COMPARISON OF DIRECT AND INDIRECT METHODS FOR DETERMINING THE COEFFICIENT OF PERMEABILITY OF CLAYS

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CHAPTER I

INTRODUCTION

The problem of permeation of a fluid through a porous medium and consequently the concept of permeability has had a special place in scientific research since the beginning of the 19th Century, although it had been discussed even before that. This problem has always been an intricate and complex one rather than a precise and well realized subject. Since the discovery by Darcy in 1856 of the proportionality of the discharge of a fluid through a porous medium to the product of the cross-sectional area of the medium and the first power of the hydraulic gradient, many engineers and scientists have investigated the variables which influence the coefficient of permeability (proportionality coefficient in Darcy's equation), and the methods of determining it. In the past fifty years, because of the growing importance of this problem in engineering works, numerous attempts have been made to solve it experimentally as well as theoretically.

Because of numerous factors involved in the problem (as will be discussed in Chapter II), no universally valid relationship has been obtained. For example, considering only the influence of void ratio on the permeability of porous media, tens of papers with conflicting results have been written.

The main reason for the great discrepancies in the results is the fact that many of the earlier works on this problem were focused on a rather limited objective, and the result proves to be valid only for

that specific problem. An obvious example of this is the extensive work which has been done on the permeability of uniform sands and gravels, which has a limited applicability to some specific applications such as filters or oil-bearing formations. Most of the earlier experiments on permeability were conducted on uniform sands, lead shots, glass beads, or steel balls of equal diameter, using various fluids such as air, oil, and water. It can be seen that for such a specific condition, the porosity of the material varies in a very limited range of 0.259 to 0.476 or a void ratio of 0.42 to 0.67. It is obvious that such a narrow range of porosity or void ratio cannot cover the wide variation of this property which might be encountered in the numerous types of porous media found in nature or made artificially. Moreover, the results of such experiments cannot be extrapolated over a wider range because many other factors are involved and their effects may be different outside of the range investigated. The best example is the case of very fine-grained clay soils, where many other major factors such as mineralogy of the particles, particle size and shape, adsorbed ions, and several other factors affect the permeability significantly. Moreover those factors which are important in one case may not be important in another, and may sometimes be ignored completely. Therefore, those data which do not include all of the controlling factors are not comparable. Another important fact is that despite all efforts made by scientists, most of the theoretically derived expressions have not been correlated to the experimental results, and most of the equations in use are those which are either empirical or were derived from some fundamental assumptions and fitted or modified for experimental data. As an example, the equations given by Blasius,

Nikuradse, Weissbach, Chezy, Manning, Hazen, and Williams, all of which are quite commonly used, are of this nature.

Despite the uncertainties and complexities involved in permeability investigations, continuing study is justified because of the increasing importance of permeability in solving such problems as:

- drainage of highways and air ports
- dewatering of construction sites
- seepage pressure causing earth slides and failure of retaining walls
- seepage through earth dams
- settlement of structures
- uplift pressure beneath concrete dams and structures below groundwater level
- rate of increase in the strength of deposits subjected to consolidation
- prediction of the rate of increase of effective stress and the rate of dissipation of pore pressure in the clay foundation soils below structures

With regard to the permeability of clay soils, there has been a considerable amount of research recently, but there are still many problems that are not well defined for which solutions are needed. One of the needs is to obtain a reliable expression for the coefficient of permeability in terms of the physical properties of the soil and permeating fluid so that the permeability of clay soils may be estimated with an acceptable accuracy. So far, the best methods for predicting the permeability of clay soils have been direct measurement in the laboratory or field. Several expressions involving various parameters and constants have been proposed as indirect methods for determining

permeability, but none of them has a proven validity. Another indirect method, which arose naturally from the theory of one-dimensional consolidation developed by Terzaghi and Fröhlich, is often used as an alternative for direct measurement. But so far, little has been reported in the literature to indicate the accuracy and applicability of this method and the comparability of the results with those of direct methods. The only available article is one by Maytas (53), who determined the air and water permeability of Sasumua clay using a triaxial cell. In a single triaxial test, Maytas determined the permeability of the clay both by direct measurement and from computations based on the results of a consolidation test, and compared the results obtained for various incremental stages of the consolidation pressure. The results found by the two methods differed by more than five hundred percent. He offers no explanation for the difference, nor does he state the void ratios corresponding to the permeability values given.

In Terzaghi's theory of consolidation, one of the basic assumptions is the validity of Darcy's law to describe the movement of water in the soil upon application of the loads. Based on this and other simplifying assumptions, Terzaghi derived a differential equation for one-dimensional consolidation of a clay layer and, through this, correlated the coefficient of permeability, k, with a coefficient of consolidation, c_v, a coefficient of compressibility, a_v, and the initial void ratio of the soil, e_o, in the following form (see Appendix A for derivation of the equation):

$$k = \frac{c_v \cdot a_v \cdot \gamma_w}{1+e_o}$$

If Darcy's law is valid for the consolidation process, and if the other assumptions used in deriving the equation are valid, there should be a close relation between values of the coefficient of permeability of two more or less identical samples of soil determined by direct measurement and by calculations using data from the consolidation test. The existence of such a close relation would demonstrate the validity of Terzaghi's assumptions and the applicability of his theoretically developed differential equation for the consolidation process. If such a correlation can be confirmed, a consolidation test would have a two-fold importance; first, for describing the settlement behavior of a clay layer, and second, for estimating hydraulic conductivity of the soil.

It is the objective of this study to determine the permeability of different clay soils by both direct and indirect (consolidation) methods using one dimensional permeation and consolidation system in separate units. Moreover, because of the numerous factors influencing the permeability, the variational effects of all factors will be minimized by keeping exactly the same conditions for both types of tests so that the results may be directly compared.

CHAPTER II

LITERATURE REVIEW

Movement of Water in Soils and Concept of Permeability

The problem of hydrodynamics in soils has been the subject of hundreds of papers and research works during the last hundred years. Many aspects of this problem have been recognized and solved, and many others are still under investigation.

Important early contributions include those of Hagen (1839), Poiseuille (1846), and Darcy (1856). More recently, Terzaghi (1925), Kozeny (1927), Carman (1937), Kirkham (1945), Muskat (1946), and many others have advanced our knowledge of this area of mechanics significantly.

Soil-water System

As soon as water, a liquid composed of bi-polar molecules, comes into contact with a wettable solid surface, its physical properties are greatly altered from those of the bulk liquid, depending upon the nature of the surface and the solutes in the liquid (50).

In the case of soil particles, the nature of the solid surface is influenced by a variety of factors including the physical, chemical, and mechanical properties of the particles. In sand or silt particles, the

predominant phenomenon is the physical adhesion of the water molecule to the soil particles while, for clay, chemical behavior of the particle has a very important role.

Grim (27) has studied adsorption and orientation of water molecules on clay mineral surfaces, and discussed its implications with respect to the properties of a clay-water system. According to his findings, the structure and organization of adsorbed water molecules depends on the clay mineral composition and its adsorbed ions. He showed that water molecules tend to group into a network around the soil particles. This initial water is adsorbed in a rigidly oriented state, and as the adsorbed water layer increases in thickness, there is a point at which this orientation is lost or greatly reduced in rigidity of organization.

The forces that cause adsorption of water molecules on soil particles are chemical or electrical in nature. They may originate from broken bond forces caused by interruption, at the particle surface, of the normal sequence and balance of the molecular or atomic force fields within the crystal lattice. These surface atoms tend to establish bonds with adsorbed atoms by sharing electrons or orbitals. The attraction may also arise from Van der Waals' forces, which cause bonding of adsorbed molecules by lowering the total energy of the system through mutual harmonic motion of the electrons in the electron clouds (71).

The manner of adsorption of water molecules around clay particles has important consequences in its effects on the physical and mechanical behavior of the clay-water system. Grim (27) has indicated that the mineral composition of the clay particles has a great influence on the thickness and rigidity of the adsorbed water layer. The thickness of

the water layer around different clay mineral particles decreases in the following sequence: montmorillonite, vermiculite, illite, chlorite, kaolinite, halloysile, and allophane. However, the type of adsorbed ions on the surface of a given clay also have a great effect on the thickness of the water layer. Among different ions, Mg^{++} and Ca^{++} tend to develop a very well oriented system of water molecules to a thickness of about two to four molecular layers; Na^{+} provides a very thick layer of water (tens of molecules), but with a very loose orientation. Other ions such as K^{+}, H^{+}, Al^{+++}, and Fe^{+++}, form a light bond between particles with very low potential for the growth of thick oriented water layers (27)(20).

Water Movement in Soil

As the amount of water in the soil is increased, more void spaces are filled and air is forced out or entrapped and compressed. So long as the soil is not saturated, air voids in the form of continuous or discontinuous channels will be present in the system. This soil-water system, like other bodies in nature, can contain energy in different quantities and forms (33). Between two principal forms of energy, namely kinetic and potential, the former is negligible because it is proportional to the square of the velocity which is commonly quite low in soils. The latter form of energy, i.e., potential energy, which is due to position or internal condition, is of primary importance in determining the state and movement of water in the soil. This potential energy of water may be different in different parts of the soil mass, causing water movement from a point of higher potential energy to a point of lower potential energy. Water moves constantly in the direction of

decreasing potential energy. The rate of decrease of potential energy with distance is in fact the motivating force causing flow. If the change in potential energy, p, in a distance, dx, is represented by dp, the force acting on water directed from a zone of higher to a zone of lower potential will be equal to the negative potential gradient, $-\frac{dp}{dx}$.

The nature of water movement under hydrostatic, thermal, and electrical potential gradients has been investigated (93). Hydrostatic potential gradient is the main form of energy that causes water movement in nature, but all three types of potential will be examined briefly.

If a soil mass without significant air voids but possessing a uniform moisture content (as in the vicinity of the plastic limit) is subjected to a temperature gradient, according to the second law of thermodynamics, a maximum amount of free energy is available for the transfer of liquid from the location of the highest temperature to that of lower temperature (29)(40)(93). Winterkorn (93) has developed a mathematical relationship, similar to Darcy's equation, expressing the velocity of water movement under a thermal potential gradient.

The highly electrical character of the mineral-water interaction phase renders soil water susceptible to movement if an electrical potential is applied. Cassagrande (8) has surveyed the general and practical aspects of this phenomenon. Factors affecting this phenomenon have been studied by Winterkorn (93). In a moist clay soil system in which the electrical charges are assymetrically distributed between the predominantly negatively charged clay particle surfaces and predominantly positively charged water phases, any electrical interference in the stable system may result in the movement of water molecules. This process, called electro-osmosis, has found considerable application

in dewatering techniques, and many references concerning it are found in the literature, e.g., (8)(91)(92)(93).

It was mentioned earlier that the water phase in soils can be found in two different forms. First, there is a rigidly adsorbed called fixed or dead water, the thickness of which depends on the nature of the soil particle. In coarse-grained soils, this layer is very thin and negligible in comparison to the particle size; in fine-grained soils such as clay, it is relatively thick and has a very important effect on the movement of water in the soil. Second, there is free water which can flow through the pores under an applied potential gradient (1). Actually, there is no distinct boundary between these two parts; the rigidity of the adsorbed water decreases with the distance from the particle surface so that at some distance, water is not under any adsorptive force from the particle surface. Some investigators believe that the thickness of the dead water (fixed layer) is a function of the applied hydraulic gradient (1)(47)(56)(71)(79)(93).

The early theories of fluid mechanics were based on the properties of a perfect fluid, i.e., one that is both frictionless and incompressible. But in the flow of real fluids, adjacent layers do impose tangential stresses (friction or drag forces). In the case of water movement in soils, boundary friction must also be taken into consideration (33)(71). The degree of interaction (fixation) between fluid and soil particles will determine the thickness of the stationary boundary layer (dead water) and, as discussed previously, it is a function of mineral composition of the soil, ions adsorbed by particles, and physical properties of the fluid (71).

Concept of Permeability

The ease with which a fluid can move through a porous medium is called permeability, and accordingly, the moving fluid is called a permeant. The medium being permeated is called a permeate, and the system is called a permeation system.

From the above definition, it is seen that the permeability of a medium indicates its ability to pass a given fluid through it; therefore, it is related to the properties of both medium and fluid. Thus, the problem is one of mutual composite interaction of both phases, and the permeability of the system cannot be investigated by analysis of the properties of each phase separately. For this reason, permeability is neither a dimensionless coefficient nor a material constant, but rather a property of the permeation system. The many factors affecting permeability will be discussed in a separate section.

Darcy's Law

Henry Darcy (1856) during his experimental studies on sand filters found that the velocity, v_x, of purely viscous flow through an element under a hydrostatic pressure difference of dp, between two points separated by a distance, dx, is given by (76):

$$v_x = - k\left(\frac{dp}{dx}\right)$$

The factor k is called coefficient of permeability. The negative sign indicates that the flow is opposite to the direction of pressure increase. The ratio of $\frac{dp}{dx}$ represents the applied pressure gradient. From the above equation, the rate of flow, Q, through an area, A, is

given by the following expression:

$$Q = - k \cdot A \cdot \frac{dp}{dx}, \text{ from which}$$

$$k = - \frac{Q}{A \cdot \frac{dp}{dx}}$$

Thus, the coefficient of permeability is simply the quantity of fluid driven through a unit area by a gradient of unity.

Since Darcy's law is valid only for laminar flow, it cannot be used for extremely coarse sediments, in which water moves at a high velocity and a turbulent condition exists (76).

Burmister (7) has indicated that Darcy's law is valid only when the following conditions exist:

1 - The flow is laminar

2 - Steady state conditions are present

3 - Soil voids are saturated

4 - Continuity conditions are satisfied and no volume change occurs during or as a result of the flow.

More recent investigations (18)(30) have shown, however, that condition number 3 is not necessary, and that Darcy's law is valid for non-saturated flow.

Stability of the permeation system as a requirement for validity of Darcy's law has been discussed by Schmid (71), who states that the following conditions should be either absent or negligible during the permeation process:

1 - Volume change of permeate or permeant

2 - Ion exchanges or dissolution and leaching of the permeate

3 - Deposition of solid, liquid, or gaseous matter by the permeant within the permeate

4 - Structural changes of the permeate due to dispersion, aggregation, or change in the crystal lattice, and

5 - Temperature changes.

When these changes take place, they generally occur at the beginning of the permeation, but sometimes may continue after an otherwise stable condition is reached.

The applicability of Darcy's law for different hydraulic gradients has also been investigated. Slepicka (74) has expressed the general form of Darcy's law as

$$v = k(i)^n$$

where

$n > 1$ for very small velocities

$n = 1$ for intermediate velocities

$n < 1$ for very high velocities

This has been confirmed by Hansbo (31), Swartzendruber (79), and Abelev (1) for very low velocities, and by Muskat (59) and Muskat and Botest (58) for high velocities. Darcy himself realized that his equation was not valid for high fluid velocities, and during the past forty years considerable effort has been directed toward a fuller understanding of the problem (19)(59)(69). It seems well established that when the hydraulic gradient exceeds a critical value, the flow velocity is no longer proportional to it, but increases less rapidly than the gradient. The reason for this is that since viscous forces no longer balance the inertia and turbulence forces, not all of the driving force of the

hydraulic gradient is used to overcome viscous resistance.

Fireman (18) has indicated, with regard to high hydraulic gradients, that so long as a laminar flow is maintained, Darcy's law applies. With regard to very low hydraulic gradients, there are many different opinions. Fishel (19) has indicated that for his experiments, Darcy's law was valid for a hydraulic gradient as low as two or three inches per mile. King (38) has reported that for flow through porous media under very low hydraulic gradients, the velocity was not proportional to the gradient, but increased more rapidly than the gradient. This is in contrast to the case of high gradients, where the velocity increases less rapidly than the gradient. A more extensive study of this subject was done by Swartzendruber (79). He explains the non-proportionality of velocity and hydraulic gradient, using the theory of Newtonian and non-Newtonian flows. According to this, there are three types of flow with respect to the relationship between the rate of shear and the shear stress in a liquid (Figure 1):

1 - Newtonian flow, which is shown as a straight line passing through the origin

2 - Non-Newtonian flow, which is a curved line passing through the origin and concaving upward

3 - Bingham flow, which is a straight line but does not pass through the origin.

Swartzendruber compared these three relationships with the velocity-hydraulic gradient relationship of fluid movement in a porous medium. He categorizes as a Newtonian liquid those for which the v-i relationship is a straight line passing through the origin; as a non-Newtonian liquid, those for which the v-i relationship is a curve passing through

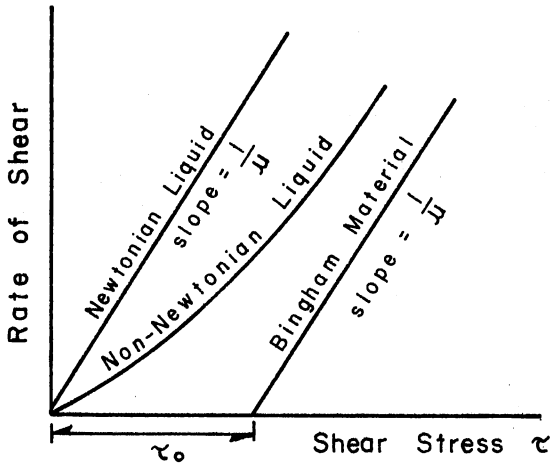


Figure 1. Shear Rate versus Shear Stress for Three Flowing Materials (79)

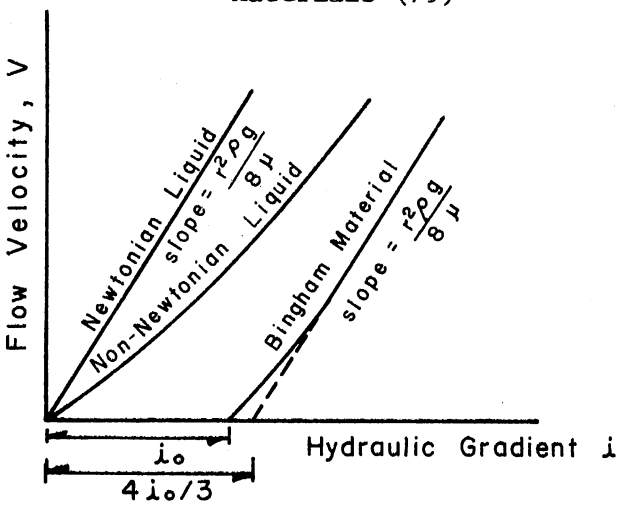


Figure 2. Flow Velocity versus Gradient for Three Fluids in Figure 1 (79)

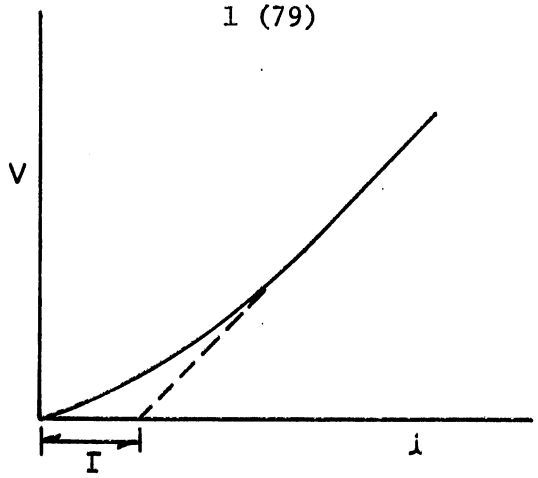


Figure 3. General Aspects of the Suggested Flow Relationship (79)

the origin and concaving upward, and as a Bingham material, those for which the v-i relationship is linear above a given velocity, and non-linear for velocities less than that (Figure 2). The slopes of straight lines are given as

$$\frac{r^2 \cdot \rho \cdot g}{8\mu}$$

where r is the radius of capillary channels, ρ is the density of the liquid, μ is the coefficient of viscosity of the fluid, and g is the acceleration of gravity. It appears that this concept can be useful for understanding the v-i relationship for low hydraulic gradients in porous media.

Hansbo (31) and Slepicka (74) have suggested that the curved part of the Bingham line be represented mathematically by a power function such as $v = k(i)^n$. This suggestion shows that they did not believe in the existence of a threshold gradient in very fine-grained soils, a topic which has been the subject of many papers. The general shape of the v-i relationship according to Hansbo's suggestion would be as shown in Figure 3. However, many workers in this area believe that there is a threshold gradient or a lower limit for i below which there is no flow and therefore v = 0 (13)(47)(50)(56)(60)(71). This problem will be discussed in more detail in a later section on factors affecting permeability.

Terzaghi (83) also observed a distinct departure from Darcy's law for very low hydraulic gradients. He reasoned that

> .....When water percolates through a clay mass under a considerable head it produces elastic and non-elastic deformations and grain displacement similar to the deformations

> produced by a stream of water forced through a system of very expansible rubber tubes, but at lower hydraulic gradients, the elastic deformations disappear and the coefficient of permeability changes accordingly (p. 834).

After Darcy formulated his empirical equation, attempts were made to derive it theoretically [Emersleben (15), Muskat and Bostest (58)]. Hall (30) has derived Darcy's equation in the following simple manner:

Considering a soil-water system, the forces which act on the system are classified as:

1 - pressure or gradient, $f_p = - \nabla P$

2 - gravity force, $f_g = - \rho \nabla gz$

3 - viscous force

It has been indicated that viscous force per unit volume is a linear vector function of the velocity, $f_v = \mu.v.T$ (T is tensor). Thus,

$$v = \left(\frac{1}{\mu}\right) K \cdot f_v$$

For equilibrium $\Sigma F = 0$, and

$$\therefore - \nabla p - \rho \nabla gz + f_v =$$

$$- \nabla p - \rho \nabla gz + \mu.v.K = 0$$

$$v = - \frac{1}{\mu} .K.(\nabla p + \rho \nabla gz)$$

where

v = velocity

μ = coefficient of viscosity

∇p = pressure gradient

g = acceleration of gravity

ρ = density

z = elevation factor

K = tensor, called permeability

For an isotropic medium, K will be the same in all directions and therefore becomes a scalar value whose magnitude may vary with other factors such as material properties, time, and location. If the continuity of the fluid is assured, this derivation is valid both for saturated and non-saturated conditions. Another form of this equation may be obtained assuming the density of the fluid to be constant. By combining the constant factor, $\frac{\rho g}{\mu}$, with the permeability term, the velocity is seen to be proportional to the gradient of the potential energy per unit weight of fluid expressed as the height of a column of fluid.

$$v = - k\nabla \left(\frac{p}{\rho g} + z\right)$$

Actually, Darcy's equation is a general form of the Poiseuille-Hagen equation for capillary flow in the following form (93):

$$V = \frac{\pi r^4}{8\mu} \left(\frac{p_1 - p_2}{l}\right) \cdot t$$

where

r = inside radius of the capillary tube (cm)

l = length of tube (cm)

t = time (seconds)

$p_1 - p_2$ = pressure head over the distance, l

μ = coefficient of viscosity (poise)

$\gamma = \rho . g$ = unit weight of the fluid (gr/cm^3)

V = volume of the fluid (cm^3)

The equation may be reduced to the following form:

$$\frac{V}{t} = Q = \frac{\pi r^2 . r^2}{8\mu}\left(\frac{\Delta p}{\Delta l}\right)$$

$$Q = A.v = \frac{\pi r^2 . r^2 . \gamma}{8\mu} \quad (i)$$

$$v = \frac{r^2 . \gamma}{8\mu} \cdot i = \frac{r^2 \rho g}{8\mu} \cdot i$$

$$k = \frac{r^2 \rho g}{8\mu} \rightarrow v = k.i$$

which is the same as Darcy's equation.

There have been two basic definitions for permeability based on direction; first, permeability measured in the direction of the flow lines, used when the length of the sample is infinitely greater than the width, and second, permeability in the direction of the gradient, used when the width of the sample is infinitely greater than the length (51) (69).

Factors Affecting Permeability

There have been many studies of factors influencing the permeability of coarse-grained soils. Most of the work has been theoretical, starting with the basic Poiseuille-Hagen equation and Darcy's law applied to capillary tubes. The best known equation relating different factors to the permeability of a given soil (most applicable to coarse-grained soils) was given by Kozeny (18) and extended by Carman, and is known as the Kozeny-Carman equation. This equation has appeared in several forms, the most common being (44):

$$k = \frac{1}{k_o S_s^2} \cdot \frac{\gamma}{\mu} \cdot \frac{e^3}{1+e} \quad \text{(for saturated soils)}$$

where

k = coefficient of permeability

k_o = constant, depending on pore shape and ratio of length of actual flow path to soil bed thickness

S_s = specific surface area

e = void ratio

γ, μ as defined previously

Mitchell (57) derived the following equation for the unsaturated condition, based on the Kozeny-Carman equation:

$$k = \left(\frac{\gamma}{\mu}\right) \frac{C_s V_s^2}{A_s^2} \cdot \frac{e^3}{1+e} \cdot S^3$$

where

S = degree of saturation

C_s = shape constant

γ = unit weight of water

V_s = volume of solids

A_s = wetted area of the capillary tubes

The following form has been suggested by Taylor (44)(45):

$$k = D_E^2 \frac{\gamma}{\mu} \cdot \frac{e^3}{1+e} \cdot C$$

where

D_E = some effective particle diameter

C = shape factor

Schmid (71), starting from the original Hagen-Poiseuille equation and considering the theory of stationary boundary layer and the effective porosity, has derived the following expression:

$$k = D_E^2 \frac{\gamma}{32\mu} \cdot n_E = \frac{\gamma}{32\mu} \cdot D_E^2 (n-n_o)$$

where

$n_E = n-n_o$ = effective porosity

n = total porosity

n_o = ineffective porosity which is related to the volume of voids occupied by dead, stationary water, as explained in the previous sections.

Following the concept of threshold saturation (percent of immovable water in the voids), and considering the porosity of the soil, Irmay (35) has derived the following equation:

$$K_L = c\, d^{-2} (S-S_o)^3 \frac{n^3}{(1-n)^2}$$

where

K_L = specific permeability = $k \frac{\mu}{\gamma}$

d = effective diameter of voids

c = numerical constant

S = percent saturation

S_o = threshold saturation

n = porosity

In addition to these equations, there are other expressions relating the permeability to the soil properties, but most of them have not been verified experimentally. Schmid (given above), which considers permeability proportional to the first power of porosity, is not capable of experimental verification.

It was mentioned earlier that most of the theoretical expressions given for permeability of soils have applicability only for coarse-grained soils, and generally are of limited use for fine-grained soils, especially for clays. The reasons for this are:

1 - difficulty of selecting the effecting "constants" and soil characteristics

2 - the fact that the various terms are not independent, but are interrelated in a very complex manner

3 - several factors which affect the permeability of fine-grained soils are not contained in the theoretical equations. Soil composition, permeant characteristics, structure of the soil, etc., are not represented.

In the following sections, the influence of various factors which might affect the permeability of soils in general and fine-grained soils in particular will be discussed. These factors are classified in four major groups:

a - factors related to the porous media (permeate)

b - factors related to the fluid (permeant)

c - factors related to the permeameter device

d - other factors

Factors Related to the Porous Media (Permeate)

Mineral Composition of the Soil

This factor has a great influence on the permeability of the soils, especially clay soils, which is the subject of this paper. The results of hundreds of experiments made by different investigators have shown that the permeability of the clay soils changes with changes in mineralogical composition of the soil, keeping all other factors the same. A major reason for this--the difference in thickness and rigidity of the water film around the clay particles--was discussed earlier (44)(50). Considering the mineral composition of the soil, the following trends are usually accepted:

k montmorillonite < k attapulgite < k kaolinite (44)

k Wyoming bentonite < k Bladen clay < k Utah bentonite < k halloysite (50)

Adsorbed Cations

The kind of adsorbed ions on fine-grained soils has a great influence on the thickness of the water film around the particles and, consequently, on the permeability of the soil.

Sodium clays are generally much less permeable to both water and electrolytes than are Ca^{++} and H^{+} clays (4)(32)(50). Still, the effect of Ca^{++} and H^{+} depends on the type of clay. For example, the permeability of various ionic forms of montmorillonite at the same void ratio varies as $k^{+} < Na^{+} < H^{+} < Ca^{++}$, and for kaolinite, varies as $Na^{+} < k^{+} < Ca^{++} < H^{+}$ (44). Sodium clay, particularly sodium montmorillonite, is

generally the least permeable soil mineral and is therefore widely used by engineers as an impermeabilizing additive for other soils. In experiments unrelated to this study, conducted at Oklahoma State University by the author, the permeability of a silty sand from Tonkawa, Oklahoma, was decreased from 1×10^{-4} cm/sec to 1×10^{-6} cm/sec, and 1×10^{-9} cm/sec by adding, respectively, 5 and 10 percent by weight of sodium montmorillonite.

Experiments have indicated that the lower the ion exchange capacity of a soil, the lower the effect of the exchangeable ion on permeability. It was observed in the previous sections that there is no term in the given expression for permeability to take into consideration the effect of such important factors as mineral compoistion or exchangeable ions of the soil.

Void Ratio

Considering the different expressions relating permeability to the void ratio of soils, one would expect a straight line relationship between k and $\frac{e^3}{1+e}$; but the experimental results given by several authors (55) do not show such a linear relationship. Instead, a number of investigators have obtained a straight line relationship between log k and e (42)(44)(81). This relationship, of course, may hold if other factors are held constant. Since a relationship between log k and e has been well substantiated, the method will be used for analyzing the results presented in Chapter IV.

Schmid (71) has shown mathematically that if k is directly proportional to the first power of n (porosity), there will be a logarithmic relationship between k and e. Some investigators have tried to

find a linearity between k and n or n^2, but only Winterkorn (90), who reported a straight line relationship for k vs n^2, has succeeded.

Soil Structure

Soil structure, i.e., orientation of the particles in a soil mass has great influence on the permeability of fine-grained soils. Two extremes of soil structure, namely dispersed and flocculated, exhibit a great difference in permeability of a given soil at the same void ratio. Generally, the more dispersed the structure, the lower will be the permeability, and a more flocculated structure provides higher permeability (45). This is one of the important factors which is not considered in the ordinary expressions for permeability. The structure of the soil in most laboratory experiments (disturbed samples) is controlled by the compaction process and depends on whether the soil is compacted on the wet or dry side of optimum. Compaction on the wet side provides a more dispersed structure (parallel orientation of particles) and lower permeability. Compaction on the dry side of optimum gives a more flocculated structure and consequently a higher permeability (44). Minimum permeability of a compacted soil occurs at a water content greater than optimum (43)(44)(57)(89).

Air permeability of a soil is indicative of its structure before wetting, and its water permeability is an indication of its structure after wetting (6). Brooks (6) has suggested that the ratio of air to water permeability be used as a measure of stability of the soil structure. The greater the difference between these two permeabilities, the more unstable the soil structure.

Soil Texture

In general, permeability decreases with an increase in clay or silt content (18). Finer particles in a soil have a high impermeabilizing effect and normally control the permeability of a mixture (7)(44). As a general rule, the more nearly homogeneous and isotropic the soil, the lower will be its permeability.

Particle Size and Shape

Normally, the effect of particle size is expressed by specific surface in permeability equations, but there is no way of considering the effect of the shape of particles, especially for fine-grained soils (4) (44). For coarse-grained soils such as sand and gravel, the effect of particle size has been expressed as an effective grain size such as in Hagen's equation (36).

Degree of Saturation

Unsaturated voids, containing some entrapped air, cannot transmit water as well as can saturated voids. There have been some attempts to take the degree of saturation into account in some theoretical relationships [see section on factors affecting permeability, Mitchell equation (71) and Irmay equation (35)]. As a general rule, the higher the degree of saturation, the higher the permeability (45), but Lambe (44) has indicated that the influence of the degree of saturation on permeability is relatively minor in comparison with the composition, structure, and void ratio.

Pore Size Distribution

The effect of pore size distribution on soil permeability has been investigated by Marshall (52) and Smith (75). The effective pore size distribution has been measured by water removal at different levels of tension forces. Smith indicates that the water-filled pores contribute to percolation approximately in proportion to their diameters, while Marshall has derived the following equation relating permeability to the pore size distribution:

$$K = \frac{n^2 N^{-2}}{8}\left[r_1^2 + 3r_2^2 + 5\,r_3^2 + \ldots\ldots(2N-1)r_N^2\right]$$

where

K = permeability in cm^2

n = porosity

N = integer numbers 1, 2, 3,.....

r = radius of pores of given size related to pore size distribution in cm. (If the mean radius of the pores of a given size related to all pores in each N equal fractions of the total pore space is represented in decreasing order of size by $r_1, r_2, {}^4_3, \ldots\ldots, r_N$

$$r_i = \frac{2T}{\rho g.h_i}$$

where

T = surface tension of water

ρ = density of water

g = acceleration of gravity

h_i = suction head for each radius r_i

Factors Related to Fluid (Permeant)

Density and Viscosity

These are the two principal fluid characteristics affecting the permeability of soils (44). These major effects of the permeant on permeability can be eliminated by using absolute or specific permeability $K = k\frac{\mu}{\gamma}$. The lower the viscosity and the higher the density, the higher will be the permeability.

Type of Fluid

Results of experiments conducted by Michaels and Lin (55) indicate that for two different fluids having the same viscosity and density and under the same conditions, the soil permeabilities were different. This discrepancy has been ascribed (44) to elector-osmotic backflow and thickness of immobilized fluid layer, both of which increase with fluid polarity.

The experimental results show that at any given void ratio, permeability decreases regularly with increasing polarity of the permeant, because of more orderly packing of the solid (55).

Chemical Composition of the Fluid

Chemical composition of the permeant and especially the concentration of ions in a solution has great effect on the permeability of the soils. For example, soils remain moderately permeable when leached with a high-sodium solution so long as the salt content remains fairly

high. The reason for this is that salt tends to maintain the flocculated structure of the soil. If the sodium concentration is lowered, the flocculated structure of the soil is destroyed and a dispersed structure with much lower permeability is produced (18).

Factors Related to the Permeameter Device

Length and Diameter of the Permeameter

The effect of the length of the permeater has been studied by Fireman (18), who found no difference in the measured permeability of samples having different lengths varying from one inch to 34 inches and different diameters of one inch to six inches. He found, however, that with increasing diameter and length, the non-uniformity in packing and particle size distribution is diminished. The larger the sample, the more uniform it will be.

Shape of the Permeameter

Fireman's experiments also indicated that a cylindrical form is the most suitable shape for a permeameter and soil sample to avoid any kind of flow restriction (18).

Effect of the Permeameter Wall

The effect of the permeameter wall depends on R (Reynold's number) as well as the $\frac{D}{d}$ ratio of the permeameter diameter to the mean particle diameter. If the $\frac{D}{d}$ ratio is greater than 40, the wall effect is of negligible magnitude (21)(67).

The main effects of the permeameter wall arise from an increased

resistance to flow along the surface area of the wall and a greater soil porosity in the immediate vicinity of the wall than in the body of the medium. These two effects are opposite, and the overall effect depends on Reynold's number for $\frac{D}{d}$ ratios of 40 or less.

Soil-retaining Screen

Normally, in any permeameter device, the sample is retained by a porous material or screen at the top and bottom. To eliminate the effect of this retaining material, its permeability should be much greater than that of the soil. Moreover, it should be so constituted that a relatively uniform water movement is maintained and soil sloughing is prevented. Fireman (18) conducted an extensive evaluation of the suitability of various materials for this purpose. Perforated brass disks covered with a thin layer of coarse asbestos, sand supported by a thin fiberglass screen, and lathing screen covered by one thickness of "fast" filter paper were found quite satisfactory.

Other Factors Affecting Permeability

Effect of Compaction

A pronounced decrease in permeability will be produced by increasing the compactive effort at any given water content. Mitchell (57) has indicated that permeability decreases more than 100-fold without a change in density or moisture content, as a result of increasing the kneading compactive effort. The effect of the higher compactive effort is to produce a more dispersed structure and consequently a lower permeability.

Effect of Method of Compaction

It has been shown that permeability is related more to the structure of the soil than to any other single variable and that the structure itself is influenced greatly by shear strains associated with the method of compaction wet of optimum moisture content (57). Different compaction methods induce different amounts of shear strain. The shear strain and, therefore, the degree of dispersion of the soil particles, is related to the method of compaction. Thus, permeability of the soil will be influenced by the method of compaction. Seed (7) has shown that shearing strain and degree of dispersion for different compaction methods increases in the following order: static, vibration, and kneading. Therefore, samples prepared by kneading compaction have lower permeability than do samples prepared by static compaction. This is true for compaction wet of optimum, but for dry of optimum, the effect will be negligible because no method of compaction induces appreciable shear strain under such conditions.

Effect of Flow Direction

Many papers have indicated that an upward direction of flow for a permeability test is more effective in removing entrapped air from the sample. Contrarily, the results of experiments conducted by Fireman (18) disclosed no difference in respect to removal of entrapped air during downward and upward flows. The measured permeabilities were different only in the early stages of the flow, and during later stages were in excellent agreement. The same result was obtained by Smith (75) and Christiansen (9).

Effect of Time

It has often been observed that permeability changes with time (5)(7)(52)(62). Generally, it is found that the decrease in permeability occurs in a short time during early stages of the experiment. In some instances, the initial decrease is followed by a gradual increase before a constant permeability is reached. The initial decrease in permeability is believed to be due to the following reasons (5):

- Increased dispersion and migration of particles. The wetting process weakens the bonds between colloidal particles and, with movement of water through the soil, such loosened particles are moved to more stable positions, decreasing the effective pore size and producing a more dispersed structure with a consequent decrease in permeability (18)(62).
- Removal of electrolytes which cause a more dispersed structure and reduced permeability (5)(18).
- Gupta (28) has suggested bacterial activity as one of the reasons for reduction in permeability.

