LARGE DEFLECTION ANALYSIS OF DISCRETE-
ELEMENT THIN PLATES

by

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Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
May, 1977

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ELEMENT THIN PLATES

Thesis Approved:

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ACKNOWLEDGMENTS

Deepest appreciation is expressed to my adviser, Dr. Allen E. Kelly, for his encouragement and guidance throughout this study. His interest, suggestions, and concern have been invaluable and are gratefully acknowledged.

Recognition is also extended to the other committee members: Dr. William P. Dawkins, Dr. John P. Lloyd, and Professor Louis O. Bass. Their encouragement and comments are sincerely appreciated.

To my dear wife, Anchalee T. Leesavan, I am greatly indebted for her sacrifice, understanding, and support. To my family in Thailand, especially my father and mother, Mr. Somboon and Mrs. Supa Leesavan, I am grateful for their spiritual strength and support. Without them I would not have been able to complete my degree.

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LIST OF SYMBOLS

a	bar in the x direction
a_{ij}	stiffness matrix coefficient
A_a	area of the elastic bar in the x direction
A_b	area of the elastic bar in the y direction
A_c	area of the elastic bar in the diagonal direction
b	bar in the y direction
b_{ij}	stiffness matrix coefficient
c	bar in the diagonal direction
c_{ij}	stiffness matrix coefficient
d_{ij}	stiffness matrix coefficient
D_x, D_y	plate bending stiffness in the x and y directions
D_{xy}	plate twisting stiffness
e_{ij}	stiffness matrix coefficient
E_x, E_y	modulus of elasticity
f_{ij}	stiffness matrix coefficient
$\{F\}$	force vector in the membrane model
F_z	force in the z direction
g_{ij}	stiffness matrix coefficient
G_{xy}	torsional stiffness of plate
h_x, h_y, h_z	discrete element length
i, j	node point identifications
I_{xx}, I_{yy}	moment of inertia about x and y axis

$[K]$	stiffness matrix in the bending model
M_x, M_y	continuum bending moment
M_{xy}, M_{yx}	continuum twisting moment
M^x, M^y	model bending moment
M^{xy}, M^{yx}	model twisting moment
MX, MY	increment length
N_x, N_y	inplane force
N_{xy}, N_{yx}	inplane shear force
p	uniform load on plate element
P_{ij}	stiffness matrix coefficient
P^x, P^y, P^{c1}, P^{c2}	inplane force in the membrane element
q	uniform distributed load on plate
q_{ij}	stiffness matrix coefficient
$\{Q\}$	vertical load vector in the bending model
Q_B	bending resistance
Q_M	membrane resistance
Q_T	total resisting force
r_{ij}	stiffness matrix coefficient
R^x, R^y	rotational stiffness
s_{ij}	stiffness matrix coefficient
$[S]$	stiffness matrix of the membrane model
S_{ij}	discrete foundation support
t	plate thickness
t_{ij}	stiffness matrix coefficient
T^x, T^y	twisting couple
u	displacement in x direction
u_{ij}	stiffness matrix coefficient

$\{U\}$	displacement vector in the membrane model
v	displacement in y direction
w	displacement in z direction
$\{W\}$	deflection vector in the bending model
σ_x, σ_y, τ_{xy}	normal and shearing stresses from the bending model
$(\sigma_x)_M$, $(\sigma_y)_M$, $(\tau_{xy})_M$	normal and shearing stresses from the membrane model
ε_x, ε_y, γ_{xy}	strains on the middle plane of the plate
α, β	constant values
κ_x, κ_y, κ_{xy}	curvature of the plate element

CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

It is usual to represent components of many modern structures, such as airplanes, ships, pressure-resisting tanks, storage bins, and glass windows, as thin elastic plates. The classical linear plate theory by Kirchhoff gives sufficiently accurate results provided the deflections of the plate are small ($w \leq 0.3t$) (1). If the deflections are not small when compared with the thickness of the plate but are small relative to other dimensions, the linear plate theory is no longer valid. Under this condition of large deflections, membrane resistance, not present for small deflections, must be added to the resistance due to bending to adequately describe the behavior of the plate structure.

The formulation of a theory which includes the effects of membrane strains yields a set of nonlinear partial differential equations. Membrane strain is related to the square of the derivative of plate deflection, and is responsible for the nonlinear equations. Therefore, interaction between the deflection and the net load taken by the plate structure is no longer linear and a numerical method is required to solve the equations for the general plate structures.

In this study a discrete element method of analysis for plates subjected to various types of load and boundary conditions which undergo large deflections is presented. Although the theory used in this study requires linear material behavior, geometric nonlinearity is included. A computer program was developed which permits the description of a general plate structure to include variations of shapes, cross section geometry, loads, and boundary conditions.

1.2 Review of Literature

Analytical solutions for general cases of load and boundary conditions are not available. Many investigators have studied this problem and proposed a variety of methods for stretching and bending of plates. For a very thin plate, the bending stiffness may be neglected, which results in a membrane structure. Equilibrium equations for a membrane were derived by Foppl (2) in 1907, and a solution procedure, based on the method of finite differences, was presented in 1920 (3). The use of energy methods for the analysis of the membrane problem was pioneered by Hencky (4). This investigator utilized the Ritz method for the analysis of the membrane.

Von Karman (5) introduced a stress function to the equilibrium equations for thin plates. The three unknown membrane stresses were replaced by the derivatives of a single function. The resulting nonlinear equations are functions of the stress function, vertical deflection, and derivatives of these variables.

Way (6) was one of several investigators who studied plates of finite dimensions. Until 1938 the research focused on thin plates which extend infinitely in the plane of the plate. The Ritz energy method was used to obtain the approximate solutions for rectangular plates having clamped edges. The rectangular plates were studied for aspect ratios of 1, 2/3, and 1/2. Way's studies also included the analysis of circular plates with a uniform edge moment but without the influence of inplane forces at the support (7).

Levy (8)(9) used Fourier series to solve the von Karman equations for simply supported and clamped rectangular plates. Rectangular clamped plates with an aspect ratio of 1/2 were studied by Levy and Greenman (10). They concluded that plates with aspect ratios less than 1/2 could be analyzed as infinite long plates.

Wang (11)(12) used finite differences approximations to solve the von Karman equations. A method of successive approximation was used to evaluate the stress function and the vertical deflections of the plates. Results are presented for a variety of rectangular plate problems to include clamped and pinned supports as well as several aspect ratios.

Kaiser (13) and Stippes (14) studied simply supported plates having edges which were permitted to move in the plane of the plates. In addition to the analytical study, Kaiser provides experimental data for this problem. Excellent agreement was noted between calculated and measured deflection. Results of Stippes compared favorably with the reported deflections of plates of Kaiser. Stippes used the Ritz method for the analysis of plates.

Berger (15) analyzed both circular and rectangular plates. In this work the strain energy due to the second invariant of strain in the middle plane of the plate was neglected. His results compared favorably with results published by other investigators.

Yang (16) applied the finite element method for the large deflections analysis of plates. He solved the problems utilizing an incremental load approach in which the stiffness of the plate structure is taken as piecewise linear for each load increment.

Although the methods described above gave acceptable results, some of these methods may be difficult to adapt to the digital computer. This is true for plates of irregular shape and stiffness with variable support conditions. The discrete element and finite element methods, however, are suitable for this problem. In this work, the discrete element method will be applied to the large deflection analysis of thin plates and is an extension of the work by Kelly and Matlock (12).

1.3 Structural Idealization

For the method of analysis presented by this work, the plate structure is represented by two discrete element models: a bending model which resists the vertical loads by bending moments, and a membrane system which resists both vertical and inplane loads by stretching or contracting. The combined action of these two models represents the behavior of the plate structure. Equilibrium and compatibility of these models are satisfied by repeated solutions of the linear equations for bending and stretching.

The method of analysis of these models starts by applying a vertical load to the bending model. From the equilibrium equation for bending resistance, vertical deflections are calculated. The vertical deflections of the membrane are set equal to those of the bending model for compatibility. The vertical displacements produce stretching in the membrane model. Equilibrium is satisfied in the plane of the membrane and vertical forces are calculated at the joints. The vertical membrane forces combine with plate bending to resist vertical load applied to the plate structure. Vertical equilibrium must be satisfied at each joint. In order to satisfy this condition, repeated calculation of the deflection and total resisting force must be performed. Once both equilibrium and compatibility are satisfied, stresses are calculated from the vertical and inplane displacements.

CHAPTER II

THEORY OF PLATES

2.1 General

Timoshenko (18) distinguishes three kinds of plates: (1) thin plates with small deflections, (2) thin plates with large deflections, and (3) thick plates.

For the thin plate with small deflections, where the deflection is small in comparison with its thickness, a very satisfactory approximate theory of bending of the plate by vertical loads can be developed by making the following assumptions:

1. There is no deformation in the middle plane of the plate.

2. Points of the plate lying initially normal to the middle surface of the plate remain normal to the middle surface of the plate after bending.

3. The normal stresses in the direction transverse to the plate can be neglected.

From these assumptions all components of stresses can be expressed as functions of the deflected shape of the plate. The deflected shape of the plate must satisfy a linear partial differential equation and necessary boundary conditions.

The strain in the middle surface must be considered for the large deflection analysis of thin plates. These supplementary strains (membrane strains) produce nonlinear equations, and the solution of these equations is available for only a limited class of problems. Owing to the curvature of the deformed middle surface of the plate, the supplementary tensile stresses which predominate act in opposition to the applied vertical load; thus, the applied load is now transmitted partly by the flexural rigidity and partly by a membrane action of the plate. Very thin plates tend to behave as membrane except near the edges.

In case the thickness of the plate is not small in comparison to other dimensions of the plate, thick plate theory must be applied. This theory considers the problem of the plate as a three-dimensional problem of elasticity.

2.2 Plate Equilibrium Equations

From a plate element shown in Figure 1 equilibrium equations of forces in the x and y directions are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = 0 \qquad (2.1)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \qquad (2.2)$$

With the higher order terms neglected, equilibrium equations for moments about the x and y axis are

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} = Q_y \qquad (2.3)$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} = -Q_x \qquad (2.4)$$

In considering equilibrium of forces in the z direction, the effect of bending on the inplane forces must be included. Due to this effect the projection of inplane force N_x on the z axis is

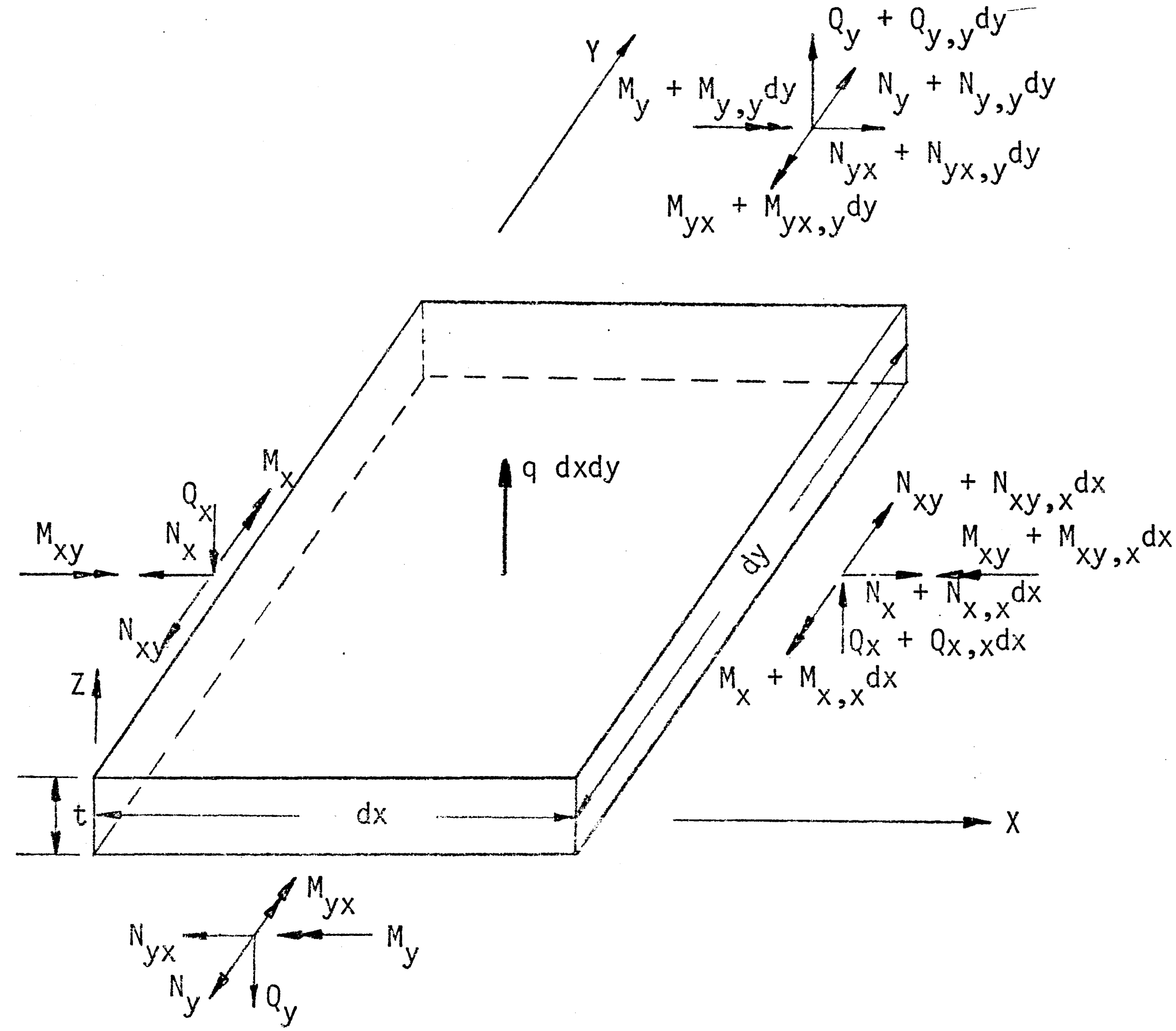


Figure 1. Illustration of Sense of Stress Resultants and Applied Load

$$-N_x \frac{\partial w}{\partial x} dy + (N_x + \frac{\partial N_x}{\partial x} dx) (\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx) dy$$

or

$$N_x \frac{\partial^2 w}{\partial x^2} dxdy + \frac{\partial N_x}{\partial x} \frac{\partial w}{\partial x} dxdy$$

By the same procedure the projection of inplane force N_y on the z axis is

$$N_y \frac{\partial^2 w}{\partial y^2} dxdy + \frac{\partial N_y}{\partial y} \frac{\partial w}{\partial y} dxdy$$

The projections of inplane shearing forces on the z axis are

$$N_{xy} \frac{\partial^2 w}{\partial x \partial y} dxdy + \frac{\partial N_{xy}}{\partial x} \frac{\partial w}{\partial y} dxdy$$

and

$$N_{yx} \frac{\partial^2 w}{\partial x \partial y} dxdy + \frac{\partial N_{yx}}{\partial y} \frac{\partial w}{\partial x} dxdy$$

Therefore, the total inplane force on the z axis is

$$[N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_{yx} \frac{\partial^2 w}{\partial x \partial y}] dxdy$$

Equilibrium equation of forces in the z direction is

$$q + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_{yx} \frac{\partial^2 w}{\partial x \partial y} = 0 \tag{2.5}$$

Substituting Q_x and Q_y from Equations (2.3) and (2.4) into Equation (2.5) gives

$$\frac{\partial^2 M_x}{\partial x^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_{yx} \frac{\partial^2 w}{\partial x \partial y} \tag{2.6}$$

From equilibrium of the plate element, it can be shown that

$$M_{yx} = -M_{xy}$$

and

$$N_{yx} = N_{xy}$$

Substituting these values into Equation (2.6) gives

$$\frac{\partial^2 M_x}{\partial x^2} - 2\,\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = q + N_x\,\frac{\partial^2 w}{\partial x^2} + N_y\,\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\,\frac{\partial^2 w}{\partial x \partial y} \tag{2.7}$$

Equations (2.1), (2.2), and (2.7) are equilibrium equations of plates subjected to lateral and inplane loads.

The moments and forces in Equation (2.7) can be related to deflection by the familiar equations:

$$\begin{aligned} M_x &= D_x\,[\kappa_x + \nu_y\,\kappa_y] \\ M_y &= D_y\,[\kappa_y + \nu_x\,\kappa_x] \\ M_{xy} &= -M_{yx} = -2D_{xy}\,\kappa_{xy} \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} N_x &= \frac{E_x t}{(1 - \nu_x \nu_y)}\,[\varepsilon_x + \nu_y \varepsilon_y] \\ N_y &= \frac{E_y t}{(1 - \nu_x \nu_y)}\,[\varepsilon_y + \nu_x \varepsilon_x] \\ N_{xy} &= N_{yx} = G_{xy} t\,\gamma_{xy} \end{aligned} \tag{2.9}$$

where

$$D_x = \frac{E_x t^3}{12(1 - \nu_x \nu_y)}$$

$$D_y = \frac{E_y t^3}{12(1 - \nu_x \nu_y)}$$

$$D_{xy} = \frac{G_{xy} t^3}{12}$$

$$\kappa_x = \frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = \frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = \frac{\partial^2 w}{\partial x \partial y}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Substituting moments in Equation (2.8) into Equation (2.7) and arranging terms yields

$$D_x \frac{\partial^4 w}{\partial x^4} + (\nu_y D_x + \nu_x D_y + 4D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}$$

$$= q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} \qquad (2.10)$$

If the material of the plate is isotropic, Equation (2.10) becomes

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} [q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y}] \qquad (2.11)$$

In the case of large deflection, where the lateral deflection of the plate is large in comparison to its thickness, the effect of lateral deflection must be included in the strains of the middle surface. Therefore, the strains in Equation (2.9) are changed to

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y}\right)^2 \qquad (2.12)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial y}$$

Substituting strains in Equation (2.12) into Equation (2.9) yields

$$N_x = \frac{E_x t}{(1 - \nu_x \nu_y)} [\frac{\partial u}{\partial x} + \frac{1}{2} (\frac{\partial w}{\partial x})^2 + \nu_y \frac{\partial v}{\partial y} + \frac{\nu_y}{2} (\frac{\partial w}{\partial y})^2]$$

$$N_y = \frac{E_y t}{(1 - \nu_x \nu_y)} [\frac{\partial v}{\partial y} + \frac{1}{2} (\frac{\partial w}{\partial y})^2 + \nu_x \frac{\partial u}{\partial x} + \frac{\nu_x}{2} (\frac{\partial w}{\partial x})^2] \qquad (2.13)$$

$$N_{xy} = G_{xy} t [\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial y}]$$

Therefore, the equilibrium equations for large deflection of a thin plate are Equations (2.1), (2.2), and (2.10) with the inplane forces expressed in Equation (2.13). Equation (2.10), with Equation (2.13), is a nonlinear partial differential equation, and solutions for general plate problems are not readily available.

2.3 Stress Resultants

The solutions of the equilibrium equations are vertical and inplane displacements. Bending and twisting moments are calculated by substituting the lateral displacements into Equation (2.8). The bending and shearing stresses, then, are calculated by substituting the moment values into the following expressions.

$$(\sigma_x)_B = \frac{M_x(t/2)}{I_{yy}} = \frac{E_x t}{2(1 - \nu_x \nu_y)} [\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2}]$$

$$(\sigma_y)_B = \frac{M_y(t/2)}{I_{xx}} = \frac{E_y t}{2(1 - \nu_x \nu_y)} [\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2}] \qquad (2.14)$$

$$(\tau_{xy})_B = -2G_{xy}(t/2) \frac{\partial^2 w}{\partial x \partial y} = -G_{xy} t \frac{\partial^2 w}{\partial x \partial y}$$

where

$(\sigma_x)_B, (\sigma_y)_B$ = normal bending stresses in x and y directions; and

$(\tau_{xy})_B$ = shearing stress due to twisting.

Membrane stresses are calculated from the inplane forces in Equation (2.13). The normal and shearing stresses are

$$(\sigma_x)_M = N_x/t = \frac{E_x}{(1 - \nu_x \nu_y)} [\frac{\partial u}{\partial x} + \frac{1}{2} (\frac{\partial w}{\partial x})^2 + \nu_y \frac{\partial v}{\partial y} + \frac{\nu_y}{2} (\frac{\partial w}{\partial y})^2]$$

$$(\sigma_y)_M = N_y/t = \frac{E_y}{(1 - \nu_x \nu_y)} [\frac{\partial v}{\partial y} + \frac{1}{2} (\frac{\partial w}{\partial y})^2 + \nu_x \frac{\partial u}{\partial x} + \frac{\nu_x}{2} (\frac{\partial w}{\partial x})^2] \qquad (2.15)$$

$$(\tau_{xy})_M = N_{xy}/t = G_{xy} [\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial y}]$$

where

$(\sigma_x)_M, (\sigma_y)_M$ = normal membrane stresses in x and y directions; and

$(\tau_{xy})_M$ = shearing membrane stress.

Total normal stresses in each direction are the summation of normal bending and membrane stresses. Shearing stress in Equation (2.14) is acting on the top and bottom surfaces while that in Equation (2.15) is on the middle surface of the plate.

CHAPTER III

DISCRETE ELEMENT MODELS

3.1 General

The general theory of plates discussed in the preceding chapter are based on infinitesimal calculus. Closed-form solutions for the large deflection problem, and for the majority of complex engineering problems, are not available. Numerical methods are most often used to solve complex engineering problems, in which the governing differential equations can be mathematically approximated by the substitution of finite difference forms for derivatives. Numerical methods can be interpreted by a physical model, in which the problem is represented by a system of finite or discrete elements whose behavior can properly be described with algebraic equations. The physical model seems preferable because it facilitates visualization of the problem and formulation of proper boundary and loading conditions.

The concept of using the discrete element model for plates can be traced to Ang and Newmark (19). Tucker (20) extended the concept for beams to grid and plate structures. Ang and Prescott (21) presented model equations for solving complex isotropic plate problems. An orthotropic plate model was developed by Hudson (22). Stelzer (23) used a direct method to solve the plate equations.

In this study two discrete element models are used. These models are similar to those described by Hudson and others (22) for the bending element, and Hrennikoff (24) for the membrane effects. They are connected to represent the combined effect of both bending and membrane behaviors of thin plates with large deflections. Each model is discussed in detail in the next sections.

3.2 Bending Model

The bending model is constructed from rigid bars, elastic joints, and torsional bars. A convenient bending model shown in Figure 2 was developed by Hudson (22). Elastic joints represent the bending property of the plate. Torsional bars simulate the twisting characteristic while rigid bars transmit the shearing forces in the plate.

Derivation of the equilibrium equation for the discrete element bending model is summarized in Appendix A. In this derivation the properties of elastic joints and torsional bars are defined by applying finite difference approximation to the moment expressions. Joint equilibrium equations summarized in Appendix A may be written in a matrix notation as

$$[K]\,\{W\} = \{Q\} \tag{3.1}$$

where

$[K]$ = stiffness matrix of the bending model;

$\{W\}$ = vertical deflection vector; and

$\{Q\}$ = vertical load vector.

3.3 Membrane Model

The membrane model shown in Figure 3 is utilized to represent the membrane behavior of the plate. This model is composed of ball and socket joints and elastic bars. The elastic bars transmit membrane

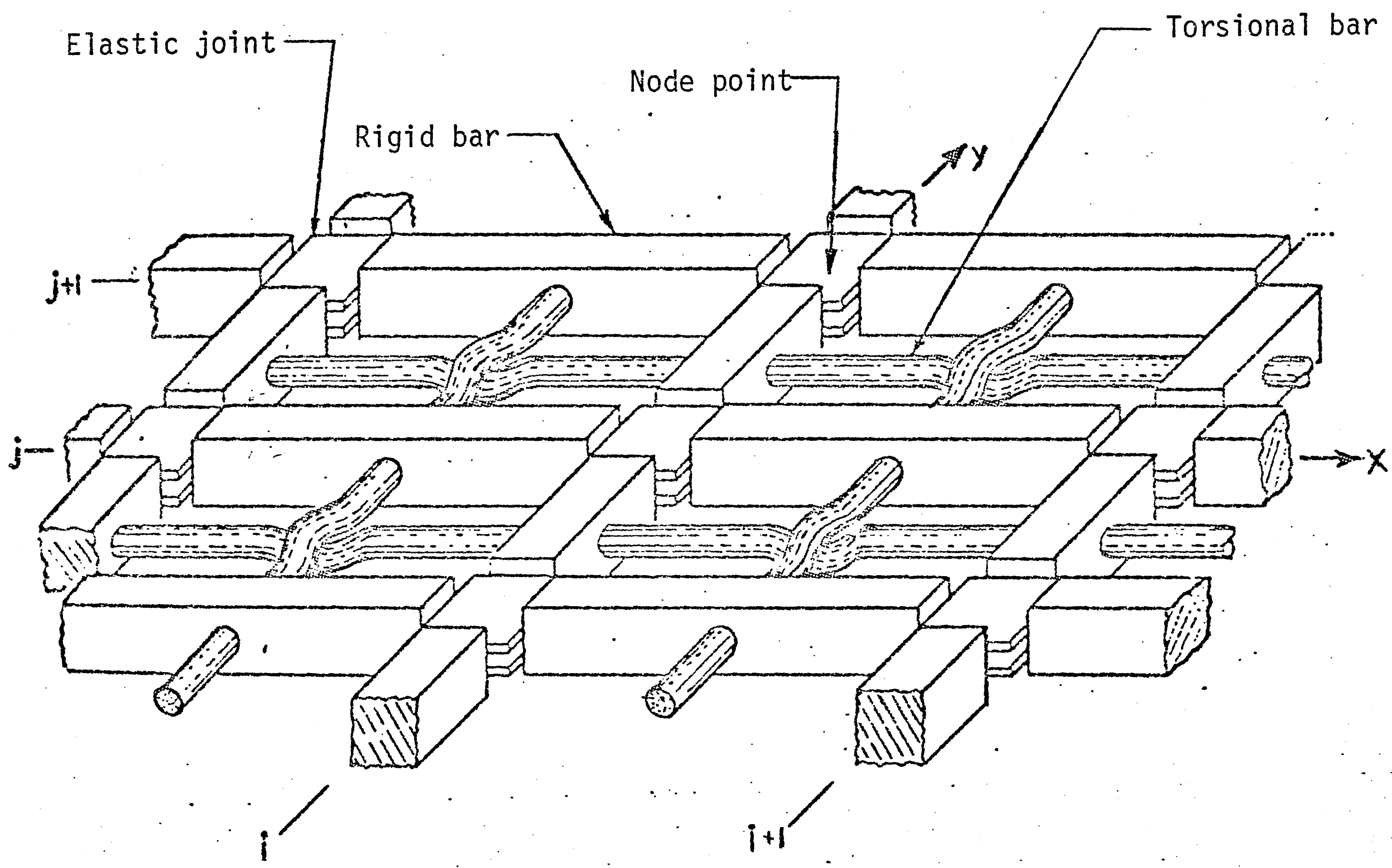


Figure 2. Discrete Element Bending Model

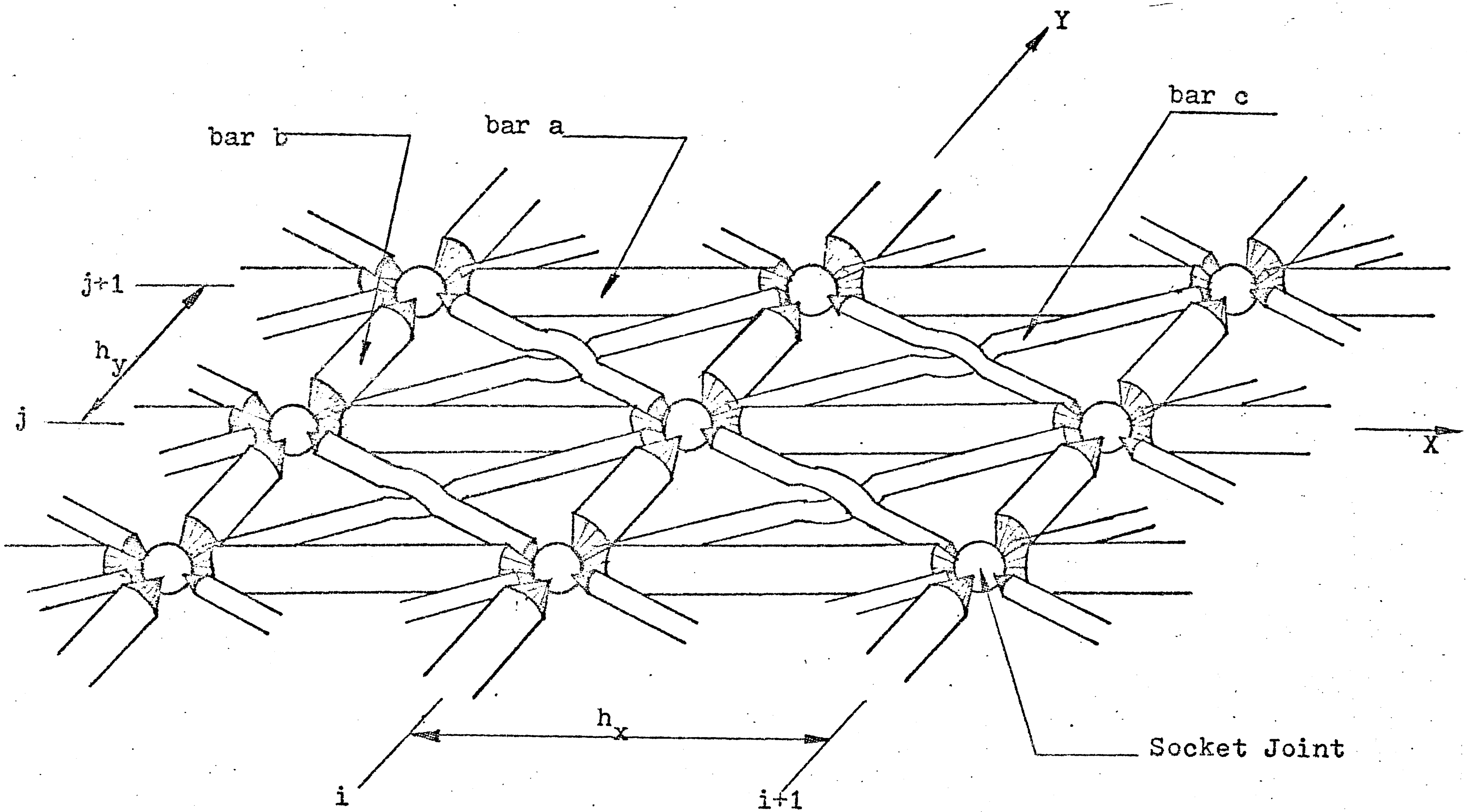


Figure 3. Discrete Element Membrane Model

forces by stretching and contracting. The properties of elastic bars, such as cross section area and the strain relation between bars and plates are discussed in Appendix B. The stiffness matrix for the membrane resembles that of the space truss structure and is presented elsewhere (25).

The equilibrium equations of this membrane model are summarized into a matrix form as

$$[S]\ \{U\} = \{F\} \tag{3.2}$$

where

[S] = stiffness matrix for the membrane model;

{U} = displacement vector of the membrane model; and

{F} = force vector of the membrane model.

3.4 Combined Model

The combined model is shown in Figure 4. Equilibrium conditions for vertical and inplane directions of the combined model are analyzed separately. Equilibrium in the vertical direction is analyzed by considering only the bending model. From this analysis vertical deflection of the combined model is calculated. Stiffness used in this calculation must include the effects of inplane loads and is discussed in detail in Appendix A.

Similarly, the membrane model is used to provide equilibrium in the plane of the model. Inplane displacements of each joint are determined utilizing forces developed as the result of vertical joint displacements. The details of this method, in which compatibility and equilibrium of the joints are determined, are discussed in the following chapter.

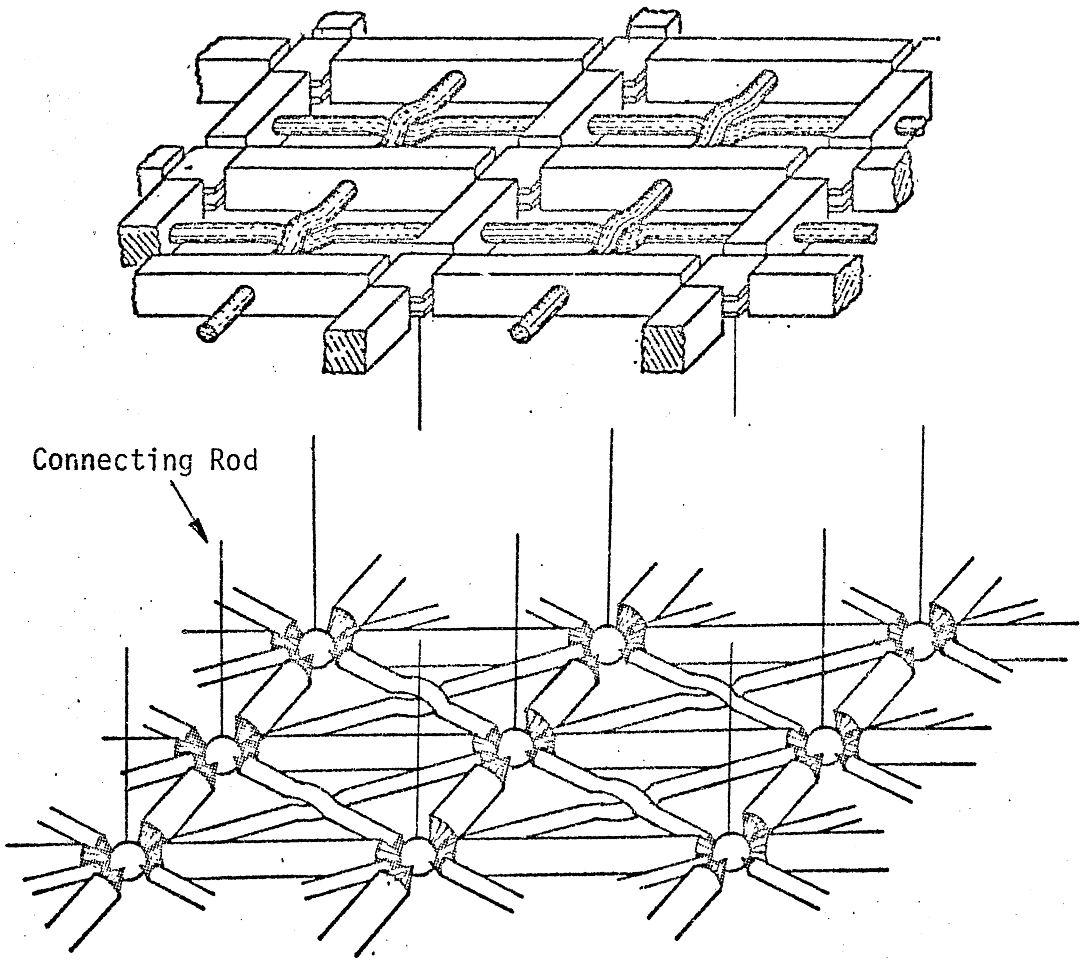


Figure 4. Discrete Element Combined Model

3.5 Model Stress Resultants

The purpose of this section is to illustrate the method used to calculate the stresses in the plate. Equations for model bending moments and inplane forces are given in Appendices A and B. These moments have the units of lb-in. instead of lb-in/in as for the continuum plate equations. Substituting these moments into Equation (2.14) yields

$$
\begin{aligned}
(\sigma_x)_{i,j} &= \frac{M^x_{i,j}\ (t/2)}{I_{yy}\ h_y} = \frac{6M^x_{i,j}}{t^2\ h_y} \\
(\sigma_y)_{i,j} &= \frac{M^y_{i,j}\ (t/2)}{I_{xx}\ h_x} = \frac{6M^y_{i,j}}{t^2\ h_x} \qquad (3.3) \\
(\tau_{xy})_{i,j} &= -\frac{6M^{xy}_{i,j}}{t^2}
\end{aligned}
$$

where

$(\sigma_x)_{i,j}, (\sigma_y)_{i,j}$ = normal stresses due to bending;

$(\tau_{xy})_{i,j}$ = shearing stress;

$M^x_{i,j}, M^y_{i,j}, M^{xy}_{i,j}$ = model bending and twisting moments;

t = plate thickness; and

h_x, h_y = increment lengths of the model.

Membrane stresses are calculated from the forces in the elastic bars of the membrane model. These bar forces are calculated from the vertical and inplane displacements of the membrane model as discussed in Appendix B. Membrane stresses, then, are expressed as

$$(\sigma_x)_M = [F_A + F_B + (F_H + F_E)\cos\theta]/th_y$$

$$(\sigma_y)_M = [F_C + F_D + (F_H + F_E)\sin\theta]/th_x$$

$$(\tau_{yx})_M = [F_H + F_E] \cos\theta / th_x$$

$$(\tau_{xy})_M = [F_H + F_E] \sin\theta / th_y \qquad (3.4)$$

where

$(\sigma_x)_M, (\sigma_y)_M$ = normal membrane stresses;

$(\tau_{xy})_M, (\tau_{yx})_M$ = membrane shearing stresses; and

$F_A, F_B, F_C, F_D, F_E, F_H$ = forces in elastic bars of the membrane model, shown in Figure 31.

The derivation of Equation (3.4) is given in Appendix B.

The total stress is determined by adding membrane and bending stresses with regard for the direction of the stress. Positive stresses are shown in Figure 5. These stresses are produced by positive stress resultants.

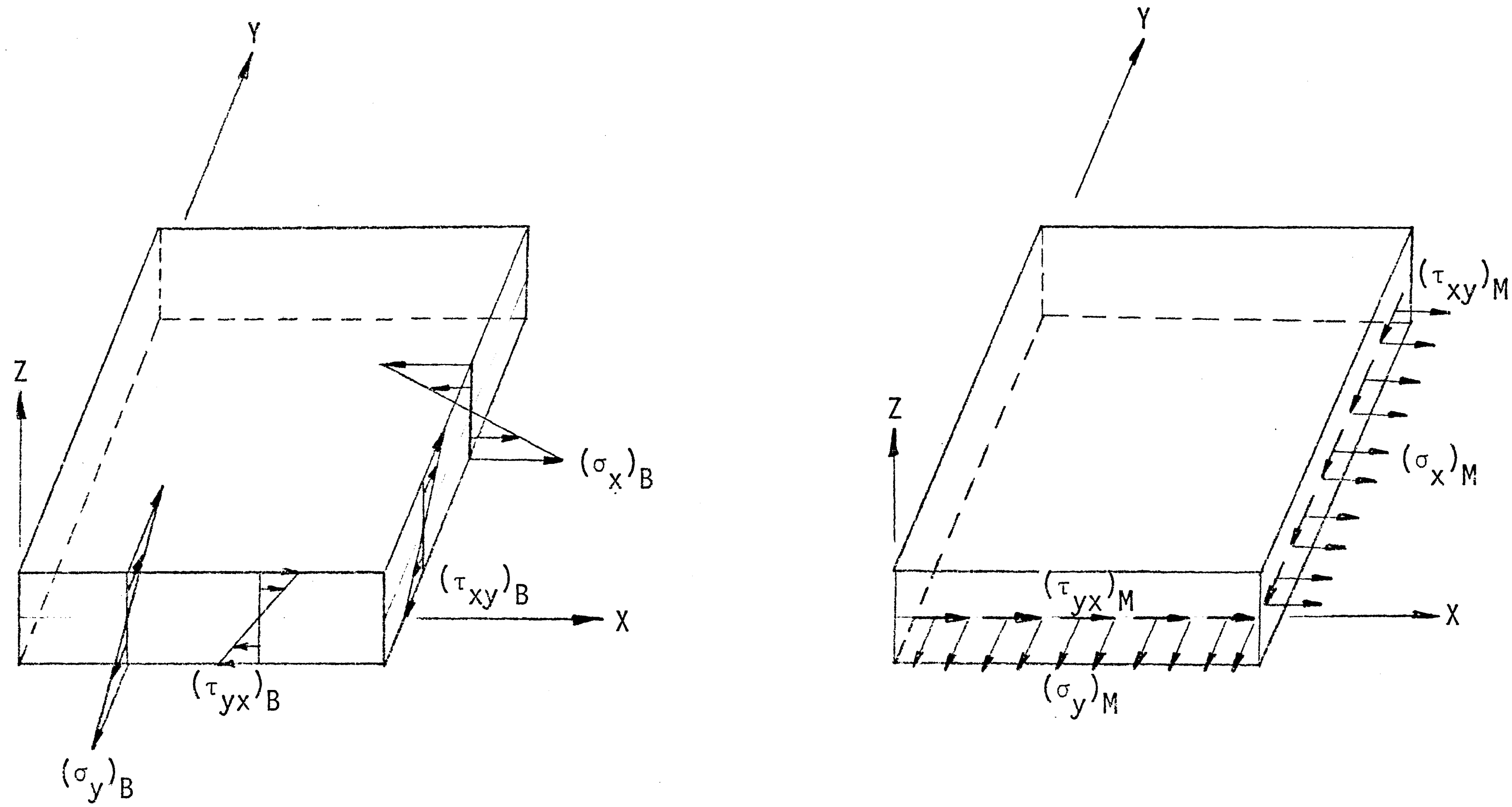


Figure 5. Positive Bending and Membrane Stresses

CHAPTER IV

NONLINEAR ANALYSIS

4.1 Method of Analysis

If the deflections of a thin elastic plate are not small in comparison with the thickness of the plate but are still small relative to other dimensions, the analysis of this plate must include the effects of the strain in the middle plane of the plate. The formulation for the equilibrium equations of this problem leads to a set of nonlinear partial differential equations as shown in Chapter II. Therefore, a numerical method has been developed by this study to solve for the large deflections of thin elastic plates.

The procedure used in this study is to provide both compatibility and equilibrium of two systems of discrete element models by iteration. Both models are connected, as shown in Figure 4, to provide deflection compatibility. Equilibrium is evaluated by the calculation of vertical resistance of both the membrane and bending models and the inplane resistance of the membrane model. If joint equilibrium is not satisfied, the joint loading is adjusted and new deflections are calculated. Details for each model and the resulting equilibrium equations are presented in Chapter III.

Two loading systems are considered in this study. Vertical loading and couples are applied to joints of the bending model whereas inplane loads are applied at joints of the membrane model. If the inplane loads are applied, they must first be distributed into the elastic bars of the membrane model. Joint displacements, u and v, are calculated due to these inplane loads and the force in each elastic bar is determined. Both vertical and inplane loads are considered in the calculation of the vertical deflections of the plate. Because the inplane displacements are small in comparison with the vertical deflections, the x and y coordinates of the unloaded plate are used for this calculation.

The solution is obtained by an iteration technique in which deflection compatibility between bending and membrane models is achieved, and static equilibrium of the joints of the models is satisfied. This analysis procedure requires repeated solutions of the linear problems of first the bending model, for vertical equilibrium, followed by an investigation of the forces developed in the membrane model to satisfy vertical deflection compatibility between the models. The membrane forces are calculated from the vertical and inplane displacements of joints in the membrane model. The total vertical resisting force, which is the summation of bending and membrane forces, is compared with the applied joint load. The difference between the applied and the resisting force at a joint must be eliminated in order to satisfy equilibrium. To eliminate this disparity in force, a new bending load is applied to the system while the inplane load is held constant. The details for the adjustment of lateral load for the bending model are presented below.

