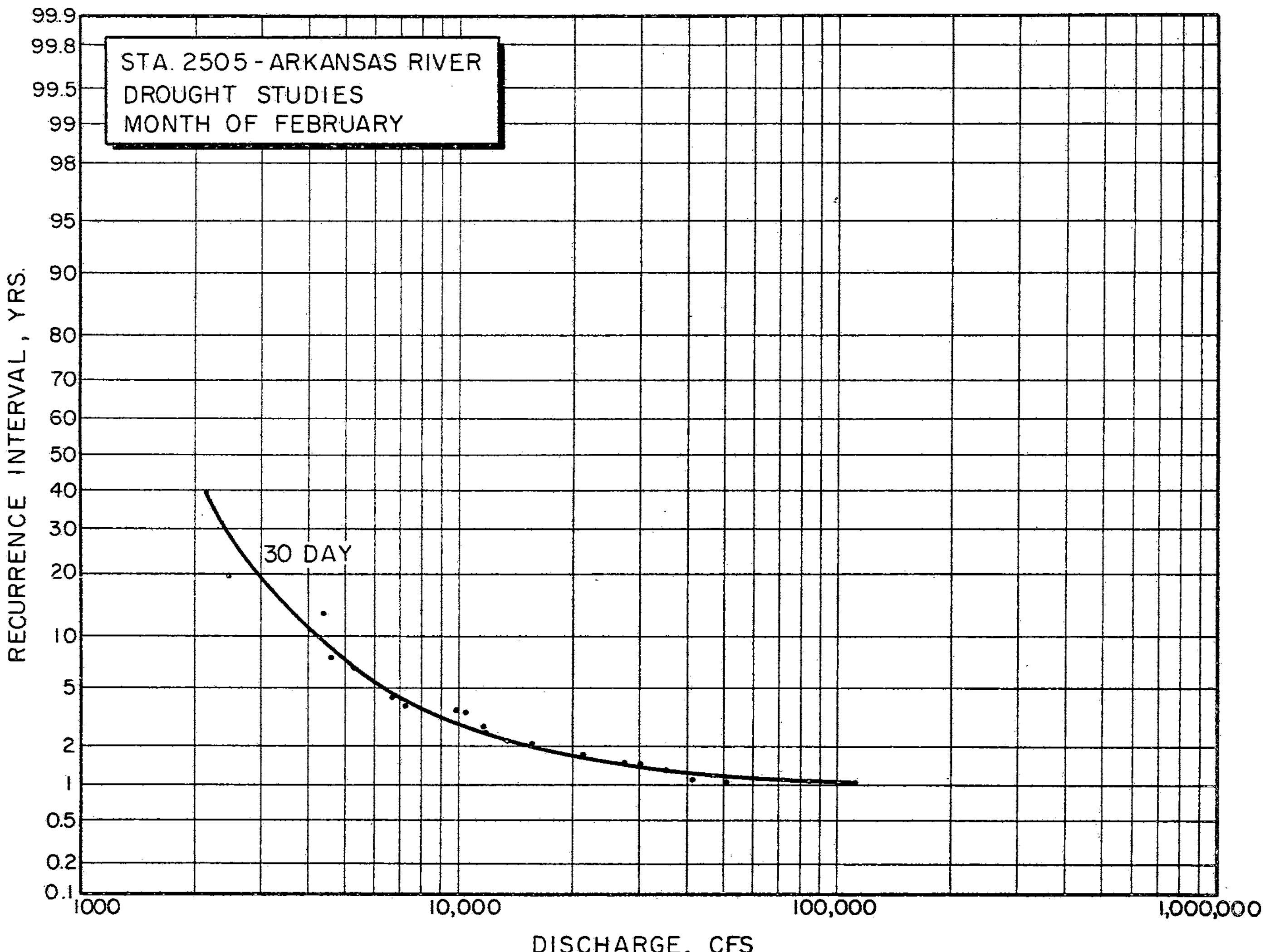


Fig. 24 Duration Curves for February at Station 2505

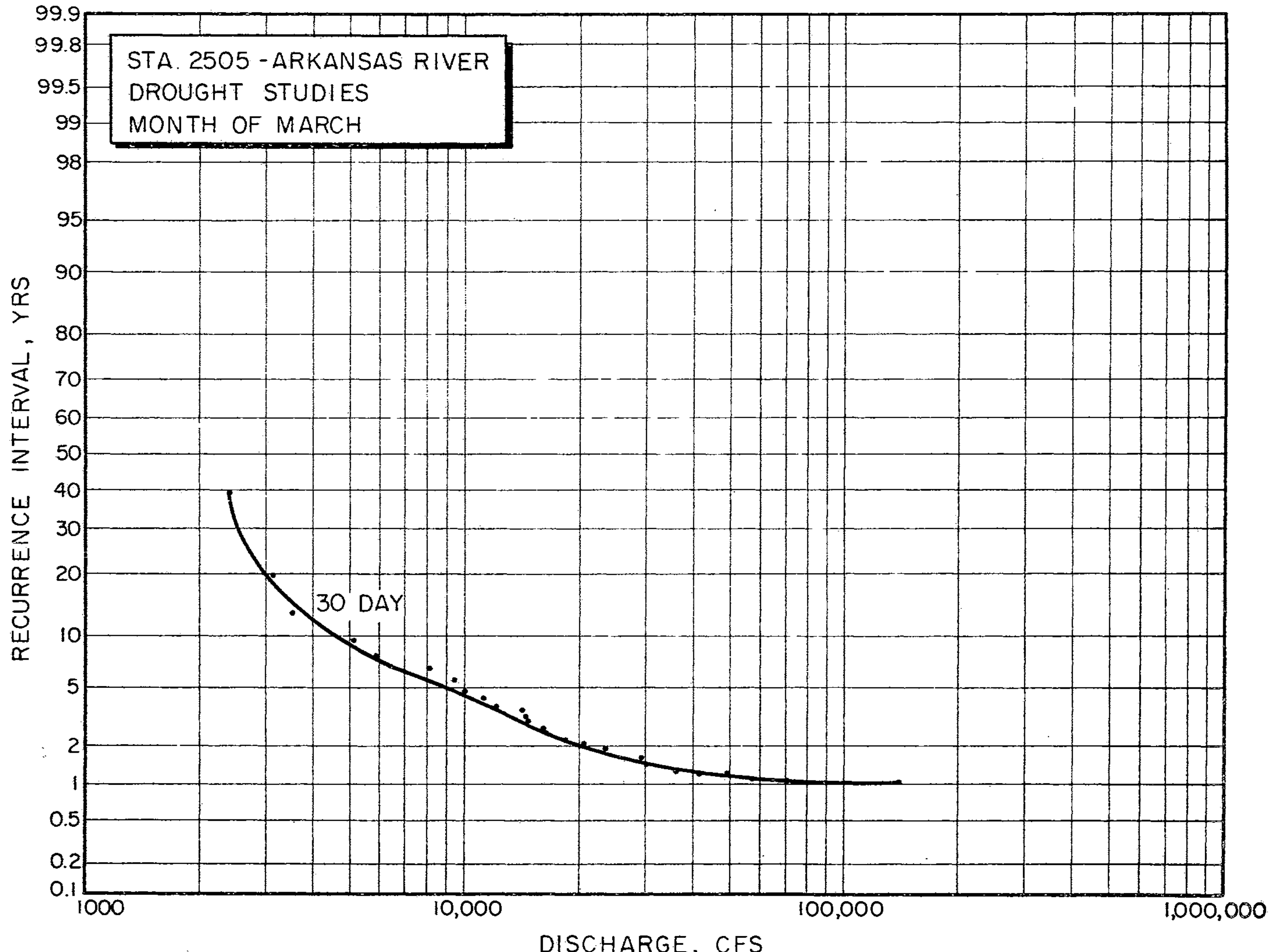


Fig. 25 Duration Curves for March at Station 2505

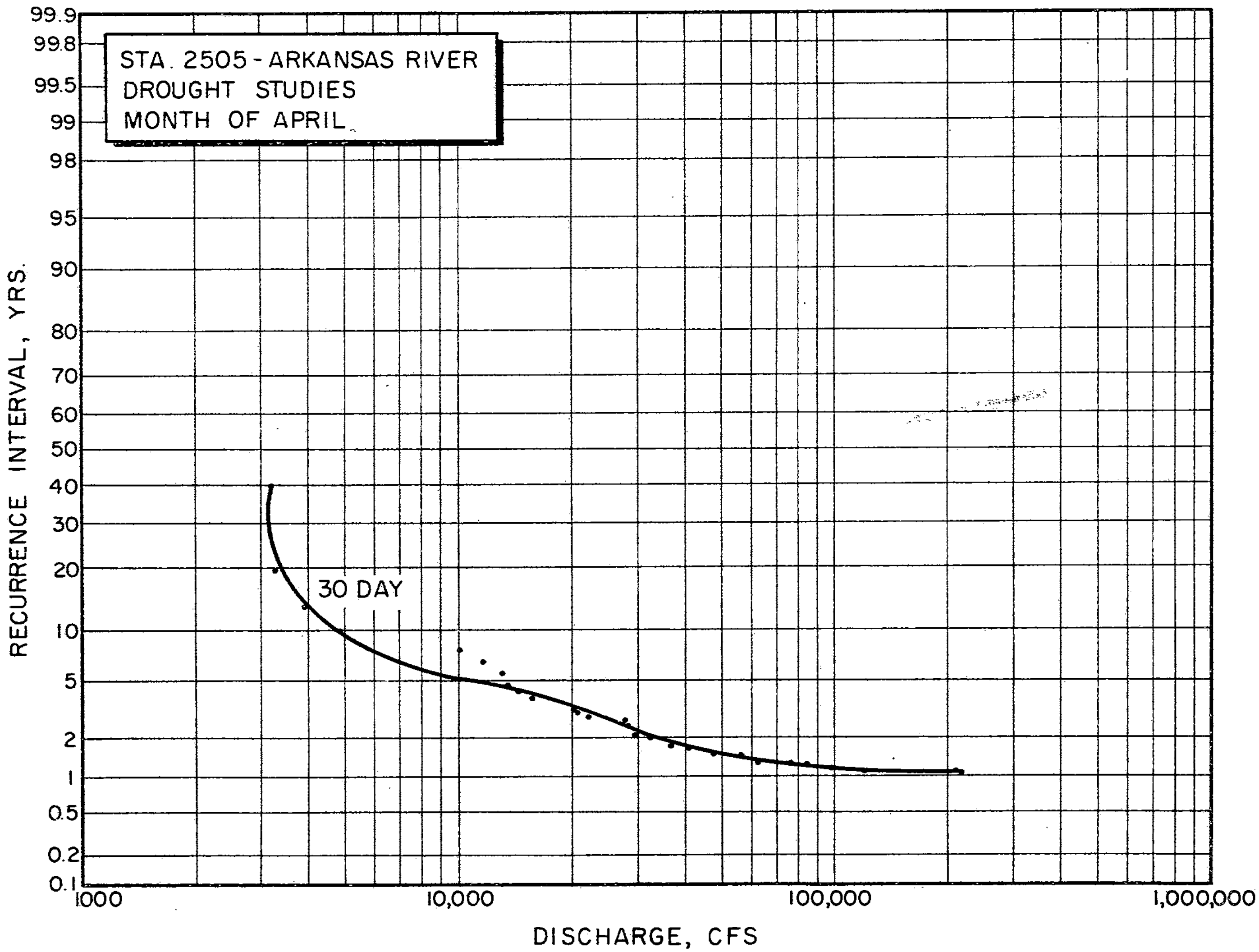


Fig. 26 Duration Curves for April at Station 2505

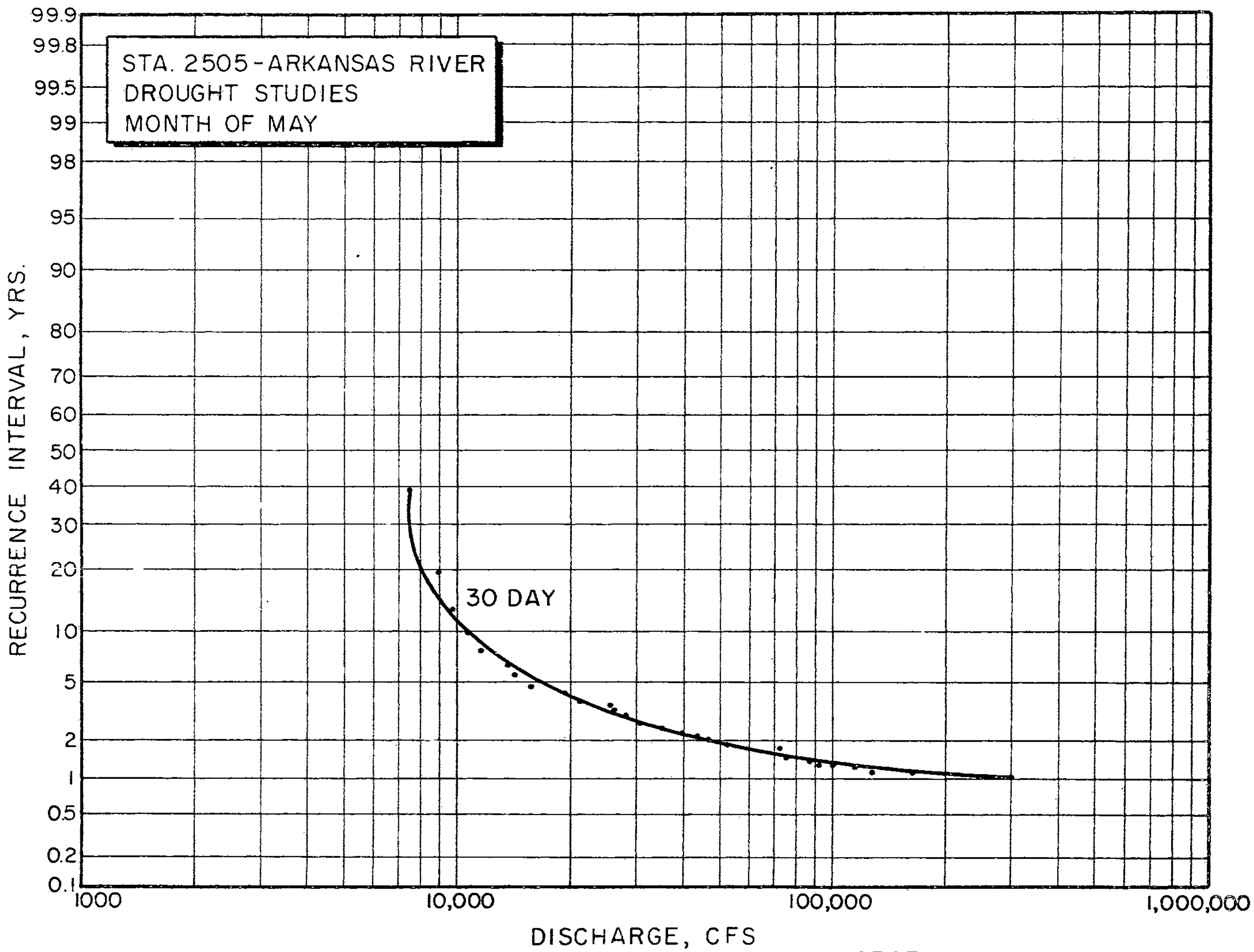


Fig. 27 Duration Curves for May at Station 2505