During the second stage of the permeability-time relationship there is generally an increase in permeability. The reasons for this phenomenon may be:

- Dissolving of air trapped in the voids of an unsaturated soil, which produces a larger effective pore size and higher permeability (44).
- Thixotropy, or a tendency toward flocculated structure. If a transition to a more flocculant structure takes place, accompanied by thixotropic hardening, the permeability should be

expected to increase (57).

An increase in permeability may well depend on whether one of the above processes takes place in a given soil or not (5).

Bodman (5) based on an experimental study, concluded that the decrease in permeability with time is related primarily to the silt content of the soil. He found no relation to clay content, moisture, or other factors which have frequently been observed to affect the permeability. However, clay content is the most influential factor governing the permeability of the soil.

Effect of Magnitude of Hydraulic Gradient

This subject has been the main topic of a number of investigations. Some studies support the theory of a dependency of permeability on hydraulic gradient, while many others do not. Threshold gradient, i.e., the minimum hydraulic gradient required to start the permeation process in a given soil, is one aspect of this problem. The problem basically relates to the non-proportionality of flow velocity and hydraulic gradient for very low and very high gradients, and to the non-linear v-i relationship for these two extremes.

Considering the theory of stationary boundary layer (a viscous or rigid layer of water adsorbed by soil particles), it is reasonable to assume that only a part of the channel voids are available for water flow (71). It has also been shown that the thickness of this fixed water layer is related to the applied pressure. Some of those who have worked on the threshold gradient problem believe that there is a hydraulic gradient below which no flow occurs, and that a linear v-i relationship, having an intercept with the i-axis at a value $i_o > 0$,

exists for values of $i > i_o$. Thus, it is assumed that Darcy's law may be applied over the entire range of flow velocities (47)(56)(93). Other investigators have found the v-i relationship to be non-linear for very low values of i, represented by a curve passing through the origin and exhibiting an upward concavity (79). For the first case, see Figure 2, and for the second case, Figure 3.

A similar non-linear v-i relationship has sometimes been reported also for very large values of hydraulic gradient. For very high values of i, the velocity increases more rapidly than i. In cases where the flow remains laminar, the non-linearity appears to be associated with a decrease in the thickness of adsorbed water film and the consequent enlargement of flow channels and increase of effective porosity resulting from the increased stress level (31)(50)(63)(79). The enlargement of pore openings may be related to:

- a reversible orientation of the particles along the stream line
- a surpassing of the yield value of the smaller pore as i increases (new pores become available to flow).

These phenomena are also consistent with the threshold gradient theory. It should be emphasized that the characteristics of the sample play an important role and that for some soil types, none of the behavior described above has been observed. The aberrant behavior is observed especially in the more clayey soils (56). As a general rule, there is a straight line relationship between v and i for intermediate values of i.

Bodman (4) states that downward flow velocities are more affected than are upward velocities by the non-linear influences existing for lower values of i. Burmister (7) reported a decrease in permeability

with an increase in hydraulic gradient. It is believed that this might be true only for very coarse-grained soils, where flow under high pressure is no longer laminar and the energy losses of turbulent flow cause a decrease of the velocity gain resulting from increased gradients.

Effect of Temperature. An increase in temperature increases permeability because of the decrease in viscosity of the permeant. If a temperature differential exists, the movement of permeant caused by the thermal gradient will also affect the measured permeability (93).

Effect of Entrapped Air. This problem has been studied by Christiansen (9) and others. The general effect of entrapped air is to decrease the permeability by plugging smaller voids and decreasing the effective porosity available for the water movement. But most of the entrapped air will dissolve in the water after a time, causing an increase in permeability (second stage of k-t curve, as discussed previously.

Effect of Disturbance. Laliberete (41) studied the effect of sample disturbance on the permeability of soils. Because sampling produces a change in the macrostructure of the soil, the permeability is always lower for disturbed samples than for corresponding undisturbed samples. Disturbance by pulverization during compaction may produce very great changes in the properties of material from a particular site.

Units of Permeability

Because of numerous factors affecting permeability, several different units have been used by investigators, depending upon their

needs and the factors which have been considered by them. The most common units for permeability have been defined and classified by Richards (66). According to him, the following points should be considered in selecting a permeability unit:

"1 - It should be a practical unit chosen for maximum convenience and usefulness in laboratory and field work.

2 - It should be simple as regards definition, defining equations, and dimensional relations so as to facilitate its use in either the metric or English system.

3 - It is desirable to have a permeability unit that is usable for all flow cases following Darcy's law."

The most common permeability units used for soils can be identified as:

a. Darcy's coefficient of permeability, which from the standpoint of historical significance and present usage is the most well known unit.

$$k = \frac{v}{i}$$

In this equation, k has a dimension of velocity and its transformation from metric to English units is very simple. In the English system, it is commonly expressed in ft/min; in the metric system, in cm/sec.

b. A different unit is obtained when the effect of viscosity of the fluid is taken into consideration. Dividing the right side of Darcy's equation by μ (coefficient of viscosity), makes the permeability independent of viscosity

$$v = k \cdot \frac{\mu_s}{\mu} \cdot i$$

where μ_s is the viscosity of a standard liquid at standard temperature, and μ is viscosity of the permeant fluid. The dimentionsl relations and conversion from metric to English system becomes more complicated.

c. Introducting the acceleration of the gravity term (g) into Darcy's equation gives

$$v = k \cdot g \cdot i$$

which has a dimension of time. Introducing the viscosity effect does not change the dimension of the unit

$$v = k \cdot \frac{\mu_s}{\mu} \cdot g \cdot i$$

and because of its simple dimension, it can be used in any system of force and length, always giving the same numerical value of permeability (because the units of time are the same for all systems.) This unit is best suited for international use.

d. For general applicability of Darcy's equation for both liquids and gases, a density term can be introduced into the equation

$$v = \frac{K}{\mu} \cdot \rho \cdot g \cdot i = K \cdot \frac{\gamma}{\mu} \cdot i$$

In this case, permeability has a unit of (L^2) and is denoted by K and called specific permeability or intrinsic permeability, proposed by Muskat (59).

e. Another permeability unit which has been proposed (59)(66) is the "darcy." If macroscopic velocity is expressed in cm/sec, viscosity in centipoise, and the pressure gradient in atm/cm, k is expressed in "darcy" units. One darcy equals 0.987×10^{-2} cm^2 (45).

Richards (66) also states that a permeability unit "to be acceptable for general use in different lines of work must be based on a flow equation which is adequate to cover a variety of flow cases." He considers the following equation to have certain advantages in this respect.

$$v = \frac{K}{\mu}.\gamma.i$$

Irmay (35) suggests that a coefficient of permeability of "hydraulic conductivity" with the dimension of velocity (Lt^{-1}) and represented by k is one of the most useful forms. It is a measure of permeability of a soil to a given fluid since there is no term containing the fluid properties. For more general use, he prefers specific or intrinsic permeability, K, as defined above. This introduces the density and viscosity of the fluid and, therefore, is a measure of the permeability of the soil to any fluid. The relationship between these two units is given by

$$K = k\frac{\mu}{\gamma}$$

Lambe (45) has given a chart for converting permeability from one unit to another.

Measure of Permeability

Because many factors affect the permeability of soils, numerous methods have been suggested and used for its determination (10)(11)(18)(25)(45)(48)(49)(52)(64)(83)(85). The various procedures are outlined and described in the following sections.

Laboratory Measurement of Soil Permeability

In this procedure, the permeability of the soil is measured either by a direct method or by the use of the results of other tests on the soil samples.

Direct Methods. Direct methods are based on measurement of the rate of flow of water through the soil, generally assuming the validity of Darcy's equation. The measurements may be conducted under one of the following conditions:

1 - Constant head method: In this method, an undisturbed or disturbed sample of soil of given dimensions is subjected to a constant hydraulic potential and the average rate of flow during a given time interval is measured. Considering Darcy's equation and assuming that Q represents the rate of flow per unit time; A, the cross-sectional area of the sample perpendicular to the flow direction; h, the total hydraulic head, and L the length of the soil sample,

$$Q = k \cdot A \cdot \frac{h}{L} = k \cdot A \cdot i$$

$$k = \frac{Q.L}{h.a}$$

where k is a coefficient of permeability for the particular conditions of the test. The results may be made more general by considering the viscosity and density of the fluid and the influence of temperature. This method is normally used for both plastic and non-plastic soils (45). It has the following advantages (18):

- The soil is not disturbed during the entire run;

- The effluent is quickly and accurately measured;

- The reservoir requires a minimum of attention (18).

Disadvantages of this method are the necessity for maintaining a constant head, which is often troublesome, and the point that a constant head is seldom encountered under natural conditions.

2 - Falling head method: In this method, a soil sample is placed under a hydraulic head which decreases in magnitude with time as water passes through the soil and the level of water in the reservoir goes down. Applying Darcy's law, it is easily shown that

$$k = \frac{a.L}{A.t} \log_e \frac{h_1}{h_2}$$

where

a = cross-sectional area of the water reservoir

h_1, h_2 = hydraulic head at the start and end of the run

t = time interval in which the water level in the reservoir drops from h_1 to h_2

Lambe (45) believes that this method should be limited to saturated soils of rather high permeability. Fireman (18) expresses an opposite opinion, and considers the method best suited for finer material of very low permeability.

The advantages of this method are the ease of control of head and the insignificant effect of evaporation on the results. Disadvantages are the greater complexity required, the possible effect of refilling the reservoir periodically, and the effect of entrapped air on the calculated permeability, because this method dissolves air less effectively in the percolating water (18).

Indirect Methods. Permeability of the soils may be determined indirectly by using properties of the soil mass and soil particles, or by the use of the results of a consolidation test.

1 - Use of the theoretically derived equations: In this method, soil permeability may be determined by analysis of the size, shape, and arrangement of the soil particles, or by some related properties such as void ratio, effective porosity, pore size distribution, etc. (18). These methods do not provide an accurate quantitative value for soil permeability but, rather, yield quantitative values which are normally used for preliminary evaluation and comparison of permeabilities of different soils. The most common relationships of this nature available in the literature are as follows (48):

Hazen equation, 1892:

$$k = cd_{10}^{2}$$

where

d_{10} = effective grain size

$c = 41$ to 146 ($c_{ave} = 100$)

For uniform sand, Hazen's equation will be $k = 150\ d^2$.

Slichter equation, 1899 (7)(48)(83):

$$k = \frac{771d^2}{c}$$

where

d = particle size in a uniform sandy soil

c = constant related to e (varies from 84.3 to 12.8 for values of e equal to 0.35 to 0.85, respectively)

Terzaghi equations, 1925 (48)(83):

$$k = \frac{c}{\mu} \cdot \left[\frac{n-0.13}{\sqrt[3]{1-n}}\right]^2 \cdot d_{10}^{\ 2} \quad \text{(for sands)}$$

where

μ = coefficient of viscosity

n = porosity

c = 800 for well-polished and rounded grains

c = 460 for irregular, rough grains

$$k = c \frac{\mu_o}{\mu_t} (e-0.15)^3 \ (1+e) d_e^{\ 2} \quad \text{(for clays)}$$

Later revised to

$$k = c \left(\frac{\mu_o}{\mu_t}\right) \frac{(e-0.15)^{11} \ (1+e)}{(e-0.15)^8 + \frac{c}{d_e^{\ 8}}} \cdot d_e^{\ 2}$$

where

d_e = effective grain size

μ_o = coefficient of viscosity at 10^oC

μ_t = coefficient of viscosity at t^oC

e = void ratio

Kozeny equation (1927 (48)(49):

$$k = \frac{\gamma_w}{k \ .\mu.S_s^{\ 2}} \cdot \frac{n^2}{(1-n^2)} \quad \text{(best used for sands)}$$

where

γ_w = unit weight of water

k' = constant ~ 5

S_s = specific surface

and other terms as previously defined.

Kozeny-Carman equation (44)(45):

$$k = \frac{1}{k_o S_s^2} \cdot \frac{\gamma}{\mu} \cdot \frac{e^3}{1+e}$$

where

k_o = constant

Taylor equation (44)(45):

$$k = d_e^2 \cdot \frac{\gamma}{\mu} \cdot \frac{e^3}{1+e} \cdot c$$

where

d_e = some effective particle diameter

c = shape factor

Rose equation, 1950 (48):

$$k = \frac{g d_e^2}{1000 \mu} \cdot \frac{1}{f(n)}$$

where

$$f(n) = 1.115\ (1-n) \cdot \frac{\left[(1-n)^2 + 0.018\right]}{n^{1.5}}$$

g = gravitational acceleration

Loudon equation, 1953 (48):

$$\log_{10} k.S_s^2 = 1.365 + 5.15n$$

where

S_s and n are as given before. This equation was obtained experimentally.

Childs-Marshall equation, 1958 (49)(52):

$$K = \frac{n^{2N-2}}{8}\left[r_1^2 + 34_2^2 + 5r_3^2 + \ldots\ldots + (2N-1)r_N^2\right]$$

This equation has been derived based on a relationship between permeability and pore size distribution which is found from the moisture content-suction curve. In this equation, N = integer number, and r_1, r_2, r_3, ... r_N are the mean radii of pores in decreasing order of size in each N equal fraction of the total pore space. To obtain values of r, the moisture content of the soil is plotted as a function of suction head on an arithmetic scale. The suction head is a measure of the square of the pore radius by the following expression (52);

$$r = \frac{1}{h}\left(\frac{2T}{\rho.g}\right)$$

where

h is the suction head

c is the density of water

T is surface tension of water

For the case of uniform pore size,

$$k = \frac{n^2 . r_u^2}{8}$$

where r_u is the radius of pores the the expression is similar to Kozeny's equation. The method has been used successfully for different materials with a wide range of permeability (2.7×10^{-11} to 1.3×10^{-6} cm/sec), which embraces the range of very fine to medium-grained materials. Marshall (52) suggests that it is best suited for coarser materials.

Use of the Results of Consolidation Tests

Results of a consolidation test can be used for an indirect determination of soil permeability. Use of this method is the main objective of this paper and because of its importance, the procedure will be discussed in more detail in the next chapter.

This method is based on Terzaghi's theory of consolidation. Consolidation is the process of volume decrease of a saturated clay layer under applied static load because of gradual movement of water out of the soil. The theory and its mathematical statement were developed by Terzaghi and Frȍhlich between 1910-1920.

For any flow condition that is accompanied by a change in volume of the voids, the law of continuity of flow is no longer valid. This is the case for compressible soils such as clays. If the voids of such a soil are filled with water, a change in the effective stress involves a decrease or increase in the water content of the soil (84). According to Terzaghi's definition (Theoretical Soil Mechanics),

> .....every process involving a decrease of the water content of a saturated soil without replacement of the water by air is called a process of consolidation. The opposite process is called a process of swelling, which involves an increase

> of the water content due to an increase of the volume of voids (p. 265).

It should be noted that in a highly compressible soil, the ease with which water can move out of the sample under the applied loads is related to the permeability of the soil mass. In clay soils of low permeability, changes in the water content due to a change in the effective stress takes place very slowly; consequently, deformation of the soil mass itself takes place gradually over a long period of time.

To understand the consolidation process, let us consider a unit of a saturated compressible clayey soil, located between drainage faces at its top and bottom. When a load is applied to the top face of the sample, the load is initially carried by the water in the pores (since water is much less compressible than the soil skeleton), but under the applied pressure gradient, the water begins to flow toward the drainage faces. As the water flows outward, the applied load is gradually transferred to the soil skeleton, and since the soil skeleton is compressible, the deformation that occurs corresponds exactly to the volume of voids which has been emptied. It should be noted that this is a continuous process and outward movement of water and, consequently, deformation of the soil mass starts with application of the load and process of load transfer from water phase to the soil phase, and continues for a long time. A further important feature of the process is that the deformation due to consolidation is not uniform throughout the soil mass because those molecules of water farthest from the drainage face take much more time to move out of the voids than do those near the boundaries, where pressure gradients are initially very high in comparison with interior parts of the mass. It is obvious that the consolidation

process starts at boundaries in contact with the drainage surfaces and progresses toward interior parts of the mass. The magnitude of pore pressure at a given time is a function of the location of the point in the soil mass (z), and at a given location is a function of time (t). Therefore, pore pressure is a function of both time and location. Terzaghi derived the governing differential equation, based on several simplifying assumptions in the following form:

$$\frac{\partial u}{\partial t} = c_v \cdot \frac{\partial^2 u}{\partial z^2}$$

where

u = pore water pressure

t = time

z = distance from drainage face

c_v = coefficient of consolidation = $\frac{k(1+e_o)}{a_v \cdot \gamma_w}$

k = average coefficient of permeability

e_o = initial void ratio of the soil mass

γ_w = unit weight of water

a_v = coefficient of compressibility = $\frac{\Delta e}{\Delta p}$

Δp = effective pressure

Δe = change in void ratio

From the given equation for c_v,

$$k = \frac{a_v \cdot c_v \cdot \gamma_w}{1+e_o}$$

which is the basic relationship used for the indirect experimental determination of k. The values of a_v and c_v are obtained from the time

deformation data of the consolidation test, and e_o is an easily determinable physical property of the sample. The following expressions give a_v and c_v:

$$a_v = \frac{\Delta e}{\Delta p} = \frac{e_{100}-e_o}{p_1-p_o}$$

where

e_o = void ratio at zero percent consolidation

e_{100} = void ratio at 100 percent consolidation

p_o = pressure before application of the new load

p_1 = total pressure after application of the new load

$$c_v = \frac{T.H^2}{t}$$

where

H = half thickness of a soil sample drained on two faces

T = time factor for a given percentage of consolidation

t = time required for the same percent of consolidation

Values of c_v and a_v can be calculated in the laboratory by measuring t, H, Δe, and Δp, and the magnitude of k can be calculated from those results. The testing procedure and method of calculation will be discussed in the next chapter.

In situ Measurement of Soil Permeability

Several methods for in situ measurement of the soil permeability have been used. Most of these methods try to measure the rate of flow of water into or out of some kind of cavity, well, or borehold. Darcy's

law is then applied to the particular conditions of the test to calculate the coefficient of permeability. Among the methods described by different authors, the following are most common.

Single Cavity Method (39). In this method, a cavity is bored into the soil to a depth below the water table. The water is pumped from the cavity and then the pumping is stopped and water is allowed to flow into the cavity for a given time period, Δt. If the height of the water rise in the cavity is represented by Δh, the radius of the cavity by r, and the depth from the natural water table to the bottom of the cavity by d,

$$k = 0.617 \frac{r}{s.d} \cdot \frac{\Delta h}{\Delta t}$$

where s is a geometrical function of h, r, and d. This equation gives the best results when there is an impervious layer at the bottom of the hole.

Piezometer Method (39). Piezometers are small-diameter pipes with porous tips of different shapes which are encased in sand and used for measurement of pore pressure at a given depth in a soil layer.

The piezometer method gives the permeability of a soil at a given point (bottom of the piezometer) in the soil mass. A piezometer pipe is installed at the desired depth, water is pumped from the piezometer system and the rate of rise of the water level in the piezometer is measured. The coefficient of permeability is found from the following expression:

$$k = \frac{\pi r^2 \left[\ln(d-h_1)/(d-h_2)\right]}{A(t_2-t_1)}$$

where h_1 and h_2 are the depth of water in the piezometer below the water table at times t_1 and t_2, r = inside radius of the piezometer pipe, d = depth of piezometer tip below the water table, ln = natural logarithm, and A = a function of the geometry of the flow system having the physical dimensions of length (39).

Tube Method (large diameter piezometer)(77). The procedure is the same as the piezometer method, but the following expression is used:

$$k = \frac{\pi r^2 \ln \frac{h_1}{h_2}}{E(\Delta t)} \times 60$$

where

k = coefficient of permeability in/hr

h_1 and h_2 = depth of water surface in the tube below original water table at times t_1 and t_2

r = radius of the tube

E = shape factor given by Spangler (77)

Single Cavity Method, when it is not extended to impervious layer [Hooghout's Method (49)]

$$k = \frac{2.3\ r.s}{(2d+r)\Delta t} \cdot \log_{10} \frac{h_1}{h_2}$$

where

$s = \frac{r.d}{0.19}$ = shape factor, and the rest of the terms are given previously

Two-Auger Hold Method [Child's Method (39)(40)]

Water is pumped at a steady rate out of one hole and carried by a hose into the other hole, creating a small hydraulic head difference between the levels of water in the holes. If Q is the pumping rate, Δh the hydraulic head difference, d the length of each well, r the radius of each well, and L the distance between their axes, k is given by:

$$k = \frac{Q}{\pi . d . \Delta H} \cdot \text{Cosh}^{-1} \frac{L}{2r}$$

This equation is used for the condition where holes are extended to an impervious layer. The procedure is applied to the steady-state condition.

Pumping Test Method (39)(46)(87)

Thiem Formula. Using the standard well methods and drawdown distance curve of a pumped well, the coefficient of the permeability of the aquifer is obtained from the following expression:

$$k = \frac{Q \log_e \frac{r_2}{r_1}}{2\pi d(s_1 - s_2)} = \frac{528\ Q \log_{10} \frac{r_2}{r_1}}{d(s_1 - s_2)}$$

where

k = coefficient of permeability in gal/day/ft^2

r_1, r_2 = distances from pumped well to the first and second observation wells in ft

s_1, s_2 = drawdown in first and second observation wells

Q = rate of discharge of the pumped well in gal/min

d = thickness of water-bearing zone in ft.

Single Well Method [Maag's equation (87)]

If the time drawdown curve of a single well is available, the coefficient of permeability can be obtained from the following formula:

$$k = \frac{r}{4\Delta t} \cdot \log_e \frac{h.t_1}{t_2}$$

where

h = magnitude of the drawdown from time t_1 to t_2

r = radius of the well

$\Delta t = t_2 - t_1$

Spherical Hydraulic Piezometer Method

This method was first suggested by Gibson (23) for a special case, but was later extended to cover additional conditions (24)(25)(88). For the case of an infinite mass of soil surrounding a spherical piezometer of radius, a, a constant pressure head, Δh, is maintained (above or below the ambient water pressure in the soil) at the piezometer tip, and the rate of flow of water into or out of the system is measured. The coefficient of permeability can be obtained from the following equations:

$$Q_t = 4\pi.a.k.\frac{\Delta u}{\gamma_w}\left[1 + \frac{1}{\sqrt{\pi T}}\right]$$

$$Q_{t=\infty} = 4\pi a.\frac{k}{\gamma_w}.\Delta u$$

$$k = \frac{Q_{t=\infty} \cdot \gamma_w}{4\pi a \cdot \Delta u}$$

where

T = time factor = $\frac{c \cdot t}{a^2}$

c = coefficient of consolidation or swelling

c and k can be obtained by plotting $Q_{(t)}$ vs. $t^{-\frac{1}{2}}$ see (23) and (88) . There is a straight line plot of Q_t vs. $t^{-\frac{1}{2}}$ and k is found from the intercept on the Q axis of the straight line. The coefficient of consolidation or swelling (c) is then estimated from the slope of this line.

Many other aspects of the soil permeability addressed in numerous papers have no application to this study, and are not discussed here. Examples are "permeability of layered soils (16)(37)," "permeability of irregular specimens (34)," "permeability and hydraulic fracturing (3)," and tens of papers pertaining to the characteristics of special devices used for permeability measurement. These and many other subjects can be found in the literature given in the references (95).

CHAPTER III

MATERIALS, SAMPLE PREPARATION, EQUIPMENT, AND TESTING PROCEDURE

This chapter provides information on the physical and engineering properties of the materials used throughout the study as well as the method of preparation of the samples for both disturbed (remolded) and undisturbed conditions. Details of the equipment used and the testing procedures are also presented. Information concerning the properties of materials is limited to that which might have some effect on the results of the experiments. In describing testing procedures, details are given only for the permeability and consolidation tests, which are the principal evaluative experiments of this investigation. Details of the tests used for determining the physical properties of the soils can be found in any soil mechanics textbook.

Materials

The soils used in this study are divided into two groups--disturbed (remolded), and undisturbed.

Disturbed soils are those which have been dug from the site, dried, pulverized, and passed through some given sieve size (normally No. 40). The materials are then stored for later use in preparing compacted or remolded samples.

Undisturbed samples were taken from the site with a thin-wall

sampling tube and preserved at their natural moisture contents. Every attempt is made to avoid unnecessary disturbance or distortion throughout the sample preparation and testing process.

Disturbed Materials

The soils that were used for disturbed or remolded samples were selected from available clay soils in the Soil Mechanics Laboratory at Oklahoma State University. These soils were used both in their natural state and as components of fabricated mixtures containing sand or silt. Seven natural soils and four mixtures were used in this part of the study. The purpose of mixing the natural clay soils with silt or sand was to change the gradation and plasticity of the soil, giving a wider range of those properties than would otherwise have been present.

The natural soils all have their origin in the state of Oklahoma. The soils are described generally in the following sections, and the physical and engineering properties of the natural soils and mixtures are given in Table I.

Union City Clay (Red Clay No. 1). This soil was taken from a clay pit two miles north of Union City, Oklahoma, in Canadian County. The material has a distinctive red color because of its high iron content, and is commonly called "Permian Red Clay." The clay fraction of this soil is entirely illite (68) and is moderately-to-low plastic.

Union City Clay (Red Clay No. 2). This material was taken from the same place as Union City Clay No. 1, at a location about 500 ft away from that of the first material. It is more clayey, but has almost the same plasticity as Red Clay No. 1.

TABLE I

PHYSICAL AND ENGINEERING PROPERTIES (Disturbed Material)

Soil	Liquid Limit (%)	Plastic Limit (%)	Plasticity Index (%)	Activity Number	Grain Size Analysis			Optimum Water Content (%)	Specific Gravity
					Sand +0.06mm	Silt -0.06mm	Clay -0.002mm		
Union City Red Clay No. 1	38.6	26.0	12.6	2.1	4.0	90.0	6.0	23.0	2/78
Union City Red Clay No. 2	33.3	21.4	11.9	0.31	3.0	59.0	38.0	22.0	2.74
Camargo Bentonite White Clay	132.7	73.9	58.9	1.96	5.0	65.0	30.0	65.0	2.82
Burleson Black Clay	64.8	24.0	40.8	0.92	6.0	50.0	44.0	30.0	2.70
Summit Green Clay	55.4	22.7	32.7	0.90	8.0	56.0	36.0	23.5	2.74
Stillwater Brown Clay	44.5	21.1	23.4	9.35	6.0	91.5	2.5	23.3	2.74
Stillwater Dark Brown Clay	40.5	19.1	21.4	10.7	8.0	90.0	2.0	17.4	2.72
60% Green Clay + 40% Loess	43.0	21.6	21.4	0.86	9.0	66.0	25.0	19.5	2.72
60% Green Clay + 40% Uniform Sand	33	16.3	16.7	0.75	44.0	34.0	22.0	16.5	2.65
60% Green Clay + 40% Well-graded sand	-	-	-	-	-	-	-	14.0	2.67
66% Red Clay + 33% Well-graded sand	-	-	-	-	-	-	-	16.0	2.66

Camargo Bentonite (White Clay). This material was obtained from Dewey County in northwestern Oklahoma. It is a calcium bentonite (68) having a white color and a very high water absorption capacity (LL>130). It is highly plastic and soapy to the touch.

Burleson Clay (Black Clay). This soil was taken from Bryan County, in southwestern Oklahoma. The soil is moderate-to-high in plasticity, soapy to the touch, and black in color. It has more than 40 percent clay size particles (-0.002 mm).

Summit Clay (Green Clay). This soil was obtained from Osage County, in northern Oklahoma, from the side of a small gully. It is a residual soil from the Summit formation, with moderate plasticity, and a distinct green color. This soil and all of those listed above are described in detail by Sabry (68).

Stillwater Brown Clay. This soil has its origin in the vast Permian beds of the state of Oklahoma. The sample was taken from a pit located in the southeast quadrant of the Oklahoma State University campus. The sample has an uncharacteristically low content of clay-size particles, being composed predominantly of silt-size grains. However, its properties are such that it must be generally classified as a clay soil. The material has a brownish color and a moderate-to-low plasticity.

Stillwater Dark Brown Clay. This Permian clay soil was taken from a pit located in the central part of the Oklahoma State University campus. Its grain-size distribution characteristics are similar to those of the brown clay described above. It has moderate-to-low plasticity,

and a dark brown color.

For simplicity, future reference to these soils will be in terms of the color codings designated above. The following mixtures of natural clay soils and natural granular soils were also utilized in the study.

Sixty Percent Summit Green Clay + 40 Percent Vicksburg Loess. This material was made by mixing 60 percent by weight of Summit Clay and 40 percent by weight of Vicksburg Loess available in the Soil Mechanics Laboratory at Oklahoma State University. Its plasticity was about 10 percent lower than that of the natural Green Clay.

Sixty Percent Summit Green Clay + 40 Percent Uniform Sand. The material was made by mixing 60 percent Summit Clay with 40 percent by weight of uniform sand passing a U. S. No. 20 sieve and retained on a U. S. No. 80 sieve. The sand, an alluvial deposit of the Cimarron River, is a well-graded material in its natural state.

Sixty Percent Summit Green Clay + 40 Percent Well-graded Sand. This mixture is similar to that above, except that the sand was used with its natural gradation without any sieving separation.

Sixty-six Percent Union City Clay (No. 1) + 34 Percent Well-graded Sand. This mixture was a combination consisting of 60 percent Union City Red Clay and 34 Percent Well-graded sand from the Cimarron River.

A summary of the physical and engineering properties of the disturbed materials appears in Table I. The listed properties include those which may be indicative of, or related to, permeability and which

may be useful for analysis of the results. Listed properties include liquid limit, plastic limit, plasticity index, activity number, specific gravity, grain-size distribution, and optimum moisture content (which was used for compacting the samples). It may be seen that a rather wide range of properties is embraced by the materials selected for study. Note that because of the presence of large sand particles in the last two materials, the Atterberg limits for them were not determined.

Undisturbed Materials

The materials which were tested in their undisturbed state were, with two exceptions, Shelby tube samples that had been preserved in the moist room of the Soil Mechanics Laboratory at the Oklahoma State University. These undisturbed samples were 2.875 inches in diameter. Two other samples were taken from test pits--one on the Oklahoma State University campus, and the other in the northeast part of Stillwater, Oklahoma, using a thin-wall sampling tube with a cutting edge. Separate samples were taken for consolidation and direct permeability tests. The materials used are listed below:

- silty clay from Ski-Island, northwest Oklahoma City, 25-26 ft depth
- silty clay from Ski-Island, northwest Oklahoma City, 9-10 ft depth
- silty clay from Hefner Dam site, northwest Oklahoma City, 44-45 ft depth
- sandy clay from Stillwater Hospital site, Stillwater, Oklahoma, 6-7 ft depth
- sandy clay containing some organic matter, from Muskogee, Oklahoma, 1.5-2.5 ft depth
- stiff clay from Black Bear Creek, Red Rock, Oklahoma, 15-16 ft depth

- silty clay containing some organic matter, from the Oklahoma State University campus, 2-3 ft depth (No. 1)

- silty clay from northeast of Stillwater, Oklahoma, 6-7 ft depth (No. 2)

The natural water contents, specific gravity, and initial void ratio were determined for these soils and are listed in Table II

TABLE II

PHYSICAL PROPERTIES OF UNDISTURBED SAMPLES

Soil	Natural Water Content	Specific Gravity	Void Ratio	Porosity	Sample Description
Ski-Island silty clay (25'-26')	24	2.69	0.734	0.423	silty clay, mod. plas.
Ski-Island silty clay (9'-10')	22.3	2.68	0.678	0.416	silty clay, mod. plas.
Hefner Dam silty clay (44'-45')	22	2.68	0.642	0.391	silty clay, mod.to high plas.
Stillwater Hospital sandy clay (6'-7')	18.2	2.68	0.607	0.378	sandy silty clay mod.to low plas.
Muskogee, Oklahoma sandy clay (1.5'-2.5')	26	2.67	0.895	0.472	sandy silty clay cont. organic matter, low plas.
Red Rock, Oklahoma stiff clay (15'-16')	24	2.75	0.707	0.414	stiff silty clay high plasticity
Stillwater, Oklahoma No. 1 silty clay (2'-3')	22	2.69	0.712	0.416	silty clay cont. organic mat.,mod. to low plasticity
Stillwater, Oklahoma No. 2 silty clay (6'-7')	20	2.70	0.680	0.405	silty clay, mod. plasticity

Sample Preparation

In an investigation such as this in which a soil property is to

be determined by two fundamentally different procedures for direct comparison of results, the greatest care must be taken in preparing test specimens. It is essential that paired specimens be closely similar, if not identical, if comparisons are to have the desired degree of validity. Many specimens were discarded during or after preparation because of discernible differences in macroscopic structural features, excessive disturbance, or other features that would clearly affect permeability. In some cases, tests already begun were abandoned because of gnawing doubts concerning some feature of the specimen or test conditions. The greatest difficulty in obtaining properly matched pairs was, of course, associated with the undisturbed soil samples.

Remolded Specimens

Remolded specimens were made by compacting the soil under desired conditions of moisture content and compactive effort. Throughout this study, the Standard Proctor method of compaction or its equivalent in terms of unit compaction energy, was employed except when it was desired to produce specimens having lower or higher void ratios than those corresponding to the standard procedure. For this reason, the moisture content-density curves for all of the samples were initially obtained using the Standard Proctor method. Test specimens were then compacted at optimum moisture content and under standard compactive effort (12,400 lb/ft^3). For consolidation tests, the soils were compacted in the standard 4"x4.5" cylindrical mold, using an electrically operated impact compactor (Model F-590 from Test Lab. Corp.). The compacted samples were pushed out of the mold for a distance of about two inches, using a hydraulic jack, then cut off very carefully. Out of each disk of sample

prepared in this way, a consolidation test specimen was obtained. A sharpened 2.5" diameter sampling cylinder was pressed into the disk manually. The dimensions of the sampler and the rings used for consolidation tests are:

- inside diameter of sampler cutting edge, 2.50"
- inside diameter of the sampler, 2.55"
- outside diameter of the sampler, 2.76"
- length of the sampler, 3.80"
- inside diameter of consolidation ring, 2.50"
- outside diameter of consolidation ring, 2.61"
- height of consolidation ring, 1.00"

It can be seen that the inside diameter of the cutting edge is somewhat smaller than the inside diameter of the sampler to minimize the frictional drag and reduce sample distortion and disturbance. Care was taken to maintain a vertical alignment of the sampler during the push. If any deviation in this respect was observed, the specimen was discarded. Acceptable specimens were carefully pressed into the consolidation rings for a distance sufficient to allow it to protrude about 0.5" from either end of the ring. Specimens were then trimmed flush with the ring, weighed, and prepared for testing.

For the direct permeability tests, sample preparation was less complicated. The samples were compacted directly in a Harvard miniature mold, which was used as a component of the permeameter. It was necessary only to trim excess soil from the top of the mold before installing it in the permeameter.

Undisturbed Specimens

Consolidation test specimens were fairly easy to prepare because consolidation rings having the same inside diameter (2.875") as the Shelby tubes used for sampling were employed. The sample was carefully pressed through the consolidation ring, which was one inch in height, and trimmed flush on the top and bottom faces. Although the diameter of the two samples obtained from the test pits was smaller (2.5"), the procedure was otherwise the same as that for the Shelby tube samples.

Preparation of the permeability test specimens was more difficult, because samples about three inches long were needed (length of the permeameter mold = 2.82"). A cylindrical stainless steel sampler with a cutting edge and the following dimensions was used to obtain specimens of the required size:

- inside diameter of cutting edge, 1.35"
- inside diameter of the sampler, 1.40"
- outside diameter of the sampler, 1.51"
- length of the sampler, 3.96"
- inside diameter of the permeameter mold, 1.35"
- outside diameter of the permeameter mold, 1.52"
- length of the permeameter mold, 2.82"

In the manner previously described, the sampler was pressed into the soil. The sample obtained was then inserted into the permeameter mold. Because of the greater length of the samples, and consequently greater possibility of disturbance, extra care was needed to prevent or minimize distortion. Although some disturbance was unavoidable, that factor is believed to be insignificant compared to the sources of error

or uncertainty involved in consolidation tests. Examples of the latter include simplifying assumptions used to derive the theoretical equation for consolidation and the use of arbitrary methods of fitting for time-deformation curves.