4.2 Bending Analysis

In the solution procedure discussed above, only the bending model is used to calculate the vertical deflection. Both vertical and inplane loads must be included in this calculation. The equilibrium of the bending model is presented in Appendix A. This equilibrium equation is written for each joint in the structure. It has been shown that satisfactory results may be obtained for simply supported plates having 64 degrees of freedom; accuracy increases with an increase of the degree of freedoms. The bending equations are solved by the method presented in Appendix C.

4.3 Membrane Analysis

Bending and membrane models are connected as shown in Figure 4. From the applied loads, vertical deflections of the bending model are calculated as discussed in the previous section. For deflection compatibility between the bending and membrane models, these vertical deflections are enforced on the membrane model and cause stretching in the elastic bars of the membrane model. Joint forces are produced in the elastic bars which cause inplane displacements. With the vertical displacements held constant, the vertical resistance of the membrane model is calculated. These forces are added to the vertical resistance provided by the bending model to determine the total resistance of the plate.

4.4 Adjustment of Load for Nonlinear Analysis

When the external loads are applied to the plate structure, vertical resistance is provided by both bending and stretching of the plate. Therefore, the total resisting force of the plate system at each joint is

$$Q_T = Q_B + Q_M \tag{4.1}$$

where

Q_T = total vertical joint force;

Q_B = force resisted by the bending model; and

Q_M = force resisted by the membrane model.

These forces are shown in Figure 6.

For the equilibrium condition in the plate system the total resisting force Q_T must be equal to the applied load at the joint. Since the vertical membrane deflection is set equal to the vertical deflection due to bending, the selection of the vertical load to apply to each joint of the bending model is critical to the iteration procedure. Therefore, it is appropriate to begin the bending analysis by applying only a portion of the total load. The repeated calculations of Q_T are performed by adjusting the applied bending force until the summation of bending resistance and calculated membrane resistance are equal to the applied load.

Due to the nonlinearity of the system, the solution procedure described above is often unstable, and the calculation may not converge to the applied vertical load. To avoid this problem, the iteration technique of Fujino and Ohsaka (26) is adopted. In this method the new value of Q_B, the vertical load on the bending model must be related to the previous value of Q_B and the corresponding value of the membrane resistance Q_M.

The new value of the vertical load for the bending model can be calculated from the following equation.

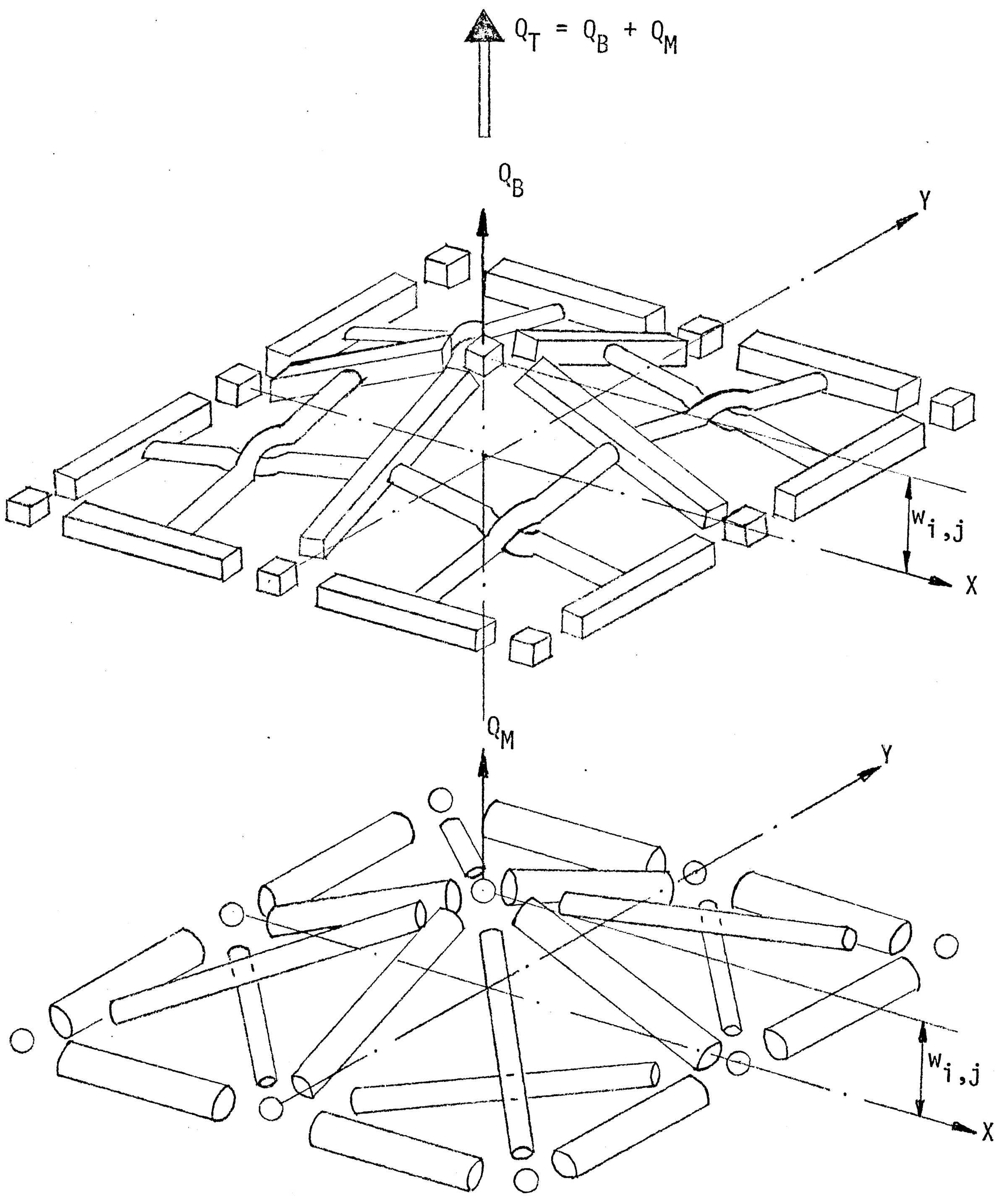


Figure 6. Distribution of Vertical Force to Bending and Membrane Models

$$(Q_B)_{i+1} = \frac{Q_T(Q_B)_i}{(Q_B)_i + (Q_M)_i} \quad (4.2)$$

where

$(Q_B)_{i+1}$ = vertical load to be applied to the bending model for iteration i+1;

$(Q_B)_i$ = vertical load for iteration i;

$(Q_M)_i$ = vertical load which must be applied to the membrane model to produced deflection compatibility for iteration i; and

Q_T = the applied vertical load.

In the nonlinear system, the load calculated from Equation (4.2) must be related to the vertical load used for the previous calculation. Fujino and Ohsaka use the following technique for the prediction of the vertical load carried by bending for the next iteration.

$$(\overline{Q}_B)_{i+1} = \alpha(Q_B)_{i+1} + (1-\alpha)(Q_B)_i \quad (4.3)$$

where

$(\overline{Q}_B)_{i+1}$ = predicting value of vertical load for iteration i+1;

α = a constant value; and

$(Q_B)_i$, $(Q_B)_{i+1}$ are defined in Equation (4.2).

Substituting $(Q_B)_{i+1}$ from Equation (4.2) into Equation (4.3) yields

$$(\overline{Q}_B)_{i+1} = \frac{\alpha\ Q_T(Q_B)_i}{(Q_B)_i + (Q_M)_i} + (1-\alpha)(Q_B)_i \quad (4.4)$$

Rearranging Equation (4.4) gives

$$(\overline{Q}_B)_{i+1} = \frac{\alpha\ (Q_B)_i}{(Q_B)_i + (Q_M)_i} \quad [Q_T - (Q_B)_i - (Q_M)_i] + (Q_B)_i \quad (4.5)$$

This value of vertical force is applied to the bending model and vertical displacements are calculated. The deflections are imposed on the membrane model and forces in the elastic bars are calculated. From the geometry of the deflected shape, the vertical load required to hold the membrane in place is found, $(Q_M)_{i+1}$, and a new estimate of the total resistance of the structure is determined.

The constant in Equation (4.3) can vary from zero to one. A value of 0.3 is recommended for the solution of the plate problem.

In some cases of loading, such as a concentrated load, the load applied to most joints will be zero. To accelerate closure for this problem, the vertical loads for unloaded joints will be predicted from the following equation:

$$(\overline{Q}_B)_{i+1} = (1 - \beta)(Q_B)_i - \beta\ (Q_M)_i \tag{4.6}$$

where

β = a constant value.

$(\overline{Q}_B)_{i+1}$, $(Q_B)_i$, $(Q_M)_i$ are defined in Equation (4.3).

A value of 0.5 for β was found suitable for the analysis of plates subjected to a single, concentrated load at the center of the plate.

CHAPTER V

COMPUTER PROGRAM

5.1 General

The analytical procedure described in the preceding chapter has been programmed for the digital computer. This computer program is similar to the linear plate program by Stelzer (23) and nonlinear analysis of Kelly (17). The program discussed in this report is written in the ASA FORTRAN language and should require only minor revisions to be used on a computer having at least 25,000 words storage capacity or equivalents. On a machine operating with a word size less than 60 binary bits (15 significant decimal figures), double precision arithmetic must be used.

This program will not only solve the large deflection of plates, but also problems of plane stress and laterally loaded membranes.

A summary flow diagram for the program is shown in Figure 7. A complete FORTRAN listing of the program is included in Appendix D.

5.2 Input Information

The program has been developed to provide for wide varieties in plate geometry, stiffness, support conditions, and loading. A technique for automatic distribution of data was utilized to minimize the amount of input data. The specific formats of the data input are given in Appendix E, Guide for Data Input.

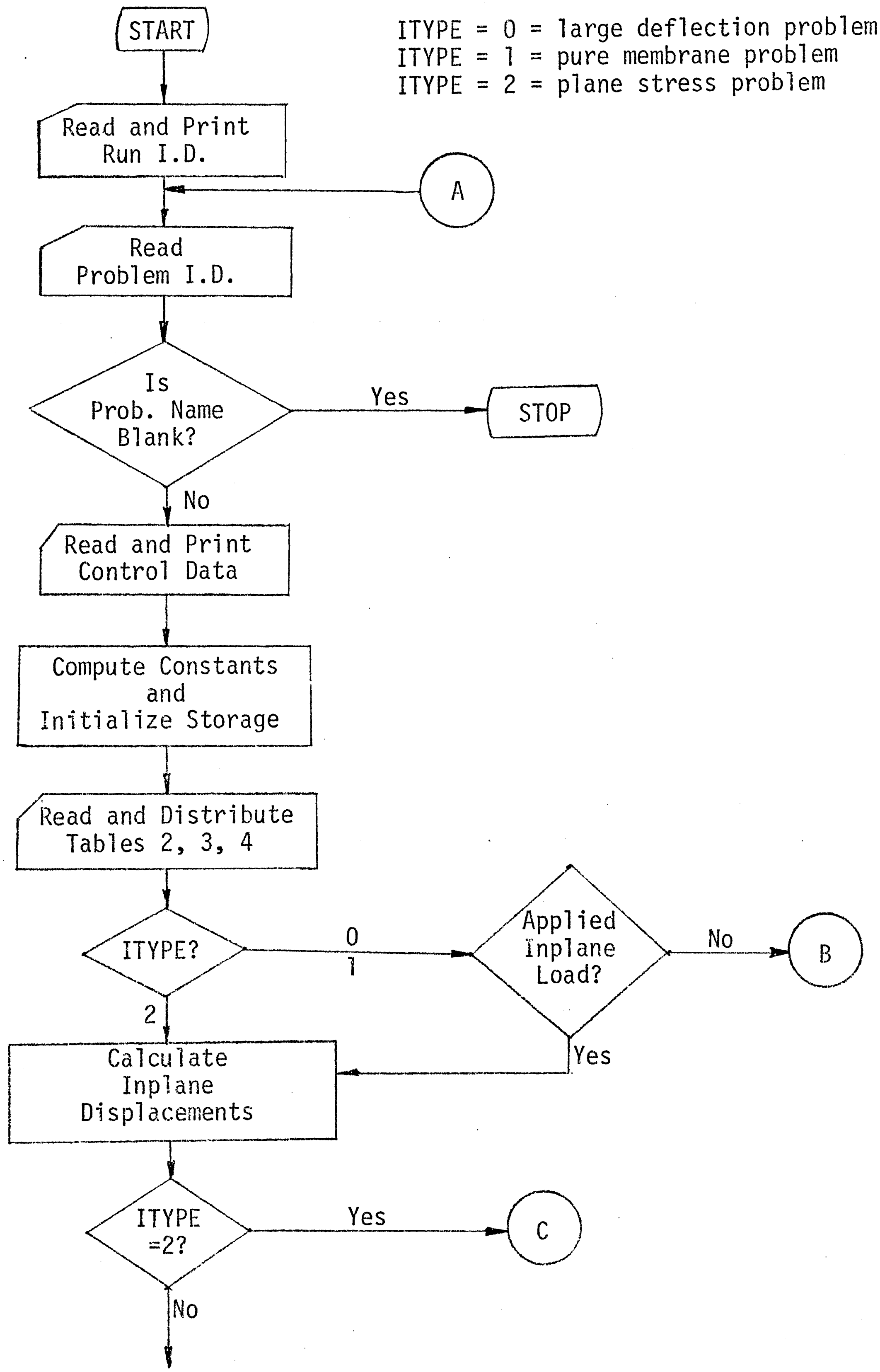


Figure 7. Summary Flow Diagram

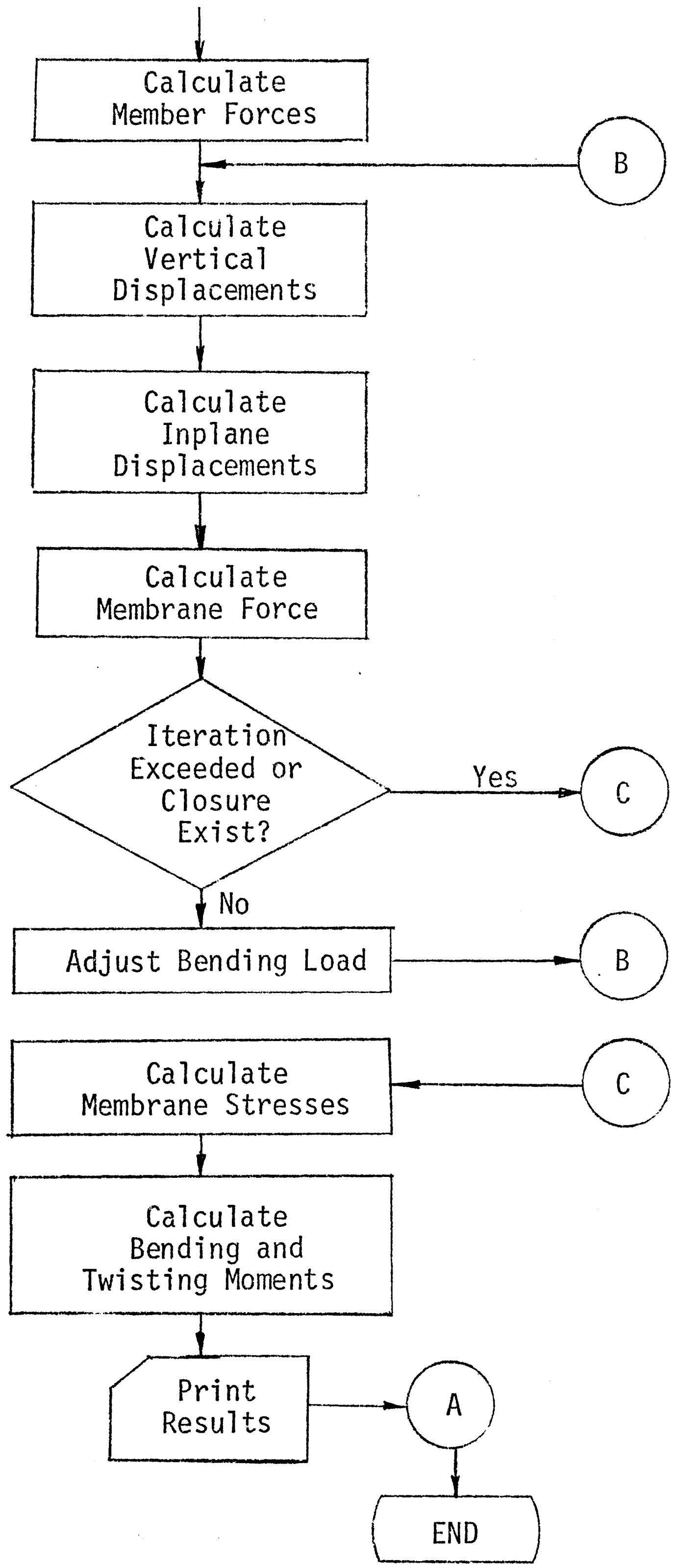


Figure 7. (Continued)

The bending and membrane models are represented by bars connected at joints. The bending model consists of flexible joints and bars which are rigid out of plane and behave as telescoping tubes inplane. The membrane model consists of ball and socket joints which connect flexible bars. While the membrane model is able to resist both inplane and out of plane loading, the bending model can resist only vertical loads.

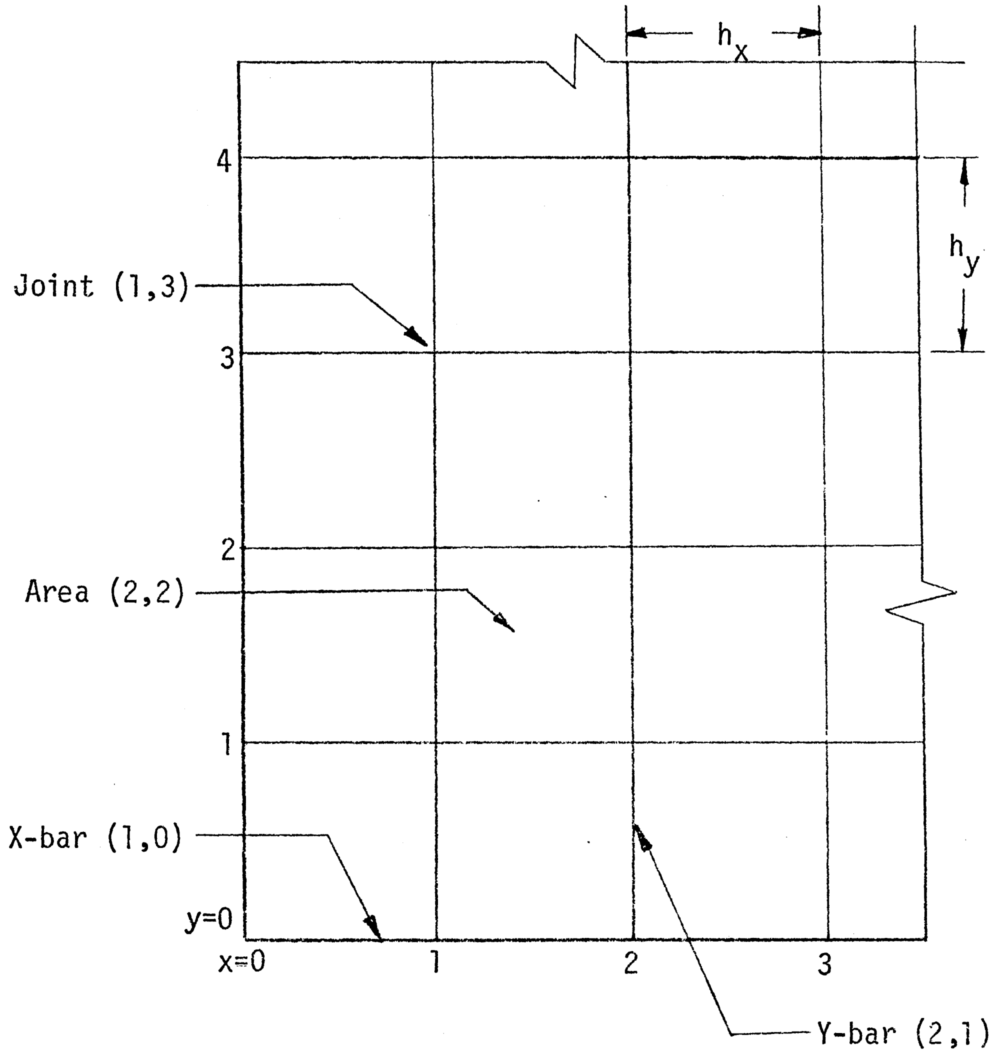
The properties of the models are defined by node or joint, bar, and area data. For the bending model, node data are plate bending stiffness (DXN, DYN), spring stiffness (SN, SUN, SVN), rotation stiffness and vertical load (RXN, RYN, QN). Bar data for the bending model are couples in the x and y directions (TXN, TYN). Area data is the twisting stiffness (CN). For the membrane model, node data are the applied inplane forces (PXN, PYN), and area data are elastic modulus (EXN, EYN). The data are arranged in the tabular form, and the general input sequence is described below (see Figure 8).

5.2.1 Identification of Run

The execution of the program starts by reading the identification of run. Two alphanumeric cards are required.

5.2.2 Identification of Problem

One alphanumeric card is required at the beginning of each problem. The program stops if the Problem Name is blank.



Node Data: DXN, DYN
SN, SUN, SVN, RXN, RYN
QN, PXN, PYN

Area Data: CN, EXN, EYN

Bar Data: TXN, TYN

Figure 8. Joint Coordinate Identification

5.2.3 Table 1: Control Data

Two cards are required for each problem. The first card specifies the number of cards in each of the following tables. The second card contains information about increment length, number of increment, and type of problem. The problem is large deflection, pure membrane, and plane stress of thin plate when ITYPE is 0, 1, and 2, respectively.

5.2.4 Table 2: Plate Stiffness

Bending, twisting, and membrane stiffnesses are organized in this table. The number of cards may vary, up to 20, depending on the problem. If the membrane stiffness is zero, the program will analyze the plate structure by small deflection theory.

5.2.5 Table 3: Supporting Spring Stiffness

Vertical and inplane spring stiffnesses as well as rotational spring stiffness are organized in this table. The maximum of cards permitted for this table is 20.

5.2.6 Table 4: Loading System

All external loads, vertical, inplane, and moments are input by this table. As many as 20 cards can be used to define the loading system.

5.2.7 End of Run

A blank card is required at the end of the data deck to terminate the program.

5.3 Output Information

The computer program prints the complete list of input data in tabular form as the data are read. Calculated results are also printed out in tabular form. Table 1 to Table 4 are the print out of the data input. Table 5 consists of the results of the vertical and inplane displacements. Table 6 indicates the bending and twisting moments which are the results from the bending model. Table 7 lists the normal and shear membrane stresses which are the results from the membrane model. Examples of some output are included in Appendix F.

CHAPTER VI

EXAMPLE PROBLEMS

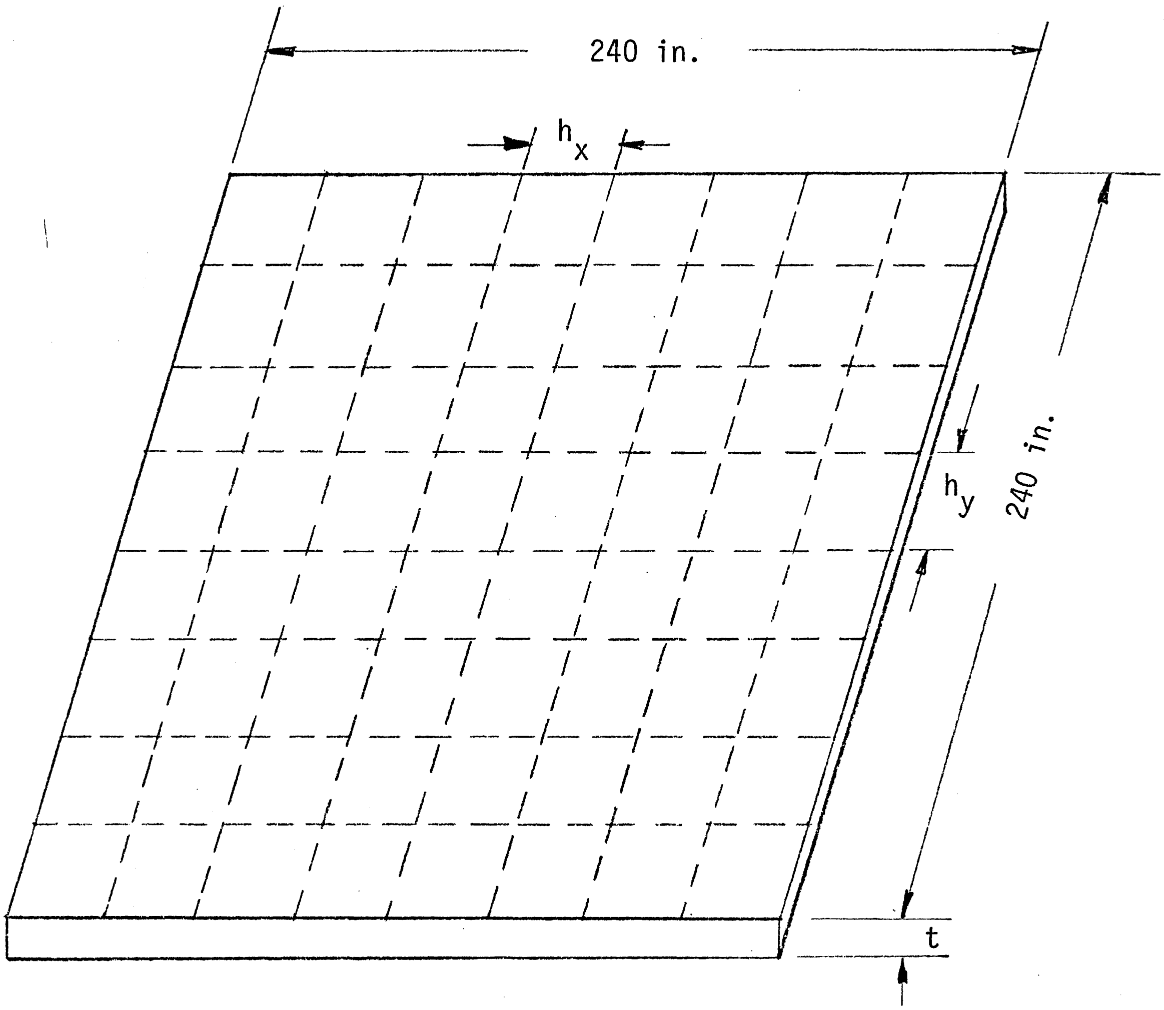
6.1 General

In this chapter the results of several problems are presented to illustrate the efficiency of the program, to demonstrate its use, and to verify the accuracy of the method of analysis. The results of these problems are compared to those obtained by other investigators. The listing of data and the computer output for these problems are shown in Appendix F.

6.2 Example Problems

A square plate shown in Figure 9 is used to demonstrate the method of analysis given in this report. The effects of plate properties, such as modulus of elasticity, length, width, and thickness of the plate are eliminated by presenting the results in terms of nondimensional constants. The plate problem is divided into increments in the x and y directions. The accuracy of the method depends on the number of discrete elements formed by the division. The accuracy increases with the number of elements. Most of the example problems in this chapter use 8 x 8 discrete elements.

There are five problems in this chapter. The first four problems are simply supported plates with different load and boundary conditions. The last problem is a rectangular plate with three edges fixed and the



$DX = DY = 1.13 \times 10^8$ lb-in.

$EX = EY = 3.60 \times 10^6$ lb/in^2

$C = 7.90\ 10^7$ lb-in.

$MX = MY = 8$

$h_x = h_y = 30.0$ in.

$t = 7.0$ in.

$\nu = 0.3$

Figure 9. Square Plate for Example Problems

other edge free, which may represent the side of a storage bin or a retaining wall. These example problems are: (1) simply supported plate with immovable edges subjected to a uniform load; (2) simply supported plate with immovable edges subjected to a concentrated load; (3) simply supported membrane with a uniform load; (4) simply supported plate with movable edges in the plane of the plate subjected to a uniform load; and (5) rectangular plate with three edges fixed and one edge free. The details of each problem are discussed separately below.

6.2.1 Simply Supported Plate Immovable Edges; Uniform Load

The first problem to be considered is the simply supported plate with edges restrained against translation in all directions. Uniform vertical loading is applied to the plate. The solutions for deflection, bending stress, and membrane stress at the center of the plate are shown in Figures 10, 11, and 12, respectively. These values compare favorably with the results by Levy's method (8). The deflections and bending stresses show very good agreements with those solutions while membrane stresses are slightly less. These membrane stresses can be improved by using smaller discrete elements in the analysis. However, the total stresses still agree very well with Levy's.

6.2.2 Simply Supported Plate Immovable Edges; Concentrated Load

In this problem, a single concentrated load is applied at the center of the plate. The support conditions remain unchanged. The results are shown in Figure 13. This problem has not been solved by

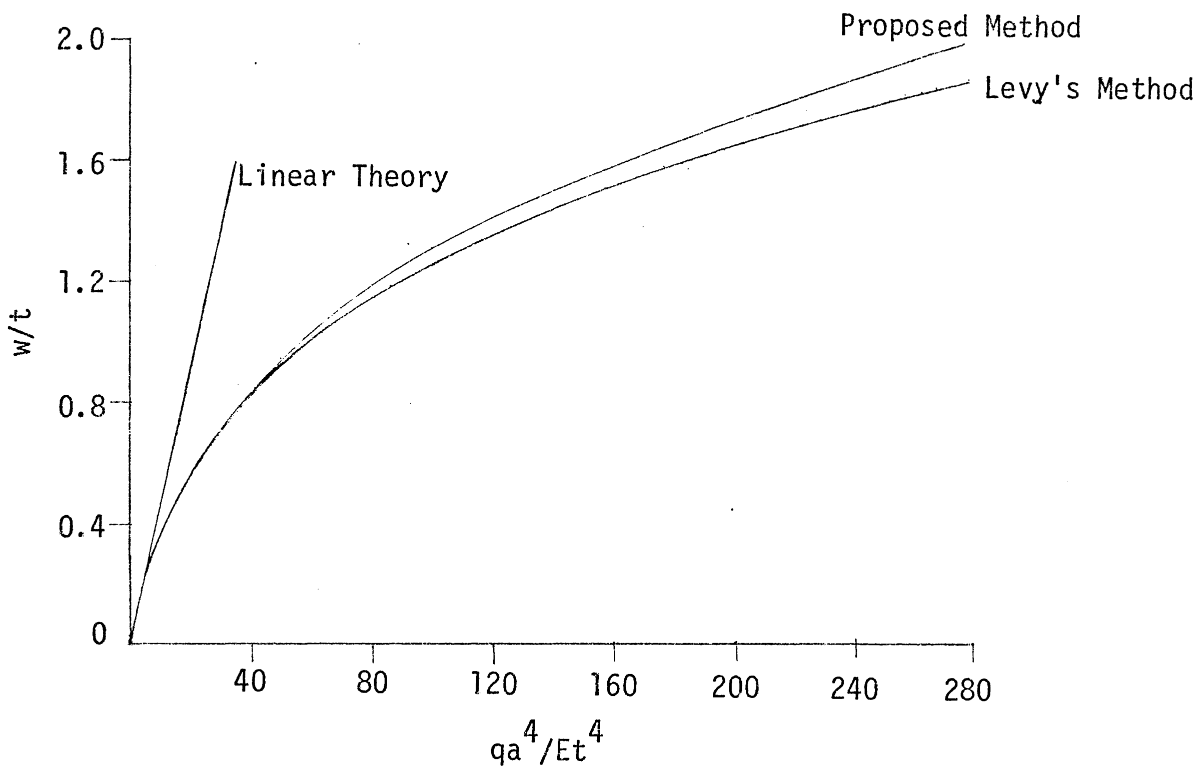


Figure 10. Relation Between Load and Maximum Deflection for a Plate With Restrained Edges and Uniform Vertical Loading

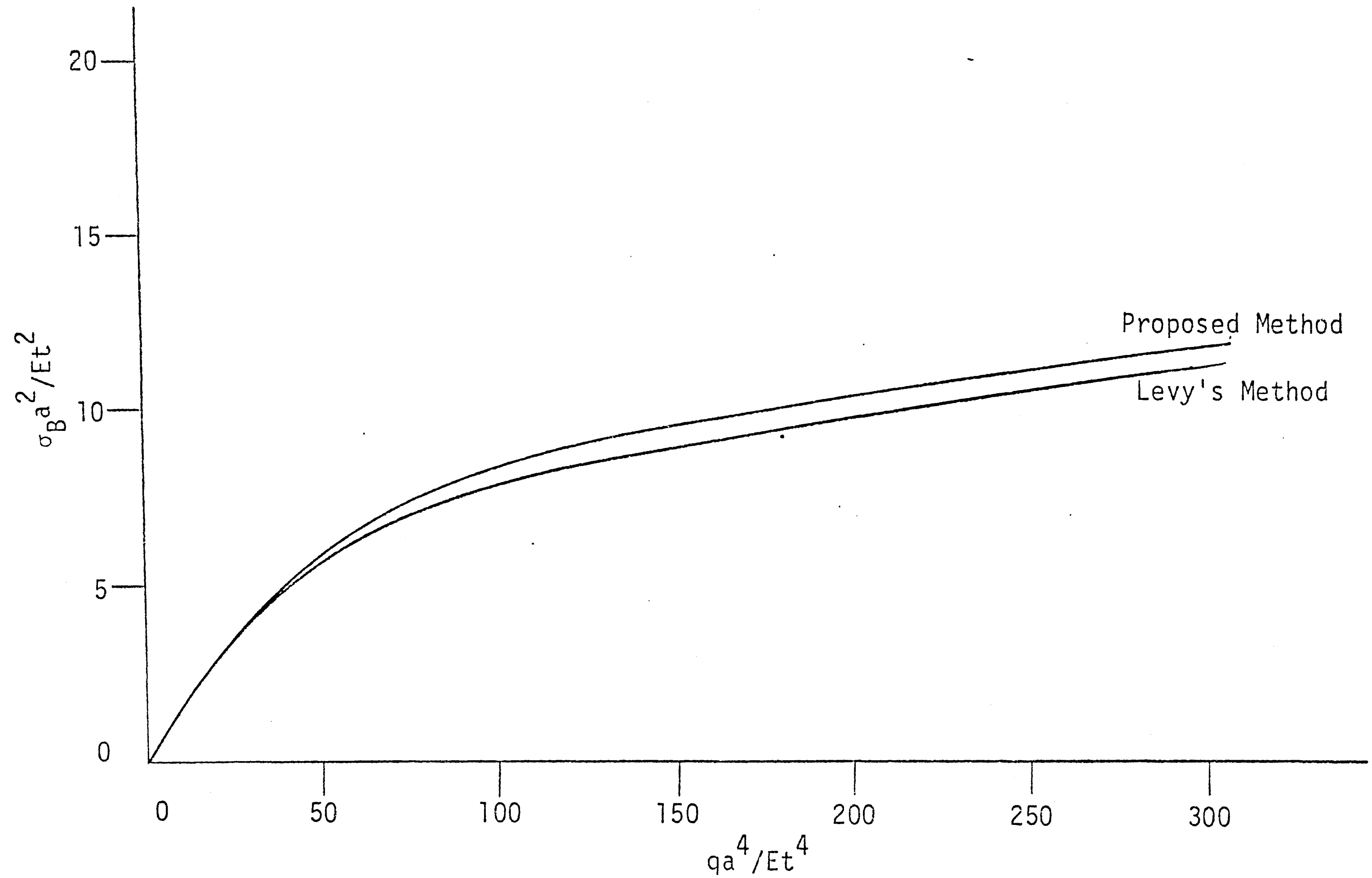


Figure 11. Relation Between Load and Bending Stress for a Plate With Restrained Edges and Uniform Vertical Loading

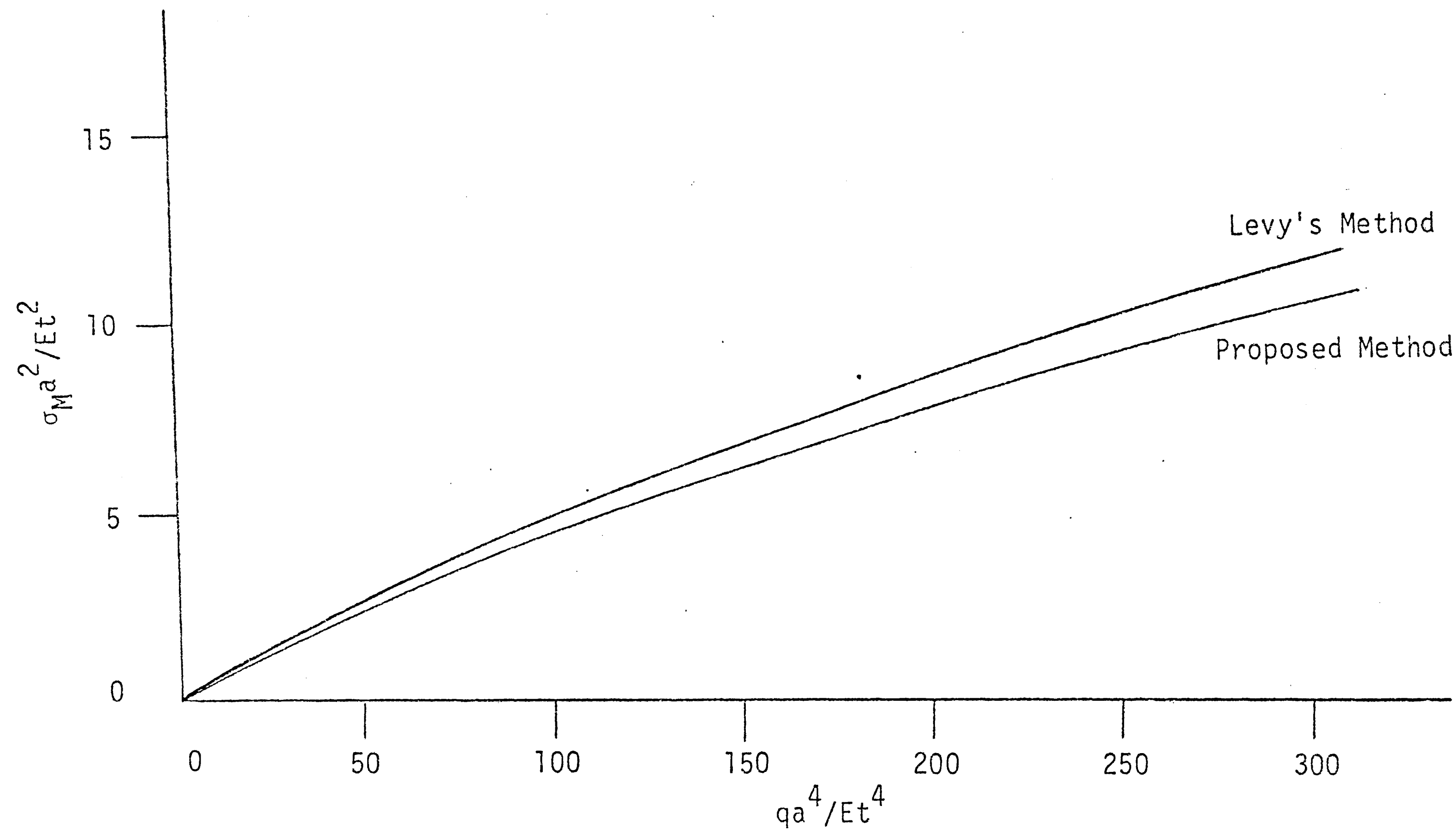


Figure 12. Relation Between Load and Membrane Stress for a Plate With Restrained Edges and Uniform Vertical Loading

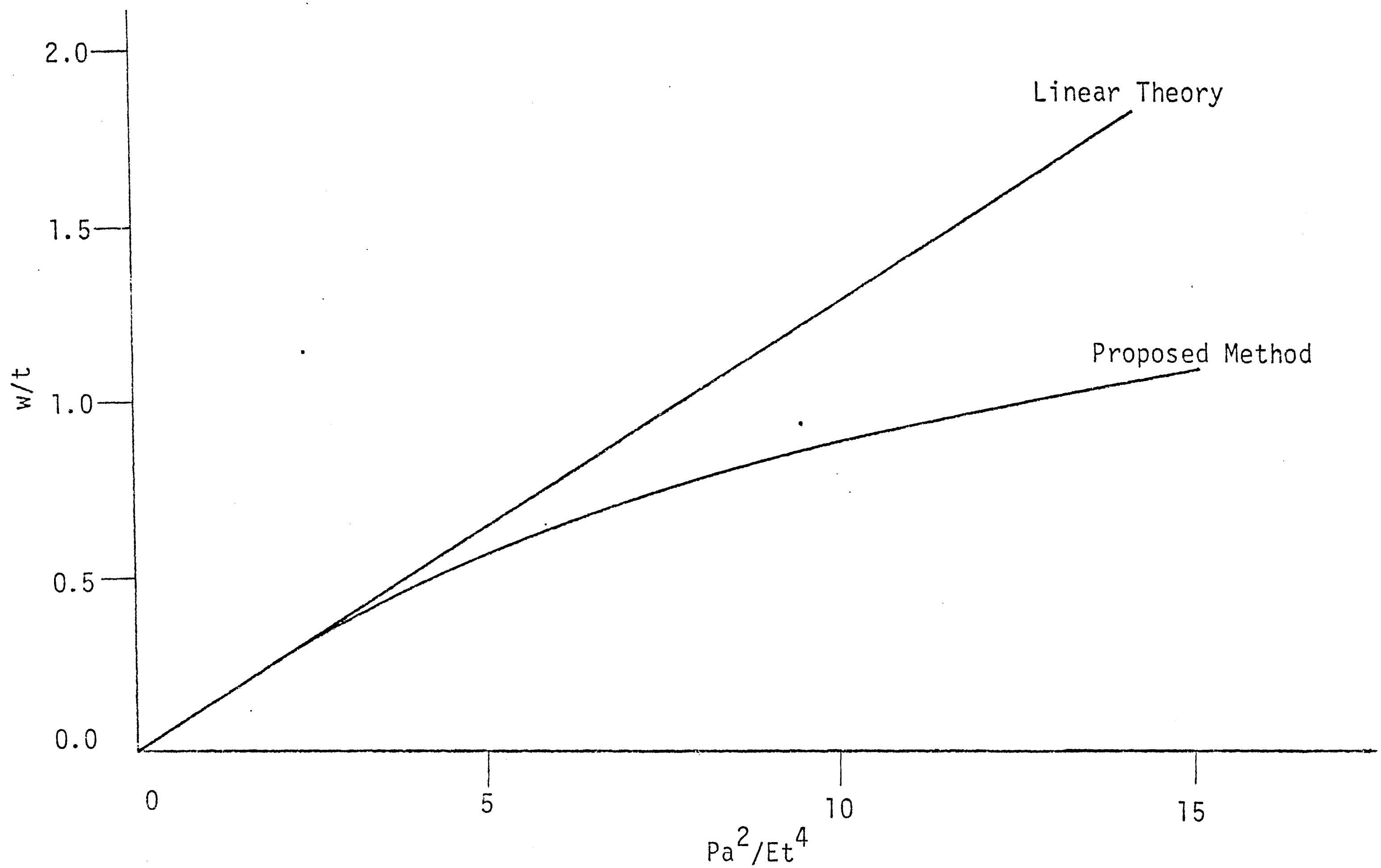


Figure 13. Maximum Deflection for a Plate With Restrained Edges and a Concentrated Loading

other investigators and is presented for information and a demonstration of the method.