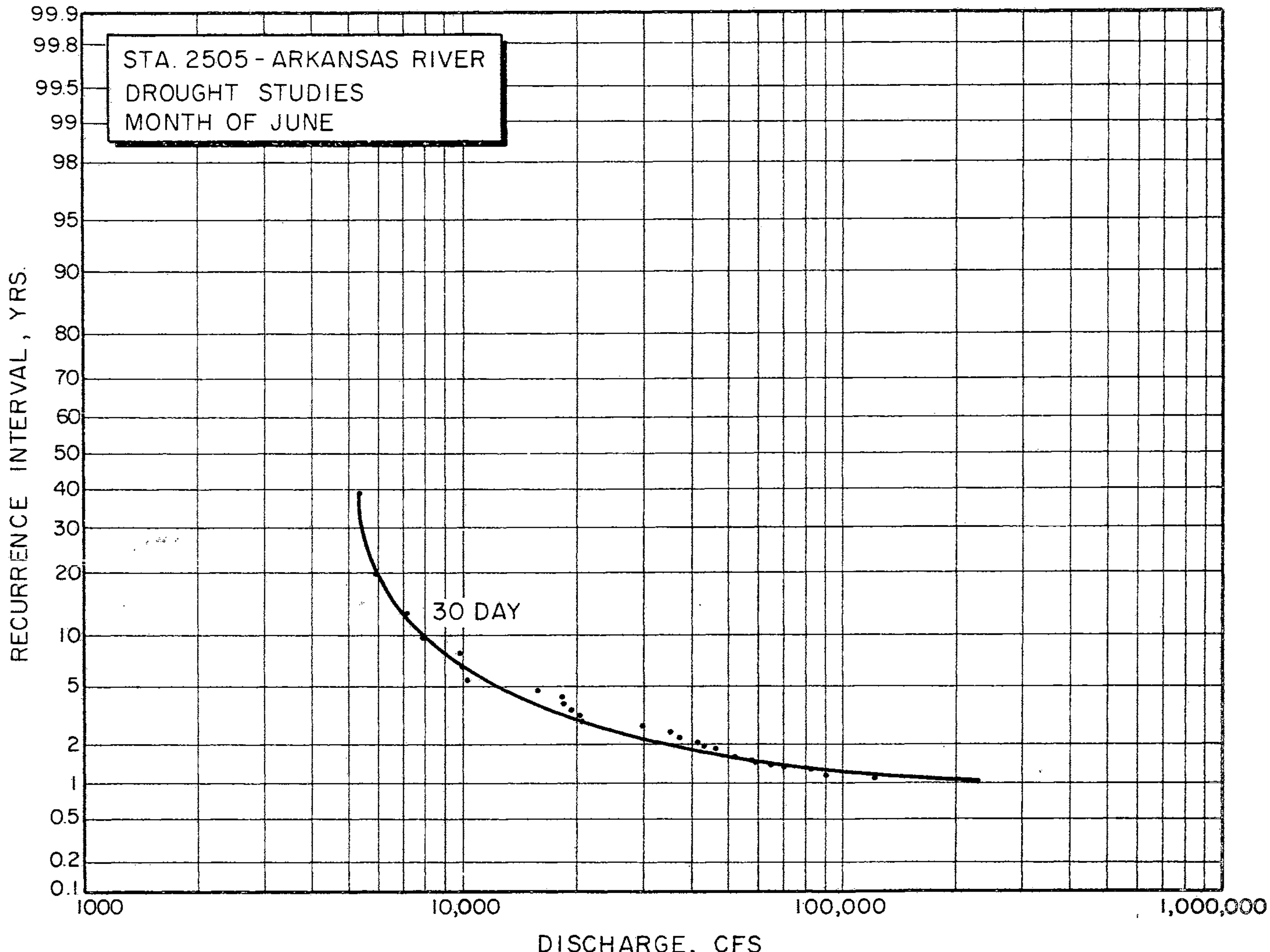


Fig. 28 Duration Curves for June at Station 2505

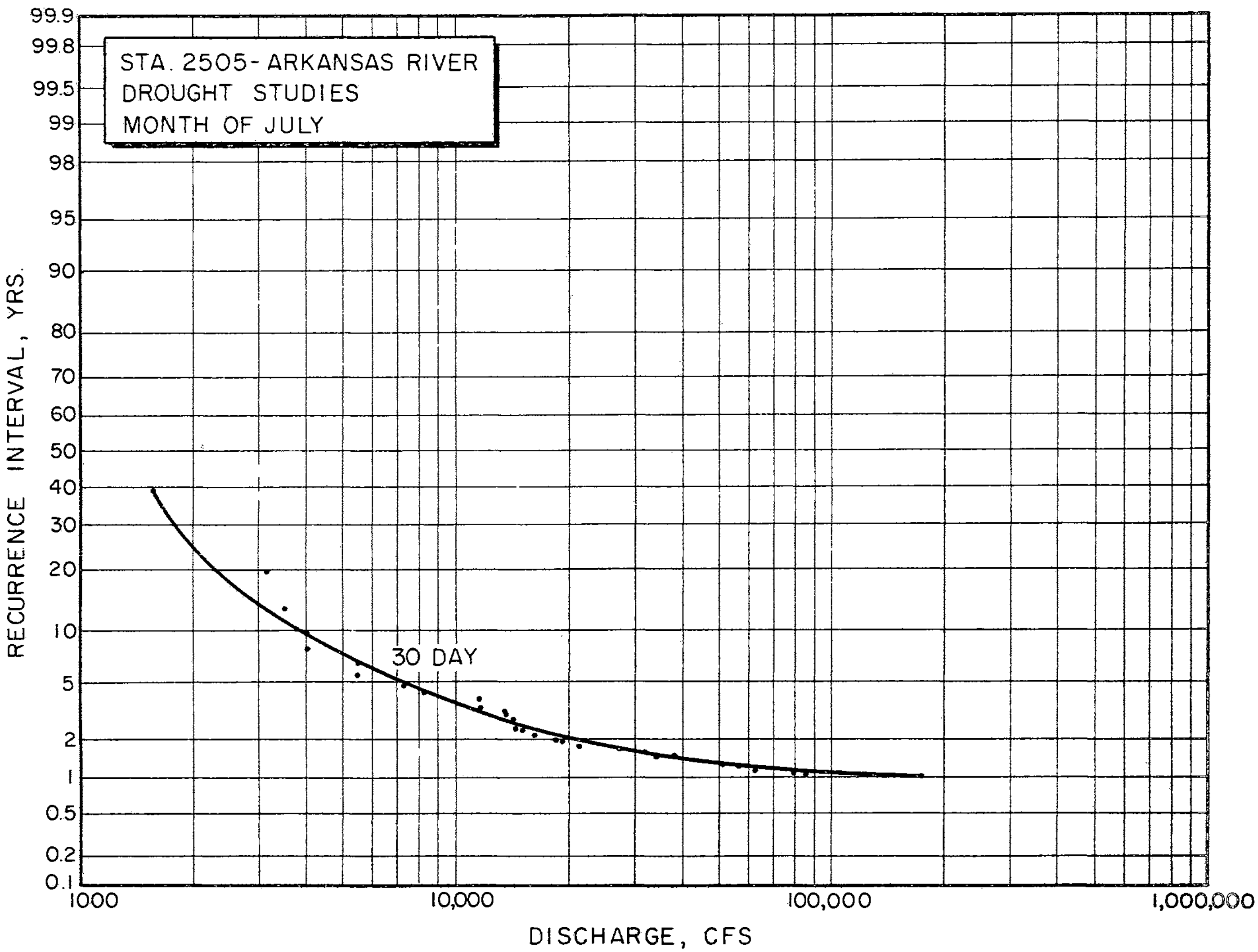


Fig. 29 Duration Curves for July at Station 2505

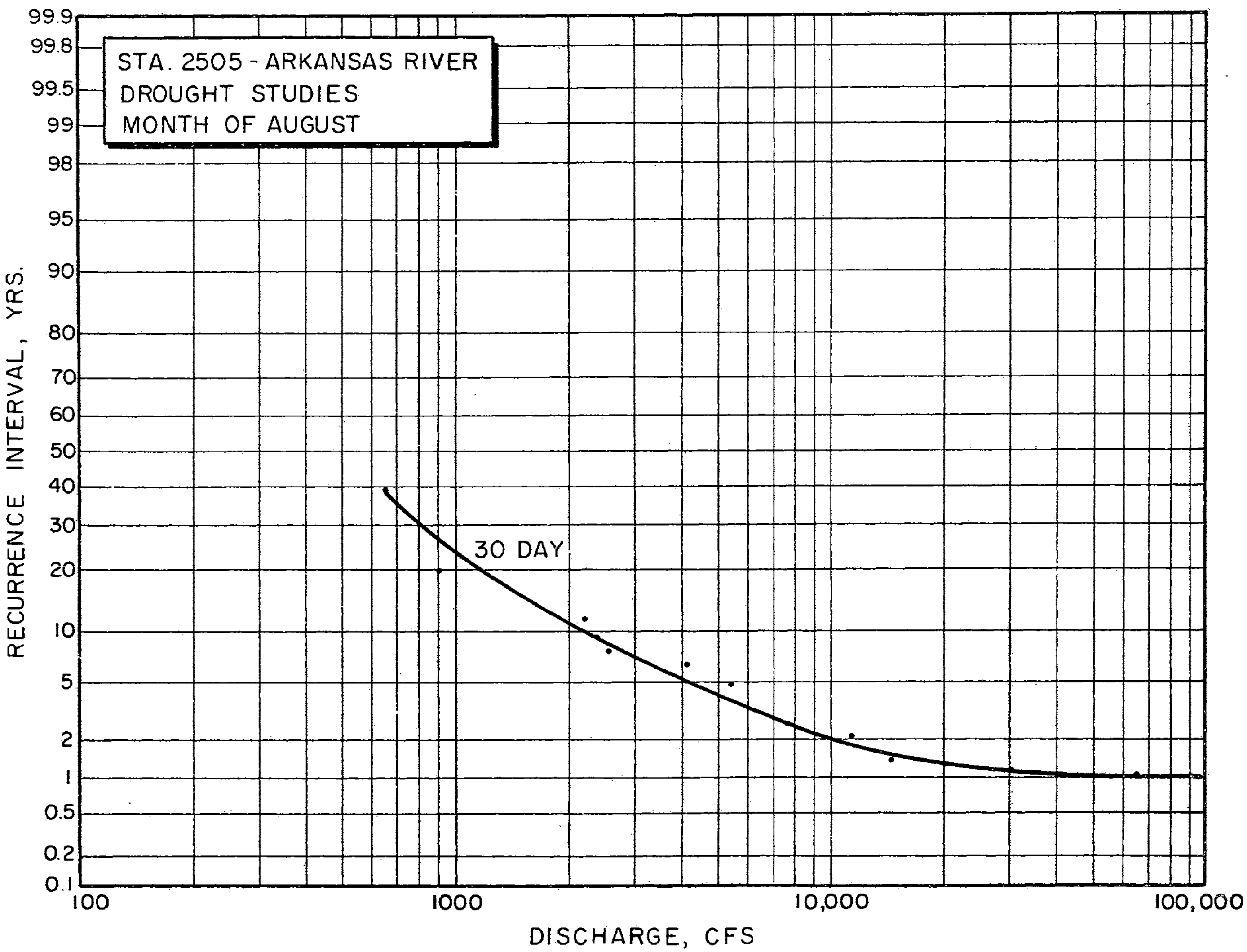


Fig. 30 Duration Curves for August at Station 2505

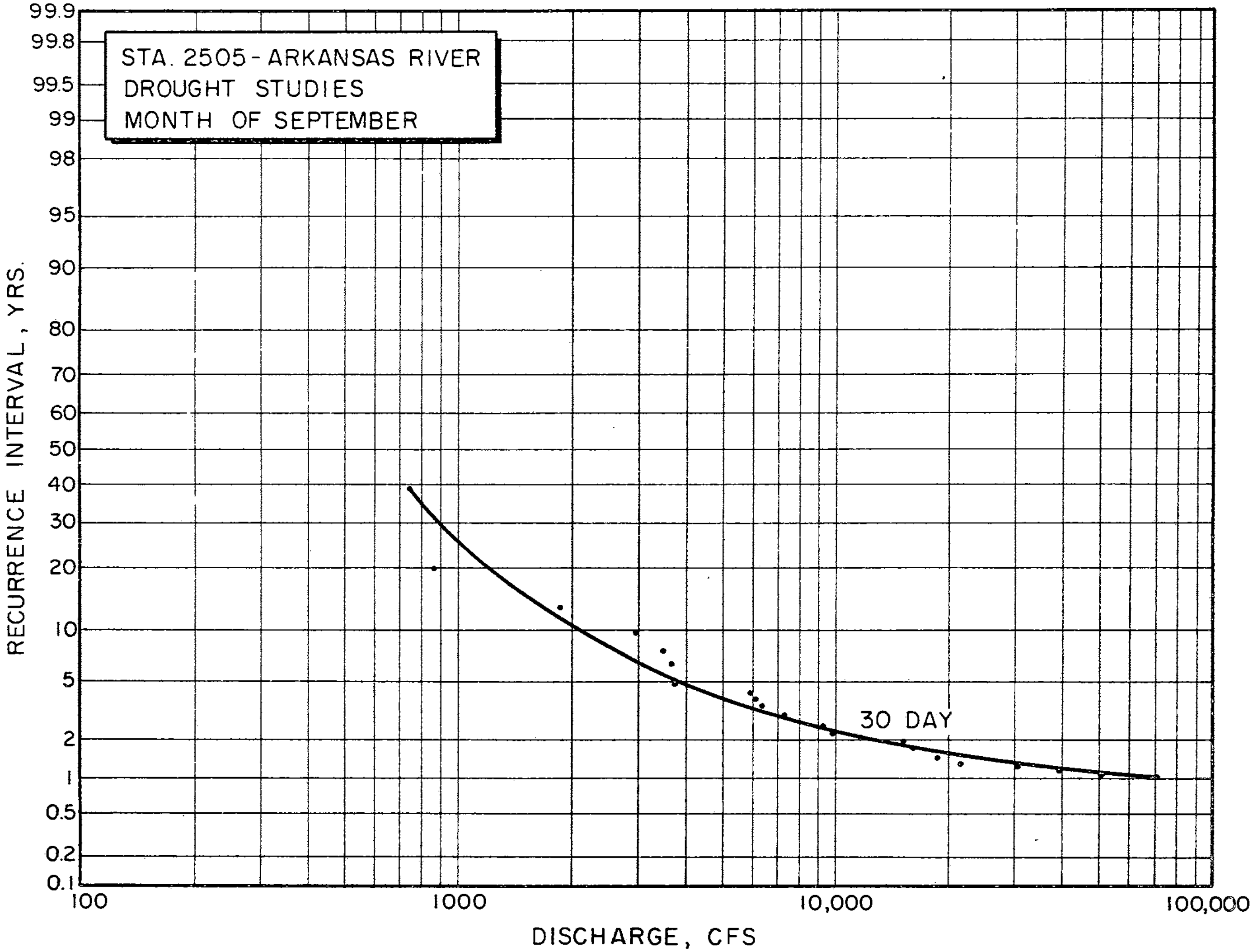