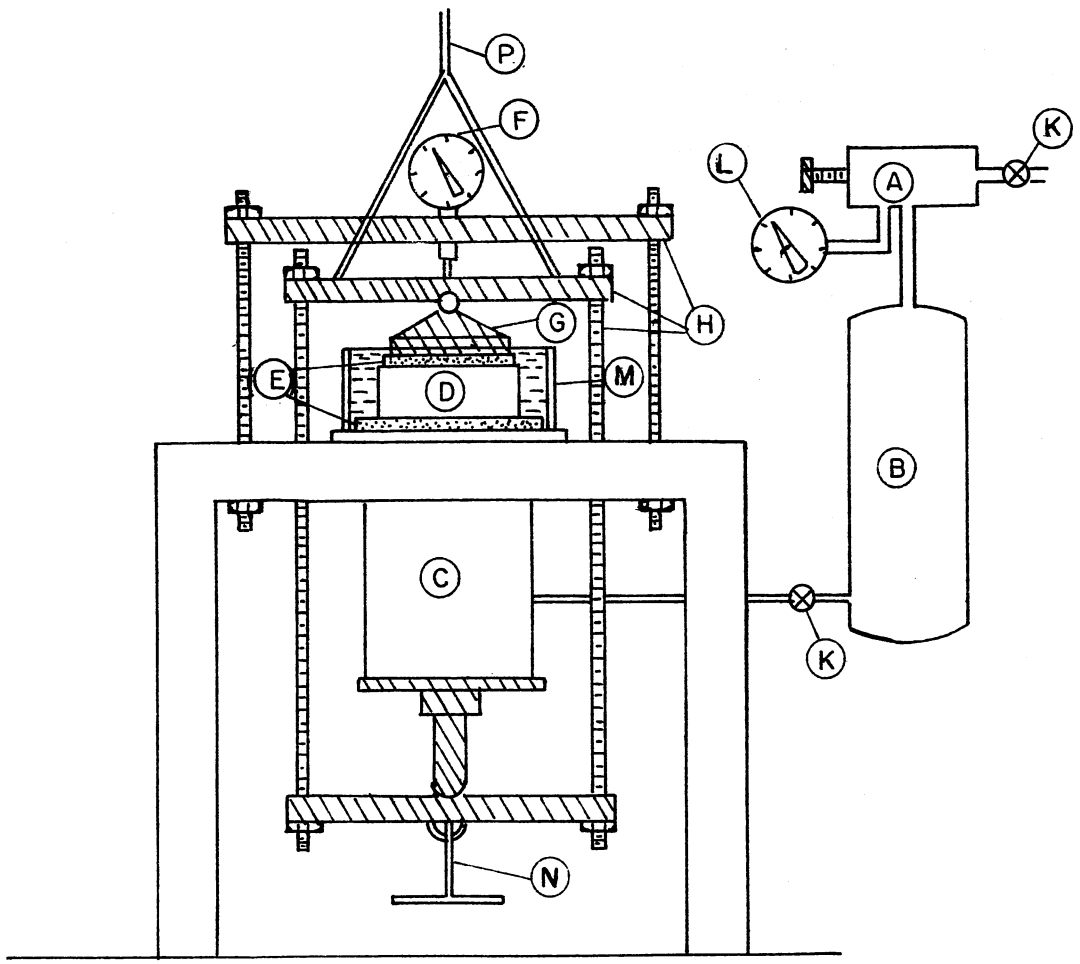
Equipment

Consolidation Machine

The consolidation setup used in this study was designed by Prof. T. A. Haliburton and built in the Soil Mechanics Laboratory at the Oklahoma State University. It is composed of eight units, any of which may be used separately. The system uses compressed air as a source of required pressure, and the desired load is applied through a hydraulic cylinder for each individual unit. Figure 4 is a schematic diagram of one unit. The air pressure is controlled through a regulator (A), and constant pressure is applied to the oil in the tank (B). This pressure is transferred through a hydraulic ram to the sample (D), and the deformation is measured by a dial gauge (F). A maximum hydraulic pressure of 120 psi can be supplied to the ram. The hydraulic pressure required to produce a loading pressure of one ton/sq ft on samples of various diameters is shown in Table III.

The load control mechanisms of this machine are remarkably reliable in maintaining a constant pressure on the sample, and in the incrementation of loads during the course of the test.

It has been determined that after adjusting the system pressure to that required for incrementation with a valve (K) closed, the new load is fully applied to the soil sample within three seconds after the



(A) Pressure Regulator
(B) Oil Tank
(C) Hydraulic Ram
(D) Ring Containing Sample
(E) Porous Stone
(F) Dial Gauge
(G) Loading Head
(H) Yoke Assembly
(K) Valve
(L) Pressure Gauge
(M) Consolidation Bowl
(N) Dead Weight Hanger
(P) To Counter Weight

Figure 4. Consolidation Test Setup

valve is opened, and no further adjustment is needed. The unloading process is done similarly by reducing the system pressure to the desired value and releasing the excess ram pressure by opening valve K. All piping in the hydraulic system consists of transparent hard plastic tubes to facilitate de-airing.

TABLE III

HYDRAULIC PRESSURE REQUIRED TO PRODUCE 1 TSF ON SAMPLE

Diameter of Sample (in.)	Hydraulic Pressure Required for 1 TSF on Sample (psi)	Maximum Sample Loading Available (TSF)
2.125	2.544	48
2.500	3.472	35
2.875	4.591	26

Each unit of the system has two separate sets of pressure regulators, one for the range of 0 to 30 psi, and the other for the range of 30 to 120 psi.

Since the regulators are unreliable for pressures less than two or three psi, an independent, dead weight loading capability is provided by a hanger suspended from the lower cross-beam of the loading yoke. This device is generally used for sample loads of less than one ton/sq ft.

Permeability Setup

One of the difficulties encountered in this study was the development of a permeability setup capable of measuring reliably the low permeability of clay soils. Since permeability coefficients of the order of 10^{-9} cm/sec must be determined, hydraulic gradients of about 500 are needed to produce an accurately measurable flow through the small cross-sectional area of the samples (8.76 cm^2). For the 2.82" sample length, a pressure differential of 50 psi was judged to be sufficient for the experiments. Such a high pressure is feasible only in a constant head system using gas pressure. For this reason, the 120 psi compressed air system of the Soil Mechanics Laboratory was used as the pressure source for the experiments. The setup shown in Figure 5 was designed for these experiments. It is made up of eight permeameter units which can be used separately or simultaneously.

Air pressure, controlled through a regulator (A) and monitored by a pressure gauge (B) is applied to the water in tank (C). Under pressure, the water flows to a glass cylinder (D), where entrapped air is removed through valve (L). From this cylinder, water is distributed through a manifold tube (E) to the individual permeameters (G). Pressure at the top of the permeameters is measured by a gauge (F). The outflow is collected in graduate cylinders (H) of 10-25 cc volume, with 0.1 cc calibration divisions. To prevent evaporation, the cylinders were covered with plastic sheets. Allowable pressure in the tank is 75 psi, which permitted a maximum pressure equivalent to about 45 meters head of water to be used in the experiments.

The permeameters are Harvard miniature size (Model K620), from the

Ⓐ Regulator
Ⓑ Tank Pressure Guage
Ⓒ Water Tank
Ⓓ Glass Cylinder
Ⓔ Manifold Tube
Ⓕ End Pressure Gauge
Ⓖ Permeameter (See Fig. 6)
Ⓗ Graduated Cylinder
Ⓚ Valve
Ⓛ Deairing Valve

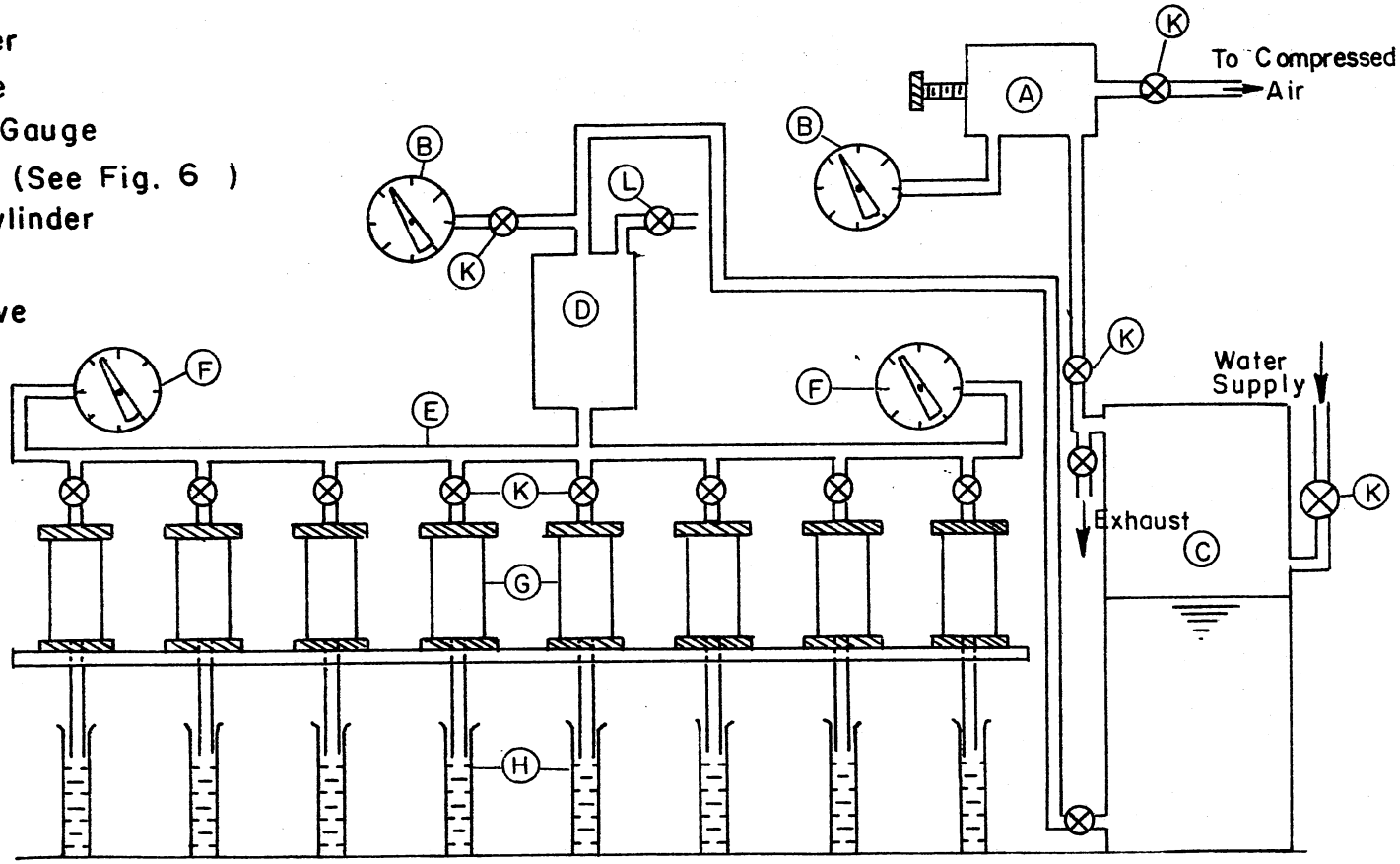


Figure 5. Direct Permeability Test Setup

Soil Test Company. They are 1.35" in diameter and 2.823 in. in length, made of stainless steel. A detailed drawing of an individual permeameter is shown in Figure 6. The soil sample is compacted in the mold (D) and placed between top and bottom plates (E) which are clamped by tension rods (F). A porous stone (H) is located at the bottom of the sample, and a very fine mesh screen (No. 325) is placed between the soil and porous stone. Water under pressure enters the permeameter through pipe (A) and air bubbles are removed from the system through pipe (C) by opening valve (B). The outflow is drained through pipe (G).

Testing Procedure

Consolidation Test

After the specimen is prepared, it is placed between two porous stones (E) (Figure 4), wetted in advance to prevent the absorption of moisture from the sample, and centered in the consolidation bowl (M). The loading head (G) is brought into contact with the top porous stone without applying an appreciable load. Dial gauge (F) is installed and adjusted to a convenient initial reading (normally zero). With valve K closed, the desired hydraulic pressure is established by opening valve (L) and adjusting the regulator.

In early tests, the initial load applied to consolidation test specimens was such as was calculated to produce an initial hydraulic gradient equal to the hydraulic gradient used in the direct permeability tests. After it was determined that the gradient had no effect on the measured permeability, this procedure was not adhered to.

To eliminate the effect of temperature, consolidation and

permeability tests of a given soil sample were conducted at the same time and place.

To begin the test, the valve (K) is opened three seconds before the desired starting time. Readings are taken from the dial gauge (F) at 5 seconds; 30 seconds; 1 minute; 2 minutes; 4 minutes; 8 minutes; 15 minutes; 30 minutes; 1 hour; 2 hours; 4 hours; 8 hours; 16 hours, and 24 hours after the starting time. Primary consolidation is generally completed within 24 hours; if not, the load is maintained for an additional 24 hours. None of the samples in this study needed more than 48 hours for the primary consolidation process. After the final dial reading is taken for a given loading, the valve (K) is closed to maintain the load on the sample while the system pressure is increased to the next desired value. The second loading cycle is then performed in the same manner as the first.

To detect any measurable effect of the hydraulic gradient on permeability, load increments were kept constant for some samples and doubled for others.

The initial and final moisture contents and dry weight of the sample are determined by weighing the sample and ring immediately before and after the test, and after 24 hours of oven drying at $40^{o}C$.

Permeability Test

The mold containing the soil sample is clamped in place between top and bottom cover plates of the permeameter unit and connected to the system through pipe (A) (Figure 6). To start the test valve (B) is opened and water under pressure is allowed to flow into the permeameter by opening valve (K). After complete removal of air bubbles through

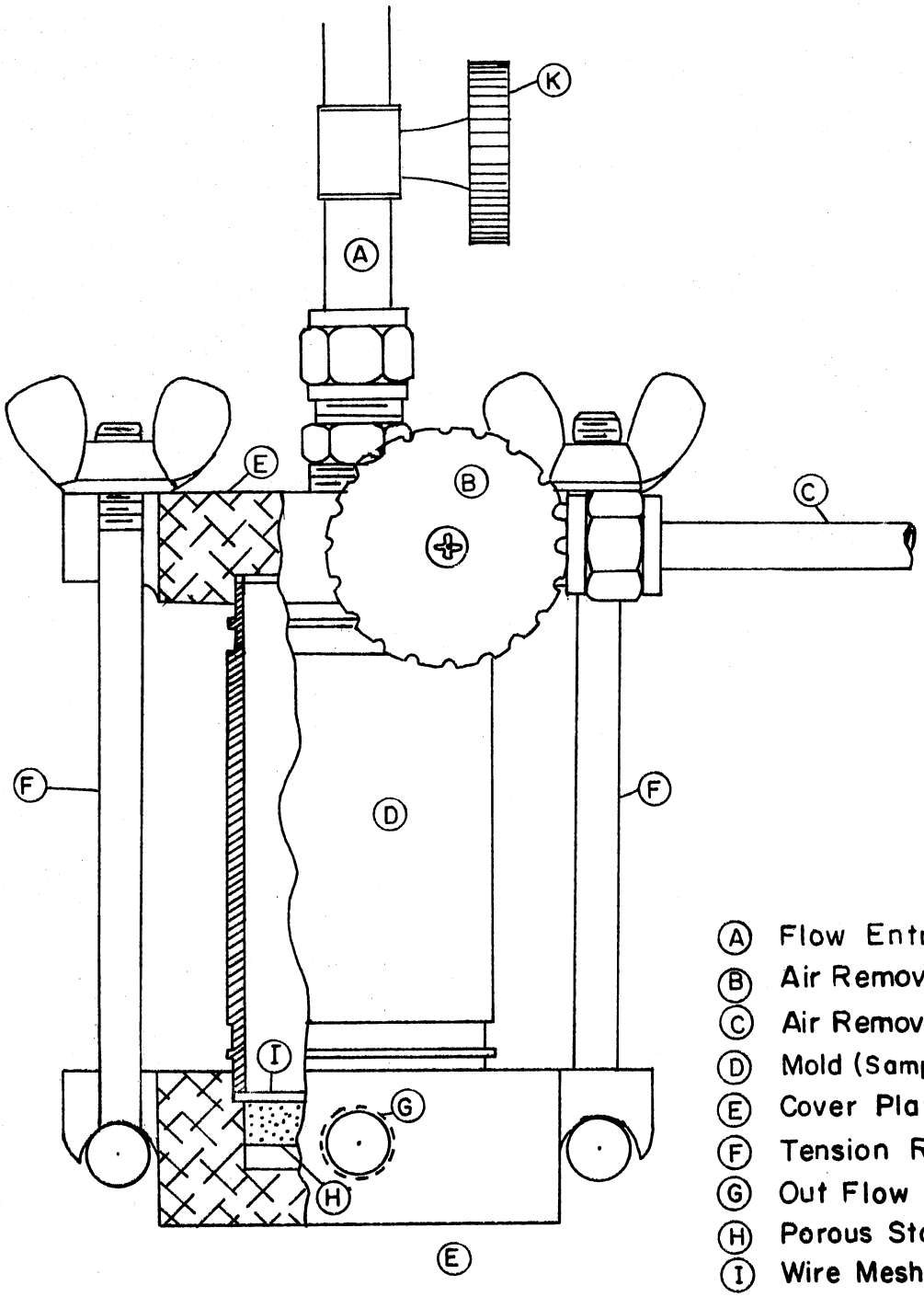


Figure 6. Detail Drawing of One Permeameter Unit

(A) Flow Entrance
(B) Air Removal Valve
(C) Air Removal Pipe
(D) Mold (Sample Holder)
(E) Cover Plate
(F) Tension Rod
(G) Out Flow Pipe
(H) Porous Stone
(I) Wire Mesh
(K) Valve

pipe (C), valve (B) is closed and the sample is allowed to become saturated. The saturation time for different samples varied between twelve hours and five days. After saturation, effluent is directed into graduated cylinders, and the amount collected was noted at 12-hour intervals. Occasionally, valve B should be opened during the experiment to expel any air bubbles from the system. Weight of the mold and sample is recorded before and after the test for determination of void ratio and water content.

Other Testing Procedures

Determination of specific gravity, Atterberg limits, and grain-size distribution followed standard procedures. The only special procedure to be mentioned is the use of a miniature mechanical impact compactor for compacting samples of Harvard miniature size for permeability tests. The device uses a freely-falling hammer lifted by a rack-and-pinion gear arrangement. To achieve a standard compactive effort, soil was compacted in four layers, applying nineteen blows per layer.

CHAPTER IV

RESULTS AND DISCUSSION

In the entire study, eleven different remolded samples and eight undisturbed samples were used. For each type of soil, three to five consolidation tests and four to six direct permeability tests were conducted, and the average results are presented in this chapter. All together, around ninety consolidation tests and one hundred permeability tests were made. The results for each method of determining permeability, i.e., indirect and direct, are presented separately in graphical form; the results of the two methods are compared in a separate section.

Results of Direct Permeability Tests

According to the procedures discussed in Chapter III, the permeability was determined directly for all undisturbed and remolded samples. Specimens of all soils were compacted at optimum moisture content--some under standard compactive effort and others under greater or less compactive effort--to produce variation in the void ratios of test specimens. The direct permeability tests were continued for at least nine days for all samples, and the average permeability of the samples during a given day was calculated from the following expression:

$$k = \frac{V}{t} \cdot \frac{L}{A.h}$$

where

k = coefficient of permeability in cm/sec

V = volume of collected water in cm^3

t = time in seconds for V cm^3 of water to be collected (for all experiments, t was taken as one day or 86,400 seconds)

L = length of permeameter in cm

A = cross-sectional area of the permeameter in cm^2

h = constant pressure applied to the water tank expressed in cm of head

The results of direct permeability tests are given in Figures 7 through 16 for remolded samples, and 17 through 24 for undisturbed samples.

To determine whether or not the size of the permeameter has a significant effect on the measured permeability, some of the samples were tested using both miniature molds (1.35" x 2.82") and standard (4" x 4") compaction molds. The results for both sizes of permeameters are shown on the same graphs in Figures 7, 8, 9, 11, and 13. It may be seen that the results are about the same for the two different mold sizes. Because of this, the rest of the tests were conducted in the miniature permeameter. Undisturbed samples were tested at natural moisture content.

In Chapter II, it was reported that a substantial decrease in permeability with time is frequently observed (5)(7)(57)(62). As shown in Figures 7 through 24, the clay soils used in this study, without exception, exhibit a decrease in permeability with time but, in most cases, the coefficient of permeability has become essentially constant by the

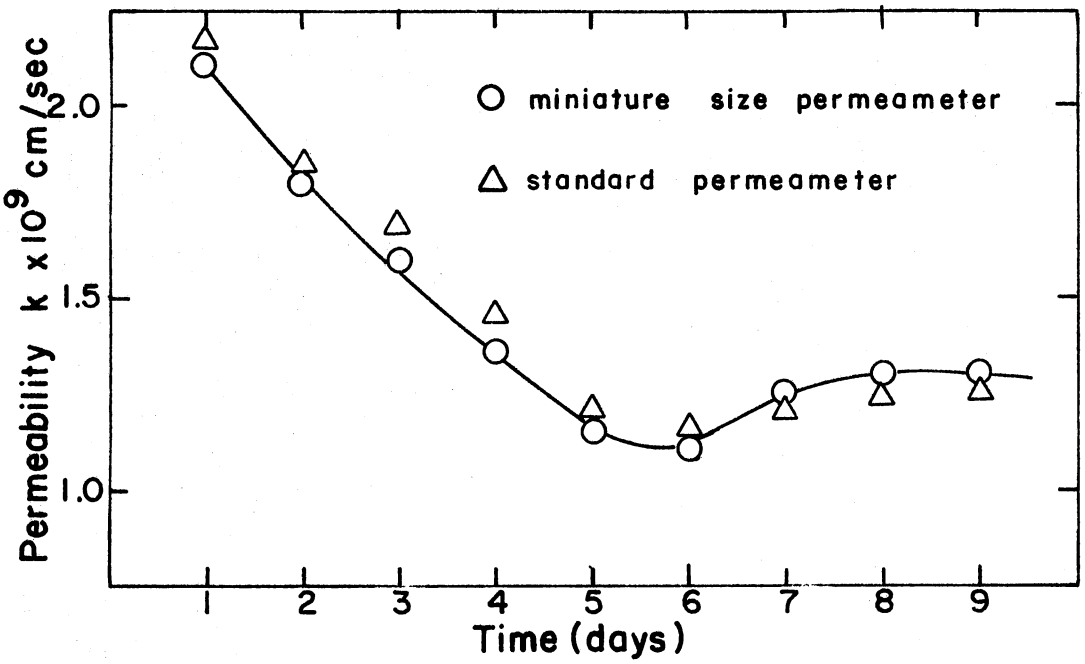


Figure 7. Permeability-Time Relationship for Compacted Black Clay

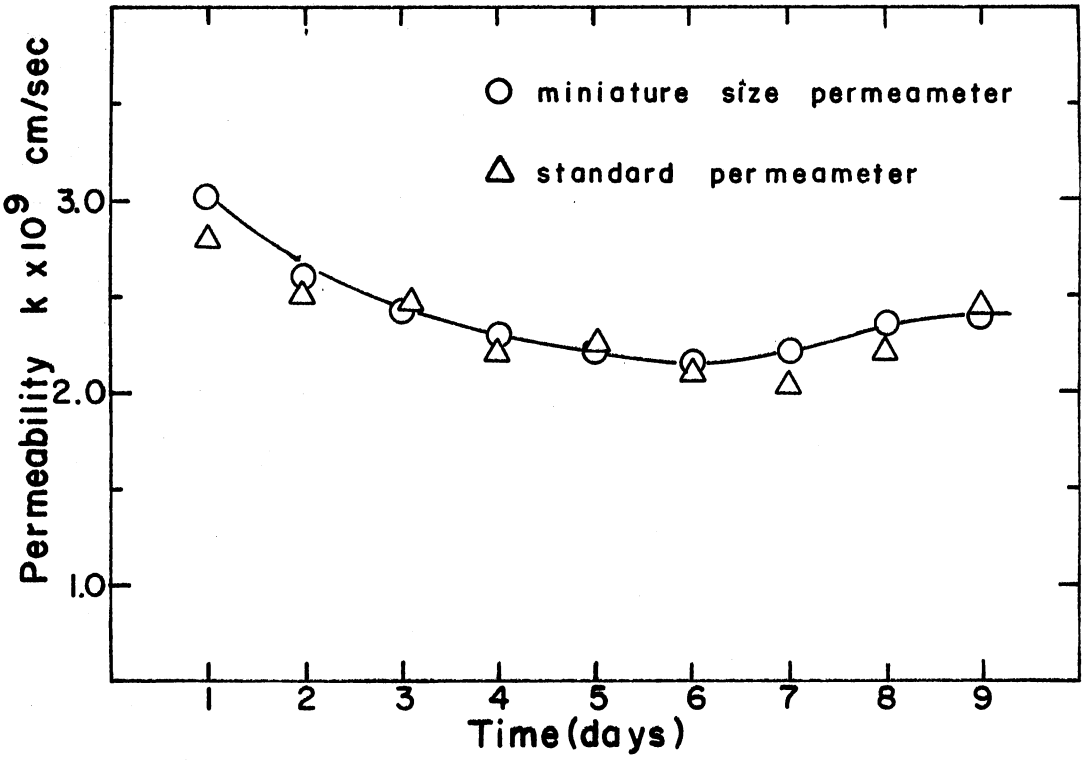


Figure 8. Permeability-Time Relationship for Compacted Green Clay

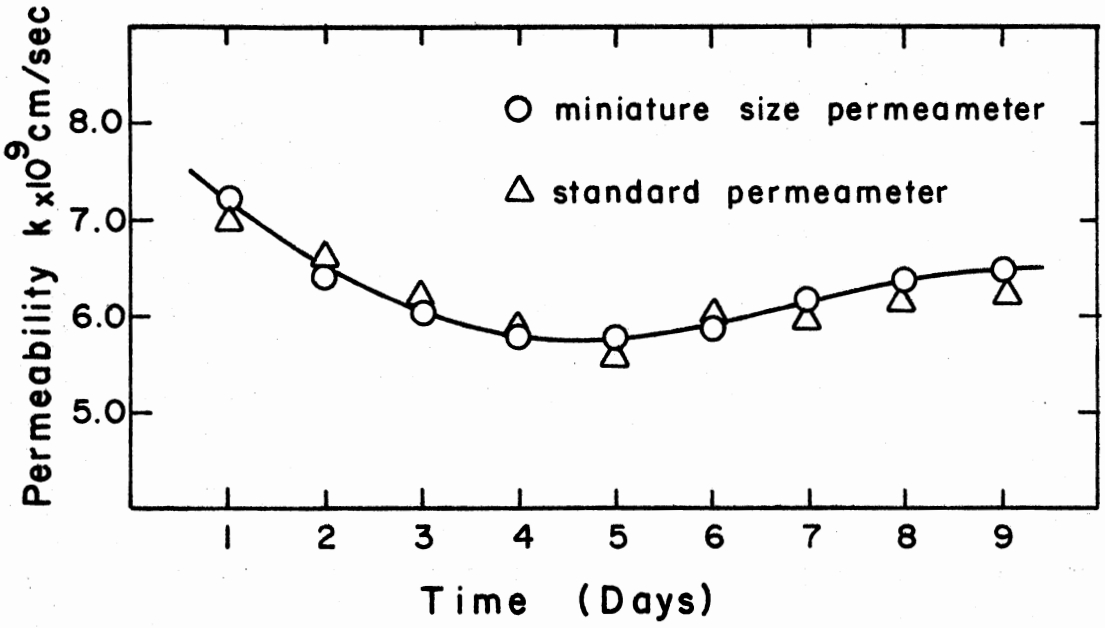


Figure 9. Permeability-Time Relationship for Compacted Red Clay No. 1

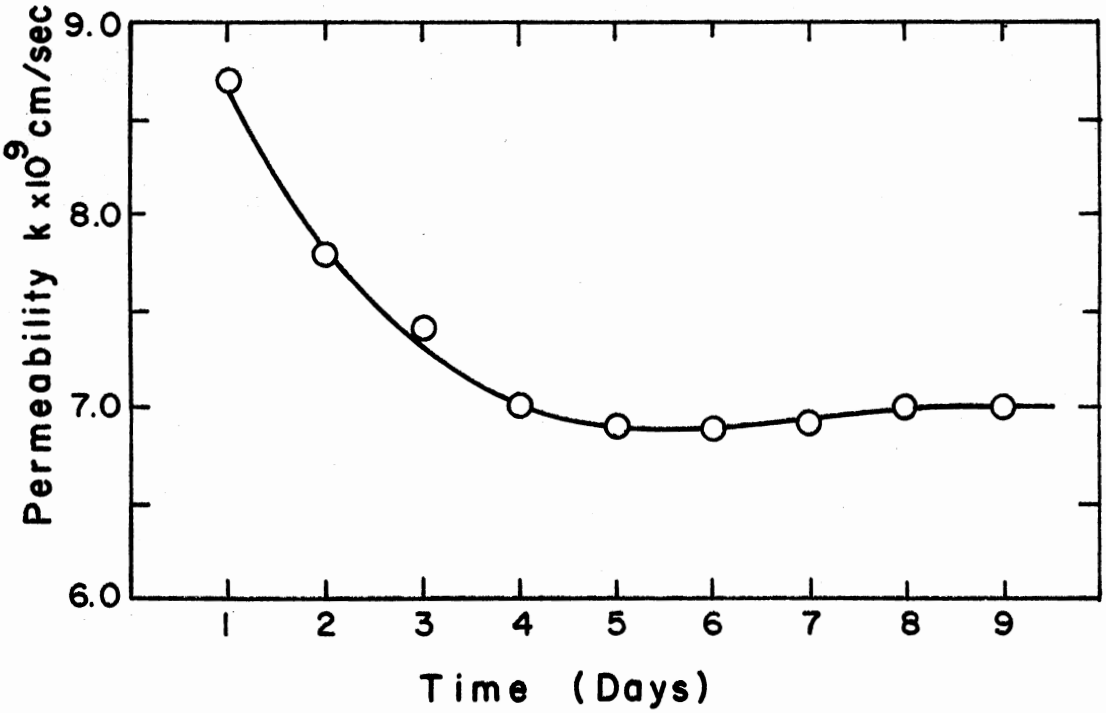


Figure 10. Permeability-Time Relationship for Compacted Red Clay No. 2

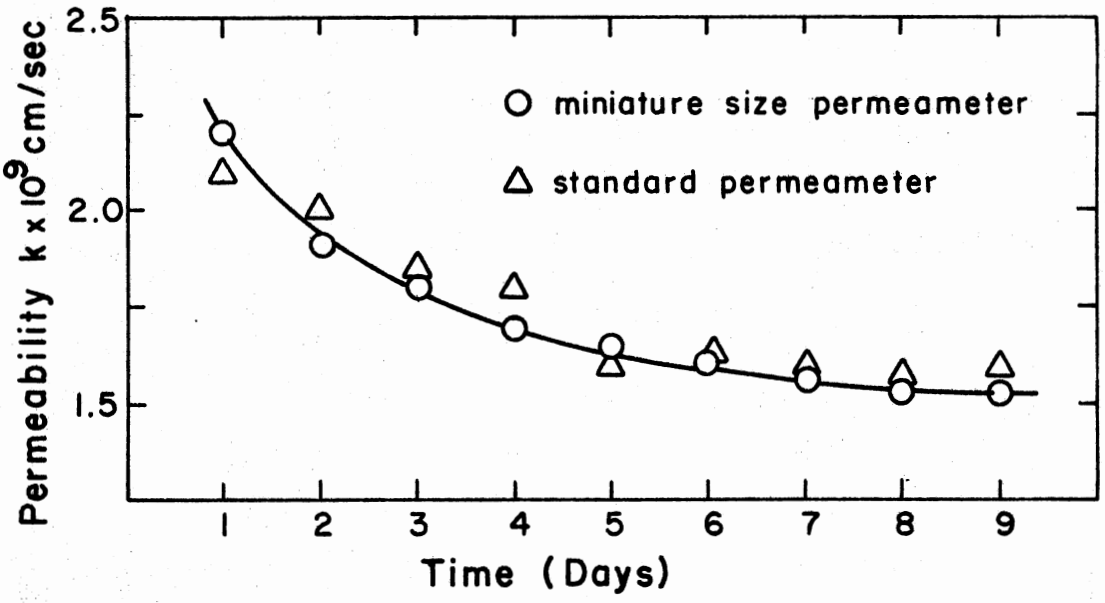


Figure 11. Permeability-Time Relationship for Compacted Brown Clay

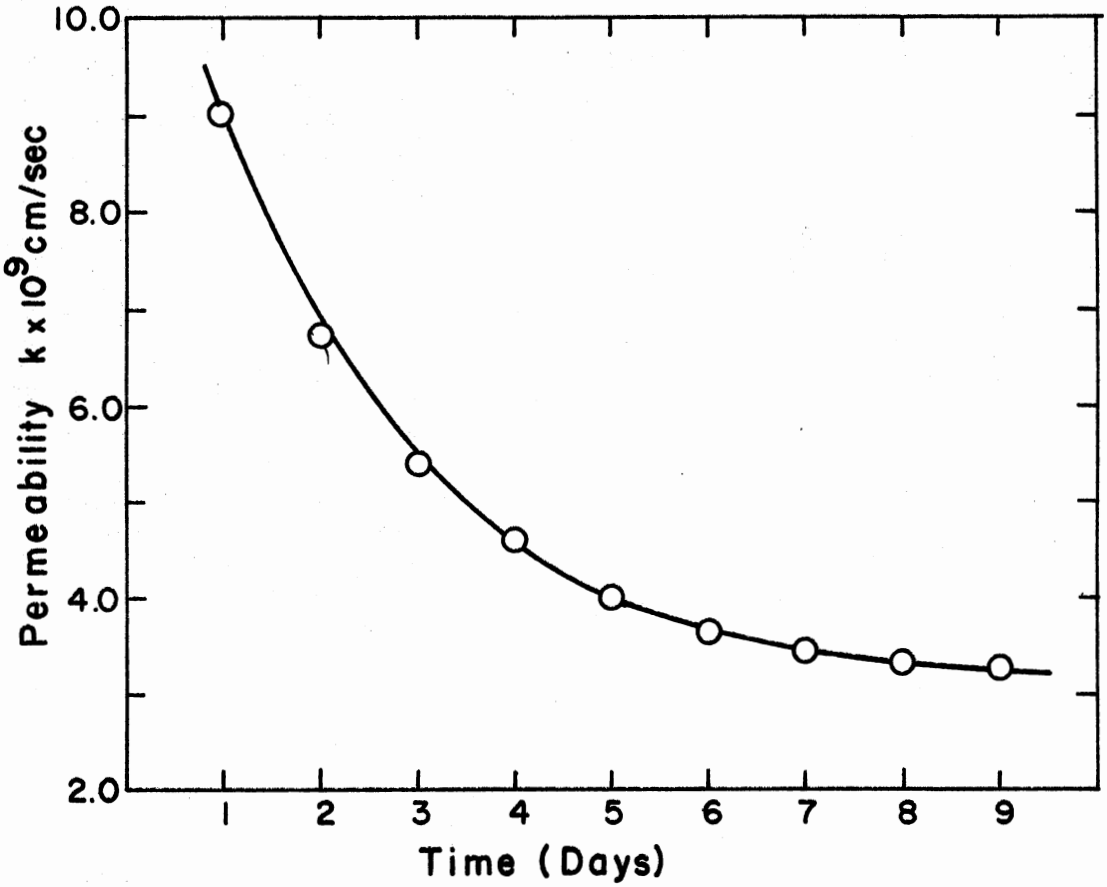


Figure 12. Permeability-Time Relationship for Compacted Dark Brown Clay

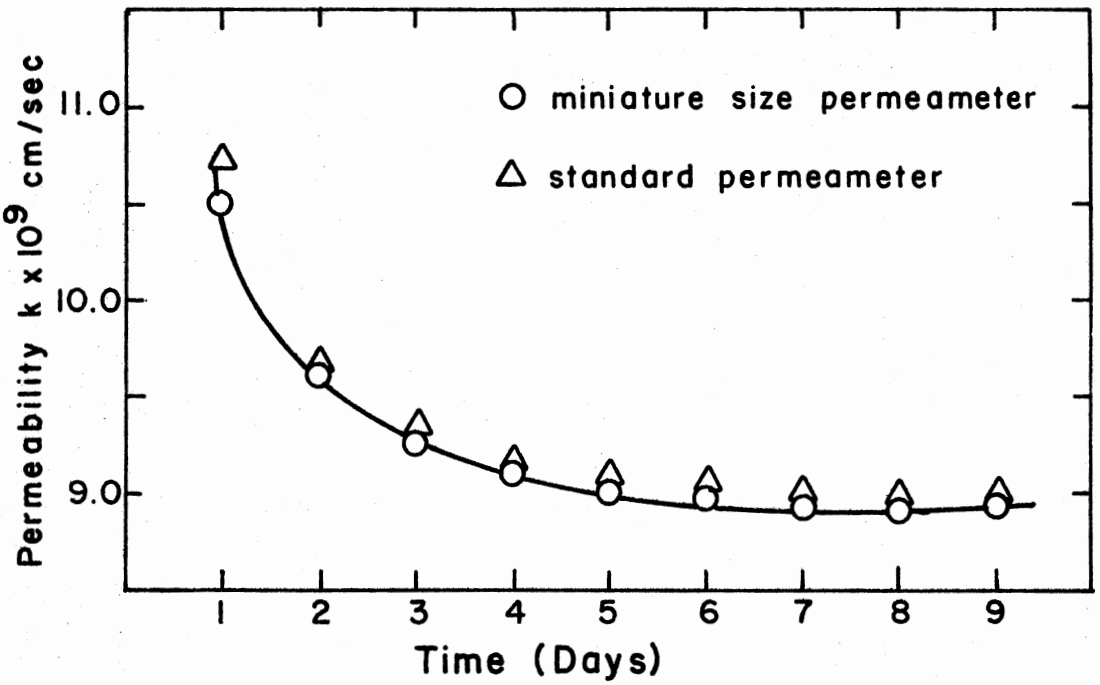


Figure 13. Permeability-Time Relationship for Compacted White Clay

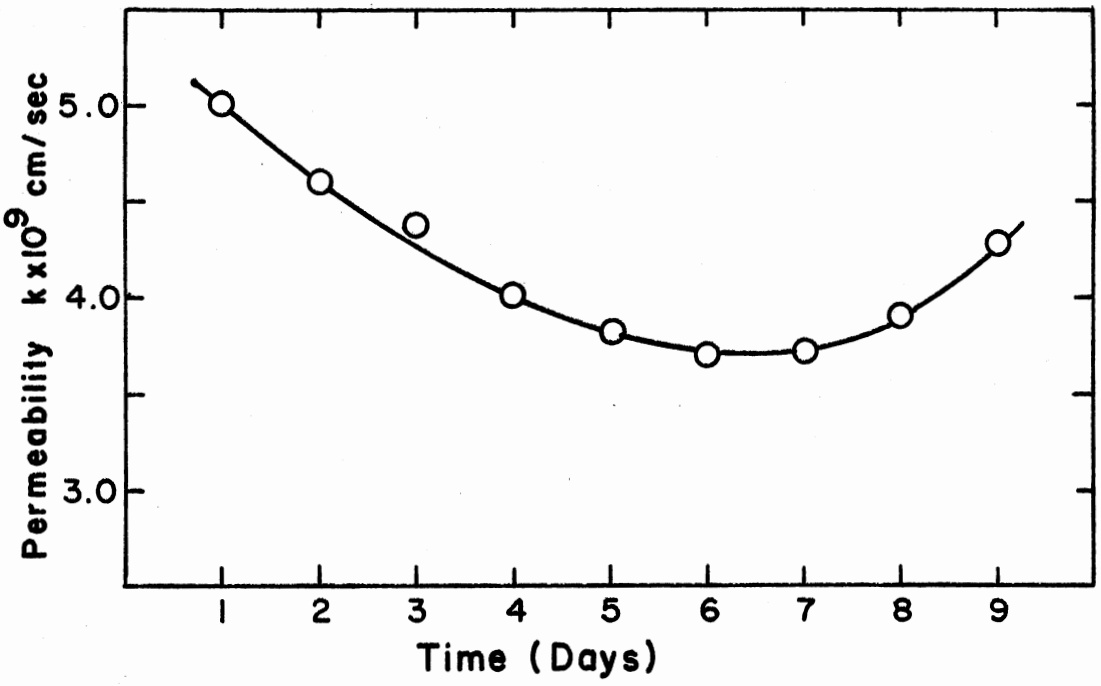


Figure 14. Permeability-Time Relationship for Compacted Sample of 60 Percent Green Clay + 40 Percent Loess