6.2.3 Membrane Problem

The dimensions of the membrane are the same as the plate of the previous examples. However, bending and twisting stiffnesses are omitted. A uniform load is applied to the plate and only the membrane resists the vertical load. The deflections for several values of loads are compared to those from Timoshenko (18) in Figure 14. Note that excellent agreement is noted between results from this work and Timoshenko.

6.2.4 Simply Supported Plate Movable Edges; Uniform Load

The plate in Figure 9 with uniform distributed load is considered in this problem. For this problem, edges are simply supported but free to move in the plane of the plate. A load of 533.33 lb/in^2 is applied to the plate. The maximum nondimensional deflection, w/t, of the plate calculated by the computer program written in this study is 2.60. This solution compares well with the nondimensional deflection calculated by Levy's method (8) for which the value is 2.54.

6.2.5 Rectangular Plate with Three Edges Fixed and One Edge Free

To demonstrate the method presented by this work the plate shown in Figure 16 will be analyzed. Three edges are fixed against all translations and rotations, while the fourth edge is free. Plates with these

$Q' = qa^4/Et^4$	Timoshenko's Method (in.)	Proposed Method (in.)
50	8.27	8.29
100	10.42	10.48
150	11.93	12.03
200	13.13	13.26

Figure 14. Comparison of Calculated Vertical Membrane Deflection With Timoshenko

boundary conditions may represent an integral part of rectangular tanks or retaining walls. The plate is 40 inches wide and 60 inches long. The bending and membrane stiffness, uniform in both x and y directions, are 2.5×10^6 lb/in^2 and 30.0×10^6 lb/in^2, respectively. Poisson's ratio is 0.3, and the thickness of the plate is 0.97 inches. The plate is divided into ten discrete elements in both x and y directions with h_x and h_y are 4.0 and 6.0 inches respectively. The load varies from top to bottom, as shown in Figure 15. This load pattern represents the effect of hydrostatic pressure which varies from zero at the top to 300 lb/in^2 at the lower edge. The solution for deflection at joint (5,10) is equal to 0.497 inches compare to 1.29 inches from the linear theory (18). Results of this problem are shown in Figures 16 through 20.

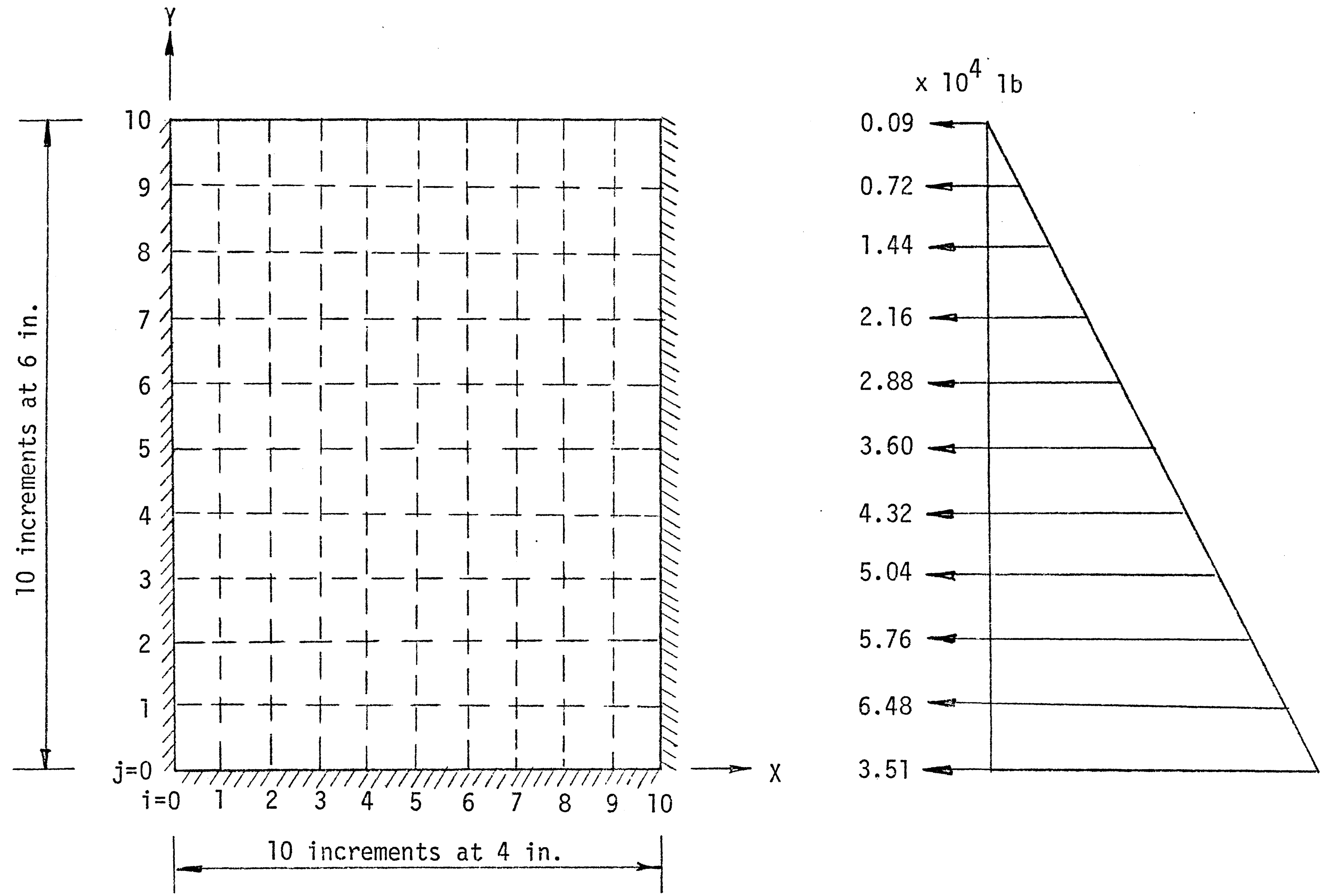


Figure 15. Rectangular Plate With Three Edges Fixed and One Edge Free

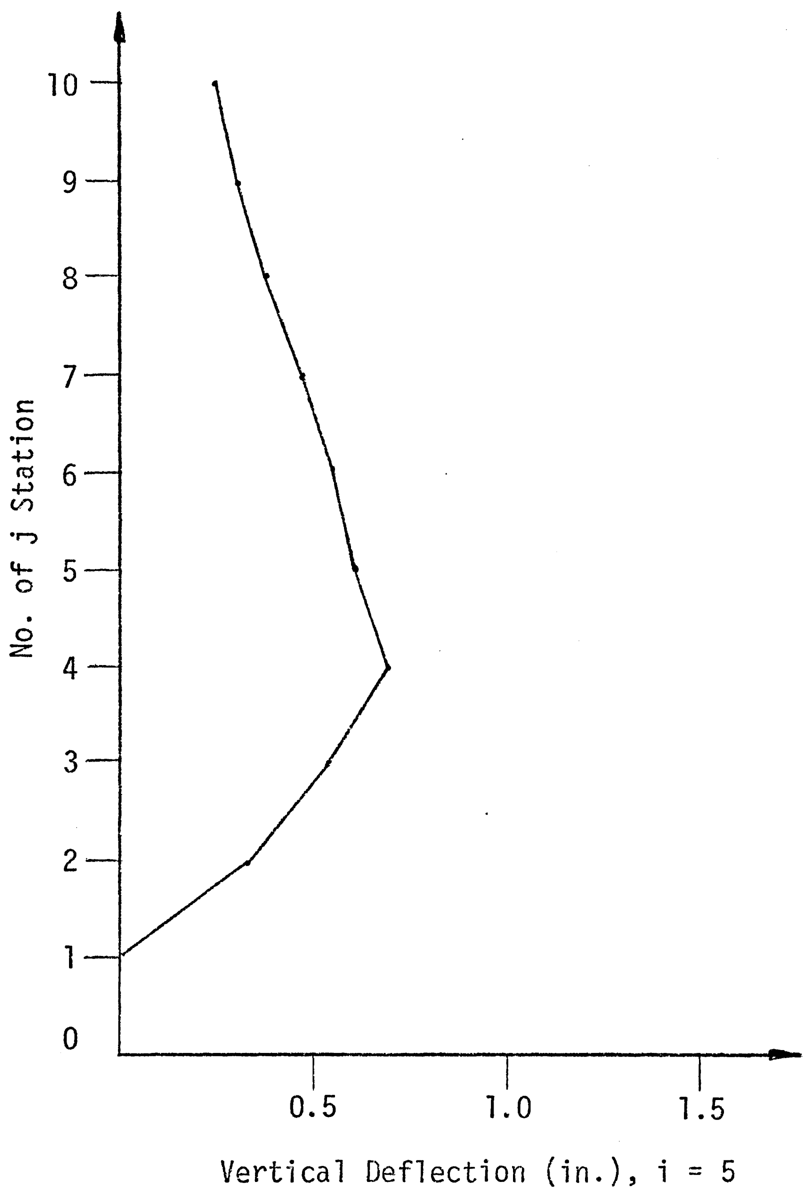


Figure 16. Vertical Deflection of Centerline of Plate

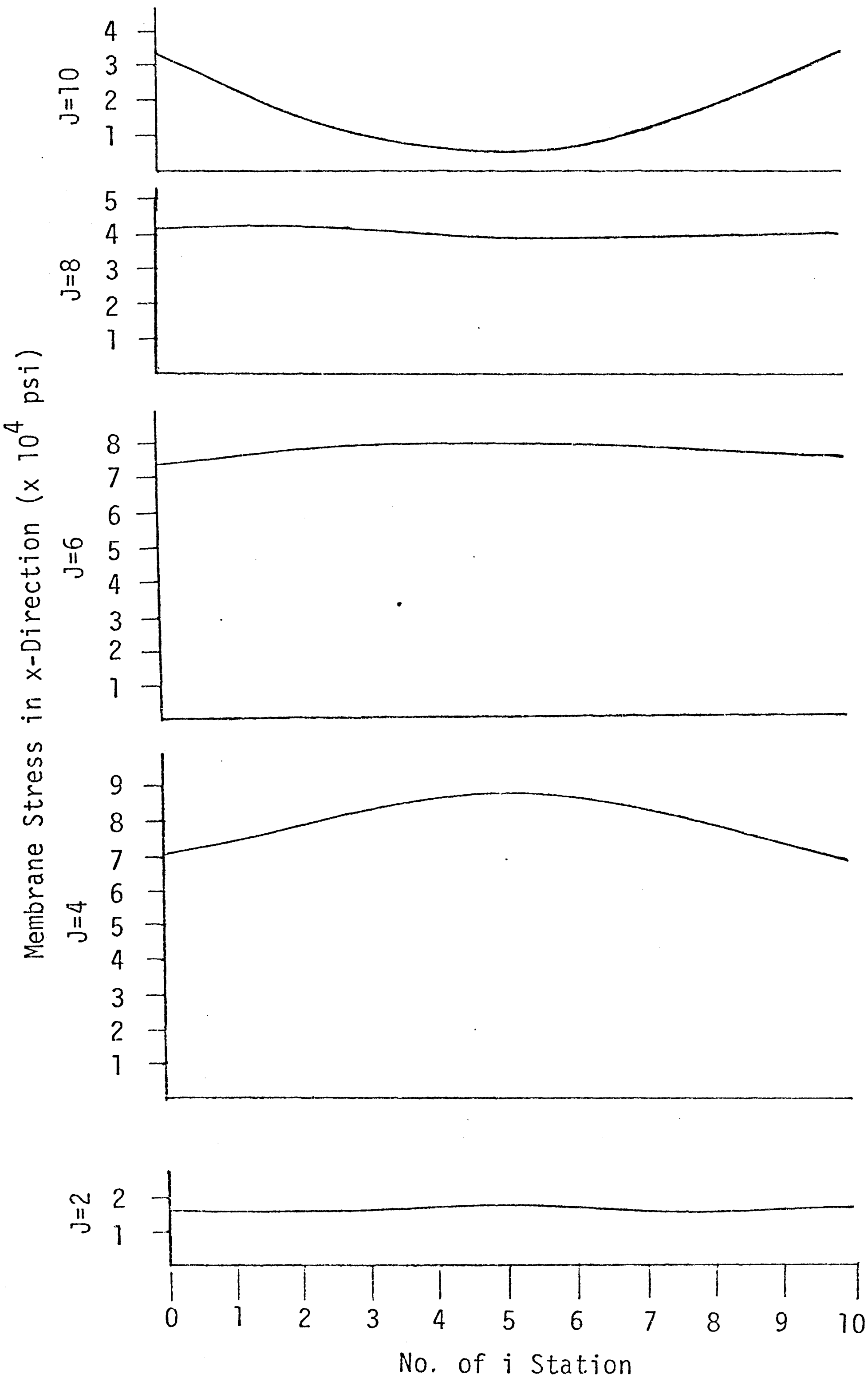


Figure 17. Membrane Stress in x-Direction

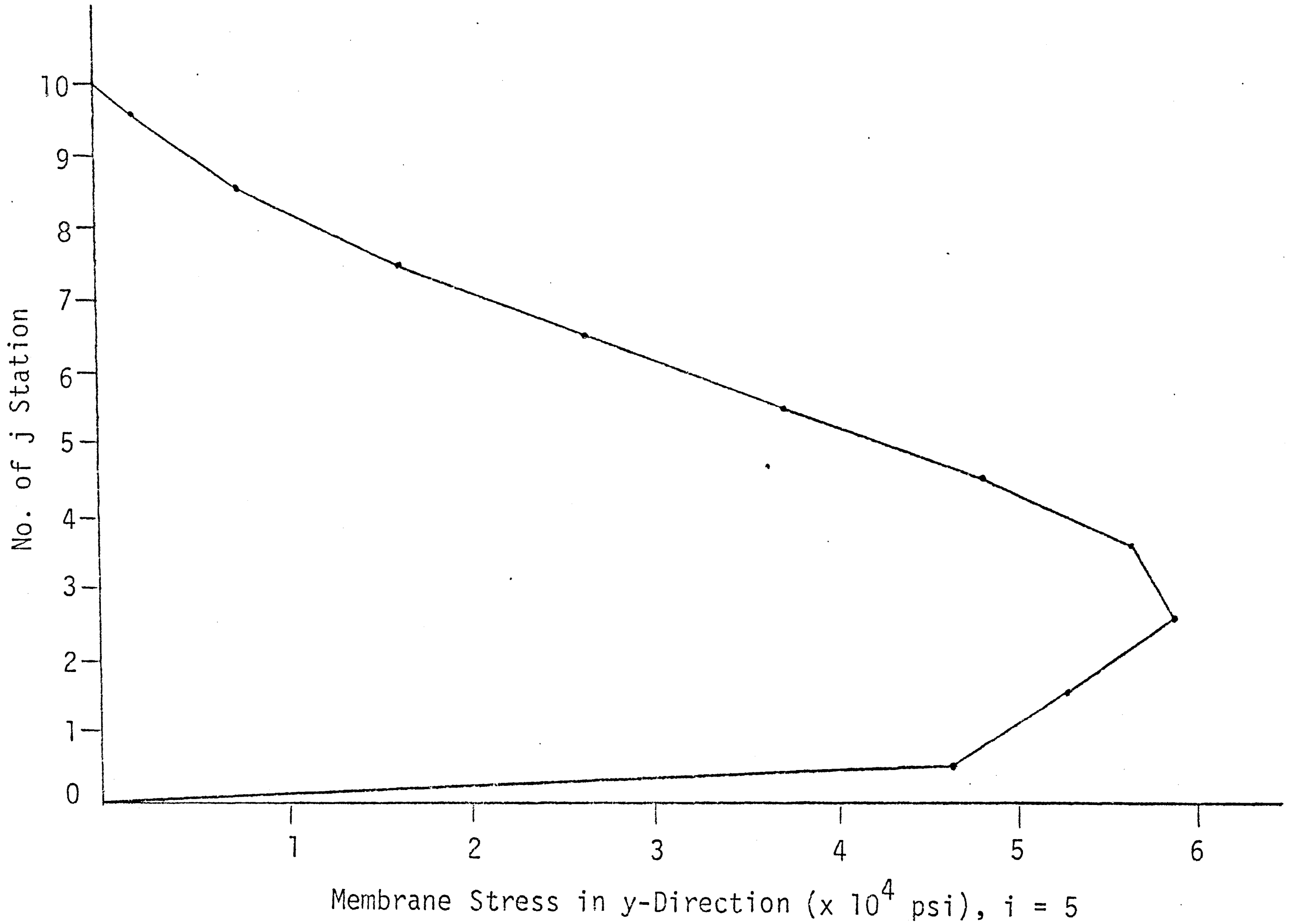


Figure 18. Membrane Stress in y-Direction Along Centerline

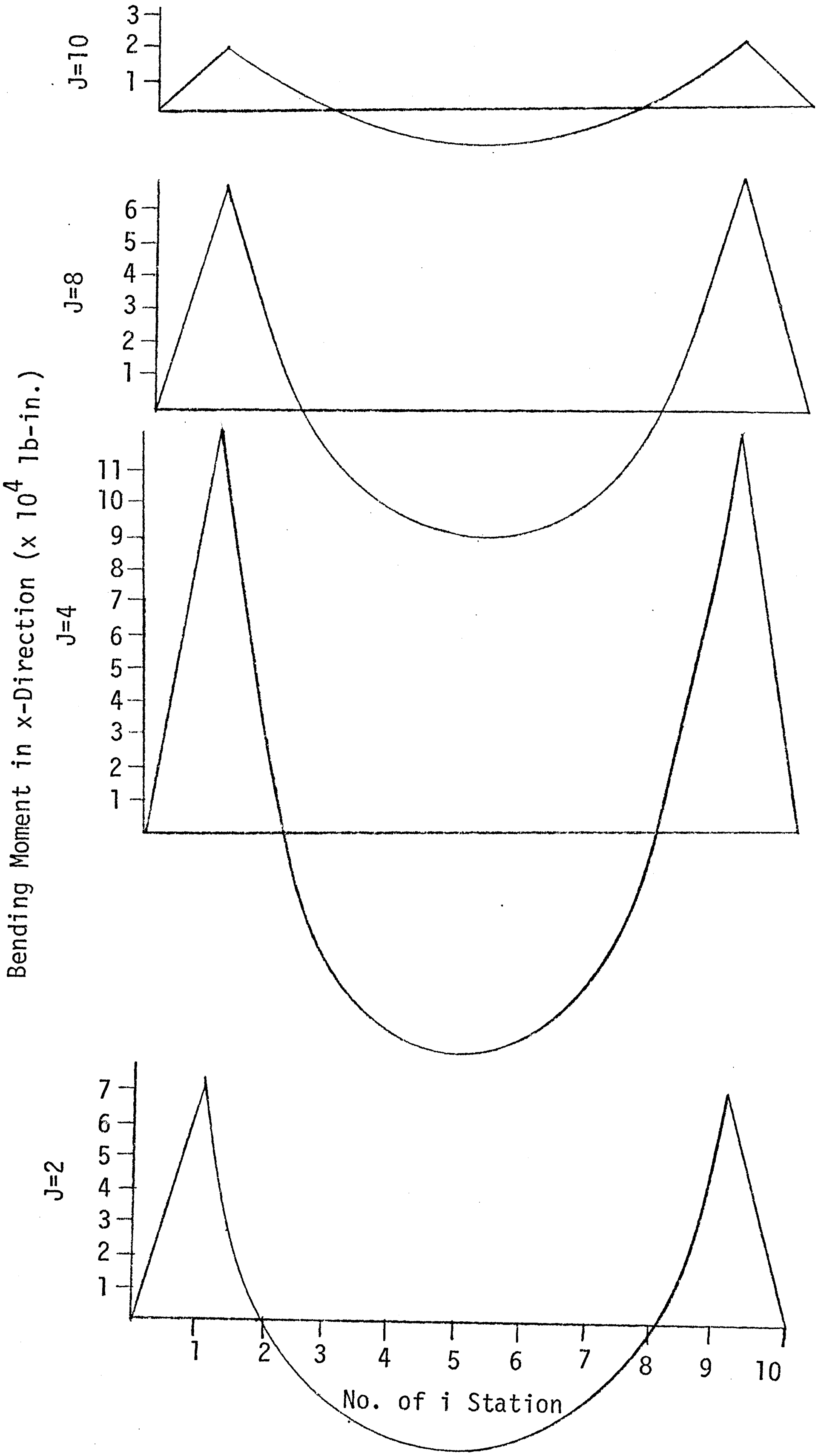


Figure 19. Bending Moment in x-Direction

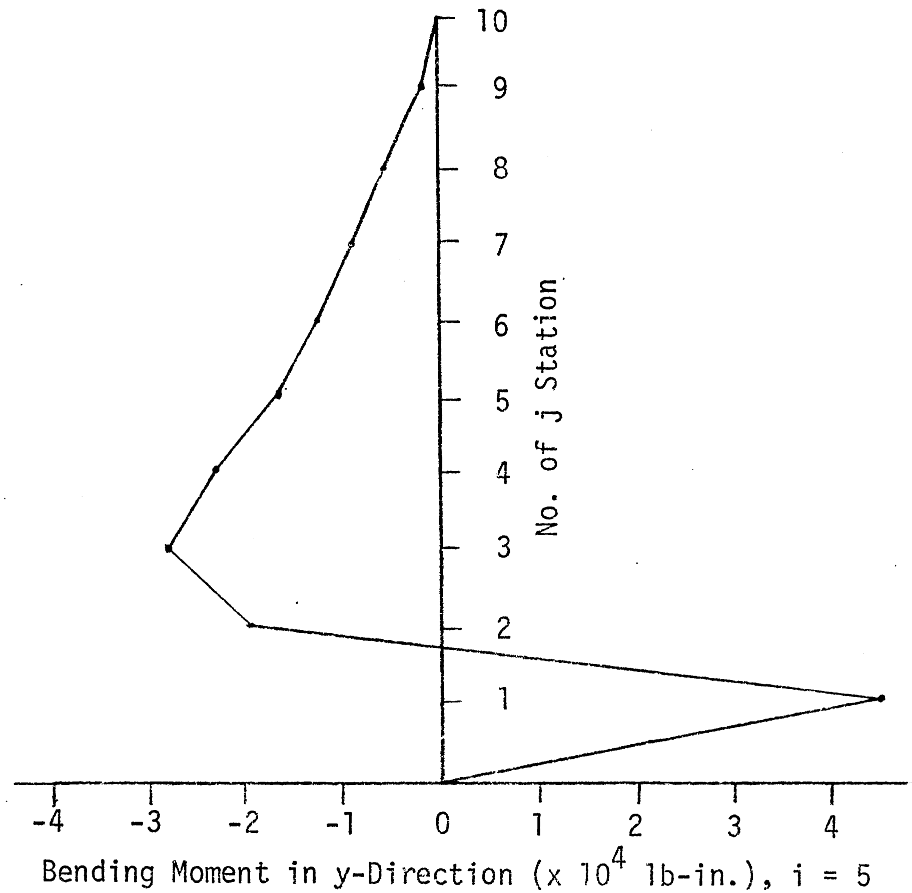


Figure 20. Bending Moment in y-Direction Along Centerline

CHAPTER VII

SUMMARY AND RECOMMENDATIONS

7.1 Summary

A method of analysis for thin elastic plates with large deflections has been developed. The plate structure is represented by two systems of discrete element models, one for bending and the other for membrane resistance. These two models are connected to enforce deflection compatibility. Linear theories are applied to calculate the resisting force in each model. Vertical deflections from the bending model produce inplane forces and vertical resistance in the membrane model. Equilibrium between the applied load and resisting forces is satisfied by using an iterative technique to distribute the applied vertical load to the bending and membrane models.

A computer program has been written to provide flexibility in problem description, to include variable plate geometry, loading and boundary conditions. A technique for automatic distribution of data is utilized to minimize the amount of input data. The program can be applied to analyze a wide variety of plate structures.

It was shown in Chapter VI that deflections and stresses calculated by the method developed in this work compared favorably with accepted values. The simply-supported square plate and membrane analyzed by the computer program gave deflections which agreed within 1.0 percent of the theoretical values. Furthermore, for the grid selected for this study (8 x 8) excellent agreement was also noted for bending moments and membrane stresses.

The program was also applied to a problem to illustrate its flexibility and demonstrate the solution procedure for various support conditions. The accuracy of the solutions can be specified by the number of iterations and the closure tolerance which also input into the computer program.

It was shown that when a thin plate undergoes large deflections, a significant membrane resisting force is developed and must be included in the analysis of the structure to accurately evaluate vertical deflection. This membrane force may be greater than the bending resistance of the plate. The method developed in this work may be applied for the analysis of complex plate structures with irregular shapes, loading and boundary conditions.

7.2 Recommendations

The work discussed in this report may be extended to include both nonlinear material behavior and dynamic response of plates. Because of large vertical deflections, there may be yielding of the middle surface. However, a small change in geometry due to inelastic behavior of the material can provide increased resistance to vertical load.

The existing model will provide a suitable presentation of the plate structure for dynamic analysis of plates undergoing large vertical displacements. Mass and damping may be added to the model and a numerical integration method, such as Newmark's Beta method (27), could be utilized to satisfy dynamic equilibrium.

The method used in this study may also be extended to determine the buckling load for plate structures. The inplane force distribution can be determined by the computer program. Vertical deflections may be calculated for small vertical loads. The buckling load would not be reached if the plate is in equilibrium at a finite deflection for a given inplane force. The buckling load could be determined by increasing the inplane force until very large vertical displacements are caused by a small vertical load.

Finally, it is recommended that the method be applied to shell structures. The model is capable of representing the behavior of types of shell structures, such as cylindrical shells. The location of the nodes or joints of the undeformed structure need not fall in the plane. By generating the grid in space, it would be possible to describe a shell. A method developed by this work can be utilized to calculate the deflection, membrane force, and moment in the shell structure.

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APPENDIX A

DERIVATION OF EQUILIBRIUM EQUATION FOR DISCRETE-ELEMENT BENDING MODEL

The discrete element model for bending shown in Figure 2 is similar to one presented by Hudson (22). It consists of rigid bars, elastic restraints at joints, and torsional bars connecting the middle of the rigid bars. In this appendix equilibrium equations will be derived.

An expanded view of a joint in the bending model is shown in Figure 21. The elastic elements are replaced by the forces and moments which are developed as the joints of the model undergo deformations. The equilibrium equation of forces in the z direction is

$$\Sigma F_z = 0 = Q_{i,j} + V^x_{i,j} + V^y_{i,j} - V^x_{i+1,j} - V^y_{i,j+1} - S_{i,j}w_{i,j} \tag{A.1}$$

This equilibrium equation, Equation (A.1), can be extended to include the effects of the rotational stiffness at each joint in the model. These rotational stiffnesses may be represented by a set of the rotational supports that are attached to the bending model. Figure 22 shows the rotational stiffness at joint i,j in the x direction which is represented by a rotational support connecting to the bending model at joints i+1,j and i-1,j. The forces developed by the rotational support are given below.

The rotational support resists rotation of the bars of the plate model. A couple, proportional to the slope at joint i,j, is produced as a result of the rotation. The couple forces are applied at joints i-1,j and i+1,j due to the attachment of these joints to the rotational restraint. The couple moment is related to the slope and rotational stiffness by

$$M^x_{i,j} = R^x_{i,j}\ (w_{i+1,j} - w_{i-1,j})/2h_x \tag{A.2}$$

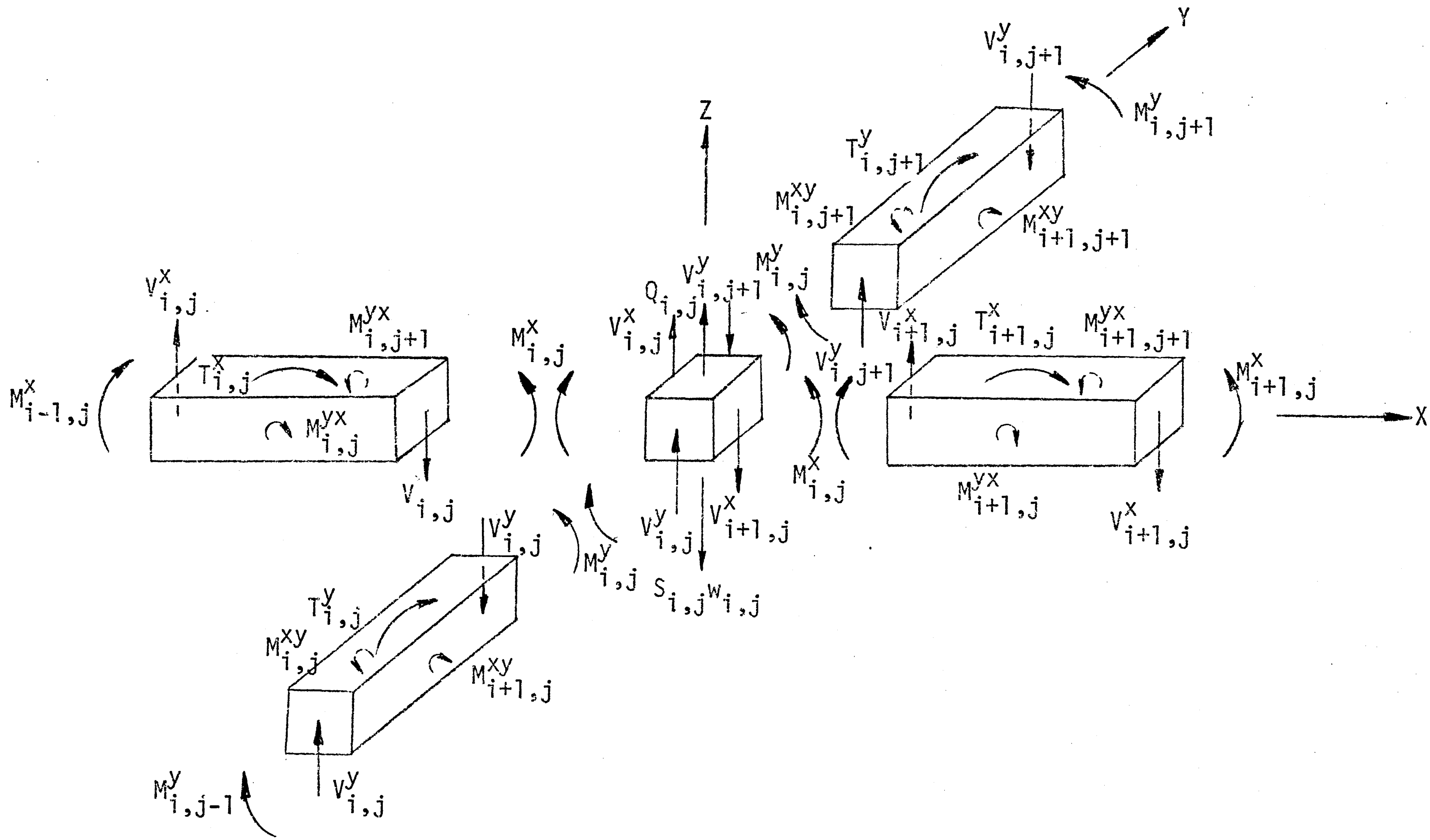


Figure 21. Expanded Joint in the Bending Model

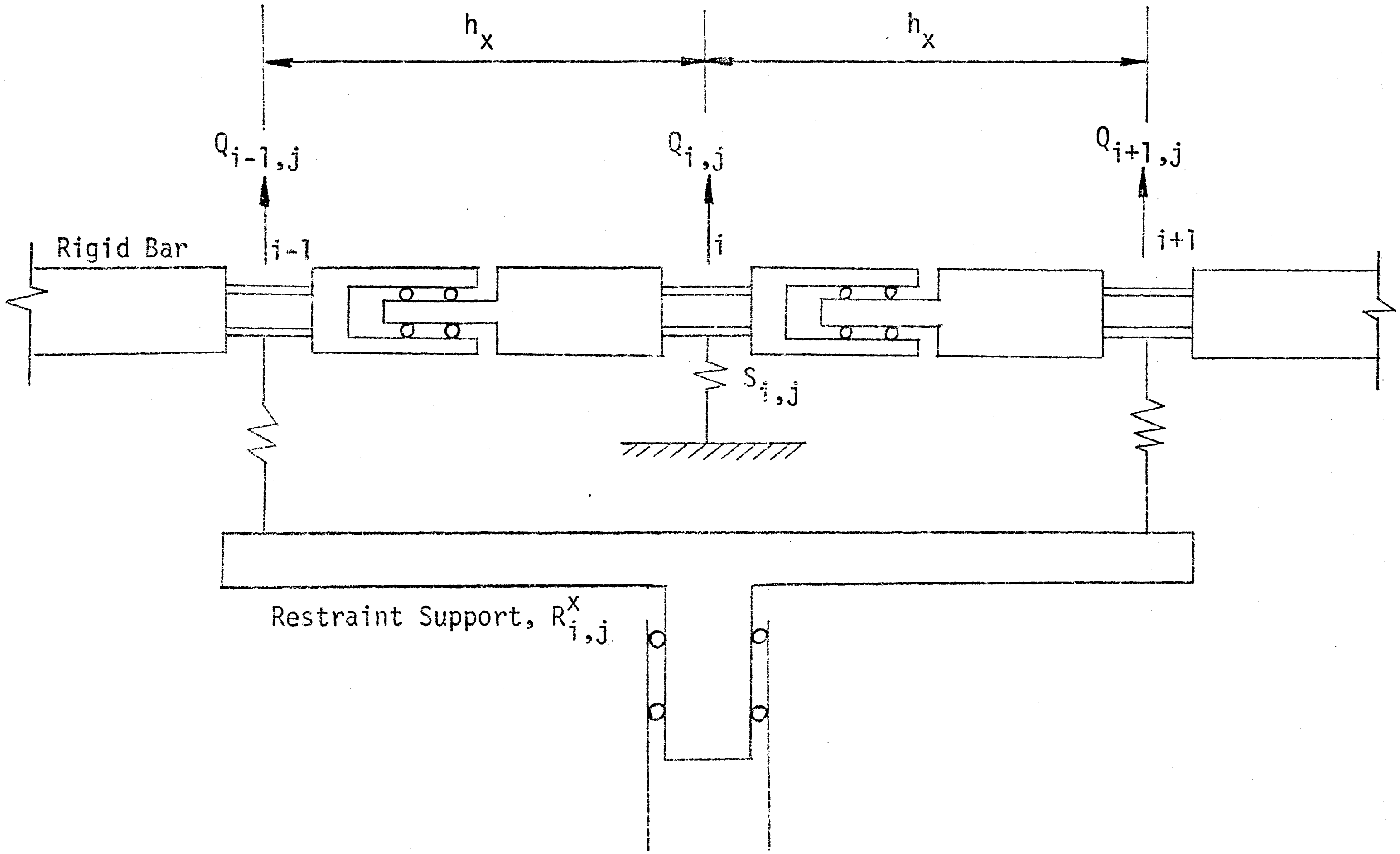


Figure 22. Typical Joint of Bending Model Showing Rotational Stiffness

where

$M^x_{i,j}$ = the couple developed at joint i,j; and

$R^x_{i,j}$ = rotational stiffness at joint i,j in the x direction.

The couple forces at joints i-1,j and i+1,j are

$$F_{i+1,j} = -R^x_{i,j}\ (w_{i+1,j} - w_{i-1,j})/4h_x^2$$

$$F_{i-1,j} = +R^x_{i,j}\ (w_{i+1,j} - w_{i-1,j})/4h_x^2$$

By applying the method mentioned above, the total forces at joint i,j due to rotational restraints at adjacent joints are

$$F_{i,j} = F_1 - F_2 + F_3 - F_4 \qquad \text{(A.3)}$$

where

$$F_1 = R^x_{i-1,j}\ (w_{i,j} - w_{i-2,j})/4h_x^2$$

$$F_2 = R^x_{i+1,j}\ (w_{i+2,j} - w_{i,j})/4h_x^2$$

$$F_3 = R^y_{i,j-1}\ (w_{i,j} - w_{i,j-2})/4h_y^2$$

$$F_4 = R^y_{i,j+1}\ (w_{i,j+2} - w_{i,j})/4h_y^2$$

Although the membrane model will be discussed later (Appendix B), it is necessary to illustrate the effect of inplane loads transmitted by the membrane model on joint equilibrium. Vertical displacement is resisted by the axial forces in the bars of the membrane model. This effect is shown in Figure 23 and the resistance due to the change in geometry is given by Equation (A.4).

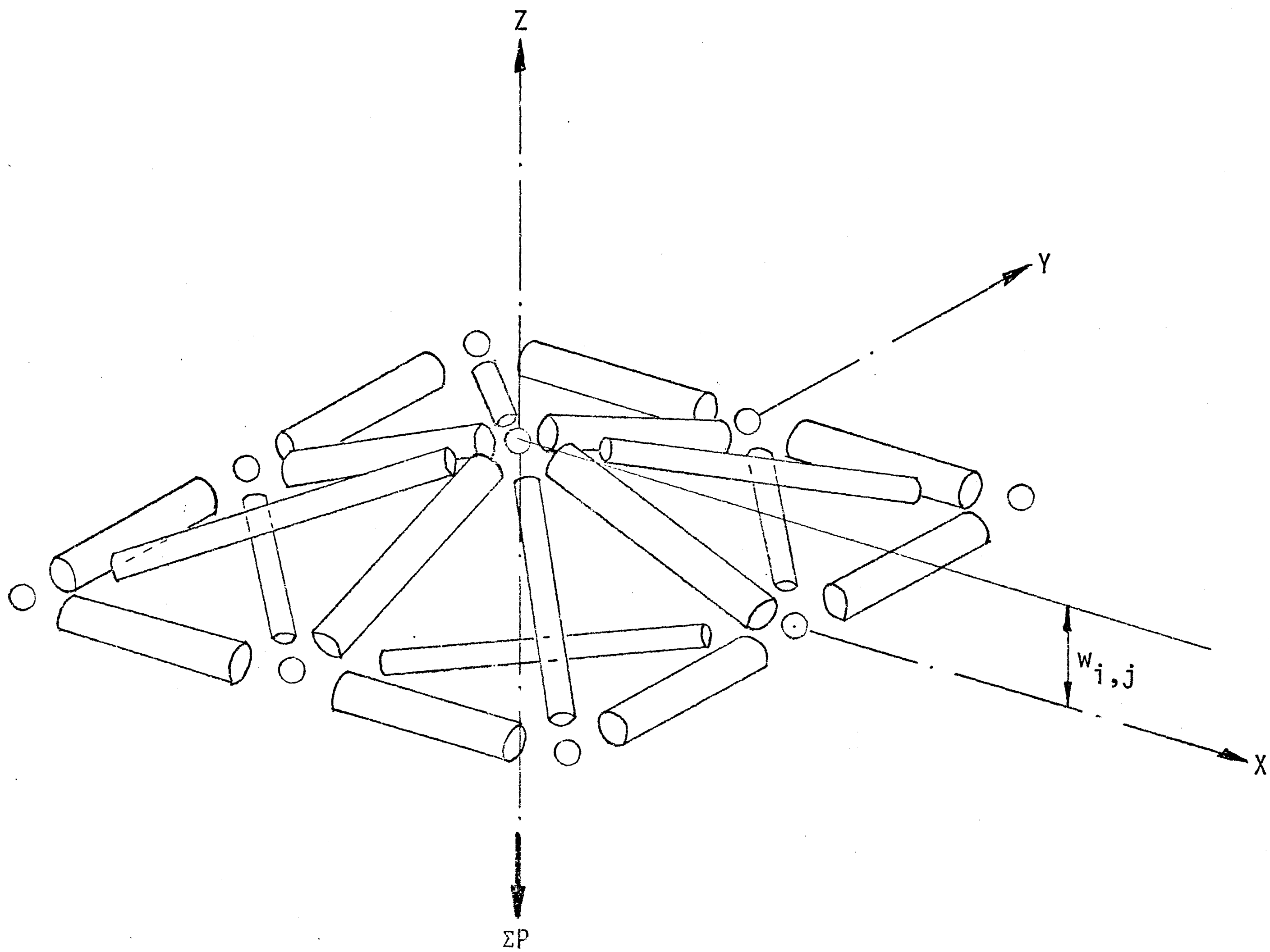


Figure 23. Vertical Resisting Force from the Membrane Model

$$\Sigma P = -P^x_{i,j}\ (w_{i,j} - w_{i-1,j})/h_x + P^x_{i+1,j}\ (w_{i+1,j} - w_{i,j})/h_x$$
$$- P^y_{i,j}\ (w_{i,j} - w_{i,j-1})/h_y + P^y_{i,j+1}\ (w_{i,j+1} - w_{i,j})/h_y$$
$$- P^{c1}_{i,j}\ (w_{i,j} - w_{i-1,j+1})/h_z + P^{c1}_{i+1,j-1}\ (w_{i+1,j-1} - w_{i,j})/h_z$$
$$- P^{c2}_{i,j}\ (w_{i,j} - w_{i-1,j-1})/h_z + P^{c2}_{i+1,j+1}\ (w_{i+1,j+1} - w_{i,j})/h_z \tag{A.4}$$

The resistance of the membrane is added to the forces acting at joint i,j, shown in Figure 24, and yields

$$\Sigma F_z = 0 = Q_{i,j} + V^x_{i,j} + V^y_{i,j} - V^x_{i+1,j} - V^y_{i,j+1} - S_{i,j}w_{i,j} - F_{i,j}$$
$$- P^x_{i,j}(w_{i,j} - w_{i-1,j})/h_x + P^x_{i+1,j}(w_{i+1,j} - w_{i,j})/h_x$$
$$- P^y_{i,j}(w_{i,j} - w_{i,j-1})/h_y + P^y_{i,j+1}(w_{i,j+1} - w_{i,j})/h_y$$
$$- P^{c1}_{i,j}(w_{i,j} - w_{i-1,j+1})/h_z + P^{c1}_{i+1,j-1}(w_{i+1,j-1} - w_{i,j})/h_z$$
$$- P^{c2}_{i,j}(w_{i,j} - w_{i-1,j-1})/h_z + P^{c2}_{i+1,j+1}(w_{i+1,j+1} - w_{i,j})/h_z \tag{A.5}$$

Satisfying moment equilibrium for the rigid bars connected to joint i,j yields

$$-h_x V^x_{i,j} = M^{yx}_{i,j} - M^{yx}_{i,j+1} + M^x_{i-1,j} - M^x_{i,j} + T^x_{i,j}$$
$$-h_x V^x_{i+1,j} = M^{yx}_{i+1,j} - M^{yx}_{i+1,j+1} + M^x_{i,j} - M^x_{i+1,j} + T^x_{i+1,j}$$
$$-h_y V^y_{i,j} = -M^{xy}_{i,j} + M^{xy}_{i+1,j} + M^y_{i,j-1} - M^y_{i,j} + T^y_{i,j}$$
$$-h_y V^y_{i,j+1} = -M^{xy}_{i,j+1} + M^{xy}_{i+1,j+1} + M^y_{ij} - M^y_{i,j+1} + T^y_{i,j+1} \tag{A.6}$$