Fig. 31 Duration Curves for September at Station 2505

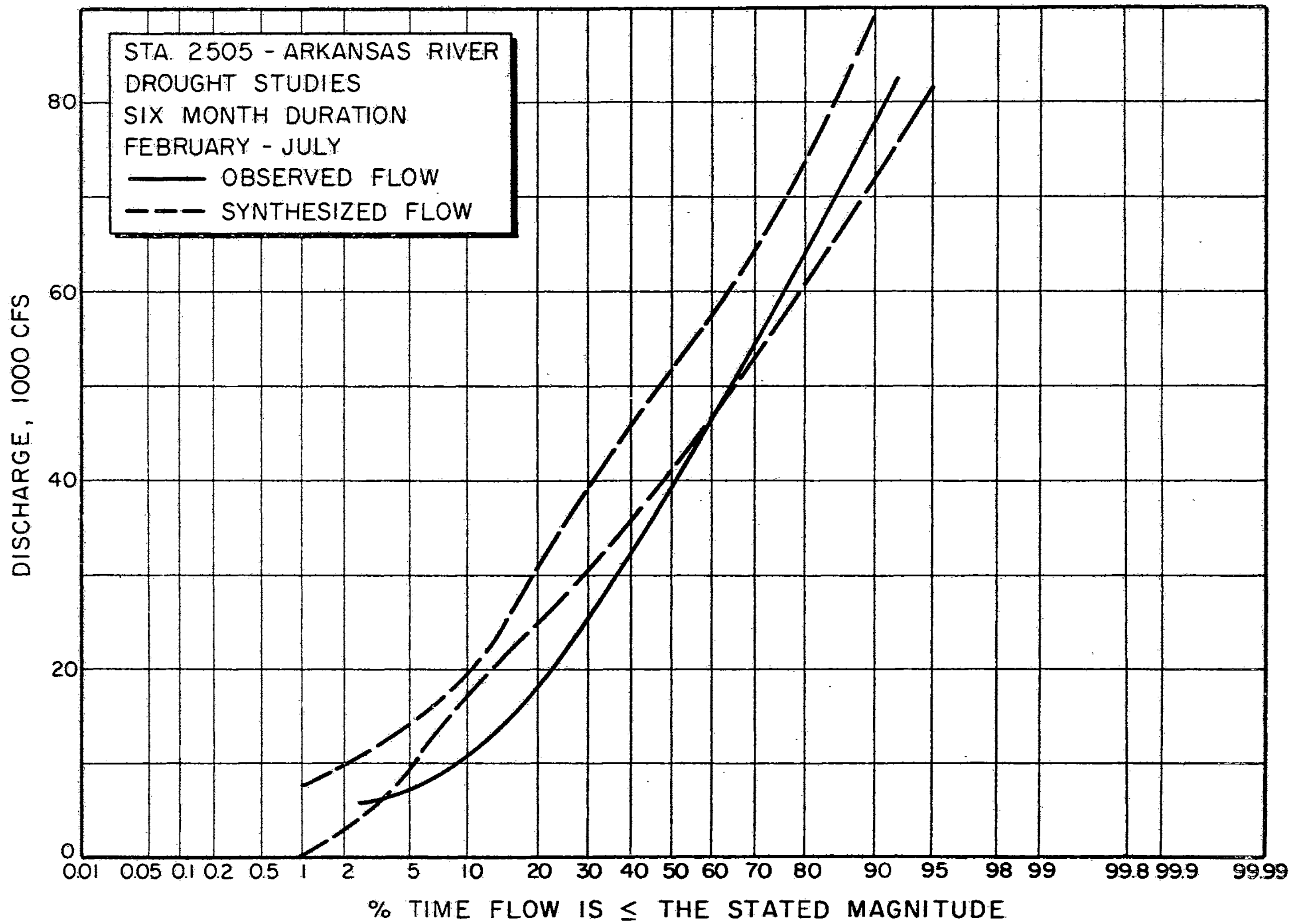


Fig. 32 Duration Curves for February - July at Station 2505

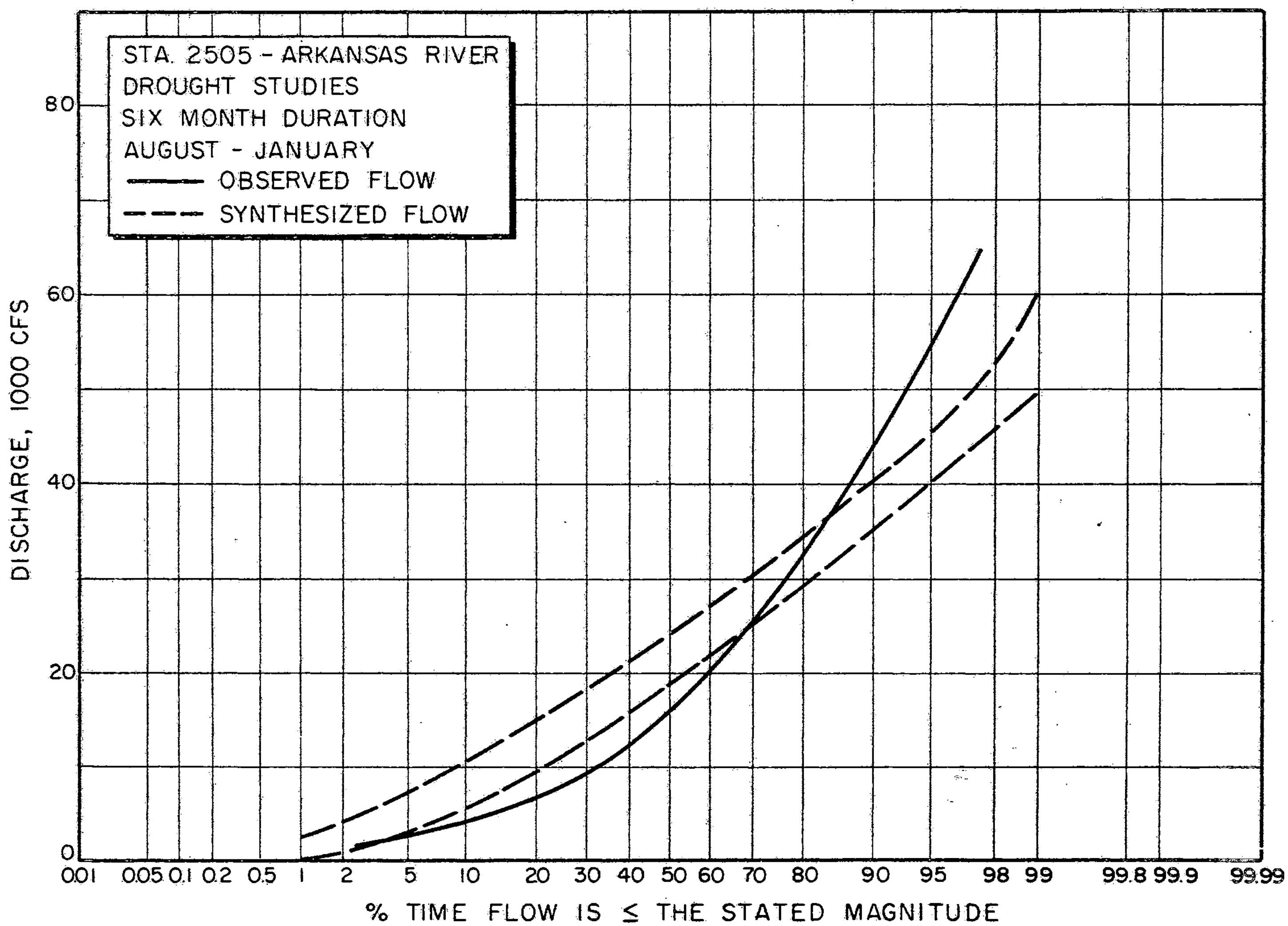


Fig. 33 Duration Curves for August - January at Station 2505

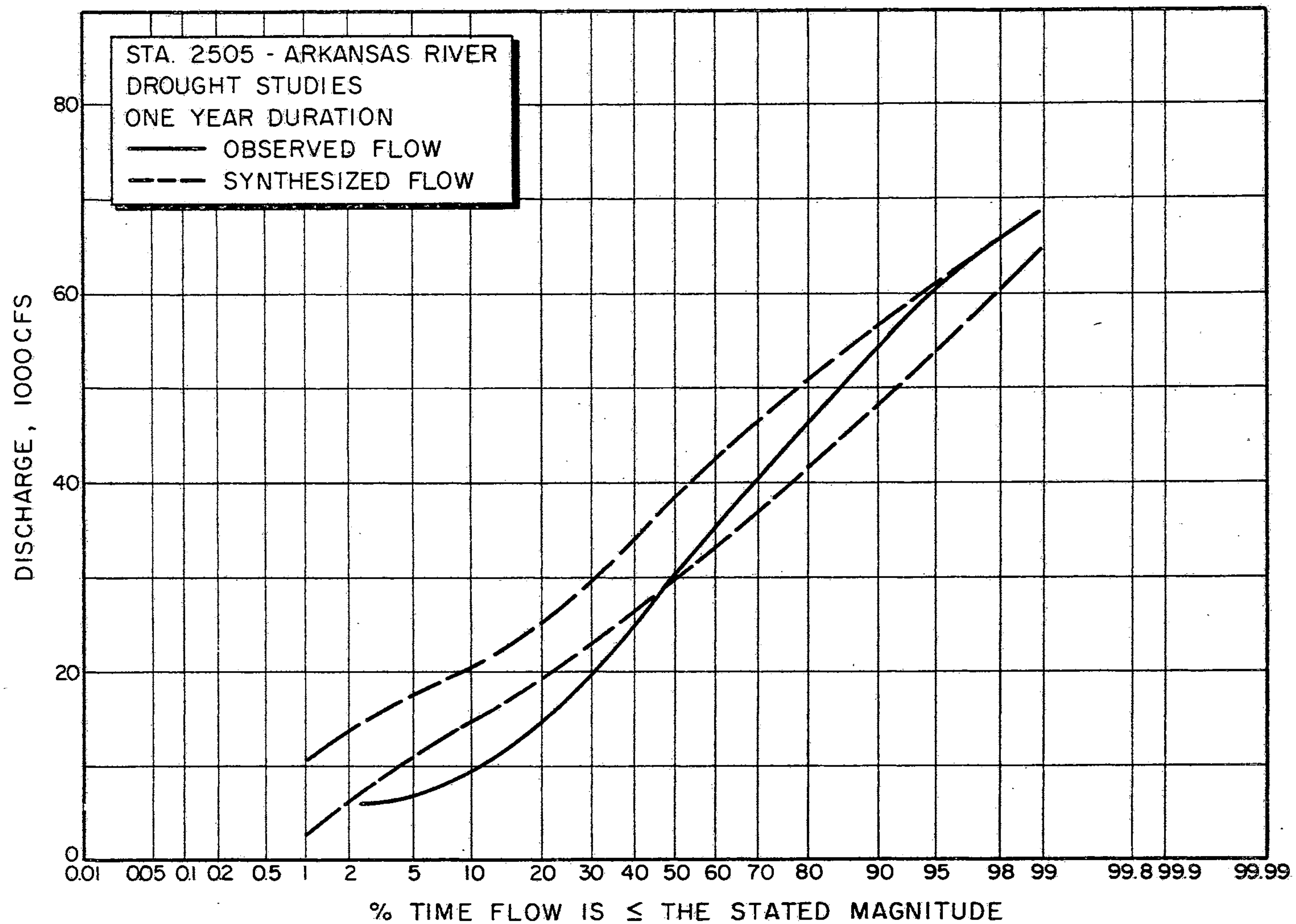


Fig. 34 Duration Curves for one year at Station 2505