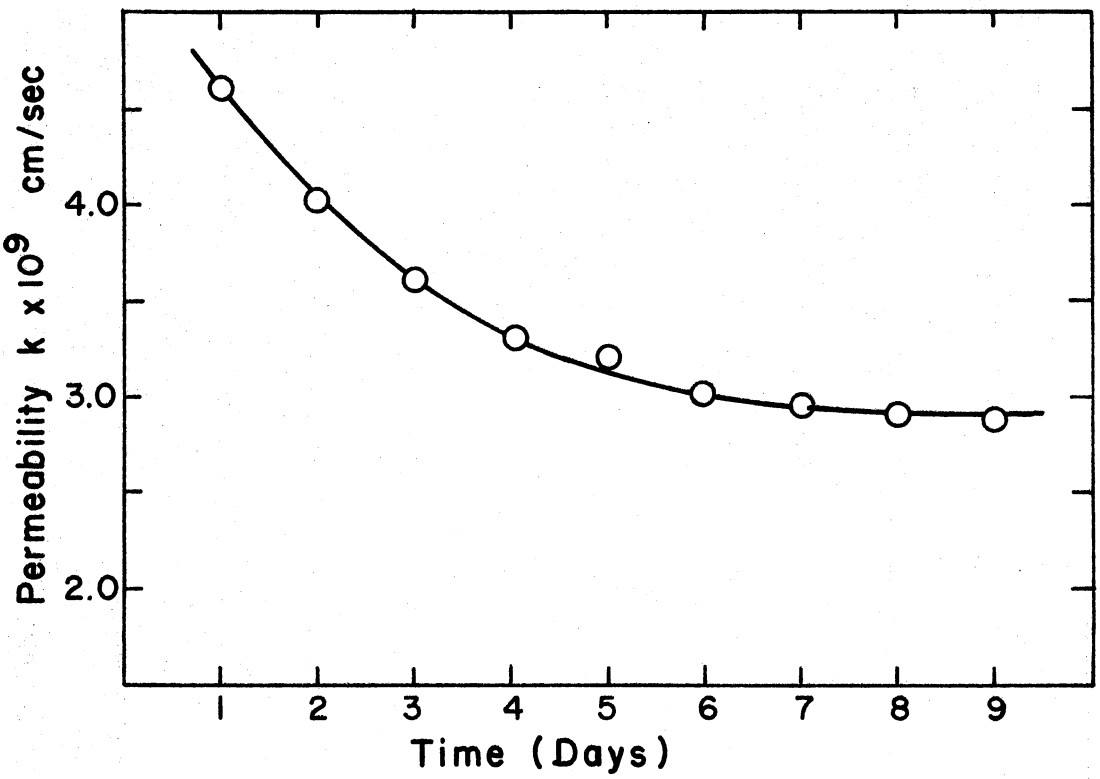


Figure 15. Permeability-Time Relationship for Compacted Sample of 66 Percent Red Clay No. 1 + 34 Percent Well-graded Sand

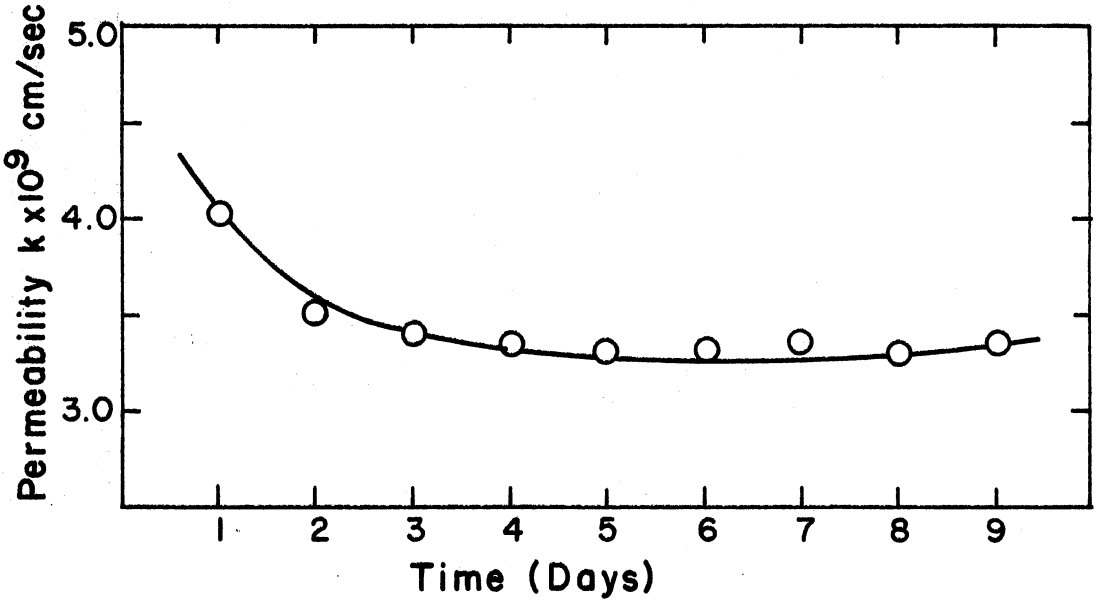


Figure 16. Permeability-Time Relationship for Compacted Sample of 60 Percent Green Clay _ 40 Percent Uniform Sand

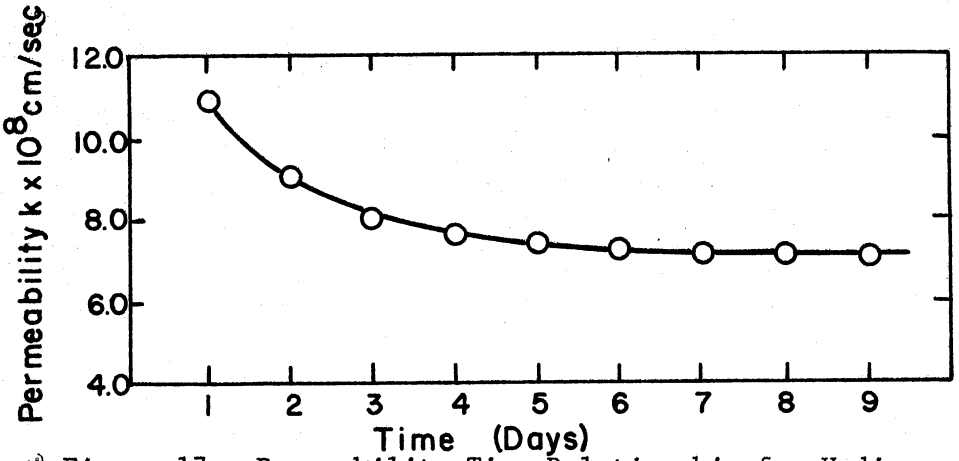


Figure 17. Permeability-Time Relationship for Undisturbed Silty Clay Sample From Ski-Island (9'-10')

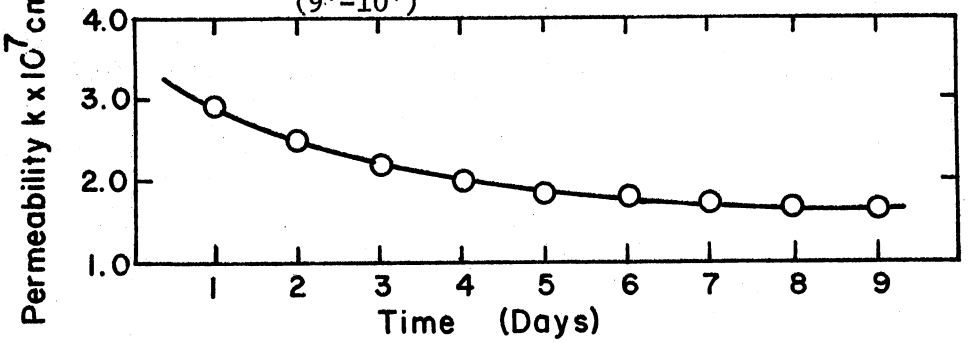


Figure 18. Permeability-Time Relationship for Undisturbed Silty Clay Sample From Ski-Island (25'-26')

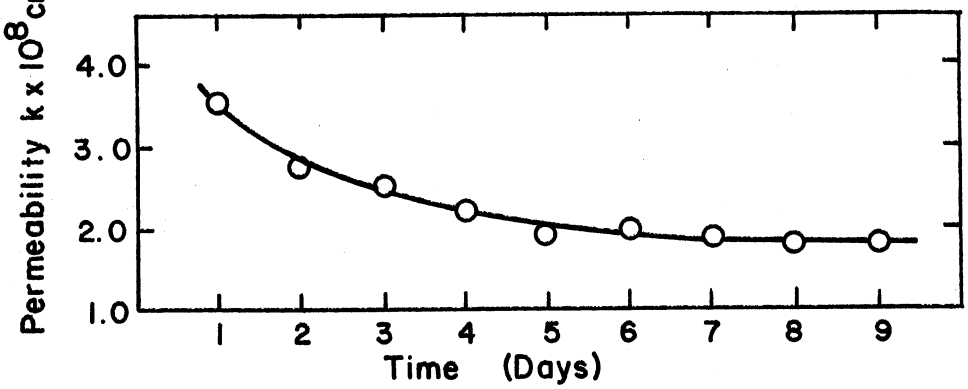


Figure 19. Permeability-Time Relationship for Undisturbed Silty Clay Sample From Oklahoma State University Campus, Stillwater

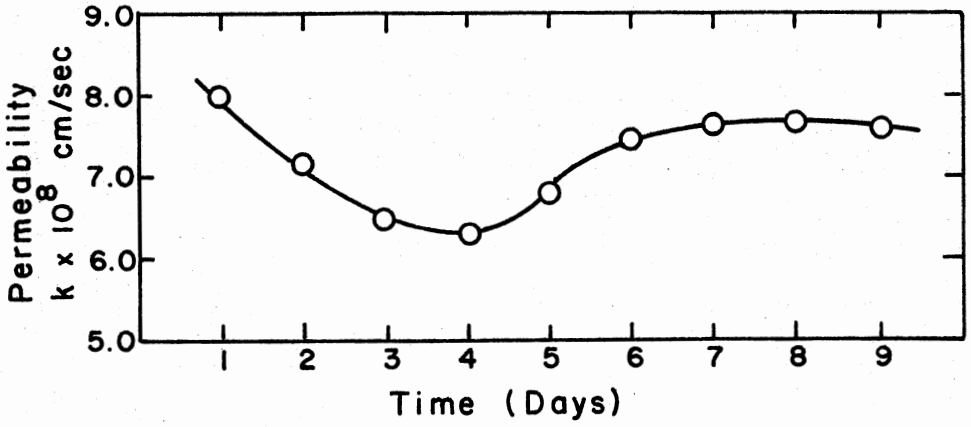


Figure 20. Permeability-Time Relationship for Undisturbed Sandy Clay From Stillwater Hospital (6'-7')

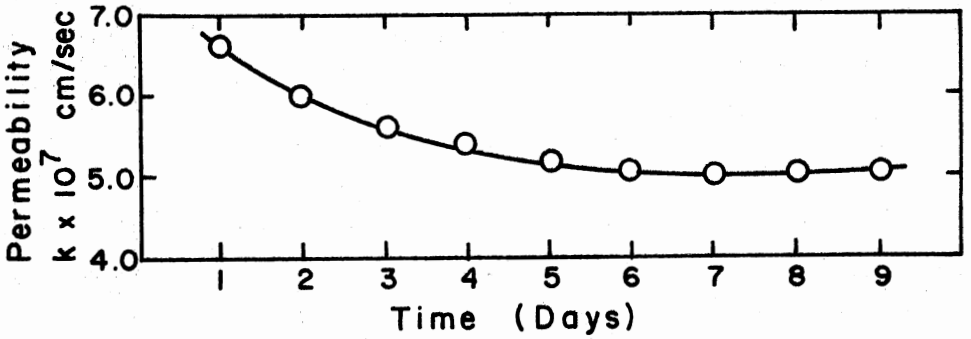


Figure 21. Permeability-Time Relationship for Undisturbed Sandy Clay From Muskogee, Oklahoma (1.5'-2.5')

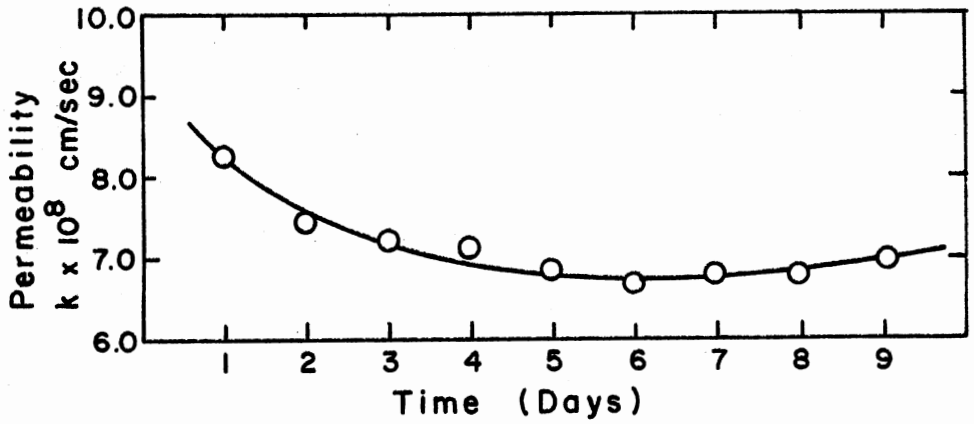


Figure 22. Permeability-Time Relationship for Undisturbed Silty Clay Sample From Northeast of Stillwater

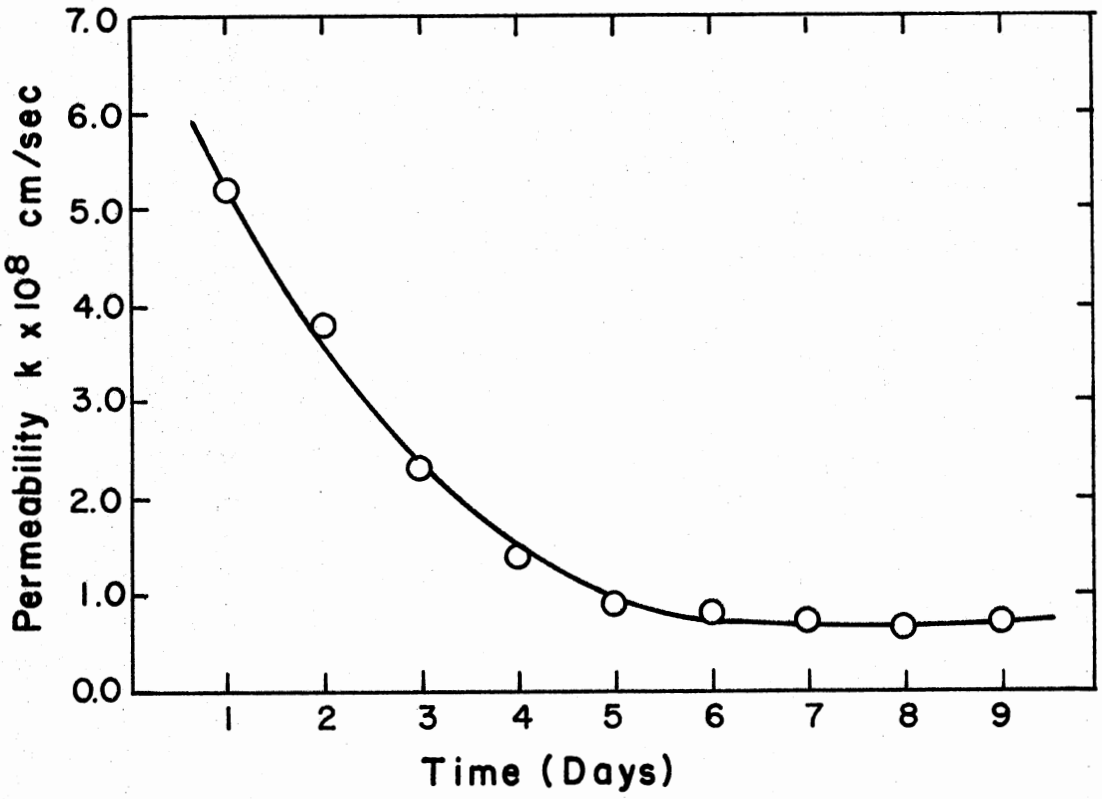


Figure 23. Permeability-Time Relationship for Undisturbed Silty Clay Sample From Hefner Dam

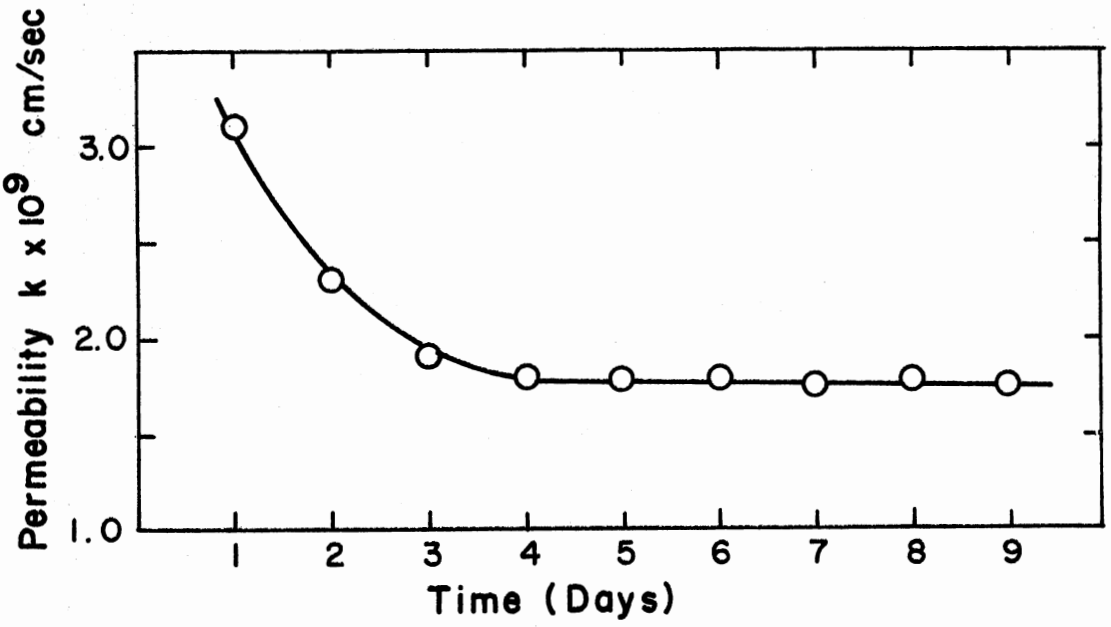


Figure 24. Permeability-Time Relationship for Undisturbed Stiff Clay Sample From Red Rock, Oklahoma

time the experiment ends. In some cases (Figures 7, 8, 9, 10, 14, and 20), the permeability passes through a minimum value and then increases slightly before attaining a constant value. The latter behavior was observed in only one of the eight undisturbed samples. It is believed that this is because of the more uniform initial saturation of the undisturbed samples in comparison with remolded samples.

This phenomenon, i.e., an early decrease in permeability followed by a later increase, is consistent with the results observed by Bodman (5), Mitchell (57), and Pillsburg (62). The main reasons for an initial decrease of permeability are:

1 - migration of the finer particles to a more stable position, plugging some of the voids. This was especially confirmed, for some of the samples, by observing a more compacted structure containing much finer material at the bottom part of the sample after extraction from the mold.

2 - removal of some of the ions, resulting in a more dispersed structure of the soil.

The phenomenon of an increased permeability during later stages of the experiment is believed to result mainly from the dissolution and removal of air trapped in the voids of unsaturated soils, which produces an increase in the effective volume of voids. The following reasons may possibly explain why this type of behavior was observed in only about half of the samples, primarily from remolded ones:

1 - differences in the initial degree of saturation

2 - test duration was perhaps insufficient to permit the phenomenon to develop.

With one exception, permeabilities had become essentially constant

by the end of the test, whether an intermediate minimum was exhibited or not.

Figures 7 through 16 show that the permeability of all compacted soil samples falls in the narrow range of $1x10^{-9}$ to $10x10^{-9}$ cm/sec. Because of the non-homogeneity, higher void ratio, and varying degrees of disturbance of undisturbed samples, their permeability varied over the wider range of $1x10^{-7}$ to $4x10^{-9}$ cm/sec.

To compare the results of the direct permeability tests with those obtained indirectly from consolidation tests, the average permeability during the first day of measurement will be used, in the belief that this more nearly corresponds to conditions during early stages of consolidation. It should be mentioned that permeability values given in the figures as a single point are an average of three to five tests of the same material.

Results of Indirect (Consolidation) Method of Permeability Measurement

According to the procedure discussed in Chapter III, the permeability was determined indirectly for all remolded and undisturbed samples. For each consolidation test, four or five loading cycles were applied and the resulting deformation, in terms of dial reading, were recorded at specified time intervals. The following relationships are used to calculate permeability (see Appendix A):

$$k = \frac{a_v \cdot c_v \cdot \gamma_w}{1+e_o}$$

$$a_v = -\frac{\Delta e}{\Delta p} = \frac{e_{100}-e_o}{\Delta p}$$

$$c_v = \frac{T_{50(90)} \cdot H^2}{t_{50(90)}}$$

where

k = coefficient of permeability in cm/sec

e_o = void ratio of the sample at zero percent consolidation

e_{100} = void ratio of the sample at 100 percent consolidation

$e_{ave} = \frac{e_o+e_{100}}{2}$

Δp = load increment in gr/cm^2

a_v = coefficient of compressibility in cm^2/gr

t_{50} = time required for 50% consolidation of the sample in sec

t_{90} = time required for 90% consolidation of the sample in sec

H = half-thickness of the sample in cm (for two face drainage)

T_{50} = constant time factor for 50% consolidation = 0.197

T_{90} = constant time factor for 90% consolidation = 0.848

c_v = coefficient of consolidation in cm^2/sec

Deterimination of c_v is the main task in this analysis, and three different methods are available in the literature to accomplish this.

1 - Square Root of Time Method, suggested by Taylor in 1942 (80), in which deformation is plotted against square root of time and t_{90} is obtained from the resulting curve.

2 - Log-time Method, suggested by Cassagrande and Fadum in 1940. In this method, deformation is plotted against logarithm of time and t_{50} is obtained from the resulting graph.

3 - Scott Method, suggested by R. F. Scott in 1960 (72) in which

c_v is calculated based on the time-deformation data for the first ten to fifteen minutes of the consolidation process. He has defined a compression ratio, C_r, as

$$C_r = \frac{d_s - d_t}{d_s - d_{Nt}}$$

where

C_r = compression ratio

d_s = dial reading at zero percent consolidation

d_t = dial reading at time t after application of load

d_{Nt} = dial reading at time Nt after application of load

N = some number between 1.5 and 4

From the value of C_r and a series of graphs of T (time factor) versus C_r for given N values, the corresponding T values are obtained. Having t and T, c_v is then found from $c_v = \frac{T.H^2}{t}$.

For a detailed discussion of these methods and their application, see Appendix B. The objective of all of the procedures is to provide a reliable value of the coefficient of consolidation, c_v, which is used both for time-settlement analyses and permeability determination. The c_v values for each sample for a given loading cycle have been determined by the three procedures described, and for comparison purposes, some of the results are given in Table IV.

Table IV shows that the values of c_v calculated by $\sqrt{t}$ and log t methods agree closely with each other, but differ by an order of magnitude from those found by the Scott method. Consequently, the latter procedure produces much higher calculated coefficients of permeability. Further analysis and comparison with permeabilities obtained by direct

measurement indicates that the Scott method cannot be used to calculate permeability if there is to be reasonable agreement between the calculated and measured permeabilities.

TABLE IV

COMPARISON OF THE VALUES OF c_v OBTAINED BY DIFFERENT METHODS

Soil	Consolidation Pressure (TSF)	c_v, $cm^2/sec \times 10^4$		
		Log t Method	$\sqrt{t}$ Method	Scott Method
Green clay	6.0	1.20	1.58	29.60
"	12.0	1.12	1.53	25.10
"	24.0	1.06	1.19	22.10
Black clay	6.0	1.10	1.27	33.60
"	12.0	0.96	1.19	28.90
"	24.0	0.59	0.67	10.80
Red clay #1	7.2	4.40	4.90	33.60
"	10.8	4.10	4.50	23.00
"	14.4	3.10	3.80	21.00
White clay	6.0	3.10	3.80	25.50
"	12.0	1.80	2.89	20.80
"	24.0	0.59	0.54	12.10
Brown clay	6.0	2.10	4.56	30.00
"	12.0	1.20	2.85	25.50
"	24.0	0.85	1.04	15.50
Red Rock stiff clay	3.6	1.70	3.40	20.80
"	7.2	0.83	0.89	8.40
Hefner Dam, silty clay	7.2	2.65	4.96	18.20
"	10.8	1.32	3.80	14.10
"	14.4	0.76	1.07	11.80

Since, in a consolidation test, the void ratio begins to decrease as soon as load is applied, the permeability of the sample will not remain constant during consolidation. This implies that permeability decreases with time at a given point and that, at a given time, the permeability is different at different points in the sample. Since consolidation progresses from the drained face inward, the permeability increases correspondingly toward the center of the test specimen. Clearly, the permeability of a soil sample during consolidation is a function of both time (t) and location (z).

$$k = f(z,t)$$

This phenomenon is analogous to the distribution of pore pressure in the soil sample during the consolidation process (see Appendix A), and can be expressed by an analogous mathematical relationship:

$$\frac{\partial k}{\partial t} = c_k \frac{\partial^2 k}{\partial z^2} \qquad (1)$$

The coefficient c_k is analogous to the coefficient of consolidation, c_v, and is similarly related to the compressibility of the soil. From a practical point of view, a relationship such as equation (1) has no application in this study and is beyond the scope of this paper. It would be useful, however, to know more precisely the average permeability of the soil sample during a given time interval, and one may speculate that equation (1) might be used to obtain a best estimate of average permeability. Since procedures for evaluating c_k have not been developed, the relationship of $\frac{a_v \cdot c_v \cdot \gamma_w}{1+e_o}$ has been assumed to yield a sufficiently reliable average value of k for use in comparison with directly

measured values. For each successive load increment, the initial void ratio, e_o, is smaller than for the preceding increment but it can be determined easily from the test data and used to compute the value of k_{ave} corresponding to a given loading cycle.

To test the consistency of results obtained by this procedure, the computed permeability will be plotted against the corresponding consolidation pressure for the different soil samples. The following analysis illustrates the nature of the relationship that should be expected between permeability and consolidation pressure.

From Terzaghi's consolidation theory:

$$k = \frac{a_v \cdot c_v \cdot \gamma_w}{1+e_o} \tag{2}$$

and

$$a_v = \frac{-\Delta e}{\Delta p} \tag{3}$$

Also, it has been found that a plot of void ratio versus logarithm of consolidation pressure for normally consolidated soils gives a straight line relationship in which the slope of the line is designated as a compression index, C_c, and is a measure of soil compressibility. By definition:

$$C_c = \frac{-de}{d(\log p)} \tag{4}$$

Equation (4) can be written in the following forms:

$$C_c = - \frac{\Delta e}{\Delta(\log p)}$$

$$\Delta e = - C_c . \Delta (\log p) \tag{5}$$

Substituting for Δe in equation (3)

$$a_v = C_c . \frac{\Delta (\log p)}{\Delta p} \tag{6}$$

Converting to the base of natural logarithms and taking the limit of the righthand side of the equation:

$$a_v = 0.435 . C_c \frac{\Delta (\log_e p)}{\Delta p} = 0.435\ C_c \frac{1}{p}$$

Limit $\Delta p \to 0$

or

$$a_v = \frac{0.435\ C_c}{p} \tag{7}$$

Substituting for a_v in equation (2)

$$k = \frac{c_v . \gamma_w}{1+e_o} . \frac{0.435\ C_c}{p} \tag{8}$$

Taking the logarithm of both sides:

$$\log k = \log \left(\frac{0.435\ C_c . c_v . \gamma_w}{1+e_o} \right) - \log p \tag{9}$$

Assuming c_v and C_c to be constant,

$$\log k = C - \log p \tag{10}$$

Equation (10) shows that there is a straight line relationship between log k and log p and that a straight line should be obtained by

plotting k as a function of p on log-log coordinates. Such graphs of the experimental data are given in Figures 25 through 34 for remolded samples, and in Figures 35 through 42 for undisturbed samples. The resulting straight lines confirm the general validity of the experimental data.

Further analysis of the experimental data (graphs) indicates that the general form of the equation of the straight lines is as follows:

$$\log k = b - a \log p \tag{11}$$

where a and b are constants for the particular soil. For the samples tested in this study, "a" varies between 0.82 and 1.66, and "b" varies between 0.64 and 1.9. This analysis suggests that "a" and "b" depend on other factors that affect the permeability-consolidation pressure relationship. It is obvious that the constants "a" and "b" are related to compressibility and other physical properties of the soil and the determination of the pertinent relationships would be an interesting objective for future research. The general form of the relationship has special advantage in predicting the permeability of layers of clay soils at different depths. In other words, having the permeabilities of two samples from different depths (by direct or indirect testing), and estimating the pressure under which they have consolidated, a straight line may be plotted through the two corresponding points for use in estimating permeability at any depth.

Comparison of the Results of Permeability Determination by Direct and Indirect Methods

If the results of the two methods are to be compared, all of the

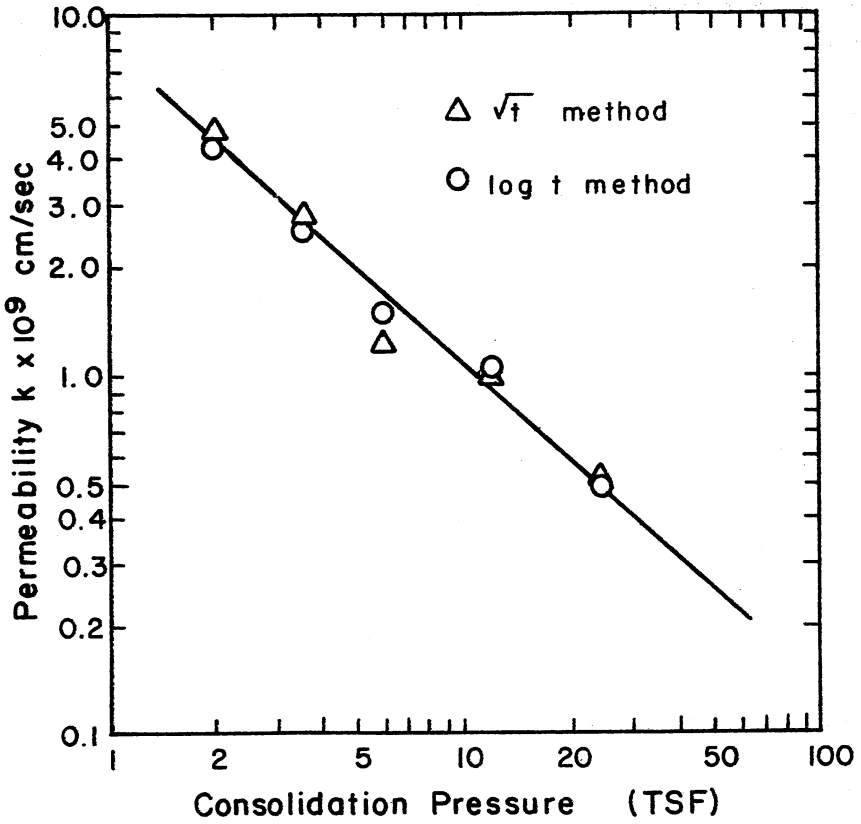


Figure 25. Permeability-Consolidation Pressure Relationship for Compacted Green Clay

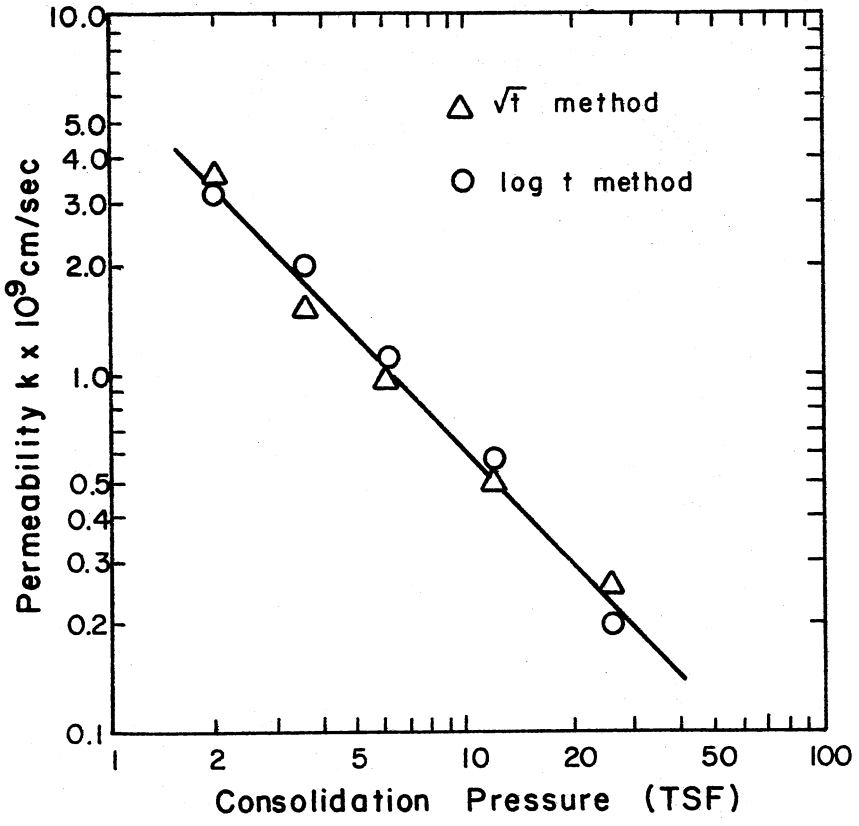


Figure 26. Permeability-Consolidation Pressure Relationship for Compacted Black Clay

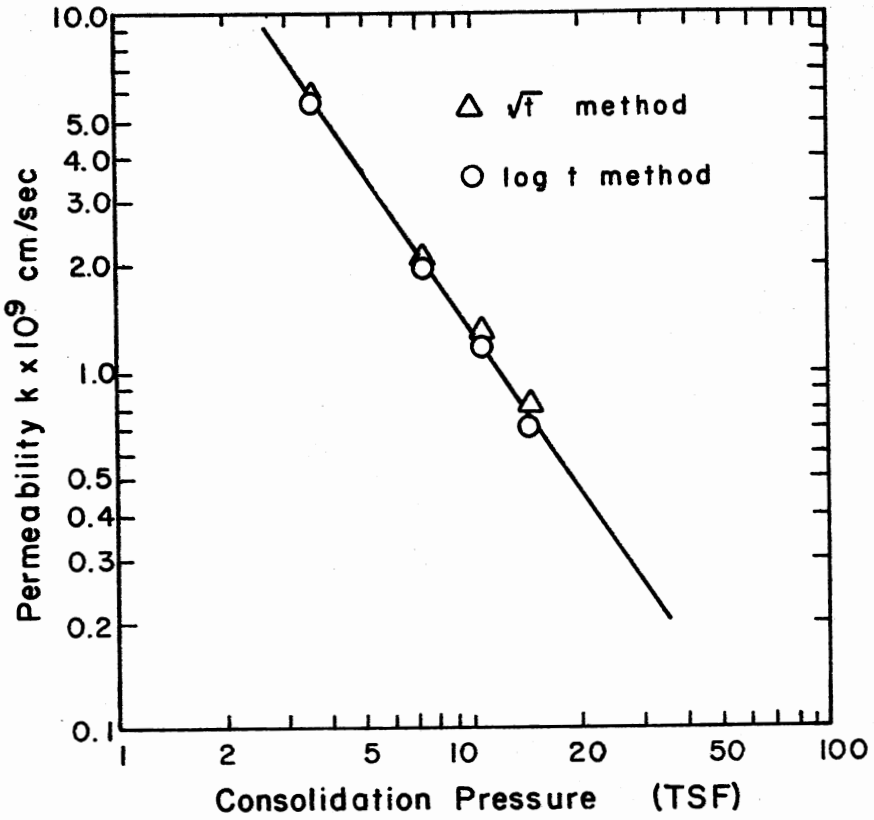


Figure 27. Permeability-Consolidation Pressure Relationship for Compacted Red Clay No. 1