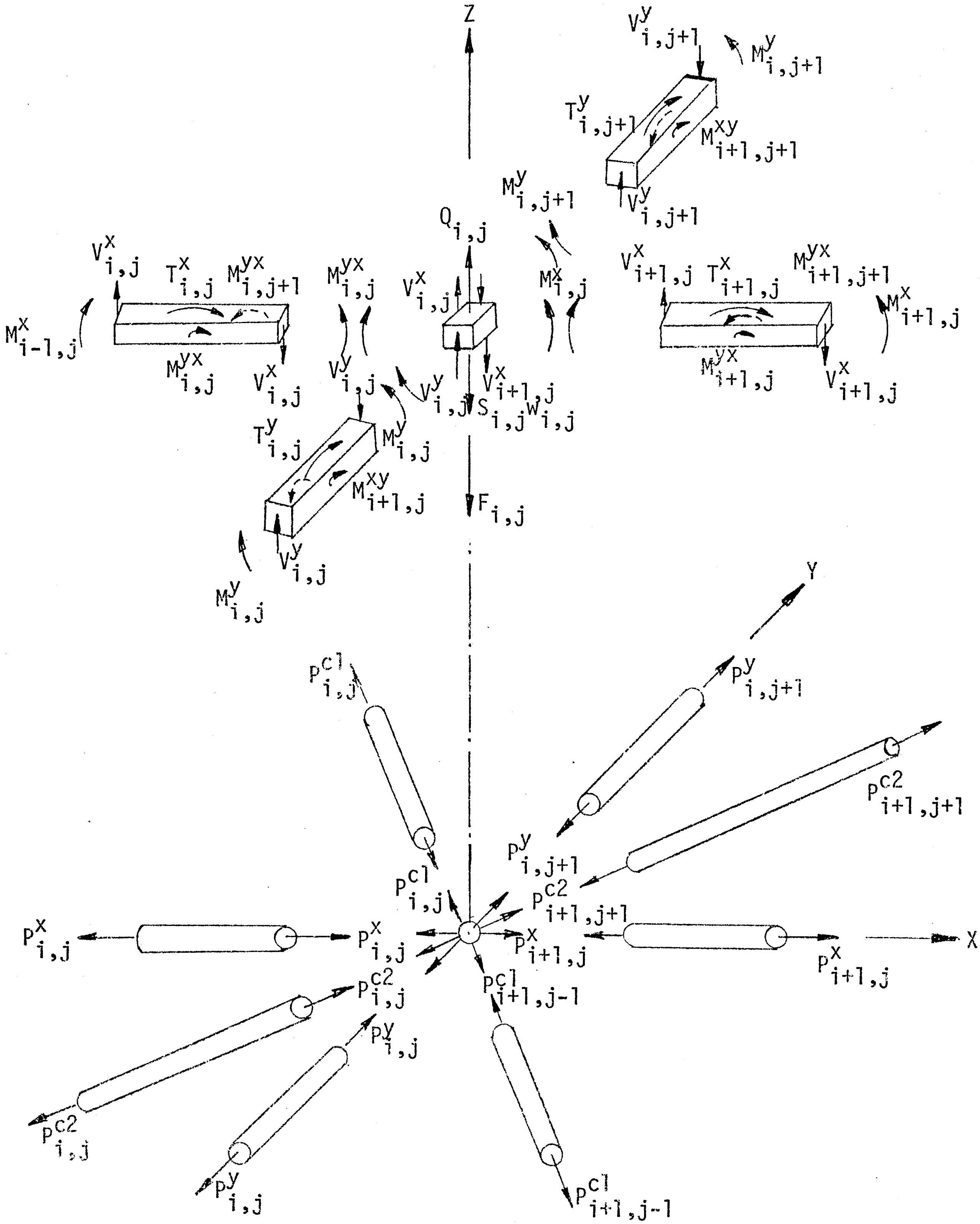


Figure 24. Expanded Joint in the Combined Model

Substituting Equations (A.3) and (A.6) into Equation (A.5) yields

$$
\begin{aligned}
Q_{i,j} - S_{i,j}w_{i,j} = \\
& - \frac{1}{4h_x^2}[R^x_{i-1,j}\, w_{i-2,j} - w_{i,j}\,(R^x_{i-1,j} + R^x_{i+1,j}) \\
& \qquad + R^x_{i+1,j}\, w_{i+2,j}] \\
& - \frac{1}{4h_y^2}[R^y_{i,j-1}\, w_{i,j-2} - w_{i,j}\,(R^y_{i,j-1} + R^y_{i,j+1}) \\
& \qquad + R^y_{i,j+1}\, w_{i,j+2}] \\
& + \frac{1}{h_x}[M^{yx}_{i,j} - M^{yx}_{i,j+1} - M^{yx}_{i+1,j} + M^{yx}_{i+1,j+1} + M^x_{i-1,j} \\
& \qquad - 2M^x_{i,j} + M^x_{i+1,j} + T^x_{i,j} - T^x_{i+1,j} \\
& \qquad + P^x_{i,j}\,(w_{i,j} - w_{i-1,j}) - P^x_{i+1,j}\,(w_{i+1,j} - w_{i,j})] \\
& + \frac{1}{h_y}[-M^{xy}_{i,j} + M^{xy}_{i+1,j} + M^{xy}_{i,j+1} - M^{xy}_{i+1,j+1} + M^y_{i,j-1} \\
& \qquad - 2M^y_{i,j} + M^y_{i,j+1} + T^y_{i,j} - T^y_{i,j+1} \\
& \qquad + P^y_{i,j}\,(w_{i,j} - w_{i,j-1}) - P^y_{i,j+1}\,(w_{i,j+1} - w_{ij})] \\
& + \frac{1}{h_z}[P^{c1}_{i,j}\,(w_{i,j} - w_{i-1,j+1}) \\
& \qquad - P^{c1}_{i+1,j-1}\,(w_{i+1,j-1} - w_{i,j}) \\
& \qquad + P^{c2}_{i,j}\,(w_{i,j} - w_{i-1,j-1}) \\
& \qquad - P^{c2}_{i+1,j+1}\,(w_{i+1,j+1} - w_{i,j})] \qquad \text{(A.7)}
\end{aligned}
$$

The bending and twisting moments in Equation (A.7) can be represented by the finite difference approximations as

$$M^{x}_{i,j} = h_y \, D^{x}_{i,j} \left[\frac{w_{i-1,j} - 2w_{i,j} + w_{i+1,j}}{h_x^2} + \nu_y \frac{w_{i,j-1} - 2w_{i,j} + w_{i,j+1}}{h_y^2} \right]$$

$$M^{y}_{i,j} = h_x \, D^{y}_{i,j} \left[\frac{w_{i,j-1} - 2w_{i,j} + w_{i,j+1}}{h_y^2} + \nu_x \frac{w_{i-1,j} - 2w_{i,j} + w_{i+1,j}}{h_x^2} \right]$$

$$M^{xy}_{i,j} = -h_y C_{i,j} \left[\frac{w_{i-1,j-1} - w_{i-1,j} - w_{i,j-1} + w_{i,j}}{h_x h_y} \right]$$

$$M^{yx}_{i,j} = +h_x C_{i,j} \left[\frac{w_{i-1,j-1} - w_{i-1,j} - w_{i,j-1} + w_{i,j}}{h_x h_y} \right] \qquad \text{(A.8)}$$

where

$$C_{i,j} = 2D^{xy}_{i,j}$$

Substituting moment values into Equation (A.7) and rearranging terms yields

$$\begin{aligned} a_{i,j}w_{i,j-2} &+ b_{i,j}w_{i-1,j-1} + c_{i,j}w_{i,j-1} + d_{i,j}w_{i+1,j-1} \\ &+ e_{i,j}w_{i-2,j} + f_{i,j}w_{i-1,j} + g_{i,j}w_{i,j} + h_{i,j}w_{i+1,j} \\ &+ p_{i,j}w_{i+2,j} + q_{i,j}w_{i-1,j+1} + r_{i,j}w_{i,j+1} \\ &+ s_{i,j}w_{i+1,j+1} + t_{i,j}w_{i,j+2} = u_{i,j} \end{aligned} \qquad \text{(A.9)}$$

where

$$a_{i,j} = h_x D^{y}_{i,j-1}/h_y^3 - R^{y}_{i,j-1}/4h_y^2$$

$$b_{i,j} = (\nu_y D^{x}_{i-1,j} + \nu_x D^{y}_{i,j-1} + 2C_{i,j})/h_x h_y - P^{c2}_{i,j}/h_z$$

$$c_{i,j} = -2h_x\,(D^y_{i,j-1} + D^y_{i,j})/h_y^3 - P^y_{i,j}/h_y$$

$$- 2(\nu_y D^x_{i,j} + \nu_x D^y_{i,j-1} + C_{i,j} + C_{i+1,j})/h_x h_y$$

$$d_{i,j} = (\nu_y D^x_{i+1,j} + \nu_x D^y_{i,j-1} + 2C_{i+1,j})/h_x h_y - P^{c1}_{i+1,j-1}/h_z$$

$$e_{i,j} = h_y D^x_{i-1,j}/h_x^3 - R^x_{i-1,j}/4h_x^2$$

$$f_{i,j} = -2h_y\,(D^x_{i-1,j} + D^x_{i,j})/h_x^3 - P^x_{i,j}/h_x$$

$$- 2(\nu_y D^x_{i-1,j} + \nu_x D^y_{i,j} + C_{i,j} + C_{i,j+1})/h_x h_y$$

$$g_{i,j} = h_y\,(D^x_{i-1,j} + 4D^x_{i,j} + D^x_{i+1,j})/h_x^3$$

$$+ h_x\,(D^y_{i,j-1} + 4D^y_{i,j} + D^y_{i,j+1})/h_y^3$$

$$+ (4\nu_y D^x_{i,j} + 4\nu_x D^y_{i,j} + 2C_{i,j} + 2C_{i+1,j} + 2C_{i,j+1}$$

$$+ 2C_{i+1,j+1})/h_x h_y + (R^x_{i-1,j} + R^x_{i+1,j})/4h_x^2$$

$$+ (R^y_{i,j-1} + R^y_{i,j+1})/4h_y^2 + (P^x_{i,j} + P^x_{i+1,j})/h_x$$

$$+ (P^y_{i,j} + P^y_{i,j+1})/h_y + S_{i,j}$$

$$+ (P^{c1}_{i,j} + P^{c1}_{i+1,j-1} + P^{c2}_{i,j} + P^{c2}_{i+1,j+1})/h_z$$

$$h_{i,j} = -2h_y(D^x_{i,j} + D^x_{i+1,j})/h_x^3 - P^x_{i+1,j}/h_x$$

$$- 2(\nu_y D^x_{i+1,j} + \nu_x D^y_{i,j} + C_{i+1,j} + C_{i+1,j+1})/h_x h_y$$

$$p_{i,j} = h_y D^x_{i+1,j}/h_x^3 - R^x_{i+1,j}/4h_x^2$$

$$q_{i,j} = (\nu_y D^x_{i-1,j} + \nu_x D^y_{i,j+1} + 2C_{i,j+1})/h_x h_y - P^{c1}_{i,j}/h_z$$

$$r_{i,j} = -2h_x \ (D^y_{i,j} + D^y_{i,j+1})/h^3_y - P^y_{i,j+1}/h_y$$

$$- 2(\nu_y D^x_{i,j} + \nu_x D^y_{i,j+1} + C_{i,j+1} + C_{i+1,j+1})h_x h_y$$

$$s_{i,j} = (\nu_y D^x_{i+1,j} + \nu_x D^y_{i,j+1} + 2C_{i+1,j+1})/h_x h_y - P^{c2}_{i+1,j+1}$$

$$t_{i,j} = h_x D^y_{i,j+1}/h^3_y - R^y_{i,j+1}/4h^2_y$$

$$u_{i,j} = Q_{i,j} - (T^x_{i,j} - T^x_{i+1,j})/h_x - (T^y_{i,j} - T^y_{i,j+1})/h_y$$

Equilibrium Equation (A.9) is written for every joint of the bending model and may be summarized into the matrix form as

$$[K] \{W\} = \{Q\} \qquad \text{(A.10)}$$

where

[K] = stiffness matrix of the plate bending model which includes the effects of inplane loads;

{W} = vertical deflection vector; and

{Q} = vertical load vector.

APPENDIX B

PROPERTIES OF MEMBRANE MODEL

B.1 Bar Cross-Sectional Bar

The inplane behavior of a plate problem is represented by a membrane model shown in Figure 3. This membrane model is composed of elastic bars connected by ball and socket joints. The areas of these bars are related to the increment lengths, plate thickness, and material properties. The evaluations of these areas are presented elsewhere (24) (28), but are included in this work for the benefit of the reader.

The areas of the elastic bars must permit the model to represent the inplane behavior of a thin plate subjected to normal and shearing stresses. The resisting inplane stresses and deformations must agree with the plane stress problem. To insure that the discrete-element model will represent the plane stress problem, the following conditions must be investigated.

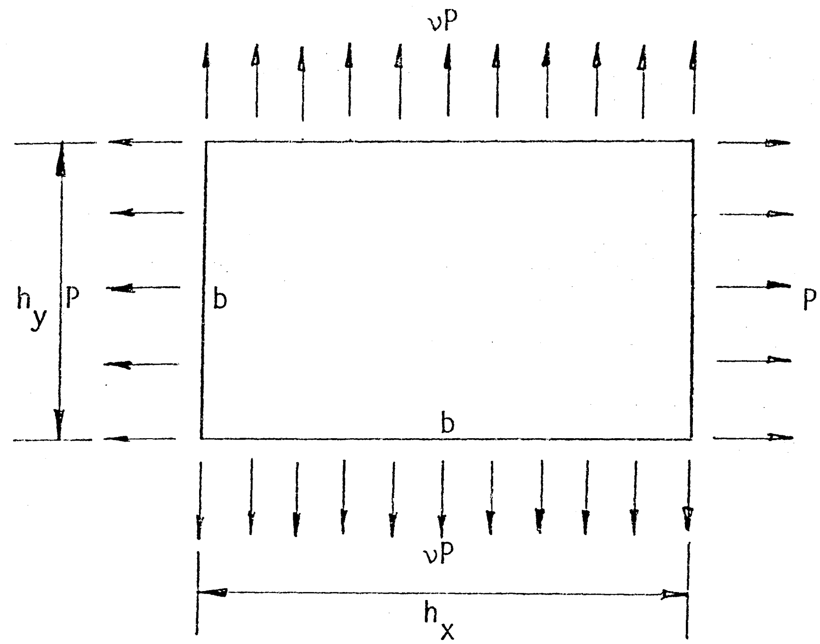
1. When the model is subjected to uniform normal load of p per unit length in the x direction and νp in the y direction (Figure 25), deformations should correspond to those found by conventional methods of elastic analysis. The strains of the model in the x and y directions must be

$$\begin{aligned} \varepsilon_x &= p(1-\nu^2)/Et \\ \varepsilon_y &= 0 \end{aligned} \qquad (B.1)$$

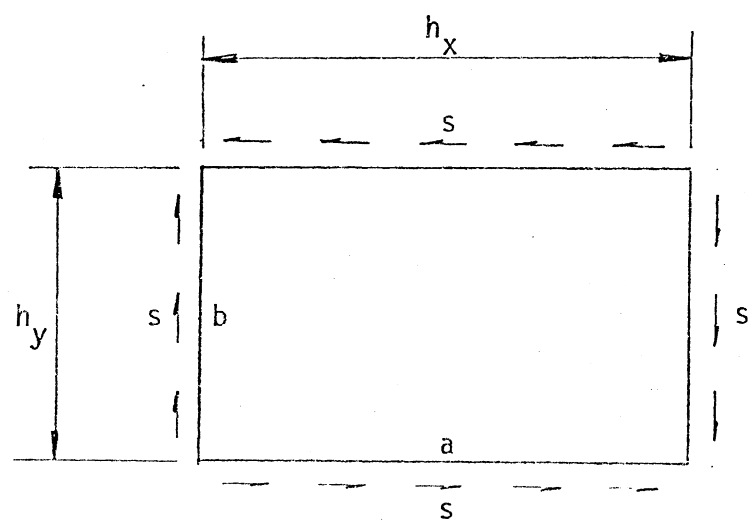
2. If the load p is applied in the y direction and νp in the x direction, the strains in the model are

$$\begin{aligned} \varepsilon_x &= 0 \\ \varepsilon_y &= p(1-\nu^2)/Et \end{aligned} \qquad (B.2)$$

3. For a uniform tangential load s per unit length, the shearing strain of the model must be



(a) Normal Loading



(b) Pure Shear

Figure 25. Plane Stress Element

$$\gamma_{xy} = 2(1+\nu)s/Et \tag{B.3}$$

The discrete-element model representing the plane stress problem is shown in Figure 26 with joint loads which are the loads applied to the element in Figure 25. Taking a section of a joint, equilibrium of the joint forces of the discrete-element model can be written as

$$F_a + F_c \cos\theta = ph_y/2 \tag{B.4a}$$

and

$$F_b + F_c \sin\theta = ph_x/2 \tag{B.4b}$$

where

F_a, F_b, F_c = forces in bars a, b, and c, respectively;

θ = angle between bar a and bar c; and

h_x, h_y, h_z = lengths of bars a, b, and c, respectively.

The deformation of the model must correspond to the strain found by elastic analysis, as shown in Equation (B.1). Since the strain in the y direction is zero, the force in bar b must be zero. From Equation (B.4) the forces in bars a and c are

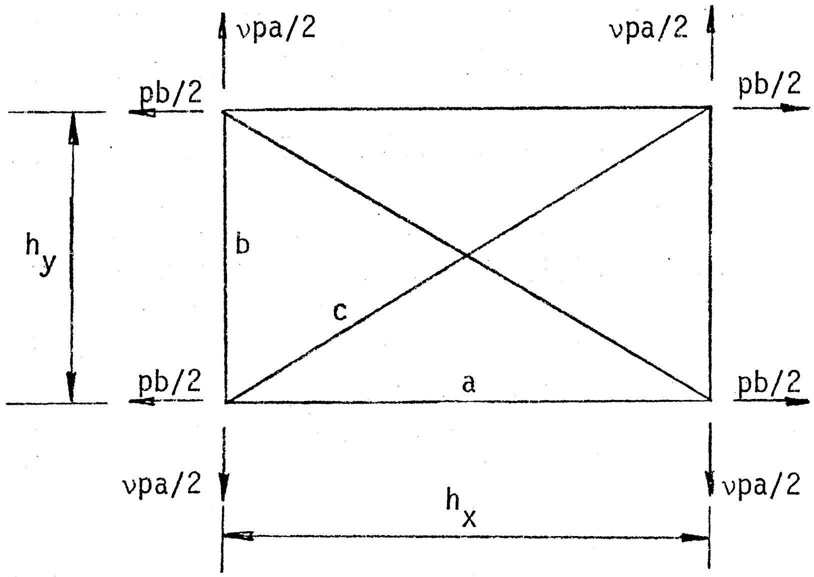
$$F_a = ph_y/2 - p(h_x)^2/2h_y \tag{B.5a}$$

$$F_c = ph_xh_z/2h_y \tag{B.5b}$$

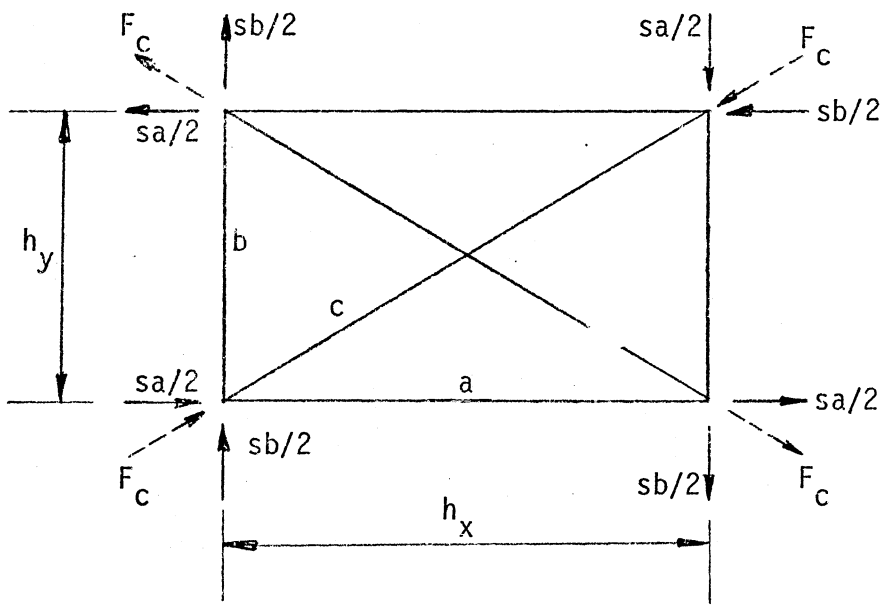
The strain in bar a is

$$\varepsilon_a = F_a/E_aA_a = (ph_y/2 - p(h_x)^2/2h_y)/E_aA_a \tag{B.6}$$

where E_a, A_a are elastic modulus and cross section area of bar a. The strain of bar a must be the same as that given in Equation (B.1). By equating these two strains and rearranging terms, the cross-sectional area of bar a is found to be



(a) Normal Loading



(b) Pure Shear

Figure 26. Member of Discrete-Element Model

$$A_a = \frac{t(h_y^2 - \nu h_x^2)}{2h_y(1 - \nu^2)} \tag{B.7}$$

To determine the cross-sectional area of bar c, the deformations of bars a, b, and c are related as follows:

$$h_y^2 = h_z^2 - h_x^2$$

Differentiating yields

$$h_y dh_y = h_z dh_z - h_x dh_x$$

Substituting $dh_x = \varepsilon_a h_x$, $dh_y = \varepsilon_b h_y$, and $dh_z = \varepsilon_c h_z$ into the above equation yields

$$h_y^2 \varepsilon_b = h_z^2 \varepsilon_c - h_x^2 \varepsilon_a$$

Since the strain in bar b is zero for the load condition under investigation, therefore

$$h_z^2 \varepsilon_c = h_x^2 \varepsilon_a$$

Replacing the strains by forces in the bars, bar areas, and elastic constants, the area of bar c is related to the area of bar a by

$$A_c = h_z^2 \, F_c A_a E_a / h_x^2 \, F_a E_c \tag{B.8}$$

The forces F_a and F_c were evaluated in Equation (B.5) and letting E_a equal E_c, the area of bar c is found to be

$$A_c = \frac{\nu t \, h_z^3}{2h_x h_y \, (1-\nu^2)} \tag{B.9}$$

The cross-sectional area of bar b can be calculated by the same procedure. The loading described in the second condition will cause bars b and c to deform while the length of bar a remains unchanged. The area of bar b is found to be

$$A_b = \frac{t(h_x^2 - \nu h_y^2)}{2h_x(1-\nu^2)} \tag{B.10}$$

In the case when the tangential load s per unit length is applied to the plate element as shown in Figure 25, and equivalent joint forces applied to the model as shown in Figure 26, all horizontal and vertical bars of the model must be unloaded. This is necessary to insure zero normal strain in the horizontal and vertical directions. The forces in the diagonal bars are equal in magnitude but opposite in sense and are

$$F_c = (sh_x \cos\theta + sh_y \sin\theta)/2 \tag{B.11}$$

The change in length of the diagonal bars (Figure 27) will be

$$\delta = F_c h_z / A_c E_c$$

$$= sh_z^2 / 2A_c E_c \tag{B.12}$$

The deformation of the element is shown in Figure 27 and the shearing strain is shown to be

$$2\tan(w) = 2\delta \sin\theta / h_x$$

$$= sh_y h_z / h_x A_c E_c$$

Equating the shearing strain in the model of theoretical shearing strain yields

$$\gamma_{xy} = sh_z h_y / h_x A_e E_e \tag{B.13}$$

Equating Equations (B.3) and (B.13), the area of bar c is found to be

$$A_e = \frac{h_z h_y t}{2h_x(1+\nu)} \tag{B.14}$$

The area of bar c can satisfy both Equations (B.9) and (B.14) only if

$$\nu = \frac{h_y^2}{h_y^2 + h_z^2} \tag{B.15}$$

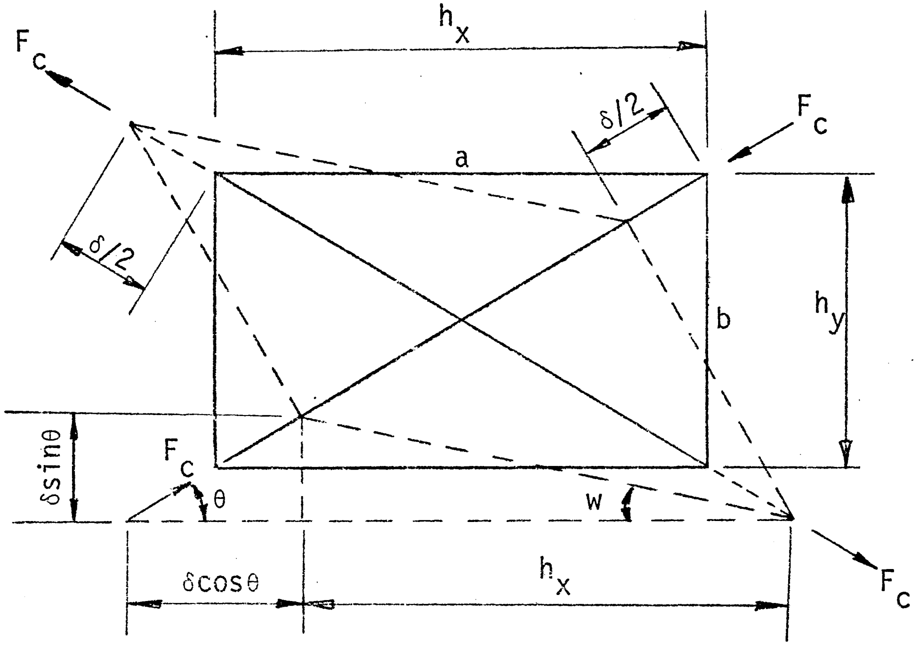


Figure 27. Shearing Deformation of Discrete Element

Therefore, the Poisson's ratio depends on the geometry of the discrete element model given by Equation (B.15). The Poisson's ratio will vary from 0.09 to 0.473 as the ratio for h_x/h_y varies from 3.0 to 0.333. However, this study has shown that the value of Poisson's ratio has little effect on the vertical displacement of the plate due to either vertical or inplane loads.

B.2 Force in an Elastic Bar

To calculate the forces in the elastic bars of the discrete-element model, displacements, both vertical and inplane, are combined and include second order effects as described below. The original length of a bar in the x-y plane (Figure 28) is

$$h_z = (h_x^2 + h_y^2)^{\frac{1}{2}} \tag{B.16}$$

Following joint displacements, the final length of the bar is

$$\bar{h}_z = [(h_x + \Delta u)^2 + (h_y + \Delta v)^2 + (\Delta w)^2]^{\frac{1}{2}} \tag{B.17}$$

where $\bar{h}_z$ = final length of the bar, and

$$\Delta u = (u_2 - u_1)$$

$$\Delta v = (v_2 - v_1)$$

$$\Delta w = (w_2 - w_1).$$

The axial strain of the bar can be written as

$$\varepsilon = (\bar{h}_z - h_z)/h_z$$

or

$$\bar{h}_z = (h_z\varepsilon + h_z) \tag{B.18}$$

Equating Equations (B.17) and (B.18) and rearranging terms gives

$$\varepsilon = \frac{h_x\Delta u + h_y\Delta v}{(h_z)^2} + \frac{(\Delta u)^2 + (\Delta v)^2 + (\Delta w)^2}{2(h_z)^2} \tag{B.19}$$

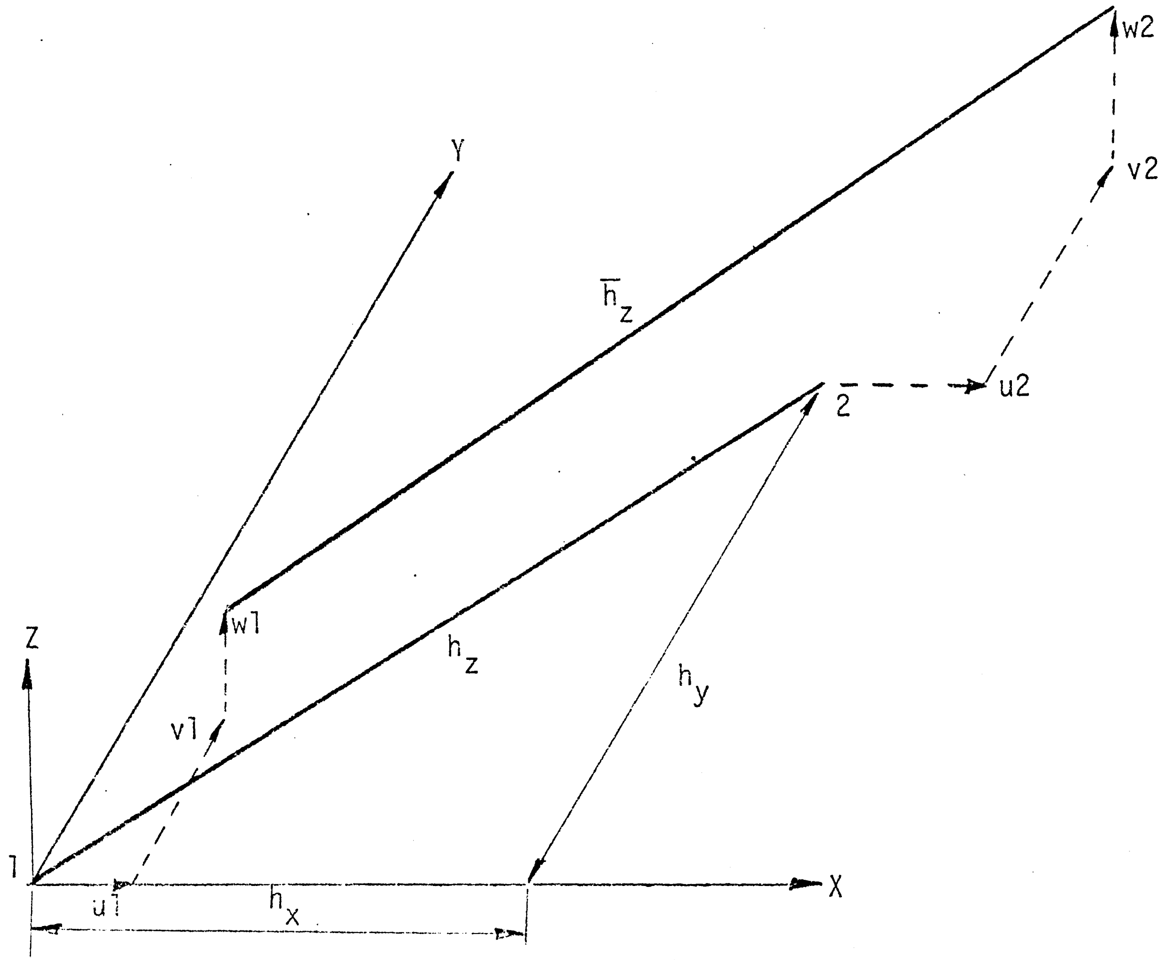


Figure 28. Deformation of an Elastic Bar of the Discrete Element

The component of this axial strain in each direction is

$$\varepsilon_x = (h_x + \Delta u)\varepsilon/\overline{h}_z \tag{B.20a}$$

$$\varepsilon_y = (h_y + \Delta v)\varepsilon/\overline{h}_z \tag{B.20b}$$

$$\varepsilon_z = \Delta w\varepsilon/\overline{h}_z \tag{B.20c}$$

Therefore, the force of this bar in each principal direction is

$$P_x = EA\varepsilon_x \tag{B.21a}$$

$$P_y = EA\varepsilon_y \tag{B.21b}$$

$$P_z = EA\varepsilon_z \tag{B.21c}$$

The same procedure can be applied to the bars that lie in the x and y directions. Equation (B.21) is used to calculate the membrane forces as described in Chapter IV.

B.3 Stresses from the Membrane Model

The purpose of this section is to demonstrate the method of calculation for the membrane stresses. Consider a plate element in Figure 29 subjected to inplane forces; the normal and shearing stresses are

$$\sigma_x = F_x/th_y \tag{B.22a}$$

$$\sigma_y = F_y/th_x \tag{B.22b}$$

$$\tau_{xy} = V_{xy}/th_y \tag{B.22c}$$

$$\tau_{yx} = V_{yx}/th_x \tag{B.22d}$$

A membrane element in Figure 30 is used to represent the plate element. The forces on the plate element are applied directly to the joints of the model. The axial force of each bar is calculated as described in section B.2 and is identified by the letters in Figure 31.

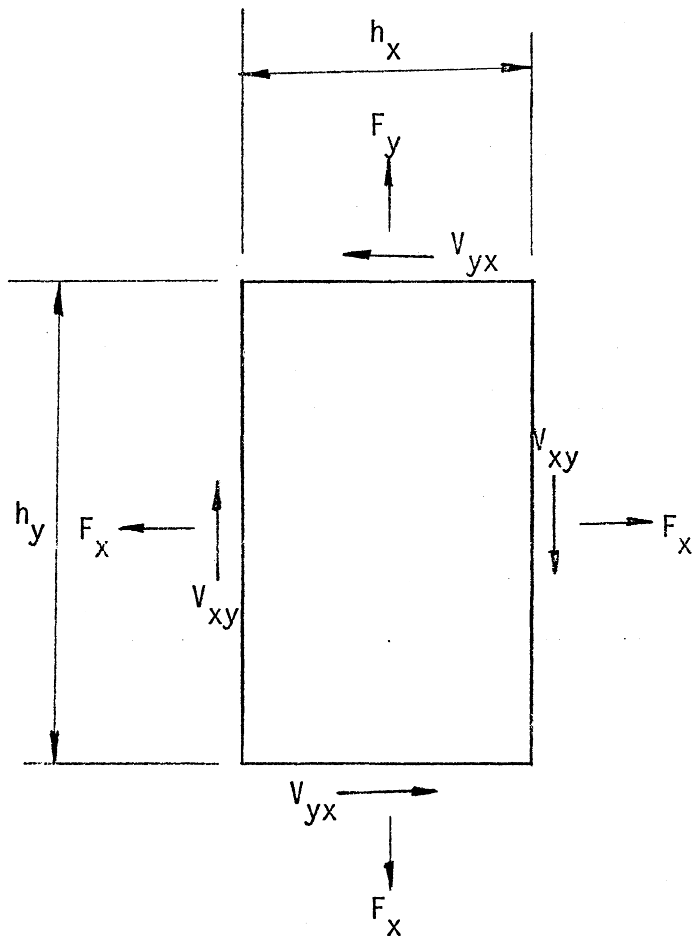


Figure 29. Plate Element with Inplane Loads

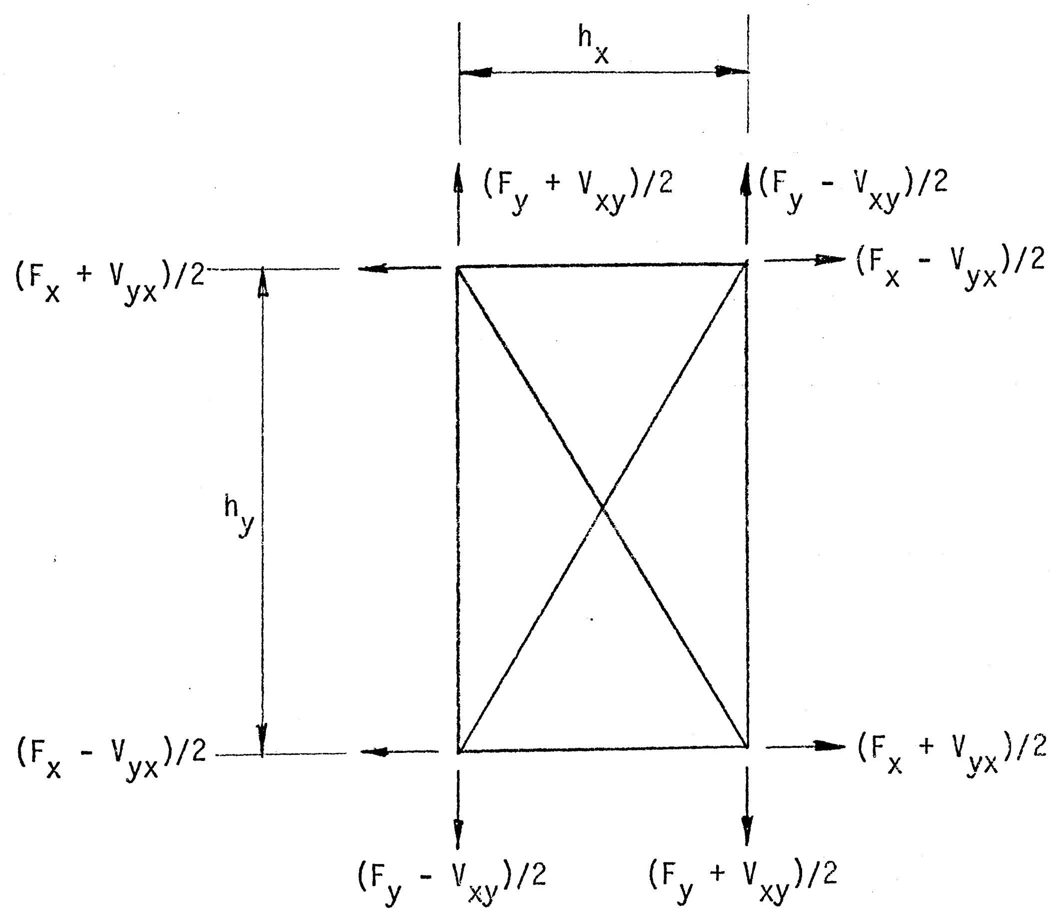


Figure 30. Discrete Element Representation of Plate Element

The forces are calculated from the known joint displacements. The joint equilibrium equations for the model are

$$
\begin{aligned}
[F_B + F_H \cos\theta] &= [F_x + V_{yx}]/2 \\
[F_C + F_H \sin\theta] &= [F_y + V_{xy}]/2 \\
[F_B + F_E \cos\theta] &= [F_x - V_{yx}]/2 \\
[F_D + F_E \sin\theta] &= [F_y - V_{xy}]/2 \\
[F_A + F_E \cos\theta] &= [F_x - V_{yx}]/2 \\
[F_C + F_E \sin\theta] &= [F_y - V_{xy}]/2 \\
[F_A + F_H \cos\theta] &= [F_x + V_{yx}]/2 \\
[F_D + F_H \sin\theta] &= [F_y + V_{xy}]/2
\end{aligned}
\tag{B.23}
$$

Solving these equations yields

$$
\begin{aligned}
F_x &= [F_A + F_B] + [F_H + F_E] \cos\theta \\
F_y &= [F_C + F_D] + [F_H + F_E] \sin\theta \\
V_{xy} &= [F_H - F_E] \sin\theta \\
V_{yx} &= [F_H - F_E] \cos\theta
\end{aligned}
\tag{B.24}
$$

Substituting these values into Equation (B.27) yields

$$
\begin{aligned}
(\sigma_x)_M &= [F_A + F_B + (F_H + F_E) \cos\theta]/th_y \\
(\sigma_y)_M &= [F_C + F_D + (F_H + F_E) \sin\theta]/th_x \\
(\tau_{xy})_M &= [F_H - F_E] \sin\theta/th_y \\
(\tau_{yx})_M &= [F_H - F_E] \cos\theta/th_x
\end{aligned}
\tag{B.25}
$$

where F_A, F_B, F_C, F_D, F_E, and F_H are shown in Figure 31.

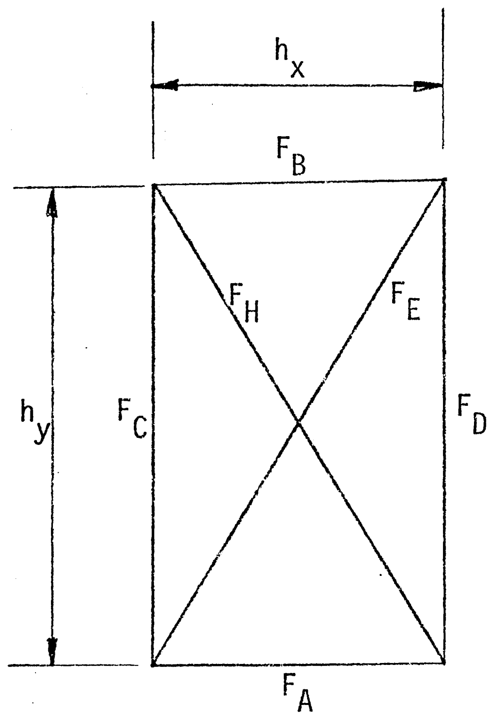


Figure 31. Forces in Elastic Bars of Membrane Element

APPENDIX C

SOLUTION METHOD FOR BENDING MODEL

C.1 Organization of Equations

The area of the plate is divided into small rectangular grids. The nodes, at the intersections of the grids, are the joints and the grid lines are the bars of the discrete element model. The number of elements or bars in the x and y directions are M and N, respectively. For this program M must be less than or equal to N. There will be (M+1) and (N+1) joints for the x and y grid line. Since two boundary condition equations are required at the end of each grid line, the total (M+3)(N+3) equations must be solved.

The equilibrium equation, Equation (A.9), of the bending model is written for each joint as shown in Figure 32. The K (stiffness) matrix is of special interest. It is both symmetrical about its major diagonal and banded. The partitioned stiffness matrix has a band width of five.

C.2 Solution Procedure

Figure 33 shows equilibrium equations of the joints on the jth grid line. In the following discussion $AA1_j, AA2_j, \ldots, AA5_j$ are submatrices of the stiffness matrix, W_j is a submatrix of the deflection matrix, and $AA6_j$ is a submatrix of the load matrix. These submatrices are shown in Figures 34 and 35.