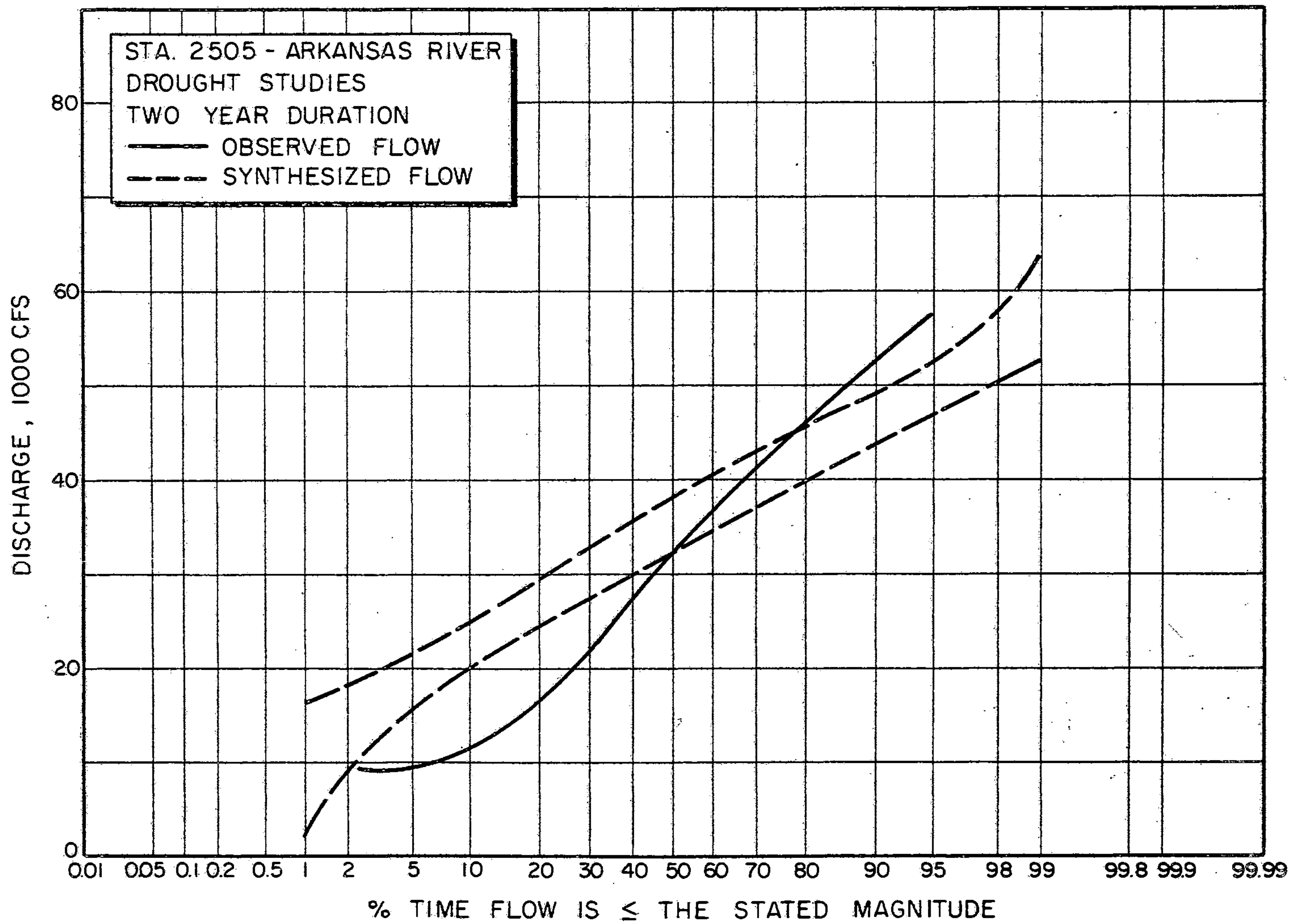


Fig. 35 Duration Curves for two years at Station 2505

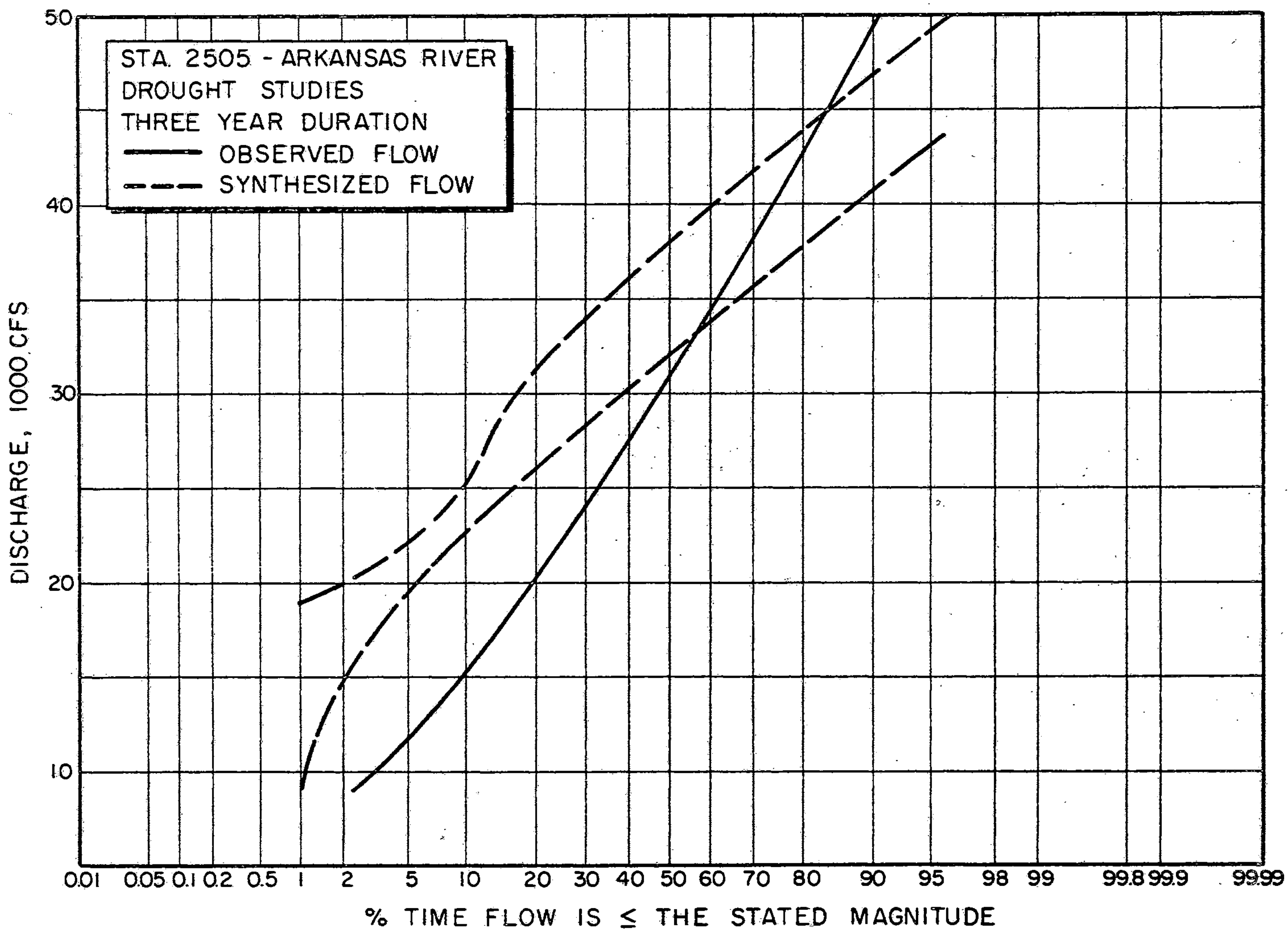


Fig. 36 Duration Curves for three years at Station 2505

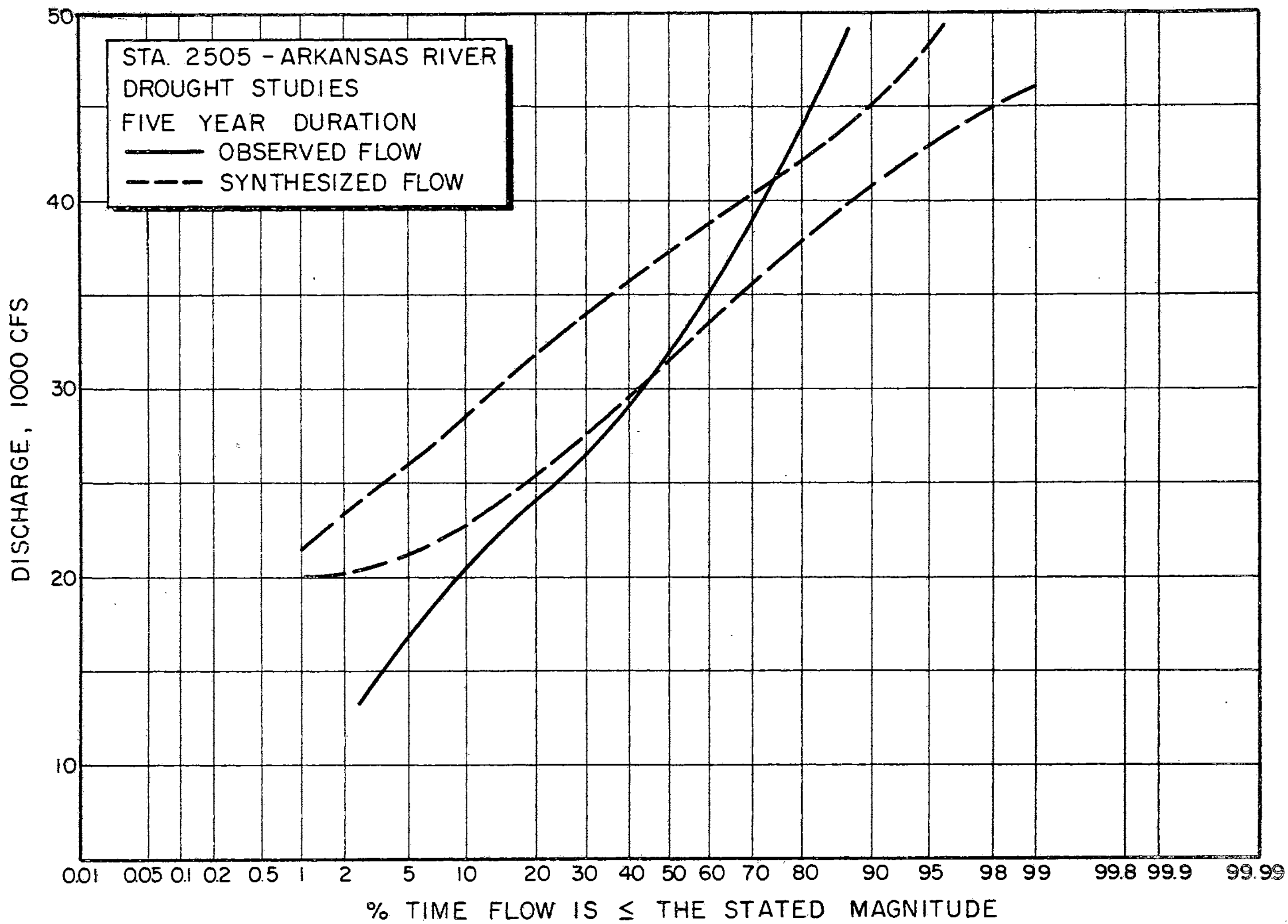


Fig. 37 Duration Curves for five years at Station 2505

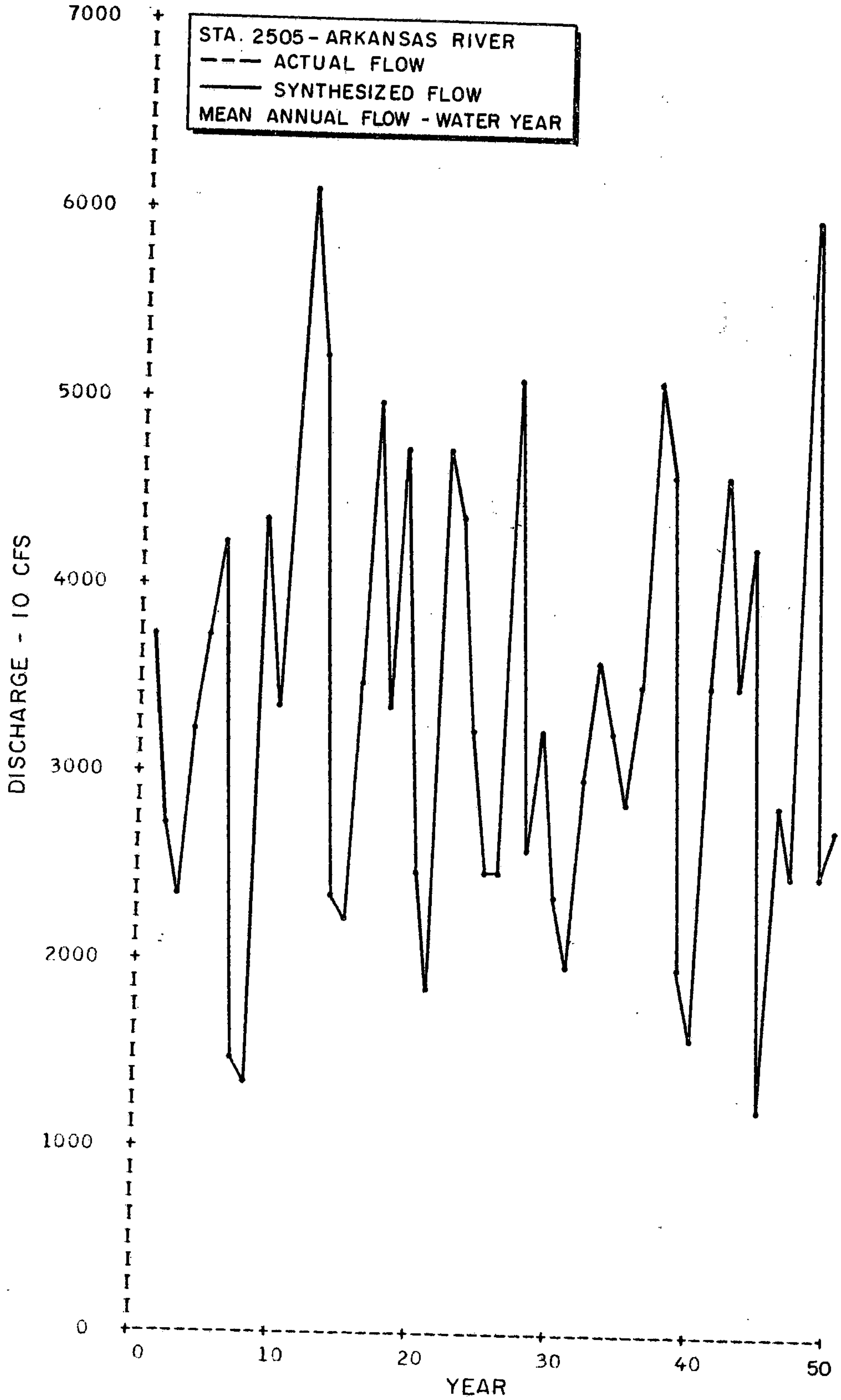
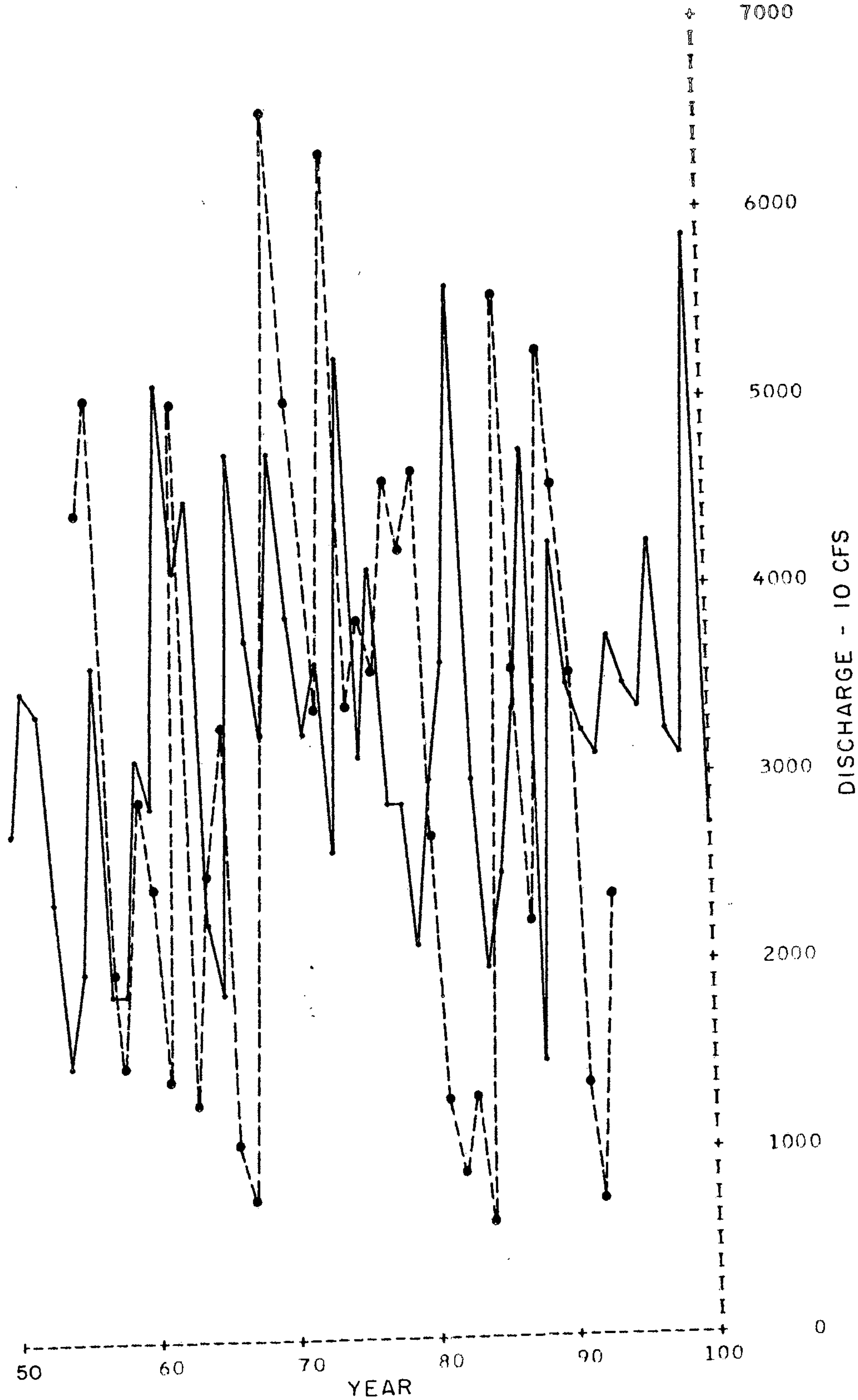