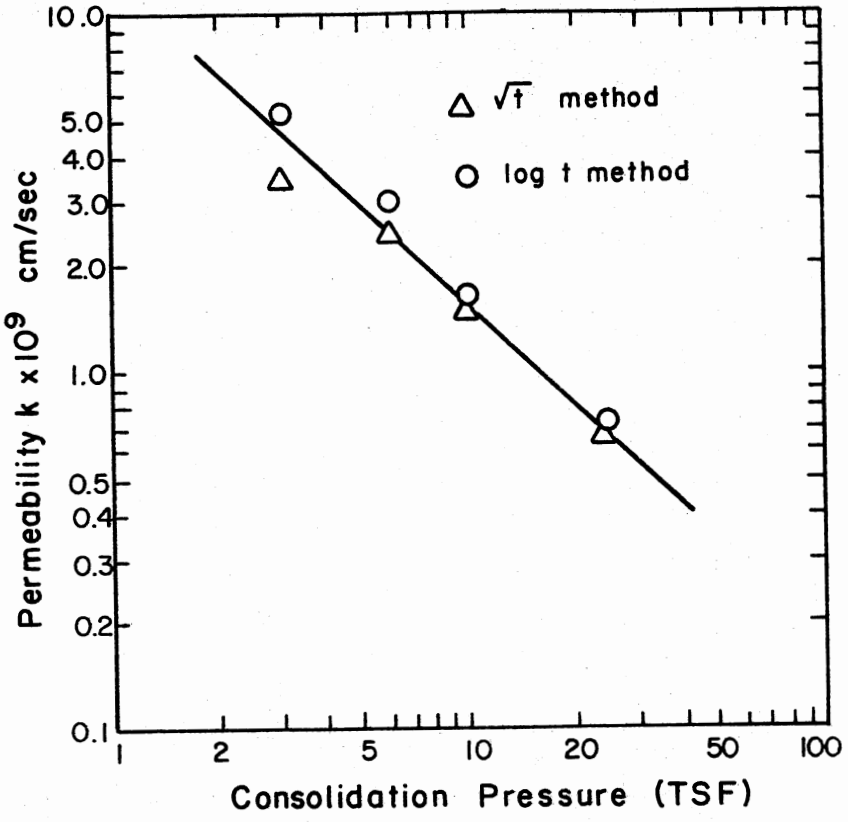


Figure 28. Permeability-Consolidation Pressure Relationship for Compacted Red Clay No. 2

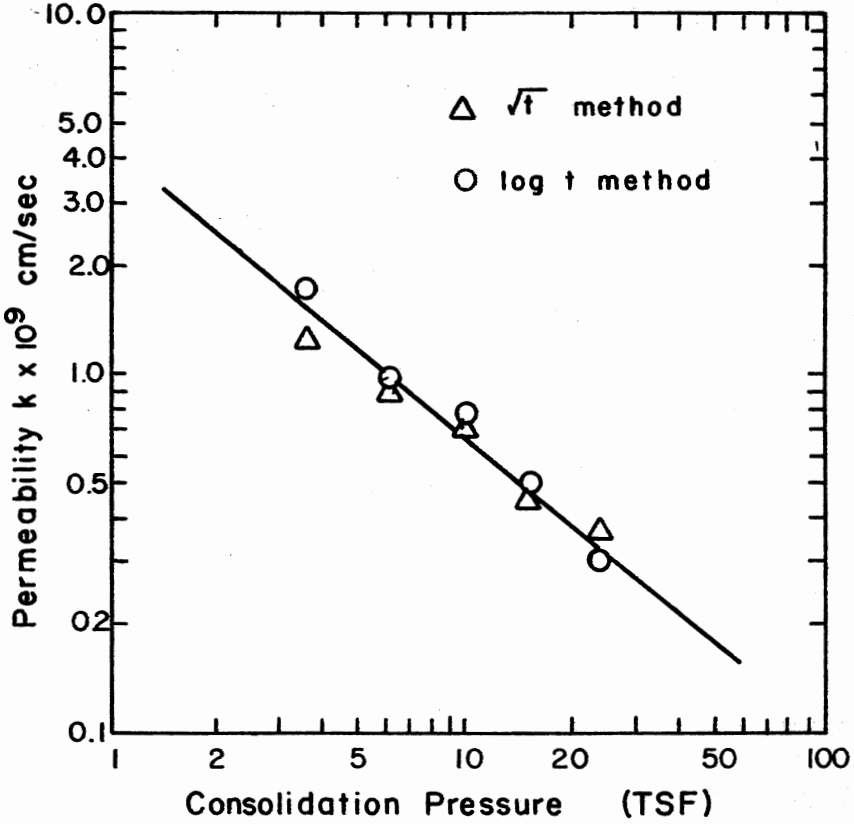


Figure 29. Permeability-Consolidation Pressure Relationship for Compacted Brown Clay

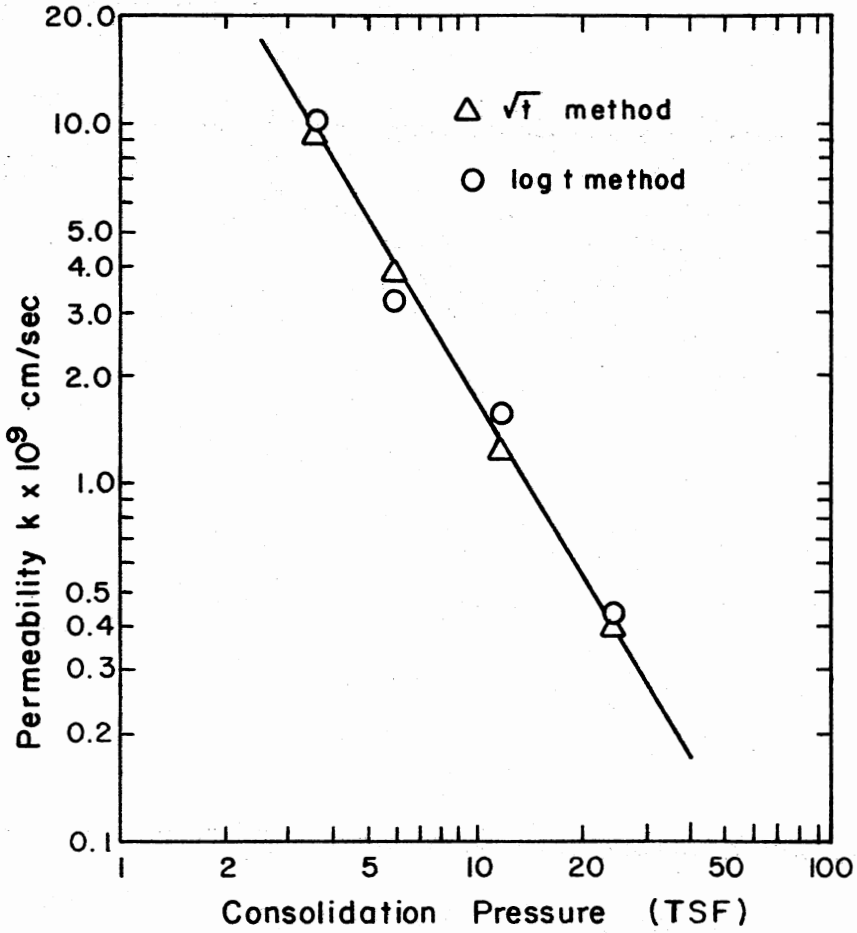


Figure 30. Permeability-Consolidation Pressure Relationship for Compacted White Clay

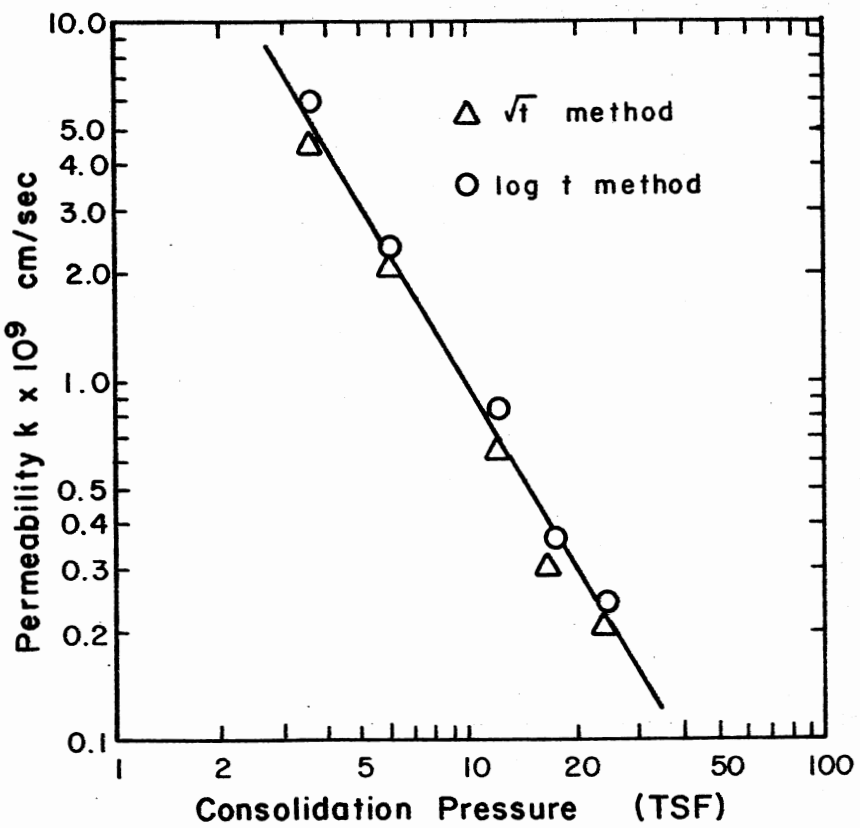


Figure 31. Permeability-Consolidation Pressure Relationship for Compacted Dark Brown Clay

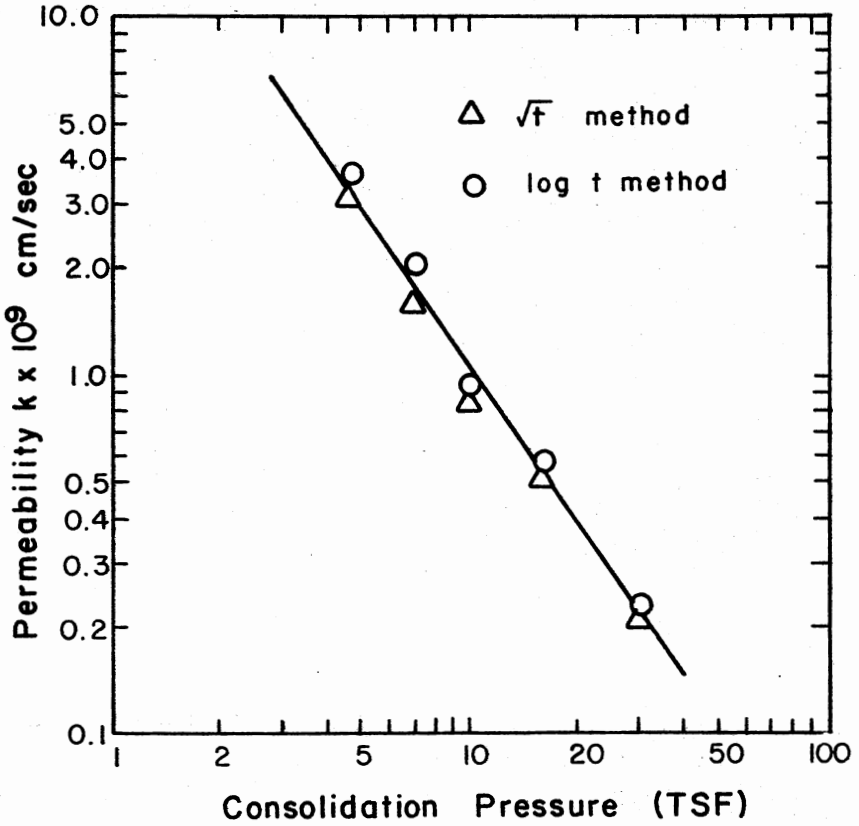


Figure 32. Permeability-Consolidation Pressure Relationship for Sample of 60 Percent Green Clay + 40 Percent Loess

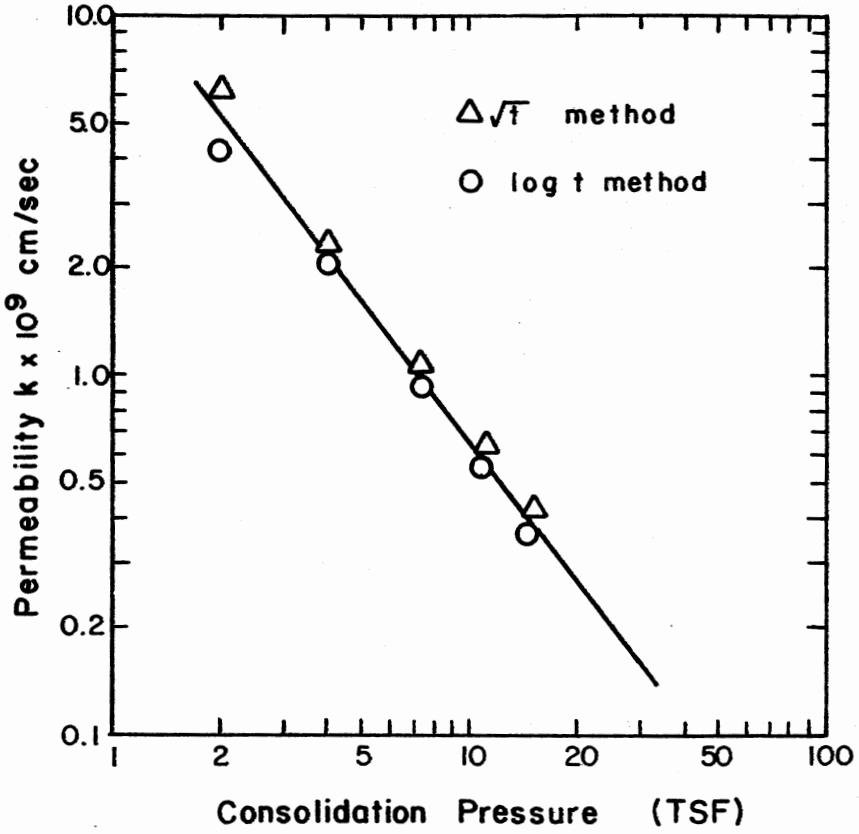


Figure 33. Permeability-Consolidation Pressure Relationship for Sample of 60 Percent Green Clay + 40 Percent Uniform Sand

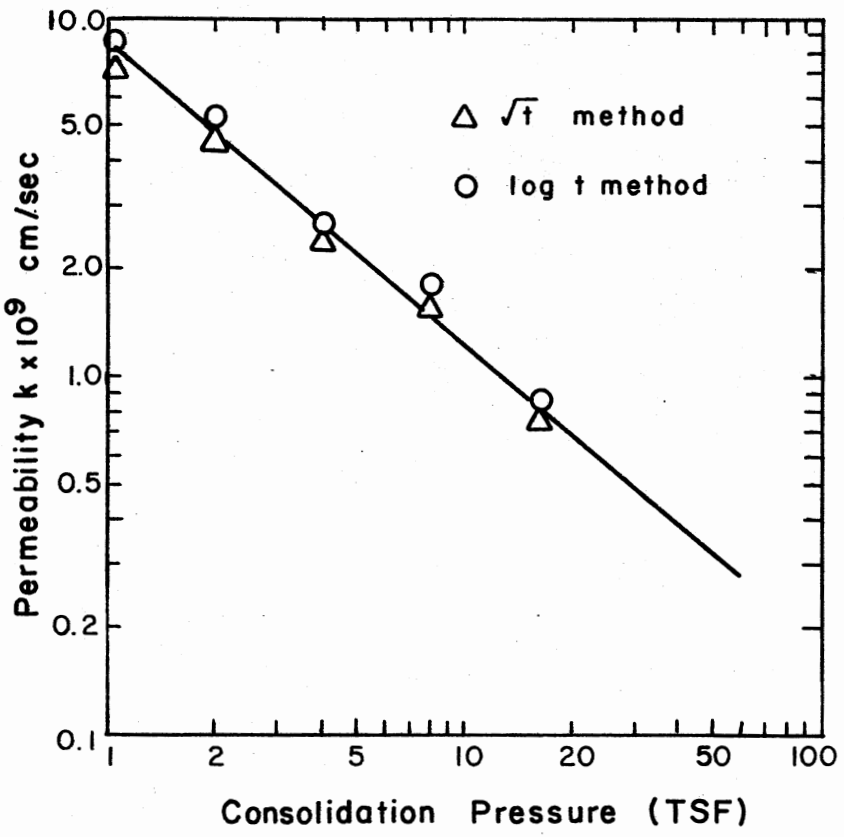


Figure 34. Permeability-Consolidation Pressure Relationship for Sample of 66 Percent Red Clay No. 1 + 34 Percent Well-graded Sand

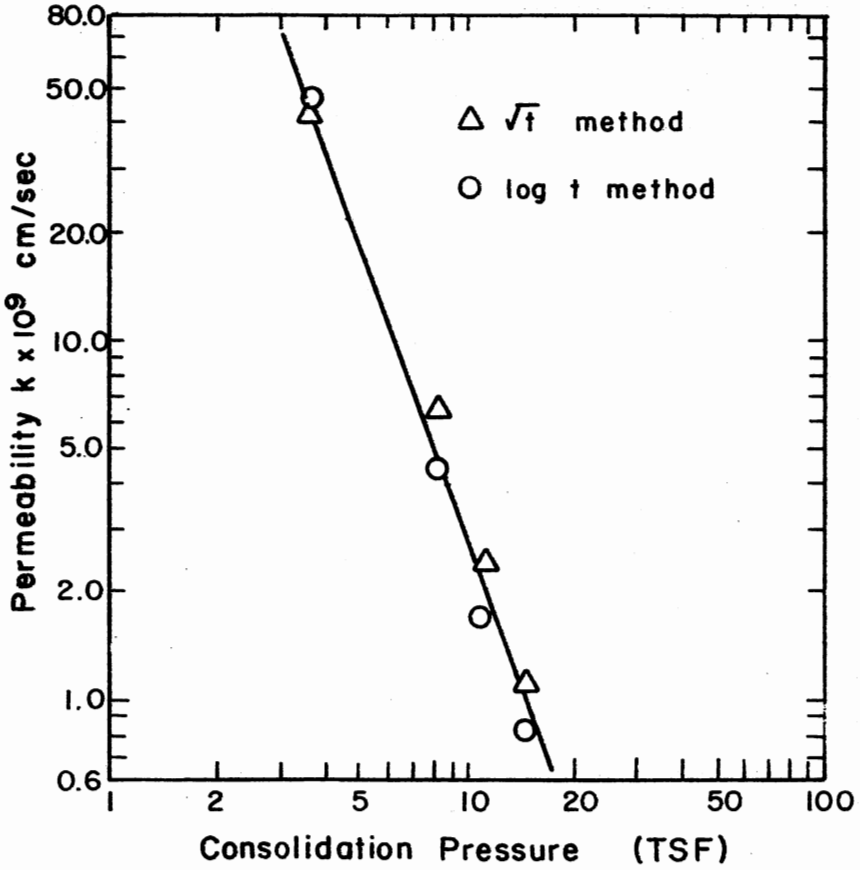


Figure 35. Permeability-Consolidation Pressure Relationship for Undisturbed Silty Clay Sample From Ski-Island (25'-26')

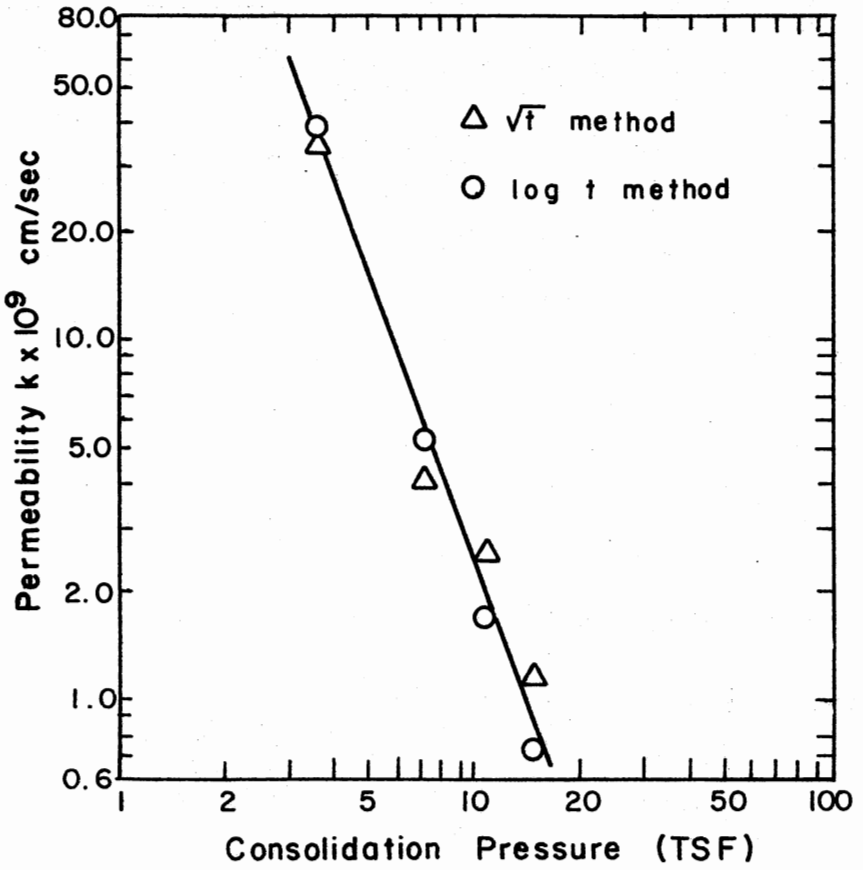


Figure 36. Permeability-Consolidation Pressure Relationship for Undisturbed Silty Clay Sample From Ski-Island (9'-10')

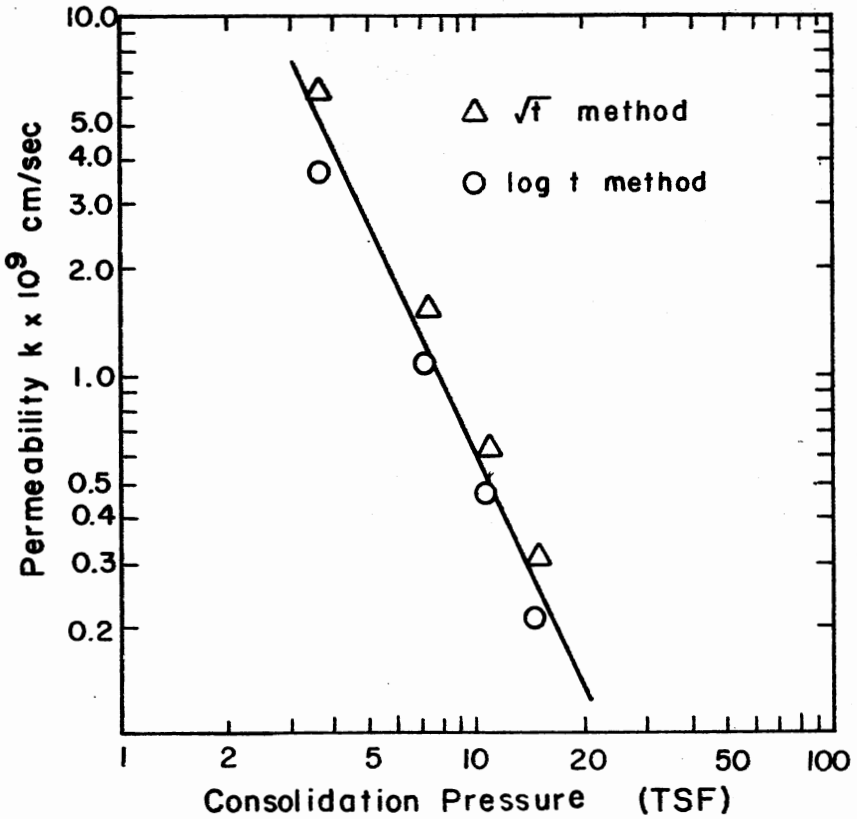


Figure 37. Permeability-Consolidation Pressure Relationship for Undisturbed Silty Clay Sample From Hefner Dam (44'-45')

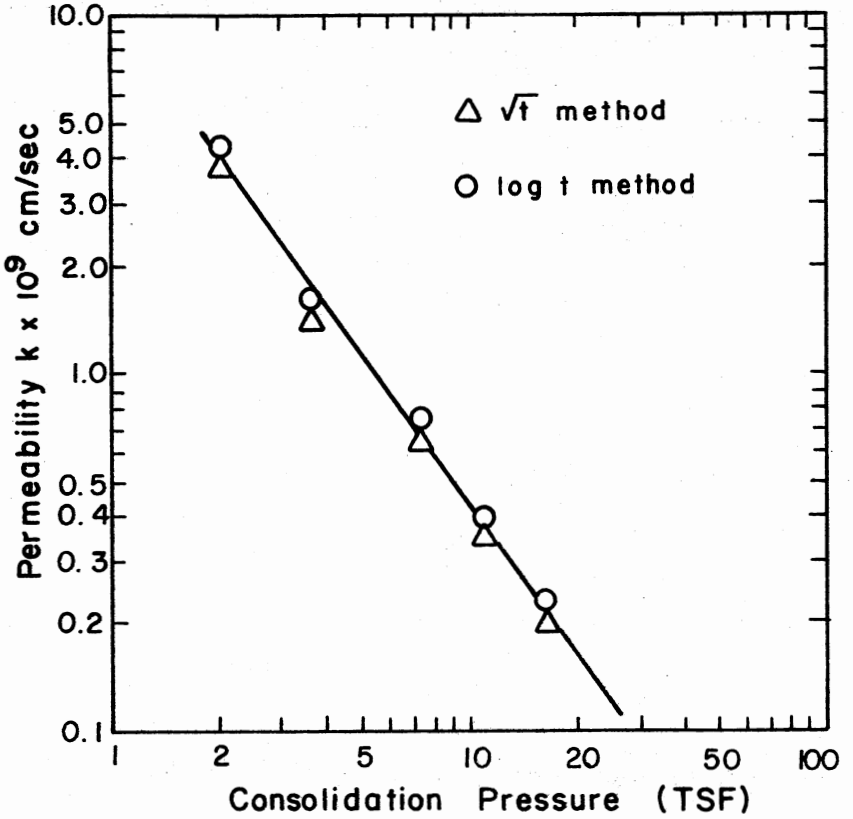


Figure 38. Permeability-Consolidation Pressure Relationship for Undisturbed Stiff Clay Sample From Red Rock, Oklahoma (15'-16')

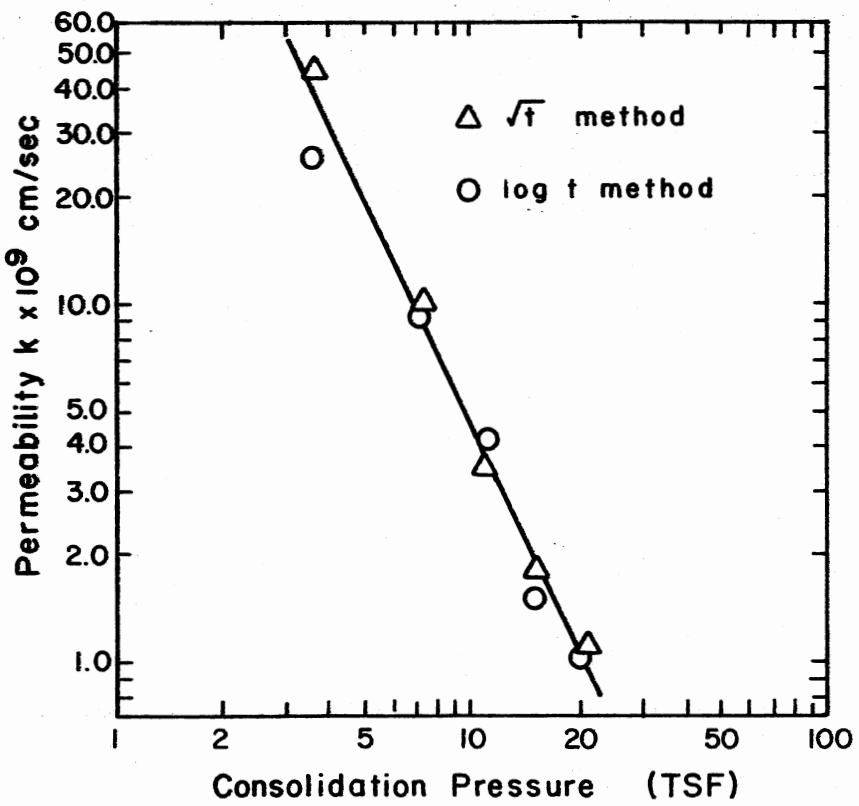


Figure 39. Permea bility-Consolidation Pressure Relationship for Undisturbed Sandy Clay Sample From Muskogee, Oklahoma (1.5'-7.5')

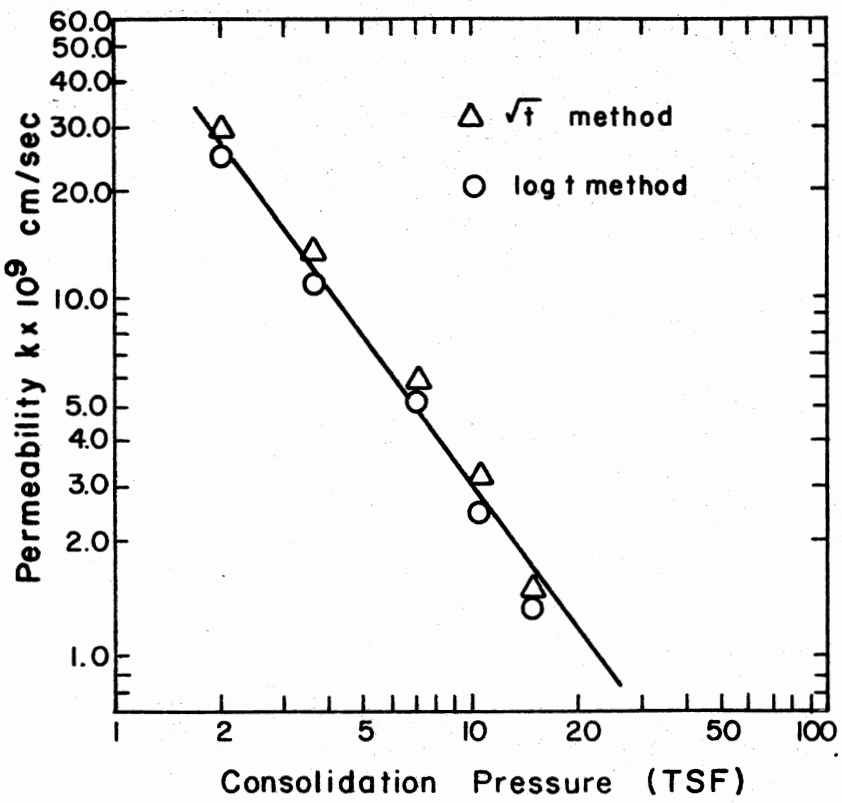


Figure 40. Permeability-Consolidation Pressure Relationship for Undisturbed Sandy Clay Sample From Stillwater Hospital (6'-7')

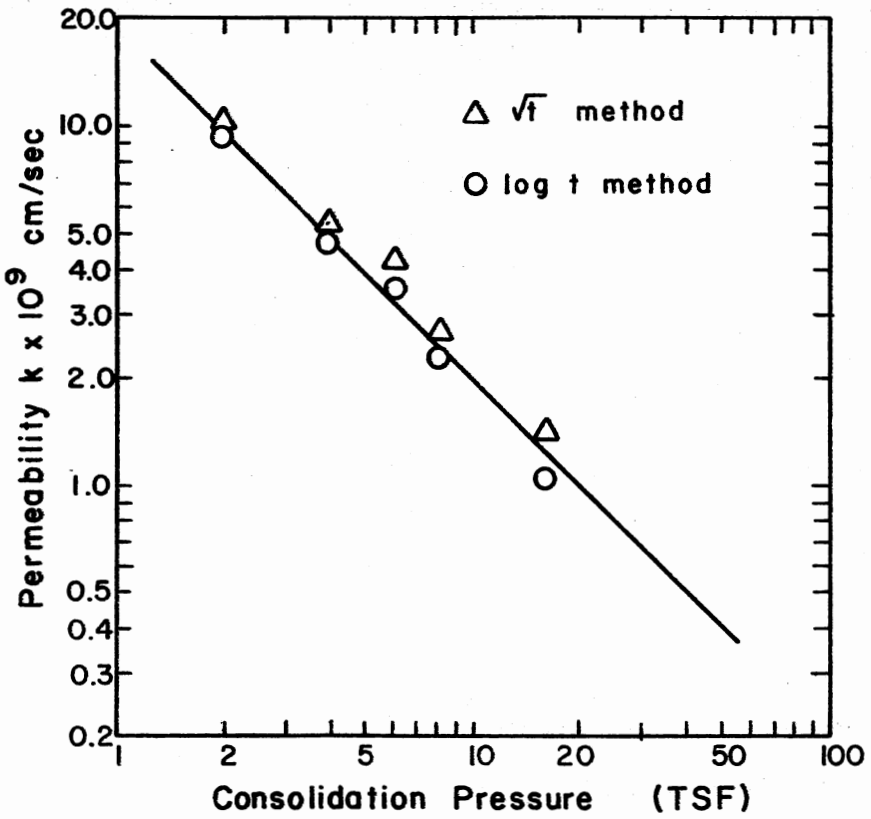


Figure 41. Permeability-Consolidation Pressure Relationship for Undisturbed Silty Clay Sample From Oklahoma State University Campus, Stillwater (2'-3')

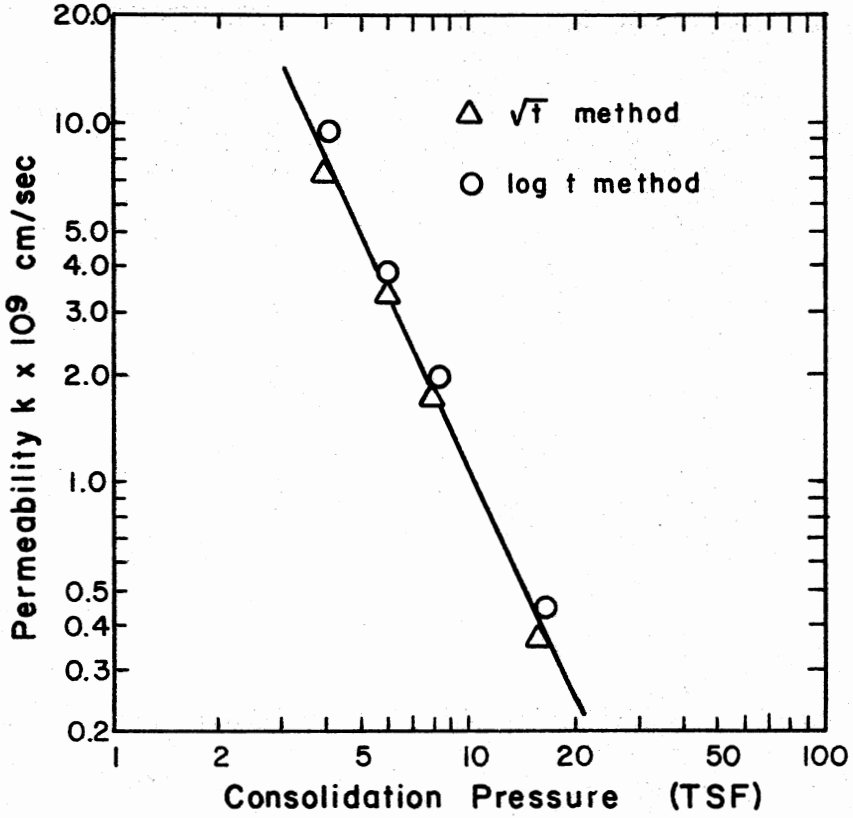


Figure 42. Permeability-Consolidation Pressure Relationship for Undisturbed Silty Clay Sample From Northeast of Stillwater (6'-7')

factors affecting permeability (see Chapter II) should be the same; otherwise the comparison may be meaningless. As previously described, all samples were prepared under the same conditions (using the same source for soil and water), and the two different tests were conducted under the same conditions of temperature and humidity. Therefore, the results of the two methods apparently should be comparable. There is, however, one major point to be considered in connection with the indirect determinations of k. It was previously pointed out that when the load is applied to a sample in a consolidation ring, its void ratio begins to decrease and the permeability decreases correspondingly. Therefore, the average effective void ratio of the soil sample during the consolidation process is less than the initial void ratio, and the average permeability must be less than that corresponding to the initial void ratio.