From Figure 34, the equilibrium equation of the jth grid line can be written as

$$AA1_j W_{j-2} + AA2_j W_{j-1} + AA3_j W_j + AA4_j W_{j+1} + AA5_j W_{j+2} = AA6_j \qquad (C.1)$$

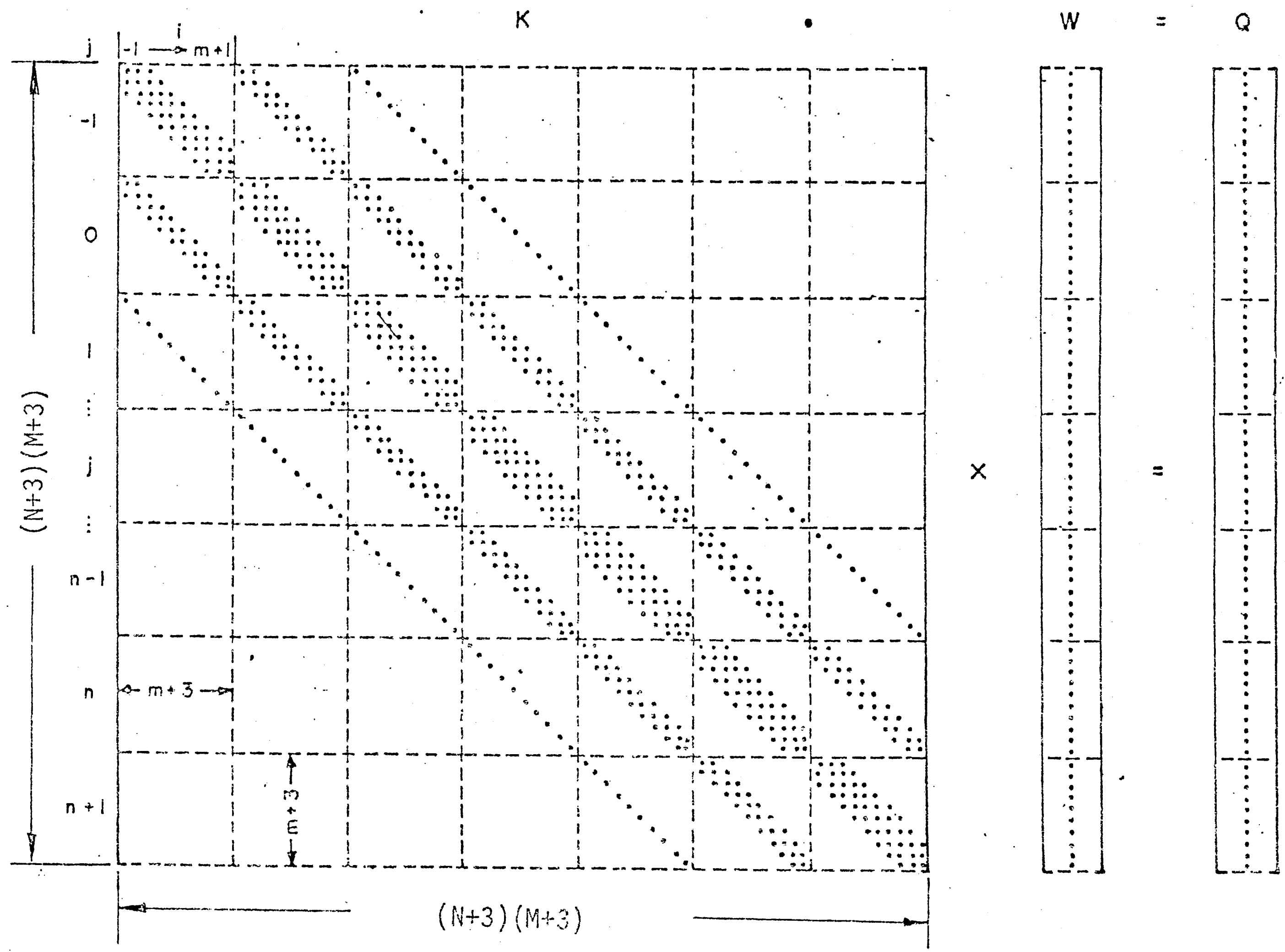


Figure 32. Equilibrium Equation of Bending Model

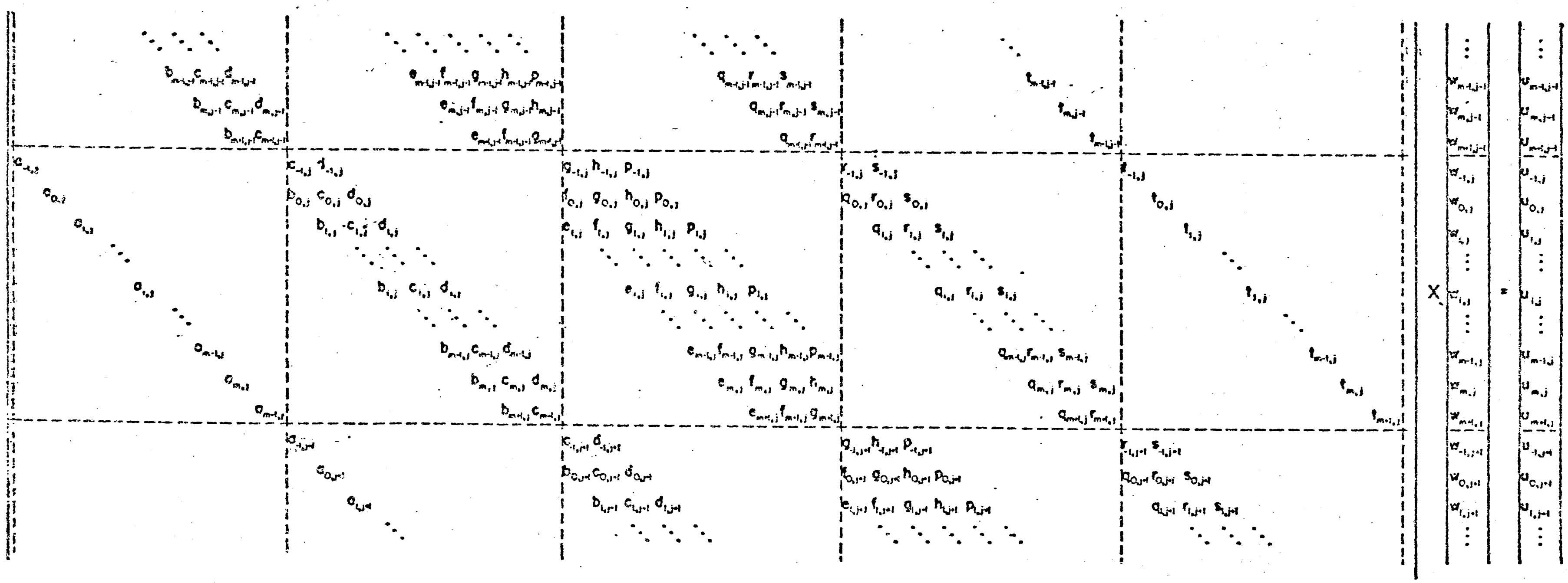


Figure 33. Equilibrium Equation of Joints on the jth Gridline

$$
\begin{bmatrix}
AA3_{-1} & AA4_{-1} & AA5_{-1} & & & & & & \\
AA2_{0} & AA3_{0} & AA4_{0} & AA5_{0} & & & & & \\
AA1_{1} & AA2_{1} & AA3_{1} & AA4_{1} & AA5_{1} & & & & \\
& \ddots & \ddots & \ddots & \ddots & \ddots & & & \\
& & AA1_{j-2} & AA2_{j-2} & AA3_{j-2} & AA4_{j-2} & AA5_{j-2} & & \\
& & & AA1_{j-1} & AA2_{j-1} & AA3_{j-1} & AA4_{j-1} & AA5_{j-1} & \\
& & & & AA1_{j} & AA2_{j} & AA3_{j} & AA4_{j} & AA5_{j} \\
& & & & & AA1_{j+1} & AA2_{j+1} & AA3_{j+1} & AA4_{j+1} \;\; AA5_{j+1} \\
& & & & & & AA1_{j+2} & AA2_{j+2} & AA3_{j+2} \;\; AA4_{j+2} \;\; AA5_{j+2} \\
& & & & & & & \ddots & \ddots \\
& & & & & & AA1_{n-1} & AA2_{n-1} & AA3_{n-1} \;\; AA4_{n-1} \;\; AA5_{n-1} \\
& & & & & & & AA1_{n} & AA2_{n} \;\; AA3_{n} \;\; AA4_{n} \\
& & & & & & & & AA1_{n+1} \;\; AA2_{n+1} \;\; AA3_{n+1}
\end{bmatrix}
\times
\begin{bmatrix}
W_{-1} \\ W_{0} \\ W_{1} \\ \vdots \\ W_{j-2} \\ W_{j-1} \\ W_{j} \\ W_{j+1} \\ W_{j+2} \\ \vdots \\ W_{n-1} \\ W_{n} \\ W_{n+1}
\end{bmatrix}
=
\begin{bmatrix}
AA6_{-1} \\ AA6_{0} \\ AA6_{1} \\ \vdots \\ AA6_{j-2} \\ AA6_{j-1} \\ AA6_{j} \\ AA6_{j+1} \\ AA6_{j+2} \\ \vdots \\ AA6_{n-1} \\ AA6_{n} \\ AA6_{n+1}
\end{bmatrix}
$$

Figure 34. Identification of Partitioned Sub-Matrices

$$AA1_j = \begin{bmatrix} a_{-1,j} & & & & & & & \\ & a_{0,j} & & & & & & \\ & & a_{1,j} & & & & & \\ & & & \ddots & & & & \\ & & & & a_{i,j} & & & \\ & & & & & \ddots & & \\ & & & & & & a_{m-1,j} & & \\ & & & & & & & a_{m,j} & \\ & & & & & & & & a_{m+1,j} \end{bmatrix}$$

$$AA2_j = \begin{bmatrix} c_{-1,j} & d_{-1,j} & & & & & \\ b_{0,j} & c_{0,j} & d_{0,j} & & & & \\ & b_{1,j} & c_{1,j} & d_{1,j} & & & \\ & & \ddots & \ddots & \ddots & & \\ & & b_{i,j} & c_{i,j} & d_{i,j} & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & b_{m-1,j} & c_{m-1,j} & d_{m-1,j} \\ & & & & & b_{m,j} & c_{m,j} & d_{m,j} \\ & & & & & & b_{m+1,j} & c_{m+1,j} \end{bmatrix}$$

$$AA3_j = \begin{bmatrix} g_{-1,j} & h_{-1,j} & p_{-1,j} & & & & & \\ f_{0,j} & g_{0,j} & h_{0,j} & p_{0,j} & & & & \\ e_{1,j} & f_{1,j} & g_{1,j} & h_{1,j} & p_{1,j} & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & e_{i,j} & f_{i,j} & g_{i,j} & h_{i,j} & p_{i,j} & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & e_{m-1,j} & f_{m-1,j} & g_{m-1,j} & h_{m-1,j} & p_{m-1,j} \\ & & & & e_{m,j} & f_{m,j} & g_{m,j} & h_{m,j} \\ & & & & & e_{m+1,j} & f_{m+1,j} & g_{m+1,j} \end{bmatrix}$$

$$AA4_j = \begin{bmatrix} r_{-1,j} & s_{-1,j} & & & & & \\ q_{0,j} & r_{0,j} & s_{0,j} & & & & \\ & q_{1,j} & r_{1,j} & s_{1,j} & & & \\ & & \ddots & \ddots & \ddots & & \\ & & q_{i,j} & r_{i,j} & s_{i,j} & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & q_{m-1,j} & r_{m-1,j} & s_{m-1,j} \\ & & & & & q_{m,j} & r_{m,j} & s_{m,j} \\ & & & & & & q_{m+1,j} & r_{m+1,j} \end{bmatrix}$$

$$AA5_j = \begin{bmatrix} t_{-1,j} & & & & & & & \\ & t_{0,j} & & & & & & \\ & & t_{1,j} & & & & & \\ & & & \ddots & & & & \\ & & & & t_{i,j} & & & \\ & & & & & \ddots & & \\ & & & & & & t_{m-1,j} & & \\ & & & & & & & t_{m,j} & \\ & & & & & & & & t_{m+1,j} \end{bmatrix} \qquad W_j = \begin{bmatrix} w_{-1,j} \\ w_{0,j} \\ w_{1,j} \\ \vdots \\ w_{i,j} \\ \vdots \\ w_{m-1,j} \\ w_{m,j} \\ w_{m+1,j} \end{bmatrix} \qquad AA6_j = \begin{bmatrix} u_{-1,j} \\ u_{0,j} \\ u_{1,j} \\ \vdots \\ u_{i,j} \\ \vdots \\ u_{m-1,j} \\ u_{m,j} \\ u_{m+1,j} \end{bmatrix}$$

Figure 35. Sub-Matrices

It is possible to develop a solution in the form

$$W_j = A_j + B_j W_{j+1} + C_j W_{j+2} \tag{C.2}$$

where the constants in Equation (C.2) are

$$A_j = D_j(E_j A_{j-1} + AA1_j A_{j-2} - AA6_j) \tag{C.3}$$

$$B_j = D_j(E_j C_{j-1} + AA4_j) \tag{C.4}$$

$$C_j = D_j \; AA5_j \tag{C.5}$$

$$D_j = -(E_j B_{j-1} + AA1_j C_{j-2} + AA3_j)^{-1} \tag{C.6}$$

$$E_j = AA1_j \; B_{j-2} + AA2_j \tag{C.7}$$

This method is described in detail by Matlock (29).