Fig. 38 - Hydrograph Comparison for Observed and Synthesized Flow at Station 2505


7000
6000
5000
4000
3000
2000
1000
0
DISCHARGE - 10 CFS
50
60
70
80
90
100
YEAR

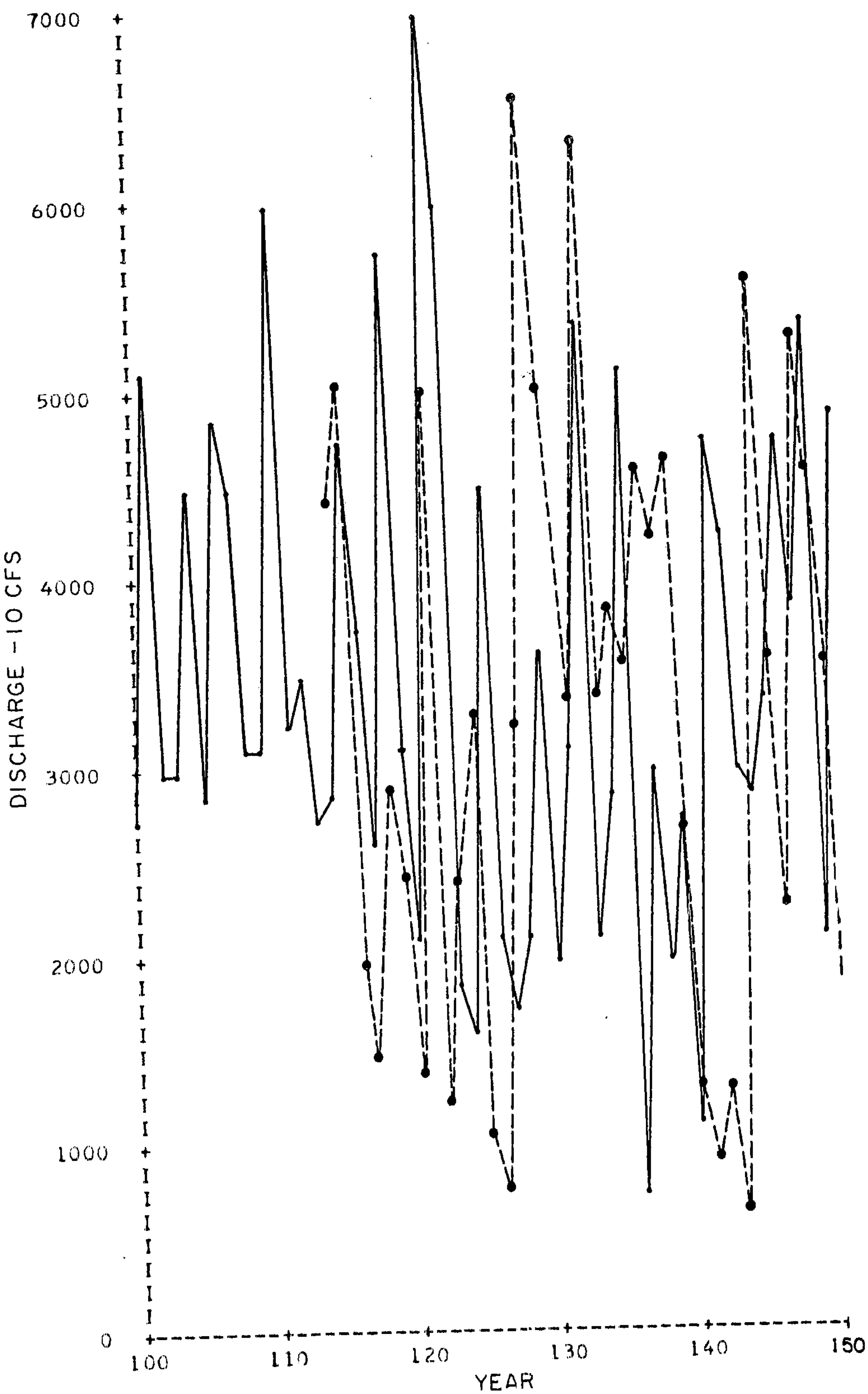
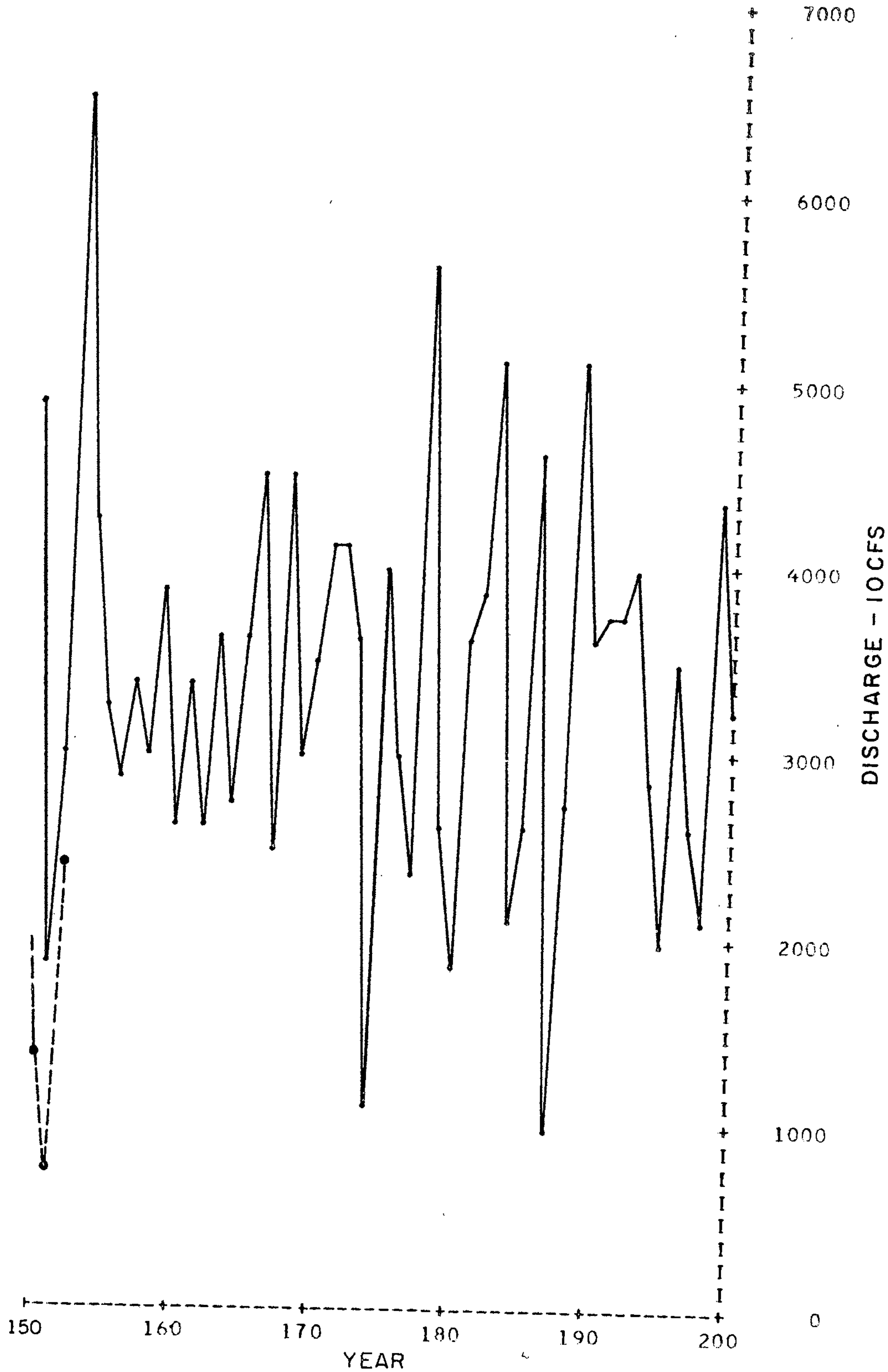