Assuming that in the permeameter used for the direct method there is no volume change of the sample (actually, under high water pressure there is a little volume change), and that the average permeability of the sample in this method corresponds to its initial void ratio, the measured permeability should be higher than the average permeability found from consolidation test data using the initial void ratio, e_o. Such a discrepancy was, in fact, observed for all of the samples tested in the study; that is, the coefficient of permeability for samples compacted to the same density and under the same conditions was found to be higher for direct measurement than for indirect determination. Therefore, the permeabilities determined by two methods are not comparable unless the average void ratio of the sample during both tests is taken into consideration. There are different opinions concerning

the influence of void ratio on permeability, and there is no unique expression defining the relationship between the two, especially for clay soils. Nevertheless, the results of many experiments have shown that a plot of logarithm of permeability against void ratio gives a straight line over a wide range of permeabilities (44)(71). The straight line relationship might be inferred by the following analysis.

It is well established (42) that there is a linear relationship between logarithm of consolidation pressure (p) and void ratio (e) for a normally consolidated soil. The slope, C_c, of this line is indicative of the compressibility of the soil samples. This relationship, given in Equation (4) may be written in the following form:

$$\log p = \frac{e}{C_c} + C' \tag{12}$$

where C' is a constant of integration.

Substituting for log p in Equation (10),

$$\log k = C + \frac{e}{C_c} - C', \text{ or}$$

in general form

$$\log k = a'.e + b' \tag{13}$$

Equation (13) establishes, theoretically, the linear relationship between log k and void ratio, and corresponds to observed behavior of soils. Thus, if the values of k computed from consolidation test data are plotted on semi-log paper against average void ratio, the result should be a straight line, and the values of k determined by the

direct method should be located more or less on the same line.

In accordance with the indicated evaluative procedure, the average values of void ratio $\left(\frac{e_o + e_{100}}{2}\right)$ of a consolidation sample for different loadings are plotted against logarithm of the computed k, and the values of k determined by direct measurement are plotted according to the initial void ratio of the permeameter specimen. The resulting plots are given in Figures 43 through 52 for remolded samples, and in Figures 53 through 59 for undisturbed samples. The points indicating the results of the direct method have been plotted using the average permeability of the samples during the first day of the test.

The graphs indicate that, for all samples except two from the remolded group, the points indicating the results of direct permeability tests fall below the linear plot of log k versus e obtained from the consolidation tests. Thus, the permeability found by the direct method is a little less, generally, than would be predicted by the indirect method. It is believed that one or more of the following reasons may account for the difference.

1 - possibility of decrease in volume of the permeameter sample under the high pressure gradient, with a corresponding decrease in average void ratio. The average effective void ratio may be a little less than the initial value used in the analysis.

2 - possibility that a sudden drop in permeability occurs during the early moments of a direct permeability test because of the migration of finer particles to positions of greater hydraulic stability.

3 - the possibility that the time-average void ratio of a sample during consolidation differs from the volumetric average,

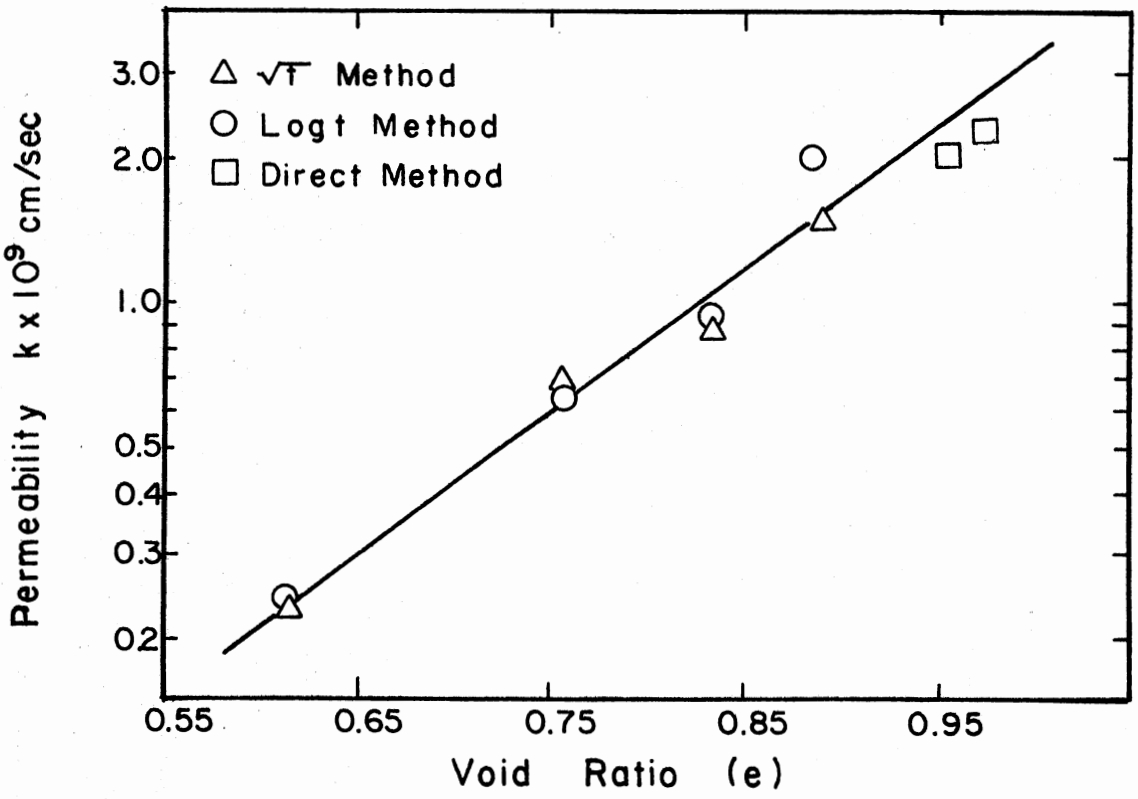


Figure 43. Permeability-Void Ratio Relationship for Compacted Black Clay

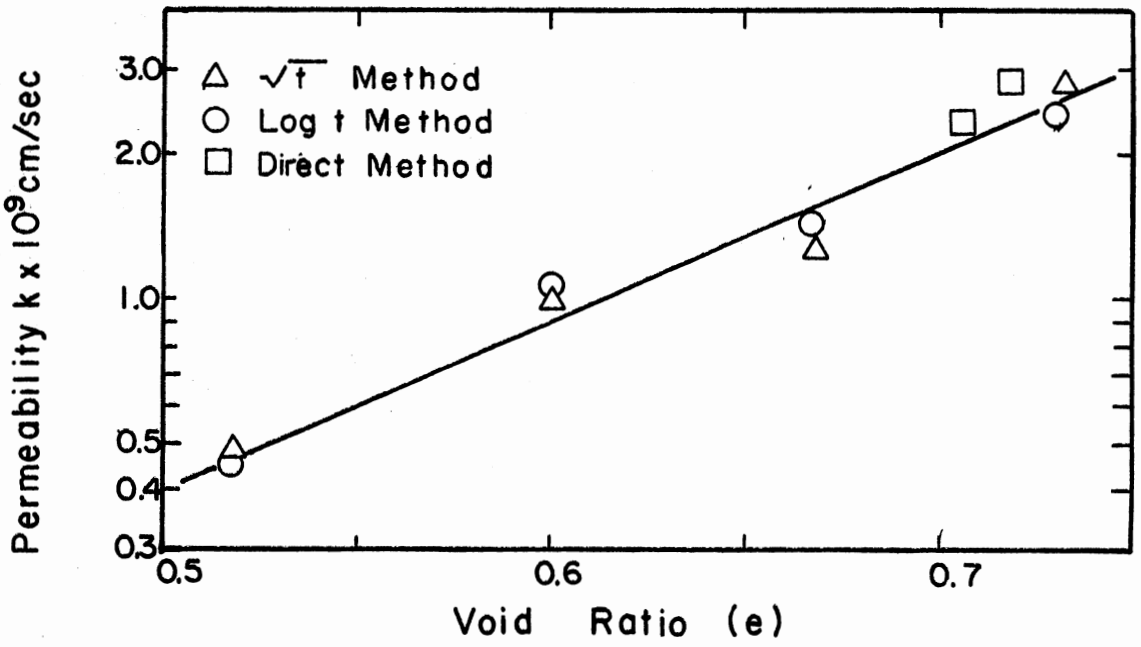


Figure 44. Permeability-Void Ratio Relationship for Compacted Green Clay

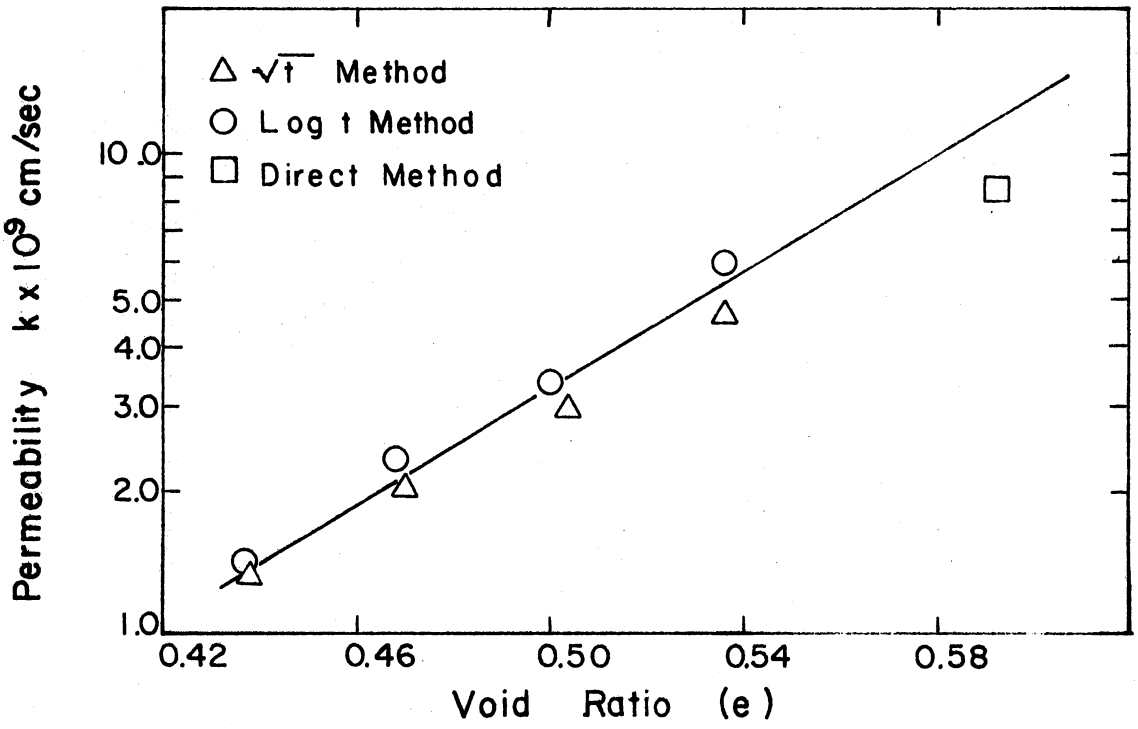


Figure 45. Permeability-Void Ratio Relationship for Compacted Dark Brown Clay

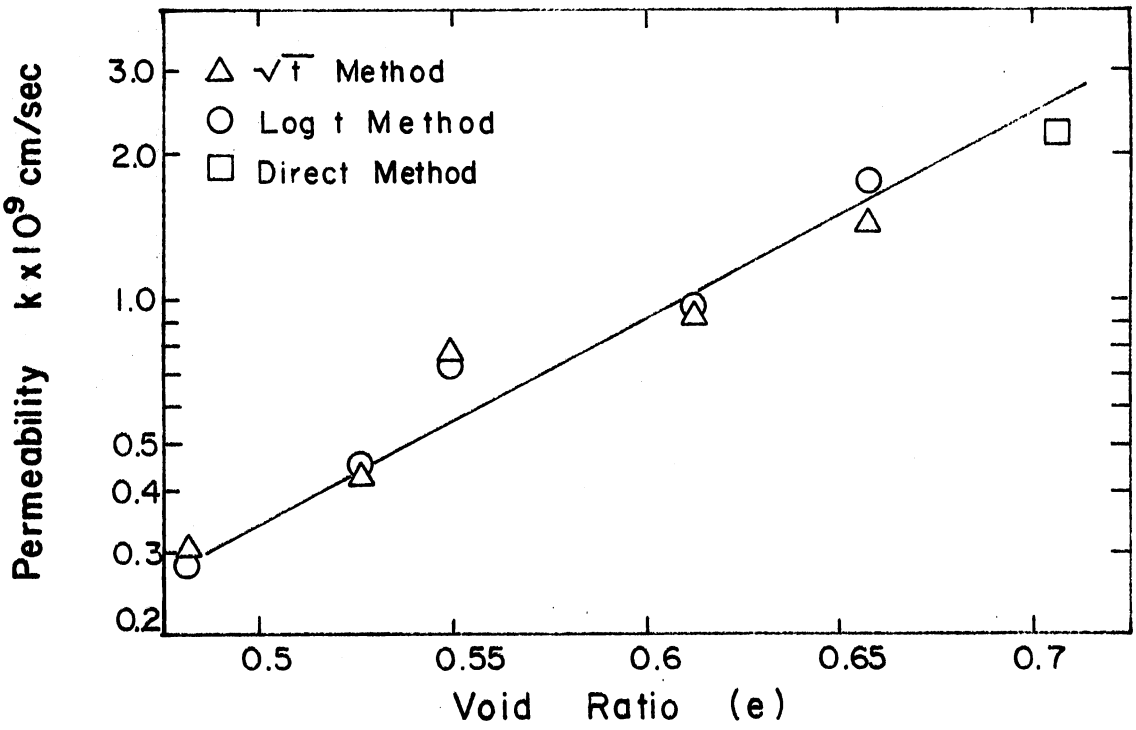


Figure 46. Permeability-Void Ratio Relationship for Compacted Brown Clay

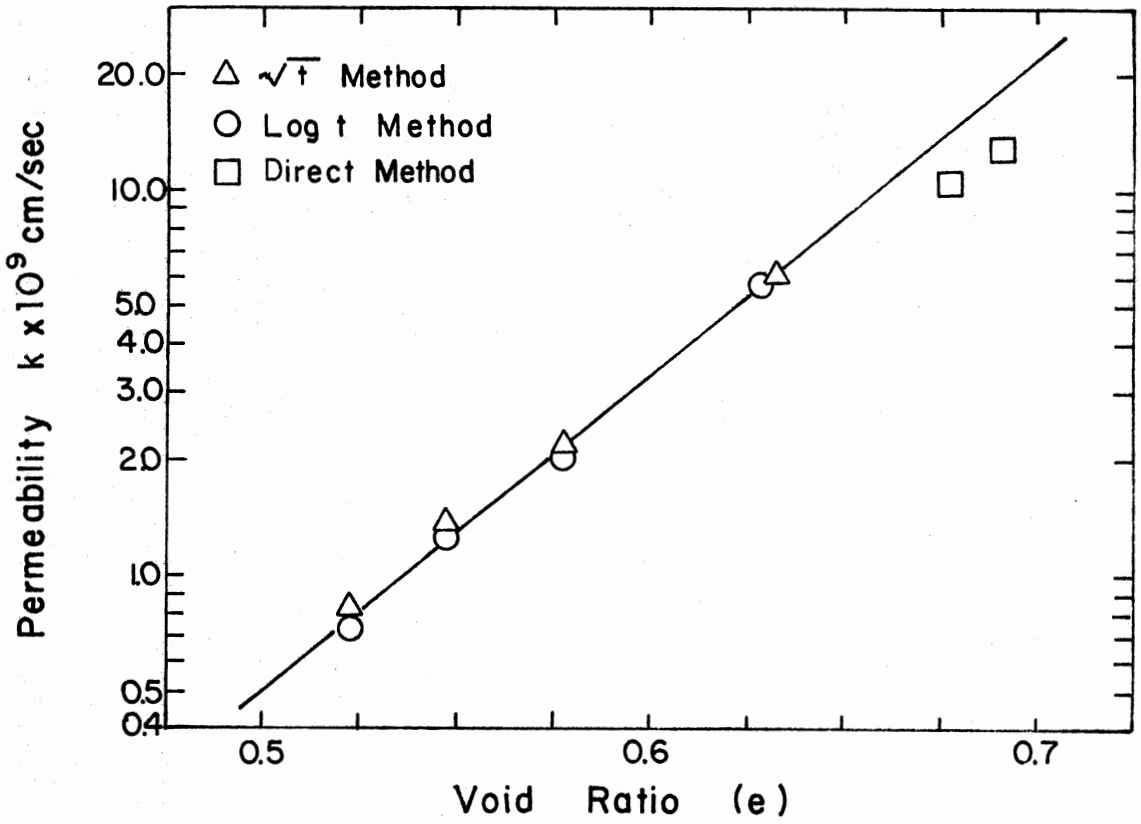


Figure 47. Permeability-Void Ratio Relationship for Compacted Red Clay No. 1

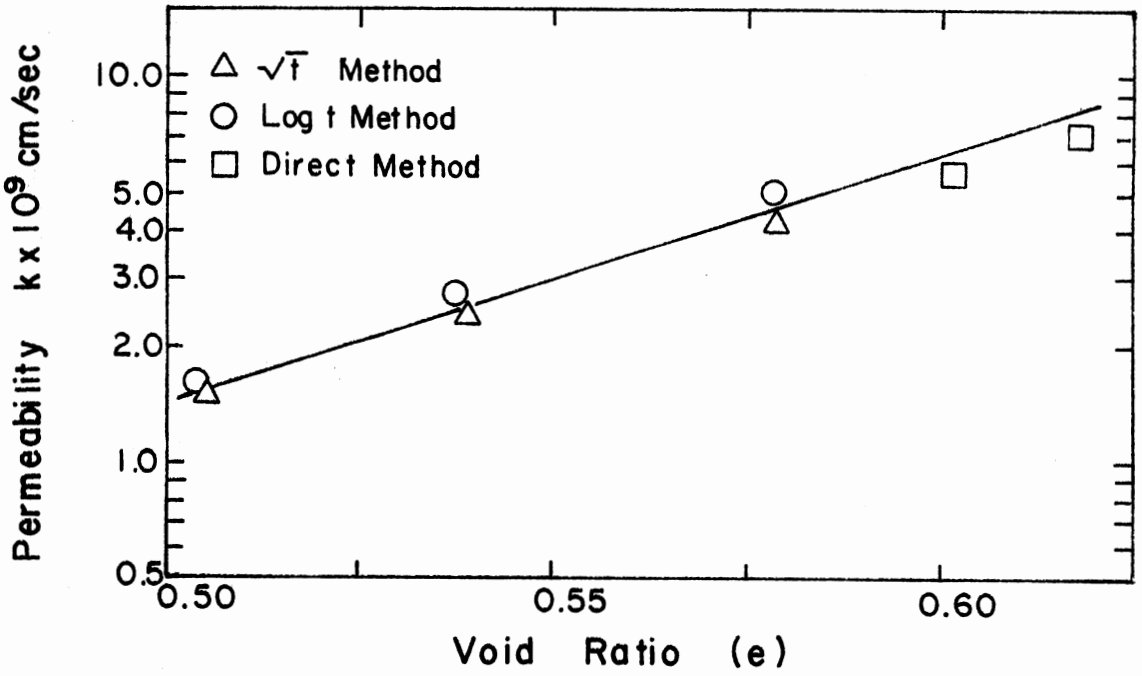


Figure 48. Permeability-Void Ratio Relationship for Compacted Red Clay No. 2

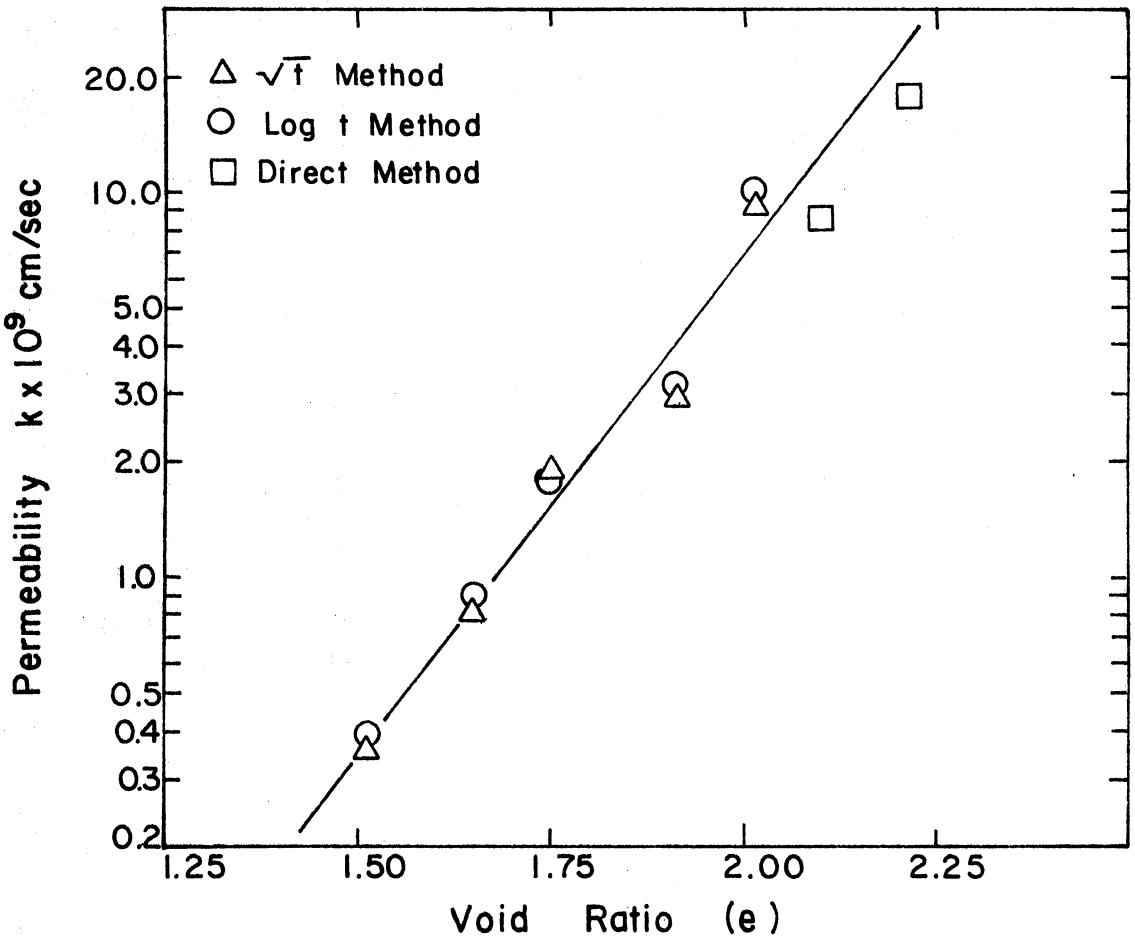


Figure 49. Permeability-Void Ratio Relationship for Compacted White Clay

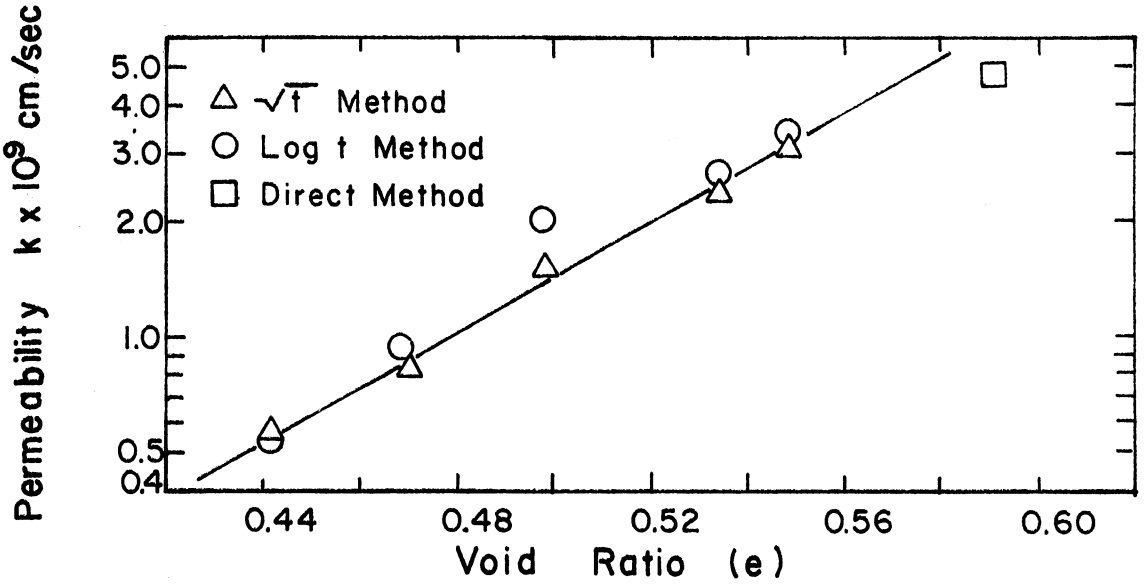


Figure 50. Permeability-Void Ratio Relationship for Compacted Sample of 60 Percent Green Clay + 40 Percent Loess

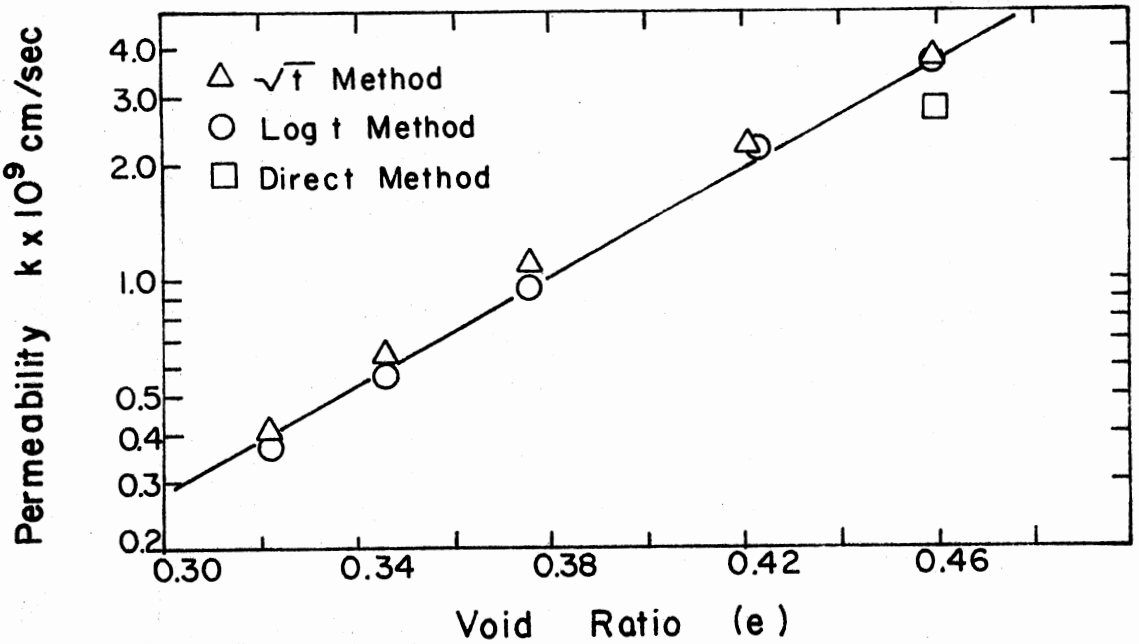


Figure 51. Permeability-Void Ratio Relationship for Compacted Sample of 60 Percent Green Clay + 40 Percent Uniform Sand

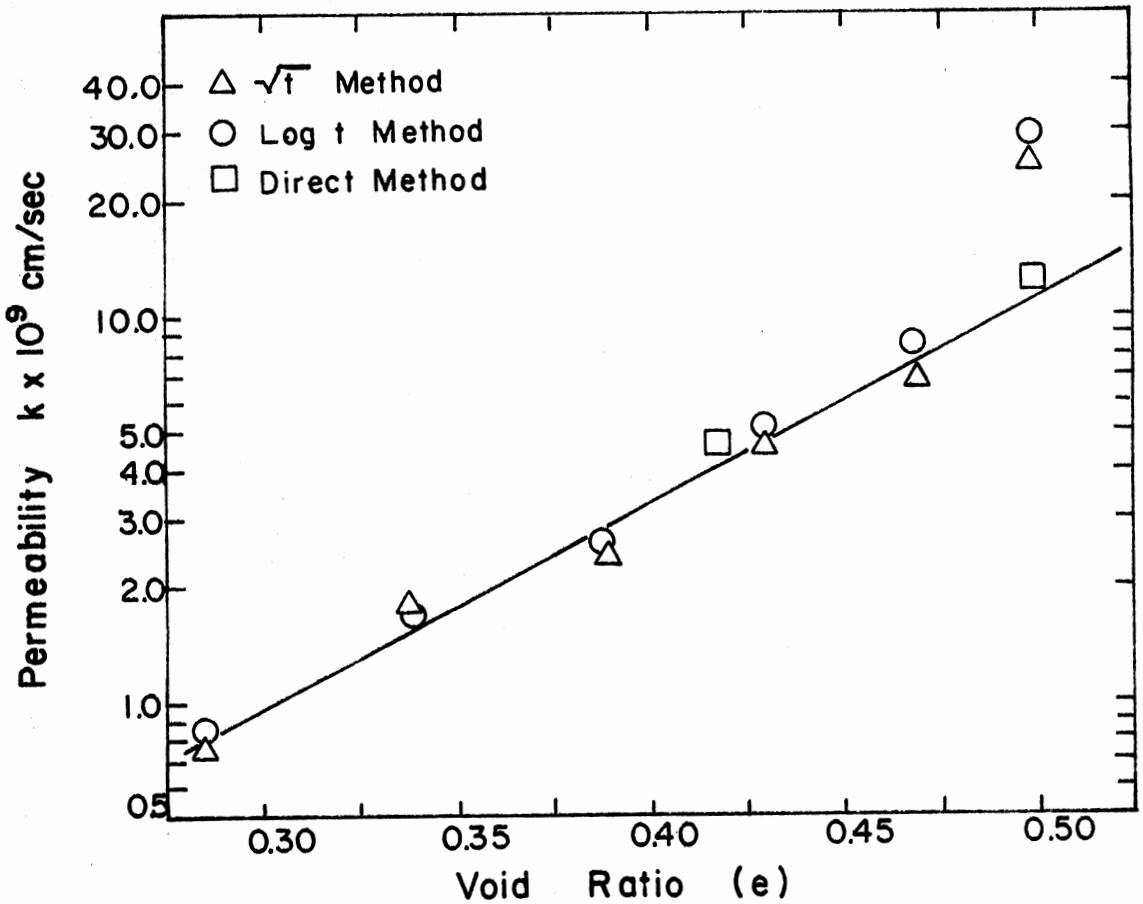


Figure 52. Permeability-Void Ratio Relationship for Compacted Sample of 66 Percent Red Clay No. 1 + 34 Percent Well-graded Sand

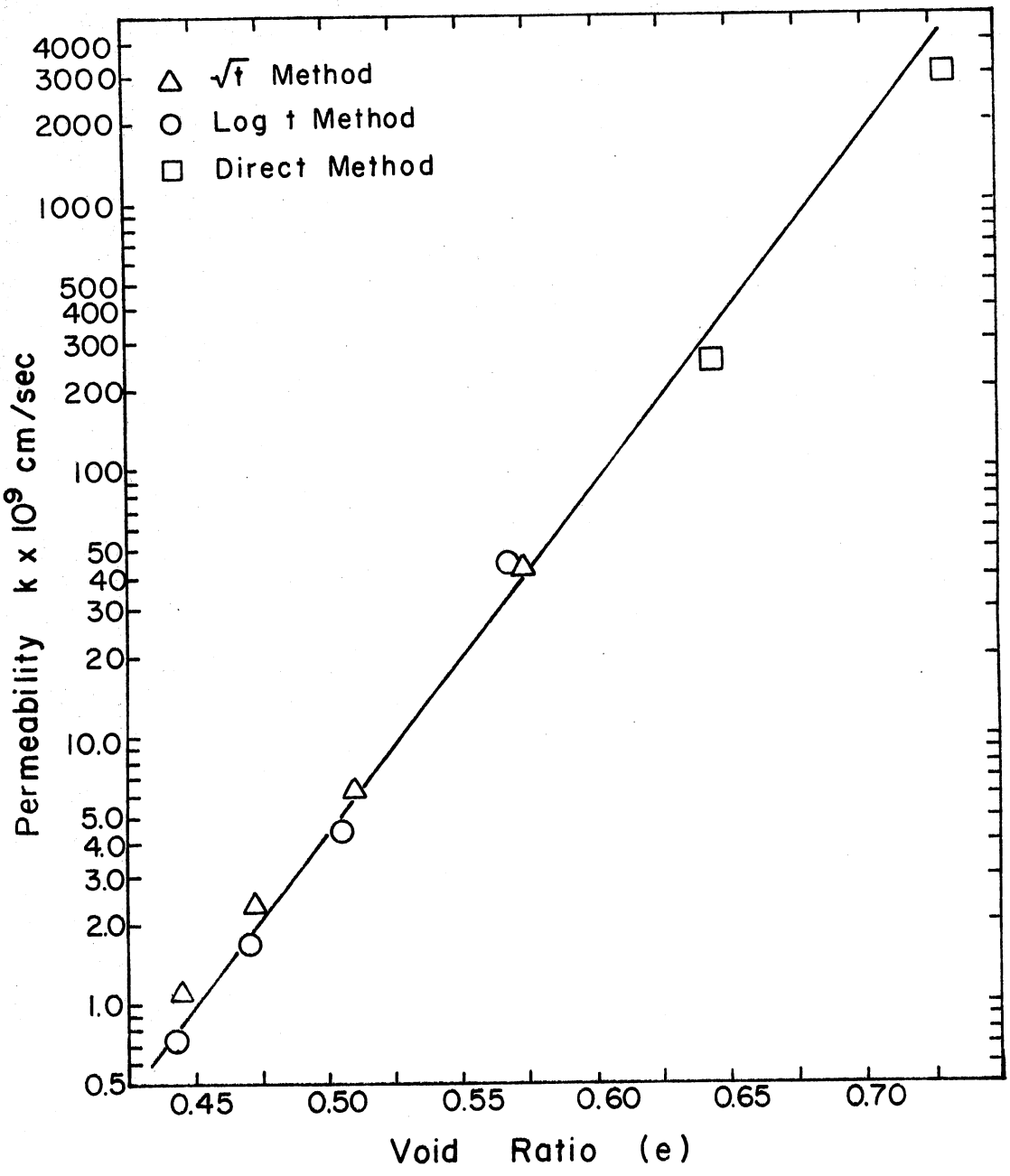


Figure 53. Permeability-Void Ratio Relationship for Undisturbed Silty Clay Sample From Ski-Island (25'-26')

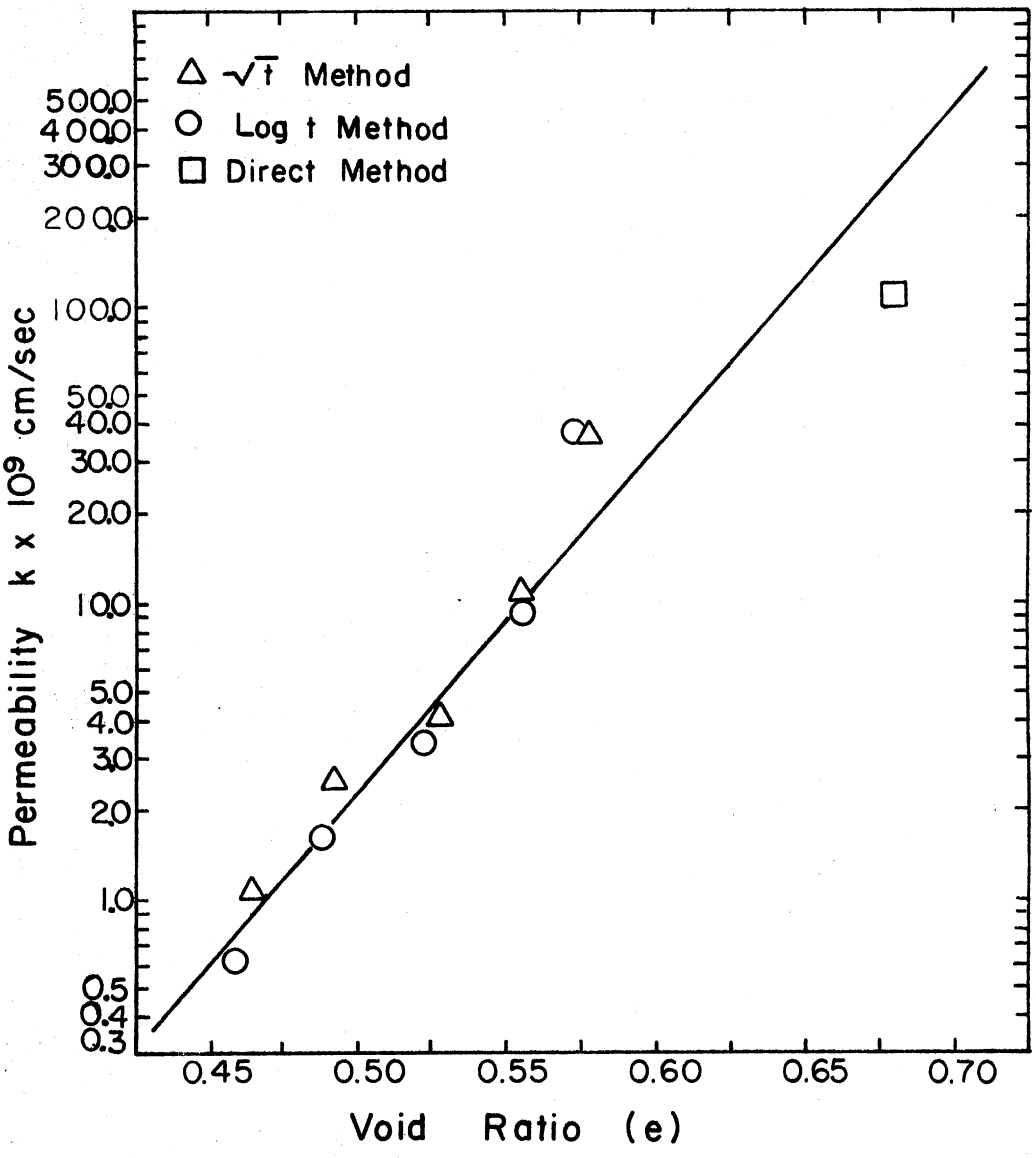


Figure 54. Permeability-Void Ratio Relationship for Undisturbed Silty Clay Sample From Ski-Island (9'-10')