APPENDIX D

LISTING OF THE COMPUTER PROGRAM

```
C---------THIS PROGRAM SOLVES ORTHOTROPIC PLATES AND PAVEMENT SLABS BY
C         A DIRECT METHOD. THE DIRECT SOLUTION IS CARRIED OUT
C         BY USING A BACK AND FORTH RECURSIVE TECHNIQUE DESCRIBED
C         BY HUDSON MATLOCK.
C
C
C
C-----FOR DIFFERENT SIZED PROBLEMS, ONLY THE DIMENSION CARDS AND THE
C           L1 CARD NEED BE CHANGED.  FOR EXAMPLE, AA1(S+3), A(S+3,L+5),
C           B(S+3,S+3,L+5)  WHERE S AND L REFER TO THE SHORT AND LONG
C           LENGTHS OF THE REAL PROBLEM.  L1 = S+3 .
C
C-----THIS PROGRAM IS NOW DIMENSIONED TO SOLVE A GRID WITH MAXIMUN SIZE
C           OF 10   BY 10   MESH POINTS
C
      IMPLICIT REAL * 8 ( A-H, O-Z )
C
C------DIMENSION STATEMENTS
C
      DIMENSION     A(13,15) ,          B(13,13,15) ,       BMX(17,17) ,
     1              BMY(17,17),         C(13,13,15),        CX(17,17),
     2                                  DX(17,17),          DY(17,17),
     4              S(17,17),           TX(17,17),          TY(17,17),
     5              Q(17,17),           W1(17,1),           W2(13,1)
      DIMENSION  RXN(20), RYN(20), RX(17,17), RY(17,17)
      DIMENSION     AN1(40) ,           AN2(35) ,           DP(6)
      DIMENSION  IN1(20), IN2(20), JN1(20), JN2(20), DXN(20), DYN(20),
     1       QN(20), SN(20), CXN(20),          TXN(20), TYN(20),
     2      PXN(20), PYN(20)
      DIMENSION  SUN(20), SVN(20)
      DIMENSION  PXX(17,17), PYY(17,17), PZZ(17,17)
      DIMENSION  EXN(20), EYN(20)
      DIMENSION  QQ(17,17), QQ1(17,17), PX(17,17), PX1(17,17),
     1          PY(17,17), PY1(17,17)
      DIMENSION  DSX(17,17), DSY(17,17), SHS(17,17)
      DIMENSION  PC1(17,17), PC2(17,17),  UU(17,17), VV(17,17)
      DIMENSION  PD1(17,17), PD2(17,17)
      DATA ITEST/4H    /
C
C-----COMMON STATEMENTS
C
      COMMON /INCR/ MX, MY, MXP2, MXP3, MXP4, MXP5, MXP7,
     1              MYP2, MYP3, MYP4, MYP5, MYP7
      COMMON /MATR/ AA1(13), AA2(13,3), AA3(13,5), AA4(13,3),
     1    AA5(13), AA6(13), AA(13,1), A1(13,1), A2(13,1),
     2    BB(13,13), BB1(13,13), BB2(13,13), CC(13,13), CC1(13,13),
     3    CC2(13,13), AAUG(13,13,2), D(13,13), E(13,13) , D1(13,13)
      COMMON /DAT/ CORDX(121), CORDY(121), CORDZ(121), EC(17,17),
     1    EX(17,17), EY(17,17), AE(420), JT(420), KT(420)  ,
     2         AX(17,17),  AY(17,17),  AC(17,17)
      COMMON /PLANE/ PX2(17,17), PY2(17,17), W(17,17), SU(17,17),
     1          SV(17,17), U(17,17), V(17,17)
C
C-----FORMAT STATEMENTS
C
   11 FORMAT (    5H1     , 80X, 10H1-----TRIM )
   12 FORMAT (  20A4  )
   13 FORMAT (     5X, 20A4  )
```

```
   14 FORMAT ( A4,A1,5X, 35A2)
   15 FORMAT (///10H     PROB , /5X, A4,A1, 5X, 35A2 )
   16 FORMAT (///17H     PROB (CONTD), /5X, A4,A1,5X, 35A2 )
   19 FORMAT (5X, 1H )
   20 FORMAT ( 5I5, 4E10.3, 2A4)
   21 FORMAT (2I5, 4E10.3, I5, E10.3, I5 )
   23 FORMAT (    4( 2X, I3),  6E10.4)
   24 FORMAT (   4( 2X, I3 ),        6E10.4 )
   30 FORMAT ( //30H      TABLE 1.  CONTROL DATA     ,              /
     1         / 30H            NUM CARDS TABLE 2   , 42X, I3,     /
     2           30H            NUM CARDS TABLE 3   , 42X, I3,     /
     3           30H            NUM CARDS TABLE 4   , 42X, I3,     /
     4           30H            NUM INCREMENTS  MX  , 42X, I3,     /
     5           30H            NUM INCREMENTS  MY  , 42X, I3,     /
     6           30H            INCR LENGTH  HX     , 35X,   E10.3,/
     7           30H            INCR LENGTH  HY     , 35X,   E10.3,/
     9           30H            POISSONS RATIO      , 35X,   E10.3,/
     1           30H            SLAB THICKNESS       , 35X,   E10.3,/
     2           33H            DEFLECTION CLOSURE TOLE,32X,E10.3,/
     3           30H            MAX NUM ITERATION   ,42X,I3,/
     3           36H            TYPE OF PROBLEM          , 35X, I4,/
     6           45H                 0 FOR LARGE DEFLECTION PROBLEM ,/
     4           40H                 1 FOR MEMBRANE PROBLEM     ,/
     5           40H                 2 FOR PLANE STRESS PROBLEM ,/
     7           40H                 3 FOR BUCKLING PROBLEM     ,/
     2           30H                                   , 35X,   E10.3 )
   33 FORMAT ( //50H      TABLE 2.  STIFFNESS DATA FOR PLATE PROBLEM    ,/
     2         / 50H        FROM    THRU      DX            DY          C
     3           45H        EX              EY                           ,/)
   37 FORMAT ( //50H      TABLE 3.  STIFFNESS FOR SUPPORTING SPRINGS    ,/
     2         / 50H        FROM    THRU      S             SU          SV
     3           45H        RX              RY                           ,/)
   38 FORMAT (//25H     TABLE 4.    LOAD DATA,/
     1/70H          FROM    THRU       Q       PX           PY           TX
     2      TY ,/)
   39 FORMAT (// 45H      TABLE 5.  RESULTS:  DEFLECTIONS               ///
     1           50H        I,J        WDEFL        UDEFL        VDEFL
     2           20H TOTREACT              / )
   40 FORMAT ( //45H      TABLE 6.  BENDING AND TWISTING MOMENTS  ,///
     1           50H        I,J        BMX          BMY          TMX
     2           20H  TMY                 ,/)
   41 FORMAT ( //48H      TABLE 7.  NORMAL & SHEAR MEMBRANE STRESSES ,///
     1           48H        I,J        MSX          MSY          SHS      ,/)
   42 FORMAT ( //48H      TABLE 8.  BUCKLING LOADS                       ,///
     1           34H        I, J,           PX            PY ,/)
   43 FORMAT  (5X,  2( 1X,  I3,  I3  ),  1P6E11.4)
   44 FORMAT  ( 5X,  2(  1X, I3, I3 ),  1P6E11.4)
   45 FORMAT  ( 7X, I2, I3, 1P9E12.3)
   46 FORMAT(/, 5X, 24H      NUM OF ITERATION  =, I5)
C
C-----PROGRAM AND PROBLEM IDENTIFICATION
C
             L1 = 13
      READ  12, ( AN1(N), N = 1, 40 )
 1010 READ  14,  NPROB, ( AN2(N), N = 1, 35 )
          IF ( NPROB - ITEST ) 1020, 9990, 1020
 1020 PRINT 11
      PRINT 13, ( AN1(N), N = 1, 40 )
```

```
      PRINT 15, NPROB, ( AN2(N), N = 1, 35 )
C
C-----INPUT TABLE 1
C
      READ 20, NCT2, NCT3, NCT4
      READ 21, MX, MY, HX, HY, PR, THK, NITERA, CLOS, ITYPE
      PRINT 30, NCT2,NCT3,NCT4,MX,MY,HX,HY,PR,THK,CLOS,NITERA,ITYPE
C
C-----COMPUTE FOR CONVENIENCE
C
               MXP1 = MX + 1
               MXP2 = MX + 2
               MXP3 = MX + 3
               MXP4 = MX + 4
               MXP5 = MX + 5
               MXP7 = MX + 7
               MYP1 = MY + 1
               MYP2 = MY + 2
               MYP3 = MY + 3
               MYP4 = MY + 4
               MYP5 = MY + 5
               MYP7 = MY + 7
               HYDHX3 = HY / HX**3
               HXDHY3 = HX / HY**3
               PDHXHY = PR / ( HY * HX )
               ODHXHY = 1.0 / ( HY * HX )
               ODHX = 1.0 / HX
               ODHY = 1.0 / HY
               ODHX2 = 1.0/HX**2
               ODHY2 = 1.0/HY**2
               WMAX1 = 0.0
               WMAX2 = 0.0
               WDIFF = 0.0
               ITERA = 0
               ITERW = 0
               ITE = 0
               BETA = 0.10
               NJT = MXP1 * MYP1
               NMEM = 4*MX*MY + MX + MY
               HZ = DSQRT( HX*HX + HY*HY )
          DO  105  J = 1, MYP7
          DO  100  I = 1, MXP7
               BMX(I,J) = 0.0
               BMY(I,J) = 0.0
               DX(I,J) = 0.0
               DY(I,J) = 0.0
                Q(I,J) = 0.0
                S(I,J) = 0.0
               CX(I,J) = 0.0
               TX(I,J) = 0.0
               TY(I,J) = 0.0
               PX(I,J) = 0.0
               PY(I,J) = 0.0
               SU(I,J) = 0.0
               SV(I,J) = 0.0
               U(I,J) = 0.0
               V(I,J) = 0.0
               W(I,J) = 0.0
```

```
                EX(I,J) = 0.0
                EY(I,J) = 0.0
                EC(I,J) = 0.0
                PXX(I,J) = 0.0
                PYY(I,J) = 0.0
                PZZ(I,J) = 0.0
                PX1(I,J) = 0.0
                PX2(I,J) = 0.0
                PY1(I,J) = 0.0
                PY2(I,J) = 0.0
                QO(I,J) = 0.0
                QO1(I,J) = 0.0
                RX(I,J) = 0.0
                RY(I,J) = 0.0
                SHS(I,J) = 0.0
                DSX(I,J) = 0.0
                DSY(I,J) = 0.0
                AX(I,J) = 0.0
                AY(I,J) = 0.0
                AC(I,J) = 0.0
                PD1(I,J) = 0.0
                PD2(I,J) = 0.0
                PC1(I,J) = 0.0
                PC2(I,J) = 0.0
                UU(I,J) = 0.0
                VV(I,J) = 0.0
  100     CONTINUE
  105     CONTINUE
          DO 120  NJ = 1, NJT
                CORDX(NJ) = 0.0
                CORDY(NJ) = 0.0
                CORDZ(NJ) = 0.0
  120     CONTINUE
          DO  140  J = 1, 2
          DO  135  II = 1, MXP3
                A(II,J) = 0.0
          DO  130  I = 1, MXP3
                B(I,II,J) = 0.0
                C(I,II,J) = 0.0
  130     CONTINUE
  135     CONTINUE
  140     CONTINUE
C
C------INPUT TABLE 2
C
          IF ( 1.GT.NCT2 )   GO TO 220
      PRINT 33
          DO 210  L = 1, NCT2
      READ 23, IN1(L), JN1(L), IN2(L), JN2(L), DXN(L), DYN(L),
     1    CXN(L),           EXN(L), EYN(L)
      PRINT 43, IN1(L), JN1(L), IN2(L), JN2(L), DXN(L), DYN(L),
     1    CXN(L),           EXN(L), EYN(L)
          IF ( ITYPE  .NE.  1)   GO TO 205
                DXN(L) = EXN(L)
                DYN(L) = EYN(L)
  205 CONTINUE
  210 CONTINUE
      CALL INTERP (IN1,JN1,IN2,JN2, DXN,NCT2,DX, 1,0,0,  17,  17,0,0)
```

```
      CALL  INTERP (IN1,JN1,IN2,JN2, DYN,NCT2,DY, 1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, CXN,NCT2,CX, 0,0,1,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, EXN,NCT2,EX, 0,0,1,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, EYN,NCT2,EY, 0,0,1,  17,  17,0,0)
  220 CONTINUE
         DO 225  I = 1, MXP7
         DO 225  J = 1, MYP7
            EC(I,J) = (HX*HX*EX(I,J) + HY*HY*EY(I,J))/(HZ*HZ)
  225 CONTINUE
C
C-----INPUT TABLE 3
C
         IF ( 1.GT.NCT3 )   GO TO 240
      PRINT 37
         DO 230  L = 1, NCT3
      READ 24, IN1(L), JN1(L), IN2(L), JN2(L),
     1   SN(L), SUN(L), SVN(L), RXN(L), RYN(L )
      PRINT 44, IN1(L), JN1(L), IN2(L), JN2(L),
     1   SN(L), SUN(L), SVN(L), RXN(L), RYN(L )
  230 CONTINUE
      CALL  INTERP (IN1,JN1,IN2,JN2, SN, NCT3,S,  1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, SUN,NCT3,SU, 1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, SVN,NCT3,SV, 1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2,RXN,NCT3,RX, 1,0,0, 17, 17, 0, 0 )
      CALL  INTERP (IN1,JN1,IN2,JN2,RYN,NCT3,RY, 1,0,0, 17, 17, 0, 0 )
  240 CONTINUE
C
C-----INPUT TABLE 4
C
         IF ( 1.GT.NCT4 )   GO TO 260
      PRINT 38
         DO 250  L = 1, NCT4
      READ 24, IN1(L), JN1(L), IN2(L), JN2(L),
     1   QN (L), PXN(L), PYN(L), TXN(L), TYN(L)
      PRINT 44, IN1(L), JN1(L), IN2(L ), JN2(L),
     1   QN (L), PXN(L), PYN(L), TXN(L), TYN(L)
  250 CONTINUE
      CALL  INTERP (IN1,JN1,IN2,JN2, QN, NCT4,Q,  1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, PXN,NCT4,PX, 1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, PYN,NCT4,PY, 1,0,0,  17,  17,0,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, TXN,NCT4,TX, 0,1,0,  17,  17,1,0)
      CALL  INTERP (IN1,JN1,IN2,JN2, TYN,NCT4,TY, 0,1,0,  17,  17,0,1)
  260 CONTINUE
C
C-----CALCULATE CROSS SECTION AREA OF MEMBRANE BARS
C
         DO 270  J = 5, MYP4
         DO 270  I = 5, MXP4
            AX(I,J) = THK*( HY*HY - PR*HX*HX*EY(I,J)/EX(I,J))
     *           /( 2*HY*(1-PR*PR))
            AY(I,J) = THK*( HX*HX - PR*HY*HY*EX(I,J)/EY(I,J))
     *           /( 2*HX*(1-PR*PR))
            AC(I,J) = THK*PR*(HZ**3)*EY(I,J)/( EC(I,J)*
     *           2*HX*HY*(1-PR*PR))
  270 CONTINUE
C
C-----CHECK PROBLEM TO DISTRIBUTE IN-PLANE LOAD
C
```

```
          IF ( ITYPE  .EQ.  1)   GO TO 380
          IF ( ITYPE  .NE.  0 )   GO TO 281
               IPL = 0
          DO 280  L = 1, NCT4
          IF( PXN(L)  .NE.  0.0)  IPL = 1
          IF( PYN(L)  .NE.  0.0)  IPL = 1
  280 CONTINUE
          IF ( IPL  .EQ.  0 )   GO TO 380
  281 CONTINUE
          DO 285  J = 4, MYP4
          DO 285  I = 4, MXP4
               PX2(I,J) = PX(I,J)
               PY2(I,J) = PY(I,J)
  285 CONTINUE
  290 CONTINUE
      CALL DATRUS ( NJT, NMEM, HX, HY )
      CALL INPLAN ( NJT, NMEM)
               EPP = 1.0E-5
          DO 310  J = 3, MYP5
          DO 310  I = 3, MXP5
          IF( DABS(U(I,J))  .LE.  EPP )  U(I,J) = 0.0
          IF( DABS(V(I,J))  .LE.  EPP )  V(I,J) = 0.0
               UU(I,J) = U(I,J)
               VV(I,J) = V(I,J)
  310 CONTINUE
          IF ( ITYPE  .EQ.  2 )  ITERA = 1
          IF (  ITYPE  .EQ.  2 )  GO TO 732
C
C-----CALCULATE DISTRIBUTION OF IN-PLANE LOADS
C
          DO 320  J = 4, MYP4
          DO 320  I = 4, MXP4
               PXX(I,J) = ( EX(I,J)*AX(I,J) + EX(I,J+1)*AX(I,J+1))
     1    *( HX+U(I,J)-U(I-1,J))*( HX*(U(I,J)-U(I-1,J)) + 0.5*(((
     2    U(I,J)-U(I-1,J))**2) + ((V(I,J)-V(I-1,J))**2))) /(HX*HX*
     3    DSQRT(((HX+U(I,J)-U(I-1,J))**2)+((V(I,J)-V(I-1,J))**2)))
               PYY(I,J) = ( EY(I,J)*AY(I,J) + EY(I+1,J)*AY(I+1,J))
     1    *(HY+V(I,J)-V(I,J-1))*(HY*(V(I,J)-V(I,J-1)) + 0.5*(((
     2    U(I,J)-U(I,J-1))**2) +((V(I,J)-V(I,J-1))**2)))/(HY*HY*
     3    DSQRT(((U(I,J)-U(I,J-1))**2)+((HY+V(I,J)-V(I,J-1))**2)))
               PC1(I,J) = EC(I,J+1)*AC(I,J+1)*(HX*(U(I,J)-U(I-1,J+1))
     1    + HY*(V(I-1,J+1)-V(I,J)) + 0.5*(((U(I,J)-U(I-1,J+1))**2)
     2    +((V(I-1,J+1)-V(I,J))**2)))/(HZ*HZ)
               PC2(I,J) = EC(I,J)*AC(I,J)*(HX*(U(I,J)-U(I-1,J-1))
     1    + HY*(V(I,J)-V(I-1,J-1)) + 0.5*(((U(I,J)-U(I-1,J-1))**2)
     2    +((V(I,J)-V(I-1,J-1))**2)))/(HZ*HZ)
  320 CONTINUE
          DO 350  J = 4, MYP4
          DO 350  I = 4, MXP4
          IF ( DABS(PXX(I,J))  .LE.  EPP)   PXX(I,J) = 0.0
          IF ( DABS(PYY(I,J))  .LE.  EPP)   PYY(I,J) = 0.0
          IF (DABS(PC1(I,J))  .LE.  EPP )   PC1(I,J) = 0.0
          IF (DABS(PC2(I,J))  .LE.  EPP )   PC2(I,J) = 0.0
               PX1(I,J) = PXX(I,J)
               PY1(I,J) = PYY(I,J)
               PD1(I,J) = PC1(I,J)
               PD2(I,J) = PC2(I,J)
  350 CONTINUE
```

```
  380 CONTINUE
C
C------PLACE SPRING AT MESH PTS BEYOND BOUNDARIES OF REAL SLAB TO MAKE
C        SOLUTION OF NON-RECTANGULAR SLABS OR SLABS WITH HOLES POSSIBLE.
C        SET INCREMENTAL LOAD VALUES
C
         DO  400  J = 3, MYP5
         DO  395  I = 3, MXP5
              QQ(I,J) = Q(I,J) -ODHX*( TX(I,J) - TX(I+1,J) )
     1              -ODHY*( TY(I,J) - TY(I,J+1) )
         IF( ITYPE  .EQ.  3)  BETA = 1.0
              QQ1(I,J) = BETA * QQ(I,J)
              SUM = DX(I-1,J) + DX(I,J) + DX(I+1,J)
     1             + DY(I,J-1) + DY(I,J) + DY(I,J+1)
         IF ( SUM ) 395, 390, 395
  390         S(I,J) = 1.0E+20
  395    CONTINUE
  400    CONTINUE
  401    CONTINUE
C
C------FORM SUB-MATRICES
C
         DO  600  J = 3, MYP5
         DO  500  I = 3, MXP5
              II = I - 2
              AA1(II) = DY(I,J-1) * HXDHY3
     1            - 0.25*ODHY2* RY(I,J-1)
              AA2(II,2)  = -2.0 * ( PDHXHY * ( DX(I,J) + DY(I,J-1) )
     1                        + HXDHY3 * ( DY(I,J-1) + DY(I,J) ) )
     2                        + ODHXHY * ( - CX(I,J) - CX(I+1,J)
     3                        - CX(I,J) - CX(I+1,J) ) - ODHY * PY1(I,J)
         IF( II - 1 )  402, 403, 402
  402    IF( II - MXP3 )  407, 408, 407
  403         AA3(II,3) = HYDHX3*DX(I+1,J) + 0.25*ODHX2*RX(I+1,J)
         GO TO 404
  408         AA3(II,3) = HYDHX3*DX(I-1,J) + 0.25*ODHX2*RX(I-1,J)
         GO TO 404
  407         AA3(II,3) = HYDHX3 * ( DX(I-1,J) + 4.0 * DX(I,J)
     1                        + DX(I+1,J) ) + HXDHY3 * ( DY(I,J-1) + 4.0
     2                        * DY(I,J) + DY(I,J+1) ) + PDHXHY * 4.0
     3                        * ( DX(I,J) + DY(I,J) ) + ODHXHY
     4                        * ( CX(I,J) + CX(I,J+1) + CX(I+1,J)
     5                        + CX(I+1,J+1) + CX(I,J) + CX(I+1,J)
     6                        + CX(I,J+1) + CX(I+1,J+1) ) + ODHX
     7                        * (PX1(I,J) +PX1(I+1,J) ) + ODHY
     8                        * (PY1(I,J) + PY1(I,J+1)) + S(I,J)
     9             + 0.25*ODHX2*( RX(I-1,J) + RX(I+1,J) )
     A             +0.25*ODHY2*( RY(I,J-1) + RY(I,J+1) )
     B        + ( PC1(I,J) + PC1(I+1,J-1) + PC2(I,J) + PC2(I+1,J+1))/HZ
  404    IF ( AA3(II,3)  .EQ.  0.0 )  AA3(II,3) = 1.0
              AA4(II,2)  = -2.0 * ( HXDHY3 * ( DY(I,J) + DY(I,J+1) )
     1                        + PDHXHY * ( DX(I,J) + DY(I,J+1) ) )
     2                        + ODHXHY * ( - CX(I,J+1) - CX(I+1,J+1)
     3                        - CX(I,J+1) - CX(I+1,J+1) ) - ODHY
     4                        * PY1(I,J+1)
              AA5(II) = HXDHY3 * DY(I,J+1)
     1            - 0.25*ODHY2*RY(I,J+1)
              AA6(II) = QQ1(I,J)
```

```
          IF(II-1) 410, 410, 405
  405          AA2(II,1)       = DX(I-1,J) * PDHXHY + DY(I,J-1) * PDHXHY +
     1                           ODHXHY * ( CX(I,J) + CX(I,J) )
     2                                - PC2(I,J)/HZ
               AA3(II,2)       = -2.0 * ( HYDHX3 * ( DX(I-1,J) + DX(I,J) )
     1                           + PDHXHY * ( DX(I-1,J) + DY(I,J) ) )
     2                           + ODHXHY * ( - CX(I,J) - CX(I,J+1)
     3                           - CX(I,J) - CX(I,J+1) ) - ODHX * PX1(I,J)
               AA4(II,1)       = PDHXHY * ( DX(I-1,J) + DY(I,J+1) )
     1                           + ODHXHY * ( CX(I,J+1) + CX(I,J+1) )
     2                                - PC1(I,J)/HZ
  410     IF(II-2) 430, 430, 420
  420          AA3(II,1)       = DX(I-1,J) * HYDHX3
     1              - 0.25*ODHX2*RX(I-1,J)
  430     IF(II-MXP3) 440, 450, 450
  440          AA2(II,3)       = PDHXHY * ( DX(I+1,J) + DY(I,J-1) )
     1                           + ODHXHY * ( CX(I+1,J) + CX(I+1,J) )
     2                                - PC1(I+1,J-1)/HZ
               AA3(II,4)       = -2.0 * ( HYDHX3 * ( DX(I,J) + DX(I+1,J) )
     1                           + PDHXHY * ( DX(I+1,J) + DY(I,J) ) )
     2                           + ODHXHY * ( - CX(I+1,J) - CX(I+1,J+1)
     3                           - CX(I+1,J) - CX(I+1,J+1) ) - ODHX
     4                           * PX1(I+1,J)
               AA4(II,3)       = PDHXHY * ( DX(I+1,J) + DY(I,J+1) )
     1                           + ODHXHY * ( CX(I+1,J+1) + CX(I+1,J+1) )
     2                                - PC2(I+1,J+1)/HZ
  450     IF(II+1-MXP3) 460, 500, 500
  460          AA3(II,5)       = HYDHX3 * DX(I+1,J)
     1              - 0.25*ODHX2*RX(I+1,J)
  500     CONTINUE
C
C-----BEGIN MAIN SOLUTION
C
          DO  515  I = 1, MXP3
               A1(I,1) = A(I,J-1)
               A2(I,1) = A(I,J-2)
          DO  510  K = 1, MXP3
               BB1(I,K) = B(I,K,J-1)
               BB2(I,K) = B(I,K,J-2)
               CC1(I,K) = C(I,K,J-1)
               CC2(I,K) = C(I,K,J-2)
  510     CONTINUE
  515     CONTINUE
               JJ = J
      CALL MATRIX  ( L1, JJ, MXP3, MY   )
          DO  585  I = 1, MXP3
               A(I,J) = AA(I,1)
          DO  580  K = 1, MXP3
               B(I,K,J) = BB(I,K)
               C(I,K,J) = CC(I,K)
  580     CONTINUE
  585     CONTINUE
  600     CONTINUE
C
C-----COMPUTE LATERAL DEFLECTION
C
          DO  650  LL = 3, MYP5
               J = MY + 8 - LL
```

```
          DO  625  I = 1, MXP3
                W1(I,1) = W(I,J+1)
                W2(I,1) = W(I,J+2)
                AA(I,1) = A(I,J)
          DO  620  K = 1, MXP3
                BB(I,K) = B(I,K,J)
                CC(I,K) = C(I,K,J)
  620     CONTINUE
  625     CONTINUE
      CALL MATMPY (L1 , MXP3 , 1 , BB , W1 , A1)
      CALL MATMPY (L1 , MXP3 , 1 , CC , W2 , A2)
          DO  630  I = 1, MXP3
                W(I,J) = AA(I,1) + A1(I,1) + A2(I,1)
  630     CONTINUE
  650     CONTINUE
                W(1,3)= 2.0 * W(1,4) - W(1,5)
                W(MXP3,3) = 2.0* W( MXP3,4) -W(MXP3,5)
                W(1,MYP5) = 2.0* W(1,MY+4) -W (1,MY+3)
                W(MXP3,MYP5) = 2.0 * W(MXP3,MY+4) -W(MXP3,MY+3)
          DO  665  J = 3, MYP5
          DO  660  I = 3, MXP5
                II = MXP5 + 3 - I
                W(II,J) = W(II-2,J)
  660     CONTINUE
  665     CONTINUE
          DO  670  J = 3, MYP5
                W(1,J) = 0.0
                W(2,J) = 0.0
                W(MX+6,J) = 0.0
                W(MX+7,J) = 0.0
  670     CONTINUE
C
C-----CHECK TO STOP PROBLEM FOR BUCKLING PROBLEM
C
          IF (ITYPE  .NE.  3)  GO TO 679
                ITERA = ITERA + 1
          DO 673  J = 4, MYP4
          DO 673  I = 4, MXP4
          IF ( DABS(W(I,J)) .GE. WMAX2)  WMAX2 = DABS(W(I,J))
  673 CONTINUE
          IF (ITERA  .EQ. 1)  GO TO 674
                WDIFF = WMAX2/WMAX1
          IF ( WDIFF  .GT. 2.0)  GO TO 732
  674     DO 675  J = 4, MYP4
          DO 675  I = 4, MXP4
                PC1(I,J) = PC1(I,J) + 0.01*PD1(I,J)
                PC2(I,J) = PC2(I,J) + 0.01*PD2(I,J)
                PX1(I,J) = PX1(I,J) + 0.01*PXX(I,J)
                PY1(I,J) = PY1(I,J) + 0.01*PYY(I,J)
  675     CONTINUE
  676           WMAX1 = WMAX2
          GO TO 401
  679     CONTINUE
C
C-----    INPLANE LOAD DUE TO LATERAL DEFLECTION
C
                EPP = 1.0E-5
          DO 680  J = 4, MYP4
```

```
          DO 680  I = 4, MXP4
C
                PX2(I,J) = EX(I+1,J+1)*AX(I+1,J+1)*((W(I,J)-W(I+1,J))**2)
     1    /(2*HX*HX)
     2    - EX(I,J+1)*AX(I,J+1)*((W(I,J)-W(I-1,J))**2)/(2*HX*HX)
     3    - EX(I,J)*AX(I,J)*((W(I,J)-W(I-1,J))**2)/(2*HX*HX)
     4    + EX(I+1,J)*AX(I+1,J)*((W(I,J)-W(I+1,J))**2)/(2*HX*HX)
     5 + EC(I+1,J+1)*AC(I+1,J+1)*HX*((W(I,J)-W(I+1,J+1))**2)/(2*HZ**3)
     6 - EC(I,J+1)*AC(I,J+1)*HX*((W(I,J)-W(I-1,J+1))**2)/(2*HZ**3)
     7 - EC(I,J)*AC(I,J)*HX*((W(I,J)-W(I-1,J-1))**2)/(2*HZ**3)
     8 + EC(I+1,J)*AC(I+1,J)*HX*((W(I,J)-W(I+1,J-1))**2)/(2*HZ**3)
                PY2(I,J) = EY(I+1,J+1)*AY(I+1,J+1)*((W(I,J)-W(I,J+1))**2)
     1    /(2*HY*HY)
     2    + EY(I,J+1)*AY(I,J+1)*((W(I,J)-W(I,J+1))**2)/(2*HY*HY)
     3    - EY(I,J)*AY(I,J)*((W(I,J)-W(I,J-1))**2)/(2*HY*HY)
     4    - EY(I+1,J)*AY(I+1,J)*((W(I,J)-W(I,J-1))**2)/(2*HY*HY)
     5 + EC(I+1,J+1)*AC(I+1,J+1)*HY*((W(I,J)-W(I+1,J+1))**2)/(2*HZ**3)
     6 + EC(I,J+1)*AC(I,J+1)*HY*((W(I,J)-W(I-1,J+1))**2)/(2*HZ**3)
     7 - EC(I,J)*AC(I,J)*HY*((W(I,J)-W(I-1,J-1))**2)/(2*HZ**3)
     8 - EC(I+1,J)*AC(I+1,J)*HY*((W(I,J)-W(I+1,J-1))**2)/(2*HZ**3)
C
          IF( DABS(PX2(I,J))  .LT.  EPP )   PX2(I,J) = 0.0
          IF( DABS(PY2(I,J))  .LT.  EPP )   PY2(I,J) = 0.0
  680     CONTINUE
  681 CONTINUE
C
C------CALCULATE INPLANE DISPLACEMENT DUE TO PX2 AND PY2
C
          IF ( ITERA  .GT.  0 )   GO TO 691
      CALL DATRUS  (NJT, NMEM, HX, HY )
  691 CALL  INPLAN  ( NJT, NMEM )
          DO 685  J = 3, MYP5
          DO 685  I = 3, MXP5
          IF( DABS(U(I,J))  .LT.  EPP )   U(I,J) = 0.0
          IF( DABS(V(I,J))  .LT.  EPP )   V(I,J) = 0.0
          IF( DABS(W(I,J))  .LE.  EPP )  W(I,J) = 0.0
  685     CONTINUE
C
C------CALCULATE PZZ VERTICAL MEMBRANE FORCE
C      PZZ IS POSITIVE IN DIRECTION QQ1
C
          DO 700  J = 4, MYP4
          DO 700  I = 4, MXP4
                PZZ(I,J) =
     1    (2*HX*(U(I+1,J)-U(I,J)) + ((W(I+1,J)-W(I,J))**2))
     2    *(W(I+1,J)-W(I,J))*(EX(I+1,J)*AX(I+1,J) + EX(I+1,J+1)*AX(I+1,
     3    J+1))/(2*HX**3)
     4    -(2*HX*(U(I,J)-U(I-1,J))+((W(I,J)-W(I-1,J))**2))*(W(I,J)-W(I-1
     5    ,J))*(EX(I,J)*AX(I,J)+EX(I,J+1)*AX(I,J+1))/(2*HX**3)
     6    +(2*HY*(V(I,J+1)-V(I,J))+((W(I,J+1)-W(I,J))**2))*(W(I,J+1)-
     7    W(I,J))*(EY(I,J+1)*AY(I,J+1)+EY(I+1,J+1)*AY(I+1,J+1))/(2*HY**3
     8    )-(2*HY*(V(I,J)-V(I,J-1))+((W(I,J)-W(I,J-1))**2))*(W(I,J)-
     9    W(I,J-1))*(EY(I+1,J)*AY(I+1,J)+EY(I,J)*AY(I,J))/(2*HY**3)
                PZZ(I,J) = PZZ(I,J) -
     1 (HX*(U(I+1,J+1)-U(I,J))+HY*(V(I+1,J+1)-V(I,J))+((W(I,J)-W(I+1,J+1
     2 ))**2)/2)*EC(I+1,J+1)*AC(I+1,J+1)*(W(I,J)-W(I+1,J+1))/(HZ**3)
     3-(HX*(U(I,J)-U(I-1,J+1))+HY*(V(I-1,J+1)-V(I,J))+((W(I,J)-W(I-1,J+1
     4 ))**2)/2)*AC(I,J+1)*EC(I,J+1)*(W(I,J)-W(I-1,J+1))/(HZ**3)
```

```
     5-(HX*(U(I+1,J-1)-U(I,J))+HY*(V(I,J)-V(I+1,J-1))+((W(I,J)-W(I+1,J-1
     6 ))**2)/2) *AC(I+1,J)*EC(I+1,J)*(W(I,J)-W(I+1,J-1))/(HZ**3)
     7-(HX*(U(I,J)-U(I-1,J-1))+HY*(V(I,J)-V(I-1,J-1))+((W(I,J)-W(I-1,J-1
     8 ))**2)/2)*AC(I,J)*EC(I,J)*(W(I,J)-W(I-1,J-1))/(HZ**3)
  700 CONTINUE
C
C-------- NEW LOAD FOR NEXT LOOP
C      QRESIT IS POSSITIVE IN NEGATIVE DIRECTION OF QQ1
C
             TOLE = 1.0E-6
      DO 710  J = 4, MYP4
      DO 710  I = 4, MXP4
      IF ( ITYPE  .NE.  1 )  GO TO 701
             QRESIT = -PZZ(I,J)
      GO TO 702
  701        QRESIT = QQ1(I,J) - PZZ(I,J)
  702        XRESIT = PXX(I,J)
             YRESIT = PYY(I,J)
      IF( DABS(QRESIT)  .LT.  TOLE)  GO TO 708
      IF ( DABS(QQ1(I,J))  .LT.  TOLE )  GO TO 705
      IF( DABS(QQ(I,J))  .LT.  TOLE )  GO TO 704
             QQ1(I,J) = 0.7*QQ1(I,J) + 0.3*QQ1(I,J)*QQ(I,J)/QRESIT
      GO TO 707
  704        QQ1(I,J) = QQ1(I,J) - 0.5*QRESIT
      GO TO 707
  705        QQ1(I,J) = -0.3*QRESIT
  707 IF ( DABS(QQ1(I,J)) .LT.  TOLE )  QQ1(I,J) = 0.0
      GO TO 710
  708        QQ1(I,J) = 0.0
  710 CONTINUE
C
C------CHECK TO STOP THE ITERATION
C
      DO 715  J = 4, MYP4
      DO 715  I = 4, MXP4
      IF( DABS(W(I,J))  .GT.  WMAX2 )  WMAX2 = DABS( W(I,J) )
  715 CONTINUE
             WDIFF = (WMAX1 - WMAX2)/WMAX2
      IF( DABS(WDIFF)  .LT.  CLOS )  GO TO 730
             ITERA = ITERA + 1
      IF ( ITERA  .GE.  NITERA )  GO TO 730
             WMAX1 = WMAX2
             WMAX2 = 0.0
      GO TO 401
  730 CONTINUE
      IF ( ITYPE  .NE.  1 )  GO TO 732
      DO 731  J = 3, MYP5
      DO 731  I = 3, MXP5
             DX(I,J) = 0.0
             DY(I,J) = 0.0
  731 CONTINUE
  732 CONTINUE
C------TOTAL INPLANE DISPLACEMENTS
C
      IF (ITYPE  .EQ.  3)  ITERA = ITERA-1
      DO 733  J = 4, MYP4
      DO 733  I = 4, MXP4
      IF ( ITYPE  .NE.  3 )  GO TO 734
```

```
              U(I,J) = 0.01*ITERA*U(I,J)
              V(I,J) = 0.01*ITERA*V(I,J)
         GO TO 733
  734    IF ( ITYPE  .EQ.  2 )  GO TO 733
              U(I,J) = U(I,J) + UU(I,J)
              V(I,J) = V(I,J) + VV(I,J)
  733    CONTINUE
C
C     CALCULATE AND PRINT MEMBRANE DIRECT AND SHEAR STRESS
C
         DO 735  J = 5, MYP4
         DO 735  I = 5, MXP4  .
C
      DSX(I,J) = AX(I,J)*EX(I,J)*HX*((((U(I,J)-U(I-1,J))/HX
     1   +(((U(I,J)-U(I-1,J))**2) +((V(I,J)-V(I-1,J))**2)
     2   +((W(I,J)-W(I-1,J))**2))/(2*HX*HX))
     3   /DSQRT((( HX+U(I,J)-U(I-1,J))**2) +((V(I,J)-V(I-1,J))**2)
     4   +((W(I,J)-W(I-1,J))**2))
     5   +((U(I,J-1)-U(I-1,J-1))/HX +(((U(I,J-1)-U(I-1,J-1))**2)
     6   +((V(I,J-1)-V(I-1,J-1))**2) +((W(I,J-1)-W(I-1,J-1))**2))
     7   /(2*HX*HX))/DSQRT(((HX+U(I,J-1)-U(I-1,J-1))**2)
     8   +((V(I,J-1)-V(I-1,J-1))**2) +((W(I,J-1)-W(I-1,J-1))**2)))
              DSX(I,J) = DSX(I,J) + AC(I,J)*EC(I,J)
     1   *(HX+U(I,J-1)-U(I-1,J))*(HX*(U(I,J-1)-U(I-1,J))
     2   +HY*(V(I-1,J)-V(I,J-1)) +0.5*(((U(I,J-1)-U(I-1,J))**2)
     3   +((V(I,J-1)-V(I-1,J))**2) + ((W(I,J-1)-W(I-1,J))**2)))
     4   /(   HZ*HZ*DSQRT(((HX+U(I,J-1)-U(I-1,J))**2)
     5   +((HY+V(I-1,J)-V(I,J-1))**2) +((W(I,J-1)-W(I-1,J))**2)))
              DSX(I,J) = DSX(I,J)
     6   +AC(I,J)*EC(I,J)*    (HX+U(I,J)-U(I-1,J-1))
     7   *(HX*(U(I,J)-U(I-1,J-1)) +HY*(V(I,J)-V(I-1,J-1))
     8   +0.5*(((U(I,J)-U(I-1,J-1))**2) +((V(I,J)-V(I-1,J-1))**2)
     9   +((W(I,J)-W(I-1,J-1))**2)))/(   HZ*HZ*DSQRT(((HX+ U(I,J)
     1   -U(I-1,J-1))**2) +((HY+V(I,J)-V(I-1,J-1))**2)
     2   +((W(I,J)-W(I-1,J-1))**2)))
              DSX(I,J) = DSX(I,J)/(THK*HY)
C
      DSY(I,J) = AY(I,J)*EY(I,J)*HY*((((V(I-1,J)-V(I-1,J-1))/HY
     1   +(((U(I-1,J)-U(I-1',J-1))**2) +((V(I-1,J)-V(I-1,J-1))**2)
     2   +((W(I-1,J)-W(I-1,J-1))**2))/(2*HY*HY))
     3   /DSQRT(((U(I-1,J)-U(I-1,J-1))**2) +((HY+V(I-1,J)
     4   -V(I-1,J-1))**2) +((W(I-1,J)-W(I-1,J-1))**2)) +
     5   ((V(I,J)-V(I,J-1))/HY +(((U(I,J)-U(I,J-1))**2)
     6   +((V(I,J)-V(I,J-1))**2) + ((W(I,J)-W(I,J-1))**2))/(2*HY*HY))
     7   /DSQRT(((U(I,J)-U(I,J-1))**2) +((HY+V(I,J)-V(I,J-1))**2)
     8   +((W(I,J)-W(I,J-1))**2)))
              DSY(I,J) = DSY(I,J)
     1   +AC(I,J)*EC(I,J)*    (HY+V(I-1,J)-V(I,J-1))
     2   *(HX*(U(I,J-1)-U(I-1,J)) +HY*(V(I-1,J)-V(I,J-1))
     3   +0.5*(((U(I,J-1)-U(I-1,J))**2) +((V(I-1,J)-V(I,J-1))**2)
     4   +((W(I-1,J)-W(I,J-1))**2)))
     5   /(   HZ*HZ*DSQRT(((HX+U(I,J-1)-U(I-1,J))**2)
     6   +((HY+V(I-1,J)-V(I,J-1))**2) +((W(I-1,J)-W(I,J-1))**2)))
              DSY(I,J) = DSY(I,J)
     1   +AC(I,J)*EC(I,J)*    (HY+V(I,J)-V(I-1,J-1))
     2   *(HX*(U(I,J)-U(I-1,J-1)) +HY*(V(I,J)-V(I-1,J-1))
     3   +0.5*(((U(I,J)-U(I-1,J-1))**2) +((V(I,J)-V(I-1,J-1))**2)
     4   +((W(I,J)-W(I-1,J-1))**2)))
```

```
     5     /(    HZ*HZ*DSQRT(((HX+U(I,J)-U(I-1,J-1))**2)
     6     +((HY+V(I,J)-V(I-1,J-1))**2) +((W(I,J)-W(I-1,J-1))**2)))
                 DSY(I,J) = DSY(I,J)/(THK*HX)
C
      SHS(I,J) = AC(I,J)*EC(I,J)*    ((HY+V(I-1,J)-V(I,J-1))
     1     *(HX*(U(I,J-1)-U(I-1,J))+HY*(V(I-1,J)-V(I,J-1))
     2     +0.5*(((U(I,J-1)-U(I-1,J))**2) +((V(I,J-1)-V(I-1,J))**2)
     3     +((W(I,J-1)-W(I-1,J))**2)))
     4     /(    HZ*HZ*DSQRT(((HX+U(I,J-1)-U(I-1,J))**2)
     5     +((HY+V(I-1,J)-V(I,J-1))**2) +((W(I-1,J)-W(I,J-1))**2))))
                 SHS(I,J) = SHS(I,J) - AC(I,J)*EC(I,J)*
     6     ((HY+V(I,J)-V(I-1,J-1))*(HX*(U(I,J)-U(I-1,J-1))
     7     +HY*(V(I,J)-V(I-1,J-1)) +0.5*(((U(I,J)-U(I-1,J-1))**2)
     8     +((V(I,J)-V(I-1,J-1))**2) +((W(I,J)-W(I-1,J-1))**2)))
     9     /(    HZ*HZ*DSQRT(((HX+U(I,J)-U(I-1,J-1))**2)
     1     +((HY+V(I,J)-V(I-1,J-1))**2) +((W(I,J)-W(I-1,J-1))**2))))
                 SHS(I,J) = SHS(I,J)/(THK*HY)
  735      CONTINUE
C
C------CALCULATE BENDING EFFECTS
C
  738 PRINT 11
      PRINT 13, ( AN1(N), N = 1, 40 )
      PRINT 16, NPROB, ( AN2(N), N = 1, 35 )
      PRINT 39
           DO 800  J = 4, MYP4
           DO 750  I = 4, MXP4
                 ISTA = I - 4
                 JSTA = J - 4
           IF (ITYPE  .EQ.  1)  GO TO 748
           IF( ITYPE  .EQ.  2)  GO TO 749
           IF (ITYPE  .EQ.  3)  GO TO 748
           DO  740  N = 1, 3
                 K = I + N - 2
                 DP(N+3) =DSQRT ( DX(K,J) * DY(K,J) )
                 BMX(K,J) = DX(K,J) * ( W(K-1,J) - W(K,J) - W(K,J)
     1                 + W(K+1,J) ) /(HX*HX)+ DP(N+3) * PR * (W(K,J-1)
     2                 - W(K,J) - W(K,J) + W(K,J+1) )/ ( HY * HY )
                 L = J + N - 2
                 DP(N) =DSQRT ( DX(I,L) * DY(I,L) )
                 BMY(I,L) = DY(I,L) * ( W(I,L-1) - 2.0 * W(I,L)
     1                 + W(I,L+1) ) /(HY * HY) + PR * DP(N)
     2                 * ( W(I-1, L) - 2.0 * W(I,L)+ W(I+1,L) )/ ( HX
     3                 * HX )
  740      CONTINUE
                 QBMX = ( BMX(I-1,J) - 2.0 * BMX(I,J) + BMX(I+1,J) )
     1                 * HY / HX
                 QBMY = ( BMY(I,J-1) - 2.0 * BMY(I,J) + BMY(I,J+1) )
     1                 * HX / HY
                 QTMX = ( W(I-1,J-1) * CX(I,J) - W(I-1,J) * ( CX(I,J)
     1                 + CX(I,J+1) ) + W(I-1,J+1) * CX(I,J+1)
     2                 - W(I,J-1) * ( CX(I,J) + CX(I+1,J))+ W(I,J)
     3                 * ( CX(I,J) + CX(I,J+1) + CX(I+1,J) + CX(I+1,J
     4                 +1) ) - W(I,J+1) * ( CX(I,J+1) + CX(I+1,J+1) )
     5                 + W(I+1,J-1) * CX(I+1,J) - W(I+1,J) * ( CX(I
     6                 +1,J) + CX(I+1,J+1) ) + W(I+1,J+1) * CX(I+1,J
     7                 +1) ) /(HY * HX)
                 QTMY = ( W(I-1,J-1) * CX(I,J) - W(I-1,J) * ( CX(I,J)
```

```
     1                  + CX(I,J+1) ) + W(I-1,J+1) * CX(I,J+1)
     2                  - W(I,J-1) * ( CX(I,J) + CX(I+1,J) ) + W(I,J)
     3                  * ( CX(I,J) + CX(I,J+1) + CX(I+1,J) + CX(I+1,J
     4                  +1) ) - W(I,J+1) * (CX(I,J+1)+CX(I+1,J+1) )
     5                  + W(I+1,J-1)
     5                  * CX(I+1,J) - W(I+1,J) * (CX(I+1,J) + CX(I+1,J
     6                  +1) ) + W(I+1,J+1) * CX(I+1,J+1) ) /( HX * HY )
              QPX = ( 1.0 / HX ) * (PX1(I,J) * W(I-1,J) - (PX1(I,J)
     1                  +PX1(I+1,J) ) * W(I,J) +PX1(I+1,J) * W(I+1,J) )
              QPY = ( 1.0 / HY ) * (PY1(I,J) * W(I,J-1) -(PY1(I,J)
     1                  +PY1(I,J+1) ) * W(I,J) +PY1(I,J+1) * W(I,J+1) )
              REACT = QBMX + QBMY + QTMX + QTMY - QPX - QPY
C
C-----TOTAL REACTION
C
              REACT = REACT - PZZ(I,J)
  748     IF( ITYPE  .EQ.  1)  REACT = -PZZ(I,J)
          IF (ITYPE  .EQ.  3)  REACT = 0.0
  749 PRINT 45, ISTA, JSTA, W(I,J), U(I,J), V(I,J), REACT
  750     CONTINUE
      PRINT 19
  800     CONTINUE
      PRINT  46,  ITERA
          IF ( ITYPE  .EQ.  1)  GO TO 961
          IF( ITYPE  .EQ.  2)  GO TO 961
          IF (ITYPE  .EQ.  3)  GO TO 976
      PRINT 11
      PRINT 16, NPROB, ( AN2(N), N = 1, 35 )
      PRINT 40
          DO 960  J = 4, MYP4
          DO 950  I = 4, MXP4
              ISTA = I - 4
              JSTA = J - 4
              TMX = ( CX(I,J) + CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1 ) )
     1                  *(-0.25)* ( W(I-1,J-1) - W(I-1,J+1) - W(I+1,J
     2                  -1) + W(I+1,J+1) ) / ( 4.0 * HX * HY )
              TMY = ( CX(I,J) + CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1) )
     1                  * ( +0.250 ) * ( W(I-1,J-1) - W(I-1,J+1)
     2                  - W(I+1,J-1) + W(I+1,J+1) ) / ( 4.0 * HX * HY)
      PRINT 45, ISTA, JSTA, BMX(I,J), BMY(I,J), TMX, TMY
  950     CONTINUE
      PRINT 19
  960     CONTINUE
  961 PRINT 11
      PRINT 16, NPROB, ( AN2(N), N = 1, 35)
      PRINT 41
          DO 975  J = 5, MYP4
          DO 970  I = 5, MXP4
              ISTA = I - 4
              JSTA = J - 4
      PRINT 45, ISTA, JSTA, DSX(I,J), DSY(I,J), SHS(I,J)
  970     CONTINUE
      PRINT 19
  975     CONTINUE
          GO TO 1010
  976 PRINT 11
      PRINT 16, NPROB, ( AN2(N), N=1, 35)
      PRINT 42
```

```
            ITERA = ITERA + 1
            CONST = 0.01*ITERA
      DO 978  J = 4, MYP4
      DO 977  I = 4, MXP4
            ISTA = I-4
            JSTA = J-4
            PX(I,J) = (1.0+CONST)*PX(I,J)
            PY(I,J) = (1.0+CONST)*PY(I,J)
      PRINT 45, ISTA, JSTA, PX(I,J), PY(I,J)
 977      CONTINUE
      PRINT 19
 978      CONTINUE
          GO TO 1010
9990      CONTINUE
9999      CONTINUE
      STOP
      END
```

```
C
      SUBROUTINE  MATRIX  ( L1, JJ, MXP3, MY  )
      IMPLICIT REAL * 8 ( A-H, O-Z )
      COMMON /MATR/ AA1(13), AA2(13,3), AA3(13,5), AA4(13,3),
     1    AA5(13), AA6(13), AA(13,1), A1(13,1), A2(13,1),
     2    BB(13,13), BB1(13,13), BB2(13,13), CC(13,13), CC1(13,13),
     3    CC2(13,13), AAUG(13,13,2), D(13,13), E(13,13) , D1(13,13)
      CALL MATMY1 (L1 , MXP3 , MXP3 , AA1 , BB2 , E)
      CALL MATA1 (L1 , MXP3 , E , AA2 , E)
      CALL MATMPY (L1 , MXP3 , MXP3 , E , BB1 , D)
      CALL MATMY1 (L1 , MXP3 , MXP3 , AA1 , CC2 , CC)
          DO  535  K = 1, MXP3
          DO  530  I = 1, MXP3
              D(I,K) = D(I,K) + CC(I,K)
 530      CONTINUE
 535      CONTINUE
      CALL MATA2 (L1 , MXP3 , D , AA3 , D)
      CALL INVR6 ( D, L1, MXP3)
          DO  545  K = 1, MXP3
          DO  540  I = 1, MXP3
              D(I,K) = -D(I,K)
 540      CONTINUE
 545      CONTINUE
      CALL MATM2 (L1 , MXP3 , D , AA5 , CC)
      CALL MATMPY (L1 , MXP3 , MXP3 , E , CC1 , BB)
      CALL MATA1 (L1 , MXP3 , BB , AA4 , BB1)
      CALL MATMPY (L1 , MXP3 , MXP3 , D , BB1 , BB)
      CALL MATMPY (L1 , MXP3 , 1 , E , A1 , AA)
      CALL MATMY1 (L1 , MXP3 , 1 , AA1 , A2 , A1)
          DO  560  I = 1, MXP3
              A2(I,1) = AA(I,1) + A1(I,1)
 560      CONTINUE
          DO  570  I = 1, MXP3
              A1(I,1) = A2(I,1) - AA6(I)
 570      CONTINUE
      CALL MATMPY (L1 , MXP3 , 1 , D , A1 , AA)
      RETURN
      END
```

```
      SUBROUTINE MATMPY (M1 , L2 , L , X , Y , Z)
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION X(M1,M1), Y(M1,L ), Z(M1,L )
          DO 100  I = 1 , L2
          DO 100  M = 1,L
               Z(I,M) = 0.0
          DO 100  K = 1 , L2
               Z(I,M) = X(I,K) * Y(K,M) + Z(I,M)
100   CONTINUE
      RETURN
      END


      SUBROUTINE MATM2 (M1 , L2 , Y , X , Z)
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION X(M1), Z(M1,M1), Y(M1,M1)
          DO 100  I = 1 , L2
          DO 100  J = 1 , L2
               Z(I,J) = X(J) * Y(I,J)
100       CONTINUE
      RETURN
      END


      SUBROUTINE MATMY1 (M1 , L2 , L , X , Y , Z)
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION X(M1), Z(M1,L ), Y(M1,L )
          DO 100  I = 1 , L2
          DO 100  J = 1,L
               Z(I,J) = X(I) * Y(I,J)
100       CONTINUE
      RETURN
      END


      SUBROUTINE MATA1 (M1 , L2 , Z , X3 , Y)
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION X3(M1,3), Z(M1,M1), Y(M1,M1)
               MM1 = L2 - 1
          DO 50  I = 1 , L2
          DO 50  J = 1 , L2
               Y(I,J) = Z(I,J)
 50       CONTINUE
          DO 100 I = 2, MM1
               Y(I,I-1) = Y(I,I-1) + X3(I,1)
               Y(I,I) = Y(I,I) + X3(I,2)
               Y(I,I+1) = Y(I,I+1) + X3(I,3)
100       CONTINUE
               Y(1,1) = Y(1,1) + X3(1,2)
               Y(1,2) = Y(1,2) + X3(1,3)
               Y(L2,L2-1) = Y(L2,L2-1) + X3(L2,1)
               Y(L2,L2) = Y(L2,L2) + X3(L2,2)
      RETURN
      END
```

```
      SUBROUTINE MATA2 (L1 , M1 , Z , X3 , Y)
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION X3(L1,5) , Z(L1,L1) , Y(L1,L1)
             MM2 = M1 - 2
         DO 100 I = 3, MM2
             Y(I,I-2) = Z(I,I-2) + X3(I,1)
             Y(I,I-1) = Z(I,I-1) + X3(I,2)
             Y(I,I) = Z(I,I) + X3(I,3)
             Y(I,I+1) = Z(I,I+1) + X3(I,4)
             Y(I,I+2) = Z(I,I+2) + X3(I,5)
100      CONTINUE
             Y(1,1) = Z(1,1) + X3(1,3)
             Y(1,2) = Z(1,2) + X3(1,4)
             Y(1,3) = Z(1,3) + X3(1,5)
             Y(2,1) = Z(2,1) + X3(2,2)
             Y(2,2) = Z(2,2) + X3(2,3)
             Y(2,3) = Z(2,3) + X3(2,4)
             Y(2,4) = Z(2,4) + X3(2,5)
             Y(M1-1,M1-3) = Z(M1-1,M1-3) + X3(M1-1,1)
             Y(M1-1,M1-2) = Z(M1-1,M1-2) + X3(M1-1,2)
             Y(M1-1,M1-1) = Z(M1-1,M1-1) + X3(M1-1,3)
             Y(M1-1,M1) = Z(M1-1,M1) + X3(M1-1,4)
             Y(M1,M1-2) = Z(M1,M1-2) + X3(M1,1)
             Y(M1,M1-1) = Z(M1,M1-1) + X3(M1,2)
             Y(M1,M1) = Z(M1,M1) + X3(M1,3)
      RETURN
      END
```

```
C
      SUBROUTINE INVR6  ( X, L1, L2 )
      IMPLICIT REAL * 8 ( A - H, O - Z )
      REAL * 8 ABSF
C******* THIS ROUTINE TAKES THE INVERSE OF A SYMMETRIC POSITIVE - DEF
C        MATRIX USING A COMPACTED CHOLESKI DECOMPOSITION PROCEDURE ,
C        A FULL DIMENSIONED MATRIX IS REQUIRED BUT ONLY THE LOWER
C        HALF IS USED BY THE 3 ROUTINES DRIVEN BY INVR6
      DIMENSION  X(L1,L1)
          IF ( L2 - 1 ) 600, 10, 20
   10     IF ( DABS ( X(1,1) ) .LT. E-10 ) GO TO 600
               X(1,1) = 1.0 / X(1,1)
          GO TO  500
   20     IF ( L2 - 2 ) 30, 30, 40
   30          S1 = X(1,1) * X(2,2) - X(1,2) * X(2,1)
          IF ( DABS ( S1 ) .LT. E-10 )  GO TO 600
               S1 = 1./ S1
               S = X(1,1)
               X(1,1) = S1 * X(2,2)
               X(2,2) = S1 * S
               X(1,2) = -S1 * X(1,2)
               X(2,1) = -S1 * X(2,1)
          GO TO 500
   40 CALL FIX1 ( X,  L1, L2 )
      CALL DCOM1 ( X, L1, L2 )
      CALL INVLT1 (X, L1, L2 )
      CALL MLTXL ( X, L1, L2 )
      CALL FIX1 ( X, L1, L2 )
          DO 100  I = 2 , L2
               KC = I - 1
          DO 50  J = 1 , KC
               X(J,I) = X(I,J)
   50     CONTINUE
  100     CONTINUE
  500 RETURN
  600 PRINT 601,((X(I,J), J=1,L2), I=1,L2)
  601 FORMAT ( 1H1,30H   SINGULAR MATRIX ENCOUNTERED ,/,2(5X,2E15.7) )
      STOP
      END
```

```
C
      SUBROUTINE DCOM1 ( X , L1 , L2 )
      IMPLICIT REAL * 8 ( A - H, O - Z )
      DIMENSION  X(L1,L1) , T(100)
   10 FORMAT ( /85X,' NON-POSITIVE DEFINITE MATRIX ENCOUNTERED ' )
   15 FORMAT ( /,5X,13E10.3 )
          DO 20  I = 1 , L2
               T(I) = X(I,I)
   20     CONTINUE
          IF ( X(1,1) .LE. 0.0 )  GO TO 4000
               X(1,1) = DSQRT(X(1,1))
               S1 = 1.0 / X(1,1)
          DO 50  I = 2 , L2
               X(I,1) = X(I,1) * S1
   50     CONTINUE
               L2M1 = L2 - 1
C                         . . . . . . . . . . . . . . . . . . . . . .
          DO 200  J = 2 , L2M1
               S = 0.0
               JM1 = J - 1
          DO 120  K = 1 , JM1
               S = S + X(J,K) * X(J,K)
  120     CONTINUE
          IF ( X(J,J) .LE. S )  GO TO 4000
               X(J,J) = DSQRT ( X(J,J) - S )
              S1 = 1.0 / X(J,J)
              JP1 = J + 1
          DO 190  I = JP1 , L2
               S = 0.0
          DO 180  K = 1 , JM1
               S = S + X(I,K) * X(J,K)
  180     CONTINUE
               X(I,J) = ( X(I,J) - S ) * S1
  190     CONTINUE
  200     CONTINUE
C              . . . . . . . . . . . . . . . . . . . . . . . . . . .
               S = 0.0
          DO 250  K = 1, L2M1
               S = S + X(L2,K) * X(L2,K)
  250     CONTINUE
               S = X(L2,L2) - S
          IF ( S .LE. 0.0 )  GO TO 4000
               X(L2,L2) = DSQRT(S)
      RETURN
 4000 PRINT 10
               X(1,1) = T(1)
          DO 400  I = 2 , L2
               K = I - 1
               X(I,I) = T(I)
          DO 350  J = 1 , K
               X(I,J) = X(J,I)
  350     CONTINUE
  400     CONTINUE
          DO 500  I = 1, L2
  500 PRINT 15, ( X(I,J), J=1,L2 )
      STOP
      END
```

```
C
      SUBROUTINE INVLT1 ( X , L1 , L2 )
      IMPLICIT REAL * 8 ( A - H, O - Z )
      DIMENSION X(L1,L1)
          DO 50  I = 1 , L2
                X(I,I) = 1.0 / X(I,I)
   50     CONTINUE
                L2M1 = L2 - 1
          DO 200  J = 1 , L2M1
                JP1 = J+1
          DO 150  I = JP1 , L2
                IM1 = I-1
                SUM = 0.0
          DO 120  K = J , IM1
                SUM = SUM - X(I,K) * X(K,J)
  120     CONTINUE
                X(I,J) = X(I,I) * SUM
  150     CONTINUE
  200     CONTINUE
      RETURN
      END


C
      SUBROUTINE MLTXL ( X , L1 , L2 )
      IMPLICIT REAL * 8 ( A - H, O - Z )
      DIMENSION X(L1,L1)
          DO 200  I = 1 , L2
          DO 150  J = 1 , I
                SUM = 0.0
          DO 100  K = I , L2
                SUM = SUM + X(K,I) * X(K,J)
  100     CONTINUE
                X(I,J) = SUM
  150     CONTINUE
  200     CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE FIX1(D,L1,NK)
      IMPLICIT REAL * 8 ( A - H, O - Z )
      DATA L3,L4/-1,-1/
      DIMENSION D(L1,L1),IFIX(100)
              L3 = L3*L4
         IF (L3.LT.0)GO TO 500
         DO 100 I = 1, NK
              IFIX(I) = 0
              XX = D(I,I)
         IF ( DABS(XX)  .LT.  1.0E+15 )  GO TO 100
         DO 50 L= 1, NK
              D(I,L) = 0.0
              D(L,I) = 0.0
 50      CONTINUE
              D(I,I) = 1.0
              IFIX(I) = 1
100      CONTINUE
         GO TO 900
500      CONTINUE
         DO 600 I = 1,NK
         IF (IFIX(I).EQ.1)  D(I,I) = 0.0
600      CONTINUE
900      CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE INTERP  ( I1, J1, I2, J2, D, ICARD, Z,
     1                     IS, IB, IG, L2, L3, ICX, ICY      )
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION  I1(20), I2(20), J1(20), J2(20), D(20), Z(L2,L3)
      COMMON /INCR/ MX, MY, MXP2, MXP3, MXP4, MXP5, MXP7,
     1              MYP2, MYP3, MYP4, MYP5, MYP7
   10 FORMAT (/,'DATA TYPE NOT PROPERLY DEFINED FOR INTERP')
   20 FORMAT (/, 'ERROR IN INPUT OF DATA FOR DISTRIBUTION')
   30 FORMAT ( / 51H     STATIONS NOT IN PROPER ORDER FOR INTERPOLATION)
              EP = 1.0E-15
C----      ZERO STORAGE BLOCK
          DO 100  J = 1, MYP7
          DO 50  I = 1, MXP7
              Z(I,J) = 0.0
   50     CONTINUE
  100     CONTINUE
          IF ( ICARD  .EQ.  0 )  GO TO 3000
              IPL = 4
          DO 2000  L = 1, ICARD
              IX1 = I1(L) + IPL
              IX2 = I2(L) + IPL
              JY1 = J1(L) + IPL
              JY2 = J2(L) + IPL
          IF ( IX2  .LT.  IX1 )  GO TO 5500
          IF ( JY2  .LT.  JY1 )  GO TO 5500
              ISW = 0
              JSW = 0
          IF ( IX2  .GT.  IX1 )   ISW = 1
          IF ( JY2  .GT.  JY1 )   JSW = 1
C-----    DISTRIBUTE DATA OVER AREA DEFINED BY IX1, JX1, IX2, JX2
C
C-----    CHECK FOR TYPE OF DATA
          IF ( IS  .GT.  0 )    GO TO 700
          IF ( IB  .GT.  0 )    GO TO 200
          IF ( IG  .GT.  0 )    GO TO 300
C-----    TYPE OF DATA NOT DEFINED ---  ERROR
      PRINT 10
          GO TO 6000
C-----    SET UP INTERPOLATION FOR GRID AND BAR TYPE DATA
  200     IF ( ICX  .EQ.  1 )    GO TO 250
          IF ( JSW  .GT.  0 )    GO TO 500
          IF ( D(L)  .EQ.  0.0 )   GO TO 2000
          GO TO 275
  250     IF ( ISW  .GT.  0 )     GO TO 500
          IF ( D(L)  .EQ.  0.0 )     GO TO 2000
  275 PRINT 10
          GO TO 6000
  300     IF ( ISW  .EQ.  1 )   GO TO 400
          IF ( D(L)  .EQ.  0.0 )     GO TO 2000
      PRINT 20
          GO TO 6000
  400     IF ( JSW  .EQ.  1 )     GO TO 450
          IF ( D(L)  .EQ.  0.0 )     GO TO 2000
      PRINT 20
          GO TO 6000
  450         IX1 = IX1 + 1
              JY1 = JY1 + 1
```

```
500       IF ( ICX   .EQ.  1 )    IX1 = IX1 + 1
          IF ( ICY   .EQ.  1 )     JY1 = JY1 + 1
              ISW = 0
              JSW = 0
700       DO 1600  J = JY1, JY2
          DO 1500  I = IX1, IX2
              CMX = 1.0
              CMY = 1.0
          IF ( ISW   .EQ.   0 )     GO TO 900
          IF ( JSW   .EQ.   0 )     GO TO 800
          IF ( J  .EQ.   JY1 )      CMY = 0.5
          IF ( J  .EQ.   JY2 )    CMY = 0.5
          IF ( I  .EQ.   IX1 )      CMX = 0.5
          IF ( I  .EQ.   IX2 )      CMX = 0.5
          GO TO 1000
800       IF ( I   .EQ.   IX1 )     CMX = 0.5
          IF ( I   .EQ.   IX2 )     CMX = 0.5
          GO TO 1000
900       IF ( JSW  .EQ. 0 )    GO TO 1000
          IF ( J  .EQ.   JY1 )      CMY = 0.5
          IF ( J  .EQ.   JY2 )      CMY = 0.5
1000          CMP = CMX * CMY
          IF (DABS(D(L))   .LT.   EP )    D(L) = 0.0
          IF (DABS(Z(I,J))   .LT.   EP )    Z(I,J) = 0.0
              Z(I,J) = Z(I,J) + CMP * D(L)
1500      CONTINUE
1600      CONTINUE
2000      CONTINUE
3000 RETURN
5500 PRINT 30
6000      CONTINUE
     STOP
     END
```

```
C
      SUBROUTINE DATRUS  (NJT, NMEM, HX, HY )
      IMPLICIT REAL * 8 ( A-H, O-Z )
      COMMON /DAT/ CORDX(121), CORDY(121), CORDZ(121), EC(17,17),
     1    EX(17,17), EY(17,17), AE(420), JT(420), KT(420)   ,
     2         AX(17,17),  AY(17,17),  AC(17,17)
      COMMON /INCR/ MX, MY, MXP2, MXP3, MXP4, MXP5, MXP7,
     1              MYP2, MYP3, MYP4, MYP5, MYP7
C
C----- CONSTANTS
C
              MXP1 = MX + 1
              MYP1 = MY + 1
C
C-----    JOINT CO-ORDINATES
          DO 50  NJ = 1, NJT
              CORDX(NJ) = 0.0
              CORDY(NJ) = 0.0
              CORDZ(NJ) = 0.0
   50     CONTINUE
C
          DO 100  J = 1, MYP1
          DO 100  I = 1, MXP1
              NJ = (J-1)*MXP1 + I
              I1 = I + 3
              J1 = J + 3
              CORDX(NJ) = (I-1)*HX
              CORDY(NJ) = (J-1)*HY
  100     CONTINUE
C
C-----    CALCULATE MEMBER CONNECTIVITY AND ITS STIFFNESS
C
C------   MEMBER IN X DIRECTION
              MN = 0
          DO 330  JJ = 1, MYP1
          DO 330  II = 1, MX
              I1 = II + 4
              J1 = JJ + 4
              MN = MN + 1
              JT(MN) = (JJ-1)*MXP1 + II
              KT(MN) = (JJ-1)*MXP1 + (II+1)
          IF ( JJ .NE. 1 )  GO TO 310
              AE(MN) = AX(I1,J1)*EX(I1,J1)
          GO TO 330
  310     IF ( JJ .NE. MYP1 )  GO TO 320
              AE(MN) = AX(I1,J1-1)*EX(I1,J1-1)
          GO TO 330
  320         AE(MN) = AX(I1,J1)*EX(I1,J1) + AX(I1,J1-1)*EX(I1,J1-1)
  330     CONTINUE
C
C-----    MEMBER IN Y DIRECTION
C
          DO 370  II = 1, MXP1
          DO 370  JJ = 1, MY
              I1 = II + 4
              J1 = JJ + 4
              MN = MN + 1
              JT(MN) = (JJ-1)*MXP1 + II
```

```
            KT(MN) = (JJ)*MXP1 + II
        IF ( II .NE. 1 )  GO TO 360
            AE(MN) = AY(I1,J1)*EY(I1,J1)
        GO TO 370
  360   IF ( II .NE. MXP1 )  GO TO 365
            AE(MN) = AY(I1-1,J1)*EY(I1-1,J1)
        GO TO 370
  365       AE(MN) = AY(I1,J1)*EY(I1,J1) + AY(I1-1,J1)*EY(I1-1,J1)
  370   CONTINUE
C
C-----  MEMBER IN XY DIRECTION
C
        DO 380  JJ = 1, MY
        DO 380  II = 1, MX
            I1 = II + 4
            J1 = JJ + 4
            MN = MN + 1
            JT(MN) = (JJ-1)*MXP1 + II
            KT(MN) = (JJ)*MXP1 + (II+1)
            AE(MN) = AC(I1,J1)*EC(I1,J1)
  380   CONTINUE
        DO 390  JJ = 2, MYP1
        DO 390  II = 1, MX
            I1 = II + 4
            J1 = JJ + 4
            MN = MN + 1
            JT(MN) = (JJ-1)*MXP1 + II
            KT(MN) = (JJ-2)*MXP1 + (II+1)
            AE(MN) = AC(I1,J1-1)*EC(I1,J1-1)
  390   CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE INPLAN  ( NJT, NMEM )
      IMPLICIT REAL * 8 ( A - H, O - Z )
C
C     THIS SUBROUTINE CALCULATE  THE RESULTANT FORCES AT EACH JOINT
C     IN THE MEMBRANE MODEL
C
      DIMENSION  SM(6,6), RT(6,6),
     1    SSU(121), SSV(121),           F(363), DF(363), U(363),
     2                 FX(121), FY(121), RZ(121)
      COMMON /STIFF/ SI(363,39)
      COMMON /DAT/ CORDX(121), CORDY(121), CORDZ(121), EC(17,17),
     1    EX(17,17), EY(17,17), AE(420), JT(420), KT(420)   ,
     2          AX(17,17),  AY(17,17),  AC(17,17)
      COMMON /PLANE/ PX(17,17), PY(17,17), W(17,17),
     1    SU(17,17), SV(17,17), UU(17,17), VV(17,17)
      COMMON /INCR/ MX, MY, MXP2, MXP3, MXP4, MXP5, MXP7,
     1               MYP2, MYP3, MYP4, MYP5, MYP7
C
C-----FORMAT STATEMENTS
C
 2090 FORMAT ( 8X, I5, 7X, 1PD10.3, 5X, D10.3, 5X, D10.3 )
 22300FORMAT (// 45H          JT.NO.       X-DISP          Y-DISP
    1             15H       Z-DISP          /      )
 2270 FORMAT (// 28H             MEM.  AXIAL FORCE   /     )
 2280 FORMAT ( 8X, I5, 2X, 1P3D11.3, 3X, 3D11.3 )
 22900FORMAT (///38H     SUPPORT REACTIONS, LOADING NUMBER, I5,
    1   //      45H          JT.NO.       X-REACT         Y-REACT
    2             15H        Z-REACT       , /  )
      REAL * 8 SQRT
               SQRT(X) = DSQRT(X)
               MXP2 = MX + 2
               MYP2 = MY + 2
               MXP1 = MX + 1
               NSIZE = 363
               NWIDE = 39
         DO 100  I = 4, MXP4
         DO 100  J = 4, MYP4
               NJ = (J-4)*MXP1 + (I-3)
               FX(NJ) = PX(I,J)
               FY(NJ) = PY(I,J)
               SSU(NJ) = SU(I,J)
               SSV(NJ) = SV(I,J)
               RZ(NJ) = W(I,J)
               CORDZ(NJ) = W(I,J)
  100    CONTINUE
C---->INITIALIZE ARRAYS
               NDF = 3 * NJT
               MWD = 3*MXP2 + 3
         DO 270  L = 1, NDF
               U(L) = 0.0
               DF(L) = 0.0
         DO 260 K = 1, MWD
               SI(L,K) = 0.0
  260 CONTINUE
  270 CONTINUE
C---->SET UP STRUCTURE STIFFNESS MATRIX
         DO 330 MN = 1, NMEM
```

```
               JMN = JT(MN)
               KMN = KT(MN)
               DX = CORDX(KMN) - CORDX(JMN)
               DY = CORDY(KMN) - CORDY(JMN)
               DZ = CORDZ(KMN) - CORDZ(JMN)
               XL = SQRT(DX*DX + DY*DY + DZ*DZ)
               CX = DX / XL
               CY = DY / XL
               CZ = DZ / XL
               AEM = AE(MN)
      CALL TRSTF ( CX, CY, CZ, XL, AEM, SM, RT )
  310 CALL RTSR ( RT, SM, SM )
C---->ADD MEMBER STIFFNESS MATRIX TO STRUCTURE STIFFNESS MATRIX
               IROW1 = 3 * ( JT(MN) - 1 )
               IROW2 = 3 * ( KT(MN) - 1 )
               JSTRT = 1
               JSTOP = 3
          DO 330  I = 1, 3
          DO 320  JS = JSTRT, JSTOP
               SI(IROW1+I,JS) = SI(IROW1+I,JS) + SM(I,JS+I-1)
               SI(IROW2+I,JS) = SI(IROW2+I,JS) + SM(I+3,JS+I+2)
  320     CONTINUE
               JSTOP = JSTOP - 1
          IF( JT(MN)  .GT.  KT(MN) )  GO TO 324
               IS = IROW2 - IROW1 + 1
          DO 322  JS = 1, 3
               SI(IROW1+I,JS+IS-I) = SM(I,JS+3)
  322 CONTINUE
          GO TO 330
  324          IS = IROW1 - IROW2 + 1
          DO 326  JS = 1, 3
               SI(IROW2+I,JS+IS-I) = SM(I+3,JS)
  326 CONTINUE
  330     CONTINUE
C---->ADD ELASTIC RESTRAINTS AND REVISE FOR SPECIFIED DIEPLACEMENTS
C      ADD ELASTIC RESTRAINTS TO STIFFNESS MATRIX
C      SET UP TEMPORARY LOAD VECTOR TO ACCOUNT FOR SPECIFIED DISPLACEMENT
C
          DO 410  I = 1, NJT
               JN = 3 * (I-1)
               JROW = JN + 3
               SI(JN+1,1) = SI(JN+1,1) + SSU(I)
               SI(JN+2,1) = SI(JN+2,1) + SSV(I)
               SI(JN+3,1) = SI(JN+3,1) + 1.0D+50
  410     CONTINUE
C---->DECOMPOSED STIFFNESS MATRIX
      CALL DCMPBD ( SI, NWIDE, NSIZE, MWD, NDF )
C---->INITIALIZE LOAD VECTORS
  420     DO 430 I = 1, NDF
               F(I) = 0.0
  430     CONTINUE
C---->READ AND ECHO JOINT LOADS
          DO 440 I = 1, NJT
               JROW = 3 * (I-1)
               F(JROW+1) = FX(I) + DF(JROW+1)
               F(JROW+2) = FY(I) + DF(JROW+2)
               F(JROW+3) = DF(JROW+3)
  440     CONTINUE
```

```
      SUBROUTINE TRSTF ( CX, CY, CZ, XL, AE, S, R )
C---->SET UP STIFFNESS AND TRANSFORMATION MATRICES FOR SPACE TRUSS
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION S(6,6), R(6,6)
      DATA ZERO, EP / 0.0D00, 1.0D-06 /, ONE / 1.0D00 /
          DO 100 I = 1, 6
          DO 100 J = 1, 6
               S(I,J) = ZERO
               R(I,J) = ZERO
  100     CONTINUE
               AEOL = AE / XL
               S(1,1) = AEOL
               S(1,4) = - AEOL
               S(4,1) = - AEOL
               S(4,4) = AEOL
               D = DSQRT ( CX * CX + CZ * CZ )
          IF ( D .LE. EP ) GO TO 120
               R(1,1) = CX
               R(1,2) = CY
               R(1,3) = CZ
               R(2,1) = - CX * CZ / D
               R(2,2) = D
               R(2,3) = - CY * CZ / D
               R(3,1) = - CZ / D
               R(3,3) = CX / D
          DO 110 I = 1, 3
          DO 110 J = 1, 3
               R(I+3 ,J+3) = R(I,J)
  110     CONTINUE
      RETURN
  120          R(1,2) = CY
               R(2,1) = - CY
               R(3,3) = ONE
               R(4,5) = CY
               R(5,4) = - CY
               R(6,6) = ONE
      RETURN
      END


C---->SOLVE FOR JOINT DISPLACEMENTS
      CALL SLVBD ( SI, F, U, NWIDE, NSIZE, MWD, NDF )
          DO 510  J = 4, MYP4
          DO 510  I = 4, MXP4
               NJ = (J-4)*MXP1 + (I-3)
               JN = 3*(NJ-1)
               UU(I,J) = U(JN+1)
               VV(I,J) = U(JN+2)
  510     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DCMPBD ( S, NWIDE, NSIZE, MWD, NDF )
C
C---->DECOMPOSE BANDED STIFFNESS MATRIX
C
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION  S(NSIZE, NWIDE)
             EP = 1.0E-6
         DO 120  I = 1, NDF
             II = NDF - I + 1
         IF ( MWD  .LT.  II )  II = MWD
         DO 120  J = 1, II
             IJ = MWD - J
         IF ( I - 1  .LT.  IJ )  IJ = I-1
         IF ( DABS(S(I,J))  .LT.  EP )   S(I,J) = 0.0
             SUM = S(I,J)
         IF ( IJ  .LT.  1 )  GO TO 102
         DO 100  K = 1, IJ
         IF ( DABS(S(I-K,K+1))  .LT.  EP)  S(I-K,K+1) = 0.0
         IF( DABS(S(I-K,K+J))  .LT.  EP )  S(I-K,K+J) = 0.0
         IF( S(I-K,K+1) .EQ. 0.0  .OR.  S(I-K,K+J) .EQ. 0.0 ) GO TO 100
             SUM = SUM - S(I-K,K+1) * S(I-K,K+J)
 100     CONTINUE
 102     IF ( J  .NE.  1 )  GO TO 110
             TEMP = 1.0 / DSQRT(SUM)
             S(I,J) = TEMP
         GO TO 120
 110         S(I,J) = SUM * TEMP
 120     CONTINUE
      RETURN
      END

      SUBROUTINE RTSR ( R, S, X )
C
C-----FORM PRODUCT ( RT * S * R = X )
C
      IMPLICIT REAL * 8 ( A - H, O - Z )
      DATA ZERO / 0.0D00 /
      DIMENSION R(6,6), S(6,6), X(6,6), T(6,6)
C
         DO 20 I = 1, 6
         DO 20 J = 1, 6
             TEMP = ZERO
         DO 10 K = 1, 6
             TEMP = TEMP + R(K,I) * S(K,J)
  10     CONTINUE
             T(I,J) = TEMP
  20     CONTINUE
C
         DO 40 I = 1, 6
         DO 40 J = 1, 6
             TEMP = ZERO
         DO 30 K = 1, 6
             TEMP = TEMP + T(I,K) * R(K,J)
  30     CONTINUE
             X(I,J) = TEMP
  40     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SLVBD ( S, F, U, NWIDE, NSIZE, MWD, NDF )
C
C---->SOLVE FOR DISPLACEMENTS USING DECOMPOSED STIFFNESS MATRIX
C
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION S(NSIZE, NWIDE), F(NSIZE), U(NSIZE)
          DO 120  I = 1, NDF
               J = I - MWD + 1
          IF ( I+1  .LE.  MWD )  J = 1
               SUM = F(I)
          IF ( I  .EQ.  1 )  GO TO 110
               JLIM = I - 1
          DO 100  K = J, JLIM
               SUM = SUM - S(K,I-K+1) * U(K)
 100      CONTINUE
 110           U(I) = SUM * S(I,1)
 120      CONTINUE
          DO 150  II = 1, NDF
               I = NDF + 1 - II
               J = I + MWD - 1
          IF ( J  .GT.  NDF )  J = NDF
               SUM = U(I)
          IF ( I + 1  .GT.  J )  GO TO 140
                KS = I + 1
          DO 130  K = KS, J
               SUM = SUM - S(I,K-I+1) * U(K)
 130      CONTINUE
 140           U(I) = SUM * S(I,1)
 150      CONTINUE
      RETURN
      END
```