Fig. 38, Continued


7000
6000
5000
4000
3000
2000
1000
0
DISCHARGE - 10CFS
150
160
170
180
190
200
YEAR

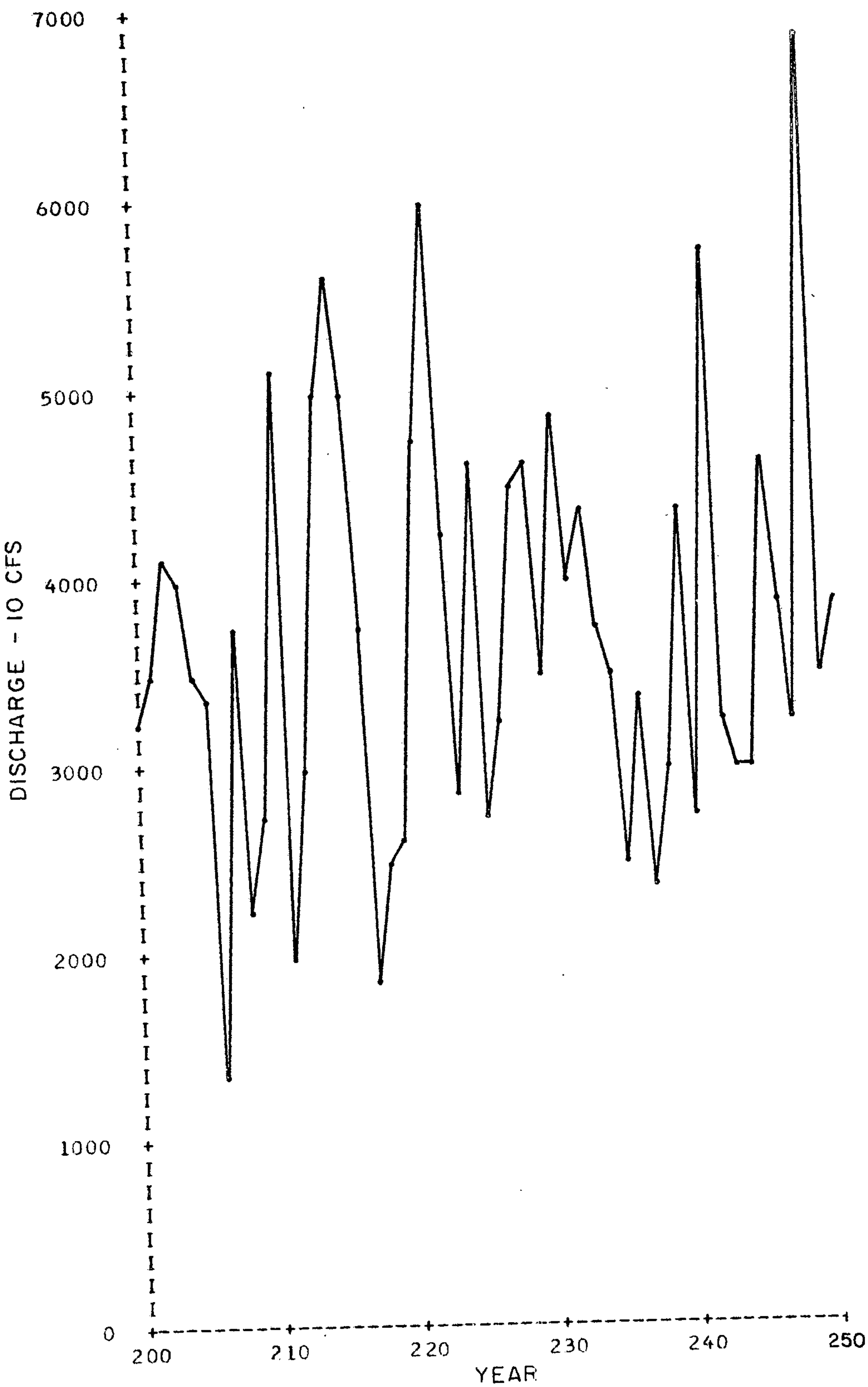
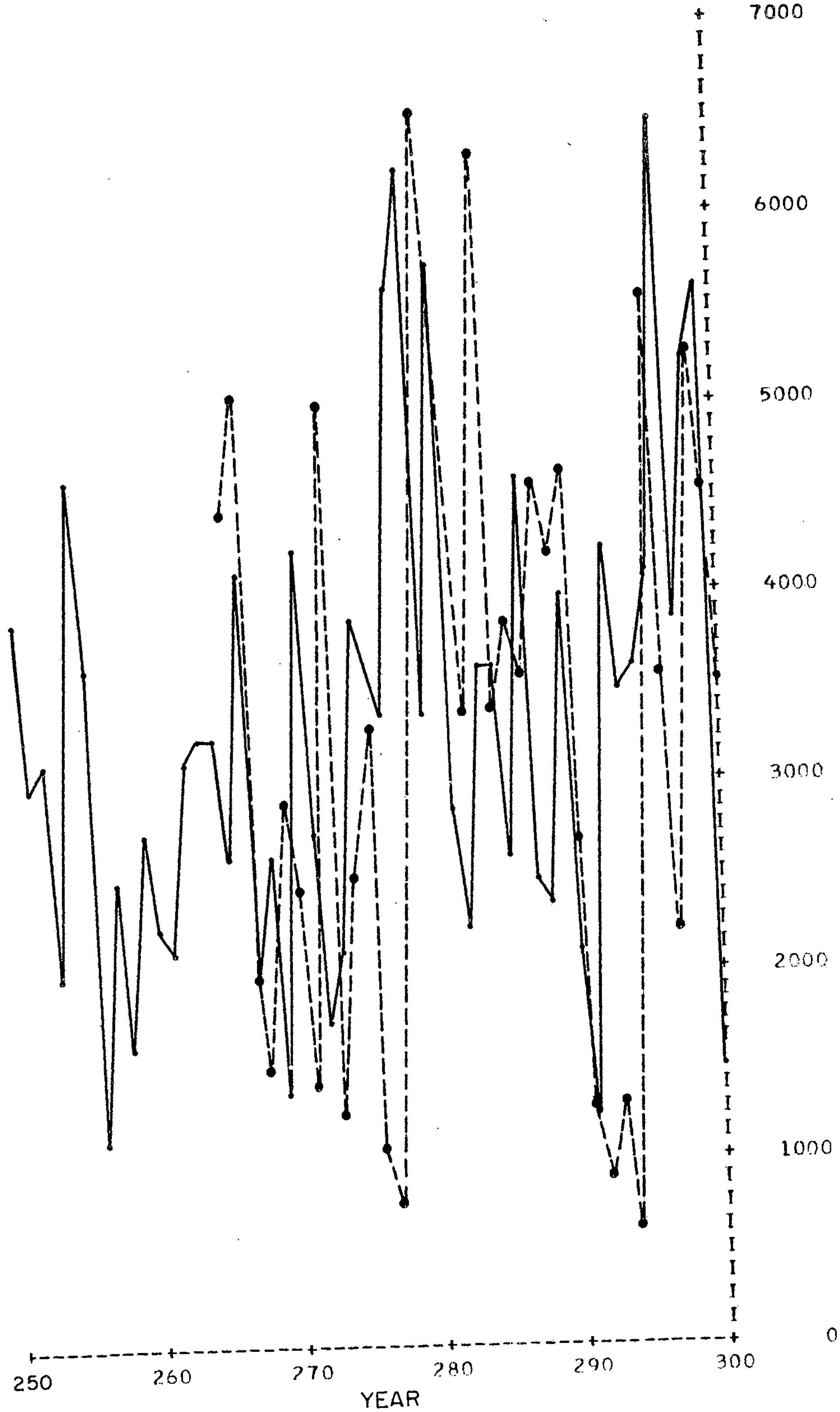


Fig. 38, Continued


7000
6000
5000
4000
3000
2000
1000
0
DISCHARGE - 10 CFS
250
260
270
280
290
300
YEAR

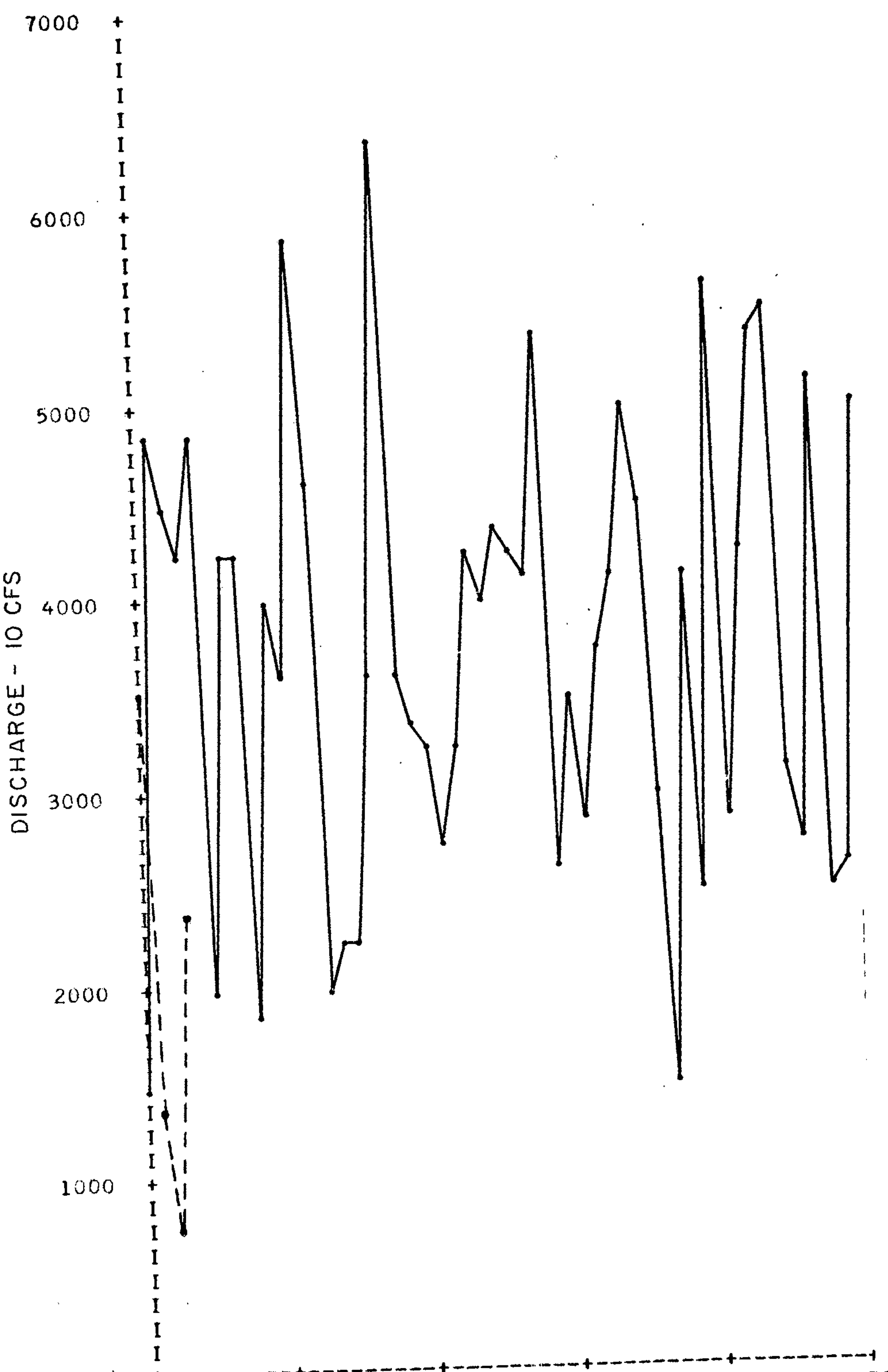
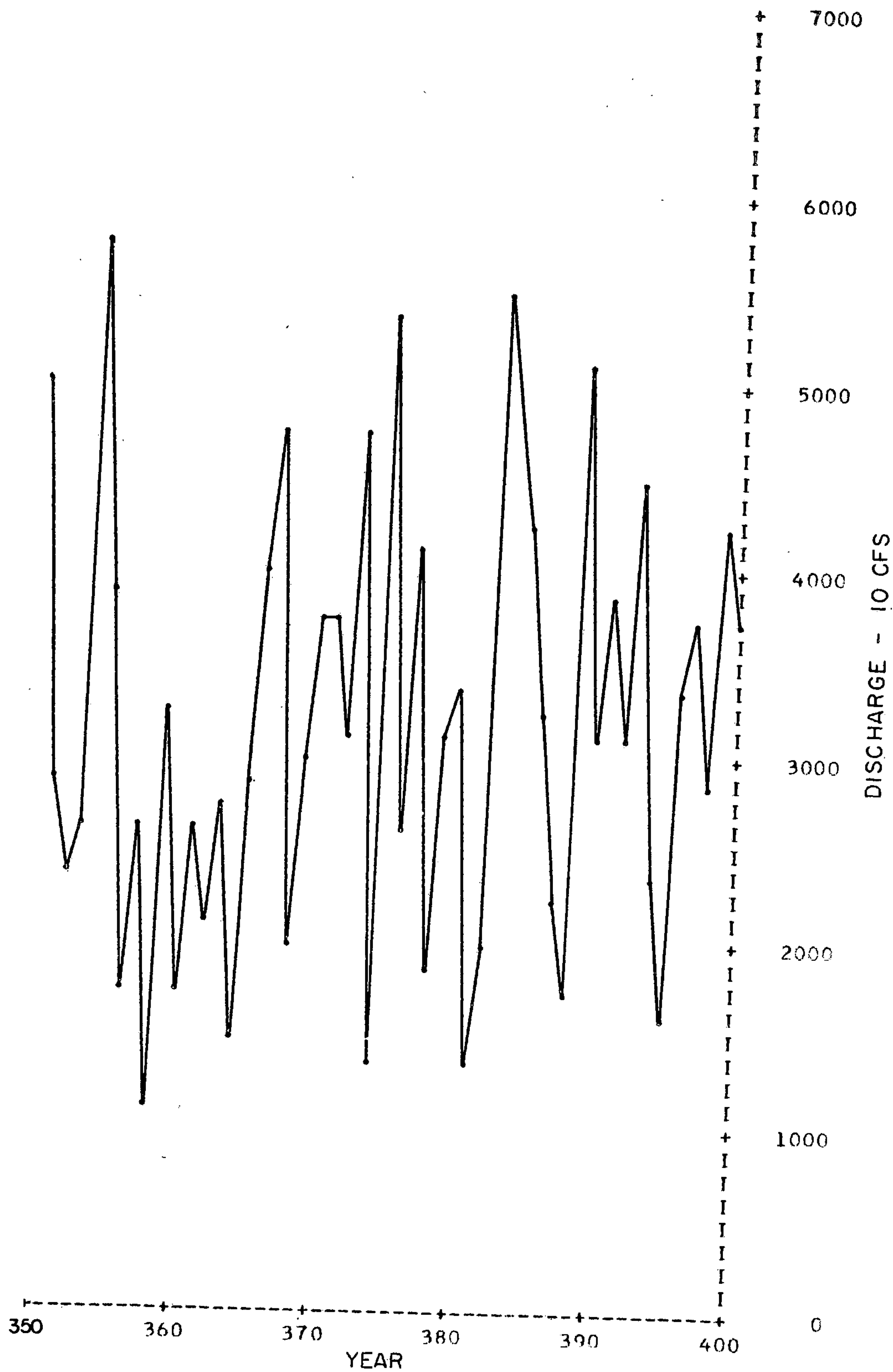