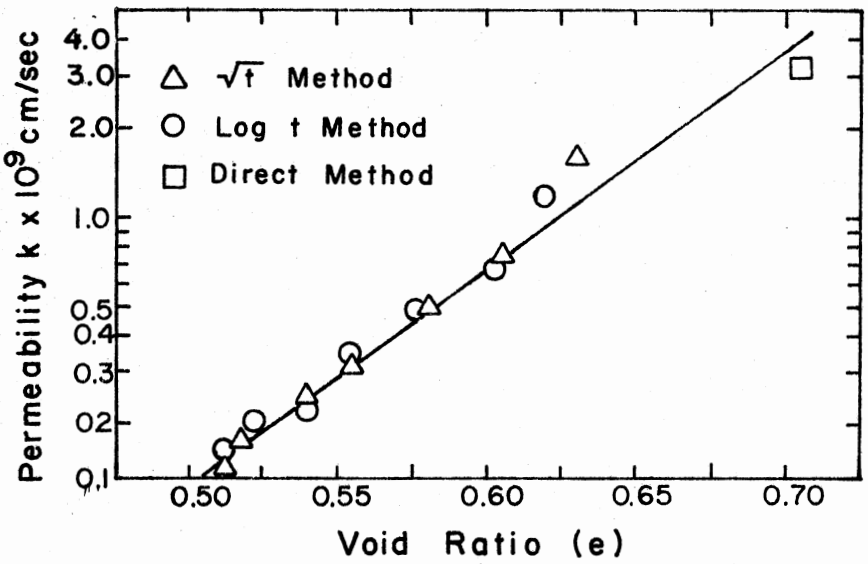


Figure 55. Permeability-Void Ratio Relationship for Undisturbed Stiff Clay Sample From Red Rock, Oklahoma

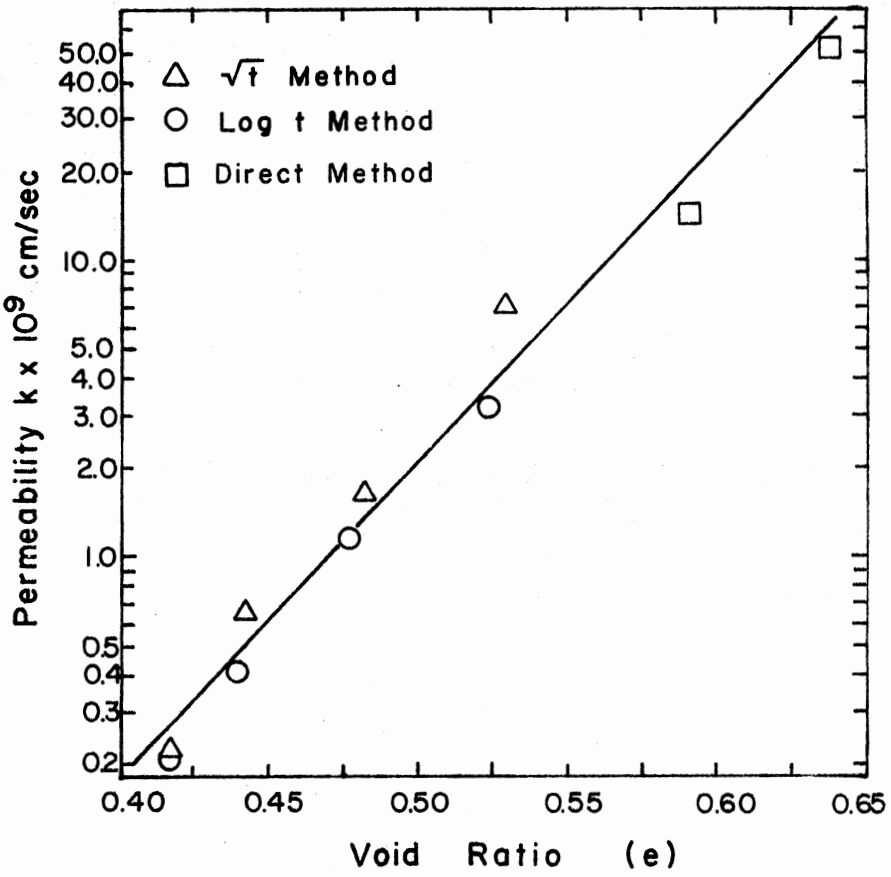


Figure 56. Permeability-Void Relationship for Undisturbed Silty Clay Sample From Hefner Dam

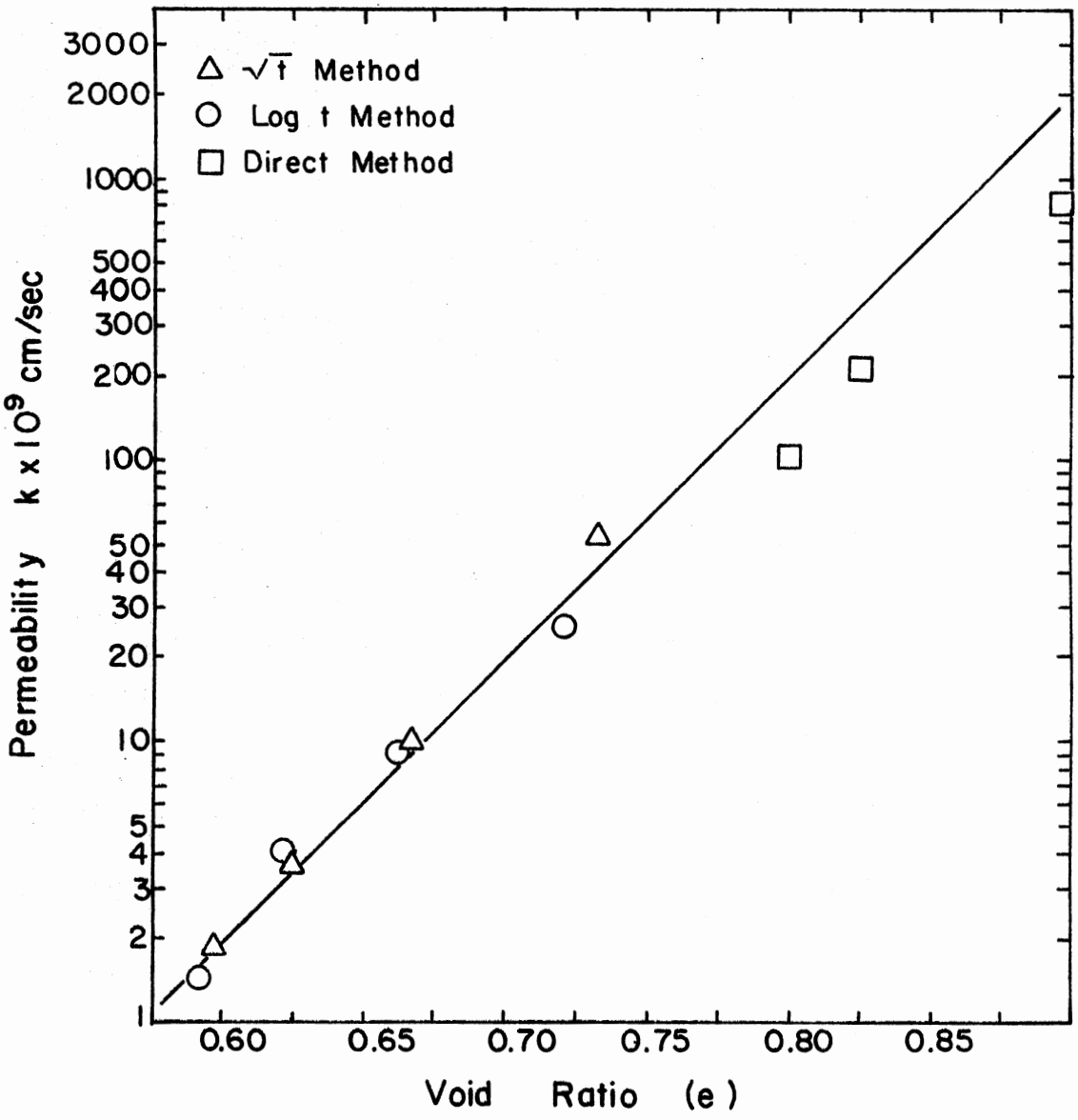


Figure 57. Permeability-Void Ratio Relationship for Undisturbed Sandy Clay Sample From Muskogee, Oklahoma

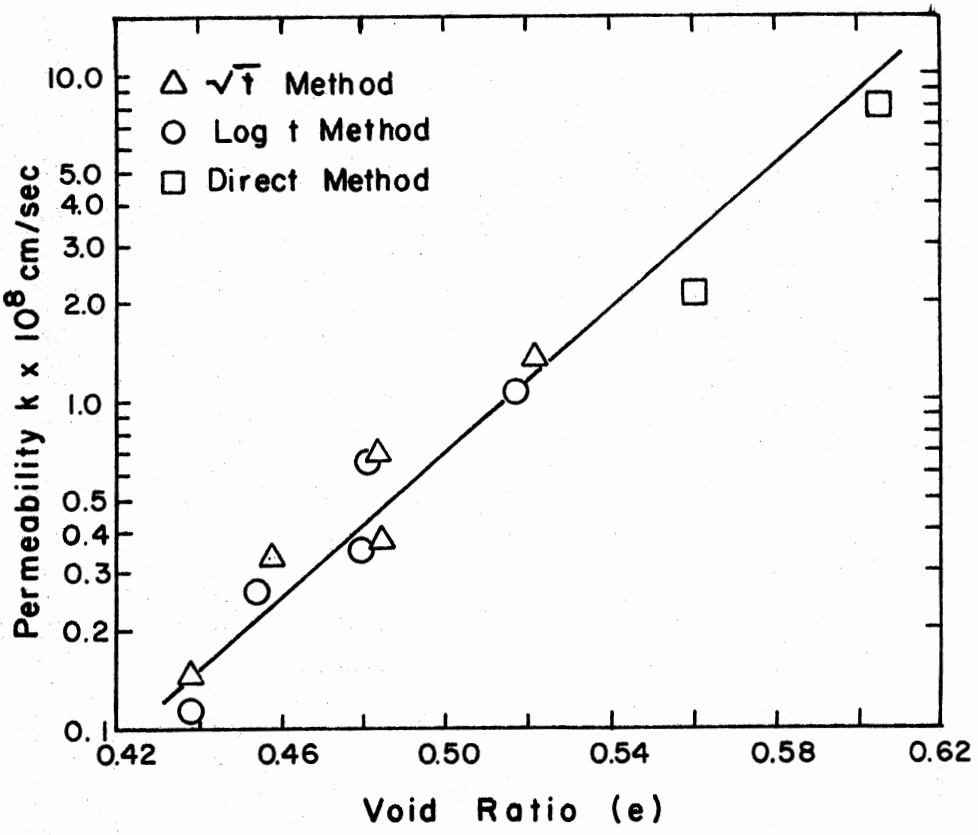


Figure 58. Permeability-Void Ratio Relationship for Undisturbed Sandy Clay Sample From Stillwater Hospital

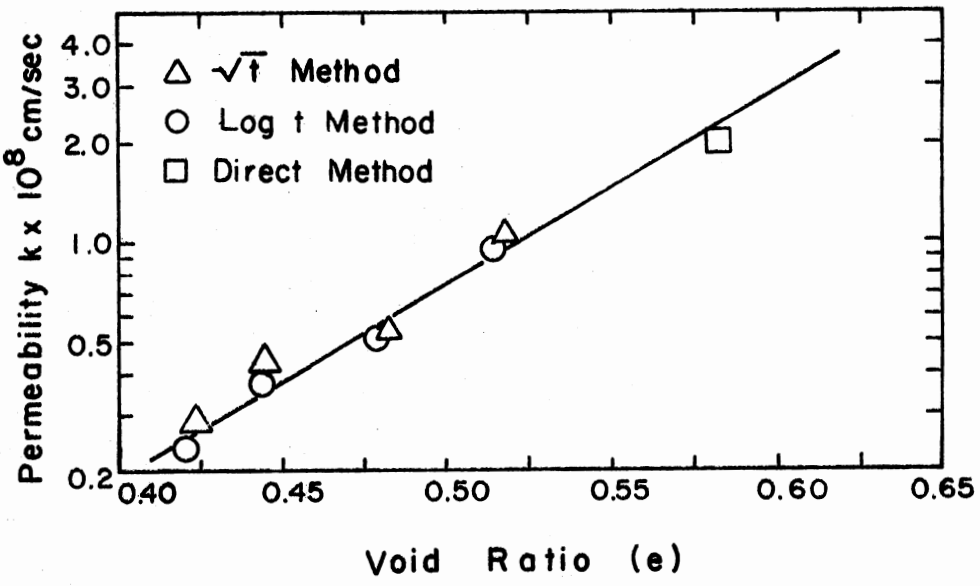


Figure 59. Permeability-Void Ratio Relationship for Undisturbed Silty Clay Sample From Oklahoma State University Campus, Stillwater

$\frac{1}{2}\left(e_{100} + e_o\right)$ used in the analysis.

Since a little volume change was sometimes found to have occurred in permeameter samples at the end of the experiment, the first statement seems to offer a reasonable explanation for at least some of the discrepancy.

A quantitative comparison of the results is given in Tables V and VI, in which the permeabilities determined by each method and the difference as a percent of the permeability from the direct method are shown for specimens having the same void ratio.

A comparison of the tabulated results shows that the difference between permeabilities found by the two methods is greater for undisturbed than for compacted samples. For reasons previously stated, this result was to be expected. But the results, in general, are in excellent agreement. The difference between the two methods does not exceed 145 percent for the undisturbed samples, or 55 percent for the compacted samples. Considering the many uncertainties, simplifying assumptions, and possible errors involved in the theory and execution of a consolidation test, this difference is quite reasonable. Thus, it is demonstrated that for a fairly broad range of clay soils, the indirect determination of permeability from the results of a consolidation test provides a very good estimate of the permeability of the soil. From a practical viewpoint, it is generally sufficient to know only the order of magnitude of the permeability, and, indeed, it is unrealistic to expect to gain any more accurate knowledge of soil permeability for use under field conditions.

It has been suggested (71) that a linear relationship exists between the porosity, n, and the permeability of soils. To evaluate

TABLE V

COMPARISON OF THE RESULTS OF DIRECT AND INDIRECT PERMEABILITY DETERMINATIONS FOR COMPACTED SAMPLES

Soil	K Direct Method (cm/sec)	K Indirect Method (cm/sec)	Percent Difference
Camargo Bentonite	8.4×10^{-9}	13.0×10^{-9}	+ 55
Burleson clay	2.9×10^{-9}	2.3×10^{-9}	- 21
Union City clay #1	13.0×10^{-0}	18.0×10^{-9}	+ 30
Union City clay #2	7.0×10^{-0}	8.2×10^{-9}	+ 17
Stillwater brown clay	2.2×10^{-9}	2.6×10^{-9}	+ 18
Stillwater dark brown clay	8.5×10^{-9}	12.0×10^{-9}	+ 41
60% Summit clay + 40% loess	4.5×10^{-9}	6.4×10^{-9}	+ 42
60% Summit clay + 40% uniform sand	2.8×10^{-9}	3.8×10^{-9}	+ 36
60% Union City clay + 34% W. G. sand	4.6×10^{-9}	4.0×10^{-9}	- 13

TABLE VI

COMPARISON OF THE RESULTS OF DIRECT AND INDIRECT PERMEABILITY DETERMINATIONS FOR UNDISTURBED SAMPLES

Soil	Direct Method (cm/sec)	Indirect Method (cm/sec)	Percent Difference
Ski Island silty clay 25'-26'	2.9×10^{-10}	4.0×10^{-6}	+ 38
Ski Island silty clay 9'-10'	1.1×10^{-7}	2.7×10^{-7}	+145
Hefner Dam silty clay 44'-45'	5.2×10^{-8}	6.4×10^{-8}	+ 23
Stillwater Hospital sandy clay 6'-7'	8.0×10^{-8}	10.0×10^{-8}	+ 25
Muskogee sandy clay 1.5'-2.5'	8.5×10^{-7}	18.0×10^{-7}	+112
Red Rock stiff clay 15'-16'	3.1×10^{-0}	3.8×10^{-9}	+ 23
O.S.U. Campus silty clay 2'-3'	2.0×10^{-8}	2.3×10^{-8}	+ 13
Northeast Stillwater silty clay 6'-7'	8.3×10^{-8}	12.0×10^{-8}	+ 43

the validity of this proposition in respect to clay soils, the permeabilities obtained from consolidation tests of compacted samples were plotted against the average porosities of the samples. The resulting graphs are given in Figures 60 through 68. None of the plots shows a straight line relationship, and all of them exhibit an upward concavity. Thus, it is seen that permeability increases more rapidly than does porosity, and that the effect becomes increasingly disproportionate as the porosity increases. It seems probable that permeability and porosity are directly proportional for sands, but this is clearly not the case for fine-grained soils, in which several quite different factors influence the permeability.

Further analysis of the plots of log k versus e indicates that the general relationship expressed in Equation (13) is valid for all samples. The values of the constants a' and b' are related to physical properties of the soil, especially to the compressibility index, C_c. Analysis of the results for eight of the compacted samples shows that a' is related to the plasticity of the soil. The higher the PI, the lower a'. It is believed that further study with more data can provide a useful approximation of the relationship between a' and b' and the physical and chemical properties of the soil. Since the scope of this study was satisfied by relatively few samples, it has not been possible to derive relationships reliably. However, a tentative relationship between a' and the PI was developed, and is illustrated in Figure 69. Future research toward establishing valid expressions for these constants in terms of the soil properties is strongly recommended.

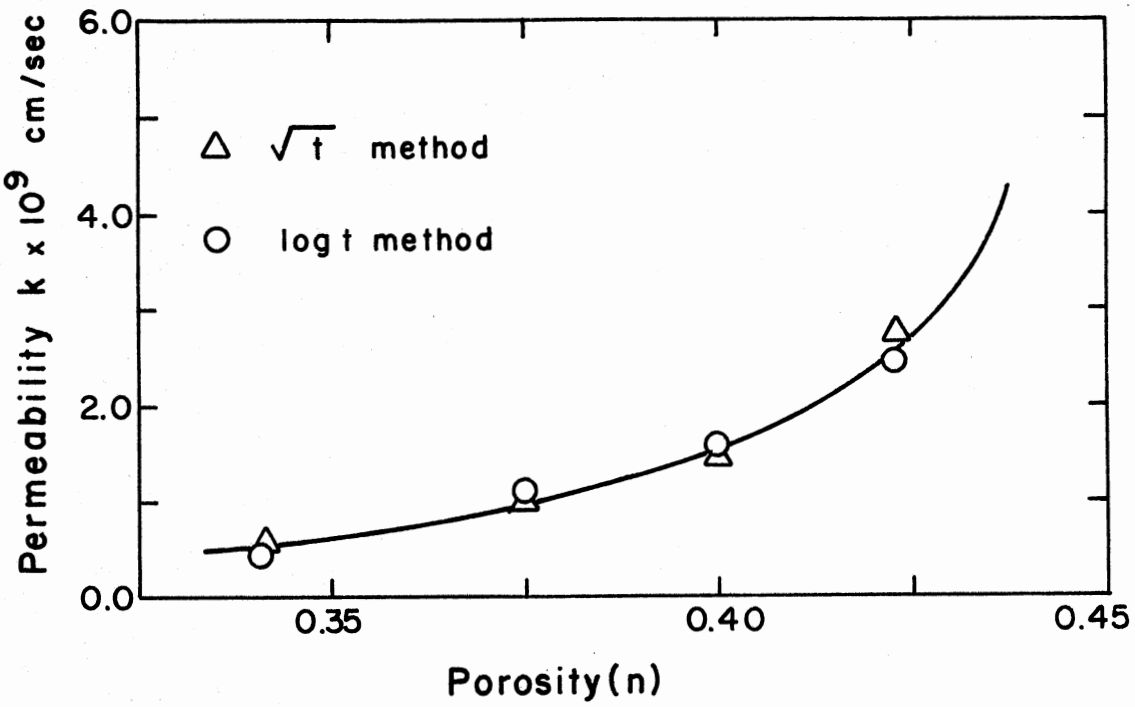


Figure 60. Permeability-Porosity Relationship for Compacted Green Clay

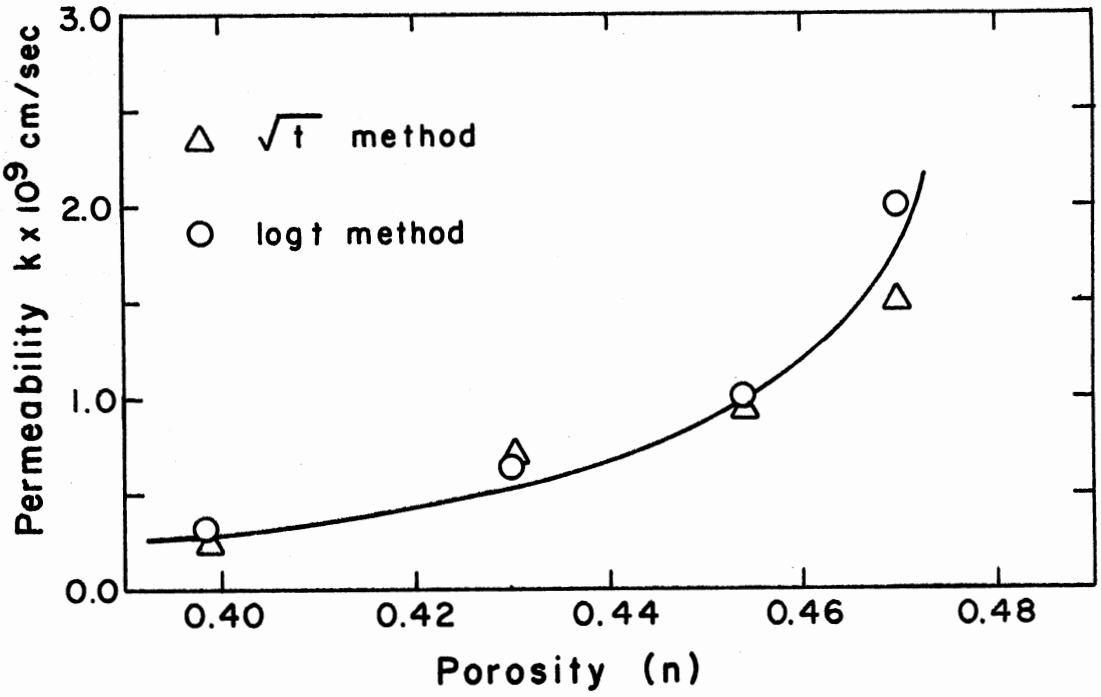


Figure 61. Permeability-Porosity Relationship for Compacted Black Clay

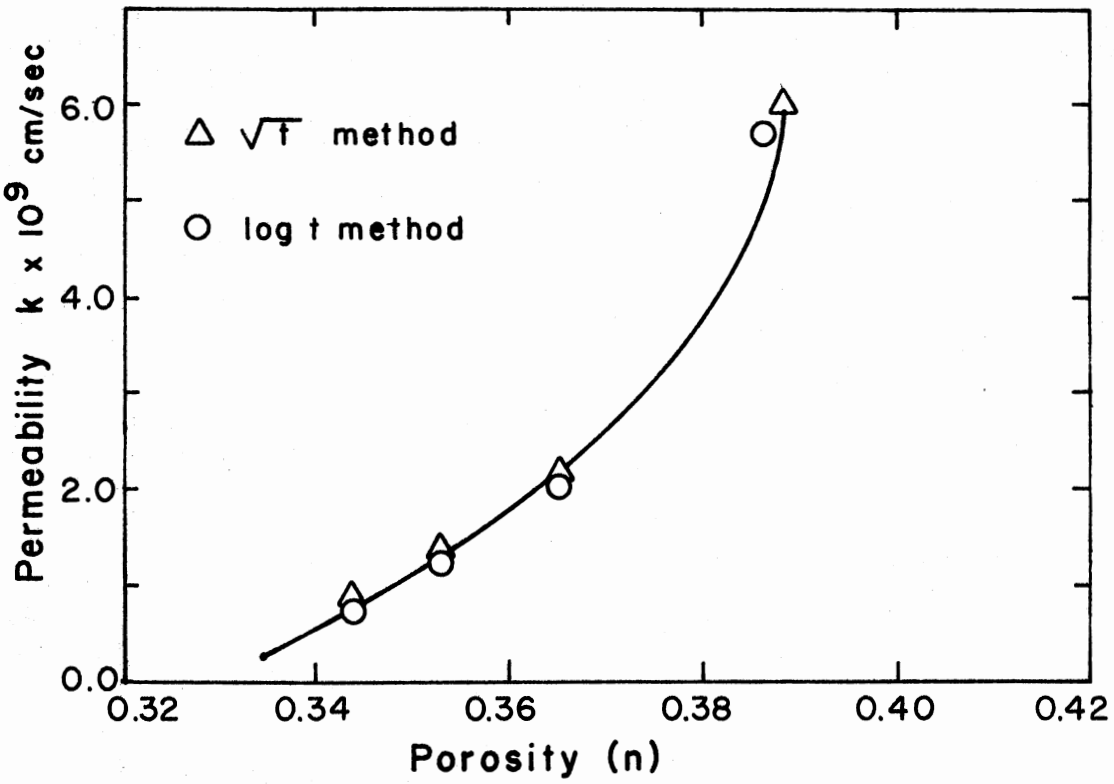


Figure 62. Permeability-Porosity Relationship for Compacted Red Clay No. 1

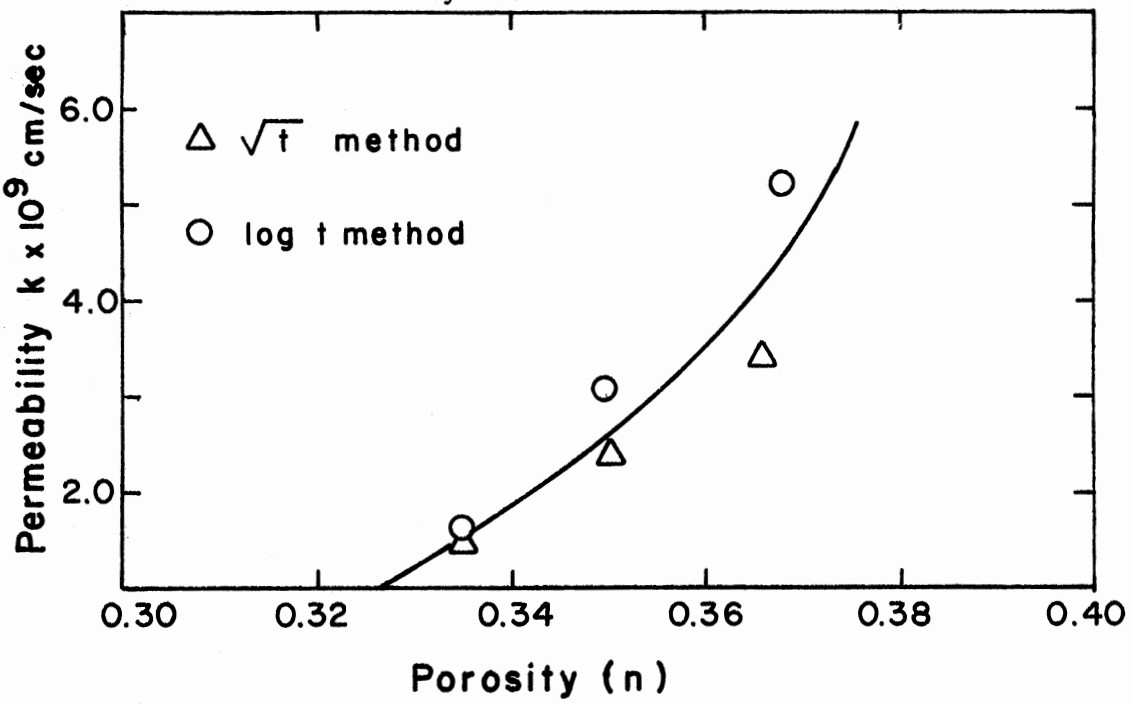


Figure 63. Permeability-Porosity Relationship for Compacted Red Clay No. 2

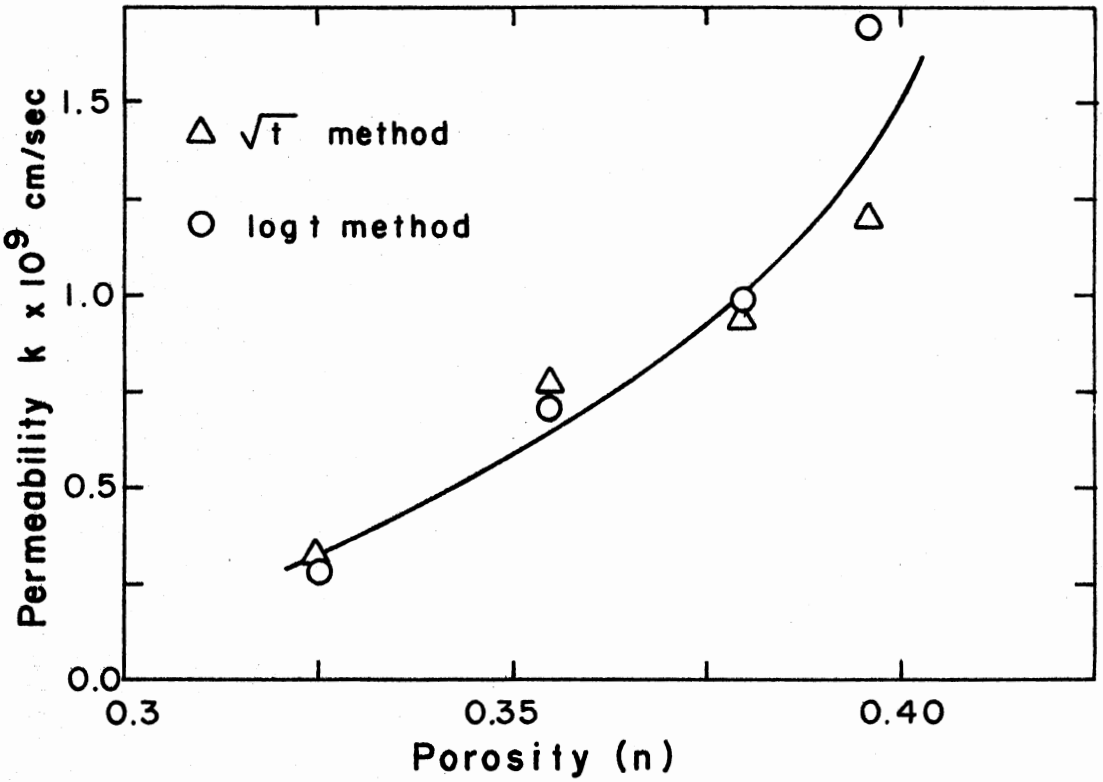


Figure 64. Permeability-Porosity Relationship for Compacted Brown Clay

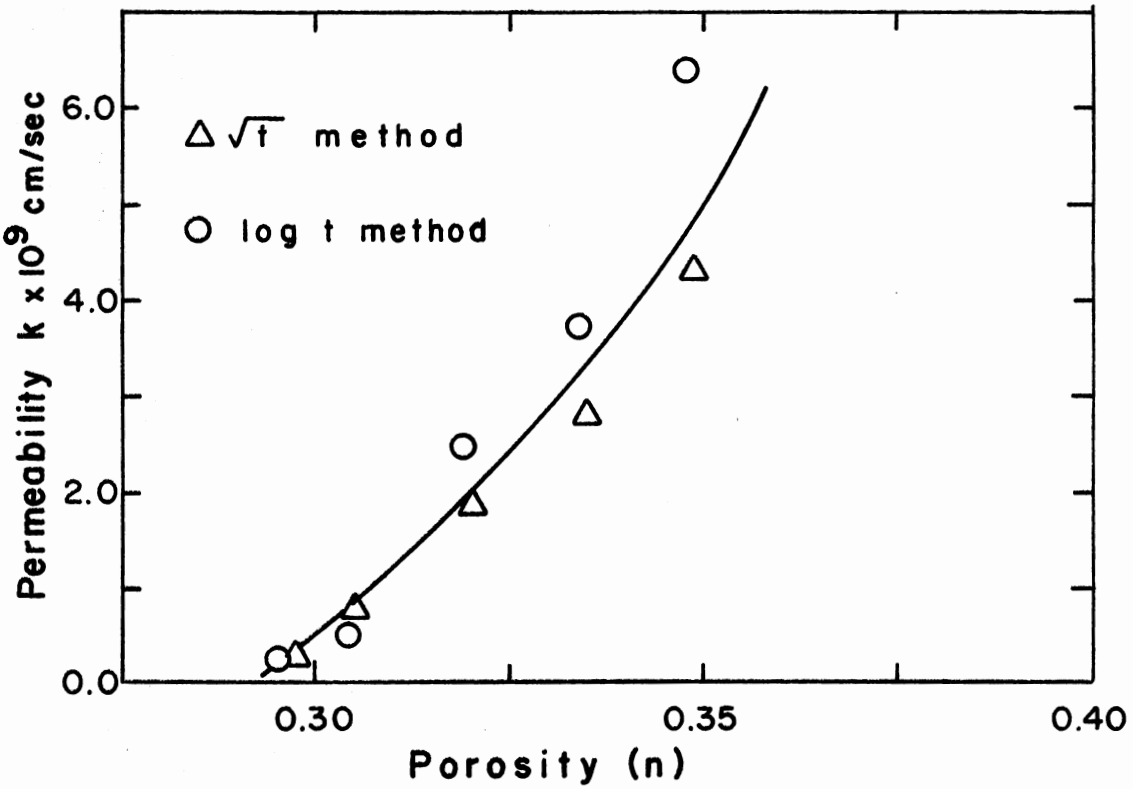


Figure 65. Permeability-Porosity Relationship for Compacted Dark Brown Clay

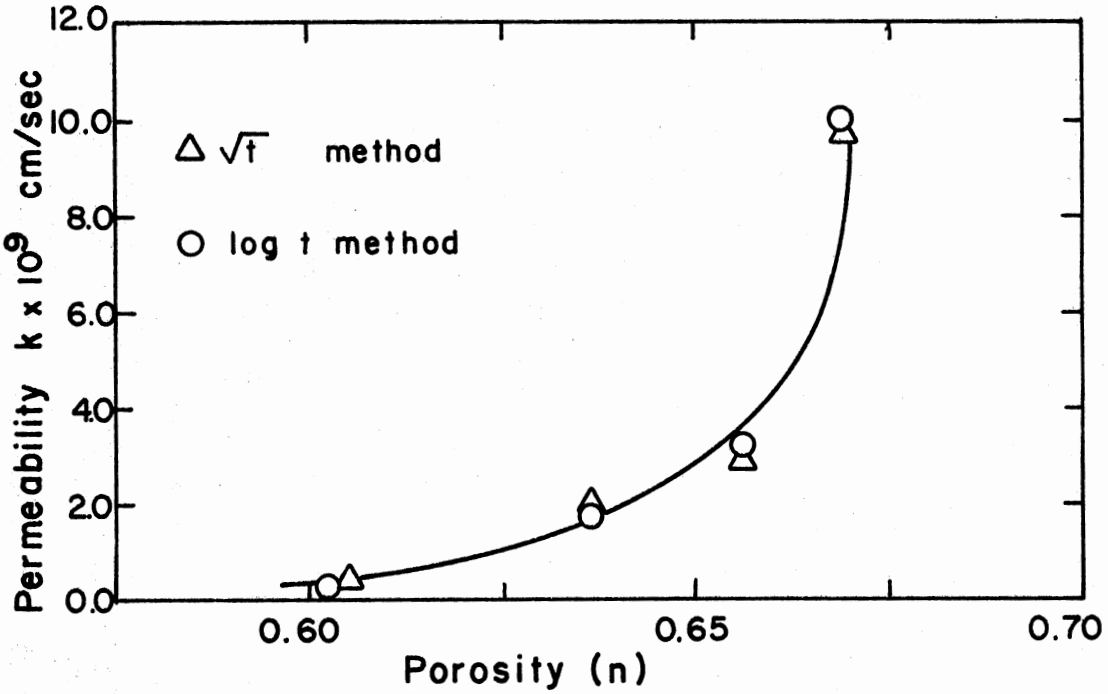


Figure 66. Permeability-Porosity Relationship for Compacted White Clay

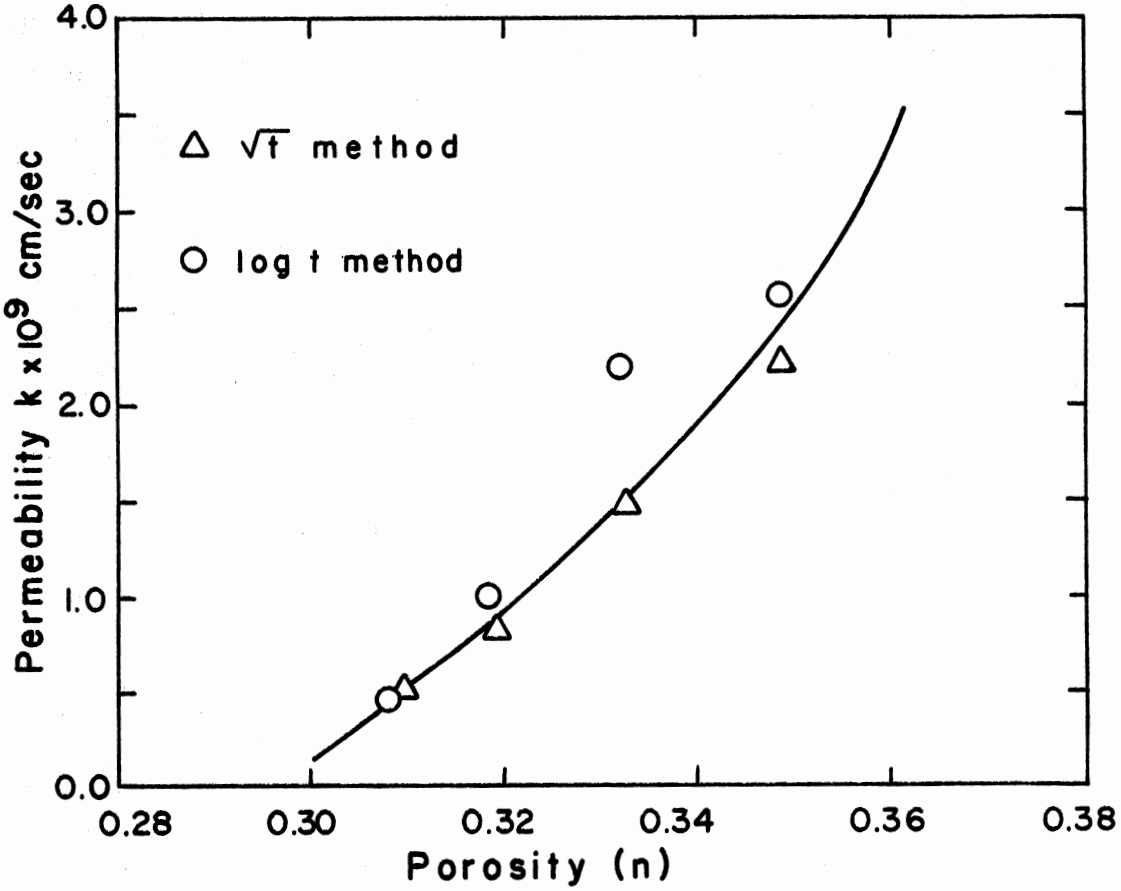


Figure 67. Permeability-Porosity Relationship for Compacted Sample of 60 Percent Green Clay + 40 Percent Loess