APPENDIX E

GUIDE FOR DATA INPUT

IDENTIFICATION OF RUN (2 Alphanumeric Cards Per run)

1 ... 80

1 ... 80

IDENTIFICATION OF PROBLEM (1 Card Each Problem)

Prob. Name		Description of Problem
1 5		11 ... 80

TABLE 1. PROGRAM CONTROL DATA (2 Cards Each Problem)

Number of Cards in Tables

2 NCT2	3 NCT3	4 NCT4
1 5	10	15

Number Increments MX	MY	Increment Length HX	HY	Maximum Poisson's Ratio PR	Plate Thickness THK	Max. No. Iter. NITER	Defl. Closure Tolerance CLOS	Type of Prob. ITYPE
1 5	10	20	30	40	50	55	65	70

TABLE 2. PLATE STIFFNESS (Maximum 20 Cards)

From Sta. IN1	JN1	Thru Sta. IN2	JN2	Bending Stiffness DXN	DYN	Twisting Stiffness CN	Elastic Modulus EXN	EYN
1 5	10	15	20	30	40	50	60	70

TABLE 3. SUPPORTING SPRING STIFFNESS (Maximum 20 Cards)

From Sta. IN1	JN1	Thru Sta. IN2	JN2	Vertical Spring SN	Inplane Springs SUN	SVN	Rotational Springs RXN	RYN
1 5	10	15	20	30	40	50	60	70

TABLE 4. LOADING SYSTEM (Maximum 20 Cards)

From Sta. IN1	JN1	Thru Sta. IN2	JN2	Vertical Loading QN	Inplane Loading PXN	PYN	External Couple TXN	TYN
1 5	10	15	20	30	40	50	60	70

END OF RUN: 1 Blank Card

1	80

General Program Notes

The data cards must be in the proper order for the program to run.

The variable identifications on the guide for data input is consistent with the FORTRAN notations of the program.

A consistent system of units must be used for all input data.

All 5-space words must be right-justified integer numbers.

+	4	3	2	1

All 10-space words are floating-point decimal numbers.

+	2	.	3	4	5	E	+	0	3

The problem name may be alphanumeric.

Table 1. Program Control Data

Number of total cards in Tables 2 to 4 are specified in the first card of this table.

All other constants are in the second card.

A single value of Poisson's ratio is input. For orthotropic plate analysis, the largest value is input.

Maximum number of iterations must be specified. The value of 25 is appropriate.

The deflection closure tolerance is the ratio of the deflection difference to the former value of those two. The value in the range of 5.0 to 0.5% is adequate to insure closure.

Type of problem can be specified as 0, 1, or 2, in which the program will solve the large deflection, membrane, or plane stress problems, respectively.

Table 2. Plate Stiffness

Variables:	DXN	DYN	CN	EXN	EYN
Units:	lb-in.	lb-in.	lb-in.	lb/in^2	lb/in^2

The maximum number of cards in Table 2 is 20.

Data are described by a node coordinate identification as shown in Figure 36.

Bending stiffness is a joint data.

Twisting stiffness and elastic modulus are area data.

Data may be distributed to every joint in an area by specifying the lower left-hand and upper right-hand coordinates. Quarter values are automatically placed at corner joints and half-values are placed at edge joints. For line specification, half-values are placed at the starting and end joints. Data for a single point will be identified by placing the same joint coordinates in both the "From Sta." and "Thru Sta." columns.

Coordinates IN2, JN2 must be equal to or greater than coordinates IN1, JN1.

Data on each card are added to preceding card values.

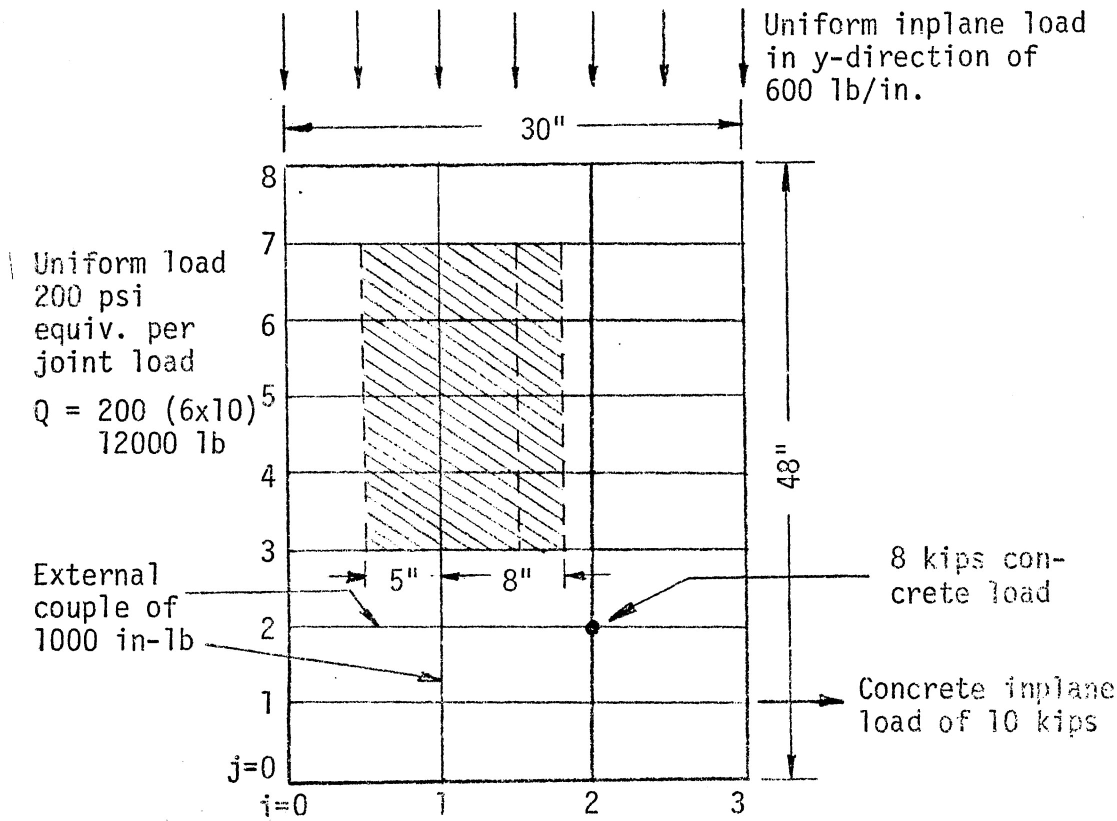
Table 3. Supporting Spring Stiffness

Variables:	SN	SUN	SVN	RXN	RYN
Units:	lb/in	lb/in	lb/in	lb-in/rad	lb-in/rad

The maximum number of cards in Table 3 is 20.

Data are distributed the same way as described in Table 2.

An unyielding support is specified by a supporting spring stiffness greater than 10^{30}.



From Sta. IN1	JN1	Thru Sta. IN2	JN2	DXN	DYN	CN	EXN	EYN
0	0	3	8	1.0E08	1.0E08	8.0E07	3.6E06	3.6E06
				SN	SUN	SVN	RXN	RYN
0	0	3	0	1.0E40	1.0E40	1.0E40	1.0E40	1.0E40
0	0	0	8	1.0E40	1.0E40	1.0E40		
3	0	3	8	1.0E40				
0	8	3	8	1.0E40				
				QN	PXN	PYN	TXN	TYN
2	2	2	2	8.0E03				
1	3	1	7	1.2E04				
2	3	2	7	3.6E03				
3	1	3	1		1.0E04			
0	8	3	8			-6.0E03		
1	2	1	2				1.0E03	1.0E03

Figure 36. Example for Data Input

Table 4. Loading System

Variables:	QN	PXN	PYN	TXN	TYN
Units:	lb	lb	lb	lb-in	lb-in

The maximum number of cards in Table 4 is 20.

Vertical and in-plane loads are applied directly at joints, and data are distributed the same way as described in Table 2.

External couples are applied to the bar elements left and below station specified.

Sign convention for the loading system is the same as for the plate problem.

APPENDIX F

LISTING OF INPUT AND OUTPUT OF EXAMPLE PROBLEMS

```
              EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
                           THIN ELASTIC PLATE
621         SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; DISTRIBUTED LOAD
  1     4     1
  8     8     .30D02    .30D02    .30D00    .70D01    25    .50D-3
  0     0     8     8   .113D09   .113D09   .790D08   .360D07   .360D07
  0     0     0     8   .10D50    .10D50    .10D50
  0     0     8     0   .10D50    .10D50    .10D50
  8     0     8     8   .10D50    .10D50    .10D50
  0     8     8     8   .10D50    .10D50    .10D50
  0     0     8     8   .24D06
622           SIMPLY SUPPORTED PLATE IMMOVABLE EADGES;   CONCENTRATED LOAD
  1     4     1
  8     8     .30D02    .30D02    .30D00    .70D01    25    .10D-2
  0     0     8     8   .113D09   .113D09   .790D08   .360D07   .360D07
  0     0     0     8   .10D50    .10D50    .10D50
  0     0     8     0   .10D50    .10D50    .10D50
  0     8     8     8   .10D50    .10D50    .10D50
  8     0     8     8   .10D50    .10D50    .10D50
  4     4     4     4   .96D06
623           MEMBRANE PROBLEM:  UNIFORM DISTRIBUTED LOAD
  1     4     1
  4     4     .60D02    .60D02    .30D00    .70D01    25    .10D-2     1
  0     0     4     4                                 .360D07   .360D07
  0     0     0     4   .10D50    .10D50    .10D50
  0     0     4     0   .10D50    .10D50    .10D50
  4     0     4     4   .10D50    .10D50    .10D50
  0     4     4     4   .10D50    .10D50    .10D50
  0     0     4     4   .96D6
624           SIMPLY SUPPORTED PLATE MOVABLE EDGES; DISTRIBUTED LOAD
  1     6     1
  4     4     .60D02    .60D02    .30D00    .70D01    35    .10D-2     0
  0     0     4     4   .113D09   .113D09   .790D09   .360E07   .360D07
  0     0     4     0   1.0E50
  0     0     0     4   1.0E50
  4     0     4     4   1.0E50
  0     4     4     4   1.0E50
  2     0     2     4             1.0E50
  0     2     4     2                       1.0E50
  0     0     4     4   1.92D06
626        RECTANGULAR PLATE WITH THREE SIDES FIXED  ANOTHER SIDE FREE
  1     3     11
 10    10     .40D01    .60D01    .30D00    .97D00    50    .10D-1     0
  0     0    10    10   2.50D06   2.50D06   1.75D06   3.00D07   3.00D07
  0     0    10     0   1.0D50    1.0D50    1.0D50              1.0D20
  0     0     0    10   1.0D50    1.0D50    1.0D50    1.0D20
 10     0    10    10   1.0D50    1.0D50    1.0D50    1.0D20
  0     0    10     0   3.51D04
  0     1    10     1   6.48D04
  0     2    10     2   5.76D04
  0     3    10     3   5.04D04
  0     4    10     4   4.32D04
  0     5    10     5   3.60D04

  0     6    10     6   2.88D04
  0     7    10     7   2.16D04
  0     8    10     8   1.44D04
  0     9    10     9   7.20D03
  0    10    10    10   9.00D02
```

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB
621 SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; DISTRIBUTED LOAD

TABLE 1. CONTROL DATA

NUM CARDS TABLE 2	1
NUM CARDS TABLE 3	4
NUM CARDS TABLE 4	1
NUM INCREMENTS MX	8
NUM INCREMENTS MY	8
INCR LENGTH HX	0.300D+02
INCR LENGTH HY	0.300D+02
POISSONS RATIO	0.300D+00
SLAB THICKNESS	0.700D+01
DEFLECTION CLOSURE TOLE	0.500D-03
MAX NUM ITERATION	25
TYPE OF PROBLEM	0
0 FOR LARGE DEFLECTION PROBLEM	
1 FOR MEMBRANE PROBLEM	
2 FOR PLANE STRESS PROBLEM	
3 FOR BUCKLING PROBLEM	

TABLE 2. STIFFNESS DATA FOR PLATE PROBLEM

FROM		THRU		DX	DY	C	EX	EY
0	0	8	8	1.1300D+08	1.1300D+08	7.9000D+07	3.6000D+06	3.6000D+05

TABLE 3. STIFFNESS FOR SUPPORTING SPRINGS

FROM		THRU		S	SU	SV	RX	RY
0	0	0	8	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0
0	0	8	0	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0
8	0	8	8	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0
0	8	8	8	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0

TABLE 4. LOAD DATA

FROM		THRU		Q	PX	PY	TX	TY
0	0	8	8	2.4000D+05	0.0	0.0	0.0	0.0

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB (CONTD)
621 SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; DISTRIBUTED LOAD

TABLE 5. RESULTS; DEFLECTIONS

I	J	WDEFL	UDEFL	VDEFL	TOTREACT
0	0	0.0	0.0	0.0	3.085D+05
1	0	0.0	0.0	0.0	-2.524D+05
2	0	0.0	0.0	0.0	-4.466D+05
3	0	0.0	0.0	0.0	-6.012D+05
4	0	0.0	0.0	0.0	-6.599D+05
5	0	0.0	0.0	0.0	-6.012D+05
6	0	0.0	0.0	0.0	-4.466D+05
7	0	0.0	0.0	0.0	-2.524D+05
8	0	0.0	0.0	0.0	3.085D+05
0	1	0.0	0.0	0.0	-2.524D+05
1	1	1.732D+00	-1.773D-02	-1.773D-02	2.400D+05
2	1	3.013D+00	-1.900D-02	-7.144D-02	2.397D+05
3	1	3.687D+00	-1.087D-02	-1.130D-01	2.391D+05
4	1	3.898D+00	0.0	-1.277D-01	2.388D+05
5	1	3.687D+00	1.087D-02	-1.130D-01	2.391D+05
6	1	3.013D+00	1.900D-02	-7.144D-02	2.397D+05
7	1	1.782D+00	1.773D-02	-1.773D-02	2.400D+05
8	1	0.0	0.0	0.0	-2.524D+05
0	2	0.0	0.0	0.0	-4.466D+05
1	2	3.013D+00	-7.144D-02	-1.900D-02	2.397D+05
2	2	5.173D+00	-7.910D-02	-7.910D-02	2.384D+05
3	2	6.395D+00	-4.684D-02	-1.292D-01	2.389D+05
4	2	6.784D+00	0.0	-1.475D-01	2.405D+05
5	2	6.395D+00	4.684D-02	-1.292D-01	2.389D+05
6	2	5.173D+00	7.910D-02	-7.910D-02	2.384D+05
7	2	3.013D+00	7.144D-02	-1.900D-02	2.397D+05
8	2	0.0	0.0	0.0	-4.466D+05
0	3	0.0	0.0	0.0	-6.012D+05
1	3	3.687D+00	-1.130D-01	-1.087D-02	2.391D+05
2	3	6.395D+00	-1.292D-01	-4.684D-02	2.389D+05
3	3	7.964D+00	-7.826D-02	-7.826D-02	2.451D+05
4	3	8.471D+00	0.0	-9.004D-02	2.499D+05
5	3	7.964D+00	7.826D-02	-7.826D-02	2.451D+05
6	3	6.395D+00	1.292D-01	-4.684D-02	2.389D+05
7	3	3.687D+00	1.130D-01	-1.087D-02	2.391D+05
8	3	0.0	0.0	0.0	-6.012D+05
0	4	0.0	0.0	0.0	-6.599D+05
1	4	3.898D+00	-1.277D-01	0.0	2.388D+05
2	4	6.784D+00	-1.475D-01	0.0	2.405D+05
3	4	8.471D+00	-9.004D-02	0.0	2.499D+05
4	4	9.019D+00	0.0	0.0	2.557D+05
5	4	8.471D+00	9.004D-02	0.0	2.499D+05
6	4	6.784D+00	1.475D-01	0.0	2.405D+05
7	4	3.898D+00	1.277D-01	0.0	2.388D+05
8	4	0.0	0.0	0.0	-6.599D+05

0	5	0.0	0.0	0.0	-6.012D+05
1	5	3.687D+00	-1.130D-01	1.087D-02	2.391D+05
2	5	6.395D+00	-1.292D-01	4.684D-02	2.389D+05
3	5	7.964D+00	-7.826D-02	7.826D-02	2.451D+05
4	5	8.471D+00	0.0	9.004D-02	2.499D+05
5	5	7.964D+00	7.826D-02	7.826D-02	2.451D+05
6	5	6.395D+00	1.292D-01	4.684D-02	2.389D+05
7	5	3.687D+00	1.130D-01	1.087D-02	2.391D+05
8	5	0.0	0.0	0.0	-6.012D+05
0	6	0.0	0.0	0.0	-4.466D+05
1	6	3.013D+00	-7.144D-02	1.900D-02	2.397D+05
2	6	5.173D+00	-7.910D-02	7.910D-02	2.384D+05
3	6	6.395D+00	-4.684D-02	1.292D-01	2.389D+05
4	6	6.784D+00	0.0	1.475D-01	2.405D+05
5	6	6.395D+00	4.684D-02	1.292D-01	2.389D+05
6	6	5.173D+00	7.910D-02	7.910D-02	2.384D+05
7	6	3.013D+00	7.144D-02	1.900D-02	2.397D+05
8	6	0.0	0.0	0.0	-4.466D+05
0	7	0.0	0.0	0.0	-2.524D+05
1	7	1.782D+00	-1.773D-02	1.773D-02	2.400D+05
2	7	3.013D+00	-1.900D-02	7.144D-02	2.397D+05
3	7	3.687D+00	-1.087D-02	1.130D-01	2.391D+05
4	7	3.898D+00	0.0	1.277D-01	2.388D+05
5	7	3.687D+00	1.087D-02	1.130D-01	2.391D+05
6	7	3.013D+00	1.900D-02	7.144D-02	2.397D+05
7	7	1.782D+00	1.773D-02	1.773D-02	2.400D+05
8	7	0.0	0.0	0.0	-2.524D+05
0	8	0.0	0.0	0.0	3.085D+05
1	8	0.0	0.0	0.0	-2.524D+05
2	8	0.0	0.0	0.0	-4.466D+05
3	8	0.0	0.0	0.0	-6.012D+05
4	8	0.0	0.0	0.0	-6.599D+05
5	8	0.0	0.0	0.0	-6.012D+05
6	8	0.0	0.0	0.0	-4.466D+05
7	8	0.0	0.0	0.0	-2.524D+05
8	8	0.0	0.0	0.0	3.085D+05

NUM OF ITERATION = 25

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TABLE 6. BENDING AND TWISTING MOMENTS

I,J		BMX	BMY	TMX	TMY
0	0	0.0	0.0	-3.910D+04	3.910D+04
1	0	1.673D-11	5.576D-11	-6.611D+04	6.611D+04
2	0	2.509D-11	8.364D-11	-4.181D+04	4.181D+04
3	0	2.927D-11	9.758D-11	-1.942D+04	1.942D+04
4	0	3.345D-11	1.115D-10	-5.360D-11	5.360D-11
5	0	2.927D-11	9.758D-11	1.942D+04	-1.942D+04
6	0	2.509D-11	8.364D-11	4.181D+04	-4.181D+04
7	0	1.673D-11	5.576D-11	6.611D+04	-6.611D+04
8	0	0.0	0.0	3.910D+04	-3.910D+04
0	1	1.394D-11	4.182D-12	-6.611D+04	6.611D+04
1	1	-8.987D+04	-8.987D+04	-1.135D+05	1.135D+05
2	1	-1.020D+05	-1.280D+05	-7.422D+04	7.422D+04
3	1	-9.508D+04	-1.404D+05	-3.534D+04	3.534D+04
4	1	-9.099D+04	-1.429D+05	-1.023D-10	1.023D-10
5	1	-9.508D+04	-1.404D+05	3.534D+04	-3.534D+04
6	1	-1.020D+05	-1.280D+05	7.422D+04	-7.422D+04
7	1	-8.987D+04	-8.987D+04	1.135D+05	-1.135D+05
8	1	1.394D-11	4.182D-12	6.611D+04	-6.611D+04
0	2	8.364D-11	2.509D-11	-4.181D+04	4.181D+04
1	2	-1.280D+05	-1.020D+05	-7.422D+04	7.422D+04
2	2	-1.532D+05	-1.532D+05	-5.205D+04	5.205D+04
3	2	-1.475D+05	-1.744D+05	-2.614D+04	2.614D+04
4	2	-1.428D+05	-1.796D+05	-7.796D-11	7.796D-11
5	2	-1.475D+05	-1.744D+05	2.614D+04	-2.614D+04
6	2	-1.532D+05	-1.532D+05	5.205D+04	-5.205D+04
7	2	-1.280D+05	-1.020D+05	7.422D+04	-7.422D+04
8	2	8.364D-11	2.509D-11	4.181D+04	-4.181D+04
0	3	9.758D-11	2.927D-11	-1.942D+04	1.942D+04
1	3	-1.404D+05	-9.508D+04	-3.534D+04	3.534D+04
2	3	-1.744D+05	-1.475D+05	-2.614D+04	2.614D+04
3	3	-1.734D+05	-1.734D+05	-1.370D+04	1.370D+04
4	3	-1.702D+05	-1.812D+05	-6.334D-11	6.334D-11
5	3	-1.734D+05	-1.734D+05	1.370D+04	-1.370D+04
6	3	-1.744D+05	-1.475D+05	2.614D+04	-2.614D+04
7	3	-1.404D+05	-9.508D+04	3.534D+04	-3.534D+04
8	3	5.576D-11	1.673D-11	1.942D+04	-1.942D+04
0	4	6.970D-11	2.091D-11	1.705D-11	-1.705D-11
1	4	-1.429D+05	-9.099D+04	-4.873D-12	4.873D-12
2	4	-1.798D+05	-1.428D+05	-3.411D-11	3.411D-11
3	4	-1.812D+05	-1.702D+05	-7.796D-11	7.796D-11
4	4	-1.788D+05	-1.788D+05	-1.023D-10	1.023D-10
5	4	-1.812D+05	-1.702D+05	-4.873D-12	4.873D-12
6	4	-1.798D+05	-1.428D+05	5.847D-11	-5.847D-11
7	4	-1.429D+05	-9.099D+04	8.771D-11	-8.771D-11
8	4	1.255D-10	3.764D-11	5.116D-11	-5.116D-11
0	5	8.364D-11	2.509D-11	1.942D+04	-1.942D+04

1	5	-1.404D+05	-9.508D+04	3.534D+04	-3.534D+04
2	5	-1.744D+05	-1.475D+05	2.614D+04	-2.614D+04
3	5	-1.734D+05	-1.734D+05	1.370D+04	-1.370D+04
4	5	-1.702D+05	-1.812D+05	-1.949D-11	1.949D-11
5	5	-1.734D+05	-1.734D+05	-1.370D+04	1.370D+04
6	5	-1.744D+05	-1.475D+05	-2.614D+04	2.614D+04
7	5	-1.404D+05	-9.508D+04	-3.534D+04	3.534D+04
8	5	9.758D-11	2.927D-11	-1.942D+04	1.942D+04
0	6	4.182D-11	1.255D-11	4.181D+04	-4.181D+04
1	6	-1.280D+05	-1.020D+05	7.422D+04	-7.422D+04
2	6	-1.532D+05	-1.532D+05	5.205D+04	-5.205D+04
3	6	-1.475D+05	-1.744D+05	2.614D+04	-2.614D+04
4	6	-1.428D+05	-1.798D+05	1.121D-10	-1.121D-10
5	6	-1.475D+05	-1.744D+05	-2.614D+04	2.614D+04
6	6	-1.532D+05	-1.532D+05	-5.205D+04	5.205D+04
7	6	-1.280D+05	-1.020D+05	-7.422D+04	7.422D+04
8	6	5.576D-11	1.673D-11	-4.181D+04	4.181D+04
0	7	4.182D-11	1.255D-11	6.611D+04	-6.611D+04
1	7	-8.987D+04	-8.987D+04	1.135D+05	-1.135D+05
2	7	-1.020D+05	-1.280D+05	7.422D+04	-7.422D+04
3	7	-9.508D+04	-1.404D+05	3.534D+04	-3.534D+04
4	7	-9.099D+04	-1.429D+05	1.852D-10	-1.852D-10
5	7	-9.508D+04	-1.404D+05	-3.534D+04	3.534D+04
6	7	-1.020D+05	-1.280D+05	-7.422D+04	7.422D+04
7	7	-8.987D+04	-8.987D+04	-1.135D+05	1.135D+05
8	7	5.576D-11	1.673D-11	-6.611D+04	6.611D+04
0	8	0.0	0.0	3.910D+04	-3.910D+04
1	8	4.182D-11	1.394D-10	6.611D+04	-6.611D+04
2	8	2.509D-11	8.364D-11	4.181D+04	-4.181D+04
3	8	3.345D-11	1.115D-10	1.942D+04	-1.942D+04
4	8	2.509D-11	8.364D-11	1.145D-10	-1.145D-10
5	8	4.600D-11	1.533D-10	-1.942D+04	1.942D+04
6	8	2.509D-11	8.364D-11	-4.181D+04	4.181D+04
7	8	1.673D-11	5.576D-11	-6.611D+04	6.611D+04
8	8	0.0	0.0	-3.910D+04	3.910D+04

PRCB (CONTD)
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TABLE 7. NORMAL & SHEAR MEMBRANE STRESSES

I	J	MSX	MSY	SHS
1	1	1.967D+03	1.967D+03	-3.449D+02
2	1	3.358D+03	7.548D+03	-1.560D+02
3	1	4.711D+03	1.288D+04	-7.449D+01
4	1	5.479D+03	1.593D+04	-2.183D+01
5	1	5.479D+03	1.593D+04	2.183D+01
6	1	4.711D+03	1.288D+04	7.449D+01
7	1	3.358D+03	7.548D+03	1.560D+02
8	1	1.967D+03	1.967D+03	3.449D+02
1	2	7.548D+03	3.358D+03	-1.568D+02
2	2	7.780D+03	7.780D+03	7.136D+02
3	2	8.186D+03	1.293D+04	6.767D+02
4	2	8.453D+03	1.598D+04	2.596D+02
5	2	8.453D+03	1.598D+04	-2.596D+02
6	2	8.186D+03	1.293D+04	-6.767D+02
7	2	7.780D+03	7.780D+03	-7.136D+02
8	2	7.548D+03	3.358D+03	1.568D+02
1	3	1.288D+04	4.711D+03	-7.552D+01
2	3	1.293D+04	8.186D+03	6.777D+02
3	3	1.273D+04	1.273D+04	6.678D+02
4	3	1.258D+04	1.556D+04	2.596D+02
5	3	1.258D+04	1.556D+04	-2.596D+02
6	3	1.273D+04	1.273D+04	-6.678D+02
7	3	1.293D+04	8.186D+03	-6.777D+02
8	3	1.288D+04	4.711D+03	7.552D+01
1	4	1.593D+04	5.479D+03	-2.224D+01
2	4	1.598D+04	8.453D+03	2.603D+02
3	4	1.556D+04	1.258D+04	2.598D+02
4	4	1.521D+04	1.521D+04	1.020D+02
5	4	1.521D+04	1.521D+04	-1.020D+02
6	4	1.556D+04	1.258D+04	-2.598D+02
7	4	1.598D+04	8.453D+03	-2.603D+02
8	4	1.593D+04	5.479D+03	2.224D+01
1	5	1.593D+04	5.479D+03	2.224D+01
2	5	1.598D+04	8.453D+03	-2.603D+02
3	5	1.556D+04	1.258D+04	-2.598D+02
4	5	1.521D+04	1.521D+04	-1.020D+02
5	5	1.521D+04	1.521D+04	1.020D+02
6	5	1.556D+04	1.258D+04	2.598D+02
7	5	1.598D+04	8.453D+03	2.603D+02
8	5	1.593D+04	5.479D+03	-2.224D+01
1	6	1.288D+04	4.711D+03	7.552D+01
2	6	1.293D+04	8.186D+03	-6.777D+02
3	6	1.273D+04	1.273D+04	-6.678D+02
4	6	1.258D+04	1.556D+04	-2.596D+02
5	6	1.258D+04	1.556D+04	2.596D+02
6	6	1.273D+04	1.273D+04	6.678D+02

7	6	1.293D+04	8.186D+03	6.777D+02
8	6	1.288D+04	4.711D+03	-7.552D+01
1	7	7.548D+03	3.358D+03	1.568D+02
2	7	7.780D+03	7.780D+03	-7.136D+02
3	7	8.186D+03	1.293D+04	-6.767D+02
4	7	8.453D+03	1.598D+04	-2.596D+02
5	7	8.453D+03	1.598D+04	2.596D+02
6	7	8.186D+03	1.293D+04	6.767D+02
7	7	7.780D+03	7.780D+03	7.136D+02
8	7	7.548D+03	3.358D+03	-1.568D+02
1	8	1.967D+03	1.967D+03	3.449D+02
2	8	3.358D+03	7.548D+03	1.560D+02
3	8	4.711D+03	1.288D+04	7.449D+01
4	8	5.479D+03	1.593D+04	2.183D+01
5	8	5.479D+03	1.593D+04	-2.183D+01
6	8	4.711D+03	1.288D+04	-7.449D+01
7	8	3.358D+03	7.548D+03	-1.560D+02
8	8	1.967D+03	1.967D+03	-3.449D+02

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PRCB
622 SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; CONCENTRATED LOAD

TABLE 1. CONTROL DATA

NUM CARDS TABLE 2	1
NUM CARDS TABLE 3	4
NUM CARDS TABLE 4	1
NUM INCREMENTS MX	8
NUM INCREMENTS MY	8
INCR LENGTH HX	0.300D+02
INCR LENGTH HY	0.300D+02
POISSONS RATIO	0.300D+00
SLAB THICKNESS	0.700D+01
DEFLECTION CLOSURE TOLE	0.100D-02
MAX NUM ITERATION	25
TYPE OF PROBLEM	0

0 FOR LARGE DEFLECTION PROBLEM
1 FOR MEMBRANE PROBLEM
2 FOR PLANE STRESS PROBLEM
3 FOR BUCKLING PROBLEM

TABLE 2. STIFFNESS DATA FOR PLATE PROBLEM

FROM		THRU		DX	DY	C	EX	EY
0	0	8	8	1.1300D+08	1.1300D+08	7.9000D+07	3.6000D+06	3.6000D+06

TABLE 3. STIFFNESS FOR SUPPORTING SPRINGS

FROM		THRU		S	SU	SV	RX	RY
0	0	0	8	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0
0	0	8	0	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0
0	8	8	8	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0
8	0	8	8	1.0000D+49	1.0000D+49	1.0000D+49	0.0	0.0

TABLE 4. LOAD DATA

FROM		THRU		Q	PX	PY	TX	TY
4	4	4	4	9.6000D+05	0.0	0.0	0.0	0.0

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB (CONTD)
622 SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; CONCENTRATED LOAD

TABLE 5. RESULTS; DEFLECTIONS

I	J	WDEFL	UDEFL	VDEFL	TOTREACT
0	0	0.0	0.0	0.0	7.872D+04
1	0	0.0	0.0	0.0	-1.723D+04
2	0	0.0	0.0	0.0	-3.890D+04
3	0	0.0	0.0	0.0	-6.371D+04
4	0	0.0	0.0	0.0	-7.852D+04
5	0	0.0	0.0	0.0	-6.371D+04
6	0	0.0	0.0	0.0	-3.890D+04
7	0	0.0	0.0	0.0	-1.723D+04
8	0	0.0	0.0	0.0	7.872D+04
0	1	0.0	0.0	0.0	-1.723D+04
1	1	4.508D-01	1.658D-03	1.658D-03	-5.205D+00
2	1	8.596D-01	1.992D-03	-1.481D-03	-1.018D+00
3	1	1.167D+00	1.317D-03	-5.760D-03	5.391D+00
4	1	1.291D+00	0.0	-7.982D-03	8.017D+00
5	1	1.167D+00	-1.317D-03	-5.760D-03	5.391D+00
6	1	8.596D-01	-1.992D-03	-1.481D-03	-1.018D+00
7	1	4.508D-01	-1.658D-03	1.658D-03	-5.205D+00
8	1	0.0	0.0	0.0	-1.723D+04
0	2	0.0	0.0	0.0	-3.890D+04
1	2	8.596D-01	-1.481D-03	1.992D-03	-1.018D+00
2	2	1.654D+00	-2.570D-03	-2.570D-03	1.125D+01
3	2	2.278D+00	-2.425D-03	-9.346D-03	3.099D+01
4	2	2.550D+00	0.0	-1.348D-02	4.457D+01
5	2	2.278D+00	2.425D-03	-9.346D-03	3.099D+01
6	2	1.654D+00	2.570D-03	-2.570D-03	1.125D+01
7	2	8.596D-01	1.481D-03	1.992D-03	-1.018D+00
8	2	0.0	0.0	0.0	-3.890D+04
0	3	0.0	0.0	0.0	-6.371D+04
1	3	1.167D+00	-5.760D-03	1.317D-03	5.391D+00
2	3	2.278D+00	-9.346D-03	-2.425D-03	3.099D+01
3	3	3.215D+00	-8.489D-03	-8.489D-03	9.971D+01
4	3	3.698D+00	0.0	-1.368D-02	2.148D+02
5	3	3.215D+00	8.489D-03	-8.489D-03	9.971D+01
6	3	2.278D+00	9.346D-03	-2.425D-03	3.099D+01
7	3	1.167D+00	5.760D-03	1.317D-03	5.391D+00
8	3	0.0	0.0	0.0	-6.371D+04
0	4	0.0	0.0	0.0	-7.852D+04
1	4	1.291D+00	-7.982D-03	0.0	8.017D+00
2	4	2.550D+00	-1.348D-02	0.0	4.457D+01
3	4	3.698D+00	-1.368D-02	0.0	2.148D+02
4	4	4.471D+00	0.0	0.0	9.561D+05
5	4	3.698D+00	1.368D-02	0.0	2.148D+02
6	4	2.550D+00	1.348D-02	0.0	4.457D+01
7	4	1.291D+00	7.982D-03	0.0	8.017D+00
8	4	0.0	0.0	0.0	-7.852D+04

0	5	0.0	0.0	0.0	-6.371D+04
1	5	1.167D+00	-5.760D-03	-1.317D-03	5.391D+00
2	5	2.278D+00	-9.346D-03	2.425D-03	3.099D+01
3	5	3.215D+00	-8.489D-03	8.489D-03	9.971D+01
4	5	3.698D+00	0.0	1.368D-02	2.148D+02
5	5	3.215D+00	8.489D-03	8.489D-03	9.971D+01
6	5	2.278D+00	9.346D-03	2.425D-03	3.099D+01
7	5	1.167D+00	5.760D-03	-1.317D-03	5.391D+00
8	5	0.0	0.0	0.0	-6.371D+04
0	6	0.0	0.0	0.0	-3.890D+04
1	6	8.596D-01	-1.481D-03	-1.992D-03	-1.018D+00
2	6	1.654D+00	-2.570D-03	2.570D-03	1.125D+01
3	6	2.278D+00	-2.425D-03	9.346D-03	3.099D+01
4	6	2.550D+00	0.0	1.348D-02	4.457D+01
5	6	2.278D+00	2.425D-03	9.346D-03	3.099D+01
6	6	1.654D+00	2.570D-03	2.570D-03	1.125D+01
7	6	8.596D-01	1.481D-03	-1.992D-03	-1.018D+00
8	6	0.0	0.0	0.0	-3.890D+04
0	7	0.0	0.0	0.0	-1.723D+04
1	7	4.508D-01	1.658D-03	-1.658D-03	-5.205D+00
2	7	8.596D-01	1.992D-03	1.481D-03	-1.018D+00
3	7	1.167D+00	1.317D-03	5.760D-03	5.391D+00
4	7	1.291D+00	0.0	7.982D-03	8.017D+00
5	7	1.167D+00	-1.317D-03	5.760D-03	5.391D+00
6	7	8.596D-01	-1.992D-03	1.481D-03	-1.018D+00
7	7	4.508D-01	-1.658D-03	-1.658D-03	-5.205D+00
8	7	0.0	0.0	0.0	-1.723D+04
0	8	0.0	0.0	0.0	7.872D+04
1	8	0.0	0.0	0.0	-1.723D+04
2	8	0.0	0.0	0.0	-3.890D+04
3	8	0.0	0.0	0.0	-6.371D+04
4	8	0.0	0.0	0.0	-7.852D+04
5	8	0.0	0.0	0.0	-6.371D+04
6	8	0.0	0.0	0.0	-3.890D+04
7	8	0.0	0.0	0.0	-1.723D+04
8	8	0.0	0.0	0.0	7.872D+04

NUM OF ITERATION = 14

PROB (CONTD)
622 SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; CONCENTRATED LOAD