Fig. 38, Continued


7000
6000
5000
4000
3000
2000
1000
0
DISCHARGE - 10 CFS
350
360
370
380
390
400
YEAR

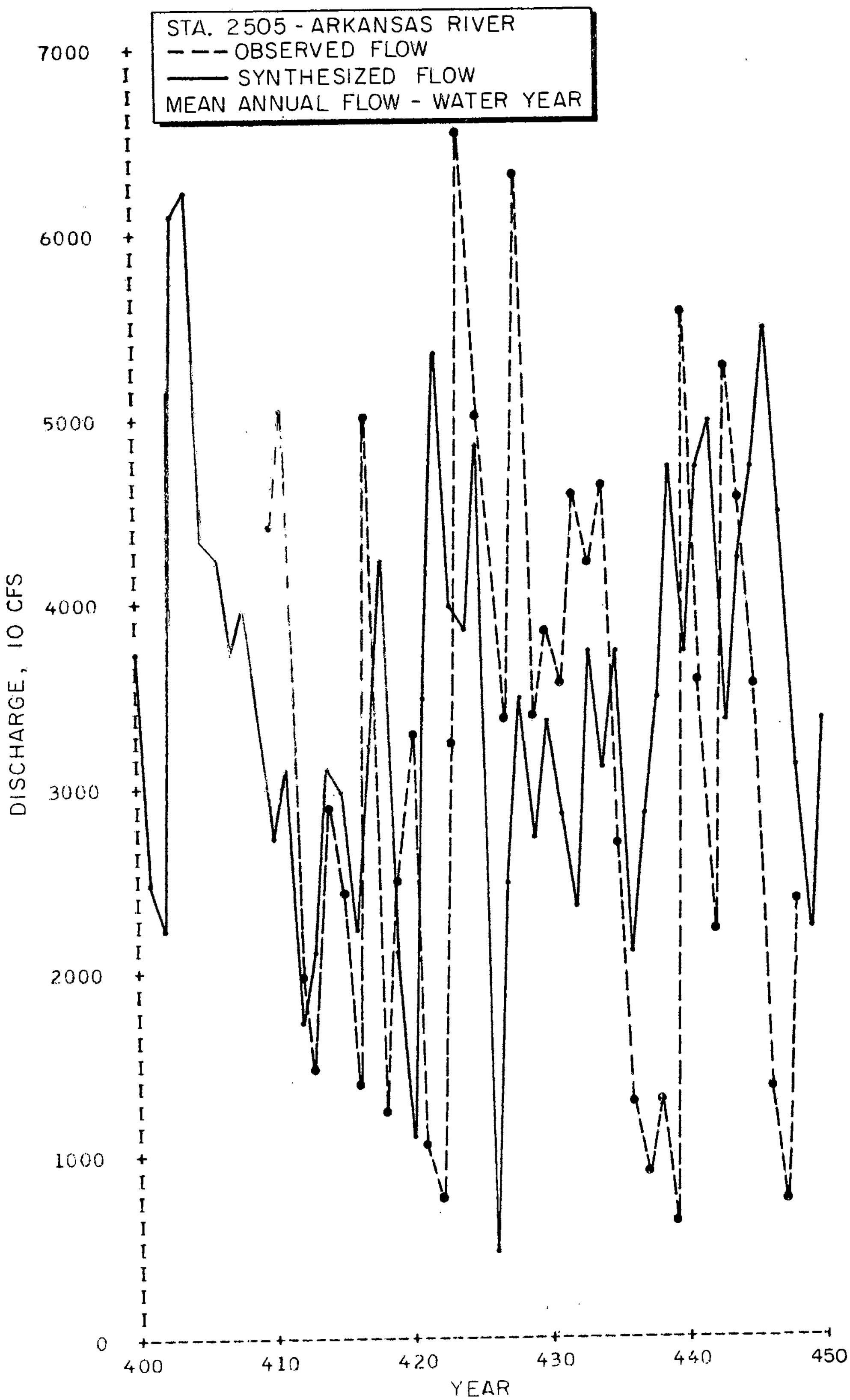
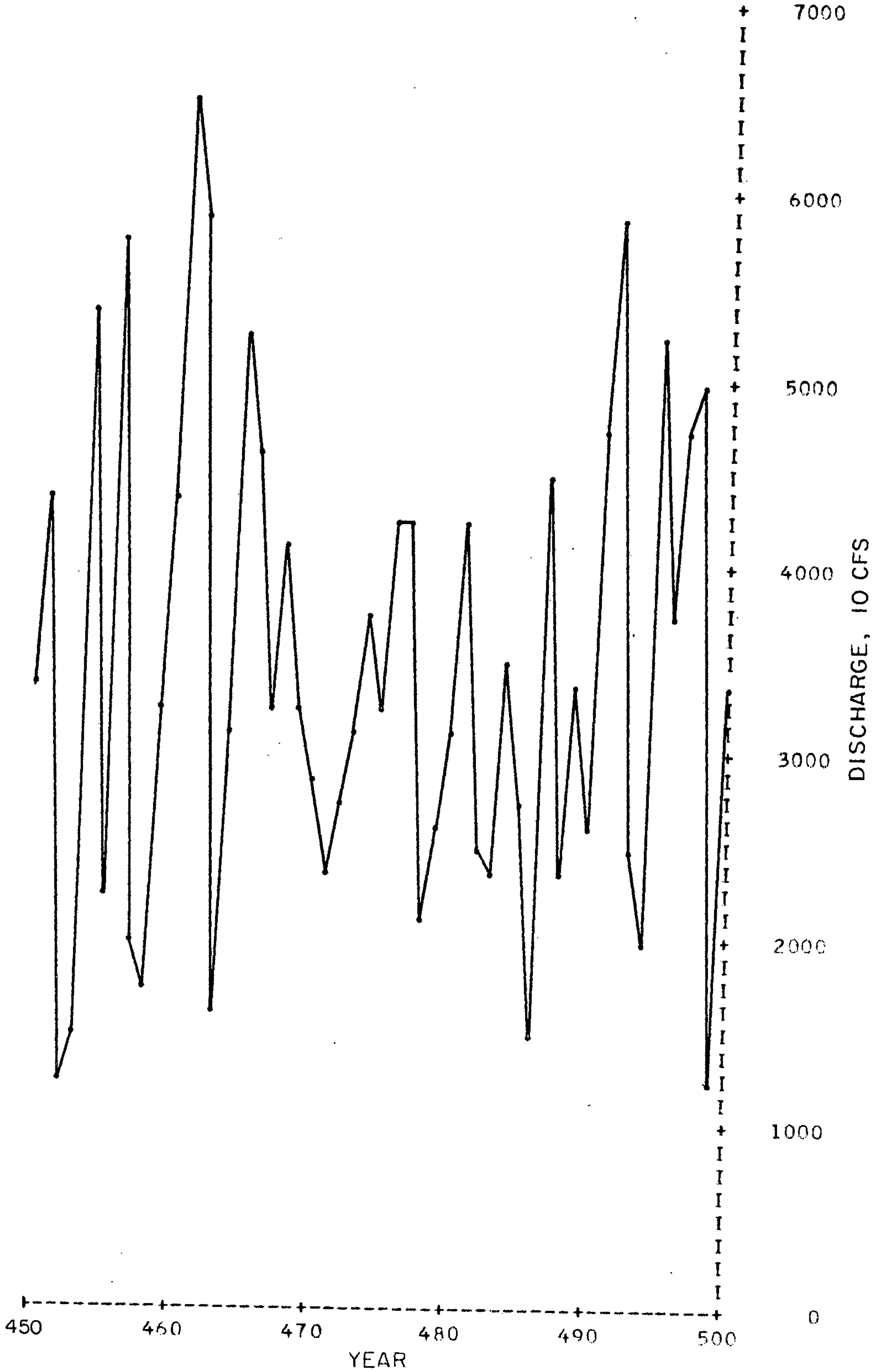


Fig. 38, Continued


7000
6000
5000
4000
3000
2000
1000
0
DISCHARGE, IO CFS
450
460
470
480
490
500
YEAR

CHAPTER VI

DISCUSSION OF RESULTS

A. Poteau River Basin at Station 2485

1. Evaluation of Synthesized Unregulated Flows

The method of deriving the synthesized monthly flows was reasonably reliable. The highest flow from the historical record was 10860 cfs. during the month of March; and the highest flow from the synthesized data was 10828 cfs. during the month of May. Also, a synthesized flow of 10784 cfs. occurred during the month of April. This indicates that the magnitude of the synthesized data falls within the range of flows as indicated by the historical record.

The mean flows for the historical (actual record) and the unregulated record were reported in Table I in Chapter V. The unregulated mean monthly flows for January, March, April, and May are higher than the mean monthly flows for the corresponding months of the historical record. The mean monthly flows for February of both records were equal for all practical purposes. These characteristics are indicative of streamflow regulation because a reservoir impounds water during periods of high flows and gradually releases the water during periods of low or moderate flows.

The results indicate very strongly that the synthesized data does reliably represent the flow characteristics for unregulated flow. Therefore, the synthesized data can be used in the analysis of low flows.

2. Analysis of Low Flows

The flow-duration curves measure the variability of runoff for the design of storage reservoir. Figures 2-13 of Chapter V gave the variability of runoff for each month over the twenty-seven years of record. These curves gave an indication of the magnitude and probability of occurrence for expected monthly flows.

The duration curves for seven days, Figures 2-13, Chapter V, gave an indication of the magnitude and probability of occurrence for seven day flows occurring during each month of the year. These "seven day" flows can be helpful in determining the amount of flow needed to abate stream pollution.

Duration curves were determined for six-month periods (wet and dry), where the wet period had high flows and the dry period had relatively low flows. The six-month period, December-May, was selected as the period of high flows. Table I, Chapter V, shows these months had high mean monthly flows. It also shows that the six-month interval, June-November, had low mean monthly flows. The six-month duration curves show that the probability of occurrence of low flows was greater in the six-month duration, June-November.

Duration curves of one, two, three, and five years gave an indication of flows occurring over long periods of time. For example, the average flow for one year was 770 cfs. (curve value) with a probability of occurrence of 30.0%. An average flow for two years was 850 cfs. for a 30.0% probability of occurrence. Also, average flows of 950 cfs. and 975 cfs., both at 30.0% probability of occurrences, were determined for three and five year durations respectively. These duration curves can be helpful in appropriating water for needs downstream from a reservoir.

3. Comparison of Unregulated and Regulated Flows

Generally speaking the duration curves for regulated and unregulated flows showed similar characteristics. The thirty-day duration curves for December, July, and August were congruent for both regulated and unregulated flows and the thirty day duration curves for October, November, January, February, March, June and September were very similar with maximum difference of 28 cfs. at a return period of 28 years. Low flows from thirty-day duration curves for April and May varied significantly. The regulated flow duration curve for April showed lower flows in contrast to the unregulated flow duration curve for May which showed lower flows. This characteristic could have been caused by the reservoir impounding flow in April and gradually releasing flow in May.

The six-month duration curves also exhibited marked differences at high flows. The regulated flows for the December-May duration curve showed lower values than the unregulated flows, while the unregulated flows for the June-November duration curve showed lower values than the regulated flows. The phenomenon was attributed to the reservoir which impounded water during the December-May period to release it during the June-November period.

The one year duration curve for unregulated flow had slightly lower flows, but the two, three, and five year duration curves were similar for regulated and unregulated flows.

In conclusion, the duration curves of unregulated flows seem to be reliable and will serve as a useful tool in formulating decisions concerning the appropriation of available waters.

B. Arkansas River Basin at Station 2505

1. Evaluation of Statistics for Mathematical Synthesis

It was assumed that a bivariant normal population existed between monthly flows for Station 2505 and by testing the distribution it was found to be slightly skewed at high flows; while transformed flows such as log and square root, showed an approximately normal distribution. However, the advantage of using transformed flows were so slight that no transformation was used in the analysis.

Table IV of Chapter V gives the pertinent statistics of serial correlation for each month for Station 2505. The correlation coefficients in this table are statistically significant for a sample size of N=38 years, the length of run-off record at Station 2505. This was indicated by the fact that the values of the correlation coefficient exceed the conventionally accepted minimum value of 0.2573 as established by R. A. Fisher's (6) table for testing the statistical significance of sample product-moment correlation coefficients at the 90 percent level of probability.

2. Effects of "Zero" Flows on the Synthesized Data

The synthesized data exhibited a distinct characteristic of negative flows. These negative flows were later changed to zero; and the total amount of negative flows added to the system by changing the negative values to zero was only 9.23 percent as shown in Table VI, Chapter V, of the total discharge during the 500 years. This was within the 90 percent confidence limit for the population mean. Therefore the elimination of negative values did not distort the population significantly.

3. Comparison of the Synthesized and the Actual Flows

Station 2505 was highly regulated from reservoirs such as Pensacola, Oologah, Keystone, Fort Gibson, Tenkiller, and others which controlled a drainage area of approximately 106,000 square miles of the total drainage area of 150,483 square miles. Minimum flows are maintained at a predetermined level which could be considered for the purposes of this study as the base flow. The author suggests that the base flow for specific months should be substituted for the zero or negative flows.

The synthesized flow and the actual flow exhibited distinct characteristics, the most important of which was the presence of synthesized monthly flows of zero magnitude; whereas, mean monthly flows of zero magnitude were not present in the actual record. Another noteworthy phenomenon was that excluding the zero flows, flows lower than the historical record for each month of the year were present in the 500 years of synthesized data as shown in

Table VII, Chapter V. The aforementioned table also shows that these low monthly synthesized flows were even lower than the corresponding recorded minimum daily flow for that month. For example, for the month of October the lowest monthly flow occurring in the synthesized data was 50 cfs., whereas, the actual flow had 492 cfs. and 306 cfs. for minimum monthly and minimum daily flows respectively.

A third characteristic was that the average of the monthly flow for each month of the five hundred years of synthesized data was greater than the corresponding average mean monthly flows over the observed record, Table VIII of Chapter V.

The largest synthesized monthly flow 236,170 cfs. occurred in May of the year 221. The largest mean monthly flow in the observed record was 302,100 cfs. occurred in May, 1943. These high flows would indicate that an unusually high synthesized flow could not have adversely affected one of the twelve average monthly flows of the synthesized data.

The hydrograph of mean annual flows for the thirty eight year observed record was superimposed on portions of a five hundred year hydrograph of yearly synthesized flows, Figure 38, in Chapter V. This figure shows the statistical characteristics of the synthesized data and those of the observed record. The observed record was compared with a period of synthesized flows which exhibited similar statistical characteristics. The statistical characteristics of the observed flows and synthesized flows showed good similarity. This similarity was exhibited in the peaks of both types of flows. However, the observed record showed lower flows over consecutive periods longer than two years.

4. Effects of "Zero" Flows on the Analysis of Low Flows

The "zero" flows had the greatest effect on six-month duration curves. The average flow over a six month period was not indicative of monthly flows averaged over a six months period as in the existing record, but rather a combination of three to five "zero" flows plus one to three positive flows. For example, the average flow for a duration curve from August to January of the year 55 was 873 cfs. with five "zero" flows and one flow 5240 cfs.

Zero flows affect the average flow for one and two year durations in the same manner. For example, the lowest actual yearly flow was 5965 cfs. as compared to 4465 cfs. year 426 of the synthesized data. The effects of "zero" flows on the average flow for three and five year durations seemed to be minimized. For three year duration curves the lowest flow occurring in the synthesized data was 9062 cfs.; compared to 9002 cfs. occurring in the existing record. The lowest average flows for a five year duration was 19940 cfs. and 13394 cfs. for the synthesized and observed records respectively.

5. Evaluation of the Duration Curves

The thirty day duration curves, Figures 20-31 in Chapter V gave an indication as to the magnitude of monthly low flows for the year. For example, the flow magnitude of a five year return period was 3600 cfs. for October, while it was 17,000 cfs. for

May. These duration curves were derived from existing records. Thirty day duration curves from the synthesized data were not derived because the high number of "zero" flows would adversely affect the suitability of the curves.

The actual flow-duration curves, Figures 32-37 in Chapter V, did not completely fall within the boundaries of the synthesized flows. This characteristic occurred during periods of low flows, that is, flows having small probabilities of occurrence. Initially the actual flow-duration curves showed smaller magnitudes than the synthesized flows for given probabilities; and as the flows increased the actual flow-duration curves finally fell within or crossed the synthesized flow boundaries. This was characteristic for the duration curves of six months, one, two, three, and five years. Possibly, the short thirty eight year observed record was not hydrologically stable and these low flows have lower probabilities than indicated. The observed flow-duration curves showed lower flows for probabilities of occurrence between 3-45 percent. However, there was no assurance that the recurrence intervals or probabilities computed for the observed flows by the equation in Chapter IV, part D, are even approximately correct. Because there was a 40% probability that a 100 year drought (or greater) would occur in any 50 year period and 22% that it would occur in a 25 year period. On the other hand there was a 36% probability that a 50 year drought would not have occurred in any 50 year period (10). Thus, it was highly probable that the low flows occurring in the thirty eight year historical record had even larger return periods (smaller probabilities of occurrence) than those derived by the previous equation.