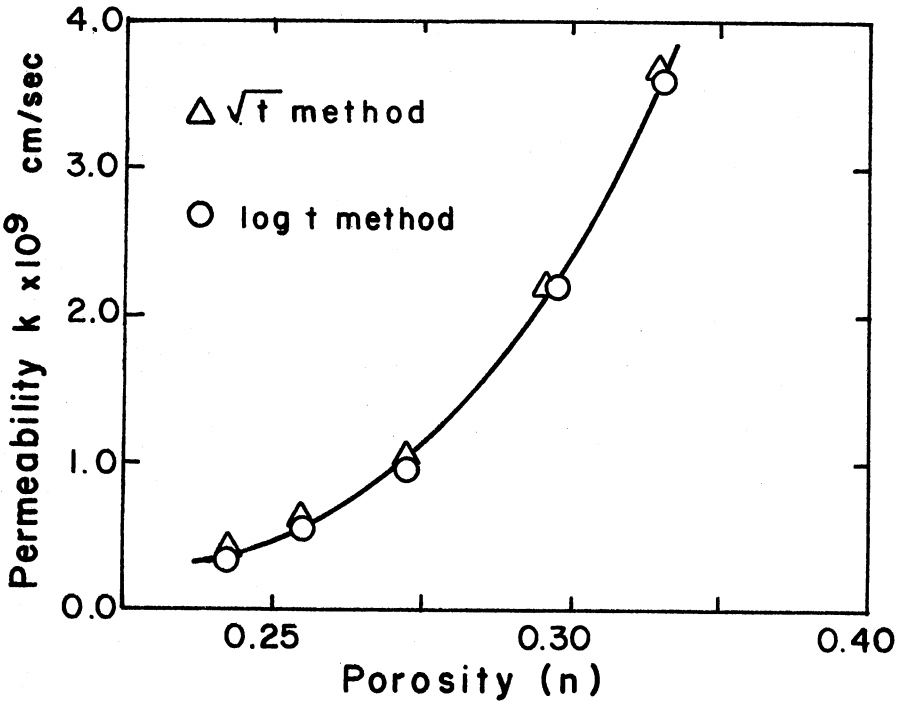


Figure 68. Permeability-Porosity Relationship for Compacted Sample of 60 Percent Green Clay + 40 Percent Uniform Sand

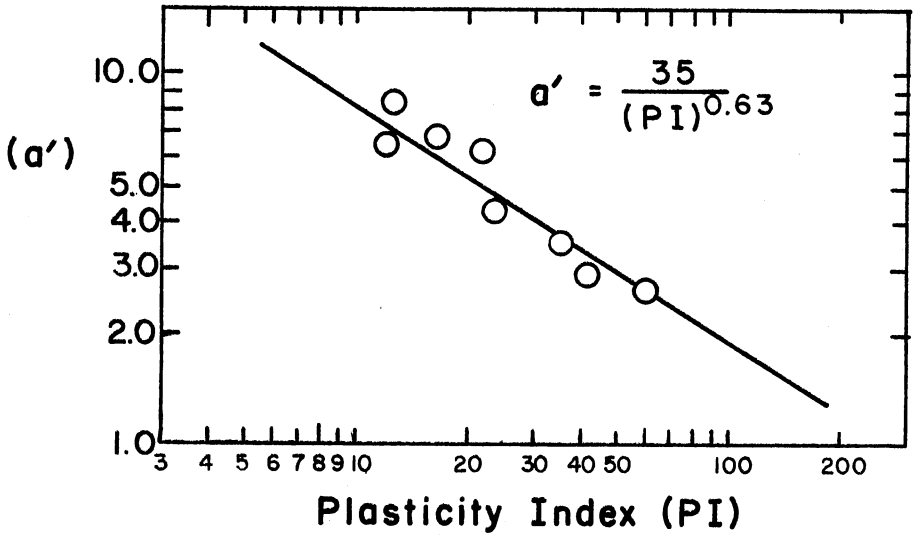


Figure 69. Relationship Between Constant a' in Equation (13) and PI

CHAPTER V

CONCLUSIONS

The primary objective of this research was to compare the results obtained from direct and indirect methods for determining the coefficient of permeability of clay soils. The materials used in the study comprised a broad variety of compacted natural and fabricated soils and undisturbed natural soils. Excellent agreement was found between the coefficient of permeability calculated from consolidation test data and that determined directly from the measured flow of water in a permeameter. It is concluded that the indirect determination of soil permeability, using consolidation test data, is as reliable as any other laboratory procedure to predict the permeability of clay soils for use in field applications.

The following specific conclusions pertaining to soil permeability and its determination follow directly from the results of this investigation.

1. The permeability of a clay soil sample in the permeameter decreases with time as the flow of water continues. This is believed to result primarily from the migration of finer particles and the creation of a layer of lower permeability in the bottom part of the sample. After the initial decrease has occurred, a subsequent increase in permeability (of lesser magnitude than the prior decrease) is frequently observed. This is believed to be due to the dissolution and

removal of entrapped air by water flow under high pressure.

2. While the permeability of a soil element in a consolidometer varies with time and position, the average premeability of the sample over a given time is a function of the applied consolidation pressure. The greater the consolidation pressure, the lower the permeability. It has been proven both analytically and experimentally that the following relationship exists between the permeability of the sample and applied consolidation pressure in a consolidation test:

$$\log k = b - a \log p \qquad (11)$$

3. The linear log-log relationship of equation (11) can be used for estimating the permeability of a layer of soil that has been consolidated under a known pressure if soil constants "a" and "b" have been experimentally evaluated.

4. Analytical and experimental results show that a straight line relationship exists between logarithm of permeability and void ratio of the sample, i.e.,

$$\log k = a'.e + b' \qquad (13)$$

This finding is not new, having been established previously from permeameter studies, but it was reinforced in this study by the use of an indirect method for determining permeability (consolidation method). Such an agreement between direct and indirect procedures indicates the validity of the indirect method for determining the permeability of clay.

5. The permeability of a given sample of compacted clay, as determined from the results of a consolidation test, is less than that of an

initially identical sample tested in a permeameter. The reason for this is that the average void ratio of the sample during consolidation is less than that of the permeameter sample, which remains essentially at its initial void ratio.

6. The results of direct and indirect permeability tests can be correlated satisfactorily on a e-log k plot by referencing the k determined indirectly to the average void ratio, $\frac{1}{2}(e_o+e_{100})$, corresponding to the pressure increment for which k was determined.

7. The coefficients of permeability obtained from direct permeability tests generally are slightly lower than those found from consolidation tests, but the differences are not significant. The maximum difference was less than 150 percent. In view of the difficulties involved in assessing the permeability of fine-grained soils, this constitutes excellent agreement and, in any case, the difference has no practical significance.

8. This study confirms the general validity of the theory of consolidation for describing the behavior of laboratory consolidometer specimens (see Appendix A).

9. Among the available methods for determining the coefficient of consolidation, c_v, the square root of time and logarithm of time curve-fitting methods both proved to be reliable in the context of this investigation, producing no significant difference in results. The Scott method, however, proved to be entirely unsuited for use in this study. Permeabilities so determined were too great by about one order of magnitude.

The two acceptable curve-fitting techniques can be applied comparably only with the greatest difficulty to the data for the first one or

two load increments of a consolidation test. The erratic and relatively small deformations of unsaturated samples under light load lead to difficulties in interpretations. For the heavier load increments, where deformations are large and the degree of saturation approaches 100 percent, the sample response is much more consistent, and both techniques can be used with confidence.

10. The data from carefully performed consolidation tests may be used independently and with considerable confidence to estimate the coefficient of permeability of homogeneous formations of fine-grained soils.

CHAPTER VI

RECOMMENDATIONS FOR FUTURE RESEARCH

1. The relationship between consolidation pressure and permeability appears to have potential for useful application, particularly in field situations involving thick deposits of normally consolidated clay. Equation (11) ($\log k = b - a \log p$) may be restated in the following form, where γ = unit weight of soil, and h = depth:

$$\log k = b - a \log \gamma h$$

or, alternatively, if γ is treated as a constant,

$$\log k = (b - a \log \gamma) - a \log h$$

or

$$\log k = c - a \log h \qquad (14)$$

It is believed that the constants a, b, and c in the above relationships can be expressed in terms of the properties of a particular soil--perhaps properties that can be determined easily or routinely. It is recommended that the further research required to establish such relationships be undertaken.

2. An investigation similar to that above is recommended in respect to the constants a' and b' in the expression $\log k = a'e + b'$. A tentative relationship between a' and the plastic index of the soil

was found in the present study; however, the conclusion is far too tenuous to be acceptable. An extensive experimental program based on direct permeability tests appears to be required.

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APPENDIX A

TERZAGHI'S THEORY OF CONSOLIDATION

"Consolidation" has been defined as the process of deformation (volume change) of a clay layer under applied loads, due to the flow of pore water out of the voids. The mathematical expression for the theory of consolidation was developed in 1910-1920 by K. Terzaghi, and was based on the following assumptions:

1. The soil layer is homogeneous.
2. Deformation of the soil layer is due entirely to volume change.
3. Volume change of the voids is due to the flow of pore water out of the soil.
4. Soil is completely saturated.
5. Water flows only in the direction of load application.
6. Load is applied in one direction, and deformation takes place in the same direction. In other words, the soil layer is restrained against lateral deformation (one dimensional consolidation).
7. The total change in thickness of the layer during the consolidation process is small in comparison with the overall thickness of the layer.
8. The boundary is defined as the drainage surface which does not resist water movement out of the soil.
9. The coefficient of consolidation is constant during the process.
10. Soil grains and water both have negligible compressibility.
11. Darcy's law is valid for flow of water in the consolidation process.
12. Pressure - void ratio relationship could be written in the form of $a_v = -\frac{de}{dp}$, where a_v is the coefficient of

compressibility, de is the change in void ratio, and dp is the change in consolidation pressure.

The validity of some of the above assumptions has often been questioned in terms of their practical applicability. Some of the assumptions are obviously not valid for layered deposits, non-homogeneous soils, or very soft, weak, and compressible materials in their natural environments. However, all of the assumptions (possibly excepting No. 9) are satisfied quite well in a competently performed laboratory consolidation test. The test is thus quite suitable for purposes of the permeability studies described in the body of this dissertation.

Mathematical Derivation of Terzaghi's Equation (81)

The basic equation for steady state flow of water in a saturated soil has been given as

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad (1)$$

The equation is valid for the case of a saturated medium in which no volume change occurs. For the consolidation process, where volume change takes place, the rate of change in the volume of water (actually volume of voids) is given by

$$\left(k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} \right) dxdydz$$

Considering the one dimensional character of the flow and deformation, the above expression is written in the following form:

$$k \frac{\partial^2 h}{\partial z^2} dxdydz \tag{2}$$

where

dx, dy, dz are the dimensions of an elemental volume of the soil mass.

k = permeability of the soil

h = potential head applied to soil-sater system

z = distance measured in the direction of flow and deformation.

The volume of the soil element is dxdydz, and the volume of the pores is represented by $dxdydz \cdot \frac{e}{1+e}$. It was assumed that the soil grains and water are incompresible; therefore, all of the deformation or volume change occurs in the voids. Thus, the expression for the time rate of change in volume of pores is given by:

$$\frac{\partial}{\partial t} dxdydz \frac{e}{1+e_o} \tag{3}$$

But $\frac{dxdydz}{1+e_o}$ is the volume of solids and is constant; therefore (3) can be written as

$$\frac{\partial e}{\partial t} \cdot \frac{dxdydz}{1+e_o} \tag{4}$$

Equating (4) and (2) gives

$$\frac{\partial e}{\partial t} \cdot \frac{dxdydz}{1+e_o} = k \frac{\partial^2 h}{\partial z^2} \cdot dxdydz$$

or

$$k \frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e_o} \cdot \frac{\partial e}{\partial t} \tag{5}$$

Converting the hydrostatic head (h) to pore pressure (u), $h = \frac{u}{\gamma_w}$, and substituting in equation (5) gives

$$\frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_o} \cdot \frac{\partial e}{\partial t} \tag{6}$$

The relation between applied consolidation pressure, p, and excess pore water pressure, u, is in the form of $dp = - du$, and considering assumption 12, it can be written as

$$de = a_v \cdot du \tag{7}$$

Substituting for de in equation (6) gives

$$\left[\frac{k\ (1+e_o)}{a_v \cdot \gamma_w}\right] \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

Letting

$$\frac{k\ (1+e_o)}{a_v \cdot \gamma_w} = c_v \tag{8}$$

which is called coefficient of consolidation, the general form of the differential equation governing the consolidation theory will be:

$$c_v = \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{9}$$

From equation (8), the coefficient of permeability can be found.

$$k = \frac{a_v \cdot c_v \cdot \gamma_w}{1+e_o} \tag{10}$$

where

$a_v = \frac{-\Delta e}{\Delta p}$ = coefficient of compressibility

c_v = coefficient of consolidation

e_o = initial void ratio of the sample

γ_w = unit weight of water

The solution of equation (9) has been given in the following form (96):

$$u = (p_2-p_1) \sum_{N=0}^{N=\infty} \left\{ \frac{4}{(2N+1)\pi} \operatorname{Sin}\left[\frac{(2N+1)\pi}{2} \cdot \frac{z}{H} \right] \right\} e^{-\left[\frac{(2N+1)^2\pi^2}{4} \cdot c_v \frac{t}{H^2} \right]} \tag{11}$$

where

N = whole numbers from zero to infinity

e = base of natural logarithms

u = excess pore pressure at time t and location z

t = time after consolidation process has started

z = vertical distance from drainage surface to any point

p_2-p_1 = applied pressure

$c_v \frac{t}{H^2}$ = T = time factor

Time factor (T) is a dimensionless factor, related to the percent of conslidation and its values are found from the following equation:

$$u\% = 100 \left\{ 1 - \sum_{N=0}^{N=\infty} \frac{8}{(2N+1)^2\Pi^2} \cdot e^{-\left[\frac{(2N+1)^2\pi^2}{4} \cdot T \right]} \right\} \tag{12}$$

Values of T for different degrees of consolidation have been calculated, and are shown in Table VII.

TABLE VII

TIME FACTOR VERSUS % CONSOLIDATION

U%	T	U%	T
0	0.000	55	0.238
10	0.008	60	0.287
15	0.018	65	0.342
20	0.031	70	0.405
25	0.049	75	0.477
30	0.071	80	0.565
35	0.096	85	0.684
40	0.126	90	0.848
45	0.159	95	1.127
50	0.197	100	∞

The important factor to be determined in a consolidation test is the coefficient of consolidation, c_v, which is used for calculation of the time required for a given percent of consolidation, and also to find the coefficient of permeability. Various proposed methods for determining c_v are discussed in Appendix B.

APPENDIX B

DETERMINATION OF COEFFICIENT OF CONSOLIDATION, c_v, AND CALCULATION OF COEFFICIENT OF PERMEABILITY, k

The significance of c_v was discussed in Appendix A, and methods for its determination will be discussed here. Three methods have been proposed for determining c_v. Two of them, namely, square root of time method and logarithm of time method, are most commonly used. The third method, suggested by T. F. Scott (72) has not been used frequently. These methods are discussed separately, and their application is illustrated by a numerical example.

Square Roof of Time Fitting Method,
suggested by Taylor

In this method, time-deformation data collected from a consolidation test are plotted with $\sqrt{t}$ on the x axis, and the dial gauge reading (deformation) on the y axis. Normally, the resulting plot is linear up to a certain time, and then becomes concave upward. The straight line portion of the curve is extended back to intersect the zero time axis and the dial reading at this point is taken as (d_s), the dial reading at zero percent consolidation. From this point (d_s), a straight line having an inverse slope of 1.15 times the first line is drawn to intersect the curve at some point d_{90}. The coordinate of this point on x axis is assumed to be the time required for 90 percent consolidation, t_{90}. Having t_{90}, c_v can be found easily from the following expression:

$$c_v = \frac{0.848H^2}{t_{90}}$$

where H is the half-thickness of the sample (for two face drainage). The distance between d_s and d_{90} is then divided into 90 divisions and d_{100} is found by direct extrapolation. Dial reading numbers can be

converted to void ratio using the known height, weight, water content, and specific gravity of the sample in consolidation ring.

Logarithm of Time Fitting Method,
suggested by Casagrande

In this method, a plot of dial reading versus logarithm of time is made from the consolidation test data. Since it is not possible on a log scale to plot a dial reading (d_s) corresponding to time zero, it is necessary to employ a special procedure to find d_s. Up to about 60 percent consolidation, the theoretical U-T curve very closely approximates a parabola; therefore, the early portion of the curve plotted from collected data should be very nearly parabolic. The general properties of a parabola are such that if a point on the curve at time t has an ordinate of y, the ordinate at time 4t will be 2y. Thus, the ordinate distance Δy between two points whose abscissas are t and 4t, is the same as the ordinate distance from the first point (t,y) to the origin. Therefore, the corresponding origin of the consolidation curve (o,d_s) is located by laying off the vertical distance between any two points with abscissas of t and 4t above the point with the abscissa of t and drawing a horizontal line through this point to intersect the y axis (dial readings). It is better to choose t in the early portion of the curve between, say, 15 seconds and one minute. To find d_{100} or the point of 100 percent consolidation, a tangent is drawn to the curve at its point of steepest slope (point of inflection) and extended to intersect the backward extension of the uniformly sloping secondary branch of the curve. The interval between d_s and d_{100} is simply divided into 100 divisions, and d_{50} is halfway between d_s and d_{100}. Having d_{50}, the

time corresponding to 50 percent conslidation, t_{50} is read directly from the time axis and c_v is calculated from the following equation:

$$c_v = \frac{0.197H^2}{t_{50}}$$

Again, the dial reading axis can be converted to void ratio, and e_o and e_{100} found easily.

Scott Method, suggested by
R. F. Scott (72)

According to Scott, only the early portion of the time-deformation curve need be used. Scott defined a factor called compression ratio, C_r, as

$$C_r = \frac{d_s - d_t}{d_s - d_{Nt}} = \frac{U(t)}{U(Nt)}$$

where d_s is the dial reading at time zero, and d_t and d_{Nt} are dial readings at time t and Nt after consolidation has started. Scott presents a series of curves which give C_r as a function of T for various values of N. See Figure 70.

The plot is entered with a known value of C_r (taken from the dial reading-time data), and the corresponding T is read from the abscissal scale. c_v is then calculated from $c_v = \frac{TH^2}{t}$.

The most important advantage of this method is its ease of application and the savings in time required for testing as well as for analysis of the data. In this method, only a few early readings of the

consolidation data are required, which reduces greatly the time needed to conduct a test. Furthermore, the entire dial reading time need not be plotted. As was shown in Table IV, the values of c_v obtained by this method are much greater than those found by the two other methods. Since the validity of the first two methods has been confirmed through long experience and, in this study, by the excellent agreement of calculated coefficients of permeability with those obtained through direct measurement, the validity of the Scott method is doubtful. Its use is not recommended under the present circumstances.

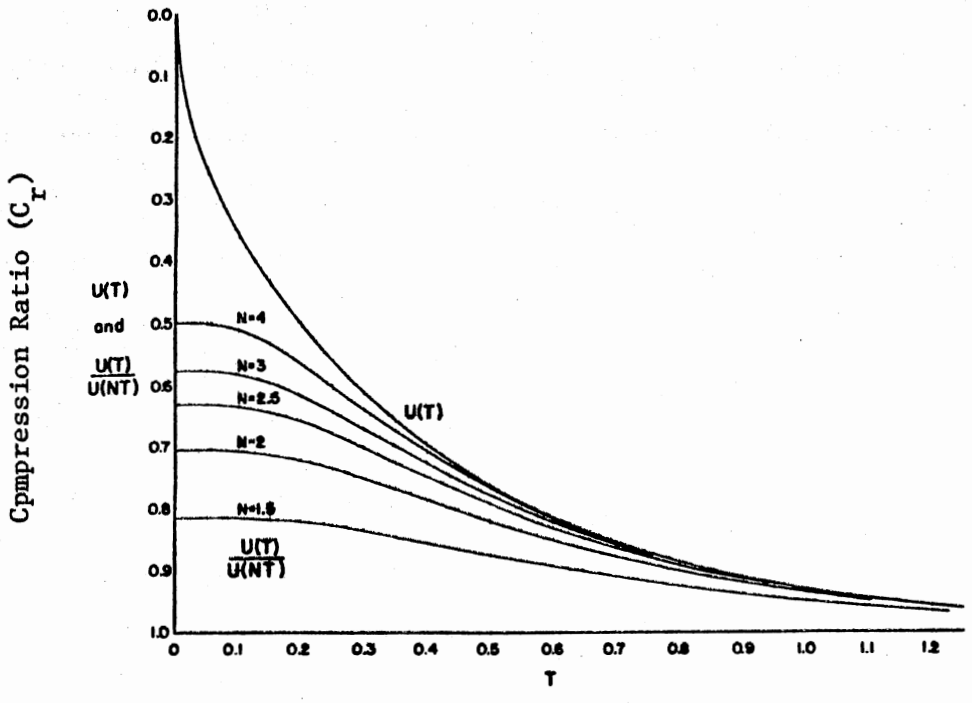

Figure 70. Degree of Consolidation and Consolidation Ratio versus Time Factor

Numerical Example

The dial-reading time data of a consolidation test on a sample of Summit clay under 12 TSF pressure (third loading cycle) is presented in Table VIII. Dial readings are in inches x 10^3. The three procedures described above will be illustrated.

TABLE VIII

DIAL READING TIME DATA FOR SUMMIT CLAY
(12 TSF)

Elapsed Time	Dial Reading x 10^3 in	Elapsed Time	Dial Reading x 10^3 in
0	688.5	15 min	669.3
5 sec	680.3	30 min	664.9
15 sec	679.4	1 hr	659.4
30 sec	678.5	2 hr	652.8
1 min	677.5	4 hr	647.4
2 min	676.3	8 hr	644.5
4 min	674.5	15 hr	643.4
8 min	672.1	24 hr	642.5

The pertinent soil properties are:

Initial water content, w_1 = 25.9%

Final water content, w_2 = 18.9%

Weight of solids, W_s = 126.05%

Specific gravity, G_s = 2.74

Initial height of the sample, H_1 = 2.54 cm

Diameter of the sample, d = 6.35 cm

Area ofthe sample, A = 31.669 cm^2

Height of solids, H_s = $\frac{W_s}{A.G_s.\gamma_w} = \frac{126.05}{31.669 \times 2.74 \times 1}$

= 1.453 cm

Height of voids, H_v = $H_1 - H_s$ = 2.54-1.453=1.087 cm

Initial void ratio, $e_o = \frac{H_v}{H_s} = \frac{1.087}{1.453} = 0.7486$

Log t Method

The data in Table VIII are plotted in Figure 71 (dial reading vs. log t). According to the procedure discussed in Chapter II, values of d_s, d_{50}, and d_{100} have been determined and are shown on the figure.

$d_s = 681.5$

$d_{50} = 663.1$

$d_{100} = 644.7$

From the figure--for $d_{50} = 663.1$, $t_{50} = 37$ min, and for $d_{100} = 644.7$, $t_{100} = 282$ min; e_o (at d_s) $= 0.586$, and e_{100} (at d_{100}) $= 0.522$. With this information, the required calculations may be made:

$$c_v = \frac{T_{50} \cdot H^2}{t_{50}} = \frac{0.197 \left(\frac{2.54}{2}\right)^2}{37 \times 60} = 1.43 \times 10^{-4} \text{ cm}^2/\text{sec}$$

$$a_v = \frac{\Delta e}{\Delta p} = \frac{0.586 - 0.522}{\frac{(12 - 6) \times 10^6}{144 \times (2.54)^2}} = 9.96 \times 10^{-6} \text{ cm}^2/\text{gr}$$

$$k_{ave} = \frac{a_c \cdot c_v \cdot \gamma_w}{1 + e_o} = \frac{9.96 \times 10^{-6} \times 1.43 \times 10^{-4} \times 1}{1 + 0.586} = 0.89 \times 10^{-9} \text{ cm/sec}$$

Square Root Method

The data given in Table VIII are plotted in Figure 72 (dial reading vs. square root of time). The following information is obtained from Figure 72.

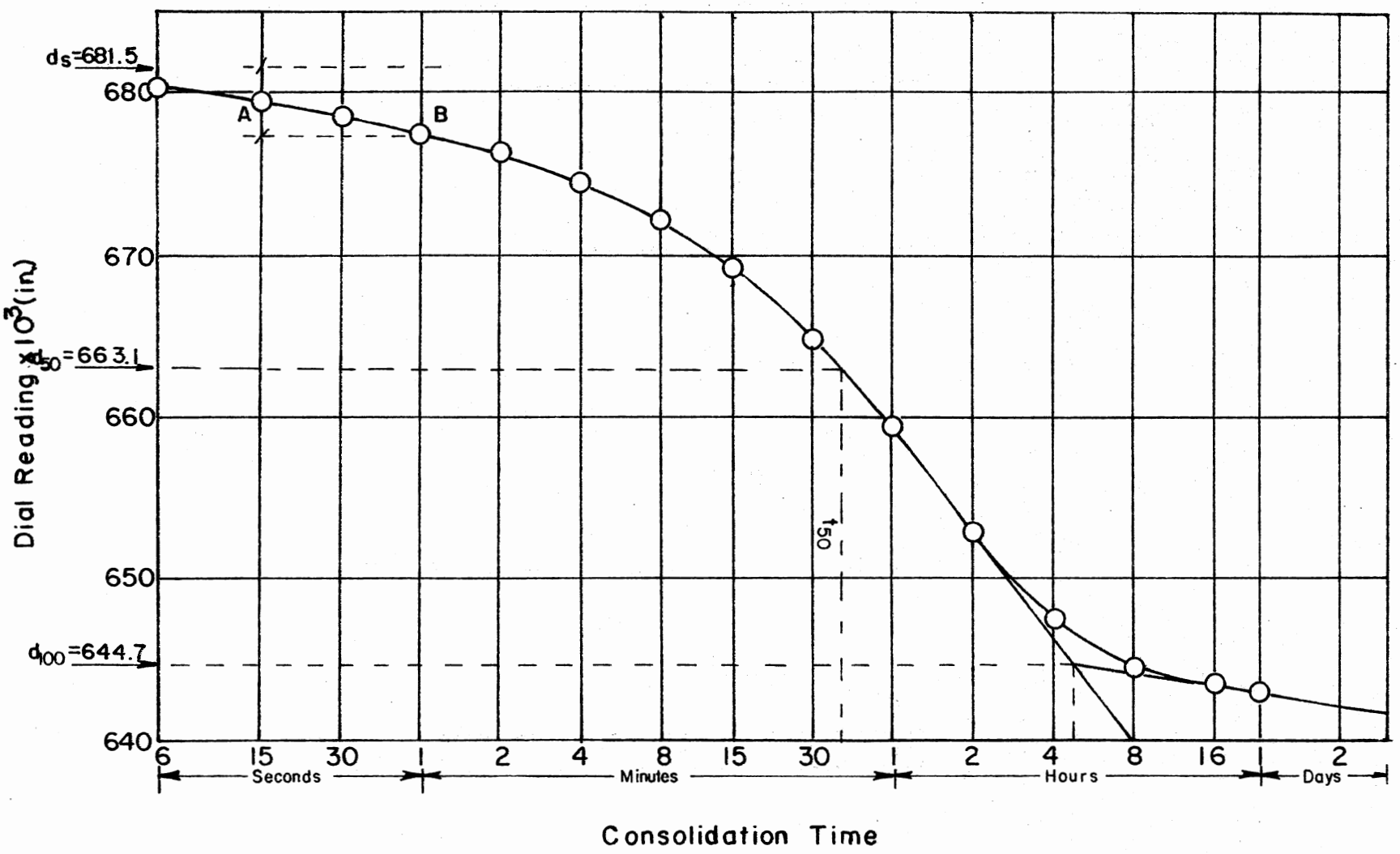

Figure 71. Consolidation Time versus Dial Reading (Log t Method)

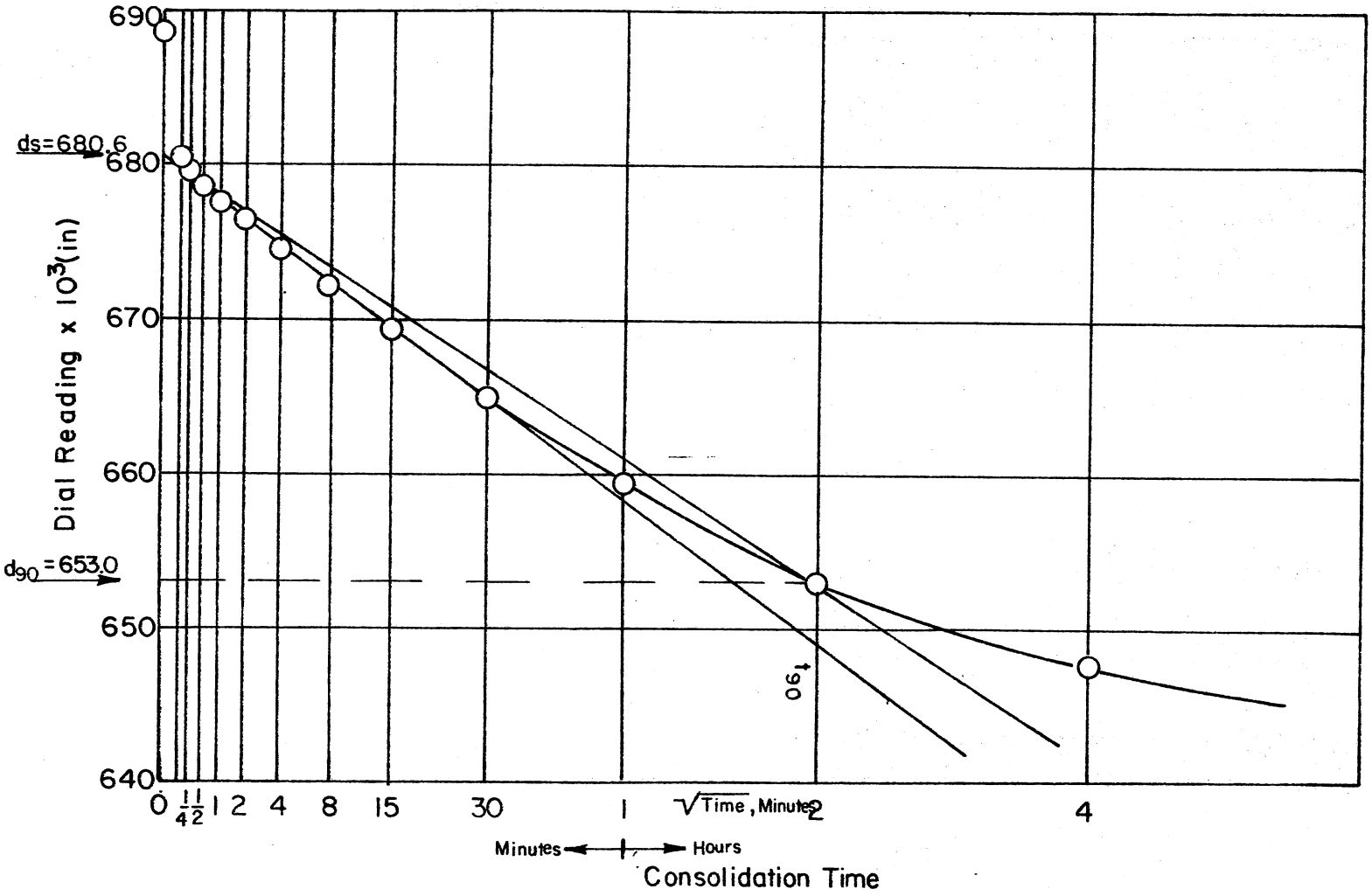

Figure 72. Consolidation Time versus Dial Reading ($\sqrt{t}$ Method)

$d_s = 680.6$ $\quad d_{90} = 653.0$ $\quad t_{90} = 120$ min

$e_o = 0.584$ $\quad e_{100} = 0.531$

Calculations are as follows:

$$a_v = \frac{\Delta e}{\Delta p} = \frac{0.584 - 0.531}{\frac{(12 - 6)\ 10^6}{144(2.54)^2}} = 8.3 \times 10^{-6}\ cm^2/gr$$

$$c_v = \frac{T_{90} \cdot H^2}{t_{90}} = \frac{0.848 \left(\frac{2.54}{2}\right)^2}{120 \times 60} = 1.9 \times 10^{-4}\ cm^2/sec$$

$$k_{ave} = \frac{a_v \cdot c_v \cdot \gamma_w}{1 + e_o} = \frac{8.3 \times 10^{-6} \times 1.9 \times 10^{-4}}{1 + 0.584} = 0.99 \times 10^{-9}\ cm/sec$$

Scott Method

According to the procedure given for the Scott Method and Table VIII, the following analysis may be made: (From Table VIII)

$d_s = 681.5$

$d_t = d_{4\ min} = 674.5$

$d_{Nt} = d_{8\ min} = 672.1$

$N = 2$

$$C_r = \frac{d_s - d_t}{d_s - d_{Nt}} = \frac{681.5 - 674.5}{681.5 - 672.1} = 0.744$$

From Figure 70, for $C_r = 0.744$ and $N = 2$, $T = 0.275$

$$C_r = \frac{T.H^2}{t} = \frac{0.275 \left(\frac{2.54}{2}\right)^2}{4 \times 60} = 18.5 \times 10^{-4} \text{ cm}^2/\text{sec}$$

and

$$k_{ave} = \frac{a_v \cdot c_v \cdot \gamma_w}{1 + e_o} = 10.4 \times 10^{-9} \text{ cm/sec}$$

VITA[2]

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