TABLE 6. BENDING AND TWISTING MOMENTS

I,	J	BMX	BMY	TMX	TMY
0	0	0.0	0.0	-9.893D+03	9.893D+03
1	0	3.659D-12	1.220D-11	-1.886D+04	1.886D+04
2	0	6.534D-12	2.178D-11	-1.572D+04	1.572D+04
3	0	1.255D-11	4.182D-11	-9.472D+03	9.472D+03
4	0	1.255D-11	4.182D-11	-4.385D-11	4.385D-11
5	0	1.255D-11	4.182D-11	9.472D+03	-9.472D+03
6	0	6.534D-12	2.178D-11	1.572D+04	-1.572D+04
7	0	3.659D-12	1.220D-11	1.886D+04	-1.886D+04
8	0	0.0	0.0	9.893D+03	-9.893D+03
0	1	1.045D-11	3.136D-12	-1.886D+04	1.886D+04
1	1	-6.856D+03	-6.856D+03	-3.630D+04	3.630D+04
2	1	-1.514D+04	-1.197D+04	-3.112D+04	3.112D+04
3	1	-2.522D+04	-1.405D+04	-1.966D+04	1.966D+04
4	1	-3.234D+04	-1.344D+04	-6.822D-11	6.822D-11
5	1	-2.522D+04	-1.405D+04	1.966D+04	-1.966D+04
6	1	-1.514D+04	-1.197D+04	3.112D+04	-3.112D+04
7	1	-6.856D+03	-6.856D+03	3.630D+04	-3.630D+04
8	1	2.614D-12	7.341D-13	1.886D+04	-1.886D+04
0	2	1.655D-11	4.966D-12	-1.572D+04	1.572D+04
1	2	-1.197D+04	-1.514D+04	-3.112D+04	3.112D+04
2	2	-2.787D+04	-2.787D+04	-2.920D+04	2.920D+04
3	2	-5.073D+04	-3.508D+04	-2.169D+04	2.169D+04
4	2	-7.245D+04	-3.437D+04	-3.411D-11	3.411D-11
5	2	-5.073D+04	-3.508D+04	2.169D+04	-2.169D+04
6	2	-2.787D+04	-2.787D+04	2.920D+04	-2.920D+04
7	2	-1.197D+04	-1.514D+04	3.112D+04	-3.112D+04
8	2	3.398D-11	1.019D-11	1.572D+04	-1.572D+04
0	3	4.182D-11	1.255D-11	-9.472D+03	9.472D+03
1	3	-1.405D+04	-2.522D+04	-1.966D+04	1.966D+04
2	3	-3.508D+04	-5.073D+04	-2.169D+04	2.169D+04
3	3	-7.402D+04	-7.402D+04	-2.250D+04	2.250D+04
4	3	-1.355D+05	-8.350D+04	-3.898D-11	3.898D-11
5	3	-7.402D+04	-7.402D+04	2.250D+04	-2.250D+04
6	3	-3.508D+04	-5.073D+04	2.169D+04	-2.169D+04
7	3	-1.405D+04	-2.522D+04	1.966D+04	-1.966D+04
8	3	2.788D-11	8.364D-12	9.472D+03	-9.472D+03
0	4	4.182D-11	1.255D-11	-1.462D-11	1.462D-11
1	4	-1.344D+04	-3.234D+04	-1.949D-11	1.949D-11
2	4	-3.437D+04	-7.245D+04	-1.462D-11	1.462D-11
3	4	-8.350D+04	-1.355D+05	-2.924D-11	2.924D-11
4	4	-2.523D+05	-2.523D+05	-4.385D-11	4.385D-11
5	4	-8.350D+04	-1.355D+05	-4.873D-12	4.873D-12
6	4	-3.437D+04	-7.245D+04	3.411D-11	-3.411D-11
7	4	-1.344D+04	-3.234D+04	5.360D-11	-5.360D-11
8	4	2.788D-11	8.364D-12	3.654D-11	-3.654D-11
0	5	1.394D-11	4.182D-12	9.472D+03	-9.472D+03

1	5	-1.405D+04	-2.522D+04	1.966D+04	-1.966D+04
2	5	-3.508D+04	-5.073D+04	2.169D+04	-2.169D+04
3	5	-7.402D+04	-7.402D+04	2.250D+04	-2.250D+04
4	5	-1.355D+05	-8.350D+04	9.745D-12	-9.745D-12
5	5	-7.402D+04	-7.402D+04	-2.250D+04	2.250D+04
6	5	-3.508D+04	-5.073D+04	-2.169D+04	2.169D+04
7	5	-1.405D+04	-2.522D+04	-1.966D+04	1.966D+04
8	5	1.394D-11	4.182D-12	-9.472D+03	9.472D+03
0	6	1.045D-11	3.136D-12	1.572D+04	-1.572D+04
1	6	-1.197D+04	-1.514D+04	3.112D+04	-3.112D+04
2	6	-2.787D+04	-2.787D+04	2.920D+04	-2.920D+04
3	6	-5.073D+04	-3.508D+04	2.169D+04	-2.169D+04
4	6	-7.245D+04	-3.437D+04	5.847D-11	-5.847D-11
5	6	-5.073D+04	-3.508D+04	-2.169D+04	2.169D+04
6	6	-2.787D+04	-2.787D+04	-2.920D+04	2.920D+04
7	6	-1.197D+04	-1.514D+04	-3.112D+04	3.112D+04
8	6	1.220D-11	3.659D-12	-1.572D+04	1.572D+04
0	7	1.133D-11	3.398D-12	1.886D+04	-1.886D+04
1	7	-6.856D+03	-6.856D+03	3.630D+04	-3.630D+04
2	7	-1.514D+04	-1.197D+04	3.112D+04	-3.112D+04
3	7	-2.522D+04	-1.405D+04	1.966D+04	-1.966D+04
4	7	-3.234D+04	-1.344D+04	9.745D-11	-9.745D-11
5	7	-2.522D+04	-1.405D+04	-1.966D+04	1.966D+04
6	7	-1.514D+04	-1.197D+04	-3.112D+04	3.112D+04
7	7	-6.856D+03	-6.856D+03	-3.630D+04	3.630D+04
8	7	1.045D-11	3.136D-12	-1.886D+04	1.886D+04
0	8	0.0	0.0	9.893D+03	-9.893D+03
1	8	6.534D-12	2.178D-11	1.886D+04	-1.886D+04
2	8	3.920D-12	1.307D-11	1.572D+04	-1.572D+04
3	8	1.673D-11	5.576D-11	9.472D+03	-9.472D+03
4	8	1.673D-11	5.576D-11	6.334D-11	-6.334D-11
5	8	1.673D-11	5.576D-11	-9.472D+03	9.472D+03
6	8	3.398D-12	1.133D-11	-1.572D+04	1.572D+04
7	8	2.875D-12	9.583D-12	-1.886D+04	1.886D+04
8	8	0.0	0.0	-9.893D+03	9.893D+03

PROB (CONTD)
622 SIMPLY SUPPORTED PLATE IMMOVABLE EADGES; CONCENTRATED LOAD

TABLE 7. NORMAL & SHEAR MEMBRANE STRESSES

I,J		MSX	MSY	SHS
1	1	3.654D+02	3.654D+02	-1.326D+02
2	1	4.646D+02	1.053D+03	-1.866D+02
3	1	5.779D+02	1.818D+03	-1.863D+02
4	1	6.519D+02	2.396D+03	-8.246D+01
5	1	6.519D+02	2.396D+03	8.246D+01
6	1	5.779D+02	1.818D+03	1.863D+02
7	1	4.646D+02	1.053D+03	1.866D+02
8	1	3.654D+02	3.654D+02	1.326D+02
1	2	1.053D+03	4.646D+02	-1.867D+02
2	2	1.027D+03	1.027D+03	-1.726D+02
3	2	9.859D+02	1.856D+03	-2.019D+02
4	2	9.128D+02	2.540D+03	-1.092D+02
5	2	9.128D+02	2.540D+03	1.092D+02
6	2	9.859D+02	1.856D+03	2.019D+02
7	2	1.027D+03	1.027D+03	1.726D+02
8	2	1.053D+03	4.646D+02	1.867D+02
1	3	1.818D+03	5.779D+02	-1.864D+02
2	3	1.856D+03	9.859D+02	-2.019D+02
3	3	1.862D+03	1.862D+03	-2.945D+02
4	3	1.779D+03	2.757D+03	-2.142D+02
5	3	1.779D+03	2.757D+03	2.142D+02
6	3	1.862D+03	1.862D+03	2.945D+02
7	3	1.856D+03	9.859D+02	2.019D+02
8	3	1.818D+03	5.779D+02	1.864D+02
1	4	2.396D+03	6.519D+02	-8.252D+01
2	4	2.540D+03	9.128D+02	-1.093D+02
3	4	2.757D+03	1.779D+03	-2.143D+02
4	4	3.059D+03	3.059D+03	-3.143D+02
5	4	3.059D+03	3.059D+03	3.143D+02
6	4	2.757D+03	1.779D+03	2.143D+02
7	4	2.540D+03	9.128D+02	1.093D+02
8	4	2.396D+03	6.519D+02	8.252D+01
1	5	2.396D+03	6.519D+02	8.252D+01
2	5	2.540D+03	9.128D+02	1.093D+02
3	5	2.757D+03	1.779D+03	2.143D+02
4	5	3.059D+03	3.059D+03	3.143D+02
5	5	3.059D+03	3.059D+03	-3.143D+02
6	5	2.757D+03	1.779D+03	-2.143D+02
7	5	2.540D+03	9.128D+02	-1.093D+02
8	5	2.396D+03	6.519D+02	-8.252D+01
1	6	1.818D+03	5.779D+02	1.864D+02
2	6	1.856D+03	9.859D+02	2.019D+02
3	6	1.862D+03	1.862D+03	2.945D+02
4	6	1.779D+03	2.757D+03	2.142D+02
5	6	1.779D+03	2.757D+03	-2.142D+02
6	6	1.862D+03	1.862D+03	-2.945D+02

7	6	1.856D+03	9.859D+02	-2.019D+02
8	6	1.818D+03	5.779D+02	-1.864D+02
1	7	1.053D+03	4.646D+02	1.867D+02
2	7	1.027D+03	1.027D+03	1.726D+02
3	7	9.859D+02	1.856D+03	2.019D+02
4	7	9.128D+02	2.540D+03	1.092D+02
5	7	9.128D+02	2.540D+03	-1.092D+02
6	7	9.859D+02	1.856D+03	-2.019D+02
7	7	1.027D+03	1.027D+03	-1.726D+02
8	7	1.053D+03	4.646D+02	-1.867D+02
1	8	3.654D+02	3.654D+02	1.326D+02
2	8	4.646D+02	1.053D+03	1.866D+02
3	8	5.779D+02	1.818D+03	1.863D+02
4	8	6.519D+02	2.396D+03	8.246D+01
5	8	6.519D+02	2.396D+03	-8.246D+01
6	8	5.779D+02	1.818D+03	-1.863D+02
7	8	4.646D+02	1.053D+03	-1.866D+02
8	8	3.654D+02	3.654D+02	-1.326D+02

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB
623 MEMBRANE PROBLEM; UNIFORM DISTRIBUTED LOAD

TABLE 1. CONTROL DATA

NUM CARDS TABLE 2	1
NUM CARDS TABLE 3	4
NUM CARDS TABLE 4	1
NUM INCREMENTS MX	4
NUM INCREMENTS MY	4
INCR LENGTH HX	0.600D+02
INCR LENGTH HY	0.600D+02
POISSONS RATIO	0.300D+00
SLAB THICKNESS	0.700D+01
DEFLECTION CLOSURE TOLE	0.100D-02
MAX NUM ITERATION	25
TYPE OF PROBLEM	1

0 FOR LARGE DEFLECTION PROBLEM
1 FOR MEMBRANE PROBLEM
2 FOR PLANE STRESS PROBLEM
3 FOR BUCKLING PROBLEM

TABLE 2. STIFFNESS DATA FOR PLATE PROBLEM

FROM		THRU		DX	DY	C	EX	EY
0	0	4	4	0.0	0.0	0.0	3.60000D+06	3.60000D+06

TABLE 3. STIFFNESS FOR SUPPORTING SPRINGS

FROM		THRU		S	SU	SV	RX	RY
0	0	0	4	1.00000D+49	1.00000D+49	1.00000D+49	0.0	0.0
0	0	4	0	1.00000D+49	1.00000D+49	1.00000D+49	0.0	0.0
4	0	4	4	1.00000D+49	1.00000D+49	1.00000D+49	0.0	0.0
0	4	4	4	1.00000D+49	1.00000D+49	1.00000D+49	0.0	0.0

TABLE 4. LOAD DATA

FROM		THRU		Q	PX	PY	TX	TY
0	0	4	4	9.60000D+05	0.0	0.0	0.0	0.0

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB (CONTD)
623 MEMBRANE PROBLEM; UNIFORM DISTRIBUTED LOAD

TABLE 5. RESULTS; DEFLECTIONS

I	J	WDEFL	UDEFL	VDEFL	TOTREACT
0	0	0.0	0.0	0.0	-3.713D+04
1	0	0.0	0.0	0.0	-6.001D+05
2	0	0.0	0.0	0.0	-1.091D+06
3	0	0.0	0.0	0.0	-6.001D+05
4	0	0.0	0.0	0.0	-3.713D+04
0	1	0.0	0.0	0.0	-6.001D+05
1	1	6.337D+00	-1.250D-01	-1.250D-01	9.249D+05
2	1	7.938D+00	0.0	-2.141D-01	1.071D+06
3	1	6.337D+00	1.250D-01	-1.250D-01	9.249D+05
4	1	0.0	0.0	0.0	-6.001D+05
0	2	0.0	0.0	0.0	-1.091D+06
1	2	7.938D+00	-2.141D-01	0.0	1.071D+06
2	2	1.048D+01	0.0	0.0	1.331D+06
3	2	7.938D+00	2.141D-01	0.0	1.071D+06
4	2	0.0	0.0	0.0	-1.091D+06
0	3	0.0	0.0	0.0	-6.001D+05
1	3	6.337D+00	-1.250D-01	1.250D-01	9.249D+05
2	3	7.938D+00	0.0	2.141D-01	1.071D+06
3	3	6.337D+00	1.250D-01	1.250D-01	9.249D+05
4	3	0.0	0.0	0.0	-6.001D+05
0	4	0.0	0.0	0.0	-3.713D+04
1	4	0.0	0.0	0.0	-6.001D+05
2	4	0.0	0.0	0.0	-1.091D+06
3	4	0.0	0.0	0.0	-6.001D+05
4	4	0.0	0.0	0.0	-3.713D+04

NUM OF ITERATION = 25

PROB (CONTD)
623 MEMBRANE PROBLEM; UNIFORM DISTRIBUTED LOAD

TABLE 7. NORMAL & SHEAR MEMBRANE STRESSES

I,J		MSX	MSY	SHS
1	1	5.664D+03	5.664D+03	-8.372D+02
2	1	9.756D+03	1.833D+04	2.355D+02
3	1	9.756D+03	1.833D+04	-2.355D+02
4	1	5.664D+03	5.664D+03	8.372D+02
1	2	1.833D+04	9.756D+03	2.384D+02
2	2	1.767D+04	1.767D+04	3.579D+02
3	2	1.767D+04	1.767D+04	-3.579D+02
4	2	1.833D+04	9.756D+03	-2.384D+02
1	3	1.833D+04	9.756D+03	-2.384D+02
2	3	1.767D+04	1.767D+04	-3.579D+02
3	3	1.767D+04	1.767D+04	3.579D+02
4	3	1.833D+04	9.756D+03	2.384D+02
1	4	5.664D+03	5.664D+03	8.372D+02
2	4	9.756D+03	1.833D+04	-2.355D+02
3	4	9.756D+03	1.833D+04	2.355D+02
4	4	5.664D+03	5.664D+03	-8.372D+02

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB
626 RECTANGULAR PLATE WITH THREE SIDES FIXED ANOTHER SIDE FREE

TABLE 1. CONTROL DATA

NUM CARDS TABLE 2	1
NUM CARDS TABLE 3	3
NUM CARDS TABLE 4	11
NUM INCREMENTS MX	10
NUM INCREMENTS MY	10
INCR LENGTH HX	0.4000D+01
INCR LENGTH HY	0.6000D+01
POISSONS RATIO	0.3000D+00
SLAB THICKNESS	0.9700D+00
DEFLECTION CLOSURE TOLE	0.1000D-01
MAX NUM ITERATION	50
TYPE OF PROBLEM	0

0 FOR LARGE DEFLECTION PROBLEM
1 FOR MEMBRANE PROBLEM
2 FOR PLANE STRESS PROBLEM
3 FOR BUCKLING PROBLEM

TABLE 2. STIFFNESS DATA FOR PLATE PROBLEM

FROM		THRU		DX	DY	C	EX	EY
0	0	10	10	2.50000D+06	2.50000D+06	1.75000D+06	3.00000D+07	3.00000D+07

TABLE 3. STIFFNESS FOR SUPPORTING SPRINGS

FROM		THRU		S	SU	SV	RX	RY
0	0	10	0	1.00000D+50	1.00000D+50	1.00000D+50	0.0	1.00000D+20
0	0	0	10	1.00000D+50	1.00000D+50	1.00000D+50	1.00000D+20	0.0
10	0	10	10	1.00000D+50	1.00000D+50	1.00000D+50	1.00000D+20	0.0

TABLE 4. LOAD DATA

FROM		THRU		Q	PX	PY	TX	TY
0	0	10	0	3.51000D+04	0.0	0.0	0.0	0.0
0	1	10	1	6.48000D+04	0.0	0.0	0.0	0.0
0	2	10	2	5.76000D+04	0.0	0.0	0.0	0.0
0	3	10	3	5.04000D+04	0.0	0.0	0.0	0.0
0	4	10	4	4.32000D+04	0.0	0.0	0.0	0.0
0	5	10	5	3.60000D+04	0.0	0.0	0.0	0.0
0	6	10	6	2.88000D+04	0.0	0.0	0.0	0.0
0	7	10	7	2.16000D+04	0.0	0.0	0.0	0.0
0	8	10	8	1.44000D+04	0.0	0.0	0.0	0.0
0	9	10	9	7.20000D+03	0.0	0.0	0.0	0.0
0	10	10	10	9.00000D+02	0.0	0.0	0.0	0.0

EXAMPLE PROBLEM FOR PROGRAM NON LINEAR SLAB
THIN ELASTIC PLATE

PROB (CONTD)
626 RECTANGULAR PLATE WITH THREE SIDES FIXED ANOTHER SIDE FREE

TABLE 5. RESULTS: DEFLECTIONS

I	J	WDEFL	UDEFL	VDEFL	TOTREACT
0	0	0.0	0.0	0.0	0.0
1	0	0.0	0.0	0.0	0.0
2	0	0.0	0.0	0.0	1.040D+04
3	0	0.0	0.0	0.0	2.081D+04
4	0	0.0	0.0	0.0	2.764D+04
5	0	0.0	0.0	0.0	2.996D+04
6	0	0.0	0.0	0.0	2.764D+04
7	0	0.0	0.0	0.0	2.081D+04
8	0	0.0	0.0	0.0	1.040D+04
9	0	0.0	0.0	0.0	0.0
10	0	0.0	0.0	0.0	0.0
0	1	0.0	0.0	0.0	0.0
1	1	0.0	9.497D-04	1.064D-03	4.603D+04
2	1	0.0	1.670D-03	2.036D-03	-2.846D+04
3	1	0.0	1.855D-03	4.860D-03	-7.461D+04
4	1	0.0	1.220D-03	8.066D-03	-1.014D+05
5	1	0.0	0.0	9.434D-03	-1.106D+05
6	1	0.0	-1.220D-03	8.066D-03	-1.014D+05
7	1	0.0	-1.855D-03	4.860D-03	-7.461D+04
8	1	0.0	-1.670D-03	2.036D-03	-2.846D+04
9	1	0.0	-9.497D-04	1.064D-03	4.603D+04
10	1	0.0	0.0	0.0	0.0
0	2	0.0	0.0	0.0	5.264D+04
1	2	0.0	3.444D-03	1.853D-03	-1.347D+05
2	2	2.246D-01	7.694D-04	-1.952D-04	5.193D+04
3	2	4.494D-01	-2.162D-03	-6.310D-03	5.301D+04
4	2	5.970D-01	-2.074D-03	-1.297D-02	5.404D+04
5	2	6.472D-01	0.0	-1.534D-02	5.448D+04
6	2	5.970D-01	2.074D-03	-1.297D-02	5.404D+04
7	2	4.494D-01	2.162D-03	-6.310D-03	5.301D+04
8	2	2.246D-01	-7.694D-04	-1.952D-04	5.193D+04
9	2	0.0	-3.444D-03	1.853D-03	-1.347D+05
10	2	0.0	0.0	0.0	5.264D+04
0	3	0.0	0.0	0.0	8.363D+04
1	3	0.0	7.941D-03	6.772D-04	-2.160D+05
2	3	3.568D-01	7.207D-04	-2.733D-03	4.612D+04
3	3	7.339D-01	-8.082D-03	-1.069D-02	4.908D+04
4	3	9.925D-01	-7.585D-03	-1.870D-02	5.326D+04
5	3	1.083D+00	0.0	-2.198D-02	5.514D+04
6	3	9.925D-01	7.585D-03	-1.870D-02	5.326D+04
7	3	7.339D-01	8.082D-03	-1.069D-02	4.908D+04
8	3	3.568D-01	-7.207D-04	-2.733D-03	4.612D+04
9	3	0.0	-7.941D-03	6.772D-04	-2.160D+05
10	3	0.0	0.0	0.0	8.363D+04
0	4	0.0	0.0	0.0	9.240D+04

1	4	0.0	1.046D-02	-1.902D-03	-2.362D+05
2	4	3.942D-01	2.289D-03	-5.545D-03	3.991D+04
3	4	8.243D-01	-9.408D-03	-1.060D-02	4.323D+04
4	4	1.128D+00	-9.588D-03	-1.528D-02	4.960D+04
5	4	1.236D+00	0.0	-1.720D-02	5.209D+04
6	4	1.128D+00	9.588D-03	-1.528D-02	4.960D+04
7	4	8.243D-01	9.403D-03	-1.060D-02	4.323D+04
8	4	3.942D-01	-2.289D-03	-5.545D-03	3.991D+04
9	4	0.0	-1.046D-02	-1.902D-03	-2.362D+05
10	4	0.0	0.0	0.0	9.240D+04
0	5	0.0	0.0	0.0	8.857D+04
1	5	0.0	1.029D-02	-4.332D-03	-2.190D+05
2	5	3.779D-01	3.283D-03	-8.041D-03	3.344D+04
3	5	7.989D-01	-8.063D-03	-1.055D-02	3.557D+04
4	5	1.101D+00	-8.854D-03	-1.210D-02	4.112D+04
5	5	1.209D+00	0.0	-1.265D-02	4.331D+04
6	5	1.101D+00	8.854D-03	-1.210D-02	4.112D+04
7	5	7.989D-01	8.063D-03	-1.055D-02	3.557D+04
8	5	3.779D-01	-3.283D-03	-8.041D-03	3.344D+04
9	5	0.0	-1.029D-02	-4.332D-03	-2.190D+05
10	5	0.0	0.0	0.0	8.857D+04
0	6	0.0	0.0	0.0	7.858D+04
1	6	0.0	8.542D-03	-5.875D-03	-1.840D+05
2	6	3.353D-01	3.309D-03	-9.751D-03	2.673D+04
3	6	7.155D-01	-6.050D-03	-1.103D-02	2.785D+04
4	6	9.918D-01	-7.055D-03	-1.097D-02	3.140D+04
5	6	1.091D+00	0.0	-1.080D-02	3.290D+04
6	6	9.918D-01	7.055D-03	-1.097D-02	3.140D+04
7	6	7.155D-01	6.050D-03	-1.103D-02	2.785D+04
8	6	3.353D-01	-3.309D-03	-9.751D-03	2.673D+04
9	6	0.0	-8.542D-03	-5.875D-03	-1.840D+05
10	6	0.0	0.0	0.0	7.858D+04
0	7	0.0	0.0	0.0	6.570D+04
1	7	0.0	6.290D-03	-6.524D-03	-1.438D+05
2	7	2.803D-01	2.831D-03	-1.068D-02	1.987D+04
3	7	6.047D-01	-4.058D-03	-1.186D-02	2.052D+04
4	7	8.431D-01	-5.038D-03	-1.149D-02	2.237D+04
5	7	9.294D-01	0.0	-1.116D-02	2.322D+04
6	7	8.431D-01	5.038D-03	-1.149D-02	2.237D+04
7	7	6.047D-01	4.058D-03	-1.186D-02	2.052D+04
8	7	2.803D-01	-2.831D-03	-1.068D-02	1.987D+04
9	7	0.0	-6.290D-03	-6.524D-03	-1.438D+05
10	7	0.0	0.0	0.0	6.570D+04
0	8	0.0	0.0	0.0	5.196D+04
1	8	0.0	4.270D-03	-6.538D-03	-1.051D+05
2	8	2.217D-01	2.344D-03	-1.106D-02	1.309D+04
3	8	4.852D-01	-2.231D-03	-1.289D-02	1.347D+04
4	8	6.822D-01	-3.148D-03	-1.303D-02	1.432D+04
5	8	7.541D-01	0.0	-1.289D-02	1.471D+04
6	8	6.822D-01	3.148D-03	-1.303D-02	1.432D+04
7	8	4.852D-01	2.231D-03	-1.289D-02	1.347D+04
8	8	2.217D-01	-2.344D-03	-1.106D-02	1.309D+04
9	8	0.0	-4.270D-03	-6.538D-03	-1.051D+05
10	8	0.0	0.0	0.0	5.196D+04
0	9	0.0	0.0	0.0	3.961D+04
1	9	0.0	3.119D-03	-6.256D-03	-7.820D+04
2	9	1.690D-01	2.514D-03	-1.139D-02	6.466D+03
3	9	3.779D-01	-3.929D-04	-1.392D-02	6.703D+03
4	9	5.383D-01	-1.460D-03	-1.475D-02	7.046D+03
5	9	5.976D-01	0.0	-1.488D-02	7.175D+03
6	9	5.383D-01	1.460D-03	-1.475D-02	7.046D+03

7	9	3.779D-01	3.929D-04	-1.392D-02	6.703D+03
8	9	1.690D-01	-2.514D-03	-1.139D-02	6.486D+03
9	9	0.0	-3.119D-03	-6.256D-03	-7.820D+04
10	9	0.0	0.0	0.0	3.961D+04
0	10	0.0	0.0	0.0	1.380D+04
1	10	0.0	4.720D-03	-7.247D-03	-1.451D+04
2	10	1.295D-01	5.233D-03	-1.197D-02	8.232D+02
3	10	3.042D-01	2.575D-03	-1.466D-02	8.527D+02
4	10	4.445D-01	4.276D-04	-1.562D-02	8.903D+02
5	10	4.974D-01	0.0	-1.581D-02	8.987D+02
6	10	4.445D-01	-4.276D-04	-1.562D-02	8.903D+02
7	10	3.042D-01	-2.575D-03	-1.466D-02	8.527D+02
8	10	1.295D-01	-5.233D-03	-1.197D-02	8.232D+02
9	10	0.0	-4.720D-03	-7.247D-03	-1.451D+04
10	10	0.0	0.0	0.0	1.380D+04

NUM OF ITERATION = 6

PROB (CONTD)
626 RECTANGULAR PLATE WITH THREE SIDES FIXED ANOTHER SIDE FREE

TABLE 6. BENDING AND TWISTING MOMENTS

I	J	BMX	BMY	TMX	TMY
0	0	0.0	0.0	0.0	0.0
1	0	0.0	0.0	0.0	0.0
2	0	0.0	0.0	0.0	0.0
3	0	0.0	0.0	0.0	0.0
4	0	0.0	0.0	0.0	0.0
5	0	0.0	0.0	0.0	0.0
6	0	0.0	0.0	0.0	0.0
7	0	0.0	0.0	0.0	0.0
8	0	0.0	0.0	0.0	0.0
9	0	0.0	0.0	0.0	0.0
10	0	0.0	0.0	0.0	0.0
0	1	0.0	0.0	0.0	0.0
1	1	0.0	0.0	-4.094D+03	4.094D+03
2	1	4.679D+03	1.560D+04	-8.193D+03	8.193D+03
3	1	9.363D+03	3.121D+04	-6.789D+03	6.789D+03
4	1	1.244D+04	4.146D+04	-3.604D+03	3.604D+03
5	1	1.348D+04	4.494D+04	-1.012D-12	1.012D-12
6	1	1.244D+04	4.146D+04	3.604D+03	-3.604D+03
7	1	9.363D+03	3.121D+04	6.789D+03	-6.789D+03
8	1	4.679D+03	1.560D+04	8.193D+03	-8.193D+03
9	1	0.0	0.0	4.094D+03	-4.094D+03
10	1	0.0	0.0	0.0	0.0
0	2	0.0	0.0	0.0	0.0
1	2	3.505D+04	1.053D+04	-6.505D+03	6.505D+03
2	2	-1.885D+03	-6.402D+03	-1.338D+04	1.338D+04
3	2	-1.551D+04	-1.508D+04	-1.159D+04	1.159D+04
4	2	-1.942D+04	-1.856D+04	-6.358D+03	6.358D+03
5	2	-2.009D+04	-1.940D+04	-2.530D-13	2.530D-13
6	2	-1.942D+04	-1.856D+04	6.358D+03	-6.358D+03
7	2	-1.551D+04	-1.508D+04	1.159D+04	-1.159D+04
8	2	-1.885D+03	-6.402D+03	1.338D+04	-1.338D+04
9	2	3.505D+04	1.053D+04	6.505D+03	-6.505D+03
10	2	0.0	0.0	0.0	0.0
0	3	0.0	0.0	0.0	0.0
1	3	5.575D+04	1.673D+04	-3.092D+03	3.092D+03
2	3	1.187D+03	-5.637D+03	-6.835D+03	6.835D+03
3	3	-2.255D+04	-1.903D+04	-6.586D+03	6.586D+03
4	3	-3.173D+04	-2.595D+04	-3.892D+03	3.892D+03
5	3	-3.408D+04	-2.808D+04	-4.048D-12	4.048D-12
6	3	-3.173D+04	-2.595D+04	3.892D+03	-3.892D+03
7	3	-2.255D+04	-1.903D+04	6.586D+03	-6.586D+03
8	3	1.187D+03	-5.637D+03	6.835D+03	-6.835D+03
9	3	5.575D+04	1.673D+04	3.092D+03	-3.092D+03
10	3	0.0	0.0	0.0	0.0
0	4	0.0	0.0	0.0	0.0
1	4	6.160D+04	1.848D+04	-3.844D+02	3.844D+02
2	4	4.491D+03	-2.047D+03	-1.185D+03	1.185D+03

3	4	-2.215D+04	-1.398D+04	-1.598D+03	1.598D+03
4	4	-3.400D+04	-2.045D+04	-1.125D+03	1.125D+03
5	4	-3.737D+04	-2.253D+04	-8.095D-12	8.095D-12
6	4	-3.400D+04	-2.045D+04	1.125D+03	-1.125D+03
7	4	-2.215D+04	-1.398D+04	1.598D+03	-1.598D+03
8	4	4.491D+03	-2.047D+03	1.185D+03	-1.135D+03
9	4	6.160D+04	1.848D+04	3.844D+02	-3.844D+02
10	4	0.0	0.0	0.0	0.0
0	5	0.0	0.0	0.0	0.0
1	5	5.905D+04	1.771D+04	1.074D+03	-1.074D+03
2	5	6.186D+03	1.931D+02	1.984D+03	-1.984D+03
3	5	-1.976D+04	-9.590D+03	1.408D+03	-1.408D+03
4	5	-3.205D+04	-1.484D+04	6.480D+02	-6.480D+02
5	5	-3.573D+04	-1.654D+04	7.589D-12	-7.589D-12
6	5	-3.205D+04	-1.484D+04	-6.480D+02	6.480D+02
7	5	-1.976D+04	-9.590D+03	-1.408D+03	1.408D+03
8	5	6.186D+03	1.931D+02	-1.984D+03	1.984D+03
9	5	5.905D+04	1.771D+04	-1.074D+03	1.074D+03
10	5	0.0	0.0	0.0	0.0
0	6	0.0	0.0	0.0	0.0
1	6	5.239D+04	1.572D+04	1.779D+03	-1.779D+03
2	6	6.769D+03	1.250D+03	3.541D+03	-3.541D+03
3	6	-1.683D+04	-6.786D+03	2.926D+03	-2.926D+03
4	6	-2.844D+04	-1.101D+04	1.564D+03	-1.564D+03
5	6	-3.199D+04	-1.235D+04	7.842D-12	-7.842D-12
6	6	-2.844D+04	-1.101D+04	-1.564D+03	1.564D+03
7	6	-1.683D+04	-6.786D+03	-2.926D+03	2.926D+03
8	6	6.769D+03	1.250D+03	-3.541D+03	3.541D+03
9	6	5.239D+04	1.572D+04	-1.779D+03	1.779D+03
10	6	0.0	0.0	0.0	0.0
0	7	0.0	0.0	0.0	0.0
1	7	4.380D+04	1.314D+04	2.071D+03	-2.071D+03
2	7	6.808D+03	1.813D+03	4.199D+03	-4.199D+03
3	7	-1.361D+04	-4.625D+03	3.573D+03	-3.573D+03
4	7	-2.402D+04	-7.978D+03	1.947D+03	-1.947D+03
5	7	-2.725D+04	-9.029D+03	-5.566D-12	5.566D-12
6	7	-2.402D+04	-7.978D+03	-1.947D+03	1.947D+03
7	7	-1.361D+04	-4.625D+03	-3.573D+03	3.573D+03
8	7	6.808D+03	1.813D+03	-4.199D+03	4.199D+03
9	7	4.380D+04	1.314D+04	-2.071D+03	2.071D+03
10	7	0.0	0.0	0.0	0.0
0	8	0.0	0.0	0.0	0.0
1	8	3.464D+04	1.039D+04	2.029D+03	-2.029D+03
2	8	6.654D+03	2.371D+03	4.134D+03	-4.134D+03
3	8	-1.014D+04	-2.274D+03	3.527D+03	-3.527D+03
4	8	-1.919D+04	-4.682D+03	1.914D+03	-1.914D+03
5	8	-2.207D+04	-5.428D+03	5.060D-13	-5.060D-13
6	8	-1.919D+04	-4.682D+03	-1.914D+03	1.914D+03
7	8	-1.014D+04	-2.274D+03	-3.527D+03	3.527D+03
8	8	6.654D+03	2.371D+03	-4.134D+03	4.134D+03
9	8	3.464D+04	1.039D+04	-2.029D+03	2.029D+03
10	8	0.0	0.0	0.0	0.0
0	9	0.0	0.0	0.0	0.0
1	9	2.641D+04	7.923D+03	1.682D+03	-1.682D+03
2	9	6.496D+03	2.777D+03	3.300D+03	-3.300D+03
3	9	-6.868D+03	6.254D+01	2.652D+03	-2.652D+03
4	9	-1.475D+04	-1.262D+03	1.379D+03	-1.379D+03
5	9	-1.737D+04	-1.663D+03	-1.518D-12	1.518D-12
6	9	-1.475D+04	-1.262D+03	-1.379D+03	1.379D+03
7	9	-6.868D+03	6.254D+01	-2.652D+03	2.652D+03
8	9	6.496D+03	2.777D+03	-3.300D+03	3.300D+03

9	9	2.641D+04	7.923D+03	-1.682D+03	1.682D+03
10	9	0.0	0.0	0.0	0.0
0	10	0.0	0.0	3.982D+02	-3.982D+02
1	10	9.203D+03	2.331D-12	9.998D+02	-9.998D+02
2	10	3.217D+03	-4.377D-12	3.358D+02	-3.358D+02
3	10	-2.445D+03	-7.901D-12	1.729D+02	-1.729D+02
4	10	-6.214D+03	-9.379D-12	4.470D+01	-4.470D+01
5	10	-7.521D+03	-1.529D-11	-3.795D-12	3.795D-12
6	10	-6.214D+03	-1.359D-11	-4.470D+01	4.470D+01
7	10	-2.445D+03	-5.684D-13	-1.729D+02	1.729D+02
8	10	3.217D+03	-4.775D-12	-3.358D+02	3.358D+02
9	10	9.203D+03	1.194D-12	-9.998D+02	9.998D+02
10	10	0.0	0.0	-3.982D+02	3.982D+02

PROB (CONTD)
626 RECTANGULAR PLATE WITH THREE SIDES FIXED ANOTHER SIDE FREE

TABLE 7. NORMAL & SHEAR MEMBRANE STRESSES

I	J	MSX	MSY	SHS
1	1	4.791D+03	4.098D+03	-2.099D+03
2	1	5.524D+03	9.410D+03	-3.361D+03
3	1	6.451D+03	1.918D+04	-6.398D+03
4	1	8.039D+03	3.474D+04	-6.503D+03
5	1	9.392D+03	4.660D+04	-2.700D+03
6	1	9.392D+03	4.660D+04	2.700D+03
7	1	8.039D+03	3.474D+04	6.503D+03
8	1	6.451D+03	1.918D+04	6.398D+03
9	1	5.524D+03	9.410D+03	3.361D+03
10	1	4.791D+03	4.098D+03	2.099D+03
1	2	1.875D+04	7.600D+03	-5.663D+03
2	2	1.673D+04	5.165D+03	-5.177D+03
3	2	1.715D+04	1.621D+04	-6.865D+03
4	2	1.890D+04	3.733D+04	-6.211D+03
5	2	2.019D+04	5.273D+04	-2.469D+03
6	2	2.019D+04	5.273D+04	2.469D+03
7	2	1.890D+04	3.733D+04	6.211D+03
8	2	1.715D+04	1.621D+04	6.865D+03
9	2	1.673D+04	5.165D+03	5.177D+03
10	2	1.875D+04	7.600D+03	5.663D+03
1	3	4.593D+04	1.085D+04	-6.835D+03
2	3	4.769D+04	6.342D+03	-4.830D+03
3	3	5.081D+04	1.662D+04	-2.908D+03
4	3	5.558D+04	4.043D+04	-1.496D+03
5	3	5.871D+04	5.847D+04	-4.716D+02
6	3	5.871D+04	5.847D+04	4.716D+02
7	3	5.558D+04	4.043D+04	1.496D+03
8	3	5.081D+04	1.662D+04	2.908D+03
9	3	4.769D+04	6.342D+03	4.830D+03
10	3	4.593D+04	1.085D+04	6.835D+03
1	4	7.364D+04	1.567D+04	-5.638D+02
2	4	7.766D+04	9.975D+03	2.463D+03
3	4	8.222D+04	1.953D+04	5.255D+03
4	4	8.765D+04	4.036D+04	5.367D+03
5	4	9.046D+04	5.632D+04	2.225D+03
6	4	9.046D+04	5.632D+04	-2.225D+03
7	4	8.765D+04	4.036D+04	-5.367D+03
8	4	8.222D+04	1.953D+04	-5.255D+03
9	4	7.766D+04	9.975D+03	-2.463D+03
10	4	7.364D+04	1.567D+04	5.638D+02
1	5	8.346D+04	1.900D+04	7.837D+03
2	5	8.684D+04	1.381D+04	9.692D+03
3	5	8.957D+04	2.099D+04	1.108D+04
4	5	9.307D+04	3.631D+04	9.245D+03
5	5	9.436D+04	4.793D+04	3.627D+03
6	5	9.436D+04	4.793D+04	-3.627D+03
7	5	9.307D+04	3.631D+04	-9.245D+03

8	5	8.957D+04	2.099D+04	-1.108D+04
9	5	8.684D+04	1.381D+04	-9.692D+03
10	5	8.346D+04	1.900D+04	-7.837D+03
1	6	7.632D+04	1.907D+04	1.406D+04
2	6	7.819D+04	1.550D+04	1.391D+04
3	6	7.904D+04	1.983D+04	1.339D+04
4	6	8.058D+04	2.968D+04	1.018D+04
5	6	8.085D+04	3.714D+04	3.848D+03
6	6	8.085D+04	3.714D+04	-3.848D+03
7	6	8.058D+04	2.968D+04	-1.018D+04
8	6	7.904D+04	1.983D+04	-1.339D+04
9	6	7.819D+04	1.550D+04	-1.391D+04
10	6	7.632D+04	1.907D+04	-1.406D+04
1	7	6.059D+04	1.658D+04	1.718D+04
2	7	6.117D+04	1.466D+04	1.565D+04
3	7	6.105D+04	1.673D+04	1.382D+04
4	7	6.144D+04	2.206D+04	9.916D+03
5	7	6.130D+04	2.623D+04	3.644D+03
6	7	6.130D+04	2.623D+04	-3.644D+03
7	7	6.144D+04	2.206D+04	-9.916D+03
8	7	6.105D+04	1.673D+04	-1.382D+04
9	7	6.117D+04	1.466D+04	-1.565D+04
10	7	6.059D+04	1.658D+04	-1.718D+04
1	8	4.353D+04	1.304D+04	1.781D+04
2	8	4.324D+04	1.226D+04	1.581D+04
3	8	4.249D+04	1.242D+04	1.339D+04
4	8	4.191D+04	1.429D+04	9.226D+03
5	8	4.139D+04	1.601D+04	3.330D+03
6	8	4.139D+04	1.601D+04	-3.330D+03
7	8	4.191D+04	1.429D+04	-9.226D+03
8	8	4.249D+04	1.242D+04	-1.339D+04
9	8	4.324D+04	1.226D+04	-1.581D+04
10	8	4.353D+04	1.304D+04	-1.781D+04
1	9	3.071D+04	9.933D+03	1.676D+04
2	9	2.955D+04	8.945D+03	1.486D+04
3	9	2.704D+04	7.276D+03	1.151D+04
4	9	2.489D+04	7.127D+03	7.539D+03
5	9	2.350D+04	7.173D+03	2.649D+03
6	9	2.350D+04	7.173D+03	-2.649D+03
7	9	2.489D+04	7.127D+03	-7.539D+03
8	9	2.704D+04	7.276D+03	-1.151D+04
9	9	2.955D+04	8.945D+03	-1.486D+04
10	9	3.071D+04	9.933D+03	-1.676D+04
1	10	3.151D+04	6.993D+03	1.537D+04
2	10	2.168D+04	2.707D+03	9.848D+03
3	10	1.453D+04	2.390D+03	6.231D+03
4	10	9.768D+03	1.815D+03	3.410D+03
5	10	7.308D+03	1.585D+03	1.080D+03
6	10	7.308D+03	1.585D+03	-1.080D+03
7	10	9.768D+03	1.815D+03	-3.410D+03
8	10	1.453D+04	2.390D+03	-6.231D+03
9	10	2.168D+04	2.707D+03	-9.848D+03
10	10	3.151D+04	6.993D+03	-1.537D+04

VITA

Saroj Leesavan

Candidate for the Degree of

Doctor of Philosophy

Thesis: LARGE DEFLECTION ANALYSIS OF DISCRETE-ELEMENT THIN PLATES

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Biographical:

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Education: Graduated from Vajiravudh College, Bangkok, Thailand 1968; received the Bachelor of Engineering degree from Chulalongkorn University, Bangkok, Thailand in 1972; received the Master of Science degree in Civil Engineering from the University of Louisville, Louisville, Kentucky in December, 1973; completed the requirements for Doctor of Philosophy degree at Oklahoma State University, in May, 1977.

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