On the whole the synthesized and actual flow duration curves did not compare favorably. The observed flow and synthesized flow-duration curves showed less similarity for long (two, three, and five year) durations. The probable cause of this was that the effect of "zero" synthesized flows were minimized by the other large monthly flows. Also, the synthesized low flows for particular durations were dependent on the "zero" flows and were not indicative of low monthly flows averaged over the selected duration.

CHAPTER VII

CONCLUSIONS

Based upon the results reported in this study, the following conclusions may be drawn:

A. Station 2485, Poteau River at Wister, Oklahoma

The synthesized monthly flows for Station 2485 represent the characteristics of unregulated flow; and the unregulated flow-duration curves can be relied upon in making decisions concerning the safe yield of a reservoir and the appropriations of water for downstream use.

B. Station 2505, Arkansas River at Van Buren, Arkansas

1. The correlation coefficients exceed the accepted minimum for testing the statistical significance of product-moment correlation coefficients at the 90 percent level of probability.

2. Although the changing of negative values to zero did not distort the population of the synthesized data significantly, the "zero" flows did affect the duration curve analysis especially the six-month duration curves. Low flows were dependent on the "zero" flows and were not a result of low monthly flows averaged over a given duration.

3. The hydrograph comparison of the synthesized and the

observed flows showed that these flows have similar statistical characteristics. A closer examination showed that the observed flows had lower values over long periods of time of two, three, and five years as was also apparent from the corresponding duration curves.

4. The observed flow-duration curves showed lower flows for probabilities of occurrence between 3-45 percent. However, there was no assurance that the recurrence intervals or probabilities of occurrence are even approximately correct, because of the instability of the short thirty eight year record. It was highly probable that these low flows had even larger return periods than those calculated using only thirty eight years of record.

The synthesized data was a statistical inference of what the future runoff would be. There can be no absolute certainity until 100 years of actual record are available for purposes of comparison.

5. The thirty day observed flow-duration curves showed the range of flows that occurred throughout the year, the six-month synthesized and observed flows gave an indication of "seasonal" flows and the one, two, three, and five year duration curves show the probabilities and magnitudes of low flows occurring over long periods of time.

6. Although the "zero" flows affected the duration curve analysis, especially the six-month duration curves, the author feels that the duration curves derived from the synthesized data served as a truer basis for selection of low flows than the historical record because the synthesized flows probabilities

were ascertained over a long period of record. However, these synthetic flows have the same statistical characteristics as the short historical record.

7. In conclusion, the synthesized flow duration curves can be very helpful in determining the safe yield of reservoirs and water appropriations along the Arkansas River and should be used as the basis for design purposes.

CHAPTER VIII

SUGGESTIONS FOR FUTURE WORK

Based on the results of this investigation, the following suggestions are made for future research in the area of low flow analysis:

1. A study on the applicability of synthesizing future run-off for different river basins using the Markov Chain.

2. A study on developing a modification of the Markov-Chain so that negative and zero flow could be eliminated. One possibility could be the selection of a random normal deviate until a positive flow was generated.

3. A study utilizing transformed flows in the Markov Chain.

4. A study synthesizing flows by the Markov Chain for periods longer than one month. For example, two or three month or even six month flows. This would eliminate successive additions of monthly flows for the duration curves and could improve results.

5. A study on the use of mass curves from synthesized data to analyze periods of low flows.

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APPENDIX A

```
340003200701360003200702490240251196361130O102
ZZJOB
ZZFORX
C       SYNTHESIS OF COMBINED FLOWS FROM TWO STATIONS
C       FIRST DATA CARD N= NO. OF YEARS OF RECORD
C       SECOND DATA CARD R = MONTHLY RATIOS
C       FIRST DATA SET = FLOWS FROM FIRST STATION BY YEARS
C       SECOND DATA SET = FLOWS FROM SECOND STATION BY YEARS
C       PUNCHES OUT SYNTHESISED MEAN MONTHLIES, BY MONTHS FOR USE IN
C       MINIMUM FLOW RANKING PROGRAM
        DIMENSION E(17,12),F(17,12),X(17,12),R(12),S(17,12)
     99 READ 100, N
        READ 102,(R(J),J=1,12)
        READ 101, ((E(I,J),J=1,12),I=1,N)
        READ 101, ((F(I,J),J=1,12),I=1,N)
        DO 10 J=1,12
        DO 10 I=1,N
        S(I,J) = E(I,J) + F(I,J)
     10 X(I,J) = S(I,J) * R(J)
        DO 50 J = 1,12
     50 PUNCH 101,  (X(I,J),I=1,N)
        PRINT 103
        GO TO 99
    100 FORMAT (I3)
    101 FORMAT (6F10.0)
    102 FORMAT (12F6.3)
    103 FORMAT (12HSET COMPLETE)
        END
ZZZZ
```

APPENDIX B

FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

```
$IBFTC MAIN
      COMMON QI(12),SUMF(12),SUMN(12),SUMC(12),NUMN(12),PCN(12),TSUMN,
     *LNCTR,NTEN,TSUMF
      X=0.
      LNCTR = 0.
      NTEN = 1
      DIMENSION        QJ(12),BJ(12),CON(12)
      COMMON /DATCOM/DATE (12)
   10 FORMAT (12F6.0)
   20 FORMAT (12F6.5)
      READ (5,10) QJ, CON,QONE,QJONE
      READ (5,20) BJ,BJONE
      DO 30 I = 1,12
      SUMF(I)=0.
      SUMN(I)=0.
      NUMN(I)=0
   30 PCN(I)=0.
      QLAST=QONE
      QJLAST=QJONE
      BJLAST=BJONE
      NNUMN = 0
      TPCN = 0.
      TSUMN= 0.
      TSUMC = 0.
      TSUMF = 0.
      DO 200 K=1,510
      DO 100 I=1,12
      CALL NORNUM (X)
      QI(I)=QJ(I)+BJ(I)*(QLAST-QJLAST)+X*CON(I)
      QLAST=QI(I)
      QJLAST=QJ(I)
  100 BJLAST=BJ(I)
      CALL REPORT
  200 CONTINUE
      WRITE(6,210)
  210 FORMAT (36H1    NEGATIVE VALUES FOR STATED MONTH//40H MONTH    NUMBE
     *R    MAGNITUDE    PERCENTAGE)
      DO 300 I = 1,12
      PCN(I)=(ABS(SUMN(I))/TSUMF   )*100.
      WRITE (6,320) DATE (I),NUMN(I),SUMN(I),PCN(I)
      NNUMN = NNUMN + NUMN(I)
  300 TPCN  = TPCN  + PCN(I)
  320 FORMAT (2H0 ,A6,3X,I4,3X,F8.0,8X,F5.2)
      WRITE (6,330) NNUMN,TSUMN,TPCN
  330 FORMAT (7HOTOTALS,4X,I4,2X,F9.0,7X,F6.2/1H )
      WRITE (6,340)
  340 FORMAT     (34H-TOTAL FLOW AND MEAN FLOW BY MONTH//18X,9HCORRECTED/
     *35H MONTH      TOTAL      TOTAL      MEAN)
      DO 400 I=1,12
      SUMC(I) = SUMF(I) - SUMN(I)
      AMEAN = SUMC(I)/500.
      TSUMC = TSUMC + SUMC(I)
      WRITE (6,350) DATE(I),SUMF(I),SUMC(I),AMEAN
  350 FORMAT (2H0 ,A6,F8.0,2X,F8.0,4X,F5.0)
  400 CONTINUE
      WRITE (6,410) TSUMF,TSUMC
  410 FORMAT (7HOTOTALS,F10.0,1X,F9.0)
      CALL EXIT
      END
```

APPENDIX C

```
JUST FOR CIV EN                                   FORTRAN SOURCE LIST
    ISN        SOURCE STATEMENT

$IBFTC
      DIMENSION C(500,12)
      DIMENSION A(500,12),AA(500,4),B(500,3)
      READ(5,1)((A(I,J),J=1,12),I=1,500)
    1 FORMAT(5X,12F6.0)
      DO 2 I=1,500
      DO 3 J=1,12
      C(I,J)=A(I,J)
      AA(I,1)=AA(I,1)+A(I,J)
    3 CONTINUE
      DO 22 J=3,8
   22 AA(I,3)=AA(I,3)+A(I,J)
      AA(I,2)=AA(I,1)-AA(I,3)
      AA(I,1)=AA(I,1)/12.
      AA(I,4)=AA(I,1)
      AA(I,2)=AA(I,2)/6.
    2 AA(I,3)=AA(I,3)/6.
      DO 9 I=1,5
      WRITE(6,6)I
      WRITE(6,5)
    5 FORMAT(25X,31HDISCHARGES OF ONE YEAR DURATION)
   11 FORMAT(1H0,5X,4HRANK,6X,9HMAGNITUDE,13X,2HTP,17X,1HP,10X,4HYEAR)
   52 FORMAT(25X,31HDISCHARGES OF TWO YEAR DURATION)
   53 FORMAT(25X,33HDISCHARGES OF THREE YEAR DURATION)
   54 FORMAT(25X,32HDISCHARGES OF FIVE YEAR DURATION)
    6 FORMAT(1H1,29X,7HFOR THE,I2,12HTH 100 YEARS)
      WRITE(6,11)
      K=I*100
      KK=K-99
      DO 9 L=1,100
      IF (L.EQ.51) WRITE(6,971)
      X=999999
      DO 8 J=KK,K
      IF (AA(J,1).GT.X) GO TO 8
      X=AA(J,1)
      KKK=J
    8 CONTINUE
      AA(KKK,1)=9999999
      TP=101./FLOAT(L)
      P=1./TP
  971 FORMAT(1H1,5X,4HRANK,6X,9HMAGNITUDE,13X,2HTP,17X,1HP,10X,4HYEAR)
   10 FORMAT(5X,I3,3X,F15.8,6X,F15.8,5X,F15.8,5X,I3)
    9 WRITE(6,10)L,X,TP,P,KKK
      DO 40 J=1,5
      K=J*100-100
      DO 13 I=1,99
      KK=I+K
   13 B(KK,1)=(AA(KK,4)+AA(KK+1,4))/2.
      DO 15 I=1,98
      KK=I+K
   15 B(KK,2)=(AA(KK,4)+AA(KK+1,4)+AA(KK+2,4))/3.
      DO 40 I=1,96
      KK=I+K
   40 B(KK,3)=(AA(KK,4)+AA(KK+1,4)+AA(KK+2,4)+AA(KK+3,4)+AA(KK+4,4))/5.
      DO 60 I=1,5
```

```
JUST FOR CIV EN                                       FORTRAN SOURCE LIST
    ISN        SOURCE STATEMENT

    115           WRITE(6,6) I
    116           WRITE(6,52)
    117           WRITE(6,11)
    120           K=I*100-1
    121           KK=K-98
    122           DO 60 L=1,99
    123           IF (L.EQ.51) WRITE(6,971)
    126           X=9999999
    127           DO 61 J=KK,K
    130           IF (B(J,1).GT.X) GO TO 61
    133           X=B(J,1)
    134           KKK=J
    135        61 CONTINUE
    137           B(KKK,1)=99999999
    140           TP=100./FLOAT(L)
    141           P=1./TP
    142        60 WRITE(6,10) L,X,TP,P,KKK
    145           DO 70 I=1,5
    146           WRITE(6,6) I
    147           WRITE(6,53)
    150           WRITE(6,11)
    151           K=I*100-2
    152           KK=K-97
    153           DO 70 L=1,98
    154           IF (L.EQ.51) WRITE(6,971)
    157           X=9999999
    160           DO 71 J=KK,K
    161           IF (B(J,2).GT.X) GO TO 71
    164           X=B(J,2)
    165           KKK=J
    166        71 CONTINUE
    170           B(KKK,2)=99999999
    171           TP= 99./FLOAT(L)
    172           P=1./TP
    173        70 WRITE(6,10) L,X,TP,P,KKK
    176           DO 80 I=1,5
    177           WRITE(6,6) I
    200           WRITE(6,54)
    201           WRITE(6,11)
    202           K=I*100-4
    203           KK=K-95
    204           DO 80 L=1,96
    205           IF (L.EQ.51) WRITE(6,971)
    210           X=9999999
    211           DO 81 J=KK,K
    212           IF (B(J,3).GT.X) GO TO 81
    215           X=B(J,3)
    216           KKK=J
    217        81 CONTINUE
    221           B(KKK,3)=99999999
    222           TP= 97./FLOAT(L)
    223           P=1./TP
    224        80 WRITE(6,10) L,X,TP,P,KKK
    227           N=5
    230           DO 29 I=1,5
```

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```
ISN        SOURCE STATEMENT

           WRITE(6,6)I
           NJ=N+5
           NK=N+6
           NI=N+11
           WRITE(6,21)
        21 FORMAT(25X,33HDISCHARGES OF SIX MONTHS DURATION,14H FEBRUARY-JULY)
        31 FORMAT(25X,33HDISCHARGES OF SIX MONTHS DURATION,15H AUGUST-JANUARY)
          1)
           WRITE(6,11)
           DO 500 II=1,500
           AA(II,3)=0.0
           AA(II,2)=0.0
           DO 501 JJ=N,NJ
       501 AA(II,2)=AA(II,2)+C(II,JJ)/6.
           DO 502 JJ=NK,12
       502 AA(II,3)=AA(II,3)+C(II,JJ) /6.
           IF (N.EQ.1) GO TO 500
           NI=NI-12
           III=II+1
           IF (II.EQ.500) GO TO 500
           DO 504 JJ=1,4
       504 AA(II,3)=AA(II,3)+C(III,JJ) /6.
       500 CONTINUE
           IF (N.NE.1) AA(500,3)=0.0
           K=I*100
           KK=K-99
           DO 29 L=1,100
           IF (L.EQ.51) WRITE(6,971)
           X=999999
           DO 28 J=KK,K
           IF (AA(J,2).GT.X) GO TO 28
           X=AA(J,2)
           KKK=J
        28 CONTINUE
           AA(KKK,2)=9999999
           TP=101./FLOAT(L)
           P=1./TP
        29 WRITE(6,10) L,X,TP,P,KKK
           DO 39 I=1,5
           WRITE(6,6)I
           IF (NK.GT.12) NK=NK-12
           WRITE(6,31)
             WRITE(6,11)
             K=I*100
           KK=K-99
           DO 39 L=1,100
           IF (L.EQ.51) WRITE(6,971)
           X=999999
           DO 38 J=KK,K
           IF (AA(J,3).GT.X) GO TO 38
           X=AA(J,3)
           KKK=J
        38 CONTINUE
           AA(KKK,3)=9999999
           TP=101./FLOAT(L)
           P=1./TP
        39 WRITE(6,10) L,X,TP,P,KKK
           STOP
           END
```

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